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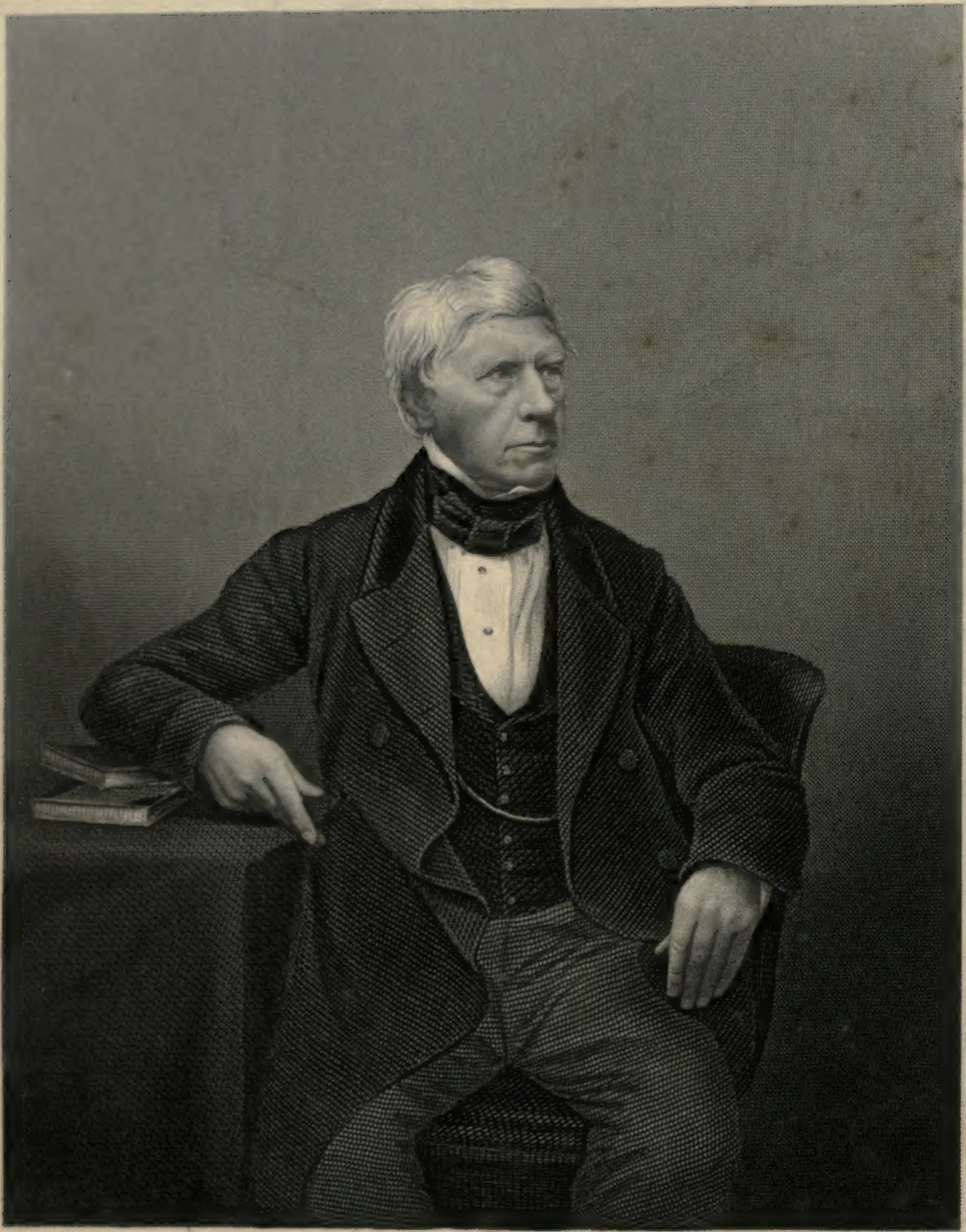

HENRY LORD BROUGHAM, D.C.L. L.L.D. F.R.S.

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THE

CIRCLE OF THE SCIENCES.

WITH

AN INTRODUCTORY DISCOURSE

OF

THE OBJECTS, PLEASURES, AND ADVANTAGES OF SCIENCE.

BY HENRY, LORD BROUGHAM.

EDITED BY

JAMES WYLDE,

AUTHOR OF THE "MAGIC OF SCIENCE," "SHELLS AND THEIR INHABITANTS," "FOSSILS," "USEFUL PLANTS," ETC., ETC.

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VOL. I.

EXPERIMENTAL, CHEMICAL, MATHEMATICAL, AND MECHANICAL PHILOSOPHY.

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PREFACE.

One of the most interesting features of our time, is the great advance which has been made in the study of the various branches of Science. But a few years only has elapsed, since philosophy was confined to those who had received a collegiate education, or were engaged in the learned professions. By slow degrees, our public schools adopted the pure mathematics as a portion of their curriculum; whilst the schools, in which the masses were educated, ignored entirely the study in every form. It was even considered that a knowledge of Science, by calling too strongly on the mental powers, would unfit the youth for commercial pursuits; and the expensive character of such studies, on the other hand, limited them to those whom ample means placed beyond such considerations.

The nation owes a deep debt of gratitude to those noble pioneers of intellectual advancement, Lord Brougham and Dr. Birkbeck, who were among the first to break down the barriers which stood between the "people" and the "learned." The former, by his untiring exertions in connection with the Society for the Diffusion of Useful Knowledge, and the latter by the establishment of Mechanics' Institutes, have done immense good to the masses, and have earned the deepest gratitude of their countrymen. It was prophesied against the issue of their labours, that the masses would soon over-ride their employers, and that sedition and infidelity would become rampant in the land. How far such prophecies have been falsified has been shown by recent events, in which we have seen hundreds of thousands of our artisans suddenly plunged from prosperity to want, by circumstances beyond their control, and bearing their afflictions with an intelligent resignation unequalled in the history of mankind. There is no impropriety in alleging the cause of this noble feeling to be due, in a great measure, to the advancement of knowledge, and to the mental culture which such individuals have acquired by the means to which we have briefly alluded.

The present state of Education, the exigencies of Agriculture and Manufactures, together with a high appreciation of the pleasures and advantages of Science, have created a great demand for works devoted to the subject. At first sight, it would seem that abundant means had been afforded to supply this demand; but a little consideration will suffice to show that this idea is erroneous. Even with all the advantages of cheap literature, there scarcely yet exists a systematic arrangement by which persons, unaided by scholastic training, may hope to arrive at a competent knowledge of the numerous branches into which the study of nature is now divided. For want of personal experience, such cannot properly select for themselves those works most fitted to instruct them; and, in the attempt so to do, they often experience great disappointment, partly from technicalities, unfortunately too prevalent in our scientific expositions; and frequently, from the abstruse or obscure style in which those works are veiled. The practical man, again, has often to supply himself with many works on the same subject, in order to acquire both the theory and applications of it; hence much labour, patience, and expense are necessarily incurred, which too often end in weariness and disgust.

These difficulties are fairly grappled with, and, it is hoped, successfully mastered, in the following Work. It includes every branch of Science; from the study of matter and forces, to the widest applications of scientific research for social purposes. To the educated person, it offers a comprehensive *résumé* of all that is known in Experimental, Chemical, Mathematical, Mechanical, and Natural Science. To the artisan, it affords a complete guide to each branch of Science, rising, by plain and easy steps, from the very elements to the higher branches of each department. He will equally find that, if his calling in life depend on the applications of Science, his wants will be fully satisfied. The home student may undertake its perusal, with the certainty of acquiring, without the aid of tutors, a general knowledge of each branch of philosophy. In the arrangement of each subject, the circumstances of both artisan and student have been specially borne in mind; and it is hoped that no person of ordinary intelligence, will find any difficulty in mastering any portion of the Work.

With respect to the authors of each section, it will be only requisite to remark, that they stand at the head of their several departments. Without an exception, they all occupy eminent positions in the world of Science, and have distinguished themselves as much by the diffusion as by the possession of scientific information.

It will be unnecessary for us here to give any detailed explanation of the plan of the Work. This will be found in the Introduction to Physics, which follows Lord Brougham's admirable Discourse of the Objects, Pleasures, and Advantages of Science. The arrangement of the subjects has been adopted with the view of affording a thorough, but graduating course of study, so that, as the reader progresses, he may be prepared to advance, by aid of that which he has already learned. Generally speaking, it has been assumed that the reader knows nothing of the subjects treated on; and this assumption will be highly appreciated by those who have to pursue Science under any class of difficulties.

In constructing an edifice, each stone adds both to its strength and height; but unless they be joined together by a suitable cement, the building will not itself be strong. So, in a work of this kind, unless each subject be braced with the rest, each may be individually complete, without rendering the work itself so. This duty the Editor has endeavoured to perform, by adopting the plan in which the Work is arranged, by notes of reference, explanatory remarks, &c.; and even, in some cases, by modifying the valuable contributions which he has had placed at his disposal. He feels assured that their authors, animated with him in a desire to spread knowledge, will readily pardon the liberty he has so taken, if their readers are in any way benefited by such a course. In Nature, we observe that the beauty of the landscape consists in the harmony of its outlines; so in Science, whose only object is to unfold the hidden laws of nature, a similar harmony of construction must exist, if we would successfully impress its students with a sense of its beauties and perfections. Its divisions must be numerous, owing to the imperfections of human intellect; yet all these have one origin, tend to one end, and are in unison in their great design.

J. W.

INTRODUCTORY DISCOURSE

OF THE

OBJECTS, PLEASURES, AND ADVANTAGES

OF

SCIENCE.

BY HENRY LORD BROUGHAM,

D.C.L., L.L.D., F.R.S.

Member of the National Institute of France, and of the Royal Academy of Naples.

INTRODUCTORY DISCOURSE

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In order fully to understand the advantages and the pleasures which are derived from an acquaintance with any Science, it is necessary to become acquainted with that Science; and it would therefore be impossible to convey a complete knowledge of the benefits conferred by a study of the various Sciences which have hitherto been cultivated by philosophers, without teaching all the branches of them. But a very distinct idea may be given of those benefits, by explaining the nature and objects of the different Sciences: it may be shown, by examples, how much use and gratification there is in learning a part of any one branch of knowledge; and it may thence be inferred, how great reason there is to learn the whole.

It may easily be demonstrated, that there is an advantage in learning, both for the usefulness and the pleasure of it. There is something positively agreeable to all men, to all at least whose nature is not most grovelling and base, in gaining knowledge for its own sake. When you see anything for the first time, you at once derive some gratification from the sight being new; your attention is awakened, and you desire to know more about it. If it be a piece of workmanship, as an instrument, a machine of any kind, you wish to know how it is made; how it works; and what use it is of. If it be an animal, you desire to know where it comes from; how it lives; what are its dispositions, and, generally, its nature and habits. You feel this desire, too, without at all considering that the machine or the animal may ever be of the least use to yourself practically; for, in all probability, you may never see them again. But you have a curiosity to learn all about them, because they are new and unknown. You accordingly make inquiries; you feel a gratification in getting answers to your questions, that is, in receiving information, and in knowing more,—in being better informed than you were before. If you happen again to see the same instrument or animal, you find it agreeable to recollect having seen it formerly, and to think that you know something about it. If you see another instrument or animal, in some respects like, but differing in other particulars, you find it pleasing to compare them together, and to note in what they agree, and in what they differ. Now, all this kind of gratification is of a pure and disinterested nature, and has no reference to any of the common purposes of life; yet it is a pleasure—an enjoyment. You are nothing the richer for it; you do not gratify your palate or any other bodily appetite, and yet it is so pleasing, that you would give something out of your pocket to obtain it, and would forego some bodily enjoyment for its sake. The pleasure derived from Science is exactly of the like nature, or, rather, it is the very same. For what has just been spoken of is, in fact, Science, which in its most comprehensive sense, only means *Knowledge*, and in its ordinary sense means *Knowledge reduced to a System;* that is, arranged in a regular order, so as to be conveniently taught, easily remembered, and readily applied.

The practical uses of any science or branch of knowledge, are undoubtedly of the highest importance; and there is hardly any man who may not gain some positive advantage in his worldly wealth and comforts, by increasing his stock of information. But there is also a pleasure in seeing the uses to which knowledge may be applied, wholly independent of the share we ourselves may have in those practical benefits. It is pleasing to examine the nature of a new instrument, or the habits of an unknown animal, without considering whether or not they may ever be of use to ourselves or to anybody. It is another gratification to extend our inquiries, and find that the instrument or animal is useful to man, even although we have no chance of ever benefiting by the information: as, to find that the natives of some distant country employ the animal in travelling;—nay, though we have no desire of benefiting by the knowledge; as, for example, to find that the instrument is useful in performing some dangerous surgical operation. The mere gratification of curiosity; the knowing more to-day than we knew yesterday; the understanding clearly what before seemed obscure and puzzling; the contemplation of general truths, and the comparing together of different things,—is an agreeable occupation of the mind; and, beside the present enjoyment, elevates the faculties above low pursuits, purifies and refines the passions, and helps our reason to assuage their violence.

It is very true, that the fundamental lessons of philosophy may to many, at first sight, wear a forbidding aspect, because to comprehend them requires an effort of the mind somewhat, though certainly not much, greater than is wanted for understanding more ordinary matters; and the most important branches of philosophy, those which are of the most general application, are for that very reason the less easily followed, and the less entertaining when apprehended, presenting as they do, few particulars or individual objects to the mind. In discoursing of them, moreover, no figures will be at present used to assist the imagination; the appeal is made to reason, without help from the senses. But be not, therefore, prejudiced against the doctrine, that the pleasure of learning the truths which philosophy unfolds is truly above all price. Lend but a patient attention to the principles explained, and giving us credit for stating nothing which has not some practical use belonging to it, or some important doctrine connected with it, you will soon perceive the value of the lessons you are learning, and begin to interest yourselves in comprehending and recollecting them; you will find that you have actually learnt something of science, while merely engaged in seeing what its end and purpose is; you will be enabled to calculate for yourselves, how far it is worth the trouble of acquiring, by examining samples of it; you will, as it were, taste a little, to try whether or not you relish it, and ought to seek after more; you will enable yourselves to go on, and enlarge your stock of it; and after having first mastered a very little, you will proceed so far, as to look back with wonder at the distance you have reached beyond your earliest acquirements.

The Sciences may be divided into three great classes: those which relate to *Number and Quantity*—those which relate to *Matter*—and those which relate to *Mind*. The first are called the *Mathematics*, and teach the property of numbers and of figures; the second are called *Natural Philosophy*, and teach the properties of the various bodies which we are acquainted with by means of our senses; the third are called *Intellectual* or *Moral Philosophy*, and teach the nature of the mind, of the existence of which we have the most perfect evidence in our own reflections; or, in other words, they teach the moral nature of man, both as an individual, and as a member of society. Connected with all the sciences, and subservient to them, though not one of their number, is *History*, or the record of facts relating to all kinds of knowledge.

I. Mathematical Science.

The two great branches of the *Mathematics*, or the two mathematical sciences, are *Arithmetic*, the science of number, from the Greek word signifying *number*, and *Geometry*, the science of figure, from the Greek words signifying *measure of the earth*,—land-measuring having first turned men's attention to it.

When we say that 2 and 2 make 4, we state an arithmetical proposition, very simple indeed, but connected with many others of a more difficult and complicated kind. Thus, it is another proposition, somewhat less simple, but still very obvious, that 5 multiplied by 10, and divided by 2, is equal to, or makes the same number with, 100 divided by 4—both results being equal to 25. So, to find how many farthings there are in 1.000*l*., and how many minutes in a year, are questions of arithmetic which we learn to work by being taught the principles of the science one after another, or, as they are commonly called, the *rules* of addition, subtraction, multiplication, and division. Arithmetic may be said to be the most simple, though among the most useful of the sciences; but it teaches only the properties of particular and known numbers, and it only enables us to add, subtract, multiply, and divide those numbers. But suppose we wish to add, subtract, multiply, or divide numbers which we have not yet ascertained, and in all respects to deal with them as if they were known, for the purpose of arriving at certain conclusions respecting them, and, among other things, of discovering what they are; or, suppose we would examine properties belonging to all numbers; this must be performed by a peculiar kind of arithmetic, called *Universal* arithmetic, or *Algebra*.* The common arithmetic, you will presently perceive, carries the seeds of this most important science in its bosom. Thus, suppose we inquire what is the number which multiplied by 5 makes 10? This is found if we divide 10 by 5,—it is 2: but suppose that, before finding this number 2, and before knowing what it is, we would add it, whatever it may turn out, to some other number; this can only be done by putting some mark, such as a letter of the alphabet, to stand for the unknown number, and adding that letter as if it were a known number. Thus, suppose we want to find two numbers which, added together, make 9, and, multiplied by one another, make 20. There are many which, added together, make 9; as 1 and 8; 2 and 7; 3 and 6; and so on. We have, therefore, occasion to use the second condition, that multiplied by one another they should make 20, and to work upon this condition before we have discovered the particular numbers. We must, therefore, suppose the numbers to be found, and put letters for them, and by reasoning upon those letters, according to both the two conditions of adding and multiplying, we find what they must each of them be in figures, in order to fulfil or answer the conditions. Algebra teaches the rules for conducting this reasoning, and obtaining this result successfully; and by means of it we are enabled to find out numbers which are unknown, and of which we only know that they stand in certain relations to known numbers, or to one another. The instance now taken is an easy one; and you could, by considering the question a little, answer it readily enough; that is, by trying different numbers, and seeing which suited the conditions; for you plainly see that 5 and 4 are the two numbers sought; but you see this by no certain or general rule applicable to all cases, and therefore you could never work more difficult questions in the same way; and even questions of a moderate degree of difficulty would take an endless number of trials or guesses to answer. Thus a shepherd sold his flock for 80*l.*; and if he had sold four sheep more for the same money, he would have received one pound less for each sheep. To find out from this, how many the flock consisted of, is a very easy question in algebra, but would require a vast many guesses, and a long time to hit upon by common arithmetic.[*] And questions infinitely more difficult can easily be solved by the rules of algebra. In like manner, by arithmetic you can tell the properties of particular numbers; as, for instance, that the number 348 is divided by 3 exactly, so as to leave nothing over: but algebra teaches us that it is only one of an infinite variety of numbers, all divisible by 3, and any one of which you can tell the moment you see it; for they all have the remarkable property, that if you add together the figures they consist of, the sum total is divisible by 3. You can easily perceive this in any one case, as in the number mentioned, for 3 added to 4 and that to 8 make 15, which is plainly divisible by 3; and if you divide 348 by 3, you find the quotient to be 116, with nothing over. But this does not at all prove that any other number, the sum of whose figures is divisible by 3, will itself also be found divisible by 3, as 741; for you must actually perform the division here, and in every other case, before you can know that it leaves nothing over. Algebra, on the contrary, both enables you to discover such general properties, and to prove them in all their generality.[†]

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* Algebra, from the Arabic words signifying the *reduction of fractions*; the Arabs having brought the knowledge of it into Europe.

By means of this science, and its various applications, the most extraordinary calculations may be performed. We shall give, as an example, the method of *Logarithms*, which proceeds upon this principle. Take a set of numbers going on by equal differences; that is to say, the third being as much greater than the second, as the second is greater than the first, and the common difference being the number you begin with; thus, 1, 2, 3, 4, 5, 6, and so on, in which the common difference is 1: then take another set of numbers, such that each is equal to twice or three times the one before it, or any number of times the one before it, but the common multiplier being the number you begin with: thus, 2, 4, 8, 16, 32, 64, 128; write this second set of numbers under the first, or side by side, so that the numbers shall stand opposite to one another, thus,

1	2	3	4	5	6	7
2	4	8	16	32	64	128

you will find, that if you add together any two of the upper or first set, and go to the number opposite their sum, in the lower or second set, you will have in this last set the number arising from multiplying together the numbers of the lower set corresponding or opposite to the numbers added together. Thus, add 2 to 4, you have 6 in the upper set, opposite to which in the lower set is 64, and multiplying the numbers 4 and 16 opposite to 2 and 4, the product is 64. In like manner, if you subtract one of the upper numbers from another, and opposite to their difference in the upper line, you look to the lower number, it is the quotient found from dividing one of the lower numbers by the other opposite the subtracted ones. Thus, take 4 from 6 and 2 remains, opposite to which you have in the lower line 4; and if you divide 64, the number opposite to 6, by 16, the number opposite to 4, the quotient is 4. The upper set are called the *logarithms*, of the lower set, which are called *natural numbers;* and tables may, with a little trouble, be constructed, giving the logarithms of all numbers from 1 to 10,000 and more: so that, instead of multiplying or dividing one number by another, you have only to add or subtract their logarithms, and then you at once find the product or the quotient in the tables. These are made applicable to numbers far higher than any actually in them, by a very simple process: so that you may at once perceive the prodigious saving of time and labour which is thus made. If you had, for instance, to multiply 7,543,283 by itself, and that product again by the original number, you would have to multiply a number of 7 places of figures by an equally large number, and then a number of 14 places of figures by one of 7 places, till at last you had a product of 21 places of figures—a very tedious operation; but, working by logarithms, you would only have to take three times the logarithm of the original number, and that gives the logarithm of the last product of 21 places of figures, without any further multiplication. So much for the time and trouble saved, which is still greater in questions of division; but by means of logarithms many questions can be worked, and of the most important kind, which no time or labour would otherwise enable us to resolve.

Geometry teaches the properties of figure, or particular portions of space, and distances of points from each other. Thus, when you see a triangle, or three-sided figure, one of whose sides is perpendicular to another side, you find, by means of geometrical reasoning respecting this kind of triangle, that if squares be drawn on its three sides, the large square upon the slanting side opposite the two perpendiculars, is exactly equal to the smaller squares upon the perpendiculars, taken together; and this is absolutely true, whatever be the size of the triangle, or the proportions of its sides to each other. Therefore, you can always find the length of any one of the three sides by knowing the lengths of the other two. Suppose one perpendicular side to be 3 feet long, the other 4, and you want to know the length of the third side opposite to the perpendicular; you have only to find a number such, that if, multiplied by itself, it shall be equal to 3 times 3, together with 4 times 4, that is 25.* (This number is 5.)

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[*] It is 16.

[†] Another class of numbers divisible by 3 is discovered in like manner by algebra. Every number of 3 places, the figures (or digits) composing which are in arithmetical progression, (or rise above each other by equal differences,) is divisible by 3: as 123, 789, 357, 159, and so on. The same is true of numbers of any amount of places, provided they are composed of 3, 6, 9, &c., numbers rising above each other by equal differences, as 289, 299, 309, or 148, 214, 280, or 307142085345648276198756, which number of 24 places is divisible by 3, being composed of 6 numbers in a series whose common difference is 1137. This property, too, is only a particular case of a much more general one.

Now only observe the great advantage of knowing this property of the triangle, or of perpendicular lines. If you want to measure a line passing over ground which you cannot reach—to know, for instance, the length of one side, covered with water, of a field, or the distance of one point on a lake or bay from another point on the opposite side—you can easily find it by measuring two lines perpendicular to one another on the dry land, and running through the two points; for the line wished to be measured, and which runs through the water, is the third side of a perpendicular-sided triangle, the other two sides of which are ascertained. But there are other properties of triangles, which enable us to know the length of two sides of any triangle, whether it has perpendicular sides or not, by measuring one side, and also measuring the inclinations of the other two sides to this side, or what is called the two *angles* made by those sides with the measured side. Therefore you can easily find the perpendicular line drawn, or supposed to be drawn, from the top of a mountain through it to the bottom, that is, the height of the mountain; for you can measure a line on level ground, and also the inclination of two lines, supposing them drawn in the air, and reaching from the two ends of the measured line to the mountain's top; and having thus found the length of the one of those lines next the mountain, and its inclination to the ground, you can at once find the perpendicular, though you cannot possibly get near it. In the same way, by measuring lines and angles on the ground, and near, you can find the length of lines at a great distance, and which you cannot approach: for instance, the length and breadth of a field on the opposite side of a lake or sea; the distance of two islands; or the space between the tops of two mountains.

Again, there are *curve-lined* figures as well as straight, and geometry teaches the properties of these also. The best known of all the curves is the *circle*, or a figure made by drawing a string round one end which is fixed, and marking where its other end traces, so that every part of the circle is equally distant from the fixed point or centre. From this fundamental property, an infinite variety of others follow by steps of reasoning more or less numerous, but all necessarily arising one out of another. To give an instance; it is proved by geometrical reasoning, that if from the two ends of any diameter of the circle you draw two lines to meet in any one point of the circle whatever, those lines are perpendicular to each other.

Another property, and a most useful one, is, that the sizes, or areas, of all circles whatever, from the greatest to the smallest, from the sun to a watch-dialplate, are in exact proportion to the squares of their distances from the centre; that is, the squares of the strings they are drawn with: so that if you draw a circle with a string 5 feet long, and another with a string 10 feet long, the large circle is four times the size of the small one, as far as the space or area inclosed is concerned; the square of 10 or 100 being four times the square of 5 or 25. But it is also true, that the lengths of the circumferences themselves, the number of feet over which the ends of the strings move, are in proportion to the lengths of the strings; so that the curve of the large circle is only twice the length of the curve of the lesser.

But the circle is only one of an infinite variety of curves, all having a regular formation and fixed properties. The *oval* or *ellipse* is, perhaps, next to the circle, the most familiar to us, although we more frequently see another curve, the line formed by the motion of bodies thrown forward. When you drop a stone, or throw it straight up, it goes in a straight line; when you throw it forward, it goes in a curve line till it reaches the ground; as you see by the figure in which water runs when forced out of a pump, or from a fire-pipe, or from the spout of a kettle or teapot. The line it moves in is called a *parabola;* every point of which bears a certain fixed relation to a certain point within it, as the circle does to its centre. Geometry teaches various properties of this curve: for example, if the direction in which the stone is thrown, or the bullet fired, or the water spouted, be half the perpendicular to the ground, that is, half way between being level with the ground and being upright, the curve will come to the ground at a greater distance than if any other direction whatever were given, with the same force. So that, to make the gun carry farthest, or the fire-pipe play to the greatest distance, they must be pointed, not, as you might suppose, level or point blank, but about half way between that direction and the perpendicular. If the air did not resist, and so somewhat disturb the calculation, the direction to give the longest range ought to be exactly half perpendicular.

The *oval*, or *ellipse*, is drawn by taking a string of any certain length, and fixing, not one end as in drawing the circle, but both ends to different points, and then carrying a point round inside the string, always keeping it stretched as far as possible. It is plain, that this figure is as regularly drawn as the circle, though it is very different from it; and you perceive that every point of its curve must be so placed that the straight lines drawn from it to the two points where the string was fixed, are, when added together, always the same; for they make together the length of the string.

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* It is a property of numbers, that every number whatever, whose last place is either 5 or 0, is, when multiplied into itself, equal to two others which are square numbers, and divisible by 3 and 4 respectively:—thus, 45×45=2025=729+1296, the squares of 27 and 36; and 60×60=3600=1296+2304, the squares of 36 and 48.

Among various properties belonging to this curve, in relation to the straight lines drawn within it, is one which gives rise to the construction of the *trammels*, or elliptic compasses, used for making figures and ornaments of this form; and also to the construction of lathes for turning oval frames, and the like.

If you wish at once to see these three curves, take a pointed sugarloaf, and cut it anywhere clean through in a direction parallel to its base or bottom; the outline or edge of the loaf where it is cut will be a *circle*. If the cut is made so as to slant, and not be parallel to the base of the loaf, the outline is an *ellipse*, provided the cut goes quite through the sides of the loaf all round, or is in such a direction that it would pass through the sides of the loaf were they extended; but if it goes slanting and parallel to the line of the loaf's side, the outline is a *parabola;* and if you cut in any direction, not through the sides all round, but through the sides and base, and not parallel to the line of the side, being nearer the perpendicular, the outline will be another curve of which we have not yet spoken, but which is called an *hyperbola*. You will see another instance of it, if you take two plates of glass, and lay them on one another; then put their edge in water, holding them upright and pressing them together; the water, which, to make it more plain, you may colour with a few drops of ink or strong tea, rises to a certain height, and its outline is this curve; which, however much it may seem to differ in form from a circle or ellipse, is found by mathematicians to resemble them very closely in many of its most remarkable properties.

These are the curve lines best known and most frequently discussed; but there are an infinite number of others all related to straight lines and other curve lines by certain fixed rules: for example, the course which any point in the circumference of a circle, as a nail in the felly of a wheel rolling along, takes through the air, is a curve called the *cycloid*, which has many remarkable properties; and, among others, this, that it is, of all lines possible, the one in which any body, not falling perpendicularly, will descend from one point to another the most quickly. Another curve often seen is that in which a rope or chain hangs when supported at both ends: it is called the *Catenary*, from the Latin for chain; and in this form some arches are built. The form of a sail filled with wind is the same curve.

II. Difference between Mathematical and Physical Truths.

You perceive, if you reflect a little, that the science which we have been considering, in both its branches, has nothing to do with matter; that is to say, it does not at all depend upon the properties or even upon the existence of any bodies or substances whatever. The distance of one point or place from another is a straight line; and whatever is proved to be true respecting this line, as, for instance, its proportion to other lines of the same kind, and its inclination towards them, what we call the *angles* it makes with them, would be equally true whether there were anything in those places, at those two points, or not. So if you find the number of yards in a square field, by measuring one side, 100 yards, and then, multiplying that by itself, which makes the whole area 10,000 square yards, this is equally true whatever the field is, whether corn, or grass, or rock, or water; it is equally true if the solid part, the earth or water, be removed, for then it will be a field of air bounded by four walls or hedges; but suppose the walls or hedges were removed, and a mark only left at each corner, still it would be true that the space inclosed or bounded by the lines supposed to be drawn between the four marks, was 10,000 square yards in size. But the marks need not be there; you only want them while measuring one side: if they were gone, it would be equally true that the lines, supposed to be drawn from the places where the marks had been, inclose 10,000 square yards of air. But if there were no air, and consequently a mere void, or empty space, it would be equally true that this space is of the size you had found it to be by measuring the distance of one point from another, of one of the space's corners or angles from another, and then multiplying that distance by itself. In the same way it would be true, that, if the space were circular, its size, compared with another circular space of half its diameter, would be four times larger: of one-third its diameter nine times larger, and of one-fourth sixteen times, and so on always in proportion to the squares of the diameters; and that the length of the circumference, the number of feet or yards in the line round the surface, would be twice the length of a circle whose diameter was one-half, thrice the circumference of one whose diameter was one-third, four times the circumference of one whose diameter was one-fourth, and so on, in the simple proportion of the diameters. Therefore, every property which is proved to belong to figures belongs to them without the smallest relation to bodies or matter of any kind, although we are accustomed only to see figures in connection with bodies; but all those properties would be equally true if no such thing as matter or bodies existed; and the same may be said of the properties of number, the other great branch of the mathematics. When we speak of twice two, and say it makes four, we affirm this without thinking of two horses, or two balls, or two trees; but we assert it concerning two of any thing and every thing equally

Nay, this branch of mathematics may be said to apply still more extensively than even the other; for it has no relation to space, which geometry has; and, therefore, it is applicable to cases where figure and size are wholly out of the question. Thus you can speak of two dreams, or two ideas, or two minds, and can calculate respecting them just as you would respecting so many bodies; and the properties you find belonging to numbers, will belong to those numbers when applied to things that have no outward or visible or perceivable existence, and cannot even be said to be in any particular place, just as much as the same numbers applied to actual bodies which may be seen and touched.

It is quite otherwise with the science which we are now going to consider, *Natural Philosophy*. This teaches the nature and properties of actually existing substances, their motions, their connections with each other, and their influence on one another. It is sometimes also called *Physics*, from the Greek word signifying *Nature*, though that word is more frequently, in common speech, confined to one particular branch of the science, that which treats of the bodily health.

We have mentioned one distinction between Mathematics and Natural Philosophy, that the former does not depend on the nature and existence of bodies, which the latter entirely does. Another distinction, and one closely connected with this, is, that the truths which Mathematics teach are *necessarily* such,—they are truths of themselves, and wholly independent of facts and experiments,—they depend only upon reasoning; and it is utterly impossible they should be otherwise than true. This is the case with all the properties which we find belong to numbers and to figures—2 and 2 must of *necessity*, and through all time, and in every place, be equal to 4: those numbers must *necessarily* be always divisible by 3, without leaving any remainder over, which have the sums of the figures they consist of divisible by 3; and circles must *necessarily*, and for ever and ever, be to one another in the exact proportion of the squares of their diameters. It cannot be otherwise; we cannot conceive it in our minds to be otherwise. No man can in his own mind suppose to himself that 2 and 2 should ever be more or less than 4; it would be an utter impossibility—a contradiction in the very ideas; and if stated in words, those words would have no sense. The other properties of number, though not so plain at first sight as this, are proved to be true by reasoning, every one step of which follows from the step immediately before, as a matter of course, and so clearly and unavoidably, that it cannot be supposed, or even imagined, to be otherwise; the mind has no means of fancying how it could be otherwise: the final conclusion, from all the steps of the reasoning or demonstration, as it is called, follows in the same way from the last of the steps, and is therefore just as evidently and necessarily true as the first step, which is always something self-evident; for instance, that 2 and 2 make 4, or that the whole is greater than any of its parts, but equal to all its parts put together. It is through this kind of reasoning, step by step, from the most plain and evident things, that we arrive at the knowledge of other things which seem at first not true, or at least not generally true; but when we do arrive at them, we perceive that they are just as true, and for the same reasons, as the first and most obvious matters; that their truth is absolute and necessary, and that it would be as absurd and self-contradictory to suppose they ever could, under any circumstances, be not true, as to suppose that 2 added to 2 could ever make 3, or 5, or 100, or anything but 4; or, which is the same thing, that 4 should ever be equal to 3, or 5, or 100, or anything but 4. To find out these reasonings, to pursue them to their consequences, and thereby to discover the truths which are not immediately evident, is what science teaches us: but when the truth is once discovered, it is as certain and plain by the reasoning, as the first truths themselves from which all the reasoning takes its rise, on which it all depends, and which require no proof, because they are self-evident at once, and must be assented to the instant they are understood.

But it is quite different with the truths which Natural Philosophy teaches. All these depend upon matter of fact; and that is learnt by observation and experiment, and never could be discovered by reasoning at all. If a man were shut up in a room with pen, ink, and paper, he might by thinking discover any of the truths in arithmetic, algebra, or geometry; it is possible at least; there would be nothing absolutely impossible in his discovering all that is now known of these sciences; and if his memory were as good as we are supposing his judgment and conception to be, he might discover it all without pen, ink, and paper, and in a dark room. But we cannot discover a single one of the fundamental properties of matter without observing what goes on around us, and trying experiments upon the nature and motion of bodies. Thus, the man whom we have supposed shut up, could not possibly find out beyond one or two of the very first properties of matter, and those only in a very few cases; so that he could not tell if these were general properties of all matter or not. He could tell that the objects he touched in the dark were hard and resisted his touch; that they were extended and were solid: that is, that they had three dimensions, length, breadth, and thickness. He might guess that other things existed besides those he felt, and that those other things resembled what he felt in these properties; but he could know nothing for certain, and could not even conjecture much beyond this very limited number of qualities. He must remain utterly ignorant of what really exists in nature, and of what properties matter in general has. These properties, therefore, we learn by experience; they are such as we know bodies to have; they happen to have them—they are so formed by Divine Providence as to have them—but they might have been otherwise formed; the great Author of Nature might have thought fit to make all bodies different in every respect.

We see that a stone dropped from our hand falls to the ground; this is a fact which we can only know by experience; before observing it, we could not have guessed it, and it is quite *conceivable* that it should be otherwise: for instance, that when we remove our hand from the body it should stand still in the air; or fly upward, or go forward, or backward, or sideways; there is nothing at all absurd, contradictory, or inconceivable in any of these suppositions; there is nothing impossible in any of them, as there would be in supposing the stone equal to half of itself, or double of itself; or both falling down and rising upwards at once; or going to the right and to the left at one and the same time. Our only reason for not at once thinking it quite conceivable that the stone should stand still in the air, or fly upwards, is that we have never seen it do so, and have become accustomed to see it do otherwise. But for that, we should at once think it as natural that the stone should fly upwards or stand still, as that it should fall down. But no degree of reflection for any length of time could accustom us to think 2 and 2 equal to anything but 4, or to believe the whole of any thing equal to a part of itself.

After we have once, by observation or experiment, ascertained certain things to exist in fact, we may then reason upon them by means of the mathematics; that is, we may apply mathematics to our experimental philosophy, and then such reasoning becomes absolutely certain, taking the fundamental facts for granted. Thus, if we find that a stone falls in one direction when dropped, and we further observe the peculiar way in which it falls, that is, quicker and quicker every instant till it reaches the ground, we learn the rule or the proportion by which the quickness goes on increasing; and we further find, that if the same stone be pushed forward on a table, it moves in the direction of the push, till it is either stopped by something, or comes to a pause by rubbing against the table and being hindered by the air. These are facts which we learn by observing and trying, and they might all have been different if matter and motion had been otherwise constituted; but supposing them to be as they are, and as we find them, we can, by reasoning mathematically from them, find out many most curious and important truths depending upon those facts, and depending upon them not accidentally, but of necessity. For example, we can find in what course the stone will move, if, instead of being dropped to the ground, it be thrown forward: it will go in the curve already mentioned, the parabola, somewhat altered by the resistance of the air, and it will run through that curve in a peculiar way, so that there will always be a certain proportion between the time it takes and the space it moves through, and the time it would have taken, and the space it would have moved through, had it dropped from the hand in a straight line to the ground. So we can prove, in like manner, what we before stated of the relation between the distance at which it will come to the ground, and the direction it is thrown in; the distance being greatest of all when the direction is half way between the level or horizontal, and the upright or perpendicular. These are mathematical truths, derived by mathematical reasoning upon physical grounds; that is, upon matter of fact found to exist by actual observation and experiment. The result, therefore, is necessarily true, and proved to be so by reasoning only, provided we have once ascertained the facts; but taken altogether, the result depends partly on the facts learned by experiment or experience, partly on the reasoning from these facts. Thus it is found to be true by reasoning, and necessarily true, that *if* the stone fall in a certain way when unsupported, it must, when thrown forward, go in the curve called a parabola, provided there be no air to resist: this is a necessary or mathematical truth, and it cannot possibly be otherwise. But when we state the matter without any supposition,—without any "*if*,"—and say, a stone thrown forward goes in a curve called a parabola, we state a truth, partly fact, and partly drawn from reasoning on the fact; and it might be otherwise if the nature of things were different. It is called a proposition or truth in Natural Philosophy; and as it is discovered and proved by mathematical reasoning upon facts in nature, it is sometimes called a proposition or truth in the *Mixed Mathematics*, so named in contradistinction to the *Pure Mathematics*, which are employed in reasoning upon figures and numbers. The man in the dark room could never discover this truth unless he had been first informed, by those who had observed the fact, in what way the stone falls when unsupported, and moves along the table when pushed. These things he never could have found out by reasoning: they are facts, and he could only reason from them after learning them by his own experience, or taking them on the credit of other people's experience. But having once so learnt them, he could discover by reasoning merely, and with as much certainty as if he lived in daylight, and saw and felt the moving body, that the motion is parabolic, and governed by certain rules. As experiment and observation are the great sources of our knowledge of Nature, and as the judicious and careful making of experiments is the only way by which her secrets can be known, Natural and Experimental Philosophy mean one and the same thing; mathematical reasoning being applied to certain branches of it, particularly those which relate to motion and pressure.

III. NATURAL OR EXPERIMENTAL SCIENCE.

NATURAL PHILOSOPHY, in its most extensive sense, has for its province the investigation of the laws of matter; that is, the properties and the motions of matter; and it may be divided into two great branches. The first and most important (which is sometimes, on that account, called *Natural Philosophy* by way of distinction, but more properly *Mechanical Philosophy*) investigates the sensible motions of bodies. The second investigates the constitution and qualities of all bodies, and has various names, according to its different objects. It is called *Chemistry*, if it teach the properties of bodies with respect to heat, mixture with one another, weight, taste, appearance, and so forth; *Anatomy* and *Animal Physiology* (from the Greek word signifying *to speak of the nature* of any thing), if it teaches the structure and functions of living bodies, especially the human; for when it shows those of other animals, we term it *Comparative Anatomy; Medicine*, if it teaches the nature of diseases, and the means of preventing them and of restoring health; *Zoology* (from the Greek words signifying *to speak of animals*), if it teach the arrangement or classification and the habits of the different lower animals; *Botany*, (from the Greek word for *herbage*), including *Vegetable Physiology*, if it teach the arrangement or classification, the structure and habits of plants; *Mineralogy*, including *Geology* (from the Greek words meaning *to speak of the earth*), if it teach the arrangement of minerals, the structure of the masses in which they are found, and of the earth composed of those masses. The term *Natural History* is given to the three last branches taken together, but chiefly as far as they teach the classification of different things, or the observation of the resemblances and differences of the various animals, plants, and inanimate and ungrowing substances in nature.

But here we may make two general observations. The *first* is, that every such distribution of the sciences is necessarily imperfect; for one runs unavoidably into another. Thus, Chemistry shows the qualities of plants with relation to other substances, and to each other; and Botany does not overlook those same qualities, though its chief object be arrangement. So Mineralogy, though principally conversant with classifying metals and earths, yet regards also their qualities in respect of heat and mixture. So, too, Zoology, besides arranging animals, describes their structure, like Comparative Anatomy. In truth, all arrangement and classifying depends upon noting the things in which the objects agree and differ; and among those things, in which animals, plants, and minerals agree, or differ, must be considered the anatomical qualities of the one and the chemical qualities of the other. From hence, in a great measure, follows the *second* observation, namely, that the sciences mutually assist each other. We have seen how Arithmetic and Algebra aid Geometry, and how both the purely Mathematical Sciences aid Mechanical Philosophy. Mechanical Philosophy, in like manner, assists, though, in the present state of our knowledge, not very considerably, both Chemistry and Anatomy, especially the latter; and Chemistry very greatly assists both Physiology, Medicine, and all the branches of Natural History.

The first great head, then, of Natural Science, is Mechanical Philosophy; and it consists of various subdivisions, each forming a science of great importance. The most essential of these, and which is indeed fundamental, and applicable to all the rest, is called *Dynamics*, from the Greek word signifying *power* or *force*, and it teaches the laws of motion in all its varieties. The case of the stone thrown forward, which we have already mentioned more than once, is an example. Another, of a more general nature, but more difficult to trace, far more important in its consequences, and of which, indeed, the former is only one particular case, relates to the motions of all bodies, which are attracted (or influenced, or drawn) by any power towards a certain point, while they are, at the same time, driven forward, by some push given to them at first, and forcing them onwards, at the same time that they are drawn towards the point. The line in which a body moves while so drawn and so driven, depends upon the force it is pushed with, the direction it is pushed in, and the kind of power that draws it towards the point; but at present, we are chiefly to regard the latter circumstance, the attraction towards the point. If this attraction be uniform, that is, the same at all distances from the point, the body will move in a circle, if one direction be given to the forward push. The case with which we are best acquainted is when the force decreases as the squares of the distances, from the centre or point of attraction, increase; that is, when the force is four times less at twice the distance, nine times less at thrice the distance, sixteen times less at four times the distance, and so on. A force of this kind acting on the body, will make it move in an oval, a parabola, or an hyperbola, according to the amount or direction of the impulse, or forward push, originally given; and there is one proportion of that force, which, if directed perpendicularly to the line in which the central force draws the body, will make it move round in a circle, as if it were a stone tied to a string and whirled round the hand. The most usual proportions in nature, are those which determine bodies to move in an oval or ellipse, the curve described by means of a cord fixed at both ends, in the way already explained. In this case, the point of attraction, the point towards which the body is drawn, will be nearer one end of the ellipse than the other, and the time the body will take to go round, compared with the time any other body would take, moving at a different distance from the same point of attraction, but drawn towards that point with a force which bears the same proportion to the distance, will bear a certain proportion, discovered by mathematicians, to the average distances of the two bodies from the point of common attraction. If you multiply the numbers expressing the times of going round, each by itself, the products will be to one another in the proportion of the average distances multiplied each by itself, and that product again by the distance. Thus, if one body take two hours, and is five yards distant, the other, being ten yards off, will take something less than five hours and forty minutes.*

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* This is expressed mathematically by saying, that the squares of the times are as the cubes of the distances. Mathematical language is not only the simplest and most easily understood of any, but the shortest also.

Now, this is one of the most important truths in the whole compass of science; for it does so happen that the force with which bodies fall towards the earth, or what is called their *gravity*, the power that draws or attracts them towards the earth, varies with the distance from the Earth's centre, exactly in the proportion of the squares, lessening as the distance increases: at two diameters from the Earth's centre, it is four times less than at one; at three diameters, nine times less; and so forth. It goes on lessening, but never is destroyed, even at the greatest distances to which we can reach by our observations, and there can be no doubt of its extending indefinitely beyond. But, by astronomical observations made upon the motion of the heavenly bodies, upon that of the moon for instance, it is proved that her movement is slower and quicker at different parts of her course, in the same manner as a body's motion on the earth would be slower and quicker, according to its distance from the point it was drawn towards, provided it was drawn by a force acting in the proportion to the squares of the distance, which we have frequently mentioned; and the proportion of the time to the distance is also observed to agree with the rule above referred to. Therefore, she is shown to be attracted towards the Earth by a force that varies according to the same proportion in which gravity varies; and she must consequently move in an ellipse round the Earth, which is placed in a point nearer the one end than the other of that curve. In like manner, it is shown that the Earth moves round the Sun in the same curve line, and is drawn towards the sun by a similar force; and that all the other planets in their courses, at various distances, follow the same rule, moving in ellipses, and drawn towards the Sun by the same kind of power. Three of them have moons like the Earth, only more numerous, for Jupiter has four, Saturn seven, and Herschel six, so very distant, that we cannot see them without the help of glasses; but all those moons move round their principal planets, as ours does round the Earth, in ovals or ellipses; while the planets, with their moons, move in their ovals round the Sun, like our own Earth with its moon.

But this power, which draws them all towards the sun, and regulates their path and their motion round him, and which draws the moons towards the principal planets, and regulates their motion and path round those planets, is the same with the gravity by which bodies fall towards the earth, being attracted by it. Therefore, the whole of the heavenly bodies are kept in their places, and wheel round the sun, by the same influence or power that makes a stone fall to the ground.

It is usual to call the sun, and the planets which with their moons move round him (eight in number, including Neptune, lately discovered; and besides these, are the asteroids), the *Solar System*, because they are a class of the heavenly bodies far apart from the innumerable fixed stars, and so near each other as to exert a perceptible influence on one another, and thus to be connected together.

The *Comets* belong to the same system, according to this manner of viewing the subject. They are bodies which move in elliptical paths, but far longer and narrower than the curves in which the earth and the other planets and their moons roll. Our curves are not much less round than circles; the paths of the comets are long and narrow, so as, in many places, to be more nearly straight lines than circles. They differ from the planets and their moons in another respect; they do not depend on the sun for the light they give, as our moon plainly does, being dark when the earth comes between her and the sun; and as the other planets do, those of them that are nearer the sun than we are, being dark when they come between us and him, appearing to pass across his surface. But the comets give light always of themselves, being apparently vast bodies heated red-hot by coming in their course far nearer the sun than the nearest of the planets ever do. Their motion, when near the sun, is much more rapid than that of the planets; they both approach him much nearer, retreat from him to much greater distances, and take much longer time in going round him, than any of the planets do. Yet even these comets are subject to the same great law of gravitation which regulates the motions of the planets. Their year, the time they take to revolve, is in some cases 75, in others 135, in others 300 of our years; their distance is a hundred times our distance when farthest off, and not a hundred and sixtieth of our distance when nearest the sun; their swiftest motion is above twelve times swifter than ours, although ours is a hundred and forty times swifter than a cannon-ball's; yet their path is a curve of the same kind with ours, though longer and flatter, differing in its formation only as one oval differs from another by the string you draw it with having the ends fixed at two points more distant from each other: consequently the sun, being in one of those points, is much nearer the end of the path the comet moves in, than he is near the end of our path. Their motion, too, follows the same rule, being swifter the nearer the sun: the attraction of the sun for them varies according to the squares of the distances, being four times less at twice the distance, nine times less at thrice, and so on; and the proportion between the times of revolving and the distances is exactly the same, in the case of those remote bodies, as in that of the moon and the earth. One law prevails over all, and regulates their motions as well as our own; it is the gravity of the comets towards the sun, and they, like our own earth and moon, wheel round him in boundless space, drawn by the same force, acting by the same rule, which makes a stone fall when dropped from the hand.

The more full and accurate our observations are upon those heavenly bodies, the better we find all their motions agreeing with this great doctrine; although, no doubt, many things are to be taken into the account besides the force that draws them to the different centres. Thus, while the moon is drawn by the earth, and the earth by the sun, the moon is also drawn directly by the sun; and while Jupiter is drawn by the sun, so are his moons; and both Jupiter and his moons are drawn by Saturn: nay, as this power of gravitation is quite universal, and as no body can attract or draw another without being itself drawn by that other, the earth is drawn by the moon, while the moon is drawn by the earth; and the sun is attracted by the planets which he draws towards himself. These mutual attractions give rise to many deviations from the simple line of the ellipse, and produce many irregularities in the simple calculation of the times and motions of the bodies that compose the system of the universe. But the extraordinary powers of investigation applied to the subject by the modern improvements in mathematics, have enabled us at length to reduce even the greatest of the irregularities to order and system, and to unfold one of the most wonderful truths in all sciences; namely, that by certain necessary consequence of the simple fact upon which the whole fabric rests, the proportion of the attractive force to the distances at which it operates,—all the irregularities which at first seemed to disturb the order of the system, and to make the appearances depart from the doctrine, are themselves subject to a certain fixed rule, and can never go beyond a particular point, but must begin to lessen when they have slowly reached that point, and must then lessen until they reach another point, when they begin again to increase; and so on, for ever. Nay, so perfect is the arrangement of the whole system, and so accurately does it depend upon mathematical principles, that irregularities, or rather apparent deviations, have been discovered by mathematical reasoning before astronomers had observed them, and then their existence has been ascertained by observation, and found to agree precisely with the results of calculation.* Thus, the planets move in ovals, from gravity, the power that attracts them towards the sun, combined with the original impulse they received forwards; and the disturbing forces are continually varying the course of the curves or ovals, making them bulge out in the middle, as it were, on the sides, though in a very small proportion to the whole length of the ellipse. The oval thus bulging, its breadth increases by a very small quantity yearly and daily; and after a certain large number of years, the bulging becomes as great as it ever can be: then the alteration takes a contrary direction, and the curve gradually flattens as it had bulged; till, in the same number of years which it took to bulge, it becomes as flat as it ever can be, and then it begins to bulge again, and so on for ever. And so, too, of every other disturbance and irregularity in the system: what at first appears to be some departure from the rule, when more fully examined, turns out to be only a consequence of it, or the result of a more general arrangement springing from the principle of gravitation; an arrangement of which the rule itself, and the apparent or supposed exception, both form parts.

The power of gravitation, which thus regulates the whole system of the universe, is found to rule each member or branch of it separately. Thus, it is demonstrated that the tides of the ocean are caused by the gravitation which attracts the water towards the sun and moon; and the figure both of our earth and of such of the other bodies as have a spinning motion round their axis, is determined by gravitation combined with that motion; they are all flattened towards the ends of the axis they spin upon, and bulge out towards the middle.

The great discoverer of the principle on which all these truths rest, Sir Isaac Newton, certainly by far the most extraordinary man that ever lived, concluded by reasoning upon the nature of motion and matter, that this flattening must take place in our globe: every one before his time had believed the earth to be a perfect sphere or globe, chiefly from observing the round shadow which it casts on the moon in eclipses; and it was many years after his death that the accuracy of his opinion was proved by measurements on the earth's surface, and by the different weight and attraction of bodies at the equator, where it bulges, and at the poles, where it is flattened. The improvement of telescopes has enabled us to ascertain the same fact with respect to the planets Jupiter and Saturn.

Besides unfolding the general laws which regulate the motions and figures of the heavenly bodies forming our Solar System, Astronomy consists in calculations of the places, times, and eclipses of those bodies, and their moons or *satellites* (from a Latin word signifying an *attendant*), and in observations of the Fixed Stars, which are innumerable assemblages of bodies, not moving round the Sun as our Earth and the other planets do, nor receiving the light they shine with, from his light; but shining, as the Sun and the Comets do, with a light of their own, and placed, to all appearance, immovable, at immense distances from our world, that is, from our Solar System. Each of them is probably the sun of some other system like our own, composed of planets and their moons or satellites; but so extremely distant from us, that they all are seen by us like one point of faint light, as you see two lamps placed a few inches asunder, only like one, when you view them a great way off. The number of the Fixed Stars is prodigious: even to the naked eye they are very numerous, about 3,000 being thus visible; but when the heavens are viewed through the telescope, stars become visible in numbers wholly incalculable: 2,000 are discovered in one of the small collections of a few visible stars called *Constellations*; nay, what appears to the naked eye only a light cloud, as the *Milky Way*, when viewed through the telescope, proves to be an assemblage of innumerable Fixed Stars, each of them in all likelihood a sun and a system like the rest, though at an immeasurable distance from ours.

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* The application of mathematics to chemistry has already produced a great change in that science, and is calculated to produce still greater improvements. It may be almost certainly reckoned upon as the source of new discoveries, made by induction after the mathematical reasoning has given the suggestion. The learned reader will perceive that we allude to the beautiful doctrine of *Definite* or *Multiple Proportions*. To take an example; the probability of an oxide of arsenic being discovered is impressed upon us, by the composition of arsenious and arsenic acids, in which the oxygen is as 2 to 3; and therefore we may expect to find a compound of the same base, with the oxygen as unity. The extraordinary action of chlorine and its compounds on light,leads us to expect some further discovery respecting its composition, perhaps respecting the matter of light.

The size, and motions, and distances of the heavenly bodies are such as to exceed the power of ordinary imagination, from any comparison with the smaller things we see around us. The Earth's diameter is nearly 8,000 miles in length; but the Sun's is above 880,000 miles, and the bulk of the Sun is above 1,300,000 times greater than that of the Earth. The planet Jupiter, which looks like a mere speck, from his vast distance, is nearly 1,300 times larger than the Earth. Our distance from the Sun is above 95 millions of miles; but Jupiter is 490 millions, and Saturn 900 millions of miles distant from the Sun. The rate at which the Earth moves round the Sun is 68,000 miles an hour, or 140 times swifter than the motion of a cannon-ball; and the planet Mercury, the nearest to the Sun, moves still quicker, nearly 110,000 miles an hour. We, upon the Earth's surface, besides being carried round the Sun, move round the Earth's axis by the rotatory or spinning motion which it has; so that every 24 hours we move in this manner near 24,000 miles, besides moving round the Sun above 1,600,000 miles. These motions and distances, however, prodigious as they are, seem as nothing compared to those of the comets, one of which, when farthest from the Sun, is 11,200 millions of miles from him; and, when nearest the Sun, flies at the amazing rate of 880,000 miles an hour. Sir Isaac Newton calculated its heat at 2,000 times that of red-hot iron; and that it would take thousands of years to cool. But the distance of the Fixed Stars is yet more vast: they have been supposed to be 400,000 times farther from us than we are from the Sun, that is 38 millions of millions of miles; so that a cannon-ball would take nearly nine millions of years to reach one of them, supposing there was nothing to hinder it from pursuing its course thither. As light takes about eight minutes and a quarter to reach us from the Sun, it would be above six years in coming from one of those stars; but the calculations of later astronomers prove some stars to be so far distant, that their light must take centuries before it can reach us; so that every particle of light which enters our eyes left the star it comes from three or four hundred years ago.

Astronomers have, by means of their excellent glasses, aided by Geometry and calculations, been able to observe not only stars, planets, and their satellites, invisible to the naked eye, but to measure the height of mountains in the Moon, by observations of the shadows which those eminences cast on her surface; the angles of these shadows being taken for that purpose.

The tables, which they have by the like means been enabled to form of the heavenly motions, are of great use in navigation. By means of the eclipses of Jupiter's satellites, and by the tables of the Moon's motions, we can ascertain the position of a ship at sea; for the observation of the Sun's height at mid-day gives the *latitude* of the place; that is, its distance from the equinoctial or equator, the line passing through the middle of the Earth's surface equally distant from both poles; and these tables, with the observations of the satellites, or moons, give the distance east and west of the observatory for which the tables are calculated—called the *longitude* of the place: consequently the mariner can thus tell, nearly, in what part of the ocean he is, how far he has sailed from his port of departure, and how far he must sail, and in what direction, to gain the port of his destination. The advantage of this knowledge is therefore manifest in the common affairs of life; but it sinks into insignificance compared with the vast extent of those views which the contemplations of the science afford, of numberless worlds filling the immensity of space, and all kept in their places, and adjusted in their prodigious motions by the same simple principle, under the guidance of an all-wise and all-powerful Creator.

We have been considering the application of Dynamics to the motions of the heavenly bodies, which forms the science of *Physical Astronomy*. The application of Dynamics to the calculation, production, and direction of motion, forms the science of *Mechanics*, sometimes called *Practical Mechanics*, to distinguish it from the more general use of the word, which comprehends every thing that relates to motion and force. The fundamental principle of the science, upon which it mainly depends, flows immediately from a property of the circle already mentioned, and which, perhaps, appeared at the moment of little value,—that the lengths of circles are in proportion to their diameters. Observe how upon this simple truth nearly the whole of those contrivances are built by which the power of man is increased as far as solid matter assists him in extending it; and nearly the whole of those doctrines, too, by which he is enabled to explain the voluntary motions of animals, as far as these depend upon their own bodies. There can be nothing more instructive in showing the importance and fruitfulness of scientific truths, however trivial and forbidding they may at first sight appear. For it is an immediate consequence of this property of the circle, that if a rod of iron, or beam of wood, or any other solid material, be placed on a point, or pivot, so that it may move as the arms of a balance do round its centre, or a see-saw board does round its prop, the two ends will go through parts of circles, each proportioned to that arm of the beam to which it belongs: the two circles will be equal if the pivot be in the centre or middle point of the beam; but if it be nearer one end than the other, say three times, that end will go through a circular space, or arch, three times shorter than the circular space the other end goes through in the same time. If, then, the end of the long beam pass through three times the space, it must move with three times the swiftness of the short beam's end, since both move in the same time; and therefore any force applied to the long end must overcome the resistance of three times that force applied at the opposite end, since the two ends move in contrary directions: hence one pound placed at the long end would balance three placed at the short end. The beam we have been supposing is called a *Lever*, and the same rule must evidently hold for all proportions of the lengths of its arms. If, then, the lever be seventeen feet long, and the pivot, or *fulcrum* (as it is called, from a Latin word signifying *support*), be a foot from one end, an ounce placed on the other end will balance a pound placed on the near end; and the least additional weight, or the slightest push or pressure on the far end, so loaded, will make the pound weight on the other move upwards. If, instead of an ounce, we place upon the end of the long arm the short arm of a second beam or lever supported by a fulcrum, one foot from it, and then place the long arm of this second lever upon the short arm of a third lever, whose fulcrum is one foot from it; and if we put on the end of this third lever's long arm an ounce weight, that ounce will move upwards a pound on the second lever's long arm, and this moving upwards will cause the short arm to force downwards sixteen pounds at the long end of the first lever, which will make the short end of the first lever move upwards, though two hundred and fifty-six pounds be laid on it: the same thing continuing, a pound on the long arm of the third lever will move a ton and three-quarters on the short arm of the first lever; that is, will balance it, so that the slightest pressure with the finger, or a touch from a child's hand, will move as much as two horses can draw. The lever is called, on this account, a *mechanical power;* and there are five other mechanical powers, of most of which its properties form the foundation; indeed they have all been resolved into combinations of levers. The pulley seems the most difficult to reduce under the principle of the lever. Thus the *wheel and axle* is only a lever moving round an axle, and always retaining the effect gained during every part of the motion, by means of a rope wound round the butt end of the axle; the spoke of the wheel being the long arm of the lever, and the half diameter of the axle its short arm. By a combination of levers, wheels, pulleys, so great an increase of force is obtained, that, but for the obstruction from friction, and the resistance of the air, there could be no bounds to the effect of the smallest force thus multiplied; and to this fundamental principle Archimedes, one of the most illustrious mathematicians of ancient times, referred, when he boasted, that if he only had a pivot or fulcrum whereon he might rest his machinery, he could move the Earth. Upon so simple a truth, assisted by the aid derived from other sources, rests the whole fabric of mechanical power, whether for raising weights, or cleaving rocks, or pumping up rivers from the bowels of the earth; or, in short, performing any of those works to which human strength, even augmented by the help of the animals whom Providence has subdued to our use, would prove altogether inadequate.

The application of Dynamics to the pressure and motions of fluids, constitutes a science which receives different appellations according as the fluids are heavy and liquid like water, or light and invisible like air. In the former case it is called *Hydrodynamics*, from the Greek words signifying *water*, and *power* or *force;* in the latter *Pneumatics*, from the Greek word signifying *breath* or *air;* and Hydrodynamics is divided into *Hydrostatics*, which treats of the weight and pressure of liquids, from the Greek words for *balancing* of *water;* and *Hydraulics*, which treats of their motion, from the Greek name for certain musical instruments played with *water* in *pipes*.

The discoveries to which experiments, aided by mathematical reasoning, have led, upon the pressure and motion of fluids, are of the greatest importance, whether we regard their application to practical purposes, or to their use for explaining the appearances in nature, or their singularity as the subjects of scientific contemplation. When it is found that the pressure of water or any other liquid upon the surface that contains it, is not in the least degree proportioned to its bulk, but only to the height at which it stands, so that a long small pipe, containing a pound or two of the fluid, will give the pressure of twenty or thirty tons; nay, of twice or thrice as much, if its length be increased and its bore lessened, without the least regard to the quantity of the liquid, we are not only astonished at so extraordinary and unexpected a property of matter, but we straightway perceive one of the great agents employed in the vast operations of nature, in which the most trifling means are used to work the mightiest effects. We likewise learn to guard against many serious mischiefs in our own works, and to apply, safely and usefully, a power calculated, according as it is directed, either to produce unbounded devastation, or to render the most beneficial service

Nor are the discoveries relating to the Air less interesting in themselves, and less applicable to important uses. It is an agent, though invisible, as powerful as Water, in the operations both of nature and of art. Experiments of a simple and decisive nature show the amount of its pressure to be between 14 and 15 pounds on every square inch; but, like all other fluids, it presses equally in every direction: so that though, on the hand, there is a pressure downwards of above 250 pounds, yet this is exactly balanced by an equal pressure upwards, from the air pressing round and getting below. If, however, the air on one side be removed, the whole pressure from the other acts unbalanced. Hence the ascent of water in pumps, which suck out the air from a barrel, and allow the pressure upon the water to force it up 32 or 33 feet, that body of water being equal to the weight of the atmosphere. Hence the ascent of the mercury in the barometer is only 28 or 29 inches, mercury being between 13 and 14 times heavier than water. Hence, too, the motion of the steam-engine; the piston of which, until the direct force of steam was applied, used to be pressed downwards by the weight of the atmosphere from above, all air being removed below by filling the cylinder with steam, and then suddenly cooling and converting that steam into water, so as to leave nothing in the space it had occupied. Hence, too, the power which some animals possess of walking along the perpendicular surfaces of walls, and even the ceilings of rooms, by squeezing out the air between the inside of their feet and the wall, and thus being supported by the pressure of the air against the outside of their feet.

The science of *Optics* (from the Greek word for *seeing*), which teaches the nature of light, and of the sensation conveyed by it, presents, of itself, a field of unbounded extent and interest. To it the arts, and the other sciences, owe those most useful instruments which have enabled us at once to examine the minutest parts of the structure of animal and vegetable bodies, and to calculate the size and the motions of the most remote of the heavenly bodies. But as an object of learned curiosity, nothing can be more singular than the fundamental truth discovered by the genius of Newton,—that the light, which we call white, is in fact composed of all the colours, blended in certain proportions; unless, perhaps, it be that astonishing conjecture of his unrivalled sagacity, by which he descried the inflammable nature of the diamond, and its belonging, against all appearance of probability, to the class of oily substances, from having observed, that it stood among them, and far removed from all crystals, in the degree of its action upon light; a conjecture turned into certainty by discoveries made a century afterwards.

To a man who, for original genius and strong natural sense, is not unworthy of being named after this illustrious sage, we owe the greater part of *Electrical* science. It treats of the peculiar substance, resembling both light and heat, which, by rubbing, is found to be produced in a certain class of bodies, as glass, wax, silk, amber; and to be conveyed easily or *conducted* through others, as wood, metals, water; and it has received the name of *Electricity*, from the Greek word for *amber*. Dr. Franklin discovered that this is the same matter which, when collected in the clouds, and conveyed from them to the earth, we call *lightning*, and whose noise, in darting through the air, is *thunder*. The observation of some movements in the limbs of a dead frog gave rise to the discovery of *Voltaic Electricity*, or *Galvanism*, as it was at first called from the name of the discoverer; and which has of late years given birth to improvements that have changed the face of chemical philosophy, affording a new proof how few there are of the processes of nature incapable of repaying the labour we bestow in patiently and diligently examining them. It is to the results of the remark accidentally made upon the twitching in the frog's leg, not, however, hastily dismissed and forgotten, but treasured up and pursued through many an elaborate experiment and calculation, that we owe our acquaintance with the extraordinary metals potassium and sodium, lighter than water, and more inflammable than phosphorus, which form, when burned by mere exposure to the air, two of the salts best known in commerce, and the principal ingredients in saltpetre and common salt.

In order to explain the nature and objects of those branches of Natural Science more or less connected with the mathematics, some details were necessary, as without them it was difficult immediately to perceive their importance, and, as it were, relish the kind of instruction which they afford. But the same course need not be pursued with respect to the other branches. The value and the interest of chemistry is at once perceived, when it is known to teach the nature of all bodies; the relations of simple substances to heat and to one another, or their combinations together; the composition of those which nature produces in a compound state; and the application of the whole to the arts and manufactures. Some branches of philosophy, again, are chiefly useful and interesting to particular classes, as surgeons and physicians. Others are easily understood by a knowledge of the principles of Mechanics and Chemistry, of which they are applications and examples; as those which teach the structure of the earth and the changes it has undergone; the motions of the muscles, and the structure of the parts of animals; the qualities of animal and vegetable substances; and that department of Agriculture which treats of soils, manure, and machinery. Other branches are only collections of facts, highly curious and useful indeed, but which any one who reads or listens, perceives as clearly, and comprehends as readily, as the professed student. To this class belongs Natural History, in so far as it describes the habits of animals and plants, and its application to that department of Agriculture which treats of cattle and their management.

IV. Application of Natural Science to the Animal and Vegetable World.

But, for the purpose of further illustrating the advantages of Philosophy, its tendency to enlarge the mind, as well as to interest it agreeably, and afford pure and solid gratification, a few instances may be given of the singular truths brought to light by the application of Mathematical, Mechanical, and Chemical knowledge to the habits of animals and plants; and some examples may be added of the more ordinary and easy, but scarcely less interesting observations, made upon those habits, without the aid of the profounder sciences.

We may remember the curve line which mathematicians call a Cycloid. It is the path which any point of a circle, moving along a plane, and round its centre, traces in the air; so that the nail on the felly of a cart-wheel moves in a Cycloid, as the cart goes along, and as the wheel itself both turns round its axle and is carried along the ground. Now this curve has certain properties of a peculiar and very singular kind with respect to motion. One is, that if any body whatever move in a cycloid by its own weight or swing, together with some other force acting upon it all the while, it will go through all distances of the same curve in exactly the same time; and, accordingly, pendulums have sometimes been contrived to swing in such a manner, that they shall describe cycloids, or curves very near cycloids, and thus move in equal times, whether they go through a long or a short part of the same curve. Again, if a body is to descend from any one point to any other, not in the perpendicular, by means of some force acting on it together with its weight, the line in which it will go the quickest of all will be the cycloid; not the straight line, though that is the shortest of all lines which can be drawn between the two points; nor any other curve whatever, though many are much flatter, and therefore shorter than the cycloid—but the cycloid, which is longer than many of them, is yet, of all curved or straight lines which can be drawn, the one the body will move through in the shortest time. Suppose, again, that the body is to move from one point to another, by its weight and some other force acting together, but to go through a certain space,—as a hundred yards,—the way it must take to do this, in the shortest time possible, is by moving in a cycloid; or the length of a hundred yards must be drawn into a cycloid, and then the body will descend through the hundred yards in a shorter time than it could go the same distance in any other path whatever. Now, it is believed that Birds, as the Eagle, which build in the rocks, drop or fly down from height to height in this course. It is impossible to make very accurate observations of their flight and path; but there is a general resemblance between the course they take and the cycloid, which has led ingenious men to adopt this opinion.

If we have a certain quantity of any substance, a pound of wood, for example, and would fashion it in the shape to take the least room, we must make a globe of it; it will in this figure have the smallest surface. But suppose we want to form the pound of wood, so that in moving through the air or water it shall meet with the least possible resistance; then we must lengthen it out for ever, till it becomes not only like a long-pointed pin, but thinner and thinner, longer and longer, till it is quite a straight line, and has no perceptible breadth or thickness at all. If we would dispose of the given quantity of matter, so that it shall have a certain length only, say a foot, and a certain breadth at the thickest part, say three inches, and move through the air or water with the smallest possible resistance which a body of those dimensions can meet, then we must form it into a figure of a peculiar kind called the *Solid of least resistance*, because, of all the shapes that can be given to the body, its length and breadth remaining the same, this is the one which will make it move with the least resistance through the air, or water, or other fluid. A very difficult chain of mathematical reasoning, by means of the highest branches of algebra, leads to a knowledge of the curve which, by revolving on its axis, makes a solid of this shape, in the same way that a circle, by so revolving, makes a sphere or globe; and the curve certainly resembles closely the face or head part of a fish. Nature, therefore (by which we always mean the Divine Author of nature), has fashioned these fishes so, that, according to mathematical principles, they swim the most easily through the element they live and move in.*

Suppose upon the face part of one of these fishes a small insect were bred, endowed with faculties sufficient to reason upon its condition, and upon the motion of the fish it belonged to, but never to have discovered the whole size and shape of the face part; it would certainly complain of the form as clumsy, and fancy that it could have made the fish so as to move with less resistance. Yet if the whole shape were disclosed to it, and it could discover the principle on which that shape was preferred, it would at once perceive, not only that what had seemed clumsy was skilfully contrived, but that, if any other shape whatever had been taken, there would have been an error committed; nay, *that there must of necessity* have been an error; and that the very best possible arrangement had been adopted. So it may be with man in the universe, where, seeing only a part of the great system, he fancies there is evil; and yet, if he were permitted to survey the whole, what had seemed imperfect might appear to be necessary for the general perfection, insomuch that any other arrangement, even of that seemingly imperfect part, must needs have rendered the whole less perfect. The common objection is, that what seems evil *might have* been avoided; but in the case of the fish's shape, it *could not* have been avoided.

It is found by optical inquiries, that the particles or rays of light, in passing through transparent substances of a certain form, are bent to a point where they make an image or picture of the shining bodies they come from, or of the dark bodies they are reflected from. Thus, if a pair of spectacles be held between a candle and the wall, they make two images of the candle upon it; and if they be held between the window and a sheet of paper when the sun is shining, they make a picture on the paper of the houses, trees, fields, sky, and clouds. The eye is found to be composed of several natural magnifiers which make a picture on a membrane at the back of it, and from this membrane there goes a nerve to the brain, conveying the impression of the picture, by means of which we see. Now, white light was discovered by Newton to consist of differently-coloured parts, which are differently bent in passing through transparent substances, so that the lights of several colours come to a point at different distances, and thus create an indistinct image at any one distance. This was long found to make our telescopes imperfect, insomuch that it became necessary to make them of reflectors or mirrors, and not of magnifying glasses, the same difference not being observed to affect the reflection of light. But another discovery was, about fifty years afterwards, made by Mr. Dollond—that, by combining different kinds of glass in a compound magnifier, the difference may be greatly corrected; and on this principle he constructed his telescopes. It is found, too, that the different natural magnifiers of the eye are combined upon a principle of the same kind. Thirty years later, a third discovery was made by Mr. Blair, of the greatly superior effect which combinations of different liquids have in correcting the imperfection; and, most wonderful to think, when the eye is examined, we find it consists of different liquids, acting naturally upon the same principle which was thus recently found out in optics, by many ingenious mechanical and chemical experiments.

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* The feathers of the wings of birds are found to be placed at the *best possible* angle for helping on the bird by their action on the air.

Again, the point to which any magnifier collects the light is more or less distant as the magnifier is flatter or rounder, so that a small globe of glass or any transparent substance makes a microscope. And this property of light depends upon the nature of lines, and is purely of a mathematical nature, after we have once ascertained by experiment, that light is bent in a certain way when it passes through transparent bodies. Now birds flying in the air, and meeting with many obstacles, as branches and leaves of trees, require to have their eyes sometimes as flat as possible for protection; but sometimes as round as possible, that they may see the small objects, flies, and other insects, which they are chasing through the air, and which they pursue with the most unerring certainty. This could only be accomplished by giving them a power of suddenly changing the form of their eyes. Accordingly, there is a set of hard scales placed on the outer coat of their eye, round the place where the light enters; and over these scales are drawn the muscles or fibres by which motion is communicated; so that, by acting with these muscles, the bird can press the scales, and squeeze the natural magnifier of the eye into a round shape when it wishes to follow an insect through the air, and can relax the scales, in order to flatten the eye again, when it would see a distant object, or move safely through leaves and twigs. This power of altering the shape of the eye is possessed by birds of prey in a very remarkable degree. They can thus see the smallest objects close to them, and can yet discern larger bodies at vast distances, as a carcass stretched upon the plain, or a dying fish afloat on the water.

A singular provision is made for keeping the surface of the bird's eye clean—for wiping the glass of the instrument, as it were—and also for protecting it, while rapidly flying through the air and through thickets, without hindering the sight. Birds are, for these purposes, furnished with a third eyelid, a fine membrane or skin, which is constantly moved very rapidly over the eyeball by two muscles placed in the back of the eye. One of the muscles ends in a loop, the other in a string which goes through the loop, and is fixed in the corner of the membrane, to pull it backward and forward. If you wish to draw a thing towards any place with the least force, you must pull directly in the line between the thing and the place; but if you wish to draw it as quickly as possible, and with the most convenience, and do not regard the loss of force, you must pull obliquely, by drawing it in two directions at once. Tie a string to a stone, and draw it straight towards you with one hand; then make a loop on another string, and running the first through it, draw one string in each hand, not towards you, but sideways, till both strings are stretched in a straight line: you will see how much more easily the stone moves quickly than it did before when pulled straight forward. Again, if you tie strings to the two ends of a rod, or slip of card, in a running groove, and bring them to meet and pass through a ring or hole, for every inch in a straight line that you draw both together below the ring, the rod will move onward two. Now this is proved, by mathematical reasoning, to be the necessary consequence of forces applied obliquely: there is a loss of power, but a great gain in velocity and convenience. This is the thing required to be gained in the third eyelid, and the contrivance is exactly that of a string and a loop, moved each by a muscle, as the two strings are by the hands in the cases we have been supposing.

A third eyelid of the same kind is found in the horse, and called the *haw;* it is moistened with a pulpy substance (or mucilage) to take hold of the dust on the eyeball, and wipe it clear off; so that the eye is hardly ever seen with anything upon it, though greatly exposed from its size and posture. The swift motion of the haw is given to it by a gristly elastic substance placed between the eyeball and the socket, and striking obliquely, so as to drive out the haw with great velocity over the eye, and then let it come back as quickly. Ignorant persons, when this haw is inflamed from cold, and swells so as to appear, which it never does in a healthy state, often mistake it for an imperfection, and cut it off: so nearly do ignorance and cruelty produce the same mischief.

If any quantity of matter, as a pound of wood or iron, be fashioned into a rod of a certain length, say one foot, the rod will be strong in proportion to its thickness; and, if the figure be the same, that thickness can only be increased by making it hollow. Therefore hollow rods or tubes, of the same length and quantity of matter, have more strength than solid ones. This is a principle so well understood now, that engineers make their axles and other parts of machinery hollow, and therefore stronger with the same weight than they would be if thinner and solid. Now the bones of animals are all more or less hollow; and are therefore stronger with the same weight and quantity of matter, than they otherwise would be. But birds have the largest bones in proportion to their weight; their bones are more hollow than those of animals which do not fly; and therefore they have the needful strength, without having to carry more weight than is absolutely necessary. Their quills derive strength from the same construction. They possess another peculiarity to help their flight. No other animals have any communication between the air-vessels of the lungs and the hollow parts of their bodies; but birds have it; and by this means they can blow out their bodies as we do a bladder, and thus become lighter when they would either make their flight towards the ground slower, or rise more swiftly, or float more easily in the air; while, by lessening their bulk and closing their wings, they can drop more speedily if they wish to chase or to escape. Fishes possess a power of the same kind, though not by the same means. They have *air-bladders* in their bodies, and can puff them out, or press them closer, at pleasure: when they want to rise in the water, they fill out the bladder, and this lightens them; when they would sink, they squeeze the bladder, pressing the air into a smaller space, and this makes them heavier. If the bladder break, the fish remains at the bottom, and can be held up only by the most laborious exertions of the fins and tail. Accordingly, flat fish, such as skaits and flounders, which have no air-bladders, seldom rise from the bottom, but are found lying on banks in the sea, or at the bottom of rivers.

If you have a certain space, as a room, to fill up with closets or little cells, all of the same size and shape, there are only three figures which will answer, and enable you to fill the room without losing any space between the cells; they must either be squares, or figures of three equal sides, or figures of six equal sides. With any other figures whatever, space would be lost between the cells. This is evident upon considering the matter; and it is proved by mathematical reasoning. The six-sided figure is by far the most convenient of those three shapes, because its corners are flatter, and any round body placed in it has therefore more space, less room being lost in the corners. This figure, too, is the strongest of the three; any pressure from without or from within will hurt it least, as it has something of the strength of an arch. A round figure would be still stronger, but then room would be lost between the circles, whereas with the six-sided figure none is lost. Now, it is a most remarkable fact, that *Bees* build their cells exactly in this shape, and thereby save both room and materials beyond what they could save if they built them in any other shape whatever. They build in the very best possible shape for their purpose, which is to save all the room and all the wax they can. So far as to the shape of the walls of each cell; but the roof and floor, or top and bottom, are built on equally true principles. It is proved by mathematicians, that, to give the greatest strength, and save the most room, the roof and floor must be made of three square planes meeting in a point; and they have further proved, by a demonstration belonging to the highest parts of Algebra, that there is one particular angle or inclination of those planes to each other where they meet, which makes a greater saving of materials and of work than any other inclination whatever could possibly do. Now, the Bees actually make the tops and bottoms of their cells of three planes meeting in a point; and the inclinations or angles at which they meet are precisely those found out by the mathematician to be the best possible for saving wax and work.* Who would dream of the bee knowing the highest branch of the Mathematics—the fruit of Newton's most wonderful discovery—a result, too, of which he was himself ignorant, one of his most celebrated followers having found it out in a later age? This little insect works with a truth and correctness which are perfect, and according to the principles at which man has arrived only after ages of slow improvement in the most difficult branch of the most difficult science. But the Mighty and All-wise Creator, who made the insect and the philosopher, bestowing reason on the latter, and giving the former to work without it—to Him all truths are known to all eternity, with an intuition that mocks even the conceptions of the sagest of human kind.

It may be recollected, that when the air is exhausted or sucked out of any vessel, there is no longer the force necessary to resist the pressure of the air on the outside; and the sides of the vessels are therefore pressed inwards with violence: a flat glass would thus be broken, unless it were very thick; a round one, having the strength of an arch, would resist better; but any soft substance, as leather or skin, would be crushed or squeezed together at once. If the air were only sucked out slowly, the squeezing would be gradual; or, if it were only half sucked out, the skin would only be partly squeezed together. This is the process by which *Bees* reach the fine dust and juices of hollow flowers, like the honeysuckle, and some kinds of long fox-glove, which are too narrow for them to enter. They fill up the mouth of the flower with their bodies, and suck out the air, or at least a large part of it; this makes the soft sides of the flower close, and squeezes the dust and juice towards the insect, as well as a hand could do, if applied to the outside.

We may remember this pressure or weight of the atmosphere as shown by the barometer and the sucking-pump. Its weight is near fifteen pounds on every square inch, so that if we could entirely squeeze out the air between our two hands, they would cling together with a force equal to the pressure of double this weight, because the air would press upon both hands; and if we could contrive to suck or squeeze out the air between one hand and the wall, the hand would stick fast to the wall, being pressed on it with the weight of above two hundredweight, that is, near fifteen pounds on every square inch of the hand. Now, by a most curious discovery made by Sir Everard Home, the distinguished anatomist, it is found that this is the very process by which *Flies* and other insects of a similar description are enabled to walk up perpendicular surfaces, however smooth, as the sides of walls, and panes of glass in windows, and to walk as easily along the ceiling of a room with their bodies downwards and their feet over head. Their feet, when examined by a microscope, are found to have flat skins or flaps, like the feet of web-footed animals, as ducks and geese; and they have, by means of strong folds, the power of drawing the flap close down upon the glass or wall the fly walks on, and thus squeezing out the air completely, so as to make a vacuum between the foot and the glass or wall. The consequence of this is, that the air presses the foot on the wall with a very considerable force compared to the weight of the fly; for if its feet are to its body in the same proportion as ours are to our bodies, since we could support by a single hand on the ceiling of the room (provided it made a vacuum) more than our whole weight, namely, a weight of above fifteen stone, the fly can easily move on four feet in the same manner by help of the vacuum made under its feet.

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* Kœnig, pupil of Bernoulli, and Maclaurin, proved by very refined investigations, carried on with the aid of the fluxional calculus, that the obtuse angle must be 109° 28′, and the acute 70° 32′, to save the most wax and work possible. Maraldi found by actual measurement, that the angles are *about* 110° and 70°. These angles never vary in any place; and it is scarcely less singular, that the breadth of all bees' cells is everywhere precisely the same, the drone or male cells being 5-18ths, and the worker or female cells 13-60ths of an inch in breadth, and this in all countries and times.

It has likewise been found that some of the larger *Sea-animals* are, by the same construction, only upon a greater scale, enabled to climb the perpendicular and smooth surfaces of the ice hills among which they live. Some kinds of *Lizard* have a like power of climbing, and of creeping with their bodies downwards along the ceiling of a room; and the means by which they are enabled to do so are the same. In the large feet of those animals, the contrivance is easily observed, of the toes and muscles, by which the skin of the foot is pinned down, and the air excluded in the act of walking or climbing; but it is the very same, only upon a larger scale, with the mechanism of a fly's or a butterfly's foot; and both operations, the climbing of the sea-horse on the ice, and the creeping of the fly on the window or the ceiling, are performed exactly by the same power, the weight of the atmosphere, which causes the quicksilver to stand in the weather-glass, the wind to whistle through a key-hole, and the piston to descend in an old steam-engine.

Although philosophers are not agreed as to the peculiar action which light exerts upon vegetation, and there is even some doubt respecting the decomposition of air and water during that process, one thing is undeniable,—the necessity of light to the growth and health of plants: without it they have neither colour, taste, nor smell; and accordingly they are for the most part so formed as to receive it at all times when it shines on them. Their cups, and the little assemblages of their leaves before they sprout, are found to be more or less affected by the light, so as to open and receive it. In several kinds of plants this is more evident than in others; their flowers close entirely at night, and open in the day. Some constantly turn round towards the light, following the sun, as it were, while he makes or seems to make his revolution, so that they receive the greatest quantity possible of his rays. Thus clover in a field follows the apparent course of the sun. But all leaves of plants turn to the sun, place them how you will, light being essential to their thriving.

The lightness of inflammable gas is well known. When bladders of any size are filled with it, they rise upwards, and float in the air. Now, taking advantage of this fact, Mr. Knight suggested that the fine dust by means of which plants are impregnated one from another, is composed of very small globules, filled with this gas—in a word, of small air-balloons. These globules thus float from the male plant through the air, and, striking against the females, are detained by a glue prepared on purpose to stop them, which no sooner moistens the globules than they explode, and their substance remains, the gas flying off which enabled them to float. A provision of a very simple kind is also, in some cases, made to prevent the male and female blossoms of the same plant from breeding together, this being found to hurt the breed of vegetables, just as breeding, in and in, spoils the race of animals. It is contrived that the dust shall be shed by the male blossom before the female of the same plant is ready to be affected by it; so that the impregnation must be performed by the dust of some other plant, and in this way the breed be crossed. The light gas with which the globules are filled is most essential to the operation, as it conveys them to great distances. A plantation of yew-trees has been known, in this way, to impregnate another several hundred yards off.

The contrivance by which some creeper plants are enabled to climb walls and fix themselves, deserves attention. The *Virginia creeper* has a small tendril, ending in a claw, each toe of which has a knob, thickly set with extremely small bristles; they grow into the invisible pores of the wall, and swelling, stick there as long as the plant grows, and prevent the branch from falling; but when the plant dies, they become thin again, and drop out, so that the branch falls down.

The *Vanilla* plant of the West Indies climbs round trees likewise by means of tendrils; but when it has fixed itself, the tendrils drop off, and leaves are formed.

It is found by chemical experiments, that the juice which is in the stomachs of animals (called the *gastric* juice, from a Greek word signifying *the belly*) has very peculiar properties. Though it is for the most part a tasteless, clear, and seemingly a very simple liquor, it nevertheless possesses extraordinary powers of dissolving substances which it touches or mixes with; and it varies in different classes of animals. In one particular it is the same in all animals; it will not attack living matter, but only dead; the consequence of which is, that its powers of eating away and dissolving are perfectly safe to the animals themselves, in whose stomachs it remains without ever hurting them. This juice differs in different animals, according to the food on which they subsist; thus, in birds of prey, as kites, hawks, owls, it only acts upon animal matter, and does not dissolve vegetables. In other birds, and in all animals feeding on plants, as oxen, sheep, hares, it dissolves vegetable matter, as grass, but will not touch flesh of any kind. This has been ascertained by making them swallow balls with meat in them, and several holes drilled through to let the gastric juice reach the meat: no effect was produced upon it. We may further observe, that there is a most curious and beautiful correspondence between this juice in the stomach of different animals and the other parts of their bodies, connected with the important operations of eating and digesting their food. The use of the juice is plainly to convert what they eat into a fluid, from which, by various other processes, all their parts, blood, bones, muscles, &c., are afterwards formed. But the food is first of all to be obtained, and then prepared by bruising, for the action of the juice. Now birds of prey have instruments, their claws and beaks, for tearing and devouring their food (that is, animals of various kinds), but those instruments are useless for picking up and crushing seeds; accordingly, they have a gastric juice which dissolves the animals they eat; while birds, which have only a beak fit for pecking, and eating seeds, have a juice that dissolves seeds, and not flesh. Nay more, it is found that the seeds must be bruised before the juice will dissolve them: this you find by trying the experiment in a vessel with the juice; and accordingly the birds have a gizzard, and animals which graze have flat teeth, which grind and bruise their food, before the gastric juice is to act upon it.

We have seen how wonderfully the *Bee* works, according to rules discovered by man thousands of years after the insect had been following them with perfect accuracy. The same little animal seems to be acquainted with principles of which we are still ignorant. We can, by crossing, vary the forms of cattle with astonishing nicety; but we have no means of altering the nature of an animal once born, by means of treatment and feeding. This power, however, is undeniably possessed by the bees. When the queen bee is lost by death or otherwise, they choose a grub from among those which are born for workers; they make three cells into one, and placing the grub there, they build a tube round it; they afterwards build another cell of a pyramidal form, into which the grub grows; they feed it with peculiar food, and tend it with extreme care. It becomes, when transformed from the worm to the fly, not a worker, but a queen bee.

These singular insects resemble our own species in one of our worst propensities, the disposition to war; but their attention to their sovereign is equally extraordinary, though of a somewhat capricious kind. In a few hours after their queen is lost, the whole hive is in a state of confusion. A singular humming is heard, and the bees are seen moving all over the surface of the combs with great rapidity. The news spreads quickly, and when the queen is restored, quiet immediately succeeds. But if another queen be put upon them, they instantly discover the trick, and, surrounding her, they either suffocate or starve her to death. This happens if the false queen be introduced within a few hours after the first is lost or removed; but if twenty-four hours have elapsed, they will receive any queen, and obey her.

The labours and the policy of the *Ants* are, when closely examined, still more wonderful, perhaps, than those of the *Bees*. Their nest is a city consisting of dwelling-places, halls, streets, and squares into which the streets open. The food they principally like is the honey which comes from another insect found in their neighbourhood, and which they, generally speaking, bring home from day to day as they want it. Late discoveries have shown that they do not eat grain, but live almost entirely on animal food and this honey. Some kinds of ants have the foresight to bring home the insects on whose honey they feed, and keep them in particular cells, where they guard them to prevent their escaping, and feed them with proper vegetable matter, which they do not eat themselves. Nay, they obtain the eggs of those insects, and superintend their hatching, and then rear the young insect until it becomes capable of supplying the desired honey. They sometimes remove them to the strongest parts of their nest, where there are cells apparently fortified for protecting them from invasion. In those cells the insects are kept to supply the wants of the whole ants which compose the population of the city. It is a most singular circumstance in the economy of nature, that the degree of cold at which the ant becomes torpid is also that at which this insect falls into the same state. It is considerably below the freezing-point; so that they require food the greater part of the winter, and if the insects on which they depend for food were not kept alive during the cold in which the ants can move about, the latter would be without the means of subsistence.

How trifling soever this little animal may appear in our climate, there are few more formidable creatures than the ant of some tropical countries. A traveller, who once filled a high station in the French government, Mr. Malouet, has described one of their cities, and, were not the account confirmed by various testimonies, it might seem exaggerated. He observed at a great distance what seemed a lofty structure, and was informed by his guide that it consisted of an ant-hill, which could not be approached without danger of being devoured. Its height was from fifteen to twenty feet, and its base thirty or forty feet square. Its sides inclined, like the lower part of a pyramid, the point being cut off. He was informed that it became necessary to destroy these nests, by raising a sufficient force to dig a trench all round, and fill it with fagots, which were afterwards set on fire; and then battering with cannon from a distance, to drive the insects out and make them run into the flames. This was in South America; and African travellers have met them in the same formidable numbers and strength.

The older writers of books upon the habits of some animals, abound with stories which may be of doubtful credit. But the facts now stated respecting the Ant and Bee, may be relied on as authentic. They are the result of very late observations, and experiments made with great accuracy by several most worthy and intelligent men; and the greater part of them have the confirmation arising from more than one observer having assisted in the inquiries.[*] The habits of *Beavers* are equally well authenticated, and, being more easily observed, are vouched by a greater number of witnesses. These animals, as if to enable them to live and move either on land or water, have two web-feet like those of ducks or water-dogs, and two like those of land animals. When they wish to construct a dwelling-place, or rather city, for it serves the whole body, they choose a level ground with a stream running through it; they then dam up the stream so as to make a pond, and perform the operation as skilfully as we could ourselves. Next they drive into the ground stakes of five or six feet long in rows, wattling each row with twigs, and puddling or filling the interstices with clay, which they ram close in, so as to make the whole solid and water-tight. This dam is likewise shaped on the truest principles; for the upper side next the water slopes, and the side below is perpendicular: the base of the dam is ten or twelve feet thick; the top or narrow part two or three, and it is sometimes as long as one hundred feet.[†] The pond being thus formed and secured, they make their houses round the edge of it; they are cells, with vaulted roofs, and upon piles: they are made of stones, earth, and sticks; the walls are two feet thick, and plastered as neatly as if the trowel had been used. Sometimes they have two or three stories for retreating to in case of floods; and they always have two doors, one towards the water and one towards the land. They keep their winter provisions in stores, and bring them out to use; they make their beds of moss; they live on the bark of trees, gum, and crawfish. Each house holds from twenty to thirty, and there may be from ten to twenty-five houses in all. Some of their communities are larger than others, but there are seldom fewer than two or three hundred inhabitants. In working, they all bear their shares; some gnaw the trees and branches with their teeth, to form stakes and beams; others roll the pieces to the water; others, diving, make holes with their teeth to place the piles in; others collect and carry stones and clay; others beat and mix the mortar; and others carry it on their broad tails, and with these beat it and plaster it. Some superintend the rest, and make signals by sharp strokes with the tail, which are carefully attended to; the beavers hastening to the place where they are wanted to work, or to repair any hole made by the water, or to defend themselves or make their escape, when attacked by an enemy.

The fitness of different animals, by their bodily structure, to the circumstances in which they are found, presents an endless subject of curious inquiry and pleasing contemplation. Thus, the *Camel*, which lives in sandy deserts, has broad spreading hoofs to support him on the loose soil; and an apparatus in his body by which water is kept for many days, to be used when no moisture can be had. As this would be useless in the neighbourhood of streams or wells, and as it would be equally so in the desert, where no water is to be found, there can be no doubt that it is intended to assist in journeying across the sands from one watered spot to another. There is a singular and beautiful provision made in this animal's foot, for enabling it to sustain the fatigue of journeys under the pressure of its great weight. Besides the yielding of the bones and ligaments, or bindings, which gives elasticity to the foot of the deer and other animals, there is in the Camel's foot, between the horny sole and the bones, a cushion, like a ball, of soft matter, almost fluid, but in which there is a mass of threads extremely elastic, interwoven with the pulpy substance. The cushion thus easily changes its shape when pressed; yet it has such an elastic spring, that the bones of the foot press on it uninjured by the heavy body which they support, and this huge animal steps as softly as a cat.

Nor need we flee to the desert in order to witness an example of skilful structure; the limbs of the *Horse* display it strikingly. The bones of the foot are not placed directly under the weight; if they were in an upright position, they would make a firm pillar, and every motion would cause a shock. They are placed slanting or oblique, and tied together by an elastic binding on their lower surfaces, so as to form springs as exact as those which we make of leather and steel for carriages. Then the flatness of the hoof, which stretches out on each side, and the frog coming down in the middle between the quarters, adds greatly to the elasticity of the machine. Ignorant of this, ill-informed farriers nail the shoe in such a manner as to fix the quarters, and cause permanent contraction of the bones, ligaments, and hoof—so that the elasticity is destroyed; every step is a shock; inflammation and lameness ensue.*

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[*] A singular circumstance occasioned this in the case of Mr. Huber, by far the most eminent of these naturalists: he was quite blind, and performed all his experiments by means of assistants.

[†] If the base be twelve, and the top three feet thick, and the height six feet, the face must be the side of a right-angled triangle whose height is eight feet. This would be the exact proportion which there ought to be, upon mathematical principles, to give the greatest resistance possible to the water in its tendency to turn the dam round, provided the materials of which it is made were lighter than water in the proportion of 44 to 100. But the materials are probably more than twice as heavy as water, and the form of so flat a dike is taken, in all likelihood, in order to guard against a more imminent danger—that of the dam being carried away by being shoved forwards. We cannot calculate what the proportions are which give the greatest possible resistance to this tendency, without knowing the tenacity of the materials, as well as their specific gravity. It may very probably be found that the construction is such as to secure the most completely, against the two pressures at the same time.

The *Rein-deer* inhabits a country covered with snow the greater part of the year. Observe how admirably its hoof is formed for going over that cold and light substance, without sinking in it or being frozen. The under side is covered entirely with hair, of a warm and close texture; and the hoof, altogether, is very broad, acting exactly like the snow-shoes which men have constructed for giving them a larger space to stand on than their feet, and thus avoid sinking. Moreover, the deer spreads the hoof as wide as possible when it touches the ground: but, as this breadth would be inconvenient in the air, by occasioning a greater resistance while he is moving along, no sooner does he lift the hoof than the two parts into which it is cloven fall together, and so lessen the surface exposed to the air, just as we may recollect the birds doing with their bodies and wings. The shape and structure of the hoof are also well adapted to scrape away the snow, and enable the animal to get at the particular kind of moss (or *lichen*) on which he feeds. This plant, unlike others, is in its full growth during the winter season; and the Rein-deer accordingly thrives, from its abundance, at the season of his greatest use to man, notwithstanding the unfavourable effects of extreme cold upon the animal system.

There are some insects, of which the males have wings, and the females are grubs or worms. Of these, the *Glow-worm* is the most remarkable: it is the female, and the male is a fly, which would be unable to find her out, creeping as she does in the dark lanes, but for the shining light which she gives to attract him.

There is a singular fish found in the Mediterranean, called the *Nautilus*, from its skill in navigation. The back of its shell resembles the hulk of a ship; on this it throws itself, and spreads two thin membranes to serve for two sails, paddling itself on with its feet or feelers, as oars.

The *Ostrich* lays and hatches her eggs in the sands: her form being ill-adapted for sitting on them, she has a natural oven furnished by the sand, and the strong heat of the sun. The *Cuckoo* is known to build no nest for herself, but to lay in the nests of other birds; but late observations show that she does not lay indiscriminately in the nests of all birds; she only chooses the nests of those which have bills of the same kind with herself, and therefore feed on the same kind of food. The *Duck*, and other birds breeding in muddy places, have a peculiar formation of the bill: it is both made so as to act like a strainer, separating the finer from the grosser parts of the liquid, and it is more furnished with nerves near the point than the bills of birds which feed on substances more exposed to the light; so that being more sensitive, it serves better to grope in the dark stream for food. The bill of the *Snipe* is covered with a curious network of nerves for the same purpose; but the most singular provision of this kind is observed in a bird called the *Toucan*, or *Egg-sucker*, which chiefly feeds on the eggs found in birds' nests, and in countries where these are very deep and dark. Its bill is broad and long; when examined, it appears completely covered with branches of nerves in all directions; so that, by groping in a deep and dark nest it can feel its way as accurately as the finest and most delicate finger could. Almost all kinds of birds build their nests of materials found where they inhabit, or use the nests of other birds; but the *Swallow of Java* lives in rocky caverns on the sea, where there are no materials at all for the purpose of building. It is therefore so formed as to secrete in its body a kind of slime, with which it makes a nest, much prized as a delicate food in Eastern countries.

Plants, in many remarkable instances, are provided for by equally wonderful and skilful contrivances. There is one, the *Muscipula*, *Fly-trap*, or *Fly-catcher*, which has small prickles in the inside of two leaves, or half-leaves, joined by a hinge; a juice or syrup is provided on their inner surface, which acts as a bait to allure flies. There are several small spines or prickles standing upright in this syrup, and upon the only part of each leaf that is sensitive to the touch. When the fly, therefore, settles upon this part, its touching, as it were, the spring of the trap, occasions the leaves to shut, and kill and squeeze the insect; whose juices, and the air arising from their rotting, serve as food to the plant.

In the West Indies, and in other hot countries of South America, where rain sometimes does not fall for a great length of time, a kind of plant called the *Wild-pine* grows upon the branches of the trees, and also on the bark of the trunk. It has hollow or bag-like leaves, so formed as to make little reservoirs of water, the rain falling into them through channels which close at the top when full, and prevent it from evaporating. The seed of this useful plant has small floating threads, by which, when carried through the air, it catches any tree in the way, and falls on it and grows. Wherever it takes root, though on the under side of a bough, it grows straight upwards, otherwise the leaves would not hold water. It holds in one leaf from a pint to a quart; and although it must be of great use to the trees it grows on, to birds and other animals its use is even greater.

"When we find these pines," says Dampier, the famous navigator, "we stick our knives into the leaves just above the root, and the water gushing out, we catch it in our hats, as I myself have frequently done, to my great relief."

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* Mr. Bracey Clarke has contrived an expanding shoe, which, by a joint in front, opens and contracts so as to obviate the evils of the common process.

Another tree, called the *Water-with*, in Jamaica, has similar uses: it is like a vine in size and shape, and though growing in parched districts, is yet so full of clear sap or water, that by cutting a piece two or three yards long, and merely holding it to the mouth, a plentiful draught is obtained. In the East, there is a plant somewhat of the same kind, called the *Bejuco*, which grows near other trees and twines round them, with its end hanging downwards, but so full of juice, that, on cutting it, a good stream of water spouts from it; and this, not only by the stalk touching the tree so closely must refresh it, but affords a supply to animals, and to the weary herdsman on the mountains. Another plant, the *Nepenthes distillatoria*, is found in the same regions, with a yet more singular structure. It has natural mugs or tankards hanging from its leaves, and holding each from a pint to a quart of very pure water. Two singular provisions are to be marked in this vegetable. There grows over the mouth of the tankard, a leaf nearly its size and shape, like a lid or cover, which prevents evaporation from the sun's rays; and the water that fills the tankard is perfectly sweet and clear, although the ground in which the plant grows is a marsh of the most muddy and unwholesome kind. The process of vegetation filtrates or distils the liquid, so as to produce from the worst, the purest water.* The *Palo de Vaca*, or cow-tree, grows in South America, upon the most dry and rocky soil, and in a climate where for months not a drop of rain falls. On piercing the trunk, however, a sweet and nourishing milk is obtained, which the natives gladly receive in large bowls. If some plants thus furnish drink, where it might least be expected, others prepare, as it were, in the desert, the food of man in abundance. A single *Tapioca* tree is said to afford, from its pith, the whole sustenance of several men for a season.

V. ADVANTAGES AND PLEASURES OF SCIENCE.

AFTER the many instances or samples which have now been given of the nature and objects of Natural Science, we might proceed to a different field, and describe in the same way the other grand branch of human knowledge, that which teaches the properties or habits of *Mind*—the *intellectual faculties* of man, or the powers of his understanding, by which he perceives, imagines, remembers, and reasons;—his *moral faculties*, or the feelings and passions which influence him;—and lastly, as a conclusion or result drawn from the whole, his *duties* both towards himself as an individual, and towards others as a member of society: which last head opens to our view the whole doctrines of *political science*, including the nature of *governments*, of *policy*, and generally of *laws*. But we shall abstain entirely from entering at all upon this field, and shall now take up the subject more particularly pointed at through the course of the foregoing observations, and to illustrate which they have been framed—namely, the Use and Pleasure of Scientific Studies.

Man is composed of two parts, body and mind, connected indeed together, but wholly different from one another. The nature of the union—the part of our outward and visible frame in which it is peculiarly formed—or whether the soul be indeed connected or not with any particular portion of the body, so as to reside there—are points as yet wholly hid from our knowledge, and which are likely to remain for ever concealed. But this we know, as certainly as we can know any truth, that there is such a thing as the *Mind;* and that we have at the least as good proof of its existence, independent of the Body, as we have of the existence of the Body itself. Each has its uses, and each has its peculiar gratifications. The bounty of Providence has given us outward senses to be employed, and has furnished the means of gratifying them in various kind, and in ample measure. As long as we only taste those pleasures according to the rules of prudence and of our duty, that is, in moderation for our own sakes, and in harmlessness towards our neighbours, we fulfil rather than thwart the purpose of our being. But the same bountiful Providence has endowed us with the higher nature also—with understandings as well as with senses—with faculties that are of a more exalted order, and admit of more refined enjoyments, than any to which the bodily frame can minister; and by pursuing such gratifications, rather than those of mere sense, we fulfil the most exalted ends of our creation, and obtain both a present and a future reward. These things are often said, but they are not therefore the less true, or the less worthy of deep attention. Let us mark their practical application to the occupations and enjoyments of all branches of society, beginning with those who form the great bulk of every community, the working classes, by what names soever their vocations may be called—professions, arts, trades, handicrafts, or common labour.

1. The first object of every man, who has to depend upon his own exertions, must needs be to provide for his daily wants. This is a high and important office; it deserves his utmost attention; it includes some of his most sacred duties, both to himself, his kindred, and his country; and although, in performing this task, he is only influenced by a regard to his own interest, or by his necessities, yet it is an employment which renders him truly the best benefactor of the community he belongs to. All other pursuits must give way to this; the hours which he devotes to learning must be after he has done his work; his independence, without which he is not fit to be called a man, requires first of all that he should have insured for himself, and those dependent on him, a comfortable subsistence, before he can have a right to

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* A specimen of this curious plant, though of a small size, is to be found in the fine collection at Wentworth, reared by Mr. Cooper.

taste any indulgence, either of his senses or of his mind; and the more he learns—the greater progress he makes in the sciences—the more will he value that independence, and the more will he prize the industry, the habits of regular labour, whereby he is enabled to secure so prime a blessing.

In one view, it is true, the progress which he makes in science may help his ordinary exertions, the main business of every man's life. There is hardly any trade or occupation in which useful lessons may not be learnt by studying one science or another. The necessity of science to the more liberal professions is self-evident; little less manifest is the use to their members of extending their knowledge beyond the branches of study with which their several pursuits are peculiarly conversant. But the other departments of industry derive hardly less benefit from the same source. To how many kinds of workmen must a knowledge of Mechanical Philosophy be useful! To how many others does Chemistry prove almost necessary! Every one must with a glance perceive that to engineers, watch-makers, instrument-makers, bleachers, and dyers, those sciences are most useful, if not necessary. But carpenters and masons are surely likely to do their work better for knowing how to measure, which Practical Mathematics teaches them, and how to estimate the strength of timber, of walls, and of arches, which they learn from Practical Mechanics; and they who work in various metals are certain to be the more skilful in their trades for knowing the nature of those substances, and their relations to both heat and other metals, and to the gases and liquids they come in contact with. Nay, the farm servant, or day-labourer, whether in his master's employ, or tending the concerns of his own cottage, must derive great practical benefit,—must be both a better servant, and a more thrifty and therefore comfortable, cottager, for knowing something of the nature of soils and manures, which Chemistry teaches, and something of the habits of animals, and the qualities and growth of plants, which he learns from Natural History and Chemistry together. In truth, though a man be neither mechanic nor peasant, but only one having a pot to boil, he is sure to learn from science lessons which will enable him to cook his morsel better, save his fuel, and both vary his dish and improve it. The art of good and cheap cookery is intimately connected with the principles of chemical philosophy, and has received much, and will yet receive more, improvement from their application. Nor is it enough to say, that philosophers may discover all that is wanted, and may invent practical methods, which it is sufficient for the working man to learn by rote without knowing the principles. He never will work so well if he be ignorant of the principles; and for a plain reason:—if he only learn his lesson by rote, the least change of circumstances puts him out. Be the method ever so general, cases will always arise in which it must be varied in order to apply; and if the workman only know the rule without knowing the reason, he must be at fault the moment he is required to make any new application of it. This, then, is the *first* use of learning the principles of science: it makes men more skilful, expert, and useful in the particular kinds of work by which they are to earn their bread, and by which they are to make it go far and taste well when earned.

2. But another use of such knowledge to handicraftsmen is equally obvious: it gives every man a chance, according to his natural talents, of becoming an improver of the art he works at, and even a discoverer in the sciences connected with it. He is daily handling the tools and materials with which new experiments are to be made; and daily witnessing the operations of nature, whether in the motions and pressures of bodies, or in their chemical actions on each other. All opportunities of making experiments must be unimproved, all appearances must pass unobserved, if he have no knowledge of the principles; but with this knowledge he is more likely than another person to strike out something new which may be useful in art, or curious or interesting in science. Very few great discoveries have been made by chance and by ignorant persons; much fewer than is generally supposed. It is commonly told of the steam-engine, that an idle boy being employed to stop and open a valve, saw that he could save himself the trouble of attending and watching it, by fixing a plug upon a part of the machine which came to the place at the proper times, in consequence of the general movement. This is possible, no doubt; though nothing very certain is known respecting the origin of the story; but improvements of any value are very seldom indeed so easily found out, and hardly another instance can be named of important discoveries so purely accidental. They are generally made by persons of competent knowledge, and who are in search of them. The improvements of the Steam-engine by Watt resulted from the most learned investigation of mathematical, mechanical, and chemical truths. Arkwright devoted many years, five at the least, to his invention of spinning-jennies, and he was a man perfectly conversant in everything that relates to the construction of machinery: he had minutely examined it, and knew the effects of each part, though he had not received anything like a scientific education. If he had, we should in all probability have been indebted to him for scientific discoveries as well as practical improvements. The most beautiful and useful invention of late times, the Safety-lamp, was the reward of a series of philosophical experiments made by one thoroughly skilled in every branch of chemical science. Again, the process of Refining Sugar, by which more money has been made in a shorter time, and with less risk and trouble, than was ever perhaps gained from an invention, was discovered by a most accomplished chemist,* and was the fruit of a long course of experiments, in the progress of which, known philosophical principles were constantly applied, and one or two new principles ascertained. But in so far as chance has anything to do with discovery, surely it is worth the while of those who are constantly

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* Edward Howard, brother of the late Duke of Norfolk.

working in particular employments to obtain the knowledge required, because their chances are greater than other people's of so applying that knowledge as to hit upon new and useful ideas: they are always in the way of perceiving what is wanting, or what is amiss in the old methods; and they have a better chance of making the improvements. In a word, to use a common expression, they are in the way of good luck; and if they possess the requisite information, they can take advantage of it when it comes to them. This, then, is the *second* great use of learning the sciences: it enables men to make improvements in the arts, and discoveries in philosophy, which may directly benefit themselves and mankind.

3. Now, these are the *practical* advantages of learning; but the *third* benefit is, when rightly considered, just as practical as the other two—the pleasure derived from mere knowledge, without any view to our own bodily enjoyments: and this applies to all classes, the idle as well as the industrious, if, indeed, it be not peculiarly applicable to those who enjoy the inestimable blessing of having time at their command. Every man is by nature endowed with the power of gaining knowledge; and the taste for it, the capacity to be pleased with it, forms equally a part of the natural constitution of his mind. It is his own fault, or the fault of his education, if he derive no gratification from it. There is a satisfaction in knowing what others know—in not being more ignorant than those we live with: there is a satisfaction in knowing what others do not know—in being more informed than they are. But this is quite independent of the pure pleasure of knowledge—of gratifying a curiosity implanted in us by Providence, to lead us towards the better understanding of the universe in which our lot is cast, and the nature wherewithal we are clothed. That every man is capable of being delighted with extending his information upon matters of science will be evident from a few plain considerations.

Reflect how many parts of the reading, even of persons ignorant of all sciences, refer to matters wholly unconnected with any interest or advantage to be derived from the knowledge acquired. Every one is amused with reading a story: a romance may divert some, and a fairy tale may entertain others; but no benefit beyond the amusement is derived from this source: the imagination is gratified; and we willingly spend a good deal of time and a little money in this gratification, rather than in resting after fatigue, or in any other bodily indulgence. So we read a newspaper, without any view to the advantage we are to gain from learning the news, but because it interests and amuses us to know what is passing. One object, no doubt, is to become acquainted with matters relating to the welfare of the country; but we also read the occurrences which do little or not at all regard the public interests, and we take a pleasure in reading them. Accidents, adventures, anecdotes, crimes, and a variety of other things amuse us, independent of the information respecting public affairs, in which we feel interested as citizens of the state, or as members of a particular body. It is of little importance to inquire how and why these things excite our attention, and wherefore the reading about them is a pleasure; the fact is certain; and it proves clearly that there is a positive enjoyment in knowing what we did not know before; and this pleasure is greatly increased when the information is such as excites our surprise, wonder, or admiration. Most persons who take delight in reading tales of ghosts, which they know to be false, and feel all the while to be silly in the extreme, are merely gratified, or rather occupied with the strong emotions of horror excited by the momentary belief, for it can only last an instant. Such reading is a degrading waste of precious time, and has even a bad effect upon the feelings and the judgment.* But true stories of horrid crimes, as murders, and pitiable misfortunes, as shipwrecks, are not much more instructive. It may be better to read these than to sit yawning and idle—much better than to sit drinking or gaming, which, when carried to the least excess, are crimes in themselves, and the fruitful parents of many more. But this is nearly as much as can be said for such vain and unprofitable reading. If it be a pleasure to gratify curiosity, to know what we were ignorant of, to have our feelings of wonder called forth, how pure a delight of this very kind does Natural Science hold out to its students! Recollect some of the extraordinary discoveries of Mechanical Philosophy. How wonderful are the laws that regulate the motions of fluids! Is there anything in all the idle books of tales and horrors more truly astonishing than the fact, that a few pounds of water may, by mere pressure, without any machinery—by merely being placed in a particular way, produce an irresistible force? What can be more strange, than that an ounce weight should balance hundreds of pounds, by the intervention of a few bars of thin iron? Observe the extraordinary truths which Optical Science discloses. Can anything surprise us more, than to find that the colour of white is a mixture of all others—that red, and blue, and green, and all the rest, merely by being blended in certain proportions, form what we had fancied rather to be no colour at all, than all colours together? Chemistry is not behind in its wonders. That the diamond should be made of the same material with coal; that water should be chiefly composed of an inflammable substance; that acids should be, for the most part, formed of different kinds of air, and that one of those acids, whose strength can dissolve almost any of the metals, should consist of the selfsame ingredients with the common air we breathe; that salts should be of a metallic nature, and composed, in great part, of metals, such as potassium, &c.,

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* *Children's books* have at all times been made upon the pernicious plan of exciting wonder, generally horror, at whatever risk. The folly and misery occasioned by this error, it would be difficult to estimate. The time may come when it will be felt and understood. At present, the inveterate habits of parents and nurses prevent the children from benefiting by the excellent lessons of many of our popular writers.

yet lighter than water, and which, without any heating, take fire upon being exposed to the air, and, by burning, form the substance so abounding in saltpetre and in the ashes of burnt wood: these, surely, are things to excite the wonder of any reflecting mind—nay, of any one but little accustomed to reflect. And yet these are trifling when compared to the prodigies which Astronomy opens to our view: the enormous masses of the heavenly bodies; their immense distances; their countless numbers, and their motions, whose swiftness mocks the uttermost efforts of the imagination.

Akin to the pleasure of contemplating new and extraordinary truths, is the gratification of a more learned curiosity, by tracing resemblances and relations between things, which, to common apprehension, seem widely different. Mathematical science, to thinking minds, affords this pleasure in a high degree. It is agreeable to know that the three angles of every triangle, whatever be its size, howsoever its sides may be inclined to each other, are always, of necessity, when taken together, the same in amount: that any regular kind of figure whatever, upon the one side of a right-angled triangle, is equal to the two figures of the same kind upon the two other sides, whatever be the size of the triangle: that the properties of an oval curve are extremely similar to those of a curve which appears the least like it of any, consisting of two branches of infinite extent, with their backs turned to each other. To trace such unexpected resemblances is, indeed, the object of all philosophy; and experimental science, in particular, is occupied with such investigations, giving us general views, and enabling us to explain the appearances of nature; that is, to show how one appearance is connected with another. But we are now considering only the gratification derived from learning these things. It is surely a satisfaction, for instance, to know that the same thing, or motion, or whatever it is, which causes the sensation of heat, causes also fluidity, and expands bodies in all directions; that electricity, the light which is seen on the back of a cat when slightly rubbed on a frosty evening, is the very same matter with the lightning of the clouds;—that plants breathe like ourselves, but differently by day and by night;—that the air which burns in our lamps enables a balloon to mount, and causes the globules of the dust of plants to rise, float through the air, and continue their race—in a word, is the immediate cause of vegetation. Nothing can at first view appear less like, or less likely to be caused by the same thing, than the processes of burning and of breathing—the rust of metals and burning—an acid and rust—the influence of a plant on the air it grows in by night, and of an animal on the same air at any time, nay, and of a body burning in that air; and yet all these are the same operation. It is an undeniable fact, that the very same thing which makes the fire burn, makes metals rust, forms acids, and enables plants and animals to breathe; that these operations, so unlike to common eyes, when examined by the light of science, are the same—the rusting of metals—the formation of acids—the burning of inflammable bodies—the breathing of animals—and the growth of plants by night. To know this is a positive gratification. Is it not pleasing to find the same substance in various situations extremely unlike each other;—to meet with fixed air as the produce of burning, of breathing, and of vegetation:—to find that it is the choke-damp of mines, the bad air in the grotto at Naples, the cause of death in neglecting brewers' vats, and of the brisk and acid flavour of Seltzer and other mineral springs? Nothing can be less like than the working of a vast steam-engine of the old construction, and the crawling of a fly upon the window. Yet we find that these two operations are performed by the same means, the weight of the atmosphere, and that a sea-horse climbs the ice-hills by no other power. Can anything be more strange to contemplate? Is there, in all the fairy-tales that ever were fancied, anything more calculated to arrest the attention and to occupy and to gratify the mind, than this most unexpected resemblance between things so unlike to the eyes of ordinary beholders? What more pleasing occupation than to see uncovered and bared before our eyes the very instrument and the process by which Nature works? Then we raise our views to the structure of the heavens; and are again gratified by tracing accurate, but most unexpected resemblances. Is it not in the highest degree interesting to find, that the power which keeps this earth in its shape, and in its path, wheeling upon its axis and round the sun, extends over all the other worlds that compose the universe, and gives to each its proper place and motion; that this same power keeps the moon in her path round our earth, and our earth in its path round the sun, and each planet in its path; that the same power causes the tides upon our globe, and the peculiar form of the globe itself; and that, after all, it is the same power which makes a stone fall to the ground? To learn these things, and to reflect upon them, occupies the faculties, fills the mind, and produces certain as well as pure gratification.

But if the knowledge of the doctrines unfolded by science be pleasing, so is the being able to trace the steps by which those doctrines are investigated, and their truth demonstrated: indeed, you cannot be said, in any sense of the word, to have learnt them, or to know them, if you have not so studied them as to perceive how they are proved. Without this you never can expect to remember them long, or to understand them accurately; and that would of itself be reason enough for examining closely the grounds they rest on. But there is the highest gratification of all, in being able to see distinctly those grounds, so as to be satisfied that a belief in the doctrines is well founded. Hence to follow a demonstration of a grand mathematical truth—to perceive how clearly and how inevitably one step succeeds another, and how the whole steps lead to the conclusion—to observe how certainly and unerringly the reasoning goes on from things perfectly self-evident, and by the smallest addition at each step, every one being as easily taken after the one before, as the first step of all was, and yet the result being something not only far from self-evident, but so general and strange, that you can hardly believe it to be true, and are only convinced of it by going over the whole reasoning—this operation of the understanding, to those who so exercise themselves, always affords the highest delight. The contemplation of experimental inquiries, and the examination of reasoning founded upon the facts which our experiments and observations disclose, is another fruitful source of enjoyment, and no other means can be devised for either imprinting the results upon our memory, or enabling us really to enjoy the whole pleasures of science. They who found the study of some branches dry and tedious at the first, have generally become more and more interested as they went on; each difficulty overcome, gives an additional relish to the pursuit, and makes us feel, as it were, that we have by our work and labour established a right of property in the subject. Let any man pass an evening in vacant idleness, or even in reading some silly tale, and compare the state of his mind when he goes to sleep or gets up next morning with its state some other day, when he has passed a few hours in going through the proofs, by facts and reasoning, of some of the great doctrines in Natural Science, learning truths wholly new to him, and satisfying himself by careful examination of the grounds on which known truths rest, so as to be not only acquainted with the doctrines themselves, but able to show why he believes them, and to prove before others that they are true;—he will find as great a difference as can exist in the same being—the difference between looking back upon time unprofitably wasted, and time spent in self-improvement: he will feel himself in the one case listless and dissatisfied, in the other comfortable and happy: in the one case, if he do not appear to himself humbled, at least he will not have earned any claim to his own respect; in the other case, he will enjoy a proud consciousness of having, by his own exertions, become a wiser and therefore a more exalted creature.

To pass our time in the study of the sciences, in learning what others have discovered, and in extending the bounds of human knowledge, has, in all ages, been reckoned the most dignified and happy of human occupations; and the name of Philosopher, or Lover of Wisdom, is given to those who lead such a life. But it is by no means necessary that a man should do nothing else than study known truths, and explore new, in order to earn this high title. Some of the greatest philosophers, in all ages, have been engaged in the pursuits of active life; and an assiduous devotion of the bulk of our time to the work which our condition requires, is an important duty, and indicates the possession of practical wisdom. This, however, does by no means hinder us from applying the rest of our time, beside what nature requires for meals and rest, to the study of science; and he who, in whatever station his lot may be cast, works his day's work, and improves his mind in the evening, as well as he who, placed above such necessity, prefers the refined and elevating pleasures of knowledge to the low gratification of the senses, richly deserves the name of a **True Philosopher.**

One of the most delightful treats which science affords us is the knowledge of the extraordinary powers with which the human mind is endowed. No man, until he has studied philosophy, can have a just idea of the great things for which Providence has fitted his understanding—the extraordinary disproportion which there is between his natural strength and the powers of his mind and the force he derives from them. When we survey the marvellous truths of Astronomy, we are first of all lost in the feeling of immense space, and of the comparative insignificance of this globe and its inhabitants. But there soon arises a sense of gratification and new wonder at perceiving how so insignificant a creature has been able to reach such a knowledge of the unbounded system of the universe—to penetrate, as it were, through all space, and become familiar with the laws of nature at distances so enormous as to baffle our imagination—to be able to say, not merely that the Sun has 329,630 times the quantity of matter which our globe has, Jupiter 308$\frac{2}{10}$, and Saturn 93$\frac{1}{2}$ times; but that a pound of lead weighs at the Sun 22 lbs. 15 ozs. 16 dwts. 8 grs. and $\frac{2}{3}$ of a grain! at Jupiter 2 lbs. 1 oz. 19 dwts. 1 gr. $\frac{20}{43}$; and at Saturn 1 lb. 3 ozs. 8 dwts. 20 grs. $\frac{1}{11}$th part of a grain! And what is far more wonderful, to discover the laws by which the whole of this vast system is held together and maintained through countless ages in perfect security and order. It is surely no mean reward of our labour to become acquainted with the prodigious genius of those who have almost exalted the nature of man above its destined sphere, when, admitted to a fellowship with these loftier minds, we discover how it comes to pass that, by universal consent, they hold a station apart, rising over all the Great Teachers of mankind, and spoken of reverently, as if NEWTON and LAPLACE were not the names of mortal men.

The highest of all our gratifications in the contemplations of science remains: we are raised by them to an understanding of the infinite wisdom and goodness which the Creator has displayed in his works. Not a step can we take in any direction without perceiving the most extraordinary traces of design; and the skill everywhere conspicuous is calculated, in so vast a proportion of instances, to promote the happiness of living creatures, and especially of our own kind, that we can feel no hesitation in concluding that, if we knew the whole scheme of Providence, every part would be found in harmony with a plan of absolute benevolence. Independently, however, of this most consoling inference, the delight is inexpressible of being able to follow, as it were, with our eyes, the marvellous works of the Great Architect of Nature—to trace the unbounded power and exquisite skill which are exhibited in the most minute, as well as the mightiest parts of his system. The pleasure derived from this study is unceasing, and so various, that it never tires the appetite. But it is unlike the low gratifications of sense in another respect: while those hurt the health, debase the understanding, and corrupt the feelings, this elevates and refines our nature, teaching us to look upon all earthly objects as insignificant, and below our notice, except the pursuit of knowledge and the cultivation of virtue; and giving a dignity and importance to the enjoyment of life, which the frivolous and the grovelling cannot even comprehend.

Let us, then, conclude, that the Pleasures of Science go hand in hand with the solid benefits derived from it; that they tend, unlike other gratifications, not only to make our lives more agreeable, but better; and that a rational being is bound by every motive of interest and of duty, to direct his mind towards pursuits which are found to be the sure path of Virtue as well as of Happiness.

THE

CIRCLE OF THE SCIENCES;

A CYCLOPÆDIA

OF

EXPERIMENTAL, CHEMICAL, MATHEMATICAL, AND MECHANICAL PHILOSOPHY,

AND NATURAL HISTORY.

INTRODUCTION TO PHYSICS.

At a time when the applications of Science are so numerous and highly appreciated, it would be a work of supererogation to attempt to urge their value on the intelligent reader. It may, however, not be out of place, to enquire by what means Experimental Philosophy has attained to its present high rank, either as an object of study, or in its applications to the various requirements of civilised life.

From the earliest ages, the phenomena of Nature must have been deeply investigated by intelligent observers. The labours of Archimedes, Ptolemy, Thales, and others, evidence that there have been those in olden times who delighted to enquire into the reasons and causes of the varied effects they witnessed. Indeed, Electricity—a branch of Science which, during the last half-century, has been applied to so many purposes—was known in some of its effects at least two thousand years ago. Of Geometry and Astronomy, the oldest records extant prove that there were many ardent followers.

But, although a branch of Science has its origin in observation, still its progress is only effected by the mental process of sound induction; and in this department of Philosophy, the ancients were eminently deficient. Having observed an effect, they, in most cases, hastily proceeded to seek a reason; and having to contend with those prejudices of which Lord Bacon speaks so eloquently, they assigned causes which met special cases only. Hence they were at last compelled to invent imaginary ones, which, as they only explained a few collateral effects, could not be generalised. The result of this was, that no branch of Science, properly so called, was ever established: guess-work and empiricism occupied the place of sound reasoning, and mists and mysticism were the final results of their researches in most instances.

The Alchemists of the middle ages fall under the same category as the ancients. They had but one object in view—the conversion of the base into the precious metals; hence every occurrence that presented itself to their notice, which they conceived to be not relevant to their pursuit, was entirely neglected, or ridiculously philosophised on. They preferred, indeed, to teach Nature, rather than to be taught by her; and thus it can be matter of no surprise that, under their auspices, Science made no progress.

Eventually, however, a brighter intellectual day dawned. Experiment became the foundation; fact the superstructure; whilst theory was but the scaffolding to the temple: and as soon as the system was adopted of tracing causes from their effects, and of arranging such, that one theory might connect many causes, Experimental Science advanced with rapid strides. From its lofty abode in the time of Copernicus and Galileo, it has since descended, and has become, as it were, the household god of even the very peasant of the land, and the proximate cause of most of the comforts and conveniences of our existence.

It is not, however, on account of the material advantages which Philosophy affords, that its pursuit is most to be admired. Next to the great object of man's existence in this world—the cultivation of religious sentiments—the expansion of the human mind immediately succeeds in value. And it is impossible to point out any other study which more fully calls out and exercises the mental powers, than that of Nature. The perceptive faculties are employed in investigation and observation; the reasoning power in arranging causes; and the reflective in focalising the rays of light continually emitted: in fact, all the powers of the mind are thus braced and made ready for answering every call which can be made on them.

In arranging the subjects included in the following pages, it has been attempted to make the simple to introduce and prepare the way for the complex. By thus progressing from known to obscure matters, it is hoped that the way of Science has been made plain, although no royal road has been opened out. The persistent attention of the reader has been assumed to have been granted by him; and it is earnestly hoped that the demonstrations herein set forth, may result in the comparatively easy solution of some of the most difficult, as well as the plainer problems in Experimental and Natural Science.

In pursuance of this plan, it will be necessary that we should enquire into those properties possessed by every kind of matter; and despite of all the sophisms of a class of metaphysicians, it will be impossible to define matter otherwise than as that which can be weighed, measured, and perceived by the senses. The speculations of Berkeley and others, however ingenious they may be, have a great tendency, if followed to their logical consequences, to restore the days of philosophical darkness, in which imagination was the *ignis fatuus* which led men astray from the true path of induction.

The leading properties of matter are *Extension*, *Impenetrability*, and *Inertia*, and they are characteristic; for if but one atom existed in the universe, it must necessarily have those. If more than one particle exist, two forces would be called into operation. By that of gravitation, each particle would be attracted mutually; by cohesion, the two would unite, and so form a mass. Thus, by the attraction of gravitation, masses affect each other's position; whilst by the attraction of cohesion, the masses are themselves formed and held together.

It will be most convenient to deal with these properties and forces individually; and to this end it will be first necessary to define the term "atom."

ATOMS.—The original signification of the term is simply, "that which cannot be divided." To such an idea it has been objected, that it is impossible to conceive any particle so small that it cannot be bi-sected or otherwise cut into parts. Mathematical reasoning and demonstration have been adduced in support of this view; and perhaps the most ingenious attempt is that wherein the mutual relationship of parallel lines and angles are employed. As, however, geometricians are compelled to assume that a point occupies no space—that a line has length, but no breadth—so it may be permitted to the physicist to assume, that an atom is such a particle of matter as to be indivisible by physical means; and in this sense the term "atom" will be in future employed by us.

Assuming, for the present, that a body is composed of a number of atoms held together by their attraction for each other, it may be interesting to consider some facts in reference to the divisibility to which a body may be subjected without losing its general and individual properties. The examples of such instances are so numerous as to make a selection most difficult. The following, however, may be adduced, on account of their recent discovery:—

Example 1.—In a cubic inch of chalk, as obtained from the rock, there are the remains of many millions of animalcules, each of which had possessed, during life, all the organs necessary to its independent existence.

Example 2.—It has been found that silver exists in a soluble state in sea-water; but in such minute quantities as to have escaped the notice of the most careful analysts, till lately, when a small portion of the solid metal was discovered as a film on the coppered bottom of a vessel, and had been collected from the immense quantity of water through which the ship had sailed during a long voyage.

The extension of metals by hammering, and in the processes of wire-drawing, are familiar illustrations of a similar nature.

EXTENSION.—This term, in its philosophical sense, defines that property of matter by which it occupies space; and, as it appears at once so evidently a quality of bodies, we pass on to another property, Impenetrability, which is not so self-evident, when considered in certain aspects.

IMPENETRABILITY.—By this is meant that no two bodies can occupy the same space at the same time. There are many *apparent* exceptions to this, which, however, neither have, nor can have, any foundation in fact. Thus, on adding strong sulphuric acid to cold water, in a long glass vessel, the two liquids, on being mixed, will occupy less space than they did separately. This fact is, however, easily explained. No state of matter is really solid; for each mass has its atoms at a distance from each other; the divisions between them being called pores or interstices. In the instance just adduced, the particles are brought closer to each other, so that whilst the *mass* may occupy *less* space, the individual atoms are impenetrable, or, in other words, could not at the same time occupy each other's place. India-rubber, cork, atmospheric air, &c., all seem to create paradoxes in reference to this doctrine; which, however, are easily explained away, and on which further remarks will be made under the head of Cohesive Attraction. Familiar instances illustrate the impenetrability of solids; such as the insertion of a nail, chisel, or saw into wood, &c., where the particles of the wood are merely thrust aside, to make way for the body introduced between them.

INERTIA.—That no inanimate mass can be set in motion spontaneously, will be readily admitted; its converse, that no body can come to rest spontaneously, may, however, be difficult to understand, owing to daily occurrences seeming to demonstrate the opposite conclusion. A little consideration, however, will easily remove any doubts on these two doctrines of the Inertia, or the immovability of matter. All matter requires the application of external force to remove it from its position; and so far as terrestrial objects are concerned, any force so applied is instantly opposed in most cases by two others—namely, those of gravitation and resistance. Taking, for instance, an extreme case of motion—such as the flight of a bullet from the muzzle of a rifle—it is found that the ball instantly falls towards the earth by virtue of gravitative force, and it is also diverted right or left, or opposed in its forward progress, by the resistance of the air to its passage. The former of these forces may be temporarily suspended, as in the case of a balloon; the latter as in the vacuum of an air-pump, wherein a penny and a feather will, owing to the absence of air, descend at equal speed.

With respect to the heavenly bodies, it is found that their rate of motion is so regular as to never vary in the course of many years; and thus they illustrate the second principle of the doctrine of Inertia—that bodies will not come to rest of their own accord. Occasionally, however, the proximity of one planet to another will disturb each in its orbit. An instance of this occurred some years ago, which led to the discovery of the planet Neptune. M. Leverrier, in France, and Mr. Adams, of Cambridge, had noticed a disturbance of the motion of Uranus, and arguing on the principle that the inertia of that planet could not be affected except by an external force, they were led to imagine that another planet had, by virtue of its attraction, been the disturbing cause; and, eventually, they were rewarded by its discovery. This result may be adduced as a wonderful triumph of Mathematical and Physical Science, and also as an evidence of the exact and undeviating character of natural laws, in obedience to which, the planetary systems present no notable change in the lapse even of ages.

Numerous instances are daily occurring wherein the inertia of matter plays a prominent part. The jolting motion of a carriage passing over a rough road; the apparent projection of a passenger forward or backward, according as a boat in which he may be at rest is suddenly moved in the reverse direction, &c.—are cases wherein the body does not, by its inertia, partake of the motion, and therefore remains still. In the case of a collision on a railway, the passengers continue to move at the velocity they had already acquired, and are not able to bring themselves to rest: but the carriage in which they were travelling loses its motion, owing to the inertia of the object against which it had been impelled, and which opposes its further progress.

Having thus mentioned those properties which are common to every kind of matter, we proceed to inquire into the powers by which two or more masses mutually act on and attract each other. Before the time of Sir Isaac Newton, to whom is accorded the merit of having first proved the universal nature of the law of gravitation, many philosophers held the opinion that there was a great attractive force acting on all matter. Hooke, a contemporary of Newton, gave expression to such views, but in an unintelligible and enigmatic form. Newton, however, availed himself of some rough observations of the moon's revolution in her orbit, to test the idea of universal gravitation. By an acute intelligence, and deep mathematical acquirements, he was enabled, despite every obstacle, to make certain of that which had previously been mere conjecture. He not only proved the fact that masses mutually attract each other, but also demonstrated the amount of such attraction, and the variations to which it was subject at different distances of the attracting bodies.

GRAVITATION.—The attraction of gravitation, so far as our earth is concerned, may be defined as that force in obedience to which all bodies tend to fall towards its centre. This force varies according to the distance from the centre; and as the poles are about thirteen miles nearer to it than the equator, it follows that a body will weigh more at the poles than at the equator.

The amount of this variation is inversely as the square of the distance; or, more plainly, if any object be attracted with a force at any specified distance from the centre of the earth, at double the distance the force will be diminished to one-fourth; at three times the distance, to one ninth; and so on.

The amount of attractive force which two bodies in space can exert will be directly proportional to the mass of matter they contain; hence the greater mass will draw the smaller to it. But each has an attraction for the other; and if their masses do not greatly differ, they will, by their mutual attraction, remove each other from any supposed position they may have occupied, on coming within the sphere of each other's action. It is literally true, that the earth is attracted by every object raised from its surface, but as such bears no assignable proportion to the mass of the earth, the effect of course is infinitely small. If, however, a ball be attached to a flexible cord, and suspended near a mass of metal, the attraction of both the earth and metal will act on it, and, instead of hanging in a line perpendicular to the earth's surface, it will be drawn to one side. By comparing the attraction which a known mass of metal has, in causing a ball thus to diverge from the perpendicular, a rough idea of the density of the earth may be arrived at. This is, however, more perfectly ascertained by means of a pendulum, which, as it is continually falling towards the earth, when in motion, must necessarily vary in the number of its vibrations, according to its distance from the centre of the earth. At London, a pendulum of the length of 39.139 inches, vibrates sixty times per minute; whilst at the Poles, the number of vibrations would be much more numerous, and at the Equator they would be less.

By thus experimenting, it has been found that the weight of the earth is equal to that of about six globes of water of the same size, or, in other words, the earth weighs nearly six times as much as its own bulk of water.

One effect of the attraction of gravitation is, that bodies falling towards the surface of the earth, move towards it with an accelerated motion, that is, the nearer they approach the earth, the more space do they pass through in each second of time. In the case of a body being projected from the surface of the earth, its upward motion is gradually retarded until it comes to rest, and again returns to the earth, by virtue of the attraction of the latter. The acceleration and retardation here spoken of, are directly consequent of the law that the force of gravitation varies inversely as the square of the distance of the attracted from the attracting body. It has been found that a mass will fall, in one second of time, through a space of $16\frac{1}{10}$ feet; but, neglecting fractions, a general rule may be ennunciated—to the effect, that the space through which a body will fall during any number of seconds, may be ascertained by multiplying 16 feet by the square of the number of seconds of time during which it has fallen.

If, for instance, it were desired to know the depth of a mine, or the distance of the top of a tower from the ground, it would be only necessary to count the number of seconds of time which a stone takes in falling to the base; and by multiplying the square of the number of seconds by 16 feet, the depth sought for would be at once ascertained.

The rule just named gives only the total space through which a body would fall towards the earth during a given number of seconds. If it be desired to know the space passed through in each second, a different course must be adopted. It has been found, that by multiplying 16 feet successively by 1, 3, 5, 7, 9, &c., the space passed through in the first, second, third, fourth, fifth, &c., second is at once obtained. The reader will therefore perceive, that the *total* space passed through increases in geometrical progression, whilst the relative space in each second increases in an arithmetical ratio only. The following table will more fully explain the rules just laid down:—

Number of Seconds during which a body has fallen.	Space passed through during each Second.	Total space passed through,
1″	16 feet.	16 feet.
2″	48 ,,	64 ,,
3″	80 ,,	144 ,,
4″	112 ,,	256 ,,
5″	144 ,,	400 ,,
6″	176 ,,	576 ,,

The application of these interesting laws to the motions of the planets and other astronomical phenomena, will be fully dealt with under the section of Astronomy. The consideration of the cause of the tides will also fall under that head.

Having thus attempted to give a general idea of the force of gravitation, we pass on to consider the aggregation of atoms, and the causes of solidity, &c. It has already been remarked, that a mass is composed of very minute particles. These are brought and held together by what is called the attraction of cohesion.

COHESION.—This force, unlike gravitation, is only exercised at extremely small distances; and when particles are violently separated, they generally remain apart, owing to the loss of their cohesive power. The ordinary processes of pounding, cutting, cleaving, &c., are instances of cohesive attraction being destroyed between particles, or, at all events, they cause its temporary suspension. In some instances, however, parts of bodies may be reunited, as in the case of india-rubber, which, on being cut by a clean knife, may be joined together again by simply pressing the clean edges closely together. A leaden bullet, if divided by a sharp knife, may be re-formed by immediately bringing the surfaces together by means of a kind of twisting motion, assisted by pressure.

Highly polished surfaces will often adhere strongly if brought into close contact. Polished plate-glass affords an excellent illustration of this; for sometimes two sheets, if placed on each other, and having nothing between them, will adhere with such force as to prevent their separation. Finely polished steel surfaces often present similar results. Generally speaking, however, an extremely small distance is required to be attained before bodies will adhere; and hence the use of adhesive substances, such as glue and pitch, which, by touching the particles of two bodies in many places, produce a temporary cohesion of great force.

It may seem paradoxical, but at the same time it is true, that although the sense of touch leads us to imagine that we actually come in contact with a body, it is extremely rare that such an event takes place. If two pieces of convex glass, such as common lenses, are pressed together with great force by means of screws, there will still remain between them a plate of air, which, by its colour, indicates the distance still separating them. A needle resting on the surface of water, has round it a film of air which prevents the liquid from at all touching it. If the hand be pressed on any surface, there will generally exist three plates, which will prevent actual contact with it—namely, one of water, grease, and air. Again, if a little lycopodium, in powder, be cast on to the surface of some water, the hand may be thrust into the liquid without becoming moistened. Some of these, and similar, effects have been described as resulting from an active principle, which has been termed "repulsion;" but they are easily explained, by presuming that distance negatives the usual effect of cohesive attraction; indeed, the slightest film on the surfaces of the divided lead bullets above-named, will prevent their re-adhesion.

The different states of bodies, such as the solid, liquid, and gaseous, are caused, it is believed, by the repulsive power of heat. A solid may be supposed to have its particles close together. On the addition of heat thereto, its particles are at once driven farther apart, and a fluid is produced. By a still further application of heat, the particles of the fluid may be separated to a greater distance, and a gas or vapour will be produced. The most common illustration of these three separate states being found in one body, is that of water, which may be successively a solid as ice, a fluid as water, and a gas in the form of vapour or steam.

Masses may be reunited, after division, by various means; solution and crystallisation being those most commonly employed. Sugar, for example, may be powdered, and no pressure will reunite the particles. On being dissolved in water, the different parts are aggregated, and the solid mass may be restored. Under the section on Crystallography, this mode of inducing cohesive attraction will be fully explained.

CHEMICAL, ELECTRICAL, and MAGNETIC attraction will necessarily be spoken of under the respective branches of Science to which they relate. It will therefore be sufficient to state that they exercise a varied influence on, and modify or control, the form and condition of bodies. The former of these forces has a universal effect throughout nature; whilst the influence of Magnetism is, comparatively speaking, restricted to the metal iron, and its compounds.

CAPILLARY ATTRACTION.—This force seems to be a modification of that of cohesion, and is manifested whenever a liquid is in contact with solids, which are placed at small distances from each other. On dipping a narrow glass tube into water, it will be found that the fluid will rise higher inside the tube than the general level of the aqueous surface. A lump of sugar absorbs and raises above its level any water in which it may be placed. This force has an astonishing effect both on animal and vegetable economy. By its action the sap is conveyed from the root of the plant or tree to the highest branches; and, generally speaking, liquids are raised by this force for a vast number of purposes in nature. To this end the pores or interstices of bodies highly contribute: and thus, whether we regard the cells of animal and vegetable structure, or the particles of sand, earth, loam, &c., this comparatively unobserved but constantly acting force, is one of the chief agents in producing an infinite variety of effects. In daily life, capillary attraction is constantly employed: as, for instance, in the conveyance of oil in lamps by the wick; in the absorption of perspiration by clothing; and in other ways too numerous to mention.

We have hitherto restricted our remarks to those properties of matter and forces in nature which are properly called universal, because we can conceive of no body which can exist unaffected by them. There are, however, others to which we must direct attention; because, although not partaken of so generally, they are discovered in most of the objects to which the researches of philosophers are directed. It will be found that some of these properties are characteristic of the body possessing them; but in others, these properties may vanish when the state of the body possessing them, is varied by the action of heat and other forces.

ELASTICITY is that property by virtue of which bodies regain a form which they had temporarily lost. The effect of Elasticity is produced when two other properties, COMPRESSIBILITY and EXPANSIBILITY, are called into exercise. Air, for instance, when *compressed*, will at once *expand* on the pressure being removed; hence air is called an *elastic* body. Some masses may be compressed, and will not regain entirely their previous form; but a perfect elastic body is one which will oscillate between its original, a larger, and smaller form—eventually, however, regaining its previous size. A common ivory billiard-ball is one of the best illustrations of this class of bodies, and wet clay may be adduced as an instance of those of an opposite kind. All gases are highly elastic, and, owing to atmospheric changes, are perpetually undergoing alteration of bulk. Liquids are sparingly so; and, in fact, seem to form an exception to these laws; but, by careful experiments, it has been ascertained that water is elastic; for, having been compressed by enormous pressure, it will regain its previous volume on the pressure being removed.

The mind at once recurs to the numerous applications of the elasticity of different materials. The springs of carriages, the buffers of the railway car, the passage of gas through the mains conveying it, the production of wind, the main-springs of clocks and watches, &c.—all are instances wherein elasticity plays an active part. But, perhaps, the most interesting object of this force is involved in the arrangement of the different parts of the human body; which, by the elasticity of their separate processes, permit any kind of motion to be safely undertaken, that would certainly destroy the construction of the most perfect of human mechanical productions.

SPECIFIC GRAVITY.—A relative property of bodies is that called Specific Gravity; and the term is employed to express the fact that the same bulk of different substances varies in weight. Thus air weighs about $\frac{1}{800}$th part of its bulk of water; whilst iron weighs more than seven times as much as an equal bulk of water. The various means used for ascertaining the relative weights, called Specific Gravities, will be described as required in the progress of the work—an arrangement which will assist the student in learning the valuable application of the law, more especially in reference to the processes of the chemist, the weight of machinery, &c.

CENTRE OF GRAVITY.—This term is used to point out that part of a body wherein all the forces acting on it are in equilibrium. In a globe or sphere made of materials of one kind only, the centre of gravity would be at the centre of the body: hence, if the sphere were suspended by that centre, all its parts would hang round it at rest. If, however, we imagine a sphere to be made partly of wood and of lead, the specific gravity of the metal is so much greater than that of the wood, that the lead, if not placed exactly in the centre of the sphere, would have the centre of gravity somewhere in itself. The common toy, the Chinese tumbler, is constructed so that its centre of gravity shall be at the lower part. The centre of gravity of an egg is so placed, that it will, if left to itself, lay sideways on a level surface. If, however, its contents are violently shaken, the yolk will settle to one end, and the egg may easily thus be made to stand upright, because its centre of gravity has been shifted from its previous position near the centre. The applications of the law will be fully considered under the head of Mechanics: it may, however, be interesting to point out here some cases where its action is prominent. In walking, we are continually shifting the centre of gravity of our bodies: in the construction of railways, the lines are so laid as that, in curves, one rail shall be higher than another, so as to prevent the train running off the line; and, in loading ships, care is taken that the cargo shall be so distributed as that the centre of gravity of the vessel shall be in the best position to insure speed and safety. When a vessel is required to sail from one port to another without cargo, a quantity of coarse gravel, called "ballast," is cast into the hold, which thus maintains the centre of gravity low down in the hull.

Having thus attempted to give an introductory view of properties and forces to which all matter is subject, we proceed to detail those branches of Science which take cognizance of special forces, qualities, and phenomena; and in so doing, we shall confine our attention to those only which will be introduced and explained in these volumes, dealing with them in the order in which it is proposed to arrange them.

It has already been stated that force is required to set matter in motion; but, beside the forces to which reference has already been made, and which affect only the external condition of matter, there are others which can modify and change it in so many ways, as to produce results which are of the highest importance in animal and vegetable existence.

HEAT.—The first which we shall notice is the power or agent called Heat—one of the most active and necessary in creation. We shall have to examine its effects on matter, in the solid, liquid, and gaseous state; its production, conveyance or conduction; and other facts relating to its diffusion and general agency. It may here be remarked, that the term CALORIC is often used instead of heat, as being more in accordance with scientific phraseology. Natural phenomena, such as Wind, Rain, Dew, Evaporation, &c., will be dealt with under this head; and the varied applications of heat, and the instruments used for its measurement, such as the Thermometer, will be fully described.

LIGHT.—Under the head of Light, a vast variety of interesting phenomena arrange themselves; such as colour, reflection, refraction, polarisation, &c. Photography, as one of the most interesting of the numerous applications of Science, will claim an extended notice; and ample directions will be given, so as to assist the reader in the pursuit of the art. Optical Instruments, employed in scientific research, and for social and amusing purposes, will be explained, both with respect to their construction and use. Among these will be included, Telescopes, Microscopes of all kinds, the Magic Lantern and Dissolving Views, Cameras, Stereoscopes, Polariscopes, &c. The production of Artificial Light will also be extensively illustrated. In this branch of philosophical research some most interesting discoveries have recently been made by Messrs. Bunsen and Kirchkoff, who, by means of the spectrum analysis, have opened out an entirely new field in Chemical and Optical Science.

ELECTRICITY.—No branch of Science has made, during the present century, such rapid strides as Electricity. Its various divisions of Frictional and Voltaic Electricity will be examined, together with numerous natural phenomena arising from their agency. Electro-Chemistry, Electrotyping, Plating, and Gilding, naturally fall under this section; and extended details of these processes will be afforded.

MAGNETISM.—To the existence of this force we are indebted for the mariner's compass, whose variation will require special notice. Terrestrial Magnetism and Dia-Magnetism will be investigated, and the leading facts and experiments on these subjects will be introduced. The action of the material and cargo of ships on the compass will be explained, so far as modern Science has brought light on that intricate subject.

ELECTRO-MAGNETISM.—This branch of Science is of very recent origin; and its chief application, the electric telegraph, has made it, so far, the most popularly known of any. The production of magnetic effects in soft iron, the mutual action of Electric and Magnetic Currents, the divergence of the Magnetic Needle, and the various applications of these effects, such as the Electric Telegraph, both on land and through water, together with Electro-Magnetic Machines, will be found in this section. Ruhmkorf's new Induction Coil, and various experiments with it, will be explained.

ACOUSTICS.—The four agents, Heat, Light, Electricity, and Magnetism, are at present supposed to be produced by motion of an undulatory nature; and much analogy exists between them and the production of sound. Under the head of ACOUSTICS, this connection will be considered; together with the conveyance and reflection of Sound, Echoes, Musical Instruments, and other allied subjects.

CHEMISTRY.—The most casual observation will at once suggest that there must exist a vast variety of substances in nature, having qualities greatly differing from each other. It is the object of Chemistry to discover these qualities, and to examine their mutual relationship. In dealing with this branch of Science, the chemical history of Inorganic Bodies will be first considered. The Chemistry of Life, both vegetable and animal, Agricultural Chemistry, and Chemical Analysis, will follow in order; and the subject will be made complete by an extended notice of the numerous applications of chemistry employed in the arts, manufactures, and in mining and metallurgy.

MATHEMATICS.—The object of Mathematical Science is to point out the different relations of Number, Time, and Space. Arithmetic, Algebra, Logarithms, and Logarithmic Tables of Natural Numbers; Geometry, Plane and Spherical; Trigonometry; and the application of these, under the heads of Practical Geometry and Mensuration, will be all fully investigated in this section: and each branch will be so treated as to enable the student to master the majority of problems which occur in the application of Mathematics to the investigations of Physical Science. Every endeavour will be made to combine simplicity of expression with scientific instruction.

MECHANICAL PHILOSOPHY, or APPLIED MATHEMATICS.—Under the head MECHANICS, the Laws of Motion and Force, as applied to Machinery, will be first considered; with mechanical contrivances for economising labour, or for producing motive power, including Steam-Engines, &c. Implements of husbandry and trade, and other similar mechanical inventions, will be fully described; and their essential peculiarities also pointed out. Rules for calculating the power of engines, the weight of machinery, its regulation, and other interesting facts, will be included, so as to make this section of value to the practical man.

HYDROSTATICS and HYDRAULICS.—The different fluids which are generally typified by water; the laws which govern them when at rest or in motion; the application of these laws, as in Water-Wheels, &c.; the flow of Water through pipes; the construction of Canals and Reservoirs; the Hydraulic Press, and many allied matters, will be dealt with under this head.

PNEUMATICS.—The laws which govern air and other gases are generally included in this branch of Science. Beside explaining the causes of many atmospheric phenomena, we shall here give descriptions of the Barometer, Pumps, Air-Engines, and other inventions which depend on the application of the laws of air or gasiform bodies.

ASTRONOMY.—The consideration of the subjects to which we have hitherto alluded, necessarily attaches the mind to objects of terrestrial interest. In the study of Astronomy, we are led beyond all mundane influence, and seem to take so high a stand in nature as to be enabled to have an extended survey of the works of the Creator. For sublimity of subject and range of observation, Astronomy stands immeasurably beyond every other branch of Science. Even a superficial view of its truths is deeply interesting to the intelligent mind. But our object will extend further. It will be our endeavour to give an insight into those laws by which the constitution and motion of the Heavenly Bodies are governed. Whilst a popular description of Astronomical Science will be afforded, the mathematical student will have an opportunity of investigating the truths and bases on which the leading theories of this noble branch of Science are founded. It is impossible to enumerate here the varied matters which we shall have to speak of. The Sun, the Planets, the Earth, and Moon; their motions in their orbits and on their axes; the causes of the Tides, of Eclipses; the nature and motion of Comets; the Constellations and Fixed Stars, &c., will be dealt with, so as to interest both the general and advanced reader; and in a way which, we trust, will add the element of intelligence to the natural admiration which all feel when they "consider the heavens," the handiwork of GOD.

NAVIGATION and NAUTICAL ASTRONOMY.—Nothing is of greater importance to the navigator than a knowledge of Astronomy. By it he is enabled to trace his path on the sea with a certainty which seems truly marvellous. The heavenly bodies are as celestial lighthouses to him; and, by observing their positions, he is enabled to calculate his exact place on the surface of the earth. The subject will be dealt with in an essentially practical manner; and ample rules will be given for calculating, arithmetically and by logarithms, the leading problems of this branch of Science; such as Latitude, Longitude, errors caused by Refraction, Departures, Courses, and allied subjects. The relations between Astronomy and Navigation will thus be fully pointed out, and varied practical matter will also be introduced in this section.

With some of the subjects named above, tables of various kinds will be required; and these have been accordingly supplied in each section, so that the student will have no occasion to refer to other works.

METEOROLOGY.—This interesting and almost new branch of Natural Science, gives an explanation of the various phenomena which occur in our atmosphere. Amongst these may be named—Wind, Water-Spouts, Rain, Hail, Snow, Thunder and Lightning, the Aurora Borealis, and Shooting Stars, Dew, Moisture, Evaporation, Clouds, &c. We shall include an ample description of the different modes adopted for the purpose of observing and registering atmospheric changes, and give directions for ascertaining the approach of storms, cyclones, &c.; referring also to the late efforts of the British government, to diffuse this valuable information amongst seafaring persons. The simplest and most economical mode of pursuing private observations will claim our attention, and special instructions will be given to assist those who are desirous of undertaking that interesting occupation.

From the brief epitome thus presented, it will be observed, that the division of our work treating on Natural Philosophy, will include the different branches of Science which share the description of the laws of inanimate nature. In our progress, a large variety of experiments will be introduced, which will enable the reader to test the truth of the principles or assertions which are laid down. As we have previously remarked, Observation and Experiment are the bases of sound philosophy; and, as a rule, nothing should be assumed but what can be proved. Our endeavour will be to avoid, as far as possible, the use of technical phraseology; and where such is absolutely necessary, sufficient explanations of the terms so employed will be given, so that the reader will not need to refer to other works for explanation.

In the second division of the work we shall deal exclusively with those branches of Science which relate to the laws of Organised Bodies. The student will find that his previously acquired knowledge of Experimental Philosophy will be called into practical exercise when he enters into the examination of the phenomena of animal and vegetable life. His acquaintance with Inorganic Chemistry will enable him to examine the variety of substances described in the section on Mineralogy; and, when collecting specimens, he will find the great value which accrues from the application of chemical analysis and the use of the blow-pipe, in arriving at a knowledge of the constituents of mineral substances. We may here observe, that immense sums have been wasted by otherwise intelligent individuals, who, having discovered what appeared at first sight to be ores of great value, in previously unknown positions, have not been able to apply the laws of Chemical and Geological Science in verification of their opinions. Thus, it has been proposed to search for coal in some parts of Kent and Surrey. The writer was once seriously informed by a lady that she had discovered a "silver mine" on her estate in Essex; and he can call to mind several instances where search has been made in different parts of England and Scotland for lead, copper, &c., which any geologist would at once have pronounced as sure to prove futile, and with a certainty which would leave no room for conjecture.

The arrangement of the division on Natural Science, will include Comparative Anatomy, Zoology, Ethnology, Botany, Crystallography, Mineralogy, Physical Geography, and Geology; and we shall take them in the order we have named. We do so, because a knowledge of Comparative Anatomy, Zoology, and Botany, is essential to the student in enabling him to arrange and classify the fossils which form the key-stone of all our geological discoveries. Any person well acquainted with these branches of Natural Science, will have no difficulty in at once ascertaining the period at which any strata of the earth's crust he may meet with has been formed. Indeed, fossils have been expressively called the "medals of creation."

Under the head of Comparative Anatomy, we shall describe the form, size, and number of the bones which are found in skeletons of different organised beings. This branch of Natural Science is of the deepest interest to every lover of nature; and to Professor Owen (the Cuvier of Great Britain) we owe a vast accession of knowledge respecting the relationship which exists between each class of extinct and living species.

The object of Ethnology is to trace the characteristics of what have been called the "races" of men. With respect to this term, Dr. Latham well remarks, "that all the varieties of man are referable to a single species;" and, for that reason, he defines Ethnology as the "Science of Varieties."

Under the head of Zoology, it will be our duty to trace the Natural History of animal creatures, from the lowest to the highest species. Their production, growth, characteristics, will be inquired into; and the subject will be made of popular interest by the introduction of anecdotes, illustrating the instinct of many of the Mammalia, &c.

From Animal Life we descend to that of the Vegetable Kingdom. In this, with but trifling exceptions, we lose all sign of the power of locomotion. We perceive a cellular structure, and the circulation of sap; each of which is analogous to the bone and blood of the animal. The student who has not already become acquainted with Botany, will at first imagine that the variety of vegetable productions is like that of the mineral kingdom, almost infinite. But Botany, like Chemistry, systematises our observations: and thus, as we find but sixty or seventy elementary bodies by the aid of chemical analysis; so, by using the arrangements which Botanical Science affords us, we are enabled to arrive at a classification of every tree, plant, and flower.

Leaving all trace of life, we descend to the lowest rank in creative results, but still find abundant objects for study and reflection. Crystallography opens out an astonishing evidence of design, by showing that inanimate substances have specified forms of their own. Thus it matters not in what part of the world we find a grain of quartz, nitre, or common salt—we shall discover that an exact and undeviating crystalline form is possessed by it. We shall find a knowledge of Geometry of essential value in this branch of our studies; indeed, we shall do no violence to truth, if we call Crystallography the geometry of inanimate matter.

Having mastered the details of the last subject, we shall be enabled to study with advantage the facts of Mineralogy—a subject of great interest, and of the highest importance in an economical and commercial point of view. This section will afford a detailed description of most known minerals; the places in which they have been discovered; their crystallographic forms, &c. Those of our readers who may have access to the British Museum, will find an account of nearly all the minerals contained in the collections, at that institution.

GEOLOGY—the last subject on which we shall treat—has for its object the explanation of the various causes which have changed or modified the appearance and nature of the crust of our earth. Of all branches of Science, except Astronomy, this has been a source of the fiercest contention on the part of speculatists of every school. The zealous but hasty Christian philosopher has called in its facts to prove his views; and the sceptic has employed the same facts for a diametrically opposite purpose. The philosophers of one school have, on certain data, "proved" theories which their opponents have as vigorously "disproved" by the same means. We might remark on this subject to our readers, that if there be any branch of Science calling for patient, calm, and philosophic thought, it is this. The imagination, at all times, unless chained by the iron fetters of sound reason, will rush violently to conclusions; and hence the bitter denunciations which geological opponents have hurled on each other. If, however, the student will follow carefully the laws of rigid induction, he cannot avoid arriving at the truth; and in his observations he will assuredly find

"Sermons in stones, and good in everything."

Throughout the divisions of the work, every endeavour will be made to render each subject as complete as possible. Those who have already advanced in scientific pursuits, will find formulæ and facts, conveying full information in each department; and, where necessary, mathematical demonstrations will be freely employed. The practical engineer, and others engaged in like pursuits, will be enabled to trace the various steps between Abstract Science and those extensive ramifications through which Experimental Philosophy and Applied Mechanics spread their influence. The general reader will, it is hoped, gather information which, whilst strictly philosophical, will be, by a little attention, easily arrived at; and, as far as possible, the difficulties which naturally present themselves in every work on Science aiming at completeness, will be modified by the use of concise description, and illustrative experiments. In conclusion, we have to express the hope that, although many omissions and shortcomings must necessarily occur in our pages, the object which has been constantly held in view—that of assisting the reader to an intelligent and practical acquaintance with the laws of Nature, and their general applications—may yet be, in some measure, attained.

Having thus laid the foundation—if we may so say—of our work, and having given a general view of its arrangement, it may be useful to our readers to place before them, in a tabular form, the various subjects to which we have alluded. They have been so arranged as to conduct the student, from the most elementary principles, to a knowledge of each branch of Science into which the study of Nature is divided, according to the present system of philosophy. Without attempting to claim for our method any special advantage beyond that afforded by others, we venture to suggest that those who follow it will be enabled gradually to advance from comparatively well-known to more abstruse subjects, in such a manner, as that they may use the knowledge acquired in each step for the elucidation of that which has to follow.

The entire work is composed of two great divisions—the first relating simply to matter and forces; the second to matter and forces as connected with vitality, &c. The sub-divisions connect allied subjects; whilst the sections, into which each volume is divided, serve to render reference easy. They also separate the comparatively minor details, by which each sub-division is characterised, in a manner which is at once convenient and philosophical.

DIVISION I.—VOLUME I.—EMBRACING EXPERIMENTAL, CHEMICAL, MATHEMATICAL, AND MECHANICAL PHILOSOPHY.

SUB-DIVISION I.—EXPERIMENTAL PHILOSOPHY—UNDULATORY FORCES.

SECTION I.—HEAT.
" II.—LIGHT.
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SECTION VI.—CRYSTALLOGRAPHY.
" VII.—MINERALOGY.
SECTION VIII.—GEOGRAPHY.
" IX. & X.—GEOLOGY.

We have already explained those general laws to which matter is obedient, whatever kind, form, or state it may be possessed of; and, with this general introduction, we conceive our readers will be quite prepared to commence the study of any of the sections. Some little difficulty, however, may be presented in the first subdivision—viz., that which treats on Experimental Philosophy, in connection with the Undulatory Forces; and we shall, therefore, endeavour to smooth the way of the student by some general remarks on their nature, laws, and correlation. As we shall see in our progress, the term is comparatively new, and has been applied to Heat, Light, &c., on account of certain theoretical views which are maintained at the present day. By means of the theory referred to, the various phenomena of each force can be viewed in a definite relation to those of the rest, and thus great assistance is rendered to the philosopher in generalising on, and to the student in acquiring, the facts which they present to our observation.

THE PHENOMENA AND LAWS OF THE UNDULATORY FORCES.

The most indifferent observer of natural phenomena cannot fail to notice the variety which presents itself on every hand; and we can scarcely imagine a more interesting occupation for an intellectual creature, than to trace the causes of those changes which are thus continually afforded us. In no department of Natural Science is there any possibility of disappointment; in fact, wherever we attempt the investigation of those laws which govern Creation, order, beauty, and precision are prominent characteristics.

It, however, is impossible that we can suddenly appreciate the perfection which Philosophy discloses to us, in all the arrangements and sequences of prime causes, unless we enter into their study with a determination to put aside every preconceived notion, and to be taught by the lessons which are but slowly unfolded to us. To all but the humble philosopher, Nature is a sealed book; but to those who enter her kingdom as little children, a storehouse of knowledge and enjoyment is at once opened.

This humility, however, is only arrived at after much experience, because there is an idea entertained by every tyro in Science, that he is at once capable of understanding the assumed simplicity of natural laws. The true man of Science has to arrive at this by slow degrees; and, after achieving a position which, to ordinary minds, seems to be one of the highest importance, he so far learns his own ignorance as to doubt whether he has attained to any knowledge. In the words of an eminent living philosopher, " I once thought that I knew all that was to be known of Science; but the longer I live, the less I seem to know of it."

These remarks are fully borne out by the fact, that all our discoveries of the causes of most of the effects observed in nature, have been arrived at by the most difficult and laborious processes. It matters not what department of Science we choose as illustrative of this truth. In every branch observation has frequently been baffled, the process of induction has been delayed, and the establishment of an adequate theory has been effected only by the most extended investigations, after apparently accurate inductions have undergone the sifting process of many independent and occasionally hostile minds.

If, however, we study the result of such labours, we cannot but admire the perseverance which has been employed, and must rejoice that human intellect holds so proud a station as that of being able, to a small extent, to understand those laws which God has impressed on all His works. Even beyond this, we now reap the fruit of the works of those who in former days devoted themselves to investigate some of the most intricate of the operations in nature. The labours of a Newton and a Leibnitz are now the very bases of our national prosperity; for without the results which those great men first opened out, our position as a maritime people would yet have to be won.

In many instances, the most valuable additions to scientific stores have been accidentally met with. We must not, however, forget, that while so-called "accidents" occur to every observer, they are only made use of by those whose minds are prepared to reflect on, and examine, any new appearance which may attract their attention. The discovery of the polarisation of light, and of electricity evolved by chemical action, are instances of the most common occurrences being turned to philosophic purposes. Every one must have observed that the light of the sun produces most beautiful colours when reflected from a thin film of oil floating on water; but it required the penetrative genius of Newton to explain their cause: and, since his time, the most talented intellects of the day have never ceased to expand the ideas whose germ Sir Isaac first gave in his experiments with the thin plates of light on the soap-bubble, and those produced between two pieces of glass closely pressed together.

In reference to processes in various kinds of manufacture, it is often found that great progress has been effected entirely independent of accurate scientific information; but such progress has been necessarily slow, owing to the want of knowledge on the part of those who had to superintend the operations. At the present time, however, a fresh discovery is scarcely made in any branch of scientific research, before its applications are instantly effected; and, possibly, the two most remarkable instances on record of this nature, are those of the Electric Telegraph and Photography—the two greatest marvels of our age.

We may, as illustrating this position, adduce the state of the glass manufacture during the Roman era, and that which it occupies at this moment. In the time of Nero a glass vessel cost an almost fabulous price, whilst now the meanest utensil is made of that material. This, and many such results, have accrued solely through a knowledge of certain natural laws being first obtained, and afterwards applied by intelligent and scientific manipulators.

Nearly all the most remarkable and beneficial inventions which have been produced during the last two centuries, have been the result of deep researches into the action of those forces which have been termed imponderable agents, or, as they are now called, " Undulatory Forces." Thus, in the applications of Heat, Light, Electricity, and Magnetism, we have remarkable illustrations of the value which is involved in the extended study of abstract Science, antecedent to applying the results so obtained in a practical form. By the study of the laws of latent heat, Dr. Black and James Watt have presented us with that mightiest of human contrivances—the Steam-Engine.

Our immense riches in iron and coal were as dross until extensive arrangements were constructed, by means of which intense and long-continued heat could be produced in our furnaces. To the earlier investigators of the laws of the refraction of light, from the time of Galileo, we are indebted for the accurate construction of our telescopes, by means of which we can penetrate into space, and discover the laws of planetary motion, which, when applied by the navigator, are of the utmost commercial importance to us. By means of the microscope, as another application of the laws of light, we are enabled to detect differences of nature, quality, &c., utterly unapproachable by the naked eye. There is perhaps no law of nature which has so magnificent a range as that of the refraction of light, for in its minute application we may detect the presence of individual insects, millions of which may be crowded into a space less than the size of a pin's point; and, in the next moment, may view by its agency groups of stars—to count which, is beyond the power of the most accomplished astronomer. Again, the generalisation of this law of light is truly marvellous; for we observe that the delicate colour of the flower, the gorgeous plumage of tropical birds, the colours of the rainbow, and the infinite variety of colours seen throughout nature, are all produced by this one cause. Proceeding to the polarisation of light, we enter into one of the most fascinating of all scientific pursuits, and are enabled to detect the most brilliant and varied appearances which the imagination can depict, in bodies which, to the unassisted eye, seem utterly destitute of beauty.

Turning to another of the imponderable forces—Electricity—we find an agent which, although almost always unperceived by our senses, is, nevertheless, only second to light in its universality: at one moment we find its presence manifested in the vivid flash of lightning; whilst, at the same instant, it is all but the animating power of our existence, and is coursing unperceived through our nerves and muscles. In its applications, we have it at times passing through wires, and enabling us, by its heating effects, to destroy masses of rocks as with the power of a giant; and yet, at our command, it silently travels by the path we choose for it, until, vibrating the little compass-needle, it conveys instantaneously the expression of our wishes from one extremity of the world to the other—commanding the march of hostile armies, or conveying words of peace and good-will towards men.

With respect to Magnetism, we need scarcely remind the reader, that, to an application of its laws, we are indebted for the mariner's compass, by which our sailors may steer safely by day or night across the otherwise trackless ocean. In the deepest darkness, without a star to be seen, in storm and tempest, the helmsman keeping his eye on the oscillation of the needle, progresses boldly on his voyage, and pursues his course as free from error as if a cloud of fire directed his path. With what a noble contrast does a modern voyage stand out when compared with that of Ulysses, who, coasting round shore after shore in search of his beloved Ithaca, had nothing to guide him in the absence of a clear sky. Besides the mariner's compass, there are various facts of the most interesting nature, which have been brought to light by the researches of Faraday and others, as some of the effects of the magnetic force. These have been arranged under the term Dia-Magnetism; and we learn that nearly every substance in nature can be made susceptible of the magnetic force, and not iron alone, as had previously been imagined. We must, however, not be misunderstood as referring here to that pseudo-scientific nonsense termed Mesmerism, &c., which is merely the product of the ignorance and evil design of some men. The effect to which we refer, and of which we shall fully treat hereafter, is the result of the agency of natural laws, which have been carefully studied both as to cause and effect, by men whose names alone are a guarantee for truth and accuracy.

We have hitherto refrained from mentioning the connection which each of the imponderable agents have with others; and the subject, of course, can only be dealt with here in the most superficial manner. It will answer our purpose, until we treat more fully on the matter in future pages, to remark, that each of the four forces, Heat, Light, Electricity, and Magnetism, are mutually producible; or, in other words, the presence of either agent may be, or generally is, united with that of another. For instance, by heating a substance to a high temperature, it becomes red or white hot, emitting rays of light just in proportion to the intensity of heat employed. By heating a few bars of antimony and bismuth, arranged in a peculiar manner, electricity and magnetism may be produced. Again, electricity continually produces heat, light, and magnetism, during its progress by conductors. Magnetism not only produces electricity, &c., but will, under certain circumstances, magnetise a ray of light; whilst a ray of light can be used to render a sewing-needle magnetic.

We may remark, that with respect to all these forces, motion of some kind is necessary to their evolution; and that, in most cases, they exist in what is called a *latent* state. Thus, for instance, the well-known article of food, sugar, may easily be made to afford heat, light, and electricity in a perceptible state, by means of chemical and mechanical action. It is also remarkable, that whenever any of these forces is set free, motion is, generally speaking, communicated to matter, as in heating a body, which causes its parts to separate or expand. Motion and these forces have therefore a mutual relation; and philosophers have endeavoured to account for the existence of the imponderable agents, and for the phenomena they produce, by supposing that an "ether," which is presumed to be far more rare than matter, and to exist in the pores of bodies, is put into motion, and by its undulations, gives rise to the varied effects we observe when heat, light, &c., become evident to our senses. Others have supposed, that each of these forces is owing to the emanation of a peculiar kind of matter from bodies, which, by radiating from any object, produces heating, lighting, and electric effects. We shall, however, not stop to discuss either of these theories, because we must take them in detail hereafter, and endeavour to satisfy, as far as possible, in the present state of scientific knowledge, all enquiries which may be made on this interesting question.

We have stated, that most of the imponderable agents are mutually producible; but we must add, that between two of them a most remarkable analogy exists in some respects. We refer to light and heat, the rays of which can be both refracted, reflected, and polarised. Thus, if a heated or luminous body be placed in the focus of a concave mirror, the rays of either force may be reflected from one, and received on another mirror placed at a considerable distance from the first. The phenomena, as produced by either force, are found to be obedient to the same laws; and the only difference we can detect, is in the nature or speciality of the force under examination.

By the term "refraction," we mean that the rays of either heat or light are bent out of their straight course when passing from a rare into a dense medium, or *vice versâ*. A very simple illustration of refraction is witnessed when a stick is put obliquely into a vessel containing water. The stick will appear bent, or broken, owing to the rays of light which pass from it being bent or refracted from a straight line. In a similar manner, if light is made to pass through a piece of glass of a triangular shape, such as a prism, the ray is refracted, and seven beautiful colours are produced. A ray of heat may be refracted or broken up by passing it through a prism of rock-crystal; and although, of course, no colours are visible to the eye of the observer, still effects exactly like those produced by the glass prism on light are at once afforded.

The effects of polarisation are not so easily understood without a lengthened description, which we shall give in its proper place. We may, however, state, that when a ray of light is passed through, or reflected from, a bundle of glass plates, or a piece of tourmalin, it is divided into two parts, one of which proceeds onwards, and the other is deflected at a certain angle. Our description would be more scientific, if we were to say that one ray is transmitted and the other reflected. The very same result happens to heat, if its rays are treated in a similar manner.

For these and many other reasons, we may arrive at the conclusion, that heat and light proceed from the same cause, modified by circumstances with which we are at present but slightly acquainted. This assumption, though most likely true, is not based, however, on an unexceptional foundation.

Each of the forces of which we are now speaking, are termed central or radial, because their effects are observed to extend in straight lines from a centre. As we shall subsequently prove, their intensity diminishes as the square of the distance from the source; and having any data of the amount of force first excited, its intensity becomes a matter of easy measurement.

With respect to the speed with which these forces travel, little is known, except in reference to light; and here we may speak with tolerable certainty. By observing the eclipses of Jupiter's satellites, it is found that their light does not appear so soon after an obscuration as it should do, according to exact astronomical calculations. Now, as the latter are so accurate as not to vary in many instances to the extent of a second, even if the event predicted shall be months after being calculated in coming to pass; and as the distance of Jupiter from the earth, and other data, are well known, it requires little stretch of the imagination to guess, that the error of observation is due to the length of time required for the light of the satellite to reach the earth after it had passed the solid body of its planet. Thus we judge that light travels at the rate of about 200,000 miles per second. To familiarise this to the mind, we may add, that a ray of light would pass round the earth eight times in one second; that it requires but one second of time for light to pass from the moon to the earth; and if the sun could suddenly be obscured on the surface of its luminous part, and again exposed, we should not perceive its light, after first losing it, for the space of about eight minutes.

We have no means of ascertaining the rate at which heat travels, except when associated with light, as in the rays of the sun; and, so far, it has been judged that heat and light travel at an equal speed.

Electricity seems to travel at a speed which varies according to the conductor through which it passes. In the earlier works on this subject, an extraordinary rate was assigned; but recent experiments by no means countenance the idea. There are so many interfering causes, as to leave the best-tried results uncertain. As far as our present knowledge goes, it would seem that, in a conductor 3,000 miles long, the electric effect is observed at its extremity in about a second of time. There is, however, a very singular result perceived when a current of voltaic electricity is sent through a very long wire. Instead of the effect being instantaneous, the delivery of the current is protracted just in proportion to the length of the conductor, after the first sign of the presence of the current is manifested. The electricity seems, in fact, to have lost its instantaneous, and to have acquired a gradual, mode of evidencing its presence.

Our means of producing magnetic effects are so small, as to preclude the chance of measuring the rate at which they are propagated. The comparatively small distance at which magnetism is perceptible from its sources in our experiments, and the impossibility of our cutting off the effects of terrestrial magnetism, present insuperable difficulties in arriving at a knowledge of the speed of the magnetic force.

After discussing the different laws which research has opened out to us, it will be our duty to present, in the following pages, some outlines of the applications of which the imponderable agents are susceptible, whether in respect to natural occurrences, or for social, economical, and domestic purposes. It will be our endeavour to impress on the minds of our readers the truths of the different branches of Science; and, presuming that those have been mastered, their varied applications will necessarily become interesting and valuable. And here we have a wide field opened for contemplation. There is scarcely a circumstance, or an object of daily life in our time, but which points to Science as having something to do with it.

Let us, for instance, take each force in the order in which we have hitherto spoken. To Heat we are indebted for our steam-engines, our metallurgical processes, the infinite variety of manufactures which employ the industrial population of our country, and also for the comforts, conveniences, and vital necessities of our social existence. Of some of these applications we shall speak under the head of Steam, and Warming and Ventilation. Many others we must defer to our Chemical section. The natural phenomena dependent on heat—such as Volcanic Agency, the heat of the Sun, Climate, &c.—we shall enter into in the subject of Terrestrial Heat, on which we shall have still further to expand when we treat on Meteorology.

The Applications of Light—such as the Cause of Colour, Optical Instruments, Artificial Illumination, Photography, &c.—will immediately follow our consideration of the Laws of Optics; and thus what has been by many considered to be a comparatively difficult subject, will be found to present a large amount of interesting and useful matter to the general reader, and valuable results to the practical man.

With respect to Electricity, we shall, after discussing the laws of that force as obtained either by friction or by chemical action, proceed to the consideration of Lightning Conductors; the effects of Free Electricity on the Animal and Vegetable Kingdom; the Production of Minerals, Ores, and Gems; that instructive and interesting process called Electrotyping, &c.; and allied matters which we cannot now stop to mention.

In the applications of Magnetism—the Mariner's Compass, the Action of Iron and of Iron Vessels thereon; Electro-Magnetism as a motive power; the construction of the Electric Telegraph; and those beautiful phenomena which have been brought to light by the apparatus invented by M. Ruhmkorf, will engage our attention.

We propose introducing, as an appendix to our discussion of the Undulatory Forces, a treatise on Acoustics, or the Laws of Sound. This may, at first sight, appear incongruous to those who are unaware of the analogy which exists between the four forces we have hitherto spoken of, and the effects produced by vibrations of the air. This difficulty will, however, immediately disappear, when we reflect that, as sound is certainly caused by undulations in the atmosphere, and as we can measure the lengths of these with great accuracy, it follows that sound actually becomes an explanatory key of all the phenomena which may arise from the undulations of a supposed ether. Indeed, the reader may instantly combine his *accurate* knowledge of the laws of atmospheric vibrations, with *theoretical* ideas of those produced in ether. We may, with advantage, dwell for a short time on their analogy. If a string be tightly stretched between two points, as in the violin and harp, and be made to vibrate, the sound produced is propagated by the vibration producing others in the surrounding air, just as successive waves are produced on casting a pebble into a still pond of water.

Again, if the length or tension of the string is varied, the sound produced will be proportionally altered from a bass to a treble. Transferring our attention to light, we equally assume that a vibration produces that force; and if we increase the number of these vibrations in a given space, the intensity and nature of the light produced is at once affected. The analogy between sound and light is therefore undeniable.

But we may go still farther: for if we cause a set of sonorous vibrations to cross each other, their sound either is doubled, or entirely disappears; just as two waves of water, when coming in contact, either increase in size, or are destroyed. So with light, we find the interference of rays, or fits of transmission and reflection, produce analogous effects, which, by the aid of modern research, may be measured with great accuracy

In connection with Acoustics, we propose to deal somewhat extendedly with the construction of several kinds of musical instruments, so far as they may relate to scientific laws. Some interesting applications of the Conduction of Sound will be fully noticed. The human voice, and that of animals, will also be treated on.

We have thus attempted to give a general epitome of the leading phenomena of what are called the Undulatory Forces; and we now proceed to discuss them in detail. Our arrangement will be, first, to investigate the abstract laws of each force; secondly, to point out and enter into the details of its applications; and, lastly, we shall furnish copious Tables, which will be equally valuable to the *savant*, the practical man, and the general reader.

We may be permitted to offer a word of advice to those who may use our pages without the aid of a competent instructor. Whilst we have endeavoured to be as plain and as concise as possible, still we cannot conceal the fact that difficulties will necessarily arise to those who have not had the advantage of a course of scientific training. On such we urge, that they leave no subject until they have mastered its details. If they do not pursue this course, they can only be compared to a general who, having passed through a hostile country, leaving behind him hosts of minor foes that he may press on to give battle to the powerful one, will, on his return, find that the petty enemies he had despised have combined against him, and are more to be dreaded than the apparently stronger foe. A general and superficial study of the laws of nature may give a smattering of knowledge, but can never afford solid pleasure. Of those who adopt that course, it may be safely said—

"A little learning is a dangerous thing."

Such persons are always foremost in the rank of inventors: they strike out sparks of light; but the genial benefit of sound advantage is never a product of their career.

Those, however, who will patiently and perseveringly devote a modicum of intelligence to Studying Nature, do not only derive great personal advantage, intellectually and pecuniarily, but they form a class of men on whom the world may reckon as its benefactors; and of whom it may be said, as of a Watt or of a Stephenson—.

"Si monumentum queris, circumspice."

The following is the order in which we shall proceed in treating on the Undulatory Forces:—

1st.—Heat and its Applications.
2nd.—Light ,, ,,
3rd.—Electricity ,, ,,
4th.—Magnetism ,, ,,
5th.—Acoustics ,, ,,

SECTION I.

HEAT, AND ITS APPLICATIONS.

In our Introductory Chapter we referred chiefly to those elementary facts which may be considered as the basis of Natural Philosophy. It will be necessary to recur frequently to what may be termed the axioms of science, on which we there treat; and, presuming that a general idea has been arrived at with respect to fundamental principles, we proceed to consider certain forces, by whose agency the external characteristics and form of all kinds of matter are modified.

To many, who are untutored in science, the heading of this chapter may be suggestive of difficulty. We therefore proceed to define the term "undulatory forces."

In producing motion by mere mechanical action—such, for instance, as in removing a body from its position, we are conscious of exercising a force, and such is put into operation by vital action. We will to do anything, and action may or does follow. The sequences of cause and effect are so immediate, that we do not attempt to trace their relationship.

When, however, we do not apply *directly* the vital force, but employ others, such as heat, &c., to produce a certain result, we necessarily inquire into the intermediate steps which exist between our will and the effectuation of its desire; and, in this intermission, we naturally seek to know what intervening causes have been called to our aid. Nothing is more simple than to remove a body by mere exercise of the muscles. When, however, we employ an agent, as in the propulsion of a ball from a cannon, we see that a variety of powers or forces are called into operation; and the object of science is to ascertain their nature, and to measure their extent and duration.

The most cursory observation will show that many natural occurrences owe their origin to a quasi-animate force. If, for instance, we carefully balance two weights at each end of a lever, we shall cause one to descend, or to weigh down one end of the lever, if we expand that end by the application of heat. An intelligent observer will not be content to ascertain the facts of the case: he will endeavour to find out the reason of such an effect; and to such inquiries, in reference to "Thermotics," or the Science of Heat, we now direct attention.

It has been already remarked, that a body is composed of infinitely small particles, called atoms. We do not know what the ultimate state of matter is. We believe, however, that its bulk, or the space any body occupies, is solely owing to the comparative presence or absence of heat or caloric.

The question, however, arises as to the nature of a force whose agency has so great an influence on natural phenomena; and to account for this, various theories have been advanced. It has been supposed that heat, like matter, has a solid nature; or may be made up of ultimate particles, just as matter is presumed to be. To this idea there seems to be a fatal objection. For if a body is what we may term *solid*, our experience teaches that its bulk is rarely enlarged, except by the introduction into its pores of another body, equal at least to it in solidity. Thus, we cannot divide a piece of wood by means of another piece of exactly the same nature as itself, unless we use also a force equivalent to overcoming the inertia of the particles of the body which we propose to divide.

Now, if we presume heat to be a material substance, we must, from our practical knowledge of its nature, admit, that being so subtle, it has yet escaped the evidence of our senses so far as solidity is concerned; and yet if it acts as matter in expanding a body, it must have a force which ordinary solid matter has not. Indeed, to state the case plainly, we must, assuming that heat is material, at once assert that a less solid body, by its own inherent power, has the ability to overcome the inertia of a mass possessing a resistance greater than the power which is exercised. The material theory of heat may thus be reduced to an *argumentum ad absurdum;* and to the general reader, it may be better understood, if we illustrate our position through the idea of dividing a mass of gold as the solid, by a film of air as the mechanical agent.

The material theory of heat is, however, often employed to illustrate some of the phenomena which occur during the abstraction from, or addition of caloric to a body. We often hear at the lecture table such expressions as "forcing or squeezing out heat;" and to the philosopher, no danger arises of misconception, because he uses and hears such terms as mere accommodations, whose object is to popularise ideas, and to make them accessible to those who have not had the advantage of scientific training. Astronomers take similar liberties with the facts of their science. We thus hear them speaking of the "daily motion of the sun," &c.; electricians speak of "currents of electricity," &c.: all of which are permissible, but, at the same time, have a great tendency to retard the progress of that rigid mental qualification called truthfulness, which it would seem that Experimental Philosophy has for the base of its operations.

Quitting the material theory of heat, or, as it has been sometimes called, the "corpuscular," because corpuscles, or small particles, of this quasi-material were supposed to emanate from heated bodies—we now proceed to investigate what has been termed the "undulatory theory," or that which explains the production and effects of Heat, Light, &c., on the supposition that an "*ether*," by its wave-like or undulatory motion, is their mutual cause.

The term undulatory has been derived from the Latin word *unda*, a wave; and the reader will understand the nature of the whole theory by performing what at first sight may seem a most childish experiment; but which has, in its application, as much effect on the scientific theories of the present day, as had the notable one observed by Newton—namely, the falling of an apple. The Deity impresses the same causes into His wise purposes, whether that a hair shall fall from our head, or that a world shall travel in its orbit—the most insignificant and the grandest effects alike owing their cause to the action of one universal law.

On casting a pebble into a still pond, it will be observed that the water forms a series of circles, all of which have the point where the stone first touched the water as their common centre. These waves, or undulations, thus created, continue to be produced, until the edge or bank of the pond prevents their further propagation. At first sight it would seem that the water really moved in a horizontal direction from the centre: but, on a careful examination, such will not be found to be the case. Each particle of water communicates its motion to that next to it; and thus each particle is scarcely disturbed horizontally: the action rather raises the particles in an upright or vertical position; and thus the apparent and actual motion afford a paradox, and also a refutation to the general idea, "that seeing is believing."

A very familiar illustration of the fact, that the *body* or *mass* of the water does not move, is found in the case of a swan or other bird floating on ruffled water. Instead of the bird moving in the apparent direction of the waves, in the absence of tide or current, its body will retain its *horizontal*, although its vertical or perpendicular position will undergo continual change.

By refining the ideas or imagination, and supposing the existence of an extremely rare substance, which has been called "*ether*," undulations in this material have been suggested as the cause of the forces of which we have now to speak; and, as we shall presently show, that although the idea or theory has no exact foundation, still it has the inestimable advantage of explaining a variety of phenomena for which we have no equally suitable expression.

Taking as a postulate the existence of an ether, we proceed to speak of its motion as the cause of forces; and we may here remark, that whilst the undulations of the ether may be *proximate* causes, we assume the existence of some unknown force, the action or suspension of which is the *ultimate* cause of the proximate. Of the ultimate cause we are ignorant; we can only assign its origin to the First Cause, of whom it has been recorded—

> "God said, Let there be light, and there was light."

When two pebbles are cast into the same still sheet of water, it will be found that the waves produced destroy each other when they come in contact in certain positions. Transferring our attention to the undulations of ether in a binary or twofold form, we can thus explain the cause of interference and polarisation; and assuming that any number of undulations may be produced by a similar number of initiative motions, we can, by applying the doctrine, explain a vast variety of the phenomena of the forces now under discussion. The reader will do well to study the effects produced by casting stones into still water, and by watching the results afforded when the radial waves intersect each other. Indeed, we cannot recommend any plan so effective for inducing an appreciative idea of the various laws to which the undulations of a fluid are subject.

Presuming that ether has been put into motion in one direction only, we can at once explain the phenomena of the conduction, convection, &c., of heat; for we have only to suppose motion to have been primarily imparted, and we can assume that it is propagated in every direction, and in every part of the body, from the centre to which motion had been first applied.

Thus, if a nail or match be struck, the percussion is supposed to set ether in motion or undulation: hence the production, radiation, and diffusion of heat, &c. If we presume a double or any number of motions to have been induced, we can understand that they may interfere with each other, and so produce double refraction, polarisation, and analogous phenomena.

Again, as all forces from a common centre must diminish in power, because they extend over a space equal to the square of any relative distance from the central point, so the undulation of an ether being at once a central and radial cause, will produce effects obedient to the same laws. In practice, we observe that a body giving off any amount of heat, will, at a distance of two feet, seem to afford but one-fourth of that which is perceived at one foot from it; at three feet, the intensity of its radiant heat seems to be but one-ninth, and so on. The calculation, therefore, of the forces of heat, light, &c., becomes one of mathematical precision; and the laws of circles and conic sections are of extreme value, and, in fact, a necessity, in arriving at accurate measurements of the intensities of these forces, and in arranging such in a tabular form.

Having thus shown the feasible, probable, and applicable nature of this undulatory hypothesis, we shall not further enter into the subject for the present. Under the heads of Light and Acoustics, we shall have to enlarge considerably on these remarks. Recapitulating our statements, and extending them for our present purpose, we presume—

1. That Heat, Light, &c., are produced by the undulations of an ether.
2. That these undulations may be propagated through space, and in the pores of a body.
3. That the rapidity of these undulations, which is exceedingly great, and that their regularity in one direction, or their mutual interference, are the causes of the intensity of the forces to which we have alluded, and also of their modifications.

Lastly. That the undulations of ether are caused by the application of a force which produces the motion of its parts, and consequently its phenomena.

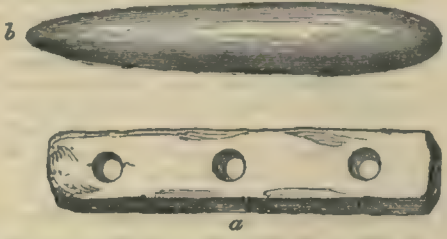
THE PRODUCTION OF HEAT.

The best known source of terrestrial heat is the sun; but the cause of its heat and light has been hitherto unascertained. By some it has been suggested that an extensive combustion is going on in that luminary. Recent observations on the solar spots, or black patches which occasionally appear on the disc, by no means countenance such views; and, indeed, it would seem that the body of the sun presents the black appearance thus observed, and that an external atmosphere has the power of either producing light and heat, or of, by some unexplained means, inducing these effects on the earth. Future researches, more especially in connexion with polarisation, may eventually lead to more correct views in respect to the cause of the forces present in the solar rays.

There is no doubt that the heat which all terrestrial bodies afford has been chiefly gathered from the sun: and thus, although generally unobserved by the senses, may be educed, by various means, to a large extent. The different processes employed for the purpose we shall now enumerate. They are chiefly Mechanical, Chemical, and Electrical.

Friction is a well-known cause of the production of heat, and, in civilised and savage nations, is equally employed for affording flame for the ignition of fuel. The common lucifer-match is well known in Europe, and has succeeded the now old-fashioned flint and steel. In Greenland, and in many tropical countries, the inhabitants produce sufficient heat, by the friction of dry wood, to ignite fuel, &c. The annexed engraving illustrates an instrument used in the arctic regions. It consists of a straight piece of dry wood (*a*), into holes in which the end of a small roller (*b*) is rapidly rubbed, in much after the same style that a hole would be bored in any solid; and thus a piece of dry moss may be easily ignited.

Fig. 1.



In the operation of boring cannon, it has been found that the friction evolves sufficient heat to boil water; and it has been stated, that a brass cannon, about seven-and-a-half inches in diameter, will, whilst the borer makes thirty revolutions in a minute, under a pressure of 10,000 pounds, produce sufficient heat to boil eighteen pounds of water in two hours and a-half.

In the Exhibition of 1851, in London, an instrument was used, which, by the friction of metal plates, afforded sufficient heat to keep various refreshments ready for use, such as coffee, &c.; and an arrangement has lately been patented, whose object is to introduce this novel mode of heating liquids into general employment.

The axles of wheeled carriages frequently become heated by their friction against their bearings. Steam and spinning machinery are often seriously injured from a like cause; and many extensive fires have been produced through neglect in oiling their different parts. The ordinary mode of grinding steel articles is an illustration of light and heat being evolved by friction; and so extended is the production of heat by such means, that two pieces of ice may be made to melt by being rapidly rubbed against each other.

Percussion is another source of caloric; and of this numerous instances are found in daily life. The cap universally employed in fire-arms, depends for its action on the sudden evolution of heat by rapid percussion, produced by the hammer falling, and compressing the fulminating mercury which the cap contains. By hammering a stout iron nail on an anvil, sufficient heat will be afforded to ignite a match; and at one period a similar plan was adopted by artisans, for the purpose of obtaining heat to light the fires of their forges. In the process of coining, the sudden percussion or compression employed, produces considerable heat. By suddenly compressing air in a suitable vessel, sufficient heat may be obtained to ignite a piece of tinder. The philosophical instrument-makers supply a syringe for the purpose of illustrating this interesting fact.

A change of bulk is generally attended with the production or loss of heat, as a natural result of the contraction or expansion of a body. Further remarks on this subject will be found under the head of Latent Heat.

Chemical Action is the source very generally resorted to, for the purpose of affording heat for social and economic purposes; and combustion, as being the most important, demands special attention. In the ordinary process of burning fuel, the natural heat contained therein is called out by intense chemical action; and just in proportion as this action is intensified, so is the amount of heat increased. The draught of a tall chimney, of the blast of the locomotive steam-jet, and of the fan or pump of the smelting-furnace, each effect the rapid promotion of chemical action between the oxygen of the air and the charcoal, &c., contained in the fuel, and so produce an intense combustion and great heat.

The production of heat by chemical action, is so often presented to the notice of the chemist, as to embarrass our attempts to make a selection. The following experiments, however, will tend to illustrate various instances of the kind:—

Experiment 1.—Mix together two parts of chlorate of potass with one part of powdered lump sugar, and drop on the mixture a little strong sulphuric acid. The chemical action which ensues will cause the rapid production of heat and light.

Experiment 2.—Dissolve some phosphorus in bi-sulphide of carbon, and pour the liquid thus obtained on some paper. In a few seconds the liquid will evaporate, and the solid phosphorus, being left in a minute state of division, will ignite, owing to the chemical action of the air; and sufficient heat will be produced to consume the paper.

Experiment 3.—Mix suddenly equal volumes of sulphuric acid and cold water. The two liquids will condense into a smaller bulk, and great heat will be evolved.

The addition of cold water to newly-made lime, affords another instance of the production of heat by chemical means. Amongst the bricklayers this plan is often used for the purpose of heating their tea. It has been patented as a method of affording heat for domestic requirements; and to so large an extent is caloric disengaged in this manner, that stables and outhouses have been destroyed by fire, owing to heavy rain-storms moistening lime which had been stored in them.

Spontaneous Combustion is often a dangerous source of heat, and is caused by the chemical action of air on oily substances, when these are in contact with fibrous matter, such as cotton and hemp. If a small piece of cotton wool is dipped into linseed oil, and then placed under gentle pressure, it will generally inflame at the ordinary temperature of spring or summer. From this cause, the destruction of cotton-waste stores and mills often takes place. This result is greatly promoted by a minute state of division, one instance of which may be found by trying the following experiment:—

Experiment 4.—Introduce some dry tartrate of lead into a narrow glass tube, having one of its ends sealed. Heat the tube until no more acid vapours pass off, and then instantly seal the open end. If the tube is broken, when quite cold, the fine particles of the lead will inflame on coming into contact with the air, chemical action being the cause of the heat thus produced.

The most intense heat which can be produced by chemical means, is that afforded by the combustion of a mixture of coal gas or pure hydrogen, and oxygen gases. This is generally effected by means of a peculiar form of blow-pipe, so arranged as to prevent risk of explosion. As this will be fully described in the chemical section, we shall not further allude to it here than by stating, that by these means the most refractory metals may be melted, and the earths fused: there are few solid substances which cannot by its agency be reduced to a fluid state, or be dissipated in vapour.

Electricity is another source of heat, to which special attention will be called in the proper place. By means of electricity, produced by chemical action, the most powerful effects of heat are afforded.

Vital Action is a source of heat with which all are familiar; and it seems chiefly to depend on the slow combustion which the food undergoes, after being absorbed by the stomach, and conveyed to the surface of the lungs, in the form of highly carbonaceous blood. The charcoal contained in the blood undergoes combustion by coming in contact with the inspired air; and in every respect the process is analogous to that which is observed in the burning of coals in the ordinary fire-place.

The intelligent reader will not fail to remark, that in every source of heat which has been enumerated, motion has been the leading cause; and a general statement may be made, that whenever motion is produced in matter, heat is simultaneously evolved. It is true that daily experience does not tend to confirm such a proposition; but that is solely because the senses, and our means of measuring heat thus evolved, are far too gross to permit its detection. Heat, Light, Electricity, and Magnetism are mutually producible: all matter seems to contain these forces; and motion, at any time, is sufficient to bring them from a latent to a sensible state.

THE EFFECTS OF HEAT.—EXPANSION.

Having spoken of the various sources of heat, we must next enquire as to its effects on matter generally; and it is necessary to bear in mind, that any results to which we shall allude, are produced, not *in* the atoms of which a body is composed, but rather *around* them, modifying and changing their relative positions. Many errors are often fallen into with respect to this point; and nothing is more injurious to true progress in scientific study, than a careless appreciation of the lessons and laws which it teaches.

There are but three states in which a body is found: the solid, the liquid, and the gaseous or as vapour; and, as it has already been observed, these states are modifications of each other, effected by the presence or comparative absence of heat. According to the generally received doctrine, it is assumed that a solid is so, because its particles are near to each other, or in close approximation—that a liquid is produced from a solid by the addition of heat, which has the effect of removing the particles of a mass from each other; and that by still further carrying on the process, a gas or vapour is formed, wherein the separation of particles is still more effected.

It would thus be seen that heat has what is called a repulsive power: in other words, it can lessen the cohesion of particles, and so enlarge the bulk they occupy in mass. As we shall presently find, the addition of heat almost always increases the size of a mass; and hence has arisen the idea that it repels the constituent particles of that mass. When, however, a body has changed its state by this expansion, other laws govern its further increase in bulk. Indeed, there is a special law of expansion incident to solids, liquids, and gases; and these considerations are of the utmost importance in physical science and its applications.

The expansion of solid bodies varies considerably on their receiving an equal addition of sensible heat; and they expand in all directions, although differing in individual extent. A very simple way of showing the expansion of a metal, both in respect to its length and breadth, may be thus tried:—

Experiment 5.—Fit a brass ring very accurately on an iron rod; having removed the ring, heat the rod nearly to a red-heat, and attempt to replace the ring. It will be found, that owing to the expansion of the metal, the ring cannot be passed on to the rod until the latter has cooled down.

Experiment 6.—Fix an iron rod, whilst cold, so as that it may exactly rest between two points touching each of its ends. On heating the rod, and attempting to replace it, its expansion lengthways will be found to prevent its resting between the two points, because they will be at less distance from each other than are the two ends of the heated metal. The rod, on cooling, will again fit between them, owing to the loss of heat and subsequent contraction which it undergoes.

Having thus illustrated the linear and general expansion of a solid by the agency of heat, we remark that different bodies expand at unequal rates; and perhaps the simplest illustration of this is afforded by the following experiment:—

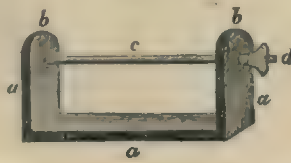
Experiment 7.—A piece of brass bar, an eighth of an inch thick, one inch wide, and eighteen inches long, is to be firmly rivetted to an iron one of the same size, care being taken that the two shall form a straight bar when completed. On heating such an arrangement, it will immediately take a curved form, because the metals expand unequally. The brass, expanding more than the iron, will form an arch, enclosing the iron. The bar will regain its straight form on again becoming cool.

A vast variety of interesting applications of this law of expansion by heat are made in daily life. The tires of railway wheels are made to embrace the spokes tightly, by making them a little too small when cold. On heating the tire, it expands, and can then be passed on to the ring of metal in which the spokes end. On cooling, the tire contracts with great force, and so remains completely fixed in its desired position. The tires of carriages and cart-wheels are affixed in a similar manner; and some parts of machinery are thus easily made to retain their position, such as in the case of rings or pulley-wheels, often secured on shafting. In constructing railway and other iron bridges, allowance is always made for expansion by heat; and in laying down the rails on the track of a railroad, sufficient space is left between each metal to permit free extension. On a railway where sufficient allowance had not been made, the rails were lifted, during a hot summer day, several inches from the ground. The rails on a line of railway of two hundred miles' nominal length, will be some hundred feet longer in summer than in winter time, presuming the absolute length of iron surface is carefully measured.

It has been noticed that a rod of metal will contract to its previous bulk, when the heat which had caused its expansion had been lost. An interesting experiment may be adduced in illustration of this fact.

Experiment 8.—In an arm of cast-iron *(a)*, represented in Fig. 2, have two holes made at *b*, so that a rod of wrought-iron *(c)* may be screwed tightly, by means of a thumbscrew, as at *d*. Remove this rod, and heat it red-hot, afterwards replacing it in this state, and screwing it firmly in the arms of the cast-iron frame. On pouring cold water on the hot iron, the metal will rapidly contract, and in so doing, will act so powerfully on the cast-iron frame as to break it into pieces, with a loud report.

Fig. 2.



An adaptation of this experiment has been occasionally employed for the purpose of restoring walls which had bulged outwards. Iron plates are fixed on the outsides of the wall, and a stout iron rod is stretched between them, and firmly screwed at its extreme ends, beyond the plates. On the rod being heated, it of course expands, and is again screwed up tight. On contracting, it draws together the plates and walls with great force. A repetition of the process generally succeeds in restoring the perpendicularity of the walls of a building, and is easy of application.

Serious accidents have happened in cases when builders, on fastening iron girders resting between stone walls, had neglected to make allowance for the expansion of the metal. In many instances, extensive erections have been thrown down: for as the iron expands in length, it forces the walls outwards; and should no immediate apparent mischief result, still the alternate expansion and contraction has the tendency of weakening the brickwork, and thus of endangering the stability of the building.

The annexed table, which has been arranged from various sources, gives the linear expansion of the substances named, on their being raised from a temperature of 32° F. to 212°; and is to be understood as expressing the ratio of their individual expansion, in the order in which that increases.

Glass Tube	0.000861
Platina	0.000884
,,	0.000992
Iron	0.001220
Gold	0.001466
,,	0.001555
Copper	0.001722
Silver	0.001909
Lead	0.002848
Zinc	0.003011

The above numbers are expressed in decimals, as giving a greater amount of accuracy than would have been arrived at by employing vulgar fractions; and the standard is unity. Hence the length of a bar of metal will be increased to the extent of its original length, multiplied by the factor in the table, on the temperature of the metal being raised the number of degrees above indicated.

Various interesting applications of the law of the expansions of solids will be mentioned in the section on Mechanical Philosophy: such as the regulation of pendula, watch-springs, and other similar matters. Their introduction in this place would tend to distract attention from the laws to which we prefer for the present to confine our remarks.

Expansion of Liquids.—As a general rule, liquids expand far more in bulk, by the same increase of temperature, than solids; and hence these changes become more evident to the senses. They have been chosen as the fittest media for the purpose of measuring the temperature of bodies within a limited range. Hence the construction of thermometers of various kinds, to which we shall now devote our consideration.

That liquids expand considerably on being heated, is often observed in daily life. If a kettle, completely filled with cold water, is heated, the liquid it contains gradually expands, and soon passes out of the vessel. A more philosophical mode of illustrating this effect, is that of fixing into a glass flask a long, narrow glass tube, and filling the vessel with coloured water, so that a portion of the liquid shall also rest in the tube. On heating the flask, the water will gradually expand, and will do so according to certain laws, indicating its expansion by rising in the tube until the boiling point is reached. On this principle the thermometer is constructed; mercury, however, being generally employed in preference to other liquids, except for special purposes.

Liquids, like solids, vary much in their relative expansion on receiving equal increments of heat; and without giving a tabular form of these, which could not be done with accuracy, we may state that mercury expands least, and alcohol (pure spirits of wine) expands most, on being exposed to the same increase of temperature. The total expansion of a solid by heat is very nearly ascertained by multiplying its linear extension by three. In the case of liquids, they naturally accommodate themselves to the shape of the vessel which contains them, and as a solid that will expand also. It is therefore necessary, in every instance, to make an allowance for this in constructing instruments for measuring changes of temperature by means of liquids. There is in every case, with but one exception, a constant contraction sustained by liquids on their losing heat; and thus they afford means of ascertaining temperatures which may be much below those experienced in daily life.

This exception is that of water; and in it we find a remarkable instance of an apparent suspension of a universal law for effecting a special end. At a temperature of 40°, water ceases to contract; and as it approaches a temperature of 32°, its freezing point, it expands considerably; so much so, indeed, that if confined in a closed vessel of great strength, it will, on freezing, expand sufficiently to rend the vessel into fragments. This singular exception has an astonishing effect in influencing the climate of the polar regions. If water contracted continuously on suffering a decrease of temperature, it would, on arriving at the solid state of ice, at once sink to the bottom of a vessel from its attaining an increased specific gravity; but through the expansion which takes place as it cools, this cannot occur. Hence we find the ice on the *surface* of the rivers and the sea, ready to receive the rays of solar heat, which will again change it to the liquid state—a result which could never occur if, obeying the usual laws, it sank, when formed, to the bottom of the ocean. This circumstance may be adduced as one of the most striking proofs of the existence of an OMNISCIENT BEING.

The ordinary thermometer consists of a glass tube, of small diameter, one of whose ends has been blown into a bulb. This and a portion of the stem is filled either with mercury, or spirits of wine which has been coloured; and after expanding the fluid until it completely fills the stem, the open end is hermetically sealed or closed. A scale is then made, whose object is to ascertain comparative temperatures. Various scales are employed; such as those of Reaumur, the Centigrade, and that of Fahrenheit. Confining our attention to the latter for the present, we shall describe the mode adopted for making the scale which is invariably used in this country.

On the bulb and stem being completed, the former is dipped into a vessel containing melting snow or ice; and when the mercury in the stem ceases to fall, a mark is made on the glass. This indicates the freezing point of water, which is denominated by Fahrenheit as 32°. Water is then heated in a *metal* vessel (one of glass, &c., will not do); and when the water, which should be quite pure, boils, another mark is made at the point to which the mercury has risen in the stem. The interval between the freezing and the boiling point is divided into 180 equal parts, or degrees. Hence, on the scale we here refer to, water boils at a temperature of 212°.

We shall not enter, at present, into various corrections which must be made for the expansion of the glass bulb and tube, each of which has the tendency of inducing errors in observation; as these points will be fully considered under the head of Meteorology. Spirits of wine is usually employed for measuring very low temperatures, as it is not liable to freeze—an occurrence which would happen to mercury, even in the temperature of the arctic regions, at some periods of the year. The scale of Reaumur has but 80° between the freezing and boiling points of water; and as this is often used in foreign scientific works, we append a rule for converting the degrees of each scale to that of the other.

1. To convert the degrees of Fahrenheit's scale to those of Reaumur, multiply the number of degrees of Fahrenheit, from which has been subtracted 32, by 4; and divide the amount by 9: the quotient will be the degrees indicated by Reaumur's scale.

Example.—What number of degrees will be shown on the scale of Reaumur, if the temperature indicated by that of Fahrenheit is 212°?

Subtracting 32 from 212, leaves 180.
Multiplying 180 by 4, gives 720; and,
Dividing 720 by 9, ,, 80°.—*Ans.* 80°.

2. To convert the degrees on Reaumur's scale to those of Fahrenheit, multiply those of Reaumur by 9, and divide by 4; to the quotient add 32, which will give the number sought.

Example.—What number of degrees on Fahrenheit's scale corresponds with 80 degrees on that of Reaumur?

Multiplying 80 by 9, gives 720.
Dividing 720 by 4, ,, 180.
Adding 32 to 180, ,, 212°—*Ans.* 212°.

The above calculations evidently rest on the fact, that nine of the degrees of Fahrenheit's scale are equivalent to four degrees of that of Reaumur.

The Centigrade scale is divided into 100° between the freezing and boiling points. Hence, five degrees on this scale correspond with nine degrees of those on Fahrenheit's scale.

3. To convert the degrees of Fahrenheit to those of the Centigrade, multiply the number of the former, less 32, by 5; and divide by 9: the quotient will afford the corresponding degrees of the Centigrade scale.

Example.—What is the number of degrees on the Centigrade corresponding to 212° Fahrenheit?

212 less 32, equals 180.
180 multiplied by 5, gives 900.
900 divided by 9, equals 100°.—*Ans.* 100°.

4. To convert the Centigrade to the Fahrenheit scale, multiply the former by 9; and, dividing by 5, add 32 to the quotient.

Example.—What number, on the scale of Fahrenheit, corresponds with 100° on the Centigrade scale?

Multiplying 100 by 9, produces 900.
Dividing 900 by 5, ,, 180.
Adding 32 to 180, gives 212°.—*Ans.* 212°.

The following skeleton table may be of assistance in many instances, and, occasionally, will save the trouble of calculation:—

Comparative indications of thermometers of the scale of—

	Fahrenheit.	Reaumur.	Centigrade.
Freezing point . .	32°	0°	0°
Boiling ,, . .	212°	80°	100°.

Various forms of instruments for measuring heat will be fully noticed in other parts of the work, as coming more especially under other subjects: among these we include the air, registering, and dew-point thermometers; as also Breguet's, and an interesting instrument, by means of which electricity is employed for ascertaining minute changes of temperature. We have omitted to give exact directions for constructing a thermometer, because long practice is necessary to produce even a tolerable instrument; and the cost of such is very slight, if purchased of the instrument-maker.

Having thus described the thermometer as an instrument which enables us, by the expansion of a fluid, to ascertain the comparative temperature of bodies, we proceed to notice some of those natural and ordinary effects which depend on the expansion of liquids by heat for their proximate cause.

In every instance in which a liquid is heated, it becomes lighter than that of a lower temperature; hence it has a tendency to ascend. This effect is magnificently illustrated in the case of the gulf stream, which sets in towards our western shores from the Gulf of Mexico. Its temperature is much higher than the surrounding ocean, and it accordingly floats on the surface of a colder, and therefore denser, fluid. In domestic life this law is taken advantage of; hence the construction of hot-water apparatus, in which the hot water rises through an upper pipe, and, when cooled, returns by one placed at a lower level. The character of these currents of heated liquid, and their tendency to rise, may easily be noticed by trying the following experiment:—

Experiment 9.—Throw a small piece of litmus, or indigo, into a glass flask nearly filled with cold water, and placing the bottom of the flask over the flame of a candle or lamp, it will be noticed, that as the lower stratum of liquid becomes heated, the colouring matter will rise with it; and, on the liquid cooling, it will return to the bottom of the flask, again to undergo a similar process.

EXPANSION OF GASES.—Gaseous bodies, such as air, expand still more than liquids, on the addition of heat; and the cause and applications of this fact, form a most interesting feature in Physical and Natural Science, the results being more definite and constant than those found in the expansion of solids and liquids.

The simplest mode of illustrating the fact, that a gas, such as air, expands by an addition of sensible heat, is by plunging the stem of a retort or thermometer tube, full of air, beneath the surface of water, and applying heat to the bulb. On the air expanding, a portion will escape in bubbles through the water; and on the source of heat being removed, the water will rise in the neck, to take the place of the air which has been expelled by heat.

Air thermometers are constructed on this principle; and a very simple one may be made by fixing a glass tube, by means of a cork, into the neck of a glass flask. If the flask is partly filled with a coloured liquid, and inverted so that the end of the tube shall rest in a cistern of water, the air remaining in the flask will expand on a slight increase of its temperature, and the liquid in the tube will be driven downwards, recovering its position on the application of heat ceasing. Differential thermometers are of this kind, but have two bulbs, into one of which the liquid passes on the expanded air pressing it downwards.

Nearly all gases expand equally on receiving an equal increase of temperature; at least, in practice, their differences may be neglected for all ordinary purposes. After careful experiments, it has been ascertained that they increase in the proportion of $\frac{1}{493}$rd part for every degree of temperature rising from the freezing point of water, or 32° Fahrenheit.

In other words, if a volume of air measure 493 parts at 32°, it would become expanded to 986 parts on attaining a temperature of 525°. Any intermediate result may be obtained for other temperatures by adding $\frac{1}{493}$rd part for each degree of increase. The following formula expresses this rule, where x represents the volume sought, a the normal volume, and t the number of degrees of increment of temperature; 32° Fahrenheit being the standard :—

$$x = a + \frac{a}{493} t$$

To illustrate this we will suppose that it is desired to ascertain the expansion which air will undergo by being heated from 32° to 212°. Supposing a is taken as 100 parts, t as 180°, being the difference between 32° and 212°, we shall have

$$x = 100 + \frac{100}{493} \times 180^\circ = 136.5$$

or, one hundred parts of air, on being heated from freezing to boiling point, expand to one hundred and thirty-six parts and a-half.

In applying this rule to other gases, an allowance has to be made for very slight variations in the ratios of expansion. We have, in our example, supposed that 100 parts of air expands to a volume of 136½, on being heated from 32° to 212° Fahrenheit. In the following table, the ratio of the different expansion is stated for a similar change of temperature in various gases, unity being the standard, and on the authority of Regnault :—

Air	1.36700
Hydrogen	1.36613
Carbonic acid . . .	1.37099

By multiplying any number of volumes by the above factors, the actual expansion which they will undergo by being heated from 32° to 212° is at once obtained.

Having thus explained the laws which govern the expansion of aëriform or gaseous bodies, we may pass on to consider some instances in which their application becomes a matter of general interest.

The various changes of the wind depend on the unequal expansion of the atmosphere by the agency of heat. Expanded air being lighter, of course tends to rise, and, consequently, air of a more dense and cooler nature rushes in on all sides to take its place. We thus have either the gentle breeze, the hurricane, or the whirlwind. To the engineer, the expansion of gases and vapours is of the utmost importance; and, within the last few years, heated air has been extensively employed as a source of motive power by Ericson and others; of which we shall speak under the head of Mechanics. In daily life we have to study and apply the laws of expansion in attempting the ventilation of public and private buildings —a subject to which we shall draw special attention hereafter. It would be impossible to enumerate all the instances wherein these laws exercise an influence. We, however, cannot fail to remark, that an astonishing range of application exists in the effects of expansion by heat; for, under the same cause, the fierce storm in one place may be spreading desolation, whilst the gentle current of air is supporting existence, in its quiet obedience thereto in other situations. We thus observe how one cause may be productive of a multiplicity of effects; and, perhaps, one of the greatest charms of experimental science, is the frequent disclosure of such interesting instances of design and adaptation of means to an end, which the attentive student is continually witnessing.

THE EFFECTS OF HEAT.—LIQUEFACTION AND FUSION.

We proceed to consider the effects of increasing the temperature of bodies by the addition of sensible heat; and we shall find that unequal results are produced on equal increments of heat being made to various substances. We have already noticed, that whilst some bodies are generally found in a solid state, others take that of the liquid, or gaseous, at ordinary temperatures; and the most cursory observation will show, that an addition or abstraction of heat would be necessary to render all bodies of a uniform condition.

The terms "liquefaction" and "fusion," although they appear at first sight almost synonymous, have, nevertheless, very different significations. We liquefy, or render liquid, ice; but, generally speaking, the term "fusion" is applied to the process of melting solids, such as lead or rock-crystal; and although ice is as much a solid as glass, still there is considerable convenience in maintaining this distinction of terms.

On increasing the sensible temperature of a substance, the effects vary considerably. On heating ice, the solid is immediately converted into a liquid; and the most accurate observations fail to trace any notable effect intervening between these two conditions. If, however, zinc is submitted to the same process, we find that its solid state is considerably modified; and, whilst brittle at first, it becomes softer and more malleable before it enters into a state of fusion. The metal tin becomes pulverisable, or, rather, easily divisible at high temperatures; and masses of iron or platina may be welded or united together at a heat below their point of fusion.

It will be thus observed, that a variety of phenomena may occur between the solid state and its conversion into that of a liquid. The separate causes which induce this chain of variation are entirely hidden from us; but we presume that the series of effects results from a peculiar arrangement of the molecules or particles of which a body is composed, and is thus owing to the relative position which they may assume whilst undergoing the process of heating.

In the conversion of the solid into the liquid state, bodies of different kinds absorb unequal amounts of caloric; and, consequently, we find that their melting points vary accordingly; or, which is the same thing, the liquid state becomes converted to the solid at different temperatures.

We shall have to consider the cause of these variations more fully when we treat on Specific and Latent Heat; but the following table will give an idea of the great difference which exists amongst solid bodies when changing their condition to that of the liquid state :—

The Melting or Fusing point, according to the scale of Fahrenheit, of—

Mercury, is	—40
Water (ice)	+32
Tallow	98
Phosphorus	110
Stearine (variable) . . .	120
Sulphur	226

It must not, however, be assumed that the solid state of bodies can be directly attained by liquids on their arriving at a temperature such as we have indicated in the foregoing skeleton table, although such is true with respect to the conversion of solids to the liquid state. The various forces of nature act together; and thus water may be cooled much below its point of congelation without becoming solid, provided it is kept perfectly still. This interference of apparently heterogeneous and yet correlative forces, may be easily observed on trying the following experiment :—

Experiment 10.—Dissolve, in boiling water, as much sulphate of soda (Glauber's salts) as the liquid will take up. This is best performed in a glass flask. Place the vessel and contents on several folds of flannel in a still place, and allow all to attain the temperature of the atmosphere, and the liquid state will be maintained. If, however, a small piece of any angular substance is dropped into the flask, or if its contents are violently shaken, the liquid will instantly pass into the solid state.

By exposing a glass vessel filled with water to a temperature below freezing point, the water will retain its liquid state for some time; but, on the least agitation, it will be converted into ice. It has been stated, that even the rays of the light of the moon will produce the same effect. We are not in a position to verify this by personal experience, and would suggest, that as there is great difficulty in maintaining a quiescent position, owing to the chances of motion being unwittingly communicated to a liquid, it is highly possible that an experiment of this kind may be vitiated by accidental circumstances.

Whilst speaking of substances which change their physical condition at comparatively low temperatures, we may mention that different bodies assume a variety of forms by the peculiar arrangement of their particles during the act of solidification. Thus we have crystals of snow, the arborescence of ice, and the crystallographic forms peculiar to each body produced at that moment. We shall devote a special article to this subject; and content ourselves, for the present, by merely mentioning the occurrences as commonly taking place.

When a solid assumes a liquid state at a temperature higher than that of the boiling point of mercury, we are obliged to resort to other means of ascertaining the temperature than the employment of the mercurial thermometer, because a temperature of 670° Fahrenheit would destroy the instrument. For such purposes *pyrometers* are employed; and although much ingenuity has been directed to construct an instrument of this class, on whose indications reliance may be placed, it is to be regretted that no great success has attended such efforts. The earliest form of pyrometer, or fire-measurer, was that of Mr. Wedgwood. It was constructed on the assumption that a piece of clay would contract in proportion to the amount of heat to which it was subjected. The student may be here surprised to learn that a body *contracts* by the application of heat; but the difficulty is only an apparent one; for when clay contracts on being heated, the effect is produced solely by the water between its particles being expelled; hence it forms no exception to the general law of expansion by heat. Owing to the variety of the texture of clay, Mr. Wedgwood's pyrometer affords unreliable results, and has, therefore, been discarded.

The late Dr. Daniell invented an ingenious arrangement, wherein the expansion of a bar of platina was made to register high temperatures. The bar is enclosed in a cylinder of black-lead crucible ware; and a piece of porcelain, which fits tightly into the cylinder, and touches one end of the platina bar, indicates, by the removal from its first position, the extent of expansion which the platina has undergone during the application of heat. By a mechanical arrangement, the expansion is registered in degrees corresponding with the scale of Fahrenheit.

Other arrangements have been made, intended to facilitate the registration of high temperatures; but, in every case, the great difficulty which has to be contended with is, that the intense heat to which the indicating substance has to be exposed, almost always vitiates the result obtained.

As it is usual, in scientific works, to give a table of the melting point of various solids, we follow that course: however, the reader must bear in mind, that some of the figures in the following table have been calculated only approximately, and are liable to the sources of error which have been pointed out.

Melting point, according to the degrees of the scale of Fahrenheit, of the metals.

Potassium	136° F.
Sodium	194
Tin	442
Bismuth	476
Lead	594
Zinc	700
Brass	1869
Silver	1280
Copper	2548
Gold	2590
Cast-Iron	3479

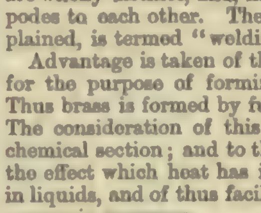
The above may be fused by means of furnaces of various kinds; but platina, and many other bodies, require the powerful heat of the oxyhydrogen blow-pipe, or of the voltaic battery, to reduce them to the liquid state.

Platina and wrought-iron, however, seem to enter into a partial fluid state long below their melting point; and hence separate masses of either of these metals may be united at a white heat, so as to form one piece. It is impossible to state definitively the reason of this property being confined to two substances; and it is the more remarkable because their physical and chemical character are wholly distinct, and, in many instances, the very antipodes to each other. The process, as we previously explained, is termed "welding."

Advantage is taken of the power of heat to fuse bodies, for the purpose of forming what are termed "alloys." Thus brass is formed by fusing together copper and zinc. The consideration of this will be duly observed in our chemical section; and to that we must also defer noticing the effect which heat has in rendering substances soluble in liquids, and of thus facilitating numerous scientific and economic processes.

A most singular effect is observed when fusion is effected by heat produced by electricity. The combination of these two forces produces the anomalous result of sometimes fusing a body so suddenly, and of permitting its reconsolidation, as not to allow time for the heat to pass to surrounding bodies; and hence, during violent thunder-storms, instances have occurred in which metallic articles, such as coins, chains, &c., worn by a person who has been struck, have been completely melted; whilst the linen or cotton on which such have rested has been entirely uninjured. We have frequently noticed that a powerful Leyden discharge will superficially melt a wire, and yet leave a piece of paper, on which the metal rested, perfectly unburnt. No adequate reason has been as yet assigned for such occurrences.

Fig. 3.



Having mentioned the leading methods of measuring intense heat by means of the expansion of solids, we must not omit to describe a very ingenious instrument, invented by M. Breguet, of Paris, for ascertaining minute changes of temperature. It is constructed on the principle that two metals, expanding unequally on receiving increments of heat, will assume a curved form—a fact which we illustrated by Experiment 7.*

The annexed engraving illustrates one of these interesting arrangements. The spiral is formed of a ribbon of silver platina and gold, soldered together. Its upper end is fixed to a stand; and to the lower one an index is attached, which, by traversing over a graduated plate, indicates a change of temperature, owing to the unequal expansion of the metal forming the spiral. This is one of the most elegant adaptations of a scientific law which has yet been made; and for ascertaining minute changes between low rates of temperature, it is a valuable instrument in the hands of the philosopher.

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* *Ante*, p. 16.

It is scarcely necessary to point out instances in which liquefaction and fusion operate to our advantage in daily life. The processes of metallurgy, such as smelting and refining, casting, moulding, and other such operations, are sufficiently familiar. We may remark, however, that the difference of temperature at which bodies cease to be solids is a matter of great importance, and leaves an extensive choice of material for various purposes, extending from the refracting crucible used by the chemist, to the more easily melted, and therefore more tractable, substances employed for economic processes.

THE EFFECTS OF HEAT.—EVAPORATION, VAPORISATION, Etc.

Continuing to restrict our remarks to the effect of *sensible* heat, we now examine the processes of Vaporisation and Evaporation. At first sight, these terms seem convertible; but in scientific phraseology they hold a distinct meaning. Vaporisation is that process in which anything is converted into a state of vapour, by the addition of a considerable amount of heat; whilst the term "evaporation" refers to a process continually and imperceptibly going on at or about the ordinary temperature of the atmosphere. Thus steam is produced by vaporisation, and clouds, &c., by evaporation.

It will be here necessary to point out the distinction which is maintained between the terms "vapour" and "gas." By vapour we understand an unstable condition in which a body exists, so long as a comparatively high temperature is maintained. Thus the steam of water, alcohol, ether, &c., fall under this category. The air, coal gas, &c., are, however, under all ordinary circumstances, in a constant state; and hence are ranked under the term "gas." It is true that some gases, as carbonic acid gas, or fixed air, may, by great pressure and intense cold, be reduced to the solid state; but these results being extraordinary, are not considered to militate against the justice of the rule which we have laid down. We may add, that vapours can easily be converted into liquids; whilst gases present great resistance to that process; hence the distinction of the terms. The consideration of a permanent gaseous state, we shall take up under the head of Latent Heat.

The process of evaporation is continually going on at the ordinary temperatures. By placing an inverted glass over grass during the heat of a summer day, it will be found that moisture soon forms at the top of the vessel. The insensible perspiration which continually exudes from the skin of animals, the gradual loss of fluid from an uncovered vessel, and other similar instances, are due to this constant process.

Whenever evaporation takes place, cold is at once produced; and hence, if a porous vessel, containing a liquid, is placed in any position where a draught of air can act on it, the result is that its sensible temperature is much diminished. The cause of these phenomena will be more fully dealt with presently.

On applying heat to liquids, and some solids, they may be made to assume a vaporous form; but, as in the conversion of a solid to the liquid state, different liquids require unequal increments of heat to effect this purpose. Ether, for instance, is converted into vapour at a temperature of 96° Fahrenheit; whilst mercury requires that of 660° Fahrenheit: both of these, however, may evaporate at lower temperatures. The temperature of ebullition, or rapid conversion into vapour, is termed the *boiling point.*

We here refer to vaporisation as taking place under ordinary circumstances; but the boiling point of liquids varies when the pressure of the air is raised or lowered. We shall take these different cases in the order of—common pressures, the vacuum of the air-pump, and extraordinary pressure, as in the high-pressure steam-boiler.

The boiling point of liquids can easily be ascertained by immersing a mercurial thermometer in them; but a metal vessel must be employed to hold them, because those of glass or porcelain have the effect of raising the apparent boiling point, perhaps through the inferior conducting power of their material, and from other causes.

Annexed is a skeleton table, which affords the boiling point of some liquids; and it is supposed that the temperature has been taken whilst the barometer stands at 30 inches.

Boiling point, on Fahrenheit's scale, of—

Sulphuric ether	96 to 98°
Pure alcohol (spirits of wine)	173
,, Water	212
,, Nitric acid	242
,, Sulphuric acid	550
,, Sulphur	560
,, Mercury	660

If liquids are heated under a lower pressure than that of the atmosphere, they will boil at a much lower temperature, as the following simple experiment will illustrate:—

Experiment 11.—Boil some water in a glass flask, and, when in full ebullition, fit a cork air-tight into the neck of the vessel. Remove the latter from the source of heat, and dip it into cold water. Because a vacuum is thus formed, and the pressure of the air removed, the water will recommence to boil.

If ether, spirits of wine, or water heated to 98°, is placed under the receiver of an air-pump, and the air is exhausted, each may be made to boil at ordinary temperatures, because the pressure of the atmosphere, of course, cannot affect them in a vacuum. This circumstance has been taken advantage of by sugar-refiners and pharmaceutists, who are in the habit of boiling their liquids in vacuo at a low temperature, and thereby prevent the formation of certain products, which might prove injurious to their processes: economy, also, is found to accrue thereby.

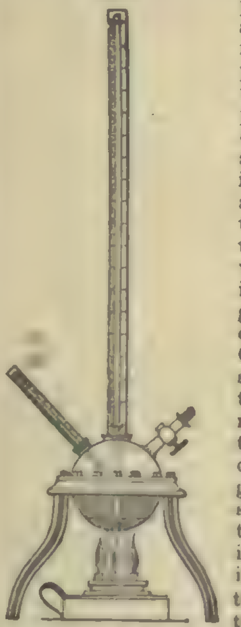
The property of liquids boiling at lower temperatures when the superincumbent pressure of the air has been removed, has been used as a means of ascertaining the heights of mountains. As we ascend in the atmosphere, the pressure decreases; the mercury in the barometer descends; and, therefore, water will boil at a lower temperature. An approximate ratio of the decrease of the boiling point, may be taken as one degree of Fahrenheit's scale to an elevation of 520 feet above the level of the sea; hence, on the top of a mountain 10,000 feet high, water would boil at a temperature of 192° Fahrenheit. Heights found in this manner are, after all, only rough calculations, inasmuch as that barometric changes may not be synchronous between the base and summit of the mountain.

The following table gives the three elements to which we have alluded—namely, the boiling point of water, the barometric height, and the corresponding elevation, in round numbers, above the surface of the earth. The barometer is presumed to stand at 30 inches at the level of the sea:—

Boiling Point of Water.	Elevation of Observer.	Height of Barometer.
212	surface	30
210	1050 feet	29
208.5	1850 ,,	28
206.5	2900 ,,	27
205 nearly	3700 ,,	26

Having occasionally employed the term "atmospheric pressure," it is necessary that we should define its meaning before proceeding further. At the level of the sea the air presses on every surface with a force of nearly fifteen pounds on a square inch. It is true that this pressure is not felt, because it acts on each and every side; and thus, on holding out the hand, we sustain as much pressure *upwards* on its under surface, as we do *downwards* on its upper side. But, if either pressure be removed, the other would at once become evident, and its existence be perceived. In dealing with pressures beyond fifteen pounds on the square inch, such are reckoned by "atmospheres;" that is, supposing air was forced into a vessel until it exercised a pressure on its sides equal to thirty pounds per square inch, such would be called a pressure of two atmospheres; if of forty-five pounds, it would be three atmospheres; and so on. This short explanation will suffice to render the succeeding remarks intelligible.

Fig. 4.



Whilst water boils at ordinary pressures at a temperature of 212°, and at about 90° in a vacuum, it is found that a much higher temperature is required to produce the same result under a pressure exceeding that of the atmosphere. An arrangement, represented in Fig. 4, is employed for the purpose of ascertaining the successive boiling points of water under such circumstances. It consists of a strong copper boiler, in which is fixed a tall glass tube, open at both ends. The bottom of the boiler contains mercury, and the vessel is half filled with water. On one side a stop-cock is fitted, and on the other a good thermometer, whose bulb enters into the metal vessel. On heat being applied, and the steam produced being allowed to escape, the thermometer will not indicate a higher temperature than 212°, and the mercury will not rise within the tall glass tube. But, on closing the stop-cock, and thus preventing the escape of the steam, the internal pressure will gradually increase, the mercury indicating this by rising in the central tube. As the mercury rises, the temperature indicated by the thermometer will also rise; showing that by an increase of pressure the boiling point is raised.

If the stop-cock is opened, the steam will escape, and the pressure will be simultaneously diminished. The mercury in the thermometer will fall to 212°, and there remain, so long as free access is allowed between the external air and the boiling water.

The following table exhibits the results obtained by an experiment of this kind :—

Pressure per Square Inch.	Height of Mercury in a Barometer Tube.	Temperature.
15 pounds	30 inches	212°
30 ,,	60 ,,	250
45 ,,	90 ,,	275
60 ,,	120 ,,	294
75 ,,	150 ,,	310
90 ,,	180 ,,	323

It will thus be observed, that the boiling point of water is increased in by no means a regular ratio under an increased pressure of vapour on its surface. Tables indicating this result have been extended to a much higher degree than that which we have presented; but the principle will be understood from what we have stated.

Liquids, on being converted into vapour, undergo great expansion. Water, when changing its liquid to the vaporous state, is expanded to about 1,700 times its previous volume. The annexed table gives the relative bulk of a cubic inch, in the liquid state, to that of the condition of vapour, of water, alcohol, and ether.

	Bulk at 60° F.	Temperature.	Bulk of Vapour at Boiling Point.
Water . .	1 cubic inch	212° F.	1690 cubic inches
Alcohol .	1 ,,	173	494 ,,
Ether . .	1 ,,	96	213 ,,

In round numbers, it is estimated that one cubic inch of water is converted into one cubic foot of steam, by being raised to the boiling point.

As the state of vapour depends essentially on the maintenance of a high temperature, it follows that, on the abstraction of heat, such vapours regain their liquid state, and, in so doing, contract from their extended to a diminished bulk. Thus steam, of a temperature indicated by the thermometer of 212°, becomes reduced to $\frac{1}{1700}$th of its bulk on returning to the condition of a liquid. The following experiment may tend to impress the fact on the mind of the student :—

Experiment 12.—In an air-tight tin vessel, having an open stop-cock, boil water until the vessel is filled with steam. A cylinder twelve inches high and three inches in diameter, made of thin metal, answers well for this purpose. When the vessel is filled with steam by the ebullition of the water, close the stop-cock, and immerse the arrangement in cold water. The steam will at once condense and return to its diminished bulk of water. The atmospheric pressure, acting on all sides, will compress the vessel, and prove that the vapour has lost its bulk by the condensation and abstraction of heat, which has thus been effected.

On these principles the condensing steam-engine is constructed, of which we shall have to speak in the section of Applied Mechanics. A vast variety of natural phenomena also depend on the laws of vaporisation and evaporation. We must defer considering these in their amplitude; for the branch of science, called Meteorology, embraces these effects.

CONDUCTION OF HEAT.

BEFORE treating on Latent Heat, we shall consider the ability which bodies have of conveying the force of heat by what has been termed "conduction."

All substances are divided by philosophers into two classes, so far as Thermotics are concerned; and these are respectively termed "conductors" and "non-conductors." Nevertheless, these distinctions are empiric; because no body can be called absolutely a good or bad conductor, the quality existing only in comparison with others relatively better or worse.

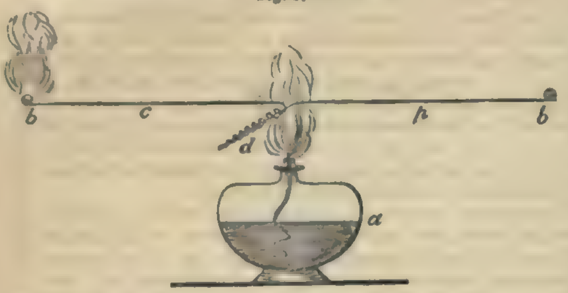
Daily life affords familiar instances wherein the properties of conductors and non-conductors are called into play. In many domestic contrivances, one part is made of a good conductor, and the other portion of a non-conducting substance. The body and handles of an urn, a tea-kettle, teapots, &c., are respectively made of metal and wood: the latter being a bad conductor, prevents the heat passing from the hot metal to the hand. An illustration of the law of conduction may be noticed in the different rate at which heat passes to the hand, from the bright metallic portion of a kettle, and that part covered with charcoal. A kettle filled with boiling water may be safely placed on the hand, if the bottom is well coated with soot, &c.

Another simple mode of illustrating the comparative power of conducting heat, possessed by different substances, is that of placing in the flame of a gas-lamp a copper wire and a rod of glass of equal size. It will be found that the heat of the flame travels rapidly to the hand by means of the metal, whilst no sensation of heat is experienced through the rod of glass. Now, because the metal conveys the force of heat rapidly, it is called a good conductor; whilst the glass, conveying the force but slowly, has been termed a bad conductor.

Even metals, which are generally excellent conductors of heat, vary very much in their power in this respect; as the following experiment will prove :—

Experiment 13.—Twist together a piece of copper and platina wire, and at the extreme free ends of each attach a piece of phosphorus. On applying heat at the point where the two metals are in contact, the heat will travel along each wire until it reaches the phosphorus. In doing so it will travel faster through the copper than through the platina wire—illustrating the fact that copper conducts heat quicker, and is, therefore, a better conductor than platina. The annexed figure illustrates the mode of carrying out the experiment.

Fig. 5.



a, the source of heat, as a spirit-lamp; *bb*, pieces of phosphorus; *c*, the copper wire; *d*, the twisted part where the metals join; *p*, the platina wire.

The wires may be held in the flame by a pair of nippers pressing the twisted part, and each wire should be of the same thickness, and, at the ends, of the same distance from the centre of the flame.

The following table illustrates the comparative conducting power of some of the metals, of which gold stands the highest, and lead the lowest, in the list:—

If Gold conducts heat at the rate of	1000
Silver ,, ,, ,,	973
Copper ,, ,, ,,	898
Platina ,, ,, ,,	381
Iron ,, ,, ,,	374
Zinc ,, ,, ,,	363
Tin ,, ,, ,,	304
Lead ,, ,, ,,	180

It will thus be found, that the conducting power of gold, silver, and copper are nearly alike; as is that of platina, iron, and zinc.

Why one solid body should conduct heat better than another, is by no means evident. It may depend on some peculiar molecular constitution of which we are ignorant. It is plain that solidity or specific gravity is not the sole cause, as, in that case, lead should be a better conductor than iron, &c. We shall have to remark on this subject when we deal with Specific Heat.

Other solids besides metals, which, as we have observed, are generally speaking good conductors, vary greatly in their powers of carrying off heat, and, for that reason, are used in daily life as economical arrangements for preventing its escape. Of these may be named earths of various sorts, charcoal, &c. Whilst gold conducts heat at a rate of 1,000, clay, in the form of brick, has but the one-hundredth part of that power, compared to the metal; hence the employment of fire-brick in the sides of furnaces, and of clay "backs" in ordinary domestic fire-grates, &c. The same principle has lately been employed in constructing fire-proof safes, which are often made of plates enclosing a comparatively non-conducting material, such as charcoal, by means of which heat is prevented from passing through and injuring the contents of such arrangements. Steam-boilers, and the cylinders and pipes of steam-engines, are covered with wood or felt, to prevent the escape of heat, and the condensation of the steam which they contain.

A very general idea of the relative conducting power of various materials, may be obtained by placing the hand consecutively on paper, wood, stone, and a metal. Each of these, although of the same temperature, will seem colder than the other, in the order we have arranged them, because they are successively better conductors of heat, and, therefore, more rapidly abstract caloric from the body. A more accurate mode of ascertaining the differences of their conducting powers, is by introducing thermometers into closed vessels coated with each material, and noticing the time required for water of the same temperature in each, in attaining one of a lower degree. Or if rods of various metals, of equal length, have one end immersed in a liquid of high temperature, and if the other end is coated with wax, the time elapsing between the melting of the wax on each end will give an approximate ratio.

The latter mode is subject to error; because, as different bodies have different capacities for heat, the length of time they require to be heated to identical temperatures will be influenced thereby.

Either method should be so tried as to avoid the effects of absorption of heat from other bodies; and unless each metal is exactly in the same state as to its external surface, error would be occasioned by its power of radiation, &c.

One of the most interesting series of facts in connection with the conducting power of solid bodies, is that which relates to the value of different sorts of clothing employed to retain animal heat; respecting which, instructive experiments may easily be tried by immersing thermometers in closed tin vessels containing water of equal temperature in each, and noticing the length of time required for each vessel, clothed externally with different materials, to come down to one standard.

It is thus found, that the following articles conduct heat in the order we have arranged them; in which the best conductor is first placed, and those of an inferior kind will be found in succession. The number indicates the time, in seconds, required for a heated body to cool to an even temperature, when coated with the substances named.

Comparative conducting power of clothing materials.

Silk twist	81″
Cotton wool	104
Wool unspun	112
Silk from the cocoon	128
Beaver, eider, and hare fur, about	130

It is thus found, that the substances with which animals are generally clothed, are the best adapted for the preservation of vital heat; and, generally speaking, the thickness of the coating is proportioned to the climate in which the animal naturally exists. The reader will not fail to notice, that the animals, birds, &c., found in northern regions, are liberally supplied with hair, wool, &c., such as the polar bear and the wild goose; whilst those existing in tropical climates, as the giraffe, &c., are supplied with but a thin coating of external non-conducting material. Man, whose intelligence enables him to adapt himself to circumstances, takes advantage of various materials for clothing purposes; and hence the employment of linen and cotton garments in the hot regions of the tropics, and the use of woollen and fur garments in colder countries.

Convection of Heat.—Liquids are, generally speaking, very bad conductors of heat, and the process of its propagation through them has been termed *convection*, because the particles move, and thereby *convey* heat with them. This is true both with respect to liquids and aëriform bodies; and we shall therefore consider them together in reference to this property.

If a solid body is heated in any part, the caloric so imparted at once travels in every direction; but that such does not occur in liquids, may at once be perceived on trying the following experiment:—

Experiment 14.—Place a metal dish, containing spirits of wine, on the surface of some water, and immerse a thermometer in the latter, so that its bulb shall rest three inches below the bottom of the metal vessel. On igniting the alcohol, the water immediately touching the copper may be made to boil, but the thermometer will not show any increase of temperature; which proves that heat scarcely passes downwards through the liquid. If heat, however, is applied at the bottom of the vessel holding the water, it will be observed that the temperature will rapidly increase, owing to the *convection* or conveyance of caloric by the particles of the expanded liquid rising to the surface.

In arranging steam or hot-water apparatus for heating purposes, this principle must always be observed. Steam and water-pipes are, therefore, placed at the lower part of a room, because the hot air they produce, being expanded, rises, and conveys with it the caloric from the heating apparatus. Ventilation is generally effected by adaptations of this principle; and, as hot air always rises, an apartment may easily be ventilated by introducing, at a low level, any source of heat. Much has been said and written in respect to the best modes of ventilation; and whilst the necessity of a supply of pure, and the removal of foul air, is universally recognised, the different means of obtaining these results have been the subjects of a large number of patented inventions.

In many of these, the projectors have lost sight of the principles, in obedience to which success can only be insured. In some instances, the supply of fresh, and the removal of foul air, are proposed to be effected by openings at the upper part of a room. Despite of the ingenious modes by which these arrangements are carried out, their success is but partial, and can never go beyond a *dilution* of the vitiated air contained in the room. We know of no experiment which more completely illustrates the true principles which should govern attempts to effect proper ventilation than the following; and a careful study of its facts will be a better guide to success than a volume of precepts :—

Experiment 15.—Place a lighted candle on the floor of a room at the open door; and, if there is no other access for air into the apartment, it will be observed that a current of cold, and therefore dense, air, on entering, will drive the flame of the candle *inwards*. By holding the candle *midway* between the top and bottom of the doorway, the flame will be quite still, because no current of any kind exists in that position. If the candle is held so as that its flame shall be near the *top* of the door, it will be observed to be driven *outwards*, because thence the hot and lighter air makes its escape.

Following these principles, it is evident that a supply of pure air must be made at the lower part of an apartment; and an escape for hot air should be arranged for at the top, or near the ceiling, of a room. Within the last few years, openings, concealed by ornamental work, have been made in most of the rooms of new houses; and, by communicating with the open air, they permit the escape of noxious products, and so ensure, to some extent, an amount of ventilation.

It is impossible here to trace the variety of interfering causes which oppose the application of such principles in general practice; but, in most cases, such may be overcome by attention to the rules we have laid down; and just so far as natural effects are imitated, so success will attend endeavours of this kind.

A very interesting series of effects has been lately brought to light; all of which relate, more or less, to the laws of the conduction of heat by liquids and vapours. We refer to the peculiar state which a liquid acquires when placed in contact with a surface heated considerably beyond its boiling point, and which has, to some extent, explained the reason of boiler explosions in instances wherein no other cause could be assigned.

If cold water is allowed to come in contact with metal heated to 200° or 300°, the liquid is speedily converted into vapour, because the heat from the metal is rapidly communicated to it. If, however, the metal is heated to a temperature approaching red-heat, the same result does not take place—the liquid, in fact, will run off the surface, just as mercury leaves a sheet of glass on being poured thereon. The effect is termed the spheroidal state, and only occurs when a stratum of steam or air exists between a surface and a globule of liquid. Something of an analogous kind occurs when water is cast on to the feathers of a bird; or, as may be often observed, in the case of the dew on leaves; which, running over them, is nevertheless not in contact with them.

If a vessel of copper, &c., is made red-hot, and cold water is poured into it, some time will elapse before the liquid will boil, because a film of steam is formed, which prevents the conduction of heat between the metal and the liquid. If, however, the vessel is allowed to cool, the liquid will flash into steam so soon as a lower temperature is arrived at. It will be seen, therefore, that vapour is a worse conductor of heat than the liquid from which it has been formed.

Now, steam-boiler explosions may, in many cases, be explained on the principle here named. In almost every instance, the water supplied to steam-boilers contains earthy matter, such as lime, &c.; and, in the process of time, this forms a "fur" on the shell of the boiler. The fur being a bad conductor of heat, permits the metal of the boiler to become red-hot, and this often causes the earthy matter to "chip" off, and thus leaves exposed the red-hot metal, apparently in contact with the water. For some time no extraordinary effect results; but if a supply of cold water is sent into the boiler, the metal gradually cools down, and the stratum of steam, which had kept the liquid from touching the metal, being removed, an immense evolution of steam takes place, and an explosion of course at once ensues. An inexperienced person would at once suggest the extinguishing of the fire as the best course in such circumstances; whereas, strange to say, the only safe plan is to increase the heat until all the water can be removed from the boiler by means of a valve, &c. By such a course the intermediate stratum of steam is maintained, and risk of explosion is diminished.

By forming a stratum of vapour by means of sulphurous acid, M. Bontigny was the first to show that water could be frozen in a red-hot platina crucible. As the repetition of this experiment is by no means easy to many persons, we suggest one which may easily be tried, and which fully illustrates the principles to which we here allude. It is that of handling red-hot lead, without risk of injury to the operator.

To do this safely, the hand must be perfectly clean, and must be coated with a "skin" of steam. This is best effected by washing the hand in some strong liquid ammonia, which, by its alkaline action, removes all grease, &c., that would otherwise prevent the moisture from completely wetting its surface. The lead should be red-hot; as, if used at a lower temperature, severe injury might be sustained. The best manner of trying this singular experiment, is to pour the liquid ammonia (liq. ammon. forts. of the shops) into a basin, whilst an assistant presents the lead in a ladle. The hand is first to be carefully washed in the liquid, and may then be introduced into the red-hot metal. Any quantity may thus be cast into a pail of water; and, having repeatedly tried the experiment, we may say, that the only inconvenience which we have sustained is, that the hand becomes gradually cold, and sometimes painfully so, if the experiment is much extended.

Few persons have nerve enough to try this; but there is no danger, providing the precautions we have named are fully attended to. If, however, the lead is not very hot, it may solidify round the fingers, and so produce great inconvenience.

A similar experiment to the above is that of walking on red-hot iron—the feet being prepared in a similar manner; or they may be coated with a skin of glue and sand, which, as a non-conductor, will effectually prevent injury to them.

Daily life presents us with innumerable instances wherein the conduction or non-conduction of heat is taken advantage of. Metallic vessels, because of their good conducting powers, are employed for the purpose of heating liquids for manufacturing and domestic purposes; whilst the interposition of non-conductors—such as the glass or wooden handles of teapots, the cover of felt to the cylinder and boiler of the steam-engine, and numerous similar contrivances, are used for an opposite purpose. In vegetable life, we have the bark of the tree; in animal life, the skin or wool over the flesh—each of which, as a comparatively bad conductor, retains the vital heat of either system. The thick coating of snow which falls during winter, has the effect of preventing the destruction of vegetable life, through being a bad conductor of heat. The cells of the plant, which would otherwise be burst by the expansion of the ice, are thus preserved uninjured, and their delicate organisation maintained till the returning spring. Thus, in this, as in every other instance of natural laws, we find one cause having a universal and beneficial operation in its varied relationships.

Having thus spoken of the effects of sensible heat, we next proceed to a subject of great interest, wherein caloric will be considered in its latent or hidden state.

LATENT HEAT.

CONSIDERABLE difficulty is often found by those not far advanced in scientific study, in understanding the doctrine of *latent* forces; and before proceeding to investigate the theories and facts respecting latent heat, it will therefore be convenient that we should define our meaning of the term.

Those powers or forces which are evident to, and perceived by, the senses, are termed *sensible*, their presence being easily recognised; but in all matter there also exist forces which are not perceptible to the senses, until proper means are employed to call them forth. Thus, no one could predicate, by simple inspection, that a common match contained the forces of heat and light, although it is well known that friction easily educes them.

It is manifest, that caloric, or its proximate cause, must, although unperceived, exist in a body; because, as we can create nothing, we can only call out that which really, although imperceptibly, is present. Now forces thus disguised are called *latent*, because they lie hid, until called forth by special means. We have chosen a familiar illustration of the meaning attached to the term latent; but we must give this a general signification, by stating that all substances contain heat, light, &c., in a hidden state; and this is true even in those least likely to afford them.

Ice would, at first sight, appear to be the most unlikely body from which heat could be obtained; but nothing is more easy than to produce such a result, as the following experiment will prove:—

Experiment 15.—Make a hole in a piece of ice, and, into this, introduce a pea of the metal potassium. Owing to the chemical action which will take place, both heat and light will be rapidly evolved.

From this instructive experiment, we learn that both heat and light must exist in a latent state; and, in this instance, chemical action is employed to render these forces sensible. Our introductory experiments on the production of heat are examples of a similar kind.*

It is owing to heat being present in a latent form, that gases exist; and hence, when gases, vapours, and liquids are reduced to the solid state, they invariably give out heat; and, *vice versâ*, when solids are converted into liquids or gases, they absorb sensible heat, which afterwards becomes latent. On compressing air in a vessel by means of a syringe, the metal of the vessel generally becomes quite hot, owing to the air forced in losing a portion of its latent heat. When the air is allowed to escape, it will be found that the reverse effect is produced, and the vessel parts with heat, and becomes cold to the touch. If a bottle containing soda-water, of the temperature of the air, is suddenly uncorked, and its contents are poured into a vessel containing a thermometer, the abstraction of heat, produced by the expansion of the carbonic acid gas, will be made evident by the mercury of the thermometer at once falling.

The amount of latent heat in various vapours differs materially; and the latent heat of steam exceeds that of any other. This will be evident from the fact, that it requires 5½ times as much heat to convert a given quantity of water into steam, as it does to heat the same quantity of water from freezing to boiling point.

Again, steam, on being reduced to the liquid state, parts with the heat it had previously received, which passes into a latent form; and hence, a pound weight of steam would heat 5½ pounds of cold water to a temperature of 212°. The reader will here perceive another proof of what has been already stated—that gases and vapours owe their condition to the force or agency of heat.

If we examine the effect of mixing water of two different temperatures in equal quantities, we shall find that the resulting temperature will be increased just half of the excess of the highest. Thus the addition of one pound of water, at a temperature of 100°, with one pound at 32°, will give a mean temperature of 66°. If, however, equal weights of ice and hot water are mixed together, the resulting temperature will not be a mean between the two; for it will not exceed 32° until the whole ice is melted. The heat of the hot water thus assumes a latent form, and is employed in liquefying the ice; or, in other words, in imparting sufficient heat to enable it to assume the liquid state. As an illustration of this, we may state that, if a pound of ice at 32° is mixed with a pound of water at 172°, the difference of 140° will be apparently lost in converting the ice into water.

The following table, collected from the investigations of Dr. Ure, represents the latent heat, at the boiling point, of

Water, as equal to	1000°
Alcohol, s. gr. 0.825 . . .	457°
Sulphuric ether	312.9

The abstraction of its latent heat, and the conversion of water into ice, may be easily effected by means of rapid evaporation, by the air-pump, or in an instrument called the Cryophorus, or "frost bearer."

Experiment 16.—Fill a watch-glass with water, and on the surface of the liquid place another watch-glass filled with ether. On introducing these into the receiver of an air-pump, and exhausting the air in the usual manner, the rapid evaporation of the ether will abstract the latent heat of the water, and will convert it into ice. Generally speaking, the two glasses become solidly frozen together.

This peculiar mode of producing cold has been lately employed on a large scale, for the purpose of producing ice in tropical countries. Special arrangements are employed for recovering the ether, which may thus be repeatedly used, and the economy of the process maintained.

Experiment 17.—The Cryophorus is made by blowing a bulb at each end of a long glass tube. One of the bulbs being filled with water, heat is applied thereto, so as to expel all air, and the vessel is then hermetically sealed. For the purpose of this experiment, the whole of the water is retained in one bulb, and the empty one is immersed in a mixture of snow or pounded ice and salt. This reduces it to a temperature of 0° Fahrenheit. In a very short time the water in the other bulb will be converted into ice, owing to the rapid evaporation and loss of latent heat which it sustains.

By the abstraction of latent heat, a great degree of cold may be produced—hence the employment of freezing mixtures, for the purpose of cooling wines, &c. Whilst these assume the liquid state, they abstract the latent heat of bodies immersed in them. A well-known instance of this is the use of salt for the purpose of removing ice and snow from foot-paths. In the employment of these means, the snow, &c., melts, and a great decrease of temperature results. The practice, for this reason, is highly to be reprehended, because it is injurious to health. Persons treading in a mixture of this kind sustain a great loss of natural heat from the feet; and to so great an extent does this take place, that the leather soles of shoes, if thus moistened, will, for the reasons we have named, become the cause of dangerous diseases, such as fever, diphtheria, &c. The following are mixtures which may be employed for the purpose of producing intense cold.

Experiment 18.—Mix together equal weights of sal-ammoniac and nitre, both in fine powder. Place the mixture in a tea-cup, and, having added cold water, stir the whole by means of a thin test-tube holding cold water. In a short time the liquid in the test-tube will be completely frozen, owing to its latent heat being abstracted during the solution of the salts external to it.

Experiment 19.—Mix equal weights of snow or pounded ice and chloride of calcium, in the dry state. If mercury, in a test-tube, is introduced into this mixture, the metal will be frozen, owing to the intense cold produced.

Experiment 20.—Place a small piece of lint or cotton wool on the bulb of a thermometer, and moisten it with a little ether or sulphide of carbon. The evaporation, which will thus take place very rapidly, causes the mercury of the thermometer to fall, sustaining, as it will do, a loss of latent heat.

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* See *ante*, p. 15.

Without multiplying instances of this kind, we may refer to an extended application of the principle in the condenser of the steam-engine, and the ordinary process of distillation. With respect to the mechanical details of the arrangement used for the former purpose, we must defer any detailed description to the section on Mechanical Philosophy, wherein the subject will be fully dealt with. The steam, after exercising its force on one side of the piston, is ejected into the condenser, which is an air-tight vessel, surrounded by a cistern of cold water. A jet of cold water plays into the condenser, and, immediately on the steam entering, its latent heat is abstracted by the cold water, and it suffers condensation, suddenly contracting to a bulk of $\frac{1}{1700}$th part of that which it occupied in the state of vapour. To the researches of Dr. Black and James Watt, this application of the laws of latent heat is chiefly due.

In the process of distillation, heat is applied for the purpose of converting a liquid into the state of vapour, and of thus separating it from others with which it had been previously mixed. The vapour thus formed is again condensed by passing it through a metal worm surrounded with cold water.

In the ordinary process of manufacturing spirits, stills of great size are employed, and they are generally made of copper. The "wash," or mixture, formed by infusing malt, and fermenting it with yeast, is introduced into the body of the still, and, heat being applied, the spirit, which boils at a lower temperature than water, passes over into the worm; and, being there condensed, it reassumes the liquid state. The stills and general arrangement vary in shape, &c.; but a common tea-kettle, to which two or three feet of metal tubing has been attached at its spout, will illustrate the whole principle of the process. The mechanical arrangements alone vary from this simple mode of distilling, when adopted by the spirit distiller.

The latent heat of vapour seems to vary as the temperature is increased; but the differences hitherto ascertained do not warrant any reliance on their accuracy.

A very common application of the laws of latent heat is that of drying and warming by means of steam conveyed through pipes. Owing to the latent heat of steam being so great, it may be made a vehicle to carry heat to a great distance from its source; and it must be borne in mind that steam will heat six times as much space (nearly) as a piece of metal of the same weight and sensible temperature. Hence its employment for manufacturing purposes, especially in bleaching, dyeing, calico-printing, paper-making, tallow-melting, &c. It is usual to cover the pipes conveying the steam with felt, until they arrive at the spot where heat is required, where of course they are left uncovered, so that the heat may freely pass off. Steam is an economical agent for warming apartments, and exceeds in this respect water, when similarly applied; because, whilst the water only gives up an amount of heat equal to the difference between its own temperature and that of the air of the apartment in which it is employed, steam affords also its latent heat, and, therefore, more entirely and conveniently answers the purpose.

On placing the hand over a jet of steam proceeding from a boiler, it will be found that the higher the pressure under which the steam escapes, the lower will its sensible temperature appear. At first sight this seems paradoxical, but the difficulty at once disappears on applying the laws of latent heat to its explanation. Of course, as the steam rushes out of the jet, it must expand. In so doing, it robs the surrounding air of heat, so as to permit of its expansion; and hence, the stronger the pressure of emission, the cooler is the blast. When tried with a pressure of 120 pounds on the square inch, the jet of steam seems to have the same effect on the hand as a jet of cool air. A vacuum is also formed during this expansion, and a ball of wood may be thus kept playing in a jet of steam for any length of time.

The abstraction of latent heat, together with pressure, has been applied by Dr. Faraday and others to the liquefaction of gases. Of all experiments of this kind, the condensation of carbonic acid gas has been the most successful. The gas is manufactured on a large scale, in a strong metallic vessel, and is then allowed to pass into a receiver. On issuing from a jet placed in the side of the receiver, the gas expands so rapidly as to produce intense cold, and undergoes congelation. A very usual course adopted to liquefy some gases, is to enclose the materials from which they are procured in a strong glass tube, bent at an angle at its centre. Each end being hermetically sealed, the gas is received in the liquid state in that end opposite to where it is being produced. By these means, carbonic acid, cyanogen, &c., have been obtained in the liquid state.

If a jet of air issuing from an orifice under a pressure of 60 to 100 pounds on the square inch, is allowed to impinge on a good conductor, its moisture will be immediately frozen. In some deep mines in Germany, where air is pressed by a high column of water for the purposes of a blast, it is no uncommon occurrence to observe snow produced from the moisture of the air issuing from the pipes, or from any crack in the arrangement; and it is the custom of the miners to place their caps against the jet, for the purpose of showing the effect to the visitor.

SPECIFIC HEAT.

It has been already stated, that if equal quantities of water, at different temperatures, be added together, the resulting temperature will be a mean between the extremes. But that result will not occur when liquids of different kinds are similarly mixed. Thus, if mercury and water, at different temperatures and in equal weights, be added to each other, the resulting temperature will be nearer that of the water, presuming that to have been the hottest of the two liquids.

This fact has given rise to the term *Specific Heat*, which means that different bodies have various capacities for heat—some requiring more, and some less, of sensible heat to raise them to an equal or standard temperature. The ratio of these capacities is found by mixing liquids, &c., at different temperatures, and noticing that which results from the mixture; and, in most cases, the specific heat, or capacity of water for heat, is adopted as the standard.

The best illustration of the mode usually adopted, may be found by taking mercury as the type of other bodies; and we shall thus be able to explain the general law which governs all matter with respect to its capacity for heat.

If one pound of mercury, at a temperature of 40°, be mixed with a pound of water at 100°, the resulting temperature of the mixture will not be a mean between the two, but will be found to be 97½°. Thus the heat of the mercury will be raised about 57½°; whilst the water will be reduced in temperature to the extent of 2½°.

Now, putting these figures into the ordinary form of proportion, we have

$$57.5^\circ : 2.5^\circ :: 1 : 0.04348.$$

In which 57.5° represents the number of degrees by which the temperature of the mercury is raised; 2.5° the loss of heat sustained by the water; and 0.04348 is the specific heat of mercury compared to water, as the standard, or 1.00000. The figures here obtained are somewhat too high; but the illustration will give an insight into the means usually employed for the purpose of obtaining the specific heat of bodies. The plan is varied with solids; but the principle is still the same.

The following table gives the specific heat of some bodies, water being the standard:—

Water	1.000
Alcohol	0.660
Iron	0.114
Copper	0.095
Zinc	0.095
Platina	0.032

The student will find considerable difficulty in reconciling the laws of latent and specific heat with those of the undulatory theory (which we entered into at the commencement of this section), more especially if he observe such phrases as "the *quantities* of heat which a body contains;" which are often employed in treatises on this subject. It is true that these are only used to popularise our ideas; but still there will be a tendency to take a material view of the nature of heat whilst experimenting with the force; and this will be, to some extent, confirmed when the effects of specific and latent heat are kept alone in view. Theories, however, are valuable only so far as they generalise numerous facts; and the undulatory theory is insisted upon by philosophers, because it collects under one broad view a chain of allied causes. In its application to the branch of Thermotics, with which we are now dealing, many difficulties arise, which cannot be cleared up until our researches have been so far extended as to give a greater insight into the nature of heat, &c., than we at present possess. When we refer to radiant and polarised heat, the value of the undulatory theory will be at once perceived, because we shall be able to range so many phenomena under one comprehensive idea.

Whatever theory is employed to connect the various facts of Thermotics, it is evident that the actual amount of caloric in any body cannot be measured; and that the specific heat of any substance is merely a term signifying its relative power of absorbing the force as compared with that possessed by other bodies. There is, however, a very singular relation existing between the atomic weight, or combining proportion, and the specific heat of many substances.

In our prefatory remarks, we stated that all matter is composed of atoms; and by examining the chemical relationship of these atoms, we find that they have a tendency to combine in certain proportions only, and every elementary body has accordingly had assigned to it a number which expresses the relative value of its power of combination. As an instance of this, the analysis of water shows that eight parts, by weight, of oxygen combine with one part of hydrogen, and, by their union, nine parts of water are produced. These numbers are called the equivalents, atomic weights, or combining proportion of these gases; and it is with these relative weights that the specific heat of a body seems to have some relation in certain instances. If, for instance, the number 3.1 is divided by the number expressing the specific heat, say of iron, zinc, or copper, we find that the quotient very nearly corresponds with the atomic weights or equivalents of those metals when expressed in a series of which that of hydrogen is unity. This will be observed by the annexed table.

Specific heat of		Calculated Equivalent.	Equivalent obtained by Chemical Analysis.
Iron	0.114	27.2	28.0
Copper	0.095	32.6	31.7
Zinc	0.095	32.6	32.0

These relationships are, however, by no means so evident in other bodies, although, generally speaking, the product of the specific heat and the atomic weight is either nearly a constant sum or one which is a multiple thereof. In this, as in many other connections of different phenomena, we meet with difficulties which prevent our affirming the existence of a law of which we have as yet but a glimpse.

We have hitherto referred solely to the specific heat of solids and liquids; and, with respect to gases, the information which has been obtained is by no means satisfactory. This is chiefly owing to the difficulty which exists in carrying out the investigations in an accurate manner, in any case, with aëriform bodies. This is generally done by passing the gas under examination, at a certain temperature, through water, also of a known temperature; and, in proportion as the heat of the liquid is raised by the same bulk of each gas, their relative capacity for heat is ascertained, and the specific heat arrived at.

RADIATION, ABSORPTION, AND REFLECTION OF HEAT.

Having directed attention to the leading facts which the study of sensible, latent, and specific heat affords, we proceed to consider those phenomena which present themselves when rays of heat are emitted from a central source.

The intensity of all radiant forces is diminished in a geometrical ratio as the centre of force is departed from; and it is found that this intensity is inversely as the square of the distance. The student will, perhaps, more readily understand this by placing before a candle-flame a square piece of cardboard, of any size. If this gives a shadow of one foot square at a distance of a foot from the source of light, then that shadow will cover an area of four square feet at double the distance, nine square at three times the distance, and so forth. The intensity or amount of light will diminish in the same ratio as the space over which it spreads is increased. The law holds good with respect to gravitation,* light, electricity, magnetism, and other central forces. So far as heat is concerned, we shall at present confine our attention to its radiation through air; because, like light, it suffers both refraction and polarisation when it is passed through certain media—subjects on which we shall subsequently dwell.

Bodies vary much in their power of radiating heat; and those having a dark and rough surface hold the highest place in the scale. Pointed surfaces especially fall under this class, whilst polished ones scarcely radiate heat at all.

Absorption is a property by which bodies are enabled to take in heat from any source, and thus have their temperature rapidly raised. Generally speaking, good radiators are also good absorbers, because each effect is due to a similar state of the body under examination. Of this class are lamp-black, cloth, and bodies of a dark colour; and the truth of this will be at once made evident by the following experiments:—

Experiment 21.—Inclose a thermometer in a metal cylinder which has a polished external surface, and introduce one into a vessel which has been blackened by means of lamp-black and glue. Fill these with water, and expose them to the action of the rays of the sun. It will be found that, owing to the blackened surface, the rays of heat will be rapidly absorbed, and the temperature of that vessel will quickly rise. In the one having a polished surface, very little change of temperature will be produced, because the rays of heat are reflected, and therefore not absorbed.

Experiment 22.—If the two vessels are exposed in the open air on a clear night, it will be found that the temperature of the blackened vessel will speedily fall, owing to the loss of heat caused by radiation, whilst that of the polished surface will scarcely be affected.

Hence we find that good radiators are good absorbers, and that bodies having a polished surface reflect heat freely, but neither radiate nor absorb it to any extent. The best radiator, such as lamp-black, will emit eight times as many rays in the same period as a polished surface, such as a silver plate would do under the same circumstances.

A great variety of interesting phenomena in nature depend on these laws of radiation, &c. Dew is formed owing to the radiating power of the earth's surface; and if a thermometer is placed at night on a grass-plot, when the sky is quite free from clouds, the temperature indicated will be much below that shown by a thermometer raised a few feet from the ground. The radiation of heat thus cools down the temperature of the surrounding air, and the moisture, being condensed, falls down and rests on the surface of plants, grass, &c.

The temperature at sea is always higher at night-time than on land, because the level surface of the water is a bad radiator. The peculiar effect of the sea and land breezes is owing to the changes resulting from the radiating and absorbing power of the earth's surface. During the day-time, the ground becomes warmed by the absorption of the rays of heat, and thus an upward current is produced, the sea air rushing in to supply the partial vacuum formed. After sunset, the reverse effect takes place; the earth then commences to radiate heat, and, becoming cooler, the air sets towards the sea, and hence the production of the land-breeze. The following extract from a work by Captain Maury, entitled *The Physical Geography of the Sea* (Jan., 1861), will be read with interest, as affording full confirmation of these applications of natural laws.

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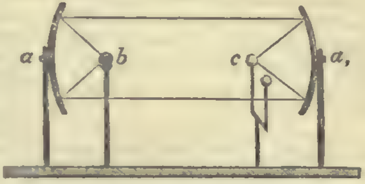
* See Introductory Chapter; p. 3, *ante*.

Speaking of the country near the coast, in hot climates, Captain Maury observes—"The inhabitants of the sea-shore, in tropical countries, wait every morning with impatience the coming of the sea-breeze. It usually sets in about ten o'clock. Then the sultry heat of the oppressive morning is dissipated, and there is a delightful freshness in the air, which seems to give new life to all for their daily labours. About sunset, there is again another calm. The sea-breeze is now done, and in a short time the land-breeze sets in. This alternation of the land and sea breeze—a wind from the sea by day, and from the land by night—is so regular in intertropical countries, that they are looked for by the people with as much confidence as the rising and setting of the sun. In extra-tropical countries, especially those on the polar side of the trade winds, this phenomena is presented only in summer and fall, when the heat of the sun is sufficiently intense to produce the requisite degree of rarefaction over the land. This depends in a measure, also, upon the character of the land upon which the sea-breeze blows; for when the surface is arid, and the soil barren, the heating power of the sun is exerted with most effect. In such cases, the sea-breeze amounts to a gale of wind. In the summer of the southern hemisphere, the sea-breeze is more powerfully developed at Valparaiso, than at any other place to which my services afloat have led me. Here, regularly in the afternoon, at this season, the sea-breeze blows furiously; pebbles are torn up from the walks, and whirled about the streets: people seek shelter; the Almendral is deserted, business interrupted, and all communication from the shipping to the shore is cut off. Suddenly the wind and the sea, as if they had again heard the voice of rebuke, are hushed, and there is a great calm." He also remarks—"Within the tropics, however, the land and sea breezes are more gentle. The oppressive heat of the sun and the climate of the sea-shore is mitigated, and made both refreshing and healthful by the alternation of those winds, which invariably come from the coolest place, the sea, which is cooler by day, and the land, which is cooler by night. About ten in the morning, the heat of the sun has played upon the land with sufficient intensity to raise its temperature above that of the water. A portion of this heat being imparted to the superincumbent air, causes it to rise, when the air, first from the beach, then from the sea, to the distance of several miles, begins to flow in with a most delightful and invigorating freshness."

The phenomena of the radiation and reflection of heat, may be conveniently examined by employing two parabolic metal mirrors, which must be highly polished. They may be from eight to twelve inches in diameter, and should be fixed on stands of exactly equal height, so that a straight line passing from one shall be in a line with both their axes or centres. They may be made of polished tin, or sheet zinc; perhaps the latter is to be preferred, as being more readily beaten into the proper shape. They may be polished by means of rottenstone and oil, and should be finished with tripoli, rubbed on by means of cotton wool. Their focus is found by placing a candle-flame some distance from the reflecting surface; and if a piece of smouldering brown paper is held so that the smoke shall rise between the light and the reflector, the focal point will be at once observed as that place where all the rays passing from the reflector meet together. In the focus of one of the mirrors, the source of heat is placed on a stand; and, in the other, the thermometer which is employed to indicate the reflected heat. Fig. 6 represents a pair of these reflectors, arranged for the purpose of an experiment.

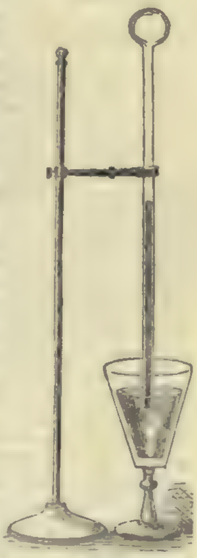
Fig. 6.



a a, represent two metallic reflectors; *b*, the source of radiant heat, as an iron ball; *c*, a differential thermometer. The rays of heat proceeding from *b* are reflected to *a*, and thence to the bulb of the thermometer at *c*, in the course indicated by the lines.

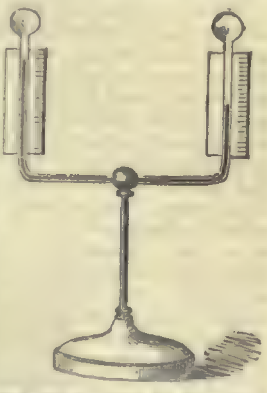
An ordinary mercurial or spirit thermometer may be used for many experiments, its bulb of course being placed in the focus of the mirror. For delicate experiments, however, an air thermometer must be employed. A simple one may be constructed by filling a thermometer tube with any coloured liquid, and then inverting the stem in a vessel of water. When so inverted, sufficient liquid should be allowed to escape, so that the bulb may be full of air, because it is by the expansion of the air that the presence of heat is manifested, and which also drives down the liquid in the stem. One of these thermoscopes, as they are generally called, is represented in Fig. 7.

Fig. 7.



For many experiments on radiated and reflected heat, an instrument, constructed on similar principles, but having two bulbs, is always employed. This is the invention of Sir J. Leslie; and it is exceedingly sensitive in indicating minute changes of temperature. It consists of two bulbs, each containing air, which are joined by means of a glass tube, nearly filled with any coloured liquid. The tube is bent into two arms, one of which is graduated. In using the instrument, one of the bulbs only is brought towards the source of heat, and that should be the graduated one. Its action is due to the expansion of the air in one bulb, and the consequent compression of that in the other; and according to the depression of the fluid in the graduated limb, so the amount of sensible heat is measured. When employed in one of the reflecting mirrors, the bulb attached to the graduated limb is so placed in the focus of the reflector, that the other may be removed to one side; the latter is thus not influenced by the reflected heat. The annexed engraving illustrates the usual form of one of these instruments; and *c*, Fig. 6, shows one in use.

Fig. 8.



The following experiments will give an idea of some of the most interesting facts, both as regards radiating and reflecting substances:—

Experiment 23.—Place in the focus of one mirror an iron ball, previously made red-hot; and in the focus of the other mirror, a thermometer bulb, which should be blackened, may be introduced. The heat reflected from one mirror to the other, and thence to the thermometer, will be evident by the mercury rising. The air thermometer is preferable for these experiments. That it is not the mere radiation of the heat from the ball which produces the effect, is certain, because the mirrors may be removed a considerable distance from each other, and yet the mercury of the thermometer will rise, under the circumstances above-named. By means of two mirrors, each six feet in diameter, and about ninety feet apart, we have seen a piece of meat cooked by the reflected heat of a gas jet.

Experiment 24.—If the bulb of the thermometer is retained, as in the last experiment, and if the mirror before which the ball of iron is placed be coated with lamp-black, it will be found that no heat is then reflected thereby, because the blackened surface absorbs the heat, and is, therefore, incapable of reflecting it.

It will thus be seen how the power of reflecting and absorbing heat may be produced in a body by merely changing the character of its external surface; and, in obedience to these laws, a great variety of instructive experiments may be tried.

By way of parenthesis, we would remark, that the abstract teachings of science are valuable only so far as they give us an insight into natural laws, and assist us to make such applications of them as may conduce to intellectual progress and social benefit. In every case, however, these lessons so taught can only be appreciated and rendered of service in proportion to the intelligence of the student. Having this object in view, we shall suggest a few experiments which, whilst they may still further elucidate the special laws to which we now refer, will also give a wide field for the exercise of the ingenuity of the experimenter, so far as their application to the purposes of daily life are concerned.

Experiment 25.—Instead of the iron ball mentioned in Experiment 23, place a tin vessel, having one side white, another polished, and one blackened, in the focus of one of the mirrors—retaining the thermometer in the focus of the opposite mirror. The vessel is then to be filled with boiling water, and must be closed at the top, so as to prevent the escape of heat. It will be found, on presenting the polished or white side to the mirror in whose focus it stands, that the vessel will scarcely radiate any heat so as to be apparent by the rise of the mercury in the thermometer; whilst the blackened side, if similarly placed, will radiate considerable heat, which, by reflection from the mirror, will immediately raise the temperature as indicated by the thermometer.

The employment of dark-coloured substances for absorbing and radiating heat is thus accounted for; and one well-known instance, where the opposite character of an absorbing and reflecting or non-radiating material is often used in one vessel, is that of the common tea-kettle, whose lower side, in contact with the fire, absorbs heat rapidly when coated with charcoal, whilst the polished surface prevents the escape of heat by its non-radiating nature. For similar reasons, a polished metal teapot will retain heat longer than a dark stone one. If, however, the latter is placed near the fire, this difference will not be so evident, because the absorbing power of the stone vessel will be exercised, and the heat apparently retained, although it really is being continually received, and so counterbalances the radiating power of the material.

Experiment 26.—Place pieces of cloth of different colours on snow, so that the sun's rays may shine directly on them. It will be observed, that the darker the colour of the cloth, the more heat will be absorbed, and the deeper the cloth will sink in the melted snow. Hence, dark-coloured cloths are warmer, as a clothing material, than those of a light colour, because they absorb more of the heating rays of the sun.

Experiment 27.—Place a thin covering of any kind over a shrub, so as to prevent the radiation of its heat during the night-time. On removing it next morning, no dew will be found beneath. The clouds act in precisely the same way, and prevent the formation of dew, by impeding radiation; hence dew is never formed during cloudy weather.

Experiment 28.—Having blackened the bulb of the thermometer employed in the focus of one mirror, place a piece of ice in the focus of the other one, and carefully adjust both, so that they shall be exactly opposite to each other. If this experiment is carefully performed, the latent heat of the mercury of the thermometer will be reflected to the opposite mirror, and will be again reflected to the ice, which it will eventually melt. This is a highly instructive experiment, and illustrates the action of the reflected latent heat of a body when its sensible heat appears to be comparatively low, or at natural temperatures.

The different effects of reflection, radiation, and absorption, are often observed in daily experience. Chalk rocks, for instance, reflect heat; whilst trap and other dark-coloured strata absorb it rapidly. In passing between walls which have been painted black, the radiation of heat, previously absorbed, is very great. Dark soils, for similar reasons, more rapidly change their temperature than those that are light-coloured, and the atmospheric heat thus varies considerably, under such circumstances, over their surfaces. Some very ingenious applications of the laws of radiant heat are made for domestic purposes, amongst which may be named the American oven. One side of this being made of rough metal, absorbs heat; whilst the other, being polished, reflects it: the conjoint action of these makes the contrivance an exceedingly valuable and economical one for baking purposes.

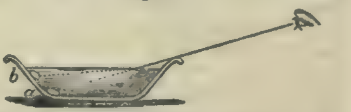
REFRACTION OF HEAT.

In our last section, we presumed that the whole of the heat radiated from a body passes to any object at a distance from it, without sustaining any loss from its absorption by the atmosphere. It is, however, found, that gases, liquids, and solids prevent the passage of a portion of the rays emitted from a heated body; and some substances have the power of bending rays of heat from what would otherwise be their natural course. Those bodies which but slightly prevent the transmission of the rays of heat, have been termed "diathermanous."

As some little difficulty may arise as to the meaning of the term "refraction," we suggest a simple experiment, which will at once illustrate its purport with respect to light; and, as heat is subject to similar laws, the student will easily perceive the nature of the refraction of either force.

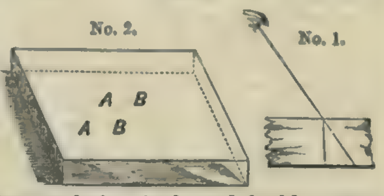
Experiment 29.—Place a shilling in a basin, and retire from it until the edge of the coin is but just visible. If an assistant now fills the basin with water, the whole of the coin will appear in sight, owing to the bending, or refraction, of the rays of light. The effect is illustrated in the annexed engraving, where *a* represents the real place of the coin, and *b* its apparent position.

Fig. 9.



The above is an instance of what is termed *single* refraction. Light and heat also undergo *double* refraction in some circumstances; and the effect with respect to light is easily observed, by placing a rhomb of Iceland spar on a line drawn across a piece of paper. The crystal, having the power of double refraction, will exhibit the line in a double appearance. Fig. 10 represents this effect, where No. 1 is a piece of spar, through which the letters appear double; and No. 2 illustrates the separated rays of light passing through the spar, and thus, also, the nature of the refractive effect.

Fig. 10.

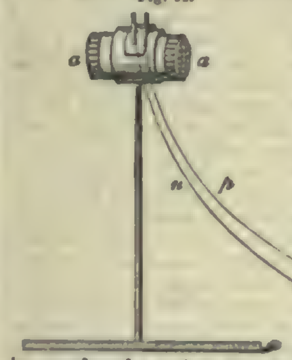


Having thus illustrated the single and double refraction of light, we proceed to examine the analogous changes which the rays of heat undergo when passing through various media.

It is necessary to bear in mind, that heat alone, and heat combined with light, pass through bodies in by no means the same degree. Thus the heat from a gas-light will pass through a plate of glass; whilst that emanating from a metal heated *below* redness, is entirely stopped in its passage. Indeed, it would seem that heat passes through solid transparent bodies just in proportion as it is accompanied by light; for the heat of red-hot coke will scarcely pass through thick glass; whilst that produced by the electric light is transmitted to a large extent. A plate of alum entirely prevents the transmission of non-luminous heat. It has been found, by the researches of Melloni and other philosophers, that rock-salt is the most permeable to rays of heat, allowing more than twice as many to pass through as plate-glass. This substance is therefore chiefly employed in experiments in connection with the refraction of heat. An instrument of a very delicate kind is employed in these experiments; which we shall but briefly describe, because the philosophy of its action will be fully investigated under the head of Electro-Magnetism.

If two metals, such as antimony and bismuth, are arranged together in bars (one of each metal being soldered to the other), and if wires are brought from the last bar of each, so as to traverse over a magnetic needle, a current of electricity will pass if either end of the bundle of bars is heated: and a current thus generated by the force of heat, will, if present to but a slight extent, affect the needle of a galvanometer, which will be instantly diverted from its usual position. This arrangement is much more sensitive of thermal changes than either the mercurial or air thermometer, and thus is invaluable in researches where small disturbances of temperature have to be measured. It is called the thermo-multiplier, and its arrangement is represented in Figs. 11 and 12.

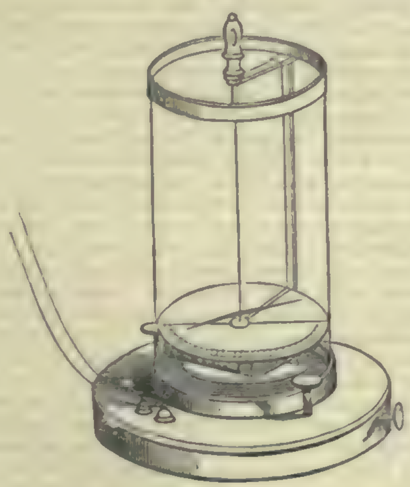
Fig. 11.



To illustrate the construction and use of the instrument, we give three engravings. In Fig. 11, we observe the thermo-electric battery—*a* consisting of bars of antimony and bismuth, from which proceed two wires, *n*, *p*. These are connected with binding screws attached to the galvanometer, represented in our next illustration.

The rays of heat being received at the end of the arrangement of antimony and bismuth bars, a current of electricity is thereby produced; and if this is allowed to pass through a wire coil, alternately wound over and under an ordinary compass-needle, and yet not touching it, the electricity will diverge the magnetic needle just in proportion to the strength of the current; and hence the amount of radiant heat may be measured by the deflections of a galvanometer. One of these is illustrated in Fig. 12, in which we observe a magnetic needle suspended over a coil placed just beneath it. The wires proceeding from the thermo-electric battery, are observed on the left hand of the engraving.

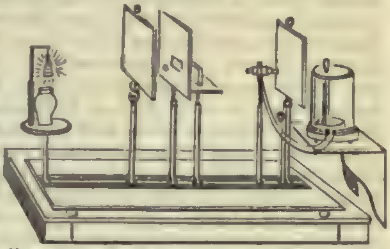
Fig. 12.



When this instrument is employed for the purpose of ascertaining the amount of heat radiating from a body, or in researches on the laws of the refraction and polarisation of heat, the arrangement illustrated in Fig. 13, is adopted; wherein we observe, on the left hand, the source of heat; in the centre, are diaphragms, through which the rays of heat may be passing; and on the right hand side are placed the thermo-electric battery, and the galvanometer, by means of which the intensity of the calorific rays is measured.

As we have already remarked, this instrument is the most sensitive which we can employ for the purposes we have named; and it has been of essential service in all researches having for their object the detection of minute changes of temperature.

Fig. 13.



The mode usually adopted of decomposing light—that of the prism—is well known; and, by similar means, heat may be divided into rays forming a spectrum, which is highly analogous to one of light. A prism of rock-crystal is used for this purpose, instead of one of glass.

A vessel filled with boiling water is the source of heat usually employed. The rays of heat radiated from it are allowed to pass through a small hole in a screen, and, impinging on a prism of rock-salt, they at once undergo refraction, just as light would do by means of the ordinary prism.

The beautiful effects produced by the decomposition of light, are of course visible to the eye; whilst those of heat are not only invisible, but are not perceived by the senses at all. The thermo-multiplier, on being placed in the heat-spectrum, at once evidences the refractive power of the rock-crystal prism, and shows that the heat-giving rays have an unequal refrangibility, just similar to that of the rays of light.

The discovery of this great similarity between heat and light, is of the deepest importance in physical science; and, as we have already stated, it goes to prove the truth of the undulatory theory of those forces. As it completes their analogy in so many instances, the mind is naturally led to infer that heat and light are merely modifications of the same cause.

The polarisation of heat may be effected in a very similar manner to that of light; and the effect takes place when a calorific ray is allowed to pass through a plate of tourmaline or mica. In certain positions of the polariser, heat will pass through, whilst in others the heat is entirely stopped. We shall reserve a fuller description of the result for the section on Polarised Light, because, as that subject has been so widely extended by the researches of modern philosophers, we shall be better able to trace both the facts and analogies of the same properties in caloric.

Heat can not only be refracted by a prism of rock-salt; but if a lens is made of that material, the rays of heat may be collected and focalised, and their effect made evident by receiving them on the thermo-multiplier, although they may be unaccompanied by light. We have here another analogy with light in this singular property. The simplest mode of trying this, is that of placing a lens of rock-salt in front of a vessel containing boiling water, having a screen before it. The end of the battery of the multiplier must then be adjusted to the focal point of the lens. The screen intercepting the rays of heat being removed, the needle of the multiplier will immediately and violently diverge, and thus at once show that the rays have been collected into a focus by the lens.

Experiments of this kind require considerable care and attention on account of their extreme delicacy, and also from the fact that the senses cannot judge of the effects produced, as can be done when light is the force under examination. Much depends on the careful construction of the thermo-multiplier to obtain successful results, and on a proper adjustment of every part of the apparatus employed.

TERRESTRIAL HEAT.

Although the subject of atmospheric temperatures, and the laws which govern their changes, are properly to be considered in detail under the section on Meteorology, it will, however, be convenient that we should enter into the examination of some of the causes and effects of the natural heat of the earth in the present section. The idea has long been held, that the centre of the earth is in a state of intense ignition. The gradual increase of temperature observed in descending deep mines, certainly gives countenance to such an opinion. It is found, that the temperature of a shaft sunk beneath the surface of the ground, increases at the rate of one degree Fahrenheit for a descent of every seventy feet; and, arguing on the assumption that this increase of temperature continues to progress at the same rate, it would follow that, at a depth of ten miles, all substances would be at a red heat, and those melting at a lower temperature than 1000° Fahrenheit would be in a liquid or gaseous condition. The existence of volcanoes, and the boiling fountains or geysers of Iceland, indicates that, at some depth beneath the average surface of the earth, the temperature must be very high.

Some very ingenious speculations have been made as to the cause of the internal heat of the earth. Many have ascribed it to chemical action; and, shortly after the discovery of the metals potassium and sodium, it was suggested that their combustion might, to some extent, account for many of the phenomena observed in volcanic countries. Intense combustion has also been suggested as another cause. Against this idea, however, a difficulty arises, inasmuch as, according to our daily experience, combustion cannot generally occur except in the presence of oxygen. The most feasible mode of explaining the matter seems to be, that at some remote period the earth's surface has been at a much higher temperature than at the present time, but that it has cooled down gradually, by the radiation of heat, into space. It is highly improbable that any definite notion of the cause of this internal heat of the earth will be ever arrived at, because the depth to which we can penetrate in mining and other operations, is very trifling compared with that which would be required to verify speculation, or even to obtain reliable data for calculation.

The application of the laws of the conduction, absorption, and radiation of heat, as applied to the earth's surface, is of the most extended kind. We have already noticed some of the phenomena which arise from the action of those laws. There are, however, two special instances of such in the gulf streams and the trade winds, which deserve further notice, inasmuch as both phenomena depend on the expansive powers of caloric acting on liquids.

There are three parts of the earth's surface where extremes of temperature naturally exist; namely, at the north and south poles, and under the equator. Bearing in mind the laws which we have mentioned in our previous pages, in reference to the expansion, and consequent diminution of density, which all gases undergo when heated, it follows that an ascending current of air should always be found in the equator, which is replaced by the flow of air from the cooler regions of temperate climes, and the still colder polar blast. As the heated air, proceeding at a great altitude, approaches the poles, its temperture is gradually diminished; and, eventually, the hot current is so cooled down, as to become successively the return cold current towards the tropics. It will thus be observed, that by the heat of the sun's rays absorbed, and afterwards radiated by the earth at the tropical regions, an enormous current of air is constantly kept in motion, having a definite direction of its own, and comparatively independent of minor disturbing causes which it may meet with during its progress. There is, however, one cause which modifies the direction of this great calorific current—which is the rotation of the earth on its axis. Now, owing to the levity of the atmosphere, it does not partake so completely of the motion of the earth as it would do if solid. The result is, that the current just spoken of does not take a due north or south direction from either pole, but divides into one passing towards the south-west from the north pole, and another blowing towards the north-west from the south pole. Hence the origin of the trade winds so well known to, and so highly prized by, navigators.

Besides the currents here referred to as the result of the agency of heat, there are an infinity of minor and modifying effects produced by various disturbing causes. The land and sea breeze, before mentioned, owe their origin to partial or local radiation of heat; and the same may be remarked in reference to the mountain storm of wind and rain, where similar causes are always operating on the small scale. The monsoons of India, and the hot winds of the arid Arabian and African deserts, simply result from the effects of the heat absorbed by the sandy plains on the air resting over their surface. A little reflection will at once point out, that the law of the convection of heat by gases, produces results of the utmost importance, and of the most beneficial character throughout nature. Without heat was thus conveyed by means of these expanded air-currents from the tropics, those regions would become a scene of desolation, and of the absence of vegetable and animal life; and were it not that such currents convey the excess of heat from the tropics to colder regions, the latter would lie in one ocean of ice, or in a winter of perpetual frost. The grand arrangement or chain of these laws of heat prevents either catastrophe, and insensibly but surely spreads an airy mantle of fertility and fruitfulness over every part of the surface of our world.

It remains for us to speak of another current which prevails in the waters of the ocean. This is called the gulf stream; and it has a great effect in modifying the annual temperature of the polar regions. An immense body of water, of a temperature of about 84°, proceeds continually from the Gulf of Mexico, in a northerly direction, eventually reaching the shores of Great Britain, where it divides into two streams—one passing northwards, and the other bearing easterly on to the coast of France. The heat of the tropics is thus conveyed towards our own coasts, and hence the prevalence of rain when the wind is in the south-west, because the air is loaded with moisture, which condenses on reaching land. The force of this great current of heated water is such, that plants and fruits, the products of tropical countries, are often cast on the shores of the Western Isles, along the coast of Scotland.

The thermal effect of the gulf stream may be judged of from the fact, that whilst the whole of the American coast, in the same latitude as our own country, is locked in frost during winter, and unbenefited by the genial warmth of this stream, our sea-shores never present the appearance of a mass of ice, through the whole length of the western sea-coast. There is even a less extreme annual range of temperature in the north-west of Scotland, than on the south-east coast of England, although the latter is about 400 miles nearer the equator. The same comparison may be instituted between the climate of Scotland and that of some portions of Denmark, situated in the same latitude.

It has been discovered that there are return currents in the ocean, corresponding to those we referred to as occurring in the atmosphere, and as producing the trade winds. The gulf stream is bounded, and, in fact, rests over such currents; and hence the analogy between it and the trade winds is complete in all respects.

Without trenching on the subjects of physical geography and meteorology, we may state, that there are certain places on the earth's surface which have an equal annual average temperature. This arises from a variety of local and general causes; and the lines drawn on a map expressing the geographical position of such places, are called isothermal lines—a term derived from two Greek words, signifying "equal heat."

The range of temperature in any country varies for the same latitude, according to the altitude of the place of observation above the level of the sea. In our own climate, the limit of perpetual snow is rarely seen, on account of our highest hills being below that point. In the north of Scotland, there is, however, one hill, Ben-Wyvis, on which, as we have been informed, snow may generally be found at all periods of the year. This, perhaps, is partly owing to the winter drifts being so deep as not to allow the high temperature of summer heat to melt them entirely. In more southern climates, the Alps, the Pyrenees, the Andes, and Himalayan range, at all times present a place of continual congelation. In some parts of India and South America, every known climate may be met with, from that of a tropical to one of a polar nature, within a few hours', or, at most, a day or two's journey. Hence the inhabitants of those hot regions of the earth's surface, are enabled to seek and enjoy comparatively temperate climes, and thus to escape the effect of that intense heat which so often proves fatal to European constitutions.

The cause of these temperatures varying with the rise above the sea-level, is easily understood, if we bear in mind the fact, that as a gas expands, its capacity for heat is increased, and that atmospheric rarefaction is always attended by a loss of heat. The reader will remember, that the loss of latent heat in the vacuum of an air-pump is rapidly effected: when ether is thus evaporated from a watch-glass placed in another containing cold water, the latter liquid is thereby easily frozen. Now, at great altitudes, the atmospheric pressure is greatly diminished, and results analogous to those producible in the vacuum of the air-pump are thus presented. Even in a small tract of country, great difference of temperature may be found. These often are owing to rapid evaporation, as in the case of marsh lands, which always maintain a comparatively low temperature. In countries abounding with chalk, such as Kent, the temperature does not vary so rapidly, although in summer-time the heat is highly reflected by chalky soils. Many tracts of land powerfully absorb and radiate heat, and hence their temperature is undergoing incessant change.

We cannot here enter into some most interesting instances of the laws of radiation and absorption, which affect the fertility of the soil. We must not, however, omit to remark, that in these inquiries we continually observe evidences of design, and complete adaptation of means to an end. In winter we have the surface of the earth covered with a mantle of snow, which, by its non-conducting power, prevents the escape of heat, and also prevents radiation. So soon as the snow melts, or the shower of rain falls, the colour of the soil is darkened, and it is thus rendered a better absorber of the genial heat of the sun's rays. As the heating process goes on, the earth dries and becomes of a lighter colour, and is made to radiate less; it is, indeed, thus changed from being a good absorber of heat in its moist state, to be a comparatively bad radiator in its dry condition. By these successive arrangements the soil is enabled to drink in the solar heat, which at the same time so changes the surface as to enable it to store up heat. The seed is thus made to germinate—to grow and to bear fruit—not by a variety of complicated natural causes, but by the application of one simple and unchangeable law.

We might add a vast variety of illustrations which come under our daily experience, in connection with the influences of heat on the vegetable and animal kingdom. As we must confine ourselves, however, in this section to those considerations which strictly belong to Thermotics, we shall reserve our further observations on these and allied matters for our section on Agricultural Chemistry.

HEAT, APPLIED FOR THE PURPOSES OF WARMING AND VENTILATION.

The continual variations of the temperature of the atmosphere in climates situated beyond the tropics, necessitate arrangements for the production of artificial heat; and the different plans adopted for this purpose are but so many accidental or intentional applications of the laws of Thermotics, which we have previously discussed in the earlier pages of this section.

The contrivances of various nations depend very much on the kind of fuel employed as a source of heat. The interior of an Esquimaux snow-hut may be considered as perhaps the rudest, but, at the same time, a very effectual mode of warming, but certainly not of ventilating, a dwelling. Access is generally obtained to the house by means of a long and narrow tunnel, through which the natives creep on the hands and knees. On emerging from the tunnel, a kind of lamp is observed in some part of the snow-house. The fuel employed is common seal oil, which produces plenty of heat and smoke during its combustion. Such lamps serve the double purpose of affording heat and light, and also are employed for that of cooking food. In the arctic regions, wood of course is very scarce; yet nature casts at the feet of these boreal natives the snow, which, as a bad conductor, makes an exceedingly warm house; and the seal, which, like the cocoa-nut of India, serves for food, clothing, fuel, and material for various purposes, to the inhabitants of these inhospitable regions.

Even in the northern parts of our own country, the cooking and warming arrangements of some classes are of the most primitive kind. We have visited many huts in the north Highlands of Scotland, where the only fire-place was a large stone, the chimney a hole in the thatched roof, and the fuel dried sticks, gathered from the neighbouring woods. In many instances, the top of the door of the bothy forms the only means of exit for the smoke of the scanty fire.

The employment of coal as fuel, necessitates that arrangements should be made for an adequate supply of air to support combustion, and also that means should be provided by which the dense and offensive smoke produced may be readily got rid of. Hence the invention of fire-grates and chimneys, having the objects just named in view. It is, however, exceedingly curious to remark, that modern arrangements, such as are employed in our houses both for cooking and heating purposes, are, generally speaking, constructed on principles diametrically opposed to those which sound science would suggest. For instance, the whole of the grate intended to contain the burning coal is made of metal, which, being a good conductor, rapidly abstracts a large portion of heat produced by the combustion of the fuel. The chimney, by means of which the smoke should escape, is often so wide at its mouth as to defeat its object, by drawing up cool air, which thus diminishes the draught; and generally, the grate itself, which is intended to supply warmth to the room, is often "set" so far back, as that a greater proportion of heat escapes up the vent than ever reaches the apartment. It is true that all these objections have been met by the construction of various stoves; but as the use of this kind of heating apparatus rarely finds favour in the majority of cases, our existing arrangement is almost universally the open fire, involving as it does loss of heat, and, therefore, of fuel, besides producing strong draughts of air, which are frequently prejudicial to health.

It is, perhaps, no exaggeration to say, that full one-half of the fuel employed in open fire-places is entirely wasted. In the grates used by the working classes, the loss may even be greater, because such are generally constructed of heavy masses of metal, very different to the more complete contrivances used by their opulent neighbours. Owing to the mass of metal containing the fuel, the heat rapidly passes off on all sides. If, however, the back and sides of the grate were made of some non-conducting material, such as fire-clay, &c., this important cause of waste would be at once removed. Although this can rarely be carried out, still, if plates of clay are so arranged as to completely enclose the burning fuel, a great saving would result. As an illustration of this, we may remark, that a rough fire-place, constructed of bricks, and only having metal in the bars to hold the fuel in front and beneath, will afford more heat for all domestic purposes, at an expenditure of one hundredweight of coal in seven days, than one of the same size, constructed entirely of metal, would give at an expenditure of more than two hundredweight of the same kind of coal, and for the like period. The reason of this is obvious, for the bricks are very imperfect conductors of heat, and, after a time, they become red-hot, and so form, if we may term it, a heat-jacket to the burning fuel, whilst the metal grate has an exactly opposite effect.

Several inventions have been patented and applied for the purpose of diffusing the heat produced from the ordinary fire-grate. Polished reflectors are sometimes placed beneath the fire-bars; and, whilst forming a receptacle for the falling ashes, they at the same time reflect a large amount of heat into the apartment. The top of the fire-place, or the metal arch over and in front of the fire, is often made of polished metal. This, of course, is in direct opposition to the laws of science; because, as this portion of the arrangement is in front of the fire, it can only receive heat by absorption, and throw it off by radiation; and, therefore, it should be of a dark or unpolished surface, which, as we have already explained, forms the best kind of radiator. It is, of course, very difficult to retain that part of the grate forming its sides, above the fire, in a polished state; but just so far as this is managed, so an additional amount of heat is reflected into the room.

It would be impossible for us to enter into detail with respect to the construction of the various kinds of stoves which have been proposed as substitutes for the open fire. In every case, their heating power is obtained at less cost than by means of the ordinary grate, because the whole of the heated surface is placed bodily in the apartment. In most instances, the employment of metal stoves is decidedly prejudicial to health; for not only is the atmosphere dried, and a high temperature induced, but there are certain obscure chemical changes which take place between the atmosphere and the metal (iron), which produce highly injurious effects. These are entirely apart from those results afforded by the products of the combustion of the fuel being allowed to escape into an apartment. Indeed, this very rarely happens in any stove to which a chimney is attached.

With respect to the employment of gas stoves, which have been lately introduced into such general use, it is impossible to speak too strongly in their condemnation. We of course refer to those from which the products of combustion, together with the heated air, are allowed to escape into the apartment. All coal gas contains arsenic, sulphur, ammonia, and other impurities. During the combustion of the gas, these substances are diffused in a gaseous form; and, being inhaled by the lungs, are eventually absorbed into the blood. To speak of the substance most commonly present—arsenic—is sufficient to remind our readers that it is a deadly poison, and one from the use of which we almost instinctively shrink. There is, however, scarcely a room in the kingdom wherein gas is burned, and no means are employed to get rid of its products, that is not more or less a poison propagator; and, by chemical tests, arsenic may easily be detected in its gaseous form, floating in the air. If a small portion of the solid poison is taken into the stomach, we at once seek for remedies with the utmost anxiety; whilst it is much to be feared, that there are thousands of our countrymen, in our warehouses and shops, &c., who are unconsciously and yet daily inhaling its gas, the effects of which, whilst manifested in the decay of health, are really ascribed to other but undue causes.

Similar objections exist to the employment of any source of heat when the products of the combustion are permitted to escape into the apartment, and so to mix with and vitiate the air. The ordinary chauffer in which charcoal is burned, and which is much employed on the continent, has been the cause of numerous fatal accidents. Whenever charcoal undergoes combustion, carbonic acid gas is produced in great quantities. This, if inspired into the lungs, produces speedy death—not perhaps from its poisonous qualities alone, but owing to its containing no free oxygen, which is so essential to animal existence. If a chauffer is used in an apartment wherein abundant means of ventilation do not exist, the carbonic acid gas, or fixed air, as it is sometimes called, at first descends by its weight to the ground, and then gradually accumulates, until, rising in the room to the level of the mouths of persons who may be sleeping, is inhaled into the lungs. It unfortunately produces no painful effects, and some have even described the sensations as really pleasurable; but a heavy sleep is eventually induced, from which the sufferer never again rises alive. It is a matter of great surprise, that, although cases of this kind are continually occurring, the use of these instruments of death largely prevails at the present day. It may, perhaps, be not out of place to name, that a person thus affected with the fumes of carbonic acid gas, may be sometimes restored by removing him immediately into the open air, and by dashing cold water in the face. Generally speaking, the patient should be treated as in cases of drowning, inasmuch as that death occurs in fatal cases from the same cause—asphyxia taking place from the gorging of the lungs with carbonic acid gas. The process lately introduced by Dr. Marshall Hall, for restoring suspended animation (and which has been adopted by the Humane Society of London), is of essential value in such instances.

Having so far noticed the ordinary modes, as adopted in our domestic arrangements, for the purpose of affording heat, we shall next proceed to mention some facts in reference to ventilation; because the two processes—that of producing artificial heat, and of obtaining a supply of pure air—are so intimately connected. Indeed, as we shall presently show, the only means of heating and ventilating the apartments of many modern houses is that of the fire-place, which at best is but a poor and ineffectual contrivance. After having explained some facts in reference to the necessity of ventilation, the products of respiration, &c., we shall treat on the modes of producing warmth by means of steam and hot-water apparatus.

VENTILATION.

We have already stated, that the ventilation of an apartment can only be carried on successfully so long as the laws of expansion by heat are implicitly obeyed. We now proceed to investigate in detail the application of those laws.

Our daily experience teaches us that a constant supply of pure air is essential to our existence; and, in the absence of this, that disease and death will take place. Like many other generally received truths, to which we readily give our assent, this, while believed in abstractedly, is rarely put into full practice; and hence, in most cases, the complete ventilation of our churches, school-rooms, houses, &c., is at present, to a great extent, a desideratum, and often a matter of chance. We have already suggested a very simple experiment,* which will materially assist at arriving at a knowledge of those general laws on which the principles of ventilation depend. We have also observed, that hot air, being light, ascends in a room, and should escape at its upper part; whilst cold air, being denser, remains, till heated, at the lower part, and must also be admitted there. However, at ordinary temperatures, and in a room in which no artificial heat is being produced, a double current of air always exists, provided that openings, such as an open door or window, are there. If any persons are present, then the heat of their bodies and of their expired breath tends to increase the flow of this current, and so to assist the ventilation of the apartment.

In ordinary circumstances, however, the doors and windows of houses are kept closed, for the purpose of

* See *ante*, p. 23—*Experiment* 15.

preventing the escape of heat; and the opening of the fire-grate into the chimney becomes almost the only means by which the foul and heated air can escape. As, however, this opening rarely extends to one-half the height of the apartment, it follows, as a matter of course, that any ventilation which is effected is of the most imperfect nature. The causes of the vitiation of air in any room are so various, and so dependent in many instances on accidental circumstances, as almost to defy enumeration. We may, however, refer to some which are found active in the majority of cases—such as the products of breathing, and of the combustion of gas, oil, and candles.

The constitution of the atmosphere is such as to afford a gas, called oxygen, which, by combining with the charcoal of the food during respiration, is expired in the form of carbonic acid gas on the expiration of the breath. In fact, the process carried on in the lungs of man is exactly identical, in its chemical character, to that which we observe in the combustion of the coal in our furnaces or domestic fire-places. The food we partake of is converted into a peculiar substance, called chyle; and this being absorbed by the vessels of the stomach, becomes converted into blood. The blood thus produced is conveyed, by means of vessels, to the surface of the lungs; and the charcoal it contains is gradually consumed, and its latent heat set free. The charcoal is thus converted into carbonic acid, which is composed of sixteen parts of oxygen to six parts of charcoal, in every twenty-two parts of gas produced. It follows, therefore, that we are continually casting a poison into the air at each expiration of the breath. Besides, having robbed the atmosphere of a portion of its vital part, we also set free at the same time its nitrogen, which forms four-fifths of its bulk, and which is a gas that will not support life. On an average, about twenty pints of air thus vitiated is expired in each minute from the lungs. Exhalations are also continually passing from the skin of the human body, and to an extent far greater than is generally known. Taking these well-ascertained facts into consideration, it is a matter of no surprise that the air of an ordinary-sized room becomes "close" and unpleasant, when even only a few persons may be living in it. Careful chemical tests discover the fact, that our bedrooms invariably contain a larger amount of carbonic acid in the morning than on our retiring to rest; and if three or four per cent. of this gas exists in the air of a room, it is sure to prove harmful, not to say dangerous, to the inhabitants.

Another cause of the vitiated state of the air in confined apartments, is the combustion of various articles employed for the purpose of giving light. Coal gas, animal and vegetable oils, parafin, and tallow, are all composed of the same substances, united in different proportions—namely, hydrogen and charcoal. These, with oxygen, produce, during combustion, carbonic acid and water. It must also be kept in mind, that a minute portion of coal gas, and some of the more volatile parts of oils, always escape unburnt, and hence increase the vitiation of the air beyond that produced by their complete combustion. The annexed table we have calculated from data supplied by Dr. Frankland; and it shows the amount of carbonic acid produced per hour, by quantities of each material sufficient to produce a light equal to that of two sperm candles of six to the pound.

Tallow candles	1.00	cubic foot.
Wax ,,	.83	,,
Sperm oil	.64	,,
Coal gas (London)	.50	,,

From which it appears that tallow candles give off one cubic foot per hour of carbonic acid gas, and ordinary coal gas half a cubic foot per hour, when these are employed to light a room to only a very moderate extent—in fact, affording a light but little better than that produced by burning two "composite" candles. As, however, when gas is used, a much greater light is always maintained, the production of carbonic acid may be reckoned as equal to four times that indicated in the above table, when two burners of the ordinary size are employed in our sitting-rooms.

The products of gas combustion are not only prejudicial to animal life, but are also injurious to furniture, books, &c., on account of the production of sulphuric acid, or oil of vitriol, by the sulphur in the gas uniting with the oxygen of the air. The effect of this acid is found in the complete destruction of the leather of bound books, and of most animal and vegetable substances with which it comes in contact. The colour of many articles, such as curtains and window blinds, is also injured by it; and hence an absence of ventilation is at once a source of both disease and expense.

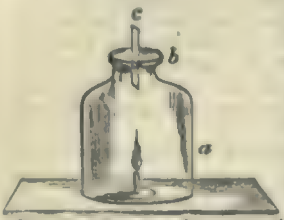
Our remarks have been confined to instances where a want of good ventilation can be chiefly ascribed to the ignorance, and not the want of means of those who suffer from it. When, however, we descend in the social scale, it is scarcely possible to exaggerate the physical and even moral evils which result from inhaling and living in an atmosphere of foul air. Those who have made it their business to visit the houses of the majority of our working classes, have no difficulty in discovering the causes which produce fevers of the typhoid class, and the consequent fearful mortality which ensues. It is by no means an uncommon case to find four or six persons cooking, eating, and sleeping in one room, of very contracted size, from which foul air can scarcely escape, and into which pure air rarely flows, except in the smallest quantity. For the purpose of economising fuel, these apartments are always kept closely shut up. In towns where gas is generally laid on even to the poorest houses, such as Glasgow, Edinburgh, &c., the room is often kept warm by the gaslight, and another source of impurity is thus continually in active operation. The expenditure incurred, and the time necessarily employed in washing, is often a barrier to cleanliness, especially in manufacturing districts, where every member of the family is ordinarily engaged at the factory. Disease of the lungs, phthisis, &c., are natural results of the sudden changes of temperature which are undergone by persons thus living in confined apartments, who are necessarily exposed to the cool air, as they pass to and from the place of their daily avocations. In a moral point of view, the results are equally lamentable. Want of pure air, and a continual breathing of impure air, induce a languor of the system, which in most cases is relieved by spirituous drink. Add to this the contrast of the handsomely fitted and often well-ventilated public-house to the hovel the man has escaped from, and we at once find a solution of the social enigma of drunkenness, poverty, and destitution.

Having stated some of the causes which tend to render the air of an apartment impure, we proceed to consider some of the different plans which may be adopted, or have been proposed, for the purpose of remedying the evil.

The simplest mode of ventilating a sitting or bedroom is that of leaving a space between the bottom of the door and the doorway, for the admission of fresh air, and of having an opening at the top of a room communicating with the outside of the house, by which the foul air may escape. The latter may easily be effected by the removal of a brick, and by covering the opening by means of an iron grid, or a piece of perforated zinc. A still better plan is that of having a large opening in the centre of the ceiling, communicating with the open air. This may be concealed by means of an ornament of any kind. Many objections have been made to admitting air at the lower part of a room, owing to the strong draughts which are thus produced. A very ingenious arrangement has lately been patented, whose principle will easily be understood on trying the following experiment:—

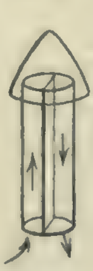
Experiment 30.—Place a glass bell-jar, as represented in Fig. 14, at (*a*), so that its lower end may rest evenly on any flat surface, and thus prevent any air entering. Introduce a lighted candle into it, and, after a short time, the candle will be extinguished for want of air, although the top of the jar (*b*) may be open. Relight and replace the candle, but now divide the neck of the glass jar by inserting into it a card (*c*) so as to form two sections or openings. It will now be found that a current of air enters the vessel on one side of the card, whilst another leaves it on the other side. These currents will be better observed, by holding a little smouldering brown paper near the neck of the vessel, when smoke will be seen to enter, and to leave by the opposite openings. With such an arrangement, the candle may be made to burn any length of time; because a continual supply of cold air is admitted, and the carbonic acid gas is removed.

Fig. 14.



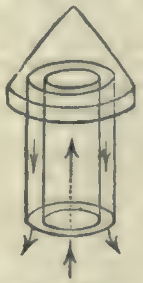
By dividing a pipe, running from any opening in a ceiling, into two parts, in a similar manner as is done by the card in the neck of the glass jar, or by inserting within each other two cylinders, separated by a distance of a few inches, and placing them in an orifice at the top of a room, a system of ventilation may be tolerably carried out. This principle was adopted by Mr. James Kite, about twenty years ago, for the purpose of ventilating ships' cabins. His arrangement consisted of an iron pipe, divided into two parts by means of a slip of metal. Mr. McKinnell, of Glasgow, has improved on this, and has been very successful in his attempts at ventilating many public buildings. The annexed engravings illustrate both of these inventions.

Fig. 15.



Kite's Ventilator.

Fig 16.



McKinnell's Ventilator.

The arrows show the direction of the currents of air.

In this, and similar modes of ventilation, an entire change of the air of the apartment is never produced; because the supply of fresh, and the escape of foul air, are never carried on to sufficient extent for that end. Dilution alone is effected, so that a considerable amount of impurity necessarily remains.

A very ingenious, but complicated plan, was adopted some years ago, by Dr. Reid, in the ventilation of the houses of parliament. Arrangements having been constructed for warming the air when necessary, it was allowed to enter through an immense number of small holes in the floor of the house; the upward draught being produced by a tall ventilator, fixed on the roof. This, whilst highly scientific in plan, was unfortunately not successful in practice. Of course every one present was necessarily in a continuous draught of air; and the result of this, in most instances, is injurious to health, and productive of cold, &c. Few are aware of the difference of being exposed to either an extreme high or low temperature in still air, and in a current of air of the same temperature. In the latter case, rapid evaporation takes place, and consequent cold is produced. Thus, in the Arctic regions, the most intense cold may easily be sustained, provided no breeze is blowing; but the same would be productive of the greatest inconvenience, and possibly of death, during a strong wind. Modifications of Dr. Reid's plan are now often adopted in many large buildings, such as churches, concert-rooms, &c. Air is admitted through openings or grids in the floor; and a number of concealed openings are made in the roof, through which the foul air makes its escape. The burning of gas materially assists this process of ventilation, by causing a continual upward current of hot air. The introduction of "sunlights," which are always placed in the ceilings of rooms, also aids in promoting ventilation.

We may here inquire into the cause of those downward cold draughts which frequently produce so much unpleasantness in churches, &c.; and which, from the ignorance of those in charge of the ventilating arrangements, are often ascribed to wrong causes. In nearly every instance, these downward draughts are chiefly found at the sides of a building; and in the case of churches, they are mostly observed near the walls and windows; and despite all attempts to make the latter air-tight, still these draughts will be felt. Their cause is easily explained. The hot air, rising in the centre of the building, impinges on the roof, and escapes, if suitable openings are made for that purpose. At the same time, the walls and windows being of a lower temperature than the air, cool it, and thus the air descends to the floor of the church, being denser than the central rising volume. If, however, the glass of the windows is covered by means of calico, blinds of any sort, or a thick coat of paint, its conducting power is lessened and the downward draught speedily diminished. Hence the value of double windows, which, by enclosing a body of air between them, prevent the cooling effect which we here speak of.

It is much more difficult to overcome the cooling effect of the walls, because their extent is too great. In many concert-rooms, this difficulty is considerably modified, owing to their being frequently occupied; whereas churches are rarely warmed more than once in a week, and never attain a genial temperature until the daily services are completed, and the requirement has ceased.

A glass window has a power of cooling the warm air of a church, twenty times greater than that of the walls; whilst a double glass window reduces this immense waste of heat to about a fifth of that caused by a single one.

Some years ago, the writer was consulted in reference to this subject, by the trustees of a church in the neighbourhood of London. Having placed a number of thermometers in different parts of the edifice, whilst the usual heating apparatus was in full operation, the cooling effect of the glass windows was at once evidenced by the lower degree of temperature registered beneath them. The officials, however, could not be persuaded that the conducting power of the glass was the cause of the obnoxious draughts, until it was made evident to their senses by filling the building with smoke from brown paper. The currents of air were then easily distinguished, and the cause was at once seen. The remedy applied was thick calico blinds, so arranged as to lie close to the glass, and the evil was thereby greatly lessened.

The employment of steam and hot water for heating purposes, at all times affords a ready means of ventilation. In summer-time, considerable difficulty is experienced in obtaining an upward draught for ventilating purposes, because the external air is often at the same, or even of a higher temperature, than that inside the building. In some instances, large furnaces have been erected on the roof. Through these, the air is drawn by the draught of the burning fuel; and thus an upward current may always be ensured. There is, however, great danger from fire, and also the expense and trouble of carrying up the fuel—objections which strongly militate against the adoption of plans of this kind.

In winter, when hot air is allowed to enter into the lower part of a large room, an ascending current is easily obtained; and thus, if proper means for its escape are furnished, ventilation may easily be effected.

Omitting to mention the use of fixed stoves on a level with the floor of any large room, &c., there are three different means at present adopted for the purpose of warming and ventilation. In each of these, the source of heat is placed beneath the audience, and warm air is admitted from it, and diffused by openings in the floor. Hot air is sometimes produced by allowing external air to pass over iron or brick surfaces enclosing a furnace. This plan is objectionable, because of the extreme dryness of the air thus produced; and also on account of smoke and other volatile products frequently escaping and mixing with the current, to the annoyance of all who have to breathe it.

Hot water, conveyed in iron pipes, is generally adopted, at the present day, as a source of artificial heat in most of our public buildings; and many points of considerable scientific interest are involved in the construction and working of the arrangement. In Mr. Perkins' apparatus, tubes, of an inch in diameter, are formed into a coil, varying in size according to the extent of cubic dimension to be heated. The pipe, rising from the surface of the coil, is extended generally over the flooring of the building, and returns to the coil at its lower part. The coil itself is fixed in a furnace of any kind. The water in the coil, on becoming heated, rises to the upper tube, and traverses the pipes throughout the edifice, so long as its temperature is higher than that of the surrounding air. Having been cooled, the expansion which it had, when heated, is lost, and becoming denser, it returns to the coil, again to pursue the same course on receiving a fresh accession of heat. The philosophy of the process is easily understood, when we bear in mind, that the hot water being light, naturally ascends; on cooling, however, it contracts, and descends to the coil.

The bore of the pipe in Mr. Perkins' arrangement rarely exceeds half an inch. Numerous modifications of this plan have been lately adopted. Cast-iron pipes, of three or four inches in the bore, and peculiarly shaped boilers, are substituted for the small-bore pipes and coil, at the present time; and the use of these arrangements is generally spoken of as effective, and requiring but little attention. By the employment of pipes of large dimensions, such as those just described, the circulation of the hot water is rapidly effected, in a manner which could not be obtained in the smaller kind. Indeed, we have frequently seen the small-pipe coils red-hot, and have observed a temperature of 400° F., at a distance of fifty feet from the furnace, in the pipe conveying the upward current of water. In the boiler and large-pipe arrangement, the temperature rarely exceeds, if it reaches, 212° F.

There is a great advantage in the employment of hot water over that of steam, for heating public buildings—in the freedom from risk of explosion. If a hot-water apparatus should actually burst, not the least danger could possibly arise; the water, in fact, would merely flow out: thus the apparatus may at all times be left to itself, and only requires that the furnace should be supplied with fuel. In the use of steam (to which we shall refer immediately), great care is required that a proper supply of water should be kept up; that the safety-valves should be in proper condition; and other details requiring intelligence and care are involved. The expansion of the water in the hot-water apparatus is always provided for, by inserting in some part of the piping an upright tube, through which the water is first introduced, on erecting the apparatus. A space containing air is left inside this, in which the water gradually rises as it is heated, and expands.

It would be beyond the plan of our work, were we to enter into those practical details involved in applying this apparatus, so far as questions of the size, &c., required for special cases is concerned. This is a matter which must be left to the engineer, and is one which depends on various local and incidental circumstances, which can only be determined as they arise. Our object has been to present the philosophy of warming and ventilation in such a manner, that the principles may be understood and carried into effect by any workman of ordinary intelligence, on the exercise of whose skill and judgment success must of course depend.

STEAM, AND ITS APPLICATIONS.

The process of converting water into steam is so familiar to all, as to require no description. The physical changes which the liquid undergoes are, however, of the highest importance, so far as the laws of its change of state are concerned.

Evaporation takes place at nearly all observed temperatures. If a piece of ice be placed in the receiver of an air-pump, and a watch-glass, filled with strong sulphuric acid, be placed near it, the ice will gradually evaporate, provided that a good vacuum is maintained. Vapour produced at even low temperatures, exerts a pressure on a column of mercury contained in a barometer tube; and as an illustration of this, the following experiment may be tried:—

Experiment 31.—Pour a little ether into a tube of glass, sealed at one end, and about thirty-two inches long. When the tube is completely moistened by the liquid, fill it with mercury. If no vapour existed in the tube, the mercury would stand as high as in the barometer, if the glass tube were inverted into a cistern of mercury, and would act as a barometer at the moment this experiment is tried. But owing to the ether being converted into vapour, and so partially occupying the upper portion of the glass, the mercury will not stand at a height of thirty inches, or thereabouts, but be depressed nearly ten inches below that point, owing to the elasticity of the vapour of the ether. If water be employed, the depression of the mercury will be very slight at low temperatures—perhaps not exceeding half an inch; because it is less volatile than ether. It will be thus perceived, that liquids not only continually evaporate at low temperatures, but that their vapour exerts a pressure on the barometric column proportional to their ready evaporation, or easy conversion into a vaporous state.

It has been already explained, that the atmosphere presses with a force of about fifteen pounds on every square inch of surface at the level of the sea, in ordinary circumstances. The vapour of water presses with an equal force when of a temperature of 212°, the barometer indicating a height of thirty inches of mercury. The annexed table gives the pressure which aqueous vapour exerts at different temperatures, calculated in parts of an atmosphere of fifteen pounds; the column of mercury which the vapour would sustain; and the pressure relative to that of the atmosphere.

Temperature.	Pressure in Atmospheres.	Inches of Mercury supported.	Pressure in Pounds per Square Inch.
60° F.	0.017	0.5	0.25
120	0.120	3.6	1.75
180	0.500	15.0	7.50
212	1.000	30.0	15.00
250	2.000	60.0	30.00
275	3.000	90.0	45.00
290	4.000	120.0	60.00
305	5.000	150.0	75.00
320	6.000	180.0	90.00

It will be thus observed, that the pressure of steam increases in proportion as the temperature is raised; but when the temperature exceeds that of the boiling point of water, the tension of the vapour rises more rapidly, for the same number of degrees, than between lower temperatures.

It is generally reckoned that one cubic inch of water, at ordinary temperatures, is expanded into 1,690 cubic inches, on being converted into steam at a temperature of 212° Fahrenheit; and so long as that temperature is maintained, the steam exerts a mechanical effect equal to fifteen pounds pressure on every square inch of any surface with which it is in contact.

If, however, steam be confined in a vessel—such, for instance, as an ordinary steam boiler—and the production of vapour is carried on continuously, high-pressure steam is produced, which exerts a greater pressure than that afforded in an open vessel. Its density is thus increased, the steam being compressed into a small bulk, in proportion as the pressure augments; and when the steam thus formed exerts a force exceeding that of the atmosphere by fifteen pounds, the vapour has been compressed by the pressure into one-half of the space it would occupy if allowed to escape freely into the open air. The bulk of the vapour is thus inversely as its density. In practice, 100 to 150 pounds per square inch is the maximum pressure used in steam boilers; and these pressures are chiefly confined to the locomotive engines employed on railways.

Besides being employed in the high-pressure engine, steam of a density exceeding that of the atmosphere, has been extensively used in what is called the expansive engine. As the mechanical details of the steam-engine will be fully entered into hereafter, we confine ourselves to stating, that in engines of this class, the steam, after being supplied to the cylinder of the engine, is allowed to expand till the end of the stroke is arrived at; and thus its pressure is gradually diminished, whilst its bulk is as gradually increased. In other forms of this class of engine, two cylinders, of different sizes, are provided, into the smaller of which, the high-pressure steam is first introduced. As soon as the stroke of the piston is completed in this cylinder, the steam is allowed to escape into one of larger size; and there exercising its expansive power, a further mechanical effect is gained; and thus the same *weight* of steam is made to do double duty whilst expanding to its normal bulk at 212° F. This employment of steam is highly economical where water can be obtained for condensing purposes. If very high-pressure steam be employed, as in locomotives, it can be similarly expanded, and is then allowed to escape at a lower pressure in the usual manner, instead of being condensed, as in the low-pressure engines. Great economy in the consumption of fuel is thus effected; and the application of this plan has materially lessened the cost of motive power on railways. In most marine engines, steam of comparatively low pressure is employed, varying from ten to twenty pounds on the square inch, above the atmospheric pressure; and after the steam has exerted its mechanical effect on one side of the piston, it is allowed to escape into the condenser, where, by coming in contact with cold water, it regains a liquid state, by the abstraction of its latent heat.

In an earlier part of this section, we have already directed attention to the doctrine of latent heat; we shall, however, inquire more fully into that law as involved in the production and condensation of steam.

If water be kept boiling in an open vessel, a thermometer immersed in the liquid will not indicate any rise of temperature beyond the boiling point of 212° F. If another thermometer be immersed in the steam which passes off, it will be found to register the same temperature. It therefore follows, that the steam, after attaining a temperature of 212°, must carry off all the caloric which passes into the liquid; and this must pass off in a latent state, because it cannot be appreciated, by any instrument, in a sensible form. The attempt to ascertain the amount of latent heat contained in steam, has received the attention of some of the most able experimenters, who have arrived at various results. It is, however, generally considered, that 1000° of heat, according to the scale of Fahrenheit, are absorbed, and become latent, during the formation of steam. As an illustration of the mode of conducting such investigations, the following experiment is suggested:—

Experiment 32.—Allow steam to pass through a pewter worm contained in a vessel holding a known weight of cold water—say at a temperature of 60° F. As soon as much condensed water, equal to an eighth part of the bulk of the cold water, has been received from the worm of the condenser, ascertain the temperature of the water surrounding the worm. It will have risen to about 188°. Now, as one part of water, in the form of steam, has heated eight parts of water from a temperature of 60° to 188°, or a difference of 128°, it follows that the latent heat of steam is not less than eight times 128°, or 1024°; but as there is a difference between the sensible heat of the water surrounding the condenser, and that of the steam which has passed through it, of 212 — 188, which is equal to 24°, this must be deducted; and hence a carefully conducted experiment gives 1000° as the latent heat of steam at 212° F.

In the condensing steam-engine, cold water is employed for the purpose of absorbing, or taking away the latent heat of the waste steam. As, however, it is necessary that the temperature added to the water should be as little as possible, so as to keep the vacuum perfect, a much larger proportion of water must be employed than we named in Experiment 32, because eight parts were there heated to a temperature of 188°. As, therefore, the temperature of the condenser is never allowed to exceed 110° if possible, about twenty times as much water is employed as the weight of steam which has to be condensed.

It will not be required that we should here enter into the question of economy in the employment of superheated steam, because we shall have to investigate that subject when treating on the steam-engine.

We shall next proceed to speak of the use of steam for heating and warming purposes; and in this we have an interesting instance of the application of the laws of latent heat.

By Experiment 32, we found that steam at 212° will raise the temperature of eight times its weight of water, at ordinary temperatures, by 128°. And here we have a most valuable means of conveying heat to any point we wish, without the necessity of erecting numerous furnaces, or other heating apparatus. Indeed, if steam-pipes are carefully covered with some non-conducting material, such as felt or wood, the steam may be conveyed to very great distances without undergoing condensation. As an instance of this, the whole of the steam-engines at the Exhibition in Hyde Park, in 1851, were worked by steam thus conveyed from boilers in a distant shed; and the same plan is now adopted at the Crystal Palace, at Sydenham. In most of the large bleach and printing-works in the north of England and in Scotland, the whole of the liquids employed are heated by means of steam conveyed into them through pipes, or by steam being admitted into an external vessel, surrounding that which has to be raised in temperature. In many cases, the waste steam passing from a high-pressure engine, is made to raise the temperature of the water supplied to the boiler; and this plan greatly economises the consumption of fuel.

In our remarks on warming and ventilation, we deferred describing the use of steam, from the idea that such would be most conveniently dealt with under this head. For such purposes, an ordinary steam boiler is used, from which iron pipes are carried. These are carefully coated with a non-conductor, until they arrive at the place where the heat of the steam is required. By the radiation of the heat of the steam, at a temperature of 200°, from the iron pipe, a room sixteen feet square by twelve feet high, and of ordinary construction, can be well warmed and ventilated by means of a steam-pipe, twenty feet long and four inches in diameter, or by any other form of metal casing exposing an external surface of twenty square feet. According to this calculation, which was made by Dr. Arnott, it follows, that in rooms or large buildings having the usual amount of window and wall surface, every square foot of steam piping, at the temperature of about 200°, will warm, to an agreeable extent, a space of 150 cubic feet, allowing for imperfect fittings, and for the admission, for the purpose of ventilation, of air, which is also heated during its passage into the apartment. This estimate supposes the external air to be at a temperature below freezing point. Of course, when the external temperature is higher, the same heating surface will have a greater warming effect on the air in the apartment. As we have already explained, the amount of window surface materially affects the success of any heating arrangement.

We have hinted at various objections which apply to the use of steam in churches, theatres, &c.: these, however, do not exist in manufactories where engineers are always employed. Hence, steam is much used in such places as a drying agent, and for various objects which our limited space will not permit us to detail.

TABLES OF THE EFFECTS OF HEAT ON BODIES.

HAVING endeavoured to lay before the reader an exposition of the laws of heat, and their general application, we conclude our section on this subject by supplying various tables, skeletons of which have already appeared in some of the previous pages.

Whilst every endeavour has been made to afford accurate statements for general reference, we must confess, that no branch of science presents so many difficulties when the tabular results of researches are arranged together. Nearly every experimenter differs as to numerical values from those arrived at by his fellow-labourers. And although the science of Heat has occupied the attention of many profound students, still it cannot boast of so great a number as the sister branch of Light;—hence their discrepancies become more glaring. Any person may soon satisfy himself that our remark has foundation in fact, by referring, for instance, to a few tables of the melting point of solids: and it is more than probable, that each work on the subject will present entirely different tabular statements of the required temperature.

Where considerable difference existed, we have supplied the results obtained from various sources, and have mostly omitted the names of their authors for the sake of preventing confusion. We may, however, here state, that we are indebted to the researches of Dalton, Ure, Delaroche, Regnault, Thomson, Daniell, Melloni, and other leading writers on the subject; of whose labours we have freely availed ourselves in constructing the following tables.

Linear Expansion of Solids.

Showing the length of a bar which has been heated from 32° to 212°.

Length of bar at 32° = 1.00000

Glass tube	1.000828
„ „	1.000861
„ (plate)	1.000890
„ (crown)	1.000897
Platina	1.000884
„	1.000991
Antimony	1.001083
Iron (cast)	1.001111
Steel	1.001189
Iron (soft)	1.001220
Iron wire	1.001440
Gold	1.001460
„ annealed	1.001513
„ unannealed	1.001551
Copper	1.001722
„	1.001918
Brass	1.001855
„	1.001918
Silver	1.001909
„	1.002082
Tin	1.002173
Zinc	1.002942
hammered	1.003011

Conducting Power.

The following table gives the conducting power of various bodies, gold being the standard:—

Gold	1000
Silver	973
Copper	898.2
Platina	381
Iron	374.3
Zinc	363
Tin	304
Lead	179.6
Marble	23.6
Porcelain	12.2
Fire-brick	11.4

Reflecting Power.

We cannot give a table whose indications of the relative reflecting power of various bodies can be depended on. The following, however, indicates the order in which the substances named stand in reference to each other; the first being considered the best reflector of rays of heat incident on its surface.

Polished gold.
„ silver.
„ brass.
Looking-glass.
Blackened glass.
„ metal.

Radiating Power of—

Lamp-black	100
Writing-paper	98
Rosin	96
Crown glass	90
China ink	88
Ice	85
Plumbago	75
Lead, oxidised	45
„ clean	19
Iron, polished	15
Tin plate	12
Gold and silver	12

Freezing Mixtures.

Mixture.	Parts.	Thermometer sinks.
Sal ammoniac	5	from + 50° to + 10°
Nitre	5	
Water	16	
Sulphate of soda	6	„ 50° to — 14°
Nitrate of ammonia	5	
Dilute nitric acid	4	
Snow	2	„ 50° to — 5°
Common salt	1	
Snow	3	„ 32° to — 23°
Cold dilute sulphuric acid	2	
Snow	2	„ 32° to — 50°
Chloride of calcium	3	

Table of Latent Heat.

It will be observed that the following numbers vary much from each other. They have been obtained from experiments by Ure, Despretz, Regnault, Andrews, and Southern:—

	Boiling Point.		Latent Heat.
Water	212	Ure	1000
„	229	Southern	942
„	212	Regnault	966
Alcohol	175	Ure	457
„	172	Andrews	364.3
Ether	104	Despretz	312.8
„	95	Andrews	162.8
Bisulphide of carbon	114.8	„	156.0

Table of Specific Heat.

Water being the standard = 1.000.

Water	1.000
Alcohol	0.660
Ether	0.517
„	0.520
Sulphur	0.202
Phosphorus	0.188
Iron, wrought	0.113
„ „	0.114
Cobalt	0.107
Gold	0.030
„	0.032
Platina	0.032
Silver	0.057
Copper	0.095
Mercury	0.033
Lead	0.031
„	0.029

Elasticity of Vapour of Water—

In inches of mercury, supported in the barometric column, from a temperature of 32° Fahrenheit; from tables by Drs. Dalton and Ure:—

Temperature.		Inches of Mercury.
32°	Both	0.200
40	Ure	0.250
40	Dalton	0.263
50	„	0.375
50	Ure	0.360
60	„	0.516
60	Dalton	0.524
70	„	0.721
70	Ure	0.726

Elasticity of Vapour of Water—(continued).

Temperature.		Inches of Mercury.
80°	Ure	1.01
80	Dalton	1.00
90	Both	1.36
100	,,	1.86
110	Dalton	2.53
120	,,	3.33
130	,,	4.34
130	Ure	4.36
140	,,	5.77
150	,,	7.53
150	Dalton	7.42
160	,,	9.46
160	Ure	9.60
170	,,	12.05
170	Dalton	12.13
180	Both	15.15
190	,,	19.00
200	,,	23.60
212	,,	30.00

Boiling Point of various Liquids.

Sulphuric ether, of an average specific gravity		96° F.
Alcohol, of an average specific gravity		174
Water		212
Saturated solution of common salt		218
Nitric acid		242
Oil of turpentine		560
Sulphuric acid		580
,, ,, of lower specific gravity		550
Linseed oil		600
Mercury	Dalton	660
,,	Irvine	672

Melting Points of various Substances.

Sulphuric Ether		— 47° F.
Mercury		— 38
Vinegar, and Milk		+ 28
Ice		32
Phosphorus		100
Potassium		136
Alloy of Cadmium, 3 parts ,, ,, Tin 4 ,, ,, ,, Lead 8 ,, ,, ,, Bismuth 15 ,,		140
Sodium		194
Alloy of Tin 3 parts ,, ,, Lead 5 ,, ,, ,, Bismuth 8 ,,		210
Sulphur		226
Tin		442
Bismuth		476
,,		493
Lead		594
,,		630
Zinc		700
Brass (with 25 per cent. of zinc)		1750
Brass (ordinary)		1869
Silver		1280
,,		1873
Copper	Daniell	1996
,,		2548
Gold	Daniell	2016
,,		2590
Cast Iron		1786
,, ,,		3479

Platina, and metals associated with it, are infusible except under the flame of the oxy-hydrogen blow-pipe.

ADDENDA ON RADIANT HEAT.

WHILST the previous pages were passing through the press, some highly interesting observations and experiments, by Dr. Tyndall, F.R.S., of the Royal Institution, have been communicated to the scientific public; a summary of which we append to this section.

Referring the reader to our remarks on Radiant and Terrestrial Heat,* he will find, that we explained, under those heads, the general laws of Radiant Heat; and also alluded to some of their applications in reference to many natural phenomena. We found that various substances had different powers of absorbing and radiating caloric. In another portion of our pages,† we stated, that some gases, liquids, and solids, prevent the passage of calorific rays more than others; and, also, that heat, radiating from any source, undergoes at times both refraction and polarisation. Our remarks also extended to the "diathermancy," or general power of various bodies, in permitting the transmission of calorific rays through their substance at unequal speed.

Dr. Tyndall has extended our knowledge of these various facts, more particularly in reference to the radiating power of the earth, and the absorbing or preventing influence which the atmosphere, resting on its surface, exerts. By numerous experiments on vapours, with respect to the amount of rays of heat of which they permit transmission, we learn from him, that non-luminous rays pass most completely through dry air; whilst some gases and vapours entirely stop their progress, such as olefiant gas and the vapour of ether.

Carbonic acid is found to a limited extent in our atmosphere; and Dr. Tyndall states, that this, in its gaseous form, permits of the rapid transmission of radiant and non-luminous heat, and the consequent cooling of the radiating surface of the earth. An antagonism to this result is found not only in the dense clouds so common in our climate, but also in the action of aqueous vapour, which is always present in the air in temperate climes, and prevents the radiation of non-luminous heat.

Viewing these facts in their practical results, we cannot help being impressed with a deep sense of their value. Under a clear sky, and in dry air, as observed so frequently in the Arctic regions, intense cold continually prevails, and, at the same time, the atmospheric temperature is such as not to allow of the presence of aqueous vapour. As, however, we approach the tropics, the average temperature of the air increases, and with it the amount of aqueous vapour existing in it. Now this, as we have already mentioned, has been pointed out by Dr. Tyndall as acting after the manner of an invisible screen, and preventing the great loss of terrestrial heat which would occur through its absence, in the ordinary process of terrestrial radiation. The general application to Meteorology, of these principles, has yet to be made; but we can easily perceive, that the production or modification of an immense variety of atmospheric phenomena may result from the action of these laws.

Having thus given a general idea of the facts which Dr. Tyndall has developed, we quote from the *Saturday Review*, of March 2nd, 1861, a more detailed account of his experiments, as the best *résumé* to which we have had access; and which will afford our readers considerable information on the subject.

"With his admirable thermo-electric apparatus, Melloni was unable to detect any trace either of the absorption or radiation of heat by air; and we possess no knowledge regarding the action of other gases or of vapours. In the Bakerian lecture, recently delivered before the Royal Society, by Professor Tyndall, the following new results in connexion with this subject were enunciated.

"Oxygen, hydrogen, nitrogen, and air, enclosed in a tube four feet long, absorb respectively about three-tenths per cent. of the calorific rays emanating from a source of 212° Fahrenheit. This is the feeblest action which has been observed. At the other end of the scale stands olefiant gas, which absorbs more than eighty per cent. of the same rays. This gas, which is perfectly transparent to rays of light, is opaque to the rays of obscure heat. Streaming into the atmosphere, it is not seen by the eye; but if a sheaf of calorific rays be caused to cross it, its presence is at once declared by a proper instrument; it intercepts such rays almost as effectually as a screen of cloth drawn across their path. Coal gas acts in a similar manner.

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* See *ante*, pp. 26, 30.

† See *ante*, p. 23; "Refraction."

"When a volume of olefiant gas, say one-thirtieth of that necessary to fill it, is sent into an exhausted tube, through which calorific rays are passing, it destroys so many of these rays, that a second equal volume does not produce anything like the same effect. In fact, the rays are not there to be destroyed by the second discharge of gas. But when small volleys of gas are used, the number of rays shot down is exactly proportional to the number of shots. For example—when a measure possessing a capacity of one-fiftieth of a cubic inch was used, five such measures destroyed exactly five times as many rays as a single measure; ten such measures destroyed ten times as much; fifteen such measures destroyed fifteen times as much; but here the rays became sensibly thinned in numbers, so that twenty measures did not produce twenty times the effect of a single measure. As the quantity of gas introduced into the tube augmented, the quantity of rays destroyed deviated more and more from the law of proportionality; until, finally, the addition of a single measure ceased to produce any appreciable effect. At this point, in fact, the vulnerable rays had been practically disposed of. Transparent gases have been hitherto considered as almost perfectly diathermanous; but this opinion can no longer be retained in presence of the fact, that a measure of olefiant gas, one-fiftieth of an inch in volume, diffused through a tube possessing a capacity of 220 cubic inches, and there possessing a tension of only $\frac{1}{11000}$ of an atmosphere, exercises a measurable absorption.

Between atmospheric air and olefiant gas, the other compound gases occupy intermediate positions. In the case of vapours, the most energetic is that of sulphuric ether; the least energetic is that of bisulphide of carbon. Comparing small volumes at equal tensions, the absorptive energy of sulphuric ether is ten times that of olefiant gas, and ten thousand times that of oxygen, hydrogen, nitrogen, or atmospheric air. This vapour, in fact, which, tested by light, is as transparent as the diamond, tested by calorific rays of low intensity, is almost as black as pitch. Between sulphuric ether and bisulphide of carbon, stand chloroform, wood-spirit, benzol, alcohol, the iodides and chlorides of ethyl, methyl, and amyl, and other vapours.

"The influence of the aqueous vapour in the atmosphere may be judged of from the following fact. On a fair November day, the quantity of transparent vapour suspended in the air produced fifteen times the absorption of the true air itself. Philosophers have long speculated on the influence exerted by a diathermanous envelope, like our atmosphere on the climates of a planet. D. Saussure, M. Pouillet, and Mr. Hopkins, have shown the effect of such an envelope in preserving the terrestrial rays from being wasted in space. But, in reality, the atmosphere itself exerts a vanishing influence compared with that of the aqueous vapour it contains. Here we have a constituent, whose changes must produce corresponding alterations of temperature at the surface of the earth. Whether it is competent to account for the variations of climate revealed by geologists, future investigations must decide; but it is a *vera causa* which can never henceforth be overlooked in speculations on the subject.

"It is known that oxygen, obtained from the decomposition of water by an electric current, contains a trace of that singular substance ozone. Such oxygen exerted four times the absorptive energy of the same substance, after it had been passed through the iodide of potassium, and thus freed from its ozone.

"The *radiative* power of gases was examined by causing them to pass over a sphere of heated metal, and to ascend in a column in front of a thermo-electric pile. It was here found that the order of radiation was precisely that of absorption. The theory of heat now universally accepted is, that it is a kind of motion of the atoms of bodies, and *radiant* heat is the propagation of this motion through an ethereal fluid in which all bodies are immersed. A popular exposition of this theory is given by Professor Tyndall, in his *Glaciers of the Alps;* and his recent experiments prove, that every atom, or molecule, which is competent to accept motion from agitated ether—or, in other words, to absorb heat—is competent, in precisely the same degree, to impart motion to still ether, or, in other words, to radiate heat. His experiments have one peculiarity which preceding ones did not possess. They deal with *free* atoms, and thus fix upon them individually the responsibility of the effects observed. In solids and liquids the atoms form joint-stock companies; considerations of cohesion and aggregation come into play, from which the molecules, in a state of gas or vapour, are free. Hence the experiments are calculated to throw light upon the physical characters of the atoms themselves.

"The lecturer pointed out, that the reciprocity of absorption and radiation is a necessary mechanical consequence of the theory of an ether. But the question here arises—'Why is one molecule competent to stop or to generate a calorific flux so much more powerfully than another? What is it, for example, which enables olefiant gas to transcend so enormously in these respects its constituent hydrogen?' To this question the experiments prompt the following reply:—The *elementary* gases which have been examined, all exhibit extremely feeble powers of absorption and radiation as compared with the *compound* ones. Now, in the former case, we have oscillating atoms, but in the latter case oscillating systems of atoms; and if we unite the atomic theory of Dalton with the perfectly independent conception of an ether, it follows that the compound molecule, or group of atoms, which furnishes *points d'appui* to the ethereal fluid, must be capable of accepting and generating motion to a far greater degree than a single atom, which we may figure to our minds as an oscillating sphere. Thus oxygen and hydrogen, which, taken separately or mixed *mechanically*, produce a scarcely sensible effect—when united *chemically* to form oscillating systems, as in aqueous vapour, produce a powerful effect. Thus, also, nitrogen and hydrogen, which, separate or mixed, produce but little action—when *combined* to form ammonia, produce a great action. So, also, nitrogen and oxygen, which, as air, are feeble absorbers and radiators—when *combined*, as in nitrous oxide, are very powerful in both capacities. Comparing small volumes at equal tensions, the action of nitrous oxide is fully 250 times that of air; and there is, perhaps, no fact on record which shows so conclusively that air is a mixture, and not a compound, as this one. Carbonic oxide is about 100 times as powerful as its constituent oxygen; carbonic acid, 150 times as powerful; while olefiant gas, as already stated, is 1,000 times as powerful as its constituent hydrogen. In the case of the hydro-carbon vapours, where the atomic groups attain a higher degree of complexity, the action is still greater than that of olefiant gas. The lecturer also referred to the influence of the rate of atomic oscillation on the amount of heat absorbed. But this subject is difficult to be rendered popular, and is, therefore, omitted here. The discourse concluded by applying considerations similar to the foregoing to the conduction of heat. It was shown how radiation, absorption, and conduction were connected together, and that the phenomena of all of them might be deduced from simple mechanical principles."

CONCLUDING REMARKS.

In the foregoing pages, we have endeavoured to present to the reader, the leading facts and principles involved in that branch of science which treats on heat, as one of the forces of nature. We have found, that whether caloric be considered either as a material substance, or as resulting from the vibrations of an ether, it has certain definite effects on matter. These, whilst characteristic of the action of an individual force, are yet analogous to those of light, as we shall see in our next section.

We preferred to call the attention of the student to caloric, as the first in our arrangement of the undulatory forces; because the principles we have enunciated, call for less mental exertion to comprehend, than those of the forces which we have now to consider. We must, however, presume, that what we have already stated has been fully understood; and that we shall only have to refer incidentally, in future, to those laws we have attempted to unfold. This will be necessary, as tending to save space, time, and useless repetitions.

It may, however, be desirable, that we should point out those special analogies which exist between the laws of heat and light, to which we shall have to give prominence in our future progress. To this end, the student should pay particular attention to the facts connected with the *production*, *conduction*, *radiation*, *absorption*, *reflection*, *refraction*, and *polarisation* of caloric; because we must go over exactly the same ground under the sections of Light and Sound, and show the nature of those phases of force under similar but non-identical circumstances. It is also advisable that a clear knowledge of the doctrine of undulations should have been acquired; and this is more easily arrived at whilst studying the effects of heat, than those of other forces; because such are highly palpable to the senses, and familiar to us in the occurrences of daily life.

In conducting the experiments which we shall adduce in the next section, in support of the principles of optical science, much delicacy of manipulation is frequently required. This, we are aware, is not to be obtained except by long practice. Still, the experiments we have already offered, will serve as an introduction to those which follow; and a frequent repetition of the past, will much assist our progress in the future. Clever experimenters often achieve more success in the study of natural philosophy than deep thinkers; and the latter class often owe the greater part of the success which attends their speculations, to the ready and accurate manipulation of their more expert, but not less valuable, co-workers in the pursuit of scientific research.

There are remote analogies existing between the *latency* of heat and light, so far as their separate or mutual production from matter, by mechanical or other agency, is concerned. The student will find examples of this in our remarks and experiments on the production of heat;* a re-perusal of which may assist him materially in understanding similar phenomena, belonging more exclusively to luminous effects.

Under the head of *double refraction* and the *polarisation* of light, we shall have an opportunity of considerably enlarging on similar effects to which caloric is subject; to which allusion has already been made.† In the course of the work, we shall continually make the remarks in one section to relate to those of the preceding one; conceiving that such a course has the twofold advantage, of impressing the truths of science on the minds of our readers, and also of maintaining that intimate and interesting connection which exists in every department of experimental philosophy.

The subject of Terrestrial Heat is of great interest, in relation to that of Physical Geography and Geology; to each of which we shall hereafter devote an entire section. Dr. Tyndall's discoveries—a *résumé* of which precedes these remarks—may be studied advantageously in connection with it; and if our readers should wish to pursue the subject more fully, we would recommend the perusal of Captain Maury's late work, *The Physical Geography of the Sea;* from which we have made a quotation.‡

Those who may be desirous of turning our remarks on Ventilation, Warming, &c., to practical purposes, will find these subjects ably discussed in works by Dr. Arnott, Mr. Hood, Mr. Tomlinson, and others. We have only attempted to explain general principles, and their immediate application, because our limits would not permit of lengthened details: we, however, hope that the suggestions and observations we have made, will tend to excite the interest of many, who may have both the means and opportunity of applying them to the physical advantage of their less favoured dependents and neighbours. We may especially urge such considerations on individuals connected with establishments wherein large numbers of persons are congregated, such as hospitals, workhouses, schools, &c.

To many, it would seem desirable that we should have included a popular description of the steam-engine; but as we intend, in another part of this work, to devote a section to that interesting subject, any further exposition of its construction, &c., would have been out of place. We shall also have to extend our remarks on the laws of vapour and gases, when treating on Pneumatics. The various forms of land, marine, and locomotive engines, will naturally fall to be considered under the head of Mechanical Philosophy.

Having thus endeavoured to assist the student in carrying out a connected system of self-instruction, we proceed, in the next section, to discuss the doctrines of Optical Science, or that branch of philosophical inquiry in which the nature, laws, and applications of Light are investigated. We shall also deal with the construction and management of Optical Instruments, Photography, and other allied subjects.

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* See *ante*, pp. 14 and 24.

† *Ante*, p. 28, *et seq.*

‡ *Ante*, p. 27

SECTION II.

LIGHT, AND ITS APPLICATIONS.

CHAPTER I.

OPTICS, AND OPTICAL INSTRUMENTS.

The phenomena which depend on the agency of light, have been ranged under the branch of science called Optics; and we shall now have to consider this most interesting and fascinating subject.

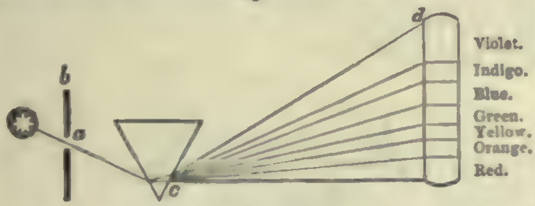
Our Introductory Chapter on the Undulatory Forces, and the remarks which we made thereon in our section on heat, will have, to a considerable extent, prepared the reader for a more enlarged view of their laws and applications in respect to light. We are, indeed, enabled to measure both the number and extent of luminous undulations with great accuracy; and thus shall have the satisfaction of dealing with substantial truth, instead of vague generalisations.

The chief source of light is the sun; and, as we have already remarked, the solar luminous rays are always accompanied by calorific rays, which can, by certain means, be separated from each other. Light, again, like heat, is found in a latent state in all bodies; and by mechanical, electrical, and chemical means, can easily be brought out so as to be perceived by the senses.

The light afforded us by the sun, is called white light; but, by a very simple arrangement, it can be decomposed, and resolved into numerous rays of various colours. For this purpose a prism is employed, which is a piece of glass, of a triangular shape; and on a ray of white light being allowed to pass through it, the component rays are easily separated from each other, and may be received on a sheet of paper, for complete examination.

The annexed figure illustrates the most convenient mode of carrying out this experiment.

Fig. 1.



A ray of sun-light (*a*) is allowed to pass into a darkened room, through a hole in a shutter (*b*), and to impinge on a glass prism (*c*). The light, on emerging from the opposite side of the prism, will be divided into numerous rays, having at least seven different colours; namely, red, orange, yellow, green, blue, indigo, and violet. They are observed in the order in which we have arranged them; the red being always the lowest in the series, and the violet at the upper end. An image of coloured light so formed, is termed a spectrum (*d*); and each of the rays varies, both in respect to its luminous and calorific effect, as we shall perceive in our further experiments.

We have stated, that seven different coloured rays may be thus produced; but, by careful examination, it will be observed, that four of the colours are produced by the overlapping of the other three. Hence, perhaps, in strict truth, we only discover *three* primary colours in a ray of light; namely, blue, yellow, and red. Thus, green is at once formed by combining yellow and blue; the orange ray, by uniting red and yellow; the violet is a combination of red and blue; and the indigo is produced by an intensification of the blue rays.

In explaining the phenomena of the prismatic spectrum, we can happily call in the doctrine of undulations to our aid; and we find that the extreme violet rays are produced by a greater number of undulations taking place at that part of the spectrum, than at the place where the red rays are evident. The subjoined table gives the length of these undulations for each ray of the spectrum.

Colour of Rays.	Length of the Waves in decimal parts of an Inch.	Number of Undulations in an Inch.	Number of Undulations per Second in Billions.
Extreme Red	0.0000266	37640	458
Red	0.0000256	39180	477
Intermediate	0.0000246	40720	495
Orange	0.0000240	41610	506
Intermediate	0.0000235	42510	517
Yellow	0.0000227	44000	535
Intermediate	0.0000219	45600	555
Green	0.0000211	47460	577
Intermediate	0.0000203	49320	600
Blue	0.0000196	51110	622
Intermediate	0.0000189	52910	644
Indigo	0.0000185	54070	658
Intermediate	0.0000181	55240	672
Violet	0.0000174	57490	699
Extreme Violet	0.0000167	59750	727

It will at once be perceived, that a great difference exists in the number of undulations which produce the different coloured rays of the spectrum; and we should here remark, that the luminous rays in which certain chemical phenomena of light are produced, are those of the violet end; whilst those in which the heating effects are most observed, are placed in the red terminal of the spectrum. This is a point of great importance, as we shall observe when we have to enter into the examination of the chemical effects of light, under the head of Photography.

If any coloured substance be placed in any of the prismatic rays, it will immediately assume the colour of that ray; and hence we perceive, that the cause of the colour of bodies is due to the absorption of all the rays of the spectrum but that which they reflect from their surface.

Whilst by means of a prism we can decompose white light, and produce its colours, an experiment may be tried by which the colours can be reunited, and white light at once be reproduced. For this purpose, another prism is to be placed on the one by which light has been decomposed, so that the two may form together a square. The ray of light will pass through without suffering much change. Indeed, the two prisms thus become equal to a solid piece of glass. A slight amount of colour may be visible; and this is owing to the refraction of light occurring when it passes through a dense from a rarer medium—a phenomenon to which we shall have to refer at length in our future pages.

Another mode of illustrating the recomposition of white light, is by painting on a circular card the different prismatic colours, in divisions extending from its centre. If the card be made to rotate rapidly on its axis, a kind of greyish-white colour is produced. This experiment is often employed at the lecture-table, for the purpose of illustrating the composition of white light.

We have already spoken of the speed with which the rays of light traverse space; the experiment, however, which we have just mentioned, is another illustration of the rapidity with which light travels. The first experiments which were made with respect to the speed of light, were conducted by means of a wheel filled with spokes, and which was made to rotate rapidly on its axis. Of course, in ordinary circumstances, all the spokes seem to be lost to sight when the wheel goes rapidly round. If, however, a flash of electric light is made to illuminate them, at the instant the light is produced, each spoke at once becomes visible, because they pass through space in a much longer period of time than does the light which has fallen on them. In passing through a railway tunnel, a red-hot cinder produces a similar optical effect. Previous to the production of its light, the divisions of the sides or bricks of the tunnel would be undistinguishable; but at the instant of the fall of the cinder, every part is immediately observed. Now these effects are due partly to the rapid flight of light, and also to the fact, that the eye cannot appreciate or separate rays of light whose interval of appearance is less than the eighth of a second of time. Hence all the rays passing from the sides of the tunnel seem to be merged together. When, however, only one powerful ray reaches the eye, it at once produces an impression on the retina; and each surface from which it has been reflected becomes instantly visible. This fact may be easily verified by turning round rapidly a piece of red-hot wood, by means of a string, in a dark room. A ring of light will thereby be formed. If, however, an assistant discharges a highly charged leyden jar, so that the light of the spark may fall on the rotating wood, it will at that instant appear stationary, for reasons we have just assigned.

Light, like all radial or central forces, diminishes in its intensity from the source of emission, at a rate equal to the square of the distance. We refer to our remarks on this subject,[*] in which we fully entered into this law, both in reference to heat and light.

Having thus pointed out some general facts respecting the nature of light, we now proceed to investigate some of those laws to which it is subject in its passage from the surface, or through the mass of various bodies, and the changes which it thereby undergoes.

All natural substances may be divided into three classes, so far as the passage of light through them is concerned. Those which prevent the passage of light, or stop the undulations of luminous ether, are called opaque. Some permit freely of its passage, such as glass, crystal, &c.; and these are termed transparent substances. Others only permit the partial transmission of light, as in the case of ground glass, milk, and other substances; and these have accordingly been termed translucent.

When light passes through a body, it undergoes various changes, such as single refraction, double refraction, and polarisation. When a ray of light falls on some substances, such as polished metals, it is reflected; but if it falls at certain angles, it may also be polarised, as we shall more fully point out when we deal with polarised light.

We shall treat on the phenomena of reflection, refraction, and polarisation, under separate heads; because each subject has so many matters of both scientific and general interest attached to it. In our progress, we shall include special applications of the various laws we have to examine, including the use and construction of various optical instruments. We shall also add some remarks on the eye and its structure, which will illustrate the application of many of the laws of the reflection and refraction of light.

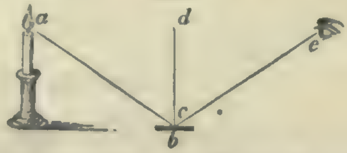
THE REFLECTION OF LIGHT, OR CATOPTRICS.

In our chapter on heat, we stated that that force, as well as light, could be reflected from a polished surface; and this may be of any form; but the shape of the reflective surface has various effects on a ray of either force. The term Catoptrics has been applied to those investigations which are connected with the laws of the reflection of light; and to this subject we now direct attention.

If a ray of light fall on a plane or level surface, such as a piece of glass or polished metal, it is reflected at an angle which has a definite relation to that by which it arrived at the reflecting surface. The angle by which the ray arrives at the reflector, is called the angle of *incidence;* and that by which it leaves the reflecting surface, is termed the angle of *reflection*.

These terms will be better understood by a reference to the annexed diagram.

Fig. 2.

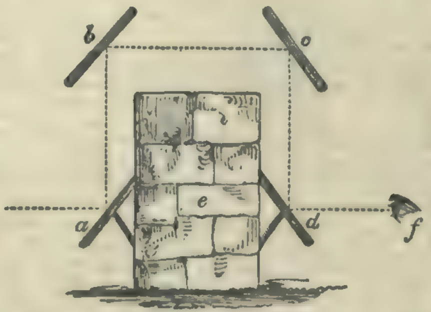


In Fig. 2, *a* being the source of light from which a ray passes to *b* (the reflecting surface), then the angle *c*, *a*, *d*, is called the angle of incidence; *c*, *e*, being the line by which the reflected ray travels to the eye of the observer, then the angle *c*, *e*, *d*, will be the angle of reflection.

Now the law of reflection from plane surfaces is, "that the angle of incidence and the angle of reflection are equal to each other;" so that by being acquainted with either of these angles, the other is known also.

Some interesting applications of this law are often employed in daily life. By means of properly arranged plane reflectors, light may be conveyed to remote places; and thus one source may be made available for a great variety of purposes. Rooms remote from daylight are often illuminated by such means. Coal and other mines have been lighted by reflectors, so arranged, that one large source of light—as, for instance, the electric light—may be made to illuminate distant passages; and thus all danger from the ignition of firedamp is entirely avoided. The whole of one end or length of a street may be observed from an apartment, by a person sitting inside it, if a mirror is placed at an angle of 45° on the outside of the window. A very ingenious contrivance was suggested for employing similar means in warfare. It was proposed to place outside the wall of a battery, an arrangement of mirrors, by which a view of a distant fort might be constantly obtained, without danger of the observer being "picked off" by the enemy's riflemen. This plan is illustrated in our next engraving; and, we believe, was employed during the Crimean war.

Fig. 3.



In Fig. 3, *a*, *b*, *c*, *d*, represent four mirrors, of which *a* receives the view of the fort. This view is successively reflected over the wall (*e*), and observed by the spectator at *f*. Thus, any person placed at *f*, under protection of the wall, will be enabled to observe all that is going on at a distance from him, and yet be unseen by the enemy.

The amount of light which is reflected from any surface, is not equal to that which impinges on it. This is owing to the absorption of some of the rays. We may here remark, that light, like heat, is absorbed in different proportions by various substances. We thus observe very little light reflected from a body of a black colour; and as no metallic body can be perfectly polished, so, from this cause, some of the rays of light are lost before reflection. But even from the most polished surface the same loss occurs; and thus light reflected from water or glass, undergoes considerable diminution in its intensity.

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[*] See *ante*, p. 14.

We cannot refrain from remarking on the value of this circumstance, when viewed in connection with natural objects. If all bodies reflected light with equal power, the whole face of nature would present a monotonous appearance. Those gradations of tint which form the beauty of the landscape, and the variety of our fields and gardens—the fine effect which a stream of water or river has in the distant prospect, would all be absent. The various reflecting power of different soils, is also of great importance both in a cosmical and agricultural point of view. We here have another instance of one law which is at once universal and beneficent in its operations.

The reflection of light exists on a grand scale in our planetary system. In the centre we have the sun, the source at once of heat and light. Around him are the planets, receiving on, and reflecting from, their surface his borrowed rays. Round the planets we have their satellites, which, like our moon, act as so many enormous reflectors, cheering, during the night season, the side of the planet turned from the sun. Hence, again, the phenomena of eclipses, which occur from the interference, by the eclipsing body, of the rays of solar light, and induce its consequent non-reflection from the eclipsed body.

Two interesting optical toys—the kaleidoscope, invented by Sir David Brewster; and the Debusscope, lately invented by M. Debus—are arranged on the principle of reflection. In the kaleidoscope, plane mirrors are employed, each of which reflect the image of an object placed between them; and thus an infinite variety of beautiful and ever-changing forms may be produced. In the Debusscope, any object placed between the mirrors is multiplied, so as to present a fourfold appearance; and it thus effects a pleasing expansion of any design.

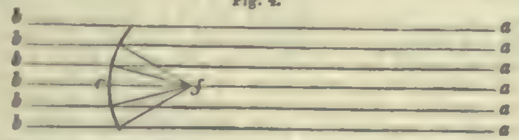
REFLECTION FROM CURVED SURFACES.

We have hitherto referred only to the effects produced by the reflection of light from plane or level surfaces; but we shall now examine the results obtained when rays of light are incident on any polished body of a curved form; and also enter into various applications of the principles we may investigate.

By means of curved reflectors, we are enabled to concentrate the rays of heat and light to a very large extent; or, in other words, we collect rays passing in a straight line on to the mirror; and by its peculiar form, all these are brought together, and joined in one point, called a focus. We may be better understood, if we state, that the object of curved reflectors, is to produce, at one point, the effects of all the rays of light, which had previously only passed side by side, or at an angle with each other.

To illustrate our meaning, we refer to the annexed diagram, Fig. 4, in which rays of light are passing from any source in parallel lines, as from *a*, *a*, *a*, &c., to *b*, *b*, &c.

Fig. 4.



If a curved reflector be placed in any part, say *c*, between *a* and *b*, the rays will not only be stopped in their passage, but will also be diverted from the original straight line in which they had previously travelled, with the exception of the middle one, which coincides with the axis, or line passing through the centre of the mirror at *c*. We shall find, that all the rays thus reflected are joined together at a point (*f*); and this is what is termed the focal point, or focus of a curved reflector.

In practice, this focus can easily be ascertained by holding some burning paper before the mirror, when the junction of the reflected rays will be detected.

It thus follows, that parallel rays of light, passing from a distant source, may, by means of a curved mirror, be brought to a focal point, and their combined effect be concentrated thereon.

If, however, the source of light be placed behind the focus (*f*, Fig. 4), and nearer the mirror, the result will be different. The rays of light will be still reflected, but will pass from the luminous object to the curved surface, and will then diverge in all directions. The student will do well to remember the following epitome of what we have stated, viz.:—When rays of light from a distant source impinge on a curved reflector, they are *converged* to a focus; but when the source of light is placed behind the focus of, and near to, such a reflector, the rays are *diverged* by the reflecting surface.

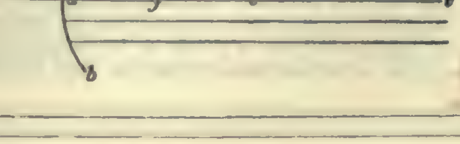
Before we enter into more minute details in reference to curved surfaces, we shall give an illustration of the use of the reflector, which will make plain the statements we have above mentioned. An ordinary reflector, such as is used in carriage-lamps, is an instance of the curved surface we have described. If such a reflector be exposed to the rays of the *sun*, as a distant source of light, it will collect the rays, and bring them to a focal point. If, however, a *candle* be placed behind the focal point, then the rays of light will be reflected in all directions, after reaching the surface of the mirror. These two modes of employing the same mirror, will thus illustrate the difference between the *convergence* and *divergence* of the rays of light; and show that the two reflective results are the reverse of each other in effect. When the rays passing from the reflector are examined beyond the focus, it is found that they have crossed each other; and if an object be placed near the focal point, and observed some distance therefrom, it will be found that an inverted view of it is obtained. We are here confining our attention to a *concave* mirror, or one in which the reflecting surface is "hollowed out," or bent inwards. We shall now examine more minutely into the results obtained in employing this form of reflecting surface, by varying the position of a source of light, or of an object placed at different distances from its focus, together with some of the practical applications which result from the principles at which we shall arrive.

We shall refer, in the following remarks, at first to reflectors of a spherical form; that is, such as would be produced by taking any portion of a globe of glass, or metal in a polished state. The best type of this class of reflector is a common watch-glass, whose *outside* surface has been covered with quicksilver, so as to present an internal or concave reflecting surface. In practice, reflectors of polished metal, of various sizes, are employed; of which, however, we shall speak more fully hereafter.

The law with respect to the reflection of light from plane surfaces, also holds good in reference to curved ones; the angle of incidence being always equal to the angle of reflection. Indeed, a curved surface may be regarded as made up of an infinite number of plane surfaces; and by reflection from each of these planes, to one point, of the rays of light impinging on them, the reflection of curved surfaces is in reality effected.

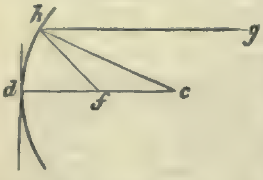
The first case of reflection from a spherical concave mirror which we shall deal with, is that in which the rays of light arrive at its surface in parallel lines. This occurs only from objects at a great distance from the reflector—as the sun, moon, and stars. In Fig. 5, we have an illustration of the effects which result on parallel rays arriving on a concave mirror.

Fig. 5.



Referring to Fig. 5, if *a b* represents a spherical concave reflector, *c* is the centre of concavity, or that point from which, if straight lines are drawn, they shall touch the mirror, and be of equal length; they are, in fact, the radii of the circle, of which the mirror is but a portion: *c d* is the axis of the mirror: *e d* represents a ray of light passing along the axis, and incident on the mirror at *d*. This ray, because passing along the line of the centre of the mirror to *d*, will be reflected back in the same straight line. But if another ray, *g h*, impinges on the mirror at *h*, it will not be reflected back in the same line. The direction which such a ray will be reflected in, will be towards *f*, and will be in the line *h f*. This will be evident if a line be drawn from *h* to *c*; for, as the angle *g h c*, which is the angle of incidence, is equal to the angle *c h f*, which is the angle of reflection, the law of reflection is at once satisfied; and the point *f*, in the line of the centre of concavity, becomes the focus of the mirror, and is situate half-way between the centre of concavity and the centre of the reflector at *d*. This may be better understood to the general reader, if we deal with the subject in the following form. (See Fig. 6, in which we employ the same letters as in the last engraving).

Fig. 6.



Supposing *g h* to be a ray of light touching an exceeding small plane, forming part of the sphere, and that the line *h c* is perpendicular to *h*. Now the angle of incidence, *g h c*, must be equalled by the angle of reflection. To find the latter, draw the line *h f*, so that *f* shall be any point on the straight line *c d*; and in so doing, let the angle *c h f* equal the angle *g h c*. It will be found, that the point *f* is necessarily half-way between *c* and *d*, and, as we before stated, is the focal point of the mirror. This illustration can easily be repeated, by the non-mathematical student, by a pencil on paper; and that plan will assist materially in relieving the difficulty of understanding this portion of the laws of reflection.

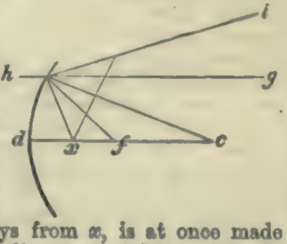
It also follows, that all *parallel* rays impinging on the mirror, except at the point *d*, will necessarily be nearly focalised at *f*; because, with every ray, the circumstances are almost exactly the same. To understand this, suppose the plane *h* to be moved nearer to *d*, and that a ray of light impinges on it in a straight line. In such case, a line perpendicular to its new position must be drawn from it to *c*, the centre of concavity; and as the same course of reasoning must be adopted to explain the results so obtained, the focus of the parallel rays will still be found at *f*.

As an application of the above remarks, we may state, that an image of a star, if its light were thus reflected by a spherical mirror, would be found at *f*.

We may, at this place, urge on those of our readers who may not be accustomed to strict scientific reasoning, that they should not pass any fact or illustration without having thoroughly mastered its details. In every branch of science in which mathematics are employed, it is necessary that succeeding facts should have a relation to those already explained. A chain of argument is thus formed; and if one link be broken, the whole arrangement becomes defective. "Slow and sure" should be the guiding maxim of every tyro in philosophy.

If the source of light be placed in the focus of such a mirror, all its rays would be collected on the surface, and be reflected therefrom in straight lines. This will be at once evident, on considering that by such an arrangement we simply invert the progress of the rays by sending them *from*, instead of *to*, the focus of the mirror. If the source of light were nearer to *d*, the centre of the mirror, than to *f*, its focal point, then the rays would not proceed in parallel lines from the reflecting surface, but would diverge outwards, because the angle of incidence would be greater than those which are produced when the luminous object is placed in the focus; and as the angle of reflection must equal it, that must be greater also. This will be evident on inspecting the following diagram.

Fig. 7.

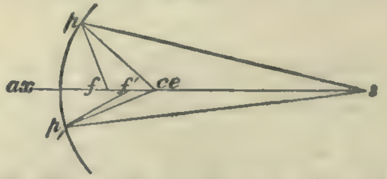


Using the same letters as before, and presuming *x* to represent the new position of the source of light between *f* and *d*, we observe, that if a perpendicular is drawn from *h* to *c*, the angle *x h c* is greater than the angle *c h f*. Now as *x h c*, the angle of incidence, must equal the angle of reflection *c h i*, it follows that the ray of reflected light will diverge as at *h i*; and thus the fact of the divergence of the rays from *x*, is at once made evident by comparing the direction of *h i* with *g h*, the parallel ray.

The foregoing illustration of the properties of rays reflected from a spherical surface, may possibly be difficult to a beginner. We have been compelled to use a circumlocutory description, because we presume that some of our readers will be unacquainted with geometry. To a mathematician, the proposition will appear self-evident; and we trust that, after a little study, others may find the matter easily understood. In every case of reflection, the student should bear in mind the laws of incident and reflecting angles, and, by strictly applying them, many of the problems of optical science will be readily apprehended.

We now proceed to investigate the results which are arrived at when rays of light fall on a spherical mirror from objects at but moderate distances from it, and in lines which are not parallel to each other. To simplify our observations, we shall suppose that a ray of light, from any source, falls on the reflecting surface, as represented in our annexed diagram. (See Fig. 8).

Fig. 8.

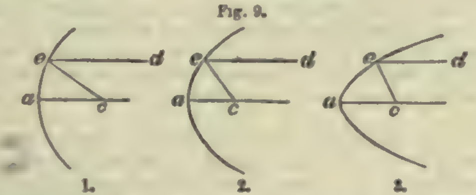


In our illustration (Fig. 5), we presumed that the source of light was at an infinite distance from the mirror, and that all these rays were consequently reflected so as to meet in one point distant from the reflector, and called its focus. In Fig. 8, we suppose the case of the source of light being placed at *s*, and *s ax* to be the axis of the mirror; *ce* the centre of curvature. Let *p* be any plane in the mirror on which a ray *s p* shall fall. The line *p ce* being perpendicular thereto, the angle *p ce f* will be the angle of reflection, and the focal point of this ray will be found at *f*. But suppose another ray of light, *s p′*, to fall on the mirror at *p′*, then *s p′ ce* will be the angle of incidence. Draw a line, *p′ f′*, so as to form an angle equal to *s p′ ce*; then, as the angle *ce p′ f′* = *s p′ ce*, which is the angle of incidence, *ce p′ f′* will be the angle of reflection. But *f′* is not at the same point, in the line *s ax*, as *f*, but is nearer to *ce*. Hence it is evident that all the rays passing from a source of light at *s* will not converge to one point, and of course, therefore, their focus will be removed from that place which would be found between *ce* and *ax* if all the incident rays were parallel. This produces what is termed *aberration*, or what we may call, in plain language, the straying of some of the reflected rays of light from one converging point, and, consequently, the production of several foci instead of one. The optical effect which this produces, we shall more fully inquire into presently.

As the reflectors to which we have referred have been assumed to be portions of a sphere, we generally term the effect of the departure of rays from a common focus as *spherical aberration*. It would take up too much of our space, and we should have to presume a greater amount of mathematical knowledge on the part of some of our readers, were we to enter into an exact analysis of the different curved surfaces which might be employed to overcome this tendency to aberration. There is, however, one curved surface, called the parabola, which, employed as the reflective curve of any mirror, enables us to get rid of aberration, and so to effect the reflection of incident rays in a far more perfect manner. The curve called an ellipse answers a similar purpose. Without, therefore, entering into any mathematical disquisition on the subject, we may state, that, obeying the law of reflection—the angle of incidence and reflection being always equal to each other—rays of light incident on a parabolic or elliptic reflector, will, owing to the properties of these curves, be converged to one focus; and, *vice versâ*, all rays passing from the foci of these curves to the reflecting surface, will be equally reflected therefrom to the other foci of similar curves. The value of this law will be at once seen when we have to refer to the use of mirrors in various optical instruments.

That the non-mathematical student may understand the difference existing between the curvature of a sphere, an ellipse, and a parabola, we annex a diagram illustrating each of these curves. (See Fig. 9).

Fig. 9.



In Fig. 9, No. 1 represents a portion of a sphere; No. 2, of an ellipse; No. 3, of a parabola. In each of these, *a c* is the axis of the curve, and *c* the focus of incident rays. The lines *d e* show the direction of a reflected ray from its source to the focal point.

Having thus endeavoured to explain the laws of reflection of concave curved surfaces, we proceed to examine the effect on a ray of light incident on surfaces of a convex nature. We may state, generally, that exactly opposite results are produced by convex to concave reflectors, the incident rays being always diverged from those of a convex form. Thus, in Fig. 10, we have a portion of a sphere, *a b c*, on which rays of light, *d e*, *f g*, are incident. Instead of converging to a point in front of the mirror, the real focal point would be found at *h*, or that place which would be the focus of rays incident on a curved surface of a concave form, having the same centre of curvature. The ray thus incident will take the direction *c i* and *g i'*, if not in the same line with the axis of the mirror. The ray in the axis of the mirror will be reflected back in the same line as that of incidence, for reasons which we have already frequently explained. The same remarks apply to a parabolic reflector, the internal or concave focus being also that point from which rays incident on its convex surface will be diverged. This is illustrated in Fig. 11, where *f* is the focus of the parabola *a b c*, and *d a* ray of light, which is incident on its convex exterior, and diverged in the direction *a e*.

Fig. 10.

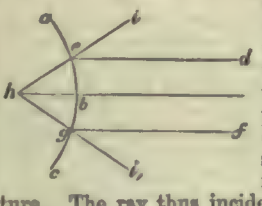
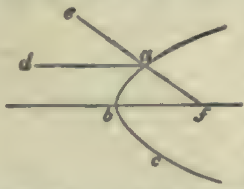


Fig. 11.



Having thus pointed out some special laws affecting the reflection of light from surfaces of different curved forms, we shall leave their extended application—as in the case of reflecting telescopes—to the subject of Optical Instruments, when the variety of uses to which reflectors are adapted will be fully dealt with. We shall next proceed to examine the phenomena of refraction, which are produced in bodies capable of allowing the passage of light through them; and shall deal with those changes to which a ray is thus subjected.

REFRACTION OF LIGHT, OR DIOPTRICS.

Having examined the results which take place when a ray of light falls on a reflecting surface, we proceed to investigate the phenomena of refraction, or the effects produced when light passes through transparent media of different densities.

If a ray of light pass perpendicularly through any medium of equal or homogeneous character in every part, it will do so in a straight line. If, however, the medium vary in its density, then the ray will be bent out of its straight course; and, to use philosophical terms, we say that it is "refracted." If the ray pass obliquely, the same effect will occur. We shall be better understood if we give instances wherein this refraction takes place. Amongst them are the following.

On looking from the banks of a river at a fish swimming in the water, we see the fish at some distance from its real place, owing to the rays of light passing from its body being bent or refracted from a straight line.

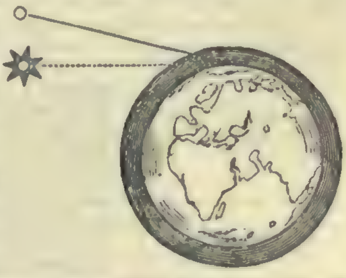
A stick introduced obliquely into a pail of water, appears broken, because the rays of light from that portion in the water, and from that remaining in the air, do not reach the eye at the same angle.

In looking down a long trough of water, the bottom of the trough seems to touch the surface of the liquid at that end furthest from the eye of the spectator. Again, if a coin be placed in a basin, and the observer retreats until he just loses sight of it, the piece of money will reappear on water being poured into the vessel; because, by the refractive power of the liquid, the rays of light are bent upwards, and so reach the eye.

On looking through a piece of window-glass of different thicknesses, all straight lines so observed will appear curved. A similar effect is observed when the eye is placed in a line with the surface of a wall strongly heated by the sun's rays. Objects viewed through the heated air arising from the surface, will appear to assume most fantastic shapes, owing to the refraction of light thus produced.

The refraction of light causes the heavenly bodies to appear, if viewed by the naked eye, much larger when near the horizon than when at the zenith. This effect is often noticed at those seasons of the year when moisture is abundant in the air, or when the atmosphere is heated. The apparent position of the sun is thus altered by refraction; and we therefore observe the disc of that body some time before it has actually risen. This effect is illustrated in the following engraving (Fig. 12), wherein the inner circle represents the earth, and the outer one the atmosphere. The upper line points to the apparent position of the sun before rising; the sun actually being below the horizon, as shown in the second line, although visible to the spectator by the refraction which its rays undergo whilst they pass through the air.

Fig. 12.



Having thus given familiar instances of the effects of the refraction of light, we proceed to investigate the laws in accordance with which its phenomena are manifested.

For this purpose, we annex diagrams which illustrate the progress of a ray of light from a rare to a comparatively dense medium, and *vice versâ*; and at the same time they indicate the refraction or bending of the rays during their progress.

In Fig. 13 we observe the progress of a ray from *c*, at an oblique position to the perpendicular line *a b*. On arriving at the surface of the water, *f g*, the ray does not proceed in the straight line *c d*, but is refracted or bent backward towards *e*; so that when the ray arrives at a certain position in the water, it will be observed at *e*. The student must carefully bear in mind the direction of the ray in our illustration, as *entering* the water.

Fig. 13.

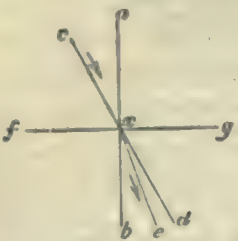
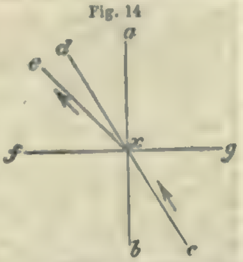


Fig. 14

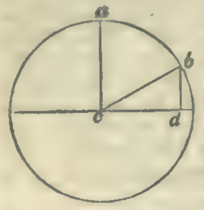


We will now examine the result of a ray passing from any object under the surface of the water; and, for this purpose, refer to Fig. 14, in which we find *c* as the point from which the ray proceeds. Instead of progressing from *c* to *d* in a straight line, we find that the ray is bent or refracted, during its progress, from *c* towards *e*; *a b* still remaining as a line perpendicular to the horizontal surface *f g*.

In Fig. 13, the angle *c x a* is the angle of incidence; and *b x e* the angle of refraction. In Fig. 14 we have the ray of light passing from *c* towards *d*, but refracted towards *e*. Hence *b x c* is the angle of incidence, and *e x a* the angle of refraction. Now if the student will carefully compare the results we here obtain, with those exhibited in the experiment with the coin and a basin of water, he will at once perceive how it is that the ray passing from the coin is refracted or bent back, as we have already mentioned. The case of the fish and the stick in the water are similarly explained; but in these cases we deal with the ray as passing from an object through a dense into a rarer medium.

We next proceed to examine the ratio which exists between the angle of incidence and the angle of refraction; and for this purpose we shall have to call into our aid some of the laws of the mathematical science called Trigonometry. In the annexed diagram we have a circle, in which we have drawn several lines. (See Fig. 15).

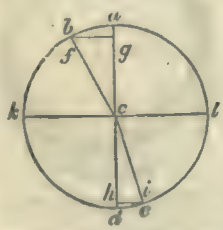
Fig. 15.



The lines *a c* and *c b* are termed radii; and stretch, like the spokes of a wheel, from the centre of the circle to its circumference. The line *b d* is called the sine of the angle *b c d*; and it is to this line that we have to call attention in reference to the index of refraction.

In the annexed diagram we have a circle, in which we have drawn two angles. (See Fig. 16).

Fig. 16.



In this, *a c b* represents the angle of incidence of a ray of light entering a dense medium; and *d c e* the angle of refraction. Now the straight line, *f g*, is the sine of the angle of incidence, and the line *h i* the sine of the angle of refraction; presuming that the line *k l* represents the surface of any refracting medium, such as water, &c. The index of refraction is found by taking the ratio which exists between the two lines or sines *f g* and *h i*; and the ratio varies for different substances, as we shall see by the following table, which gives the index of refraction for several bodies. We shall give a more extended series in an appendix.

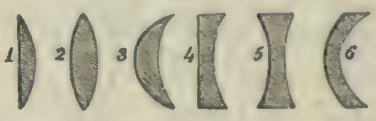
Table of Indices of Refraction.

Refracting Medium.	Index of Refraction.
Vacuum	1.000000
The atmosphere	1.000294
Ice	1.308500
Water	1.336000
Alcohol	1.370000
,,	1.374000
Florence oil	1.485000
Camphor	1.500000
Plate glass	1.514000
Crown ,,	1.526000
,, ,,	1.533000
Brazil pebble	1.532000
Rock crystal	1.547000
,, ,,	1.568000
Amber	1.547000
Flint glass	1.578000
,, ,,	1.601000
Sulphur	2.148000
Phosphorus	2.260000
Diamond	2.439000
,,	2.470000

In looking over the above table of the index of refraction for various bodies, it will be noticed that they differ very much with respect to their bending a ray of light passing through them from its original straight course; and we shall observe that inflammable bodies possess this power to the greatest extent. Sir Isaac Newton noticed this fact, and he guessed that the diamond was composed of some inflammable substance, on account of its possessing so high a refractive power—an opinion which has been verified by chemists, who have shown that that valuable gem is simply a pure form of carbon or charcoal.

Having thus pointed out the nature of refraction, we proceed to explain the principles on which lenses are constructed. Of these there are several kinds, the nature of which will be at once observed, as they are represented in Fig. 17.

Fig. 17.



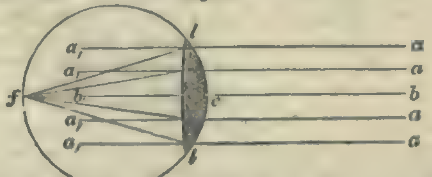
In the above diagram, the various lenses illustrated have the following names:—

No. 1 is a plano-convex lens.
,, 2 ,, double convex lens.
,, 3 ,, meniscus lens.
,, 4 ,, plano-concave lens.
,, 5 ,, double concave lens.
,, 6 ,, concavo-convex lens.

In the plano-convex lens, we observe that one side is a plane surface, whilst the other is convex, and forming part of a sphere. In the double convex lens, both surfaces are curved. The meniscus lens has a similar appearance to a watch-glass, and has its opposite surfaces concave and convex. In the plano-concave, we have one plane, and one inwardly curved surface. In the double concave lens, both surfaces are concave; and in the concavo-convex glass we have two curved surfaces, in some respects like to the meniscus glass, with the exception, however, that, in the concavo-convex, the two curves are about equi-distant in every part.

We now proceed to trace the course of the rays of light passing through the lenses of the different forms above described, commencing with the plano-convex lens, as, perhaps, the simplest for our purpose.

Fig. 18.



Let *a a a a* be a number of parallel rays of light passing through the lens *l l*. Now, it is evident, according to what we have previously stated, that all these rays would continue to pass in a straight line and parallel position, provided that $l\,l$ is a homogeneous substance, or of equal size in every part. But $l\,l$ forms part of a spherical refracting medium, such as glass, &c.; and, as such, the rays passing through it will, with the exception of b', which is in its axis, all fall upon it obliquely, and will therefore be refracted, or bent, from their straight course. Instead, therefore, of continuing on towards a' a', &c., they will incline towards each other, and at last meet in one focus f, which is the focus of the lens under examination. This focal point is found, in the case of the plano-convex lens, to be of a distance, from the centre of the lens $l\,l$, equal to the diameter of the sphere or circle, of which the curved surface of the lens forms part. This will be observed in our diagram, where f is placed at the extremity of the diameter of the circle including the curvature of the lens.

In the double convex lens, the focus is found at the centre of the circle of curvature, and will, therefore, be at only half the length of the diameter of the circle, from the centre of the lens; in other words, the extremity of the radius of the circle, drawn from the glass, will be the focal point of this class of lens. As an illustration of this and the previous rule of finding the foci of these two kinds of lenses, we may add, that presuming the curvature of a *plano-convex* lens to be part of a circle three feet in circumference, then its focal point would be found about twelve inches behind it. If, however, we take the case of a *double convex* lens, we should find the focus six inches behind it, which is about the length of the radius of a circle thirty-six inches in circumference.

As we are confining our attention to the course of parallel rays through lenses, we will next examine the effect thereon when they pass through a double concave lens.

Fig. 19.

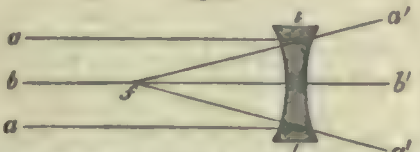


Fig. 19 represents a lens where both surfaces are concave; a b a are three parallel rays of light, incident on one surface of the lens $l\,l$. We shall find that the centre ray b will continue its straight course to b', being in the axis of the lens, and passing through its centre. In the case of the rays a a, we find that they are refracted during their passage, falling as they do on the curved surface of the lens; and on emission on its opposite side, we observe that they diverge, as shown in the lines a' a'. The focus of this lens will be found at f, or at such a point as would be arrived at on the line b, by the refracted rays a' and a', if they were produced backwards through the lens to f, which is the centre of the cavity of the glass.

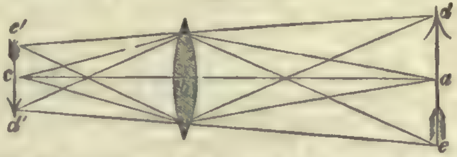
It will be found that f will be distant from the centre of the lens to the extent of the length of the radius of the circle of curvature. If, however, the lens is plano-concave, then, like the plano-convex, its focus will be distant from the centre of the glass to the extent of the diameter of the circle, of which the curve of the lens is a portion.

By following the above rules, the foci of a meniscus and concavo-convex lens may be found: as, however, these are not likely to engage the attention of the reader, we shall not enlarge on the matter.

Having thus examined the effects produced on rays of light passing through lenses of various forms, we shall now refer to other results, which are more especially connected with their practical uses.

If the rays of light passing from an object placed at some distance beyond the focal point of a convex lens, are viewed after their passage through the glass, an inverted image of the object may be seen by placing a piece of ground glass, oiled paper, &c., so as to receive the transmitted rays near the focus, on the opposite side to that of the object. The inversion of the image is caused by the rays crossing each other. This is illustrated in Fig. 20, where a represents the object, b the lens, and c the inverted image.

Fig. 20.



Now, if the rays pass from d in the object a, they will be found refracted as passing obliquely through the lens, and their image will be seen at d', on the screen c; and so with the rays passing from the object at e, their image will appear on the screen c at e', of course inverted.

One of the simplest methods of illustrating this property of convex lenses, is that of allowing the rays of a candle, whose flame is placed beyond the focus of a double convex lens, to pass through the glass on to a sheet of paper placed on the other side. The inverted image of the flame will be at once observed. If a lens, again, is held towards a window, and the rays of light passing from external objects be received on paper or ground-glass, as before mentioned, a complete but inverted representation of them will be seen. On this principle the camera-obscura is constructed, an instrument of which we shall give a full description under the head of Optical Instruments.

On placing a plano or double convex lens between the eye and an object, the latter will present a magnified image, if at about a distance from the lens equal to its focal length: this effect is called the magnifying power of a lens, and varies in proportion to the difference of focal lengths in lenses, when compared with each other. The shorter the distance of the focus from the centre of the lens, the greater will be the magnifying power of the instrument. This effect in convex lenses is owing to our being enabled to view any object at a larger angle, by their assistance, than could be done by the eye alone.

Perhaps the most ready mode of illustrating this law, will be that of referring the student to the apparent diminution of an object as we recede from it. The further we are from any form, the less angle does it subtend to the eye: the nearer we approach to it, the larger this angle of vision becomes; and thus the object appears of greater size. Now the effect of a lens is to collect the rays passing from an object placed at its focal point; and, by refraction, these rays reach the eye, subtending a greater angle on the retina than that existing from the centre of the unassisted eye itself; hence, of course, the image is enlarged. The effect of concave lenses is directly the reverse of this, for they diminish the angle at which an object is seen; and hence the apparent size is also lessened.

We have thus endeavoured to give an explanation of the laws of lenses, and of many phases of single refraction. We shall enlarge on the various uses of lenses under the head of Optical Instruments, in respect of the microscope, telescope, &c. We shall defer the consideration of double refraction to the subject of Polarisation, with which it stands so intimately connected; and shall now attempt to show the application of these laws of lenses, so far as they refer to the structure and use of the eye.

THE EYE AND ITS STRUCTURE.

Fig. 21.

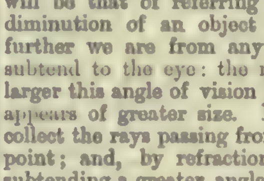
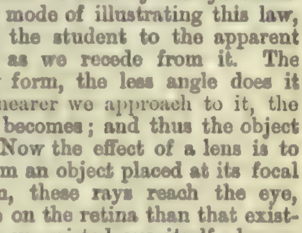


Fig. 22.



a, the cornea, containing the aqueous humour; b, the pupil; c, the crystalline lens; $e\,f$, the iris; g, chamber of the vitreous humour; h, the optic nerve; $i\,i$, the sclerotica; $k\,k$, the choroides; $l\,l$, the retina.

Presuming that our readers have become familiar with the laws of reflection and refraction, as well as of the construction and use of different kinds of lenses, we now purpose to apply our knowledge in investigating the structure of the eye. We shall also inquire into the different modifications which are found to exist in the eyes of various creatures, and which are essential to the particular conditions in which they exist.

We shall look in vain for any greater instance of perfect adaptation of means to an end, than is found in the construction of the organs of sight. As we proceed in our inquiries, we shall find that every necessity has been provided for; each part is exactly adapted to its purpose; and that simplicity of construction is, after all, the characteristic of this masterpiece of the handiwork of the Deity.

At the head of this section we have introduced two engravings, which give a sectional and general view of the human eye; and we now proceed to describe the uses of each part.

We need scarcely state that one engraving represents the eye as seen from the front. By means of Fig. 22, we shall be able to examine each division, and to ascertain its separate purpose.

We commence with that part which forms the external visual portion of the eye, termed the cornea. This is a transparent horny substance, and encloses what is called the aqueous or watery humour. It forms the first lens in the eye, and refracts rays of light incident on its surface. Referring to Fig. 22, it will be found as marked *a* therein. On the other side of the cornea, the pupil *b* is next observed, which is an aperture through which light passes into the eye from the cornea. A most admirable contrivance exists in front of the pupil—called the iris, *e f*. This is a movable diaphragm, which lets in or closes out light from the eye; and which, by its contraction or expansion, regulates the amount of incident rays in their passage to the retina. The student may easily satisfy himself as to the object of the iris, by observing the eye of the domestic cat during dusk and in full sunshine. When but a dim light exists, the iris will be expanded, and the pupil will appear large. In full daylight, however, the iris is contracted, and the pupil of the eye presents only the appearance of a narrow slit—almost, in fact, shutting out light from the inner portion of the eye.

Beyond the pupil *b*, we next observe the crystalline lens *c*, which has a double convex form. This lens is filled with a fluid of mixed chemical character, and of higher refractive power than that found in the cornea. The centre, *g*, of the eye forms a cell of a nearly circular form, filled with what has been termed the vitreous humour.

At the back of the eye, facing the crystalline lens, we find the retina, so called from its net-like construction. This is at *l l* in our illustration. It is on this membrane that the images of objects, whose rays pass through *a b c*, are depicted. It, in fact, forms a kind of screen, very similar in its use to the ordinary screen employed in the exhibition of dissolving views, &c.

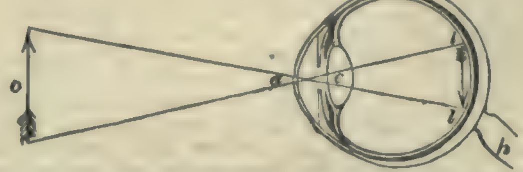
Surrounding, and next to the retina, is the choroid membrane *k k*, which has a black colour, whose office is to absorb the rays of light after they have passed on to the retina. An external coat, the sclerotica, encloses this, and will be observed at *i i*. This extends to the cornea, and forms what is called the "white of the eye."

At the back of the ball of the eye, we observe a projection extending from it. This is called the optic nerve, *h*, and, by its action, the image painted on the retina becomes perceived by the mind. In fact, it forms the connecting link between mind and matter, and is the channel of nervous sensation between the eye and the brain.

The reader will thus perceive that there are three different cells, each filled with fluids. These differ in their refractive power; the index of that of the cornea, *a*, being 1.337; that of the crystalline lens, *c*, 1.384; and of the vitreous or centre of the ball, *g*, 1.339.

Having thus described each portion of the eye, we proceed to examine the course of a ray of light from an object incident on the cornea.

Fig. 23.



In Fig. 23, retaining the same letters as those we employed in describing the eye in Fig. 22, we observe the rays of an object, *o*, proceeding to the cornea, *a*, and passing to the crystalline lens, *c*. Here they undergo refraction, just as they would if passing through a double convex lens made of glass; and eventually we find that an image is painted on the retina, as seen at *l l*. It may be more satisfactory to the student to try an experiment with the eye of an ox. For this purpose he should remove the external coatings of the eye, including the retina; and by placing a piece of ground-glass against the cell, which contains the vitreous humour, in the same position that the retina had previously occupied, he will at once perceive an inverted image of any object whose rays have passed through the cornea, the crystalline lens, and the vitreous cell.

The reader will at once perceive that the image painted on the retina is inverted, and will, doubtless, inquire how it is that we see all objects in an apparently vertical position? The answer hitherto given to this question is at least unsatisfactory. It is generally stated, that the distinctions between top and bottom, and such expressions, are merely the results of habitual experience; or, in other terms, we express ourselves in such a manner as accords with the impression of our minds. "Seeing properly," therefore, becomes a mere question of education, just as the sense of touch, &c., may be similarly explained. This, however, is a subject on which it is useless to spend space and time in discussing, and any attempt at analogical illustration would by no means clear away the difficulty of the question.

One of the most astonishing capabilities of the eye, is its power of adapting itself to view objects at different distances. It is evident that some change must take place in the shape of the lens of the eye, when rays of greatly different positions as to distance pass through it. The mode of action may be twofold. The crystalline lens may be enabled to alter its shape, and therefore its convexity, of its own accord, or the entire ball of the eye may perform the same operation by the contraction of the muscles surrounding it. Each view has had its advocates and its opponents. It has been properly remarked, that the crystalline lens has no muscular structure sufficient for the purpose; and, on the other hand, that the contraction of the muscles surrounding the eyeball would, if exerted sufficiently, tend to disorganise the whole structure of the organ. This subject is one of great difficulty, and not to be easily decided. We are at present much in the same position on the question, as the man who, although his blindness had been removed, could not explain the chain of causes existing between the fiat of his Great Physician and its result. We may explain our position safely and correctly by quoting his words, as one of the most logical and satisfactory of conclusions: "One thing I know, that whereas I was blind, now I see;" and thus we likewise must be content with a simple statement of facts.

Having confined our attention to the structure, &c., common to both eyes, we have now to speak of another subject, which also presents some difficulty; namely, how is it that we see singly by means of two eyes? The readiest answer to this is, that if any image be presented to each eye, at such a distance from either that the two images overlap or join each other, then the object will be viewed singly. The student will understand what we mean by trying a very simple experiment. Place the fore-finger between the eyes, parallel to the nose, and not exceeding an inch in distance therefrom—the finger will thus be seen double. Now, move the finger gradually from the face, and observe the effect produced. It will be seen that, as the finger retrogrades, the two images, previously separated, will gradually merge into one, so that, at a distance of six inches from the nose, only one image will be observed. In fact, a gradual junction of the two images is thus effected.

Again, bring the eyeballs into such a position that all distant objects appear double—just, in fact, so that the axes of both eyes shall be directed to a point close to the line parallel with the nose. If the finger be now held at a distance of two feet from the eyes, it will appear double: if, however, it be gradually brought so as nearly to touch the nose, its two images will gradually coalesce, and one image will be formed in the same place as that in which two images were observed in our last experiment. In this case we have brought, by the process of "squinting" (as it is sometimes termed), the two axes or lines of vision of the eye, so as to join each other at a distance much less than that adopted in ordinary sight, and have so produced an inversion of effect. In either case, however, the cause was the same; namely, the overlapping of the two images impressed on the retina, but at different distances from a point situated midway between the two eyes.

We may here explain two instances of defective sight, which we observe many persons to be troubled with. They are those of long and short sight. In aged persons, we find that the shape of the cornea has become altered, and less convex than is the case in a healthy state of the organ. The remedy for this is very simple; for what is deficient in the eye is easily supplied by means of convex lenses;—hence the general use of spectacles by old persons and others, in whom the cornea is deficient in convexity. This state of the eye permits a person to see well enough any distant object, because the rays proceeding thence are comparatively parallel to those emanating from an object nearer to the eye.

In the case of "short-sighted" persons, the cornea is far more convex than in ordinary cases. The use of concave glasses is therefore essential to such, and they at once supply the requisite remedy.

We shall speak more fully on this subject when we treat on optical instruments; and shall defer the consideration of some peculiarities of vision, in reference to colour, to our section on the phenomena of colour.

In many of the mammalia, the uses of the eye being, to some extent, similar to those of the human subject, its construction is mostly similar. Considerable susceptibility to the action of light, in some animals, is generally attended by a greater irritability of the iris; and thus we find those creatures which have their type in the common cat, possess better power of sight during dusk than at other times. As we have already remarked, the pupil of the eye contracts during strong daylight, and expands as the light becomes less intense. The same adaptation of organism to special circumstances is found in the owl, &c., amongst birds.

The eye of the fish is peculiarly adapted to the condition of its existence, and may be taken as a kind of antitype of the organs of sight of insects. The fish naturally exists in a liquid whose refractive power is scarcely different to that of its organs of sight. In the insect we find a most remarkable construction of the eye; and of this class we may refer to the eyes of the bee and the fly, which are formed of a multitude of lenses. In general, we may state, that the shape of the cornea is made to depend on the circumstances in which the creature has to maintain its existence. Thus, fishes have the cornea flat, and the crystalline lens more spherical; whilst the cornea of the eye of the bird has greater convexity, and the crystalline lens is less convex than that of other creatures.

There are some interesting facts relating to the construction of the human eye as indicating design, which we must not omit to place prominently before our readers. Our remarks on aberration* will prepare us to expect that special provision has been made to prevent the consequences of that defect which causes so much loss and diffusion of light in spherical lenses. In the human eye, the curvature of the cornea and crystalline lens is such as entirely to prevent any chance of such a defect existing; they, in fact, being elliptic, and hence exactly of that form as would be required for a non-aberrant lens. The voluntary motion of the iris regulates the amount of light which can enter the eye. The change of the convexity of the cornea provides for a distant or close view of any object at pleasure. The mobility of the eyeball enables us to instantly change the direction of sight. The refractive powers of the humours of the eye are exactly such as are required for the medium in which we live. External to the eye, we have the protection of the eyelashes, which serve as a screen to preserve the organ from the intrusion of foreign matter on its surface. The prevalent colour of both the air and the surface of the landscape, is just that which agrees with the optical necessities of sight. Whilst the ear cannot be closed to external sound, the eye, which is an infinitely more delicate organ, can be shut off from external impression, by the motion of the eyelids, at any moment, and thus injury to its delicate organisation may be almost entirely prevented. In no point of view, therefore, is perfection and completeness more evident than in the arrangement of the different parts of the eye of man.

It may also be remarked, that the eye is less subject to disease, in general, than any other organ of the human system. Despite its constant use, and apparent exposure to external injury, we find, on an average of instances, that it stands the wear and tear of life to an astonishing extent. The changes of the healthy organ are very gradual; and thus it is only towards the close of life, when all the faculties and senses become dulled by age, that the power of sight becomes seriously affected. Even then, science gives to a "green old age" a "second sight"—the very disease, indeed, suggesting its own remedy. Whilst, during life, the eye is the exponent of the sentiments of the mind—alternately beaming with pleasure or dimmed with pain—it generally maintains vitality to the closing scene, and, by its last glance, indicates the farewell of its possessor to the changes and shadows of earthly existence.

THE PHENOMENA OF COLOUR.

Having investigated the principal laws of the reflection and refraction of light, we shall now enter on the consideration of a most interesting branch of optical science; namely, the cause of colour.

We have already stated that white light is composed of certain rays which have been called primary colours; and we shall presume that the reader has become acquainted with these, by examining the spectrum produced from a ray of light refracted by means of a glass prism. We, however, must remark, that the characteristics of a spectrum vary according to the substances employed as a prism; and by having a triangular vessel arranged so that it can be filled with liquids of various refracting powers, the student will soon discover, by its use, the results to which we here refer.

The chief effect produced by employing various substances in place of glass as a prism, is, that some have a greater dispersive power over the rays of light passing through them than others. Thus, a prism of flint-glass increases the extent of the image of the coloured rays much beyond that produced by one of crown-glass. And the same may be stated of many other substances. Some have the power of extending certain of the spectral rays in preference to others, and of thus having a special action on one or other end of the prismatic image.

We have already stated that the different rays of the spectrum may be divided into three kinds—the luminous, the calorific, and those having a chemical action on certain bodies. The chief of the luminous rays are found in the yellow; the greatest heating effect in the red; and the chemical power exists entirely in or beyond the blue band. By placing a thermometer in the various bands of colour, the student will be enabled at once to perceive how much more heat is to be found in the red than in the blue rays; and if a piece of note-paper be well soaked in a solution of common salt, and, when dry, moistened with a solution of nitrate of silver, it will be a test of the power of the chemical rays. If placed in the blue or violet band, it will be speedily darkened, owing to the chemical action of the violet ray on the salt of silver on its surface. In the red ray it will scarcely undergo any perceptible change.

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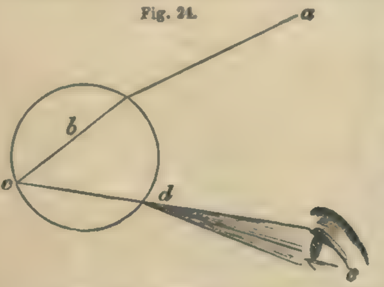
* See *ante*, p. 44.

The most convenient mode of conducting this instructive experiment, is by stretching a piece of paper, prepared as above, along the entire length of the spectrum, so that all the rays from the red to the extreme violet may fall on it. The paper will be darkened just in proportion to the energy of the blue rays, and unaffected in the yellow and red. Both the calorific and chemical rays extend, however, beyond the visible spectrum, and are more energetic than at the point where light ceases to be perceived: of this, however, we shall speak fully hereafter.

The well-known phenomenon of the rainbow is easily explained, on principles similar to those involved in the production of the prismatic spectrum. Indeed, each drop of rain, by refracting the rays of light incident on it, acts as a prism, and produces the coloured image which is impressed on the retina of the eye. The reason of this will be better understood if we examine the effect produced on a ray of sunlight falling on the rain-drops, as illustrated in the annexed engraving. (Fig. 24.)

Fig. 24.



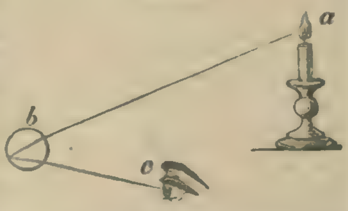
We will suppose *a* to be a ray of light falling on the rain-drop *b*, the sun being behind the spectator, and the rain-drop facing him, and at a certain angle with the sun and his eye. On entering the drop, the ray is, of course, immediately refracted, passing as it does, in an oblique direction, from a rare into a dense medium. It will arrive at some point in the drop (say *c*), and a portion of it will be transmitted. Another part, however, will be reflected to *d*; and as it leaves the drop the emerging ray will be divided, and its colours will be evident to the spectator at *e*. It will be at once perceived, therefore, that the rain-drop acts precisely as a prism; and in like manner the different coloured rays reach the eye, in the same order as in the prismatic spectrum; the blue rays being observed at the top, and the red rays at the lower part of the rainbow. Of course, what is true of one drop is equally applicable to all, and their great number combines to produce the unparalleled splendour which the "bow in the heavens" presents to the eye of the admiring observer. With respect to the order in which the colours of the rainbow are seen, we must add, that each coloured ray has a special angle of its own; and as these emerge from each drop of water, they successively reach the eye in the order of the least refrangible or red rays in an inferior, and the more refrangible or blue rays in the superior, position. The student will thus perceive a beautiful illustration of all the laws of reflection and refraction to which we have already alluded in the phenomena of the bow, whether produced by the shower of rain, or by any means where water, in a minute state of division, can be placed in the proper angle of light incident on it.

A most beautiful effect is produced by casting the prismatic spectrum on steam rising from hot water, which affords an appearance much resembling a rainbow. Artificial means can easily be employed for producing the appearances of a "bow," by casting light on water issuing under pressure from an orifice, so as to form spray. In a similar manner a "rainbow" may often be observed over a waterfall, and at the bows of a steamer, or near the paddle-wheels when they are in motion, because the water thus cast up is in a favourable state of minute division for affording the bow-like appearance on a small scale.

The rich variety of colours observed during sunrise and sunset, is equally due to the refraction of light produced by the moisture held in the state of vapour in the atmosphere. Generally speaking, the more dense the state of the vapour or clouds, the more inclined to a red appearance is the effect produced; and by a gradation in the amount of moisture present, every variety of tint is thus produced. Hence, we invariably find, that the colour of the clouds towards sunset is of a deep red tint, because the refraction is greater at the horizon than at the zenith, whether in the presence or absence of cloud. There are many other appearances of coloured light observed round the sun and moon during cloudy weather; and all these are easily explained, on the principle of the refraction of light, by its passage through, or reflection from, globules of aqueous vapour.

If the student place a round and solid ball of glass so that the light of a candle may impinge on it in a manner similar to that observed when the sun produces the rainbow, he may easily observe all the effects of coloured light, produced by the rays from the candle being refracted by the glass.

Fig. 25.



The best mode of carrying out this simple experiment, is that represented in Fig. 25, where *a* represents the flame of a candle, *b* a solid globe of glass an inch in diameter, and *c* the position of the eye, some way between the glass and the candle, so that the latter may be behind the spectator. The ray of light will pass to the back of the glass globe, and, having been refracted, the observer, if placed at angles varying from 40° to 50° with a straight line drawn from the glass globe, between the eye and the candle, will perceive the coloured rays of light in their natural order of succession. As the head is moved towards the light, the blue rays will be perceived as reflected from the glass; but, on gently moving the eye from the candle, all the prismatic colours will gradually present themselves. An easily understood experiment will be thus afforded, giving a good insight into the laws which govern the production of coloured light in atmospheric phenomena. Even a glance at an ordinary chandelier pendant, will give the same effects, and is a still more common illustration of the points we have mentioned.

On similar causes to those we have here explained, the beautiful play of colours seen when a brilliant is viewed by reflected light, depend. The light is refracted by entering the substance of the diamond, and, on subsequent reflection, the coloured rays are perceived by the eye.

The colour of all bodies is simply caused by their absorbing every ray of the prismatic spectrum, except that which reaches the eye. Thus, if we look at a red article, we state, as the reason of its colour, that all the rays of every other coloured light are absorbed by the material, that of red only being reflected from its surface. Of course, the same argument is applied in reference to all colours which can be perceived. We must here refer to those admirable researches instituted by Sir Isaac Newton, which have thrown great light on this most interesting subject.

Every one must have noticed, that when oil, tar, and other liquids are dropped on the surface of water, a variety of beautiful colours is produced. Sir Isaac Newton, by forming thin plates of air through pressing two glass surfaces together, was enabled to even measure the thickness of these plates for each colour afforded. As these results are of great importance, we give directions, by following which the student may easily reproduce them at his convenience.

Experiment 1.—Place a glass lens, of a focal length of ten or twelve feet, on a flat plate of glass, and press the two surfaces together. It will be observed, that if great pressure be employed, a small dark spot will be produced at the point where the two surfaces are nearest each other; and round this point a series of concentric circles will be formed, each being of a definite colour, and varying from violet to red.

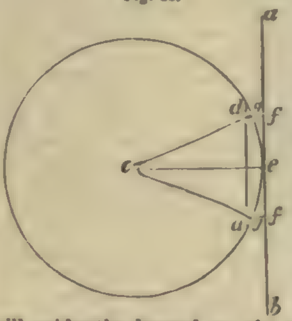
Experiment 2.—Boil some water in a Florence flask, or other glass vessel, and throw into it a pea of Castille soap. When steam issues from the neck of the vessel, and all the air has been expelled, cork the flask so that it shall be air-tight. On shaking up its contents, thin films will be formed, presenting a similar set of appearances to those named in the last experiment.

Now, the production of these colours is due to the fact, that the plate of air or film of the soap-bubble is of exactly the thickness required to produce the colours observed. If the plate be exceedingly thin, then no light will be reflected, and the spot will be black. If somewhat thicker, then violet light will be perceived; and so on until the plate becomes of greater substance, when red light will be reflected from the surface, and so reach the eye.

If, however, the observer look *through*, instead of *at*, the plate of glass, he will see what are called the *complementary* colours, which, combined with the colour observed, would produce white light. These experiments enable us to tell the thickness of a plate of air; and, therefore, the length of an undulation of light of any colour; for we have only to learn the distance between the lens and the flat glass at any point where we notice a colour whose undulation we wish to study, when the length of the undulation is of course obtained. This is easily done; for, knowing the size of the circle of which the lens forms part, we can at once learn the length of any arc of that circle, and, taking the length of the tangent of that arc in which we observe the undulation existing, we can obtain the thickness of air between the tangent and the curve of the circle.

This will be easily understood by referring to Fig. 26, in which *a b* represents a flat glass plate, resting against a lens *d d*, which forms part of a sphere whose centre is *c*. Let *f f* be the extremities of a ring of coloured light distant from *e*, which is a point where we will suppose a black spot, or place where no light is reflected; *e f* is, of course, the tangent of the angle *f c e*. Now *f g*, the thickness of the plate of air, will evidently depend on the angle which *f* forms with the radius of the circle *c e*. The length of *c e*, *c g*, being easily obtained by ordinary measurement, *f g* is, of course, as easily found. It follows, that the smaller the angle existing with respect to the centre of the circle *c*, the thinner will be the plate of air between the lens and the glass plate *a b*. As the measurement of all intermediate distances involves only the ordinary rules of mathematics, the student will at once perceive that we can thus calculate the thickness of any coloured plate, and, therefore, infer the length of an undulation of any ray of light under examination.

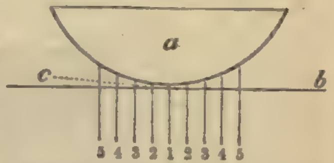
Fig. 26.



We have thus endeavoured to explain how we are enabled to measure the thickness and length of these plates of air; and having once ascertained the facts of the case, their general application is easily arrived at. Thus, if we observe that a soap or oil film presents the same colour as that which we find at any part between the lens and the glass plate, we at once infer that the thickness of each is equal. There is, however, a limitation of the law, which we must carefully bear in mind. It is, that the thickness of plates of different materials reflecting the same coloured light, is inversely as the ratio of their index of refraction. Hence plates of various transparent media, such as air, water, glass, oil of cassia, &c., are of different thicknesses when producing similar optical coloured effects.

We append a rough illustration of the position of the colours as observed between the lens and the plate, and a table of the thickness of the coloured plates; but must here state, that for the sake of simplicity, we have confined the attention of the reader to the production of only one order of rings. There are, however, several of these produced. The relative thicknesses and positions of the first order are pointed out below, and represented in the annexed engraving. In Fig. 27, *a* represents the lens placed on the glass plate *b*, and *c* is the interval existing between the two. In the annexed table, the numbers in the figure above indicate the position of the reflected colours placed before them; and the thicknesses of these are also given for the series of rings, commencing from the nearest or central point of contact.

Fig. 27.



	Colour.	Thickness in parts of an inch.
No. 1	Black	.000005
„ 2	Blue	.000024
„ 3	White	.000052
„ 4	Yellow	.000071
„ 5	Red	.000090

The above being the reflected colours, it follows that their complementaries will be seen by transmitted light.

The means of measuring the length of an undulation being obtained, we can next proceed to calculate their speed. We have already given a table of these;[*] to which we now refer, whilst we enter into their calculation in detail.

Having found the length, in parts of an inch, of an undulation, and knowing by the eclipses of Jupiter's satellites, that light travels at the rate of 192,000 miles per second, in round numbers—it follows that there will be as many undulations of any coloured light, in a second of time, as there are parts of an inch equal to the length of one wave in a distance of 192,000 miles, which the ray has travelled during one second. (See also p. 168.)

We have previously mentioned,[†] that an extreme red ray has a length of undulation of 0.0000266 of an inch; and if we divide 12165120000, the number of inches in 192,000 miles, by this length of one undulation, we obtain the result of about 458 billions of vibrations in one second of time, as producing the extreme red ray of the prismatic spectrum. The same result is afforded by multiplying the number of undulations per inch by the number of inches in 192,000 miles. Thus:—37640, the number of undulations per inch for the red ray, multiplied by 12165120000, the number of inches in 192,000 miles, gives 457,898,116,800,000 as the number of undulations per second of a ray of red light.

We have been particular in detailing the mode employed in calculating this result, because we are not aware of any work in which the entire process has yet been mentioned. The student will not fail to be impressed with the astonishing minuteness and rapidity of the undulations which produce the magnificent effects of colour; and perhaps we can scarcely point out any instance in the range of science, in which so great a number of proximate causes is constantly required to produce one well-known, and therefore, perhaps, often unappreciated result.

We need scarcely add that the undulations of each of the other coloured rays are calculated in a similar manner for every band in the spectrum. As a matter of practice, and to impress the laws which we have been here illustrating, the student will do well to continue these calculations of the speed of each undulation, taking the table before mentioned as a guide, and as affording the requisite data.

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[*] See *ante*, p. 41.

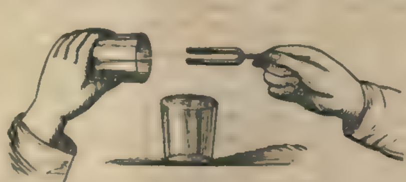
[†] *Ante*, p. 41.

An immense variety of similar effects of colour to those to which we have alluded, is found to occur from a variety of causes. Thus we see the curious play of colours on the surface of melted lead, and heated copper and steel; the beautiful appearance of mother-of-pearl, fringes in thick glass, which, after heating, has been dipped in cold water, &c.: but we must enter more deeply into the philosophy of the question, by inquiring into the laws of what is termed the *interference of light*.

This subject may present some little difficulty at first sight: we will, however, by illustration and analogy, endeavour to make it as plain as we can; and we shall connect the remarks we have often made in reference to the undulation of an ether, as illustrated by sound, and the circles produced in water when a pebble is cast therein. If two sets of waves are produced on the surface of still water, it will be found, that at such places where the waves are of different depths, there they will naturally destroy each other. This is easily seen by casting two pebbles into a pond, at some little distance from each other, and watching the intersection of the waves thus produced. If the two sets of waves are of a similar nature, or in a like phase, their resulting effect will be double that of each taken separately. Now, in the same way as in the cases we have instanced, when an undulation of ether is interfered with by what we may call a half undulation, the result is, that either darkness, that is, absence of light, is produced, or colours varying in their nature, as observed in the spectrum, occur.

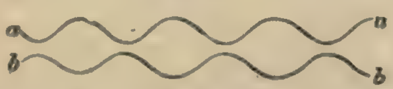
The subject may be still further illustrated by means of a tuning-fork and two glass cylinders. On striking the tuning-fork, and placing it over one of the glasses standing in a vertical position, the air in the glass, and the vessel itself, will also vibrate. If, however, the other glass is placed so that its mouth is at right angles with that of the vertical glass, all sound will immediately cease, owing to the *interference* of the two sets of waves of sound thus coming in opposition to each other. The mode of conducting this experiment is represented in Fig. 28, wherein the glasses are found in that position in which the interference of sound can be effected.

Fig. 28.



Now, substituting ether for the air (so far as light goes) and water in the two instances we have just selected, the student may imagine two sets of waves thus interfering with each other, one of these being represented by the line *a* in Fig. 29, and the other by the line *b*.

Fig. 29.



We here find, instead of the depression and altitude of each of the waves being synchronous, or co-incident with each other, they are actually opposed to each other, and that the difference of the position of each curve of the wave is equal to half an undulation. The result is, that at the end of an undulation, the two waves are in different phases or positions, and *interference* results therefrom.

We might extend our analogical illustration still further if necessary, and compare the different coloured plates produced by the interference of light with the chords which a skilful player on a stringed instrument is enabled to produce by the combination of vibrations of different lengths. We trust, however, that our readers have understood the drift of our remarks, and we shall, therefore, not pursue our explanations to any further extent.

To apply these doctrines, we may add, that the colours of thin plates, &c., are the result of the interference of two sets of *reflected* rays with each other, when such are viewed from the surfaces of the glass plate and lens; and the complementary colours observed by looking *through* the lens and glass, are the effect of the interference of the *transmitted* rays with each other.

We may here explain more fully than we have hitherto done, the meaning of the term "complementary colours." If the eye watch one colour intensely for some time, on removing this from the sight, an image of the object, of an entirely different colour, will be perceived. A simple way of trying this, is by placing a red wafer on a sheet of paper. After looking at this intensely for a few minutes, the eye will perceive a green image of the wafer, if the sight be directed to another place. Sheets of deeply coloured paper answer a similar purpose; and thus, if a person look intently for any length of time on the back of a book or other object of a deep red or yellow colour, a spectral image will be observed, of a green or violet colour respectively.

The colours so observed are complementary to those of the object; or, in other words, are those which, with that of the object, would together produce white light. For the information of the student, we append a table of the different colours of the spectrum, or of any object, and of their corresponding complementaries.

Colour of Object.	Its Complementary.
Violet	Yellow.
Indigo Blue	Orange.
Green	Red.
Yellow	Violet.
Orange	Blue.
Red	Green.

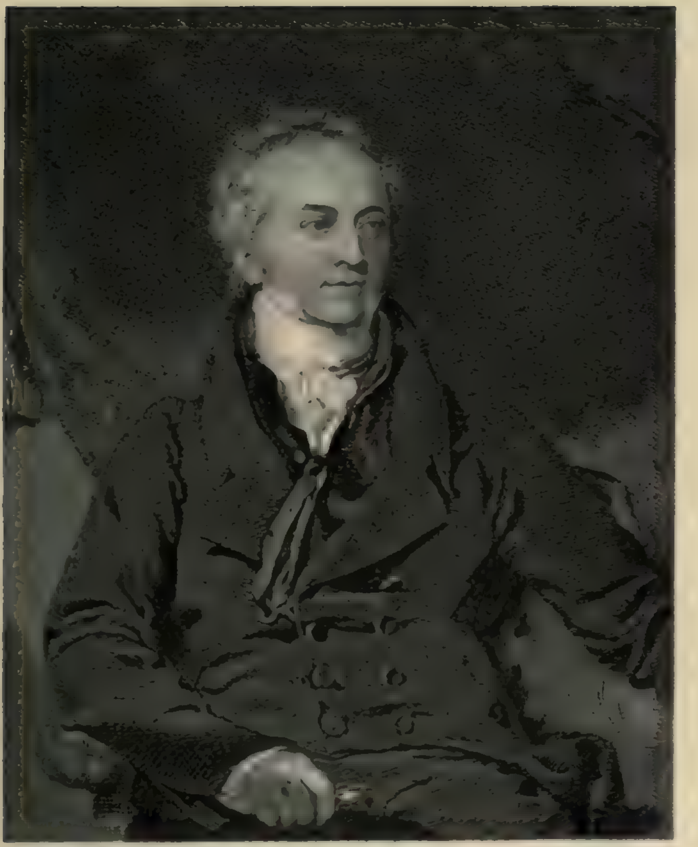
A very interesting application of this property of colours is often made in economic and commercial affairs. Coloured fabrics are generally arranged in covers, or papers of such colours as are complementary to the articles they enclose. Thus a green silk or cotton fabric appears of a far richer colour when wrapped with, or placed on, a sheet of reddish paper, than when placed on a white one. By judiciously choosing such arrangements the saleable nature of the article is often enhanced. This is another instance of scientific laws being often applied by persons ignorant of their existence, who, however, by long practice have blindly arrived at truth. Great taste is often exhibited by ladies in the arrangement of the colours of the different parts of their dress, in their unwittingly following the principles we have laid down. We may here, perhaps, be permitted to become the apologists for some of the fair sex, who do not seem to be characterised with correct taste in this respect. We do so by stating, that it frequently occurs that such have not the optical power of judging of such matters, and therefore are not to be blamed for what, at first sight, would appear to indicate an absence of refinement or judgment. We shall enter more fully into this question hereafter, when we consider certain physical defects in the construction of the eye, which incapacitate many persons, more or less, in discriminating coloured objects.

The law of contrasts of colours, as applied to dress and to commercial purposes, although of great interest, falls scarcely within the scope of our plan. We have, however, much pleasure in recommending the perusal of a work entitled *Colours and their Contrasts*, by M. Chévreul, a French chemist; in which the subject is treated in a complete and masterly manner, and in which all the laws of complementary colours are fully entered into, and their applications judiciously pointed out.

DIFFRACTION.

IN connection with the interference of the undulations of light, we must now refer to the phenomena of *diffraction*, or the production of coloured fringes by means of special arrangements acting on a diverging ray of light.

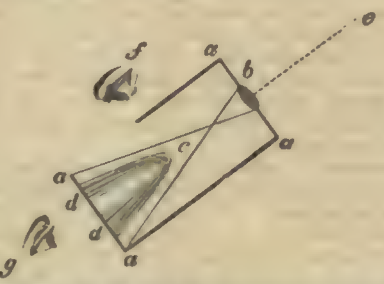
A very easy mode of observing the effects of the diffraction of light, is by admitting into a dark room a ray of light through a short-focus lens, and placing a knitting-



Thomas Young MD
Sec. R.S.

needle at any place beyond the focus whence the rays diverge. If the shadow of the needle be received on a sheet of paper, fringes of coloured light will be observed thereon. A convenient instrument for the purpose may easily be constructed out of a common camera, by removing the ground-glass plate and the mirror, and by replacing the convex lens in front by one of a very short focus—say not exceeding an inch. The opposite end is covered with a sheet of white paper; and the object whose shadow is to be obtained, must be placed between the paper and the focal point of the rays passing from the sun and through the lens. The observer can thus perceive the coloured fringes on the paper by looking through the open door generally found in the old form of the camera. Our annexed illustration will give an idea of the construction of this apparatus.

Fig. 30.



a a a a, represent the usual form of the camera-box; *b*, a lens of short focus; *c*, a needle producing the shadow; *d d*, the coloured fringes received on a paper screen, resting on the end of the box; *e*, a ray of light entering the lens *b*; *f*, position of the eye required to observe the rings by reflection.

The above arrangement, whilst extremely simple, has the advantage of being easily held in such a position as to be at any time in the direct rays of the sun, or of an artificial light. The end of the box, *a d d a*, may be taken out and replaced by a sheet of ground glass, when the fringes may be observed through it, as at *g*, instead of on the surface of the paper screen. The observer will perceive that the white light has been completely decomposed, and the fringes present the same colours as if produced by the glass prism. The first set of fringes exhibits the colours violet, blue, green, yellow, and red; the next, blue, yellow, and red; and the third, the same colours as the second, but fainter in their tint. If the object by which the diverging rays pass be moved to and fro between the light and the screen receiving the fringes, the latter increase in width.

The effects thus observed are to be explained on identical principles with those which we have already referred to when describing the phenomena of the interference of two sets of undulations. And the fringes of colour, or the dark shadows, are simply due to undulations of different lengths interfering with each other. Having already entered fully into the philosophy of the question,* we pass on to consider some modification of these experiments.

Experiment 3.—In the diverging beam place a piece of metal through which a very fine hole has been made by means of a pin's point, and between this and the eye place a convex lens. A circle of beautifully coloured rings will be at once observed, which will expand or contract as the lens is shifted to and fro between the eye and the hole in the metal plate. A piece of thick tinfoil answers as well for this experiment as the metal plate, if a hole be pierced through it, as already described.

A great variety of arrangements, wherein two surfaces placed close together and introduced into the diverging beam, may be employed. Amongst these may be two knives, whose edges are together at one end, and slightly separated at the other; gauze-work, either of vegetable fibre or of metal wires; perforations in any opaque object; and, indeed, any object which by interfering with the continuity of the rays of the diverging beam, will tend to produce the phenomena we have mentioned.

In certain instances, the centre of the shadow will present a black line, or point, where the undulations have destroyed each other by their mutual interference. In other cases a line of white light is produced. The student will here perceive a practical illustration of our previous remarks on the production of light, or its absence, by the motion of the waves of ether being destroyed in certain cases, and intensified in others, just as these undulations do or do not interfere with each other.

It will be noticed that the rays passing from an interfering object are not in straight lines. They follow the curve of a hyperbola, and so reach the screen, as mentioned in our description of the simple apparatus whose use we suggested when introducing this subject.

DISPERSION AND ACHROMATISM.

We have already mentioned, that different transparent media have varied powers in dispersing or spreading the rays of light transmitted through them. We now propose to investigate the subject more fully, and also to refer to a very valuable application of the laws we shall meet with; namely, the construction of achromatic lenses.

If solar light be transmitted through two lenses, one of crown and the other of flint glass, being similar in every respect excepting in the material of which they are made, it will be found that the spectra they produce differ very materially. They affect the length and position of the spectrum produced—that of flint glass acting to the greatest extent in these respects. This is owing to the fact, that flint glass has a greater dispersive power on the rays of light than crown glass. Moreover, it is found, that certain substances extend one end of the spectrum more than others, some having a special influence on the blue, and others on the red rays; and thus either of these predominate over the other, according to the material of which the prism is composed; hence arises that which has been called the irrationality of the spectrum.

As we have already remarked, two prisms of different material may be so arranged, that the light passing through them shall not be separated into a spectrum, and but comparatively little coloured light will thereby be produced. By an analogical process, achromatic lenses are constructed; that is, lenses which do not give coloured images such as those which one material always affords.

To understand what is called "chromatic aberration," or the cause of these coloured images when ordinary lenses are employed, the student may receive the rays of the sun through a lens on a sheet of white paper. If the light reach the paper at a less distance than that at which the general focal point exists from the lens, then he will perceive a ring of a blue colour. If the paper be placed further from the lens than the focal point, then a red ring will be observed. This effect may often be noticed when the lenses of a common magic lantern are being focussed, the light proceeding therefrom being received on a white screen. A red, or blue hazy colour is thus produced, and is an instance illustrating the fact, that the red and blue rays focus at different points after proceeding from the lens, owing to unequal refrangibility. This effect is what is called *chromatic aberration*.

The student will observe the general effect of chromatic aberration by looking at an object through a common lens, when he will perceive the external parts of the image to be coated with coloured fringes.

This gives rise to great inconvenience when such lenses are employed in telescopes, microscopes, photographic cameras, and other optical instruments. However, a knowledge of the cause of any defect very often suggests its remedy; and this occurs in reference to the subject with which we are now dealing.

It has been found that, by combining lenses composed of materials having different dispersive powers, a single lens almost achromatic—that is, giving no coloured images—may be produced which affords a focus where all the rays meet simultaneously. In practice, this is done by combining a flint with a crown or plate glass lens. In some instances more than two lenses are employed;

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* See *ante*, p. 52.

and the result is, that the coloured fringes are almost entirely neutralised, the lens becoming perfect for all the purposes of the optician.

Fig. 31.

We shall defer entering into the great value and essential nature of these improvements in the construction of lenses, until we deal with optical instruments. It will be desirable, however, that the student should fully master the principles which we have here explained, so as to avoid the necessity of circumlocution, on our part, when we have to refer to their applications.

Fig. 31 represents a section of an achromatic lens; wherein a double concave lens of flint glass, *a*, and a double convex one of crown glass, *b*, are united together.

EPIPOLIC DISPERSION.

A PECULIAR kind of dispersion of light is effected by some substances both in a solid and liquid state; which, from its connexion with the chemical rays of the spectrum, has become a subject of considerable interest in the science of Optics.

If a solution of sulphate of quinine in dilute sulphuric acid be poured into a glass of water, it will be observed that a peculiar blue tint pervades the liquid, whether it is viewed by reflected or transmitted light. The attention of scientific men was first called to this effect by Sir J. F. W. Herschel; and it has been since fully investigated by several physicists.

Similar effects to those produced by the solution of quinine may be observed in an infusion of the bark of the horse chesnut; the green colouring matter of leaves, obtained by leaving them some considerable time in spirits of wine; a solution of the colouring matter of some kinds of sea-weed; glass coloured by the oxide of uranium, &c. We have lately noticed the epipolic appearance whilst pouring semi-congealed oil from a glass bottle, and also when the juices of some kinds of lemon are squeezed into a glass of cold water. The cause of this phenomenon is a peculiar and internal dispersion of the refrangible rays of light. Indeed, Professor Stokes shows that the refrangibility of some rays is changed. It is found, that light which has thus been affected, has not the power of transmission through a solution similar to that which has been employed to produce it. Thus, a solution of quinine placed in a glass vessel, and viewed through another vessel containing a similar solution, does not present the blue appearance which may be seen when it is viewed alone. In other words, the passage of the epipolised rays is completely stopped by the second solution. The investigations which have been made on this subject have much assisted in illustrating the nature of the rays of light existing *beyond* the visible portion of the spectrum, where the chemical power of light is found to be most active. To this we shall further allude under the head of Photography, when we enter into the philosophy of the various processes involved in that interesting application of the effects of light.

Having thus pointed out some of the chromatic effects resulting from the interference of rays of light and the undulations of ether, as producing these effects, we now proceed to investigate a series of singular and interesting phenomena connected with mono-chromatism, and the production of Fraünhofer's lines or spectral bands. We shall also introduce an account of a very interesting discovery made by continental chemists; from which we shall not fail to observe how close is the connexion between the matter with which we are surrounded, and those forces which are at once latent in it.

MONO-CHROMATIC LIGHT.

IN previous pages we have supposed that ordinary white light has been employed for our experiments and investigations. Under certain circumstances, however, a variety of interesting phenomena is presented when light of a single colour is used. Mono-chromatic light, as its name indicates, simply presents one colour to any object on which it falls; and such objects, therefore, only reflect that colour in its simple or partially modified state, to the eye viewing them.

As all the objects generally perceived by the eye are seen by the aid of white light, which (with comparatively trifling exceptions) is composed of the colours presented by the spectrum; it may be well that we should afford some instances which will illustrate the peculiar phenomena arising when single-coloured light is similarly employed. For this purpose the following experiments may be tried.

Experiment 4.—Put a table-spoonful of common salt into a glass bottle containing two ounces of strong spirits of wine, and agitate them together until as much as possible of the salt is dissolved. Pour some of the clear solution on a piece of tow or on a sponge, and inflame it in a room where no other light is present. All objects of a light colour will present a pale yellow tinge; whilst those of a dark tint, such as deep blue, &c., will appear of a gray or black colour. The human face presents a most ghastly appearance when viewed by this light, simply because it reflects no colour except that afforded by the burning spirit.

Experiment 5.—Hang a few light-coloured ribbons on a stick, and view them by means of the light produced in the last experiment. If light prints, yellow, and orange are used, each ribbon will present the same colour, owing to the mono-chromatic nature of the light employed.

Experiment 6.—After observing for a short time the effect of this colour on the ribbons, introduce the light of a candle. This source of light being nearly equal to white light, will represent the original and natural colours of the ribbons.

As some of our readers may be desirous of repeating this experiment in public, or as a class illustration, we may mention, that the best mode of so doing is, by burning a large quantity of the spirit on a piece of tow; and instead of employing ribbons, a very effective mode is that of clothing the upper part of the experimenter with folds of glazed calico of different colours. The effect on the face and the clothing is thus perceived simultaneously. If the oxy-hydrogen light be then suddenly thrown on the operator, or the light of a powerful gas flame, the original colours will be at once restored. They are, of course, immediately lost on the extinction of the white light, and subsequent employment of the mono-chromatic. This, if well managed, forms an excellent experiment for the lecture-table; and is a good illustration of the effects produced by either the presence or absence of those rays which are essential for viewing bodies in their ordinary state.

Other solutions of salts in alcohol produce flames which have characteristic colours. Thus a solution of nitrate of baryta or boracic acid in spirits, produces a flame of a green colour; and nitrate of strontia gives one of a rich red colour. In the same manner these salts afford rich green or red flames when burned with charcoal; and, under the names of green and red fire, are largely employed in theatrical representations for numerous scenic effects. By the use of these mixtures the transformation scenes of the pantomimes are always illuminated. We have already referred to the effects produced on the natural colour of bodies by the spectrum at a previous page;* and that, with the experiment we have just mentioned, afford illustrations of the effect of single-coloured light.

We have supposed that ordinary artificial light is identical in its physical character with that of the sun; but such is not actually the case. The nearest approach which we have to solar light is that produced by means of charcoal points, forming the terminals of the voltaic battery. This light (commonly called the electric light) is one of great intensity; and every coloured body appears, as near as possible, of the same tint as when viewed by means of sunlight. The lime light comes next in order, with respect to the purity of its rays; but is much inferior to the electric light. A very beautiful white light is produced by burning phosphorus in a jar of oxygen gas; which has a similarly optical effect to that produced from the two previously named sources.

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* *Ante*, p. 49, *et seq.*

When certain colours, especially blue and green, are viewed by means of a gas or candle flame, it is found that, in many instances, the actual tint is quite lost; for some blues when thus seen appear of a green hue. This is owing to the predominance of the *yellow* rays in the source of light; which, by mixing with the colour reflected from the object, produce a green hue. We have already explained the reason of this when speaking of the prismatic spectrum, as a result of the overlapping of two colours. In commerce, it is therefore impossible to judge of natural colours by ordinary artificial light; and this is well appreciated by ladies in purchasing articles of any intermediate spectral tint. This fact is another, though partial, illustration of the laws of mono-chromatic light.

In connection with this subject, and also with a discovery made by Fraünhofer, we shall introduce some remarkable facts lately published by MM. Bunsen and Kirchkoff. These gentlemen, by means of the prism, have been enabled to identify the peculiar colour of the vapour of each metal whilst the solid is undergoing combustion. Most of our readers will be familiar with the colours produced by some metals when undergoing combustion—as seen in the blue flame of zinc, the red of strontium, and the green of barium. Instances of the two latter we mentioned in the previous page. Now these are easily distinguished from each other when separately in combustion; but the eye might fail to detect their presence when they are burned in contact with other substances. As, however, each coloured or mono-chromatic light has a peculiar effect of its own in producing a special spectrum when refracted by a prism, MM. Bunsen and Kirchkoff have availed themselves of the fact, and have thus made the prism an instrument which may be employed as an agent in chemical analysis. As the discovery has only been made public but recently, we cannot, at present, afford more information than that which we append, and which is a condensation of their published experiments, extracted from the *London Review*, of January and March, 1861.

"MM. Bunsen and Kirchkoff have, by a complicated arrangement of prisms and telescopes, arrived at such certainty and delicacy in their results, as to be able to see, by a glance at the most complicated mixture (of burning metals), through the eye-piece of their instrument, its constituents ranged side by side, each individual metal glowing with its own peculiar colour. By thus removing the interfering action which coloured fires would naturally have if all were united and burnt together, they were at once placed in possession of a very delicate method of chemical analysis. Not only could they have visible proof of the existence, in any mixture, of a body capable of giving a colour to flame; but this effect was produced by such an inconceivably minute quantity of the element as the mind almost fails to comprehend—the one hundred millionth part of a grain being thus capable of detection. During their examination of these phenomena, every mineral substance which came to hand was optically interrogated in this searching manner, and in all cases with remarkable results—bodies which had hitherto been looked upon as so rare as to be merely chemical curiosities, being seen to form constituent parts of most rocks, soils, and waters. Some mineral waters, however, gave results which could not be accounted for by any of the previously observed facts; a new colour starting up before their eyes, and occupying a hitherto blank space in the glowing row of metals. Evidently this was a new element, which has thus revealed itself to an optical inspection.

"The next point was to collect some of the unknown body. * * * * After boiling down twenty tons of mineral water, and submitting the residue to various chemical agents, the latest accounts report that they have at their disposal upwards of one hundred grains of cæsium; this being the name under which the new metal enters the domain of science, given to it to recall the colour which it communicates to flame."

In another number of the same periodical (March 30), a further account has been published respecting these interesting facts; from which we make the following extracts:—

"Mr. Crookes, the editor of the *Chemical News*, has just discovered that the sulphur group of elements contains another member. The appearance shown by the spectrum of this new body, when a little of it is burned in a gas flame, is very remarkable; and, as the discoverer has repeated before us many of his experiments in his laboratory, we are enabled to give a full account of the grounds upon which he bases his claim for the elementary character of this newly discovered body.

"Nearly every elementary substance in nature (and recent developments of this branch of science would almost warrant us in saying *every* element) possesses the property of imparting certain colours to a flame, provided that the latter is hot enough to volatilise the body; and this holds good not only with the elements in the free state, but to all their compounds. When these colours are examined by a prism, in an appropriate spectrum apparatus, they are, in most cases, resolved into a series of brilliantly luminous lines of every conceivable tint and colour—some sharp and defined as if ruled by hand, others shaded off at the edges, and having a nebulous appearance; but each system of coloured bands being invariably associated with the same element, and thus being as accurate a test of its presence as the most rigid chemical analysis could devise: in fact, far better; for chemists, being human, may err; whilst the laws of light and refraction can never fail." * * * * * *

"The new body which we have mentioned above as having been just discovered, came to light in this way. A new mineral from the Hartz mountains having given some very discordant results in ordinary chemical analysis, a portion was subjected to spectrum examination, when the disturbing cause was at once seen in the shape of a beautiful green line, glowing on a pale background of the same colour. Our experimentalist having an intimate knowledge of the appearances which the bodies present ought to give, at once pronounced this line to be an interloper, and set to work to isolate its cause. After numerous trials and failures, constantly appealing to the spectrum for information as to how the stranger behaved under the various and unusual methods of treatment to which it was being subjected, success at last crowned his effort, and Mr. Crookes is now the happy possessor of a whole grain of the new element. The appearance of its spectrum is very striking. The discoverer stationing himself at the gas-burner, placed us in front of a powerful telescope, with which the instrument is fitted. At first, only the green and blue bands due to the gases in the burner were visible; but suddenly a bright flash of green light illuminated the field of view, and the new element was glowing before us as a sharply-cut brilliant green line, on a jet-black ground. The only elements which at all give a similarly striking spectrum, are sodium and lithium; and the equally beautiful lines of these, which were afterwards exhibited simultaneously with the green line, are in entirely different portions of the spectrum, besides being of different colours—one being crimson, and the other yellow."

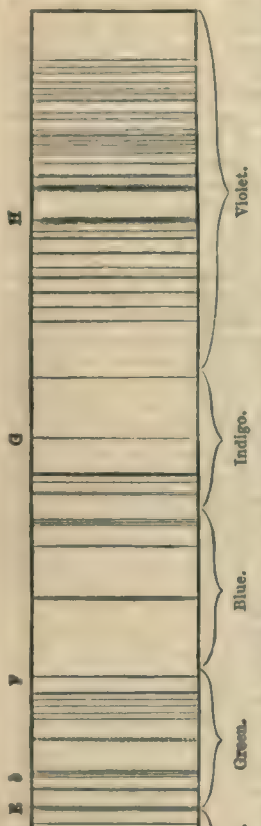
These results are obtained by viewing the spectrum produced from the flame by means of an achromatic telescope, which is employed in a similar way to that we shall presently describe as adopted for viewing Fraünhofer's lines. We need scarcely add, that this discovery, although still in a crude state, is of the highest importance in experimental science, and promises to add largely to our stock of knowledge of the chemical characters of elementary bodies, and also to add to their number.

FRAUNHOFER'S LINES.

It was discovered by Dr. Wollaston, and independently by M. Fraünhofer, that there exists, throughout the length of the spectrum obtained from any source of light by means of a flint-glass prism, a number of black bands or lines, which cross the spectrum perpendicularly to it. One of the most surprising facts connected with these lines is, that their number and position vary for every kind of light. Thus, there is a set of lines for solar, lunar, and planetary light; which being obtained from the same independent source, the sun, are productive of identical bands; whilst the light of the fixed stars, of coloured flames, &c., each presents these bands in different numbers or positions when compared with each other, or with solar light.

Bearing in mind the remarks which have been made in reference to the discoveries we mentioned in the previous section, the reader will not fail to perceive, that this phenomenon of spectral lines is of great importance both in physical and chemical researches: we shall, therefore, endeavour to explain their characteristics, and to afford an opportunity for the student to observe them, by describing an apparatus which will answer that purpose.

Fig. 32.



In the annexed engraving we have an illustration of the appearance of these dark lines, as obtained from solar light. We have, however, represented but a small number of them, because the limited size of our page would not admit of a greater extension.

We are indebted for the annexed cut to Sir J. F. W. Herschel's article on light, in the *Encyclopædia Metropolitana;* and it affords the reader a representation of the mutual position and comparative frequency of these bands in a spectrum of solar light; and showing, at the same time, their connexion with the different coloured rays. The Roman capitals at the upper part of the diagram show the position of the larger and more defined lines; the italics represent narrower lines placed close to each other. Between the capital letters numerous fine lines are exhibited; and their position in reference to the coloured rays is indicated by the names of those rays at the foot of the engraving. In all, about 1,000 lines have been counted as existing between each end of the spectrum.

Our remarks on interference* will prepare the student to understand the cause of these lines. We there stated, that when undulations of different phases, either of sound or light, met each other, or rather, perhaps, united together, the result would be their mutual destruction. Now there is no doubt that these black lines arise from a somewhat similar cause; and as they show the absence of light at any particular point where they exist, we can but conclude, that certain undulations which would, if unimpeded, have produced light in such places, must have been destroyed, lost, or absorbed from the rays incident on the refracting prism during their passage from their source.

We have already remarked that every light presents a different series of these lines. In some they are comparatively absent, or are differently placed. On reflection, we cannot but perceive a most astonishing instance of design in that in which we could least have expected to have discerned such. Every ray of light can be identified from its fellow; and there is not a greater distinction between human faces in different nations than exists here. The generic likeness is complete, but the individual characteristic and difference are not less so.

We have, at a previous page,† entered fully into the laws of single refraction; and also explained how the index of refraction may be found for various bodies. Fraünhofer applied, however, the position of the black spectral lines to ascertain still more accurately the refractive indices of different media. The principle of this system will be easily understood, if the student bear in mind that, if the rays of the *same kind of light* are refracted through *different transparent media*, then the black lines thus produced by each medium differ in their distance from each other according to a fixed law. Now, by comparing these mutual distances in terms of each colour of the spectrum, the refractive and dispersive power of any body can be ascertained, and measured with mathematical accuracy.

It would, of course, be impossible for us to illustrate, by means of engravings, this most interesting fact. Such is entirely a work for exact experimenting, and can only be conducted by those who, through long experience, are prepared to enter into such delicate researches. Having, however, given an exposition of the principles on which these investigations are conducted, we proceed to describe the arrangement which may be employed for the purpose of viewing these lines, and in measuring the distances between them.

Sir John F. W. Herschel recommends that the lines should be viewed by means of a good achromatic telescope. For this purpose, by means of which a magnified view of the spectrum is gained, the colours obtained by the use of the prism‡ are allowed to fall on to the object-glass of the telescope. The lines, which are too close together to be seen by the naked eye, being magnified, may thus be easily perceived, and those in each colour of the spectrum may be separately examined. In using the telescope, the object-glass takes the place of the screen usually employed to receive the spectrum rays.

Mr. Ladd, of Beak-street, London, has lately manufactured an apparatus, constructed on the same principles as that which was employed by MM. Bunsen and Kirchkoff, which can be used also to discover the presence of M. Fraünhofer's lines in various spectra. It is a modification of that which we have just described, and will, doubtless, prove of great value as an instrument of research in the new branch of what we may call Optical Chemistry.

THE EFFECTS OF LIGHT ON ORGANISED BODIES.

In a former page§ we have noticed, generally, the effect which light has on the colour of vegetable and animal bodies. We here propose to enlarge on that interesting subject, and to suggest some experiments which may easily be repeated by the student.

The most cursory observation of the appearance of plants in a growing state, is sufficient to show that light has a great influence on them. This may be well observed in a garden whose walls run east and west. Plants growing on that side which is shaded from the solar rays will be found generally in a sickly state, and their flowers, fruits, &c., are few, and imperfect in their development; whilst those growing beneath the north wall, with a southern aspect, and thus continually exposed to the full rays of the sun, reach their proper maturity and size.

We may also notice that plants, trees, &c., invariably shoot out more branches on the side on which they are exposed to the sun. This may be well illustrated by placing a geranium or other quick-growing plant in a window with a southern aspect, and retaining it in one position for two or three weeks. It will be found that numerous shoots spring out towards the window; whilst that portion of the plant which is not exposed to the external light, scarcely evinces any sign of growth. If, however, the shaded portion be turned round, it will, owing to the action of light, soon show active signs of life, and will rapidly bud forth. The common hollyhock, which has a long stem, may be thus twisted into a screw-like shape by exposing its sides alternately to light. Its tall stem gradually bends towards the sun during its growth, and it may thus be cited as another and excellent illustration of the effects of light on the growth of plants.

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* See *ante*, p. 52.

† See *ante*, pp. 45, 46.

‡ *Ante*, p. 49.

§ *Ante*, p. 50.

Even by means of artificial light, plants may be grown which will present nearly the same colours as they would if freely exposed to the rays of the sun. A geranium plant may thus be brought up by confining it in a place excluding the light of day, if a powerful light, such as from a paraffin or gas lamp, be employed. This interesting experiment may easily be repeated by the student if a little care is exercised. To show the effect of even artificial light, two equally healthy plants should be chosen, and whilst one is exposed to the light of the lamp, the other should, for an equal space of time, be entirely secluded from light of any kind. The latter will speedily fade, whilst the former will flourish pretty well.

Now, it is found that certain of the rays of light have more effect on germination and growth than others. Mr. Hunt tried a very ingenious experiment for proving this. He cast the rays of coloured light obtained in the usual way from a prism upon some cress, and found that that portion of the plant growing under the *blue* rays grew vigorously, whilst that under the red rays bent away from them as from the face of an enemy.

One of the simplest modes of trying this experiment, is that of exposing cress seed to coloured light, produced by passing white light through glasses of different colours. Three portions of common cress may be placed behind a piece of red, yellow, and blue glass respectively, and the relative effect may thus be readily observed. Since the discovery of the chemical effect of the blue rays of light, the principles to which we have alluded have been put into practice in the construction of greenhouses. Instead of using white glass, which permits the flow equally of all the rays of light and heat, greenish or blue-coloured glass is now much employed, and great advantage thus arises in respect to the growth of the plant, and the production of its flowers and fruits.

The spectrum, however, as we have already remarked, presents rays having three characteristics—the red being associated with the heating, the yellow with the luminous, and the blue with the chemical effects of light; and each of these emanations has a specific office in connexion with vegetable life. It has been stated, as the result of experiments by Dr. Draper, of New York, that the yellow rays assist the nutrition of plants, whilst the blue rays are essential to their movements or expansion in size or direction.

Without entering, at present, into the department of organic chemistry, we may state, that all plants thrive through the chemical changes which their organisms effect on the carbonic acid contained in the atmosphere surrounding them. It may appear paradoxical to those unacquainted with the many astonishing facts presented to the philosopher, when we state, that the greater part of plants, from the creeping moss on our garden walls, to the stately oak tree of our forests, is obtained not from the earth by their roots, but is actually received from the air, by the decomposition of the carbonic acid, and the assimilation of its carbon or charcoal to the body of the vegetable. During the day-time, the rays of light assist and give vital energy to the plant. Its leaves absorb the carbon from the carbonic acid, and set free the oxygen which is combined with it. They thus literally live on air. In the daylight, all plants give off oxygen; whilst at night, their vital force being diminished in intensity, they are found to undergo a partial decomposition, and to *give off* carbonic acid gas.

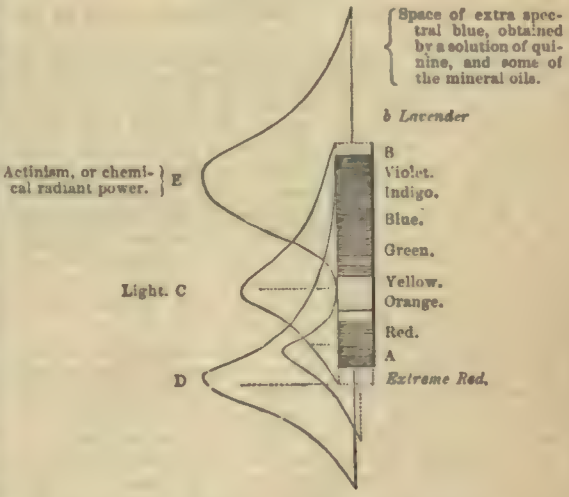
To popularise still further these interesting principles, we may state, that by excluding light entirely from a growing vegetable, we as effectually destroy its chance of vitality and growth, as we should do in the case of a human being by sinking him in a pond of water. Indeed, the analogy is quite complete, because the cause of death is identical; namely, the asphyxiating effect of an undecomposable gas on the "lungs" of either object.

We cannot omit here to mention a beautiful illustration which is found in this instance, of the universality of design in nature. All animals expire from their lungs carbonic acid gas, which is to them a deadly poison. The lungs—that is, the leaves of trees—actually breathe this poison, and, assisted by the rays of light, they decompose it, returning to the air the oxygen, which is to us its vital portion, and retaining the solid part, or carbon, for their own subsistence. Thus, what we poison, the plant purifies, and a grand compensating action is continually kept up between the products and requirements of the animal and vegetable kingdoms.

The cause of those peculiar effects produced on vegetable and other objects by the blue rays of light, and to which the chemical changes observed in the photographic process are due, has been termed *actinism;* and when we refer to the *actinic rays* of light in our future pages, the reader will understand that we speak of those which have the powers to which we have here alluded, in contradistinction to those of the luminous and calorific rays of the spectrum, which do not possess them.

To render our remarks better understood, we here introduce a diagram, which will give a clear idea of the relative position and space occupied by those three classes of rays in the spectral image, as received on a screen; as also of those rays which extend beyond the visible spectrum, both at the red and blue apparent terminals. (See Fig. 33).

Fig. 33.


Space of extra spectral blue, obtained by a solution of quinine, and some of the mineral oils.
b Lavender
B
Violet.
Indigo.
Blue.
Green.
Yellow.
Orange.
Red.
A
Extreme Red.
Actinism, or chemical radiant power. E
Light. C
D

In this figure we observe, on the right-hand side, at the top, the place of the epipolic rays, to which we have already alluded.[*] Below these we have the lavender, violet, indigo, and blue, which are the chief seat of the actinic, or chemical rays of the spectrum. This is indicated at E, on the left of our engraving. The green rays, which seem to form the middle of the spectrum both in a physical and in actual position, are succeeded by the yellow and orange, which are those most luminous in their character, as shown at C. Following these are the red rays; in which, as at D, the heating effects of the prism are most manifest. The student will thus, by studying the above engraving, be enabled to experimentalise directly with any of these three classes of rays independently; and of thus repeating some of the interesting effects of coloured light on plants, &c., which we have already named. With respect to the extension of the rays beyond the spectrum at B, it is found, that the actinic or chemical effect is active, as are the calorific effects beyond A. These represent the position of rays which cannot be observed by the eye, but which are respectively discovered by placing paper coated with chloride of silver beyond B, and a thermometer beyond A, when these peculiar properties will become evident. Progressing below the spectrum at A, and still further from the red rays, we shall gradually arrive at another spectrum, wherein the red of the first will merge into the extreme blue of the second; and hence, after passing the last heating in red rays, these are necessarily succeeded by others of an actinic nature, or capable of producing chemical changes.

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[*] See *ante*, p. 54.

In our section on Photography we shall have to avail ourselves of these principles to a large extent. We must, however, to save space, presume that our readers will at this point fully master the details of our analysis of the spectrum, and thus render continual repetition on our part unnecessary.

Before leaving this interesting subject, we recommend the student to try the following experiments, which will tend to impress the truths we have stated on his mind.

Experiment 7.—Prepare a strong solution of sulphate of copper in cold water, and add liquid ammonia to it until all the precipitate is dissolved, and a rich blue colour produced. Pour this into a flat glass vessel, and allow the rays of sunlight to pass through the liquid, and to impinge on a piece of paper coated with chloride of silver. This is made by dipping one surface of a sheet of white paper into a solution of common salt, and, after drying it, the salted side is placed in a solution of nitrate of silver, and subsequently dried. The actinic rays of light pass freely through the solution, and will speedily turn the paper black.

Experiment 8.—Repeat this experiment, but employ a weak solution of bichromate of potass in cold water. The prepared paper will scarcely change at all; because, in this instance, the liquid permits the luminous and calorific rays to pass, but entirely stops those of actinism.

Experiment 9.—Prepare three solutions—one as in Experiment 7; another as in the last experiment, so as to be of a rich yellow colour; and another with the same salt, so as to give a deep orange or red tint on looking through it. Expose at the back of each of these solutions a small thermometer, so that its bulb may receive the transmitted rays. That behind the blue solution will exhibit no increase of temperature; and that behind the yellow one will not indicate any extraordinary change; while that which receives the rays from the red solution will speedily evidence that heat easily passes, and that to a very much greater extent than through the two other liquids.

It will thus be perceived, that the three effects of light may be separated from each other; or, in other words, that which we conventionally call "light" may be divided into three phases, so far as its effects are concerned; namely, those of actinism, luminosity, and caloric.

We here append a few instances in which the action of the rays of light effect either physical or chemical changes on some substances; indicating, at the same time, the nature of the change which is produced. We, however, need scarcely remark, that our list is only illustrative, and, of course, far from exhausting the almost infinite number of such changes which are constantly occurring in nature, although they are rarely observed by us, because they take place so gradually. We shall omit those which more particularly pertain to photography, because such will come under our consideration when we treat on that subject.

The Human Skin.—Every one is familiar with the fact that the natives of the tropical regions are generally of a colour approaching to black. There are many causes which tend to produce this appearance besides that of light; and as such pertain rather to physiology, we cannot, of course, enter into their consideration in this place. We shall, however, deal with them in our section under that head, and also under Ethnology. We need scarcely remind our readers, that a lengthened exposure to strong sunlight, of inhabitants of temperate regions, has the effect of darkening the colour of the skin. This takes place most readily with persons whose complexion is fair. Light, however, has not only a colouring effect, but also must be regarded as having a decided sanitary influence on all constitutions. We hence find the ruddy and healthy appearance of out-door labourers, of sailors, and locomotive engineers. Instances are on record of persons who, having acquired deformity and disease through close confinement from daylight, have been speedily restored to health by subsequent removal to places where free exposure to the sun was obtained. Our experience of the beneficial effects of a temporary residence at the sea-side, is another illustration of this result; and such arises not simply from a change of air, but also from the fact, that the rays of the sun shine purer, and more free from loss and absorption of their actinism, when passing through an unclouded atmosphere, than through the smoke of our large cities. We have repeatedly tried experiments, during the same day, in different parts of London and Glasgow, with photographic apparatus; and have been surprised to find the variation of time required to obtain a good image on a sensitive plate, at places respectively free from, or beclouded with, the smoke of the city. Any one, especially of a fair complexion, may repeat these trials in a rough but interesting manner, by walking for an equal length of time, on two clear days, during the heat of summer—on one in the streets of a large city, and subsequently in the open country adjacent to it. In the former instance the skin will be scarcely affected; whilst, out of town, an exposure to sunlight will often prove extremely painful, from its action on the skin of the face and hands. That such is the effect of light, and not of heat, is evident, inasmuch as the red heating rays are more prevalent under the smoky sky than in a clear one. The former absorbs, whilst the latter permits the free passage of actinic rays; hence the results to which we refer. We may, perhaps, be permitted to adduce an illustration which we afford, individually, of this effect. Whilst experimenting, some years ago, with the substance found in the Russian "infernal machines," we had the misfortune to receive serious injury from the sudden combustion of several pounds' weight of chlorate of potass and sugar; by which the face and hands were severely burnt. Although not the least external evidence exists of this at the present time, an exposure of a few hours to a summer's sun induces extreme pain, whilst we never find the least inconvenience from the intense heat of a wind or steam-boiler furnace. We have also observed, that the electric light from a large voltaic battery has a similar effect, even although we may not perceive the least sensation of heat. We have named this as an interesting physiological fact, and one which we trust that none of our readers may ever be in a position to verify.

It has been stated by Dr. Edwards, that tadpoles, if entirely deprived of light, are never changed into complete frogs, but remain growing in what we may term the tadpole state. Without affirming the exactness of this statement, we adduce it as one which is frequently quoted in illustration of the effects of light.

We have already referred to the fact, that the changes of colour both of the leaves and flowers of plants, are due solely to the action of the actinic rays. We may here add, that some gum-resins are much acted on by the same agency: amongst them, as a good example, we may name guiacum, which becomes of a green colour when exposed to the light of the sun.

So long as flowers retain their vital state, the brilliancy and richness of their colour is enhanced by light. When, however, their tint is conveyed to fabrics, as in the process of dyeing, such colours speedily fade on exposure to the solar rays. Pink and rose-red colours, produced from safflower, and the blue colours afforded by litmus, are instances of this kind. Tincture of litmus, which is often employed in thermometers, and which although in a vacuum where no ordinary chemical change could take place, often loses its colour. This, however, is speedily restored by exposing the litmus to the action of the oxygen in the atmosphere.

Many chemical compounds change their colour or character from the action of light. Nitric acid, if very strong, will thus be decomposed, and become of a deep yellow tint. Chlorine and hydrogen gases will rapidly unite when exposed to the sun's rays. White paint is often darkened only in those places on which solar light has long impinged; and, similarly, the effects of light are observed on nearly every surface which has been artificially coated with the colours usually employed by house-painters, their hue being either increased or diminished. Some of these effects, however, are due partly to other agencies, which modify, although they do not prevent, that to which we have alluded. White-lead paint thus always becomes of a dark colour, in the presence of sulphuretted hydrogen gas, which, with the lead, forms a sulphide of that metal.

Besides being affected by the purely chemical or actinic rays, many plants seem susceptible of what we may almost call a nervous influence, from the presence or absence of light. Some flowers droop and close during full sunshine, and others close in its absence. Instances of this kind are common in our gardens, and may be observed in the flowers of the evening primrose, the marvel of Peru, the marigold, the convolvulus, and several others. Of all these, however, the sunflower is the most extraordinary for its apparent affection for solar light; and hence its name.

OCULAR DEFECTS IN RESPECT TO COLOUR.

Individuals are occasionally met with who are quite incapable of distinguishing the difference between colours, and who ascribe names different to those applied to any colour by persons unaffected by this peculiarity of vision. One of the most remarkable instances of this kind was that of Dr. Dalton, the eminent chemist; of whom it is amusingly related, that having on one occasion purchased, as he thought, a pair of gaiters of a *drab* colour (he being a member of the "Society of Friends"), was remonstrated with by a "friend," who found him walking through Manchester with those of a *flaming red*.

Some interesting experiments are detailed in Sir J. F. Herschel's *Treatise on Light*, which were tried on a person thus affected. So as to secure the true complementary colour to each presented to the eye in succession, polarised light, transmitted through various thicknesses of plates of mica, was employed. By these means, it was found that the blue and yellow colours were those which were best distinguished; whilst all others, including pure red and compounded colours, were either confounded with each other, or named differently to their usual titles. Sir J. F. Herschel sums up the results of this in the following manner:—"It appears that the eyes of the individual in question are only capable of fully appreciating blue and yellow tints, and that these names uniformly correspond, in his nomenclature, to the more or less refrangible rays generally; all which belong to the former indifferently exciting a sense of 'blueness,' and to the latter of 'yellowness.'" In some cases red is confounded with green, green with brown and red, and violet with blue.

It is difficult to give a reason for this strange affection; but, perhaps, it can be accounted for on the supposition, that the defect lies not in the mechanical construction or chemical composition of the humours of the eye, but rather in some affection of the brain, so far as its optical sense exists. Instances have occurred in which disease has produced the effect. We met with a case, some years ago, of a friend who lost the power of distinguishing colours, and the sense of smelling, owing to a fall which injured the back of the head. The injury, however, produced no other permanent result than those we have named. We regret that, owing to the death of this gentleman, we cannot furnish more accurate and extended statements of the facts of his case. Considerable benefit would accrue if individuals thus affected would forward statements of their peculiarities of vision to scientific men. Such would not only add to our own stock of facts, but, by the variety of cases, it might happen that some remedy would eventually be discovered. This might be done without the publication of the names of individuals; their statements being guaranteed by a respectable medical practitioner, as a precaution against incorrect detail or wilful misrepresentation.

OPTICAL INSTRUMENTS.

We now proceed to apply the laws which we have been previously explaining; and if our readers have succeeded in mastering the statements and doctrines we have been investigating, they will have little difficulty in understanding the use of the philosophical instruments we are now about to describe. Although in these we shall find alike the toy of the child and the splendid instrument of discovery of the astronomer, still each and all depend, both in their use and construction, on the laws of the reflection and refraction of light.

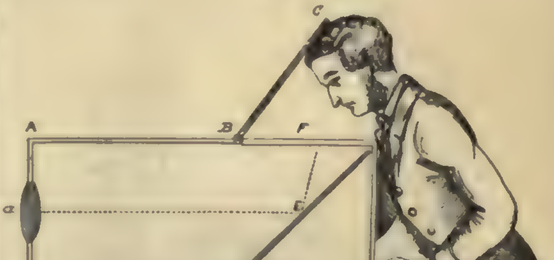
Still pursuing our plan of passing from known to unknown matters, we commence with the Camera Obscura, as one of the simplest of optical instruments. We shall then pass on to the Magic Lantern, Phantasmagoria, Dissolving Views, &c.; thence to Microscopes, simple and compound—the oxy-hydrogen and solar; Telescopes, refracting and reflecting; and shall then describe minor arrangements, such as the Kaleidoscope, Debusscope, Kalotrope, the use of concave and convex mirrors, with others which we need not mention here.

Besides detailing the construction of the instruments, we shall give extended directions for their general management, repair, &c.; affording our readers the results of some experience in the practical use of each, under the belief that many persons who may peruse our pages will be desirous of availing themselves of as much information as we can offer on the subject.

THE CAMERA.

One of the most simple and easily constructed of all optical instruments, is the camera obscura; and of this there are several kinds. The higher class of instrument is chiefly employed for photographic purposes; and as great attention must be paid to the character and make of the lenses and their adjustment; and as its value also is only to be properly appreciated when employed for the purposes we have named, we reserve its consideration for that branch of our remarks included under the head of Photography.

Fig. 34.



A camera obscura, for ordinary purposes, may be thus made:—Into one end of an oblong box, A, say a foot long, eight inches wide, and six inches deep, fix a double convex lens as at *a*, in Fig. 34, which gives a sectional view of the instrument. A piece of British plate looking-glass must be fixed at an angle of 45°, as at E, so as to receive the rays of light passing from the lens *a*, and at a distance dependent on the focal length of the lens. The top of the box must have a flap fitted into it, and fixed on a hinge as at B C; and between B and the edge of the box a piece of ground glass or thin paper must rest, so as to receive the rays passing from the reflecting mirror E, and which are thus spread over the glass at F. The inside of the box should be blackened, so as to prevent the reflection of its sides from marring the picture. If an arrangement of this kind be placed with the lens *a* turned towards any object, the image of the latter will at once appear on the ground-glass plate at F; and if a piece of paper be placed thereon, an excellent outline or drawing of the image may easily be taken, by simply tracing the lines and shading by means of a blacklead pencil.

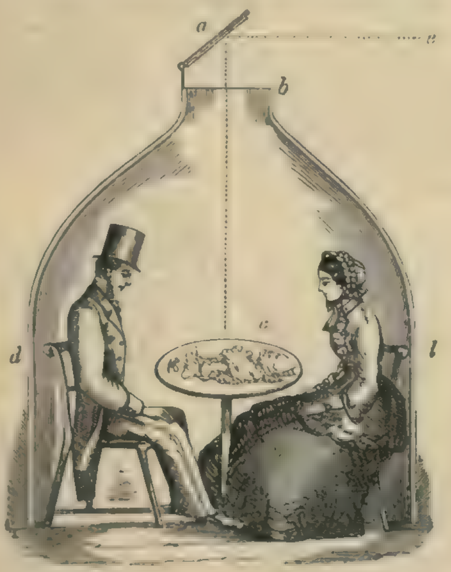
No difficulty need be experienced by any one in constructing a camera of this kind, because the position of each part of the apparatus can easily be arrived at by trial. We may, however, point out two chief matters which should be attended to; namely, let the distance of the lens *a* from the mirror be a little beyond that which would permit its focal point to touch the glass;* and secondly, care must be taken that the glass mirror is properly adjusted, both for receiving the image from the lens, and of transmitting it to the ground-glass plate placed over it.

The above instrument is an exceedingly valuable one for teaching the art of landscape sketching to young persons. The idea of distance and perspective is thus easily acquired; and as they have not to trust to their imagination, either in sketching or shading, they speedily acquire considerable accuracy in delineating objects, and to an extent far greater than would be at first supposed.

Our remarks on aberration † will prepare the intelligent reader to expect that there will be considerable error in all the external parts of the picture. This may be avoided by employing a lens of the meniscus form, in place of a convex one. It is, however, more expensive, and less easily obtained than a double convex one.

A very ingenious application of the camera, on a large scale, is often employed for the purpose of obtaining, on a level surface, extensive views of distant scenery; and some of these answer the purpose extremely well. One of the best we have yet seen is that of the observatory and museum of Dumfries, in Scotland. By it, views extending over a radius of ten or more miles are accurately represented on the receiving-surface of the table. One of these arrangements is represented in Fig. 35.

Fig. 35.

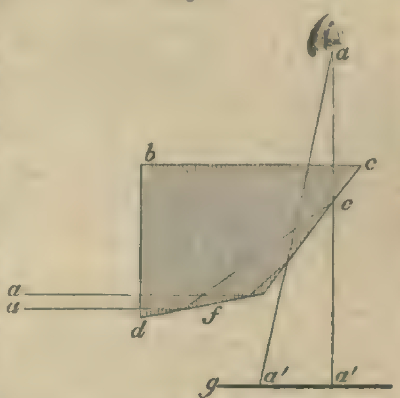


The arrangement is very different to the simple camera we have just described. In Fig. 35, *a* represents a reflector, on which the distant view is first received; the light is thence transmitted through the lens *b*, and at last received on a table covered evenly with white paper, as at *c*. The spectators should be placed, with the table, in a room or tent from which all light is excluded but that which passes through the lens. An arrangement of this sort can be easily fitted in the roof of an attic; and if the reflector be made so as that it can be turned round in any direction, or the angle of its elevation altered, the spectators may have either a view of distant or proximate objects in any direction around them.

This form of the camera is highly applicable to the purposes of the artist; and if the lenses and arrangements are carefully fitted, the picture produced on the paper screen is of great beauty and accuracy. As an illustration of this, we may observe, that by means of the camera to which we have already alluded, we were enabled to trace the ridges of hills and the windings of a very distant river, in a far more distinct manner than the painter or sketcher ever attempts to delineate them either on canvas or paper.

The camera lucida, which consists of a four-sided glass prism, is a very ingenious and simple contrivance, which may also be used by artists for sketching purposes. One of these is represented in the succeeding cut.

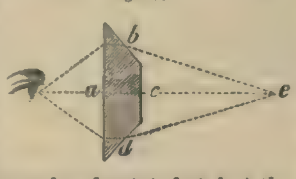
Fig. 36.



In Fig. 36, *a a*, *a' a'*, represent the direction of the rays proceeding from an object, and their subsequent reflection both to the eye and, apparently, to a piece of paper placed beneath the instrument; *b c* and *b d* are two sides of the glass prism; the angle *c b d* being a right angle. The other two faces of the glass are cut at different angles; *e* being at one of 135°, and *f* at an angle of 67½°. The rays *a a* first fall on the face *f*, and being thence reflected to *e*, reach the eye, which seems to observe their image on the sheet of paper, *g*, placed beneath the prism. The spectator, looking through the glass, is thus enabled to sketch an image on a surface placed parallel to the rays of light proceeding from the object. On careful inspection, the reader will perceive some similarity to the camera obscura which we previously described—the eye, in fact, receiving the rays as did the ground-glass plate.

We shall here include a description of a well-known optical toy, called the *Multiplying Glass*. It has one flat surface, on the other side of which numerous facets or planes are cut. It may be described as a plano-convex lens, whose curved surface is formed of a variety of planes; and, like that instrument, the multiplying glass produces its effects by the refraction of the rays of light incident on one of its surfaces. Fig. 37 represents one of these arrangements with three planes. *a* represents the flat or entire plane surface of the glass, and *b c d* three planes cut on the opposite surface. If a ray of light pass from an object *e* to the eye of the spectator, through *a c*, it will not undergo refraction, for reasons already stated;‡ but the rays *e b* and *e d*, falling obliquely on the planes *b* and *d*, will suffer refraction, and will be therefore viewed at a different angle by the eye than that which passes perpendicularly through, as at *a c e*. Of course, the same reasoning applies to any number of facets cut on the glass; and as the number of these will increase the number of angles under which one object will appear to the eye of the observer, so the appearance of the object will be proportionably multiplied.

Fig. 37.



THE MAGIC LANTERN, PHANTASMAGORIA, DISSOLVING VIEWS, CHROMOTROPES, AND PHYSIOSCOPE.

THERE are few optical instruments which are of so amusing a character as those named at the head of this section; and as they are all constructed on similar principles, we have deemed it desirable to treat on them collectively.

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* See *ante*, p. 47.

† *Ante*, p. 44.

‡ See *ante*, p. 45.

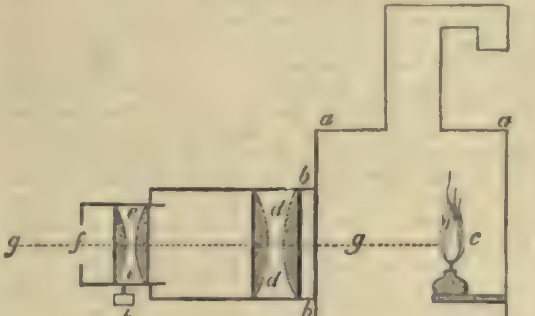
In all, except the physioscope, the object is strongly illuminated by light from a lamp or other arrangement, and the rays passing *through* the picture are subsequently magnified, and the image received on a white screen. In the physioscope, the rays *reflected* from the object are magnified, and their image received in a similar manner.

Commencing with the magic lantern, with whose pretty effects we are all so well acquainted, we may remark, that those sold for the amusement of young persons, are constructed on exactly the same principles as the dissolving views employed in the larger arrangements of public institutions; and, indeed, only differ in their size and quality of workmanship.

The best form of an ordinary magic lantern is that of two sets of lenses, each of which generally consists of two plano-convex lenses. The larger of these two sets is placed nearest the source of light; and is called the condenser. This receives and condenses the light, which then passes through the painted glass or other object; and is afterwards transmitted to the next set, whose object is to magnify the rays so travelling. The relative distance between the two sets of lenses can easily be altered; because, whilst the condenser is fixed, the magnifying glasses are easily moved to and fro in a tube by means of rack-work; or, being in a tube which fits tightly in that holding the condenser, they are easily shifted by the fingers. This operation is called "focussing;" and on care, in this respect, depends the accuracy of outline of the picture on the screen.

We need scarcely say, that unless the axis or line passing from each lens proceeds through each exactly in their centres, the instrument will not give successful results. The source of light must be so placed, that its most brilliant portion is also in the same straight line. In other words, a horizontal line passing from the centre of the light to the outer lens, must pass exactly through the centre of all the lenses. The following engraving illustrates the section of a magic lantern of the most improved form.

Fig. 38.



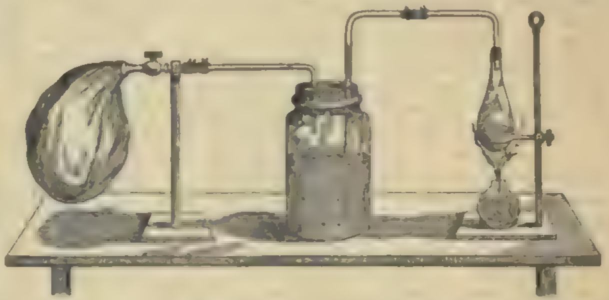
In Fig. 38, *a a* represents the box of the lantern; *b*, the groove in which the painted slides are placed; *c*, the source of light; *d*, the condensing lenses; *e*, the magnifying lenses; *f*, a stop to cut off the superfluous rays, which would diminish the brightness of the picture; *g g* represents an imaginary straight line, passing through the centre of the light and of all the lenses; *h* is a nut, by which the lenses are properly focussed. In purchasing this instrument, care should be taken that all the lenses are free from streaks or hollow specks, as these mar considerably the good effect of the picture on the screen. In occasionally using the electric light, we have observed, that the light passing from bad lenses presents almost a mass of lines of shadow. The worst glasses, however, rarely show this to any great extent on the screen when ordinary artificial light is employed.

Everything depends on a good strong light being employed to illuminate the objects exhibited in any of the arrangements to which we are now referring; and this, therefore, demands our attention before we proceed to describe their separate details.

For instruments of a small size, an oil lamp, or gas flame may be employed; but when large arrangements are used, the oxy-calcium, oxy-hydrogen, or electric light becomes necessary.

The oxy-calcium light is a very simple and useful contrivance; by means of which, considerable illuminating power may be obtained. It depends on the fact, that almost any flame, on being urged by a jet of pure oxygen gas on to a piece of burnt lime, will produce an intensely white light. Various arrangements are sold by the instrument-makers for this purpose. The oxygen is produced from chlorate of potass mixed with half its weight of black oxide of manganese. The mixture is placed in a glass retort, to the stem of which an india-rubber tube is attached, connecting the retort with a gas-bag or bladder. On applying heat to the mixture, by means of a spirit or gas lamp, abundance of pure oxygen is afforded; which passing into the bag, soon fills it. It is advisable to allow the gas to run through some water in a bottle, so as to prevent the passage of injurious substances, which may injure both the brass connections and the texture of the bag itself. The following engraving illustrates an apparatus of this kind; which we adopt ourselves for making oxygen on a small scale, and for filling a bladder.

Fig. 39.

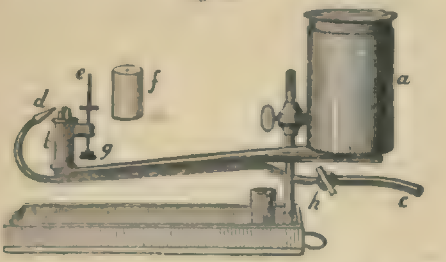


This arrangement is thus constructed and managed. After tying a bladder to a stopcock, drive out all the air it contains by gentle pressure, and leave it moist. Then tie to the end of the stopcock a piece of india-rubber tubing about two inches long. Into a wide-mouth bottle holding about a pint of water fit a cork which has had two holes bored through it, just large enough to receive two pieces of pewter gas-pipe. One of these must reach to the bottom of the bottle, and the other should just pass into its inside through the cork. The bottle, which we will call the purifier, should then be three parts filled with water; and having the retort or flask ready for making oxygen, attach its stem, by means of another piece of india-rubber pipe, to the pewter tube which reaches to the *bottom* of the bottle. The *short pipe* is to be attached to the *stopcock* of the bladder. On the right we have the Florence flask, in which the oxygen mixture is contained; in the centre the purifier, whose object is to retain any manganese or chlorine which may pass over; and, on the left, the bladder is observed in the process of filling. After all is prepared run a little melted cobbler's wax over the cork of the bottle, to keep it air tight; then open the stopcock of the bladder. On lighting the spirit-lamp, the gas will pass from the retort through the water in the bottle, and, proceeding through the stopcock, will gradually fill the bladder; as soon as this is filled, remove the retort before taking the spirit-lamp away, lest the water might pass into it and destroy it. The above is a very simple and yet effective arrangement; and its cost is trifling.

Having thus obtained a supply of oxygen, the next point of importance is its application to the required purpose; and for this, a blowpipe and a lamp of some kind are requisite. Almost every optical instrument-maker manufactures these in what we may call a special form. We shall therefore, we trust, be not considered as making an invidious selection, if we state, that as a matter of choice we prefer that illustrated in the annexed engraving; which is made and sold by Mr. J. J. Griffin, of Bunhill-row, London.

In Fig. 40, *a* represents the spirit-holder; *b* is the wick-holder, in which there are three tubes for separate wicks; *c* is the tube which brings oxygen gas from the gas-bag, the supply of the gas being regulated by the stopcock *h*; *d* is the jet by which the gas is directed into the flame of the spirit-lamp, the wicks of which must be so arranged that the gas does not strike upon the cotton and produce a ragged flame, to which end the wicks must be carefully spread on the two sides; *e* is a wire for supporting the lime cylinder, which is shown apart by *f*; and *g* is a milled head for turning the cylinder, so as to present different sides of it to the action of the flame. The front end of the apparatus is put bodily into the lantern, through an opening made for that purpose in the back of the lantern. The cylinders of lime, fixed at *e*, may easily be made by filing a piece of hard fresh-burnt lime into a cylindrical form; they may be about an inch long, and half an inch in diameter. In using this arrangement the spirit-lamp is first lit, and the lime brought into its flame. This drives off the moisture which the lime always contains; and which would cause it to split into pieces if it were suddenly heated. The stopcock, *h*, is then opened, and oxygen gas is allowed to pass through the jet *d*, so as to force the flame on the lime cylinder. Weights should be placed on a board covering the gas-bag or bladder, which may lay on the floor, and be connected with *c* by means of an india-rubber tube. A most beautiful white light will be thus obtained, which gives the excellent results when applied to the magic lantern and similar arrangements. Of course, the light may be kept up for any length of time, by maintaining the supply of gas through the jet, and of spirit in the lamp. The lime cylinder should be continually turned round, so as to prevent it burning into holes, which would speedily lessen the amount of light; and sometimes might throw back the flame, and crack the glasses of the condenser.

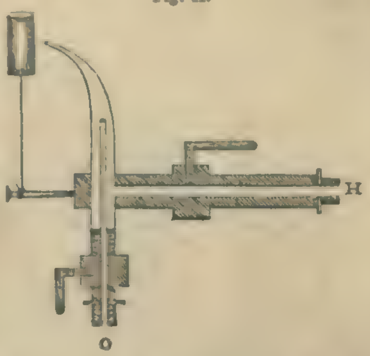
Fig. 40.



In using the oxy-calcium light no danger of any kind can arise; and as such, it is eminently fit for home use. This cannot be said of the next arrangement; which should never be employed except by those well acquainted with the risks which are incurred. Despite an experience of some years, and although from such we naturally have become careful during its employment, we have had several accidents with the oxy-hydrogen arrangement, which might have proved serious; and which, if occurring to inexperienced persons, would have possibly ended in fatal results. The force of our remarks will be appreciated as the student peruses the following description of this apparatus.

The oxy-hydrogen, or lime light, is always employed when large dissolving-view lanterns are used; and whilst not so powerful or so pure as the electric light, it has the advantages of constancy, steadiness, and greater convenience of application. An intensely hot flame is produced by the combustion of hydrogen or coal gas, together with oxygen gas; and for this purpose a peculiar form of blowpipe is required, whose object is to keep the two gases separate until the moment of their combustion. If this be successfully done, all danger will be obviated: in the absence, however, of this precaution, the most dangerous explosion will certainly ensue. We shall not enter into a description of various kinds of these blowpipes, but at once recommend that which we prefer for our own use. (See Fig. 41).

Fig. 41.



In the annexed engraving, H represents the passage by means of which the hydrogen or coal gas enters the blowpipe, and passing on, it at last escapes at the jet. O shows the opening by which the oxygen gas enters a pipe, which is entirely isolated from the hydrogen compartment until the two gases mix at the small orifice at the top of the oxygen tube; and this being near the mouth of the jet, prevents any previous admixture of the two gases. Stopcocks are attached to each tube to regulate the supply of the gases, and the cylinder of lime is placed in front of the jet, as represented above. The lime cylinder is almost always mounted on a pin connected with clock-work, which, by its rotation, causes different surfaces of the cylinder to be successively presented to the jet.

In using the blowpipe, the following directions should be carefully attended to, both as regards questions of safety and success:—1st. To fill the bags with gases. The oxygen bag may be filled as already described, for the oxy-calcium light. Care should be taken always to use the same bag for the same gas; and never to employ the bag containing hydrogen left from a previous exhibition if a few hours have elapsed, lest any air may have entered, and have so formed a dangerous and explosive mixture. The student should always remember, that both bladders and india-rubber bags are porous; and that gases have an astonishing tendency to mix together, by what is called endosomic action. Coal gas is best employed, as being easily obtained; and the bag may be filled by connecting it with an ordinary gas-pipe. Before doing this the bag should be completely emptied of air, by rolling it up and pressing it. It may then be left to expand by itself, through the pressure of the incoming gas. Pure hydrogen may be used; and is produced by adding one part of strong sulphuric acid to six parts of water, which, when cool, should be poured on zinc cuttings, or iron nails, contained in a stoneware or leaden bottle. Water is thus decomposed, and its hydrogen set free. The gas should be allowed to pass through a bottle of water, to purify it,[*] before being received in the bag. This will prevent corrosion of the brass and other metal parts of the stopcocks and blowpipe. —2nd. In attaching the bags by the india-rubber pipes to the blowpipe, care should be taken that each should be affixed to the proper connection. The hydrogen should pass through that tube which surrounds the inner one supplying oxygen; this is a matter of great importance. Care should also be taken to have all the connections perfectly air-tight, or loss of gas, and risk of explosion may occur.—3rd. The hydrogen stopcock being opened, the gas should be lit, and allowed to burn gently on the lime cylinder till it is quite dry.—4th. When this is done, the light may be produced by the hydrogen cock being first turned on full. The oxygen cock may then be opened, and afterwards the supply of hydrogen diminished, until a full, brilliant, and white light is obtained. The operator should always make it a rule to turn on the *hydrogen* tap fully first, when *producing* the light; and to turn off the *oxygen tap first* when he desires to extinguish the light. It is also a safe precaution always to have a little excess of hydrogen over the oxygen supply. This slightly lessens the light, but adds to the safety of the arrangement.—5th. When india-rubber bags are employed, the weights pressing each should be equal on both bags. This will ensure a uniformity of supply. In large institutions, gasometers or gas-holders are always used, and these are sometimes of considerable size. They are, however, unfitted for private use, for which nothing is better than the bags to which we have alluded.

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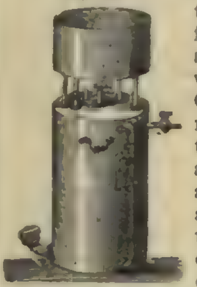
[*] See *ante*, p. 61, Fig. 39.

Fig. 42.



In Fig. 42, a very convenient form for a small gas-holder is illustrated. It consists of a trough to hold a supply of water, from which runs a pipe fitted with a stopcock, into the bottom cylinder which is intended to contain the gas. On the right-hand side is a cock, by means of which the gas passes when the holder is in use. When such an arrangement is to be filled with gas, all the cocks are first to be opened, and water must be poured into the trough. This will run into the cylinder and fill it. Each cock is then closed, and a screw, observed at the left hand of the lower part of the holder, is removed. If the vessel be quite air-tight, only a few drops of water will escape. Into the opening thus made, the end of a retort from which oxygen is being made, is introduced, so that the gas may pass into the holder. The water will escape as the gas passes in, and it is therefore advisable to place the vessel over a tub during the process of filling. Of course, two gas-holders will be required for the purposes we have referred to—one for oxygen, and another for hydrogen. The latter is filled by passing in the tube from the hydrogen bottle; or, if coal gas be used, an india-rubber pipe may be attached to the stopcock on the side, the other end being connected with an ordinary gas jet. The holder must first be filled with water, and the lower screw afterwards removed, as when filling with oxygen.

In public institutions, where large gas-holders are used, a sufficient quantity of gas is prepared to last for a few days; and the slight admixture of air, through accidental leakage or otherwise, which always occurs, is a matter neither of importance or of danger. In using small holders, such should be first entirely emptied of all gas by means of water, and then they may be refilled at as short a time as possible before their contents are required for use: an equality of pressure should also be maintained in each, so that the gases shall pass from their respective vessels at an equal speed and pressure. A stock of lime cylinders may easily be made by means of a file and some good hard lime; and that obtained from Ireland is best for the purpose. The cylinders, which may average an inch and a-half in height, and three-quarters in diameter, should have a hole pierced half-way, or wholly through them, to permit of their resting on the wire-holder which sustains them before the jet; and until required for use, they must be kept wrapt up in paper, and in a glass-stoppered or tightly corked bottle, to prevent access of moisture, which would speedily destroy them. As they are of next to no cost if made at home, a fresh one should be used on each occasion. If purchased of the optician, they become rather an expensive affair; and yet these have no advantage, in any respect, beyond such as may be made by any person in the manner we have explained.

Having thus given sufficient directions respecting the management of the light, we proceed to speak of the cleaning and general care of the lenses.

Most persons attempting to show views by means of any of the instruments to which we are now directing attention, have to experience the annoyance of finding the lenses covered with moisture at certain seasons of the year—a circumstance which almost entirely prevents the appearance of the picture on the screen. This partly arises from the vapour evolved by the combustion of the materials from which the light is produced; indeed, water is formed in abundance when an oil or gas flame, and the lime light, are employed. This evil is, however, easily remedied by having the light lit for some time before the exhibition takes place; and, in the case of the lime light, by allowing the hydrogen to burn gently against the lime cylinder, and thus drying it also at the same time. In every case the lenses should always be a little warmer than the air of the apartment; and thus, of course, all chance of the condensation of moisture is at once removed. It is a good plan to cover the whole of the lantern by means of a thick sheet of baize, so as to prevent the deposition of dust or dirt on the lenses when they are not in use.

The lime light causes a considerable quantity of volatised lime to settle on the lenses, and soon dims their surface. This requires that they should be frequently cleaned. Unless care is taken in this operation, the polished surfaces soon become scratched, and of course are so far deteriorated. The plan we have always adopted, is that of first removing all the lenses from their seats, and wiping each perfectly with a damp old linen towel, to take off the "rough coat" of dirt. Each lens should then be carefully washed, by means of cotton wool, with spirits of wine (not methylated spirits, because these contain a resin in solution), and afterwards it may be polished with a soft chamois leather, or what is still better, an old silk handkerchief. No polishing powder, or anything of a solid nature should be used, because these almost invariably contain gritty hard matter, which soon spoils the glass. The towels should be kept in a place where no dust or dirt can settle on them. Almost every one has an idea that nothing is more difficult than to scratch glass; but chemists, opticians, and photographers know, to their cost, that the difficulty lies in preventing, rather than in producing, that result. The lanterns should afterwards be well cleansed from all dust, both inside and out; and the lenses may then be replaced in the exact position which they had previously occupied. The slides or views should also be kept scrupulously clean, or ugly black spots of dust may appear on the screen; and when not in use they should be kept in racked shelves, or in boxes in which they may be kept separate, and thus any risk of fracture be avoided.

We have thus given minute directions in reference to the general management of the magic lantern, &c.; and may here remark, that the only difference between it and the phantasmagoria, is, that the former is so used as to show the pictures on the same side of the screen as the spectators are placed; whilst, in the latter, the views are cast on a transparent screen, and the lantern is on the side opposite to the position of the spectators. By this arrangement the size of views is *diminished* by bringing the lantern nearer to the screen; and *increased* by removing it therefrom. This is an advantage when the instrument is employed to amaze young folks, who are greatly puzzled to know where the views come from; and also at the sight of the figures apparently retreating or advancing. An opaque screen or thick linen sheet is best for the magic lantern; whilst a thin calico sheet, moistened with water, answers best for the exhibition of the phantasmagoria. In either case the screen should be kept tightly stretched at each side, and at the corners, and should be in such a position that draughts of air may not cause it to swing to and fro.

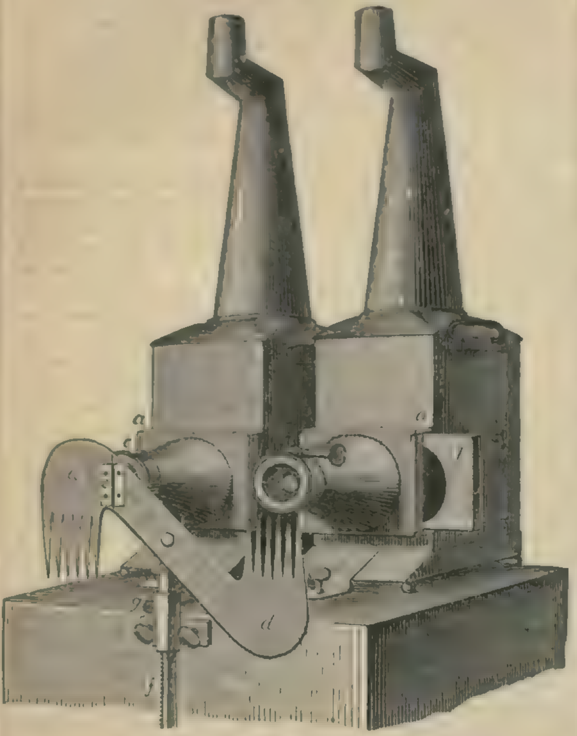
DISSOLVING VIEWS, AND CHROMOTROPES.

At one time the exhibition of dissolving views was confined to the London Polytechnic Institution; and they have invariably proved a most interesting and instructive addition to entertainments uniting science and amusement.

Two magic lanterns placed side by side, and throwing an equal-sized picture on the screen, are the apparatus required. There are additions, which we proceed to describe, by means of which the "dissolving" effect is produced. All our previous remarks on the magic lantern are therefore equally applicable to this arrangement in every other respect. The following engraving illustrates a very usual form of the dissolving view apparatus.

In Fig. 43, letter *a* represents the slider-holder; *b* is a slider partly inserted into the holder; c is part of the rack-work for adjusting the lens to the picture. The adjustment is regulated by the appearance of the picture on the screen. *d d* is the *dissolving apparatus;* that is to say, the apparatus for making one picture gradually disappear while another gradually comes into view before the spectators. This change of pictures is managed thus:—The bar *f*, which is attached to the screen *d e d*, ascends and descends in a collar; this is screwed to the front of the box on which the apparatus is mounted. It is provided with a rack, which is acted on by a pinion marked *g*, and the pinion is turned by a handle. Supposing the pair of lanterns to be in action, and each lantern to contain a slider, one of these sliders will give a picture upon the white screen, while the other is stopped by the upper limb of the screen *d*. If, then, the screen *d e d* is gradually raised, by the turning of the handle *h*, which causes the racked bar *f* to ascend, the acting slider will be gradually covered by the rising comb of one end of the dissolver, while the other will, in the same proportion, become uncovered by the rising comb of the other end of the dissolver; so that, when the first picture is quite obscured, the second will be quite developed. In that manner, the first picture is made to dissolve or gradually fade into the second picture. At *e*, in Fig. 43, the dissolving screen, *d d*, has a hinge, the use of which is to admit of suddenly cutting off the action of the picture in the corresponding lantern. This action is required for the exhibition of *lightning* and other *sudden effects*. The change of sliders in the lanterns is made when each nozzle is alternately obscured. The turning of the rack-work, by which the dissolving is effected, must be slow, regular, and noiseless.

Fig. 43.

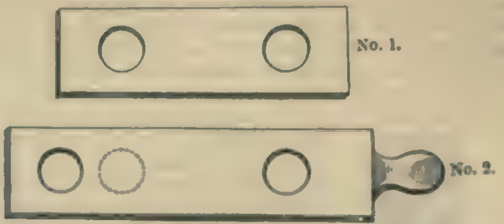


We have here given the plan usually employed in the dissolving-view lanterns. We, however, will give another one, which we used ourselves in the lanterns of the Glasgow Polytechnic, which is extremely simple in its construction. (See Fig. 44).

No. 1 is a piece of wood with holes, which are placed exactly opposite to the ends of the lantern lenses.

No. 2 is a slip of wood made to slide at the back of No. 1, also having holes, but in different positions. It is evident that only one of the holes, as cut in this, can be open at the same time with those of No. 1. Thus those on the right hand are open; but supposing that the slip is moved to the right hand, then the right hole will be closed, and the left one opened. Thus by sliding No. 2 to and fro at the back of No. 1, the views are dissolved.

Fig. 44.



This arrangement is an exceedingly useful and effective one; and any person having a pair of lanterns, can easily arrange them as dissolving-view apparatus, by having two slips cut as illustrated in the engraving, and by fixing them in a frame in front of the nozzles of the lanterns.

In arranging the lanterns, they must be placed at a slight angle with each other, so that the light they cast on the screen shall fall equally on it. In other words, the centre of each picture, cast by each lantern, must fall on the centre of the screen: this is technically termed "centre-ing" the lanterns.

Much care is required to focus the lenses, so that a clear and distinct picture be obtained. As, however, this can only be arrived at by practice, we forbear giving special directions on the subject, except so far as to state, that the pictures should be so focussed as that their *centres* may present the best possible image on the screen.

It will be beyond our plan to give any directions in reference to painting the slides, as that belongs to the art of the glass painter. An unlimited supply, either on hire or for purchase, is kept of all kinds, whether views, astronomical diagrams, chromotropes, &c., by most opticians in our large towns. Chromotropes are simply two pictures so arranged that they may revolve over each other on a common centre; and the lines painted on the glass thus intersecting each other, produce a very pleasing optical effect.

In concluding these directions and remarks, we would mention, that the magic lantern and dissolving-view apparatus, may be made an excellent medium for the instruction of youth; and they certainly should be an adjunct to the means of communicating knowledge to every public school. We shall not, however, enlarge here on this subject, because we propose to do so when describing the construction and arrangement of the oxy-hydrogen microscope.

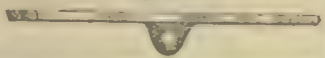
The physioscope is a modification of the magic lantern; but instead of the light being transmitted through the object whose image is to be represented, a powerful light is cast thereon: the reflected rays are collected by lenses, as in the magic lantern, and the magnified image is cast on the screen. The human face may thus be presented in a highly magnified state; as, indeed, any other solid object may be. The difficulties of the arrangement are, however, very great; and the comparative rarity of its use will not demand from us a more detailed account of its construction.

MICROSCOPES.

ONE of the most popular of all optical instruments is the microscope; and perhaps throughout the whole of the different applications of the laws of the refraction of light, we cannot discover one to which we are so indebted for a knowledge of the beauties and wonders of natural history, and the varied appearances of almost every kind and form of matter. We shall first describe a considerable variety of microscopes, commencing with the most simple kinds, and passing to those of a compound and achromatic form; and afterwards illustrate the arrangement of the oxy-hydrogen microscope, by which a large number of spectators are enabled to see the magnified image of an object simultaneously. With our remarks on these instruments, we propose giving instructions to the student, which will assist him in collecting, preparing, and mounting objects of all sorts; and thus we hope to make our subject complete and useful to all our readers.

Perhaps the simplest microscopic arrangement we can mention, is that illustrated in the annexed engraving, which represents a piece of window-glass, under whose lower surface a drop of water or oil depends. The drop, of course, assumes a spherical form, and thus produces a plano-convex lens. In cases where no other kind of microscope can be procured, this is at once a ready and tolerably efficient instrument. We need scarcely remind the reader who has perused our previous remarks on lenses,* that almost any spherical and transparent substance will, as it forms a lens, answer to some extent for microscopic purposes. Even globules of melted glass may be thus employed. For fine compound instruments, lenses of precious stones, such as the diamond, have been applied. These are valuable on account of their purity, homogeneity of structure, and high powers of refraction; but through their extreme hardness they are difficult to polish, and hence are exceedingly expensive. Rock crystal, and various kinds of "paste," or glass used in making artificial gems, have been also made into lenses with varied success. The best kind of glass is almost always, however, as a rule, alone used in the manufacture of lenses for microscopic purposes.

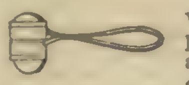
Fig. 45.



In all ordinary spherical lenses there exists an objection to their utility, owing to the aberration of light which is a necessary consequence of their form. We have already explained the reason of this;† and to remedy it, and other evils, two very useful, simple microscopes are constructed; namely, the Stanhope and Coddington lenses.

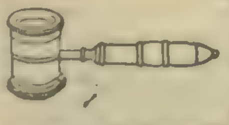
The Stanhope microscope consists of one lens, whose two curved surfaces are so ground as to have different foci, and the focal point of each terminates near the surface of the other. Hence, in using the arrangement, the object to be examined is placed on the surface of one of the ends of the lens, and is viewed through the other. This is a most useful and complete contrivance; and, in the absence of more complicated instruments, will be found of great value to the student. It is of small size, and but of slight cost; and as a field companion to the naturalist, it is of considerable advantage. Fig. 46 represents a Stanhope lens of the ordinary form, and is about half the size of the instrument itself.

Fig. 46.



The Coddington lens is an equally valuable little microscope; and its peculiarity consists in the fact, that a portion of the lens between its two curved surfaces is removed by grinding, which prevents many of the annoyances occasioned by spherical aberration. This makes the instrument of a much more complete kind than the simple lenses to which we at first referred. Fig. 47 represents a Coddington lens of about half the usual size, as sold by the instrument-makers.

Fig. 47.



Either of the above instruments may be purchased for two or three shillings; and as some of those who peruse our pages may not have the means to procure instruments of a high class, we can recommend no better microscopic arrangements for ordinary purposes. By means of the Stanhope lens the student will be enabled to examine a great variety of beautiful objects, both of an animal and vegetable nature. The structure of flowers and leaves, the wings, down, and other parts of insects, may be easily studied thereby. We know of few more interesting amusements than a morning spent in the microscopic investigation of the infinite variety of objects which the bountiful hand of Nature spreads in all directions about us. Whilst we may view, with awe and admiration, those immense results of the Creator's power which the telescope reveals to us, we can, by means of the sister instrument, the microscope, dive into the minutiæ of Creation, and reap an immense amount of intellectual pleasure therefrom.

Having drawn attention to some of the best kinds of simple microscopes, we shall now pass on to consider the construction and uses of those called "compound," in which an extensive power of examining the most minute objects is afforded by means of a combination of lenses, and other apparatus.

COMPOUND MICROSCOPES.

From the remarks which we have made in our previous pages, when speaking of lenses, the reader will have observed that the magnifying power of a lens is inversely as its focal length; or, more plainly, the shorter the focal length, the greater is the magnifying power of the lens employed. It would follow, therefore, theoretically, that a single lens may be manufactured which would afford any magnifying power which might be desired. In practice, however, this is impossible; because, when the focal length of a lens is exceedingly small, the eye, lens, and object would require to be so near each other for the purposes of microscopic investigation, as to entirely prevent every chance of the observer using such an arrangement.

Lenses of almost the shortest possible focal length may, however, readily be employed, when combined with another lens which magnifies the image produced by the former. Such is the combination found in the compound microscope, in which the *object-glass*, or lens of short focal length, receives the rays passing from the object under examination, and produces an image thereof; such image is viewed by means of another lens of far less magnifying power, and therefore of greater focal length, which, being next the eye of the observer, is called the *eye-glass*.

By such an arrangement an extended field of view is obtained, and the magnifying power which may be employed is only limited, practically, by the skill of the instrument-maker.

We must, however, remind the reader, that if ordinary glass lenses are employed, all the effects of chromatic and spherical aberration (of which we have already spoken so frequently) are called into play; and hence all the rays except those passing nearest to the centre of the lenses of the instrument must be cut off by means of stops, which of course would cause a great loss of light. This difficulty has, however, been remedied in another way, by using achromatic lenses; and such instruments as the compound achromatic microscopes, manufactured by Ross, Smith and Beck, and other eminent makers, are perhaps amongst the most perfect productions of human skill and ingenuity.

It, of course, will be out of our power to recommend the best form of microscope for the student, varying as they do in so many particulars. As, however, the means of many of our readers may not permit the purchase of a high-priced instrument, we may state, that for ordinary purposes, a good microscope, containing most of the modern improvements, may be obtained for from three to five pounds, from the maker. And sometimes, at sales by auction, a very good instrument may be purchased at a comparatively low cost. In such cases, however, the purchaser should not trust to his own judgment, if inexperienced, but should first have the arrangement examined by an optician, so as to prevent disappointment. We may state, that in nearly all our large towns, philosophical instruments are made expressly for public sales, at which both the article and the purchaser are frequently "sold" together. As an instance of this, we may mention the case of a friend who purchased, in one of the large towns in Scotland, what he thought to be a first-rate galvanic coil machine. On attempting to obtain a shock from it in the usual manner, he discovered, that instead of the machine containing 300 or 400 yards of covered copper wire, there were only a few inches connecting highly-polished brass binding-screws; the remainder of the apparatus being what is technically called a "dummy."

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* See *ante*, p. 46, *et seq.*

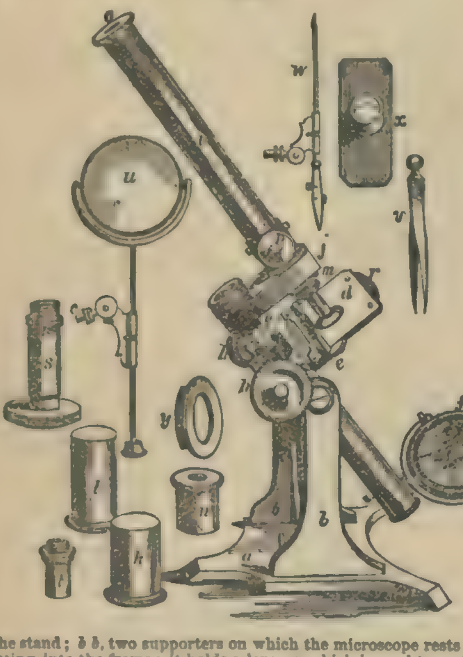
† *Ante* p. 44.

As we cannot properly recommend any particular form of microscope, or the production of individual makers, consistently with fairness to all—we have, therefore, an equal difficulty in presenting an illustration of one of the most approved kind.

That represented in the annexed engraving will, however, give a general idea of the most essential parts of compound microscopes, and will also assist the tyro in putting any of the ordinary class of instruments together. In almost every instance the essential parts are similarly constructed by the different makers, the improvements or special characteristics being generally connected with comparatively minor details of the instrument.

In Fig. 48 we have an illustration of an excellent form of compound microscope.

Fig. 48.



a, the stand; *b b*, two supporters on which the microscope rests; *c*, a peg fitting into the frame; it holds a lens, *u*, which is used to cast light on opaque objects; *d e*, the stage on which the object is placed; *f*, a mirror which throws light through transparent objects; *g*, a rack-work for raising and lowering the tube *l*; *h h*, two thumb-screws for adjusting the focus; *i*, a branch on which the tube *l* is fixed; *l*, the brass tube holding the eye-piece, *o*.—The other parts here represented will be more fully described as we proceed.

We may now refer to certain modifications of detail, which cause some instruments to be highly valued by microscopists; and for this purpose we commence with the lower part of the microscope. The use of the mirror, *f*, is to illuminate objects placed on the stage, *d e*. In this stage is a circular hole, through which the light reflected from *f* passes. Of course the reflector is employed only for transparent objects. During day-time, the mirror should be so placed as to reflect as much light as possible to the stage, and this is best effected by exposing the reflector to daylight at a window. At night-time, an argand, oil, or gas-lamp may be employed, the light of which should be shaded by means of a chimney-glass, of a very light-blue colour. This corrects the yellow colour of artificial light, and so renders it whiter than it otherwise would be. The mirror is so fixed to the arm which supports it as to allow of its being placed at any favourable angle with the stage, and thus to secure at all times sufficient reflected light.

A very ingenious arrangement has been proposed, in which a piece of pure white unglazed stone or ceramic ware is employed as a reflector. Such an arrangement prevents that unpleasant glare which light reflected from a polished surface generally has. The ordinary reflector may easily be converted into one of these by covering the surface of the glass with any white powder, of which, perhaps, sulphate of lime (plaster of Paris) is the best.

The stage, *d*, on which the object rests, is of considerable importance in the microscope. It is generally made in two parts, which easily and yet firmly slide on each other. By means of these combined motions, the objects placed thereon can be readily shifted to and fro, so as to be in any required position with the object-glass. An accurate and readily shifting stage is of great use when high powers are employed. Its motions are generally guided by means of a small lever, working in a universal joint; various arrangements, however, are employed by different makers.

On one side of the stage is fixed, by means of a peg, *c*, a lens, represented at *u*, in Fig. 48, whose object is to illumine opaque bodies whilst under examination, and for which the mirror would be useless. The focal point of the lens is brought to bear on the object, so as to afford as much light as possible. The student, however, must not employ the direct solar rays for the purpose; for not only will the light be too powerful, but it is more than probable that the heat so concentrated would immediately destroy the object, if of an organic nature. Next, above the stage, and screwed into the long brass tube, *l*, is the object-glass, by which the rays passing from the object are first received, and subsequently transmitted, by means of the eye-piece, to the eye of the observer. Several of these, having different focal lengths, are supplied with the compound microscope; and, as we have already intimated, it is essential that such should be achromatic. The image formed by the object-glass is viewed by means of the eye-piece, *o*, of which two or three are generally provided, each having different magnifying powers. By means of the latter, the image afforded by the object-glass is magnified and presented to the eye.

Arrangements are required for focussing the lenses; or, in other words, to enable them to present the best possible image to the eye of the observer, by regulating their mutual distance. This is generally done by means of two sets of screws and a rack-work motion; the latter of which is represented at *g* in Fig. 48, the thumb-screw being seen at *h*. This is what we may call the coarse adjustment, which is still further improved by means of a similar but much finer arrangement represented at *p*. On the exactness of adjustment of the lenses, and their distance from the object under examination, depends the excellence of the image as seen by the observer.

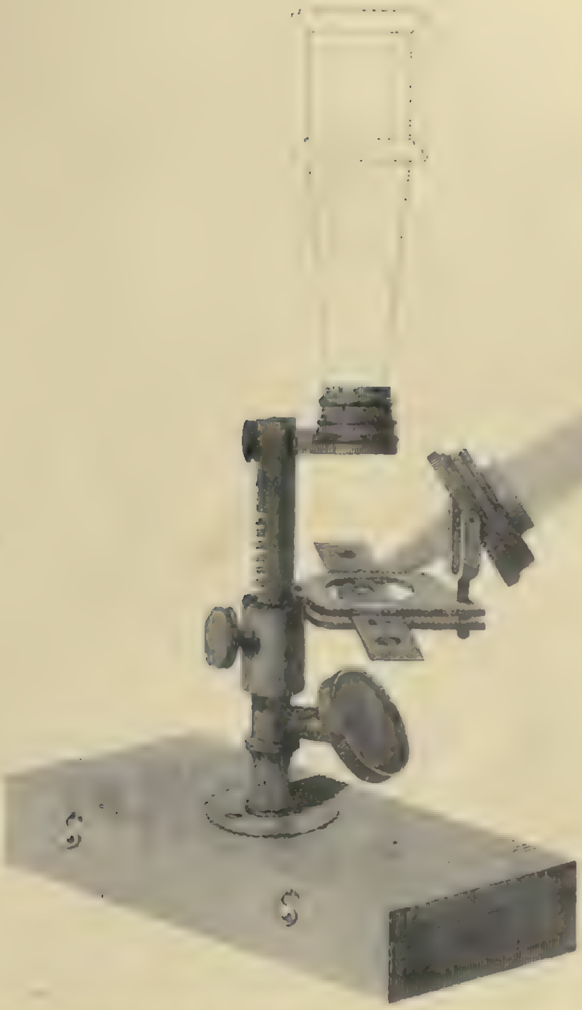
Objects may be mounted, if intended to be preserved, under thin squares of glass, secured at their edges by means of pasted paper. The mode of doing this is described under the head of "Oxy-hydrogen Microscope," &c. Many objects may, however, be temporarily placed in the microscope by means of pincers or forceps, respectively illustrated at *v* and *w*. It is often necessary to examine water insects in a living state, and for this purpose a trough containing the liquid must be used: of this kind one is represented at *x*.

We have thus endeavoured to illustrate, for the use of the student and our readers generally, the chief parts of modern compound microscopes. Some excellent works have been published on the subject, in which all details are fully treated; and, perhaps, the best that we can recommend for those of our readers who are disposed to enter fully into microscopic investigation, is that by Dr. Carpenter, entitled *The Microscope and its Revelations*; in which the subject is almost exhausted. Other valuable treatises, of a more elementary character, such as *Half-hours with the Microscope*, and similar productions, form excellent hand-books to the student.

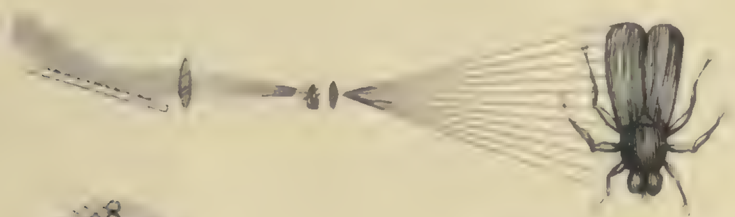
We may here describe a very simple form of microscope, which can be frequently employed in the stead of larger instruments; and it can be purchased of most optical instrument-makers for a moderate cost.

Fig. 49 represents one of these. Its principal parts

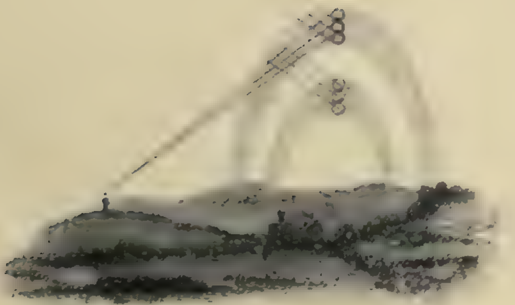
Optics.



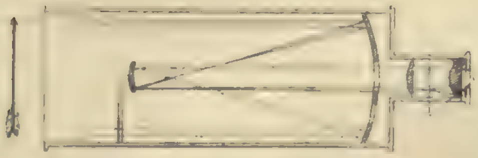
The compound Microscope, Showing the use of the Condenser for Artificial Light. See Page 66.



The Solar Microscope. See Page 69.

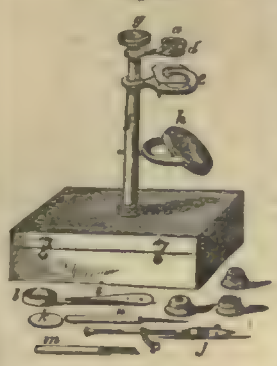


Cause of the Rain-bow. See Page 50.



The Gregorian Telescope. See Page 71.

Fig. 49.



are—*a*, the magnifying lens; *e*, the frame on which the objects are supported; *f*, an upright stand; *g*, a regulating screw; *h*, a mirror casting light through transparent objects. The other parts in Fig. 49 represent lenses of different magnifying powers, which are fixed in the holder, *d*; and forceps, &c., for holding the objects to be examined. In observing an opaque object, light may be thrown upon it by holding a double convex lens or burning-glass, so that its focal rays shall be cast on the object, in the manner we have already mentioned. It will be observed, that this arrangement somewhat resembles the compound microscope, but differs in this respect—that the *object* is viewed by the eye through the eye-piece, which is, at the same time, the power, and not the *image*, as is the case in the compound arrangement.

THE OXY-HYDROGEN MICROSCOPE.

The oxy-hydrogen microscope is an instrument of great value when employed for the purpose of exhibiting, in a popular manner, some of the most interesting facts brought to light by means of the compound microscope. It is quite unnecessary for us to remind our readers, that the use of the latter-named instrument is necessarily confined to one person at a time. Now, the advantage accruing in the use of the oxy-hydrogen microscope, is that of permitting any number of persons to observe, at the same moment, almost any object, in a magnified state, which can become the subject of microscopical investigation. Of course there is a limit to its uses, arising from the colour, opaqueness, or other affections of the objects; but still, these being of rare occurrence, do not interfere with its general utility. The extent of magnifying power which may be employed, is only limited by the amount of light which can be obtained for the purpose of illuminating the objects; and for this, the electric and oxy-hydrogen lights afford every assistance.

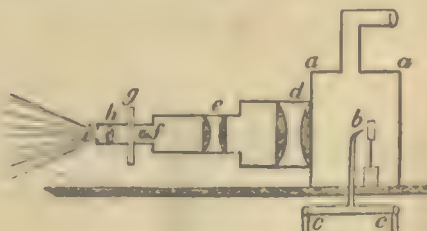
In its general construction the instrument resembles the apparatus employed for exhibiting the ordinary dissolving views; one lantern, however, only being used. Inasmuch as the magnifying power is much superior to that of the magic lantern, greater care is required in the manufacture of its lenses, and the adjustment of every part must be very exact. We may here remark, in reference to the light employed, that M. Duboscq has been very successful, so far as intensity goes, in applying the electric light for the microscope. We may, however, state, that in most cases, any advantages so gained are more than counterbalanced by the trouble and expense required to keep a galvanic battery at work; and that the lime light is decidedly to be preferred beyond any other for all practical purposes. We have already given full directions for the management both of the oxy-calcium and lime lights.* We therefore proceed to speak of the general arrangement and management of other parts of this microscope.

The lenses placed nearest to the source of light are termed the condenser; and their office, as in the magic lantern or dissolving-view arrangement, is to collect the light, and so to illuminate the object whose image is subsequently magnified. Another pair of lenses is generally placed between the "power" (that is, the last lens in front of the arrangement); and by these the light is still better concentrated on the object, which in the microscope is always of very small size, and requires strong illumination. The object is not placed, as in the magic lantern, near the condenser, but close to the focal point of the last magnifying lens; which, in future, we shall term the power. In an arrangement which the writer had constructed for the Glasgow Polytechnic (which, however, has since been destroyed by fire), the first condenser, next to the light, had a diameter of twenty-eight inches; and between it and the power, three sets of achromatic lenses, of twelve, eight, and six inches diameter respectively, were placed. By this instrument an ordinary flea was magnified so as to present a very highly illuminated image, only a portion of which covered a screen thirty-two feet square. If, therefore, the insect is supposed to be a quarter of an inch long, and an eighth of an inch broad, the superficial magnifying power, in this instance, was equal to a total of some millions beyond the natural size. Practically, we reckoned that, with a power of a quarter of an inch focal distance, the magnified result was equal, in round numbers, to five million times, superficial measurement. This microscope was the most perfect which has yet been constructed; and was an adaptation of the dissolving-view arrangement employed in the institution, of which a further account will be found under the heading "Magic Lantern," &c.

With respect to the powers employed in oxy-hydrogen microscopes, we may observe, that several should be provided, ranging from about 20,000 to 1,000,000 times or more. It must, however, be remembered, that the illumination or distinctness of the image on the screen decreases as the magnifying power is increased; and this result limits the power of the instrument. Again, unless the glasses are truly achromatic, spherical and chromatic aberration interfere seriously with the employment of high magnifying powers, and also destroy that vivid and distinct appearance so essential in microscopic illustrations. It is, above all things, essential that each lens should be *exactly* in the line of centres;† and to neglect on this point, the frequent failure of many persons in their attempts to use the instrument are due. These and other incidents occurring in manipulation, have greatly restricted the use of this interesting instrument; although, if properly managed, we know of no apparatus which can be so amusingly or more instructively employed, admitting, as it does, of a popular illustration of some of the most minute, beautiful, and complete objects of the Creative Power.

In the annexed engraving we have a section of one of the most usual forms of oxy-hydrogen microscopes. The arrangement of each part is varied by almost every maker of the instrument. Those manufactured by the British opticians are similar, in their external appearance, to the best forms of magic lanterns; indeed, the microscope is often added to a good dissolving-view arrangement, in which form it is of great value to Mechanic Institutions. The instruments made in France, by M. Duboscq and others, have all the lenses fixed to an arm of brass extending from the body of the lantern, beyond the condensers. This form is liable to derangement of the line of centres, should the brass arm receive an accidental blow.

Fig. 50.



a a, represents the box of lantern; *b*, the lime light; *c c*, the india-rubber tubes bringing the gases; *d*, the first pair of condensers; *e*, the second pair of condensers; *f*, the rack-work used for focussing; *g*, the object to be magnified; *h*, the power, or magnifying lens; *i*, a stop or diaphragm used to cut off superfluous rays.

We need give no further directions, in respect to the general management of the microscope, than those already afforded in respect to dissolving views, &c.* In cleaning the lenses, great care should be observed in replacing them exactly in their proper position in reference to each other, and in the seat or frame. It is a good plan to introduce a small piece of thin paper round the rim of each lens, so that if the lantern be accidentally shaken the lens shall not be shifted.

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* See *ante*, p. 62.

† See *ante*, p. 61.

With respect to the objects which may be used, we may remark, that those that can be examined by the compound microscope, are equally available for the oxy-hydrogen instrument. It must be remembered, however, that the objects are inverted thereby; and, to be viewed in their ordinary appearance, must be introduced, into the slide which receives them, in an inverted condition.

As our pages may fall into the hands of managers and others connected with public societies, we shall suggest a few matters which, unless they have been previously seen, might not be thought of. Amongst these, the following form extremely interesting applications of the powers of the instrument.

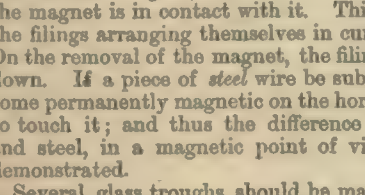
1. *The Crystallisation of Salts.*—To show this, a hot saturated solution of Epsom salts, sal-ammoniac, common salt, &c., should be kept in separate watch-glasses, close by the operator. One drop of each of these may be successively introduced on a slip of glass into the slide. The heat of the lime light will cause speedy evaporation, and the gradual and sometimes rapid formation of the crystals produces a beautiful object on the screen.

2. *Magnetic Attraction.*—This is easily exhibited by placing a few iron filings in a glass trough, such as we shall presently describe, and introducing amongst them a piece of *soft iron* wire, whose end should hang outside the trough. If this be occasionally touched by means of a horse-shoe magnet, it will become magnetic so long as the magnet is in contact with it. This will be shown by the filings arranging themselves in curves on its surface. On the removal of the magnet, the filings will at once fall down. If a piece of *steel* wire be substituted, it will become permanently magnetic on the horse-shoe being made to touch it; and thus the difference between soft iron and steel, in a magnetic point of view, may easily be demonstrated.

Several glass troughs should be made for this and the following purposes. These are easily constructed by cementing two pieces of thin, clear window-glass on each side of a frame of wood left open at the top. One of these is represented in the next engraving (Fig. 51), in which *a a a* are the wooden sides and bottom; *b b*, the pieces of window-glass; and *c*, the opening of the trough into which the objects are introduced. We scarcely dare offer any cement which we can recommend as certain to keep the trough water-tight. We have used solutions of shell-lac, india-rubber, gutta-percha, wax of all kinds, white lead, &c., but have never succeeded in producing ourselves, or of obtaining from the opticians, a trough that would long hold water. However, they are so easily repaired as to make this a matter of secondary importance.

Of course, the size of the trough must depend on that of the slit in the microscope in which it is to be placed. One, two inches square, and three-eighths of an inch wide, is convenient for most purposes.

Fig. 51.

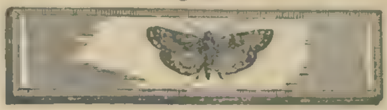


3. *Live Insects, Larvæ, &c.*—Perhaps the most amusing objects for the oxy-hydrogen microscope are found in the insects which abound in ponds, ditches, and the edges of rivers. A few of these may be placed in a trough with some water, and introduced into the slit. The insects being largely magnified, and also stimulated by the heat and light, will be observed swimming rapidly about the trough, biting, fighting, and otherwise amusing themselves. The larvæ of the water-beetle is a most beautiful object; and if two of these are introduced into the same vessel, a fierce battle generally ensues, which often ends in the destruction of one, and perhaps of both the combatants. Water-beetles, water-fleas, worms, and many other insects inhabiting the water, may thus be exhibited to great advantage; and the internal construction of their bodies, together with the circulation of the blood, may be observed at the same time. These insects are easily collected by employing a gauze net mounted on an iron ring. One of these is represented in Fig. 52.

Fig. 52.

4. *Land Insects,* in the living state, may also be shown in a trough, if the top is covered by means of a piece of card-board to prevent their escape. Dead insects may easily be mounted, by placing them between two pieces of glass whose edges are pasted together, as shown in Fig. 53.

Fig. 53.



We need not here extend our list of animated or other objects which may be exhibited, inasmuch as there are few matters to be met with in daily life, of this kind, which will be otherwise than interesting: and we shall enlarge on the subject further on.

5. *The Decomposition of Water by Galvanism.*—To exhibit this, a little zinc and mercury should be placed at the bottom of a trough, which must then be filled by some dilute sulphuric acid, and then introduced into the slit. If a piece of copper wire be dipped into the water, no gas will be given off until it touches the zinc. When this occurs, a small voltaic battery is formed, and torrents of hydrogen rise from the wire surface. On destroying the connexion between the two metals, the effect immediately ceases. By this simple arrangement the action of voltaic batteries may be readily explained.

Having thus given general hints as to the variety of uses to which the oxy-hydrogen microscope may be applied, we shall now point out two most interesting applications, which were first introduced by M. Duboscq, of Paris—namely, the exhibition of photographs, and of photographic copies of blood globules.

Photographs of landscapes, likenesses, &c., may be introduced into the slide, and magnified by a low power. It is essential that they should be transparent, and yet that the lights and shades should be very distinct. The best of this kind are those taken by the albumen process on glass plates; and of all we have yet seen, we have found none to equal those supplied from Paris. The magnified image, under a power of 20,000 times, is of the most interesting character. Some which we have tried, of a size of about two inches square, reproduced most perfectly the inscription on monuments and tombstones; whilst every crevice or crack in the stone of a building was vividly depicted on the screen.

The photographs of the blood globules of animals, &c., are of the deepest interest to the physiologist. Each living creature has a distinct blood globule for the species it represents. Hence photographs of them, when magnified by the oxy-hydrogen microscope, represent the distinctive features of the class of animal from which it has been taken, both in respect to size and shape. It is thus perfectly easy to name any animal by its blood, if a few globules are examined, and compared with what we may call standard specimens. The blood of different animals of the same order, genus, or species, may even be distinguished from each other. This interesting fact has a singular application in respect to jurisprudence; for in cases of murder, nothing is more easy than to examine, microscopically, the nature of blood-spots on the dress of the suspected person, and to be thus enabled to state definitely whether such have belonged to animals or to the deceased person. The investigations and researches of optical science have thus advanced another claim on the esteem of civilised society; and may be called in to solve doubts, which are often the cause of intense anxiety to those who, as jurymen, have to hold in their hands the balance between the life and death of a fellow-creature.

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* See *ante*, p. 64.

Another application to which we have often applied the instrument, is that of exhibiting the effects of chemical tests, when the quantity of a substance under examination is exceedingly minute. Of course this is a matter requiring delicacy of manipulation; but a careful operator may easily show, at the same time, to any number of students, the operation of any test affording a coloured liquid or precipitate. The best mode of doing this is to brush, over a clean glass plate, a portion of the solid or liquid to be tested, and to introduce this into the slide. A drop of the test solution is then placed on the plate, and allowed gradually to run into the solid or liquid already on its surface. The characteristic colour is immediately produced. By these means we have often exhibited the mode of testing for strychnine, by the bichromate of potass, and sulphuric acid, using but a fraction of a grain of the poison for the purpose. The acid, salt, and strychnine solution were placed in three different parts of a glass plate resting in the slide; and on being stirred together by means of a glass rod, the beautiful and characteristic purple colour was at once produced; and, of course, was easily observed by all present.

In concluding our remarks on this instrument, we would strongly urge its adoption by establishments, either public or private, as an excellent means of conveying information, on an immense variety of subjects, to a large number of individuals simultaneously. By means of photographs, or by original objects, diagrams, &c., the principles of arithmetic, algebra, geometry, trigonometry, botany, crystallography, astronomy, physics, mechanics, chemistry—indeed, nearly every branch of human knowledge may be communicated. We have frequently adopted it in public for illustrating all these subjects. With respect to young persons, there exists a great advantage in its use. The whole of the illustrations must necessarily be shown in the dark; and hence there are no objects to distract or attract the attention, excepting the teacher and the image on the screen. From the latter sufficient light is always reflected to enable the speaker to watch any class, however large, and so retain order therein. We have seen numbers of the very dregs of society, and some of its choicest ornaments, equally, and as deeply interested; the former bursting from perfect quiet into enthusiasm at the wonders presented to the eye of their minds; whilst their educated and intellectual superiors have attested a refined interest as each development of the minutiæ of nature has successfully been illustrated, or exhibited before them.

THE SOLAR MICROSCOPE.

We have hitherto deferred to mention the solar microscope, as it is but seldom employed at the present time, having been, in most respects, superseded by the oxy-hydrogen arrangement.

The light of the sun is the illuminating agent, which, being reflected by a plane mirror, is transmitted through the lenses and object; the image being received on a suitable screen, in a like manner to that adopted in exhibiting dissolving views, &c. The arrangement of the lenses being in all respects similar to that of the oxy-hydrogen microscope, it will be unnecessary for us to describe more fully its different parts.

OBJECTS FOR THE MICROSCOPE.

As many of our readers may be desirous of learning the best plan of obtaining, preparing, and mounting objects for the microscope, we shall now give some general instructions, which may be of value, and assist those who are unacquainted with certain minutiæ of the *modus operandi*.

It would be impossible to enumerate the various objects which come under microscopic observation, as of course there is no limit thereto. We may, however, select some which may serve as a general type of others, and thus guide the student in many respects. In carrying out this idea, it may be perhaps as well that we take our subjects separately from the mineral, vegetable, and animal kingdoms; and we shall thereby be enabled to offer considerable variety of illustration. We may, however, remark to all our readers, that the interest of microscopic study does not consist solely in following the beaten track of previous observations. Indeed, one of the greatest charms which the microscope presents as an instrument of investigation, is the opportunity of engaging in entirely novel and independent researches by those who use it. The field it opens out is unlimited; and thus every form or kind of existent matter, whether animate or inanimate, presses into the service of the student and observer of nature.

Our aim will therefore be, to point out "how to observe," rather than "what to observe;" and in this attempt we shall endeavour to be as practical and definite as possible.

Presuming that the reader has furnished himself with a good achromatic compound instrument, we will now proceed to describe the best methods of collecting, examining, and preserving objects.

We have given special directions respecting some objects in the previous page, to which our present remarks are supplementary. Perhaps the most interesting objects of microscopic study are found amongst the insect tribe of animal life. It cannot fail to astonish every one, especially those unaccustomed to scientific observation, when they discover the extreme minuteness, and yet perfection of organs, which are noticed amongst the invisible but animate beings which exist both in the air and in water. In spring, summer, and autumn, we cannot inspire a breath of air without inhaling numbers of germs floating therein; and the refreshing draught of water taken from the river or brook, equally abounds with living creatures. Even the very dust of the ground, and of our houses, and many articles of our daily food, present living organisms, thousands, and even millions of which may be covered with a pin's point. Some of them are so small as not to exceed the twenty-thousandth of an inch in length; and yet they are provided with mouths, stomachs, &c. Their process of feeding, and their delicate organisation, may be easily detected by means of a good microscope, and the peculiarities of their construction observed, which alike puzzle the naturalist to explain, and interest the observer in their examination.

Ascending in the scale of life, we find the various parts of insect organisation to become a subject of the deepest interest; and we may state, that the same laws as to generation, reproduction, growth, and death, alike affect the existence of the smallest creatures as are found to govern the same occurrences in our own lives. Indeed, we perceive by our researches, that unity of design, simplicity of causes, and identity of effects, are alike operating on all forms of existence, modified occasionally in instances where special purposes have to be fulfilled.

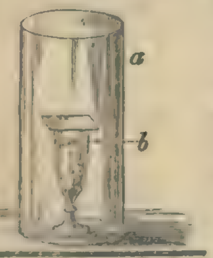
Amongst the smallest objects of insect life to which we can direct attention, are those included in the infusoria. These abound in many places, and may often be observed covering water in brooks and ditches, with an appearance resembling oil.

A stagnant pond, if examined during a hot day in spring or summer, will be found to be literally alive with insects. These should be carefully collected by dipping with a basin from the pond; and must then be transferred to a tall glass vessel, filled with water. Into this a few plants should be introduced; and those growing beneath the surface of the pond from which the insects have been withdrawn, will perhaps be the best. A little of the earth should be removed with them, and the plants will thus thrive in the vessel; and by their action on the water they will tend to keep it fresh, and fit for animal existence. Indeed, such an arrangement forms a small aquarium. By similar means we have been able to keep a good stock of insects throughout the year; and have often surprised our microscopical friends by exhibiting many curious and interesting little creatures in the depth of winter, by aid of the oxy-hydrogen microscope. Fresh water should be occasionally added; and the temperature should never be allowed to go below 60° Fah. In such a collection the student will be surprised to find an immense variety of objects. We shall speak of many of these under the head of Natural History; to which, or to any work of a similar kind, we must for the present refer our readers.

The unlimited number of land insects, places at the disposal of the microscopist a most interesting field for investigation. As it would be impossible for us to enumerate or classify such objects, we shall content ourselves by pointing out only those parts of most insects which require special notice. We must first say a few words as to catching, killing, and mounting.

Generally speaking, insects of the moth kind are best caught about dusk; and this should be done by means of a fine gauze net, similar to one already described and illustrated;* the sack, however, being larger, and of lighter materials. By this method, the wings, antennæ, &c., are uninjured. With respect to killing insects, the best plan we know of combines the advantages of producing instant death, and retaining the colours, &c., quite uninjured—which is by means of carbonic acid. In endeavouring to simplify the process of making the gas, we suggest the following plan:—Place an ale-glass in a confectioner's show-glass; and into the former put a teaspoonful of chalk or whiting. On this pour some vinegar and water, or a little hydrochloric acid, diluted with five times its bulk of cold water. On the top of the ale-glass place a small card, so as to entirely cover it. The carbonic acid will gradually escape, and, being heavy, will flow out of the ale-glass, and fill the larger vessel. Any insect dropped into the latter will instantly fall down dead to the bottom, with wings extended; and, in fact, will preserve in death every external appearance of life. The card of course prevents the insect from falling into the mixture; and none of this should be allowed to flow into the outer vessel. Fig. 54 represents this simple and effective arrangement.

Fig. 54.



a, the outer glass vessel; *b*, the ale-glass, containing the mixture producing carbonic acid gas.

The insect, if small, may then be placed between two slips of window-glass; but this should not be done until all the moisture has been dried out of the body. This is best done by leaving the insect for some days in the open air, and in sunlight. The mode of mounting, and the actual size of the glass plate, are already described.†

Considerable delicacy is required in separating the various parts of insects, such as the antennæ, eyes, wings, legs, stings, &c.; but such can only be gained by long practice, and great care. Instruments, such as forceps and small lancets, are often supplied with compound microscopes for such purposes; and whilst attempting these operations, a magnifying glass should be used, which will enable the operator to proceed with greater certainty. When the part has been removed, it should be placed on the centre of a piece-glass, half an inch wide and three inches long, and on it a drop of Canada balsam may be poured. This preserves the object from injury by shifting, &c.: the arrangement is completed by placing another slip of glass over the balsam when dry, and the edges of the two slips may then be pasted together, as already mentioned. In a similar manner, the hair, down, &c., of insects, birds, or animals; and the scales, &c., of fishes, may be preserved from injury, and placed in a fit condition for microscopic purposes.

The ova or eggs of insects are generally found in crevices or slits in wood, &c.; and from their extreme minuteness they actually require, in many instances, a microscope to discover them. Dust and dirt, so called (which, however, has been defined as "matter in the wrong place"), are often prolific with these objects. Indeed, we would venture to suggest the use of the microscope to many housewives, who might thus discover the cause of the appearance of "such swarms" of certain insects; and being so forewarned, might take the necessary steps for preventing their subsequent presence in a lively and predatory state of existence. So extremely small, indeed, are some of these ova, that they have not been seen by aid of even the most powerful microscopes. Hence has arisen the absurd theory of "spontaneous generation," an idea as discreditable to the reasoning powers of a man, as it is an evidence of the crudeness of his imagination. We need but further say here, on this subject, that if *any* creature could create itself, a Creator would be unnecessary; for, as the most minute living thing that has yet been examined, has organs which, in their construction, functions, and use, are analogous with those of the largest size of animals, it follows that the creative power should exist in every being, small or great; the result being only modified by the individual ability or strength of self-creation in each species—an argument too ridiculous to require refutation. Moreover, the advocates of the theory must admit, that inanimate matter must also have been self-creative, to be consistent in all their propositions: of this idea, however, they decline to take the responsibility; and thus they attempt to build an edifice, but forget first to lay its foundation.

An interesting class of objects is found in insects which are bred, grow, and feed in various animal substances. Amongst these we may mention the mites in cheese, the maggot, and parasitic creatures, such as the louse, the tick, &c. These are generally regarded with a degree of disgust, which, however, should not be shared in by any student of science. Duties of the most offensive kind are necessarily undertaken by those experienced in scientific pursuits, for the purposes of jurisprudence, which require the exercise of great nerve in their pursuit. The student, however, is not called to these, and may not object to examine any of the productions of nature because, under certain circumstances, these may prove offending—bearing in mind, that whatever the Deity has made is worthy of man's investigation.

Directing our attention to the organs of animals, we may mention, that a most instructive branch of inquiry is found in comparative physiology and anatomy. Commencing with the heart, we may remark, that that organ has its representative in every kind of breathing creatures. The circulation of the blood, the nervous and muscular systems, the perspiratory ducts, the skin, bone, fibre, hair, down, feathers, are mostly found in actual existence, or substituted in each order; and an intense interest is created in the search for these in different species, or the discovery of their modifications. Under the description of the oxy-hydrogen microscope and its uses, we have made special mention of the nature of blood globules, and their characteristic differences in animals; a subject of importance in a philosophical and even legal point of view.

We have thus indicated some of the interesting objects to be found in various departments of the animal kingdom, and must leave our readers to complete that which we cannot further extend without encroaching on the limits of our plan.

A very interesting field is opened out in the vegetable kingdom for microscopic study; and in it the analogies between vegetable and animal structure hold a prominent place. As we have already remarked, it will be impossible for us to give more than general hints; and the directions we have given in reference to mounting animal objects, are equally applicable to those obtained from plants, &c.

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* See *ante*, p. 68, Fig. 52.

† *Ante*, p. 68, Fig. 53.

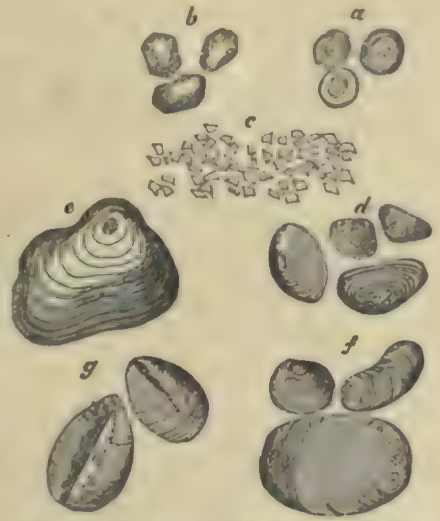
The fungi so continually formed on decayed substances vary in size, from those requiring a microscope to detect their presence, to those of the mushroom and larger kinds, with which all are familiar. Some of these present very beautiful appearances under the microscope, and are therefore well worthy of the attention of the student.

Ascending to the higher classes of vegetables, we observe the most interesting traces of analogy between plants and animals. The leaves take the place of lungs; the solid fibre that of bones; the sap is as the blood; and we might thus continue to connect the two kinds of existence by an almost continuous chain. It will be sufficient for our purpose to indicate some of the leading points of resemblance between the two kinds of vital existence; and we therefore append a synopsis of the most easily recognised subjects.

The cells of all plants perform a similar office to that of the veins of animals, as the sap—which is the blood of plants—is propagated to every part by their agency. The existence and appearance of the cells may be observed by cutting a thin slice off any branch or stem. The form of the cell, and the arrangement of the successive rings, vary in each tree; so that they are really characteristic, and form beautiful microscopic objects.

The woody fibre or solid portion of plants is, as it were, their flesh, bone, and muscle, and differs, in its physical character, in every production of the vegetable kind. Thus we have the strong and close fibre of the oak and ash, and the delicate fibres of the cotton plant. The fibrous matter of grasses is easily obtained by soaking the leaf in a little solution of carbonate of soda, and then immersing it in some very dilute sulphuric acid. Each filament will split from its neighbour, and thus may be separately examined.

Fig. 55.



a represents granules of rice starch; *b*, granules of sago starch; *c*, the granules of wheat starch; *d*, granules of potato starch; *e*, the same, but more magnified; *f*, the granules of *tous les mois* starch; *g*, is the same starch after being heated, by which the granules are burst open; *h*, is a specimen of pea starch.

The leaves of trees, &c., present a very interesting appearance, and the green matter may easily be separated from the fibre, and so produce the net-work in a separate state. There are few more beautiful objects than these for exhibition in the oxy-hydrogen microscope; and for that purpose the currant tribe of leaves is to be preferred.

The cuticle or skin of plants is analogous, in its office, to the animal skin. Hairs, down, pores, &c., may be detected in the leaves of most plants, the pores being very numerous on them, and serving the double office of respiratory and perspiratory organs.

In some, or most grasses, in wheat, barley, and oats, a silicious skin is observed, which is composed of minute crystals of silica or flint. A coarse and rank grass, growing in stagnant ponds, affords it in considerable quantities.

Starch is a substance abounding in all plants, and presents a variety of appearances when the granules are obtained from vegetables. Fig. 55 illustrates the form of some varieties.

We need not remind our readers, that flowers are interesting objects for microscopic investigation, and their variety affords an almost endless field for research. The seeds and fruits of all kinds of plants are equally interesting; and perhaps the most instructive matter to which we can direct the attention of the student, is that of the process of germination, from the first change of the starch into sugar, until the radicle has made its appearance on the outside of the seed. We must not, however, enlarge here on this subject, because we shall have to enter fully into the consideration of these points in our section on Botany.

In the mineral kingdom, we find less variety of objects than in those on which we have been treating. This is partly owing to what we venture to term a comparative monotony of external form which exists therein, in contrast to the infinite variations of animated structures. The mineral, in fact, forms the raw material, which, in the hands of the Creator, has been fashioned into life. Organised existence, indeed, depends on the arrangement of but few elementary bodies. The means are surprisingly limited; whilst the number of products is only bounded by the skill of Him who in wisdom hath made them all.

There are, however, many points of interest to be discovered in the mineral kingdom, more especially in reference to the molecular and crystalline structure of bodies. It is true that we cannot perceive those ultimate atoms of which a mass is composed; but, as the power of our microscope is increased, so we can often detect the existence of definite forms utterly unperceivable by the naked eye. A most remarkable instance of this is found in the discovery of Professor Ehrenberg, who has detected the presence of exceedingly minute shells in common chalk; which indicates that that substance is simply the remains of an immense destruction of animalcules, whose exuviæ are now the only testimony of their previous existence. Similar facts have been brought to light during the examination of the bed of the Atlantic, preparatory to laying the electric cable between this country and America. Quantities of what appeared at first to have been sand, turned out to be the shells of marine insects, which had gradually accumulated in such quantities, as to present a bed thousands of miles in superficial area, and of considerable depth.

In geological researches, the microscope is of great value in determining the character of some strata, by the examination of the materials of which it is composed; and similarly, in the study of mineralogy, the amorphous or crystalline nature of masses can be detected. We shall, however, defer for the present any details on these subjects, reserving our remarks until we deal with them in their proper sections.

We have thus endeavoured to give some general idea of the diversified objects which fall under our notice, as subjects of microscopical examination in the three kingdoms—the animal, vegetable, and mineral; and conclude by saying, that our observations can only be regarded as indicating the nature, rather than the extent, of valuable knowledge which the microscope opens out to our understanding and examination.

TELESCOPES.

The next optical instrument to which we shall call attention is the telescope, with its various modifications of form, &c., which arise from the different purposes to which it is applied. Before entering on this subject, the student should have become familiar with the principles on which lenses and curved reflectors are constructed, and which we fully explained in the earlier pages of this work.*

Telescopes are of two kinds; namely, refracting and reflecting. In the former, the rays of light passing from the object are successively refracted by two or more lenses before reaching the eye. Of these, again, there are two classes, the ordinary and achromatic; in the latter, the coloured fringes produced by the employment of lenses of one kind of glass, as in common telescopes, are avoided by the use of lenses composed of two kinds; namely, crown and flint, which, by their different dispersive powers, destroy their natural tendency to produce coloured images. The principles on which achromatic lenses are constructed have been fully explained in our previous pages;† to which, for the sake of saving space, we must refer our readers.—Reflecting telescopes, on the other hand, receive the rays passing from a distant object on a peculiarly curved reflector, called a speculum; and these are reflected to the eye through an eye-piece. Hence, generally speaking, reflecting telescopes combine the principles of both reflection and refraction in their construction. Their forms have been varied according to the special views or requirements of their inventors. We shall select those most known and appreciated as the subject of our explanation; and give a general outline of the means employed to cast and to grind the speculum to a proper shape.

REFRACTING TELESCOPES.

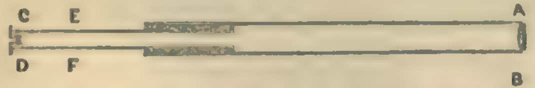
From the time of Galileo, to whom is generally accorded the merit of their discovery, telescopes of this kind have been the object of great care on the part of the instrument-makers. The value of a telescope depends, essentially, on its affording a well-defined, colourless, and correct image of the object viewed by it; and these results are only obtained by the employment of lenses as free as possible from either spherical or chromatic aberration.‡

For ordinary purposes a combination of lenses is required, which will present the image, to the eye of an observer, in a vertical position; or, in other words, the object must appear, in all respects, with the exception of size, as if it were viewed by the naked eye. In instruments used for astronomical purposes, this precaution or peculiar arrangement is unnecessary, because size and position are the chief objects which are sought after; hence, two lenses are sufficient in many cases.

Before proceeding to describe various forms of the refracting telescope, it may be as well that we should call the attention of the reader to the names of the two chief lenses—the *eye-piece* and the *object-glass*. The former term is applied to the lens which is placed *next to the eye* of the observer; whilst the *object-glass* is that lens furthest from the eye, and *next to the object;* it, indeed, first receives the rays of light entering the instrument. The definition of these terms will save us much circumlocution hereafter.

As nothing conduces more to interest the student in physical science than assisting him to the personal application of its laws and principles, we shall now give directions, by means of which any one may construct a simple telescope at a trifling expense, and yet of sufficient power to enable him to view many interesting astronomical appearances. We are indebted to Dr. Dick for the following description of an instrument of this kind, which we have slightly modified.

Fig. 56.



"Procure a double convex lens of about thirty-six inches focal length, and place it at the end of a tube, as at A B, Fig. 56. At thirty-seven inches distant from this lens fix another, as at C D, whose focal length is one inch. This should be fixed in a separate short tube, E F, which should be so made as to slide backwards and forwards in the large tube, for the purpose of adjusting the focus of the telescope to the eye. The following particulars require to be attended to. The aperture or opening which lets in the light at A B (that is, in front of the lens), should not exceed one inch in diameter, otherwise the image of the object will be somewhat confused. There should be a hole (or stop) in the focus of the eye-glass, C D, rather less in diameter than the breadth of the lens, for the purpose of excluding extraneous rays."

The tubes may be made of tin or zinc, and the insides should be blackened by painting them with a mixture of lamp-black and size. The stops at each end of the tube are easily made out of circular pieces of zinc, from the centre of each of which a neat round hole must be cut; that for the object-glass being an inch in diameter; and that for the eye-glass may be about half an inch. These can easily be soldered into their places on the tube. Such an instrument can be made for five shillings, and it will have a magnifying power of thirty-six times. Of the question of magnifying power, however, we shall speak more fully as we proceed. Simple as this form of telescope is, we may inform our readers that, in using it, they will be enabled to observe Saturn's ring, Jupiter's satellites and his belts, the lunar mountains, and the spots on the surface of the sun. For the latter purpose, however, the eye should be protected by means of a piece of dark-blue or blackened glass, placed between the eye and the eye-piece of the telescope, or the intensity of light and heat would be seriously injurious. We must also impress on all who may attempt to make such an instrument, the necessity of having the axes of the two lenses in exactly the same line. In other words, a line drawn through the centre of the object-glass A B, must, in a properly-constructed instrument, also pass through the centre of the eye-piece, C D. In the construction of optical instruments of all kinds, wherein two or more lenses are employed, this is certainly the "*pons asinorum;*" and neglect of this simple rule often involves a vast amount of annoyance and expense. Of this we met with an amusing instance in the case of a friend, who had expended upwards of thirty pounds in remedying a vast number of imaginary defects in an oxy-hydrogen microscope. On inspection, we found that the only fault existing was, that a brass arm, at each end of which a lens was fixed, required to be straightened, an operation which was performed in a few seconds by the aid of a vice.

We must now explain the cause and extent of the magnifying power of telescopes; and the simple one which we have just described will enable us to do so without danger of distracting the attention of the student by unnecessary detail, because the principles involved in its construction are equally so in every other form of telescope. In this telescope, the eye-piece is the medium by which the image produced by the object-glass is viewed and magnified; hence the magnifying power of any telescope of this kind will be in the ratio which exists between the focal distances of these two lenses. For instance, the focal length of the eye-piece, in the instrument we have described, is supposed to be one inch, whilst that of the object-glass is thirty-six inches; we thus obtain a magnifying power of thirty-six times. It also appears, that if the focal length of the object-glass is increased, and that of the eye-piece diminished, the magnifying power of the telescope is increased in the ratio which these two measurements bear to each other. We should here observe, that when we speak of the power of a telescope as magnifying so many times, it is meant that the diameter, or linear measurement, is increased. Of course, the total superficial result will equal the square of the linear one. Our telescope, for instance, is supposed to have a linear magnifying power of thirty-six times; but its superficial power will be equal to the square of 36, or 1,296 times.

In the following diagram, we have an illustration of the optical effects produced in the employment of telescopes (See Fig. 57). The larger lens at L being the object-glass, receives the rays of light, which, for our present purpose, we shall suppose to proceed from a star, A B; and are therefore, practically speaking, parallel to each other. The image of these rays is found at *a b*, and is, of course, inverted. The lens, or eye-piece, L′, magnifies the image, *a b*, in the manner represented at *b′ a′*.

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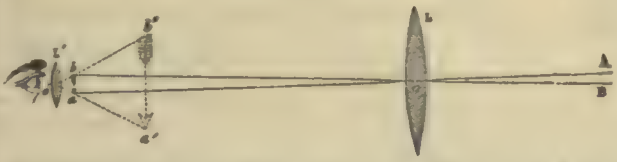
* See *ante*, p. 43, *et seq.*

† *Ante*, p. 53.

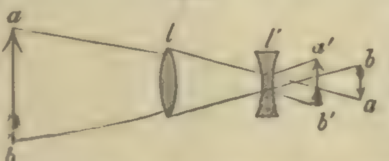
‡ *Ibid.*

Fig. 57.



In all telescopes, arrangements are made by means of which the relative distances between the object-glass and the eye-piece can be varied. This is for the purpose of adapting the instrument to the optical peculiarities of each observer; and is necessary, because the point of distinct vision varies considerably in different individuals, owing to the convexity of the eye in some, and its comparative concavity in others. We have already spoken fully of this subject in our remarks on the human eye;* to which we refer those of our readers who may be desirous of observing their application in reference to the use of telescopes, and other instruments generally. If two eye-pieces, of equal focal length, are placed successively between the object-glass and the eye-piece of the telescope represented in Fig. 57, the image will be presented to the eye in a non inverted form. This is, in fact, the arrangement employed in all ordinary telescopes used for viewing terrestrial objects; and the magnifying power of such instruments, as in the one we have been considering, is found by dividing the focal length of the object-glass by that of one of the eye glasses. The non inversion, however, of the image may be obtained by using a double concave eye-piece, instead of a convex one; the lens being placed nearer to the object-glass than at the point at which the rays passing therefrom would form an image. Fig. 58 represents an arrangement of this kind, in which *a b* is the distant object; *l*, the object-glass; *l′*, the concave lens; *b a*, the place at which the inverted image produced from the object-glass would be formed; and *a′ b′* is the non-inverted image, as observed by the eye of the spectator. The length of the instrument is much diminished by this arrangement; and hence it is a convenient form for opera-glasses, and other similar instruments, in which a moderate magnifying power, and limited field of view, only are required. In this instrument the two lenses are placed at a point equal to the *difference* of their focal lengths; whilst, when two convex glasses are employed, they are separated to a distance equal to the *sum* of their focal lengths. This instrument is generally called the Galilean telescope, from its having been employed by Galileo, the celebrated Italian philosopher.

Fig. 58.



Having thus described the ordinary forms of the telescope, we proceed to those of a higher class, in which chromatic aberration is almost wholly destroyed by the use of what are termed achromatic lenses. We have already explained the principles on which they are made, when treating on dispersion and achromatism;† it therefore only remains for us to describe the advantages derived from their adoption in telescopes. *Mathematically* speaking, two lenses may be so arranged in ordinary telescopes, that one, being the object-glass, shall have a focal distance of any assignable length, whilst the other, or eye-piece, may equally have one of the smallest extent; and the magnifying power of a telescope constructed of two such lenses, will, as we have already explained, be just in the ratio of the large focal distance of the object-glass divided by the short one of the eye-piece. But *practically*, there are limits, both of construction and use, which forbid the employment of lenses beyond a certain size and focal length. In the earlier days of telescopic discovery, object-glasses were used which had focal lengths varying from three to three hundred feet. Of these it will be unnecessary to do more than state, that their use was highly inconvenient. But to employ lenses of ordinary glass for high magnifying power, and at but a manageable distance from each other, was an unsolved problem until Mr. Dollond discovered, that a combination of lenses of two kinds of glass, whose dispersive and refractive powers would correct each other, effected that purpose. An object-glass composed of a concave lens of flint glass, into which is fixed a double convex lens of crown glass, meets the difficulty; and telescopes fitted with such, give not only a clear and distinct image, owing to an almost entire freedom from both chromatic and spherical aberration, but also supply another great boon to the astronomer, arising from this effect—namely, that of permitting the use of a telescope not exceeding a few feet in length, in place of those of the extraordinary size to which we have just alluded. In such arrangements, magnifying powers of astonishing extent may be employed; and thus the astronomer is supplied with an instrument whose capabilities are, practically, only limited by the means of its owner, and the skill of the workman employed in its construction.

REFLECTING TELESCOPES.

We now proceed to examine another kind of telescope, whose use is almost entirely confined to astronomical purposes, and in the construction of which great ingenuity has been manifested.

A reflecting telescope consists chiefly of three parts—namely, the *speculum*, or reflector, which receives the rays from the object (this takes the place of the object-glass in the refracting telescope); the *smaller reflector*, which is sometimes employed to receive the image from the large one; and the *eye-piece*, by means of which the image is viewed and magnified. These arrangements either vary or are modified in different telescopes, as we shall presently see.

It will be unnecessary that we should go over the same ground again in describing the general character, &c., of various forms of curved reflectors. This we have already done in a former part of this work, somewhat minutely; and we therefore refer our readers thereto,‡ and will proceed to describe each of the parts of the telescope in detail.

The speculum is generally made of an alloy composed of variable quantities of copper and tin; in addition to which, small proportions of other metals have been used. It is impossible to give the exact proportions in which these metals should be employed to obtain the best result. The object sought after is an alloy which shall be able to receive the highest polish; to retain this as long as possible; and yet, despite the hardness necessary to these ends, it must be sufficiently yielding to undergo the process of grinding. After being cast in a mould of suitable form, the alloy is annealed, and its brittle nature is thus tempered down. The process of grinding the casting into a proper shape then follows, and is effected by pressing on its surface a wooden mould which has been turned to a shape convex to the concave one which the speculum has to receive. Whilst the speculum is made to revolve, the mould, or grinder, is continually being covered with the powder employed in grinding. In some cases the speculum is stationary whilst the polishing mould or tool rotates against it. This operation is one of great delicacy, as on the perfection of shape and polish, the value of the speculum entirely depends. The largest speculum yet made is that belonging to Lord Ross. It has a diameter of six feet; weighs about three tons; and its focal length is about fifty-two feet. This reflector is composed of copper and tin united, in what, in chemical language, would be termed atomic proportions; or, in other words, such quantities of each metal were employed as are represented by those in which they would combine with acids, &c., in forming definite compounds.

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* See ante, p. 43.

† Ante, p. 53.

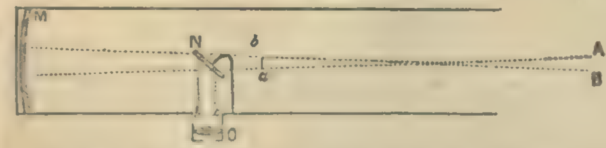
‡ See ante, p. 43.

Recurring to the proper shape of the speculum, we must refer to our remarks on spherical aberration,* and also to those on the properties of reflectors which have curves of one of the conic sections. In obedience to the laws of reflection from curved surfaces, the specula used in reflecting telescopes are usually made in the shape of a parabola; and thus, as far as possible, the evils of spherical aberration are avoided.

It will, perhaps, be more convenient that we should now illustrate the principal forms of reflecting telescopes which have been employed; and include, at the same time, the description of the small reflector and eyepieces specially adapted to their construction.

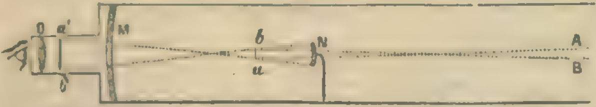
In the annexed figure we have a representation of Sir Isaac Newton's reflecting telescope. In this (see Fig. 59), M is the speculum on which the rays from a distant object, A B, impinge. These are reflected from M to N, which is a plane reflector, placed at an angle of 45° to the axis of the mirror. The image reflected from N is received and magnified by an eye-glass placed at O. The eye of the observer is of course placed at the opening beneath O, in a position which is extremely inconvenient. The mirror is placed between the focus *a b* and the speculum, and the image is formed at *a′ b′*.

Fig. 59.



The Gregorian telescope—so called after its inventor, Dr. Gregory—has a very different and much more convenient arrangement. In using an instrument of this kind, the observer views the image in a line which forms the axis, instead of being at right angles thereto, as in Sir Isaac Newton's telescope. A hole is bored in the large speculum, at its centre; and the rays of light reflected from the surface are received by a smaller concave mirror, which again reflects them to the eye-piece, and so they reach the eye of the spectator. Fig. 60 illustrates the construction of a Gregorian telescope; and the same letters are employed as in our last engraving, which will, therefore, at once point out the different parts, their position only being varied.

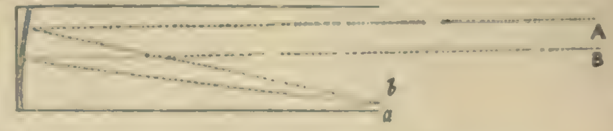
Fig. 60.



The Cassegranian telescope has a *convex* mirror, placed at N, instead of a concave one, as employed in the Gregorian. Both these forms of instrument have been variously modified, by astronomers and others, since the time of their invention.

The celebrated Sir William Herschel introduced another form of reflecting telescope, in which the observer looked down the tube enclosing the speculum, and of course with his back to the object. The speculum was placed at an angle with the axis of the instrument, and so cast the reflected rays towards one edge of the tube, at which they were viewed by means of an eye-piece. By this arrangement a second reflector, as employed in the previously described instruments, was rendered unnecessary. Fig. 61 represents a section of this form of telescope, in which the rays proceeding from A B are received by the angularly placed speculum, and reflected to *a b*, at which point they are viewed.

Fig. 61.



MISCELLANEOUS OPTICAL INSTRUMENTS.

HAVING mentioned the principal optical instruments, their construction and their uses, we now proceed to speak of those which, whilst highly interesting and useful, are still of comparatively minor importance, and, as such, will not require so extended notice.

Amongst these we may include the following: namely, Concave and Convex Mirrors, Spectacles, Magnifying Glasses, Burning Glasses, the Kaleidoscope, Debusscope, Thaumatrope, Kalotrope, &c.; of each of which we shall give a short explanation.

We have already explained the principles of concave and convex mirrors, and have, to some extent, alluded to their various uses. In doing so, however, we confined ourselves to strictly scientific illustration, and now propose to show a variety of amusing uses to which many of them can be applied. The same may be said of the other instruments to which we have alluded; all of which are constructed on the principles already laid down—modified, however, by peculiarity of construction and object.

As a variety of interesting experiments may be tried with both concave and convex mirrors, we may here enter more fully into the practical part of their construction, than we have in our previous pages.† We there stated, that unless a parabolic or elliptic form be employed, great loss of light and heat is sustained during reflection. For the purpose, however, of the following experiments, those of a spherical shape will answer very well. We may add, that such will be equally useful to the student during his studies of heat and sound; because they, like light, obey the laws of radiant forces.

Where the means of the student will permit, we advise the purchase of reflectors having a polished silver surface, which may be obtained of most of the optical instrument-makers. As, however, some of our readers may not be in a position so to do, and it being our desire to facilitate the repetition of all our experiments, the following directions may be useful; and if carefully carried out, they will enable almost any person to become practically acquainted with the laws and amusing applications of curved reflectors.

Either metal or glass may be used for the purpose. Generally speaking, metallic reflectors are preferred. A sheet of copper is to be hammered into a hollow shape, which can be readily done by any copper or tin smith. Instead of copper, tin or zinc may be used, as either of these metals will receive a good polish. They are, however, liable to tarnish; and just in proportion as that takes place, the value of the reflector for experimental purposes is diminished. The size of the reflectors may vary from eight to twelve inches in diameter, the reflecting power increasing in the ratio of the diameter. If copper reflectors be preferred, they should have their surface polished as brightly as possible, and be afterwards silvered by means of the electrotype process, or in the ordinary manner of plating. They then require repolishing; and such have the advantage of retaining the polish for a long time, and also reflect a larger amount of incident rays of light than any other metal.

If glass be used, the *outside* surface may be "silvered" in a similar manner as is adopted for ordinary looking-glass if a *convex* mirror is required, and the *inside* for a *concave*: in either case, the side opposite to the reflecting surface is that which is to be coated with mercury. Reflectors of this kind must form a portion of a sphere; and a watch-glass, of the ordinary form, illustrates the shape of such a reflector. The "silvering" process is easily performed; and for this purpose the glass must be carefully cleaned, so as to be quite free from dust, spot, or dirt: a piece of clean tinfoil is to be pressed on one of its surfaces, so as to evenly cover every part; a little mercury is then poured on to the tinfoil, and evenly spread by means of a hare's foot, the hairs of which answer the purpose extremely well: a piece of thin tissue-paper may then be laid on the foil and mercury; and gentle pressure is to be applied, so as to force off the superfluous metal. The amalgam will adhere to the glass surface, and thus form a good reflecting coat. Either concave or convex reflectors are thus made; and although inferior to metals in their reflecting power, they will illustrate many interesting laws and applications of light. The most convenient way of arranging two reflectors is by placing them on stands, allowing them to slide on a rod, to which they can be fixed in any desired position by means of a binding-screw. Two should be obtained; and their appearance is illustrated in the annexed engraving. Referring to our remarks on curved reflection,[*] it will be found that we there enter fully into the explanation of such matters as focal distance, &c.; and these data, in reference to any mirror which the student may employ, must be ascertained by him experimentally, according to the rules we have laid down, and which we shall not repeat here. Such having been found, the following uses of curved reflectors may easily be examined, and several interesting experiments may be tried.

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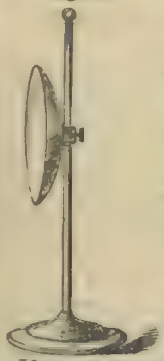
* See *ante*, p. 44.

† See *ante*, p. 43, *et seq.*

Fig. 62.



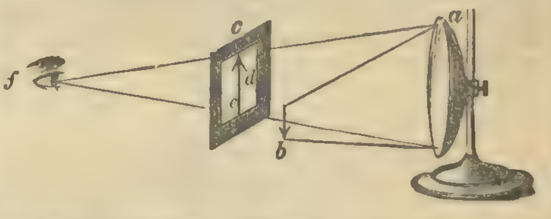
If a concave mirror be so placed as to receive the rays of light and heat from the sun, such will be condensed at the focal point, and powerful heat and light will be there produced. It does not matter whether the reflector is entirely spherical; for if made-up of a number of reflecting planes, the same effect is produced. Very powerful "burning-glasses" have been thus constructed, by building up a number of small plane reflectors in a concave curved form. By means of these the most refractory metal may be fused; and other substances which resist the heat of the most powerful furnaces are converted into the liquid state. Such reflectors have been in constant use in lighthouses, for the purpose of casting the rays of a lamp to a considerable distance. Perhaps the most remarkable instance of this sort was that of the reflection of the lime light in Scotland, during the survey made. On one occasion, the rays reflected by a concave mirror were distinctly perceived at a distance of ninety miles from their source. By similar means, the light produced by Holmes' magnetic arrangement was cast across the English Channel from the South Foreland lighthouse, and seen on the French coast, shining with great brilliancy. Concave reflectors, combined with lenses, are similarly employed on railways, as "signal lights;" and in any case, where a powerful and concentrated light is required, their use is of great value.

If an object be placed somewhat beyond the centre of curvature of a concave mirror, an inverted image of it may be seen in the air. This effect can be applied for a variety of amusing purposes. Thus, a jug of water, if its contents be poured out, will seem to be filling. An excellent plan of illustrating this amusing fact, is that of allowing thin smoke, as from smouldering brown paper, to pass between the observer and the mirror, when the inverted image will be distinctly perceived. The object is to be placed a little beyond the focus of the mirror in such experiments: the proper position is easily ascertained, however, by a trial or two. A person standing before a concave mirror, beyond the centre of curvature, will see an inverted and smaller image of himself; and by moving to and fro, the image will appear to advance till the centre of curvature is reached: passing that, the image will seem to recede from the observer.

Some amusing optical illusions may be effected by means of concave mirrors; one of which we shall here illustrate, and leave to the ingenuity of the reader to extend them in any other form he pleases.

If an object be placed, as we have just described, before a concave mirror, the spectator will see an inverted image, when placed in certain positions. If, however, the object be inverted, then an upright or vertical image will be produced. Availing ourselves of this fact, and placing before the mirror a screen having an opening of less diameter than the mirror, a spectator will see an image of any object placed between the screen and the reflector, apparently suspended in the air. Cameos, statuary, and other matters may be employed for the purpose; and for the amusement of young people, objects of a comic character may be employed. The arrangement will be fully understood by inspecting the annexed engraving; in which *a* represents the concave reflector; *b*, the object which must be inverted; *c*, the screen; *d*, the aperture therein; *e*, the image formed in the air; and *f*, the eye of the spectator. (See Fig. 63). The use of smoke, as before recommended, improves the effect; and if the desire be to startle or surprise, the mirror and object should be placed in one room, and the observer should look through an opening in a doorway, from the apartment in which he is placed, adjacent to that containing the mirror. The cause of the image will thus be entirely concealed from him; and the effect on his imagination will be proportionably increased.

Fig. 63.



From our observations on convex mirrors,[†] the reader will be prepared to find, that an image cast thereon will be magnified, at certain positions, instead of being diminished, as is the case when concave reflectors are similarly employed. On bringing an object near to a convex reflector, its image increases in size until the object and the reflector come in contact. This may be observed by watching the effect on the image of the face, whilst approaching or receding from a polished teapot, whose body, generally speaking, is of a convex form. Some concave and convex mirrors, which are sold by the opticians, have an amusing effect on the minds of persons looking thereon, especially if they are ignorant of the laws of optics. These mirrors are constructed so as to be viewed both widthways and lengthwise; and they respectively produce a ludicrously enlarged image of the face and features generally. These were often used as mirrors for the drawing-room, some years ago; and, perhaps, many of our readers may have seen them in houses furnished after the old fashion.

By means of concave mirrors of a cylindrical form, some very curious optical effects are produced, similar to those we have just named. Objects are painted, say of a landscape, &c., presenting an irregular or distorted form when seen by the eye. If, however, such are viewed by means of the mirrors we have been describing, they at once present a definite form. Such are called *anamorphoses*, on account of the peculiar change of effect which is produced by the arrangement.

SPECTACLES.

We have already mentioned, incidentally, instruments of this kind:[‡] we shall, however, enlarge on the subject, so that our readers may more fully understand the principles on which spectacles, &c., are constructed.

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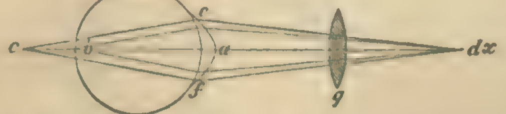
[*] See *ante*, p. 44, *et seq.*

[†] See *ante*, p. 45.

[‡] *Ante*, p. 49.

In our remarks on the structure of the eye,* we mentioned that the cornea is of a convex form, like an ordinary convex glass lens. It performs that office to the eye which a lens does to a microscope or telescope; that is, it collects the rays of light passing from any object. Each part of the eye, however, is exactly adapted to its special purpose, and any disarrangement, no matter how trifling, is followed by inconvenience. Even in early life, the eyes of some persons have the defect of being either too convex or too flat; and such defects are remedied by employing glasses or spectacles of an opposite character. Thus a highly convex cornea requires a concave lens; and one too flat is remedied by the employment of a convex glass. When the defect arises from the cornea not having sufficient convexity, then the rays of light received and refracted by it, have their focus behind the retina. The effect is very similar to that found when the lenses of the magic lantern, &c., are out of focus. The result, of course, is, that the person so affected suffers from great indistinctness of vision. This will be better understood by reference to the annexed diagram, in which it is supposed that the cornea has not sufficient convexity for the ordinary purposes of vision.

Fig. 64.

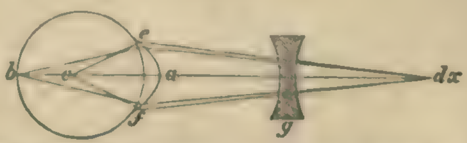


In Fig. 64, *a* represents the cornea, to which rays of light are supposed to be passing, unimpeded, from *dx*. Owing to the want of proper convexity, they pass on from *e f*, and do not reach the focus till they arrive at *c*, a point beyond the retina, *b*. If, however, a lens, *g*, be interposed between *a* and *dx*, then, by its convexity, it brings to a focus all the rays on the retina of the eye at *b*. The difficulty experienced by all persons so affected, is to find what is called their "sight;" that is, the focal distance of the lens, *g*. This can only be done by trial; and when successful, the result will be due to a choice of a glass whose focal point exactly falls on the retina when the spectacles are placed in their proper position before the eyes. The use of any pair of spectacles is, however, limited to special purposes; because, unlike the eye, they cannot accommodate themselves to the change of position or distance, in which objects are ordinarily found. A very ingenious arrangement, however, is sometimes employed. Instead of using one glass in each eye-hole of the spectacles, two, of different focal distances, are placed, so that four, instead of two lenses are employed. The two at the bottom of the spectacles are used for reading, sewing, and other purposes requiring close vision. The upper lenses are useful for viewing distant objects; and such a combination is of great value to elderly persons. We cannot help remarking on the imperfection of all human contrivances when put into comparison with those of the Creator. The eye, in its perfect state, can instantaneously adapt itself to any purpose for which we require it; all the intermediate steps of its arrangement are unperceived by us. But despite the skill of our best workmen, all attempts at imitating this are eminently faulty, partly from our ignorance of the necessary mechanical arrangement, and still more so from the absence of vitality in the instruments we employ.

We may now examine into the opposite defect of sight—namely, that in which the eye is too convex; and we need scarcely say, that the remedy is exactly the contrary to that we have just proposed. This will be at once understood by inspecting the following engraving.

In Fig. 65, we presume that the rays are proceeding from an object, *dx*, to the cornea, *a*. The cornea being too convex, converges the rays too rapidly, and they therefore come to a focus at *c*, between the cornea, *a*, and retina, *b*. If, however, a double concave lens, *g*, be interposed between *a* and *dx*, then the rays impinging on *e f* will not focalise except at *b* (which is the retina), providing the lens is chosen of the proper focal length.

Fig. 65.



In the absence of glasses, individuals affected in the manner to which we have been adverting, endeavour to accommodate either objects or the eyes, so as to partially overcome the inconvenience to which they are subject. Thus a short-sighted person, or one whose eye is too convex, sees better if the objects are placed close to the eye; whilst those whose cornea is too flat, remove the object to some distance from the eye. In either case, a temporary alteration of the visual focus is thus effected.

MAGNIFYING AND BURNING GLASSES.

We have classed under this head two instruments, whose properties arise from a similar cause—that of simultaneously concentrating the rays of light and heat when these two forces are present at the same moment. Of course our remarks are now chiefly directed to those circumstances in which solar or analogous influences are experimented on. In no case do we find the light of the sun unassociated with calorific effects; and as each force is subject to the same laws, these effects may be considered together.

In a former page,† we stated, that light, emanating from a distant source, does so in what may practically be considered as parallel rays. As such, the results are nearly coincident in a point called the focus. The effect varies according to the distance between the centre of the lens and the focus of the lens. If, however, the lens have a large curvature, and consequent distant focus, it will have the power of collecting from a greater area a larger amount of each radial force. We therefore find that the magnifying and burning power of any lens are not coincident. Added to this, we must take into account the effects of spherical aberration. The influences which may control the latter source of disturbance have been already discussed in our previous pages,‡ to which we must refer our readers.

Some extraordinary statements have been made in reference to the power of different kinds of reflectors and refractors, which have been employed for the purpose of concentrating the solar rays of light and heat. By large mirrors reflecting these forces, platina, quartz, and other refracting substances have been easily melted. The most successful experiments have been tried by means of reflectors which have been built up in segments, forming part of a sphere or ellipse. Such are most effective when arranged as reflectors of a parabolic form. Our own experience leads us to believe that there is a limit which entirely prevents the employment of solid glass lenses for such purposes. Experiments we have tried with a double convex glass, twenty-eight inches in diameter, and about six inches thick, showed a loss of light and heat, by absorption, &c., which rendered the lenses practically useless. In such instances, reflectors of the same size produced a far greater effect.

It has often been stated, that Archimedes, by means of burning-glasses, destroyed the fleet of his enemies. Whilst we congratulate him on his supposititious victory, we may prudently doubt its cause. Suffice it to say, that modern science, with all its assistants and adjuncts, has not, as yet, means at all equivalent to those which would be essential to the success of the hero we have named.

If convex glass lenses of large size are employed, it is advisable that they should be formed of blown glass—of

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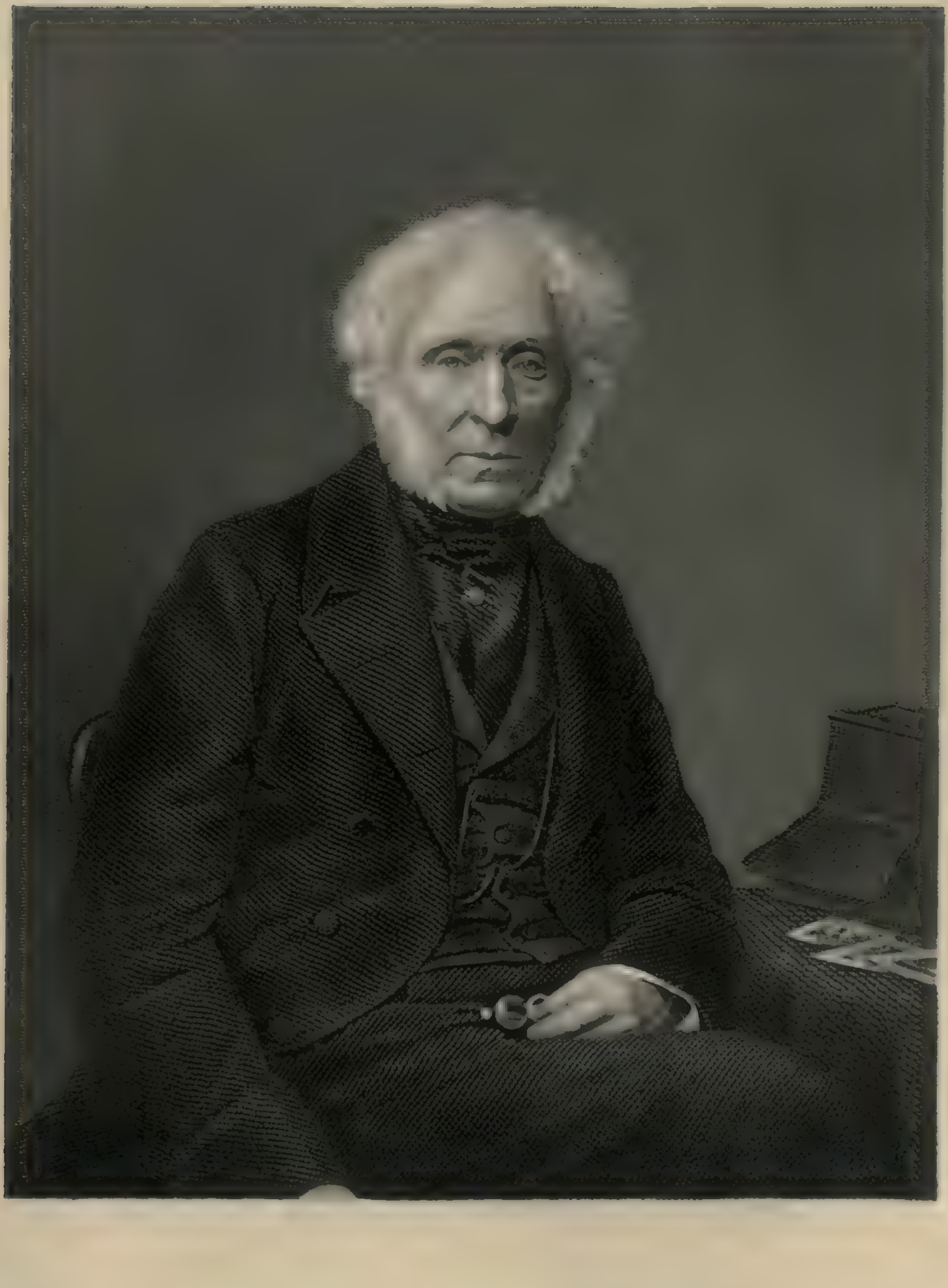
* See *ante*, p. 48.

† See *ante*, p. 44.

‡ *Ante*, p. 44.



[illegible]

[illegible]

course enclosing a cavity, which may be filled with a refracting liquid. This may vary from distilled water to strong hydrochloric acid, the latter having a strong refractive power, and also a peculiar action on the rays of heat and light. In naming this, we may presume that the majority of our readers will not have the opportunity of trying experiments on the large scale: we may, however, suggest the employment of two watch-glasses, or other glasses of large size, properly joined together, so as to hold a fluid. By such an arrangement a great variety of interesting results may be obtained; and, by a little patience, the student will greatly enlarge his knowledge of the effects of various liquids on the dispersion and refraction of the forces of light and heat. Any ordinary single or double convex lens answers the purpose of both a magnifying and burning-glass; one of the simplest of the kind being small blown-glass globes filled with water, which are often sold in the streets for employment of a microscope. It may not be out of place here to warn our readers of the risk run when water-bottles, or some kinds of window-glass, are in such a position that they can concentrate, lens fashion, the rays of heat and light. In the manufacture of crown glass, a thick piece is generally left in the centre of the plate, which, if suitably placed, will act as a lens. If either a water-bottle, or such a piece of glass, happen to be at their focal distance from any inflammable substance, it may just occur that they may bring the rays of the sun to a focus thereon. By such singular means toilette tables have been set on fire, and much destruction of property has ensued. Granaries and warehouses have, in a similar manner, suffered through this lens-like glass placed in the windows. These instances show how desirable it is that a general knowledge of scientific laws should be spread as widely as possible.

THE KALEIDOSCOPE.

The kaleidoscope, to which we shall first direct attention, was invented by Sir David Brewster; and has, for many years, been a source of pleasure and amusement, owing to the infinite variety and beauty of the forms it presents to the eye.

Externally it has the form of a cylinder; inside which are placed pieces of looking-glass, which act as reflectors. At one end of the instrument is an eye-piece, through which the observer looks; and at the other is a piece of ground glass, on the inner side of which rest the objects, which may be small pieces of coloured glass, or other coloured and transparent bodies. In whatever position the latter may be placed, they are continually and successively reflected by means of the mirrors; and hence any single object will undergo reflection, first from its own surface, and then from its image in one of the mirrors to that placed at an angle thereto. The result is, that, with several little objects, an immense variety of pleasing forms are produced; and the slightest change in their position instantly produces a change of form in the design. This, whilst being an essential character in the instrument, and productive of the most pleasing results, detracts from its value in an artistic point of view, as we shall explain when describing the Debusscope.

THE DEBUSSCOPE.

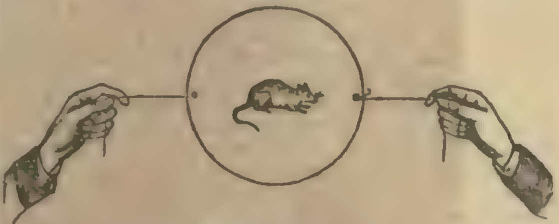
This ingenious contrivance is constructed on similar principles to those involved in the kaleidoscope. It, however, has the advantage of presenting permanently the patterns and forms it produces; and hence may become of great use to manufacturers and others who may avail themselves of it, in order to design patterns. It consists of two reflectors, set at an angle with each other; between which, the object whose appearance is to be multiplied is placed. Successive reflections produce, from the most heterogeneous object, the most pleasing and regular results. Thus, a piece of paper on which the most irregular scrawl may be drawn, will, owing to the composition of the rough design, produce a pattern of a most pleasing character. Even a piece of string, carelessly placed between the reflectors, may be made to assume any variety of form, and, simultaneously, to produce an equal variety of designs. Instead of having to look through the instrument, as in the kaleidoscope, the observer simply has to look on it between the two reflectors; and hence convenience and permanency are at once obtained. When the Debusscope becomes fully known, there is no doubt but that it will be highly valued. We are indebted to M. Debus for its invention; and hence its name.

THE THAUMATROPE, PHANTASMASCOPE, AND ROSE'S KALOTROPE.

In the earlier part of this section* we referred to the fact, that light travels with such speed as not to permit the eye to observe successive impulses on the retina, if such are more frequent than about eight in a second. Thus, the spokes of a wheel, if in rapid motion, present an indistinguishable appearance to the eye. If, however, a strong light is cast on them instantaneously, they will appear, for the moment, quite stationary; even the act of suddenly shutting and opening the eyes will sometimes afford the same result.

By means of mechanical arrangements this principle is capable of many interesting applications, in which what is termed the persistency of vision is employed, together with an intermission of light passing from the object to the eye. Generally speaking, the object is caused to revolve rapidly; and, by means of suitable contrivances, its surface is alternately, but rapidly, exposed or shut off; and so the effects of the thaumatrope, &c., are produced.

Fig. 66.



The thaumatrope, illustrated in the above cut, is made by painting, on the opposite sides of a round card, any two objects—say a mouse and a trap, or cage. Through the sides of the card a string is passed, each end of which is held by means of the finger and thumb. On twisting the card rapidly round, the mouse and cage come successively to view; but because the rays reflected from the two surfaces do not arrive at the same time on the retina, the effect produced leads a person to suppose that the mouse is running in and out of the cage. This arrangement often affords considerable amusement to young people; and, of course, the subjects may be varied to an almost unlimited extent.

The phantasmascope acts on similar principles; but the arrangement is very different. A circular piece of cardboard, of any diameter, is fixed on a handle by means of a pin at its centre, on which it can freely turn round. On one side of the circle, figures, in different attitudes, are pasted or painted, and over each a small slit is made. On turning the card rapidly round, and looking through the slits on a looking-glass placed in front, the reflection of the figures, which seem to be in rapid motion and all alive, will be seen. Amongst many of the illustrations used are horses and men leaping gates, acrobats, &c.; all of which afford great amusement. The following engraving (Fig. 67) illustrates the construction of this singular toy.

Mr. Rose, of Glasgow, has invented a very interesting instrument, which he has called the kalotrope. It consists of a circular disc, on which are drawn, in deep lines, various curved figures. Before this is placed a kind of screen, perforated with divisions. Several of these are furnished, and thus the effect is varied. The disc is made to revolve rapidly by means of wheel-work; whilst the screen also revolves in front of the disc, either at a different speed or in a contrary direction. The result is, that the eye perceives the lines on the disc at variable intervals; and the intersection of the perforations on the screen, has the effect of producing to the eye the appearance of every form of geometrical curves, conic sections, &c. The results are exceedingly pleasing, especially when viewed by means of a powerful light. The arrangement is not susceptible of illustration by means of an engraving; and the instrument has not commanded that attention which its simplicity and beauty entitle it to.

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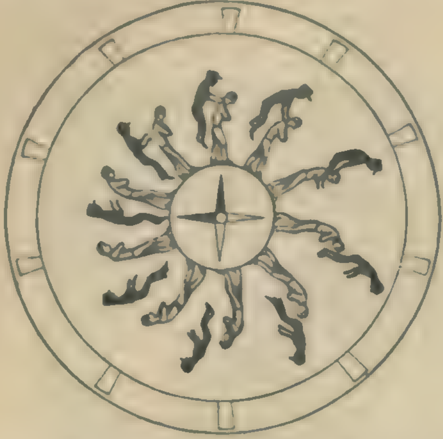
* See *ante*, p. 42

Fig. 67



With this we must conclude our description of optical instruments, and pass on to another and important branch of the science of light, in which we shall have to deal with double refraction and polarisation. We have preferred to describe the application of light, so far as we have gone, in this portion of our work, because the instruments we have named are constructed and used in obedience to the principles and doctrines already defined, and have but a remote connection with the phenomena developed by double refraction. It may, perhaps, be as well if we give a kind of general *résumé* of some of the chief points of the affections of light to which we have hitherto directed the attention of our readers, so that, at all events, the student may retain a knowledge of the subject sufficient to guide him to those which we have yet to bring to his notice, each of which consists of special applications of previously enunciated truths, in an extended and practical form. It so happens that we shall have to introduce many matters, in connection with chemical science, with which some of our readers may be unacquainted. We shall, however, do so in an elementary manner, and, therefore, will not have to call for anything beyond the use of an ordinary intelligence in making our remarks understood.

At the commencement of this section, we first remarked on light as a latent force, and found that, although its chief source is the sun, still, by proper means, it could be educed from a latent state in bodies by certain processes; it is, therefore, in this, and in other respects, analogous to heat and electricity. We then proceeded to show that white light is compounded of coloured rays, which may easily be obtained by aid of the prism. We observed that each of these rays, or waves of light, varied from the rest, both with respect to the length of its undulations, their number within a given time, and their refrangibility.

Reflection, and its laws, were next reviewed; and the properties of plane, spherical, parabolic, and elliptical reflectors were investigated—this branch of the science being that of Catoptrics. We next observed, that a ray of light, passing into or out of media of different densities, undergoes refraction, or an alteration of its progress from a straight line, if the ray be incident on the refracting body at any angle from a perpendicular direction. As an expansion and application of this law, we next entered into an examination of various forms of lenses, and the direction which a ray of light takes through them. That perfection of lenticular arrangement, the human eye, next came under our notice, together with some peculiarities of vision to which some persons are subject.

The phenomena of colour, we stated, were produced by the absorption of all the rays of light except that which was reflected by the surface of the body to the eye; and we noticed that, whether such effects of colour were produced by reflection from a body, or by transmission through its substance, still the colour observed was due to the retention of every other ray except that perceived by us. The colours of thin plates of air assisted us to measure the length and frequency of the undulations of each ray presented by the prismatic spectrum. We then entered into the question of complementary colours, to which we shall have constantly to refer when dealing with polarisation.

Diffraction, dispersion, and achromatism then came under review, together with a peculiar form of dispersion, called epipolic.

Under the head of mono-chromatic light, we were enabled to discuss the effect of a single coloured light on surrounding objects, whereby we noticed that the colour of a body depended on that of the light incident on its surface.

An interesting discovery of Fraünhofer, that there are certain marks or bands in the spectrum, and an extension of these facts in the recent discoveries of Messrs. Bunsen and Kirchkoff, next engaged our attention. We shall add a supplemental chapter on this subject, as the best mode of giving a full account of the interesting facts which have been daily discovered, since those named in the remarks we made went to press.

We then described some of the effects of light on organised bodies; showing how animal and vegetable life are variously and powerfully influenced by that agent.

Having noticed some special ocular defects from which some persons suffer in respect to distinguishing colours, we subsequently explained, as an application of our previous remarks, the construction of the following optical instruments: namely, the camera obscura and lucida; the multiplying glass; the magic lantern, dissolving views, &c.; microscopes, and interesting objects to which they can be applied; telescopes, both refracting and reflecting; concave and convex mirrors; spectacles; magnifying and burning-glasses; the kaleidoscope; the Debusscope; the thaumatrope; the phantasmascope; and Rose's kalotrope.

We may here make a few remarks on the peculiar phenomena which we shall more particularly deal with in our section on chemistry: we refer to phosphorescence and fluorescence. Phosphorus and fluor-spar, with many other bodies, have the power of spontaneously emitting light; and in the case of phosphorus, this is owing to a slow combustion, or its union with the oxygen contained in the atmosphere. This effect, however, may be observed in many animal substances; hence the luminous appearance of crabs, &c., and the phosphorescence of the sea, which proceeds from extremely minute creatures swimming on its surface.

Fluorescence has obtained its name from the spar we have named; and is developed by heating that substance. The two terms are frequently applied to appearances similar to those we have referred to, but arising from different causes; hence we speak of the phosphorescence of the electric discharge. We can fully understand that the student may find some difficulty, whilst considering the phenomena of which we are taking notice, in reconciling the fact of the spontaneous production of light, with the remarks we have previously made on the doctrine of latent forces. Indeed, it would seem, at first sight, that light, in such cases, produced itself; but we must bear in mind, that chemical action is really a form of motion, that arises from the attraction which one element may have for another; and just as the sun and the earth have a mutual attraction of gravitation, and so tend to approximate—the chemical attraction subsisting in bodies, tends also to bring them into intimate union; and, as such, is the promoting cause of phosphorescent effects. Heat, of course, produces the appearance of fluorescence; and, in that case, we therefore have one force producing another; and a sufficient cause is therefore existing towards the production of its consequent effect.

We shall now pass on to the subjects of Double Refraction and Polarisation, in which the reader will find that all his previously acquired knowledge will be called into requisition. These will be succeeded by articles on Artificial Illumination and Photography—two subjects of the deepest interest as applications of the force or agency of light.

CHAPTER II.

DOUBLE REFRACTION AND POLARISATION OF LIGHT.

We have now to enter on a most interesting branch of scientific inquiry, which, although of comparatively recent origin, has attracted the profound attention of the most eminent living philosophers.

The reader will have already become acquainted with many of the changes which a ray of light is subject to when it is either reflected from, transmitted through, or refracted by various substances; and it will be essential to our progress that he should bear in mind the various laws on which we have already dilated. This will be more fully seen when we have to deal with phenomena which depend both on reflection and refraction, yet in other conditions than those which we have previously inquired into.

In a former part of this section, we fully entered into the consideration of the single refraction of light. We then observed, that when a ray of light passes from a rare through a dense medium, and *vice versâ*, it does so in a line varying from that in which it had previously travelled; in other words, it suffers refraction: still, however, no visible change in its nature is apparent; its direction alone being affected. There are, however, a great variety of substances which have the power of not only refracting light singly, but also of dividing a ray into two parts—an effect called *double refraction*. The best known instance of this kind of bodies is the common Iceland spar. If a piece of it be placed on a line drawn on paper, or on a wafer, &c., a double image of the object will be perceived. The rays proceeding from the object are divided into two pencils by the peculiar structure of this doubly refracting body; and hence the effect on the eye is to cause the perception of a double image. To make this more easily understood, we annex an engraving illustrating the case of double refraction by a rhomb of Iceland spar, together with one showing the direction of the rays of light passing through a rhomb of that substance.

Fig. 68.

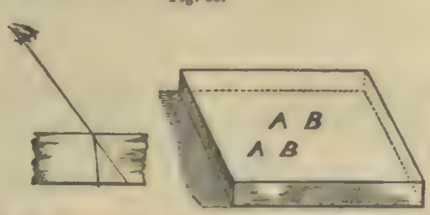
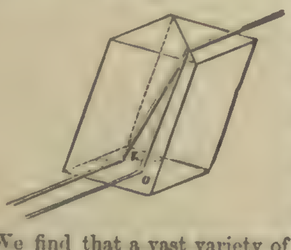


Fig. 68 represents a piece of Iceland spar, placed on a paper on which the letters A B are printed. These, when viewed through the spar, appear double, as illustrated in our diagram. Fig. 69 shows the division of the ray of light into two pencils, *e o*, when passing through a doubly refracting substance. The effect of double refraction must be owing to some peculiar arrangement of the particles of the refracting body; or, in other words, to its molecular structure.

Fig. 69.

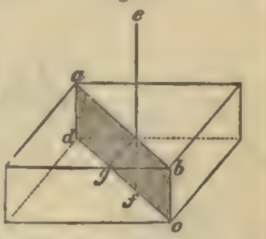


We find that a vast variety of bodies, which, in their ordinary state, do not possess this property, may be made doubly refracting by means of pressure, by changes of their form by mechanical force, by suddenly heating and cooling them, and through any cause which alters the relative arrangement of their particles. Glass may be taken as an illustration of this class of substances; for although but single refracting in its ordinary state, it affords the phenomena of double refraction on being suddenly heated or cooled, or when under pressure or distortion.

We have already stated that the phenomena of double refraction is caused by a ray of light, on entering certain bodies, being split into two rays or pencils, which are separated thereby at an angle from each other. In bodies of a crystalline character, there are what are called optic *axes*, or fixed lines; and on the position and nature of these, the direction, and consequent refraction, of the ray of light depends. Some bodies have but one, and others two axes of double refraction. As an instance of a substance possessing one axis, we may name Iceland spar; and one possessing two axes we find in crystallised nitre. These two classes of bodies are respectively termed uniaxial and biaxial crystals.

We must now explain more clearly what we mean by an optic axis; and for this purpose we refer to the annexed engraving, which represents a rhomb of Iceland spar.

Fig 70.



In Fig. 70, we have a representation of a piece of Iceland spar. If a line is supposed to be drawn from *a b* to *b c*, this would represent the axis of the mass. Let *e* be a ray of light falling perpendicularly on the surface of the rhomb. Now if this were of glass, the ray would, according to the laws of single refraction, pass through unchanged, because perpendicularly incident thereon.* But when falling on a rhomb of Iceland spar, the ray is divided into two parts, of which *e f* is the ordinary ray, and *e g* the extraordinary ray. If the ray of incident light fall obliquely, still the ordinary ray is obedient to the laws of single refraction, whilst the extraordinary pursues a course divergent from the optic axis of the crystal, and at various angles of position thereto, according to the substance under examination. In observing the two images presented by a piece of Iceland spar whilst it is turned round over a spot, it will be found that the extraordinary ray moves in a circle, whilst the ordinary remains stationary. The relation of these two rays in reference to the optic axis of the crystal, may thus be examined by the student, and a knowledge of their mutual divergence and position in various circumstances is at once obtained. The optic axes are respectively positive and negative when the extraordinary ray is bent either to or from the geometrical axis of the crystal. As an instance of some bodies possessing positive axes, we may mention quartz, ice, &c.; whilst Iceland spar, the tourmaline, prussiate of potass, &c., have negative axes. We shall be able to see the application of this law more fully when we enter on the consideration of polarised light, and its resulting phenomena of the production of colours, &c., under certain circumstances.

We must here state, that the velocities of the ordinary and extraordinary rays in a refracting medium of the kind we are now dealing with, differ from each other; that of the extraordinary ray varying according to the angle it makes with the axis of the crystal, whilst that of the ordinary is constant. The larger the angle which the extraordinary ray makes with the axis of the crystal. the greater is its velocity; and its relation to that of the ordinary ray is dependent on the positive or negative character of the crystal.

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* See *ante*, p. 45.

POLARISATION OF LIGHT.

PERHAPS there are no phenomena of which it is more difficult to convey a popular description, than those connected with the polarisation of light. We shall, however, endeavour to meet the difficulty by affording simple means whereby a ray of light may be polarised; give directions for constructing apparatus called polariscopes; and after thus conveying a general idea of the effects of polarisation, which the student may easily become acquainted with, we shall go more deeply into the facts and principles of this interesting branch of physical science.

The student will naturally ask, "What do you mean by polarised light?" And we will endeavour to answer the question, by pointing out certain effects which occur when a ray of common light is reflected or refracted at certain angles.

When speaking of the reflection of light,* we stated, that a ray, incident on any reflecting surface, was reflected at an angle equal to that of incidence. If, however, a ray of light fall on a plate of glass (which should be blackened on the back), placed in such a position that the ray is incident at an angle of about 57°, then the ray is divided into two parts, one of which is reflected, and the other is absorbed. If the reflected ray be received by another glass plate, placed so that the ray reflected from the first is received at an angle of 57° on the second plate, the ray will be polarised; and it has thus acquired certain properties greatly differing from those of ordinary white light. The student will understand this better by constructing the simple apparatus represented in the annexed engraving, which will afford him an elementary polariscope, or instrument by which rays of polarised light may be obtained.

Fig. 71.

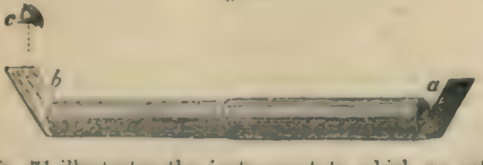


Fig. 71 illustrates the instrument to which we refer. To construct it obtain two tin tubes, each six inches long, and about two inches in diameter; and one of these should be so much narrower than the other that it may slide easily and yet firmly therein. The insides of the tubes should be blackened by coating them with a mixture of lampblack and size. This will prevent any reflection of light internally, which would interfere with the desired result. At one end of each tube a piece of plate glass, blackened also at the back, must be placed; and each of these is fixed at an angle of 33°, so that a ray of incident light may fall, at an angle of 57°, on the surface of either.

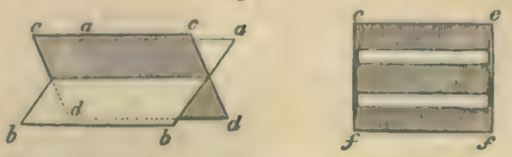
If the eye of the spectator be placed as at *c* in the engraving, he will perceive the reflection of the ray from *a*; but if the glass, *a*, be turned round to the extent of a quarter of a circle—that is, 90°—he will find that the reflection of light from it is almost entirely lost. On turning *a* round to one-half of the circle, or 180°, so that *a* and *b* shall be in entirely opposite positions, it will be observed that the light reflected from *a* will again appear. If the glass be further moved so as to have passed through three-quarters of the circle, the light will be again cut off. On continuing the motion of *a* until restored to its primary condition, the reflected ray will again appear; and thus we observe, at each quarter of the circle, a reflection or loss of the ray incident on *a*, as viewed from the reflected surface, *b*. Now this only happens when each of these glasses is placed at a certain angle, as already remarked; and such is therefore called the *polarising angle* of the substance employed for that purpose. It is at this angle that the greatest polarising effect takes place, although such also occurs at other angles, but, of course, to a diminished extent.

Another mode of producing similar effects, is that of using two plates of a mineral called tourmaline, which may be obtained of the optical instrument-makers. Ordinary light—as of the sun, a candle, or a gas flame—is easily seen through one of these plates, or through two if placed over each other in certain positions. If, however, one plate be held before any source of common light, and the other be gradually revolved through a complete circle in front of it, and next to the eye of the observer, it will be noticed, that at every quarter of a circle, or 90°, the light will be alternately exhibited or extinguished, just as we stated would be found in using the simple apparatus already described. There is, however, this difference between the two processes—the first produces the result of polarisation by *reflection*, and the latter by the *transmission* of the rays through a refracting substance. The student will thus see, that what is called polarisation, is equally effected on ordinary light by reflection and refraction.

Having thus shown how light may be polarised, we proceed to describe the nature of the effect produced; and this, perhaps, can be best done by pointing out the difference which exists between a polarised and a non-polarised ray. An *ordinary* ray will be equally transmitted and reflected through or from the body on which it may fall; whilst a *polarised* ray is not thus indifferent to the physical nature of such body; and, moreover, will only pass in such directions through it, or be reflected from its surface, at certain angles varying for each material employed. Hence, in polarisation, a ray of ordinary light acquires such properties as connect it essential with the physical nature of the substances on which it may fall, or pass through; and this is evidenced from the fact, that it will not pass through some substances at certain angles; that it will similarly not be reflected at some angles of incidence; and lastly, its relationship to doubly refractive structures is eminently different to that it maintained previous to polarisation. On the latter property we shall have to enlarge more fully hereafter.

A variety of illustrations have been offered, in scientific works, for the purpose of familiarising the eye and the mind of the student with these facts. One of the simplest of these is represented in the annexed illustration. (See Fig. 72).

Fig. 72.



In Fig. 72, let *a b* represent a piece of cardboard, fixed so as to form a cross, with another piece, *c d*; and let *e f* represent a flat piece of card, through which parallel openings have been made from one side to the other. Now, supposing either end of *a b c d* be presented to *e f*, only that portion parallel to the openings of *e f* could be introduced; the other part of the cross, being at right angles to these openings, could not of course pass through. Now, in a similar manner does the plate of tourmaline act on the ray of light; and in the above we have illustrated the effect observed by turning round the tourmaline plates—namely, that of the rays being alternately transmitted or extinguished. Our remarks and illustrations are, for the present, entirely confined to the consideration of what is called *plane polarised light*; or, in other words, we refer to the absorption or transmission of light, solely in connection with imaginary rectilinear planes, in or about the bodies employed for our purpose.

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* See *ante*, p. 41.

There are no natural analogies which we can offer as illustrative of such phenomena. It is true that we have magnetic polarity, or that power by which a magnetised needle arranges itself in reference to the magnetic poles of the earth. But such considerations fail to assist us; and it will perhaps be better, after the introduction which we have here given, to describe various polariscopes, by means of which we may be able to investigate the subject experimentally, before proceeding to consider the more intricate parts of our subject.

TO CONSTRUCT POLARISCOPES.

It is generally supposed that expensive apparatus is required for the purpose of studying the phenomena of polarisation. This, like many other ideas of matters required in the pursuit of science, is entirely erroneous. It is true that extreme accuracy, such as is required in exact research, can never be obtained except by employing instruments of the most careful construction. We have been at some pains to find the cheapest and readiest form of polariscope, of which any of our readers can avail themselves; and after trying several experiments with this view, we can recommend the following, which will, at least, serve as an introductory arrangement to the beauties of this branch of experimental science. In every polariscope there are two essential parts; namely, the *polariser* and the *analyser;* the former receives and polarises the incident rays of light, and the latter presents to the eye the polarised ray either by reflection or by refraction.

A polarising plate is easily made by coating a sheet of level window-glass, on one side, by means of lampblack and size, or by holding it over the flame of a candle or gas-light till completely blackened. We have found that the flame of a "composite" candle answers best for this purpose. The glass should be about eight inches square, and should be placed on another piece of blackened glass; the two being separated by means of thin slips of card at their edges, and the black surfaces should face each other inside. This will prevent the black from being rubbed from their surfaces whilst experimenting with it. We may here remark, that so easy is it to obtain a polarising plate, that we have often employed the slates of a roof when wet by rain, or the horsehair covering of a parlour chair, for the purpose; although the latter would scarcely claim the dignity of a scientific instrument. To prevent the glasses from slipping when in use, their edges should be pasted together by means of narrow slips of black paper.

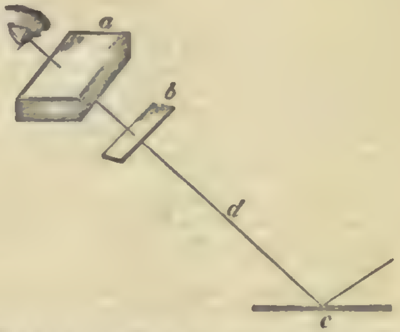
The analysing plate is made by piling up twelve pieces of perfectly clean thin window or microscopic glass, which must be closely packed together. They may be three inches long, by about two inches wide; and their edges should be pasted by means of paper, &c., so that they may form a compact and immovable mass, as represented in the next figure. This also excludes dust and dirt.

Fig. 73.

To use these arrangements for the purpose of polarising light, place the blackened glass plate at a window, so that they may reflect the image of a white cloud; or place before the plate a sheet of ground glass, so that the light falling on the polariser may have a uniform white tint. The analyser should then be held in the right hand, and the eye should view the plate through it at a distance of about four feet. On looking through the analyser in one direction, the blackened plate will appear much of the same colour as if viewed by the naked eye; but if the analyser be turned round through a quarter of a circle (90°), and the blackened plate be still observed, it will be found, that nearly all the light from its surface is lost. Thus, one of the phenomena of polarised light may be noticed. If the analyser be turned round through half a circle, the light will again appear; if through three-quarters, or 270°, it again vanishes, until the analyser has regained its original position between the eye and the observer, when the light reflected and polarised by the black plate will again be observed in its original intensity. If a piece of mica be now interposed at a distance of a few inches from the analyser, and between it and the polarising plate, it will present, if placed at the proper angle, a beautiful appearance. The most magnificent colours will be seen; and if the analysing plate be steadily turned round, to the extent of 90°, whilst the mica is held in the same position, the original colours will entirely disappear, and their complementary colours succeed them.* Instead of a piece of mica, a thin plate of the yellow prussiate of potass may be used; and the effect is, if possible, still more brilliant.

To enable the student to employ this simple piece of apparatus successfully, we append an engraving, showing as near as possible the relative position of the different parts. We are aware that a little difficulty may be experienced, at first, in placing each at its proper angle; but after acquiring this, the student will find the arrangement extremely convenient for many purposes.

Fig. 74.



In Fig. 74, *a* represents the bundle of glass plates forming the analyser; *b*, the plate of mica under examination; *c*, the polarising plate. The direction of the polarised ray is from *c*, through *d*, the mica plate, the analyser, and thus at last it reaches the eye. In looking through the analyser, the long edge of the glass plates should be held as to make such an angle with the nose as that the observer may see the line of light in a plane passing about an inch from the lower end of the top surface of the analysing glass; and which, if carried through the bundle of plates, would almost touch the extremity of the under surface of the bundle, which will thus be at an angle of about 57° with the horizon. A most beautiful play of colours may be produced by gently moving the plate of mica about in various directions. This, however, the student will soon discover. By splitting pieces of mica up into plates of different thickness, each piece may be made of such a size as to reflect one colour only, and its complementary. A sheet of mica, therefore, presents a great number of beautiful appearances, owing to the various thicknesses of which it is composed. One of the readiest ways of splitting that substance is by making it red-hot; a great number of parallel plates are thus formed.

Having described this apparatus, which may be made for a few pence, and which will enable any one to become acquainted with some of the magnificent effects of polarised light—we proceed to point out other arrangements answering the same purpose, but which are more complete in their construction.

The most simple of the kind that we have yet seen is that constructed by Mr. Bestall, of Kennington; one of which is exhibited at the South Kensington Museum. The following engraving illustrates its arrangement; and the description we give is in the inventor's own words.

Description of Bestall's Polarising Apparatus.—This apparatus for showing the beautiful phenomena of polarised light in glass, acids, salts, crystals, &c.—adapted for night or day—consists of a polarising mirror, a glass stand to give the proper angle, and an analysing tube; all of the most simple and cheap form, with all the properties of the most expensive apparatus.

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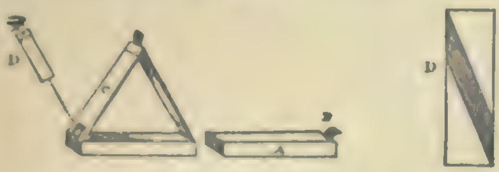
* See *ante*, p. 52.

Fig. 75.



Directions for using Bestall's Polarising Apparatus.—Hold the red box, A, Fig. 75, in one hand; pull the red tongue, B, with the other—that will place the mirror and the glass at the proper angle. Place the black mirror in the box, A, with the reflecting part upwards; and the glass for giving the angle on the mirror, as in the drawing. Place the object on glass, C, and fix the ground glass to the light. Place the apparatus on a table by the window, for day; for night, a light to reflect about half-way up the ground glass should be used. Hold the analysing box, D, to the eye, turn it gently round, and sit about two or three feet from the apparatus; look at the object, and so obtain the phenomena of polarised light.

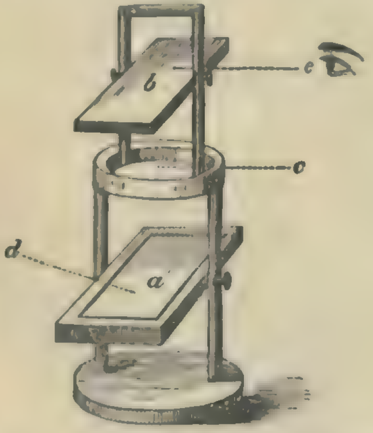
By turning the analysing tube round, the primary and complementary colours will be afforded.

The analysing plates can be taken out of the tube to clean them; but be particular in replacing them in their former position.

Having tried the above-described arrangement, we can unhesitatingly state that it is a most useful instrument. It has, also, the recommendation of being as cheap as it is effective.

The most commonly used form of polariscope is that represented in the annexed figure. In this apparatus, polarisation takes place by reflection, and the analyser is also a reflector. It is an extremely convenient arrangement for the purpose of showing, at alternate angles of 90°, the phenomena of complementary colours. We observe, that at the lower part of this polariscope (Fig. 76), a square frame, containing a few glass plates (the last of which has been well blackened), is fixed. On this the ray of common light is allowed to fall at an angle of 33°. Being reflected from the surface, the polarised ray reaches the analyser, which consists, also, of a number of glass plates (the bottom one being blackened). The object to be viewed is placed on an intermediate stage, and the eye of the observer sees the effects of polarised light by means of the top mirror.

Fig. 76.



In Fig. 76, *a* represents the polarising plate; *b* is the analysing plate; *c* is the stage on which the object to be examined by polarised light is placed. The frame holding the analysing plate, *b*, is movable, so that it may be turned round, and be so arranged at any angle with the polarising plate. The angles are measured by a graduated rim, as observed in the engraving; *d* shows the direction of a ray of light incident on the polarising plate, *a*, which, passing to *b*, is reflected to *e*. This is the position in which the eye of the observer is placed. Both the analysing and polarising plates are placed on pivots similar to those employed in common looking-glasses. This arrangement permits of their being placed at any angle. In employing this instrument, the polarising plate should receive a ray of light at an angle of about 57°; and the analysing plate should be similarly arranged, and receive the ray from *a*, at an angle also of 57°. A considerable beam of polarised light may be thus obtained. We may remark, that such an apparatus may easily be constructed by any cabinet or looking glass manufacturer, by following the detailed instructions we have here given.

The arrangement of the polariser and analyser, in our engraving, throws the two at such an angle as will afford the ordinary phenomena of polarised light. By turning the analyser to an angle of 90° with the polariser, the complementary colours to those seen in any object by the represented arrangement will be observed most readily. We may remark, that 0°, 180°, and 360°, are the relative positions of the two plates for the *reflection* of the polarised ray, and 90° and 270° for that of its *absorption* or *extinction*. At these the complementary colours become visible. We have hitherto confined ourselves to the description of the simplest forms of polariscopes. Others of a more exact character, in which tourmaline plates, Nicol's prism, &c., are employed, will be described as we proceed.

GENERAL PHENOMENA OF POLARISED LIGHT.

From what we have stated in reference to the construction of polariscopes, our readers will have perceived, that all the effects of polarised light depend on the peculiar molecular construction of bodies. Indeed, one of the most valuable uses, in a philosophic point of view, to which the polariscope can be applied, is that of detecting differences which exist in the arrangement of the atoms* of which a mass is composed. We are enabled to determine, with a most astonishing precision, the characteristics of two or more bodies having exactly the same chemical constitution, and yet differing in their purely physical character. An interesting illustration of this fact is found in the case of sugar, and many kinds of starch, which, whilst presenting the same results as to quantity and proportion when subjected to the test of chemical analysis, exhibit astonishing differences when examined by means of the polariscope.

To those who take a pleasure in studying natural laws, and their special application, this fact must necessarily be one of the most interesting kind. A number of bodies is taken, each member of which has the same elements for its constitution; and each of these may be found exactly in the same ratio in one as in the other; yet the whole group differs individually. Nothing can seem more heterogeneous than sugar and starch; and yet, chemically, their constitution is the same. Then, again, we have the diversity of character in the same family of starches, as potato, rice, arrowroot, tapioca, flour, &c.; each of which differs from its fellow in many of its qualities; such difference, however, almost entirely escaping the keen eye of the practical chemist. Again, some bodies have so singular a relation to, or power over, a ray of light passing through them, as not only to alter its course, as in the ordinary case of single and double refraction, but also to divide or split it, and so to produce a series of phenomena of the most extraordinary kind. Hence we perceive the effects of plane and circular polarisation.

We may here remark on the fact, that whilst the generally received idea invests the qualities of the taste, smell, &c., of bodies with a chemical character, the laws of polarisation tend to enlarge our views in reference to such qualities of bodies, and to give us an insight into other causes than those which we assign to many effects observed in daily life. We should, perhaps, trench too greatly on the doctrines of chemical science, were we to fully express our views on this subject, and illustrate them by the countless instances which we could adduce, wherein a change in the molecular constitution of a body modifies its character. We may, however, take the case of alcohol, and its products. Retaining the same elements—alcohol, ether, acetic acid, oxalic acid, &c., are all producible from one substance; namely, common sugar. By means of what has been termed catalysis, alcohol is easily converted into acetic acid; and yet the only agent required is one—platina, which does not undergo the least perceptible change during the process: similarly, the same metal can effect the union of oxygen and hydrogen, especially when presented to a mixture of those gases when it is in a state of minute division. Instances of the rearrangement of atoms (purely physical in its character) may be adduced, wherein no chemical change is perceived, no external alteration made manifest; and yet no sooner do we employ a ray of polarised light, than we are enabled to detect differences and effects which otherwise would entirely escape our senses. Annealed and unannealed glass, Iceland spar, and Arragonite, the different kinds of oils, &c., are illustrations of such occurrences. We must, however, not stop at a general view of the matter: our desire has been to give the student a distant but comprehensive view of the almost unlimited nature of our subject; and having attempted this, we shall endeavour to lead him, step by step, among the arcana of this interesting branch of physical research.

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* See Introduction, p. 1.

We shall presume that the student has provided himself with some form of polariscope such as we have already described; or, in the absence of this, the following will be necessary; namely, two bundles of glass plates, each perfectly clean, and to the number of ten in each bundle. These should be fastened together, as illustrated previously.* A large piece of plate or window-glass, which has been blackened on one side, either by lampblack or the black varnish used by photographers, and a few pieces of mica, on which to try various experiments, will be required. The latter may be obtained in most of our large towns, as it is largely employed for the purpose of making lamp-protectors, and for other purposes.

Furnished with these, our readers may readily study all the leading phenomena of polarised light. Indeed, the whole of the results we shall for the present refer to, may be produced by the simple arrangement we have named. This is another illustration of the fact, that science, even in its most refined branches, may be experimentally studied at a trifling cost.

Having placed the polarising black plate with the polished surface upwards, so as to receive light thereon from a white cloud, or from a lamp screened by means of a piece of ground glass, its surface may be viewed through the bundle of glass plates, or by means of another piece of blackened glass placed above it. If the latter be employed, the polariser and it must be at similar angles (about 57°) with the horizon. The angle at which the bundle of glass plates is to be held, depends on their number. Their position, however, has already been illustrated.† The observer will find, that if either the bundle of glass plates or blackened analyser be held in the proper position, then the light passing from the polariser will alternately be visible or be shut off, as the analyser is turned through each quarter of a circle; so that through 360°, there will be, alternately, a phase of light, and one in which the light entirely disappears.

As we have already remarked, this is due to the peculiar change which light undergoes when it suffers repeated reflection, refraction, or double refraction. The result is obtained by reflection simply when the polariser and analyser are two pieces of blackened plate, such as we have described. The result occurs, partly by reflection and by single refraction, when the polariser is a blackened glass plate, and the analyser a bundle of plates. The same result is arrived at by single refraction solely when two bundles of glass plates are used, placed at the proper angles with each other. When, however, a piece of Iceland spar, or tourmaline, is employed, then these crystals produce the same result by double refraction only.

As a matter of convenience (although it involves expense), we should recommend those of our readers who can afford it, to purchase a Nicol's prism, which is merely a rhomb of Iceland spar, divided and rejoined in a wedge-like form, for the purpose of removing, as far as possible, the extraordinary and ordinary rays from each other. The angle of vision through it is of course always the same, its optical character having been attended to during its construction by the optician. It forms the best kind of analyser. It is also desirable that the student should possess two pieces of tourmaline, as they are the readiest means of observing several of the phenomena which we shall mention. In the absence of these, however, the simple apparatus we have named will answer the purpose; and will have also the advantage of calling out both the skill, patience, and close attention of the inquirer.

If pieces of mica, of varying thickness, are interposed between the analyser and the polariser, they will present appearances of colour which vary according to the thickness of the substance. Those comparatively thick afford red; and the thinner kinds give tints of various shades of blue: all the intermediate colours of the spectrum, and their numerous combinations, being thus obtained, their complementaries come in view on the analyser being moved through an angle of 90° with its previous position in reference to the polariser. If, however, the mica be slightly shifted, the play of colours will be continually varied; because other planes or laminæ come within the line of view.

Now these colours correspond in all respects as to order, &c. (excepting the thickness of plates producing them), with those obtained by means of a lens pressed on glass, the prism, the soap-bubble, &c., on which we have already dilated.‡ Their immediate cause, however, is polarisation; whilst those we have referred to previously, are the result of interferences, produced either by simple reflection or refraction. However, the doctrine of the interference of light, is equally applicable to that which has been polarised as to light in its ordinary state; and pursuing the plan we have hitherto adopted, we shall now only refer to our remarks on interference,§ leaving our readers to extend such in their application to the subject with which we are now dealing. The experimental proofs and mathematical investigations which would be required to fully elucidate this subject of polarised interference, are at once too delicate and abstruse for most of those for whom our work is intended. The advanced physicist may consult, with advantage, papers by Fresnel, Arago, Herschel, and others; of which we regret to say that no general synopsis exists—a fact partly due to the narrow limits in which the study of the subject is confined by its abstract character; and on account of the necessity of high mathematical attainments on the part of those who would fully investigate it.

We shall now pass on to consider some magnificent appearances which certain substances present when examined by means of polarised light: but, before doing so, we may just name, that the coloured phenomena to which we have just alluded may be obtained in selenite, the yellow prussiate of potass, &c.; all of which, like mica, vary in the tints they produce, in proportion to their thickness. Selenite, which is a crystallised form of sulphate of lime (plaster of Paris), is frequently arranged in the form of animals, church windows, &c.; and this affords a pleasing and popular illustration of polarised light and its effects.

Referring to our remarks at a previous page,‖ whilst speaking of double refraction and optic axes, we noticed that some crystals are uniaxial, and others biaxial. The latter class, when viewed under the polariscope, presents some interesting phenomena, and of a kind to which there is found no analogy in any other department of physical science.

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* See *ante*, p. 81, Fig. 73.

† See *ante*, p. 81, Fig. 74.

‡ See *ante*, p. 51.

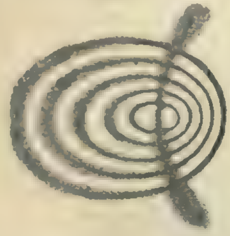
§ *Ante*, p. 52.

‖ *Ante*, p. 79.

If a piece of mica be placed close to the analyser, and carefully examined, a proper specimen will present two black spots, round which extend five series of rings, of the most beautiful colours. These are best seen by means of a tourmaline plate; but the simple apparatus which we have described answers the same purpose. An addition of an eye-piece, which Mr. Bestall has made to his polariscope at our suggestion, shows these fine effects exceedingly well.

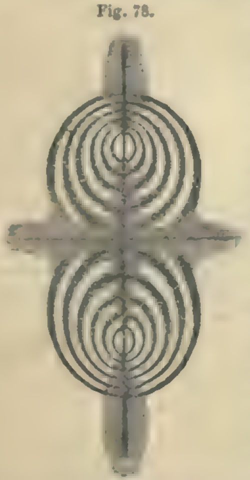
In the centre of the rings, the black spot indicates the position of the optic axes of the mica;* and besides the rings and spots, a curved line, cutting the oval rings, may be observed. The annexed engraving gives a very faint idea of the beauty of these effects; each of the curves, of course, being coloured exactly like the Newtonian rings, but far exceeding them in size. A similar effect is observed at both axes; and if the plate be revolved round its centre, the curved band will assume a longitudinal appearance.

Fig. 77.



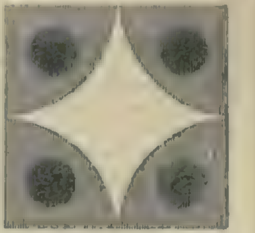
Nitre, which is also a biaxial crystal, exhibits this phenomenon in a more perfect manner, because the axes of its two crystals are close to each other. A perfect crystal is to be chosen, and a section made of it at right angles to its length, and of about the thickness of a sixth part of an inch. On this being polished, and examined in the polariscope, the appearances presented are exceedingly beautiful. The two axes, rings, &c., are nearer together than in the case of mica; and their appearance is represented in the annexed engraving, together with a cross, which extends between the two axes. The cross may be made to assume various positions, by causing the piece of nitre to revolve between the analyser and the polariser, on the geometrical axis of the crystal. Particular attention is required to the polish, thickness, and other points. This, and many other crystals, however, may be purchased, ready mounted, of the opticians. Amongst such, we may mention Iceland spar, which presents a very fine appearance when of proper thickness.

Fig. 78.



Some very beautiful effects are observed when unannealed glass is viewed in the polariscope. In our early remarks on the influence of the molecular constitution of a mass on a ray of light passing through it, we stated, that many differences could be noticed, by aid of polarised light, which are entirely unperceived by common light. Glass, whose parts are homogeneous, shows no effects; if, however, such is suddenly heated or cooled, then its constitution is altered, and colour, &c., may be perceived therein. A piece of glass, a quarter of an inch thick, and an inch and a-half square, if suddenly cooled from the liquid mass, presents a very pretty effect if placed some distance from the analyser. This is represented in the following engraving, which but crudely illustrates the beautiful results produced. In the piece from which we have drawn our cut, each of the circular parts are dots of a rich blue colour, separated by a white cross. The latter changes to a black cross, with reddish-orange dots, on revolving the analyser through 90°.

Fig. 79.



This singular appearance and effect has a very valuable application. In the manufacture of glass, the greatest care must be taken to have it properly annealed before it is introduced into commerce, if the manufacturer has the least regard to his own character or the interests of his customers. These considerations, however, have very little weight with those whose consciences are in the inverse ratio of their love of money; and hence an immense quantity of glass vessels are annually manufactured and sold, which, for the ordinary purposes of life, are utterly valueless. This arises from the fact that they are badly annealed. The process of annealing is intended to render glass less liable to break on being subjected to sudden variations of temperature; and is effected by first submitting the vessels to a great heat, and then allowing them to cool very gradually. Glass, like steel, may thus be rendered more fit for daily use; and, indeed, annealing and tempering are similar processes, and produce similar results. Well-annealed glass produces none of the appearances we have been describing; whilst unannealed readily answers that purpose. If, therefore, any suspected glass vessel, lens, &c., be examined by the polariscope, its character may at once be ascertained. This fact is of considerable use in daily life. To the philosopher it affords a means of testing the nature of the glass of which his lenses and chemical apparatus are made. To our housewives we may equally offer the use of the polariscope in the purchase of glass vessels of any kind; and a judicious application of the principles we have laid down, may have the double effect of preventing annoyance by the sudden fracture of badly annealed vessels, and an infraction on that patience of which, it is said, the fairer sex are the chief possessors.

CIRCULAR POLARISATION.

Our remarks have hitherto been confined to that kind of polarisation which takes place in certain planes existing between the axes of any objects and the direction of the polarised ray. We proceed now to notice briefly some phenomena which are presented by solids and liquids, in which we shall find that the effects depend on a rotation, right or left respectively, between the axis of the body and the direction of the polarised light. A plate or section of rock-crystal is an object which presents this peculiarity to the eye of the observer. This is cut perpendicularly to its axis of refraction; and it affords, when viewed by means of an analyser (which may be a Nicol's prism), the appearance of rings round its axes, the colours of which change according to the direction in which the analyser is moved. The colours succeed each other in regular order, just as we find them presented by the prism:† and supposing the analyser to be turned towards the RIGHT hand, then red, orange, yellow, green, blue, and violet, successively appear; and such bodies have what is called RIGHT-handed polarisation: if, however, these colours succeed each other in any body when the analyser is turned towards the LEFT hand, then such is said to have a LEFT-handed polarisation.

It may be as well, before we proceed further, to state, that an ordinary polarising plate, and a Nicol's prism, will answer well for solid transparent bodies, such as rock-crystal, starch, &c.; but for fluids (of which we shall presently speak), an arrangement is required of a form suitable for the purpose; one of which is represented in the following engraving, and of the best form we have tried.

In Fig. 80, *a* represents a Nicol's prism, fixed, by means of a ring and screw, on a common ring-stand; *b* is a tube in which the liquid is placed, which may be partly covered with black paper, to keep out

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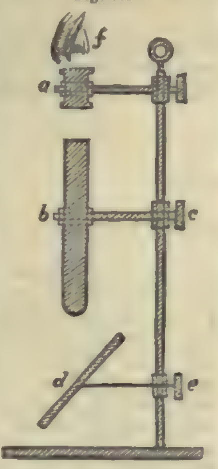
* See *ante*, p. 79.

† See *ante*, p. 41.

Fig. 80.



extraneous rays. These tubes must be of various lengths, to suit the different liquids which may be employed; and they are held upright between the eyepiece and the polariser, by means of a ring, which is again attached, by the screw, *c*, to the ring-stand: *d* is an ordinary polarising plate, attached to the stand by the screw, *e*; and *f* is the eye of the observer. We need scarcely state, that the analyser must so rest in its socket, as to be readily turned round through the circular ring, as by means of its rotation over the tube, *b*, the phenomena are observed. In complete instruments, *a* rests in a graduated frame, which is essential for accurate experiments, because the angle of rotation varies for each liquid examined; and their differences, with respect to polarised light, are so pointed out. The bottom of the tube, *b*, should be as flat as possible, so as to prevent the loss of light on account of its external curvature. With such an apparatus, which is easily constructed, the student will have a wide field for research. He will be enabled to detect physical characteristics in bodies which no other means can point out; his ideas in reference to the constitution of such will be extended; and from merely examining the external peculiarities of the various substances presented to him, he will be able, as it were, to acquaint himself with their internal constitution, in a manner as astonishing as it is instructive.

We may here shortly point out the essential difference which exists between plane and circular polarisation. In the former, the chief effects are witnessed when the planes of observation are at *right* angles to each other. In circular polarisation these angles vary, and depend on circumstances, which, in their turn, arise directly from the peculiar or individual constitution of the body under examination. We should remark, that the best plan of observing the phenomena of circular polarisation, is that of using homogeneous or single-coloured light;[*] and this is easily managed by interposing a plate of red glass between the eye and the analysing prism. This prevents a variety of disturbing causes; and it also ensures the disappearance of the extraordinary ray.[†]

We may mention, that some little difficulty occurs to the beginner in observing the direction of rotation; because, as we have just stated, different liquids require greater lengths before the phenomenon is exhibited. The following, however, we may adduce respectively, as instances in which the effects may be observed; and, as they have all been tried in an equal length of tube—namely, in one of about six inches—they will afford a starting-point, from which the student may commence his progress. We have also given the angle, as indicated by the index on the analyser, which completes the arc of rotation for each.

Possessing Left-handed Polarisation.	
Oil of turpentine	45°
Naphtha	12° 40′
Possessing Right-handed Polarisation.	
Oil of bergamot	29°
Weak solution of cane sugar in water .	23° 5′
„ „ of milk „ „ „ .	10° 3′

It will thus be observed, that the angle of the arc of rotation, and its direction, become definite physical characteristics of the various bodies in solution to which we may apply the test of polarised light. These researches have been of the greatest advantage to the chemist as a means of analysis; and, as such, we cannot too highly value the results which have been obtained. We must, however, defer entering fully into the consideration of these interesting facts, to our section on Chemistry, in which we shall more completely show their special applications.

If our space, and the character of our work permitted, we might examine a great variety of subjects incidentally connected with, or elucidating, the phenomena of polarisation. Some of these we shall have to consider when we deal with what we may term the Magnetic Relations of Light. Others will come under our notice, under the sections of Mineralogy and Crystallography; and, as such, we shall have a better opportunity of dealing individually with special cases, than we have at the present time, whilst endeavouring to lay down and illustrate laws which have a general rather than a particular application.

Before concluding, however, our remarks on the Polarisation of Light, we may refer our readers again to the analogy which exists between the same phenomena in connexion with heat. The observations we made previously,[‡] will have prepared the student for the more complex phenomena of luminous over calorific rays; and what we have just finished discussing, will, in a similar manner, elucidate and extend the doctrines of calorific refraction. We may also add, that the analogies of the two forces of heat and light, thus seem, at each step, to converge together; and, like the vanishing points in a landscape, their differences seem to grow less in proportion as our view of them becomes more extended and comprehensive.

As we here must end our examination into the laws of light, we may remind the student, that many of the facts we have been investigating will again require our attention when we inquire into the phenomena produced by electricity, magnetism, dia-magnetism, and sound. Each of the forces ranked under the term "undulatory," has necessarily a mutual connection. In fact, they sometimes seem as if one force, modified by special circumstances, or the affections of matter, would satisfy the conditions implied in what we now recognise as so many separate and independent agents.

We have already seen how far heat and light are similarly subject to the same results when undergoing reflection, refraction, and polarisation. We shall find that sound is partly subject to the same influences; and that electricity and magnetism have a connection with both the agents to which we have already directed attention.

The attentive reader and student will reap great advantage in following, as closely as possible, the various relations in which these questions stand to each other. The nature of our plan forbids our entering into minute detail; but the student can, at his leisure, ponder over and collect these analogies, and thereby he will find that each subject and section, as we proceed, will form a kind of exposition to others. One of the greatest benefits, in an intellectual point of view, which attends the study of experimental science, is its tendency to expand our powers of generalisation. By giving us a broad and expansive view of creation in general, we are insensibly raised above that narrow individuality of character which is so natural to us all. Whilst we mentally roam through the vast expanse of truth, we become enlarged in our ideas, chastened in our imagination, and strengthened in our reasoning powers.

Having thus considered the chief laws of light, we now proceed to some of its applications, in which we shall find the theories already explained put more or less into practical purposes. In these branches of our subject, the reader will not fail to notice how much more rapidly improvements in matters of daily life take place, when under the fostering guide of sound science, than under the uncertain *dicta* of crude and ancient empiricism. Yet we find that, although much progress has been made in many manufactures by men utterly ignorant of scientific laws—still such persons, generally speaking, gradually acquire an amount of information which is a partial substitute for sounder acquirements. Of this fact, many instances might be quoted; such as the production of the Tyrian purple, the staining of ancient glass and porcelain, &c. We may, however, possibly err in our notions with respect to the state of science in the early ages. But few works of those times have been handed down; and the destruction of the Alexandrian library, may have entirely shut from our view the results of investigations which might possibly have rivalled the productions of our own age. Whilst, therefore, we may rejoice in that which we possess, we should do so with modesty; for as long as science is studied, we shall ever have to acknowledge, that some of the ancients—Euclid and his co-workers—have done more to assist us in the means of pursuing exact investigations, than any of their successors.

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[*] See *ante*, p. 54.

[†] See *ante*, p. 79.

[‡] See *ante*, p. 29.

CHAPTER III.

ARTIFICIAL ILLUMINATION.

We shall now proceed to consider some of the applications of light, commencing with that of Artificial Illumination; a paper on which has been furnished by a gentleman highly conversant with the subject.[*]

After the article on Artificial Illumination, we shall direct the attention of our readers to the interesting subject of Photography; and endeavour to put them in possession of both the practice and theory of that singular application of light.

ARTIFICIAL ILLUMINATION.

Historical Notice.—The advantages of artificial light are so numerous and important, that they must have claimed attention from the very earliest period of history: in fact, long before man had commenced his work of civilisation, the light of the cheerful fire must have been a source of comfort and security; for, perhaps, nothing assists so powerfully in dispelling the gloom of darkness, and banishing the terrors of night, as the lustre of a cheerful fire. And then, as civilisation advanced, the usefulness of artificial light must have been perceived in a thousand ways. True, it might at first have been employed merely as a luxury among the rich; but, in process of time, it became a necessary even with the poor; and were we now to deprive man of all means of obtaining artificial light, the consequences would be terrible—we should cripple his energies and impoverish his intellect—a great portion of his time would be lost to him—every industrial occupation would be hindered—the very safety of the community would be endangered—and, in fact, the wealth and commerce of every nation would be seriously affected. These considerations endow our subject with an interest of no ordinary character; and we may truly say, that few inquiries would furnish so profitable a result as a complete history of Domestic Illumination, "tracing its gradual development from the solitary watch-lantern, graven on the pyramid, through the graceful but very imperfect lamps of the Greek and Roman period, as exhibited in our museums, and the clumsy contrivances of the middle ages, up to the productions of modern times, satisfying the demands both of taste and science. In such a narrative might be shown the progress of light, in the literal signification of the word, by a careful examination of the various forms in which it has been at different times employed —as lamp, lantern, torch, flambeau, falot or cresset, candle, and gas—whether for the celebration of religious ceremonies, for increasing domestic comfort, adding to the security of the streets, or forming a beacon to guide the mariner at night. It would be, at the same time, a history glancing at the advancement which the improvements in illumination have given to the social condition of mankind, and at the advantages which science has derived from the study of this subject." Besides which, there would be mixed up with such an historical account many of the fanciful hypotheses and highly poetical conceits in respect of the nature of fire, which have at various times occupied the attention of man.

Perhaps the earliest source of artificial light was the wood fire, blazing in the recesses of the forest, or in the hut of the savage; and then, as experience led to the discovery of the fact that different kinds of wood burn with different degrees of splendour, it must have been perceived that certain vegetable substances might be employed, in preference to others, as a means of obtaining light. In this way a splinter of pine, the resinous bark of a tree, or even the oily kernel of a nut, might early have been resorted to for such a purpose; indeed, we are told that the inhabitants of Tortuga had long used the wood of yellow sandal; those of New England made choice of a resinous splinter of pine; the natives of British Guiana selected the wood of an amyris; and in Otaheite, the candle was a row of nuts fastened upon a skewer. Again, accident must soon have taught mankind that the resinous exudations of trees, the fat of animals, and the bitumen and naphtha of the mineral kingdom, were not only highly combustible, but that, while burning, they were also highly luminous. Having learnt this fact, it required but little ingenuity to suggest the use of some porous material upon which the combustible might be smeared, or into which it might be dipped, before it was burnt. In this way the torch, the candle, and the lamp, were doubtless invented.

The first authentic evidence that we have of the use of candles is furnished by Pliny in the 13th Book of his *Natural History.* He there says, in speaking of the lost books of Numa, that when Terentius, the scribe, discovered the sepulchre of the king, he found in it a parcel of books tied round in every way with *candles*, after the manner of the cere-cloth. This story is quoted from Piso, Tuditanus, Varro, Antias, and others: it is also repeated by Livy, who states that the candles were two in number. Now as Numa was the successor of Romulus, and reigned about 700 years before the birth of Christ, it is evident that candles were used in the earliest days of Roman history: judging, also, from the use to which they had been put, it would appear that they were composed of string or rope covered with some combustible, perhaps pitch or wax. It is to be regretted, however, that Pliny has not given us a more particular account of the construction of those candles; indeed, he has not informed us how any of the candles of his time were manufactured; he merely alludes to the subject incidentally. Thus, in his chapter on Willows, he says that the pith of the brittle rushes, which grow in marshy places, is used for making wicks for watch-candles and funeral-lights, to burn by a dead body while it lieth above the ground; and, in a subsequent chapter on Flax, he states that the part of the reed which is outermost and nearest to the peel, or rind, is called tow, and is good for nothing but to make lamp-match or candle-wick: nothing whatever being said respecting the material which was put about those wicks. The inference, however, is that the watch-candles in Pliny's time were like our rushlights, and that the others were similar to the pitched rope which we employ, after the fashion of a torch, for common illuminating purposes. Fosbrooke informs us, in his *Encyclopædia of Antiquities*, that the candles of the ancients were sometimes made of the leaves of the papyrus covered with wax or tallow: and he remarks, that common kitchen-stuff was used for such purposes as far back as the days of Augustine. A proof of this is to be found in the writings of Apuleius, who speaks of two kinds of candles—namely, the *cerei*, or wax, and the *sebacei*, or tallow.

During the middle ages, wax was extensively employed for purposes of illumination; and, according to Fosbrooke, the candles were not made by regular craftsmen, but by the monks, and the servants of the nobility. An illustration of this is to be found in Asser's *Annals*, where an account is given of the manner in which King Alfred directed his candles to be formed. "He commanded his chaplain to supply wax in sufficient quantity; and he caused it to be weighed in such a manner, that when there was so much of it as would equal the weight of seventy-two pence, he caused the chaplain to make six candles thereof, each of equal length, so that each candle might have twelve divisions marked across it." These candles, when burnt in succession, lasted for twenty-four hours, and each division indicated the third of an hour.

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[*] As some of the facts included in the original have been already discussed, the editor has omitted such, and has made additions on matters which have come under his own observation.

Up to that time, the use of candles was chiefly confined to the churches, the monasteries, and the houses of the nobility; but in the 15th century the employment of candles had become very general; and the trade of making them had acquired so much importance, that the chandlers of London had obtained an act of incorporation. The candles of those days were all made by dipping the wick into the melted wax or tallow; but in the 18th century, the Sieur de Brez of Paris invented the plan which is now practised, of casting them in metal moulds; and, later still, the wax candles were made by rolling the wax around the wick. All subsequent improvements in the manufacture of candles have resulted from the very elaborate investigations of Chévreul into the composition of fats; indeed, most of his results have been made the basis of the several patents of modern time; as, for example, those of Bolts, Frémy, De Milly, Gwynne, Wilson, and others.

Lamps are also contrivances of very ancient date. They are frequently mentioned in the sacred writings; and there can be no doubt that they were much better known and more generally used than candles. Clemens Alexandrinus, and Eusebius, ascribe their invention to the Egyptians; but it is rather a singular fact that they were not well known in Greece during the time of Homer—at least, he has not referred to them. In his story of Penelope, he says, as most classical scholars will remember, that the suitors of Penelope paid homage to her with torches and odoriferous wood laid in a brazier. Lamps, however, were common enough in Rome during the early period of her history. Pliny frequently refers to them, and even describes the oil and the wicks that were burnt in them. The cities of Herculaneum and Pompeii have furnished us with excellent examples of both the form and the material of ancient lamps. It would appear that at first they were made of baked clay (terra-cotta), and that the design was simple in the extreme—an oval or elongated vessel, having a lip at one end for the wick; but in process of time, as the habits of the Romans became more luxurious and expensive, the material was changed for gold, silver, or Corinthian brass, and the design became more complicated. These lamps were either suspended from the ceiling or arranged in rows on a stand or candelabrum, the designs of some of which are exceedingly beautiful. The light which they furnished must have been dim and unsteady, for the construction of the lamp was always the same—namely, a solid wick immersed in a vegetable oil. Pliny says that the inhabitants of Sicily burned a kind of bitumen, resembling an unctuous or oily liquor; and that they collected it from the surface of a spring in the territory of Agrigentum. In other cases the oil which they used was of vegetable origin: it was extracted from the fruits of castor or olive; and, in more recent times, animal fats were employed. This was the condition of things until a very modern period, when M. Argand, of Geneva, effected a complete change in the art of illumination. Every one is acquainted with the lamp that bears his name; the principle of which is, that the oil burns at a high temperature, with a plentiful supply of atmospheric air. This is accomplished by means of a hollow cylindrical wick, and a glass chimney which surrounds the flame. Few improvements of any practical importance have been made on this principle, notwithstanding that many contrivances have been originated, and various combustible liquids resorted to.

The history of street-lighting furnishes many examples of the slow progress with which the art of domestic illumination has advanced. At first, the only lights in the public highways were those of the cautious citizen, who deemed it prudent to make his nocturnal visits under the protection of a link, a flambeau, or a lantern. We are told that the streets of Rome, even in her palmiest days, rarely exhibited more than one or two lanterns, which were suspended over the baths and other places of public resort. Now and then they were illuminated for a festival, and sometimes the forum was lighted up for a midnight exhibition; but, with these few exceptions, the city was a city of darkness. In the 4th century, the streets of Antioch and Edessa were furnished with public lamps. Libanius, in his panegyric of the former, says—"The light of the sun is succeeded by other lights, which are far superior to the lamps lighted by the Egyptians on the festival of Minerva or Sais. The night with us differs from the day, only in the appearance of the light: and with regard to labour and employment, everything goes on well; for some work continually, while others laugh and amuse themselves with singing." This fact is confirmed by Jerome, who tells us of a serious dispute that was maintained for some hours in the streets of Antioch, between a disciple of Lucifer and one of the orthodox: he says that the dispute was kept up until the streets were lighted, and then the disputants spat in each other's face and retired. In the history of Jesue Stylites, we are informed that Eulogius, the governor of Edessa in Syria, ordered lamps to be kept burning in the streets during the night; and that he employed for that purpose a part of the oil which was before given to the churches and monasteries.

It is worthy of notice, however, that public illuminations, either on account of religious festivals or general rejoicings, were very common with the ancients, and are of great antiquity. Herodotus states that the Egyptians had a festival of much solemnity, during which lamps were placed before the houses, and kept burning throughout the night; the Jews, also, celebrated their *festum encæniorum* in like manner. According to Æschylus, it would appear that the Greeks had their nights of public rejoicing; and there can be no doubt that the Romans were continually in the habit of lighting up their streets with lamps and torches, whenever an event of public importance commanded their attention. In some instances these displays were wholly unpremeditated, as when an orator distinguished himself in the senate, or a soldier in the camp. Cicero was thus honoured when he defeated the conspiracy of Cataline; and many a Roman general has been encouraged in his march by a like display of public enthusiasm.

Until very recently, the modern cities of Europe were no better provided for in this respect than the ancient. It is true that statutes were made, and orders proclaimed, to the effect that every citizen should contribute his share to a system of general illumination. This was effected by placing a candle in each of the lower windows of the house, and keeping it burning, from nightfall to the hour of twelve. At first the performance of this duty was optional, but at last it became compulsory; nevertheless, it was at all times so sadly neglected, that the thief and the assassin had abundant opportunities for mischief. Paris was the first city to improve on this condition of things; for in the year 1558, huge contrivances, called *falots*, were erected in the principal thoroughfares. The *falot* was a sort of vase filled with pitch, resin, and such-like things, in a state of combustion; but it was soon found that this mode of lighting the streets was expensive, dangerous, and inconvenient, and consequently the *falot* was quickly displaced by the lantern, which was a rude frame covered with horn or varnished leather. For more than a hundred years this was the plan of illumination generally adopted; and, as may be supposed, the light was too feeble for any useful purpose: indeed, no one of importance ventured abroad after dark without his torch or flambeau. The latter, therefore, became so indispensable to the midnight traveller, that an ingenious Italian, named Laudati, conceived the idea of opening stalls for their hire. He started his business in Paris in the month of March, 1662; and he managed it so well that he obtained the entire monopoly of the whole city: his charge for a link was from three to five sous the quarter of an hour, according to the rank of his customer. In 1667, Nicholas de Reynio, the first lieutenant-general of police, introduced a still better system of street-lighting. He invented lamps of glass, which, from their resemblance to a bucket, were called *lanternes a seau*. These he fixed in the middle of the streets, exactly in the same way as they are now suspended in many parts of France, by means of ropes or wires fixed at each side of the street, the lamp being suspended in the centre.

We have no means of showing when London was first lighted with lamps; though Maitland says, in his *History of the Metropolis*, that an order was issued as early as the year 1414, commanding the inhabitants to hang out lanterns for the benefit of passers-by. This information is derived from Stow, who, in his *Survey of London*, remarks that, "in 1417, Sir Henry Barton, the mayor, ordained lanthorns with lights to be hanged out in the winter evenings, betwixt Hallontide and Candlemasse." It does not appear, however, that these orders were much attended to, for we find that they were repeated again and again over a period of three hundred years. At the expiration of that time the corporation of London determined on removing the service altogether out of the hands of the inhabitants; they, therefore, entered into contract with a person to set up the public lights, and to attend properly to them; for which they gave him permission to charge six shillings a-year to every householder whose annual rent exceeded ten pounds. In 1736, the lord mayor and common-council applied to parliament for power to light the streets in a better manner. This power was further increased in 1744; and, from that time, the illumination of the City has been gradually improving.

Most of the preceding facts have been derived from Beckmann, who tells us that the following are the dates when public lamps were introduced into the other cities of Europe:—Amsterdam, 1669; Hamburg, 1675; Copenhagen, 1681; Berlin, 1682; Hanover, 1696; Leipsic, 1702; Vienna, 1704; Dresden, 1705; Halle, 1728; Birmingham, 1783; Brunswick, 1765; Nantes and Versailles, 1777; Zurich, 1778; and Strasburg in the year following.

In very recent times the greatest of all improvements in street-lighting has been effected by the use of gas; and those who can remember the old-fashioned lamp, with its miserable glimmer, and the dangers which constantly beset the traveller after nightfall, will have no hesitation in saying that the employment of gas for illuminating purposes has been one of the most important events of modern time. In truth, it has not only been the means of effecting a wonderful change in the whole system of artificial illumination, but it has also produced an equally important change in the domestic concerns of the people: it has encouraged industry, developed the arts, protected property, diminished crime, and operated in a thousand ways as a medium of wealth, prosperity, and social improvement. All this has been accomplished in less than half a century; for Mr. Clegg tells us, in his work on gas-lighting, that he, in 1813, was the first to put gas into the lamps of Westminster; and he has also given us a graphic account of the fear and wonder with which it was contemplated. Now, however, it is one of the most familiar objects of daily life; for there are almost as many gas-lights in this metropolis as there are human individuals. What would Beckmann say of all this, if, in the year 1786, he imagined the lighting of London to be perfect, and thought that the appearance of the City, after dark, when seen from a distance, was noble and magnificent! Of a verity, his admiration would be boundless. One thing, however, is still wanted to make gas the greatest boon of the 19th century—that is, a guarantee in respect of its purity and harmless qualities. Already the demand for these has become urgent; and it will not be long before the gas companies of England will find it to their advantage to yield to it. In fact, the City of London has, as usual, taken the initiative in this matter, and has appointed an officer to test the quality of the gas supplied within its boundaries. The results are so important to the community, that all other cities and towns will, ere long, follow the example; and then we shall hope to see the art of domestic illumination made more perfect than it is at present.

ON COMBUSTION AND FLAME.

General Remarks.—Artificial light is due in almost every instance to chemical action—that is, to a rapid or energetic union of two or more substances, and the formation of new compounds. At one time it was thought that matter in the act of burning was destroyed, and thus the term combustion was used to designate the phenomena: but we now know that matter is indestructible, and that substances, while burning, merely change their form. In proof of this, it may be mentioned that the chemist is enabled to ascertain the exact composition of an organic substance, by simply collecting and weighing the products of its combustion. This, indeed, constitutes the principle of every organic analysis.

Those substances which evolve light and heat during their chemical union, are generally distinguished by the terms "combustible" and "supporter"—the former term being applied to the body which burns, and the latter to that which permits of the burning. We speak, for example, of wood, tallow, and coal as combustibles, and of atmospheric air, oxygen, and chlorine as supporters; but a very little reflection will show that those expressions are altogether arbitrary, and that they might in most cases be reversed without destroying their sense. This will be clear from what follows. When coal gas is burnt in atmospheric air or oxygen, we call the gas the combustible, and the air or oxygen the supporter; but by changing this condition of things, and igniting a jet of air in a vessel of coal gas, we should then call the air the combustible, and the gas the supporter; from which it must be evident that the phenomena of combustion are due to a reciprocal play of affinities, in which one element takes as large a share as the other; and that the distinction of property, as implied by the terms combustible and supporter, is not founded on fact: nevertheless, we still continue to make use of those terms, because of the convenience they afford in description.

Theories of Combustion.—For a long time the phenomena of combustion were thought to be dependent on the evolution of a peculiar subtle principle, called *phlogiston*. This hypothesis was originated by Becher in the 17th century, but it was not elaborated until the days of Stahl, when it was universally accepted among chemists as the Phlogistic theory. For more than a hundred years the theory held a prominent place in the science of chemistry; and, although it was founded in error, or rather in a total misapprehension of facts, yet its truthfulness was not doubted until the time of Lavoisier, when, by an accurate examination of the facts, he exposed its fallacy. According to the Stahlian theory, there was something—namely, phlogiston—always given out during combustion, from which it might reasonably be inferred that the body became lighter; but, by collecting the burnt products, and weighing them, Lavoisier showed that it actually became heavier, and that something must, therefore, have been absorbed or taken in. This something he soon found was derived from the atmosphere, and was the gas just then discovered by Scheele and Priestly. Having determined those facts, and tested their truthfulness in every possible manner, he boldly advanced his new theory of combustion, the Antiphlogistic; in which he said that the phenomena of combustion were *at all times* due to the rapid chemical union of oxygen with a combustible. Had Lavoisier been content to say that the phenomena were *usually* dependent on such a combination, it is probable that his theory would have existed in a modified form until the present time; but, in his attempts to make it too general in its application, he effected its ruin; for, in the course of a few years, the investigations of Sir Humphry Davy into the properties and elementary nature of chlorine gas, demonstrated the fact, that combustion might be effected without the aid of oxygen at all. This was proved by the action of chlorine on antimony or copper; and, from that time, the phlogistic theories were put aside; and chemists have ever since regarded the phenomena in question, not as the result of any particular kind of combination, but merely as the energetic display of ordinary chemical action. And here we may remark, that there is not, perhaps, in the whole range of chemical science, a subject that offers so many interesting facts for contemplation, as that to which we have just alluded; for it teaches us that the progress of human knowledge is often dependent, not so much on the discovery of great and important truths, as on the manner in which they are contemplated; and if men's minds are not prepared for the reception of those truths, they will either be disregarded, or else made the means of propagating error. This was the case with the theory of Becher and Stahl, which had the effect of distorting every fact that came under its influence. In this way, for years, it delayed the progress of science, and checked the development of truth. History tells us, that for more than a century before the discoveries of Lavoisier, the main facts of his investigations were broadly set forth in the writings of Hooke and Mayew; but from the circumstance that those facts were premature—that men's minds were not prepared to receive them, and that chemists were infatuated and led astray by the false doctrines of Becher and Stahl—the more humble but important truths of Hooke and Mayew were allowed to pass unnoticed; and they lay dormant for more than a hundred years.

Manner in which Combustion may go on.—An examination of this question will show that most substances have the power of burning in three ways; namely, by slow oxydation, when little or no light is evolved; by a more rapid combination, when the burning becomes so hot as to render itself luminous; and by a still more energetic action when it bursts into flame. We have examples of the first of these processes in the phenomena termed *Eremacausis*, or slow burning, as is witnessed in the glowing of phosphorus, and in the luminosity of decaying wood or putrefying fish. In most of these cases, the heat and light evolved at any given moment are not very considerable; and few persons would be disposed to regard the phenomena as those of combustion; but when it becomes known that the total amount of heat, and perhaps also of light, generated during this slow kind of oxydation, is exactly the same as that evolved during the most rapid combustion of the same substances, there will be no difficulty in understanding that the phenomena in the two cases are referable to the same kind of chemical action, and belong to the same category.

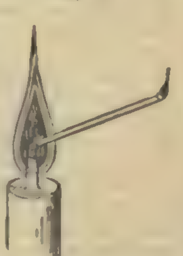
The second mode of combustion is observed when coal gas, or the vapour of ether, alcohol, or wood-spirit, is mixed with air, and brought under the influence of spongy platinum or fine platinum wire. This is best effected by suspending a coil of the wire in the flame of either of the combustibles, then blowing the flame out, and allowing the vapour or gas to play on the surface of the metal: in this way the platinum will keep up the combustion, and will continue to glow, although the vapour or gas will not be inflamed.

The third kind of combustion is produced whenever a sufficiently high temperature is applied to any kind of vaporous matter so as to inflame it. The temperature at which this is effected varies with different combustibles; some take fire at ordinary temperatures, as finely-divided phosphorus, and phosphuretted hydrogen; whereas, solid or massive phosphorus requires a temperature of 140° to inflame it; sulphur takes fire at about 500°; hydrogen and carbonic oxide at 1000° (which is a red-heat); and coal gas, ether, turpentine, alcohol, oil, tallow, and wood, at about 2000° (or an incipient white-heat); but when once inflamed, they all continue to burn at a very exalted temperature.

The Nature of Flame.—The preceding remarks will be sufficient to show that flame is nothing more than gaseous matter burning at a very high temperature. We may prove this by experiment. If we take a coil of iron wire, or a piece of watch-spring, arm it at one end with a fragment of burning wood, and then introduce it into a jar of oxygen gas, the metal will take fire and burn with the most brilliant scintillations, but it will not produce flame. Again, if we expose wood to the action of heat in a closed vessel, so as to drive off all volatile matters and obtain its fixed solid constituent, charcoal, we may burn it in an atmosphere of oxygen without producing flame. In both of these examples, the absence of flame is entirely due to the absence of gaseous matter in the combustibles; and to show that a high temperature is necessary to produce flame, we have only to burn coal gas, alcohol, ether, or wood-spirit, at the low temperature of glowing platinum, and they will not inflame.

It may be thought, perhaps, that in the case of wood, tallow, oil, phosphorus, &c., the flame cannot result from gaseous matter, because those bodies are either solid or liquid in their nature; but a little consideration will teach us, that, in the act of burning, they all become converted into gases or vapours. This may be easily demonstrated in the case of a common candle or oil-lamp, the wick of which conveys the fluid combustible to the flame, where it is decomposed and converted into vapour. Now, if we hold a small glass tube, or even a piece of tobacco-pipe, in the flame of the candle, so that the end of the tube shall be exactly over, and almost in contact with, the top of the wick, it will collect the combustible vapour and convey it out of the flame, whence it may be burnt by applying a light to the opposite end of the tube (Fig. 81). This experiment also proves that flame is hollow: it consists, in fact, of three cones placed one over the other. The inner cone, which is of a dark colour, and surrounds the wick, is formed by the vapour of the decomposed tallow. The second cone is the luminous one; it envelops the preceding on all sides, and consists of the ignited particles of the gas making its way outwards to the air. The third, or outermost cone, is nearly invisible; it constitutes that pale blue film which everywhere surrounds the luminous part of the flame. We may make this cone more evident by bringing a piece of thread moistened with salt into the lower edge of the flame, when the cone will be instantly lighted up with a deep yellow tinge. Another mode of discovering it is to screen off the luminous cone by means of the hand; or it may be rendered visible by intersecting the flame with a piece of fine platinum wire, or wire gauze, the ignition of which discovers the exact boundary of the cone. This cone is made up of gas in an actual state of combustion; for it is here, and here only, that the process of oxydation is rapidly going on.

Fig. 81



Cause of Light in Flame.—A little experience will inform us that different combustible substances burn with different degrees of intensity (Fig. 82). We will endeavour to show that this is wholly dependent on the number of solid particles that are ignited within the flame. If we take a jet of hydrogen gas and fire it, or make use of a flame of alcohol from a common spirit lamp, we shall notice that neither of these bodies burn with any great amount of light; the case is very different, however, with a candle, or even with a jet of coal gas. Now, in the former examples the hydrogen and the spirit do not contain any solid particles, and consequently do not contain any species of matter that can, by its ignition, evolve light: but in the latter examples

Fig. 82.

the case is very different; for by intersecting the flame with a cold plate, we shall find that there are myriads of solid particles in the form of soot, which, by their ignition, produce that intensity of light for which such combustibles are valued. Again, when sulphur is burnt in oxygen gas it evolves a faint blue spectral light, that is hardly sufficient to illuminate the dial of a watch; but when phosphorus is treated in this manner, it emits a volume of light that rivals in intensity that of the sun. It will be noticed that the products of the former are wanting in solid particles, whereas those of the latter are made up of myriads of white flakes of solid phosphoric acid. It is on this account that carbonic oxide and wood-spirit, as well as hydrogen, sulphur, and alcohol, burn with little or no light; while zinc, antimony, ether, turpentine, oil, tallow, and coal gas, burn with more or less splendour. But we may communicate a great degree of brilliancy to some of the former, by giving them a large proportion of solid particles; for example, when we sift a little magnesia into a jet of hydrogen, its illuminating power is at once raised to a high standard; and the same thing happens if we employ lime, oxide of zinc, or white antimony, instead of the magnesia. So, also, if we naphthalise the gas by passing it through a chamber containing ether, turpentine, benzole, or coal naphtha, its light is increased to that of the best description of coal gas. In fact, coal gas consists, in great part, of hydrogen and carburetted hydrogen; both of which are, as it were, naphthalised with other compounds that give it illuminating power. But the most striking example that we can refer to in illustration of the fact that the light of flame is dependent on the presence of solid particles, is afforded by the results obtained with the oxy-hydrogen blowpipe.* When the flame of the mixed oxygen and hydrogen gases is seen as it issues from the jet without impinging on any solid substance, it strikes the observer as being an insignificant and almost invisible object; but directly it is thrown on a piece of lime or other material that will give it solid particles, it instantly becomes one of the most splendid lights with which we are acquainted: in fact, the intensity of the oxy-hydrogen light is so great, that when its rays are reflected by means of a concave mirror, they are distinctly visible at a distance of sixty-eight miles. We can easily understand, therefore, why it was so earnestly proposed by Lieutenant Drummond, as a means of illuminating lighthouses. So, again, with the electric light, the brilliancy of which is dependent on a number of minute particles of charcoal that are intensely heated by the galvanic current. The splendour of this light is scarcely inferior to that of the sun; for, on one occasion when it was tested, it was found to be equal to that of 300,000 wax candles, and was distinctly seen at a distance of several miles.

Lastly, it may be stated, as the converse of the preceding, that we can always reduce the illuminating power of flame by diminishing the amount of solid material contained in it. To take coal gas, by way of example, we shall find that the light may be kept down, or even destroyed altogether, by blowing atmospheric air into it. This sometimes happens with the lights in the streets, and then the flame is reduced to an insignificant blue flicker. The same thing occurs if we mix atmospheric air with the gas before it is consumed, or if we employ a glass chimney that is too tall for the size of the flame. In all these instances, the solid particles of charcoal contained in the gas are burnt too speedily, and, consequently, there is no time for their previous ignition. This teaches us that there is one great point always to be attended to in the management of gas for illuminating purposes: we should take care that the supply of atmospheric air is so regulated that, on the one hand, the solid particles shall not be instantly consumed; and, on the other, that they shall not escape as uncombined soot. Many gas-burners are constructed so as to effect this of themselves; this is the case with the fish-tail and the bat's-wing, which present a thin stratum of flame for the action of atmospheric oxygen. Other burners, as the Argand and its modifications, require a glass or chimney for the purpose of regulating a proper supply of air. The chimney acts by creating a draught, and so causing the air to play into the body of the flame; but, if the chimney be too high, the draught will be too great, and the gas will be overburnt; whereas, if it be too low, the current of air will not be sufficiently strong, and the carbon of the gas will escape in the form of soot or smoke. As a rule, the flame should be kept at about one inch below the top of the chimney. Mr. Billow, of London, has contrived a burner, which not only demonstrates the nature of flame, and the cause of its light, but also shows the effects of an under supply of atmospheric air. In ordinary burners the air is admitted to the flame on all sides, and it even passes up into its interior; but in the burner contrived by Mr. Billow, the inner supply of atmospheric air is cut off, and, consequently, the gas is but imperfectly consumed. The result of this is that carbon is liberated, and, by the construction of the burner, it is made to gyrate round and round, until it collects into little solid masses, which fall by reason of their own weight: directly this happens they come into contact with a part of the flame where combustion is actively going on, and they are instantly projected into the atmosphere like so many diminutive rockets.†

Quantity and Intensity of Flame.—It must have been noticed again and again, by those who are in the habit of pursuing their avocations by the aid of artificial light, that there is a great difference in the practical value of different kinds of illuminating agents: for example, the eye is often most painfully excited with the strong glare of certain varieties of light, although there is not enough of it to produce the necessary illumination of surrounding objects; on the other hand, we sometimes perceive that the light is particularly inoffensive, notwithstanding that everything about us is brightly and sufficiently illuminated. It is probable that these two conditions are entirely dependent on two states of light, which have not yet been properly appreciated. To the one we apply the term *intensity*, and to the other *quantity*. We have examples of the former in the electric light, in the Drummond light, and in the vivid combustion which is going on in the burners of Leslie and Wingfield; of the latter in the diffused light of day, and in the illumination that is produced by a number of separated gas jets. In the one case we have comparatively few solid particles, but they are heated to a high degree of intensity; in the other we have a much larger number of ignited points, but their ignition is not carried to so high a temperature. These facts have not yet received so much attention as they deserve; and in all our endeavours to improve the quality of a burner, we should never lose sight of the fact, that the human eye requires far more of quantity than it does of intensity for agreeable vision.

The Colour of Flame.—Our preceding remarks have gone to show that the light of flame is dependent on the *number* of solid particles present in it: we will now demonstrate that the colour of the light is dependent on the *nature* of those particles. The flame of alcohol or wood-spirit is naturally colourless; but we may give it various tints by saturating the spirit with different kinds of saline substances. Chloride of potassium will make it violet; boracic acid or chloride of barium, green; chloride of copper, blue; common salt, yellow; chloride of strontium, red; and chloride of lithium makes it of a carmine tint. The same substances produce a similar effect when they are sifted into the flame of hydrogen; and the pyrotechnist relies upon these properties in producing the various coloured devices which are the glory of his art. Some years ago there was an exhibition in London of philosophical fireworks; and it was described at the time as a very great novelty. It was said to be effected by the combustion of gases charged with various substances, and ignited from jets arranged in different forms, as spirals, wheels, stars, &c., the tints being chiefly red, blue, green, and purple. It is not known what were the materials used; but from the strong smell of ether and spirit in the room, there is no doubt that these liquids were among the constituents of the gases, and that they were employed as the solvents of different saline compounds; in fact, the chromatic cloud which used to attract so much attention a few years since, was nothing more than the burning vapour of spirit charged with the above-named salts.

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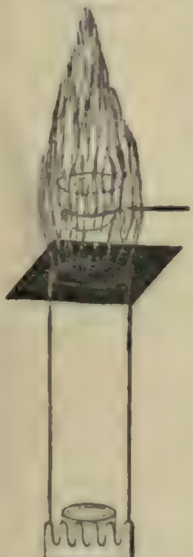
* See *ante*, p. 62.

† Our readers will find a description of this burner at page 257 of the second volume of the *Journal of Gas-Lighting*.

Fig. 83.



The Heat of Flame.—Under ordinary circumstances, the combustion of gaseous matter takes place only at the exterior of a flame—that is, where the surface of the luminous cone is in immediate contact with the atmosphere; but we may increase these points of contact, and therefore the combustion and heat, by throwing air into the interior of the flame, or by mixing the combustible with the atmosphere before it is burnt. In this way we can increase the heat of flame to almost any extent; indeed, the principle of the common blowpipe, of which there are numerous varieties, is founded on this fact; and if we blow a jet of oxygen into a flame of coal gas by means of a double tube, we obtain a temperature that is sufficiently high to melt platinum. A mixture of air and coal gas may also be burnt over a chimney covered with wire gauze, and then the heat produced is sufficient to keep a small crucible at a bright red-heat (Fig. 83); but the highest temperature of all is generated by the combustion of a mixture of two parts, by measure, of hydrogen gas, and one of oxygen. This constitutes the mixture of the oxy-hydrogen blowpipe, which has been so useful an instrument in the hands of the chemist. Different combustible substances do not, however, produce the same amount of heat, or consume the same relative proportion of oxygen. This is an important fact, and it should always be remembered in considering the effect produced by different illuminating agents in the atmosphere of the room in which they are burnt. The following table exhibits, in approximate numbers, the relative values of different combustibles in this respect:—

Name of Combustible.	Weight of Water raised 1° by 1 of body.	Weight of Oxygen consumed by 1 of body.	Weight of Carbonic Acid produced by 1 of body.
Carbonic oxide	3650	0·57	1·57
Cyanogen gas	9380	1·23	1·70
Olefiant gas	21630	3·41	3·14
Marsh gas	23610	4·00	2·75
Hydrogen gas	61230	8·00	0·00
Wood-spirit	9930	1·50	1·37
Alcohol	12630	2·09	1·87
Rape oil	16770	2·89	2·81
Sperm oil	16950	2·90	2·82
Olive oil	17130	2·90	2·82
Turpentine	19110	3·29	3·27
Tallow	14430	2·90	2·91
Wax	16230	3·00	2·92
Spermaceti	17130	3·10	3·00
Sulphur	4530	1·00	
Phosphorus	10830	1·25	

Coal gas is chiefly composed of hydrogen and light carburetted hydrogen, with which there is a small proportion of olefiant gas; it therefore evolves enough heat to raise about forty thousand times its weight of water one degree, and it produces about twice its weight of carbonic acid.

In order to show the advantage of well-purified coal gas over all other materials for artificial illumination, as regards the injury done to the atmosphere of the room in which their combustion is going on, Dr. Frankland has given the following table, which exhibits the amount of carbonic acid produced by a number of illuminating agents, burnt in such quantity as to give a light, for ten hours, equal to that of twenty sperm candles, each consuming 120 grains per hour:—

Tallow	10·1	cubic feet.
Wax	8·3	,,
Spermaceti	8·3	,,
Sperm oil	6·4	,,
Common London gas	5·0	,,
Manchester gas	4·0	,,
London cannel gas	3·0	,,
Hydro-carbon gas, with Boghead coal gas	2·6	,,
Hydro-carbon gas, with Lesmahago coal gas	2·3	,,

The Products of Combustion.—One of these—namely, carbonic acid—has been noticed in the preceding table; the other product of combustion is water. Occasionally there is also produced a small quantity of sulphurous acid; but, as a rule, it will be found that combustible substances are composed of only carbon, hydrogen, and oxygen. Coal gas, however, contains a small proportion of bisulphuret of carbon, and sometimes a very perceptible quantity of sulphuretted hydrogen, both of which, in burning, produce sulphurous acid—the acid of the burning match; and this, by further oxydation of the air, becomes sulphuric acid, or oil of vitriol, a product which has been found to exert the most destructive influence on textile fabrics, as the covers of books, &c. Again, when coal gas is not perfectly consumed, it generates aldehyde, volatile oil, and a few other compounds which are exceedingly offensive.* This is experienced in every kind of gas-stove, where the flame is allowed to play on a large cooling or radiating surface; and hence it is advisable that gas should always be burnt in such a manner that the products of combustion may easily escape into the external atmosphere. Other illuminating agents, as tallow, oil, turpentine, &c., likewise give rise to the formation of unpleasant compounds when they are burnt at a low temperature. Every one must be acquainted with the odour of a smouldering candle or a badly-trimmed lamp; and when alcohol, ether, or wood-spirit are burnt at the temperature of glowing platinum, they produce a number of most irritating compounds, as aldehyde, acetic acid, formic acid, &c. This teaches us that combustion should at all times be kept up with as much energy as is compatible with the required effect, and that the product of its action should be disposed of as speedily as possible.

Effect of Cooling Influences on Flame.—Every circumstance that tends to lower the temperature of flame, operates to a like extent in diminishing the intensity of its light. We see this when a large "snuff" collects on the wick of a tallow-candle—the carbon of which radiates the heat so fast, that the temperature and light of the flame are considerably reduced; and by bringing a mass of metal in contact with a small flame, the latter is immediately extinguished. Sir Humphry Davy was the first to investigate facts of this description; and he showed, by his masterly researches, that flame cannot exist below a certain temperature. This is easily proved by intersecting the flame of a candle with a piece of wire gauze, having about thirty or forty meshes to the inch, by endeavouring to pass the flame through a ring of stout copper wire, or through a small hole punched in a sheet of that metal. In all these cases the flame will be extinguished above that point where the metal intersects it; and, although the inflammable vapour will continue to pass through the holes in the form of smoke, yet the cooling influence of the metal is so great, that the combustible gases will not be inflamed. The same process occurs when we place a layer of wire gauze over the glass or chimney of an ordinary gas-burner; the wire will, by its conducting power, so far reduce the temperature of the flame, that it will not pass through to the apertures of the burner. Sir Humphry Davy took advantage of this important fact, and applied it in the construction of the safety-lamp, which has been of such essential service to the miner. "The Davy," as it is termed, is nothing more than a common oil-lamp, with a cylinder or cage of wire gauze surrounding the flame (Fig. 84). This gauze permits the passage of light, but it will not allow the flame to traverse it; and hence it may be exposed to the most inflammable mixtures without danger of their explosion. The miner knows when the atmosphere of the mine is explosive, by the enlargement of the flame and the burning of the fire-damp within the cylinder of wire gauze, and he perceives a struggle, as it were, of the flame to pass out; but such a result is not possible, unless the cooling influence of the wire is prevented by its becoming almost red-hot. In this case there is danger; and the fire-damp of the mine may be exploded. These lamps are also employed in spirit warehouses, gas manufactories, in the sewers, and in many other localities where there is risk from explosion. The same principle was employed by Mr. Hemming in the manufacture of his safety jets for the oxy-hydrogen blowpipe. We have already said that the mixed gases which are used in this apparatus are exceedingly explosive; and if some contrivance were not adopted to prevent the backing of the flame, they would assuredly take fire. In fact, in the earlier experiments with the oxy-hydrogen blowpipe, this was a result of frequent occurrence; and Mr. Clarke tells us, in his work on the subject, that he often narrowly escaped being killed by the bursting of his apparatus. On one occasion, when he was experimenting in the presence of some friends, "the reservoir for the compression of the gases, although made of thick copper, was torn in pieces; and the fragments flew with the force of cannon-shot in all directions, like the bursting of a bomb." To guard against the danger of such terrible consequences, it was found necessary to protect the operator by means of a thick oaken screen. Mr. Hemming saw the disadvantage of this; and adopting the facts which Sir Humphry Davy had recently brought to light, he contrived a safety jet that enabled the operator to do away with all the paraphernalia that had hitherto protected his life. The jet consists of a tube of brass, about six inches long and three-quarters of an inch in diameter, packed full of fine copper or brass wire: the wire is cut into lengths of six inches, then packed as close as possible in the tube, and finally wedged together by means of a central bar of metal. In this way a number of fine apertures are left for the passage of the gas; and the conducting or cooling power of the wire is so great, that the flame cannot pass back to the reservoir of mixed gases.

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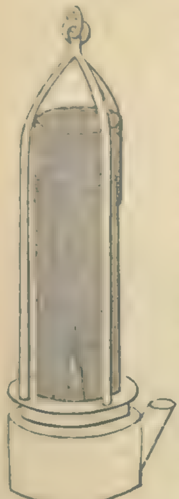
* See *ante*, p. 32; article, "Heat."

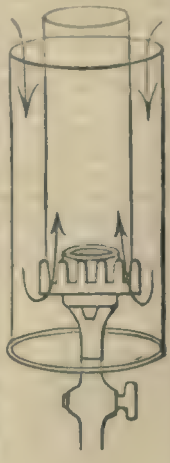
Fig. 84.



Another circumstance that operates in reducing the light and heat of flame is its rarefaction. When a flame is put under the exhausted receiver of an air-pump, it first enlarges, then the light diminishes, and finally it is extinguished. A flame of alcohol, ether, wood-spirit, wax, tallow, oil, or spermaceti, is extinguished when the rarefaction is carried to a fifth or sixth degree; hydrogen is extinguished when it is carried to a seventh or eighth degree; sulphur when it reaches the eighteenth degree; and phosphorus when it arrives at the thirtieth degree. "Slow combustion on the surface of platinum is exhibited by marsh-gas, down to a fourfold rarefaction of the air; by carbonic oxide, to sixfold; by vapour of alcohol, ether, or wax, to eightfold; by olefiant gas, to ten or elevenfold; by hydrogen gas, to thirteenfold; and by vapour of sulphur, down to a twentyfold rarefaction of the air."[*] From this it would follow, that the light emitted by any illuminating agent is not so great at the top of a mountain as at its base, or in the tropics as at the poles of the earth.

Lastly, it should be mentioned, that the supply of very cold air to a flame always diminishes its light by the abstraction of heat; and hence we may improve the illuminating power of any combustible substance by supplying it with atmospheric air that has been previously warmed. The Rev. Mr. Bowditch, of Wakefield, has contrived an apparatus whereby this may be effected in the case of common gas. He places a disc of metal or a cup of glass, having a hole in its centre, on to the screw which receives the burner. This disc, or cup, is made the means of supporting an outer glass, and thus of directing the air down over the surface of the hot chimney before it enters the flame from below (Fig. 85). It is stated that the light is increased about 25 per cent. by this arrangement.

Fig. 85.



PHOTOMETRY.

Intensity of Light and Heat.—We know that the power of light and heat diminishes as we recede from any object that produces it, and the decrease is in proportion to the square of the distance: for example, if we have a certain amount of light; at a foot from its source, the intensity will be one-fourth at two feet, because the square of two is four; one-ninth at three feet ($3\times3=9$), one-sixteenth at four feet ($4\times4=16$), one twenty-fifth at five feet ($5\times5=25$), and so on. Upon this law, as it is termed, is founded all the methods now employed for estimating the relative value of different illuminating agents. The practice is called *Photometry;* and there are various modes of conducting it. Count Rumford adopted the plan of adjusting the lights at such a distance from an upright rod, that the shadows produced by each on a white screen should be equal; he then measured the distance of each light from the rod, and, on squaring the number of inches, he obtained two sets of figures, which represented the relative values of the lights: by dividing the larger sum by the smaller, we obtain a product that expresses the fact in simpler terms. The late Mr. Ritchie contrived an apparatus that consisted of two pieces of glass, placed at right angles to each other, and so disposed, that the light from two opposite sources should be reflected upwards upon a piece of tissue-paper that covered two holes in a darkened chamber. When he had adjusted the instrument so as to obtain two reflections of equal intensity, he then measured the distances, and proceeded as before. Leslie and Wheatstone have also invented photometers, which are useful enough for certain purposes; but they have all given place to the very simple contrivance of Professor Bunsen. It will be noticed, that if a piece of white filtering-paper is painted over with melted wax or spermaceti, it acquires a greasy appearance, and becomes translucent: if this be done so as to leave a spot or disc, about the size of a shilling, untouched in the centre of the paper, we shall find that the apparatus will have the following properties:—When examined by reflected light—that is, with the light on the same side of the paper as the observer is—the disc will look white, and the surrounding greasy part dark; but by altering this condition of things, and looking at the paper by transmitted light—that is, with the light on the opposite side of the instrument—the disc will appear dark, and the surrounding greasy portion light and translucent. Lastly, if two lights of equal intensity are placed one on each side of the paper, the disc will disappear entirely; for then the light of one side neutralises that of the other, and there is no disposition to produce either effect. Upon this is founded the principle of Bunsen's photometer. The instrument consists of a graduated rod about five feet in length, having a support at each end. The prepared paper is held by a frame which slides upon the rod between the lights. This frame is usually enclosed in a darkened chamber, so as to exclude all light but that emanating from the object to be tested. The frame containing the paper is slid to one side or the other until the disc entirely disappears, and thus we read off the relative values of two lights. In making experiments on coal gas, it is usual to burn it at the rate of five cubic feet an hour, from an Argand burner having fifteen holes and a seven-inch chimney. The candle, which is used as a standard for comparison, is generally of sperm, and the rate of consumption is fixed at 120 grains per hour. When it happens that the consumption is greater than this, it may be reduced to the normal standard by a very simple calculation: thus—suppose that, on experiment, it was found that the light of the gas was equal to 11 candles of 140 grains' consumption; then, as 120 grains is to 140, so are 11 candles to the standard value—namely, 12·8 candles.

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[*] Gmelin.

In this way the following results have been obtained; and, in order that an estimate might be made of the relative values of different illuminating agents in respect of their consumption and heating power, other results have been appended:—

	Consumption of each candle or burner per hour.	Number required to produce the same light.	Total consumption per hour.	Illuminating power in standard candles.	Heating power in lbs. of water raised 1°
Cannel coal gas . .	3·5 cub. ft.	2 burners	7 cub. ft.	28	4550
Common " . .	5 " "	" "	10 " "	"	6500
Sperm oil in Argand	450 grs.	5 lamps	2250 grs.	"	5448
Sperm candles, 6 to lb.	134 "	25 candles	3350 "	"	8223
Wax " " .	167 "	23 "	3841 "	"	8904
Cocoa-nut " .	143 "	26 "	3718 "	"	8664
British sperm " .	140 "	29 "	4060 "	"	8743
Composition " .	144 "	30 "	4320 "	"	8906
Oil in common lamp	133 "	36 lamps	4788 "	"	11505
Tallow moulds, 6 to lb	143 "	40 candles	5720 "	"	11793

The amount of heat, in these cases, has been determined by burning the lamp, or candle, under a vessel containing a given quantity of water, or in a space surrounded by a certain amount of that liquid, at a known temperature: from which it will be evident that, for the same amount of light, cannel gas produces the least heat; then follows sperm oil in an Argand or carcel-lamp; then common coal gas; then wax, sperm, and other candles; and, finally, oil in a common lamp, and tallow candles—both of which produce an enormous amount of heat in proportion to the light evolved.

CANDLES.

Materials employed in Candle-making.—It has been already remarked that candles are among the most ancient of illuminating agents, and it cannot be doubted that originally they were manufactured in a very rude way, by simply smearing a porous combustible solid with animal fat; but, at the present time, a number of fatty substances are employed in their manufacture, and great pains taken to have them as pure as possible. We have, for example, tallow and its derivatives stearine and stearic acid, margarine and margaric acid; palm oil and its constituents, palmitin and palmitic acid; cocoa-nut oil, cocinin and cocinic acid; spermaceti, wax, paraffin, &c. All these require to be purified before they can be converted into candles; and hence the necessity for describing them in detail.

The *wicks of candles* are of three kinds; namely, the pith of a rush (*Juncus effusus*), which grows abundantly in marshy places; and of cotton, which is either rolled into skeins or plaited into strands. The first is used for the common rushlight, the second for ordinary dips and for wax-lights, and the third for spermaceti, stearic acid, stearine, and other candles which do not require snuffing. In many cases it is necessary to prepare the wicks with a salt of ammonia, as the phosphate, borate, or sulphate, in order to keep them from clogging, and to give them a sufficient degree of rigidity to enable them to stand firmly up when they are burning. This was an improvement introduced in 1836, by M. De Milly, of Paris. Dr. Ure says that the best wicks are still imported from Turkey, and they are composed of skeins of unbleached cotton. The wick of wax mortars and night-lights are made of flax, as cotton is not able to resist the long-continued action of the high temperature of the flame.

Varieties of Candles, and their Manufacture.—Two sorts of candles are commonly met with in commerce—namely, *dips* and *moulds*. The former are made by repeatedly dipping the wicks into melted fat, allowing a sufficient time between each of the dippings for the tallow or fat to cool. Usually the wicks are cut into proper lengths by a machine, according to the sort of candle to be made, and then suspended from a rod or frame, called a port; by this means the workman is enabled to dip a number of candles at the same time. In large establishments the ports or frames are attached to a revolving beam, so that, without much exertion, the workman can successively dip one port after another, and thus make from seven to eight thousand candles in a day. The tallow is kept in the dipping-vessel, at a temperature just over the point of solidification.

Mould candles are made in a tubular mould, which is either of pewter or glass. The mould consists of two parts—namely, the cylinder and the cap. The former is of the full length of the candle, and is highly polished in the interior, so as to allow the candle to slip easily out of it. The latter is a small cup, having a hole in the centre for the passage of the wick; it is fixed on to the bottom of the cylinder, and serves to give the taper-form to the top of the candle. Sometimes there is a third piece, called the foot, which is a sort of funnel, that is screwed on to the opposite end of the cylinder, and serves for the guidance of the melted fat into the mould. Eight or ten of these cylinders are usually fitted into one frame, which has the upper part formed into a trough for the reception of the melted tallow. The wicks are drawn into the cylinders by a hooked wire, and kept in their places by a wedge, which fixes them in the cap of the mould; the other end being held by a rod which passes across the frame. When all is ready, the liquid fat is run into the trough, and thence into the moulds, until each cylinder is full. After it has thoroughly cooled, the surplus fat is scraped out of the trough, and, the wedges being removed from the hole in the cap, each candle is drawn out of its mould by means of a hooked wire, which catches hold of the loose end of the wick. The candles are then cut of one uniform length, and trimmed up for the market.

The cheaper sort of mould candles are those manufactured by Price and Company, and are made by machinery, eighteen candles being moulded at one time. The wicks are made in lengths of sixty yards; eighteen of these wicks are wound off upon eighteen separate rollers, and a roller is placed over each mould. The wicks having been passed through the cylinder, are seized at the lower end by a set of eighteen forceps, which draws them tight, and fixes them in their places. The moulds, which during the operation have remained in a horizontal position, are now turned in a vertical direction, the small end downwards, and are passed upon a railway to the person who fills them. In their course they have traversed a hot closet, and have thus acquired a proper temperature for receiving the tallow. When full, they are pushed on to other railways, and allowed to cool. This being accomplished, they are brought back in succession, by means of turn-tables, to their former places. The frame is then placed in a horizontal position; and eighteen plungers or pistons press forward the loose caps of the moulds, and thus push out the candles, and deliver the wicks for another operation.

Wax and Paraffine candles are usually moulded by hand, in consequence of their sticking so tight to the sides of the mould, that they cannot be drawn out of it. The operation for wax candles is conducted as follows:—The wicks are first warmed in a stove, and then suspended to a hoop over the vessel of melted wax. The latter is then poured from a ladle over each wick in succession; and to prevent the wax from accumulating more on one side of the wick than the other, the wick is sharply rotated between the finger and thumb. When the candles are thus coated to about one-third their proper size, they are allowed to cool for a short time, and the operation is then repeated until they are half-made. This being accomplished, they are removed from the hoop, and rolled between two marble slabs until they are of one uniform thickness. The upper end of each candle is now formed by cutting it down to a metal tag, which covers one end of the wick. The candles are then again suspended to the hoops, but in a reversed position; and the operations of basting and rolling are repeated as often as necessary. Finally, the lower ends of the candles are cut square, so as to make them of the desired length.

The large wax candles used in churches are formed by rolling the wick in a thin layer of wax, and then, after adding layer after layer, it is finished off in the way just described.

Long wax tapers are made by winding the wick on a drum, and then leading it, by means of a guide-roller, into a trough of melted wax, from which it passes through a series of holes progressively smaller and smaller, on to a second drum, where it is wound up—the operation being somewhat like that of wire-drawing. A little turpentine is added to the wax to render it pliable, so that it may not crack as it bends over the drum.

We proceed now to speak of the different kinds of fats, and of the candles made from them.

Tallow is obtained from animals; it is the fat which is located under the skin, about the intestines, in the bones and muscles, and around the kidneys. It is extracted by cutting the tissue into small pieces, then submitting it to heat, and finally to pressure. The tallow or fat of bones and the dressings of skins is obtained by boiling them in a large caldron, and skimming off the fat as it rises to the surface. A large quantity of tallow is obtained from meat during the ordinary operations of cooking; and this is known in commerce by the name of kitchen-stuff. Tallow, when first obtained, has a very disagreeable smell; indeed, it contains a great number of impurities, as blood, animal tissue, water, &c.; and in this condition it is not suited for the manufacture of candles. To render it pure, it is submitted to various operations, which are called *rendering*. One of these is as follows:—The tallow is melted in a large copper set upon an open fire, and the water contained in the impure fat is boiled away; during this stage of the operation the animal tissues and bloody matters coagulate and rise to the surface. These are strained off and pressed, the residue is sold under the name of greaves or cracklings, and is used for feeding dogs. Another mode of rendering, is to submit the melted tallow to the action of steam, which is blown into it from a pipe pierced with a number of holes. But the most effective process of all is that recommended by M. D'Arcet: the tallow is melted by steam-heat, and then treated with very dilute sulphuric acid; this destroys the colouring matter, and separates the animal tissue in the form of black flakes, which speedily settle to the bottom of the vessel. After drawing off the tallow, and washing it well with warm water, it is allowed to stand until it rises to the surface and is cold. In this state it is nearly free from colour and smell. Some years ago Mr. Watt obtained a patent for purifying tallow by a mixture of acids, as sulphuric, oxalic, nitric, and chromic; the two latter of which give out oxygen gas, and thus bleach the fat.

Tallow consists of several fats; one of which (oleine) is liquid at ordinary temperatures; and the others (margarine and stearine) are solid. Occasionally, also, when it has been obtained from particular animals, it contains a few other fats, as hircine, butyrine, &c. The relative proportions of these fats give to tallow its different consistence; for example, beef-marrow, or bone-fat, which is rather hard, contains 76 of stearine to 24 of oleine; mutton fat, 62 to 38; beef fat, 54 to 46; butter, 40 to 60; hog's-lard, 38 to 62; goose fat, 32 to 68; and turkey fat, 26 to 14. Again, the melting-point of different tallows varies with the proportions of these constituents; for, while common tallow melts at from 94° to 104° Fah., marrow fat requires a temperature of 115° to liquefy it, and hog's-lard is fluid at 81°.

Three sorts of candles are made out of ordinary tallow; namely, rushlights, dips, and moulds. The former are not much employed in towns, except for watch-lights; but they are still the favourite candle with the poor in country villages; and it is very probable that they are chosen because of their not guttering so much as the common dip when they are exposed to currents of air: besides which, they burn longer, and are, therefore, more economical. In many cases these candles are made by the peasants themselves. Mr. Gilbert White has given an account, in his *Natural History of Selbourne*, of the manner in which they are produced by the cottagers of Hampshire. They take a quantity of rushes, which have been previously peeled, and dip them into the melted tallow, so that six pounds of tallow shall serve for 1,600 rushes. In this way they produce candles which require 228 to make a pound. The rushes are peeled on three sides for the best lights, and on two only for watch-lights, and which, says Mr. Gilbert White, only shed a dismal light—darkness visible. Of the other kind, a good one, which measured 2 feet 4½ inches in length, burned 57 minutes; and he was assured by an experienced old housekeeper, that 1½lb. of rushes, after having been coated with tallow, completely supplied her family for a year. The cost of lighting with rushes he estimated at one farthing for 5½ hours; whilst a halfpenny candle, in the blowing, open rooms of the poorer classes, only lasted two hours. The rushlights that are sold in London vary from ten to eighteen in the pound. A specimen of ten to the pound, which measured 11½ inches in length, burnt at the rate of 1½ inches in the hour, equal to a consumption of 74 grains; and it took five of them to give the light of one sperm of 120 grains per hour.

Ordinary dips burn with great irregularity—even the same candle will vary from 120 to 180 grains per hour; and, when consumed at the standard rate, they give a light which is just one-third that of sperm. Dips are made of all sizes, from six to sixteen in the pound; and they are sold at from 5*d.*, and upwards, per pound, according to the price of tallow.

Mould candles, of tallow, are generally made of the better description of fat; they are, therefore, whiter than dips, and melt at a little higher temperature. A mould candle, of six to the pound, burns at the rate of about 143 grains per hour, and its illuminating power is rather more than half that of a sperm of 120 grains; so that, weight for weight, its power is about half that of sperm.

The disadvantages attending the use of tallow candles are, that they melt at a very low temperature, and are, therefore, not suited for warm climates or hot rooms; they gutter when they are exposed to draughts; they require constant snuffing; they give out an unpleasant smell, both before burning and after; they soil the hands, and grease everything with which they come into contact; and, in warm weather, they are very apt to break. Some of these disadvantages have been overcome by the chandlers of Dublin, many of whom have acquired a high reputation for the superior quality of their mould candles; and it has been found, that by waxing the wicks before they are used for making the candles, the disposition to gutter, which is so objectionable, is in a great degree prevented. Candles of this description generally fetch from a halfpenny to three-farthings a pound more than the commoner sort.

Palm oil is the produce of the *Elais Guiniensis*, or *Avoira elais*, a palm that grows very abundantly in the tropical parts of Africa. Most of the palm oil that we receive into this country comes from the western coasts of that continent—chiefly from Guinea. It is obtained by crushing the fruit, then submitting it to the heat of boiling water, and, finally, to pressure. The oil has a buttery consistence, is of an orange-yellow colour, has a sweetish taste, and an agreeable odour. When new, it melts at a temperature of 81° Fah.; but, if old, it requires a heat of from 90° to 96° to liquefy it. It consists of about 69 of oleine, and 31 of a solid fat named palmitine. The crude oil is readily bleached by exposing it to light and air, or to chlorine gas, and it may also be purified in the same way as common tallow. Until it is deprived of its great excess of oleine, it is not fit for the manufacture of candles.

Galam or Ghea butter is another fat oil, that closely resembles palm oil: indeed, it is very often mistaken for it. It comes from the western parts of Africa, and is the produce of a palm, the *Micadenia*, or *Bassia Parkii*—a tree that is very similar in its appearance to the *Bassia latifolia*, and the other species of *Bassia* that are indigenous to the province of Hindostan. According to Park, the tree is very abundant in Bambara; and the oil is obtained from the fruit in the same way as palm oil. Ghea is of a greyish-white colour; it melts at a temperature of 97° Fah., and consists of about 68 of oleine, and 32 of a solid fat like palmitine.

Cocoa-nut oil or fat is extracted from the kernel of the common cocoa-nut, which is the fruit of a palm named *Cocos nucifera*. The oil is semi-fluid, or rather buttery, in its consistence. It melts at a temperature of 68°, and contains two or more fatty principles, one of which, namely, the oleine, amounts to about 71 per cent., and the other, cocine or cocinine, to 29. The oil is not fit for the manufacture of candles until it has been submitted to cold and pressure. We import this oil from several parts of India, from the islands of the Pacific, from Australia, and from Borneo.

Several kinds of vegetable fats or butters, well suited for the manufacture of candles, have at various times been sent to this country; but the demand for them has not been sufficiently great to encourage a trade. Among these may be mentioned the solid fats obtained from three species of *Bassia*, indigenous to India. These are, *Ilpa oil*, or *Eloopei unnay*, which is expressed from the seeds of the Illupie tree, or *Bassia longifolia*—a tree that is very abundant in the Madras Presidency, and in the southern parts of Hindostan. This oil is white, and it requires a temperature of from 70° to 80° to melt it. Similar fats are obtained from the seeds of *Bassia latifolia*, of the Bengal Presidency, and *Bassia butyracea*, of the province of Dotee—the former is named *Epie*, or *Mahowa seed oil*, and the latter *Phoolwa*, or *vegetable butter*. A solid oil, of a pale greenish colour, is obtained from the tallow-tree of Java—probably a species of *Bassia*, which is common not only to Java, but also to the western countries of the Archipelago. In fact, it appears, from the observations of Mr. Lowe, that several kinds of solid oil are obtained in the islands of the Archipelago from different species of *Dipterocarpus*. These oils are hard, yellowish-green, and brittle, and they melt at about 90° Fah. An oil named *Piney tallow* is expressed from the fruit of the panoe tree (*Vateria indica*), which grows abundantly in Malabar and Canara. The oil is white, solid, and fusible at 97°; it makes excellent candles, which do not give out any unpleasant smell in burning. *Cocum oil*, or *Kokum butter*, is obtained from the seeds of a kind of mangosteen (*Garcinia purpurea*), which is common in several parts of the peninsula. It is of a pale greenish-yellow colour, and it melts at 95°. *Kali-ziri*, or *Khatzum butter*, another variety of fat oil, is in all probability the produce of the seeds of *Vernonia anthelmintica*, or of *Salvadora Persica*, both of which are common in Guzerat and Concan Ghats; it is a bright green oil, of the same fusibility as the last-named. The seeds of *Carapa Guianensis*, of Guiana and Acagie, yield a semi-solid oil, named *Crab* or *Carapa oil*; and, lastly a solid fat called *Neem oil*, or *Vaypum unnay*, is obtained from the ripe fruit of the margosa tree (*Melia azadirachta*). All these fats might, if necessary, be largely supplied to this country, and thus be the means of keeping in check the high price of tallow, or even of taking its place altogether.

Stearine, Margarine, Palmitine, Cocine, or Cocinine, &c., are the names applied to the solid fats contained in tallow, suet, lard, palm oil, ghea butter, cocoa-nut oil, &c. It has been already stated, that none of the raw fats, except tallow, can be applied at once to the manufacture of candles; but they must be subjected to certain processes in order to remove the oleine or liquid constituent, and so obtain the fats in a more solid state.

As far back as the year 1799, Mr. William Bolt took out a patent in this country for the manufacture of candles from compressed tallow. This, perhaps, was the first attempt ever made to improve the quality of candles by fabricating them from the solid constituents of fats and oils. About twenty years after this, the researches of Chévreul gave an impetus to the discovery of Bolt, by showing that all the fats consisted of at least two proximate elements—namely, an oily or liquid portion, which he named oleine, and one or more solid constituents, which he called stearine, margarine, &c. Taking advantage of this fact, the candle-makers soon adopted a process whereby they were enabled to separate the one fat from the other, and thus to procure a material which would not melt at so low a temperature as ordinary tallow. To effect this, the tallow, palm oil, or cocoa-nut oil is melted, and then allowed to cool as slowly as possible, care being taken that it is constantly agitated during the whole time that it is solidifying. When the mass has acquired a pasty consistence, it is transferred to horsehair or linen bags, and submitted to great pressure. In this manner the oleine is squeezed out, and the solid fats are left in nearly a pure state By repeating the process of liquefaction and pressure, the stearine, margarine, palmitine, and cocinine are obtained still purer. In this condition they are perfectly white, hard, and nearly free from greasiness, and they melt at a much higher temperature than the original fats: for example, common tallow melts at from 99° to 104° of Fah.; whereas stearine, which is obtained from it, melts at 144°, and margarine at 117°. Palm oil is fluid at from 90° to 96°; but palmitine requires a heat of 124° to melt it; and lastly, cocoa-nut oil is liquid at 68°, while its solid constituent, cocine or cocinine, is fusible at 110°. These fats are employed in the manufacture of *composition* and *Palmer's candles*. The former burn at the rate of from 140 to 155 grains per hour, and they furnish a light which is scarcely inferior to sperm; the latter have an average consumption of about 160 grains per hour for each wick, and the light emitted is in the same ratio as the former.

Stearic, Margaric, Palmitic, Cocinic, and other solid Fatty Acids, are obtained from the oils, &c., already named; indeed, the solid fats which have just been described contain in each case a still more solid compound, which bears the name of the fat from which it is derived. These acids were discovered by Chévreul in the year 1823; and, two years afterwards, he thought them of so much importance, that he allied himself with Gay Lussac, and took out patents in this country and in France for their manufacture. But, notwithstanding that they furnished candles of the very best description, and have since been made the basis of most profitable speculation, they did not realise any advantage whatever to the original patentees; in fact, the processes of Chévreul and Gay Lussac were so complicated and expensive, that they could not be followed out with profit or advantage: in addition to which, it was found that the candles manufactured from the fatty acids would not burn with an ordinary wick; and hence the necessity for still further improvement in this direction. Cambacérès, however, overcame this difficulty by inventing the present description of plaited wick; and this was afterwards improved by De Milly, who suggested the use of boracic acid and the salts of ammonia for impregnating the fibres of the cotton. Another obstacle to the use of the fatty acids was the pertinacity with which they crystallised; thus rendering the candles brittle, unsightly, and irregular of combustion. An attempt was made to remedy this by introducing fine powders into the melted acids, so as to break their grain; but then it was necessary to use powders of a volatile nature which would not clog the wick, and in an unfortunate moment they made choice of white arsenic. The introduction of this deadly poison into candles soon created alarm, for they produced very injurious effects on those who inhaled the vapours of combustion; and, in fact, it brought the discoveries of Chévreul into so great disrepute, that it almost annihilated the infant art of stearine candle-making. "It is true," say the jurors in their report on the candles of the Great Exhibition, "this deleterious substance was added in very minute quantities, yet it was entirely incompatible with health, and was soon prohibited on the continent by authority, and in England by equally powerful public opinion." Here commenced all the manufacturer's troubles anew. In all directions he sought a substitute, and yet found none: at last, after innumerable experiments, and when almost driven to despair, he hit on two simple expedients that answered the purpose in the most admirable manner. They were the addition of a very minute quantity of wax to the melted fatty acids, or allowing the acids to cool almost to the point of congelation before they were poured into the moulds. By this means a sort of liquid pulp is obtained, which sets in the moulds without crystallising. This is the plan adopted at the present time; the melted acids being constantly stirred during their congelation, and the moulds warmed to a temperature of about 110° Fah. before the semi-fluid mass is poured into them.

Two, if not three, distinct processes are employed at the present time, for the preparation of the fatty acids. *The saponification process* is founded on the discoveries made by Chévreul in the year 1823; namely, that when fatty matters are boiled with potash, soda, or lime, they are converted into soap by the union of the fatty acids with the alkali; and that from this soap the acids may be obtained in a more solid condition by the action of a stronger acid, as the sulphuric or hydrochloric. Chévreul saw the importance of his discovery, and took out a patent for the manufacture of the acids from soaps made with potash or soda. These, however, were found to be far too expensive; and in the year 1831, De Milly directed attention to the use of lime.

The following is an outline of the process as it is conducted at the works of the Apollo Company at Vienna; and of De Milly in Paris, Brussels, and Vienna. The process is divided into three stages: namely, the saponification of the fat; the decomposition of the soap by a strong acid; and the removal of the liquid oil and other impurities by means of pressure.

(*a.*) *The saponification of the fat* is effected by melting the tallow, or the palm oil, in a large vat, by means of steam which issues from a pipe perforated with holes, and then stirring in a quantity of lime in the state of thin cream; ten or fifteen parts of dry lime being used for every 100 of fat. The mixture is kept in a state of ebullition for five or six hours, or until it is completely saponified. It is then allowed to stand, in order that the impurities may settle to the bottom, and the melted soap rise to the surface and cool. The soap thus formed is very hard, and is generally called rock. It is ground to a coarse powder, and is then ready for the next operation—

(*b.*) *The decomposition of the soap by a strong acid.* The ground rock-soap is placed in wooden vats lined with lead, and, after being drenched with water, it is raised to a boiling temperature—the heat being applied in the same way as before. At this moment the soap is treated with dilute sulphuric acid, using about twenty-five of the strongest acid to every 100 of tallow or palm oil. In this way the soap is decomposed; its lime unites to the acid of the vitriol to form sulphate of lime, or plaster of Paris, which partly dissolves in the water, and settles to the bottom of the vat, while the fatty acids are set free and float to the surface. These are called "yellow matter;" and, after having been well washed with warm water, they are poured off into vessels called "jacks," and from thence into shallow tin pans, where they cool.

(*c.*) The cakes thus obtained consist of the solid and liquid acids of the fat; to separate which they are placed in horsehair bags, and subjected to hydraulic pressure, gradually increased up to about six hundred tons. This is extended over a period of six or eight hours, during which the liquid oleic acid runs out, and carries with it the brown colouring matter of the fat, leaving the solid fatty acids in a nearly pure condition. It is generally thought necessary to re-melt the acids, and to treat them with a little dilute sulphuric acid, so as to remove iron and other impurities that may have been acquired in the act of pressing. After this they are again cast in shallow pans, then placed in bags, and submitted to a more moderate pressure of about forty tons. During the last operation the cakes are slightly warmed in an atmosphere of steam, so as to facilitate the removal of the last portion of oil. In this way a solid mass is obtained which has all the appearance of ivory; and the product amounts to about thirty per cent. of the fat originally used.

The second or vitriolic acid process for obtaining the fatty acids, was originated in 1836, by M. Frémy, who discovered that these acids have the power of combining with concentrated sulphuric acid, to produce compounds which are named sulpho-stearic, sulpho-oleic, sulpho-margaric, sulpho-palmitic, and sulpho-cocinic acids. These are readily decomposed when they are brought into contact with boiling water, the sulphuric acid being dissolved by the latter, while the fatty acids are set free and float to the surface. This fact was made the basis of two patents, which appeared in the year 1840. One of these was taken out by Mr. George Gwynne, who proposed that the fats should be separated from the acids by distillation; and the other was obtained by Mr. George Clark, who advised that the acids should be set free by simply washing the product. But neither of these patents produced successful results; and it was not until the year 1844 that the process was so perfected by the labours of Jones, Wilson, and Gwynne, as to be applicable to the purposes intended. In that year the names of Wilson and Gwynne are found associated in a patent which still continues to be in force. The process adopted by those patentees is now extensively practised at the works of Price's Candle Company, at Vauxhall and Battersea; and at those of MM. Masse and Tribonillet, at Paris; of Motard, at Berlin; of Bert, at Gijon, in Spain; and at one of the works of the Milly Company at Vienna.

The fats usually employed for this purpose are, palm oil, refuse grease from glue-making and bone-boiling and ordinary tallow. The process is as follows:—

1st. *The Decomposition of the Fat with Sulphuric Acid.*—This is accomplished by first melting the oil or fat in a leaden vessel by the aid of steam, then allowing it to stand for a short time, in order that the mechanical impurities may subside. The liquid fat is now pumped into another vessel, and heated to a temperature of 350° Fah. While in this state it is subjected to the action of strong sulphuric acid—using about 6 lbs. of acid to 112 lbs. of palm oil. The acid quickly decomposes the oil, and gives it a black colour. The latter is now drawn off and transferred to a washing-tank, where it is boiled up with water by means of a jet of steam. After two or three washings it is ready for the next operation.

2nd. *The Distillation of the Fatty Acids by means of superheated Steam.*—The dark liquid fat of the washing-tank is conveyed into a copper still, where it is heated to a temperature of 560°, and submitted to the action of steam that has been heated to a very high degree by passing through a system of pipes set in a furnace. The hot steam raises the fatty acids into vapour, and carries them over into a series of vertical pipes, which act as condensers. These are kept at a temperature of 212°; and consequently they retain the fatty acids, but do not arrest the steam. Towards the end of the process it is necessary to elevate the temperature of the furnace in which the still is set, and also to raise the heat of the steam, in order that the whole of the fatty matter may be distilled over. The residue in the still, after this operation, looks like pitch, and may be employed for the same purposes.

3rd. *The cold and hot pressing of the Fatty Acids.*—These are effected in the same way as that described in the saponification process.

The other process to which we have referred—namely, that portion at the works of Jailon, Monier and Co., at La Villette, near Paris—is a modification of De Milly's lime process. It consists in passing a rapid current of sulphurous acid into the lime vat while the rock mass is forming; and thus of increasing the amount of solid fat by the conversion of the liquid oleic acid into solid elaidic. In this way the very commonest tallow may be used, for the sulphurous acid acts as a bleaching and deodorising agent. The subsequent stages of the process are the same as De Milly's.

Lastly, it may be said that a large quantity of the fatty acids which enter into the composition of ordinary soap, is obtained at the grease-works of Mr. Banwens, at Wakefield, from the waste lyes and suds of the woollen, silk, and cotton manufactories. It is calculated that about 11,000 tons of fat and oil are annually expended in this country in the cleansing and preparing of the fabrics in question. Hitherto all this had been allowed to run to waste; but the energies of Mr. Banwens having been directed to the subject, it is hoped that the larger portion of the fatty acids contained in the soap and oil will be retrieved. These acids are worth about £15 a ton; and if only half of them are annually recoverable, as much as £93,500 will be annually secured. The acids are obtained by treating the boiling suds with a little oil of vitriol or spirits of salt: the liquefied fats rise to the surface, and, when cold, may be skimmed off, or otherwise collected. The fats are then cast into blocks, and pressed in the usual manner.

In whatever way the solid fatty acids are obtained, they always present the following characters:—They have the appearance of ivory, or of fine spermaceti: they are inodorous, and do not communicate a greasy stain to paper, or soil the fingers: they melt at a much higher temperature than the original fats: indeed, the melting-point of pure stearic acid is 167°; that of bassic acid, which comes from the palm fats of India, is 159°; that of palmitic and margaric acids, 140°; and that of cocinic acid, from cocoa-nut oil, is 110°. It rarely happens, however, that these acids are obtained perfectly free from a small proportion of liquid oleic acid; and hence the melting-points of the acids, as found in commerce, are a little below the preceding. Stearic candles generally melt at from 130° to 132° Fah.

These candles, of six to the pound, burn at the rate of from 140 to 144 grains per hour, and they give a light, as nearly as possible, the same as sperm. If there be any difference, the light of sperm is a little greater, and that of stearic acid a little whiter. When calculated into an average consumption of 120 grains per hour, it will be found that, for equal weights consumed, 15 sperm candles will give the light of 16·5 stearic. The advantages which are attendant on the use of these candles are, the great regularity of their burning, the dryness of the cup below the flame, the absence of all disposition to gutter, of not softening in warm climates or hot rooms, and of not soiling the fingers or clothes. It is very probable that stearic candles will, ere long, supersede every other description of *bougie*.

The candles which are manufactured by Price and Co., from the fatty acids obtained by the distillation of *Chinese vegetable tallow*—the product of *Stillingia sebifera*—are even more infusible than the last; for they require a temperature of 136° to liquefy them. A patent has been taken out by Messrs. Wilson, Gwynne, and Wilson, for the manufacture of candles from this material.

Spermaceti.—This remarkable fat is chiefly obtained from the great spermaceti whale, or great-headed cachalot (*Physeter macrocephalus*), which inhabits the Pacific Ocean, the Indian Ocean, and the China Sea. It is also obtained from other species of cachalot, as the *Physeter cetadon*, *trumpo*, *cylindricus*, *microps*, &c.; and from the two kinds of dolphin, viz., the *Delphinus turno* and *edentulus*. The spermaceti is found in all parts of the bodies of these animals, mixed with the common fat or blubber; but the great receptacle for it, in the *Physeter macrocephalus*, is a large excavation or case, situated in the upper jaw, directly in front of the skull, and above the nostrils. This receptacle is opened by the whalers directly the animal is captured; and the liquid contents, consisting of oil, spermaceti, and cellular matter, are dipped out. The dense mass of cellular tissue, called junk, which lies immediately beneath the case, is also removed; and when boiled, it furnishes an inferior kind of oil and spermaceti. The contents of the case are carefully boiled, and then strained off into casks. In this state it goes by the name of "head-matter;" and is composed of spermaceti and sperm oil. After standing for some time at a temperature of from 40° to 50° Fah., the spermaceti solidifies as a dirty-brown crystalline mass. This is separated from the oil by straining through bags, and pressing. The crude spermaceti is melted by the aid of steam, and then allowed to cool very slowly; after which it is ground to powder, placed in bags, and subjected to enormous pressure—a pressure of six hundred tons. In this way the residue of the oil is squeezed out of it, and the spermaceti which remains is nearly white. To purify it still more, it is melted in a large iron vessel, and boiled for some time with a solution of caustic soda. This has the effect of converting all the oil with which it is contaminated into a soap, which dissolves in the water, while the spermaceti floats to the surface. It is now run into tin pans and allowed to cool. The mass so obtained is crushed to a powder a second time, and then pressed as before; the operation being conducted in an atmosphere heated by steam. Finally, the spermaceti is boiled with a strong solution of potash; and when it is perfectly limpid and colourless like water, it is cast into square blocks.

In this condition it is named *cetine:* it is a white crystalline solid, with a pearly lustre and greasy feel. It melts at a temperature of 120°; and it consists of a fatty acid (*cetylic*), which fuses at 131°, and a species of alcohol, named *ethal*, which melts at 118°. Impure kinds of spermaceti liquefy at from 112° to 160°.

Spermaceti candles contain about three per cent. of wax, which is added to break the grain, or to prevent crystallisation. A sperm candle of six to the pound burns with great regularity if it be properly made; but those in commerce at the present time, are but poor examples of what such candles should be, for they range in combustion between 120 and 146 grains per hour. A spermaceti candle of six to the pound, burning at the rate of 120 grains per hour, is generally taken as the standard of comparison for all other illuminating agents, for the light emitted is clear, white, and very brilliant. The candles which are sold under the name of *transparent wax*, are mostly sperm candles, coloured with a little gamboge.

Wax is obtained from several sources: it is secreted between the abdominal scales of the honey-bee, and formed by that insect into honey-comb. The insect wax of China is produced by the male of the *Coccus ceriferus*, which deposits it on the trees on which it feeds, especially the *Rhus succedaneum*. A soft, tenacious, mahogany-coloured wax is obtained in great quantity at the Brazils, and is the product of a black bee which hives under ground. Vegetable wax is procured from the berries of several myrtles (the *Myrica cerifera*, *augustifolia*, *latifolia*, &c.), which grow abundantly at the Cape of Good Hope, and in South America: besides which, there are many trees, in Japan and St. Domingo, which yield substances resembling wax; these are the *Croton sebiferum*, *Celastrus ceriferus*, and *Ceroxylon audicola*. A wax-like substance, named *cerosine* by Dumas, is also obtained from the surface of many species of sugar-cane; and Mulder informs us, that the skins of apples, and the berries of the mountain-ash, yield abundance of wax: in fact, wax is a very common product of the vegetable kingdom: it forms the bloom of fruits and of young leaves, and it is a large constituent of the green and yellow colouring matter of plants (*Chlorophylle* and *Xanthophylle*).

The wax of English commerce is procured from the honey-comb; the comb being first allowed to drain in order to remove the honey, after which it is boiled up with water, and permitted to stand until cold, when the wax solidifies upon the surface as a brownish-yellow cake. This is purified and bleached by the following processes:—The wax is cut by machinery into very small shavings, and put into a vat with water, and a little concentrated sulphuric acid—the proportion of acid being a pint to a ton. Steam is then blown into the vat by means of a coiled pipe pierced with holes; and the mixture is kept in constant agitation. After a time it is allowed to stand quiet, when the impurities subside to the bottom, and the wax floats to the surface as a clear and almost colourless liquid. It is now run into a trough named a cradle, which has a number of holes about the size of a quill in the bottom of it. This trough is placed over a drum or wooden cylinder, which revolves in a tank of water: the melted wax, running in small streams upon the revolving wet drum, is floated off upon the surface of the water in the form of exceedingly thin strips or flakes, called ribbons. These are collected at the opposite end of the tank; and, after being drained in baskets, they are spread out upon tables, and exposed to the air and light to bleach. This occupies a period of from five to ten weeks, during which the flakes are frequently wetted and turned. They are also melted and ribboned once or twice during the process, in order that fresh surfaces may be exposed, and the whole acted on in a uniform manner. When the wax is sufficiently white it is melted in vats, and cast into thin cakes.

Different kinds of wax bleach with different degrees of facility: thus the wax of England, Hamburg, Odessa, Portugal, Mogador, Zanzibar, the East and West Indies, and North America, bleach very rapidly; while those of Cuba, Dantzic, Königsberg, Gabon, and Gambia, bleach with difficulty; and the soft mahogany-coloured wax of the Brazils cannot be bleached at all.

Pure wax is solid up to the temperature of 148°, but unbleached wax melts at from 144° to 146°. It contains about twenty-two per cent. of a peculiar fatty acid (*cerotic*) which was formerly named *cerine;* and the remainder consists of a compound substance named *myricine*, which Mr. Brodie says is composed of the solid acid of palm oil (*palmitic*), and a species of alcohol, which he has named *oxide of melissyl*. Chinese wax is entirely free from myricine, and is made up of cerotic acid and an alcohol named *oxide of cerotyl*. Cerine and myricine are distinguished from each other by the following properties:—Cerine is soluble in boiling alcohol, from which it is deposited as the spirit cools; but myricine is wholly insoluble in that menstruum. Cerine melts at from 162° to 172°, while myricine fuses at as low a temperature as 147°.

Wax candles are rarely composed of pure wax, but consist of wax and stearine, or stearic acid, in various proportions. The candles of commerce are very irregular in combustion; in fact, they burn from 135 to 175 grains of wax per hour; and they give a light which varies from one up to three, according to the condition of the wick. At the best of times the light of wax is, weight for weight consumed, about one-sixth less than that of spermaceti.

We do not employ much wax in this country for candle-making, as preference is now given to sperm and stearic candles; but, on the continent, it is still made use of to a great extent in the fabrication of candles for religious purposes, and also for holiday and *fête* occasions. In the Greek church, wax and oil are the only illuminating agents allowed; and, in the Romish church, large wax tapers are also employed during divine service. These, as well as the candles for weddings, *fêtes*, and saint-days, are generally ornamented either with spirals of gold, or with different devices in colours—the colours being artificial ultramarine for blue; a mixture of verdigris and emerald green, or verdigris alone, for green; chromate of lead, or gamboge, for yellow; vermillion for red; and madder lake, or alkanet root, for pink. The best description of candle manufactured from wax is the *mortar-light*, which is used either for night watching or for heating dishes on the table. On the continent these lights are termed *veilleuses*.

Paraffine.—About twelve years ago, the sagacity of Liebig led him to remark, that "it would certainly be esteemed one of the greatest discoveries of the age, if any one could succeed in condensing coal gas into a white, dry, solid, odourless substance, portable, and capable of being placed upon a candlestick or burned in a lamp."[*]—In the course of the last few years this grand result has been effected; indeed, we may say that the germ of the discovery was brought to light more than twenty years ago, by the researches of Reichenbach and Christison. The former of these chemists showed that the thick heavy oil procured by the distillation of tar from beech-wood, contained a solid crystalline body, which he named *paraffine;* and the latter, in examining Rangoon petroleum, discovered a substance which he called *petroline*. Both of these compounds are identical in their chemical composition with the illuminating principles of coal gas, and they are now known to be one and the same thing. The term paraffine is still employed to designate the substance, in consequence of its weak affinities (from *parum*, little, and *affinis*, affinity). Since that time, it has been procured by Mr. Rees Reece and Sir Robert Kane, from peat; by M. Ettling and Mr. Brodie, from wax; by Mr. Young, from Boghead coal; and by others, from the bituminous schists of England, France and Germany. In all cases it is obtained by destructive distillation; and the following are the processes which are commonly employed.

The plan proposed by Reichenbach was to distil wood-tar to dryness, and to separate the heavy oil which is thus obtained, from the water and light oil that also come over during the process. This is distilled a second time, and only the last portions are retained. These are treated with concentrated sulphuric acid, then with water, and finally with boiling alcohol, from which the paraffine separates in the form of crystalline plates as the spirit cools.

Mr. Reece's process is to distil peat in a sort of blast furnace, to the top of which a condensing worm is attached. The air supplied at the bottom of the furnace keeps up an imperfect combustion of the peat, and the products are conveyed into the worm, where they are condensed. The greasy tar which subsides is collected and heated to a temperature of 100°; it is then submitted to the action of strong sulphuric acid, and afterwards boiled in water. The paraffine collects on the surface, and solidifies as the water cools. It is now ready for distillation; and when so treated, it furnishes three products—namely, a very light oil (*eupion*), which first comes over; then a heavy oil; and finally paraffine. The light oil is separated from the mixture of the other two by decantation. When the latter is allowed to cool, the paraffine separates in the form of very minute grains; and on submitting the pasty mass to pressure in linen bags, the oil is squeezed out, and the paraffine obtained in an impure condition. The next step of the process is to deprive the paraffine of its smell and colour by the aid of chlorine and chromic acid; it is then washed, redistilled, cooled, and pressed a second time. Finally, it is submitted to the action of steam until it is perfectly white and free from odour. It is said that a thousand parts of Irish peat will furnish from ten to twelve of paraffine.

Mr. Young's process is somewhat similar to the last, but he employs Boghead coal in the place of peat. The coal is distilled from a closed retort at a low red-heat, and the products are treated in the same way as the last. One hundred parts of Bathgate coal will yield about forty of oil which is fit for lubricating machinery, and ten of paraffine.

Two years ago, a patent was obtained by Mr. Brown for the preparation of this substance from bituminous coal and bituminous schist. The process which he adopts is not essentially different from Mr. Young's.

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[*] *Familiar Letters on Chemistry*, p. 158.

The schist is heated in an iron retort, and then exposed to a current of very hot steam. The steam carries over all the volatile matters—namely, the oils, the tar, and the paraffine. These are condensed in a proper receiver; and the semi-solid tar and paraffine are afterwards distilled, and then purified by means of sulphuric acid and oxide of manganese, or bichromate of potash. After the residue has been washed with boiling water and weak soda, it is distilled a second time; then cooled, strained, and pressed. The last purification is effected by heating it to a temperature of 400° with strong sulphuric acid. By this means, all foreign matters are destroyed; and when washed with boiling water and a solution of soda, it is allowed to set, and is then ready for the market.

Pure paraffine is obtained in the form of brilliant silvery scales when it crystallises out of boiling alcohol; but the paraffine of commerce is an amorphous substance, having the appearance of a very transparent wax. It is entirely without odour, and it does not communicate a greasy stain to paper. It is slightly flexible, and when warm may be moulded into any shape. It melts at a temperature of 110°, and distils at a red-heat, unchanged. Ether, naphtha, the volatile and fixed oils, dissolve it very readily; and it mixes freely with wax, stearine, and the fatty acids. The light emitted from it varies with the size of the wick: when a small plaited wick is used the light is clear, brilliant, and free from smoke; but when the large cotton strands of wax candles are employed, the flame is large, yellow, and very smoky. The experiments which have hitherto been made with this substance, in order to determine its illuminating power, are not sufficiently complete to enable us to judge of its value; but the candles which we have burnt are consumed at the rate of from 166 to 185 grains per hour; and the light emitted has, weight for weight of candle consumed, been exactly one-twelfth less than that of sperm.

LAMP-OILS AND SPIRITS.

Classification.—Oils are divided by chemists into two kinds—namely, *fixed* and *volatile*. The former communicate a permanently greasy stain to paper or cloth, and the latter do not. Olive oil may be mentioned as an example of one, and turpentine of the other. Again, the fixed oils are subdivided into those which become thick or gelatinous on exposure to the air (*drying oils*), as linseed and poppy; and those which do not (*fat oils*), as olive and sperm. The cause of this change will be referred to directly; but it may be remarked, in a general way, that drying oils are not adapted for combustion in lamps, on account of this disposition to become thick, and so to clog the wick.

The composition of oils is very much the same as that of the solid fats or butters: they consist, for example, of two or more ingredients; namely, *liquid oleine*, which is always present in very large proportion, as from 70 to 90 per cent.; and *solid margarine* or *stearine*. In addition to this, many of the animal oils, as sperm, whale, seal, fish, &c., contain a volatile fat, which gives them their peculiar odour. In the case of whale and seal oils, this is called *phocenine*. The relative proportion of the solid and liquid constituents of different oils is subject to great variation, and hence the difference in the property of solidifying, or becoming thick during cold weather. It must be stated, however, that all oils deposit a solid tallow-like material when they are subjected to cold; and when this deposit is very considerable in amount, as is the case with the oils of cocoa-nut, lard, olive, &c., such are not well adapted for burning in lamps unless they are used in rather warm places.

The sources of oils are very extensive; in fact, they are found in both kingdoms of nature. Animals yield to us lard and tallow-oil, sperm, whale, porpoise, seal, walrus, herring, cod, ling, and other such oils; while vegetables furnish us with the oils of almond, olive, rape, colza, cocoa-nut, linseed, hemp, mustard, poppy, cotton, teal, sesama or gingilie, castor, &c. In the former case the oil is obtained either from the cellular tissue which lies immediately beneath the skin, or else from the liver; and in the latter it is procured from the fruit, or the cotyledons of the seeds.

The modes of extraction vary with different circumstances; so much so, that it is scarcely possible to give a general description of the numerous processes adopted. In the case of lard, tallow, and cocoa-nut oils, the solid fats are slightly warmed, then enclosed in linen or horse-hair bags, and submitted to enormous pressure—a pressure of from four hundred to six hundred tons. The common fish-oils, as those of the herring, pilchard, and sprat, are obtained by piling the fish into heaps or walls, and then weighting them with stones. The oils from the livers of fish, as the cod, skate, ling, &c., are procured by boiling the livers and skimming off the oil, or else by crushing and pressing, or by allowing them to putrefy, so that the tissue may break up and let out the oil. The fat or blubber of different species of whale, porpoise, seal, dolphin, walrus, &c., is subjected to heat in iron vessels, and then submitted to pressure; and, lastly, vegetable oils are procured by first crushing the seeds, then enclosing the bruised mass in bags, and exposing to enormous pressure: this is obtained either by hydraulic power, by screws, or by wedges driven by heavy stamps. "Linseed, rape-seed, poppy-seed, and other oliferous seeds," as we are informed by Dr. Ure, "were formerly treated for the extraction of their oil by pounding them in wooden mortars with pestles shod with iron, set in motion by cams driven by a shaft, turned with horse or water-power; then the triturated seed was put into woollen bags, which were wrapped up in hair-cloths and squeezed between upright wedges in press-boxes, by the impulsion of vertical rams, driven also by a cam mechanism. In the best mills, upon the old construction, the cakes obtained by this first wedge-pressure were thrown upon the bed of an edge-mill, ground anew, and subjected to a second pressure, aided by heat now as in the first case. These mortars and press-boxes constitute what are called Dutch mills. They are still in very general use, both in this country and on the continent; and are by many persons supposed to be preferable to the hydraulic pressure."

Sometimes the crushed seeds are exposed to the action of heat and a little moisture during the operation of pressing. This facilitates the flow of the oil, and consequently makes the seeds yield a larger produce; but the oil is never so good under these circumstances, as it contains much colouring matter, together with mucilage or vegetable mucus and sugar; all of which diminish the combustibility of the oil, and render it very liable to become rancid. In the former case the oil is said to be *cold drawn*, and in the latter *hot*. The amount of oil obtained in this manner varies with different seeds, and even with the same seeds in different countries and seasons. Walnuts and hazel-nuts usually furnish about half their weight of oil; poppy seeds, nearly half; olives, about one-third; rape-seed, a third; and that variety named colza, about two-fifths; hemp-seed, a fourth; almonds, a fourth; linseed, from one-fourth to one-fifth; and the seeds of grape, or wine-stones, about one-tenth.

Olives are sometimes allowed to ferment, so as to become soft and pulpy before they are crushed and pressed. In this way they yield a larger proportion of oil; but the oil is not good, and is only fit for lamps or machinery.

We shall take occasion to notice more particularly the details of all these processes when we come to the subject of the individual oils.

Refining or Purifying.—In the state in which oils are first obtained from plants and animals, they always contain impurities, as albumen, vegetable mucus, colouring matter, sugar, rancid acids, &c. These must be removed before the oil is fit for combustion in lamps; and several processes are adopted for this purpose. Sometimes the oil is merely exposed to the action of steam or boiling water; and, after having been well agitated therewith, it is allowed to stand until the oil floats to the surface in a clear or pellucid condition: it is then drawn off by means of a syphon or tap, and so separated from the water which holds the impurities. At other times it is boiled with weak alkaline solutions, or with lime water, and allowed to repose. Dossie's process for purifying fish-oils is as follows:—To each gallon of oil an ounce of powdered chalk, and half an ounce of recently slaked lime, are added: after standing a short time, about half a pint of water is carefully stirred in, and the mixture is agitated at intervals of two or three hours, for a period of several days. At the expiration of that time, a solution of one ounce of pearlash in four ounces of water is stirred in, and the stirring is kept up at intervals for several hours: after this, two ounces of salt dissolved in a pint of water are poured in, and the mixture is agitated occasionally for the next two days. It is then allowed to stand until it becomes clear; and if the oil be not sufficiently pure and free from odour, the process is to be repeated, taking care that the quantities of the ingredients are reduced one-half.

The plan that is usually adopted for the purification of oils, is founded on the fact, that concentrated sulphuric acid, in small quantity, has the power of decomposing the impurities of oil without touching the oil itself: but in carrying out the process, care must be taken that oil of vitriol is not added in too great quantity, or allowed to act for too long a period. The process is generally conducted in the following way:—About one per cent. of commercial sulphuric acid is cautiously added, little by little, to the impure oil; in those cases where the oil clots with strong acid, as happens with linseed oil, the vitriol acid must be diluted with about half its bulk of water before it is added. The mixture is then stirred for several hours, in order that the acid may be brought into contact with all the impurities in the oil; the stirring is generally effected by means of a sort of churn or spindle with cross pieces, which is kept slowly revolving in the vat containing the mixture. In the course of a few minutes after the vitriol is added, the oil becomes discoloured; and after the agitation has been kept up for some time, the impurities clot together in the form of flakes. When this happens the oil is allowed to stand quiet; and in the course of a few hours the flakes subside, and leave the oil in a clear and almost colourless condition. The oil is now run off into vats, and boiled up with about half its bulk of water; this removes any acid that may be adhering to the oil: the fire is then withdrawn, and in a short time the oil floats upon the water, and may be run off into casks ready for the market.

Other processes have been recommended for the purification of oils; as, for example, the filtration of them through animal or peat charcoal, or the exposure of them to the action of light, or to the chemical influence of a weak solution of chloride of lime; but all these processes are more or less difficult of management, and consequently they have given place to the more speedy and certain method of refining by means of oil of vitriol. The filtration of oil through charcoal is, however, a very effectual plan for removing bad colour or bad odour.

General Properties of Oil.—Every variety of fixed oil communicates a greasy stain to paper or cloth, and has an unctuous feel. It is also perfectly insoluble in water; and when mixed therewith, speedily rises to the surface. This shows that it is lighter than water; in fact, a bottle which, when full, holds 1,000 grains of distilled water, will contain only 884 of sperm oil, or 965 of castor oil. These are the two extremes, for sperm oil is the lightest of all fixed oils, and castor the heaviest. Chemists have applied the term *specific gravity* to these numbers; and as it is found that each oil has its own particular weight, specific gravity is made the means of discovering the nature and quality of any doubtful oil.

With respect to the mode of taking specific gravity, we may state, that solids are weighed first in air, and afterwards in distilled water. The weight in air is then divided by the loss of weight sustained by the body on being weighed in water; and the quotient is the specific gravity. Liquids, such as water, spirits, ether, oils, &c., may have their specific gravity taken by weighing them in a glass globe with a narrow neck, in the manner already mentioned.

The following table represents the relative weights or specific gravities of different fixed oils:—

Sperm	884	Cherry-stone . . .	924
Tallow	900	Fish	924
Rape-seed	913	Cameline	925
Plum-kernel . . .	913	Nightshade . . .	925
Henbane-seed . . .	913	Sunflower-seed . .	926
Colza	914	Hemp-seed	926
Ground-nut . . .	915	Cocoa-nut	926
Olive	916	Walnut	926
Sessamum	916	Filbert	926
Almond	917	Anda	927
White mustard-seed .	918	Horse-chesnut . .	927
Radish-seed . . .	919	Cod-liver	928
Grape-seed . . .	920	Seal	929
Poppy-seed . . .	922	Linseed (new) . . .	930
Whale (train) . . .	923	Weld-seed	936
Black mustard-seed .	923	Orange-seed . . .	940
Walnut	923	Linseed (old) . . .	960
Cucumber-seed . .	923	Castor	965
Tobacco-seed . . .	923		

Most of the fixed oils are affected by the atmosphere—the oxygen of which they slowly absorb. In some cases the oil becomes thick; or, if exposed in a thin layer, it dries. In this manner a skin is frequently formed over the surface of the oil; and if it be left for some time in a lamp, it will assume a jelly-like appearance. Other oils do not become so thick by exposure to the air; but they acquire an unpleasant smell, and get rancid. The former is the property of the drying, and the latter of the fat or unctuous oils. These changes are due to the action of atmospheric oxygen; indeed, Saussure found that a layer of nut-oil, one-fourth of an inch in thickness, absorbed as much as 145 times its bulk of oxygen in the course of eleven months, of which quantity 142 volumes were taken up during three months' exposure to the sun. It appears, also, that it is the carbon of the oil which undergoes oxidation; and, as it were, slowly burns, evolving carbonic acid. This change is always accompanied with an elevation of temperature; and hence it sometimes happens that rags, tow, or cotton, that have been smeared over with oil, and then thrown aside as useless, have; in the course of a few days, generated heat enough to produce spontaneous combustion. Fires have frequently originated in this manner in warehouses and dockyards, where such materials have been allowed to accumulate. This points to the danger that is attendant on the careless and slovenly trick of throwing greasy rags, and other such matters, into out-of-the-way corners. Those oils which absorb oxygen with great facility, and thereby become thick, are not well-suited for combustion in lamps, unless the lamp is cleaned out every day, and a fresh wick adapted to it. The quick-drying oils are linseed, poppy, walnut, hemp-seed, and nut—all of which are employed by painters on this very account; while rape, colza, sessama or gingilie, cocoa-nut, grape-seed, cameline, sunflower, cotton, mustard, &c., only dry after very long exposure to the air; and sperm, olive, almond, seal, and whale oils are not much disposed to dry at all.

Heat and cold act upon the oils, and produce changes in them which are more or less hurtful. In cold weather—that is, at a temperature below 32°—most of the oils become thick, from the congelation of their solid constituents; and we then find great difficulty in making the oil burn in a lamp. At a temperature of 600°, or thereabouts, the several fats begin to boil; and if the temperature be carried a little higher, the oil undergoes decomposition, and gives forth a most irritating and unpleasant vapour, which is called *acroline*. At still higher temperatures, the fixed oils are resolved into combustible gases, which burn with a very bright light and a sooty flame: indeed, the object of all the arrangements for the combustion of oil in lamps for illuminating purposes, is that of bringing the oil slowly into contact with the burning wick, so as to generate the gas in question, the supply of which ought to be duly apportioned to that of the air which consumes it. When oils are heated to near their boiling-point, they frequently become thick, and acquire an increased disposition to absorb oxygen. It is on this account that inferior drying oils cannot be employed for any length of time in a lamp, without closing the channels that lead to the wick; and hence it is found necessary to clean the lamp very frequently, or else to pour away the residue of the last consumption before fresh oil is supplied. The same circumstance leads to the stoppage of the pores of the wick, and finally to a total closing of all the fine capillary channels by which the oil ascends to the flame; so that a new wick and a clean lamp are indispensable to the perfect combustion of an inferior oil.

Although the fixed oils are not soluble in water, yet they combine very readily with fluids which contain alkaline matters, as soda or potash. In this way they form white emulsions of the nature of soap; and the alkalies, or their carbonates, may be at all times resorted to for the purpose of cleaning out the half-gelatinised oils from old lamps. Ammonia, also, or strong sal volatile, is a very effective agent for removing grease from the clothes.

Concentrated sulphuric acid acts on the fixed oils in the same way as it operates on the solid fats: it first combines with the fatty acids, and then with the basic constituent to form composite acids, named sulpho-oleic, sulpho-margaric, sulpho-steric, and sulpho-glyceric acids. These are not permanent compounds, and have not, therefore, been isolated; but when they are treated with hot water, they are decomposed; and as the sulphuric acid separates, the other fatty constituents resolve themselves into four new acids—namely, metamargaric, hydromargaritic, metoleic, and hydroleic. The first two are solid, crystallisable, and partly volatile, while the last two are liquid and oily. Besides these changes, there are others effected by the action of the sulphuric acid on the sugar, mucilage, colouring matter, &c., contained in the oils; and these are so peculiar and characteristic, that they have been made the means of discovering the nature of a fixed oil. This fact was first recognised by Heidenreich of Strasburg, in 1841; but it has been greatly developed by the investigations of Penot, Marchand, and others. The mode of proceeding is as follows:—Put eight or ten drops of the oil on a white plate, and then let fall into the centre of the oil a single drop of concentrated sulphuric acid: in the course of a few seconds or minutes, the characteristic tints will be observed. Another mode of conducting the experiment is to stir the drop of acid so as to mix it with the oil. The following are the changes observed:—

(*a.*) With *olive oil* there is a yellow colour produced at the points of contact; this rapidly passes into orange, and at last into a bright chesnut-brown, while the surrounding parts of the oil gradually acquire a dirty gray, and finally a smoky tint; but there is never any shade of blue or lilac. If the mixture be stirred, it speedily becomes dirty-brown or brownish-gray.

(*b.*) *Poppy oil* immediately acquires a fine lemon-yellow colour, which soon becomes darker in some parts. The surrounding portion gradually assumes a rose tint, which speedily passes into violet, and then into a violet-blue. This requires a period of from half-an-hour to three-quarters; and finally the yellow colour becomes dirty-brown. If the oil has been kept for some months, or if it has been expressed by the aid of heat, it will then assume a greenish tint. When stirred it acquires a brownish-olive colour.

(*c.*) *Nut oil* produces almost exactly the same tints as olive oil, but it becomes brown more rapidly, so that within ten minutes it acquires a chesnut-brown colour, and the gray border changes into olive-green. If it be stirred, the oil clots and becomes dirty-brown.

(*d.*) *Castor oil* assumes a yellow tint, which slowly passes into grayish-red, and at last, after many hours, into purple-brown. When the oil is very old it becomes brown at once; and if the oil and acid be stirred, the tint is pale yellow, and then brown.

(*e.*) *Almond oil* produces nearly the same reactions as olive oil; but the surrounding tint is pale grayish, and the yellow has a greenish tint. If stirred it becomes dirty-green.

(*f.*) *Sessama, gingilie,* or *teal oil,* assumes a yellowish colour, which, in the course of a few minutes, becomes orange, and then chesnut-brown, the surrounding oil acquiring, in the course of half-an-hour, a purplish tint. If it be stirred it acquires an orange-brown colour directly.

(*g.*) *Orange-seed oil* takes on a yellow shade, which rapidly passes into brown, and then into black. If stirred, it becomes black almost immediately.

(*h.*) *The oil of black and white mustard-seed* is first yellow, then rich yellow-orange, and finally, in the course of a few minutes, brownish-black, the surrounding oil being of a greenish tint. By stirring, the dark tint is brought out almost immediately.

(*i.*) *Linseed oil* becomes rich chesnut-brown almost immediately, and it soon coagulates, or clots into a hard tenacious spot. When stirred it immediately thickens, and acquires a brownish-black appearance.

(*k.*) *Hemp-seed oil* presents nearly the same character as linseed, except that this oil assumes a greenish-yellow tint at the edges of the acid; and if stirred it acquires a greenish-brown colour.

(*l.*) *Cocoa-nut oil* acquires a pale purplish-brown colour, which gradually darkens; and when stirred it is first ochre-brown, and then deep violet-brown.

(*m.*) *Refined rape-seed oil* assumes the same appearance as the last; and it is charred, as it were, by the acid.

(*n.*) *Fresh-drawn,* or *raw rape-seed oil* becomes bright green; and then, after the lapse of ten minutes or a quarter of an hour, the tint deepens, and finally becomes olive-green or dirty greenish-brown; the surrounding edges retain a bright green colour. If the oil and acid be stirred, the tint is blueish-green, and at last olive-green.

(*o.*) *Oleic acid* changes into a sepia brown, which at length darkens almost to a black.

(*p.*) *Neat's-foot oil* becomes yellow, and then, after some time, brownish. If stirred, it changes into a dirty-brown at once.

(*q.*) *Whale* or *train oil* acquires a reddish-brown colour, with the edges of a violet tint; and, if stirred, it passes into a dark violet-brown, like that of the lees of wine.

(*r.*) *Cod* and *other liver oil* instantly changes to a rich violet, with the edges of a carmine colour; in the course of a few minutes this passes into orange, and, finally, into dark-brown. When stirred, the violet tint is very remarkable, but very transient.

(*s.*) *Seal oil* assumes a yellow colour, which passes into rich orange, and then into a blackish-brown, having a number of purple spots about the mixture. When stirred, it acquires a lively yellow tint, which soon changes to orange-brown.

If the acid be diluted with one-third its bulk of water, the reactions are slower, and the effect, in some cases, is more marked. With such a mixture, olive, almond, and castor oils show but little action; while poppy, orange, and mustard become dirty-brown; gingilie, yellowish with a pink border; linseed, brown; rape, green; cocoa-nut and refined rape, pale purple-brown; sessama, lavender; oleic acid, dirty-brown; sperm, pale lavender; cod-liver, rose, passing into rich violet, and then into brown; common whale, black-brown; seal, dirty-brown; and tallow-oil, blackish-brown.

Mixtures of the above-mentioned oils give reactions which are compounded of the preceding; and thus the fraud of adulteration may be easily detected.

Another reaction which has been noticed during the admixture of the oil with concentrated sulphuric acid, is that of a great elevation of temperature; and, as each oil produces its own amount of heat, M. Maumené has proposed that this reaction should be used as a means of discovering the quality of an oil. Maumené's results have been confirmed in the laboratory of Professor Fehling, by Faisst and Knauss, who state, that when 225 grains of oil are quickly mixed in a thin glass vessel, with 75 grains of the strongest sulphuric acid, the following are the number of degrees raised by different oils :—

Olive oil . .	68° Fah.	Rape oil . .	100° Fah.
Lucca oil . .	72° ,,	Poppy oil . .	127° ,,
Almond oil . .	72° ,,		

With an acid of 90 per cent., the rise is :—

Lucca oil . .	54° Fah.	Linseed oil . .	133° Fah.
Rape oil . . .	67° ,,		

Mixtures of these oils produce intermediate results.

Aqua-fortis, or *nitric acid*, acts with more or less energy on the fats, and converts them into other acids; one of which, namely, suberic acid, is a very constant product. But the most remarkable change is brought about in all the fats by means of *nitrous acid*, or a solution of *nitrate of mercury*. Either of these compounds will cause the fat oils to become solid; the oleine of the oil being converted into a semi-transparent jelly-like mass, named *elaidine*. This curious change is effected in olive, almond, rape-seed, hazel-nut, and other non-drying oils; but the drying oils, as linseed, hempseed, walnut, poppy, &c., are not affected by these compounds. This circumstance, and the facility with which the change is brought about, offer a means of distinguishing one kind of oil from another. M. Poutet recommends the following method for the application of this test:—Dissolve six parts of mercury in seven and a-half parts of nitric acid (sp. gr. 1·35); add two parts of this solution to ninety-six parts of oil, and shake the mixture every now and then for half-an-hour or so. If the experiment be made on pure olive oil, it will congeal in the course of seventeen hours in summer, or four hours in winter. Other vegetable oils do not combine so quickly with nitrate of mercury; and the mixture either remains fluid, or else the olive oil congeals, and the other oil separates into a distinct layer. If the oil has been adulterated with animal fat, the mixture congeals in five hours, whilst the olive oil floats on the surface, and may be decanted. MM. Boudet and Fauré have shown that this change is brought about by the *hyponitrous acid* contained in the nitrate of mercury. They therefore recommend a solution of that acid in aqua-fortis —*common nitrous acid* does very well for the purpose; and they say that each of the fat oils takes its own time to solidify, and develops its own colour. When one part of hyponitrous acid is dissolved in nine of aqua-fortis, and then added to a hundred parts of oil, the colour produced, and the times of solidification, are as follow :—

	Colour.	Time.
Olive oil . .	Greenish-blue . .	73 minutes.
Almond . .	Dingy-white . . .	160 ,,
Filbert . .	Greenish-blue . .	103 ,,
Acorn . . .	Lemon-yellow . .	40 ,,
Castor . .	Golden-yellow . .	603 ,,
Colza . . .	Yellowish-brown .	2400 ,,

Oil of poppies retards the solidifying effect, and this to so great an extent, that when present in either of the above, in no larger portion than 1 per cent., it delays the action for forty minutes.

A mixture of *equal parts of nitric and sulphuric acids* also effects the coloration of the fixed oils; and this has been shown by M. Behrens, as well as by MM. Guibourt and Reveil, to be a good means for detecting the adulteration of oils. About 150 grains of the mixed acids are added to a like quantity of the oil, and the colour, which is instantly produced, is as follows :—

Oil of Sessama	dark grass-green.
,, Olive	light yellow.
,, Linseed	brownish-red.
,, Almond	peach-blossom.
,, Castor	little changed.
,, Colza	reddish-brown.
,, Poppy	brick-red.
,, Gingilie	rich orange-brown.
,, Orange-seed . . .	rich chesnut-brown.
,, Mustard	orange-brown.
,, Rape-seed (raw) . .	orange-red, and then dirty green.
,, ,, (refined) .	yellow-red, and then purple-brown.
Oil of Cocoa-nut	pale orange-red.
,, Sperm whale . . .	orange.
,, Seal	orange-brown.
,, Cod-liver	pinkish-violet.
,, Tallow	dirty brown.
,, Neat's-foot . . .	,, ,,

M. Behrens states, that when olive oil is mixed with a fourth part of its weight of sessama oil, it takes a beautiful green colour; but Guibourt and Reveil assert, that it will discover one-tenth part of sessama in oil of olive.

Other oxydising agents, as *chromic acid*, or *a saturated solution of bichromate of potash, in oil of vitriol*, also act in a very characteristic manner on the fixed oils. This fact was first observed by M. Penot, who employed a single drop of the latter solution with twenty drops of oil, stirring them together on a white plate. The following are the effects produced :—

Almond oil .	becomes	yellowish in small lumps.
Hemp-seed oil	,,	yellow in clots on a green ground.
Rape-seed oil .	,,	,, ,, ,, ,,
Poppy oil . .	,,	,, ,, ,, ,,
Ditto (cold-drawn)		,, ,, white ground.
Castor oil . .	,,	slightly green.
Linseed oil .	,,	brown clots on a white or greenish ground.
Nut oil . .	,,	brown clots.
Olive oil . .	,,	brown.
Cod-liver oil	,,	dark red.
Whale oil . .	,,	brownish-red clots on brown ground.
Tallow oil .	,,	reddish-brown.
Neat's-foot oil	,,	brownish-red spots on brown ground.

The illuminating power of different oils varies with different circumstances, as with the temperature of the room, the size and form of the wick, and the freedom with which the oil is supplied to the flame; so that it is difficult to arrive at anything more than an approximation to the value of different oils as illuminating agents. Count Rumford estimated the relative illuminating power of different oils when burnt in Argand and in common lamps, thus—the numbers being the weights consumed to get an equal amount of light :—

Good wax, well-snuffed	100 grains.
Olive oil in an Argand lamp . . .	100 ,,
Olive oil in a common lamp	129 ,,
Rape oil ,, ,, ,, ...	125 ,,
Linseed oil ,, ,, ,, ...	120 ,,

From some experiments which have been conducted for the purpose of arriving at an approximation to the relative value of different oils, we have drawn up the following table, which represents the proportions of oil to be consumed in order to obtain a light of given value —namely, that of thirteen sperm candles, each consuming 120 grains per hour. In these experiments the same lamps were used for each of the oils, and the wick was in each case of precisely the same description; a new wick being employed for each experiment. The consumption of oil in the Argand ranged between 316 and 378 grains per hour, and that of the common lamp between 87 and 123.

	Argand Lamp.	Common Lamp.
Tallow . . .	1008 grains. . . .	1300 grains.
Sperm . . .	1029 ,, . . .	1751 ,,
Train . . .	1067 ,, . . .	1941 ,,
Olive . . .	1080 ,, . . .	2014 ,,
Seal . . .	1107 ,, . . .	1892 ,,
Sessama . .	1113 ,, . . .	1776 ,,
Poppy . . .	1119 ,, . . .	1660 ,,
Refined rape .	1134 ,, . . .	1881 ,,
Refined linseed	1231 ,, . . .	2376 ,,
Raw rape . .	1236 ,, . . .	1940 ,,
Brown rape .	1343 ,, . . .	1827 ,,
Mustard-seed .	1354 ,, . . .	2058 ,,
Fish	1362 ,, . . .	1976 ,,

From this it will appear that there is no direct relation between the illuminating power of the oil and its consumption; for in some cases the oil burns better in the common lamp than it does in the Argand, and *vice versâ:*

showing that the illuminating power is modified by circumstances. This table has been constructed on the scale of thirteen sperm candles of standard value, because they represent the light of an Argand burner consuming five cubic feet of coal gas per hour.

VARIETIES OF OILS EMPLOYED FOR ILLUMINATING PURPOSES.

Animal Oils.—(*a.*) *Sperm oil.*—This, as we have already said, is the fluid portion of the head-matter which is removed from the cranial cavity of the spermaceti whale (*Physeter macrocephalus*). The oil is usually separated from the spermaceti by cold and pressure; and after having been purified by means of a little weak sulphuric acid, or alkaline lees and water, it is sent into commerce. Sperm oil is of a pale colour; it has but little odour, and it shows no disposition to congeal in cold weather, or to resinify on exposure to the air; it is, therefore, well suited for illuminating purposes.

(*b.*) *Common whale oil*, or, as it is sometimes termed, *whale train-oil*, is derived from several species of whale; as the common whales (*Balænæ*), the finned whales (*Balænopteræ*), and the narwhales (*Monodons*). Most of the oil consumed in this country is furnished by the common Greenland whale, the *Balæna mysticetus*, large numbers of which are annually captured by the whalers of England and America in several of the Arctic Seas. They are taken in the Greenland seas; in Davis' Straits; along the coasts of Spitzbergen, Iceland, and Norway; off Labrador; in the Gulf of St. Lawrence; around Newfoundland; in Baffin's and Hudson's Bays; and in the seas northward of Behring's Straits. The *balæna* is also found in more congenial climates, as on the coast of Ceylon, and in the China Sea.

In the early days of whaling, when the animal was found in great numbers immediately around the shores of Spitzbergen, the Dutch formed a settlement on that island, and performed there all the operations of preparing the bone and extracting the oil. They gave the name of Smeerenberg (from *smeeren* to melt) to their settlement; and to so flourishing a state did the fishery arrive, that, during the busy season, every species of luxury could be obtained at the village, although it was situated within a few degrees of the pole. This was the condition of things towards the close of the 17th century. But it appears from a narrative of the voyage of Ohthere the Dane, given by King Alfred in his translation of Orosius, that the pursuit of the whale was practised by the people of Norway at least as early as the 9th century. We have no account, however, of the way in which the animal was captured, nor are we informed as to the object of the pursuit—whether it was for mere sport, or for some useful purpose. It is very probable the business of hunting the whale was not carried out upon any systematic plan, but was confined to such accidental encounters as opportunity offered. As early as the 12th century, the inhabitants of the coast surrounding the Bay of Biscay were undoubtedly engaged in the capture of the whale for commercial purposes; and they are generally regarded as the founders of this species of enterprise. At first their operations were confined entirely to the neighbouring bay; but as the whale became scarce, the Biscayan mariners extended their search farther and farther from their shores, until they reached the coasts of Iceland, Greenland, and Newfoundland. "Thus," says Mr. Dewhurst, "was commenced, in the course of the 16th century, the northern whale-fishery, as pursued in modern times. In 1594, the English made their first whaling voyage; and four years afterwards the merchants of Hull fitted out several ships for the purpose: about the same time the Dutch were tempted to engage in the trade; and soon afterwards the Hamburgers, the French, and the Danes were occupied in the same pursuit."

The whale is captured by means of harpoons, and, when dead, it is drawn to the side of the ship, and flensed or cut up. This is effected by means of spades and powerful knives, the blubber being cut into cross pieces of about half a ton each, which, after being hoisted on deck, are subdivided into small strips, and then forced into the bungholes of the storing casks. The blubber is the true skin of the animal; and it consists of a network of intersecting fibres, which enclose the liquid fat. It encompasses the whole of the body: its thickness varying from eight to twenty inches, though one foot is about the average; and a single whale will yield from twenty to eighty tons of it. The lips of the animal furnish the best kind of blubber; and they weigh from one and a-half to three tons each. The quantity of oil so obtained, amounts to about three-fourths of the entire weight of the raw blubber.

Formerly, it was the custom to extract the oil from the blubber at the places where the whales were caught, but now this process is effected after the fat arrives in this country. The operation is thus managed:—The half-putrefied fat is thrown into vats which have a wire grating at the bottom. The tissue is then broken up by pressure, and the oil runs out into a reservoir, which is placed below the vats: that which remains behind is called *finks*. After the oil has settled for two or three days, it is poured off into another vessel, and heated to a temperature of 225°. This causes the albuminous matter to coagulate, which, with the other impurities, soon subsides to the bottom of the boiler. The fire is now withdrawn, and water is poured into the vessel, in order that the dregs may be more easily separated from the oil, and may not stick to the bottom of the boiler. After standing for some time, the oil becomes clear, and then it is run off into casks. Another mode of extracting the oil is to boil the blubber with water, and to skim off the oil as it rises. The oil may also be obtained by allowing the blubber to putrefy, and thus to release the fluid fat from the cells in which it is contained. In this manner, by suspending the blubber in bags over casks, the oil gradually drips out, and is collected.

We have already said that the train-oil of commerce is chiefly derived from the common Greenland whale (*Balæna mysticetus*); but when this creature is scarce, another species of balæna—namely, the *Balæna Icelandica*, or nord-caper—is sought for; and under certain circumstances the different species of finned whale, as the *Balænoptera gibbar*, *boops*, *rorqual*, &c., and even the *Monodons* or *narwhales* may be made to yield train-oil. It is not often, however, that the whaler has an opportunity of capturing these creatures; for they are so swift and shy, that there is great difficulty as well as danger in approaching them. Occasionally they are cast ashore on the northern and western coasts of Europe; and then they become a source of great profit from the quantity of oil which they yield. Some idea may be formed of the value of these animals by the following fact:—In the month of August, 1827, a large specimen of the *Balænoptera rorqual* was found floating off the port of Ostend. It was towed into the harbour by some fishermen; and when cut up, it yielded 40,000 lbs., or 4,000 gallons of oil. The specimen measured ninety-five feet in length, and weighed about 240 tons.

Lastly, it may be stated that the several species of *dolphin* furnish abundance of oil. They belong to the whale tribe, and inhabit the seas of all latitudes; for they are found in the Arctic Ocean, the Mediterranean, the Gulf of Messina, the Adriatic, and on the coasts of China: in many of which places there are large establishments for their capture, in consequence of their furnishing an excellent oil for illuminating purposes. To this order of animals belong the common porpoise (*Phocæna vulgaris*), the round-headed porpoise, or ca'ing whale (*Phocæna melas*), and the white whale (*Phocæna leucas*); all of which yield oil in considerable quantity. The common porpoise is captured by the inhabitants of the western islands of Scotland, where it abounds; and we are told, that about eight gallons of oil are obtained from each individual. The round-headed porpoise is taken in the Shetland Islands, the Orkneys, and in Iceland; and the white whale in several localities on the shores of the North Sea.

The oil obtained from these sources, when properly purified, is very little inferior to sperm oil: it does not clog the wick, or congeal in cold weather, and it burns with a clear white flame, which is tolerably free from smell. About 20,000 tons of whale oil and spermaceti are annually imported into this country.

(*c.*) *Seal oil* is procured from several species of *phocidæ*. The common seal (*Phoca vitulina*) is captured in large numbers on the shores of Newfoundland: indeed, we are informed that, during a good year, hundreds of thousands are taken in that locality, for the sake of the oil which they yield. They are also killed on the northern coast of Scotland, in the Orkneys, Shetlands, and on the shores of Greenland. To the inhabitants of the last-named locality the seal is invaluable; for it furnishes them with food, raiment, and oil. The animal is usually captured with spears or harpoons, and sometimes it is shot. As soon as it is dead, the skin is stripped off, and then the fat is removed and boiled down in copper or iron vessels. Seal oil, like the preceding, is not much disposed to thicken; and hence it is well suited for combustion in lamps.

(*d.*) *Walrus oil* is obtained from the morse or seacow (*Tricheeus Rosmarus*), many of which are annually destroyed at Spitzbergen, and elsewhere, for the sake of the skin, oil, and teeth. We do not receive the oil into commerce in this country, and consequently have little or no opportunity of testing its value; but it appears that its qualities are not inferior to those of the last-mentioned oil.

(*e.*) *Fish oils* are extracted from the bodies and livers of fish. As examples of the former, we may mention the oils obtained from the herring (*Clupea harengus*), pilchard (*Clupea pilchardus*), and sprat (*Clupea sprattus*): all of which are procured by submitting the fish to great pressure at the time that they are undergoing the process of salting. Of the latter may be mentioned the oils of cod (*Morrhua vulgaris*), ling (*Gadus molva*), skate (*Raia batis, &c.*), turbot (*Lota vulgaris*), torsk (*Brosmius vulgaris*), &c. The oil is obtained by placing the livers in a tub which has a perforated bottom covered with small branches of trees. As the livers putrefy, the oil drips out, and is caught in a vessel placed underneath. At other times the oil is extracted by boiling the livers in an iron pot, and then squeezing them in linen bags. Cod-liver oil is not much used for purposes of illumination, as it is a valuable remedy for the cure of many diseases; but the oil of ling is extensively prepared by the poor of the Orkneys and western islands of Scotland, where it is employed as a common lamp-oil. All these oils are somewhat of a drying nature, and therefore become thick after a time; besides which, unless great care has been taken in their preparation, they are sure to have a most unpleasant, fishy, or putrid odour.

(*f.*) *Lard oil, and the oleine from tallow and other animal fats*, is obtained from the solid fats by slightly warming them, and then submitting to pressure. This oil is apt to deposit solid matter in cold weather, and thus to become thick; but the properties of the oil are otherwise very good; and, consequently, it is well suited for combustion in lamps. The *oleic acid*, which is procured during the manufacture of stearic and margaric acids for candles, is not fit for illuminating purposes; for, although it gives out a very good light during its combustion, yet it is so apt to clog the wick from the impurities which it contains, that, in the course of a very short time, the lamp ceases to burn. Were it not for this, oleic acid might be extensively employed as an illuminating agent.

Vegetable Oils.—(*a.*) *Olive oil.*—This is furnished by the fruit of *Olea Europæa*, of which there are two varieties —namely, the *longifolia* of France and Italy, and the *latifolia* of Spain. The olives are gathered as soon as they are ripe, and this takes place early in November. In France, where the best oil is prepared, the fruit is bruised in a mill directly it is gathered; it is then wrapped in a sort of matting, and submitted to pressure. The oil which runs out is called virgin oil, and is kept separate for table and dietetical purposes. The cake is removed from the press. broken up by hand, moistened with boiling-water, and re-pressed; in this manner a second quality of oil is obtained, which, on standing, separates from the water with which it is mixed. The cake that is left from this operation is called *grignon*, and, generally, it is set aside to dry, in order that it may be used for fuel; but sometimes it is submitted to fermentation, then wetted with boiling-water, and pressed a third time, by which means a third quality of oil, called *gorgon*, is procured, which is used for lamps and machinery.

In Spain the olives are allowed to ferment for a period of ten days or a fortnight before they are crushed and pressed. In this way a larger supply of oil is obtained, but the quality is very inferior to that prepared from the fresh nut. One of the reasons why this delay takes place, is, that there are not sufficient presses in the oil districts to perform the necessary work; and hence the several growers are obliged to wait their turn, and keep their olives ready for the mills. But, within the last few years, considerable improvement has been effected in this respect by the introduction of hydraulic presses; and now a large portion of the oil obtained from Spain is equal, or nearly equal, in quality to that of France and Italy.

The machinery employed by the Neapolitan peasants in the preparation of Gallipoli oil, is of the rudest kind. The olives are allowed to ripen to the fullest extent on the trees; and, as they fall off, they are collected by women and children, and carried to the mill. The oil which is expressed is put into sheep or goat-skin bags, and conveyed on the backs of mules to Gallipoli, where it is allowed to clarify by standing in cisterns which are cut out of the rock on which the town is built. When it has become sufficiently clear by the deposition of mucilage, water, and other impurities, it is run off into skins, and conveyed to oil-basins which are situated near to the sea-shore. From these it is put into casks, and exported.

According to Sieuve, olives furnish about thirty-two per cent. of oil—twenty-one of which come from the pulp (or pericarp) of the fruit, four from the seed, and seven from the woody matter.

In whatever way the oil is obtained, it must be clarified and freed from mucilage, &c., before it is fit for use. This is usually accomplished by allowing the oil to stand in a warm place for a fortnight or three weeks, during which time it deposits impurities, and becomes clear. It may also be refined by heating it for a short time with a weak solution of potash or soda, and then allowing it to stand; or it may be deprived of its acid congealable matter, by exposing it for some time to the action of a piece of lead—the bottle containing the oil and the lead being placed in a window, or other place where it will receive the direct rays of the sun. In this way the oil used by watchmakers and machinists is refined. The best variety of olive oil is called Florence oil, which is the produce of Aix, in France; while the worst is the Spanish.

In this country the price of olive oil renders it too costly for lamps; but in Italy, Spain, and France, it is extensively employed for such purposes. It burns with a clear white light, and does not emit any unpleasant odour; besides which, it is not a drying oil, and is therefore not likely to clog the wick. When it is adulterated with poppy, nut, or sessama oils, its properties are very much deteriorated.

(*b.*) *Almond oil* is extracted from the kernels of the common almond (*amygdalus communis*), of which, as in the case of the olive, there are two varieties; namely, the sweet (*dulcis*) and bitter (*amara*), both of which yield the oil of commerce. The almonds are agitated in bags, so as to separate a portion of their brown skin, then crushed in a mill, and, after being folded in canvas bags, they are subjected to pressure between cast-iron plates. That which runs over first is the best. The residue is then heated and again pressed, by which means an oil of inferior quality is procured. When first obtained, the oil is thick and discoloured; but by repose in a warm place, or by filtration through paper or sand, it becomes clear. Almonds yield from twenty-two to twenty-four per cent. of oil. It is too expensive for lamps, though its flame is very brilliant, and its other qualities are remarkably good.

(*c.*) *Rape oil* is extracted from the seeds of several species of brassica (the cabbage and turnip tribe), as the *Brassica oleracea*, *campestris*, *napus*, &c.; all of which are cultivated in this and other countries for the oil which they yield. In France a very superior rape oil, termed *colza oil*, is obtained from a variety of *Brassica campestris*, named *oleifera*. In every case the oil is procured by crushing or grinding, and pressing the seed in the way already described, and it is refined by the addition of one or two per cent. of sulphuric acid. The seeds yield from twenty-eight to thirty per cent. of oil. Most of the rape oil employed in this country is expressed here. Occasionally we receive small shipments of the oil from Belgium, France, and the East Indies; but by far the larger proportion is obtained from the seed itself, of which we import large quantities. Mr. Brotherton, who is a large oil-presser, states, that good English-grown rape yields the best kind of oil; and he recommends this fact to the notice of agriculturists, saying that as much as five quarters of seed, worth fifty shillings a quarter, may be obtained from an acre of land.

Rape oil is extensively used for illuminating purposes, both here and on the continent: in fact, it is now the usual lamp-oil of commerce. Its properties are but little inferior to those of sperm; and the only objection that can be urged against it is, that after it has once been heated in the lamp it is apt to thicken and to clog the wick. Colza oil is generally consumed in the Carcel or French lamp; but it has no very great advantages over the commoner kinds of rape oil. The crude or raw oil is not suited for such purposes, on account of the mucilage which it contains: it is of a greenish-brown colour, and has somewhat the odour of linseed oil; but when it is refined it loses both of these objectionable properties, and becomes as pale and limpid as sperm.

(*d.*) *Cocoa-nut oil* is the produce of the *Cocos nucifera*, or common cocoa-nut palm; the fruit of which is decorticated, crushed, heated, and pressed. We have already stated that the oil is imported into this country in a buttery or tallow-like condition; and that after submitting it to pressure between warm plates, the liquid oil runs out, leaving the cocinine or cocoa-nut stearine for the manufacture of candles. The elaine or oleine of *palm oil* may be obtained in a similar way; and both of the oils may be purified by means of common sulphuric acid. Cocoa-nut oil has rather a pleasant odour, and it burns exceedingly well in lamps, provided it is not exposed to too low a temperature; for it is apt to congeal by cold.

(*e.*) *Sessamum or Gingilie oil* is procured from the *Sessamum orientale*, of which there are several varieties cultivated in India for the oil which they yield. These are the *suffed-til*, or white-seeded variety; the *kala-til*, or party-coloured; and the *tillee*, or black. It is from the latter that the oil is chiefly obtained. A large quantity of the oil and seed is imported into this country, and into France for the purpose of adulterating other oils; but in India it is used very extensively as an article of diet, and also for lamps. The oil is extracted and refined in the usual way.

(*f.*) *Ground-nut oil* is obtained in large quantity from the ground-nut or seed of the Bhoe moong (*Arachis hypogæa*), a plant that is pretty extensively cultivated in various parts of India. The seeds furnish about forty-four per cent. of a clear pale-yellow oil, which is largely used as food and for lamps. Two varieties of the plant are cultivated in Malacca—namely, the white seed and the brown, both of which yield a very good oil. About eighty or ninety tons of this oil are imported into this country every year.

(*g.*) *Common nut oil* is derived from two sources—namely, the *Corylus avellana*, or common hazel-nut; and the *Juglans regia*, or common walnut. The former produces about half its weight of oil, and the latter about one-third. The oil is not much used for lamps on account of its energetic drying properties: indeed, it is more apt to resinify and clog the wick than linseed; but it is largely employed for adulterating other oils.

(*h.*) *Poppy oil* is procured from the seeds of several species of poppy, as *Papaver somniferum*, *bracteatum*, *orientale*, &c. The plant that yields the largest amount of oil, and which is usually cultivated for this material, is a variety of the *somniferum*, named *nigrum*, from the black colour of the seeds. Large quantities of this oil are expressed every year for the purpose of adulterating other oils. It is clear, sweet, limpid, and almost colourless; but the great objection to its use as a lamp-oil, is its disposition to dry.

(*i.*) *Linseed oil* is extracted from the seeds of the flax plant (*Linum usitatissimum*), which yield from twenty-two to twenty-seven per cent. of oil. If the seeds be crushed and pressed at an ordinary temperature, they yield not more than eighteen or twenty per cent.; but the oil is of a pale colour, and is thought by some to be of superior quality to the hot-drawn. A large quantity of linseed is cultivated in this country, but the great bulk of the seed used by the oil-presser is imported. The seed is crushed, ground, and pressed in the usual way; and the oil is refined by means of dilute sulphuric acid. Linseed oil is not usually burnt in lamps, on account of its drying properties; but if the wick be changed every day, and no more oil is placed in the lamp than is necessary for one night's consumption, it will be found to burn very well, and to give a very clear light. By boiling or heating, it acquires increased consistence, and is then more apt to dry.

(*k.*) *Hemp-seed oil.*—This oil is produced from the Indian hemp (*Cannabis sativa*), the seeds of which yield about one-third their weight of oil. The oil has a disagreeable smell, and is not much employed for illuminating purposes, except by the poorer classes of India.

(*l.*) *Cameline or Dodder oil* is extracted from the seeds of the *Camelina sativa*, a plant that grows abundantly in Canada, where the oil is used as a common lamp-oil. It is also employed for the same purpose in several parts of Germany.

(*m.*) *Cotton-seed oil* is thought to be as good as rape for lighting purposes: indeed, small quantities of the oil have been expressed for several years past, and used in this way; but the value of the material has not been fully realised until within the last few years. At the Exhibition of 1851, specimens of the oil and cake were shown by Mr. Burn of Edinburgh, and by M. De Géminy of Marseilles, to both of whom prize-medals were awarded. It appears that, as early as 1785, the importance of this material was perceived by the Society for the Encouragement of Arts and Commerce, for they offered a prize for its manufacture on a large scale; but it does not seem to have been taken up extensively, perhaps because of the difficulty in purifying the oil. It has, however, been extracted for some time in India, America, and Egypt. Of late years the oil has attracted attention, and means have been devised for its purification. This is of importance, for very large quantities of cotton-seed are destroyed every year: in fact, more seed is always produced than is required for the next year's crop, and hitherto this excess has been thrown away as useless. At present it is exported to this country, or to France, where it is crushed and pressed. Mr. Burn of Edinburgh, and M. De Géminy of Marseilles, have each large mills for the expression and purification of the oil. When first expressed it has a dirty-brown colour; but, by rectification with sulphuric acid, it becomes clear, and assumes a pale amber tint, in which condition it is well suited for combustion in lamps. The botanical name of the plant which furnishes the seed is *Gossypium herbaceum*. It is cultivated in India, Syria, Asia Minor, along the Mediterranean, and in America.

(*n.*) *Mustard oil.*—This is procured from the dross or siftings of black and white mustard-seed (*Sinapis alba* and *Sinapis nigra*), both of which are cultivated very extensively in this and other countries, for the manufacture of mustard-flour. The siftings furnish about forty per cent. of a dark-brown oil; the seeds themselves yield from eighteen to thirty-six per cent. In India an excellent oil, called *shersha*, is expressed from several species of *sinapis*, as the *toria*, *glauca*, *nigra*, &c. All these varieties of mustard oil are very dark-coloured when first expressed, and they have the peculiar pungent odour of mustard. Both of these properties are, however, removed by the process of refining; and then the oil may be used in the place of rape or colza for illuminating purposes. Usually the oil is employed for the adulteration of the latter.

(*o.*) Besides these oils, many others are employed in various parts of the world, for the purpose of giving light; thus, the oil of plum-stones (*Prunus domestica*), and of raisin-stones, or wine-stones (*Vitis vinifera*), are used in Spain, Germany, and France. The oils of belladonna-seed (*Atropa belladonna*), tobacco-seed (*Nicotiana tabacum et rusticum*), and henbane-seed (*Hyoscyamus niger*), are used in Swabia and Würtemberg. Oils are also extracted from the beech-nut (*Fagus sylvatica*), sunflower-seed (*Helianthus annuus*), weld-seed (*Reseda luteola*), orange seed (*Citrus aurantium*), cucumber-seed (*Cucurbita pepo*), &c.; and in India there are numerous plants, which yield abundance of oil that is well suited for illuminating purposes. Among these may be mentioned *ramtil oil*, or, as it is sometimes named, *teal oil*, from several varieties of *Gurzotia*, as *Gurzotia oleifera* and *Abyssinica*, both of which yield about eighty-four per cent. of oil that is very similar to sessamum oil; *Poon-seed oil*, or *Pinnacottay oil*, from the seeds of *Calophyllum inophyllum*, which furnish about sixty per cent. of it; *Napala oil*, from the seeds of *Jatropha curcas; Mulu unnay oil*, from the seeds of *Argemone Mexicana; Cheeroojee oil*, from the fruit of *Chirongia sapida*, or *Buchanania latifolia*; oil of *Kossumba*, or *Koosm oil*, from the seeds of the safflower (*Carthamus tinctorius*), which yield about twenty-eight per cent. of it; *Kanagu nune*, or *Kurrunj oil*, from the seeds of *Pongamia glabra*, or *Galedupa arborea; Mooncela oil*, from the seeds of *Dolichos biflorus* (?); *Caju apple oil*, from the seeds of *Anacardium occidentale; Lambolee oil*, from the seeds of *Bergera koenigii; common jungle oil*, from the seeds of *Ricinus communis;* and several other varieties, the sources of which are not well known. Many of these oils are admirably well suited for combustion in lamps; and, if there were a sufficient demand for them, they might be furnished to commerce in considerable quantity. "The knowledge of this circumstance," say the jurors, in their report on the products of the Great Exhibition, "is of great practical value, because, not only is it possible that by the introduction of improved machinery, or by increased facilities of conveyance, their price may be reduced; but the very existence of such substances tends to equalise the market value of those oils now generally employed. And should, at any time, accidental circumstances cause the price of the latter to advance, these substances would then be most advantageously introduced, and would, probably, ere long, altogether supersede the oils in the place of which they had been originally imported."

Volatile Oils.—(*a.*) *Oil of turpentine, or camphine*, may be obtained from the oleo resinous exudation of various species of pine, larch, &c., as the Scotch fir (*Pinus sylvestris*), the cluster pine of Bordeaux (*Pinus pinaster*), the swamp pine of America (*Pinus palustris*), the frankincense pine of Virginia (*Pinus tæda*), the silver fir of Germany, Siberia, and Switzerland (*Abies picea*), the common larch of the continent (*Larix Europæa*), and the turpentine pistacia of Syria and Greece (*Pistacia terebinthus*). The oleo-resin is imported into this country under the names of common turpentine, Bordeaux turpentine, Strasburg turpentine, and Venice turpentine. The first of these yields the great bulk of the turpentine of commerce; it is the produce of the *Pinus palustris*, and perhaps, also, of the *Pinus tæda*, and is chiefly imported from the United States of America. The method which is generally adopted for procuring this oleo-resin is as follows:—The tree is selected, and a hollow is cut into it, a few inches from the ground; the bark is then removed for a space of eighteen or twenty inches above the hollow; and for several months—namely, from March to October—the turpentine flows from the divided sap-vessels into the excavation. At convenient times the semi-fluid matter is scooped out, and put into casks; and, when these are full, they are sent away for exportation.

Volatile oil of turpentine is procured from the oleo-resin by distilling the latter with a due proportion of water. The turpentine and water come over together, forming a milky liquor, which, on standing, separates into two layers, of which the turpentine is the uppermost. These are easily decanted or poured off one from the other; that which remains in the still is resin. Common American turpentine yields from fourteen to sixteen per cent. of spirits.

The turpentine which is thus obtained is not sufficiently pure for combustion in the camphine lamp, for it contains a small proportion of resin, which is very apt to clog the wick. This impurity is easily removed by a second distillation, and the product which is thus obtained is sent into commerce under the name of *camphine*. It is a colourless, limpid, and very inflammable liquid, that burns with a remarkably sooty flame. Its specific gravity is 870, and it boils at a temperature of 314° Fah.; though, if water be present, it will distil at as low a temperature as 212°. Turpentine freely absorbs oxygen from the air, and is converted into an oleo-resin. In the course of four months it will take in about twenty times its bulk of atmospheric oxygen. The change which is thus produced in the liquid, renders it unfit for combustion in the camphine lamp, in consequence of the resinous oxide having a tendency to clog the wick. To remedy this evil, the liquid must be redistilled, and the camphine should be preserved in well-corked vessels.

The light that is emitted from turpentine when it is properly burnt, is remarkably vivid and white; in fact, the illuminating power of camphine is nearly twice as great as that of sperm oil; and if it were not for the liability of the combustible to evolve smoke, it would be one of the most valuable of all illuminating agents. This, indeed, is the great objection to its use; for it is found that the slightest mismanagement of the flame causes the production of a cloud of blacks, which settle upon the furniture and dress, and damage them irreparably. To obviate this as far as possible, the chimney of the camphine lamp is made very tall, and thus a strong current of atmospheric air is secured to the flame.

A mixture of turpentine and alcohol has been used in France for some time past, under the name of "*Eclairage au Gaz Liquide.*" The lamp which is employed for the combustion of this material, was originally contrived by Lüdersdorff; it is called a vapour-lamp, because it is constructed so as to convert the volatile liquid into vapour, which burns as it escapes through a number of fine orifices. By diluting the turpentine with alcohol, its liability to smoke is considerably diminished; but still there is a large quantity of soot evolved when the mixture is burnt in an open lamp without a glass. The French liquid has a very peculiar odour; it is clear and limpid like water, and has a density of 823. Its boiling point is 190° Fah. When mixed with water, it becomes turbid and milky from the separation of the turpentine, which soon floats to the surface and forms an oily layer, the bulk of which is about half that of the original liquid. From this it would appear that it consists of about equal parts of strong alcohol and turpentine, the mixture being doubtless effected by distilling the two liquids together; for if alcohol and turpentine are merely shaken up together, they will not unite in this proportion: indeed, 100 parts of spirits of wine, of specific gravity 840, will only take up 13½ of turpentine; and alcohol of much less density (830) will not take up more than 20 per cent. of it.

When the French liquid is burnt in an ordinary open lamp, at the rate of 138 grains per hour, it gives a light which is about two-thirds as great as that of a standard sperm candle.

The great objection to the use of this liquid is its liability to explode when its vapour becomes mixed with atmospheric air. In consequence of this property, the greatest caution is necessary in manipulating with the fluid; for should an explosion take place, the most dangerous results might follow. We are not likely to employ the mixture in this country, on account of the high price of spirits of wine; but in France and Germany, where alcohol is comparatively cheap, the fluid is often used as an illuminating agent at railway stations in country towns.

(*b.*) *Coal-naphtha.*—When coal is distilled for the manufacture of gas, a tar is obtained which is the source of common naphtha. The tar itself is a very complex material, for it contains a number of oily acids, alkalies, and neutral bodies. As it comes from the gas-works, it is a thick, dark liquid, of a most offensive odour. To extract from it its various constituents, it is put into large iron retorts or stills, and submitted to distillation; that which comes over first is of an aqueous nature, and smells strongly of ammonia; then follows a brownish oil, which floats on the preceding. After a time, a denser or heavier oil begins to make its appearance; and when this happens, the receiver is changed, and the first product is set aside for the manufacture of light oil or crude naphtha. The coal-tar generally yields from four to five per cent. of this fluid. As the distillation of the tar proceeds, a heavy oil, which falls to the bottom of water, and is hence termed dead oil, comes over. This is used, under the name of creosote, for the preservation of timber. After this a yellowish semi-crystalline fat, called naphthaline, makes its appearance; and, finally, a more solid material, named paranaphthaline, distils over. That which remains in the retort is pitch.

The crude coal-naphtha is rectified either by distilling it a second time, or by driving steam through it and collecting the condensed products. In this way it is separated from another portion of heavy oil which remains in the still.

The light naphtha thus obtained is sent into commerce, and sold for about 2*s.* 4*d.* per gallon, for combustion in the common vapour-lamps which are so frequently to be seen in the streets of London, lighting up the stalls of the poor tradesmen. In this condition it is an amber-coloured liquid, of a powerful gas-like odour, and spirituous appearance. It has a density of from 860 to 900—usually it is about 887. It floats on water, like turpentine, and becomes darker coloured by exposure to the air. It mixes very freely with wood-spirit or with spirits of wine; and may thus be burnt like the last-named liquid, in an ordinary lamp.

The light naphtha is further purified for commerce by agitating it with a little oil of vitriol, then washing with water, and redistilling. In this condition it is sold as rectified naphtha, and is used for combustion in the naphtha spirit-lamps which have a flat wick and oval glass.

Mr. Mansfield has shown that light coal-naphtha contains a number of volatile oils, which may be separated from it by fractional distillation. One of these—namely, benzole—is of great importance. It is procured by boiling the naphtha in a retort, to which there is adapted a worm which coils through a vessel of boiling water; the worm is so constructed that all the vapour which condenses in it shall run back again into the still, while the uncondensed vapour (that of benzole) passes on into another receiver, where it is cooled and collected. The benzole thus obtained is rectified a second time in a similar apparatus, the temperature of the worm being kept at about 176° Fah. In this way a large proportion of volatile oil is obtained, which is further purified by agitating it with one-fourth its bulk of strong sulphuric acid, or, better still, with about one-tenth of strong nitric acid; and then, after separating the nitric acid, it is agitated with oil of vitriol as before. The naphtha is now to be decanted and distilled a third time. If it be required to have the benzole perfectly pure, it is submitted to a cold of 4° Fah. This is produced by mixing salt and snow together. The benzole freezes, and leaves its impurities in a fluid condition, from which it may be separated by means of a filter.

The use of sulphuric acid in this process is to remove all the basic substances, and to oxydise the brown colouring matter of the naphtha; the nitric acid assists the oxydation, and at the same time forms a small quantity of nitro-benzole, which gives a fragrant, almond-like odour to the product.

Benzole is a limpid colourless liquid, of a rather ethereal odour; its density is 850—consequently it floats on water. It boils at a temperature of 177° Fah., and gives off a vapour which is very inflammable, burning with a sooty flame. It solidifies at the freezing-point of water, and then looks like camphor. So volatile and combustible is the liquid, that when a current of hydrogen gas is passed through the fluid, or through a sponge moistened with it, the gas will burn with an intensely white light. Atmospheric air charged with the vapour also burns with a smoky flame and a bright light: the flame is sometimes of a violet-blue colour when the apertures of the jet are very small.

Benzole mixes freely with alcohol or with wood-spirit, and the compound so formed burns in common lamps with a very powerful light. It is necessary that the mixture should be made with proper proportions, or else the light of the flame is not good; for if there be too much spirit the light is blue, and if too little it is smoky. The mixture which is found to give the best results, is about one part benzole and two of spirit, of specific gravity 840. This mixture, when burning at the rate of 160 grains an hour, gives a light of from one-and-a-half to two sperm candles.

The extreme volatility of benzole gives to coal-naphtha the property of naphthalising air or bad gas; for if a little of the liquid be placed in the gas-meter, or in a chamber containing some pieces of sponge through which the gas passes, it will acquire increased illuminating powers. Mr. Lowe, of the Chartered Gas Company of London, has taken out a patent for this mode of naphthalising gas. Beale's lamp is also a contrivance for naphthalising atmospheric air. It consists of a cup of naphtha, through which a stream of air is made to pass; and to facilitate the volatilisation of the naphtha, a hot cap is placed over the cup, so as to communicate its heat to the air and liquid. The other constituents of coal-naphtha are not so volatile as benzole, and hence they are not fit for the purpose of naphthalising. Mansfield states, that the oil which distils over from the crude naphtha at a temperature of 230°, will take fire at its surface; but it yields so little vapour to cold air, that the latter, when passed through it, burns with but a feeble blue flame; and the oil which distils at a temperature of 300° is still less inflammable, for it will not take fire at the surface, or furnish any combustible vapour to atmospheric air. In these respects it resembles turpentine, which requires a heat of 314° Fah. to boil it.

A fluid like coal-naphtha is also obtained from the distillation of certain oily matters, or petroleums, which exude from the earth. In many places in the neighbourhood of the Caspian Sea, in Ava, at the Tegernsee in Bavaria, at Amiano in Italy, at Neufchatel, at Saint Zibio in the Grand Duchy of Modena, at Clermont and Gabian in France, at Val di Noto in Sicily, at Rangoon, Barbadoes, Trinidad, Lake Geneka in New York, and many other places, an oily matter called rock-oil oozes out of the ground, and is collected in pits dug in the earth to receive it. When distilled, it furnishes a volatile oil called naphtha, of which Persian naphtha may be taken as a good example. It is colourless, limpid, very combustible, and burns with a sooty flame. In some places it is used for illuminating purposes.

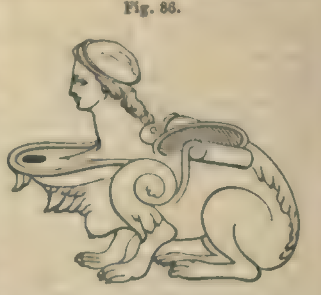
(*c.*) *The Oil of Fermented Liquor, Oil of Grain, or Fusel-oil.*—In the process of fermentation, all the saccharine fluids produce a volatile oil, which can be separated from the spirit by distillation. Pellitan, in 1825, first noticed this fact; and as he obtained the oil from spirit of potatoes, he called it potato-spirit oil. It was subsequently examined by Dumas (1834); and in 1839 it was investigated by Cahours. More recently Buchner obtained it from corn-spirit. For a long time it was obtained as a waste product by Mr. Bowerbank, a rectifier of London, who used it in his manufactory as an illuminating agent. The oil is procured at the end of the process for rectifying spirit. In its raw and impure condition it is usually called *faints*. It has the odour of bad whisky, and contains alcohol, and various fatty acids. By washing with water, then distilling with carbonate of potash, and, finally, with chloride of calcium, it is obtained in a tolerably pure condition. Five hundred gallons of corn-spirit yield about one gallon of oil. In its pure state, the oil has a peculiar ethereal odour, which, when inhaled, is rather unpleasant and irritating to the throat. Its specific gravity varies from 823 to 840; it boils at 268°, and emits a combustible vapour. The oil is not soluble in water, but it mixes with alcohol and wood-spirit in all proportions. It burns in an ordinary open lamp, with a clear and smokeless flame—the light of which is tolerably intense. A lamp that consumed the oil at the rate of 278 grains per hour, gave a light that was about half as good again as that of a standard sperm candle. At present the oil is in demand for the manufacture of artificial essences, and consequently it is too expensive for combustion in lamps.

LAMPS.

History and General Principles of the Subject.—It has been already stated that the employment of lamps can be dated back to a very early period; indeed, it is generally thought that they were invented by the Egyptians, who not only used them for common illuminating purposes, but also placed them in the tombs of the dead as emblems of mortality. The ancient Greeks were likewise accustomed to the use of lamps, which we have every reason to believe were fed with a vegetable oil. Herodotus alludes to this fact; and it is further evidenced in many devices that we find sculptured in some of the most ancient Greek vases; but it was centuries after that before the Romans began to employ lamps, and then they were only used in the houses of the rich, or upon occasions of special festivity. That most of the classical nations have been accustomed to place lamps in the sepulchres of the dead is an instructive fact; for, although various motives have been assigned for the custom, there can be no doubt that it was intimately connected with their belief in the existence of a soul, and that it was meant to typify the departure of the spirit or vital fire from its frail tenement of clay. This is clearly set forth in many of the beautiful devices which adorn the funereal lamps of the early Greeks, where the immortality of the soul, and its departure from the body, is represented by the escape of a butterfly from an apparently dead chrysalis. The testimony of Pliny, St. Augustine, and others, has induced a belief, that in many cases the sepulchral lamps were constructed so as to burn for ever; and some remarkable instances have been cited in which the lamps were said to have been found burning centuries after the tomb had been closed up; but none of these are sufficiently well authenticated, notwithstanding that Liceto and other authors have taken great pains to establish their truthfulness.

It is very probable that the earliest lamps were not made of any set form, but that the fat or oil was placed in any convenient vessel, and burned by means of a bundle of rushes or dried moss. As civilisation advanced, and the necessity for artificial light increased, attention would naturally be directed to the form best suited to the wants of the people: and it is very likely that at first the lamp was nothing more than a circular vessel or saucer containing the combustible material. Lamps of this description are still employed on the continent for purposes of general illumination. It is thought by some persons that the lamps of the virgins alluded to in the Gospel of St. Matthew (chap. xxv.) were merely rods of porcelain or iron covered with cloth, and steeped in oil or fat, and that the same kind of lamp or torch was used by the soldiers of Gideon; but we have no positive testimony in support of such an opinion, although there is plenty of evidence to show that lamps trimmed with oil were in use long before that time.

Fig. 86.



The next improvement in the form and construction of the lamp is to be seen in the ancient lamps of Herculaneum and Pompeii. Examples of these are to be found at the Louvre, the British Museum, the Vatican, and, indeed, in almost every considerable museum in Europe; but the finest specimens belonged to the king of Naples, who had a collection of such things at Portici; in fact, in that museum there is a large collection of lamps taken from the buried cities of Pompeii and Herculaneum. Besides which, Liceto, Bartoli, and Passeri have published descriptions and drawings of many hundreds of such lamps that were in the museums of Italy during the 16th century. The common form of all of them is that of an elongated vessel, like a boat, having the wick at one end (Figs. 86, 87, and 88); at other times it was a simple disc, with a hole for the wick on one or both sides, and an aperture in the centre for supplying the oil (Fig. 89). Lamps of the former description are still used by the poor of the Orkney and Shetland Isles.

Fig. 87.

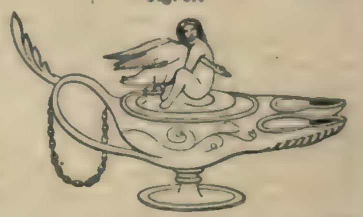
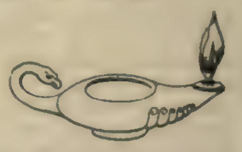
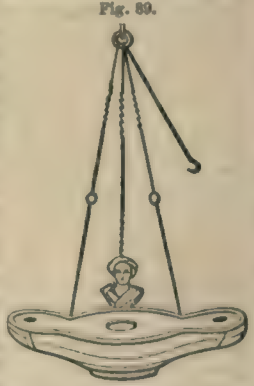


Fig. 88.



The material of which the Greek and Roman lamps were composed, was chiefly *terra-cotta*, though some of the better sort were made of bronze, and even of silver and gold. A few ancient lamps of iron have also been discovered, but they are comparatively rare, perhaps because of the perishable nature of the metal. In the museum at Portici there are several iron lamps, together with one of glass, all of which were taken from the ruins of Herculaneum.

Fig. 89.



Much ingenuity was exhibited by the Greeks and Romans in the construction and ornamentation of the supports (the λυκνουκοι, *lampadaria* or *candelabra*) which held the lamps. At first these were composed of cane; and the Greeks, in all their designs, never lost sight of this fact; consequently we find a variety of vegetable forms blended with the stiff reed, giving it lightness and elegance; the twining ivy and the graceful acanthus were frequently associated in this manner. With the Romans, however, the usual support was a tripod resting on lions' feet.

At a very early period it must have been observed, that when an attempt was made to enlarge the wick beyond a certain point, the flame became dull and smoky. To remedy this, it was customary to split the wick up into a number of small flames, each of which would allow the atmosphere to play freely around it, and thus to keep up a tolerably good combustion. There is no doubt that this device was well known to the ancients (Fig. 88), though it is generally considered, that Dr. Franklin was the first to show, that the same quantity of cotton divided into two wicks, gives a better light than when it is used as one; but the difficulty which presents itself in the employment of such a contrivance, is that of keeping the wicks equally well trimmed. To obviate this, the flat wick was invented; and thus a thin sheet of flame was produced, which allowed of much more perfect combustion. Nevertheless, with all these improvements, the light of the lamp could not be increased beyond a certain point, without its becoming dull and the wick sooty; and consequently, for many years the lamp was rarely used in the apartments of the rich, for fear of its doing harm to the walls and furniture by its great tendency to smoke.

In 1780, however, as we have already said, a great change was effected in the art of illumination by the discovery of M. Argand of Geneva, who found, that whenever a due supply of atmospheric air was furnished to the inside as well as the outside of a flame, the combustion of the oil might be kept up to any amount without danger from smoke or bad smell. The plan which he adopted for accomplishing this was very simple. He made the wick circular, and placed a glass around it so as to secure a strong current of atmospheric air to both sides of the flame. We shall have occasion to describe the details of his contrivance when we come to the subject of the various Argand lamps at present in use. All subsequent improvements on this invention of M. Argand, have been with the view of adjusting the wick, of regulating the supply of oil, of doing away with the shadow cast by the reservoir, and of directing the current of atmospheric air into the body of the flame. The first object was accomplished by means of a rack and pinion, or by a point working in the thread of a screw; the second by various forms of fountains, syphons, and, in the Carcel lamp, by means of a piston which is worked by machinery. The wick has also been modified in its construction so as to suit the condition of the oil; it is made of fine web when the oil is very fluid, and of coarse web when it is thick. The third object has been accomplished by giving the reservoir an annular form—this was Count Rumford's invention—or by placing it above the level of the flame, as in Parker's lamps; and the fourth, by contracting the glass immediately around the wick, or by putting a metallic disc into the centre of the flame, as in the Vesta lamp; or by dropping a brass nipple over it, as is the case with the solar lamp. All these contrivances increase the supply of air; and, by breaking its course, they cause it to impinge upon the body of the flame.

For the combustion of volatile oils and naphtha, lamps of very different construction are required; especially in those cases where the oil or spirit is to be consumed in a gaseous form without the aid of a wick. We shall describe these lamps, which are of comparatively modern invention, when we speak of Beale's, Holladay's, and Lüdersdorff's lamps.

With all lamps, however, the great principle that is to be kept in view is, so to adjust the supply of atmospheric air to the combustible, that, on the one hand, the flame shall not evolve smoke, and, on the other, it shall not be cooled or over-burnt; for, in the one case there is too little atmospheric air, and in the other there is too much.

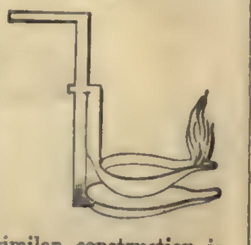
Management of Lamps.—Lamps will not burn in a satisfactory manner if they are not kept clean and well trimmed; for, in the first place, nearly all the oils which are made use of at the present time are, more or less, drying in their nature, and consequently they are apt to become thick in the lamp, and to clog its several apertures. Whenever this occurs, the lamp ceases to burn; and there is no help for it but a good cleaning. This is accomplished by draining out all the oil as completely as possible, then charging it with a strong solution of soda or pearlash, which combines with the oil and forms soap. This solution ought to remain in the lamp for twenty-four hours, and it should be frequently agitated; but care should be taken not to spill any of the liquor over the paint or lacquering of the lamp, for fear of dissolving it off. After it has stood in this way for the necessary time, the soapy liquid may be run out, then washed clean away with warm water; and, finally, the lamp is to be well dried. Some of the very common oils are so liable to resinify, that it is necessary to change the wick every day: this is the case with the oils that are used in Parker's hot-oil lamp, and consequently it is trimmed with a very short wick. In the second place, the wick should be properly attended to: if a solid wick is used it should not be twisted too tight, for fear of stopping the capillarity of the oil; nor should it be too loose, for then it is apt to accumulate soot. If the Argand, or hollow wick, be employed, it should be selected with due regard to the quality of the oil; for a thick or fatty oil requires a coarse texture, and a very fluid oil a fine one. The top, or carbonised portion of the wick, should always be removed immediately before the lamp is lighted; for this is so changed by the action of heat, that the oil will not rise in it; indeed, the common oils are so disposed to char and clog this portion of the wick, that it sometimes requires removal several times in the course of an evening. The wick should be cut perfectly level, or the flame will be irregular, and will smoke. Lastly, in cold weather it is advisable to warm the oil before the lamp is lighted.

The relative Illuminating Power and Economy of different Lamps.—This is a subject which has not been well investigated; for the difficulties connected with it are extremely great. At the Exhibition of 1851, there were forty-nine lamps sent for examination; but the jurors declared that it was a matter of impossibility to test their value. M. Peclet is nearly the only person who has devoted attention to this subject; and the following table is constructed in great part from his investigations:—

Lamps.	Consumption per hour.	Luminosity in sperm candles of 120 grains.	Relative power for equal weight.
1. Vesta lamp without button (camphine)	140 grains	2·6	185
2. Vesta lamp without button (with coal-naphtha)	136 ,,	2·0	147
3. Common Argand . .	350 ,,	4·0	114
4. Carcel lamp	630 ,,	7·0	111
5. Sinumbra, with lateral fountain	270 ,,	2·9	109
6. Thilorier's or Parker's lamp	767 ,,	7·6	99
7. Sinumbra, with fountain above	645 ,,	6·3	98
8. Common sinumbra . .	645 ,,	6·0	91
9. Gerard's hydrostatic .	521 ,,	4·5	86
10. Common open lamp . .	103 ,,	0·8	77
11. Eclairage au gaz . . .	343 ,,	2·1	61
12. Fountain lamp, with flat wick	165 ,,	0·9	54
13. Dome Argand . . .	400 ,,	2·1	52

Varieties of Lamps.—These are so exceedingly numerous, that it is not possible, in a work like the present, to give anything more than a very general account of the most important. Indeed, there are but few really distinct principles involved in the construction of lamps, notwithstanding that there are so many modifications in their form and arrangement; we shall have no difficulty, therefore, in understanding the construction of any lamp, after we have become acquainted with the following varieties. It is proper to add, that we are indebted to the *Encyclopædia of Domestic Economy* for many of the illustrations which we are about to offer, and that the reader will therein find a very good description of the lamps now in use.

Fig. 90.



1. *Common Oil Lamp, without any Glass.*—Of these there are several:—

(*a.*) *The Common Lamp of the Shetland and Orkney Islands* (Fig. 90).—This is contrived for the combustion of common fish-oil, and the wick that is used is nothing more than a bundle of dried rushes. A lamp of similar construction is used by the Esquimaux, who employ a wick of dry moss.

(*b.*) *The Common Street Lamp* (Fig. 91) was once to be seen at the stalls of poor tradesmen; though it is now almost entirely displaced by the common naphtha lamp. It is a tin vessel, with one or two spouts emerging from the sides close to the bottom. These are packed pretty tightly with cotton wick; and being constructed on the principle of the bird-fountain, the oil flows very freely to the top of the wick, but does not run over.

Fig. 91

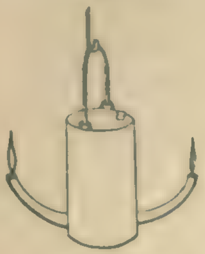
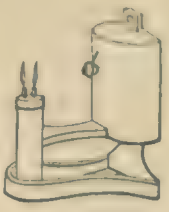


Fig. 92.

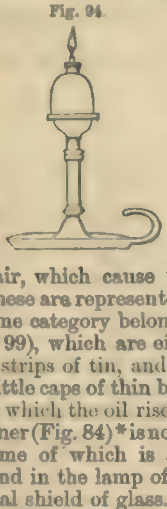


(*c.*) *The Fountain Lamp* (Fig. 92) is constructed on nearly the same plan as the last; and, as the reservoir for the oil is above the level of the wicks, the flame burns with the same brilliancy as long as any oil remains.

(*d.*) *The Common Domestic Lamp* (Figs. 93 and 94) is made to fit a candlestick. It is a very economical lamp; though, from the circumstance that the flame is situated at a considerable distance above the level of the oil, there is some difficulty in getting the lamp to burn when the oil is at all thick, or when the wick is clogged by age.

Fig. 93.

Fig. 94.



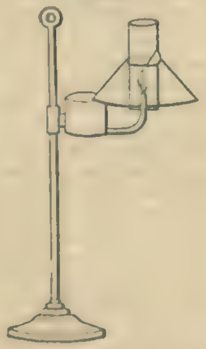
2. *Common Lamp with a Glass or Shade.*—Most of the preceding may be improved by surrounding the flame with a glass, to keep off the currents of atmospheric air, which cause the light to flicker. The principal of these are represented in Figs. 95, 96, and 97; and to the same category belong the floating night-lights (Figs. 98 and 99), which are either pieces of waxed wick supported on strips of tin, and kept floating by means of cork, or else little caps of thin brass, pierced with a hollow glass tube, in which the oil rises and burns. The safety-lamp of the miner (Fig. 84)* is nothing more than a common lamp, the flame of which is surrounded by a shield of wire gauze; and in the lamp of Upton and Roberts there is an additional shield of glass.

3. *Common Lamp, with Oxydator.*—It is found that when a current of atmospheric air is made to impinge on the flame of a common lamp, the light is much more steady and brilliant. Various contrivances have, therefore, been adopted for the purpose of effecting this; as, for example, the bending in or contracting of the glass immediately around the flame (Fig. 100); or the fixing of a metal or mica disc around it (Fig. 101); or, better still, the dropping of a perforated nipple over the flame, as is seen in Fig. 102. The last constitutes the principle of the solar lamp, to which we shall have occasion to refer by-and-by. It was, we believe, first contrived by Mr. Roberts, the miner, and was applied to his form of the improved Davy.

Fig. 95.

Fig. 96.

Fig. 97.



It will be evident, on inspecting the different lamps we have yet described, that the air supplied has access only to the exterior of the flame. Hence the interior of the flame is necessarily cooler than the exterior, and therefore tends to diminish the intensity of the whole light produced. This effect may be well noticed in a common tallow candle, when the wick has been allowed to burn for a long time without being snuffed. If, however, a supply of air could be afforded in the centre, the light would be at once increased. A patent was taken out for a plan of this kind, applicable to candles; but it was never extensively adopted. It was on the same principles as those on which the following lamps are constructed.

4. *The Argand Lamp.*—This differs from all others, in the circumstance that the wick as well as the flame is hollow; and it is so contrived that a current of atmospheric air plays on both sides of the flame, and so increases its brilliancy. There are several modifications of this lamp, of which the following are the most important:—

Fig. 98.

Fig. 99.

(*a.*) *Common Argand Lamp* (Fig. 103), which consists of a vase, *a*, to hold the oil; a cistern, *b*, to supply the burner; and an arrangement, *c*, for adjusting the hollow wick, and allowing a supply of atmospheric air on both sides of the flame. The oil is put into the vase *a*, by unscrewing it from *b*, and then running in the oil through the hole *d*, or else through the aperture in the bottom, which is usually closed by the plug *f*. The hole *d* is then closed by drawing up the handle *g*, which communicates with a sliding tube *h*, and the vase is returned to its place on *b*. When the lamp is lighted, the handle *g* is to be depressed: this causes the oil to escape through the hole *d* into the cistern *b*; and from this it runs by the side tube *i i* into the conical chamber *o*, which contains the wick *n*. This chamber consists of two tubes, *k k*, *l l*, one within the other, and joined at *m m*, so as to make a closed receptacle for the oil. In this receptacle the wick *n* plays freely up and down; and it is kept constantly immersed in oil, as high as the aperture *d*, in the cistern *b*. *p* is a cup placed at the bottom of the chamber to receive any oil that may run over from the wick. Atmospheric air rises freely through *c* into the centre of the flame, and it also blows upon the exterior of it: the cause of the current of air is the glass chimney which rests on the top of the burner.

Fig. 100.

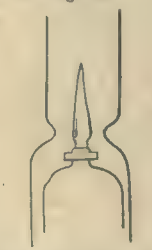


Fig. 101.

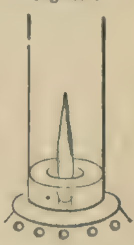


Fig. 102.

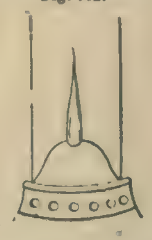
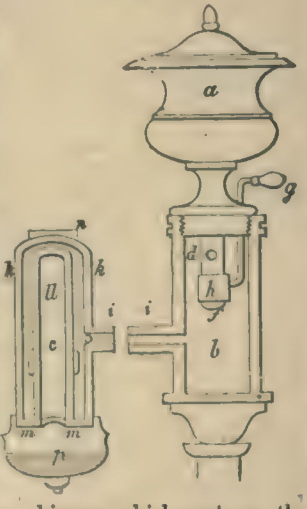


Fig. 103.



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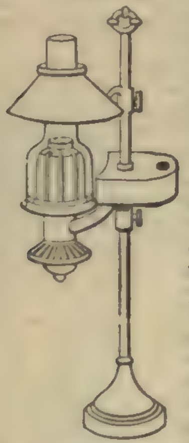
* See *ante*, p. 92.

Many forms have been given to this lamp of M. Argand, according to the purpose which it is intended to serve: thus, there is the reading or Cambridge lamp (Fig. 104), the well-known table lamp (Fig. 105), and the suspended lamp.

Fig. 104.



(*b.*) *The Rumford or Annular Table Lamp* is an improvement on the last. It was contrived by Count Rumford, for the purpose of avoiding the deep shadows which are produced by the reservoirs of the preceding; and it became so great a favourite, that even to this day it is occasionally used. The oil is contained in a hollow ring, which is placed a little below the level of the flame; and the cistern is fed by means of two tubes, which also serve as supports. In order to diffuse the light still more, the burner is surrounded by a ground-glass shade, which almost entirely conceals the ring (Fig. 106).

(*c.*) *Parker's Sinumbra Lamp* was patented in 1820. It was called the shadowless lamp, because it did away with the slight shadow which is always perceptible in the last. The improvement is threefold:—First, the annular reservoir is bevelled off from above, so as to present a very thin edge on the outside; secondly, the glass is shaped in such a manner as to diffuse the light over the edge of the ring, and under it; and, thirdly, a conical reflector is placed around the inner glass a little above the flame. All these arrangements are represented in Fig. 107, which shows the lamp in section.

Fig. 105.

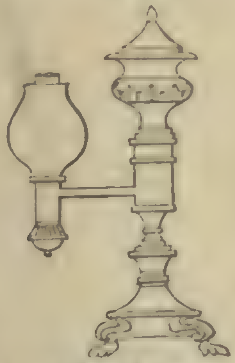


Fig. 106.

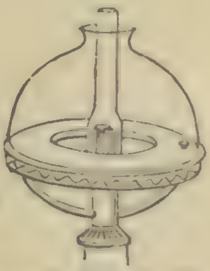
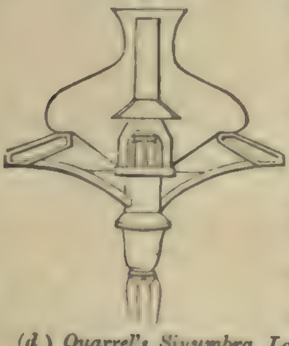
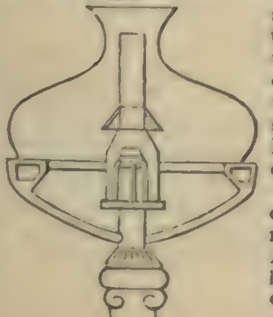


Fig. 107.



(*d.*) *Quarrel's Sinumbra Lamp* (Fig. 108) is another contrivance for effecting the same object as Parker's; but the bevelling of the reservoir is on the under surface of the ring, instead of the upper; and the ground-glass shade is made sufficiently large to reach to the very outer edge of the ring.

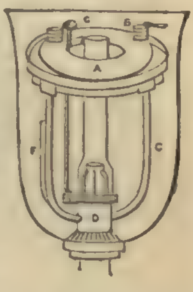
Fig. 108.



(*e.*) *The Iris Lamp* is one of the most recent improvements of the Rumford and Argand: it differs very little from the preceding, except that the outer edge of the annular chamber is reduced to a mere bead; and the ground-glass shade is not only brought to the very front of the ring, but its body is formed in such a way as to bulge over it. By these contrivances the shadow of the reservoir is reduced to a minimum.

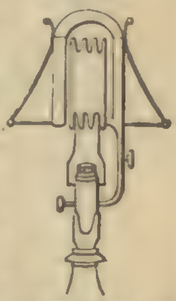
(*f.*) *Quarrel's Albion Lamp.*—This marks the next attempt to improve the Argand, by carrying the reservoir for the oil over the flame, and, therefore, out of the way of the shadow, instead of having it, as in Rumford's contrivance, around it and a little below it. The construction of this lamp is represented in Fig. 109. A is the reservoir for holding the oil, which is introduced through the two valve-cocks B; C is the tube that conducts it to the cistern D; and F is a syphon-valve for admitting atmospheric air to the reservoir, so as to supply the place of the consumed oil. G is a pinion for regulating the height of the wick. The whole is enclosed in a tulip-shaped glass; and, with the exception of the side-tubes, there is nothing to produce a shadow.

Fig. 109.



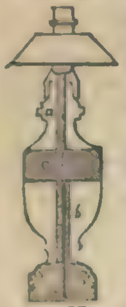
(*g.*) *Parker's Hot-oil Lamp* is constructed somewhat like the preceding, though a contrivance is adopted for making the oil hot before it reaches the wick; by which means it becomes more fluid, and burns with greater facility. Fig. 110 shows the plan of this lamp. The reservoir is above the flame, and the top of the glass is fitted into an iron chimney, which radiates heat very freely to the reservoir. The oil passes down through a lateral tube to the cistern, and there is a stopcock in the tube to cut off the supply of oil when necessary. The wicks are very short; and instead of being adjusted by the rack and pinion, a movement is given to the glass, which, by its position, regulates the intensity of the flame. A painted shade is put over the whole, in order that the reservoir may be hidden. The advantage of this lamp is, that it will consume the very commonest oil without producing an unpleasant smell. One caution, however, is necessary in the management of it—namely, that the reservoir be filled quite up with oil before the lamp is lighted. If this be not attended to, the air contained in the chamber will expand by the action of the heat, and the oil will be forced out over the wick, and will run about.

Fig. 110.



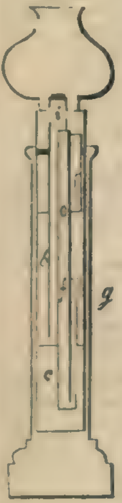
(*h.*) *Keir's Fountain Lamp.*—In all the preceding contrivances, the supply of oil to the flame is chiefly effected by the capillarity of the wick. In some cases it is assisted by the gravitation of the oil from a reservoir situated above the level of the burner. But it has always been thought desirable to have the reservoir in the stem or body of the lamp, so that the unsightly appearance of the chamber might be avoided, and the shadow which it invariably casts to a greater or less extent upon surrounding objects, entirely prevented. This, however, can only be accomplished by means of some power whereby the oil shall be pressed up from the well in which it is contained, to the level of the flame. "To effect this, two methods have been resorted to: one is on a hydrostatic principle, in the manner of Hiero's fountain, where the oil is placed in the body of the stem, and is raised to the wick, as it is wanted, by the pressure of a column of some fluid: in the other method, the oil is forced by clock-work mechanism, as in the lamp of Carcel of Paris. The first successful attempt of this kind in England, was in the lamp invented by Mr. Keir, about forty years ago, upon a hydrostatical principle; and although it is not used at present, being superseded by contrivances of a similar kind by other manufacturers, yet it will serve to illustrate the general nature of these lamps, of which several varieties have been brought partially into use."* Fig. 111 represents Keir's lamp, the vase and pedestal of which are hollow. *a* is a tube into which a certain quantity of salt and water, having three times the specific gravity of oil, is put. Upon this is poured the oil until the tube is full. The brine, or solution of salt in water, runs down into the pedestal of the lamp; and when the oil is poured upon it, the latter, by its weight, forces up the former through a second tube *b*, into a chamber *c*, in the upper part of the body of the lamp, and the oil takes the place of the brine in the pedestal. It will be noticed that the tube *a* reaches only to the top of the chamber in the pedestal, while the tube *b* passes down to the bottom of it. The brine is represented by diagonal lines, and the oil by dots. Now, in consequence of the great weight of the brine, as compared with that of the oil, the latter is forced up to the burner as fast as it is consumed; and thus, by a sort of natural spring, the flow of oil to the wick is constantly maintained. After a supply of salt and water has once been introduced into the lamp, there is no necessity for a further addition of it.

Fig. 111.



Similar lamps have been constructed on the same principle by King, Barber, and others; but they are all difficult to manage, and hence they are not much in vogue, although they generally have a very light and elegant appearance.

Fig. 112.

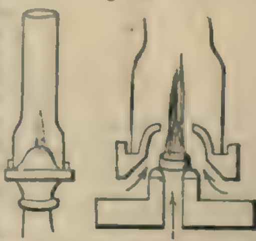


(*i.*) *Parker's Fountain Lamp* (Fig. 112) is a very complicated apparatus, which, like the fountain of Hiero, owes its action to compressed air. Externally it presents the appearance of a column surmounted by the lamp; but within this column there is another cylinder which contains the oil. This cylinder must be removed before the lamp can be charged. It is divided crosswise into three compartments, *a b c*, which have no direct communication with each other. Through the centre of the whole there passes a tube *f*, which is open at its top, and at the bottom it communicates by a sort of valve with the compartment *c*; it also communicates with the compartment *a*, by means of a hole which is seen near the top of the tube. The three compartments or chambers are therefore in indirect communication with each other; thus *a* communicates through the hole just mentioned with the tube *f*; this communicates by its bottom valve with compartment *c*; and this, by means of a tube and valve *g*, with the middle compartment *b*; and *b* communicates by means of an ascending tube with the burner *i*, in which the wick is placed: so that if oil be poured into the upper opening of the tube *f*, it will fill the compartment *c*; and then, on turning the apparatus upside-down, the oil will flow through *g* into the middle compartment *b*. On restoring the cylinder to its proper position, the oil cannot return to *c*, because of the peculiar form of the contrivance *g*—consequently it remains in the middle chamber; and now, on refilling *c* through the tube *f*, the air in the lowest chamber is compressed, and it forces the oil in *b* up through the lateral tube to the burner *i*. The chamber *a* is filled at the same time as *f*, and its contents flow as fast as they are wanted through the hole in the tube *f*, and thus keep up a supply to the pressure-column. Every time the lamp is charged it is inverted, in order that the oil in *c* may flow into *b*; and then it is returned to its original position, and recharged with oil. This lamp was reported on by the French Academy of Science, and it was formerly much used in India; though now it is superseded by the Iris lamp.

(*k.*) *The Carcel Lamp* presents much of the appearance of the last described; but instead of the flow of oil being effected by means of atmospheric elasticity, it is accomplished by the aid of machinery moved by clock-work. In this way the oil is raised, or rather pumped up, to the wick, so as to keep up a constant supply by continually overflowing it. The oil drips back into the cistern below; whence it is drawn up repeatedly, until it is all consumed.

(*l.*) *The Solar Lamp.*—This is Roberts's great improvement on the Argand, although it was patented by Mr. Bynner. It is a contrivance for increasing the supply of atmospheric air to the flame, and so enabling it to consume a larger proportion of oil, and thus to give out a greater body of light; besides which, it effects a more complete combustion of the oil, and therefore produces a more intense light. In all the common forms of Argand, the air passes straight up through the burner, and only slightly impinges on the two sides of the flame; but in this contrivance the current of air is broken, and made to blow in upon the flame. The apparatus which effects this is a small cone or nipple, that is dropped down over the flame (Fig. 113).

Fig. 113.



A still greater improvement on this principle is that of Quarrel, in which he causes a second current of air to enter under the glass, and thus to assist the other in blowing on the flame (Fig. 114); and by introducing a button into the centre of the flame (as was also originally proposed by Mr. Roberts), this inner current of atmospheric air is likewise deflected, and thus we get the greatest possible amount of oxydation (Fig. 115). By the adoption of such contrivances as these, almost any kind of oil, even the commonest fish-oil, may be burnt without producing any unpleasant smell.

Fig. 114.

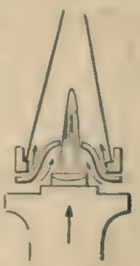
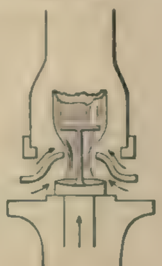


Fig. 115.



(*m.*) *The Bude Light* of Mr. Goldsworthy Gurney is only an extension of the preceding principle. Roberts increased the supply of atmospheric oxygen by means of the nipple, which others patented and applied to the solar lamp. Mr. Gurney, however, uses a jet of pure oxygen, or of an atmosphere rich in oxygen, which he throws into the centre of the flame by mechanical power; and by this means he increases combustion and light to an enormous extent. This light requires complicated apparatus for its production, and hence it has never come into general use.

(*n.*) *The Argand with Concentric Wicks.*—Many years since, Mr. Webster suggested that Argands might be made with two circular wicks, one within the other; and Sir Humphry Davy referred to the invention as one well adapted for obtaining heat for chemical purposes. A lamp on the same principle, with four concentric wicks, has since been constructed by M. Fresnel, for lighthouses; and, according to Dr. Brewster, it gives a light equal to forty common Argands; but the heat produced by the lamp is very intense.

5. *Lamps for Burning Solid Fats.*—The commonest form of these is the saucer and central-wick lamp, which may be seen so frequently in France during the nights of illumination; but a more agreeable form has been given to this kind of lamp by the Hon. G. Cochrane, who took out a patent for it a few years ago. His lamp is very much like an ordinary Argand, but it has a piece of metal over the flame which communicates with the chamber containing the fat, and thus keeps it warm and liquid (Fig. 116). The fats which are best suited for this lamp are cocoa-nut and palm; but tallow and kitchen-stuff may also be used.

6. *Camphine or Vesta Spirit Lamps.*—These are the

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* Webster and Parker, p. 159.

names given to the lamps which are constructed to burn the highly rectified oil of turpentine—a liquid which Mr. English originally prepared and patented under the name of camphine. Mr. Young, of Queen-street, Cheapside, has devoted great attention to the manufacture of these lamps, which he calls *Vesta spirit lamps*. The reservoir for holding the camphine is made of glass, in order that the heat from the burner may not be communicated to it: in fact, every precaution is taken to guard against such a result; for the wick, instead of being supported on a long metal tube, which passes down into the body of the oil, as is the case with the common Argand lamps, is merely held at the top by a narrow ring, while the remaining portion floats freely in the spirit; besides which, the burner is insulated from the lamp by a collar of wood. The next point of importance in their construction is that which ensures an abundant supply of atmospheric air; for if this is not attended to, the turpentine burns with a remarkably sooty flame, and evolves a large quantity of black smoke. This is effected by having a slit in the side of the wick so as to allow the atmosphere to pass freely into its interior; and the nipple, or cap, of the solar lamp is used to break the outer current of air, and blow it into the flame; besides which, the chimney is very tall, and it is bulged in, or contracted, at the point where it reaches the flame. In the larger kinds of camphine lamps, the button of Mr. Roberts is also introduced into the interior of the flame, so as to break the inner current of atmospheric air (Fig. 117). And then, in addition to all this, a cylinder of perforated brass is put around the vent-holes of the burner, so as to prevent the ill effects of extreme draughts. The principles, therefore, which are kept in view in the construction of these, are—1st, to prevent the heating of the liquid; 2nd, to ensure a large supply of atmospheric air; and 3rd, to guard against the influence of external draughts. One of the great advantages of the Vesta lamp, is the intensity and purity of the flame. Dr. Ure says, that a lamp which consumes two ounces of camphine in an hour, gives the light of nearly twelve sperm or wax candles of three or four to the pound. Our own experiments show, that the smaller lamp, which has no button, consumes about 140 grains per hour, and gives the light of two-and-a-half sperm candles, each burning at the rate of 120 grains per hour; while the larger lamp, which has the button, consumes about 540 grains of camphine per hour, and gives the light of seven sperm candles.

Fig. 116.

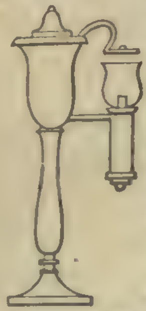
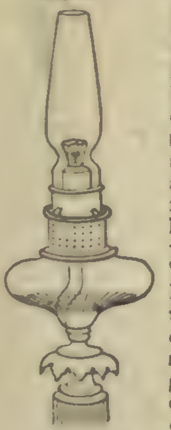
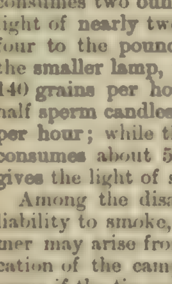


Fig. 117



Among the disadvantages of the Vesta lamp, are its liability to smoke, and its disagreeable smell. The former may arise from bad management, or from a resinification of the camphine—a circumstance that is sure to occur if the tin-can containing the liquid is not kept well corked, or if too much liquid is put into the lamp for an evening's consumption.

Fig. 118.



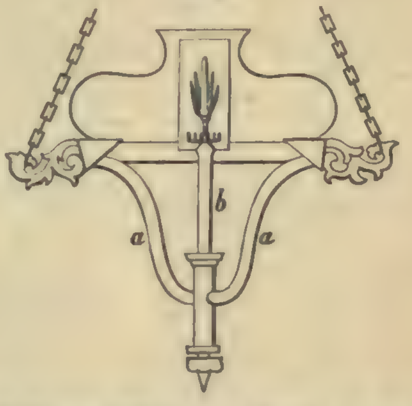
7. *Naphtha Lamps.*—Of these there are two kinds—viz., the Argand naphtha, which is constructed exactly like the preceding; and the flat-wicked lamp, which is represented in Fig. 118. In the latter, the air is directed upon the exterior of the flame by means of two lateral pieces of tin, which incline inwards as they ascend to the wick; and the supply of air is regulated by an external opening or valve, which may be shut up to any extent. When naphtha is burnt in a small Vesta lamp, it is consumed at the rate of about 136 grains per hour, and it gives the light of two standard sperm candles; when burnt in the large one, its consumption is 486 grains per hour, and its light is equal to nine sperm candles. The disadvantages of these lamps are, the unpleasant smell of the naphtha, the liability to smoke, and the danger of explosion; for naphtha, being much more volatile than camphine, is likely to give off vapour, which, with the air, forms an explosive mixture. Great caution is therefore necessary in managing these lamps.

8. *Gas or Vapour Lamps.*—Several attempts have been made at various times to burn inflammable liquids without a wick. The earliest and simplest of these is—

(*a.*) *The Common Floating Night-lamp*, which is nothing more than a small cup of metal pierced in the middle with a small glass tube. The oil rises by capillary attraction in the tube, and may be ignited.

(*b.*) Another form of *self-generating lamp* is represented in Fig. 119, where two tubes, *a a*, bring the oil down from the annular reservoir, and convey it into a cistern, whence it rises by the tube *b* into a cup which has a cylinder filled with it, pierced with a number of small holes that serve for jets. When the lamp is trimmed, alcohol is poured into the cup, and set fire to. This, in the act of burning, makes the cylinder so hot that the oil within it is converted into gas; and this escaping through the hole or jets, is fired, and burns with a brilliant light. The heat produced by the combustion of the gas, keeps up the supply. A glass is placed around the burner to protect it from external currents, and to prevent the flame from smoking.

Fig. 119.



(*c.*) A mixture of camphine or highly rectified spirits of turpentine with alcohol, is burnt on the continent in a lamp that was originally designed by Lüdersdorff, but which has been patented in France, by Ribot, under the name of "*Eclairage au Gaz Liquide.*" The French lamp is represented in Fig. 120. It is an elegant-looking arrangement, the body and stem of which are composed of cut glass. Fig. 121 exhibits the mechanism by which the spirit and turpentine are converted into vapour. *a* is the wick which floats in the liquid, and conveys it up through the tube *b* into *c*, which is the vapour chamber; *d* is a cap of brass, which, when heated

Fig. 120.

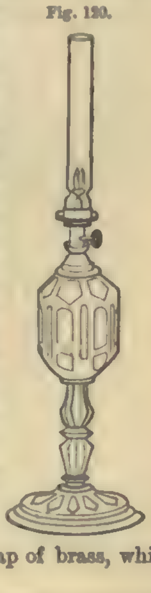
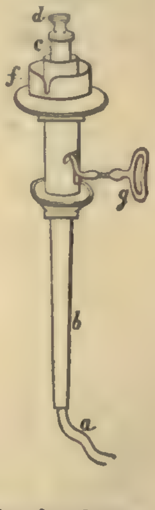


Fig. 121.



effects the vaporisation of the liquid, and causes the gas to escape through three little holes at its base; *f* is the collar for holding the glass; and *g* is a handle for turning the liquid off or on. When the lamp is to be lighted, it is filled up with mixed spirit; and the glass being removed, the handle *g* is to be turned from the left to the right. A ring of wire gauze, saturated with the spirit, is then lighted and brought down over the cap *d*, as low as *e*. This is to be kept burning until the metal cap and chamber are sufficiently heated to generate gas, which will escape through the holes and take fire. When this has occurred, the glass is to be replaced, and the burning jets will keep up the desired temperature. Instead of turpentine and alcohol, a mixture of equal parts of spirits of wine and coal-naphtha, or even coal-naphtha itself, may be used. The disadvantages of the lamp are its liability to go out with the least draught of air, and its danger of exploding.

(*d.*) *The Common Naphtha Lamp* is now extensively employed by poor tradespeople who keep their stalls out of doors. The construction of this lamp will be understood from Fig. 122: *a* is the reservoir for the naphtha, from the bottom of which there passes a tube *b*, which supplies the fluid to the burner *c*. A stopcock is inserted into the middle of the tube, in order that the supply of naphtha may be regulated, or even cut off altogether, when it is not wanted at the burner. Fig. 123 exhibits the construction of the latter: the naphtha enters by the tube *a*, and it flows out of a small hole which is in the lower arch of the burner, and trickles down into a small cup *b*; here it is lighted, and soon the combustion of the naphtha produces so much heat that it makes the whole body of the burner very hot. This causes the naphtha to assume the form of vapour or gas; and as it issues out of the small hole, it is forced up against the bottom of the disc *c*; and thence it comes spreading out in a star-like form all round the burner. This form is given to it by the little breaks of perpendicular wires which descend from the bottom of *c*; another small disc is placed below this to prevent the flame from bounding back upon the jet whence it issues from the tube; *d* is a small wire, which is withdrawn when the interior of the burner wants cleaning.

Fig. 122.

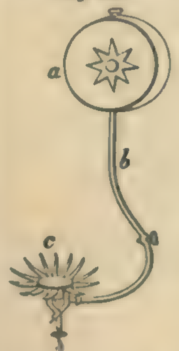
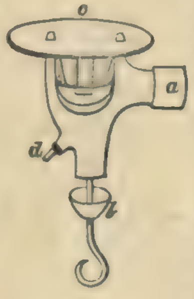
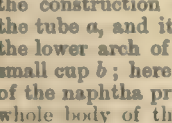


Fig. 123.



(*e.*) *Beale's Naphtha Lamp.*—This form of lamp was invented some years ago by Mr. Beale, of London: it consists of a vessel *a* (Fig. 124), in which the naphtha is placed; thence it flows by a lateral tube into the cup-shaped cistern *b*, and it is prevented from overflowing by reason of its being constructed in the same way as a bird-fountain. A tube *c* passes up through the bottom of the cistern, and reaches a little above the level of the liquid: this tube is placed in communication with a reservoir or gasometer containing air. Lastly, a brass cap *d*, having a hole in the top, is placed upon the cistern; the lamp is put into action by removing the cap and setting fire to the naphtha; air is then blown through the tube *c*, and the cap is gradually brought down into its place in the cistern, care being taken that it is sufficiently heated during its descent to keep up the volatilisation of the naphtha. When this is properly managed, the air which passes through the tube *c* carries with it so much naphtha vapour as to become inflammable, and burns with a brilliant jet of flame.

Fig. 124.



We have not thought it necessary to give a description of the different ornamental contrivances for setting up and supporting lamps, &c.—as, for example, candlesticks, candelabra, chandeliers, &c.; for this would entail a very elaborate account of the principles of ornamentation, which is not suited to the object of this work.

ON GAS.

General Remarks.—Long before gas was manufactured artificially, it was generated in the great laboratory of nature by the action of terrestrial heat on large accumulations of vegetable matter, as beds of coal. The products of this action made their way to the surface of the earth, and escaped in the form of gas and petroleum; both of which are highly inflammable. The fires thus generated, commanded attention at a very early period, and altars dedicated to the gods were erected over them. At the time that the Persians, under the command of Mardonius, overran Greece, there were innumerable altars lighted up in this manner; and so much were they reverenced by the Greeks, that when the Persians were defeated at the battle of Platæa, and driven from the country, the two victorious generals, Pausanias and Aristides, were directed by the Oracle of Delphi to build an altar to Jupiter, their deliverer, and not to offer any sacrifice upon it until they had extinguished all the fires throughout the country which had been polluted by the Persians, and had relighted them with the sacred fire from Delphi. In Plutarch's *Life of Alexander*, we are told, that when that monarch arrived at Ecbatana, "he was particularly struck with a gulf of fire, which streamed continually, as from an inexhaustible source. He admired also a stream of naphtha not far from the gulf, which flowed in such abundance that it formed a lake. The naphtha in many respects resembles bitumen, but it is much more inflammable; before any fire touches it, it catches light from a flame at some distance, and often kindles all the intermediate air. The barbarians, to show the king its force and the subtlety of its nature, scattered some drops of it in the street which led to his lodgings; and standing at one end, they applied their torches to some of the first drops, for it was night. The flame communicated itself swifter than thought, and the street was instantaneously all on fire."

For several thousand years the Chinese province of Se-tschuan has been celebrated for the quantity of inflammable gas that issues from the earth; and it has been stated, that the gas which escapes from the ground in the neighbourhood of Pekin, is collected by the inhabitants, and used for lighting the streets and houses. The holy fires of Baku, near the Caspian Sea, have a similar origin; and jets of inflammable air are evolved at Pietra Mala, not far from the road between Florence and Bologna; at Maina, which is a few miles from Modena; at Lycia, in Asia Minor; and at the Artesian wells of Lichweg, in Schauenburg. In this country the gas is abundantly produced from the fissures of coal mines, where it is known by the name of *fire-damp;* and it is also evolved from stagnant pools, when it is termed *marsh gas.* At the village of Wigmore in Herefordshire, inflammable gas has on several occasions escaped from the earth to such an extent, that it has been made the means of lighting the neighbourhood. The same has been the case at Charlemont in Staffordshire, and at a place near Glasgow. Not long since, a considerable jet of gas was discharged from the Chat Moss, near to the Manchester and Liverpool Railway, and it was used by a neighbouring farmer for the purpose of working a small steam-engine. In the village of Fredonia, in the state of New York, the gas issues from the earth in such abundance, that the inhabitants collect it, and employ it for lighting the streets.

History of Gas Lighting.—It would naturally be supposed that facts like the preceding would, at a very early period, have commanded the attention of practical men of science, and that some effort would have been made to imitate the process which they saw going on in nature. Some, indeed, have thought that the ancient

Greeks were really acquainted with a mode of generating gas, and that the chief priests took advantage of such knowledge in exciting the veneration of the people. The altar in the temple of Ægina is described by Dr. Dodwell as having a round hole, thirteen inches in diameter, cut out of the top of it. This hole communicates with another which passes down through the solid stone to the depth of several feet, and there it opens into a cavity which is supposed to have contained fire that was always burning. He says that nothing more was necessary than to pour oil into the upper opening; and as it trickled down, it would be converted into gas, which would burst forth as flame, and appear to have a miraculous origin. But however possible, or even probable, the truth of this supposition may appear to be, it is certain that nothing practical was undertaken in Europe until the beginning of the 17th century, when Van Helmont, the physician and alchymist of Vilvorden, was led, in the course of his investigations, to expose a quantity of animal and vegetable matter to the action of heat in a closed vessel. By these means he obtained a "vapour," or "spirit," that burnt with a bright flame; but he little imagined that this *gaz fuliginosum*, as he termed the vapour, would one day become an agent of general illumination. Nor even at a much later period—namely, in the year 1726—when Dr. Hales informed the chemists of his time, that by distilling a few grains of Newcastle coal, he had obtained an equal number of cubic inches of inflammable air, could it be supposed that a similar experiment, on a very gigantic scale, would be daily performed in every city in Europe, and that millions of cubic feet of that subtle, inflammable material would be made to traverse, unseen, along the highways of the land, and be the means of lighting them up into an almost perpetual day.

Again, we may take up the history of our subject from another important discovery. Some time before the death of the Honourable Robert Boyle (1691), a letter was written to him by the Rev. Dr. Clayton, on the subject of distilling pit-coal. That letter was published many years afterwards—in 1739—in the Transactions of the Royal Society; and the author states, that from an examination of some inflammable vapours which were given off from a ditch near Wigan in Lancashire—vapours which had been collected and examined by Thomas Shirley in 1659—he was led to conclude that they came from the coal of the neighbourhood, which was acted on by terrestrial heat. Accordingly, he obtained a portion of the coal, and distilled it in a retort over an open fire. By this means he obtained a "phlegm," which first passed over; then a "black ore;" and lastly a "spirit," which he could nowise condense, for it forced the luting of his apparatus, and broke his glasses. This "spirit" was coal gas; and on discovering its inflammable nature, he was in the habit of collecting it in bladders and oiled silk bags, and thus preserving it for the amusement of his friends.

As yet, however, he had not learnt to burn it from a metallic jet, for he was accustomed to prick a hole in the bag, and then to set fire to the gas as it issued forth. For a period of more than fifty years these interesting facts were allowed to slumber, and no one entertained the idea of applying them to any useful purpose. At length, in the year 1792, just one hundred years after Boyle's death, an ingenious engineer and miner of Cornwall, whose name was Murdoch, conceived the notion that gas might be conveyed through pipes to a distance, and be thus employed as an illuminating agent. Acting upon this idea, it was not long before he had the satisfaction of seeing his house and offices at Redruth lighted up with Dr. Clayton's "subtle spirit." He also collected the gas in bladders, and used it as a means of lighting himself along the road between the mines and his own house; for which piece of ingenuity he acquired the reputation of a wizard. Soon after this, Mr. Murdoch was employed in the establishment of the celebrated engineers, Messrs. Bolton and Watt. There he erected apparatus for the manufacture of gas; and at the peace of Amiens, in 1802, he lighted up their factory for the first time with this agent. About a year afterwards, gas was generally employed in all the workshops of the factory; and in 1804 he set up a similar apparatus in several of the large cotton-mills of Lancashire—one of the earliest of which was the establishment of Messrs. Phillips and Lee at Manchester.

Ten years after this—in 1813—the manufacture of gas had extended to London, and in that year Westminster-bridge was lighted with it. Mr. Clegg gives an account of the horrors of the lamplighters when they first beheld the burning gas, and how he was obliged to light the lamps himself for some time, on account of the fears of the people. Even such men as Sir Humphry Davy and Sir Joseph Banks were unable, for many years after this, to overcome the prejudices which existed in their minds concerning it; and they thought the scheme a wild and dangerous one. The public, however, soon became reconciled to it; and in 1814 the oil-lamps were removed from the streets of St. Margaret, Westminster, and gas-lights were put into their places. This was the first parish that entered into a contract to have the streets lighted with gas.

In the same year the allied sovereigns came to this country; and as they were to be fêted and feasted in no ordinary manner, a great opportunity occurred for the use of gas, where no other means of illumination could be employed. On the ornamental water of St. James's Park a magnificent pagoda was erected; it was furnished with thousands of jets of gas; and in an instant, as if by magic, they all burst forth into flame, and gave the building the aspect of a brilliant fountain of fire. At one of the City feasts, the Guildhall was lighted up in a similar manner; and we are told by one of the journals of the time, that the light was "clear as summer's noon, but soft and undazzling as moonlight, forming a magnificent combination worthy the inauguration of the presiding citizen of the great city." Up to that time the gas was marvellously impure, and its fœtid odour proved an insurmountable barrier to its use in private houses. Eventually the attention of chemists began to be directed to this point; and as the processes of manufacture and purification were perfected, the use of gas became more general; so that in 1822 there were four great companies established in London, having forty-seven gasometers, supplied by 1,315 retorts, which generated upwards of 397,000,000 cubic feet of gas annually, supplying about 61,000 private lamps, and 7,268 public ones. In five years this quantity had nearly doubled itself; and in ten years more it was doubled again; so that in the year 1837 it had acquired so much importance as to become a subject for parliamentary investigation. In that year a paper was laid before a committee of the House of Commons, by Mr. Hedley; from which we gather, that for lighting London and its suburbs, a capital of £2,800,000 was employed. This yielded a revenue of £450,000, and furnished an annual supply of 1,460,000,000 cubic feet of gas. Twelve years after this, we are told by Mr. Croll, in his evidence before the committee of the House of Commons on the Great Central Bill, that the consumption of gas in the metropolis, during the year 1849, was more than double that of the preceding estimate; for it amounted to 3,200,000,000 cubic feet annually, of which the City alone consumed 500,000,000. This was distributed to 2,678 public lights, and to a multitude of private consumers. The area of the metropolis is about sixty-six square miles, and that of the City, one mile. In the former space there are, according to Mr. Barlow, about 2,400 miles of main-pipes, which run along 1,500 miles of streets; and in the latter there are about 110 miles of main-pipes, which light up 75 miles of streets. The surveyor to the Corporation states, in one of his reports, that the length of the public way in the City is only 51 miles, and that the lamps average 54 to each linear mile, or 1 in every 33 yards. But perhaps the best estimate of the enormous extent to which this branch of industry is carried on, may be formed from a statement made by Dr. Hoffman, on the authority of Mr. Lowe, who is one of the oldest gas engineers of the present time. He says, that about 6,000,000 tons of coal are annually consumed in England in the manufacture of gas; and as each ton of coal, on the average, produces about 10,000 cubic feet of gas, we have the almost incredible quantity of 60,000,000,000 cubic feet of gas produced yearly.

Action of Heat on Organic Matter.—As a preliminary to the study of gas-making, it is necessary that something should be known of the changes which occur when animal or vegetable matter is subjected to the action of heat. Chemists have shown that the effects vary with the temperature. At first, when the heat is not considerable, the matters evolved consist of aqueous vapour, organic acids, ammonia, and various combustible fluids which are soluble in water. In the second period, when the heat is somewhat higher, the products are carbonic acid, carbonic oxide, water, and a number of oleaginous or empyreumatic compounds, which are not soluble in water; and lastly, when the temperature is still higher, the products of the decomposition are hydrogen, marsh gas, and sundry hydro-carbons, which retain their gaseous condition. In the case of non-nitrogenous bodies, as wood, resin, fat, oil, &c., the chief products of distillation are water, acetic acid, naphtha or wood-spirit, volatile oil, tar, paraffine, creosote, carbonic acid, carbonic oxide, olefiant gas, super-olefiant gas, marsh gas, hydrogen, &c.; and when the substance contains nitrogen and sulphur, as is the case with coal, there are evolved ammonia, aniline, leukol, picoline, lutidine, &c., together with cyanogen, sulpho-cyanogen, and most of the compounds just named. In every case there remains in the retort a quantity of carbonaceous matter, which goes by the name of coke.

Of the gases which are thus evolved, the most important are hydrogen, carbonic oxide, marsh gas, olefiant gas, and various hydro-carbons which give to gas its high illuminating power: all the others are positively injurious, and ought, therefore, to be got rid of before the gas is supplied to the public. An examination of the quality of these gases will readily convince us that they may be divided into three kinds—namely, the light-giving gases, the diluters, and the positive impurities. Of the *light-giving gases*, the following are the most important:—

(*a.*) *Olefiant gas* (C^4H^4): a compound that was discovered in the year 1795, by the associated Dutch chemists, and was so named from the property which it has of forming an oily-looking fluid when it combines with chlorine. It is an odourless gas, and has a specific gravity of 0·97. It burns with a bright yellow flame, and consumes three times its bulk of oxygen, or nearly fifteen times its bulk of atmospheric air, producing twice its volume of carbonic acid. The gas is readily condensed by chlorine, bromine, and anhydrous sulphuric acid; it is also absorbed to a slight extent by water.

(*b.*) *Other hydro-carbons*, as *Propylene* (C^6H^6), or the super-olefiant gas of Dalton and Henry, *Etherene* (C^8H^8), or the volatile gas of Faraday, and perhaps some others of a like atomic composition, are met with in most of the illuminating gases of commerce. These, like the last, consist of equal proportions of carbon and hydrogen. They are very condensible by chlorine, bromine, and anhydrous sulphuric acid, and they burn with a very bright sooty flame.

The *diluting gases* are marsh gas, hydrogen, and carbonic oxide. These are important constituents of common gas, because they serve as the purveyors of the rich illuminating hydro-carbons, which could not be burnt alone.

(*a.*) *Marsh gas, or light carburetted hydrogen*, is a compound of one atom of carbon and two of hydrogen (CH^2). It is about half as heavy as atmospheric air, and it burns with a bluish flame, tipped with yellow. It consumes twice its bulk of oxygen, or nearly ten times its bulk of air; and it produces its own volume of carbonic acid.

(*b.*) *Hydrogen* is the lightest of all known substances. It weighs but a fifteenth of its bulk of atmospheric air. It burns with a pale blue flame, and consumes only half its bulk of oxygen, or two-and-a-half times its bulk of air; the sole product of its combustion being aqueous vapour.

(*c.*) *Carbonic oxide* (CO) is a little lighter than atmospheric air. It burns like the preceding, with a blue flame, and consumes only half its volume of oxygen. The product of its combustion is its own bulk of carbonic acid.

The *impurities of gas* are carbonic acid, ammonia, sulphuretted hydrogen, bisulphuret of carbon, tarry matter, and various compounds of cyanogen and sulphur.

(*a.*) *Carbonic acid* (CO^2) is a very heavy gas—its density being about 1·5. It is not only incombustible, but has the power of checking the combustion of all inflammable gases. It is freely absorbed by lime and alkalies. Water takes up about its own bulk of the gas.

(*b.*) *Ammoniacal gas* (NH^3) is about half as heavy as atmospheric air. It is not combustible, unless it is decomposed by the heat of some other burning body; and then the hydrogen of the gas burns in the usual manner. Ammonia is readily absorbed by water, and by solutions of acids and metallic salts. It is known by its communicating a red colour to turmeric paper, and by fuming with muriatic acid.

(*c.*) *Sulphuretted hydrogen* (HS) is a most unpleasant-smelling compound; it is a little heavier than atmospheric air, and burns with a pale-blue flame that evolves the odour of a burning match. When plenty of atmospheric air is present, the products of its combustion are water and sulphurous acid; but if the supply is limited, water alone is formed, and the sulphur is precipitated. The sulphurous acid, when produced, quickly absorbs more oxygen, and becomes sulphuric acid—a compound that exerts a most destructive influence on every kind of textile fabric. When sulphuretted hydrogen escapes into the air without burning, it discolours lead paint, and tarnishes silver. On these accounts, sulphuretted hydrogen is regarded as one of the most injurious compounds of ordinary gas. It is absorbed by lime, and by the salts of iron, zinc, copper, and lead; and the test for it is a piece of white paper dipped in a solution of sugar of lead.

On exposing such paper, while damp, to the action of the gas, the lead-salt is quickly discoloured; and thus the smallest trace of sulphuretted hydrogen may be easily recognised.

(*d.*) *Bisulphuret of carbon* (CS^2) is even a more serious impurity than the last, for it not only produces the same acid compound by its combustion, but is also more difficult of detection; and then, again, chemists are not yet acquainted with any process for the removal of this noxious body from the gas of commerce. All these circumstances give it an importance that it would not otherwise possess. Bisulphuret of carbon, when pure, is an oily-looking liquid, that sinks in water. It evolves the unpleasant odour of putrid cabbage, and boils at a temperature of 106° Fah.; it burns with a blue flame, and its vapour consumes twice-and-a-half its bulk of oxygen, or nearly twelve-and-a-half times its bulk of atmospheric air, producing twice its volume of sulphurous acid, and half its volume of carbonic acid. This compound is best recognised by burning the gas, and collecting the products, in which sulphurous or sulphuric acid will be discovered.

(*e.*) The *tarry matters of inflammable gas* are of a very complex nature; they appear to be held in solution by ammonia, and to be precipitated in the form of dark flakes, when the gas is made to pass through a vessel containing flints moistened with acid. It is very probable that these tarry matters are of an acid nature, and that, in their union with ammonia, they produce compounds that are sufficiently volatile to be suspended in the gas.

(*f.*) The *cyanogen compounds* are not likely to be found in the gas of commerce, for they are readily absorbed by the lime made use of in its purification. Of these, cyanogen, hydrocyanic acid, and sulpho-cyanogen are the most important.

The Manufacture of Gas.—In all cases, the destructive distillation of the organic substance which yields the gas is effected in a vessel called a retort, which is set in a furnace; and the gaseous products are purified by transmitting them through a series of vessels named condensers and purifiers. The details of the process vary with each particular gas, and, consequently, it is necessary to describe them under different heads.

Coal Gas.—The *apparatus* which is employed for the manufacture of coal gas consists, first, of a *retort*, made of iron or clay, set in a furnace in such a manner that it may be heated throughout to a tolerably uniform temperature; an *outlet pipe* ascends from the end of the retort, and terminates in another pipe, called the *hydraulic main*. This part of the apparatus is so constructed, that the pipe which delivers the gas from the retort dips down, to the extent of three or four inches, into the liquid contained in the hydraulic main. By this means the end of it is sealed or closed by a water-valve, and no gas can return from the other parts of the apparatus into the retort during the time that the latter is being charged with coal. The *hydraulic main* is a large horizontal pipe, which runs from one end of the building to the other; it receives all the exit-pipes from the retort, and discharges its contents into a series of smaller pipes, which are so arranged in water and air, as to form a large cooling surface for the condensation of the liquid matters contained in the raw gas. Here it is that water and tar are deposited; and the apparatus is so constructed that the fluid matters run off, as fast as they are condensed, into a tank which is conveniently placed to receive them. This part of the apparatus is called the *condenser*. From the condenser the gas passes to the *purifiers*, which are vessels charged with lime and other substances that have the power of absorbing the various impurities of coal gas. To assist the flow of gas through these vessels, an instrument called *an exhauster* is sometimes employed: it is a kind of air-pump worked by a steam-engine, which draws the gas away from the retorts, and so relieves them from that enormous amount of pressure that they would otherwise have to encounter while their gaseous contents are being forced on through the various obstructions that intervene between them and the gasometer. Leaving the purifiers, the gas passes into the large receiver or gasometer, where it is stored.

Materials employed in the Manufacture of Coal Gas.—Of the three varieties of coal, known to chemists, the black or bituminous is the only one which is employed to any extent for the manufacture of gas. Lignite, or brown coal, is not sufficiently plentiful for this purpose; and glance-coal, or anthracite, is not rich enough in hydrogen to be of any use to the gas manufacturer. A small quantity of a peculiar bituminous shale, named Boghead coal, has of late been employed in London and elsewhere; but its nature is not sufficiently well determined to enable us to say whether it is to be regarded as a coal or not.*

The varieties of black coal are exceedingly numerous: in a general way, however, they may be divided into four kinds—namely, caking coal, which has the property of melting when it is heated, and of running together; splint coal, which is so named from its splintery fracture; cherry coal, which burns without caking at all; and cannel coal, which is exceedingly hard, compact, and bituminous. The first of these occurs abundantly in the neighbourhood of Newcastle, Northumberland, and Durham; the second in South Wales; and the last in Scotland and in Lancashire. Although these varieties of coal differ very considerably in their value for gas-producing purposes, yet regard must always be paid to the convenience or facility with which they are obtained; and hence we find that, in London, the coals of Newcastle, with certain cannels of Lancashire and Scotland, are employed. In Bristol and its neighbourhood, the coals of Gloucestershire and Wales are used; in Birmingham, those of Staffordshire and Wigan; in Leicester, Nottingham, and Derby, the coals of Derbyshire; in Leeds and Sheffield, those of Yorkshire; in Liverpool, Salford, and Manchester, those of Wigan; in Edinburgh and the north of Scotland, the coals are chiefly derived from the Lothians and from Fifeshire; in Glasgow they are obtained from Lesmahago, Kelvinside, Wilsontown, &c.; and in Greenock they are procured from Monkland and Skaterig.

The temperature at which the carbonisation of the coal is effected, and the manner in which the heat is applied, have an important influence on the quality of the gas and other products obtained. If the heat is too low, the quantity of gas produced is small, while that of the tar is large: on the contrary, if it is too high, the latter is sacrificed for the generation of the former. In the one case the gas is too rich in hydro-carbons, and in the other it is too poor. In practice, therefore, it is necessary to hit the happy medium; and it is generally thought that Newcastle coal requires a temperature of a dull red heat (1300° Fah.) for its distillation, and that the richer cannels will bear a temperature of 1800° Fah., or a bright cherry red.

Some idea may be formed of the relative value of the different kinds of coal, by reference to the following table, which exhibits the proportions of volatile matter, coke, and ash, produced by each description of coal, as well as the per-centage of sulphur contained in the coal, the coke, and the gas. The coals are arranged in the order of their gas-producing properties; and it will be seen that the amount of volatile matter ranges between 23 and 68 per cent., and that the quantity of sulphur contained in the volatile matter is from 1 to 5 per cent.:—

Name of Coal.	Vol. Matt.	Coke.	Ash.	Sulphur in		
				Coal.	Coke.	Vol. Matt.
Boghead	68·4	31·6	22·8	0·53	0·08	0·45
New Brunswick cannel	66·3	33·7	0·6	0·07	0·00	0·07
Kirkness	60·0	40·0	13·5	1·40	0·58	0·82
Capeldrae	54·5	45·5	10·5	0·65	0·20	0·45
Old Wemyss	52·5	47·5	15·1	1·30	0·60	0·70
Staffordshire cannel	50·0	50·0	2·9	1·30	0·52	0·78
Lesmahago	49·6	50·4	9·1	2·25	1·14	1·09
Knightswood	48·5	51·5	2·4	1·10	0·61	0·49
Arniston	45·5	54·5	4·2	1·70	0·95	0·75
Heathern (Stafford)	42·9	57·1	1·8	1·50	0·70	0·80
Ruabon main (N. Wales)	41·5	58·5	1·0	0·85	0·45	0·40
Staveley (Derby)	40·9	59·1	2·7	1·20	0·80	0·40
Radstock (Somerset)	38·3	61·7	3·5	3·10	1·80	1·30
Silkstone (Yorkshire)	38·0	62·0	2·6	1·10	0·60	0·50
Blenkinsopp	38·0	62·0	5·1	1·60	0·80	0·80
Wigan (Lancashire)	37·0	63·0	3·0	1·25	0·60	0·65
Mortomly (S. York)	37·0	63·0	1·6	1·10	0·60	0·50
Elsecar (Yorkshire)	37·0	63·0	1·1	1·20	0·63	0·57
Ramsay (Newcastle)	36·8	63·2	6·6	1·75	0·94	0·81
Hastings' Hartley	36·5	63·4	2·0	0·95	0·50	0·45
South Tyne	36·3	63·7	3·9	2·10	1·10	1·00
West Hartley	35·8	64·2	4·7	1·10	0·60	0·50
Grigleston Cliff	35·6	64·4	1·6	1·40	0·75	0·65
Gosforth	35·0	65·0	1·0	1·10	0·50	0·60
Soap-house Pit	35·0	65·0	0·8	0·75	0·40	0·35
Nailsea (Somerset)	34·9	65·1	3·0	2·85	1·50	1·35
Leverson's Wallsend	34·9	65·1	4·9	1·30	0·65	0·65
Arley (Lancashire)	33·7	66·3	3·6	1·20	0·60	0·60
Lochgelly cannel	33·5	66·5	13·1	0·75	0·25	0·50
Woodthorpe (S. York)	33·1	66·9	10·5	1·20	0·70	0·50
Pelton Main cannel	31·5	68·5	9·4	0·95	0·49	0·46
Washington	31·3	68·7	2·2	1·30	0·67	0·63
Pelaw main	30·3	69·7	2·6	1·20	0·70	0·50
New Pelton	30·2	69·8	1·8	1·10	0·56	0·54
Coal-pit Heath	30·1	69·9	5·8	4·10	2·20	1·90
Garesfield	29·4	70·6	1·0	0·85	0·40	0·45
Dean's Primrose	29·3	70·7	2·4	1·40	0·71	0·69
Urpeth	28·7	71·3	1·4	1·00	0·60	0·40
Pelton main	28·4	71·6	1·4	1·10	0·62	0·48
South Peareth	27·8	72·2	1·8	1·20	0·60	0·60
Cumberland	25·6	74·4	1·4	1·10	0·60	0·50
Rhonda (S. Wales)	23·1	76·9	2·1	2·20	1·10	1·10

With regard to the manner in which heat is to be applied, it may be said, that, within certain limits, the more quickly it is so, the greater the quantity, and the better the quality of the gas obtained; for too slow a heat generates volatile matter, which condenses in tar; and too quick a heat decomposes the gas, and destroys its illuminating powers. Lastly, it may be remarked, that the duration of the heat ought not to exceed five hours; for at the expiration of that time the gases which are evolved are of little use for illuminating purposes, and the sulphur which is contained in the coke begins to distil over as bisulphuret of carbon, which is a most objectionable impurity.

Purification of Gas.—This is effected in various ways; some of the contrivances being mechanical in their action, and others chemical. When the volatile matters quit the retort, they consist of aqueous vapour, tar, olefiant gas, and other rich hydro-carbons, light carburetted hydrogen, hydrogen, carbonic acid, carbonic oxide, sulphuretted hydrogen, sulphuret of carbon, cyanogen, and ammonia. Only a few of these are required for illuminating purposes, and the rest must be got rid of. Foremost in the order of purification is that which takes place in the hydraulic main. There the most condensible of the empyreumatic vapours are deposited, and they run away into the neighbouring tank as a most fœtid mixture of tar and watery matters. Next to this is the condenser—an apparatus which exposes the gas to a large extent of cooling surface. We have already described the form of the apparatus, and said that it causes the condensation of the liquifiable matters contained in the gas. It precipitates tar, water, and ammonia, in combination with sulphuretted hydrogen, carbonic acid, and cyanogen. In some cases, the gas is made to pass through a vessel containing pieces of coke, over which a stream of water is constantly running. The gas enters at the bottom of the vessel, and, having made its way between the fissures of the coke, it escapes at the top. A stream of water runs over the coke in an opposite direction; and thus the gas is, as it were, washed and scrubbed by the two materials with which it is brought into contact. This apparatus is, therefore, very appropriately named a *scrubber*. The only impurities then left in the gas are ammonia, carbonic acid, sulphuretted hydrogen, cyanogen, and bisulphuret of carbon. These can only be removed by the aid of chemical absorbents; and the gas is, therefore, made to pass through a set of vessels which are named *par excellence*, the purifiers. These contain milk of lime, or lime that has been recently slaked. In the former case it is named a wet-lime purifier, and in the latter a dry. In both cases the gas enters at the bottom of the vessel, and it is either distributed in a stream of small bubbles through the liquid, or else it pursues its way between the moist particles of the recently slaked lime. By this contrivance, carbonic acid, cyanogen, and sulphuretted hydrogen are extracted from the gas; these combine with the lime, and produce a most unpleasant-smelling compound, which is technically termed *blue-billy*. In consequence of the loss of ammonia by this process of purifying, and the disgusting nature of the refuse materials, a number of patents have been taken out at various times for the purification of coal gas by other means than that of lime alone. The oxides of iron, in various conditions, have been patented by Messrs. Croll, Hills, Laming, and Lowe; the common salts of lead have been patented by Mr. Lowe and Mr. Losh; oxy-chloride of antimony, by Mr. Kirkham; sulphate of iron, with common salt and charcoal, by Mr. Cormack; sulphate of lime and magnesia, by Mr. Hills; superphosphate of lime, by Mr. Johnson; muriate of lime, by Mr. Laming; muriate of manganese, by Mr. Croll; and clay, by Mr. Bowditch.[*] Indeed, it would appear as if all the refuse matters of the arts had been successively tried and patented, in the hope of their becoming a means of extracting the impurities from coal gas. In most cases these substances merely absorb ammonia, and in a few instances they take up sulphuretted hydrogen also. Quitting the purifiers, which are charged with one or more of the preceding compounds, the gas enters the gasometer, and is in the condition in which it is to be supplied to the public.

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* At the present time (July, 1861), an action is pending in the Scotch law courts to decide this question, between the lessors and lessees of the Torbanehill property.—*Editor.*

The products of these operations are more or less valuable in every stage of the process. The coke which is drawn from the retort after the extraction of the gas, meets with a ready sale; the ammoniacal liquor which floats upon the tar in the tanks of the condenser and hydraulic main, contains enough ammonia to make about sixteen ounces of sulphate, twelve-and-a-half of carbonate, or eleven-and-a-half of muriate, from each imperial gallon; and as each ton of Newcastle coals produces from ten to twelve gallons of this liquor, there is a large amount of valuable matter generated in the process. The tar also is made to give up its wealth by the all-powerful aid of chemistry. As it leaves the manufactory, it is a dark-coloured, heavy liquid, of a most unpleasant odour: but, by distillation in rude iron boilers, it furnishes naphtha for lamps, dead-oil or creosote for railway timbers, and pitch or asphaltum for a variety of purposes. When the richer varieties of cannel coal have been used for the generation of gas, the tar also contains paraffine, which has already been described; and it likewise yields, by distillation, an oil which is largely employed for lubricating machinery. Within the last few years a number of patents have been taken out for the management of coal-tar; but as yet we have only begun to have an insight into the nature of the many valuable compounds that are locked up in it. Nevertheless, three classes of bodies have already been discovered in coal-tar; namely, neutral principles, acid substances, and alkaline matters. Among the first are benzole ($C^{12}H^{6}$), toluole ($C^{14}H^{8}$), cumole ($C^{18}H^{12}$), cymole ($C^{20}H^{14}$), naphthaline ($C^{20}H^{8}$), paranaphthaline ($C^{30}H^{12}$), pyrene ($C^{15}H^{4}$), chrysene ($C^{12}H^{6}$), paraffine ($C^{20}H^{21}$), and various liquid hydro-carbons, which nave not yet been isolated Among the acid substances, the most important is the acid of creosote, or carbolic acid ($C^{12}H^{6}O^{2}$); and of the alkaline matters there are pyridine ($C^{10}H^{5}N$), pycoline ($C^{12}H^{7}N$), and its homologue, aniline ($C^{12}H^{7}N$), lutidine ($C^{14}H^{9}N$), leucoline ($C^{18}H^{7}N$), a new base ($C^{16}H^{11}N$), and parvoline ($C^{18}H^{13}N$): in addition to which, there are, in all probability, other compounds which are isomeric with the preceding. Now the great and interesting fact which has been developed by the study of these compounds is, that they all contain but one element of nitrogen, and that, with one or two exceptions, they rise by regular gradations of two of carbon and two of hydrogen: first we have $10 + 5$, then $12 + 7$, then $14 + 9$, and so on. Besides which they are all isomeric, or have exactly the same composition with another series of bases called the aniline series. To the chemist these facts are of the greatest importance, not merely because of their individual interest, but because of their influence on the doctrines of science. At one time it was thought that whenever two things differed in their chemical and physical properties, they must be different also in their composition; and when Professor Faraday showed that one of the constituents of the fluid obtained by the condensation of oil gas, was identical, in its chemical composition, with another body (olefiant gas), notwithstanding that it differed from it in all its physical properties, chemists were hardly able to comprehend it; but ere long the fact was recognised as one of the most common occurrences in the whole range of chemistry. And thus an examination of the most insignificant of materials, will often furnish results that not only affect the ancient land-marks of science, but also open up a new way to the practice of industry and the acquirement of wealth. Who, for example, could have supposed that so disgusting a liquid as coal-tar—a liquid which, a few years since, the manufacturers of gas knew not how to dispose of—would, through the golden key of chemistry, be made the means of changing the aspect of chemical science, and of opening up new and profitable branches of industry? Take the crude tar, and submit it to distillation at a low temperature—the temperature of boiling-water: it yields a light volatile oil, which is commonly called naphtha. This consists, in great part, of a valuable ethereal liquid, which we have already described under the name of benzole—a liquid which may be used for burning in lamps, for dissolving resins, and for manufacturing a rich perfume (nitro-benzole), which has the delicious odour of the essential oil of bitter almonds. By continuing the distillation at a somewhat higher temperature, there passes over an oil which is heavier than water. This is called dead-oil; it contains the naphthaline, paraffine, creosote, and various liquid hydro-carbons, which have not been sufficiently well studied, but which perhaps contain mines of chemical wealth. The dead-oil is largely employed for the preservation of timber, and for lubricating machinery: and lastly, that which remains in the still is asphalt or pitch. As might be expected, there is a difference in the quality of these products, according as the tar is obtained from common Newcastle coal, or from the richer sorts of cannel.*

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[*] Mr. Leslie employs sulphate of copper and acetate of lead for such purposes, in his purifiers at the houses of the consumers.—*Editor.*

Of the other impurities or products of gas-making, cyanogen is the most important; already it has been extracted from the impure gas, and converted into Prussian blue. It is said that a ton of Newcastle coal will yield enough cyanogen to produce seven pounds of Prussian blue—a quantity that will, at the present market-price of the pigment, almost cover the original cost of the coal.

The mode of obtaining this compound is very simple. When the raw or impure gas is purified by hydrated oxide of iron, according to the patents of Croll, Laming, and Hills, the cyanogen combines with the iron, and produces the pigment in question. But when thus made it is largely contaminated with sulphuret of iron and other impurities. These may be got rid of by washing the mixture with dilute sulphuric or muriatic acid. Or if the iron compound is treated with a solution of potash, it gives up its ferro-cyanogen, and produces prussiate of potash, which is an equally valuable compound.

The following table, which has been constructed from the experiments of Messrs. Barlow and Wright, will afford some idea of the relative proportions of gas, tar, ammoniacal liquor, and coke, furnished, per ton, by different varieties of coal:—

Coal.	Gas in cubic feet.	Tar in lbs.	Ammo. liquor in lbs.	Coke in lbs.
Pelton main	9650	102	102	1,543
Newcastle cannel	9630	98	60	1,426
Wigan (Ince Hall) . . .	10850	248	162	1,332
Lochgelly cannel	8381	225	340	1,245
Boghead	13340	733	0	715
Lesmahago	10779	506	41	1,077
Ramsay cannel	9016	295	61	1,435
Derbyshire deep main . .	9400	219	179	1,335
Wemyss	10564	210	very little	1,156

Tests for the Impurities in Coal Gas.—Notwithstanding that so much trouble is taken with the purification of coal gas, yet it is found that a greater or less proportion of the several noxious compounds already described, will escape absorption, and will find their way into the street mains. The most important of these are ammonia, carbonic acid, sulphuretted hydrogen, bisulphuret of carbon, and tarry matter.

The first of these is objectionable from the circumstance that it attacks the fittings, corrodes the meters, and fixes the stopcocks; besides which, it has the property of holding tar in suspension. Ammonia ought not to exist in coal gas to a greater extent than one part in about 50,000: that is, 100 cubic feet of gas ought not to contain more than about 3·5 cubic inches, or rather more than half a grain, of ammoniacal vapour. Anything approaching to this quantity may be readily discovered by means of turmeric paper, which is immediately reddened by the impure gas.

The presence of carbonic acid may be known by collecting a bottleful of the gas, and shaking it up with a little lime-water; if the impurity be present, the lime-water will be rendered milky. The great objection to carbonic acid is, that it reduces the illuminating power of the gas, and thus lowers its value. Mr. Wright and Mr. Lewis Thompson say, that every one part of carbonic acid in a hundred of gas, reduces its illuminating power to the extent of eight or ten per cent.

Sulphuretted hydrogen may be discovered by a solution of sugar of lead, a little of which ought to be dropped on a strip of white paper, and then held in the gas for a period of not less than ten minutes or a quarter of an hour. If the paper becomes discoloured, sulphuretted hydrogen is present. There are several reasons why this impurity should not exist in coal gas: it gives to the gas a fœtid odour; it tarnishes silver, and destroys the beauty of paint; and in the act of burning, it generates sulphuric acid, which acts injuriously on books, linen goods, and other textile fabrics.

Bisulphuret of carbon is not so easily detected, for the gas must be burnt under a platinum rosette, and the products of combustion collected in a vessel containing a little ammonia. Mr. Wright, of the Western Gas Company, has contrived an apparatus for this purpose. Having consumed about twelve cubic feet of gas, at the rate of half a cubic foot per hour, it will be found that six or seven ounces of water will have condensed in the receiver. This is to be treated with a solution of nitrate of baryta, that has been rendered acid by a little nitric acid. If, in the course of a few hours, a white powder settles to the bottom of the liquid, then bisulphuret of carbon was present in the gas; and if the precipitate is collected and weighed, the quantity of impurity present can be determined; for every 234 grains of sulphate of baryta represent 38 grains, or nearly 46 cubic inches, of the vapour of bisulphuret of carbon. By proceeding in this way, it will be found, that 100 cubic feet of coal gas yield from 50 to 300 grains of sulphate of baryta—quantities that represent from 8·1 to 48·6 grains of bisulphuret of carbon. The presence of this impurity in coal gas is a most serious affair: for it has been shown by Dr. Letheby, in several of his reports to the Corporation of London, that bisulphuret of carbon, in the act of burning and oxydising, forms sulphuric acid; a great portion of which escapes in a corrosive form, and does enormous damage to every kind of textile fabric. He states, that the books in almost every library in the kingdom where gas is used, are falling to pieces from the action of this acid upon the covers; and he makes reference to the libraries of the Athenæum Club-house, the London Institution, the Royal College of Surgeons, the Portico Library at Manchester, and that of the Literary Society at Newcastle-upon-Tyne, for examples of the mischief done. In most of these places, the injury has been so great that a remedy has been called for. This remedy consists in burning the gas in such a way, that the products of combustion shall be carried off as soon as they are generated.

As regards the quantity of sulphuric acid which is produced in this manner from bisulphuret of carbon, we may say that there is some discrepancy of opinion. Mr. Lewis Thompson states that coal gas rarely contains less than the one-thousandth of its bulk of bisulphuret of carbon; and if so, 100 cubic feet of the gas can rarely produce less than 29·8 grains of anhydrous sulphuric acid. In another place he states, that he has never obtained less than 40 grains of such acid from 100 cubic feet of gas. On the other hand, Dr. Letheby asserts that he rarely obtains more than 20 grains of the acid. But let the quantity be what it may, it is evident that mischief must arise from its presence, and that the attention of chemists and gas manufacturers ought to be especially directed to the subject. The quantity of this impurity is small when the gas has been generated at a low heat; but it is large when the temperature of the retort has approached a full red, or when the charges have been kept in beyond five hours. If, therefore, no means can be adopted for the removal of the impurity after it has once been formed, it is manifest that care should be taken not to produce it.

Tarry matter is held in suspension in coal gas by means of ammonia; consequently, if we pass the gas through a small bottle or tube containing fragments of flint, moistened with dilute sulphuric acid, we shall absorb the ammonia, and arrest the tar. When coal gas, containing much tar in suspension, escapes through the fissures in the street-pipes, it impregnates the soil of the neighbourhood, and gives it a most offensive odour.

Atmospheric air is sometimes present in coal gas when the exhauster has been doing its work too energetically, or when there have been leakages in the vessels or pipes between the retorts and the exhauster. This is recognised by the blueness or thinness of the flame, and by its low illuminating power.

The Analysis of Coal Gas is rather a complicated undertaking; though a general idea may be formed of its composition by the following process:—Collect a quantity of gas in a graduated tube over quicksilver; introduce a solution of caustic potash, and agitate it with the gas for some minutes; then allow the tube to stand for a short time, and observe how much carbonic acid has been absorbed. After this has been done, pass up a concentrated solution of pyrogallic acid in potash, or, better still, a small quantity of the powdery crystals of pyrogallic acid itself; shake the tube again, and having allowed it to stand for a few minutes, read off the bulk of oxygen that has disappeared; transfer the gas to another tube over water, and by means of a syringe, or other contrivance, introduce a small quantity of bromine, or its strong solution; shake the tube for a minute or so, and observe that the bromine is in sufficient quantity to give the gas an orange-red colour; after the lapse of four or five minutes, pass up a solution of potash, and shake again. In this way the excess of bromine will be absorbed; and on allowing the tube to stand for a short time, the amount of condensible hydro-carbon may be determined by the loss in bulk. Transfer the gas to another tube, and agitate it with a solution of dichloride of copper in muriatic acid (this is made at once by mixing equal parts of black oxide of copper and recently precipitated copper, with hydrochloric acid). After a few minutes, the solution is to be withdrawn, and the gas washed with potash; the loss in bulk indicates the quantity of carbonic oxide present. Lastly, a portion of the residual gas is to be transferred to an eudiometer, and mixed with about twice its bulk of oxygen; and the mixture is to be fired by the aid of an electric spark. After standing for a few minutes, the loss in bulk is to be observed; a solution of caustic potash is then to be passed up into the gas, and the absorption of carbonic acid noted. This indicates the amount of light carburetted hydrogen present; and then by subtracting twice this volume from the total amount of diminution caused by the detonation, we obtain a number, two-thirds of which represent the hydrogen of the gas. Lastly, the residual gas, from which the portion for the oxygen experiment was taken, is to be mixed with about four times its bulk of pure chlorine, and exposed for some hours to daylight, or for a moment or two to sunlight, and then washed with potash—the residue is nitrogen. In this way we may obtain an estimate of the proportions of the chief constituents of coal gas. These, however, vary to the following extent:—

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* Gas tar is largely employed by calico printers and dyers, in producing mauve, Magenta, and other colours.—*Editor.*

Light carburetted hydrogen,	from 35	to 52	per cent.
Hydrogen	,, 25	,, 52	,,
Carbonic oxide . . .	,, 7	,, 9	,,
Carbonic acid . . .	,, 0	,, 4	,,
Oxygen	,, 0	,, 2	,,
Nitrogen	,, 0	,, 8	,,
Condensible hydro-carbons.	,, 3	,, 20	,,

The approximative or commercial value of coal gas is determined in several ways: thus—

1st. *By means of the Photometer.*—This we have already alluded to; and it is only necessary to say here, that the methods employed are four-fold—namely, Count Rumford's plan, with shadows; Ritchie's, with his instrument that has two reflecting surfaces and a screen in a dark chamber; Wheatstone's, which consists of a mechanical contrivance, whereby a silver bead is made to revolve in such a manner as to produce a geometrical figure with two outlines; and, lastly, the plan usually employed, which is that of Bunsen's, with a waxed screen and a graduated rod—a description of which has already been given. Some doubt exists as to the kind of illuminating standard which ought to be adopted. Originally the standard was a mould candle of six to the pound; but there are so many objections to its use that it has long been discontinued. Dr. Frankland employs a composite candle; Dr. Fyfe prefers one of wax; and the standard which is specified in most of the Acts of Parliament relating to the subject, is a wax candle of six to the pound, burning at the rate of 120 grains per hour. In general, however, it is found, that there are more irregularities with the combustion of wax than with sperm; and, consequently, the latter is now almost always employed. Sometimes the comparison is made with one candle; Dr. Letheby uses two, and Mr. Evans employs three. A jet of gas composed of hydrogen and olefiant gases, mixed in the proportion of nine of the former to one of the latter, has been proposed as a standard of constant value; but the use of it is very inconvenient: and, lastly, Mr. Lewis Thompson has suggested the employment of paraffine.

Finally, it ought to be stated, that whenever the quality of gas is estimated by any of these methods, it ought to be consumed from the burner or jet which is best fitted for its combustion. If this is not attended to, a discrepancy to a large extent may arise between two different experimenters. As a rule, common gas requires a larger aperture for combustion than cannel; and high glasses, or chimneys, are apt to lower the illuminating power of the former, and to raise that of the latter. Different experimenters have exhibited different fancies with regard to this part of the subject: for example, Mr. Wright prefers a single jet, one-eighth of an inch in diameter, for his investigations; Dr. Fyfe makes use of a Winfield or Aberdeen burner; and in London, the burner fixed by Act of Parliament is an Argand of sixteen holes, with a seven-inch chimney, consuming five cubic feet per hour. There is no doubt, however, that in many cases a fish-tail or bat's-wing will afford the best light for testing purposes. We append the following table, to show how much the character of the burner affects the quality of the results obtained:—

Burner.	Consumption per hour in cubic feet.	Illuminating power.	Illuminating power per foot.
Jet five inches high . .	1·00 . .	1·00 . .	1·00
Small fish-tail	1·98 . .	2·89 . .	1·45
Large ditto	2 60 . .	4 00 . .	1·53
Small bat's-wing . . .	3·00 . .	4·40 . .	1·46
Large ditto	4·60 . .	8·40 . .	1·87
Argand of forty holes .	4·50 . .	7·84 . .	1·74

These results were obtained by Dr. Fyfe with cannel gas; and they show that the large bat's-wing produces a flame that, for equal consumption, is nearly twice as powerful as that with the single jet.

Again, the quantity of gas consumed per hour in the same burner will affect the results; thus—

Burner.	Consumption per hour in cubic feet.	Power of gas per foot.
Argand, with seventy-two holes . .	7·0 . .	5·57
,, ,, ,, ,, . .	5·0 . .	6·60
,, ,, ,, ,, . .	3·3 . .	3·40

From which it is manifest, that in ascertaining the illuminating power of gas, great judgment is necessary, both in the selection of the burner and in the rate of consumption, in order to obtain fair and proper results.

2nd. *The Chlorine test* is very much appreciated by Dr. Fyfe. It was originally proposed by Dr. Henry, and in his hands it afforded very accurate results. The objection to the test is its inconvenience; for chlorine takes a long time to prepare, and we are never certain of its being pure. Besides which, it is an unpleasant gas to inhale; and, escaping into the laboratory, it produces the most serious injury to the brass and iron-work of chemical apparatus. The mode of conducting the experiment is this:—A quantity of gas is to be let up into a graduated tube over water; the tube is then to be covered so as to exclude light, and chlorine is to be passed up into it. After standing for ten or fifteen minutes in the dark, the excess of chlorine is to be absorbed by potash, and the amount of absorption read off. The larger the quantity absorbed, the better the gas. This will range from three to twenty per cent. The matters absorbed by the chlorine are the condensible hydro-carbons, which are the illuminating principles of the gas.

3rd. *The Bromine Test.*—Many years ago, M. Balard showed that bromine had the power of absorbing olefiant gas, and that in this respect, as in most others, it was like chlorine. Lately, Mr. Lewis Thompson has taken advantage of this property, and has made it the means of discovering the quality of coal gas. The reactions of bromine on gas are exactly the same as those of chlorine; but it has an advantage over the latter, in the circumstance that it is much more manageable, that it is more likely to be pure, and that the admission of light does not affect the results. In manipulating with this liquid we fill a graduated vessel, called a Cooper's tube, with gas, and then pour into the shorter leg of the instrument a small quantity of a saturated solution of bromine in water, taking care to use enough to give the gas an orange-red colour. After the mixture has been shaken, the tube is allowed to stand for about ten minutes, and then the excess of bromine is to be absorbed by means of potash; after which, the amount of absorption is noted. As in the last case, this will range from three to twenty per cent., according to the quality of the gas.

4th. *The Sulphuric Acid Test.*—Professor Faraday long since observed, that when concentrated sulphuric acid was brought into contact with the condensible hydrocarbons, it speedily absorbed them. Relying upon this fact, Professor Bunsen has recommended that fuming or anhydrous acid should be employed for the purpose of ascertaining how much of these agents is present in coal gas. Messrs. Leigh and Frankland have spoken well of the results obtained in this manner; but however successful the process may be in their hands, it is open to many fallacies, and cannot, therefore, be recommended to the unskilful operator. The mode of experimenting is this:—The gas is to be collected in a graduated tube over mercury; and then a piece of coke or pumice-stone, fastened to a platinum wire and moistened, or rather saturated, with the acid, is to be passed up into the gas; after remaining in contact with it for ten minutes or a quarter of an hour, the coke is to be withdrawn; and, as a small quantity of sulphurous acid will have been formed by the action of the coke on the mercury, the gas is to be washed with a little potash, and then the amount of absorption noted. Sulphuric acid does not, however, attack all the hydro-carbons; for it is found that chlorine or bromine will effect a further condensation after the action of the acid. This, with other circumstances, renders the process objectionable.

5th. *The Explosion Test.*—Dr. Henry noticed that there was a direct relation between the value of a gas for illuminating purposes, and the quantity of oxygen required to burn it, or of carbonic acid produced thereby. In fact, as the illuminating power of any gas is dependent on the quantity of carbon contained in a given bulk of it, it follows that the products of its combustion must furnish a sure indication of its value. This will be manifest from the following table:—

One volume of	Oxygen required to burn.	Carbonic acid produced.
Marsh gas (CH^2)	2 volumes .	1 volume.
Olefiant gas (C^4H^4) . . .	3 „ . .	2 „
Super-olefiant gas (C^6H^6) .	4·5 „ . .	3 „
Faraday's gas (C^8H^8) . .	6 „ . .	4 „
Bicarburetted hyd. (C^6H^3) .	7·5 „ . .	6 „

So that if we mix a known quantity of any gas with about three times its bulk of oxygen, and explode them in an eudiometer by means of electricity, or make them combine by the aid of spongy platinum, as Dr. Henry suggested, the amount of oxygen consumed, and of carbonic acid produced, will serve as indications of the quality of the gas. Dr. Henry found that the best description of coal gas requires two-and-a-quarter times its bulk of oxygen for combustion, and gives one-and-a-quarter of carbonic acid; while the worst gas of his time took only eight-tenths of its bulk of oxygen, and gave but three-tenths of carbonic acid. The amount of carbonic acid produced is to be determined in the usual way by means of a solution of caustic potass.

6th. *The Specific Gravity Test.*—This is founded on the fact that the rich hydro-carbons are much heavier than the poor ones; for example, if a given bulk of marsh gas, or light carburetted hydrogen, weighs 10 grains, the same bulk of olefiant gas will weigh 17½ grains; and, in the case of the other hydro-carbons, the increase in weight is still greater. A knowledge of this fact will enable us to ascertain the value of any description of gas. We take a glass globe, or flask, fitted air-tight to a stopcock, and exhaust it with great care, by means of an air-pump; then let in pure and dry hydrogen, and again exhaust. Do this a third or even a fourth time, so as to get the flask as empty of air as possible; then weigh in a delicate balance, and note the amount: pure and dry atmospheric air is now to be admitted, and the flask is to be weighed again. In this manner we ascertain how much of pure dry air, at a temperature of 60° Fah. and a pressure of 30 inches of the barometer, it contains. When we wish to take the specific gravity of any gas, the globe is to be exhausted as before, then filled with the gas, and weighed; corrections are to be made for any abnormal difference of temperature or pressure; and then we say, as the weight of the vesselful of air is to 1, so is the weight of the gas to its specific gravity. In practice it will be found convenient to have a globe with two stopcocks, one opposite the other; so that after the first exhaustion and weighing, the globe can be easily filled with gas without the aid of an air-pump, by simply allowing the gas to pass through it for about a quarter of an hour. Mr. Wright has constructed an apparatus which still further simplifies this calculation. It consists of an oiled silk balloon that holds 1,000 cubic inches of gas; and, as coal gas is lighter than air, he determines its specific gravity by ascertaining the number of grains which the balloon will carry up. A book that accompanies the apparatus contains instructions for the management of the experiment. The specific gravity of coal gas ranges between 390 and 750. The former is about the weight of the worst gas from Garefield coals, and the latter of the best from Boghead cannel; a good average of specific gravity is 450. In conducting experiments of this kind, it must be ascertained that the gas does not contain carbonic acid, carbonic oxide, or atmospheric air; for if it does, the specific gravity of the gas is sure to be high, notwithstanding that the illuminating power may be very low.

Before we leave this part of the subject, it may be remarked, that if the specific gravity of a gas be taken before its condensation by bromine, and then again afterwards, the difference in weight will afford a means of ascertaining the specific gravity of the condensed portion; and if this be multiplied by the amount of condensation, we obtain a number that represents very nearly the illuminating power of the gas in sperm candles, as it is usually expressed. For example: a gas of specific gravity 447, has a condensation of 5 per cent., and the residual gas has a specific gravity of 328. Now since 100 cubic inches of the former weigh 13·63 grains, and 95 cubic inches of the latter 9·5 grains, the 5 cubic inches of the condensed portion must have weighed 4·13 grains, and it must have had a specific gravity of 2·7. This, multiplied by 5, the amount of condensation, gives 13·5 as the illuminating power of gas. Experiment showed it to be 14.

7th.—*The Durability Test.*—Dr. Fyfe, who is an authority in matters of this description, is accustomed to estimate the value of a gas, not only by noting its amount of condensation with chlorine, but also by observing the time that it takes to burn a given bulk of it from a jet of a given size, with a flame of a given height. The jet which he employs has an aperture of the $\frac{1}{33}$rd of an inch in diameter, and the flame is four inches in height. The first of these tests he calls the *quality* test, and the latter the *durability*. "I consider," he says, "both of these circumstances absolutely necessary; for, though some have insisted only on the one, and some on the other, yet, unless both be taken into account, we do not arrive at the true *value* of the gases, and, consequently, cannot compare one with another for the purpose of illumination." He thinks it possible that two gases may afford, by combustion, the same amount of light for the same height of flame, but that one may burn away half as fast again as the other; and, consequently, if no regard is paid to this circumstance, there will be a false estimate of their relative values. His mode of amalgamating these two powers is to multiply the per-centage of the amount of condensation by the time required to burn a cubic foot; and in this way he obtains a result that may be said to represent the true value of the gas. Suppose that one gas, which he takes as a standard, has a condensation of 4·33, and a durability of 50·5 minutes—these multiplied together make 218·7, which may be called the value of the gas. Another gas has a condensation of 7·55, and a durability of 57 minutes; these multiplied together give a value of 430·3. Now if we call the standard number of the former 1, that of the second will be 1·95; and thus we obtain their relative values. The following table is constructed on this principle:—

Gas from	Condensation by chlorine.	Durability of a cubic foot.	Relative value.
English caking coal (Newcastle)	4·33	50′ 30″	1·00
Peareth and Pelton	6·50	50′ 40″	1·50
Yorkshire Parrot	7·66	52′ 30″	1·85
English cannel (Wigan)	7·55	57′ 0″	1·93
Ramsay Parrot	12·00	62′ 0″	3·40
Mid-lothian	13·00	60′ 0″	3·56
Lesmahago	17·10	65′ 0″	5·07
Scottish Parrot	15·00	80′ 0″	5·46
Wemyss	19·50	75′ 0″	6·69
Kirkness	20·70	80′ 0″	7·75
Boghead	22·40	81′ 3″	8·32

It will be noticed, that when the value of a gas is tested in this manner, it indicates a higher quality than we can obtain by means of the photometer; but this Dr. Fyfe regards not as an error in his process, but rather as an evidence that we do not take the best means to burn the gas to the best advantage; and consequently that the illuminating power is, in the case of the richer cannels, a little below its true value.

As a corollary to the preceding, we append a table, which shows the quantity of gas furnished by different varieties of coal, together with the specific gravity of the gas, its illuminating power in sperm candles of 120 grains each, its per-centage of condensation by bromine, and its actual value in sperm candles. This table has been compiled from the results obtained by Mr. Evans, Mr. Thompson, Dr. Letheby, and others.

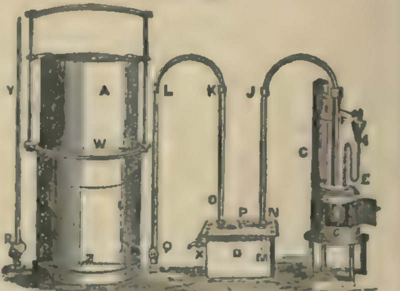
Coals.	Cubic feet of gas per ton.	Illuminating power.	Specific gravity.	Condens. by Brom.	Value in grains of Sperm
Boghead cannel	15,000	37·75	752	30·0	113,250
Lesmahago, No. 1	13,500	27·10	642	16·0	73,170
Wemyss	14,300	24·50	580	14·0	70,070
Lesmahago, No. 2	13,200	24·80	618	17·0	65,472
Capeldrae	14,400	19·75	577	16·5	56,880
Armiston	12,600	22·50	626	17·0	56,600
Kirkness	12,800	21·20	562	10·2	54,272
Knightswood	12,200	19·00	550	9·5	50,160
Wigan (Ince Hall)	11,400	20·00	528	11·5	45,600
Ramsay	10,300	21·40	548	12·5	44,084
Pelton cannel	11,500	18·50	520	10·5	42,750
Leverson cannel	11,600	18·00	525	10·0	41,720
Washington cannel	10,500	18·00	500	10·5	37,800
Brymbo main	10,500	15·00	540	6·8	31,500
Pelton main	11,000	14·00	430	4·5	30,800
Dean's Primrose	10,500	12·00	430	5·0	28,350
Washington	10,000	14·00	430	5·0	28,000
Pelaw	11,000	12·75	420	4·5	28,050
Brymbo cannel	6,650	20·00	504	11·5	27,160
Blenkinsopp	9,700	14·00	450	6·0	27,160
Leverson	10,800	12·50	425	4·0	27,000
West Hartley	10,500	12·50	420	4·2	26,250
Hasting's Hartley	10,300	12·50	421	4·3	25,750
New Pelton	10,500	12·00	415	4·8	25,200
Garefield	10,500	11·50	398	3·8	24,150
Gosforth	10,000	12·00	402	4·0	24,000

Oil Gas.—Originally this was the gas most generally in use for illuminating purposes; but the cost of its manufacture was found to be too great for its continued employment.

As far back as the year 1805, Dr. Henry published an account of some experiments which he made on a gas obtained from sperm oil, and he showed that in illuminating power it was the only one which could compete with olefiant gas. Soon after that, Messrs. J. and P. Taylor contrived an apparatus for procuring it on a large scale. Their apparatus consisted of a furnace in which was placed a twisted iron tube, containing fragments of coke. The object of this arrangement was to increase, as much as possible, the extent of the heating and decomposing surface. When the tube was red-hot, oil was suffered to run into it in a small but continuous stream. In this way the oil was decomposed and resolved into inflammable vapour, which escaped, and a fine spongy charcoal, which remained behind. The gases evolved were merely washed with water, and then collected in a gasometer. By this process it was found, that a gallon of whale-oil yielded about ninety cubic feet of gas, which had a specific gravity of 900, and was about twice as rich in illuminating principles as the best description of coal gas. Other patentees, as Mr. English, Mr. Booth, and others, followed in the same course, and endeavoured to perfect the process so as to work it on an economical scale; but notwithstanding that every means were taken to accomplish this, and that the commonest oils were used for the production of the gas, it was found that it could not compete with the cheaper gas obtained from coal: and therefore the process was abandoned.

An attempt was made to revive it by a company, which adopted the name of the Vegetable Gas Company; but the attempt has not been successful. Nevertheless, it may be said, that where coal gas is very dear, or not to be obtained at all, or where, for sanitary or other purposes, an unusually pure gas is required, then the process of making oil-gas may be practised with advantage. To meet such cases, a small apparatus has been contrived by Mr. Skelton; and a somewhat similar one is sold by Messrs. Burgess and Key, of Little Britain. The latter consists of a small cylinder B (Fig. 125), fitted into a furnace C, so that it may be made red-hot. A reservoir I, containing oil, or refuse fat, is suspended near the chimney of the furnace G, and when the fat is perfectly liquid it is allowed to drip from a small tap into a syphon pipe E, whence it runs into the red-hot cylinder, and is decomposed. The evolved gases escape by the tube F, and are conveyed by J into a purifier D, which contains water; the delivery tube M dips an inch or so under water, in order that the gas may be washed and cooled. The tube K L transmits the gas from the purifier into the gasometer A. The gas which is produced in this manner is of high illuminating power; and it appears, from the statements made by the venders of the apparatus, that a pound of kitchen-stuff yields from ten to eleven cubic feet of gas. Now a pound of this material weighs 7,000 grains; and as the gas produced weighs only about 5,000 grains, there is manifestly a loss of nearly one-third. This occurs in the form of charcoal, which remains in the retort; and hence the necessity for a frequent cleaning out of this part of the apparatus. Dr. Fyfe states that the loss in his experiments was much greater—that it amounted to nearly one-half of the fat used; and the same remark is also made by Mr. Wright, who has reported upon the comparative economy of the vegetable-gas scheme. The latter gentleman states, that for a gallon of 9 lbs., the waste amounts to 2 lbs. 10 oz.

Fig. 125.



The gas which is produced from oil is of high specific gravity, and ranges between 700 and 900. It contains from 30 to 40 per cent. of rich hydro-carbons, which may be condensed by chlorine and bromine: of these about 20 per cent. are absorbed by anhydrous sulphuric acid: 100 volumes of the gas require about 260 volumes of oxygen for their consumption, and they yield 158 volumes of carbonic acid. The durability of the gas, with Dr. Fyfe's four-inch flame from a jet the $\frac{1}{33}$rd of an inch in diameter, is 68′ 20″; consequently, its value is ten times as great as that of his standard of gas from ordinary Newcastle coal. As might be expected, the illuminating power of this gas is very high: indeed, Mr. Wright has stated that it is about four times as great as that of common gas. He reports that, with a consumption of one foot and a quarter per hour, it gives the light of twelve sperm candles, each burning at the rate of 120 grains per hour. Our own experiments, however, are not so satisfactory: they are recorded in the following table:—

Burner used.	Consumption per hour.	Illuminating power in sperm candles of 120 grs. per hour.
Small fish-tail	1·50	8
Leslie's Argand	2·25	11
Common Argand, 15 holes	3·50	14
Fine Argand, 70 holes	2·75	12
Winfield Argand	3·50	15

The advantages of this gas are, its freedom from smell; its not containing any sulphur compound; its not producing so much heat as ordinary gas for an equal amount of light; and its easy production in an apparatus which does not occupy much room. The chief disadvantage is its cost; for a shilling's worth of kitchen-stuff will only produce about 40 cubic feet of gas, which will not go farther than six or eight pennyworth of coal gas.

Portable Gas.—When oil-gas is compressed with a power of from fifteen to twenty atmospheres, it is forced into a very small bulk; and the vessels containing it may be moved about, and placed in any situation where a light is required. A company was formed some years ago for carrying out this object; and they condensed the gas in globes and cylinders which were subsequently placed in vases, columns, and other ornamental devices, so as to be completely hidden from view. Burners were fixed to the apparatus, and the gas was let out by means of a stopcock of a peculiar construction. In the act of compressing the gas, about one-fifth of its volume underwent condensation into an oily liquid; indeed, a thousand cubic feet of good oil-gas yielded about one gallon of it. This was made the subject of investigation by Professor Faraday. It is a thin oily fluid, lighter than water, sometimes transparent and colourless, occasionally opalescent, having a red tint by transmitted, and a green by reflected, light. Its specific gravity is ·821. This fluid Mr. Faraday found to be a mixture of various hydro-carbons of different degrees of volatility; and taking advantage of this circumstance, he was enabled to separate them. The great bulk of the liquid distilled at a temperature of from 176° to 190°; but a portion also came over at a temperature of 98°, and another portion required a heat of about 200° to volatilise it. These liquids consisted of carbon and hydrogen in various proportions. The most volatile was composed of equal parts of carbon and hydrogen (C^4H^4), and a part of the intermediate portion consisted of two of carbon to one of hydrogen (C^6H^3). The others were not sufficiently examined to determine their exact composition, although there was every reason to believe that they were composed, like the first, of equal atomic particles.

M. Couerbe has also examined the liquor which is produced by the compression of resin gas, and his results are quite as interesting as those of Professor Faraday. He finds that the liquid contains six different fluid hydro-carbons; five of which consist of four atoms of hydrogen and a proportion of carbon that rises in arithmetical progression from four to eight. The sixth is a compound of twenty-eight atoms of carbon and twenty-two of hydrogen.

Resin Gas.—When it was found that oil could not be profitably used for the manufacture of gas, attention was naturally directed to other and cheaper gas-producing substances. Resin was therefore selected; but it was noticed, that when this body was decomposed in the apparatus commonly employed for the generation of oil-gas, it produced a quantity of thick bituminous matter which choked the exit-pipe. This, at first, was regarded as an insurmountable difficulty; but it was overcome by the late Professor Daniell, who contrived an arrangement whereby the gas was delivered from the bottom of the retort by means of a descending pipe. His patent was taken out for this contrivance about thirty years ago, and Messrs. Taylor and Martineau were the first to put it into operation on a large scale. The retort was set in the furnace in the usual manner, and was charged with coke in order to increase the heating-surface. Across the extreme end of the retort there was a diaphragm, which was open at the top; and beyond this was the communication with the down-going delivery-pipe. The diaphragm served to prevent any of the coke from falling into the exit-tube. The resin was dissolved, or rather liquefied, by the aid of heat, in a small quantity of oil of turpentine, or of the volatile oil produced by the decomposition of the resin in a former process. The proportions were about twenty pounds of resin to a gallon of oil. This mixture was kept hot and fluid, in a reservoir placed over the fire above the retort; and when the contents of the latter were red-hot, they were allowed to run through a syphon tube, and to drop upon the incandescent coke at the nearest end of the retort. The vapours which were thus produced passed along the whole length of the retort, and were completely decomposed before they escaped over the diaphragm into the descending pipe. A part of these vapours was liquefied by the cold of the condenser, and such appeared as a limpid volatile oil; the other portion passed on as gas; and, after having been purified by washing with lime-water, was collected in the gasometer. This process of Daniell's was considered to be very complete, though it has been modified in certain particulars by subsequent patentees: thus, Mr. Richardson has proposed that the resin should be mixed with sawdust and some alkali, as lime, potash, or soda; and that it should be put into the retort in thin sheet-iron vessels of a cylindrical form. In these the first distillation takes place, and the volatile products are conveyed into other vessels, or retorts, charged with coke, lime, or broken bricks, kept at a red heat. By these means it is thought that more gas, and less volatile oil, are obtained. Mr. Webster generates the gas in another way. He passes the vapour of coal-naphtha over a bed of heated coke, and thence over a quantity of melted resin. The volatile matters so obtained, traverse another vessel filled with incandescent coke, and there they undergo decomposition.

A pound of resin will yield about ten cubic feet of gas; or, more correctly speaking, a hundredweight of resin will furnish from 1,000 to 1,200 cubic feet of gas, and about three gallons of oil. The gas has a specific gravity of from 660 to 850; and its illuminating power is but a little inferior to that of oil-gas. When it is imperfectly made, it does not contain more than eight or ten per cent. of condensible hydro-carbons; and it was found by Dr. Fyfe to have a durability of fifty-three minutes and twenty seconds: its value, therefore, is about the same as that of gas from Wigan cannel. As in the case of oil-gas, this illuminating agent does not contain a particle of any sulphur impurity.

Although the production of gas from resin is not practised in this country, in consequence of the more economical source of coals, yet in many parts of America, where resin and crude turpentine are cheap and abundant, the process is still worked with advantage.

Hydro-carbon Gas.—This name is given to the mixed gases which are generated from water, together with substances that are rich in hydro-carbons, as tar, resin, fats, oils, and the better kinds of cannel coal.

Although this description of gas has but recently been made on the large scale, yet the principles of its manufacture were known and appreciated many years ago. In the year 1830, Mr. Donovan took out a patent for the generation of gas from steam, by passing it over red-hot coke or charcoal, and subsequently naphthalising it at the burner, by means of turpentine or coal-tar. Since that time many improvements have been made in the process: and we may refer to the names of Manby, Val-Marino

Radley, Lowe, White, Croll, Webster, Barlow, and Gore, in proof of the number of persons who have taken out patents in this country for the generation and naphthalisation of water-gas. In every case the water, or rather the steam, is decomposed by passing it over red-hot coke or charcoal; and it is subsequently naphthalised by passing it over rich hydro-carbons.

The details of the process vary with the nature of the materials used. When fat and resin are employed, the process, as described by Mr. White in his patent, is as follows:—Two retorts, about seven feet long and nine inches in diameter, are set vertically in a furnace so that they may be heated to full redness, or even to an incipient white heat. A flue passes through the centre of each of the retorts, in order that the contents may be thoroughly heated. They are packed full of coke, charcoal, or anthracite, and a few scraps of iron. When the retorts are red-hot, water is allowed to flow through a syphon upon the top of the ignited mass. The steam which is thus generated, passes over the red-hot coke, and is decomposed into hydrogen and carbonic oxide, which is incidentally attended with more or less of carbonic acid. The gases and vapours thus produced, escape from the bottom of the retorts into two others placed horizontally in the same furnace. These are about six feet long, and are divided into two or more compartments by longitudinal divisions which reach nearly to the end. The compartments are filled with coke, chains, or coils of iron wire, so as to increase the surface; and into the first of these, tar, fat, or resin is allowed to flow, so as to undergo decomposition. The evolved gases are rapidly carried off and mixed with the gaseous products of the first retort; and after they have passed through a condenser and a lime purifier, they go on to the gasometer. In working the process, the supply of the materials should be so managed, that the gaseous mixture should consist of about four parts of water to six of the material. When resin is used, the proportions of the various materials are, one hundredweight of resin, seven-and-a-quarter gallons of resin-oil, fifteen pints of water, a sixth of a bushel of charcoal, and a few ounces of scrap iron. These produce from 1,500 to 1,600 cubic feet of gas, and about three gallons of oil. Dr. Frankland has entered into a very detailed examination of this process; and it appears from his report, that a hundredweight of resin, with from thirty to forty pints of water, will yield from 1,500 to 1,900 cubic feet of gas, and from two to four gallons of oil. The temperature at which the best results are obtained is that of dull redness; for if the heat be carried much above this point, the richer gases are decomposed, and changed into several gases of low illuminating powers. He found that two distinct changes were effected in the water-retorts. In one case the steam was decomposed by the red-hot charcoal, and equal volumes of hydrogen and carbonic *oxide* were indirectly produced; in the other, it was converted into two volumes of hydrogen, one of carbonic *acid* being also evolved. These mixed gases, together with some undecomposed steam, pass into the resin retort, where they mix with the vapours of the decomposing resin, and pass twice along the length of the red-hot retort. Here the steam is again subjected to decomposition by the agency of the fuliginous matter evolved from the resin. This change is a very important one, for it prevents the accumulation of carbon in the exit-pipe of the retort. If the gas be examined at this stage of its manufacture, it will be found to contain a very large proportion of carbonic acid; and one of the greatest difficulties in the subsequent management of it, is the removal of this impurity. Ordinary wet lime will only remove a portion; and hence Dr. Frankland has recommended the use of caustic soda, which may be produced by the admixture of carbonate of soda with cream of lime.

The gas which is thus produced has a specific gravity of from 600 to 660. It contains from seven to eight per cent. of condensible hydro-carbons; and its illuminating power is about the same as that of ordinary cannel gas. Its composition is as follows:—

	Ordinary gas.	Pure gas.
Condensible hydro-carbons	7 41	8 13
Light carburetted hydrogen	26·50	29·71
Hydrogen	40·27	43·38
Carbonic oxide	18·55	18·78
Carbonic acid	7·27	0·00
	100·00	100·00

According to Dr. Frankland's report, hydro-carbon gas, from steam and resin, may be manufactured at from 9½*d.* to 1*s.* 1½*d.* per 1,000 cubic feet. This is irrespective of the cost of the apparatus, or of the charge for wear and tear.

When the gas is made from rich cannel coal, the arrangement of the apparatus is a little different from the preceding. In such cases the patentee adopts the following plan:—

Both retorts are placed horizontally, and they are divided into two compartments by means of a longitudinal septum that reaches nearly to the end of each. These, called the water-retorts, are charged with coke; and after being raised to a high temperature, water is allowed to fall gently through a syphon-pipe upon the red-hot coke at the outer end of the upper compartment. The steam which is thus generated passes through the ignited contents of the upper compartment, and thence descending behind the septum, it comes forward through the lower. By these means it is exposed to a large surface of red-hot charcoal, and is in great part decomposed, and hydrogen, carbonic oxide, and a little carbonic acid are produced, all of which enter the lower compartment of another retort, which is charged with cannel coal. Here they mix with the various hydro-carbons that are evolved from the coal; and after traversing the retort from one end to the other and back again, they pass into the condenser and wet-lime purifier, and thence into the gasometer. Dr. Frankland states, that by this mode of working, all the carbonic acid which is generated in the water-retort is decomposed by the fuliginous matter of the coal, and converted into twice its bulk of carbonic oxide. It is thought, also, that the water-gases exert a conservative influence on the rich hydro-carbons which are evolved from the coal; and that by sweeping them away as fast as they are produced, and so protecting them from the destructive agency of heat, a much larger quantity of illuminating matter is obtained.

Dr. Frankland has examined the value of this process, as applied to different varieties of cannel coal. He worked with one hundredweight of each of the following coals, and he obtained the following amounts of mixed hydro-carbon gases. The coals were placed in each of the compartments of the second retort, and they were distilled at a low red-heat. The water-gas was obtained in the usual manner:—

Name of coal.	Yield per ton.	Illuminating power of the gas in sperm candles.
Wigan (Ince Hall)	16,120	20·0
Ditto (Balcarres)	15,500	19·1
Newcastle cannel	15,020	18·8
Methyl	26,400	21·0
Lesmahago	29,180	28·7
Boghead	51,720	17·9

The illuminating power was tested in the usual way with a No. 4 fish-tail burner, consuming five cubic feet per hour, at a pressure of half an inch of water; and the candle was a sperm candle, calculated to the amount of 120 grains per hour. According to Dr. Fyfe's mode of estimating the value of the gas, an average sample has a durability of 43 minutes and 20 seconds, and its amount of condensation by chlorine was 11·44 per cent.; so that its quality was about 2¼ that of ordinary gas from Newcastle coal.

Mr. Clegg speaks highly of the value of this process. He says that a ton of Wigan coal yields, under ordinary circumstances, about 10,000 cubic feet of twenty-candle gas; while with the hydro-carbon process it furnishes 16,000 cubic feet of gas of the same quality, and 26,000 cubic feet of twelve-candle gas. A ton of Lesmahago cannel produces, in the usual way, 10,500 cubic feet of forty-candle gas; but with the hydro-carbon process it will yield 36,000 cubic feet of twenty-candle gas, or 58,000 cubic feet of twelve-candle power; and a ton of Boghead, which generally furnishes about 13,500 cubic feet of forty-candle gas, will by this process give as much as 52,000 cubic feet of twenty-candle gas, or 75,000 of twelve-candle. From which it is manifest that, although there is a comparative reduction of the illuminating power in consequence of the diluting power of the water-gas, yet, on the whole, there is a considerable increase in the amount of illuminating material. This increase varies from 50 to 100 per cent.; and it is, no doubt, due to the property which water-gas has of suspending the volatile hydro-carbons which are generally condensed in the tar.

Mr. Clegg states, that the cost of working the process is not very considerable, for gas of twenty-candle power may be obtained from Wigan coal at the rate of 1*s.* 3½*d.* per 1,000; from Lesmahago at 11¾*d.*; and from Boghead at 11*d.*: besides which, the gas is remarkably free from every kind of sulphur compound. The conclusions at which Dr. Frankland has arrived, after having made a very careful examination of Mr. White's process, are the following:—

1st. That it greatly increases the produce of gas from a given weight of coal, the increase being from 46 to 290 per cent., according to the nature of the material operated upon.

2nd. That it greatly increases the total amount of illuminating power, the increase being from 12 to 108 per cent.

3rd. That it diminishes the quantity of tar formed, by converting it into gases of high illuminating power.

4th. That it affords a means of reducing the illuminating power of gases which could not be profitably burnt alone: and,

5th. It may be easily applied to any of the apparatus or modes of working now in use.

In opposition to these conclusions must be placed the results obtained by Messrs. Brande and Cooper, who found that little or no advantage was gained by the process; and Dr. Fyfe, who is much opposed to the whole scheme, states that, when all things are considered, it is a process which ought at once to be abandoned. Mr. Barlow has, however, recently examined the merits of the process; and he believes that, with some few alterations in the way of working, it may be profitably carried out. His suggestion is to collect the water-gas in separate gasometers, and to convey it from them, instead of from the retorts, into the compartments where the coal is distilling.

Water-gas alone.—Several patents have been taken out, at various times, for the production of gas from water, and for the employment of such gas either alone, as when it is wanted for heating purposes, or in combination with the vapour of naphtha. Donovan's patent, of 1830, we have already alluded to. M. Floret has contrived an arrangement for decomposing steam, by means of coke heated to a high temperature in an atmosphere of oxygen. Mr. Paine, of America, says he decomposes water by the aid of electro-magnets; Mr. Adams, of Boston, effects it by the force of a galvanic battery; and Mr. Shepard assists the action by using a salt of ammonia, or a vegetable acid, as the oxalic. In Mr. Gillard's process, steam is decomposed by passing it over incandescent charcoal, in such a manner as to form hydrogen and carbonic acid; the latter being absorbed in the usual way by means of caustic lime. All these patents, however, are but so many examples of fruitless speculation; and they may be pointed at as illustrations of so-called unsound chemical knowledge, combined with the worst kind of practical experience.

As the hydrogen and carbonic oxide produced by either of these processes does not possess the smallest amount of illuminating power, it must be brought up to the required standard, either by passing it through a volatile hydro-carbon, or by burning it in such a way as to make a solid incombustible body white-hot. Mr. Gillard adopts the latter contrivance; and his plan is to consume the gas from an Argand burner, over which there is suspended a rosette of platinum wire. The wire becomes intensely heated; and, when the gas is burning at the rate of six-and-a-half cubic feet an hour, the light emitted is about equal to five composite candles, or to that of a fish-tail consuming three feet of ordinary gas per hour.

Wood Gas.—When wood is subjected to distillation in closed iron vessels, it evolves a large quantity of inflammable vapour, some of which condenses into a liquid form, while the rest passes off as combustible gas. The liquid consists of water, pyroligneous acid, naphtha, and a complex heavy tar; and the gas is composed of hydrogen, carbonic oxide, carbonic acid, light carburetted hydrogen, and a variable proportion of olefiant gas. When these are consumed in the ordinary way, they do not produce much light; but if they are passed through a volatile hydro-carbon, they may be brought up to any degree of illuminating power. It is possible, also, that by certain modifications of the process whereby they are obtained, a sufficient amount of rich hydro-carbons may be generated, to do away with the necessity for subsequent naphthalisation. Mr. Hills proposed to mix the gases which are obtained during the manufacture of crude pyroligneous, with those which result from the destructive distillation of coal-tar; and Mr. Lowe has taken out a patent for the use of tar, oil, resin, or cannel coal, as a means of giving illuminating power to the gases evolved from sawdust, spent bark, or peat. Other patentees have suggested that the sawdust and tar should be mixed together before they are distilled; but it does not appear that any of these schemes have been successful in practice, or have been able to compete with the more economical mode of obtaining gas from coal. On the continent, however, where wood is very abundant, and coal scarce, it is frequently more profitable to make gas from the former than from the latter. At Bâle, for example, and some other towns of Switzerland, Norway, and Sweden, the process of distilling gas from wood is practised on a very large scale. We are told that the old city of Heilbronn has recently been lighted up with wood-gas; the manufacture of which is under the management of M. Schäufelen, who produces a gas of very good illuminating power. It is said that a fish-tail burner, consuming four-and-a-half cubic feet per hour, gives the light of thirteen wax candles; and that one of five cubic feet an hour, produces the light of sixteen candles; while an Argand burner, on Dumas' construction, consuming the same amount of gas, evolves the light of twenty candles. If this be so, it is evident that wood-gas can be obtained of a quality superior to that of ordinary coal gas.

Peat Gas.—This is nearly of the same composition as the gas evolved from wood; and the collateral products are nearly of the same description and value. Many chemists have, at different times, devoted their attention to this subject; but it is only very recently that precise information has been obtained concerning it. Sir Robert Kane in Ireland, and Dr. Letheby in England, have each reported on the quality of the products generated during the destructive distillation of peat; and from their statements it appears, that 100 parts of peat will furnish from 10 to 40 of charcoal, from 2 to 5 of tar, from 11 to 38 of aqueous matters, and from 25 to 58 of gas. The charcoal is very valuable as a decoloriser and disinfectant; the tar is rich in paraffine and creosote; the watery fluids contain ammonia, acetic acid, and wood-naphtha; and the gases consist of hydrogen, carbonic oxide, carbonic acid, and various hydro-carbons. One thousand parts of peat will yield, on an average, 2·7 of ammonia (equal to 10·4 sulphate), 1·9 acetic acid, 1·4 wood-naphtha, 7·9 volatile oils, 5·5 fixed oils, and 1·4 paraffine. From Dr. Letheby's experiments, it appears that a ton of peat will furnish about 13,000 cubic feet of gas, the illuminating power of which is equal to seven standard candles, when the gas is burned from an Argand that consumes five cubic feet per hour; but the intensity of the light may be brought up to any degree by the usual process of naphthalisation. As peat is very abundant, its products valuable, and its gas entirely free from sulphur, it is very probable that it may be used with great advantage as a source of inflammable gas: in fact, a patent has been taken out by Mr. Hansor of London, for the manufacture of an illuminating gas from a mixture of 12 parts of peat, 12 of resin, 8 of coal-tar, and 16 of oil. This is effected by distilling the mixture from perforated iron boxes, placed in a furnace heated to a cherry-red; the condensible vapours so produced are then passed through another furnace divided by diaphragms, and raised to a bright red-heat. By this they are decomposed, and rendered permanently gaseous. The gas so obtained, after having been purified by means of lime, has a density of ·626, and its illuminating power is a little higher than that of common cannel gas, or about twice as great as that of ordinary coal gas. A fish-tail burner consuming 2·5 cubic feet per hour, gives the light of 9·2 standard sperm candles; and an Argand consuming 3·5 feet, gives the light of 15·5 candles. One of the great advantages of the gas is its perfect freedom from every kind of sulphur impurity. A company has been formed, under the name of Hansor's Olefiant Gas Company, for carrying the patent into full operation.

Gas from Wine Lees and Grape Skins.—These substances are said to produce a very good gas. One pound weight of either of the materials will furnish seven cubic feet of gas of more than ordinary quality. This fact was demonstrated in the year 1849, by M. Levenair of Bordeaux, and Dr. Berhardt of Paris, in one of their public lectures before the Faculty of Sciences. If the results be uniform, the process affords an easy means of obtaining gas, and other valuable products, from a material that has hitherto been used only as a manure; besides which, the residue of the distillation is just as valuable to the wine-grower as the lees themselves.

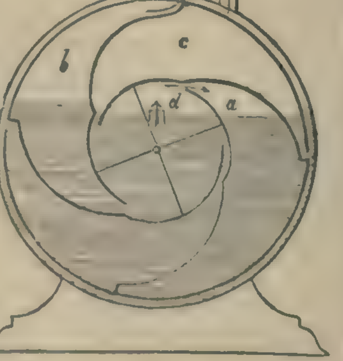
Gas from Coal Tar.—This liquid has very frequently been made the subject of experiment, in the hope that the rich hydro-carbons which it contains might be converted into permanent gases of high illuminating power; but, hitherto, all attempts at effecting this desirable result have signally failed. In the year 1820, Mr. Lowe contrived an apparatus for decomposing coal-tar, and resolving it into permanent gases. It consisted of a furnace with five retorts, three of which were placed below, and two above. These were charged in the ordinary way with coal; and when the carbonisation had gone on for three hours, the tar was allowed to flow through a syphon-pipe into the back part of the upper retorts—these being the hottest. Here the tar was converted into vapour, and it passed over the incandescent surface of the charge to the front of the retort, where it escaped through the exit-pipe to the hydraulic main. According to Mr. Lowe, he was enabled to produce, by this arrangement, a maximum quantity of gas of high illuminating power; but, although the process was worked upon a very large scale under the immediate superintendence of the patentee, it did not succeed, and it was soon abandoned. Since that time a number of patents have been taken out for a like purpose; but none have been successful. In some cases the tar has been mixed with small coals; in others with peat; in others with sawdust; and, very recently, Mr. Way has proposed the use of a porous stone, which he saturates with tar. Some have distilled the material at a low heat, others at a moderate temperature, and others at a very high one: so that every species of ingenuity has been exhausted in endeavouring to convert this rich inflammable fluid into a permanent gas.

Other organic substances have been resorted to as a source of gas. M. De Cavaillon has proposed a mixture of bones, suet, oleaginous seeds, spent bark, sawdust, molasses, and small coal. Mr. Booth has claimed the use of resinous woods and oily seeds. Mr. Witty has patented a mixture of vegetable oils, and refuse vegetable matters, such as spent hops, dry peat, or dry sawdust, which he presses into blocks, and distils in earthenware retorts; and then, again, it may be said that Mr. Lowe has obtained a patent for naphthalising gases from any source, by passing them through a vessel containing coal-naphtha—a process that was suggested many years ago by Dr. Henry, of Manchester. Lastly, Mr. Archibald has gone so far as to naphthalise atmospheric air in a somewhat similar way. He first saturates the air with moisture, and then conveys it through benzole, which is the most volatile of the coal-naphthas. He also passes the air through a mixture of one part of benzole, two of alcohol or wood-spirit, and one-and-a-half of water. By this means the air acquires inflammability, and may be burnt from an ordinary gas jet: it may even be stored in gasometers, and distributed in the same way as coal gas. If the air is not saturated with moisture, so much cold is produced by the volatilisation of the benzole as to stop the process of naphthalisation; and hence the necessity for adding water to the hydro-carbons.

ON THE APPARATUS REQUIRED FOR THE CONSUMPTION OF GAS.

The Gas-meter.—When gas was first used by the public, it was sold at an annual rent, at so much per burner; but it soon became apparent that this was not a fair mode of dealing with the article, and hence the ingenuity of the mechanic was taxed to contrive a means whereby the gas might be accurately measured. This was accomplished by Mr. Clegg, who, in the year 1815, constructed the first meter. In the following year he improved it so far as to obtain a patent for it. At first Mr. Clegg attempted to register the gas by means of two small gasometers, which rose and fell alternately, one receiving the gas while the other was delivering it. But this plan was not successful, and it was, therefore, abandoned for another which constitutes the basis of all the wet-meters that have been contrived since Mr. Clegg's time. It consisted of a drum, which revolved in a chamber half filled with water. The drum was divided into two compartments, one of which received the gas while the other delivered it. The gas entered through the hollow axis of the instrument; and as the drum revolved and submersed the compartment, the gas was forced through a lateral opening into the outer chamber, and thence to the burners. By means of valvular contrivances, two of which were closed by water and two by springs, the gas was made to flow only in one direction; but as the spring-valves were easily thrown out of order, and the water-valves were of a clumsy form, the instrument was open to very great improvement. Mr. Malam, therefore, in 1819 reconstructed the apparatus. He divided the drum into five compartments—one of which was central, and the others around it (Fig. 126). As in Mr. Clegg's instrument, the gas entered the apparatus through the axis of the drum; but in order that there should be no friction or impediment to its movement, he did away with the stuffing-box in which it worked, and brought the central tube or axis, by means of a rectangular bend, up above the level of the water in the central chamber *d*. He also put aside the clumsy water-valves and the too delicate spring-valves of Clegg's instrument, and adopted a simple contrivance whereby the delivery apertures were made to act of themselves by simply rising above the level of the water. These apertures were in the form of slits, which communicated first between the central chamber and the circumferential ones, and then between the latter and the outer case. On entering the central compartment, the gas escapes through whichever of the first set of slits

Fig. 126.



happens to be above the level of the water. In the figure it is passing from *d* into *a*. This gives a buoyancy to that chamber; and as it rises or floats, it turns the drum round from right to left, causing the gas which is in the upper and descending compartments to escape through their outer slits into the outer chamber, and thence to the burner. It will be noticed that, as the drum revolves, the entrance-slits between the middle and outer chambers are successively carried under the water, and that as soon as this happens, the exit-slits in the circumference of the drum are each in their turn brought out of it. This is shown in the figure as about to occur with compartment *c*, whose entrance-slit is just dipping under the water, while the exit-slit is rising out of it. Since the time of Mr. Malam, other improvements have been effected in the details of this meter; by means of which the revolution of the drum has been made steadier, the amount of friction lessened, and the water kept at one uniform level. Among the names of those who have devoted attention to the subject, are Crosley, Wright, Evans, Hulet, Smith, Paddon, and Ford; all of whom have, in some particular or another, improved the instrument.

Fig. 127.

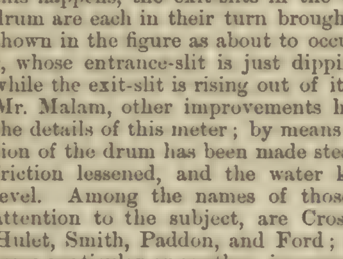
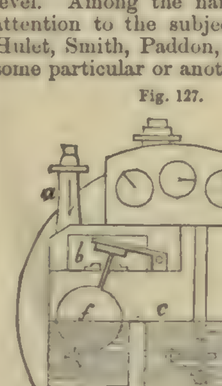


Fig. 128.



Figs. 127 and 128 represent a meter of modern construction, with most of the improvements adopted. The gas passes by the inlet-pipe *a* into the chamber *b*, whence there is a communication by means of a flat valve with the compartment *c*; from this it passes through the bent pipe, forming the axle of the drum, into the compartment *d*, and thence, in the manner already described in Malam's meter, it gets into one or other of the four compartments, and so escapes by an exit-slit into the space formed by the surrounding case: *f* is a ball which floats on the water, and keeps the flat valve open. If the water gets below its proper level, the ball falls; and by bringing down the valve, shuts off the gas. *g* is a tube for passing water into the cistern, and *i* is a waste cistern for receiving the water when it gets above the proper level and flows over: *h* is a syphon-tube for drawing off the water which has been added in excess. By a train of wheelwork, the revolutions of the drum are communicated to the hands on the dial-plate; and thus the quantity of gas consumed is registered.

There are two objections to the wet meter, which are insurmountable. These are, that the instrument registers the vapour of water as well as gas; and, secondly, that it is very liable to stop, at an unexpected moment, by the deficiency of water and the sinking of the float. When a meter is placed in a warm situation, the quantity of aqueous vapour that is registered by the instrument, and set down as gas, is very considerable; and if it should so happen that the gas, after passing the meter, has to traverse a cold locality, the aqueous vapour will be recondensed as water, and will cause a flickering of the flame, or even a total closure of the pipe.

At one time it was possible for the gas companies to derive an advantage from the overfilling of the water-cistern, whereby the capacity of each measuring chamber was somewhat diminished; and thus a revolution of the drum did not record a proper quantity of gas. It was also possible for the company to be defrauded by a deficiency in the amount of water; but now both of these objections are guarded against by the waste-tube and cistern on the one hand, and by the float-valve on the other.

It will be seen, by an examination of the meter represented in Fig. 126, that, by tilting the meter a little on one side, the gas will pass through the slits of the uppermost chamber without turning the drum at all, and frauds are sometimes committed in this manner; but the experiment is a very dangerous one, and is not likely to be practised by any but the most determined rogue.

The *dry-meter* is a more complicated apparatus, though it consists in all cases of a chamber, or set of chambers, with flexible leather sides. The action is very similar to that of a pair of easy-going bellows; and the movement is communicated to levers and rack-work, which not only register the amount of gas that passes, but, by a set of sliding-valves like those of a steam-engine, they also cut off the supply of gas from one chamber, and turn it on into another.

The first dry-meter was patented by Mr. Malam in 1820: it consisted of a set of six bellows placed in a radiating direction around a common centre, the whole being inclosed in an air-tight chamber. These bellows worked successively one after the other, and they communicated their motion to a set of wheels, which served to register the quantity of the gas delivered. This machine was, however, not perfect—it had many radical faults; and hence it did not come into general use. Next after this was the instrument constructed by the Dry-Meter Company. The measuring chamber was formed of leather, which was found to be liable to many objections; for if the meter were used constantly, the leather expanded under the influence of the gas, and so it recorded against the interest of the company; whereas, if it were not used so constantly, it underwent contraction, and then it registered against the consumer: besides which, the valve moved so suddenly from one side to the other as to produce an unsteady flame, and hence the meter was not much patronised. Sullivan's meter, which followed upon this, was open to the same objections; for it consisted of two loose leather diaphragms and a rotatory valve. In 1833, an American, by the name of Berry, patented an instrument which was invented by a person named Bojardin. The meter was a hollow chamber, which had a movable diaphragm or partition that divided it into two compartments, and, by means of sliding-valves, the gas was alternately let in or out of them. In 1836, Bojardin invented a still better machine; and he is looked upon as the originator of the present form of dry-meter. Since that time, Mr. Defries, Mr. Edge, and Mr. Croll have improved the instrument, and brought it into its present condition. Mr. Defries' meter contains three measuring chambers, which are separated from each other by a flexible partition formed of leather, and partially protected from the action of the gas by four triangular metal plates, which almost cover the diaphragm. This flexible partition is raised or depressed by the gas, so as to form a pyramid or a flat surface; and the rise and fall of the partition is communicated to the wheelwork of the apparatus; but, as in all cases where leather enters into the construction of the measuring chamber, the instrument is very liable to register incorrectly. The meter which is manufactured by Messrs. Croll and Glover is said to be free from this objection: its construction will, perhaps, be understood from the following account, taken from Mr. Croll's paper, which was read before the Society of Civil Engineers in 1845. "If we imagine a steam-engine measuring its steam by the movements of the piston, we shall have some idea of the principle of the instrument. The steam enters the cylinder over the piston, and forces it down through a certain space; the supply is then cut off, and the action is reversed. Now, suppose the piston to be of a given area, and the distance through which it moves at every stroke to be constant, it can readily be conceived how the actual quantity of steam employed could be indicated and calculated. The meter in question bears a strong resemblance to a double engine: it consists of a cylinder divided by a plate in the centre into two separate cylindrical compartments, which are closed at the opposite ends by metal discs; these discs serve the purpose of pistons, and they are kept in their places by a kind of universal joint adapted to each. The space through which the discs move, and consequently the means of measurement, is governed by metal arms and rods; which space, when once adjusted, cannot vary. To avoid the friction occasioned by a piston working in a cylinder, a band of leather is attached so as to act as a hinge. This folds with the motion of the discs, but it is not in any way concerned in measuring the gas; so that if it were to contract or expand, the registration of the proper quantity of gas would not be affected; for such a change would only decrease or increase the capacity of the hinge, leaving the disc still at liberty to move through the required space. The leather is also attached in such a manner as to avoid folds, and thus to render it more durable.

Gas-burners.—Scarcely anything connected with the subject of gas illumination has commanded more attention than the means whereby gas may be burnt to the best advantage; and although the greatest ingenuity has been displayed in the construction of many of the burners which have at different times been invented, yet none of them possesses that universal applicability for which they have in most cases been so highly vaunted. The reason of this is obvious: different kinds of gas require different forms of burners, in order to effect perfect combustion. As a rule, it may be stated, that the rich cannel gases are best consumed from burners with very fine apertures: while the poorer gases—namely, those which contain less than five or six per cent. of condensible hydrocarbons—are burnt with most advantage from larger apertures. Again, in the former case, provision should be made for a large supply of atmospheric air, as by spreading out the flame by means of an internal button, or by using tall glasses; whereas, in the latter case, the very opposite condition should be observed. It is manifest, therefore, that no single burner can be constructed so as to secure both of these requirements; and, consequently, that any burner which is well suited for one kind of gas, is altogether unfit for the other.

Another point of importance to which we may refer in speaking of this subject, is the following:—that when several jets issue from the same burner, and blend together or coalesce, the light is always improved; for it is the property of one jet to assist another by exalting its temperature; and thus a greater heat and a brighter flame are the result of the union—more light being given out than is the sum of the individual jets. It is on this account that the Argand burner, the fish-tail, the Gaumont, and the Gardner, have obtained preference over many other forms of burners.

Lastly, it may be stated, that in whatever way gas is consumed, the maximum effect, as regards its illuminating power, is always produced by burning the gas just short of its smoking-point: for if it be burnt with too much air, the particles of carbon are consumed; and we thus obtain a diminishing light, until the flame becomes of a pale-blue colour. On the other hand, if it be burnt with too small a supply of atmospheric air, the particles of carbon will not be sufficiently consumed, and they will escape as soot—thereby cooling the flame, and making it of a dingy yellow tint. Our object, therefore, should always be to burn the gas in such a manner that the particles of carbon may be first intensely heated, so as to produce a white light; and then, as they reach the exterior of the flame, they ought to be consumed entirely, so as to avoid the evolution of soot. Of the different varieties of burners now in use, the following are the most important:—

1. *The simple jet* is produced from a burner pierced with a single hole. This mode of consuming gas is not considered to be cleanly or economical; and, except for certain purposes of illumination, as where we wish to produce different kinds of devices, it is rarely employed. Occasionally it is used in the laboratory of the chemist for experiments with the blowpipe, as it is found to give a much hotter and clearer flame with this instrument than any other form of jet. The average consumption of a jet the thirty-third of an inch in diameter, with a flame four inches high, is about one cubic foot per hour. With richer gases the quantity is a little less, and with very poor gases it is somewhat more. As we have already said, Dr. Fyfe prefers a jet of this description for the purpose of estimating the illuminating power of gas; and, in a general way, it is equal to from one to one-and-a-half, or two sperm candles.

When it is necessary to produce a greatly diffused light, it may be accomplished by means of one or more rows of single jets. These may be distributed around the cornice or mouldings of a room, and the effect is remarkably pleasant and agreeable, for there is a flood of light without the least glare or shadow. The only objection to it is its great cost and its liability to evolve smoke; but if these were overcome, and our rooms constructed so as to have a row of lights around a panel in the ceiling, there is no doubt that the effect would be much more pleasing than that which results from our present mode of burning gas; besides which, the panel might be so arranged as to convey away the products of combustion, and thus the effect would also be more salutary.

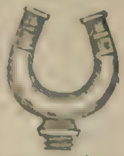
2. *The Cockspur Burner* is a burner with three or more jets radiating off from it, and burning separately. The light from such a burner is only equal to the sum of the individual jets; for as they do not coalesce, they cannot in any way assist each other. It is one of the worst forms of burner that can be employed. A burner with three jets, consuming three cubic feet per hour of ordinary London gas, gives a light of from five to six candles.

3. *The Fish-tail* or *Union Burner* is so named because of the form of its flame, and because of the manner in which it is produced by the union of two jets. It is formed by drilling two holes at an angle of about 60°. The jets are directed into each other; and as they coalesce, they spread out so as to produce a flat sheet of flame of the form of a fish-tail: by this means the intensity of the light is greatly increased. The holes are drilled large or small, according to the quality of the gas to be employed. In the case of cannel gas, the holes are small; and for common London gas they are rather large. The former are known by the name of Lancashire or Scotch fish-tails, and the latter as London jets. Each sort is numbered 1, 2, 3, or 4, according to the size of the holes; and these numbers are indicated by means of little rings turned on the body of the jet. No. 1 is the smallest. These burners consume from two to four cubic feet of gas per hour; and with cannel gas they give the light of from eight to fourteen candles, and with common gas, of from four to ten.

4. *Johnson's Burner* is a fish-tail with four converging holes; and there is an aperture through the centre of the burner for the admission of atmospheric air into the flame.

5. *Billow's* or *Gardner's Burner.*—This is a combination of two fish-tail or bat's-wing burners, arranged so as to produce one flat flame (Fig. 129). The flames impinge on each other, and thus increase the illuminating power by about thirty per cent. These burners are usually constructed of two fish-tails; and occasionally, for experimental purposes, they are attached to a hinge-joint, so that the flames may be burnt separately or together. As in the case of the ordinary fish-tails, they are numbered 1, 2, 3, and 4. No. 1 consumes about three cubic feet per hour, and No. 4 about five.

Fig. 129.



6. *The Bat's-wing Burner* is so named on account of the shape of the flame. It is one of the oldest forms of gas-burners; it is constructed with a slit instead of two holes for the exit of gas; and the flame is broader, and not so high as the fish-tail. These burners are easily managed, and on this account they are generally supplied to the public lamps. They burn from three to five cubic feet per hour; and, with a consumption of four cubic feet per hour, they give the light of about nine sperm candles of standard quality.

7. *The Gaumont Burner* is a bat's-wing burner with two or three slits, instead of one. By means of this contrivance there are two or three flames amalgamated into one; and, as is always the case under these circumstances, the total amount of light is increased. A Gaumont, which consumes three cubic feet per hour, gives the light of seven sperm candles.

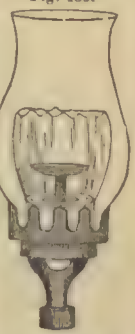
8. *The Sun Burner* is a cluster of fish-tail burners, usually nine in number, placed round a common axis, and spreading out in a horizontal direction, so as to produce the figure of a flower or of the sun. This burner is so constructed, that the products of combustion are carried out of the room by means of a ventilating funnel and tube placed immediately over it; and the air which supplies the flame is made to descend through the ventilating tube, and thus to become very hot before it reaches the burner. By this arrangement there is a great saving of gas. In most cases the sun-burner consists of seven clusters of nine fish-tails each. The burners are supplied with gas by a descending-pipe, which branches to each cluster; and, surrounding the whole, is a sheet-iron cone, with a tube attached to the top, for carrying off the products of combustion. In this tube there is placed a butterfly-valve, for the purpose of regulating the current of air, so that the draught may not be too great, and the lights may burn in a horizontal direction. Around the cone are placed three other sheet-iron cases, which not only serve for ventilation and for the supply of air to the burners, but also insulate the inner cone, and by their cooling effect, prevent the intense heat of the latter from being communicated to the woodwork of the ceiling. These cylinders are not connected with each other, or the cone, and, therefore, distinct currents of air pass between each of them; and such is the cooling effect of these currents, that, while the cone is red-hot, the two outside cases are of the same temperature as the atmosphere of the room. On the upper part of these cylinders there is an inverted cone, with a pipe passing through the ceiling and roof, and protected on the outside by a wind-guard, which allows the hot air and products of combustion to escape. By this contrivance the gas is consumed in a very heated atmosphere, and thus there is less necessity for combustion in order to obtain a given amount of light; for, in the generality of cases, a large portion of the gas consumed is employed in raising the air to the temperature necessary for sustaining the ignition of the small particles of carbon contained in the flame. It has been stated by Mr. Edwards, of Liverpool, that the burners, in this position, do not consume more than half the usual amount of gas; besides which, the intensity of the light is very great.

9. *The Common Argand Burner* produces a flame which is exactly like that of an ordinary Argand oil-lamp. The burner consists of a circular disc of iron, pierced with a number of holes. It is hollow in the middle, for the purpose of allowing a supply of air to the interior of the flame; and the jets of gas coalesce, so as to form a hollow cylindrical flame. A glass chimney is placed around the burner, in order that the supply of atmospheric air may be copious and steady. The number of holes or jets varies from ten to thirty for ordinary gas, and from thirty to ninety for cannel. In the former case the holes are comparatively large, and in the latter they are very small. When common gas is consumed from an Argand burner, the chimney ought not to be above seven inches in height; but, when cannel gas is burnt, it may be increased to nine or ten inches. If the chimney be too high, the supply of atmospheric air is too great, and the gas is overburnt; whereas, if it be too low, the supply is not sufficient, and then the gas smokes: in either case the intensity of the light is diminished. For ordinary London gas, a burner with fifteen holes, and a seven-inch chimney, is considered to be the best. Such a burner will consume about five cubic feet of gas per hour, and will give the light of fifteen sperm candles.

Several patents have been taken out, during the last few years, for improvements in this form of burner: they have chiefly been directed to the lessening of the shadow which is cast by the ring and body of the apparatus.

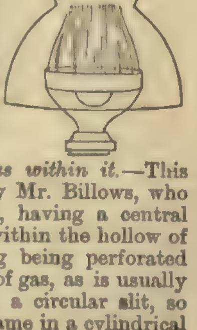
10. *The Argand with a Button or Deflecting Disc.*—This form of burner has received various names, according as it has been modified by different patentees. It is called the Winfield burner, Young's burner, Guize's burner, and the Aberdeen burner. The button or disc is composed of copper or iron, and it is placed in the centre of the flame, a little above the level of the burner. It acts as a break to the inner current of air, and deflects it outwards so as to enlarge the upper part of the flame, and to give it the form of a tulip (Fig. 130). This kind of burner is only suited for the richer kinds of gas, as cannel gas and naphthalised gas. In some cases the air is deflected to the outside as well as inside of the flame. This is the principle of Guize's burner, and the deflection is effected below the button by means of a bend or constriction in the glass chimney, or else by the aid of a metal cone like that of the solar lamp. In Young's burner there is a series of discs placed one above the other, the discs being successively larger and larger from below upwards; by this means the air suffers a series of deflections, and causes the flame to be most vivid. A burner of this description is particularly well suited for the combustion of very rich cannel gas.

Fig. 130.



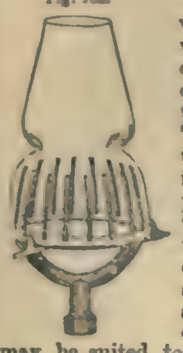
11. *The Argand Burner with two or more Rings of Flame, one within the other.*—This is the principle of the Boccius burner; and, from the circumstance that one flame always assists in promoting the combustion of another, the light from this burner is very considerable. Mr. Carter has obtained a patent for a burner of this description, modified somewhat in its form, so as to provide for a due supply of atmospheric air, and also to carry off the products of combustion. The burner consists of a series of Argand flames concentric to each other, with only just so much space between the rings as will serve for the transmission of the necessary quantity of atmospheric air; and over the flames there is placed a conical chimney, which carries away the products of combustion. In the concentric burners hitherto used, the object has been simply to obtain the effect of two or more concentric flames, without having due regard to the proper and uniform supply of air to each flame. The consequence of this is, that great and unnecessary quantities of air enter such burners; and the flame is thereby cooled much below the temperature requisite for the perfect combustion of the gas. On this account the intensity of the light is very much below what it ought to be. In Mr. Carter's burners this objection is overcome; for the space between the rings is properly graduated, and the current of air is made to compress the flame, and blow in upon it by means of a contraction in the glass chimney—the point of contraction being at such a height above the burner as to produce a bright and steady flame (Fig. 131). The ventilating-shaft is placed over the burner, and communicates above the ceiling with the chimney. It consists of a funnel, and a tube, which may be either of metal or glass. These are arranged so as to regulate the supply of air to the flames, and they are enclosed in a chandelier of glass pendants.

Fig. 131.



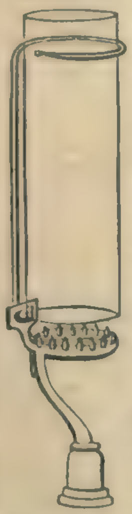
12. *An Argand with a Jet of Gas within it.*—This form of burner has been patented by Mr. Billows, who describes it as an ordinary Argand, having a central tube with a single jet, which burns within the hollow of the flame: and, instead of the ring being perforated with a number of holes for the issue of gas, as is usually the case, the ring is contrived with a circular slit, so that there is a continuous sheet of flame in a cylindrical form.

Fig. 132.



13. *Leslie's Argand* is the very reverse of the last; for the principle on which it is constructed is to allow a current of air to pass up between each of the jets, and so to destroy, to a certain extent, the continuity of the flame. This is effected by a number of small tubes, which rise to the height of an inch or so above the ring which supports them. These tubes are made to converge a little as they advance upwards, and thus they form a truncated cone (Fig. 132). The glasses are constricted, so as to deflect the air into the flame; and they are of different heights, in order that they may be suited to different amounts of consumption. These burners are very well suited for the combustion of cannel gas, but they destroy the light of common gas by overburning it. The flame should be always managed so as to reach nearly to the top of the glass: for if it pass above it, soot will be deposited; and if it do not reach almost to the level of the glass, the gas will be overburnt, and light sacrificed.

Fig. 133.



14. *The Pinnacle Burner*, of Messrs. Baldwin and Neal, is constructed on somewhat the same principle as the last, only the tubes are very short: they are, in fact, but mere nipples on the top of the ring, and the glass is not constricted (Fig. 133).

Many other varieties of gas-burners have been invented; and as the orifices of the jets are very liable to corrosion from the ammonia and sulphur contained in the gas, Mr. Hallen has proposed that clay or porcelain nozzles should be employed.

The relative value of the several kinds of burners, as employed in the combustion of ordinary London gas, as well as that from cannel coal, may be perceived from the following table. The value is represented as per cubic foot, in sperm candles of 120 grains consumption:—

Burner.	Cannel gas: candles per foot.	Common gas: candles per foot.
Single jet . . .	2·6 . .	1·5
Cockspur . . .	3·0 . .	1·5
Fish-tail, No. 1 .	3·0 . .	1·9
,, No. 2 .	3·6 . .	2·0
,, No. 3 .	4·0 . .	2·3
,, No. 4 .	4·3 . .	2·4
Bat's-wing, nearly the same as fish-tail.		
Leslie, with 28 tubes	3·6 . .	2·7
Gaumont . . .	4·0 . .	2·3
Common Argand 15 holes	4·0 . .	2·8
Bynner's 28 ,,	4·0 . .	3·2
Platow's 16 ,,	4·2 . .	2·9
Guize 26 ,,	4·2 . .	3·0
Winfield 58 ,,	4·3 . .	2·8
Billows', with 3 slits .	4·1 . .	2·6

From this it will be seen, that the simple jet is the worst kind of burner that can be used for the consumption of gas; next come the smaller-sized fish-tails, then the bat-wings, and lastly the Argands. The Gaumont or double fish-tail, and the Billows' or compound bat's-wing, are also very good burners. But it will be evident from the preceding table, that the burner which is best suited for common gas, is not always the one that can be most economically employed for cannel.

The pressure at which gas ought to be consumed is another point of considerable importance: for if the amount of pressure be high, the gas will burn with a roaring noise, and will be consumed wastefully; whereas if it be low, the fish-tail and bat's-wing flames will not be sufficiently spread out, and the light will be dim and smoky. Dr. Letheby states, in his Ninth Report to the Corporation of London, that gas ought to be delivered to the public at not less than half an inch of water pressure; and it may be said that, in practice, this is found to be the best pressure at which gas can be consumed.

Again, it is a matter of importance that the pressure at which the gas is supplied to the burner should be as uniform as possible; for if at one time the pressure be great, and at another low, the burner will require constant attention, in order that the flame shall be of one uniform height.

Experiments have been made to determine the rate at which gas burns under different pressures; and as the results are somewhat important, they are tabulated below. In a general way, it may be said, that by doubling the amount of pressure, we increase the consumption of gas by about half.

Burner.	Pressure in inch of water.	Consumption per hour.
Single jet . .	0·30 . .	2·6 cubic feet.
,, ,, . .	0·60 . .	3·9 ,, ,,
,, ,, . .	1·20 . .	5·2 ,, ,,
Small fish-tail .	0·34 . .	1·4 ,, ,,
,, ,, ,, .	0·77 . .	2·2 ,, ,,
Large ,, ,, .	0·48 . .	2·3 ,, ,,
,, ,, ,, .	0·97 . .	3·3 ,, ,,
Large bat's-wing .	0·70 . .	3·1 ,, ,,
,, ,, ,, .	1·40 . .	4·5 ,, ,,

Dr. Fyfe has also observed, that there are certain constant relations between *the specific gravity* of a gas (that is, equivalent to its goodness) and *the pressure* at which it is burnt, and *the time* required to consume it—that is, provided we use a jet of a given size, and take care that the flame is of a given height. The jet which he prefers, is one having a hole the fortieth of an inch in diameter; and the height of the flame should be five inches. These relations are as follow:—

1st. The consumption of gas in a given time is as the square root of the pressure; and, consequently, the time required for the consumption of equal volumes, is inversely as the square root of the pressures.

2nd. The specific gravity of the gas is also inversely as the square root of the pressures.

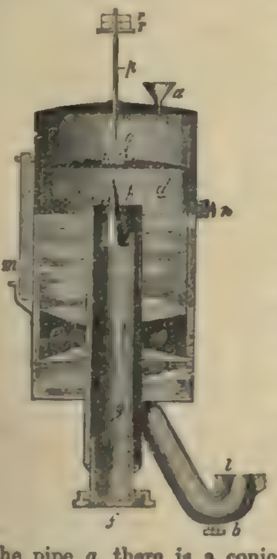
So that if we determine, by experiment, what time it takes for a given volume of gas, of known specific gravity, to burn from a jet of the given size, with a flame of the given height, we are then in a condition to tell the specific gravity, or the rate of consumption, of any other gas, provided it be burnt under the same circumstances, and we observe the pressure. This will be manifest from the following table:—

Pressure in inch of water.	Consumption per hour.	Specific gravity.
0·6 . . .	0·67 . . .	·841
0·7 . . .	0·72 . . .	·779
0·8 . . .	0·77 . . .	·729
0·9 . . .	0·81 . . .	·687
1·0 . . .	0·86 . . .	·652
1·1 . . .	0·90 . . .	·622
1·2 . . .	0·94 . . .	·595
1·3 . . .	0·98 . . .	·572
1·4 . . .	1·02 . . .	·551
1·5 . . .	1·05 . . .	·532
1·6 . . .	1·09 . . .	·515
1·7 . . .	1·12 . . .	·500
1·8 . . .	1·15 . . .	·486
1·9 . . .	1·18 . . .	·472
2·0 . . .	1·21 . . .	·461

By means of this table, we are able to determine the rate at which gas is burning, or its specific gravity, by merely observing the pressure which is necessary to obtain a flame of the given height. In conducting the experiment, the pressure-gauge must of course be on the jet side of the tap. Dr. Fyfe suggests that we may, by operating in this manner, do away with the necessity for a meter or a photometer, or both, and that we may arrive at results which are approximately correct. Of course it must be understood that the gas is of the usual quality, free from carbonic acid and atmospheric air.

When coal gas is supplied to the consumer at pressures which are variable, or inconveniently large, the difficulty may be overcome by using instruments which are called *governors* or *regulators*. In some small towns, the only means that are adopted for regulating the pressure of gas, is that of taking off or putting on the weights of the gasometer. But as this is a very inconvenient, and at the same time ineffective mode of regulating the supply of gas, it is usual, in all large manufactories, to employ a governor at the works themselves. This is a small gasometer, working very easily and truly in a tank or cistern of water. It is furnished with two pipes—one for the inlet of the gas, and the other for the outlet. The former is situated in the centre of the apparatus; and it is provided with a conical or parabolic valve. This consists of a conical or parabolic cavity or socket, in which a solid plug of the same form fits very accurately. The plug is attached to the top of the gasometer, and the inlet-pipe, containing the socket, is fixed to the bottom of the instrument: consequently, as the gasholder rises and falls, it carries the plug with it, and so opens and shuts the conical valve; by which means the supply of gas is regulated. Any amount of pressure short of that at which the gas is received into the apparatus, may be communicated to the delivery-pipe by means of weights that are put upon the top of the gasholder; and, as the governor is always placed between the large gasometer at the works, and the pipes constituting the street mains, the pressure of the gas in the latter is regulated in a tolerably effective manner. It would be very effective, indeed, if it were not that there is a rapid taking-off of pressure from the gas in the mains by the turning-on of the consumers' lights. This causes such a sudden and enormous abstraction of gas from the mains that the pressure immediately falls. For example:—In the early part of the evening, just before the lights of the neighbouring shops are turned on, the gas in the mains has a pressure of from one inch to one inch and a-half of water; but as soon as the supply is opened to the several burners of the locality, the pressure immediately drops to less than half an inch of water; and then again, when the lights are in great part turned off, the pressure quickly rises. This occasions much inconvenience to those who are burning gas in an economical manner; for it demands constant attention to keep the flame at its proper level. To remedy this, a smaller kind of governor is often used by the consumer himself. There are various forms of this instrument, but they are nearly always constructed on the same principle as the large one. They contain a valvular aperture, which opens or shuts according to the supply of the burners. This is effected either by the plug and socket already described, or by means of a disc which rises and falls over a small aperture. The first mode of constructing the governor was patented by Mr. Clegg, as long ago as the year 1815, and it constitutes the principle of most of the wet governors of the present time. Of these, one of the simplest in its construction is that manufactured by Mr. Glover. It is represented in Fig. 134. *f* is the connection by which it is fixed to the meter, and the gas passes through the pipe *g* into the gasometer *d*. At the top of the pipe *g*, there is a conical valve *i*, *h*, the plug of which is fixed to the top of the little gasometer *d*; and, consequently, as the gas enters the latter and lifts it out of the water in which it floats, the valve is closed. *p q* is a rod fixed to the top of the gasometer, and projecting beyond the cover of the instrument; it serves for holding weights, *r r*, which press down the gasometer, and communicate any amount of pressure that may be required. The gas escapes from *d*, through the outlet pipe *l*—water is poured into the apparatus through the funnel *a*, and it is kept at a proper level by the tube and plug *n*, while the gauge *m* enables the observer to note at what height the level of the water stands. Lastly, there is a plug at *b* to draw off any water which may happen to pass over or condense in the outlet pipe *l*. When the gas enters the chamber *d*, it lifts the gasometer, and with it the plug *h*, so that the aperture *i* is then closed. Directly the gas is let out from the chamber to the burners by the delivery tube *l*, the weights *r r* on the rod *p* immediately depress the gasometer, and then the valve is again opened for the entry of more gas; and thus, according to the weights at *r r*, and the supply to the burners, will the gasometer be rising or falling, and so will keep up a uniform supply and pressure.

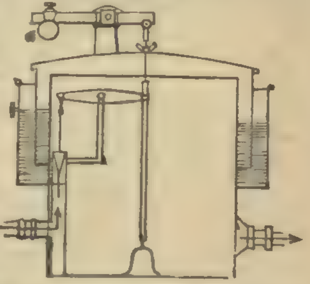
Fig. 134.



In the governor which has been constructed by Mr. King of Liverpool, the gasometer is made rather heavy, and it is suspended to one arm of a very nicely moving beam, like that of a pair of scales, while weights are attached to the other arm so as to regulate the degree of pressure.

In Mutrel's governor the beam is within the body of the instrument (Fig. 135), and the gasometer floats in an exterior vessel or chamber of water, while the pressure is regulated by a weight *a*, which slides backwards and forwards on an external beam.

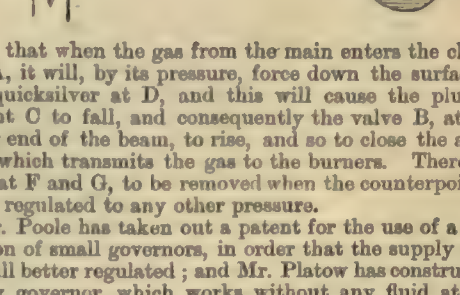
Fig. 135.



The American regulator, invented by Dr. Kidders, is constructed on the same principle, though the gasometer floats in quicksilver instead of water, and the valve is discoid in lieu of being conical.

The governor which is patented by Messrs. Hulett and Paddon contains the beam within a horizontal tube, and the weight floats upon the surface of quicksilver, so that the pressure of the gas may be made to regulate itself. This instrument is shown in Fig. 136. It will be seen, that when the gas from the main enters the chamber A, it will, by its pressure, force down the surface of the quicksilver at D, and this will cause the plug or weight C to fall, and consequently the valve B, at the other end of the beam, to rise, and so to close the aperture which transmits the gas to the burners. There are caps at F and G, to be removed when the counterpoise is to be regulated to any other pressure.

Fig. 136.



Mr. Poole has taken out a patent for the use of a succession of small governors, in order that the supply may be still better regulated; and Mr. Platow has constructed a dry governor, which works without any fluid at all. Lastly, Mr. Biddel has contrived a burner which regulates itself. It consists of an ordinary Argand, with a compound bar of brass and iron. The bar is formed of a brass tube with a central rod of iron; the latter is attached to the top of the former, and plays freely within it. When the burner is lighted, the bar stands up in the middle or hollow of the flame; and as these metals expand unequally for the same temperature, the motion of the longer rod is communicated to a small lever so as to move a plug which opens or shuts the aperture that delivers the gas. The temperature of the flame is thus made available as a means of regulating the supply of gas, and so of keeping the flame at one uniform height. After the burner has been once set, so as to produce the necessary amount of light, any increase in the supply of gas will immediately increase the size of the flame, and this will cause the bar to expand and act on the plug so as to shut off a portion of the gas. On the contrary, if the supply is not sufficient, the bar contracts, and then the aperture is opened.

ON THE GENERAL MANAGEMENT OF GAS, VENTILATION, Etc.

General Management of Gas.—On the continent, the government or municipal authorities enforce certain regulations for the management and distribution of gas. In Hamburg, for instance, the gas-fitters are obliged to perform their work in a certain manner: they must use tubes of wrought iron, brass, or copper; and in cases where these are not very easily adapted, tubes of drawn-tin may be employed. The joinings must be made in a durable and solid manner, either by means of sockets ground in and cemented with iron cement, or by screwing up, or by soldering. Any other mode of connection is forbidden. The tubes must be placed in localities where they are accessible, so that in case of leakage the mischief may be easily remedied. The cocks must be arranged in such a manner that they only make the fourth part of a turn, and they must be fixed so as not to be removable from the barrel. The gas-meters are to be furnished by the gas company, and examined and stamped by the adjusting commissioner. All the tubes of one inch or more in diameter must have a stopcock, so that they may be shut off from the main-pipe, in case of fire. No one is allowed to make use of his gas-fittings until the gas-fitter has tested their soundness, by means of a pressure of one inch of mercury or fourteen inches of water. All the gas-fitters are sworn to adhere to these instructions; and in case of any damage resulting from negligence on the part of the fitter, private persons are entitled to enforce their claims against him before a civil court of justice.

In a supplemental notice attached to the contract made between the English Gas Company in Paris and its consumers, the following regulations are recommended:—"To prevent any inconvenience in the use of gas, it is requisite that the burners should not allow any gas to escape in an unconsumed or imperfectly-consumed state. This result is obtained by maintaining the flame at a moderate height—three inches and a quarter at most, and confining it within a glass chimney of eight inches in height. The rooms lighted should be carefully ventilated, even during the cessation of lighting, by openings in the ceiling, through which the gas and its burnt products may escape. The plugs of the cocks should be greased from time to time, so as to prevent oxydation, and to facilitate their working.

"In *lighting* the gas, it is essential to open, in the first instance, the main-cock, and then to apply the light successively to each burner at the moment of opening its cock, so as to prevent the escape of any unconsumed gas.

"In *extinguishing the light*, it is better first to close the main-cock, and then to shut off the taps at each of the burners. Whenever a smell of gas indicates that there is a leak in the pipes, the doors and windows should be opened, so as to cause a current of air through the room; and the main-tap should be closed. The consumer should abstain from searching for any leak with a light, but he should rather give notice to the company and the gas-fitter. In case, whether by imprudence or by accident, the escape shall have been ignited, the best means of extinguishing it is to cover the aperture with a wet cloth."

These rules are of great importance, for very serious accidents have resulted from the explosion of gas mixed with atmospheric air. A few years ago—namely, in the month of August, 1848—a fearful disaster of this description occurred in Albany-street. The gas accumulated in the shop for a very short time only—in fact, it had been escaping no longer than one hour and twenty minutes from a crack in the meter; the area of the room was about 1,620 cubic feet; but when the gaseous mixture ignited, it blew out the entire front of the premises, carrying two persons through a window into a back-yard, and forcing another, by the violence of the shock, on to the pavement on the opposite side of the street, where she was picked up dead. For more than a quarter of a mile, on each side of the house, the effects of the explosion were severely felt, and the glass in most of the windows of the neighbourhood was shattered. But the most extraordinary evidence of its enormous power was exhibited in the condition of the premises which immediately faced the house that was destroyed: in one of these the iron railings around the area were snapped asunder, and in another a part of the roof and back windows were carried to a distance of from 200 to 300 yards; besides which, the pavement was torn up for a considerable length. According to the official reports which were made to the insurance offices, it appears that 103 houses were injured by the explosion, and that the damage done amounted to £20,000.

Another accident, of a similar kind, occurred in the month of July, 1850, at the Ilford toll-gate, whereby three persons were severely injured. The space in which the gas accumulated had an area of eighty cubic feet, and the gas had been escaping into it at the rate of forty cubic feet an hour for a period of fifteen minutes. A piece of lighted paper was incautiously introduced for the purpose of seeing where the leak was, and the mixture immediately exploded, breaking up the flooring of the room, blowing out the window, and knocking down a large portion of the front and partition walls of the building. These accidents were made the subject of scientific investigation, and reports were furnished to the journals by Dr. Arnott, Dr. Letheby, and Dr. Alfred Taylor. In speaking of the latter, Dr. Letheby says—"It is difficult to form an estimate of the total explosive force exerted by the gas on this occasion; but I am led to think that it was probably equal to about twenty tons; for when a column of mixed gas, consisting of one part of coal gas and seven of air (the proportion in this case), is fired, it expands to about five times its bulk, and exerts a pressure of about four pounds on the square inch." In the report given by Dr. Arnott on the explosion in Albany-street, it is stated, that the strongest explosive mixture consists of one part gas and ten atmospheric air, the expansion being in that case tenfold; and in a report furnished by M. Tourdes on the explosion which took place at Strasburg in 1848, it is stated, that the greatest force results from a mixture of one part of gas and eleven of air. These discrepancies doubtless arise from the variations in the composition of coal gas; but it may be stated, in a general way, that from seven to eleven parts of air to one of gas constitute the most dangerous proportions; for if the gas or the air be much in excess over these, the force of the explosion will be very much diminished. This is exemplified in a very striking manner by the admirable researches of Sir Humphry Davy into the explosive properties of light carburetted hydrogen, or *fire-damp*; for they show, that while seven or eight parts of air to one of gas produce the greatest explosive effect, other proportions are less dangerous: in fact, a mixture of equal parts of gas and air will burn, but it will not explode. The same is the case with a mixture of two of air, or even three of air, and one of gas; whereas four of air and one of gas begin to show an explosive tendency; and this becomes more and more marked up to seven or eight of air to one of gas. His experiments also prove, that one part of gas to ten, eleven, twelve, thirteen, or fourteen of air were also inflamed, but the violence of the combustion became less and less; and when the mixture consisted of fifteen parts of air to one of gas, there was no explosion at all.

These results indicate, that the most easy way of destroying the inflammability of coal gas, is that of mixing it with a large proportion of atmospheric air; and hence the necessity for good and effective ventilation wherever there is an escape of gas. We can recognise the odour of gas long before the mixture acquires explosive properties. Dr. Alfred Taylor states, that the smell of gas is perceptible when it is mixed with five hundred parts of atmospheric air; and that it is very manifest when it forms one part in a hundred and fifty of air. We have, therefore, a ready means of discovering the danger; and, indeed, the offensive odour of coal gas is one of its most valuable properties; for if it were to be deprived entirely of its odour, accidents would be far more frequent than they are at present.

As gas is lighter than atmospheric air, it is always disposed to accumulate in the upper part of the room; and here it is that ventilation will be most effective. Still, however, there is a strong diffusive power possessed by all gases, by virtue of which they rapidly commingle; and hence the necessity for a complete displacement of all the atmosphere of a room in which coal gas has been escaping.

Dr. Taylor attaches importance to the poisonous properties of coal gas; saying that there are reports of six deaths on record, where persons have been killed by sleeping in rooms near to which there was a leakage of gas. M. Tourdes found, that an atmosphere containing one-thirtieth, or even one-fiftieth, part of coal gas seriously affected animals. It cannot, therefore, be too strongly impressed upon the minds of those who use gas in dwelling-houses, that where a smell is perceptible, the defect should be immediately found out and remedied. When the leakage is comparatively slight it may endanger the lives of those who sleep in or near the spot; and when it has reached a higher point, it may lead to a serious accident by explosion. The effects which it produces on the human system are those of depression, headache, sickness, and general prostration of the vital powers, followed by deep coma.

Gas Ventilation.—It has been already stated that the products of gas combustion are very pernicious—that they not only cause discomfort to the feelings, and perhaps injury to the health of those who inhale them, but they are also very destructive to property; besides which, the high temperature which is produced in rooms where gas is burnt in a wasteful manner, is very objectionable. All these circumstances render it necessary that the products of combustion, as well as the heated atmosphere, should be removed as speedily as possible. In fact, it is of the greatest importance that gas should be consumed in such a manner as not to affect the atmosphere of the room at all. Several contrivances have been suggested for the purpose of effecting this.

Many years ago, when the books in the library of the Athenæum Club-house were found to have been impaired by the gas used in the building, Professor Faraday invented an apparatus which was found to remedy the evil. It consists of an ordinary bat's-wing, or fish-tail burner, enclosed in a globe or bell-glass, closed at the top with mica; the interior of the globe communicates with a tube, which surrounds the gas-fitting, and passes away through a condensing cistern to a ventilating shaft. The tubes are so arranged as not to be visible, and the burners are suspended in the ordinary way from a chandelier. By this contrivance the gas burns in a closed chamber, and the products of combustion are at once carried away. This plan of consuming gas is practised at the club-house before mentioned, and also at Buckingham and Windsor palaces; but the great objection to it is the frequent breaking of the glasses, and the necessity for a ventilating shaft with a strong upward current.

The *sun-burner* has already been described, and so also has the ventilating burner of Mr. Carter.* Messrs. Whichcord and Rosser have also patented a contrivance for effecting the same purpose; it consists of a ventilating-bell and draught-tube placed over the burner. If the bell is lowered so as to be a little below the level of the glass chimney, and the whole surrounded by a glass globe which is open only at the top, a current of atmospheric air passes down over the tube and chimney, and thence to the flame, where it is consumed, after which it passes away through the draught-tube. By this means the air is heated before it reaches the flame, and therefore the intensity of light is augmented; besides which, the cooling influence of the air on the draught-tube and chimney prevents the heat of the gas from being communicated to the room.

Another mode of effecting ventilation, is to place a simple catch-tube, or funnel, over the gas, and thus to carry away the products of combustion into a neighbouring chimney, or to the outside of the house. The draught-tube need not be very large, and it may be hidden above the ceiling. When this plan is not available, a less perfect mode of ventilation may be adopted by boring a number of holes through the ceiling immediately over the chandelier or burner. The holes should be about half an inch in diameter, and they should communicate with the space above the ceiling. A few ventilating bricks should also be introduced into the wall, bounding the space on each side of the house, so as to carry off the warm air. The holes in the ceiling may be hidden from view by means of a perforated or open rosette.

Lastly, where gas can be burnt out of the room altogether, it is of the greatest importance that no plan of internal combustion should be adopted. Shop-windows, for example, are best lighted by means of an external jet with a reflector. On the continent this is the plan very generally employed; and it is found to produce a much more pleasing effect, than when the burners are placed on the inside of the windows. There is no reason why gas may not be used in a somewhat similar way for illuminating private rooms. At present we generally receive the light from the ceiling, and the combustion takes place in the atmosphere of the room; but it is quite possible to burn the gas in a closed chamber at the side of the room; say, for instance, in a recess which might be formed by removing a portion of the wall in some convenient situation, as between two windows. This recess might have a number of small gas-burners with reflectors behind them; and it might be covered in front with ground glass, ornamented with some device. It should, of course, communicate with the external atmosphere, and be shut off from the room. By day it might be covered by a mirror; and at night the mirror might be slid to one side, so as to form a shutter for a window. Again, in some situations where the space between the ceiling and the upper floor is very considerable, a ring of gas jets, or a sun-burner, might be introduced into a closed chamber in the ceiling, and the light might be reflected or diffused in a very agreeable manner. In short, there are many plans which might be suggested for a more perfect and wholesome mode of burning gas for illuminating purposes than that which we generally employ; and there is no doubt that if attention were sufficiently aroused to the importance of this subject, many improvements would be adopted.

Vitiating Effects of Illuminating Agents.—Although gas is generally regarded as the most injurious of all light-giving bodies, yet this is not true when we consume it in moderation; that is, when we burn it in such a manner as to obtain the same amount of light as we are accustomed to have from other agents. This will be made evident from what follows. A very little consideration will show that we have become exceedingly wasteful in the use of gas, burning it in situations where we obtain the least possible advantage from its luminous effects, and demanding such a strong glare from it, that there is a much larger consumption of the material than need be; and consequently there is a larger vitiation of the atmosphere than would occur with any other illuminating agent. Take the case of an ordinary sitting-room. With two candles on the table, at a distance of a foot from the observer, he can see well enough to read, write, or work. But suppose that gas is introduced into the room: it is placed in a chandelier four or five feet from the table, and then, according to the law of intensity, it requires twenty-five candles' worth of light to give the same amount of luminosity on the table; and so two or three gas-burners, consuming in all about ten cubic feet of gas per hour, are fitted up to do the work of two candles. It is this profusion of light and heat which has occasioned so strong a prejudice against the employment of gas, on the score of its heating and vitiating effects; but the prejudice is not well-founded. Dr. Frankland has made experiments to determine the relative amounts of carbonic acid produced by the usual illuminating agents, and he states that the following proportions of carbonic acid are produced per hour during the combustion of a sufficient quantity of each of the materials to get the light of twenty sperm candles, each burning at the rate of one hundred and twenty grains per hour:—

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* See *ante*, p. 129; Fig. 131.

Tallow	10·1 cubic feet.
Wax	8·3 ,, ,,
Spermaceti	8·3 ,, ,,
Sperm oil	6·4 ,, ,,
Common London gas . .	5·0 ,, ,,
Manchester gas . . .	4·0 ,, ,,
London cannel gas . .	3·0 ,, ,,
Hydro-carbon Boghead gas	2·6 ,, ,,
Hydro-carbon Lesmahago gas	2·3 ,, ,,

Now, if we bear in mind that each cubic foot of carbonic acid involves the destruction of nearly five cubic feet of air, and that, according to toxicologists, a proportion of five per cent. of carbonic acid in the atmosphere is dangerous to animal life, we shall perceive that there is an enormous amount of atmospheric air vitiated and rendered irrespirable from this cause alone; but to this must likewise be added an almost equally large quantity of oxygen which is consumed by the hydrogen of these illuminating agents, and which in coal gas amounts in many cases to nearly fifty per cent. It will be manifest, therefore, that in obtaining artificial light by any of these means, we destroy a large quantity of atmospheric air; and hence provision should be made for an ample supply of it by means of good ventilation. This will be still more evident from what follows.

Mr. Lewis Thompson has instituted a set of experiments for the purpose of ascertaining how long a flame of a given intensity, obtained from different illuminating agents, will burn in a given bulk of atmospheric air. In all cases the value of the light emitted was the same—namely, that of thirteen standard sperm candles, each of one hundred and twenty grains' consumption per hour; and a distinct experiment was made with each agent. The times that elapsed before the flames were extinguished were as follows:—

Rape or Colza oil . . .	71 minutes.
Olive oil	72 ,,
Russian tallow . . .	75 ,,
Town tallow	76 ,,
Sperm oil	76 ,,
Stearic acid	77 ,,
Wax candles	79 ,,
Spermaceti candles . . .	83 ,,
Common coal gas . . .	98 ,,
Cannel coal gas . . .	152 ,,

These times are inversely as the salubrity of the illuminating agent; and hence it follows that common rape oil is the most destructive of the atmosphere, and rich cannel gas the least.

The same is true of the heating effects of these bodies. Already we have alluded to this fact; but as it has been made the subject of special experiment by Mr. Lewis Thompson, we will again refer to it. He says, that when the following materials are burnt for an hour, in such quantity as to give the light of one sperm candle of one hundred and twenty grains' consumption, they raised the following amounts of water from the temperature of 60° to 212° Fah.:—

Cannel gas	raised	4074	grains	of water	152°
Common gas	,,	6840	,,	,,	,,
Sperm candle	,,	7505	,,	,,	,,
Tallow candle	,,	7534	,,	,,	,,
Colza oil	,,	7870	,,	,,	,,

"The impossibility of maintaining one uniform rate of consumption in the case of the candle and oil, detracts slightly from the value of the results; but the indications are too decisive to permit the general conclusion to be doubted—that, light for light, the inconvenience arising from heat is much less with gas than with any of the ordinary agents employed to give light; and in the case of cannel gas the advantage is very great."

ILLUMINATING AGENTS WHICH DO NOT VITIATE THE ATMOSPHERE.

Of these there are two which are especially deserving of notice: they are the oxy-hydrogen, or Drummond light, and the electric light.

The Oxy-hydrogen Light.—This was first introduced to public notice by Lieutenant Drummond. It consists of a jet of oxygen and hydrogen gases, or of alcohol and oxygen, burning so as to ignite a piece of lime or magnesia; and the high temperature which is thus produced, renders the earthy body so incandescent as to be intensely luminous.

The apparatus which is employed for the production of this light has, at various times, undergone considerable alteration and improvement. Originally the mixed gases, consisting of two parts, by measure, of hydrogen and one of oxygen, were condensed, by means of a syringe worked at great pressure, into a square metal box, from which there issued a long jet, of very small bore; this jet passed through a thick oak partition, in order that the operators might be protected from the danger which was incidental to the bursting of the metal box from explosion. This was the form of apparatus originally contrived by Clarke and Newman. After this the safety-jets of Gurney, Hemming, and others, were adopted; and at the present time, it is customary to burn the mixed gases by means of the latter, or else to deliver the gases separately into a double jet or nozzle, where they mix immediately before they are consumed. Both of these plans are very manageable, though the latter is thought to be less open to the risk of explosion than the former.

The hydrogen gas is obtained by acting on zinc with dilute sulphuric acid (one of acid to ten or twelve of water); and the oxygen, by heating a pulverulent mixture of four parts of chlorate of potash and one of peroxide of manganese in a glass retort. In each case the gas is to be collected in a gasometer, or else in bladders fixed to receivers over a pneumatic trough. Mr. Watson has obtained a patent for procuring the gases already mixed in proper portions by the decomposition of water by galvanic agency; but the process is an expensive one.

When the mixed gases are burnt, the flame is projected upon a small cylinder of lime or magnesia, which is from time to time turned round, so as to expose a fresh surface to the action of the flame.

Another mode of obtaining this light is to throw a jet of oxygen into a flame of spirit of wine or ether, or to mix the oxygen with coal gas instead of with hydrogen.

The light which is obtained by either of these plans is very intense. When concentrated by means of a concave mirror, it is distinctly visible at a distance of sixty-five miles; and when compared with the light of a wax candle, the mixture of oxygen and coal gas is equal to twenty-nine of such candles; that of alcohol and oxygen to sixty-nine; that of ether and oxygen to seventy-six; and that of hydrogen and oxygen to a hundred and fifty-three. In consequence of the great intensity of the oxy-hydrogen light, it is generally employed for the phantasmagoria, the dissolving views, and solar microscope, and for theatrical illuminations, and experiments in optics; besides which, it has been recommended for lighthouses and signal-lights. The light differs from all others which have been described, in the circumstance of its being exceedingly white, and therefore well suited for the display of bright and delicate colours. With this light the various shades and tints of a picture or dress are as plainly discernible as they are by the diffused light of day.

The Electric Light.—Within the last few years, public expectation has been raised to a great height, by the announcement that voltaic electricity might be made the means of producing a very pure and intense light; and several patents have been taken out for the purpose of accomplishing this. Indeed, many exhibitions of the light have been made at several public institutions, and also in the open air, by which the sanguine hopes of its originators have been apparently increased. Of late, however, little has been heard of the subject beyond the few remarks which occasionally fall from men of science in their discourse upon it.

The idea of employing electricity for this purpose is not novel; for the power of the voltaic light was thoroughly investigated in the time of Sir Humphry Davy; but the difficulties which electricians had then to contend with in the inconstancy of the galvanic battery, offered an insurmountable barrier to its use. A part of this difficulty was overcome by the late Professor Daniell, who was the first to contrive a constant battery. Since then, the voltaic arrangements, by which a current of electricity of great power may be sustained for any length of time, have been improved by Mr. Smee, Professor Grove, M. Bunsen, and the Rev. Professor Callen, of Maynooth; so that, at the present time, there are abundant facilities for producing the light.

The first patent which was taken out for the use of electricity for illuminating purposes, was that of Mr. Staite, in November, 1846. In July, 1848, he obtained another patent for an improved form of battery. This he called the *perfluent* battery, in contradistinction to the *percolating*, which was already in use. It was thus named on account of the arrangement which he adopted for keeping up a supply of acid. The troughs, or cells of the battery, communicated with each other by means of elastic syphons or cross tubes, and the acid was made to flow from cell to cell throughout the entire length of the series; so that, when it arrived at the last cell, it was completely exhausted of its exciting power, and saturated with zinc. It was thought that such an arrangement would be the means of economising power and material; but it happens to be the very worst that could possibly have been contrived for such a purpose. The electrician knows, that if he requires the greatest power from his battery, each of the cells must be acting alike, and to their fullest extent; for it is the condition of electricity to multiply itself, not after the ratio of the most powerful cell, but after that of the weakest. If, therefore, we have in the arrangement a number of cells working unequally, as is the case in the perfluent battery, the activity of the one set of cells will be of no avail in raising the inactivity of the other; for the least powerful cell governs all the others, and reduces them to its own standard. It follows, therefore, that the last cells in Staite's arrangement must act as clogs on the first; for, as they are almost without action, they must reduce the power of the first cells to little or nothing. Another alteration, proposed by Mr. Staite, was, that a liquid amalgamation of zinc and mercury should be used in bags, or porous cells, in lieu of the ordinary form of amalgamated zinc. He also suggested the use of a solid amalgam, consisting of 5 of zinc and 1 of mercury. He likewise proposed that lead should be employed instead of zinc, in order that valuable products might be obtained. All these suggestions, however, are of no value; for they are contrary to the principles on which a good galvanic battery ought to be constructed.

In July, 1848, the Chevalier Alexandre Edouard Lemolt also took out a patent for improvements in the batteries and apparatus used in obtaining the electric light. His improvements in the battery were but modifications of those long since adopted by Professor Bunsen. He employed plates of carbon, which were either cut out of the hard coke which lines the interior of gas retorts, or else formed by powdering the coke, mixing it with a little coal-tar, then pressing it into a mould, and baking it at a low red-heat. Since then—namely, in the month of February, 1849—Mr. Charles Thomas Pearce obtained another patent for improvements of a like description. His battery was a perfluent one, and it differed from Staite's in the circumstance that the flow took place separately in each cell, and not from cell to cell; so that the amount of power generated in each cell was the same, and his porous diaphragms were made of sycamore wood, soaked in dilute acid. Another of his claims was the use of alkaline salts in solution, which did away with the necessity for amalgamating the zinc. Lastly, another patent has since this been obtained by Messrs. Staite and Petrie, wherein they recommend a modification of their former arrangement, so as to keep the liquids in all the cells of the same degree of strength. Each of these patentees has likewise secured to himself the use of certain contrivances, whereby the charcoal-points which emit the light shall be removed and kept at a proper distance from each other. These, in fact, constitute the real claims to consideration; for the batteries which they have invented are not worthy of the least attention.

Batteries used in obtaining the Electric Light.—(*a.*) *Daniell's Battery*, which consists of a jar containing a cylinder of sheet copper, and a saturated solution of its sulphate; within this there is placed a porous cell (composed of brown paper, unglazed earthenware, or bladder), which holds a cylinder of sheet zinc, and dilute sulphuric acid. These are arranged so as to obtain both quantity and intensity of electricity.

(*b.*) *Grove's Battery* is constructed in a very different manner. The outer cell is of an oblong form, and it may be made of glass, earthenware, or gutta-percha. It contains a piece of sheet zinc bent into the form of the letter U, one leg being a little longer than the other. The zinc is amalgamated according to Mr. Smee's plan, by dipping it into dilute sulphuric acid, and then covering it with quicksilver. Within the bend of the zinc there is placed a porous oblong cell of unglazed earthenware, and within this a sheet of platinum. The battery is set in action by pouring dilute sulphuric acid (in the proportion of one acid to seven water) into the zinc compartment, and strong nitric acid into the platinum cell. As before, the battery may be arranged for quantity as well as intensity.

(*c.*) *Bunsen's Battery*, as well as that modification of it proposed by M. Lemolt, is truly a Grove's battery, with a piece of charcoal instead of a sheet of platinum.

(*d.*) *The Maynooth Battery*, which was contrived by the Rev. Professor Callen, of Maynooth, is also a modification of Grove's principle; for a plate of cast-iron is used instead of platinum; or a cast-iron cell is charged with a mixture of two parts of strong sulphuric acid, one-and-a-half of nitric acid, and the same of water; and it is better to excite the zinc with a strong solution of muriate of ammonia instead of with dilute sulphuric acid, for this does away with the necessity for amalgamating the metal.

Professor Grove says that the result of his experience is, that the nitric acid battery, in one or other of the preceding forms, is the only one, hitherto invented, which offers anything like a practical means of applying this power to illuminating purposes; and the best arrangement that can be adopted for obtaining the greatest amount of power, is to use about forty or sixty cells, arranged in two series of twenty or thirty each. By this means we have the quantity of two cells, and the intensity of twenty or thirty. If we go beyond this in the intensity arrangement, the fluids in the cells begin to boil, and quickly to evaporate. This is objectionable; for it not only renders the atmosphere of the room irrespirable, but it frequently brings the action of the battery to a standstill. The source of power in all cases is the chemical action which takes place in the cells.

Mode of obtaining the Light.—The wires which convey the electricity from each of the series must be connected, so that the two positive poles are brought together, and the two negatives. The wires should be of large size, so as to conduct the electricity with ease; and they should be covered with gutta-percha, so as to insulate them. If this precaution be not taken, they may touch each other, and so cut off the current; or they may become unmanageably hot; or they may communicate a shock to the operator. It is usual to have the galvanic batteries in any convenient place at a distance from the experimenter, and to convey the electricity, by means of the insulated wires, to the point where it is wanted. Light may be obtained from the battery in two ways: either by bringing the poles into contact with a yard or so of platinum or iridium wire, wound into the form of a spiral; or by terminating them with cylinders of charcoal, and then bringing them into contact. In each case the light is produced by the ignition of the conducting medium. Platinum or iridium is not so well suited for the purpose as charcoal, because, in the first place, the light is never so vivid, and, in the second place, the metal is very likely to fuse, and put a stop to the experiment. When charcoal is employed, it is found that the greatest intensity of light is produced by drawing the points apart to the extent of from a quarter to half an inch; and then there is a stream of finely-powdered charcoal, in a most intensely ignited state, projected from one pole to the other, forming an arc of flame. If the charcoal-points are too close together, we do not obtain the maximum effect; and if they are too far apart, the arc is broken, and the light extinguished. This it is which constitutes the difficulty in keeping up the electric light, and which gives to the flame its unsteady, flickering character. Consequently, all the contrivances which have been adopted for overcoming this, have been made the bases of the several patents to which we have alluded. These we shall now proceed to describe.

Fig. 137.

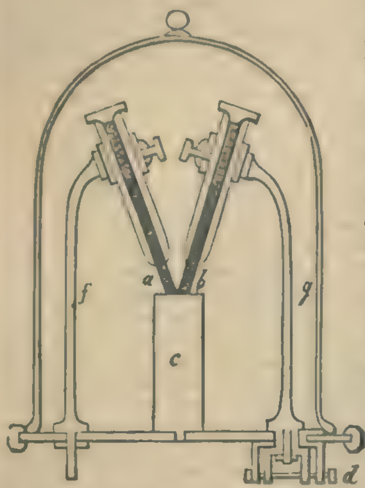
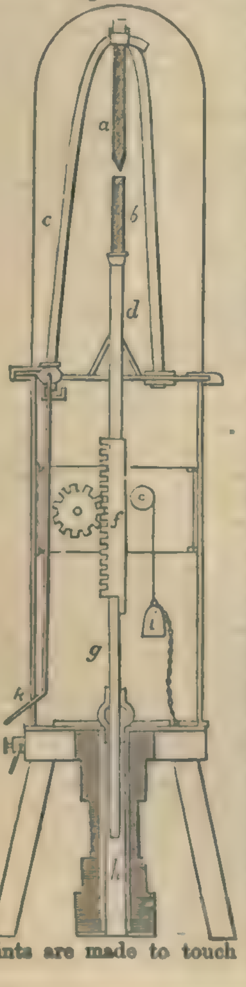


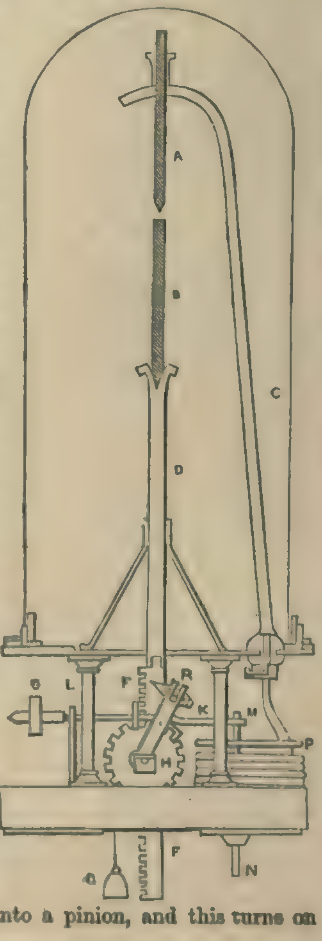
Fig. 138.



Apparatus for sustaining the Electric Light.—The first of Mr. Staite's patents was for a contrivance or method for maintaining the charcoal-points at a uniform distance. One of these is represented in Fig. 137: *a* and *b* are the charcoal-points; they slide easily in a brass tube which holds them, and their free ends rest upon a solid cylinder of plaster of Paris, *c*. At the opposite ends they are pressed upon by a spiral spring, which is contained within the brass tube; and by which means they are always forced down very firmly on the plaster of Paris cylinder. There is an adjusting screw at *d* for regulating the distance of the points, and the arms *f g* convey the current. To set it in action, the two charcoal-points are made to touch by means of the adjusting screw *d*, and then they are separated to the required distance, so as to get the maximum amount of light. As the points burn away, the springs keep up a fresh supply by forcing them down on the plaster of Paris cylinder, and maintaining them at their proper distance. It was at first thought that this arrangement would meet all the difficulties of the case; but it was soon found that there were irregularities in the action of the battery, as well as a projection of the charcoal from one pole to the other, which demanded a constant motion of the points; and, therefore, a few months afterwards the patentee adopted another invention, whereby the points were adjusted by the aid of an electro-magnet. A third patent, with still further improvements in this respect, was obtained in the course of the same year. In this last patent there are three distinct kinds of apparatus described for the management of the electric light—namely, one for obtaining a regularly intermitting light, another for procuring a constant and uniform light, and a third for developing a constant light by the ignition of a metallic wire.

The apparatus for producing the first effect is represented in Fig. 138: *a b* are the two charcoal-points; one of them is fixed to the conductor *c*, and the other to the movable rod *d*. The lower part of *d* is fixed to a rack *f*, and this to an iron rod *g*, which moves freely up and down in a tube *h*, that is surrounded by a coil of wire in the form of a helix: a weight *i* is attached to the rod *f g* by means of a string, which passes over a pulley. The object of this is to counterbalance the weight of the rod, rack, and charcoal-holder; and there is a small piece of chain also attached to the weight, so that the balance may be equalised as the rod is pulled down. The action of the apparatus is as follows:—The conductor *k*, and the wire of the helix around *h*, are brought into connection with the galvanic battery, and the circuit is closed by bringing the two charcoal-points into contact. At this moment the iron rod *g* is rendered magnetic, and is drawn down into the hollow tube within the helix. By this means the charcoal-points are separated, and a most intense light is produced; but soon the separation takes place too far, and then the circuit is broken and the light extinguished. The iron bar now loses its magnetism, and the weight draws it up again, so as to bring the charcoal-points once more into contact, when the same phenomenon is repeated. It is obvious that by this contrivance a succession of flashes at regular intervals is produced, and the weight may be so regulated as to maintain any period for the duration of the light and its intermission.

Fig. 139



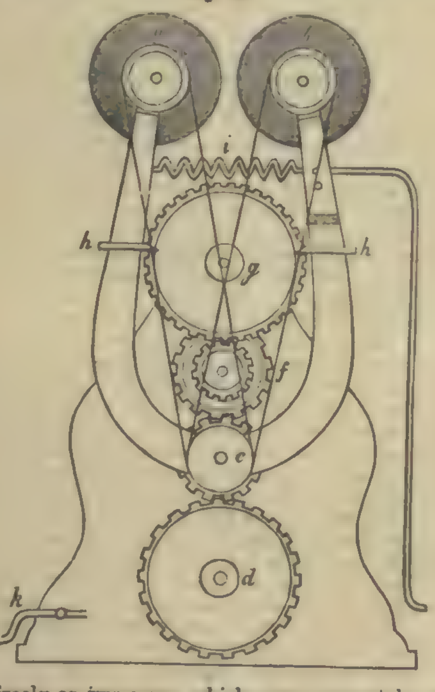
The apparatus for producing a constant and uniform light is shown in Fig. 139. The charcoal-points A B are attached as before—one to a fixed conductor C, and the other to a moveable conductor D; the latter is moved by a rack F F, which works into a pinion, and this turns on a spindle with fixed supports. A barrel is attached to the pinion, and over this there passes a string, to which the weight G is fastened: by this means the rod and rack D F are counterpoised. To the spindle there is fixed a cogged wheel H and a lever I; the latter carries a double paul, which locks into the cogs of the wheel in either direction. A long horizontal lever, K, passes over the paul, and moves on the fulcrum L; while it supports at one end a rod M, which is fixed to an iron bar N, and at the other a movable counterpoise O. The iron bar travels freely up and down in the coil P.

The paul and its lever I are kept in a state of slow vibration from side to side by means of a crank R, which works in a fork at the end of the lever I. This crank is made to revolve by an ordinary train of wheelwork, furnished with an escapement or fly-wheel, and driven by spring-power or by weights. The object of this movement is to elevate or depress the rack F by turning the wheel H.

The mode of action of the apparatus is as follows:—The negative pole of the battery is brought into contact with the rack and rod D, F, and the positive pole with the coil P, and thence with the upper charcoal support C. Immediately the points touch, and the circuit is completed, the coil raises the iron rod N. This acts on the lever K, and causes the lower charcoal-point to descend and separate from the upper one. Directly the separation has taken place to the maximum extent, there is a contrivance (not shown in the drawing) which arrests the movement of the rack, and so keeps the electrodes stationary. Before the apparatus is set to work, the proper distance for the charcoal-points is adjusted by means of the counterpoise O, which screws backwards or forwards on the short arm of the lever K, and thus regulates the movement of the rod M, N. If the light should go out, and the circuit be broken, the rod N immediately falls, pulling down with it the long end of the lever K. This presses upon one arm of the paul, and gives motion to the wheel H, whereby the rack is elevated, and the charcoal-points again brought into contact.

M. Lemolt's apparatus for adjusting the charcoal electrodes is somewhat different from the last. In the first place, the electrodes are not cylindrical, but are in the form of circular discs, *a*, *b*, Fig. 140; and they revolve freely on two arms, which move separately on one common axis, *c*. This axis also carries a pinion, the square cogs of which are in communication below with the driving-wheel *d*, and above with another pinion *f*, the inner cogs of which are in gear with a large wheel *g*. Over the drum of the pinion at *c*, there pass two endless pulleys, which give motion to the charcoal discs, *a*, *b*. In order that the discs may be at a proper distance for the production of a good light, there are two adjusting stops *h h*, fixed to the arms on which the discs rotate. These stops are brought into close contact with two cams, which are situated in the periphery of the wheel *g*; and as this wheel rotates, the charcoal discs are brought closer and closer together, so as to compensate for the wear of the electrodes. There is a spiral spring at *i* to keep the arms and their stops tight against the cams. The apparatus is thus set in motion—the terminal wires of the battery are brought into connection with the charcoal discs by means of the conductors *k*, *l*. The driving-wheel *d* is then set in motion by clockwork within the case of the instrument, and its motion is communicated to the other wheels, and also to the charcoal discs. The discs are then brought together so as to complete the circuit, and the stops are adjusted in the cams, so as to produce the necessary arc of flame. By the revolution of the discs, new surfaces are constantly presented to each other, and the old or worn edges are cleaned down by the sides of the stirrups in which they revolve.

Fig. 140.



In Mr. Pearce's arrangement, there is a prismatic or cylindrical bar-electrode in contact with one, two, or three of a discoid form. These are moved by clockwork; and it is so managed, that while the first advances or rises through its sheath, the others revolve at a proper distance from it, and the edges are kept clean by means of iridium cutters. The charcoal bar is also held by the iridium conductors, which grasp it close to the ignited point. The advantage of this mode of mounting the electrodes is, that if from any cause, such as unequal waste of carbon, or irregularity of surface, the light should be extinguished at one point, the others remain burning, until by the further revolution of the disc, or the advance of the bar, the electrodes are again brought into contact, and the light restored. Besides which, in presenting two or three centres of light, so merged as to appear as one in a reflector, there is greater brilliancy and greater steadiness of flame.

Another apparatus has been patented by Mr. Pearce, which does away with the necessity of clockwork altogether. It consists of two bar-electrodes, which are approximated by means of springs or weights and pulleys; and the necessary distance is preserved by the intervention of slips of non-conducting charcoal. He has also contrived a plan for relighting the electrodes in case the arc of flame is blown out. This he effects by aid of a wedge-shaped piece of charcoal, which falls down between the electrodes directly the light is extinguished, and thus re-establishes the contact. The charcoal is fixed at one end of a lever, and to the other end there is a soft iron armature: this is placed opposite to a soft iron magnet, enclosed in a coil which carries the current. While the electricity is passing, the electro-magnet draws the armature to it, and so keeps the wedge-shaped piece away from the electrodes; but directly contact is broken, the armature retires, and the wedge-shaped piece falls down between the poles and re-establishes the arc.

Mr. Staite has patented a plan for obtaining a constant and uniform light by the ignition of a loop of platinum or iridium wire, enclosed in a glass globe from which the air has been exhausted. But there is no novelty whatever in this contrivance; and, consequently, it is undeserving of further notice.

Some little attention is necessary to the quality of the electrodes, in order to get the best effect. *Common charcoal* is unsuited for the purpose, because of its being in most cases a non-conductor of electricity; the *charcoal of porous wood* is also objectionable, from the circumstance of its burning away very rapidly: therefore it is that electricians make choice of the densest varieties of carbon. One of these is the *charcoal from boxwood*, which is obtained by cutting the wood into pieces of nearly the required form: they are then put into a crucible, filled up with fine sand, so as to secure the exclusion of atmospheric air, and exposed for two or three hours to a bright red or white heat. The crucible is to be well covered, and allowed to cool before the charcoal is taken out.

Hard coke has been employed with considerable advantage by many persons. The objection to it is, that it frequently scintillates, from the large quantity of iron which is contained in it. This difficulty may be overcome by powdering the coke, then stirring it about with a magnet so as to attract the iron, or else digesting it in nitro-muriatic acid, and washing very well before drying. The powdered coke is then to be moistened with a little syrup or coal-tar, and rammed into a cylindrical mould: the bar which is thus made must be ignited in a crucible covered with sand, in the same way as that already described for the preparation of boxwood charcoal. These cylinders will be found exceedingly hard, and they give a light which is superior in whiteness and intensity to that from any other kind of charcoal. If the electrodes are soaked in a strong solution of common salt, and then dried, they give a still more brilliant effect; and by using a salt of copper, chloride of strontium, &c., we obtain green, red, and other coloured flames.

Again, it has been noticed that the intensity of the light is increased by removing the atmospheric air from the vessel containing the electrodes, and effecting contact in a rarefied atmosphere, or in a vacuum. The *effects* which are produced in this manner are very remarkable; for the purity and intensity of the light are greater than those from any other source. In the first place we find that it simulates the light of the sun, in the circumstance of its affording a means of distinguishing the most delicate tints of colour: blues, yellows, and whites, which are not to be seen in a pure state by ordinary artificial light, are recognisable by this mode of illumination as if they were seen by day. Again, when the light is decomposed by a prism, we obtain a spectrum which is similar to that afforded by the rays of the sun.

The intensity of the light has been variously estimated. That which was exhibited by Mr. Staite on the 30th of May, 1849, from the summit of one of the towers of the Hungerford Suspension-bridge, was said by him to be equal to 750 wax candles. Professor Grove, in experimenting with a battery of thirty cells, the platinum being four inches by two, obtained a light which equalled that of 1,444 wax candles; and Dr. Letheby states, that in some experiments which were made by Mr. Hearder, of Plymouth, in the month of April, 1849, with a Maynooth battery of eighty cells, each four inches square, arranged in two sets of forty each—the light, when concentrated by a parabolic mirror, and sent over the country for a space of 5,490 yards, gave a light equal to that of a candle at thirty feet distance; so that the intensity of the focussed light was equal to that of 301,401 candles. It is probable that Mr. Grove's estimate is nearest to the truth, as great pains were taken in his investigations to arrive at correct results.

It is very natural that a light of so great brilliancy should command a large share of public attention, and should likewise be made the subject of frequent experiment. Hence it is that Mr. Staite and others have often exhibited it in London and elsewhere. When it was shown from the top of the Hungerford Suspension-bridge, the light was sufficiently intense to illuminate the water-frontage of Somerset House; and when it was cast to the opposite side of the river, it lighted up all the buildings on the Surrey shore. Exhibitions of it have also been made at the Hanover-square Rooms, from the top of the Duke of York's Column, and from the portico of the National Gallery; but the most marvellous illustration of its power was afforded by Mr. Hearder, of Plymouth, who placed the light at the top of the Devonport Column, and first experimented with it at Trematon Castle, which is distant about 18,266 feet; and then at Bovisand, which is 16,470 feet from the column. At the former place the light was sufficiently strong to mark the time on the seconds-hand of a small watch, and the walks of the castle were distinctly visible at a distance of half a mile; besides which, the ivy-leaves over the gateway of the building were plainly seen when the observer was sixty feet away from them. Its intensity, says Mr. Tucker, who reported upon it, was magnificently brilliant. At Bovisand the light was sufficiently strong to cast a shadow of objects on a yellow wall, and it was thought to be about equal in intensity to that of the full moon when at its meridian in a calm clear night. At that distance, without the reflector, it looked like the planet Venus when seen through a telescope.

The following are the accounts given of these effects, as seen at Bovisand and Trematon, by Messrs. Walker and Tucker, the two gentlemen who were deputed by Mr. Hearder to make the necessary observations. The accounts are extracted from the *Plymouth Herald* for April 21st and May 5th, 1849. Mr. Walker writes thus:—

"With regard to our observations at Bovisand. About half-past eight o'clock we saw flashes and glimmerings of a bluish light from the column at Devonport, which, to all but myself, were unsatisfactory. The people thought that something had gone wrong. The light was then, in all probability, on the Trematon side of the column, the column being directly between us and the light itself. After waiting for some time, we finally saw the electric light outshining all the other visible lights, and we sent up a rocket or two to indicate our satisfaction. Our personal shadows were projected upon a boat-house door (painted yellow, and itself illuminated) by the electric light, and pronounced equal to that of the full moon when on the meridian in a calm clear night.

"A candle (six to a pound) projected a similar shadow upon the same door, illuminated in the same way, at a distance of 30 feet from the person whose shadow was thrown upon the door. That is to say, the shadow projected by the electric light at a distance of 5,490 yards, was just equal to the shadow projected by a single candle at a distance of 10 yards. Consequently, their illuminating powers were as 10 (squared) : 5490 (squared), or as 1 : 301,401. This comparison is rather a 'stunning' one! I am of opinion that the electric light possesses a space-penetrating power infinitely superior to any light we can obtain by combustion, since it is free from all carbonaceous or solid particles!

"The Breakwater light may probably be equal to twenty-four Argand lamps, each equal to half-a-dozen candles; yet *this* light projected a shadow *only* slightly visible at a distance of 500 yards; while the electric light projected shadows, at lucid intervals, strong enough for children to make *rabbits* with their fingers upon a wall more than 500 yards distant from the light itself; that is to say, ten times farther than from a lighthouse that could only throw a shadow 'faintly visible.' The electric light, as seen in the open air, may be compared (as far as colour is concerned) to that of the planet Venus when seen through a telescope, or to the light of a glow-worm, or to those brilliant flashes of light we sometimes see within the tropics, from the waves when surcharged with phosphorescent matter."

And Mr. Tucker's account is as follows:—"Sir,—A variety of occupations have prevented my sooner informing you of the effect produced here by the electric light, on the evening of the 12th inst. You are aware that this castle is 18,266 feet distant to the N.N.W. from the column at Devonport. The reflector which you used was one from which the rays diverge. The evening was very favourable; but little breeze (E.N.E.) was stirring, yet sufficient to blow off the smoke from the town. The atmosphere was so clear that the Devonport lamps were very distinctly visible, and the stars which appeared shone very brightly; but they were few, and the moon had not yet arisen. The instant the light shone, the lamps of Devonport were all but totally eclipsed: as it fell upon a sail spread over the walls of the keep, we immediately perceived that a shadow was cast by the fingers of a hand upon the sail, by a twig of ivy, and by the stem of an ivy-leaf. We could clearly see what o'clock it was by our watches. I ascertained the light to be polarised, and the whole effect was very striking, for the light upon the column was exquisitely beautiful—its brightness was magnificently brilliant. The interposition of the red shade then informed us that the light would be changed. When that change, whatever it were, was made, the light sensibly increased; we then could see the time by the seconds-hand of a small watch; the usual hand-writing was read; and the ivy-leaves on the gateway tower were seen at the distance of ninety feet. Persons more than half a mile behind the keep could plainly see the walks around it; and we all were very much pleased by the striking effect produced by your turning the reflector upwards towards the clouds, which we clearly saw, the light then having the appearance of the tail of a huge comet, the reflector being the nucleus. I think I may state that the breadth of the intensity of the light was at least three-quarters of a mile; and I believe that persons standing on the brick-field (close to Devonport) saw the light reflected to them from a looking-glass which was suspended from the keep."

If the electric light is exhibited in a room where there are gas or other lights, the intensity of the former is so great that it actually produces shadows of the flames of the others. At the Polytechnic Institution, the electric light is used for the purpose of illustrating the optical effects of refraction and reflection in a stream of water—as seen in Duboscq's fountain. The chromatic effects are produced by means of coloured glass, which is rapidly shifted before the light.

As to the economy of the light, little can be said. Professor Grove stated, that in his experiments with the battery which gave the light of 1,444 wax candles, the cost was at the rate of about 3*s.* or 3*s.* 6*d.* per hour; and Mr. Ward, who has devoted attention to this part of the subject, states, that to obtain a given light with 100 pairs of Smee, 55 of Daniell, or 34 of Grove, each cell consuming 60 grains of zinc per hour, the cost would be about 6*d.*, 7½*d.*, or 8*d.* per hour respectively. But we apprehend that this does not include the original cost of the battery, or the charge for attendance. At the Royal Opera House, where the light was exhibited for several nights in a new ballet, it cost the manager £2 per night, although the exhibition was not of long duration. In this case a Maynooth battery of seventy-five cells was employed.

This clearly indicates that the light can never be economically employed as an ordinary illuminating agent: indeed, it is manifest, that as the source of power is really the same as that of other artificial lights—namely, the oxydation of a combustible body—it resolves itself into this: whether it is cheaper to burn gas, oil, or tallow, by means of atmospheric oxygen, or to consume zinc by the aid of water and very expensive acids. Of course some allowance must be made for the value of the products in the latter case; but then, as a set-off to this, we have to consider the expense of constructing the batteries, and of attending to them. It will, however, be freely admitted, that although it is not an economical light for ordinary purposes, it may be advantageously employed whenever a vast amount of illuminating power is required, and there are facilities for charging the batteries. It might be employed, for example, in light-houses, perhaps also in mines, in theatres, and for public exhibitions of various kinds. It has been stated that the electric light has been used to illuminate the works of the Napoleon Docks in Paris, where the men are employed night and day in their work. The light has been employed there for four months, at a cost of thirty-six francs per night: and as it served for the use of 800 men, the cost was exactly four-and-a-half centimes, or less than a halfpenny, per man. This does not look like a very expensive mode of illumination; and it is very probable that it may be resorted to in such cases as this with considerable advantage. Again, it is not altogether impracticable to have an electric light in some convenient part of a mine, and, by the aid of reflectors, to throw the light along the different galleries, and into the workings. By this means all danger from explosion in those localities where the fire-damp abounds, would be completely obviated.

The Steel Mill of the Miner is the last form of apparatus for the production of artificial light to which we shall allude. Before the introduction of the Davy lamp into the coal-mines of this country, a rude instrument was employed by the miner for the generation of light in those localities where the fire-damp rendered the atmosphere unusually dangerous. It consisted of a small steel wheel, which was made to revolve very fast by means of a small pinion that was turned by hand; and while the wheel was revolving, a piece of flint was held against it, so as to emit a brilliant shower of sparks. These gave out sufficient light for the miner to work by. It was thought that the heat from this apparatus was less dangerous than that from a candle; but Dr. Pereira succeeded in firing explosive gas with it, and thus demonstrated to the Parliamentary Committee that it was just as dangerous in a mine as a candle. At the present time the instrument is quite out of use.

Although the subject of artificial illumination has been fully dealt with in the preceding pages, the following remarks may prove interesting and useful to some of our readers. We have taken each subject separately, and supplementary to what has been already advanced.

Young's Paraffine Lamps.—Mr. James Young, of Bathgate, near Edinburgh, has, for some considerable time past, been engaged in producing an oil for illuminating and other purposes, from a shale known as the Torbanehill mineral. With respect to this substance, we may observe, that there is hardly anything in the way of scientific discovery which has proved so difficult to classify. Nearly every eminent chemist in Europe has been called on to decide as to its nature; the question being, whether it should be regarded as coal, or as a mineral substance. Owing to the terms of the lease, the lessors reserved their rights to the minerals on the property; whilst the lessees object and resist all claim on them, by asserting that the substance from which the oil is obtained does not fall within that category. Advertisements have been freely distributed in the leading papers by the lessors, since the commencement of this year (1861), inviting all competent persons to afford their advice and assistance; and as the means of warfare are not wanting on either side, and as the prize is of almost fabulous value, we may expect to find a most exciting contest before the Court of Session, in Scotland, at the impending trial of the case. The object is to invalidate Mr. Young's title in the question, so far as his patents are concerned—the right to use the material having been compromised by the parties.

Paraffine itself is so well known as to require no description; and Mr. Young uses a form of lamp similar to the one already illustrated.* Such are, however, modified as to size, ornament, &c.; and the paraffine lamp may now be seen in the humblest cottage, and in the residences of our nobility, so popular and general is its use.

The Fitzmaurice Light.—This invention caused a considerable sensation amongst naval circles, on account of its brilliancy and portability. For some time its arrangements were kept secret; but it is now stated that it is simply a modification of the lime light, the gases being kept in small reservoirs, into which they are pumped to a high pressure.

The Lime Light.—This light has lately been partially used for street purposes; and for the sake of introducing it into popular notice, one side of Westminster-bridge was lighted up with it. A company was formed to carry out its general adoption. We believe, however, that nothing further has been done than what we have mentioned; and until a cheap and ready mode of obtaining oxygen is discovered, it is very unlikely that the lime light will be generally adopted.

The Electric Light.—This subject we shall fully enter into hereafter; but we may just mention, that Mr. Holmes has, within the last few years, made a considerable improvement in obtaining electricity, by constructing a large magnetic machine, which is driven by steam-power. Coils of wire, enclosing bars of soft iron, are opposed to powerful magnets. These rotate rapidly, and, each time of passing the coils, a current of electricity is produced. In 1856, we were present at a trial of its powers, at the premises of the Trinity Board, Blackwall; and we can, perhaps, give no better illustration of the effect of this light, than by stating, that we could easily detect the divisions or panes of a window in a house a mile distant, when the light was concentrated by means of the ordinary lighthouse lenses. The apparatus was subsequently transferred to the South Foreland; and nightly, for six months, without a moment's interruption, the light was maintained, casting its brilliancy across the Channel, and being distinctly seen on the French coast.

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* See *ante*, p. 113; Fig. 117.

We shall reserve our further remarks on the electric light for the section of Electricity, under which it will more conveniently be discussed.

CHAPTER IV

PHOTOGRAPHY.

Having, in our former pages, entered fully into the examination of the phenomena and laws of light, we are prepared to deal with one of the most interesting applications of physical and chemical science—namely, the art of taking pictures by the agency of light. We shall have to introduce the subject by referring to some of the laws and facts of chemistry; because, although photography is an application of the effects of light as an agent, the process is an entirely chemical one so far as the operator is concerned.

The power of light in changing the ordinary colour of objects, is far more universal than is generally supposed. Most of our readers must have noticed that some delicate colours speedily fade when exposed to the full light of the sun. This effect is undoubtedly due to some action produced by the rays of light on the substances with which a material has been dyed. Light-blue and pink silks are thus easily faded. Sir John Herschel, in some masterly researches, published a few years ago in the Philosophical Transactions of the Royal Society, pointed out, that all the juices of any flower are thus acted on by light. Indeed, we may state generally, that the cause of the varied colours of our landscapes and flowers, is simply that of a process analogous to those employed in photography. One of the simplest illustrations of the kind we can offer, is that of the different appearances of the inside and outside of some vegetables, such as celery, lettuce, and the cabbage. The external leaves which have been freely exposed to the direct solar rays, have a deep green, or, in celery, a red colour; whilst the inner leaves, which have been protected from light, always present a pale or blanched appearance. Even the human face may be taken as another instance of the same kind; for we observe the ruddy glow and deep colour in the rustic, presenting a bright contrast to the sallow, pale, and unhealthy appearance of the factory operative, or the resident of our large cities. Light, indeed, has a colourising and a sanitary effect, to an extent little dreamt of by those unacquainted with the science of physiology.

Photographers, however, avail themselves almost entirely of the metal silver as the great medium of their operations. When that well-known metal is united with certain substances, the resulting compounds are rapidly affected by light; and on exposure thereto, become changed to a dark colour. The whole art of photography may be almost summed up as existing in the choice of the salts of silver, their proper application, and immediate removal when the desired effect is gained. It is true that many other substances may be similarly employed; but, in practice, they are of comparatively little independent worth.

It will be impossible to go into the entire history of this most interesting art, for its inventors and improvers are indeed "legion." Suffice it to say, that we are indebted to Mr. Wedgwood and Sir Humphry Davy for the initiative of our processes; and, since their day, MM. Niepce and Daguerre, in France, and Messrs. Talbot, Archer, Hunt, Sir John Herschel, and a host of other eminent men, have, by their intelligence and practical skill, raised to the highest rank as a philosophical study, and as a social and artistic occupation, that which, twenty years ago, was but barely named in ordinary scientific works.

It will be our endeavour to discuss and illustrate the principles and practice of photography in as plain a manner as possible. For the sake of saving space, we shall assume that our readers, through our previous pages, have become acquainted with the various optical laws, instruments, &c., to which we have alluded. This will prevent tedious and unnecessary repetition. The chemical processes we shall describe as if we addressed those who are unacquainted with chemical science; and whilst giving a variety of details as to processes employed or recommended by different individuals, we shall mostly keep in view those which are simple, effective, and easy of prosecution by persons of ordinary intelligence, and limited scientific attainments.

INTRODUCTORY EXPERIMENTS.

We commence by introducing the experimenter to a few illustrations, which will give a general idea of the nature of photographic processes, and which he will do well to repeat until a certain amount of ease in manipulation is gained. We must, however, preface our remarks by urging on all a great point, without attention to which all attempts at photographing must surely fail: we refer to *scrupulous cleanliness*. Neglect of this, and carelessness, are the two chief causes of all the disappointments both of the tyro and practised operator; and to impress the necessity of our advice being attended to, we shall suggest an experiment, by which the cleanest hands may be proved to be photographically dirty on many occasions.

Experiment 1.—After washing and drying the hands in the usual manner, allow them to remain some time in distilled water, afterwards frequently rinsing them therein. Add to the water a solution of nitrate of silver: in a short time the water will assume a milky appearance, owing to the common salt exuded with the perspiration from the skin, forming a substance with the silver, called chloride of silver. If a little of the solution of nitrate of silver be added to pure water, no salt of the kind we have named will be produced.

It will thus be seen how, in every case, the photographer carries with him the instruments of his own destruction. Whenever a piece of paper or glass is touched by the fingers, then a spot of common salt, grease, and organic matter is left: these acting on the silver salts, produce patches and spots which spoil any photograph. We trust that our readers will be by this, fully persuaded of the necessity of care in this respect, and thus they may prevent the numerous annoyances to which their neglect would otherwise expose them.

The salt formed during Experiment 1, is called the chloride of silver, because the chlorine in the common salt from the hand, has joined with the silver contained in the nitrate of that metal. We must here state, that the term "salt" is applied by the chemist to a vast range of substances; and generally, whenever an acid is united with an earth or an alkali, a "salt" is formed. Thus nitre, or saltpetre, is composed of nitric acid and potass, and is called a salt of potass. Its name is partly derived from the acid, the nitric, and partly from the base, the potass. Hence the chemist terms it the nitrate of potass; and by this system of naming salts, he instantly sees, by the name, what the salt is composed of. Our common table-salt is called, in chemical language, chloride of sodium, because it is composed of a gas called chlorine, which produces the smell of bleaching powder, with a metal called sodium, found in soda, &c. The reader having mastered this system, will at once see its great value; and although perhaps experiencing at first some difficulty in acquiring names of salts, he will speedily reap the advantage of the general nomenclature.

To save repetition of terms, we shall, in future, call a solution of the salts of silver, *the silver solution;* and solution of common salt, *the salt solution;* and for our introductory experiments, these may be made as follows:—

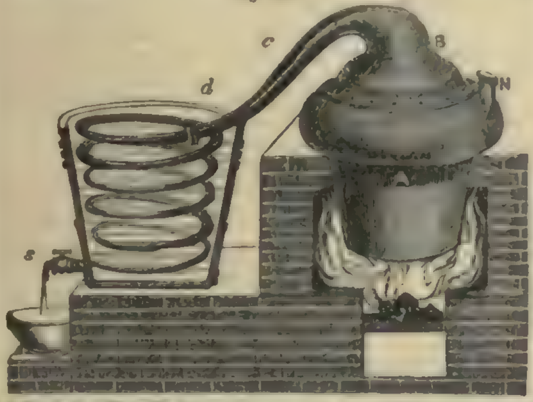
To make the "Silver Solution."—Dissolve fifty grains of crystallised nitrate of silver in one ounce of distilled water.

To make the "Salt Solution."—Dissolve sixty grains of common salt in four ounces of clean water.

It will be observed that we have mentioned, that *distilled* water must be employed to dissolve the nitrate of silver. This is because river and spring water contain common salt. Distilled water may be procured of the chemist and druggist, or may be made by affixing a long pewter tube to the spout of an ordinary tea-kettle. As the steam passes from the kettle it will be condensed in the tube, if the latter be kept cold by means of wet rags, or by being immersed in cold water. The distilled water may be received in clean glass vessels, which should be kept stoppered and free from dust.

A useful still may easily be constructed by an intelligent smith; and such as is represented in Fig. 141 is a convenient form.

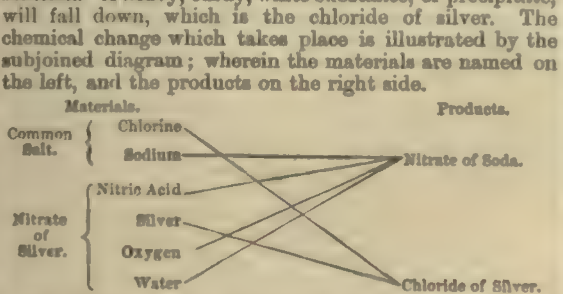
Fig. 141



A is the body of the still, into which the water to be distilled is introduced. Round this vessel the fire is placed. B is the head of the still, through which the vapour passes by means of *c*, until it arrives at *d*. *d* is the worm, surrounded by cold water. In this part the steam is condensed, and escapes, as distilled water, at the tap *s*, where it may be collected in bottles, which should be kept carefully corked.

The solution of the salts of silver should always be kept in a glass-stoppered bottle, and surrounded with a piece of black paper, or kept in a dark place, so as to prevent the action of light.

Experiment 2.—Pour some of the salt solution into a wine-glass, and then add a few drops of the silver solution. A heavy, curdy, white substance, or precipitate, will fall down, which is the chloride of silver. The chemical change which takes place is illustrated by the subjoined diagram; wherein the materials are named on the left, and the products on the right side.



The result of the mixture is, that the chlorine and silver unite together to form the chloride of that metal; and the nitric acid, sodium, and water form the salt, nitrate of soda, which is dissolved in the liquid. We have thus an instance of chemical decomposition, and the formation of new compounds.

Experiment 3.—Divide the white powder obtained in the last experiment into two portions, and keep one part in a dark place, and expose the other to the rays of the sun. In a short time, the portion left in the light will turn to a dark mulberry tint, owing to the action of the chemical rays on the salt; whilst that kept in a dark place will remain entirely unchanged.

By these experiments the student will learn the elementary principles of the art of photography; and may now proceed to the next step, that of preparing paper which will be sensitive to the action of light.

Experiment 4.—*To prepare Sensitive Paper.*—Pour the salt solution into a clean plate, and place on its surface a piece of white writing-paper, leaving it there for some time, so that the solution may enter the pores of the paper. The salted side should then be marked with a black-lead pencil, so as to distinguish it from that which is unsalted. The paper may then be hung up to dry. The next process is to render the paper sensitive to the action of light; and is effected by placing the salted side, face downwards, on the silver solution contained in a clean plate, and leaving it there for about ten minutes. The paper is then to be dried in a place to which light has no access. This part of the preparation of the paper should be done by candlelight; and several sheets may thus be made and kept, till required, in a portfolio or book.

Paper thus prepared will at once change its colour on being exposed to diffuse daylight, or in the sun's rays; and if any opaque object be placed over it, a complete copy of its form may be procured. If a penny-piece, for instance, be placed on a sheet of this paper, and exposed to sunlight, all the uncovered portion will be turned to nearly a black colour; whilst that under the coin will retain a white and unchanged appearance, because there no light has reached, and therefore no chemical effect could take place. On removing the coin, the white part will become changed, if exposed to the light; and this introduces us to another process—namely, that of "fixing" the picture obtained as above.

The fixing process is intended to dissolve away that portion of the silver salt which has not been acted on by the light; and it will thus prevent a picture, when obtained, from being spoilt or lost. The solution used for this purpose is that of a salt called hyposulphite of sodium, an ounce of which may be dissolved in four ounces of water. The picture is to be left on this solution, face downwards, for a short time, and afterwards well washed in abundance of cold water. After being dried, it may be exposed, without any danger of undergoing further change; in fact, it becomes permanent and unalterable.—If, instead of a coin, a piece of lace, a fern-leaf, or other such object be placed on the paper, and pressed thereon by means of a piece of plate-glass, a complete copy of it may be taken: a child's transparent slate is a very convenient arrangement for this purpose; but the ground-glass plate must be replaced by a piece of window or plate-glass, so that the light may freely pass through. By this simple plan a vast number of objects may be copied; the solid parts being represented by white spaces, and the transparent portions by dark marks on the paper. This process is so exceedingly easy, that any young person may readily follow the directions we have given. We may here remark on the advantages which elementary science affords in training and disciplining the young mind. The ordinary course of juvenile instruction, has the twofold object of teaching facts, and preparing the mental powers for future and more exact duties. If, however, we can find any means by which amusement, interest, and instruction can be combined, such become a most valuable adjunct to an educational course.

We have now to notice that there are two classes of photographs; namely, *negative* and *positive*. By the former, the dark portions of the object are represented by light parts in the picture; but in a positive, the appearances are the same as in the object which has been copied. Fig. 142 represents a negative, and Fig. 143 a positive, of the same object.

Fig. 142.

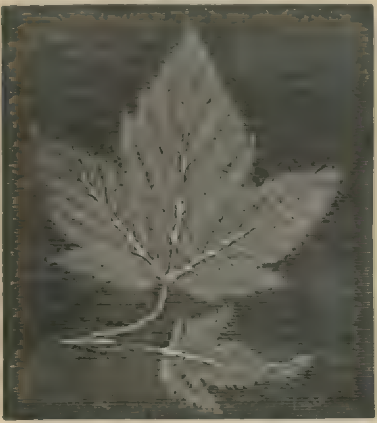
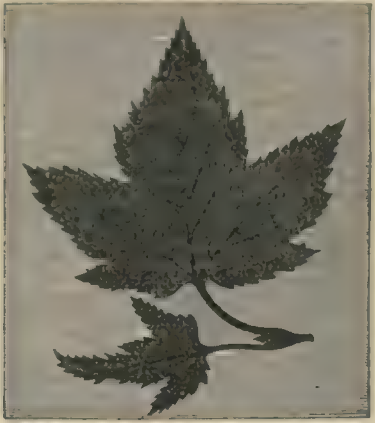


Fig. 143.



A positive picture may easily be obtained by placing a negative, face downwards, on another piece of prepared paper, and proceeding as before. The light will thus pass through the unchanged portions of the negative, and so produce a darkening of the paper beneath it; whilst the dark portions of the negative, of course, prevent any change in that part of the paper beneath them. The positive, thus obtained, may be fixed by the process previously described.

We have purposely restricted these experiments to the simplest form of photographic practice. The materials are easily obtained; and if the process be repeatedly performed, the student will gradually acquire an expertness, and, at the same time, meet with difficulties which he may easily overcome. If, for instance, he observe brown marks on the picture, he may guess that his fingers have been their cause. White, round spots indicate places where neither the salt or silver solution have touched the surface of the paper, owing to the presence of air-bubbles. Blotches, and large patches on the surface, show that the paper is not suitable for the purpose. This may easily be avoided by the purchase of an article specially prepared for photography; but we still recommend that the tyro should begin in the simplest manner, and so arrive, step by step, at success, by becoming gradually acquainted with obstacles as they accidentally present themselves.

To save trouble, and to simplify our instructions in these elementary attempts, it may perhaps be as well that we give a recapitulation of the whole of the process; and also special directions for obtaining positive copies from negative pictures.

1. Salt one side of the paper, taking care that no air-bubbles are left between its surface and the liquid.

2. Mark the salted side.

3. Dry the paper.

4. Render the salted side of the paper sensitive by placing that surface in the silver solution for about ten minutes, taking precautions against the presence of air-bubbles, by drawing the paper over the solution whilst holding it at one corner. This must be done by candle-light.

5. Dry the paper in a dark place, and, when dry, keep it out of the light.

6. To obtain a negative picture, place the object on the sensitive surface of the paper, and press thereon a piece of plate-glass, so that the object and paper may be in the closest possible contact.

7. Expose to day or sun light so long that the paper may acquire a dark mulberry tint in those places where the light has free access.

8. When the picture is finished, remove it into a dark room, and fix by means of a solution of hyposulphite of sodium.

9. Wash abundantly in cold water, so as to remove all trace of the salts. This should be done by placing the picture in a basin of water for some hours, and meanwhile frequently moving it in the liquid.

10. The picture may then be dried, and pasted or gummed on a piece of cardboard.

11. To obtain a positive picture from a negative, place the negative, face downwards, on a piece of sensitive paper, the two being pressed together by means of plate-glass, or otherwise. When the picture is finished, proceed as recommended in Nos. 8, 9, and 10.

If resident in any of our large towns, the operator may purchase paper which is ready salted, and also covered with albumen. This only requires to be excited by placing the albumenised side on the silver solution, as named in No. 4. This paper may be prepared by dissolving fifty grains of salt or sal-ammoniac in two ounces of water, to which twice as much of the clear white or albumen of an egg has been added. The mixture must be well shaken together, and then strained through muslin. The paper may then be laid on its surface after the manner before recommended for the salting process. The albumen gives a nice gloss to the picture, and so greatly improves its appearance.

We need scarcely state, that an ingenious person may employ the above process to a vast variety of interesting and amusing purposes. Leaves of trees, flowers, ferns, engravings, lace patterns, and an immense number of similar subjects, may thus be easily copied: indeed, we know of no branch of scientific application so readily acquired, and more varied or interesting in its pursuit.

We may here mention, that the ordinary process of marking linen by means of common "marking-ink," and various modes of dyeing hair, are really but applications of photographic principles and practice. In most of these cases, solutions of the nitrate of silver are employed; and the results obtained are identical with those we have named, except so far as the objects to which they are applied are concerned.

Having thus attempted to introduce the reader to some of the most elementary instances of the practice of photography, we now proceed to describe more intricate processes, by means of which the highest class of effects are obtained; and which will of course require greater attention, care, and perseverance in their pursuit, than those to which we have hitherto referred.

PHOTOGRAPHIC APPARATUS AND CHEMICALS, Etc.

Before proceeding to detail the various processes now in use, we shall enter into a description of the apparatus which will be required by the student, and also mention the different chemicals employed by photographers.

The chief photographic instrument is the camera. We described some forms of this instrument under the head of Optical Instruments; but such are of far too incomplete a kind to be employed for photographic purposes, in which the utmost accuracy of construction hitherto attained has yet left many defects to be overcome. We shall therefore endeavour to point out, not only the best form of the instrument for various purposes, but also shall suggest contrivances which may assist the beginner in many circumstances in which he may be placed.

A camera of an ordinary kind, suitable for a beginner, may be procured for a moderate sum; but where the means of the student will permit, none but the best kind, as to construction and character of the lenses, should be employed. Many persons commencing the practice of photography, are content to do so with an inferior instrument. Having thus to contend with difficulties which, from their want of knowledge of the science of optics, they can neither overcome nor control, they often entirely fail in obtaining good results, and, consequently are soon discouraged by failure. Our remarks on aberration[*] will at once suggest to the intelligent reader, that the construction of an accurate camera depends on an extended acquaintance with the laws of optics, combined with a high degree of skill on the part of the maker; and of course such acquirements both demand and deserve an adequate pecuniary recompense; hence the high price of some cameras employed in the art. If a lens could be ground of such an exact shape as to overcome the occurrence of spherical aberration, as easily as the difficulty of chromatic aberration has been dealt with, such would, of course, almost arrive at perfection. We here observe how exact are the works of the Deity compared to the best productions of human skill. In the eye we have every change of circumstance at once provided for, as we have already explained when treating on the eye and its structure. Now, the lens of the camera has to fulfil the same object as the crystalline lens of the eye; and to extend our analogy, we may add, that the prepared plate of the photographer is the analogue of the retina of the eye; for, like it, it has to receive and communicate impressions of the rays of light incident on its surface.

It would, of course, be highly improper for us to recommend any instrument-maker, in preference to others, of whom the student might purchase his camera, and other photographic apparatus. We, however, shall attempt to explain the advantages and improvements which have been introduced by various persons; and giving the student an opportunity of judging of the essential points to which his attention should be directed, we must leave to his own judgment and preference, the particular form, &c., of the instruments he may desire to obtain. Two classes of cameras, or at least two sets of lenses for the camera, are necessary, for the separate purposes of taking likenesses and landscape views. This will be evident when it is remembered that, in the first instance, the object may be placed close to the instrument; whereas, when taking a photograph of a country scene, &c., the different objects will be at all possible distances from the centre of the lens. The human eye is capable of adapting itself instantaneously to the variations of distance in respect to objects seen by it; and to a certain, but not entire extent, the same result is effected in the position of the lenses of a camera by a mechanical arrangement—the operation being termed "*focussing.*"

In the annexed engraving, the general arrangement of the photographic camera is represented. In Fig. 144, A represents the box of the camera; B a brass tube in which the lenses are placed; C a slide or dark frame, in which the sensitive plate is fixed. In Fig. 145, D shows this frame in its details; the back being opened, so that its internal construction may be observed.

Fig. 144.

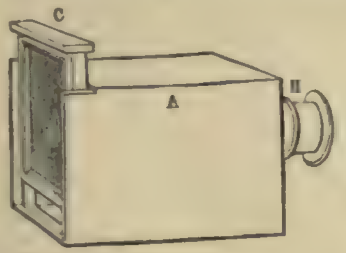
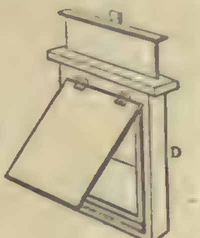


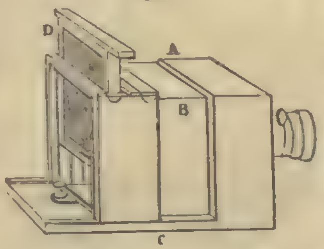
Fig. 145.



The image received on the ground-glass plate at C, in Fig. 144, is focussed in this form of camera, by moving or adjusting the lens contained in the tube B in front of the camera; hence its use will be necessarily confined to occasions wherein such limited adjustment is admissible or required. Another form of the camera, in which the distance between the ground-glass plate for viewing the image, and the tube in which the lenses are placed, is illustrated in the following engraving.

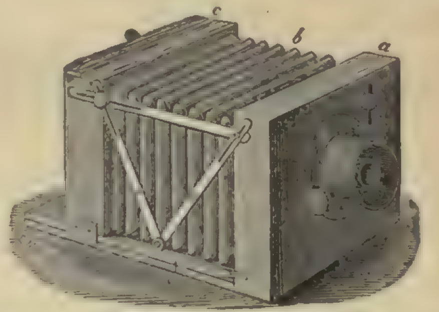
In Fig. 146, we observe that the box of the camera is divided into two parts, of which B is so constructed as to slide easily in and out of A. Thus the ground-glass plate represented in the frame D, may be extended to and fro from the lens, and a greater focal arrangement can be arrived at.

Fig. 146.



A very convenient form of camera, for travelling purposes, is represented in Fig. 147. The body of the box, instead of being solid, is made exactly like the bellows of the concertina. The back, *c*, holding the plate, can thus, by means of the folding construction of *b*, be easily shifted any distance from *a*, the part of the arrangement holding the lenses; and all parts are maintained in a firm condition, when in use, by means of the crossbars *d d*. These are secured in any desired position by means of screws working through slots in the bars.

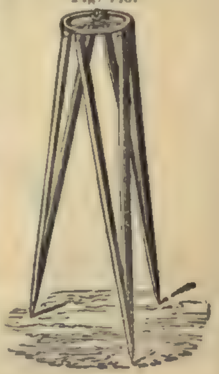
Fig. 147.



We need not, however, occupy further space by details of those forms of the camera which result from the peculiar choice, requirements, or means of the operator. Any, or most of these, may be inspected at the optical instrument-makers, who, according to their individual opinions on the subject, will be able to give more precise and effective descriptions than it falls to our province to attempt. We may, however, offer a word of advice to those who may be desirous of purchasing an instrument; and, in so doing, we need scarcely state, that the purchaser will do wisely to seek his apparatus at any house whose reputation for skill and respectability is well attested. Achromatic and properly ground lenses, their transparency, firmness of workmanship in the material of the box, are the essential points. We have seen cameras made of unseasoned material, in which some of the parts of the woodwork have contracted by change of weather, which rendered them extremely inconvenient, not to say useless. In such cases, the dark frame, by fitting too tightly or loosely in the slide receiving it, will often cause the camera to be shifted whilst introducing the sensitive plate, and so entirely remove the camera itself out of position, and destroy the focus. We can only add, "*caveat emptor.*"

The camera requires a steady support, and one which can be adapted so that it may be placed in any position. One of the most convenient of these is that represented in Fig. 148; and one of this kind is equally available for house and out-of-door use. Great care should be observed in purchasing or making a stand of any shape, because the use of unseasoned wood soon

Fig. 148.



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[*] See *ante*, p. 53.

causes them to become unsteady, and makes them utterly valueless.

Amongst the minor, but essential apparatus of the photographer, are the following.

Fig. 149 represents the dipping-bath, and frame for holding the plate in the silver solution contained in the bath.

Fig. 149.

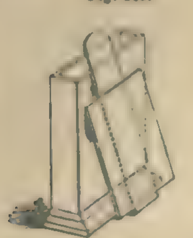


Fig. 150

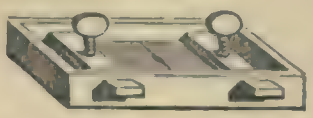


Fig. 150 is a representation of a copying frame, by means of which positive copies are obtained from negative pictures. It is an improvement on the simple arrangement of the child's transparent slate, to which, and its use, we have previously alluded.*

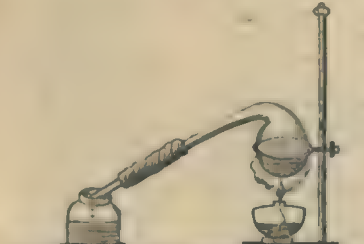
We have thus referred to some of the most essential arrangements required by the reader when commencing the study of photography. We shall allude to, and illustrate, various contrivances which have been invented for the purpose of facilitating several operations, as we proceed.

In our humble opinion, it is better that every student of science, whatever his means may be, should commence with as few "assistants" as is consistent with his progress. He is thus thrown on his own resources; he sees his wants; his ingenuity is stimulated to supply them; and his progress is infinitely more satisfactory to himself, and advantageous to others in every department of scientific research, than when he is helped over every little obstacle that may drop across his path. Both morally and mentally, "he that chiefly owes himself unto himself, is the substantial man."

Besides the different apparatus we have described, the tyro will require some glass-stoppered bottles of about four ounces capacity; square dishes of glass or porcelain for holding developing solutions, &c.; two or three graduated glass measures, of two ounce, one ounce, and one drachm capacity; some glass rods for stirrers; and a dozen or so of test-glasses, which, however, may be replaced by ordinary uncut wine or ale-glasses. He should also possess a spirit-lamp, of four or eight ounce size, and a small ring-stand, for supporting glass vessels in preparing solutions by heat; some glass funnels for filtering solutions, &c.; some filtering papers, or, what is nearly as good, some stout white blotting-paper; and last, and by no means the least of his requirements, is that of an unlimited supply of clean, soft towels, and of rain or soft river water. The quality of ordinary water differs very much, according to the source whence it is obtained: when hard, distilled water should always be used.

We have mentioned that distilled water is essential in many of the operations of photography; and we refer our readers to the remarks made in reference to this subject.† When required in but small quantities, it can be made by using a pint-glass retort, supported on a ring-stand. A spirit-lamp may be employed as a source of heat; and the stem of the retort should be kept cool by means of wet cloths. (See Fig. 151).

Fig. 151.



PHOTOGRAPHIC CHEMISTRY.

In the ordinary arrangement of a scientific work, it is almost a question of choice as to whether photography might be considered an application of chemical or of optical science. As we have adopted the latter idea, we shall necessarily have to introduce our readers to so much of the science of chemistry as will enable them to understand the rationale of the processes in which they must engage. This will by no means prove any hindrance; indeed, had the student to forage out, on his own account, such a knowledge of chemistry as he will require, we imagine that he would thus effect a much larger expenditure of time and means, and would, most likely, obtain a much less satisfactory result, than if he simply studied and followed the *excerpta* and directions of those who are more fitted by experience to point out and prepare the way for him.

With this end in view, we shall now direct briefly the reader's attention to some of the leading applications of chemistry, so far as they are required in the photographic art; referring him, however, to the remarks which we have already made on this subject in our introductory experiments.‡

To one or more of three substances (chlorine, iodine, and bromine) the photographer is indebted for his most successful results; and the student may become acquainted with their specific characters by trying the following experiments:—

Experiment 5.—Chlorine may be obtained in various ways. It is a constituent of common salt, &c. Place a teaspoonful of chloride of lime (bleaching-powder) in a glass bottle or jar; pour on the powder a little hydrochloric (muriatic) acid and water; in a little time, gas will be given off, and will gradually fill the jar. It has a greenish-yellow colour; hence the name, chlorine, from the Greek. Its smell is very penetrating; and if a piece of paper, coloured by litmus or any other vegetable, be dipped therein, it will be immediately bleached. Hence the employment of chloride of lime in the manufacturing districts, for bleaching cotton and linen goods.

Experiment 6.—Iodine. This substance is obtained from sea-water, and was first discovered by its action on the pans in which salt was evaporated. In its ordinary state, it is a dark-coloured powder. Heat a little of this on a plate, and observe the colour of the vapour, which will be of a rich purple; and thence has arisen the name of the substance. Combined with potassium or potass, it forms the iodide of potassium, or hydriodate of potass; names which are used for the same salt. Iodide of potassium is largely employed for a variety of purposes in photography.

Experiment 7.—Bromine. This is always sold in a liquid state; and, like iodine, is obtained from sea-water. It has a deep orange colour, and most offensive smell. Expose a little of the liquid bromine to heat; its characteristic vapour will be at once noticed: the fumes, however, which are extremely irritating, should be avoided. Bromide of potassium—that is, bromine and potassium united together—is the combination most used. Bromine and iodine are used, together with collodion, in the liquid state for the collodion process; whilst, as vapour, they are employed for making the silver plates of the Daguerreotype process sensitive, before being placed in the camera.

These three substances, chlorine, iodine, and bromine, have been termed elementary bodies, because they have resisted all attempts at decomposition; that is, by the exercise of all our knowledge of chemical science, we have been unable to find any other substance in them different to their own nature or characters. This fact leads us to remark, that the union of such elementary bodies with others, forms a class called compounds. Of such are common salt, composed of chlorine and sodium, and the iodide and bromide of potassium, which we have just referred to in our experiments.

The most essential element of photographic art, is the metal silver. This is never used in the state we observe it in our silver coins, because it is therein alloyed with copper, to give it sufficient hardness to withstand the wear and tear of daily use.

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* See *ante*, p. 141.

† *Ante*, p. 141.

‡ See *ante*, p. 140, *et seq.*

Silver is employed both in the metallic state and in the form of nitrate of silver: the former is used in plates for the Daguerreotype process, or as plating or covering a sheet of copper; and the salt is universally used in the calotype, the collodion, albumen, and other processes of photography generally.

The salt, nitrate of silver, is easily formed, by putting a piece of pure silver in a little nitric acid diluted with water. A violent action takes place; the metal is dissolved; and by evaporating the solution, crystals of the salt are easily obtained. It, however, is far better that the operator should purchase the salt at a photographic chemist's or instrument-maker; as such prepare it more especially for photographic purposes, and thus avoid a variety of impurities which are ordinarily found in the nitrate of silver of commerce. It may perhaps be as well for us to remark, that all photographic chemicals may be procured by post from any of the leading towns in Great Britain and Ireland. This may be valuable information to those resident at a distance from such places. In our own journeys, we have met with several "towns" so called, wherein we could not purchase a chemical of any kind; and while writing this, we have in our mind's eye two places in Scotland, containing each over two thousand inhabitants, wherein neither medical man or chemist and druggist could be met with, or could their professed wares be obtained at any price—a fact which unutterably astonished us, considering the prevalence of "medicinal taste" amongst us as a people.

On adding a salt containing chlorine, iodine, and bromine, to nitrate of silver, other substances are formed, being respectively the chloride, iodide, and bromide of that metal. The chloride of silver is a white powder, turning to a dark mulberry tint on being exposed to the action of light. The iodide of silver is of a yellow colour, as is also the bromide; each being produced on adding either the iodide or bromide of potassium to nitrate of silver. These salts are formed when a plate, covered with iodised and bromised collodion, is dipped into a bath of nitrate of silver—a process into which we shall extensively enter when we consider and describe the usual method of taking likenesses, &c., by means of collodion.

In chlorine, iodine, bromine, and their salts, or salt-like combinations, we have described the chief active agents employed in photography; and in silver we have the substance acted on by them. The resulting products of their mutual action are the media which the photographer employs directly to obtain his result; but these have to be placed or used in combination with others. His chemicals are as the colours of the painter; whilst his collodion, &c., take the place of the painter's canvas.

Collodion, like its basis, gun-cotton, is of very recent discovery; but it has done more to extend and render popular the pursuit of photography, than any other invention which has been connected with the art; and the process deriving its name from the substance, at once affords beautiful and satisfactory results. Whilst we urge on our readers the wisdom of purchasing, rather than of attempting to make their collodion, we shall give a general direction as to its manufacture, so that its composition, &c., may be fully understood; and to this end we suggest an experiment by which gun-cotton may easily be made.

Experiment 8.—Put some perfectly clean jewellers' cotton, or cotton wool, into a glass jar, or a common tumbler. Mix in a porcelain vessel equal parts of strong sulphuric and the strongest nitric acid; and, after stirring these together, allow the mixture to cool. We name a *porcelain* instead of a glass vessel, because of the great heat produced, which would almost certainly destroy glass. The mixture should also be made in the open air, or in a place where a draught would carry off the suffocating fumes which are given off. When the acids have cooled, pour them on to the cotton wool, and press them together repeatedly for about five minutes by means of a glass rod, so that every part may be well moistened with the acids. It then should be removed by means of the glass rod, and washed under a tap of running water, until every vestige of acid is removed. This can be judged of by the taste of the water as it runs away. The cotton may then be pressed between a clean cloth; and afterwards being opened out, so as to expose plenty of surface to the air, it may thus be dried either in the sun or in a warm room. Artificial heat should only be used with great care, because an explosion might ensue if too great heat be applied. Blotting-paper, sawdust, &c., may be treated in a similar manner, and they will undergo identical chemical changes.

If the cotton thus prepared be ignited, it will be found to explode suddenly like gunpowder, and not in the slow fashion that takes place when common cotton wool is placed in a flame. Indeed, the substance has been frequently employed instead of gunpowder for blasting purposes. It, however, undergoes too rapid a combustion to be of use in rifles, cannon, &c.; and is a dangerous agent if so employed, generally causing the bursting of the piece. We have lately been informed, by a friend, that in the summer of the past year (1860), it was successfully adopted by the Austrian service to a considerable extent.

Having thus obtained the gun-cotton, or, as it is chemically termed, "pyroxyline"—the next step is to dissolve it, and so to form collodion. This is done by adding a few grains of the cotton to a mixture of six parts of ether with three parts of alcohol, highly rectified; or the following proportions may be taken:—

Gun-cotton . .	10 grains.
Ether . . .	half an ounce.
Rectified alcohol .	quarter of an ounce.

These should be added together in a glass-stoppered bottle, and, after a short time, the greater portion of the gun-cotton will dissolve, and this solution will be collodion.

If a little of this solution is dropped on to a glass plate, the spirit will soon evaporate, and a fine film will form on its surface. It is this film that is employed to hold the chemical agents which are subsequently placed on its surface, and by whose changes, under the action of light, the photographer effects his beautiful results.

The collodion thus prepared, requires, however, other additions to fit it for use; and these consist of substances which, by their action on the silver solution in which they are afterwards placed, render the coating sensitive. For this purpose, iodide of potassium, iodide of ammonium, or that of cadmium, may be added to the collodion. The iodising of the collodion is a question on which almost every operator differs; and it is, to a large extent, an empirical subject. We shall, therefore, state generally, that the addition of a few grains of either of the three salts, iodide of potassium, ammonium, or cadmium, to the quantity of collodion manufactured as we have just directed, may be employed by the student. We still however, only mention this that he may understand the constitution of iodised collodion; and, as before, advise its purchase as ready prepared. The uses and modifications of this liquid we shall fully consider hereafter.

We have thus dealt with most of the substances employed for the ordinary processes so far as preparing the plate for the camera is concerned. We now proceed to speak of others which are employed for developing and fixing the pictures on the removal of the plate from the camera after its exposure to light, referring chiefly to those of the collodion process. Omitting alcohol and ether, we shall chiefly refer to those which are most in demand; and shall thus avoid distracting the attention of the operator by too extended a description.

The following acids and salts are variously employed for developing solutions; namely: acids—acetic, gallic, pyrogallic, and nitric; salts—protosulphate, and protonitrate of iron. Acetic acid is the basis of the vinegar of household use; but for the purposes of the photographer, a pure and strong kind is required. It is generally sold by the photographic chemists, under the name of glacial acetic acid, which is its strongest state, and which will assume the solid form when at a temperature near the freezing-point of water.

Gallic acid is produced by the decomposition of gall-nuts, which are freely exposed for a few weeks to the action of the air and water. Pyrogallic acid is obtained by the action of heat on gallic acid; and is largely used in the collodion process; whilst gallic acid is equally so for the calotype.

Nitric acid is employed for various purposes as a photographic agent. Its usual source is nitre, whence it is obtained by distilling that salt with sulphuric acid and water.

Protosulphate of iron is well known under the name of green copperas, although not a particle of copper enters into its composition. It is composed of iron and sulphuric acid, and may be obtained by adding these together. In a pure state it is sold in the form of rich green crystals, and is much employed for developing-solution. Protonitrate of iron, which is also similarly used, is composed of iron and nitric acid, and may be made by mixing together a solution of sulphate of iron, and one of nitrate of baryta. For making small quantities of this salt in solution, 100 grains of each salt should be dissolved in water in separate vessels. On the two solutions being mixed, a white powder, the sulphate of barytes, will fall down; and being filtered from this, the solution will contain the protonitrate of iron.

Having thus disposed of those acids and salts mostly employed in the collodion process, we conclude by referring to two substances used for the purpose of fixing the picture, or, in other words, to remove the unchanged salts of silver, and so to prevent any further chemical change being produced on its subsequent exposure to light. Hyposulphite of soda is largely prepared for photographic "fixing," and is accordingly a cheap and easily procured salt. The strength of solution required will be mentioned when we further refer to its use. It is obtained by transmitting sulphurous acid through a solution of carbonate of soda. Sulphurous acid is a product of the combustion of sulphur, and may be abundantly obtained by heating sulphuric acid with the metal mercury in a glass retort.

Cyanide of Potassium.—This salt has lately come into great use as a fixing agent, and is also largely employed in electro-plating and gilding. It is generally procured by heating together, in fine powder, one part of dry carbonate of potass with two parts of ferrocyanide of potassium (yellow prussiate of potass): a white deliquescent powder is the result. We may here warn the student, that this salt is a most deadly poison, owing to the presence in it of cyanogen (the base of prussic acid); and he will act wisely in avoiding the smell which arises from its solution: he should also be cautious in immersing the hands, especially if they are at all wounded, or the skin abraded. This salt should be kept in close-stoppered bottles, as it readily absorbs moisture from the air.

We have thus endeavoured to give a general idea of the nature and uses of those substances employed as chemical agents; and as the collodion process is one which is easily managed by a little care and patience, we shall commence our directions in practical photography therewith, reserving any further remarks, both on apparatus, chemicals, and other matters, to such opportunities as may more pertinently be presented as we proceed through the different branches of our subject.

THE COLLODION PROCESS.

In many existing treatises on this and other branches of the art, it has been the custom to introduce a vast variety of processes, which have been from time to time altered, improved, or modified by their authors. In our humble opinion, this course, whilst extremely valuable to those who have somewhat advanced, is of a most perplexing character to the student who may be commencing his attempts. We shall, therefore, employ the same method as we have adopted in all our previous pages, endeavouring to place before our readers plain and successive directions, that they may, in obedience thereto, gain a satisfactory conclusion. To avoid distraction of our arrangement, we shall omit mentioning the names of those individuals to whom photography is indebted for its advance; earnestly hoping, that if such peruse our pages, they will, with us, share the desire in which we write them—that of spreading, by all possible means, the elements and principles of knowledge to as great an extent as possible, in a disinterested and truly philosophic spirit. We shall give all the hints and practical details which our own experiments will allow; and only hope that our readers may reap any or every advantage from the facts which we have gathered from numerous sources.

Assuming that the student has provided himself with all the apparatus, chemicals, &c., which we have already mentioned, we shall suppose that, under our direction, he is about to take the portrait of a friend, or a *positive* picture of any stationary object. We commence with positive pictures, because, to the beginner, they afford immediate fruit of his early endeavours.

TO TAKE A POSITIVE PICTURE.

Our process naturally divides itself into the following operations; namely—

1. Cleaning the glass plate.
2. Coating it with collodion.
3. Rendering it sensitive.
4. Exposing it in the camera.
5. Developing the latent picture.
6. Fixing the picture.

The above have been arranged in the order in which they succeed each other; and neglect in carrying out each process individually, will, without the shadow of a doubt, bring inevitable failure as its result: whilst, if conducted in a quiet, persevering, and cleanly manner, success may most justly be expected to reward the operator's attempts. The glass plates are sold, of various sizes, by the instrument-makers; and the material should have a high polish—be perfectly free from specks or blemishes of any kind. For this reason common window-glass is rarely good enough; and a better kind, manufactured expressly for the purpose, must therefore be purchased.

1. *To Clean the Glass Plate.*—The methods proposed for this purpose are endless in number. We, however, find the following to succeed as well as any we have tried. Place the plate on a soft linen towel, and alternately rub each side till all free dust and dirt are removed. Then, by means of a little soft cotton wool, wash the two sides with a mixture of spirits of wine and liquid ammonia, in equal quantities; this will remove all extraneous solid matter: afterwards wash with distilled water. The plate being dried, must then be held by means of a finger and thumb placed at opposite corners, over a spirit-lamp, until the surface of the glass presents a brilliant polished appearance, and the whole becomes very hot: this removes a film of air which rests on every polished surface. We have never seen this plan recommended by any photographer; but have no doubt that in many cases it will prove a remedy for a common evil—that of the non-adhesion of the collodion film. We were first led to try it from long practice with Grove's voltaic batteries. The platina in such arrangements is always coated with a film of air; and unless this is removed, the nitric acid never comes in contact therewith. Having observed the effect of removing this aeriform coat in increasing the battery power, we tried it on the glass plate for photographic purposes, and have found, with many whom we have induced to try it, great advantage therefrom. After thus heating the plate, place it on its edges, so that it may rest at an angle against any support; and do not attempt to use it till quite cool, but do so immediately after it has regained the temperature of the surrounding

atmosphere, because another film of air or moisture will speedily form on its surface.

By such means a glass plate may be thoroughly cleansed from every kind of matter prejudicial to its employment for photographic purposes; presuming, of course, that the chemical nature of the glass is such as to present no other difficulty. We may here mention, that although glass, in a practical point of view, may be considered perfectly insoluble in any agent ordinarily employed to cleanse it—such is not strictly true. We are inclined to believe, that solutions of potash, nitric acid, &c., often employed by photographers to cleanse their glass plates, act in an unexpected manner in producing many of the difficulties which frequently present themselves. We are the more inclined to urge this point on our readers, from having found great difficulty in cleansing glass for the purpose of several experiments on polarised light, introduced in our previous pages. Even the mere mechanical action used in rubbing a piece of good glass, by means of a towel, silk handkerchief, chamois leather, &c., is quite sufficient to form minute lines, which are worked thus into its surface, and, therefore, render such uneven. Whilst visiting several mills in the north of England and Scotland, we were surprised to find in a large "doubling" establishment—that of Messrs. Coats, of Paisley—that thick glass rods, which have been employed as rollers, over which the cotton thread had to pass whilst being wound on the spindles, were cut through, in some places, to the depth of a quarter of an inch, by the soft cotton filaments. The two following experiments will impress this fact on the minds of our readers; and, at the same time, will show the necessity of great caution in rubbing or cleansing glass surfaces, whether in the form of lenses in the camera, or of plates intended to receive pictures on their surface.

Experiment 9.—Into a perfectly clean test-glass or wine-glass, pour a weak solution of carbonate of potass; then add a solution of tartaric acid—both being made by means of distilled water. Immediately after mixing the liquid, rub the sides of the glass, by means of the smooth end of a glass stirring-rod. After a short time, minute crystals of bitartrate of potass will be deposited only on those places which have been rubbed with the rod. Now, as we know that crystals always require an angular projection to cause them to be deposited on a surface, we may justly conclude that such has been produced on the inside of the glass, although by means seemingly inadequate to produce the effect.

Experiment 10.—Rub a piece of plate or window-glass repeatedly in the same direction, by means of an apparently soft towel. Allow it to cool from the production of heat which the friction will evolve. If the rubbed surface be then breathed on, a series of minute lines may generally be observed on holding the glass at an angle, and by viewing it by the light reflected from its surface.

We thus observe, that the polished surface of any body, no matter how hard it may appear, is really affected by others of a much softer nature, by rapid or peculiar mechanical action. As an expansion of the illustrations we have offered, we may state, that a piece of soft iron revolving rapidly, will cut a piece of hard steel as readily as a knife would a loaf of bread. We may, therefore, urge on our readers an excellent old saying of the Greeks—"hasten slowly"—as a motto to be held in mind when they are preparing plates for the camera.

One of the best tests of the perfect cleanliness of a glass plate, is the even condensation of moisture on every part of its surface. This indicates that its external condition is uniform in every place; and such is exactly the condition which an aspirant to success in the art should at all times endeavour to obtain.

After the plate is thus prepared, be careful to touch it in no part, except at those edges which are at the extremes of its diagonals. This rule should be observed carefully throughout all the subsequent processes: the only exception allowable, is that in which the plate must be held in certain positions during the time the collodion is poured on its surface—a process which we have now to enter upon.

2. *Coating the Plate with Collodion.*—The next step in our process is that of coating the cleaned plate with collodion. But before describing this, we shall give some directions with respect to arranging the camera, and thus of having it ready to receive the plate made sensitive by a subsequent process. The professional photographer generally has an assistant, who cleans the plate whilst the operator is getting the camera ready for it. As, however, we presume that our readers will work alone, we need scarcely state, that the camera should be adjusted before the plate is coated and made sensitive.

The person whose likeness is to be taken should put himself into a natural position, free from constraint, and in such a posture that all the muscles of the body may be, comparatively speaking, at rest. In our opinion, there is a vast deal more art in *sitting* to have a portrait taken, than in *taking* it. Almost every one experiences a degree of nervousness; and each determines to maintain a strict and grave deportment, which, by its affectation, increases, or at all events disposes to beget, nervous irritability. Hence, most persons complain that their "likenesses" are not like them; nor, indeed, can they be, when individuals practically determine, on such occasions, to assume airs and graces which, like ladies' best dresses and drawing-room furniture, are only used on "state occasions."

A support of some kind is required to maintain the head in its position; because the muscles of the neck are apt to become fatigued if a long "sitting" be necessary.—Contrivances of this kind are of various shapes; two of which are represented in the annexed figure. One of these (Fig. 152), as will be observed, may be attached to the back of a chair; whilst the other (Fig. 153) stands on its own pedestal. It will be found, in practice, that the latter is the most convenient, because it is easily adjusted to any position. The rest for the head is the curved portion at the top of each figure; and should be so arranged as to support the head of the sitter just behind each ear; and, of course, out of sight with respect to the camera, or they will be included in the portrait.

Fig. 152.

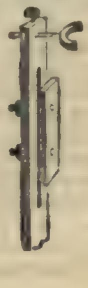
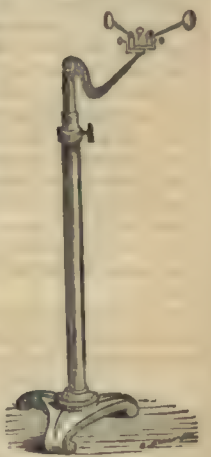


Fig. 153.



In arranging either of these supports, let them be employed solely as rests to support the head, and to prevent fatigue; and not as a framework, by which a rigidity of position would be induced, and stiffness of carriage promoted.

Having adjusted the position of the sitter, place the camera in front of him; and after removing the brass cap from the front of the lens, adjust the camera in such a position, that if you are taking the bust only, its image shall occupy the centre of the ground-glass plate. This is of course done by looking at the image on the ground glass; by shifting the camera to and fro; and focussing the lenses until a clear and distinct picture is observed.

In obtaining a good focal view, pay especial attention to the forehead and face, which of course form the essential points of the picture: and, in getting these correctly, the value of your lenses will at once be tested, and your full skill and judgment called into exercise; on which, in fact, success will depend. These points having been attended to, the ground glass is still to be left in the frame, so that the focus may be readjusted just previous to introducing the sensitive plate into its place.

Although a plate coated with collodion only, is unacted upon by light, still we shall presume that all the subsequent processes, except that of taking the likeness, are conducted in what is called a dark room. This room, in the language of photography, is considered to be a place into which not a ray of light of any sort, except of a deep *yellow* colour, is allowed to enter; such ray, as we have already remarked,* having little or no action of a chemical nature on the sensitive plate. The professional or amateur photographer often goes to considerable expense in arranging this "*sanctus sanctorum.*" We have had the good fortune to see a considerable variety, even from a room fitted with every convenience which science and means would permit, to one wherein a crazy four-post bedstead, covered with extremely ancient bed-furniture, did duty in the waggon of a travelling operator. Leaving the choice of position or place to our reader, we may remark, that an abundant supply of soft water running from a tap, a sink to carry away waste water, a large firm table, and complete absence of daylight, are the essentials of such a place. A sitting-room may be used in which the windows are completely closed from external light by means of thick brown paper pasted over them; and the water-supply may be that of a jug and pails, with such other contrivances, when circumstances render greater conveniences unattainable.

Of course light of some kind must be used to enable the operator to proceed. Some persons employ a tallow candle, others a low gas-flame; but both of these have a prejudicial effect on sensitive plates. In some instances a little daylight is admitted through thick orange-coloured glass. We have always preferred to use a spirit-lamp, charged with spirits of wine, in which *dried* table-salt has been dissolved. This affords a flame of a dingy yellow colour, perfectly inactive, from the absence of chemical rays—affording both sufficient light to work by, and also a ready and necessary source of heat when the plate has to be dried, &c., after fixing and washing. It is essential that the dark room should be kept scrupulously free from dust and dirt, otherwise it is more than likely that such will settle on the delicate collodion film, and spoil the plate. It is a good plan to wet the floor of the room, especially in summer-time, which will prevent the fine particles of dust rising as the operator moves to and fro.

A cleaned glass plate being held in the hands, the iodised collodion may be poured on it in the manner represented in the annexed engraving. The plate should be held quite horizontally in the left hand; the collodion bottle being placed over the centre, so that the liquid may spread evenly over every part until the edges are reached without its overflowing them. The object is to give a thin and perfectly even film to the entire surface of the glass; and when such is obtained, the superfluous collodion is to be poured off by holding the plate over a wide-mouthed glass bottle, so that its diagonal may rest on the neck. (See Fig. 155). The plate is then to be restored to an horizontal position for a few seconds, in order that the regularity of the film may be regained. This operation is one requiring both care and practice; and can only be successfully performed after the operator has had some little experience. It should, however, be done calmly, and without haste, to ensure good results; and it is advisable not to proceed further with any plate which does not present an even coating. Such are easily cleaned for another trial, in the manner already described.† Great care is to be observed in the mode of removing superfluous collodion after the glass plate has been covered with the liquid.

Fig. 154.

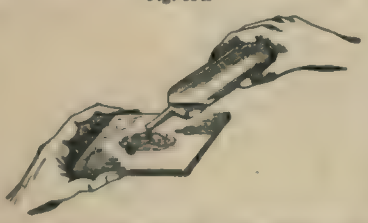
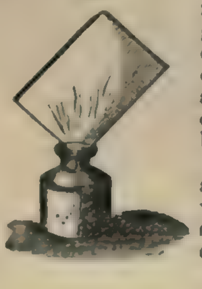


Fig. 155.



Presuming that an even, spotless, and otherwise perfect film of iodised collodion has been obtained, the next step is to render this sensitive to light, which we will now proceed to describe.

3. *To render the Coated Plate Sensitive.*—The choice of a "bath," as it is termed, for rendering a coated plate sensitive, has been a subject of great discussion amongst photographers; and almost every authority has propounded views and directions, which, from their diversity, are eminently qualified to perplex a beginner.

Although a middle course is at all times the safest in which to steer, it is most difficult to find that position. Some have recommended a proportion of nitrate of silver in solution as the most sensitising, which is much less than half the quantity that others consider essential to success. It has hence been found necessary to include various recipes in all of the works which have been produced that deal with this subject; and the recommendations, both scientific and empiric, which have thus been published, may safely be called "legion."

We confess to a difficulty in recommending any specific course to our readers; but we shall do well to inquire into those contingencies which may affect the best *formulæ;* and afterwards we shall endeavour to suggest, not *ex cathedrâ*, but in deference to the opinion of every one who may differ from us, as to what is the best course for the student to pursue.

The object which we have to attain, is that of making the coated plate sensitive to the action of the chemical rays of light; and this should be so effected as to require the shortest possible exposure in the camera to produce the picture. For this end, a salt of silver must be formed on the surface of the iodised collodion plate, which will meet our demands. The difficulty lies in satisfying all the conditions which are essential to success.

It would appear, at first sight, that the simplest mode of rendering a plate sensitive, would be that of immersing it in a solution of nitrate of silver, of any strength within certain limits; but it is found in practice, that the result of developing depends very much on the strength of the silver solution employed. Again, unless the "bath" is prepared by the addition of certain substances, the sensitive character of the film will be impaired, owing to a loss of a portion of its iodine when it comes in contact with the silver solution. Even the constitution (if we may so call it) of the film itself, in a mechanical point of view, may be either modified or injured by neglecting certain precautions.

The student must not suppose that we are presenting imaginary difficulties in his way. We are only anxious to anticipate circumstances with which he may have to contend; and thus by giving him preparatory hints, we hope to enable him to avoid both the Scylla and Charybdis of photographic manipulation.

From our own experiments, we recommend the following silver solution as an average one:—

Nitrate of silver	60 grains.
Distilled water.	2 ounces
Iodide of potassium	1 grain.

Dissolve the nitrate of silver first in an ounce and a-half, by measure, of distilled water; the iodide of potassium may then be dissolved in the remaining half-ounce of water. The two solutions should then be mixed and agitated together in a clean glass-stoppered bottle, and subsequently filtered through clean filtering-paper. This solution we have found to act well with good positive collodion. When, through repeated use, it has become replete with iodide of silver, a further addition of a solution of the nitrate of silver, without the iodide of potassium, should be made; and the whole is then to be filtered. A few grains of kaolin, or pipe-clay, may be added to neutralise an excess of free acid which will occasionally be present.

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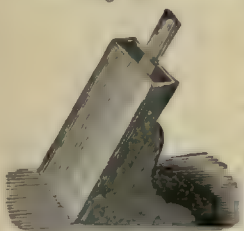
* See *ante*, p. 50.

† See *ante*, p. 146.

Many writers have recommended a less proportion of nitrate of silver; others have advised the addition of more iodide of potassium: some recommend the use of a little acetic acid and alcohol to such a solution. We can only state, that our own opinion is, that all these additions are, and have been, recommended to meet cases which may, or may not, occur to other experimenters. For instance, the nature of the collodion incessantly varies, by reason of the chemical changes to which it is subject; and with every desire to assist the beginner, we must state, that experience, patience, and watchfulness, are the sole guides or assistants on which he can at all depend. We shall, however, give a variety of recipes for these and other solutions which have been proposed by various experimenters, in another part of the work, leaving to the student the chance of trying each until an average of success attends his labours.

Fig. 156.



The solution having been prepared, it should be poured into an upright bath or trough, such as is illustrated in the annexed engraving. These are generally made of gutta-percha; but an upright glass, or porcelain trough, will answer just as well. The length and breadth, of course, must depend on the size of the glass. If the vessel be upright, and not oblique, it should not be less than half an inch wide. Sufficient silver solution should be employed as will nearly fill the vessel; and, of course, it should be in such a quantity as will cover the whole of the glass when introduced therein. Some operators use flat dishes for this purpose; but they run risks of spoiling the plate, owing to dust, &c., settling on the surface of the solution.

When an upright trough is employed, the plate should be placed in a frame, for the purpose of supporting it whilst remaining in the solution. This arrangement prevents the necessity of touching the plate, or of wetting the finger; and, as such, is conducive to cleanliness and convenience. One of these frames is represented on the outside of the trough in Fig. 156.

The plate having been coated with collodion, should, whilst still wet, be placed in the frame, and must then be *steadily* and *continuously* introduced into the silver bath. This should not be hastily done, lest the collodion film may be removed or injured; and no stoppage should be allowed, because that would produce a distinct and damaging mark on the surface. Of course this must be done in the dark room. After leaving the plate in the bath for about a minute, it should be gradually withdrawn; and after remaining a few seconds in the air, is again to be introduced, and may be left there for about two minutes longer. On removal, it should present an even opal appearance, and be perfectly free from spots, lines, &c., or otherwise it will be quite useless. The length of time during which the plate should be in the bath, depends on a variety of circumstances; and hence we have only mentioned an average, leaving to the experience of the operator to decide in special instances. It should then be placed in the dark frame, the lid of which should be carefully closed; and no time must be lost in transferring it to the camera, lest its sensitiveness may become lessened.

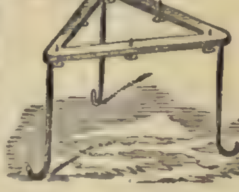
4. *Exposing the Plate in the Camera.*—The plate having been prepared, and placed in the dark frame, is now ready to be transferred to the camera; and in removing it, a cover of black cloth or velvet should be thrown over the whole before removal from the dark room. This precaution prevents any chance of a ray of light entering through any imperfection in the construction of the frame itself. The operator should again examine the image formed in the camera; for although only two or three minutes may have elapsed since he focussed his lenses, still persons are very apt to shift from their position; and therefore, of course, by so doing they will lessen the sharp appearance of their image on the ground glass. The sitter must be requested to remain perfectly still; and having arranged him in the most advantageous position, and refocussed the lenses, the brass cap in front of these should be put on to the tube, so as to prevent the ingress of any light into the body of the camera. The ground-glass frame is now to be removed, and the prepared plate must take its place. It is best put into position by lifting it up with the black cloth, which should be dexterously covered over the top of the camera, whilst the frame containing the plate is inserted in the slide. The lid or flap in front of the plate is now to be removed; and after again enjoining complete stillness on all present, gradually remove the brass cap in front of the lenses. The rays of light will now fall on the sensitive plate, and produce their chemical effect.

The student will expect that we should here give him minute directions as to the length of time during which he must expose the plate; but this it is impossible for us to do. The weather, both with respect to the amount and colour of daylight, has a great effect: then, again, the nature of the collodion employed, the colour of the face and dress of the sitter, and various other contingencies, render the question of great uncertainty. The same collodion used on two different days, or at different periods in the same day, may produce different results. As a general rule for the beginner, an exposure of from fifteen to thirty seconds may be adopted; and as he gains experience, he will soon be enabled to estimate the value of the different causes which tend to render a picture more or less perfect. We shall have to speak more fully of this presently, when we refer to the developing process. Presuming that the plate has been sufficiently exposed, the brass cap should now be replaced in front of the lenses; the flap is to be shut down in front of the plate; and the frame should be removed from the camera, so that the black cloth shall completely cover it. The frame, &c., are now to be removed into the dark room, for the purpose of development.

5. *Developing the Picture.*—The tyro will very likely imagine, that on removing the plate from the frame, he will observe the picture at once on the prepared surface. Such, however, never occurs: the plate presents, as near as possible, the same appearance after, as before, its exposure to light; and the latent picture must, therefore, be developed—that is, brought out into a sensible form by means of further chemical action. The philosophy of this process is easy of explanation: the iodide of silver, formed on the surface of the collodion when it was immersed in the silver bath,[*] has now become changed. That portion which has received the rays of light, will be differently acted on, by the addition of certain agents, to that on which no change has taken place by luminous rays. On pouring the developing solution on to the surface, this difference gradually becomes apparent as the picture unfolds itself on the glass surface. But we now proceed to the practical part of the operation.

The door of the dark room being closed, and only a faint yellow light employed, the lid of the frame should be removed, and the glass plate is to be carefully taken out, and placed, with the uncoated surface downwards, on a level stand, resting in a large porcelain or glass dish. Such an arrangement as this is represented in the annexed engraving.

Fig. 157.



In the absence of a stand of this sort, two glass rods may be placed across the edges of the vessel; and on these the plate may rest; or even an inverted tumbler or wine-glass may be employed. The object is to keep the plate quite horizontal during the development, and to receive, in the dish beneath it, the superfluous developing fluid which will run from the plate.

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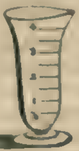
[*] See *ante*, p. 148.

We would here urge on the operator to employ each vessel in his photographic practice for its special use only. By so doing, he will avoid any risk of mixing solutions, and of thus rendering them useless. Every chemical agent employed throughout the processes we have already mentioned, and are yet to describe, must only be brought together when wanted. If mixed carelessly, each solution, if present in only minute proportions, will certainly impair, if not destroy, the utility of any other. A place (or vessel) for everything, and everything in its place, should be a golden rule in photography.

Fig. 158.



The plate being thus arranged, the developing solution should now be poured on to it from a glass measure, similar to that represented in the annexed figure; and this should be done in a careful manner, so that the whole of the plate surface may be covered evenly with the liquid. The latent image will now gradually appear; and much care is required that the process is not carried too far, for fear of spoiling the picture. We must, of course, therefore leave the study of its indications to the judgment of the operator, who, although perhaps failing at first to arrive at the *exact* point at which development should stop, will eventually, after a few trials and failures, acquire the necessary experience. As soon as the picture is sufficiently brought out, all further action should be stopped by washing the plate well with cold water. This may be done by holding it cornerwise under a gentle stream of water running from a tap, or by pouring water in abundance from a jug with a narrow lip. It is extremely essential that this washing should be as complete as possible, so that every trace of soluble matter may be removed.—This being effected, the next step will be that of "fixing" the picture. But before we proceed to that, we must give our readers recipes for making the developing solutions to which we have been referring.

It will at once be perceived, by looking over the annexed list of solutions, that considerable latitude is assumed by various photographers. This is partly owing to the varying nature of the collodion employed; to the manipulation of the operator; and, in many instances, to the peculiar views of those who follow the art. We have, however, deemed it desirable, for the purpose of making our subject as complete as possible, to lay before the student ample opportunities for the exercise of his skill, judgment, and choice, with respect to solutions.

Developing Solutions for Positive Pictures.

Protosulphate of iron	40 grains.
Nitric acid	3 drops.
Acetic acid	30 ,,
Alcohol	20 ,,
Distilled water	1 ounce.

Or—

Protosulphate of iron	8 grains.
Acetic acid	1 drachm.
Nitric acid	2 drops.
Water	2 ounces.

6. *Fixing the Picture.*—The last-described process having fully developed the latent picture, it only remains to secure this from further change by the action of light. The result produced is simply that of removing every trace of those salts of silver which would become changed if white light were allowed to impinge on their surface.

There are two fixing agents which are chiefly employed by photographers, each of which has the power of removing the sensitive silver salts: these are the hyposulphite of sodium, and the cyanide of potassium.—The first-named of these salts was largely used in the early stages of photographic practice; but at the present time, the cyanide of potassium is mostly employed as a fixing agent; and as such, we have, from our own experiments, to add our testimony to its value.

The process of fixing is extremely simple. Either of the following solutions being employed—they are to be poured over the developed picture, and a portion is to be retained on its surface; or the plate may be dipped into a dish containing the solution, and be gently moved about in it. The soluble and changeable salts of silver are thus removed, and any risk of injury is entirely prevented when the picture is subsequently exposed to daylight. After remaining in the fixing solution for about two minutes, the plate should be abundantly washed by means of soft water. We always employ distilled water in the last washing; because river and spring water contain salts, which are left, to a minute extent, on the surface of the plate, and so dim its lustre, and, consequently, lessen the brilliancy and purity of the picture.

After the picture is thus thoroughly cleansed from every impurity, it may be left to drain, and to dry gradually. The usual method, however, is to hold it for a short time over a spirit-lamp or gas-flame; which, of course, speedily causes the evaporation of all moisture from its surface.

For all ordinary purposes the picture may now be considered finished, with the exception that it requires coating with a black varnish; which has the twofold effect of preserving the collodion film from injury, and of also increasing the effect of light and shadow. This varnish is sold by all photographic chemists; or it may be made by dissolving a little black asphalte in mineral naphtha. This should be poured on to the collodion surface, and left to dry. Some persons varnish the collodion surface with the best white mastic varnish, before applying the black coating.

Either of the following solutions may be employed to fix the picture. In that of the cyanide solution we have given considerable latitude; but it is advisable that the student should commence with a weak solution, and continue its use until his experience is sufficient to guide him safely in the employment of stronger kinds.

Fixing Solutions.

Hyposulphite of sodium	4 ounces
Water	16 ,,

Or—

Cyanide of potassium	6 to 12 grains.
Water	1 ounce.

It will be unnecessary for us to enter, at present, into a description of the modes of colouring positive collodion pictures, because we shall refer to this in a separate article. We may, however, mention a few causes which lead to failure in the early attempts of those unacquainted with the delicacy of photographic practice; and we shall discuss these in detail.

First, we may refer to the cleaning of the glass plate. This, we already have stated, is a matter of the highest importance, and sometimes one of some difficulty, because of the nature of the glass itself. The electrician is well aware, that no two circular glass plates, in the plate machine, will give off an equal amount of electricity when similarly employed, although they may be of exactly the same size. This is chiefly owing to the chemical constitution of the glass. Now, similarly in photography, the quality of the glass, to some extent, influences the photographic result; and such being the case, the operator cannot be too cautious in the purchase and choice of the plates he employs. In cleaning the plates, every precaution should be taken to prevent the presence of grease, grit, or dirt of any kind; and the least touch of the finger on any part of the cleansed surface, will most certainly produce an ugly spot at that place. Cloths, &c., used from time to time, gradually acquire grease and animal matter from the hands of the experimenter, and so communicate objectionable matter to the plate. Hence, an abundant supply of clean cloths is essential to success in the earlier steps of taking collodion pictures.

The collodion itself varies, according to the source whence it is obtained. Each maker considers his to be the best; but we advise the student, after making repeated trials, to keep to that which he finds to suit his purpose, and not to heed the various recommendations of his friends, or others, unless they are backed by decided proofs of superiority. Generally speaking, each maker maintains the regularity of the article he sells; and when once the student has so far succeeded in adapting his bath and developing solutions to one kind of collodion, he will only have the trouble of going over the old ground again, in attempting to suit his practice to the production of a new maker. Much, therefore, depends on making a first and proper choice of the collodion.

For the silver bath, a thousand-and-one advices and recipes have been given: indeed, their name is legion. The object of course is, that when the collodionised plate is immersed in the bath, its surface shall be made as highly sensitive as possible, and that each part should be equally so when compared with another. Sometimes the bath is too acid, and at others too alkaline. The former evil is remedied by the addition of a small piece of kaolin; and the latter by the addition of a drop or two of nitric or acetic acid. Now, without going far into practical chemistry, we may state, that a piece of litmus paper will turn red in an acid; and a piece of turmeric paper turns to a deep brown in an alkaline solution. These may easily be obtained; and by them the bath may at once be tested. The strength of the bath is subject to continual variations; because, every time a collodionised plate is introduced, a portion of silver is withdrawn, and a small portion of iodine is left behind. Now these facts must be borne in mind, because they necessarily influence the process to a great extent. Fogging is the result of an improper state of the bath; and from whatever cause such may arise, it is of importance that the remedy should be instantly applied. This tendency is generally removed by the addition of a small portion of nitric acid, so that the super-alkalinity of the bath may be destroyed, and a very slight acidity be produced. This is a point which, having named, we must leave to the judgment and skill of the operator.

Besides the alterations of the bath, produced by the causes to which we have referred, the student must bear in mind, that each time a plate is introduced, a small quantity of ether and alcohol is left behind. This produces a decided harmful effect in the end; and the only plan is to renew the bath entirely.

We may here state, that the silver can easily be recovered, in the form of chloride, from used-up baths, by precipitating with a solution of common salt. It may be reduced, by means of heat in a crucible, to the metallic state, if mixed with cyanide of potassium; which, however, is perhaps too troublesome a process to most of our readers. Photographic chemists and metallurgists, however, will purchase it at its commercial value; and this hint may therefore be of use to those whose means do not permit of extravagance in their photographic pursuits.

The developing solutions, like the sensitising bath, have been the subject of enormous variation in the proportion of their ingredients. We shall not enlarge thereon, because we shall only fall into the error of those who have preceded us, by giving advice for circumstances over which we can have no control, not being present to observe them. The student should watch the effect of the different strengths of the solutions he employs, and he will thus gain more knowledge by a few trials, than he could do by perusing any number of works on the subject.

In using the fixing solution of the cyanide of potassium, care should be taken that it is of not too great strength: it will be therefore advisable to use a weak one, and incur the expenditure of a little extra time, rather than to spoil the picture by employing one of greater strength for the purpose of speed. The motto of "hasten slowly," is, generally speaking, a piece of sound philosophy in the practice of photography.

Abundance of washing removes almost every chance of subsequent change in a positive picture. We strongly advise that the last washings should always be performed by means of *distilled* water.

In choosing the transparent varnish for the collodion surface, care should be taken that it is such as not to crack when dry. The same may be said of the black varnish used for backing-up the picture. Each of these can, generally speaking, be depended on when purchased of the photographic chemists, as they specially prepare these varnishes for the use of photographers.

Having thus detailed the processes and the difficulties, so far as obtaining positive collodion pictures is concerned, we pass on to give directions for obtaining negatives; and shall then describe the best modes of printing paper positives from them.

NEGATIVE PICTURES BY THE COLLODION PROCESS.

We shall now describe the methods which must be adopted to obtain negative pictures, or those in which the lights and shadows are the reverse of what we find in nature. The chief utility found in these pictures is, that they enable us to obtain any number of positive copies from them; and so, to a large extent, they increase the value of photographic processes, and approximate their application to a similar purpose of reproduction, as that obtained by the ordinary process of engraving.

The remarks, advice, and precaution which we have already afforded, are equally applicable in reference to this part of our subject. We, however, shall find, that in some respects, the materials employed, and the manipulative processes, vary from those which are followed in producing positive pictures; and it will now be our duty to point such out to our readers.

The plates are to be prepared with scrupulous care after the manner we have already stated, and are then to be coated with collodion prepared expressly for the purpose. This is generally sold under the name of "negative collodion," and contains more pure collodion, in each ounce of the liquid, than that employed for positive pictures. The following recipe for negative collodion is recommended by Mr. Hardwich :—

Purified ether, sp. gr. ·720	5 drachms.
,, alcohol, sp. gr. ·825 . . .	3 ,,
Soluble pyroxyline	3 to 6 grains.
,, iodide of ammonium . . .	3 to 4 ,,

Although we have introduced the above recipe, we still advise the purchase of the collodion by the student, as being the best course he can pursue.

After having obtained a good coating on the surface of the glass plate, it must be made sensitive by means of the silver bath; and in this, considerable variety of constitution has been recommended. The student must bear in mind, that a positive picture is required to give a distinct appearance of surface by light *reflected* to the eye; and this requires but a moderately thin coating of collodion. Now a negative picture must have depth and body, because its value and effects depend, in use, on the differences of tone produced by the light *transmitted* through it when paper copies are taken from it. Hence the various modifications required to produce these results in negative pictures during the preparation of the plates.

We annex some recipes for making the silver solution, or bath, which we have gathered from various sources; and the student will do well first to try them in the proportions named, which, however, he can modify as his experience permits, and circumstances require. We need scarcely repeat, that the ever-varying qualities of the collodion he may employ will frequently require such modifications in practice. A knowledge of this can only be gained by experiment under special circumstances; hence our duty cannot go further than to indicate, as complete direction to ensure constant success is entirely impossible.

Silver Bath for Negative Pictures.

Nitrate of silver	30 grains.
Iodide of potassium . . .	½ grain.
Distilled water	1 ounce.

This solution may be made by first dissolving the nitrate in a portion of the distilled water, and the iodide in the remainder. The two solutions may then be mixed together for use, and be filtered if necessary: or, the bath may be made by dissolving 200 grains of nitrate of silver in six ounces of distilled water. Into this place a collodionised plate, and leave it in the bath for twelve hours. Sufficient iodide will be removed from its surface to iodise the bath.

The plate being made sensitive, is then to be exposed in the camera, as when positives are being produced; but, as a general rule, the exposure should be of longer duration. Here, again, definite directions are impossible, because of the great variety of contingencies, such as weather, amount of light, temperature, &c., which have to be taken into account.

After due exposure, the plate is taken into the dark room to be developed. A different kind of solution is employed for this purpose, to that used in developing positives, although the process, so far as manipulation is concerned, is similar. Much care, however, must be taken to produce the best result. The process should be conducted slowly, and the effect of the developing solution must be constantly watched. When the operator conceives that a good depth of tone, and decision of light and shade is obtained, the process must be immediately stopped, and the plate should then be abundantly washed with cold water, preparatory to fixing. We annex recipes for various developing solutions as follows:—

Developing Solutions for Negative Pictures.

Protosulphate of iron . . .	1 ounce.
Acetic acid	12 drops.
Distilled water	1 pint.

Or—

Pyrogallic acid	5 grains.
Glacial acetic acid	40 drops.
Distilled water	10 ounces.

This solution requires filtration before use.

Another solution, as follows, may be used:—

Protosulphate of iron . . .	6 grains.
Glacial acetic acid	5 drops.
Distilled water	1 ounce.

Or—

Pyrogallic acid	4 grains.
Glacial acetic acid	60 drops.
Distilled water	4 ounces.

Our readers will here perceive the extraordinary latitude which is given in the choice of developing solutions. We must, however, repeat, that whilst we have employed each of the above, and scores of others, we do not venture to say which is the best, because the circumstances under which they are employed vary so much.

Presuming that the operator has succeeded in properly developing the picture, the next process, after careful washing, is that of fixing it. This is done by immersing it in a bath, or pouring over it a solution of hyposulphite of soda, which may be of the strength of—

Hyposulphite	3 ounces.
Distilled water	16 ,,

The picture must then be well washed in cold water, as already recommended, for positives, and so all traces of soluble matter must be removed.

As a negative plate has to undergo what we call comparatively rough usage, its collodion side must be protected in some way to prevent its destruction. This is done by coating it with a thin layer of varnish. The plate must first be gently heated, and the varnish is then to be poured on so as to cover the coated side on every part, most evenly. After the whole surface is covered with the liquid, the superfluous quantity should be drained off, by inclining one end of the plate until it is pretty well cleaned. The plate is then restored to its horizontal position, and left to dry, being carefully covered over to prevent dust or dirt falling on its surface.

The following varnishes have been recommended for this purpose, as suitable on account of their transparency and other qualities; but the operator can purchase those of a proper kind at the photographic chemists. The object to be kept in view is, to have a varnish which shall be transparent, and not liable to crack when it becomes dry.

Negative Varnishes.

Mastic varnish, diluted till of a thin creamy consistency.

Schnee's varnish, which consists of white lac dissolved in spirits of wine.

Copal varnish, of the consistency of thin syrup.

It sometimes happens that the depth of shade, in some negatives, does not sufficiently contrast with the lighter portions of the picture: the result is, that a good print can scarcely be obtained from such. To deepen those tones, we have tried baths or solutions of the bichloride of mercury, sulphate of iron, gallo-sulphate of iron, iodide of potassium, &c., of various strengths. A writer on the subject, in the *Photographic News* Almanac, recommends the following solution, which can be applied after the picture has been fixed: namely—

Iodine	1 grain.
Iodide of potassium	1 ,,
Water	1 ounce.

The picture is to be covered with this solution, and to be left for five minutes: the process may be conducted in daylight. A fresh development in the dark is then required, by means of the following solution:—

Pyrogallic acid	2 grains.
Glacial acetic acid	15 ,,
Distilled water	1 ounce.

This is to be poured over the plate a few times. To this solution may then be added a few drops of a twenty-grain solution, to the ounce of water, of nitrate of siver. This new solution is then to be poured over the plate. The iodine and pyrogallic solutions may be alternately used, until sufficient intensity has been obtained. The picture is then to be well washed, and afterwards varnished. The inventor of the plan states, that, as an intensifying process, it is to be preferred to that of the bichloride of mercury; and our own experiments confirm that view. The student will arrive at some successful results by taking the observations we have made, rather as a guide to his own experiments, than as final dicta. In hurried and unregistered trials with various solutions, we have occasionally hit upon successful results. We, however, do not recommend such a course to our younger readers; because, although permissible in those having experience, it engenders a careless habit in the beginner, which, of all things, is most to be avoided in scientific pursuits.

We have thus given, so far as we can judge, the most definite directions for obtaining good negatives. Our next course will be to show how such may produce positives by means of prepared paper of various kinds.

PRINTING ON PAPER FROM NEGATIVES.

We have stated, that the chief use to which negative pictures are applied, is that of producing an almost indefinite number of positive copies; and this is effected by means of sensitive paper, which the solid parts of the negative picture protect from the action of the chemical rays of light, whilst the lighter portions permit that influence to have a greater effect. The transparent parts of the negative allow the freer transmission of light, and thus we can obtain a picture presenting, so far as light and shade are concerned, an exact copy of the original. We regret to say, that all attempts to copy the natural colours of objects have hitherto failed. It has often been announced that such has been accomplished; but hitherto no success has actually been attained, although some approximation has been realised. A positive copy, although precisely the reverse of a negative original, yet has all its blemishes and excellences: hence it follows, that unless a good negative is employed, a good positive can never be printed from it. This fact the tyro in photography will do well to bear in mind, as it will induce care and caution in his attempts to produce the negative in the first instance.

In our earlier pages we gave some elementary experiments, illustrating the process of paper-printing. They were, however, necessarily of rather a crude character, because they were only intended as an introduction to photographic practice. We must now enter more fully into the minutiæ of the process, and so assist the operator in overcoming many difficulties which arise in its pursuit. We shall, therefore, examine each point in detail, so that the successive steps may be fully understood by all our readers. For this purpose, it will be proper that we should divide the process into separate heads; and in its pursuits, we must remind our readers of the necessity of care and complete cleanliness, as of the last importance. The following are the different points to which we shall separately call the attention of the experimenter:—

1. The choice of paper.
2. Its preparation.
3. The copying process.
4. The fixing.
5. The toning.

1. *The Choice of the Paper.*—Perhaps this is the most difficult, because it is the most uncertain, part of the whole affair. However much care is exercised in the selection, still some hidden defect is sure to appear in almost every specimen when it is submitted to the photographic process. At the present time, however, there are some paper manufacturers who produce an article intended solely for photographic use; and this, to a considerable extent, modifies the difficulties with which the photographer will have to contend. This is sold under different names, according to that of its maker. There is, however, a general title of French and English prepared paper. Either may be obtained, ready salted and albumenised, of the photographic chemist; and we shall, therefore, dismiss them without further remark, and devote some space for the benefit of those who may be unable easily to procure them. We may observe, that the two kinds of paper most esteemed by photographers, are the *Saxe* paper and the *papier Rive*.

In the manufacture of all sorts of writing-paper, numerous impurities, in a photographic point of view, are always present. The water employed in washing the rags, in reducing them to pulp, and in subsequent processes, contain common salt, sulphates, &c.; and in two cases within our knowledge, an appreciable amount of lead, or its sulphide, has been detected, owing to the impregnation of the water by that metal as it flows past the mines. Oxide of iron is almost invariably present, and no doubt exercises a decided effect on the results obtained in paper. We do not wish to multiply the difficulties which present themselves to the tyro; on the contrary, we prefer that he should know the chances of failure which may continually present themselves; and, by being forewarned, he may, by patience and care, be equally forearmed.

During the manufacture of paper, a portion of size or starch, and occasionally both, are introduced, for the purpose of giving a smoothness and evenness of surface to the texture. Our readers will be prepared to find that such matters have a decided effect on the nitrate of silver, which is the salt usually employed to sensitise the paper. A drop of the solution of that salt immediately produces a black spot on the hands, owing to the increased or peculiar action which occurs from the presence of animal matter. Generally speaking, animal or vegetable matter is always present in some form during photographic processes; but when so in circumstances beyond our control, it is often a source of great inconvenience.

With respect to ordinary sizing, there is no remedy; for blotting, filtering, or bibulous paper, are all objectionable; and the patent paper parchment which is prepared from these by means of sulphuric acid, is, from other causes, as unfit for photographic purposes.

We have found, that the best paper of an ordinary kind, and which is generally to be procured in most places, is common drawing-paper; that is, such as is used for lead-pencil drawing. All cream and blue-laid paper are utterly useless. The plain Bath or satin post may be employed; subject, however, to the difficulties and objections to which we have already alluded. The points requiring attention are—that the surface should be even; not too highly glazed; free from spots; and the paper itself as chemically pure as possible. These results can only be practically arrived at by repeated and careful trials.

Albumenised paper is generally employed by photographers, because the addition of the animal matter has a very good effect on the appearance of the positive, both during the process and after its completion. The mode of applying this, together with the salting process, we shall presently explain. Gelatine, starch, &c., are also used for the same purpose; but they are not so convenient in manipulation, or generally so successful in the result, as the white of an egg, which is nearly pure albumen.

Before proceeding to describe the preparation of the paper, we may state, that there are a variety of causes which, independent of the manufacturing faults, produce inconvenience to the experimenter.

In cutting, sorting, and packing writing-paper, &c., it necessarily passes literally through many hands; and each time it is fingered, it acquires, in some part, a portion of grease and common salt. Despite, therefore, of all possible care in selecting a good paper, these accidental circumstances tend to spoil it in some parts. Hence, we often find unaccountable spots, and what are called "stars" on the surface; these being generally brought to light during the fixing process.—For these we can offer no remedy; only let the operator be careful that he does not increase their number by his own carelessness.

2. *The Preparation of the Paper.*—If our readers can supply themselves with any of the different sorts of paper, ready salted and albumenised, the remarks we are about to make will be superfluous. But we should not be carrying out the general plan of our work, did we not prevent, as far as possible, the chance of our readers being left to their own resources, without any aid which we can afford them. We shall, therefore, give plain directions for the salting and albumenising process, and then deal with the modes adopted for making the paper so prepared sensitive to the action of light. The latter process is, of course, as equally necessary in its application to paper which may be purchased readily prepared, as for that which may be salted, &c., by the student at home.

The albumenisation and salting of the paper may be thus effected. Break a sufficient number of fresh eggs as to produce two fluid ounces of the white, which must be poured into a clean vessel as it passes out from the shell. Care should be taken not to mix any of the yolks with it. To the clear albumen so obtained add an equal quantity of distilled water, in which has been dissolved eighty grains of chloride of ammonium (sal-ammoniac). This salt is to be preferred to any other, for many reasons, and its action is generally to be depended on. Shake up the albumen and the water together, and when well mixed, pour them into a tall glass jar or bottle, and leave the mixture at rest for several hours. The clear liquid may then be poured into a plate or other flat receptacle, and after adding to it ten drops of liquid ammonia, it is ready for use. Then turn up one corner of a sheet of paper, so as to serve as a kind of handle, and placing the part next to the corner first on the surface of the liquid, gradually draw the whole of the paper so that each portion shall be equally moistened. If this be properly managed there will be no air-bubbles left on the paper surface. It may be left on the liquid for a few minutes, and is then to be hung up to dry in a moderately warm place. The object to be attained is that of an even coat of the albumen, so that when the paper is dry, each portion shall present a regular and even appearance. A good deal must be left to the tact of the operator: if, however, our instructions are carefully followed, there need be no risk of failure. Any quantity may be so prepared; and if it is properly dried, and kept in a dry place, it will remain without change for any length of time. We need scarcely say that it is not liable to alter by the action of light.

The sensitising process, or that by means of which the salted paper is rendered fit for the action of light, should be performed shortly before the paper is required for use, because animal matters, as we have already remarked,* have a spontaneous action on salts of silver. The solution may be as follows, viz.—

Nitrate of silver	80 to 100 grains.
Distilled water	1 fluid ounce.

This is to be poured into a clean glass or porcelain vessel; and a piece of the paper, prepared as we have described, is to be placed on its surface in the same manner as that recommended for its albumenisation, care being taken that no air-bubbles are left on the surface, and that no part be touched by the finger, except the turned-up corner. The paper may be left in the solution for a few minutes, and is then to be dried in a dark place, because, owing to the formation of the chloride of silver on the surface, it has become sensitive to light. Of course, this part of the process should, for such reasons, be entirely carried on by candlelight.

The strength of the silver solution gradually diminishes as each sheet is taken from it, and therefore an occasional addition of some more nitrate of silver, or of a strong solution of that salt, will be required. When the solution becomes of a brown tint, its colour may be restored by adding to it a small portion of pipe-clay. This is to be shaken up with the liquid, which can afterwards be filtered through some blotting-paper, if requisite.

3. *The Copying Process.*—To effect this successfully, a copying frame should be employed. It consists of an outer frame, which holds a sheet of plate-glass. The negative is placed on the glass with the collodion side next to the paper. The prepared paper is placed with its albumenised side next to the negative; and to keep the two in close contact, a back fits into the frame, and is secured by means of two cross pieces therein. Folds of blotting-paper, or soft cloth, may be placed between the paper and back of the frame, so as to press the paper firmly against the negative plate. This is of great consequence, because any crease in the paper would at once spoil the print, and produce general indistinctness of outline. The back of the frame is made to open and shut; so that whilst one part still firmly presses on the plate and paper, another part may be raised, and the progress of the printing examined.

We can give no definite directions as to the length of time required for exposure. We have already often remarked on this point. To illustrate this, we may instance the numerous experiments which have been made for the purpose of testing the question. In the morning of a summer day, in clear sunshine, the printing process has been effected in a few minutes; whilst after four o'clock, it has required an exposure of from half to three-quarters of an hour to produce the same effect. Occasionally we have noticed an almost entire suspension of actinic effect; but, a few moments afterwards, whilst the sun has been obscured by a thin cloud, the printing seemed to have gone on with increased vigour.

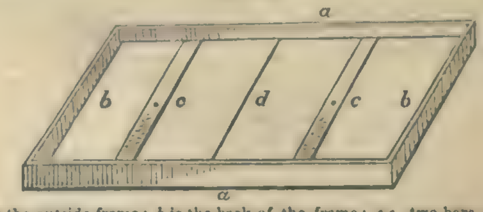
In the hope of giving something like an idea on this point, we have had collodion positives taken during every quarter of an hour, from ten in the forenoon, with little intermission, till six in the evening, in the early part of June, choosing alternately a clear sunshine and dark cloudy weather; and despite all attempts at uniformity in every circumstance, so far as manipulation and chemicals were concerned, our results entirely baffled any hope of affording constant rules.

Generally speaking, it is better that the positive should be over, rather than under-printed, because, when placed in the fixing solution, it undergoes considerable modification, and every part becomes of a lighter tint. Of course, much depends on the depth of shade, &c., in the negative, other circumstances being equal. We must therefore leave this point to the care and judgment of the reader, who will, by practice, arrive at something like certainty of result, and which experience alone can give.

During the printing process, the paper may be examined in the way we have mentioned, or a piece of the same paper as that in the frame, may be exposed under an equally thick piece of plate-glass, and the process of colouring be watched; unless so placed it will deceive the operator, because the glass of the frame exercises a retarding effect on the progress of the chemical changes.

A usual form of copying frame is illustrated in the following engraving:—

Fig. 159.



a is the outside frame; *b* is the back of the frame; *c c*, two bars, which fit into the sides of the frame, and so press the paper and negative together; *d* is a hinge, by means of which, either end of the back may be opened, so as to allow of the positive being examined during the progress of printing.

4. *The Fixing Process.*—Presuming that the positive has been properly printed, the next step is to prevent further action of light thereon, and so to render the picture permanent. This is effected by immersing it in a bath of hyposulphite of soda, which dissolves away all the unchanged chloride of silver. The proportions of the solution are as follow:—

Hyposulphite of soda . . .	1 ounce.
Distilled water	4 ounces.

On removing the print from the copying frame, it should be abundantly washed with cold water, which removes all substances soluble in that liquid. A solution of half an ounce of common salt, and a small quantity of bicarbonate of soda, is then made in a pint of water, into which the print is to be dipped, to free it from all traces of nitrate of silver and nitric acid. It should then be again washed in abundance of water, to free it from the soluble salts it may contain. The next step is to immerse it, face downwards, in the hyposulphite bath, in which it may remain for a quarter of an hour. It, however, should be continually examined, because the fixing solution has a tendency to destroy the distinctness of outline and gradation of shadow which gives the beauty to a good print. During this process, stains occasionally appear, which are owing to the accidental presence of free nitrate of silver. This should have been prevented by the salt and soda bath; or they may have been caused by carelessness in touching the print with the fingers. As hyposulphite of soda is a cheap material, it is better to use a fresh solution for every other picture. This plan often prevents the production of stains, and generally ensures good results.

It only remains that the print should be afterwards well cleansed from all hyposulphite of soda, which is done by placing it on the surface of clean water, and pouring a gentle stream on it for some time. The print is then left for an entire day in a vessel containing plenty of water, which should be occasionally changed. This having been effectually performed, the picture may be considered as permanently fixed.

5. *The Toning Process.*—We have as yet described what may be termed the ordinary process of printing: there, however, remain for us to mention various plans, by means of which the brilliancy of the positive may be increased. This is usually effected by means of a solution of chloride of gold, in the manner which we proceed to describe. We must specially observe, that great care is required to produce the best results; and the tyro must not expect, at a first trial, to arrive at perfection. For this reason we have deferred introducing the subject of toning until we supposed, by the practice of previous processes, some amount of proficiency had been acquired.

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* See *ante*, p. 140.

We need scarcely observe, that nothing but a really good positive print is worthy of the toning process; and more than this, it should have been printed with that object in view. In giving instructions as to this process, we cannot do better than quote from the *Photographic News* Almanac, in which the writer lucidly points out each step. We can confirm, from our own experience, the statements which he makes; and shall presume that our previous instructions as to sensitising the paper have been fully carried out. The author of the paper referred to, says:—

"The printing should be effected as soon after exciting the paper as possible: considerable over-printing is generally desirable. The lights should be of a delicate lavender tint, and the shadows deeply bronzed, before the print leaves the pressure frame.

"The prints should be washed in two or three changes of water just before toning; and the final washing-water may have a little salt added. Long soaking in salt and water renders the toning more difficult. All the processes in connection with printing and toning, with the exception of the exposure, should be conducted in the dark, or in yellow light; and the most scrupulous cleanliness, as to fingers and dishes, must be observed. In regard to the latter, it is desirable that each one be kept to its own purpose, and never used for any other; and the utmost care should be taken not to put the fingers which have just touched the hyposulphite of soda, into the gold solution.

"Several different forms of the alkaline gold toning bath have been proposed, all of which have been practised with more or less success. We shall only refer to those with the results of which we are familiar, and can speak with personal confidence. The first is the process of Mr. Maxwell Lyte, with which most photographers are already familiar. It stands thus:—

"Chloride of gold	1 grain.
Phosphate of soda	18 grains.
Distilled water	*quantum suff.*

"The chloride of gold, and phosphate of soda, should each be kept in solutions of given strength—say one grain of gold to an ounce of water; they can then be well mixed in any quantity required for use, and diluted with sufficient water to float the prints easily. The only effect of using a large quantity of water, is to make the process of toning somewhat slower, which is frequently an advantage. Sufficient water tends, moreover, to secure regular toning. The prints should be kept moving, and prevented from sticking together, or red, imperfectly-toned patches will be the result. The water used for diluting the bath may be previously made warm, as it materially facilitates the process, especially in cold weather.

"The process of toning is more rapid in some cases than others; but in most cases, a few minutes—from two minutes to ten—is sufficient. It is, in all cases, desirable to allow the print to assume a little deeper tint than is required in the finished picture, as a little will be lost in the hypo fixing bath. If a sepia or brown be required, a decided purple tint should be obtained in the toning bath. If a decided purple be desired, then the tint in the toning bath should be black.

"For the phosphate of soda, the bicarbonate or biborate may be substituted, but in less proportions in regard to the carbonate especially, as an excess of it has a tendency to dissolve the size of the print, and, in some cases, even the albumen. Five grains will, therefore, be generally found sufficient; the only object being to neutralise the free acid in the chloride of gold.

"We have seen very fine prints produced by the toning process recommended by Legray; which is as follows:—

"Chloride of gold	1 grain.
,, ,, lime	1 ,,
,, ,, sodium; *i.e.*, common salt	1 ,,
Distilled water	4 ounces.

The treatment is similar to the first-mentioned; but as there is a strong bleaching tendency, it is necessary that the prints be considerably over-printed.

"The last process of gold toning to which we shall refer, is one published during the past year (1860), as being the suggestion of M. L'Abbé Laborde. We may mention, however, that its advantage had been pointed out some months before by Mr. Hannaford, who states, that a peculiarly rich purple bloom is secured, which is absent when any other salt of soda is used with the gold, with the exception of the citrate as proposed by Mr. Hardwich, which has a similar effect. Mr. Hannaford states, that it has another advantage, which the Abbé confirms—namely, that it may be used more than once; all other alkaline toning baths being useless for toning purposes after once being used for toning. The formula stands as follows:—

"Chloride of gold	15 grains.
Acetate of soda	7½ drachms.
Water	35 ounces.

"The solution becomes colourless by degrees, and, at the expiration of twenty-four hours, it is ready for use. On removing the positive from the printing frame, it is washed in two or three waters, to remove the free nitrate of silver; it is then immersed in the gold bath, where it must be allowed to remain not more than twenty-five to thirty seconds, when the bath is first used.

"If the gold bath has been used before, its action is slower. Experience will enable the operator to see, by the successive changes in tone the proof assumes, when he should remove it from the bath. If it be removed too soon, the proof assumes a disagreeable red hue after it is fixed with the hypo; on the other hand, if it be allowed to remain too long in the gold bath, the proof assumes a cold blue tint. Between these two extremes there are a variety of tones of sepia and violet, which can be secured by removing the proof at the proper moment. In proportion to the length of time the gold bath has been used, so we must prolong the toning until the required tint appears. The strength of the gold bath can be restored by adding fresh chloride of gold; but, before making the addition, we must take care that the solution is nearly colourless; for if the chloride of gold is not in combination, it will, like other chlorides, weaken the proof.

"In all cases the prints should be washed in two or three changes of water, and then immersed in the hyposulphite of soda; which, it is desirable, should possess a strength of at least twenty per cent., or four ounces to a pint of water. It is better used fresh each time, and should be neutral or slightly alkaline. Mr. Lyte recommends the addition of a little Spanish white, or chalk, for that purpose. Prints on thin paper are more rapidly fixed than those on thick; but they should not, in any case, be removed from the fixing bath in a shorter time than ten minutes. They should then be washed in several plentiful changes of water, within the first half-hour; and subsequently soaked and washed, so as to thoroughly remove all traces of hyposulphite of soda."

The above instructions will be of great value to all who desire to produce the best results obtainable in the printing process; and, as we have already observed, our own experiments affirm their useful and reliable character.

Having thus entered fully into the entire process of printing, &c., we trust that our readers will find no extraordinary difficulty in pursuing these experiments: and we shall now pass on to consider a variety of photographic processes; which, although not employed so universally as those we have minutely described—still, either afford interesting results, or, in an historical point of view, are not to be overlooked by us.

Our previous pages have been an exposition of processes which are universally adopted at the present time for likeness and landscape taking. Our limits have, of course, forbidden us to enter into every minute detail; and we must refer those of our readers who desire to make a special study of the philosophy of the processes, to the works of Messrs. Hunt, Hardwich, Price, and many others; in which they will find an almost exhaustion of the subject.

THE DAGUERREOTYPE PROCESS.

We shall now proceed to describe, in general terms, the process which, by the use of silver plates rendered sensitive by means of iodine, &c., gives similar results to those to which we have already alluded. We may, however, observe, that this mode of taking portraits, &c., has been, of late years, but rarely employed, for the collodion process has almost entirely superseded it.

We owe to M. Daguerre the first steps which were taken to render practically useful the fact that salts of silver are applicable to retain the impression of solar influences. It is true that Sir Humphry Davy and Mr. Wedgwood were well aware that copies of objects might be obtained by the agency of light; but those eminent men never proceeded beyond the threshold of their discoveries. M. Niepce and M. Daguerre, however, applied themselves to the subject; and the result has been, that every part of the civilised world has become familiar with the fact that light may be made a most valuable servant.

About twenty years ago, M. Daguerre discovered, that plates of silver, coated with iodine, were sensitive to the action of light, and that those portions so acted on, might afford a surface on which distinct pictures could be procured. This discovery produced a profound sensation in scientific circles; and the French government were so impressed with the novelty and value of the process, that they rewarded its discoverer in the most handsome manner. He had voted to him a pension of 6,000 francs; and France took to herself the credit of having given to the world one of the most surprising of discoveries.

That portion of the world which forms our native land, seems, however, to have been an exception; for a patent was taken out in this country, by which the employment of Daguerre's process was limited to one person and his licensees. If we compare the likenesses taken in the year 1840, with those of the present day, we shall certainly find that we hold the palm in every respect. This fact, however, must not deter us from doing justice to the valuable discoveries made by M. Daguerre.

One great objection which exists against this process is, that portraits, &c., produced by it, can only be viewed in one light, owing to the reflection from the silver surface. This is a source of considerable inconvenience, and, indeed, prevents any chance of representing some objects successfully.

We now proceed to detail each part of the process. In doing so, we shall not be so minute in our description as when speaking of the collodion process, for the reasons which we have already given. We may conveniently consider, separately, the following particulars, in the order in which we have arranged them :—

1. Cleaning the silver plate.
2. Iodising it, and exposing it to bromine vapour, &c.
3. Exposure in the camera.
4. The mercury process.
5. The fixing process.
6. The toning process.

1. *Cleaning the Silver Plate.*—We may here observe, that various sorts of plates may be employed, providing a surface of pure silver is formed thereon. Hence, copper, plated with silver by the ordinary method, or by electrotyping pure silver plates, or standard silver electrotyped over with pure silver, may be employed. In either case the surface requires to be highly polished; and the directions which have been given to secure this desirable result, are innumerable. We presume that a perfectly level plate has been chosen, free from scratches, &c. Its surface should first be roughly cleaned from oxide by dipping it into, or by rubbing it over, with a very dilute nitric acid; this removes a thin coat of the silver, but ensures, at the same time, more chance of success in the process of cleaning. The plate is then to be washed in plenty of cold water, so as to remove all trace of acid. This being effected, it may be polished by the following means :—Place the plate on a sheet of clean white paper, much larger than itself, and, by means of a pellet of the best cotton wool, which has been previously dipped in some of the best olive oil, commence rubbing the plate in a circular manner, from the centre to its outside. Having done this until the surface is covered with oil, dust over it some of the finest tripoli, and pursue the same plan of rubbing circularly, from the centre to the outside, for some little time. By such means all the coarse irregularities will be removed, and a tolerably level surface will be obtained.

The plan which we have adopted, after having given the plate the rough polish, is to wash it carefully with a mixture of equal parts of spirits of wine and liquid ammonia: this has the effect of dissolving and clearing away the minute portions of oil which would otherwise remain in the fine lines left on the surface of the plate; which, although invisible to the naked eye, may easily be detected by means of a microscope. After thus having cleansed the plate, it is to be held over the flame of a spirit or gas-lamp, which has the effect of driving away a thin film of air which always rests on polished surfaces, and of which we have already spoken.[*]

A little clean cotton wool, made into a pellet, is then to be moistened with spirits of wine. Having dipped it into some of the finest tripoli powder, the plate is to be rubbed with it until a dark jet-like appearance is produced: a dry pellet of cotton is then to be employed, to remove all trace of the powder. The last operation is that of buffing, or rubbing the plate on some velvet fixed tightly over a piece of wood. This is generally made, in shape and appearance, like a razor strop; and the plate is to be briskly moved, under pressure, to and fro on the surface of the velvet. By these means a brilliant polish is obtained; and, practically speaking, we may suppose the plate as fit for all subsequent processes.

We must here warn the tyro against carelessness in respect to the cotton wool, tripoli, and buffer, each of which are liable to catch small portions of dust, sand, or grit. Neglect of this is sure to afford a scratched and uneven surface on the plate. The smallest particle of any hard substance, repeatedly rubbed on a comparatively soft body like silver, produces an infinity of fine lines, and renders the plate useless. Our remarks on cleanliness, in a former page, have an equal application to the Daguerreotype process; and, indeed, too much care cannot be bestowed at each step of our progress.

When the operator has a lathe in his possession, much labour is saved by its use in the operation of polishing Daguerreotype plates, because both buffer and plate may readily be fitted in it. The action is much more regular than that obtained by manual labour, and there is less risk of producing a scratched surface through over-pressure on any part. We need scarcely add, that the time required for polishing the plate is also materially lessened.

2. *Iodising the Plate.*—After thus producing a clean and polished surface on the plate, it must then be made sensitive to the action of light by means of the vapour of iodine and bromine. In the earlier days of photography, iodine alone was employed; but the addition of bromine has many advantages, as we shall presently see.

The philosophical instrument-makers sell boxes expressly made for the iodising process. The student, however, will succeed very well by using a square box of about four inches deep, and a little longer and broader than the plate. The latter should be fixed in a frame so that it may be exposed with the silver face downwards. The frame itself may form the lid of the box. In the bottom of the box a glass tray must be placed, to hold the iodine, &c. A similar box will be required for the bromising process, if the operations are carried on separately. Of this, however, we shall speak more fully as we proceed.

To iodise the plate, put a small portion of iodine into the glass tray at the bottom of one of the boxes; and having fitted the silver plate, face outwards, in the frame, place it horizontally over the tray containing the iodine. The vapour which arises will reach the silver surface, forming an iodide of silver, which should be of a rose colour. The time required to effect this varies according to temperature and other circumstances; and so, like many other photographic processes, must be left to the judgment of the operator. As soon as the rose tint is obtained, the plate may then have its sensitiveness increased by means of the vapour of bromine, &c.; and such have been termed accelerators, from their power of rendering the subsequent processes more rapid of execution. After being iodised, the plate becomes sensitive to the action of light, but not to any great extent. It is, however, desirable that iodising should be carried on in the dark room; whilst it is essential that the accelerating process should be so conducted. Great care should be taken that as little as possible of the vapour of iodine and bromine should escape into the room, because such not only may have a generally prejudicial action on the plates, but also on various metals, chemicals, &c.; besides the personal inconvenience and injury which would occur to the operator.

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[*] See *ante*, p. 146.

The accelerating effect is produced by exposing the plate, previously iodised, in another box, which has a glass tray, into which the accelerating solution is to be poured. The exposure is conducted in precisely the same manner as in the process of iodising, and a rich, but fuller, dark rose-red surface is obtained on the silver plate, which will then be nearly ready for exposure in the camera.

Accelerating Solutions.—A solution of bromine in water may be employed for this purpose; and it is made by adding some bromine to distilled water, and shaking the two together until as much bromine as possible has been taken up. One pint of this saturated solution is to be diluted with distilled water, until it presents a pale straw colour; and then poured, to the depth of half an inch, in the glass tray.

Another solution may be prepared by adding a few drops of hydrochloride acid to a drachm of bromine, which is then to be diluted with six ounces of distilled water.

By using the above solutions as accelerators, a rich red rose-colour is produced on the plate. The latter must afterwards be exposed again to the iodine vapour, until the silver acquires a very deep rose-red, or, occasionally, a blue colour, resembling that produced on steel when it has been heated to a temperature of between 600 and 700 degrees Fah.; and the iodising process is then finished.

An accelerator recommended by M. Claudet, has the advantage of preventing the necessity of a second exposure to the iodine vapour. A portion of it is poured into the glass tray of the accelerating box, and after the silver has been exposed to the action of the vapour for several minutes (even to the extent of half-an-hour occasionally), it is ready for the camera. The mixture consists of a saturated solution of iodine in bromine; which is easily done by adding as much iodine to the liquid bromine as it will take up. A bromide of iodine is thus formed, and as much of it is added to distilled water as will impart to the latter a full yellow colour. It is stated, that this plan is always to be depended upon; and it has one advantage for the tyro—which is, that he can scarcely err in its employment.

We must here point out a few matters by way of caution. The iodine should be spread evenly on the glass tray; because, wherever it is in masses, it will produce a deeper tint over that part. Care must be taken not to disturb the particles of iodine by hastily placing the plate on the top of the box, as this might possibly cause the air inside to be disturbed, and so drive particles on to the surface of the plate, where it would produce spots, and entirely destroy its fitness for subsequent processes.

3. *Exposure in the Camera.*—A sensitive plate having been produced, the next step is its exposure in the camera. On this head we have already made several remarks whilst directing the reader in conducting the collodion process. The same precautions and arrangements are required. The silver plate, however, is not so sensitive to the action of light as those of highly sensitised collodion. The time of exposure varies according to a variety of circumstances, each of which, as it presents itself, becomes the subject of annoyance to the operator, but yet adds to his knowledge and experience. We shall, therefore, not further deal with this part of the whole process, but refer our readers to the remarks we have made at a previous page.* We may, however, remark, that great care is required to prevent the effect called *solarisation*, which is due to over-exposure. This entirely destroys the contrast of light and shade, on which depends the beauty of all photographed productions. However, to prevent this, the operator must be well acquainted with the power of his lens, and the excellency of the plates and chemicals; matters which, of course, can only be gained by practice and experience.

4. *The Mercury Process.*—On removing the plate from the camera into the dark room, no picture will be observed on its surface; it must therefore be exposed to the vapour of mercury, which, settling on the face of the plate, rests on those parts which have been affected by the action of light. For this purpose a peculiarly shaped box is employed. In the lower part of it is a cistern, containing about two ounces of quicksilver; and in this the bulb of a thermometer is placed, which indicates the temperature of the mercury when heated by means of a spirit-lamp placed beneath it. This temperature should never be allowed to rise beyond 150° or 160° Fah. The plate is placed at an angle of 45° at the top of the box.

Presuming that a plate has been duly exposed to light, and is allowed to receive the vapour of mercury in the manner we have explained, we shall soon perceive that the parts which have been chiefly acted on by light, are most deeply brought out by the mercurial vapour, and every other part receives its due proportion, and produces a corresponding effect.

By these means the latent picture becomes developed; the effects of light and shade are produced on the surface of the plate, and a representation of the object copied is afforded, correct and brilliant, just in proportion to the care which has been exercised by the operator in each process, and in the actinic power of the light which has produced the picture.

5. *Fixing the Picture.*—The next part of the process is that of fixing the picture, and of so preventing the further action of light on the plate. The only thing required is to remove the remainder of the sensitive silver salt from the face of the plate; and this is effected by immersing the plate, when cool, in a solution of one part of hyposulphite of soda, to from six to ten parts of water. It should be left in the solution for a few minutes, and then must be abundantly washed with cold water. This is best effected by allowing a gentle stream, from a water-tap, to impinge on the plate: by these means all the soluble salts are completely removed. As, however, common water contains lime, &c., the plate should be finally washed with distilled water; and it may then be dried by holding it at some distance over the flame of a spirit-lamp. In drying, great care should be taken that no drops of water are left on the surface, as they would be sure to form spots. As the plate heats, the water has a tendency to run off from its surface; and an attentive operator will take advantage of this fact, so as to prevent the occurrence of spots, &c., thereon.

The appearance of all Daguerreotypes is remarkably improved by submitting them to the action of chloride of gold, after they are removed from the hypo bath. The contrast of light and shade is thereby greatly enhanced, and the brilliancy of the picture is of course increased. The process is very simple, and may be performed as follows:—

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* See *ante*, p. 149.

6. *The Toning Process.*—After washing the plate, place it on a ring-stand, so that it may be quite level; and there is to be poured on it the following toning mixture, so that every part of the picture may be well covered. It is made as follows:—

Chloride of gold . . .	8 grains.
Distilled water . . .	4 ounces.

The salt of gold, having been dissolved, is then added to the following solution of—

Hyposulphite of soda . .	25 grains.
Distilled water . . .	4 ounces.

The two solutions, when mixed together, have a slight yellow tint, which, however, passes off. Some of this being poured on the picture, the heat of a spirit-lamp is cautiously applied beneath each part, until minute bubbles appear. In a short time, the improving effect of the toning process becomes visible, in the intensity and depth of contrast produced between the light and dark parts of the picture. The plate may now be left to cool, and the gold solution is then to be washed off, by means of distilled water, until the plate is perfectly cleansed. The spirit-lamp may now be used to dry it; and on this being carefully performed, the whole process may be considered as complete.

We have thus given a detailed account of the successive processes which enable us to obtain pictures on silver plates by means of the Daguerreotype process; but have omitted numerous modifications and improvements which have been from time to time suggested by various operators, because the process is rapidly becoming obsolete, in consequence of the greater value of, and the better results afforded by, the various processes on glass. It requires a considerable amount of care, and some photographic experience, to obtain even tolerable results; and, as we before remarked, the silver plate has the great objection of reflecting light, so that the pictures can only be seen at a certain angle. We are not aware of any photographer who professes to take portraits by this process at the present time; and therefore we have introduced the subject more as a curiosity, and as an illustration of the earlier attempts at picture-taking by the agency of light, than as one of any utility or profit to the experimenter.

We may conclude by stating, that if any of our readers attempt the process, and arrive at the very common results of obtaining "spoilt plates," such may be employed again by heating the surface to remove the mercury, and then by repeating the process of cleaning, according to the directions given in a previous page.*

THE CHROMOTYPE PROCESS.

This process is so easily conducted as to be at the command of any one, however slightly acquainted with science or experimental manipulation. It depends on the fact, that certain salts of chromic acid are easily acted on by actinic rays; and the most proper for the purpose is the bichromate of potass.

A saturated solution of this salt is made in cold water, in which any suitable-sized sheet of paper may be immersed. The salt will soon penetrate its pores; and the paper, after draining, is to be dried between folds of common blotting-paper. It will thus have acquired an orange-yellow tint, and it may be preserved for any length of time if kept in a dark place. It is used in exactly the same way as the positive paper (described at a previous page†) in the copying frame. The object is placed on the plate-glass: on this the chromatised paper; and the back of the frame is then fitted into its place; or the object may be simply kept in contact with the prepared paper, by means of a sheet of plate-glass.

The time required for exposure, of course, varies according to the chemical power of the daylight. The exposed parts should, however, present a dark-brown tint before the object is to be removed.

The process of fixing is remarkably simple. The picture is soaked for some time in warm water, so as to dissolve out any unchanged bichromate of potass. The paper will thus be rendered nearly white, or, at all events, of a light primrose colour, in those parts which have been shaded by the object copied; whilst the other parts present a dark-brown hue.

This process may be readily employed for copying any object whose outlines are well defined—such as lace, ferns, the shape of leaves, &c. Indeed, with a child's transparent slate, and a few sheets of paper, the botanist, and others, may copy a vast variety of objects; and, from the simplicity of the whole affair, it may afford an instructive and interesting amusement to young people.

THE CALOTYPE PROCESS.

One of the most ardent and successful of all those who have devoted their attention to photographic investigations, has been Mr. Fox Talbot. From the earliest days of the art he has continually made contributions to our stock of information; and to him we are indebted for a great variety of improvements on the processes proposed by other experimenters.

So early as the year 1839, Mr. Talbot communicated his views to the Royal Society, in reference to "the art of photogenic drawing, or the process by which natural objects may be made to delineate themselves without the aid of the artist's pencil." He also pointed out how such impressions might be fixed or rendered unchangeable by the further action of light.

The earlier attempts at producing and retaining photographic copies were but of a comparatively valueless character. The numerous causes of failure to which we have already so fully alluded, were only distantly guessed at; and the eagerness with which each new discovery was hailed, frequently prevented that close and philosophic examination into the chain of circumstances which gave them birth, or into the causes of failure, which almost invariably produced disappointment in those who attempted to repeat the experiments of others. We can well remember the obscure accounts which were published from time to time, in reference to the different processes, as they were given to the world. This, coupled with the various opinions held by discoverers, by no means assisted even the most ardent experimenter, but rather left him in what has expressively been termed "a fog." The mists of uncertainty, however, were gradually dispelled; and at the present time, photography stands out as one of the most complete applications, both in rationale and practice, of any which have been made of the various branches of science.

These remarks are here made, because we regard the calotype as the first elegant and complete branch of photographic manipulation. The discoveries of Daguerre were of great value; but those of Mr. Talbot at least equal them in that respect, and, in practical utility, often exceed them.

Having thus given a slight sketch of early photographic history, so far as it bears on our subject, we now proceed to detail Mr. Talbot's method of producing his calotype pictures; and shall introduce the subject in nearly the words used by the inventor whilst describing his earlier processes.

We may, however, impress on our readers the remarks which have been made in reference to the choice of paper. It matters not what process is undertaken in photography—the same chemical laws must be observed, and chemical changes must be looked for or guarded against.

The following, therefore, will guide the operator into what we may call the elements of the calotype process.

A proper paper having been chosen according to the general instructions we have already given, such is to be placed in a solution of nitrate of silver, dissolved in the proportions of—

Nitrate of silver . . .	100 grains.
Distilled water . . .	6 ounces.

In the manner described at a previous page.‡ The side so placed should be marked by means of a pencil, that it may thereby be easily recognised. The paper is afterwards to be dried in a dark place, and, if not immediately used (which is advisable), it should be kept out of the rays of the sun, or of diffused daylight. As, however, organic matters tend to decompose silver salts, even in the dark,* it is better that this paper should only be prepared in such quantities as may be consistent with the immediate requirements of the operator.

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* See *ante*, p. 156.

† See *ante*, p. 154.

‡ See *ante*, p. 154.

The paper is then to be placed, with its silver side downwards, on a solution of iodide of potassium (hydriodate of potass), in the proportion of—

Iodide of potassium . .	100 grains.
Distilled water . . .	3 ounces.

By these means the iodide of silver is formed on its surface, which gives the paper a kind of primrose appearance. It is then dried, first being pressed between folds of clean white blotting-paper. In this state it is barely sensitive to the action of light; but the process should be conducted in a dark place, such as by candlelight, or in a dark room. This we may term iodised paper; and, if properly prepared, and all the precautions with respect to cleanliness, &c., having been observed, it may be kept for some time without danger of undergoing any very serious change. However, in our repetition of Mr. Talbot's experiments, we have found that the almost certain presence of organic matter in the size of almost every kind of paper, has a tendency to produce changes which eventually exhibit their effects in spots, &c., on the surface of the prepared paper, if such be made for any length of time before being used.

The reader will thus observe, that the calotype process depends on a formation of an iodide, in place of the chloride of silver, on the paper surface; and on this depend some of the peculiarities of Mr. Talbot's calotype process.

Before exposure in the camera, this iodised paper must be rendered more highly sensitive; and this is done by placing the prepared surface on a solution of—

Nitrate of silver . . .	100 grains.
Distilled water . . .	2 ounces.
Acetic acid	3 drachms.

To which is to be added an equal quantity of a saturated solution of gallic acid. We may here observe, that gallic acid is but sparingly soluble in water. Mr. Talbot calls the mixture the gallo-nitrate of silver. This solution should, whilst prepared in the proportions we have named, be only made in quantities required for immediate use, because it spoils by keeping.

After the iodised surface has been rendered sensitive by these means, the paper is to be dried; and this process should be conducted shortly before the exposure of the paper in the camera. It may be used with advantage in a slightly moist state, which somewhat facilitates the subsequent process.

The paper may then be used in the camera, in the manner already pointed out for the collodion and Daguerreotype processes; and the time of exposure will vary according to the sensitiveness of the surface, and the intensity of the light reflected from the object about to be copied. A latent image will thereby be formed; and this is to be rendered visible by placing the paper on, or by pouring over its surface, a solution of the nitrate of silver, acetic, and gallic acids just named. The photograph or calotype is thereby developed; and this should be done in a gradual manner. Sometimes the development takes place somewhat irregularly, and spots and stains appear. If the exposure has not been sufficiently long in the camera, the picture takes a considerable time before it appears; and, in such cases, the aid of a gentle heat may be called in. The picture may then be fixed by immersing it in a solution of—

Bromide of potassium . . .	100 grains.
Distilled water	10 ounces.

After which it is to be abundantly washed with water, and subsequently dried. By these means a negative is produced; and by dipping the prepared paper in a solution of iodide of potassium, a positive may be produced, providing the gallo-nitrate immersion has been adopted. After exposure in the camera, the gallo-nitrate solution is then used for development in the way already described, and the fixing process immediately follows.

Such is an outline of the earlier attempts of Mr. Talbot; which, however, have been modified by numerous subsequent experimenters. The same principles generally are involved, and the process is conducted more rapidly, owing to the more sensitive nature of the prepared paper.

The calotype process, like many others, is not so much in favour as that of the collodion, owing to the numerous details which have to be observed, and the tediousness of its manipulation. The effects, however, are very pleasing; and we may almost call it the most elegant and refined of photographic processes, when it is properly carried out.

THE WAX-PAPER PROCESS.

Amongst other materials which have been used as a photographic material, we may briefly notice that of waxed paper, which has the advantage of considerable sensitiveness, and of affording a pleasing effect when the picture is complete. This, like all other photographic processes, has been the subject of numerous modifications; and we shall, therefore, only give a general outline of it.

A thin kind of paper is to be soaked in the best melted white wax, until that substance is well absorbed into its pores. The waxed paper is then placed between folds of blotting-paper, and a hot iron is passed over the whole, so that the wax may be equally diffused, and any superfluity be removed. The paper, so prepared, is next placed in a bath of iodide of potassium, to which a little iodine has been added; and, after remaining some time, it is carefully dried. It is sensitised by means of a weak nitrate of silver bath, to which some acetic acid has been added; and if thus dried, it will keep for some time, provided it has been well washed with distilled water, to remove the superfluous nitrate from its surface.

The length of exposure in the camera varies from one minute upwards; and the latent picture is developed by means of gallic acid and nitrate of silver. The fixing is effected by a solution of the hyposulphite of sodium, and subsequent repeated washings in water. The picture may then be dried before a fire, and a hot iron should again be passed over it to equalise the wax in each part of the paper.

This plan is not much employed at the present time; and, like most others, has been superseded by the collodion process

THE ALBUMEN PROCESS.

A considerable variety of organic substances have been tried by photographers, as media by which the sensitive salts of silver may be retained on glass and paper surfaces. Amongst these, albumen has been frequently employed; and we have already drawn attention to one of its uses when speaking of positive printing-paper.† It has been applied with great success to glass plates; and some of the productions of French artists are extremely beautiful. Strange to say, few of the pictures produced exclusively by English photographers have arrived at such excellence.

Albumen, in this process, is used in place of collodion. The manipulation, generally speaking, is similar; great precaution, however, being paid to the delicate nature of the film, lest it should be injured. A picture prepared thus has a very transparent appearance; and the best of the kind we have seen are those of the blood globules, to which reference has already been made. Pictures by the albumen process are extremely valuable for use in the oxy-hydrogen microscope,‡ having a considerable transparency, and yet precision of detail. We shall now give full directions for pursuing the collodio-albumen process.

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* See *ante*, p. 140.

† See *ante*, p. 153

‡ See *ante*, p. 63.

THE DRY COLLODION PROCESS.

Great inconvenience is felt by photographers in taking views by means of the ordinary collodion process, owing to the necessity of exciting the plate immediately previous to its being introduced into the camera; and numerous attempts have been made to prepare plates which would retain their sensitive condition for a length of time after they have been excited. These have resulted more or less successfully, but they still leave room for improvement. It was at first proposed to use a salt which would deliquesce, or absorb moisture from the atmosphere, and so retain the collodion surface in a proper condition for the camera. This plan was superseded by others, which seemed more convenient in practice. We can only refer to some of the latest improvements, which are, more or less, modifications of previously published processes: and we are indebted for the following *résumé* of recent inventions, to the pages of the *Photographic News* Almanac (1861).

THE COLLODIO-ALBUMEN PROCESS.

"The plates should be very carefully cleaned for this and all dry processes, and especial care taken that they are perfectly dry before coating with collodion. The quality of the collodion used is not of importance, so far as the question of sensitiveness is concerned; but for the better prevention of blisters, &c., an old or powdery collodion is desirable. The plate is coated and excited in the regular way, and washed thoroughly in a running stream, to remove free nitrate of silver. It is then coated with a solution, prepared as follows:—

" Albumen		2 ounces.
Distilled water		¼ ounce.
Iodide of potassium		10 grains.
Bromide of potassium		2 ,,
Liquid ammonia	about	20 minims.

"The iodide and bromide should be dissolved in the water, which should then, together with the ammonia, be added to the albumen, and the whole thoroughly beaten to a froth. When it has settled, the clear portion should be poured off, and kept in a bottle for use.

"The washed collodion plate is to be coated with this whilst it is still wet. After it has drained a moment or two, pour over its surface the albumen twice. The same albumen may be used for two or three plates; but after that, it is better to take a fresh quantity. The plate must now drain on one corner a few minutes; then dry it rapidly before a clear, bright fire, and make it quite hot. In this state it is not sensitive to light, and will keep for many months. To render the plate sensitive for use, it must be again immersed in a bath, prepared as follows:—

" Nitrate of silver	40 grains.
Glacial acetic acid	½ drachm.
Water	1 ounce.

"After taking the plate out of the bath, drain a moment, and then wash well under a stream of water. A plate well washed always keeps longer, and develops cleaner, than one washed insufficiently. After washing, drain, and place it on blotting-paper to dry. The plate may be dried artificially; but will do so spontaneously in about ten minutes.

"The plates will keep ready for the camera two or three weeks in summer, and as many months in winter; but it is better to use them fresh.

"The development of the latent image is the most important operation in this process. It is slow; but, on that account, very manageable. If the plate is well exposed, and free from blisters, there is no doubt of getting a picture. There are two methods of development; with pyrogallic acid and with gallic acid. To develop with pyrogallic acid, take the exposed plate, and place it on a levelling-stand; pour a little water over the surface; then take a sufficient quantity of the following developing solution:—

" Pyrogallic acid	2 grains.
Glacial acetic acid	½ drachm.
Water	1 ounce.

And pour it over the plate repeatedly. When the sky and high lights appear, add a few drops of a ten-grain solution of nitrate of silver. This will bring out all the details of the picture; but when held up to the light, it will appear weak and transparent. In this state more silver should be added, until sufficient intensity is gained.

"During the development, it is more than probable that the surface may be marked by streaks or stains, or a deposit may cover the whole plate. If this should occur, stop the development; wash with water; and with a piece of fine cotton wool, rub away these defects, and go on again with the development; the horny surface of the albumen allowing this to be done without fear of damaging the negative. This is the great advantage the process has over every other. The plate can be developed for hours or days; because, though a deposit falls, it can be wiped over and over again. This is an immense advantage when the picture is under-exposed, as it can be frequently brought out by long development.

"To develop with gallic acid, take the exposed plate, and put it, face upwards, into a glass or other dish, with a sufficient quantity of a saturated solution of gallic acid to cover it. When it has remained five or ten minutes, add a few drops of a ten-grain silver solution, and mix well in the dish; the picture will gradually appear. When all the details are out, add more silver, till the development is complete.

"Whichever process of development be adopted, great care is required to attain just the proper amount of intensity, and no more. In all processes where albumen is used, the real printing intensity is much greater than it at first appears, owing to the light-resisting colour of the deposit. There is great danger of over-development, as some photographers are not content until the sky is quite black; at which time, in an artistic point of view, the picture is quite ruined.

"The picture is fixed with hyposulphite of soda, in the proportion of five or six ounces of the salt to the pint of water. Cyanide of potassium would act rapidly on the organic deposit, and is therefore unfit for use."

THE FOTHERGILL PROCESS.

From the same authority, we quote the following particulars of a modification of the Fothergill process; so called after its inventor.

"For coating the plates, take of old iodised collodion, containing one grain of a bromide, in addition to four of iodide, one part; and of a newly iodised collodion, containing iodide only, one part. The glasses must be carefully cleaned, and quite dry when coated: a thirty-five grain nitrate bath, with a very slightly acid reaction, is the best sensitising solution. The plates, when excited, must be washed in a limited quantity of distilled water. A solution is then required; prepared as follows:—Of albumen, from fresh eggs, one ounce; of water, three ounces; of the strongest liquid ammonia, a few drops to each ounce of the preparation; and also a few grains of citric acid. The plate is coated with this preparation, which is left on it a few minutes; it is then to be thoroughly washed in water, and dried. Plates so prepared are very sensitive, and possess good keeping properties."

We have chosen the two processes above described, because they are comparatively easy of prosecution. A variety of substances, of a vegetable kind, have been employed as preservatives; such as linseed, malt, glucose, &c.; but our limits forbid us entering into special details respecting them.

Our readers will perceive, that many precautions have to be observed to ensure successful results; and their patience and perseverance will doubtless be well tested. At the same time, a really good negative is an ample reward for all the pains we can bestow; whilst an inferior one is, at all times, valueless for printing purposes.

PHOTOGRAPHIC ENGRAVING.

The usual process of copying from negatives is necessarily somewhat slow, and therefore greatly limits the production of paper positives. A great variety of attempts has accordingly been made, the object of which has been to produce blocks, on which photographic pictures could be engraved, and printed from.

Some of the earliest experiments for this purpose were made on Daguerreotype plates. These were etched out by chemical means, or electrotyped, so as to present an uneven surface, which would afford the effects of light and shade when engraved from.

One of the best prints which was made by this process, was produced many years ago, by Mr. Grove, at the London Institution; being an outline of that building, obtained from a Daguerreotype. However, no great success attended the efforts then made.

Mr. Fox Talbot and others have, since then, made considerable advance in this department of photographic practice. Pictures have been transferred to stone and wood; and, from these, engravings have been produced. We shall confine our remarks to two processes; the first being that of photozincography, for an account of which we are indebted to the *Photographic News* Almanac; and M. Pretsch's process, which the inventor kindly explained to us himself, and the whole of which we have frequently repeated.

The photozincographic process is described as follows:—

"The most successful method of carbon printing, is that perfected by Colonel Sir Henry James, R.E., Director of the Ordnance Survey of Great Britain; which has led to the most important results, both in an artistic and economical point of view. The singular advantage which this art possesses, consists in the fact, that we can now produce authentic copies of any of the numerous rare manuscripts, which are carefully preserved in different parts of the world; and print any number of copies of them that may be required, at a cost which will not exceed one penny for a folio sheet, and this without ever touching the original document; or, if required, without even being in the same room with it, provided we have a hole in the wall, to place the lens of the camera in.

"Photozincography is the name given to this process, which is the art of producing a photographic *fac simile* of any subject; such as a manuscript, a map, a line engraving, and transferring the photograph to a zinc plate; thereby obtaining the power of multiplying copies in the same manner as is done from a drawing on a lithographic stone, or on a zinc plate.

"The process is based on that first suggested by M. Aser, of Amsterdam; viz., the obtaining a photographic picture on paper prepared with the bichromate of potass; washing off the unchanged salt; inking the image left on the paper with photographic ink; and then transferring the same to a prepared zinc plate, from which any number of impressions may be printed.

"The steps to be taken to obtain this important result, are—first, to produce a collodion negative on glass, in the usual manner, of the document to be copied; this negative may be identical in size with the original, or larger or smaller, as may be most desirable. The greatest care is required in obtaining this negative perfect, as any defects it may contain, will necessarily be reproduced in the copies. The lenses employed must be in due relation with the size of the object copied, so as to avoid distortion.

"The negative will require to be intensified, in order that whiteness of the paper may be reproduced in the positive. This intensity is readily obtained by immersing the fixed negative in a saturated solution of bichloride of mercury, washing it with water, and then putting it into a solution of commercial hydrosulphate of ammonia; one part to ten parts of water.

"In this manner the ground of the negative is rendered extremely dense, without the clearness of the details being in the least affected. When dried and varnished it is ready for use.

"With this negative we proceed to obtain positives on paper. The quality of this paper is a point of much importance; the best is that known as 'engravers' tracing-paper.'

"A solution of gum-arabic is made by dissolving three ounces of the gum in four ounces of water.

"Boiling water is saturated with bichromate of potass; and two parts of the solution is mixed with one part of the gum-water, both being kept at a temperature of 200° Fah.

"The tracing-paper is evenly coated with the hot mixture, by means of a flat camel-hair brush, and dried; it is then exposed to light, under the negative, in the usual way. The time for printing may vary from two minutes in the sunshine, to ten minutes in clear diffused light. If a good image is not obtained in that length of time, the operation had better be deferred to a more favourable opportunity. The period of exposure is determined by the appearance of the print; when all the details have become distinctly impressed, it may be removed.

"The next thing is to coat the entire surface of the print with an even and thin layer of a greasy ink, which may be composed of the following ingredients:—

" Middle linseed oil varnish . .	4½ ounces.
Wax	4 "
Tallow	½ ounce.
Venice turpentine . . .	½ "
Mastic resin	¼ "
Lampblack	3½ ounces.

"A portion of these is dissolved in turpentine sufficient to form a thin cream, which is applied to the surface of the positive print with a brush.

"The turpentine is allowed to evaporate for half-an-hour, and the positive is then floated, face upwards, on hot water for a few minutes; then removed, and laid, in the same position, on a porcelain slab.

"The surface is then gently rubbed with a sponge, dipped in warm gum-water, when the ink readily leaves the surface at those parts which have been unacted on by light; while it adheres tenaciously to the details of the image.

"As soon as the lines are quite clear, the print is placed on a flat dish, and washed, first with warm, and finally with cold water. When dry, it is ready for transferring to zinc or stone.

"There are two methods of transferring to zinc, varying according to the quantity of ink in the photograph.

"If a very small quantity has been applied on account of the closeness of the lines in the subject, the print is transferred by the anastatic process.

"If a larger quantity of ink has been applied, the process is different.

"The positive is laid for ten minutes between sheets of paper, damped as uniformly as possible with water; it is then laid face downwards on the plate, covered with two or three sheets of paper, and passed once through an ordinary lithographic press. The sheets of paper being removed, it is damped at the back with gum-water, till its adhesion to the plate is so lessened that it can easily be pulled off. After the transfer has been gummed, brought up, and etched, the ink is cleared off with turpentine, and the design is rolled up with printing-ink. Impressions can then be taken from it.

"The quantity of ink necessary to be applied to a photographic positive, to make a successful transfer to grained zinc, is greater than that which is necessary for stone; and the anastatic process requires the least of all.

"The action of the warm water in which the positive is immersed on the insoluble gum, is to cause it to swell; and the ink which overlies the lines formed of insoluble gum, expands likewise. It is evident, therefore, that if the subject photographed be of a close nature—as a fine engraving—the amount of enlargement of the inked lines may be sufficient to bring them into contact while the print is in the water; and when they have once coalesced, they will not separate again when the gum shrinks in drying, and there will consequently be a continuous shade of ink instead of lines. In such a case, the quantity of ink applied should be as small as possible; and to enable a light but even coating to be laid on it, must be thin. As a consequence of the small quantity of ink used, the transfer must be made on a smooth plate by the anastatic process; because, to make a successful transfer to a grained plate or stone, a larger quantity of ink is necessary. On the other hand, as impressions taken from a grained plate or from stone, are, as a rule, better than those from a smooth plate, and as a larger number of good impressions can be struck off if the subject photographed be so open that there does not appear to be any likelihood of the lines coalescing in the water, it is better to apply the ink in greater proportion to the print, as a certain quantity is indispensable for the employment of the latter mode of transfer. By the application of this process to the reduction of the plans of the Ordnance survey, an immense saving of time is effected; while the total money saving will amount to no less a sum than £35,000 on the cost of the survey."

Having thus given our readers the above account of the photozincographic process, we shall now proceed to describe that of M. Paul Pretcsh; which, as already observed, we have repeatedly examined in all its details.

M. PAUL PRETCSH'S PROCESS FOR ENGRAVING BY LIGHT AND ELECTRICITY.

PERHAPS one of the most ingenious and successful inventions which has yet been made in attempting to adapt photography to the purposes of the engraver, is that invented by M. Paul Pretcsh, formerly manager of the Imperial Press at Vienna. That gentleman courteously explained the entire process to us; and we shall give our readers a general outline of it.

The inventor avails himself of the fact, that animal matter, combined with bichromate of potass, is peculiarly acted on by light, and becomes thereby considerably hardened. A mixture of good glue or gelatine is made with a solution of bichromate of potass, aided by the application of heat. It is of such a consistency as to become solid when cool. Whilst in the liquid state, solutions of nitrate of silver and iodide of potassium are added to it; and after the whole has been well incorporated together, the liquid is poured on to a clean glass plate, and allowed to cool in a dark place.

Presuming that an engraving is to be copied, it is to be placed, face downwards, on one of these prepared plates, and pressed closely on the glued surface. This is best effected by the copying frame referred to, and illustrated, in a previous page.* It is then to be exposed in the usual manner, varying from a few hours in direct sunlight, to a day or two in diffused light. On removing the plate from the frame, it will present no great visible change, or, at all events, its surface will still be perfectly level, even if some effect of shadowed outlines may be perceived. It is then to be immersed in water, or, as we believe the inventor recommends, a solution of common salt in water. An immediate effect takes place: those parts which have been exposed to solar action retain their previous condition; whilst those which have been shaded by the dark part of the engraving from the action of light, immediately rise up beyond the ordinary level, and form a series of *raised* lines, exactly resembling the shaded lines in the original. The glue surface, therefore, assumes exactly the opposite condition of that of an engraved plate in which the shaded portions are *hollow* lines. The plate is then dried; and when again each part has become firm, a cast is taken from it by means of any suitable substance, such as plaster of Paris, which, of course, produces an *intaglio* impression. From this a cast is taken by means of gutta-percha, &c., which becomes the mould, from which a solid engraved copper-plate may be obtained. For this purpose the electrotype process is employed; to explain which, we may observe, that a current of voltaic electricity passing from one pole of a battery through a metallic solution, conveys at the same time portions of that pole to the other extremity or terminal of the battery. A galvanic battery consists of a sheet of zinc, and one of copper, silver, or platina. To each metal a wire is attached, and the two sheets of metal are placed in a jar, into which acid and water are poured. So long as the two metals do not touch each other, or the wires attached to them are apart, no action takes place in the battery. But on contact being made, then a current of electricity passes from the zinc, through the liquid to the copper, and thence by the wire, back to the zinc. If, however, the two wires are placed in another vessel containing a solution, say of sulphate of copper, then the electricity will pass from the wire attached to the copper-plate, through the solution, to the wire proceeding from the zinc plate, and the latter wire will receive continually the metal from the opposite one, till it is entirely dissolved away. If the gutta-percha mould is first blackleaded carefully so as to make it a conductor of electricity, and it is then attached to the wire from the zinc end of the battery, whilst a copper-plate is hung from that wire proceeding from the copper end of the battery—both being immersed in a jar containing a solution of sulphate of copper, and dilute sulphuric acid—the mould will be completely covered with the metal transferred from the plate by the electric action; and in the course of twenty-four hours' action, a complete copper copy will be produced. This is next gently removed from the face of the mould; its edges are trimmed; and it may then be employed in the printing-press, in exactly the same manner as an ordinary engraved copper-plate.

It will be perceived that, by this curious process, a complete and perfect engraved copper-plate may be produced by the sole action of light and electricity; and viewed in a philosophical point of view, it presents the most astonishing application of two forces with which we are acquainted.

For the sake of simplicity, we have only supposed the case of the copying of an engraving. Photographs, &c., can, however, be easily reproduced by the same means; and thus an exact copy of any object can be almost indefinitely produced. The inventor, however, finds, that, owing to the softness of copper deposited by the electrotype process, not more than six or eight hundred copies can be printed from one plate. Any number of plates, however, may be produced from the original mould. We have seen numerous specimens of engravings so procured; and have no hesitation in pronouncing them to be of the greatest accuracy and perfection in their detail and general effect. The ingenious inventor has applied the process to a variety of purposes; amongst which we may mention that of producing engravings from personal photographs; by means of which, linen or cotton may be marked with the likeness of the owner, intead of by the ordinary plan of marking-ink. Our readers of the fair sex will instantly appreciate this application, because it prevents that constant interchange of the ownership of property, to which the laundry-women of our large towns are so prone. Similarity of initials is common enough; but a photograph on wearing apparel at once stamps an unerring title as to the proper right of possession.

We are not aware that this invention has received that encouragement which its value deserves. Several illustrated works, however, have been published, in which the cuts have been produced by the process. Its value greatly consists in the accurate reproduction of natural scenes, &c., represented by it.

STEREOSCOPIC PHOTOGRAPHY.

THE stereoscope is so well known as not to require any particular description on our part. It gives to the eye the effect of solidity from two pictures, placed in a proper position, the images overlapping each other. We have already entered into an exposition of the principles of the laws of vision;† and our remarks on the cause of single vision by two eyes, may be taken as descriptive of the effect of the stereoscope. The particular form of the instrument varies according to the peculiar method of construction of each maker; and into such we shall not enter, but rather confine ourselves to the method of taking stereoscopic pictures, and the instrument required for that purpose.

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* See *ante*, p. 154.

† See *ante*, pp. 44 and 49.

In a single camera, the rays of light passing from an object to the ground glass or sensitive plate, do so exactly as occurs when any object is seen by one eye. If another camera be placed next to the first, the rays will not fall exactly to the centre of its plate, because they arrive at two different angles from the source. Again, if an object, at a short distance from the eye of the observer, is viewed alternately by each eye, it will not appear in the same position as it would if viewed by both eyes simultaneously. The distance between the two eyes, of course produces a separate angle of vision for each; and hence the right eye can see further to the left than can the left eye; and, *vice versâ*, the left eye can see more to the right of any object.

Now stereoscopic pictures are always taken in duplicate, each at an angle varying with the other, as the angle of vision of the two eyes differs. It follows, therefore, that either two cameras must be employed, or that, if the picture is taken by one camera, then the instrument must be successively placed in two different angles to take the picture in a proper manner. An inspection of any stereoscopic print will illustrate the remarks we have here made; for it will be found, that one either exceeds or falls short of the other in the representation of the width of its field of view.

In practice, a double camera is always employed; for the operator, having the two instruments arranged permanently at the proper angle, has no trouble in the mechanical part of the process. Besides this advantage, the person desiring a likeness has not to undergo the fatigue of a double sitting, and the chance of placing the head and body in such a position as would be required to produce a second and corresponding one to that first procured by the single arrangement.

The entire manipulation, beyond the management of the camera, is exactly the same, in all respects, as that we have described under the collodion process; two plates being employed, or rather one plate, on which two pictures are produced.

The stereoscope has done much towards popularising the art of photography; and the favour with which the simple instrument has been received by the public, has induced the invention of a great variety of arrangements, by means of which, a number of pictures may be successively viewed by one or more persons. The pictures are generally mounted on an endless band, between two stereoscopes placed opposite to each other; and thus the number of pictures is limited only by the length of the band, or the size of the whole arrangement. A handle, affixed to the roller on which the band revolves, brings the objects successively in view to either stereoscope.

MICROSCOPIC PHOTOGRAPHY.

Some very interesting results have been obtained by applying the photographic process to obtaining pictures of microscopic objects. This is easily effected by receiving, either through or without the eye-piece of an achromatic compound microscope, the image of the object on a plate by means of the usual collodion process. In fact, the lenses of the microscope answer in place of the lens of the camera: the plate, however, must be so arranged as to be entirely protected from external light. This is easily done by placing it in a small dark box at the end of the eye-piece, from which the image proceeds to the prepared plate.

Such pictures can afterwards be submitted to the microscope, and may of course be magnified to any desired extent. One great advantage arising from this use of photography is, that perishable objects can be copied exactly as they appear at any moment, and so all their peculiarities of construction or constitution may be permanently retained; and if negatives are taken, any number of copies can be subsequently produced either on glass or paper.

Perhaps the most interesting kind of photographs of this kind, are those which have been taken of the blood globules of animals, of which we have already spoken in connexion with the oxy-hydrogen microscope.* We believe that these were first exhibited by M. Duboscq, of Paris, at a meeting of the British Association, some six years ago, at Glasgow. Since then, an immense variety of microscopic photographs have been produced; and they are now a common article of sale at the opticians, &c.

PHOTOGRAPHIC REGISTRATION.

Amongst the various uses to which photography has been applied, is that of registering magnetic, astronomical, barometrical, and other observations. As each of its applications is made in a form suitable to the instrument, the indications of which have to be noted, we shall confine our remarks to the explanation of the general plan, rather than to separate details.

A piece of paper rendered sensitive to the action of light by any of the processes mentioned in the previous pages, is fixed round a cylinder. This is moved by clock-work, so as to rotate once in twelve or twenty-four hours; and the paper is divided into parts, so that each hour or part of an hour is marked on its surface. The whole is excluded from light, except at a small aperture, through which the rays of a lamp, concentrated by means of a reflector, alone can pass. Now the light so admitted will at once act on the paper opposed thereto; and as the paper is continually revolving as the day passes on, any ray falling on the surface will produce a black mark at a place on the paper, indicating the time of day.

When the paper is removed, a long and irregular line will be observed on the surface; and the position of this line will thus indicate—say the vibration of a magnetic needle, the rise or fall of the mercury in the barometer tube, &c., according to the mechanical and other arrangements to which it has been adapted.

It will be at once seen that this plan is of the greatest possible value to the philosopher. Day or night, the registration can be carried on with the most extreme accuracy: no chance of error need creep in if proper precautions are taken; and the results, printed by the hand of nature, are permanent, and not like the efforts of the human mind, which are intermittent from the necessity of rest, or uncertain from their infirmity. The plan is extensively employed in most of the public and private observatories of our own and other countries

COLOURING PHOTOGRAPHS.

We regret to say, that all attempts to obtain pictures with natural colours, by means of photography, have, as yet, been unsuccessful. The photographic artist has, therefore, endeavoured to supply this want by applying colours to photographs, and so producing a picture resembling that produced by the painter.

The practice was first commenced with Daguerreotypes, which, from their heavy metallic hue, were anything but flattering likenesses in many cases. After the collodion process was introduced, the art of colouring was still more largely practised, and has now arrived, in many instances, to a high artistic rank; although, when in the hands of the majority of those who employ it, the results are scarcely better than hideous daubs, in which the value (if any) of the photograph is entirely concealed, and its defects strongly magnified.

The colours, in a state of impalpable powder, are applied by means of a camel-hair pencil, moistened with a little water. Some highly-finished oil portraits have been produced by leading photographers; which have the advantage of being exact likenesses, together with that peculiarly pleasing expression which the taste of a thorough artist can alone give.

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* See *ante*, p. 68.

We have frequently employed photography for another purpose, to which we will refer; and our remarks may be useful to the managers of Mechanics' Institutions, &c. The ordinary mode of producing dissolving views, is that of first sketching on glass by means of Indian ink, and then of filling up the outlines with transparent colours. Instead of this plan, we have had portraits and landscapes first photographed, and afterwards coloured by the artist. The result is a pleasing and accurate picture, which, when magnified by means of the dissolving views or magic lantern, presents an exact illustration of an object on the screen. We have employed magnifying powers, varying from one to sixteen thousand times superficial, and can strongly recommend our plan to all who are desirous of having sets of views painted.

Care must be taken that the colours employed are quite transparent; that the collodion film is not too thick; and it is also essential that the lenses of the lanterns should be as free as possible from spherical aberration. The effects of this are avoided by the artist, as he paints all straight lines at a slight curvature: of course this is not allowed for in the photograph, which strictly represents objects as they actually appear.

PHOTOGRAPHY BY ARTIFICIAL LIGHT.

The comparative absence of the chemical or actinic rays in most kinds of artificial light, render them unsuitable for the purposes of photography in many cases. It is, however, possible to obtain pictures by means of the oxy-hydrogen light, the electric light, and the combustion of some substances; and as there has been but little published on the subject, we shall confine our remarks to our own researches and experiments.

We have succeeded in taking likenesses by means of the following arrangements:—

1. *The Electric Light.*—For this purpose a powerful voltaic battery of fifty cells of Grove's battery was employed, together with a modification of Duboscq's electric lamp. The cells were arranged in single series, and each platina exposed an active surface of twenty-eight square inches. The length of flame was about half an inch, and, as nearly as could be judged, about a quarter of an inch thick. No reflector was employed, and the sitter was placed at about three feet from the charcoal points. Ordinary collodionised plates, which, on an average, required from twenty to fifty seconds' exposure in the camera by daylight, were used; the manipulation, &c., being identical with the usual mode pursued in daylight photography.

An equally good photograph was produced by an exposure to the electric light for about thirty to forty seconds, so far as the intensity of the light and shade of a positive was concerned. But, strange to say, the same objection exists to such photographs as is found in the use of the electric light for illuminating purposes—namely, that of the excessive depth of light and shadows, all half tones seeming to be entirely lost.

To remedy this, we tried a variety of arrangements, but entirely without success. The face of the sitter presented a deadly or ghastly white appearance on the plate; whilst any shaded or dark part of the dress or skin, was equally black. The picture itself was therefore divided into two appearances only, and was entirely deficient of pleasing expression. In no trial with a varying strength of the light, did we succeed in tempering this excessive contrast of effect.

2. *The Oxy-hydrogen Light.*—Owing to a less amount of actinic rays in this light, the solarising effect is not so intense. Longer time of exposure is required, even to the extent of one minute; depending, of course, on the sensitive nature of the plate.

3. *The Combustion of Phosphorus in Oxygen.*—By this plan a most intense light is produced; but owing to its short continuance, it is of little or no avail. The pictures are, however, softer than those produced by the means already referred to, and have a more intimate blending of light and shade.

4. *Moule's Photogenic Arrangement.*—Unaware of Mr. Moule's invention, we tried, in 1856, some experiments with a compound called blue fire, composed of the following ingredients:—

Nitre	4 parts.
Sulphur	2 ,,
Sulphide of antimony	1 part.

And found that a portrait, nearly equal to that obtained by daylight, can be produced. Mr. Moule, we believe, adopts some chemical compound similar to that we have named; but in addition he uses a lamp, in which the substance is burned. Photographs taken by such means are very good, and well defined as regards half tones, &c. The process, however, has not been largely employed; and, so far as we know, has never been used commercially, having been confined chiefly to lecture-table purposes.

Ordinary gas, oil, and paraffine lights, have a very slight action on collodionised plates; and we shall therefore not enter into details respecting them.

The electric and oxy-hydrogen lights, but especially the former, are of great value in certain circumstances. In many of the museums of this and other countries, there are objects which, from their size, &c., are necessarily placed in positions where sufficient daylight can scarcely impinge on them for the purposes of the photographer. In such cases, the electric light has been frequently employed with great success; and photographs of many interesting objects have been obtained. As the objections which exist to portraits taken thus, are rather advantageous when the pictures of inanimate articles are required, the electric and oxy-hydrogen lights afford the most eligible means of illumination in such cases.

CELESTIAL PHOTOGRAPHY.

One of the most surprising achievements of modern science, is that of taking views of the heavenly bodies; which, in the case of the moon, are so accurate, as to permit our tracing the valleys and mountains represented on photographs of her surface. The difficulty incident to such attempts, will be better estimated when we compare the perfect stillness required of a person sitting for a likeness, with the circumstances which have to be allowed for in celestial photography. Some of these we may name, in connection with lunar photography. In the first place, the earth is rotating on its axis, at the rate of 1,000 miles per hour. At the same time, we are travelling through space at a rapid rate. The moon, again, is revolving round the earth at the rate of about 2,000 miles per hour. The difficulty, therefore, at once presents itself, as to how sufficient absence of motion, or its equivalent, can be obtained for a few seconds of time. This is managed by so arranging the telescope, &c., by means of clockwork, as that it shall travel an angular distance equal to that traversed by the heavenly body in any space of time.

Our readers will understand the principle of this, if we afford them a familiar illustration. We will suppose the case of a steam-boat in motion, towards which a telescope is being pointed by an observer on shore. The instrument, if gradually shifted, will apparently present the steamer to the eye at a state of rest; and the angular motion of the telescope will require to be less, exactly in proportion to the distance of the steam-boat.

Now the heavenly bodies are in a similar position; and as we know the exact rate of their, and our own, passage through space, it becomes a matter of pure mechanical arrangement to accommodate the angular motion of our instruments to them. This is done by means of clockwork; and the arrangements lately employed have afforded some very interesting results.

The most recent instances of this application of photography, is that effected during the solar eclipse of July 18th, 1860. To those of our readers who are unacquainted with astronomy, we may explain, that some peculiar appearances which are presented during an eclipse of the sun, have afforded matter of deep discussion on the part of astronomers. Of these, the beads, red flames, &c., which are frequently seen in an annular or total eclipse, are the most prominent. The intense excitement which naturally affects the coolest observer when phenomena of the deepest interest, and lasting but a few seconds of time, take place, is enough to unfit any one for taking accurate observations. Besides this, the power and peculiarities of vision of each person so engaged, vary to so great an extent, as to give an entirely different complexion to the observations taken. Photography here, therefore, steps in. The inanimate paper is subject to no emotion, or disturbing cause, and it affords a ready and efficient means of registering an observation, which cannot possibly be otherwise than accurate.

For the purpose of obtaining varied and positive results, some of the leading philosophers of our time betook themselves to various places over which the shadow of the moon would pass. They were abundantly provided with instruments for ordinary and photographic observation; and when possible, the duties of observing were divided amongst a number of persons, so that each might make one phenomenon his special duty to attend to. We must confine ourselves entirely to the photographic results; and are indebted for such to the *Photographic News* Almanac, from which the following extracts have been made:—

"M. Foucault studied the corona, both optically and photographically. Immediately after the disappearance of the last solar ray of direct light, a prepared plate was exposed ten seconds; then a second plate was exposed twenty seconds; and lastly, a third plate was exposed during sixty seconds: they were developed by sulphate of iron, and fixed with cyanide of potassium, with the object of obtaining direct positives. Owing to a vibration accidentally given to the apparatus, four images were impressed on the first sensitive plate, in a very short space of time; three at least in less than a quarter of a second each: these possessed a peculiar interest and value; they did not give a complete picture of the aureola, but only of a circumference of the circle surrounding the obscure disc of the moon. On the side where the interior contact took place, the circular contour presented greater intensity, which confirmed the impression entertained by M. Leverrier. Irregularities, similarly situated, on these three images were remarked, which appeared to be an exaggerated representation of the lunar contour. When the proofs were placed in the actual position of the planets, it was seen, that among the indentations, two were principal and contiguous, situated at the inferior and eastern extremity of a diameter inclined at forty-five degrees. The three other proofs gave an extension to the aureola, which increased with the duration of the exposure. The aureola diminished in proportion as it receded from the planet, and its line of demarcation became lost in the tint of the ground which represented the sky. In the proof exposed sixty seconds, the aureola extended sensibly to a distance equal to thrice the radius of the central disc. But, in certain particular directions, the aureola exhibited positive and negative variations in its intensity, producing the representation of a glory. One of them, better defined than the other, was prolonged, in all the proofs, beyond the rest of the aureola, and seemed to emanate exactly from the point occupied by the indentation on the edge of the moon, already described.

"Such are the results of an experiment undertaken solely to ascertain the photogenic activity of the aureola on wet collodion. The harmony prevailing among the six proofs, supply positive elements for the discussion of the nature of the aureola."

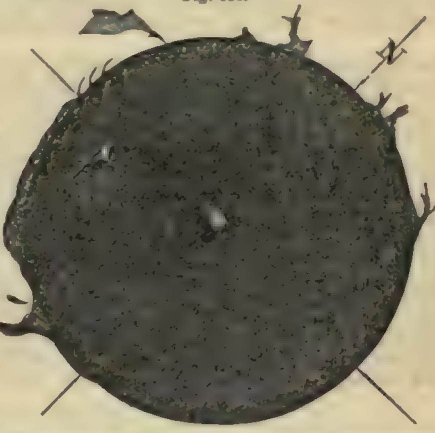
The Astronomer Royal, Dr. Airy, proceeded to Spain; and the following contains the substance of a paper read by him before the British Association, at the meeting held in Manchester, in September, 1861.

In the early part of his discourse, Dr. Airy gave a general account of the cause and phenomena of eclipses; entering into a great variety of astronomical data, which it would be foreign to our purpose to repeat. Having disposed of the general nature of eclipses, he called attention to some peculiar appearances, termed "Baily's Beads"—a number of bright spots, which have been noticed on the edge of the sun just before an eclipse became total. In 1842, red flames were noticed as protuberances from the edge of the sun; but the surprise and unprepared state of the spectators, unfitted them to make accurate observations or detailed reports. In 1850, during a solar eclipse, similar, and additional, phenomena were noticed; and due preparation having been made to observe the same in 1851, several philosophers, including Dr. Airy, proceeded to Sweden, and other places, for the purpose of making a fresh series of observations. A variety of interesting results were obtained; but still, the uncertainty of human power, and the mental excitement under which all the observers suffered, invalidated any conclusions at which they arrived, and still left much to be desired in the way of truthful result.

Accordingly, in 1860, a large party of astronomers, and others, proceeded from this country to Spain, well provided with apparatus of all kinds, and also with a complete set of instruments, by means of which the photographic art was put into full operation; and the entire proceedings were attended with the greatest success. Similar arrangements of clockwork, to which we have already alluded, were employed, and an accuracy of result was obtained which left scarce anything to be desired.

Amongst the most successful of the operations were those by Mr. De la Rue; which he conducted by means of apparatus he had fitted specially for the purpose. The annexed engraving represents a photograph of the eclipse at its totality, and the red flames issuing from the outer edge of the sun.

Fig. 160.



The photographs thus taken were subsequently compared with those effected by other operators, and their exact similarity was so great, as to prove the extreme value of photography in its application to astronomical observations. Indeed, many occurrences unperceived by the observer were thus registered, and additional light was thrown on a variety of previously obscure phenomena.

CONCLUDING REMARKS ON PHOTOGRAPHY.

In the preceding pages, we have endeavoured to present the reader with a *résumé* of most of the photographic processes in use at the present time, together with others which have been superseded by various experiments. In doing so, our desire has been to assist those who may be almost entirely ignorant of the minutiæ of scientific researches, and the difficulties which attend their pursuit. We have attempted first to introduce to the elements; and, subsequently, to expand to matters of great difficulty and importance in some branches of the art.

We must, however, remark, that the art of photography, perhaps, of all others, is continually undergoing rapid change and improvement; and if any of our readers have been so far interested in our experiments, &c., as to be induced to repeat them, we may state, that the various photographic periodicals may, at all times, be advantageously consulted; because, by them, the new processes, as they are made public, may be learned, and considerable saving of time and money effected.

Despite the elaborate treatises which have been published, the chemistry of photography is as yet but imperfectly understood. The varying character of the numerous recipes which we have given for solutions, baths, &c., proves that, to a great extent, empiricism, rather than strict scientific rule, is the order of the day. Indeed, if our readers had the patience and time to wade through the various processes which are continually being published, and would compare the directions given with such processes, for instance, as those of the electrotype, they would at once perceive the absence of exactness and precision which should always be sought after in the application of experimental philosophy. Many a discovery in the art has been made by an accidental mixture of chemicals; and frequently pure accident has been the best assistant to photographic progress. We may state, that photography stands now in a somewhat similar position to what alchymy does to chemistry; and it is almost certain, that all our successes hitherto will form but the introduction to others of a far more enlarged kind. This, however, is a matter of great importance and of encouragement to every attentive student, because an accident may just as readily lead him to new and valuable discoveries as it would the most practised hand. In mathematics a new discovery is almost impossible, so well has that branch of human knowledge been investigated; but in experimental science, we are continually stumbling over a novelty which often only requires elaboration to convert into a valuable addition to our stock of well-ascertained facts.

Addressing ourselves to our young readers, or to self-taught students, we would observe, that however often a process or experiment be repeated, something new may be detected if close and keen observation be employed. There are hundreds of class experiments that have been long well known to every scientific inquirer, which present new features on each repetition. Indeed, many of our most important discoveries have been made from occurrences which are familiar to every human being. The invention and perfection of the steam-engine, by James Watt, is a remarkable instance of the kind. Applying these remarks to photography, we may state, that no two collodionised plates ever present the same appearances; the effect of developing solutions equally varies; and so on. Now, as each of these variations has its own cause, and cannot be the result of accident—as such must take place according to some definite, although, perhaps, unknown law—we cannot but perceive that a failure always contains a germ of some future success; whilst success itself, like prosperity in life, often hides that which would become a great advantage.

We make these remarks with another object: it is that of urging on all, the duty of at once making public any fact of importance which they may meet with. This is easily done by communicating the particulars in a terse, pithy manner to any of the photographic societies or periodicals, each of which is at all times open to the reception of useful information. The discovery of a new principle or fact, does not constitute a right of property in its discoverer, however justly he may make its application a matter of profit. The laws of nature are the property of every man; but as the application of each requires special training and character of mind, it is only just that those who have such acquirements (and they are only to be obtained by great patience and perseverance) should reap a proper reward for their exercise. This spirit, we are happy to say, has been eminently observed throughout the history of photography, and is as creditable to its possessors as it has proved beneficial to society at large. By pursuing the course we are recommending, the experimenter, whilst assisting his co-workers, will put forward, by his generosity, a claim on them for reciprocal conduct, and the result will be a rapid advance in every branch of this interesting art.

Having thus investigated the laws of light, and their applications, we shall now direct attention to the phenomena presented by another of the undulatory forces—namely, that of Electricity. We shall find many analogies between it and heat and light; some of which we have already pointed out in our general introduction to physical science.* The subject on which we are about to enter, affords a vast variety of interesting and brilliant experiments, and will so far relieve the mind of the tyro by exciting simultaneously the eye and the mind. The science of light is necessarily of considerable difficulty, although of deep interest: it involves an intelligent and deep study, especially in connexion with mathematical science, of which it is a most important application. For the sake of simplifying the subject, we have avoided, as far as possible, all difficulties of the kind, and have therefore given results rather than extended details.

Electricity, on the other hand, is a purely experimental subject: many of its phenomena are familiar to every one; and we trust that our exposition of them will suffice to give the attentive reader a general knowledge and insight into its laws and applications.

* See *ante*, p. 1, *et seq.*

ADDENDA ON THE SPECTRUM ANALYSIS.

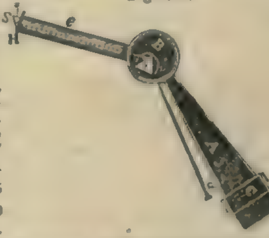
At the time our remarks on the new discoveries of Messrs. Bunsen and Kirchkoff were written,† the published information on the subject was very imperfect. We have, therefore, availed ourselves of a most interesting paper, read by Dr. Miller, at the meeting of the British Association in Manchester, in September, 1861; and are indebted for a *résumé* and engravings to the *London Review*, of Oct. 5th, in which a full report may be found.

The learned professor commenced by giving historical details of the various discoveries which had been made in connection with the subject, from the time of Newton to the present day; to which we have previously referred incidentally. Many facts bearing on the question have already been mentioned in the previous pages; we shall, therefore, confine our extracts to the practical details and results obtained by recent experimenters on the Photographic Spectra of the Electric Light, in contrast with those of the solar rays.

† See *ante*, p. 55.

ON THE PHOTOGRAPHIC SPECTRA OF THE ELECTRIC LIGHT.

Fig. 161.



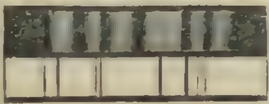
Apparatus for Photographing Spectra.

"A plan of the apparatus by which the spectra may be photographed is shown in the figure (Fig. 161). It consists of an ordinary camera-obscura, C, attached to the end of a long wooden tube, A, which opens into a long cylindrical box, B, within which is a prism of heavy flint glass, or of bisulphide of carbon, *i*. At *l* is a lens of 15 inches focal length, and at *s* is a slit parallel to the axis of the prism. The slit is adjusted so that it shall be distant from the lens 30 inches or twice its focal length, and the screen of the camera is at an equal distance from the lens. The prism can be turned round its own axis by the lever, *d*, and the tube, A, can be adjusted so as to vary the angle with the brass tube, *e*; a small reflecting prism is, when necessary, placed so as to cover half the slit, *s*, and to reflect light from a second source so as to form a second spectrum on the plate, as in Steinheil's apparatus.

"If the prism be so adjusted as to throw the solar rays reflected from a heliostat upon the screen of the camera, the wires which transmit the sparks from a Rhumkorff's coil are placed in front of the uncovered portion of the slit, and the two spectra are simultaneously impressed. The solar beam is easily intercepted at the proper time by means of a small screen, and the electric spectrum is allowed to continue its action for two, three, or six minutes, as may be necessary.

"Although with each of the metals—including platina, gold, silver, copper, iron, bismuth, cadmium, zinc, aluminum, magnesium—when the spark was taken in air, decided photographs were obtained, it appeared that in each case the impressed spectrum was very nearly the same, proving that the lines produced were not those which were characteristic of the metal, but that they were the lines due to the incandescence of the air. These bright lines, it is important to observe, do not correspond to any black lines in the solar spectrum, apparently indicating the absence of nitrogen in the solar atmosphere.

Fig. 162.



Platina and Solar Spectra.

"The peculiar lines of the metal seem to be chiefly confined to the *visible* portion of the spectrum, and these have little or no photographic power.

"*Metallic Spectra, showing Corresponding Bands.*

Fig. 163.—Gold Spectrum.

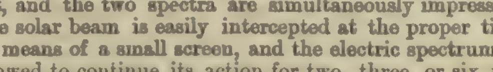


Fig. 164.—Silver Spectrum.

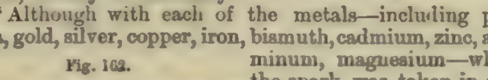


Fig. 165.—Copper and Solar Spectra.†

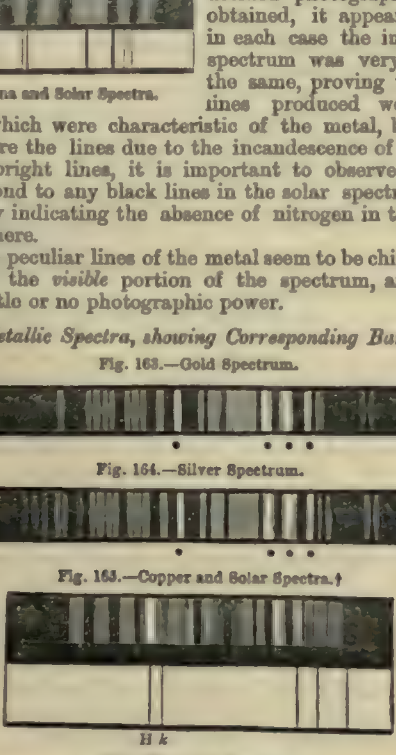


Fig. 166.—Iron Spectrum.

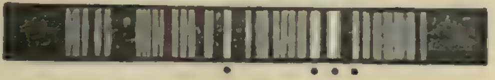
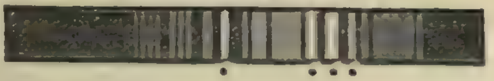
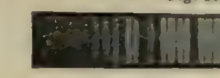


Fig. 167.—Zinc Spectrum.



"This was singularly exemplified by repeating the experiments upon the same metal in air, and in a continuous current of pure hydrogen. In these experiments two iron or platina wires were sealed into a bulb blown upon a long narrow tube which conveyed the gas under trial. Under these circumstances, for example, iron gave in hydrogen a spectrum in which a bright orange and a strong green band were visible, besides a few faint lines in the blue. Although the light produced by the action of the coil was allowed to fall for ten minutes upon a sensitive collodion surface, scarcely a trace of any action was perceived (Fig. 168); whilst in five minutes in the air, a powerful impression with numerous bands was obtained (Fig. 166).

Fig. 168.



Iron Spectrum in Hydrogen.

"Similar results were obtained with platina (Figs. 169 and 162). When carbonic acid was substituted for hydrogen, maintaining a continuous current of the pure gas during the whole experiment, the effect upon the sensitive plate was scarcely more marked.

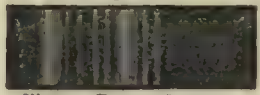
Fig. 169.

Platina Spectrum in Hydrogen.

"No photographic impression was obtained in ten minutes of the brilliant red line produced by sending the discharge of Rhumkorff's coil through one of Geissler's hydrogen vacua.

"Fig. 170 shows a copy of the photographic spectrum produced by the violet line of light obtained from one of Geissler's nitrogen vacua. The lines produced by the different metals which are characteristic of them, are best displayed by using, as Wheatstone did originally, somewhat feeble electrical discharges. When, for instance, the Leyden jar is introduced into the secondary current of the Rhumkorff's coil, as practised by Grove, the light is principally produced, as Alter and Angström have shown, by the ignition of the atmospheric air, and particularly by the nitrogen. The spectrum will appear to be filled with brilliant lines, which are nearly the same, whatever electrodes are used; but if the Leyden jar be removed, the characteristic bands of the metals may be distinctly seen. Those of iron are only well seen when the metal is red-hot, but not actually burning.

Fig. 170.



Nitrogen Vacuum Spectrum.

"Most of the metals give, as was remarked by Masson, a feeble continuous spectrum streaked by bright bands of varying intensity; and this sombre ground is not due to the ignition of the electrodes, as may be at once seen by the instant change presented in the appearance of the spectrum, if one of the electrodes does become incandescent.

"It was early remarked by Mr. Talbot, that, in the spectra of coloured flames, the nature of the acid did not influence the position of the bright lines of the spectrum, which he found was dependent upon the metal employed; and this remark has been confirmed by all subsequent observers.

"But the case is very different in the absorption bands produced by the vapours of coloured bodies. There the nature of both constituents of the compound is essentially connected with the production of absorption bands. Chlorine combined with hydrogen gives no bands by absorption in any moderate thickness.

"Chlorous acid and peroxide of chlorine both produce the same set of bands; while hypochlorous acid, although a strongly coloured vapour, and containing the same elements, oxygen and chlorine, produces no absorption bands. Again, the brownish-red vapours of perchloride of iron produce no absorption bands; but iron, when converted into vapour in a flame, gives out bands which are independent of the form in which it occurs combined.

"These anomalies appear, however, to admit of an easy explanation, if we suppose that in every case the compound is decomposed in the flame, either simply by the high temperature, just as Grove has shown that water is. In other cases of the production of bright lines by the introduction of a metallic salt into the flame of burning bodies, the reducing influence of the hydrogen, and other combustible constituents of the burning body, would decompose the salt, liberating the metal, which, after producing its characteristic lines, would immediately become oxidised, and be carried off in the ascending current. In the voltaic arc this decomposition must of necessity take place by electrolytic action.

"The compound gases, protoxide and deutoxide of nitrogen, give, when electrified, the same series of bright bands as Plücker has shown, which their constituents when uncombined furnish. Aqueous vapour always gives the bright lines due to hydrogen, and hydrochloric acid the mixed system of lines which would be produced by incandescent hydrogen and chlorine.

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† Focus rather shorter than that of other photographs.

"There is obviously a marked difference between the effect of intense ignition upon most of the metallic and non-metallic bodies.

"The observations of Plücker upon the spectra of iodine, bromine, and chlorine, show that they give, when ignited, a very different series of bands to those which they furnish by absorption, as Dr. Gladstone has already pointed out; but it is interesting to remark, that in the case of hydrogen, which, chemically, is so like a metal, we have a comparatively simple spectrum, in which the three principal bright lines correspond to Fraunhofer's dark lines, C, F, and G (Plücker). It is, however, to be specially noted, that hydrogen occasions no perceptible absorption bands at ordinary temperatures in such thicknesses as we can command in our experiments; and the vapour of boiling mercury is also destitute of any absorptive action, although mercury, when ignited by the electric spark, gives a characteristic and brilliant series.

"The following experiment suggested itself to the author as a direct test of Kirchkoff's theory:—Two gas-burners, into which were introduced chloride of sodium on the wick of a spirit-lamp, were placed so as to illuminate equally the opposite sides of a sheet of paper partially greased; the rays of the electric light, screened from the photometric surface, suitably protected, were made to traverse one of the flames. If the yellow rays of the light were absorbed by the sodium flame, the light emitted laterally by the flame should be sensibly increased. The experiment, however, failed to indicate any such increase in the brilliancy of the flame, possibly because the eye is not sufficiently sensitive to detect the slight difference which was to be expected."

From a perusal of the interesting facts contained in the preceding remarks and experiments, our readers will perceive that every solid body, when brought into a state of vapour and incandescence, produces definite and decided effects in reference to the bands of the spectrum. Each body, in fact, has a spectrum of its own; the perfection and constant identity of which is preserved, provided no other interfering cause be present. Great precautions, however, are required in performing the experiments, especially by those persons who may not be accustomed to delicate investigations. Even solid but invisible bodies floating in the atmosphere may invalidate the result; and, under certain circumstances, the minute portion of common salt often present in the air, has produced a spectrum, of the sodium kind, much to the perplexity of the observer. It must be borne in mind that the new method of analysis is so excessively refined and exact, that every possible precaution may fail in affording results which might be expected; and substances in proportions so small as to defy all other means for their detection, may intrude themselves, and so modify the spectral appearances.

The spectrum analysis has been extensively investigated and employed by many eminent observers; and its indications have been used in numerous branches of research in various departments of optical science. All luminous bodies may be examined by it; and if the results are compared with certain standards previously obtained, inferences may be drawn with respect to the colour of light and its cause, which would be utterly unattainable by any other means. For instance, much dispute respecting the nature of the solar rays and atmosphere has been prevalent, and numerous theories have been advanced to account for the production of both the heat and light of the sun. By means of the spectrum analysis, however, Professor Airy, the Astronomer Royal, does not hesitate to affirm that the surface of the sun is in a state of strong incandescence; that immense bodies of vapour are constantly arising therefrom; and even the spectrum produced by iron in the state of vapour has been distinctly noticed in the solar light.

In a similar manner, the light of the planets, fixed stars, comets, &c, is readily examined, and compared with that of the solar rays. A new means of investigation is thus put into the hands of the philosopher. The astronomer, the mathematician, and the chemist, are thus jointly called in to determine some of the most interesting problems of astronomy. The theories and facts of each branch of science are thus correlated; the deductions of each are checked or modified by each other; and both the chances and sources of error are diminished in a wonderful and satisfactory manner.

To those of our readers who may be desirous of repeating the experiments, we may mention, that complete apparatus for the purpose is now made by the leading philosophical instrument-makers in London, in which all the requisite minutiæ of construction, &c., are fully attended to. This method of analysis is, therefore, within reach of any one possessing a tolerable acquaintance with optical and chemical science; and will doubtless prove a source of extensive discovery in physical science generally.

Like all new inventions, its use may at first lead to crude and indigested theories. In the hands of many observers, however, that evil will meet with its own correction; and as soon as a sufficient number of facts has been collected, we shall be able to make safe deductions. It is more than likely that the spectrum analysis will prove, in a physical point of view, as beneficial and useful an adjunct as did the invention of fluxions, or the differential calculus in the mathematical department of astronomy and allied sciences.

SECTION III.

ELECTRICITY, AND ITS APPLICATIONS.

CHAPTER I.

FRICTIONAL ELECTRICITY.

Having already considered separately the forces of Heat and Light, we shall now undertake the investigation of phenomena connected with that of Electricity.

As a branch of experimental science, electricity is of comparatively modern date. It is true that the ancients were acquainted with what we should now term elementary facts. Thus, Thales was aware that certain resinous substances, especially amber, would, on being sharply rubbed, attract light bodies to its surface. It was not, however, till within the last century, that such, and similar results, were philosophically investigated, and the foundation of electrical science was properly laid.

We shall not expend any space in a separate historical sketch of our subject; such will naturally fall into our pages as we describe various apparatus which have been invented with the object of illustrating the electrical phenomena. The unpleasant office will thus be avoided, of deciding the relative claims which have been advanced on the part of individuals, as to their just rights to the merits of discoveries they have made; which are at all times liable to be unsatisfactorily dealt with, and to call up the criticisms of a host of partisans.

The term "electricity" has been derived from the Greek word "electron," which signifies amber; and as such, the science and its name are associated with the early experiments of Thales, the Milesian philosopher.

We have endeavoured to familiarise the minds of our readers with the doctrines of latent forces in our two previous sections.* Electricity, like light and heat, is obedient to the same law; and, so far as our experience goes, there is no body with which we are acquainted, which cannot afford us, if proper means are employed, some evidence of the existence of electricity.

There are, however, chiefly two sources which we can employ for the purpose of disturbing the electrical equilibrium; or, in other words, of educing electric forces: these are motion, or mechanical action, which is chiefly employed under the form of friction; and chemical action, when it takes place in the union or division of two or more bodies. There is, however, another source—namely, that of magnetism, which, by a peculiar arrangement, affords us what has been frequently termed the electric fluid.

It is proper that we should here explain some terms which are continually employed by writers on electricity. These are, "current," "fluid," &c.; and which had their origin in the speculations of early experimenters. Even in the *parlance* of daily life these terms are often employed; and thus it is frequently stated, "that the electric fluid" struck a building.

These terms, and expressions involving them, have great convenience in conveying the idea of a passing or moving force. Hence, strictly philosophical definitions often contain such remarks as, that "the electric current" passes in a certain direction. Now, our modern views classify electricity with the undulatory forces; and we equally believe in the idea of electric, as in that of calorific and luminous undulations. If, therefore, in the following pages, we employ the terms to which we have referred, our readers must understand that we do so with the limitations we have mentioned, and that we simply consult the convenience of expression rather than peculiar exactness of speech.

Having thus guarded our future progress against any charge of looseness of expression, we shall now proceed to examine generally the various means by which the electric force is excited, the machines which are employed to illustrate its effects, and the laws which have been found as governing the occurrence of electrical effects. In doing this, we shall afford a vast variety of experiments, each of which may be readily tried by the student; and shall also put him in possession of means by which he may pursue his investigations at a comparatively moderate cost. We must defer the consideration of electricity, as produced by magnetic action, to a separate section, and confine our attention in this to the subjects of—

1. Frictional Electricity.
2. Voltaic Electricity.

Frictional Electricity.—When the particles of any body are disturbed from their quiet, and put in motion, there always results the production of heat, light, or electricity. They seem associated with matter in all its varied forms; and thus we find them in every state or condition of all bodies surrounding us.

The most ready means of evolving these forces is that of friction, of which daily life affords us numerous instances. The simple experiment of striking a lucifer-match will produce heat and light; and by means of a piece of apparatus, which we shall describe as we proceed, the evolution of electricity is also detected.

Former writers on electricity divided bodies into two classes—namely, electrics, and non-electrics. In the former, glass, resins, fur, silk, &c., were included; whilst the metals, because they did not seem to evolve electricity by friction, were termed non-electrics. At the present time, however, metals are the chief sources whence we obtain the force, the rationale of which we shall explain under the head of Voltaic Electricity.

The following experiments will illustrate the production of electricity by friction, and also exhibit some of the phenomena which attend its presence in a free state.

Experiment 1.—Dry a glass tube about eighteen inches long and an inch in diameter, by means of a cloth, warming it occasionally before a fire. Having also dried an old silk handkerchief, rub it briskly on the tube, and then hold the latter near to some feathers or small pieces of paper. These will be attracted to the tube, on which they will adhere for some time; they will, however, fly from it, returning to the table, again to fly to the tube. This motion they will continue until the electrical state of the tube is restored to the same condition as that of surrounding bodies, when the effect will cease.

Experiment 2.—Repeat the same experiment, but employ a roll of sealing-wax or of sulphur, a piece of amber, and a sheet of gutta-percha. The same effects will be produced.

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* See *ante*, pp. 13 and 41.

Experiment 3.—Dry a sheet of paper before a fire until all moisture is removed from it, and then place it flat on a wooden table. Rub it briskly by means of a piece of India-rubber, and then suddenly lift it from the table surface. It will adhere thereto; but on being removed, and held vertically by the hand, it will approach to the fingers, if they are held near to it; and if the experiment be tried in a dark room, a small spark will be seen to pass when the fingers and the paper are nearly in contact. This experiment shows both an instance of *electrical attraction* and the production of the *electric spark*.

Experiment 4.—The fur of animals is also capable of affording considerable electrical effects. A piece of prepared skin may be rubbed, on the fur side, by means of a dry silk handkerchief. But the best plan is to rub the fur of a black cat, from the head towards the tail, either by means of the dry hand, or by a silk handkerchief. It will be found that each hair separates from its neighbour—giving an illustration of what is called *electrical repulsion*. And if the experiment be tried in a dark place, abundant sparks may be obtained by presenting the fingers to the fur; a sharp crackling sound being noticed at the same time.

The above will afford a sufficient variety of illustrations, showing how friction disturbs the natural electrical state of bodies; and an elementary idea will also be gained of what is meant by the terms "attraction" and "repulsion," in connexion with electricity; on which we shall treat more fully hereafter, as our subject expands to our view.

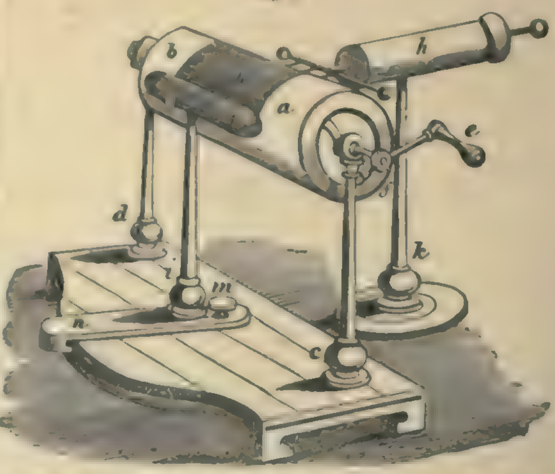
ELECTRICAL MACHINES.

We now proceed to describe a variety of arrangements, by means of which we obtain a considerable amount of free electricity. We consider this as the best course, because the student must be provided with some instrument of the kind before he can perform any experiments elucidating the law of the science. We shall describe those in ordinary use, and afterwards suggest simpler and cheaper arrangements, which any person, with an ordinary mechanical turn of mind, may, at a slight cost, construct for his own use.

Glass is universally employed in the usual form of electrical machines; and these are of two classes—the cylindrical and the plate. Each kind of machine has its advocate; but for all practical purposes, we recommend the use of the plate machine.

The cylindrical machine consists of a glass cylinder, mounted on two upright supports. On one side a rubber, which, by its friction, produces electricity, is supported on a glass rod, so as to press firmly on the surface of the glass. On the opposite side of the cylinder, a ball or cylinder of brass is placed; its surface, next to the cylinder, being coated with metal points. This is called the conductor, and, like the rubber, is also mounted on glass legs. We may here, without entering at present into the laws of conduction, observe, that glass, resin, gutta-percha, &c., convey electricity very slowly over their surface; and hence have been termed non-conductors. Now, as it is requisite that the force produced by the machine should be guided in one particular direction, the "conductor" itself is formed of a good conducting substance, which is always a metal; whilst it and the rubber are insulated electrically by means of the glass rods on which they stand. At the end of the conductor a brass knob is placed, from which the spark is received. Points being useful in collecting electricity, are, therefore, as we have stated, exposed to the excited glass surface; whilst the brass knob, which has a kind of conserving power, forms the terminal of the conductor at that point furthest from the machine. The cylinder is turned, by means of a handle, in the direction of from the rubber towards the conductor. The following engraving illustrates the various parts of a cylindrical machine.

Fig. 1.

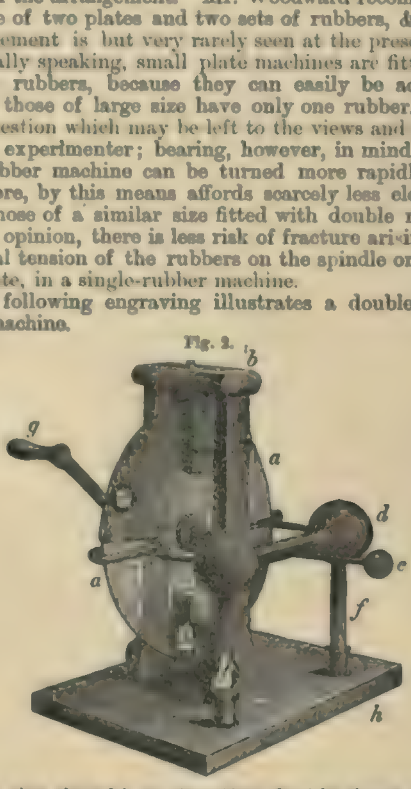


In the above diagram, *a b* represent the glass cylinder; *c d*, the legs by which it is supported; *e f*, the winch handle, by which the cylinder is turned; *g*, the rubber; *h*, the conductor; *i*, the glass support to the rubber; *k*, the glass support to the conductor; *m n*, a movable foot, by which the distance between the rubber and the cylinder can be adjusted.—These arrangements vary slightly, according to the views of the maker; but the different parts are common to every form of the machine.

As each kind of glass machine is similarly prepared for experimental purposes, we shall defer entering into such particulars till after the description of the plate machine. These have been made in various forms. Occasionally two rubbers are used, which are placed at the top and bottom of the plate. In another form, the rubber is placed on one side, and the conductor on the opposite side, of the arrangement. Mr. Woodward recommended the use of two plates and two sets of rubbers, &c.: this arrangement is but very rarely seen at the present day. Generally speaking, small plate machines are fitted with double rubbers, because they can easily be adjusted; whilst those of large size have only one rubber. This is a question which may be left to the views and decision of the experimenter; bearing, however, in mind, that a one-rubber machine can be turned more rapidly, and, therefore, by this means affords scarcely less electricity than those of a similar size fitted with double rubbers. In our opinion, there is less risk of fracture arising from unequal tension of the rubbers on the spindle or axis of the plate, in a single-rubber machine.

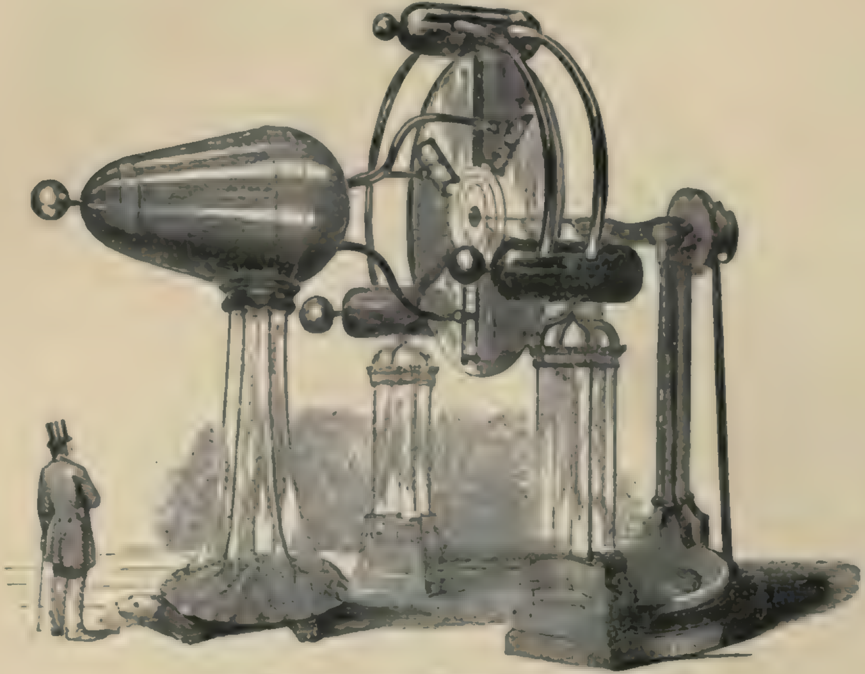
The following engraving illustrates a double-rubber plate machine.

Fig. 2.

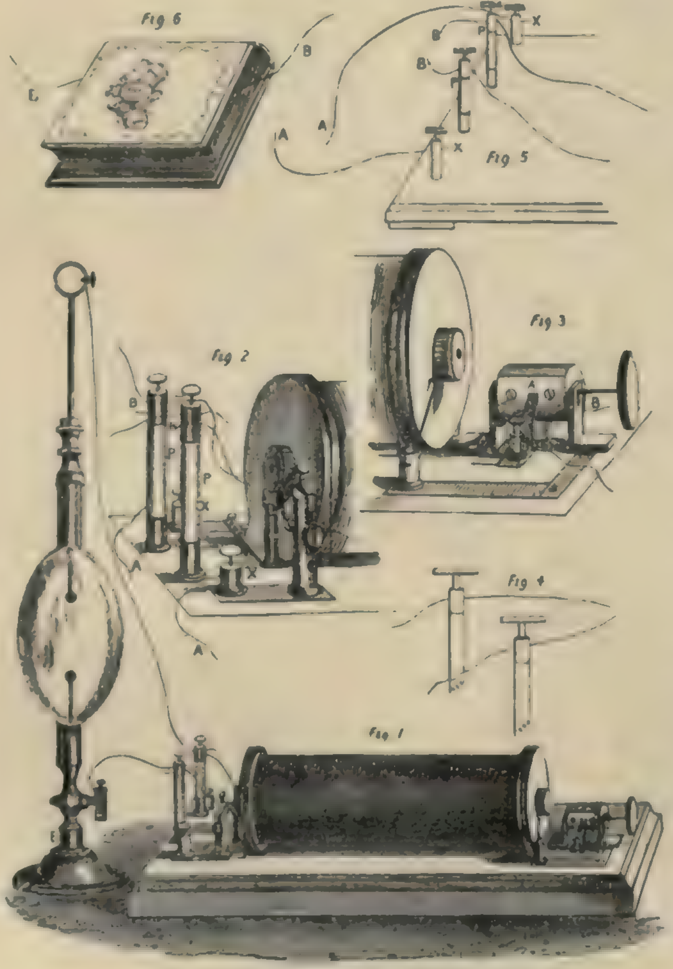


a a is the glass plate; *b b*, are the rubbers, fitted by clamps to an upright frame; *c* is a flap of black silk, which covers the glass plate; *d* is the conductor, which extends, by means of two arms covered with points, to the two sides of the plate: they terminate at the ends of the silk flaps, and so collect the electricity from the excited glass surface; *e* is the brass knob of the conductor; *f f*, are its two glass insulating supports; *g* is the handle, by means of which the plate is turned between the rubbers; *h* is the wooden table, on which the different parts of the machine are fixed.

ELECTRICITY AND ELECTRO-MAGNETISM.—PLATE I.



Ten-feet Glass Plate Electrical Machine. (See pages 171 and 242).



The Rhumkorf-coil Apparatus. (See pages 255—257).

An extended description of the Rhumkorf coil has been given at pages 255—257; but as its arrangement and management require great care, we have added, in the annexed cuts, an illustration of various minutiæ of detail, which will require the attention of the experimenter. It will be seen, that in Fig. 1, an ordinary exhausted receiver, such as is employed in Frictional Electricity, is represented as attached to the secondary wire of the coil; and the phenomena thus obtained, are extremely beautiful, if a good vacuum be produced by means of the air-pump.

Fig. 1. Rhumkorf's coil—E, exhausted receiver. (See Experiment 16, page 256).

Fig. 2. B, B, the poles of the fine coil; P, P, the glass pillars supporting them; A, A, wires from the battery, connecting it, by X, X, with the primary coil. (See also Fig. 5).

Fig. 3. A, the contact-breaker; B, its handle.

Fig. 4. The terminal wires of the secondary coil, arranged for the spark. (See Experiments 12—14, page 255).

Fig. 5. Outline view of Fig. 2.

Fig. 6. Book arranged for piercing the leaves, by spark from the secondary coil (See Experiment 17, page 256).

In a modification of the plate machine invented by Mr. Winter, an arm, extending to a height of three feet above the conductor, is placed, which, by its inductive effect, has the power of increasing the length and intensity of the spark to an astonishing degree.

Plate machines are made of various sizes. Indeed, we have experimented with them from those of one foot to one of seven feet diameter, of glass surface. The latter instrument may be seen at the Royal Polytechnic, Regent-street, London; and from it we have obtained sparks from six inches to two feet in length, depending on the dryness of the atmosphere and other circumstances, to which we shall have to allude. One of still larger size was daily used at the Panopticon, Leicester-square, some years ago. It had a plate ten feet in diameter, and was furnished with double rubbers and conductors. It is now in private hands, and is the finest specimen of a plate electrical machine yet produced.

We may here give a caution to those intending to purchase an instrument of this kind. The quality of glass used very materially affects the results which can be obtained. On one occasion, a twelve-inch plate which we possessed was cracked by being over-heated. On fitting the machine with a new plate, we found that we could not obtain anything like so good a spark as from that previously used. This was owing to a peculiar "greasiness" on the new glass. Some plates will attract moisture more than others; which, of all things, is most to be avoided in electrical experiments. The only general rule we can give is, that the more highly polished the glass, the fitter it is for the purposes of the electrician. Plates having specks, streaks, or spots on their surfaces, should be rejected; and the rim of the glass should have as complete a polish as the flat surface. It is, also, of the utmost importance, that the spindle which holds the plate, and on which it turns, should be fixed exactly in the centre. If this be not attended to, there will be an unequal pressure on the glass as it revolves, and risk of fracture will be inevitable.

Attention should also be paid to the construction of the clamp holding the rubbers, and to the mode in which the conductor is fixed on its supports, especially if placed on one glass rod only, as is frequently the case.

Before proceeding to the description of other forms of electrical machines, we will give directions for the management of those we have illustrated and explained; and we must impress on the mind of our readers, the necessity of a strict adherence to the precautions, &c., we shall mention, if they would obtain successful results in their electrical studies.

Some old silk handkerchiefs should be provided, which are much to be preferred to any other material, because silk does not afford fine fibres, as do cotton and linen. A few such fibres, if they get on the glass surface, adhere to it with great pertinacity, and have an astonishing effect in dispersing electricity.

The cylinder or plate machines are prepared for use as follows:—

1. Wipe the whole machine, glass, legs, handle, and foot, by means of a warm, dry handkerchief, so as to remove all dust or dirt which may have accumulated on its surface

2. Wash the cylinder or plate by means of a rag moistened with spirits of wine—having first loosened or removed the rubbers. This will cleanse away all hard matter which may have remained from previous experiments. The cylinder or plate is then to be rubbed with a dry handkerchief, until it has a high polish, and is quite free from every extraneous substance.

3. The rubbers should be carefully cleaned, if they have been previously in use, especially if any hard amalgam has been left on their surface. The more evenly they press on the glass, the greater will be the amount of electricity which will be produced.

After thus cleansing them, they may be slightly smeared with good tallow, and the amalgam is then to be spread evenly over the surface by means of a palette-knife. The amalgam is made by pounding together in a mortar—one ounce of zinc shavings, with two ounces of clean mercury. The two metals will soon combine; and the amalgam should be kept in a box or bottle, out of the air, because it would soon otherwise become oxidised and spoilt.

4. The rubbers having been thus amalgamated, are to be replaced and screwed up, so as to press evenly and firmly on the surface of the cylinder or plate; the silk flap being thrown back, lest it should become soiled. The handle of the machine is now to be turned gently, and the grease, &c., must be wiped off the machine until the rubbers no longer soil the surface. The silk flaps may now be placed over the glass, and the machine is ready for use.

5. As, however, all glass and metal surfaces receive a coating of moisture on being used in a room of higher temperature than their own, it is advisable to warm the whole machine before a fire, until its temperature exceeds that of the surrounding atmosphere. By this, all chance of the deposition of moisture is prevented. A silk handkerchief should be thrown over the machine whilst it is being warmed, so as to keep off any dust which might lodge on it.

6. As dryness and cleanliness are absolutely essential for success, the legs, and all other parts of the machine, should be continually wiped with warm, dry handkerchiefs, during the interval between the experiments.

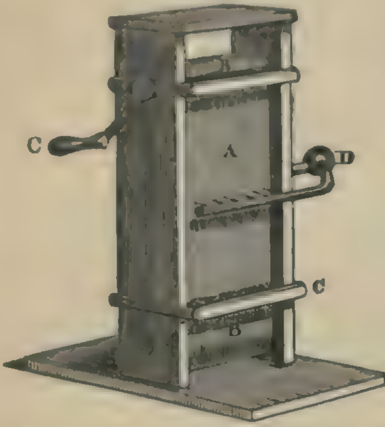
7. The rubbers of either machine should be connected, by means of a copper wire, with a gas or water-pipe; or, in the absence of these, the end of the wire should be allowed to rest on the floor, so as to keep up a continual production of electricity. In many works, brass chains are recommended; we, however, strongly urge the use of wire, which, being continuous, produces a far better effect than any chain.

A close observance of the above cautions, &c., will generally ensure successful experiments; but much depends on the state of the weather; for we have seen the spark of powerful machines, in the best order, reduced to one-fourth its usual length when a change has occurred from an east to a south-west wind. A simple indication of the state of the atmosphere, as suitable or otherwise for electrical experiments, is to be found by the fur on the back of the cat. If this produces a decided repulsion between the hairs on being rubbed, the electrician may expect success in his attempts with the machine; but sometimes the air is so loaded with moisture as to render it next to impossible to exhibit any effective results. We have seen more than one eminent lecturer risk his reputation with his audience by attempting electrical experiments in a crowded room. Of course, from ignorance of the cause of failure, the *vox populi* votes that the experimenter has neglected taking proper precautions. Whilst such may be partly true, our readers will soon become convinced, by experience, that every allowance should be made, under such circumstances, for "the pursuit of science under difficulties."

We may now proceed to describe machines constructed of resinous materials, which may easily be put together by any of our readers, and in which gutta-percha forms a chief material. Mr. Simmonds, of the west of England, has exhibited a machine composed of gutta-percha and resin, in the proportion of two of the former to one of the latter substances. In this, the plate (for it was a complete copy of the plate machine), the rubbers, handle, foot supports, and conductor, were all made of the same material; the points of the conductor were pins; and it was covered with tinfoil, to make it a good conducting surface. The rubbers were covered with rabbit-skin, and required no amalgam. A machine of this kind, which he favoured us with some six years ago, could be worked in any weather, provided it was first carefully wiped from dust. It seemed to have a very slight attraction for moisture; and, although the plate was no more than twelve inches in diameter, we have given a shock to twenty persons by means of a Leyden jar, under circumstances in which no glass plate machine could be made to work. If the plate machine, illustrated in Fig. 2, be taken as a pattern, and the material of which we gave the recipe, be used, any person may, at a trifling expense, put themselves in possession of one of these ingenious electrical contrivances.

Another form of gutta-percha machine was invented some years ago, and was, we believe, exhibited at the Exhibition in Hyde-park, in 1851. It consists of an endless band of gutta-percha, running over two rollers. The rubbers are formed of hard hair-brushes; and its general construction is illustrated in the following figure.

Fig. 3.



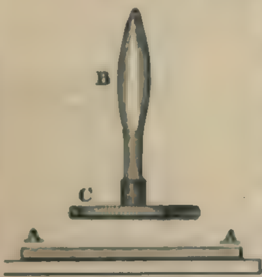
A represents the band of gutta-percha; B B are the two rubbers; C is the handle which turns the upper roller; the under one moving by the friction of the gutta-percha against its surface; D is the conductor, which extends in two arms, ended with points facing the gutta-percha band.

As we have never tried this form of machine, we can give no particulars with respect to its value. It, however, has the advantages of ready construction, slight cost, and, of course, does not require amalgamation or extended preparation before use, beyond that of cleaning all the surfaces from dust and dirt.

Perhaps the simplest form of electrical machine is that of the electrophorus, which, at a most trifling expenditure of trouble and expense, presents an opportunity for any one to try a vast number of electrical experiments. It may be constructed as follows:—

Into a tray of sheet-tin or zinc, half an inch deep, and twelve inches in diameter, pour melted yellow resin till the liquid reaches the top. This forms the sole of the electrophorus. A circular piece of zinc plate, nine inches in diameter, should have fixed to its centre a gutta-percha handle nine inches long. The plate should be quite level, and its edges smooth. The arrangement is represented in the annexed figure.

Fig. 4.



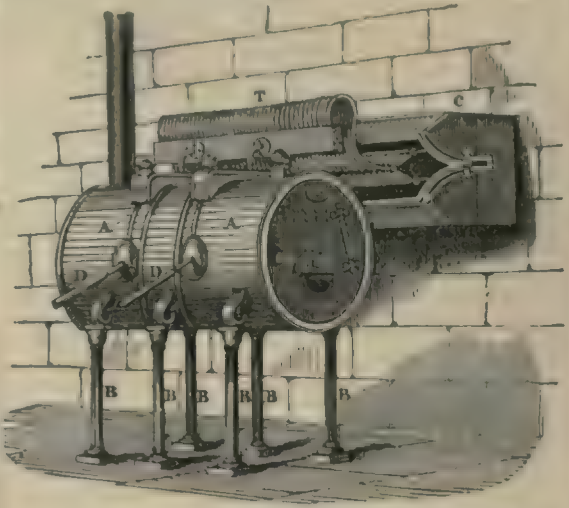
A represents the resin sole; B the gutta-percha handle; C the zinc plate.

The mode of exciting the electrophorus is very simple. The resin is, of course, allowed to cool, and it may then be rubbed briskly, by means of warm, dry flannel. The metal plate, held by the handle, is then placed on the resin plate. The *top* of the zinc is then to be touched by the finger, and it is then to be raised a few inches. On presenting the knuckle, a spark will pass to it. This process may be repeated for a long time, as the plate only requires occasional rubbing by means of the flannel. A continual supply of electricity may thus be obtained; the only inconvenience which is experienced, is that of continually lifting the zinc plate to obtain the spark.

We recommend our readers, especially the younger portion of them, to make one of these useful little instruments, as, in the absence of those of larger size, a great number of the experiments we shall mention in our future pages may be tried.

Another, and singular source of electricity, is that from the friction of pure water; which is effected by means of steam as it condenses at its exit from the boiler. Machines constructed for this purpose are called hydro-electric machines. The largest of the kind is that at the Polytechnic Institution, Regent-street; and as we there had it in daily use for several months together, we shall be somewhat particular in describing its construction, and the effects which may be obtained from it. The following engraving illustrates the external appearance of this instrument.

Fig. 5.



A A represents the boiler, which is about eight feet long, including the smoke-box. B B are six glass legs, which support and insulate the boiler. C is a box fitted with iron spikes or points. This box can be removed nearer or further from the boiler, and, by its proximity thereto, influences the length and intensity of the spark. D D are the handles, by means of which the steam is permitted to pass into, or to be shut from, a series of bent tubes, which are curved into the form of the letter S, and are represented at E E. These tubes are terminated by circular knobs, which enclose jets of partridge-wood, shaped of a cylindrical form, and having a jet somewhat resembling a bat's-wing gas-burner. As the steam passes from the boiler through the series of bent tubes to which we have alluded, it is partially condensed, and arrives at the jets in the state of minute particles of water. The pressure of the steam drives them with great force against the side of the jet, and thus the sides of the jet take the place of the rubber in this machine, and the pure water that of the glass. The action of the hydro-electric is therefore analogous to that of the glass machine in most respects. It has been stated, that the water used in the boiler should be almost, if not entirely, free from mineral substance. We may, however, state, that having used this machine for illustrating lectures on electricity for a long time, in daily succession, it was always supplied with water abounding with lime and oxide of iron. Care, however, was always taken to prevent "priming;" and the neglect of that may have given rise to the opinion in respect to the necessity of pure water, to which we have referred.

The effects produced by this machine vary with the pressure, all other circumstances being equal. We have employed pressure varying from thirty pounds on the square inch to that of 120; and up to ninety pounds the spark was in direct proportion to the pressure. Thus, if a spark of six inches was afforded by a pressure of thirty pounds, one of eighteen inches in length was produced by a pressure of ninety pounds; but beyond that pressure the results were uncertain; and, indeed, no further benefit could be observed in that respect.

It must, however, be borne in mind, that the steam was necessarily shut off and thrown on suddenly; and, of course, such tends to increase the evils arising from "priming," in direct proportion to the pressure employed. Full allowance must therefore be made for errors which may thus arise in computing the effect of an increase of pressure. By careful watching, we arrived at the opinion—whether correct or not—that an excess of pressure beyond ninety pounds on the square inch, did not produce any advantageous results.

We have previously remarked, that the length and intensity of spark were modified by shifting the box of metal points nearer to, or further from, the steam issuing from the jets. The effect of these points is simply that of induction—a term which we shall have to explain more fully hereafter. The closer the box of points is to the jet of steam, the longer and thinner is the spark taken from the boiler. When the box is removed to its greatest distance from the jets, then the spark assumes its maximum of density and minimum of length; and in this state is most fitted to charge the Leyden jar, and to produce other effects of what we shall venture to call "condensed" electricity.

A great loss is, of course, sustained by the ascent of hot air from the furnace, which forms the cellular tube of the boiler. This is effected by means of a chimney, which is divided into two parts; one of which is attached to the boiler, and the other is so arranged that it can slide, telescope fashion, into that rising from the boiler. The fuel which is ordinarily used is coke, which has the advantage of producing no smoke, and an intense heat; and the furnace is situated in the centre of the boiler, in an internal flue. Safety-valves and a water-gauge are attached.

The electrical effects produced by this powerful arrangement are of the most astonishing kind. The length of the spark is sometimes nearly two feet, depending on the state of the weather; and during an east wind and intense frost, when the atmosphere is almost destitute of moisture, the maximum effects are produced.

The spark may be obtained from any portion of the boiler, and is received by means of a large brass ball attached to a copper rod. This is connected, by means of wires, with the earth or drain beneath the boiler. A person taking a spark from the boiler, by means of the rod, &c., receives no shock, unless his shoes contain iron nails, when the effect is painful. Few persons would repeat a trial of receiving a spark of two inches in length; and a strong man, who accidentally received one when the steam was issuing at a pressure of ninety pounds, was immediately prostrated, and for some time remained insensible.

In damp weather, it is essential that the glass legs, or supports, be continually wiped between the intervals of the experiments, with dry cloths. This precaution we have already alluded to as necessary with all kinds of electrical apparatus.

Hydro-electric machines, of all sizes less than that we have described, have been constructed. They are, however, dangerous instruments in the hands of incautious persons. The brilliant effects they produce have the tendency of inducing a prolongation of the experiments, until the water in the boiler may be reduced below its proper level; and, in such cases, an explosion is imminent.

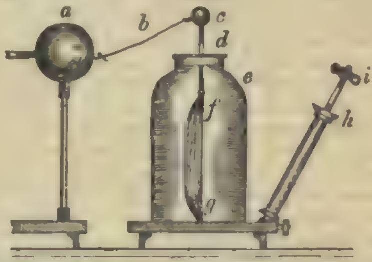
Having described a variety of forms of electrical machines, we shall proceed to investigate the numerous phenomena which may be illustrated by them. As the whole of the experiments we shall introduce are such as we have repeatedly tried ourselves, the reader will have no difficulty in producing the results to which reference will be made, if he follow the directions we shall give him.

THE ELECTRIC SPARK AND BRUSH.

We shall now enter into some descriptions of the different appearances which the electric light presents in the form of a spark or brush, as produced by electrical machines of any kind that we have already referred to. When a metallic wire is placed in contact with the conductor of the machine, all the electrical effects pass off silently, because the wire has sufficient power to convey them away—being what is called a good conductor. If, however, an interval of air be allowed between the wire and the conductor, the charge produced by the working of the machine will not expend itself noiselessly; but owing to the resistance which the air affords to the passage of the electricity, a brilliant spark is produced, the size of which depends on the power of the machine from which it is obtained: a loud snapping sound is also afforded, which increases in intensity according to the quantity of electricity set free. From a small machine, having a plate of a diameter of twelve inches, a spark of one inch in length may be obtained; from one of three feet in diameter, a spark of six inches is produced; and the large plate machine of seven feet diameter, and the hydro-electric, already described, afford sparks varying from fifteen to thirty inches in length, depending on the dryness of the surrounding air.

When a large quantity of electricity is in motion, the spark has a deep-blue colour, and a jagged shape, exactly like a flash of lightning; which it, indeed, really is in miniature. The colour varies according to the medium in which the spark is produced, and also from the substances from which it passes. Some beautiful experiments may be tried by means of an air-pump; and a convenient arrangement is illustrated in the following engraving.

Fig. 6.



a represents the conductor of the machine; *b* is a wire connecting the conductor with the rod *c*; *c*, a rod which fits air-tight in a brass collar, *d*; *d* forms the upper covering of the glass receiver, *e*; *f* is the other extremity of the brass rod, between which and *g* the discharge takes place; *h* is the cylinder of an air-pump, of which *i* is the handle.

With this apparatus the following experiments may be tried:—

Experiment 5.—Having attached the conductor *a* to *c*, draw the rod *c f* so as to leave a space of six or eight inches between its end *f*, and *g*, and exhaust the air from the receiver so as to produce a good vacuum; connecting the air-pump, by means of a wire, with a gas or water-pipe; then commence to work the machine. A spark will pass between *f* and *g*, of a colour which varies according to extent of vacuum in the receiver, *e*. If that be good, a fine purple tint is perceptible: if the vacuum be imperfect, the spark, instead of presenting a continuous discharge, will be intermittent, and of a red colour, attended with sudden star-like flashes and a snapping noise. When the vacuum is good, and the machine of large size, this forms one of the most beautiful experiments in physical science. The spark from the hydro-electric machine, when passed through a tube five feet long and three inches in diameter, produced a magnificent appearance of purple flame, which completely filled the tube so long as the vacuum was maintained. On diminishing this by a gradual admission of air, the continuity of the spark ceased, and intermittent discharges, of a blue colour, were produced. The engraving (Fig. 6) gives an idea of this appearance, so far as shape is concerned, and as it may be obtained by means of a small-sized air-pump.

Experiment 6.—After the air has been exhausted from the receiver, admit, by means of the small screws, a small portion of air, together with the vapour of bisulphite of carbon. This is easily done, by placing a little cotton wool, moistened with liquid, on the screw. The air, rushing in, will take a portion of the vapour with it. The receiver is again to be exhausted, after closing the screw. If the spark be then passed, a fine lavender-coloured discharge will take place; a portion of the vapour will be decomposed; a bright flash of blue light will be seen; and sulphur will be deposited on the inside of the glass receiver.

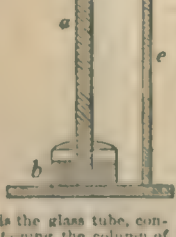
Experiment 7.—By similar means, the vapour of ether, benzole, camphine, alcohol, and other volatile liquids, may be introduced; and each will produce a characteristically coloured spark.

Experiment 8.—Besides volatile liquids, various gases, such as oxygen, nitrogen, carbonic acid, hydrogen, &c., may be employed. This is best done by filling a bladder, provided with a good stopcock, with the gas, and screwing the worm into the screw-hole of the air-pump, through which air is admitted into the receiver. The stopcock of the bladder should at first be closed, until the air in the receiver has been exhausted; a small quantity of gas may then be admitted by opening the stopcock. The spark is then passed as before, and some pleasing effects may thus be obtained.

We must caution the experimenter, however, as to the employment of coal gas, or pure hydrogen; as, if the receiver happen to contain air, or if the gas be mixed with air in the bag, the electric spark may cause its ignition, and an explosion may ensue.

Experiment 9.—A very ready mode of trying these experiments in a vacuum, may be found in an ordinary barometer tube. A glass tube, thirty-six inches long, and a quarter of an inch in diameter, is to be sealed at one end, and through this a platinum wire is to be passed, so that one end may be inside, and the other outside of the tube. The latter is to be filled, at its open end, with mercury; and the finger being placed on the hole, so as to prevent the escape of the metal, the tube is to be inserted into a small vessel containing mercury. The finger may then be removed from the tube, and a portion of the metal will run out into the cistern. The height of mercury remaining in the tube, will correspond with that of an ordinary barometer, and a vacuum will be left at the top of the tube. The platina wire is then to be connected with the conductor of the machine, and any liquid or gas can easily be passed up to the vacuum portion of the tube, by means of its open end. We first employed this arrangement for experiments with a Rhumkorff coil machine, and found it extremely convenient; because, excepting the presence of a slight amount of mercurial vapour, a perfect vacuum is readily obtainable. Fig. 7 represents an apparatus of this kind.

Fig. 7.



a is the glass tube, containing the column of mercury; *b*, the cistern of that metal; *c*, the vacuum; *d*, the platinum wire; and *e e*, a ring-stand to support the tube in a vertical position.

We shall have again to refer to this arrangement when we explain the cause of the Aurora Borealis; but we may just remark, that the electrical discharge in a vacuum, presents many points of resemblance to the phenomena we have named.

The reader, on repeating the experiments we have suggested, will at once perceive the truth of what we before stated—that the colour and intensity of the electric spark depend on the density of the medium through which it passes: and having illustrated this fact, we may now suggest a variety of experiments, which present the spark as it passes between conductors. A great number of ingenious devices have been arranged for this purpose.

Experiment 10.—Paste, on a piece of window-glass, some small dots of tinfoil, in the manner illustrated in Fig. 8, so that a continuous and equi-distant series may pass from one end to the other; attach to one end a wire, connected with the earth, and present the opposite end to the conductor of the machine. At each interval between the pieces of tinfoil, a spark will pass as the electricity breaks from one to the other through the air. Any device may thus be made luminous by the electric discharge; and an illustration of one of these arrangements is found in Fig. 9.

Fig. 8.

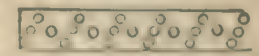


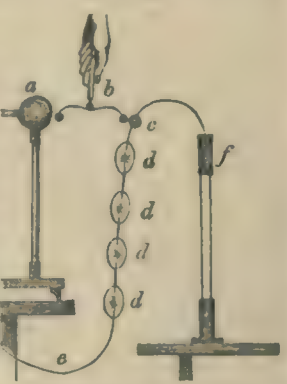
Fig. 9

A brass knob should be attached to that end which is presented to the conductor, for reasons we shall presently explain.

Experiment 11.—A very pretty mode of showing the electric spark, is that of threading, alternately, glass beads, with pieces of cork covered with tinfoil, on a silk thread previously varnished. One end of this is to be connected with the earth; and the opposite one, being terminated with a brass ball, is to be presented to the conductor of the machine. As the spark passes, each bead will appear luminous. A thread of upwards of 100 feet long was frequently used with the hydro-electric machine, when many hundred sparks, passing over the whole length, produced a most brilliant effect.

Experiment 12.—Those of our readers who possess a three-feet plate machine, may make a very elegant arrangement for exhibiting the spark passing inside egg-shells. Four or six egg-shells are attached to each other, so that a copper wire may pass through the end of each. The wires are separated, at the centre of the egg, by a thin thread of silk being tied to each, so as to preserve the mechanical continuity of the chain whilst the electrical continuity is broken. On presenting one end of the arrangement, by means of a knob, to the conductor, each egg will be beautifully illuminated as the spark passes between the wires inside it. The arrangement is illustrated in Fig. 10.

Fig. 10.



a is the conductor of the machine; *b* is a discharger, through which the spark passes to *c*; *d d d d*, are four eggs, through which the spark passes; *e* is a wire connecting them with the earth; *f*, a stand of glass rod, to support the eggs.

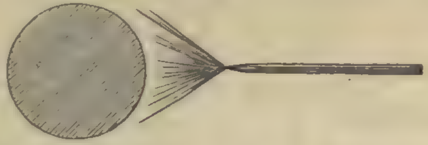
In each of the experiments suggested, we have recommended, that one end of the apparatus should have a brass ball attached to it; or, what will do equally as well, one of cork, covered with tinfoil; and that this ball should be held at a small distance from the conductor. This plan has the effect, apparently, of (what we shall call, for want of a better term) giving an impetus to the charge. If any of the arrangements we have described are placed in direct contact with the conductor, the effect is very feeble.

This leads us to remark on the result which points have on the passage and character of the electric spark. When any pointed object is presented to an electrised surface, the spark does not pass in the manner we have previously spoken of, but becomes converted into a brush-like form; hence the term "electric brush."

Experiment 13.—Present a penknife, or pointed brass rod, to within a short distance of the conductor of the machine. A peculiar hairy-like spark will be produced, having an appearance illustrated in Fig. 11.

Points have exactly an opposite effect on the electric force, to that of round and even substances. They tend to diffuse the force; and hence the conductor of the electric machine is always well rounded and polished in every part, to prevent the loss of electricity. For the opposite reason, lightning conductors (of which we shall speak fully hereafter) have their extreme end, towards the sky, always pointed. A very instructive experiment, illustrating the property of points, may be tried as follows :—

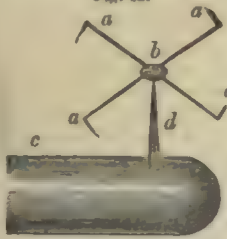
Fig. 11.



Experiment 14.—The machine being in good action, and affording large sparks between the conductor and a brass knob, place a sod of grass on the conductor itself. On presenting the knob, no further spark can be obtained, because the points of the grass leaves radiate and diffuse the electricity. The grass of the fields thus exercises a wonderful effect on the electrical state of the earth.

An ingenious arrangement of points, which are fixed to a common centre, and free to rotate on a point fixed in the conductor of the machine, exhibits another effect of pointed surfaces; which is, that a current of air is driven from them on the electric discharge taking place. One of these is represented in Fig. 12, in which *a a a a* are the pointed wires, fixed in a common centre *b*. *c* is the conductor of the machine; and *d* a pivot on which *b* can rotate. Now, not only will such an arrangement afford a current of air at each point—which may be tried by holding a lighted candle thereto—but the wheel will speedily turn round, owing to the mechanical action which is generated by the electric force against the surrounding air; and at each point a brush-like flame will be seen. This arrangement has been termed the "electric windmill."

Fig. 12.



Experiment 15.—The current of air produced from a pointed body attached to an electrical surface, may be noticed by fixing a pointed wire to any part of the conductor. If the machine be then set into action, and a lighted candle be held before the point, the flame will be rapidly driven away.

The reader, having thus seen the diffusive power which pointed bodies have on electrified substances, will at once perceive the necessity which exists for all the parts of the machine being kept as free as possible from dust; because each particle acts precisely in the same way as the pointed wires, &c., of which we have been speaking.

By other arrangements, many interesting experiments may be tried with the electric spark, as taken from the human body, &c. For this purpose, an electric stool will be required; which is easily made, by fixing into a piece of wood, previously well dried by heat, four rods of glass, to serve as legs, in place of wooden ones. Another mode is, that of resting a square piece of wood on four glass wine-bottles, which, as bad conductors, prevent the escape of electricity from the person standing on the stool. A sheet of gutta-percha, which also is a bad conductor, may be placed on a block of dry wood; and such often answers the purpose of an electric stool; the object being to interpose, between the earth and the individual, some non-conducting and insulating body. A wire is then connected with the conductor of the machine, and its other end is held in the hand of the person standing on the stool. The machine is then to be worked; and the following experiments may be tried :—

Experiment 16.—Instead of presenting a brass ball to the conductor, as mentioned in Experiment 14, hold one near to the hand of the electrified person, and a bright spark will pass. Indeed, the body becomes, for the time, an extension of the conductor of the machine.

Experiment 17.—Present a metal spoon, filled with ether, to a brass ball, held in the hand of the person on the stool. When the spark passes, the ether will be inflamed.

These experiments may be varied in many ways, and they will be sufficient to show the manner of conducting others. We must, however, caution our readers not to take strong sparks from the eyes or head, as such may prove exceedingly harmful. A spark passing from the body of a person electrified by the hydro-electric, or large plate machine, has so powerful an effect on the system, as to produce a shock which would prevent the individual from choosing a repetition of the experiment.

We must, of course, here confine our attention to the spark produced by frictional electricity. We shall find, when we treat on voltaic and magneto-electricity, that the light, and general luminous effects produced thereby, are much different from those to which we have been referring, owing to causes which we shall then explain.

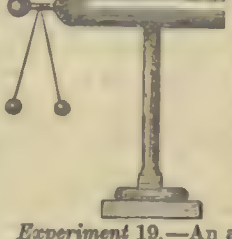
ATTRACTION AND REPULSION.

HAVING chosen first to speak of the electric spark, because it naturally attracts the attention of the experimenter, and paves the way for more intricate and interesting electrical phenomena; we shall now consider two effects which present themselves when bodies are electrified—namely, those of attraction and repulsion.

It has already been mentioned, that an electrified body has the power of attracting light substances to its surface.* Thus, if a piece of dry window-glass be placed at a distance of half an inch from, and over, some pieces of paper or feathers, and its upper surface be briskly rubbed by means of a silk handkerchief, the feathers will dart up, and adhere for a short time to the under surface, until, being electrified equally with the glass, they will be repelled from it, and fall again to the place on which they had previously rested. We shall suggest a few experiments illustrating both attraction and repulsion, before entering into the laws which govern the phenomena.

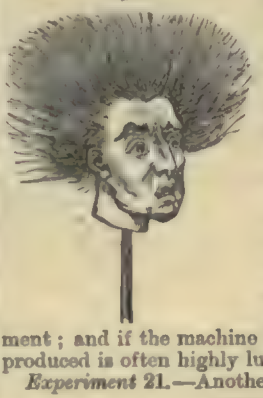
Experiment 18.—Hang two small pieces of pith, obtained from the elder-tree, by means of a silk thread, from the conductor of the machine, so that they may rest against each other. On working the machine, they will be similarly electrified, and will repel each other so long as they continue in that state. On presenting the knuckle, or a brass ball to either, they will fall together again, owing to their electric equilibrium being restored. The mode of conducting this experiment is illustrated in Fig. 13.

Fig. 13.



Experiment 19.—An amusing arrangement is sold by the instrument-maker; in which a head of hair is employed. It is to be fixed by means of a brass rod, in an upright position, on the conductor of the machine. So long as it is unelectrified, the hair rests naturally about the head; but on working the machine, each hair becomes similarly electrified, and so repels its neighbour in the manner shown in Fig. 14.

Fig. 14.



Experiment 20.—Let a person stand on the glass stool; and having combed out the hair of his head, connect him, by means of a wire, with the conductor of the machine. Each hair will separate, as in the last experiment; and if the machine be in good action, the effect produced is often highly ludicrous.

Experiment 21.—Another mode of showing the effect of repulsion, is that of attaching a number of narrow strips of tissue-paper to the conductor of the machine. When electrified, they will all separate, as in the two last experiments.

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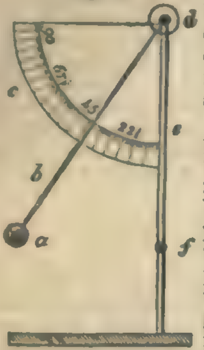
* See *ante*, p. 167.

Fig. 15.

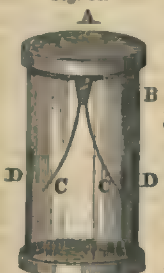


a is a pith ball suspended by means of a straw, *b*, to a pivot at *d*, on the upright stand, *e*; *c* is a quadrant graduated to 90°; *f* is a small ball attached to the upright stand.

Electroscopes have been constructed, which are intended to show the presence of free electricity. They are chiefly of two kinds—the pith ball and gold-leaf electrometers, as they are sometimes also called. The former alone deserves that name, because it registers, by the extent of repulsion indicated on a graduated quadrant, the amount of free electricity present. Its construction is as follows. (See Fig. 15).

Now, when this electrometer is placed in contact with a non-electrised body, the ball, *a*, will rest in contact with *f*; but if the body be electrified, then *a* will be repelled from *f*, and will rise towards the top of the quadrant, just in proportion to the amount of free electricity present: the number of degrees marked on the quadrant, by means of the straw attached to the pith ball, of course indicates the extent of electrisation.

Fig. 16.



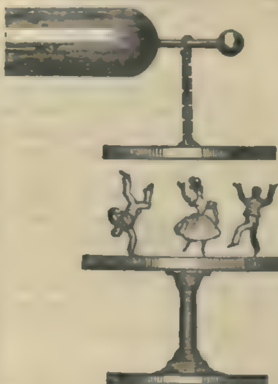
Such an instrument, whilst acting tolerably well when a large amount of free electricity can be observed, is of far too coarse a nature for detecting minute electrical disturbances; and, for this purpose, the gold-leaf electroscope is of great value. One of these is represented in Fig. 16; in which A is a brass cap, fitting on a cylindrical glass vessel, B; C C are two pieces of gold-leaf, suspended from the brass cap; D D are pieces of tinfoil, pasted on the opposite sides of the glass cylinder. Now, so long as the brass cap is not electrified, the gold leaves, C C, hang together in contact; but on presenting any electrified body to A, the leaves immediately repel each other, even although the free electricity is extremely slight in quantity. We should, however, state, that the indications of this instrument entirely depend on the state of the glass cylinder. If this is in the least dirty or moist, the electricity at once passes away. This electroscope must therefore be carefully dried before use, and should be slightly elevated in temperature beyond that of the surrounding air.

An instrument of this kind is easily made by employing an ordinary gas chimney-glass. In the inside, to about half its length, two narrow strips of tinfoil are to be pasted opposite to each other; the bottom of the glass should then be closed with a cork. Into the top, another cork, *entirely covered with tinfoil*, may be fitted; and to the inside of this, two strips of gold-leaf are to be attached, so that they may hang inside the glass with the lower end a little below the top of the tinfoil strips, as shown in Fig. 16. Such an instrument, if carefully made, will answer every purpose which can be required for many delicate electrical experiments.

Various forms of electroscopes and electrometers have been devised, each being suited to some special purpose. Amongst the most delicate of these is that invented by Professor Thomson, of Glasgow; which was used during the laying of the Atlantic telegraph. Its peculiar construction prevents the possibility of illustrating it by an engraving: we may observe, however, that its indications of the feeblest electrical disturbance, makes it of great value in many investigations.

Generally speaking, electrical attraction and repulsion are coincident. The reason of this we shall understand better when we examine into the opposite electrified conditions which bodies assume, and which have been termed positive and negative. For the present, we shall only illustrate the phenomena, without entering deeply into their causes; and adduce some experiments illustrative of attraction.

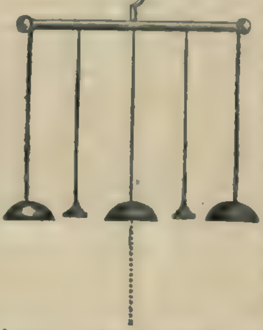
Fig. 17.



If a metal plate be hung from the conductor of the electrical machine, and some pith balls be placed within two or three inches of it, they will be attracted towards the plate. An amusing mode of showing this is illustrated in Fig. 17; in which we observe a stand for holding figures of men, &c., cut out of pith, for the sake of lightness. At a distance of three or more inches, the plate from the conductor is suspended. On working the machine, the figures will commence dancing between the plates, owing to the electrical attraction and repulsion which ensues.

Another mode of showing the same effects, is that of the electrical bells. The arrangement consists of three bells and two clappers, suspended from a beam, which is to be attached to the conductor of the machine. The bells are electrified whilst the clappers oscillate between them; and so long as the machine is worked, a continual ringing of the bells takes place. A chain is hung from the centre bell, so as to communicate with the earth. The ordinary form of this apparatus is shown in Fig. 18.

Fig. 18.



Electrical see-saws, &c., are constructed by the instrument-makers, for illustrating the general laws of attraction and repulsion.

The intensity of these forces varies inversely as the square of the distance between the electrified body and that on which the attractive or repulsive effect takes place; and this law agrees with that governing all radiating forces, as we have previously explained in our sections on heat and light. There are, however, so many interfering causes which disturb the exactitude of an experimental illustration of this fact, so far as electricity is concerned, that whilst, as an inference, we may entirely trust to its truth, we have not such rigidly-founded premises and conclusions as we can obtain when investigating the other undulatory forces.

CONDUCTION.

We have already stated, that all bodies have, electrically speaking, been divided into two classes—namely, conductors and non-conductors; or, what is equivalent thereto, electrics and non-electrics. This distinction, however, has been made without any title to correctness; for we are acquainted with no body which will not, to some extent, however trifling, conduct electricity; and, on the other hand, we can never call any body incapable of affording evidences of free electricity, when proper means are employed to that end. We shall therefore use such terms with the limitations we have named, and as expressions which, whilst they assist to convey definite ideas to the minds of our readers, are not to be considered as incorporating the real facts of the case.

By the term "conduction," we mean that ability which some substances have, of exhibiting, at a distance from its source, certain effects of any force which seem to be conveyed on their surface, or by their mass. The relative power of such substances is measured by the rapidity with which these effects are shown when one body is compared with another in that respect. Thus, a piece of wood, six inches long, may be in a rapid state of combustion at one of its extremities, whilst, at the other, not the slightest sensation of heat can be experienced. A piece of copper, of the same size, will, however, produce a very different result; for if one end be heated but moderately, the calorific effect will be speedily evidenced at the other end—showing that the principles of heat, or its effects, travel much more rapidly from one extremity of the metal to the other, than is found in the case of wood. Now, the metal is therefore termed a good conductor, and the wood a bad one; or sometimes, erroneously, a non-conductor.

Generally speaking, there is no great difficulty experienced in deciding on the relative electric conducting power of a body. The effects of free electricity can be easily observed by means of electroscopes, &c., even when the amount present is but very slight. In the case of heat and sound, our means of detection are not so delicate; and, therefore, a list of conductors is less to be relied on. On the other hand, the electrical indications rapidly exhibit themselves and pass away, and, for that reason, are sometimes but slightly appreciable at a distance from the source of disturbance. With respect to electricity obtained by friction—glass, resins, sulphur gutta-percha, fur, hair, and similar bodies, are included amongst bad conductors, because they have very little tendency to permit the passage of electricity over them. Possibly, for this reason, they most readily exhibit electrical effects when submitted to friction; and hence have been termed electrics. The metals, in the order of gold, silver, copper, platina, brass, iron, &c., are excellent conductors of electricity, and, accordingly, are employed whenever its effects are required to be shown at a distance from its exciting cause. Another class, standing intermediate between good and bad conductors, embraces fluids of various kinds, moisture, &c.; which, according to circumstances, more or less readily permit of the passage of electricity. We have often spoken of the necessity which exists of keeping every part of electrical apparatus free from moisture; and this is simply because its presence on the surface of non-conductors, such as glass, &c., actually has the effect of rendering them, superficially, tolerable conductors.

By way of parenthesis, we may observe, that it is a good plan to cover the legs and supports of electrical apparatus with a solution of shell-lac, or best red sealing-wax, in spirits of wine. This forms a coating, which has not so great a tendency to condense moisture as glass; and, in damp weather, its advantages are greatly felt by the electrician. This resinous coating is a worse conductor of heat than glass; and it and gutta-percha, for that reason, may at all times be employed for insulating purposes.

With respect to the mode in which electrical effects travel over, or are conveyed by, conductors, we may observe, that, unlike heat, which travels in the mass of a body, electricity seems conveyed only by their outer surface. Hence, within certain limits, a large surface of a comparatively bad conductor, may act as efficiently as a small mass of a good conductor. We have thus observed all the expected effects of a seven-feet plate machine, or of the hydro-electric, entirely dissipated by a damp glass surface; whilst the same quantity of electricity, sent through a solid but fine metallic wire, would have instantly reduced it to a melted state, or dispersed it into fragments. It is thus of extreme importance, in employing conductors for protecting buildings from lightning, that the double condition of surface and solidity should be attended to. In experiments with large electric and voltaic batteries, we have, for many years, preferred the use of flat copper strips to that of wire of the same weight, but, of course, exposing less superficies. And, at the present day, flat copper bands are preferred as lightning conductors, in the navy, to copper rods which had been previously used.

The continuity of a conductor, if broken even to the smallest extent, destroys its efficiency. We shall, however, defer considering this element, until we examine the mechanical and other effects of intense charges of electricity, which will come under our notice when we speak of the Leyden jar. We have already stated our preference, at all times, for the use of wires to that of chains, in connection with electrical experiments; and this arises, experimentally and theoretically, from the facts which we are here mentioning. As a powerful charge passes over either a brass or iron chain, a spark may be noticed at every link, indicating, that there the continuity of conduction is broken, and that the force has, so to speak, to "leap over" an obstacle. Besides, at each break there is a tendency to diffuse electricity, and hence a decided loss of power ensues.

The conducting power of bad conductors is modified by heat; hence, heated glass and air become better conductors than when they are at ordinary temperatures. This fact should be borne in mind in warming and drying electrical apparatus; and their temperature should only slightly exceed that of the surrounding air, so as to prevent the deposition of the moisture, by condensation, on their surface.

Some very remarkable effects may be noticed when a powerful electric charge passes by a good conductor. We have observed, that the external surface of a metal may be actually fused, whilst the internal mass is entirely unaffected. It often happens that metallic objects, in the pockets, &c., of persons struck with lightning, have been externally fused; but this generally occurs where no continuous conductor for either heat or electricity is present. We can only explain this by suggesting, that the development of each force is very sudden, and yet only affects the extreme outer portion of the metal, the mass of which speedily absorbs the intense heat which has been generated.

Circumstances occur in which a tolerable conductor may become an excellent electric; and the most extreme case we can point out is that of water. This liquid, in the state of fine particles, spread dew-like over a bad conductor, as we have already seen, improves its conducting power; yet, in exactly the same state at the nozzles of the hydro-electric machine tubes, it affords, by friction, an enormous amount of free electricity.

From these facts, our readers will at once see, that the idea of conduction and non-conduction can only be sustained in a relative or comparative point of view, and that special circumstances can entirely modify, and occasionally change, the character of any substance or liquid. There, however, exists great analogy between heat and electricity in this respect.

ELECTRICAL THEORIES.

For the sake of simplicity, we have hitherto restricted our observations to such facts as could be explained, under the idea that electricity, like heat, could be conveyed to any place; was diffusible, and, generally, had the characteristics of what we may term a single acting force. We carefully abstained from mentioning, that certain theories which have been, and are strongly maintained, entirely militate against that notion. This course has been adopted because it permits the reader to become acquainted with facts sufficiently plain to explain themselves, and so a kind of foundation has been laid for the more complicated structure of phenomena and laws which we shall now begin to examine.

The earlier electricians were of opinion, that an electrified body differed from a non-electrified one, by containing a greater quantity of electricity in a free state. In other words, an electrified object had an excess, whilst the non-electrified had a deficiency, of the electric force. Hence, the terms positive and negative were continually employed, in their strictly mathematical sense, to express such an idea; and we may, perhaps, still further illustrate the position by referring to the ordinary process of addition in algebra, whereby numbers of different signs, on being added together, give a total value midway between their individual amount. Thus, the addition, algebraically, of $+a$ to $-a$, would afford nothing in amount. In the same way, electric equilibrium was similarly considered as existing in bodies at zero, whilst its disturbance was simply an addition to the electrical quantity of bodies in a positive electrical state, and an abstraction from those which assumed the negative condition.

This has been termed the single theory; and so extensive was its employment at one time, that all its terms are still to be found in electrical works and lectures. Indeed, it is often a matter of considerable amusement to a strictly philosophical person, to hear those who strenuously adopt the double theory, using constantly such terms as "current," "passage," and the like, which can only properly be used under the supposition that a single electric force exists.

We shall find, however, that although the single theory has the advantage of great simplicity, and ready adaptation to *nearly* all the circumstances of electric disturbance, it yet fails to explain some of the most important phenomena, especially such as are involved in electrical induction.

The double theory supposes the existence of two electricities, exactly opposed to each other in every respect, just as we observe in the opposite poles of a magnet. When electricity is excited, this theory supposes that the two electricities are parted; that they have a mutual tendency to recombine; and that all electric phenomena result from one or other of these tendencies, and, generally speaking, from the combined action of the two.

In the double theory, the terms "positive" and "negative" are used in a *physical* sense only. They express no idea of quantity, but rather of position. The opposite electricities, indeed, are supposed to be exactly equal in quantity and intensity; and this is one chief cause of their constant tendency towards recombination, and, consequently, to reproduce the natural or normal state of all bodies in an unexcited state, electrically speaking. The distinction has been carried even still further; and the terms "vitreous," or "glass," and "resinous," or "rosin-produced" electricities, are employed, for reasons which will be best explained by our readers trying the following experiment.

Experiment 22.—Suspend a feather, by means of a silk thread, so that it may hang freely, and at a distance from any other substance. Let an excited glass tube be brought near it, when the feather will be immediately attracted, and afterwards repelled. Remove the glass tube, and then present a roll of sealing-wax, which has been rubbed with a dry flannel, and the feather will immediately fly towards the wax.

This would be explained, under the *single theory*, by saying that the glass had positively electrified the feather, and that the wax, being in negative or deficient electrical state, had attracted, and so the excess of electricity of the feather had combined with the deficiency of the wax, and an electrical equilibrium had thereby been restored.

Under the *double theory*, we should say, that the *vitreous kind* of electricity of the excited feather combined with the *resinous kind* of the wax, and hence the result. Our readers will thus perceive, that the most simple way of placing before them the antagonism of the two theories, is that of assigning quantity as the leading feature of the single, and kind, or quality, as that of the double theory. That similarly electrified bodies repel each other, is, of course, admitted by the advocates of each theory as a fact; but the cause each assigns is in accordance with that adopted to explain the results of our last experiment.

We do not intend to take our readers through the almost interminable arguments which have been used on both sides. It has been our good fortune to have read most of the works, and to have heard the public lectures, of the leading electricians of the day; and the only conclusion at which we have arrived, is, that whilst the subject in dispute has strong and able arguments on either side, those who take part in it are almost constantly led to compromise their views as some occasional or accidental circumstance arises. As we before observed, the employment of the terms of each theory, indifferently at least, indicates extreme uncertainty as to the real state of the case.

However, the double theory is the most usually recognised at the present day; and we shall use it in explaining many of our future experiments. The strongest objection which suggests itself to our mind is, that it has no analogy to the theories of the other undulatory forces. It is true that magnetic phenomena countenance the idea; but we must bear in mind, that the connection of magnetic and electric affections is so intimate, as to leave us scarcely room to doubt whether any difference really exists. If, however, we compare the double theory of electricity with our theories of heat and light, the analogy at once fails; and no one has yet been bold enough to suggest the idea that there are two kinds of heat or two of light. The investigations which have lately been made into the correlation of forces, certainly tend to induce the adoption of the single theory. However, this is one of a class of subjects which can only be properly understood by a much more extended knowledge of force, motion, and matter, than we at present possess. We shall, therefore, refrain from urging any further views on the subject upon our readers; and having endeavoured to state the case fairly, shall make free use of each theory, in our future pages, as convenience of expression or illustration may render desirable. We shall thus stand free from the charge of inconsistency or dogmatism.

It will now be proper that we should examine the doctrine of electrical induction, and use the Leyden jar and battery for the purpose of illustrating it.

INDUCTION—THE LEYDEN JAR AND BATTERY.

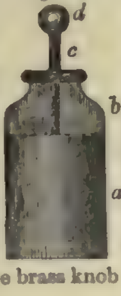
In attempting to convey an adequate idea of the phenomena included in electrical induction, considerable difficulty arises, because there is no analogy to which we can refer, except that of magnetic phenomena, which can assist us only to a limited extent. We shall, therefore, rely on a careful and attentive study on the part of our readers, if we endeavour to present to them, as far as possible, a logical exposition of the doctrines or laws which modern science has determined to be the basis of this department of natural philosophy. The term "induction" has been chosen by electricians as expressing a similar idea, in electric science, to that which the same word would convey in the conversation of daily life.

We endeavour to induce a person to fall into our ideas by presenting him such facts or arguments as will be sufficient to bring him into our mode or course of thoughts. Similarly, when we wish to induce a certain electric state in any body, we present such an electrified condition as will produce the effect we desire. There is, however, this difference—we produce an exactly opposite state to that we present; because, if we electrify any body positively, we intend to produce on its surface a negative electrical state, and *vice versâ*. Hence, we depend on the fact, that action and reaction are not only equal, but that they, in certain cases, are naturally producible. Our readers will more fully understand our statements when they reduce them to a practical form by actual experiment.

The most eligible instrument we can choose for the purpose of illustrating our subject, is that of the Leyden jar and battery. The early experimenters in electrical science, were not acquainted with the fact, that a greatly increased effect might be produced by means of an arrangement such as that to which we allude. It is stated, that the discovery was accidentally made by M. Muschenbroeck, who, having allowed a chain from the conductor of an electrical machine to rest inside a glass vessel partly filled with water, whilst his hand encompassed the outside, was astonished at receiving a powerful shock on attempting to remove the chain from the vessel. The occurrence, or, at all events, its repetition, in the town of Leyden, has given a name to the arrangement.

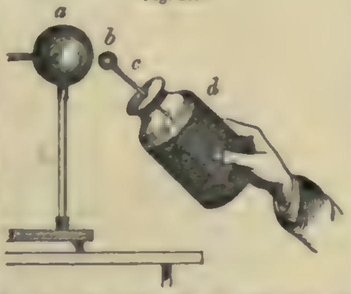
A Leyden jar is easily made by coating any kind of thin glass vessel with tinfoil, on both sides. They are usually cylindrical glass jars, on the outside and inside of which tinfoil is pasted to about two-thirds the height. The reason the coating is not carried higher, is, that the charge would rapidly discharge itself if the two coatings were not separated to some distance. The bottom of both, in and outside, should be also covered with foil. The coatings are simply required as distributors of the electricity, and, as such, only are they essential. It is well that the reader should bear this in mind, because many persons imagine that the charge rests in or on the tinfoil, instead of the glass. So far from this being the case, a jar may be fitted with movable coatings, which, after charging the jar, may be removed by means of silk threads, and it will be found that the charge is actually retained on the surface of the vessel. On the top of the jar a wooden cover is placed, the object of which is to keep out dust, &c. Through the cover a wire is passed, which communicates with the inside metal lining by means of a chain. The wire also extends outwards, beyond the top of the cover; and at its extremity a brass knob is placed, at which the charge is taken from the conductor of the electrical machine. Fig. 19 represents the usual form of the Leyden jar: *a* is the outside coating of tinfoil; *b* is the interval of glass between the inside and outside coatings; *c* is the wire which extends into the jar; and *d* is the brass knob at which the charge is received.

Fig. 19.



The annexed engraving (Fig. 20) illustrates the mode of charging the jar; *a* being the conductor of the electrical machine; *b* the knob of the jar; *c* the wire reaching into the inside; and *d* the outside coating. The jar may be held in the hand whilst being charged, and a wire should be attached to its outside so as to maintain connexion with the earth, for an insulated jar cannot be charged.

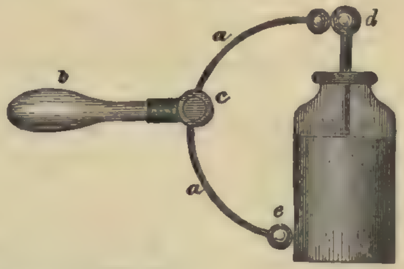
Fig. 20.



The length of time required to charge the jar, depends on its size and the power of the electrical machine; for a small one—say of twelve inches in the plate—will not fully charge a jar of large size. A seven-feet plate machine, or the hydro-electric, will easily charge thirty jars, two feet high and twelve inches in diameter, in one minute or less; and a single jar, of the same size, can be charged several times per minute, by either the plate or steam machine.

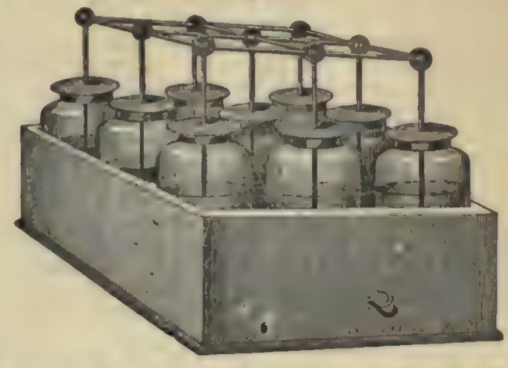
If a jar be presented too long to the conductor of a machine, it will discharge itself; a brilliant spark, attended with a loud, snapping noise, will take place; and it is more than likely that the discharge will force itself through some portion of the glass, and so render the jar useless for further experiments.

Fig. 21.



The usual mode of discharging a jar is represented in the following engraving. An instrument, called a discharger, is employed, which consists of two brass knobs, fixed to a bent wire, *a a*. The instrument is held by the glass handle *b*, and the wire is fixed on a joint *c*, by means of which the two balls can be removed from each other to any extent which may be desired. The jar being charged, the balls of the discharger are brought into contact with the knob, *d*, of the jar and its outside coating, *e*, when a spark and a snap will be produced; and the electric equilibrium will be restored in a small jar, and nearly so in one of large size. In using the latter, a slight second discharge may occasionally be observed, which arises from what is called the residual charge. In very dry, frosty weather, we have found that very large jars will actually charge themselves—a fact which we first became acquainted with by removing one whilst holding the knob and the outside coating at the same time. This, however, is a rare occurrence, because, unless a Leyden jar is kept very clean and dry, dust and moisture soon settle on the glass surface, and so prevent the electrical separation of the two coatings. In frosty weather, and during a dry east wind, some curious results are often obtained from Leyden jars; and it is a good rule to apply the discharging-rod to every part of an insulated coating before setting the apparatus aside, because the charge will sometimes remain for days on its surface. Large jars have the necessary disadvantage of being made with thick glass, which prevents their being readily charged. They, however, must be used for large machines. It is usual to connect a number of either the large or small size together, when powerful electrical effects are desired. Such an arrangement is called a Leyden Battery; and by its use some most interesting natural phenomena may be readily illustrated. One of these is represented in the following engraving; in which it will be observed, that all the insides are connected together by means of rods which are fastened to the knobs of the jars, and the outsides rest in a box lined with tinfoil, by means of which they are all, electrically speaking, joined together. It is advisable, however, to fix a wire so that it may pass round each jar, and it should be extended to a gas or water-pipe, so as to ensure the ready charging of the jars. In Fig. 22, this is effected by means of a wire attached to the hook in the side of the box. A rod should extend from the knobs of the jar to the conductor of the machine; and thus the battery is to be charged. The mode of discharging it will be noticed as we proceed with our experiments. We should here state, that unless the single jar, or each one in a Leyden battery, is kept perfectly free from dust and moisture, it is impossible to charge them. They should be carefully wiped with a warm silk handkerchief before use, and heated to a little beyond the temperature of the room in which the experiments are to be tried; and it is also frequently

Fig. 22.

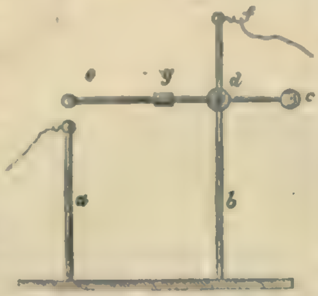


necessary to wipe them when in use, as they often gather dust on their surfaces by the electrical attraction which is produced as they are charged.

It is often a matter of surprise to observe the quantity of dust which collects on a charged surface; the effect of which is to much diminish the power of both the machine and battery.

A very convenient form of discharger for powerful batteries is represented in Fig. 23. It has the advantage of discharging the battery at any period, or at any amount of charge the experimenter may choose. It consists of two upright glass rods, *a* and *b*. On *a*, a brass knob and hook are fixed, from which extends a wire to the outside of the battery, or to any intervening piece of apparatus, and thence to the outside of the battery. *c e* is a rod of brass, moving readily on a pivot, *d*; and *g* is a sliding weight, by means of which the balance between the knobs, *c* and *e*, can be adjusted. The knob, *c*, tends to raise *e* upwards; but by shifting *g* from *d* to *e*, the amount of charge in the battery can be regulated. The inside of the battery is attached by a wire to the hook on the knob *f*. The engraving represents the arrangement as if a battery were being charged. When it is fully so, the knob, *e*, will fall down towards that on the rod, *a*, and the discharge will at once take place. We need scarcely state, that the principle on which this arrangement works, is that of electrical attraction between the charged knob, *e*, and that fixed on *a*. Besides the convenient character of this apparatus, it prevents the unpleasant and, possibly, dangerous effect which might arise when a large battery is discharged by means of the discharger previously described.*

g. 23



It is, however, occasionally necessary to use a hand discharger; and the handle for that purpose should be a glass rod, not less than two feet in length, so as to prevent risk of accident.

We know of no better mode of regulating an electric charge in a battery, than that we have just described, and which we always prefer to use. Many others have been proposed, but as they present no special advantage, we shall refrain from describing them.

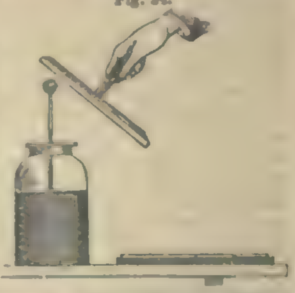
To assist those who may not be able to purchase extensive apparatus, we shall give directions by which they may succeed in constructing a Leyden jar; and by using the electrophorus, previously described,† they may easily try several of the experiments we are about to suggest.

A Leyden jar may be made by half filling a thin, round glass phial with brass or iron filings. A cork is then to be fitted into its neck, and a wire should be fixed in the cork, so that one end may enter the filings, and the other should extend two inches beyond the neck of the bottle. On the outer end of the wire a brass knob is to be fixed, or, what answers equally well, a round cork covered with tinfoil. The outside of the phial is to be coated with tinfoil to half its height, and the bottom should be similarly covered. The arrangement, though somewhat rude in its construction, answers remarkably well.

Such a jar may easily be charged by means of the electrophorus, which must be first excited, as already directed.‡ The cover is then to be brought to the knob of the jar, when a spark will pass. This is to be repeated until the jar is charged, which may speedily be done with one of the kind we have just described. The method of thus charging a jar with the electrophorus, is illustrated in the following engraving.

Having described the apparatus which will be required for illustrating the laws of induction, &c., we shall enter into an experimental investigation of them, and also illustrate the luminous, calorific, mechanical, and other effects of a high charge of electricity.

Fig. 24.

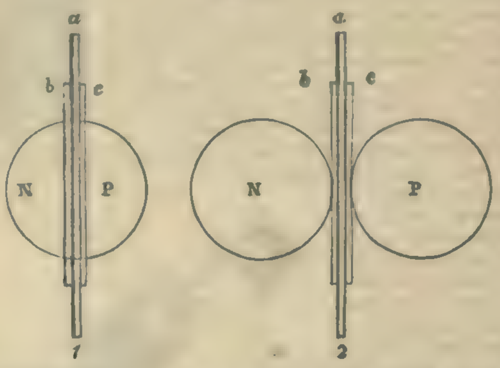


When a Leyden jar is charged by presenting the knob to the conductor of an electrical machine in action, the natural equilibrium of the latent electric force is at once disturbed, and the surfaces acquire what may be termed a polar state. By this we mean, that a new and active condition of force is induced, by which the hitherto quiescent and inactive conditions of the latent electricities are placed in opposite positions to each other, and yet having a tendency to recombine.

This may be better understood if we employ a diagram illustrating the changes which take place on the glass surfaces; and instead of using the ordinary Leyden jar, we shall suppose that we have, coated on each side with tinfoil, a piece of window-glass. A Leyden jar, in fact, may be regarded as such—a piece of glass bent into a cylindrical form. We shall also confine our attention to one small point on its surface, as such, of course, illustrates all that takes place in the entire jar whilst being charged and discharged.

In Fig. 25 we have two diagrams, of which No. 1 is intended to represent a jar or piece of coated glass, in its natural state; and No. 2 illustrates the electrical change which takes place when the jar is charged. *a* is the glass plate, and *b c* are the tinfoil coatings. In No. 1, we observe a circle illustrating an electric equilibrium, in which the negative and positive, or vitreous and resinous electricities, are in a state of combination. In No. 2 we observe the effect of charging the jar, which results in the separation of the two electricities, by which a polar state is induced, and each side of the glass plate becomes in an opposite electrical state.

Fig. 25.



If a metallic or other good conducting communication be made between *b* and *c*, in No. 2, the two electricities will recombine, the polar state be lost, and the coated surfaces return to a state of equilibrium, such as at first was spoken of, and shown in No. 1.

We therefore observe, that the primary effect of charging a glass surface, is to put the natural electric state of repose into an active condition; and this results in the separation of the two electricities, and their individual appearance on each side of the glass plate, which we have supposed to be infinitely thin, and to have undergone no electrical change of any kind in its mass or substance.

We need scarcely observe, that the explanation we have given is that arising from the employment of the double electric theory. If, however, we suppose the existence of only one kind of electricity, then we shall explain the changes which occur by stating, that each side of the charged plate becomes polar; but that the positive side has an excess of electricity, whilst the negative one becomes deficient in that respect. These two states are occasionally represented by the mathematical signs of + and — (*plus* and *minus*).

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* See *ante*, p. 178. † See *ante*, p. 171. ‡ See *ante*, p. 172.

We have confined our attention to one sheet or plate of glass only; and before proceeding further, we may observe, that the thickness of the glass has a decided effect on the readiness with which the two sides may be polarised. This arises from two causes—namely, the distance between the sides, and the partial polarisation which the material itself undergoes. It is therefore essential, in making a Leyden jar or battery, that the glass employed should be as thin as possible, as on that depend both the facility of charging and discharging the arrangement.

We shall now examine the effect produced on a number of particles, with respect to the electrical arrangement which they assume when in an excited state; and we shall represent the quiescent and active condition by means of a diagram, which will assist in conveying the subject in an intelligible manner to our readers. For this purpose, we shall suppose the following arrangement to represent the twofold state to which we have alluded. Of course, we do not intend to assume that the particles we speak of are those of electricity. We presume that they are the atoms of a body, in which electricity is respectively in a latent and active form.

No. 1. NP NP NP NP

No. 2. N PN PN PN P

We here suppose the existence of four atoms of a body, whose sides are respectively negative and positive; but in No. 1, such states are those of *latent* or combined electricity. If, however, by means of the electrical machine, we induce an *active* state, then the previous electrical equilibrium is at once disturbed; and, as we observe in No. 2, the electricities arrange themselves so that at each end of the series of atoms we have a free electricity; and, intermediately, the electricity of one atom instantly acts on, and induces action on, that of its neighbour. If a connection be made between the opposite electrical states, at the extremities of the series of excited atoms, then they are at once restored to the condition shown in No. 1, and the equilibrium is thereby reproduced.

It will thus be perceived, that electrical induction takes place when an excited body is brought near to one in a quiescent state; that the effect produced is that of arranging the electricities of the previously unexcited body in a polar state; and these laws hold good, no matter how many particles may be affected, provided that the exciting power is of sufficient energy.

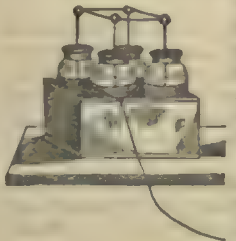
It will be presumed, in all future experiments, that our readers will bear these facts in mind, and that when we speak of induction, they will fully understand the term in all its bearings.

We shall next proceed to illustrate the various effects which a powerful charge of electricity, as afforded by means of the Leyden battery, produces; and shall divide these into mechanical, luminous, calorific, and physiological phenomena; although, occasionally, two or more of these classes are simultaneously produced by the same experiment.

THE MECHANICAL, LUMINOUS, AND OTHER EFFECTS OF ELECTRICITY.

We shall now suggest numerous experiments which can be tried by the Leyden jars or battery, in which the various effects of a powerful charge of electricity will be exhibited; and shall also examine some natural phenomena in which that force is the active agent.

Fig. 26.

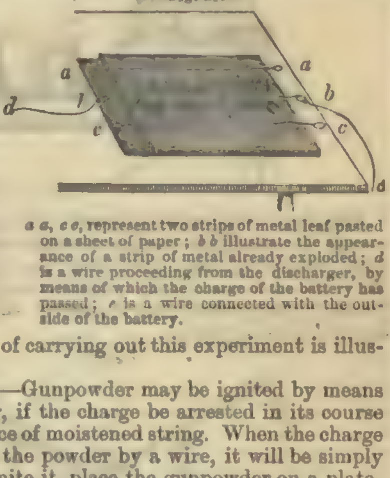


Most of the experiments we shall introduce, may be tried by means of a plate machine of from twelve to eighteen inches in diameter in the glass plate, and with a battery of from two to four jars. To facilitate the construction of an arrangement of the kind, we refer to Fig. 26, which represents a battery of four jars, of about three pints capacity each. Such is sufficient for a vast variety of experiments, and each jar may be of the simple kind we have already described.*

The different parts of this Leyden battery correspond in every respect, except as to size, with that illustrated at a previous page.† Before commencing any experiment with the battery, care should be taken that the outside of the jars be connected with some good conductor, such as a gas, drain, or water-pipe, as on that will depend much of the success which may be desired. As it is impossible to separate the mechanical, luminous, and calorific effects of frictional electricity, we shall introduce our experiments without reference to such distinctions; pointing out, however, as we proceed, any notable matter which may arise.

Experiment 23.—*Combustion of Metal Leaves.*—For this purpose, a narrow strip of gold or silver leaf is to be pasted on paper. The width of the strip of metal, as well as the length, depends on the power of the battery; but a strip of an eighth of an inch wide, and eight inches long, may be used. The paper should be well dried; and strips of several metals may be then pasted on it. The paper so covered is to be fastened on to a well-dried board, and one end of the metal strip is to be attached, by means of a wire, with the outside of the battery. The other end of the strip may be connected with the knob of the discharger at *a*—illustrated already.‡ To the other knob, marked *f*, the inside of the battery must be attached by a wire. By such means the charge will spontaneously discharge itself. In the absence of such an arrangement, the ordinary discharger may be used, one of its ends being brought into contact with the inside of the battery by means of a wire; and its other end, after the battery is charged, is to be brought in contact with the strip of metal. On the charge being thus directed over the metal strip, a loud explosion will take place, and the metal foil will be destroyed. The effect on the paper is often very pretty, owing to the dispersion and oxydation of the metal in all directions. The mode of carrying out this experiment is illustrated in Fig. 27.

Fig. 27.



a a, *c c*, represent two strips of metal leaf pasted on a sheet of paper; *b b* illustrate the appearance of a strip of metal already exploded; *d* is a wire proceeding from the discharger, by means of which the charge of the battery has passed; *e* is a wire connected with the outside of the battery.

Experiment 24.—Gunpowder may be ignited by means of the Leyden jar, if the charge be arrested in its course by means of a piece of moistened string. When the charge is sent directly to the powder by a wire, it will be simply dispersed. To ignite it, place the gunpowder on a plate, and dip into it a wire proceeding from the outside of two jars. Attach a piece of wet string to one knob of the discharger, and to the end of the string fix a piece of wire, which is to rest on the gunpowder, at a short distance from the end of the wire proceeding from the outside of the jars. When they are fully charged, bring the other knob of the discharger into contact with their inside coatings in the usual manner, and the powder will be at once ignited. Fig. 28 shows the proper method of conducting the experiment.

Experiment 25.—Two very singular effects may be produced by causing the Leyden discharge to pass between two plates of glass. A narrow strip of gold leaf is to be placed between two slips of common window-glass, which may be an inch wide and four inches long. These should be tied tightly together, and the metal leaf should extend about half an inch beyond each end. One end of the leaf is to be attached to the outside of the battery, and the plates should be insulated by placing them on two glasses.

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* See *ante*, p. 180. † *Ante*, p. 179. ‡ *Ante*, p. 180; Fig. 23.

When the battery is charged, it is to be discharged by means of the usual discharger; and the mechanical effect will be such that portions of the gold leaf will be driven into the glass, and the two plates shivered into numerous small pieces. This experiment requires great care, as the broken pieces are exceedingly sharp, and fly to a considerable distance on all sides. It is advisable that a kind of screen be placed over them, to prevent accident to the experimenter. The following engraving illustrates the whole arrangement.

Fig. 28.

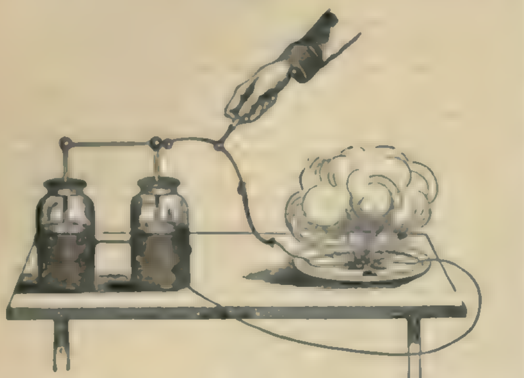
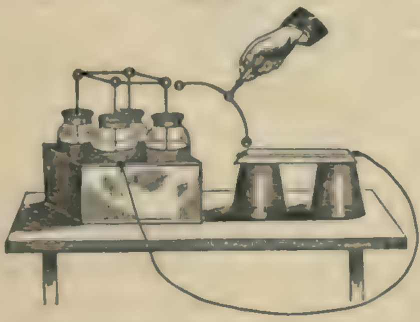
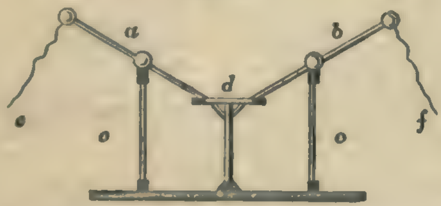


Fig. 29.



We have frequently shivered into almost dust, by means of the hydro-electric machine and twelve large jars, pieces of glass a foot long and four inches wide. For this and many other similar experiments, a convenient arrangement may be made, which is illustrated in the following figure.

Fig. 30.



In the engraving, *a b* are two brass rods, which are easily moved on universal joints, secured to two glass pillars, *c c*; *d* is a small table, on which the object to be experimented on is to be placed; *e* and *f* are two wires for conducting the electricity from the battery. One of these is to be attached to the outside of the battery, and the other to the discharging rod, so that when the battery is charged, the electricity may be thereby conveyed to the table of the apparatus. We shall suppose that the student will supply himself with an arrangement of this kind, which, in fact, he can easily construct, in a rough way, himself; and which he will find a matter of great convenience. We shall in future call it the "universal discharger." We may add, that a similar arrangement will be required for many experiments in voltaic electricity. In the engraving, we have represented the ends of the discharger as terminating in points; two brass balls will, however, be occasionally required in some of the experiments.

Experiment 26.—Place a few sheets of paper between the two points of the universal discharger, and pass a charge through them. The mechanical effect will be so great as to perforate them with ease.

Experiment 27.—Place a piece of loaf-sugar on the table, and bring the ends of the discharging rod in contact with it on its opposite sides. The charge will pass through the sugar if the points be closely pressed on it, and it will be shivered into pieces. A beautiful phosphorescence will be noticed if the experiment be tried in the dark. A piece of dry wood, if similarly placed, may easily be broken to pieces.

Experiment 28.—Cork tightly one end of a narrow glass tube, and insert a wire in the cork, so that its two ends may be respectively inside and outside; and having filled the tube with distilled water, cork up the other end, with a wire also inserted in the cork, and let the two wires be separated to a short distance in the tube from each other. Place this on the table of the discharger, and bring the two rods in contact with the two wires of the tube. If a charge from the battery be now effected, the tube will be burst into pieces, owing partly to the mechanical expansion of the liquid, the effects of the heat generated, and the partial decomposition which the water undergoes. We annex an engraving, which will assist the student in carrying out the experiment. (See Fig. 31).

Fig. 31.

We must here warn our readers, that care should be taken lest the pieces of glass be driven into the face of the experimenter: a small iron cage placed over the tube is a prudent precaution.

Experiment 29.—A very pretty luminous effect is produced by sending the battery charge through a piece of alum. This is best done by boring a hole on each side of the salt, so that when the points of the rod of the discharger are inserted, they may be at a distance of a quarter of an inch from each other. On the discharge being effected, the alum will become beautifully illuminated, and be broken to pieces.

In the preceding experiments we have chiefly observed the mechanical and luminous effects of frictional electricity; we will, therefore, give some instances in which the calorific effects of the force are manifested.

Experiment 30.—Connect the two rods of the universal discharger by means of a very fine piece of steel wire, about six or eight inches in length, and send the charge of the battery through it. The wire will be instantly melted, and will fall down in fine globules. Steel and other metallic wires may be thus melted. This result may be obtained by a single Leyden jar, if of about three pints capacity: the wire is to be attached to one knob of the common discharger, and so may be brought into contact with that of the inside of the jar; the other knob of the discharger must rest against the outside of the Leyden jar.

Experiment 31.—A gas-lamp, or newly-blown-out candle, may easily be ignited by means of the battery. The ordinary discharger must have one knob in contact with one belonging to the jars of the battery, and the other knob is to be brought nearly into contact with the gas jet. A wire should stretch from the metal pipe to the outside of the battery.

These experiments will be sufficient to illustrate some of the mechanical and other effects of a powerful electric charge. We shall now proceed to put our remarks to a practical purpose, by referring to the use of lightning conductors in protecting tall buildings, chimneys, steeples, masts, &c.

ATMOSPHERIC ELECTRICITY—LIGHTNING CONDUCTORS—THE ELECTRIC KITE, Etc.

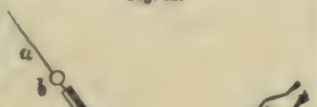
There are, perhaps, none of our readers who are not familiar with the destructive results which are occasioned by severe thunder-storms; and we shall now enter into some examination of the phenomena of atmospheric electricity, and explain the inventions of various kinds which have been brought out for the purpose of averting the danger arising from an over-accumulation of electricity in the air.

Franklin was the first to discover that atmospheric and frictional electricity are identical in all respects. He was also the first "to draw fire from the clouds;" except we give that honour to Prometheus, whom some have considered to have been the earliest inventor of lightning conductors. Franklin, for the purpose of ascertaining the presence of electricity in the air, raised a small kite; having chosen, for that purpose, a day when a thunder-storm was expected. For some little time no success attended his experiment; but when the string became wet, and so was a better conductor, he was enabled to draw sparks from it by means of a key, and so solved at once the problems of atmospheric electrical disturbances. Perhaps some of our readers may be desirous of repeating Franklin's experiment; and it will, therefore, be as well for us to give a little advice on the subject, and so to diminish the chance of danger which always arises in such attempts.

A kite, from four to six feet long, may be employed; and, at the top, a sharp metal wire should project about two inches. This wire is to be connected with the long string by which the kite is to be flown. This string should have a fine copper wire twisted inside the external hemp, which is easily done; and, of course, highly improves its conducting power.

Great care is to be observed in flying the kite. After raising it a short distance, a dry silk handkerchief should be slipped round the string, noose-fashion, and the extreme end of the string may be fastened round a tree. The kite is raised by holding the string by means of the silk handkerchief, so that the former may readily pass through it. Of course, the object of the silk is to act as a non-conductor, and so to prevent the passage of the electricity to the body of the person holding it. The elevation of the kite may be from 200 to 1,000 feet. The portion of the cord wound round the tree may be loosed when the kite is sufficiently elevated, and its end is to be attached to the silk handkerchief, and it will be ready for experimenting with. It is well to have the rod on which the string is wound made of copper, with a ring at each end. By means of this ring and the silk handkerchief, the string may be fastened to a well-dried stick thrust into the ground, as shown in the above figure; and so the conducting link will be broken with the earth, and sparks may be obtained from the metal rod. In Fig. 32, *a* is the metallic string proceeding from the kite, and fastened on the ring *b*; *c* is the metal rod; *d* is the ring to which the silk handkerchief, *e*, is attached; *f* is the ring of a post of dried wood, fixed in the ground; *g* is a discharger, the glass handle of which should be at least two feet long; and *h* is a wire conducting the electricity from *d* to the ground.

Fig. 32.

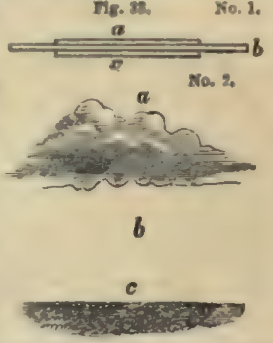


The kite experiment should never be tried during stormy or hazy weather, because such would most likely prove highly dangerous. Fatal accidents, indeed, have occurred from want of caution; and we should not have given directions for carrying out the trial, except under the idea that some of our readers might resolve to attempt it for themselves, and, in the absence of proper precautions, might have sustained injury. We have frequently adopted the plan we have described, and therefore recommend it with confidence. It is, however, advisable that no tyro should ever attempt to fly the kite under any circumstances.

The most celebrated electrical experiments of this kind, of modern times, are those which were conducted by Mr. Crosse, of Bromfield, near Taunton, who was a complete devotee to electrical science. This gentleman erected, on ground near his house, a large number of metallic rods, which were insulated at their lower end, and joined together by a rod, the terminal of which entered a room specially arranged for electrical experiments. Opposite to the terminal was placed a rod of metal, which, by sliding to and fro, could be fixed at any required distance from the rod communicating with the external conductors. By these means he was able to conduct a thunder-storm and its effects into a small space, and the results he obtained were of the most extraordinary character. During stormy or hazy weather, brilliant flashes of light were observed between the conductors, accompanied with terrific crashes of sound, and violent reports. The entire effects were sometimes quite appalling, and cannot be conceived by any one who has not seen them.

Now, it is plain, from these experiments and results, that, by means of proper arrangements, electricity may be conducted (to use an ordinary phrase) from the clouds. But we must now be more philosophical in our remarks, and explain the phenomena of thunder-storms, &c., in accordance with the principles already laid down. In doing this, we shall materially assist the student in understanding the circumstances which attend the development of atmospheric electricity, by comparing the arrangement of the Leyden jar, or piece of flat-coated plate, with that of the clouds, air, and earth. We shall consider, therefore, that the air corresponds to the glass of the jar, the clouds to its inside, and the earth to its outside coating. Lightning conductors we shall hold in the light of a discharger, such as is usually employed in our experiments. This may be better understood by the illustration which the annexed engraving affords.

Fig. 33.



In Fig. 33, No. 1, we have a piece of window-glass, *b*, coated at *a* and *c* with tinfoil. Now, we have already shown, that if *a* or *c* be positively charged, the opposite will become negatively so by induction. Similarly, in No. 2, we shall suppose the clouds, *a*, to be charged positively; the air, *b*, which corresponds to the glass, *b*, in No. 1, to be intermediate between the clouds and the earth; and the earth, *c*, by induction, will necessarily be in a negative state.

In a former page,* we have stated, that when a jar is overcharged, it will generally discharge itself; and this is exactly what occurs in nature. The high charge of the clouds is spontaneously discharged to the earth; and just as we have the spark and snap of the jar, so, in the large scale in nature, we have the flash of lightning and the clap of thunder. It will thus be seen that the analogy is complete.

We may, however, go a step further, and remind our readers, that if a wire be stretched from *a* to *c* (No. 1) whilst the former is charged, the discharge takes place without any external manifestation. In the very same way if a tall metallic rod be raised from the ground, *c*, towards the clouds, *a* (No. 2), a silent discharge at once takes place; and this is precisely the way in which lightning conductors act. The common notion is, that lightning conductors *attract* the lightning—an idea which is entirely erroneous. They simply form the readiest and most effectual means of restoring the electric equilibrium of the earth and clouds, and only so far are they of any use or purpose.

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* See *ante*, p. 179.

We may briefly glance at some practical matters in connection with this subject, which may be useful to any of our readers desirous of putting our remarks into active use.

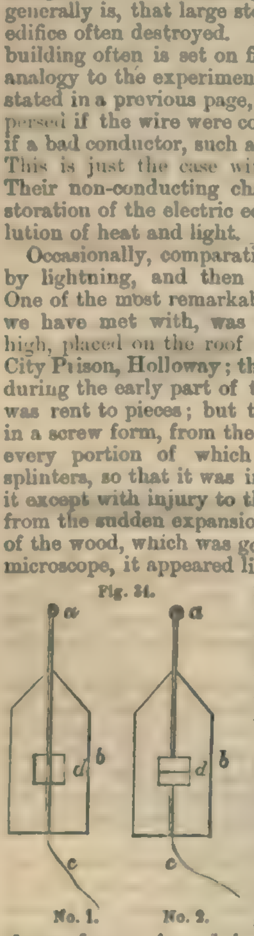
The conductor should have its upper end pointed, for reasons previously explained.* Its lower end should be not less than from three to four feet imbedded in the earth, for the same reason that we connect the outside of a battery with a gas or water-pipe.† Copper, being a better conductor than iron, is to be preferred; but galvanised iron answers very well.

All the adjacent metallic objects, such as the leaden or zinc roof of a house, should be connected with the conductor, because they might possibly act in independence of it, and produce antagonistic results. The top of the rod should extend much beyond the highest part of the building, and should not be attached to the chimneys of private houses, because the fine soot they contain is, to some extent, a conductor.

In many large buildings, such as churches, the stones of the steeple, &c., are tied together by means of iron clamps. In the absence of a conductor, these produce the most serious results in a storm. The weathercock is first struck as the most prominent metallic object, and then every metallic tie in the building succeeds as a conductor. These being at a distance from each other, do not permit of a continuity of conduction; and the result generally is, that large stones are thrown down, and the edifice often destroyed. Under such circumstances, the building often is set on fire; and here we have an exact analogy to the experiment of igniting gunpowder. We stated in a previous page,‡ that the powder would be dispersed if the wire were continuous, but would be ignited if a bad conductor, such as a wet string, were interposed. This is just the case with the stones of the building. Their non-conducting character prevents the rapid restoration of the electric equilibrium; and hence the evolution of heat and light.

Occasionally, comparatively bad conductors are struck by lightning, and then destruction is sure to follow. One of the most remarkable instances of this kind which we have met with, was that of a flag-staff, forty feet high, placed on the roof of a large inn opposite to the City Prison, Holloway; the destruction of which occurred during the early part of the summer of 1861. The staff was rent to pieces; but the course of the lightning was in a screw form, from the top to the bottom of the wood, every portion of which was divided into the finest splinters, so that it was impossible to touch any piece of it except with injury to the hand. This seemed to arise from the sudden expansion of the moisture in the fibres of the wood, which was good pine. When viewed by the microscope, it appeared like a bundle of fine hemp fibres rather than solid wood. At the time of the occurrence, the entire house seemed filled with flame, but no further damage was effected. A strong sulphurous smell was noticed for some time afterwards.

Fig. 34.



An experimental illustration of the effect of non-conductors, or of breaks in the conductor, may be made by the arrangement represented in the annexed engraving.

In the cut, the same letters are used for each figure. *a* is a brass knob for receiving the charge from a jar; *b* is a piece of dried wood, in the shape of the gable of a house; *c* is a wire conducting to the earth; and *d* is a small square of wood, with tinfoil pasted on it, and movable in the larger frame. Now, in No. 1, the conductor is continuous from *a* to *c*, and so the piece of wood, *d*, would not be disturbed by a charge. But if a jar be discharged on No. 2, then the wood, *d*, will be rapidly ejected from its place, because the conducting course is broken by its position and that of the tinfoil being changed.

Trees being moderately good conductors, are often similarly injured; but unless they are very dry, they are not, generally speaking, cloven into pieces. The rain which usually falls in a thunder-storm, improves the conducting power, and so renders them exceedingly dangerous as places of shelter for man or animals. A singular circumstance came under our notice, three years ago, at Glasgow. Three persons had taken shelter under a tree, which was subsequently struck with lightning. They were all prostrated and rendered insensible for a time, from which they soon recovered; but, singular to say, the heels of their boots were completely torn off, owing to the conducting power of the nails in them. This is another illustration of the remarks we have just made, in reference to the necessity of continuous conductors from the different metallic arrangements in buildings.

It seems a tolerably well ascertained fact, that persons struck by lightning fatally, undergo rapid decomposition. The cause of this is by no means evident: it is stated, however, in such instances, that the albumen of the blood does not coagulate, as is usual in ordinary cases of death.

When injury by lightning has taken place, it is well to treat the person in the same manner as would be adopted for drowning; the object being to restore, as speedily as possible, suspended animation. The latter is caused by a cessation of the action of the heart, and consequent asphyxia. Occasionally the application of electricity may be tried when the means of doing so are within reach; and, on breathing and feeling being restored, recourse should at once be had to the warm bath, and subsequent rubbing with very coarse towels.

Before quitting the subject of thunder-storms, we may make a few observations on the cause of the rolling sound of thunder. This is entirely owing to the echo produced by the clouds overhead. The noise of the discharge would, like that of the Leyden jar, be single were it not for this cause. On a cloudy day, if the report produced by the discharge of cannon be noticed, a precisely similar effect is produced, and successive, instead of single sounds, are heard.

Another atmospheric phenomenon, most commonly observed in the northern regions, is the aurora borealis, the cause of which we ascribe to a discharge of electricity in a highly rarefied medium. We have already given an experiment illustrating its appearance in a previous page,§ and shall not further enter into the subject here, as it more properly belongs to the department of meteorology.

PHYSIOLOGICAL EFFECTS, AND ANIMAL ELECTRICITY.

UNDER the above heading we shall include some of the physiological effects of electricity, together with a short account of fishes, which have the power of transmitting electric shocks from their bodies. We shall have to extend considerably on this subject in our next chapter: we shall therefore here confine our remarks to those effects which are due to electricity excited by friction, or its analogues.

Physiological Effects.—If the knuckle be presented to the conductor of an electrical machine, a tingling sensation will be experienced as the spark passes. If, however, one hand be placed on the knob of a charged jar, whilst the other grasps its external coating, a powerful shock will be felt; which will possibly, if from a pint jar, extend as far as the chest. This effect is produced by the sudden and powerful contraction of the muscles. Any number of persons may readily take a shock simultaneously (if the jar be large enough), by joining hands, and by the last person at each end respectively touching, one the knob of the inside, and the other the outer coating of a charged jar. A convenient way of doing this, is illustrated in the following engraving.

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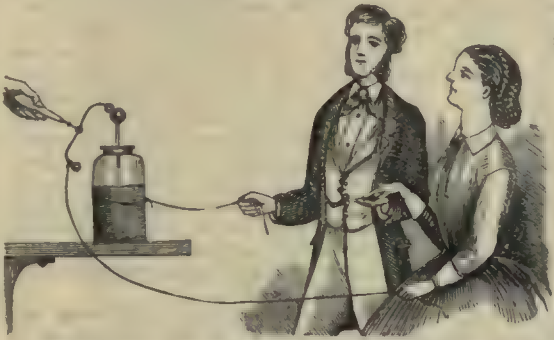
* See *ante*, p. 175.

† *Ante*, p. 179.

‡ *Ante*, p. 181.

§ See *ante*, p. 173; Fig. 6.

Fig. 35.



By this mode, one person holds a wire proceeding from the outside coating of the jar: the last person in the series, holds the wire which is attached to the discharger; and an assistant connects it with the knob of the charged jar, and so gives the shock.

Individuals vary much with respect to their susceptibility of electric shocks. If several persons take a shock together, those having a moist skin on the hands will feel it most acutely. If, however, a shock be taken so that it may pass over the whole body, then the moisture on the skin diminishes the pain by making the body a better conductor. Persons with a harsh, dry skin, feel a shock, with a number of persons, less than they do when taking it singly; and owing to this, a few persons having a dry skin, placed amongst others, will retard and diminish the effect on those with whom they may be joined by the hands. For that reason, the hands are sometimes wetted to improve the conducting power. We have frequently taken, whilst in a violent perspiration, with impunity, a two-inch spark from the hydro-electric; and have also been amused at others, whose skin was entirely dry, calling out, in severe pain, with a shock of the same intensity, which they were induced to try from observing its trifling effect on ourselves.

Considerable care should be observed in giving an electric shock, in any form, to females and young children, as one too severe might be productive of serious injury, in consequence of its effect on the nervous system. A half-pint charged jar is quite large enough for one person; whilst one from a quart to three pints capacity, will give a strong shock to twenty or more persons joining hands together.

Electricity, in the shape of sparks, was, at one time, in high favour in medical circles, for relieving acute pain in neuralgia and nervous complaints generally; and, at its first application, like all other novelties, it was held out as a panacea for "all the ills that flesh is heir to." In the present day, the coil machine (which we shall describe in a future page) is chiefly employed for that purpose, because it is easily set into action, and the strength of the shock, which may be continued for any length of time, is entirely under control.

There is no doubt that electricity, in animal bodies, is an active agent in sustaining life. The nerves and the muscles are not only affected thereby, but always exhibit a current of electricity if proper means be employed to detect it. This is a subject, however, on which we shall have to expand hereafter, as it more particularly belongs to that of voltaic electricity.

Electrical Fishes.—Some animals can set free from their bodies, spontaneously, a large amount of electricity: amongst these are the *torpedo*, the *gymnotus electricus*, and the *silurus electricus*.

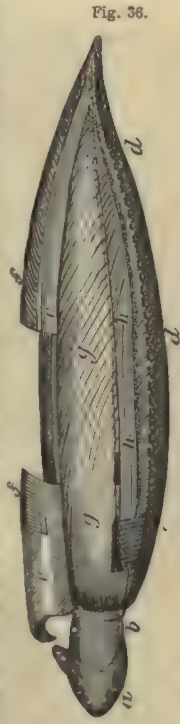
Each of these fishes has the power of giving an electric shock. The *torpedo* is a native of Europe; and its electric organs are placed in its body in a manner which is analogous to the arrangement of a Leyden jar; for the shock is obtained by placing one hand on the belly, and the other on the back of the fish.

The *gymnotus* is a fish of the eel kind, and is chiefly found in the rivers of South America. It has a very powerful effect on other animals, and can not only stun them, but even destroy life by its shock. One of these was exhibited, some years ago, at the London Polytechnic. It was about three feet long, and was kept in a circular tank of water. Two wires were laid inside; and these extended to the outside of the tank, and ended in two basins of water. If the hands were dipped in the basins, a powerful shock was experienced, about equal to that of a fully-charged half-pint Leyden jar. The strength of the shock varied, however, from day to day, according to the activity of the fish.

The *silurus electricus* is a native of Africa, and is an inhabitant of swamps, &c. Its shock is similar to that of the Gymnotus, but its natural history is not so well known.

In the annexed engraving, a representation of the various internal organs of the *gymnotus electricus* will be observed.

Fig. 36.



In the above, *a* represents the head; *b*, the cavity of the body; *d d*, the ventral fin; *e e*, the skin; *f f*, the muscles of the fin; *g g*, the electrical organ; and *h h*, the smaller organ.

CHEMICAL EFFECTS.

The chemical effects produced by frictional electricity are so feeble as to be scarce worthy of notice, were it not that they complete a chain of analogy with similar results produced on a much larger scale by other modes of developing the electric force. This is owing to the fact, that electricity obtained by friction, has great intensity or power of overcoming obstacles presented by bad conductors, such as the air, &c.; but has little or no quantity, or that which would tend to act on a large surface or mass energetically. A few plates of zinc and copper, arranged as a voltaic battery, rapidly decompose water which has been rendered a better conductor by the addition of some sulphuric acid; whereas, pure water, which would scarcely be acted on by voltaic electricity, must be employed for the purpose of decomposition by electricity obtained by friction.

The mode usually adopted for this end, is that of employing two gold wires, the extreme points of which are alone exposed to the liquid. A continuous current from a large plate machine, the conductor of which is connected with one wire, may be used; the other gold wire is to be connected with the earth; and so a continued current passes between the points of the wires, which should be near to each other. Minute bubbles of gas will be seen to rise, which are those of oxygen and hydrogen. The process is extremely slow, difficult of pursuit, and requires care in the entire arrangement. Minute quantities of metallic solutions, such as of sulphate of copper, have been decomposed by the action of the hydro-electric machine, which was evidenced by the deposition of metallic copper on one of the wires. Feeble, but decided evidence has been adduced, that the *gymnotus electricus* has the power of decomposing water—a result which, we believe, was first obtained by Dr. Faraday. This fact goes to prove the identity of the electricity obtained from the fish, with that produced by friction.

CHAPTER II.

VOLTAIC ELECTRICITY

It may, perhaps, be no exaggeration to say, that scarcely any branch of experimental science has produced so great a social revolution as that which we are about to investigate. Within the short space of some sixty or seventy years, that which was, comparatively speaking, a philosophic toy, has become the cement of national, social, and commercial relations. The simple experiment which, according to the generally received opinion, first gave birth to the idea that electricity could be set free by chemical action, has expanded to an extent far exceeding the most sanguine hopes of its discoverers, and has become, in its applications, a positive necessity of daily life.

Most of the remarks and experiments we have given in connection with frictional electricity, have a decided bearing on the subject with which we now have to deal. Indeed, we shall presume that the student and general reader has entirely mastered all the laws and phenomena (so far as the present state of our knowledge will permit) to which reference has already been made. Even the terms and expressions we have employed will have to be constantly repeated in our future pages, and, as such, they will form the groundwork of further progress.

The history of voltaic electricity, or, as it is sometimes termed, galvanism, is connected with the early researches of two Italian philosophers. It is stated, that Galvani first noticed how the contact of a metal with the nerves of the leg of a frog, produced muscular contractions in the limb; and that Volta, extending the idea, found that the "contact of two metals" was sufficient to produce various electric phenomena. We have hitherto carefully abstained from entering into any discussion as to the relative merits of individual discoverers or inventors. The natural pride and conceit of humanity is always prostituting to its own purposes the novelties of scientific research which are continually arising; and we can refer to no fact more degrading to the dignity of human intelligence, than that its chief possessors have, from the days of Newton and Leibnitz, been found engaging in personal disputes as to the priority of discovery, where humility of mind and a pure love of science ought to have been the reigning principle. Unfortunately, it is too true, that to err is human; and yet we may, and ought not to be too Utopian in our hopes, if we predict that the day will arrive when the consciousness of having extended the limits of our knowledge, and of having been the instruments of benefiting our fellow-creatures, shall be the highest reward to which "philosophers" can aim at.

It is useless for us to describe the contests which took place, many years ago, between those who maintained the theory of contact, and others who advocated the idea of chemical action, as the cause of the development of voltaic electricity. The former imagined that the mere contact of two dissimilar metals was the sole exciting cause. Dr. Faraday, however, by a series of masterly researches, proved, that on such contact taking place, chemical action instantly ensued; and, further, that electricity was evolved by chemical action in circumstances where contact was impossible. We may add, that just in proportion to the chemical action in a voltaic battery, so the quantity of electricity set free is increased; and, at the present time, we are not aware that any advocates of the contact theory are to be found. We shall at once proceed with our subject, and commence our experiments with voltaic batteries.

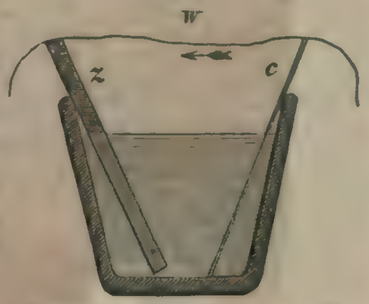
VOLTAIC BATTERIES.

It is an unfortunate circumstance, that many of the terms employed in science do not sufficiently indicate the meaning and object they are intended to express; and hence the beginner incessantly finds obstacles to his progress. An amusing illustration of the ignorance of even well-educated persons, with respect to this point, came under our notice whilst effecting a fire insurance some years ago. The agent of an office gravely objected to the presence of voltaic batteries on the premises; expressing an opinion that they were highly dangerous, and enforcing his views by stating that they never issued a policy when *gunpowder* was kept in a building. He had confounded the term *battery* with its military signification, and was utterly surprised to see fifty cells of Grove's "batteries," "charged" with acid and water, instead of the *matériel* of warfare.

A voltaic battery is a combination of cells, each containing the materials requisite to the production of electricity by chemical action; and we shall now proceed to examine such arrangements theoretically and practically.

If a piece of zinc and a silver coin are placed on either side of the tongue, no sensation of any sort is experienced until they come in contact in some part: but on that taking place, a peculiar metallic taste is perceived in the mouth. The elements of a voltaic battery are employed in this experiment. A current of electricity is excited by the action of the moisture of the tongue on the zinc; and it passes from the zinc, through the liquid to the silver, at their point of junction, and so back to the zinc again.

Fig. 37.



A more satisfactory mode of illustrating this interesting fact, is represented in the annexed engraving.

Experiment 32.—Place a sheet of zinc which has been cleaned in a little sulphuric acid and water, and rubbed over with mercury till it is quite bright, in a glass vessel, and opposite to it a sheet of copper, so that they do not touch. The glass is then to be nearly filled with a mixture of one part of sulphuric acid to eight parts of water. No action of any sort will be perceived. But if a wire be placed across both plates, as represented in Fig. 37, so that it may join the two, then a current of electricity will pass from the zinc plate, *z*, through the liquid to the copper plate, *c*, and, passing by the wire, *w*, in the direction indicated by the arrow, it will at last return to the zinc plate; abundance of hydrogen will be given off from the copper plate. If the wire be broken in any part, and its extremities be kept in contact with each plate, the tongue will feel the same sensation when in contact with the ends of the wire, as that which we have already alluded to; and thus the existence and passage of an electric current will be at once made evident to the senses. Each of the plates of the battery is named in accordance with the theories we have already discussed in the last chapter.[*] The zinc plate is called the positive, and the copper the negative, element of the battery; and we may here lay down, as a general rule, that that metal which is most acted upon in a voltaic battery, is positive to the other metal.

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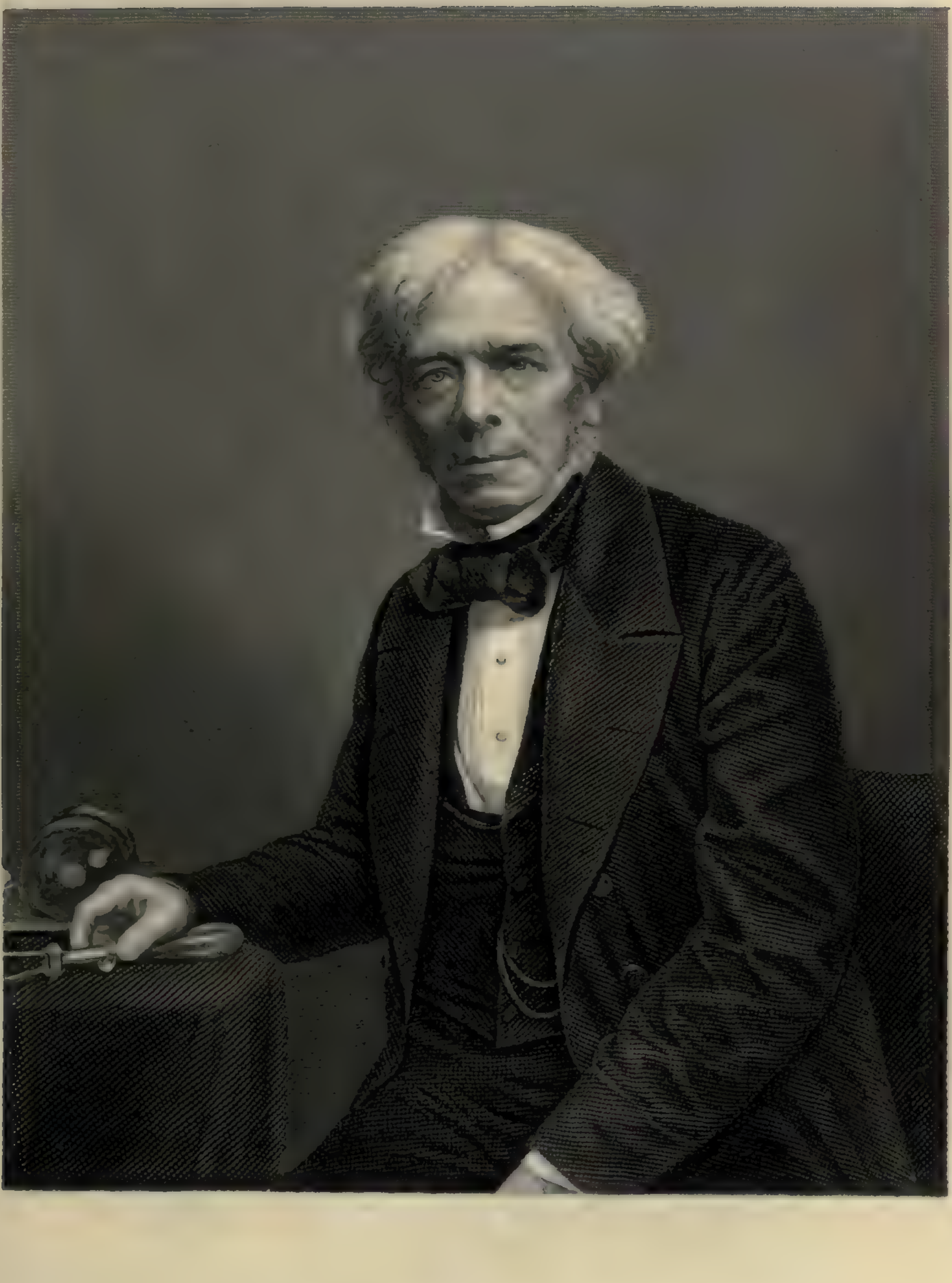
[*] See *ante*, p. 178.



MICHAEL FARADAY, ESQ., F.R.S., D.C.L.

[illegible]

There are several circumstances into which we shall here inquire, which modify, increase, or diminish the power of voltaic batteries. They may be shortly summed up as follows :—

1. The extent and rapidity of chemical action.
2. The size of the plates.
3. The conducting power of the liquid used as an excitant; and of the wire, &c., joining the plates, external to the cell.

We have already mentioned, that there have been those who have strongly advocated the idea, that the production of electricity by the voltaic battery, is due to the contact of dissimilar metals. The causes of the modification of the power of the voltaic battery, which we have given above, recognise no such an opinion. Indeed, it is not necessary that we should bring into contact the two metals at all; for in all chemical decompositions affected by voltaic electricity, the metals are never connected by a wire—as shown in Fig. 37;* but merely by the liquid which is the subject of the electric action.

1. That the power of a cell depends on the rapidity of the chemical action which proceeds on one of the metals, is made evident by the following experiment :—

Experiment 33.—Connect a piece of zinc and copper plate, each four inches square, by means of a very fine piece of steel wire. This is best done by soldering a thick copper wire to each plate, round which the steel wire is to be twisted, so as to leave about half an inch of the fine wire between them. Dip the plates into common water, and no effect will be perceived on the fine wire. Now replace the water by strong sulphuric acid, diluted with six times its bulk of water, and introduce the plate into it. The steel wire between the two copper wires will be instantly melted.

2. The size of the plates has the effect of increasing the power of a cell, in respect to the quantity of electricity which is set free. Thus, in the last experiment, a very thin wire was recommended, because the plates were small. If, however, large plates be employed, then just in proportion to the extent of the surface of the plates, so will greater thicknesses of wire of the same kind be melted. It is hence usual to adjust the size of all voltaic batteries to the work which they are expected to perform. This will be particularly noticed when we have to refer to the chemical decomposition, &c., carried on during the electrotype process.

3. The conducting power of the liquid in each cell, and that of the wire, &c., external to it, are matters of the highest importance. Liquids, in an electrical point of view, are, generally speaking, bad conductors; but, in a voltaic battery, this fact is modified by means of mixing acids, &c., with the water used as an excitant. Pure water has so little chemical action on zinc, and is so bad a conductor, as to afford only the most feeble evidences of disturbing electricity; whereas, if a little acid be added thereto, the result is, that rapid chemical action may be speedily induced. The conducting power of a liquid is inversely as its length; hence it is essential that the two plates of a cell should be placed as near as possible to each other without touching. As an illustration of this, we may state, that twelve sets of copper and zinc plates, each at a distance of an inch from each other, do not afford a tenth part so much effect as when they are so arranged as to be in as close proximity as possible without touching. The addition of sulphuric, nitric, and hydrochloric acids, of salts, &c., to the water of a cell, increases its conducting power almost in proportion to the extent of their presence; hence strong solutions of the above are always employed when brilliant effects are desired from voltaic batteries. The wires forming the terminals of a battery or single cell, always, like the liquid in the cell, afford a resistance to the passage of electricity; and this depends on their thickness and length. Different metals vary in their conducting power—gold, silver, and copper standing the highest, and lead, tin, zinc, &c., lowest in the scale. For all practical purposes, copper wire is employed as a conductor in all voltaic batteries; and we suggest the use of flat ribbons of the metal in place of wire, as being more convenient where long conductors have to be arranged.

A voltaic battery consists simply of a number of single cells joined together. The zinc of one cell is attached, by means of a wire, to the copper of the next; and so on, throughout the series. By this arrangement, one zinc and one copper, at the extreme ends of the battery, will be unconnected; and it is to these that the terminal wires, by means of which the experiments are carried on, are attached.

The *intensity*, or power of a battery to overcome resistance offered by bad conductors, such as air and water, depends on the *number* of cells employed.

The *quantity*, or power to overcome the resistance of good conductors, such as metallic wires, in melting them, depends on the *size* of the plates. On the regulation of these two particulars in the construction of voltaic arrangements, their efficiency for all purposes is governed. It may be well that we should here give some general directions as to the management of voltaic batteries generally, before proceeding to describe the various kinds which have been invented. And first, the zinc should always be amalgamated. This prevents the acid acting on the metal, except when the battery is in action; and it is easily performed. The zinc plate should be dipped into some strong sulphuric acid and water, until abundance of gas arises from its surface. It may then be laid on a plate, and mercury is to be rubbed over it until the whole presents a highly polished appearance on both sides. If the metal be now dipped again into the acid water, no gas will arise from its surface, because the mercury has covered over a vast number of impurities, consisting of iron, &c., which are in the zinc, and which formed little batteries on its surface. This process has been of the utmost importance in the application of voltaic batteries; and diminishes to a great extent their working cost, by economising the zinc.

Every junction of metals, wires, binding-screws, &c., should be kept quite bright; and, wherever possible, soldering should be employed, as complete contact of all connecting arrangements is very essential. We have seen batteries in a dirty condition which would scarcely melt an inch of wire; and yet, with the same charge, they would melt five or six feet, after all the connections were carefully cleaned. For this purpose common sand-paper should be used, or sand may be rubbed on to the metals, by means of a damp cloth, until they are quite polished. In winter-time the action of a battery will quite cease, if the weather be very cold. Of course, the only remedy is, to carry on the experiments in a warm room, or to heat the solutions before charging the battery.

In using those batteries having porous cells and two liquids (Daniell's and Grove's), great care should be taken not to allow the two liquids to mix, as this would ruin the zinc: and the porous cells, when done with, should be completely emptied, and left to soak in water, so as to remove all salts, which would crystallise and break them to pieces.

It will occasionally happen, in large batteries, that the middle cells will get very hot, and the liquid will evaporate from them. This of course must be watched, and, if necessary, they must be refilled. The student should bear in mind, that the power of the battery is regulated by the action of the *feeblest* cell of the whole series. It is therefore essential that each should have the same charge, equal-sized plates, and that the connection should be perfect throughout. After using wires for voltaic experiments, they sometimes become completely rotten, and break with the least touch. This is remedied by heating them red-hot, which at once softens them.

DIFFERENT FORMS OF VOLTAIC BATTERIES

HAVING explained the construction and principles of an elementary battery, we now proceed to notice the various forms which have been suggested as convenient modes of obtaining voltaic electricity for experimental purposes. In so doing, we shall be very brief with those which are rarely used, and shall omit many, the only claim of which to notice would be their novelty, or the ignorance of their inventors.

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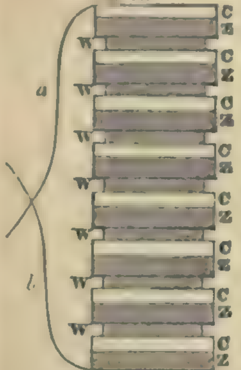
* See *ante*, p. 186.

The first we shall examine is the pile, which was the invention of the celebrated Volta. It is easily constructed by means of zinc and copper plates, of each of which there should be about forty, of a size of two or four inches square. Forty pieces of woollen cloth or brown paper are required, which are to be soaked in strong salt and water, and left to drain. The pile is made by building up the series in the order of zinc, copper, cloth, and so on; a wire is attached to the last plate at each end; and so the pile is completed. One of these is represented in the annexed engraving; in which the letters or initials indicate the position of the metals and the cloth. (See Fig. 38).

Fig. 38.

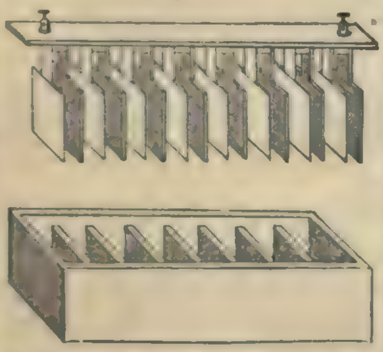


If the two wires, *a b*, be crossed, then a current of electricity is at once produced; and if one of these points be scratched over a file, the other being in contact with it, bright little sparks are afforded. A shock will be felt if the hands are moistened, and grasp the terminal wires.

We need not say that such an arrangement is extremely inconvenient; for it almost surely happens, that the first experiment determines the existence of the pile, by upsetting it altogether: if not, the salt drips from each cloth, and so makes a continuous conductor—destroying the power of the battery. Mr. Cruikshank improved on it by placing the plates, soldered together, in a water-tight box, which was charged with acid and water; and a modification of this form of battery—the cells being filled with wet sand, to prevent spilling the liquid—is still employed by the telegraph companies at the present day.

The next form of battery which came into favour was that of Wollaston's; in which a porcelain trough, to hold the acid water, was employed; and the plates were screwed into a frame. The zinc of one cell was connected with the copper of the next, by means of a copper band, and nuts and screws; and at each end of the frame a binding-screw was placed, which served to hold the conducting wires. One of these batteries is represented in Fig. 39.

Fig. 39.



We have constructed an arrangement of a similar kind; in which each plate was less than the twelfth of an inch from each other; and forty of these will enable the student to try numerous experiments at little cost. A wooden trough, pitched inside, and without divisions, will do to hold the acid.

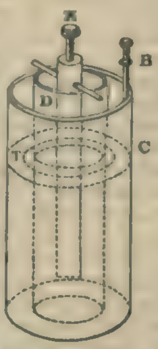
But to all these forms of batteries there is the fatal objection, that the power diminishes from the moment the arrangement is put into action. This arises from a variety of causes, but chiefly from the adhesion of the hydrogen to the surface of the copper plate, which as effectually injures the conducting power as if it were varnished over. For many years electricians combated with the difficulty. Sir Humphry Davy's brilliant discoveries were all made with the instrument we last described. Batteries of immense extent were constructed, which, by their number of cells or the size of the plate, were intended to compensate for these deficiencies. Eventually, Dr. Daniell, of King's College, London, discovered that the adhering hydrogen may be readily removed by presenting the oxide of a metal, in a state of solution, to it; and by the extension of this principle, we can now repeat all the usual class experiments by a battery a few inches long and broad, instead of the expensive and cumbersome arrangements previously in use.

If a piece of common zinc or iron be dipped in sulphuric acid and water, abundance of hydrogen is given off from their surface. But on dissolving a quantity of sulphate of copper (blue vitriol) in the liquid, it will be found that the gas will cease to rise, and a coating of pure copper will be deposited on the metal plate. Now this is an instance of what is called *secondary* action, and may be thus explained. The sulphate of copper contains that metal in combination with oxygen; the hydrogen, rising from the zinc, seizes the oxygen, and forms water, and the metal being set free, falls down as a powder, or is deposited on the iron plate. Omitting, for the present, the action which takes place on the iron or zinc, the following diagram illustrates the changes which occur.

Materials.		Products.
Hydrogen rising from the zinc plate		Water.
Sulphate of copper, composed of	Oxygen	
	Copper	Copper.
	Acid	Acid.

A Daniell's battery is constructed on the above principles. It consists of an outer case of copper, a porous earthenware pot, and a rod of zinc; and one of them is represented in Fig. 40: in which C is the outer copper vessel; D the porous pot; Z the zinc rod, on which is a binding-screw—one also, B, being fitted to the copper cylinder. The zinc being amalgamated, is placed in the porous pot, which is filled with a mixture of one part of sulphuric acid to eight parts of water. The copper vessel is filled with the same acid mixture, which, however, has been first saturated with sulphate of copper; crystals of which are hung on a kind of tray, T, to replenish the solution with copper. If the binding-screws, Z and B, are connected together, a current of electricity is generated; and, after a short time, the inside of the copper vessel is coated with a pink metallic deposit, which is pure copper. This continues as long as the battery is in action, and the strength of the copper solution is kept up. Any number of these cells may be joined together by means of wires inserted in the binding-screws, the copper of one being connected to the zinc of the next, in the usual manner, throughout the series.

Fig. 40.



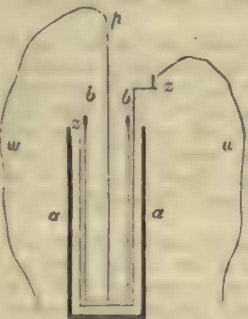
We have tried many modifications of this form of battery; and, for constancy of action, have found it to exceed any other. A common gallipot, a sheet of copper with a wire soldered to it, and a porous pot made of brown paper, with a slip of zinc in the centre, will answer very well for many experiments. In fact, twenty such cells will enable the student to perform many experiments on the small scale, and forms a cheap and effective little battery. Dr. Daniell produced some most brilliant effects with seventy cells, each twenty-one inches high. Some years ago, we fitted up eighty flat cells, each exposing a copper surface of two square feet; which, of course, was a much larger arrangement than that we have just named. The light between charcoal points, and other effects, were of the most astonishing kind.

In using Daniell's battery, it is essential that none of the copper solution should get into the pot holding the zinc; for it would immediately attack the metal, destroy its amalgamation, and lower the power of the battery.

The accidental occurrence of this is common, and is the chief objection to the use of the arrangement.

Grove's Battery.—The next arrangement to which we have to direct attention, is that invented by Professor Grove; which is always employed at the present time for lecture-table purposes, because it is the most powerful of all arrangements yet discovered. The same principles are involved in its construction as in that of Daniell's; but the hydrogen is removed by means of strong nitric acid; and platina, instead of copper, is employed. There is a great advantage in the use of platina, because it is quite unacted on; and the further the affinity of the two metals forming a battery, are apart, the greater is the power of the combination.

Fig. 41.



The annexed engraving illustrates a flat cell of a Grove's battery; in which *a* represents the outer porcelain pot; *z z* a plate of zinc, bent so as to fit into the pot, and rise up on each side; *b b* is a porous pot; and *p* a platina plate; *w w* are wires from each plate. The outer pot next the zinc is charged with sulphuric acid and water; and the porous pot with strong nitric acid. A powerful current of electricity is thus generated, which lasts so long as any nitric acid remains. Fumes of nitrous acid are given off; and this is a serious objection to the battery; for they destroy everything they come in contact with, making it necessary to place the instrument in some position where a strong draught can be obtained.

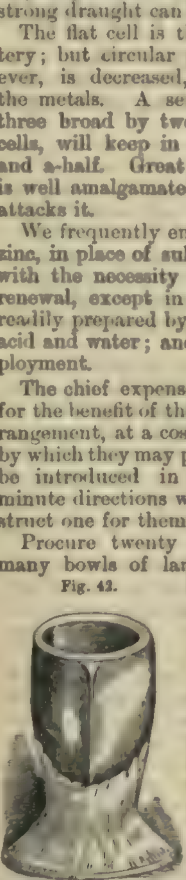
The flat cell is the usual form of arranging the battery; but circular pots may be used: the power, however, is decreased, owing to the increased distance of the metals. A series of flat cells four inches deep, three broad by two wide, with the usual-sized porous cells, will keep in action for about an hour to an hour and a-half. Great care must be taken that the zinc is well amalgamated before use, as the nitric acid soon attacks it.

We frequently employ a strong solution of chloride of zinc, in place of sulphuric acid and water. It dispenses with the necessity of amalgamation, and never requires renewal, except in the addition of more water. It is readily prepared by adding common zinc to hydrochloric acid and water; and many conveniences attend its employment.

The chief expense of this battery is the platina; but for the benefit of those who are desirous of a powerful arrangement, at a cost of not more than twenty shillings—by which they may perform all the experiments which will be introduced in our future pages—we shall give minute directions which will enable our readers to construct one for themselves.

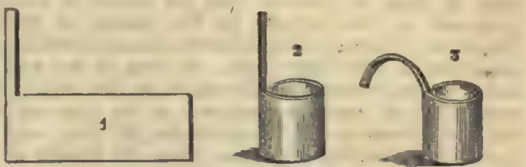
Procure twenty gallipots, two inches high, and as many bowls of large tobacco-pipes. The latter must have the small hole at the lower end stopped up by means of plaster of Paris, and a kind of stand made of the same material, to support the bowls upright, as seen in Fig. 42. It is as well to keep a few of these simple porous pots ready, in case of accident to those in use in the battery. Have cut, by a tin-smith, twenty pieces of zinc, six inches long and two inches deep. (See Fig. 43). The long thin strip may either be cut from the sheet, or be a piece of copper soldered on the zinc; it should be four inches long and half an inch wide. (1.) The sheet must be then bent into the form of a cylinder (2), and the copper or zinc strip is so bent as to form a curve (3).

Fig. 42.



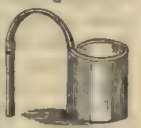
The zinc, when bent into form No. 3, must be amalgamated.

Fig. 43.



There must be soldered on to each of nineteen of the zincs, a piece of platina, a quarter of an inch wide and two inches long. This is easily done by wetting the end of the copper strip with a little muriatic acid, and then dipping it into a little soft solder melted in a ladle. Some of the solder will adhere to the copper; and by now holding it in the flame of the spirit-lamp, and placing the edge of the platina with it in the flame, the tin will melt and the platina will adhere, and must be allowed to cool. The whole, when finished, will appear as in Fig. 44.

Fig. 44.



By this easy plan, all the connections are complete without binding-screws. To one zinc, and to one platina (the first and twentieth of the series), a copper wire of No. 16 gauge, and thirty inches long, must be soldered. These will form the last plates of the series, from each of which the wires will serve to conduct the electricity.

The next step is to put the battery together. First, fill each of the tobacco-bowls with strong nitric acid, and place them in the centre of each gallipot. Take care not to spill the acid over into the outer pot, as that would speedily destroy the zincs. Then place the zinc cylinder, to which the wire is attached, in the first pot; and, in the second, put another cylinder of zinc, with its platina immersed in the tobacco-pipe bowl of the first one, the platina of the third zinc in the bowl of the second pot, and so on throughout the series.

Having thus arranged that the platina of each zinc shall dip into the bowl of the pot immediately preceding it, on arriving at the last pot, the platina, with the wire attached, must be placed in its bowl, and the wires kept steady by driving into a board two nails, round which each of them should be twisted, but kept separate.

The outside pots, containing the zincs, should each now be filled with sulphuric acid and water, in the proportion of one of the former to eight of the latter by measure. The mixture should be cooled before being poured into the cells.

A battery of this description will melt ten inches of thin iron wire; will give a spark of an eighth of an inch long between charcoal points; decompose water rapidly, potass, soda, &c.; and, in short, will answer every purpose which the student can require it for. We have frequently used such an arrangement for purposes of research, extending the number of cells to one hundred, from which some interesting results may be obtained.

Several modifications of Mr. Grove's battery have been proposed, the object having been to do away with the platina. Considerable misunderstanding seems to rest on this point. The first cost of platina is its only cost; and if it be carefully dealt with, and frequently heated red-hot, it will last almost any length of time. We may here observe, that we invariably heat the platina to a red heat before using it, so as to drive away a film of air which always rests on polished surfaces, and which wonderfully lessens the power of the battery by preventing the direct access of the acid to the platina surface.

Bunsen's carbon battery is exactly the same as Grove's, with the exception that some form of charcoal is employed in place of platina. The plate may be cut out of the carbon which forms the inside lining of gas-retorts, or may be made by pounding coke and making it into a paste, with sugar, &c., and pressing it into a mould to obtain the proper shape. The mass is afterwards heated to a red heat, and allowed to cool gradually.

Iron, especially in the cast state, has the singular power of becoming passive—that is, of resisting the action of nitric acid. It may, therefore, sometimes be used in place of platina; and we believe it was first proposed for this purpose by Dr. Callan, of Dublin. There are, however, several inconveniences attending its employment with nitric acid. So long as the acid remains strong, no action takes place on the metal; but, eventually, as water is produced in the iron cell, and the acid is weakened, it often attacks the iron, and the prodigious quantity of fumes given off render it impossible to come near the battery.

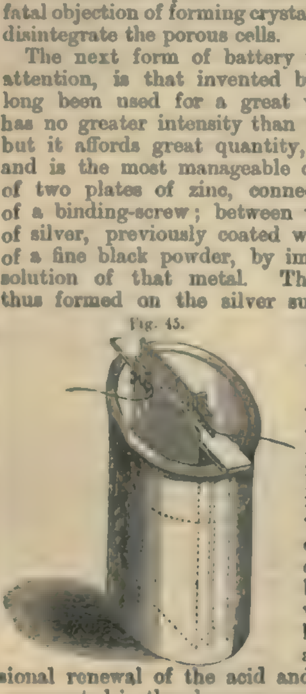
After all the modifications proposed, no arrangement equals Grove's platina battery, in any respect, where great power is required in a small space.

We noticed some very singular results, on employing a plate of copper instead of platina, in the nitric acid cell of a Grove's battery. The heating effects were quite equal to those of the platina arrangement; but, as might have been expected, a most violent action took place on the copper surface; still it maintained a negative state, in respect to the zinc, when tested with a galvanometer, and afforded a strong current for a considerable period. When the nitric acid was saturated with the metal, of course a kind of Daniell's battery was produced. The point which we chiefly noticed, was the fact, that the metal most powerfully acted on still retained its negative character, and the action was neither modified or suspended when the zinc and copper were joined by means of a wire.

In using a Grove's battery, it is of great importance that the platina surface should be perfectly clean. We have already noticed, that the film of air which always coats polished surfaces, frequently prevents the contact between the acid and the metal. Many persons use a charge in the porous parts of equal measures of sulphuric and nitric acid. This, although increasing the power temporarily, does not permit of so great constancy in the action of the battery, owing to the diminished quality of nitric acid.—The mixture must be cooled before charging the cells. Mixtures of nitrate of soda and common nitre (nitrate of potash), with sulphuric acid, have been recommended in place of nitric acid; but they have the fatal objection of forming crystallised salts, which speedily disintegrate the porous cells.

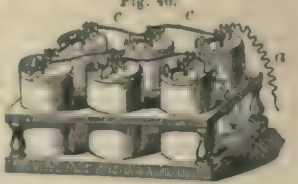
The next form of battery to which we shall direct attention, is that invented by Mr. Smee; which has long been used for a great variety of purposes. It has no greater intensity than the old form of battery; but it affords great quantity, requires but one fluid, and is the most manageable of any kind. It consists of two plates of zinc, connected together by means of a binding-screw; between which is placed a plate of silver, previously coated with platina in the form of a fine black powder, by immersing it in a chloride solution of that metal. The extremely fine points thus formed on the silver surface, throw off rapidly the hydrogen, which otherwise would adhere to the negative metal. The battery is charged with sulphuric acid and water, in variable proportions, according to the power required. This form of battery is perhaps the most suitable for electrotype experiments, especially for beginners; because, if the zincs are well amalgamated, the battery requires no further attention beyond an occasional renewal of the acid and water. A single cell is represented in the above engraving.

Fig. 45.



A number of Smee's batteries may be connected together, and they so form an efficient arrangement for various experiments. The mode of doing this is illustrated in the following cut; in which *a b* represent the end-wires of the battery; and *c c* copper bands, connecting the zinc of one cell with the silver plate of the succeeding one. As a guide to any of our readers who may desire to know the relative power of this and other arrangements, we may observe, that twenty cells of Smee's battery, each plate being four inches square, give the same heating, chemical, and luminous effects as a Grove's battery of four cells, with each platina four inches long and two inches wide. In each instance our measurement refers to the actually immersed surfaces in the exciting fluid.

Fig. 46.



The following results of experiments we have made with various batteries, may prove generally interesting.

1st. Discarding all the old forms of battery, we fitted up, amongst many other arrangements, forty-eight cells of Grove's flat-cell battery, each cell containing sixteen square inches of active platina surface. This battery gave an arch of light between charcoal points of the length of nearly two inches, and nearly three-eighths of an inch thick.

2nd. Seventy-two cells of Daniell's battery in *flat* cells. Each copper exposed a surface of two square feet, and was distant from the zinc about an inch on each side From this we obtained a splendid light of about an inch and a quarter long, and three-quarters of an inch thick.

3rd. Twenty cells of Grove's battery, with platinas in a central porous pot, surrounded by zinc. The platinas were fourteen inches long, and two inches wide. This battery gave a light of about three-eighths of an inch wide, and half an inch long; and made a piece of platina foil, of the same thickness as that used in the cells—one inch wide, and sixteen inches long—red-hot through its entire extent.

4th. Fifty cells of Bunsen's carbon battery. The carbons were one inch and a-half wide, and six inches long. This gave a flame half an inch long, and an eighth of an inch wide.

5th. Fifty cells of Grove's round pots, of the same size as the last, and arranged in a similar manner, but with platina instead of carbon, afforded, for nearly six hours, a light of one inch in length, and rather thicker than the last.

6th. One hundred cells of Grove's battery, of the same as that we have recommended[*] as to size, with tobacco-pipe bowls, &c., gave a light an inch and a-half long, and about as thick as an ordinary bell-wire. The shock from the battery was very severe.

Mr. Gassiot, of Clapham, obtained an arch of light three inches long, by using a battery of 100 cells, of the same size as those mentioned in No. 1, above.

In all these experiments, the effect on the eyes and face was very severe. The battery No. 2 produced a shadow at nine o'clock in the morning, when a piece of stick was held in the direct rays of the sun, shining clearly; and the experiment was tried about the early part of May, 1849.

There have been a vast variety of modifications proposed of the different forms of battery we have described; one of the latest being the use of cast-iron, in place of the copper of the old form of battery. We had twenty of these fitted up; the cast-iron being in the form of a flat cell, six inches high, four inches broad, and half an inch wide. Into each, a slip of amalgamated zinc was placed, which was covered with brown paper to prevent contact with the iron; and the iron of one cell was connected, by copper strips, with the zinc of the next in the usual manner. The charge we used was very strong; namely, one part of sulphuric acid to two of water. The quantity of electricity produced was sufficient to render a piece of No. 16 gauge copper wire, six inches long, quite red-hot; but the intolerable stench of sulphuretted hydrogen given off from the battery, compelled each present at the experiment to beat a hasty retreat. The intensity, or chemical effects, were exceedingly feeble.

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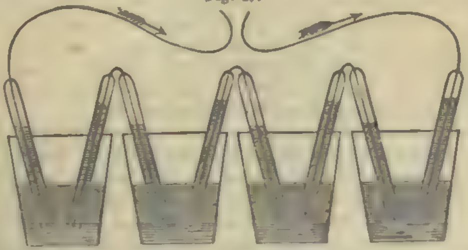
[*] See *ante*, p. 189.

In all the batteries we have described, the effects produced have resulted from the *decomposition* of water. Professor Grove, however, has constructed a gas battery, in which the combination of oxygen with hydrogen, and the consequent production of water, is sufficient to afford a current of electricity. One of these arrangements is represented in the following engraving (Fig. 47); in which each glass contains two tubes, one filled with hydrogen, and one with oxygen. A platina plate is soldered so as to expose its surface to the gas inside the glass. A wire passes from each tube; and the oxygen of one glass tube is thereby connected to the hydrogen of the next, and so on. A wire is brought from the two end-tubes of the series, by means of which the current of electricity is guided to any desired place. The tubes are filled with the gases, in the same proportions, by bulk, as are required to form water; and during the action of the battery, they diminish through their conversion into water. The liquid, which should be dilute sulphuric acid, in each glass vessel, rises, for the same reason, in the tubes. By means of this arrangement, all the ordinary effects of the voltaic battery are readily obtained. Fig. 47 represents one of these singular arrangements.

Fig. 47.

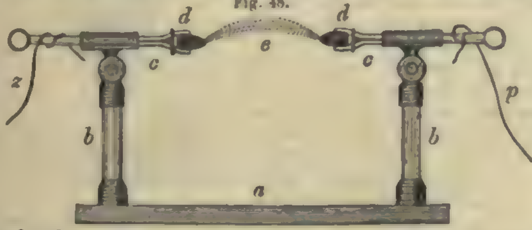


Having thus described the leading forms of voltaic batteries, we shall pass on to examine the various effects which may be produced by them.

THE LUMINOUS EFFECTS OF VOLTAIC BATTERIES.

In the whole range of experimental science, we can find no results of so brilliant a kind as those produced by a powerful voltaic battery. It will be necessary to have at least twenty of either Daniell's, or forty of Smee's arrangements, to repeat the experiments we shall name. To those who can afford larger apparatus, we may mention, that forty cells of Grove's battery, having platinas six inches long and two inches wide, afford some splendid effects; but, for general use, we recommend the simple modification of Grove's, which we fully described in a previous page,* and which will answer every purpose. In all the following experiments, each part of the apparatus and wires, and all the connections, should be kept perfectly clean and polished by a constant and liberal use of sand-paper; and some fine copper wire is useful to tie and connect metallic points, &c. The student should provide himself with such a stand as we described in our remarks on frictional electricity,† or he may easily construct one out of a piece of mahogany; and on this, two uprights should be placed, so as to support two crayon-holders, in which charcoal and other points can be held. One of these arrangements is represented in the above engraving.

Fig. 48.



In Fig. 48, *a* is the table of the apparatus; *b b*, the two uprights; *c c* the crayon-holders; *d d* the charcoal, or other points; *e* the arc of light; *z* the wire from the zinc; and *p*, the wire from the platina of the battery. With such an instrument, the following experiments may be tried.

Experiment 34.—Insert in each crayon-holder a piece of pointed boxwood charcoal, or of the hard carbon formed inside gas-retorts; and having attached the wires of the battery to the crayon-holders, bring the two charcoal points into contact. They will immediately become red-hot; and if they be separated for a short distance, a brilliant arc of flame will extend between the points. This light, which, *par excellence*, is called the Electric Light, far exceeds, in intensity, any other kind of artificial light we can obtain. It is not due to combustion, for it will take place equally well beneath the surface of water, or even in the vacuum of an air-pump. It is due to the fact that fine particles of carbon are transferred rapidly from one pole to the other. This is easily noticed when a powerful battery is employed; for one point gradually increases in size, whilst the other becomes perforated with holes.

We have already given some account of the effects to be obtained by using powerful batteries, at a previous page:‡ but we may remark, that a light produced by fifty cells of a Grove's battery, with platinas four inches long and two wide, is equal to about 1,000 wax candles, of six to the pound, all burning at the same time.

The objections to the use of this light, are its great unsteadiness, and the intensely black shadow which it casts. The latter difficulty cannot be overcome; and every attempt has been made to compensate for the irregularity and intermittence which occurs, by the construction of electric lamps, some of which have already been illustrated in our chapter on artificial illumination.§ We have tried some of the best lamps yet constructed by the most celebrated makers, and others which we have had made ourselves; and we cannot but unhesitatingly observe, that they are all failures in every respect. The objection of expense is of no great force, because the intensity of the light is so great.

The application of the electric light to lighthouse purposes, has, however, been highly successful; and for many months, the South Foreland shone brilliantly with the light produced by the magneto-electric machine, which we shall describe under the head of Electro-Magnetism. At the present time, some parts of Paris are thus lit, and the effect is stated to be very fine. The electric light seems to have what we may call a space-penetrating power, and it can accordingly be seen further in a fog than any other kind of artificial light; its value for lighthouses, on this ground alone, is therefore very great. The colour of the electric light varies according to the materials used as points, between which the arc of flame is produced, as the following experiments will illustrate.

Experiment 35.—Remove the charcoal points, and in their place insert two of zinc. On bringing these into contact, and again separating them, a most beautiful effect is produced. A vivid blue flame, tinged with ruby red, is afforded, and the zinc becomes oxidised, producing, in abundance, a flocky-white powder, which has been called "philosophers' wool."

Experiment 36.—Use two points of thick copper wire: a rich green-coloured light is produced, which has a singular optical effect on all surrounding objects, giving them a deep-green tint. The face of the experimenter presents a most peculiar appearance, which has often a most ludicrous effect on an audience, at the lecture-table. Silver affords a rich green colour, of a darker hue than that of copper.

Experiment 37.—Fix two pieces of stout iron wire in the crayon-holders, and bring their points together, taking care that they do not fuse, and so become solidly united. On separating them, a splendid arc of flame will be produced, and the iron will burn with the most brilliant coruscations. Steel affords a still finer appearance; and if a powerful battery be employed, showers of sparks will sometimes rise from the metal to a height of several feet.

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* See *ante*, p. 189.

† See *ante*, p. 182; Fig. 30.

‡ See *ante*, p. 190.

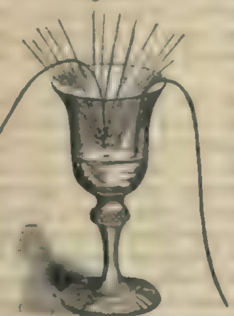
§ See *ante*, p. 136.

Experiment 38.—Pour some mercury into a wine or other glass, and insert one of the wires of the battery. Introduce the other wire of the battery, and then withdraw it to a distance of a quarter of an inch from the mercury surface. A very brilliant flame will be produced, of a bluish-green colour; and the mercury will burn rapidly away, affording dense fumes, which should be carefully avoided.

A modification of this experiment has been applied by Mr. Way in constructing an apparatus, by means of which he obtains a tolerably continuous electric light. Two surfaces of mercury, or one of platina and mercury, are used; and the fluid nature of the latter metal permits of an almost constant distance being kept up between the two terminals of the battery. By this means, one of the great objections to the electric light—its want of constancy—is overcome.

Experiment 39.—Proceed as in the last experiment; but introduce a steel wire, or watch-spring, in place of the copper wire of the battery, and withdraw it a short distance from the mercury. A splendid combustion will ensue, and the sparks of melted steel will be driven about in all directions, affording one of the most beautiful experiments with which we are acquainted. Great care should be taken in the experiment, lest particles of the melted metal enter the eyes—an occurrence which may take place, and which might be a source of injury. The annexed engraving illustrates the mode of carrying out this beautiful experiment.

Fig. 49.



Experiment 40.—Another plan of burning steel, is that of holding one of the wires of the battery in contact with a piece of coke, whilst a file or knife, to which is attached the other wire of the battery, is brought into contact with it, and then separated from it. The combustion is often so powerful as speedily to destroy a good-sized file; and the effect, with a large battery, is frequently of the most brilliant description. The preceding experiments illustrate what is called the *disruptive discharge* of the voltaic battery; and whilst we have selected a few only of the metals, the rest afford very beautiful results.

Some of the experiments we have named may be easily tried under water, by introducing one of the wires into a vessel full of that liquid, and containing some mercury at the lower part, in which one wire should rest. The other wire is to be dipped into the water, and then into the mercury, and to be withdrawn, as in the previous experiments. The discharge will thus take place beneath the liquid, and the singular appearance of flame under water will be presented.

Experiment 41.—Fill a basin with iron or steel filings, and dip one of the wires of the battery into them. Then introduce the other wire, and stir it about roughly for some time. The filings will gradually become heated, and at last red-hot. It is advisable to try this experiment on an iron tray, to prevent any risk of accident, in case the basin may break. Occasionally the filings will become almost fused into a solid mass, owing to the intense heat which is produced.

Experiment 42.—The heat afforded by the disruptive discharge is very great, as may be shown by introducing a piece of platina between the charcoal points of a battery in powerful action, when it will be speedily melted. The diamond may thus be easily burned, or even evaporated away in this manner.

The foregoing experiments will illustrate some of the most interesting of the luminous effects of voltaic electricity. If conducted with a battery of great power, we should warn the reader that the intense light produced is very injurious to the eyes. Indeed, the pain sometimes is intolerable when severe inflammation ensues. Green shades, or spectacles, should always be employed whilst conducting these experiments.

THE CALORIFIC EFFECTS OF VOLTAIC ELECTRICITY.

Having investigated the luminous effects which may be obtained from the voltaic battery, we shall now proceed to describe the heating effects which may be produced from the same source; and shall presume that a battery, similar to that we have already described,* or one of larger size, will be employed.

It will be observed, that the thick wires which convey the electricity from the battery, are not easily heated, except at the points, after they have been in contact. This is because they are sufficiently large to carry the current without affording any considerable resistance to its passage. If, however, these wires are joined together by means of a thin iron wire which is too small to carry the current, and which accordingly affords great resistance to its passage, then a large amount of heat will be set free, and the wire will either be greatly heated, or if the battery be very powerful, the heat will be sufficient to melt even a platina wire. The length of wire heated by any battery depends on the number of cells; whilst the thickness of the wire which may be melted, depends on the size of the plates.

Different metallic wires, of the same thickness, vary in their power of carrying a voltaic current. Copper, as the best conductor, is the most difficult to melt; and iron or platina, which are much inferior in their conducting power, are very readily heated by the voltaic current, and hence are most suitable for illustrating these effects.

The relative conducting power of wires of different metals, or of different thicknesses, may be conveniently illustrated by making a chain of them. Thus, a chain of silver or copper wire, with links of iron wire, will, when made to form part of the conducting course of a powerful battery, present the appearance of an alternation of cold and red-hot wires. Or the thick wires may be nailed to the edge of a table, and each joined by means of finer wire, when the latter, on conveying the current, will be instantly melted. A plan of arranging this is illustrated in the annexed engraving; and the effect, if properly managed, is both instructive and pleasing. The heat thus generated may be made to boil water. The fine wire should be coiled into a screw-like form; and after being attached to the large conducting wires of the battery, it may be introduced into a glassful of water. After a short time, the liquid will commence to boil, and will soon be evaporated off. The best mode of carrying out the experiment is illustrated in the annexed engraving.

Fig. 50.

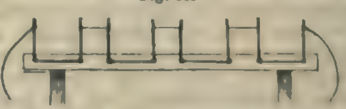
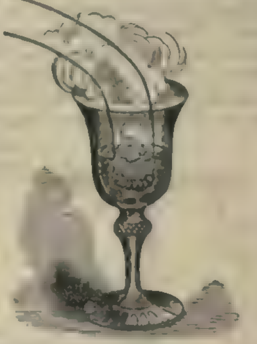


Fig. 51.



Experiment 43.—The electricity of a powerful battery may be conveyed to any distance by means of properly insulated wires; and for this purpose, gutta-percha answers admirably. The heating powers of the voltaic battery have thus been applied for blasting, military, engineering, and numerous other purposes; and we shall suggest an experiment which will illustrate the plans usually adopted in applying electricity in this manner.

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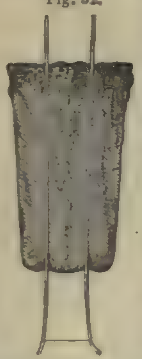
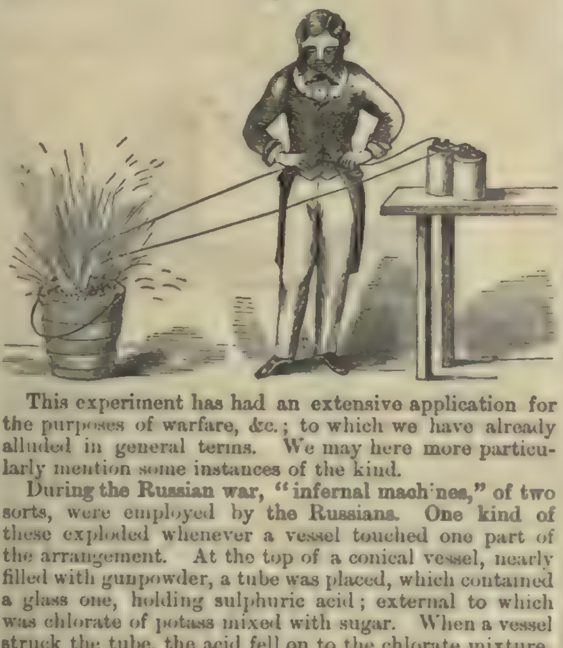
* See *ante*, p. 189.

Fig. 52.



Experiment 44.—A cartridge, closed at one end, and well pitched over, to render it water-tight, is to be filled with gunpowder, and into it may be fitted a cork, through which two thick copper wires have been passed, their inner ends being joined together by a fine iron or platina wire, as represented in Fig. 52. The iron wire thus rests in contact with the powder. The conducting wires are attached to the coarse wires of the cork, and their other ends are laid near the battery. The cartridge is then to be sunk into a pail of water, or a pond, and one of the wires may be connected with one end of the battery. On the other wire being brought into contact with the battery, the current at once passes; the fine wire in the gunpowder becomes ignited, and an explosion at once ensues. The mode of carrying out this experiment is illustrated in the following engraving; in which each part of the arrangement is fully shown.

Fig. 53.



This experiment has had an extensive application for the purposes of warfare, &c.; to which we have already alluded in general terms. We may here more particularly mention some instances of the kind.

During the Russian war, "infernal machines," of two sorts, were employed by the Russians. One kind of these exploded whenever a vessel touched one part of the arrangement. At the top of a conical vessel, nearly filled with gunpowder, a tube was placed, which contained a glass one, holding sulphuric acid; external to which was chlorate of potass mixed with sugar. When a vessel struck the tube, the acid fell on to the chlorate mixture, and, igniting it, soon exploded the powder. These machines were, however, very uncertain in their action, and were superseded by an arrangement in which voltaic electricity was employed as the igniting agent. The plan adopted was precisely similar to that described in our last experiment. The "infernal machine" was sunk in the course which vessels were expected to take; and inside, next to the gunpowder, were two wires, across which was stretched a finer wire. From the main wires two long conducting wires extended to the shore, the ends being placed in close proximity to a voltaic battery. A person was kept watching the progress of any vessel; and when the latter had arrived over the "infernal machine," contact was made with the battery, and an explosion either actually took place or ought to have done so. There is, however, a wonderful difference between trying experiments at the lecture-table, and carrying them out on a large scale. This fact the Russians found out to their cost; and, practically speaking, these voltaic "infernal machines" were utterly worthless.

For engineering and mining purposes, in which every arrangement can be made as complete as possible beforehand, and not necessarily exposed to any chance of disarrangement, this plan of igniting gunpowder for blasting, &c., is of the utmost value. Any amount of powder may be discharged simultaneously. There is no risk of the explosion occurring before proper precautions have been taken, because the whole arrangement is subject to the will of the superintendent. The instant the command to "fire" has been given, the effect takes place; and hence no accident need occur.

As an instance of such an application of voltaic electricity, we may mention the removal of many thousands of tons of chalk from the cliffs of Dover, during the formation of the South-Coast line of railway. Chambers were made in various parts of the rock, and these were filled with gunpowder. Each was connected, by means of wires, with the large battery placed at a considerable distance from the charge: when connexion was made with the battery, the explosion instantly took place.

Colonel Pasley first proposed to apply voltaic electricity for the purpose of blowing up sunken vessels; and this plan has been most successfully adopted in many cases. It is much to be regretted that our miners adopt the old and uncertain plan of the common fuse, which is highly dangerous, and is the frequent cause of loss of life. Generally speaking, nothing is more difficult than the introduction of any new invention; and just in proportion to its value, so the prejudice against it increases. We once met with an old miner, whose chief objection to the use of voltaic electricity, for blasting purposes, was its *great safety;* for he argued, that a diminished risk would tend to lower the wages of the men.

A very ingenious application of voltaic electricity has been made by Mr. Hart, an optician, and philosophical instrument-maker, of Edinburgh, for the purpose of lighting chandeliers, and other gas arrangements which are inaccessible by ladders. He arranges over each burner two main conducting wires, connected by means of a fine platina wire. On the ignition of the latter by the voltaic battery, the gas is simultaneously lit in each burner; and by means of an electro-magnet, the platina wire is instantly removed from the flame when the latter is produced. This, of course, prevents the destruction of the platina by the impurities which coal gas always contains. The experiment may be easily repeated by means of a single gas-burner: indeed, a tallow candle may be readily ignited by a wire heated white-hot by means of the voltaic battery

THE CHEMICAL EFFECTS OF VOLTAIC ELECTRICITY.

As this subject will be fully dealt with under the head of Electrotyping, we shall confine our remarks to the pure philosophy of the question; and so prepare our readers to enter into those details which the applications of any branch of science always afford.

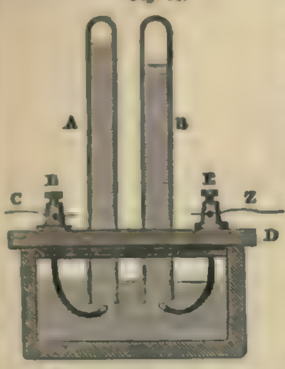
We have already stated, that a voltaic battery, or the single cell, depends for its action on the fact that chemical decomposition takes place in some part of the arrangement; and to this is due the evolution of free electricity. If the terminals of a single pair of plates, or of one cell of any form of the voltaic battery, be immersed in a solution of any salt, the acid always passes to one side, and the base to the other; and this is so universally the case, that definite laws, of what has been termed electro-chemical decomposition, have been discovered, which govern every case in which chemical changes take place in the presence of free electricity.

The simplest instance of the chemical effects of a voltaic battery to which we can refer, is that of the decomposition of water, which is a type of all electro-chemical changes. For the purpose of trying the following experiments, four cells of Grove's battery, or ten of Daniell's or Smee's, will be required; and presuming that the student has supplied himself with any of these arrangements, we shall proceed to investigate some interesting phenomena. The wires proceeding from the battery should be connected with a piece of platina foil; because, as that is unacted on by most of the substances to which we shall refer, no secondary or disturbing cause will interfere with the effects resulting from the action of electricity on the chemical compounds we shall examine.

If the wires proceeding from a battery furnished with platina poles, are dipped into a solution of sulphuric acid in water, the platina surface alone being exposed, abundance of gas will be given off from each plate. If these gases are collected separately, it will be found, that one is pure oxygen, and the other hydrogen gas; and these are given off from the platina terminals in the proportions required in their reunion to form water. A convenient arrangement for collecting the gases separately is represented in the following figure. By such an instrument, the two gases, which when combined produce water, may be collected apart from each other, and their characters may be readily examined. The left-hand tube, in Fig. 54, is supposed to be receiving oxygen gas; and that on the right hand is being filled with hydrogen, the other constituent of pure water. By bulk, one of oxygen to two of hydrogen is given off; but, by weight, one pound of hydrogen to eight pounds of oxygen are required to produce nine pounds of water.

If the gas (oxygen) received in the left-hand tube be tested, it will be seen that the combustion of a taper or match is rapidly increased by it. On the contrary, the gas (hydrogen) received in the right-hand tube, will not support combustion; and, indeed, it will at once catch fire if a light be presented to it.

Fig. 54.



A and B represent the two tubes intended to receive the gases; C and Z the wires coming from the copper or platina, and zinc ends of the battery, and tipped with platina; D, a cover in which the tubes rest; E E two binding-screws, through which the battery wires are passed.

It is found, that these gases, evolved during the decomposition of water by electricity, always present themselves in the same part of the instrument. For instance, hydrogen is invariably found at that terminal or pole (as it is frequently called) which ends the wire proceeding from the *zinc* end of the battery; whilst oxygen is always noticed as proceeding from that terminal, or pole, which is connected with the *platina*, or *copper* plate of the battery. It hence follows, that that pole which proceeds from, or ends the positive plate of a battery, is negative to that which terminates or proceeds from the negative plate of the battery itself. The wire or plate connected with the negative plate of a battery, is positive in any solution undergoing decomposition. We may simply state, that the poles or terminals of a battery, when immersed in any solution undergoing decomposition, have opposite electrical states to the plate of the battery from which they proceed; and this results from the laws of induction, already explained in a previous page.*

These poles, or terminals, have had other names applied to them. Dr. Faraday has suggested the term "electrodes," from the Greek "odos," a way; and the negative pole, or that terminating the wire from the zinc end of the battery, has been called the "zincode," or "cathode;" while that proceeding from, and ending the wire of the copper or platina plate of a battery, has been termed the "platinode," or "anode." These terms include the idea of the "passage" of a "current" from the copper pole to that of the zinc—a question on which we have already made some remarks in our earlier pages.†

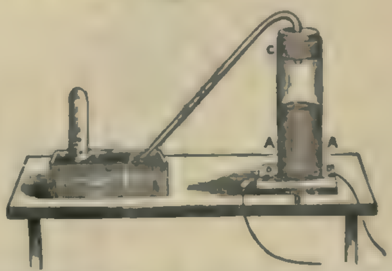
The changes which occur, both electrically and chemically, may be represented as in the following diagram:—

No. 1	OH	OH	OH	
No. 2	O	HO	HO	H

In No. 1, we observe the constitution of pure water represented as a combination of oxygen (O) with hydrogen (H); but when the poles of a voltaic battery are placed in the liquid, then the oxygen of one atom is set free, and its hydrogen combines with the oxygen of that next to it; and so on, until hydrogen is given off from the last atom in the series. The two gases obtained from water may be collected together by a very simple piece of apparatus. Into a glass tube or a gas chimney, fit two corks, so that when finally fixed they shall be water-tight. On each side of the inside of the glass put a small piece of platina foil facing each other, and extending half an inch outside of the glass. These pieces will serve the double purpose of conductors both inside and outside. Now press one of the corks into the glass, and so keep the foil in its place. The small pieces of platina on the outside may be bent outwards, and are intended to receive the conducting wires of the battery. This will complete the bottom part of the arrangement. Into the top of the glass fit the other cork, in which has been fixed a bent pewter or glass tube, and through which the gas produced will afterwards be carried away. The glass should previously be half filled with some acid water, and may then be cemented, by means of some sealing-wax, to a piece of wood, which will serve as a stand. Fig. 55 represents the arrangement we have described.

On bringing the wires into contact with D D, the water inside the glass will be decomposed; and if the bent tube be dipped beneath the water in a pneumatic trough, the two gases may be collected in any vessel, after the usual manner. When the tube is filled with gas, apply a light, and an explosion will at once take place, and water will be formed.

Fig. 55.



A A, two pieces of platina foil, part of each extending outside the glass; B, the bottom cork, to keep A A in their place; C, a cork holding a tube through which the gases are to pass; D D, the two outside pieces of platina foil, on which the wires of the battery are to be placed, and by which the electricity enters and leaves the water.

The philosophical instrument-makers sell an arrangement of a similar, but more complete kind: the student, however, who has to consult economy, will find that which we have described to answer every purpose. Some patents have been taken out, the object of which has been to apply the mixed gases thus obtained to illuminating purposes. The mixed gases may be produced at the rate of 200 cubic inches per minute, by means of a Grove's battery of five cells, each platina exposing one hundred square inches of active surface; and if the gases so afforded are passed through a Hemming safety-jet, and their flame be thrown on a cylinder of lime, the ordinary lime or Drummond light will be produced. We have frequently tried an arrangement of this kind; but much prefer the ordinary mode of producing oxygen and hydrogen, and burning them from gasometers or gas-bags. Independent of any other consideration, the expense of the voltaic battery is a serious item, in a commercial point of view, in this application.

If two pieces of platina, attached to each end of the battery, be immersed in a solution of any neutral salt, chemical decomposition at once ensues, and the acid and the base are readily separated from each other. An ordinary tumbler, or any other glass vessel, may be employed. It should be divided by means of a stout card, as shown in Fig. 56; in which the two platina plates, and the wires proceeding from the battery, are also represented.

Experiment 45.—A solution of common salt in water, coloured by a little litmus, may be used. The chlorine, set free by the voltaic current, will bleach that side of the liquid in which the wire from the platina of the battery is immersed, whilst the other side will retain its blue appearance.

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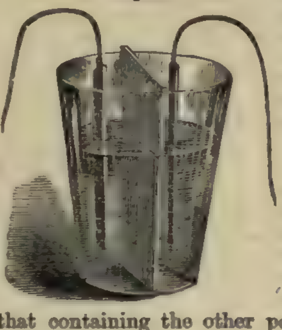
* See *ante*, p. 178, *et seq.*

† See *ante*, p. 160.

Fig. 56.



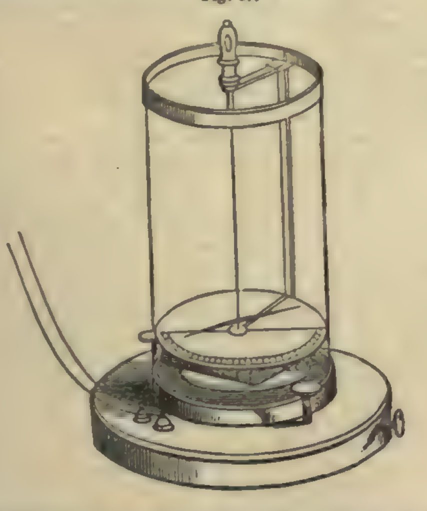
Experiment 46.—Employ a solution of sulphate of soda, also coloured with litmus. The salt will be decomposed; the side in which the wire from the zinc is immersed, will retain a blue colour; whilst that containing the other pole, will present a red appearance, from the liberated sulphuric acid acting on the litmus.

Experiment 47.—Dissolve some iodide of potassium in water, and add a little arrowroot (nearly pure starch). On inserting the two poles of the battery, the iodide will be decomposed, and one side of the liquid will turn to a deep-blue colour, owing to the liberation of free iodine.

The above are instances of chemical decomposition effected by means of the current of electricity passing through the liquid; and we may here remark that liquidity is essential for such changes. We shall, perhaps, draw too largely on the chemical knowledge of some of our readers, if we state, that a direct connection exists between the equivalents of the metal dissolved in the battery, with the amount of the elements set free in the glass decomposing-vessel. What we mean is simply this—for every thirty-two grains of zinc dissolved in *one* cell of a battery arrangement, one grain of hydrogen, and eight grains of oxygen, are set free during the decomposition of water. If five battery cells are used, then five times as much zinc is consumed during the evolution of one grain of hydrogen and eight of oxygen; so that the quantity of gases evolved by the action of a battery in decomposing water, is a measure of the quantity of electricity which is set free.

There are other modes of ascertaining the strength of a voltaic current; and although such would be better described under the head of Electro-Magnetism, we may just state, that the galvanometer, by the deflection of a magnetic needle, when a current of voltaic electricity passes over it, is often employed as a test of the presence, and also as a measure of the amount, of electricity passing over its wires. One of these instruments is represented in the following engraving.

Fig. 57.



Two wires may be observed on the left-hand of the cut: these convey the electricity from the battery. A coil of fine covered wire is wound repeatedly over, but not touching, a magnetic needle; and just in proportion as the quantity of electricity in the current is great, so the needle is deflected from zero, or 0°. This deflection rarely exceeds 70°, even when a powerful battery is employed. But neither this instrument or the decomposing apparatus give any measurement of the intensity of the current. This is best arrived at by interposing a length of some bad conductor, such as pure water, between one of the wires and the galvanometer, or the decomposing apparatus. The latter has been somewhat ambiguously termed, by way of distinction, a voltameter.

The secondary results of voltaic action in the decomposition of metallic solutions, is so elaborately and practically dealt with in the next chapter, by Mr. Gore, of Birmingham, that we shall not enter into any explanation of the subject. Our readers will have the opportunity of studying the theory and practice of electrotyping, plating, and gilding, in all their details. We have been compelled to make some alterations in the original paper, because we have already explained many points in our previous pages. All essential matters, however, have been retained; and we have made several additions on processes which late discoveries and inventions have introduced to public notice.

In the early part of this century, Sir Humphry Davy created a great sensation in scientific circles, by the publication of his researches in electro-chemistry. He found that, on bringing the two poles of a voltaic battery into contact with a piece of potash, that substance was decomposed, and a metal was liberated which he called potassium. He extended his investigations, and eventually obtained sodium, barium, and other metals by a similar process. The student may try the following experiments, which will illustrate this interesting discovery.

Experiment 48.—Place a small piece of fused potass on some platina foil, which must be connected with one pole of the battery; and bring the other pole, which should be a piece of platina wire, into contact with the potass. Chemical decomposition will soon take place; hydrogen will be given off, and potassium will be set free. If a little mercury be used instead of the platina foil, an amalgam of that metal with the potassium will be produced; and a little of this, if placed in cold water, will show the presence of potassium by the production of hydrogen gas, through the metal decomposing the water, and seizing its oxygen to form potass.—A similar experiment may be performed with soda, when the metal sodium will be produced. If the air be very dry, the potass should be breathed on, so as to make it a conductor by the moisture which it absorbs, as, in the dry state, its power of conduction is very slight.

By these experiments, it is shown, that potass, soda, &c., are oxides of metals, similar in constitution to the rust of iron, red lead, &c. At first these results were chiefly of interest to philosophers only; but during the last few years, the cheap production of sodium has led to that of aluminium, a metal obtained from common clay, which has now come into extensive use for a great variety of purposes. Our readers will not fail to observe how valuable are those philosophical researches which at first seem to have no practical result, but which, by patience and skill, may eventually turn out to be of the utmost commercial and social benefit.

Electricity, it is believed, has a great influence in producing many of those valuable gems which adorn jewellery; and many attempts have been made to produce the diamond, &c., by electro-chemical means. It has been ascertained that a current of free electricity always exists in mines from which metals are obtained. This, however, may be caused by the chemical changes which air and moisture always produce on metals and their oxides. Mr. Crosse and others have succeeded in producing minute crystals of some minerals, by long-continued, intense voltaic currents; and for this purpose small quantity is required. Indeed, a battery composed of small plates in large numbers, charged with common water, is as effective as any of the powerful arrangements to which we have already alluded. Such an arrangement is termed a water battery; and, for the purposes we have named, its two poles, which should be platina wires, are to be immersed in a mineral solution. As natural processes are always slow, so the results of the action of the electric current, in these cases, must be greatly prolonged. A microscope is often required to detect the first formation of crystals; and much patience and care are required to obtain a successful result.

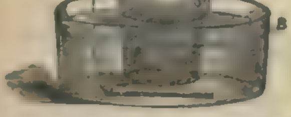
By means of a moderate battery power, some very interesting results may be obtained from various metallic solutions. Amongst them we may mention the production of brilliant metallic rings on a steel plate; and the deposition of metals in veins similar to those we observe in nature. The two following experiments illustrate each process.

Experiment 49.—*Nobili's Rings.*—These are most beautiful rings, formed on a piece of steel by means of a small galvanic battery. To produce them, place a sheet of polished steel in a plate containing a solution of sugar of lead, and attach the steel plate to the wire coming from the *platina* end of three cells of Grove's battery. Hold the wire from the zinc end in the solution, within an eighth of an inch over the centre of the steel plate. Beautiful rings of the oxide of lead will be at once produced; and in a few seconds the steel should be removed, and carefully washed in distilled water. This is one of the prettiest effects produced by a galvanic battery, and the colours obtained are similar to those observed on mother-of-pearl. Care must be taken to make no mistake with reference to the wires in placing them in the solution.

The next experiment, if carefully performed, illustrates the formation of metallic veins, such as are found in copper mines.

Experiment 50.—*To produce Minerals by means of Electricity.*—For this purpose, half-fill a garden-pot, six inches deep, with plaster of Paris, by stirring some of this powder in a basin of water, and pouring the mixture into the porous pot, where it must remain until it becomes solid. Solder a copper wire to a piece of zinc and a piece of copper, so that the two metals may be placed, one in the inside of the pot, and the other so that the pot may rest on it, and cover the intermediate wire with sealing-wax. Fill a basin, say to a depth of four inches, with a strong solution of sulphate of copper, adding to it a little sulphuric acid. In this basin place a piece of copper, and on the metal place the pot. Bend the wire so that the zinc may rest inside the pot, and touching the plaster of Paris; and fill the inside with a solution of common salt. Having done this, throw a cloth over the whole arrangement, and leave it for two or three weeks, replacing the loss of liquid, both in the basin and pot, by occasionally adding some water. At the end of the time named, on breaking the pot and plaster in half, it will be seen that the copper has crept in and assumed various beautiful forms. In some places it will be observed in crystals; in others as a layer of a rich blue or purple colour; and if the lump be dried, it will form a complete counterpart of a natural mineral. We have succeeded with iron and other metals, by using them instead of copper, which, however, we have given as being the easiest to commence with. To assist in constructing one of these interesting arrangements, we annex an engraving, and a description of the various parts to which we have referred.

Fig. 56.



B is the basin holding the copper solution; C, the plate of copper on which G, the garden-pot, rests; P, plaster of Paris in the pot; W, the copper wire covered with sealing-wax; Z, zinc plate inside the pot, hanging in the solution of salt, S.

Considerable sensation was produced, some years ago, by the statement, that insects of the acarus kind had been produced by Mr. Crosse, through long-continued voltaic action. Mineral solutions, after being electrised a few weeks, seemed to produce an abundance of these insects; although great care had been taken to prevent extraneous circumstances from interfering with the experiment. We believe that Mr. Weekes, of Sandwich, repeated these experiments, and obtained partial success. The idea of such a method of producing living creatures was so repugnant to theory and common sense, that it has been supposed some eggs of the insect had been accidentally present, and that time, electricity, and heat had eventually caused them to be hatched. We repeated the experiments exactly as they were described by Mr. Crosse, with the exception that every possible care was taken to prevent the access of fresh atmospheric air, which is always loaded with germs of all kinds; and, like most other experimenters, we met with no success. We can only hint, to say the least, that it was a most strange occurrence that electricity should produce a tolerably perfect organised insect, such as those of the acarus kind. One would naturally suppose, in the absence of all extraneous interfering causes, that nature would have presented us with creatures of a more elementary kind of organisation as the result of electric action. We must, however, dismiss the subject, and leave our readers to draw their own inferences as to the results and the circumstances which were connected with the experiments.

The "results," published by Mr. Crosse, have actually been quoted as instances of spontaneous generation, by some who, however, have been more noted for the boldness than the exactitude of their speculations.

THE PHYSIOLOGICAL EFFECTS OF VOLTAIC ELECTRICITY.

In our remarks on the effects of frictional electricity on animals, or on the human frame, it will be observed that the shock obtained from the Leyden jar is sudden, and only occurs each time the jar is discharged. The shock from the voltaic battery, on the contrary, is continuous as long as the arrangement is kept in action, and its power depends conjointly on the number and size of the plates, together with the amount of chemical action which is going on in each cell. The sensation produced, also materially differs from that of the Leyden jar: an intense cramp of the muscles is felt; and, if a very extensive series be employed, it will be impossible for any one to leave hold of the wires proceeding from the battery, owing to the contraction of the muscles of the arm which ensues.

Forty or fifty pairs of plates in the pile, or the same number of Smee's or Daniell's arrangement, produce considerable effect on the muscular system of one person. But if a number of persons be joined together, the power of the shock will be greatly diminished, unless the hands are well moistened with salt and water. Fifty cells of Grove's battery produce a severe shock; and it is necessary that that portion of the wires which have to be touched during the experiments, should be covered by some non-conducting substance, than which nothing is better than brown paper.

Voltaic electricity has been frequently employed for medical purposes, in cases of drowning, asphyxia, from inhaling carbonic acid gas, paralysis, &c.; and it has the advantage of being easily regulated to any extent desirable, with respect to the power of the shock. The coil machine, which will be described under the head of Electro-Magnetism, is almost invariably employed for this purpose, however, at the present time.

Some remarkable experiments have been tried on the bodies of men and animals immediately after death, by means of powerful voltaic batteries; and the most curious which we have met with, is that related by Dr. Ure, in his *Dictionary of Chemistry*, under the head of Galvanism. The body of a murderer, recently executed, was the subject in this case; and by sending the current through various parts of the body, many of the appearances of life were produced; in fact, had the body not been drained of blood, there is no doubt that life would have been restored. We must refer our readers to the work named, for full particulars; but may remark, that the contortions of the face, and the contractions of various muscles so produced, were of such a horrible character, that most of the spectators had to leave the room from sickness or faintness. Such experiments may easily be repeated on rabbits, cats, and dogs, immediately after death. In one experiment which we tried by a powerful battery, a rabbit leaped several feet from the table on which it was placed, which was caused by the powerful contraction of the muscles of the leg. By a judicious insertion of the poles of the battery in different parts of the body, any set of muscles may be called into play, and the effects almost invariably present the appearance of restored life in the subject.

ANIMAL ELECTRICITY.

In the previous chapter, we confined our remarks on this subject to describing some particulars in reference to electrical fishes. We shall now place before our readers some facts relating to the effects on, and the production in, the bodies of animals, of free electricity.

Electricity and magnetism have always been charged with being the cause of various phenomena observed in animals, to which no other could be assigned : hence, we have had the vagaries and absurdities of "mesmerism" and "animal magnetism" continually thrust on public notice. We can only spare space to remark, that those who have been most prominent in support of such subjects, have generally been equally eminent for their ignorance of true science, and have, according to the old adage, found plenty to admire them. It is a most unfortunate failing of our mental powers, when in an untrained state, that we are always ready to believe that which is hardly credible, in preference to well-substantiated facts and conclusions which may be readily demonstrated—the imagination frequently overruling the power of our reasoning faculties.

It has long been known, that electrical may, in some cases, be substituted for nervous action ; and thus, many years ago, it was shown that a piece of zinc, and one of platina, might be inserted between the previously divided nerves of the stomach of a rabbit, and that digestion, and other vital processes, continued just as if no alteration of the continuity of the nerves had occurred.

Aldini has made numerous researches in this branch of science ; and the fact of free electricity in the current form being always present in the body, and passing between the nerves and muscles, has been satisfactorily proved. Indeed, a voltaic battery may be made, and many experiments can be tried, by means of the nerve and muscle recently obtained from the living animal, if such are arranged in a similar form to that of the voltaic pile.*

Many persons may exhibit a kind of double set of phenomena in reference to electricity. Some, indeed, may have the hair and skin excited by friction, so as to afford sparks in abundance. This, however, is chiefly observed in persons of a phlegmatic temperament, having a harsh skin, and a freedom from excessive perspiration of the body. Others, of a sanguine and nervous temperament, more readily exhibit free electricity in the nervous system, arising, as far as we know, from chemical action. Thus we may rank individuals under the separate classes of those presenting the phenomena of frictional and voltaic electricity respectively.

That electricity has a decided influence on the general health, is admitted on all hands. Every one experiences in foggy weather, or at times when the atmosphere is surcharged with electricity, a peculiar heavy and depressed feeling. The sanitary tables, now so freely published, when taken in connection with simultaneous electrical observations, indicate that electricity has a definite relationship, in some way or another, with the health and diseases of a community.

This fact, however, has been sometimes extended far beyond its legitimate limits ; and there have been those having some claim to a scientific reputation, who have not hesitated to avow their belief that electricity is the vital principle. Had they affirmed the same of caloric, their views would have had a more feasible or plausible appearance ; for it is an admitted fact, that life cannot be maintained unless a certain amount of animal heat is kept up. It is, however, unnecessary for us to discuss this question. The common sense and experience of our daily lives, apart from the more exact teachings of science, are sufficient to show, that however active and necessary certain agents may be for the purpose of producing definite results, it by no means follows that such are primary causes. The celebrated Stephenson once asked a friend what it was that caused the rapid motion of one of his locomotives. The reply was characteristic of an experimentalist—the combustion of the fuel in the furnace was assigned as the reason. Stephenson, however, most expressively corrected this ; and referred to the fact, that all the heat and effect produced, were due to those rays of heat and light which had first been absorbed from the sun, and were given out again with their full force in the furnace ; although, perhaps for thousands of years, they had laid dormant in the bowels of the earth. In the same spirit, the true philosopher, whilst investigating and admiring the chain of second causes, constantly operating in the course of natural phenomena, refers all for their first origin, to that Great First Cause who called them into being. As glimpse after glimpse is obtained of the infinity of creative power, which our limited intelligence can but feebly comprehend—the humble but earnest inquirer is compelled, in reverence, to exclaim—"Oh Lord, how manifold are thy works ; in wisdom hast thou made them all."

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* See *ante*, p. 188 ; Fig. 38.

CHAPTER III.

ELECTRO-METALLURGY.

The following paper, written by Mr. Gore, of Birmingham, affords full details of the theory and practice of the arts of electrotyping, gilding, plating, &c. As many of the subjects which were discussed in the original, have already been mentioned in the preceding pages, the editor has omitted such portions, and has made various alterations and additions in reference to numerous processes, and their practical details, which he trusts may be of value both to the student and general reader.

THE THEORY OF ELECTRO-DEPOSITION.

Introductory Remarks.—As the ultimate object of this chapter is to enable the reader to work in an electro-plate manufactory in a commercially successful manner, we shall endeavour to include within its pages, as far as the limited space will allow, every portion of the subject calculated to assist him in obtaining that result, excluding from it every other which does not contribute towards that object.

With this view we shall explain the *principles* or *theory* of electro-deposition, because every workman in an electro-plate manufactory is certain to meet with difficulties, which no amount of practical knowledge or experience will enable him to overcome without a perfect knowledge of the theoretical principles. These difficulties may be new ones, such probably as he has never seen before ; and no doubt, in some cases, such as no one else has ever seen : a knowledge of the theory will here enable him to apply principles, and suggest remedies, some of which are almost sure to be successful.

We shall also include the *practice* of the subject ;

because, after all, its success depends on careful manipulation; for with ever so perfect a knowledge of principles, without a perfect knowledge of the application of those principles in the form of practical rules and manipulation, success cannot possibly be attained. With the same end in view, we shall avoid saying anything about the history of the subject, or the claims of rival discoverers or inventors—these being subjects for the historian. Neither shall we say much about the electro-deposition of rare metals, or about any collateral branches of the subject, excepting only so far as they are capable of illustrating the subject in a direct manner, or of otherwise furthering the object in view.

To enable the reader to master the subject as he proceeds, we have so arranged it that every portion shall be, so far as it goes, complete in itself, requiring no anticipatory knowledge of more advanced parts to enable him to understand it.

The only arrangement which admits of this important object being attained, is to explain the theory before treating of the practice, and by arranging the theory in an *inductive* order. In other words, we shall commence with the various classes of facts on which electro-deposition is founded, and ascend from these to the general laws or principles, chemical or electrical, which govern them. Proceeding from the theory to the practice, arranging all in a *deductive* order, and applying theoretical principles in the form of practical rules, the results cannot be other than successful. Beginning with the more general rules which apply to all electro-deposition processes, and to the electro-deposition of all metals, and proceeding, step by step, to those more special rules of manipulation which are required for the working of particular metals and solutions, the necessary requirements for the production of the more difficult substances and more complicated works of art will be attained. Through the whole chapter, the reader will thus be led gradually from the most common and well-known facts, to the most complex and difficult applications of electro-deposition.

In accordance with this plan, the subject will commence with a review of such facts of electro-deposition as every man possessing the few necessary materials, which are easily procurable, may readily verify for himself. On these facts the whole subject throughout will be based. From them we shall proceed to the circumstances or *conditions* under which they occur; namely, the *causes* of electro-deposition. The principles will be inferred from the facts as we proceed, until we arrive at the more abstract laws of the phenomena.

The facts will be based on numerous experiments, in which instances where deposition does occur, as well as experiments in which it does not occur, will be cited. These investigations will satisfy the reader, that in all cases where deposition does occur, certain conditions are invariably present; and where it does not occur, one or more of them is invariably absent; and, therefore, that the conditions observed are essential to the production of the phenomena. Another and more ultimate reason for mentioning negative as well as positive instances is, that, in practical working, it is nearly as important to know what will prevent deposition, as to know what will produce it.

The following table exhibits the phenomena of electro-deposition, arranged in an inductive order, suitable for teaching the subject theoretically, and without immediate reference to its practical applications. The first portion of the table contains the *facts* of electro-deposition, divided into seven classes, under which may be ranged the whole known facts of the art. The second portion contains the *principles* or *conditions* under which those facts are manifested; these are also divided into seven classes, which are capable of including all the known conditions or causes of electro-deposition.

A.—FACTS.

1. Deposition by one metal and one liquid.
2. Deposition by two metals and one liquid.
3. Deposition by one metal and two liquids.
4. Deposition by two metals and two liquids.
5. Deposition by connecting either of the foregoing arrangements (except the first) with a separate depositing liquid.
6. Deposition by connecting other sources of depositing power with a separate depositing liquid.
7. Deposition by combinations of the foregoing.

B.—PRINCIPLES.

1. Chemical conditions of deposition.
2. Electrical conditions of deposition.
3. Thermal conditions of deposition.
4. Mechanical conditions of deposition.
5. Mathematical conditions of deposition.
6. Logical conditions of deposition.
7. Ontological conditions of deposition.

This arrangement has been used with much success in teaching the theoretical part of electro-deposition, as it enabled the pupils to understand each portion clearly as they proceeded. The plan adopted was—

First, to exhibit before the pupils numerous experiments of each class of facts in succession, including positive cases in which deposition did occur, as well as negative ones in which it did not occur.

Second, to place each of the theoretical principles in succession before them in the form of a hypothetical question, referring them to the various facts on which it is founded, and leaving them to observe for themselves whether or not the principle there stated was borne out, allowing them to draw their own conclusions. By this method they were soon led to observe, that, wherever deposition occurred, certain conditions were present, and that where it did not occur those conditions were absent.

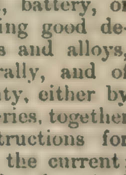
When treating of the laws and principles of deposition, the reader will be referred back to the facts upon which they are based; so, when describing its practical applications, he will in like manner be referred to the laws or principles for his guidance: his knowledge of the practice will thus be based in a great measure upon the principles, as the principles will be deduced from the facts, which, as we have said before, are within the reach of every one to repeat and prove for himself.

The practical part will treat, in succession, of the general rules for working all the different processes of washing or dipping, whether by single cell, battery, or other process. The requisites for preparing good depositing solutions, both for simple metals and alloys, with methods of making solutions generally, and of working them; suitable sources of electricity, either by the magneto-electric machine or other batteries, together with their construction; as well as instructions for regulating the quantity and intensity of the current; regulating the quantity and quality of the deposited metal; cleaning and preparing metallic surfaces for receiving adhesive and non-adhesive deposits; copying works of art in various substances; elastic moulding; preparation of non-conducting surfaces to receive a deposit; multiplication of works of art by deposition; deposition, by the various processes, of such metals and their alloys, as antimony, bismuth, zinc, cadmium, tin, lead, iron, cobalt, nickel, copper, brass, German silver, mercury, silver, gold, platina, and palladium; chemical relations of the cyanides of gold and silver; manufacture of cyanide of potassium; recovery of gold and silver from damaged solutions; and a full list of patents upon electro-deposition will be fully treated on as we proceed.

THEORETICAL DIVISION OF THE SUBJECT.

1. *Facts.—One Metal in One Liquid.*—There are various modes in which deposition of one metal upon another may take place; and they may be classed as follows:—

Fig. 59.



1st. By the simple immersion of *one* metal in *one* liquid (Fig 59); namely, by putting the metal to be coated into a solution of the metal to be deposited, and allowing it to remain a longer or shorter period of time, the liquid being at a suitable temperature; for instance, if we immerse a piece of clean iron in a solution of sulphate of copper, it will become coated with copper; but if we immerse a piece of silver in that liquid, it will not become so coated.

2nd. *Two Metals in One Liquid.*—By the immersion of *two* metals in *one* liquid (Fig. 60), the two metals being in contact with each other: for instance, if we connect a piece of silver A and a piece of iron B together, and immerse them in a solution of sulphate of copper C, the silver will become coated with copper as well as the iron; but if a piece of silver in contact with a piece of gold or platina be immersed in the same liquid, it will not become coated. We have already seen, that silver immersed alone in such a liquid, will not receive a deposit of copper.

Fig. 60.

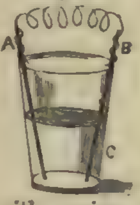
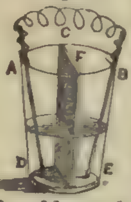
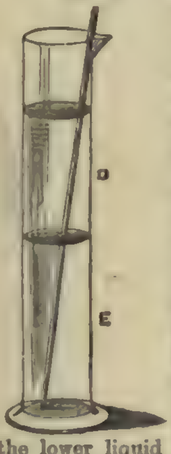


Fig. 61.



3rd. *One Metal in Two Liquids.*—By the immersion of *one* metal (*i.e.*, one *kind* of metal) in *two* liquids D and E (Figs. 61 and 62), the liquids being prevented from mixing with each other either by a porous partition F (Fig. 61) of bladder, thin wood, unglazed earthenware, or other porous material which will allow the two liquids to touch each other through its pores; the piece of metal being either bent so as to dip into each liquid, or cut into two portions, and its two ends united by a wire C, the end or piece to receive the deposit being immersed in one liquid, and the other piece in the other liquid (Fig. 61); or the two liquids being put in a deep narrow vessel, the heavier one being poured in first, and the lighter one poured carefully above it so as not to mix them together, and the piece of metal being in the form of a rod or wire placed vertically in the two liquids (Fig. 62); for instance, if the lower liquid consist of a solution of sulphate of copper, and the other of dilute sulphuric acid, and a piece of copper be immersed in both liquids, that part of it which is in the sulphate solution will become coated with copper, whilst that in the acid liquid will be partly dissolved; but if, instead of copper, we use a piece of platina, it will neither be dissolved nor receive a metallic deposit.

Fig. 62.



4th. *Two Metals in Two Liquids.*—By the immersion of *two* metals A and B (Fig. 63) in *two* liquids, D and E, the two being, as in the last arrangement, either separated by a porous diaphragm F, or poured one above the other, the two metals being immersed one in each liquid, and connected together by a wire C: for instance, if one liquid be dilute sulphuric acid, and the other a solution of sulphate of copper, and a piece of copper be immersed in the dilute acid, and a piece of silver in the metallic solution, the two metals being thus in mutual contact, the piece of copper will dissolve, and the silver receive a deposit of copper; but if we immerse a piece of platina in the dilute acid with the silver in the sulphate solution, the platina will not dissolve, nor the silver receive a metallic deposit.

Fig. 63.

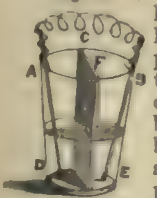
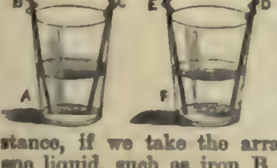


Fig. 64.



5th. *Separate Depositing Liquid with all the others.*—By connecting any one of the foregoing arrangements by means of wires with two separate pieces of metal of a similar kind immersed in a separate and suitable liquid (Fig. 64): for instance, if we take the arrangement of two metals in one liquid, such as iron B and copper C, in a solution of sulphate of copper A; or zinc B and silver C in dilute sulphuric acid A, and connect them by two separate wires, D and E, with two pieces of copper, D and E, immersed in a solution of sulphate of copper F contained in a separate vessel, the piece of copper E connected with the silver will dissolve, whilst the other piece D which is connected with the zinc will receive a deposit of copper; but if we substitute a solution of sulphate of zinc, freely acidulated with sulphuric acid, for the solution of sulphate of copper F, and two pieces of platina for the pieces of copper, the one piece of platina will not dissolve, nor the other receive a metallic deposit.

6th. *Separate Depositing Liquid with any other Source of Power.*—By connecting the pieces of metal in the separate depositing liquid with any other source of depositing power, such as a magneto-electric machine, cell, or battery.

2. In these arrangements, it will be observed that we have—1st, deposition by one metal and one liquid; 2nd, by two metals and one liquid; 3rd, by one metal and two liquids; 4th, by two metals and two liquids; 5th, by a separate depositing liquid and metals connected with either of these; and 6th, by a separate depositing liquid and metals connected with any other source of depositing power. These six classes, and their combinations, are capable of including all the known cases of electro-deposition.

3. Under the head of each of these classes will be mentioned a number of experiments with various metals and liquids; and it would be advisable for the student to try a few, as he proceeds, both of deposition and non-deposition of each class, in order to fix the facts more firmly in his memory, and give him a fuller comprehension of the principles.

4. *Depositing Arrangement No. 1.*—Deposition by one metal and one liquid (Fig. 65) takes place in the following instances:—

Hydrochlorate of Terchloride of Antimony.—In a solution of hydrochlorate of terchloride of antimony (the ordinary chloride of antimony, as prepared for pharmaceutical purposes), bismuth, zinc, tin, lead, brass, and German silver, become coated with antimony; whilst antimony, iron, nickel, copper, silver, gold, and platina do not become coated.

Fig. 65.

Chloride of Bismuth.—In a solution of acid hydrochlorate of bismuth oxide (chloride of bismuth), zinc, tin, lead, and iron, deposit the bismuth upon themselves; whilst antimony, bismuth, copper, brass, German silver, gold, and platina do not.

Sulphate, Chloride, Nitrate, or Acetate of Zinc.—In a solution of either sulphate, chloride, nitrate, or acetate of zinc, neither antimony, bismuth, zinc, tin, lead, iron, nickel, copper, brass, German silver, silver, gold, or platina become coated with zinc.

Protochloride of Tin.—In a solution of protochloride of tin, zinc and lead become tinned; whilst antimony, bismuth, tin, iron, nickel, copper, brass, German silver, silver, gold, and platina receive no deposit.

Hyponitrate, Nitrate, or Acetate of Lead.—In a solution of hyponitrate, nitrate, or acetate of lead, zinc receives a coating of lead; whilst antimony, bismuth, tin, lead, iron, nickel, copper, brass, German silver, silver, gold, and platina receive no deposit.

Ferrous Sulphate.—"Zinc," as Fischer says, "immersed in a perfectly neutral solution of ferrous sulphate (protosulphate of iron) contained in a stoppered bottle, throws down metallic iron, which is deposited partly on the zinc;" but in this solution neither antimony, bismuth, tin, lead, iron, nickel, copper, brass, German silver, silver, gold, or platina receive any metallic deposit.

Sulphate of Copper.—In a solution of sulphate of copper, zinc, tin, lead, and iron become coated with copper; whilst antimony, bismuth, nickel, copper, silver, gold, and platina do not.

Chloride of Copper.—In a solution of chloride of copper, bismuth, zinc, tin, lead, and iron receive a copper deposit: whilst antimony, nickel, copper, silver, gold, and platina do not.

Nitrate of Copper.—In a solution of nitrate of copper, zinc, tin, lead, and iron become coated; whilst antimony, bismuth, nickel, copper, silver, gold, and platina receive no deposit.

Dichloride of Copper.—With a solution of dichloride of copper in liquid ammonia, or of oxide of copper in a solution of sal-ammoniac, zinc receives a deposit; whilst antimony, bismuth, tin, lead, iron, nickel, copper, silver, gold, or platina do not.

Mercurial Salts.—Solutions of mercurial salts have their metal deposited by arsenic, antimony, bismuth, zinc, cadmium, tin, lead, iron, copper, and brass; also by the alloys of silver with zinc, tin, lead, or copper.

Nitrate of Mercury.—A solution of nitrate of mercury yields its metal to bismuth, zinc, cadmium, lead, iron, or copper, and, if acidulated with nitric acid, to antimony also; but not to silver, gold, or platina.

Acetate of Mercury.—Iron deposits mercury from a solution of acetate of mercury.

Silver Solutions.—The following metals—viz., manganese, arsenic, antimony, bismuth, zinc, cadmium, tin, lead, iron, copper, and mercury, deposit silver from its solutions in the metallic state; an aqueous solution of nitrate of silver yields its metal to manganese, arsenic, antimony, bismuth, zinc, tin, lead, iron, nickel, copper, brass, and German silver; but not to silver, gold, or platina. Lead and tin deposit the silver most quickly; then follow the other metals in this order—cadmium, zinc, copper, bismuth, antimony, arsenic, mercury. Arsenic deposits silver from the alcoholic solution of nitrate of silver; antimony receives a coating of silver either in the aqueous sulphate or alcoholic nitrate; bismuth deposits silver from the alcoholic nitrate, but not from the aqueous sulphate; zinc receives a silver deposit in the alcoholic nitrate; tin becomes silvered in the alcoholic nitrate, but more quickly in the aqueous sulphate; iron deposits silver from the sulphate of silver, but not from the alcoholic nitrate; copper deposits it from the aqueous sulphate or alcoholic nitrate; brass and the alloys of silver, with zinc, tin, or lead, deposit silver from silver solutions completely. In a solution of the double cyanide of silver and potassium (the ordinary plating liquid), zinc, lead, and copper become silvered; also brass and German silver, but more slowly; whilst antimony, bismuth, tin, iron, nickel, silver, gold, and platina do not.

Gold Solutions.—From an acid solution of terchloride of gold, most of the base metals, likewise mercury, silver, platina, and palladium deposit gold, generally in the metallic state, but not always; arsenic rapidly deposits gold from this solution; antimony, tellurium, and bismuth become gilded; zinc, cadmium, lead, iron, cobalt, mercury, silver, platina, and palladium deposit the gold; whilst titanium, tungsten, molybdenum, and chromium do not. In a solution of the double cyanide of gold and potassium, zinc quickly becomes gilded, and copper, brass, and German silver slowly; whilst antimony, bismuth, tin, lead, iron, nickel, silver, gold, and platina do not.

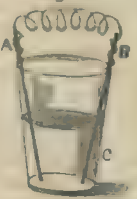
Bichloride of Platina.—Platina is deposited from a solution of its bichloride by arsenic, antimony, tellurium, bismuth, zinc, cadmium, tin, lead, iron, cobalt, nickel, copper, brass, German silver, mercury, and silver; but not by gold or platina.

5. *Observations upon Class of Instances, No.* 1.—In reviewing all these instances, we may make the following observations:—1st, that various metals, by mere immersion in solutions of other metals, at the ordinary temperature of the atmosphere, sometimes become coated with a deposit of metal, and sometimes not; 2nd, that no metal becomes coated by mere immersion in a solution of the same metal—for instance, zinc does not become coated with zinc in a solution of sulphate of zinc; copper with copper, in a solution of its sulphate, gold with gold in its chloride; 3rd, that the baser metals, especially zinc, cadmium, tin, lead, and iron, become coated more frequently than the noble metals, especially gold and platina; 4th, that solutions of base metals, especially of zinc and iron, yield their metal less frequently than those of the noble metals, especially those of gold and platina; 5th, that of all the ordinary metals mentioned in the foregoing instances, zinc deposits metal from the greatest number of solutions, and appears to have the strongest depositing power; 6th, that the coherent and adhesive deposits obtained, are, in all cases, exceedingly thin; and 7th, that frequently the deposited metal, whatever its kind may be, has the appearance of a black or dark-coloured powder on its surface, especially when it has been deposited very rapidly; but sometimes it exhibits its ordinary colour and appearance, especially if its outer portion be rubbed off.

6. To this mode of depositing belongs the process of tinning brass articles (wash tinning), by boiling them in water containing a salt of tin and bitartrate of potash; the process of silvering brass nails, buttons, hooks and eyes, buckles, &c., by rubbing them with any of the well-known silvering compositions moistened with water; also the water-gilding process, &c.

7. *Depositing Arrangement No.* 2.—*Deposition by Two Metals and One Liquid.*—The following instances belong to the class of deposition by two metals and one liquid, the two metals being either in mutual contact (touching each other either above or beneath the liquid), or connected together by a wire.

Fig. 66.



Chloride of Antimony.—If we immerse a piece of antimony A, in contact with a piece of zinc B, in a solution of the ordinary chloride of antimony C, it will receive a coating of antimony; or if we immerse a piece of platina in contact with a piece of tin in this liquid, it will receive a deposit of antimony; but if we immerse a piece of antimony in contact with a piece of platina, or a piece of platina in contact with a piece of silver in this liquid, it will receive no metallic deposit.

Chloride of Bismuth.—In a solution of chloride of bismuth, brass in contact with a piece of zinc, copper in contact with tin, or German silver with iron, receive a deposit of bismuth; but brass in contact with a piece of gold, gold in contact with silver, or German silver with platina, receive no deposit.

Sulphate, Chloride, or Nitrate of Zinc.—With a solution of either sulphate, chloride, or nitrate of zinc, no metal of any pair of metals selected from amongst the following, will receive a deposit of zinc: antimony, bismuth, zinc, tin, lead, iron, nickel, copper, mercury, silver, gold, platina, or palladium.

Protochloride of Tin.—With a solution of protochloride of tin, either antimony, tin, or copper, immersed in contact with zinc or lead, will receive a coating of tin; but antimony in contact with tin, tin with silver, copper with iron, or either gold or platina with copper, will not receive a deposit.

Hyponitrite of Lead.—With a solution of hyponitrite of lead, either tin, copper, or brass, in contact with a piece of zinc, will receive a deposit of lead; but tin in contact with copper, copper with lead, or brass with platina, receive no deposit.

Nitrate of Lead.—With a solution of nitrate of lead, either copper, brass, or silver, in contact with zinc, receive a coating of lead; but copper in contact with iron, brass with tin, or silver with copper, receive no such coating.

Protosulphate of Iron.—With a saturated solution of protosulphate of iron, platina in contact with zinc receives a deposit of iron; but in contact with copper it receives no metallic deposit.

Chloride of Nickel and Ammonia.—In a solution of the double chloride of nickel and ammonia, copper in contact with zinc receives a deposit of nickel; but in contact with silver it does not receive such a deposit.

Sulphate of Copper.—In a solution of sulphate of copper, brass in contact with zinc; or tin, German silver, silver, or platina, in contact with iron, receive a deposit of copper; whilst silver in contact with antimony, or platina in contact with brass, receive no deposit.

Oxide of Copper in Ammonia.—In a solution of oxide of copper in ammonia, platina in contact with zinc receives a deposit; but silver in contact with iron does not.

Nitrate of Mercury.—In a solution of nitrate of mercury, silver in contact with either zinc or iron, or platina in contact with copper, receive a metallic deposit; but platina in contact with silver does not.

Nitrate of Silver.—In a solution of nitrate of silver, gold in contact with zinc receives a deposit of silver; but in contact with platina it does not.

Bichloride of Platina.—In a solution of bichloride of platina, platina in contact with zinc becomes coated with platina; but in contact with gold it receives no such coating.

8. *Observations upon Class of Instances No. 2.*—The following general observations may be made upon the foregoing facts:—1st, that in some instances deposition does, and in others it does not, occur; 2nd, that no metal will cause another metal to be coated by this method, unless it can coat itself in the same liquid by simple immersion—for instance, zinc cannot coat itself with zinc in solutions of zinc, neither can it cause other metals to become coated with that metal in those solutions; copper cannot coat itself with zinc in a solution of sulphate of zinc, or with tin in a solution of chloride of tin; neither can it cause silver, gold, or any other metal, to become coated with zinc or tin, in those liquids; 3rd, that one of the two metals which receive a deposit by this method, derives its power of receiving the deposit by virtue of its contact with the other metal; 4th, that any metal which has the power of coating itself by simple immersion in a given liquid, can, by this method, cause other metals which do not coat themselves by simple immersion in that liquid to become coated—for instance, zinc, tin, and iron coat themselves with copper by simple immersion in a solution of sulphate of copper; and silver, gold, and platina do not; but if either of the former metals be connected with either of the latter, and the two are immersed together in that liquid, the latter metals, as well as the former, will become coated with copper; 5th, that base metals, and especially zinc, have generally the power of causing other metals to become coated by this method; whilst the noble metals, and especially gold and platina, rarely possess this power; 6th, that by this method, metal is deposited much more frequently from solutions of the noble metals, than from those of the base ones; and 7th, that thick deposits of metal may be obtained by this method, provided the action is continued sufficiently long, and the liquid properly renewed.

Fig. 67.

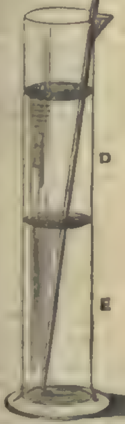
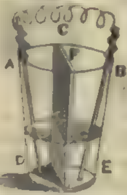


Fig. 68.



9. *Depositing Arrangement No. 3.—Deposition by One Metal and Two Liquids—Chloride of Antimony.*—The following instances belong to deposition by the immersion of one metal in two liquids, D and E (Fig. 68), separated by a porous diaphragm F, the metal being either in two pieces, connected together by a wire or wires C, or in one piece, and bent so as to dip into both liquids: the diaphragm may be dispensed with, as already explained (1), by pouring the lighter liquid carefully above the other, and placing the piece of metal vertically, as seen in Fig. 67. If two pieces of antimony, A and B, connected together by a wire or wires C, are immersed, one in dilute nitric acid D, and the other in a solution of chloride of antimony E, the piece in the dilute acid will dissolve, whilst that in the chloride solution will receive a metallic deposit.

Chloride of Bismuth.—If two pieces of antimony are immersed in the previous manner, one in hydrochloric acid, and the other in a solution of chloride of bismuth, that in the acid will dissolve, and the other receive a coating of bismuth.

Sulphate of Copper.—With antimony, in dilute hydrochloric acid on one side, and in a solution of sulphate of copper on the other, a deposit of copper is obtained.

Chloride of Bismuth.—With bismuth in hydrochloric acid on one side, and in a solution of chloride of bismuth on the other, a free deposit of bismuth is soon obtained.

Chloride of Zinc.—If a piece of zinc be bent so as to dip into dilute hydrochloric acid on one side, and into a neutral solution of chloride of zinc on the other, a free deposit of zinc will be found upon the end in the metallic solution after a period of twelve hours.

Solution of Acetate of Zinc.—With zinc in a solution of acetate of zinc on one side, and in dilute sulphuric acid on the other, that in the dilute acid will dissolve, whilst the other end will receive a metallic deposit.

Iron in Chloride of Antimony.—With iron in dilute sulphuric acid on one side, and in a solution of chloride of antimony on the other, the end in the metallic solution will receive a deposit of antimony, whilst that in the dilute acid will dissolve.

Iron in Sulphate of Zinc.—With iron in dilute sulphuric acid on one side, and in a solution of sulphate of zinc on the other, no deposit of zinc is obtained in twelve hours; similarly with iron, dilute sulphuric acid, and a solution of protosulphate of iron, no deposit occurs in twelve hours.

Tin in Chloride of Tin.—With tin in dilute hydrochloric acid on one side, and in a solution of chloride of tin on the other, a deposit of tin is obtained.

Zinc in Sulphate of Zinc.—With zinc in dilute sulphuric acid, and in a solution of sulphate of zinc, a free deposit of zinc occurs in twelve hours.

Bismuth in Nitrate of Bismuth.—With bismuth in dilute nitric acid, and in a solution of acid nitrate of bismuth, a thin deposit of bismuth is found in twelve hours.

Copper in Sulphate of Zinc.—With copper in dilute sulphuric or dilute nitric acid on one side, and in a solution of sulphate of zinc on the other, no deposit of zinc occurs in twelve hours.

Brass or Copper in Sulphate of Copper.—With brass or copper in dilute sulphuric acid on one side, and in a solution of sulphate of copper on the other, a deposit of copper is obtained in twelve hours; similarly with copper in dilute hydrochloric acid, and in a solution of chloride of copper, a metallic deposit occurs.

Silver in Plating Liquid.—With silver in either dilute sulphuric or dilute nitric acid on one side, and in a solution of sulphate of copper on the other, no deposit of copper takes place in twelve hours; but with silver in a solution of cyanide of potassium on one side, and in the double cyanide of potassium and silver on the other, a free deposit of silver takes place upon the end or piece in the latter solution.

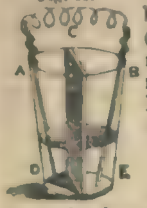
Platina in Nitrate of Copper.—With platina in aqua regia on one side, and in either a solution of nitrate of copper, the ordinary cyanide gilding solution, or a solution of bichloride of platina on the other, no deposit of copper, gold, or platina occurs.

10. *Observations on Class of Instances No. 3.*—1st, it appears, that in this class also we obtain negative as well as positive instances; 2nd, that by this arrangement, unlike the previous classes, almost any metal may cause the same metal to be deposited—for instance, zinc may deposit zinc, copper deposit copper, and silver deposit silver; 3rd, that by it even a noble metal may cause the deposition of a base metal, provided we have a suitable combination of liquids—for instance, if a piece of gold or silver be immersed in a strong solution of cyanide of potassium on one side, and in a solution of sulphate of copper or chloride of antimony on the other, the end in the free cyanide solution will dissolve, whilst that in the copper or antimony solution will receive a deposit; 4th, that the metal, or end which receives a deposit,

derives that power from its contact with the metal in the other liquid; 5th, that, as a general rule, base metals have a greater power of causing deposition by this method than the noble ones; 6th, that the noble metals are more readily and more often deposited than the base ones; and 7th, that we may produce thick and coherent deposits by this method.

11. *Depositing Arrangement No. 4.—Deposition by Two Metals and Two Liquids.*—The following instances belong to the class of deposition produced by the immersion of two metals, A and B (Fig. 69), in two liquids, D and E, the metals being in mutual contact, or connected together by a wire C, and the liquids separated by a porous partition F.

Fig. 69.



Zinc Depositing Antimony.—If a piece of antimony A be immersed in a solution of chloride of antimony D, and a piece of zinc B be immersed in dilute sulphuric acid E, and the two metals are connected together by a wire or wires C, a free deposit of antimony upon the metal A will take place in twelve hours.

Tin Depositing Zinc.—With tin in hydrochloric acid, and zinc in a neutral solution of sulphate of zinc, a deposit of zinc is obtained in the metallic solution.

Iron Depositing Antimony.—With iron in dilute hydrochloric acid, and antimony in chloride of antimony, a copious deposit of antimony takes place in twelve hours.

Copper Depositing Zinc.—With zinc in dilute sulphuric acid, and zinc in a solution of sulphate of zinc, a deposit of zinc occurs.

Zinc Depositing Copper.—With zinc in dilute sulphuric acid, and brass in a solution of sulphate of copper, copper is deposited.

Bismuth and Chloride of Antimony.—With bismuth in dilute hydrochloric acid, and antimony in chloride of antimony, no deposit of the latter takes place in twenty-four hours.

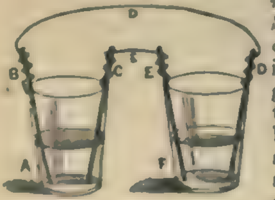
Iron and Chloride of Tin.—With iron in dilute hydrochloric acid, and tin in a solution of chloride of tin, no deposit of tin took place in eighteen hours.

Copper and Chloride of Antimony.—With copper in dilute hydrochloric acid, and antimony in chloride of antimony, or tin in chloride of tin, no deposit of antimony or tin took place in twenty hours.

12. *Observations upon Class of Instances No. 4.*—1st, it appears that negative as well as positive instances occur in this arrangement in common with the others; 2nd, that by using suitable metals and liquids, deposition may be effected more rapidly by this method than by the preceding ones; 3rd, that the metal which receives the deposit derives its power from its contact with the other metal; 4th, that base metals in strong acids have the greatest power of causing a deposit upon the other metals, and noble metals the least; 5th, that the noble metals are more readily deposited than the base ones; and 6th, that thick and coherent deposit may be obtained.

In all the above instances, instead of using one vessel divided into two parts by a porous diaphragm, it will be found convenient to put one of the liquids in an unglazed earthenware porous cell, and immerse the cell in the other liquid. (See vessel A, Fig. 71). In this case, either liquid may be in the outer vessel. This last arrangement (No. 4) is usually termed the "single cell" process.

Fig. 70.

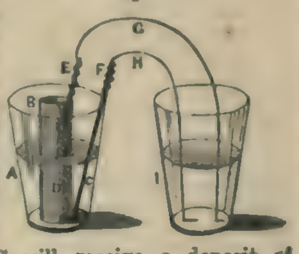


13. *Depositing Arrangement No. 5.—Deposition by Separate Liquid.*—The next class of instances are those in which either of the foregoing arrangements, except the first, may be connected by wires with two pieces of similar metal immersed in a separate liquid. For instance :—

1st. *With Two Metals and One Liquid* (Fig. 70).—If we take a vessel A, containing either dilute sulphuric acid or a solution of sulphate of copper, and immerse in it a piece of zinc B and copper C, with copper wires D and E attached to them, and either immerse the free ends of those wires in a separate solution of sulphate of copper F, or connect them with two pieces of copper immersed in that liquid, the piece of copper E in liquid F will dissolve, whilst the opposite piece D, connected with the zinc, will receive a deposit of copper.

2nd. *With One Metal and Two Liquids.*—If we take a vessel A (Fig. 71) containing a porous cell B, with a neutral solution of sulphate of zinc C in the outer vessel, and dilute sulphuric acid D in the inner, and immerse two pieces of zinc E and F, with copper wires G and H attached, into D and C respectively, and immerse the ends of those wires in a separate solution of sulphate of copper I, the end of the wire H will dissolve, whilst that of G will receive a deposit of metallic copper.

Fig. 71.

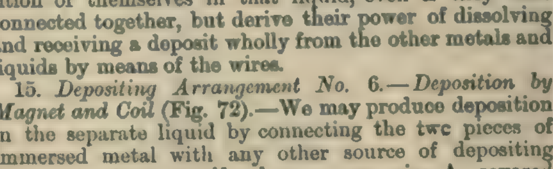


3rd. *With Two Metals and Two Liquids* ("single cell" arrangement, Fig. 71).—If we substitute a piece of copper for the piece of zinc F in the last-mentioned instance, and a solution of sulphate of copper for that of sulphate of zinc, similar effects will take place at the ends of the wires in the liquid I, except that the action will be much more rapid; but if in either of these three instances we use a solution of sulphate of zinc freely acidulated with sulphuric acid, instead of the solution of sulphate of copper I, and platina wires in place of the copper ones to be immersed, neither of the pieces of platina will dissolve or receive a metallic deposit.

14. *Remarks upon Class of Instances No. 5.*—In this class of instances, the method or arrangement differs from the three preceding ones, simply by the wires which connect the two pieces of metal being cut in two, and their free ends either immersed in a separate liquid, or connected with two pieces of metal dipping into that liquid. It is not necessary to have the depositing-vessel perfectly separated; it may even be attached to the same piece of apparatus, provided the liquid in it is perfectly separated from the other liquids and metals. The pieces of metal in the separate liquid possess no power of deposition of themselves in that liquid, even if they were connected together, but derive their power of dissolving and receiving a deposit wholly from the other metals and liquids by means of the wires.

15. *Depositing Arrangement No. 6.—Deposition by Magnet and Coil* (Fig. 72).—We may produce deposition in the separate liquid by connecting the two pieces of immersed metal with any other source of depositing power—for instance, if a long copper wire A, covered with silk or cotton, be coiled upon a large bar of pure soft iron B, and its ends C and D are immersed in a solution of sulphate of copper E, and the poles of a powerful horse-shoe magnet F are brought in contact very many times with the end of the bar, and every time before removing the magnet from the bar one of the ends of the wire be taken out of the liquid, and replaced before returning the magnet, one end of the copper will slightly dissolve, and the other receive a thin copper deposit; but if each of the ends be allowed to remain constantly in the liquid, no such effects will occur.

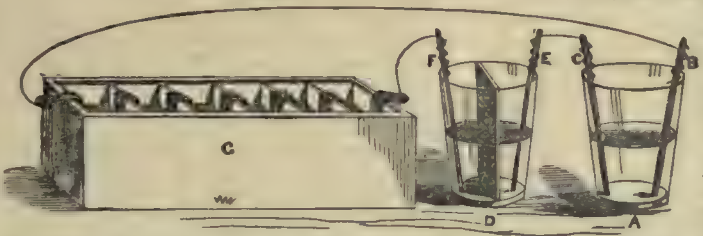
Fig. 72.



16. *Compound Depositing Arrangement No. 7.*—Any of the foregoing combinations of liquids and metals (except the first), or the magnetic arrangement, with or without

separate depositing liquids, may be connected together in a *series* of any number, and may include each of the arrangements in the same series, or include any number of depositing liquids, and deposition may be obtained either in the whole or in any portion of them at the same time: for instance, the vessel A (Fig. 73) contains a piece of zinc B, and copper C immersed in dilute sulphuric acid; vessel D contains zinc E, in dilute sulphuric acid, and copper F, in a solution of sulphate of copper; vessel G contains a series of separate depositing liquids, consisting of solutions of sulphate of copper, connected together by bent pieces of copper; the extreme pieces being attached to zinc B and copper F; here deposition takes place upon every alternate piece of copper throughout the whole series, except that in vessel A.

Fig. 73.



17. *General Observations.*—We may make the following general observations upon the whole of the foregoing facts:—1st, that negative as well as positive instances occur in all classes of facts of electro-deposition; 2nd, that almost any of the ordinary metals, both noble and base, may be deposited by each of the methods or arrangements described; 3rd, that the particular result of deposition or non-deposition occurring, appears to depend chiefly upon the particular combination of liquids and metals, their arrangement and connections; 4th, that the size or shape of the containing vessels, the bulk or depths of the liquids, the size, form, or position of the metals, appear to exercise little or no influence upon the result; 5th, that in all cases of deposition there is a difference in *kind*, either of the metal, of the liquid, or of both; 6th, that in all such cases a metal dissolves in a liquid; for instance, in Arrangement No. 1, with a piece of iron immersed in a solution of sulphate of copper, a portion of the iron is dissolved as the copper is deposited; in Arrangement No. 2, with iron and copper together in a solution of sulphate of copper, the iron dissolves and the copper receives a deposit; in Arrangement No. 3, with copper in dilute sulphuric acid, and in a solution of sulphate of copper, the copper in the acid dissolves, whilst that in the metallic solution receives a deposit; in Arrangement No. 4, with zinc in dilute sulphuric acid, and copper in sulphate of copper, the former dissolves, whilst the latter receives a deposit; in Arrangements Nos. 5 and 6, one piece of metal in the separate depositing liquid dissolves, whilst the other receives a metallic deposit, and the same with the compound Arrangement No. 7; 7th, that in Arrangements Nos. 1 and 2, the same piece of metal which dissolves also receives a metallic deposit; and in Arrangements 3, 4, 5, 6, and 7, the pieces of metal which receive a deposit do not dissolve; 8th, that, as a general rule, in all methods, and in all solutions, except alkaline metallic cyanides, zinc, among common metals, has the greatest, and platina the least power of producing deposition; 9th, that zinc generally deposits metals most rapidly from their solutions, and most frequently in the state of a dark-coloured or black powder; and 10th, that among solutions of the salts of ordinary metals, those of the salts of noble metals yield their metal most easily, and those of the base metals, zinc especially, with the greatest difficulty.

18. In looking over the foregoing instances, we also observe:—1st, that when several metals are used, they must either *touch* each other or be connected together by wires or other pieces of *metal;* 2nd, that when several liquids are used, they also must *touch* each other, either by means of a porous diaphragm, or otherwise; 3rd, that when a series of metals and liquids are used, they must together form a *complete circuit*, and all their points of contact be perfectly clean; 4th, that a separate depositing liquid possesses no power of deposition by itself, but derives its power by means of the wires from the other arrangements with which it is connected; and 5th, that the length of the connecting wire, if not great, has no influence on the result. These observations have led us to conclude that deposition is caused by some *force* which is generated in some part of the apparatus, and circulates through the liquids, metals, and wires, which compose the circuit.

19. *Principles.—Conditions of Electro-Deposition.*—From the simple *facts* of electro-deposition, and the general observations made upon them, we proceed to consider the *causes* of deposition, and the conditions or circumstances under which deposition occurs, in the following order:—

1st. *The Chemical Conditions.*—If we immerse a clean iron wire in a solution of nitrate of mercury, it receives a deposit of that metal; but if we immerse it in perfectly dry metallic mercury, it receives no deposit, because in the former case the necessary chemical conditions of deposition are present, whilst in the latter case they are absent.

2nd. *The Electrical Conditions.*—If we connect together a piece of iron and a piece of copper, by means of a metal wire, and immerse them in a solution of sulphate of copper, the copper will receive a metallic deposit; but if we connect them together by a cord of gutta-percha, or rod of glass, no deposit will take place, because, in the former case, all the electrical conditions are present, whilst in the latter case, one of them—viz., a complete conducting circuit, is absent.

3rd. *The Thermal Conditions* remain almost unknown.

4th. *The Mechanical Conditions.*—If a piece of iron be immersed in a solution of sulphate of copper, it receives a copper deposit; but if a piece of platina be so immersed it receives no such deposit, because, for one reason, in the former instance the mechanical conditions of attraction and repulsion at the dissolving and depositing surfaces are present, but in the latter they are absent.

5th. *The Mathematical Conditions.*—If we immerse two pieces of carbon in fused *proto*-chloride of tin, and connect them with a voltaic battery, tin will be deposited: but if we immerse them in fused *bi*-chloride of tin, no deposition will occur, because, in the first instance, all the mathematical conditions are present; the fluid salt contains *one* atom of chlorine for each atom of tin; whilst in the latter case one of them is absent, the salt contains *two* atoms of chlorine to one atom of tin; and, according to Faraday's law, "only those substances of the first order are directly decomposable which contain one atom of one of their elements for each atom of the other."

These several heads are capable of including all the known circumstances or conditions under which deposition occurs; and under each of them will be given a few instances, both of deposition and non-deposition, to illustrate the principle. It would be advisable for the reader to try for himself most of the experiments given, in order to fix the principles firmly in his memory.

20. *Chemical Conditions of Deposition.*—The first chemical condition to be observed is, that in every case of deposition, the depositing liquid contains acid and basic elements—namely, a salt, the acid of which is to dissolve or combine with one metal, and its metal or base to be deposited upon the other.

1st. *Deposition by One Metal and One Liquid.*—With the first class of facts, if we immerse a piece of clean iron in a solution of nitrate of mercury, it will receive a deposit of that metal; but if we immerse it in mercury alone, it will receive no deposit: in the first instance an acid as well as a basic substance was present, and deposition took place; but in the second instance the metal or base alone was present, and no deposition occurred.

2nd. *With Two Metals and One Liquid.*—If zinc and platina are immersed in mutual contact in a solution of nitrate of mercury, the platina will receive a metallic deposit; but if they are immersed in pure dry mercury, no deposit will occur.

3rd. *With One Metal and Two Liquids.*—If we immerse one end of a platina wire in a strong solution of cyanide of potassium, and its other end in a solution of nitrate of mercury, the two liquids being in mutual contact by means of a porous partition, the end in the metallic solution will soon receive a deposit of mercury; but if perfectly dry mercury be substituted for the nitrate solution, no such deposit will occur.

4th. *With Two Metals and Two Liquids.*—If we immerse zinc in dilute sulphuric acid, and platina in a solution of nitrate of mercury, the two liquids being separated by a porous partition, and the metals connected together by a wire, the platina will quickly receive a deposit of mercury; but with dry mercury instead of the metallic solution, the platina will not receive a deposit.

5th. *With a Separate Depositing Liquid.*—If we take two pieces of platina wire, connect them, as already described (13), with either of the foregoing Arrangements, or with a magnet and coil (15), and immerse their free ends in a solution of nitrate of mercury, one of the wires will receive a deposit of mercury; but if the separate liquid consist only of dry mercury, no deposit will be obtained.

It is evident from these facts, that in every case where deposition occurs, the depositing liquid contains both acid and basic substances, and that without the presence of both no deposition will take place.

21. *Degrees of Chemical Affinity of Metals and Liquids.*—At the present point it is necessary to mention a few instances of the different degrees of chemical affinity manifested by different metals and liquids, that the reader may be able to understand their general chemical relations in electro-deposition more clearly.

1st. If we immerse a piece of potassium in almost any liquid, very violent chemical action takes place, which is stronger in mineral and vegetable acids than in water or organic liquids; if we place a small piece of it upon water, violent chemical action occurs, the water is decomposed, heat is produced, gas is evolved which takes fire, the metal melts and rolls about on the surface of the water, oxidates and dissolves.

2nd. If we immerse a piece of zinc in any of the strong mineral acids (sulphuric, hydrochloric, hydrofluoric, or nitric acids), strong chemical action takes place, gas is freely evolved, and the metal oxidates and dissolves; with solutions of the ordinary vegetable acids, *i.e.*, oxalic, tartaric, citric, formic, and acetic acids, the same effects occur in a much weaker degree; but with pure water there is no visible decomposition, no gas evolved, nor any perceptible chemical action. These instances show that potassium has a much stronger affinity for water or its oxygen than zinc, and that both potassium and zinc have a stronger affinity for acids, especially mineral ones, than for water.

3rd. If we immerse a piece of zinc successively in each of the ordinary mineral and vegetable acids diluted with water, it will be quickly dissolved, with evolution of hydrogen gas in nearly all of them; but if we immerse a piece of copper in those liquids, it will be quickly dissolved in only one of them—viz., nitric acid; and from this we conclude that copper has generally a much weaker affinity for acids than zinc.

4th. If we immerse either gold or platina in any of the strong mineral or vegetable acids, or even in cold aqua regia, it will be quite unaffected in all of them; whilst copper would be rapidly acted upon by nitric acid or by aqua regia, and slowly by several of the others, thereby showing that the affinity of gold or platina for acids, is generally much weaker than that of copper.

From the foregoing, and many other instances that might be mentioned, we conclude that the general order of affinity of the metals for acids occurs in the following order—namely, potassium, zinc, copper, gold, and platina. Hydrocyanic acid and cyanogen appear to differ in one respect, in their chemical relations towards ordinary metals, from oxygen, chlorine, and the ordinary mineral and vegetable acids, in having a much stronger affinity for noble metals, and a weaker for base metals: this is probably one of the chief reasons for the extensive adoption of cyanogen compounds in electro-deposition. Those compounds are highly suitable for the deposition of noble metals, because of the great affinity of cyanogen for them; but not, as some persons assert, for the deposition of many of the base metals, on account of a comparatively weak affinity for them.

22. Potassium is an *alkali* metal; zinc, cadmium, tin, lead, iron, cobalt, nickel, and copper, are *base* metals; and mercury, silver, gold, platina, palladium, &c., are *noble* metals; and of these three classes, the alkaline metals possess the strongest affinity for acids, base metals intermediate, and noble metals the least. The same order prevails in their degrees of depositing power; potassium and the alkaline metals generally deposit nearly all others from their solutions; zinc, and the base metals generally, deposit a smaller number; and gold, platina, and the other noble metals deposit a very few from their solutions. Thus we perceive that those metals which have the strongest chemical affinity for acids, possess the greatest depositing power; and those which have the least affinity for acids, have also the least depositing power.

23. *Difference of Chemical Affinity necessary to Deposition.*—The second chemical condition which we have to observe upon is, that, in every case of *deposition*, there is a *difference* of chemical affinity at the dissolving and receiving surfaces for the different elements of the liquid, and that the dissolving metal has a stronger affinity for the acid elements of the liquid than either the metal in solution or the receiving metal. For instance—

1st. *With One Metal and One Liquid.*—If we immerse a piece of iron in a solution of sulphate of copper, a deposit takes place upon it; but if we immerse a piece of platina in the liquid, it receives no deposit: in the first case, the iron having a stronger affinity for the sulphuric acid of the salt than copper, combines with it and dissolves, and the copper thereby set free from the acid is deposited upon the iron; whilst in the second case, platina having a much weaker affinity for the acid than the copper, cannot separate the acid and copper, and therefore cannot cause deposition.

2nd. *With Two Metals and One Liquid.*—If we immerse copper and iron in mutual contact, in a solution of sulphate of copper, the iron dissolves, and deposition of copper takes place upon both metals; but if we immerse copper and platina in mutual contact in this solution, no deposition will occur. In the first instance, the iron possessing a stronger chemical affinity for sulphuric acid than the copper, combines with it, and sets the copper free; by this action, a current of depositing force is generated, which circulates through the iron, liquid, and copper, at their points of contact, and causes the metal of the liquid to be deposited upon the piece of copper; but in the second case, neither the copper nor platina possessing a stronger affinity for the acid of the salt than its associated metal, there is no copper set free, no current of depositing force generated, and consequently no deposition takes place.

3rd. *With One Metal and Two Liquids.*—If we immerse one end of a piece of copper in dilute sulphuric acid, and the other in a solution of sulphate of copper, the two liquids touching each other, copper will be deposited upon the end immersed in the metallic solution, whilst the other end will combine with the acid and dissolve; but if a piece of platina or gold be substituted for the copper, neither of its ends will dissolve or receive a metallic deposit: in the first instance, the dilute sulphuric acid, having a stronger affinity for copper than a solution of sulphate of copper, combines with it, causes it to dissolve, develops a current of depositing force which circulates through the metals and liquids, and a deposit of copper is produced; whilst in the second instance, the platina or gold having a weaker affinity for the acid of one liquid than copper for the acid of the other liquid, cannot separate the copper, or cause deposition.

4th. *With Two Metals and Two Liquids.*—If we immerse a piece of silver in a strong solution of cyanide of potassium, and a piece of copper in a solution of the double cyanide of copper and potassium, the liquids being separated by a porous partition, and the metals connected by a wire, the silver will dissolve, and the copper receive a metallic deposit; but if a piece of iron be substituted for the silver, no deposit will occur. In the first instance, the one solution has a stronger affinity for the silver than the other has for the copper—consequently the former is dissolved, a current or depositing force is generated, and copper deposited; but in the second case, the one liquid has a weaker affinity for iron than the other has for copper, and therefore no iron is dissolved, no depositing force generated, and no copper deposited.

5th. *With a Separate Depositing Liquid connected with any Source of Depositing Power.*—If we connect two pieces of silver with any of those sources of power, and immerse them in a solution of the double cyanide of silver and potassium, one piece will quickly dissolve, and the other receive a deposit of silver; but if pieces of iron are substituted for those of silver, neither will dissolve or receive a metallic deposit.

In every case where a separate depositing liquid is used, the two pieces of metal immersed in it have a *difference* of chemical affinity imparted to them by virtue of their connection with some arrangement which develops depositing force; and this difference of affinity is manifested most when the liquid has a strong affinity for the immersed metal, and least when it has a weak affinity for that metal: so in the first of the two immediately preceding instances, the liquid having a strong affinity for silver, allows this difference of affinity to be freely exercised at the immersed surfaces of the two pieces of metal, and consequently one dissolves, and the other receives a deposit; but in the second of these instances, the liquid having a very weak affinity for iron, does not admit of the exercise of this difference of affinity, and hence neither piece dissolves or receives a deposit. From these instances it is manifest, that whenever deposition occurs, there is a difference of chemical affinity between the dissolving and receiving surfaces for the different elements of the liquid; that the dissolving metal has a stronger affinity for the acid elements than the receiving one; and without this condition no deposition will occur.

24. It must be mentioned, also, that the metals which have the greatest difference in their degrees of affinity for acids, are those which evolve the greatest strength of depositing power. For instance—

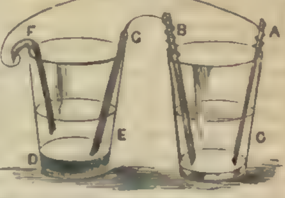
1st. *With One Metal and One Liquid.*—If we immerse a piece of silver in a solution of terchloride of gold, it slowly becomes gilded; but if we immerse a piece of zinc in it, gold is almost instantaneously deposited; because, in the former case, the difference of affinity between gold and the immersed metal for the acid of the liquid is very much smaller than in the latter case.

2nd. *With Two Metals and One Liquid.*—If we immerse a piece of platina and a piece of copper, in mutual contact, in a solution of nitrate of silver, the platina will become silvered, but much more slowly than if iron or zinc were used in place of the copper, because there is a greater difference of affinity between platina and iron or zinc for the acid of the liquid, than between platina and copper; or, if we immerse a piece of zinc and a piece of platina or platinised silver in dilute sulphuric acid, and connect them separately with two pieces of copper immersed in a separate solution of sulphate of copper, copper will be dissolved and deposited in the separate liquid more rapidly than if we used zinc and copper in place of zinc and platina, and much more rapidly than if we used iron and copper, because the difference of affinity between zinc, platina, or platinised silver for dilute sulphuric acid, is more than zinc and copper for that liquid, and much more than between iron and copper.

3rd. *With Two Metals and Two Liquids.*—If we immerse a piece of zinc in dilute sulphuric acid, and a piece of copper in a solution of sulphate of copper, the two liquids touching each other, and the two metals connected with two pieces of copper in a separate solution of sulphate of copper, the amount of metal dissolved and deposited in a given time in the separate liquid, will be much smaller than if we used zinc in dilute sulphuric acid, and platina in strong nitric acid; because the difference of affinity between the two metals in the two liquids, in the first instance, is less than in the second instance.

25. *Acid and Basic Affinities necessary.*—Chemical affinity differs not only in degree; but also in kind; basic substances, such as metals, alkalies, alkaloids, and most metallic oxides, have a great tendency to combine with acids; whilst oxygen, sulphur, phosphorus, chlorine, iodine, bromine, fluorine, mineral and vegetable acids, and some metallic oxides, tend to combine with bases. In a similar manner, the difference of affinity between the dissolving and receiving surfaces in electro-deposition is not wholly one of degree, but is also one of kind: the dissolving metal, in a separate depositing liquid (13), acquires, by virtue of its connection with some source of depositing power, an affinity for the *acid* elements of the liquid; whilst the receiving metal acquires, by the same means, an affinity of an opposite kind: for instance (Fig. 74), if a piece of platina A, and a piece of zinc B, are immersed in dilute sulphuric acid C, some mercury D placed at the bottom of a separate solution of protosulphate of iron E, a piece of iron F immersed in this liquid and connected by a wire with the platina, and the mercury connected by an iron or platina wire G, which is prevented from touching the liquid by a tube of glass or gutta-percha, the immersed piece of iron will exercise one kind of affinity, combine with the acid of the dissolved salt, and form a definite chemical compound (protosulphate of iron), containing one equivalent of iron and one equivalent of sulphuric acid; while the mercury will exercise an opposite kind of affinity, and combine with the metal or base of the salt, forming likewise a definite chemical compound (Fe Hg), containing one equivalent of iron and one of mercury. If a solution of sulphate of copper be substituted for that of sulphate of iron, and a piece of copper for the piece of iron, a similar definite compound of copper and sulphuric acid will be formed at the dissolving plate, and of copper and mercury (Cu Hg) at the receiving metal.

Fig. 74.



These experiments prove, in a most satisfactory manner, that, in the act of deposition, the surface of the dissolving metal possesses one kind of affinity by virtue of which it tends to attract *acid* substances, and combine with them in definite proportions; and the surface of the receiving metal possesses an opposite kind of affinity, by virtue of which it tends to attract and combine with *basic* substances, also in definite proportions. Mercury is the only metal which has been observed to manifest this definite affinity at the receiving surface, probably because it is the only metal fluid at ordinary temperatures, fluidity being an essential condition of such affinity; but it is likely that other metals would also manifest this tendency, if kept in a melted state in contact with suitable fused salts, and properly acted upon by depositing force.

26. *Fluidity essential to Electro-Deposition.*—The affinities of electro-deposition, like those of ordinary chemical action, require, generally speaking, one at least of the combining bodies to be in the liquid state; and they act, like them, wholly at insensible distances, being confined in their exercise to the immediate surfaces of mutual contact in the opposed substances. The compound formed at those surfaces becomes diffused through the fluid masses by capillary attraction and mechanical

mixture. This affords a reasonable explanation why fluidity of the receiving metal is essential to the formation of definite compounds at its surface, as well as why fluidity of the metallic salt is essential to its decomposition, and for the formation of definite compounds at the dissolving surface. If chemical action took place at sensible and considerable distances, *i.e.*, throughout the whole mass of the opposed fluid bodies, combination would probably be in all cases violent and instantaneous; and if fluidity were not essential to combination, the substances deposited upon the receiving surface would probably, in most cases, enter into the mass of the receiving metal, and combine with it.

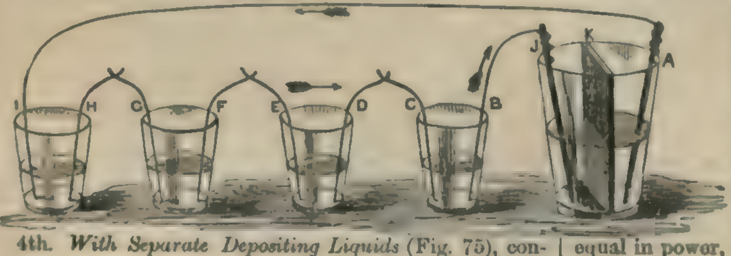
27. *Alternation and Circulation of Chemical Affinities necessary.*—The next chemical condition to be mentioned is, that in every case of deposition, the surfaces at which the acid and basic affinities are manifested, *alternate* with each other in the circuit, and that the acid affinity *circulates* one way, while the basic affinity *circulates* in the opposite direction. For instance—

1st. *With One Metal and One Liquid.*—If we immerse a piece of copper in a solution of double cyanide of silver and potassium, it becomes silvered; but if we immerse a piece of iron therein, it receives no deposit. In the first of these instances, it is considered that, immediately upon the immersion of the metal, the superior affinity of copper for cyanogen over that of silver, causes it to combine with that compound, and sets the silver free; at the same time, an immense number of minute currents of depositing force or chemical affinity are developed all over the immersed surface of the piece of copper, leaving it at innumerable minute points, passing a very small distance into the liquid, and re-entering the copper at numberless other points; and thus the affinities circulate, the copper dissolves, and receives a deposit simultaneously. In each of these atomic circuits, as they are termed, acid affinity is exercised where the depositing force leaves the metal, and basic affinity where it re-enters it; but in the second instance, where iron is used, there is no circulation of those affinities, no dissolving of metal at one point, or deposition of it at another.

2nd. *With Two Metals and One Liquid.*—If we immerse a piece of iron and a piece of copper, in mutual contact, in a solution of sulphate of copper, a deposit of copper will take place upon the iron and upon the copper; but if we immerse a piece of gold in place of the iron, no deposit will occur upon either. In the first of these instances, in addition to the circulation of atomic currents of affinity all over the immersed surface of the iron, as already explained, and which causes it to dissolve and receive a deposit, there are separate and distinct currents of the same force circulating through the liquid; and the two metals, by their points of mutual contact, cause the surface of the iron to combine with the acid, and that of the copper to receive a metallic deposit; but in the second case there is no circulation of affinities, and no solution of the gold, or deposition of the copper.

3rd. *With One Metal and Two Liquids.*—If we immerse two pieces of silver, one in a solution of cyanide of potassium, and the other in a solution of double cyanide of silver and potassium, the two pieces being connected together by a wire, and the liquids being divided only by a porous partition, chemical affinities will circulate by the metals, wire, and liquids, and silver will be dissolved and deposited; but if we substitute pieces of iron for the pieces of silver, there will be no circulation of affinities and no deposition.

Fig. 75.



4th. *With Separate Depositing Liquids* (Fig. 75), consisting of solutions of sulphate of copper, with separate pairs of copper wires C D, E F, G H, immersed in them, and with end pieces of copper B and I, B being connected with a piece of zinc J immersed in dilute sulphuric acid, and I connected with a piece of copper A immersed in a solution of sulphate of copper, the two liquids being separated by a porous diaphragm K, chemical affinities will circulate in opposite directions through the whole of the circuit, every alternate piece of immersed metal, J, I, G, E, and C, will exercise one kind of affinity and dissolve, and every other alternate piece, A, B, D, F, and H, will exercise an opposite kind of affinity, and receive a deposit of copper. If in either of the foregoing instances, where deposition occurs, we break the continuity of the circuit, either by separating the metals from each other at their points of contact, lifting them out of the liquid, or by cutting through the connecting wires, no deposition will occur; but if we immerse the free ends of the divided wire in a suitable liquid, such as a solution of sulphate of copper, if the wires are of copper, deposition will immediately recommence throughout the circuit.

28. A consideration of such facts as these, leads us to conclude that the process which causes every alternate metal in a series to combine with *acids* and dissolve, and every other alternate metal in the circuit either to combine with bases, or receive a deposit, is of a chemical character, and occurs in opposite directions through the circuit, which has led to the application of the term "current affinity," to designate the depositing force, when viewed only in a *chemical* aspect.

29. Two or any other *even* number of vessels containing combinations of liquids and metals, each similar, and capable of generating current affinity, may be so arranged in a complete conducting circuit, that no current affinity will circulate or deposition occur: for instance, if we take two separate vessels A and B (Fig. 76), each containing dilute sulphuric acid, and a separate piece of zinc Z, and silver (S), and connect them together thus, zinc-silver silver-zinc, no depositing power will be manifested if we immerse the free ends of their copper wires C C in a separate solution of sulphate of copper D; but if they are connected thus, zinc-silver zinc-silver (Fig. 77), and the copper wires immersed as before, current affinity will circulate, and deposition will proceed in the separate liquid. In the first of these cases the affinities set in motion by the metals and liquid in the vessel A are opposite in direction to those generated in vessel B, and the two arrangements, being equal in power, exactly neutralise each other, preventing

Fig. 76.

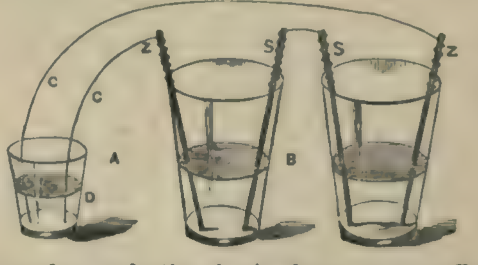
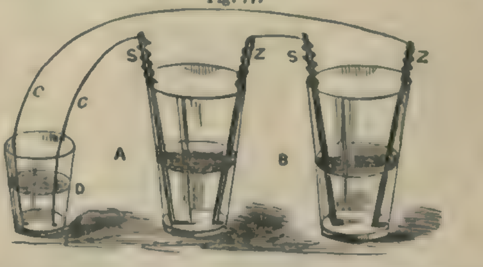


Fig. 77.



the currents of affinity and their effects; but, in the second case, the direction of the affinities evolved by the vessel A coincide with that of those from vessel B, the currents circulate, and deposition is effected. In the case of an unequal portion of the metals of a series being connected in the opposite way, those which are wrongly connected will neutralise and be neutralised by an equal number of the remainder, provided that all the pairs of metals and the liquids are similar: for instance, if three out of twelve are wrongly connected, they will neutralise the power of three more, and only the remaining six will act in the desired direction.

30. During the act of deposition, a salt is generally formed at the dissolving surface by the union of the metal with the acid elements of the liquid, which is dissolved in the liquid; at the same time the acid, which combines with the dissolving surface, is generally set free at the receiving surface by the deposition of the metal. For instance—

1st. *With One Metal and One Liquid.*—When iron coats itself with mercury by simple immersion in a solution of nitrate of mercury, nitrate of iron is formed by the union of the iron with the nitric acid of the nitrate of mercury, and nitric acid is at the same time set free by the deposition of the mercury.

2nd. *With Two Metals and One Liquid.*—When copper receives a deposit of copper by immersion, in contact with a piece of iron in a solution of sulphate of copper, sulphate of iron is formed at the immersed surface of the iron, by the union of the sulphuric acid of the sulphate copper salt, with the iron; at the same time sulphuric acid is set free at the copper surface by the deposition of the copper.

3rd. *With One Metal and Two Liquids.*—When copper receives a deposit of copper in a solution of sulphate of copper, by connection with another piece of copper immersed in dilute sulphuric acid, the two liquids being separated by means of a porous partition or otherwise, sulphate of copper is formed at the surface of the metal in the dilute acid, by the union of the copper with the acid, and, at the same time, sulphuric acid is set free at the surface of the other piece by the deposition of the copper.

4th. *Silver.*—When silver or any other metal receives a coating of silver in a solution of double cyanide of silver and potassium, by connection with a piece of zinc in dilute sulphuric acid, the two liquids being separated by a porous diaphragm, sulphate of zinc is formed at the surface of the zinc, by the union of the acid and zinc, and cyanogen (a substance of an acid character) is at the same time set free at the receiving surface, by the deposition of the silver.

5th. *With a Separate Depositing Liquid*, connected with any source of current affinity: for instance, when two silver plates, immersed in a solution of the double cyanide of silver and potassium, are connected by wires with a piece of zinc, and a piece of copper immersed in dilute sulphuric acid in a separate vessel, cyanide of silver is formed at the surface of one of the pieces of silver by the union of that metal with the cyanogen of the liquid; and, at the same time, either cyanogen or hydro-cyanic acid is set free at the surface of the other piece by the deposition of the silver.

31. *Proper Proportion of Free Acid in the Depositing Liquid.*—If a solution contain a large excess of uncombined acid, metallic deposition will not always occur: for instance, if two pieces of zinc are immersed in a neutral solution of sulphate of zinc, and connected by wires with another piece of zinc, and a piece of silver immersed in dilute sulphuric acid, in a separate vessel, one piece of zinc will dissolve, and the other receive a deposit of metal; but if a rather large quantity of sulphuric acid be added to the depositing liquid, no deposit of zinc will occur. In the same manner, if we connect two pieces of silver, immersed in a strong solution of cyanide of potassium, with the zinc and silver in dilute sulphuric acid, as just described, one of the pieces of silver will combine with the cyanogen of the cyanide of potassium, and form cyanide of silver, which will combine with a portion of the remaining cyanide and then dissolve; but the other piece of silver will not receive a deposit of silver, until the remaining uncombined cyanide of potassium has decreased to a certain proportion by the working of the process. If, on the other hand, a depositing solution contain no *free* combining substance, deposition will either proceed very slowly, or be entirely stopped, in consequence of an insoluble salt being formed upon the surface of the dissolving metal, which will impede the action: for instance, when two silver plates, immersed in a solution of double cyanide of silver and potassium, are connected with some source of depositing power, one of the plates will receive a deposit of silver, whilst the other will gradually become covered with a white layer of insoluble cyanide of silver, which eventually stops deposition.

32. *Necessity of a Proper Proportion of Water.*—If a depositing solution be diluted with water to a very large extent, deposition will progress very slowly; but if, on the other hand, it contain insufficient water, crystals of metallic salts will collect upon the dissolving metal, and gradually stop the action: for instance, if two pieces of copper, immersed in a *saturated* solution of sulphate of copper containing free acid are connected with a piece of zinc, and a piece of silver immersed in dilute sulphuric acid in a separate vessel, one piece will receive a deposit of copper, whilst the other will slowly dissolve and gradually become covered with crystals of sulphate of copper, first at its lower part and then at the edges, which will gradually stop the action.

33. All the foregoing facts prove that chemical affinity plays a very important part in the phenomena of electro-deposition.

34. *Electrical Conditions of Deposition—Positive and Negative Substances necessary.*—The first electrical condition to be observed is, that in every case of deposition, the liquid contains both substances of an electro-positive and of an electro-negative character; metals and alkalies are electro-positive, and metalloids (oxygen, sulphur, chlorine, iodine, bromine, &c.) and acids are electro-negative; cyanogen is also electro-negative.*

1st. *With One Metal and One Liquid.*—In the first instance, if we immerse a piece of copper in a solution of nitrate of mercury, deposition occurs, mercury being thrown down; but if we immerse it in dry mercury, there is no deposition: in the former case the liquid contains both electro-positive mercury and negative nitric acid, but in the latter case it only contains the positive mercury.

2nd. *With Two Metals and One Liquid.*—Immerse zinc and platina in mutual contact in a solution of nitrate of mercury, the platina will receive a deposit; but if we immerse them in dry mercury, it will receive no deposit.

3rd. *With One Metal and Two Liquids.*—If one end of a platina wire be immersed in a solution of cyanide of potassium, and the other end in a solution of nitrate of mercury, the two liquids being separated by a porous partition, the end in the mercurial solution will receive a metallic deposit; but if dry mercury be substituted for the nitrate solution, no deposition will occur.

4th. *With Two Metals and Two Liquids.*—If we immerse zinc in dilute sulphuric acid, and platina in a solution of nitrate of mercury, the metals touching each other, and the liquids separated by a porous partition, the platina will receive a mercurial deposit; but when dry mercury is substituted for the nitrate solution, it will receive no deposit.

5th. *With a Separate Depositing Liquid.*—If we immerse two platina wires in a solution of nitrate of mercury, and connect them with any source of depositing power, one of the wires will receive a metallic deposit; but if we use dry mercury instead of the nitrate solution, there will be no deposit.

These and many other instances prove, that unless the depositing liquid contain both electro-positive and electro-negative substances, no deposition will occur.

35. *Electric Polarity of the Dissolving and Receiving*

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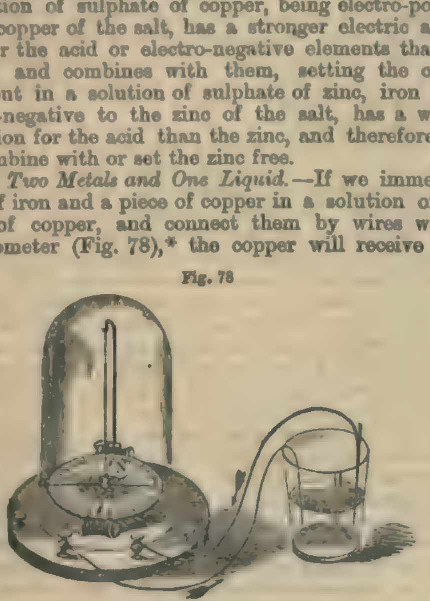
* See *ante*, p. 194.

Metals.—The second electrical condition to be observed is, that in every case of deposition there is an electrical difference between the dissolving and receiving metals, and that the former is always electro-positive, the latter being electro-negative, relatively to each other; the dissolving metal, consequently, has the strongest electrical attraction for the electro-negative or acid elements of the liquid, and the receiving metal has the strongest electrical attraction for the electro-positive or metallic elements. For instance—

1st. *With One Metal and One Liquid.*—Iron placed in a solution of sulphate of copper, being electro-positive to the copper of the salt, has a stronger electric attraction for the acid or electro-negative elements than the copper, and combines with them, setting the copper free; but in a solution of sulphate of zinc, iron being electro-negative to the zinc of the salt, has a weaker attraction for the acid than the zinc, and therefore does not combine with or set the zinc free.

2nd. *Two Metals and One Liquid.*—If we immerse a piece of iron and a piece of copper in a solution of sulphate of copper, and connect them by wires with a galvanometer (Fig. 78),* the copper will receive a deposit, and the needle N of the instrument will be strongly deflected in such a direction as to indicate that the iron is positive, and the copper negative; but if a piece of iron and a piece of zinc are immersed in a solution of sulphate of zinc, and connected with the instrument, no deposit will take place upon either metal, and scarcely any deflection of the needles will be produced.

Fig. 78



3rd. *With One Metal and Two Liquids.*—If one piece of copper be immersed in dilute sulphuric acid, and another in a neutral solution of sulphate of copper, the two liquids being separated by a porous partition, and the pieces of copper connected by wires with the galvanometer, deposition will take place upon the piece of copper in the metallic solution, and the instrument will indicate, by the direction of the deflection of its needles, that the dissolving piece of metal is positive, and the receiving piece negative; but if we substitute two pieces of platina for the pieces of copper, no deposition of copper will occur, and scarcely any difference of electric condition between the two pieces of metal will be indicated by the galvanometer.

4th. *With Two Metals and Two Liquids.*—If a piece of zinc be immersed in dilute sulphuric acid, and a piece of copper in a solution of sulphate of copper, the two liquids being separated by a porous partition, and the metals connected with the galvanometer, deposition will take place freely upon the copper, and the needles of the instrument will be powerfully deflected, indicating, by the amount and direction of their movement, the zinc to be strongly positive and the copper negative; but if a piece of platina be substituted for the zinc, there will be no deposition, and scarcely any deflection of the needle.

5th. *With a Separate Depositing Liquid.*—If we immerse two pieces of silver in a solution of the double cyanide of silver and potassium, and connect them with any source of depositing power, interposing a galvanometer in the circuit, deposition of silver will occur, and the needles will be strongly deflected in such a direction as to indicate the dissolving piece of silver to be positive, and the receiving piece negative; but if we substitute two pieces of iron for the pieces of silver, there will be scarcely any deposition of silver, and very little deflection of the needle, the receiving piece being feebly negative. From these and many other similar cases, we conclude, that in every case where deposition occurs, the dissolving metal is electro-positive, and the receiving metal electro-negative, and that the former has the strongest electric attraction for the negative elements of the liquid, and the latter for the basic or metallic element. In all electric phenomena, positive substances repel positive and attract negative, and negative substances repel negative and attract positive.†

36. *Alternation and Circulation of Electro-Polarities necessary.*—The third electric condition is, that in every case of deposition, the positive and negative surfaces, or metals, *alternate* with each other in the circuit, and electric forces *circulate* through the circuit. For instance—

1st. *With One Metal and One Liquid.*—When a piece of iron is immersed in a solution of sulphate of copper, innumerable minute electric currents are generated upon the surface of the metal, and circulate to a very minute depth within the opposed surface of the liquid; while the positive electricity, passing out of innumerable points of the iron into the solution, causes it to dissolve—pass through the solution, re-enter the iron at innumerable other points, and deposit the copper; but when a piece of gold is immersed in this solution, there are no currents of electricity generated, and the gold is neither dissolved nor does it receive a deposit.

2nd. *With Two Metals and One Liquid.*—When a piece of zinc and a piece of silver are immersed in mutual contact in a solution of nitrate of silver, the zinc dissolves and the silver receives a deposit, and an electric current is developed which circulates through the metals and liquids, and may be rendered further evident by connecting the metals with a galvanometer (Fig. 78): the zinc is positive, and the silver negative.

3rd. *With One Metal and Two Liquids.*—When two pieces of copper are immersed, one in dilute sulphuric acid, and the other in a solution of sulphate of copper, the two liquids being separated by a porous diaphragm, and the pieces of copper connected with a galvanometer, an electric current circulates, the piece of copper in the acid is positive and dissolves, and the piece in the metallic solution is negative and receives a deposit.

4th. *With Two Metals and Two Liquids.*—When a piece of zinc is immersed in dilute sulphuric acid, and a piece of silver in a solution of double cyanide of silver and potassium, and the two are connected with a galvanometer, the two liquids being separated by a porous partition, electricity circulates, the zinc is positive and dissolves, and the silver is negative and receives a deposit.

5th. *With a Separate Depositing Liquid.*—When two pieces of antimony, immersed in the ordinary chloride of antimony, are connected with any source of electricity, one piece becomes positive and dissolves, and the other negative and receives a deposit, and an electric current circulates, as may easily be proved by the galvanometer, in the foregoing manner.

6th. *With a Series of Depositing Liquids.*—If a series of depositing vessels (Figs. 73 and 75), containing solutions of sulphate of copper, and pieces of copper, are connected by their extreme pieces with a sheet of zinc, and a sheet of copper be immersed in a solution of sulphate of copper, every alternate piece of metal in the series will be electro-positive and dissolve, and every other alternate piece electro-negative and receive a deposit, and a current of electricity will circulate through the whole series.

37. *Electrical Conducting Circuit necessary.*—In every case where deposition is proceeding, the whole of the circuit must be capable of conducting voltaic electricity; and this is one important condition of the result; for if, in any such case, we interpose an imperfect electric conductor in the circuit such as a long iron wire, or a short

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* See *ante*, p. 195.

† See *ante*, p. 178.

column of water, the process will be greatly impeded; and if we interpose a non-conductor of electricity, such as a rod of glass or of gutta-percha, or otherwise break the continuity of the circuit, deposition will be completely arrested.

38. *Electric Conductivity.*—The following is the general order of the conducting power of metals and alloys for voltaic electricity at 60° Fah., beginning with those which conduct most freely—viz., silver, copper, gold, cadmium, zinc, brass, tin, palladium, iron, steel, lead, platina, German silver, antimony, mercury, bismuth, potassium. The order is somewhat different at other temperatures.

39. *Direction of the Electric Currents.*—In every case where a current of electricity is developed by the mutual contact of liquids and metals, or where it merely passes through them, as in a separate depositing vessel, the current of *positive* electricity invariably passes from the positive or dissolving metal, *through the liquid*, to the negative or receiving one; and the negative electricity passes similarly in the opposite direction. When we speak of "*the current*," without stating which is meant—the positive or negative electricity—it is always intended, for the sake of simplicity of expression, to indicate the *positive* electricity: when we speak of the positive *metal* or *plate*, the metal which is positive and dissolves is meant; but when the positive *pole* is mentioned, the metal from which the positive electricity proceeds out of the arrangement or apparatus into the wires, and which is invariably the negative or receiving metal, is intended: for instance, when a piece of zinc and a piece of silver or copper are immersed in dilute sulphuric acid, the zinc is the *negative pole* and the *positive plate*, whilst the copper or silver is the *positive pole* and the *negative plate*.*

40. In a separate depositing vessel it is the dissolving metal which is called the positive plate, because it corresponds to the zinc or dissolving plate of the battery; while the receiving metal is called the negative plate: the term pole is also sometimes applied to those plates, but in an irregular manner.

41. *Electrical Decomposition of Liquids.*—In all cases of electro-deposition, the elements of the liquids are split asunder by electric action at the surfaces of the metals; the electro-negative elements, such as metalloids and acids, either combine with, or are set free at the surface of the dissolving or positive metal, and the electro-positive elements, such as metals and alkalies, either combine with, are set free, or deposited at the surface of the receiving or negative metal: for instance, if a piece of silver and a piece of copper are immersed in a solution of sulphate of copper, and a piece of zinc and a piece of platina are placed in dilute sulphuric acid, the silver connected with the zinc by one wire, and the copper with the platina by another wire, the *negative* elements of the liquid—namely, the sulphuric acid of the sulphate of copper solution, will be split from its associated copper, and will *combine* with the positive metal, the copper, causing it to dissolve in the liquid; while the *positive* element of the liquid—namely, the copper of the salt, will be deposited at the surface of the negative or receiving metal, the silver, but will *not combine* with it; but if we substitute a piece of platina for the piece of copper, and mercury for the silver (Fig. 74), the effects will be reversed, the acid or negative element will collect around the positive platina, but will *not combine* with it; whilst the positive element of the liquid, the copper, will be deposited and *combine* with the negative mercury. Fused salts yield the same substances by electric decomposition as the same salts do when dissolved in water.

42. *Electrical Terms in Deposition.*—For the more clear remembrance of the different parts of the circuit and of the direction of the electric forces, and for the better understanding of the action of the currents, Faraday has proposed the following terms, which have come into general use:—The liquid undergoing decomposition he terms an "*electrolyte*," from two Greek words, "electron," meaning "electric," and "luo," to "set free;" the act of electric decomposition he terms "*electrolysis*;" the metallic or other surfaces at which the electric forces enter and leave a liquid, he terms "*electrodes*," from two Greek words, "electron" and "odos," meaning a "way;" the positive electrode, or that point at which the positive electricity enters a liquid, is termed "*anode*," from two Greek words, "ana," meaning "upwards," and "odos," a "way"—the way in which the sun rises; and the negative electrode, or that by which the positive electricity leaves a liquid, is termed "*cathode*," from two Greek words, "cata," meaning "downwards," and "odos," a "way"—the way in which the sun sets; the elements of the liquid set free by electrolysis, he terms "*ions*," from a Greek word meaning "going;" those which combine with, or are set free at the anode, are termed "*anions*;" and those which combine with, or are set free at the cathode, are termed "*cations*."†

43. *Anions and Cations.*—Under the head of Anions may be classed—oxygen, fluorine, chlorine, bromine, iodine, and cyanogen; probably, also, sulpho-cyanogen, and the various mineral acids. Cations include—hydrogen (and ammonium), the alkaline metals, magnesium, manganese, arsenic, antimony, bismuth, zinc, cadmium, tin, lead, iron, cobalt, nickel, copper, mercury, silver, gold, platina, palladium, and the salifiable bases.

44. *Electro-chemical Scale.*—The various elementary substances have been arranged by Berzelius according to their relative degrees of positive and negative electro-chemical character, in a table or scale like the accompanying one, commencing with those substances possessing the strongest electro-positive properties, and ending with those of the strongest electro negative properties:—

POSITIVE END.

NAME.	SYMBOL.
Potassium	K.
Sodium	Na.
Lithium	L.
Barium	Ba.
Strontium	Sr.
Calcium	Ca.
Magnesium	Mg.
Glucinum	G.
Yttrium	Y.
Aluminum	Al.
Zirconium	Zr.
Thorinum	Th.
Cadmium	Cd.
Manganese	Mn.
Zinc	Zn.
Iron	Fe.
Nickel	Ni.
Cobalt	Co.
Cerium	Ce.
Lead	Pb.
Tin	Sn.
Bismuth	Bi.
Uranium	U.
Copper	Cu.
Silver	Ag.
Mercury	Hg.
Palladium	Pd.
Rhodium	R.
Platina	Pt.
Iridium	Ir.
Osmium	Os.
Gold	Au.

NAME.	SYMBOL.
Hydrogen	H.
Silicon	Si.
Titanium	Tit.
Tantalum	Ta.
Tellurium	Te.
Antimony	Sb.
Carbon	C.
Boron	B.
Tungsten	W.
Molybdenum	Mo.
Vanadium	Va.
Chromium	Cr.
Arsenic	As.
Phosphorus	P.
Selenium	Se.
Iodine	I.
Bromine	Br.
Chlorine	Cl.
Fluorine	Fl.
Nitrogen	N.
Sulphur	S.
Oxygen	O.

NEGATIVE END.

It will be observed, that the division indicated in the above table between gold and hydrogen, is, in a great measure, an arbitrary one; useful to assist the memory in recollecting the general electro-chemical character of the substances, but not really existing in nature: for instance, sulphur and chlorine, two of the most negative of substances, must be viewed as positive in relation to oxygen. A still more negative substance, when combined with that element, is hyposulphurous or hypochlorous acid, which is negative in relation to arsenic, hydrogen, zinc, potassium, when combined with those elements in the various metallic sulphides and chlorides: in fact, each substance throughout the scale may be viewed as both positive and negative—positive in relation to those below it, and negative in relation to those above it; those of the upper end being strongly positive and feebly negative, and those of the lower end strongly negative and weakly positive. It has been objected that sulphur and nitrogen occupy a position too near the negative end of the scale, they being generally less negative than chlorine and fluorine; also that hydrogen should be placed higher up in the positive division.

45. A consideration of all the foregoing facts leads us to conclude, that current electricity acts a most important part in all the phenomena of electro-deposition.

46. *Thermal Conditions.*—Under this head comparatively little can be said, because it is that portion of the subject which has been the least investigated; but it is highly probable that, as heat is generally evolved by the chemical combination of metals with metalloids or

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* See *ante*, p. 194.

† See *ante*, p. 194.

with acids, there exist thermal as well as electrical and chemical conditions of deposition. In a series of experiments with two plates of antimony immersed in a conducting liquid, the two plates being maintained at different temperatures, and numerous liquids being tried, a weak current of electricity was developed, which passed from the hot metal through the liquid to the cold one, the hot metal thus being positive. This occurred in all cases except with liquids, containing uncombined nitric acid.

47. *Influence of Temperature on the Depositing Liquid.*—The strength of affinity between the different elements of a liquid undergoing electrolysis, varies with the temperature of the liquid; being almost invariably diminished by elevation of temperature. Rise of temperature increases the electric conductibility of an electrolyte, and decreases that of the metal plates immersed in it; but the decrease of conductibility of the latter is small in proportion, at moderate elevations of temperature, compared with the increase of the former; consequently, the general effect of heating a depositing liquid, is to increase the rapidity of deposition.

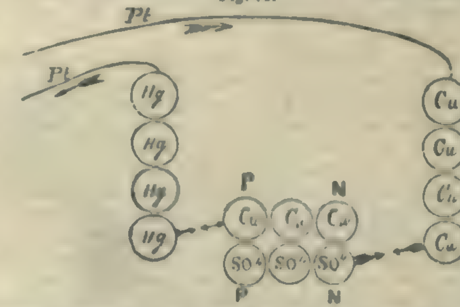
48. We have repeatedly observed, that with some solutions used at a high temperature for depositing, the cathode being immersed in the liquid at the ordinary atmospheric temperature, and the liquid then heated to the desired point, no conduction or deposition took place; nor did it occur when the receiving metal was taken out, washed in cold water, and re-immersed; but if the temperature of the liquid were first raised, and then the cold cathode suddenly immersed, deposition took place freely, and the liquid might be cooled down many degrees without stopping the action. In coating iron with tin in some solutions, the iron being immersed before heating the liquid, no deposition took place even at 150° Fah.; but if the liquid were first heated, deposition occurred below 100° Fah.

49. *Influence of Light upon Deposition.*—Light appears to exercise much less influence upon electro-deposition than heat: in some cases, however, where the elements of a depositing liquid are held together by unstable affinities, it decomposes the liquid, and renders it unfit for deposition. For instance, a solution formed by dissolving hyposulphate of silver in a solution of hyposulphite of soda, has a tendency to be decomposed in this way.

50. *Dynamic or Mechanical Conditions.*—The various phenomena occurring in a liquid undergoing electrolysis, may be viewed, in a mechanical or dynamic aspect, as a series of minute movements (attractions and repulsions) occurring between the various particles of matter composing the opposed surfaces of the liquids and of the metals immersed in them. For instance, if we immerse a piece of zinc in a solution of sulphate of copper, and connect it with some mercury in the same liquid, by a platina wire protected from the solution by a tube of glass or gutta-percha, the particles composing the surface of the zinc, being all electro-positive, will tend to repel each other, and the particles of the liquid surface in contact with it, being rendered electro-negative, will also tend to repel each other; whilst the particles of acid near, being electro-negative, will attract the particles of zinc, which are electro-positive, and the two will combine together and form a salt; at the same time, the particles of the liquid surface in contact with the mercury, being made electro-positive, will tend to repel each other. The particles of the opposed mercury surface, being rendered electro-negative, will also tend to repel each other; whilst the particles of the mercury, being electro-negative, and the contiguous particles of copper in the liquid being electro-positive, the two will combine together and form an alloy. The deposition of copper upon the *zinc*, in this case, must be wholly disregarded, because it is quite a separate and distinct phenomenon.

To put this in a clearer form—suppose (Fig. 79) the vertical row of particles Cu, Cu, Cu, Cu, to represent the copper anode of a sulphate of copper depositing liquid, and the row of particles Hg, Hg, Hg, Hg, to be the mercury cathode, Pt, Pt being the connecting wires from the battery, and the double row representing the particles of acid SO^4. and copper Cu, composing the intervening liquid. The particles of the anode surface, being all electro-positive, tend to repel each other; and the contiguous particles of Cu and SO^4, being all negative, also tend to repel each other; whilst the particles of the copper anode, being positive, attract the nearest particles of negative acid, and combine with them, and form a salt. At the same time, the particles of Cu and SO^4 nearest the mercury, being all positive, tend to repel each other; whilst the contiguous particles of mercury being negative, and the copper in the liquid being positive, attract and combine with each other and form an alloy. By this combination of simultaneous movements, the copper anode dissolves, the mercury (or any other conducting substance which forms the cathode) receives a deposit, and the particles of copper of the liquid are gradually removed and replaced by those from the anode.

Fig. 79.



51. These attractions and repulsions, like ordinary chemical actions, are all supposed to take place at insensible distances, at the mutually opposed surfaces of the liquids and metals, and not to extend into their masses, except so far as they are mixed with each other by capillary attraction or ordinary mechanical motion, and can therefore only take place where one or both of the substances are in a liquid state; if it were otherwise, the combinations and decompositions of the whole masses would probably occur instantaneously.

52. In addition to these minute and invisible movements of the particles, there are other and sometimes visible movements produced by capillary attraction and by difference of specific gravity in the liquids. For instance, the salt formed at the anode, if it be soluble in the liquid, will be dissolved and gradually diffused through it by capillary attraction or adhesion; whilst from its greater specific gravity than the remainder of the liquid when dissolved, it tends to sink towards the bottom; at the same time, the acid set free at the cathode is likewise gradually diffused through the liquid by similar means, and from its less specific gravity tends to rise to the surface. These movements are of general occurrence during deposition; and in some solutions, especially if they are very dense and possess a colour, they are plainly visible to the unassisted eye. Their occurrence explains why the substances set free by deposition are not instantaneously transferred from one electrode to the other, but occupy, especially in dense liquids and electrodes far asunder, a considerable period of time in their transference; it also explains why, if depositing solutions are not occasionally stirred, their upper portions become exhausted of metal, whilst their lower parts become deficient of acid. Motion of the cathode is generally considered necessary to make the deposited metal harder.

53. *Position of the Electrodes.*—The position of the electrodes has a considerable influence upon the phenomena of electro-deposition. For instance—1st. If the two electrodes in a depositing liquid are horizontal, with the anode above and the cathode below, the salt formed at the anode will, by virtue of its greater specific gravity, sink in the liquid; whilst the acid set free at the cathode will, by its less specific gravity, rise upwards, and thus the anode will be constantly supplied with fresh uncom-

bined acid, the cathode will receive a constant supply of metallic salt, and deposition will continue without interruption. 2nd. If the two electrodes are vertical in the liquid, and opposite to each other, similar differences of specific gravity will cause the lower part of the liquid to become saturated with metallic salt, and its upper part to consist of free acid mixed with the water: in consequence of this, the current of electricity will almost wholly pass from the upper part of the anode diagonally downwards through the liquid, to the lower part of the cathode, and thus the upper part of the anode will dissolve rapidly, whilst its lower part will dissolve but slowly, and the cathode will receive a rapid deposit at its lower part, and but very little at its upper part. In this position, vertical lines, and even deep grooves, are sometimes produced in the deposit (especially if the position of the cathode be slightly overhanging), by the ascent of streams of the lighter acid liquid from which the metal has been exhausted by deposition. If the solution be nearly a saturated one, and has been freely worked without stirring or disturbance for some time, crystals of the metallic salt are apt to form all over the lower part of the anode, which will be dissolved very rapidly at the surface of the liquid, and appear as if cut by a knife. In addition to these effects, if the solution be a very deep one, with much free acid, two independent currents of electricity will be developed, one in each electrode, by the unequal action of the two different strata of liquid upon their upper and lower parts. An instance of this will be found in "One Metal and Two Liquids" (9), in which these currents leave them at their upper parts, pass down through the liquid, and re-enter them at their lower extremities.

54. *Form of the Electrodes.*—If either of the electrodes be of an irregular form, or have unequal projections or hollows, the anode will dissolve most freely, and the cathode receive the greatest deposit of metal at those parts at which they are nearest each other, and least in the hollows and more distant parts. If the surface of the cathode be rough, it greatly increases the tendency of the deposit to become crystalline.

55. *Mathematical Conditions.—Definite Chemical Action.*—In the combination of different elementary and compound substances with each other, by ordinary chemical affinity, it has been observed, and accurately proved by analysis, that their combinations invariably take place in certain definite proportions, or in very simple multiples of those proportions; and as in the first of these cases it is inferred that their combinations with each other occur, atom with atom, the numbers which represent those simple proportions represent also the relative weights of those atoms to each other. The following table contains the names, symbols, and atomic weights of most of the simple or elementary substances:—

Name.	Symbol.	Atomic weight.
Hydrogen	H	1·0
Beryllium	Be	4·7
Carbon	C	6·0
Lithium	Li	6·5
Oxygen	O	8·0
Boron	B	10·9
Magnesium	Mg	12·2
Aluminum	Al	13·7
Nitrogen	N	14·0
Sulphur	S	16·0
Fluorine	Fl	18·9
Calcium	Ca	20·0
Silicium	Si	21·3
Zirconium	Zr	22·4
Sodium	Na	23·0
Titanium	Ti	25·0
Chromium	Cr	26·7
Manganese	Mn	27·6
Iron	Fe	28·0
Cobalt	Co	29·5
Nickel	Ni	29·6
Copper	Cu	31·7
Phosphorus	P	32·0
Zinc	Zn	32·6
Chlorine	Cl	35·5
Potassium	K	39·2
Selenium	Se	39·5
Strontium	Sr	40·8
Molybdenum	Mo	46·0
Lanthanum	La	47·0
Cerium	Ce	47·0
Didymium	D	50·0
Rhodium	R	52·2
Ruthenium	Ru	52·2
Palladium	Pd	53·3
Cadmium	Cd	56·0
Tin	Sn	59·0
Thorinum	Th	59·6
Uranium	U	60·0
Tellurium	Te	64·2
Barium	Ba	68·5
Vanadium	Va	68·6
Arsenic	As	75·0
Bromine	Br	80·0
Tungsten	W	95·0
Platina	Pt	98·7
Iridium	Ir	99·0
Osmium	Os	99·6
Mercury	Hg	100·0
Lead	Pb	103·7
Silver	Ag	108·1
Iodine	I	127·1
Antimony	Sb	129·0
Tantalum	Ta	184·0
Gold	Au	197·0
Bismuth	Bi	213·0

56. As there is a number of compound substances used in electro-deposition, such as sulphuric acid, cyanide of potassium, &c., and it will be useful to the practical depositor to know their combining proportions in making the different salts, used in the art, we have selected those which are likely to be required, and give their names, symbols, and atomic weights or combining proportions:—

Water, HO; 9·0.

Common Oil of Vitriol, specific gravity, 1·848; SO^3, HO; 49·0.

Strongest Hydrochloric Acid, sp. gr. 1·21; HCl, 6HO; 90·5 (*i.e.*, 90 and 5-tenths).

Strongest Nitric Acid, sp. gr. 1·52; NO^5, 2HO; 72·0.

Sesquicarbonate of Ammonia (Sal-volatile); 2 NH^3, $3CO^2$, 2HO; 118·0.

Hydrochlorate of Ammonia (Sal-ammoniac); NH^3, HCl; 53·5.

Hydrate of Potash (fused Caustic Potash); KO, HO; 56·2.

Crystallised Carbonate of Potash; KO, CO^2, HO; 78·2.

Carbonate of Soda (ordinary washing Soda); NaO, CO^2, 10HO; 143·0.

Chloride of Sodium; Na Cl; 58·5.

Caustic Lime; CaO; 28·0.

Calcined Magnesia; MgO; 20·2.

Ordinary Carbonate of Magnesia; MgO, CO^2, HO; 51·2.

Oxide of Zinc; *Zn*O; 40·6.

Commercial Sulphate of Zinc (White Vitriol); ZnO, SO^3, 7HO; 143·6.

Sesquioxide of Iron (Crocus, Colcothar); Fe^2O^3, 80·0.

Commercial Sulphate of Iron (Green Vitriol); FeO, SO^3, 7HO; 139·0.

Protoxide of Copper (Black Oxide of Copper); CuO; 39·7.

Oxide of Silver; AgO; 116·1.

Commercial Sulphate of Copper (Blue Vitriol); CuO, SO^3, 5HO; 124·7.

Chloride of Silver; Ag Cl; 143·6.

Nitrate of Silver; AgO NO^5; 170·1.

Oxide of Gold; AuO; 205·0.

Terchloride of Gold; Au Cl^3; 303·5.

Bichloride of Platinum; Pt Cl^2; 169·7.

Cyanogen; Cy; 26·0.

Cyanide of Potassium; KCy; 65·2.

Cyanide of Zinc; Zn Cy; 58·6.

Sesquicyanide of Copper; Cu^3, Cy^2; 147·1.

Cyanide of Mercury; Hg Cy; 126·0.

Cyanide of Silver; Ag Cy; 134·1.

Cyanide of Gold; Au Cy; 223·0.

57. *Definite Electro-Chemical Action.*—The chief mathematical condition of electro-deposition is, that in every case, all the actions, both of combination and decomposition, take place in certain mathematical proportions; that is, according to the relative atomic weights of the substances combining or being decomposed. For instance—

1st. *With One Metal and One Liquid.*—If a piece of pure iron be immersed in a solution of sulphate of copper, it will be dissolved, and copper deposited; and for every 28 parts, or one atomic combining equivalents of iron dissolved, 31·7 parts, or one atom, of copper are deposited, and 49 parts (one equivalent) of hydrated sulphuric acid (common oil of vitriol) are separated from the copper, and combine with the iron, forming therewith one equivalent of protosulphate of iron.

2nd. *With Two Metals and One Liquid.*—If a piece of zinc and a piece of silver in mutual contact are immersed in a solution of nitrate of silver, the zinc will dissolve and the silver receive a deposit; and for every 108·1 parts of silver deposited, 32·6 parts of zinc will be dissolved.

3rd. *With One Metal and Two Liquids.*—If one piece of copper be immersed in a solution of sulphate of copper, and another in dilute sulphuric acid, the two being connected together by a wire, and the liquids separated by a porous partition, copper will be dissolved in the dilute acid in the proportion of 31·7 parts for every 31·7 parts of copper deposited in the metallic solutions; and for every 49 parts, or one equivalent, of hydrated sulphuric acid set free at the cathode by the deposition of the equivalent of copper, one equivalent of acid will combine with a like amount of copper at the anode.

4th. *With Two Metals and Two Liquids.*—If we immerse a piece of zinc in dilute sulphuric acid, and a piece of silver in a solution of double cyanide of silver and potassium, the two metals being connected by a wire, and the liquids separated by a porous partition, one equivalent, or 32·6 parts, of zinc will combine with one equivalent of the acid, and one equivalent, or 108·1 parts, of silver will be deposited, setting one equivalent, or 26 parts, of cyanogen free.

5th. *With a Separate Depositing Liquid.*—If the plates of a battery are connected by wires with two pieces of gold in a hot solution of double cyanide of gold and potassium, for every atomic equivalent, or 197 parts of gold deposited, and one equivalent of cyanogen set free, one equivalent of gold will combine with one equivalent of cyanogen, and will be dissolved: and not only this; but for each of these actions, one equivalent of zinc will combine with one equivalent of acid, and one equivalent of hydrogen will be evolved in each of the battery cells supplying the current of electricity.

6th. *With a whole Series of Depositing Liquids.*—Such, for instance, as solutions of sulphate of copper with electrodes of copper—arranged as in Figs. 73 and 75—connected with a piece of zinc immersed in dilute sulphuric acid, and a piece of copper immersed in a solution of sulphate of copper, the last two liquids separated by a porous diaphragm, the whole of the combinations, decompositions, and depositions throughout the series, will take place in the proportions of their atomic weights or chemical equivalents. These, and many other instances which might be adduced, prove that all the electro-chemical actions taking place in any given circuit, occur in certain definite proportions, and that this definite electro-chemical action is one very important condition of electro-deposition.

58. *Binary Theory of Electrolysis.*—The law of definite electro-chemical action was first established by Faraday; and, in addition, he has advanced what is termed the binary theory of electrolysis—that "only those compounds of the first order are *directly* decomposible by the electric current, which contain one atom of one of their elements for each atom of the other: for instance, compounds containing one atom of hydrogen or metal with one atom of oxygen, iodine, bromine, chlorine, fluorine, or cyanogen;" whilst "boracic acid ($B\,O^3$), sulphurous acid ($S\,O^2$), sulphuric acid ($S\,O^3$), iodide of sulphur, chloride of phosphorus ($P\,Cl^3$) and ($P\,Cl^5$), chloride of sulphur ($S^2\,Cl$), chloride of carbon ($C^4\,Cl^6$), bichloride of tin ($Sn\,Cl^2$), terchloride of arsenic ($As\,Cl^3$), quintochloride of antimony ($Sb\,Cl^5$)," are non-conductors of electricity, and incapable of electrolysis. Some substances, which are not of the simple binary character mentioned, are decomposed by current electricity, and yield their positive and negative elements in equivalent proportions at the respective electrodes; but, according to this theory, they are *indirectly* decomposed, *i.e.*, they are decomposed by the chemical action of some of the elements set free by the direct action of the current upon other substances present. For instance, "fused borax (biborate of soda $NaO, 2BO^3$) yields oxygen gas at the anode, and boron at the cathode: now, since fused borax is not decomposible by the electric current, the separation of the boron must be attributed to indirect action; the current resolves the soda (NaO) into oxygen and sodium, and the latter separates boron from the boracic acid."* Again, an aqueous solution of ammonia (HO, NH^3) yields, by electrolysis, nitrogen gas at the anode, and hydrogen at the cathode. In this case, according to the theory, it may be supposed that only the water (HO) is *directly* decomposed, and that its oxygen, set free at the anode, combines chemically with some of the hydrogen of the ammonia, again forming water, and thus indirectly the nitrogen is set free.

59. *Mathematical Idea of Electro-depositing Force.*—A consideration of the law of definite electro-chemical action, and the binary theory of electrolysis, leads us to view the electric current or depositing force in a mathematical aspect, as "an axis of forces equal in power, but opposite in direction;" because, for every atom of an electro-positive substance attracted or transferred in one direction, an atom of an electro-negative character is attracted in the opposite direction. It also suggests to us the idea that an intimate connection exists between those equivalents or mathematical relations of matter and the development of current electricity by chemical action, and its transference through liquids by electrolysis, because these phenomena only occur when the mathematical conditions are present.

60. *Sizes of Electrodes, Liquids, and Wires.*—The rapidity of deposition is influenced by the area of the electrodes, the length and area of the intervening liquid, and of the connecting wire; the larger the immersed surfaces of the metals, the shorter the length and the greater the transverse area of the liquid between them; and the shorter and thicker their connecting wires, the more rapid is the process of deposition. If the anode be very large, and immersed in the lower part of the liquid, and the cathode very small and suspended near the surface, much more metal will be dissolved than is deposited, gas being generally evolved at the cathode in place of some of the metal being deposited.

61. *Rapidity of Deposition.*—The character of the deposited metal is very much influenced by the rapidity of deposition: if it be deposited very rapidly, it will be in the state of a perfectly black, soft, non-coherent powder; if deposited more slowly, it will possess the ordinary characters of the particular metal; and if deposited very slowly, it will be crystalline, because the atoms are then allowed sufficient time to arrange themselves in the crystalline form.

62. *Logical Conditions.*—The logical conditions are—1st, that there is a *difference* at the dissolving and receiving surfaces, either in the "*material substratum*" of metal, of liquid, in the *forces* involved, or in all combined, both of chemical affinity and electricity—probably, also, of heat and of motion; and, 2nd, that in all cases of deposition there are certain chemical, electrical, thermal, dynamical, and mathematical conditions invariably present, and certain other conditions of each of those kinds invariably absent, both of which classes of conditions are necessary to the production of the phenomena; the whole of the necessarily present circumstances constituting its causes, and all the necessarily absent ones constituting its preventives. Thus, if *all* the causes of deposition are present, and *all* its preventives absent, deposition will invariably occur; but if only one of its causes is absent, or one of its preventives present, deposition cannot take place.

63. *Ontological Condition.*—The last, the most necessary, most evident, and most simple condition of all deposition is—that metals, liquids, and forces are required in order to produce it.

THE PRACTICE OF ELECTRO-DEPOSITION.

64. *Objects of Practical Deposition.*—In the theoretical division we have brought forward a large number of instances, both of deposition and non-deposition; and from a consideration of them, we have drawn conclusions both as to what substances, and what conditions and arrangements of those substances, really exist in all cases of deposition, and in similar cases of non-deposition; and we have pointed out in what respect the conditions varied. Our object in pursuing this course was to impress the reader with an extended knowledge of the theoretical principles on which deposition proceeds, that he may feel himself perfectly able to apply his knowledge to remove the difficulties certain to arise in his practice. Now, however, our object is different; it is to instruct the reader how he is to apply those principles in daily working; to give him practical rules, recipes, and directions for carrying out the various minute points in workshop manipulation; and to enable him to obtain the greatest degree of practical success.

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* Faraday.

65. *General Arrangement of Electro-Deposition.*—The practical part will be best arranged by beginning at the very commencement; and assuming the reader to be without materials or apparatus of any kind—that he has to provide a workshop, prepare his solutions, batteries, scouring and cleaning apparatus; that he has to clean his articles for receiving deposits, and prepare his materials for moulds, before he can commence the process of deposition, gradually acquiring a knowledge, as he proceeds, of the kind of depositing process best fitted for his purpose; of the most suitable source of depositing power; the best solutions; the best recipes for making solutions; the construction of voltaic batteries and of magneto-electric machines; the rules for regulating the currents of electricity, and the character of the deposited metals; rules for depositing metals generally, as well as for cleaning and preparing metal articles to receive deposits; copying works of art by moulding; preparation of moulds for receiving deposits; making solutions for ordinary coppering, silvering, and gilding; and for the management of those solutions;—information on all these points it will be our task to supply.

66. *First Considerations.*—The first step in practice is, to consider the probable magnitude of the operations to be carried on, and to provide rooms of suitable size. These should be upon the ground-floor (except for electro-gilding), well lighted and ventilated, with conveniences for the erection of boilers and drying flues, for placing washing-troughs, depositing vats and batteries, and for the escape of unwholesome vapours; there should also be ready access to a plentiful supply of clean water. The establishment should consist of at least three rooms, and an open yard with an outhouse; an upper or more private room for gilding, a ground-floor room for silvering, and another ground-floor room for the coarser work, such as coppering, brassing, and the preparation of the larger and coarser articles for receiving deposits. The outhouse is for the batteries, and the yard for washing the battery cells. If a magneto-electric machine be employed, an additional small, dry, and clean apartment will be required, which should be reserved for it alone.

67. *Boilers, Furnaces.*—For the purpose of general deposition, several large iron boilers, with furnaces beneath, either in the coppering-room or in close proximity to the silvering-room, are required; these are to contain solutions of caustic potash for cleaning articles. A low furnace should be erected between those rooms, having a long horizontal flue covered with plates of iron for drying deposited or plated articles upon; the room for coppering should be furnished either with a low furnace or stove for heating the solutions used for coppering or brassing iron. Each room, whether for coppering, silvering, or gilding, should be provided with a tap of running water, and a leaden trough beneath, for washing the smaller articles; and the coppering-room should be furnished with one or two large wooden tubs or troughs, filled with water, for washing articles of larger size. This room and the outhouse should contain a number of large stoneware pans and jars, oval and round, of different sizes and proportions, to receive the various "pickling" and "dipping" liquids, acids, or "spent solutions." Several large iron trays, filled with sawdust, should also be provided and fitted to the furnace flue, for drying plated and deposited articles upon. Each of the rooms should be provided with a "scratch-brush lathe," for scouring the various articles. The gilding-room should have several small stoves for heating gilding solutions, or, in lieu thereof, several iron tripods, with large gas-burners beneath. The silvering and coppering-rooms should each be provided with one or two pairs of large and well-insulated copper wires, proceeding from the depositing vats to the batteries outside. The gilding-room will not require these, small batteries only being used in it, which are kept in the same room.

68. *Sources of Electricity.*—A point for early consideration will be, whether a magneto-electric machine or voltaic batteries are to be used as the source of depositing power. The choice of either will depend very much upon the degree of confidence the operator possesses in each source of electricity, upon its expense, and whether or not motive power to drive the former, is readily and constantly available at a moderate cost. Voltaic batteries are readily obtained, and worked in almost any situation. We will suppose, for the purpose of explanation, that the operator has resolved to use both a magneto-electric machine and voltaic batteries.

69. *Construction of Magneto-Electric Machine.*—The simplest form of apparatus for generating current electricity by the joint influence of magnetism and motion, has been already described (15); but the apparatus required for practical purposes is far more elaborate and costly in its construction. A (Fig 80) is a strong framework of wood; B, B, B, B, are four bundles of powerful horse-shoe magnets, firmly fixed to the wooden frame; C is an axle driven very rapidly by steam-power; the axle carries two brass circles, upon which, at right angles, are firmly fixed four round bars of pure soft iron, B, C, D, E (Fig. 82), equal in length to the distance asunder of the poles of each magnet; upon each of these is coiled a long piece of thick copper wire, the wire being covered with cotton to insulate its coils from each other; the ends of these wires are connected with four semicircular pieces, formed of a piece of brass tube, fixed upon the axle C, but insulated from it and from each other (Fig. 81) by a tube of hard wood or gutta-percha: by rotation of the iron armatures past the poles of the magnets, currents of electricity are generated in the wires in one direction as they are approaching the magnets, and in an opposite direction as they recede from them: to collect those currents and convey them to the depositing vat, also to throw them all into one uniform direction, two brass springs, D E (Fig. 81), press against the semicircles of brass during their revolution, and are so arranged at their points of mutual contact, that, just at the moment that the currents are changed in direction by the armatures passing the magnets, the revolution of the axle causes the points of contact of the springs to pass from one pair of the semicircles to the other; and thus, by reversing the connection at the moment the direction of the current is changed, a uniform direction of currents is obtained in the wires beyond.

Fig. 80

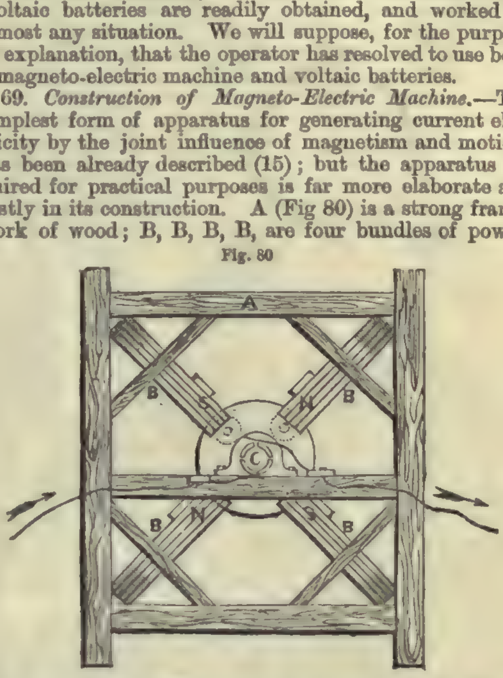
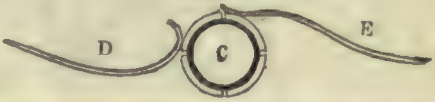
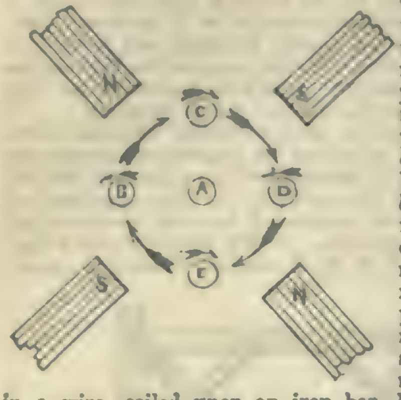


Fig. 81.



To explain the action of this machine more fully, we will suppose NS, NS (Fig. 82) to represent those poles of the compound horse-shoe magnets which are placed facing the observer in the elevation (Fig. 80); A in Fig. 82, is the axle, and B, C, D, and E are the ends of the four horizontal round iron bars or armatures, moving in the direction of the large arrows. When a bar of soft iron, having a coil of insulated wire wound upon it, is moving *towards* a pole of a magnet, a current of electricity is developed in the wire in *one* direction by the piece of iron within it *gaining* magnetism; and when the bar is moving *from* the pole of the magnet, a current is produced in an *opposite* direction by the iron bar *losing* its magnetism; and these currents are both reversed in direction, if the pole of the magnet be reversed; so that the current developed in a wire, coiled upon an iron bar, by moving that bar *towards* the *south* pole of a magnet, is the same in direction as that produced by moving it *from* the *north* pole. We will now suppose that the current produced in the coil of wire upon bar B, moving from S to N, is in the direction of the small arrow surrounding that bar—viz., *left*-handed motion; if so, the current of the wire of bar D will also be *left*-handed in direction, because it also is moving from a south to a north pole; whilst the currents in the wires of bars C and E will be *right*-handed, because both those bars and coils are moving from north to south poles.

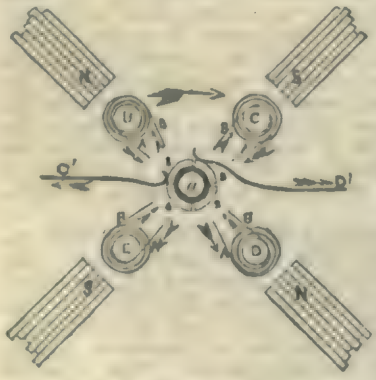
Fig. 82.



From these remarks, it may be perceived—1st, that the currents of electricity, in all the coils, are reversed in direction every time the bars pass the centres of the poles of the magnets, *i.e.*, four times in every revolution; 2nd, that to obtain a current of one uniform direction from all the four coils (by conducting the whole four into one stream), during *only one quarter* of a revolution, *i.e.*, from one pole to the next one, it is necessary to connect the ends of the wires of the coils of B and D in an opposite manner with the semi-cylinders of the commutator or break-piece (Fig. 81), to those of the coils of C and E; and, 3rd, that as the currents in all the coils are reversed in direction every time the bars pass the centre of the poles of the magnets, the two springs which press upon the semi-cylinders, must, by some means or other, be reversed in their order of connection with all the wires every time, and at the same moment that the bars pass the centres of the poles, in order to throw the whole of the currents, during a complete revolution, into one uniform direction in the springs and in the wires which proceed to the depositing vessel.

To enable us to understand how the opposite currents of the different coils are thrown into one uniform direction during one quarter of a revolution, and how the whole of the currents are alternately conducted into one uniform stream during rapid revolution, we will suppose *a* (Fig. 83) to be the axle, and 1, 2, 3, and 4, the semi-cylinders insulated from the axle by a tube of gutta-percha. In the first place 1 and 2 are connected together by a short piece of thick copper or brass wire, or a strip of sheet copper behind (not shown in the cut), and 3 and 4 are connected together by another and similar piece of metal; next the ends of the wires A, A, A, A are gathered into one bundle, and connected by soldering with the wire or strip of metal of the semi-cylinders 1 and 2, and the ends B, B, B, B are connected in like manner with the semi-cylinders 3 and 4; and by this arrangement, on careful attention, it will be perceived that the whole of the currents are thrown into one direction during *one quarter* of a revolution. In the sketch, the bars are just approaching the centres of the poles, and the semi-cylinder 1 is just about breaking contact with spring C′, and semi-cylinder 4 is about making contact with it; semi-cylinder 3 is about passing from spring D′, and semicircle 1 is about making contact with it. In this position of the bars, it will be perceived that all the A ends of the wires, the semi-cylinders 1 and 2, and the spring C′, are positive, as indicated by the arrows; whilst the B ends, the semi-cylinders 3 and 4, and the spring D′ are negative: but immediately the bars pass the centres of the poles, the currents are all reversed in direction—all the B ends of the wires, and the semi-cylinders 3 and 4 become positive, and the A ends, and the semi-cylinders 1 and 2 negative; but as the rotation of the axle shifts the points of contact of semi-cylinder 1 to spring D′, and of semi-cylinder 4 to spring C′, at the same moment, the spring C′ still remains positive, and D′ negative, as before. The connections of the springs with the semi-cylinders in the other two quarters of the revolution, alternate in a similar manner; and these alternations take place regularly, and synchronously with the reversals of the currents, during the most rapid revolutions; and notwithstanding the incessant and indefinitely rapid changes of these currents, one uniform stream of electricity is obtained. In the practical machine, the bars of iron rotate as closely as possible to the ends of the magnets without absolutely touching them, in order to obtain the greatest amount of power; and the power is regulated by placing the soft iron keepers which are upon the magnets (Fig. 80) nearer or further from the poles; the keepers are secured to the magnet, that they may not fall off by the vibration of the machine. The wires proceeding from the springs are connected, one with the dissolving plates, and the other with the receiving articles in the vat. The machine may contain eight revolving armatures, and either four or eight magnets, according to the amount of work to be effected.

Fig. 83.



70. *Voltaic Batteries.*—Each of the arrangements of metals and liquids which have been described under the head of "Facts," and which develop a current of electricity, constitutes an elementary voltaic battery—a battery in all its essential parts, but not in outward form. All voltaic batteries consist of one or other of those theoretical arrangements modified and adapted for practical use, and are composed either of two metals and one liquid, or two metals and two liquids; because these two arrangements develop the greatest amount of electricity, and are most convenient in use. In the theoretical form, any kind of metal, of any size or shape, with almost any conducting liquid, and with any kind, shape, or size of containing vessel, will develop a current of electricity and produce deposition; but a true voltaic battery consists of particular metals and liquids, of particular sizes, shapes, and proportions, and at certain distances apart, with suitable screws attached for connections, and with containing-vessels made of particular materials, and of special forms and sizes.

71. Arrangements of various voltaic batteries have already been fully described in our previous pages.*

72. *Battery Cells.*—The form of cells for these batteries is generally either round or square; for *small* batteries, either of the old zinc and copper kind, or of Mr. Smee's arrangement, square ones are generally used; but for small Daniell's batteries, or for large batteries of either of these kinds, round vessels are almost invariably adopted. They are made of stoneware, glass, or gutta-percha; the first of these is universally used for large batteries of all kinds: glass is too expensive for large vessels, but it possesses the great advantage of enabling the operator to watch the action of the

* See *ante*, p. 187, *et seq.*

batteries, and is now being moulded into large vessels for the use of some of the electro-plate manufacturers: gutta-percha has been also used, but possesses rather less advantage, upon the whole, than the other materials, being opaque and expensive, while the zinc salt of the battery liquid passes rather rapidly over its edges by capillary action.

73. *Porous Cells.*—When Daniell's or any other battery with two liquids is used, porous vessels are required also, to allow the two liquids to touch each other without mixing: they are of three kinds—unglazed earthenware, wood, and bladder. The first of these is the only kind in use by manufacturers; they should always be kept in clean water when not in use, to remove the salts of the battery liquids from them, to prevent their cracking, and to preserve them always fit for immediate use.

74. *Zinc for Batteries.*—The best kind of zinc for batteries, and the kind chiefly in use by electro-platers, is the German or Liége zinc, known as "Mosselman's," from the name of a former manufacturer of it. The thickness of the plate should vary with the size of the battery; the smallest should not be less than one-eighth of an inch thick, on account of its brittleness when amalgamated; large ones are generally about one quarter or three-eighths of an inch in thickness. Zinc bolts for Daniell's batteries are generally made by melting together a number of old worn-out pieces of battery plates, and casting in a suitable mould.

75. *Amalgamation of Zinc.*—Zinc plates or bolts are best amalgamated by immersing them, during a quarter of an hour, in a mixture of about one part of sulphuric acid and ten or twenty parts of water; then pouring mercury upon them, and rubbing it all over them with a hare's foot or piece of old cloth, using a small hard brush for the refractory places; they are then washed in water, and drained for half-an-hour, and brushed, to recover the superfluous mercury.

76. *Copper and Platinised Silver for Batteries.*—Ordinary sheet copper answers very well for this purpose, and platinised silver may be obtained of most philosophical instrument-makers, or it may be easily prepared by any one by the following means:—Immerse a piece of zinc in dilute sulphuric acid contained in a porous cell; place the cell in an outer vessel, and fill the outer space with water, to which a few drops of sulphuric acid have been added; add to this a sufficient quantity of a solution of bichloride of platina to render it of a brown colour; immerse the piece of silver to be platinised in the outer liquid, and connect it by a wire with the piece of zinc; gas will soon be evolved from the surface of the silver, when the silver will gradually become black with a deposit of platina; it may then be removed, dipped several times in water, and afterwards dried, care being taken not to rub off the platina. The porous cell requires to be immersed a short time beforehand. The solution of bichloride of platina may be easily and cheaply made by adding scraps of the metal foil to a hot mixture of one measure of nitric acid and two-and-a-half measures of hydrochloric acid, as long as gas is evolved from them; the liquid will then be of a deep red colour.

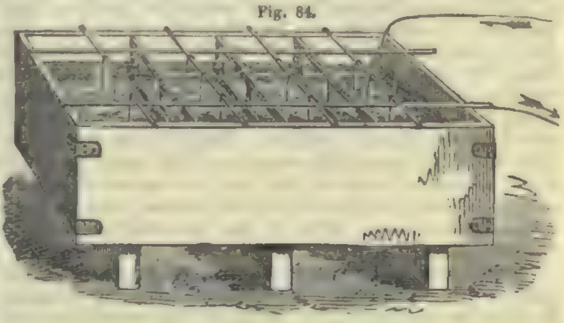
Silver alone is not nearly so effective for the negative metal of voltaic batteries as platinised silver, because the hydrogen gas evolved adheres very strongly to it, and greatly reduces the amount of its surface in contact with the liquid; whereas the platina being a very negative metal, and being deposited in the state of a fine powder, causes the hydrogen to be thrown off very rapidly from its surface, and thus increases the action.

Copper is still less effective than silver, because the battery liquid acts chemically upon it, and forms a salt of copper, which dissolves in the liquid and reacts upon the zinc plates, causing them to waste rapidly; for this reason they cannot, like silver, be safely left in the liquid any great length of time when the battery is not at work. In addition to this, when they are taken out and exposed to the air, they soon become covered with a film of oxide, which considerably weakens the electric current on their reimmersion.

77. *Depositing Vessels, Vats, &c.*—The depositing vessels are made of various materials. For small operations, nothing is so suitable as glass vessels or a stoneware pan; but for ordinary manufacturing purposes, vats containing from twenty to several thousand gallons are used: they are generally made of wood, lined with sheet lead; but very large ones, for containing sulphate of copper solution, such as are used for depositing life-sized figures in copper, have in some instances been built of bricks, coated with cement, and lined with gutta-percha. Vats used to contain cyanide solutions should not be lined with this substance, because the cyanide of potassium acts upon it. The vats used for ordinary silver-plating are about twenty-four or thirty inches deep, from two to three feet wide, and from three to twenty feet long; their dimensions vary greatly in different manufactories, and depend upon the number and size of the articles to be plated in them. Some electro-depositors use vats formed of sheets of wrought-iron riveted together; but there is always a slight salt or sediment found on their sides, which settles at the bottom of the liquid.

78. *Arrangement of Dissolving Plates in Vat.*—In the vats used for silvering general articles, such as spoons, knives, forks, teapots, plates, &c., the dissolving plates are sometimes fixed all round the sides of the vessel just beneath the surface of the liquid; in addition to this, vertical wooden frames are fixed at intervals of about two feet, across the vat (Fig. 84), with dissolving plates upon them, and all the dissolving plates are connected together metallically. The articles to be plated are suspended by small copper wires, from brass or copper tubes resting across the vessel upon two other and longer tubes passing along the upper edges of the vat. These are connected by a large copper wire with the negative pole of the battery; whilst the dissolving plates are connected by another large copper wire with the positive pole: by this arrangement, each row of articles has dissolving plates all round it, which greatly facilitate the rapidity of deposition. The wooden cross-frames are movable, so that when large articles are to be plated, one or more of them may be removed to make more room.

Fig. 84.

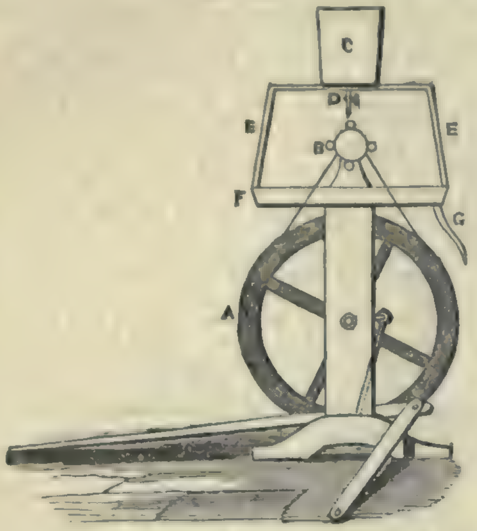


79. The large vessels used for depositing solutions which require to be worked *hot*, such as the cyanide coppering or brassing liquids, are formed either of cast-iron, wrought-iron, or iron coated with enamel; and the smaller vessels, such as are used for gilding, are oftentimes of stoneware or glass; enamelled iron pans are also used for this purpose.

80. "*Scratch-brush Lathe.*"—The deposition will require several "scratch-brush" lathes, one in each depositing-room, for scouring and preparing the surfaces of metal articles to receive a deposit. This instrument (Fig. 85) consists simply of an ordinary lathe A, with a wooden chuck B, to the sides of which are firmly secured four horizontal bundles of fine brass wire; above it is a vessel C, containing stale beer, which is allowed to drop constantly, by the pipe and tap D, upon the revolving brushes whilst working; the sides E E are to prevent splashing, and the tray F, and pipe G, are to collect and remove the waste liquid. The workman stands opposite the end of the machine in using it, working the treadle with his foot, and pressing the article against the ends of

the revolving wire brushes, exposing in succession different parts of the article to their action. Wire of different degrees of fineness is used with different articles.

Fig. 85



81. *Connecting Wires.*—The depositor should provide a large number (several pounds weight) of pieces of copper wire, of about the size No. 18 or 20, of the Birmingham brass wire gauge, and about fifteen or twenty inches long, for suspending the smaller and more numerous articles to be coated in the depositing solutions; a few other pieces of a larger and stronger kind should be provided for the heavier articles. Copper is the most suitable metal for connecting-wires, and the most generally used, because it is one of the best conductors of electricity, being also flexible and not expensive; next to it we should select brass: silver is the best conductor, but is too expensive.

82. *Dipping Liquids, Pickling Liquids, &c.*—The depositor will next prepare his various liquids for cleansing articles for plating. For cleaning iron articles he will require large stoneware pans and jars, containing a mixture of one part of sulphuric acid, and twenty parts of water; a weaker solution may be used for some kinds of metal; smooth wrought-iron requires a weaker liquid than rough cast-iron. For cleaning either copper, brass, or German silver, he will require several stoneware pans, one containing strong nitric acid, another filled with "dipping" liquid (a mixture of 64 parts of water, 64 parts sulphuric acid, 32 parts nitric acid, and 1 part of hydrochloric acid), and a third containing "spent" liquid, *i.e.*, either nitric acid or dipping liquid, the power of which has been partly exhausted. In addition to these, he will require some glass-cutters' fine sand; with several small hand-brushes and pieces of old cloth, for brushing and rubbing the sand upon the more rusty and refractory parts of the metals to be cleaned; also a file and scraper to further assist in cleaning them. Hydrofluoric acid, contained in a small leaden or gutta-percha bottle, should be at hand, to apply to the "glazed" patches occasionally met with upon cast-iron.

83. *Battery Liquids.*—The only kind of acid used by electro-platers, to excite their batteries, is sulphuric acid. For the negative solution of Daniell's batteries, a stock of sulphate of copper should be provided.

84. *Liquids for Causing Adhesive Deposits.*—Solutions of nitrate, or of cyanide of mercury, will be required for preparing the surfaces of copper, brass, and German silver, for receiving adhesive deposits of silver. The nitrate solution is prepared by adding one ounce of mercury to sufficient nitric acid, diluted with three times its bulk of distilled water to dissolve it: no more mercury must be added than the liquid will dissolve: when thoroughly dissolved, dilute it with about one gallon of water. To prepare the cyanide solution, dissolve one ounce of mercury as stated, dilute it with water, and add a solution of cyanide of potassium to it, just as long as a precipitate is produced; filter it, add a small quantity of water to the precipitate in the filter, and, when thoroughly drained, take out the precipitate, and add to it a strong solution of cyanide of potassium until it is all dissolved; then add a little more cyanide solution, and finally dilute it with water until the whole measures one gallon. The solution, when prepared, is kept in a large stoneware pan, a pan of dipping liquid and another of water being near it, and each placed near the scratch-brush lathe and depositing-vats in the silvering-room.

85. *Materials for Moulding.*—The electro-depositor who includes in his business not only the ordinary electroplating, but also the manufacture of works of art by deposition, requires a number of substances for moulding, and preparing the surface of the moulds to receive a deposit. For moulding ordinary metal objects, he often uses gutta-percha or a composition consisting of equal parts of white wax and spermaceti; but one of the best substances we have used for this purpose, has been a composition of our own, consisting of two parts of gutta-percha, and one part of Jeffery's marine glue: the glue is cut up into small pieces, and melted at a gentle heat in an iron ladle; the gutta-percha, also cut very small, is then added, and the mixture constantly and vigorously stirred at a gentle heat until the two are thoroughly incorporated. This substance possesses several important advantages over gutta-percha alone as a moulding material; it is softer when heated, and takes a sharper impression; it contracts more in cooling, and is therefore more easily removed from the original; and in taking the blacklead it is very superior to gutta-percha. With ordinary care many copies may be taken by deposition off one of these moulds; we have taken upwards of ten from one of them.

86. *Elastic Moulding Composition.*—When the objects to be copied are much under-cut, or when we wish to take a mould of a bust all in one piece, elastic moulding composition is required. The best substance of this kind, and almost the only one used, is composed of four parts of best Russian glue and one part of treacle; the glue is broken into small pieces, and soaked for one or two hours, or until it is quite soft, in sufficient *cold* water to cover it; when it is soft the superfluous water is thrown away, and the glue, together with the treacle, is heated in a common glue-pot, like ordinary glue, to nearly a boiling heat, and stirred until the two substances are thoroughly mixed: the use of the treacle is to prevent the mould drying and shrinking too rapidly.

87. *Blacklead, Phosphorus, Liquid, &c.*—For rendering the surfaces of non-conducting substances, such as gutta-percha, wax, marine glue, &c., conductible, the following substances are used:—

1st. The common powder blacklead for ordinary non-elastic moulds: there is the greatest difference, however, between various specimens of blacklead in their value for this purpose, some causing the deposit to spread over the moulds very quickly, whilst others scarcely cause it to spread at all; the best we have found, and it has been very good, is "Dix's," sold in twopenny packets, one or two of which will serve the operator a long time for this purpose.

2nd. For moulds of elastic composition (86), the depositor will require the following liquids, patented by Mr. Alexander Parkes:—A, the phosphorus solution: to make nearly three ounces of which, melt 64 grains of bees-wax or tallow; then dissolve eight grains of india-rubber cut up very small, in 160 grains of bisulphide of carbon; and when it is dissolved, add to it very carefully (as it is highly inflammable) the melted wax, and shake the mixture thoroughly. Then dissolve 64 grains of phosphorus in 960 grains (about 2¼ ounces) of bisulphide of carbon, and add to it 80 grains of spirit of turpentine, and 64 grains of asphalte in fine powder: when dissolved, add this solution to the previous one of india-rubber and wax, and thoroughly mix them by shaking. B, the silver solution: to make twenty ounces (one pint) of this liquid, dissolve about 18 or 19 grains of pure silver in about 20 or 25 grains of the strongest nitric acid, and then dilute it to the required volume with *distilled* water. And C, the gold solution: to make twenty ounces of which, dissolve about 5 or 6 grains of pure gold in about 20 or 25 grains of a hot mixture of one measure of nitric acid, and about two or three measures of hydrochloric acid; when dissolved, dilute the solution with 20 ounces of *distilled* water.

The same patentee includes in his patent a phosphorus moulding composition, by the use of which the immersion in the phosphorus solution is dispensed with, the moulds themselves containing the required amount of phosphorus. To make about one pound of this composition, melt together half a pound each of wax and deers' fat; then dissolve about 19 or 20 grains of phosphorus in about 300 grains of bisulphide of carbon; keep the wax mixture barely melted, and add the phosphorus solution slowly to it, and with brisk stirring of the fat, pouring it in at the bottom of the melted mixture by a vessel with a long spout, to prevent its inflaming. It is highly dangerous to have split portions of the phosphorus composition where it can come in contact with wood, paper, rags, &c., as, after a lapse of some time (even hours), they will often burst into flame.

88. *Selection of Depositing Processes.*—For very small articles, of which there are a great number, such as buttons, hooks-and-eyes, pins, &c., and which require only a very thin deposit, the simple immersion or wash process will answer very well, being both easy of execution and cheap. For the multiplication of numerous small articles in copper, such as medallions, &c., the single cell process is very advantageous; it is quicker than the battery process; and considering the time occupied, and the other elements of expense, it is to be preferred to that method. But for all ordinary deposits, plating, &c., the battery process is by far the best, because coatings of any thickness, in all ordinary metals, may be obtained by it, and the solutions do not, as in other processes, require renewal.

89. *Methods of Making Depositing Solutions.*—The operator will next consider about making depositing liquids. This may be done by two methods; the one called the chemical, and the other the battery process. The chemical process consists in mixing the various ingredients by the usual means, and in suitable proportions to form the liquids: for instance—1st, the ordinary sulphate of copper solution is prepared by dissolving a certain proportion of commercial sulphate of copper in water, and adding to it a definite quantity of sulphuric acid to afford *free* acid; and, 2nd, to form the ordinary cyanide of silver and potassium plating liquid, silver is dissolved in dilute nitric acid; the solution of nitrate of silver formed is precipitated by addition of a solution of cyanide of potassium; the white precipitate of cyanide of silver is washed, and then added, as much of it as will dissolve, to a solution of cyanide of potassium; after that, an additional portion of cyanide of potassium is added to afford *free* cyanide. The battery process consists in taking some water, and dissolving in it a certain proportion of acid or salt, as the case may be; then placing a large anode of the given metal at the lower part of the liquid, and a small bright cathode at the upper part, and, if necessary, applying heat. Connection is to be made with a suitable battery until the required quantity of metal is dissolved, which is indicated by the cathode receiving a good deposit: in making gold solutions the cathode is generally placed in a small porous cell filled with the same liquid, and immersed nearly to its edge in the outer liquid; and by transferring the cathode occasionally to the gold solution, and observing if it receive a good deposit, we may know that sufficient metal is dissolved; the liquid of the cell may then be added to the outer solution. If it be wished to make sulphate of copper solution by this method (which we should not advise, however, the salt being so cheap), take the same quantity of water as we prescribed for the chemical method, and add to it as much acid as was contained in the salt of copper with the free acid as before, and then pass a current from a battery of one or two pairs by a large anode and small cathode, until sufficient metal is dissolved; or if it be desired to make some cyanide of silver and potassium solution by this method, which is sometimes done, take the same proportions of water, cyanide of potassium, and free cyanide as in the chemical process, and pass the electric current by a large silver anode, until the same proportion of silver is dissolved as required by the chemical method.

90. *Selection of Depositing Liquids.*—The following rules should be observed in selecting a suitable depositing liquid for the battery process.

1st. It should act strongly upon the anode, and hold abundance of metal in solution.

2nd. It should possess good electrical conducting power.

3rd. It should yield its metal freely, and in a reguline state.

4th. It should not act chemically to any great extent upon the base metals, because it is those that we generally wish to coat, and chemical action upon them would endanger the adhesion of the deposited metal.

5th. It should not decompose by contact with the atmosphere, nor should light influence it in such a way as to injure it for depositing purposes.

6th. It is better if it do not evolve gas at the surface of the receiving article whilst depositing, because that generally indicates a waste of battery power, attended by oxidation of the liquid.

91. *Testing a Depositing Liquid.*—To test a depositing liquid, pass a current of electricity through it, from about two pairs of Smee's batteries, with a suitable clean anode of proper size, and a clean piece of iron, brass, or copper, of about the same size, to receive a deposit, observing how much gas is evolved in the battery; if the deposit appear quickly, and of a bright and proper colour; and if it adhere to the metal, the cathode evolving gas from its surface, and the anode dissolving freely, cleanly, and without escape of gas, work it at intervals, keeping it exposed to light and air; observe if it continue to work well, or whether, on the contrary, it shows a decrease of conductibility, deposits a sediment, or if the anode become covered with an insoluble crust (this last may arise either from deficiency of free acid, or from impurities in the metal). If but little gas be evolved in the battery, the liquid in the cell is a bad conductor, and will neither dissolve nor deposit the metal freely at that temperature, or it is deficient in *free* acid or *free* salt. If the deposited metal be of a bad colour, either the battery is too strong, the receiving article too small, or the liquid is incapable of yielding good metal. If the immersed metal or article be coated by simple immersion without the aid of the battery, it shows that, to adapt the liquid to articles made of that particular metal or alloy, they must receive some previous preparation, in order to make the deposit adhere. If it deposit a sediment, or alter in conductibility by exposure to air and light, the probability is that those influences alter either its chemical composition, or the arrangement of its particles. If it evolve gas at the receiving surface during deposition, it shows either that there is too much battery power, too little metal in solution, too much *free* acid, or that it is a wasteful liquid, in which one part only of the current is employed in depositing metal, whilst another part of it is employed in evolving gas and oxidating the liquid.

92. *Testing Solutions for Depositing Alloys.*—With solutions in which alloys are to be deposited, the most important condition is, that neither of the metals to be deposited should be electro-positive to each other in that liquid. This is best tested by taking a wire of each metal, connecting them with a galvanometer, and simultaneously immersing their free ends in the liquid; if either be electro-positive, the needles of the instrument will be deflected, and the direction of the deflection will indicate which is positive, while the amount of deflection will indicate the amount of their electric difference in that liquid. It may also be tested by immersing a wire of each metal (not in mutual contact)

in the liquid; if either become coated with metal in an hour, that one is positive; but if neither become coated in six hours, there is no perceptible electric difference between them.

93. The following experiments bear upon this part of the subject, and show that if a liquid contain two metals in solution, and a wire or other piece of each of those metals be immersed in the liquid, and one becomes covered with a deposit of metal, while the other does not, the one so covered is electro-positive to the other in that liquid, and the solution will only yield the same metal which is deposited by simple immersion.

Experiment 51.—With an alloy solution consisting of equal measures of a strong solution of protochloride of tin, and terchloride of antimony, with an anode either of tin or antimony (the latter is the most proper, because it does not coat itself by simple immersion in the liquid), a copper cathode, and two small cells of Smee's battery, only antimony was deposited; tin coated itself with antimony in this solution by simple immersion, and was found by the galvanometer to be strongly positive to that metal.

Experiment 52.—With a liquid composed of equal measures of a solution of protochloride of tin and chloride of bismuth, and either a bismuth or tin anode (the former is the best), a brass cathode, and two cells of Smee's battery, only bismuth was deposited; tin was positive to bismuth in this liquid by the galvanometer, and coated itself quickly with that metal by simple immersion.

Experiment 53.—With a mixture of equal measures of terchloride of antimony, and chloride of bismuth, antimony anode, copper cathode, and a feeble Smee's battery, only antimony was deposited; bismuth coated itself slowly with the antimony in solution by simple immersion, and was found by the galvanometer to be moderately positive to the latter metal.

Experiment 54.—With 100 grains each of protochloride of tin and chloride of zinc dissolved together in an ounce of distilled water, tin anode, copper cathode, and one cell of Smee's battery, only tin was deposited; zinc was positive to tin in this liquid by the galvanometer, and deposited tin upon itself by simple immersion.

Experiment 55.—With equal measures of strong solutions of nitrate of zinc, and ternitrate of bismuth, and a little nitric acid, bismuth anode, copper cathode, and a feeble one-pair battery, only bismuth was deposited; zinc was strongly positive to bismuth in this liquid by the galvanometer, and coated itself quickly with that metal by simple immersion.

Experiment 56.—With a solution of the mixed sulphates of zinc and copper, copper anode and cathode, and a single small battery, copper alone was deposited; zinc was strongly positive to copper in this liquid by the galvanometer, and coated itself immediately with copper in it by simple immersion.

94. Further, if we take some distilled water, and caustic potash be dissolved in it, on passing a moderately strong current through it by platina electrodes, hydrogen gas will alone be set free at the cathode: in this case, also, hydrogen—the least positive of the two positive elements of the liquid—potassium and hydrogen—is set free or deposited. If we now add a little sulphuric acid to the liquid to convert it into a solution of sulphate of potash, with some sulphate of zinc besides, and pass a weak current through, we shall obtain a deposit of zinc on the cathode, but no hydrogen or potassium: in this case we cannot determine by the galvanometer which is the most positive in this liquid, hydrogen or zinc, because the former is a gas; but it is probable that hydrogen is the most positive, because zinc does not evolve it by simple immersion in this liquid. If we further add to the liquid a small quantity of sulphate of copper, and treat it as before, neither potassium, hydrogen, nor zinc will be deposited, but only copper; and we find by the galvanometer, that copper is less positive than zinc in such a liquid, and that zinc coats itself with copper in it by simple immersion: in this case, also, the least positive of the positive elements of the liquid is alone deposited. From these and many other experiments, which we have tried with similar results, we deduce the following rule:—If a liquid contain several metals or other electro-positive substances dissolved, and a *weak* electric current be passed through it, only that substance which is the least electro-positive will be deposited.

95. With regard to the influence exercised by the proportions of the ingredients of the liquid, and the strength of the current, we may observe, that, if a liquid contain several metals dissolved in equal quantities, and only one is being deposited by the passage of a weak current, a considerable increase in the strength of the current will cause a portion of the next more positive metal to be deposited along with the less positive one; but this alloy deposit will not be very coherent, because the power required to deposit the second metal in the reguline state will be so great as to deposit the first as a soft powder. This holds most true when the difference of electric power required is at a maximum. For instance—1st. If small and equal quantities of sulphate of zinc and sulphate of copper are dissolved together in a large quantity of water, and a feeble current be passed through the solution, only reguline copper will be deposited; but if the battery power be considerably increased, either by a greater number or larger surface of the battery plates, the deposit of copper will cease to be reguline, and zinc will be deposited with it. If the power be still further increased, hydrogen gas will also be evolved at the surface of the deposited metals. 2nd. If we dissolve a small quantity of sulphate of copper, and a great quantity of sulphate of zinc, in a large quantity of water, and pass a strong current through the solution, copper, zinc, and hydrogen will be set free at the cathode. 3rd. If we slightly moisten a lump of caustic potash with pure water, and pass a weak electric current through it by platina electrodes, hydrogen alone will be set free at the cathode; but if a very powerful current be employed, potassium also will be deposited. In each of these cases we find, that when the current is weak, the least positive of the positive substances is alone deposited; but if the power be sufficiently increased, and there is only a small proportion of the less positive substance present, the more positive substances, even though they are much more positive, will also be deposited. Thus the weaker affinities are overcome first, and to the greatest extent; the current of electricity exercising its influence first, and in the greatest proportions, upon the salt of the least positive metals.

96. *Depositing Liquids.*—For the benefit of the practical depositor, to whom a general knowledge of all solutions from which ordinary metals may be deposited, with their respective advantages and disadvantages, is of considerable importance, we will give a description of those solutions in regular order, making such remarks in our progress as will be likely to assist him in the selection of those most suitable for his particular purposes.

97. *Antimony Solutions.*—The most common salts of antimony are the sulphide, terchloride, and potassio-tartrate. The hydrochlorate of terchloride of antimony, i.e., the ordinary chloride or butter of antimony, as prepared for pharmaceutical purposes, is formed thus:—Take one pound of black sulphide of antimony; add to it four pints of hydrochloric acid; gently heat the mixture with constant stirring, until the gas evolved decreases; then boil it slowly down to two pints, keeping it partly covered all the time; cool, and filter through calico, and keep it in a stoppered bottle. It is a yellowish-red liquid, of specific gravity 1·47, but becomes nearly colourless by depositing antimony by the battery process.

A similar solution may be made by the battery method; this consists in passing a current from several pairs of batteries through strong hydrochloric acid by a large anode of antimony, until a good deposit is obtained: this solution is nearly colourless. The chloride of antimony is an excellent conductor of electricity; it dissolves the anode freely; yields plenty of bright reguline metal if the battery power be sufficiently weak; and its depositing power does not deteriorate by exposure to light or the atmosphere, but it appears to be gradually exhausted by working; it is also decomposed more or less rapidly by zinc, tin, lead, iron, brass, copper, and German silver, each of which coat themselves in it with antimony by simple immersion; and articles immersed in it require to be washed with hydrochloric acid before washing them with water, otherwise the latter decomposes the adhering liquid, and covers them with a white insoluble powder.

98. The mixed chlorides of antimony and ammonia form a very good depositing liquid. It may be produced either by the battery process, by mixing one measure of a saturated solution of sal-ammoniac with one measure of hydrochloric acid, and working antimony into it by means of a battery and a large antimony anode; or by simply mixing together equal measures of a saturated solution of sal-ammoniac and commercial chloride of antimony. This solution conducts easily, yields its metal freely and of good quality, and does not act so strongly upon base metals as chloride of antimony alone; but in other respects it is similar to the chloride. The mixed chlorides of antimony and manganese, or of antimony and bismuth, yield a reguline deposit easily, but do not appear to possess any special advantages.

99. The potassio-tartrate of antimony is a salt not very soluble in water; its aqueous solution is a very bad conductor of electricity, and is not to be compared to the chloride for depositing purposes. We have never been able, either with strong or weak batteries, to deposit from it anything better than a small quantity of antimony in the state of a perfectly black powder: on the other hand, its solution in hydrochloric acid (which dissolves it very freely), or hydrochloric acid and water, is by far the best that we have tried for depositing antimony: it is an excellent conductor of electricity; it is not impaired by long working or exposure to light or the atmosphere (we have deposited antimony from it constantly during many months); it will bear a great amount of battery power without the deposit passing into the state of a loose powder; it deposits reguline metal very rapidly and in great thickness. We have obtained such deposits from it upwards of two inches in thickness; articles immersed in it wash clean in water alone, without the previous use of hydrochloric acid. It may be made by mixing together about two pounds of water, four pounds of hydrochloric acid, and eight pounds of potassio-tartrate of antimony; a greater proportion of water may be used if desired.

100. Both the black and red sulphides of antimony dissolve in cold hydro-sulphate of ammonia, and the resulting solutions conduct very freely with an antimony anode and one cell of Smee's battery, but yield no deposit of metal even with a battery of twenty-five cells' intensity. Aqueous solutions either of caustic potash, tartrate of potash, or oxalate of potash, scarcely conduct at all with an anode of antimony, and a battery consisting of one or two cells. Cyanide of antimony, dissolved in a solution of cyanide of potassium, has been proposed as a depositing liquid; but we have found a solution of cyanide of potassium to be a very bad conductor with an anode of antimony.

101. Antimony is one of the easiest metals to deposit in the reguline state; its appearance when deposited from the chloride solutions, and from the solution of the potassio-tartrate in hydrochloric acid, is very beautiful; and when deposited slowly, it has much of the lustre of highly-polished steel. Some of its properties when thus deposited are very peculiar and interesting, especially with regard to heat. If, during any part of the time the deposit is progressing, the deposited antimony be taken out and struck gently, or rubbed with any hard substance, such as metal or glass, an explosion will occur, accompanied with a small cloud of white vapour—sometimes with a flash of light, and nearly always with considerable heat, sufficient to burn the fingers, melt gutta-percha, burn paper and even scorch deal wood to a brown colour, especially if the deposit be thick. This is invariably accompanied by fracture of the deposited metal; sometimes, when the process of deposition has been interrupted, and the deposited metal is not homogeneous, the fracture extends through the metal to upwards of one-eighth of an inch in depth. This phenomenon has been observed many times, both before and since its first publication; in several instances the explosion was produced even in the liquid, by striking the deposit against the glass vessel which contained it; and in one instance it occurred after the metal had been well washed with dilute hydrochloric acid, dried, and had remained out of the liquid several hours.

On one occasion, a deposit had been well washed, dried, and out of the solution many hours; and a friend, in course of conversation, was unconsciously breaking small portions off it with his fingers, when it became suddenly heated, and exploded, causing a slight noise like the lighting of a congreve match, and burning his fingers. In other cases, a deposit has been progressing, and has been removed an instant for examination, and the battery liquid strengthened by the addition of acid: upon examining the deposit a few hours afterwards, it has been found cracked in various directions, as if an explosion had occurred in the interval, although the apparatus had been undisturbed. A French writer has suggested that this deposit is a compound of antimony and hydrogen; and from the fact that the explosions occurred when the metal was depositing rather rapidly, we are inclined to think his explanation correct; the extra power, as we have seen in other cases (94, 95), causing hydrogen deposit, which, in its nascent state, instead of being evolved, might combine with the metal and form an explosive compound. Another suggestion we would make is, that the metal may be deposited in a peculiar condition of unequal mechanical tension, similar to that of unannealed glass, and that, by breaking, the closer aggregation of the particles may develop light and heat.

102. Another peculiarity in depositing antimony from the potassio-tartrate solution is, that if the solution be a very dense one, and the process long continued without disturbance of the liquid, the deposit occurring upon the cathode will slowly spread out in the form of a thin sheet upon the surface of the liquid, until it touches the anode; whilst the deposit beneath progresses very slowly. We have a button of antimony formed in this way upon a vertical copper wire, one and five-eighths inches in diameter, the deposit beneath the surface of the liquid having been only half an inch thick; it occupied about eighteen days, with a small cell of Smee's battery, in forming. Deposits of antimony formed in the above solution do not spread over blackleaded surfaces of gutta-percha, nor do they adhere with any great degree of firmness to copper, brass, or iron.

103. *Bismuth Salts.*—The most usual compounds of bismuth are the chloride, mononitrate (pearl white), and ternitrate. The chloride is formed by digesting bismuth filings a long time in a warm place with hydrochloric acid. The mononitrate is formed by dissolving bismuth to saturation in warm dilute nitric acid, and then adding a large quantity, say fifty or a hundred times its volume, of water; the precipitate produced, when well washed with water, is the substance required. The ternitrate is formed by dissolving the metal in hot nitric acid, evaporating the solution, and leaving it in a cold place to crystallise. Bismuth may easily be deposited from a solution formed by dissolving either mononitrate or ternitrate of bismuth in dilute nitric acid, but it requires an exceedingly feeble current to deposit it in a reguline state; its appearance when so deposited is very beautiful—white with a faint pinkish tint, and with a fine silky lustre; it does not spread over black-leaded surfaces of gutta-percha in this liquid. A bismuth anode does not dissolve readily in a hot solution of cyanide of potassium.

104. *Zinc Salts.*—There are a variety of salts of this metal in ordinary use, the most common of which are

the sulphate, chloride, nitrate, and acetate. The sulphate may be formed by dissolving zinc to saturation in a mixture of sulphuric acid and water, filtering and evaporating the liquid, and setting it in a cold place to crystallise. The chloride is formed by saturating hydrochloric acid with zinc, filtering, evaporating, and crystallising. The acetate may be made either by dissolving zinc in strong acetic acid to saturation, then evaporating and crystallising the solution; or by adding a solution of acetate of lead to a solution of sulphate of zinc as long as it produces a precipitate; then filter, evaporate, and crystallise the liquid.

105. *Zinc Solution.*—The sulphate of zinc solution for depositing may be formed by dissolving two pounds of the salt in a gallon of water, and filtering the mixture; but the best sulphate depositing solution we have used has been the spent liquid taken from a cell of a Smee's battery, in which there had occurred a very good deposit of zinc upon the platinised silver plate; but with this and other solutions of zinc there is a great tendency to the evolution of hydrogen gas at the cathode during deposition; they require, therefore, to be worked very carefully, and with very feeble battery power. Zinc may be readily deposited, either by the single cell, or by the battery process, from a neutral solution of the sulphate; but the single cell is less adapted for its deposition than the battery, because the acid, set free by the deposition of the metal, reacts upon the deposit, and diminishes its cohesion. The other solutions, such as the chloride, nitrate, acetate, or the various double salts of zinc, with ammonia, or potash, do not appear to possess any general advantages over the sulphate.

Amongst other liquids, that of cyanide of zinc dissolved in a solution of cyanide of potassium, has been recommended; but it is a bad conductor with a zinc anode, and requires to be used hot to make it conduct at all freely, or make the anode dissolve. This might easily have been foreseen from a knowledge of the fact, that the affinity of cyanogen for all, or nearly all, the base metals is comparatively feeble. We have found by experiment, that a solution of cyanide of potassium will dissolve only about one-half as much cyanide of zinc as it will of cyanide of copper. Zinc oxide dissolves somewhat freely in a boiling solution of cyanide of potassium. Cyanide of zinc dissolves freely in a solution of sesquicarbonate of ammonia. Ferro-cyanide of zinc is but feebly soluble in a boiling solution either of ferro-cyanide (yellow prussiate) or of ferri-cyanide (red prussiate) of potassium, but it is freely soluble in a boiling solution of cyanide of potassium. Zinc deposits spread over blackleaded surfaces by the battery process, in the same manner as copper.

106. *Cadmium Solution.*—A patent was taken out, March 19th, 1849, by Messrs. Russell and Woolrich, for the electro-deposition of cadmium; and the following is their description of the process:—"Take cadmium and dissolve it in nitric acid diluted with five or six times its bulk of water, at a temperature of about 80° or 100° Fah., adding the dilute acid by degrees until the metal is all dissolved; to this solution of cadmium a solution of carbonate of soda (made by dissolving one pound of the ordinary crystals of soda in one gallon of water) is to be added until the cadmium is all precipitated; the precipitate thus obtained is to be washed four or five times with tepid water; next add as much of a solution of cyanide of potassium as will dissolve the precipitate; after which, one-tenth more of the solution of cyanide of potassium is to be added, to form *free* cyanide. The strength of this solution may vary; but the patentees prefer a solution containing six troy ounces of metal to the gallon. The solution is worked at about 100° Fah., with a plate of cadmium as an anode."

107. *Tin Salts.*—The most common salts of tin are the peroxide and protochloride: in addition to these there are two others, used extensively in Manchester, and the cotton-printing districts—viz., the bichloride and the stannate of soda, *i.e.*, oxide of tin combined with caustic soda. Peroxide of tin is formed by dissolving protochloride of tin in water containing a few drops of hydrochloric acid, and then adding liquid ammonia, or a solution of carbonate of potash, as long as a precipitate can be produced; the precipitated peroxide of tin should be washed and dried. Protochloride of tin is easily made by adding grain tin to strong hydrochloric acid, and keeping it at 150° or 200° Fah., until gas ceases to be evolved from the metal; the resulting solution should then be evaporated and crystallised. Aqueous bichloride of tin may be made by dissolving tin in aqua regia not containing too much nitric acid; a mixture of nitric acid with sal-ammoniac or common salt may likewise be used. Stannate of potash may be formed by fusing together one equivalent (75 parts) of freshly precipitated peroxide of tin, and one equivalent either of caustic potass (56·2 parts), or of crystallised carbonate of potash (87·2 parts).

108. *Tin Solutions.*—M. Roseleur has patented the following liquids for the deposition of tin:—

1st. For simple immersion or wash process, which may be used for small articles generally—dissolve 17½ ounces of ammoniacal alum in 22 pounds of boiling water, and, when dissolved, add one ounce of protochloride of tin: the articles to be coated should be well cleaned, and then immersed in the liquid, and moved about in it until they are sufficiently white.

2nd. For depositing tin upon lead, iron, steel, copper, or brass, by connecting the articles with a piece of zinc, and immersing them in the solution—dissolve 10½ ounces of bitartrate of potash in 17½ pints of water; then add three-quarters of an ounce of protochloride of tin, and boil it a few minutes: the articles to be coated are immersed in the solution in contact with a piece of zinc of proportionate size.

3rd. For coating zinc, iron, copper, and many other metals by the battery process—dissolve 11 ounces of pyrophosphate of potash or soda in 17½ pounds of water; then add 4½ ounces of protochloride of tin, and operate by the battery process with an anode of tin. By this process, M. Roseleur states that he can tin metals beautifully and to any thickness. Pyrophosphate of soda is easily formed by heating to redness the common diphosphate of soda.

109. A protochloride of tin depositing liquid, may easily be formed by dissolving the ordinary commercial protochloride in water, and adding a little hydrochloric acid to remove any cloudiness or white precipitate which may be formed; a similar liquid may be made by the battery process, by passing a current through dilute hydrochloric acid by means of a large tin anode, until sufficient metal is dissolved. This or any other chloride of tin is not a good solution to obtain reguline metal from; it has a very great tendency to deposit the tin in the form of long crystalline needles, of a fernlike appearance, which often project from the corners and edges of the cathode to a distance of upwards of half an inch. A solution composed of eleven ounces of water, one ounce of hydrochloric acid, and eighty grains of protochloride of tin, admits of this effect being produced in a striking degree: nearly all the compounds of tin, and especially those formed with mineral acids, exhibit this tendency in a greater or less degree when acted upon by electrolysis, rendering the deposition of tin in thick layers of fine white coherent metal a matter of considerable difficulty.

110. The stannate of potash solution is easily formed either by dissolving the crystallised salt in water, or by dissolving freshly precipitated peroxide of tin (109), whilst still moist, in a boiling solution of caustic potash. It may also be easily formed by the battery process, by passing a strong current of electricity, by a large tin anode, through a strong and boiling solution of caustic potash, until the immersed cathode receives a free white deposit. This solution, if worked at 150° Fah., yields a good deposit of fine white metal; but it decomposes by exposure to the atmosphere, and soon deposits all its metal as oxide of tin. A solution of cyanide of potassium and tin has been proposed as a depositing liquid; but it is a bad conductor with a tin anode, even if hot, and does not dissolve the metal freely.

111. Mr. Joseph Steele coats zinc, iron, steel, copper, and brass, with tin, in his patent solution, by the battery process, thus:—Dissolve 60 pounds of common soda, 15 pounds of pearl-ash, 5 pounds of caustic potash, and 2 ounces of cyanide of potassium, in 75 gallons of water, at 75° Fah., and filter the mixture so made; then add 2 ounces of acetate of zinc, and 16 pounds of binoxide of tin; stir the resulting solution until all is dissolved; it is then ready for use. Work it by the battery process with an anode of zinc or tin, and with the liquid at 75° Fah.

112. *Electrical Relations of Tin and Iron.*—Tin is feebly negative to iron at all temperatures between 62° and 203° Fah. in distilled water, and positive to it at 212° Fah. It is positive to iron at all temperatures between 62° and 212° Fah. in a saturated solution of boracic acid; also the same between those temperatures in a strong solution of phosphoric acid in distilled water; or in one measure of oil of vitriol mixed with either nine or ninety-six measures of distilled water; or in a mixture of one measure of this acid, and 192 measures of distilled water, from 73° to 158° Fah., and negative to iron above that to 212° Fah. It is positive to iron from 72° to 212° Fah. in a mixture of equal measures of hydrochloric acid and water; it is negative to iron from 70° to 77° Fah., and positive above that to 212° Fah., in a mixture of one measure of hydrochloric acid and nine measures of distilled water; it is negative to iron from 70° to 212° Fah. in a mixture of one measure of hydrochloric acid, and ninety measures of distilled water; and positive to iron from 68° to 212° Fah. in one measure of hydrofluoric acid and nine measures of water; it is positive to iron in one measure of nitric acid and nine measures of water from 70° to 111° Fah., and negative from 111° to 212° Fah.; and it is positive to iron from 82° to 212° Fah. in a mixture of one measure of nitric acid and ninety-six measures of water.

113. *Lead Salts.*—The most common salts of lead are the nitrate, and the acetate. The nitrate is formed by dissolving lead in dilute nitric acid, taking care that no more lead is added than the acid will dissolve; the resulting solution must be filtered, evaporated, and crystallised; it is a hard, white salt, soluble in water. Acetate of lead is made by digesting oxide of lead in vinegar or acetic acid; filtering, evaporating, and crystallising the liquid: it is soluble in water.

114. *Lead Solution.*—Lead may be deposited from an aqueous solution, either of nitrate or acetate of lead, or from a solution of plumbite of potash: the latter is formed by dissolving litharge in a boiling solution of caustic potash. Zinc and tin articles (but not iron) decompose this liquid, and coat themselves with lead in it, by simple immersion. It is difficult to deposit any considerable thickness of reguline metal from either of these liquids.

115. *Salts of Iron.*—Among the salts of iron in most common use, are the sulphate, chloride, and nitrate: they may be respectively formed by dissolving metallic iron in dilute sulphuric, hydrochloric, or nitric acids, evaporating and crystallising the solution as much as possible out of contact with the atmosphere.

116. *Iron Solutions.*—Iron may be reduced from a solution of its protosulphate (green copperas), made by dissolving metallic iron in dilute sulphuric acid; or from its protochloride, which is preferable, and which is made by dissolving iron in hydrochloric acid. We have deposited it in the state of reguline white metal, by passing a current of considerable intensity (15 or 20 pairs) for one hour, through an anode of iron immersed in a saturated aqueous solution of sal-ammoniac; its appearance when deposited from this liquid is rather white, and very similar to that of freshly broken cast-iron. By the same means it may also be deposited, using a saturated solution, either of carbonate of ammonia, acetate of ammonia, or acetate of potash. Good metal may be obtained from a saturated aqueous solution of a mixture of two parts of protosulphate of iron and one part of sal-ammoniac. We have deposited it from an aqueous solution of ferrate of potash, which may be formed either by igniting peroxide of iron (crocus) very strongly for some minutes with caustic potash and saltpetre. Or make a very strong solution of caustic potash, immerse it in a large iron or steel anode, and a small copper or platina cathode, and pass a strong current from fifteen or twenty pairs of Smee's batteries through it until it acquires a deep amethyst or purple colour; by that time the cathode will have obtained a coating of iron, which will be in the state of a dark powder if the power have been too great, or it will have the appearance of white cast-iron, or intermediate between that and the appearance of reguline deposited zinc, if the power have been sufficiently weak. This solution rapidly decomposes without any very apparent cause, becoming colourless, and depositing all its metal in the state of peroxide at the bottom of the vessel. Iron may be very easily deposited from its sulphate, thus:—Dissolve a little crystalline sulphate of iron in water, and add a few drops of sulphuric acid to the solution; one pair of Smee's batteries may be used to deposit the iron upon copper or brass. The metal in this pure state has a very bright and beautiful silvery appearance. An aqueous solution of cyanide of potassium is a very bad conductor with an iron anode, even if it be maintained hot.

117. *Cobalt Solution.*—A solution of protochloride of cobalt may be easily formed by digesting commercial brown oxide of cobalt in hot hydrochloric acid; it is a deep-blue liquid, but changes to a reddish-brown colour on the addition of water; by slow cooling, fine red crystals of the chloride may be obtained. We have not tried this liquid for electro-deposition.

118. *Nickel Solutions.*—The nitrate of nickel solution may be formed by dissolving nickel in nitric acid slightly diluted with water, and, when dissolved, diluting with additional water; it is a solution which does not yield its metal freely. We have deposited nickel in the state of reguline white metal, from a solution of the double chloride of nickel and ammonia, by making a lump of metallic nickel the anode in a strong aqueous solution of sal-ammoniac, and passing a strong current of electricity through it for several hours, until the liquid acquired a pale greenish-blue colour. We have also obtained a similar deposit by treating a solution of one part of arseniate of potash and five parts of water in a similar manner. It has also been deposited from a solution formed by dissolving pure nickel in nitric acid, then diluting and precipitating it by a solution of carbonate of potash, or cyanide of potassium; washing the precipitate, and dissolving it nearly to saturation, in a solution of cyanide of potassium, and operating upon this liquid, by the battery process, with an anode of pure metal. Its appearance, when deposited from this solution, is said to be nearly equal in whiteness to silver, and its deposition has been proposed to be applied to the production of an inferior class of plated articles.

119. *Copper Salts.*—The ordinary salts, &c., of copper, are the protoxide (black oxide of copper), sulphate, chloride, nitrate, acetate, and cyanide. To make the protoxide, heat either the carbonate or nitrate to a moderate red heat, or the sulphate to intense redness. The sulphate may be formed by heating one equivalent (31·7 parts) of copper filings, and at most two equivalents (98 parts) of oil of vitriol, until the residue is quite dry; then dissolving the product in water, filtering, evaporating, and crystallising the solution; but it is much more convenient to purchase it, on account of its low price. The chloride and nitrate may be formed—the first by dissolving copper in aqua regia, or by saturating hydrochloric acid with protoxide of copper, and evaporating and crystallising the liquids; and the second, by dissolving copper in nitric acid, evaporating and crystallising the solution. Acetate of copper is most conveniently purchased; its commercial name is crystallised verdigris. Cyanide of copper may be made by adding a solution of cyanide of potassium to a solution of sulphate of copper (each liquid being cold), as long as a precipitate can be produced—filtering and washing the precipitate, which is the required compound; it is a fine powder, of a pale-green colour. In the operation, a large quantity of

cyanogen gas is evolved, which, if freely inhaled, is dangerous to the health: in consequence, also, of this escape of cyanogen, the cyanide of copper is not a *proto*-cyanide, but contains two equivalents of cyanogen for every three equivalents of copper; it is freely soluble in a solution of cyanide of potassium; it is also soluble in aqueous ammonia and in a solution of carbonate of ammonia. The proportions of materials we have used in making it, have been 65 parts of cyanide of potassium, and 125 parts of sulphate of copper. The precipitating solution is invariably greenish-blue, and contains much dissolved copper after all precipitation ceases; but no use has hitherto been made of this remainder.

120. *Copper Solutions.*—Copper may be easily deposited either by simple immersion (wash process), by the single cell, or by the battery process. According to Reinsch, iron may be coated with a durable and polishable layer of copper of any thickness (?). The process by simple immersion is thus:—Mix together one measure of hydrochloric acid, three measures of water, and a few drops of a solution of sulphate of copper; clean and immerse the iron; wash it, rub it with the copper solution, and reimmerse it repeatedly, adding a few drops of the copper solution occasionally. In depositing copper by the single cell process, a nearly saturated solution of sulphate of copper answers very well; but for the battery process, an excellent solution may be made by dissolving four parts, by weight, of finely divided sulphate of copper (best quality), and one part of sulphuric acid, in about eighteen or twenty parts of water, and then filtering it; neither of these solutions, however, is fit to deposit copper upon iron, steel, or zinc; because the electrical relations of these metals in the solution are unsuitable: these metals decompose the solutions rapidly, and deposit the copper upon themselves by simple immersion. To effect an adhesive deposit of copper upon iron, a solution composed of cyanide of copper dissolved in a solution of cyanide of potassium may be used. It is formed thus:—Dissolve cyanide of copper to saturation in water containing about two pounds of cyanide of potassium to the gallon, and then add about one-eighth more of the cyanide of potassium solution to form *free* cyanide; the liquid is then ready, and should be used at a temperature of about 150° Fah.

121. Copper is electro-positive to iron in the following liquids at 60° Fah.:—Powerfully in a solution of hydrosulphuret of ammonia; feebly in a saturated solution of ammonia; in a solution of oxide of copper in liquid ammonia; in aqueous ammonia, or in a saturated solution of ferro-cyanide of potassium, each but for a short time; it then becomes negative. In a saturated solution of bichromate of potash; in a strong aqueous solution of sulphide of potassium, it is increasingly positive up to the boiling point of the liquid. This last liquid has a similar effect on brass.

122. *Brassing Solutions.*—Much interest and importance was long attached to the discovery of solutions whereby alloys, and especially brass, might be deposited in the reguline state, and various liquids have been used and patented for this purpose. M. de Ruolz, in 1841, deposited brass from the cyanides of zinc and copper, dissolved together in a solution of cyanide of potassium. Copper articles may be superficially brassed by boiling them in a solution of bitartrate of potash with zinc amalgam, or by boiling them in dilute hydrochloric acid with some bitartrate of potash and zinc amalgam.

123. The same object was effected by Russell and Woolrich's patent, dated March 19th, 1849; which is as follows:—Take 10 pounds of acetate of copper, 1 pound of acetate of zinc, 10 pounds of acetate of potash, and 5 gallons of hot water; dissolve the salts in the water, add as much of a solution of cyanide of potassium as will precipitate the mixture, and redissolve the precipitate; in addition, add about one-tenth of cyanide of potassium. Use a brass anode, or else two anodes—one of zinc and one of copper.

124. Joseph Steele's patent, dated August 9th, 1850.—Dissolve 2¼ pounds of American potash in 6 gallons of hot water, and filter the solution; also dissolve 2¼ ounces of acetate of copper in half a pint of strong liquid ammonia, and add it to the first solution with stirring; also add 4 or 5 ounces of sulphate of zinc, and stir till dissolved; and, finally, add 2 ounces of cyanide of potassium, filter the resulting solution, and use it at 100° Fah., with a brass anode. To obtain a dark-coloured brass add more acetate of copper; and to obtain it of a lighter colour, add more sulphate of zinc.

125. Salzede's patent, dated September 30th, 1847.—Take 5,000 parts of water, dissolve 12 parts of cyanide of potassium in 120 parts of it, then add 610 parts of sub-carbonate of potash, 48 parts of sulphate of zinc, and 25 parts of chloride of copper to the remainder of the water, and heat the mixture from 144° to 172° Fah.; and when the salts are entirely dissolved, add 305 parts of nitrate of ammonia, allow the liquid to remain undisturbed for twenty hours, and then add the solution of cyanide of potassium; allow it to remain again till clear, and then draw off the transparent liquid, which is ready for use; work it with a large brass anode and a strong battery. Another liquid which he uses for brassing, consists of 5,000 parts of water, 500 parts of sub-carbonate of potash, 35 parts of sulphate of zinc, 15 parts of chloride of copper, and 50 parts of cyanide of potassium. For a bronzing solution, he uses 25 parts of chloride of tin in place of the sulphate of zinc of the first brassing liquid, and proceeds as with that process; for a second bronzing solution, he uses 12 parts of chloride of tin in place of the sulphate of zinc of the second brassing liquid, using the solution from 77° to 97° Fah.

126. Brunel, Bisson, and Gaugain's formula consists of 50 parts of carbonate of potash, 2 parts of chloride of copper, 4 parts of sulphate of zinc, and 25 parts of nitrate of ammonia, dissolved together in cold water, which are used with a brass anode and a strong battery.

127. Morris and Johnson's patent, dated December 11th, 1852.—According to this patent, dissolve 1 pound of cyanide of potassium, 1 pound of commercial carbonate of ammonia, 2 ounces of cyanide of copper, and 1 ounce of cyanide of zinc, in 1 gallon of water, and use the solution at 150° Fah., with a large anode of brass and a powerful battery. Or a solution may be taken of 1 pound of cyanide of potassium and 1 pound of carbonate of ammonia, dissolved in 1 gallon of water, and saturated with copper and zinc to the requisite degree by means of a strong battery, a large brass anode, and small cathode, until the latter receives a good deposit of brass; the solution being at a temperature of 150° Fah. To increase the proportion of copper in the deposit, either add cyanide of potassium, or raise the temperature of the liquid; and to increase the proportion of zinc in it, either add carbonate of ammonia, or lower the temperature.

128. Of the numerous solutions that have been tried for depositing brass, the one just mentioned is much the best. By it, reguline and thick deposits of brass, of uniform colour, and of any desired composition, may be obtained. It is not an expensive liquid; it acts with average strength upon the anode; it holds a sufficient quantity of the alloy in solution; it conducts electricity with moderate facility; and it yields its metal in the reguline state very uniformly; it bears a great variation in the electric power without injury to the character of the deposit, and is, therefore, very easily managed; it does not act perceptibly upon cast-iron, wrought-iron, steel, or even zinc, so as to injure the adhesion of the deposit; and it is not decomposed by exposure to the atmosphere, to light, or heat, in such a way as to affect its depositing power. Its defects are, that it requires to be worked hot, and with considerable battery power, in order to make the anode dissolve rapidly, the solution conduct copiously, and to cause a rapid deposit; it also evolves an abundance of gas at the cathode when working, whether the solution be hot or cold, which indicates that part of the battery power is expended in decomposing the water of the liquid, depositing its hydrogen with the metallic alloy, and oxidising the solution.

But all the brassing solutions are, in a greater or less degree, imperfect.

Deposition of German Silver.—The same patentees also deposit German silver by the following process:—Dissolve 1 pound of cyanide of potassium and 1 pound of carbonate of ammonia in 1 gallon of water; heat the solution to 150° Fah.; immerse a large anode of German silver in the liquid, and a small cathode of any suitable metal; connect the two with a powerful battery, and pass the current of electricity until considerable metal is dissolved and a bright cathode receives a deposit of good colour: the solution is then ready for use.

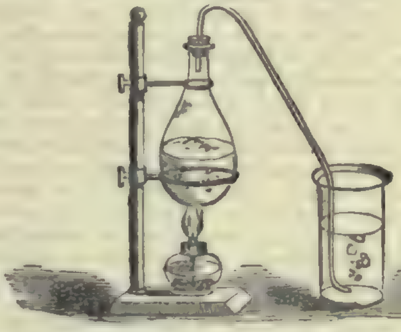
129. *Mercury Solutions.*—The ordinary compounds of mercury are the binoxide (red precipitate), bisulphide (vermilion), bichloride (corrosive sublimate), nitrate, and bicyanide. The nitrate is formed by dissolving mercury in nitric acid diluted with three times its bulk of water, the mixture being cold, and no more metal added than the acid will dissolve. The solution, when diluted with water, may be used for depositing by the battery process—a layer of mercury at the bottom of the liquid being used as the anode, and connected with the battery by a platina wire passing through a tube of glass or gutta-percha. The bicyanide is made by taking 8 parts of Prussian blue and 16 parts of peroxide of mercury, both in the state of fine powder, in 30 parts of water, boiling the mixture for about a quarter of an hour, filtering the liquid, and evaporating, and crystallising the solution; the resulting salt is the bicyanide; and to form it into a solution fit for depositing, it must be dissolved in a solution of cyanide of potassium: the solution may be used with a mercury anode and battery, as already described.

130. *Silver Salts.*—The most common salts, &c., of silver are the oxide, chloride, nitrate, and cyanide. Oxide of silver is prepared by adding a solution of caustic potash to a solution of nitrate of silver, as long as a precipitate can be produced, which is oxide of silver. Chloride of silver is made by adding either hydrochloric acid or a solution of common salt to a solution of nitrate of silver, until a precipitate ceases to be formed: the white precipitate of chloride of silver should be washed, dried, and preserved out of the influence of light. Nitrate of silver is easily formed by adding grain silver, in small quantities at a time, to a warm mixture of one measure of distilled water and four measures of the strongest nitric acid: if the liquid be too hot, or too much silver be added at a time, the action will be very strong, and loss of materials may be occasioned; in such a case add a small quantity of cold distilled water. When the liquid ceases to dissolve more metal, it should be evaporated and crystallised, or else kept protected from the light until required to be used: nearly all the compounds of silver are formed by means of this salt. Acetate of silver is made either by adding a solution of acetate of potash or acetate of soda, to a solution of nitrate of silver, as long as a precipitate occurs, or by digesting the oxide or the carbonate of silver in hot and strong acetic acid; it is freely dissolved by a solution of cyanide of potassium. Cyanide of silver is generally prepared by adding a solution of cyanide of potassium to one of nitrate of silver as long as a precipitate occurs; the white precipitate, which is cyanide of silver, is insoluble in water, and is not perceptibly soluble in commercial hydrocyanic acid; it dissolves very freely in a solution of cyanide of ammonium, potassium, or sodium, and in hyposulphite of soda; it is also said to be soluble in solutions of ammonia, carbonate of ammonia, salammoniac, nitrate of ammonia, and ferro-cyanide of potassium.

131. M. Brandeley, a French experimentalist, makes the following remarks upon the preparation of cyanide of silver:—"To obtain a beautiful and easy deposit of silver, we choose, among all the salts of silver, the cyanide, as giving the best results; but as the dealers sell this at a high price, both amateurs and manufacturers reject it. Others, for the sake of economy in procuring it, purchase hydrocyanic acid; but this, also, is of too high a price; and independently of our being obliged to use it immediately, it is both dangerous and difficult to preserve, as the air and light decompose it. If we take commercial hydrocyanic acid which has been prepared fifteen days, and pour it into a solution of nitrate of silver, consisting of one part of the nitrate to six parts of water, cyanide of silver will be formed; but it is more or less yellow, and much ammonia and hydrocyanic gases are evolved. On the other hand, if we make a solution of cyanide of potassium, filter it, and dissolve cyanide of silver in it, this solution, which was clear and colourless, immediately becomes troubled and black, and betrays an odour of ammonia and hydrocyanic acid. The odour will continue as long as the solution exists, and a deposit of carbon will be found in the vessel containing it. This sediment arises from the decomposition of one part of the cyanide of potassium, caused by the presence of the cyanide of silver. Having occasion to use considerable quantities of the cyanide of silver, I dissolve pure silver in pure nitric acid; evaporate just to dryness; dissolve the nitrate of silver thus obtained in distilled water, and pass hydrocyanic gas (prussic acid) through it, from a mixture of pounded ferro-cyanide of potassium and sulphuric acid, diluted with twice its weight of water—continuing this as long as a precipitate will form. Wash the cyanide of silver, and preserve it below water, away from the light. Thus precipitated, the salt dissolves without residuum or colour, and gives splendid results."

With regard to this process, we may remark, that six parts of sulphuric acid should be mixed with from thirty to forty parts of water, and the mixture allowed to cool; then put it into a glass vessel (Fig. 86), together with ten parts of coarsely-powdered ferro-cyanide of potassium; heat must be applied until gas is evolved from the mixture, and continued as long as any is given off, or as long as a precipitate is produced in the silver solution, the gas being passed into the liquid by a suitable tube. (See Fig. 86). This process may yield a purer product than when the nitrate solution is precipitated by cyanide of potassium; but it cannot be very economical, because only half of the cyanogen of the ferro-cyanide passes over as hydrocyanic acid; the remainder is left behind, and is contained in the yellowish-white residuum in the gas-generating vessel.

Fig. 86.



132. *Silver Solutions.*—Silver has been deposited by the ordinary dipping or wash process, by the single cell, and by the battery process. The following are recipes for solutions, taken from various sources, adapted for silvering articles by the simple immersion or wash process, chiefly applicable to small articles, such as pins, buttons, buckles, coffin-nails, hooks-and-eyes, &c., where only a very thin coating of silver is required. The solutions in the proportions indicated, are used by adding a small quantity of water, sufficient to form the ingredients into a pasty liquid of the consistence of cream, stirring the articles thoroughly about in it, or rubbing them over with it until they have acquired the desired degree of whiteness:—1st, take equal parts of chloride of silver and bitartrate of potash; 2nd, take chloride of silver one part, alum two parts, common salt eight parts, and tartar eight parts; 3rd, take chloride of silver one part, prepared chalk one part, common salt one-and-a-quarter parts, and pearlash three parts; 4th, a "novargent" solution for resilvering old plated goods, consists of one

hundred parts of hyposulphite of soda and chloride, or any other salt of silver, fifteen parts. Compounds of this description are also used for silvering clock-faces, thermometer and barometer plates, and many other articles of copper and brass.

133. *Silvering by Immersing.*—Mr. Joseph Steele took out a patent, dated August 9th, 1850, for silvering articles, by immersing them in a silver solution in contact with a piece of zinc of proper size. The process is as follows:—Dissolve four ounces of pure silver in twenty ounces of nitric acid; also dissolve, separately, one-and-a-half pounds of common salt in one-and-a-half gallons of water; mix the two solutions together, allow the mixture to remain till clear, pour away the clear liquid, and wash the precipitate, which is chloride of silver; next fuse together twenty-four ounces of ferro-cyanide of potassium, and twelve ounces of carbonate of potash; and when the mass is cold, add it, together with the chloride of silver, to one gallon and a-half of water; boil the mixture, and filter it; it is then ready for use.

134. *Silvering Solution for Battery Process.*—Many solutions have been proposed and tried for depositing silver by the battery process, but none have stood the test of time and experience like the one composed of double cyanide of silver and potassium dissolved in water, and a little *free* cyanide of potassium added. It may be made of various strengths, from half an ounce of silver to the gallon of water, to two, four, six, or more ounces; and from an ounce of cyanide of potassium to several pounds per gallon, and still be effective in working. The formula of M. de Ruolz is as follows:—Dissolve one part of cyanide of silver and ten parts of cyanide of potassium in one hundred parts of water, and dilute the resulting liquid with water to the required strength.

135. *Silver Plating Solution.*—The following is the most practical method of making a large quantity of the ordinary cyanide of silver plating liquid:—Take four parts of grain silver, add it, in small portions at a time, to a warm mixture of about five parts by weight of strong commercial nitric acid (the acid varies greatly in strength), and one part of water, contained either in a glass or stoneware vessel. Gas will be evolved from the surfaces of the pieces of silver, and brown fumes of nitrous acid will arise from the mixture, which should be conveyed out of the apartment by means of the chimney. The action should be maintained moderate and uniform, and if it should become too strong, a little cold water should be added, and the mixture kept more cool; when the whole of the metal is dissolved, apply a greater heat, and evaporate the solution nearly to dryness, which will drive off any excess of acid that may be present; the resulting salt, nitrate of silver, may then be dissolved in a large quantity of water, in the proportion of half a gallon (more or less) to each ounce of the silver used; at the same time a solution should be made of from 3 to 3½ parts (according to its quality) of cyanide of potassium, in 30 or 40 parts of water, which is to be added gradually to the solution of nitrate of silver as long as it produces a precipitate; if too much be added, it will cause some of the precipitate to redissolve and be wasted; it will also make the liquid appear clear where it passes: in such a case the liquid should be stirred, then allowed to settle clear, and a small quantity of nitrate of silver, dissolved in *distilled* water, should be added as long as it produces a white cloud. By conducting the operation in a glass vessel, adding the liquid towards the latter period in small quantities at a time, and at intervals of a few minutes each, with gentle stirring immediately upon each addition, carefully observing when it ceases to produce a precipitate, the point of neutralisation may be very accurately determined. The liquid must now be allowed to remain undisturbed until quite clear, the clear portion poured steadily away from the precipitate of cyanide of silver, and the precipitate washed five or six times in a large quantity of water, by simply adding the water briskly to it, allowing it to settle, and then pouring off the clear portion. Next dissolve from 3 to 3½ parts (according to its quality) of cyanide of potassium in 20 parts of water, adding it, in portions at a time, to the wet cyanide of silver, with free stirring, until the whole is dissolved; then add about 3 parts more of cyanide of potassium to form *free* cyanide, and sufficient water to reduce the whole to the proportion of about one ounce of silver to the gallon: finally, when all the free cyanide is dissolved, filter the solution through a piece of unglazed calico. On the small scale, distilled water is used in the various parts of the process, except the washing; but, on the large scale, clean rain-water or spring-water is used in all the operations.

136. *Another Solution.*—The cyanide of silver plating solution may be made by other modifications of the chemical method than the one described: for instance, some electrotypists make the solutions by dissolving oxide, carbonate, or even chloride of silver in a solution of cyanide of potassium to saturation, and then adding an amount of free cyanide; by this process we are enabled to use caustic potash, carbonate of potash, hydrochloric acid, or common salt, instead of cyanide of potassium, for precipitating the nitrate of silver; nevertheless it still requires two equivalents of cyanide of potassium to be used as before—viz., one to convert the salt of silver into cyanide, and the other to dissolve the cyanide of silver formed; because, in all such cases, according to the researches of Messrs. Glassford and Napier,* when any salt of silver is added to a solution of cyanide of potassium, it is first converted into cyanide of silver at the expense of one portion of the cyanide of potassium; it then combines with the remaining cyanide to form double cyanide of silver and potassium, which dissolves in the water: therefore, by this modification of the chemical method, no cyanide of potassium is saved, and the carbonate of potash, hydrochloric acid, &c., are wasted. This modification has a still greater disadvantage; it introduces substances into the depositing liquid which are injurious. We have before said (90) that a good depositing solution should dissolve the anode freely, hold abundance of metal in solution, and not act chemically upon base metals, because it is such metals we generally wish to coat. Now, if instead of cyanide of silver, we add oxide of silver to the cyanide of potassium liquid, it converts part of the cyanide into caustic potash; if we add carbonate of silver, it converts it into carbonate of potash; and if chloride of silver, it converts it into chloride of potassium; and each of these substances, especially the last, diminishes the action of the liquid upon the dissolving plate, decreases its solvent power for cyanide of silver, makes its particles less mobile, and causes it to act in some degree upon base metals, and thus endangers the adhesion of the deposits upon them. Some electro-platers think the presence of these salts not injurious, but many consider them highly detrimental.

137. *Solution by Battery Process.*—The same silver solution may be formed by the battery process (89) as well as by the chemical method; and this process has its advantages and disadvantages: it is very convenient in making a small quantity of liquid, because it enables the operator to do so quickly, to avoid the trouble of making the nitrate solution, of precipitation, of washing, and of the attendant risk of loss of materials; but it has the disadvantage of converting a large proportion of the cyanide of potassium into caustic potash, by taking its cyanogen to form cyanide of silver, and setting the potassium free, which immediately combines with the oxygen of the water, forming caustic potash, which dissolves in the liquid. The hydrogen of the water is evolved at the cathode, and the dissolved potash gradually becomes converted into carbonate of potash by absorption of carbonic acid from the atmosphere. Neither caustic potash nor carbonate of potash are so injurious in the liquid as chloride of potassium; still they diminish the action of the liquid upon the dissolving plate, render it a worse conductor, reduce its solvent power for cyanide of silver, and make its particles less mobile.

138. *Solid Deposition of Silver.*—Mr. Alexander Parkes took out a patent, March 29th, 1841, for improvements in the solid deposition of silver. He converts an ounce of silver into oxide of silver, by first dissolving it in

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* *Philosophical Magazine*, 1844.

nitric acid, and then precipitating it by caustic potash; he then dissolves the oxide, together with sixteen ounces of cyanide of potassium, in two gallons of water, and uses the resulting liquid for depositing solid articles in silver.

139. Mr. Edmund Tuck took out a patent, June 4th, 1842, for "improvements in depositing silver upon German silver." For plating the commoner qualities of this alloy, he uses a solution composed of sulphate of silver dissolved in a solution of carbonate of ammonia; and for the best quality, he uses cyanide of silver dissolved in a solution of carbonate of ammonia. The solutions are formed by dissolving 70 parts of carbonate of ammonia in distilled water, then adding 156 parts of sulphate of silver, or 134 parts of cyanide of silver, and boiling the liquid until the salt is dissolved: for coating common German silver, he adds half an ounce of sulphate of silver to 107 grains of bicarbonate of ammonia.

140. For depositing purposes, a solution composed of water 20 parts, cyanide of potassium four parts, and acetate of silver one part, conducts very freely, and yields a fine white deposit of silver. A solution composed of water 25 parts, prussic acid 65 parts, "black" cyanide of potassium 12 parts, and cyanide of silver 10 parts, is also a very good one.

141. Many electro-platers use a cyanide solution containing about half an ounce of silver to the gallon, and add a very large proportion of free cyanide to make it conduct freely: such a solution has the advantage of being comparatively inexpensive in its first formation, quick in working, and yields metal of an average character; but it is rather difficult to manage in hot weather, and dissolves the anode very rapidly, on account of the large proportion of free cyanide. In practice, the amount of silver to the gallon varies from half an ounce to about four ounces, but ordinary solutions contain about one or two ounces; the amount of *free* cyanide of potassium also varies from about half the weight of the silver dissolved in the liquid to five or ten times this quantity: a very good proportion is about three-fourths of the weight of the dissolved silver; but there is no rule generally recognised in the trade upon this point; some manufacturers use a very large and others a very small proportion.

142. A good plating liquid should contain one equivalent (65 parts) of pure cyanide of potassium, and one equivalent (134 parts) of cyanide of silver, besides *free* cyanide, and sufficient water to form a thin liquid. It is necessary to have *free* cyanide, because, in working the solution, insoluble cyanide of silver is formed, and requires *free* cyanide of potassium to combine with it and form the soluble double cyanide; at the same time, cyanogen and cyanide of potassium are set free at the cathode or receiving surface by the deposition of the silver; and as it requires some time for those substances to mix with the liquid and reach the dissolving plate, *free* cyanide must be provided. The necessity of having sufficient water to form a *thin* liquid, arises from the double cyanide formed at the dissolving plate, being specifically heavier than the liquid, having a tendency to sink to the bottom; whilst the cyanogen and cyanide of potassium set free at the surface of the articles, being specifically lighter, tend to rise to the surface. At the same time, each of them mixes more or less with the surrounding liquid by capillary attraction or adhesion; and the more dilute the liquid is, the more mobile are its particles, and the more rapidly does this mixture take place. This explains why strong silver solutions require more frequent stirring than weak ones to keep them uniform. In some manufactories, where they have steam-power at command, the articles are kept in constant motion by machinery swinging them gently to and fro; but in ordinary electro-plating establishments, the silver solutions are stirred every evening.

143. If a solution contain but little water and a large supply of free cyanide, and, from any cause, the battery current become suddenly weak towards the evening, the silver deposited upon the articles will be redissolved, in consequence of the liquid about the dissolving sheets having, by the day's work, become saturated with silver, and that about the articles become full of free cyanide; the two electrodes (*i.e.*, the dissolving plates and the articles) form a kind of voltaic battery (one metal in two liquids), which develops a current of electricity in an opposite direction to the original one, and thus redissolves the deposited silver.

144. *Bright Silver Solution.*—Much practical interest was for a long time attached to the anticipated discovery of a solution by which silver might be deposited in a *bright* condition, and the labour of burnishing be thereby avoided or lessened. This discovery was at last effected as follows:—Some operators at the electro-plate works of Messrs. Elkington and Mason, Birmingham, were engaged in experiments on moulds containing bisulphide of carbon: whilst these moulds were being coated with silver in the depositing vats, very peculiar appearances upon the various articles receiving a deposit in the vat were noticed, some having very bright patches upon them like burnished metal. From the known presence of bisulphide of carbon, experiments were tried of adding that substance to a quantity of silvering liquid, which ultimately resulted in success, and a patent was taken out by Messrs. Lyons and Millward, March 23rd, 1847; in which they give the following instructions for forming a "bright solution:"—"Add to the usual solution of silver in cyanide of potassium, bisulphide of carbon, terchloride or other chloride of carbon, sesquichloride of sulphur, or hyposulphite of either potash or soda. The bisulphide of carbon may be used alone or dissolved in sulphuric ether; or it may be used in conjunction with any of the other substances mentioned above; but the patentees prefer using it as follows: six ounces of bisulphide of carbon are put into a stoppered bottle, and one gallon of the usual plating liquid added to it; the mixture is then shaken and set aside for twenty-four hours; two ounces of the resulting solution are then added to every twenty gallons of the ordinary plating solution in the vat, and the whole stirred together: this proportion must be added every day, on account of the loss by evaporation; but when the mixture has been made several days, less than this proportion may be used at a time; when hydrocarbons are used instead of the bisulphide, a much larger quantity must be added. This proportion gives a bright deposit; but, by adding a larger proportion, a dead surface may be obtained, very different to the ordinary dead surface. This substance is generally employed throughout the trade, although few are licensed to use it. Other compounds are also used, but to a very limited extent: among these are sulphur and collodion. A solution of iodine and gutta-percha in chloroform is said to be more permanent in its effect than the bisulphide of carbon. The liquid is generally added to the vat in the evening, after the work has been taken out. A method of bright *gilding* has also been recently brought into use in the trade.

145. *Method of Making Cyanide of Potassium.*—As nearly all the solutions which are used for electro-silvering and gilding contain cyanide of potassium, and as this substance is used extensively in electro-deposition generally, it will be necessary for the practical plater to understand how it is made, and to possess information respecting its impurities, and the method of testing its quality. It is nearly always made by the following process:—Take ferro-cyanide of potassium (yellow prussiate of potash), pound it fine, and gently heat it in an iron pan, with constant stirring until quite dry; treat a quantity of the best quality of carbonate of potash in a similar manner. When they are perfectly dry, add about three parts of the carbonate to eight parts of the ferro-cyanide, and thoroughly mix them; heat the mixture rapidly in an iron ladle or crucible, until it melts into a clear liquid, when gas will be evolved from its surface. It should be maintained at a moderate or dull red heat about fifteen or twenty minutes, or until the end of a cold iron rod dipped into it shows a white sample. The fusion should not be continued until the evolution of gas ceases, or the product will be of a gray

colour. It should be kept covered as much as possible. By allowing it to stand undisturbed a few minutes towards the latter part of the operation, and occasionally tapping the sides of the ladle or crucible, the iron of the ferro-cyanide will settle at the bottom as a fine black powder; the colourless cyanide of potassium may then be poured off into a cold iron pan, or upon a thick and cold iron plate; it should be broken up whilst still warm, and preserved in a well-stopped jar.

The black sediment, which contains much cyanide of potassium, should be scraped out of the vessel while still hot, and preserved, as water will at any time dissolve the cyanide. If the process be well conducted, the product will be of a clear white colour, or, at most, but very slightly gray. A larger proportion of cyanide of potassium is obtained by this process than when ferro-cyanide alone is employed, because, in the former case, one-third of the cyanogen (that which was combined with the iron) combines with the potassium of the carbonate of potash, whilst in the latter case it is lost; the cyanide produced by the fusion of the ferro-cyanide of potassium alone is of a grayish-black colour, and is termed "black cyanide."

46. *Cyanide of Potassium.*—Commercial cyanide of potassium varies very much in price and quality. By dissolving several specimens of commercial black cyanide in water, and filtering the solution, we found the proportion of black impurity in them varied from one-fourth to one-sixth of their weight; and by experiments with the commercial white cyanide, we found that one part (200 grains) of it dissolved in about one-and-a-quarter parts (230 grains) of distilled water at 60° Fah., and that it dissolved much more freely in water containing hydrocyanic acid.

147. *Impurities in Cyanide of Potassium.*—According to the researches of Messrs. Glassford and Napier,[*] commercial white cyanide generally contains about thirty-five per cent. of impurities, and often as much as fifty per cent., in the form of carbonate and sulphate of potash, chloride of potassium, cyanate of potash, ferro-cyanide of potassium, and silica; and if the mixture of salts from which it is made be not dry, ammoniacal compounds will be also formed. The sulphate of potash and chloride of potassium occur in the commercial carbonate of potash; the silica is present when we operate with an earthen crucible: and even when the process is well conducted and pure materials used, the product contains twenty per cent. of cyanate of potash, produced partly by the contact of the air with the melted mixture.

148. *Testing Cyanide of Potassium.*—According to the same experimentalists, the quantity of pure cyanide in any given sample of cyanide of potassium may be correctly ascertained thus:—Make two solutions, one of the given cyanide and one of nitrate of silver, each containing known weights of the salts—say one ounce of the cyanide dissolved in six ounces of distilled water in a graduated glass vessel, and 175 grains of the crystallised nitrate dissolved in about two or three ounces of distilled water; add the cyanide solution carefully and slowly to the nitrate of silver liquid, until the precipitate first formed is all redissolved. The quantity of the cyanide solution required to effect this (with the above quantity of nitrate of silver) will have contained 130 grains of pure cyanide; and, from the quantity used, we may easily calculate the amount of pure cyanide in the whole ounce. It is said, that "when nitrate of silver is added to a solution of cyanide of potassium, so long as the precipitate formed is all redissolved, we obtain the *whole* of the cyanide of potassium in combination with the silver: none of the other salts in solution take any part in the action, even though they be present in a large proportion. This enables us to test the exact quantity of cyanide of potassium in any sample."

149. *Chemical Characters of Cyanide of Silver.*—In the presence of cyanide of potassium, as we are informed by Messrs. Glassford and Napier, cyanogen has a greater affinity for silver than anything else has—decomposing every salt of silver except the sulphide, and forming cyanide of silver. In dissolving the oxide, carbonate, chloride, or ferro-cyanide of silver, in a solution of cyanide of potassium, they are all decomposed, and cyanide of silver always formed. Cyanide of silver should be dried below 260° Fah.: hydrochloric acid decomposes it with a solution of hydrocyanic acid gas; cold nitric acid has no action upon it; a boiling mixture of sulphuric acid and water decomposes it, with escape of hydrocyanic acid gas, and formation of sulphate of silver; it is soluble in the alkaline chlorides, but its best solvent is an aqueous solution of cyanide of potassium, of each of which one equivalent is required. The resulting solution, when evaporated, yields crystals of double cyanide of silver and potassium, which are soluble in eight parts of cold and in one part of boiling-water. The solution of this double salt, which is nearly the same as the ordinary plating solution, may be boiled for any length of time without being decomposed, and it is very little affected by light; it is decomposed by all acids, and they precipitate the silver as cyanide of silver: the hydro-acids—hydrochloric acid, for example—decompose the cyanide of silver also; sulphuretted hydrogen precipitates the silver as sulphide of silver.

150. *Gold Solution.*—Various salts of gold have been used for electro-deposition; among which are the hyposulphite, sulphite, iodide, bromide, terchloride, cyanide, and sulpho-cyanide. Finely-divided gold, which is sometimes used for dissolving, may be obtained by adding a solution of protosulphate of iron to a solution of terchloride of gold, as long as a greenish-brown precipitate occurs; this is gold in a state of very minute division. Oxide of gold is obtained by adding to a solution of terchloride of gold a cold solution of caustic potash, until it ceases to produce a precipitate; or by digesting terchloride of gold with magnesia, washing the fresh precipitate, first with dilute nitric acid, and then with water only. Iodide of gold is formed, either by digesting oxide of gold in hydriodic acid, or by adding a solution of iodide of potassium to a solution of terchloride of gold as long as a precipitate is produced—washing the precipitate with water; it is of a yellow colour, insoluble in cold water, but freely soluble in a solution of iodide of potassium. Bromide of gold may be formed either by digesting finely-divided gold, or oxide of gold, in liquid bromine contained in a stoppered bottle. It is a salt of a rich red colour, and is soluble in water.

151. *Chloride of Gold.*—Terchloride of gold, commonly known as chloride of gold, is the most usual salt of the metal, and its preparation requires separate explanation. It is formed as follows:—Take a mixture of two or three measures of hydrochloric acid, and one measure of nitric acid; make the mixture hot, and add pure grain gold to it as long as it will dissolve: it evolves gas whilst dissolving. When it is all dissolved, slowly evaporate the liquid until it crystallises to a dark ruby-red mass; or it may be of a yellow colour, according to the proportions of the ingredients. This is terchloride of gold, and contains one equivalent (197 parts) of gold, and three equivalents (106·5 parts) of chlorine; if it be too much evaporated, chlorine gas will be evolved, the gold will be set free and be mixed with the salt, and will precipitate on dissolving the salt in water. To produce the *yellow* chloride, mix together, in a glass or stoneware vessel, one part, by weight, of nitric acid, three parts of hydrochloric acid, one part of fine gold, and one part of water: cover the vessel with a glass dish, make the liquid quite hot, and maintain the heat until the red vapours cease; if some of the gold remain undissolved, add more of the liquid mixture, and treat as before: when the vapours cease, remove the glass cover, and replace it by folds of blotting-paper, and evaporate until it crystallises, on *cooling*, into yellow chloride of gold. The red chloride is formed in the same manner, except that the liquid mixture should be composed of one part of nitric, and two parts of hydrochloric acid, more being added than is necessary to dissolve all the gold. One ounce of gold will dissolve in

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[*] *Philosophical Magazine*, 1844.

about four ounces of the mixture, and when crystallised into the red mass, will weigh about one ounce and 165 grains.

152. *Cyanide of Gold.*—"Cyanide of gold is formed by cautiously adding a solution of cyanide of potassium in six parts of water, to a normal solution (*i.e.*, not containing any *free* acids) of terchloride of gold, consisting of one part of the chloride and five parts of water, until a copious yellow precipitate settles down; if more cyanide of potassium be added, the precipitate becomes dirty yellow, and is more quickly deposited; a still larger quantity renders it orange yellow. It is a crystalline powder, permanent in the air. By ignition it is resolved into gold and cyanogen gas; it is not decomposed by sulphuric, hydrochloric, or nitric acids, nor by aqua regia, unless freshly precipitated, and then only slowly. It is not decomposed by sulphuretted hydrogen; hydrosulphate of ammonia dissolves it slowly but completely, forming a colourless solution, from which, by the addition of acids, sulphide of gold is precipitated. It dissolves in aqueous solutions of ammonia, hyposulphite of soda, or alkaline cyanides; but not in water, alcohol, or ether."

153. "Gold precipitated from a solution of chloride of gold by protosulphate of iron, dissolves in a boiling solution of cyanide of potassium; a hot solution of cyanide of potassium will also dissolve ordinary metallic gold if air be present. Both oxide of gold and aurate of ammonia, dissolve completely in a solution of cyanide of potassium, and form double cyanide of gold and potassium. Cyanide of gold requires 23 parts of a solution of cyanide of potassium in water to dissolve it. For every one part of gold to be dissolved by the battery process, a solution of six parts of cyanide of potassium, in from two to four times their quantity of water, at 100° Fah., is required; two electrodes of gold, connected with a suitable battery, are to be immersed in it until the required quantity of gold is dissolved."—"The crystallised cyanide of gold and potassium dissolves in seven parts of cold, and in half a part of hot water;[*] in four parts of cold and in 0·8 parts of hot water.[†] It dissolves very sparingly in alcohol. Its aqueous solution gilds copper and silver by simple immersion, especially if hot, and those metals are dissolved."

154. *Gilding by Immersion.*—The following solutions have been used for gilding by the simple immersion, or "water-gilding" process:—First, dissolve five troy ounces of grain gold in fifty-two avoirdupois ounces of hot aqua regia, until vapours cease to be evolved. Decant the clear liquid when cool; dilute it with four gallons of distilled water; add twenty pounds of purified bicarbonate of potash, and boil it for two hours. The articles to be gilt are immersed in the liquid from a few seconds to one minute, according to the kind of metal immersed, and the temperature and newness of the liquid; warmth assisting the action, and a new liquid acting more quickly than an old one. Second: for gilding silver articles, dissolve equal parts of bichloride of mercury (corrosive sublimate) and sal-ammoniac in nitric acid; add pure gold to it, and evaporate the liquid by heat to half its volume. Apply the liquid, while hot, to the surface of the article.

155. Joseph Steele's patent, dated August 9th, 1855.—Dissolve an ounce of gold in a mixture of four ounces of hydrochloric acid and eight ounces of nitric acid, and evaporate the solution to dryness. Fuse together 24 ounces of ferro-cyanide of potassium, and 12 ounces of carbonate of potash: when this is nearly cold, dissolve in two or three gallons of pure boiling-water; cool and filter the solution; then add the chloride of gold, and boil for a quarter of an hour. The articles to be coated are connected with a piece of zinc, of suitable size, and immersed in the liquid, the latter being at a temperature of 80° or 85° Fah.

156. *Gold Solution for the Battery Process.*—For the battery process there are many gold solutions, though but few good ones:—1st. The hyposulphite of gold and soda—formed by dissolving chloride of gold in a solution of hyposulphite of soda; which is not considered a good liquid for practical purposes. 2nd. The sulphite of gold and potash, used by Mr. Woolrich: his solution is made by adding sulphite of potash to water, saturating five-sixths of the resulting liquid with oxide of gold, and then adding the other portion of solution to form *free* sulphite. 3rd. The terchloride of gold dissolved in water is a very inferior liquid for practical purposes, because all the common metals decompose it. 4th. The bromide of gold, proposed by Mr. Spencer, is made as follows:—"Make a mixture of equal parts of bromine and alcohol; and of this mixture take one part, of acetic acid one part, and four parts of water containing a few drops of sulphuric acid." The resulting liquid is then nearly saturated with gold by suspending in it two electrodes of that metal, and connecting them a sufficient time with a suitable battery; when nearly saturated, add to the solution three times its volume of water containing a few drops of sulphuric acid.

157. *Electro-Gilding Liquid.*—The best liquid that has yet been tried for practical electro-gilding, consists of the double cyanide of gold and potassium dissolved in water. It was first patented by Messrs. Elkington, and may be formed, either by dissolving finely-divided gold, or any salt of gold, in a solution of cyanide of potassium; or by the battery process, by suspending two electrodes of gold in a solution of cyanide of potassium, and passing a current from a small battery until the cathode receives a proper deposit, the liquid being at about 100° or 150° Fah.

158. *Cyanide of Gold by Chemical Process.*—The cyanide of gold and potassium gilding solution is often made by the chemical method as follows:—Form some terchloride of gold (151), and dissolve it in water; then either add a cold solution of caustic potash as long as a precipitate is produced, filtering and washing the precipitate with distilled water, or by digesting the chloride solution with magnesia. Filter, and wash the precipitate first with nitric acid, and then with distilled water; or add to the chloride liquid a solution of carbonate of ammonia, until a precipitate ceases to be formed; filter, and wash the precipitate with water The precipitates produced by potash or by magnesia, consist of oxide of gold; whilst those produced by ammonia, or its carbonate, are aurate of ammonia (fulminate of gold), which is a very explosive compound. The precipitate, after being well washed by the successive additions of clean water, should be added, whilst still wet, to a solution of cyanide of potassium, containing the proportion of one pound of cyanide to one gallon of water; and then about one-fifth more of the same solution should be added to form *free* cyanide. A very good proportion of the ingredients is, one ounce of gold, one pound of cyanide of potassium, and one gallon of water. The wash-waters should not be thrown away without being tested for gold, by immersing a piece of bright zinc in them, and observing if it receive a yellow deposit; in that case, a solution of protosulphate of iron should be added as long as a precipitate of a greenish-brown powder, which is metallic gold, is produced. If this fail to precipitate the whole of the gold, a sheet of bright zinc should be immersed in the liquid, taken out occasionally, and the deposit of gold brushed off by a hard brush in water containing a little sulphuric acid. The greater the quantity of *free* acid contained in the original chloride solution, and the larger the excess of potash, ammonia, or carbonate of ammonia added, the greater is the amount of gold dissolved in the wash-waters. If, when we dissolve the terchloride of gold in water, a yellow powder remain undissolved at the bottom of the vessel, it indicates that there is no free acid in the salt; and it may be redissolved by the addition of a small quantity of the mixture of nitric and hydrochloric acids, and the application of heat.

159. *Solution by Battery Process.*—The same solution may be made by the battery process, thus:—Dissolve some cyanide of potassium in hot distilled water, in the proportion of from one to two pounds to the gallon; nearly fill a small porous cell with the liquid, and immerse it nearly to its edge in the solution; place a large gold anode in the outer liquid, and a small bright copper cathode in the liquid of the porous cell, and

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[*] Himly.

[†] Glassford and Napier.

connect them with about three pairs of Smee's batteries—the gold anode with the platinised silver by one wire, and the copper cathode with the zinc by another wire, allowing the current to pass, until, by transferring the cathode for a short time to the outer liquid, it receives a good deposit of gold, the solution being maintained at a temperature of about 150° Fah.; the liquid of the porous cell should then be transferred to the outer solution, and the process stopped. The amount of gold dissolved is not of material consequence provided the deposit be good, as a solution may contain from half an ounce to four ounces to the gallon, and be a good depositing liquid.

160. *Gilding Solution of M. Ruolz.*—"Dissolve ten parts of cyanide of potassium in 100 parts of distilled water; filter it, and add one part of cyanide of gold, prepared with care, well washed, and dried out of the influence of light; keep the mixture in a closed glass vessel, at the temperature of 60° to 77° Fah., for two or three days, out of the presence of light, and frequently stir it."

161. *M. Becquerel's Gilding Liquid.*—"Dissolve one part of terchloride of gold and ten parts of ferro-cyanide of potassium in 100 parts of water; filter the liquid, to remove the separated cyanide of iron; add 100 parts of a saturated solution of ferro-cyanide of potassium, and dilute the mixture with one or two volumes of water. In general, the tone of the gilding varies according as the solution is more or less dilute; the colour is most beautiful when the liquid is most dilute, and most free from iron. To make the surface bright, it is sufficient to wash the article in water, acidulated with sulphuric acid, rubbing it gently with a piece of linen cloth."

162. *Gilding Liquids of M. Fizeau.*—"1st. Dissolve one part of dry chloride of gold in 160 parts of distilled water; then add, little by little, a solution of carbonate of potash in distilled water until the liquid begins to become cloudy; we may use this liquid immediately." And, 2nd, used by M. Lerebour:—"Dissolve one gramme[*] of chloride of gold, and four grammes of hyposulphate of soda, in one litre of distilled water."

163. *M. Levol's Solution for Gilding Silver.*—"Dissolve neutral chloride of gold, then add an aqueous solution of sulpho-cyanide of potassium, until the precipitate first formed is redissolved. The liquid will retain a slightly acid reaction; if it lose this, it must be renewed by adding a few drops of hydrochloric acid."

164. *By M. de Briant.*—"Dissolve thirty-four grammes of gold in aqua regia, and evaporate the solution until it becomes neutral chloride of gold; then dissolve the chloride in four kilogrammes of warm water; and add to it 200 grammes of commercial magnesia, carefully sifted. The gold is precipitated in union with the magnesia; filter and wash with pure water; digest the precipitate in forty parts of water mixed with three parts of nitric acid to remove the magnesia; then wash the oxide of gold remaining with water, until the wash-water exhibits no acid reaction. Next dissolve 400 grammes of ferro-cyanide of potassium, and 100 grammes of caustic potash, in four litres of water; add the oxide of gold, and boil the solution about twenty minutes. When the gold is dissolved there remains a small amount of iron precipitated, which may be removed by filtration, and the liquid, of a fine gold-yellow colour, is ready for use. It may be used either hot or cold."

165. *Formulas of M. J. L.*—"1st. Take thirty-one grammes and twenty-five centigrammes of oxide of gold, five hectogrammes of cyanide of potassium, and four litres of water, and boil them together half-an-hour. The resulting solution must be worked hot, and may be used to gild copper, brass, and silver. 2nd. Dissolve ten parts of ferro-cyanide of potassium and one part of dry terchloride of gold in 100 parts of water; oxide of iron will be precipitated. Boil the solution two or three hours, in a porcelain or glass vessel, until a precipitate collects at the bottom and the supernatant liquid is transparent and of a canary-yellow colour; filter the solution, and dilute it with three times its volume of water."

166. A process or branch of trade, termed "solid depositing," has of late years been gradually extending itself. It consists in making solid articles of gold and silver, by electro-deposition, upon gutta-percha or other moulds—such, for instance, as watch and clock faces, ornamental snuff-boxes, and other articles elaborately chased or engraved, or which have very complex or under-cut ornaments upon them; the expense of multiplying these by the electro-process being less than by the ordinary means. Mr. Alexander Parkes took out a patent, dated March, 1841, for a solution for depositing solid articles in gold. It is formed thus:—Dissolve one ounce of pure gold in aqua regia, and evaporate the solution to dryness; then add two gallons of water and sixteen ounces of cyanide of potassium, and work the resulting liquid at a temperature of about 120° or 130° Fah.

167. *Salts of Platina.*—The only common salt of platina is the bichloride: it is formed by adding pieces of platina foil to hot aqua regia as long as gas is evolved from them; the solution, which is then of a deep-red colour, should be evaporated nearly to dryness, and left to cool.

168. *Platina Solutions.*—For platinising silver by simple immersion process, we may use a solution consisting of bichloride of platina, dissolved in water containing one-fourth its volume of nitric acid, or we may use simply a very hot aqueous solution of the bichloride alone. Nearly all metals decompose the bichloride solution, and become coated with platina in it by simple immersion. For the battery process we may use solutions of the iodide, bromide, or bichloride, or the double chloride of platina and sodium. The solution of the double chloride of platina and sodium is made by dissolving one equivalent (169·7 parts) of bichloride of platina and one equivalent (58·5 parts) of common salt, in water; it requires a small anode of platina and a very weak battery to obtain a reguline deposit. A good solution for depositing reguline platina may also be made by dissolving bichloride of platina and common salt in a solution of caustic potash.

169. *Palladium Solutions.*—A solution of double cyanide of palladium and potassium may be used for depositing palladium. It may be made by chemical means, by dissolving palladium in nitric acid, precipitating the solution by a solution of cyanide of potassium, washing the precipitate, and dissolving it in a solution of cyanide of potassium to saturation, and then adding a little free cyanide; or it may be easily made by the battery process, by passing a current through a large palladium anode in a solution of cyanide of potassium, until a clean smooth cathode receives a good deposit. This is an excellent solution for depositing reguline metal; it acts upon the anode with uncommon energy, conducts freely, and deposits plenty of reguline metal: a thin deposit of palladium obtained in this solution has been used for fixing Daguerreotype pictures, instead of gold, and is said to give them a finer tone.

170. *Selecting Practical Liquids.*—Having enumerated nearly all the solutions which have been used for depositing different metals, and described the modes of forming them, we now offer a selection of those which are in most general use in the trade.

1st. For depositing zinc, which is not very often attempted, the sulphate solution (105) may be used.

2nd. For depositing copper upon all ordinary metals (except zinc, tin, lead, iron, and steel), and upon gutta-percha, wax, and elastic moulds, after being made

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[*] The following table of some French weights, &c., may be of use to the student. We may observe, that French weights and measures have an invariable relation to the *metre*, which is equal to about 39 inches British. A gramme, which is the unit of French weights, is equal to about 15½ grains British, and represents the weight of a cube of water at 40° Fah., the sides of which measure the 1-100th of a metre each.

FRENCH.	BRITISH.
The Gramme	15½ grains nearly.
,, Kilogramme (1,000 grammes)	2⅕ lbs. ,,
,, Litre (1 kilogramme)	2⅕ ,, ,,

This being the unit of French liquid measure, equals the weight of a cube of water, each of the sides of which measures the tenth part of a metre.

The Centigramme is the one-hundredth part of a gramme.
The Hectogramme is equal to one hundred grammes.

conductible by the battery process, the sulphate solution (120) is in general use; but for surfaces of tin, lead, iron, or steel, the solution of cyanide of copper and cyanide of potassium (120) is used.

3rd. For the deposition of brass upon all ordinary metals, the patented solution of Messrs. Morriss and Johnson (127) is very successfully used; the solutions of Brunel (126) and Salzede (125) are also in practical use; and, in the United States of America, a solution very similar to that of Messrs. Morriss and Co. is employed.

4th. For depositing silver upon all common metals, the solution of double cyanide of silver and potassium (135) is almost universally used; the only exception is where the double sulphite of silver and potash is used, but only to a small extent, the magneto-machine being in some cases employed.

5th. For gilding all ordinary metals, almost the only liquid used is the double cyanide of gold and potassium (157 and 158); the only exceptions being a few minor modifications for operations on the small scale.

6th. For platinising, the only solution in practical use is the bichloride (167).

7th. For *solid* deposition of silver and gold, the patented solutions of Mr. Alexander Parkes (138 and 166) may be employed.

171. *Preparing Metal for Receiving a Deposit.*—The electrotypist having acquired a general knowledge of depositing liquids, as well as of the necessary manipulation, and having made the several solutions necessary for his ordinary use, will require to prepare his articles for receiving a deposit. All metallic articles must be cleaned, and otherwise prepared, before they are fit to receive a deposit; this preparation differs, of course, according to the nature of the article, whether the deposit be required to adhere firmly to the surface, or merely to be used as a matrix, from which the deposited metal is to be removed.

172. *Cast Iron and Zinc.*—Articles formed of zinc, wrought-iron, cast-iron, or steel, are first immersed for a few minutes in a boiling solution of caustic potash, to remove any grease, tar, or oily substance which may be upon them; they are then washed in water; and those of wrought or cast-iron are immersed in "pickling liquid" (82), until the acid acts rather freely upon them; rough cast-iron requires a stronger liquid than smooth wrought-iron: after being again washed in water, they are "scratched" at the brush (80). If they are very coarse castings or rusty articles, they may require several soakings in the dilute acid, and scourings with sand and a harsh brush, and even filing, to make them quite clean.

173. *Copper, Brass, and German Silver.*—Those formed of copper, brass, or German silver, should be boiled in the potash liquid, washed in water, and dipped into nitric acid, or into the "dipping liquid" (82). They are then to be washed in water, and dipped into a solution of nitrate or cyanide of mercury (84), before immersing them in the silvering solution.

174. Sometimes, in order to assist in cleaning the articles, they are suspended for a short time in the depositing liquid, in contact with the negative pole of the battery; this dissolves the surface, and loosens their impurities, unless they are very foul, or the solution is too valuable. In every case they should be well rinsed with water to remove the adhering acid, before dipping them into the mercury solution, or immersing them in the depositing vat. All objects which are to have a definite weight of metal deposited upon them, are weighed, and their weight noted down after they have been cleaned.

175. *Wiring Articles.*—The articles having been cleaned thoroughly, have wires of copper attached to them, to suspend them in the vat. The wires differ in size with different articles; with small ones, such as spoons, knives, forks, snuffers, teapots, jugs, and such articles, size No. 20 or 22 of the Birmingham brass wire gauge, and about eighteen or twenty inches long, are used. Very large articles, such as fire-irons, fenders, hat-stands, and articles of ornamental iron-work, are suspended in the solution by strong copper or brass hooks; in some cases where a powerful and certain connection is required, the wires are soldered to the articles.

176. *Preparing Articles for Adhesive Deposits.*—We have already explained how necessary it is that all articles intended for the depositing vat should be cleaned in the most perfect manner possible, before being immersed in the depositing liquid, otherwise the deposit will not adhere. Articles of copper, brass, or German silver, which are to be silver-plated, should also be dipped into one of the solutions of mercury (84); otherwise the deposit will not adhere at all, or will vary in appearance in different parts; and in consequence of this perfect degree of cleanliness required, the cleaning of them often involves more trouble than the depositing. All articles should be plunged, while still wet from the cleaning process, into the depositing vat. The practical minutiæ of preparing the surfaces of different metals for receiving adhesive deposits of other metals, vary in almost every manufactory, and much information yet remains to be developed upon this point For want of this knowledge, the most skilful operators sometimes fail in producing perfect adhesion, especially upon zinc, cast-iron, steel, and Britannia metal.

177. *Preparing Metals for Non-adhesive Deposits.*—Metal articles which are to receive non-adhesive deposits, such as medallions, of which copies are desired in copper, should be allowed to remain a sufficient time to slightly oxidate after being cleaned, before being plunged in the depositing vat, the oxide preventing adhesion. In some cases they are rubbed over with cotton wool slightly moistened with a very weak solution of bees-wax dissolved in camphine, in the proportion of a piece of wax of the size of a small pea, in a quarter of a pint of the spirit; others use a little sweet oil, which is immediately wiped off with a fresh piece of dry cotton wool.

178. *"Stopping off" to Prevent Deposition.*—Many articles which are to receive deposits require to have portions of their surface "stopped off," to prevent the deposit spreading over those parts: for instance, in taking a copy of one side of a metal medallion, the opposite side must be coated with some kind of varnish, wax, or fat, to prevent deposition; or in gilding the inside of a cream jug which has been silvered on the outside, varnish must be applied all round the edge on the outside for the same reason. For gilding and other hot solutions, copal varnish is generally used; but for cold liquids and common work, an ordinary varnish, such as engravers use for a similar purpose, will do very well. In the absence of other substances, a coating of sealing-wax dissolved in naphtha will answer every purpose.

179. *Moulding and Copying Works of Art.*—The electrotypist who includes in his business the multiplication of works of art, as well as the simple plating of metal articles, will require a knowledge of the art of moulding. To copy both sides of a metallic coin or medal in the mixture of gutta-percha and marine glue recommended (85), take a strip of thin sheet copper, brass, or tinned iron, about an inch wide; wind it closely round the edge of the medal, and solder its ends together; wipe the medal and take two balls of the composition, quite hot and soft, and press them simultaneously against the two faces of the medal, working the material from the centre towards the circumference to exclude bubbles of air; place two thick plates of cold metal, one on each side, and gradually screw up the whole in a vice, or screw-press, gently at first, but increasing the pressure to a high degree as the materials become hard. When it is quite cold, which will be in about two hours, the two copies may be easily removed from the original, by inserting the end of a gimlet in their backs, and drawing them out; they are easily removed, because the composition slightly contracts in cooling. They will present fine impressions of the original, and be perfectly free from air-bubbles, when the operation has been carefully performed.

180. *Elastic Moulding.*—If the medallion be under-cut, it must be copied in "elastic moulding" (86), thus :—

Encircle its edge by a strip of stout paper, and pour the mixture upon its surface quite hot, and of the consistency of treacle, to the depth of half an inch or more, according to the size of the medal and the depth of its hollow parts, brushing its surface beneath the liquid with a brush having fine and long hairs, to remove air-bubbles. Allow the mixture to remain until it is quite firm, which will be from two to twenty-four hours, according to its bulk; take off the paper, and remove the mould very gently, carefully stretching and drawing it at the same time in the direction of the overhanging parts, to prevent injury. Should the object to be copied be a hollow metallic bust, proceed as follows:—Partly fill it with sand, to make it heavy and thus prevent its rising in the liquid, and cover its opening by sticking a piece of millboard strongly over it; then place the bust in the centre of a cylindrical and tapering vessel, a few inches deeper and wider than itself, and pour the melted composition in steadily, until it is a few inches above the top of the head, tapping the bust, and inclining the outer vessel, to facilitate the escape of air-bubbles. When the composition is quite firm, which it will be in about twenty hours, it may easily be removed from the vessel by shaking, the vessel having been previously well oiled; the mould may then be removed from the bust by previously marking on its lower end the position of the face, passing a knife carefully up the back of the bust nearly to the crown of the head, and opening the elastic mould with the hands, whilst a second person lifts out the bust. If the original bust be composed of plaster, it must be previously saturated with oil, to prevent the melted composition adhering to it.

181. *Rendering Moulds Conductible.*—To render the surfaces of non-metallic moulds conductible, there are two methods in use: first, to cover them with a thin film of blacklead by brushing; and, second, to coat them with a minute film of gold or silver by chemical means. The first of these methods is generally used for moulds composed of gutta-percha, wax, resinous composition, or plaster saturated with oil, where the parts are not much under-cut; and the second for elastic moulds, because the blacklead cannot be readily applied to all their recesses.

182. *Blackleading.*—To apply blacklead to a small round or oval medallion formed of gutta-percha and marine glue, first insert the sharp end of a piece of copper wire, size No. 16 or 18, and about fifteen or twenty inches long, into the edge of the mould near its face; then pass a piece of fine copper wire, size No. 28 or 30, once tightly round the edge of the mould, close to its face, securing its ends to the other wire. Fix a strip of paper about one inch wide, by means of sealing-wax, tightly round the edge, to prevent the blacklead passing anywhere except upon the face of the mould. Apply the blacklead by a soft camel's-hair brush with a large and thick body of short hairs, breathing upon the face of the mould occasionally to facilitate the adhesion of the material. When the medal is perfectly black and bright, blow off the superfluous blacklead, and remove the paper; it is then ready for receiving a deposit, the whole operation occupying about ten or fifteen minutes with a small object the first time of preparing it, but less in subsequent operations. If the mould be very large, and especially if it have deep hollows in its surface, it will require, after being blackleaded, to have several short and fine copper "guiding-wires" carefully attached to the main wire, and their free ends slightly inserted in the face of the mould, in the most hollow and distant parts, or to lie in contact with them, in order to cause the deposit to commence and spread there as well as round the edge. If this precaution be not taken, the deposit will be much thinner over those parts than upon the nearer and more prominent places, and sometimes will not spread over them at all.

183 *Preparing the Surfaces of Elastic Moulds.*—Elastic moulds are treated in a different manner. First, a stout connecting wire is attached; then a number of fine copper "guiding-wires" are twisted round it, and their free ends slightly inserted in the face of the mould, in all the hollow and distant parts; the mould is then either dipped into the phosphorus solution (87), or its surface is covered with that liquid; and, after it has been drained clean, it is allowed to remain until perfectly dry; the silver solution is next applied to it (87), in like manner, for several minutes, until it appears black, with a metallic lustre like black china; it is then gently rinsed with distilled water, and the gold solution applied in the same way (87), which gives it a yellowish aspect; after another rinsing in distilled water, it is ready for receiving a deposit.

184. *Moulding by Phosphorus Composition.*—Some objects which are not much under-cut, are moulded in the phosphorus moulding composition (87); and, in some cases, where they are under-cut, as well as busts, they are first copied in the elastic moulding, and then the elastic mould is copied in this material, the composition being but barely melted that it may not dissolve the elastic moulding. In either of these cases, instead of immersing the mould in the phosphorus, silver, and gold solutions, it is only immersed in the two latter, in the manner already described, the phosphorus contained in the mould itself serving to reduce the silver and gold.

185. *Preparing Surfaces of Glass for Deposition.*—Glass surfaces may be prepared for receiving a deposit by means of the phosphorus, silver, and gold solutions, but not very satisfactorily. A better method, which we have tried, has been silvering them by Drayton's patent process, thus:—Take one part, by weight, of liquid ammonia, three parts of alcohol, two parts of nitrate of silver, and three parts of distilled water; dissolve the silver salt in the water, add the liquid ammonia and the alcohol, shake the mixture, allow it to remain until quite clear, and pour off the clear part into the glass vessel to be silvered, which must be perfectly clean: then add to it one quarter of a part of grape-sugar dissolved in weak spirits of wine, mix the liquid, and heat it to about 150° or 160° Fah. for about twenty or thirty minutes, and the glass vessel will become silvered. The liquid may then be removed; the vessel gently rinsed with distilled water; a connection is to be formed by means of a fine copper wire with the film of silver and the battery; the vessel filled with a suitable depositing liquid, an anode immersed, and the surface deposited upon in the ordinary way. If the process be successfully performed, the deposit, whether of silver or copper, will spread instantaneously over the silvered surface. The only way by which we have been able to form an adhesive deposit upon glass or porcelain, has been to send the articles to a glass and porcelain gilder, and have gold-leaf burnt into their surfaces, and then depositing upon them in the usual manner may be successfully effected.

186. *Immersion of Articles in the Vat.*—The operator having prepared the various articles and moulds, will immerse them in the depositing solution, having previously introduced the anode and completed all the connections, taking care always to connect the article to receive the deposit with the zinc of the battery, and the metal to be dissolved with the copper or silver plate.

187. *Regulation of Battery Power.*—In depositing all metals, it is of the greatest importance that the battery power be properly regulated; and this may be done in a variety of ways, which collectively consist either in making alterations in the battery, in the depositing vessel, or in the wires connecting them. The *intensity* of the current is increased by increasing the number of alternations of the battery plates. Resistance to the current is diminished by increasing the conductibility of the battery liquid or depositing solution, which may be effected either by the addition of free acid to the battery or more salt to the solution; or by increasing the thickness and decreasing the length of the connecting wires. The *quantity* of the current is increased by immersing all the battery plates deeper in their liquids; also by some of those means which increase the intensity. Both the intensity and the quantity may be decreased by separating the electrodes in the depositing vessel farther asunder, or by interposing a long and fine iron wire in

the circuit. The most usual means adopted of increasing the *intensity* of the current, is to add to the number of the batteries; and for increasing the *quantity*, to immerse all the battery plates deeper in their liquids.* Sometimes, to increase the power, the temperature of the liquid may be raised, or the article placed nearer the dissolving metal; and when a smaller quantity of current is required than the ordinary batteries can conveniently supply, a piece of sheet metal may be hung in the depositing solution with the receiving article, to receive a portion of the deposit, and thus transfer some of the power from the article to itself

188. *Intensity and Quantity of the Current.*—The intensity of the current obtained from a series of batteries, depends upon the *number of alternations* of the metals; whilst its *quantity* depends upon the *amount of immersed surface in each alternation*; and it makes no difference whether that amount of surface is in one battery plate or in many, or in one or many containing vessels. A series of similar batteries may be so connected together by wires or other conductors, as to give either an intensity or a quantity of current, provided they have screws or other conveniences for attaching the wires to the plates. For instance—1st, if we have four pairs of plates, in four separate cells, and connect them alternately, thus (Fig. 87), zinc, silver—zinc, silver—zinc, silver—zinc, silver, with wires from the terminal plates, we obtain a current possessing the intensity of four pairs, and the quantity of one; but if, 2nd, they are connected, all the zincs by one wire, and all the silvers by another wire (Fig. 88), with one portion of each wire left convenient for making connections, we obtain a current possessing the quantity of four pairs, and the intensity of one; and, 3rd, if they are arranged in two series or rows, two pairs in each, each series being connected intensity-fashion, the end silvers of each row facing one way and connected by one wire, and the terminal zincs facing the opposite way and connected by another wire, and these wires left free for connection with the vat (Fig. 89), we obtain a current possessing the intensity of two pairs and the quantity of two. By contrivances of connection like these, any number of batteries, provided they are similar in kind, charged alike, and have suitable connecting screws attached, may be connected together so as to give any desired quantity or intensity of current within the limits of their power: thus, a battery of one hundred pairs may be arranged to yield a current having the intensity of one hundred pairs, and the quantity of one; the quantity of one hundred, and the intensity of one; the intensity of fifty, and the quantity of fifty; or any intermediate degree of each.

Fig. 87.

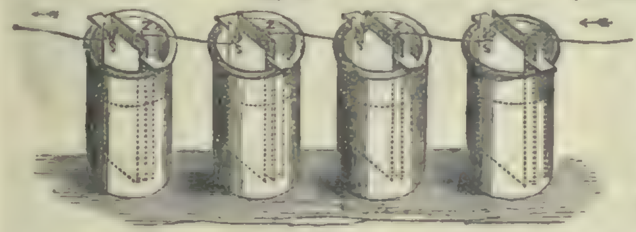


Fig. 88.

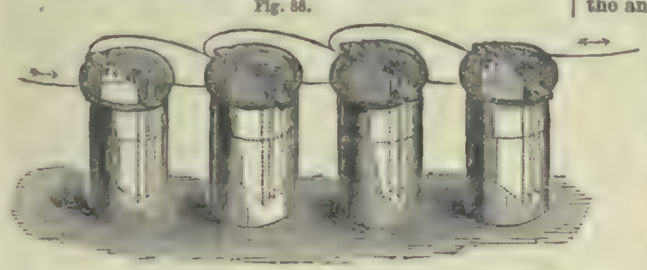
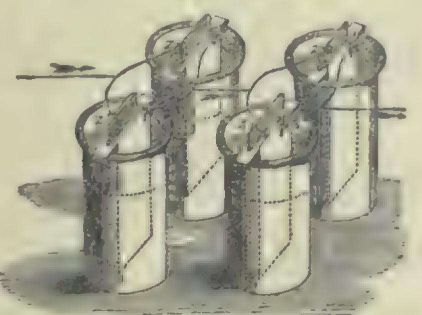


Fig. 89.



189. *Regulating the Quantity of Deposited Metal.*—The quantity of metal dissolved and deposited in the vat, is in direct proportion to the quantity of zinc dissolved and acid consumed in each alternation of the battery. With a perfect depositing liquid, good battery arrangements, and pure materials, for every equivalent of zinc dissolved in each alternation of the battery, an equivalent of metal is dissolved on one side, and an equivalent deposited on the other, in the depositing vessel. For instance—for every equivalent (32·6 parts) of zinc so dissolved, and 49 parts, or one equivalent, of oil of vitriol consumed in the battery, an equivalent (31·7 parts) of copper is deposited in the sulphate of copper solution, or an equivalent (108 parts) of silver in the cyanide silver plating liquid, and similar quantities of copper or silver dissolved at the anode. But in practical working, the materials are rarely or ever pure, or the arrangements perfect; the zinc nearly always contains a small proportion of other substances; the mercury contains tin or lead; and the sulphuric acid contains a little nitric acid. The acid liquid of the battery is often too strong: much acid liquid is frequently wasted, being thrown away before it is completely exhausted: the zinc plates have not been kept well amalgamated, or the silver well platinised, or the plates have been suffered to remain too long in the liquid when not in use. The metal of the anode is frequently impure also; occasionally some of the deposit is allowed to redissolve, in consequence of the battery power becoming low, and from not stirring the solution; in some solutions, a part of the battery strength is expended in evolving gas at the cathode; and, finally, the repeated operation of "scratching" removes some of the deposit. Allowing for all these and other unavoidable sources of loss in practical working, about one pound only of copper can be deposited in the ordinary sulphate solution, by the consumption of from one-and-a-quarter to one-and-a-half pounds of zinc, and an equivalent quantity of acid, in each alternation of the battery. To increase the quantity of metal deposited in a given period of time, the battery plates should be sunk deeper in their liquids.

190. *Regulation of the Quality of the Deposit.*—The quality of the deposited metal, *i.e.*, its degree of cohesion, hardness, flexibility, &c., depends upon the intensity of the current. As a general rule, the greater the intensity and the smaller the quantity of the current, the harder and brighter is the deposited metal; and the greater the quantity and the smaller the intensity, the less coherent and the softer it is. To obtain a very hard, bright, and crystalline deposit, we should use a current of small quantity and high intensity; and to obtain a soft black powder deposit, we should employ large quantity and low intensity; the combination of moderate quantity and moderate intensity, produces a coherent reguline deposit, possessing all the ordinary characteristics of the particular metal. These results can only be obtained with a good depositing liquid, and with metals, such as copper, silver, gold, &c., which are known to exist in a tough reguline state; and not with those—such as bismuth or antimony, crystalline metals—which do not exist in that state.

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* See *ante*, p. 137.

191. If we are producing a reguline deposit, with a one-cell arrangement, in a good depositing liquid, with electrodes of the same size as the immersed portion of the battery plates, and it is wished to change the deposit to a soft black powder, the plates of the battery must be immersed considerably deeper in the liquid, and a very much larger dissolving plate in the vat used. If we wish to change the deposit to a crystalline one, several more pairs of battery should be put on, connected intensity-fashion, and the plates must be immersed to an exceedingly small depth in their liquids, and a very small anode employed. These results have a direct reference to the size of the receiving surface: for if, with any given battery and anode, we are producing a black powder deposit upon a very small article, a larger article would receive a reguline deposit, and a much larger one would have thrown down on it a deposit bordering upon the crystalline state. Thus it will be perceived that the black powder deposit is a result of too rapid, and the crystalline one of too slow action.

192. *Spread of Deposit.*—If we wish to make a deposit spread rapidly over a metal of inferior conductibility, such as a long iron rod, we must use a current of high intensity and rather small quantity; this will drive it over the surface without causing it to become soft or non-coherent. The action of such a current appears to consist in conferring upon the particles a kind of polarity, a power of grouping themselves into separate warty nodules or crystals, each of which, as it becomes larger, appears to powerfully repel its neighbour, and thus causes the metal to spread rapidly; when this action is continued to a considerable thickness of deposit, especially in cold weather, the metal is exceedingly hard, and easily broken into a number of distinct grains or nodules, which are in the form of warty lumps with rounded edges. When the action has been rather too quick, or the liquid not sufficiently cold, and composed of more or less perfect crystals, with edges sometimes beautifully defined, or when the action has been very slow, and the liquid very cold and undisturbed, the same occurs. With the intensity of one hundred pairs of Smee's battery, acting for a long period of time in cold weather, and the quantity of the current kept down to the lowest possible degree, we have seen a tough deposit of zinc spread over several square inches of clean gutta-percha; and in depositing copper with a current of rather high intensity, and small quantity, upon blackleaded gutta-percha medallions, we have repeatedly observed, that where there was a sunk boundary line near the edge, the deposit remained quite thin, as if powerfully repelled, whilst on each side of the line it was very thick, and on the outside edge, accumulated in large warty masses, hard and distinctly separate, and containing as much metal as the whole of the medallion besides.

193. *Management of Batteries.*—The most suitable strength of liquid for filling the battery cells, consists of one measure of sulphuric acid and about twenty measures of water; a stronger liquid may be used, even to one part of acid to ten or twelve parts of water; but then the zinc plates require constant watching and frequent amalgamating, to prevent waste. If the acid liquid be very strong, and the electric action energetic, the zinc plates will require to be examined every day, to see that there is no local action, *i.e.*, chemical action in particular places; and when gas is found to be freely rising from them, as well as from the silver or copper, or when any dull patches appear where the acid has acted too strongly upon them, they should be taken out and amalgamated. They should be frequently amalgamated when new; and afterwards, if much worked, they should be amalgamated every few days: when they become old they should be rarely amalgamated, because it very much weakens the power. When they become so thin as to fall to pieces on handling, new plates should be substituted, and the old should either be melted at as low a degree of heat as possible, to prevent loss of mercury, and cast into rods for Daniell's batteries; or be broken up, put in an iron retort, and the mercury distilled from them at a strong red heat into a vessel of water.

194. The zinc plates should be taken out of the liquid every evening, unless deposition is required to continue all night. After the battery has been at work a few days, a little more acid should be added, and the liquid stirred; and this should be done as often as the power gets low, until at length the liquid becomes thick and nearly saturated with zinc salt, which crystallises about the edges of the cells. It is then time to throw it away and put fresh, or it may be filtered, evaporated, and crystallised, and the resulting salt (sulphate of zinc) preserved for depositing purposes. If any of the silver or copper plates of the batteries become covered with a deposit of zinc whilst working, it shows that all the acid in that cell is exhausted, and that more should be added, or fresh liquid put in. This only happens in a Smee's battery, or in the old zinc and copper arrangement. The deposit of zinc may be easily and quickly removed by the addition of acid to that cell, by a fresh liquid, or by immersing the coated plate in dilute sulphuric acid as long as gas is evolved from it.

195. If copper plates are employed in the batteries, they should be heated red-hot all over every week or ten days, and quenched in water, and then dipped in "dipping liquid" (82), or in nitric acid; and if platinised silver plates are used, they should be re-platinised as often as they become light in colour, or their power becomes low, which will happen once in two or three months with constant working. The re-platinising greatly increases the power of the battery. Great care must be taken that the zincs never touch the silver or copper plates when wet, otherwise the mercury will get upon the latter, and much weaken the battery power; and the silver plates will drop to pieces if they are very thin.

196. *General Rules for Working Solutions.*—In working any depositing liquid—1st, avoid doing anything which will alter the chemical composition of the liquid, or even the proportions of its ingredients, except the water—that may be altered, in proportion, in most liquids without much inconvenience; 2nd, adapt your electric power to the liquid, rather than the liquid to the power, and regulate the deposit rather by alterations in the battery than by alterations in the depositing vessel, except as regards the distances of the electrodes or the temperature of the liquid—these may be altered with safety, and, sometimes, with convenience; and 3rd, as a general rule, let your dissolving metal, the anode, expose a larger immersed surface than the receiving article, the cathode.

197. *Position of Articles and Dissolving Plates.*—The best practical position for the dissolving plate is the vertical, the dissolving plate and the receiving article being suspended in the liquid facing each other, the latter being rather the lowest of the two, and both wholly immersed. The horizontal position, with the dissolving metal above, although the most philosophical arrangement, does not succeed in practical working, because the metal used for dissolving, is never quite pure (with copper, especially), and the impurities from it fall upon the surface of the receiving article beneath, and make it rough; in addition to this, the position of the article prevents our being able to examine it easily or remove it conveniently. If the article to be coated have a very irregular outline, either the dissolving plate should be bent somewhat to its form, so that the two may be nearly equi-distant at all parts; or the article should be often shifted in its position, so as to produce a nearly uniform thickness of deposit all over. The nearer the receiving article is to the dissolving plate, the more rapid is the deposit; and a large body of liquid deposits more rapidly and more evenly than a small one; large connecting wires are more favourable to quick deposits than small ones. The greatest thickness of deposit always takes place upon the most prominent places, *i.e.*, upon those parts nearest the dissolving metal.

198. *Motion of Articles in Vat.*—In some solutions—for instance, the double sulphite of silver and potash—if the current be too strong to produce a good deposit, motion of the articles will prevent it becoming bad. In some plating establishments, the articles in the vats are

kept in constant motion, gently swinging to and fro—the metal frame upon which they are suspended having four small wheels running upon four small inclined planes fixed upon the edges of the vat, which are kept in constant motion by steam or other available power.

199. *Temperature of Solution.*—Several solutions, such as the cyanide of copper and potassium, the cyanide of gold and potassium, &c., require to be kept hot, in order to make them conduct freely, and yield suitable metal; not that they cannot be worked cold, but that they work much better and quicker at a high temperature, which more than compensates for the expense of heating them.

200. *Protection of Depositing Liquids from Light.*—Some liquids, such as the double sulphite of silver and potash, and the hyposulphite of silver and potash, require to be protected as much as possible from the influence of light; and even the ordinary cyanide of silver liquid is better screened from an excess of this agent.

201. *Clean Connections necessary.*—In every case we must be very careful to observe that the circuit is complete, and that it is capable of conducting the current freely throughout; that the articles to be coated are conductors of electricity, or have been made so by any of the means already recommended; and that their surfaces, as well as all the ends of the wires, at their various points of contact, are perfectly clean.

202. *Management of Coppering Liquids.*—With the sulphate of copper solution (120) no particular management is required, beyond the general rules already laid down (196). It is not suitable for depositing direct upon zinc, tin, lead, iron, or steel. Articles formed of these metals are first coated with a thin layer of copper, in the cyanide of copper and potassium solution (120); then well washed, and transferred immediately to the sulphate solution, and the remainder of the required thickness of copper deposited upon it.

203. *Uses of Copper Deposition.*—Among the many uses to which the electro-deposition of copper has been applied, we may mention the following:—To make copper cells for Daniell's batteries; making copies of stereotype plates, engraved copper plates, and engraved rollers; coppering the surface of printing-type; coppering steel pens (patented); to protect iron and steel goods from rusting, coating telegraph wires, ship's bolts, &c.; to make copies of Daguerreotype pictures; to make coppered cloth; to coat glass chemical vessels; to coat and protect metal and plaster statues, busts, and sculptured works; to preserve the form of flowers, fruits, ferns, seaweed, insects, reptiles, &c.; to make medallions, busts, and various figures and ornaments in copper; it has also been applied in the arts of glyphography and electro-tint printing.

204. *Making Objects in Copper, Coppering, &c.*—To make a cell of Daniell's battery in copper, coat the inside of a glass jar or earthen jelly-pot with wax, resin, or stearine, by making the vessel hot; then either blacklead it thoroughly, or treat it with the phosphorus, silver, and gold solutions; or, what is more simple, coat it uniformly all over the inside with the phosphorus moulding composition (87), and then treat it with the gold and silver liquids. Make a connection by a fine copper wire with the lower part of the coating; fill the vessel nearly full of the sulphate of copper depositing solution (120), suspend in it a sheet of copper, and connect the sheet of copper and the fine copper wire with a small battery of one or two pairs. If, instead of the battery process, we adopt the single cell arrangement, containing dilute sulphuric acid and a piece of zinc, placing a porous cell in the sulphate solution, and connecting the fine copper wire with the piece of zinc; a deposit of copper will soon spread over the entire inside surface of the jar, when that has been previously prepared by the phosphorus method. Engraved steel plates are copied by stopping-off the back with copal varnish, allowing it to become perfectly dry. They are then immersed in the cyanide coppering liquid, and a thin film of copper is thrown down: after washing well, they are at once immersed in the sulphate of copper solution, and the required thickness of copper is deposited; this will require from 24 to 48 hours. The surface of the steel should be previously prepared for a non-adhesive deposit, otherwise the two metals cannot be separated.

205. *Copying Wood-cuts in Copper.*—For printing purposes, where a large number of impressions of a particular wood-cut is required, the plan of taking copies of the engraved wooden block in copper by the electro-process, and using those copies instead of the original block to print from, has been gradually extending itself for some years, and has now attained a considerable degree of importance; the vignette at the head of the title-page of the *Illustrated News*, the title-page of *Punch*, many of the large engravings in the *Illustrated News*, and even the illustrations of some of the penny periodicals, are regularly produced in this way. To copy an engraved wooden block, the engraved surface is first moistened with water, and firmly enclosed by a shallow frame; a thick piece of gutta-percha, more than sufficient to fill the enclosed space, and made quite soft by heat, is then laid upon it, commencing its contact at the centre of the engraving, and proceeding outwards, so as to exclude all air-bubbles; a plate of cold iron is then laid upon the gutta-percha, and the whole subjected to pressure, gentle at first, but increased to a high degree as the substance cools. The block and copy are then separated, and the figured surface of the gutta-percha (with connecting and guiding-wires previously attached) is treated in the usual manner with blacklead or with the phosphorus, silver, and gold solutions; copper is then deposited upon it in a solution of sulphate of copper, until a moderate thickness of deposit is obtained, which will occupy at least twelve or eighteen hours; when sufficiently thick the deposit is removed, its back made rigid by a layer of solder or type-metal (the surface being previously moistened with a solution of chloride of zinc to make the solder adhere), the back is planed flat, and mounted upon a block of wood to the height of the type. In London this process is carried on upon a large scale, some of the copies being upwards of two feet square. Engravings upon steel are copied in an exactly similar manner to those upon wood.

We have recently tried some experiments with a view of making the deposit of copper upon gutta-percha and marine glue spread more rapidly than it does, by preparing the surface with blacklead; it being a matter of some importance, in copying wood engravings for periodicals of such large circulation as the *Illustrated News*, that the time occupied in the process be reduced. Our experiments have succeeded to some extent, and we give the following results of them for the benefit of the printing trade.

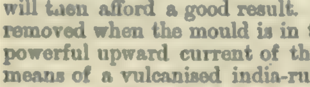
After having formed a reverse copy, in gutta-percha and marine glue, of the engraved wooden block, and affixed the conducting wires to it, take a mixture of one measure of spirit varnish, and either four or five measures of vegetable naphtha, and apply it very sparingly in a thin layer, by a soft camel's-hair brush, over the whole surface of the mould where the deposit is desired to be spread. Whilst the surface is still wet, cover it with a mixture of three parts of yellow and one part of white bronze powder, and bring the powder in thorough contact with the whole of the moistened surface, by striking it all over with a dry, soft brush; then gently brush off all the superfluous powder. The bronzed mould may now be immersed in the ordinary sulphate of copper solution, and the following actions will occur:—The particles of white bronze powder being composed almost wholly of tin, and those of yellow bronze containing much copper, those of tin will dissolve and coat themselves with copper by the simple immersion process (4), and those of brass or copper (the yellow ones) will become coated by their contact with those of tin (Two Metals and One Liquid Process, 7), and thus a thin deposit of copper will almost instantaneously spread all over the bronzed surface. This effect will of course take place without connecting the mould with the battery; but they may be immediately connected together, and a deposit will spread almost instantaneously over the whole of the bronzed surface by the ordinary battery process, through the medium of the bronze and the thin deposit already mentioned, which may be continued to any required thickness in the usual way. In our earlier experiments with this method, yellow bronze alone was used, which did not reduce the copper by simple immersion; but even then, medallions were repeatedly covered with a deposit of copper in from two to five minutes, which would occupy from twenty to forty-five minutes when prepared by blacklead in the usual manner. The addition of white or tin bronze causes the deposit to spread as rapidly as when the surface is prepared by the phosphorus solution, but without the disadvantage which occurs in using the latter, of making the deposited metal brittle. The effect of using white bronze alone was not so satisfactory.

The surface of the copper copy so obtained is bright and clean, and the character of the deposited metal is good, but the surface obtained is hardly so smooth and fine as that obtained with blacklead; the difference, however, is very slight, and it is sufficiently smooth for all ordinary purposes, and for the object sought, if care be taken to blow off or otherwise remove all superfluous bronze powder before immersing the mould in the vat. We hope that those who may have the opportunity will try it upon a larger scale.

206. *Copying Set-up Type in Copper.*—The process of electrotyping has also been gradually encroaching upon that of stereotyping, and has, we are informed, almost superseded that process in America. The plan adopted is similar to that of copying wood-cuts—viz., to lay a sheet of softened gutta-percha upon the surface of the page of type, and subject it to increasing pressure until it is cold; the gutta-percha copy is then removed, and treated as in copying wood engravings. It would be advisable to try a somewhat softer material for this purpose, such as the mixture of gutta-percha and marine glue, which we have recommended (85). This material takes a sharper and smoother impression than gutta-percha alone, and the deposit spreads over it more rapidly; and, being softer, it enters more freely and with less pressure between the fine lines of the letters, and still is not sufficiently soft to penetrate the minute crevices between the body of the types. If a solution of grape-sugar (as used in Drayton's patent process for silvering glass), aldehyde, or other reducing agent, were substituted for the phosphorus solution, for reducing the silver upon the surface of the mould, it would be an advantage, as, besides the dangerous character of the phosphorus, it has an offensive odour, and the copper deposited upon surfaces prepared by it, moreover, is invariably brittle.

The mould may also be prepared for a deposit by blackleading; it will require a first-rate quality of blacklead, and prolonged and attentive brushing, but will then afford a good result. The air-bubbles may be removed when the mould is in the liquid, by directing a powerful upward current of the liquid against them by means of a vulcanised india-rubber bladder, with a long and curved glass tube having a fine orifice (Fig. 90) attached to it; but the liquid should be free from sediment.

Fig. 90.



The advantages of electrotyping over stereotyping are numerous; the metal is harder, takes a sharper impression of the mould, and delivers the ink much more readily than type metal, besides being a cleaner process; it also takes up less ink, and consequently the printed pages dry more quickly. Both wood-cuts and letter-press have also been copied in plaster of Paris, and the deposit of copper formed upon that; but this material is much inferior to gutta-percha for the process. Messrs. Bradbury and Evans are considered the most successful manipulators in this branch of electro-deposition, and their apparatus the most perfect of its kind. In this establishment the temperature of the room is carefully attended to, and the vessels containing the solutions have glass covers. The result of this careful manipulation has been, that, in some instances, successful deposits of large *Illustrated News* engravings have been formed and taken off in eight hours; this can only have been effected by the most perfect blackleading, keeping the solution in excellent condition, and working with the maximum of battery power. Gutta-percha and marine glue is well worthy of a trial, and the use of bitumen with gutta-percha is also a good idea; the marine glue would be better, because it is tougher.

Iron and steel wire may be coated with an adhesive deposit of copper, by first immersing them, with their surfaces perfectly clean, in the cyanide coppering liquid, and completing the deposit in the ordinary sulphate solution. The coils should be kept separate from each other in the liquid by suspending them upon a horizontal brass rod, turning it occasionally to cause a uniform deposit. Iron screws and nails may be treated in a similar manner, except that they should be contained in a wicker basket, and shook about occasionally to produce a uniform deposit.

207. *Copying Daguerreotype Pictures in Copper.*—The most interesting and beautiful application of the deposition of copper, and, at the same time, one of the easiest to be effected, is that of copying Daguerreotype pictures. First solder a wire to the back of the plate near the edge; paint over the back and edges, and allow it to dry; hang it in a clean sulphate of copper solution, which is perfectly free from dust or grease on its surface; and, in the course of twenty or thirty hours, if about two cells of small Smee's batteries have been used, the deposit will be sufficiently thick to be removed; it should then be taken out, well washed, wiped perfectly dry, and a narrow strip be cut off its edges with a strong pair of scissors; the two may then be easily separated by inserting the point of a knife, or the end of a thin wedge of hard wood, between them at the edges. If the process have been carefully managed, and the original picture be a strong one, a most beautiful and vivid copy will be obtained; and if the picture be not only a strong one, but has also been well fixed by Fizeau's process, a number of successive copies may be taken from it; but their intensity, as well as that of the original, appears to diminish in each succeeding trial. With a vivid original picture, clear solution, very regular and undisturbed action of the battery, and a fine deposit, we have observed a most strange phenomenon—viz., the picture has not entirely disappeared, even in twenty hours, although the coating of copper has constantly increased in thickness; the image has penetrated quite through the deposited metal, and appeared upon the back, even upon deposits as thick as an address-card. In some cases the figure was optically positive, and in others negative.

208. *Coating Plaster Models, Flowers, and Clay Figures with Copper.*—Busts and other similar objects may be coated by saturating them with linseed oil, then well blackleading, or treating them with the phosphorus, silver, and gold solutions, and attaching a number of guiding-wires, connected with all the most hollow and distant parts. The object is then to be immersed in a sulphate of copper solution containing no free acid, and just sufficient copper is to be deposited upon them by the battery process to protect them, but not to obliterate the fine lines or features. Flowers, fruits, ferns, seaweed, insects, &c., may be prepared by the phosphorus, silver, and gold liquids, and the copper deposited upon them, either by the single cell or battery process, in a neutral sulphate of copper solution.

209. *Coppering Cloth.*—To copper cloth, first stretch it upon a sheet of copper slightly curved, so that it may be in close contact with the metal all over; then varnish the back or hollow side of the copper, and deposit on the opposite side, by the battery process, from a sulphate solution not containing much free acid, until the meshes of the cloth are quite filled with copper, and the metal and cloth firmly united together; the deposit may then be removed and well washed: the original sheet of copper should of course be properly prepared for a non-adhesive deposit. Mr. J. Schottlaender took out a

patent, December 8th, 1843, for depositing either plain or figured copy upon felted fabrics. He passes the cloth under either a plain or an engraved copper roller, horizontally immersed in a sulphate of copper solution not containing much free acid, and a deposit takes place upon the roller as it slowly revolves; the meshes of the cloth are thus filled with metal, and the design of the roller copied upon it. The coppered cloth is slowly rolled off, and passed through a second and closely contiguous vessel filled with clean water; the roller is properly prepared for a non-adhesive deposit.

210. *Etching Copper.*—In etching a copper plate by galvanism, we first solder a wire on the back, then varnish the back, and cover the front with a thin layer of engravers' etching-ground; draw the design upon the front surface with an etching-needle, cutting through this material to the clean surface of the copper. Having completed the etching, hang the plate as an anode in the ordinary sulphate of copper solution, opposite a suitable cathode of brass or copper. The current of electricity, in passing out of the engraved lines into the liquid, causes the copper in them to dissolve, and thus etches the design in the plate. The different gradations of light and shade are produced by suspending cathodes of different forms and sizes opposite the plate to be etched, in different positions and at different distances from it, thus causing the etching to be of different depths in different parts, the deepest action being always at the parts of the electrodes nearest together.

211. *Glyphography.*—This art consists in varnishing the back of a flat and smooth copper plate, laying first a thin coating of white etching-ground upon its front side, and then a layer of black etching-ground upon that; engraving the design upon the coating with different engraving-tools; then blackleading the whole of the engraved surface, and depositing a thick sheet of copper upon it in a sulphate solution by the battery process. The deposited plate is then removed, its defects corrected, and fixed upon a block of wood in the same manner as a stereotype plate, ready for printing by the ordinary hand-press. This process has been patented, and the patent is worked by Mr. Hawkins, Hatton Garden.

212. *Management of Silver Solutions.*—Silver plating liquids require much more care and attention than the sulphate of copper solution. Articles formed of zinc, iron, or steel, require to be coated with a thin film of copper in the cyanide of copper liquid, before being immersed in the cyanide of silver solution. Those formed of Britannia metal, tin, or pewter, are taken direct from the hot potash liquid without rinsing in water, and immersed a short time in a cyanide of silver solution containing considerable supplies of free cyanide. A large anode is used, and a current of considerable intensity from a strong battery passed through for several minutes, until the articles receive a thin deposit of silver; they are then transferred to the ordinary vat to receive the full amount of deposit. Those of lead are first scraped, or otherwise made quite clean and bright by mechanical means, and then treated in the same manner as those of Britannia metal. Articles of copper, brass, or German silver, after being properly cleansed, are dipped into the solution of nitrate of mercury (84), or of cyanide of mercury and potassium (84), then rinsed in a vessel of water, and immediately suspended in the depositing vat. The preparation of those articles by immersion in a bath of cyanide of mercury, was patented by Dr. H. B. Leeson, June 4th, 1842, and is in use by the electroplaters of Birmingham. If they are immersed without this preparation, the deposited silver will not always adhere firmly.

213. *Peculiarities in Practical Silver Deposition.*—Peculiar phenomena often occur in the electro-deposition of silver, not only upon different metals, but also upon the same metals in different forms or in different conditions of surface. For instance—

1st. If two perfectly similar pieces of thin sheet brass are taken (except that one is perforated all over with small holes), and both be simultaneously immersed in the same solution to be silvered, and with the same battery power applied to each, the latter, although its amount of surface is reduced by the perforations, will become coated with silver much more slowly than the former.

2nd. If a wire-gauze cylinder of a Davy lamp be suspended side by side with a piece of thin tubing of the same metal and of the same dimensions, the latter will become coated much more rapidly than the former.

3rd. If two pieces of the same metal—iron for instance—be immersed, to be silvered, in the ordinary cyanide solution, or to be coppered in the hot cyanide of copper and potassium liquid, each containing exactly the same amount of surface to be coated, but one being in the form of a thin sheet, and the other in that of a thick plate or solid block of metal, the former will become coated much more rapidly than the latter.

4th. The edges and points of articles, whilst being plated, exhibit a greater tendency to a crystalline deposit than the flat parts, and this tendency is sometimes manifested in depositing silver upon table-knives and forks. It is the knowledge of these and many other peculiarities of different metals and articles met with in practical working, and of the means of overcoming their attendant difficulties, which constitutes one of the chief differences between the practical operator and the scientific man.

214. *Management of "Bright Solution."*—A bright solution is much more difficult to manage than the ordinary silvering liquid; if it be not worked constantly and in a uniform manner, it will lose its power of yielding bright metal. If any of the articles which are being plated in it are disturbed, or removed from the liquid and replaced, that one will not now receive a bright deposit; and the disturbance of the liquid, by removing it, will oftentimes cause all the neighbouring articles to lose their brightness. If too much "brightening liquid" (144) be added, the solution will be considerably injured: many silver solutions have been irretrievably damaged in this way. A bright solution requires a battery current of large quantity and low intensity to work it, and the dissolving plates in it are generally of a darker colour than those in the ordinary silvering liquid; the silver deposited from it is much harder than that deposited from the ordinary plating solution, and has very much the appearance of fused metal; the bright appearance commences at the upper part of the articles, and travels downwards; it soon after commences also at their lower extremities and travels upwards, until the bright portions meet each other. If there are very small holes in the surface of the articles, dull streaks appear above them.

215. *Adding Cyanide of Potassium to Plating Liquids.*—It is necessary to add a little cyanide of potassium occasionally to every cyanide of silver plating liquid, probably because the solutions absorb carbonic acid from the atmosphere, which converts some of the cyanide of potassium into carbonate of potash, and sets hydrocyanic acid gas free. A further portion of the potassium salt may also be decomposed by some means, with formation and escape of ammonia; the necessity of adding a little fresh cyanide is indicated when the dissolving plate begins to change from its ordinary pure white appearance to a dull yellowish-gray colour; it is best added in the evening after plating—about half-an-hour before stirring the solution.

216. If the solution be too strong, *i.e.*, if it contain insufficient water, but has silver and cyanide of potassium in their proper relative proportions, it conducts freely, deposits rapidly, and gives a rich deposit of a fine silky lustre; but it is more difficult to manage than a weaker liquid, especially in hot weather, because, from the less mobility of its particles, it is very apt to settle, by working, into strata of different densities—its upper part becoming exhausted of silver and full of free cyanide, and its lower part becoming nearly saturated with that metal, and destitute of free cyanide; the consequence of this is, that the upper parts of the dissolving plates waste rapidly, whilst the upper parts of the article receive either very little deposit, or one of a

bad quality, being gray, brown, or yellowish, sometimes of a lilac hue, and generally in dull streaky vertical lines: all these evils may be mitigated by stirring the solution well every night after having finished plating, or it may be entirely prevented by diluting the liquid with water to a proper extent, stirring it every evening, and working it uniformly. All silvering and other depositing liquids exhibit this tendency to settle into strata in working, especially if worked rapidly; but the most dilute ones show it in the least degree.

If the solution be deficient in water, and contain a great excess of free cyanide, the foregoing evils are all greatly aggravated. In hot weather it becomes very unmanageable, and the vapours of ammonia and hydrocyanic acid arising from it are quite overpowering. In this case, the best way to improve it is to add cyanide of silver and water in sufficient quantities to make it of a good composition, keep it in a cool place, stir it daily, and work it constantly in a uniform and careful manner. New solutions, or old ones which have been injured, often improve by daily stirring, with regular and judicious working. An excess of cyanide of potassium is indicated when the dissolving plates are very strongly acted upon, and the deposit is at the same time either very sparing or of a bad colour.

If the solution be too weak, *i.e.*, if it contain too much water, it conducts sparingly, deposits slowly, and the deposit has a dead-white appearance. This may be easily remedied by adding cyanide of silver and cyanide of potassium to it in proper proportions, and working it uniformly a few days with daily stirring.

217. *Washing, Drying, and Ornamenting Silver-plated Articles.*—Articles that have been plated with silver are always washed in a running stream of water until every trace of the depositing liquid is removed from them; they are then immersed in hot, dry sawdust, moved about in it, and gently rubbed with it until they are perfectly dry. After drying, they are, if necessary, weighed, to ascertain how much silver has been put upon them; then "scratched" (80), and finally finished by burnishing, polishing, &c.

Sometimes, for the purpose of ornament, portions of their surface are "oxidised." This is done by applying a hot solution of bichloride of platina (167) to them, and allowing it to dry. The more platina the solution contains, and the hotter it is, the deeper black does it produce; or it may be effected by the application of the solution of "liver of sulphur." To produce a brownish colour, apply a solution of equal weights of sulphate of copper and sal-ammoniac in vinegar; and to produce a "dead" appearance, like frosted silver, deposit a mere trace of copper upon it in a copper solution, then well wash it, and deposit a very thin layer of silver upon this. In each of these cases the parts which are to remain bright must be stopped-off with varnish.

218. *"Stripping" Silver from Copper and Copper from Silver.*—Occasionally the electro-plater has sent to him, to be replated, old worn-out articles formed of "Sheffield plate," in which the outer layer of silver has been worn away, and portions of the copper base beneath are exposed; these articles generally require to have the remaining portions of silver removed, in order to obtain a uniform surface to deposit upon. The removal of the silver is termed "stripping." To effect this, add a little nitrate of potash (saltpetre) to a quantity of strong oil of vitriol, and apply heat, until it is all dissolved; then immerse the articles in the hot liquid, and allow them to remain until all the silver is dissolved. If the action become slow, apply more heat or add more saltpetre; the copper will not be much acted upon if the articles are not allowed to remain in too long. A number of such articles are generally done together, and are afterwards washed, and prepared in the usual manner for receiving a deposit. The silver may be recovered from the liquid, in the form of chloride of silver, by diluting it with much water, then adding a solution of common salt to it as long as a precipitate is produced: the precipitate, when washed and dried, is chloride of silver. By fusing this with carbonate of potash, the metallic silver is obtained.

To remove copper from silver (which is but rarely required), boil it in dilute hydrochloric acid, or immerse it in a hot solution of perchloride of iron. The latter solution may be made by adding peroxide of iron (crocus, jeweller's rouge) to hydrochloric acid as long as it will dissolve; it will remove either tin, lead, or copper, from either gold or silver, without affecting those metals. A solution of chloride of zinc has been used for the same purpose. Copper may also be completely removed from silver or gold, by making it the anode in a sulphate of copper solution until all the copper is dissolved; the silver will remain unaffected, if the current employed be feeble and has not a greater intensity than one or two cells.

219. *Testing the Purity of Silver.*—M. Runge adopts the following method of testing the purity of silver:—He immerses the article in a mixture of 32 parts of water, 4 parts of sulphuric acid, and 3 parts of chromate of potash. If the metal be pure, the immersed part quickly assumes a purple colour, which is less deep and lively in proportion to the amount of alloy contained in the silver. No other metal exhibits the same colour with this liquid.

220. *Testing the Amount of Silver and of Free Cyanide in Silver Solutions.*—To ascertain the amount of silver in a cyanide plating solution, add dilute sulphuric acid to a known quantity of the liquid as long as it produces a precipitate; wash and dry the precipitate, which is cyanide of silver—containing in every 134 parts, 108 parts of metallic silver. To test the amount of *free* cyanide of potassium, add a solution of crystallised nitrate of silver in distilled water to a known quantity of the plating liquid, as long as the precipitate formed continues to be redissolved, and note how much crystallised nitrate is used; every 175 parts expended indicate 130 parts of free cyanide, or about three parts of free cyanide to four parts of nitrate of silver.

221. *Management of Gilding Solutions.*—Cyanide gilding solution is generally kept in a glazed iron vessel, and heated either by a stove, or by gas jets beneath; or in a stoneware or glass pan immersed in boiling-water. On account of the general smallness of the articles to be gilded, the thinness of the deposit required, and the rapidity of the action in a hot liquid, the articles only need to be immersed in the solution for a few minutes; when a thicker deposit is required, they should be taken out several times, brushed, and reimmersed. Articles formed of iron or steel must be coated with a thin film of copper in the cyanide coppering liquid before gilding. The strength of battery used for gilding is generally about two cells of Smee's, of different sizes, according to the magnitude of the articles to be gilded. The loss of water by evaporation is generally made good by adding a little distilled water, after having finished gilding.

222. *Regulation of Colour in Electro-gilding.*—The general method now adopted for regulating the colour of electro-gilding is as follows:—After having prepared the solution, work it with a large copper anode until the deposited metal begins to deteriorate in colour; then replace the copper by a small gold anode. With the copper anode can be obtained a rich full colour, becoming deeper as the temperature of the liquid is higher; to produce a paler yellow, use a small gold anode with the liquid at a lower temperature.

223. *Recovery of Gold and Silver from Depositing Liquids.*—As both silver and gold solutions occasionally get out of working condition, and become quite unfit for use, it is very desirable that the operator should understand the chemical action of different substances upon them, and how to recover the metal. Those of silver generally get out of order, either from the addition of too much "brightening liquid;" from excess of cyanide of potassium, together with the heat of the weather, and injudicious management; from unsuccessful attempts to improve the condition of the liquid; from the accidental introduction of impurities; or from the liquid having been improperly made. Supposing it to be the usual

cyanide liquid, the silver may be recovered in the metallic state thus:—Evaporate the solution nearly to dryness; reduce the resulting salt to powder; mix it with its own weight of a mixture of one part of nitrate of potash and two parts of common salt; and roast the whole in an iron pan to dryness. Fuse the dried mixture at a bright red heat in an earthen crucible, until the silver collects at the bottom of the vessel in a melted state; then pour it slowly into a large quantity of water. The resulting granules of silver should not be used in making a new plating liquid, because they generally contain copper derived from the articles suspended in the plating solution (see 151), but should be exchanged at a silver refiner's for pure silver.

224. "The crystallised double cyanide of gold and potassium fuses and effervesces by heat, and is resolved into cyanogen gas, ammonia, and cyanide of potassium, if air be present; its complete decomposition requires a strong red heat. When it is strongly ignited, and mixed with an equal weight of carbonate of potash, a button of metallic gold is obtained. When heated with sulphuric acid, it gives off hydrocyanic acid gas, and, after ignition, leaves a mixture of gold and sulphate of potash. Iodine sets free cyanogen gas, forms iodide of potassium, and throws down the cyanide of gold. The aqueous solution of cyanide of gold and potassium gilding liquid, when mixed with sulphuric, hydrochloric, or nitric acid, slowly deposits cyanide of gold; and, when boiled with hydrochloric acid, it is completely resolved into cyanide of gold and chloride of potassium. Similar effects are produced by sulphuric or nitric acid, and even by oxalic, tartaric, and acetic acid."

225. *Extraction of Gold and Silver from Exhausted Solutions.*—"To obtain the remaining gold from gilding solutions which have become inactive, they should be evaporated to dryness, the residue finely powdered, and intimately mixed with an equal weight of litharge, fused at a strong red heat, and the lead extracted from the alloy button of gold and lead by warm nitric acid; the gold will then remain as a loose yellowish-brown spongy mass."

226. The following extracts are from the works of Bœttger, J. Pr. Chem., 36, 169; Elsner, Redtel, Hessenberg, J. Pr. Chem., and other foreign writers:—"I have undertaken a series of researches upon this subject, and hasten to communicate the results to the public; but, before proceeding to the communication, I think it necessary to mention the results of the experiments upon which are based the methods given further on for extracting both the silver and the gold of old cyanide of potassium liquids.

227. "1st. If we add hydrochloric acid to a solution of silver in cyanide of potassium, until the liquid exhibits an acid reaction, we obtain a white precipitate of chloride of silver, which, when submitted to heat, melts into a yellow mass. If this were cyanide of silver, the application of a red heat would have left a regulus of silver. The addition of the hydrochloric acid precipitates all the silver present in the liquid in the form of chloride of silver.

228. "2nd. If we evaporate a solution of silver in cyanide of potassium to dryness, and heat the residue to redness until the mass is in a state of quiet fusion, and has assumed a brown colour, there remains, when we wash the mass with water, metallic and porous silver. The wash-waters, when filtered, still contain a little silver in solution; because, if hydrochloric acid be added to them, it will produce a precipitate of chloride of silver. In evaporating and calcining a solution of gold in cyanide of potassium, the result is the same—we obtain metallic gold. The wash-waters, acidulated with hydrochloric acid, give, when treated with sulphuretted hydrogen, a brown precipitate of sulphide of gold; and, with the salt of tin, a violet precipitate purple of Cassius—a proof that these liquids still contain a little gold in solution.

229. "3rd. If we pour upon finely-divided silver—for instance, silver-leaf, or silver precipitated in the porous state by zinc from a solution of silver—a concentrated solution of cyanide of potassium, at the ordinary temperature, and shake it frequently, the liquid, at the end of a certain time, exhibits silver in solution; and by adding hydrochloric acid to it, we produce an abundant precipitate of chloride of silver. This experiment explains why, in the wash-waters of the various combinations of gold or silver with cyanide of potassium, we can still demonstrate the presence of gold and silver, after the most minute separation.

230. "4th. When hydrochloric acid, or ordinary sulphuric acid, is added to a solution of cyanide of copper and cyanide of potassium, until the liquid exhibits an acid reaction, there results a reddish-white precipitate, which is a cyanide of copper in the anhydrous state. If the precipitate be well washed and boiled in potash lye, protoxide of copper will be separated, of a beautiful red colour; and if to the filtered alkaline liquid we add a solution of green copperas (sulphate of iron), a dirty-blue precepitate will be obtained. A solution of carbonate of soda furnishes the same results, and yields, with the copperas, the same dirty-blue precipitate. If the reddish-white precipitate be dissolved in pure nitric acid, and a solution of nitrate of silver added to it, an abundant white precipitate will be produced, which, when washed, dried, and calcined, yields silver in the metallic state—a proof that the precipitate is cyanide of silver.

231. "The reddish-white precipitate is soluble in an excess of hydrochloric acid, in nitric acid, and in aqua regia; it is also soluble in aqueous ammonia, and in a solution of cyanide of potassium.

232. "5th. If we pour hydrochloric acid into a very pure solution of gold in cyanide of potassium, there will be slowly formed, at ordinary temperatures, and immediately on the application of heat, a yellow precipitate, which is cyanide of gold; the filtered liquid which has given this precipitate, still contains a little gold in solution. In evaporating to dryness, fusing, dissolving, and filtering afresh, there is found upon the filter the remainder of the gold.

233. "When a solution of silver prepared for silvering articles of bronze or of brass, has been employed a certain time for that purpose, the precipitate produced in it by the addition of hydrochloric acid, is not pure white, but reddish, in consequence of the reddish-white cyanide of copper which is precipitated with it; for we know that those silvering liquids which have been used for some time, contain copper in solution. The same thing occurs with the solutions for gilding, in which articles of silver, copper, bronze, and brass, have been gilded for a long time; the liquid contains, after a certain time of service, not only gold, but also silver and copper. This case presents itself especially when gilded articles of silver, containing copper or other alloys of silver, are in the solution of gold; then the precipitate of cyanide of gold, produced by the addition of hydrochloric acid, does not possess its proper pure yellow colour. It has happened to me to observe a precipitate of this kind, which, instead of being yellow, was green; and, in fact, articles of iron have been gilded in a solution, and the precipitate contained, besides cyanide of gold, Prussian blue, so as to be demonstrated in an examination, which consisted in boiling the green precipitate in aqua regia, filtering to separate the dirty-green residue, evaporating the filtered liquid to dryness, and dissolving the dry salt in water acidulated with hydrochloric acid; the addition of sulphate of iron to this new liquid gave a brown precipitate, and the salts of tin a reddish-brown precipitate. In treating by aqua regia, the cyanide of gold was then decomposed, and converted into chloride of gold.

234. "Based upon the preceding facts, we may found several methods for recovering all the silver and gold of old cyanide of potassium solutions. The extraction of these precious metals may be effected either by the wet or by the dry process.

235. "*Extraction of Silver by the Wet Method.*—Add hydrochloric acid until the liquid exhibits a strongly acid reaction (230). The precipitate of chloride of silver which is thus obtained, will be, as we have already said, of a reddish-white colour, because of the cyanide of copper which is precipitated with it when the solution has

been used a long time for silvering objects containing copper. In this precipitation by hydrochloric acid, there is hydrocyanic acid gas set free, therefore the operation should only be performed in the open air, or in a place where there is good ventilation. If the precipitate be very red, it must be treated with hot hydrochloric acid, which will dissolve the cyanide of copper. The chloride of silver having been washed with water, must be dried, fused with potash in a Hessian crucible, and then coated with borax, in the ordinary manner for obtaining metallic silver.

236. "This method is very simple in its application, and very economical, considering that, by the aid of the hydrochloric acid, all the silver contained in the solution of cyanide of potassium is precipitated, and there remains no trace of it in the liquid. But the large quantity of hydrocyanic acid gas which is disengaged, is a circumstance which must be taken into serious consideration when operating on large quantities of silver solution, the vapour of which is most deleterious; and nothing but the most perfect ventilation, combined with arrangements for the escape of the poisonous gases, will admit of the process being carried on without danger to the workmen: when, however, we have taken the precautions dictated by prudence, the method in question may be considered as perfectly practical. The liquid should be poured into very capacious vessels, because the addition of the acid produces a large amount of vapour.

237. "*Extraction of Silver by the Dry Method.*—The solution of cyanide of silver and potassium is evaporated to dryness, the residue fused at a red heat, and the resulting mass, when cold, is washed with water. The remainder is the silver in a porous metallic condition. There still remains in the wash-waters a little silver, which may be precipitated by the addition of hydrochloric acid.

238. "*Extraction of Gold by the Wet Method.*—A solution of gold and cyanide of potassium, which has long served for gilding articles of silver alloyed with copper, may still contain, as we have already remarked, independently of the gold, both silver and copper, and perhaps iron. In order to obtain these metals we operate in the following manner:—

"The liquid, the same as with the solution of silver, is acidulated with hydrochloric acid; in which case there is produced hydrocyanic acid gas, which requires the same careful ventilation. This addition of hydrochloric acid causes a precipitate, which may, according to circumstances, consist of cyanide of gold, cyanide of copper, and chloride of silver. The precipitate, washed and dried, is boiled in aqua regia, which dissolves the gold and copper in the form of metallic chlorides, and leaves the chloride of silver unaffected. The solution, containing the gold and the copper, is evaporated nearly to dryness, in order to drive off any excess of acid; it is then dissolved in a small quantity of water, and the gold precipitated from it by the addition of protosulphate of iron, in the state of a brown powder. The chloride of silver is reduced to the metallic state by the usual means. The liquid from which we have precipitated the cyanide of gold, &c., by hydrochloric acid, may yet contain a little gold in solution. I refer to section 232 for its further treatment.

239. "This method is distinguished by the great simplicity of the operation, and we may repeat for it all that we have already said respecting the extraction of silver by the wet method.

240. "*Extraction of Gold by the Dry Method.*—The solution of cyanide of potassium, which contains gold, silver, and copper, is evaporated to dryness; the residue fused at a red heat, cooled and washed (the wash-waters still containing a little gold and silver; and this occurs most often when the solution of either metal contains a very large excess of cyanide of potassium). The residue, after washing, consists of gold and silver in a metallic porous state, and carbide of copper resulting from the decomposition of cyanide of copper by the heat. The metallic residue is treated by aqua regia, which forms insoluble chloride of silver, and contains the chlorides of gold and copper in solution. In order to obtain these metals in the metallic state, we must proceed in the manner previously indicated.

241. "If we operate according to the method of Professor Bœttger, *i.e.*, if we fuse the dried residue with its own volume of litharge, in a covered crucible, the regulus we obtain in this case consists of gold, silver, and lead. In treating this alloy by nitric acid of specific gravity 1·2, and applying heat, the gold remains in the form of a brown powder, whilst the lead and the silver are dissolved in the acid. This solution, after having been diluted with distilled water, may have the silver separated from the lead, by the addition of hydrochloric acid.

242. "These methods of extracting the silver and gold from old solutions of cyanide of potassium by the dry process, present this advantage, that the operator is not incommoded, while working, by the disengagement of vapours of hydrocyanic acid. In these operations the poisonous gases are not developed as they are in the process for extracting the metals by the wet process.

243. "After the experiments here reported, those who are interested in the subject may choose for themselves which of these methods appears the most suitable to the circumstances in which they are placed, and the object which they wish to attain.

244. "*Means of Recovering Gold or Silver by M. Bolley.*—Cyanide of gold dissolved in an excess of cyanide of potassium, resists all the means which we have tried to separate them; and hydrosulphuric acid, for example, does not produce a precipitate. By the wet way we cannot always precipitate the gold completely, and for that reason MM. Bœttger, Hessenberg, Elsner, and others, propose to evaporate the liquid to dryness; mix the residue with its own weight of litharge; fuse the mixture at a strong red heat; then dissolve the lead from the alloy by boiling it a long time with dilute nitric acid, which leaves the gold in the form of a light sponge.

245. "M. Wimmer has more recently proposed to evaporate the solution to dryness in a water bath, then mix the residue with one-and-a-half times its weight of saltpetre, and introduce the mixture by small portions at a time into a Hessian crucible, heated to redness in order to cause explosions; and to continue this until the entire mass is in a state of quiet fusion.

246. "The first of these two processes does not give room for any objection, except the employment of a great heat, and the use of nitric acid; the second process is, on the contrary, disagreeable, and very uncertain. We know that saltpetre never explodes with more violence than with cyanide of potassium; and, notwithstanding that the inventor of the process advises us not to add more than small portions of the mixture at a time, the explosions are so powerful that they cannot be caused without loss of materials.

247. "The following process is applicable, on the small scale, with a spirit-lamp and a crucible of platina:—Evaporate the solution to dryness; mix the saline mass with its own weight of sal-ammoniac, and heat it gently: ammoniacal salts decompose, as we have said, the metallic cyanides, and form cyanide of ammonium, which is itself decomposed by the heat and volatilised, whilst the acid of the ammoniacal salt (the body which salifies the ammonia) combines with the metals (passed to the state of oxides), which were previously united to the cyanogen. The sal-ammoniac, then, in this case forms chloride of potassium and chloride of gold, and, if the salt contain ferro-cyanide of potassium, chloride of iron in addition. The chloride of gold is easily decomposed; the chloride of iron is partly decomposed, and leaves oxide of iron in beautiful crystalline spangles. The undecomposed portion of the chloride of iron, like the chloride of potassium, may, after the decomposition is finished (which only requires a low red heat), be washed away with water, leaving the gold in the form of a light coherent mass, and the iron in small spangles, which may be removed by mechanical means.

248. "If we fear that a little of the gold remains mixed with the iron in a pulverulent state, we may dissolve it in hot aqua regia, and precipitate the gold from

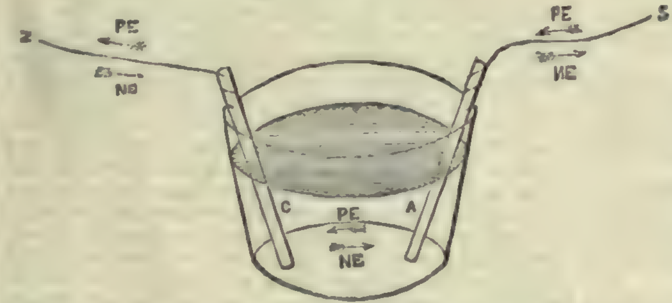
the resulting solution by adding to it a solution of protosulphate of iron; but this appears superfluous: and I am assured, by evaporation of given volumes of the same solution of gold, the evaporation and calcination of the sal-ammoniac, and other operations, that we have collected in a sufficiently exact manner all the gold of these solutions.

249. "The same process is applicable to the solution of silver; and independently of the oxide of iron (of the ferro-cyanide of potassium), we obtain chloride of silver, which is soluble in aqueous ammonia."

250. To further assist the reader in remembering the terms applied to the different parts of the electric circuit, and in understanding the action of electricity upon liquids, we repeat those terms, and append a diagram of the anode, cathode, &c.

S (Fig. 91) is a wire proceeding from the silver plate or positive plate of a battery; Z is another wire from the zinc plate or negative plate; the two wires are attached to two pieces of metal immersed in a depositing liquid. By the direction of the arrows, it will be perceived that the positive electricity circulates in one direction through the circuit, and the negative electricity in the opposite direction.

Fig. 91,



The immersed pieces of metal are termed "electrodes," meaning ways by which the electricity enters and leaves the liquid; the piece A is called the "anode," and is that by which the positive electricity enters the liquid; and C is the "cathode," or that by which the positive electricity leaves the liquid. In the ordinary depositing vat, the anode is that piece of metal which dissolves and supplies the solution; whilst the article or articles which are receiving a metallic coating or deposit, constitute the cathode. The liquid is termed an "electrolyte;" and whilst the electric current is passing through, it is said to be undergoing "electrolysis," to be "electrolysed," or suffering "electro-chemical decomposition." The elements of the liquid are termed "ions;" those of them which combine with, or are set free at the "anode," are called "anions;" and those which combine with, or are set free at the "cathode," are termed "cathions:" for instance—1st, in the electrolysis of a solution of sulphate of copper with copper electrodes, sulphuric acid is viewed as an anion, because it is the product which combines with the anode, and copper the cathion, because it is the element set free at the cathode; and, 2nd, in the electrolysis of the ordinary cyanide silver-plating liquid, cyanogen is the anion, and silver the cathion.

251. *List of Patents upon Electro-Deposition.*—The following is a chronological list of some important patents which have been taken out for subjects connected with electro-deposition.

Elkington's Patent.—June 24th, 1836: gilding copper, brass, and other metals; G. R. Elkington.

Elkington's Patent.—February 17th, 1837: gilding metals, and coating with platinum—apparatus described; H. Elkington.

Elkington's Patent.—December 4th, 1837: gilding and silvering certain metals—apparatus described; H. Elkington.

Elkington and Barratt's Patent.—July 24th, 1838: coating and colouring certain metals; G. R. Elkington and O. W. Barratt.

Elkington's Patent.—March 25th, 1840: coating or plating certain metals; G. R. Elkington and H. Elkington.

Parkes' Patent.—March 20th, 1841: production of works of art by electro-deposition; Alexander Parkes.

Barratt's Patent.—September 8th, 1841: deposition of metals; O. W. Barratt.

Fox Talbot's Patent.—December 9th, 1841: coating and colouring metals with other metals; H. H. Fox Talbot.

Zeeson's Patent.—June 1st, 1842: electro-deposition—with apparatus; H. B. Zeeson.

Tuck's Patent.—June 4th, 1842: coating metals with silver by electro-deposition; E. Tuck.

Woolrich's Patent.—August 1st, 1842: coating metals and alloys with metal; J. S. Woolrich.

Sturges' Patent.—August 10th, 1842: making plated articles; R. F. Sturges.

Fox Talbot's Patent.—November 25th, 1842: coating metals with other metals; H. H. Fox Talbot.

Moses Poole's Patent.—May 25th, 1843: deposition of metals—with apparatus; Moses Poole.

Barratt's Patent.—June 15th, 1843: gilding and plating metals; O. W. Barratt.

Schottlaender's Patent.—December 8th, 1843: deposition of metal upon felted fabrics; J. Schottlaender.

Parkes' Patent.—February 21st, 1844: deposition of metals and alloys; A. Parkes.

Parkes' Patent.—October 24th, 1844: depositing metals and their alloys; A. Parkes.

Parkes' Patent.—October 9th, 1845: coating metals and alloys; A. Parkes.

Lyons and Millward's Patent.—March 23rd, 1847: deposition of metals; Lyons and Millward.

Salzède's Patent.—September 30th, 1847: brassing and bronzing steel, iron, zinc, lead, and tin; Charles de la Salzède.

Fontainemoreau's Patent.—March 14th, 1849: coating metals and non-metallic substances; J. Fontainemoreau.

Russell and Woolrich's Patent.—March 19th, 1849: coating iron and other metals with metals and alloys; Russell and Woolrich.

Parkes' Patent.—March 26th, 1849: deposition of certain metals and alloys; A. Parkes.

Smith's Patent.—June 7th, 1849: depositing metals; S. B. Smith.

Roseleur's Patent.—March 23rd, 1850: tinning metals; A. G. Roseleur.

Steele's Patent.—August 9th, 1850: coating metals; Joseph Steele.

Ridgway's Patent.—April 20th, 1852: coating glass with metal by battery process; J. Ridgway.

Lyons' Patent.—October 7th, 1852: coating surfaces of iron; M. Lyons.

Morriss and Johnson's Patent.—December 11th, 1852: deposition of brass and alloys; Morriss and Johnson.

Junot's Patent.—December 28th, 1852: reducing metals by electricity and plating; C. J. E. Junot.

Power's Patent.—December 29th, 1852: silvering metals and glass; J. Power.

Newton's Patent.—July 29th, 1853: improvements in depositing metals and alloys of metals; W. E. Newton.

Newton's Patent.—August 5th, 1853: coating cast-iron with metals and alloys; W. E. Newton.

Person's Patent.—April 27th, 1854: electro-coating with zinc; C. C. Person.

Joubert's Patent for the Acierage of Engraved Plates.—In the preceding pages, we have repeatedly referred to the applications of electro-metallurgy in the art of engraving and printing. To them, M. Joubert has made a most valuable addition, in his method of coating engraved copper-plates with iron, by means of electro-chemical action. As the invention is of the highest importance, and as it has been extensively adopted by many engravers, we shall not hesitate to give an extended description of it, availing ourselves of the report furnished in the *Journal of the Society of Arts*, for

November 26th, 1852. M. Joubert describes his process as follows :—

"The last century produced many engravers of great merit ; and, in this country, foremost amongst them are Hogarth, Sir Robert Strange, and James Heath. The excellence of their works gave rise to such a demand for print impressions of engravings, that some forty years ago, when it was found that a copper plate could not yield a sufficient number of impressions for the demand, steel plates were introduced, for small plates only at first, and several editions of books were published containing plates most elaborately engraved on steel.

"Subsequently, when means were found to obtain a large surface of steel of pure quality, this metal was adopted for the style of engraving known as mezzotinto, which is now practised on steel plates, the result being a much larger number of impressions obtainable as compared with the old copper plates ; but historical or line engraving, for important subjects, was still entirely practised on copper, when the discovery of the art of electrotyping took place, about the year 1838.

"Several line engravings on copper were then multiplied by this process ; the result, however, was only attended with partial success, in consequence of the soft quality of the copper so deposited, which will yield but a very limited number of good impressions, and soon wears away : this caused the process to be almost abandoned for artistic engraving ; but for commercial purposes it is still practised extensively, and has been often successfully applied in cases where a large number of impressions is not required.

"Under the circumstances which I have described, it had become a desideratum to harden, if possible, the surface of a copper plate, and to protect it from wear while printing ; but it is only lately that this important object has been attained.

"In March last, my friend M. Jacquin, of Paris, took out a patent in this country for a method of coating plates with iron, which had already been successfully applied in France, and of which the merit is due to my friend, M. Henri Garnier, of Paris.

"I have myself had the advantage of co-operating with M. Garnier in the development of the invention, the principles of which I shall now proceed to describe.

"If the two wires of a galvanic battery be plunged separately into a solution of iron, having ammonia for its basis, the wire of the positive pole is immediately acted upon, while that of the negative pole receives a deposit of the metal of the solution. This is the principle of the process which we have named 'acierage.'

"The operation takes place in this way :—By placing at the positive pole a plate or sheet of iron, and immersing it in a proper iron solution, the metal will be dissolved under the action of the battery, and will form hydrochlorate of iron, which, being combined with the hydrochlorate of ammonia of the solution, will become a bichloride of ammonia and iron : if a copper plate be placed at the opposite pole, and likewise immersed, the solution being properly saturated, a deposit of iron, bright and perfectly smooth, is thrown upon the copper plate, from this principle :—

"Water being composed of hydrogen and oxygen ;

"Sal-ammoniac being composed of—

"1st. Hydrochloric acid containing chlorine and hydrogen ;

"2nd. Ammonia, containing hydrogen, nitrogen, and oxygen ;

"The water is decomposed under the galvanic action, and the oxygen fixes itself on the iron plate, forming an oxide of iron ; the hydrochloric acid of the solution acting upon this oxide, forms a hydrochlorate of iron, whilst the hydrogen precipitates itself upon the plate of the negative pole, and, unable to combine with it, comes up to the surface of the solution in bubbles.

"My invention has for its object certain means of preparing printing-surfaces, whether for intaglio or surface printing, so as to give them the property of yielding a considerably greater number of impressions than they are capable of doing in their ordinary or natural state. And the invention consists in covering the printing-surfaces, whether intaglio or relief, and whether of copper or other soft metal, with a very thin and uniform coating of iron, by means of electro-metallurgical processes. The invention is applicable whether the device to be printed from be produced by engraving by hand, or by machinery, or by chemical means, and whether the surface printed from be the original or an electrotype surface produced therefrom. I would remark, that I am aware that it has been before proposed to coat type and stereotypes with a coating of copper, to enable their surfaces to print a larger number of impressions than they otherwise would do ; I therefore lay no claim to the general application of a coating of harder metal on to the surface of a softer one, but my claim to invention is confined to the application of a coating of iron by means of electricity on to copper and other metallic printing surfaces.

"In carrying out the invention, the solutions of iron employed may be varied ; and such is the case in respect to the arrangement of the galvanic battery, or other source of electric currents used. I do not, therefore, limit the invention to the means hereinafter described, but I believe they will be found to be the best for the purpose.

"I would further remark, that it is important that a ferric solution should be employed which will not dissolve or corrode the plate intended to be coated ; for if it be attempted to use such a solution, though the iron will be precipitated, it will not only be in a non-coherent state, but the engraved surface itself will be liable to be attacked and injured. It may also be remarked, that the coating of iron admits of being removed from a printing-surface of copper without injury to the original plate ; hence the original plate may, after being coated and used for some time, have the worn coating removed, and then be recovered with an iron coating as often as may be required ; and if care be taken to remove the coating of iron before it has been entirely worn away, the engraved copper or other plate may be made to print a vast number of impressions, and yet remain in the original state it was in when it left the hands of the engraver, or was otherwise first produced : the only limit appears to be in the gradual change which takes place in the body of the printing-surface, by the compression to which it is subjected in the process of printing. Heretofore, in respect to plates engraved in intaglio, if of steel, they each yield, on the average, about 3,000 impressions without retouching : if of copper, they each yield, on an average, not more than 800 without retouching ; whilst electro casts of copper, obtained from the originals, will not, on an average, each yield even 200 impressions without retouching : in fact, such printing-surfaces are so easily worn, that, after the first 100 or 150 impressions, there is a considerable deterioration in the quality of the work produced. Therefore, for the supply of the number of impressions often required by art associations and others, it has been found necessary to multiply the electro casts very considerably. In such cases the invention is applicable with considerable advantage ; for I find that an electro plate, 40 × 22 inches, covered or coated with iron, has yielded 2,000 impressions without its being necessary to remove and renew the iron coating, there being no perceptible difference between the first and last impression ; the work on the plate appearing not to have suffered in the slightest degree. Hence, in future, by the application of the invention, it will only be necessary to multiply electro casts to such an extent as may be necessary to ensure the production of prints or impressions with the requisite speed, on paper, calico, or other fabrics. At the same time, an original engraving on copper would become, when treated according to the invention, more lasting than if engraved on steel. Although original surfaces engraved in relief, and also electro and other casts taken from them, yield a considerably greater number of impressions than those I have mentioned as obtained from plates engraved in intaglio—to which the invention has not been applied—nevertheless the invention is applicable, with great advantage, to such relief printing surfaces, whether of copper or other soft metal; for if they be coated with iron, according to the invention, they will yield almost an indefinite number of impressions, provided the iron surface be renewed as often as may be necessary, and the printing surfaces be again recoated.

"In carrying out the invention, I prefer to use that modification of Grove's battery, known as Bunsen's; and I do so because it is desirable to have what is called an intensity arrangement. The trough I use for containing the solution of iron, in which the engraved printing surface is to be immersed, in order to be coated, is lined with gutta-percha, and it is 45 inches long, 22 inches wide, and 32 inches deep. In proceeding to prepare for work, the trough, whether of the size above-mentioned or otherwise, is filled with water in combination with hydrochlorate of ammonia (sal-ammoniac), in the proportion of one thousand lbs. by weight of water, to one hundred lbs. of hydrochlorate of ammonia. A plate of sheet iron, nearly as long, and as deep as the trough, is attached to the positive pole of the battery, and immersed in the solution. Another plate of sheet iron, about half the size of the other, is attached to the negative pole of the battery, and immersed in the solution; and when the solution has arrived at the proper condition, which will require several days, the plate of iron attached to the negative pole is removed, and the printing surface to be coated is attached to such pole, and then immersed in the bath till the required coating of iron is obtained thereto. If, on immersing the copper plate in the solution, it be not immediately coated with a bright coating of iron all over, the bath is not in a proper condition, and the copper plate is to be removed, and the iron plate attached, and returned into the solution. The time occupied in obtaining a proper coating of iron to a printing surface, varies from a variety of causes; but a workman, after some experience, and by careful attention, will readily know when to remove the plate from the solution; and it is desirable to state, that a copper plate should not be allowed to remain in the bath, and attached to the negative pole of the battery, after the bright coating of iron begins to show a blackish appearance at the edges. Immediately on taking a copper plate from the bath, great care is to be observed in washing off the solution from all parts; and this I believe may be most conveniently done by causing jets of water forcibly to strike against all parts of the surface. The plate is then dried and washed with spirits of turpentine, when it is ready for being printed from in the ordinary manner.

"If an engraved copper plate be prepared by this process, instead of a comparatively limited number of impressions being obtained, and the plate wearing out gradually, a very large number can be printed off without any sign of wear in the plate, the iron coating protecting it effectually; the operation of coating can be repeated as many times as required, so that an almost unlimited number of impressions can be obtained from one plate, and that a copper one.

"This process will be found extremely valuable for electrotype plates, and also for photogalvanic plates, since they can be so protected as to acquire the durability of steel; and more so, for a steel plate will require repairing from time to time; these will not, but simply recoating, whenever it is found necessary: by these means one electro-copper plate has yielded more than 12,000 impressions, and was found quite unimpaired when examined minutely.

"It is easy to appreciate the importance of this invention, as applied to artistic or line engraving more especially; for a copper plate being once engraved, if submitted to the acierage process, will become a lasting property, not liable to deterioration by printing, and the public may expect to be supplied with the very best impressions at a more moderate charge; whilst, to the numerous branches of commercial engraving, for the ceramic manufactures and others, as well as to the vast number of old engraved copper plates existing in this country, this process is likely to confer an immense additional value.

"I need not say that copper is by no means the only metal to which the process is applicable, for the same principle will be found to answer in the case of other soft metals used for printing purposes; and I shall only add, in conclusion, that although the principle of electrotyping has been applied, up to the present date, in a variety of ways since it was organised by Thomas Spencer, in 1837; this is, I believe, the first time that an attempt has been successfully made to prepare an engraved copper plate with harder metal, with the view of increasing its printing capabilities; and I feel happy to have been the first to introduce so valuable a discovery into this, my adopted country."

THERMO-ELECTRICITY.

We have yet to draw attention to another source of electrical power, which of late years has attracted considerable attention from scientific men. In the previous pages, we have repeatedly noticed how frequently the disturbance of electrical equilibrium is attended with the production of other forces, such as heat, light, &c. These and other facts have led to the idea that each of the undulatory forces has a common origin—a theory which has already been enlarged on. We must now explain the methods by which electricity may be evolved by the action of heat on certain metals, and describe some of the phenomena which result therefrom.

As we have seen, electrical effects result, in some cases, chiefly from the action of intense electricity, or that which has the quality of exerting its effect at a distance from the source of power. To some extent, thermo-electricity falls under this category, but little quantity being noticed in its action. Our thermo-electric batteries, however, have hitherto been on a very small scale, and therefore we may yet perceive greater effects when larger arrangements are made.

A thermo-electric battery is easily constructed. It is composed of a bar of two different metals, generally bismuth and antimony. The bars may be united at one end, as represented in Fig. 92; whilst the others are left free. In the cut, *a* is supposed to represent the bar of antimony, and *b* that of bismuth. Now, if heat be applied at the point, *c*, a current of electricity will be produced; and if a number of these arrangements be connected together, as in Fig. 93, they will form a thermo-electric battery; the current induced passing in the direction indicated by the arrow.

Fig. 92.

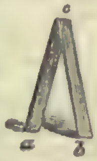
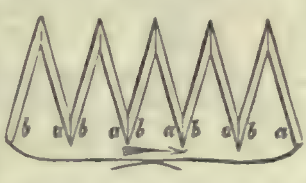
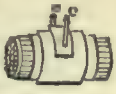


Fig. 93.



Another method of making a thermo-electric battery, is that of soldering, end to end, bars of bismuth and antimony, and of rolling them into a bundle, such as is represented in the annexed figure, in which *c c* represent the wires proceeding respectively from the last bar of each metal. The principle is precisely the same as that described already, but the arrangement is more convenient for many experiments. If wires be conducted from the extremities of the coil, they can be then attached to apparatus just as is done in some voltaic batteries, &c.

Fig. 94.



The cause of electric disturbance, although evidently dependent on heat, is not easily investigated in all its details. Extremes of temperature favour the development of the current; thus, if one metal be heated whilst the other is cooled by ice, a more powerful current is produced than by heating alone. It has been stated that difference of conducting power is partly the cause; and this is borne out, apparently, by the order in which the conducting power of metals is connected with that of thermo-electric production. The following table shows the thermo-electric relationship of some metals in reference to this question; the greater differences between them in position, the more powerful is the battery formed by their union; and hence the choice of the metals, bismuth and antimony, for constructing batteries.

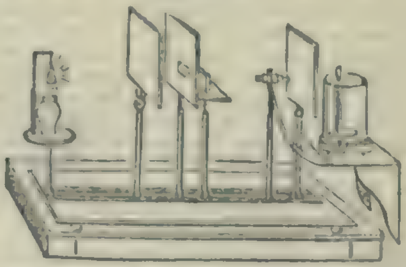
TABLE OF THERMO-ELECTRIC METALS.

Bismuth.
Lead.
Tin.
Copper.
Zinc.
Iron.
Antimony.

This singular method of producing electric currents has been of great value in many respects. In the section on *Heat*, we have already pointed out how it may be employed in measuring changes of temperature, far too minute to be appreciated by ordinary thermometric arrangements. But, besides these philosophical applications, thermo-electric batteries have been arranged with other apparatus, for a variety of practical purposes in daily life, one or more of which we shall have to allude to after our remarks on the Electric Telegraph.

The best method of showing the existence of a thermo-current, is to attach the wires of the battery to a galvanometer. As soon as heat is applied, in the way we have described, the needle is instantly diverged, just as if an electric current from a voltaic battery were passing. The method of applying a thermo-electric current for many experiments, in connection with radiant heat, is illustrated in the following cut; to the left of which we see the source of heat, which, passing through screens of substances under experiment, arrives at the battery. The current thus produced acts on the galvanometer, seen on the right hand of the engraving.

Fig. 95.



ADDENDA ON ELECTRICAL MACHINES.

We have already mentioned, at page 171, a plate electrical machine, which was, at one time, in the Royal Panopticon, London; and Mr. Warner, under whose superintendence it was constructed and worked, has favoured us with some interesting particulars respecting it.

The plate itself was ten feet in diameter, and the machine was provided with double rubbers and conductors. For the purpose of keeping the glass surface free from moisture, steam was allowed to pass into the rubbers, which were constructed of sheet iron. The entire machine was supported by plate-glass columns, so as to insure complete insulation; and the plate was driven by steam power. The number of revolutions averaged about forty per minute, for ordinary experiments; and the rubbing force, or friction, was regulated in the usual manner by screws, &c.

With respect to the force developed, we may observe that this machine never had a fair trial. It was, unavoidably, placed in the centre of an immense hall, round which currents of moist air were continually passing; and in front of the machine was a basin, from which many hundred gallons of water were forced per minute, twice each day, during the exhibition of the Chromatic Fountain. With all these disadvantages, the effects were surprising. Above two hundred feet of Leyden jar surface could be charged in a few seconds; twenty feet of steel wire could be rapidly deflagrated by the battery discharge; and other electrical phenomena were produced on an equally large scale. Under the best circumstances, in dry weather, a spark of upwards of two feet in length, and of great density, could be obtained from the prime conductor; and the usual class experiments with the direct spark, afforded brilliant results. On the breaking-up of the institution, this machine passed into the hands of its constructer; but it is to be hoped that, eventually, its powers will be more fully tested than they have yet been. Such would, no doubt, result in our acquiring additional facts in reference to the powers and effects of electricity produced by friction, and possibly lead to discoveries of great interest.

SECTION IV.

MAGNETISM, AND ITS APPLICATIONS.

CHAPTER I.

The subjects on which we have already treated, will have prepared the minds of our readers for the study of a very interesting class of phenomena, which, although of a restricted character, have yet the most important relations, both in a philosophical and social point of view.

There are few persons who have watched the oscillations of the mariner's compass, but who must have reflected that some unseen agent is incessantly at work maintaining the needle true in its polarity. Amongst all the curious facts which science or daily experience presents to our notice, none seem more extraordinary than that a small piece of metal should constantly point in one direction, if it have been previously endued with the magnetic force. Day and night it ceaselessly obeys some unperceived power: if undisturbed, it truly guides its possessor through the trackless ocean; and in it he puts an implicit faith, which can only be justified by the fact that Nature is never at fault.

It may not perhaps be too great an exaggeration if we remark, that many of the political, commercial, and social circumstances of the present age are due, to a large extent, to this application of magnetism. Without it our ships would be compelled to confine their voyages to the coast-line; our colonies in Australia, New Zealand, &c., would be inaccessible; the construction of railroads and the progress of mines, &c., would be most difficult; the intercourse of nations, separated by the ocean, would be impossible; and commerce itself, with our manufactures, would be confined, for want of raw material, to the narrowest limits.

Another application of the magnetic needle, to which we may here refer, has become of the most extreme importance in every respect. Thirty years ago, the earliest intelligence which could be obtained between our manufacturing districts and the metropolis, required at least twelve to eighteen hours for its transmission. At the present time, it is possible to receive intelligence of any event which has taken place in any capital of Europe, within a few minutes of its occurrence; and within the space of two or three years, there seems every probability that the whole of our Indian possessions will be, in respect to the power of the electric telegraph, within an hour's time of communication with London, or any city in Great Britain. We may also remark on the extreme importance which exists in those discoveries of science that have often been made without any apparent or immediate practical result. Scarcely forty years have elapsed since it was first noticed, that if a current of voltaic electricity were passed by a wire placed over and parallel to a magnetised needle, the latter would be diverged from its usual position, and, according to the direction of the current, the needle deflected to the right or left hand. But we are anticipating the subject on which we shall have to dilate more fully; and, without further preface, we shall proceed to investigate the laws of that force called Magnetism, which we shall subsequently find to be analogous to the other undulatory forces.

As a matter of convenience, we shall adopt the following plan in treating on the subject; and shall describe—

1. Natural Magnets.
2. Artificial Magnets.
3. Magnetic Attraction, Repulsion, and Polarity.

After which we shall enter into a variety of phenomena which are presented by terrestrial magnetism; and other phases of the force.

1. Natural Magnets.—The ancients were well acquainted with the fact, that a stone found in Magnesia had the power of attracting metallic iron; and from this substance, which has since been called the loadstone, the term "Magnetism" has arisen. It may readily be procured from the philosophical instrument-makers. It has a black colour, and a close texture; and if placed amongst iron filings, it immediately attracts them. If a piece of this natural magnet, which is an oxide of iron, be rubbed on a rod of steel, the latter immediately acquires the properties of the loadstone, and becomes, as it is termed, magnetic. There is a singular peculiarity in reference to the permanence of this force, when it is communicated to different kinds of iron. Soft iron is rendered magnetic only so long as it is in communication with a body permanently magnetic. On the removal of such influence, the magnetism immediately ceases. Steel, which is a compound of iron, carbon, and possibly of nitrogen also, has the power of retaining the magnetic influence, and thus of becoming a permanent magnet. Its power of retaining magnetism, depends, however, on its quality. If the steel be badly tempered, or of an inferior quality—that is, approaching the character of soft iron—its retentive power is greatly diminished. In fact, this power of retaining the magnetic force may, to some extent, be employed as a test of the quality of steel; it being borne in mind, that the degree of hardness of the metal also affects the result.

Cast-iron—which is what we may term an analogue of steel—may be often rendered permanently magnetic. Its composition is, however, so varied as to leave us in constant doubt as to its character. Indeed, some specimens of cast-iron far exceed in hardness those of steel. This is owing to a process of cementation, or partial conversion into steel, which the external surface undergoes when in contact with the coal and coke used as moulds in casting the metal. We have seen specimens of this nature which were hard enough to cut glass. An artificial loadstone may be prepared by making a paste of oxide of iron: this, however, deserves notice simply as a philosophical curiosity.—We shall now proceed to the second and more practical division of our subject.

2. Artificial Magnets.—The most simple manner by which an artificial magnet can be produced, is that of rubbing a piece of the loadstone on a common sewing-needle. It will thus acquire two properties: first, that of attracting any piece of iron; and, secondly, of possessing polarity, or a tendency to point in the direction *nearly* of the pole star, in the constellation of Ursa Major. It hence appears that magnetism can be communicated to any extent; and not only from a piece of

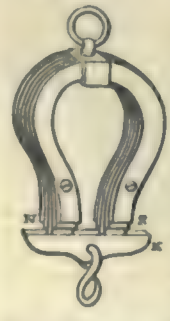
natural loadstone—but if the magnetised needle be rubbed against another one, the latter will also exhibit similar properties to that from which it acquired them.

The extent of magnetic force so communicated, depends on the size of the loadstone and of the piece of steel which is magnetised. In every instance in which force of any kind is communicated to any body, there is a definite relation between the body possessing and that receiving a force, with respect to their mass. This is a natural consequence of the constitution of any mass, because its atoms have, either chemically or dynamically, a fixed amount of attractive power, which they can exert on any proximate atom within their influence. This power can be mathematically calculated. The circumstances may vary considerably, but the laws are in every case constant; and the attraction of gravitation is perhaps the best illustration which we can adduce to prove our position. For instance, we may suppose that any body exerts an attractive power on another near to it. Such a power must necessarily have some relation to the mass or other qualities of the body which it attracts, and the distance existing between. This relation or comparative effect of force may be ascertained in a variety of ways. In the case of gravitation, the time occupied by any body in falling from any height, may become a measure of the force. But in the case of magnetism there is no such opportunity of calculating the value or amount of its force. There exists a kind of antagonism in the magnetic effect, which the earth, and the electric currents which circulate around it, exert. It is true that we can, by the oscillations of a magnetic needle, approximate towards an idea of the amount of magnetic attraction which may exist between terrestrial magnetism, and an artificially induced magnetic arrangement. But still we are left in doubt as to the exact proportion; and the varied affections which may exist between the two phases of the same force, interfere considerably. For instance, there is a daily variation in the polar direction of the magnetised needle. This alone is sufficient to show that a considerable uncertainty exists between the indications afforded by our best arrangements, and the permanent, or continuous and regular, indications which the earth affords of its magnetic intensity.

These remarks may at first sight seem impertinent to the subject on which we are treating: but such is not the case; because we shall have to examine into the comparative power of various magnetic arrangements, and the causes which influence it.

We may here remark, that a straight or bar magnet has two ends, which exhibit the greatest amount of magnetic force; and that the point half-way between the extremities seems entirely destitute of magnetism. Hence, in making an artificial magnet, special attention is paid to these extremities, or poles, as they are frequently termed. A horse-shoe magnet is simply a bar magnet bent into the form of a horse-shoe; and the advantages which this form presents is, that the two poles are brought near to each other, and can so exert simultaneously their power on any object presented to them.

Fig. 1.



When a number of horse-shoe magnets are bound together, they form what is called a compound magnet. One of these is illustrated in the annexed engraving, in which N and S represent respectively the north and south poles of the magnet, and K is what is called the *keeper*. This is a piece of soft iron placed at the poles of a horse-shoe magnet, for the purpose of preventing any loss of power. We may here remark, that either straight or horse-shoe magnets should always have a piece of soft iron, or a quantity of iron filings, at the poles; which, by their reaction on the magnet, prevent its losing its attractive force.

There are two modes of making magnets;—the one by touch we have already alluded to: that by means of voltaic electricity we shall describe under the head of Electro-Magnetism. The simplest mode of making a bar magnet, is that of rubbing a piece of good steel, continually, by means of a compound horse-shoe magnet, such as we have just described. This, however, must be done in a proper manner; the steel being placed on a level table so as to be rubbed from its centre towards its extremity by one pole of the horse-shoe magnet; and this will induce in it a magnetism of an opposite character. The other extremity is then to be rubbed in a similar manner, from the centre of the steel, by the other pole of the horse-shoe magnet; and this is to be repeated until the bar is rendered as magnetic as possible. This, as we have already mentioned, will depend on the quality of steel, and the strength of the magnet employed. Bar magnets are best kept in pairs, and the north pole should be marked on each. In laying them aside, the north pole of one should be in contact with the south pole of the other—a plan which, like that of the keeper, prevents a loss of power.

3. Magnetic Attraction, Repulsion, and Polarity.—We have already had to refer, in our previous observations, to the fact, that a magnet has the power of attracting pieces of iron or steel towards it. We have also mentioned, that the extremities of a magnet are termed "poles." We shall now enlarge on these matters, and investigate some interesting phenomena, and applications depending on them.

If a piece of steel, in the shape of a long rod or bar, be magnetised, and then hung by a silk thread so that it may move freely, it will be found to take up a position pointing nearly north and south, if no disturbing cause be present. This result indicates that some force must act on the magnetised needle; and such is the magnetism of the earth itself. If, while the bar is thus hanging, another magnet of equal power and size be presented to it, pole to pole, the first needle will be either repelled or attracted. If a north pole of one be presented to the south pole of the other, or *vice versâ*, then these will attract each other. If, however, the north poles or the south poles be presented to each other, then they will repel instead of attracting, as in the last case.

From this experiment we learn that similar poles repel, whilst dissimilar poles attract each other; and this law holds good in every shape or form which a magnet may possess.

The simplest mode of trying this is by magnetising two sewing-needles, and floating one of them in water, which is easily done by gently laying it on the surface. The film of air always attached to polished surfaces enables it to swim on the liquid. The poles of the other needle may be alternately presented to either end of that swimming on the water, and the phenomena of attraction, repulsion, and polarity conveniently studied.

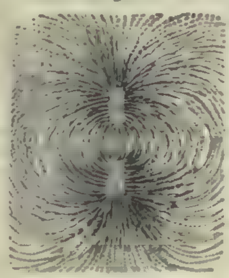
It will be found, that as the distance between the two needles is increased, the effect they mutually exercise is also decreased; and the ratio of this decrease will, for any two points in each magnet, be, like all radial forces, as the square of distances. It must, however, be borne in mind, that we refer here to two mathematical points, rather than to the extension of a magnetised surface: for it is evident that the magnets, being of considerable length and breadth, the force they exercise will be given off in many directions. Could we obtain a magnet in the form of a sphere, then we should doubtless find that the force obeyed the laws above-mentioned; but as, in all ordinary forms of magnets, the length exceeds the breadth—and hence the laws of force must be of unequal length—so the law of radial forces will not have an exact application practically. These observations apply equally to the attracting and attracted body.

It must also be kept in mind, that the poles, or chief attracting portions of a magnet, are not at its extremities. They exist at a short distance nearer to the centre, and equi-distant from that point; and thus, if the two ends of a horse-shoe or bar magnet were slightly hollowed out between the two sides, the chief centre of attraction would be more nearly reached by any body exposed to their action.

The attractive and repulsive powers of a magnet are equal to each other; and, indeed, they may be regarded as the opposite conditions of one force. In this consists the analogy between magnetism and the double electrical theory adopted in reference to frictional electricity. Just as we presume that the inside and outside of a charged Leyden jar[*] are in opposite states of electrical excitation, and yet that such are exactly equal in amount; and, again, as similarly electrical bodies repel, and dissimilar attract, each other[†]—so we have identical conditions and affections of the magnet. We may also illustrate the condition of a magnetised body in a similar manner to that adopted in instances of electrical excitation; for as the alternate sides of the atoms of an electrified body are supposed to be in opposite states of electricity, so the particles of steel composing a magnet, have respectively, on their opposite sides, a north and a south pole—the whole magnet being built up of such until the two extremities form the two opposite poles.

A very beautiful illustration of the effects of magnetic attraction, is afforded by placing a sheet of writing-paper on the poles of a horse-shoe magnet, and sifting over it some fine steel or iron filings. The paper is then to be gently tapped, when the filings will arrange themselves in various forms and at different distances, producing what are termed magnetic curves, the appearance of which is illustrated in the annexed engraving. The same effect may be produced by a bar magnet. Artificial magnets produced by the rubbing process, never exhibit so great an attractive power as those produced by soft iron when surrounded by a voltaic current, and hence termed an electro-magnet. We shall, however, defer speaking on this point until we treat on the subject of Electro-Magnetism.

Fig. 2.



Pieces of steel acquire the power of magnetic attraction, repulsion, and polarity, if they are kept in some positions for a considerable time. A poker will thus often be found magnetic if it be kept in a position nearly due north and south. Old iron window-frames are similarly affected. In fact, masses of cast-iron and steel are frequently found to have acquired magnetic properties from such causes. This results from the inductive action of the magnetism of the earth, which, like electricity, can induce its effects in bodies at a distance from its source.

Nickel and cobalt have decided magnetic properties under certain circumstances; but such are extremely feeble compared with those possessed by iron and its compounds. The magnetic force exercises its influence through bodies which are incapable of being acted on by it. Thus, if a magnetised needle be placed on the surface of water in a basin, and a horse-shoe or other magnet be brought near to it whilst the material of the vessel intervenes, it will be found that the attraction and repulsion are as actively exhibited, as would occur supposing that no substance was placed between the two magnets. Most solids, liquids, and even gases, may, however, be affected by the magnetic force. This we shall more fully explain when we treat on Dia-Magnetism.

THE MARINER'S COMPASS AND THE DIPPING-NEEDLE.

WE have preferred to introduce our readers to a general knowledge of magnetic attraction and polarity, before directing their attention to the very interesting subjects of the Mariner's Compass and the Dipping-Needle, because these depend on a special application of the laws we have just explained.

We have, however, hitherto confined our attention to the artificial magnet, or the loadstone: we must now extend our ideas beyond such limits, and transfer all the powers and laws we have examined, to the earth, as the great source of magnetic attraction. We may conceive, in fact, the earth to be an immense magnet, having polarity, attraction, &c., as have the feeble instruments we employ in our researches into the nature of this force. Indeed, all development of magnetism which we observe, is due to the inductive power of the earth on the bodies we find magnetic. The magnetic poles of the earth vary, from its natural polarity. The north magnetic pole is situated in long. 96° 45′, and lat. 70° 5′. The magnetic equator is therefore, of course, not coincident with that of the earth.

The mariner's compass consists of a bar of steel which has been well magnetised—mounted on a pivot, so that it can move as freely as possible in a horizontal direction. Attached to, or placed beneath it, is a circular card, on which are engraved the different points of the compass. This card is divided into thirty-two parts. Each of these parts, or points, are subdivisions of the four cardinal points—north, east, south, and west; and the intermediate points between any of them, acquire their joint names; such as north-east, south-west, south-south-east, &c.

Generally speaking, at least two compasses are carried on board a vessel, so that in case of any damage to either, another may at once supply its place. The greatest care is observed to place the compass in a position in which any external object shall have as little effect as possible upon it, as any derangement might lead to the most serious results; of which we shall speak more fully as we proceed.

The needle, as we have already stated, does not point due north; that is, it does not lie in a plane parallel to the meridian of a place—which, of course, is due north and south. This deviation is termed the *variation* of the compass; and it is undergoing constant, although gradual change. This variation also differs for different places on the earth's surface. About 300 years ago, the needle pointed 11° east of north. In 1657, it pointed due north and south; and at the present time its variation is about 23° west of north; each of the above referring to observations made in London. Tables are constructed, giving the variation of the needle for different places; and we need scarcely remark that such are of great importance to the mariner.

But besides the horizontal motion of the magnetic needle, a bar of steel, if properly balanced, so as to be capable of a vertical motion, will, on being magnetised, lose its balance, and bend at an angle towards the earth. Such an arrangement is called a *dipping-needle;* and the variation from a horizontal line is called the dip. At the present time, the dip for London is about 67°; and, like the horizontal direction of a needle, is constantly varying. If a needle were placed in lat. 70° 5′ north, and long. 96° 45′ west, which is the position of the magnetic pole of the earth, it would then assume a vertical position.

Apart from any accidental influence, the needle varies daily in its indications; the maximum being during the day, and the minimum during the night, in any locality. It has hence been assumed that the sun has a definite action on it; and some have gone the length to suggest that that luminary is the great source of magnetic force. When, however, we discuss the influence of electric currents on a magnetised needle, we shall find abundant reasons for the effects we have named. There are other disturbing causes which act on a magnetised needle occasionally. It has been noticed, for instance, that the appearance of the aurora borealis has a decided influence on the direction of a needle; causing it, at times, to fluctuate considerably. Indeed, the needle indicates the presence of that splendid phenomenon, when, from any cause, it is invisible to the eye.

The diurnal variations of the needle have been made the subject of anxious investigation by most of the leading astronomers in the observatories of Europe and

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[*] See *ante*, p. 179.

[†] See *ante*, p. 175.

America; and a vast fund of valuable information has been acquired. At the present time the observations are registered by means of photography;* and thus the incessant care required on the part of observers, and the chance of erroneous results, are entirely avoided. There is also the great advantage of no intermission being required, as, by means of properly prepared paper, a lamp with a reflector, and a clockwork arrangement, the indications print themselves, and the photograph, on removal from the instrument, shows precisely all the variations which have taken place, and the exact moment of the occurrence. One of the best instruments of this kind is that which was constructed by Mr. Brooke, for the Greenwich Observatory.

Returning to the subject of the compasses of a ship, we may notice some special influences to which they are subject, and the remedies which have been employed to obviate such. They may be divided into accidental, local, and, possibly, atmospheric.

It would be impossible to enumerate the accidental circumstances which may arise, to cause the derangement of a ship's compasses. Articles of iron placed near them are a frequent cause; and an intelligent captain once informed us, that, for four hours, he had been steering, on a dark night, out of his course, owing to the stupidity of a cabin-boy, who had placed *a box of knives* close to the compass.

The local influences to which the needle is liable on board ship are of great importance, and depend on the nature of the material of which the ship is built, the cargo, the engines of a steam-vessel, and the position of the two latter in reference to the compass.—As soon as the ship is completely loaded, she is taken to any convenient place in which she can turn round in an entire circle—being held by her anchor—as the tide rises and falls. In this process the direction of the needle is carefully watched in reference to any distant and fixed object, and the variation, if any, is noted. Her compasses are then adjusted—that is, magnets are placed near, so as to entirely counteract all the influences which may affect them on board, and to which we have just alluded. If the cargo be undisturbed until the vessel reach her final destination, and no accidental or atmospheric influence be exerted, such adjustment would be a sufficient safeguard for the whole voyage. If, however, a portion of the cargo be discharged at intermediate ports, it may happen that the adjustment previously effected will become a source of danger; because the disturbing element being removed, its intended remedy necessarily becomes a source of disturbance itself. This is a matter of common occurrence on board coasting vessels: we observed an instance of this, some years ago, in a steam-vessel which had left port with a large cargo of iron on board. This was discharged at an intermediate port; and the needle was thereby affected to the extent of between two and three degrees. It is highly probable that many of the shipwrecks which annually occur round our coast, may owe their origin to this constantly-occurring circumstance.

With respect to atmospheric influences, especially as regards fog, great diversity of opinion exists. Many persons insist that the needle ceases to act in a dense fog; whilst others equally deny its influence: and thus when any vessel is lost during foggy weather, the evidence offered as to the cause, is always accompanied with the special views of the witnesses on each side. In the absence of any reliable data, we shall mention two instances of our own observation, taken within a few days, on the east and west coasts of England and Scotland.

In a voyage from London to Edinburgh, in an iron screw steamer of about 600 tons burthen, and with a general cargo, a dense fog set in immediately after leaving the Pool. This continued for twenty-eight hours; during which, for safety's sake, the speed was never allowed to exceed five miles per hour, and the anchor was cast frequently, owing to the positive danger of proceeding at all. There were three compasses on board—one at the stern, another on the hurricane deck, and one in the cabin. The writer compared their indications, repeatedly, each hour during the time mentioned; and in no case did either, or all of the compasses, vary from each other to the extent of one degree. A pocket compass, the needle of which was delicately hung, agreed with the average of the ship's compasses. There was not the slightest motion of the vessel, except from the vibration of the screw. So far, no indication of the action of fog was afforded. In another instance, however, which occurred in a voyage between Ayr and Campbelton, at the mouth of the Clyde, very different results were noticed. The voyage was commenced in a dense fog, and no special cause of disturbance was on board, the vessel being built of wood, and no iron was contained as cargo. After steering direct westward towards the Isle of Arran for about an hour, with the intention of calling at a port at its *southern* end, it was discovered that the vessel had been steered gradually towards the north, although the compass had indicated a westerly course. The needle, in fact, varied about 90° from its proper indication; and this increased until the setting sun appeared in the north-east, instead of slightly south of west. During the whole time the compass had been shaken so as to cause the needle to move freely; besides, the vibration from the paddles ought also to have been quite sufficient for that purpose. At the end of about two hours' time from the commencement of the voyage, a brisk wind sprang up, and almost immediately afterwards the compass-needle veered round to its proper direction, and gave no more cause for anxiety. In the former, and in this instance, the circumstances were exactly the same; for a breeze sprung up and cleared away the fog at the end of the twenty-eight hours, yet the compasses showed no change of direction from that which they had indicated previous to the clearing-up of the weather. The observations in each case were conducted with great care, and with that scrupulous hesitancy as to theoretical opinion, which the study of experimental science makes a habit; and we could not find, on free conversation with the captain of each vessel, that any extraordinary circumstance of any kind was present which could vitiate the result.

We may, however, state, that three or four iron vessels have been lost between the Isle of Arran and the west side of the coast of Argyllshire, near Campbelton, since the above observations were made. In each case the cause was assigned to the fog acting on the compasses. We may venture to suggest that there may be some local cause in the constituents of the rocks, &c., of that neighbourhood, especially as we learned from an intelligent scientific observer, that one or two masses of rock near Campbelton, have a decided effect on the magnetic needle. However, we merely throw out the idea as a suggestion rather than as expressing any opinion on the matter, which, of course, can only be decided by minute and scrupulously exact observations.

The effect of electrical currents we have already referred to; and we shall have to enlarge considerably on the subject in the next chapter. They are scarcely of much moment during the voyage of a vessel, because, being local with respect to extraordinary occurrence, and the position of the vessel continually varying, their force and intensity can be practically, except in very special cases, of little consequence.

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* See *ante*, p. 163.

CHAPTER II.

ELECTRO-MAGNETISM.

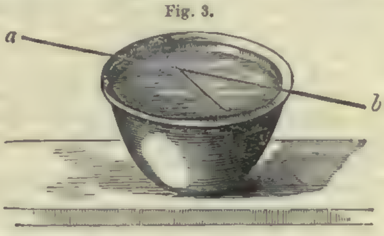
No branch of experimental science has made such rapid progress as that to which we shall now direct attention. This is chiefly owing to the discovery of voltaic electricity, by means of which we have the power of obtaining any amount of current force of great quantity, which is an essential point in all electro-magnetic researches. For most of the experiments which we shall adduce, but little apparatus is required, and such can easily be constructed by attention to the directions we shall give. A voltaic battery of either Daniell's, Smee's, or Grove's form, will be required; one cell, however, being sufficient for most purposes. When larger arrangements are needed, we shall mention the fact as we proceed. A quantity of copper wire, covered with cotton or silk, of No. 16 and 22 gauge; a strong horse-shoe magnet, and some good steel needles, will also be requisite for the majority of the experiments. With these the student may make an extended investigation of the leading laws and phenomena of electro-magnetism, and thus acquire a competent knowledge of the subject at a trifling expense.

In studying the facts of which we shall have to speak, several difficulties may arise in the minds of those of our readers who are not prepared to take an independent view of the action of two forces, which, whilst seeming to be antagonistic, are really due to the same cause. For instance, we shall show that a current of electricity which produces magnetic phenomena, in some cases acts in opposition to the force it may generate. This difficulty, however, if carefully examined, will prove rather an advantage than otherwise to a careful and attentive student, because it will call out his reasoning and observing powers, and so add, as far as he is concerned, an additional interest and value to this branch of philosophical pursuit. We must, however, remark, that all such opposition of the forces to which we refer, is more apparent than real, and results from the direction of the electro-current being modified in reference to the plane of the magnetic force: this, however, we shall fully inquire into as we proceed.

Perhaps the most convenient and instructive course which we can take in introducing our readers to the laws of electro-magnetism, is that of proving, in the first place, that a current of voltaic electricity affects the magnetic needle when certain particulars are observed in carrying out the experiments which illustrate this fact. This may readily be ascertained by means of the following arrangement, which we suggest as one of the simplest which can be used for the purpose:—

Experiment 1.—Fill a basin nearly to its brim with water, and float on its surface a magnetised sewing-needle.* When the needle has ceased to vibrate, and is therefore quite at rest, place over, and exactly parallel to it, a straight copper wire, at a distance of about a quarter of an inch from the needle, so that the two shall lie on the same straight line. Now, so long as no current of electricity passes by the copper wire, the needle will not be deflected from its position; but if a current of electricity be sent from one cell of any form of the voltaic battery, then the needle will instantly be moved from its position, and be deflected right or left, according to the direction of the current. A voltaic battery, for this experiment, is easily made by soldering a wire to a small piece each of zinc and copper. These metals are then to be placed in a vessel containing water, to which a tenth part of its bulk of sulphuric acid has been added. On the terminal wires being brought in contact with the wire placed over the needle, the deflection of the latter will be immediately observed. Of course the two plates of metal must not be allowed to touch each other. The following engraving illustrates the arrangement we have described, and shows the deflection of the needle during the passage of the current by the wire.

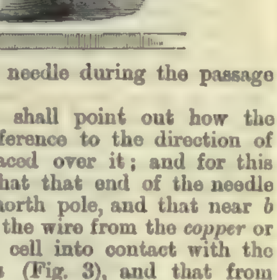
Fig. 3.



Experiment 2.—We now shall point out how the deflection takes place in reference to the direction of the current by the wire placed over it; and for this purpose we shall consider that that end of the needle nearest *a*, in Fig. 3, is its north pole, and that near *b* to be its south pole. Bring the wire from the *copper* or *platina* plate of the battery cell into contact with the wire over the needle at *a* (Fig. 3), and that from the *zinc* end of the cell into contact with the wire at *b*. The current of electricity passes *in* the battery from the zinc to the copper, and thence by the wire from *a* to *b*. In this direction it will deflect the needle towards the *right* hand of the observer, supposing that he looks at the needle from its south, towards its north pole. Afterwards, remove the wires of the battery, and adjust the needle so that it may again be at rest, parallel to the wire passing over it on the basin. Then allow the current from the battery to pass from *b* to *a* by placing the wire from the copper plate at *b*, and that from the zinc plate at *a*. The needle will now be deflected towards the observer's left hand, the current passing from *b* to *a*.

This will be still further illustrated by means of the annexed diagram; in which No. 1, Fig. 4, represents the current as passing downwards from *a* to *b*, in the direction of the arrows; whilst, in No. 2, it is passing in the direction from *b* to *a*. The north pole of the needle is thus diverged to the right in No. 1, and to the left in No. 2, owing to the reversal of the current in the latter case. It is of the greatest importance that the student should bear in mind these laws of deflection, as they exercise great influence in all the phenomena of electro-magnetism; and, although our engravings are explicit enough to explain the facts, it is far better that the experiment should be frequently repeated in the simple manner we have just described, as that will certainly impress the truth more firmly on the memory than any amount of study could do. We may here add, that when we describe the form of electric telegraphs in common use in this country, we shall presume that the facts we have thus illustrated have been fully understood by our readers.

Fig. 4.



No. 1. No. 2.

From these experiments we can readily deduce the first law of electro-magnetism; which is, "That a current of electricity affects the magnetic force at right angles to the direction of the current itself." In applying the law practically to the two last experiments, we observe, that as the current passes by the wire, the magnetic needle assumes a direction approaching that of a right angle to the wire. With a powerful battery the right angle is nearly completed.*

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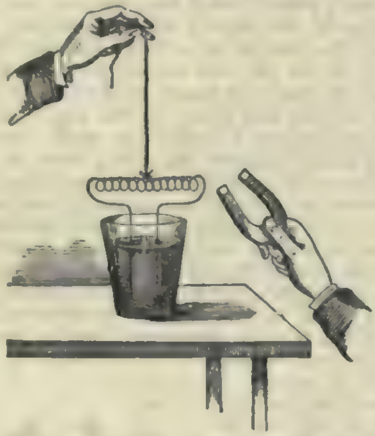
* See *ante*, p. 244.

Whenever a wire conducts a voltaic current, it manifests magnetic tendencies in a direction at right angles to that of the current: in other words, supposing that a current of electricity be sent through a copper wire in a direction from north to south, or *vice versâ*, the magnetic effect will be observed on its east and west sides, and, in fact, all round the wire in any position at right angles to the course of the electric current. By extending this principle practically, we may make a voltaic conducting wire, although of copper, to present all the phenomena afforded by an ordinary piece of magnetised steel; that is, it will be subject to the laws of attraction, repulsion, and polarity,† just as if it were an ordinary magnet. The following experiments will illustrate the fact. We shall choose two simple pieces of apparatus, which may readily be constructed by the reader, and for which we shall give full directions.

Experiment 3.—Solder to a piece of covered copper wire, of about eighteen inches in length, a narrow strip of zinc and one of copper—one at each end—and then twist the wire into a spiral form, so as to leave the strips of metal hanging beneath it. Let these dip into some sulphuric acid and water, and suspend the spiral by means of a piece of thread, so that the whole arrangement may freely move in any direction. Its entire appearance is represented in the following engraving.

Fig. 5.



It will be found that the spiral wire will have become powerfully magnetic. It will assume a direction of nearly north and south, just as a magnetic needle would do; and if a horse-shoe magnet be presented to it, the poles of the spiral and the magnet will either repel or attract each other, dependent on the poles which are experimented on, and according to the laws which have been previously explained when treating on magnetism.‡

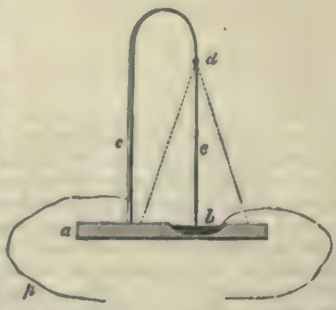
It is thus evident that a most intimate connection exists between magnetic force and a current of electricity; and the next experiment will still more familiarly prove the fact.

Experiment 4.—Immerse a wire conveying the voltaic current in some iron filings. It will be found that so long as electricity passes, some of the filings will adhere to the wire—indicating the existence of magnetic attraction on its surface.

A great variety of apparatus has been invented for the purpose of illustrating this mutual action of electric currents and magnetised bodies, by means of which motion may be readily produced by their opposition or attraction of each other. We may select two of these as still further illustrative of the laws we have been speaking of; and we have chosen such as may be readily constructed by our readers.

Experiment 5.—Fix in a wooden foot, which may be six inches long and three broad (*a*—Fig. 6), an upright brass or copper wire, *c*, which must be bent round at the top, so as to hang over a hollow trough cut out of the foot, as seen at *b*. From the wire *c* suspend another, so that it can move freely in all directions, and fill the trough *b* with mercury, so that it may touch the extremity of the wire *e*, suspended from *d*. Now, of course, as such has no force acting on it, it will remain perpendicular. Attach one of the wires of a cell of a voltaic battery to *c*, and dip the other wire into the cistern of mercury *b*. A current of electricity will now pass through the wires—say from *p* to *d*, thence by *e* to *b*, and so back to the battery. If a horse-shoe magnet be now placed on the wooden foot, so that one of its poles shall be on each side of the suspended wire *e*, that will be instantly put into motion, and will be rapidly jerked out of the mercury. By this, contact with the battery will be broken; and as all effects are thus stopped, the wire will fall back into the mercury. Contact and a complete current will be again established; and the wire will be again jerked from the cistern. The direction of this motion, as indicated by the dotted lines, will depend on that of the current and on the position of the poles of the magnet, and it may be continued so long as the battery power is sufficiently maintained.

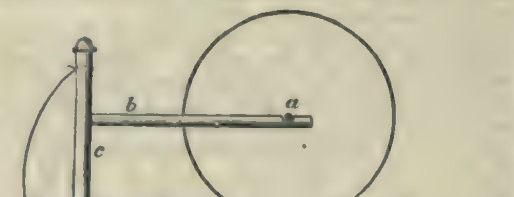
Fig. 6.



We may just observe, that for the success of all electro-magnetic experiments, it is highly necessary that the ends, &c., of the wires be kept perfectly clean. This is best effected by dipping them into a little nitric acid, and then into mercury. By this they will be slightly amalgamated, and much disappointment will be prevented, which, at all times, arises from carelessness as to this particular.

Experiment 6.—The arrangement we are about to describe, actson precisely the same principles as the last one; but instead of a jerking motion, a circular one is obtained. In fact, we may regard the wheel we shall mention, as made up of an infinite number of wires, or radii, proceeding from a common centre, instead of being suspended from a hook, as shown in our last engraving. To construct this arrangement, cut a circular wheel out of thin sheet copper, and through its centre solder an axle of copper wire, as at *a*, Fig. 7, so that it may freely turn thereon; and supported by a brass arm, *b*, which is to be fixed to an upright, also of metal, *c*. This may stand on a wooden foot, hollowed out at *e*, as in our last arrangement; and the rim of the copper wheel should just touch some mercury contained in the trough *e*. One wire of a battery is to be attached to the upright *c*, and the other is to be immersed in the mercury in the cistern. On placing a horse-shoe magnet, with its poles resting on the foot, on each side of the wheel, the latter will begin to rotate, and will maintain its motion so long as the current of electricity passes.

Fig. 7.



This arrangement may be modified by using a wheel with spokes, instead of the circular form we have described. Such will combine the principles of construction illustrated in our two last engravings; for, whilst it has a rotary motion, each spoke, as it leaves the mercury, is, as it were, similarly jerked out in the way we mentioned in the case of the wire. The annexed cut will assist our readers in making a wheel of this kind: all other matters are the same as in that form of instrument we have just noticed.

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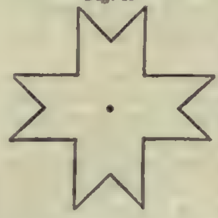
* See *ante*, p. 195. † *Ante*, p. 244. ‡ *Ante*, p. 244.

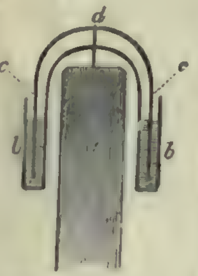
Fig. 8.



If a small voltaic battery be suspended round each of the poles of a horse-shoe magnet, the metals of which it is composed will revolve rapidly round the magnet, and in reverse directions at each pole. These arrangements are exceedingly ingenious, but very troublesome in use. In the following engraving one of them is illustrated. They are sold at the instrument-makers; and give an additional proof, when in action, of the mutual influence of electric currents and magnets.

In Fig. 9, *a* represents a pole of a powerful circular horse-shoe magnet; *b* is a copper bucket, suspended at *d* on the top of the magnet, round which it may freely rotate; *c* is a zinc cylinder which hangs inside the copper one. The latter is to be filled with some acid and water, when a voltaic battery is at once formed. As soon as this is charged the cylinders turn round rapidly on their axes at *d*, which action is due to the same laws as those we mentioned in connection with our three last experiments.

Fig. 9.



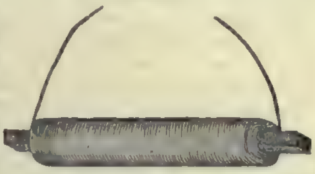
Whilst referring to the magnetism of the earth, we spoke of electric currents as affecting the magnetic needle;* and also, when describing the dipping-needle, we mentioned that there existed magnetic poles. Mr. Barlow, many years ago, invented a most ingenious arrangement, which consisted of a number of wires (corresponding with the circles of latitude on a globe), which were wound round a wooden ball, and were intended to represent the direction of electric currents as they pass round the earth. If a current of electricity be sent from a voltaic battery round a ball so arranged, and a vertical magnetic needle be presented to it in different parts, the various inclinations or dips are at once produced. Such an arrangement forms a curious and instructive illustration of the production of magnetic phenomena by means of a voltaic current, and also affords to the mind an idea of the effects which occur on the large scale in nature.

ELECTRO-MAGNETS.

We have hitherto confined our attention to simple instances of the mutual action of electric and magnetic currents; and shall now proceed to point out how, by applying the laws already discussed, we may produce an immense development of magnetic attraction in soft iron, by means of properly directed voltaic currents. If a piece of steel be rubbed by means of an ordinary magnet, of any form, it retains a portion of the magnetic force; but soft iron does not; and it ceases to exhibit any sign of attraction so soon as the magnet which had induced it is withdrawn. In fact, the result is simply one of induction, and, therefore, ceases on the inducing power being removed. The same holds good in reference to the magnetic force induced by electricity; as we shall discover in trying any of the following experiments:—

Experiment 7.—Twist some covered copper wire round a roller, so as to form a spiral of a few inches in length, and, removing the roller, insert in its place a rod of soft iron. Connect the two ends of the spiral with the wires proceeding from a voltaic battery. Immediately that the current of electricity passes round the iron, it induces, at right angles to its direction,† magnetism in the rod; and if some nails or iron filings are presented to the extreme ends, they will be powerfully attracted. The annexed cut illustrates this simple arrangement. At each end of the coil the iron rod is seen projecting; and the wires terminating the coil are also shown.

Fig. 10.

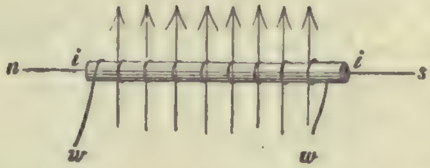


Experiment 8.—So long only as the current of electricity passes, does the iron exhibit magnetic attraction; for, on removing either of the battery wires, the iron nails will at once fall down, and the iron rod returns to its normal condition.

We must here note the fact, that the magnetic effects are induced at right angles to the direction of the electrical current; and to assist the student, we introduce the following cut.

Fig. 11 represents a skeleton view of the spiral coil and iron rod: *n* and *s* show the poles of magnetic force induced in the bar of iron *i i*. Now, the direction of the current of electricity from one of the wires, *w*, to the other on its return to the battery, is indicated by the arrows; and if the spiral be wound quite straight round the rod, then it and the current will necessarily be at right angles to the iron which they encircle. We have already seen that the sides of a spiral are magnetic when it conveys a voltaic current;‡ and it follows that the iron enclosed therein partakes of that change or effect. Indeed, the simple arrangement illustrated in Fig. 11 is quite of sufficient power, when in action, to induce magnetism in a small soft iron wire, if such be placed in its centre; and thus the application of the laws of which we have so repeatedly spoken, is evident.

Fig. 11.



This law, however, may be made more special in the instances to which we are now referring, by stating, "that a current of electricity passing round soft iron, converts the metal into what is termed an electro-magnet, when its direction is at right angles to the iron.

We have seen that the magnetic effect produced on soft iron is but temporary; that is, provided that the metal be perfectly pure. But this is rarely the case, because iron is mixed with other substances which affect its nature. Carbon is invariably present to a small extent, and hence the metal does not lose its magnetism instantly on the current being withdrawn. The magnetic effect remains for some time; and this is called *residuary magnetism*, which frequently becomes a source of great difficulty and annoyance in many of the applications of electro-magnetism. A perceptible interval of time is required to produce the highest effect of magnetic induction; and as in the case of ordinary magnets, the poles do not exist exactly at the extremity of the metal rod. We shall now examine the effect produced on steel by the voltaic current as it passes round a rod of that metal.

Experiment 9.—Place inside a coil of covered copper wire (which, for this purpose, may be wound round a paper or gutta-percha tube), a bar of good steel, which has been previously tested, so that the absence of all magnetism has been proved. As soon as a current of electricity passes over it, the steel will become magnetic, as may be shown by holding any piece of iron near it. But it will be noticed, on the current ceasing to pass by the removal of the wires from the battery, that the metal does *not lose* its magnetism. On the contrary, it has been converted into a permanent magnet. Herein, therefore, exists a similar difference between soft iron and steel in reference to electric as in magnetic induction; and both, doubtless, are due to the peculiar molecular constitution of steel, arising from the presence of carbon, and most probably, also, of nitrogen in it. Knives, needles, and any other steel object having two ends, may thus be at once rendered magnetic.

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* See *ante*, p. 245.

† See *ante*, p. 247.

‡ See *ante*, p. 248.

Before describing electro-magnets of large size, and electro-magnetic engines, which have been constructed with a view to the production of motive power, we will inquire into the circumstances which tend to increase or diminish the effects which can thus be produced by electricity on iron or steel.

These may be epitomised as follows:—

1st. The length of the coil, and the thickness of the wire composing it.

2nd. The number and size of the plates of the voltaic battery employed.

3rd. The purity of the metals used in respect to the copper wire and the iron core.

1st. *The Length, &c., of the Coil.*—It is a rule in electrical science, that the longer the conductor, the greater is the resistance afforded to the passage of a current; and it is equally true, that the greater the transverse area of the conductor—or, what is the same thing, its thickness—the less the resistance afforded by the same length of wire. It has been found, that a copper wire, of No. 16 gauge, has a conducting power of 100; whilst a thinner one, of No. 18 gauge, has only a power of 65. The following table, for which we are indebted to an able work by T. Shaffner, Esq., of Kentucky, *On the Electric Telegraph*, illustrates the fact of the difference of conducting power in wires of various thicknesses. The experiments were tried at the Gutta-Percha Company's Works, in London, on wires each 100 miles long; and these were submerged in the Regent's Canal—every care having been taken that the external coat of gutta-percha was faultless. The figures indicate the deflection of the needle of a galvanometer* for each gauge of wire employed.

No. of Battery Plates.	No. 18 Gauge.	No. 16 Gauge.
3 pairs	29°	39°
6 ,,	50°	59°

We thus observe how much the resistance is increased by a deficiency in the size of the conducting wire; and it follows that a great length may be, to some extent, compensated for by an increased thickness of conducting material.

2nd. *The Number and Size of Battery Plates.*—The remarks we have just made, and the table quoted, lead us to consider the effects produced by the size and number of the voltaic batteries which may be employed; and we must here point out that *quantity* of electricity is produced by *large plates*, irrespective of the number; whilst *intensity* results from the *number* of plates, rather than their size.

Now, the amount of magnetic effect produced on iron by electricity, depends, *theoretically*, on the quantity of the current alone; but this is only true on the supposition that the conductor offers not the slightest resistance to the passage of the current. But, in practice, we know that such never can occur; for the longer the wire, the greater the resistance afforded; and as each successive coil increases the amount of inductive effect, we are necessarily compelled to employ long wires in constructing all electro-magnetic arrangements. To overcome the resistance of the wires as to their length, we are therefore under the necessity of using, not only large plates, but a number of them. The size of the plate affords the necessary quantity of electricity; and the number of the plates assists that quantity, by their intensity, to give out the full force which one large plate would do were no resistance to be overcome. It therefore follows, in practice, that both size and number of plates must be considered in all attempts to produce the greatest amount of electro-magnetic induction under most circumstances. We always presume that copper wire is used in our experiments; as, for various reasons, none other, in most cases, is admissible.

3rd. *The Purity of the Metal used in both Coil and Core.*—This is a point which has not received that attention which it deserves. Indeed, the experiments necessary to afford accurate data, would require to be of the most elaborate and extensive kind; and the results, although of the greatest interest in a philosophical point of view, could rarely have a practical application for telegraphic and other purposes, owing to the great expense and trouble which would have to be incurred. Almost each length of copper wire which we may accidentally meet with, varies in this respect. Indeed, we have found, that the longer the same wire has been used as a conductor for powerful batteries, the worse is its power of conduction. It undergoes a molecular change; becomes hard, and frequently breaks into pieces with the slightest touch. Iron is subject to the same influence; and, like copper, it becomes completely "rotten" after having undergone lengthened electric action. We have frequently had occasion to remark these facts whilst experimenting with apparatus in which an intense current was employed; and shall refer again to the subject as we proceed.

HORSE-SHOE ELECTRO-MAGNETS, AND ELECTRO-MAGNETIC MACHINES.

Having explained the principles on which electro-magnets are constructed, and the conditions which affect their power, we proceed to make a practical application of our remarks in reference to horse-shoe electro-magnets and electro-magnetic machines, which have been invented for the purpose of obtaining a motive power by employing voltaic electricity. It will not be necessary for us to enter into any further theoretical considerations: we shall therefore confine ourselves chiefly to the mechanical details of the instruments to be described.

A horse-shoe electro-magnet has a similar advantage to that we mentioned as possessed by the permanent steel horse-shoe magnet;† for it has its two poles in close proximity, which, therefore, act in concert, and so produce a large development of attractive power. The ends should always be planed or ground as level as possible, so that any object presented to them may come into as close contact as possible. We shall give directions for making them, and leave the question of size to the choice and means of our readers. The copper wire must either be covered with cotton, worsted, or silk; the first material being the cheapest, and answering every purpose. It is also necessary that the wire should always be as soft as possible, so that it may accommodate itself to the curves of the iron. Each turn should be brought tight up to that preceding it, so that there may be no loose or "kinked" places throughout the convolutions. The iron part of a small electro-magnet may be a piece of rod iron, eighteen inches long, and an inch and a-half in diameter. It is to be bent so that its two ends may be brought within from two to three inches of each other, in the manner represented in the following engraving. The winding-on of the wire is to be commenced at the top of one end or pole, and to be continued for a depth of four or five inches towards the bend. It may then be wound back towards the top of the pole again, and returned until the lower end is again reached. One side will thus receive a coating of three coils of wire. This may then be passed over to the end of the rod nearest the bend, care being taken to bind the wire on it in the same direction as on the first arm; in fact, just as if the wire had been continuously wound on a straight rod. Now, because the centre point between the two extremities of a magnet does not exhibit externally any magnetic effect, it is unnecessary to wind any wire on it. Indeed, great advantage accrues by "condensing" the wire on the poles of the magnet, or, rather, near to the outer extremities of the iron, by winding extra coils on those parts. It is there that the greatest inductive power is manifested, and where, therefore, the greater effect should be produced. It may here be remarked, that electro-magnets, like ordinary magnets, have their poles at a distance from their extremities; hence the coating of covered wire is never carried to the extreme end of each arm.

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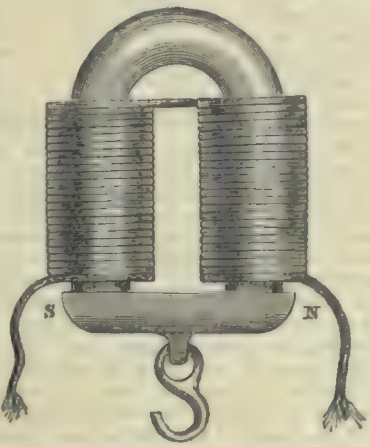
* See *ante*, p. 195.

† See *ante*, p. 244.

The following engraving illustrates the appearance of an electro-magnet when complete. Right and left are its poles, N and S. Beneath them is the keeper, to which a hook is attached, and to which heavy weights may be hung. From N and S proceed the two wires, which are to be attached to the wires of a voltaic battery when the electro-magnet is in use; and by means of which the current passes to the coils, round the iron. When very large magnets are made, it is usual to divide the coil into separate portions, and to collect the various ends of the coils together, so as to form one terminal for each end of all the coils. By these means the current of electricity circulates as many times round the iron as there are convolutions of wire placed round it, but has not to travel in one continuous length for the whole distance. By this arrangement the resistance of great lengths of wire is done away with, and yet the full effect of the quantity of the battery power is advantageously employed.* The end wires of the electro-magnet should never be twisted about, as they soon get very brittle; and if the wire be once broken close to the iron, and inside the coil, the magnet becomes useless, and must be recoiled by the wire. It is a good plan to fold some coarse brown paper, or a thin sheet of gutta-percha, on the iron arms before coiling on the covered copper wire, as this prevents contact between the metals, in case the covering is in any part defective. The coils may be bound on wooden cases, and these can then be slipped on to the iron arms of the electro-magnet, and removed when done with. As, however, the thickness of the wood removes the covered wire from the iron, and so increases the distance of the inductive power from its surface, a loss of power necessarily results from this arrangement. The attractive, or holding power of an electro-magnet, depends on its size, and on the strength of the battery employed. Such an arrangement as we have described, although small, will sustain at least one hundredweight, when a current from twenty cells of Smee's, or four to eight cells of Grove's battery, is sent through the wires.

Fig. 12



The largest electro-magnet which we have yet seen, and perhaps the most powerful which has been made, was one which we purchased from the Panopticon, at the public sale of effects, for the Polytechnic Institution in Glasgow. The wire and iron had been made of the purest metal which could be procured, and both were well annealed. The wire was a quarter of an inch thick; and the total quantity employed weighed nearly two hundred weight. The iron, which was of the usual horseshoe form, weighed about half a ton; and each arm was three feet high, and six inches in diameter: the remaining length of the iron rod formed the bend. We have repeatedly employed fifty and 100 cells of Grove's battery as the source of the voltaic current; and, under such circumstances, we have attempted in vain to remove the keeper from the poles by opposing a weight of over three tons. When two hundredweight of iron nails, each of which measured six inches long, and of the usual substance, were cast on the pole, every nail was so powerfully attracted, as that they could scarcely be pulled apart, and could not be pushed away from each other.

But there is a most remarkable fact in reference to all magnets; which is, that the attractive power decreases in a most rapid ratio. In the case of the large magnet we have just described, the attractive force, at three inches from either pole, was very trifling—not exceeding a few ounces. Thus we observe that the decrease in three inches distance from its source, was, beyond all comparison, greater in ratio than that of any other force with which we are acquainted; and this, unfortunately, has to a large extent prevented this agent from being employed as a motive power. In all kinds of magnets, the attractive power is greatest in those which have the longest arms; and this we should naturally expect, from the fact that the force diminishes to nothing as we approach to a point equi-distant between the poles. It is therefore better, in making an electro-magnet, to employ the wire so that it may cover the poles, rather than the whole length of the arms; and the neglect of this in the construction of the large magnet of which we have been speaking, doubtless prevented its maximum of power being obtained.

Electro-Magnetic Engines.—We shall now give a general outline of some of the attempts which have been made to obtain a power for driving machinery, &c., by means of electro-magnets: but before doing so, it is to be remarked, that the best-constructed arrangements have hitherto been complete failures in an engineering and economic point of view. This has arisen from three causes.—1st, the extremely small distance at which the attraction acts, and its rapid ratio of decrease, which we have already discussed; 2nd, the interference of residuary magnetism,† which prevents an instantaneous change in the direction of the attractive power; and, 3rd, the expense of voltaic batteries—not in their first cost, but in maintaining their power. In days such as our own, when inductive and experimental science, together with mechanical skill, are found in so high a degree of perfection, it seems most extraordinary that no success has as yet attended the numerous efforts which have been made to procure it.

When we compare the progress which has been made in steam machinery, with that which has attended that of the electro-magnetic arrangements, we can find no approximation in the latter. The mechanical difficulty results chiefly from the fact, that the rapid diminution of the attractive power of an electro-magnet, does not permit of any of the ordinary appliances for converting force into motion to be used. For instance, the stroke of the piston of the steam-engine may be of any length which the engineer chooses to employ. All he need do is to lengthen the cylinder, or supply the steam—each of which can be effected to any extent which may be desired; and the piston can be driven to and fro, acting on a crank of half its length. Now, in electro-magnetic engines, the greatest direct stroke which can be employed between two electro-magnets, cannot exceed (without the intervention of levers) a quarter to half an inch; and, of course, a crank of half that length, and of the necessary strength, is a physical impossibility in an engineering point of view.

It does not matter in what form the motion is obtained by any of the inventions which have been made, for still the rapid ratio of the decrease of force is constantly active. We fear, that unless some means may be discovered by which this power can be extended in a greater linear or radiating influence from its source, a good and useful engine, driven by electricity, will remain to be sought after.

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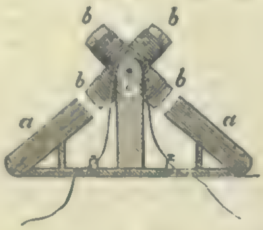
* See *ante*, p. 250.

† See *ante*, p. 249.

Having stated these difficulties, we will now describe some of the engines which the sanguine but disappointed inventors have patented, and brought more or less into public notice. In doing so, we shall not pay any heed to their chronological order, but rather to that of the principles on which they are constructed. We may divide them into two classes—namely, those, 1st, in which the motion is as a tangent in its direction to the lines of force; and, 2nd, such as, like the ordinary power of the steam-engine, have a reciprocating motion. There are other forms, to which we shall also allude in general terms, which claim attention rather for their ingenuity than their value.

Those machines in which motion is obtained in a tangential arrangement of permanent and electro-magnets, have been of numerous forms, according to the ideas of their inventors. The permanent magnets are always stationary; and the electro-magnets are made to revolve so that their poles may be nearly in contact with those of the former kind. The annexed engraving gives a view, in section, of this kind of machine, and shows the relative position of each kind of magnets. The less important parts are omitted, as is also the arrangement, by means of which contact is broken between the electro-magnets and the battery; for such varies so much in form, whilst the principle is the same in every instance.

Fig. 13.



In the above figure, *a a* are two permanent and compound horse-shoe magnets,* which are fixed on a wooden foot; *b b b b* are the four terminals, or poles, of as many electro-magnets, made in the usual manner. The wires of the ends of each electro-magnet are attached to a suitable arrangement, by means of which a current of electricity from a few cells of a voltaic battery is conveyed by them. As each pole of the electro-magnets approaches those of the permanent magnet, the current of electricity, producing magnetism in the soft iron, causes a powerful attraction. As the pole passes a little beyond the centre of the permanent magnet, the supply of electricity is cut off, and the soft iron thus returns to its normal state. Of course, the same happens to each electro-magnet as it approaches either of the permanent magnets, and thus a rapid revolution is produced. Very little power is afforded by this arrangement, which may be taken as a type of its class. In some of these machines a permanent magnet is placed, so as to be opposite each electro-magnet—one during each revolution; and the number of both may be increased to any extent. These may be arranged to revolve on their axis (*c*, in Fig. 13)—as often as from 300 to 500 times per minute; but owing to the attractive force diminishing in so rapid a ratio as each magnet passes on, the motion of the machine may be stopped by the slightest obstacle applied to the arms of the electro-magnets. When machines of this kind are constructed on the large scale, and used for driving lathes, &c., the speed of the driving-wheel, or that which communicates its motion to the machinery, is diminished below that of the electro-magnets by the interposition of variously sized cog-wheels.

The next form of electro-magnet to which we shall direct attention, is that wherein a crank is employed, which is moved by means of levers placed over the electro-magnet, and attached, at their other extremity, to a fulcrum. These machines are generally constructed with two sets of electro-magnets, &c., and the cranks are placed at right angles to each other on the crank-shaft; so that when one is on the dead points, the other may be in full action. This form of machine, like that which we last described, has been the subject of numerous modifications: its type, however, is illustrated in the annexed engraving. In Fig. 14, the general arrangement will be seen of one side of these machines: *a* is the electro-magnet; *b* is a soft iron keeper, attached to the lever *c*—this having a fulcrum at *d*. The lever is attached to the crank at *e*, which is again fixed on an axle of the fly-wheel, *g*, at *f*. The other cranks, &c., are not shown, so as to avoid confusion of detail. When a current of electricity passes round either electro-magnet, the attraction induced draws down the keeper, and so, by means of the lever, produces a rotary motion through the crank fixed on the shaft. As one keeper rises, the other one falls, and thus the general action resembles that of a double-cranked, &c., steam-engine. The contact between the electro-magnet and the battery is broken by means of any form of contact-breaker, attached either to the crank or lever. By means of this form of machine, a greater amount of power is obtained than that produced by the one we last described; but still the result, in an engineering point of view, is comparatively trifling. An effective modification of this kind of electro-magnetic engine—the invention of M. Froment, of Paris—is represented in the following engraving.

Fig. 14.

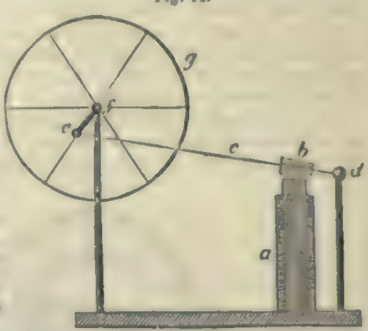
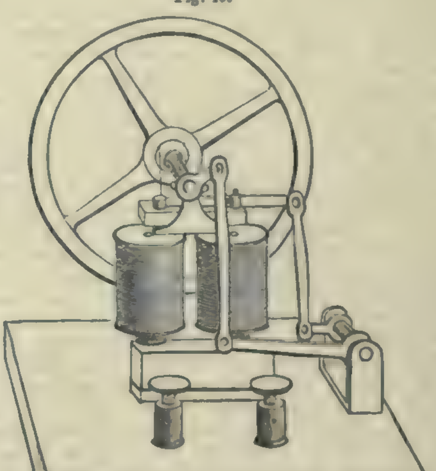


Fig. 15.



It will be impossible for us to describe the various arrangements which have been proposed by numerous inventors. M. Jacobi, of St. Petersburg, Messrs. Joule, Sturgeon, Davenport, Davidson, and many of the leading electricians and philosophical instrument-makers of this and other countries, have applied an immense amount of theoretical and practical knowledge to the subject. One of the best arrangements we have seen, is that invented by Mr. Allan, of the Adelphi, London; who has devoted himself, with great ardour and success, to the pursuit of every branch of electrical science. He employs a number of circular plates, placed over as many electro-magnets. Through each of these plates a rod runs; at the lower part of which is a pivot. The upper end of the rod is connected to a crank; and a set of plates and electro-magnets are placed vertically over

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* See *ante*, p. 244.

Fig. 1

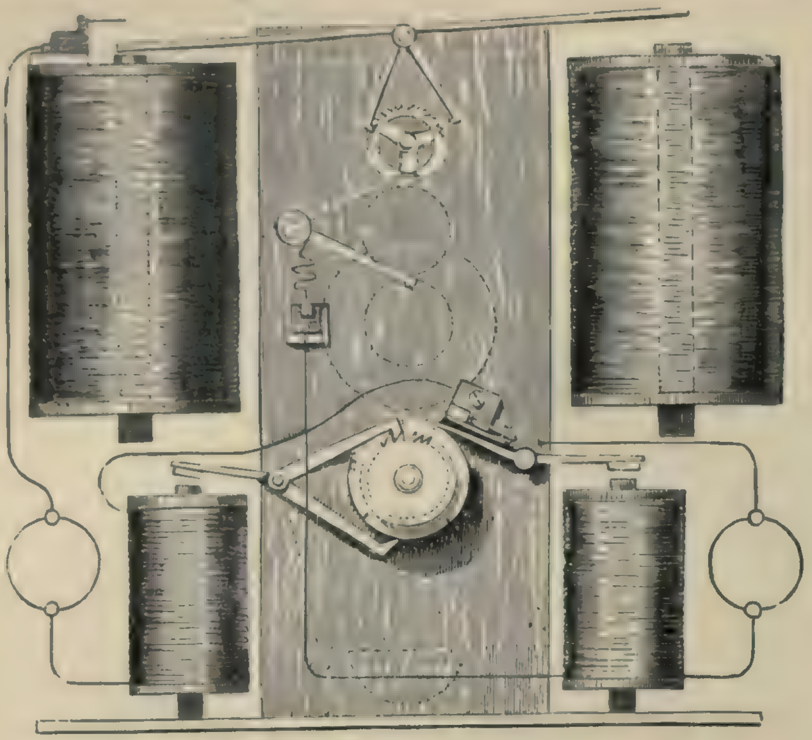
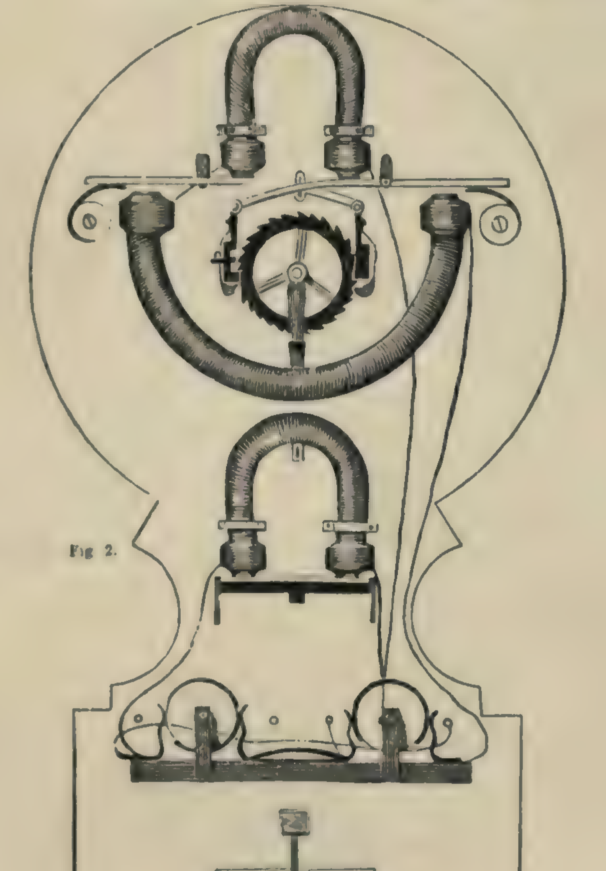


Fig 2.



Figs. 1 and 2.—Electro-magnetic Clock-work. (See page 253).

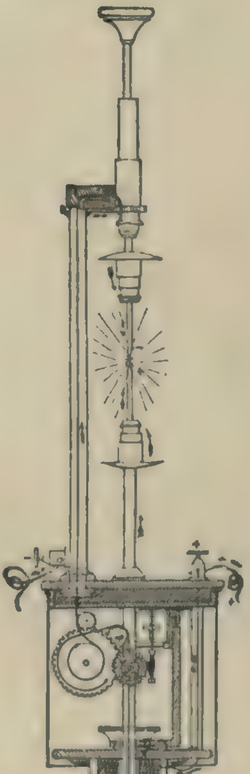
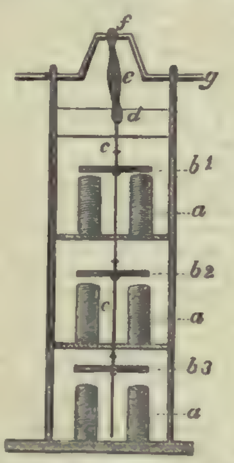


Fig. 3.—Duboscq's Electric Lamp.
(See pages 139 and 191).

each other. The electric current is successively admitted to each electro-magnet, commencing with the top one. This pulls one plate down, and so moves the crank-shaft a little: each electro-magnet does the same to its super-imposed plate; and so the crank is gradually moved. Five series of these arrangements are placed side by side; and, by their successive action, a rotatory motion of a shaft is produced. Each plate has to move not more than an eighth of an inch, and is, therefore, always in close proximity to the electro-magnet beneath it. As soon as these have done their portion of the work, the current is cut off and directed to the neighbouring series. One of these machines produces a considerable amount of force, and, in our opinion, is a decided improvement on their predecessors. Fig. 16 will give an idea of Mr. Allan's arrangement; but we have only shown one frame out of four or five, which are placed side by side, and connected with one crank-shaft. Mr. Allan also uses five sets of electro-magnets instead of three; at least, so his model, which was deposited in the London Polytechnic, was constructed: *a a a* are three electro-magnets, firmly fixed in a frame; b^1, b^2, and b^3, are three soft iron circular plates, loosely resting on pins, which are fixed on the vertical rod, *c*. It will be observed, that each plate varies in the distance from its electro-magnet. The rod *c* is attracted at *d* to another, *e*, which again is connected with the crank *f*, fixed on the shaft *g*. When a current of electricity is sent round each electro-magnet successively, they attract, one after another, the plates over them, and so, by a succession of pulls, produce a rotary motion. In this, as in every other electro-motive machine, the residual magnetism[*] is a constant source of difficulty; and if a powerful battery be employed, some of the plates become fixed to their subjacent electro-magnets, after the current has been cut off. The mode of breaking contact we have not shown: it is, however, extremely simple—cutting off the current from each electro-magnet with great precision.

Fig. 16.



Having thus endeavoured to acquaint our readers with the principles on which electro-magnetic machines are constructed, we shall dismiss the subject, with an expression of regret, that so little success has, as yet, rewarded the efforts of those who have devoted their attention to it.

Electro-Magnetic Clocks.—Numerous attempts have been made to accommodate clockwork to the arrangements which afford electric and induced magnetic currents. These inventions have, in almost all cases, proved to be failures. Such, however, have not been owing to a fault in the principle, so much as the defects which occurred in the practical details. We need scarcely say that electro-magnetism supplies a power which is constant so long as a voltaic battery gives what we may term the initiative; but it is not simply on the electric current that the success of the clock arrangement depends. There are many other details which interfere; and although minor in their character, their effect is none the less felt. For instance, we will presume the case of a clock of the highest class of manufacture, which has the necessary arrangements for carrying out the purpose to which we have referred; that is, the indication of exact meridional time at any place within reach, by means of conducting wires of an electro-voltaic current. Such an instrument will only afford accurate indications of time so long as the wheel, which is ordinarily connected with the detent of the pendulum, vibrates or passes any fixed point on the clock-face, corresponding to the original whence it first obtained its motive power. Now, this is carried out by means of a wheel, the spokes of which are alternately made of a conductor and non-conductor. A current of electricity is conveyed to the wheel; and from it is stretched a wire, which reaches a similar wheel in a distant clock. Another wire from this or an earth-current arrangement (which we shall describe hereafter) is also employed, so that the current from a battery may circulate completely between the two clocks. Each clock is furnished with an electro-magnet; and as a current passes from one to the other, it simultaneously sets each magnet in action. By so doing, a detent acts on the usual clockwork arrangement, just in a similar manner as is observed in the arrangement of the pendulum and first wheel of the common clock. Now, supposing there are sixty alternate conductors on the wheel of the first or primary clock, it is evident, that as each of these sends a current of electricity to the distant clock, its electro-magnet will be acted on, and its mechanism will be put in motion. Each minute is thus indicated on the clock-face by a movement of the minute-hand, which corresponds with that of the wheel moved by the electro-magnet. It is also evident that any number of clocks may be similarly worked by one primary; and so exact time could be indicated at any station distant from a central point.

This, however, whilst appearing perfectly easy on paper, is a very different affair in practice. It was proposed thus to regulate all the clocks on the railways of the United Kingdom, by an extension of wire from a central clock at Greenwich Observatory. This was, in part, carried into practice at the telegraph station at Charing-cross, London. Like many preceding attempts, however, it failed. In the folio plate, Figs. 1 and 2 will give a general idea of the construction of electro-magnetic clocks. The arrangement of vibrating levers, electro-magnets, detents, toothed-wheels, connecting wires, &c., will easily be understood, in connection with the above remarks. In our future pages, we shall refer to other inventions, which will be better included with the telegraph, the electric time-ball, &c.

SECONDARY INDUCTION—THE COIL MACHINE.

In all the instruments we have hitherto described in connection with electro-magnetism, we have merely noticed the magnetic effect which is produced when a current of electricity is conveyed by a wire. We shall now investigate a different class of phenomena, in which a second current of electricity is induced by the primary current in any wire near to, but not in contact with, that attached to a voltaic battery.

For the purpose of trying the experiments we shall introduce, the reader will require two or three cells of any form of voltaic battery, and a supply of covered copper wire, of two sizes—namely, No. 16 and 22, or a finer gauge; and, to prevent circumlocution, we may here state, that when we employ the term *primary* to a wire, we mean that which *conveys* the current of electricity from the battery; and the *secondary* wire, is that in which a current is *induced* by its proximity to the primary one. Electro-magnetic induction is analogous to that of frictional electricity; but differs from it in one respect—namely, that the two phases of electrical conditions are simultaneously produced by the first inducing cause. Thus, if any glass surface be charged with frictional electricity,[†] its opposite side assumes one opposite electrical state only; but when a wire suffers electro-magnetic, or secondary induction, one of its ends exhibits a positive, and the other a negative electrical state; and, so far, this phenomenon much resembles that of ordinary magnetic induction. There is, however, this peculiarity about that of electro-magnetism—which is, that it is instantaneous in its existence; and occurs, in the secondary wire, only at the moment that contact is made or broken between the ends of the primary wire and the poles of the voltaic battery, which is its source of electricity. The following experiment will introduce the reader to some facts in connection with the subject.

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[*] See *ante*, pp. 249 and 251.

[†] See *ante*, p. 180.

Experiment 10.—Wind about ten yards of No. 16 gauge covered copper wire on a hollow tube of gutta-percha, or paper, which may be an inch in diameter, and about eight inches long. On this coil wind one of the fine wires above-mentioned, in the same direction, but of a length of thirty or forty feet; and let the ends of each coil be kept separate, and extended about a foot or eighteen inches from the coil. Two coils will thus be formed, one on the other—the primary one being inside, and the secondary, or finer coil, covering it. The two ends of the secondary coil are then to be attached to the terminals of a galvanometer.* If the ends of the *primary* coil are then placed in contact with the plates of a voltaic battery—at the moment they touch, a current will be induced in the secondary wire, which will be evidenced by the deflection of the needle of the galvanometer. This deflection will speedily cease; but if contact between the primary coil and the wire be broken, then the needle of the galvanometer will again deflect; and, as often as these processes are repeated, a current of induced electricity will be produced in the secondary wire, the direction of which will depend on, but be contrary to, that of the primary one.

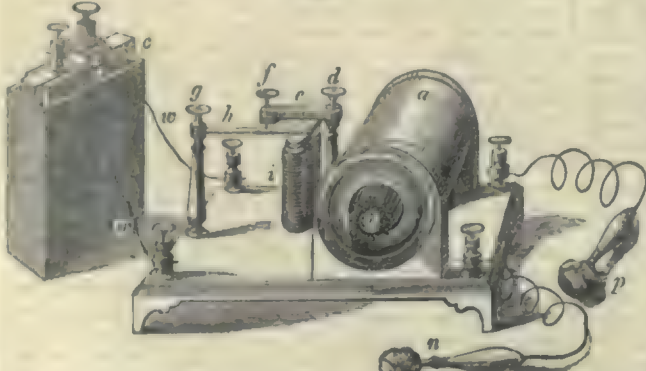
Experiment 11.—If a rod of soft iron be placed inside the hollow cylinder, surrounded by the coils, and the same process be repeated, it will be found that the deflection of the needle in the galvanometer will be much greater, and the iron itself will become magnetic. A double phenomena is thus presented—namely, the induction of a secondary electric current, and also magnetic effects.

It will thus be seen that a current of voltaic electricity, when passing by means of a conducting wire, has the power of producing another current in a conductor lying near it; and this takes place when the two wires are stretched straight, side by side, equally as well as when they are coiled up in the form we have named.

There, however, exists a great advantage in the coil form; which is, that each successive layer or coil of wire increases the intensity of the current produced. Indeed, by extending the length of the secondary coil, most astonishing effects of intense electricity can be produced. In Rhumkorf's coil (which we shall presently describe), a length of two or three miles of wire is used; and all the phenomena of frictional electricity can be procured from it.

Perhaps the most convenient mode of illustrating the various effects of electro-magnetic induction, will be that of explaining the construction, &c., of the coil machine; a common form of which is illustrated in the following engraving.

Fig. 17



The apparatus consists essentially of two parts—the coils and the contact-breaker. In Fig. 17, *a* represents the cylinder on which the primary and secondary coils are wound; *b* is a bundle of iron wires, placed in the centre of the coil; and *n* and *p* are the two terminals of the secondary coil. The primary generally contains about 100 feet of No. 16 gauge covered copper wire; and the secondary has often so much as 300 or 500 feet of No. 22 or 24 wire. One wire of the primary coil is attached to one of the binding-screws, in which a wire, *w*, from the battery *c* is fixed; and the other wires, of both the battery and coil, are in connection with the contact-breaker, which next deserves our attention.

By means of this apparatus, the current proceeding from the battery is suddenly passed to, and cut off from, the primary coil. This is effected by means of a small electro-magnet, *i*: over this is placed, at a small distance, a soft iron keeper, which is connected, by means of a steel spring, *h*, with a brass pillar, *g*. A wire is fixed under the instrument—passing from one of the binding-screws; which holds one of the battery wires. The other end of the primary coil is attached to the pillar, *d*; and from this stretches an arm, *e*; at the end of which is a screw terminating in a metallic point, *f*. The electro-magnet, *i*, however, forms part of the circuit, by means of wires arranged beneath the foot of the instrument. Now, if a current of electricity be made to circulate through the coil by the means we have named, the electro-magnet immediately attracts the keeper placed over it, and, by drawing it down, removes the spring to which it is attached from being in contact with the metallic point terminating the screw, *f*. As the coil and battery are, electrically speaking, only brought into connection when the spring, *h*, and the point, *f*, are in contact—it follows, that the moment they part, the current is at once cut off from the coil. On this happening, the electro-magnet, *i*, loses its magnetism, and, therefore, ceases to attract the keeper over it. The spring, *h*, then acts, and again brings itself in contact with the end of the screw, *f*. The coil is thus again in connection with the battery, and the electro-magnet is magnetised. The previous conditions are repeated; and thus the contact between the battery and the coil may be made and broken with great rapidity.

As we have already stated, a secondary current is induced in the fine coil each time when contact is made and broken between the primary coil and the battery; and as this is easily done 500 times per minute, as many induced currents are simultaneously produced, and a torrent of electricity afforded. There is, however, a great difference between the character of the two currents. That of the primary, which is derived from the battery, has all the characteristics of the voltaic current; whilst that afforded by the secondary coil presents effects precisely similar, in many respects, to the phenomena of frictional electricity; and this identity is developed just in proportion to the length of the coil employed—all other circumstances remaining the same.

In our engraving, the terminals of the fine coil are attached to two handles; and if these be grasped by the hands, previously moistened with salt and water, a most powerful shock may be obtained. If the secondary coil be very long, and the battery attached to the primary be in good action, the effects on the muscles of the arm are so powerful, as to make it impossible for the person receiving the shock to leave hold of the handles; and the pain produced is most excruciating. In fact, considerable caution should be observed in this respect, especially with young persons and females, as serious consequences may result from inadvertence. It is better to place the end wires of the secondary coil in two basins of water, into which persons desirous of receiving a shock can immerse the hands; and from which they, generally speaking, far more quickly withdraw them. Two or more persons can jointly receive the shock by joining hands together. The effect, however, rapidly diminishes as the number is increased; and varies very much according to susceptibility of each individual.

This form of machine is largely used for the purpose of administering electricity in rheumatic, paralytic, and other nervous diseases; in cases of asphyxia, from inhaling carbonic acid gas, drowning, &c. The power of the shock may be regulated by the removal from, or gradual introduction of, the iron rod into the hollow of the cylinder on which the coil is wound; by means of which the magnetic effects of induction are lessened or increased. Another plan, having the same result, is that of interposing a column of water between one of the wires of the secondary coil and the handle; for as the liquid conducts to a much less extent than the metal, the intensity of the shock received is readily proportioned, by the length of the liquid column, to the condition of the patient.

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* See *ante*, p. 193.

If one of the wires of the secondary coil be attached to a file, and the other be drawn quickly over its surface, bright sparks will be produced; and if the ends of the wires are immersed in acid water, or a solution of sulphate of copper, decomposition of the liquid ensues. But both the chemical and luminous effects of the secondary current are very feeble; its characteristics being great intensity and the absence of quantity.*

We have confined our description to one form of the coil machine; but may observe, that they are made in various ways by the instrument-makers, both as to the arrangement of the coil itself, and the other parts of which it is composed. We shall now proceed to describe a most valuable modification of this instrument, which has been invented by M. Rhumkorf, and by whose name it is generally known in scientific circles.

RHUMKORF'S COIL.

PERHAPS one of the most interesting instruments which have been introduced to the notice of the experimental philosopher during the present century, is the coil constructed by M. Rhumkorf, of Paris, and improved lately by many eminent electricians. It has afforded us a complete connecting link between frictional and voltaic electricity, and so has removed many difficulties which had previously existed between those two branches of electrical science, in respect to their general laws and facts.

The description we have already given of the ordinary coil machine, will prevent the necessity of describing some portions of M. Rhumkorf's instrument. It has a primary and secondary coil, and a contact-breaker, which, however, is of a peculiar construction. The principal points of difference between the two machines are, that the Rhumkorf coil has a very great length of secondary wire, extending to many thousand feet—it being of extreme fineness; and also a "condenser," which is not found in the common instrument.

Each layer of the secondary coil is insulated from its neighbour by a thin sheet of gutta-percha, or other non-conducting material. This prevents a great loss of intense electricity, which otherwise would occur. The condenser consists of a large sheet of tinfoil, enclosed in sheets of oiled silk. It is attached to the primary coil; and has the effect of increasing the intense effects of the secondary coil, by modifying the quantitive nature of the primary one.

At one end of the instrument two glass upright pillars are placed; into the top of each of these the terminals of the secondary coil are fixed. From these, wires may be extended, which can be connected with any apparatus to be employed in the course of experiments.

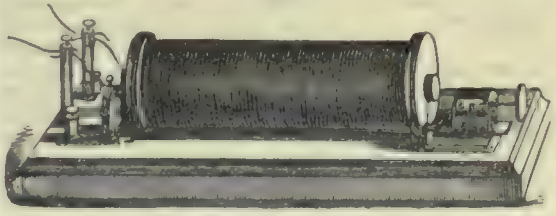
We may remark, that in all our experiments with Rhumkorf's coil, or its modification, the greatest effect has been obtained when the contact-breaker has been so regulated as to give a minimum of currents per minute. We have previously remarked, that it requires an appreciable interval of time to render a rod of iron magnetic by means of the voltaic current;† and in the coil of which we are speaking, another element must be considered; which is, the great length of the secondary coil, and the resistance which it may afford to induction, &c.

Four cells of Grove's battery, with platinas, each exposing an active surface of sixteen square inches, afford, so far as we have seen, the best effects with the instrument. Care must be taken not to use too great a battery power, lest the primary coil be heated. This frequently occurs if a large battery be employed with the coil already described;‡ and it would certainly injure a Rhumkorf arrangement, and perhaps render it useless. In the ordinary coil, the battery is directly connected with it; but in M. Rhumkorf's apparatus, a commutator is provided, by means of which the current can be turned on and off as required, or be reversed in its direction, which is frequently necessary in some experiments. Another advantage is secured, which is of no mean importance. As the handle of the commutator is insulated from the battery and coil, the experimenter runs no risk of receiving any shock. Those who have accidentally received one from a Rhumkorf, will not easily forget its effects. The writer was twice unfortunate in this respect, and suffered for some days after their occurrence. The outside of the coil is as highly charged as the terminals; and long sparks may be obtained from it, just as may be taken from the conductor of an electrical machine. If a wire proceeding from the outer extremity of the secondary coil, or from its external surface, should happen to fall on the table on which the instrument may be standing, a powerful shock is often experienced if the operator touch any part: indeed, it is impossible for us to urge too strongly, that the greatest care be observed in handling the instrument, or in avoiding neglect with respect to the management of the wires proceeding from it.

Since the first introduction of the Rhumkorf coil, it has undergone numerous modifications, which have consisted in the extension of the condenser, improved insulation of the coils, &c. A spark of half an inch in length was considered a great achievement; but we lately observed in a periodical, that M. Rhumkorf had succeeded in obtaining one seventeen inches long, between the terminals of the secondary coil.

Having thus explained the construction of M. Rhumkorf's coil, and of its modifications, we may suggest a few experiments, which will illustrate some of its peculiar powers; and for this purpose we shall choose such as will show how far it connects the phenomena respectively afforded by frictional and voltaic electricity. The following engraving represents a Rhumkorf of the form usually sold at the instrument-makers.

Fig. 18.



Experiment 12.—Having attached the wires of the battery to the coil at the proper binding-screws, fix, in the two pillars holding the terminal wires of the secondary coil, a platina wire in each, so that their two points may be within ⅜ths of an inch of each other; turn on the commutator, and a constant stream of purple-coloured fire will be produced between the two points.

Experiment 13.—It will be noticed that one wire will become red-hot, whilst the other remains comparatively cool. But reverse the direction of the commutator, and immediately the effects of heat will also be changed—that wire which had been previously cool, becoming heated.

Experiment 14.—Separate the two wires to a distance of an inch from each other; taking care, however, in this, as in all other experiments, that the current from the battery has first been turned off. On again turning-on the primary current, no sparks will pass between the secondary wires; but if the flame of a spirit-lamp be placed so that each wire may enter, then the spark will at once dart through the heated air, although it will not through the atmosphere in its ordinary state.

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* See *ante*, p. 251.

† See *ante*, p. 249.

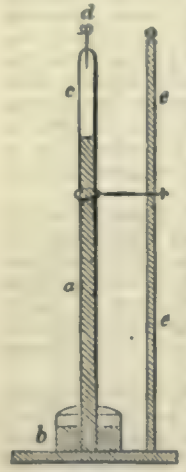
‡ See *ante*, p. 254; Fig. 17.

Experiment 15.—Employ the apparatus used for sending the electric spark in a vacuum, described under the section of Electricity[*]—attaching the outer wire of the secondary coil to the ring of the rod, and the inner wire to the metal plate of the pump. A rich purple-coloured flame will pass, which, however, will depend for its length and beauty on the degree of exhaustion of the air in the receiver. If a good vacuum be maintained, then the flame will be of a most delicate purple or violet tint. If air be slowly admitted, the colour of the flame becomes gradually redder, and, eventually, a red wire-like spark will be produced.

We have repeatedly tried, at the same time, this experiment with the Rhumkorf, and a similar one with the large hydro-electric machine at the London Polytechnic; and the vacuum being equal, the effect in each receiver was precisely the same.

In 1855 and 1856, being engaged in a series of experiments with Rhumkorf's arrangement, we found the barometer tube of great service, as affording a complete vacuum; but, for this purpose, a platina wire must be soldered into the top of the tube before it is filled with mercury. This arrangement has already been referred to:[†] when it is used with the Rhumkorf, the outer wire of the secondary coil is to be attached, by means of a fine wire, to *d*, the platina wire; and the inner wire of the coil is to be dipped into the mercury of the cistern *b*. On passing the secondary current through the vacuum *c*, a beautiful lavender-coloured flame is produced.

Fig. 19.



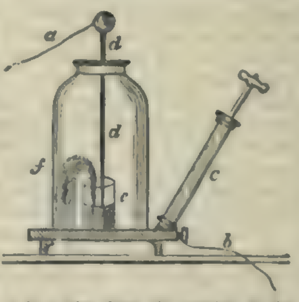
a is the glass tube, containing the column of mercury; *b*, the cistern of that metal; *c*, the vacuum; *d*, the platinum wire; and *e e*, a ring-stand to support the tube in a vertical position.

Experiment 16.—Amongst other results obtained, were many curious appearances, produced by passing small portions of the vapour of ether, bisulphide of carbon, &c., into the vacuum of either the air-pump or the barometrical arrangement. This is easily effected in the former by loosening the screw which admits air into the receiver, and holding against it a little cotton wool moistened with the liquid. The air must be again exhausted; the current may then be passed, when the diffused vapour left in the receiver will at once affect the normal colours of the discharge. Bisulphide of carbon is generally at once decomposed—a bright flash of flame being produced at first; after which sulphur is deposited on the surface of the receiver. Ether, naphtha, &c., produce a flame of a greyish-lavender tint. Some very interesting effects are also produced by passing the spark through rarefied atmospheres of hydrogen, carbonic acid, &c.

In trying these experiments, it will be noticed, that one pole presents a beautiful blue spangle of flame; whilst the other is of a red hue—the position of these appearances being altered on a reversal of the primary current being effected by means of the commutator; and we may add, that the appearances undergo constant variation as the vacuum of the receiver varies. We have met with few air-pumps which have been so well constructed as to permit of maintaining a sufficiently good vacuum for any length of time; indeed, to a practised eye, the colour of the discharge in vacuo we have been describing, is as good an indicator of the extent of exhaustion as is the barometric column.

A very pretty mode of exhibiting these results, is that of placing inside the receiver a small glass vessel, the inside of which has been partly lined with tinfoil. All the arrangements are to be carried out in a similar manner to those already described, with the exception that the conducting-rod fitting into the top of the receiver, is forced down so that its lower part may rest inside of the small coated jar, and with its end touching the coating. (See Fig. 20). A spark passed in such a manner, produces a fine cascade of flame, which presents a singularly beautiful appearance. We are indebted to Mr. Gassiot for this interesting mode of carrying out the experiment. Tables, and vessels of various shapes and materials, are sold by the instrument-makers, for the purpose of exhibiting these luminous effects in a variety of ways.

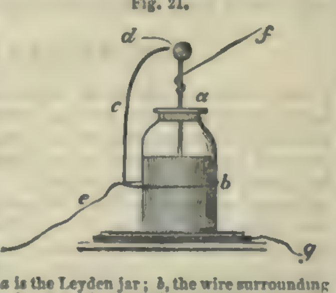
Fig. 20.



a is a wire from the outside of the secondary coil; *b*, a wire from the inside of the coil; *c*, the air-pump; *d d*, a brass rod fitting air-tight in the receiver, and resting on the coating of the jar *e*; *e*, the semi-coated jar, out of which the cascade of flame proceeds to the plate of the pump.

A Leyden jar may be easily charged and discharged, by attaching it to a Rhumkorf. For this purpose, a stout wire should be twisted round the outside coating of a jar; and one of its ends is to be bent up so as to be nearly in contact with the knob of the inside of the jar. The outside of the jar is to be attached by a wire to the inside of the coil, and the outside of the coil to the rod rising from the inside of the jar. (See Fig. 21). The latter is to be insulated by placing it on a sheet of glass, gutta-percha, or other non-conducting material. On passing the battery current through the primary coil, a series of brilliant discharges will take place from the secondary coil, between the knob of the jar and the external wire: these are attended with a loud snapping noise, and are repeated incessantly.

Fig. 21.



a is the Leyden jar; *b*, the wire surrounding it; *c*, a wire reaching nearly to the knob of the jar, but does not touch it; *d* is the interval between which the spark passes; *e*, the wire attached to the inside of the secondary coil; *f*, the wire attached to the outside of the coil; *g*, a gutta-percha plate, on which the jar rests.

Experiment 17.—If a card, or a few folds of paper, be interposed between the knob and the wire, they will be instantly perforated; and a thin sheet may be easily inflamed. A little naphtha, ether, or spirits of wine, if poured on to the knob of the jar, may also be readily ignited.

Many of the leading experiments of frictional electricity may be repeated by means of the Rhumkorf coil; amongst which are the following.

Experiment 18.—Suspend two pith balls, by means of a silk thread, from a glass tube, and attach to the top of the thread the wire from the outer end of the coil: the pith balls will at once diverge, owing to the charge of intense electricity—just as may be observed when a pair of pith balls are hung on the conductor of the ordinary electrical machine.

Experiment 19.—Hold a pith ball, suspended from a well-dried glass tube by means of a silk thread, near to the outside of the coil, in any part of its surface. The ball will be attracted to it, as if it were a charged conductor; and if a metallic ball, attached to a chain or wire, be also presented, sparks may be readily obtained from the coil. In trying this experiment, some risk occurs of injuring the silk covering of the wire of the secondary coil. In connection with the experiment, we may mention, that, in very dry weather, sparks often dart from the outside of the secondary coil, to the wires in different parts of the instrument; and a powerful shock can be obtained from that part as easily as from the terminal wires or poles. The reader will perceive, that in this fact exists another analogy between the electricity produced by a Rhumkorf's coil and that by friction.

Most of the other class. experiments exhibited in connection with frictional electricity, may be performed similarly by means of the Rhumkorf coil; and we shall leave the choice of such to those of our readers who may feel disposed to try them—referring them to those we have mentioned under the head of Frictional Electricity.

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[*] See *ante*, p. 173; Fig. 6.

[†] See *ante*, p. 174.

We may here make a remark on some peculiarities in reference to the appearance, &c., of the spark produced on charging a Leyden jar by means of a Rhumkorf coil. The brilliancy of the spark, and the noise of the discharge, are affected by the size of the jar, to some extent, but in no comparison to that witnessed when frictional electricity is employed. We have charged, successively, twelve Leyden jars, each 28 inches high, and 50 inches in circumference, by the hydro-electric machine* and a Rhumkorf coil. The noise produced by the discharge of the jars, as excited by the machine, was truly deafening; whilst that of the Rhumkorf discharge scarcely exceeded a sharp rapping sound. Of course, the discharges and charges of the Rhumkorf are produced continuously and successively, at a very rapid rate—at least 200 per minute; whilst the machine charge did not exceed two in number during that time. We have also remarked, that the larger the jar charged by a Rhumkorf, the shorter is the length of the spark; but its lateral extension is, at the same time, increased. Its mechanical and calorific effects, however, do not seem to at all increase with the size of the jars; for a half-pint jar will perforate about the same thickness of card as the large battery we have just named.

Numerous experiments, analogous to those included under the head of Voltaic Electricity, may be repeated, but on a very small scale with a Rhumkorf. We have only succeeded in obtaining a most minute but brilliant spark from carbon terminals—similar to that produced by charcoal points, forming the poles of a powerful voltaic battery. We have rendered soft iron electro-magnets attractive by the coil current; and feeble, but decided decomposition of water, or of a solution of sulphate of copper, may be produced. In this experiment we employed two pieces of platina wire as the poles of the secondary coil. In repeating experiments of this kind, the ordinary voltaic apparatus, like the frictional, may be generally employed; but, as we have already remarked, care must be taken that the experimenter shall not receive an accidental shock.

As there are some interesting particulars attached, generally speaking, to the physiological effects of electricity, which are especially attractive to the general reader, we will make a few observations on the subject. The shock produced by frictional electricity has a sudden and, of course, transitory effect; because, the instant the jar is discharged through the body of the person, the effect can only be repeated by a fresh-charged jar. The shock from a voltaic battery, of high power, acts continuously on the muscles so long as the hands are in contact with the conducting wires; and if from 100 cells of Grove's battery, the effect is very painful. The ordinary coil machine acts in a similar manner; but being more intense, the effect is still more powerful. The physiological effects produced by the Rhumkorf coil, are, of course, far greater. The writer accidentally received a shock by the wire from the outer end of the coil falling on his chest. The sensation experienced was such as to create the idea that the whole of his body was a mass of light: no pain was felt; for the shock was so powerful as to produce almost instantaneous insensibility. After recovery, the digestive organs were considerably disordered for several days, and great languor and sickness were felt for some hours.

The best mode of receiving the shock from a Rhumkorf, is to immerse two wires proceeding from the terminals of the secondary coil, in separate basins filled with river or spring-water. The hands can be dipped in these; and the force of the shock is much diminished by the inferior conducting power of the liquid.

In reference to further comparisons, or analogies, between the effects generally produced by frictional and voltaic electricity, and those afforded by Rhumkorf's coil, we have already remarked that the latter affords a kind of link for the two former; and it may be useful to the student if we enter into further details.

The voltaic spark is continuous after being once produced (within certain limits); whilst that of the electrical machine is intermittent. The spark of the Rhumkorf holds a middle place. Again, the spark of the machine exhibits but trifling heating effects; whilst that of the voltaic battery affords us a greater amount of heat than any other source with which we are acquainted. Now, the two terminals of the secondary coil of a Rhumkorf exhibit very different results. One is always intensely heated, whilst the other remains comparatively cool.† A voltaic battery, unless of enormous size as to the number of its plates, rarely exhibits decided attractive power on pith balls, or on the electroscope. The Rhumkorf effects this result with ease, in a manner similar to the electrical machine. The latter but feebly produces chemical decomposition, as is the case with the Rhumkorf; which, however, depends for its action on the chemical decomposition going on in the battery, whence it obtains its primary current.

The electrical machine, even when of the largest size, affects but slightly any piece of iron or steel, in respect to the magnetic force; whilst the voltaic battery affords us a means of inducing magnetism to almost any extent we please. The Rhumkorf holds an intermediate position in this respect; and in this and many other results which our space prevents us enumerating, we may consider this coil as the connecting tie between the various phenomena and sources of electrical excitation and its effects; and as clearing away many difficulties which have hitherto stood in the way of their complete generalisation.

CHAPTER III.

MAGNETO-ELECTRIC INDUCTION.

We have considered it better to draw the attention of our readers to the production of magnetic effects by electric currents, before entering into the discussion of magneto-electric induction, because the apparatus employed, and the phenomena manifested, will be more readily understood by the perusal of our previous remarks. Hitherto, we have been drawing attention to the fact, that currents of great quantity, such as are produced by the voltaic battery, have, under certain conditions, the power of inducing magnetic force. We shall now show that magnetism can, by certain arrangements, produce all the calorific, luminous, chemical, and physiological results which may be obtained from the voltaic battery, the electrical machine, or Rhumkorf's coil.

Magnetism and electricity have a mutual action on each other. We noticed this when experimenting with a spiral, at the end of which a voltaic battery was placed, and to which a horse-shoe magnet was presented.‡ The electrified spiral exhibits magnetic effects just so long as the current from the battery passes by it; and such effects are exactly identical with those possessed by any kind of magnetised body. We have shown, that if a current of electricity be passed, by means of a wire, round a rod of iron, the latter at once exhibits magnetic properties; and, further, that these are only produced when the current passes in a certain direction.§ We shall now describe how, by using a similar kind of apparatus, magnetism may induce an electric current. In this experiment, as in that to which we have alluded, we shall find that the effects of the forces are produced at right angles to the direction of each.

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* See *ante*, p. 179.

† See *ante*, p. 255.

‡ *Ante*, p. 248.

§ *Ante*, p. 249.

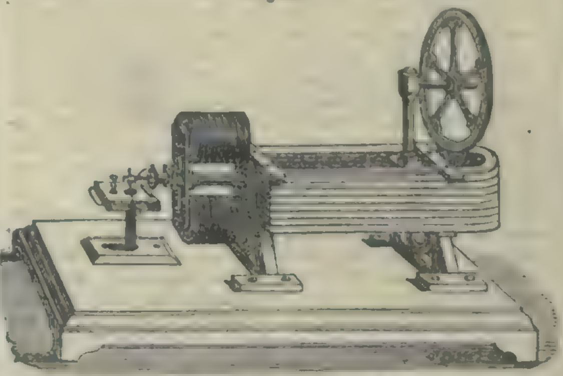
Experiment 20.—Coil about one hundred feet of No. 22 or 24 covered copper wire, round a paper or gutta-percha tube, ten inches long, and one inch in diameter; and attach its two terminals to the binding-screws of a galvanometer.* Then introduce one end of a rod, or bar magnet, suddenly; and at the moment this is done, it will be noticed that the needle of the galvanometer diverges. It will, however, speedily regain its old position. If the magnet be now suddenly withdrawn from the coil, the needle will immediately diverge in an opposite direction to that which it previously took.

This experiment shows that a current of electricity is immediately induced in the wire by the magnet, on its introduction into, and withdrawal from, the coil; and, further, that the effects are produced only when these movements take place; for no result follows if the magnet be kept permanently in the coil after its introduction is effected. This is highly analogous to the phenomena of secondary electro-induction, for we notice that the current was produced in the secondary coil, only when contact was made and broken. This fact, which was first discovered by Dr. Faraday, is of the most interesting kind; it establishes a chain of analogies between each branch of electrical science, and that of magnetism; it opens out a wide field for the exercise of inductive generalisation, and establishes a definite connection between the causes which tend to produce all the varieties of electric and magnetic disturbances, and other phenomena, whether as witnessed in our laboratories, or in the more magnificent and boundless one of nature.

The experiment which we have named will be a sufficient introduction to our subject, inasmuch as the student can easily repeat it, and personally ascertain the results we have called attention to. We shall, therefore, proceed to describe the best forms of magneto-electric machines, such as are employed at the present day. The simplest of the kind is that sold at the instrument-makers, as an American invention, for the purpose of giving shocks. It consists of a single horse-shoe permanent magnet; opposite to which are two small rods of iron, which revolve close to the poles of the magnet—being turned on their axis by the ordinary multiplying-wheel arrangement. In this instrument the magnet is not inserted in the coils, but the iron in them becomes magnetic by induction, as it passes either pole of the magnet. If these rods, or armatures, as they are called, were made of steel, they would become permanently magnetic; but being of soft iron, they instantly lose their magnetism on getting beyond the range of induction; and their rapid revolution and consequent successive appearance before, and on leaving the permanent magnet, answer exactly the same purpose as that of inserting and removing the bar magnet, as explained in our elementary experiment. These armatures are covered with a great length of fine covered copper wire, which is coiled in a similar manner to that we have already described in connection with various electro-magnetic instruments. Two sets of armatures are provided in large machines; one being of thick wire, for the purpose of producing the effects of quantity (light and heat)—the finer being employed for various effects of intensity, such as giving of shocks, &c. The ends of these wires are attached to a contact-breaker, which consists of alternate pieces of wood and brass, against which two springs rest. From these springs the two wires proceed, ended by binding-screws; and the various experiments are conducted by means of other wires attached to these screws.

The larger kinds of these machines have had chiefly two forms; in one of which the armatures revolve at the ends, and, in the other, at the face of the poles of the magnet; and, in such, the magnet consists of a number of the horse-shoe kind, forming a compound arrangement of great power. The following engraving illustrates the appearance of one of these instruments.

Fig. 22.



The power of these machines depends on the size and number of the magnets; the distance of the armatures from them; the length and thickness of the wire wound on the armatures; and the rapidity with which these are made to revolve in face of the magnets. The form of instrument used for the purposes of electro-plating has been already illustrated and described;† and as its several parts are the same, in principle, in all instruments, although they vary in form, we shall refer our readers to what we have already stated on this part of the subject.

Mr. Holmes, who has been very successful in applying magneto-electricity for the purpose of obtaining the electric light, uses a very different arrangement to any we have described. His armatures are fixed with their coils on a cylinder; over these the magnets are made to revolve face to face; and they are driven by steam power. The appearance of his machine is represented in the folio plate. It has been engraved to illustrated one which was shown in operation at the International Exhibition of 1862. To the left of the cut an ordinary electric lamp is seen; and on the right, a polyzonal lens, such as was employed for some time at the South Foreland lighthouse, with one of these engines, in place of the ordinary oil lamps, which it far exceeded, in the clearness and brilliancy of its illuminating power.

It will be unnecessary for us to suggest any experiments with the magneto-electric machine. It will be sufficient for us to state, that all those we have mentioned under the head of Voltaic Electricity, Electro-Metallurgy, and Electro-Magnetism, may be as readily performed by it as by voltaic batteries; the success and brilliancy of course depending on the size of the instrument. Its chief use, at the present time, is for electro-plating, &c.; medical purposes; and, to a limited extent, as a substitute for the voltaic battery in electro-telegraphy. This application we shall enter into in our next chapter.

As a philosophical instrument, the magneto-electric machine is of great interest: in its commercial application, it is also of great value; for as nothing but motion is required to put it into action, all the expenses and annoyance of liquids, &c., used in exciting voltaic batteries, are obviated.

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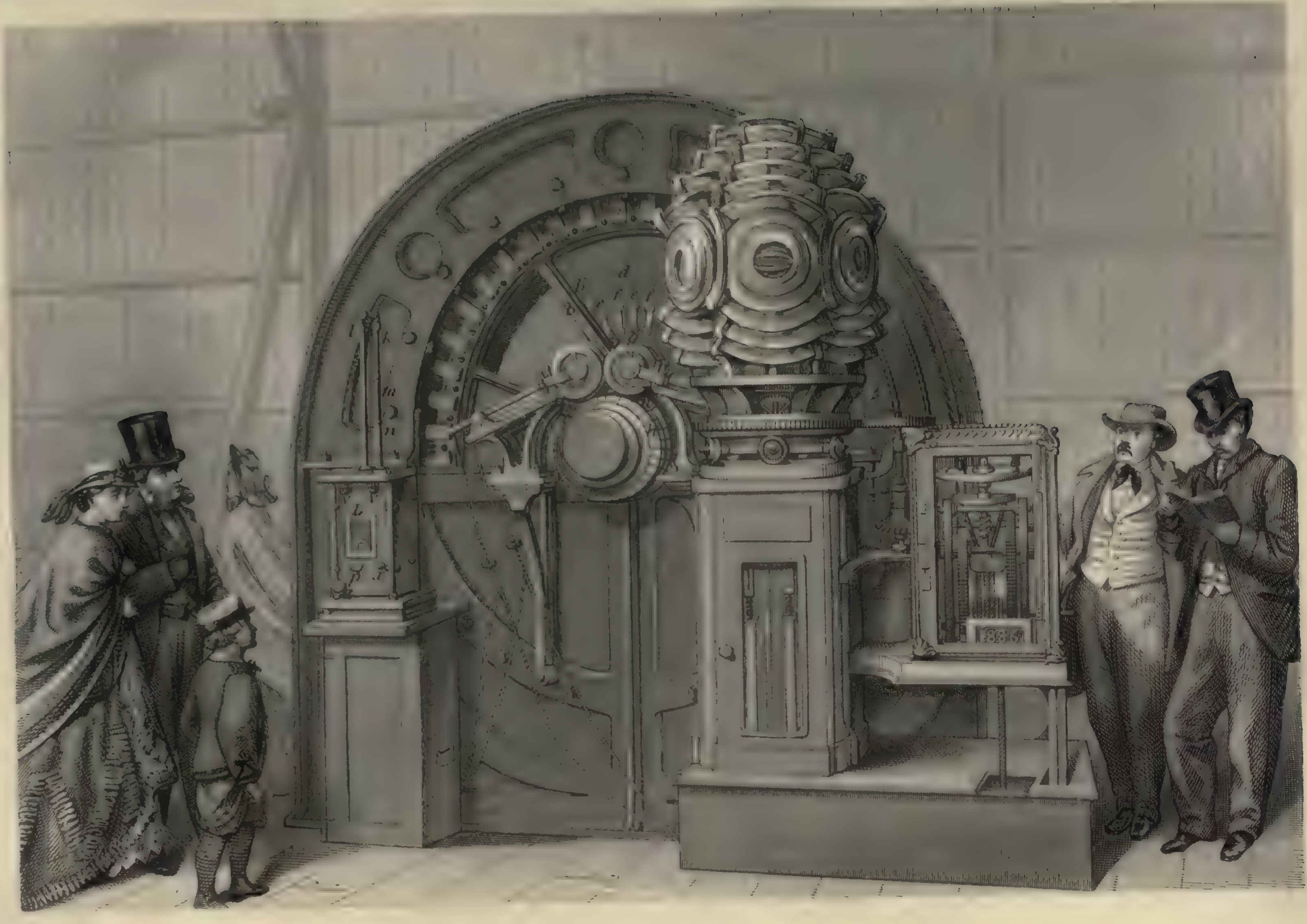
* See *ante*, p. 195.

† See p. 213, *et seq.*; and Figs. 80, 81, 82, and 83.



HOLMES' MAGNETO-ELECTRIC LIGHT APPARATUS.

CHAPTER IV.

THE ELECTRIC TELEGRAPH.

Having explained the chief laws and facts in connection with magnetism and electro-magnetism, together with the lesser branch of magneto-electricity, we shall now lay before our readers an exposition of the principles of Electro-Telegraphy, and include a description of some of the instruments which have been most generally adopted, in different countries, for telegraphic purposes.

It will be only necessary for us here to call the attention of our readers to the remarks, experiments, and illustrations which we have already made in connection with this interesting subject. We before stated, that we presume the student to have mastered the various facts on which we have treated. This will prevent unnecessary repetition on our part, and will save both time and trouble to our readers.

The earlier modes of telegraphy were of the simplest description. Fires lit on the tops of high hills, at different stations, served, by means of preconcerted signals, to convey intelligence of any event to a great distance. In more modern times, the semaphore answered a similar purpose; and even birds—as the carrier-pigeon—were employed to carry messages between correspondents. There is scarcely a doubt that our railway system has been the chief means of perfecting the present mode of telegraphy. Without it, it would be impossible that we could attain that regularity and high speed, combined with safety, which are the characteristics of railway travelling and conveyance at the present time. In fact, the discovery of Oersted laid dormant for nearly twenty years before it was used in its present application; and we are indebted to Messrs. Cooke and Wheatstone for the perfection to which our telegraphic system has arrived.

We shall not attempt to give any historical details of the subject, as our pages are intended chiefly to convey and explain facts and laws, rather than to uphold the views, opinions, and interests of those persons to whose ingenuity we owe so great a debt. Hence, we shall only introduce their names and inventions as may best suit our plan, leaving our readers to consult other works which are specially devoted to the subject, and its numerous and minute details. We shall commence with the needle telegraph—that being the most common form of the instrument employed in this country; and, afterwards, shall proceed to describe other arrangements which are in use on the continent and in America.

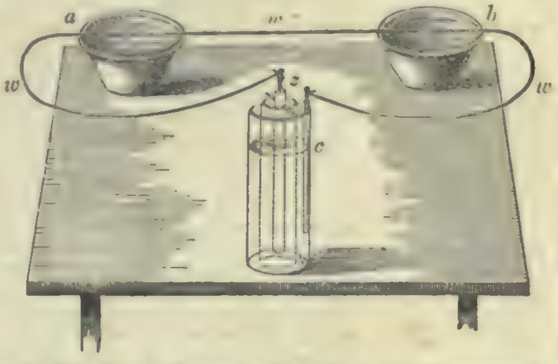
We have shown,* that when a current of voltaic electricity is passed, by means of a conducting wire, over and parallel to a magnetic needle, the latter is diverged right or left of its meridional plane, according to the direction of the current; and we also have seen, that that direction may readily be reversed, or, regulated, by the most simple means. But whilst our readers will easily understand the experiment and engraving which illustrated our remarks, they may not be able to understand how the divergence of the needle, and the consequent transmission of a message, is effected, when the station to which it is sent is at a distance from the source of power; and, further, they will also have a difficulty in finding out, how these divergences can be so managed as to indicate the wishes or instructions which they are intended to convey. It will be our business, therefore, to explain both these points; and we shall thus, we hope, in a clear and simple manner, remove all mystery in reference to this most interesting subject.

1. The Conveyance of a Message.—The first thing required for conveying a message between any two stations, is a sufficient conducting medium, by means of which the current of electricity can pass, so as to effect a divergence of a needle, at a distant station which shall agree with that produced at the place from which the signal was sent. We cannot adopt a better plan to explain the principle than that of an experiment; and shall therefore introduce a plan of illustration which we have generally used, and which may be easily arranged by the student.

Experiment 21.—Place, at the ends of a table, a basin of water, and on the surface of the liquid of each, float a magnetised sewing-needle.† Over, and parallel to the needles, place a copper wire; and, at either end of the table, have ready a small voltaic battery of any sort in action. Bring the ends of the wire close to the battery. Now, so long as no current of electricity passes, the needles will maintain their polarity, and suffer no divergence. But if the ends of the wire are connected with the zinc and copper of the battery, then a current of electricity will pass from the copper plate, by the wire, over the needles, and back to the zinc; and, during its passage, the needles will *simultaneously* be diverged from their previous direction, according to that of the current, in a manner we have already explained.‡

Now, it matters not how long the conducting wire may be. For instance, the experiment may be tried with one basin and needle, at one end of a garden, with the other at any other place; or from room to room in the house. In its passage, the wire may be twisted, or fastened, in any part, so that it be continuous throughout its length; because no mechanical action, like drawing or pulling the wire, is required. All that is wanted is, that the wire shall be of sufficient size to conduct the current freely, and that the current itself shall be sufficiently powerful to traverse the entire length of the conductor. The mode of carrying out this experiment is illustrated in the following engraving.

Fig. 23.



In Fig. 23, we observe each part of the arrangement. The vessels *a* and *b* have, floating on the water, each a magnetised sewing-needle. Over these, and parallel, are portions of the wire, *w w w*, the ends of which are connected with the zinc *z*, and the copper *c*, of a single cell of a Daniell's battery.§ Any other form of battery, or even a slip of zinc and one of copper,|| will answer equally as well.

After having thus noticed the simultaneous divergence of the two needles in one direction, they should then be allowed to come to rest—first removing the wires from the battery, so as to stop the passage of the electric current, or the divergence of the needles would be maintained. The wires should then be changed as to their connection with the plates of the battery. That which had been previously in contact with the copper should be applied to the zinc, and *vice versâ*. On this being completed, the

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* See *ante*, p. 247.

† *Ante*, p. 241.

‡ *Ante*, p. 247; Fig. 3.

§ *Ante*, p. 188; Fig. 40.

|| *Ante*, p. 186; Fig. 37.

needles will instantly diverge in a direction contrary to that which they had previously done.

We may sum up, as follows, the facts taught us by the last experiment, and those to which we have also referred in our previous pages: for we learn—

1st. That an electric current diverges a magnetic needle, under certain conditions.

2nd. That the direction of the divergence depends on the direction of the electric current.

3rd. That these results can be obtained at any distance in two instruments, providing a sufficient conductor intervenes between them.

4th. That the divergence occurs at the same moment (in almost every instance) in each instrument. Practically, there is scarcely any exception to this rule in land telegraphs; and its occurrence in submarine telegraphs will be duly noticed.

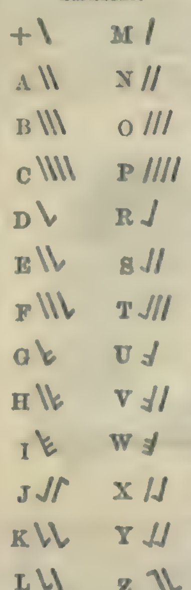
Having thus established the principle on which a message may be conveyed, we shall now explain how the divergence of the needle is used as a means of indicating the message sent by the wires. For the sake of simplicity, we shall omit all other considerations as to mode of conducting, obstacles, earth currents, &c., &c., until we have made the second question clear to our readers.

2. **The Decipherment of the Message.**—We stated that there are only two divergences from which the message can be read, if one needle alone be used at each station; and, for the present, we shall confine ourselves to the examination of the single needle arrangement. It is obvious, however, that even with the deflections of one needle, numerous signals may be transmitted between any two stations, if such deflections are taken singly or together.

The following diagram illustrates the code which has long been established; and which, although at first sight it may present some difficulty to the uninitiated, is, after all, very simple in its construction. The lines indicate the direction in which each divergence of the needle takes place; and the letters next them are those represented by the deflection. The small lines, in some signals, indicate that a combination of deflections, in different directions, is used. Thus, two to the left represent the letter A; whilst one to the left and one to the right \ represent the letter D; and so on. For the annexed, and some of our future illustrations, we are indebted to Mr. Shaffner's work on the Electric Telegraph.

It is thus evident that this combination of deflections may be employed to spell out any message which may be required to be sent from one station to the other. It is true that such a simple arrangement as we have described in our previous pages,* would be quite unfit for this purpose; because, the needle, after having been once deflected, would require a length of time before gaining its primary polar position. At the same time, our readers may easily try to effect the deflections in a leisurely manner, and so gain a practical knowledge of the working of the more perfect kind of instruments, which we shall presently describe.

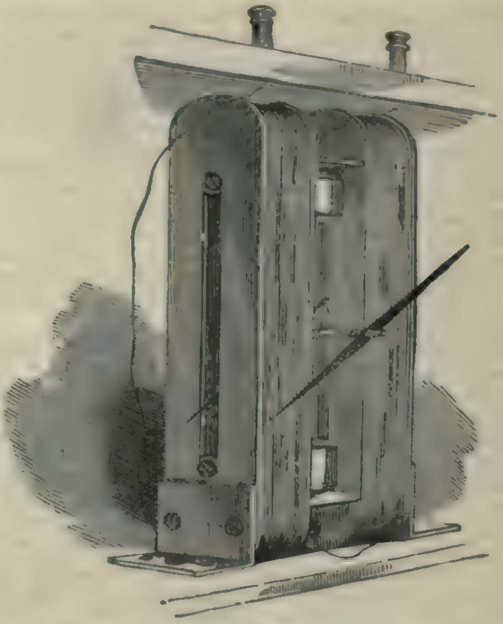
THE SINGLE NEEDLE ALPHABET.



We have gone so far as to show, that a message, or a series of signals, may be transmitted, by means of a conducting wire, from one place to another; and that such signs may, by a preconcerted arrangement of signals, be subsequently deciphered, and rendered available for telegraphic purposes; the deflections of the needle, in any specified direction, being employed for that purpose. We shall now advance another step, and explain the instruments which are actually used; commencing with the single needle telegraph, and afterwards describing the double needle, which is most generally adopted throughout Great Britain.

Instead of a needle floating on the surface of a liquid, one is suspended on pivots in a suitable frame; and, over this, wire is coiled, so as that each convolution may be parallel, and in as close proximity as possible, to the magnetised needle, in the manner illustrated in the following engraving.

Fig. 24.

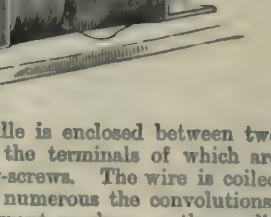


In the above cut, the needle is enclosed between two coils of fine covered wire, the terminals of which are connected with two binding-screws. The wire is coiled on a frame; and the more numerous the convolutions, the greater effect will the current produce on the needle itself. The reason of this we have already explained when we described the arrangement of the ordinary electro-magnet.†

The external needle is simply an index, which oscillates to and fro before a card on which the symbols or deflections are represented; and thus the person watching the instrument can at once see the deflections of the needle, and the code of signals which he has to follow.

As we are anxious to make our readers practically acquainted with the working of the telegraph, we shall suggest the construction of a model, which, for a trifling expense, will enable them to actually practise the telegraphic code. The following engraving illustrates a simple instrument, which we have often used for public illustrations of this subject. It consists of an upright board, on which is fixed a small coil of covered copper wire, to the ends of which are attached two binding-screws, at the foot of the instrument. Inside the coil a magnetised steel needle is suspended on pivots, so that it may freely move right or left. It should be hung a little out of its centre, so that the lower part may, by its weight, restore the needle to a perpendicular position after a signal has been sent, and the current withdrawn. Two of these instruments are required; and they can be made at an expense of three or four shillings each.

Fig. 25.



A wire must be attached to one of the binding-screws of one instrument, and be extended to that of the other; and a wire from each instrument is to be brought to the battery, which may be a single cell of any kind.

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* See *ante*, p. 259.

† See *ante*, p. 250.

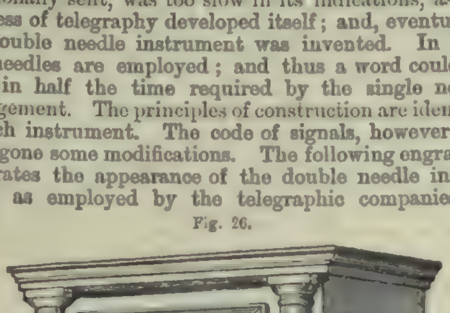
The connection of the wires is precisely similar to that illustrated in the arrangement shown in a previous page.* The ends of the wire proceeding from the coil, may be immersed in two cups containing a little mercury; and the battery wires being held in the hand, they can be dipped in either cup, so as to send the current in any direction that may be desired. Now, by this arrangement, the needles of each instrument can be simultaneously deflected; and if the code of signals already given be pasted on either side of the coils of each model, a complete single telegraph arrangement will be produced.

If one instrument be placed in any part of a house, and the battery and the other model be placed in another apartment, and two wires be extended between them, telegraphic signals can be just as easily transmitted, as is done, in the usual way, by the ordinary telegraphic arrangements; and our readers will learn more practical knowledge of the subject in half-an-hour, than a perusal of works on telegraphy can possibly afford them.

Many persons are much surprised and puzzled at the extraordinary rapidity with which the signals are read by the persons in charge of the telegraphic instruments at a station. We may, however, easily illustrate the readiness thus exhibited, by reminding our readers, that any musical instrument is rapidly performed on, solely from continued practice of the musician. He has had at first to learn the notes—a slow and laborious process: eventually, however, he learns to read and play music at sight. In the same way, the telegraph clerk gradually acquires a proficiency in reading the indications which the deflections of a needle afford. Practice in this, as in all other matters, alone makes perfect; and we may venture on a paraphrase of a well-known quotation, to illustrate our remarks—"No one suddenly became an expert telegraphist."

It was found that the single needle apparatus, although sufficient for stations between which messages were only occasionally sent, was too slow in its indications, as the business of telegraphy developed itself; and, eventually, the double needle instrument was invented. In this, two needles are employed; and thus a word could be spelt in half the time required by the single needle arrangement. The principles of construction are identical in each instrument. The code of signals, however, has undergone some modifications. The following engraving illustrates the appearance of the double needle instrument, as employed by the telegraphic companies on most of our railway lines—omitting the code of signals on the face of the instrument.

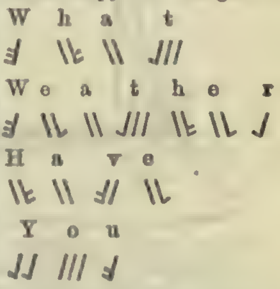
Fig. 26.



In the above engraving, we observe the two indexes which are connected with the needles inside the instrument. Pegs are placed on each side of them, to restrain their deflections; and at the lower part are two handles, by the movement of which the current of electricity is readily sent in any direction.† These handles have wires attached to them, which come in contact with those of the battery and the telegraph instrument; and according as the current is directed by them over the coils, so the needles are deflected right and left. On some lines of railway an alarum is used, which, by means of an electro-magnet, sets a bell in action, and so calls the attention of the telegraph clerk. A portion of the current is diverted from the needle coils; which, by moving a detent on clockwork, rings the bell. This continues until stopped by the attendant.

Before proceeding to explain a variety of practical details, we will illustrate the manner in which a message sent from one station is indicated and deciphered; and, for this purpose, shall employ the code of signals adopted in the single needle instrument already shown at a previous page; because, by so doing, we shall avoid any confusion which might arise in employing the two-needle code for our purpose. We shall presume that each attendant is at his station, and in front of the instrument; that the bell has been rung by the sender, and heard by the receiver. For example's sake, we may suppose the inquiry made—"What weather have you?" The sender moves the handle of his instrument so as to produce the proper deflection of the needle for each letter of the message, as seen beneath.

Now, to illustrate this more fully, we will give a literal explanation of the first word of the message—"What." For the "W," there are one deflection to the right, and three to the left. For "h," two to the left and two to the right. For "a," two to the left: and, for "t," three to the right, and one to the left.—Now, in the double needle instrument, the combinations of both needles are used at the same moment, and so much time is saved.



It is customary to shorten the message as much as possible by leaving out unnecessary words and letters; but these are practical details which scarcely concern the general reader. Each station on the line has a definite signal; so that, although the bells may be ringing, and the needles working at intermediate stations, the attendant at the intended station can at once perceive that *his* attention alone is required. The clerks have also an abbreviated code of signals for special and constantly recurring circumstances; and it is usual to transmit a special signal from and to each station when a word or sentence is completed, to signify that the same is, or is not, understood.

With respect to the rapidity of transmission of a message, we may observe, that it depends on the quickness of perception and intelligence of both receiver and sender (except under circumstances over which neither have any control, and of which we shall presently speak). From 60 to 100 letters may easily be sent and read per minute.

Most persons are curious to know how long it takes to send a message by means of the electric telegraph; and independent of the number of words, there are other circumstances to which we must now draw attention. As we have sufficiently explained the construction and use of the needle instrument, we shall now refer to the conducting wires; and the observations we are about to make, are equally applicable to every form of telegraph which we shall hereafter describe. There are two methods of connecting distant stations by means of wires. The Electric Telegraph Company suspend theirs on poles raised at the side of the railway. The wire, which is of thick galvanised iron, rests, not on the wood of the poles, but on glass or porcelain supports, which are contrived to shoot off the water deposited by rain, dew, or fog, so as to prevent a loss of power by deficient insulation of the wires. The Magnetic Telegraph Company enclose their wires—first covered with gutta-percha, so as to keep them electrically separate—in iron tubes: these are deposited on the sides of the main road; and at different parts, the wires are raised above ground, in an iron post, so that they may be tested for any defect. This precaution is, of course, unnecessary when the wires are suspended in the air, because any fault can be at once both seen and remedied.

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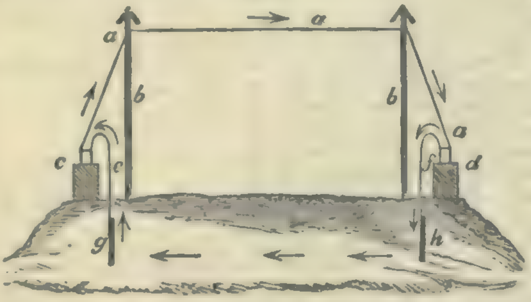
* See *ante*, p. 259; Fig. 23.

† *Ante*, p. 260

We must here remind our readers, that we have hitherto supposed that the circuit, or course of the electric current, is passed from one station to another, and back again, by means of a continuous or double conducting wire; but such is not done in practice. The *earth* is made to act as a conductor; and thus the expense of a second wire is saved. It is a most singular fact, that moist earth acts just as well as a solid wire in this respect; and if any number of instruments be working at the same time, and each using the earth for the purpose referred to, still each receives and indicates its message without the least confusion, providing an independent wire also joins the separated but corresponding instrument.

The following engraving illustrates the mode in which the earth is thus employed.

Fig. 27.



In the above engraving, *a a a* represent the conducting wire suspended on the poles, *b b*; *c* and *d* are two telegraph instruments, each of which is connected with the wire, *a*; *e* and *f* are wires which lead respectively from the instruments *c* and *d*; and at the ends of each of these wires, plates of metal, *g* and *h*, are affixed, which are buried at some distance beneath the surface of the ground, in moist earth. Now, supposing a current is passed from the station *c*, to a distant one, *d*—its course will be by the wire *a a a*, suspended on the poles; proceeding from *c* until it reaches *d*; and thence leaving the instrument by the wire, *f*, it passes into the earth by the plate, *h*: from this it then returns in the direction of the arrows, until it arrives at the plate *g*; by means of which it reaches the first station again by the wire *e*, into the instrument, *c*. A continuous circuit is thus established between the two, or, indeed, any number of stations; and, as we before remarked, it answers just as well as if a second conducting wire were employed.

Our readers will at once perceive the necessity which exists for maintaining a complete insulation of the wire from the earth; and in this is found one of the difficulties of telegraphing. It is almost impossible to keep the wires and poles completely dry during wet weather, although the glass or porcelain holders intervene. Now, moisture is a conductor of electricity; and as such, a portion of the current is often diverted from the wire towards the earth, in wet weather, by means of the wet poles. This circumstance is one, therefore, which interferes both with the speed of transmission and the distinct indication of a message, when rain, fog, or dew are prevalent.

It has been already mentioned, that the electric currents of the earth have a manifest influence on the magnetic needle; and in this is found another and serious source of annoyance to telegraphers. We have frequently noticed the needles of instruments, at stations far apart, working with all the vigour which would be seen if the clerks were engaged at the moment in sending a message by the wires. The needles oscillate in a most irregular manner, and not with that decision of appearance which is noticed when a message is actually being sent. The currents of atmospheric and terrestrial electricity vary constantly in their intensity; and, in some cases, are of extreme violence. We have noticed them as especially prevalent on the Edinburgh and Glasgow line, which runs nearly due east and west; and have heard some curious details of such occurrences, from the operators on the instruments at the extreme stations. The same effects, however, are experienced on all telegraphic lines, in a greater or less degree.

Atmospheric electricity, when present in excess—as in the case of a thunder-storm—is another source of difficulty, and possibly of danger, to the operator; this, however, is chiefly confined to lines in which the wires are suspended on poles.

In a great extent of country, of course, atmospheric changes are constantly occurring; and, to some extent, these changes may, by means of the conducting wires of the telegraph, compensate with each other, so far as that instrument is concerned. For instance—we may readily suppose, that in a distance of 40 or 50 miles, the atmosphere may occasionally be charged, at different places, with electricity of opposite characters;* and if the amount of these be equal, no effect would result on the telegraphic wires extended between any two distant stations. But it may so happen, that at or near one of these, a local electrical disturbance may occur; and such is sure to be followed by derangement of the needles of a telegraphic apparatus, if no provision be made to obviate it. Sometimes the electricity, during a thunder-storm, has passed by the conducting wires into a station, and completely destroyed the whole apparatus; or, at all events, so affected the needles as to render them useless. At times their polarity is reversed; and instances have been met with in which the persons in attendance have narrowly escaped with their lives. The remedy, however, is comparatively easy of execution. It is unnecessary for us to remark, that the intensity of the voltaic current is infinitely less than that of the electricity set free during a thunder-storm. For whilst the former requires a continuous and good conductor, the latter will readily pass through a stratum of air, between two conductors, and so reach the ground, if proper arrangements are made for that purpose. This fact is taken advantage of to prevent the accidents to which we have alluded. In some instances the poles are provided with lightning conductors; a plan adopted by the London District Telegraph Company. In others, temporary arrangements are contrived; so that, in the event of a thunder-storm occurring near a station, the atmospheric electricity can be readily directed to the earth, by means of a conductor of sufficient size; in which, however, a slight break is made—enough to prevent the escape of a current passing from a voltaic battery. Some ingenious inventions have been applied for this purpose; which, however, are of special interest only to those engaged in telegraphic matters.

It is a remarkable fact, that although the greater portion of the telegraphic wires in this country are suspended on poles, and entirely unprotected, it rarely, if ever, happens that they are subjected to intentional or malicious injury. With all possible charity for the qualities of human nature, we can scarcely suppose that the immunity of the wires is due to the good feeling of all classes, especially of the juvenile members. We believe, so far as our experience goes, that a general impression exists, to the effect that a powerful shock would be obtained on touching or meddling with the wires; and most young people throughout the country, have, at one time or another, made acquaintance with this kind of electrical phenomena. There is, at all events, a general opinion entertained, that birds flying against the wires are instantly "killed by the electric shock;" although it has not the least foundation in fact; for it would be impossible for any creature to feel the slightest electrical influence under such circumstances. This, however, is an instance of many cases in which ignorance becomes a protection rather than a bane. Except in extraordinary cases, storms of wind do comparatively little damage to the wires; and as any accident can be at once reported by the guards and engine-drivers continually passing, little or no hindrance ever arises from this cause.

There are many other circumstances which tend to the annoyance of the operators on a line of telegraph; but such are not of general interest; and we shall, therefore, omit to notice them.

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* See *ante*, p. 183.

With respect to the various uses to which the electric telegraph is ordinarily applied, it would be impossible for us to enter. Confining our attention to the operations of the companies working the instrument on land, we may observe, that there is no event occurring in daily life which is not constantly made the subject of a message. Politically, commercially, and socially, the electric telegraph has become an institution of the age. So great has been its development, that, at the present time, all London is intersected with underground wires, and the roofs of the houses passed over by those suspended on poles, for the purpose of accommodating the universal desire for speedy transmission of messages. Nearly every line of railway has its public and private telegraphic arrangements. Warehouses, in different parts of the metropolis and other cities, and the various parts of warehouses and other large buildings, are connected together by electric wires. A company has lately been formed for the purpose of supplying instruments and wire communication between any two or more private establishments in London. Indeed, we may sum up our remarks on this point by stating, that electricity is as effectually "laid on" in most of our large cities in Great Britain, as are gas and water; although its uses and advantages are not so largely availed of, or required, as the other articles we have named.

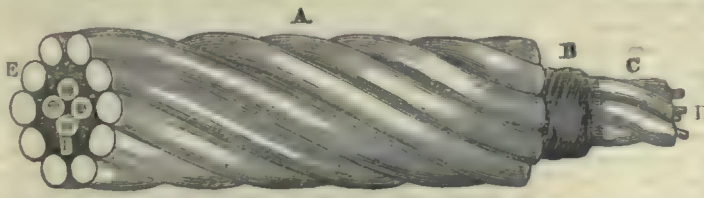
SUBMARINE TELEGRAPHY.

After it had been proved that communication could be satisfactorily and readily employed between any two or more points on land, it was but natural that an attempt should be made to connect countries separated by seas, &c., so that electric telegraphic messages might be dispatched from one to another. Of course, the difficulty of carrying out the idea chiefly rested in that of insulating the wire, and of giving it sufficient strength to resist all the accidents to which it might be subjected. The first obstacle—that respecting insulation—was overcome by using gutta-percha as an external coating to the wire; and the latter was conquered, in most instances, by the exercise of that mechanical ingenuity for which this country is generally so greatly renowned. We shall proceed to detail the various results which have been obtained in this interesting branch of electro-magnetic application, and so enable our readers to estimate and understand the triumph which science and skilled labour have accomplished, in this respect, during the last few years. For the purpose of making this subject as clear as possible to our readers, we shall divide it into three parts. Having no reference to the instruments which may be used to indicate the message, we shall chiefly confine our attention to the cable, which is, of course, the specialty of submarine telegraphy. We shall consider—

1. The construction of the cable.
2. Its submersion in the sea or rivers, &c.
3. The circumstances which interfere with its constant and successful working.

1. The Construction of the Cable.—A submarine cable requires that it should be sufficiently protected from any chance of injury by being rubbed against any hard or pointed substance after submersion; that it should have strength enough to resist the great tension it undergoes during submersion; and that the conducting wire should be so covered as to entirely insulate the current which passes by it, that such may not be diverted from its intended course. These points are secured in various ways, which will be better understood by the foregoing engraving, which is an illustration of the cable laid between Dover and Calais.

Fig. 28.



A, the iron wires surrounding the cable; B, the tarred rope surrounding the gutta-percha covered wires; C, the gutta-percha covering the wires; D, the four wires through which the message is sent; E, an end view of the covering of iron wire; F, an end view of the gutta-percha covering, and of the conducting wires.

In Fig. 28, we observe, that the outer portion of the cable consists of a coating of thick iron wires. These are twisted on by means of powerful machinery; and when this is well performed, they afford a sufficient protection to all that they enclose within their folds. The size of these wires varies according to the strength required, and the circumstances in which the cable may be placed. In shallow water, the external coat requires to be much stronger than in deep sea, because of the pebbles and broken rock against which it may rub. After a cable has laid for some time beneath the surface of sea-water, it acquires a curious coating, which has the power of affording it additional protection. This consists of the shells of small fish which attach themselves to it. It gradually increases, and sand, &c., soon become completely attached to the wires, forming an external surface of great solidity and strength. This, which, at first sight, seems to be a source of danger to a cable, becomes one of its most valuable protectors; and, instead of decreasing and wearing away, it accumulates continually. The dangers, then, of injury from such sources, peculiar to immersion in sea-water, occur only for some time after the cable has been laid, and decrease as the coating we have described accumulates. We observe, in Fig. 28, next to the external metallic coating, a covering of hemp, which is wound round the next coating of gutta-percha, enclosing the wire. The object of this coat is to allow of play between the covering of the conducting wires and that which forms the outside of the cable. The hemp is saturated with a mixture of grease and pitch, which, to some extent, tends to protect the gutta-percha from the attacks of marine insects; for any greasy substance is always avoided by them. Should the cable be accidentally bent at too acute an angle, the coat, by its elasticity, prevents the snapping of the fine internal wires.

We next observe a gutta-percha coat, which is, perhaps, the most essential of the three; for, being a non-conductor, it insulates the conducting wires which it encloses. When more than one wire is employed—and this, with few exceptions, is always the case—each wire is individually coated with gutta-percha; and thus the insulation of the entire number is more effectually arrived at. Much of the success which has been obtained in the use of submarine cables, is due to the application of gutta-percha: and the chief manufacture of the material, and its application to telegraphic purposes, is conducted by the Gutta-Percha Company, of Wharf-road, City-road, London; which has expended a large capital in perfecting this branch of the business. Whilst mentioning names of manufacturers, we must also include Messrs. Newall, of Newcastle-on-Tyne, and Messrs. Glasse and Elliot, of London, as those who have individually or collectively had the manufacture of all the cables which have been laid round our coasts.

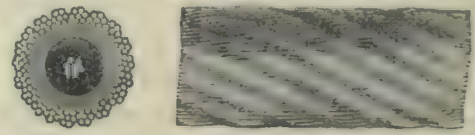
From late experiments—more especially since the failure of the Atlantic telegraph, of which we shall presently speak more fully—the opinion has gained ground, that gutta-percha is not so good an insulator as is to be desired. In fact, the process of its manufacture is such as to introduce chances of failure by the porosity of individual portions. Attention has therefore been directed to other materials. Amongst these is india-rubber, which has lately been highly recommended as a substitute for gutta-percha, for its insulating and non-porous character. Each material has its advocates; and extended trials are still required to afford accurate information on the subject. There is, perhaps, hardly any other substance in nature with which we are acquainted, so suitable to telegraphic purposes as those we have named; and the only advantage which has arisen, hitherto, in the search after substitutes for them, has accrued to the patent offices, in the shape of fees paid by inventors, who started forth in legions as soon as any defect in a submarine cable was discovered. On this point we may remark, that it would be a most merciful provision, on the part of the government, if a kind of committee were appointed, whose duty it would be to investigate and report on the various projects which are continually being patented. An immense amount of money and litigation would be saved; really good inventions would be reported as of value; and our insolvent courts and lunatic asylums cut off from a supply of broken-down and disappointed, but too sanguine inventors, who unwittingly, but surely, flock thither.

We next arrive at the conducting wires which always form the centre of a cable, and on which its value entirely depends. They are always made of copper, which combines elasticity, cheapness, and high conducting power. Various sizes are employed; but, generally speaking, that of No. 14 or 16 gauge is preferred, and is about equal to the tenth part of an inch in diameter. Occasionally, as in the Atlantic cable, a number of fine wires are twisted together, so as, electrically speaking, to form one; but we are not aware that anything is gained thereby, except, perhaps, a slight additional strength. We may here mention a circumstance which, we believe, often causes the failure in submarine cables. We have already noticed the fact, that copper wire becomes hardened, and eventually "rotten," after having long conveyed an electric current.* If it be enclosed in gutta-percha, the fact that the wire is broken cannot be discovered externally; and hence, during the laying of a cable, it is essential that its conducting power should be constantly tested. But the defect to which we allude does not occur until the cable has been some time in use, and its wire injured by the electric current passing by it. Our attention was first directed towards this point in 1856, when attempting to illustrate the explosion of the Russian infernal machines by means of the voltaic batteries.† After a few times of using the gutta-percha covered wires, which were the same as those employed for submarine telegraphs, they invariably broke on being bent to the least degree. The explosion of the powder would, of course, be prevented. But, strange to say, a broken wire would at times convey the current perfectly. This, however, is easily explained; for if, by accident, the broken wire were so bent whilst in use, as that the fractured edges were again brought into contact, the current would, for the moment, find a complete conductor, and so the explosion would take place. Now this is what frequently occurs in submarine cables. At times the current affords no indication of its passage: and we would suggest, that the cause which so continually hindered the success of our experiment, may just as likely be active in the case of submarine cables, more especially as far more intense currents are sent through their wires than those we employed. We observed, that, on submitting the edges of the fractured wires to a microscope, the metal had a decided crystalline appearance—proving that it had undergone a molecular change.

To those of our readers who may be engaged in practical telegraphy, we may state, that the "rottenness" of copper wire is produced the more rapidly as the intensity of current is increased; and, from our experiments, we do not consider that it matters of what thickness the wire is. We have employed copper wire, of a quarter of an inch in thickness, for conveying the current of from 50 to 100 cells (large size) of Grove's batteries; and the action on any portion of the wire may easily be noticed after they have been *constantly* in use for twenty or thirty hours. The readiest way of trying the experiment, is that of making some No. 22 gauge copper wire the terminals of a common or Rhumkorf coil. They will soon become as brittle as a piece of zinc or lead wire; and we have, in rare cases, seen a piece, ten inches long, break by its own weight after having been so employed.

The number of wires placed in any one cable, is regulated according to the amount of business expected to occur between any two stations. That laid between Ireland and Scotland, contains six wires: the Dover and Ostend cable, laid in 1853, has also six. The cable laid between the coast of Suffolk and Holland, has seven; which, however, have each an external coat of iron wire—the whole being twisted together to form one cable; whilst the temporary cable laid between Varna and Balaclava during the Russian war, and which served so many valuable purposes, was of small size, and contained but one wire, without any external covering. It will be unnecessary for us to enter into any special description of those we have named above; but we may here more fully describe the Atlantic cable, because so intense an interest has been attached to it. As we have already stated, it consisted of an external covering of iron wire, and a coating of gutta-percha; and, in the centre, seven fine copper wires, twisted together, formed the conducting medium. Its appearance is indicated in the annexed cut, both in section and in length.

Fig. 29.



Our readers will, doubtless, be surprised to find that such a comparatively slender cable (its actual size is represented in the cut) should have been employed. It must, however, be borne in mind, that, at great depths, the ocean is not liable to disturbance, as is observed on its surface. A dead and imperturbable calm always exists: so, if the cable stand the trial it has to undergo during submersion, it is liable to little subsequent injury from causes external to it.

The following table may be of value to many of our readers who take a general interest in the subject of submarine telegraphy. We have selected our information from various sources; and have chosen the leading cables now at work between different stations, or which have been worked and have failed.

Situation.	Date of laying.	Length in miles.	Number of wires.	Weight per mile, in tons.	Total weight in tons.
Dover and Calais......	1851	25	4	7	175
" " Ostend ...	1853	70	6	7	500 nearly.
Varna and Balaclava	1855	150	1	—	This wire had no metallic coating.
Varna and Constantinople................	1855	200	1	1	200
Atlantic Telegraph: Ireland and America	1858	2050	1	—	About 2,500 miles were used, and lost, in laying this cable. Its weight was about a ton per mile.

In the above table, the Red Sea and Mediterranean cables have not been included. The former has lately failed to work; and those laid in the Mediterranean are of various lengths and sizes. The cable between Malta and Alexandria is but just completed.

We may remark, in reference to the comparative weight of the different cables, that, in many cases, considerable material has been saved by laying a light cable in deep water, and attaching to it a stronger one near the shores where it terminates. The reason of this is obvious from our previous remarks; but we may add, that occasionally, cables are liable to accident from vessels anchoring near them. As the anchor is weighed, its fluke may catch the cable; indeed, some time ago, one of the Channel cables was thus accidentally fished up. The captain of the vessel whose anchor had caught

* See *ante*, p. 250.

† *Ante*, p. 193; Fig. 53.

it (being ignorant of its nature, it is to be hoped), was about ridding himself of the incumbrance by cleaving it with a hatchet; but one of the crew, more sensible than his commander, induced him to cast it overboard, and so to return it to its former resting-place.

2. THE SUBMERSION OF THE CABLE.—One of the most serious undertakings in connection with the submarine telegraph cable, is that of "laying it," as the operation is termed. Even when the finest and calmest weather is experienced, many difficulties arise. The cable is coiled in the hold of the vessel, and passed through an opening on deck. It is then wound two or three times round a large cylinder, to which is attached a break. The object of this is, that the cable may be "payed out" in an even manner; for, if it were allowed to pass from the stern of the vessel in an irregular manner, it would soon form knots and kinks, and most probably break. The regulation of its speed is a matter of great delicacy; for sufficient strain must ke kept on it to lay it straight, and yet care must be taken that it is not broken by too great tension, partly arising from the speed of the vessel, and from the weight of the cable itself. An ordinary speed of laying the cable is about four to five miles per hour. When, however, a storm occurs, the difficulties are enormously increased. As the vessel rises and sinks in the trough of the waves, the tension on the cable is suddenly increased or diminished; and, unless the greatest care be taken, its fracture is inevitable. Even while the cable may appear externally to be sound, it is possible that one or all of the wires inside it may be injured; hence telegraphic signals are constantly kept up between the vessel and the station whence it proceeded. In laying the Atlantic telegraph, the two vessels started from a place equi-distant from Ireland and America. The cables in each vessel were spliced together, and the portion between the two was gradually sunk. The steamers sailed then in opposite directions, until each reached its appointed port; and, during the entire voyage, the electricians kept up constant signals, acknowledged and received on each vessel, which, of course, proved that the conducting wire was perfect throughout. In the first attempt to lay the cable, the signals were interrupted, and could not be recovered, so that it at once became evident that the sunken portion had been by some means injured. This, of course, necessitated its loss, and both vessels returned to the port whence they had started.

We need not remind our readers, that the bed of the ocean, like the surface of the land, is often exceedingly irregular. Sometimes deep ravines are encountered, in which the cable quickly and suddenly falls. In other instances great elevations are met with, which diminish the rate of the cable's passage. All these, and many other circumstances, have to be minutely attended to; and without entering more fully into such details, we have pointed out sufficient to prove, that the laying of a submarine cable is a work of no ordinary difficulty and importance.

3. THE CIRCUMSTANCES WHICH INTERFERE WITH THE REGULAR WORKING OF A SUBMARINE CABLE.—It would be impossible for us to enter into a minute examination of all the circumstances which affect the regular working of a submarine cable. We shall therefore only touch on those which are either very important, or ordinarily found to be present. The numerous accidents, of a mechanical nature, to which all submarine cables are subject, have been already frequently noticed, and will not require further observation. There are, however, others of an electrical nature, which often produce serious consequences; and, at times, almost forbid all chance of communication between the two termini of the cable.

It was found, some time since, that if a short length of submerged cable formed part of a circuit between two land-lines, more disturbance occurred, in the regular indication of signals, than would have been the case had the entire line been on land, and suspended on poles in the usual manner. At first it was considered that this arose from defective insulation, or from injury to the conducting wire. Eventually, however, the fact was discovered, that an ordinary submarine cable acquired the character and qualities of a charged Leyden jar, whenever a current was passed through it: indeed, its construction is analogous to that arrangement; for we may suppose the wire to represent the inside of a charged Leyden jar; the gutta-percha to correspond with the glass of the vessel; and the external wires, or the water surrounding them, to answer the outer coating of a jar.* Now, the effect of this inductive action is not only to retard the passage of a current of electricity sent from a voltaic battery, but even to create an opposing force. In some instances, the current is all but destroyed: generally, its indications are irregular; and if a printing telegraph be employed, such as we shall shortly describe—it will be found, that, instead of an instantaneous signal being registered, such is, generally speaking, given in a slow and continuous manner, just as if the current were gradually conducted by the wire. This evil increases with the length of the cable. As we have already stated, even a short one shows such results; but in the Atlantic wire, the effect was, as might be expected, on a very large, and even alarming scale. It is a difficult matter to give a popular illustration of the actual results produced; but we may compare the retarded and opposed passage of a current to the position in which a vessel is placed on entering the mouth of a narrow river. The vessel may be urged forward by a fair or strong wind, whilst the water of the river is rushing towards the sea. The stream is urged into swelling waves, and on these the vessel tosses, making but little progress towards her destination. Now, similarly, the wave of voltaic current meets that of the induced cable-current, and the effect is that which we have described.

The question next arises as to what means can be adopted to overcome this difficulty; and the leading doctrine of Homœopathy—"*similia similibus curantur*" (*Anglice*—"Like cures like")—was called in to the aid of the telegrapher. It was considered that, if an intense current were opposed to an intense current, although produced from different sources, the strongest would gain the advantage, and more speedy signals would result. This plan was accordingly adopted on the Atlantic cable soon after it gave the first sign of defect. Powerful voltaic batteries, magneto-electric machines, and Rhumkorf's coil, or its modifications, were successively tried; but, alas! with no effect: and it is certain, that if the slightest break in the cable had existed, and a current had been transmitted, even for a few moments, from a Rhumkorf in good action, the fate of the cable would be for ever sealed by the combustion of the two edges of the fractured parts. The reason of this will be at once perceived by reference to our remarks on that coil, at a previous page.†

One effect of a fresh intense current, besides that of overcoming the resistance of a long conducting wire, is to neutralise the induction of the wire current, produced by a previously passing current; hence the employment of such means in the way we have just mentioned. Their want of success, in the case of the Atlantic telegraph, must not be ascribed to a defect in the principle of the doctrine, but rather to the peculiar circumstances in which it was applied. A section of the map of the bottom of the Atlantic, as prepared by Captain Maury and others, exhibited, as near as can be, just such a bed as would be found for a cable, if one were laid from Lombardy over the Alps, at the top of Mont Blanc—then into the valleys and plains of France, across the English channel, and so continued over the hilly parts of England and Scotland—concluding its career on the north side of Ben Nevis. For 500 miles from the Irish and American coasts, the cable was suspended on the tops of marine mountains, and hung in the deep hollow which intervened. Between these two localities, a level bed of sand, and the remains of shells, of nearly 1,200 miles in length, existed, on which the cable might have rested in undisturbed repose. The depth of the ocean there, was sufficient to prevent the least risk of injury from any cause whatever.

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* See *ante*, p. 180.

† *Ante*, p. 255; Experiment 13.

The effect of ordinary terrestrial electric currents on the submarine cable, cannot be properly ascertained; for the position of a cable submerged in a liquid of tolerable conducting power, includes so many elements of disturbance, as to prevent the possibility of our acquiring accurate or reliable data.

The remarks which we have just made, apply equally to all submarine cables; and are modified by the difference of their length, the thickness of the conducting wire, and the number of them enclosed in the cable. It would be very desirable if a series of simultaneous observations were made, as to the retardation which is exercised on a current at the same moment, on cables laid in different directions, with respect to the compass. It would thus be seen how far electric or magnetic currents affected the conducting power, and whether such had a definite direction and action under all circumstances.

We shall now describe the construction of various kinds of telegraphic instruments which have been invented during the last few years, on principles entirely different to that of the needle apparatus.

ELECTRO-PRINTING TELEGRAPHIC INSTRUMENTS.

The remarks we have hitherto made, have referred entirely to the needle instrument, or to circumstances which affect every form of telegraph apparatus, so far as the conducting medium is concerned. We shall, therefore, pass on to examine a variety of arrangements in use in different countries, and also, to a limited extent, in our own.

The needle instrument only indicates, and does not record, a message. As we have fully explained, the signals are effected by means of alternate or combined deflections of a magnetised needle; and its use is practically confined to those who are accustomed to it, and can, therefore, readily translate its signs. This, to some extent, is an objection, and limits its use. If letters and words were imprinted by it, then its indications would be intelligible to any one; and its use would doubtless be more general. It is also liable to many derangements. The needles may lose their magnetic power, and, in certain instances, perhaps, at the very moment when they may be most required: a sudden flash of lightning inverts their polarity, and all attempts to employ them are futile. Earth currents, and atmospheric influences, also combine to effect their derangement.

These, and other reasons, have led to the invention of *printing telegraphs*; by means of which, the words of which a message is composed are spelt and printed on cloth or paper; or, which answers the same purpose, dots and dashes of various lengths are printed, according to a preconcerted code of signals.

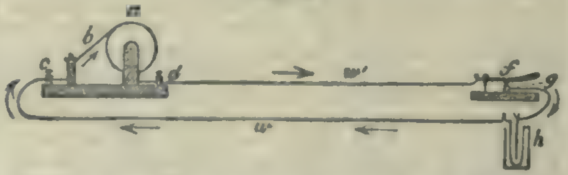
We shall first describe the general principles on which chemical printing telegraphs are constructed; then refer to the mechanism by which the plan is carried out; and shall best illustrate our subject by suggesting some experiments bearing on it.

Experiment 22.—Dissolve a little yellow prussiate of potass in warm water; and also, in a separate glass, some crystals of sulphate of iron (green copperas). Add these two solutions together: a blue precipitate will be produced, which is commonly called Prussian blue. Now, it is by the production of this colour on paper or cloth, that the printing of dots, dashes, or long lines, is effected in some kinds of telegraphs. The mode of printing may be illustrated as follows.

Experiment 23.—Moisten a piece of white calico with a solution of sulphate of soda; and, after wringing it, dip it into a weak solution of the prussiate of potass. Place the cloth, whilst damp, on a sheet of tinfoil, and to this attach the wire proceeding from the copper plate of a voltaic battery. Attach the wire from the zinc plate to a knitting-needle, and draw the end of the needle over the moist cloth. Wherever it touches the cloth, a blue stain will be left. If the needle just touch the cloth by the point, then a dot is produced; but if the point be drawn over the cloth, then a dash or long line is afforded. Now, on exactly similar principles, and by a simple modification of the plan, messages may be readily transmitted between any two stations—the printed paper or cloth kept, and the marks on it translated into ordinary language.

To familiarise the mind of our readers with the principal mechanical part of the printing apparatus, we will suppose that a pianoforte key, made of metal, and with one of its ends attached to the wire of the copper plate of a voltaic battery, is so arranged, as that, when pressed down, it shall enter a small cup containing mercury. From this cup we will suppose a wire to extend to a needle, placed so as to rest with its point against a metal roller covered with cloth, which has been impregnated with the prussiate of potass; and that the roller is connected with another wire proceeding from the zinc plate of the voltaic battery. It is evident, that when the pianoforte key is pressed into the mercury cup, a circuit will be established; a current will pass; and the point of the needle will instantly impress a blue dot on the cloth—thus, . If, however, the roller be gently moved, then, instead of a dot, a dash ——, or long line ————, will be produced; the length depending on the length of time the key is kept pressed down, so as to continue the passage of the current. The instant the key is removed from the mercury cup, the current will cease to pass, and no effect will be produced on the distant roller. Now, this is exactly what happens in most of the chemical printing telegraphs: but we may be better understood if we refer to the following engraving, which affords an illustration of the most essential part of the apparatus.

Fig. 30.



In the above cut, we have *a* representing the roller on which the prepared cloth is wound; *b* is the needle which imprints a dot, dash, or line; *c* is a binding-screw, to which is attached a wire, *w*, extending to the copper plate of the battery, *h*. A wire is attached to the zinc of the battery, and reaches to a small peg, *g*: *f* is a key, which, when pressed down on *g*, completes the circuit. The current will then travel from the battery by the wire *w*, to the needle, and thence by the roller to the wire, *w'*; returning again, through the key arrangement, to the zinc of the battery. Of course, no mark is made on the roller, except when the key, *f*, is brought into contact with the peg, *g*. The roller is gently turned by clockwork; or by hand, if for elementary experiments.

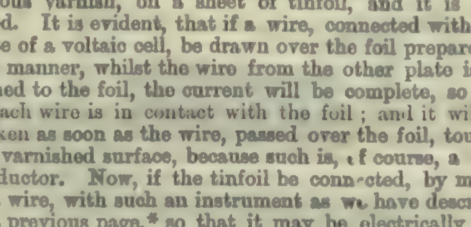
We have entirely avoided all mechanism used in the complete apparatus, so that our readers might gather a general knowledge of the *modus operandi*, and, if they so please, construct a model for their own use. Having thus illustrated the principle, we shall now describe the various forms of apparatus which have been, or are in use, constructed on such plans as we have explained.

Before doing so, we may call attention to a very ingenious piece of apparatus invented by Mr. Whitehouse, of Brighton, formerly the chief electrician to the Atlantic Telegraph Company; by which music could be printed as it was played on a pianoforte or harmonium, at any distance from the instrument, and accurate as to notes and time. Under each key of the piano a wire was placed, which extended to a needle fixed to an apparatus similar to that which we have just illustrated. On the roller a long strip of cloth was placed, on which as many needles pressed as there were keys on the piano. The cloth was marked with the lines and spaces for bass and treble, as is common in printed music; and by means of clockwork, or by the hand, it was gradually unwound from the roller. A current of electricity travelled from the piano to the needles; and as each key was pressed down, a mark was imprinted on the cloth by its corresponding needle; the length of the mark indicating the value of the note—such as a quaver, a crotchet, and so on. We have frequently played rapidly-executed music on a piano so fitted, and every note was thus printed as quickly and as accurately as it was performed. When the cloth was removed from the roller, the lines might immediately be read, or transposed into the ordinary form of musical notation. The instrument would be of great value to composers, because every idea, as it found expression on the piano, &c., would simultaneously be printed by the electric current. We believe that Mr. Whitehouse has not carried out his invention beyond making the model we have described.

BAKEWELL'S COPYING TELEGRAPH.

Perhaps the simplest and most interesting instrument constructed on the principles we have explained, is that invented by Mr. Bakewell; by means of which the handwriting of the sender of the message is faithfully copied by the electric current, at any distance from the station whence the message is forwarded. This certainly, at first sight, seems beyond belief; but by a little investigation, we shall find that it is only the result of a most ingenious application of principles, to which we have already very frequently drawn attention. The whole secret of the art consists in so arranging a conductor with a partial non-conducting surface, that, by means of the latter, the current of electricity may be intercepted; and this is managed after the following manner:—The message is first written, by means of a weak spirituous varnish, on a sheet of tinfoil, and it is then dried. It is evident, that if a wire, connected with one plate of a voltaic cell, be drawn over the foil prepared in this manner, whilst the wire from the other plate is attached to the foil, the current will be complete, so long as each wire is in contact with the foil; and it will be broken as soon as the wire, passed over the foil, touches the varnished surface, because such is, of course, a non-conductor. Now, if the tinfoil be connected, by means of a wire, with such an instrument as we have described in a previous page,* so that it may be electrically connected with the needle, and one wire of the battery be in contact with the roller of that apparatus—the wire from the other plate being passed over the written tinfoil surface—no mark will be made on the prepared cloth when the battery wire comes in contact with the varnish; but whenever it touches the tinfoil, a blue mark will be printed on the cloth. If the needle, instead of being stationary, is allowed to traverse from one side of the roller to the other, whilst the roller itself is quickly moved round by clockwork, the continuous straight lines will be printed when the circuit is completed; which, however, will be interspersed with white spaces whenever the circuit is broken by the interposition of the varnished surface. Supposing the varnished surface to be a straight line across the tinfoil, then the printed copy would present a number of blue lines, crossed by a white one—as is shown in the above diagram.

Fig. 31.

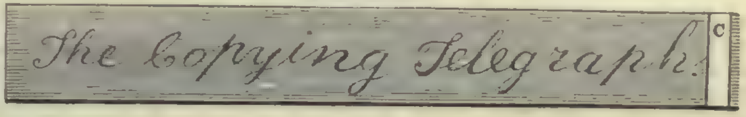


No. 1. No. 2.

No. 1, in Fig. 31, represents the tinfoil sheet, *a*, from which the signal is transmitted, crossed by the varnished line, *b*, which is the non-conductor; No. 2, *c*, is the cloth in which the signal is received and printed; *d* being the white space, unmarked with lines, and corresponding to the varnished surface of the tinfoil plate; and *c c* are the blue lines printed by the traversing needle, which receives the electric current from the foil. No. 2, in fact, gives a representation of the cloth as it appears when taken off the rollers.

Now, the principle being established, it does not matter whether the line on the foil be straight, curved, or of any form; hence, if such be ordinary writing, that would be copied on to the cloth in an exact fac-simile of the original, just as we observe in the copy shown in No. 2. This is just the plan adopted, and the result arrived at, by Mr. Bakewell's arrangement, provided that certain mechanical details are observed, to which we shall presently allude. The following diagram shows the appearance of the original, and printed, copy of a message, as illustrated by Mr. Bakewell, in his work on the subject.

Fig. 32.—No. 1.



No. 2.

No. 1, Fig. 32, represents the original, with the letters in black on the tinfoil; and No. 2 is the printed copy, having the letters in white intercepting the line produced by the needle. Our readers will thus perceive, that by this plan, an exact copy of the handwriting of any person may be transmitted by telegraph—a fact that makes Mr. Bakewell's invention rank very high as an instance of inventive genius. But, unfortunately, there is a very serious drawback to its utility, which is owing to the necessity that exists for both the transmitting and receiving-rollers to move at the same speed and at the same moment; and, consequently, through equal spaces in the same period of time. To effect this, Mr. Bakewell introduced regulating apparatus, worked by an electro-magnet, so that one roller could only proceed at the same rate as the other. In fact, in such an arrangement, the magnet acts in a similar manner to the pendulum of a clock. It is well known, that the weight or spring of a clock would quickly "run down" were it not for the action which a pendulum exerts on the seconds, or first wheel. A pendulum, thirty-nine inches long, beats about once in a second of time; and the spring of the clock is only required to keep up that motion. But two pendula never beat exactly together: thus they would not be effective as regulating arrangements in Mr. Bakewell's apparatus. Hence the employment of the electro-magnets, which necessarily act together, because their motion is mutually dependent. We are precluded from illustrating the mechanical parts which have been adopted; and this is not of so much consequence, because, although the invention is highly ingenious, it has scarcely ever been practically employed to any extent. Besides the difficulty to which we have alluded, there are others which also interfere with the result;—the receiving cloth may be unequally stretched; the two instruments may not commence work together, &c. Messages, however, have been sent accurately through a distance of fifty miles; and as many as 300 letters per minute can be printed by this interesting instrument.

Whilst referring to electro-chemical printing telegraphs, we may mention a very ingenious arrangement employed by Mr. Whitehouse, whilst he was engaged in trying experiments in connection with the Atlantic cable. The object he had in view, was to ascertain the speed with which a current of definite intensity and quantity passed through various lengths of conducting wire; and also to discover the retardation which the current suffered from the resistance arising from wires of different thicknesses. He employed an arrangement very similar to that we have already described in a previous page, and which is used in Bakewell's instrument. By these means, he found that the same current produced a dot through—say 200 miles; a dash through 500 miles; and a longer line through a circuit of 900 miles; and so on. It was to these experiments we referred when speaking of the influence which a gutta-percha coated wire has on the speed of signals transmitted through long cables.* We have first introduced Mr. Bakewell's copying telegraph, as being comparatively simple to some we shall have to mention; and shall now proceed to describe the inventions of Mr. Bain, and also of Mr. Morse, with others, which have been very extensively used for various purposes.

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* See *ante*, p. 266; Fig. 30.

BAIN'S ELECTRO-CHEMICAL TELEGRAPH.

Amongst the earliest inventors in connection with electro-telegraphy, Mr. Bain is well known. We believe that he was one of the first to produce a printing telegraph, which, however, was constructed with ordinary types. These were moved by means of mechanism actuated by electro-magnets, and each letter was successively brought to its proper place by complicated arrangements. After being inked, it was pressed on a receiving surface, which, when finished, presented a similar appearance to ordinary printed matter. A model of this arrangement was first publicly produced and explained by its inventor, many years ago, at the Polytechnic Institution, Regent-street, London; but we are not aware that it ever came into extensive use. Mr. Bain was also well known for his ingenious arrangement of electro-magnetic clocks, one of which was for some time in operation at the above-named institution. His electro-chemical telegraph is constructed on somewhat similar principles to those we have illustrated already.† As in Bakewell's apparatus, Mr. Bain obtains marks on prepared cloth, intercepting the surface of the transmitting arrangement by means of non-conductors; producing, however, dots and dashes, instead of continuous lines. Combinations of the marks produce letters, as the following instance will illustrate.

A - —   D - - —
B - — -   E -
C - - -   F - — — — &c., &c.

It will be unnecessary for us to enter into detail with respect to the mechanical arrangement of the apparatus; for the principle is so fully illustrated by the elementary instrument already referred to: but we may add, that the continuity of the current is intercepted, and the letters represented, by means of perforated paper, each of which corresponds, in its perforation, to, and produces marks, such as those represented above. The message, in fact, is perforated in paper or cardboard by means of punches; and is then placed in the transmitting arrangement; after which it is sent, in the manner we have already described, to the receiving instrument at the distant station, and is then printed on the prepared cloth.

Various inventions have been brought into public notice—being improvements, modifications, or original productions—in the form of electro-chemical printing telegraphs; but we shall not enter into any investigation of them, as their use has not been adopted, to any extent. for telegraphic purposes.

MORSE'S TELEGRAPH.

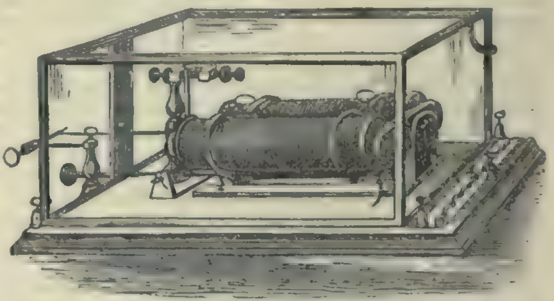
Amongst the great variety of telegraphic instruments which have been employed in foreign countries, that invented by Mr. Morse, of America, has been, perhaps, the most extensively adopted. In it, electro-magnets are employed in a similar manner to that of Mr. Bain's invention: in fact, the similarity of the two instruments is so great, as to have given rise to litigation in the American courts, which have had to decide on the respective rights of the inventors. Mr. Morse, however, seems to have been original in his ideas; and he has worked them out to a most successful issue; for nearly the whole of the American, and some continental, lines are worked under his patent.

In speaking of the needle instrument, we mentioned that its indications were effected by its deflections. Mr. Morse's arrangement is constructed on entirely different principles; and, except in some parts of the mechanism, the details are just such as we have described in Bain's instrument. The signals are transmitted in a similar manner, and the notation is that of dots and dashes of unequal length or interval. The following is a portion of the telegraphic alphabet employed by Mr. Morse.

A ·—   E ·   I ··
B —···   F ·—·   J —·—·
C ·· ·   G ——·   K —·—
D —··   H ····   L — &c., &c.

A larger amount of electric force is required to work Morse's arrangement than the needle telegraph, and hence the employment of "relay" magnets, by means of which the first current transmitted from a distant station, brings into operation another current at the station at which the message is received. The necessity of this will be at once seen if it be borne in mind, that, whilst a weak current will readily deflect a magnetised needle, it requires one of considerable force to convert a piece of soft iron into a magnet.‡ The arrangement of a relay magnet is illustrated in the following engraving.

Fig. 33.



With respect to the speed at which signals may be sent by this plan, its advocates maintain that it equals that of the needle system. It has other advantages; for, owing to the nature of its arrangement, and considering that the electro-magnetic force is generated at the moment it is required, there can be no risk of interference by atmospheric influence, which is almost always operating on the apparatus of the needle instrument.

Numerous modifications of the Morse principle have been made in America, and on the continent; but as we cannot enter into details, which, whilst interesting, invite no special novelty in principle, we shall not further refer to them than to state, that our readers may obtain full information in Mr. Shaffner's *Telegraph Manual*, from which our two last illustrations have, by his kind permission, been copied.

THE MAGNETO-ELECTRIC TELEGRAPH.

Amongst other instruments involving no great novelty in the indicating part of the arrangement, is the magneto-electric telegraph. Some years ago, a company was formed for the purpose of carrying out a system of telegraphy, in which a current of electricity, produced by magnetic induction, was to be employed in place of the old plan of voltaic batteries. It was urged that the new plan offered all the advantages of the old one, and saved the cost of chemicals, the wear and tear of plates, battery cells, &c.; that the current produced by magnetism had greater intensity; and, lastly, that the first was the only cost of the source whence electricity was to be obtained. Our readers have already been acquainted with the magneto-electric machine;* and we shall, therefore, not enter into any further detail, as the indicating instrument is simply a modification of the ordinary needle instrument. We were informed, two years ago, by the superintendent of one of the chief stations on the Glasgow and South-Western line, that the magneto-electric machine had been given up, and that the old plan of the voltaic battery had superseded it. If so, the distinctive character of the company no longer exists.

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* See *ante*, p. 265.

† *Ante*, p. 266; Fig. 30.

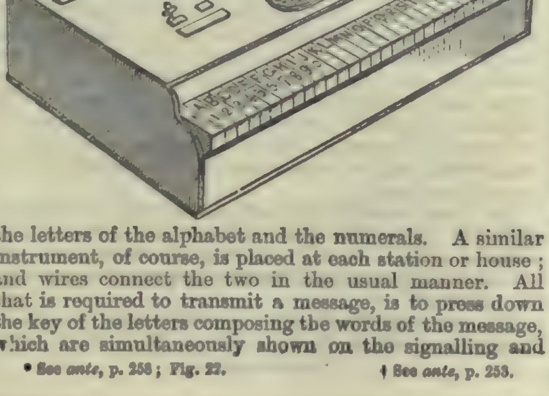
‡ See *ante*, p. 251.

ALPHABETICAL TELEGRAPHS.

ONE of the earliest forms of electric telegraph, was that of a circular plate, around the edge of which were engraved the letters of the alphabet. An index moved in the centre; and, in fact, the arrangement was exactly similar to that of a watch, with the exception that letters were employed in place of figures. The index-hand was moved by electro-magnets; and, of course, the receiving and signalling instruments were constructed, in all respects, precisely alike—in the manner already described in connection with electric clocks.† The index had to be moved from letter to letter—passing over those which were not required in the word, and resting opposite to that which was included in it. It thus happened, in many words, that the index had to travel frequently over the dial-plate before a word could be spelt. Thus, supposing there were two "e's" in a word, the index would have to pass twenty-five letters before the "e" would be again reached; and so on. The plan, however, is remarkably simple, for any one can read the indications; and it requires no previous instruction for its successful employment. At the present time some of its modifications are in use; and an invention, by Professor Wheatstone, has lately been adopted by the Universal Private Telegraph Company—the object being to maintain electrical communication between, and in, private dwelling-houses.

Froment's alphabetical telegraph will assist our readers in understanding the principle of the arrangement of these alphabetical instruments. In front is a key-board, constructed similar to that of an ordinary pianoforte; and connected with this is a dial, on which are engraved the letters of the alphabet and the numerals. A similar instrument, of course, is placed at each station or house; and wires connect the two in the usual manner. All that is required to transmit a message, is to press down the key of the letters composing the words of the message, which are simultaneously shown on the signalling and receiving instrument. The apparatus, except with respect to its internal mechanism, is remarkably simple; and even a child may soon be taught its use. Fig. 34 illustrates the appearance of one of these instruments.

Fig. 34.



The alphabetical system of telegraphs has been partially applied to the purpose of railway signals; but instead of the entire alphabet being printed on the disc, separated signals, such as "train started," &c., are employed. An instrument is placed at each station on the line; and thus a system of constant signalling may be readily maintained. There is, however, considerable difficulty in keeping the signals identical at the same moment in each instrument; and in experiments we have tried with two such instruments, exactly opposite indications were received to those sent; a result which would be most serious in practice. On many of the railway lines, especially at the extremities of tunnels, the old needle system has been introduced; and as the number of signals is small, it is found the most convenient and certain plan.

The limits of this work prevent us extending to further telegraphic details; and we shall, therefore, now draw attention to some minor applications of the instruments we have described, and of the principles already explained; amongst which are the following.

WIND AND WEATHER REPORTS.—A complete system of telegraphy is now established, between the Admiralty Office and the Royal Exchange, London, with all the outposts and chief towns in this country and foreign parts; and it is impossible to over-value the results which have accrued. Since the discovery of the general law of storms, it has been most desirable to make simultaneous communication from a common centre, so that information may subsequently be diffused as to the kind of weather which may probably be expected. We have not yet arrived at such precision of information as will permit us to foretel, to an exactitude, the atmospheric changes which are continually occurring in a climate like ours; but we can, to some extent, approximate to truth; and, in many instances, the conclusions which have been drawn from the facts collected by the telegraph, have resulted in saving many lives, through timely notice which has been sent to different parts of our coasts.

Our readers will understand this better if we give a few facts in reference to the cause and indications of a coming storm. The most dangerous kind of tempest is that called the "cyclone;" which is an immense eddy of wind, having a calm centre and circumference. Vessels in any intermediate place are exposed to the full violence of the storm; but, from their position, the captain is unable to judge of its extent. Now, if we learn the direction of the wind over an extensive area of country, and also the rise or fall of the barometer (which is, at all times, a sure indication of coming atmospheric changes), we obtain data on which tolerably accurate conclusions may be drawn. Amongst those who have taken a lively interest in this matter is Admiral Fitzroy, whose remarks on the subject we shall quote. "I do not hesitate to affirm (and it is more than forty years since I bought Daniell's first work on Meteorology), that the peculiarities of our winds and climate are now sufficiently known to make it practicable to foresee every extensive storm; and, when no such change is impending, to give the negative information—often valuable—that no tempest need be feared."

On this matter we may remark, that much useful information might be afforded by private individuals undertaking the duty of keeping a register of the barometric indications, the direction of the wind, and other meteorological phenomena. These might be forwarded, periodically, to the secretary of the Meteorological Society, or to the Admiralty Office, London: and if a tolerable amount of accuracy were maintained by observers, the extent of our general information would be largely increased.

The necessary facts having been gained by means of the telegraph, the next step is to use that instrument for the purpose of diffusing the instructions from head-quarters; and this is followed by the employment of signals, which are hoisted at some prominent place on the coast, near each port. The following are the storm signals adopted by the Board of Trade.

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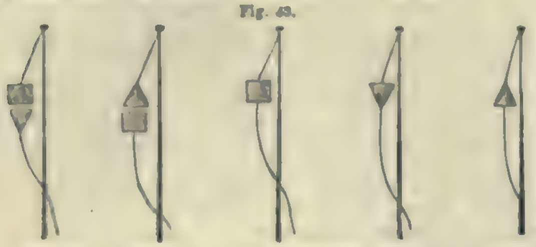
* See *ante*, p. 258; Fig. 22.

† See *ante*, p. 253.

Fig. 63.



No. 1. No. 2. No. 3. No. 4. No. 5.

No. 1. Dangerous wind probably at first from the southward.—No. 2. Dangerous wind probably at first from the northward.—No. 3. Dangerous winds may be expected from nearly opposite quarters successively.—No. 4. Gale probably from the southward.—No. 5. Gale probably from the northward.

We will suppose, for instance, that the daily reports from all parts of the kingdom have been received at the usual hour of 9 A.M. A consultation is then held as to the chances of danger, and the probable direction of a storm, if one be likely to occur. Instructions are immediately telegraphed to those ports nearly adjacent to the probable locality of the expected tempest, to hang out the proper signals, so as to warn vessels within sight of their impending danger. In this way, at least, all that human foresight and intelligence can do has been attended to; and the rest can only be left to the discretion of the captains, who have thus been forewarned.

Electric Time-Balls.—Another valuable application of telegraphs, has been that of indicating the exact moment when the sun has passed the meridian of any place; or, what answers the same purpose, any exact interval of time which has elapsed. It is usual to hoist a large ball on the top of the Observatory of Greenwich, a little before one o'clock daily; and at the instant that the sun has passed one hour over the meridian (*i.e.*, one o'clock), the time-ball is lowered. The captains of the vessels in the Thames can thus daily check the rate of the chronometers they have on board, and so gain the most valuable information; because, on a correct knowledge of time, depends their discovery of the longitude during a long voyage. Within the last few years the system has been widely extended; and by an arrangement of telegraphic signals between Greenwich, London (Charing-cross), Liverpool, Glasgow, and other shipping ports, the time-balls at each of those places are lowered synchronously with that of Greenwich, and correct time is thus indicated. The telegraph has also been employed for ascertaining some interesting facts in connection with astronomical science—such as the longitude of two different places, &c.

Modifications of telegraphic apparatus have been employed for domestic purposes; amongst which we may notice the fire-alarum and house-bell. If wires be connected with the doors and windows of a house, and arrangements be made so that, on any of these being opened, contact shall be made between the wire proceeding from a small battery and an electro-magnetic bell, such may be readily applied for the purpose of warning the inhabitants of the ingress of persons; and as a protection against thieves. By means of a most ingenious arrangement of a thermometer and electro-magnet, a fire-alarum has been constructed. The thermometer is of peculiar construction; and the mercury it contains, is, in ordinary circumstances, at some distance from a little arm, which can connect an electro-magnet with an alarum-bell. If, however, the temperature of the room rapidly and greatly increase, then the expansion of the mercury in the thermometer raises the arm, and completes the connection between the battery and the electro-magnet of the alarum; and on this taking place, a detent is removed, and the bell at once commences to sound, and so gives the alarm to the inmates.

With this, we must now draw our observations and experiments, in connection with electro-telegraphy, to a conclusion. We have attempted to put our readers in possession of the leading facts of the subject; but have been compelled to omit many interesting particulars. It is impossible to regard the discovery of this mode of communication otherwise than as the greatest that has been made in this century, if all points are viewed. It has but one rival—photography; but that rivalry is more in the nature, rather than in the extent and value, of its applications. We may, for a moment, take a hasty glance at the advantages which accrue to residents in large towns; and, for example, we shall confine our attention to London, and its telegraphic connections. For a few pence, we can communicate our wishes to any part of the metropolis, and receive an instantaneous reply; the amenities of friendship, and the necessities of the household, can be simultaneously attended to, and communicated, over our housetops; and a conversation can easily be held between two friends, at any distance from each other, whilst both may be within their houses, if advantage be taken of the arrangements made by one of the telegraphic companies. The professional man can be called to the sick chamber at a moment's notice, or the relations and family friends suddenly summoned from all parts of the metropolis, to take a last farewell of a beloved one. The man of business has scarcely to conceive a new scheme, than all the arrangements necessary for its success can, in a few minutes, be made by communicating through the telegraph, to his agents in any part of Europe. Does he desire to receive even an amount of money, this the telegraph will send him. If he be anxious to change the intended destination of a vessel, to take advantage in a change of markets, this the telegraph can effect for him. A man can now as readily converse with his representatives in St. Petersburg, Vienna, Constantinople, Berlin, Madrid, and Paris, as he can with his neighbour in the next house to him. The politician can summon his adherents simultaneously from all parts of the kingdom and the continent, in cases of important divisions in the Houses of Parliament. The diplomatist may converse with his antagonist, and, at the same moment, receive instructions from his government, hundreds of miles away. Armies and navies, at the most distant places, can be commanded by the home authorities with as much readiness as by their commander on the spot; and, as we have lately seen, the two potentates of the two greatest nations on the earth—her Majesty the Queen, and the President of the United States—were enabled, by means of a slender wire stretched under 3,000 miles of the stormy ocean, to speak words of peace and good-will, as the representatives of two nations, using the same language, having brotherly ties, and divided, not in sentiment, but by position only. We may almost imagine that the triumphs of science can proceed no further. But we cannot forget that the realm of nature is boundless; and, despite all our successes and achievements, we have but entered the threshold of that storehouse in which are yet buried, but only to be discovered, wonders and truths, which, when brought to light, shall eclipse all our past discoveries.

DIA-MAGNETISM.

We shall now make a few remarks on an interesting branch of physical science, for which we are indebted to the researches of Dr. Faraday. In the early part of this work, we called frequent attention to the fact, that each of the undulatory forces had a distinct relation and connection with the rest; and the last link in this singular chain, is supplied by the facts of dia-magnetic phenomena. We shall find that even a ray of light may be subjected to magnetic influence; and thus our ideas of magnetism, and the objects to which it has affinity, will be much enlarged.

We have already explained, that a ray of polarised light has properties highly different to those found in common light. It will not obey the same laws; and presents, under certain circumstances, some very peculiar phenomena.*

If a ray of polarised light be sent through a mass of heavy glass—which, for this purpose, is made of borate of lead—it will, in ordinary circumstances, be transmitted; and if the glass be placed between the poles of an electro-magnet in an unexcited state, of course no change in any direction will be effected. The analysing plate is, however, to be so turned that the transmitted ray is nearly extinguished; and if the electro-magnet be then brought into action by the circulation of a voltaic current, the ray will instantly appear in full force. Or, supposing that, in the first instance, the ray had been in full force during transmission, and the electro-magnet be then excited, then the ray of polarised light would nearly suffer extinction. This fact at once shows that magnetism has a direct action on light, which, of all other agents, seems to have the least claim to be considered material.

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* See *ante*, p. 80.

Dr. Faraday, however, extended his experiments, and was led to try how far a powerful magnetic force affected matter generally; and, in pursuit of this object, he discovered magnetic affection in numerous gases, liquids, and solids. These had a tendency of placing themselves in certain positions in reference to the poles of a powerful magnet, and the direction was definite for each substance tried. Gases and liquids were enclosed in glass tubes; whilst solids were chosen of such forms as were found to be most suitable for the purpose. By these investigations, we are enabled to classify all bodies, in reference to magnetism—iron, nickel, manganese, cobalt, cerium, chromium, platina, palladium, titanium, and osmium—having directly magnetic properties; whilst most others seemed to indicate an opposition to the magnetic force; or rather, under certain circumstances, range themselves in a direction at right angles to the lines of polar force in a powerful magnet—the metal bismuth being the most prominent in this respect. Such bodies have, from this fact, been termed *dia-magnetics.*

If some substances in the form of powder are placed between the poles of an electro-magnet, it will be found that the powder will rise up into a conical form, evidencing a peculiar effect of the magnetic force in relation to the forces of gravitation and cohesion. Another singular experiment, having the same result in a different form, may also be tried. A circular copper disc is mounted on an axle, and fitted with a winch handle: of course, it will readily revolve thereon in the usual manner. If, however, the rim of the disc be placed vertically between the poles of a powerful electro-magnet, the moment a current of electricity is passed, and the iron becomes magnetised, great resistance is offered to the revolution of the wheel. Now, generally speaking, we exclude the possibility of any magnetic effects being inducible on copper; but this experiment at once shows that the magnetic force can act on that metal in the method indicated; and herein we have another illustration of the universality of its action.

The subject of Dia-Magnetism is at present in its infancy. The results already obtained go towards proving that magnetism has a direct connection with the forces of gravity, cohesion, electricity, and light; and that every kind of matter is susceptible of its influence. These facts alone make the subject of deep interest, and extend materially our views of the possibility of the various kinds of forces being but modifications of one prime agent.

ADDENDA ON TELEGRAPHY.

Amongst a host of other objects which the Exhibition of 1862 brought prominently out to public notice, the numerous inventions connected with Electro-Telegraphy rank high in importance. We have made the following selections, as being of the most practical and highly improved kind. In our previous pages, we have frequently alluded to the difficulties of submarine telegraphy, the imperfect insulation of the wire, weight of the cable, &c. The failure of the Atlantic Telegraph has caused a vast number of inventions to be brought out; but, in most respects, that of Mr. Allan, whose electro-magnetic engine we have already described, is the most feasible. Its value was acknowledged by the jurors in their award of the prize medal; and, as the jury was composed of the most eminent men in connection with telegraphic affairs, we have no hesitation in endorsing their decision, by giving an extended description of Mr. Allan's cable, instruments, &c. We have to acknowledge his kind assistance in the matter.

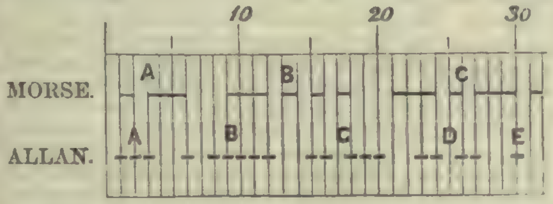
First, then, as to Mr. Allan's signal code. He rejects that of Morse, and adheres to the dot symbol alone for signalling and receiving. At the signalling extremity, a ribbon of paper has the dots separately punched by a machine; and in this prepared message, when passed into the apparatus for transmission, and the automatic action once commenced, each single punch-hole, as it passes over the tooth of a small rotating spur-wheel, permits of electrical contact being made within the sending-machine, and of thereby inducing a perfectly synchronous movement at the receiving extremity of the line wire, and of causing a puncture in the ribbon passing through the automatic apparatus there; so that, at the same instant of time, what is hole at one extremity is puncture or "emboss" at the other. Reverting, then, to the code of signals, and to the method by which the message is punched or marked on the paper ribbon. The apparatus employed is called a "Composing Machine." It consists of an oblong mahogany case, 11 inches by 8 inches, by 7 inches deep, wherein are mounted a set of keys, 30 in number, resembling those of a *pianoforte.* The letters of the Roman alphabet are printed on the touch-part of 26 of these, leaving four for purposes incidental to the working of the machine. Within the box, and movable by the key-levers, are eight punches. A ribbon of paper is made to pass through the composing machine. Now, any person who can read a message, has simply to spell out its words upon the keys by depressing each letter of the words, and its symbol in the code is punched out at a single stroke. This operation, which is perfectly simple, resembles the act of playing on the piano. The message, on being punched, is passed into the telegraphic instrument, which, from its automatic character, continues to work on, till the whole passes through; and the corresponding automatonism of the receiving apparatus is brought into play at the same instant. Further, any one may have his own "composing" or punching "machine," and adopt his own code of signals, by the very simple device of transposing the Roman characters on the touch-part of the keys. Secrecy is secured by this means; and the message on being punched, it may be, in a government or business establishment, if dispatched to the telegraph office, can be at once sent through to the confidential receiver of the same, who is in possession of the cipher to read the message, whilst the telegraph office-clerks know nothing whatever of its import. Further, this "composing machine," taken with the system of telegraphy of which it forms a part, possesses considerable interest to the newspaper press, as a probable means of cheapening what is a heavy cost in their annual expenses—namely, the telegraph despatches. One single punched ribbon may be carried continuously and simultaneously over any number of sending machines, and the same political news or parliamentary speech may be sent to half-a-dozen distant places from the same centre—say from London—at the same time; and all this can be done by so many boys as there are receiving places. Again, to receive messages depending upon either visual or audible signals, the clerk must always be in attendance; and when the distance is great, over land or water, so as to embrace many degrees of longitude, that attendance must be unremitting night and day. The clerk, in such a case, is a part of the instrument, and the complement of it; he can no more withdraw himself an instant than he can safely remove from its connections the telegraphic portion of the message system. By this automatic scheme, the attendant need not be always present. He may even go to bed; and when he awakes he may find the communications sent in the meantime in the message basket. Further, one attendant at each end of the line wire is all that is now needed; and the sender need be no other than a person of average intelligence, of little experience, and who can, as has been already said, read well. A boy or girl of twelve or fourteen years may do the sending work; and a moderate education and experience are all that is needed to receive, interpret, extend in writing, and distribute correspondence. A uniform charge for messages within a wide circuit belongs to the economy of this system, upon the basis of the plan adopted for our post-office. The Telegraph Company, according to Mr. Allan's scheme, should issue blank and file books; and any person possessing one such book, has only to write his message on a blank leaf, tear it out, envelope, and drop it into the office-box, when it has instant attention given to it. All this appears quite plain and possible, but the distinguishing features of the mechanism, conjointly with the electrical influence by which it is produced, is not so obvious. According to Allan's plan of signalling, the punctures or "embossings," on the paper at the receiving points, are made by the method of reverse currents, or the positive and negative electric currents are both made effective for marking; and the receiving instrument itself, without any expense of power, makes the distinguishing spaces between the letters; whereas, in other instruments of this kind, small spiral springs are introduced, for the quick and certain recovery of the deflected needle to the neutral position, in order to make spaces. This latter plan is also followed by Digney, a French maker of good repute; but it is objectionable for various reasons; and one of these is the superfluous demand that is made on the electric force to overcome a mechanical resistance, and the possible fluctuations and depressions of battery force, which, from the resistance of the spring, might destroy the signalling power. In the instances of the telegraphic instruments of other makers, reverse currents are employed instead of springs; but, with them, one of the currents is devoted to making the spaces, whereas Allan utilises both, as has been already stated, for marking. By this means he diminishes the number of currents sent through the line wire by upwards of 30 per cent.; and, on the other hand, the gain in point of time, by adopting altogether the "dot" system of marking, is about 50 per cent. This particular will be at once appreciated by the intelligent telegraphist at its proper importance, because, by the arrangement in question, greater signalling power is attained with relatively less expense of electrical force; and besides this, and with especial reference to the application of this plan to long-range submarine telegraphy, greater speed of transmission is also attained. But further, the effective workings of such instruments depend, amongst other things, upon the nice adjustment and suitable condition of the metallic surfaces, which are required to act by making and breaking electrical contact with each other. First, the sending or reversing break is here made to preserve clean contact by a rubbing action, which effectually wipes off any abraded matter which the spark carries over from one terminal point, and deposits upon the other and opposite point. In Allan's "relays," or pole changers, as these instruments may be called, this contingency is very ingeniously provided against. The relays are "sparkless;" and the small disruptive discharge of electricity which takes place on breaking contact is not produced in the relay at all, but in the recording instrument, where the mutual attrition of the metallic surfaces constantly preserves the parts clean. The importance that attaches to this particular will be apparent, when it is remembered that foul contact-points in the relay would soon reduce the action of the battery, which should work it, to inefficiency. By Allan's plan the power of the battery is preserved, and the machinery of the whole line kept open for the transmission of correspondence. Moreover, the absence of the spark in the relay admits of a great increase in the power of the local battery, and therefore of more efficient action in working; whereas the sparkling relay of other makers compels them to use only a very moderate power, so as to injure the operations of the instrument as little and as slowly as possible. This inconvenience, therefore, carries with it a limit upon the energy with which a telegraph line can be worked, without the risk of delaying its action for cleaning the relay at very short intervals of time.

The above gives a general description of Mr. Allan's signal arrangements: we shall now refer to his improvements in the construction of cables, especially those intended for deep-sea working. Near shore, Mr. Allan uses cables of considerable comparative thickness, so as to avoid the danger which always arises from the abrasion by rocks, and other sources of injury. The following observations, for which we are indebted to a valuable periodical, the *Electrician*, will afford the reader an insight into the improvements claimed for Mr. Allan's cable.

"Mr. Allan claims for his principle of deep-sea telegraph cable, that it combines the maximum of conducting power, insulation, strength, flexibility, and durability, with the minimum of weight, bulk, cost, and risk in submergence. He adopts the solid copper wire as giving the greatest amount of conducting power, with the least amount of inductive surface. The strand of copper wire used for ordinary forms of cable, presents considerably more surface than would be obtained with the same amount of conductivity in a solid copper wire; but it is found safer to sacrifice a small amount of speed, owing to the increased surface, than to run the risk of a fracture in the solid copper conductor. Having obtained the best soft solid copper wire, he surrounds it with small steel wires of the best quality, laid on with a slight spiral; and this forms the inextensible core or conductor, and also the strength of the cable. This core, or backbone of the cable, is then insulated with separate layers of gutta-percha, or india-rubber, or any other combination of the two; but it is Mr. Allan's opinion that none of the failures in deep-sea cables ought to injure the reputation of gutta-percha as an insulator; and his confidence in its suitability for submarine telegraphy remains unshaken, provided it be applied to the conductor with the necessary amount of care, and not submitted to any crushing or stretching process before it reaches its ocean bed. The insulated core is then clothed with a slight covering of hemp, or any other cheap material, saturated with marine paint; and this completes the deep-sea cable.

"The true mechanical test of a submarine cable is to ascertain the number of fathoms of its own length it will sustain in water without elongating to any serious extent. If the conductor, or metallic portion of the cable, be small, then the cable may sustain an enormous length of its own weight in water, and still not be at all suited for a long distance, because of the insufficiency of conducting power. In this respect Mr. Allan's cables were excellent examples, with a much larger conductor than most of the other light cables sent in for testing. Mr. Allan's bore a weight nearly equal to 7,500 fathoms without elongating one per cent.; and this may be taken as the weakest link in the chain, for joins in the steel wire abounded. This cable was also tested round a 12-inch wheel, to see if the core would cut through the gutta-percha; and although this ordeal was at least ten times more severe than it could be subjected to in submergence, not the slightest deviation from the centre could be detected."—The following is the report after the experiments were conducted (*Government Report*, page 28). "The results shown in the above table for Mr. Allan's cables are remarkable. In these cables the strength is given by steel wires laid spirally round the copper conductor, thus using the strength-giving material to form part of the conductor; the area of the conductor is therefore increased; and there is no danger of the conductor knuckling through the gutta-percha from the resiliency of the latter. This addition to the conducting area increases the induction of the cable in the full ratio of the increased area, but it only increases the conducting power of the cable in the proportion which the conductivity of steel bears to that of copper, which is about one-seventh. To obtain the same capacity for the transmission of messages in a cable of this kind, as in a cable in which copper forms the sole conductor, the dimensions of the conductor must be increased to give the same ratio of conduction to induction. The insulating material, of course, requires protection in these as in other cables." The latter portion of this report we consider scarcely correct. We have now before us a specimen, in which the compound conductor has no greater surface than the ordinary copper strand of the same conducting power. We intended giving an illustration of this principle of cable, but it is so few in parts, that we can readily convey it to our readers, by simply saying that it is composed of a No. 8 soft copper wire, surrounded with 25 No. 25 best steel wires; this is insulated with four separate coatings of best gutta-percha, and then finished in a skin of hemp, merely to protect it during submergence; the conducting power of the core is equal to 520 lbs. of copper, and it is insulated with 550 lbs. of gutta-percha per nautical mile; in diameter, it measures five-eighths of an inch; weight per naut. in ship is 13 cwt., with a specific gravity of 1·7; and it is calculated to bear 7,500 fathoms of itself in water, without elongating one per cent. "Mr. Allan considers this cable suitable for a distance of 2,000 nautical miles, and a depth of 3,000 fathoms, the shore ends of course to be heavily protected. A cable of this description has the great advantage of being very simple to manufacture; in fact, it could be entirely finished by the manufacturer of telegraph cores. There are many instances of the gutta-percha core receiving injury during the process of iron wire covering, which have only been detected after the cable has been submerged."

The following cut illustrates Mr. Allan's method of recording messages, for use either on land or submarine telegraphs. As previously mentioned, the dot system, which he prefers, saves considerable time in transmitting the message.



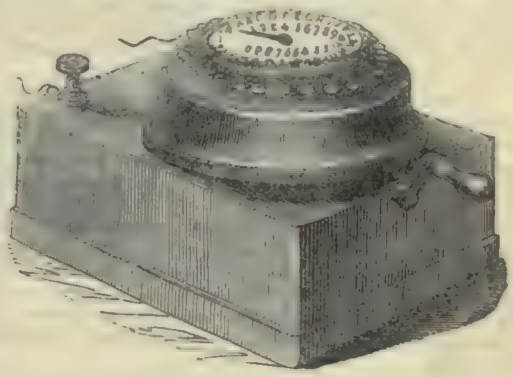
In the cut, Morse's system is compared with that of Mr. Allan's; and it will be seen that the latter has the advantage, in respect to space occupied on the receiving-tape, and that the code of signals is entirely different. The length of the dash is done away with, and the space between them economised. In Allan's arrangement, 126 currents produce the alphabet; against 164 required in the instrument employed in the system of Morse. The proportion of time is as 196 to 290 for the whole alphabet; and as 33 to 49 for a word; or generally about fifty per cent. is saved.

Tyer's Train Telegraph.—The numerous accidents which have taken place, owing to trains overtaking each other on the same line of rail, have given rise to numerous inventions, intended to prevent the recurrence of such casualties. Amongst them, we may notice Tyer's patent train telegraph. In this arrangement, a dial-face is placed at each station, on which needles are worked by electro-magnetism. A bell and gong are also provided, for calling the attendant's attention. The dial-face is divided into two parts—one for the up, and another for the down line; and on the right and the left of the face, the words "Train on Line," and "Line Clear," are engraved. According to the system adopted in this plan, each signal station communicates with the signal station on each side of it, so as to announce both the approach and departure of each train. The receiver of a signal cannot alter it; and a signal once given remains fixed, until the next signal is sent. The line of railway is divided into sections by the signal stations; and no train is permitted to pass one of the stations until the signal has been received that the next station in advance is quite clear. The mechanical arrangements are extremely simple, and the system itself is equally so. Its universal adoption would doubtless go far to prevent the possibility of collision by trains overtaking each other during a long journey. On short station-lines, such a plan would seem to be indispensable.

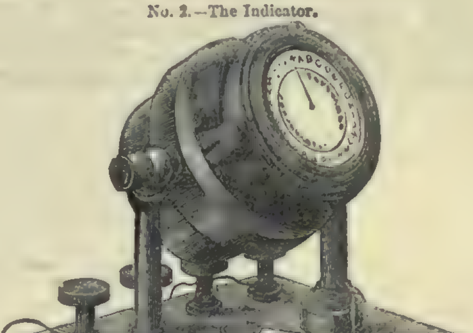
Wheatstone's Private Telegraph.—Much of the credit of all our telegraphic arrangements in this country is due to Professor Wheatstone, who, for many years, has been indefatigable in his exertions to bring the system into a state of perfection. The first attempts were confined to connecting our large cities together by means of telegraphic wires. Another step was that of extending the telegraph, by means of submarine cables, to foreign countries. Eventually, large warehouses were furnished with telegraphic apparatus, by which much time was saved by the telegraph doing the work of messengers. District telegraphs were another development of the system; and, at last, the Universal Private Telegraph Company has stepped in to give private individuals the opportunity of using the electric current for the expression of their wishes in any direction, and under their sole control. We can well remember hearing it said, somewhat derisively, some years ago, by a learned but crotchety professor, "that some day, electric wires would be as common in London as gas and water-pipes." This prophecy has been exceeded in its fulfilment; for the number of electric wires in many streets of the metropolis, far exceeds that of the pipes of both classes we have named.

It is, of course, requisite that the instruments employed in private houses should be so simple, both in their working and indications, as not to be liable to get out of order, or to require much practical knowledge in

No. 1.—The Communicator.



No. 2.—The Indicator.



reading them. These two desiderata have been accomplished by Mr. Wheatstone. The instrument consists of two parts—the communicator and indicator—on each of which the letters of the alphabet, and numerals are engraved, in the form of a disc. By means of a needle, actuated by electro-magnetic machinery, the words of the message sent by the communicator are spelt, letter by letter, on the indicator; the former instrument being in possession of the sender, and the latter with the receiver; the current is conveyed by wires, suspended in the air—in one bundle or rope, which may be made to enclose almost any number, from their small sectional area. Round the discs of both instruments, as many keys are placed as there are letters; and being pressed down, as each letter required is arrived at, in the rotation of the needle, they indicate the letter which has been telegraphed. The electric power is obtained by a modification of the magneto-electric machine—that is, by magnetic induction; and thus, the expense and nuisance of a voltaic battery are obviated. The preceding engravings will give a general idea of the instrument.

Special instructions are given as to the arrangement of the apparatus before sending a message; and the earth is used for the return current, as in the ordinary method of telegraphy.

The instruments, and use of the wires, are charged for by an annual rental. And thus, any two individuals, having agreed to employ the apparatus, and having had it suitably fixed, may carry on a correspondence, either in letter or cipher, entirely between themselves, and without the possibility of their secrecy being invaded by curious telegraph clerks, or interested parties. This is, of course, ensured by the use of separate and isolated sets of wires for each set of telegraph instruments.

Such inventions as these cannot fail to be of great benefit, in a social, commercial, and political point of view. In great cities, it is of extreme importance that police, military, and fire-engine stations, should have instantaneous means of communication; and these are at once afforded by the private telegraph system.

BALLOON TELEGRAPHS.—During the long wars of France, Napoleon conceived the happy idea of employing balloons for the purpose of surveying the enemy's lines; and the same plan was adopted by the Federals during the unfortunate internecine struggle between the American States in 1862. A most important addition, however, was made in the application of the electric telegraph. The balloon was connected by means of wires with the earth, and thus a continued communication was kept up between the observers and their attendants on *terra firma.* By this ingenious plan most valuable information was obtained, and the course of the campaign materially influenced.

SIEMENS' ELECTRIC RESISTANCE THERMOMETER.—This peculiar arrangement, primarily intended to test the temperature of large coils of submarine telegraph wires, in cases where spontaneous heating was feared, is an adaptation of the laws of resistance in conducting-wires, the resistance being measured by means of an ordinary galvanometer. It is equally applicable to the discovery of heating in masses of hemp, &c., which have frequently caused disastrous conflagrations. We are indebted to the *London Review* for a description of its arrangement.

The apparatus consists of a differential galvanometer and of a bath of water (or oil), the temperature of which can be changed at will by opening one or the other of two cocks, the one bringing a supply of cold, and the other a supply of hot liquid, an overflow pipe being provided to prevent accumulation. A battery of from four to eight cells is also provided, besides a number of electric coils, consisting each of a certain length of thin insulated platinum wire inclosed in a sealed metal tube.

These coils having been carefully adjusted, in the first instance, so as to offer equal resistance at one temperature, are connected to insulated copper leading wires of comparatively large sectional area (No. 16, B. W. G.), the ends of which are brought to the binding screws of the apparatus, to be inserted, when required, into a circuit including the battery and the galvanometer. These "thermometer coils" are deposited at the places where the temperature is to be observed, excepting one which has to be reserved for comparison with the others. This last-mentioned coil is, through its leading wires, so connected as to form an electric circuit with the battery and the second coil of the differential galvanometer, and is immersed in the bath before mentioned. It is evident, that if the temperature of the bath should be the same as that of the place where the thermometer coil under examination is deposited, the two electric currents proceeding from the battery will meet with equal resistance in the two circuits, and, in passing two spirals of the differential galvanometer in opposite directions, will produce no visible effect upon the needle. If, however, the temperatures of thermometer and balancing coil should be unequal, the needle will be deflected by the preponderance of current in the cooler circuit, showing, by the direction of its deflection, whether cold or hot water should be added to the bath to establish equilibrium of currents. When this equilibrium is obtained, the temperature of the bath is observed by means of an ordinary mercury thermometer, which temperature must be identical with the temperature at the distant place.

In dividing the "thermometer coils" into two portions, the apparatus is rendered applicable for observing wider ranges of temperature than can be directly attained by the mercury thermometer, and in this modified form it may be used for pyrometrical purposes. In measuring ordinary temperatures, the employment of equal and undivided coils is, however, not only the most simple arrangement, but it has the advantage in its favour, that the accuracy of the observation is not dependent upon the rate of variation if resistance be uniform, or even accurately determined. The heat produced in the coils by the electric currents employed, affecting the two coils equally, is also completely compensated in using equal coils.

SECTION V.

ACOUSTICS, AND ITS APPLICATIONS.

CHAPTER I.

THE PRODUCTION OF SOUND.

Although we can scarcely consider the phenomena which are usually ranged under the term "Acoustics," as resulting from a purely undulatory force—the analogy which exists between the production of sound, and light and heat, is so great, as to permit of our including its laws and applications under that head, if we do so only by way of analogy.

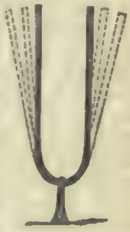
We have previously observed, that the vibrations of the atmosphere are really typical of those of the supposititious ether to whose undulations we assign the production of light; and our readers who have perused our previous remarks, will therefore be prepared to enter on a careful examination of the various conditions to which sound is subject.

In the experiments and remarks we suggested in connection with the subjects of Heat and Light, we had occasion to discuss the effects of reflection, radiation, and conduction on those agents; and, to a large extent, we shall find sound obedient to the same laws. Whenever motion is conveyed to a body, it partakes of the force imparted to it; and the result is, that heat, light, electricity, magnetism, and, almost universally, sound, is individually produced. If we take, for instance, the common process of striking a lucifer-match, we find sound first perceptible to the senses; then heat and light are observed; and, lastly, electricity may be discovered as having been set free, if the electroscope be employed to detect its presence.* Our present subject will therefore form a kind of corollary on all that have preceded it; and as such we shall treat it, including, however, many applications specially incident thereto.

The production of sound is so familiar to all our readers, that we need scarcely draw their attention to it. We cannot, however, fail to notice, that there exists a vast diversity in the sounds we hear. Thus, if a piece of wood or stone be struck, we simply notice a dull, heavy sound, entirely devoid of musical intonation, providing the bodies be struck in large irregular masses. If, however, they have certain definite shapes, and their lengths and thicknesses increase and diminish in definite ratios, then we hear graduated sounds. These may be produced by percussion—as in the case of the pianoforte, the strings of which afford "music" on being struck by the hammer or by simple vibration; as we observe in the cornet, the flute, and other wind instruments.

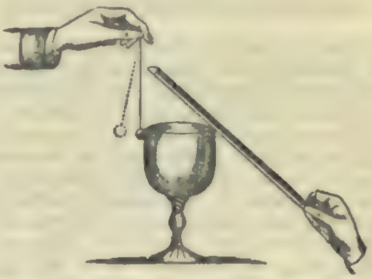
Now, in any case where sound is produced, such results simply from the vibration of the particles of the body which gives it; and the common tuning-fork, perhaps, affords us the best illustration of this fact. If the fork be struck against any hard body, then its prong at once vibrates, and, in so doing, it causes the air next to it to vibrate also. We hence have a successive series of waves afforded, just as we observe the same effect produced on casting a stone into still water. Now, just as a succession of circles passes from the spot at which the stone arrived on the surface of the water, so the same result occurs when a sounding body is put into motion. This is illustrated, so far as sound is concerned, in the annexed engraving, which shows, by the dotted lines, the vibrations which the metal of the tuning-fork undergoes after percussion. The body, however, must be elastic to produce sound: thus a piece of clay scarcely affords sonorous waves, because it has but a very slight amount of elasticity. If, however, a glass vessel of any kind, which is elastic, be struck, it at once produces and propagates in the air a series of waves, or vibrations; and these reaching the ear, so far act on that organ as to make it sensible to the sounds produced.

Fig. 1.



That an elastic body really vibrates when it is put into sufficient motion to produce waves of sound, is most readily proved: and the following engraving shows a very neat method of proving this fact. For this purpose, place a thin glass tumbler on any firm support, and hang a small wooden ball, by means of a thread, from any support, or by the hand, so that the ball may just touch the rim of the glass; then draw a violin bow against the rim of the glass. The vessel will thus be set into vibration; and the ball, resting against the rim, will be repelled, and oscillate for some considerable time. The mode of carrying out this interesting experiment is illustrated in Fig. 2; in which the straight line represents the cord holding the ball as suspended from any support, and the dotted line shows its vibration. The success of the experiment will depend on the glass rim being quite clean; and the bow should be well rubbed with resin, to afford as much friction as possible. Confining our attention for the present to the vibration of solid bodies, we may suggest a very instructive and pleasing mode of illustrating the production of musical sounds by means of glasses of various sizes. If eight finger-glasses, or tumblers, are placed on a hollow square box, and they be chosen so that they may have a relative size (of which we shall speak more fully hereafter), a series of musical sounds may be readily produced, which accord with an octave, or eight successive notes of any musical instrument. The sounds are given off on the edges of the glasses being rubbed by the finger, which has

Fig. 2.



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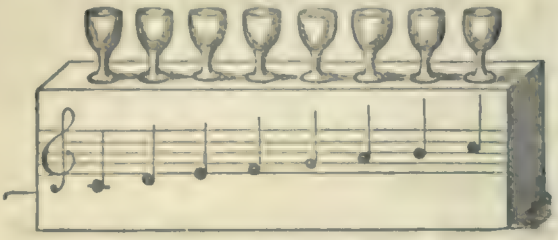
* See *ante*, p. 176; Fig. 16.

previously been wetted by a little lemon-juice, to give it a degree of roughness. The finger should be gently but firmly rubbed round the rim of each glass; and the arrangement of which we speak is illustrated in the following engraving—the octave succession of notes being also indicated. The effect produced by the action of the finger, is exactly similar to that afforded by the bow in our last illustration. The glasses are caused to vibrate, and the production of sound is the natural result.

Fig. 3.



We might add, to a great extent, various illustrations of the production of sound of a musical kind, by means of vibrations of the atmosphere and solid bodies. The Æolian harp is an instance in which the air, moving through the intervals between wires of different thicknesses, produces such results. Even the wires extended on the poles observed by the sides of our railways, for telegraphic purposes, often afford, by their vibration, what we may venture to call a wild kind of music. The striking of a gong, and similar instruments, on the other hand, afford instances in which sound is produced by the vibration of solid bodies, subsequently propagated through the air. Wind instruments of all kinds produce these effects on the ear, by a combination of the vibrations of their solid parts with those of the surrounding atmosphere. Having thus spoken generally of the production of sound, we shall now refer to its conduction through various media.

THE CONDUCTION OF SOUND.

We have already remarked, that whenever vibrations are produced, affording sound, the waves of sound are propagated by the surrounding bodies. But this depends on the elasticity of those bodies; as many substances—to use a common phrase—stop sound; that is, they prevent its passage, being incapable of continuing the vibrations which have arrived at them. This is taken advantage of in the erection of astronomical instruments, which, from the vibrations of adjacent bodies, would become so unsteady as to be perfectly useless. But if a telescope, of high magnifying power, be mounted on a clay and leaden foundation, the vibrations of surrounding bodies are stopped, and cease. The same materials, with many others, equally oppose the propagation of soniferous waves; and may, therefore, to some extent, be termed non-conductors of sound. The atmosphere permits the passage of sound at the rate of about 1,140 feet per second; that is, the sonorous vibrations of any body will not be perceived by the ear in less than a second of time, if 1,140 feet of air interpose. We find, however, that water, and many solids, convey sound much more rapidly than air; and if we make the rate of conduction in the air the standard of comparison, the following will show the relative powers of various substances:—

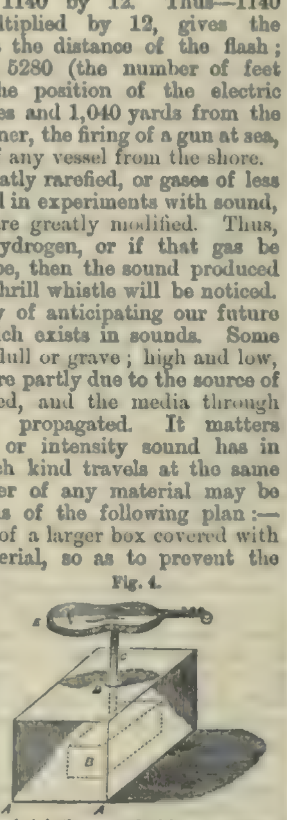
Air	1
Water	4
Tin	7½
Silver	9
Copper	12
Iron	17
Glass	
Various kinds of woods, 11 to	

Most of our readers must have noticed, in daily experience, how differently bodies conduct sound. Thus, if a long row of iron railings be struck at one extremity, the sound produced is heard at the other end long before it would be conveyed by the atmosphere.

The temperature and density of any medium, have the effect of modifying its conducting power; and this is most readily observed in the atmosphere. The rate, however, which we have mentioned, is pretty nearly uniform for all ordinary atmospheric changes; and, for this reason, the conduction of sound may be employed as a means of measuring distances. This depends on the fact that light travels infinitely faster than sound; requiring, as it does, no more than one second of time for traversing a distance of 192,000 miles. To illustrate this mode of ascertaining distances, we will suppose a flash of lightning to be seen, and that twelve seconds elapsed before the sound of thunder was heard. Presuming 1,140 feet per second to represent the rate at which the atmosphere conveys sonorous waves, we shall ascertain the distance of the place at which the flash occurred, by multiplying 1140 by 12. Thus—1140 (the rate of sound) multiplied by 12, gives the number of feet (13680) as the distance of the flash; and this sum, divided by 5280 (the number of feet in a mile), shows that the position of the electric discharge must be two miles and 1,040 yards from the observer. In a similar manner, the firing of a gun at sea, may indicate the distance of any vessel from the shore.

If the atmosphere be greatly rarefied, or gases of less specific gravity are employed in experiments with sound, then the resulting effects are greatly modified. Thus, if a person breathe pure hydrogen, or if that gas be blown through an organ-tube, then the sound produced will lose its volume, and a shrill whistle will be noticed. We may here state, by way of anticipating our future remarks, the difference which exists in sounds. Some are sharp or acute; others dull or grave; high and low, &c.: and these differences are partly due to the source of vibration, the body vibrated, and the media through which the vibrations are propagated. It matters not, however, what pitch or intensity sound has in respect to its speed, as each kind travels at the same rate. The conducting power of any material may be readily illustrated by means of the following plan:—Place a musical box inside of a larger box covered with baize, or any woollen material, so as to prevent the passage of sound from its inside. A hole should be bored in the top. Into this insert a wooden rod, so that it may rest on the musical box. If a violin, or empty box of any kind, be then placed on the outer end of the wooden rod, the sounds produced by the box will be distinctly heard. On removing the violin from the rod, the sound apparently ceases. This is owing to the wood conveying sonorous vibrations from the musical box, and the subsequent diffusion of them through the air by means of the violin. The mode of carrying out this interesting experiment, is illustrated in the above engraving.

Fig. 4.



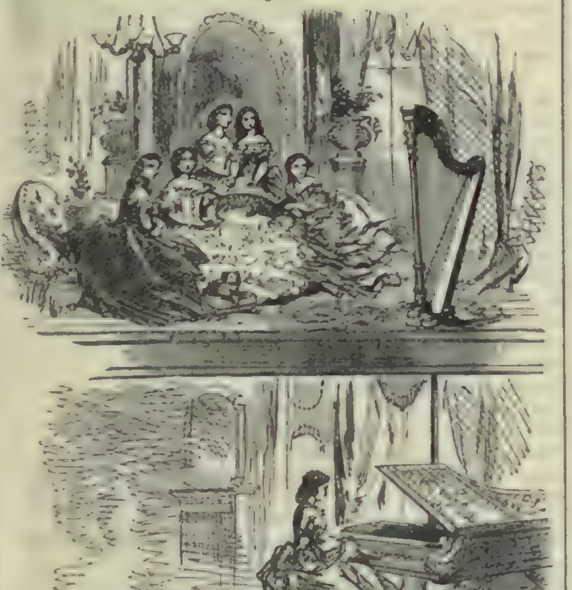
A A is the outer deal box, enclosing B, the musical box; C is a deal rod, through the hole D, resting on the musical box; E is a violin for diffusing sound.

This experiment also illustrates the use of sounding-boards, as seen in the piano, harp, violin, &c.; for it is not only essential that sound be conducted by a sufficient material, but also that an adequate diffuser be also present.

The stethoscope is constructed so as to convey the sounds of the beating of the heart, &c., to the ear of the person employing it: and a very common plan of ascertaining the regular supply of steam to each end of the cylinder of a steam-engine, is that of placing a wooden rod against the cylinder—biting it with the teeth, and closing the ears. The vibrations are thus conveyed to the ear by means of the conducting-rod. If a musical box be placed at a considerable distance from the ear, so that its tones can scarcely be heard, and afterwards a deal rod be placed with one end on the box and the other between the teeth, the sounds previously inaudible will be readily perceived, through the conducting power of the wood. In a similar manner, a poker, if hung by means of a piece of string held in the teeth, will produce a powerful sound when struck by a hammer or other body.

A very ingenious application of these principles was made, some years ago, by Professor Wheatstone, of King's College, London. He conceived the idea of conveying musical sounds from any instrument into a distant apartment; so that a "supply" of music might readily be laid on in each room of a house, if rods were attached to any instrument, and fitted with diffusers, such as we have illustrated in the last engraving. The plan was practically carried out at the London Polytechnic. In the basement of the building, a pianoforte, cornet, and violin were played; and from each a deal rod was carried, to a height of 40 feet, into the upper lecture theatre. On each deal rod a harp was placed; and by these means the sound produced by each instrument was at once diffused. The effect was singular and interesting, for each instrument was distinctly heard by itself, or in concert with the rest; the music, however, all proceeding from the sound-diffusing harps. In a similar manner the sound of human voices was also conveyed; the singers being placed with their mouths close to the sounding-board of the piano in the basement, and the harp answering as the diffuser in the lecture theatre. The following engraving shows how the same plan may be carried out in any two apartments in the same house.

Fig. 5.



The pianoforte and the performer are to be placed in a lower apartment, and the audience and diffusing-harp in the upper one. A deal rod rests on the sounding-board of the piano, near C treble; and the harp rests on its upper extremity. So long as this arrangement is maintained the performance is distinctly heard; but if the harp in the upper apartment be removed from the deal rod, the sound instantly ceases. A violin or empty deal box answers just as well as the harp for diffusing the sound.

If a musical box or bell be placed in the receiver of an air-pump, their sounds will be audible until the air be exhausted; but then, owing to the absence of any medium by means of which their vibrations may be conducted, the sounds are entirely lost; but if the air be allowed again to pass into the receiver, the sound becomes immediately perceptible.

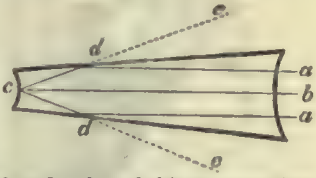
With respect to the greatest distance at which sounds can be heard in air, much depends on the state of the atmosphere. Bells of churches on shore, have been heard many miles out at sea; and the report of cannon has been noticed at astonishing distances. Sound is also well conducted by rocks; and the noise of the pickaxe of the miner may be heard by another person at a long distance off, owing to the vibrations being conveyed by the solid rock.

THE REFLECTION OF SOUND.

THE analogies between heat and sound are very evident, for they can each be conducted, and also reflected, by proper means; and having spoken of the conduction of sound, we shall now proceed to show by what means it may be reflected: and thereby explain the cause of echoes, and the construction and use of some acoustic instruments. Two concave reflectors, made of tin or zinc, will be required for some of the experiments we shall mention. They may be of a similar shape and size to those illustrated in a previous page.*

It may facilitate our progress, if, in future, we employ the term "ray" to the emanations, or waves of sound, proceeding from any source. This will also enable our readers to connect the laws of the reflection of heat and light with those of sound, and also to become acquainted with the analogy existing between them. If a ray of sound impinge on a plane reflector, such as an even wall or a sheet of glass, it will be reflected therefrom in a similar manner to one of heat or light; and the angle formed by the incident ray will be equal to that of the reflected ray. By pursuing this law, a variety of applications of it may be made in the construction of such instruments as the speaking, hearing-trumpet, &c. The annexed engraving illustrates the mode in which the voice of a man may thus be sent to a great distance by means of the former instrument. The external part of the figure represents the speaking-trumpet, usually employed on board ships: *a a* are two rays of sound, which have been reflected from *d* after proceeding from the mouth, *c*. Now, had not the reflecting surface of the trumpet been present at *d*, then the sound would have radiated off towards *e*, and only that ray proceeding in the direction *c b* would have reached the ear of a person at a distance. But by means of this tube, all rays passing from the mouth, *c*, otherwise than in a line parallel to *c b*, impinge on the sides of the tube, and are thence reflected therefrom in lines which are parallel to *c b*; and so a larger number reach the ear by the use of this arrangement.

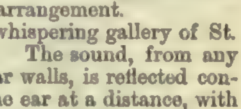
Fig. 6.



On the same principle, the whispering gallery of St. Paul's Cathedral is constructed. The sound, from any source, impinging on the circular walls, is reflected continuously, and at last reaches the ear at a distance, with a very slight diminution of effect. Echoes are similarly caused by the successive reflection of any sound from the walls or rocks, placed in such positions with each other, that any ray impinging on them may be readily reflected. Just in proportion to the length of such reflecting surfaces, and the number of incidences and reflections the sound makes, so the number of echoes occurs.

The principle of the successive reflection of sound is illustrated by the annexed engraving, which represents the half of a whispering gallery. We will suppose a sound to proceed from *a* to *c*; it may then be again reflected from *c* to *d*, or to A. Similarly, a ray of sound, proceeding from *b*, may be heard at *c b*, and reflected again to *d* or A. If the whole circle be formed, then the reflections, whether short or long, will be produced over the entire circular

Fig. 7.



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* See *ante*, p. 27; Fig. 6—Section, "Heat."

wall; and an ear, placed in any portion of it, will hear them distinctly. If a concave reflector be so placed as to receive the rays of sound from a source at its focus, and another reflector be so arranged at a considerable distance from the first, but so that their foci shall be in one straight line—then all the sounds proceeding from the first reflector will be received on the face of the second, and be again reflected to its focus. If the ear be placed at that spot, all the sounds proceeding from the distant reflector will be distinctly heard, and yet none will be audible at any place intermediate between the two foci. Again, even if much noise be made on all sides of the reflectors, still the sounds at the focus will be readily distinguished, although they may have passed through a considerable distance of intermediate air. A very similar result is produced when heat is reflected, as has been shown under that section.*

On the principle of reflection, speaking-tubes are constructed, which simply consist of pipes having an even internal surface. The sound is reflected from side to side until it reaches the other extremity of the tube. These arrangements are largely employed in the warehouses, &c., of our cities, and are a great saving of time and trouble. An amusing effect may be produced by a modification of the speaking-tube. Two tubes may be inserted in a head of wood, or plaster—one communicating with the mouth, and the other with the ear of the figure, in the manner illustrated in the following engraving. A tube is carried into an adjacent room, and, of course, a person must there listen and reply to any inquiries whispered in the ear of the figure. The reply is received by listening at the mouth of the bust. This resembles what has been termed the "invisible lady," which, however, was first introduced to the public as an acoustic toy, in the following form.

Fig. 8.

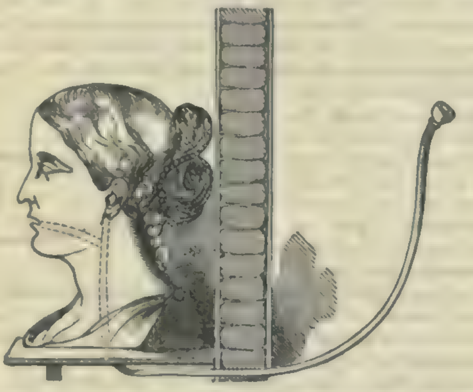
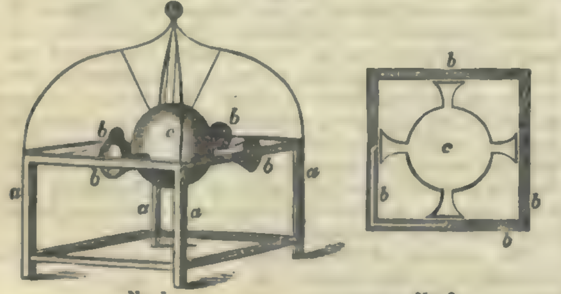


Fig. 9.



No. 1. No. 2.

No. 1, Fig. 9, represents the apparatus in its general appearance. It consists of a wooden frame, *a a*, supported on four pillars, from one of which a tube passes (as in our last illustration) to an adjacent room; *c* is a copper ball from which the tube proceeds; and *b b b b* are four openings in the side of the ball. These are shown in section, in No. 2. The mode of using the arrangement is precisely similar to that we have just mentioned in connection with the preceding illustration.

MUSICAL SOUNDS.

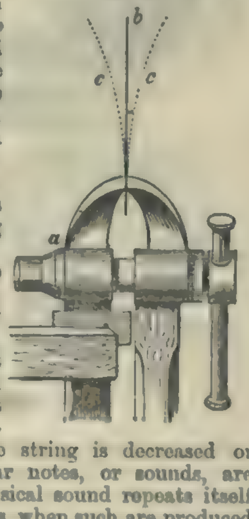
We shall now call attention to some of the laws observed in connection with the production of musical sounds; having already explained the general results obtained by the vibration of the air, and the conduction and reflection of those vibrations by means of suitable media and surfaces.

Of all the pleasures derived by our senses, those of music are the most refining and elevating. They appeal alike to the mind and the sentiments; at times, soothing the troubled spirit, and yet often rousing into action the military ardour of the soldier. It is, indeed, surprising to observe the extensive influence which musical sounds exert, whether they are produced by instruments, or by the voice of man or birds. The ancients were so impressed with this, that, amongst all their leading allegories and mythological fables, many relating to the power of music are included. The well-known tale of Orpheus, who was said to have charmed the very stones; and the influence ascribed to Apollo, the god of Music, over the gods and men—are instances, out of many others, of the kind.

We shall divide this portion of our subject into two parts. The first will deal with musical sounds as produced by instruments: and this will be followed by an account of the physiology of animal voice; for which we are indebted to Dr. Bushnan. It has already been remarked, that all sounds are caused by vibrations, but that such differ according to the body from which they proceed. Besides, the sounds so produced vary from each other in tone, and other particulars, according to certain definite laws, which we will now inquire into.

If a piece of steel be tightly held at one extremity, and then struck, it will, of course, vibrate, and, in so doing, will produce sound. If the length of the steel beyond the part at which it is held be decreased, then such sounds will become sharper. On the other hand, if the length be increased, then the sounds produced are duller, and deeper in tone. A very convenient way of trying this experiment is represented in the annexed engraving; in which *a* is an ordinary vice; *b* a piece of steel ribbon; and *c c* are dotted lines, showing the vibrations produced on the steel. The steel extremity, beyond the top of the vice, can be lengthened or shortened, by passing it, more or less, through its sides or cheeks, as may be required for altering the sound.

Fig. 10.



In trying this experiment, the reader may, by a little care, produce not only an apparently extensive range of sounds, but, in so doing, will notice, that at certain intervals the same kind of sound is reproduced; but either sharper or lower, according as the length of the string is decreased or increased. Now, these similar notes, or sounds, are termed octaves; that is, a musical sound repeats itself at every eighth set of vibrations, when such are produced according to certain rules. But intermediate to the notes forming the extremities of the octave, there are others which, to some extent, harmonise with them: such are termed thirds and fifths, owing to their relative position with that of the first or last octave note. This may readily be tried on a pianoforte, by placing the fingers on two successive C's, and then striking the following E and G. If the three notes be then struck together, the effect of their harmony will be at once apparent.

Another mode of trying the effect of increasing or lengthening a vibrating cord, is that of stretching tightly a piece of catgut between any two points. If this be vibrated it will produce sound, as usual. If it be halved in its vibrating length, by placing the finger on it and pressing it down, midway between its two extremities, then a shriller note will be produced, which will be an octave higher than that afforded by the long cord. If, again, a piece of steel be pressed against a rapidly revolving tooth-wheel, a certain sound will be produced; but if the tooth-wheel be made to revolve twice as fast, then the previous sound will be raised an octave.

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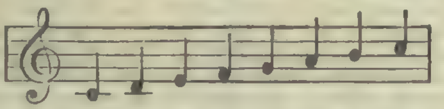
* *See ante*, p. 27; Fig. 6.

From these experiments we learn, that the tone of any sound—that is, its shrillness or dullness—depends on the number of vibrations produced in any similar period of time; and that these, again, depend on the length of the vibrating body—the number of vibrations being inversely as the length of the vibrating body. This will be more easily understood if our readers will notice the length of the different strings in a harp, or pianoforte. It will be found, that those which produce shrill sounds, are invariably the shortest; and the deeper the sound, the longer the string which produces the vibration. Violin players are thus enabled to obtain any variety of the sounds of other musical instruments, by means of the rapid movement of their fingers over the strings; for they are enabled to shorten their lengths, so far as the production of sound is concerned.

We shall confine our illustration of these principles to the eight notes in the treble clef forming an octave—from C below the lines, to C on the third space, as represented below. To each of the notes we have appended their names, for the assistance of those of our readers who do not understand music. When, however, an instrument, especially a pianoforte or harp, can be had access to, such will materially aid in making our remarks understood.

Fig. 11.



English names ... C D E F G A B C
Italian do. .....do re mi fa sol la si do

Now, the sounds of each of these letters, in Fig. 11, are produced by a definite number of vibrations, and the length of each string. Thus, that affording C vibrates 240 times per second; whilst the upper, or octave C, will vibrate 480 times per second. The length of the string producing the low C, may be 45 inches; whilst that producing the upper C, will be but 22½ inches. Each of the other notes will proceed from an intermediate length of string and number of vibrations. The following table gives the value of each of these—commencing with the lower C. Some consider that 256 vibrations equal C. We have chosen the lower number, for the purpose of avoiding fractions; but the reader can easily calculate a table on that standard, by using the table, or ratio, which follows this.

Name of note.	Length of cord in inches.	Number of vibrations per second.
C	45	240
D	40	270
E	36	300
F	33¾	320
G	30	360
A	27	400
B	24	450
C	22½	480

The law of the relation existing between the number of vibrations, the length of the cord, and the nature of the sounds produced, is thus at once made evident. The ratio existing between each of these is shown in the following table—commencing with low C, as before, it being the standard.

Name of note	C	D	E	F	G	A	B	C
Ratio of the length of the cord	1	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$
Ratio of vibrations .	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

From which we perceive, as before stated, that the length of the cord is inversely as the number of vibrations, and *vice versâ*. We have already observed that harmonies are afforded when certain sounds are produced together. This occurs when C, and G (below), which are five notes apart, C, and E (above), which are three notes apart, and C, E, and G, are struck together—forming, as they do, a chord in which thirds and fifths are combined. The effects of such combinations are naturally pleasing to the ear: but this is not a pure matter of taste only; for if some pieces of paper are allowed to fall on strings thus vibrating, they will arrange themselves in such positions as will accord with those of the length of the octave, the third, and fifth, pointed out in our last table. We shall refer to this again when speaking of acoustic figures.

We have confined these remarks to stringed instruments only; but they are equally applicable to those in which wind is alone employed. Thus, in the organ, the length of the pipes corresponds to the length of the string in the pianoforte, because they enclose a similar length of air; and it is the vibration of this which causes the sound of different notes. In the flute, cornopean, &c., the length of the column of air is regulated by the skill of the performer; hence the difficulty experienced in acquiring the art of blowing those instruments properly. Many instruments, of entirely different kind of construction, such as the harmonium, concertina, the pan-pipes, bells, &c., all owe the production of their various notes to the laws we have here explained.

The diffusion of sound, from an instrument producing it, is effected by means of a readily vibrating body. Of such kind is the sounding-board of the piano, harp, violin, &c. We have already mentioned various facts relating to this subject, and must therefore refer our readers to our previous remarks.* Great variety exists in the power, sweetness, and other qualities of instruments. Thus the flute and the cornopean, although each producing their sounds directly by the vibration of the air, convey entirely different effects to the ear. The same may be said of the violin and harp, amongst stringed instruments. These results chiefly arise from the nature and quality of the material employed, and the skill exhibited in their construction. A fastidious, or rather educated musical ear, can detect great differences in this respect; and even to persons with no special musical taste, it is by no means difficult to distinguish the tone of the same kind of instruments made by different manufacturers. It is, however, no part of our plan to enter into such details; our object being solely to illustrate the principles of which we have been speaking.

Despite the perfection of many of the musical instruments of our day, we still cannot arrive at the production of that richness, body, and flexibility of tone which is found in the human voice; and we shall now avail ourselves of Dr. Bushnan's remarks in illustration of that portion of our subject; in which he also explains the organs on which the sound-producing powers of animals and birds depend.

THE PRODUCTION OF ANIMAL SOUNDS.

Organs of Voice and Speech in Man.—The organs concerned in voice and speech may be described as the chest and lungs, the windpipe, the larynx, the posterior cavity of the mouth, the nostrils, which communicate with that posterior cavity, the palate, the tongue, the teeth, and the lips. The sounds which constitute voice, belong to the order of musical sounds, independently altogether of the singing voice. All that is rightly termed voice, takes place in the larynx, which is properly the instrument of voice. But even independently of the modifications by which voice is changed into articulate speech, the voice is variously affected by the other parts which have been enumerated: by the chest, as regulating the force of the air; by the windpipe, as susceptible of several degrees of length and tension; by the posterior cavity of the mouth, as offering an expanded vault; by the nostril, as affording a double passage of exit for the breath; and by the various conditions of the tongue, the palate, the teeth, the lips, according to the position in which they happen to be at the moment.

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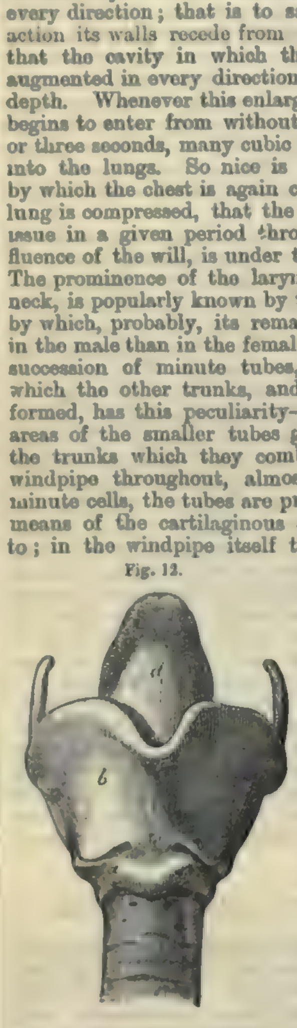
* See *ante*, p. 275.

The chest and lungs together constitute, in reference to the voice, a musical bellows, capable of supplying air with more or less force to the organs of voice. The peculiarity of these bellows is, that by them the air must be renewed at short intervals, entering from without by the same passage by which it is expelled when the voice is exercised. It can, however, supply air without interruption, in a continued stream, for about fifteen seconds. The lung consists of two large bags of air, and does not materially differ from the wind-box of an organ, or rather from the bag of a bagpipe. No air can enter the lung, or escape from it, except through the windpipe. The walls of the chest are everywhere in contact with the outer surface of the lung, and close in, around the point at which the windpipe rises upwards to the larynx. The chest is capable of expansion in every direction; that is to say, by means of musqular action its walls recede from the surface of the lung, so that the cavity in which this air-bag is contained, is augmented in every direction—in length, in breadth, in depth. Whenever this enlargement commences, the air begins to enter from without. By this process, in two or three seconds, many cubic inches of air can be drawn into the lungs. So nice is the action of the muscles, by which the chest is again contracted in size, and the lung is compressed, that the stream of air which shall issue in a given period through the larynx, by the influence of the will, is under the most complete control. The prominence of the larynx on the fore part of the neck, is popularly known by the name of Adam's apple; by which, probably, its remarkably greater prominence in the male than in the female, is referred to. The long succession of minute tubes, by the gradual union of which the other trunks, and, finally, the windpipe, are formed, has this peculiarity—that the aggregate of the areas of the smaller tubes greatly exceeds the area of the trunks which they combine to form. From the windpipe throughout, almost to their origin in the minute cells, the tubes are provided with tense walls, by means of the cartilaginous appendages before referred to; in the windpipe itself these cartilages assuming a more definite form. They are in complete rings of cartilage, being deficient posteriorly; that is, each ring of the windpipe traverses about two-thirds of its circumference, leaving the remaining one-third, on its posterior aspect, destitute of this support. The number of rings in the windpipe is from fifteen to twenty; in other respects the tube is chiefly membranous, yet provided with muscular fibres capable of diminishing its calibre, by drawing together the extremities of the rings. It has been proved, by sufficient experiments, that when the larynx is raised by the powerful muscles attached to it, the windpipe is drawn up from the chest in a corresponding extent, and that at the same time its diameter is diminished by about one-third.

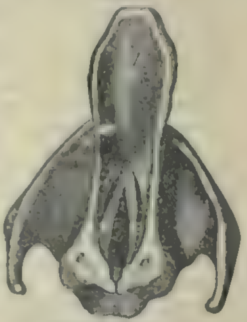
Fig. 12.



a, basement or cricoid cartilage, resting on the cylindrical windpipe; b, protecting cartilage or thyroid cartilage; d, valve-like cartilage or epiglottis.

The base, or lowest part of the larynx, rests on the upper part of the windpipe (Fig. 12); and this base consists of a ring, somewhat more developed than any of the rings of the windpipe, yet not so different from these but that it might be regarded as the summit of that tube. This ring differs from the rings of the windpipe in being complete all round; it is not, however, of a uniform breadth in the direction of from below upwards, being broader at the posterior part. It may be likened, then, to a ring, with a stone or a seal, the expansion behind corresponding to the stone or seal. On the upper edge of the expanded portion of this ring, at the base of the larynx, are set two slender bodies of a pyramidal form, which bear the most important part in the mechanism of the larynx as an organ of voice. These two bodies are exactly alike, and are placed almost close together, like two miniature obelisks set on end. The connection of their inferior extremities with the basement ring of the larynx, is by articulation—viz., by a true joint, like the shoulder-joint; that is to say, they are movable on the cartilaginous ring which supports them. From the one to the other, on their posterior aspect, muscular fibres extend, by the contraction of which, these two minute pyramids are made to approximate together. From the fore part of each, near their bases, an elastic substance proceeds forwards, converging, to interlace with its fellow at the anterior part of the larynx; that is to say, a minute, somewhat triangular, space is formed by two portions of elastic tissue, which cross the basement ring of the larynx from behind forwards, the base of this triangle being the space between the two pyramidal bodies just spoken of and its apex behind—a portion of the larynx to be presently alluded to. This triangular space between these two portions of tissue, or vocal ligaments, as they are called, is the aperture by which the breath enters and issues in respiration, and by which, when contracted to a narrow chink, the air is forced through in the exercise of voice. (See Fig. 13). These, then, are the most essential parts of the larynx; the two pyramidal bodies each resting on the posterior part of the basement ring, while the two ligaments proceed forwards, each from the base of one of these pyramids, to form a triangle, the apex of which is so directed as to be over the anterior part of the aperture of that basement ring. It is manifest, that when these two minute pyramids are drawn close together by the action of the muscular fibres, the base of the triangular opening is diminished, so that the posterior, or wider part of the opening, becomes obliterated; also, if the apex of this triangle be drawn forwards, that the sides formed by the two vocal ligaments will still further approximate. Such, then, are the two actions by which the triangular aperture is reduced to a minute chink—namely, by the points to which its base is attached being made to approximate, and its apex being drawn forwards.

Fig. 13.

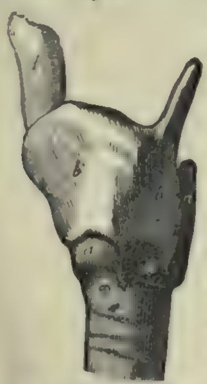


c c, Pyramidal or arytenoid cartilages; a, cricoid or basement cartilage; e e, the tongues or proper vocal cords, called also vocal ligaments; also arytenotheroid ligaments and inferior vocal ligaments; f f, the ventricles of the larynx; d, the epiglottis.

Other muscular fibres are so disposed as to antagonise the forces which close the aperture; two sets of fibres on each side extend from the basement ring inwards, to be attached to the pyramidal cartilages, by which they are drawn asunder, and the base of their rectangular aperture again restored to its former extent.

Several important, yet less essential parts of the larynx, remain to be described. The anterior narrow part of the basement ring supports that great prominence which constitutes Adam's apple. This is by far the largest portion of the larynx, but may be regarded merely as a defensive plate guarding the essential parts of the organ from injury. When the finger is placed upon its upper margin, and directed a little upwards, a hard, wire-like circle is felt; this is the convexity of the hyoid bone, or bone of the tongue, which has intimate connections, by ligaments and muscular fibres, with many adjacent parts, so that it is rendered, as it were, a centre of motion. Hence, when the hyoid bone is raised, many of the adjacent parts follow its movements. The hyoid bone is described as having the shape of the Greek upsilon, the convexity being directed forwards, and it may be felt immediately above the great cartilage of the larynx. This great cartilage, then, termed the thyroid cartilage (see Fig. 12), may be described as a quadrilateral sheet of cartilage, with appendages at its four angles, named its cornua, or horns. This quadrilateral sheet is bent along its perpendicular middle line, and this bending constitutes the angle which is felt in Adam's apple: the upper horns are attached to the hyoid bone; the under horns to the basement ring before spoken of. Thus the thyroid cartilage is wrapped round the essential parts of the larynx, covering them in front, leaving them exposed behind. The prominent angle in front corresponds to an interior angle on its posterior aspect; and to the middle part of this interior angle extends the apex of the triangle formed by the vocal ligaments, and there obtains an attachment.

Fig. 14.



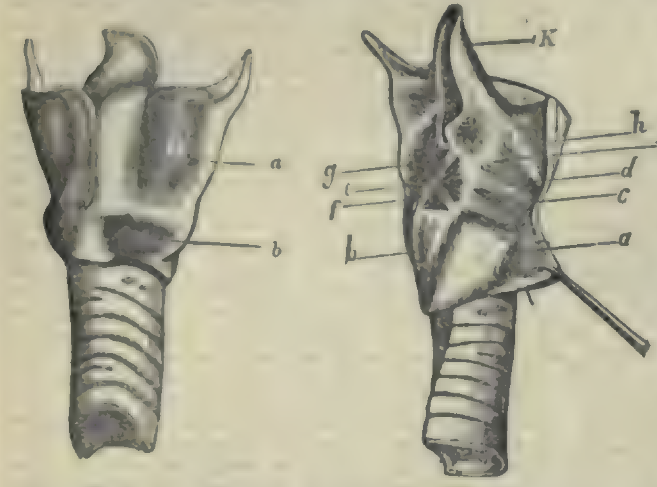
a, the cricoid; *b*. the thyroid; *d*, the epiglottis.

A movable valve, like a little tongue, its apex directed backwards, is attached to the same interior angle of the protecting cartilage, a little higher up, overhanging the cavity of the larynx. Between the same interior angle of the anterior protecting cartilage, and the two movable posterior cartilages before described, muscular fibres proceed, by the contraction of which those triangular movable cartilages are drawn forwards, so as to relax the elastic cords, already termed vocal ligaments. From the same interior angle of the protecting cartilage, an elastic substance proceeds, radiating in different directions, so as to close in the parts otherwise unoccupied, above the lateral portions of the basement ring of the larynx. This elastic substance, in particular, forms two cords, extending between the superior points of the movable triangular cartilages, and the sides of the tongue-like valve before spoken of. These cords constitute what have been termed, somewhat improperly, the superior vocal ligaments. (See Fig. 13). When the space beneath these so-called superior vocal ligaments and the true vocal cords, or vocal ligaments, is examined, a cavity is found on each side of considerable extent; and the two cavities are called the ventricles of the larynx. The mucous membrane, descending from the mouth and nostrils, covers and forms a lining to these parts in its passage downwards into the windpipe and lungs; so that the so-called superior ligaments of the larynx are often described as mere folds of the mucous membrane, extending between the posterior pyramidal cartilages and the protecting cartilage. It appears, however, from more minute investigation, that these folds of the mucous membrane do actually contain an elastic substance, not less capable, under certain circumstances, of vibratory action than the true vocal cords, or true vocal ligaments.

To recapitulate, then, the prominent points in the conformation of the larynx—the windpipe, called by anatomists the trachea, is surmounted by a complete cartilaginous ring, about an inch in diameter. This ring is the only outlet of the lungs by which air can issue from their numerous cavities, and is the only inlet by which air can penetrate from the atmosphere into the same cavities. This ring, being of a firm cartilaginous structure, is plainly incapable, under any ordinary circumstances, of dilatation and contraction. But the air is permitted neither to pass inwards, nor to come forth through the whole area of this cartilaginous ring, whether in respiration or in the exercise of voice. Its area is closed up on each side by impervious texture, so as to permit a passage to the air only by a chink, variable in its size, extending in the direction of its antero-posterior diameter. This chink is bounded, according to the common descriptions, by the vocal ligaments, one on each side; and at its posterior part by the cartilaginous processes of the base of the movable pyramidal cartilages to which these cords are connected. This chink, when most expanded lengthways, is about eleven lines in length; and of this space seven lines lie between the vocal ligaments, and four between the opposite cartilaginous bases of the pyramidal cartilages, above spoken of, to which anatomists give the name of arytenoid. (See Fig. 13). This chink, as above stated, is usually described as triangular, with its base between the two arytenoid cartilages, and its apex attached to the anterior angle of the protecting cartilage, above spoken of, to which anatomists give the name of thyroid. (See Fig. 14). More correctly, at its greatest dilatation, it has a lozenge shape, with the posterior angle truncated. Thus the chink commences narrow immediately behind the thyroid cartilage, expands between the vocal ligaments to their attachment at the base of the arytenoid cartilages, and then contracts in the space between the cartilaginous bases of these two bodies, not to a point, but to a truncated angle. The widest part of the chink, in its greatest state of dilatation, is about five lines and a-half—nearly half an inch. This greatest degree of dilatation takes place during inspiration; during expiration, the chink undergoes a slight contraction. But during the exercise of voice, the posterior part, bounded by cartilaginous margins, as being between the base of the arytenoid cartilages, is entirely obliterated. Thus it is correctly stated, that the chink, concerned in the exercise of voice between the true vocal ligaments, at its greatest dilatation, is of a triangular shape, being entirely bounded on the sides by the vocal ligaments, and its base corresponding to the points between their attachment to the arytenoid cartilages. As before stated, the arytenoid cartilages are attached to the upper surface of the posterior part of the basement cartilage of the larynx, or cricoid cartilage (see Fig. 14); and the vocal ligaments being attached to the bases of these arytenoid cartilages, it is manifest, that when these cartilages are drawn together, the vocal cords must approximate; that when these cartilages are drawn asunder, the vocal cords must recede from each other at their posterior part; that when the arytenoid cartilages are drawn backwards, the vocal cords must be put on the stretch; that when the thyroid cartilage, to the interior of which the apex of the triangle, formed with cords, is attached, is drawn forwards, they must also be put on the stretch. All these changes are known to occur by the action of particular sets of muscles. The thyroid cartilage, which forms "Adam's apple," is connected to the basement, or cricoid cartilage, by an elastic membrane, which of itself tends to keep the thyroid cartilage nearly in the same perpendicular line with the cricoid, so as, in some degree, to stretch the vocal ligaments. But there are two muscles extending between the cricoid cartilage and the thyroid, by which the thyroid cartilage is drawn forwards, so as distinctly to stretch the vocal ligaments. There are also two muscles extending between the posterior part of the cricoid cartilage, and the posterior surface of the arytenoid, by which the arytenoid cartilages are drawn back. These two pairs of muscles, when they act concurrently, must very much stretch the vocal ligaments. A set of muscular fibres, before spoken of, passing between the arytenoid cartilages on their posterior aspect, by their contraction causes the cartilages to approximate. Two other muscles, extending from the sides of the cricoid cartilage to the arytenoid, draw them asunder. Some other muscular fibres are found, connecting the cartilages of the larynx; but the account of these, owing to their less importance, may be omitted.

The small muscles of the larynx are represented in the following figure. The crico-thyroidei (*b*, No. 1; *a*, No. 2), and the crico-arytenoidei postici (*b*, No. 2), extend the vocal cords in the direction of their length, and, at the same time, narrow the glottis. The crico-arytenoidei laterales (*c*, No. 2), and the thyro-arytenoidei (*d*, No. 2), rather relax the vocal cords. The oblique and transverse fibres of the arytenoideus (*e*, and *f g*, No. 2) close the posterior half of the glottis. The epiglottis (*K*, No. 2) forms a valve, which can be brought over the glottis by fine muscular fibres attached at *b* and *i* (No. 2).

Fig. 15.



No. 1. No. 2.

Such, then, are the parts of the larynx which must be explained to render the phenomena of voice intelligible.

The larynx, like other organs of the body, is largely supplied with blood by the common blood-vessels. Two nerves, on each side, are devoted to the actions of the larynx. These nerves are from the eighth cerebral pair: the superior laryngeal nerve, is expended chiefly on the mucous lining of the larynx; the inferior laryngeal nerve, derived from the recurrent of the eighth pair, sends minute filaments to the several muscles concerned in the movements of the larynx.

Besides the movements of the component cartilages of the larynx on each other, attention must be paid to the motion of the whole larynx upwards and downwards. This motion takes place constantly in the act of deglutition, but also, on many occasions, when the voice is exercised, particularly in singing. When the whole larynx is raised, the windpipe is drawn proportionately upwards from the chest, and so put on the stretch. This movement has unquestionably some effect in extending the compass of the voice. It was before stated,* that the hyoid bone is connected to the protecting cartilage of the larynx, and that when this bone is drawn upwards, the larynx is drawn upwards along with it. The hyoid bone is drawn upwards by muscles attached in particular to the lower jaw, and also to the temporal bone of the skull. The hyoid bone is drawn downwards by muscles attached to the superior part of the breast-bone and to the shoulder-blade. From the upper part of the breast-bone a pair of muscles ascends, to be attached to the thyroid cartilage of the larynx, and a pair of muscles also extends between the thyroid cartilage and the hyoid bone. Thus, ample provision is made for the movement of the whole larynx in concert with the movements of the hyoid bone.

A few words must next be devoted to the cavities through which the air passes outwards, after issuing from the larynx. Behind the larynx, is the cavity of the pharynx, situated in front of the cervical vertebræ, and ascending to the inferior portion of the base of the skull. This cavity, not improperly termed the posterior cavity of the mouth, communicates with the nostril above, and on either side, by a narrow canal, called the "Eustachian tube," with the cavity of the drum of the ear. This posterior cavity of the mouth, is divided from the anterior cavity by the veil of the palate, a musculo-membranous movable curtain, which, by its motions, more or less completely divides the anterior from the posterior cavity of the mouth. The movable, tongue-like valve, before spoken of, termed by anatomists the epiglottis (see Fig. 15, *K*), overhangs the orifice of the larynx; the arches of the palate descend on either side, and are possessed of a muscular character. From the union of these above, the uvula hangs down. The tongue, free and movable in its anterior part, forms the floor of the whole passage between the root of the epiglottis and the incisor teeth of the lower jaw. The muscular mass forming the cheeks, contracts the cavity of the mouth on the sides; and the lips, by their mobility, variously modify the aperture by which the air issues.

Thus the air issuing from the larynx, may pass out either by the nostrils or the mouth. It passes out by the nostrils when the mouth is closed, or even when the veil of the palate descends. When the veil of the palate is raised, and the mouth is opened, a free passage is afforded, through what has been called the oral canal, outwards. The oral canal is manifestly capable of much greater modification, as to size, than the passage of the nostrils.

> "The tongue, the lips, articulate; the throat,
> With soft vibration, modulates the note."—DARWIN.

On the Human Voice.—In the investigation of the cause of the human voice, two points in particular deserve attention: first, the precise seat of the sounds; and secondly, the mode in which these sounds are produced.

As to the first question, it is now determined, beyond all doubt, that the sound of the voice is generated in the glottis, and neither above nor below that point. Before going further, it should be remarked, that this word glottis has not always been used in exactly the same sense. "By turns," says the eminent French physiologist, Adelon, "the superior aperture of the larynx, its inferior aperture, and the intermediate space between these two apertures, have borne the name of glottis; but, according to the etymology of the word, derived from γλωσσα—the tongue, the speech—no other part of the larynx should be called by that name but that where the vocal sound is formed; and we shall see that that part is the inferior aperture or chink."† In this sense alone, then, the word glottis is here employed—namely, to signify the aperture between the two vocal ligaments; that is, between the two inferior vocal cords, as they are sometimes called.

Among the proofs that this chink, or glottis, is the seat of voice, it may be mentioned, that if an aperture exist in the windpipe, the sound of the voice ceases. Such an aperture is frequently formed in man as a surgical operation, and an opening has often been made in the same situation in animals, for the purpose of experiment. When an opening exists above the glottis, the voice is not lost. Again, though the epiglottis, the superior vocal ligaments of the larynx, and the upper part of the arytenoid cartilages, be injured, the voice is not lost: moreover, in living animals, when the glottis is laid bare, it is seen that the inferior ligaments of the larynx which form the boundaries of the fissure termed the glottis, are thrown into vibration. It is known, too, that the division of the laryngeal nerves supplying the muscles, which regulate the states of the aperture, and make the vocal cords tense, destroys the power of producing vocal sounds. It is also found, that sounds can be produced in the dead human body, by forcing a current of air from the windpipe through the larynx, provided the vocal cords be in some degree tense, and the glottis narrow. The larynx has been cut from the body, and freed from all the parts in front of the glottis; thus, the epiglottis, the upper vocal ligaments, and the ventricles of the larynx between the superior and inferior, or vocal ligaments, the greater part of the arytenoid cartilages—namely, their upper part—may be removed; in short, if nothing remain but the inferior ligaments, or vocal cords, and these be so approximated that the glottis shall be narrow, clear tones will be produced by forcing air through it from the windpipe.

Such facts as these entitle us to regard the glottis and the vocal cords, which form its immediate boundaries, as the essential source of voice; while the windpipe simply conveys air; and the cavities above the glottis, comprehending the upper part of the larynx and the air passages through the mouth and nostrils, correspond to the tube of a musical instrument, by which the sound is modified, but not generated.

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* *See ante*, p. 280.

† *Physiologie de L'Homme*, ii., 256.

It has been already remarked,[*] that the vocal ligaments are composed of elastic tissue; and it is owing to this elasticity that they are adapted to the office which they perform. While, then, it is quite certain that no proper vocal sounds can be produced, except in the glottis, it seems manifest that the adjacent and somewhat abundant tissue of the same kind, is susceptible of a vibration and resonance in unison, so as at least to modify the sounds of the voice.

In reference to the second question—what is the nature of the change produced in the glottis during the formation of voice?—no inconsiderable difficulty is met with. The points of debate which have arisen on this subject are, whether the vocal ligaments be a set of membranous cords obeying the laws of musical strings; if the aperture of the glottis be a reeded instrument, in which the vocal ligaments play the part of vibrating tongues; or even whether the real source of the sounds of the voice be not a molecular vibration of the air, produced by its passage through the narrow aperture of the glottis; and, lastly, whether the organ of the voice does not, in part, combine all these three sources of sound, so as to be at once, in some respects, a stringed instrument, a tongued instrument, and a simple wind instrument.

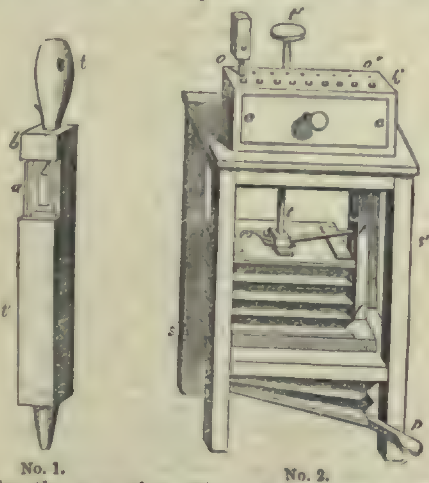
The ancients regarded the sounds of the voice as analogous to those of a flute. According to this view, the vibrations of the larynx are of little account, the actual sounds being produced by a molecular undulation of the air. That the organ of voice is, in some degree, analogous to this kind of musical instrument, is not to be absolutely denied; but it is certain that this is not the principal mode in which the sounds are produced.

One of the earliest ideas of modern times on the subject of the voice is, that the larynx is analogous to a horn; that is to say, to a wind instrument, in which the vocal cords act the same part as the lips of the performer on a horn. Not much more than a hundred years ago arose the idea that the larynx is a set of musical cords—namely, that the vibrations of these cords, on the same principle as a stringed instrument, produce the sound, which was then conveyed outwards by the air.

The prevailing opinion of the present day is, that the larynx is a wind instrument, but a reeded one.

This common view may be expressed as follows:—The expired air is thrown into the larynx through the windpipe, by the muscular action of the chest; and the proper muscles of the larynx being contracted, create a sufficient tension of the vocal cords to permit them to be thrown into vibration by the impulse of the air. The sound so produced is conveyed through the mouth and nasal passages, undergoing various modifications in its passage outwards.

Fig. 16.



No. 1. No. 2.

Let us consider, then, in the first place, what evidence there is that the organ of the voice is a reeded instrument, with a double membranous tongue.

In short, the action of the organ of voice may be best explained, in general terms, by comparing it with the pipe of an organ. Let us suppose *t'* (No. 1, Fig. 16) to be the wind-tube, into which the air is driven from below; *b*, the stopper, in which is placed the tongue; *a* and *t*, the body-tube; and let there be a pipe, *o* (No. 2), to the wind-box, *c c*, and the air be driven from the bellows, *f f p*, through *t*. The air throws the tongue, *a* (No. 1), into a state of vibration, and passes out in undulating movements from the body-tube. Such is a general illustration of the nature of voice.

An experiment may be made which illustrates the effect of an elastic organic tissue, like that of the vocal ligaments, in producing sound on the principle of a double tongue. The extremity of a tube is closed by two bands of moist elastic tissue—for example, arterial tissue—so applied as to cover the whole end of the tube, with the exception of a slight fissure between the bands. India-rubber, or leather, may be employed for this purpose. Both these substances produce a similar effect; but it appears that the middle arterial coat, being composed of the same tissue as the vocal ligaments, and having the same physical properties, forms the best kind of artificial larynx. When this tube is blown through at the free extremity, the tongues not only vibrate readily, but produce a range of musical tones. To obtain a pure quality of tone, it is necessary that the two membranous bands should be of equal weight and breadth, and subject to equal tension, otherwise they cannot vibrate equally in equal parts of time.

If the human larynx be dissected out, and the vocal cords be stretched, they will vibrate like a piece of artificial tissue, such as india-rubber or leather, in a current of air. In conducting these experiments, the same conditions must be secured as are required in the experiment with the tube, and the two membranous laminæ, before referred to. For example, the inner edges of the glottis—that is to say, of the vocal ligaments—must be turned outwards towards each other, so that they shall be in the same plane and parallel to each other, otherwise they will not produce any sound. Hence it may be inferred, that when the tension of the vocal ligaments takes place in the living animal, they turn upon their axis, till their planes, which, in the state of relaxtion, are inclined to the axis of the vocal tube, become perpendicular to it; and as the edges of the glottis approximate, its chink is nearly or entirely closed, and they acquire the true vibrating position. The production of the most simple tones of the voice requires the associated action of a most extensive range of organs; for it is calculated that, in the ordinary modulation of the voice, more than one hundred muscles are brought into action at the same time.

As the air rushes upwards from the windpipe, a portion of each edge of the glottis yields to its pressure, and is curved upwards, so as to form an angle with the axis of the vocal tube, and leave between the two edges a narrow aperture, through which the air escapes. The tension and elasticity of the vocal ligaments tend to restore them to the plane of their former position. The air having been rarefied below the glottis during their elevation, becomes dense from their depression, and the necessary force being again accumulated, they are re-elevated, and thus an oscillating movement, consisting of an opening and closing of the glottis, takes place; which, being communicated to the contiguous air, the sounds of the voice are produced.

The vibrating edge of the glottis varies in length, according to the pressure of the column of air in the windpipe, and the resistance of the vocal ligaments. When other circumstances are alike, the intensity of the voice is determined by the pressure of the column of air in the windpipe, and the range of movement described by the vibrating edges of the glottis. The pitch of the voice does not depend solely on the tension of the vocal ligaments, but jointly on the variations which they undergo in length and tension. Magendie observed, in the larynx of a dog, that a longer portion of the vocal ligaments vibrated while grave tones were produced, and that a diminution of length accompanied the succession of acute tones. Mayo has described the movements of the glottis in a man who had attempted to destroy himself by cutting his throat. The larynx in this case was cut through just above the vocal cords; and, owing to the oblique direction of the wound, an injury of the arytenoid cartilage, and of the vocal cord on one side, had occurred. When respiration was going on, the glottis was seen to be of a triangular form; but when the voice was exerted, the vocal cords passed into a parallel direction, and the glottis itself had a linear form. The posterior part of the aperture appeared to remain unclosed.

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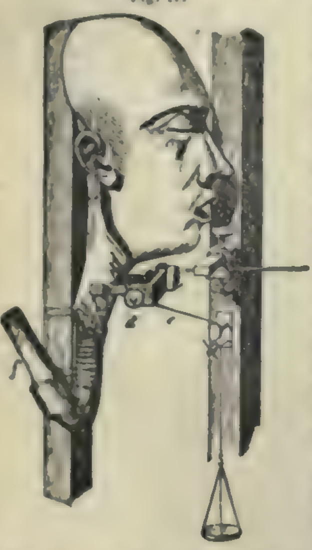
[*] See *ante*, p. 280.

Fig. 17.



The cut (Fig. 17) represents the prepared head of a corpse, after Müller. A thread *e*, which passes over a roller to a scale, is so applied to the larynx that the tension of the vocal cords can be increased by placing a greater weight on the scale. The action of the muscles is thereby imitated. The compressing apparatus seen on the wood-cut, brings the vocal cords nearer to each other, and thus produces the requisite diminution in the width of the vocal fissure. The tube *f* serves to convey the wind, which throws the tongue-apparatus into action. And thus, if we use the human head, or the head of the dog, the pig, or of any other animal, we can imitate the voice of man, the bark of the dog, the grunt of the pig, &c.

Membranous tongues, like those in the larynx, differ widely from a metal tongue, shutting up the aperture, and necessarily opening and closing as the air issues.

Objections have been taken to the view which represents the voice as the result of sounds produced by membranous tongues set in motion by air—1st, that the vibration of tongues consists in the periodical opening and shutting of the orifice through which the stream of air passes—this not being the case in the glottis; and, 2nd, that had it the structure of a reed, the edges of the vocal ligaments which open the chink would be alternately separated by the column of air in the larynx, and drawn together by their tension; while it has been found by experiment, that air transmitted through the glottis gives rise to sound, notwithstanding that its edges are from one-sixth to one-fourth of an inch asunder. In these objections, however, there is a mistake as to the essential principle of reeds; for those of the clarionet, bassoon, hautboy, &c., fail to close entirely the passages through which the breath escapes; and the case is not otherwise with the natural reed, which the lips of players on the flute and horn represent. In short, a sound can be produced by a tongue apart from the surrounding framework, indicating, beyond doubt, that so much importance should not be ascribed to the usual mode of forming reeded and tongued instruments, and to the circumstance of the air passing between the tongue and its frame. It has been shown, that the law by which the variation in the notes yielded by the tongue of a mouthpiece or reed is regulated, is the same when the tongue is made to vibrate by a current of air, as when it is thrown into vibrations by being struck or inflected. By the same law are regulated the vibrations of vibrating rods; the frequency of the vibrations of two rods of the same texture and thickness being in the inverse ratio of the squares of their length. The note afforded by a reed without a tube, is of the same pitch, whether it be the result of a current of air, or if produced by striking the tongue. The strength of the blast does not, for the most part, determine the pitch or sharpness of the note; but when the force of the blowing is increased, the strength of the tones is augmented. The size of the fissure between the tongue and the frame within which it vibrates, is of little consequence; when the opening is large there is a greater difficulty in obtaining the tone, but its pitch is not altered.

Some slight difficulties may still exist in the explanation of the theory of the voice as considered to be chiefly the result of a double vibrating tongue; but, altogether, as close a resemblance has been proved to exist, between that kind of artificial musical arrangement and the structure of the living larynx, as can reasonably be expected in such a case.

It has been hinted, that the vocal ligaments may possibly act not only as vibrating tongues in the production of voice, but also on the principle of musical strings. On this point a few words must be added. It may seem, at first sight, that the remark of so distinguished a philosopher as Biot, when he says, "What is there in the larynx that resembles a vibrating string? Where is the space for such a string of sufficient length to yield the lower notes of the voice? How could sounds, of the compass which the human voice represents, be produced by a string which the larynx would contain?"—would suffice altogether to set aside the idea of the vocal cords acting as musical strings. But Biot here seems to have fallen into error. Deep notes are still produced by a string greatly shortened, if it retain, after a sufficient amount of relaxation, the elasticity required for vibration. His attention does not seem to have been drawn sufficiently to the nature of organic membranes; strips of india-rubber, and elastic animal membranes, still retaining enough of elasticity for this purpose, after being much relaxed. There is, therefore, a perfect agreement between the vocal cords and vibrating strings, though their vibrations, whether as strings or as tongues, are produced not by the direct impulse of a solid body, but by the momentum of air. When the ordinary principles to which musical strings are subject are applied to the vocal ligaments, there is found to be a very close agreement, if allowance be made for the peculiarities of animal substances, as respects their elasticity and the like.

In their ordinary state, the vocal cords must be regarded as subject to a considerable tension, which, however, admits of being diminished, so as to add to the range of the lower notes. At the ordinary pitch of the voice, the glottis may be regarded as partially closed, and becoming more open as graver tones are produced; this opening of the glottis coinciding with the relaxation of the vocal cords, a double cause is afforded of the lowering of tone. When higher notes are uttered, the glottis closes, assuming more of a linear form; while, at the same time, the vocal ligaments, though elongated, are thrown into a much higher state of tension. In the words, then, of Mr. Bishop, "Since the vocal ligaments have been proved to extend and contract for acute and grave sounds respectively, and after death vibrate, in a great measure, like musical strings, we think it may be fairly inferred, that they likewise obey, to a certain extent, during life, the laws of the vibrations of such strings. * * * * It is, moreover, observable, that the extension and relaxation of the vocal cord, which, as we have seen, are analogous to those of a musical string, produce a corresponding shortening and elongation of its axis, if regarded as a tongue; and, lastly, since one tone only is produced at a time, the vibrations resulting from the double action which appears to exist in the vocal apparatus must be synchronous. * * * * It might possibly be objected to the idea of this twofold action, that the production of sound by the vocal cords is sufficiently accounted for by supposing them to vibrate merely as elastic tongues; but then it is found by experiment, that by artificially dividing their length into two ventral segments, there results the octave of the fundamental note, which proves that at all events they vibrate as cords. In conclusion, we must bear in mind the vast difference between natural and artificial mechanism; and however complicated a problem it may be to determine that constitution of the vocal apparatus, by which the thyro-arytenoid ligaments may simultaneously obey the laws of cords and tongues, yet, to a physiologist who is accustomed to meet with the most admirable contrivances and combinations in the animal frame, the difficulty of finding a strictly mathematical solution is, in such a case, no objection to its truth, when the facts, as far as they have been observed, are decidedly favourable to its reality."*

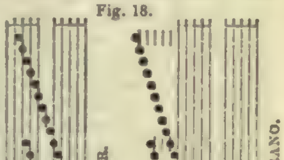
It was before hinted at, that the vibrations of the walls of the tubes through which the voice is conducted, may, in some degree, influence its sound. In rigid tubes, the vibrations depend on the nature of the impulse propagated in the air within, jointly with the length of the pipe. So long, then, as the length of the pipe remains the same, and no change takes place on the material of its walls, the pitch of the sound produced by the undulations of the air within, remains unaffected. The dimensions of the windpipe, such as its length and diameter, are invariable; and, were the height of the larynx, and the dimensions of the bifid tube (the nose and mouth), through which the air issues after the formation of voice, equally invariable, the vibrations of these parts would produce no change on the pitch of the voice, the quantities being constant for each tone produced in the glottis. It has been found, that by taking tubes composed of layers of paper, of constant length, but varied in thickness, graver sounds were produced as the parietes became thinner, and that the gravity of the sound was increased by moistening and relaxing the sides of the tubes. It was before noticed, that the windpipe is capable of being drawn upwards from the chest, to a small extent, while the larynx is elevated; and that this tube admits of being diminished in its diameter by about one-third part. Moreover, the pharynx, the mouth, and the nasal cavities are also susceptible of various modifications of diameter; so that the pipe, so to speak, near the middle of which the vocal sound is produced, is in a very different condition from a rigid tube. Hence, it has been concluded that provision is made for an invariable adaptation between the amount of tension, the vibrating length of the vocal ligaments, and the walls of the vocal tube, for the production of the ordinary tones of the voice. It appears, indeed, to have been proved that the vocal tube is so short, that were it rigid, it could not influence the pitch of the note which the glottis originates. But its want of length is compensated for by the relaxation of its walls, so that it comes to vibrate synchronously, and so to give forth sounds equally grave with those of the glottis. Its effect, therefore, is to add to the force of the tone, which, without its aid, would have been found to possess less intensity.

After considering this subject in every possible light, the conclusion appears to be that to which Mr. Bishop has come; namely, that the evidence shows "the vocal apparatus to be influenced by the air expelled from the chest, in precisely the same way as if it were a stretched cord, a reed, or a vibrating tube. Why, then," he continues, "should we hesitate to adopt the obvious conclusion, that the vocal organs do, in fact, combine the properties of these various instruments, and are thus the perfect types of which these instruments are only imperfect imitations?"

Singing.—The notes of the human voice are capable of being produced in three separate kinds of sequence. In ordinary speaking, the successive notes have nearly all the same pitch. This kind of succession, then, is properly termed the monotonous. Some deviation from this monotony occasionally arises, as when certain syllables receive a higher intonation for the sake of accent, and when, in reading or reciting poetry, rhythm is added to the accent. In these cases, however, the deviation from monotony of pitch is too slight to require a separate head. In the expression of passion, accompanied by vehement exercise of the voice, there is heard a sudden transition from high to low notes, or the reverse. This, then, constitutes the second kind of sequence in the notes of the human voice. Musical notes constitute the third mode of sequence. In music, the sound has the requisite number of vibrations; and as the sounds succeed each other, they exhibit that relative proportion in the number of vibrations which jointly characterise the notes of the musical scale. Of the adaptation of one sound to succeed another, so as to preserve the musical character of the succession, the human ear is the only original standard.

Compass of the Voice.—In singers, the compass of the voice extends through two or three octaves. When the male and female voices are taken together, the entire scale of the human voice includes four octaves. The lowest note of the female voice is about an octave higher than the lowest of the male voice; the highest of the female voice is about an octave higher than the highest of the male. The first four notes of all voices are most commonly weak. There are two kinds of male voice, the bass and tenor; and two kinds of female voice, the contralto and soprano. The essential distinction between these voices does not consist in their difference of pitch. The bass voice commonly reaches lower than the tenor, and its strength lies in the low notes; while the tenor voice extends higher than the bass. The contralto voice has most commonly lower notes than the soprano, and is strongest in the lower notes of the female voice; while the soprano voice reaches higher in the scale. It is found, however, that bass singers can sometimes go very high, and the contralto not unfrequently sings the high notes like soprano singers. The difference between the bass and tenor voice, and between the contralto and soprano, is plainly, then, not one of pitch, but consists in the peculiar timbre or quality of the notes; for these several voices are distinguished from each other, even when sounding the same note. The qualities of the baritone and mezzo-soprano voices are less marked; the baritone being intermediate between the bass and tenor, the mezzo-soprano between the alto and soprano.

Fig. 18.



Scales of Notes of the different Voices, showing the range of each.

The difference of pitch between the male and female voice, is connected with the different length of the vocal ligaments in the two sexes. It appears that the lengths of the male and female vocal cords in repose, are nearly as 7 to 5, and in tension as 3 to 2; in boys at the age of fourteen, the length is to that of females, after puberty, as 6·25 to 7—so that the pitch of the voice is nearly the same. The difference in the quality of the female voice, as compared with that of the male, is owing to the considerable difference presented by the two sexes in the walls of the larynx; the male larynx being much more expanded, and forming a much more acute angle in front. It is not yet clearly understood what is the cause of the different qualities of voice, as exhibited in the tenor and bass, and the contralto and soprano. As Müller remarks—"We may form an idea of the cause of these differences of timbre, from recollecting that musical instruments, made of different materials, as metallic and gut-strings, metallic, wooden, and membranous tongues, metallic, wooden, and paper pipes, or flutes, may be tuned to the same note, but that each will give it with a peculiar quality or timbre."

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* *Cyclopædia of Physiology:* article "Voice;" p. 1481.

In short, when the variations of the larynx in different individuals of both sexes, and at different ages, under the various circumstances more or less favourable to the development of the respiratory organs, are considered, as well as the remarkable fact that every human being is characterised by a speaking voice peculiar to himself, we shall be at no loss to understand why the singing voice should vary in different persons, not only in pitch, but also in quality.

The voice termed falsetto has much engaged the attention of physiologists. Most singers, particularly males, besides their natural voice falling under one or other of the before-mentioned characters, have the power of producing a double series of notes, of a different description. To the second series of notes the name of falsetto is applied. The notes of the natural voice—called also chest-notes—are fuller, and distinctly indicate a stronger vibration and resonance; while the falsetto voice has more of a humming character. It is only with the natural voice that the deep notes can be produced; while the highest notes of a male voice are falsetto. The notes accompanying a middle pitch may belong either to the natural or the falsetto voice. Thus the two registers, as they are termed, of the voice are not bounded in such a manner that the one ends where the other begins, as, through a certain compass, they run side by side. It is remarked that the bass voice becomes falsetto lower in the scale than the tenor. In the female voice there is less seldom presented a very marked distinction between the natural and falsetto registers.

In a human larynx, detached from the body, two distinct series of tones can be produced, when the tension of the vocal cords is very slight. One of these series corresponds to the tones of the ordinary voice, the other to the tones of the falsetto voice. With a certain degree of tension of the vocal cords, both these kinds of tones may be produced—sometimes the one kind, sometimes the other, being heard. With a different kind of tension of the cords, notes of the falsetto character are constantly produced, whether the current of air passing through the glottis be forcible or feeble. If the vocal ligaments be much relaxed, the sounds of the ordinary voice always result, whether the current be feeble or forcible. When a slight tension of the ligaments is kept up, the falsetto is most easily produced by blowing very gently; while, if the blowing be more energetic, the sound belongs to the ordinary voice. Thus, two different notes may be produced, under the same degree of tension of the ligaments, by a different force in the blowing; and the distance of these two notes from each other may be as much as an octave. "The real cause," says Müller, "of the difference between the falsetto and the notes of the natural voice is, that, for the former, the thin aperture only of the lips of the glottis vibrates; while, for the latter, the whole breadth of the cords is thrown into strong vibrations, which traverse a larger sphere." The peculiarities of the voice in different individuals, must be chiefly dependent on the particular form of their air-passages and of the lining membranes, and the consequent differences in their mode of resonance. That such causes are adequate to produce all the varieties of the voice in individuals, appears from the circumstance, that many persons, by altering the form of their vocal organs, can imitate the various tones of the voices of other individuals.

The nasal quality of the voice is determined by like causes. This nasal tone appears to be given to the voice in two ways: thus a nasal sound is produced, though the external openings of the nostrils be closed when the arches of the palate approach each other, and the larynx ascends higher than in the natural voice. When the nostrils are obstructed by mucus, a nasal sound is produced; this obstruction having the same effect as the voluntary closure of the anterior opening of the nostrils. In the second mode by which the nasal sound is produced, the nostrils are open, the larynx ascends considerably, the arches of the palate contract, the upper surface of the tongue ascends towards the palate, so that the air passes between the narrowed arches of the palate, and receives the resonance of the nasal cavities without that of the cavity of the mouth. The deficiency of tone in the voice of old people, arises from the ossification of the cartilages of the larynx, and the altered state of the vocal cords. It is unsteady, owing to the loss of nervous command over the muscles.

The strength of the voice depends partly on the extent to which the vocal cords are capable of vibration, and partly on the great capacity of the chest, and the fitness of the various parts, over which the air passes, for communicating resonance. The intensity or loudness of a given note cannot be rendered greater by the mere augmentation of the force of the current through the glottis. Such an increase of force in the current, will raise the pitch both of the natural and falsetto notes. It is therefore concluded that the variation in the intensity of a note, without the alteration of its pitch, must depend on some other cause than the mere change in the force of the current. Such a provision plainly lies in the power of modifying the tension of the vocal cords. To render a note more intense, without increasing its pitch, the vocal cords must be relaxed in proportion as the force of the current of the breath through the glottis has increased. When it is desired to render a note fainter, an opposite mode of action must be adopted.

The failure of perfectness in the notes of the human voice may arise from many causes. Variations in the temperature of the atmosphere, and in its states of humidity, have a powerful influence on the pitch of the voice. During a cold, moist state of the atmosphere prevalent in these islands, the voices of singers become lower by two or three notes; while they regain their usual pitch when the air becomes dry. Mr. Bishop mentions, that when Grassini came to this country, owing to the change of the air from that of Italy, her voice became one octave lower. After singing for two or three seasons, her natural voice returned, but it had lost its attractions with the loss of the low tones which had gained her so great applause. After long singing, dissonance of the voice is apt to arise; this is easily accounted for by the slight changes produced on the vocal cords in consequence of repeated tension, together with the fatigue of the muscles concerned, which, as in other cases of muscular contraction, at length cease accurately to obey the will, and hence afford unsteady movements.

Whistling.—Before leaving the subject of the human voice, whistling deserves a few words. The sound in whistling does not arise from the vibrations of the lips. Several experiments prove that the lips are not thrown into vibrations. They may be touched, covered, or may have a disc of cork with a central hole placed between them, and yet the same sounds will be produced. It has been supposed, then, that the air is thrown into sonorous vibration by friction against the borders of the opening. According to Müller, the cause of the vibration is the same friction of the air; but the vibration produced upon the borders of the opening, throws the whole column of air in the mouth into vibrations, and the vibrations of this column of air, by a reciprocal influence, determine the rapidity of the vibrations of the air at the orifice. The only difference, according to him, between whistling and the sounds of a pipe is, that in whistling, the whole column of air is in constant progressive motion through the tube and orifice; while, in a pipe, the air in the tube merely vibrates, and does not move as a current.

Speech.—Speech is peculiar to man. Because speech is not possessed by individuals deprived of the organs of voice or hearing, it is not, therefore, to be concluded that it originates in the mere possession of these organs. Inferior animals are fully provided with the organs both of hearing and of voice, and yet, in all essential respects, they are destitute of speech. Speech, therefore, must be considered under the light of a potentiality of man's intelligence, the condition of the exercise of which is the presence of the organs of hearing and of voice. That is to say, man is born susceptible, by the development of his nervous system, of the acquisition of speech, provided his organs of hearing and voice are perfect. But if man be born susceptible of speech, it may be asked, why does not the deaf-mute invent a language? He does invent a language, but it is a language of expression independently of speech; he fails to express his inward feelings by the use of speech, because the defect of hearing prevents him from discovering the sounds which his voice is capable of producing. His language, therefore, is confined to the other modes of expression by which an intercommunication, however imperfect, can be carried on between men. The deaf-mute might undoubtedly carry the use of natural signs of expression much further, were he not overwhelmed and overpowered by the multitude of ideas which his fellow-men around him possess, and are continually striving to make him understand. It may also be asked if, owing to this natural susceptibility of speech, every infant should not invent a language? Indeed, every infant does invent a language; but as, long before any progress is made in its language, the sounds which it continually hears are caught up, it is impossible to judge to what extent each individual is capable of carrying such an invention.

There is no more interesting speculation than to consider the several steps by which language must have arisen among men. It is easy to understand how, in the rudest community of mankind, conventional signs must have arisen of every description; nor is it difficult to perceive, that those sounds of speech which are most easily produced, would quickly form a large share of those conventional signs. But it forms no part of our present design to investigate the origin of languages; it is more to the purpose to consider, in a few words, how men came to understand the several acts concerned in speech. At a certain period, then, in the progress of mankind, it appears that languages of no inconsiderable extent had already been formed, and yet that no attention could have been paid to the individual sounds composing those languages. Men spoke, and in that speech employed words without the least reference to letters, and, perhaps, with none even to syllables. The curious inquiry, then, which arises is, in the first place, how men were led to reduce speech to letters; that is, to analyse words into their elementary sounds.

We may suppose that the difficulty of pronouncing certain sounds, such as the words of a foreign language, must have been the first circumstance which would lead men to reflect on the modes by which speech is produced. Man's natural curiosity would not fail to engage him more largely in this inquiry as soon as the subject was suggested. Little progress, however, could be made in this pursuit till some method of fixing the sounds by name, and of representing them to one's self, or to others, at periods more or less distant from their first recognition, was invented. It may be supposed that men had already acquired the art of depicting objects of sight, were it no more than rude representations made with a rod on the sands left by a receding sea. The idea, however, of representing a sound by such a symbol is plainly not of the same kind. To think of representing a sound by a symbol is manifestly a fresh step in discovery. It required, in short, an effort of invention to produce such a stretch of thought. But the moment the idea arose, all difficulty must have vanished. Nothing was easier than to observe the similar simple sounds, occurring in the compound sounds, which constitute speech. The mere observation of the form of the mouth, as certain simple sounds are uttered, would be sufficient to afford a foundation for this kind of knowledge. What the original symbols, corresponding to our modern alphabets were, is of little moment. The first alphabets, doubtless, consisted of the representatives of but a small number of sounds. It is easy, however, to perceive, that as soon as this kind of investigation was fairly commenced, it would make rapid progress, there being no great difficulty in discovering the collocation of the several parts of the mouth concerned in the production of most of the simple sounds. Thus, by an easy analysis, syllable-sounds would be reduced to letter-sounds, and each letter would quickly come to be marked by a particular symbol. The most remarkable effect of this great discovery, simple as it seems to us, would unquestionably be the rapid multiplication ot sound-symbols—that is to say, the vast extension of language. The greatness of the discovery hardly strikes us at the first sight. Some idea of the character of it is obtained from the fable of words spoken becoming frozen at the moment in their fixed forms, and not reaching the ear until the return of a more genial temperature. Letters, in short, are the pictures of sounds, by which any sound now pronounced is perpetuated, while the picture itself, or a copy of it, shall endure.

Speech, then, consists of combinations of sounds produced in the larynx, and variously modified in their transition through the oral or nasal passages outwards. No language exhausts all the sounds which can be produced in the passage of the voice outwards in this manner. Languages may be described as composed of those sounds which are most easily produced in the passage of the voice from the larynx outwards into the atmosphere. And languages differ from each other, chiefly, by presenting various predominant groups of such sounds. The chief distinction of the sounds of speech depends on the way they are transmitted through the oral canal before spoken of, or the nasal passage. Another important distinction between articulate sounds is, that some are only of momentary duration, taking place during a sudden change in the conformation of the mouth, and are not capable of prolongation by a continued effusion of the breath; while others can be prolonged as long as a particular disposition of the mouth and a constant expiration are maintained.

The same sound produced in the larynx is converted into any one of the vowel-sounds merely by a modification of the parts of the mouth through which it passes. The parts of the mouth concerned, have been termed the oral canal, and the oral opening. The oral canal, it is to be remembered, is the space between the tongue and the palate; the oral opening is the aperture formed by the lips. Some physiologists have described five degrees of size in each of these two parts—that is, five degrees of size in the oral canal, and five degrees of size in the oral opening. One sound, then, produced in the larynx is converted into *a*, *e*, *i*, *o*, *u*, according to the modifications in the size of these two parts. Thus, when the size of the oral canal is in the third degree, and the size of the oral opening is in the fifth or highest degree, the act of voice is converted into the sound of the English *a* in *far*. When the size of the oral canal is in the second degree, and that of the oral opening in the fourth degree, the sound of the English *a* in *name* is produced. When the size of the oral canal is in the first or lowest degree, and that of the oral opening in the third degree, the sound of the English *e* in *theme* is produced. When the size of the oral canal is in the fourth degree, and that of the oral opening in the second degree, the sound of the English *o* is produced. When the size of the oral canal is in the fifth or highest degree, and that of the oral opening in the first or lowest degree, the sound of *u* like *oo* in *cool* is produced. Of the general truth of this statement, any person may satisfy himself by remarking, when he utters the broad *a*, how much he opens his mouth, simply breathing forth the voice with open mouth. When, on the contrary, he, with the same breath, attempts to pronounce *e*, he finds the mouth close considerably, and the tongue rise towards the roof of the mouth, so as to contract the oral canal. In pronouncing *o*, he will observe how the lips are thrown into the form of the letter, the tongue at the same time raised from the bottom of the mouth. The form of the vowel o, in most languages, points to one source of origin of those representations of sounds which we call alphabets.

Some consonants, like vowels, can be pronounced with an uninterrupted sound, which continues as long as the expiration can be prolonged, the disposition of the parts within the mouth remaining throughout as at the commencement of the sound. Of these, the aspirate *h* is pronounced with the whole oral canal open; no interruption is offered to the passage of the breath; its sound is the simple result of the resonance of the walls of the cavity during expiration. Others of the same class, such as *m*, *n*, and *ng*, are produced by continuous expiration through the nasal canal, the aperture of the mouth being closed either by the lips or by the tongue being pressed against the palate. The mouth is closed by the lips while *m* is pronounced, the sound being produced by the simple passage of the air through the nasal cavity. When *n* is pronounced, the mouth is closed by the extremity of the tongue being pressed against the fore part of the palate. *Ng* is regarded as a simple sound in the words *sing* and *bang*. It is produced also by the passage of sound through the nostril, while the posterior part of the tongue is pressed against the palate. Other consonants, again, of the same class, are continuous sounds developed by the valve-like application of different parts of the mouth to each other, such as *f*, *s*, *r*, *l*. *F* is pronounced by the application of the lower lip to the teeth. In pronouncing *s*, the teeth are brought into contact with each other, while the point of the tongue touches the lower teeth; in the sound of *r*, the tongue vibrates against the palate; in the sound of *l*, the point of the tongue is applied close to the palate, and the air escapes between the tongue and the cheeks. The English *th* is a modification of *s*.

The mute consonants, with explosive sounds, come next to be spoken of. The organs of speech, by which these sounds are formed, undergo a sudden change of position during their production. The sound commences with the closing of the mouth, and terminates when it opens—that is to say, these consonants cannot be prolonged at pleasure; *b*, *g*, *d*, of which *p*, *k*, *t* are modifications, coming under this head. In sounding *b*, the lips are brought together, and close the mouth, while they separate again at the moment the air is expired. In sounding *d*, the tongue is applied to the anterior part of the palate, or to the upper teeth, so as to close the mouth, which opens with the escape of the breath. In sounding *g*—that is, the hard *g*, as in *gold*—the momentary closure of the passage through the mounh takes place, more posteriorly, by the application of the back part of the tongue to the palate. In sounding *p*, *t*, and *k*, the requisite modifications of *b*, *d*, and *g* are produced, by a stronger aspiration during the opening of the mouth, which was previously closed. All the sounds hitherto mentioned are capable of being pronounced in whispered speech. The English *y* and *z* cannot be uttered without an accompanying vocal sound. Thus, when an attempt is made to sound the English *y* in a whisper, the German *ch* is produced in its stead. All the vowels are capable of being produced equally in whispered speech, and with a vocal tone. Many consonants also, as *f*, *s*, *r*, *l*, *m*, *n*, *ng*, can be pronounced either with mute sounds or with vocal intonations. The continuous consonant *h* can only be pronounced in whispered voice; and it is quite impossible to combine the sounds of the explosive consonants, *b*, *d*, *g*, and their modifications, *p*, *t*, *k*, with an intonation of the voice.

Besides the ordinary sounds of consonants which enter into the formation of languages, other sounds are capable of being produced in the mouth and throat. The smacking sounds produced by the separation of the teeth from the tongue or palate, are reported by travellers to occur in the language of the Hottentots, and in those of other African tribes.

The several sounds and tones of language can even be imitated by artificial contrivances. When the sound of the voice is made to pass into a cylindrical tube, before which the hand is held, and then withdrawn, the sound of *b* is produced; and if the tube be a pipe with a membranous tongue, the sound of *v* is produced. Various speaking machines, by attention to such principles, have been produced: the most perfect of these is that contrived by Faber. The automaton, invented by Faber, has a singing voice, extending over twelve notes. The difference in the height of the notes is made by varying the width of the glottis without tension of the cords. In this respect, it is hardly an exact model of the organ of the human voice.

The singular faculty possessed by ventriloquists has engaged much of the attention of physiologists. Many different views as to the nature of this kind of speech have been at various times brought forward.

One of the oldest and most common ideas on this subject is, that ventriloquism consists in speech produced during inspiration. It is unquestionably possible, though not without difficulty, to articulate during inspiration; and the sounds so produced have some resemblance to the tones uttered by a ventriloquist.

A more recent view of the nature of ventriloquism is, that it is a mere imitation, produced in the larynx, of the various modifications which the voice ordinarily suffers from distance, by the interposition of a partition, as if the individual were enclosed in a narrow space—in a trunk, a cask, or the like. This view has been supported with much ingenuity by Magendie.

The distinguished German physiologist, Muller, has adopted an idea on this subject which coincides better with the original name of this artifice. He says that the notes of ventriloquism are produced by inspiring very deeply, so as to protrude the abdominal contents by the deep descent of the diaphragm; and the diaphragm being retained in this position, by speaking through a very narrow glottis, expiration is performed very slowly by the lateral walls of the chest alone. He affirms, that the quality which the voice has, in speaking through an expiration thus performed, is that peculiar to ventriloquism, and that sounds may be thus uttered which resemble the voice of a person calling from a distance.

A very large share of the artifice practised by the ventriloquist, particularly in the imitation of voices coming from particular directions, lies in the deception of other senses besides the hearing. The directions in which sounds reach the ear are never very easily distinguished; and when the attention is drawn to a different point, the imagination is very apt to regard the sounds produced as coming from that quarter.

Of the imperfections of speech, stammering is that which has been chiefly investigated; and it lies in a momentary inability to pronounce a consonant or vowel, or to connect it with the preceding sounds. This impediment may occur either in the commencement or in the middle of a word. When the impediment arises in the middle of a word, the commencement of the word is often several times repeated. Hence stammering is apt to be defined as the successive repetition of one sound. The repetition of the commencement of the word, however, is not the essential defect which constitutes stammering; it merely marks repeated attempts to overcome the difficulty. Neither is it correct to say that the difficulty in stammering lies chiefly in pronouncing the consonants, for the impediment most frequently occurs in the case of vowels. The best account which has been given of the nature of stammering is, that it depends on the momentary closure of the glottis, so that the passage of the air necessary to the pronunciation of the particular sound is arrested. In persons severely affected with this impediment, there are manifest indications of the struggle at the glottis, occasioned by the impediment to expiration, in congestion of blood in the head and in the veins of the neck. The real cause of stammering, therefore, must be described as an unusual movement in the larynx, associated with the articulate movements. In short, stammering is a temporary spasmodic affection of the glottis. For the prevention of stammering, the proper plan is to endeavour to bring the associated movements of the larynx with the organs of speech more under the command of the will. To sing words is one method of obtaining this effect; since in singing, more attention is directed to the action of the larynx than in ordinary speaking. Moreover, it is observed, that persons who stammer, pronounce better in singing than in mere speaking. The raising of the point of the tongue towards the palate has some effect in counteracting this habit; and this elevation of the tongue seems to have been the object of the plan practised by the ancients, of placing bodies, such as pebbles, under the tongue. Müller recommends, for the cure of stammering, that the patient should practise himself in reading sentences in which all the letters which cannot be pronounced without a vowel sound—namely, the explosive consonants, *b*, *d*, *g*, *p*, *t*, and *k*—are omitted, and only those consonants included which are susceptible of an accompanying intonation of the voice. He also directs that all those letters should be very much prolonged. He says that, by this means, a mode of pronunciation will be attained, in which the articulation is constantly combined with vocalization, and the glottis, consequently, never closed.

As already mentioned, dumbness is dependent, not on the defect of the organs of speech, but on the absence of hearing. By assiduous efforts, deaf-mutes learn the movements of articulation by means of their sight. The speech which they acquire is most commonly harsh, owing to the want of the sense of hearing to regulate their articulation. There was no discovery hailed with greater interest than that of teaching the dumb to speak; and undoubtedly, harsh though the sounds be—and yet they are not always disagreeably harsh—there can hardly be a greater triumph of human art. It will hardly be believed that some innovators on the education of the deaf and dumb, seek to abolish the practice of teaching them to articulate, on the ground that their harsh speech is unfitted for the uses of society, and that they can communicate with their fellows sufficiently by other means, as by speaking on the fingers and by writing. This is a most unwarrantable view of the case of these unhappy persons, particularly when they belong, as by far the major part of them must do, to the labouring classes of society. We have only to consider how many persons there are in the condition of a labourer who cannot write, or read writing, to be satisfied that this innovation on the education of the deaf and dumb, should be at once put down in every institution in which it has gained a footing. There is every reason to believe, that in proportion as a knowledge of the mode in which the sounds of the human voice in speech are produced, becomes better understood, the artificial articulation of the deaf and dumb will become gradually less harsh and disagreeable.

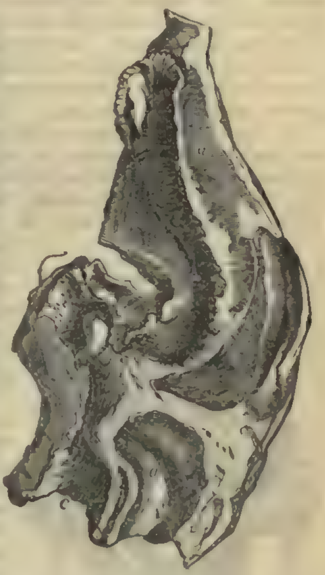
Comparative Physiology of Voice.—Organs of voice occur among inferior animals, in the mammalian tribes, birds, and reptiles. In mammals, the organs of voice bear a close resemblance to those of man. In birds, considerable modifications occur in these organs. In reptiles, the apparatus of voice is of greater simplicity.

Voice of Mammals.—Among mammals, some are mute; and yet these are not always deficient in those parts of the larynx which are most essential to voice.

Among the orders which compose the class mammalia, the cetaceans, consisting chiefly of the whale tribe, are often described as mute. These animals, however, are not mute altogether, but possess only a single lowing note, or, at the utmost, they have the power of simply bellowing. There are two distinct sections of cetaceans. The first includes what have been termed the herbivorous cetaceans, such as the sea-cow, *the representative of the popular mermaid*, and the dugong. The second order includes the common cetaceans, popularly known as blowers. The act of blowing, from which they derive their name, consists in the expulsion of water by the nostrils; that is, along with their prey they receive a large quantity of water into the mouth, and, while the mouth remains closed, they blow out this superfluous water by a hole in the upper part of the head.

This expulsion of water is produced by means of a peculiar arrangement of the veil of the palate. The water accumulates in a sac situated at the external orifice of the cavity of the nose, whence, by the compression of powerful muscles, it is violently expelled through a narrow aperture pierced on the summit of the head. By this contrivance these animals throw forth those jets of water which are seen by mariners at a great distance. The larynx has a pyramidal form (see Fig. 19), and penetrates into the posterior portion of the nostrils to receive air, and conducts it to the lungs, without the animal being obliged to raise its head and mouth above water for the purpose. As there are no projecting laminæ in the glottis, they can hardly be said to have the proper organs of voice; and thus the noise they make may be described as a simple vehemence of expiration.

Fig. 19.



SECTION OF TONGUE, PHARYNX, AND LARYNX OF PORPOISE.—*Museum of College of Surgeons of London.*

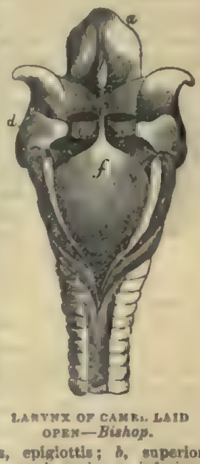
a, pyramidal position of larynx; *c*, pharynx; *d*, laryngeal cavities laid open.

The larynx, however, in these animals is highly developed in other respects.

Among the animals commonly described as mute, is the giraffe or cameleopard, termed by naturalists *Cameleo-pardalis giraffa.* In the giraffe the vocal ligaments appear to be absent.

The armadillo (*Dasypus*) is another of the mammalians described as mute. The only peculiarity of the larynx which has been observed is, that the epiglottis, or valve-like cartilage of the larynx, is bi-lobed. The armadillo, it will be remembered, is remarkable among mammals for the scaly, hard, bony shell, composed of pavement-like compartments, which cover the head, the body, and even the tail. These animals belong to the order termed Edentata. They live in burrows, which they excavate. To the Edentata also belong the ant-eaters (*Myrmecophagæ*), which are regarded as mute. In the same order is found the sloth (*Bradypus tridactylus*). In this animal, however, vocal ligaments are found, and the windpipe is convoluted. The voice is a plaintive melody, consisting of an ascending and descending scale of the hexachord.

Fig. 20.



LARYNX OF CAMEL LAID OPEN.—*Bishop.*

a, epiglottis; *b*, superior vocal cords; *c*, inferior; *d*, arytenoid cartilages; *e*, vertical ridge; *h*, tubercle; *f*, trachea.

Among the Rodentia, or gnawers, the common porcupine of Europe is mute. In this animal it has been ascertained that there are no vocal ligaments.

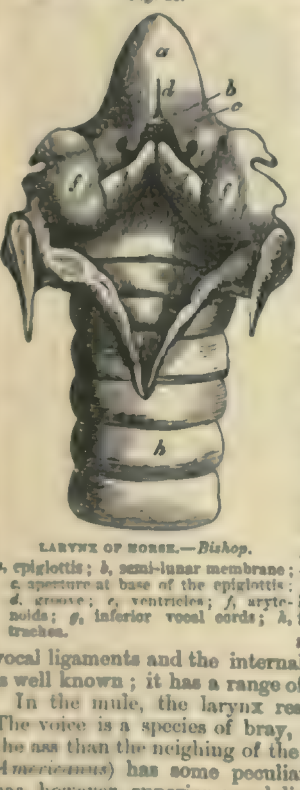
Such, then, are a few examples of the animals in the class Mammalia which are mute, or nearly so.

In the order Ruminantia, we find animals possessed of a sonorous voice, exemplified particularly in the ox. In the ox, the larynx is well developed; there are no superior vocal ligaments, but the inferior or true vocal ligaments are strong, and nearly an inch in length; the windpipe consists of fifty-two cartilaginous rings; that is, nearly three times as many as their number in man. The voice is sonorous, intense—pitched in C=256 vibrations in a second.*

The sheep belongs to the same order of quadrupeds. The larynx differs from that of the ox only in dimensions. The voice is guttural—pitched in F=341 vibrations in a second.—To the same order belongs the camel (*Camelus Bactrianus*). In the camel, the larynx is well developed; the superior vocal ligaments are present, and the inferior vocal ligaments are strong. The voice is grave, but seldom exercised. (See Fig. 20).

In the Pachydermata, or thick-skinned animals, there are many species possessed of a sonorous voice. Among these are the horse, the ass, the hog, the rhinoceros, and the elephant. In the horse, the larynx is highly developed (see Fig. 21), and the windpipe has as many cartilaginous rings as that of the ox. The superior vocal ligaments are not prominent. Above the junction of the proper vocal ligaments, between that and the epiglottis, there is an oval cavity; and on the posterior surface of the epiglottis there is a groove, furnished at its base with a semi-lunar membrane. To this membrane much effect has been ascribed in the production of the peculiar neighing of the horse. It is doubtful, however, if this peculiar sound be so much dependent on this membrane as has been believed.

Fig. 21.



LARYNX OF HORSE.—*Bishop.*
a, epiglottis; b, semi-lunar membrane; c, aperture at base of the epiglottis; d, groove; e, ventricles; f, arytenoids; g, inferior vocal cords; h, trachea.

In the ass the larynx is also well developed. In the windpipe the rings are spiral. The bray of the ass—which seems greatly to depend upon the presence of two large sacs placed between the vocal ligaments and the internal surface of the thyroid—is well known; it has a range of about five tones.

In the mule, the larynx resembles that of the ass. The voice is a species of bray, more resembling that of the ass than the neighing of the horse. The tapir (*Tapir Americanus*) has some peculiarities in its larynx. It has, however, superior vocal ligaments, which are short and distinct, and inferior vocal ligaments, which are strong. The voice is a species of whistle.

A. Fig. 22. B.

A. LATERAL VIEW OF LARYNX OF DIDELPHIS OPOSSUM.
a, thyroid cartilage; b, cricoid; c, crico-thyroid ligament; d, trachea.
B. POSTERIOR VIEW OF THE SAME.
e, cricoid; c, laryngo-tracheal ligament; d, trachea.

The hog (*Sus scrofa*) has also some peculiarities in its larynx: its voice, as is well known, is a grunting, discordant sound.

The rhinoceros is remarkable for having the superior vocal cords very prominent.

In elephants the larynx is largely developed. The superior vocal ligaments are indistinct; the inferior or proper vocal ligaments are strong. The windpipe exhibits thirty rings, which are often partially subdivided, as in the case of the bronchial ramifications. The voice, aided by the trunk, is intense, and of a grave pitch.

Under the head of Marsupial animals, we find the kangaroo and the opossum.

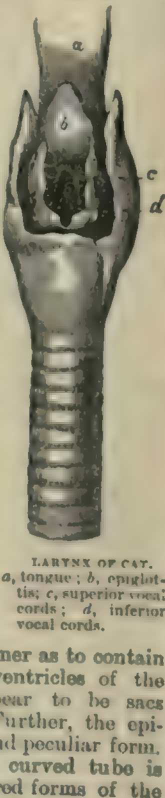
In the kangaroo (*Macropus major*), several peculiarities occur in the larynx. In particular, the vocal cords are membranous, and fold upon themselves, so that they cannot be stretched by the arytenoids. The voice, when in pain, consists of a piteous moan. In the opossum (*Didelphis opossum*), the vocal ligaments are very short; hence the voice is acute. The opossum purrs like a cat. (See Fig. 22).

In the order Carnivora, we find examples of animals with intense voice.

In the lion (*Felis leo*), the larynx is well developed; the vocal ligaments, both superior and inferior, are present; the superior being prominent. The ventricles of the larynx are deep, forming a sac between the upper and under vocal ligaments. The windpipe is possessed of fifty cartilaginous rings. The voice is grave—highly intense; the roar terrific.

The tiger (*Felis tigris*) has a larynx resembling that of the lion, the superior vocal ligaments being very prominent. The voice of the tiger is more acute than that of the lion. It purrs like the cat. The leopard and the cat belong to the same genus; *Felis leopardus* and *Felis catus*. These two animals, like the rest of the feline tribe, have the superior vocal ligaments well developed. It is supposed that by these superior vocal ligaments the purring sound is produced. The voice in both animals is a mewing; and they have by night a melancholy cry. (See Fig. 23).

Fig. 23.

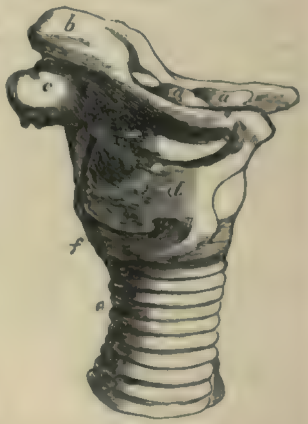


LARYNX OF CAT.
a, tongue; b, epiglottis; c, superior vocal cords; d, inferior vocal cords.

In the order Quadrumana, to which the apes and monkeys belong, the essential form of the organ of voice does not vary much, but peculiarities occur in the resounding walls. Thus, in the ourang-outang, a sac exists between the thyroid cartilage and hyoid bone; and in the mandril, pavian, and macacos, membranous sacs are observed below the hyoid bone. In the Mycetes, or howling apes of the New World, the apparatus for the resonance of the voice is greatest. In these the hyoid bone, and the thyroid cartilage, are expanded in such a manner as to contain large cavities, which open into the ventricles of the larynx; and besides this, there appear to be sacs common to the larynx and pharynx. Further, the epiglottis, in these apes, has a very large and peculiar form. In the Sapajous (Ateles and Cebus), a curved tube is formed by the increased size and altered forms of the epiglottis, and some adjacent structures. The voice of these animals has a whistling character.

In the chimpanzee the true vocal ligaments are prominent. The windpipe has sixteen rings. The voice is more acute than in women; its quality inferior, owing probably to the sacculated larynx. (See Fig. 24). In the ourang-outang the inferior vocal ligaments are prominent, but not so long as in the families of the human race. The ventricles are valvular, so that the inflation of the peculiar sacs is under the control of the animal.

Fig. 24.



LATERAL VIEW OF LARYNX OF CHIMPANZEE.
aa, sac connected with the lateral ventricle; b, hyoid bone, with c, sac protruding at its base; d, thyroid; e, trachea; f, cricoid.

In the Gibbons, the ventricles are deep, and communicate with a sac. The voice is acute; the cry "bow wow."

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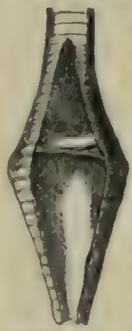
* See *ante*, p. 279.

In the monkeys of the old continent, there are also laryngeal sacs. These sacs modify the quality of the voice, giving to it, even when acute, a peculiar hoarseness. In the *Simia appella* and *Simia capucina*, there are some peculiarities in the structure of the channel for the passage of air. The voice, in quality, is like that of a flute; hence these are called whistling apes; and, from the peculiar expression of this whistle, which is a plaintive melody, they are termed weeping apes.

Voice of Birds.—The great peculiarity in the organs of voice, among birds, is the inferior larynx; that is, birds, in addition to the larynx corresponding to that possessed by mammals, have one peculiar to themselves, at the inferior extremity of the windpipe. Even the superior larynx of birds differs considerably from the larynx in mammals. The superior larynx, like that of mammals, is placed just below the hyoid bone. It is partly cartilaginous and partly osseous. In the superior larynx of birds there are cartilages corresponding to the thyroid and the cricoid, the two arytenoid, and the epiglottis. The cricoid is much less developed than in mammals; it forms but a small portion of a ring, occupying the posterior part of the larynx, and supporting, as in mammals, the two arytenoid cartilages. The thyroid cartilage, consequently, rests on the first ring of the windpipe. To the posterior margins of the thyroid cartilage are connected two quadrilateral bones, by which the extent of the protection afforded by the wings of the thyroid cartilage is much increased. The arytenoid cartilages are long, and taper upwards and downwards; they form, by their inner margins, the chink of the glottis. They are generally ossified; their external margins are bounded by the thyroid cartilage. The epiglottis, in most birds, is rudimentary, and generally is osseous. The chink of the glottis in birds is triangular, the apex being directed upwards. It is bounded in front by the thyroid cartilage; on each side by the arytenoid cartilages; and behind by the cricoid cartilages; but it has no salient membranous laminæ, such as the vocal ligaments in man and mammals have. It is capable of expansion and contraction under the action of several muscles. The inferior larynx is, as we have seen, peculiar to birds. (See Fig. 25). It varies very much in form and structure. This larynx, the vocal instrument of birds, is a tube, at the opening of which is a membranous tongue. This tongue is a doubling of the interior lining of the bronchus, its free margin being directed upwards; and birds have, for the most part, a smaller or greater number of muscles, capable of shortening this tongue, or of lengthening it in the direction of its height, and of rendering it tense or lax in a transverse direction.

Fig. 25.



SECTION OF INFERIOR LARYNX OF BIRDS.

In general, the inferior larynx of birds is produced by a membrane which makes a projection on each side of the inferior orifice of the windpipe; this orifice is divided into two apertures, sometimes by an osseous bar passing from before backwards, sometimes merely by the angle of union between the two bronchial divisions of the windpipe. The bronchi are not composed, like the windpipe, of complete rings, but merely of osseous or cartilaginous segments of rings, of a greater or smaller number of degrees in extent, each having a proper curvature in the state of rest, which curvature may vary, to a certain amount, by the action of voluntary muscles.

It hence follows, that the portions of the walls of the two bronchial divisions of the windpipe, adjacent to (that is, looking towards) each other, are, for a greater or smaller extent, membranous, being there destitute of any osseous or cartilaginous structure; and it is to this usually large portion of the wall of each bronchus that Cuvier gives the name, tympaniform membrane. Thus two tympaniform membranes descend, looking towards each other from the angle at which the windpipe divides, forming the interior wall of each of its subdivisions, and being extended transversely between the anterior and posterior extremities of the upper osseous segments of the same subdivisions; these osseous segments extending only along the posterior, the external, and anterior part of their wall, so as to leave the inner part of each bronchus simply of a membranous structure.

The first osseous segment of each bronchus has much the same curvature as the windpipe itself; but the second and third are portions of larger circles, and are less convex exteriorly than the first, so that the latter project on the inner side of the tube.

On this interior projecting part, the lining membrane forms a fold; and it is this fold, half shutting one of the inferior apertures of the windpipe, which offers to the air issuing forth, a tongue capable of vibrating and of producing sound.

The inferior larynx of singing and some other birds, in which the voice is far from musical, is very complicated. The last rings of the windpipe unite into a structure two or three lines in length, nearly cylindrical above, and expanded below, where it has two obtuse points—one anterior, another posterior—joined by the bony bar passing from before backwards, already spoken of more than once. This bar is so placed that the windpipe opens below by two oval holes, making with each other an obtuse angle; and each of these holes communicates with one of the bronchi.

The three first osseous segments of each bronchus are more near to each other, and flatter, than those which succeed them. From the first to the third there is a gradual elongation behind, so that the posterior extremity of the last makes a sort of projection, owing to the sudden diminution of the fourth segment. The arc which these segments form hardly exceeds 60°; and in each bronchus, the cord of this arc, so to speak, is the tympaniform membrane. The first segment of each bronchus curves its anterior extremity towards the inner surface of the tube, where it articulates with a small oval cartilage which is fixed to the tympaniform membrane; while it forms within, a prominence which is the vibrating lamina of the larynx on that side. Thus the transverse section of each bronchus is below—nearly circular; the section higher up becomes the segment of a circle, which diminishes in one direction while it enlarges in another; and the passage of the air, upwards into the windpipe, takes place by two oval holes, each furnished at its anterior border with a salient lamina. This apparatus is supplied with ten muscles, five on each side.

Of these, one descends from the interior of the windpipe to the anterior extremity of the third segment of the bronchus; and, by its contraction, draws that point upwards, thereby making the vibrating lamina project further inwards, and, at the same time, rendering tense, lengthwise, all that part of the tympaniform membrane lying below the segment to which the muscle is attached. Another muscle, parallel to this, has nearly the same attachments, and a like office. A third muscle, much smaller, extends from the inferior and posterior part of the windpipe, and is inserted into the posterior extremity of the second bronchial segment. Its action is similar to that of the former. A fourth muscle passes obliquely from the windpipe to the posterior extremity of the second bronchial segment. It draws that segment upwards and outwards, so as to aid the action of the muscles already referred to, and of that which follows. The fifth muscle is no longer than the preceding, but is much thicker. Taking its origin from the last ring of the windpipe, it passes downwards and forwards, and is inserted into the anterior extremity of the first bronchial segment, and particularly into the minute cartilage already mentioned as being articulated with that point. Its chief action is to draw forward the small cartilage, and consequently to put forcibly on the stretch, in a transverse direction, the upper part of the tympaniform membrane.

Such a complex structure of the inferior larynx belongs, as was hinted at, not only to singing birds—such as the nightingale, the wren, the blackbird, the goldfinch, the lark, the linnet, the canary, and chaffinch—and to those with a monotonous cry, like the swallow, the sparrow, the starling; but also to some with a decidedly disagreeable cry, such as the jay, the magpie, the crow, the raven. Thus, not only is a complex organ necessary to the musical singing of birds, but also a fine general organisation and a singing instinct.

The windpipe in birds presents some very singular modifications. As the voice is produced in the inferior larynx of birds, situated at the lower part of the windpipe, this tube comes to form, together with the mouth, the tube or pipe placed in front of the organ of voice. In short, the windpipe of birds comes to occupy a place corresponding to the situation of the organs of speech in the human body. It is capable of being shortened, not only by the diminution of the spaces between the rings themselves, but also by the rings being received one within the other. In many birds, the windpipe is much longer than the neck, being thrown into convolutions. This structure is observed in the cock-of-the-wood, the stork, and crane. In the wild swan, the convolutions of the windpipe are lodged in a cavity of the breast bone. Nor is the windpipe always cylindrical, for in herons and cormorants it has a conical figure, becoming gradually wider and wider towards the mouth. In some species of ducks it presents a sudden dilatation; while in the goosander, and some members of the duck family, it undergoes gradual dilatations.

That the inferior larynx of birds is the true organ of voice has been proved by many experiments. For example, anatomists have divided the windpipe in singing birds, such as the blackbird, about the middle of its length, so that the air could no longer pass through the superior larynx, and yet the bird would continue to sing, though with feebler tones than before. Similar experiments have been made on magpies and on ducks. After such an experiment, the magpie is found to cry with as great intensity of tone, and with the same acuteness, as before the operation. Again, if air be blown into the bronchial divisions of a duck, after their separation, together with the inferior larynx from the body, a sound exactly similar to the natural cry of the bird is obtained. Even after the bronchi have been cut away, by blowing into the trachea the same sound is produced.

It is not, however, to be concluded that the superior larynx exerts no modifying influence on the voice in most birds. It is manifestly opened and closed rapidly in singing birds, so that it is impossible to doubt that it takes an active part in the production of melody. In the song of the canary and the linnet, its simultaneous movements with those of the mouth are readily observed. Its effect, however, on the pitch of the voice, is not supposed to exceed a semi-tone. Physiologists still doubt whether the sounds of the voice in birds are the result, as in man, of the vibrations of a reed or tongue; or, as in mere flute-pipes, of the vibrations of a column of air excited by friction against the lips of an opening. There is unquestionably a great difference in the mode in which voice is produced in different birds. It seems certain that the simple organ of voice in the duck, the goose, and the like, is a reed instrument. In these the vocal cords, or bands, which form the exterior margin of the opening of the larynx, can be seen to vibrate strongly; while the sound produced, closely resembles that arising from the vibrations of membranes. But it is by no means so clear that the piping and whistling sounds of singing birds are produced in the same manner; and it is not impossible that these may be effected in the same mode as whistling by the mouth in man.

Several reasons, however, may be urged in favour of the opinion, that the sounds uttered by singing birds are the effect of the vibrations of tongues, as well as the voice of the duck and the goose. For example, the vocal cords under muscular action can hardly escape being thrown into vibrations; and even though the friction of the air may be in part concerned in the production of the sounds, a compensation must arise between the vibrations of the air and those of the vocal ligaments. If this be correct, the organ of voice in birds would not be entirely analogous to a whistle or pipe; but would, in part, possess the constitution of a reed instrument. It is found that the length of the windpipe has but a very slight influence on the note produced by the larynx; and that fact corresponds with the slight effect on the pitch of the notes produced by placing a tube in front of the human larynx. It is also found, that sounds produced by blowing, by means of a tube inserted in a bronchus, through the lower larynx of some birds after its separation from the windpipe, are not perceptibly altered in pitch by holding a tube in front of the larynx: thus is confirmed the resemblance of the lower larynx in birds to the character of the larynx in man. It may be added, that the greater number of the notes of birds can be obtained from the inferior larynx by varying the force of the blast, which, at first sight, seems to point to a resemblance with the effect of blowing by varying force upon the notes of flute-pipes, of the same size as the windpipe of small singing birds. But it is to be remembered, that the same variations of notes, by varying the strength of the blast, may be produced in reed instruments with membranous tongues, and even in reeds with very delicate metallic tongues.

The influence of the windpipe on the notes, may be either the same as that of the notes of flute-pipes, or it may merely affect them in the manner of the tube of reed instruments. Contraction of the upper opening of the windpipe, at the superior larynx, may lower the note, as in pipes and reed instruments.

An influence may be exerted on the sounds produced in the lower larynx by the tympaniform membrane, which vibrates strongly at the time. Between the internal vocal cord, the semi-lunar membrane, and the tympaniform membrane, there is a relation of compensation, the latter being analogous to the membrane formed of a reed stalk.

The muscles which vary the tension of the walls of the vocal pipe are in continual action during the modulation of the voice, in order to adjust the tube of the windpipe to the pitch of the glottis; but the number of vibrations may be determined by the glottis, reinforced by the walls of the pipe, as in mammals.

The voice of birds, as of other animals, is also in a minor key. The range of notes is commonly within an octave, though some birds can greatly exceed it. In the parrots, which have a voice of great power, the inferior larynx is single. The two membranes of the larynx leave a narrow chink between them, through which the air is forced from the lungs. These membranes, vibrating in all their dimensions, produce that harsh and disagreeable quality of sound peculiar to them. They can also whistle, during which the glottis is probably silent, and the column of air vibrates as in a flute, when a vibratory movement is communicated by the air traversing the elastic walls of the tube. Besides the power of speech possessed by some birds, many can imitate almost every sound they hear: the blackbird has been known to imitate the sound of the nightingale, the crowing of the common cock, and the cackle of the hen. The jay is said to mock the notes of the greenfinch and the neighing of a horse so closely, that it was scarcely believed to be a bird by those who heard it; it can also imitate the calling of fowls to their food, and the barking of the house-dog.

The variety in the song of singing birds is a subject of the greatest interest. The songsters, properly so called, include the skylark, the woodlark, the thrush, the blackbird, and the nightingale. A slight notice of the notes of each of these follows.

The skylark is one of our most agreeable songsters. Its song is composed of several strains, each consisting of trilling and warbling notes variously modulated, occasionally interrupted by a powerful whistling. Sometimes the lark sings on the ground, perched on a clod, or crouched among the grass; but generally, in commencing its song, it starts off, rises perpendicularly or obliquely in the air, with a fluttering motion, and continues it till it has attained its highest elevation, which not unfrequently is such as to render the bird scarcely perceptible. Even then, as remarked by a distinguished naturalist, if the weather be calm, its warble can be heard coming faintly on the ear at intervals. The lark is also a bird of singular capacity; the young learn the notes of any other bird which hangs near them in confinement; and some full-grown birds are observed to possess a like facility. There is, however, a considerable difference among larks in the strength and melodiousness of the note. In confinement, some larks begin to sing as early as November, and go on singing until moulting time; others begin in March, and cease as early as August. In the wild state their period of singing is much shortened.

The woodlark is considerably less than the skylark; but, of all the larks, it is the sweetest songster. Its voice has all the melody of the flute, marked at times by a tender, and even somewhat melancholy strain. It sings sometimes in the air, sometimes on the top of a tree. When singing in the air, it is frequently seen flying in large, irregular circles. The woodlark sings late in the evening, so as sometimes to be mistaken for the nightingale. The female woodlark, like the female of larks in general, is not destitute of song; but all that it can reach is a few strophes, which are much interrupted.

The thrush has a clear and beautiful song. On the tops of the highest trees it welcomes the approach of spring, and sings throughout the whole summer, especially in the morning dawn, and the evening twilight. It is kept in a cage by bird-fanciers, whence often on a morning, even as early as February, it will delight a whole street by its pleasing song, outside the window, or even inside, provided the window be a little open. The thrush, in its wild state, is fond of bathing. In September and October they are often caught at the places where they water, before sunrise and after sunset, and even so late that they cannot be seen, but are only heard. At the time of bathing they have a peculiar call-note. When a thrush finds water, or when it is flying towards a known watering-place, it pipes loudly—*sik, sik, sik, sik, siki, tsak, tsak!* and immediately all the thrushes in the neighbourhood reply, and come on.

The blackbird has a song rich in melody, containing some deep notes, like those of the nightingale, yet varied with some which are unpleasantly harsh. When at liberty, it sings from March to July, particularly at night. In the cage it sings throughout the whole year, except at moulting time. Its note is pure, distinct, and clear. It has a good memory, and will learn several airs or melodies without confusing them. It is even able to imitate words.

The nightingale, by the fineness of its voice, surpasses every other bird. The variety and peculiarity of its tones express its varying emotions. When the male is alone, its most significant note is the pipe-note *witt.* But if the harsh syllable, *krr*, be added, it forms the call of the male to the female. To express anger or fear, the note *witt* is repeated with great loudness and rapidity before the termination *krr* is added. When happy and contented, the nightingale utters a deep *tack.* Under the excitement of anger, jealousy, or alarm, the nightingale utters an unpleasant shrieking tone, which resembles the cry of the jay. When they sport and chase each other, which they frequently do in pairing-time, they utter a very short chirping sound. Such notes belong to both sexes; but the power and the brilliancy of its song distinguishes the male. Its vocal organ is of striking power; the muscles of its throat are more robust than those of any other singing bird. Besides the strength of its voice, the nightingale is remarkable for the force, the agreeable transitions, and the beautiful harmony of its song. Commencing softly, it warbles for a moment a succession of low melancholy notes, which gradually increase in strength, and at last die away upon the ear. A variety of sharp notes follows, and then are uttered numerous hurried sharp notes, intermingled with some detached ascending notes, with which it generally closes its strain. In the song of a fine nightingale, without reference to slighter variations, there are at least four-and-twenty different strains.

Among the sparrow and finch tribes there are many much-prized singing birds.

The bullfinch has naturally a harsh, creaking tone; but young birds learn all kinds of songs, airs, and melodies. If it be desired that a bullfinch should sing perfectly, it ought never to be taught more than one melody, in addition to the fanfare, which is always added by way of surplus.

The chaffinch has a variety of notes expressive of its wants and desires. There is one delicate note, expressed *treef, treef*, by which it appears to remark a change of temperature. The call-note, which it uses chiefly on its migration, is a repeated *yack, yack.* A spontaneous sound appears to be *fink, fink*, which it reiterates, and from which perhaps the root of its name is derived. More remarkable than these notes is its clear and trilling song; as approaching more to distinct articulation, it is termed a quaver. Each bird has one, two, three, and often as many as four different songs, each of which lasts a couple of seconds, and consists of several strophes.*

The linnet has a very remarkable, loud, and flute-like song, consisting of many connected strophes; which is the more beautiful the oftener it utters some high-sounding notes, which are termed its crowing, from the resemblance to the crowing of a cock. From its natural flute-like voice, this bird surpasses all others in its capacity for imitating melodies in a beautiful and pure style. A young linnet, taught by a nightingale, has an exceedingly pleasing song.

The goldfinch has a shrill, agreeable song, heard during all seasons, except at the period of moulting. It contains many warbling and twittering notes, on which it dwells more or less; and the oftener the syllable *fink* is repeated, the more it is admired. Some birds utter these notes only once or twice in their song; while others give them forth four or five times in succession. The goldfinch does not acquire the song of other birds with so much ease as the linnet and the canary.

The canary is distinguished by correctness of ear; by the remarkable skill it possesses of imitating all tones; and by an excellent memory. While canaries imitate the notes of other birds, they mix them with their own, so as greatly to improve the song. In different countries, canaries exhibit a different character of melody. Those birds which intermix in their melodies several strophes of the song of the nightingale, are called Tyrolese canaries. The English canaries, on the contrary, imitate the song of the lark.

Even birds of prey often exhibit no small extent of voice. The kestrel has a bell-like, ringing voice—*kli, kli, kli*, which he often repeats in rapid succession. The white owl utters a plaintive cry, which, by the superstitious, has been regarded as a sign of death. The raven has a hoarse croak, resembling the syllable *crock* or *cruck;* but it also utters a note not unlike the sound of a sudden gulp, or the syllable *cluck*, which it seems to utter when in a sportive mood. The rooks have a considerable variety of sounds. Their chief cry resembles the syllable *khraa*, more or less harsh or soft, according to occasion. There is great diversity in the voice of individuals; the notes of some being much louder and clearer than those of others. Their cries, separately, are monotonous and disagreeable; yet, when at some distance, and uttered by a large flock, they become by no means unpleasant. Mr. M'Gillivray describes the sounds proceeding from a rookery at night, as consisting of a variety of soft, clear, modulated notes, very unlike their usual cry. He regarded these sounds as expressive of affection, and was persuaded that the mothers were fondling and coaxing the newly-hatched young.

The jackdaw is extremely clamorous, with a loud and clear note, resembling the syllable *kae* or *caw*, variously modulated. The noise produced by a large flock, though in no degree musical, is far from being disagreeable. The jay can even learn to speak—uttering, however, nothing but solitary words. They may be taught also the fanfare of a trumpet, and other melodies of single bars, as well as little airs, and the notes of many birds. The magpie imitates all striking sounds, and can be taught to speak more easily than any other of the crow tribe. The cry of the cuckoo is universally welcomed as the harbinger of spring. Its principal sound is nothing but *hu-hu* or *coo-coo*, repeated at short intervals: when attention is given, however, it is found that these two loud and mellow notes are preceded by a kind of churring or chuckling sound, which consists of a low and guttural inflexion of the voice, during which the throat seems distended.

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* Those who desire a particular account of the different songs of the chaffinch, may consult *Chamber Birds*, by Bechstein; translated from the German by Mr. Shuckard: London, 1848.

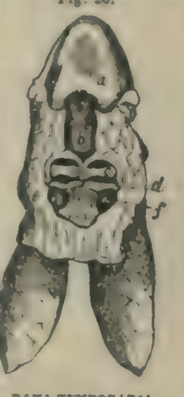
The parrot tribe are most remarkable for their power of imitating human speech. The cockatoo shrieks its own name, *cockatoo;* and calls loudly, in a trumpet-like tone, *derdeny*. It acquires the cries of all animals; particularly those of the domestic cock and hen. It rarely, however, acquires the power of articulating words. There are numerous species of cockatoo parrots having much the same character of voice. Among the commonest of the parrot tribe in Europe, is the ash-coloured parrot. This parrot readily learns to speak, and to pipe. It has not the unpleasant wild shriek of some of the parrot tribe. It takes no small delight in imitating the voice of children; hence children are its best instructors. If its education be begun early, it will sometimes acquire entire verses, and even axioms.

The grey woodpecker has a note which resembles a loud shout of laughter, whence some of its popular names are derived; this note is never varied, except by its more clamorous repetition during the spring and early summer months, and by the peculiar cry, *plui, plui, plui*, which has been supposed to indicate the approach of rain. The wryneck, in spring, frequently and loudly utters *gigigigi*, which is the call whereby it attracts its mate. The nuthatch utters a loud call, which may be heard at a considerable distance, resembling *grew, deck, deck*. The ring-dove, or cushat, has a loud and particularly pleasing cooing, during which it makes very grotesque motions, which may be backwards and forwards, or from side to side, moving the head in every direction. The turtle-dove has a peculiar cry, and bows its head while it is uttered.

Voice of Reptiles.—The sounds uttered by reptiles and amphibious animals have their source in the larynx, like the voice of mammals. In frogs, as well as in the crocodile, there are vocal cords. In the crocodile, the larynx, though more simple than in mammals, still retains something of the same character. There is one large, long-shaped cartilage, to which are attached two movable cartilages. The mucous membrane, descending from these movable cartilages into a deep pouch beneath, leaves a free fold on each side, which, when the movable cartilages approximate, becomes a vocal cord. In the gecko and the chameleon the vocal cords are more developed than in the crocodile; nevertheless they are formed on the same plan. The lizard has an acute, chirping voice, which has been supposed to depend on a peculiar membranous fold attached to the larynx; but it really seems to depend on a vibration of the margins of the glottis. In the turtle tribe there are no vocal cords, nor is their larynx adapted to a perfect intonation of the breath.

In the true serpents there are no vocal cords; the hissing sound, which constitutes their imperfect voice, is a mere forcible breathing. In the male frog, membranous sacs at the side of the neck become distended in the utterance of the voice, and serve to increase its intensity. In the *Rana pipa*, in which the larynx, as in all other frogs, receives the bronchi directly, without the intervention of a windpipe, there is a large cartilaginous box, within which are two solid reed-like bodies, nearly as long as the larynx itself. The anterior extremities of those bodies are fixed; their posterior extremity is free, and projects on each side towards the opening of the bronchus. The vocal sound is produced by the vibrations of reed-shaped tongues, which act like a tuning-fork; while in other animals of the same class, the parts which produce the sound are membranous. If a small piece of cartilage, a few lines in length, be fixed by one end, and a current of air be thrown from a small tube upon its edge at the other extremity, a humming sound will be heard. In the *Rana pipa*, also, the movable cartilages are convex externally, and concave internally; so that when the entrance to the larynx is closed, they form a dome over the windpipe, which has been compared to a kettle-drum. In the *Rana temporaria*, *R. esculenta*, and *R. hyla*, the larynx opens into two sacs on either side of the lower jaw; and these, during the cry of the animal, are filled with air. (See Fig. 26).

Fig. 26.



RANA TEMPORARIA (COMMON FROG).—*Bishop.*

a, tongue; *b*, hyoid bone; *c*, superior vocal cords; *d*, inferior vocal cords; *e*, pharynx; *f*, right bronchus.

Sounds Produced by Fishes.—A very few fishes are known to utter sounds, such as the trigla, cottus, pogonias.

The trigla utters a grunting sound when it is taken out of the water. It has been supposed that the peculiar muscle of the air-bladder, in these animals, has a share in causing the sound. The cottus, however, from which a sound is heard to proceed when pressure is made upon its body, has no air-bladder. The pogonias, on account of the sounds which it produces, has been named the tambour. These fishes produce continued sounds under the water. The air-bladder is very large, and is covered by strong muscles; further, it has appendages, which, according to Cuvier, pass between the ribs, and become imbedded in the muscles.

Sounds produced by Insects.—Most insects are mute; others produce sounds merely by friction; others, again, by the passage of air through their spiracles. The sounds produced by friction come under the head of stridulation; those produced by air from the spiracles, purring or humming. In the orthoptera, and some of the coleoptera, there are parts adapted to produce stridulation.

In the cricket, the muscular apparatus may be described as consisting of a serrated string like a file, which, in the movement of the wings, is drawn rapidly over a firm, transparent, and nearly triangular disc, or sounding-plate, surrounded by a string; and by this act the sound is produced. The pitch of the sound of the house-cricket is very acute, being equivalent to about 4,096 vibrations in a second.

The cicadæ, termed sometimes the "*chanteuses*," or singers, are so called because the males produce, in the hottest part of the day, a kind of monotonous and noisy music:—

"Et cantu querulæ rumpent arbusta cicadæ."—VIRG.

The music of the grasshopper has from early times attracted attention. Archias sung of it; and his verse has been thus translated from the Greek:—

"Erst on the fir's green blooming branch, O grasshopper! 'twas thine
To sit—or on the shady spray of the dusky, tufted pine;
And from thy hollow, well-winged sides, to sound the blythesome strain,
Sweeter than music of the lyre to the simple shepherd swain."

Those, too, who loved these "living lyres in the olive groves, sounding all the summer long," have celebrated the locust:—

"Soother of loves, encourager of sleep,
O locust! mystic muse, shrill wing'd;"—

And the cicada:—

"Cicada! thou, who, tipsy with the dews
Of weeping skies, on the tall poplar tree
Perch'd swayingly, thyself dost still amuse,
And the hush'd grove, with thy sweet minstrelsy."

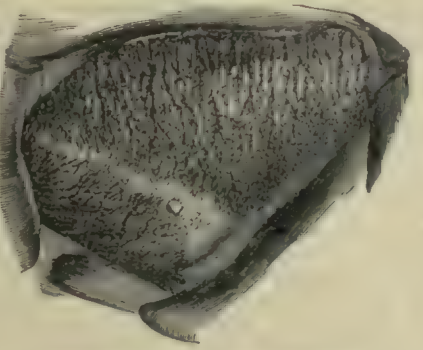
Melanger, alluding to the buzzing of insects, says, "Excute facundas pedibus titubantibus alas":—

"Striking thine own speaking wings with thy feet;"

but their real organs of sound are placed on the side of the base of the abdomen, internal, and covered by a cartilaginous plate, like a shutter, which is an appendage of the under side of the meta-thorax or posterior thorax. The cavity which incloses these instruments is divided into two partitions by a scaly and triangular edge. Seen from the under side of the body, each cell exhibits, anteriorly, a white and folded membrane, and, in the hollow part, a stretched-out, slender membrane, called the mirror. If this part of the body be opened from above on each side, there is seen another folded membrane, which is moved by a very powerful muscle, composed of a great number of straight and parallel fibres, extending from the scaly ridge; this membrane is called the *timbale*. The muscles, by contracting and relaxing with quickness, act upon the *timbales*, stretching them out or bringing them into their natural state, whereby the sounds are produced, and which, even after the death of the animal, may be repeated by moving the parts over each other in the manner they act whilst alive. The cicadæ occur chiefly in warmer countries of the world. One species, the *Cicada Anglica*, the only English species, is found in the New Forest.

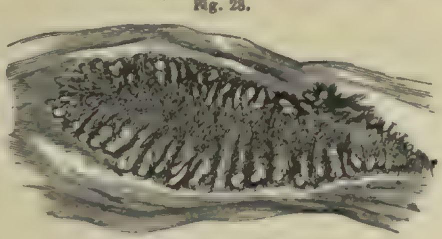
It is a common belief that the buzzing of insects is produced by the oscillations of their wings during flight. This idea has been often called in question. John Hunter found that insects emitted sounds after their wings were cut off. More recently, it has been stated that the sounds produced by many insects are the effect of a rapid transmission of air through the thoracic air-holes as they dash through space. Mr. Bishop has observed a peculiar mechanism for this purpose in the blue-bottle fly, and humble bee. Figures 27 and 28, show one of the large thoracic spiracles in each of these insects —the *Musca vomitoria* and the *Bombus terrestris*.

Fig. 27.



THORACIC SPIRACLE OF BLUE-BOTTLE FLY.
(*Musca vomitoria.*)

Fig. 28.



THORACIC SPIRACLE OF HUMBLE BEE.
(*Bombus terrestris*).

Having thus availed ourselves of Dr. Bushnan's elaborate paper on animal sounds, we shall now proceed to speak of other laws than those we have named; and also shall refer to various matters connected with daily life, in which the principles we have investigated are put into practical operation, or are more or less involved.

PRACTICAL ACOUSTICS.

UNDER the head of Practical Acoustics, we shall consider those applications of the science which relate to the construction of buildings specially devoted to concerts and public meetings; including, at the same time, some remarks on churches, and places of worship generally. Other matters incidental to the subject will fall under review.

Speaking from personal experience, we do not hesitate to affirm, that there are few buildings properly constructed on the principles of acoustics. In some places of small size, a great deal more exertion is required on the part of a public speaker, than in those capable of containing a very large audience; but which, by their shape, are more fitted for the purposes for which they have been erected. It is, however, a matter of congratulation to all whose profession or position requires of them to address an audience, that architects are, at the present time, inclined to sacrifice their own views, with respect to the appearance and ornamentation of an edifice, to considerations of utility. It is a custom, in China, to nail a baker by his ear to the doorway of his shop, if he have been found guilty of selling an adulterated article; similarly, it might conduce to the comfort of both speaker and audience, were each architect compelled to practise public speaking in a building of his own construction, until such were fit for the purpose for which it had been designed.

Sound is a radial force; that is, it proceeds from a point, in straight lines in all directions, like the radii or spokes of a wheel. Whatever object intercepts these rays of sound, either reflects or absorbs them. If such rays are emitted in the centre of a circular, or at the foci of an elliptic or parabolic shaped hall or other place, then they are reflected in exactly the same manner as rays of heat and light; and if such reflections take place from any of the curved forms we have named, then they are always inclined again to their source, and each reflection harmonises with another. In a building so constructed, we have the principles of acoustic science attended to. In the majority of modern edifices, however, we do not find such principles consulted. An eligible piece of ground has been purchased for a sum of money, and the cost of a building on it is regulated by the number of persons which can be crammed into it. Each corner must be utilised; peculiar modes of ingress, egress, and ventilation are adopted; and the only question generally solved, is that of remuneration. This evil, although great, may have a palliative in the fact, that places of public amusement are notoriously a bad speculation. But in the case of churches, &c., which, generally speaking, are utterly independent of speculative considerations, it is to be expected that some deference would be paid to the rights of those who contribute to the expenses, and to the manner in which the objects of their erection are carried out.

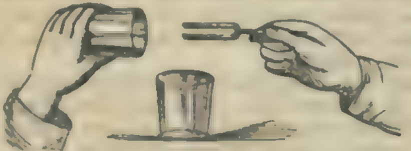
In most modern churches we find a cruciform structure. The body of the building is of great length; and, at some part, recesses are formed, which tend to produce an elegant appearance at the exterior. Now, as sound radiates from its source, it follows the laws of all radial forces—namely, that its intensity decreases as the square of the distance. Hence, persons sitting at the extremity of many of our churches, are utterly unable to hear sufficient of the discourse to understand its purport, because the person addressing them is compelled to modulate his voice, so that he may be heard distinctly by the mass of his audience near to him, and that he may avoid that undue straining of the voice which would degenerate into "screaming," were he to attempt to make himself heard in all parts. In many instances, our churches have a length equal to three times their breadth; and as it is necessary that an audience should be ranged in curves, having successive and gradually increased distance from the centre or focus, to be in accordance with the principles of acoustics—it follows, that all buildings of the class we have referred to, will render the words of the speaker inaudible to many, or he must exert himself to a most painful and harmful degree, and to the annoyance of those near to him.

There are other circumstances, independent of form, which also affect the acoustic properties of public edifices. For instance, in churches we find massive stone pillars arranged in each long aisle, from which the voice of the speaker is reflected in all directions. The stone walls, with their innumerable angles, have exactly the same effect; and, what is still worse, the glass of the windows has so great a reflective power, as to produce numerous echoes, which, on their part, again induce an annihilation of sound by the interference of its waves.

In our remarks on the interference of light,* we pointed out, that when two waves of that force were in certain phases, they combined and destroyed each other; and what happens to waves of light, equally does so to waves of sound, as a simple experiment will prove.

Place a glass cylinder on a table, and having struck a tuning-fork, hold it over the rim of the vessel, when that will at once vibrate, and produce a sound identical with that of the fork. Then bring another glass vessel, held horizontally near the first, in the manner represented in the following figure, when the sound will instantly cease.

Fig. 29.



This result of the destruction of sound arises solely from the interference of two waves, which neutralise each other. In a similar manner, the same effect is produced in a building when two or more waves of sound act on each other after reflection, or by a reflected wave interfering with one transmitted directly from the speaker. For this reason, there are many places in our public buildings in which it is impossible to distinguish the different words spoken. They are intermingled with each other, and so mutually destroy their intended effect.

The roofs of buildings have also a prejudicial effect; and this arises from the unequal reflective powers which they have. In many instances, we may notice deep recesses, which, whilst they add to the elegance of the room, materially injure its qualities in an acoustic point of view. Indeed, in every case, the walls and roof of a building should be as level as possible, provided their curves are constructed on proper principles. The reflection is thus made equal and regular; and the production of half, or irregular undulations, which are the cause of the interference of sound, is thus avoided.

It is a singular fact, that every room, hall, church, &c., has a peculiar "note" of its own; that is, if a speaker would be well heard therein, he must so pitch his voice as to suit the peculiarity of the edifice. To persons in the habit of addressing audiences in different places, in rapid succession—say, night after night—this is often a source of great annoyance. It thus frequently happens, that a really proficient rhetorician utterly fails to impress an audience in some places, whilst he may exceed their most sanguine expectations in others; and this is sure to occur if the speaker be addressing an audience in a hall in which he had never before spoken. It is true, that a singular circumstance eventually tends to remedy this evil; which is, that a speaker can always tell whether his listeners hear him, after he has spoken for a short time. If he have calmness and self-possession, he may gradually raise or lower his tone, to suit the circumstances in which he finds himself placed. This, however, requires a skill and discretion which few persons possess. Again, it often happens that the speaker himself is incapable, from physical defect, to remedy the evil to which we refer.

A common fault of public speakers, which no application of acoustic principles can obviate, is that of too rapid an utterance. Such persons produce the waves of sound so quickly, that each interferes with, and destroys the effect of the preceding one, and a confused jumble of sound results. We cannot here discuss those rules to which strict attention must be paid by those who wish to please their hearers; but may express an opinion, that however zealous and desirous of promoting the benefit of his fellow-creatures a man may be, he forgets half his duty if he convey not his ideas and opinions in an intelligible manner. It is a matter of congratulation to find, that public authorities, in religious and educational circles, are now actively turning their attention to this subject, and insisting that a good delivery shall be an essential condition to preferment.

In a concert, the arrangement of vocal and instrumental performers is of the greatest moment. In a previous page,† we pointed out the fact that each note was produced by a definite amount of vibrations. Now, in the concert-room, the bass, tenor, contralto, and soprano voices, singing in concert, are throwing off rays of sound, each of which is produced by a different number of vibrations; and the effect of the harmony results in all these producing a unison of effect on the ear. It is, therefore, the business of a skilful conductor so to arrange the voices or instruments as that the harmony shall be, as near as possible, perfect; and his success will entirely depend on attention to those principles of which we have just been speaking. Neglect in this respect will destroy the efficiency of the best choir; whilst attention thereto will frequently improve the effect of a choir of otherwise moderate pretensions. It is a remarkable fact, that almost the only public edifices in our country which have been properly constructed for speaking and musical purposes, are our theatres; and it is more than likely that such results from the fact, that they rarely present, or need any external embellishment, or that their owners depend for their existence or profit on their audiences being able to hear *all* that is said or sung in the place. Much of the charm of dramatic representation depends on the elocution of the actor; and as this would be wasted if delivered in an unsuitable building, the internal construction is considered as of the utmost importance. We forbear to extend comparisons which strike the mind on considering this fact, with respect to the relative importance of the objects for which our other buildings are erected. On this point we are sure that our readers will agree; and "a word to the wise" will doubtless be sufficient.

Having thus spoken of some of the acoustic defects which prevail in public edifices, we may suggest certain practical conclusions for the attention of those who have to do with such matters; and may divide them into two heads—namely, that of architectural construction, and of internal arrangement. A perfectly constructed building will be one in which the audience form a number of circumferences round a common centre, which is the speaker; or when they are so arranged as to assume an elliptic or parabolic form—the speaker in this case being the focus, and the walls, in either instance, the periphery of the circle, or other curve. The reason of this will be better understood if our readers peruse our remarks on the laws of the reflection of light from spherical and other curved reflectors;‡ which, as we have already observed, are equally to be applied in the case of sound. But, for the purpose of their special application to the subject we have in hand, we shall offer an illustration showing the directions of the waves of sound produced by the human voice, or otherwise, in a building constructed after acoustic principles, and compare such with the oblong edifices so common in this country.

In the following figure, we shall suppose that the solid external curved line represents the semi-circumference of a circular hall, and the cross-like figure illustrates the construction of a church, or other building, after the usual modern plan of building. In the circular building, the speaker being placed at *S*, it follows that every person placed on the curve *a a*, will be equally distant from *S*; and, as sound obeys the law of radial forces, it follows that each individual seated in that curve, hears equally well. The same holds good with the external curves, *b b* and *c c*, with the exception, that the intensity of the sound diminishes as the distance from the speaker increases; but still every person on the same curve has the same amount of sound arriving at his ears. This is a necessary consequence of the geometrical law—that the circumference of a circle is equi-distant in every part from its centre.

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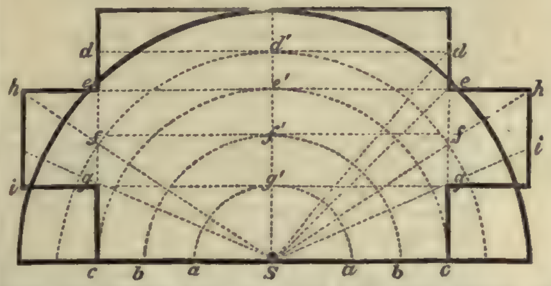
* See *ante*, p. 52.

† See *ante*, p. 279.

‡ *Ante*, p. 13, *et seq.*

Fig. 30.



But this law is not applicable to the case of the oblong building; for we will suppose two persons placed, one at *g'* and another at *g*. The first one is nearer the speaker, and at the circumference of the first curve of sound *a a*, whose radius is *S*, *g'*; whereas, the other person at *g* is at the extremity of the third curve, *c c*, although level with him at *g'*; but because the curve *c c* is three times the distance from *S*, the speaker, than is *a a*—it follows, according to the law of radial forces, that he at *g* hears sound of only one-ninth the intensity of the listener at *g'*. The same reasoning holds good with the successive positions of *d*, *e*, and *f*, the parallel lines of which touch the curves at *d'*, *e'*, and *f'*. Many of our readers will have noticed how much more difficult it is to hear a speaker when they are placed at the sides instead of the middle of a building, although both positions may be in a similar parallel line with the speaker; yet perhaps few would expect that so great a difference existed between the chances of hearing at each place. But we will now transfer our attention to persons situated in the recesses on either side, and placed at *h* and *i*. Those on *h* are on the same parallel with others situated at *e'*; but being on a curve, the circumference of which is double the distance from *S*, they will only hear one-fourth as well. The same reasoning holds good with respect to persons situated at *i*, &c. Now, all the rays of sound impinging on the walls *c h*, on each side, are either lost or reflected back: and the same occurs with all the glass surfaces of the oblong building, which can be easily traced by our readers. The effect of this is twofold; for an echo is produced, and the interference and destruction of sound also ensues.

It will be therefore readily perceived, that considerable loss of acoustic effect is experienced in all buildings of the oblong form; and that the remedy which presents itself, is that of constructing new edifices in obedience to the principles we have been indicating. We have confined our attention solely to circular erections; but those of the elliptic or parabolic form may easily be studied in the manner already mentioned.

The faults of the internal arrangement of halls, &c., may often be remedied by the most simple means. They chiefly arise from the echoes produced by the reflection of sound from plane surfaces in oblong or square buildings, and especially from glass windows, which, having a very level surface, act as good reflectors. If the seats of a building are arranged in concentric circles, rising one above the other, the echo from the wall is at once destroyed, because the audience is arranged between them and the speaker; and this plan conduces highly to the comfort of both parties. The echo produced by windows, or other plane surfaces, is at once destroyed by covering them with cloth; indeed, ordinary cotton blinds will, generally speaking, answer the purpose perfectly well. If the walls of a hall are papered, instead of being left bare, or painted, the echo is almost destroyed. Those who have not had the opportunity of speaking in a building in which echoes abound, can have no idea of the painful effect experienced by the public speaker. Each word is reverberated to his ear, as its successor passes from his mouth; and the effect is most confusing to the mind if an extemporaneous address is being delivered by him.

Sounding-boards are a contrivance by means of which the voice of the speaker can be reflected towards the audience; and here again we perceive an analogy existing between sound and light. By means of a reflector, we can send the rays of light from their source in any direction we please; and the sounding-board answers the same purpose. In some buildings, the wall before which a speaker may stand, is made of a curved shape; and if the roof be contrived so that a regular geometrical figure be formed, such becomes an excellent reflector, and adds much to the acoustic character of the place. Besides acting as a reflector, it also serves as a diffuser of sound,* and, as such, tends to spread the vibrations in all directions. The acoustic effects of a solid reflector, and a sounding-board which is made hollow, are very different: for, in the former case, the sonorous waves are merely sent forward from the reflecting surface; but, in the latter, the body itself partakes of the vibrations, and so assists in their general diffusion. That such arrangements themselves vibrate, is easily proved by throwing a little sand on a drum-head, or placing on a piano a sheet of glass, over which some sand has been strewn. If any instrument be played near the drum, the sand will speedily arrange itself in various forms. The same will occur with the glass on the piano, if that instrument be performed on; and what are termed *acoustic figures* are produced. Another mode of obtaining the same appearances, is that of placing fine sand on a piece of thin sheet glass, and drawing over one of its edges the bow of a violin; the glass will vibrate, and the sand will assume star-like and other figures, owing to its partaking of the vibration. To so great an extent can these effects be produced, that a solid body, such as a thin glass cylinder, may be fractured by producing near it more powerful sounds in unison with the note which it affords on being struck.

THE EAR.

We shall now give a short description of the human ear, as the organ by means of which sounds are perceived. Its external part is of such a form as to receive the vibrations of the air, as they proceed from any sounding body. From the outer part there is a tortuous passage, by which the sounds arrive at the tympanum or drum. This is a thin membrane; and it is stretched over bones in a similar manner to that of the parchment covering of the musical instrument of the same name. The vibrations which have arrived thus far, set the drum in motion: and here we may mention a peculiar provision that is made, by which the intensity of sound is modified. When describing the eye,† we mentioned that the iris could be either opened or closed, so as to admit or restrain the rays of light, and prevent injury to the organ of sight. Now, similarly, the drum of the ear may either be tightened or extended, so as to increase or diminish the force of communicated vibrations; and by this beautiful arrangement, the ear itself is preserved from the injurious effect of too powerful a sound, or is enabled to catch the slightest whisper. From the drum of the ear, a channel, called the *Eustachian tube*, passes towards the mouth. This tube serves as an exit, by means of the air it contains, for the vibrations which have been communicated by the drum. It is, in fact, analogous in its office to the holes in the sounding-board of the piano, the harp, violin, &c.; and deafness always ensues when, by any means, this tube is closed, which occurs from cold, the deposition of wax, &c.

A cavity, called the vestibule of the ear, also covered with a membrane, is provided with nerves; and these, like the optic nerve,‡ connect the external vibrations of matter with the sensorium of the brain; and thus the sense of hearing is produced. The entrance of the ear is protected from the intrusion of insects and foreign matter, by means of hairs, and a peculiar wax-like secretion. This, in excess, often becomes hardened, and is thus the cause of difficulty of hearing, which some persons suffer from. It is, however, readily removed by syringing the ear with warm water. The following cut will give an idea of the different parts of the human ear.

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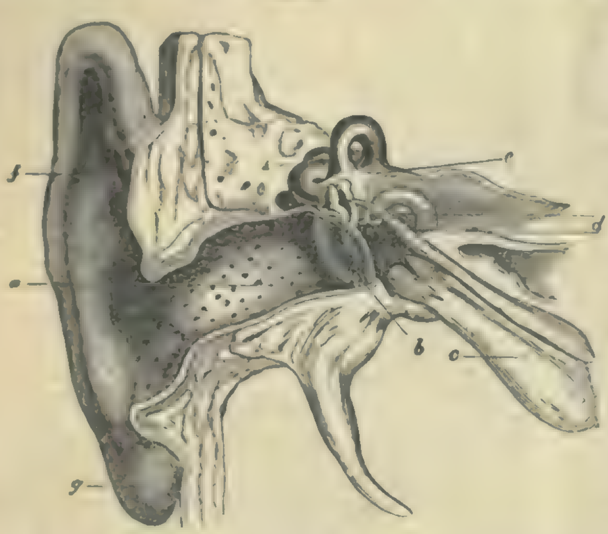
* See *ante*, p. 276.

† See *ante*, p. 48.

‡ See *ante*, p. 43.

Fig. 31.



a, is the canal by which sounds reach the drum, *b*; *c* is the Eustachian tube; *d* and *e* represent the connections of the auditory nerves with the brain; *f* and *g* are the upper and lower portions of the external fleshy part of the ear.

The organs of hearing in the lower animals are all constructed on similar principles to those of the human being, modified, however, so as to meet the special circumstances of the individual.

Amongst the mammalia, we find the ear, generally speaking, defended externally by a large flap, which serves to protect it from being injured by insects or the twigs of trees, as the animal passes through a thicket: this may be best noticed in the elephant and the dog. All animals of prey are possessed of great powers of hearing, on which they partly depend for success in the search for food. Amphibious animals can readily adapt their organs of hearing to the medium in which they may for the moment dwell. Without such a provision, these animals would sustain much inconvenience, and suffer extreme pain, when beneath the surface of water. This is chiefly owing to the great conducting power of that liquid, by which sounds act more strongly on the tympanum than when conveyed by air; and if the organ were not protected, under such circumstances, its sensitiveness would speedily be destroyed. The ears of birds are not easily distinguished externally, owing to the outer orifice being covered with feathers. As we descend in the scale of nature, we find that these organs become of simpler construction; and, passing by fishes, we at last find their development to be but rudimentary in the tribe of insects. On this subject, we refer our readers to the excellent remarks of Dr. Bushnan, which will be found at page 36 in the second volume, in the section of Comparative Physiology.

CONCLUDING REMARKS ON THE UNDULATORY FORCES.

We have now completed our exposition of the laws of those forces which, although, so far as we at present know, have not that extensive application possessed by gravitation and cohesion, are yet only second to them in that respect. As we proceed, the student will observe how large is their range; and more particularly, in the next section, will he notice their importance in the general economy of nature. Their consideration, in fact, forms a fitting and necessary introduction to the study of chemistry, on the limits of which we have often had to tread. All the operations in the laboratory of nature, as in that of the chemist, depend on the exertion of one or more of the forces we have so extendedly treated on; and it will therefore be desirable that our readers shall have become well acquainted with the principles already expounded, before they attempt to make farther progress.

In the sections on Applied Mechanics and Meteorology, we shall have frequently to refer to what has already been enunciated. This affords an additional reason for the remarks we here make, and points out to the student the necessity of a close investigation of each of the preceding sections, as a preparation for such as are to follow.

SECTION VI.

CHEMISTRY AND ITS APPLICATIONS.

INTRODUCTION.

In our previous pages, we have endeavoured to explain and illustrate those laws which are found to be connected with every form and kind of matter. The relations in which the undulatory forces stand are universal, for they do not depend for their action on specific natures, but are rather affections of all bodies which have an existence. Second only to, and yet concomitant with, the forces of gravitation and cohesion, they are constantly to be found either in a sensible or latent state; and, as second causes, they produce all the phenomena which we can witness in natural occurrences.

We have now to narrow the scope of our ideas. In one sense, we have to descend to particulars; but, at the same time, our opportunities of generalisation will be greatly enlarged. Instead of viewing the entire range of the animal, vegetable, and mineral kingdom, we shall investigate the nature and properties of the materials of which they are composed. We shall have to inquire into the specific character of the different substances which afford all the variety of appearance and quality observed around us, and to trace, as far as possible, the affinities or attractions which each body has for another—availing ourselves of such characteristics to produce what often appear to us as new products.

It is impossible to overrate the value of Chemistry, either as a branch of natural and experimental philosophy, or in its applications to social and commercial purposes. But a few centuries ago, alchemy—a congeries of imposture and deceit—was its only representative. The search after the so-called philosopher's stone, which was to turn all the "base" metals into gold, was the only type of chemical science; and, as in every instance in which philosophy is followed simply for its pecuniary reward, that and every other recompense eluded the grasp of its devotees.

In every department of human knowledge, the inductive system has been productive of advantage. It checks an exuberance of imagination; restrains and guides the reasoning powers; and, in fact, makes the intellect of man of real use to its possessor. The science of Chemistry stands prominently forth as a proof of the truth of this opinion.

In former days, no theory was considered too crude to be received, and none so ridiculous as to want believers. Let it only have the sanction of high-sounding names, and its success over the *soi-disant* philosopher was certain. If water were raised by a pump, the presumed reason was, that "nature abhorred a vacuum." The process of digestion was considered as the work of an evil but active spirit. Imaginary powers were assigned to sequences and events; and to sum up all we can say on the matter, we may state generally, that the causes suggested to account for any effect, were only limited in their character and absurdity by the narrow intellect of their inventors. The evil result of this course was not confined to the authors of it; it spread over the whole of society; and, as the natural consequence of superstition, it was considered profanity to dispute the dicta of those who assumed to be authorities in such matters. We need scarcely state, that the applications of Chemistry—such as it was—were of the slightest character. When, however, systematic experiment and rigid induction became the guiding-stars of philosophical research—Chemistry, which is, *par excellence*, an experimental science, speedily made most rapid progress; and in the hands of Boyle, Priestley, and Lavoisier, it approached towards a higher and more useful position. It has been no part of our plan, hitherto, to enter into historical details; we shall not, therefore, here commence that course; but would simply remark, that in our own day, there is scarcely a branch of manufacture, commerce, agriculture, or, indeed, any occupation, which has not benefited by the application of the truth of Chemistry; and much of our position as a first-rate power in the world at large, is due to the stimulus and aid which our manufactures have derived from chemical discoveries.

In the following pages, we shall first attempt to give our readers an exposition of the general laws of chemical attraction, and other fundamental principles. We shall then proceed to investigate the chemical history of elementary and compound bodies; and, subsequently, speak of analysis, and the varied applications of chemical facts to the purposes of manufacture, agriculture, and the like.

Before entering on this extensive range of inquiry, we must remark, that some considerable knowledge of the subjects which have been treated on in the previous pages will be necessary, if the student desire to make rapid progress in his chemical studies. All the operations of nature, and those conducted in our laboratories, depend on the exercise of one or more of the agents of Heat, Light, or Electricity; hence, ignorance of their laws would entail great embarrassment on those who enter into an experimental examination of chemical phenomena. Independently of this, we need scarcely observe, that a repetition of what we have already stated, would much encroach on the limits we have necessarily to assign to each subject, and cause considerable confusion. Chemistry calls for but a moderate knowledge of mathematical science; in fact, scarcely any but the common rules of arithmetic are involved in its calculations. We have endeavoured to smooth, as far as possible, the various difficulties which may lie in the path of the student; and, to this end, have given him extended directions for the construction and use of apparatus, with a view both to economy and efficiency. On this head, we would particularly warn all our readers against incurring unnecessary expense, by purchasing a large quantity of apparatus, at the commencement of their chemical course. To the young student, in any branch of knowledge, nothing is so useful as moderate but surmountable obstacles to his progress. These, whilst checking undue ardour, give him time for reflection and thought. The attempt to overcome them, induces readiness of resources, and calls out powers which he otherwise would hardly know that he possessed. The experiments suggested are such as may be easily conducted; and have been introduced so as to give the reader a means of verifying the statements with which they are connected. They are equally suitable for private study, as for exhibition at the lecture-table.

CHAPTER I.

CHEMICAL ATTRACTION, OR AFFINITY.

We have had frequent occasion, in our previous pages, to refer to attraction of various kinds. The attraction of gravitation we found to subsist between masses; that of cohesion between the particles of bodies, operating at insensible distances; whilst the attractions exhibited by free electricity and magnetism, were special or individual in their character. But, in chemical attraction, we find an entirely different class of phenomena to occur when compared with those we have described. Hitherto, it has not been of much consequence as to the qualities or specific character of the substances we have experimented with; it will now be our business to investigate those points as closely as possible.

By chemical attraction, we mean that power by virtue of which two or more dissimilar bodies are brought together, united, and ultimately produce another body totally different, in most respects, to those from which it was derived. One of the simplest instances of this kind which we can adduce, is that of common salt. It is composed of two substances only; one of which, chlorine, is a poisonous gas, that acts with great and destructive effect on animal tissues: the other constituent is a metal called sodium, which spontaneously takes fire on being brought into contact with water, or paper moistened with that liquid.

We must here point out the difference which exists between a chemically constituted substance and a mere mechanical mixture. Common salt is a compound, in which the parts are held together by so powerful an attraction, that they cannot be divided by any but strictly chemical means. If, however, we dissolve sugar in water, we do not consider the product to be a chemical compound, but simply a solution or mixture, because we can readily take the water away from the sugar by the application of heat, and the sugar will be again present, unaltered in any of its characters. Indeed, during the whole process of solution, no change, except a mechanical one, occurs. If, however, we expose sodium to chlorine, a very different result is afforded us: then chemical action ensues; a combination of the two substances is effected, and a new compound is the result.

Now, it is by the union of different substances, effected by chemical attraction, that the whole of the materials composing the animal, vegetable, and mineral kingdoms is composed. The combinations are held together by varying amounts of force; and the causes which modify the attraction which each body has for those united with it, are modified by several circumstances to which we must now direct attention.

Amongst all the agents with which we are acquainted, that of heat is the most powerful in producing mechanical and chemical changes; and to it the chemist is chiefly indebted for the means by which he is enabled to separate existing compounds, and to form others. The mechanical effects of heat we have already inquired into in our first section, and we shall now more particularly investigate its action on the chemical composition of a body.

So far as we can judge, the mechanical properties of caloric always precede the development of its chemical agency; and this will be no matter of surprise if we bear in mind, that the slightest change from ordinary temperature is sufficient to produce expansion, &c. Now, chemical action is rarely promoted by caloric, except when present in great excess, beyond its ordinary sensible indications. We may find a very familiar instance of these facts in the ignition of gunpowder. On applying heat to that substance, the sulphur is first converted into a liquid state; then the nitre; a red-heat is next produced; and at that moment a chemical action is generated between the charcoal and one of the constituents of the saltpetre—an explosion immediately ensuing.

Heat has not only the power of producing such rapid chemical combinations, but it also assists, in many instances, to produce those conditions most favourable to such results, by altering the molecular construction of bodies. Thus, in making solutions, heat is constantly employed to render bodies more soluble; and, in many instances, heat produces this result to an astonishing extent. By such means, a much larger amount of the substance is dissolved than can be held in solution when the liquid again becomes cool; and the solid substance, on cooling, separates in the form of a powder, or, more generally, it assumes regular forms, and crystallisation ensues. In almost every instance in which we desire to separate bodies from each other, the liquid state is essential for that purpose; and when solvents cannot conveniently be employed, heat is applied until the body has been brought to the fluid condition. The same plan is adopted when the union of two solid bodies is required. Hence the use of a class of substances called fluxes—such as carbonate of soda, borax, and the like. Take, for instance, a piece of common granite, which is composed of several substances. It may be powdered, and then submitted to the action of every known liquid; but the chemical attraction of each of its constituents is such as to defy those means for their separation. If, however, after powdering, it be mixed with some carbonate of soda, and exposed to an intense heat, it is fused, and combines with the flux. It may, when cool, be readily dissolved in acids, &c.; and then its constituents can be at once separated from each other by the usual methods.

If, again, it be desired to cause the union of solid bodies, heat readily assists us. Thus, in making an alloy, or mixture of two or more metals, such could never be effected whilst they are in the solid state, because the distance of their particles is too great from each other. But by the aid of intense heat, the particles of each are made mobile; and although the distances between the atoms of a body are increased, still the readiness with which they are detached from each other, enables them more readily to combine with those of another body; and thus the state of fusion is favourable to chemical action, as well as to mechanical mixture. The processes of evaporation by the application of heat to liquids, is in common use by the chemist; and he thus is enabled to obtain, in a solid form, salts, &c., from a liquid, although they are present to but a slight extent. Thus, in a gallon of ordinary river water, only two to ten grains of solid matter may be dissolved. By evaporating the liquid, such are brought together, and are so fit for the experiments on them.

We have previously referred to the effects of heat in producing rapid combination in mixtures of solids: we may now add a few words on the explosion of gases, &c., by the aid of caloric. There are some substances in which the chemical attraction is so slight, as to be readily destroyed even when they are heated to but a trifling extent: amongst these, are fulminating mercury and silver, and certain compounds of nitrogen with iodine and chlorine. Now such afford instances in which heat is antagonistic to chemical attraction; and many others of a similar kind might be easily adduced. In such cases, gases are always formed, when the substance is either a solid or liquid, before the explosion; and it is owing to the rapid expansion of these gases, that the noise usually heard is produced. We shall generally find that the presence of one body, nitrogen, is mostly to be detected in such instances.

As we proceed, many other applications of heat will be noticed, which will be more properly dealt with as they occur.

The agency of light, in producing chemical changes, is more restricted than that of heat, and has been already enlarged on under the head of Photography. The recent discoveries of Messrs. Bunsen and Kirchoff, which have been also described, have opened a new connection between the relation of light and chemical phenomena.

Electricity has great influence on the attraction or affinity which one body has for another. It will be unnecessary for us to enter into that question, however, as it has been fully discussed under the head of Voltaic Electricity and Electro-Metallurgy; and the same remarks apply equally to magnetism. Such being the relation which the undulatory forces have to chemical attraction, we may now more fully inquire into its different phases and laws, and so prepare our readers to consider the numerous instances to which we shall have to draw their attention as we proceed. It may be explained, that certain terms are used to express the changes produced by chemical action. By *decomposition*, we mean the separation of existing compounds; and by *composition*, the union of bodies to form new compounds is expressed. When the chemist attempts to ascertain the constituent parts of body, the process is termed *analysis;* but the act of producing new combinations, is termed *synthesis.*

It has been stated that different substances have various powers of affinity for each other; and we shall now suggest some experiments illustrating this fact. We must, however, point out, that there are two classes of affinity; or, more properly, a double act of affinity takes place under some circumstances. Thus, if a piece of polished iron be exposed to the action of air, its oxygen unites with the metal to form oxide, or rust; and here we have only an action of simple affinity. If, however, we add together solutions of common salt and nitrate of silver, then a double affinity operates, for the silver unites with the chlorine of the common salt; whilst the nitric acid combines with the soda; and so two new substances—the chloride of silver, and the nitrate of soda—are produced. The following are instances in which the chemical affinity existing between two or more substances, attended with the removal or separation of another by decomposition, is involved.

Experiment 1.—Add a few drops of a solution of nitrate of silver to one of common salt. The silver, uniting with the chlorine, as already mentioned, will fall down as a white powder; and a solution of nitrate of soda will form the liquid.

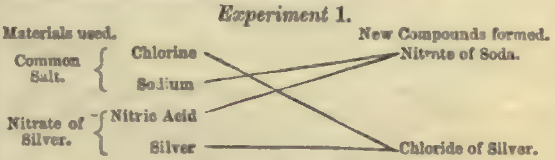
Experiment 2.—Pour off the clear solution into a clean glass vessel, and add to it a few drops of sulphuric acid. The nitric acid will be driven away from the soda, with which the sulphuric acid combines to form sulphate of soda.

Experiment 3.—Pour the solution of sulphate of soda, obtained in the last experiment, into a glass vessel, and add to it a little nitrate of baryta in solution. The sulphuric acid will now leave the soda, and combine with the baryta, forming sulphate of baryta; whilst the soda will combine with the nitric acid set free from the nitrate of baryta, and so form nitrate of soda.

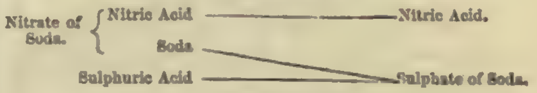
In these three experiments, we observe one substance, the soda, changing its state thrice. First, we found it in union with chlorine, forming common salt, which is left, owing to the superior affinity for it which nitric acid has. In the second experiment, we find that it left the nitric acid for the sulphuric acid, owing to the more powerful attraction which the latter exercised. But, in the third experiment, we noticed the paradoxical fact, that it united with nitric acid in preference to sulphuric acid. But this is easily explained. The baryta in the nitrate of that earth has a greater affinity for sulphuric acid than any other body, and so it decomposed the sulphate of soda—seizing the acid. Now, the soda being set free in the presence of the nitric acid, at once combined with it; and in this we have an instance of double decomposition, and a twofold exertion of chemical attraction.

Our readers will do wisely to master the principles involved in these experiments before proceeding further; and we have chosen the substances named that they may trace the progress of the soda in each of its combinations. To make the matter still clearer, we add diagrams representing the results, &c., of each experiment, in which the materials employed, and the new compounds formed, are mentioned.

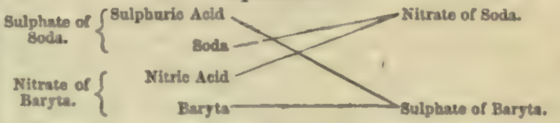
Experiment 1.



Experiment 2.



Experiment 3.



We thus perceive how much the affinities which one body has for another vary, and how the affinities of a third may destroy those exhibited in previous compounds when it is presented in proper circumstances.

In the above instances, we have confined our attention to changes easily understood. As we advance, however, into those which are produced when vitality assists the process, the conditions are so varied, and the states of bodies involved are so numerous, as to render the matter far more complex. Take, for instance, the ordinary process of digestion, and the conversion of bread and meat into blood and muscular fibre. Here we have a delicate and astonishing adjustment of affinities. The food contains charcoal, oxygen, hydrogen, and nitrogen, besides mineral substances. Part of the charcoal goes *separately* to form blood, fat, muscular fibre, nails, hair, skin, &c. With some of the oxygen it affords carbonic acid, which passes off from the lungs. The hydrogen, nitrogen, and oxygen are also employed to form various parts of the animal substance; and all these bodies, when in excess, are driven off in mutual combination as water, ammonia, or carbonic acid. Yet, complex though these changes may be, still the same law governs them as is found active in our experiments. Each body has a definite attraction for another; and the knowledge of this enables the chemist to trace, by the eye of science, the most obscure and difficult details.

There are some minor circumstances which powerfully assist in promoting the exercise of chemical affinity, to which we shall now allude; and they refer chiefly to peculiar conditions in which bodies may be placed with respect to their external surface, or relative mechanical conditions. As an instance of this, we have the peculiar action which the metal platina exercises on a mixture of oxygen and hydrogen gases; the action of light on a mixture of chlorine and hydrogen; the effect of minutely divided substances, as spongy platina, powdered charcoal, metal in a minute state of division; as in the case of finely divided lead in pyrophorus, &c., &c. Such, and many other instances we might name, affect, vary, and modify many of the exercises of chemical affinity; and all such must be taken into account in chemical researches, if truth and accuracy of result are hoped for. It will be thus perceived, that a thorough chemist must exercise the greatest care and penetration in all cases that are brought before his notice; and the science, therefore, is not only valuable for its philosophical and commercial results, but equally so as a system or means of mental training.

In cases of extremely low temperature, chemical attraction is often entirely suspended. Thus, on Mont St. Bernard, the bodies of deceased travellers have been kept, entirely unchanged, for many years. We have already mentioned that voltaic batteries, and electro-metallurgical processes, require certain temperatures for their successful working.* In many instances, however, the chemist avails himself of extreme cold to maintain the existence of many substances; as is the case with chloride of nitrogen, peroxide of hydrogen, and other compounds. In these there is a continual tendency towards decomposition if a low temperature be not maintained; and hence the employment of the means named.

The rapidity with which chemical action ensues between bodies, depends on the intensity of attraction they have for each other. Thus, if nitric acid be poured on silver or copper, the action is most violent; whereas none is perceived on platina. If, again, the acid be diluted with water, the intensity of its action is diminished, for fewer particles of the metal and acid come in contact. In some cases, a sudden suspension of action takes place: thus, nitric acid will act violently on the metal iron, and then all chemical action will cease, the iron having assumed a passive state. At times, the attraction is so powerfully exerted as to produce a large amount of heat and light—as when sulphuric acid is poured on chlorate of potass and sugar; at other times, intense cold is produced—as when freezing mixtures are dissolved in water. Indeed, it would be impossible for us to sum up the great variety of instances in which the exercise of chemical attraction is manifested, or to enumerate the peculiar effect which it produces. These, however, we shall frequently notice in our future progress—our desire having been just to put the student in possession of leading facts, which may serve as a guide to further details.

ELEMENTARY BODIES, AND THEIR COMBINING PROPORTIONS OR EQUIVALENTS.

To prevent any complication of subjects, in our previous remarks we have avoided any mention of elementary substances, and the proportions in which they unite together. These matters must now form the subject of our investigations.

All substances which have hitherto resisted our utmost endeavours to procure others from them of a different character or quality to themselves, have been termed elementary; and the number has been largely increased during the present century. For this we are largely indebted to the researches of Sir Humphry Davy, who, by aid of the voltaic battery, was enabled to decompose the alkalies potass and soda, and some of the earths. Since his time, other discoveries have been made, especially in connection with metallic bodies.

In most chemical works these bodies are arranged in groups; but, unfortunately, this arrangement is either modified or altered by each authority on the subject. The difficulty of obtaining a settled plan in this respect, lies in the fact, that the constant accessions which are obtained to chemical science, invalidate many of the conclusions which had been previously made. The point, however, is not of extreme importance; for as each element has its own distinctive character, it follows that the compounds it forms never exactly resemble those of other elements, although a great analogy may exist between them. We take hydrogen as an instance of the kind; for, in many of its chemical relationships, it behaves precisely like a metal; and yet, considering its physical characters, it seems the farthest removed of any from that class of bodies. Again, ammonia, which is known to be a compound substance, presents, under certain circumstances, every indication of metallic character; and, indeed, the existence of ammonium is theoretically admitted at the present day: yet no analogy exists between it and the metals in the characters which, of all others, distinguish them from other substances. Our present position, in these respects, is, therefore, one of great doubt and perplexity; for, whilst we see distant glimmerings of new facts by their analogies, the rigid system of philosophy cannot allow us to jump at conclusions which seem true, but are incapable of present proof. In our future pages, we shall therefore use the license which this condition of doubt permits; and after dealing with general principles, shall proceed to give the natural history of each of the elements, and of the compounds producible from them.

To Dr. Dalton, of Manchester, we owe the discovery that bodies combine in relative proportions; or, in other words, that every substance, elementary or compound, has a numerical ratio, in which alone it will form compounds with others. This ratio has had applied to it the terms, "equivalent," "combining proportion," and "atomic weight;" but, in our future pages, we shall only use that of equivalent, as expressing the facts of the case most clearly. What is meant, is simply that each substance must be taken in definite quantities to form a definite compound with others. Thus, eight grains, ounces, or pounds, of oxygen, must be united with one part of hydrogen, to form nine parts of water. If an excess or deficiency in these quantities be employed, then only so much water will be produced as the proportion of eight of oxygen and one of hydrogen would afford, and the element in excess of this ratio would be left free. Again, ten parts of oxygen would not unite with one of hydrogen, because the ratio we have named is not maintained. Sixteen parts, however, of oxygen, and one of hydrogen, may unite, because in that we have the ratio existing. A compound very different to water, however, would be the result, which, of course, might be expected from the union of two bodies in proportion, different to those producing water.—We may take another illustration in the case of oil of vitriol, or, as it is chemically termed, sulphuric acid. This is composed of three parts of oxygen, and one part of sulphur, if it be entirely free from water. In its commercial state, when pure, it is always combined with one part of water. Now the equivalent of sulphuric acid is obtained by adding together that of its constituents, thus:—

Quantity of each material.	Equivalent of each.	Total.
1 part of sulphur	16	16
3 parts of oxygen	8	24
1 part of water, composed of oxygen, 8: 1 part of water, composed of hydrogen, 1 .	9	9
Equivalent of sulphuric acid =		49

Or, in other words, we may say, that one equivalent of sulphur, three of oxygen, and one of water, form one equivalent of sulphuric acid, which, with reference to other compounds, has an amount equal to 49.

We will now take an instance in which both sulphuric acid and water are united with a metal which, however, has first combined with oxygen, and so formed what is called an oxide; and for this purpose we will choose the green copperas, or sulphate of iron. Oxide of iron is that metal united, in this case, with one equivalent of oxygen; and as the equivalent of iron is 28, it follows that the equivalent of its oxide will be 36. Now this salt of iron is composed of one part of oxide, one of sulphuric acid, and seven of water; and its equivalent will be as follows:—

Quantity.	Equivalent.	Total.
1 of oxide of iron	36	36
1 of sulphuric acid, free from water	40	40
7 of water	9	63
Equivalent of sulphate of iron =		139

These three instances of water, sulphuric acid, and sulphate of iron, will illustrate the mode in which the equivalents of compounds are formed. The last shows how that of two compounds, united together, is obtained. If the student fully comprehend these facts, he will have got over all difficulties presented in the study of the numerical value of equivalents; and have made a good advance in the study of chemical science. In the case of organic compounds, precisely the same plan is adopted; but as they present theoretical difficulties, we shall not enter into their discussion for the present.

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* *See ante, p.* 210.

It will be asked, however, how these equivalents are found, with respect to the elementary bodies; and although the practical part of the question is one requiring the greatest skill and experience, its explanation will not be difficult. If, as we have shown, every body has a definite combining proportion—it follows, whenever we examine it, we shall procure its constituents in their proper proportion of combination. For instance, if we decompose water by any means, we always obtain one part of hydrogen gas from nine of water, provided the operation has been completely performed: we therefore infer the presence of eight parts of oxygen. Or, if we prefer to take the oxygen, and weigh it, in combination, say with iron, we find that, if pure iron had been used to the extent of 28 grains, it would weigh, after the operation, 36 grains, from which we should infer that eight grains of oxygen had been added to it. But even during the process of finding the equivalent of oxygen, we gain that of the iron; for, supposing we used 50 grains of pure iron in decomposing nine grains of water, we should find that the iron only weighed 58 grains at the end of the experiment; and as we could, by means we need not describe, separate the iron which had, from that which had not been oxidised, we should discover, that out of the 58 grains, there would be 22 which had escaped oxidation. This number deducted from 58, would leave 36 as the equivalent of oxide of iron; and subtracting eight, the equivalent of oxygen, we should obtain 28 as the equivalent of iron. This mode of finding metallic equivalents is not that practically adopted in all cases; but we have availed ourselves of it because it exhibits the facts with which we are dealing. We can, however, illustrate the method of finding equivalents by another process, which is that of decomposition; and this is the plan almost always adopted; and one which our readers can roughly try for themselves, after the following manner. Of course they must not expect to obtain anything like accurate results; for it requires great care and readiness of manipulation to succeed in such delicate investigations.

Experiment 4.—Dissolve 110 grains of *pure* silver (ordinary coin will not do) in some dilute nitric acid; and when the whole is in solution, add common salt, also dissolved in water, until no more white powder falls down to the bottom of the vessel. When this has settled, pour away the water, and add fresh until the last washing does not in the least taste of salt, taking great care that not a particle of the white powder is lost. Then pour the whole of the powder on to a piece of white blotting-paper so as to filter it, taking the precaution of weighing the paper accurately beforehand. The whole is then to be carefully dried, and, whilst still warm, must be weighed. It will be found, that the powder, after deducting the weight of the paper, will weigh about 144 grains, of which 36 parts are chlorine, leaving 108 parts of pure silver as the remaining constituent of the powder. Thus, knowing the equivalent of chlorine, we infer that of silver to be 108.

These results are controlled by experiments with other combinations of the same body; and, by pursuing a similar course, the equivalents of any element may be ascertained. The numerous precautions which have to be observed, and the difficulty of obtaining the substances in a state of sufficient purity, together with other circumstances into which we shall not enter, render the attempts to obtain exact and reliable results one of the most difficult operations in experimental science. We hence find, that tables of equivalents, compiled by different experimenters, vary somewhat in the numbers given to each element.

Of course some standard is required to commence with, the multiples of which shall form the equivalents of all other bodies. Hydrogen has been chosen, and adopted, by nearly every English chemist; and if no other be named in chemical works, the reader may always assume such to be employed. Some have proposed oxygen as the unit, from the importance of its combinations; but such do not compensate for many of the advantages of the hydrogen scale. Hydrogen is the lightest body in nature, and has the lowest combining proportion. There is, however, one difficulty which presents itself; and it is, that although hydrogen and oxygen combine in the proportion of one to eight by *weight*, they do in that of two to one by measure. This, however, only affects the gases; but a late writer has proposed to double the equivalent of oxygen, so as to meet the difficulty. It is improbable that such a proposition will be generally adopted.

CHEMICAL MANIPULATION AND APPARATUS.

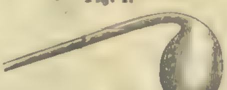
As we shall suggest a large number of experiments in future pages, with the intention that our readers may become practically acquainted with chemistry, it will be advisable that we should give some description of the apparatus which will be required, and the best mode of using and managing it. We shall, as far as possible, study the means of our readers; and therefore suggest the most economical and effective mode of carrying out the experiments. Many persons have an idea that the study of chemistry is a very expensive one. Such is not the case; for all the leading class experiments, teaching the elements of the science, may be performed with apparatus not costing more than ten shillings; and the chemicals are exceedingly cheap. If, however, the student enter into an extended course of original research, and the applications of chemistry to the arts and manufactures, and become, as many persons do, enamoured of the study, we should certainly decline to mention the extent of means required; for any amount of money may be so expended.

Much of the success which may be obtained in experimenting, depends on neatness, care, and cleanliness. A man may become an expert chemist, and yet may be but a poor philosopher; for it frequently happens, that whilst the experimenter may make the most valuable discoveries of *facts*, he has not the mental power of generalisation. On the other hand, a highly cultivated mind will avail itself of these facts, and so generate theories which shall embrace a multitude of effects. Some persons combine both these qualifications; yet such are rare accomplishments. But let none of our readers be discouraged by these statements. "*Suum Cuique*" is a motto which may be safely adopted in philosophy as in other matters; and during the last few years, the boundaries of science have been so extended, as to render subdivision of labour a question not of choice, but of absolute necessity. A schoolboy of our time, knows more of natural philosophy, so far as facts are concerned, than did all the philosophers of the days of Newton and his compeers; and the philosophers of our day are but the schoolboys of those who shall succeed them.

As we shall commence with the history of the gases, the various apparatus required for manipulating with them will first claim our attention.

Gases, such as oxygen, require the application of heat for their production; and, generally speaking, retorts, test-tubes, or flasks are employed for that purpose. The following engravings illustrate the appearance of each of these. Fig. 1 is a retort, in the bulb of which the materials to be heated are placed. They are made of various sizes. The student may provide himself with a few, varying from one ounce to a pint capacity. They require heating gently at first, to prevent the danger of cracking them. They should never be cleaned until quite cold, and are then to be placed stem downwards, so that the water employed for that purpose may drain off, and leave the inside quite dry.

Fig. 1.



A good substitute for a retort, when small quantities of materials are used, is a test-tube with a bent tube. A cork is first fitted in the neck; and a hole having been made in it, a glass tube, which has been previously bent into the required shape, is then introduced. The arrangement is represented in the annexed engraving (Fig. 2). When larger quantities have to be used, a Florence flask, fitted with a bent tube, may be employed. These are extremely useful for a great variety of chemical experiments. They are very cheap, and may be cleaned by pouring into them some strong sulphuric acid to remove the oil, and, subsequently, washing them out with hot soda and water. The student should provide himself with several of these, and various sizes of the test-tubes last described. They are both used for making solutions, and a variety of other purposes. A test-tube may be held over a lamp by means of a proper holder; but in the absence of this, a piece of brown paper held round it, will prevent the passage of heat to the fingers. Fig. 4 represents the mode of making hot solutions after this plan.

Fig. 2.

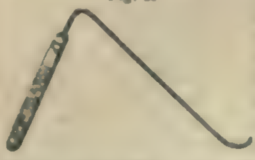


Fig. 3.

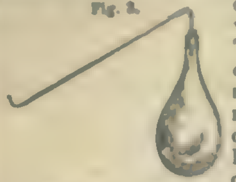
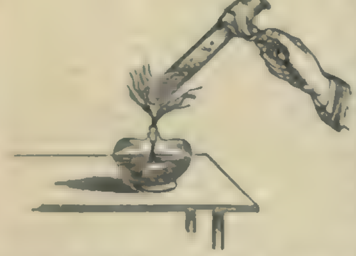
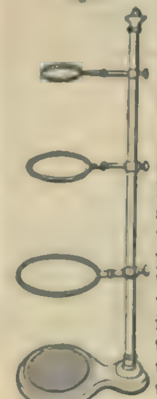


Fig. 4.



Retorts, flasks, &c., require a support to hold them over the flame of a lamp whilst being heated. The ordinary retort-stand answers this purpose; and one is easily made by fastening an iron rod in a wooden foot, and twisting round the rod rings of iron wire of various sizes. One of these stands is represented in the annexed engraving (Fig. 5).

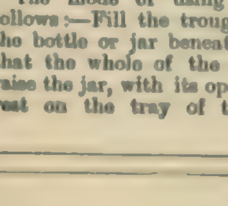
Fig. 5.



The application of heat is so frequently required in chemical experiments, that the student must supply himself with lamps for that purpose. Gas may be so employed; but the most generally useful instrument is the spirit-lamp, represented in Fig. 6. Spirits of wine, or, what is cheaper, methylated spirits, are used as fuel. An ordinary cotton wick is employed; and a cap, which fits over the wick-holder, is placed on it when the lamp is not in use, so as to prevent the loss of spirit by evaporation.

A trough is required for the purpose of collecting gases. This may be an oblong tin box, having a tray at one end, on which gas jars can be placed. The tray is perforated with holes, so that the gas may pass through them into the jars. A pneumatic trough, as they are called, is represented in Fig. 7. In the absence of such an arrangement, a basin, pan, or pail, will answer every purpose. The gas jars are made of various sizes and shapes; two of which are represented in Fig. 8.

Fig. 6.



The mode of using the pneumatic trough is as follows:—Fill the trough nearly full of water, and dip the bottle or jar beneath the surface of the liquid, so that the whole of the air may be driven out. Then raise the jar, with its open part downwards, and let that rest on the tray of the trough. The jar will thus remain full of water, and, as the gas from a retort passes into it, the water will gradually escape. The following engraving illustrates the mode of making oxygen gas, a small furnace being used in place of the spirit-lamp. A jar, when filled with gas can be removed from the tray of the trough by passing beneath it a plate. This, by holding a little water, prevents the escape of gas, and so retains the latter inside the glass vessel.

Fig. 7.

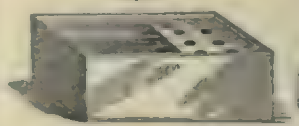


Fig. 8.

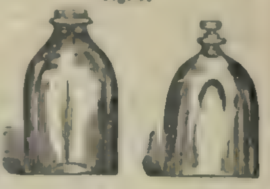
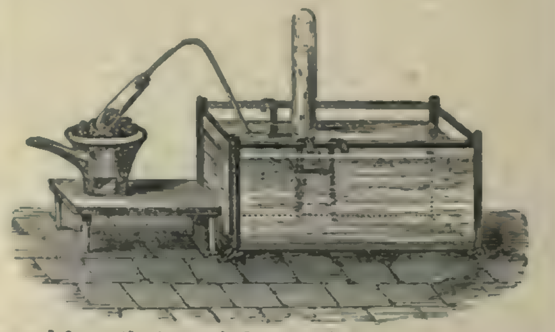


Fig. 9.



For many purposes, gas jars, open at both ends, are required; and when being filled with gas, the narrow end must be closed by means of a stopper or cork. Fig. 10 represents one of these jars; and inside it is a holder, useful in experiments with oxygen and chlorine. The holder consists of an iron wire, with a collar at one end, and a brass cup at the other. The material is placed in the cup, and, after being ignited, it is introduced into the gas jar. The student can easily make one of these, after the pattern shown in the annexed engraving. Gas-holders are very useful for holding quantities of oxygen, &c.; and one of the best kind is that represented in Fig. 12. It consists of a cylindrical metal vessel, having at the lower part an opening, which can be closed by means of a cap. At the upper part of the lower cylinder a stopcock is placed, through which the gas passes when it is in use. Between the cylinder and the trough over it are two pipes, fitted with stopcocks, through which water passes from the trough, when the gas inside the holder is required. These are filled by allowing water to pass from the trough into the cylinder until all air is driven out by the side stopcocks. All the stopcocks are then to be closed, and the bottom cap must be opened. The stem of a retort is then inserted, from which the gas passes into the cylinder, driving out the water by the bottom hole. When the holder is thus filled with gas, the retort must be removed, the cap screwed on, and the upper trough filled with water. One of the stopcocks, between the trough and the cylinder, may be opened, through which the water will pass, and force out the gas through the side stopcock. An india-rubber tube is to be fitted to this, to conduct the gas to any place it may be required. In making large quantities of oxygen, which

Fig. 10.

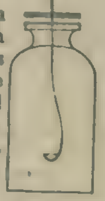


Fig. 11.

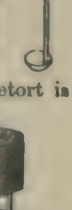
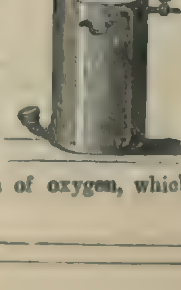
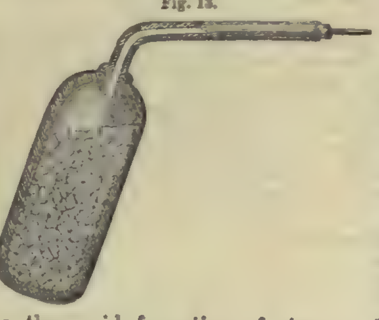


Fig. 12.



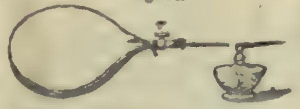
is frequently required in some experiments, a mercury bottle is used, into which a bent iron tube has been screwed. The bottle should be about half filled with the material, and great care should be taken to remove its outer orifice from water, through which the gas has to pass to purify it, lest, at the close of its manufacture, the liquid should rush back, and so endanger an explosion, owing to the rapid formation of steam. One of these arrangements is represented in Fig. 13.

Fig. 13.



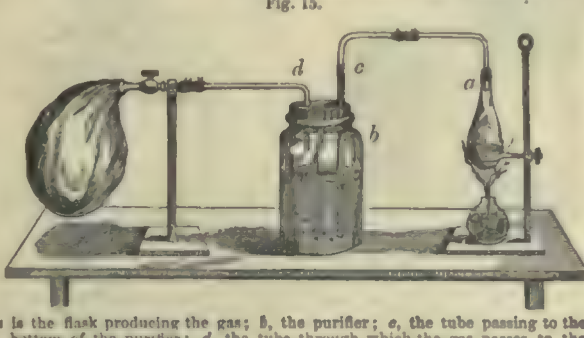
In many instances, bladders may be used to retain small quantities of gas. A large bullock's bladder should be chosen, and it must be carefully cleaned by means of cloths. The neck is then to be cut off, and a stopcock introduced into it. The neck of the bladder, and that of the stopcock, are then to be tied tightly together. One of these, fitted with a jet, useful in blowpipe experiments, is represented in Fig. 14.

Fig. 14.



In making some gases, powder, &c., is often given off from the flask, which would injure the bladder and stopcock, and also render the gas impure. This is easily prevented by means of a purifier, which may be a wide-mouth bottle with a cork in its neck. Into this two tubes are fitted: that from the vessel making the gas is to reach nearly to the bottom of the purifier, which must be three-parts filled with water; whilst the tube through which the gas passes into the bladder, is only just to reach inside the cork and neck of the vessel. Connections between the flask, &c., may be made by means of short india-rubber piping, which should be a little smaller than the tubes themselves; a little string will fasten the india-rubber tightly round the glass tubes. As the gas passes from the flask it will bubble through the water, which will retain impurities, and the pure gas will pass on to the bladder.

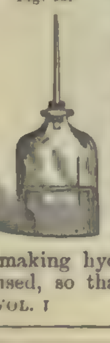
Fig. 15.



a is the flask producing the gas; *b*, the purifier; *c*, the tube passing to the bottom of the purifier; *d*, the tube through which the gas passes to the bladder.

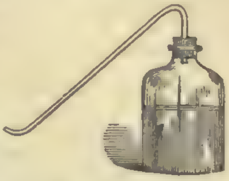
The above engraving, Fig. 15, illustrates this simple and yet effective plan of purifying gases.

Fig. 16.



Some gases, such as hydrogen, do not require heat for their manufacture; and gas bottles, of two kinds, or one bottle fitted with two sorts of tubes, are used. For experimenting with hydrogen, with respect to its flame, a straight glass tube, fitted with a cork, and ending in a jet, may be employed, such as is shown in the annexed engraving. But in making hydrogen, carbonic acid, &c., a bent tube is used, so that the lower end may be placed under water, or in any receiving-vessel. A convenient kind of this arrangement is shown in Fig. 17. Almost any kind of bottle answers for this purpose. They should only be about half filled with the materials, or the contents might run over, through the tube, and so spoil the experiment.

Fig. 17.



The apparatus we have described will be sufficient for most experiments on the gases: we shall therefore proceed to mention such as will be required for other purposes. Many substances require to be pulverised; and for this purpose a pestle and mortar is necessary. Those of Berlin porcelain, and unglazed, are generally useful; whilst some, for special purposes, should be well glazed, in order that no loss may be sustained by the powder resting in crevices or cracks of the material. Many substances which cannot be powdered in their natural state, may first be heated, and then thrown into cold water. Glass and flint are substances of this kind. Occasionally, as is the case with zinc, some substances must be heated before being powdered.

Flasks and test-tubes are always employed for heating and effecting solutions. But for the purposes of crystallisation, during which it is desirable that as much as possible of the surface of the liquid be exposed to the air, evaporating dishes are employed. One of these is represented in the annexed figure. The best material for the purpose is Berlin ware; but for ordinary experiments, stone-ware, or even watch-glasses, which are very useful, will answer very well. Those having a lip are to be preferred, as that facilitates the pouring of liquids. For some experiments, the inside of the dish should be highly glazed; whilst, for crystallisation, a rough surface is far better.

Fig. 18.

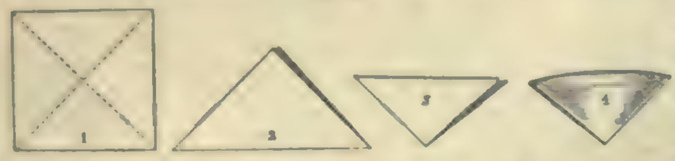
A water bath is often required for the purpose of drying precipitates and organic substances; the object being that their temperature should never rise beyond that of boiling-water. One is easily made by placing a small evaporating dish inside a larger one containing water. The outer one is heated by means of a spirit-lamp; whilst the inner one contains the substance to be dried. A tin saucepan answers extremely well for the water vessel, as on its rim almost any vessel can be supported. The instrument-makers supply such, properly constructed for the purpose; but these possess no advantage over the simple arrangement we have described.

After solution has been completed, it is often necessary that the liquid should be filtered. This is effected by means of glass or stone-ware funnels, of the ordinary shape, through filters made of proper paper; or, what answers generally as well, common white blotting-paper. Filters, ready cut, may be purchased at the operative chemists; but are easily made in the following manner:—Take a square sheet of white blotting-paper, as in No. 1, Fig. 19. Then fold its corners together, as in No. 2; again fold its corners, as in No. 3; and then open out one fold, cutting off the superfluous edges, as represented in No. 4. The filter will then be complete, but should be moistened with water when placed in the funnel, which facilitates the passage of the liquid to be filtered through it. Ribbed filters are sometimes employed; and they are made by plaiting together the folds of paper in a manner which we cannot properly describe, or illustrate by means of an engraving.

After a solution is made, the next process is that of testing, or adding to it certain solutions, which will bring out appearances by which the chemist can judge of the nature of the substance under examination. This is effected by means of test-glasses, which are made of various shapes. For many experiments, ale and wine glasses answer very well; for delicate experiments, however, lipped-glasses are needed. Generally speaking, any glass vessel answers the purpose.

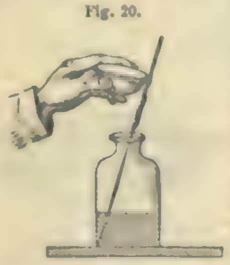
Dr. Faraday suggested, many years ago, a very ingenious mode of testing, boiling, and otherwise dealing with solutions. It is that of using slips of window-glass. Even the backs of plates, and sheets of paper, may be thus used. Test-tubes are also of value; and are, perhaps, especially useful to the beginner, as he can use them for so many purposes.

Fig. 19.



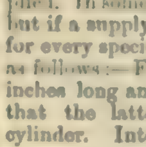
We may here suggest a very simple contrivance, which will aid the student in pouring liquids between unlipped vessels, such as flasks and bottles. If a liquid be poured out of such, it has a tendency to run down the side; but if a glass rod be placed so as to rest against the edge of the vessel out of which the liquid is being poured, and with its other extremity inserted inside the receiving-vessel, but not touching it, the liquid may then be passed from one to the other without the loss of a drop. The mode of doing this is illustrated in the annexed engraving.

Fig. 20.



In some cases an intense heat is required, as in the fusion of substances containing silica or flinty matter. For this purpose some form of furnace is needful. An ordinary fire-place may be used; coke being employed as a fuel. A most convenient furnace, suggested by Dr. Faraday, answers admirably; and having constantly employed it, we recommend its use to our readers. It is made by boring holes, an inch in diameter, at equal distances, through the outside of a blue pot, such as is used by refiners. The holes are easily made, and should be neatly finished off. An iron grid is dropped in so as to rest two inches above the lower series of holes, and the fuel may be small pieces of coke or charcoal. The furnace should be bound round by iron wire, so as to prevent it falling to pieces on being heated. One of these will last for a long time, and is useful for almost every purpose which a chemist can require. Its cost is very slight, being not more than a couple of shillings altogether. The annexed engraving represents one of these simple contrivances.

Fig. 21.

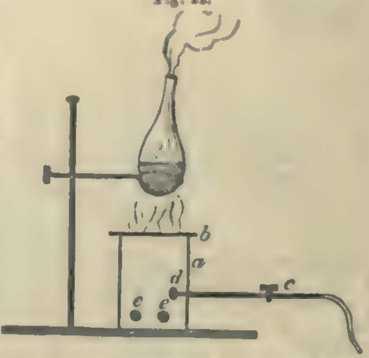


The instrument-makers sell furnaces of every variety of form, suitable for the purposes to which they may be applied. In some cases, an Argand spirit-lamp may be used; but if a supply of gas be accessible, a convenient furnace for every species of heating purpose may be readily made as follows:—Fix on the end of a cylinder of iron, eight inches long and three inches wide, some wire gauze, so that the latter may entirely close one end of the cylinder. Into the lower part of the cylinder introduce a gas-pipe, having first bored several holes beneath that one through which the pipe passes. These will admit a supply of air, which will mix with the gas as it issues from the gauze. If the gas be ignited, it will burn with a blue flame, affording little light, but an intense heat. The heat can be regulated with the greatest nicety; and the most gentle evaporation, or the fusion of siliceous compounds, can be carried on with the same arrangement. Fig. 22 represents the usual way of carrying out this useful mode of heating.

In most chemical laboratories, the gas furnace is considered indispensable; not simply because the heat produced by it is so easily regulated, but also on account of its cleanliness, and from not producing dust or sooty matter.

A very ingenious arrangement has lately been produced by Mr. Griffin, of Bunhill-row, London, which has come largely into use for chemical purposes, and is also employed by metallurgists in cases wherein an intense heat is frequently required. The principles of its construction are similar to those of the gas furnaces which we have already described; but as it can afford a temperature sufficient to melt cast iron, and to soften almost to the fusing-point, both wrought iron and platina, we shall introduce a description of its arrangement.

Fig. 22.

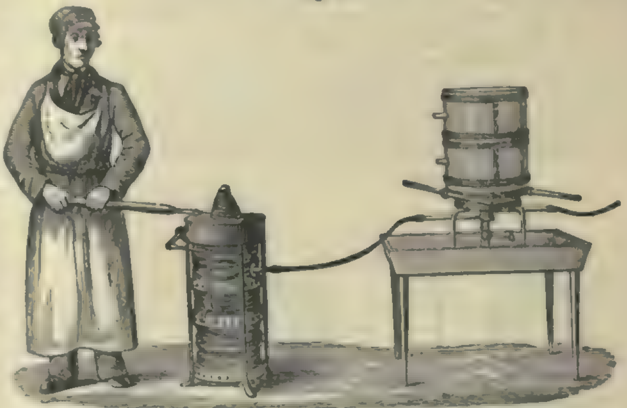


a is the iron cylinder; *b*, the sheet of wire gauze; *c*, the pipe supplying gas; *d*, the orifice whence the gas escapes into the cylinder; *e e*, are holes through which air enters the cylinder.

Our illustration (Fig. 23) will give an idea of this arrangement when in use. On the right is the gas-furnace placed on a table; in the centre we have the bellows which supply the air-blast, driven by a man, who at the same time can superintend the entire operation.

Fig. 24 is that portion through which the air and gas pass together. Fig. 25 shows the holes at the top of Fig. 24, which only represents them in section. These, in fact, form the heating portion of the furnace.

Fig. 23.



Mr. Griffin has adopted two different methods of applying heat. In some cases, he prefers to drive the heat downwards; in others, the heat is applied from beneath. The following engravings illustrate the plans by which the heat is applied from above.

Gas Furnace heated at the top, exhibited in Section by Fig. 26.—*a* is the gas-burner; *b* is the support for it, when used below the furnace; *c* is the iron tripod support for the furnace; *d d* are two perforated clay plates, adapted to the gas-burner, *a*; *e e* are two clay cylinders.

The interior of the furnace, as represented by Fig. 26, is built up as follows:—The clay plate, *d*, is put upon the tripod, *c*. Over the central hole in *d*, the clay cylinder (Fig. 27) is placed, and upon that cylinder two or three of the clay plates represented by Fig. 28. Upon these a porcelain or platina crucible is placed. If it be of platina, a piece of platina foil may be put between the crucible and the uppermost clay plate, to protect the crucible from contact with particles of iron, or against fusion with the clay. The crucible is to be covered by the plumbago jacket, Fig. 29. The space between this pile in the centre of the furnace and the two cylinders, *e e*, which form the walls of the furnace, is to be filled with flint stones, or gravel, washed clean and dried. The stones which answer best are rounded, water-worn pebbles, of half an inch to one inch diameter. These may be piled up to the top edge of the jacket, Fig. 29. The number of clay plates must be such as to bring the top of the crucible to the distance of two inches, or two-and-a-half inches at the utmost, from the flat face of the gas-burner, *a*. In some cases, merely one of the furnace cylinders, *e*, is necessary; in which case the crucible and its jacket is placed directly upon the cylinder, Fig. 27; and when only a moderate heat is required, even the packing with pebbles may be dispensed with. Another means of diminishing the heat is that of increasing the distance between the gas-burner and the crucible.

Fig. 24.

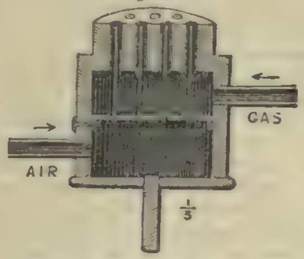


Fig. 25.

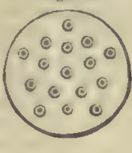


Fig. 26.

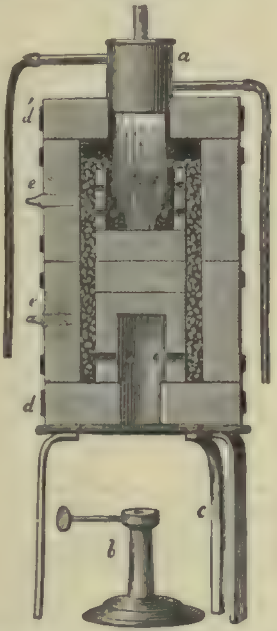


Fig. 27

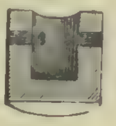


Fig. 28.

Fig. 29.

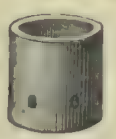


Fig. 30.

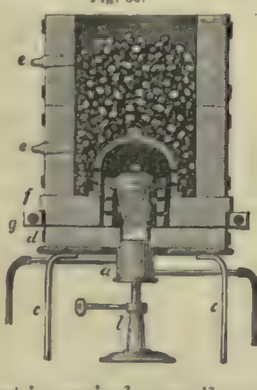


Fig. 31.

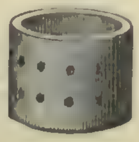
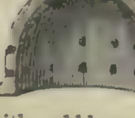


Fig. 32.

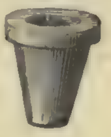


In Fig. 26, it will be seen that the gas-flame is driven, by means of the blast, directly on to the crucible. Another illustration represents that form of the furnace in which the heat is produced beneath the crucible. (See Fig. 30).

The packing of this variety of furnace is performed as follows:—The clay plate, *d*, is placed upon the tripod-stand. The crucible jacket, Fig. 31, or one similar, but of larger size, is placed upon the plate, *d*. The crucible and its cover is then put into its place, and is covered with the dome, Fig. 32, which must rest upon the lifter, *f*, and must be of such a width as to clear the crucible easily when lifted. The internal height of the dome should be such as just to clear the top of the crucible cover. Consequently, where crucibles of different sizes are used, domes of different sizes are also necessary. Observe, distinctly, that the crucible and its support are to rest upon the plate, *d*, and the dome upon the lifter, *f*. The furnace cylinders, *e e*, are now to be superposed, and the space between the dome and the cylinders, and that above the dome, are to be filled with small pebbles, as already directed. The gas may then be lighted; the blast of air set on; and the operation be allowed to proceed.

Fig. 33 represents a plumbago crucible, which is used to suspend the inner crucible over the gas-flame.—By means of such a furnace, six inches in diameter, three pounds of copper or cast iron can be fused in a quarter of an hour. From eight to ten pounds of copper or cast iron can be melted in an hour's time; and thus a portable and almost instantaneous source of intense heat is afforded.

Fig. 33.



The chemist requires various kinds of crucibles for his purposes. The student may supply himself with a few of the Hessian and English kind; and, for more delicate experiments, those of Berlin porcelain are to be preferred. Occasionally, a platina crucible is used; and it should always be placed in a Hessian one if employed in a furnace in which coke or charcoal is used. This precaution is unnecessary if a gas furnace be employed.

Besides the apparatus already mentioned, bottles for containing chemicals, stirring-rods, and a variety of minor articles, which will suggest themselves, will be required, which can be supplied at leisure. We must not omit to name, that some glass tubing, from an eighth to a quarter of an inch in diameter, is extremely useful, and can be readily converted to many purposes. The following are directions for bending it into any form that may be required.—For this purpose, soft tubing should be used. A piece of the proper length is to be cut off by means of a file; and the part that is to be bent must be held in the flame of a spirit or gas-lamp by the ends, until it is sufficiently softened to bend easily. The further ends are then to be gradually pressed downwards, care being taken that the red-hot portion is not bent into an angular form, but that a round edge is preserved, in shape similar to the letter U. If this be not attended to, the bent tube will almost certainly break at that part. Pieces of glass tube may be joined together by means of india-rubber connections;* or if the end of one piece be heated red-hot, and slightly inlaid by means of a piece of iron run round it, the end of the other piece may then be inserted, and the two fused together at a red heat, or by means of a common blowpipe. After a little practice, the student will be able to work readily with glass, providing it be of the soft kind. German glass tubes cannot be so easily managed, because they do not soften except at high temperatures.

We shall conclude our remarks on manipulation and apparatus, by urging on our readers the following precautions, if they desire to pursue chemistry successfully and economically. And first, it is of great importance that cleanliness be observed. An apparently clean glass may be chemically speaking, very dirty; for some tests are so delicate, that one part in 50,000 may be readily detected, of any substance with which the test affords a colour or precipitate. Indeed, extreme danger may ensue from neglect of this. For instance, were a drop of chloride of nitrogen to be placed on a greasy or oily dish, it would instantly explode, and break the vessel into fragments. Neatness and care should be observed in all experiments. This will induce a general habit, which will be of value to the student in every pursuit he takes in hand; and without them a person can never succeed in chemical science.

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* See *ante*. p. 305.

Beginners are often tempted to use large quantities of chemicals, where experienced persons would employ but a small portion. This error often produces an exactly opposite effect to that intended. Thus, if a small quantity of iodide of potassium be added to a solution of bichloride of mercury, a yellow precipitate will be produced; a still larger quantity will afford a red precipitate; and one in excess will redissolve all the precipitate, and leave a transparent and colourless solution.

Heat should be applied gradually to all glass vessels, lest they should be fractured. A sand-bath is sometimes employed between the vessel and the source of heat. An iron or copper basin containing sand, answers well for this purpose, by allowing the heat to pass gradually, and so avoiding the risk which would be run by applying the flame directly. As we proceed, we shall suggest other precautions when needed, and thus endeavour, as far as possible, to ensure the successful performance of all the experiments we shall introduce illustrative of our subject.

CHEMICAL TERMS AND SYMBOLS.

Before entering on a description of the natural history of the elements and their compounds, we shall make a few remarks in reference to terms employed to designate various substances; and the symbols, the use of which tends to materially assist the student in his progress, and saves much time and space.

In the early days of chemical science, every one "did that which was right in his own eyes," when naming newly-discovered substances. Hence arose such terms as "Glauber's" and "Epsom salts," "cream of tartar," "aqua regia," and the like. In fact, the names then adopted, just illustrated the mental character of their inventors, who, at all times, leaned more to their imagination than to their reasoning powers for a guide. As the number of compounds increased, or became known with respect to their constitution, great inconvenience was felt, and the necessity of some system of nomenclature was recognised.

Without entering into historical details on this subject, we state that the present plan has the advantage over all others which have been proposed, by giving an insight into the composition of the object designated. This we shall perceive as we proceed, and the student will soon appreciate its value. In naming elementary bodies, the old plan is still maintained to some extent; but this is of slight consequence, because each has but little relation to the others, and, therefore, a distinctive title is advantageous, no matter whence its source. When, however, we speak of bodies compounded of elements, it is desirable that we should know of what they are composed. We will take, for instance, the classes of bodies called oxides, &c., acids, and salts. The term *oxide* is applied to any substance containing a metal united with such a proportion of oxygen as is not sufficient to make it an acid in its properties. Thus, the rust of iron, commonly so called, is termed *oxide of iron* in chemical language. If it have but one equivalent of oxygen to one of metal, it is termed the *protoxide;* if two of oxygen, the *binoxide;* and so on—Greek prefixes being used to indicate the relative proportions existing between the metal and the oxygen.

Combinations of metals and chlorine are called *chlorides.* Thus, common salt, which contains chlorine and the metal sodium, is termed *chloride of sodium.* Combinations of iodine, bromine, and fluorine, are similarly termed *iodides, bromides,* and *fluorides.* By this arrangement we at once perceive the nature of the compound from its title. We may proceed a step further with respect to acids. Generally speaking, the names indicate their origin; thus, sulphuric acid is obtained from sulphur, by its combination with oxygen; nitric acid from nitre; hydrochloric acid from chlorine and hydrogen; but still the system to which we have referred, is not so strictly carried out with many of these compounds, as is shown in the names oxalic acid, acetic acid, &c. When, however, we deal with salts, we find no exception to the rule; and we here must explain what we mean by that term. A salt is always composed of an acid and a base. Thus, in carbonate of soda, we have the *acid* as the carbonic, and the *base* is the soda. In all the salts, both base and acid are indicated. In this application of the present system of nomenclature, we find the greatest convenience; as the following instances will show, when we compare the new with the old names. Thus, "copperas" is now called sulphate of iron, because it contains sulphuric acid and iron; "white vitriol" is the sulphate of zinc; "Glauber's salts," sulphate of soda; "Epsom salts," sulphate of magnesia; "blue vitriol," sulphate of copper. Combinations of nitric acid with a base, are similarly named. Thus, nitre is called nitrate of potass; and so each salt bears its own character in its name.

There are other combinations which differ from acids, &c., to which we must also direct attention. Thus, the combinations of sulphur with metals are termed sulphides; of phosphorus, phosphides; of carbon, carbides. In earlier chemical works, the terms sulphuret, phosphuret, and carburet, were used; but these have been changed, because of the analogy existing between these substances and the chlorides, &c. Other terms analogous to these will be met with when we study the chemistry of organic bodies; but such will be duly noticed in their proper place.

When the proportion between acid and base varies, their prefixes are employed to signify the fact. Thus, we have *proto*-sulphate of iron, containing one equivalent of acid to one of metal. *Bi*-carbonate of soda, containing two equivalents of acid to one of base; and thus, not only are the names of the constituents afforded, but their relative proportions are also indicated.

These instances will be sufficient to introduce our readers to the principles of chemical names; and we shall now proceed to refer to symbols, by which elements, oxides, acids, salts, &c., are distinguished. It is desirable that the student should acquaint himself with that of each element, as it will much facilitate his progress. A table of symbols will be found prefixed to the next chapter, in which we shall commence the description of each element and its compounds.

Generally speaking, each element is distinguished by the initial letters of its name. Sometimes those of the Latin term are used in preference to the English. The following are examples of each:—

Hydrogen has the symbol of . . .	H
Oxygen	O
Sulphur	S
Iron—Ferrum	Fe
Tin—Stannum	Sn
Silver—Argentum	Ag

By such an arrangement, each element, having a separate symbol, is readily distinguished from all others. When compounds are designated, then the symbols of their constituents are used together, thus—

HO, hydrogen and oxygen, represent water.
Ag O, silver and oxygen ,, oxide of silver.
Ag Cl, silver and chlorine ,, chloride of silver.

If, however, there are more than one equivalent of either element, then such is shown by means of small figures; as—$S\ O_3$, sulphuric acid, which, when free from water, is a compound of one equivalent of sulphur to three of oxygen.

Occasionally, a combination of symbols is required; and, in that case, the symbols of each compound are arranged together, by means of which the entire constitution of the compound is shown: thus, sulphate of iron is represented as—$Fe O + SO_3 + 7HO$; that is, it is composed of one equivalent of oxide of iron, one of sulphuric acid, and seven of water.

In many chemical works, the sign + is omitted, and the symbols are written thus—$Cu O, SO_3, 5HO$, which is that of the sulphate of copper, or "blue vitriol." This, of course, saves space, and is equally as intelligible as the method above named.

We may now show how these symbols are employed in connection with the actual value of the separate equivalents of a compound; and we would advise the student to commit to memory the different values assigned thereto. He will thus readily become acquainted with that of any compound. A table of these equivalents will be found prefixed to the next chapter. The following skeleton table will serve the purposes of illustration. In it we have used whole numbers, so as to save any difficulty with fractional parts.

Skeleton Table of Equivalents and Symbols.

Name.	Symbol.	Equivalent.
Hydrogen	H	1
Oxygen	O	8
Carbon	C	6
Nitrogen	N	14
Sulphur	S	16
Calcium	Ca	20
Chlorine	Cl	36
Sodium	Na (Natrium)	23
Silver	Ag (Argentum)	108

In the above we have included the elements only; but in the next table, we have the equivalents of bodies which are composed of some of these elements.

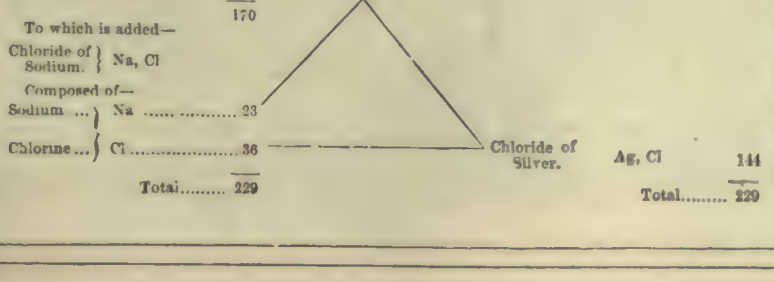
Skeleton Table of the Symbols and Equivalents of some Compounds.

Name.	Symbol.	Equivalent.
Water	HO	9
Sulphuric acid (dry)	SO_3	40
Sulphuric acid, with one equivalent of water, as found in commerce	SO_3, HO	49
Carbonic acid	CO_2	22
Chalk Carbonate of lime	$Ca O, CO_2$	50
Common salt Chloride of sodium	Na, Cl	59
Chloride of silver	Ag, Cl	144

It will thus be seen that the symbols and equivalents of a compound are composed of those of their elementary constituents.

We may now proceed to illustrate the method adopted for showing the results obtained in adding compounds in solution together, by which decomposition and the production of new compounds are effected.

Decomposition of Nitrate of Silver by means of Common Salt.


Materials used.
Name. Symbol. Equivalent.
Nitrate of Silver. Ag O, NO₅
Composed of—
Silver ...... Ag 108
Oxygen ... O..................... 8
Nitric Acid.
Nitrogen ... N 14
5 of Oxygen O₅ 40
170
To which is added—
Chloride of Sodium. Na, Cl
Composed of—
Sodium ... Na 23
Chlorine ... Cl 36
Total......... 229
Products.
Name. Symbol. Equivalent.
Nitrate of Soda. Na O, NO₅ 85
Chloride of Silver. Ag, Cl 144
Total......... 229

We have already done this to some extent in a previous page;* but did not there include the equivalents and symbols of the substances mentioned. In the preceding example, in which the decomposition of nitrate of silver is exhibited, each point is illustrated: and we recommend the student to study them carefully, so that he may have no difficulty in following our remarks in future pages. We have introduced the name of the materials employed, their symbols and equivalents, and those of the resulting compounds.

From the diagram, illustrating the decomposition of nitrate of silver by common salt, we find that 36 parts of chlorine leave the salt, and combine with 108 parts of silver, forming 144 parts of chloride of silver. Also that 54 parts of nitric acid, and one of oxygen, leave the silver in the nitrate, to combine with 23 parts of sodium, and so form 85 parts of dry nitrate of soda. We thus learn, that whilst the total amount of parts employed and obtained are equal—viz., 229 grains, ounces, or pounds—yet that the materials used, and the products of their mixture, are entirely different in every respect.

There are some theoretical considerations which might be noticed in reference to the formation of salts, by the addition of an acid to the oxide of a metal. Some would transfer the oxygen of the oxide to the acid. Thus, instead of indicating sulphate of copper, as $Cu, O + SO_3 + 5HO$; it would, according to such views, be $Cu + SO_4 + 5HO$. On this point, however, we shall have to remark hereafter. Dr. Daniell, of King's College, London, the inventor of the voltaic battery bearing his name, was, we believe, the first to broach the doctrine.

SPECIFIC GRAVITY, ISOMORPHISM, DIMORPHISM, ISOMERISM, ETC.

WE shall now refer to some matters connected with the physics of chemistry, in which the relative weight and forms of bodies are involved. We shall first explain how the specific gravities of solids, liquids, and gases are obtained; and then refer to some physical peculiarities noticed in crystalline and other bodies.

By the term "specific gravity," we mean that the same bulk of different bodies weighs differently; or, in other words, some substances contain, within any specific limit, more matter than others. For instance, if we fill a bottle of any size with mercury, and weigh it, and then pour away the metal and fill the bottle with water, we shall find, on reweighing it, that only one-fourteenth of the weight of the mercury is required to balance the water. In other words, mercury weighs, bulk for bulk, fourteen times nearly that of water. Almost every substance we meet with has thus a weight of its own, and this weight is called its specific gravity. With respect to solids, the difference is characteristic in all cases.

Now there is no difficulty in comparing the weights of equal bulks of water and mercury, because, as each is a liquid, it is easy to weigh equal bulks one against another; but this would be impossible if we were to attempt to ascertain the specific gravity of gold, iron, &c., in a similar manner; because, to render them fluid, we should have to employ so intense a heat as would instantly destroy our apparatus. This difficulty is easily overcome, in a manner we shall proceed to explain.

It is scarcely necessary to remark, that some standard of comparison is required, by means of which a uniform table of specific gravities may be constructed; and, by universal consent, water at a temperature of 60° Fah., in as pure a state as possible, has been fixed on. For this purpose it must be carefully distilled in glass vessels, and should then be kept out of contact with the atmosphere, to prevent absorption of air, which would lessen its own specific gravity.

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* See *ante*, p. 301.

The principle on which the specific gravity of solid bodies is taken, is, that any body immersed in a fluid, displaces as much as is equal to the bulk of the solid, and that a ratio exists between the respective weights of each in reference to that bulk. Thus, we will suppose equal bulks of gold and iron to be immersed in distilled water. They will each displace exactly the same *quantity* of water; but if the equal bulks of water, iron, and gold, be each weighed, they will bear a ratio to each other of 1, 7, and 19 nearly. Now, these ratios are, in round numbers, the actual specific gravities of the substances named; and hence, if we can ascertain the ratio existing between the weight of any body, and that of the bulk of water displaced by it when immersed therein, we have at once the specific gravity.

Now, this may be done in two ways. The first we mention will only illustrate the principle; whilst the second is that always employed. First, a vessel may be exactly filled with distilled water, and a body, previously weighed, may be immersed completely in it. Of course, as much water as is equal to the bulk of the body will run over the sides of the vessel. If all this could be collected and weighed, then the weight of the immersed solid, divided by that of the displaced fluid, would be the specific gravity. But, practically, this is impossible; because a considerable portion of the fluid would be lost by adhering to the vessel as it runs over the side.

Secondly, the specific gravity of any body may thus be accurately found. Suspend the substance, by means of a horse-hair or filament of silk, to the pan of a pair of scales, and weigh it carefully. Then immerse the body in a vessel containing distilled water, taking care that all air-bubbles are removed, and that the substance is entirely below the surface of the liquid. It will be found that the solid does not weigh so much when in the water as in the air. Balance it by means of weights placed in the scale from which the substance is suspended, and, having ascertained how much is required, divide the weight of the body as obtained in the air, by the loss it sustained when weighed in water, and the quotient will be the specific gravity. The loss of weight represents that of the bulk of water, equal to the bulk of the solid by which it was displaced.

To illustrate this plan, the following instance will suffice:—A piece of lead weighed in the air just 45 grains. On being weighed in water it lost four grains. The 45 grains is thus to be divided by the loss of four grains; and the result, 11·25, is the specific gravity of the lead. In this way the specific gravity of all substances weighing *more* than their own bulk of water, may be readily taken.

The practical value of this process is very great; for by consulting a table of specific gravities of bodies, we can often have a guide to the proportion in which they may exist in any substance containing two or more of them. For instance, a piece of quartz may have a greater specific gravity, in its mass, than any part of it possesses. This indicates an admixture with some other substance. Supposing that to be gold, a certain ratio will exist between the two specific gravities; that is, of the mass and of the part; and this ratio will increase directly in proportion as the quantity of gold contained therein. When a rough approximation only is required as to the value of auriferous quartz, this plan may be readily employed; and, in some instances, we have found, on subsequent analysis, that the result thus obtained is very near the truth. Of course, the presence of other foreign bodies will entirely vitiate the result. In the case of alloys, the method may be also adopted, provided that not more than two metals are present.

When the specific gravity is required of a body lighter than water, the same plan is adopted, with the addition, however, that the body must be sunk in the fluid by having attached to it a heavier substance, the weight of which in air, and loss when weighed in water, have been first ascertained. The loss of weight of the heavy body is then to be subtracted from their joint loss when weighed in water, and the weight of the light body in air is divided by the amount so obtained; the quotient is the specific gravity of the light body.

The specific gravities of liquids may be ascertained in various ways; for all that is required, is the ratio of weight of equal bulks of the different fluids. A common plan is that of using a bottle capable of containing exactly 1,000 grains of distilled water at a temperature of 60° Fah. The bottle is marked at the place at which the surface of the liquid reaches the neck of the vessel, and the liquid whose specific gravity is sought, is poured into it until it reaches that point. The weight of the whole is then ascertained; and that of the bottle having been deducted from the total, the balance is that of the liquid, and its specific gravity also. The plan is remarkably simple; and all liquids may thus have their specific gravity taken, whether they be lighter or heavier than their own bulk of water. Thus, a bottle holding 1,000 grains of water, will contain 1,840 grains of sulphuric acid, 790 of pure alcohol, and so on; and thus these weights give the respective specific gravities of the liquids named—as 1·84 and ·79 compared to water, the standard, which equals 1.00

There is, however, one point to which we must draw attention; and that is, the fact that all bodies expand by heat. This may be neglected in reference to taking the specific gravity of solids; but the case is different with liquids, having a rate of expansion much greater; hence their specific gravities must either be taken at a temperature exactly that of 60°, or allowance must be made for their expansion for higher, or contraction through lower temperatures.

Another mode of taking liquid specific gravities is that of the hydrometer, of which there are numerous forms. It is constructed on the principle that a body will rise in a liquid of greater specific gravity, and sink in one the specific gravity of which is less than its own. These instruments are generally constructed of glass, which is so weighted, that one portion of the scale enclosed in it shall be level with the surface of distilled water when the hydrometer is immersed therein. If the liquid be heavier than water, then the stem of the hydrometer will rise, as would be the case if salt and water were the subject of experiment. If the liquid be lighter, then the hydrometer would sink; and so, by reaching the scale at the level of the liquid, the specific gravity is at once obtained.

The following cut illustrates the appearance of one form of the instrument; but, as we have already remarked, they vary much in shape, size, and according to the uses to which they are to be applied.

Fig. 34.

Occasionally, glass balls, which have been so weighted as to swim in any part of a liquid of the same specific gravity as their own, are used. This plan, however, requires a separate ball for each possible specific gravity; and hence its use is confined within very narrow limits. There are other modes of ascertaining liquid specific gravities, such as weighing solids in them, &c.; but the above are the best practical methods.

The specific gravity of gases is taken by very similar means to those we have described in reference to liquids. But special care is required, because the variations of atmospheric temperature and pressure have the effect of interfering with the accuracy of the results obtained. The rate of expansion in gases far exceeds that of either liquids or solids; and hence, it is absolutely necessary that the temperature and pressure of the atmosphere should be very carefully attended to.

In practice, a glass globe, or other vessel, is first weighed as filled with air; it is then exhausted of air by means of a good pump, and again weighed, so as that the weight of the vessel itself may be known

This is a very delicate operation, because so many circumstances interfere with the accuracy of the result. When the atmosphere is at a high temperature, it contains a large amount of moisture, and so vitiates the calculations which have to be obtained, unless due allowance be subsequently made. The same remarks apply to temperatures below that of 60°, which is the usual standard; because, in that case, the absence of moisture in the air below 60°, would also give false results. The gases are, therefore, carefully dried by passing them through a tube containing chloride of calcium, or through strong sulphuric acid, either of which has so great an attraction for aqueous vapour as to abstract it readily.

The weight of the exhausted flask, deducted from that obtained when the vessel is full of air, gives the weight of the air which the flask would contain. The gas which is to be weighed, after having been dried, is then passed into the flask, which should be again exhausted, and subsequently refilled with the gas. By these means any chance of admixture with air left in the flask by the first exhaustion is avoided. Great care, however, must be taken that the gas is in a perfectly pure state, otherwise its accurate specific gravity cannot be obtained. An allowance must be made, in the subsequent calculation, for the expansion or contraction of the gas, if its temperature be either above or below 60°. The necessary data for this purpose have already been given.* The flask, thus filled with gas, is then carefully weighed, and if its capacity be exactly 100 cubic inches, the ratio of its weight to that of 100 cubic inches of air will at once be found. If the flask be of greater or smaller capacity, its reduction to that of 100 cubic inches, or the ratio of the weight of air to that of any gas, and, therefore, its specific gravity, is simply an ordinary arithmetical question.

We must, however, add another element of interference with respect to the accuracy of the results thus obtained. It is that of atmospheric pressure, which may be entirely neglected in solids, and in most cases in liquids As the atmospheric pressure lessens, all gases proportionally expand; and as it increases, gases contract in bulk. Now the usual standard of the specific gravity of gases is dry air, at a temperature of 60°, and, under a mean pressure, equal to that which supports 30 inches of mercury in the barometric column. Any excess or deficiency of pressure is allowed for, and proper corrections are made. The subject has already been referred to;† and we shall not extend our remarks thereon.

The student will do well to repeatedly practise himself in taking the specific gravities of solids and liquids. With respect to gases, we may observe, that he cannot hope to obtain anything like accurate results until he is well used to experimental manipulation. If, however, he possess the necessary apparatus, he can try the weight of different gases, comparing them with that of the air: and for such purposes, it may be useful if we state, that dry air, at 60° Fah., and 30 inches barometric column, weighs 31·0117 grains for each 100 cubic inches. There are certain phenomena observed in some compounds, which are partly the result of physical and chemical actions, such as those of isomorphism, isomerism, and dimorphism. By the term isomorphism, we mean that bodies may have similar external forms, such as crystals, &c., and yet have entirely different chemical constitution. Isomeric bodies have similar chemical constituents in the same proportions, and yet their external form may differ, as in sugar and starch. Some bodies have two different forms, or one dimorphic, under different circumstances. Thus phosphorus has a rich yellow, and partly crystalline appearance at ordinary temperature; but if it be considerably heated, it loses all external regularity of structure, and becomes a shapeless red mass. These varieties of form, doubtless, depend on a peculiar arrangement of the atoms of a body, produced by the action of two or more forces. We shall leave a fuller consideration of these subjects to the next chapter, when we shall meet with numerous instances which will fully illustrate these peculiarities.

In the following page will be found a table of the elements, their symbols, equivalents, &c.; to which the attention of our readers is especially directed, as forming the basis of all the calculations involved in chemical science, and to which we have already referred in our previous pages.

CHAPTER II.

THE CHEMICAL CHARACTERS OF THE ELEMENTS, AND THEIR COMPOUNDS.

HAVING reviewed the various agencies and circumstances which govern chemical phenomena in every species of matter, we shall now proceed to describe the elementary bodies, and the compounds to which their union gives rise. We have already remarked that almost every writer on chemical science adopts a different arrangement in treating the subject; and we shall, therefore, not hesitate to adopt a similar course, more especially as our previous pages have contained ample remarks and experiments on subjects which ordinarily form the introductory matter of chemical treatises. For these, we refer our readers to the sections on Heat, Light, Electricity, and Magnetism; to which we shall frequently allude, and the subjects of which we shall presume that the student has become fully acquainted with.

We shall commence this portion of the subject by describing the properties of such of the elements as, by combining with others as bases, produce definite chemical compounds. Of these, are Oxygen, Chlorine, Iodine, Bromine, and Fluorine.

We shall next take up Hydrogen and Nitrogen, which will be succeeded by another group, including Carbon, Sulphur, Phosphorus, &c. The metallic elements will then be dealt with; and we shall divide these into three classes; namely, Alkaligeneous, Calcigeneous, and Metals proper. The combinations of Oxygen, Chlorine, &c., will be considered in connection with each metallic element; and the salts produced by the combination of such bases with acids, will be ranged under the description of the base itself.

By this plan, we hope first to give a general idea of those substances which, by their general action, have the power of making numerous and analogous compounds; and afterwards we shall deal with those bodies which act as bases to the former, and so either become the subjects of synthesis or analysis.

In this chapter we shall principally confine our attention to inorganic substances, or those in which vital action is not present. We shall, however, touch incidentally on many organic compounds, or those with which vital action is connected. In the following chapter, we shall chiefly deal with what has been called Organic Chemistry; and in those which succeed it, we shall take up the subjects of Analysis, Agricultural Chemistry, and the general applications of Chemistry to Arts and Manufactures.

Many of our future remarks will be almost unintelligible to the student, unless he have become acquainted with all the subjects which have already been treated on. As we have remarked in the various preceding sections, each has a direct connection with the other. This arrangement prevents needless repetition, and saves the time and trouble of both reader and writer. At the present time, each branch of science is so linked with others, as to make it impossible to separate them.

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* See *ante*—Section 1, "Heat."

† See *ante*—Section 1, "Heat."

TABLE *showing the Symbols ; Equivalents, both on the Hydrogen and Oxygen Scale ; and the Specific Gravities of the Elementary Bodies.*

Name.	Symbol.	Equi. H = 1·00	Equi. O = 100·0	Specific Gravity.
Aluminium	Al	13·7	171·25	(W) 2·60
Antimony (Stibium)	Sb	129·0	1612·50	(W) 6·80
Arsenic	As	75·0	937·50	(W) 5·88
Barium	Ba	68·5	856·25	
Beryllium	Be	6·9	86·25	
Bismuth	Bi	213·0	2662·50	(W) 9·82
Boron	Bo	10·9	136·25	
Bromine	Br	80·0	1000·00	(W) 2·97
Cadmium	Cd	56·0	700·00	(W) 8·60
Cœsium*				
Calcium	Ca	20·0	250·00	
Carbon	C	6·0	75·00	(W) Diamond 3·50 Plumbago 1·86
Cerium	Ce	47·0	587·50	
Chlorine	Cl	35·5	443·75	(A) 2·47
Chromium	Cr	26·7	333·75	(W) 6·00
Cobalt	Co	29·5	368·75	(W) 8·54
Copper (Cuprum)	Cu	31·7	396·25	(W) 8·89
Didymium	Dy	50·0	625·00	
Erbium	Er			
Fluorine	F	19·0	237·50	
Gold (Aurum)	Au	197·0	2462·50	(W) 19·26
HYDROGEN	H	1·0	12·50	(A) 0·069
Iodine	I	127·0	1587·50	(W) 4·95
Iridium	Ir	99·0	1237·50	
Iron (Ferrum)	Fe	28·0	350·00	(W) 7·50
Lanthanum	Ln	47·0	587·50	
Lead (Plumbum)	Pb	103·7	1296·25	(W) 11·45
Lithium	L	6·5	81·25	
Magnesium	Mg	12·0	150·00	(W) 1·70
Manganese	Mn	27·6	345·00	(W) 7· to 8·
Mercury (Hydrargyrum)	Hg	100·0	1250·00	(W) 13·50
Molybdenum	Mo	46·0	575·00	(W) 7·40

Name.	Symbol.	Equi. H = 1·00	Equi. O = 100·0	Specific Gravity.
Nickel	Ni	29·6	370·00	(W) 8·70
Niobium	Nb			
Nitrogen	N	14·0	175·00	(A) 0·97
Norium	No			
Osmium	Os	99·6	1245·00	(W) 10·0
OXYGEN	O	8·0	100·00	(A) 1·1057
Palladium	Pd	53·3	666·25	(W) 11·5 to 11·8
Pelopium	Pe			
Phosphorus	P	32·0	400·00	(W) 1·75
Platinum	Pt	98·7	1233·75	(W) 21·00
Potassium (Kalium)	K	39·0	487·50	(W) 0·86
Rhodium	R	52·2	652·50	
Rubidium†				
Ruthenium	Ru	52·2	652·50	(W) 8·60
Selenium	Se	39·5	493·75	(W) 4·30
Silicium	Si	21·3	266·25	
Silver (Argentum)	Ag	108·0	1350·00	(W) 10·50
Sodium (Natrium)	Na	23·0	287·50	(W) 0·97
Strontium	Sr	43·8	547·50	
Sulphur	S	16·0	200·00	(W) 1·98 to 2·0
Tantalum	Ta	184·0	2300·00	
Tellurium	Te	64·2	802·50	(W) 6·10
Terbium	Tb			
Thorium	Th	59·6	745·00	
Tin (Stannum)	Sn	58·0	725·00	(W) 7·25
Titanium	Ti	25·0	312·50	
Tungsten (Wolframium)	W	92·0	1150·00	(W) 17·60
Uranium	U	60·0	750·00	(W) 9·00
Vanadium	V	68·6	857·50	
Yttrium	Y			
Zinc	Zn	32·6	407·50	(W) 7·15
Zirconium	Zr	33·6	420·00	

* and † are new metals, discovered by means of the spectrum analysis, and, at present, have been obtained in quantities too small to permit of their equivalents and specific gravities being given. Glucinum and Thallium are referred to under their respective heads.

Note.—The specific gravities above are distinguished by the letters A and W, which signify, respectively, that the standards are—

Air = A = 1·000.
Water = W = 1·000.

In consulting the above table, the reader may perceive some slight differences, when the equivalents, &c., are compared with others adopted in the following pages. These have been given as the nearest approximations to truth ; and where any degree of uncertainty existed, or information could not be afforded, as in erbium, niobium, &c., the spaces have been left unfilled ; such substances being exceedingly rare.

OXYGEN.

THIS elementary body is of the utmost importance in the economy of nature. It forms $\frac{8}{9}$ths of all water on the face of the globe. The atmosphere which we breathe contains one-fifth of its bulk of oxygen; and all the earths, oxides of metals, organic substances of nearly every kind, have it as one of their constituents. In its entire absence, the very existence of animated nature would cease. The earth would become a barren wilderness; every one of its productions would fail, and chaos would reign over the fair face of nature once more. To its gradual combination with other substances, we owe the natural heat of plants and animals. When such combinations take place more rapidly, we obtain our artificial heat, as in the combustion of wood, coals, and other fuel.

To its action on metals is due the development of that wonderful agent, Electricity, as seen in voltaic batteries, and in the vast laboratory of the mineral kingdom. All our sources of artificial light depend on it; and, indeed, it is the "one thing needful" in almost every phase of our existence, happiness, and comfort.

The chemist recognises its properties in the form of a gas; and in this state oxygen is heavier than the air—its specific gravity being 1·111 air = 1·000; it hence follows that 100 cubic inches will weigh 34 grains. Its equivalent, hydrogen being 1·00, is 8·00; and it combines, by volume, with that gas in producing water in the proportion of one of oxygen to two of hydrogen. It assumes the liquid or solid state when in combination with other bodies, as in water and the metallic oxides.

In the following pages, we shall not follow the plan usually adopted in scientific works, because we shall introduce a considerable variety of experiments, having for their object the elucidation of the facts we mention. In so doing, we expect that our readers wish to become practically acquainted with the subject, and we shall endeavour to assist them as much as possible in that respect. We have already described many kinds of apparatus which will be required, and shall afford descriptions of others which may be necessary for special purposes.

Oxygen may be procured, for experimental purposes, from various sources, some of which we shall fully describe. Amongst these are the following:—

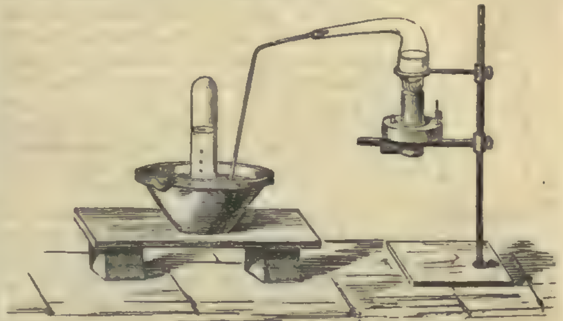
Experiment 5.—Introduce some powdered black oxide of manganese into an iron tube closed at one end, or into a gun-barrel, and into the open end of either vessel fit a pewter tube, so that its other end may dip beneath water. Apply heat, as of a common fire-place, to that part of the iron tube containing the manganese. After a short time, a quantity of gas will be given off, which, on being collected by means of the pneumatic trough and gas jars,[*] will be ready for experimental purposes.

When a large quantity of oxygen is required, a mercury bottle and bent tube are used; the vessel being but half filled with the oxide of manganese. The arrangement has already been described, together with a convenient kind of gas-holder.[†]

Experiment 6.—Into a small-sized glass retort,[‡] introduce some of the black oxide of manganese, and then add sufficient strong sulphuric acid to make the whole of a thick creamy consistence. Rest the bulb of the retort on a ring-stand, and allow its stem to dip below the water in a pneumatic trough. Apply a gentle heat to the mixture, and in a short time oxygen will be given off, which may be collected in the usual manner. The mode of conducting this and similar processes for making gases, is illustrated in the following engraving; in which, however, a bent tube is added to the stem of the retort, instead of dipping it beneath the water in the trough.

Oxygen may be obtained by heating other metallic oxides, &c.; but the following mode is always adopted when very pure oxygen is required; and unless the gas is wanted in very large quantities, it is altogether, perhaps, the best source for all purposes.

Fig 35.



Experiment 7.—Mix one part of chlorate of potass with half its weight of black oxide of manganese: introduce this into a retort, and apply the heat of a spirit-lamp or gas-furnace.[§] In a short time the whole mass will become incandescent, and oxygen will be rapidly given off, which may be collected in the usual way. Some little care is required, as if too great a heat be suddenly applied, the gas is given off with so much violence as to become unmanageable. By means of the mercury bottle and this mixture, we have frequently made thirty cubic feet in less than half-an-hour.

We should here warn those of our readers who are unacquainted with the nature of chlorate of potass, that although by itself it is a perfectly harmless substance, it gives rise to dangerously explosive substances when mixed with other articles. Amongst these are sulphur, phosphorus, and sulphuric acid; and, to a slighter extent, many organic substances, such as sugar, &c., have a similar effect. It should therefore be kept by itself; and to prevent any risk of admixture, the experimentalist should make a considerable quantity of the mixture of the salt and oxide of manganese, which he may keep by him, ready for use, in a glass bottle. This may be labled "oxygen mixture;" and when we use this term in our future pages, it will be in reference to this compound.

In heating the mixture, the oxide of manganese is not decomposed. It merely affords points of contact, by means of which the heat spreads more rapidly through the salt, and saves time. That such is effected, may be noticed by the following experiment.

Experiment 8.—Heat some chlorate of potass alone in a small retort or test-tube, fitted with a bent tube, by means of the spirit-lamp. The salt will melt, and will give off oxygen slowly.

Instead of retorts, a Florence flask and bent tube[‖] may always be employed. This is the most economical mode of procedure, as the fracture of the flask is of no consequence, from its trifling value. It is well that our readers should diminish, as much as possible, the expenses of their experiments; and, at the same time, thereby acquire a knack of substituting one arrangement for another.

By heating nitre in a similar manner to that adopted with chlorate of potass, oxygen may be obtained. There is, however, one objection to this plan; which is, that a considerable quantity of nitrogen is given off at the same time; which, of course, injures its purity. Another source of oxygen, which is necessarily quite pure, is that of the decomposition of water by means of voltaic electricity, which has been already fully described;[¶] but the practical difficulties and expenses preclude its use in ordinary circumstances.

A sufficient quantity of oxygen having been prepared, and collected in gas jars or bottles,[**] the following experiments may be tried, which will exhibit some of its leading properties.

Experiment 9.—Dip a lighted taper into a jar of oxygen. The light will be greatly increased, and the wick will burn much faster than in the open air.

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[*] See *ante*, p. 304; Fig. 9.

[†] *Ante*, p. 304; Figs. 12 and 13.

[‡] *Ante*, p. 303; Fig. 1.

[§] *Ante*, p. 306; Fig. 22.

[‖] See *ante*, p. 304; Fig. 3.

[¶] *Ante*—Section 3, "Electricity."

[**] *Ante*, p. 304; Fig. 8.

Experiment 10.—Blow out the light of a taper so as to leave a little of the wick red-hot. Whilst in this state dip it into a jar of oxygen, and it will immediately re-light. This may be repeated many times with the same jar of gas. The tapers best suited for this experiment are those used for lighting gas; but a common tallow candle answers nearly as well. (See Fig. 36).

Fig. 36.

Experiment 11.—Tie a small piece of charcoal to the end of a piece of thin brass or copper wire; make the charcoal red-hot, by holding it in the spirit-lamp flame. When red-hot, immerse it in a jar of oxygen; a most beautiful combustion will take place, sparks flying off on all sides. Boxwood charcoal answers best for this purpose.

Experiment 12.—Put a small piece of phosphorus, not larger than a pea, into the phosphorus cup or holder.* Remove the glass stopper or cork from one of the jars of gas, and set fire to the phosphorus by means of a taper. Dip the cup into the jar immediately, and a splendid light will be produced. This forms one of the most brilliant experiments in chemistry. The heat often cracks the jar; and as there is no sure way of preventing this, we recommend the use of the common preserve or sweetmeat bottle, as being of little cost in case of breakage. Fig. 37 shows the method of this experiment.

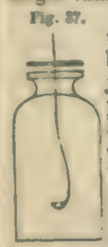


Fig. 37.

Experiment 13.—Put a small piece of sulphur into another holder, first lighting it, by heating the cup in the flame of a candle; introduce it into a jar of oxygen, and a fine blue light will be given out.

The two last experiments afford instances of the production of acids by burning the two substances in oxygen. These acids may easily be detected by tasting the water which is left on the sides of the glass whilst making the gas; this absorbs the fumes, and will be found strongly acid. The name of oxygen is, in fact, derived from two Greek words—"oxus," acid, and "gennao," to produce; the early chemists imagining that it was the sole acid-producer. The acids we refer to as produced above, are the phosphoric and sulphurous.

A distinction is made between acids produced by combinations of oxygen, and those produced by other substances. Sulphuric, nitric, and many other acids, depend for their qualities on the union of oxygen gas with other substances. The hydrochloric and other acids, however, have no oxygen in their composition; and hence it is proved, that oxygen is not the only element on which the production of acids depends. Its importance, in this respect, is of the highest kind. Of this, however, we shall have to speak more fully hereafter.

Oxides, as we have frequently mentioned, are combinations of oxygen with a metallic substance, by which both alkalies and earths are also produced. These distinctions, which have existed from the earliest days of chemical science, have a great convenience, as they classify the oxides according to their special characters. Thus, if oxide of iron, potass, soda, baryta, magnesia, lime, &c., were spoken of simply as oxides, their special characteristics and qualities would not be so evidently pointed out. Nevertheless, it is a rare case to find the earths in a state of absolute freedom from those substances to which the term is most usually applied; for, as in the case of clay, lime, &c., oxide of iron is generally present; and, indeed, the only source of all our metals is from rocks and sands, to which the term "earths" is usually applied instead of that of oxides.

There are also numerous instances in which an oxide may, by an increase of oxygen, be converted into a substance having acid properties: or, perhaps, we may put the question in another form, and state, that some substances will combine in such proportions with oxygen as to form both oxides and acids. The following may be selected as instances of these combinations:—Iron forms oxides, and also an acid—the ferric—when combined with variable proportions of oxygen; the same occurs with the metals arsenic, chromium, &c. Even with chlorine an acid is formed by oxygen;—hence the chloric and perchloric acids. But into these, and similar combinations, we shall have to make an extended examination.

Confining our attention to the production of ordinary metallic oxides, we may observe, that a vast variety is found in their appearance and general characteristics. Some metals, such as gold, silver, and platina, have very little attraction for oxygen; hence they may be exposed to atmospheric and aqueous influence, and yet remain unchanged for centuries. On the other hand, the metals potassium, sodium, &c., have so powerful affinity for it, as to abstract it from almost any of its compounds; as may be seen by the following experiment.

Experiment 14.—Throw a small piece of potassium on to some water. The liquid will be rapidly decomposed; and so violent will be the action, that a large amount of light and heat is liberated. The result is the production of potass, the alkali, or oxide of potassium, which will be dissolved in the water.

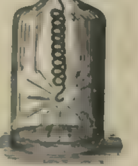
Between potassium and gold, the affinities of the metals for oxygen have a very extensive gradation; and yet such is modified by circumstances. Thus, not only are air and moisture favourable to the production of oxide; but heat and electricity have a very powerful effect, although that of the second force is generally subsidiary to, or derived from, that of heat. Hence, the process of oxidation is either slow or rapid, according to the circumstances in which the oxidising body is placed. The following experiments will fully illustrate this fact.

Experiment 15.—Hang a piece of polished and thin watch-spring in a jar containing some strong sulphuric acid. The metal must not touch the acid, and the vessel must be tightly corked. The polish will remain on the metal for almost any length of time, because the acid, by absorbing the moisture of the air in the vessel, prevents the formation of an oxide on its surface.

Experiment 16.—Expose the same piece of spring to air and moisture by placing it in a damp place. In the course of a day or two, its surface will become covered with oxide, owing to the oxidising action of air and moisture together.

Experiment 17.—Heat a piece of polished iron or steel in the flame of a spirit-lamp. Its surface will become rapidly covered with rich colours, owing to the formation of various oxides.

Experiment 18.—To one end of a piece of steel spring, or twisted steel wire, as thin as possible, and about ten inches long, tie tightly a piece of wick about an inch long, taken from the inside of a composite candle or a common taper. Pass the other end of the spring through a bung or cork, which will form a holder. On lighting the taper and dipping it into a jar of oxygen, the steel will catch fire, burning most brilliantly, and filling the interior of the jar with a red powder. Now here we have the rapid production of the oxide, which is caused by the influence of heat in promoting the chemical action of the pure gas on the metal. But the rapidity of this is exceeded in that which takes place when a powerful charge from a Leyden jar is sent through a fine steel or brass wire. By that means the metal is *instantaneously* converted into an oxide.

Fig. 38.


The reader will notice, by the last few experiments, how much chemical action is modified by extraneous circumstances. In fact, we have a double object in introducing these experiments here. One being to show the modifications which chemical action sustains in respect to oxidation; and the other, to introduce to the notice of the student, the necessity which always exists for the exercise of a comprehensive view when we attempt to explain or predicate the result of chemical affinities. The metal zinc also illustrates the varying rapidity and effects of oxidation; for if it be exposed to air and moisture, a superficial coat of oxide will form, which will protect the mass of the metal from further action; whilst if the same metal be burned in a jar of oxygen, or thrown into a ladle containing melted nitre, its oxidation is most rapid, and dense fumes of the oxide are given off from its surface. Even if some melted zinc be cast into a fire, its combustion will be very vivid, and the oxide will be produced in abundance, being attended by the production of a brilliant blue flame.

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* See *ante*, p. 304; Fig. 10.

The process of combustion generally results in the production of an acid, when carbonaceous or organic matter is present. Thus, in experiment 11, in which charcoal was burned in oxygen gas, the resulting product would be carbonic acid; and, in most cases, it matters not whether the combination takes place rapidly or otherwise. Thus the act of breathing, or of inspiring air into the lungs, is attended by the combustion of carbon in the blood of the individual, as presented by the surface of the lung to the oxygen of the inspired air. In this instance, a very slow but complete combustion results, and carbonic acid is afforded. The act of breathing, and the rapid combustion of fuel, may be taken as extreme instances of the kind; but between these a gradation exists, in which the production of heat and carbonic acid is afforded. Thus, when the atmosphere has access to substances containing carbon of the fatty or oily kind, and in contact with fibrous matter, a slow combustion progresses. If circumstances are favourable, the rate of this combustion rapidly increases; and, eventually, spontaneous combustion ensues. To this we owe the frequent destruction of buildings in which hemp, jute, and cotton waste are stored. Even if these substances be in a damp state, the same result will occur. During fermentation or germination, the changes produced are of a similar nature to those we have mentioned. Oxidation, in fact, proceeds; and, as is well known, such is always, in these instances, attended with the production of heat. The heat set free at the dung-heap, and in the processes of malting, are cases which illustrate these observations.

We have previously remarked, that all sources of artificial heat depend on the presence of oxygen as necessary to combustion. Almost every material employed by man for the production of heat, contains carbon in some form or other; and, according to the rapidity with which the oxygen in the air is caused to unite therewith, so the intensity of combustion is increased. This is effected in the stove, fire-place, and steam-engine furnace, by means of chimnies which produce a strong draught, and so cause a rapid flow of oxygen with the air through the fire-bars. In the locomotive engine, the combustion of fuel is rapidly effected by means of the blast-pipe of the cylinder which conveys the waste steam into the funnel. In the iron smelting furnace, and in other metallurgical processes, intense heat is produced by an engine, &c.; and the domestic bellows is a similar class of article, and used for the same purpose. In some phases of combustion, oxygen is supplied in a latent state. This occurs in gunpowder, in which the nitre is the source of oxygen, and affords it in great quantities. In such cases, bodies can burn without access to the atmosphere; and, in most mixtures of the chlorates or nitrates with dry organic bodies, this readily takes place on heat being applied.

The general effect of oxygen in nature, is that of a life-giving principle. It breaks up the rocks, and converts the sterile land into a fruitful country. Through its action on the organic substances in plants, they bud forth into leaf, and produce the flower and seed. Every leaf is as a lung to a tree, and thereby plants are enabled to breathe and to exist. In animals, oxygen oxidises the carbon of the food, producing animal heat, as we have already shown; and thus its agency is so universal as to leave no object out of its influence.—It is desirable that the student should become fully acquainted with its general properties and character, as throughout the following pages we shall have frequently to refer thereto. A careful repetition of the experiments we have named, will therefore prove of great use in this respect.

OZONE.

It was discovered some years ago, by M. Schonbein, that a peculiar result was produced when water was decomposed by means of voltaic electricity; and many were the guesses of chemists and others as to its causes. When a little dilute sulphuric acid is submitted to the action of a voltaic battery having two platina poles, a strong odour is soon produced, which has almost exactly the same smell as may be noticed when a few sticks of phosphorus are put into a glass bottle with some water.

Ozone—so called from its smell—is thus produced, and it has lately been considered as having an individual character and properties. It has, however, never been detected in such a form as to be submitted to any process except that of testing for its presence; and hence, although we are acquainted with some of its chemical effects, those of a physical character have yet to be discovered. It has decided bleaching properties; promotes putrefaction, and produces chemical decomposition. It is considered to be either a product of oxygen, the active principle of oxygen, or a new element.

The tests for the presence of ozone are salts of manganese, and the iodide of potassium. The best mode of detecting its presence, is that of employing white blotting or filtering paper, which has been moistened with a solution of iodide of potassium and starch, such as arrow-root, &c. If free ozone be present, the iodide is at once decomposed, the iodine is set free, and the starch becomes of a blue colour.

Although but little is known of ozone, it has considerable interest attached to it, in a sanitary point of view. It has been found, that air charged with animal matter never exhibits ozone in a free state; and that, generally speaking, the more there is of free ozone in an atmosphere, the healthier it is for the purpose of animal existence. Any one who has once smelt ozone, will readily detect its presence in the air which chiefly occurs during clear and frosty weather. Occasionally, it may be noticed before the change of weather, and in small patches or districts of country. As we speak of our own experience only, it may happen that many of our readers may not be so sensitive to its action on the nasal organs. Such, however, may easily resort to the starch-paper test.

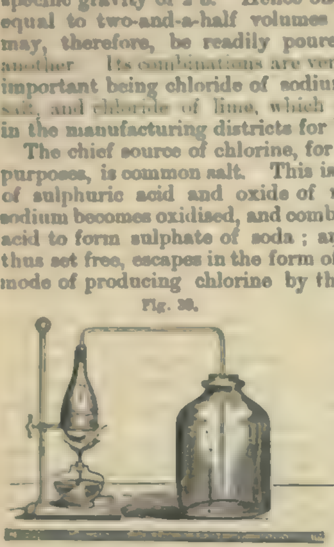
Some very interesting experiments were made, some time ago, in reference to the presence of free ozone. Whilst it could be instantly detected by the manganese test, in the middle of an open thoroughfare; the sides of the road next to houses, and courts and narrow openings running therefrom, indicated an absence of ozone. This fact shows that some intimate relationship must exist between the functions and products of animal vitality, and the presence of the substance. At the present time, the amount of free ozone present in the air, is registered at most meteorological observatories; and the reports of medical officers also embrace the same subject. The indications are measured by the relative discoloration which the iodide and starch paper undergo; a set of test-papers, of graduated tints, being kept as a standard of comparison. This subject will be again referred to, under the head of Meteorology.

CHLORINE.

For a long time, it was considered by chemists, that hydrochloric or muriatic acid had oxygen as one of its constituents; and until Sir Humphry Davy discovered chlorine to be an elementary, and, indeed, for some time afterwards, that opinion was maintained. At the present day, however, chlorine is placed amongst the elementary bodies, and is one of the most active agents in its chemical relations, next to oxygen, with which we are acquainted.

Chlorine, in its pure and uncombined state, is a gas of a yellow-greenish colour. It has an exceedingly strong, and, to some persons, offensive smell. If inhaled, it causes asphyxia, and acts violently on the animal membranes in the throat and lungs. Its equivalent is 35·5; but for ordinary calculations, it is assumed, in round numbers, to be 36, and is much denser than air, having a specific gravity of 2·5. Hence one volume of chlorine is equal to two-and-a-half volumes in weight of air. It may, therefore, be readily poured from one vessel to another. Its combinations are very numerous; the most important being chloride of sodium, the common table salt, and chloride of lime, which is so extensively used in the manufacturing districts for bleaching purposes.

Fig. 39.



The chief source of chlorine, for commercial and other purposes, is common salt. This is decomposed by means of sulphuric acid and oxide of manganese, when the sodium becomes oxidised, and combines with the sulphuric acid to form sulphate of soda; and the chlorine being thus set free, escapes in the form of a gas. A convenient mode of producing chlorine by this or other means, is represented in the annexed cut. The materials are put into a Florence flask, fitted with a bent tube; the latter passing to the bottom of a clean jar, the neck of which should be loosely fitted with a cork. The flask is supported in position by means of a ring-stand; and a gentle heat is to be applied by means of a spirit-lamp. We should here observe, that as chlorine is rapidly absorbed by water, it cannot be collected in the pneumatic trough; and mercury must not be used instead of water, because it is readily acted on by the gas. We may here caution our readers that the inhalation of chlorine gas produces the most painful effects. An assistant left some bottles unlabelled, and being unaware that such were filled with chlorine, we had the misfortune to find the evil effects of inhaling a considerable quantity, which resulted in loss of voice, and other severe and unpleasant symptoms. Should any of our readers be similarly unfortunate, their remedy will consist in immediately swallowing some unboiled white of an egg, and applying to the nose a moderately strong solution of ammoniacal gas (liquid ammonia of the shops).

Of course, the materials for producing chlorine from common salt, &c., should be added together in their equivalent proportions, to obtain the proper amount of chlorine by their mutual decomposition; but as they are exceedingly cheap, this is a point of no great importance when the gas is employed for laboratory or private experiments. There are, however, more convenient modes of obtaining chlorine on the small scale than that we have mentioned; and to such we must now refer.

Hydrochloric acid, to which we have already referred in our former pages, is a compound of chlorine with hydrogen gas; and if we can separate these elements, of course we may obtain the chlorine or hydrogen free. To separate the former is an easy matter. Some oxide of manganese may be introduced into the flask, represented in Fig. 39; and then sufficient hydrochloric acid is to be added to moisten the powder. The two should be well mixed, otherwise the oxide of manganese may remain as a dry powder on the bottom of the flask; and when heat is applied, the acid may by chance run down, and, touching the heated portion of the flask, will fracture it. After thus forming a kind of paste, more acid is to be added until a cream-like mixture is produced. The bent tube is to be replaced, and a gentle heat may be applied to the flask, when the gas will be speedily produced. Care should be taken that the materials in the flask do not froth up and pass over into the jar receiving the gas. This may arise either from too great a heat being applied, or from the materials being too dry from a deficiency of acid. The heat may gradually be raised towards the close of the process. The chlorine thus produced, being heavier than the atmospheric air, will displace the latter from the glass vessel into which it is passing, and, except for special experiments, it will be pure enough. If it be received in wide-mouthed stoppered bottles, the neck should be slightly greased with tallow, and the glass stopper may be then put in its place, being at the same time turned once or twice in the neck, so as to ensure a good distribution of the grease. The bottles, as with all others containing gases, should be distinctly labelled to prevent mistakes; and thus any quantity of chlorine may be prepared, and kept for experiments. Chlorine is frequently required in the chemical laboratory. The student may try the following experiments, for the purpose of becoming acquainted with the properties of the gas.

Experiment 19.—Suspend a piece of cloth or paper, dyed with any vegetable colour, and in a moist state, or a flower previously drenched with water, in a jar of chlorine. The colour will be speedily discharged, owing to the bleaching powers of the gas.

Chlorine, as we have already remarked, is largely employed for the purposes of bleaching, and in some departments of calico-printing, for discharging colours. It is used in the form of chloride of lime, which is prepared by passing gaseous chlorine over freshly slaked lime. We shall have to refer more fully to the use of chloride of lime, when we treat of the applications of chemistry to arts and manufactures.

Experiment 20.—Make a strong solution of litmus or indigo in cold water, and pour this into a jar of chlorine gas. The chlorine will be rapidly absorbed, and the coloured liquid bleached.

Water or moisture is essential to the bleaching powers of chlorine; hence the *dry* gas does not bleach vegetable colours.

Experiment 21.—Throw a few pieces of fused chloride of calcium into a jar of chlorine, and leave them for some time, so that they may absorb all moisture. Into the dry gas introduce a piece of dry coloured cloth or paper, but no bleaching effects will result. Indeed, it is generally supposed that the bleaching effects of chlorine really result from the decomposition of water. The chlorine seizes the hydrogen of the water, and sets the oxygen free; and this acting in a latent state on the colouring matter present, at once bleaches it. In former times, before the discovery of chlorine, all cotton and linen goods were bleached by exposing them to the action of the sun, air, and moisture—a process, however, which was exceedingly slow and tedious.

Chlorine has a limited power of supporting combustion, as may be seen by trying the following experiment.

Experiment 22.—From a wire fixed on a cork, suspend a small piece of taper; and having ignited it, immerse it in a jar of chlorine. The length of the flame will be increased, and it will become dull and smoky, depositing a sooty matter on the inside of the vessel.

The gas has a very great attraction for some substances; and the production of light and heat frequently results from some of its combinations as they take place.

Experiment 23.—Introduce some Dutch foil into a jar of chlorine. The leaves will immediately catch fire, and a chloride of copper will be produced. Antimony, &c., in a state of fine powder, which should previously be warmed, will also ignite in a jar of chlorine, and produce a chloride of antimony.

Chlorine forms chlorides when combined with metals, &c.; of which we shall speak when treating on those bodies. It also combines with nitrogen, iodine, and bromine, forming compounds therewith, which will be fully described hereafter. With hydrogen it forms the well-known hydrochloric acid, also called the "muriatic" and "marine" acid, in the old chemical works.

Hydrochloric acid, in its pure state, is a gas composed of one equivalent of chlorine, united with one of hydrogen. Its equivalent is therefore 36·5; symbol, H, Cl. In commerce, and in the laboratory, this gas is employed as a liquid acid, being in combination with water. In this state, and having a specific gravity of 1·21, it is composed of the gas united with six equivalents of water; its symbol is H, Cl, 6HO; the equivalent being 90·5.

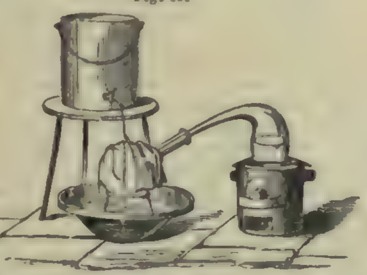
Hydrochloric acid gas is readily procured from common salt, by adding dilute sulphuric acid to that substance in a retort. A gentle heat is to be applied, and the gas will be given off in abundance. Owing to its great attraction for water, it seizes the aqueous vapour present in the air, producing white fumes. It may be collected in bottles, in a similar manner to chlorine. On pouring cold water, or putting a piece of ice into a jar of the gas, it is rapidly absorbed, and considerable heat is evolved. Its presence may readily be detected by means of a rod dipped in liquid ammonia, when dense white fumes of sal-ammoniac are afforded. Hence, if a bottle of the gas be taken into a stable, the presence of ammonia will be instantly detected by the production of the fumes named.

Hydrochloric acid is produced on a large scale during the manufacture of soda. The common salt is decomposed by means of sulphuric acid, and the gas given off is condensed by water.

For laboratory use, the acid may be prepared as follows:—

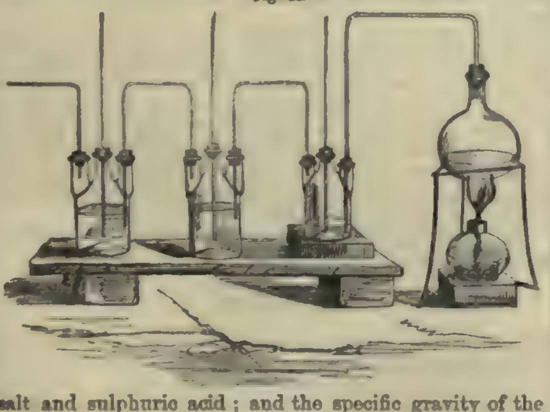
Experiment 24.—Introduce into a glass or earthen retort some common salt and water, and add to this nearly as much sulphuric acid. Apply heat by means of a spirit-lamp or furnace, and receive the gas in a Florence flask or globular receiver containing cold water, the stem of the retort just touching the surface of the water. Keep the outside of the receiver as cold as possible, by covering it with a cloth on which water is kept dropping. The gas will be rapidly absorbed, and the aqueous acid produced. The acid will be sufficiently strong for all laboratory purposes. The reason the stem of the retort is not placed *below* the surface of the water in the receiver, is, that the absorption of the gas takes place so rapidly that the liquid would rush into the retort, and perhaps break it. The following cut shows a convenient mode of carrying out this plan.

Fig. 40.

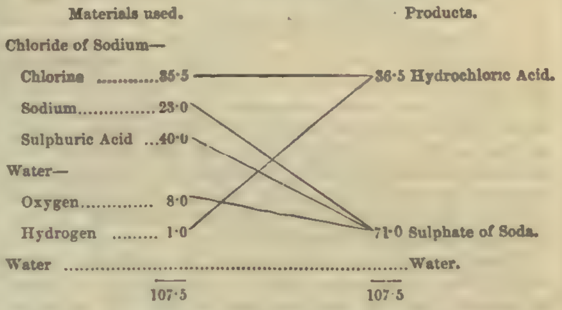


A different arrangement is used when a large quantity of the gas is prepared. Bottles with two or three necks and connecting pipes are used. These are made of glass or earthenware, and are of large size. The gas is produced, in globular vessels of iron lined with lead, from salt and sulphuric acid; and the specific gravity of the acid so prepared, will be greater than that produced by the process we last described, because the condensation of the gas is more perfect, a larger portion being dissolved in the water placed in the receiving vessels. The preceding engraving illustrates this process.

Fig. 41.



The chemical changes produced during the manufacture of hydrochloric acid may be thus represented:—



In the above diagram, the proportion of water forming crystallised sulphate of soda is omitted, so as to avoid confusion of figures to the uninitiated student. We have only, therefore, represented absolutely dry sulphate of soda as the product. We therefore notice, that 58½ parts of common salt should afford 36½ parts of hydrochloric acid gas, which, during absorption, combines with at least 54 parts of water, to form the strong aqueous acid.

We have hitherto simply spoken of acids in a very general manner; and as hydrochloric acid is the first to which we have drawn special attention, we shall avail ourselves of the opportunity of more particularly referring to the general characteristics of these substances.

Generally speaking, the test for the presence of a free acid, is a sour taste when it is applied to the palate; the power of converting vegetable colours of blue tint to a reddish hue, and of forming salts or salt-like bodies. In some cases, the acid, or sour taste, is scarcely distinguishable: the prussic acid is an instance of the kind. Carbonic acid has but feeble and transitory effects on vegetable blues; but, in every case, an acid has the power of forming a salt when combined with a base; or, what is equivalent thereto, a compound is formed, which answers in its analogies to a salt. The following experiment with hydrochloric acid, will serve the double purpose of illustrating the general properties of acids, and the special characteristics of hydrochloric acid.

Experiment 25.—Make a solution of litmus by boiling some of that substance in water. After the liquid has obtained a deep blue or purple colour, add a very small portion of liquid ammonia, to remove any free acid; and then dip into the solution some pieces of white blotting or filtering paper. They will, of course, become of a blue colour, and will produce the general test employed by chemists for detecting the presence of a free acid. Drop some hydrochloric acid on the blue paper, and it will at once become of a red tint. If such paper be brought into contact with almost any acid, the same effect will be produced. In the case of carbonic acid, the effect is slight, because that has but feeble acid properties, and is readily removed by evaporation. Most of the blue colours of vegetables are thus affected; but indigo is an exception. The acids found in fruit, &c., generally called organic—such as the acetic, tartaric, oxalic, citric, &c.—are also detected by their producing a red tint when tested by infusions of litmus or red cabbage. We may here suggest a very interesting occupation in connection with plants, fruit, &c. It is, that the student should supply himself with blue litmus paper, and the same coloured red, by means of a little weak vinegar. By the blue paper he may discover the presence of acid in vegetable juices; and, by the red. the alkaline character of others may also be detected.

We may, after this digression, return to consider some of the properties of hydrochloric acid. With potassium, sodium, and ammonia, it respectively forms the chlorides of those substances, the two best known being the common salt and sal-ammoniac. It forms chlorides with metallic bases, uniting with them in various proportions—in that of one equivalent, as in chloride of silver, or in three, as in the terchloride of gold. The proportions existing between that of chlorine to bases is similarly distinguished to those of oxygen.* Generally speaking, the chlorides are soluble in water; but this property varies considerably: thus chloride of potassium dissolves readily; whilst the chlorides of mercury are almost, or comparatively insoluble. The affinities of chlorine for its bases are generally very active, and second only to those of oxygen, which, therefore, mostly has the power of seizing the bases of the chlorides, and of setting the chlorine free, or leaving it to combine with some other substance. The various combinations of chlorine, forming chlorides, will be described in connection with their different bases.

Chlorine combines with oxygen to produce various compounds; such as enchlorine, chlorous, hypochlorous, hypochloric, chloric, and perchloric acids. Of these, the chloric is the most important in all respects; for it gives rise to the chlorates, some of which are largely employed for the manufacture of lucifer-matches, &c. Hypochloric acid is prepared by adding sulphuric acid to chlorate of potass; but as this product is dangerously explosive, only very small quantities (grains) of the ingredients should be employed; and then only in an open vessel. The acid is liberated in the form of a gas, which may be condensed by frigorific mixtures. We would warn our readers against attempting to use a test-tube for the purpose, having nearly lost the use of one eye whilst doing so ourselves.

Chloric acid is a compound of one equivalent of chlorine with five of oxygen; symbol—$Cl\,O_5$; equivalent = 75·5. It may be obtained by adding sulphuric acid to a solution of chlorate of baryta, which, like the chlorate of potash, is obtained by passing gaseous chlorine through a solution of the earth; the chlorate of potash being produced by passing the gas through a solution of that alkali. We have to speak more fully of these salts hereafter, both as to their manufacture and uses.

Perchloric acid is a combination of seven equivalents of oxygen with one of chlorine; and as it is of no great interest, we shall omit details of its manufacture.

A compound acid is produced by adding together two parts, in volume, of hydrochloric acid, with one of nitric acid, both in as high a degree of concentration as possible. The mixture is called *aqua regia*, and it is much used by metallurgists for the solution of gold. Platina and other metals are soluble in it; and it produces their chlorides, which will be duly noticed hereafter.

IODINE.

THE discovery of iodine at first arose from an accidental circumstance, and is of comparatively recent occurrence. It was noticed that the pans in which barilla had been manufactured, became very much corroded in some parts; and, eventually, chemical research accounted for the fact by showing the presence of a substance previously unknown as a distinct body. Iodine, as usually met with in commerce, is a substance in small metal-like scales, of a dark colour. On being heated, it affords a vapour of a rich purple tint—hence its name. The vapour condenses, and re-forms scaly particles. The symbol is I, and the equivalent = 127. As has been already named, its combinations are termed iodides.

For many years the uses of iodine were entirely confined to medical purposes, and especially as an antidote or preventative for the troublesome disease called *goitre*. Its presence in cod-liver oil has, doubtless, been highly beneficial in scrofulous and phthisical complaints. It is often administered in the form of its combination with oxygen and potass, as the hydriodate of potass. Since the discovery of photography, it has become of great importance; and, as such, is now a considerable product of the operative chemist's manufactures. The chief of its combinations, employed in photography, are the iodides of potassium, ammonium, and cadmium. Most of the compounds of iodine are readily decomposed by chlorine, to which it ranks next in importance with respect to its chemical characters and affinities. The following experiments will give a general idea of some of its leading properties.

Experiment 26.—Put a small quantity of commercial iodine into a clean Florence flask, and apply a gentle heat by means of the spirit-lamp. The iodine will rise as a purple vapour, and will eventually condense on the cold sides or neck of the flask, in minute, dark-coloured crystals or plates.

Experiment 27.—Dissolve a little iodide of potassium in *cold* water, and add a small portion of starch. No change will take place; but, on pouring into the solution a little sulphuric acid, the iodine will be set free, and a dark-blue colour will be observed. Starch is generally used as a test for the presence of free iodine; but the effects of the test are not evident if the solution be hot. Similarly, the presence of starch in vegetable matter may be detected by means of iodine.

Iodine combines readily with some of the metals, either in its state of vapour, or when in combination with other bodies, as the following experiment will show.

Experiment 28.—Expose a plate of silver to the vapour of iodine; a yellow iodide of silver will soon be formed on its surface. In this manner iodine was extensively employed on silver plates for the Daguerreotype process, which was, at one time, the chief method of obtaining pictures by the action of light.

Experiment 29.—Add a little of the solution of iodide of potassium to one of the bichloride of mercury in water; a yellow precipitate of iodide of mercury will be produced. On the addition of more of the iodide of potassium, the yellow colour will change to a deep red; and on a still further addition, the precipitate will be entirely redissolved; the iodide of mercury being soluble in an excess of that of the iodide of potassium.

Iodine is but slightly soluble in water; but is readily dissolved by alcohol, which thus forms what is termed the tincture of iodine, much used in medicine. It unites with chlorine, bromine, nitrogen, and hydrogen; forming with the latter an acid—the hydriodic. With nitrogen, it produces an explosive compound, which is analogous to the chloride of nitrogen, of which we shall treat on hereafter. It unites also with sulphur and phosphorus, producing light and heat at the moment of its combination with the latter substance: an explosion is afforded; hence great care is required when these substances are brought into contact. Iodine may readily be set free from its combinations by means of chlorine, as is illustrated in the following experiment.

Experiment 30.—Dissolve some iodide of potassium in water, and add a little starch or arrowroot. Pour into this a solution of chlorine in water.† The iodine will be at once set free, and the starch be converted to a blue colour.

Hydriodic acid is a combination of one equivalent each of iodine and hydrogen; symbol, IH; equivalent, 128. It is prepared from iodide of phosphorus. Iodine unites with oxygen in three forms; the most important of which is iodic acid, which is composed of one equivalent of iodine to five of oxygen; symbol, IO_5; equivalent, 167. It forms iodates with bases, which are analogous to the chlorates and bromates; they have no special application in the arts and manufactures.

The remaining combinations of iodine will be noticed in connection with their bases.

BROMINE.

BROMINE, like iodine, is a product of kelp, and is found in sea-water, in the salt called bromide of sodium, &c. It is a dense orange-coloured liquid; has an offensive smell—hence its name; and is corrosive in its action on animal substances. It is soluble in water and alcohol in almost any proportion; and being highly volatile, must be kept in well-stoppered bottles. Its symbol is Br, and the equivalent=80·0.

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* See *ante*, p. 314.

† See *ante*, p. 316.

Combined with metals and other bases, it forms the substances called bromides. Like iodine, its chief use is in connection with photography, in which it has been employed in the state of vapour, to increase the sensitiveness of the iodide of silver on Daguerreotype plates; and, in the form of bromide of potassium, it is considerably used in paper-printing processes.

It is prepared from hydrobromic acid, which is a combination of hydrogen and bromine; symbol, HBr; equivalent, 81. This acid is analogous in its constitution to the hydrochloric, hydriodic, and hydrofluoric. Its preparation is somewhat difficult; bromide of phosphorus being decomposed by means of water, when phosphoric acid and the hydrobromic are produced.

Bromic acid is a compound of bromine with oxygen, in the proportion of one equivalent of the former to five of the latter. Its symbol is Br, O_5; equivalent=120. We shall describe other combinations of bromine under the head of their bases.

FLUORINE.

Fluorine is a substance which has but little commercial value, and is never met with in a free state. It is obtained from Fluor or Derbyshire spar. Its symbol is Fl, and the equivalent=18·9. Its most known combination is that of hydrofluoric acid, in which it is combined with hydrogen.

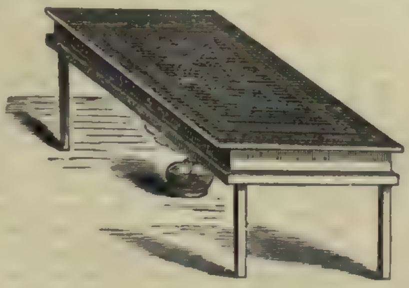
Hydrofluoric acid may be readily obtained by heating some coarsely pounded fluor spar, together with strong sulphuric acid, in a leaden vessel. A glass one cannot be used, because the acid acts powerfully on that substance. The fumes of the acid and the liquid should be carefully avoided, as they produce the most painful wounds if they come in contact with the skin.

Hydrofluoric acid, besides its occasional use in the laboratory, is much employed for etching glass; and the method of conducting this process may be seen in the following experiment.

Experiment 31.—Make a tray of lead by bending up the sides of a sheet of that metal so as to form a rim on all sides. Support this tray on a stand, and throw inside it some coarsely pounded fluor spar, to which some strong sulphuric acid may be added. Cover a glass plate with beeswax; and on this, when cold, make any sketch by means of a needle-point, so as to expose the glass surface. Then place the plate, with the sketched part face downwards, over the mixture of fluor spar and acid last mentioned, so that its edges may rest on the edges of the tray. Apply a gentle heat by means of a spirit-lamp, when the hydrofluoric acid will rise in vapour and attack the exposed surface of the glass. By these means such portions will be completely etched out; and on the wax being removed, the design will appear beautifully traced on the glass surface.

The best wax for covering the glass is that obtained by melting wax candle-ends. It should be run over both sides of the glass, lest the vapour of the acid might attack both sides. Some pretty effects may be produced by first sketching the design, with a soft black-lead pencil, on paper. This drawing is easily transferred to the wax surface by pressing the pencilled paper thereon. This will form a guide for the subsequent tracing by the needle-point. By such means the figures are produced on the ornamental sheets of glass so much used for sashes, in place of Venetian blinds. The preceding figure illustrates the mode of carrying out the experiment we have been describing.

Fig. 42.



HYDROGEN.

Next to oxygen, hydrogen stands of the greatest importance in the list of elements. By the combination of the two, we have the water of the ocean, of our rivers, &c. Combined with carbon, hydrogen enters into the composition of most vegetable and animal substances: as carburetted hydrogen, a product from coals, it is extensively used as fuel and for illuminating purposes; and the same results are obtained from vegetable and animal oils, through its presence in them. Its other numerous compounds will be noticed as we proceed.

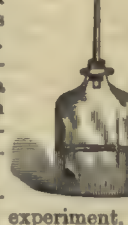
Hydrogen is the lightest known body in nature, its specific gravity being 0·069; that of air being 1·000; it therefore weighs but a little over one-fifteenth part of its bulk of atmospheric air. For this reason it has been employed to fill balloons, both in its pure and compound states.

Hydrogen may be procured, in various shades of purity, from water, coal, oils, &c. The following experiments will afford instances of the kind.

Experiment 32.—Pour some dilute sulphuric acid on some iron nails contained in a saucer or evaporating dish. Large bubbles of gas will speedily arise; and if a lighted match be brought near them, they will at once ignite.

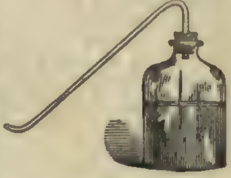
Experiment 33.—Put a few iron nails, or some shavings of zinc, into a glass bottle, and fit into its neck a cork containing a glass tube, one of the edges of which has been drawn to a jet. Add some dilute sulphuric acid to the metal, and fit in the tube. Gas will be given off; but this should be allowed to escape, because, being mixed with air, it would explode, and perhaps burst the bottle if the gas were ignited. After sufficient time has elapsed to remove all the air from the bottle, apply a light to the jet, when the gas will burn with a yellowish-coloured flame. The annexed figure shows the kind of bottle required for this experiment.

Fig. 43.



When a considerable quantity of hydrogen is required for the purposes of experiment, a bottle with a bent tube attached to it, such as is represented in the annexed engraving, may be employed; and the gas is to be collected by the pneumatic trough, in a similar manner to that mentioned for oxygen. It must, however, be borne in mind, that hydrogen is so much lighter than air, that it will at once escape if the vessel containing it be held mouth upwards. The following experiments, illustrating the levity of the gas, may impress the fact on the memory of the student.

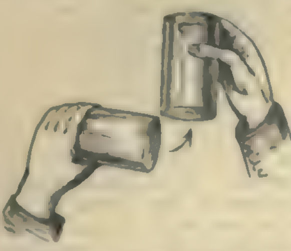
Fig. 44.



Experiment 34.—Hold a glass vessel with its mouth over the jet of the bottle described in Experiment 33. The gas will rise into the vessel, and so expel the air. This may easily be proved to have taken place by applying a light to the mouth of the vessel, when the gas will at once catch fire.

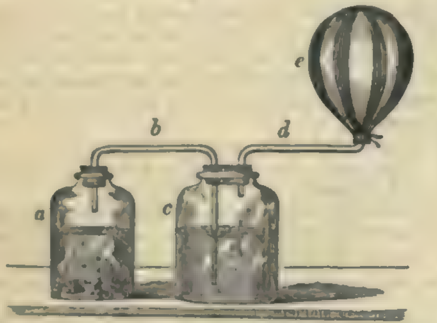
Experiment 35.—Fill a glass vessel, as in the last experiment, with hydrogen, and then bring near to it another vessel, also with its mouth downwards. Incline that already filled with gas beneath the mouth of that filled with air, and gradually allow the gas to pass from one vessel to the other, in the way illustrated in the following engraving (Fig. 45). That the gas is really transferred by these means will be at once proved by applying a light to each vessel, when that which has previously contained air will at once be seen to contain hydrogen by its contents being ignited.

Fig. 45.



The light specific gravity of hydrogen may be pleasingly illustrated by means of small balloons made of gold-beater's skin. These, if filled with hydrogen, will, on being liberated, at once ascend in the air. As the material of which they are composed is, however, easily acted on by the acid employed to generate the hydrogen, the following arrangement is recommended to purify the gas before it is allowed to pass into the balloon.

Fig. 46



In the above engraving, *a* represents the vessel in which the hydrogen is generated; *b* is a bent tube fitted into the neck of *a*, and running nearly to the bottom of the vessel *c*, which is to be three-parts filled with water. This tube, and *d*, which rises out of the neck of the vessel *c*, and to which the balloon, *e*, is attached, are to be fitted air-tight in the purifier by means of a cork. The gas will thus pass from *a* through *b*, into the vessel containing the water. It will here be cleansed from acid, and, rising in bubbles to the surface, will escape by *d*, which only just passes into its neck. The balloon, when filled, is to be tied at its neck, to prevent the escape of the hydrogen. In balloons of large size coal gas is always employed; which, although much heavier than pure hydrogen, answers well for the purpose.

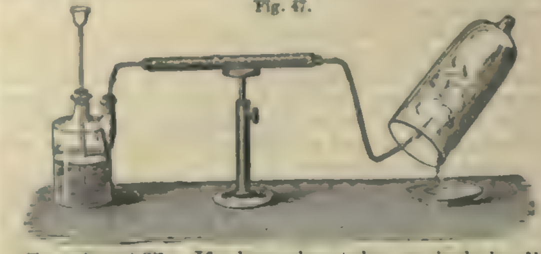
Hydrogen may be obtained from water by other means than those we have suggested. If steam from water be passed through an iron tube, heated to redness, it will be decomposed, and its oxygen uniting with the iron to form an oxide, the hydrogen may be collected in the usual way at the pneumatic trough, by fitting a bent pewter tube to the end of the iron one.

Hydrogen is obtained in the purest state by decomposing water by means of voltaic electricity. The best mode of doing this has already been described in the section of Electricity.*

The most important compound of hydrogen is the well-known liquid water, which, in its purest state, consists of eight parts of oxygen, united with one of hydrogen by weight. The gases unite, by bulk, in the proportion of two volumes of hydrogen to one of oxygen. The symbol of water is HO, and its equivalent = 9. The following experiments illustrate the production of water by the union of its elementary constituents.

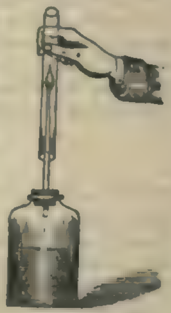
Experiment 36.—Hold a cold glass vessel over the jet of the bottle described in Experiment 33, whilst the gas is burning. The vessel will soon become coated with dew, owing to water being produced by the combustion of the hydrogen in contact with the oxygen contained in the surrounding air. Another mode of conducting the experiment, is that illustrated in the following engraving, in which the gas is first conducted through a tube containing chloride of calcium, to free it from moisture.

Fig. 47.



Experiment 37.—If a long glass tube, an inch in diameter, be held so as to inclose the jet of the hydrogen bottle in the way shown in the annexed engraving, some singular musical sounds are produced. The pitch of these sounds depends on the diameter of the tube; the bass being produced by those of a large, and the shriller tones by those of a small diameter. Any intermediate sound can be produced; and this plan has been proposed as a means of forming a new kind of musical instrument. This effect results from a series of vibrations being produced,† which arise from numerous explosions caused by the combination of the hydrogen with the oxygen of the acid. Of this, the next experiment gives a better illustration.

Fig. 48.



Experiment 38.—Into a strong glass vessel, introduce, by means of the pneumatic trough, one part of oxygen and two parts of hydrogen, by volume. For this purpose, an ordinary soda-water bottle answers exceedingly well. Apply a lighted taper to the mixture, when an explosion will instantly take place. After the expansion consequent thereon has ceased, it will be seen that the gases have lost their gaseous condition, and the result will be found as pure water in the form of vapour, or condensed in dew-like drops on the side of the vessel. This is simply a more rapid way of producing water by the combination of its elements, than those we have described in the preceding experiments.

The oxy-hydrogen blowpipe is a means of producing the most intense heat by the combustion, through a proper jet, of a mixture of oxygen and hydrogen gases, in the same proportion as that in which they form water. For the sake of safety, the two gases are kept in separate reservoirs, and are not allowed to mix, except at nearly the point of their combustion. The application of this arrangement to the purposes of illumination, and to optical instruments, has been already fully explained under the section on Light;‡ and a drawing of the most improved form of jet which can be used for the purpose, has also been given. We may here add, however, that by means of the oxy-hydrogen jet, the otherwise most infusible substances may be rendered liquid—such as porcelain, platina, wrought iron, &c.

The general uses of water are too well known to require any lengthened remarks on our part. In the laboratory it is employed as a general solvent, and pure water is therefore an object of constant requirement. This is obtained by the distillation of river or spring water in glass retorts, when great purity, as in toxicological analyses, is desired. In ordinary cases, the steam proceeding from any metal vessel, as a kettle, &c., may be condensed by means of a pewter tube, which should be kept cool by damp cloths. In most large towns, distilled water can be readily obtained; and, in many cases, ordinary rain or river water may be employed. Water of the ocean always contains many soluble matters, such as chloride of sodium, chloride of magnesium, iodide and bromide of potassium, &c. This on evaporation, forms clouds; and these again, being condensed as rain, fall in fertilising showers, to eventually form rivers which run to the ocean. Rain-water, when collected at a distance from towns, is generally almost pure; but near those places it becomes contaminated by matters suspended in the air—such as nitric acid, sulphur, and ammonia. After rain-water has percolated through strata of earth, and is again seen as rain or spring water, it will have dissolved both organic and inorganic matter, and this to a very large extent. The inorganic matter may vary from 2 to 100 grains per imperial gallon; and that of an organic nature differs similarly in quantity. The purest water, in a chemical sense, supplied in Great Britain, is that obtained from Loch Katrine, in Scotland; which is conveyed to Glasgow, a distance of forty miles; whilst the wells in the neighbourhood of London, are, generally speaking, highly contaminated with all kinds of impurity. The following table shows an analysis of various sources of the water-supply of London, &c.; and has been extracted from the report of the medical officer of the city of London, to the Common Council.

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* See *ante*, p. 194; Fig. 54.

† See *ante*, p. 275; Section, "Acoustics."

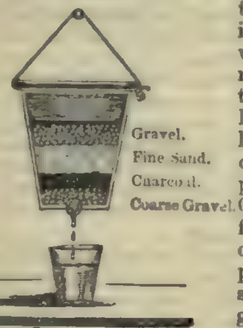
‡ *Ante*, p. 62.

Table of Impurities in Water-Supply from various sources.

—	Total per gallon.	Organic per gallon.
	gr. or deg.	gr. or deg
Distilled water	0·0	0·0
Loch Katrine water, new supply to Glasgow (March)	2·35	·605
Manchester water supply (March)	3·33	·680
Portland-road well	58·03	7·84
Chelsea (Thames Water)	19·72	1·76
Southwark ,, ,,	21 96	2·40
Lambeth ,, ,,	20·44	2·64
West Middlesex ,, ,,	21·76	2·40
Grand Junction ,, ,,	21·36	1·80
East London (River Lea)	21·03	1·36
New River	19·72	1·72
Kent (from the Chalk strata)	25·16	2·64

Nearly all the water supplied to London, is either obtained from the New River, which rises in Hertfordshire, or from the Thames near Hampton Court. Being now always filtered before it is supplied for domestic use, it never contains any notable quantity of suspended matter. In many districts, however, turf, sand, &c., are found in considerable quantities: it is necessary to filter the water before use; and we shall describe an arrangement that may be employed for the purpose—an engraving of which is subjoined.

Fig. 49



To construct one of these simple and effective filters, the common garden-pot is the only necessary vessel, into the bottom hole of which a piece of sponge or rag may be placed. On this, inside the pot, a layer of coarse gravel is laid; over this, one of charcoal, broken into small pieces, but not powdered. On the charcoal a layer of fine sand is placed; and, over it, one of gravel. A piece of calico or flannel should be placed over the gravel, so that when water is poured thereon, it may not run in channels through the different substances. A string may be tied round the rim of the pot, and the arrangement may thus be hung from a nail. Besides the filtering effect, the charcoal has the power of retaining offensive animal and vegetable matter; so that the results are both mechanical and chemical in the removal of the impurities. The preceding engraving will fully illustrate the method of making one of these filters; and shows the succession and position of each of the substances employed.

We may here make a few observations on the impurities of water, as generally found in that supplied for domestic and other such purposes, so far as they affect sanitary and commercial matters. A table already given, states the proportions of organic and inorganic substances present in water from various sources; and the nature of these substances we may inquire into.

It is very essential, for culinary and manufacturing purposes, that water should be as free from lime and earthy matter as possible; and the presence of these is the cause of the hardness of some kinds of water. Lime itself is but sparingly soluble in water; but when the latter contains carbonic acid, then chalk or carbonate of lime is readily dissolved in considerable quantities, as the following experiment will prove.

Experiment 39.—Pour an ounce of lime-water—that is, water in which slaked lime has been dissolved—into a glass vessel, and add to it some common "soda-water," which contains a large amount of carbonic acid. A white precipitate of chalk is first formed; but eventually, the excess of carbonic acid present is so great, that the chalk so formed will be dissolved and held in solution.

If water thus impregnated with lime be afterwards boiled, the excess of carbonic acid will be driven off, and the chalk will be deposited in the vessel. Hence hard waters are mostly rendered softer by the process of boiling, their earthy matters being thus separated from them. It is owing to the presence of earths, &c., in water, that the tea-kettle becomes so constantly furred. The same result takes place, on a very large scale, in boilers used for steam-engines, and becomes a source of great annoyance and positive danger at times. The earthy matter settles on the bottom and sides of the boiler, and there produces a solid and stone-like mass. This prevents the passage of heat from the fire through the metal of the boiler to the water, and thus the iron occasionally becomes red-hot, when the earthy crust is thereby detached, allowing the water to touch the overheated surface. On this occurring, a sudden and rapid production of steam ensues, which, having no adequate means of escape, frequently causes dangerous explosions.

Hard water can be rendered soft by the addition of lime to it, provided its hardness arise from the presence of an excess of carbonic acid, as we have already mentioned. This seems at first sight paradoxical, but is easily explained. By the addition of lime to water containing a large quantity of lime in solution, a portion of the carbonic acid united thereto is abstracted; and the result is, that some of the lime already present is thrown down in the solid state. The following experiment will illustrate this fact.

Experiment 40.—To the water saturated with lime, produced in the last experiment, add some more *lime-water*. A white precipitate of insoluble chalk will be produced, and a portion of lime previously existing in solution will be removed.

In a similar manner, hard water may be rendered soft, on the large scale, by the addition of lime-water to it. Dr. Clark, of Aberdeen, patented this process some years ago, for its application to water before its supply to towns, when much lime was present. We believe that the plan was long employed at the Charlton and Woolwich water-works, the water being drawn from a chalky soil, and largely impregnated with lime thereby.

Sulphate of lime, or plaster of Paris, is often met with. It is always present in both the Thames and New River waters, and in most rivers in Great Britain. Its removal is not possible by any chemical means on the large scale; and the only plan to soften water of this kind is to boil it, or to add small portions of carbonate of soda when it is used for domestic purposes.

The remaining inorganic impurities, such as oxide of iron, sulphate of magnesia, &c., are either present in such small quantities as to be of no consequence; or, on the other hand, abound so much as to constitute what are termed mineral waters.

Mineral springs are frequently found in rocky and other districts; and the water is charged with gases, salts, &c., of various kinds. Amongst the gases are carbonic acid and sulphuretted hydrogen. Oxide, or generally carbonate of iron, is frequently present in considerable quantities; so much so as to deposit a deep-red coating on all the objects it comes in contact with. Sulphate of magnesia, or Epsom salts, is a common constituent of mineral waters; and common salt is generally found also in various proportions. The water of the ocean contains numerous salts, as we have already mentioned; and it has been lately asserted that silver is also present to a slight extent, which, however, in the aggregate, would form a prodigious quantity of that metal in the ocean. In some parts of Cornwall, the water flowing from the mines is largely impregnated with copper; and the metal is extracted therefrom by throwing into the streams scraps of old iron, such as the *débris* of the pots and pans of London, which are sent to the mineral districts in large quantities for that purpose.

On the continent of Europe, especially in some parts of Germany, mineral springs abound; and such places are sought for by invalids for the purpose of partaking of the waters. We may remark, that chemical analysis has so far succeeded in detecting the presence of all the salts contained in such waters, as to permit of their manufacture in any place; and there is scarcely a doubt that more Seltzer water, &c., is manufactured yearly in London than has ever been imported from the original and recognised source.

With respect to the value of mineral waters, in a medicinal point of view, much exaggeration has doubtless been made. Change of scene and society, pure mountain air and exercise, have a more beneficial effect on the human system than any medicament which can be administered; and to such may be rather assigned the improved state of health which the valetudinarian experiences on visiting a spa, than to the effect of its waters of which he partakes, although they may exert an assistant office.

The organic impurities of water is a subject of the deepest importance, in a sanitary point of view. In direct proportion as organic matter of any kind is present, so the fitness of such water for drinking purposes diminishes. In many districts moss is gathered as the water runs down the mountain sides, and along the banks of the stream; and this is readily noticed if the water be allowed to drop on a rough stone surface, which will then become covered with a green coat of vegetable matter. Such generally occurs in mountainous districts. In towns, the wells often contain animal matter, and its products in solution; and in the city of London, there are wells which are simply the result of the drainings of the churchyards to which they are adjacent. The water obtained from such sources has a very pleasant and cooling taste, from the presence of nitre; this arising from the nitrogen afforded during the decomposition of animal matter. If such water be exposed to the atmosphere during summer-time for a few days, it will give off a most offensive smell. Indeed, persons partaking of it are simply imbibing the products of the decomposition of the bodies of their ancestors, in a soluble form; and the effect is often manifest in the production of diseases, such as diarrhœa and cholera. Districts supplied with water from such sources are certain to be the most unhealthy during summer-time, and are fitting fields for the ravages of the complaints we have mentioned.

We may here call attention to what we believe will eventually become a source of great danger to the health of large towns. We refer to the position of cemeteries, so commonly found in the suburbs. London itself is encircled with these burial-places, nearly every one of which is located on a sandy soil. Tons of human remains are yearly deposited in them; and, as these decompose, the soluble matter will be carried away with the percolating water, afterwards to be pumped up for drinking and other purposes. Of course, many years will be required to pass away before the results we have hinted at take place; but eventually, chemical analysis and sanitary reports will more than likely show, that the proximity of burial-grounds to large cities will produce an evil of no ordinary, and, perhaps, of incurable magnitude.

In animal and vegetable life, we need not remind our readers that water exercises the most important functions. It fills the cells of the tree with sap, and courses, in the form of blood, through the veins of the animal. By means of it, the food is conveyed, in a suitable form, to afford flesh and muscle; the salts, which produce bone, are dissolved by it, and are presented in a suitable condition for that purpose. We must, however, dismiss this, and many other such matters, until we more fully treat on organic and agricultural chemistry.

Hydrogen combines with oxygen, in the proportion of one equivalent of the former to two of the latter, to form what is called the peroxide of hydrogen; symbol, HO_2; equivalent = 17. This, like water, is a liquid; and, although a very interesting production in a chemical point of view, has no general use or interest. It is prepared by a somewhat intricate and difficult process, by means of peroxide of barium and hydrochloric acid. The peroxide of hydrogen and chloride of barium are produced, and the baryta is precipitated by sulphuric acid, the chlorine being removed by means of a salt of silver. The combinations of hydrogen with chlorine, iodine, bromine, and fluorine, have been already noticed: those with other substances will be duly mentioned as we proceed.

NITROGEN.

In our former pages, we have spoken at some length on the qualities of one of the gases present in the atmosphere, and showed that oxygen is a powerful supporter of combustion, and essential to the existence of all animate bodies. The air, however, contains another gas, by means of which the active properties of oxygen are modified, so as to render them suitable to the organisation and circumstances of vitality, and to the properties of this gas. Nitrogen, and its leading compounds, we must now draw attention to. Its symbol is N; equivalent = 14; and its specific = 0·972; air = 1·000.

The properties of nitrogen are almost entirely of a negative character. It does not support combustion; has no smell or taste; is scarcely absorbable by water. It derives its name from nitre, of which it is a constituent; it is occasionally called *azote*, from the fact that animals cannot breathe it in a pure state. It has, however, no direct poisonous qualities; and fatal effects resulting from inspiring it, arise from the absence of life-sustaining powers. It, however, exerts great influence in animal life, as it forms a constituent of flesh, and various compounds produced or consumed by animals. It is a chief constituent of ammonia and nitric acid; and many products of the vegetable kingdom contain it as an essential element.

The readiest mode of procuring nitrogen for the purposes of experiment, is that of removing the oxygen of atmospheric air by means of burning phosphorus. The atmosphere is not a chemical compound in the strict sense of the term, but rather a mixture of oxygen and nitrogen gases; and hence, phosphorus and other burning bodies readily remove the oxygen by combining with it, leaving the nitrogen free.

Experiment 41.—Place a small piece of phosphorus on a piece of cork floating in a basin of water; and having ignited the phosphorus, invert over it a tall glass jar full of air, pressing the mouth of the vessel beneath the surface of the liquid. A few bubbles of air will at first escape, owing to the expansion of the air by heat. Eventually, however, the water will rise inside the glass jar to the extent of about one-fifth of its capacity. The white fumes of phosphoric acid produced by the combination of the phosphorus with the oxygen of the air, will be gradually absorbed, and a colourless gas will be left, which can then be removed into another and smaller glass jar, by means of the pneumatic trough, in the ordinary manner. There are other means of procuring nitrogen, which, however, have no advantage, in a practical point of view, over that above described.

Experiment 42.—Having filled a small glass jar with nitrogen, remove it, mouth downwards, from the trough, and introduce into it a lighted taper, which will be immediately extinguished. Afterwards, raise the jar, mouth upwards, and try its contents again, after a few seconds, by means of a lighted taper. The gas, owing to its levity, will escape, and the taper will burn as in the atmosphere, because the external air has taken the place of the nitrogen.

These experiments illustrate the fact that nitrogen does not support combustion; and that it has, like hydrogen, a less specific gravity than the atmosphere.

The Atmosphere.—The most important use of nitrogen is that found in the air we breathe. Were it composed entirely of pure oxygen, it would be impossible to extinguish a fire which had once been kindled. All animal and vegetable life would be too highly stimulated, and our present state of existence would be rendered impossible. A hundred cubic inches of air weigh about 31 grains; and as it has been already often stated, dry air at a temperature of 60°, and a barometrical column, equal to support 30 inches of mercury, is taken as a standard for the specific gravity of all other gases. The following tables give an approximate statement of the composition of atmospheric air:—

Composition of the Atmosphere.

1. By weight—	
Oxygen, the active element	23
Nitrogen, the negative element	77
	100
2. By measure—	
Oxygen	21
Nitrogen	79
	100

But in the above, we suppose that absolutely pure air is estimated: such, however, is never found in nature; carbonic acid and other gases being always present with aqueous vapour, in variable quantities.

We cannot enter here into any extended account of the physical character of the atmosphere. We may, however, state, that it is considered to encircle our earth to the extent of 45 miles from its surface; hence a column of air of that height, one of water 32 to 34 feet high, and one of mercury of about 30 inches high, on the average, balance each other. The barometer—which will be fully described under the head of Pneumatics—is employed to measure the variations of atmospheric pressure, which, at the surface of the earth, at the level of the sea, is equal to about 15 lbs. on the square inch; diminishing as the height of the atmosphere from the surface increases. Hence, the higher the atmosphere, the less the pressure it exerts on any surface. The causes of the barometric variations, or of the pressure of the atmosphere, are very numerous; and, until lately, had defied accurate or approximate rules. Owing, however, to extensive observations taken in all parts of the world, the science of meteorology, in which are included such subjects, is rapidly advancing to a high position; and it is even now possible to foretel, with tolerable accuracy, the occurrence of any extraordinary atmospheric fluctuation.

It is a remarkable fact, that although we cannot view the atmosphere in the light of a chemical compound, its constitution never varies to any great extent. The air obtained from the top of a mountain, and that on the plains, at first is identical in composition. Local and accidental circumstances cause some variations. Thus, in crowded cities, more carbonic acid and other gases are present than in the open country. Again, after a thunder-storm, small quantities of nitric acid have been detected, that acid being generally formed when the electric spark is sent through air. Sulphuric, sulphurous, and hydrochloric acids, ammonia, common salt, sulphuretted hydrogen, &c., may also be noticed; but the presence of these depends on causes which are entirely local and controllable.

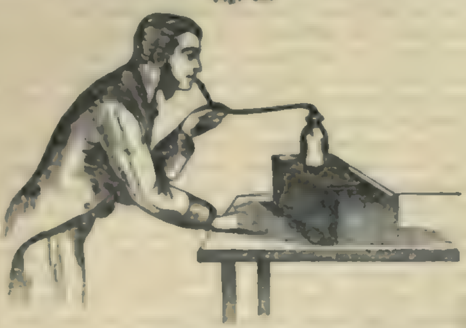
There are various methods employed to analyse the air; but of these we shall have more fully to speak when we consider the subject of chemical analysis. The method we recommended for procuring nitrogen for the purposes of experiment, illustrates these. The object always is, to remove the oxygen either by combustion or absorption. The former is generally done by means of hydrogen, when water is formed by the combination of that gas, with the oxygen contained in the air undergoing examination. A solution of protosulphate of iron, with a gas we shall presently describe—the binoxide of nitrogen—is employed when the process of absorption is adopted; such a solution having great attraction for free oxygen, and thus removing it from the air.

We need scarcely here dilate on the necessity of pure air for the maintenance of health, as we have already made extended remarks on that subject when speaking of ventilation, under the section of Heat. We may, however, point out the remarkable fact, that gases of very different specific gravities, have the extraordinary power of rapidly diffusing themselves with each other. Thus, if a tube be divided by a thin bladder, or diaphragm of plaster of Paris, and one end be filled with hydrogen, and the other with carbonic acid gas—the latter, which has a great specific gravity, weighing twenty times at least as much as its bulk of hydrogen, will, if placed in the lower end of the tube, gradually rise and diffuse itself with that gas, which again will descend, and equally diffuse itself with the carbonic acid gas. Thus the constitution of the atmosphere, whilst kept uniform, under ordinary circumstances, by this law of gaseous diffusion, is, as we have already intimated, deteriorated by other gases. Hence arise miasma, and those evil results produced on the animal economy by vitiated air and noxious gases. These subjects have been the study of many leading British and continental chemists; and although chemical science has not largely benefited by their researches, still society at large has reaped great advantages in a sanitary point of view. The general effect of the atmosphere on the animal system, in reference to the processes of digestion, &c., will be considered when we refer to the nature and uses of food.

We now proceed to speak of the combinations of nitrogen with other substances. With oxygen it forms the protoxide, binoxide; hyponitrous, nitrous, and nitric acids. The protoxide of nitrogen is, as its name implies, a compound of one equivalent of nitrogen to one of oxygen; its equivalent = 22. It is a colourless gas; has a sweet taste, and is distinguished by its peculiar effects in exciting laughter on being inhaled into the lungs; hence its common name of "laughing gas." As many of our readers may feel inclined to try experiments with it, we give full directions for its preparation and inhalation.

Experiment 43.—Put two ounces of nitrate of ammonia into a retort, or Florence flask with a bent tube. Place this on the ring-stand, and let the stem of the retort or flask dip into the pneumatic trough, or in a basin. Instead of using cold water in these, a solution of two ounces of sulphate of iron or green vitriol, dissolved in a gallon of warm water, should be employed, the object being to purify the gas as it passes through the liquid. A gas jar, open at one end, and corked at the other, is now to be filled with the water in the trough, and arranged so that the stem of the retort shall dip under its lower edge. A gentle heat is to be applied to the retort, by means of the spirit-lamp, till the salt contained in it melts. Laughing gas will then pass over, and will gradually fill the glass jar—of course driving out the water. When no more gas passes over, remove the retort from the ring-stand, and let the jar of gas stand over the water for some time, so that any gases present, which, if breathed, might do serious injury, may be absorbed. The gas may be collected in a bladder, if one be screwed to the neck of the gas jar; but a simpler plan is as follows:—Have ready a cork, which will just fit the neck of the glass jar filled with gas, and into this cork fit a piece of pewter tube, which must pass through a hole bored in the cork. To this a piece of india-rubber piping, about three feet long, should be attached, ending in a small piece of metal tube, which will serve as a mouth-piece. Our diagram, Fig. 50, will explain the construction of the arrangement. On the right is the pneumatic trough, in which the jar of gas stands. From the jar the india-rubber tube passes, and through this the person taking the gas inhales it. After removing the cork from the jar, and substituting the one just now described, the person taking the gas should place the mouth-piece in his mouth, close the nostrils with the fingers, and inhale the gas into his lungs, until a peculiar appearance is seen in his eyes—say after a few seconds of time. The effects produced on different constitutions vary materially. Some persons indulge in immoderate fits of laughter, whilst others have a tendency to fight. Great care should be taken to restrain the person from doing injury to himself or others, as he is utterly incapable of controlling his actions. Females should never have this gas administered to them, for obvious reasons; and, in every case, great caution should be observed. The effects, however, speedily pass away, and the individual suffers no inconvenience from the temporary excitement of which he had been the subject.

Fig. 50.



The binoxide of nitrogen consists of two equivalents of oxygen to one of nitrogen; equivalent = 30. It is sometimes called the nitric oxide, in old chemical works. It is prepared by dissolving copper filings in dilute nitric acid in a retort. The gas may be collected in the usual manner in ordinary gas jars; and if perfectly free from air, it is colourless. On the admixture of air or free oxygen, however, it readily combines therewith, producing, at the same time, a deep red gas, which is hyponitrous acid. The latter is composed of three equivalents of oxygen to one of nitrogen; equivalent = 38.

Nitrous acid contains four equivalents of oxygen to one of nitrogen; equivalent = 46. It may be prepared by distilling nitrate of lead in a retort; and is frequently a product of the decomposition of nitric acid, which then assumes a yellow or reddish colour. Nitrous acid, by itself, is an orange-coloured liquid, and its salts, produced by combination with bases, are termed nitrites. Its compounds are unimportant, and uncertain as to their constitution.

Nitric acid is the most important combination of nitrogen with oxygen. It contains five equivalents of the latter element to one of the former. Its symbol is NO_5; equivalent = 54, when in a pure state. In its ordinary condition it is united with water, and forms the ordinary *aqua-fortis*, when in a comparatively diluted condition. Nitric acid of the greatest strength has a specific gravity of 1·520; when quite pure, it has but a very faint yellow colour. The light of day, or the solar rays, speedily decompose it, producing a mixture of nitric and nitrous acids, by which its colour is deepened. It is obtained by distilling a mixture of nitre, sulphuric acid, and a little water, in a retort, receiving the vapour which passes over in a receiver, which should be kept cool, so as to condense the acid. A similar arrangement to that already described for preparing hydrochloric acid, may be used.* It has an intensely sour taste; turns the human skin to a deep yellow colour; reddens litmus; and, with bases, forms the salts called nitrates. Nitric acid is largely used in commerce for a variety of purposes. With hydrochloric acid it forms aqua regia, which is used to dissolve gold and platina. The chemists employ it in the laboratory, for the purpose of oxidising bodies, and as a general solvent, as it ordinarily produces salts soluble in water. It may be distinguished from other acids by its smell, and by bleaching a solution of indigo. Nitric acid is occasionally present in the atmosphere after a thunder-storm; and it may be produced by passing repeated electric sparks through a mixture of oxygen and nitrogen. It is often found in water, &c., through which animal matter has passed in a state of decomposition; but is so discovered in combination with potass or soda. With potass it forms the ordinary saltpetre of commerce, which is largely employed for the manufacture of gunpowder. When cotton wool is digested in strong nitric acid, or a mixture of nitric and sulphuric acids (which must be cooled before being used), the character of the cotton is completely changed; for, after having been well washed with water, and dried, it becomes highly explosive; and, by these means, *gun-cotton* is produced. In most of its characteristics, it is analogous to chloric acid; and is similarly constituted to the iodic and bromic acids. Its various salts we shall describe in connection with their bases.

Ammonia—symbol, NH_3; equivalent = 17.—This compound of nitrogen and hydrogen is of great importance in many respects. It enters largely into the composition of animal substances, or is the product of their decomposition; and the various salts it produces, with acids, such as the sulphate, carbonate, and hydrochlorate (sal-ammoniac), have large uses in the arts, commerce, and manufactures. Ammoniacal gas, which is the form in which ammonia is produced, may be readily obtained by heating the carbonate or sal-ammoniac with lime, in a test-tube or retort. The gas so afforded has a strong pungent smell, which is highly characteristic; is rapidly absorbed by water; reddens vegetable yellows, such as turmeric, &c.; and affords dense white fumes of sal-ammoniac on the vapour of hydrochloric acid coming in contact with it, by which its presence may readily be detected. Thus ammonia is continually given off in stables, and other places in which animal matter is putrefying; and if a stopper or rod, moistened with the acid, be exposed therein, the hydrochlorate is at once produced in the form of a white smoke.

Whilst the composition of ammonia is well known to be that of three equivalents of hydrogen to one of nitrogen, some peculiar effects, produced by the following experiment, have led to the idea that a metallic substance, to which the name of ammonium has been given, really exists.

Experiment 44.—Put a small piece of sodium into a little mercury, and pour thereon a solution of sal-ammoniac. The alloy will greatly increase in bulk, and yet remain solid.

Now, of late years, it has been strongly urged, that the result so produced arises from a combination of mercury with the hypothetic metal ammonium. We may, however, state, that all attempts to obtain the latter in a free state, have been fruitless; and if the metal really do exist, it has hitherto eluded the grasp of our most able chemists. Ammonia, in every case, behaves with acids much after the manner of a metal, and all the proofs of its existence resolve into analogies only, at the present time.

Ammonia may be obtained from such substances as horse-hair, and other animal matters, by heating them. This especially occurs when hartshorn shavings are strongly heated; hence the term "hartshorn," as frequently applied to it in commerce.

Experiment 45.—Heat some hartshorn shavings in a test-tube, and hold over the mouth a rod moistened with hydrochloric acid. Fumes of sal-ammoniac will at once appear. The same result may be produced by heating hair, flesh, or feathers; for in all these, hydrogen and nitrogen are present, and by their combination ammonia is produced.

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* See *ante*, p. 317; Fig. 40.

A large quantity of ammonia is afforded at our gas-works, by the distillation of coal in producing coal gas. In fact, the ammoniacal liquor of the gas-works goes largely to swell the dividends of the gas companies. Coal contains but a small per-centage of nitrogen; but this uniting with the hydrogen, produces the ammonia, which partly condenses, as a dark-coloured liquor, with the water evaporated from the coals. The presence of ammonia in coal gas may be readily detected by holding a piece of turmeric paper over an open gas-tap, when the paper will speedily turn to a brown colour. An immense quantity of ammonia is thus annually wasted; and during its passage through brass tubes and connections, it exerts a very prejudicial effect. Besides being thus produced from coal, ammonia may be obtained from some kinds of vegetables—amongst the fungus or mushroom kind, is an instance.

When we enter on the discussion of the constitution of animal substances, and the uses of nitrogen in vital organisation, we shall perceive how important a part that element plays in those conditions. As we have already observed, there are few animal bodies but what contain it, and most of them produce ammonia on being subjected to destructive distillation.

The chief salts of ammonia are the sulphate, nitrate, hydrochlorate, and carbonate. The sulphate is largely used as a manure by farmers, and is always a component part of common soot. Guano contains both free and combined ammonia; and hence one of its great advantages for agricultural purposes. The nitrate has not much use in ordinary life: it is, as we have already mentioned, employed to manufacture laughing gas.

Sal-ammoniac, chloride of ammonium, or hydrochlorate of ammonia, as the same salt is indifferently called, has numerous uses in the arts and manufactures. It is employed by metallurgists, dyers, and others, for various purposes, and is an important article of commerce. The carbonate forms what is commonly called "smelling salts." Both sal-ammoniac and the carbonate may be readily sublimed by means of heat; and this plan is employed for the purpose of their purification.

The other salts of ammonia, such as the acetate, oxalate, &c., are of chief use in the laboratory—the last being employed as a test for lime, with which it forms an insoluble salt.

Nitrogen forms other compounds with hydrogen; namely, imidogen, which contains one equivalent—symbol, NH; amidogen, which contains two—symbol, NH_2; and the hypothetic ammonium, which we have already noticed—symbol, NH_4.

Chloride of Nitrogen.—This is the most explosive substance with which we are acquainted. Its composition is that of one equivalent of nitrogen to three of chlorine; symbol, $N\,Cl_3$. Its preparation is attended with extreme danger, unless the utmost precaution be observed. The best plan is to make a tray of sheet lead, which must be perfectly free from dirt or grease, and into it a strong solution of sal-ammoniac is to be poured. A jar of chlorine gas, mouth downwards, is then to be placed in the solution, when the chloride of nitrogen will gradually form as small globules. After a little has been formed, the jar may be removed, when the chloride will be found at the bottom of the dish. If it be touched by any greasy substance, it will instantly explode with a loud report, and the dish will most probably be bent or broken in all directions. It is impossible to be too careful in preparing this substance; and we strongly recommend all our readers to be satisfied with the description rather than the investigation of the substance.

Nitrogen forms also an explosive compound with iodine, which is analogous to the chloride; its symbol is NI_3. This iodide of nitrogen is prepared by adding some liquid ammonia to iodine—the same precaution being employed as in the last instance.

The fulminates of different metals will be considered under their respective heads; and the various organic compounds in which nitrogen plays a conspicuous part, will be dealt with hereafter. The union of carbon and nitrogen, producing cyanogen and its compounds, will be described in the section on Carbon.

CARBON.

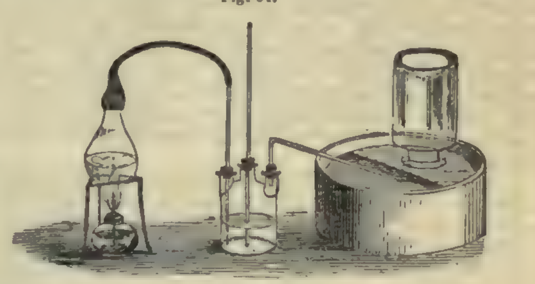
Carbon, in its ordinary state, is universally distributed in nature. We find it in nearly all animal and vegetable substances. It is familiar to us in our coals; and in this form is one of the most valuable products of our country. It floats, as carbonic acid, in the air we breathe; is part of ourselves, and constitutes the major portion of the plants of all kinds which surround us. Strange to say, that most valued of all gems, the diamond, is but carbon in a pure and crystallised state; and exists in striking contrast with another form of carbon—the soot which defiles the appearance of our houses and persons. Combined with hydrogen, it affords us coal gas; with hydrogen and oxygen, it forms sugar and starch; with oxygen, in various proportions, it produces acids; and to its other combinations we are indebted for alcohol, ether, animal and vegetable oils, with numerous other substances we cannot here stop to enumerate.

The purest form of carbon with which we are acquainted, is the diamond; and next to this is the well-known substance, "black-lead;" which, however, does not contain a particle of lead in its composition. The most ordinary form of carbon is that of wood charcoal, which is, however, mixed with earthy and saline matters. Its symbol is C; equivalent = 6.

Carbon combines with oxygen in two proportions, to form carbonic oxide and carbonic acid. Carbonic oxide, CO, equivalent 14, is often produced during the combustion of substances containing carbon. It burns with a pale-blue flame, and is the cause of the lambent flame which is often noticed arising from brick-kilns. It can only be found in the gaseous form; and, during combustion, produces carbonic acid gas by uniting with the oxygen of the atmosphere. It may be prepared as follows:—

Experiment 46.—Heat, in a test tube, or Florence flask fitted with a bent tube, some oxalic acid in crystals, to which some strong sulphuric acid is to be added. Carbonic oxide will be produced, together with carbonic acid. The gas may be received in the ordinary manner, in jars, over the pneumatic trough; which, however, should be filled with water containing a solution of potass. A better plan is to pass the gas through a solution of potass, in an intermediate vessel containing potass in solution, in the manner represented in the following engraving.

Fig. 51.

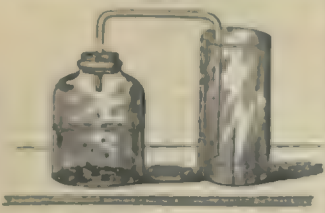


When sufficient of the gas has been collected, a taper may be applied to the mouth of the jar, with its rim held downwards; as the gas has a specific gravity of 0·97, or less than that of the air. The colour of the flame may thus be readily noticed.

Carbonic acid results from the union of one equivalent of carbon with two of oxygen: its symbol is CO_2; equivalent = 22. This gas has a greater specific gravity than air, it being equal to 1·527. It is not poisonous, but destroys life if respired, through the absence of oxygen. It is produced in large quantities during respiration, combustion, fermentation, &c.; and we shall suggest a few experiments illustrating these facts, and its general characteristics.

Experiment 47.—*Its production.*—Use the hydrogen bottle illustrated in Fig. 44; and introduce into it a quantity of marble, or chalk, broken into small pieces, but not powdered; on this pour a little water, and add half as much hydrochloric acid, so as to half fill the bottle. Fit in the bent tube, and let it dip into a tall glass jar, so as to reach to the bottom, as in Fig. 52. The acid will take the lime away from the marble, and the gas will pass into and fill the jar. This gas is very heavy, and will not escape until the glass is full: at the same time a small piece of card may just be placed over the mouth of the vessel, to prevent the action of currents of air.

Fig. 52.

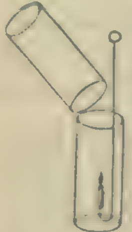


Experiment 48.—Dip a lighted taper into the jar; the flame will be immediately extinguished.

Experiment 49.—Into a glass jar put a small piece of taper or candle, by tying it to a wire (Fig. 53), and let the candle rest at the bottom of the jar, after having first lit it. Now take another jar filled with the gas, and pour its contents into the first one, in the same way as if water were being poured out. Owing to its great weight the gas will pass into the vessel holding the taper, and extinguish the flame (Fig. 54). These two experiments illustrate the deadly nature and weight of the gas; and suggest very simple means of preventing loss of life, to persons who have to descend into mines, vats, or other places where this gas is often found.

Fig. 53.

Fig. 54.



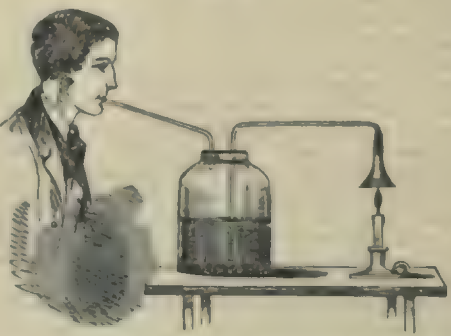
Experiment 50.—Half-empty a jar of this gas by tilting it on one side for a few seconds; then dip in a lighted taper. It will be found that the taper will burn until it reaches the part where the heavy gas is, when it will at once go out. This simple experiment shows how it often occurs that human lives are lost, by persons descending deep wells or vats, which frequently contain large quantities of carbonic acid gas. In the upper part no gas will be found; whilst, owing to its density, it may be in quantities in the lower part. Every deep vessel or place should thus be tried by means of a lighted candle, before any person is allowed to descend into it. If the flame be not extinguished, then little danger can arise; on the other hand, if it be extinguished, no attempt should be made to descend until some means have been adopted to ventilate the place. This may be done by letting down large chauffers of burning coal, which will produce currents of air; or quantities of slaked lime may be cast in, so as to absorb the carbonic acid gas.

A very large amount of carbonic acid is produced by the combustion of candles, coal, coke, and charcoal; and that produced from the last-named material has often been employed for suicidal purposes. Lime-water is the best test for the presence of carbonic acid, as the gas instantly forms chalk, or carbonate of lime, on coming in contact with the lime held in solution. Lime-water is readily made by shaking up some fresh-burnt lime with cold water, and decanting off the clear transparent solution, which alone must be used. The following experiments will illustrate the fact that carbonic acid is produced by respiration and combustion.

Experiment 51.—Pour into a small glass a little lime-water, and then, by means of a glass tube, blow from the mouth into the water. After a short time, the lime-water will become milky, owing to the carbonic acid driven from the lungs combining with the lime, and forming chalk.

Experiment 52.—Arrange a piece of apparatus as in the diagram, Fig. 55. On the right is a small funnel suspended over a candle; the funnel is connected with the purifier used in making oxygen (see Fig. 15); and the air is drawn from over the lighted candle through lime-water contained in the purifier, by sucking at the small tube. The gas produced by the burning candle will form chalk in the purifier with the lime-water; and thus the products of combustion are shown to be the same as those obtained from the breath as expired from the lungs.

Fig. 55.



Similar results obtained as above from the products of respiration and combustion, may not at first sight appear easy of comprehension by some of our readers; we, however, can readily explain them. The candle contains carbon as one of its constituents; and during combustion, that carbon becomes converted into carbonic acid. The food we partake of also contains carbon; and this, in the blood, comes in contact with the oxygen of the air on the surface of the lungs. It there undergoes a slow process of combustion, and produces carbonic acid gas. Indeed, there is every possible analogy between the production of heat in our furnaces and in our lungs, by an identical process carried on at different degrees of intensity. The following experiment will prove the presence of charcoal in sugar.

Experiment 53.—Make a strong solution of sugar in hot water, and add to it some concentrated sulphuric acid. The latter will seize the water in the sugar, and set free its charcoal, as a dense, spongy, black mass.

We have already noticed that carbonic acid is present in the atmosphere; we may here, therefore, call attention to the fact, that whilst poisonous to animals, it forms the food of plants. The leaves of trees act as lungs, and, by means of them, the carbon of the carbonic acid in the air is absorbed into their structure. Thus, three-fourths of a tree are gathered from the air. What is poison to us, becomes the food of plants: they purify what we poison, and return to us the oxygen, so essential to our existence, from the carbonic acid which we have cast off from our lungs.

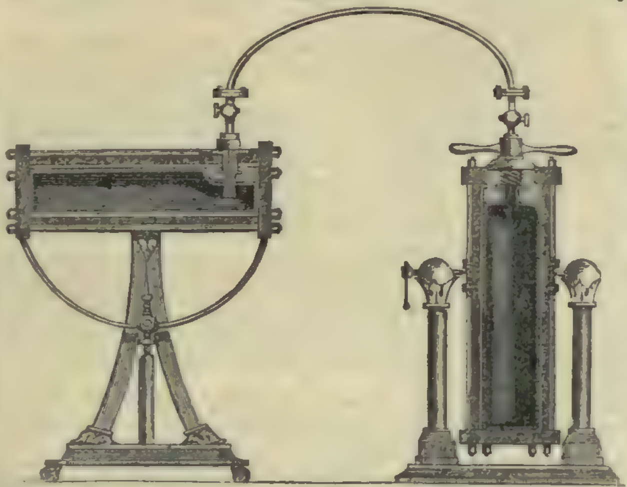
Carbonic acid is the cause of the effervescence of malt liquors, sparkling wines, soda-water, &c.; and although deathly to the lungs, acts as a healthy stimulant to the stomach: hence arises the invigorating and freshening effect of the beverages to which we have referred.

By means of considerable pressure, carbonic acid may be liquefied, and even brought into a solid state. This may be effected in a strong bent glass tube, by placing the materials at one end, and receiving the gas at the other. It is, however, more conveniently managed by means of a strong metal apparatus, constructed for the purpose; which is illustrated in the following engraving. It consists of two parts, joined by means of a pipe. The vessel on the right-hand side of the cut is that into which the materials are introduced; and the gas passes thence by means of the tube, into the receiver.

If the gas be allowed to pass out of the receiver while in a liquid state, it evaporates so rapidly as to be converted into a powdery substance, resembling snow. If this solid carbonic acid be mixed with ether, it produces the most intense cold obtainable by any means, owing to the rapid evaporation which takes place.

We need scarcely state, that great danger always attends the performance of such an experiment as the condensation of carbonic acid gas; and some lives have been lost thereby, through the explosion of the apparatus. It has been proposed to employ carbonic acid gas as a motive agent, in place of steam; but the application has never yet been practically adopted.

Fig. 56.



The composition of carbonic acid gas may readily be ascertained, and the presence of carbon therein proved, by filling a Florence flask with the gas, into which a piece of potassium, of the size of a pea, may then be dropped; the heat of the spirit-lamp may be applied, when the metal will seize the oxygen of the acid, setting the carbon free as a black powder. On the other hand, the presence of carbon may be proved in the diamond, by burning that gem in oxygen, when carbonic acid will be produced, which will at once form chalk with lime-water.

Carbon has peculiar properties of absorbing into its pores gases of all kinds; and, in every case, it has highly anti-septic properties. For this purpose, it may be procured from wood, cones, peat, &c.: all that is required is, that it should be highly porous. Putrefying animal matter may be covered, to a depth of a few inches, with small pieces of wood charcoal, and no smell whatever will arise from it. It has been employed by Messrs. Stenhouse and Watson, as a purifier of air to hospitals, &c., when such buidings are situated in unhealthy positions. Its use as a dentifrice is well known. Charcoal enters largely into the composition of gunpowder; for which purpose the best qualities are sought for. The explosive effect of gunpowder and gun-cotton is due to the sudden liberation of carbonic acid and other gases, during the combustion of the solid materials.

With sulphur, carbon forms a peculiar liquid, called bisulphide of carbon. This is prepared by passing the vapour of sulphur over pieces of charcoal heated red-hot, and contained in an iron tube. Bisulphide of carbon is a common product of the distillation of coals; to which reference has already been made in the article on artificial illumination.*

We must also refer our readers, for a full description of the composition of fatty matters, both animal and vegetable, and the nature and composition of the products of their distillation, and that of coal in producing the various combinations of carbon and hydrogen, olefiant gas, carburetted hydrogen, paraffin, &c., to the same subject; in which they will find the fullest details. We may, however, add, that since that portion of this work has gone to press, a new source of mineral oil has been discovered in North America, which seems inexhaustible, and which promises to reduce the cost of illuminating agents to a great extent.

Carbon and Nitrogen.—A very interesting series of compounds is produced by the union of carbon and nitrogen, and bases: of these we shall first mention cyanogen.

Cyanogen is a bicarbide of nitrogen; being composed of one equivalent of nitrogen united to two of carbon. Its symbol is Cy; equivalent = 26. It is obtained by heating cyanide of mercury in a test-tube, fitted with a bent glass tube; and it may be collected in the usual way by means of the pneumatic trough. This gas has a smell like prussic acid, of which it is a constituent; it burns readily, and is highly poisonous.

Cyanogen produces hydrocyanic, or prussic acid, by uniting with one equivalent of hydrogen. Hydrocyanic acid is generally prepared from the ordinary prussiate of potass, by means of sulphuric acid. These materials, with water, are placed in a retort, and the acid is distilled over in a dilute state. It is highly poisonous; has a smell like bitter almonds; and forms salts, which are commonly termed prussiates, some of which we shall have to notice hereafter. Cyanogen unites with metals forming the substances called cyanides. These are largely employed in electrotyping processes; and their manufacture and uses have already been extensively dealt with under the head of Electro-Metallurgy. Cyanogen also unites with sulphur, forming what is termed sulpho-cyanogen and the sulpho-cyanic acid.

The union of carbon with iron, producing steel and other combinations—already referred to, but not yet described—will be taken up in connection with the substances, as we examine each separately.

OXYGEN, HYDROGEN, NITROGEN, CARBON, ETC., AND THEIR FUNCTIONS IN ANIMAL STRUCTURE AND SUSTENANCE.

In our previous pages, we have frequently remarked on the different compounds on which animal life depends; and have shown that oxygen, hydrogen, nitrogen, and carbon, are each necessary to the formation of animal substances. We shall now proceed, by way of illustrating the laws already explained, to call attention to the nature of animal organisation, and the functions which the elements, already considered, exercise thereon; and for that purpose shall avail ourselves of an able article, translated by Dr. Bronner, from the works of Professor Moleschott, of Zurich. We have ventured to make some alterations in those parts in the original, which are more fitted for perusal by professional persons only, or have been previously considered; but have retained all which bear on the questions we have already discussed. The general reader will perceive the intimate relationship which exists between vitality and chemical action, and will also acquire a knowledge of the constituents of organised substances. It will also be noticed, how few are the elements which are employed to produce a multitude of results. These, with the exception of two or three mineral substances, have already passed under review.

This subject will be again referred to when we enter on the question of organic analysis; and the perusal of the succeeding article will prepare the reader for a close examination of chemical facts, whilst it assists in the generalisation of them. The nature of the blood is first considered, with its uses in producing animal tissues, &c. The author then proceeds to point out the substances producing healthy and life-giving blood in each kind of food; concluding with general observations on the different kinds of diet required to meet the varied wants of mankind. His remarks are as follow:—

§ 1. *On the Formation of the Blood.*—In former times it was conjectured, and, as it were, instinctively asserted, that the human body changes its material and properties at certain epochs of life; and this has been proved to demonstration by the investigations of physiologists during the present century. Though we cannot determine whether the body is renewed entirely in seven years, as is popularly believed, or in a much shorter period, as science has endeavoured to prove; yet we know that all nutriment passes only through the body. By this, however, is not meant, that what has been received by the mouth is immediately voided. On the contrary, the different ingredients of which our daily aliment is composed, become, in reality, essential constituents of the body itself. We may, therefore, consider that which is rejected to be only the dross which has gradually become unfitted for the functions of life in the different organs of the body.

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* See *ante*, p. 119.

The essence of the ingested nutriment passes from the digestive canal into the circulation, during which process it is transformed into blood, and thus conveyed to the several organs of the body. There it undergoes certain alterations, and returns as waste material into the venous blood, from which certain organs secrete and discharge it externally.

§ 2. The first change, therefore, which nutriment undergoes, after leaving the alimentary canal, is sanguification. The aggregate of the changes which solid and liquid foods undergo up to sanguification, is called digestion. The various organs of the body which unitedly produce these changes, are termed by physiologists, the digestive organs.

From the mouth, into which the saliva is poured by a number of glands through corresponding ducts, the aliment descends into the stomach. In the coats of the stomach are glands, smaller and simpler in construction, but far more numerous than the salivary glands, and which, during the process of digestion, secrete a peculiar fluid, termed gastric juice. Next after the stomach come the small intestines, into which the liver and pancreas, two large glandular organs, pour their secretions—the former the bile, the latter a fluid somewhat similar to saliva. In the coats of the intestines, also, are numerous small glands supplying the intestinal juice. The large intestines follow; the lowermost, a tenth part of which is called the rectum, terminating in the anus.

Besides the secretions enumerated above, there is found a considerable quantity of mucus throughout the whole intestinal canal. This mucus is materially different in different places, owing to the admixtures which it receives in the several parts of the canal.

Saliva, gastric juice, bile, pancreatic and intestinal juice, combined with mucus, are themselves products of the blood; from which, having been elaborated by various glandular organs, they flow, as before stated, into the digestive canal. Thus do the fluid combinations, which have themselves been furnished by the blood, contribute, in return, to the accomplishment of all those transformations of the nutriment, which are necessary for sanguification.

§ 3. *The Alimentary Principles.*—In order to understand these various alterations, it is necessary to make ourselves acquainted with the constituent parts of the nutriment we employ. Let us imagine, for instance, a kind of aliment so compounded that it alone shall be sufficient to maintain life. Nature furnishes such an example in milk. This article contains salts, sugar, fat, and a substance very nearly resembling albumen or the white of egg, which is the principal constituent of cheese, and is hence called caseine.

These four principles constitute by far the greater proportion of all nutritious aliment. We say the greater proportion, not the whole; for either fat or sugar may be absent singly in nutritious food, but not both together. Salts, fat, and albumen, or salts, sugar, and albumen, are three components which, together, are absolutely necessary for the nutrition of the body. They are examples of three groups of alimentary substances, which represent the essential part of all solid and liquid food. To express these groups in more general terms, we have—

1. The inorganic.
2. The organic destitute of nitrogen.
3. The organic possessing nitrogen.

The essential distinction between these three classes, lies in the particular nature of their composition.

The more important elements of nutriment are—

Potassium.
Sodium.
Calcium.
Magnesium.
Iron.
Fluorine.
Chlorine.
Phosphorus.
Sulphur.
Oxygen.
Hydrogen.
Carbon.
Nitrogen.

The seven elements, from potassium to chlorine, belong principally to the mineral kingdom; they compose the inorganic aliments—that is to say, those which exist without any mediate or immediate intervention of living beings; as, for instance, common salt. The instruments of living beings are called organs; and all those substances are called organic, the production of which depends upon the mediate or immediate activity of plants or animals; for example, sugar and alcohol are organic substances.

Nitrogen, carbon, and hydrogen are to be found in all living bodies, whilst entirely absent in many minerals. Therefore they may be denominated, in a stricter sense, organic elements. It is true they appear also in the inorganic world, just as inorganic elements appear also in the organic.

Intermediate between these two groups are oxygen, phosphorus, and sulphur, being nearly as much organic as inorganic elements.

§ 4. There are combinations of two inorganic elements in our nutriment; amongst these common salt, which consists of sodium and chlorine, holds the principal place. A combination very similar to common salt is that of chlorine and potassium, the latter of which represents the chief element of potash. We shall hereafter speak of common salt and chloride of potassium, under the appellation of compounds of chlorine. Both are distinguished by their free solubility in water.

To these combinations, each consisting of two elements, fluoride of calcium may be added, which is to be found in very minute quantities in wheat, in milk, and in blood, and is known to most readers as fluor spar. It is composed of fluorine and calcium. Water dissolves it only at a high temperature, and even then very sparingly.

It may seem strange that common salt is not called a salt by chemists; the salts of chemists are even not similar to common salt in taste. Therefore, as we mentioned before, salts are only an example of the class of inorganic elements. In chemical language, a salt is a combination which does not consist of two elements, but of a combination of two parts, each containing two elements.*

The number of salts is exceedingly large. In treating on the subject of food, it will only be important to refer to the oxy-salts; these consist of different elements of a metal forming, with a little oxygen, a base called an alkali, and of a non-metallic substance, with a large quantity of oxygen, forming an acid.

Almost all bases have a corrosive taste, like lime; almost all acids a sour one, as sulphuric acid; but this is not an essential quality either of bases or acids. A really significant characteristic of bases is their attraction for acids; and of acids, their tendency to unite with bases. These duplex combinations of bases with acids constitute salts. The matter which is commonly employed as a test of both acids and alkalies, is litmus, which is a blue pigment, prepared from a hard, granulated, grayish-white plant, belonging to the class of lichens, which, under the name of Swedish moss, is imported principally from Sweden. Acids turn the litmus red; while bases or alkalies restore its natural colour. The general reader may at any time aid his imagination by thinking of lime as the representative of the bases, sulphuric acid of the acids.†

If, in the combination of a base and an acid, the base predominate in quantity, therefore communicating to the compound its peculiar properties, the salt is called basic, and acid if the acid have the preponderance; if neither predominate, the salt is called neutral salt.

The bases of the salts of our nutriment are constituted out of the first four elements, which are called above, in a stricter sense, the inorganic. They are all metals, and,

* See *ante*, p. 306.

† *Ante*, p. 303.

in forming bases, they combine with a small quantity of oxygen. We pass now briefly to enumerate these bases.

The combination of potassium with oxygen is potash; soda is composed of sodium and oxygen, just as common salt is composed of sodium and chlorine: soda is often elicited from common salt by the introduction of oxygen, which expels the chlorine. Potassa and soda are called alkalies in a stricter sense; they readily dissolve in water. Calcium with oxygen constitutes lime; and magnesia, a substance very similar to lime, is formed from oxygen and magnesium. Magnesia is the base of the well-known medicine, carbonate of magnesia. For the future we shall include lime and magnesia under the common name of earths; they differ from alkalies principally in being much less soluble in water, and in having a slighter affinity for acids.

Iron combined with a higher proportion of oxygen than that which with calcium forms lime, is called oxide of iron, which, in combination with water, produces iron-rust. It differs from the earths and alkalies in being insoluble in water.

All the above-named bases appear in aliment, only in union with acids, forming salts. The most important acids are constituted of those elements which we have called intermediate between inorganic and organic substances.

Sulphur forms, with a large proportion of oxygen, sulphuric acid, generally known by the name of oil of vitriol.* Phosphorus, combined with still more of oxygen than sulphuric acid contains, is called phosphoric acid, which constitutes with lime the solid material of bones.† To these may be added carbonic acid, which produces the sparkling bubbles of champagne, and is a compound of carbon with a smaller proportion of oxygen than is found in sulphuric acid.‡

The alkalies, potash, and soda appear in our food, compounded with the three above-mentioned acids. They form neutral salts with sulphuric acid, sometimes acid salts with carbonic acid, and always basic salts with phosphoric acid. Of these three classes, the last-mentioned enter by far the most largely into the composition of our food.

Amongst the earthy salts, those formed with phosphoric acid again prevail. These are always basic, and can only be dissolved with difficulty in pure water, but become easily soluble if a little acid be added.

The sulphate of magnesia, known to everybody as Epsom salts, is soluble in water; but sulphate of lime or gypsum can only be dissolved with great difficulty.

Finally, the oxide of iron enters into our food in combination with phosphoric acid; and the salt thus formed, though insoluble in water, is dissolved by acids with tolerable ease.

The inorganic principles of nutriment are therefore as follows:—Compounds of chlorine and alkali salts, both soluble in water; earthy salts and phosphates of iron, which dissolve in water with difficulty, or not at all.

§ 5. The organic alimentary substances without nitrogen are in part compounds of carbon, hydrogen, and oxygen, which are capable of being transformed into fat, and we therefore call them *constituents of fat.* In part they consist of different kinds of fat itself, all of which are compounded of the same three elements.

In most constituents of fat, these elements are combined in the same relative proportions; and two of them, hydrogen and oxygen, in the same as that in which they are found in water.

All fatty substances contain the same proportion of oxygen, and proportions of carbon and hydrogen mutually equal, though varying in different kinds of fat. In comparison with hydrogen, fat itself contains much less oxygen than water or the constituents of fat.

The most important constituents of fat are starch, gum, and sugar. We shall often include these under the name of *starchy substances.* Starch is only soluble in boiling-water, but the other two easily in cold.

Of the fatty substances we must here mention *oleine*, *margarine*, and *stearine.*§ Oleine is the chief component of all oils, and denotes their characteristic part, which slowly coagulates by cold. Besides oleine, oils contain a fat which hardens more readily, and may be obtained in crystals, glittering like mother-of-pearl. On this account, and not because it is actually found in mother-of-pearl, it is called mother-of-pearl fat, or margarine. Stearine is the firmest of all fats. It is the principal solid substance in the fat of mutton and beef, where it is combined with margarine and oleine. Stearine candles contain stearine and margarine.‖

While most constituents of fat are identically composed, sugar only containing more hydrogen and oxygen than the others, there is in oleine, with the same quantity of oxygen, more carbon and hydrogen than in stearine, and in stearine more than in margarine

The above-named fatty substances are not soluble in water, and cannot as such be united with other substances in soluble combinations. As though saturated in themselves—that is to say, without any inclination to combine with other substances, and, so to speak, standing impartially between acids and bases—we may call them *neutral fats*, just as the salts in which acids and bases are in equilibrium are termed neutral salts. By the admixture of alkalies, neutral fats are separated into two substances, one of which is called the sweet principle of oil, or *glycerine*, a substance compounded of carbon, hydrogen, and oxygen; the other, and by far the greater part, is a *fatty acid*, which combines with the alkalies to form a *soap*. Thus oleine yields oleic acid; stearine, stearic acid; and margarine, margaric acid. As glycerine, which has been separated from neutral fats by potash or soda, is of the same composition, from whatever fat it is extracted—it follows that the remaining acids retain the same proportion of carbon and hydrogen, as the primitive neutral fats; that is to say, oleic acid contains more carbon and hydrogen than stearic acid, and stearic acid more than margaric acid.

The compounds of fatty acids with potash are called potash-soaps; and those of soda and fatty acids, soda-soaps. Soaps, therefore, may be regarded as analogous to salts, the acids being an organic substance destitute of nitrogen, the basis an inorganic body.

§ 6. Another group of *organic alimentary principles* are those *containing nitrogen*, which are compounded of a greater number of elements than fat and its constituents contain. Of these constituents of food, we have only to mention the albuminous substances. All albuminous aliments contain nitrogen, carbon, hydrogen, oxygen, and sulphur, and most of them phosphorus also. The white of a hen's egg gave its name to these substances, which have been properly united into one group. The proportions of nitrogen, carbon, hydrogen, and oxygen, of which they chiefly consist, show a most surprising, if not complete agreement; recent chemical investigations having proved, that, with respect to the four latter elements, their composition is almost identical. Before this, however, the great similarity of their properties had led to their being comprised under one general name. Four of them, the caseine of milk, the principle of the yolk of eggs, the gluten of grain, and the globuline in the crystalline lens, as well as in the globules which float in the blood, are distinguished by not containing any phosphorus. The quantity of sulphur is different in these several instances, just as that of phosphorus is in those which contain this element. In the following list the albuminous substances are given in order, according to the quantity of sulphur they contain; those which possess the higher proportion being placed first:—The albumen of eggs, the albumen of blood, fibrine and globuline of blood—both of which contain an equal quantity of sulphur; the principle of the yolk of eggs, gluten, caseine, soluble vegetable albumen, coagulated vegetable albumen, and legumine. The largest proportion of phosphorus exists in legumine; the albumen of eggs stands next, which, however, contains scarcely a quarter of the phosphorus of the former; and last of all, fibrine and albumen of blood. The amount of phosphorus in

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[*] See *ante*, p. 308.

[†] *Ante*, p. 311.

[‡] *Ante*, p. 326.

[§] See *ante*, p. 93.

[‖] *Ante*, p. 95.

soluble and coagulated vegetable albumen has not yet been determined; but the quantity of sulphur and phosphorus is always very small in comparison with that of the other elements.

Amongst the albuminous principles, soluble vegetable albumen, legumine, albumen of the blood, albumen of eggs, globuline and caseine, are, when fresh, soluble in water; the principle of the yolk of eggs dissolves with great difficulty; coagulated vegetable albumen, gluten, and fibrine of the blood, not at all. The different kinds of albumen, in a stricter sense, coagulate if boiled in water, as everybody has seen in hard-boiled eggs. The globuline of blood coagulates by warmth alone; legumine and caseine by acids in a warm temperature. We know how quickly milk curdles and turns sour in warm weather.

All albuminous principles, insoluble as well as soluble, are, in a coagulated state, dissolved at a somewhat higher temperature by a solution of potash, and are, from this solution, precipitated by acids in a solid form.

§ 7. *Digestion.*—After this brief description of the most important alimentary principles comprising our nutriment, we shall resume the account of digestion.

The whole of digestion is included under two grand processes. First, the alimentary principles must be dissolved, or very minutely divided; and secondly, if different from the constituents of blood, they must be transformed into these. Let us now see how these changes take place by the action of saliva, gastric juice, bile, pancreatic, and intestinal juice, upon the compounds of chlorine, the salts, the constituents of fat, the fatty substances, and albuminous matter.

All fluids which flow to the digestive organs contain a relatively large quantity of water, at a temperature of 98° Fah. Common salt and chloride of potassium are easily dissolved in this water, as well as the phosphoric, sulphuric, and carbonic alkaline salts.

In the liquid of the gastric juice is a free acid. The earthy salts, therefore, which we before described as hardly or not at all soluble in pure water, but easily so in acid, are by this means reduced to a dissolved state.

Owing to the higher temperature peculiar to all the fluids of our body, a part of fluoride of calcium is dissolved; but the greater part of it, and of the oxide of iron, remains undissolved. This explains why the alvine evacuations always contain so much iron. Here also the gastric acid plays an important part, inasmuch as it dissolves a portion of the iron, which is thoroughly indispensable to the blood.

Starch by itself is not soluble in the water of the digestive fluids; but saliva, the mucus of the mouth, pancreatic and intestinal juice, have, in combination with each other, in a remarkable degree, the property of transmuting starch into gum, and gum into sugar. Thus, starch is not only indirectly dissolved, but is subjected in that process to such changes as render it capable of being assimilated with the substances of the blood; for sugar is transformed into lactic acid by the action of the bile; lactic acid into butyric acid while progressing through the alimentary canal; and butyric acid is the first link in the series of fatty principles which are found in living bodies. Oleic and margaric acid differ in their composition from butyric acid only by containing more carbon and hydrogen in proportion to their oxygen.

The last effect of digestion upon all the constituents of fat is transmutation into fat itself. Starch and gum are transformed into sugar, sugar into lactic acid, lactic acid into butyric acid, and butyric acid into other fatty substances.

Neutral fats are principally digested by the action of pancreatic juice, assisted by the bile. These fluids divide the fat into particles so small, as to be capable of easily penetrating through the membranes moistened by the bile. Another smaller portion of the fat is actually dissolved, for the carbonic alkali of the bile first effects a saponification of the fat, then the continuous action of the pancreatic juice transforms it into fatty acids and glycerine: thus stearic acid and glycerine arise from stearine; oleic acid and glycerine from oleine. The fatty acids combine with the alkalies to form soaps, which are soluble.

All the fluids of the digestive canal act as solvents upon the albuminous substances; and to this dissolving action almost all the components of these fluids contribute. It is especially so with the free acid of the gastric juice; in the next place, with the predominating alkali of saliva, bile, pancreatic, and intestinal juice; then with organic substances, water, and the salts of all digestive juices. By the action of the acid of the gastric juice, the soluble albuminous substances are first coagulated; but they are again dissolved by degrees, by means of a power which the gastric acid exercises upon undissolved albuminous substances, supported by the alkalies of the intestinal and pancreatic juice. The most important agents in the dissolution of albuminous matter are, however, the organic components of the gastric and intestinal juices.

By the dissolving action of the digestive juices, and by the peristaltic grinding motion of the walls of the digestive canal, food is changed in the stomach into a pulp called *chyme.* This, by degrees, is rendered more liquid, until it becomes a thick, milky juice, which physiologists call *chyle.*

This chyle is essentially a mixture of dissolved compounds of chlorine, of salts, sugar not yet entirely transmuted into fat, of lactic and butyric acids, of finely divided and saponified fats, and of soluble albumen. Here, also, the latter may be regarded as representing the remaining class of albuminous substances; and this so much the more, as these after-liquefactions have become, in the highest degree, similar in their properties to soluble albumen; but still they retain their original composition.

§ 8. *The Chyle.*—Thus the liquefied alimentary principles flow over the inner surfaces of the intestinal walls, in which are numerous vessels, long and narrow canals with thin walls, into which the greater part of this milky chyle exudes. These vessels are, therefore, called *lacteals.* Not only their walls, but also those of the blood-vessels, are permeable by liquids; and, therefore, a considerable part of the chyle also exudes into the blood-vessels of the intestines.

When digestion is nearly finished, the lacteals teem with a white milky juice, which owes its colour to the fat they have taken up; for if no fat have been digested, we find in these vessels only a clear, transparent juice, scarcely deserving the name of chyle.

The very numerous and smallest lacteals gradually unite themselves into larger trunks, which, at certain points, run very near each other, repeatedly widening and contracting, something like a string of beads, and pursuing irregular windings, held together by cellular tissue, and forming, as it were, lumps or knots, which have been improperly called intestinal glands.

The lacteal vessels, after having passed through these knots, by uniting in gradually larger trunks, form at length one duct. By the junction of this trunk with two vessels, which convey a white, watery fluid from other parts of the body, and are called *absorbents,* another canal is formed, which passes through the cavity of the abdomen and chest near the vertebral column, and is called the *thoracic duct;* through this the chyle flows into the blood.

The composition of the chyle, which has been absorbed by the lacteals, differs very little from the liquid which, during digestion, is contained in the lower part of the small intestines. Before these vessels, in their convolutions, have become united into knots, the properties of their contents are almost entirely the same as they were in that part of the intestines from which they were taken. The quantity, however, of solid matter held in solution in the liquids varies; accordingly, the chyle contains more water in the lacteals than in the intestinal canal. This facilitates the transition of matter from the intestinal canal into the lacteals. The water in the latter has a great affinity to the substances which, in a dissolved or liquefied state, flow over the interior walls of the intestines. The chyle of a person fasting is more watery and thin than that of one who has had a plentiful meal.

Up to the formation of the knots just mentioned, the lacteals commonly contain an alkaline solution of compounds of chlorine and salts, among which common salt and phosphate of soda predominate, with a certain quantity of sugar; also, if any amylaceous substances have been present in the food, neutral fats, as well as oleic and margaric soaps—that is to say, compounds of oleic and margaric acid with soda and potash; and, lastly, albumen and fibrine.

The last-named substance is not yet quite identical with the proper fibrine of the blood, of which the characteristic property is that of spontaneously coagulating without the application of heat or acid when withdrawn from the living body. The fibrine of the chyle, which has in other respects the same properties, acquires this quality only on its way from the intestinal canal to the blood-vessels, and after having left that part of the lacteals which is convoluted into knots.

In the lacteals also begins the development of a red colouring matter, which, besides nitrogen, carbon, hydrogen, and oxygen, contains some iron. The quantity of the latter is as constant as it is small in proportion to that of the four first-named elements. This substance must necessarily be formed from one of the albuminous principles, and an iron salt of the chyle. It is the occasion of the light-red colour so often remarked in the superior part of the thoracic duct. This constituent of the chyle, containing iron and nitrogen, is called *colouring matter of the blood*, because blood possesses no other constituent of this colour.

How far the formation of blood has now advanced, will best be seen if we pass to the description of the blood itself.

§ 9. *The Blood.*—In the blood of man, vesicles are suspended containing a red substance, and white, granulated corpuscles. The former are called *coloured*, the latter *uncoloured blood globules.*

The fluid, in which the blood globules are continually in motion in the living body, is always impregnated with three different kinds of gas—oxygen, carbonic acid, and nitrogen. Apart from these gases, the blood consists of a solution of salts, albuminous substances, fat, and sugar. When withdrawn from the veins or arteries, the blood deposits a red dot, leaving a yellowish liquid. The former contains principally the fibrine and coloured matter of the blood; the latter is albumen and salts. Fat is equally distributed in both. This separation into clot and liquid arises from the property of fibrine to coagulate, as soon as the blood is removed from the influence of the living body. While coagulating, the fibrine incloses the blood globules, and therefore the clot retains the red colour. In the vessels of the living body, fibrine always exists in the dissolved state. This is in part effected by the salt and the predominance of alkali in the blood; but only in part, because the salts and alkalies are not able to keep the fibrine in a soluble state out of the body. The supposition may, therefore, be justified, that the fibrine leaves the blood and blood-vessels of the living body before it has become ripe for coagulation. Out of the body, fibrine undergoes this development in the blood itself, and coagulates, even where temperature, motion, and protection from the external air have been maintained, artificially, as much as possible as it had been during life. We consider the oxygen contained in the blood itself to be the chief cause of this change.

The coloured globules consist of a white capsule, containing a red liquid: this capsule being of an albuminous substance, is so transparent that it becomes entirely invisible when the colouring matter has been washed out of the globules by a sufficient quantity of water. The red colouring matter, which is contained in the blood globules, besides several salts, fat, and albumen, is invisible through its pellucid enclosure. Iron is the element which, in the colouring matter of the blood, is united with nitrogen, carbon, hydrogen, and oxygen, just as sulphur or phosphorus is in the albuminous substances. Its presence is indispensable for the formation of the colouring matter; therefore is iron so efficacious in many diseases, when the blood contained in the finest vessels, and shining through the skin, has lost its colouring matter. This red pigment of the blood, when fresh, is soluble in water, and is still more so in alkalies, which are even capable of dissolving it in a dry state. The alkali in the blood facilitates, therefore, the solution of the colouring matter.

No albuminous substance is so abundantly dissolved in the blood as pure albumen, which, remaining in the liquor after its separation from the clot, coagulates only at the boiling-point, just like the white of an egg. Traces of caseine are found with the albumen in the blood.

Fresh blood manifests an alkaline action with litmus. This is to be attributed to the presence of a basic phosphate of soda. Amongst the salts of the blood, phosphate of soda and bicarbonate of soda predominate. To these salts may be added the compound of potash with the same acids: besides the sulphates of soda and potash, the phosphates of lime, magnesia, and oxide of iron, are all conveyed to the blood with the chyle.

In our own times chemists have succeeded in detecting fluoride of calcium among the constituents of the blood.

Just as of all other albuminous matters existing in the serum of the blood, albumen, strictly so called, is the most abundantly represented; so also common salt, or chloride of sodium, predominates among the inorganic substances. Chloride of potassium is present in relatively smaller quantity.

Blood contains, if examined when taken quite fresh from the living body, oleine and margarine. But they are very rapidly decomposed into oleic and margaric soaps, by the influence of the albuminous substances and the carbonates of potash and soda in the blood. With them another fatty substance is also present, remarkable for the amount of nitrogen and phosphorus which it contains; and a second, distinguished from most vegetable and animal fats by its phosphorus.

Analogous to the neutral fats, consisting only of oxygen, hydrogen, and carbon, we find a compound intermediate between fat and wax, containing less oxygen than the former, and incapable of being saponified, bearing the name of bile-fat, only because it had first been discovered in the bile.

Sugar is always found, only in a small quantity, in the blood. Before it reaches the circulation, it has been for the greatest part transformed into fat.

In order to give an idea of the proportionate quantity of the constituents of the blood, in round numbers, we add the following summary review. A thousand parts of blood contain—

Fibrine	2
Blood globules . . .	131
Albumen	71
Compound of chlorine and salts .	5
Fat	2
Water	789
Total	1000

ORIGIN OF THE SOLID SUBSTANCES OF THE HUMAN BODY.—§ 10. The blood is conveyed from the heart to all parts of the body. The heart propels it, by regular contractions, into the vessels, which have also a regular pulsation, and are called *arteries.* These ramify in smaller and smaller vessels, which at length, from their minute size, are called *capillaries*, and pervade in very great number all the organs of our frame. By means of the arteries, every part of the body receives arterial blood.

The dissolved substances of the blood pass everywhere through the walls of the capillaries, forming thus what is called the *juice of nutrition.* As the fluid from which crystals are precipitated is called mother-lye, so we may call this juice of nutrition mother-juice of the solid constituents of the body; for all vesicles or cellules, fibres, and formless deposits, composing the several solid parts or tissues, are formed from the substances dissolved in the juice of nutrition. This mother-juice, at the same time, owes its immediate origin to the blood.

§ 11. As the transudation of the blood is dependent on its 789-thousandths of water, all solid parts of our body are, and most of them very abundantly, saturated with this liquid. But the water of the blood never passes through the capillaries without salts.

A certain quantity of the inorganic constituents of the blood is therefore to be found in all tissues. Some of them are, however, remarkable for a very especial affinity to certain tissues. Thus, in the bones, basic phosphate of lime predominates not only above all other inorganic substances, but also above all their other components. The affinity of common salt to the cartilages is equally conspicuous. The muscles, on the other hand, contain an equally large portion of chloride of potassium, the quantity of common salt being but small. Thus, while the blood contains much more common salt than chloride of potassium, the muscles possess much more of the latter than of the former.

Fluoride of calcium is another inorganic constituent, which, whilst contained in the blood only in a small quantity, forms a necessary material for certain tissues. Bones and teeth regularly attract this substance from the blood. Herein, also, we may recognise one of those peculiar affinities of tissues for certain inorganic substances of the blood, which are so frequently met with.

Carbonate of lime, or chalk, which of itself is insoluble in water, has not yet been discovered in the blood; but it is possible that it may pass as such through the circulation into the bones; for the carbonic acid and chloride of potassium of the blood, have the power of keeping small quantities of carbonate of lime in a dissolved state; and one part of the carbonate of lime is formed out of the sulphate of lime, which reaches the blood with the water and other nutriments we take. This sulphate of lime is decomposed by carbonate of soda, so as to form sulphate of soda and carbonate of lime. This new compound is a characteristic constituent of the bones.

§ 12. With a small quantity of most of the inorganic compounds in the juice of nutriment, a part also of the albumen of the blood passes into all tissues.

But certain laws of affinity prevail also amongst the albuminous substances, which determine their appearance in a regular way. Thus the crystalline lens of the eye appropriates the globuline of the blood; the walls of the blood-vessels, caseine; and the muscular tissue, fibrine.

It is true the latter is not entirely identical with the fibrine of the blood; for it is not in the muscles in a dissolved state, but forms fibres, which are characteristic of flesh. And yet this fibrine of the muscles is more easily dissolved in water containing a few drops of hydrochloric acid, than coagulated fibrine of the blood.

The influence of these laws of affinity is still more extensive with regard to those tissues which contain modified albuminous substances. To these belong *horn*, *gelatine*, and *the substance of the fibrous tissues*.

Horn is composed of nitrogen, carbon, hydrogen, and oxygen, in proportions altogether similar to those of the albuminous substances. It covers the outward surfaces, it lines the walls of all cavities of the body; for the epidermis, nails, hair, the coating of the mucous membranes lining the cavities of the intestines, the windpipe, mouth, and other interior parts of our body, consist of horny substances.

But though these parts, so different externally, are all called horny substances, on account of their affinity in composition with the horns of cattle, they are by no means identical; for, apart from the differences of their qualities, which certainly are not very considerable, they contain certain quantities of sulphur, which element, besides nitrogen, carbon, hydrogen, and oxygen, is to be found in all of them. The smallest quantity of sulphur is contained in the epidermis, and the coating of the mucous membranes; the nails contain more, the hair most. The two latter are with very great difficulty soluble in acetic acid. All horny substances agree with each other, and with the albuminous substances, in their solubility in potash, from which solution they are precipitated by acids.

We have to distinguish two kinds of gelatine: the one is prepared from the cartilages, the other from the bones.

Bone-gelatine is obtained by boiling bones in water. A saturated hot solution, which has been thus prepared, coagulates, when cooling, into a thick gelatine, which swells by the addition of cold water, and is dissolved again when boiled. In short, this substance is the well-known glue, which plays such an important part in the ordinary affairs of life; it is the same substance, better known as jelly, into which veal broth coagulates in cooling. But gelatine is not, as such, contained in the bones; it is produced by a slight transmutation, which this organic basis of the bones undergoes in boiling. Therefore bones are called a gelatine-yielding tissue, which appellation is justly extended to many other component parts of our body; that is to say, to those fibres which, accumulated in bundles, invest most instruments of our body, and connect them with each other. For the last-mentioned property they are sometimes called binding tissues, though generally known as cellular. The stratum of the skin beneath the epidermis, the exterior covering of the lungs, intestines, muscles, nerves, bones, and of many other organs, consist, for the greatest part, of fibres, which, as well as the bones themselves, yield gelatine when boiled. The bones furnish a clear evidence of the affinity of the gelatinous tissues for phosphate of lime.

Nitrogen, carbon, hydrogen, and oxygen, with a very small quantity of sulphur, compose the bone-gelatine. This is soluble in hot water; and has the important property of not being precipitated from this solution by acetic acid.

Similarly to the bones, the cartilages yield, when boiled for a length of time, a peculiar gelatine, called gelatine of cartilage. This contains the same elements as the bone-gelatine, only in other proportions. While it agrees with the latter in stiffening to a jelly when the hot solution cools, it is the distinctive character of gelatine of cartilage—that strong acetic acid produces a precipitation from a solution of it in water. We refer here again to the above-mentioned affinity of the tissue yielding gelatine of cartilage with common salt.

Whilst horn and gelatine possess an essential similarity in their composition to the albuminous substances of the blood, by containing sulphur, the elastic fibres are without this ingredient. But the latter still contain nitrogen, carbon, hydrogen, and oxygen, in such a proportion as to remind us directly of the albuminous substances in spite of this difference. Their absolute insolubility in water, as well as their resistance to acetic acid and potash, prove, however, that with respect to their properties they deviate the farthest from the albuminous substances. In these dissolving media they keep for days entirely undissolved.

Elastic fibres are found in an abundant quantity in the ligaments, which unite together the different bones of the spine; besides being found in the lungs, the walls of the arteries, and isolated in many other parts of the body.

That horn, gelatine, and the substance of the elastic fibres, are justly to be regarded as modified albuminous substances, is proved by the fact, that, with the exception of a very small quantity of fat containing nitrogen and phosphorus, no other compounds of nitrogen are present in the blood than the albuminous matters, which also alone contain sulphur. Horny substances and gelatine, both of which contain sulphur, can thus only have their origin in the albuminous principles of the blood. Perhaps it might be suggested, that the fat of the blood, which contains nitrogen, may produce the substance of the elastic fibres; but the quantity of this fat is much too small for the considerable quantity of elastic fibres in our body to be derived from it. The conclusion that the albuminous compounds are the only originating source of the horny substances, gelatine and the elastic fibres, is rendered unquestionable by the fact, that none of these substances are found as such in the blood.

§ 13. In fresh tissues, fatty substances are only present in the form of neutral fats, as oleine and margarine.

There is scarcely any part of the body from which these neutral fats are entirely absent. There is, however, very little fat in the lungs, and less in the teeth. It exists in great abundance under the skin; in the bones as marrow; in the mesentery, by which the intestines in the abdominal cavity are, as it were, suspended; in the orbit of the eye; and in the female breasts. The smoothness and roundness of the forms of females and children are chiefly attributable to the layers of fat, which cause the soft convexities of their skin. This fat is much less abundant under the skin of men; and the forms, therefore, of their bones and muscles are much more strongly defined.

Oleine and margarine are mingled in very different proportions in different parts of the body; and margarine coagulates more readily than oleine. It follows, that the greater the predominance of oleine and margarine in any part of our body, the greater also will be the lubricity and fluidity of the fat; as, for instance, in the marrow of the bones, and in the fat under the skin. The firmer fat again, which, for instance, surrounds the kidneys, contains more margarine than oleine.

We find the bile-fat of the blood unchanged in the brain; but the latter contains, besides this, oleine, margarine, and another kind of fat, in which carbon, hydrogen, and oxygen, are combined with nitrogen and phosphorus. Without this peculiar fat the brain cannot exist. It is remarkable that this brain-fat has been detected not only in the blood, but also in the yolk of eggs.

§ 14. Sugar is a regular constituent of the liver; the muscles contain also a peculiar kind of sugar, called sugar of the muscles. Lactic acid—a substance soluble in water, and compounded of carbon, hydrogen, and oxygen, exactly in the same proportion as in sugar of grapes—is found in the muscles in considerable quantities. In the brief description of digestion, we saw that, by the influence of the bile, lactic acid is formed from sugar. It is more than probable, that, in the muscles also, lactic acid is produced from their peculiar sugar; in short, the constituents of fat are also present in the different tissues, though in a very small quantity.

§ 15. *Organs of Secretion.*—The sum of the processes which we have described in a former section, in order to explain the origin of the solid constituents of our body, is designated by physiologists as nutrition in the strictest sense. Not all matters, however, which transude through the capillaries from the blood are to be considered as having a direct share in the origination of the solid constituents. A very great proportion of the transuded matters does not contribute to the nutrition of the tissues, but remain as a liquid; which, having been attracted or prepared by certain organs from the blood, is collected into receptacles, and out of these discharged into certain cavities of the body, or altogether out of it. These receptacles are often simply canals, called secretory ducts of the organs; whilst the organs themselves are called glands. In other cases, the secretory ducts lead, before they discharge the liquid, into wider reservoirs, called bladders. Thus we have a seminal bladder, a gall bladder, and a urinary bladder.

The fluids, which are attracted and collected from the blood by the glands, are, in some respects, of a special and immediate use in relation to the body, by assisting the functions of reproduction and digestion. All fluids which co-operate for these two essential functions, in preserving the species and the individual, are included under the general head of secretions. In all secretory fluids there is contained more or less water, which is a necessary condition of transudation generally.

§ 16. We begin with the ovum, not only on account of its importance in the preservation of the species, but because, next to the tissues, it is the firmest of the products of secretion. It forms, therefore, a kind of transition from the tissues themselves to the secretions. As the human ovum, in the earliest state of its development, can only be perceived by a high magnifying power, its investigation has been incomplete. We know certainly only this, that it consists of an albuminous substance, combined with oleine, margarine, brain-fat containing phosphorus, bile-fat, sugar, and the inorganic constituents of the blood.

§ 17. The seminal fluid can also be subjected only to a very imperfect analysis; but its chief constituent, the seminal principle (spermatine), is, in its essential properties, so nearly analogous to albuminous substances, that we may, without hesitation, classify it with them. Should future investigations confirm the slight distinction already observed, the seminal principle will still be proved to be a compound very similar to the albuminous substances; but it is not real albumen, as it does not coagulate by boiling. The seminal fluid contains, moreover, some true albumen combined with soda. Some fat and inorganic substances are also found in it; but their properties and proportionate quantities have not been accurately investigated.

§ 18. An albuminous compound, which as such is found in the blood, though in small quantity, is abundantly collected towards the end of pregnancy and after confinement, in the breast or lacteal gland of the female. It is the caseine, which, as contained in milk, forms an essential alimentary principle for the infant.

Caseine is the substance which coagulates at the surface of boiling milk, in the form of folded, wrinkled membranes, which, as often as they are taken off, are succeeded by others. It is completely and rapidly separated in thick flakes, if milk, which when fresh is alkaline, is boiled with an acid—for instance, with acetic acid. In this way milk is separated into a firm part, which contains, besides caseine, a great quantity of fat; and into a liquid, in which are sugar of milk, the rest of the milk-fat or butter, certain inorganic substances, and the acid which has been added; this last liquid represents the well-known whey.

The sugar of milk is a constituent of fat, like the sugar of grapes, into which the former can easily be transformed by acids. Sugar of milk differs from the sugar of grapes in being incapable of vinous fermentation by the addition of yeast. This fermentation consists in the change of sugar of grapes into spirit of wine and carbonic acid; but as acids transform sugar of milk into sugar of grapes, the former also is mediately fermentable. Sugar of milk is soluble in water, and consists, like lactic acid, of carbon, hydrogen, and oxygen, in the same proportions as sugar of grapes.

It is peculiar to sugar of milk, that its sweet taste is very slight. Here also we see, as formerly, when speaking of acids and salts, that taste is a very deceptive test of chemical distinctions. As sugar of grapes is found in the chyle and blood, and corresponds in its composition and properties very nearly with sugar of milk, the latter is therefore to be derived from the sugar of grapes, and also from the amylaceous substances of our nutriment, which are transformable into sugar.

When we speak in ordinary life of butter, this has a much wider signification than chemists attach to the name of butter-fats, or butyrine. In the latter sense, we understand by it a neutral fat—fluid at ordinary temperatures—which, when combined with alkalies, produces a soap, the acid of which, when free, is very volatile, and possesses a strong acute smell and taste of butter. This acid is, therefore, called butyric; but in butter, butyrine is mixed with oleine and margarine; and by the saponification of butter, we not only obtain alkali-salts of oleic, margaric, and butyric acids, but also of three other fatty acids, which, like butyric acid, are remarkable for their volatility and pungent smell. We shall call them caseic, sudoric, and capric acids (capronic, caprylic, and caprinic acids of other authors)

While butyric acid, with other fatty acids found in our bodies compounded with glycerine or alkalies, contains, in proportion to oxygen, the least quantity of carbon and hydrogen, the two latter elements are more abundantly represented in the three other volatile acids of the butter—namely, caseic, sudoric, and capric acids; the first containing the smaller quantities.

As milk contains, among its inorganic constituents, common salt and chloride of potassium, which is similar to it, as well as the compounds of potash, lime, magnesia, and oxide of iron, with phosphoric acid, it is apparent that the most important base, and the most important acids, together with the compounds of chlorine of the blood, are represented in milk; that is to say, all the inorganic substances to which different tissues possess a distinct and peculiar affinity, are there represented.

§ 19. Saliva, gastric juice, bile, pancreatic and intestinal juice, have already been discussed, according to their several parts in the digestive process. We have briefly to review them here also, as they take an important place among the substances transuding from the blood, through the walls of the capillaries. An albuminous compound, which, from its presence in saliva, has been called the saliva principle (ptyaline), oleic and margaric alkalies, bile-fat, a fat containing phosphorus, and all the inorganic substances which are contained in the blood, clearly bespeak the origin of saliva, which appears in the mouth, mixed with mucus.

In the saliva, as it flows from the secretory canals of the salivary glands, alkali predominates. Why, at the same time, the fluids of the mouth are so seldom found acid, is not yet thoroughly known; as, according to the most recent researches, even pure mucus of the mouth, unmixed with saliva, appears to possess alkaline properties.

§ 20. Gastric juice, furnished by the small glands of the stomach, also contains an organic substance, which, like the albuminous matters, is neither alkaline nor acid, but consists of nitrogen, carbon, hydrogen, and oxygen; and is, at all events, closely allied with the albuminous matters, though it differs from them in some less essential properties. As it cannot be denied that this combination has a highly important influence in dissolving the albuminous substances, the digestion of which is one of the first conditions of the formation of blood—it has, therefore, not improperly been called, *par excellence*, the digestive principle, or pepsin. The digestive principle is, however, very vigorously supported in its dissolving action by a free acid, to which the gastric juice owes the property of reddening litmus. This is to be considered, according to the most recent investigations, as hydrochloric acid.[*]

Among the inorganic substances of gastric juice, we have besides to mention the compounds of chlorine with sodium and potassium, calcium, magnesium, and iron, and with the phosphoric salts.

§ 21. In the secretion of the liver, there are two peculiar organic acids, one of which, containing only nitrogen, carbon, hydrogen, and oxygen, is called *cholic acid;* the other, which, besides the four elements mentioned, contains a very inconsiderable quantity of sulphur, may be called sulphurised cholic acid. Apart from the liver, no trace of these acids is to be found in the blood. They are therefore not elaborated in the general circulation, but by the action of the liver.

A sweetish-bitter taste is not only peculiar to these acids, but also to their combinations with sodium. In the latter form they are soluble in water.

The combination of both these acids leaves no doubt that the albuminous substances of the blood are essential to their formation. Oleine, margarine, and gall-fat, accompany the combinations of potash with cholic acid and sulphuretted cholic acid.

The bile owes its yellowish-green colour, sometimes inclining to brown and sometimes to green, to several organic colouring matters containing nitrogen; and its alkaline character to phosphate of soda, a salt in which the alkaline ingredient predominates. The other inorganic substances are common salt, chloride of potassium, carbonated alkalies, and the phosphorates of lime, magnesia, and oxide of iron.

§ 22. The pancreatic juice is a fluid containing so much albumen, that it coagulates by heat almost as completely as the white of eggs itself. The alkaline property of this secretion, certainly, greatly contributes to its carrying with it so much albumen from the blood into the cells of the pancreas. The albumen of the pancreatic juice, however, does not entirely correspond to that of the blood. Both are precipitated by alcohol from their solution in water; but if we dry the albumen of the blood which has been precipitated by the alcohol, we cannot dissolve it again in water; while the albumen of the pancreatic juice is even then soluble.

Besides margarine, and some other organic substances which have been very imperfectly examined, because of the difficulty of procuring pure pancreatic juice, we find in this liquid common salt, chloride of potassium, phosphorates, carbonates, and sulphates of the alkalies, and lime combined with carbonic and phosphoric acids.

§ 23. With all digestive fluids is mixed a certain quantity of mucus, which is alkaline in the mouth, and is said to be neither acid nor alkaline in the empty stomach. In the intestines this mucus is not only mixed with all the above-mentioned digestive fluids, saliva, gastric juice, bile and pancreatic juice, but contains also a peculiar secretion attracted from the blood by certain small glands in the walls of the intestines. These glands, however, are small, and the quantity of their secretion is scanty; but the quantity of the other digestive fluids mixed with it is abundant. Therefore we know of this intestinal juice nothing more than that it possesses alkaline properties.

Mucus contains, first, a peculiar element of which little is known, but from which it derives its property of cohering in threads; secondly, a number of horny cellules, detached from the interior coating of the intestinal canal; and finally, a combination of carbonate, phosphate, and sulphate of soda, phosphate and carbonate of lime, and oxide of iron.

§ 24. *Decomposition and Transition.*—It is a fundamental property of the organic substances of our body, that their combinations possess very little permanence. A perpetual process of transition from one state of being to another, constitutes a grand circle, of which the nourishment of man himself forms but one little segment. The life of plants and animals is not only a direct result of this constant change of matter, but all organic life is a continuous process of alternate combination and decomposition, to which higher and lower influences, cooperating in essential harmony, contribute. What plants absorb from the soil and the air, they transform into substances on which the herbivorous animals feed; and by the latter they are retransformed into animal food for the support of carnivorous classes, as well as for those which feed upon a mixed diet. To the latter class, man himself belongs. Then that which is given off in an uninterrupted series of various kinds of decomposition by men, animals, and plants, during life and after death, returns again into the air and soil as material for the growth of another generation of plants. Here again the process of transition recommences, and a perpetual and ceaseless revolution is pursued.

It is during life itself that this decomposition takes place. In the very tissues, a regressive metamorphosis, so to speak, has already begun; for the oxygen of the air, which we incessantly inhale, is conveyed to all parts of our body. And no element exerts a mightier influence in the formation and dissolution of organic combinations than oxygen.[†] Exposed to the continual action of this element, which all kinds of vegetation are unceasingly giving forth into the atmosphere,[‡] no organic combination of our bodies can maintain its permanence. Albumen, gelatine, and fat, fibrine and sugar, all are gradually rearranged into compounds, containing at every step an increased quantity of oxygen. These revert from the tissues into the blood, from which they are attracted by certain glands, collected into receptacles, and at last ejected externally. This ejection is called excretion.

§ 25. *Connection of Excrementitious Bodies with the Tissues.*—Decomposition continually goes on in the tissues; for what physiologists of a former age had rather conjectured than proved, has been triumphantly demonstrated by more recent investigations; and it is now known that the constituents of the tissues gradually deposit a sediment, which, being unfit for the functions of the organs in which it accumulates, is conveyed back to the blood, in order to be excreted from it.

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[*] See *ante*, p. 317.

[†] See *ante*, p. 313.

[‡] See *ante*, p. 315.

The most important substances which are excreted, have, in the course of physiological science, been traced from the blood into the tissues. The blood contains urea; the vitreous humour of the eye does also; carbonic acid is present in a large portion in the blood, and is likewise to be found in every solid part of our bodies; and carbonic acid and urea are the principal products of decomposition, which the lungs and kidneys abstract from the blood, in order to remove them as waste material from the body.*

A process which ceases not even for a second, and in which one development is continually following another without any cessation, can evidently be marked at certain stages of the transformation only. Therefore we cannot make out, in an uninterrupted chain, the whole series of intermediate conditions through which the albuminous and adipose substances pass, before being wholly decomposed into those simple products, urea, carbonic acid, and water. But since, through the increasing number of investigators, aided by instruments of greater delicacy, more and more points of transition have been found in the series conducting from the constituents of the blood, through the tissues, to the excretions, we are able confidently to assert, that we are pursuing our investigations in the right direction.

We cannot, at present, draw any distinct limit between substances belonging to the development, and those belonging to the tissues. But no greater praise can be bestowed upon the physiologist, than that he breaks through the barrier by which it has too often been attempted to enclose certain parts of nature, and to blockade the understanding of man. The transitions of nature are numberless. The boundary-lines between the several classes of natural bodies lose their distinctness in proportion to the extent and depth of our scientific knowledge.

To the substances transitional between development and regressive transformation, belongs a combination consisting of nitrogen, carbon, hydrogen, and oxygen, which is found in the juice of flesh, and which may be called the *flesh-principle*, or kreatine. It is not yet known what intermediate substances connect this principle with the albuminous compounds from which it originates; we only know, that by the mere presence of acids, which are never absent from the flesh, kreatine is transformed into an alkaline substance, which is to be found, not only in the flesh, but also in the urine. This substance may be called the flesh-basis, or kreatinine, and considered as decidedly an excrementitious body, which has passed into that stage in the tissues themselves; being formed from the albuminous matters, and having passed beyond the intermediate state of the kreatine. The acid peculiar to the flesh, technically called inosinic acid, which is to be found in the muscles, associated with lactic acid, most probably belongs also to the products of regressive transformation, although it has not hitherto been discovered in any excretion, but only in the flesh itself.

Like the flesh-principle, so are the peculiar basis and acid of the flesh, compounded of nitrogen, carbon, hydrogen, and oxygen. The flesh-principle is transformed into flesh-basis by the subtraction of water only. Flesh-acid is remarkable for containing a high proportion of oxygen. While the flesh-principle, as such, combines neither with bases nor acids, the flesh-basis unites, as its name indicates, with acids, and the flesh-acid with alkalies. Flesh-basis and flesh-acid are easily dissolved in cold water; the flesh-principle with more difficulty, requiring boiling-water, which dissolves it in great abundance.

These illustrative examples may be sufficient to establish the general proposition, that the formation of the excrementitious bodies begins in the tissues themselves, and that the function of the excretive glands is mainly to attract and elaborate these substances from the body; for the products of the regressive metamorphosis pass from the tissues into the blood, and from the blood into the excretive organs.

§ 26. *Excreting Glands.*—If among these glands the most important place is ascribed to the respiratory organs, the conviction is founded upon the fact, that the functions of the lungs can least of all bear interruption without disturbing, or altogether destroying, the activities essential to life. That element in the surrounding atmosphere, therefore, which is absolutely necessary for respiration, may be called vital air. This vital air is the oxygen, which is the mightiest agent in producing the metamorphosis of tissues.

The air we breathe is a combination of oxygen with a large proportion of nitrogen, a little aqueous vapour, and still less carbonic acid. While the oxygen at every inspiration enters the lungs in order to impart to the blood its proper composition, the carbonic acid of the atmosphere, on the other hand, originates from the breath given out at every expiration from the lungs of men and animals. Valuable researches have shown that this carbonic acid penetrates from the air into the leaves of plants, in order to supply the principal nutriment to these numberless green and flowering products of nature, which form such an essential condition of animal life: not as nutriment alone, for it is the vegetation of the earth which gradually decomposes the carbonic acid which it has absorbed, and whilst it keeps the carbon for the formation of its own tissues, exhales the greater part of the oxygen; the latter being called, as we have seen, and with the greatest propriety, the vital air of men and animals.†

The carbonic acid which we breathe out is, as before mentioned, to be traced so far back as the tissues themselves; from these it penetrates into the capillaries in order to pass into certain canals, in which the blood flows in a direction opposite to that of the arteries. These canals are called *veins*, and are distinguished from the latter by not possessing pulsation.

The blood which the arteries have conveyed from the heart to the remotest organs of our body, having been changed by the substances which it has lost, and by others which it has taken up during its course, flows through the veins back again to the heart, thus forming a true circulation. This circle is completed by the capillaries, which connect the arteries with the veins.

The venous blood contains an abundant quantity of carbonic acid; but this product of decomposition does not originate exclusively from the tissues, and pass therefrom into the veins, inasmuch as the arterial blood itself is already impregnated with a certain quantity of carbonic acid, formed in this part of the circulation. The venous blood, however, contains more carbonic acid than the arterial, the former receiving an additional supply of this gas from the tissues.

Besides the great circulation from the heart, through the tissues, and back again to the heart, there exists also a small circulation of the blood; for all the blood of the veins, which flows into a special compartment of the heart, is propelled therefrom into the lungs, and returns from these into the heart, after having been changed in a peculiar way. The blood flowing from the heart to the lungs is dark-red, even brown-red, containing but little oxygen and much carbonic acid; and this is the character of the venous blood in general. But in the lungs, to which an abundant quantity of oxygen is conveyed with the inhaled air, carbonic acid and aqueous vapour pass from the venous blood into this organ, and are compensated by the inhaled oxygen which transudes into these vessels. By this process the blood becomes light-red, containing less carbonic acid and water than that of the veins. The blood thus transformed, returning from the lungs, is now called arterial, as it corresponds with that of the arteries. It is the blood of the arteries themselves, because it is conveyed to them by the heart, in order to permeate all tissues again with nutritious juice.

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* See *ante*, p. 326.

† See *ante*, p. 315.

In its course from the heart through the tissues, the venous blood is therefore formed, which in the great circulation flows back again to the heart; and this blood, which in nourishing the tissues has undergone such important changes, is supplied again with oxygen while passing through the lungs, and thus adapted, in return, for the nutrition of the tissues; in short, it is transformed again into arterial blood.

As the ovum represents, among the secretions, a transition stage between these and the tissues, the lungs may be considered as organs, holding, in respect of their functions, an intermediate position between reception and separation. Carbonic acid and water are the excretions which they give out, and in return they take in oxygen. Without oxygen, no arterial blood, and without arterial blood, no nutrition could be produced. If the nutrition be interrupted, the functions of all tissues are deranged. The muscles cannot contract themselves, the nerves lose their irritability, the reasoning faculty of the brain is disturbed, if the blood cease to furnish these organs with their peculiar combinations; and this latter process is dependent on the supply of oxygen; for all organic constituents of the blood are gradually transformed by oxygen. The albuminous and fatty substances have, without any exception, so great an affinity for oxygen, that they are gradually decomposed into combinations containing it. Oxygen passes also through the capillaries into the tissues themselves; therefore the venous blood contains less oxygen than the arterial, and the manifold decomposition becomes possible to which we have already stated the tissues are subjected.

We recognise in the flesh-principle, the flesh-basis, and the flesh-acid, a few of the intermediate stages through which the albuminous substances pass by the influence of oxygen. The last results of this influence are the urine-principle, or *urea*, carbonic acid, and water. The different fats are also transmuted into carbonic acid and water, with even greater readiness than albumen, as their greater amount of carbon and hydrogen facilitates their combustion with oxygen. Thus, we daily exhale through the lungs, in form of carbonic acid and water, about one-third part in weight of the nutriment we have taken. As this combustion depends upon nothing else than a combination of oxygen with other elements, it follows clearly, that the inhaled oxygen, the affinity of which acts slowly but without interruption, burns down entirely all the organic substances of the blood.[*] For this process four or five days only are requisite.

This act of combustion, in great part, explains why the temperature of the human body continually exceeds that of the surrounding atmosphere. The difference between the temperature of the latter and of our own body is, in scientific language, designated as *proper heat.* This proper heat varies according to the temperature of the external air, while the actual heat always remains the same. The average amount of the latter is 98° F., without fluctuating a single degree during health.

§ 27. Though a part of the albuminous matters of the blood is lost from the body with the exhaled air, in the form of carbonic acid and water, still the kidneys are the principal organs which withdraw the waste albuminous bodies from the blood; for the urine, which is excreted by the kidneys, collected in the bladder, and voided externally by the urethra, is a solution in which urea, uric acid, flesh-basis, and flesh-principle are contained; all being substances whose amount of nitrogen, carbon, hydrogen, and oxygen, evinces their origin from the albuminous substances.

The flesh-basis and flesh-principle we have already described. They are present in the urine, but in very small quantities. This excretion contains also, in a somewhat greater quantity, uric acid—a substance scantily soluble in cold water, which, however, is dissolved in the urine as a urate of soda. Urea is easily soluble in water, and is contained in the urine in the most abundant quantity.

Besides the uric acid, the human urine sometimes also contains some lactic and butyric acids, with another acid nitrogenised, and peculiar to the urine of herbivorous animals, which chemists call uric acid of horses, or hippuric acid, as having been first detected in the urine of those animals.

But none of these acids have been found in the urine in an uncombined state. The acid reaction, however, of this excretion, is produced by an inorganic salt, termed acid phosphate of soda, in which the acid predominates. The more important concomitants of this salt in the urine, are common salt and alkaline sulphates; but chloride of potassium, and the phosphates of lime and magnesia, are also found in the urine, which sometimes also contains traces of iron and of fluoride of calcium.

The quantity of urine voided in twenty-four hours, amounts to about one-third of the weight of the nutriment taken in the same time.

§ 28. *Constituents of the Secretions.*—It is a very general popular belief, that the excrements are only formed from the undissolved remains of food. Though these constitute a not inconsiderable portion, it is quite a mistake to suppose that this excretion is not essentially mixed with other constituents, owing their origin to the blood; for how could we doubt that the capillaries, which exist in the intestinal walls in such abundance, would allow any substances to transude into the intestinal cavity, as this transudation is the indispensable condition of any transition of dissolved substances into these blood-vessels? All animal membranes, which are moistened on each side by different fluids, permit substances to pass through in such a manner, that the matters transuding from one side are replaced by those which similarly pass from the other. The excrements are also mixed with a proportion of digestive fluids, mucus, products of decomposition from the bile, horny cellules from the mucous coating of the intestines, and other substances, which, when once secreted, do not return again to the blood.

The undissolved remains of our nutriment, which form the excrements with the substances just mentioned, are in part the insoluble, or hardly soluble constituents of the food, as the elastic fibres of animal food, and the cellular principles of vegetable nutriments, of which we shall treat in detail hereafter. There will also be found in the excrements such aliments as of themselves are soluble in the digestive fluids, if the quantity of the latter have been too small in proportion to the nutriment to effect its solution. Thus, a very great variety exists in the composition of the organic constituents of the excrements.

Of the inorganic constituents of our food, the rectum discharges, more especially the earths, the lime and magnesia salts, together with a considerable quantity of iron, on which, as well as on the constituents of the bile, the colour of the excretions is principally dependent. The soluble salts of the alkalies are also to be found in these excretions, partly transuded from the capillaries of the intestines.[†]

§ 29. *Excreting Powers of the Skin.*—Besides the lungs, the kidneys, and the rectum, the skin has to be mentioned as another most important organ of excretion. Not only do the capillaries of the skin keep up an uninterrupted discharge of carbonic acid, and take in oxygen in return; but this organ is also abundantly provided with small glands of two kind, excreting from the blood the perspiration and the sebaceous or greasy substance of the skin, which are therefore called sudoriferous and sebiperous glands.

The perspiration contains many regularly detached scales of the epidermis. Besides a small quantity of fat, several volatile organic acids, consisting only of carbon, hydrogen, and oxygen, are the principal constituents of the perspiration. The chemist calls these butyric, butyro acetic, and formic acids, and classes them in one group as being in their composition very similar to the fatty acids, to which butyric acid belongs; also with respect to their various properties. These acids occasion the sourness of the perspired fluid. Common salt, chloride of potassium, sulphates and phosphates of the alkalies, and traces of phosphates of lime and iron, are also dissolved in the perspiration.

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[*] See *ante*, p. 315.

[†] It has been lately discovered, that an excess of fibrine, when taken as food, is always excreted from the body, by the rectum, in an unchanged state.—Ed.

To the perspiration may be added the tears. These are continually secreted in a very small quantity by a gland situated in the external corner of the eye, underneath the skin. From hence they flow over the surface of the eyeball to the internal corner, where they are reabsorbed by an aperture in each eyelid, leading by a minute canal into the nasal cavity, to be excreted with the nasal mucus.

The tears are a very diluted solution of common salt, mixed with some detached cellules from the external coating of the eyeball.

The sebaceous matter of the skin is also a mixture of detached epidermic cellules with some other substances, of which fat and some salts are the principal components. It has, however, a different composition in different parts of the skin. The ear-wax, for example, is remarkable for a bitter yellow substance, soluble in spirits of wine, and for a portion of gall-fat.

§ 30. *External Excretions.*—Just as a part of the intestinal mucus is voided as an excrementitious matter, so the mucus of the other parts of our body—of the nose, windpipe, lungs, urinary ducts, and the several organs principally of women—is to be considered an excrementitious substance.

As a considerable part of the mucus is represented by *horny substances* which coat the mucous membranes, with these excretions may be included those parts of the hair and nails which, being cut off and growing again—as if the younger parts pushed the older parts upwards—form a regular part of the expenditure of the body. It is the same with the epidermis, which is continually scaling off.

The organic origin of all these horny substances is to be derived from the albuminous compounds, which are, by their transformation into the former, separated from the blood; one part of a tissue is thus directly lost without having passed again through the blood from which it originated.

ON HUNGER AND THIRST.

§ 31. *Principle of Reparation.*—It has been mentioned, in a previous section, that one-third in weight of the food we take during twenty-four hours is lost with the urine, and another third with the air exhaled. Another third of the consumed aliments daily leaves our bodies in the form of excrements, perspiration, mucus, fat, tears, and detached horny matter.

It is evident that this does not mean an immediate excretion of alimentary principles as such; for, excepting the undissolved remains of food which the rectum expels with the excrements, there is no substance in our excretions which had not passed from the mouth through the blood to the tissues, and from these back again through the blood to the organs of excretion. Thus, for example, the carbonic acid and water which we exhale, have once been fat or albumen, and the urea has nourished the tissues in the form of an albuminous constituent of the blood, before the kidneys convey it as a useless sediment of the body to the expelling bladder.

This main result is not affected by the indirect mode of the excretion of our nutriment. The weight of a healthy grown-up person does not undergo any sensible alteration from one day to another; for as much as is subtracted from the body by excretion, is conveyed to it again by the food which is digested.

Thus experience teaches, when the total amount of excretion diminishes, the weight of food we take decreases also. If the proposition could also be transposed—if it were true that, with a diminished quantity of food, a proportionate diminution of the secretions succeeded, then we could learn the art of fasting, and the horse of the well-known tale would not have died the day before he promised to crown the hopes of his parsimonious master with the most brilliant realisation.

It is not, however, thus. Even when abstaining from all solid and liquid food, we exhale carbonic acid and water; the excretion of urine and excrements continues all the same, the hair and nails grow, and perspiration and mucus hourly subtract from the body its most essential constituents; and this abstinence, if continued, is only too soon followed by considerable diminution in the weight of our body.

Food only is able to obviate this decrease of weight; and as excretion takes place even if we take no food, it is less correct to say that we excrete again the aliments we have taken, than that the food restores what has been lost by the excretions. For carbonic acid, urea, salts, water; we take in exchange amylaceous and fatty matters, albumen, and inorganic substances. On this process of exchange, the metamorphosis of tissues hinges; the alimentary principles are, therefore, very often, and quite correctly, denominated the matters of reparation.

§ 32. *Effects of Inadequate Aliment.*—If the supply cease while the expenditure continues, the composition of the tissue is immediately changed; and the blood, which receives aliment not only for the tissues, but also for itself, fails in a few days, or at the longest, in a few weeks—for the oxygen we inhale wastes the blood when supplies are stopped. The constituents of the body continue, as before, to succumb to the influence of that mighty agent in the decomposition of organic matters.

The changes in composition produced in the blood and the tissues by a deficient supply, can only be distinctly observed after some time. Then we first of all notice the fat wasting away—a proof that the fats are more accessible to the action of oxygen than the albuminous substances. Carbon and hydrogen are the elements which combine with oxygen the most readily; and upon this depends their remarkable combustibility. The emaciation is, therefore, explained by the fact, that the fat substances surpass the albuminous in their quantity of carbon and hydrogen.

Next to the fats, those organs lose the most rapidly in weight which are remarkable for their great amount of albuminous substances. The muscles, the heart, the spleen, and the liver thus waste away.

From this speedy decomposition is excepted only one part of the human body, and that the part of which we should be least of all disposed to predicate a slow process of change; for, although the brain and the nerves consist almost exclusively of fat and albumen, two of the most mutable substances of our body, we know from investigation in cases of animals which have died of starvation, and of men who have succumbed to protracted diseases, that these organs have suffered the least loss of weight. The reason of this is an unresolved enigma. There is but one probable explanation for this strange circumstance—in the peculiar manner in which albumen and fat are combined in the brain and the nerves. If this combination, however, and its resistance to the influence of oxygen, cannot be sufficiently explained, we still find in this fact the best explanation of the late decay of the mental faculty, which we see so often flame up again with fallacious vivacity in the last moments of expiring life.

Slower than the fat and the muscles, but faster than the brain and the nerves, the bones, the cartilages, the skin, and the lungs waste away; including, in a word, all parts composed of gelatine, horn, and elastic fibres. These tissues owe their greater tenacity of existence to the difficulty with which they dissolve, which gives them greater power to resist the action of oxygen; for though the assertion of former chemists—that only dissolved bodies act upon each other—cannot be admitted without exceptions, yet there is scarcely any condition more favourable to chemical combination and decomposition than a state of solution.

§ 33. *Effects of Abstinence from Food.*—From a consideration of the intimate reciprocal action by which the blood is combined with the tissues, secretions, and excretions, the fact that, with a failing supply, the composition of the tissues still changes and the excretions continue, necessarily indicates an altered composition and diminished quantity of the blood. But while science endeavours to find the laws upon which this alteration in the composition of the blood is based, it has already ascertained a diminution of the colourless blood-globules in proportion to the coloured; and the decrease of the secretions after a protracted abstinence is an admitted fact. The activity of the glands of digestion, however, does not cease entirely; but the saliva, and the gastric and pancreatic juices, are decidedly diminished and altered; the saliva becomes viscid and saltish. Of the secretions, bile is present in the greatest proportion, though it also is diminished. This fact deserves so much the more attention, inasmuch as we have to consider the bile partly as a secretion and as an excretion.

It cannot surprise us that a diminution of the excretions should accompany the decay of the tissues, the impoverishment of the blood, and the impeded secretions. But although less and badly smelling air, scanty and stinking urine and perspiration, and a smaller quantity of excrements and mucus are voided, these excretions are just sufficiently abundant to prove clearly that they effect the decay of the tissues as a necessary consequence. In a smaller quantity of urine, in proportion to its contents of water, a greater quantity of urea is excreted, which explains the considerable loss of albumen in the tissues.

The proper-heat diminishes during fasting, and this completes the harmony existing between all these relative combinations; for when we exhale less carbonic acid, less carbon of the tissues is consumed; and by the smaller quantity of combustible substances which combine with oxygen, a decrease of the temperature produced in our bodies takes place.

A common tie unites matter, form, and function. The composition, the form, and the activity of the organs of our body form a chain, of which no link can be affected without a simultaneous alteration of the other two. There can likewise be no activity without continual transformation in the composition, without a perpetual becoming and ceasing to be.

During abstinence from solid and liquid food, as the composition is altered and the forms decay, the activity of all functions must necessarily deviate from that of the organs of a well-fed body. In the formation of the blood, the nutrition, the secretion and excretion, this deviation is easily to be understood, and has been, as far as necessary, examined in preceding paragraphs.

But the series of the altered functions is by no means thus closed. The muscle, lighter by having left its fat and albumen, appears a flabby flesh, which contracts slowly; the heart is inert; the number of the pulsations in a minute is considerably diminished; a sighing respiration takes place; frequent yawning, a hoarse voice, and languid movements, are all, more or less, direct consequences of the deficient nutrition of the muscles.

A bitter taste, often complained of by starving persons, comes from the bile, which is still pretty abundantly secreted, and passes from the intestines into the blood, by which it is carried to the nerves of the tongue. As the respiratory action is diminished, the combustion of that part of the bile transuding into the blood is less perfect than usual: so that a portion remains and acts upon the gustatory nerves.

In this condition, slight stimulants exercise a great effect. The light is painful, a slightly elevated sound is insupportable, and a touch excites peevishness. Perception is therefore distracted; and as memory also refuses its service, both sources of our faculty of judgment are exhausted; for what we have before met with, or now meet, induces the action of thought and judgment. But, in order to judge rightly, we must see clearly, hear distinctly, and feel without disturbance. During sleepless nights, the starving person is tormented by eagerness, the powerful lever of so many passions. He who grasps at carrion and corpses, at the flesh of his friends, or of his own body, shows a wilder longing than the imagination even of poets has ventured to conceive.

There is another instinct by which the vigour of the mind is vanquished in a more melancholy way. Hunger desolates head and heart. Though the craving for nutriment may be lessened to a surprising degree during mental exertion, there exists nothing more hostile to the cheerfulness of an active, thoughtful mind, than the deprivation of liquid and solid food. To the starving man, every pressure becomes an intolerable burden; for this reason, hunger has effected more revolutions than the ambition of disaffected subjects. It is not, then, the dictate of cupidity, or the claim of idleness, which prompts the belief in a natural human right to work and food—a motive to the force of which Christian charity itself must sooner or later give way.

All honour, indeed, to the charity which, in so many noble-minded people, anticipates and mitigates the sternness of law. We are, indeed, far from retaliating the contumacious censure with which the asserters of this right are often met by their opponents. It is the part of wisdom and comprehensive tolerance fairly to balance the merits of all opinions, and to do justice to their respective beneficial result. So much the more, however, do we consider it a duty to oppose the cogent power of convincing facts, to the rigorous sentence which makes a human right dependent upon a human grace.

Cold and rigid, the muscles quivering in the paralysed limbs; sighing, with troubled cloudy eyes, blunted sensation, and unsound judgment; the tortured wretch suffering the horrors of starvation struggles in agony, which often closes in a swoon; while sometimes a furious delirium will precede the final scene of death from exhaustion.

§ 34. *Effects of Hunger.*—We have thought it useful to describe the very extreme consequences of a deficient supply of nutriment, in order to explain by them the sensations which in general remind us to counteract the impoverishment of the blood, by a proper supply of food. The description of these sensations is the real science of hunger and thirst. For, if the blood be deficient either in its composition, in quantity, in its constituent parts, or in the rapidity with which it circulates through the organs of our body, the tissues must be nourished by other means; so likewise must the nerves, for their composition changes also with the combination of their mother-juices. But as the action, and therefore the sensation of a nerve is dependent on its composition, it follows that it must feel differently if deriving its nutriment from any other source.

If abstinence have lasted only a short time, all the phenomena occur which most people observe in themselves when awaking in the morning. The tongue is parched—*i.e.*, the coat of horny cellules lining the mucous membrane is thickened; and this condensation goes as far as the stomach. The saliva and mucus contain less water, and have often an unpleasant taste and a bad smell, as may be observed in fasting-people. Most persons, therefore, cannot eat anything in the morning until they have cleared their mouth and drunk some water. In feeble constitutions, and also in some strong ones, after a longer fast than usual, the quantity of blood present in the mucous membrane of the mouth and stomach, is perceived by a vague, unpleasant sensation in the mouth and throat, or a pressure, tension, and emptiness in the stomach; rumbling in the belly, which is hollow and drawn in. Yawning, a sensation of compression of the forehead even to headache and weakness, are the general indications of more constituents having been abstracted from the blood than is compatible with a sufficient nutrition of the nerves. Irritable persons become then so sensitive, that the least resistance, unexpectedly touching their bodies, or even an innocent word, is sufficient to put them out of humour. The total of all these phenomena, which are more or less conspicuous in different people, represents hunger.

If hunger remain for an unusually long time unallayed, the oppression of the stomach increases to pain and sickness. In autumn and winter the body becomes more sensible to cold; the general uneasiness disturbs the mental activity; unsteady thoughts are wasted in unquiet observation; and complete exhaustion is preceded by an excitement which only too often impels to unjustifiable acts.

§ 35. *Effects of Thirst.*—Many instances prove that hunger can be borne for several days before it causes death. If the fasting person drink water, this time is considerably prolonged. While, on an average, man cannot live longer than a fortnight altogether without food; there was, in 1831, an instance at Toulouse, of a convict who, preferring death by starvation to a public execution, survived for as many as sixty-three days on water alone.

Similar cases prove, that it is much more difficult to overcome thirst than hunger; and this is confirmed by daily experience. If it be alleged, on the other hand, that many people drink exceedingly seldom, and that females especially can do without drinking for many days together, neither must we forget that all food, even the driest, contains a great proportionate quantity of water.

Upon this reception of water into the system all depends. The blood and most of the tissues, with all secretions and excretions, contain an abundant quantity of water, as an indispensable condition of their right composition and proper action.

In the excretions from the skin and lungs, more than one-third of the weight of our food is daily lost in the form of water. To this a considerable quantity of water must be added besides, which is voided in the urine.

A deficient supply of water is, therefore, very soon felt; dry lips and cheeks, dryness of the tongue and throat, speedily betray the lack of that water of the mucus and saliva which ordinarily keeps the cavity of the mouth in a moist state. On the increase of thirst, the mucous membrane reddens and swells, and, after awhile, the inflamed tongue cleaves to the roof of the mouth, and the glowing breath escapes in sighs; the skin burns, because the perspiration continually diminishes; the urine becomes acid, as it contains the usual salts and less water; the slackened muscles move the limbs listlessly, and without power; mere talking or deglutition becomes an effort; groaning and quick respirations accelerate the pulse. In the action of the brain and organs of the senses, there is the same irritation as in hunger. Restlessness and despair torment with the continual representation of water and other beverages, which would allay his craving. All liquids—sea-water, tincture for the teeth, even his own urine—are swallowed, or attempted to be swallowed, by thirst-distracted man, with passionate eagerness. If no relief be given, a violent inflammation of the mouth and throat supervenes, with all the symptoms of an acute fever; sometimes the inflammation causes even gangrene of the throat, a quick, sighing respiration, and a pulse proportionally more rapid; a loose, lolling tongue, a whispering delirium and rage, ending in death, if the sufferer do not expire senseless in a swoon.

OF FOOD.—§ 36. *Definition of Alimentary Principles and Nutriment.*—From the blood are formed the tissues and the constituents of the secretions; to the blood the materials of the tissues return after having undergone a certain degree of decomposition, which renders them unfit for the peculiar functions of the different organs; and these products of decomposition are abstracted from the blood by the excretory glands, which expel them from the body.

The excretions diminish the weight of the body, and alter the nutrition of the tissues; their function depends upon their composition; and throughout a great division of the nerves of our bodies this function consists in sensation. The sensitive nerves are the media for the perception of all internal and external impressions. In the brain we become conscious of these impressions.

Hunger and thirst are the sensations which announce to the brain, through the medium of the nerves, the impoverishment of the blood. In popular language, every substance is called a nutriment which is able to quench hunger and thirst. The scientific definition of nutriment results from the cause of these sensations. Whatever substance possesses the power of restoring to the blood the essential constituents which it has lost, and begins the circulation from the blood through the tissues, may be considered a nutriment in the widest sense.

All the aliments which restore to the blood the compounds of chlorine, the salts, the fat, and the albumen, satisfy the sensation of hunger; that of thirst is quenched by the restoration to the blood of the deficient water.

§ 37. *Alimentary Principles.*—All nutriment is composed of certain *alimentary principles.* Under the latter term we comprehend all those compounds which are either identical with the essential constituents of the blood, or sufficiently similar to be transformed into them by digestion. Essential constituents of the blood are all those which do not originate from the regressive transformation of the tissues.

To this general definition we have to add, as a special one, the difference between alimentary principles and aliment. By ordinary dissolving media, like water, alcohol, and ether, simple constituents can be separated from the aliments, but not from the alimentary principles. These simpler constituents of the aliments, which cannot be separated by common dissolving media into any further parts, are the alimentary principles themselves.

Though the alimentary principles are often denominated the elementary constituents of the aliments, they are by no means to be confounded with the elements in a chemical sense. The latter cannot be decomposed into other substances manifesting, abstracted from form and colour, any essential differences; the alimentary principles can be decomposed, but only by agencies which act more powerfully than simple dissolving media, like water, alcohol, and ether.

No alimentary principle is composed of fewer than two elements; the simplest alimentary principles, like water and common salt, are combinations of two elements. A great many alimentary principles, as fat, sugar, and most of the vegetable acids, contain carbon, hydrogen, and oxygen; the inorganic salts consist of oxygen combined in the basis, and the acid with two different elements. All these alimentary principles are formed of three elements.

Four elements are found in the soaps which, in addition to the carbon, hydrogen, and oxygen of these fatty acids, contain a metal, forming with oxygen the basis alkali of the soap.

Finally, those organic alimentary principles, in which nitrogen and sulphur are associated with carbon, hydrogen, and oxygen, are composed of five elements; and some others, containing phosphorus, of six. To the former belongs gelatine; to the latter, albumen.

§ 38. *Digestive Principles.*—Before discussing the process of digestion, we classified the alimentary principles into inorganic, organic without nitrogen, and organic with nitrogen.

To the inorganic alimentary principles belong the compounds of chlorine, as common salt, and the combinations of inorganic acids and bases, which chemists, abandoning the ordinary use of language, designate by the name of salts.

The starchy matters and fatty substances, with by far the greater part of the acids of our aliments, are all alimentary principles without nitrogen.

Of the organic alimentary principles, the albuminous substances, the colouring matter of the blood, and the gelatine, all contain nitrogen.

The aliments which are composed of various alimentary principles cannot be classified, in a similar way, on any chemical basis into different groups; for the same alimentary principles appear in the most different aliments, and their distinction depends either upon the quantitative proportion of the corresponding constituents, or upon some freshly supervening substances; but there is so great a variety, both in the proportion in which the ordinary alimentary substances are combined in the aliments, and in the nature and qualities of the superadded substances, that the aliments classified with respect to their chemical properties could be united into very small groups only—a circumstance which would render difficult the review of the whole.

We have not, therefore, departed from the classification which has hitherto been popularly employed, but shall treat in succession on solid and liquid food, and or condiments. None of these sections need any further definition.

We must, however, guard against one error, not seldom met with. In believing that solid food only satisfies the hunger, while liquids only quench the thirst, we forget that there is only one alimentary principle—namely, water—that affects that condition of the blood which brings on thirst. But water is contained in all aliments in such abundance, that, on an average, it constitutes more than half of their weight. All beverages, on the other hand, contain also some other alimentary principles than water; for even in what we are accustomed to call pure water, there are always some of the compounds of chlorine and salts; while in milk, we have all classes of simple alimentary substances together, it being composed of water, compounds of chlorine, salts, caseine, fat, sugar, &c.

The condiments are also, in the restricted sense in which we shall employ the word, principally composed of alimentary principles; only a few of them contain substances which, although exciting the gustatory nerves, and stimulating the digestive organs to a greater activity, may not also be considered as restoring some of the losses of the body.

§ 39. As the alimentary principles are not found in nature in their simplest form, so also none of them is sufficient of itself to nourish the body. Not even a variety of these substances, if belonging only to one of the three groups before-mentioned, is able, if exclusively eaten, to maintain life: we may go further, and add that life cannot be sustained by two of these sections, if the third be lacking.

Not sugar, salts, nor albumen, if taken without any substances of the two other groups, are able to destroy the consequences which the change of matter effects, when the process of excretion goes on, without the accompanying one of nutrition.

Without phosphate of lime the bones cannot be formed, whatever quantity of albumen and fat we may consume; no muscular tissue could grow without albumen, however we might overload the stomach with sugar and salts; and, finally, without fat, no brain could be formed. But the bones, the brain, and the muscles are the most essential organs of the human body.

No element can be transformed into another: this is the whole solution of the secret. Phosphorus does not turn into oxygen, nor oxygen into carbon, nor carbon into nitrogen, nor nitrogen into sulphur: no power short of the miraculous is able to effect an exception from this rule. As something cannot be created from nothing, no power of the body is able to transform iron into hydrogen, or chlorine into calcium.

The organic alimentary principles, destitute of nitrogen, cannot be changed into those containing nitrogen, nor the nitrogenised organic substances into inorganic salts, containing any other elements than nitrogen, carbon, hydrogen, oxygen, sulphur, and phosphorus.

It might be supposed that albumen could be transformed into fat, because albumen, as well as fat, contains carbon, hydrogen, and oxygen; it could also be suggested that carbonates of alkalies and water, as containing carbon, hydrogen, and oxygen, could form sugar: but experience has shown that the animal body cannot accomplish the former transformation in a sufficient quantity, nor the latter at all.

Complete substitutes, therefore, for the nutriments which maintain the life constantly, can only be formed by a combination of all three groups of the alimentary principles.

§ 40. *Tests of Digestibility.*—The more easily these alimentary principles are dissolved in the digestive fluids, and changed into the constituents of the blood, the greater is their digestibility; for digestion consists not in the solution of bodies only, but also in their transformation into the essential constituents of the blood. Both conditions are of equal importance.

If two substances, therefore, are dissolved with equal ease, that will be the more digestible which has the greater similarity to some constituent of the blood. Stearine and margarine, for instance, are almost equally soluble in the digestive juices; but as margarine is present in the blood, and stearine is not, it follows that margarine surpasses stearine in digestibility.

But if the conformity of two alimentary principles with the constituents of the blood be equal, the more soluble is the more digestible. Soluble albumen and fibrine stand equally near to the blood, both being contained in it. As the soluble albumen, however, is more readily dissolved in the digestive juices than fibrine, the digestion of the latter is more difficult.

Hence it results, that the difficulty with which alimentary principles are dissolved, is, in many cases, compensated by their conformity with the ingredients of the blood. Although gum, for example, is much more easily dissolved than fat, the latter, if not taken in too great a quantity, is digested by a healthy stomach, under certain circumstances, more readily than gum; for gum is not contained in the blood, while fat belongs to its essential constituents. Gum has first to be transformed into sugar, then into lactic acid, and finally into butyric acid and some other fatty substances; while fat, on the other hand, as containing oleine and margarine, is represented in the blood itself. In the formation of fat from starch, the latter must first be changed into gum, then into sugar; the sugar into lactic acid, and this again into butyric acid. It hence appears, that of all these substances lactic acid is the most digestible, even if all were equally soluble in water; sugar would be next to it, then gum, and finally starch. Lactic acid and sugar are, besides, more soluble than gum, and gum than starch. The latter is, therefore, for two reasons, the least digestible of the above-mentioned alimentary principles. It follows, again, from the same law, that a substance which is both more easily dissolved in water than another, and more readily transformed into the constituents of the blood, greatly surpasses the latter in digestibility.

Of the aliments, those are the most digestible which contain the greatest number of alimentary principles easily soluble, and easily turned into the constituents of the blood.

§ 41. *Tests of Nutritiveness.*—The nutritiveness of any kind of solid or liquid food depends upon three circumstances: its digestibility, the quantity, and the proportions of the alimentary principles contained in it.

If an aliment contain many indigestible substances, which are voided again in an undissolved state with the excrements, it must lose as much of its nutritiveness; for only that which passes as an essential constituent into the blood, is to be considered as an alimentary principle. The more digestible an aliment is, therefore, other things being equal, the more nourishing it becomes.

In testing the value of a nutriment with respect to the alimentary principles contained in it, no regard is paid to the amount of water contained. Water is, in general, so easily to be procured, that we have not to take it into consideration, when judging of the value of solid or liquid food, with respect to its nutritious properties. In a dry arid desert, however, water would become the most important nutriment; and an aliment containing much water would be the most nutritious food. Where no deficiency in water exists, that aliment is the most nutritious which contains the greatest proportion of alimentary principles, and conveys, therefore, to the blood the greatest quantity of its essential constituents.

Besides the digestibility and the abundance of solid constituents, the combination of an aliment is also of the greatest importance. As the blood contains a greater proportion of albumen than of salts, and more salts than fat, the proportion of these alimentary principles will determine whether any particular food be nutritious or otherwise. Just as alimentary principles of equal solubility are the more digestible, the more completely they correspond with the several constituents of the blood; so an aliment is generally more nutritious, if the combination of its alimentary principles correspond more exactly with the composition of the blood. Thus, nutritious aliment is characterised by containing, of nitrogenised substances, more organic than inorganic, and of substances not nitrogenised, more inorganic than organic matter. Lean meat is the food which most completely answers these conditions.

Substances from each of the other groups, however, as we have just mentioned, are equally indispensable, though requisite in different quantities. Compounds of chlorine, salt, fat, and constituents of fat, are as absolutely necessary as albumen or gelatine. It always betokens a narrowness of view, therefore, to speak of any aliment as innutritious. It is true, potatoes are less nutritious than meat, because the latter is very similar to the blood; while potatoes contain very little albumen and a large proportion of starch: but in the unqualified assertion that potatoes do not nourish at all, we forget that starch is transformed into fat by the digestive process, and that this represents an essential constituent of the blood. Potatoes, combined with a certain proportion of white of egg, become as nourishing as milk or meat.

To distinguish, in one word, between digestibility and nutritiveness, we may say, that the former expresses the rapidity with which the alimentary principles of an aliment are transformed into constituents of the blood; the latter, the quantity of alimentary principles conveyed to the blood by solid or liquid food. Hence it follows, that we can speak of digestibility with respect to the alimentary principles as well as the compounded aliments, but of nutritiveness only with respect to the latter.

ON SOLID FOOD.—§ 42. *Meat and Eggs.*—In regarding the different kinds of food, from the whale-oil of the Greenlanders, and the bear's-grease which forms an article of diet in a Mogul tribe, up to the refined cookery of the wealthy in the more civilised countries of the globe, where, for instance, oysters and trepang (a small tubular animal belonging to the class of the Radiata) are favourite dainties, we soon find out that there exists no class of animals which does not furnish some contribution to the food of man. By all civilised nations, however, the flesh of herbivorous animals is preferred.

In this respect we may assert that the plants first prepare the food of man. It is an art in which they excel—that of preparing excellent meals from very simple materials. It is true that those plants which prepare the most nutritious food, take also up some compounded organic aliments; but it is also undeniable that plants can live exclusively on carbonic acid, ammonia, water, and some inorganic substances, and in every case derive from these simple alimentary principles the chief bulk of their body. The carbonic acid, the ammonia (a very simple combination of nitrogen with hydrogen),* and the water, are all three constituents of the air. "While the animal swallows bodies already formed," says Forster, who, before all others, deserves to be called the naturalist of the people, "these fine tubular and cellular formations eagerly take up the simplest elements from the air. Woven by sunshine and ethereal fire, such as formerly poets only dared to dream of, the soft green of the woods and fields smiles upon our sight; and, behold! in the infinitely delicate network of the blooms and ripening fruits, glows the sevenfold beam, and adorns the vegetable world with the manifold brilliancy of its hues!"

Animals living on plants not unnaturally convey the impression of a greater purity upon our fastidious senses, which are offended by the rancid smell of the carnivorous Mammalia, and by the oily flavour of the birds of prey. Hence our predilection for the herbivorous animals; and among these the Ruminants and the many-hoofed occupy the first place. Throughout the whole of Central Europe, no meat is consumed in greater abundance than beef and pork.

§ 43. *Constituents of Beef.*—The flesh of oxen, or beef, illustrates the composition of all other kinds of meat. We may take the composition of beef as the standard, with which, the varieties of the other most common animal aliments may be compared.

In beef, as in all aliments which, when taken to the exclusion of all other food, are able to maintain human life, the three groups of simple alimentary principles are represented. A combination of albuminous and fatty matters, of compounds of chlorine and of salts, abundantly diluted with water, is all that is necessary to sustain life.

The albuminous substances of beef are the fibrine of the muscles and albumen proper. The former constitutes the finest fibres of flesh; the latter is the essential body of the nutritious juice which occupies the space between the solid parts. The flesh owes its red colour to the blood which it contains in very numerous vessels; and this blood comprises albumen, globuline, fibrine, and traces of caseine; and in addition to these albuminous substances, some colouring matter containing iron.

The albuminous substances which animal food thus contains, are not the only supply it affords, to compensate for the albuminous matters of the blood lost with the excretions; for the finest muscular fibres are enclosed in the cellular or binding tissue which unites them into bundles. This tissue, when boiled, yields gelatine, which is soluble in water. In the body, gelatine is again transformed into albumen. Convalescents very often regain their strength by the almost exclusive use of jelly from bones; and gelatine is the only nitrogenised body contained in any large amount in this jelly. As the blood of man, however, does not contain gelatine, as a necessary condition of its regular combination, but albuminous matter, we must infer that the gelatine is transformed into albuminous compounds.

That the cellular tissue in the muscles is intermingled with elastic fibres, does not in any way affect the nutritious properties of animal food; for these elastic fibres are not dissolved in the digestive fluids. They constitute a part of those alimentary remnants which, united with certain excretory substances of the blood, compose the excrements.

Like gelatine, the horny cells lining the walls of the muscular blood-vessels are entitled to the name of constituents of albumen. They are, however, of small importance; for their quantity is so insignificant, that even if soluble in the alkaline secretions of some digestive glands, they are less digestive than the fibres yielding gelatine.

Kreatine, flesh-basis, and flesh-acid, are regular nitrogenised constituents of beef. Are they to be considered as alimentary principles? There is scarcely any doubt that they pass into the blood, and from the blood into the muscles. But is mercury a nutriment because it passes through the blood into the cellular tissue of the bones, and is found there in the form of small globules? Against this opinion the judgment of even the unprofessional reader protests. And the kreatine, the flesh-basis, and the flesh-acids, we cannot consider to be, strictly speaking, alimentary principles, as they belong to the regressive transformation, which process, by the action of oxygen, conveys all organic matters of our tissues to the excretory glands. The organic compounds without nitrogen, contained in beef, are much less various than the nitrogenised. The former consist of several fats, sugar, and lactic acid. We have often mentioned that the fat of the Ruminants owes its hardness to the stearine. This is associated with margarine and oleine; whilst the peculiar fat containing phosphorus, as well as the gall-fat of the nerves and the blood, scarcely deserve any consideration, on account of their small quantity.

Chloride of potassium, and phosphate of potash, are the peculiar inorganic substances of the flesh. They alone would be sufficient to distinguish the flesh from the blood. While the blood contains seventeen times more soda than potash, there is in the flesh of the ox about three times more potash than soda. Phosphates of soda, lime, magnesia, and oxide of iron are contained in beef, in somewhat considerable quantities; but there are only few traces of sulphates of the alkalies attributable to the blood of the muscles. The amount of water contained is so considerable, that, on an average, it amounts to more than three-quarters of the whole mass.

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* See *ante*, p. 323.

§ 44. *Effects of Cooking.*—The foregoing description refers to the raw flesh. What becomes of this when boiled or roasted? If we put a piece of meat into boiling-water, and leave it long enough exposed to the boiling temperature, the soluble albuminous substances coagulate; the fibrine is transformed into two new compounds, each of which is richer in oxygen than the fibrine itself, and only one of which resembles fibrine in its difficulty of solution. The other, which may also be obtained from albumen, is easily dissolved in water.

The colouring matter of the blood turns brown from being decomposed at the boiling-point. By this process it also loses its solubility in water.

The cellular tissues dissolve, as by boiling they are transmuted into gelatine.

The fatty substances melt. Those of the inorganic compounds which are more easily soluble, are dissolved in the water. The same takes place with the sugar, the lactic acid, to which the flesh-juice owes its acidity, the kreatine, flesh-basis, and flesh-acid.

The albumen being coagulated by the boiling-water in the external parts of the meat, forms around the internal part a cover or outside, penetrable only with difficulty. A large proportion of the soluble alimentary principles is consequently retained, which the water would otherwise have abstracted from the meat. The heat, however, penetrates to the interior; each flesh-bundle gets surrounded, as it were, by a sheath of coagulated albumen. This protects the gelatine and the salts, the lactic acid and the kreatine, but, most of all, the real flesh-fibre, which, by the direct action of the boiling-water, would gradually become harder and more tenacious. Accordingly, the flesh yields only very little of its constituents to the water, by quick boiling, and preserves with the alimentary principles, not only its nutritiveness, but also its flavour.

If, therefore, the cook wish to prepare a juicy, savoury, and rich food, she puts the meat into the water raw when the latter is in a state of ebullition; but if a strong broth is to be prepared, she puts the meat into cold water, and heats it gradually. The soluble alimentary principles are, by this process, abstracted from the meat, and dissolved by the water before the albumen can coagulate. It is true, that when the solution boils, a quantity of small albuminous scales are formed, which are skimmed off with the colouring matter of the blood, which has now turned brown, and other adhering substances. But when the water boils, another part of the albumen is transformed into a combination containing more oxygen, and soluble in water. The same takes place with the fibrine of the muscles, which comes into direct contact with the boiling-water. Thus, a solution is formed consisting of transformed albuminous substances, kreatine, flesh-basis, and flesh-acids, gelatine and lactic acid, besides some salts; while small scales of albumen and melted fats—the well-known fat-bubbles—float on the liquid in an undissolved state. The broth thus prepared is savoury, but the meat all the poorer and tougher, in proportion as the water has been more slowly heated.

This explains why in Germany, in the households of the middle classes, where meat is put on the fire with cold water, boiled meat is seldom or never taken without broth; for the boiled meat, and its broth, contain the constituents of the raw meat. In other countries, the Netherlands for instance, boiled meat without broth is very common food. The Dutch cooks, therefore, put the meat which has not to furnish any soup, on the fire with boiling-water.

By roasting, as in the former case, a kind of sheath is formed around the meat. The albumen coagulates on the external surface, which becomes of a deep brown by the decomposition of the colouring matter; and afterwards by the formation of several empyreumatic-smelling substances. This surface retains the greater proportion of the soluble substances in the meat, from which a thick substantial juice oozes out in a proportionately scanty quantity; a part of the fatty substance is decomposed—stearic acid, for instance, being changed into margaric acid; and finally, an important substance is produced—namely, acetic acid, which originates in the action of dry heat used in roasting, and facilitates the solution of the albuminous matters.

Vinegar has the effect of rendering the meat more digestible, like common salt, by dissolving the albuminous substances, as has been mentioned in discussing the process of digestion. This is the explanation of an expression common in Germany—that vinegar makes the meat short; short meat being easy of digestion.

Large pieces of meat, which after boiling or roasting contain liquid blood in the interior, are, in Germany, considered not to be sufficiently done; whereas, in England, this is in many cases required as a proof of good cooking. What is the cause of the blood being thus retained? It is because the colouring matter of the blood is not decomposed, excepting where the temperature has been increased to more than 158° Fah. The internal parts which retain the crimson hue, have not attained this temperature.

§ 45. *Difference of Taste.*—Although the difference of taste in different kinds of meat would indicate material distinctions in their compositions, yet the ascertained differences are very trifling in the case of the flesh of the sheep and roe; these belong, like the ox, to the class of Ruminants, and correspond most exactly with beef in the alimentary principles which they contain. The fat of sheep, however, is generally harder—that is to say, richer in stearine; and the fat of the roe is more acid than that of beef.

Pork is richer in fat, but poorer in albuminous substances than beef, although even here the difference is not considerable.

All wild Mammalia are remarkable beyond our domesticated animals for the quantity of kreatine which they contain; the latter, however, surpass the former in their proportion of fat. The reason of this is obvious. Whilst, in the ox fattening for food, its even and quiet existence promotes the formation of fat, and moderates the tissue change—the roe, leading in the woods a free and active life, takes in a greater quantity of oxygen, which transforms the nitrogenised basis of the tissues into kreatine. It is the same with birds; their rapid flight, and their bones filled with air, augment their reception of oxygen. The higher temperature of the bird proves that it consumes oxygen faster than the reptiles crawling on the earth; even faster than the Mammalia, the only class of animals with which it shares the appellation of warm-blooded. Hence the more rapid decomposition of its albuminous matter, of which the proportion of soluble albumen is, besides, greater than in the other Mammalia; hence, also, the abundance of kreatine in the muscles of birds.

While between the Mammalia and birds, no great difference has been observed in the amount of water they contain, the three-quarters in weight of water contained in the flesh of warm-blooded animals, is increased to four-fifths and more in the flesh of the fishes. Little blood, and therefore mostly a white colour; much less muscular fibrine; a larger quantity of tissue yielding gelatine; and, above all, a fat containing phosphorus, which is not limited to the contents of the blood-vessels, are regular peculiarities in the flesh of fishes. And this phosphorous fat does not constitute the only difference in their properties; for the soluble albumen of the fish, which nearly corresponds in quantity with that of the flesh of birds, coagulates much more rapidly in a somewhat higher temperature, than the soluble albumen of the warm-blooded animals. The soluble albumen differs also from the latter by not containing any phosphorus.

§ 46. *Comparative Qualities of Flesh*—We have to take into consideration not alone the species of the animals whose muscles furnish our meat, but also numerous other circumstances affecting the flesh, before it appears on the table.

Thus the flesh of young animals is poorer in fibrine than that of the mature, but richer in soluble albumen, gelatinous fibres, and water: it is therefore more tender. This preponderance of gelatine explains why the broth of veal and lamb sooner coagulates than that of beef and mutton; for it is the gelatine diffused through the broth which converts the whole, when cool, into a jelly.

Is the difference in the taste of the flesh of different animals, so familiar to us in ordinary life, traceable to certain differences in the composition of their food? The taste of fieldfares, for instance, after eating juniper-berries, and that of some ducks and other aquatic birds which feed on snails, crabs, and fish, and whose flesh resembles train-oil, evidently settles this question in the affirmative. In Tahiti the swine were fed on fruits only; and while their fat had nothing of the rich taste of European pork, Forster compared their flesh to veal. Science, however, answers the question with special regard to the fats; by its exact comparisons, it correctly explains the every-day experience of domestic life. There can now be no longer any doubt that the substances with which we feed our oxen, swine, and fowls, and stuff our geese—namely, turnips, potatoes, sawdust, maize, rice, with several other seeds—are remarkable for abundantly containing the constituents of fat; for in these kinds of food, the substances contained in greatest quantities are starch and sugar, with other similar matters, which the animal body transforms into lactic and butyric acids, and some other fats. Game, again, owes its savoury taste in great part to its large proportion of kreatine. Fattening causes the quantity of kreatine to decrease. Partridges lose their taste if cooped-up and fed like domestic fowls, as is sometimes done in severe winters, to protect them from cold. Tame ducks, if left at liberty, become *lean*, but acquire the agreeable flavour of game.

Although it does not require a sentimental mind to regard the cramming of geese with a degree of aversion, yet man has not disdained much more cruel encroachments on animal life, with the intention of procuring a new titillation to the palate. Not only has he made capons and poulards of cocks and hens by mutilation, but he has mutilated several Mammalia, as well as carps, in the same manner. Chemical research has not yet discovered why the flesh should become by this process more tender and savoury; but the fact is undoubted. Cows grow fat more rapidly after the loss of their ovaries. The practice of baiting, carried on in former times, but at present only existing in the form of hunting, liquefies the fibrine, which is the least readily soluble alimentary principle of flesh, and renders the meat more tender.

§ 47. The intestines of some animals, the entrails in which sausages are made, the brain, the liver, the kidney, the spleen, and the sweet-bread, are very similar to flesh in the properties of their constituents; the differences being chiefly in their quantitative proportions. Liver, spleen, sweet-bread, brain, and kidneys, contain soluble albumen in remarkable abundance. In addition to this quantity of albumen, the sweet-bread contains much gelatinous tissue, and an extremely small proportion of fat; while in the brain and liver, a considerable quantity of phosphorus fat is intermingled with the ordinary alimentary principles.

In the bones, the nitrogenised alimentary principles are almost exclusively represented by the gelatinous tissues. Though there exists no doubt that gelatine is capable of being transformed into albumen, yet this transmutation takes place so slowly that the bones alone cannot yield a proper nutriment. Since gelatine, as such, is not present in the blood, it has therefore to be transformed into albumen in order to become one of its constituents; it is much more difficult of digestion than albumen, in spite of its facile solubility in the gastric juice. The bones are not deficient in fat and the more important salts; but as these pass only in part into the cakes made of the gelatine of bones for the preparation of broth, the use of these cakes as a principal supply of food is to be reprobated. In England, which excels all other countries in the preference of its people for a substantial nutriment, these cakes are prepared from the broth of meat inspissated by boiling to a jelly. Sometimes, indeed, bones and other parings may be used in addition; but fresh beef furnishes the essential constituent of the inspissated juice, which is poured into small cake-moulds. These cakes, when dissolved, closely correspond with real broth; and they therefore justly claim for themselves the name of portable soups. What in France is sold under the name of bouillon-cakes, is nothing but gelatine; a production difficult of digestion, only slightly nutritious, and therefore objectionable. No economy is so nearly related to extravagance, as that of preparing soups from such cakes in order to effect a saving in meat; for this broth does not repair the expenditure of the body; and while impoverishing the purse, fails to enrich the blood.

§ 48. *Eggs of Birds as Food.*—No other aliment unites so completely the advantages of meat, as the eggs of our domestic birds. The yolk and white consist chiefly of albuminous matters—the yolk, of caseine and albumen; the white, of soluble albumen, containing more sulphur than blood—and of an albuminous body containing a large quantity of sulphur, scantily soluble, and forming cellules in the shape of small membranes, which include the soluble albumen.

We have before explained why eggs coagulate when boiled. The heat of the boiling-water being communicated through the shell to the thick albumen solution, the albumen becomes hard.

The white of egg contains more water than the yolk; somewhat more than one-half of the latter, but only four-fifths of the former, consisting of water. The yolk is also, in a nearly similar proportion, richer than the white in fat and in albuminous substances. The yellow oil of the yolk contains a large quantity of oleine, with a little margarine, less phosphorous fat, and still less gall-fat.

What further does the egg require to represent a complete nutriment? Nothing but the salts and chlorine compounds of the blood; and all these are found in the inorganic constituents both of the white and yolk.

§ 49. *Animal Food Considered.*—"Flesh makes flesh." Thus says a popular proverb. We readily agree with this view, and that so much the more when we regard diet in its direct relation to our body, since the view is much more correct than if it had been said—"Flesh makes blood." It is not only the potash predominating in the flesh which distinguishes it from the blood, the latter containing soda in a greater proportion; for while in the blood more albumen than fibrine is to be found, the fibrine predominates over the albumen in the muscles. Flesh, therefore, is much more capable of repairing the waste of our muscles than blood.

Is this discordant with the fact, that all nutriments find their way to the tissues only through the blood? The chyle, originating from meat, is, like all the rest, mingled with the blood, to which it is conveyed through the thoracic duct. But as our flesh in great part consists, not by chance, but according to a necessary law of attraction, of chloride of potassium, phosphate of potash, and fibrine, an abundance of these constituents in the blood must be to the benefit of our muscles.

Under a predominant, and still more under an exclusive animal diet, the fibrine does, in fact, pass in greater quantity into the blood. The natural result of this is great vigour of muscle, as may be illustrated by the strong muscular structure and fiery movements of the Indian tribes, in North and South America, who snatch a livelihood from the chase. Has not breeding cattle the same influence upon the Calmucs and Tartars, the pastoral tribes of the Alps, and the mountaineers of the Scotch Highlands? Who does not know the superiority of an English labourer, who is strengthened by his roast beef, over an Italian lazarone, whose predominant vegetable diet explains in great measure his inclination to idleness? And, finally, the slighter strength of the Laplanders and Samoyedes, of the Greenlanders and Kamtschatdales, whose food consists almost wholly of fish, in which scarcely more than three-fourths of the fibrine found in the birds and animals is to be traced, is an additional proof of the truth of the sentence—"Flesh makes flesh."

The richer the meats are in soluble albumen, the poorer are they in fibrine and fat, the more easily digestible will they be, if other constituents do not neutralise this property. Thus the flesh of pigeons and fowls is more digestible than veal; veal more digestible than the muscles of oxen, sheep, and deer. Horse-flesh is, doubtless, very similar to the latter; and the Mongols and Patagonians, and even some Christian nations of Europe in times of famine, have proved its advantages as a nutriment; while recent investigations of the French physicians have scientifically established them. Unjustly, therefore, was the eating of horse-flesh declared by the holy Boniface to be a heathen indulgence of appetite, interdicted by Pope Gregory III. and other authorities. The quantity of fat places the muscles of swine and geese among the most heavily digestible meats; while, on the contrary, most of the different kinds of game owe their superiority to the small proportion of fat they possess, which distinguishes them from the meat of our high-fed domestic animals.

The flesh of fishes, according to their proportionately small quantity of fibrine and their abundance in soluble albumen, might be supposed easily digestible; but the quantity of phosphorous fat renders them difficult of solution in the digestive juices—a remark which is applicable, not only to fishes, but to the brain and liver of all vertebrate animals. Of the intestines, there is no aliment more easily digestible than the sweet-bread of calves, which is remarkable for its slight quantity of fat and fibrine, as well as for its abundance of soluble albumen.

On account of the formation of acetic acid in the external parts, and of the less complete coagulation of the interior, roast meat, which does not contain too much fat, is easier of digestion than boiled.

Soft-boiled eggs are, on the whole, dissolved in the stomach with greater facility than hard ones; but as dissolved albumen is coagulated by the acidity of the gastric juice, and is afterwards dissolved again, the process of boiling the eggs hard, if not carried to excess, does not materially interfere with their digestibility.

As all kinds of flesh contain a sufficient proportion of fat and salts to restore to the human body the inorganic constituents of the excretions and the transformed fats, that kind of flesh, however, which contains the albuminous matters in the greatest abundance is to be considered the most nutritious. Venison and beef are, therefore, more nutritious than veal, and veal more nutritious than fish. Pigeons and fowls, however, surpass beef in nutritiveness, being not inferior in their proportion of albuminous matters, but superior in digestibility. Pork, also, is less nutritious than beef, from the twofold reason of its possessing fewer albuminous compounds, and of being, on account of its preponderant quantity of fat, less digestible.

When fibrine is abundant in the blood, the heart, being a tissue of muscular fibres, is more fully nourished; the activity of the circulation is consequently increased. The excitation of this activity, observed after a copious meal of venison, is due not only to the abundance of albuminous matters contained in the venison, but also probably to its proportionately large quantity of kreatine.

The formation of blood and the nutrition are increased by animal diet, as all these facts prove. Shall we therefore be surprised to find that, after a hearty meal of meat and eggs, the secretions and excretions containing albuminous matters, or compounds derived from them, likewise increase in quantity, and especially in their proportion of nitrogenised matters? That the milk flows more copiously, and the quantity of urea and uric acid, in twenty-four hours, is considerably increased? Is it to be wondered at that the blood circulates more rapidly, that the muscles contract with greater vigour? Thus, it is true that an abundant formation of blood gives the first impulse to an active metamorphosis of tissue; thus it is true that all increased activity has to be referred to a corresponding supply of new material.

§ 50. *Bread and Cakes.*—All kinds of corn, including rice, maize, oats, barley, rye, and wheat, contain in their seeds a large quantity of undissolved vegetable albumen, combined with a little vegetable gelatine. The latter compound, belonging, like the former, to the albuminous matters, is a glutinous substance, which communicates its property to the whole combination; from this circumstance it is called gluten. In addition to the gluten, formerly erroneously supposed to be a simple body, most kinds of corn contain a small quantity of soluble vegetable albumen.

The constituents of fat are also abundantly represented in all kinds of flour, for all kinds of corn contain a quantity of starch so considerable, as by far to surpass their amount of albuminous substances. Besides the starch, a proportion of gum is always present. Formerly, also, sugar was supposed to exist in these seeds; recent investigations, however, have shown this supposition to be incorrect, as far, at least, as regards wheaten flour in a fresh condition.

A small proportion of ready-formed fat accompanies the constituents of fat; and, finally, all the inorganic constituents of the human body are present in the seeds of the cerealia—namely, soda and potash, magnesia and lime, iron and chlorine, fluorine, phosphoric and sulphuric acids. The phosphates of the alkalies and earths predominate amongst the salts; and magnesia amongst the earths.

§ 51. *Composition of the Cerealia.*—The difference in composition which exists amongst the seeds of the cerealia, is much greater than one would expect from the natural affinity of their parent plants.

Gluten and starch are the constituents of grain, in the proportion of which the greatest differences are found, the quantity of the one generally varying inversely as that of the other. Thus, wheat contains the greatest quantity of gluten and the smallest of starch; rye a medium proportion of both; while in rice and barley, in oats and maize, the largest proportion of starch and the smallest of gluten are to be found.

Potash considerably predominates over soda in the seeds of the cerealia; this has been established by the most recent completed investigations on wheat and barley, oats and rice.

Maize is remarkable for its considerable proportion of fattening matter. In the external covering of all kinds of grain, there are contained much more gluten and fat than in the interior. Peeled rice and pearled barley have, therefore, lost a great deal of their nutritiveness; and bread containing the bran is much more nourishing than that prepared from sifted flour; but, unfortunately, the former is rendered, by the hard cellular tissue which it contains, much more difficult of digestion than the latter, and excites an injurious irritation, causing diarrhœa in weak digestive organs. Sifted flour is, therefore, for general use, preferable to the unsifted.

If we consider that the breeding of cattle, and agriculture, are among the most ancient pursuits of mankind, we must not be astonished that, in the methods of increasing the produce of both, experience had anticipated science by some thousands of years. In many cases, therefore, nothing was left to the latter, but correctly to interpret the results of the former, and explain its principles. Often, too, the scientific explanation of a customary practice has proved the safeguard against those errors, which would have been sure to arise in reasoning from solitary instances experimentally established, without the sure ground of a general principle. If the agriculturist were but ready to acknowledge the services of the chemist, and the latter willing to cooperate with the former, then the power of mind and that of the purse, which have, in fact, but one common object in view, would not divide their forces; but to the obstinate pride of the theorist, who seeks to overthrow all that agricultural experience has established, there is only too often opposed, the equally obstinate narrow-mindedness of the agriculturist, who has no faith out of the circle of his experience, or beyond the limit of his fields; the one forgetting that old customs have been confirmed by new reasons, at least as often as they have been set aside.

This state of things is, however, happily disappearing: at least, in this country, agriculturists are now aware of the value of the chemical appliances to agriculture, and are not found to be backward in developing its resources. They are now aware, that what feeding accomplishes in the stable, manure effects in the field; and while with cattle an abundant formation of fat is principally sought, the chief endeavours of the agriculturist are directed to the production of gluten. The greater the amount of nitrogen incorporated into the soil by means of manure in the form of ammonia, the more considerable is the yield of gluten, which by means of the seeds of the cerealia renews the blood of man.

Besides manure, the temperature exercises a most important influence upon the quantity of gluten produced by the gramineous plants of the fields. In summer, and in warm climates, more gluten is formed than in the corn of winter and of the rough north.

§ 52. *Constituents of Bread.*—Although bread is made of different cereals in different countries—as of rice by the Hindoos, and of maize in Tyrol and at the Gold-coast—wheat and rye furnish the flour chiefly used in the preparation of bread in Europe.

Our ordinary bread is prepared with leaven, and is, therefore, called leavened or fermented bread. Leaven is nothing else than a part of the common dough preserved until the next baking, during which time it has become sour. By the process of fermentation in this preserved dough, lactic and acetic acids are formed. Yeast may be substituted for leaven with precisely the same effect. In both, an albuminous compound is the occasion of the sugar formed in the dough turning it into a vinous fermentation. By this process the sugar is decomposed into alcohol, which evaporates, and into carbonic acid, which, inclosed by the tough gluten, is retained in the bread.

Flour, the ferment water, and salt, form the dough. In this a part of the starch has already been transformed into sugar; and by the action of leaven or yeast, this sugar is transmuted into alcohol and carbonic acid. The carbonic acid, which is prevented from escaping by the tenacity of the gluten, produces the vesicles which give to the bread its ordinary lightness. By the process of baking, a portion of the starch in the external layer of the bread is transformed into gum and sugar. The soluble albumen coagulates the process; the alcohol evaporates.

By exposure to heat, this crust becomes brown; a compound of an agreeable bitter taste is thus formed, similar to that produced by the roasting of different other organic compounds. This peculiar bitter principle is called "roast-bitter," or "Assamar." Bread is so easily soluble in water, that it is liquefied even by the humidity of the air.

Good wheaten bread is white; the real brown bread, as, for example, the well-known Westphalia rye bread called "Pumpernickel," is from rye. As wheaten flour contains more gluten than rye flour, the same proportion is not found in white and brown bread; and as it is gluten which causes the sugar to undergo fermentation and retain the carbonic acid, the reason is apparent why rye bread, which contains a smaller proportion of gluten, is less spongy than wheaten bread. Stale bread is scarcely drier than fresh. In five days fresh bread loses one-hundredth only of its amount of water, and becomes stale even if it has cooled in an atmosphere saturated with moisture. But stale bread may be retransformed into fresh if put again into the oven, whereby a considerable amount of water is necessarily lost. Both high and low temperatures cause a change in the smallest particles, which has yet more closely to be investigated by science. It is, however, a fact, that stale bread is hard and firm, but not dry.

§ 53. *Nutritive Qualities of Bread.*—Bread is not, on the whole, so nutritious as meat, so far as the albuminous substances are concerned; for even the richest bread contains only about two-thirds of the quantity of albuminous matters present in beef.

The digestibility, moreover, of bread and flesh is not to be considered equal; for gluten is more difficult of solution by our digestive juices than the fibres of the muscles, and corresponds less intimately with the albuminous substances of the blood; therefore it is more slowly transformed into its constituents.

The starch, so abundant in bread, has to be transformed into fat. The inferior solubility peculiar to the ready-formed fat of the flesh is thus compensated.

Of the fat which the excretions subtract from the blood, bread is a much more productive source than meat; for more than one-third of wheaten bread consists of starch; while one-tenth in weight of gum, and a small quantity of sugar, is also contained in it. This predominance of the constituents of fat, explains why bread contains much more of solid substance than flesh. In the former, the proportion of water scarcely amounts to one-third of the whole.

This abundance of the constituents of fat does not at all correspond in proportion to the small quantity of fat to be found in the blood; and in comparing the nutritiveness of flesh with that of bread, we must, therefore, decide in favour of the former.

The nutritiveness of the different kinds of grain is dependent upon their comparative proportions of gluten; for in all of them the constituents of fat are present in abundance. Wheat, rye, oats, barley, rice, and maize, form a series, in which wheat takes the highest, and maize the lowest place with respect to nutritiveness. Thus in rice and maize there is scarcely to be found one-seventh of the amount of gluten contained in wheat. A corresponding proportion of nutritiveness consequently exists in the bread prepared of these different kinds of grain. Chemical knowledge thus justifies the old usage which prefers wheat and rye-bread to all other kinds of bread.

§ 54. *Indigestibility of Cakes.*—Like stewed hare, or any other composite preparation of meat, cakes belong more to a cookery-book than to a treatise on food. Eggs, fat, sugar, different condiments, almonds, dry or fresh fruit, are mixed with the dough of different kinds of flour; and all these constituents will be spoken of in their respective places. Why are cakes less conducive to health than bread? This question is so important to that large class of housewives who are proud of their pastries, that I will not pass it without a reply.

Sugar is the constituent considered, by most people, the most dangerous, while it is in reality the least so. If not added in too great a quantity to cakes and tarts, sugar is transformed into lactic acid, and assists the stomach in digesting. It is the fat, so abundantly mixed with many tarts in the form of butter, and as a constituent of the eggs and almonds, which renders many kinds of pastry so difficult of digestion. The more these fats are transformed by heat into the products of their respective decomposition, the more will this be the case. For this reason, macaroons, almond-tarts, or chocolate-cakes, containing the fat of cocoa, are more indigestible than other fruit-cakes and pastry which contain neither almonds nor cocoa.

The indigestibility of the latter, however, corresponds exactly to the quantity of butter and yolks of eggs used in their preparation; for in the yolk the fat of the eggs is principally to be found. Cakes, therefore, containing only a small proportion of butter and eggs, are the most innoxious.

The roast-bitter, produced by baking, in the crust of bread, originates in all farinaceous food in the same way. It is the substance which principally takes up water from the air, or the interior of the cakes, and causes the crust to be moist.

But why are cakes preserved in a moister condition, when kept in tin boxes secluded from the air? Because in a closed place the water of the cake evaporates less, and the cake therefore is less desiccated. The sugar, with which the surface is usually sprinkled, and the roast-bitter principle, attract the water from the interior of the cake; and after a time the sugar is found melted, and the crust humid.

§ 55. *Peas, Beans, and Lintels.*—Peas, beans, and lintels are comprised under the name of leguminous seeds. If we wished to divide all aliments into two principal groups, the former containing those of a higher nutritive quality, we should in this class the leguminous seeds together with meat and bread. For the legumin, which is a constituent of all leguminous seeds, is contained in peas, beans, and lentils, in so abundant a quantity, that this albuminous substance not seldom surpasses both the proportion of gluten in bread, and that of fibrine in meat. Legumin is soluble in water. Acetic acid precipitates a sediment in the solution, which does not dissolve again if a larger quantity of this acid be added. By mere boiling, the legumin does not coagulate; but this effect is produced on the soluble vegetable albumen, which, as in cereal grains, is also found in the leguminous seeds in a small proportionate quantity.

A considerable proportion of starch, accompanied by a good quantity of gum, and in the sugar-peas, for instance, by sugar also, takes the place of the constituents of fat, in the interior of the seeds of peas, beans, and lentils. The husk of these seeds, as well as the shell of those beans which are eaten whole, consist in great part of a substance which forms in all plants the young cellular walls, and is therefore called the cell principle, or cellulose. This belongs to the constituents of fat; it is identical in composition with starch, and like this is transformed into sugar by acids. But this process of transformation is so slow, that the cellulose, which does not dissolve in water, belongs to the alimentary substances, which are very difficult of digestion; so much the more so, as even the alkali of our saliva, bile, pancreatic and intestinal juices, is only able to dissolve a very small quantity of cellulose.

The ready-formed fat associated in the leguminous seeds with the constituents of fat, is present in so small a quantity, that, without their abundant proportion of starch and gum, the peas, beans, and lentils would not be able to restore the fat of the body, which, after being decomposed by combustion into carbonic acid and water, we constantly exhale.

All the compounds of chlorine and salts of the blood are to be found in the leguminous seeds; the most abundant salts of the blood, the phosphates of the alkalies and earths, existing there in the greatest abundance.

§ 56. *Alimentary Principles.*—Legumin owes its name to the circumstance of its representing the most important alimentary principle of the leguminous seeds; for it represents the albuminous substances in the peas, the beans, and the lentils, which contain only a small proportion of soluble albumen besides.

How is it that peas, beans, and lentils, become hard by boiling, as the legumin does not coagulate under the influence of boiling-water, and the proportion of the coagulating soluble albumen is so small! Because the chemist and the cook employ a water altogether different. The chemist operates only with water free from all dissolved substances. He first causes the rain or spring-water to evaporate by heat, and condenses again the vapour so formed in a low temperature; the water thus distilled containing none of those compounds of lime which are found in all the water used by the cook. But, in boiling, the lime unites with the legumin, and transforms it into a very hard substance; and as rain-water contains a smaller quantity of lime than spring-water, the peas remain softer in the former than in the latter.

By the process of boiling in rain-water, which contains but a small proportion of lime, a considerable part of the legumin is dissolved. It is better, therefore, both for the blood and the purse, to eat peas, beans, and lentils, not in a dry form, but prepared as soup. Just as boiled meat should be partaken together with its broth, in order to get the most digestible and nutritive part of the meat, so in pea and lentil soups, the fluid part is better than the undissolved portion. A great part, therefore, of the nutritious substance is thrown away if we only eat the peas which have been boiled in water, and throw the liquor away.

§ 57. *Digestibility of Leguminous Seeds.*—In respect of digestibility, the leguminous seeds, if taken without husks or skins, hold an intermediate place between meat and bread; for the fibrine and albumen have an advantage over legumin in their greater conformity with the constituents of the blood, while legumin surpasses the gluten of the bread in solubility.

But if water which contains much lime, as is often the case, renders the legumin hard, and if the seeds are rendered more difficult of solution by the presence of their husks, peas, beans, and lentils become dishes difficult of digestion, and occasion flatulence. This shows the advantage of preparing the soups of the leguminous seeds in rain-water, and straining them through a hair-sieve after the bursting of the husks by boiling. Peas, beans, and lentils, with their husks, require very vigorous organs to digest them.

But the leguminous seeds, though inferior to meat in digestibility, are superior in the abundance of solid constituents which they contain; for the proportion of water in peas, beans, and lentils, is scarcely one-sixth of their weight; and while the quantity of albuminous substances may surpass that contained in meat by one-half, the constituents of fat and the salts are also present in the leguminous seeds in a greater abundance.

Blood and flesh, and milk, are therefore abundantly formed by peas, beans, and lentils; and for this reason they are the comfort of poor people, to whom meat is so seldom and so sparingly allotted.

We have already mentioned the considerable proportion of phosphorus contained in legumin; but the brain cannot exist without the peculiar fat containing phosphorus, which owes its origin to the albumen and fibrine of the blood. Phosphorus cannot be formed from any other element. It necessarily follows, therefore, that meat, bread, and leguminous seeds are serviceable in supporting the nutrition to the brain; and that aliments like fish and eggs, which contain a ready-formed phosphuretted fat, will facilitate the conveyance of this peculiar constituent of the brain. The formation, and therefore the activity of the brain, also, is dependent on this fat containing phosphorus; and it has been said, as a mere pleasantry, that a sagacious man has much phosphorus in his brain. The chemical combination of an organ is as much impaired by containing too much of a constituent, as by possessing too little. There is no danger that the laws of regular attraction, upon which the nutrition of the tissues is dependent, should allow the excessive supply of a single constituent; but the function suffers if this constituent be present in too small a proportion. We cannot, therefore, presume an abundance of phosphorus in great thinkers; while yet it remains true, that with no phosphorus, there is no thought active in the brain.

§ 58. *Vegetables.*—Meat and vegetables are aliments so generally used together, as inevitably to suggest the presumption that one is complemented by the other; and on investigating the composition of the different kinds of vegetables, such as cabbage, cauliflower, salad, spinach, sorrel, and asparagus, nine-tenths of their weight will be found to consist of water, with scarcely half-a-hundredth part of soluble albumen. Although the proportion of the constituents of fat, especially of cellulose and gum, in addition to starch, wax, and the green nitrogenised matter of the plants, exceeds the albumen in quantity, yet it is evident, from the considerable abundance of water, that the constituents of fat form only a very small proportionate part of the above-mentioned vegetables.

But the leaves and young shoots which are used as dishes of vegetables, contain several organic acids; asparagus, for instance, with all the cabbage tribe, possesses acid of apples, or malic acid; the sorrel, oxalic acid; rhubarb contains both malic and oxalic acids. Malic acid consists of carbon, hydrogen, and a large proportion of oxygen; while oxalic acid, like carbonic acid, is composed of carbon and oxygen only, but with a smaller proportion of oxygen than is contained in carbonic acid. Both these vegetable acids have the power of keeping the soluble albumen of meat in a dissolved state. In asparagus there is associated with the malic acid another body, the asparagine, neither acid nor alkaline, but remarkable for its amount of nitrogen. Finally, the saurkrout, the dainty of a German citizen, and the relief of an English sailor, derives its name from a sourish taste occasioned by lactic acid, produced by the following process:—The cabbage, which is cut into slices and placed in a barrel, intermingled with a certain quantity of common salt, is pressed down, and in this state it undergoes fermentation; lactic acid, together with a small quantity of butyric acid, is thus produced.

The dissolving action of these acids—for they act also upon the fibrine of flesh—is assisted by their abundant proportion of compounds of chlorine and of salt. In cabbage and asparagus, lettuce and cabbage sprouts, potash considerably predominates; in spinach, soda and potash are nearly equal; while in rhubarb, a considerable amount of lime is contained. Cabbage sprouts are remarkable for their amounts of lime and magnesia; and in the stalks and leaves of lettuce, in asparagus and cauliflower, traces of manganese, a metal very similar to iron, have been found.

§ 59. *Digestibility of Vegetables.*—Are we, then, to express surprise that those vegetables should be praised, and put in comparison with meat, as aliments which dilute the blood, when we know that their solid substance often forms scarcely one-tenth in weight of the whole; and, moreover, that it contains more salts than albumen, without any undissolved albuminous substance at all, but merely a small proportion of organic acid? Whilst by themselves conveying very little nutriment to the blood, as may be seen in the feeble muscles of the inhabitants of the tropics, who live exclusively on herbs, vegetables are of service, first in the digestive canal, by dissolving the albuminous substances of the meat, and afterwards in the blood itself, by keeping the albumen and fibrine in a liquefied state, but not as nutritious food.

The cellulose, which in several kinds of cabbage, especially in the stem, is present in a greater abundance, belongs to an alimentary principle difficult of digestion; and this explains the flatulence which cabbages not unusually occasion in weak digestive organs. Of all other kinds of cabbage, the white cabbage used for saurkrout is remarkable for its amount of lactic acid, which renders it a readily digestible aliment. This useful vegetable is sometimes charged with indigestibility—a charge which should be brought against the pork and hard-boiled mashed peas so often taken with it.

If it be, therefore, apparent that as vegetables alone furnish a very insufficient compensation for the excreted substances, the fact that an exclusively vegetable diet affords but a defective nutrition to the tissue, is at once explained. Not only do the muscles become powerless, but a smaller quantity of nutriment is conveyed to the brain. Hence results an irresolute will, and a cowardly relinquishment of independence, as illustrated in the timid and slavish Hindoos, and other inhabitants of the tropics, who feed almost exclusively on vegetables.

Since, moreover, the alimentary principle of vegetables requires less oxygen than meat, bread, or the leguminous seeds, for the production of the same quantity of carbonic acid and water; while, at the same time, the amount of oxygen we inhale remains undiminished, it becomes evident why we exhale more carbonic acid while living on vegetables, than when supported by a more invigorating diet of meat, bread, and leguminous seeds. Precisely for this reason do we become satiated in a short time by an exclusively vegetable diet; for the blood and the tissues are only scantily supplied with albumen. But these tissues, containing a scanty amount of albumen, furnish fewer nitrogenised products of decomposition; and the excretion of urea and uric acid is, during the twenty-four hours, considerably diminished. The amount of urine, however, is increased by the salts and acids of the vegetables; and everybody knows how quickly the constituents of the asparagus, conveyed to the blood, and attracted therefrom by the kidneys, manifest themselves to the sense of smelling in the urine.

By substituting vegetables which are poor in albumen, for one part of the meat we take as a meal, the supply of albuminous bodies is diminished, and the digestion of what we have taken facilitated. Thus a combination is performed intermediate between meat and vegetables; the blood contains more water than the meat, more solid substance than the vegetables; and, by adding together the amount of albuminous substances in the meat and vegetables, and dividing the total into two equal parts, the average quantity of albumen in the composition may be found approximately corresponding with that in the blood. Thus the seeming chance becomes an approved law; and, instead of an arbitrary taste being left to dictate the choice of aliments, the necessity of attending to the law is apparent, clear light being thus thrown upon the connection between nutriment and blood; and the night in which the realms of knowledge were peopled with nebulous spectres and conjectural dreams, tending to show that all things existed for a settled purpose, is turned to day and conviction by the discoveries of science.

§ 60. *Potatoes and Edible Roots.*—Although in ordinary life we class the potato and other edible roots in the same category as the "vegetables," we must not forget that these roots and tubers considerably surpass the latter in the quantity of solid substances which they contain; on an average, the proportion of water they contain does not greatly exceed that of meat.

Are, then, potatoes and edible roots to be compared with meat in their nutritive qualities? Here it appears how important it is to compare the connection between the several groups of alimentary principles represented by the solid substances, with the composition of the blood, if we would accurately estimate their nutritive quality. By such comparison it will be found, that in potatoes, turnips, carrots, beetroot, parsnips, and Jerusalem artichokes, in leeks and celery, shallots, onions, and radishes, the constituents of fat surpass the albumen in precisely that proportion in which the latter ought to have exceeded the former, had these edible roots been intended of themselves to sustain the composition of the blood; thus the soluble albumen, which alone represents the albuminous substances in these roots, often constitutes less than the one-hundredth, and sometimes only the two-hundredth parts in weight; while the constituents of fat fluctuate between one-fourth and one-fifth.

In the several roots and tubers, the constituents of fat are very different in kind. Cellulose and gum are indeed found in all of them; but while the potato is remarkable for its amount of starch, we find in carrots, beetroot, and Jerusalem artichokes, an abundance of sugar also.

The roots just mentioned as abundant in sugar, and also turnips, contain a new compound of carbon, hydrogen, and oxygen; which contains more oxygen, in proportion to its hydrogen, than the constituents of fat. This substance, in the unripe fruit, thickens the walls of the cells, consisting of cellular tissue, and is denominated vegetable jelly or *pectose.* By boiling, it is transformed into a gelatinous substance called pectic acid, which might, therefore, be called gelatine acid. But since this gelatine acid is not transmuted into sugar, either by our digestive juices or by any artificial means, the vegetable jelly and the gelatine acid cannot be classed with the constituents of fat.

Fat is contained in potatoes and carrots in very small quantity; while in artichokes, besides fat, a trace of wax has also been found. Rush-nuts contain, with a great amount of starch, a considerable proportion of fatty oil.

The pungent taste of leeks and garlic, of radishes and horse-radish, of onions and parsley root, is produced by certain volatile oils; for these oils, if extracted from the roots, will be found to possess exactly the same peculiar pungent smell and acid taste as the roots themselves. The garlic oil, which exceeds all other oils in pungency of taste and smell, is composed of carbon, hydrogen, and sulphur, and is imperfectly soluble in water.

Organic acids, which were mentioned as characteristic of vegetables, are also to be found in these roots: thus the malic acid in potatoes and carrots; the citric acid, composed of the same ingredients as the former; and tartaric acid, containing a greater proportion of oxygen than the former, are found in the Jerusalem artichoke.

Citric acid has been observed in several roots, which are remarkable for the above-mentioned volatile oils.

Asparagine is found in potatoes, which also remind us of the composition of asparagus, by their containing malic acid.

While all the inorganic elements of our bodies, with the exception of fluorine, are found in the potato, it is characteristic of turnips that they contain no soda, and only very little iron. By way of compensation, they possess a large quantity of potash, which is also contained in the ashes of potatoes, in a greater proportion than all the other inorganic constituents put together.

§ 61. *Comparison of Roots and Vegetables.*—In comparing potatoes and other edible roots with esculent vegetables, we find the latter inferior in digestibility and nutritiveness to the former; for, in the roots, not only are the different constituents of fat—namely, starch, gum, and especially sugar—more readily soluble than the cellulose of the vegetables; but these constituents of fat and albumen, however small the proportion of the latter may be, are also much more abundantly present in roots than in leaves and stalks.

But if we contrast the roots with the group of the more nutritious aliments, as meat, cereal grains, and leguminous seeds, we find even the least substantial aliment of this group more nutritious than any edible roots. Rice and maize are not only richer in albuminous substances than roots, but exceed them nearly fourfold in the amount of starch which they contain.

The edible roots, as well as vegetables and fruit, belong, therefore, to the less nutritious aliments, and may be united into a second group, the first being formed of meat, bread, and leguminous seeds.

In this second group, potatoes and carrots, it is true, are distinguished from the other members by their superior nutritiveness, as well as by their digestibility. But, notwithstanding this, what must we think of the value of a nutriment in which the albumen and the constituents of fat are contained in an exactly inverse proportion to that which exists in the blood? It may, indeed, overload the blood and the tissues with fat; but inasmuch as it provides the blood only scantily with albumen, it conveys no fibrine, and, therefore, no strength to the muscles; nor can it furnish any albumen or phosphuretted fat to the brain. Now what would be the consequence if we were to accustom ourselves to eat such a quantity of potatoes as would suffice for providing the blood with the requisite proportion of albumen by their means alone? It is very obvious if, through this excessive supply, the process of digestion did not come to an entire stand-still, a superabundance of fat would be formed beyond the power of the oxygen to consume; and this fat would successively absorb from the albuminous substances a part of this vital element. A barrier would thus be raised against the endless change of matter in the tissues, against nutrition and regressive transformation; and only a portion of the material, and that the less important one, would take a part in that course of decay and reparation, by which the will and the energy, the movements and the thoughts, are brought to act.

It is this which so immensely aggravates the weight of poverty. The ill-satisfied want may be tolerated for a time; the strength of the arm may by strenuous efforts, and with the hope of obtaining better food, for a time uphold the fainting courage; but can a lazy potato-fed-blood confer on the muscle the power necessary for this labour; or on the brain the necessary vivifying impulses of hope?

§ 62. *Taste.*—It is not only the peculiar taste which is produced by the volatile oils existing, as above stated, in the several roots; that taste is but the first link to a long series of impressions effected by these oils upon the body. The different kinds of leeks, onions, and radishes, accelerate the pulse; the oil is conveyed to the nerves with the blood, and the irritability is increased; the smell of the oils is communicated to the breath; and, apart from the flatulent risings caused by the radish and horse-radish, it may be perceived immediately, from the exhaled air, that onions, leeks, or garlic, have been eaten. Through the action of these oils the kidneys abstract much water from the blood; for the diuretic effect of these roots is well known.

§ 63. *Fruit.*—Vying in scent and colour with the flowers of the fields, fragrant fruits decorate our orchards; and though the savoury mango, and other tropical fruits do not tempt the tongue in our climate, still the appliances of horticultural skill have given an exquisite delicacy to our apples and pears, and transplanted the cherry and peach from Asia Minor; while active commerce provides us with the orange, and the lemon, and pine-apple.

The nutritive and refreshing qualities of so many noble fruits have not escaped the researches of chemists. The cause, however, of so many subtle varieties of taste and odour has not yet received a satisfactory chemical elucidation. It is true we know the volatile oil which causes the peel of lemons and oranges to throw out afar their peculiar fragrance; it is true we know that some fine sorts of ether reside in raspberries and peaches, which are now abundantly produced by chemical means, and used for flavouring purposes. Yet if the causes of variety are to be as lucid to our mind as the variety itself is delightful to our palate, much remains to be investigated.

Cellulose, gum, and sugar—constituents of fat, therefore, both of difficult and of easy digestion—are to be found in all fruits, as in apples, in stone-fruit, in berries, in oranges, and in melons. These components are associated with but a small proportion of albumen; it is even smaller here than in vegetables, though here and there, as in apricots, it is greater. The proportion of water contained in fruits, is intermediate between that contained in edible roots and vegetables.

Fruit-marrow, or pectose, a substance we have already specified as found in several roots, is abundantly present in all unripe fruits. While the fruits are ripening, this is gradually transformed into the constituent of vegetable gelatine (*pectine*), which, by boiling, becomes gelatine acid.

Several peculiar colouring matters, together with wax, produce the brilliancy of colour in the peel of cherries and apples. Like a film, they cover the skin; a touch of the finger deprives the peach of that tender waxy bloom, which in fragrance surrounds the fruit with a virgin-like adornment.

Different acids, supported by several salts, cool and refresh our thirsty tongues; as, for instance, malic acid, present in almost every fruit, especially in apricots and peaches, in apples and pears, in gooseberries and currants; citric acid, present in lemons and raspberries, grapes and pine-apples; and tartaric acid, in grapes and figs. The skin of grapes derives its astringent taste from the tannic acid, which, in many other fruits, as in acorns, produces a taste entirely bitter.

Almonds and nuts, as well as the kernels of stone-fruit, contain an albuminous compound (emulsine), which we may call *ferment of almonds*. In a temperature somewhat raised, this ferment of almonds causes another nitrogenised substance, contained in bitter almonds and in kernels of peaches, and called the *almond-principle* (amygdaline), to undergo a fermentation, by which the volatile oil of bitter almonds and hydrocyanic acid are formed.

As chestnuts are distinguished by their amount of starch, so almonds and nuts are characterised by their abundance of an oil consisting of oleine and margarine. It is only from compliance with custom that we place almonds, nuts, and chestnuts in the group of fruits, as their very slight proportionate amount of water decidedly approximates them to the group of the more nutritive aliments.

§ 64. *Degrees of Acidity.*—Ripe and sweet, sour and unripe, are, in popular language, words of the same signification;—wrongly, if we ascribe the degree of acidity to the proportionate abundance of sourness; for in the ripe fruit, the proportion of acid is very often increased; but the sugar, which has augmented in a still greater proportion, moderates the sourness, which in the unripe fruit predominates over the sweet, though the quantity of acid is actually smaller.

In ripe fruit the acid is enveloped in the sugar, just as in stewed fruit it is by the jelly; for the vegetable jelly of the raw fruit deserves this name only when stewed. It is true, by this process, a new acid, the gelatinous or pectic acid, is formed. The latter, however, in the form of a mucous jelly, weakens the other acids. Stewed fruits and fruit-jelly, prepared with sugar, are therefore less injurious than raw fruits, so far as concerns the irritating power of the acids and salts; for the interior coating of the digestive tube is protected by the gelatinous acid.

Apples and berries, cherries and plums, apricots and peaches, melons and cucumbers, with all other similar fruits, dissolve the albuminous matters, and exercise a cooling influence upon the blood. While more nutritious than vegetables, and less so than potatoes, they have a great advantage over the latter in not over-loading the blood with fat. A thinner blood circulates with much vivacity through the vessels of the inhabitants of the South Seas; "these blessed countries," to use the words of Forster; "the cradle of the younger human race, where, not yet condemned to slavery, it enjoyed all the rights of free men, without being compelled to purchase at too high a price the happiness of existence in sweat and exhaustion."

ON LIQUID FOOD.—§ 65. *Water.*—If life consist in a metamorphosis of the tissue, fluids are an indispensable condition of life; for the combinations and decompositions in its substance, produced by the activities of our body, cannot take place without the agency of water. The simplest beverage is therefore, at the same time, the most necessary of all.

It is true, the same combination of hydrogen with oxygen, which constitutes the essential ingredient of all drinkable water, is contained in a small proportion even in the driest food; but neither meat nor bread, still less the leguminous seeds, are so abundant in water as to be capable of maintaining the proper composition of the blood. And who is not aware, from his own experience, that the most nutritious aliments are precisely those which most stimulate our thirst?

But it is impossible, in our climate, with our activity, with the vigour of our changing tissues, to live only on fruit and vegetables, meat and bread, and peas and beans. Our most important aliments must be intermingled with a food abounding in water, if the water, which is abstracted from the body through the skin and lungs, the intestines and kidneys, is to be restored by a process of uninterrupted attraction.

Our drinking-water is a nutriment which contains the greatest proportion of liquid; for is that not a nutriment which conveys to the blood the substance that renders possible the motion of all other combinations? Is not the water to be considered a restorative substance, inasmuch as the water, which constitutes three-fourths of the blood, is continually being withdrawn by all excretive organs, without exception?

And yet drinking-water is nutritious not alone because it contains water. The warmth of the earth indefatigably raises water into the air. From brooks, rivers, and seas, from plants also, and animals, there arise uninterruptedly, through the action of heat, vapours, which are condensed into clouds in the higher strata of the atmosphere. If that were true, which in a coward passion for repose has so often been imputed to nature, that its activity never deviates from the strictest method of quiet development; evaporating water, if falling down as rain, would provide us with water only. But the dashing of the sea, and the storms of the air surrounding the earth, the pressure of the atmosphere, and the power of fire, very often impel the vapour upwards with so furious a violence, that all substances dissolved in the water are carried with it. Hence, even the purest water falling down from the clouds, is impregnated with salts. Common salt and chloride of potassium, lime and magnesia combined with sulphuric and carbonic acids, magnesium with chlorine, even iron and manganese, have been found in rain-water; and however slight the proportionate amount of these fire-proof constituents may be, the regularity of their indications establishes the law.

In a greater abundance, however, than with these solid bodies, rain-water is impregnated with air. Oxygen and nitrogen, and the most important nutriments of the plants, carbonic acid and ammonia, are taken up by the falling rain watering the thirsty earth, and fertilising the fields; and even the lightning renders its assistance in enriching the verdant clothing of the ground. Its spark unites nitrogen and oxygen into nitric acid, and the thunder-shower supplies to the plant the nitrate of ammonia.

To ammonia especially, rain-water owes its softness; while lime renders the water, containing a greater proportion of salts, hard, as we have it in our wells and springs. The carbonic acid of the water dissolves the chalk of the earth, and the water itself dissolves the gypsum, or sulphate of lime, which forms the deposit when, in the process of boiling, a considerable quantity of water is evaporated. This is well known as occurring in kettles, boilers, &c.

The quality of spring-water is as various as the earth through which it oozes. Earths and alkalies, combined with chlorine or sulphuric acid, with carbonic or nitric acid, with iron and manganese, are contained in the water of wells and springs in the most varying proportions. One or the other of these substances is often absent. Phosphoric acid rarely, if ever, forms a constituent of water, notwithstanding the predominance of the lime-salts in wells and fountains.

That which causes the water of swamps and lakes, of rivers and seas, to be almost always unfit for drinking, is the admixture of putrefying organic substances, besides the common salt of the sea, which impart a putrid taste to it. Now it is the bottom of the sea, which, like a retort, sends the water up into the air; from which, as if distilled into rain, it streams down again in a potable condition: now it is the earth, forming, as it were, a strainer, through which the water, though not entirely purified, bubbles out of the wells. Even the skin of man possesses some of the properties of a filter. Cast upon a rock where not a drop of fresh water refreshes the languishing tongue, the shipwrecked man bathes in the sea in order to escape the most tormenting death from thirst; and from the sea-water itself, a refreshing moisture, with a smaller proportion of salt, penetrates the thirsty body.

§ 66. *Water Essential to Digestion.*—Since the last result of the whole process of digestion is a liquefaction of the alimentary principles, the formation of blood is impossible without water; but not only the formation, but the continual exercise also of the organs, depends upon their receiving a due amount of water. Without it, no digestion or formation of blood, no nutrition or excretion can exist. Even this statement, however, by no means exhausts the importance of water; for it is essential, not only as the medium for the movement of all dissolved substances, not only as the humidity necessary for the organs, of which the most active, the brain and the muscles, contain the greatest proportion of water; but the hydrogen and oxygen which we take in water, enter into the composition of many alimentary principles by being changed into the constituents of the blood. When starch or gum becomes sugar, the transmutation is effected by the absorption of water; for, with regard to their composition, a greater proportion only of water distinguishes sugar from starch; and separation of oxygen from sugar, causes the latter to be converted into fat.

No alimentary principle is so easily removed from the body as water, if taken in excess; for as no decomposition is required in order to its being attracted by the lungs, the kidneys, the skin, and the sudoriferous glands, it is not even necessary for this process that a greater proportion of oxygen be taken in. Water, taken in abundance, increases, at a low temperature, the activity of the kidneys; at a high one, the perspiration. He, therefore, who takes more water than necessary for nutrition and excretion, produces, in winter-time, a proportionately greater quantity of urine; in summer, an excessive perspiration.

§ 67. *Milk.*—A nutriment like milk, which during the whole period of life is of itself capable of sustaining the formation of blood, is, as it were, a ready answer of nature to the question—What alimentary principles are required for the formation of a complete nutrition? When treating on digestion, we took milk as the prototype of a perfect nutriment. It is at once a solid and a liquid food; a source of albumen and of fatty substances, of sugar and the salts—in one word, it is the most perfect of nutriments.

While more abundant in water than our blood, more abundant than bread and meat, milk possesses, in caseine, a representative of the albuminous substances, accompanied by the ready-formed fat of the butter, by a constituent of fat in the sugar of milk, and by the most important salts of the blood.

Contained in small cellules, which it dilates into glittering globules, the fat rises to the surface of the milk which stands undisturbed, forming the cream; while the lower, and far more abundant part, contains the chief quantity of the caseine, the milk of sugar, and the salts. More than one-third of these salts consists of phosphate of lime, associated with the phosphates of potash and magnesia, with some traces of oxide of iron, chloride of sodium, and a large proportion of chloride of potassium.

§ 68. *Qualities of Milk.*—Although the milk of goats and sheep may derive their peculiar smells from the uncombined state of one of the volatile fatty acids, which in the milk of the cow are united with glycerine into neutral fats, the more important difference between the milk of women and that of animals is exclusively dependent on the different proportion of caseine and sugar of milk, of butter and salts, to be found in these several sorts of milk. Thus, the quantity of caseine in human milk is only half as large as in that of the cow; and while, in the latter, butter and salts are likewise contained in greater proportion, human milk exceeds this in the amount of sugar of milk, by more than one-half. However slight the sweetening property of sugar of milk may be, if compared with cane-sugar, the babe does not fail to distinguish the milk of its mother, from that of the cow, by its sweeter taste. The peculiar butter-fat, or butyrine of the chemist, which, at ordinary temperatures, is more fluid than margarine or oleine, must be contained in the human milk in a greater abundance than in that of the cow, as the butter of the latter is of a firmer consistence.

Immediately after confinement, human milk contains a much greater proportion of solid substances, particularly of butter, than it does some days afterwards. After a striking diminution of these contents of the milk, which has already become remarkable the fourth day after confinement, the proportionate quantities of caseine and salts increase again. After weaning, the milk is again impoverished with equal rapidity.

§ 69. *Formation of Acids in Milk.*—By the influence of the caseine of the milk, its sugar is transformed into lactic acid, especially at a higher temperature, or in thunder-storms. As oxygen promotes the formation of lactic acid, boiling protects the milk for some time from turning sour—the oxygen which is contained in the milk being expelled at the boiling-point. The production of this acid takes place so much the more easily, inasmuch as lactic acid is identical with sugar of milk in its composition. The sour milk becomes thick from the caseine being coagulated by the lactic acid which has been formed; curdled milk is that in which caseine has been coagulated by the spontaneously transformed lactic acid. If from the curdled milk, the caseine, to which a great part of the butter adheres, be taken away, the whey remains, which is a solution of lactic acid, salts, and sugar, with a much smaller proportion of caseine and butter than is contained in pure milk.

In butter-milk the proportionate quantity of butter especially is diminished; for it is the remaining part of the cream from which, by the process of churning, the fats have been taken off. It always contains, however, some traces of butter, almost all the sugar of milk, the salts, and the caseine, very little of which passes into the butter.

§ 70. *Digestibility of Milk.*—It is natural to suppose that the food of the infant must be readily digestible and nutritious; and both suppositions are verified by experience: for while caseine belongs to the most soluble of albuminous bodies, the more soluble fats are represented in the butter. Next to cane-sugar, the sugar of milk is the most readily digestible of all constituents of fat, and its transformation into fat is facilitated by the butter and caseine associated with it. With the easy digestibility of the milk its nutritiveness is also proved; nor need we be astonished at the abundance of water in an aliment which, at the same time, furnishes the infant with solid and liquid food. And if the weight of the caseine scarcely amounts to one-sixth of that of albuminous bodies in beef, we must not forget that we commonly take the latter diluted—as in the form of soups, or associated with vegetables or edible roots; combinations by which the proportionate amount of albumen is considerably diminished. But the quantity of sugar of milk in the human milk is twice as great as that of caseine, and the butter is intermediate between both.

The milk of cows is often assimilated with difficulty by weak digestive organs, from the large proportion of butter it contains; but skimmed milk is in such cases usually readily digested, and this we find in a natural state in asses' milk. Poverty in fat, and abundance in sugar, make asses' milk an invaluable nutriment, maintaining and cheering the life of many invalids. Several excellent rules are drawn from nature, and none more so than is to be found in the food which nature supplies for the nourishment of the infant. We may consider the health of the child to be the surest and largest experience which enlightens us as to the right composition of solid and liquid food which man has to partake. This experience confirms the truth, that meat and bread cannot be taken without water; and that in the best nutriment of mankind, the albuminous and fatty matters, the constituents of fat, the compounds of chlorine, and the salts, must be represented without exception.

"Whether food has a yet higher influence," says Forster, in his dissertation on the utility of the bread-fruit tree—"whether mind and heart may directly or indirectly be tuned by it, will be left to our grandchildren for discussion. We only know that gentleness, love, and sensibility of heart, are the prominent traits of the people living on bread-fruit." And if we consider that, even now, the pastoral tribes are the gentlest, that the character of the beasts of prey is softened by getting accustomed to a mixed or a vegetable diet, why should it sound fabulous if we connect the noble tenderness of heart and pacific meekness of those tribes with the milk and the fruits upon which they live?*

§ 71. *Coffee, Tea, and Chocolate.*—The general opinion that coffee, tea, and chocolate, are beverages which, to a certain degree, may be substituted for each other, is based on a good chemical reason, for all three contain a nitrogenised basis, to which they owe some of their most important chemical properties. Tea and coffee even contain the self-same basis, denominated therefore, indiscriminately, thein or caffein. The basis of chocolate we will call cocoa-principle. The cocoa-principle, or theobromin, is richer in nitrogen than the thein, which very nearly corresponds in its composition with flesh-basis. In cold water, thein is only with difficulty dissolved, while of cocoa-principle but a small proportion is soluble even in hot water, in which the former readily dissolves.

While in tea the basis is combined with common tannic acid, in coffee it forms a salt with a peculiar tannic acid containing a greater proportion of nitrogen, which, together with tanno-caffeic acid, are united with potash into a so-called double salt. Tanno-caffeic acid, when roasted, develops the agreeable smell of coffee.

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* Some curious researches have been made in reference to this by continental chemists; and it has been established as a fact, that diet has an essential connection with moral and mental development.—Ed

Not only the same basis, but also two similar organic acids—one contained in tea, the other in coffee—increase the conformity between the leaves of the former and the beans of the latter.

Legumin, cellulose, gum, sugar, citric acid, in addition to oleine and to the peculiar palm-fat, which derives its name from being found in the fruit of several species of palms, accompany the organic acids and the thein of the coffee-beans.

But the tea-leaves, apart from the basis and the acids, are composed of albumen, cellulose, gum, and wax, the green pigment of the plant, and the volatile oil of tea. This peculiar oil is the principal source of the aroma of tea, by which, in spite of the conformity between tea and coffee, it essentially differs from the latter.

The inorganic constituents of tea and coffee are, moreover, different. While in coffee, chlorine, phosphoric and sulphuric acids are combined with potash, lime, magnesia, and oxide of iron; tea contains another inorganic acid besides, consisting of manganese and a large proportion of oxygen.

In the kernels of cocoa, of which chocolate is prepared, there are to be found, besides the cocoa-principle, stearine and oleine, cellulose, gum, and starch, a red colouring matter, and a small proportion of water.

Chicory, so commonly used as a substitute or accompaniment of coffee, contains neither caffein nor any other peculiar constituent of the latter. No albuminous substance ever has, up to this time, been found in chicory. As albumen is not wholly wanting in any vegetable product, it must be present in chicory, though in a very small quantity. But the constituents of fat are represented in a larger proportion; for in addition to cellulose, sugar, and gum, it contains a pretty considerable quantity of a substance of the same composition as starch, which, by simply boiling in water, is transformed into sugar. Chloride of sodium, a compound of chlorine with ammonia (the so-called sal-ammoniac), sulphate and nitrate of potash, are its inorganic constituents.

As yet, therefore, no great importance as a nutriment can be attributed to chicory, nor any similarity with coffee. Unless a peculiar organic substance contained in it, but not yet exactly investigated, shall at a future time be proved to be peculiarly valuable, the infusion of chicory possesses no advantage over sugared water, to which a brown colour and bitter taste have been imparted; but we do not understand this bitter substance. Whilst, then, chicory cannot, on scientific grounds, be made out to be a true substitute for coffee, it has, on the other hand, been too partially investigated to justify an absolute condemnation.

§ 72. *Preparation of Teas.*—Roasted coffee differs from raw by its lighter weight and greater bulk. Several empyreumatic substances, produced by roasting, occasion the reddish or dark-brown colour; and the tanno-caffeic acid, altered by roasting, produces the aroma; the sugar loses a part of its amount of hydrogen and oxygen, and is thus decomposed into burnt sugar or caramel. In the infusion, the caffein and the acids, the gum and caramel, the melted fat and the salts, with legumin in a very slight proportion, are to be found.

Between green and black tea, erroneously so often supposed to be produced by different plants, a similar difference exists to that between raw and roasted coffee. The leaves are turned black by being dried at a higher temperature than that to which the green tea has been subjected. The latter is first brought into a withered condition by vapour, and then dried in iron kettles; while black tea is prepared by heating over an open fire. As heat exercises a decomposing action, black contains less tea-oil, less tannic acid, and less green colouring matter than green tea. By the process of drying, however, a dark product of decomposition is formed, few traces of which are found in green tea, but a considerable quantity in black. In China, tea is frequently coloured with curcume, and indigo or Prussian blue.

By the effect of the open fire, in black tea the albumen is more completely coagulated than in green; by this coagulation the tea-oil becomes disengaged. Partly from this cause the black tea has lost a greater proportion of its tea-oil than green; and for the same reason, tea possesses more flavour if infused with perfectly boiling-water, which coagulates all the soluble albumen, and therefore dissolves the tea-oil with greater ease. But the tannate of thein, also, is only sufficiently extracted by boiling-water: this compound is even separated at a low temperature; and hence the turbid appearance of a well-prepared infusion of tea when cooling, which everybody knows who has ever taken his tea out of a tumbler, as the Russians generally do. Thein and tea-oil, however, are the most important constituents of tea; boiling-water is therefore an indispensable condition, if we wish to partake really of tea, and not of a brown solution of gum. The tea-leaves have only to be infused with boiling-water, and after this the process of boiling must not be continued, or the tea-oil escapes, and the formation of a bitter solution of tannic acid takes place, a decoction which could as well be prepared from oak-apples. The first infusion contains four to six times more of the peculiar constituent of tea than the second.

Finally, the two principal kinds of chocolate are distinguished similarly to green and black tea. It is true, the cocoa seeds, of which chocolate is prepared, are always roasted. Besides the decomposed fat, an aromatic empyreumatic substance of dark colour is formed from the starch and gum; but the proportion of the latter substance is larger in the more strongly roasted, dark-brown Italian chocolate, than in the Spanish, which, being less roasted, holds more starch and butter of cocoa in an unaltered state. The colour of the Spanish chocolate is brown-red; its taste not so bitter and aromatic as the Italian.

§ 73. *Nutritive Qualities.*—From its very large proportion of albumen, chocolate is the most nutritious of these three beverages; but, at the same time, the most difficult of digestion from its quantity of fat. But as its aromatic substances strengthen the digestion, a cup of chocolate is an excellent restorative, and invigorating refreshment even for weak persons, provided their digestive organs are not too delicate. Cardinal Richelieu was indebted, in his later years, for his health and hilarity to chocolate.

Tea and coffee do not afford this advantage. Albumen in tea-leaves, and legumin in coffee-berries, are represented in very scanty proportions, and still more scantily in their infusions: for while in the former the albumen is coagulated by boiling-water, in the latter the legumin is prevented from being dissolved by the lime with which it is combined.

It is true that coffee and tea have been praised, even by chemists, as nutritious substances, at a time when "nitrogenised" and "nutritious" were considered identical terms. Since then, however, we have learned that, as restoratives for the body, not the elements but the alimentary principles are to be taken into account. The latter appellation cannot be ascribed to thein, which is excreted again as urea with surprising rapidity. To this swift transformation tea and coffee owe their diuretic action, which is considerably assisted by the warm water of the infusion; the kidneys attract the urea, with which the thein charges the blood.

Tea and coffee, though of themselves not difficult of digestion, tend to disturb the digestion of albuminous substances by precipitating them from their dissolved state. Milk, therefore, if mixed with tea or coffee, is more difficult of digestion than if taken alone; and coffee alone, without cream, promotes digestion after dinner, by increasing the secretion of the dissolving juices. No Italian takes cream with his coffee after dinner.

The volatile oil of coffee, and the empyreumatic and aromatic matters of chocolate, accelerate the circulation, which, on the other hand, is calmed by the tea-oil.

Tea and coffee excite the activity of the brain and the nerves. Tea increases the power of digesting the impressions we have received: "we become disposed for thoughtful meditation, and, in spite of the movements of thought, the attention can be more easily fixed upon a certain object; a sensation of comfort and cheerfulness ensues, and the creative activity of the brain is set in motion: through the greater collectedness and the more closely confined attention, the thoughts are not so apt to degenerate into desultoriness. Educated persons will assemble to tea for the purpose of investigating a certain subject by a regular conversation; and the higher spirits produced by the tea, tend to secure with more facility a successful result. If tea be taken in excess, an increased irritability of the nerves takes place, characterised by sleepiness, by a general feeling of restlessness, with trembling of the limbs. Spasmodic attacks even, in the cardiac region, may arise. The volatile oil of the tea produces heaviness in the head; and, in fact, a real tea-intoxication, first manifesting itself in dizziness, and finally in stupefaction, takes place. Green tea, which contains much more of the volatile oil than black, produces these obnoxious effects in a far higher degree than the latter."

While tea principally revives the faculty of judgment, and adds to this activity a sensation of cheerfulness; coffee acts also on the reasoning faculty, but communicates to the imagination a much higher degree of liveliness. Susceptibility to sensuous impressions is intensified by coffee; the faculty of observation is therefore increased, while that of judgment is sharpened, and the enlivened imagination causes the perceptions more quickly to adopt certain forms; an activity of thoughts and ideas is manifested; a mobility and ardour of wishes and ideals, which are more favourable to the shaping and combination of already premeditated ideas, than to a calm examination of newly originated thoughts.

"Coffee, taken in excess, causes sleeplessness and a state of excitement similar to intoxication, in which images, thoughts, and wishes rapidly succeed each other. A sensation of restlessness and heat ensues, together with anxiety and dizziness, trembling of the limbs, and a strong desire to go into the open air. Fresh air is commonly the best means of throwing off this condition, which, whilst it continues, exercises a really consuming power over man."

In Constantinople, the first coffee-houses used to be called schools of knowledge. Poets and philosophers assembled in these schools; their discussions and judgment alarmed even the ruling authorities; and the "schools of knowledge" were at last closed. In the seventeenth century, the coffee-houses in London had a similar fate; but the habit grew too strong for the state.

§ 74. *Wine, Beer, and Spirits.*—The immense variety of intoxicating liquors forms a striking illustration of the pains which men have always taken to excite the brain. Alcohol is the magic name with which the action of all these liquors is associated. Alcohol, a volatile substance, composed of carbon, hydrogen, and oxygen, may be produced from sugar; and thus, wherever the chemist finds an abundance of sugar, this discovery is ever closely followed by the proposition to transform the saccharine substance into alcohol. But long before the chemist could give his directions, palm-wine was in favour in Babylon; the Phœnicians and Greeks drank their wine, which their poets celebrated in songs. The Tartar sought intoxication in his kumiss, and Ossian praised the mead as "the power and glory of the shells"—the drinking-vessels of the Caledonians. In the juice of the palms and grapes, in milk and honey, in corn and potatoes, as well as in many fruits, this fermentable substance was found before the principle of fermentation and the nature of the process had even been divined.

Now we know that sugar is the fermentable substance, and that all intoxicating liquors must contain either sugar itself, or a constituent of sugar. When potatoes furnish alcohol, the starch has previously been converted into sugar; and before the kumiss is produced from milk, the sugar of milk has been changed into grape-sugar; for the latter only is directly fermentable. Starch and gum, cane-sugar and sugar of milk, are only indirectly so, being turned by the action of acids into grape-sugar. An albuminous body stimulates the fermentation: it is yeast. At a high temperature, sugar ferments under the influence of yeast. Air-bubbles rising in the juice, and a vinous smell, betoken that the effect of this agent has begun. The air is filled with carbonic acid, and the smell is occasioned by the spirit of wine, called by chemists alcohol, if free from water. The sugar is transformed into alcohol and carbonic acid, upon which the fermentation ceases. Diluted alcohol, or spirit of wine, is the intoxicating substance from which beer, wine, and brandy derive the name of spirituous beverages. Spirit of wine is contained in all fermented liquors. The proportionate quantity only is different; for while the lightest beer scarcely contains more than one-hundredth part, and the strong English ale scarcely more than eight-hundredths of alcohol, in wine its proportion rises to from seven to twenty-six per cent.; and in the strongest brandy this quantity is surpassed twofold. In the latter the alcohol of the fermented juice is concentrated. A high temperature evaporates the spirit of wine, which is collected in the vessel of a still. The name of brandy (burnt wine) attests the assistance of fire, by which it was carried over into the vessel.

First of all, therefore, beer, wine, and brandy are different in their respective strengths, which is due to their proportionate amount of alcohol. The several concomitants dissolved in spirit of wine explain the rest.

§ 75. *Alcohol in Wines and Beer.*—The weak alcoholic solution of beer contains nearly the same proportion of albumen as is found in fruits, some sugar and gum, malic acid from the hop, sometimes also some lactic and acetic acid, products of decomposition of the sugar and alcohol; and in addition to these a peculiar constituent of the hops, soluble in water, and consisting of carbon, hydrogen, and oxygen, which from its bitter taste is called the bitter principle of the hop (lupulin), together with some essential volatile oil of hops. The combinations of potash, lime, and magnesia, with sulphuric and phosphoric acids, are associated with carbonic acid, which is contained in the beer the more abundantly, as the quantity of sugar which had undergone fermentation was smaller when the beer was put into the barrel. A frothy beer is often prepared by a certain addition of sugar in bottling, and better still by adding sweet fruits—raisins for instance—in which both the ferment and the principle which promotes fermentation, sugar and yeast, are present. High-dried malt gives to the beer a more or less brown colour; hence the difference of colour between ale and porter.

In wine, more alcohol and less water than in beer, are combined with sugar and gum, with resin and colouring matter, acids and salts. Even the white wines contain a peculiar colouring matter, yellow-like oil, whitened by acids, browned by alkalies. The yellow tint of white wines, which after having been frozen have thawed again, is produced by the partial disengagement of the acid salts, which caused the lighter colour before the wine had frozen. The different kinds of grapes of a light or high yellow or copper colour, of a blue, red, or dark-brown tint, produce as many different colours in wine. The number of the peculiar colouring matters is probably smaller, as the acids and salts of the grapes alter the colour. By the action of acid salts, the blue colouring matter turns red. The particular colour distinguishing very old Rhine wines is occasioned by wax, which is united with the colouring matter.

The acids of the wine are the acids of the grape. Tartaric and malic acids—perhaps, too, citric acid—are sometimes accompanied by the tannic acid of the husk and by grape acid, being of the same composition as tartaric acid, with which it is associated in several kinds of grapes. As the fermentation has not ceased when the wine is put into the casks, all wines contain a certain proportion of carbonic acid; and the strong effervescence of sparkling champagne, depends upon the intentionally interrupted fermentation, which continues in the bottles.

The following are the salts contained in wine:—Cream of tartar, or acid tartrate of potash; tartrate of alumina, combined with tartrate of lime into a double salt; malate of lime; lime combined with phosphoric and carbonic acids, in addition to magnesia, iron, and manganese. In red wine there is a greater proportion of salts than in white; in both of them, the alcohol, augmented by the fermentation which still goes on in the casks, separates a part of the salts.

In all wines a peculiar ether is contained, upon which the odour generally depends. In this ether, the common chloric ether exists, in combination with a peculiar organic acid, which we may call the acid of wine-ether. The perfume, or "bouquet," is something different from the odour of wine produced by the wine-ether. This bouquet is extremely various. It distinguishes the Rhine wine, but in several wines it is not to be found at all. It is also produced by different kinds of ether, by ether combined with butyric, or valerianic, or acetic acid. The latter compound is said to be found in most of the Bordeaux wines or clarets. Like ether itself, which in its composition differs very little from alcohol, but surpasses the latter in volatility, the combinations of ether with organic acids are very volatile. Wine-ether and the bouquet are therefore recognised by smell.

The colouring matter and bouquet, the acids and salts, cause, by the difference of their composition, the variety of wines. The strength of wine depends upon its proportion of alcohol, and the sweetness of sack upon that of sugar. Spirits are obtained from corn and potatoes, from lees of wine and juniper-berries, from sugar and rice, from milk and fruits. The spirit of corn, or whisky, contains wine-ether and margaric-ether, oil of grain, and potato-spirit oil, which increases the pungent odour of spirits. Potato-spirit contains the same constituents, with the exception of oil of grain and margaric-ether. Brandy, or Cognac, is a mixture of water and alcohol with wine-ether and acetic-ether. The inferior sorts of Cognac contain also some potato-spirit oil. The fragrance of rum, which is prepared from the juice of the sugar-cane, is produced by a combination of butyric acid and ether. The molasses which remain after the preparation of sugar from the juice of the cane, yield the finest rum, called tafia, or ratafia. Arrack is obtained from rice; gin or Geneva from juniper-berries; kumiss (as already stated) from milk; and mead from honey.

§ 76. *Alcohol in Spirits.*—A good beer is as nutritious as fruit; the nutritious quality of wine is scarcely equal to that of sugared water; and spirits, in their proportionate amount of nutritious matter, do not even bear comparison with sugared water. Is it, therefore, a mere illusion that spirit compensates for the scanty food of the poor? And is the zeal, therefore, justifiable which attempts to check its use as an intoxicating poison—a destroyer of domestic peace and well-being? Such questions we often hear. Even medical men, occupied with the study of human physiology, have answered them in the affirmative: we shall reply to these questions by facts.

Alcohol, the essential constituent of spirits, and the most important substance in beer and wine, is not transformed into an essential constituent of the blood. It does not, therefore, effect any direct restitution, nor deserves the name of an alimentary principle. Still, it passes into the blood. By the oxygen we inhale, it is burnt in the blood into acetic acid and water, and finally into water and carbonic acid. But the oxygen, which decomposes the alcohol, is withdrawn from the albuminous and fatty substances of the blood. Alcohol, from its greater combustibility, protects these constituents of the blood from being burnt. And, if in addition to this, both special experiment and ordinary experience prove that alcoholic beverages diminish the quantity of carbonic acid we exhale—apparently from a great part of the inhaled oxygen combining with the hydrogen of the alcohol to produce water—there is a twofold argument for believing that alcohol moderates the combustion of the constituents of the blood, and therefore diminishes the first cause of the need of restitution.

He who possesses little, must spend little, if he wish to retain as much as another who unites abundance with liberality. Alcohol may be considered a savings-box of the tissues; but let the reader understand this expression rightly. He who eats little, and drinks a moderate quantity of spirit, retains as much in the blood and tissues as a person who eats proportionally more without drinking any beer, wine, or spirits. Hence it follows that it would be cruelty to deprive the working man, who earns his frugal meal by the sweat of his brow, of those means by which he is enabled to prolong the nourishment yielded by his scanty food. Or are we to abolish a custom which renders abuse possible, to degrade man by demanding of him to renounce an enjoyment, in order that he may not succumb to a bestial instinct? The monk who requires the vow of chastity, does not greater violence to the true nature of man, than the physician who would prohibit spirits because some men are drunkards. Goëthe has given to the new contemplation of the world the noble watchword—"*Memento vivere.*" He who preaches the renunciation of spirits, transfers us to the Christendom of the middle ages, which, by its motto of "*Memento mori,*" has stifled the fairest blossoms of humanity.

§ 77. *Fermented Liquors.*—Fermented liquors, taken in moderation, increase the secretion of the digestive juices, and promote the solution of the food. Taken in excess, they cause induration of the stomach; which destroys, together with the digestive powers, the formation of blood.

Beer, wine, and spirits accelerate the circulation, or, as people say, heat the blood. This acceleration is produced in a higher degree by wine than by beer—more by spirits than by wine; not merely because wine contains a greater proportion of alcohol than beer; and spirits still more than wine;—but in wine and spirits, the different kinds of ether, the oil of grain and the potato-spirit oil, all tend to quicken the circulation of the blood. The glowing cheeks and glistening eyes are produced by the capillaries of the cheeks and eyeballs being more filled, in consequence of the increased activity of the heart.

Together with the blood, the alcohol passes into the brain, which beyond all other organs is affected by its action. Spirituous liquors stimulate the imagination in particular. "In consequence of this faculty, the association of ideas is facilitated, and the memory sharpened. The susceptibility of the senses is also rendered more keen; the impressions are quickly and distinctly perceived. The judgment is formed with greater ease, as, from the lively idea and the excited memory, the facts from which it is derived are brought nearer together. Hence a frequent and surprising clearness and precision of judgment in matters not requiring a long and close examination. We speak foreign languages with more than usual adroitness. The facilitated movements of thought, the versatility of ideas, are associated with greater ease in the movement of all the voluntary muscles; the voice becomes fuller and stronger; the weariness and relaxation following the exertions of the body vanish. Thus arises a feeling of comfort and delight, of increased strength and freshly-armed courage, scaring away ill-humour, sorrow, and fear. The affairs of others find a greater sympathy and forbearance, which, in return, are likewise expected of those around us. We therefore talk with a greater self-confidence of ourselves; and not only of what has already been performed, but even future undertakings are spoken of with unreserved self-complacency.

"When wine or other spirituous liquors are taken in excess, hallucinations of the senses ensue; objects appear to the intoxicated person obliterated, blurred, or double; dark spots resembling flies and sparks appear before the eyes; there is ringing in the ears; he cannot hear distinctly his own voice nor the voice of others; crying instead of speaking, and involuntarily singing out of tune. At the same time the imagination produces uncertain, variegated, unsteady images, combined without regularity; the memory refuses its functions in the very act of speaking; the intoxicated man forgets what he intended to say, and thus the judgment is troubled and confused. Then follow the ebullitions of an unjust passion, and a sensibility to opposition, which is so much the oftener irritated as the disturbed action of the brain, over-loaded with alcohol, impairs the correctness of the judgment.

"Excessive indulgence in wine and all other spirituous liquors, causes sleep; if increased to a complete intoxication, the mental functions are disturbed to such a degree, that a condition of temporary insanity supervenes; the senses become blunted; the imagination produces the most various and irregular images, which the judgment is unable to examine, distinguish, or combine; all self-possession vanishes; at length even consciousness is lost, the intoxicated man becomes giddy, and finally falls into a profound slumber.

"But, before this, a sensation of lassitude and exhaustion arises; the muscles lose their elasticity; the features become flabby; the angles of the mouth are depressed; the pupils widen; and urine and excrements are involuntarily voided. The muscular coat of the stomach, urged at the same time by a pressure of the diaphragm and the abdominal muscles, often acts in an inverse direction; violent vomiting is a common phenomenon. Even the respiratory muscles are weakened, and often act irregularly, occasioning sighs or groans; the pulse becomes languid and slow. In addition to this, an increasing reluctance and uncertainty in all voluntary movements is observed; the tongue stammers, or speech entirely ceases; the head sinks, the arms hang down, the legs cross each other when walking; the drunken man staggers—his paces cannot maintain a steady direction—he totters and falls."*

We hasten from this disgusting picture, to describe, in the words of Hettner, the condition of a more moderate inebriation, intermediate between the former and perfect sobriety. "A beautiful, vigorous, full body," says Hettner, of an ancient work of art, "in the first freshness of a still juvenile manhood, with great, firm, swelling limbs; there he lies at his ease, partly leaning against a rock, his limbs partly outstretched. In this state of happy inebriation he has chosen this place of rest; now he has fallen asleep—arms and legs are hanging down; in his features are the indescribable expression of a half-intoxicated slumber. In a confused dream, the wakeful imagination is still dwelling with delight upon the past hours; but the weary, unnerved limbs cannot follow this sweet ecstasy. The eyes closed, the mouth widely open, this sleep is the state of that 'seedy' half-slumber, in which the internal heat and a thirst panting for water urge to awake; while the limbs, craving repose, desire to continue sleeping, and the mind is unconsciously apprehensive of the dreary, unrefreshed awakening."

ON CONDIMENTS.—§ 78. *Common Salt.*—When Cook and Forster, in 1772, visited the island of Otaheite, the natives were astonished, on account of the crew at dinner dipping every piece into "a white powder." At the same time they took their fishes and meat with a sauce of sea-water, containing common salt in abundance. It was a similar case to that of the Greeks and Romans, who were well acquainted with the alcohol in wine, but did not know the art of obtaining it pure and concentrated, by the process of distillation.

The addition of common salt to the food, whether in the form of sea-water, as a product from this, or of rock-salt, as dug from the mines, is usual everywhere. We must not wonder at this, seeing that common salt is contained in our blood in as great a proportion as it is scarce in the natural aliments; for in animal food the blood and cartilages are provided with the greatest proportion of common salt, and these are just the parts of flesh-meat which are taken in the smallest comparative quantity. In our vegetable food, potash so generally predominates over soda, that only a few plants of the sea-shore form an exception by their large proportionate amount of soda. Some vegetable aliments, turnips for instance, often contain no soda at all.

The addition, therefore, of common salt, consisting of chlorine and sodium, is usually an indispensable necessity; and as the vegetable aliments contain a smaller proportion of common salt than animal, the latter is generally mixed with less salt than the former. Hence, it becomes possible to some nations, as the Samoyedes, the Kamtschatdales, and some Indian tribes of North America, to eat meat or fish without any addition of salt. The greater the proportion of blood contained in meat, the more easily the addition of common salt may be dispensed with.

The common salt, however, of our kitchens is not a pure chloride of sodium. Rock-salt is generally the purest, as containing only some traces of chloride of potassium and chloride of magnesium, in addition to a somewhat greater proportion of sulphate of lime. But with bay-salt, a much greater proportion of chloride of magnesium and sulphate of lime, with a not inconsiderable quantity, besides, of sulphate of magnesia, or Epsom salts, are mixed. In common salt prepared from sea-water, chloride of potassium is entirely absent, or only small traces of it are found, which sometimes coexist with as small a proportion of iodine.

Common salt is as digestible as nutritious; for water dissolves it with great ease: and while no tissue of the human body can exist without common salt, the blood and the cartilages cannot keep up their regular composition without a considerable proportion of it.

If we also bear in mind that common salt promotes the digestion of the albuminous substances, and that the scantily soluble fats, when salted, become less difficult of digestion, the chloride of sodium appears to be the most important addition to food, rightfully meriting to stand at the head of the series of condiments. On account of its greater proportionate quantity of chloride of magnesium, sea-salt dissolves the albuminous substances more readily than rock-salt; for the temperature of the body is of itself sufficient to transform the chloride of magnesium into hydrochloric acid and magnesia; and every diluted mixture of hydrochloric acid and water is able to dissolve the albuminous substances.

Common salt is very useful for the preservation of aliments, by attracting their water, which more than any other constituent occasions putrefaction; yet it is important to know, that meat, by being salted, loses a part of its best alimentary principles. Together with the water of the flesh, albumen and kreatine, the lactic acid and the salts are extracted by the common salt. The brine thus formed is thrown away, and with it a part of the most soluble and most important substances of the flesh. This loss is somewhat compensated by the fibrine of salted meat being more soluble than fresh muscular fibres, as well as by the use of bread and vegetables containing chloride of potassium and phosphate of potash, which have been extracted from the meat, and substituted by the common salt.

By the excretions, common salt is abstracted from the blood. In the same proportion as the urine and the excrements, the mucus and the perspiration, the tears and the horny substances, have deprived the blood of common salt—the blood nourishing the nerves of the tongue is poorer in this salt. This is the reason why unsalted food tastes insipid. And even here the necessity of a law governs the seeming play of arbitrariness. It is the strictest consequence of cause and effect which unites the taste of the nutriment with the reception of the right alimentary principles. In former times, it was conceived to be a result of wise calculation, that just those aliments should have a better taste which contain the matters most requisite for the blood, and regarded as an inherent inclination in man to salt his aliments. "If it be considered more edifying," says Charles Snell, "to conceive of the Creator, so to speak, to be like a watchmaker who calculates exactly, than to regard Him as the organic, acting, productive reason of nature, we will not dispute any more." But then we renounce all certainty of knowledge; for it can scarcely be proved that it is part of the original tendency of our being to supply our blood with common salt; but the connection between differently mixed blood and differently feeling nerves, is perspicuous and evident.

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* *Physiology of Food*—Moleschott.

§ 79. *Butter and Olive Oil.*—In milk left undisturbed for some time, cells containing butter rise to the surface. From the stratum thus produced, by beating or stirring at a higher temperature, butter is formed. The force of stirring, supported by a moderate heat, causes the cells in which the butter has been included to burst; the disengaged fat collects into flakes, conglomerating into larger masses.

This origin explains why butter cannot consist of pure fat; the proportion of the fat in it does not amount to more than from six-sevenths to four-fifths of the whole, one-seventh to one-fifth consisting of water combined with some caseine and sugar of milk. The fat of the butter, however, is for the greater part real butter-fat, or butyrine. By this name the chemist designates a fat peculiar to butter, which, for a long time, was supposed to be the only one present in it; but at the most, this amounts to but two-hundredths of its weight. Butyrine becomes liquid at a very low temperature. It consists of glycerine and butyric acid, which is volatile in a free state, and possesses strongly the characteristic odour of butter. Three fatty acids besides, all volatile in an uncombined state, are, like butyric acid, united with glycerine, and therefore without odour in the fresh butter—namely, the caseic, sudoric, and capric acids.

The other fats of butter are oleine and margarine. As oleine is more fluid than the oil of almonds, the solidity of butter depends exclusively upon the margarine. At a very low temperature the oleine coagulates, and accordingly produces the greater firmness of butter in winter; which is also partly to be explained by the fact, that in the latter, oleine and margarine are contained in the proportion of one to three; while in the butter made in summer, the inverse proportion exists. Margarine melts at the temperature of 118° Fah., and therefore winter butter also liquefies at a higher temperature. But while melted butter is cooling again, the liquid fats separate from the coagulating margarine, and hence the butter assumes a taste disagreeable to most persons. But much more so is the decomposition of real butyrine, and the other neutral fats composed of the volatile acids and of glycerine; for if butyric, caseic, sudoric, and capric acids become disengaged, the butter assumes an offensive, pungent odour, and an acid taste, liked only by the Icelanders. The butter is then called rancid.

This decomposition is produced by the caseine and the water of the butter. Common salt, intermingled with butter, has the advantage of attracting the water and causing the caseine to be ineffectual; while by the process of melting, the caseine is skimmed off from the surface of the butter as a gray scum, and the water evaporates. For preservation, therefore, butter is generally melted and salted.

Olive-oil is commonly used as a condiment with salad. It contains nearly three-fourths of oleine, and very little more than one-fourth of margarine. In the oil of almonds, the oleine is more than three-fourths, and the margarine rather less than a quarter, of the whole.

Although these oils do not afford any neutral fat with volatile acids, yet they have the property of becoming rancid like butter; for, by the process of decomposition, both caseic and capric acids are formed out of oleic acid. This transformation, however, caused by the influence of oxygen, takes place only very slowly in olive-oil, as consisting entirely of mere fat; and olive-oil, as well as the oil of almonds, may be preserved for a very long time without becoming in the least rancid.

§ 80. *Digestibility of the Fats.*—Only that small proportion of the fats which can be saponified by the alkali of the bile, the pancreatic and intestinal juices, is to be considered soluble in the digestive juices. By far the larger part is so finely subdivided by the pancreatic juice and the bile, as to be able to penetrate the walls of the lacteals and the veins. Moreover, with respect to this fact, the fats are to be considered as difficult of digestion; they are far inferior to the constituents of fat in the capability of being dissolved, but superior to them in the greater conformity with the actual constituents of the blood.

The observation, however, which was made a little time ago, that starch is much more easily transformed into fat if taken with some fat than if alone, proves butter, as well as olive-oil, to be particularly useful. In bread-and-butter, therefore, we have the satisfaction of discovering a fresh and striking illustration of the truth, that ancient experience often leaves to science nothing to do but to establish and explain its laws.

Quite unjustly, therefore, are butter and oil said, without qualification, to be difficult of digestion; and the poor man has adopted the best substitute when he tries to supply the want of butter by grease. Butter on bread is a necessity recognised by impartial science; salad without oil is digestible only by herbivorous animals.

§ 81. *Cheese.*—Cream, milk, skimmed milk, and whey furnish severally cream-cheese, rich, meagre, and green cheese. In its abundance of fat, cream-cheese is the richest: while green cheese, prepared from whey, which contains only a very small proportion of fat, is the poorest of all.

The principal constituent of cheese is the caseine of the milk; according to the means by which this caseine has been coagulated, whether by rennet or by the lactic acid formed in the milk itself, the cheese is called sweet or sour-milk cheese. The action of the rennet (*i.e.*, the stomach of calves) is aided by a warm temperature; a high temperature causes cheese to become hard, just as the abundance of fat makes cream-cheese soft.

In milk, the fat closely adheres to the caseine; in cheese, therefore, the proportion of fat exactly corresponds with the relative quantity of butter. In addition to butter, the salts of the milk, and a part of the sugar of milk, are to be found in cheese. In real cheese the caseine and the butter are partly decomposed. Of the caseine is formed a nitrogenised substance, the "leacin," which we will denominate "cheese-white," as it crystallises in glittering white threads, and as this word recalls the chemical technical expression. Besides this substance, an oily acid is developed, not very readily soluble in water, which corresponds with the peculiar acid of the root of valerian, not only by its penetrating odour, and its strong, pungent taste, but also in its composition and all other qualities; it is therefore called valerianic acid. Some butyric acid besides is formed of the caseine; and hence we find in cheese which is several months old, more butyric acid than in fresh.

But the greatest proportion of the butyric acid originates from the decomposing butyrine; and the same is the case with the caseic, sudoric, and capric acids, associated in cheese with the valerianic acid.

A part of the whey is very often retained in the cheese; the sugar of milk of the whey is decomposed into butyric and carbonic acids, and the latter causes the holes to be observed in the Swiss and American cheese.

The odour and taste of the cheese is produced by the fatty acids, as well as by the valerianic, which is similar to them. Common salt decisively checks the formation of these acids from caseine and butter. The taste of many Dutch cheeses is less aromatic, from their being mixed with an abundant proportion of common salt; and as the decomposition of caseine and butter increases with their age, the odour and taste of the cheese is much stronger the longer it has been kept.

Though butyric acid may originate from three different substances of the milk—namely, from butter, sugar, and caseine—yet the butter contributes by far the greatest proportion of volatile acids. A rich cheese, like that of Limburg, smells therefore much stronger than the poor Marzalino, or the still poorer green cheese.

§ 82. *Digestibility of Cheese.*—In speaking of the fat contained in confectionery, we have already remarked, that the digestion of it is the more difficult in proportion as it has been transformed by a high temperature into its products of decomposition. The abundance of these products of decomposition is the cause of the difficult digestion which characterises cheese itself. But cheese impels the digestive glands to a greater activity; a greater quantity of saliva and bile, of gastric and pancreatic juices, is secreted and carried into the digestive canal; and hence the cheese, notwithstanding its difficult solubility, may be considered, in a restricted degree, as promoting digestion. And not for this reason alone; for the caseine, while in a state of decomposition, promotes also the conversion of starch and sugar into lactic acid and fat. Like butter, cheese increases the digestibility of the bread; and dry bread is, therefore, less nutritious than bread with butter or cheese.

With regard to the inorganic alimentary principles, the cheese made from sweet milk is superior to that from sour: in the latter the earthy salts have been dissolved; while the phosphate of lime, contained in milk, is abundantly represented in the former. Only where the breeding of cattle flourishes, and milk is produced in abundance, cheese is prepared; where cheese is prepared, meat cannot be deficient; a rich blood produces, together with the vigour of the muscles, the noble mind and the ardent courage of liberty. This is the association of thoughts which made John von Müller say, that liberty thrives where cheese is prepared.

§ 83. *Vinegar.*—That which renders vinegar so favourite an addition to food is a peculiar acid, consisting of carbon, hydrogen, and oxygen, formed from alcohol by the absorption of an additional quantity of oxygen. From *acetum*, the Latin name of vinegar, this acid is called the acetic, and is to be obtained from all spirituous beverages. But as alcohol yields, in addition to the acetic acid, a quantity of water, both malt and wine-vinegar contain, in proportion to the acetic acid, more water than is to be found in either beer or wine, in proportion to the quantity of alcohol. Of wine or French vinegar, about one-twentieth in weight consists of pure acetic acid. In an inferior vinegar this acid does not even amount to more than one-fiftieth to one five-and-twentieth in weight.

Vinegar, therefore, is always a considerably diluted solution of acetic acid, containing in addition a small proportion of albumen and sugar, of gum, and of several other organic substances, especially of some colouring matters, which differ according to the liquor from which the vinegar has been prepared. Thus, in wine-vinegar there is some acid tartrate and sulphate of potash to be found; and in wine and fruit-vinegars, a proportion of tannic acid, to be attributed to the husks and skins of the fruits. Acetic-ether, as passing over into wine-vinegar from some kinds of wine, communicates a fine agreeable fragrance to the vinegar.

If the acetous fermentation has not entirely ceased, the vinegar still contains a small proportion of alcohol, which, by a further accession of oxygen, becomes decomposed into water and acetic acid.

In order to increase the sour taste of the vinegar, the admixture of one-thousandth part of sulphuric acid is permitted in England. If added in a larger quantity the sulphuric acid is injurious.

§ 84. *Dissolving Action of Vinegar.*—The vinegar assists digestion. With the exception of legumin it dissolves the albuminous substances, transforming, in a short time, even the gluten and the fibrine into a gelatinous mass. Hence, vinegar and butter are useful concomitants of fish; and vinegar promotes, as we have before mentioned, the digestion of meat.

As acids are capable of transforming cellulose and starch into sugar, the vinegar added to salad is likewise to be regarded as an admixture promoting digestion. Thus, in the majority of cases, the use of vinegar is a custom founded on good reasons. Only in soups of peas, beans, and lentils, vinegar is to be rejected, as by it, even if added in excess, the legumin is brought into an undissolved state.

The dissolving action of the vinegar upon the other albuminous substances goes even as far as to the blood. Beverages containing vinegar have a dissolving effect on blood, and are cooling; and in milk, the proportion of caseine-cells, containing the butter, decreases if the mother take much vinegar.

And because of this solution of the most important constituents of the blood, manifesting itself by a greater liquefaction, it would appear an unpardonable frivolity, or a lamentable ignorance, in young girls, to endeavour, from vanity, to produce by means of vinegar an artificial thinness; only too often, in attaining their aim, they incur dangerous and deeply-rooting diseases, which deprive them of their more beautiful maiden bloom.

§ 85. *Sugar.*—Sugar has a similar history to that of alcohol and common salt. As many fermented beverages were known before the art of distilling, and the salt taste of the sea-water before the art of inspissation by boiling or crystallising; so honey and sweet juices were familiarly known before the sugar, which produces their sweet taste, was extracted. It was not until the fifteenth century that the inspissation of the cane-sugar was learned; and it was much later before this process was carried to perfection.

Not only, moreover, has the method of preparing the sugar made great advance in later days, but also the knowledge of the sources whence it is derived. The temperate zone possesses in the red-beet a substitute for sugar-cane, though far inferior to it; and for the same purpose the sugar-maple is used in North America. Although the red-beet, the sugar-maple, and many other plants possess the same description of sugar as the cane itself, no other plant has been found equal to the sugar-cane; and the denomination of cane-sugar will maintain itself as descriptive of the sweetest kind of sugar. Cane-sugar is sweeter, and poorer in hydrogen and oxygen, than grape-sugar, which is distinguished from the former, and from sugar of milk, by its direct capability of fermenting. Acids, however, transform the sugar of milk, as well as cane-sugar, into sugar of grapes; and hence, the two former can indirectly be brought into a state of fermentation.

By boiling with lime, cane-sugar is purified; for the saccharine vegetable juice contains a portion of soluble albumen, which is coagulated by this process, and then skimmed off. A liquid is thus formed, containing uncrystallisable sugar, consisting, for the most part, of the brown syrup of commerce, called molasses or treacle. In addition to this liquid, crystals of sugar are obtained, which, while depositing themselves, become mixed with uncrystallisable sugar, and represent the brown, the raw-sugar, or Muscovado. In a purer state of its crystalline grains, the cane-sugar is called "loaf" or "lump-sugar;" while sugar-candy consists of regular and larger-sized crystals, formed by a slower process. A part of the lime enters into combination with the sugar; and the Muscovado, particularly, is mixed with some phosphate of lime, and a viscous substance, probably gum.

In honey we find grape-sugar, an uncrystallisable kind of sugar, and "mannite," or manna-sugar. The latter is distinguished from the other kinds of sugar by not being fermentable, and by containing a smaller proportion of oxygen than hydrogen; these two elements are present in the other sorts of sugar just in the same proportion as that in which they are contained in water. With these different kinds of sugar there is, in honey, some lactic acid most probably associated, which is formed by the decomposition of the grape-sugar, and some wax possessing a great similarity to the fatty substances.

§ 86. *Nutritive Qualities of Sugar.*—As by salts and acids, so likewise by sugar and honey, is the quantity of the digestive juices increased, and the digestion promoted. And the sugar, while being digested, enriches the gastric juice with a substance which assists in dissolving the aliments; for the sugar, on coming into contact with the saliva, has been partly converted into lactic acid, which acts upon the alimentary principles in the same manner as the hydrochloric acid of the gastric juice.

For this reason, sugar at once appears infinitely better than its reputation. Since the composition of the milk has been recognised, the sugar ought to have been acquitted of the bad repute which adhered to it for many centuries. A slander always leaves something behind; and even to the present time, the popular belief that sugar injures the teeth, is as widely spread as, on the counter-testimony of both experience and science, the opposite doctrine ought to be: for the teeth of the negroes in the West Indian colonies are of a bright white; and that which is thus proved by a whole community, remarkable for the abundance of sugar consumed among them, the example of many individuals nearer home amply confirms. Phosphate of lime is the chief constituent of the bones and teeth, but not before adult age; and an increase of the phosphate of lime is the essential characteristic of the development of the bones of children. Lactic acid dissolves the phosphate of lime of the food; and as sugar indirectly supports this solution, it facilitates the conveyance of the lime to the teeth. To this it must not be objected that sugar causes pain in a hollow tooth. Like sugar, a thousand other substances irritate the nerve; but who seriously believes that that is necessarily injurious to the healthy, which gives pain to the diseased?

The prohibition of sugar to children, therefore, is indefensible. Sugar is not dangerous to the teeth, but, on the contrary, assists in providing them with lime; it is useful to the stomach, if it do not, by being taken in excess, produce too great a quantity of lactic acid.

§ 87. *Spices.*—Under the name of spices we shall comprise mustard and carraway seeds, common and Spanish pepper, cinnamon and cloves, nutmegs and saffron. In addition to albumen and wax, cellulose and gum, starch and resins, acids and salts, which manifest unimportant differences with respect to their proportionate quantity and properties, all these spices contain a volatile oil of a pungent odour, and an acrid or aromatic taste.

It is only in the mustard, as we take it, that this oil is a product of fermentation. Just as the ferment of almonds (emulsine) transforms the principle of almonds (amygdaline) into volatile oil of bitter almonds, so a peculiar ferment of the black mustard decomposes a certain constituent of the latter, which is not to be found in the white mustard, into the volatile oil of mustard, which contains, besides carbon and hydrogen, a proportion of sulphur and nitrogen. Of a light yellow colour and a pungent odour, the oil of mustard is soluble only in a large proportionate quantity of water. To this oil the mustard owes its peculiar taste; the mother-body of the oil of mustard is of a bitter taste. The black mustard contains a portion of stearic acid and two other fatty acids; which last are likewise to be found in white mustard, but without stearic acid.

The oils of the different spices above enumerated are to be found, ready formed, in the vegetable substances; rather aromatic than acid, they consist partly of carbon and hydrogen, and partly of a combination of these two elements with oxygen. With fatty oils, the volatile are by no means to be confounded. The latter, though scantily soluble in water, are not entirely insoluble in it; cannot be saponified with alkalies; and are especially remarkable for their disposition to turn into resins by the absorption of oxygen. There exists no fat without oxygen; while several of the volatile oils contain nothing of this element.

Cloves are furnished with the greatest proportion of volatile oil; while its relative quantity in the more acrid nutmeg scarcely amounts to one-third; in the very acrid Spanish pepper not even to one-fourth; in the hot French cinnamon to one five-and-twentieth only of the proportion contained in cloves. Hence it follows, that in these spices, the degree of pungency of taste is dependent, not on the quantity of the volatile oil, but on its quality.

§ 88. *Heating Effects of Spices.*—In the first instance, it was, perhaps, only the pungent taste of the aromatic oils to which the heating effect of spices was ascribed; but the excitement and palpitation produced by their use, show that they accelerate the circulation. Not only, moreover, are the palate and stomach heated by direct irritation, but the cheeks also become tinged with a rosy glow under their influence.

These spices, by irritating the digestive glands, are also capable of promoting, to a certain degree, the solution of the aliments. Hence, the blood is not only provided with a heating oil, but also with an abundant supply of restorative substances; and the nutrition is increased at the same time.

Thus sleep also is prevented, and the passions are aroused. All nutriments causing sleeplessness irritate the brain, and accelerate the movements of thought. It is, therefore, not a mere fiction, that mustard for the moment quickens the action of the memory: just like the movement of the muscles, memory is an action of the brain; it is capable of being called into exercise like every other manifestation of bodily power. But the action, as an attribute of matter, is confounded with the instrument itself, if we say that mustard produces a good memory. It is true the instrument cannot exist without matter; moreover, it is not the bulk of the substance, but the celerity and force of its motion, which is to be considered the essence of increased action. The motion ceases with the matter which excites it. When, therefore, the volatile oil has disappeared from the blood, the excitement produced in the brain is over likewise; the mustard cannot, therefore, be an instrument of the memory. We repeat it—memory is a form in which the action of the brain manifests itself. But to call the brain an instrument of memory would lead to a greater confusion of terms, than if we were to denominate the muscle the instrument of speed. The brain thinks just as the muscle moves; sensation and will, recollection and judgment, are different forms of thinking, just as stretching and shrugging, trilling and grasping, are different forms of motion.

The spices cannot be called restoratives, as their most essential substance does not convey to the blood an essential constituent; they are stimulants; and whatever stimulates, renders over-excitement possible. An excess of stimulants, in the strictest sense, is therefore much more dangerous than a superabundance of nutriments. Sugar and common salt, butter and cheese, oil and vinegar, are far superior to the spices with regard to the service they afford to the body. When we consider the restless passion, the choler and insidious jealousy of those inhabitants of the tropics who take so great an abundance of spices with their food, we find it impossible to forgive the cruelty with which the Europeans of a former day sought to enrich their own part of the world with pepper and cinnamon, cloves and nutmegs. Had they never possessed any of these spices, the people of Europe would have one superfluous and often obnoxious addition to their food the less; and the Spaniards, Portuguese, and Dutch would be able to erase a bloody page from their history.

In the preceding pages, we have given so much of Professor Moleschott's remarks as exhibit the astonishing variety of combinations and their uses, of which oxygen, chlorine, fluorine, nitrogen, hydrogen, and carbon, are the subjects in the animal economy. Our readers will have learned how numerous are the changes of chemical compounds and products which a few elements are capable of producing; and how rapid and unceasing those changes occur in connection with vitality. In future pages we shall extend our observations to similar results observed in connection with vegetable chemistry; which, although having less direct connection with the chemical laws of our own existence, are only second to them in philosophical and practical interest. It would, however, be unjust were we to withhold that portion of the professor's paper which shows the application of the principles already laid down. It will assist to popularise the subject of chemistry with those of our readers who simply study the science as a branch of a literal education, or as a subject of general interest and importance. After introducing Dr. Moleschott's remarks on Diet, we shall then proceed to give the natural history of the remainder of the elements; the relationship of which generally exists in connection with the mineral kingdom, with the exception of instances referred to in our previous pages. Dr. Moleschott continues as follows:—

Of Diet.—§ 89. *Conditions and Definition of Diet.*—Real equality is shown in differences. We all are equal, only because even the slightest difference of circumstance produces a difference in the composition and functions of our bodies: another mode of life, another kind of food, another climate and soil, produce a different blood and brain. And, indeed, when from the icy north, up to the pure glowing sky of the tropics, so rich in flowers, we pursue the manifold gradations which act upon man by the boldness of contrast, as well as by the abundance of varying transitions—heat and cold—mountain and valley—sea and land—forest and steppes—animals and plants—we are no longer astonished that many refuse to recognise, in the changeful and multiform human species, the children of one ancestor, seeing that form and colour, as well as mind and morals, multiply distinctions in such prodigal abundance, as to render the very conception difficult.

Whatever, in their numberless shades of form and composition, the earth and the water produce, exercises its influence upon the food of man. We no longer fear to give offence by designating food itself as one of the most important sources of differences in our species;—so much the stronger emphasis, however, must we give to the assertion, that no influence exists, as an isolated one, in the history of our eternally progressing life.

Goëthe, with reference to the important axiom of Haman—that everything isolated is objectionable—says, in a manner at once so true and consolatory in all the incompleteness of human intelligence and expression—"In every communication of thought, if not a poetical one, there exists one great difficulty; for the word must detach itself, must become isolated, in order to say or to signify something. By speaking, man necessarily for the moment becomes one-sided; there exists no communication, no doctrine without separation." On the ground of this necessity, it is to be explained why we have left in the background the equally powerful influences of the air and the soil; of Nature as it surrounds us in the solitary forest, in the undulating mountain-region, in the wearisome monotony of vast plains, or in the awe-inspiring boundlessness of the tossing sea; why we have omitted to mention the intimate relation between plants and animals, the effect of the intercourse of man with man—now elevating, now degrading, often the source of the highest pleasures, but often, also, of the keenest wounds; why we have not specified the power of oral instruction, the magic of song—in order that we might speak with a greater emphasis of the far-reaching influence of food. Under the united influence of all these agencies, man becomes of necessity the being he is, comprehending the external world in just so many various aspects as the modes in which the latter acts upon him.

We have treated one part only of this series of food; but certainly not with the intention to remove a single link from the chain. The sea bordering the earth, and the earth the sea, both affect the condition of plants and animals; and these, again, occasion infinite diversities in the aliments of man, according to the climate. The effects produced by food upon man determine the commerce and the character of the people, as well as the individual. But commerce again alters man, man the food, and food the fields; everywhere action and reaction.

And to this reaction, the power of which is the surest and most concise expression of the heaven-storming reason of the race of Prometheus, man owes the pliability, so tenacious and yet so flexible, by which he becomes indigenous in every climate of the vast domain of nature. If he approach the north pole he contents himself with fish; while in the tropics he enjoys the fragrant fruit in addition to the savoury game; and if in the plains of North America, the huntsman lives on the flesh of the buffalo alone; while the New Hollanders, possessing on their vast island no edible fruit, not even the size of a cherry, take animal food exclusively: in the tropics, where meat taken in abundance is injurious to health, a vegetable diet prevails, to which, indeed, the inhabitants of Pegu and Malabar, from superstitious motives, restrict themselves.

Our blood is intermediate in its properties between that of the carnivorous and the herbivorous animals; but it is not in the blood that the difference begins, in the first instance, which distinguishes us from the animals that live on an exclusive diet. In our digestive organs themselves, a compromise has already been effected; and those differences present themselves, which are so sharply marked in the formation of these organs in the animals feeding exclusively on animal or vegetable food. If the herbivorous animal be enabled, by numerous and variously-formed teeth, to gnaw and grind, and by a long digestive canal and large salivary glands, to digest substances which could not be sufficiently comminuted by the less-developed and sharper teeth of the beast of prey, nor dissolved by their smaller salivary glands and shorter intestinal canal; in man we observe, in the structure of the teeth and of the jawbone, of the stomach and of the intestinal canal, of the salivary glands and the muscles of mastication, a proportion intermediate between the two.

Man, therefore, is able to digest both animal and vegetable food; and as there exist vegetable as well as animal aliments in which all three groups of the simple alimentary principles are to be found—as both bread and meat are able to convey to the blood its requisite elements—the native of Pegu gets accustomed to an exclusive vegetable diet, just as many North Americans live on animal food, and the Greenlanders almost entirely on fish.

Where both these products of nature—the gifts of the field and the gifts of the wood—exist in abundance, the taste selects from both sections some important representatives of nutriment; and if we denominate cattle and swine the domestic animals of man, corn and leguminous plants ought, with equal justice, to be designated his domestic fruits.

The more civilised that nations are, the more perfectly is the cultivation of domestic animals and fruits developed among them. Where agriculture as well as the breeding of cattle flourishes—or, to speak more correctly, where a thriving flock increases the fertility of the fields, and a productive agriculture the opulence of the stables—man takes for his nourishment meat and bread, milk and fruits. While digesting both with ease, the composition of his blood conforms to the digestion, just as digestion depends upon the structure of its organs; and if in man the digestion and formation of blood corresponds with his mixed food from plants and animals, will not a nutrition afforded by this mixed diet, just as it produces muscles and bones peculiar to man, develop also the brain, which, as really human, thinks and feels?

Equally remote from the savage hunting-tribes, kindred with that of beasts of prey; and the vegetable-eating Hindoos, with their greater mental indolence—not digesting in order to live, but living almost in order to digest—stands the cultivated European, who digests his mixed diet with ease, and from whose blood is evolved a brain, the action of which we admire in those forms where human beauty and wisdom are embodied.

This circulating mutual action, which on all sides unites man with nature, is continually recurring: the difference produced by the degrees of this mutual action causes the peculiarity of the individual. After a brief description of the change of matter in the tissues, as it generally takes place in the human species, we passed to the description, in general outlines, of the action of the several aliments. As sex and age, position and mode of life, habit and climate, exert various influences, we know but one-half of the science of food, so long as our acquaintance is limited to the metamorphosis of tissue in the species, and to the general action only of the aliment. The remaining half consists in describing the manner in which the tissue-change, as modified in the individual, determines the selection of the aliments. This selection constitutes diet; and hence there remains, in this book, for us the task of determining the diet in the principal circumstances of individual life.

§ 90. *Breakfast, Dinner, and Supper.*—Although during sleep the metamorphosis of tissue is less rapid, yet the eight or ten hours which commonly elapse between supper and breakfast, are sufficient to produce all those phenomena of sanguineous impoverishment comprised under the name of hunger. Hence, the meal breaking this state of fasting is called breakfast.

To the breakfast succeed the anxieties and labours of the day. Bread, therefore, or bread-and-butter, in the morning is a very proper food, as it is digested with sufficient ease, and yet so slowly as not too suddenly to overload the blood and brain with alimentary principles. Tea or coffee suitably prepares for the attention and exercise of judgment, and is more or less required by the labour of all classes. And from the consideration that tea more especially promotes the combination and the development of thought, while coffee excites the imagination likewise, a rational explanation is obtained for the fact, that in some countries, as for instance in Holland, those classes of society whose labour is chiefly that of the intellect, take tea for their breakfast, while those which employ both judgment and imagination take coffee.

§ 91. *Combination of Meat with Vegetables.*—Soup, vegetables, and meat, are the regular constituents of a European citizen's dinner. We have already explained how the daily taking of soup, together with the meat, is necessitated by the general custom of preparing soup from the same meat which is eaten when boiled. In the same section we showed the combination of meat with vegetables or edible roots to be a custom founded upon reason. Meat furnishes what is wanting in vegetables, and the vegetables dilute what meat contains in too great an abundance.

It is therefore an unfortunate custom, or a still more unfortunate necessity, that by so many poor families, potatoes, exclusively, should be eaten for dinner. When circumstances render it impossible to eat any meat with vegetables, the dinner ought to consist, as often as practicable, of leguminous seeds, the best of soups being from peas, beans, or lentils. Hence it follows again, that the distress of the poor might be essentially alleviated by the cultivation of the leguminous plants on so many acres of soil, where potatoes now grow or perish by a destructive disease.

In those families, however, which have meat on their table some days of the week at least, the meat ought on such occasions to be dished up together with potatoes or vegetables; while on the other days, the nutritious soups of peas, beans, or lentils, should be used. Such an alternation ought to be laid down as a general rule; for however correct the taste of certain housewives may be, in the selection of viands, it happens very often, in the more opulent households of the middle classes, that the meals are very improperly composed. How often does it occur that a meagre soup is followed by nothing but fish and potatoes! Or, how often is some kind of soup, together with a farinaceous dish, eaten without any addition of meat! Or a soup from some of the leguminous seeds is taken, and leguminous seeds or farinaceous dishes follow.

Whenever possible, meat ought not to be absent from the dinner-table; or if the less readily digestible and less nutritious fish be substituted, an easily digestible and substantial broth, or a nutritious soup of peas or lentils, ought to compensate for the defect. But if beans and farinaceous dishes, or soups of the leguminous seeds, constitute the whole meal, these most nutritious aliments will, by their difficulty of digestion, oppress the stomach, overload the blood, heat the head, and render it less fit for the ensuing work.

Pea-soup, therefore, with fish and potatoes, or fish followed by farinaceous dishes, or meat and vegetables, or broth, leguminous seeds and potatoes, or roast meat and salad, are suitable combinations for a good meal. In short, the more easily digestible albuminous substances ought always to predominate; and when aliments which are more difficult of digestion form the chief constituent of a meal—when peas, beans, or farinaceous dishes are taken—then soups containing vegetables or fruit, ought at least to be added, in order to promote digestion by their admixture of acids or salts.

But not alone are the right representation and combination of alimentary principles to be considered in our meals. Whatever impels our body to a new activity is a stimulant; and hence, in a wider sense, not only the heating spices, but solid and liquid food in general, are to be regarded as stimulants. It is characteristic of a stimulant, that its effect ceases when its use is repeated at too frequent intervals; and the slighter the irritation, the more readily will its effect become blunted, if the regularity of its influence render the peculiar effect less perceptible; a diminished susceptibility to the irritation, in consequence of repeated use, establishes the habit; while, with many kinds of solid and liquid food—for instance, with bread, meat, potatoes, water, milk—our insensibility to the stimulus does not attain a higher degree than that of becoming accustomed to it; and while even stronger stimulants, like tea and coffee, beer and wine, are overcome by habit; with other aliments, the insensibility overpowers the habit, and almost produces disgust.

While soup and meat, if taken day after day in the same form, at length become repugnant, it is equally impossible to eat every day the same kind of vegetables, the same farinaceous seeds; if carrots or saurkrout have to be partaken of even for two or three days in succession, the housewife may expect to meet with dissatisfied faces. This does not proceed perhaps from any bad state of the organs of taste; but taste is a very appropriate index of the irritation which the brain and nerves receive from the blood. We know accurately, that meat taken in abundance, conveys more fibrine to the blood, and the starchy aliments more sugar; and that the volatile oils of the superior kinds of fruit and of spices, the organic acids and the several combinations of the different salts of the vegetables, pass into the mother-juice of the tissues. And if in animals the monotonous manifestations of strength correspond with their uniform food; if even in plants the want of sensation harmonises with the simple composition of their not very numerous alimentary principles; must not the strictest logical sequence of cause and effect trace the agitated life of man, with his actions and passions, and all the numberless shades of sensation and thought, in great part to his variety of solid and liquid food and condiments? That there exists individualism in character, becomes possible only through the co-operation of the various aliments, together with the numberless moral and sensible influences which the surrounding material world exercises upon him.

And as the uniformity of the stimulant, even if repeated at longer intervals, is prejudicial to its effect, a regular arrangement of dishes, repeated certain days every week, is a custom not to be commended. If a stiff regularity only too clearly betray a commonplace narrowness of mind, such a regular repetition becomes a source of petty formalism; insensibly, but all the more dangerously, repressing the free movements of the mind. Whoever has watched himself with attention, will often enough have experienced how the refreshing and stimulating effect of a walk is evidently lost if taken for a long time daily at the same hour. It is the same with uniformity in viands; and while the ancient physicians used actually to assert it to be useful sometimes to throw the body out of order, in accordance with this doctrine it is perfectly true, that an inflexible regularity of life is by no means compatible with a genial freedom.

Not seldom do we hear the opinion advanced, that drinking during a meal is an obnoxious habit; but quite wrongfully; for the gastric juice may be diluted with a considerable quantity of water without losing its dissolving power in the slightest degree. Only a superabundance of water would diminish or arrest the peculiar action of the matters contained in the digestive fluids.

Large draughts of water, therefore, would be the most injurious with aliments difficult of digestion, like the fats; and hence, drinking of too much water after fat pork, for instance, is properly avoided; but, in countries where soup does not constitute a regular part of the meal, drinking water is positively to be recommended.

Beer and wine at dinner are also hurtful only if taken in excess; for, in the latter case, the alcohol coagulates the albuminous substances not only of the food, but also of the digestive fluids, and thus disturbs digestion. If taken in a moderate quantity, such beverages are calculated to cause the meal to hold out longer; for the fact that we are not so soon hungry again after taking a meal with wine, than if we have taken only water with it, is to be accounted for by the slower combustion of the constituents of our body, because the alcohol we have imbibed, takes possession of the inhaled oxygen. Hence, wine with a meal is extremely useful when a long journey, or work in hand, renders it impossible to take food again at the usual time; so much the more so, as such detention from food itself usually causes an acceleration of the metamorphosis of the tissues, which beer and wine efficiently obviate.

The general preference of warm food at dinner is based on rational grounds; for a low temperature coagulates gelatine, and the fats coagulate, which are much more readily digested in a liquid state. But when the food is much cooler than the liquids of the stomach and intestines—that is to say, much lower than 98° Fah.—it subtracts from the latter a part of their warmth, and the colder combination effects the solution less readily. Ice, therefore, as well as very cold water, are injurious, particularly if the nutriments contain a large proportion of fat or gelatine. As great changes of temperature in our body in general are not easily endured, a sudden transition from warm food to cold, or *vice versâ*, is injurious. By the sudden refrigeration of the heated cavity of the mouth, the enamel of the teeth is said sometimes to have actually cracked.

From the different hour at which the principal meal is taken by different individuals and classes, it sufficiently follows that no absolute rule can be given for it. This time is of little consequence to the classes having an occupation chiefly mental, and of none to those who do not work at all, provided, at least, they do not so utterly subvert the natural order of life, that night becomes day, and day night. Those classes, however, which are engaged in fatiguing, bodily exercise, expend, during the first six hours of the day, so great a quantity of material, that almost everywhere they observe the custom of taking at noon, or at least not much later, their largest portion of restoratives.

§ 92. *Suppers.*—The German custom of taking supper two or three hours, or even longer, before going to bed, has this great advantage—that digestion is almost at an end before going to sleep; for digestion disturbs sleep, and sleep digestion. Hence, supper should consist, as much as possible, of easily digestible aliments, as soups, salad with little meat, and not of fish or leguminous seeds. It is only when supper is taken very early, that the less digestible bread, or, still better, bread-and-butter, and meat, are proper food. When tea is taken in addition, those who are accustomed to perform close mental labour after supper, are agreeably excited. Intemperance is, most of all, to be avoided in the evening; for, apart from the sleep being disturbed by digestion as well as by hunger, an overloaded state of the blood is less easily adjusted during the night. In the night, less carbonic acid is exhaled, and the tissue-change in general retracted. Hence, an overladen state of the tissues, and especially of the brain, is very often manifested in the night by heavy dreams and nightmare, and in the morning by headache and a general state of ill-humour.

§ 93. *Diet of Childhood.*—In the form of a general proposition, and with application to special cases, we have repeatedly insisted upon the principle, that the amount of restoratives needed by the system, depends upon the extent of the expenditure. An abundant supply, accompanied by considerable expenditure, occasions an active metamorphosis of tissues.

Although the differences in human beings, as modified by age and sex, generally confirm this proposition, so that in a healthy man the activity of the tissue-change may be estimated by the quantity of the outgoings, still it is precisely here that we find, at the first glance, the most important exceptions. In the adult, the weight of the whole body remains from day to day the same, allowance being made for the augmentation or diminution most recently produced either on the one hand by a meal, or on the other by excretions of different kinds; for here, in general, the restoratives exactly repair the loss of the excretitious parts.

Not so with the child: for that the infant grows to a boy, the boy to a youth, is entirely and solely effected by the receipt surpassing the expenditure. The balance is not even in the exchange of the body. We have not to deal with a simple tissue-change. Growth consists in nothing else but in the proportion in which the formation of tissue exceeds the products of regressive transformation. The superior activity, however, of nutrition over excretion is the cause only of growth. The assimilation of the body in a greater proportion than it expends, is the condition without which it cannot grow. Strictly speaking, this greater appropriation of alimentary principles is growth itself. But its cause has to be more profoundly sought. The blood and the tissues of the infant are of a different composition from those of the adult. When treating of animal food, we mentioned that the muscles of young animals contain a greater proportion of albumen, but less of fibrine, than those of the full-grown. The real skin, in the adult consisting of fibres yielding gelatine, is formed in the infant from an albuminous substance, which, however, is not entirely absent from the skin of the adult. The bones of the infant yield gelatine of cartilages or "chondrogen;" and the organic basis is by degrees only changed into the tissue yielding gelatine of bones, or "collagen." Thus, while in youth, the solid parts contain a greater proportion of water, but less in weight of the formative substance deposited by the mother-juice; the tissues of the adult are, on the contrary, remarkable for their abundance of solid substance. As these solid substances are mostly heavier than water, their augmentation during the period of growth, explains the increase of the body in weight.

But if the body of the child have another composition than that of the youth, we must not be satisfied with the fact that the quantity by which the receipts surpass the expenditure, effects the growth. Tissues of a different composition likewise possess a different kind of attraction for the matters conveying the nutriment to the blood.

The muscles of the child attract fibrine from the blood, and their albuminous substances decompose into kreatine and urea; and the important consequence of their composition differing from the muscles of the adult is, that the fibrine attracted, surpasess in quantity the products of decomposition taken from them by the retransformative process. The same change takes place with the skin, as its albuminous substance is decomposed and lost with the urine and the exhaled air; while, with a superior attractive power, it forms from the blood, and assimilates the organic matter of its gelatinous fibres. But the remark is chiefly applicable to the bones; for just as these, more than any other tissue, cause the increase in weight of the whole body, so do they attract, in return for the wasting matter (known by yielding, when boiled, gelatine of cartilage instead of bone-gelatine), such an abundance of the true bone-constituents, that in a very short time their composition entirely changes. While gelatine of cartilage possesses a strong affinity for common salt, the tissue yielding bone-gelatine has the greatest tendency to combine with the phosphate and carbonate of lime. With the growing preponderance, therefore, of the bone-gelatine over the gelatine of cartilage, the lime-salts are continually increasing, and that during the whole life. The bones readily collect the lime-salts and the fluoride of calcium from the blood, a function shared in equal degree by the teeth only.

This strong attractive power, peculiar to the infant and child, is more than the condition—it is the real cause of its growth. It shows how the nutrition is in proportion to the excretions; a circumstance which we have before shown to be the condition of growth and increase of weight.

In the infant, therefore, change of matter only follows the same law observed in the adult. In both, the attractive power of the tissues towards the constituents of the blood induces nutrition. If, from the peculiarity of their composition, the tissues attract and detain a greater proportion of the blood than is lost by excretion, the blood, in order to maintain its composition, has to receive a larger quantity of the alimentary principle than would have been necessary if the activity of the nutrition had been simply equal to that of the excretion.

The direct expenditure of the blood upon the tissues, therefore, determines its supply; only the different composition confers upon the freshly-formed constituents of the tissues a greater constancy; and for this reason the body appropriates more than it eliminates.

This explains why children take proportionally more food, and especially why they take it oftener than adults, although the latter discharge a greater quantity of carbonic acid and urea than the former; and thus the regularity of the phenomena establishes the law—as it can be proved that the exception was only apparent. A law has its full validity only when the exception which withstands its authority merges in the law, as soon as the conditions of the seeming anomaly have been more exactly ascertained.

Philosophically, we come to the conclusion that the supply is regulated by the expenditure, and that in the infant, as in the adult, the exchange of the constituents of the blood is rightly denominated the change of matter. Practically, it results that the child's frequently recurring desire for food does not arise from mere *ennui* or restlessness, but from processes going on in the interior of the body. Hence the rule that the infant should not be trained by the power of habit to a diminished and less frequently manifested desire for food, but that its instinct ought to be satisfied; for such is the general and natural expression of a law regulating the peculiar change of matter, which causes the inclinations of the child to differ from those of the adult.

It is, therefore, quite proper that, at first, the mother should give the breast to the nursling as often as it awakes. Gradually the child desires milk less frequently; and in the distribution of the meals for her child, the wakeful eye of the loving parent may trust to her own observation. On an average, the child, after being suckled, can wait three to four hours until the next meal. Very healthy and strong children frequently sleep even for six or seven hours together, without being awakened by the want of food.

The child, after weaning, does not require any food at night; and the same rule holds good for boyhood. During healthy sleep, the change of matter in children is moderate. This explains why even those whose want of food is generally so great, can easily wait from supper to breakfast without eating anything; while, during the day, the desire of food has to be frequently satisfied. It is a very wholesome custom to give to children a slight repast between breakfast and dinner, and another between dinner and supper—nothing, indeed, tends better to temperate habits; for, in order to thrive, they have to convey to the blood the same quantity of nutriment, whether they take it at three or five times. In the latter case, the stomach is less gorged at any one time; digestion and the formation of blood go on with greater ease; and in the process of nutrition there is less danger that too great a quantity of nutriment should be conveyed to the tissues, which might have an injurious effect, especially upon the brain.

On the other hand, children are very materially injured by being gratified whenever their excitable palate asks for food or dainties; for, as the formation of all secretions requires due time, just as the development of the ovum, &c., and the accumulation of milk, are bound to certain periods, so the digestive fluids—the saliva, for instance, the gastric juice, the bile and pancreatic juice—are only secreted in a sufficient quantity for the usual meals. We must, therefore, allow the digestive glands the needful time to prepare and collect them, between one meal and another. If not, the requisite power of digestive action is deficient, just at the time when the most nutritious aliments, soup and meat, are taken. At table, if children complain of want of appetite, and if, accordingly, they do not eat, the blood becomes deficient in the best alimentary principles; if compelled to eat, on the other hand, imperfect digestion makes them languid and thin.

§ 94. *Milk as Food for Children.*—That milk is the most appropriate food for children is so generally recognised, that science has nothing to do but to confirm and interpret the fact.

Combining in a due proportion solid and liquid food, milk contains not only in the caseine an albuminous substance, which is transformed into albumen and fibrine, afterwards into gelatine, horn, and elastic fibre; but it has also, in its peculiar sugar, one of the most easily digestible constituents of fat; and in butter there are the ready-formed fats, which assist in forming the soft cushion of round limbs and full cheeks in children.

But it is the phosphate of lime, so abundantly contained in milk, which particularly constitutes this the most fitting nutriment of the babe. In milk, more than in any other food, the conditions are furnished for the conversion of the cartilages of the infant into bones. The phosphate of lime, so constantly associated with the caseine, is easily dissolved by the lactic acid, into which the sugar of the milk has been transformed by the bile; and thus the dissolved lime-salt passes from the digestive canal through the blood into the bones. The phosphate of potash performs the same office for the growing muscles.

In the milk of animals we find the same constituents as in human milk. Can, therefore, the latter be substituted for the former?

A direct comparison of the milk of the woman with that of the Mammalia, decides the question in the negative; that of woman contains a much smaller proportion of caseine, a smaller proportion of butter and of salts, but a greater quantity of sugar of milk and of water than is found in the milk of cows.

But whoever has attentively followed us in the description we have given of the change of matter, will not see anything fortuitous in this difference of composition. Here, as in many other cases, the manifold links are wanting, which enable us, step by step, to prove the differences of power to be nothing but differences of matter: occasionally we have a better knowledge of the matter than of the force, and sometimes the reverse. In a thousand instances, however, the power has been so distinctly proved to be a necessary quality of the matter, that, like all other qualities, it is an inseparable characteristic of the body, conditioned by the peculiar composition of its substance.

It is no empty prejudice to state, nay, it is a real belief in the general prevalence of a demonstrated natural law, that the nature of the mother is communicated by the milk to the child. And there is no thought more natural than the belief, that on the breast of its mother the infant may imbibe, together with the milk, her nobleness of mind, with the love which devotes that food to the most sacred purpose, and fastens still more strongly around the feeble child and the tender mother, the ties of their endeared relationship.

There are certain differences, though slight, between the milk of one woman and that of another; and as in successive months after confinement the milk alters in composition, the difference is so much the greater, as the children of the mother and those of the wet-nurse differ from each other in age. Since, in any case, the milk of the wet-nurse will be dissimilar to that of the mother, the closest possible conformity ought to be secured, where a wet-nurse is sought, between the age of the two children. Under any circumstances, however, the milk of a wet-nurse more resembles that of the mother than cows' milk; and therefore the milk of a wet-nurse is to be preferred to bottle-feeding. It is true that, in large towns, healthy wet-nurses are scarce; and still rarer, also, is that devoted love which nourishes a stranger's child as carefully as is desirable; for the effort is, indeed, not slight, by which a woman has to restrain her own will and inclination to such a degree, as to prevent indisposition of body or any bewildering passion from becoming hurtful to the babe; the easily excitable nature of the woman is often influenced to the great disadvantage of the child. Out of the manifold inconveniences arising from these circumstances, mixed food produces a more frequent uneasiness of body, than where uniformity of food is observed.

Hence, in many cases of difficulty, bottle-feeding will be preferable to the employment of a wet-nurse. By diluting cows' milk with one-third of water, and adding to five-and-twenty parts of milk about one part of sugar, a sufficient similarity with the milk of the mother is obtained. If asses' milk can be got, which is seldom the case on account of its high price, the dilution of the whole, and the increase of the proportionate amount of sugar, are very simply effected by mixing about two parts of asses' milk, which is remarkable for its abundance of water and sugar, with one part of cows' milk. As the milk in the first days after confinement contains more solid constituents than it does from about the fourth day afterwards, the cows' milk should for the first days be diluted with a smaller proportion of water; but after the fourth day, more than one-third of water is to be added to good cows' milk—a proportion which must gradually be diminished again, as the milk of the mother, after having become considerably thinner on the fourth or fifth day, increases again by degrees in the amount it contains of caseine and salts. Like all liquids of the body, the milk in the mother's breast has a temperature of 98° Fah. At this temperature, all aliments are most easily transformed by the digestive fluids. In bottle-feeding, therefore, the mixture employed ought always to be warmed as nearly as possible to 98°.

A long time before the period for weaning arrives, and earlier if the mother have not sufficient milk, more solid food may be gradually given. From rusks, fine wheaten flour, potato-starch, or arrowroot, first combined with water, milk, and sugar, and afterwards with broth of different kinds, a spoon-meat is prepared, usually much relished by infants. While in rusks and wheaten flour the albuminous substances themselves are present, in the mixture prepared of potato-starch or arrowroot, these are only contained in the milk, or in the juice of meat; for it is to be taken into consideration that arrowroot differs from potato-starch only in being composed of smaller granules, and by its forming with boiling-water a thinner paste. Arrowroot, in fact, is nothing but starch; but starch represents only the group of organic alimentary principles destitute of nitrogen; and among these the constituents of fat: now as the constituents of fat alone are not able to maintain life, arrowroot boiled with water only, cannot restore and supply the various essential constituents of the blood. With a paste of arrowroot and water, children may be fed to death, but they cannot be nourished; and many a child has succumbed to this diet, an unhappy victim of the lamentable error that the highly-commended arrowroot was of itself a complete nutriment.

Even after weaning, soft mixtures of a good bread, with milk and sugar, or with the juices of meat; also, the more readily-digestible roots and vegetables, together with soups prepared from the meat of young animals, may be considered the best food. After the teeth are cut, meat and bread, in their simple form, may also be given. Aliments difficult of digestion, fat meat, heavy bread, rich pastry, unripe wheaten grains, leguminous seeds, and heating condiments, are carefully to be avoided for children; and of beverages, water, milk, and a light beer only deserve commendation.

§ 95. *Diet of Youth, Maturity, and Old Age.*—While the boy is growing into the youth, the composition of his blood and tissues gradually approaches that of the mature body. As the muscles acquire more fibrine, the skin and bones become richer in gelatinous substances; in the bones and teeth, the lime-salts increasingly predominate over the common salt; and the whole activity of the change of matter is altered.

As at other times, so during this period, the expenditure of the body furnishes the best means of estimating the extent of these changes. The proportion of carbonic acid excreted by the lungs, and that of urea voided by the kidneys, increases up to maturity, attaining its height at about the thirtieth year; but at about the fortieth year, the activity of the tissue-change already begins to decrease, and the most important alterations gradually take place, of which those only of the tissues of the bones have been investigated with any degree of accuracy.

Of the solid parts, it is known in general, that in old age the proportions of water and fat decrease. Hence, the tissues of the eye become drier, and the light is less refracted through the crystalline lens, which becomes poorer in water; hence, also, the shrivelled hands and the wrinkled brow. That the hair is altered in its material composition is proved by its gray colour, the venerable decoration of old age; but of the bones, we know that they become more and more brittle, because, in proportion to their organic basis, the quantity of salts astonishingly increases, and it is particularly the phosphate of lime which more and more predominates; for although the salts, on the whole, increase so considerably, the carbonate of lime decreases relatively to the phosphate of lime; which latter, therefore, in a stricter sense, is so often denominated the bone earth.

Nor is the organ of thought without its share in alterations so important. In advanced age, the proportionate amount of fat in the brain diminishes; while, unlike all other organs, its amount of water augments.

Is it, therefore, to be wondered at, if the excretions undergo essential alterations? or, should the tissues, exposed to the same influences as before, although themselves differently composed, yield equal quantities of the same products of decomposition? We repeat, that in difference only lies the equality of men; only by the variety of circumstances, affecting both the matter and the force of our organs, can we be different; we are all equally dependent on air and soil, on men and animals, on plants and minerals; we could not have been primitively all equal, could we maintain equality under the different influences to which we are exposed: it is upon the difference of circumstances that our own differences are based.

Age is an essential element worthy of notice in this difference. The tissues have now another composition; in old age, less carbonic acid is breathed out, less urea formed. The drier tissues, containing a greater proportion of salts, undergo their normal transformations more slowly; the diminished decomposition proves a diminished activity; that renewal of matter, which is so abundantly induced by the decomposing action of oxygen, is gradually weakened; a smaller proportion is decomposed, and a still smaller portion formed.

Thus the susceptibility to all sensible and mental impressions gradually becomes enfeebled: that which formerly would have made a powerful impression, scarcely now leaves any trace in the form of memory. But a rich life lies behind the old man. Though now receiving less, the crown of a maturer experience adorns him. Ripe fruit which he has collected for himself and others, gladden the last years of his life, and increase the cordial and elevating sense of gratitude, with which we render homage to the venerableness of old age. "In the late evening of a much-agitated life," Alexander von Humboldt, whose very name illustrates the idea, lays before the world his Cosmos, "the idea of which, for almost half a century, had been floating in uncertain outlines before his mind;" and with it he enkindles the spirit of every one, whose view is not narrowed by those barriers which only too often separate all sublime enthusiasm regarding the universal, from the arid, and relatively sterile, assiduity which stolidly confines itself to mere details.

Not in every case, and not for very long together, can such a noble picture of man be exhibited. With the diminishing vigour of the change of matter, the torch of life gradually sinks. More and more slowly the matter moves from the digestive organs to the blood, from the blood to the brain and muscles. At length these activities wholly cease; for all that lives, bears in itself the germ of death. Those laws of gravitation which alter at every stage of life in a necessary succession, lead through growth to perfection, and from the blooming prime, by a regressive transformation, to decay. Not only do fat and water disappear, the bones also lose in circumference. While the latter become richer in lime, harder and more brittle, the wrinkling skin shrivels up, the cartilages ossify, and the watery brain loses more and more of its essential fat. Mentally, as the retrograde course continues in unbroken progress even to death, a kind of revolution is usually completed. The memory becomes dull, even for youthful recollections; and to the other infirmities of old age, a hebetation of the senses finally supervenes, which distorts the judgment, and destroys the memory. The old man who was before so venerable in his unbroken strength, bringing out the results of the experience of his former life, gradually becomes the image of a helpless child. Dissolution then becomes a blessing, and, with respectful awe, we drop the veil over that last proof of feebleness from which no mortal being can escape.

Matter, however, is immortal. Into the grave we let fall the noblest seed, with the certain knowledge that the transiency of one form "which has been bleached by the abundance of years," will yield to flowering and fragrant plants of the fields, to rise again, after numberless transformations, in fresh and youthful vigour, and still co-operate in the task, in which the spirit of all human works continues to live amongst us, visible to our senses. For the spirit is eternal, as manifesting itself in matter; while the terrestrial constantly changes, the earth and its inhabitants are as constantly renewed.

§ 96. *Requirements of Youth and Age.*—As growth continues during the time of youth, so the young not only require to satisfy their appetite oftener than adults, but need also a more nutritious diet. The attractive power towards the alimentary principles is greater, and the blood yields more to the tissues in spite of the excretions, subtracting altogether a smaller proportion from the body than in mature age. If the adult, therefore, require meat and bread, or leguminous seeds, to keep up the necessary strength for the tissue-change, this necessity is so much more peremptory with youth.

At the time of transition from boyhood to youth, care must be taken not to carry the nutritious diet to excess; for it is essentially characteristic of healthy development, that precipitance leads only to an unhealthy precocity in instincts, thoughts, and actions.

Granting him all adequate gratification, the growing youth should be provided with cooling aliments, fruits and vegetables, water and acidulated beverages; and if then his mind be sufficiently occupied, and he take abundant exercise in the open air, the tissue-change will harmoniously promote the proportionate development of every part, and youth will enjoy the pleasures of youth.

In mature age a fixed diet is the least needful; the full-grown man discharges as much carbonic acid and water, urea and uric acid, with other excretitious matters, as he takes up of food and inhaled oxygen. His wants he can measure by his appetite; the latter recurs less frequently, and is sooner satisfied than in youth. Every kind of excess, however, must be avoided; and this is infallibly secured by always leaving off before appetite has disappeared. Without this precaution excess is easily possible, because the limited size of our lungs, and the definite power of our movements, restrict the power of the change of matter. If man eat more than he excretes, the tissues become overloaded, which endangers their activity as much as the impoverishment of the blood and the consequent defective nutrition could do; fat is collected which the oxygen does not consume, and the albuminous substances, with the salts, assume a fixity which at once enfeebles the intellect, and destroys the pleasure of thinking; while it diminishes both the strength of the muscles and the inclination for movement. Thus do gluttons gradually grow lazy, impatient of thought, anxious only for repose, and unfitted, by their unwieldy frames, to embody the moral and intellectual greatness which characterises noble-minded men.

Apart from the mode of life and the degree of temperature of the surrounding air, the influence of which latter we shall afterwards describe more in detail, the peculiar constitution of the individual deserves a special consideration. It is true that here the grades of difference are as numerous as the population of the globe. The natural disposition, with a certain tendency to firmness in the functions of the brain, gradually produced by the school of life, form the character. The greater this firmness is, and the higher the courage with which it is displayed, the more naturally do we call the man a "character" in the good sense of the word; but, inasmuch as every individual holds a necessarily defined position in his relations to the external world, every one has more or less of distinctive character. On account of the innumerable degrees of transition, however, this rule can only be applied to the extreme representatives of one class or another.

The more vivacious the disposition, the more easily the man is excited by stimulants; the greater, also, will be the tissue-change. Generally speaking, such individuals will need food more frequently than those of a less excitable nature. With individuals of this habit, too nutritious or exciting food is to be avoided, because, as a stronger stimulant, it increases their nervous irritability. Vehement, passionate natures become still more ardent from partaking of game, heavy bread, leguminous seeds, or any considerable quantity of beer, wine, or spirits, coffee or tea. By these more stimulating aliments the circulation is accelerated; the tissues, especially the brain, are overladen with blood; and the skin, which in such persons is very easily filled with blood, turns red, especially in the cheeks. This heat is moderated by cooling aliments and beverages. Fruit and vegetables, therefore, with lemonade and similar drinks, are more advantageous for irritable constitutions than spirituous and aromatic beverages.

The latter are more appropriate to persons whose activity of the brain is disproportionately great; while their weak digestive organs, their slow formation of blood and nutrition, occasion a disposition to melancholy. Such persons require a stimulating diet. On account of their slow digestion, they have to select such nutritious aliments as are easily digestible; such, for instance, as the flesh of fowls and pigeons, veal, mutton, and veal-broth, in conjunction with a small quantity of light, well-baked bread, or of vegetables. As heating spices, wines taken in moderate quantity, as well as strong tea and coffee, accelerate digestion, and with this, indirectly, the tissue-change; they produce a greater uniformity in the functions of the different organs, and thus exercise a beneficial influence upon disposition and character.

And, finally, where a slow tissue-change characterises also the main parts of the nervous system, the brain, and spinal cord—where slight irritability is united with flabby muscles, a pale, flaccid, puffy skin, an inert digestion, and a deficient formation of blood, as in phlegmatic persons—a nutritious animal diet is to be combined with strong spices, strong beer, and wine. Vegetable aliments, especially roots containing much starch and sugar, must be avoided by all such; and for this reason, that in such persons there commonly exists already an excessive disposition to deposit fat, which, being itself the consequence of a less vigorous respiration, becomes a check to the tissue-change. The large quantity of fat withdraws from the other constituents of the blood, the oxygen necessary to transformation.

With the last-named characteristics, those of old age are chiefly to be classed; for here, also, the enfeebled activity of the digestive organs is the salient fact. If the diminished activity of the tissue-change be manifested by a languid appetite, it is so much the more necessary for the aged man to choose the most easily-digestible food. Lean meat, venison, strong broths, young vegetables, and roots, as carrots, containing abundant sugar, deserve the preference; while stimulating condiments, coffee, tea, a good old wine, a strong bitter beer, taken in small quantities, serviceably promote digestion.

§ 97. *Diet of Woman.*—The general laws of the change of matter are fulfilled in the case of women through another circle of developments than in man; for at the moment when sexual maturity begins to manifest itself, that continual augmentation in the exhaled carbonic acid, which in the female, as in the other sex, lasts as long as the child is growing into the maiden, ceases. Till the age of about forty-five years a uniformity is maintained, which yields to a slight increase only after that period; and in old age, as in males, gradually declines. At every age, however, the quantity, not only of the exhaled carbonic acid, but also of the urea daily voided, is much smaller in the female than in the male sex.

While woman, therefore, is remarkable for a less active tissue-change, a slighter muscular power, and a calm, contemplative activity of the brain, which is less easily aroused to greater exertion of thought, she is also less subject to wild passion: self-control appears to be the necessary consequence of material conditions; although the whole position of woman in life, and particularly the relation of the mother to her child, combine to ingrain these mental peculiarities into the true feminine nature. The material basis of the female body is the best proof that neither an arbitrary conventionalism, nor the unutterable longings of the youth indulging in sweet hopes, have evoked the attention and the cheerful support which everywhere men have devoted to the fairer sex. Although, in many uncivilised tribes, the strength which women gain from endurance encourages the men to burden them with correspondingly heavy work; still the woman enjoys everywhere the assistance which she necessarily wants. And when, with a higher civilisation, the consciousness of the dependence of woman increases, together with that of the dignity of motherhood, the superior power of man affords protection, his assistance becomes homage. The maiden's loveliness, added to the dignity of the wife and mother, helps to adorn the life of more rugged and ambitious man, her gentleness of manner producing that love of art which so often represents the wife in a noble ideal. This is the enjoyment which indemnifies the wife for great sacrifices: owing to a less direct contact with the rough storms of external life, she is able to cultivate a purer humanity, out of which the characteristic of womanhood is born.

The secluded life usually led by woman produces that perfect symmetry of nature—that fascinating serenity of manner, by which the matron is distinguished and adorned. But inasmuch as her life is less agitated, her manners more subdued, and her whole nature more accustomed to self-denial; so are the contrasts less marked between one woman and another. Their characteristics are less distinctive, and their peculiarities manifested in more delicate and minute distinctions—distinctions only to be accurately discriminated by the most acute observers. Even their features partake of this delicacy; and the painter finds it especially difficult to catch the more delicate peculiarities of expression in the female face, or infuse the characteristic expression into the less strongly-marked features. In woman only is found that charming harmony in which it is difficult to point out any salient part, because the equable and cultivated tone does not permit any particular point to be prominent. The least favourable aspect of this harmony lies in the greater monotony which distinguishes the sex from man. Of this peculiarity, the greater equability of the change of matter is at once the consequence and the cause. The variations in the quantity of the exhaled carbonic acid, so great in man, disappear almost entirely if we compare woman with woman. With the excretions, moreover, all the corporeal activities are moderated to a greater uniformity, as, on the other hand, the narrower sphere in which the life of the woman moves, reduces the change of matter also to a more equable course.

With this fact, the greater nervous irritability of the female sex in general is quite consistent, for the slighter vivacity of the change of matter produces a diminished demand for nutriment; but nutriment, as well as the mental impressions, belong to the stimulants, in the wider sense of the word. And the more sparingly the stimulants are used, the stronger in proportion is their effect, and the more quickly also does that effect again cease. As the change of matter specifically stamps its impression upon all the functions of the body, so most women are easily excited, but soon regain their self-command; in man, on the contrary, the stronger impression usually produces more lasting effect.

§ 98. *Appetite of Women.*—From this description of the change of matter in the female sex, it follows that the restorative substances are required in a smaller quantity than in man. It accords with a general result of experience, that a less nutritious diet, or nutritious food in smaller quantities, suffices to satisfy the want of women; it is, therefore, no prudish or dainty affectation, but a natural abstemiousness, which leads girls and women to partake sparingly of food.

On account of the greater irritability peculiar to women, they commonly have a disinclination to strong condiments and exciting beverages. To coffee and tea only do they accustom themselves, and that to such a degree, that they do not suffer any injurious effect from them. Both beverages are much relished, especially by aged women. In general, however, neither tea, coffee, wine, nor any of the heating condiments, are wholesome for the female sex; of these aliments, tea and coffee only ought to be taken, and that in great moderation, diluted with a large proportion of milk.

§ 99. *Diet of the Artisan.*—An active change of matter increases the muscular power; but, on the other hand, the exertions of the muscles augment the activity of the excretions. He who digs the soil with the spade, or wields the hammer, who breaks-in horses, or wanders in the open field—not only perspires more, but also breathes out a greater quantity of carbonic acid, and voids more urea, than the man who, by luxurious and indolent tranquillity, encourages the fat of his body, and an increase of the albuminous substances.

A real development of strength is inseparably bound up with an active change of matter. But change of matter consists of excretion and restitution; for he who exchanges, receives while he gives. The permanence of the substance does not contribute to the increase of activity. We have already several times expressly mentioned, that, though the presence of matter is the condition of all activity, yet the rapid movement of this matter only intensifies the manifestation of power. The limbs, therefore, if kept at rest, relax; and in return, the whole advantage of exercise consists in the excretion of the muscles being augmented by their exertion—the nutrition of the tissue being increased by that augmented excretion, and the appetite being roused, correspondently with the necessity of forming new blood occasioned by the accelerated excretion and nutrition. The rapid change of matter increases the power of the organs, just as the exertion of the limbs promotes the change of matter, and the increase of muscular power.

But to attain this, an abundant restitution is an indispensable condition. It is, therefore, equally consistent with economy as humanity, for those who employ labourers on heavy work, to satisfy the more frequent and strongly recurring appetite of their workmen; for it is only when the workman thrives, that the work can prosper. Insufficient food causes feebleness and inertness; and the master who scantily feeds his workmen, loses more in the decreased strength of their arms, than the aliments would cost him which would raise at once the value of their achievements, and the dignity of their nature.

By every one engaged in heavy bodily labour, a nutritious diet is required; and as, in a rapid change of matter, the digestion participates in the activity of the other functions, the appropriate aliments here are meat with bread, containing a large proportion of gluten and leguminous seeds; while the preferable meat is beef, containing a good quantity of fibrine: in a word, the most serviceable aliments here are those which are also the most difficult of digestion. And these more especially deserve preference, if circumstances or the kind of employment render it impossible immediately to satisfy the want of food recurring at short intervals; the aliments in this case are rendered more difficult of digestion, through their being more slowly transformed into the constituents of the blood and tissues; they also undergo less rapidly that decomposition which transmutes them into the excretitious substances. By this strong food, the labourer is enabled to resist the calls of hunger for a longer period; and this—in spite of the obstinate denial of mock-philosophers, who are deluded by their own luxurious life into a contrary belief—is the lurking enemy whose attacks continually irritate the anxious pater-familias among the poor. In these circumstances the leguminous seeds are an essential alleviation to sufferings arising from hunger.

Corporeal exertion increases not only the expenditure of carbonic acid and urea, the voidance of water as well as the excretion by the lungs and kidneys, but the expenditure by the skin and sudoriparous gland is also augmented. The loss of water, particularly, is increased by those actions which directly produce a more abundant evaporation from the mucous membrane of the tongue, throat, and lungs; as, for instance, by talking and reading, by singing and blowing. Hence, teachers and lecturers, singers and trumpeters, glass-blowers and public criers, are subject to extreme thirst; so also are all workmen who from their avocations are exposed to a high temperature, which stimulates the activity of the skin; and in a high degree augments the perspiration. This excites in glass-blowers the desire for drinking, and is the main cause of the often-returning thirst in smiths and iron-melters, sugar-boilers and cooks, bakers, and gardeners working in hot-houses. On the contrary, thirst is at a *minimum* in fishermen and sailors, who ordinarily live in a moist atmosphere, into which a smaller quantity of water escapes from the human skin and lungs. That sailors are partial to brandy and other spirits, arises from another cause; for, on the one hand, through the alcohol, which demands the inhaled oxygen for its own consumption, the food of these labourers, which may be limited to a scanty meal, enables them to hold out for a longer time; and, on the other, the fat, which the alcohol assists to preserve, frequently protects and comforts the mariner in the rough weather to which he is exposed.

A good beer partakes in all the advantage of the alcoholic beverages, and at the same time usefully quenches the thirst by its more abundant amount of water. Hence, this beverage is particularly adapted to satisfy the frequent thirst caused by bodily exertion; it is, therefore, a laudable custom to refresh artisans, who have to work hard, in the morning and afternoon with a glass of ale; this beverage, by its proportionate amount of albumen, which is equal to that of fruit, supplies even a direct substitute for food.

§ 100. *Diet of the Artist and Literary Man.*—If the brain, like any other part of the human body, be subject to the processes of tissue-change, it is evident that a greater activity of this main organ of the nervous system will augment the products of decomposition. Thus all the sensations and passions which increase the mental action—as hope and joy, anger and ambition, suspended expectance and happy love—excite the instinct for food. By vigorous exertions of the will, active imagination, sustained thought, the change of matter is likewise accelerated. It is true, this does not prevent one sensation from being able, as it were, to neutralise another. How often do we observe, that from joy or love, anger or suspense, a person cannot eat—or, in other words, the sensation of hunger is not perceived—while the intellectual power of the brain is over-exerted. But the desire for eating and drinking is only momentarily blunted; and after some time, hunger and thirst reassert their claims with a double force.

The far-spread error, that mental activity does not increase the consumption of matter, originates in our repugnance to admit the fact, so strongly forced upon our observation—that the power is inseparable from the matter; for how many, even of those who have exclusively devoted themselves to the observation of nature, frequently engage in vain speculations, about an essence of bodies, external to or hovering over them. And how few have a clear insight into the position which, since Spinosa, can no more be banished from science—that the total of all qualities makes the essence of a body. Only too often we may observe, that in an unguarded moment, intelligent men freely, and without prejudice, assert in some single instance the doctrine, that their mental actions are conditioned by the matter which by the nutriment is conveyed to the brain. But as soon as this seeming casualty is raised to a general law, they are terrified by their own significant presentiments, and fly from the clearness of conviction, in which only they could find satisfaction. This vague separation, however, of matter from power, which, pursued to the extreme, would lead us to ascribe an immaterial spirit to steel and amber, and other material substances, is not the only cause of the frequent, but false doctrine, that the matter is not wasted by the pictures of the imagination and combinations of thought. Commonly the artist or literary man is compared, not with persons who live quietly and in indolence as regards their thoughts and sensations, but with the corporally active artisans. In this case it is too readily forgotten, that while in the intelligent workman the brain is not inactive, the change of matter excited by mental exertion in most artists and literary men, is moderated by their sedentary lives; and yet an increase of the temperature of the body and of the desire for food does actually take place in consequence of mental effort. Artists and literary men, as well as the artisan, have to compensate, by a greater supply of nutriment, for the increased consumption, which transforms the essential constituents of their brain into the decomposed ingredients of their excretions. It is well known, that, notwithstanding their sedentary life, artists and literary men rarely suffer from corpulence; and no one expects to find that store of the more constant constituents of the tissues, so remarkable in good living and in active persons as characterising men of mental activity, excelling by their accomplishments in art or science. A large body and fleshy face may be suitable for monks and gluttons longing for repose, but it is not adapted for men of intellect. Abundance of fat in the blood of the brain paralyses thought, and hangs lead upon the wings of the imagination.

As a sedentary life renders digestion and the formation of blood difficult, and moderates the secretion preceding decomposition, which is increased by the activity of the brain, the artist and literary men have to choose, within the limits of a nutritious diet, the more easily digestible aliments. A well-baked bread and lean meat, combined with young vegetables, and such roots as are easy of digestion, and contain a considerable proportion of sugar, form a wholesome diet for thinkers and poets.

Well-seasoned aliments and stimulating beverages, if not taken in excess, are to be recommended to all men whose toil is principally mental, for a twofold reason. Spices, beer and wine, tea and coffee, if moderately taken, stimulate the different digestive glands to an abundant secretion, and therefore promote digestion, which, in a sedentary life, is so apt to become sluggish: this is one aspect of their usefulness. The other aspect bears directly on the brain: as the activity of this organ predominates in poets and thinkers, it requires a constantly-renewed stimulus; hence the heating spices, with wine, coffee, and tea, are appropriate and useful, imparting a bolder sweep to the mental activity, which creates images and combines thoughts into judgments. The oftener the stimulant is used, the greater is the quantity required on every subsequent occasion, in order to produce the same degree of excitation. This continually augmenting waste originates a habit, and impairs sensibility; for the frequent repetition of the excitement produces exhaustion, and the exhausted organ is only able to perform its peculiar function when the stimulant acts again; this would indicate that a frequent change in the mode of living is necessary. There are many literary men who cannot write successfully until they have increased the activity of their brain by tea and coffee, and in some rarer cases by wine. We say in rarer cases by wine; for while, from its action upon the judgment, tea is especially proper for the literary man; and while coffee, by animating simultaneously both the judgment and the imagination, belongs alike to the literary man and the artist; wine is the most appropriate beverage for the latter, who demands, above all, that his diet may stimulate the imagination, sharpen the senses and the memory, and facilitate the combination of ideas.

Provided only that a certain limit be observed, there results no disadvantage from the use of stimulants; but if they are taken to such a degree that exhaustion sinks into a complete numbness, the greatest quantity of the strongest stimulant becomes at length ineffective. When by habit, however, the stimulant has become a necessity, an enervating relaxation infallibly follows, as sometimes mournfully illustrated by less prudent literary men. The stimulant ceases to excite—the debilitated organs have already been indebted to it for all the activity it can give. In this case the victim continues to seek his refuge, until dangerous diseases of the stomach cripple the digestive powers; with the decay of the digestive organs the formation of blood and nutrition are disturbed; and with the digestion vanish clearness of thought, acuteness of the senses, and the elasticity of the muscles.

§ 101. *Diet in Summer and Winter.*—In considering the wide diversity between the modes of life of the inhabitants of the tropics and the temperate and frigid zones, we are induced to suppose that temperature must exercise an essential influence upon the change of matter. This supposition has become a certainty. It has long been known, that in a high temperature, the evaporation from the skin predominates; while in cold weather a greater proportion is excreted by the urine. This increase of urine, which in health is maintained during the whole winter, is accompanied by an abundant formation of urea. From these observations, it would appear that a higher temperature diminishes the quantity of carbonic acid exhaled in a certain time, and consequently that, by warmth, the two most important products of decomposition are diminished; it becomes indubitable that in summer the excretion is weakened, and in winter increased. To the excretions correspond the nutrition, the formation of blood, and the digestion; hence results the necessary conclusion, confirmed by daily experience, that in summer we digest a smaller quantity, and that more slowly.

The relaxation caused by frequent and abundant respiration renders digestion more inactive. Inasmuch as the formation of blood and the nutrition of the tissues progress more slowly in summer, we are then less fit for any kind of exertion than in winter; and since we observe this difference the most strikingly when a rapid change has produced a great alteration in our excretions, it is apparent why a walk or any violent movement so readily tires, when the rough, damp, and cold weather, amid which the winter so often takes its leave, has suddenly yielded to a sunny spring day. It is only when we have become gradually accustomed to the less active change of matter, that the expanding life of the vegetable world, awakening in the freshness of its verdure, delights us—that the alluring strains of the songsters of the grove exert their full influence upon our nature; but, with the retarded movements of the change, these impressions, effective as they are, invite us rather to tranquil enjoyment than to a vigorous activity. If the "*dolce far niente*" first exhibited itself under Italy's warm and serene sky, with us the sweet delight of calm sensations, the intensity of which is seldom disturbed by energy, is the privilege of spring: May is the month of love and enjoyment.

But if in autumn, when the oppressive summer days have passed, there suddenly appear much cooler but more genial weather, we feel impelled to a cheerful assiduity. The quicker movement of matter carries us along, as it were, with the flowing stream; and we often see achieved in a few days what had lain in our mind for weeks without our having been able to find the right disposition to attempt its accomplishment.

These differences, however, are most observable in labourers with the brain. Those who rely on their bodily exertions, and are never out of exercise, are as active in spring as at other times. And naturally so; for in the restless circle of cause and effect, which the change of matter embraces, they increase by their energy the movement of the matter. While the artisan continues, with an industrious hand, to labour for a living for himself and his family—while the farmer looks forward to the encouraging reward of his labour in a rich harvest, which he extorts from the soil by his activity, and for this shuns neither heat nor labour—the changing matter of his system remains equable and well balanced.

No influence, however, passes without some effect. While, therefore, in summer, as at other times, occupation and age, habitude and sex, unite with temperature in determining the choice of food, certain rules may be clearly established, if in this series of combined and mutually reactive influences we separately consider heat and cold.

In summer the desire for food is diminished, because the consumption of the tissues is less; a smaller quantity of nutritious food, or less nutritious but easily digestible aliments, are to be taken. The meat of young animals, young vegetables, edible roots containing abundant sugar, fruits and salad, are also much relished in summer. The predilection for cooling beverages at this season has a perfectly rational basis, as the heat frequently produces palpitation; while the less rapid decomposition of the tissues, which is peculiar to the warm season, retards the transformation of the blood. Cooling and diluting beverages containing vinegar, currant-juice, raspberry-vinegar, and water, operate against this retardation; but the palpitations are only increased by heating beverages and spices. An abundance of spirituous liquors is, therefore, doubly injurious in summer; for the alcohol deprives the constituents of the body of the inhaled oxygen which is necessary for this decomposition, and also for the animation of the corporeal functions. For this reason, in summer, the lighter sorts only of beer and wine, which contain a small proportion of alcohol, ought to be chosen. It is also important to be moderate in the use of heating spices, or to allow them only where a certain excitement of the digestive activity, within the limits of health, is wished for.

The greater expenditure which distinguishes the winter, necessarily requires greater supplies. In this season, therefore, the increased wants of the appetite have to be satisfied, and the more nutritious aliments, with those which are more difficult of digestion, are with good reason selected. The latter are easily mastered by the augmented digestive power; while the greater proportion of alimentary principles, contained in the former class, is gradually transformed into the constituents of the blood, and slowly conveyed into the circulation. Farinaceous food, containing a large proportion of gluten, dry leguminous seeds, and fatty pork, are in winter much more easily digested than in summer. In winter, the fat is much more easily burnt into carbonic acid and water, as is proved by the increase of the carbonic acid which we breathe out during that season. The oxygen which is taken in also acts more vigorously. Although the fats, on account of their abundantly proportionate amount of carbon and hydrogen, are easily combustible, yet they differ from the albuminous substances, as well as from the constituents of fat, in requiring a greater proportion of oxygen in order to be really burnt. In summer, therefore, and in hot climates, where the oxygen is less efficient, the fat of the food is digested with much greater difficulty; hence the preference given in the tropics to the constituents of fat over the fats themselves, is a circumstance arising from natural laws. So long back as the times of Herodotus, we know that in hot climates people lived chiefly upon a vegetable diet. The inhabitants of the islands of the Pacific Ocean very seldom eat fish of any kind, and pork and fowl only at festivals. The food of the negro is rice, millet-seeds, maize, and roots containing a great quantity of starch. The chief meal of all inhabitants of the tropics, consists of rice. Here we meet also with an illustration of the not unfrequent fact, that a custom, apparently traceable to superstition, is really founded on a deeper and more rational foundation; for inasmuch as at a high temperature fat is less easily decomposed through the diminished activity of our lungs, a correct interpretation of natural phenomena has led to the entire prohibition of pork during summer to the inhabitants of Southern Italy, and altogether to the Jews in Palestine. Obedience becomes superstition only when it continues to exist after the reason for the law has ceased.

If in summer everything must be avoided which absorbs the oxygen so necessary for the organic constituents of the body; so, on the other hand, aliments may be taken in winter, which moderate the decomposition of our tissues, produced by the oxygen; hence we can bear in the winter not only a richer diet than in summer, but those beverages are also relished which by their abundance of alcohol retard the consumption of the constituents of our body. It is inconceivable how physiologists can preach the renunciation of brandy, when we consider the simple fact, that the further north the greater its consumption. While the inhabitant of the south of Germany is satisfied with the proportion of alcohol which he takes in beer and wine; in Northern Germany, Holland, and England, brandy, gin, and other spirits are taken; but the Russians, Swedes, and Norwegians consume a still greater abundance of spirits. In a very instructive public lecture at Mayence, Von Kittlitz stated, that the Kamtschatdales, who are generally remarkable for their great respect of property, often steal brandy; but afterwards, with child-like simplicity, confess their theft, avowing that they could not do otherwise: they only steal what they consider a necessary of life. This regular increase in the consumption of alcohol, corresponding to the nature of the climate, cannot but suggest the existence of some valid reason for this popular custom, which has been completely confirmed by scientific investigation. The alcohol which has been taken, is a new source of the development of warmth, by which, on one hand, the food is more slowly consumed; and, on the other, the cushion of fat under the skin, which is a bad conductor of heat, and sufficiently protects the system against external cold, is kept from wasting. Travellers who have visited the Polar Seas, assert, that in such voyages Europeans could not exist without spirituous beverages. In low, cold, and damp countries, experience has always proved spirituous beverages, taken in moderate quantity, to be useful.[*]

It is true, science, as well as experience, warns against excess. When spirituous beverages have been taken in abundance, the venous and arterial blood have been found alike in composition; and in animals even fits of suffocation have been observed. It is apparent, that the oxygen which alcohol takes up in order to be transformed into acetic acid and water, and then into carbonic acid and water, is withheld from the constituents of the blood; while it is upon the combination of these matters with oxygen, one of the most important conditions on which a healthy change of matter depends, that the transmutation of the venous blood into the arterial takes place.

This circumstance ought always to be kept in mind; for then only do we restrain our fellow-creatures from intemperance, when we distinctly inform them that its injurious consequences are founded upon a necessary natural law, and one therefore infallible. A rational intelligence is the sole basis of all real morality. By total abstinence, and under the form of a solemn undertaking, we create slaves of an irrational promise, and treat men no better than animals, which we shut up in a stable that they may not stray too far away.

Inasmuch as man is formed by all the circumstances conjointly, the influence of which upon diet we have endeavoured to describe in this treatise, the rules given must be separately considered. The nature of man is the product, or rather the sum of all those effects of parentage and country, age and sex, position and habits, and even of the time of the day and year, which we have referred to. It is for the reader to determine the choice of food, according to the individual influences—bearing in mind that other circumstances to which we have not alluded still remain for consideration. These combinations of circumstances are nearly as numerous as the men themselves; and it must be left to the judgment of the individual to accommodate his diet to his own particular case.

All that we have written bears exclusively on man in a healthy state. "But in disease?" we hear you ask. It is just when the activity of the bodily functions deviates from the ordinary state of health, that the question of diet is the most important; and should not the book contain some instruction bearing upon this condition?

As truly as we are convinced of the usefulness to the people of a treatise on food for the healthy, so certainly do we know that we should only do mischief by giving here rules for the selection of food, of beverages and condiments, in disease. One point only is of the highest value in daily life; intemperance may become the source of the various diseases. This affirmation, in its widest sense, bears on the use of aliments of every description. And this general statement renders superfluous any attempt to explain why in autumn, for instance, the abuse of fruit will cause diarrhœa or dysentery; why meat immoderately taken may occasion the formation of calculus; why from an excessive use of spirits, gout or cancer of the stomach may originate. Such an attempt would be without any bearing in practical life, neither could it be justified on scientific grounds; for in the present state of our knowledge, it is absolutely impossible popularly to describe, with a proper chain of deduction, intemperance as cause, and the various diseases as effect. Such a description would require so thorough a scrutiny of each particular case, with so detailed a consideration of the condition of the body, that either superficiality would annihilate its value, or profundity destroy its clearness. The doctrine of the causes of disease, pre-supposes a thorough knowledge of disease itself; and this requires a considerable familiarity with all the more important laws of science.

If this be the case as regards the knowledge of disease, it is much more so with regard to diet when under its power. Only the man who has devoted his whole life, with all his powers of sense and thought, to the investigation of the condition of man in health and disease, can unite with the necessary knowledge the more subtilely discriminating judgment under such circumstances; and that only after the most accurate account of country and training, of climate and weather, of age and sex, of habits, occupation, and of parents. No one would require this if he consider that every action has its necessary consequences; that the widest experience only, purified by the clearest judgment, and after careful personal examination, is able to secure in the individual case the desired effect with any degree of certainty. Render, therefore, to the medical man that which is his, yet without neglecting, for the healthy state, the study of those rational rules of life which are given by science; for man is only obedient to such laws as he understands.

While solid and liquid food furnish the matter which is decomposed in our body and circulated through it, and by which we think and feel, nature and mankind exercise so continuous an influence on our senses, that the material of our body is never for a moment at rest. In never-ceasing motion, sensation and thought, will and action, are hurrying on; and if all this activity be only sustained by matter, all effects being but the result of the forces of matter acting upon matter, nevertheless the less material impressions of sound which elude our grasp, with those of light and colour which are altogether intangible, are no less important, and to many persons, indeed, much greater, than the alterations which food so infallibly produces in us. It becomes the wise man to recognise this dependence; and it is true piety heartily to feel this connection with the mighty Creator of all. Justly has Schleiermacher designated the sense of dependence in the individual, the true essence of all religion.

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[*] It may be stated, however, that some Arctic voyagers have urged an abstinence from the use of spirituous liquors as being essential to health in high latitudes.—ED.

CHAPTER III.

SULPHUR—SELENIUM—PHOSPHORUS.

Sulphur, as an element, partakes to some extent of the characteristics of others we have already described. Like chlorine, iodine, &c., it has the power of uniting with other bodies, and of so forming compounds, which are termed *sulphides*. Its external appearance and general properties are exceedingly varied; generally speaking, it is sold in commerce in rolls, under the name of brimstone. It is found in a crystallised state; and the neighbourhood of volcanoes, such as Vesuvius, Etna, and Hecla, are the sources whence we derive our supply of this article. Sulphur, however, is very generally diffused throughout the animal, vegetable, and mineral kingdom. As we have already shown, it is a constituent of our bodies; in many vegetables it forms a component part; and in the mineral kingdom, we find it associated with iron and copper, in the form of pyrites, with lead as galena; and in combination with oxygen as sulphuric acid, it is exceedingly abundant throughout nature. Its symbol is S; the equivalent being 16.

Sulphur is readily melted and sublimed. At a moderate heat, a little beyond that of boiling water, it is converted into a fluid state. Beyond that temperature, it assumes a more solid state, and a darker colour; and in this form it is employed as a moulding substance, for the purpose of taking what are known as sulphur moulds. It is also much employed for the purpose of bleaching animal and vegetable substances. The following experiments will illustrate some of its qualities and applications.

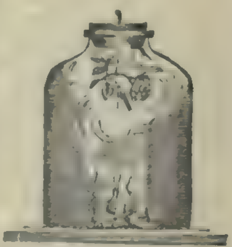
Experiment 54.—Place a little brimstone in a Florence flask, and apply a gentle heat to it by means of the spirit lamp. The sulphur will sublime, or rise to the top or cool side of the glass, in a yellow-coloured powder, formed of very minute crystals.

Experiment 55.—*To obtain Crystals of Sulphur.*—Melt some pieces of roll brimstone in an iron ladle, at a very gentle heat. When the whole is melted, remove the ladle from the fire, and allow it to cool until a crust is formed on the top. Make a hole in the crust, and let the melted sulphur inside run out. When cool, the mass in the ladle will present some beautiful crystals of a needle shape, if broken in half.

Experiment 56.—*To make Moulds from Coins.*—Melt some sulphur in a ladle, taking care not to overheat it. Pour this over any coin (except silver, which would be spoiled), first fixing the coin in a paper rim, such as a pill-box, to prevent the sulphur running away. The coin should be slightly oiled, to avoid the sulphur adhering to its surface.

Experiment 57.—*To Bleach by means of Sulphur.*—Wool, hair, straw, &c., may easily be bleached; and this plan is adopted by wool scourers, and straw bonnet manufacturers. Take a jar open at both ends, and in the upper part fix the article to be bleached by means of a string tied to a cork; as, for instance, a flower or a piece of straw bonnet, first moistened with water. Then place on a piece of hot iron a little sulphur, which must be set on fire, the jar being placed over the fumes. The jar should be slightly raised on one side, to allow a little air to pass in. In a short time the colour of the flower will be completely destroyed by the sulphurous acid thus formed. The arrangement is represented in Fig. 57.

Fig. 57.

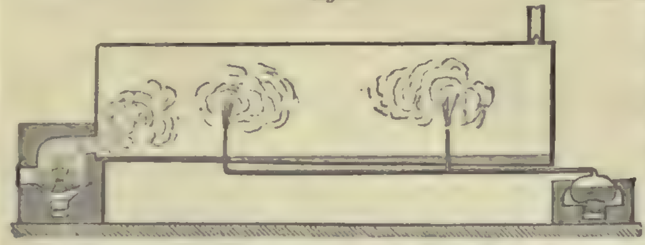


As an article of commerce, sulphur is largely imported for the manufacture of gunpowder, lucifer-matches, sulphuric acid, and as a medicinal agent. In its ordinary commercial condition, it contains numerous impurities, from which it is freed by sublimation or distillation, by means of which the non-volatilisable portion is retained, and the sulphur obtained as in Experiment 54.

Sulphurous acid is a compound containing one equivalent of sulphur united with two of oxygen. Symbol SO_2; equivalent = 32. It is at once produced when sulphur is ignited in the open air, and is the cause of the penetrating smell of the ordinary sulphur-match whilst burning. It may be produced as in Experiment 57; or, still better, by boiling sulphuric acid in a retort, with charcoal, mercury, &c., by means of which the acid is decomposed, forming an oxide of the substance, and setting sulphurous acid free. In this way sulphurous acid is produced in a gaseous form; and by extreme cold it may be obtained as a liquid, which, from its high volatility, produces great cold during evaporation. Sulphurous acid is readily absorbed by water; and, united with various bases, forms the substances called *sulphites*. It is owing to the production of sulphurous acid during the combustion of sulphur, that its bleaching powers are manifested, as shown in Experiment 57.

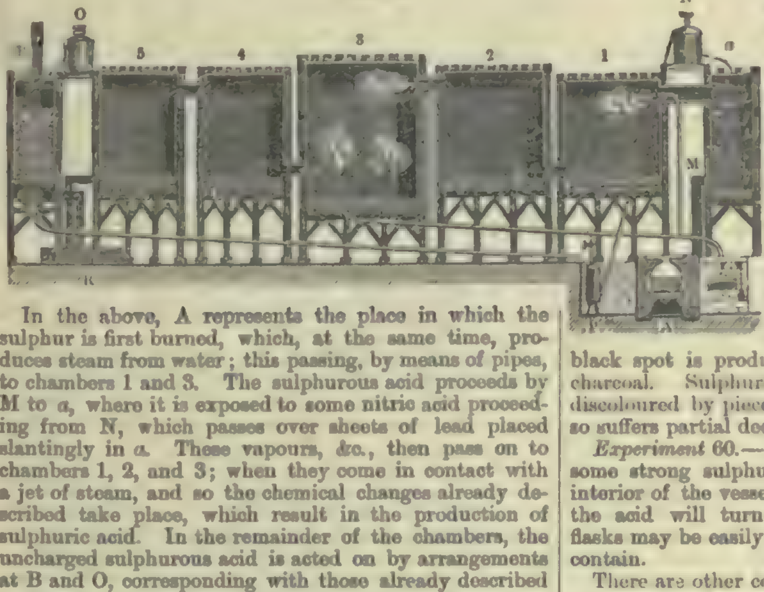
Sulphuric Acid is a liquid well known in the arts and commerce. It is a compound of one equivalent of sulphur with three of oxygen; and in the anhydrous or perfectly dry state, its symbol would be SO_3, and the equivalent = 40. In its ordinary condition, and at a specific gravity of 1·850, sulphuric acid is combined with one equivalent of water; hence its symbol will be SO_3, HO, and the equivalent 49. The manufacture of this acid has become of the highest importance in this country, as it is largely employed in producing the soda of commerce, and in bleaching and calico-printing. Its source is the combustion of sulphur, which may be in the ordinary state, or as produced from iron and copper pyrites. The latter source of the sulphur is much used in Glasgow, where the acid is produced in large quantities by Messrs. Tennant and other manufacturers. The process depends on the mutual action of sulphurous acid, the vapour of nitric acid, and the steam from water. The sulphurous acid, produced by the slow combustion of the sulphur, is allowed, together with the vapour of water, to meet that of nitric acid, produced from nitre in the usual manner, and being allowed to pass in at the same time. By these means the nitric acid is partly decomposed, and binoxide of nitrogen is produced. The latter gas seizes oxygen from the air, and so the sulphurous acid is gradually oxidised; and by its acquiring another equivalent of oxygen, sulphuric acid is produced. The process is carried on in large leaden chambers, at the bottom of which water is placed, which condenses the acid fumes. The dilute acid thus afforded is afterwards concentrated by boiling; and if required purer and freer from lead, it is subsequently distilled in platina stills, whence it issues as an oil-like fluid, having considerable gravity. From this appearance, sulphuric acid often is termed oil of vitriol.

Fig. 58.



Our diagram, Fig. 58, represents a section of a sulphuric acid chamber. On the left is the furnace supplying the vapours of sulphur and nitric acid; in the centre, we have the large leaden chambers in which the two vapours combine together with steam produced from a boiler, represented on the right of the diagram. In practice, a larger number of leaden chambers is employed; and the following engraving represents an arrangement often used on the large scale for the manufacture of sulphuric acid.

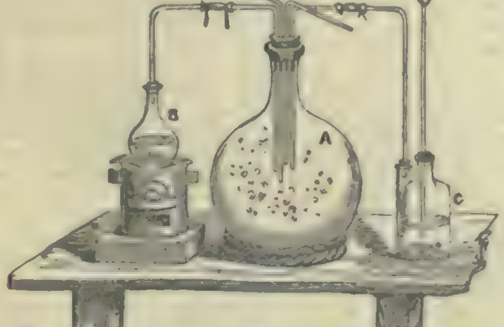
Fig. 59.



In the above, A represents the place in which the sulphur is first burned, which, at the same time, produces steam from water; this passing, by means of pipes, to chambers 1 and 3. The sulphurous acid proceeds by M to *a*, where it is exposed to some nitric acid proceeding from N, which passes over sheets of lead placed slantingly in *a*. These vapours, &c., then pass on to chambers 1, 2, and 3; when they come in contact with a jet of steam, and so the chemical changes already described take place, which result in the production of sulphuric acid. In the remainder of the chambers, the uncharged sulphurous acid is acted on by arrangements at B and O, corresponding with those already described at N and *a* at the right of the engraving. By such an arrangement all the materials are utilised; none are suffered to go to waste; and the manufacture of the acid goes on without intermission, so long as a supply of the materials is kept up.

The student may easily master the details of the process by means of the following plan:—Put some mercury

Fig. 60.



and a little sulphuric acid into a glass flask, and connect this, by means of a bent tube, with a larger flask, as seen in Fig. 60, in which A is the vessel receiving the vapours of sulphurous acid, produced by boiling the sulphuric acid and mercury in B. Pass some binoxide of nitrogen from another vessel, C, containing some copper filings and nitric acid, into the large flask, A, which should be moistened internally with water. As the sulphurous acid vapour comes in contact with the binoxide of nitrogen and air in vessel A, the decomposition, &c., already described, proceed just as they take place on the large scale in the leaden chambers, and sulphuric acid is accordingly produced. The preceding engraving illustrates this arrangement.

Sulphuric acid, in a concentrated state, is highly corrosive, acting with great vigour on all organised bodies, chiefly by the great attraction it has for water, which it will seize wherever it can be found. It unites readily with bases forming sulphates. It requires great heat to evaporate it; and the operation should only be conducted in platina vessels, on the large scale, because the vapour is disengaged so irregularly in glass vessels as to be almost explosive in its action. It may, however, be effected in small glass retorts, if a few pieces of platina foil be cast therein to facilitate the disengagement of the vapour. The following experiments will illustrate some of the properties of the acid.

Experiment 58.—Pour some strong sulphuric acid suddenly into a vessel containing an equal bulk of cold water. Great heat will be disengaged, which will sometimes rise so high as 300° Fah.

Experiment 59.—Into a strong solution of sugar pour some concentrated sulphuric acid, which will have so great an attraction for the water as to set free the charcoal of the sugar as a black spongy mass. In a similar manner, if the strong acid be dropped on wood, &c., a black spot is produced, owing to the liberation of its charcoal. Sulphuric acid of commerce is thus often discoloured by pieces of straw, &c., falling into it, and so suffers partial decomposition.

Experiment 60.—Into an oily Florence flask pour some strong sulphuric acid, and run it about over the interior of the vessel. The oil will be decomposed, and the acid will turn black. By these means Florence flasks may be easily cleansed from the oil they usually contain.

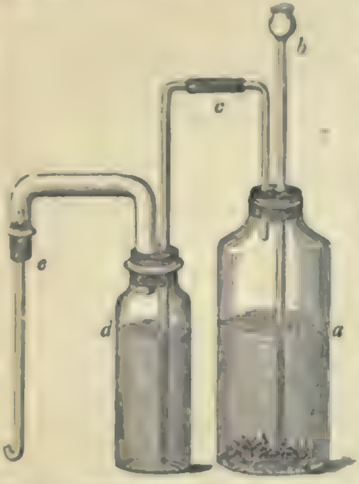
There are other combinations of sulphur with oxygen, which, with the exception of hyposulphurous acid, are of no importance. The only combination of this acid worthy of notice, is the hyposulphite of soda, which is so largely employed as a "fixing" agent in photography, and is produced by boiling soda with sulphur in excess.

Hydrosulphuric Acid.—This is generally termed, in chemical works, sulphuretted hydrogen; and it is a compound of one equivalent each of sulphur and hydrogen; symbol, HS; equivalent = 17. This gas is well known as being the cause of the offensive smell of house-drains, and arises therefrom, owing to the putrefactive decomposition of matter containing sulphur and hydrogen. It is a gaseous body, easily condensed by water, and is readily procured by pouring dilute hydrochloric acid on any metallic sulphide, such as the sulphide of iron or antimony. Its solution in water is much employed in the laboratory as a test for the presence of metals; and the following plan may be adopted for thus procuring it.

Into a glass jar fit a cork, through which pass two tubes, one of which should reach to the bottom of the vessel, whilst the other just passes within its neck. Attach this tube to another vessel, by means of india-rubber connections, so that it may be continued to the bottom of the second vessel: from this another pipe should pass to act as a safety-tube.

The arrangement is represented in the following engraving.

Fig. 61.



In the annexed figure, *a* represents the vessel in which the gas is manufactured; *b* is a funnel, by means of which more acid can be continually added to the sulphide; *c* is the pipe through which the gas passes to the vessel *d*, in which water is placed, which absorbs and forms a watery solution of the gas; *e* is a bent tube, which allows of the escape of uncondensible gas. We need scarcely remark, that the cork in each vessel should be air-tight in their necks, so as to prevent the escape of gas into the air. Instead of water, a solution of ammonia may be used in the vessel *d*, by which the hydrosulphate of ammonia is formed—a usefuland necessary test in the laboratory. Sulphuretted hydrogen gas is a deadly poison, and is partly the cause of many diseases common in badly-drained districts. It may be detected by means of lead in the form of a solution of the acetate. It is readily decomposed by chlorine; hence the use of chloride of lime in places wherever it abounds. The following experiments will impress these facts on the minds of our readers.

Experiment 61.—*To detect Sulphuretted Hydrogen.*—Dissolve a little sugar of lead in water; into this dip some white blotting-paper. On holding a strip of this paper in the jar of gas, or over a foul drain, it will turn black at once.

Experiment 62.—Into a jar of the gas introduce a little chlorine. The sulphur will be immediately separated, and all smell will vanish.

Experiment 63.—*Magic Writing.*—On a piece of paper write anything, using as ink a little solution of sugar of lead, and a quill pen. Let it dry, and the writing will be invisible. On moistening it with water, and holding it in a jar of sulphuretted hydrogen gas, or over a drain, the writing will at once be evident, and in black characters.

Sulphur unites with many substances, forming what are termed sulphides. Of these are the sulphide of antimony, iron and copper pyrites, the sulphide of lead, commonly called galena, &c. The combination of iron and sulphur is readily produced by holding a roll of brimstone against a red-hot iron rod, when the latter will fall down in globules of the sulphide of the metal. Many metallic ores, besides those we have named, contain the metal united with sulphur. The combination of sulphur and carbon, known as the bisulphide of carbon, will be further noticed as a solvent of phosphorus, when we describe that element.

SELENIUM.

We shall here call attention to an element which is rarely met with. It chiefly resembles sulphur in its relations: symbol, Se; equivalent, 39·5. With oxygen it forms selenic acid; and, combined with hydrogen, it affords seleniuretted hydrogen gas. It may be reckoned only as a chemical curiosity, for as yet it has had no application of any value whatever; and, indeed, its scarcity renders such, for the present, impossible.

PHOSPHORUS.

But a few years ago phosphorus was merely a chemical curiosity, or an object for the amusement of young persons, by its peculiar properties. It has, however, lately become an important article of commerce; and in the manufacture of lucifer-matches thousands of persons find employment.

Phosphorus is obtained chiefly from bones, which contain a large amount of phosphate of lime.* The bones are first calcined, and sulphuric acid added thereto. This decomposes the phosphate of lime. The liquid is evaporated to the consistence of a syrup, and mixed with charcoal. The mixture is then strongly heated in a retort, by which the phosphoric acid is decomposed, and the phosphorus distilled over as a wax-like substance. It is received in vessels containing cold water. Its symbol is P, and the equivalent 32.

At ordinary temperatures phosphorus undergoes a slow combustion if exposed to the air; and hence it should always be kept in bottles containing water, so that no part of the substance be above the surface of the liquid. We would impress this precaution strongly on our readers, as we have witnessed some serious results arising from its neglect: in one instance, an extensive laboratory was nearly destroyed by fire, through a stick of phosphorus having been left carelessly on a wooden table. It should never be cut except under water, nor touched by the finger, as the burn it produces is very serious, and difficult to cure. It takes fire at a temperature of about 150°. It has, however, the power of assuming what is termed an allotropic condition; for after having been heated till it obtain a red appearance, it loses its power of spontaneous inflammability, and becomes entirely changed in many of its properties. In this respect it much resembles sulphur, which has also the power of becoming allotropic. It combines with oxygen, hydrogen, &c., in a manner of which we shall presently speak. The following experiments will illustrate some of its ordinary properties.

Experiment 64.—Rub a small piece, by means of a pair of iron nippers, against a wall in a dark room; every mark will appear of a beautiful white light. This effect is called phosphorescence.

Experiment 65.—Boil a few small pieces of phosphorus in a flask, with some water; sparks will rise in all directions through the liquid; and if, at the conclusion of the experiment, you throw out the water into the open air on the ground, a splendid flash of light will be shown.

Experiment 66.—Into a test-tube pour half an ounce of olive oil, and add one small piece of phosphorus; heat *gently* over the spirit-lamp. A small quantity of the phosphorus will dissolve; and by rubbing the oil over the hands and face they will become quite luminous. This is a harmless experiment, and the oil can be easily removed by soap and water.

It is owing to the presence of phosphorus that many animal bodies afford a lambent light at ordinary temperatures. This is often noticed at sea, and during summer evenings: the wake of a steam-vessel, and the back-water cast from the paddles or screw, frequently present a most beautiful appearance, owing to the millions of small insects emitting a phosphorescent light. Crabs, lobsters, and other animal substances, the glow-worm, and fire-fly, owe their luminosity to the same cause. Oyster and other shells often afford a phosphorescent light after having been heated.

A very curious application of phosphorus may be made by its solution in the bisulphide of carbon, to which a little wax may be added. If the liquid so produced be poured upon paper, cloth, &c., after a short time such will burst into flame. Captain Norton has proposed to employ the liquid for the purposes of warfare. He encloses it in shells, which may be fired in the usual manner. On such coming in contact with the sails or hull of a ship, they will at once break, and, spreading the liquid on all sides, cause the destruction of the vessel. We cannot too strongly caution our readers as to the use of this liquid, as whatever combustible substance it touches it at once inflames. Even pouring water over it will not ensure subsequent safety.

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* See *ante*, p. 360, *et seq.*

By means of a mixture of phosphorus, sulphur, and chlorate of potass, the ordinary lucifer-match is made, which, by mere friction, affords, as is well known, our chief source of artificial light and heat. The following experiment illustrates the production of light and heat beneath water, by those materials.

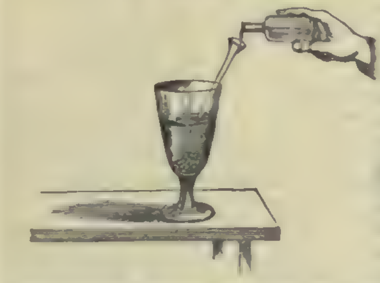


Fig. 62.

Experiment 67.—Into a wine-glass put a few grains of chlorate of potass; fill up with cold water, and then add a few pieces of phosphorus. Through a glass tube pour a little sulphuric acid; when this reaches the bottom of the glass, flame will burst out. This experiment requires caution, as sometimes the phosphorus flies out of the glass, and might injure the eye.

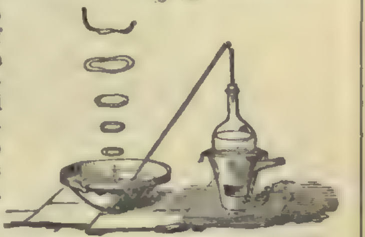
The most important combination of phosphorus with oxygen is that known as phosphoric acid: symbol, PO_5; equivalent, 72. This is produced by the combustion of phosphorus in oxygen gas, and is of great importance in animal economy. Combined with lime, it forms the phosphate of that substance which, as we have already shown, is an important constituent of the bones of animals. Phosphoric acid combines with other bases forming phosphates, such as those of soda, silver, &c. The phosphates of lime are found in the excrements and urine of animals; and to their presence in guano, together with nitrogen, the value of that substance as a manure is due. With three equivalents of oxygen, phosphorus forms phosphorous acid: symbol, PO_3; equivalent = 56. This is produced when phosphorus undergoes slow combustion in a limited amount of air.

Phosphorus combines also with chlorine and iodine, forming definite compounds; which, however, do not possess any special interest.

Phosphoretted hydrogen gas is a compound of one equivalent of phosphorus to three of hydrogen: symbol, PH_3; equivalent = 35; and has the singular property of spontaneous combustion. It may be readily prepared from the phosphide of calcium, the latter being produced by heating together fresh-slaked lime with phosphorus, in a crucible or test-tube. If a piece of the phosphide be put into a little warm water, beautiful rings of smoke are formed, as represented in the annexed figure.

Fig. 63.

Phosphoretted hydrogen may also be produced by boiling a solution of potass and phosphorus together in a Florence flask, to which a bent tube is attached. The end of the tube should dip below the surface of water; and as the smoke arises therefrom in rings, it will catch fire. An arrangement suitable for this purpose is shown in the annexed engraving.

Fig. 64.


BORON.

Boron is an element of which little is known, in a free state. Its symbol is B; equivalent equals 10·9 Combined with oxygen, however, it assumes considerable commercial importance as boracic acid; its combination with soda forming the well-known borax. Boron is prepared by heating boracic with potassium in a test-tube, and, as such, is a green powder, which on combustion again produces boracic acid. This acid may also be procured by decomposing borax, by means of sulphuric acid, which unites with the soda, setting the boracic acid free. When this substance is dissolved in spirits of wine, it affords a green colour during combustion. Borax is largely employed by metallurgists and chemists as a flux, and is particularly useful in blow-pipe experiments. Boron unites with chlorine to form the chloride of boron, and, with fluorine, it forms the fluoride.

CHAPTER IV.

THE METALS PROPER, KALIGENEOUS, AND TERRIGENEOUS.

One of the most difficult subjects in chemical science is that of defining the exact character of a metal. So long as the number of metallic bodies was confined to those known by the ancients, no difficulty existed; for the properties of gold, silver, &c., are so distinct and marked as to leave no doubt respecting them. During the early part of this century, however, the discoveries of Sir Humphry Davy brought to light metals which differed entirely in their chief characters from those previously known. Thus, for instance, potassium and sodium have very low specific gravities compared with lead, iron, &c.; they have very little tenacity; cannot be drawn out into a wire, or beaten into sheets; and yet, from their various qualities, no doubt can be entertained as to their being properly termed metals. The same may be remarked in reference to calcium, magnesium, &c., which form the basis of the earths, called lime, magnesia, &c. In classifying bodies as metallic, we must have regard to general rather than special properties; and amongst the former we may notice the tendency which all metals have to combine with oxygen, chlorine, &c.; and of forming salts with oxygen and acids. Generally speaking, the metals are susceptible of receiving a high polish, which they retain in proportion as they have little affinity for the oxygen of air and water. Thus, platina, gold, and silver, will maintain a polish on the surface for almost any length of time; whilst iron, lead, and zinc soon become tarnished by oxidation. The tenacity of metals differs exceedingly; for whilst steel, gold, and silver may be drawn into wire finer than a human hair; lead, tin, &c., can scarcely be drawn out at all.

The uses of metals are so numerous and so well known that we need scarcely refer to them; but, perhaps, amongst all, iron is of the greatest value to mankind. Without it nearly every implement of manufacture would be wanting; and, strange to say, it is the most abundant metal. It is a constituent of our bodies, affording the colouring matter of our blood: it abounds in plants and minerals; and the colour of the earth and rocks is frequently due to its presence. Within the last few years a valuable addition has been made to our list of useful metals, in the production of aluminum, which is a constituent of common clay. For years its existence was recognised by chemists; but to M. St. Claire Deville, of Paris, and to the liberal patronage and aid of the Emperor of the French, we are indebted for the means of producing this metal in quantities. It is highly probable that other metals, such as calcium, magnesium, &c., which at the present time are mere chemical curiosities, will eventually, like aluminum, become of use in the arts and manufactures.

Considerable difficulty arises in properly classifying metals for the purposes of describing them in groups. This will be evident from the remarks we have already made on their individual properties. In the following pages we shall divide them into three classes; namely, metals proper, kaligeneous, and terrigeneous. Amongst the first are such as gold, platina, silver, &c.: in the second class we have potassium and sodium; and in the third, or those which as oxides produce the earths, we have barium, strontium, &c. Amongst these we shall include silicon, or silicium, which, although not properly a metal, will be more conveniently considered with the earths than apart by itself.

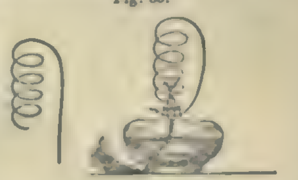
With respect to the oxides of metals, we may mention, that although, properly speaking, all the oxides may be considered to be earths, the latter term is confined to those oxides which produce the constituents of rocks, &c., in reference to mineralogical and geological doctrines. Thus, oxide of iron is a component part of nearly every earth, yet it is not reckoned of itself to be an earth. Such distinctions may be considered to be unnecessary, but yet they are required in the present state of scientific arrangement; because although we may properly hope for an extension of our knowledge in reference to the constitution of elementary bodies, we are compelled to follow the analogies which existing facts point out. The oxide of barium, or of calcium, is perfectly analogous to that of zinc or silver; yet we designate the latter as ores, whilst the two former are termed earths. Hence arises what we may term a conventionalism of science, in which its principles are recognised, but not absolutely insisted on. We admit the general laws, but also make exceptions as to their details. In this consists the humility of philosophy; for we must admit our ignorance, whilst we use the facts of which we are possessed. In the words of Newton, we pick up one pebble, whilst we reject that we previously gained, leaving the ocean of truth still unexplored before us. In the following pages we shall first treat on what are termed metals proper; we shall then describe those which by oxidation produce what are termed alkalies; and, lastly, we shall deal with the metals which, in combination with oxygen, afford us the well-known substances called earths, including, as we have already stated, the substance called silicon, and its oxide, silica, or flint.

PLATINA.

In many works on chemical and metallurgical science, the term "noble" is applied to platina, gold, and silver, from the fact that they are almost unalterable by any of the numerous agents which readily affect the appearances of other metals. Platina eminently sustains this character, inasmuch as it withstands the agency of heat, being nearly infusible; and none of the acids affect it, except aqua regia, which is employed as a solvent for it. Platina is, comparatively speaking, a rare metal, and is generally found in the form of small grains; and in this state occasionally contains iridium, osmium, &c. It is obtained pure by dissolving the native metal in aqua regia, and afterwards it is precipitated as a yellow powder, by means of chloride of potassium. It is then heated with carbonate of potash; the mass so obtained being again dissolved by means of aqua regia; and the platina is then thrown down by a solution of sal-ammoniac, which affords what is termed spongy platina, after being strongly heated. The powder is made into ingots, by being rammed into iron moulds, and heated in a wind-furnace. This process agglutinates the mass, which, when cool, is hammered. Platina has the property of welding at a white heat, like iron; and it may afterwards be rolled out into sheets, foil, &c., or drawn out into wire.

It resists the action of the most intense heat of our furnaces, being only melted by the oxy-hydrogen blow-pipe, or by means of voltaic electricity. It has a colour somewhat resembling, but darker than, silver, and is susceptible of a high polish, which it long retains. When obtained in a spongy state, as already mentioned, it has the power of igniting a mixture of oxygen and hydrogen gases, causing their combinations. For this purpose platina is used in the Dobereiner lamp, and occasionally in the laboratory. Employed in a similar manner with alcohol, it maintains a slow combustion, which results in the conversion of the spirit into acetic acid: even in the form of wire it may be made to maintain combustion without the aid of a wick; and is so used in lamps constructed to diffuse perfumes in an apartment. A piece of platina foil, twisted in the manner shown in the annexed engraving, and surrounding the wick of the spirit-lamp, will continue red-hot, if the wick be first ignited, and then gently extinguished by blowing on it.

Fig. 65.



The chemical symbol of platina is Pt; its equivalent 98·7; and its specific gravity varies from 21 to 22; that of water being 1.

In this country the uses of platina are almost exclusively confined to chemical vessels for laboratory purposes, and for the distillation of sulphuric acid. In Russia it is used as coin; and occasionally it is fashioned into snuff-boxes, &c. Spread in a state of fine powder on silver plate, it forms the negative plate of the Smee voltaic battery. Its solution in aqua regia is also employed by workers in silver, &c., to produce a dark bronze-like appearance. From the difficulty with which it is acted on by chemical agents, its combinations are not so numerous as those of some other metals. It combines with oxygen to form an oxide and binoxide. The protoxide is produced when a solution of potass is added to one of the bichloride of platina, and is a black powder, which is readily decomposed by heat. The binoxide is similarly produced from the bichloride.

The chief combinations of platina are the two chlorides. The bichloride is obtained by dissolving the pure metal in nitro-hydrochloric acid, when a deep yellow or orange-coloured liquid is produced; and the chloride may be obtained from this by heating the bichloride to a temperature of about 400° Fah. These two chlorides are readily distinguishable; the bichloride affording a yellow-coloured solution in water, whilst the chloride is insoluble. With ammonia, the bichloride of platina affords a double salt, which is obtained by the addition of a solution of sal-ammoniac to that of the metallic salt. This double salt is often produced in the course of organic analysis, and affords a ready means of estimating the amount of nitrogen present in the substance undergoing examination. For this purpose the ammonia is converted, by means of hydrochloric acid, into the hydrochlorate, which is subsequently precipitated, by means of the chloride of platina, as a yellow-coloured powder; and the constitution of the salt so produced, leaves the calculation of the ammonia an easy matter.

GOLD.

This metal is so well known, and so extensively employed for numerous purposes, that it requires but little description. It has a specific gravity of about 19·25; its symbol is Au; and its equivalent is 197. Like platina, gold undergoes no change with the most intense heat of our furnaces; it is simply melted, being quite unvolatilisable.

Although so precious a metal, it is widely diffused throughout nature, and is discovered in those rocks of which the mineral quartz is a component part. It is also found in grains or nuggets, of sizes varying from a grain to some hundred ounces in weight, in the beds of many rivers.

Generally speaking, it may be expected in any place where the primitive rocks abound, and in the granite, and allied formations; it has been discovered in many parts of Scotland, amongst which we may mention the Isle of Arran. In the south-west of England, as in Cornwall, gold has often been met with; but so far as our experience goes, at the present time there is no source in Great Britain which would repay the expense of obtaining the metal.

In Russia, Germany, Africa, and Peru, gold mines are worked to a profit; but we need scarcely state, that Australia and California are the chief places from which Europe obtains a supply of the metal.

The dust and nuggets picked up or mixed in the sand of auriferous rivers, require little more than melting before being introduced into commerce: when found attached to quartz, the stone is broken by means of powerful machinery; and the powder thus obtained is washed by an abundance of water until all the earthy particles are removed. This is sometimes done by means of a cradle, which being kept in continual motion, permits the earth to be washed away; whilst the heavy gold falls to the bottom, and is subsequently collected by the miners. In cases where the quantity of gold is comparatively small, the process of amalgamation is adopted; mercury is mixed with the golden sands, and it thus dissolves the gold, just as sugar would be dissolved by water; the gold and mercury, forming the amalgam, are put into a suitable vessel. Heat is employed, and the mercury distilling over, leaves the gold behind in an almost pure state: gold, thus procured, is generally cast into blocks called ingots, and is then ready for the various purposes to which we shall presently refer.

Gold may be distinguished from other metals by the following characteristics:—In its pure state it is easily cut by a knife; it is insoluble in almost every liquid, with the exception of nitro-hydrochloric acid: if it be thus dissolved, it affords a rich purple colour on a solution of tin being added thereto; and sulphate of iron throws the gold down, in the form of a dark-coloured powder, in the metallic state: it may be distinguished from brass by means of nitric acid, which dissolves the base metal, and leaves the gold untouched.

The uses to which gold is applied are very numerous; wherever magnificence, endurance, and costliness are involved, the metal is called on to take a part. Pure gold is never used for commercial purposes, being far too soft. In the current coin of the realm, it is alloyed in the proportion of twenty-two parts of gold to two of copper. By these means it acquires sufficient hardness to withstand the wear and tear to which it is exposed. It is alloyed in various proportions, for the manufacture of jewellery, watches, &c. Metals are readily gilded by being covered with an amalgam of gold; which is made by adding that metal to mercury. The latter is driven off by heat, leaving the gold as a thin coating. Gold is highly ductile and malleable; it may be beaten out into leaves which are so thin as to require between two and three hundred thousand of them to form a pile one inch thick. This is effected by beating the metal with heavy hammers, between skins of parchment. Gold may be drawn into wire as fine as a human hair.

Like platina, its combinations are few, owing to the weak affinity which subsists between it and chemical reagents. It forms two combinations with oxygen: the sub-oxide is obtained by precipitating the sub-chloride, by means of a solution of potash. Auric acid, or the peroxide, is a compound of gold and oxygen; and is obtained from the perchloride by digesting it with magnesia, and boiling the resulting mixture in nitric acid; which affords a yellow-coloured powder.

The most important compounds of gold are the chlorides and cyanides. The chloride is obtained by dissolving gold in nitro-hydrochloric acid; and the dichloride by heating the chloride so obtained to a temperature of 500° Fah. The cyanide is chiefly employed for the purposes of electrotyping; and for its description, manufacture, and uses, we refer our readers to the section on Electricity (article Metallurgy); in which a full account may be found.

A kind of fulminating gold—different to that which is sometimes termed the fulminate of gold—is produced by adding a solution of strong ammonia to one of the chloride, and by pouring more ammonia on the precipitate so produced. This powder explodes on being rubbed by a hard substance. The fulminates proper we shall describe hereafter. Besides the alloys with copper, which we have already mentioned, gold unites with silver, lead, &c., in various proportions. It may be obtained from such alloys by the process termed cupellation, which we have already described. The purple powder, produced by precipitating a solution of chloride of gold by means of the chloride of tin, is employed for the purpose of colouring china ware. It is termed in the arts, the "purple powder of Cassius."

As it is often desired to distinguish gold readily from the baser metals resembling it in colour, we may mention that this may be done by rubbing the suspected article on a rough hard stone, for which the "bloodstone" is usually employed. A very minute portion of the metal is thus removed to the stone; and, on the streak, a little nitric acid is to be dropped. If the metallic appearance be lost, it is evident that gold is not present, because that metal will not dissolve in nitric acid.

Steel articles may be easily gilded by dipping them into a liquid, made by adding ether to the solution of the chloride of gold. The ether takes up a portion of the metal, from which the steel article removes it to its own surface. Care should be taken that the metal does not touch the acid solution resting beneath the ether.

SILVER.

Silver, as we have previously remarked, belongs to a class of metals not readily acted on by ordinary reagents; but being more so than those we have already described, we have taken it last of the three. Its symbol is Ag; its equivalent 108·1; and its specific gravity = 10·5. It is found in a pure state in nature; and by a late discovery, which was accidentally made during an analysis of a sheet of copper, taken from the hull of a vessel which had been long at sea, it has been shown that the ocean contains many millions of tons of the metal. Silver is found in most of the lead ore of Great Britain, and is separated therefrom as much as possible, because even if not present in sufficient quantities to pay for extraction, it injures the lead by making that metal brittle. The chief sources of silver for commercial purposes, are Mexico, Chili, and Peru. It is sometimes found combined with other metals, and occasionally with sulphur. It is obtained from the ore by amalgamation, in a similar manner to that we have described in respect to gold.

Silver is easily distinguished from other metals. In a pure state it has a white colour, and is susceptible of a high polish; it, however, is readily acted on by air containing sulphur, and hence soon becomes tarnished in rooms wherein coal-gas is burned. It is readily soluble in nitric acid (*aqua fortis*); and if a solution of common salt be added thereto, a white curdy powder falls down, which turns to a dark colour on exposure to light.

It is readily separated from other metals by cupellation; and it forms alloys with copper, gold, silver, &c.; and an amalgam with mercury. The latter mixture is much employed for the purposes of plating, which, however, has been now almost entirely superseded by electro-plating processes, which we have fully described in the section on Electricity; article Metallurgy.

Our remarks in reference to the uses of gold are equally applicable to silver, which, to a considerable extent, possesses the same qualities. For the purposes of coin, silver is alloyed with copper; one pound of standard silver being coined into sixty-six shillings. It is largely employed for making various domestic utensils and ornaments.

It may be beaten into leaves of a thickness not exceeding $\frac{1}{100000}$th of an inch each. It is also easily drawn into fine wire; and in this state is, like gold, much used for embroidering drapery, &c. From being unacted upon by many chemical substances, it has been employed to line cylinders in which coffee is roasted, and also in vessels used in the manufacture of lemonade and soda-water.

Pure silver may be obtained by dissolving common silver coin in nitric acid. To this solution, common salt is to be added as long as the chloride falls down. The precipitate is then to be dried and mixed with carbonate of soda, and a small portion of powdered charcoal. It is then to be heated in a crucible, in a wind-furnace, by which the silver will be reduced from the chloride to a metallic state. Other methods for obtaining the same result in an economical manner, have already been referred to in the article on Electro-Metallurgy.

The salts, &c., of silver, are of considerable importance. In fact, at the present time, they have become an extensive article of commerce, for the purposes of photography. Their uses in this respect have already been fully described under the head of Photography, in the section on Light; to which we must direct our readers for information on that interesting application of optical and chemical science.

Silver combines with oxygen to form three oxides. The sub-oxide, $Ag_2 O$, is of little importance. The protoxide, $Ag O$, is the base of the salts of the metal, and is produced by adding a solution of potass or lime to the solution of the nitrate of silver; when a brown-coloured powder is produced. The peroxide, $Ag O_2$, is the result of electric action in bad-conducting solutions of the nitrate: its production often interferes with the regularity of electro-plating processes. The chloride, $Ag Cl$, has been frequently referred to in our previous pages; both under the head of Photography, and as an illustration of chemical changes. The cyanide of silver has also been fully described, as employed in electro-plating, and with respect to its constitution and manufacture. The iodide and bromide of silver have no interest except in connection with photography; and their uses and nature in that respect have been described in connection with that subject.

The nitrate of silver, as we have already remarked, is a substance of great importance. Besides its uses in photography, it is employed in medicine in a fused state, as the "lunar caustic" of the shops; and most kinds of ink, used for marking linen, have this salt as their basis. Nitrate of silver is produced by dissolving pure silver, by heat, in dilute nitric acid; from this solution it may be readily crystallised. Its solutions turn black on exposure to light, if they contain any animal matter; hence their employment as hair-dyes. The sulphate of silver is produced by boiling silver with strong sulphuric acid. The carbonate results from a mixture of carbonate of soda with nitrate of silver, both being in solution. The acetate may be similarly produced by substituting the acetate of soda. We have mentioned that polished silver readily tarnishes when exposed to the action of sulphur; hence plate, spoons, &c., soon lose their polish; and for a similar reason, silver spoons, if not gilded, become black on being used with eggs or mustard; each of which contains sulphur as one of their constituents.

Fulminating silver is a compound of the metal with nitrogen and hydrogen; and it is produced by adding alcohol to silver dissolved in strong nitric acid, when a powder is produced, which explodes violently on the slightest friction. We shall describe the process more fully when speaking of the fulminate of mercury. An explosive compound of silver is produced by adding liquid ammonia to the oxide of silver; forming what is termed the ammoniuretted oxide. This, however, is not a fulminate of the metal.

MERCURY.

THIS well-known metal forms an exception to the general rule that such bodies are solid, inasmuch, as at ordinary temperatures, it is in a fluid condition. In the Arctic regions, however, it is frequently observed in the solid state, owing to the abstraction of its latent heat by means of the intense cold prevalent in those latitudes. From the readiness with which this metal moves about, it has acquired the name of "quicksilver," by which it is generally known in the arts and in commerce. Its specific gravity is about 13·5; its symbol, Hg; and the equivalent = 100.

Our chief sources of mercury are South America and Spain, from which countries it is imported in wrought-iron bottles. It is obtained from its ores by the process of distillation, just as spirits or water are distilled over by the application of heat. The oxides of mercury are, in fact, readily decomposed by heat, the attraction between the metal and the oxygen being comparatively slight. It boils at a temperature of about 650° Fah, and its vapour has a highly expansive power. This circumstance affords a ready means of separating it from various impurities with which it is often contaminated. The mercury is introduced into a retort, and a moderate heat is applied. The quicksilver is by this plan volatilised, and condenses into small globules, which run into a receiver. The impurities, such as lead, tin, and bismuth, are left behind in the body of the retort, and are so separated from the mercury.

The uses of the metal are very numerous. In Brazil, California, Australia, and other countries in which gold-sands abound, mercury is used to dissolve out the fine grains, which are too small to be seen by the naked eye. The amalgam of the gold so formed is then introduced into a retort, and heat is applied. The mercury distils over, and leaves the gold pure. The metal silver is often separated from its ores by similar means, and the process is termed "amalgamation." There are many gold mines in several parts of the world, which would be utterly valueless but for this singular property of quicksilver.

We may here caution our readers to be careful lest any gold or silver ornament or coin should come in contact with the metal, for it soaks into them just as water enters the pores of loaf-sugar. They are thereby rendered very brittle; and if exposed to any mechanical action, are sure to break. In the case of mercury being absorbed by either of the metals named, the best plan is to place them on a piece of iron over a common fire, by which the mercury will be at once driven off, and the article restored to its old state.

Quicksilver is largely employed for "silvering" looking-glasses. A sheet of plate-glass, previously well cleaned, is placed on a board having a slight inclination. A piece of tin-foil is then pressed on to the face of the glass, and over this some quicksilver is spread by means of a hare's foot. A sheet of stout paper is then placed on the foil; and on this heavy weights are put for the purpose of driving off any superfluity of mercury. After a day or two's time the weights are removed, when it is found that the amalgam has adhered to the glass, and has so afforded it a reflecting surface.

As we have already mentioned, amalgams are formed by mixing mercury and other metals together. The amalgams of gold and silver are used for plating purposes. The metal to be plated is coated with the amalgam, and afterwards exposed to heat, by which the mercury is driven off, and the gold or silver left on the surface. This process is called water-gilding and plating. It has, however, gone much out of use since the introduction of the electrotype process. An amalgam of silver is employed by dentists for the purpose of filling hollow teeth.

The metal is also much used in making barometers and thermometers. It is of great value for the former purpose, owing to its fluidity and great specific gravity. Indeed, a column of mercury of a height of thirty inches, balances one of water thirty-two feet high. In thermometers, it shows the increase of temperature from very intense cold to its boiling point, by a gradual and almost regular expansion—more so, in fact, than any other liquid with which we are acquainted.

The combinations of mercury are both numerous and valuable, and of considerable use in the arts, medicine, and commerce. It forms two definite combinations with oxygen. The sub, or black oxide, is produced by adding a solution of potash or soda to those of the sub-nitrate or sub-chloride of the metal, and is so obtained as a dark-brown or black powder. The protoxide is produced by adding the solutions of the alkalies or alkaline earths to those of chloride of mercury, or corrosive sublimate, as it is frequently termed. It is of a red colour; and the oxygen may be separated from the metal by distillation, when the latter passes over in a free state. Mercury resembles the metals we have previously described in this respect. The protoxide may also be obtained by heating the nitrate. The sulphate is readily produced by boiling mercury with concentrated sulphuric acid; during which process large quantities of sulphurous acid, in a gaseous state, are evolved. Nitric acid dissolves mercury, by which the proto-nitrate is produced. On diluting the concentrated solution, a sub-nitrate falls down in the form of a white precipitate. The solution of the nitrate is often sold by unprincipled persons in the streets of our large towns, as a polishing liquid. If applied to a silver or copper surface, it at once produces a brilliant polish; but penetrating into the pores of those metals, and forming an amalgam, their tenacity is at once destroyed, and the article rendered useless.

The chlorides of mercury are of great importance as forming two well-known substances used in medicine. The sub-chloride is the well-known calomel. It is a white substance, and, when closely examined, has a crystalline structure. It may be produced by adding a solution of common salt to one of the sub-nitrate of mercury, or by rubbing together, in a mortar, metallic mercury, common salt, and corrosive sublimate. The product is then sublimed, and the sub-chloride, or calomel, is so obtained in a pure form. It is but sparingly soluble in water, and may be so distinguished from the other chloride. Corrosive sublimate is well known as a deadly poison to animals. In minute quantities, it is, however, employed in medicine; and both it and the sub-chloride have the property of producing a great flow of saliva, and for this purpose even moderate doses are sufficient. Corrosive sublimate is used as a preservative agent for both animal and vegetable substances. It may be prepared by burning mercury in a jar of chlorine gas, or by subliming common salt and the sulphate of mercury together. We may here remark, that in case of accidental poisoning, or of suicide, the best antidote which can be administered is the white of eggs, which, by coagulation with the salt, prevents any action on the intestines.

There are two iodides of mercury. The sub-iodide is a yellow powder, which may easily be procured by adding a little iodide of potassium in solution, to one of corrosive sublimate; if more iodide be added, then a red powder is produced, which is the proto-iodide: if, however, too much of the iodide solution be employed, the iodide of mercury will be dissolved, and the liquid will become transparent. We have frequently produced both iodides by placing iodine in powder on metallic mercury, and inserting the poles of a powerful voltaic battery in the metal: one is then to be withdrawn, so as to produce the disruptive discharge. A combination of the metal and the iodine will then take place, and the two iodides are produced. In many chemical works these iodides are respectively termed the proto, and biniodide. It has been proposed to use these substances as pigments; but they are liable to decomposition, and so defeat that purpose.

There are two sulphides of mercury. One, the sub-sulphide, is readily produced by passing sulphuretted hydrogen gas through a solution of the sub nitrate, by which a black powder is produced. The sulphide proper is well known in the substances termed cinnabar and vermilion; the latter of which has a beautiful red colour, and is much employed as a pigment. Cinnabar is a common ore of the metal. The sulphide, produced by passing sulphuretted hydrogen through the nitrate of mercury, is of a deep black colour; but on being heated, it becomes reddish brown; and the substance, if heated with a solution of sulphide of potassium, produces the pigment vermilion.

Mercury forms a compound with cyanogen as the cyanide of the metal, which is the usual source of cyanogen for experimental purposes. The cyanide may be procured by boiling a solution of prussiate of potash with sulphate of mercury, and subsequently evaporating the clear solution, when crystals of the white cyanide will be obtained.

One of the most interesting and useful compounds of the metal is that termed fulminating mercury, which is extensively employed in the manufacture of percussion-caps; an invention which has completely changed the construction of muskets, fowling-pieces, &c., and has made them infinitely more serviceable for the purpose they are intended for. In the old form of such guns, the powder was ignited by means of a flint and steel placed over the barrel, and the fracture of the flint rendered the musket useless. It was necessary to pour a little powder into the pan, which communicated, by means of a small hole, with the charge in the interior of the barrel; and it frequently happened that a gust of wind or a shower of rain rendered it impossible to ignite the charge. By means of the percussion-cap, the "nipple" is kept quite covered and protected, and, at the instant the hammer is allowed to fall, the powder in the cap explodes, and the charge is inflamed. The following method is that usually adopted in manufacturing the fulminating mercury, which is a combination of fulminic acid with the metal. Into a tall glass or porcelain vessel introduce one ounce of mercury, and on this pour gradually four ounces, by measure, of nitric acid, of a specific gravity of 1·45 to 1·50. Considerable heat will be evolved during the solution of the metal; and when all is dissolved, and the mixture still hot, it must be poured into six ounces of strong alcohol. As the mixture instantly froths up to a great degree, and abundance of vapour is given off, this should be done in a vessel of considerable height and capacity, to prevent the loss of its contents. A powder will soon be produced, which is the fulminate. This may be allowed to settle, and as much as possible of the clear liquid is then to be poured away. The vessel is to be filled up with cold water, and the contents are then to be gradually poured on to filtering-paper, on which the powder will be retained. This is to be well washed with cold water until all trace of acid is removed. The powder itself should always be kept wetted until required for use, as, if dry, the slightest friction would cause it to explode violently. It is best to remove the filtering-paper whilst wet, and to place it in a dish kept full of water; and when any portion of the fulminate is wanted, such should be removed in as small a quantity as possible, by means of a paper scoop, and allowed to dry by evaporation in the open air. We cannot too strongly caution our readers in respect to the manufacture of this dangerous compound. The quantities we have given are such as would be employed for making it on a large scale; but if the student desire to acquaint himself with the properties of fulminating mercury, he should use but a few grains of the metal, and the acid and alcohol in the relative proportions already indicated. To impress the necessity of caution, we may observe, that a few years ago, an able and experienced chemist, employed in manufacturing the article at the Apothecaries' Hall, London, was shattered to pieces, and the fragments of his body blown to an astonishing distance, by an explosion of but a moderate quantity of this dangerous substance. The tyro in chemical science should never attempt to make it at all. We have already named the fulminates of gold and silver, which are producible by a similar process to that just described.

Some of the salts of mercury are extensively used for the purpose of amalgamating the zinc plates of voltaic batteries; and for this purpose the nitrate is the most convenient. The same end is also obtained by brushing metallic mercury over the zinc, the latter being immersed in dilute sulphuric acid.

The detection of mercury, in a combined state, is generally very easy. The sulphate, nitrate, &c., readily part with the metal, if they are moistened with water, and placed on a sheet of zinc. A gold or copper wire is then made to touch the metal, the wire being passed through the moistened salt. The latter is at once decomposed; and metallic mercury, if present, will turn the end of the wire to a white colour. If a gold wire be employed, the mercury is at once removed by the heat of a spirit-lamp. Mercury has a peculiar effect on the human system; and persons employed in the mines, or in working with the metal, are generally afflicted with paralysis.

COPPER.

This metal was well known to the ancients, and is mentioned in the works of biblical and general history. Its specific gravity is 8·78; its symbol, Cu; and the equivalent = 31·7.

Copper is found in the rocky districts of most parts of the world, either in the metallic state or combined with sulphur and carbon. In some parts it is found in the metallic state in immense masses; a specimen weighing above one ton, and containing 95 per cent. of pure copper, may be seen in the Polytechnic Institution, Regent-street, London. When combined with sulphur, it is called *copper pyrites*; and as a carbonate, it forms that beautiful substance called *malachite*. The chief source of copper in this country is in the mines in Cornwall. Australia, New Zealand, America, Western Africa, Russia, and Sweden also produce it in large quantities. A very ingenious use of old iron pots and pans is effected in the water running from the copper mines. The old iron is collected in London, and shipped to Cornwall, and on being thrown into water containing copper in solution, the iron is dissolved, and the copper is thrown down in the state of a brown powder. It is afterwards melted and cast into bars. This plan is easily illustrated by dipping any iron article into a solution of blue vitriol (sulphate of copper), when the metal will be coated with copper, a portion of the iron having been dissolved away. When the ores of copper are employed, they are first roasted, to drive off the sulphur; and after smelting, tolerably pure copper is obtained, in quantities proportional to the richness of the ore.

In the metallic state, copper is recognised by its reddish-yellow colour. It can be highly polished, but soon tarnishes. It is soluble in many acids, and may be detected in solution, by the addition of a little smelling-salts (carbonate of ammonia), which produces a blue colour.

It is largely employed in a variety of ways. As a metal, it is fashioned into vessels for domestic use, which are, however, highly dangerous if they are not coated *completely* with tin, because a portion of the metal dissolves and forms a poisonous salt. In the manufacture of pickles and preserves, this often occurs; but copper may be detected by the means already named. In the absence of medical advice, abundance of sugar or the *unboiled* white of an egg may be taken as an antidote.

Alloyed with zinc, the well-known metal brass is produced. Bronze and gun-metal are a mixture of copper with about one-tenth of its weight of tin. Bell-metal is composed of four or five parts of copper to one of tin. An alloy of copper, instead of the metal itself, called Muntz's metal, is extensively employed for sheathing ships' bottoms.

The combinations of copper are numerous, and many of them are extensively employed in the arts and manufactures. It combines with oxygen in various proportions. Two equivalents of the metal with one of oxygen ($Cu_{,,}O$) produce the sub-oxide, which may be procured by heating the chloride with carbonate of soda in a crucible. The compound so produced by fusion, is then to be plunged in cold water, when the sub-oxide separates. It is chiefly used to colour glass, and a singular property exists in reference to it. If it be burned in at a comparatively low temperature, and for a short period, but a slight red tint is afforded. If the heat be increased, and the period of burning extended, the colour is deepened. In an instance which came under our notice some years ago, resulting from the destruction of the Glasgow Polytechnic by fire, a piece of cut-glass of about four cubic inches' capacity, which had previously been of but a light ruby tint, was converted into so dark a colour by the heat, that when chipped into the thinnest pieces, it appeared of the deepest ruby red.

The protoxide of copper ($Cu\,O$), which is extensively employed in organic analysis, may be readily obtained by heating strongly the nitrate in a crucible, when a black powder will be produced. Full directions for its use for that purpose will be subsequently given. A binoxide of copper, and an acid called cupric acid, have been discovered; but they are not of special chemical importance, except that the latter is somewhat analogous to the ferric acid, of which we shall speak hereafter.

There are two compounds of copper with chlorine—a sub and the proto-chloride. The latter may be procured by dissolving the oxide of copper in hydrochloric acid, when a pale-green solution is produced. This is often used as a sympathetic ink, and for the purpose of paintings, which, by being heated, change from a winterly to a summer-like appearance, by the production of the green colour which heat evolves. The sub-chloride may be prepared from the chloride, by heating the chloride with copper filings in a crucible.

The disulphide of copper is found naturally, and is one of the ores of copper, as we have already stated. It may be produced artificially, by heating sulphur and copper in a crucible. The ore is largely employed in Glasgow and other places, as a source of sulphur for the manufacture of sulphuric acid. The proto-sulphide is readily produced by passing sulphuretted hydrogen through a solution of a copper salt, when a black powder is thrown down. With cyanogen, copper forms the cyanide, which is chiefly used for the purposes of electrotyping.

The most important salt of copper is the sulphate, which may be obtained by boiling copper in sulphuric acid. It is a product of many copper mines; and when crystallised, affords large blue crystals, which are known in commerce as blue vitriol. This salt has become of great importance since the discovery of the electrotype process; and by means of this ingenious application of science, copies of expensive works of art, such as statues, engraved plates, &c., are easily obtained, at a trifling cost compared with that of the original.

The nitrate of copper is readily procured by dissolving the metal in nitric acid. Its chief use is that of affording the oxide of copper used in organic analysis, to which we have already referred.

The native carbonate of copper, called malachite, has been named previously. It is often fashioned into various ornamental articles, owing to the beautiful appearances which it presents. A compound of carbonic acid, copper, and water is readily produced on adding carbonate of soda to the sulphate of the metal, both being in solution, when a light-green powder is precipitated. Native carbonates are very common in the specimens of ores obtained from the copper mines.

On adding ammonia to a solution of a cupreous salt, a deep-blue colour is produced, which is a combination of the metal with the alkali, often termed by old writers the ammoniuret of copper.

One of the most valuable salts of the metal is the acetate, which is familiarly known as "verdigris," and extensively employed as a pigment. The pure acetate is readily obtained by dissolving oxide of copper in acetic acid. The ordinary commercial product is obtained by means of the waste of the vineyards and metallic copper, the external surface of the metal being converted into an impure acetate. This is removed, and the process is repeated until the plate is completely used up. "Scheele's green" is a compound of arsenic and copper, and is also used as a pigment.

IRON.

This metal is so well known, and used in such a variety of ways in civilised life, as to require no description on our part. It is universally distributed, and is one of the most abundant substances in nature. Its specific gravity is 7·5; symbol, Fe; and the equivalent = 28.

Its ores are chiefly the carbonate, oxide, and sulphide; the latter being a compound of the metal with sulphur, is called iron pyrites: this, with the copper pyrites, has been used by sulphuric acid-makers as a source of sulphur.*

The iron district of this country embraces a very extended area. In Scotland, a band of ironstone is found, which reaches from the German Ocean to the west coast; and the whole of the country from Ayr to Edinburgh is dotted with iron furnaces, producing thousands of tons of the metal annually.

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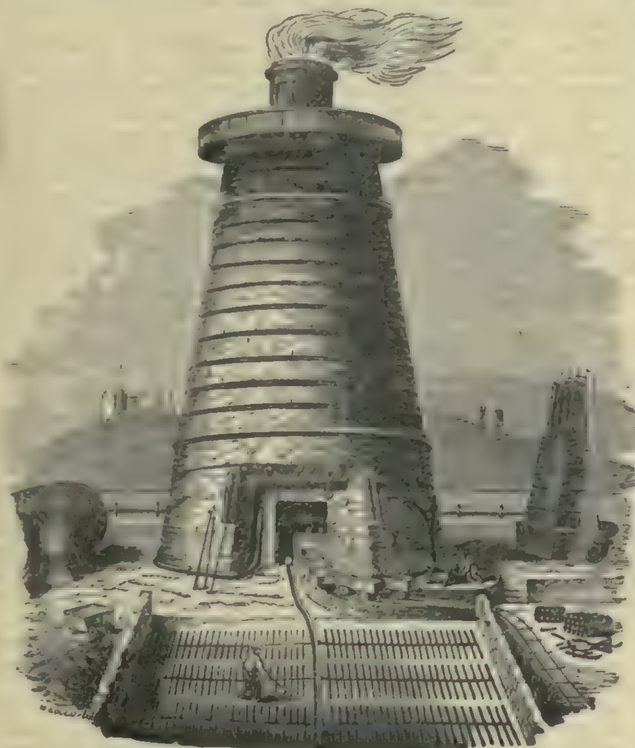
* See *ante*, p. 368.

In England, Staffordshire and the neighbouring counties produce immense quantities of the ore; and in most cases abundance of coal is obtained from mines or seams, above and below those of the mineral.

The ore, on being brought to the surface, is mixed with small coal and roasted for many hours; the mass is then broken up and taken to the smelting furnace, which is generally about thirty feet high; mixtures of the ore, coal, and limestone, which acts as a flux, are thrown into the top of the furnace, and an intense heat is maintained by means of powerful air-engines propelled by steam power. Every twelve hours the bottom of the furnace is tapped, and the melted metal is run into long furrows called "sows," adjacent to which are smaller ones called "pigs;" when cool these are sent to market as pig-iron. The following engraving represents the ordinary kind of smelting furnace.

Fig. 66.



But the metal in this form is very brittle, and would be almost useless unless further processes were brought to bear on it. Being broken up into small pieces, the pig-iron is then cast into a puddling furnace, in which it is continually stirred about by means of a rake. After a time it acquires a peculiar appearance, becomes soft and agglutinated, and is then removed to a block, on which it is hammered by steam hammers until all the impurities are driven out. It has now become fit for rolling, and is thus fashioned at a white heat into sheet, bar, or rod-iron.

Iron may be protected from rust by either tinning its surface or by covering it with a thin coating of zinc—a process which has been termed "galvanising." It may be drawn into very fine wire, being very tenacious. It has the singular property of welding; that is, two pieces at a red heat may be united together if well hammered. This property is only possessed by iron and platina.

Iron may be cast into moulds of any form, and thus castings of any size may be obtained. Cast-iron is of inferior strength to bar or rod-iron, owing to its containing several impurities. Mr. Bessemer has introduced an ingenious process for the purpose of making "wrought" iron from the ore. This he effects by driving air into the furnace whilst the iron is in a melted state. By these means its impurities are burnt away.

The most important combination of iron is that with carbon; and, as a late writer has remarked, most probably nitrogen is also present in minute quantities, inasmuch as a direct combination of carbon alone with iron does not afford a substance corresponding in all its properties with that known as steel. In many of their qualities cast-iron and steel are similar; but the latter has an elasticity, ductility, &c., which, as physical characteristics, constitute an enormous difference. Iron and steel may be easily distinguished by dropping on each a little nitric acid, when the pure iron will present only a white appearance, whilst the steel will afford a black mark, indicating the presence of carbon.

The qualities and value of wrought-iron and steel vary exceedingly, owing to the admixture of various substances, such as phosphorus, silica, &c. In England, the best is that produced in Staffordshire; but this is exceeded by that obtained from Sweden. Iron produced by the cold and hot blast also varies in its characteristics. By the latter mode, the iron has induced in it greater tenacity, the air being heated to a temperature of 600 to 700 degrees before it is admitted into the smelting furnace. Iron produced by charcoal is far better, in all respects, than that produced by coke or coal.

Of late years, the uses of iron and steel have been enormously extended. With respect to steel, we may remark, that steam-ships are now entirely constructed of it; even boilers and the masts of vessels have been made from it: some pieces of ordnance have been also manufactured from steel; and their extraordinary tenacity, we need not state, presents a great advantage.

Iron combines with oxygen in four proportions, the symbols and properties of which are as follows:—The protoxide ($Fe\ O$) is produced by adding a solution of potash to one of the protoxide of iron, when a green precipitate is produced. When this is boiled, a black powder is the result. The peroxide (Fe_2, O_3) is readily obtained by adding an alkaline solution to the nitrate of iron, and is a red-coloured substance. The presence of this in nature gives the rich red tint to the rocks of the old red sandstone, and is the cause of the colour of blood, &c. The black oxide (Fe_3, O_4) has, like the metal, decided magnetic properties: it is familiarly known in the scales so often seen when wrought-iron has been heated to redness, and is a common product of the blacksmith's shop. Ferric acid ($Fe\ O_3$) is the state in which iron is found combined with the largest proportion of oxygen.

Chlorine unites with iron in two proportions, forming a proto-chloride ($Fe\ Cl$) when iron is dissolved in hydrochloric acid; and a sesqui-chloride ($Fe_2.\ Cl_3$) when chlorine is passed over iron filings at a red heat. There exist two sulphides of iron. That which is best known constitutes the iron pyrites ($Fe\ S_2$), so common as an ore of the metal. The proto-sulphide is readily produced by bringing a roll of sulphur in contact with a bar of iron heated to redness. The metal will immediately be seen to melt away; and the dross which falls down, and which should be received in dry sand, forms the substance required.

Iron and sulphuric acid readily combine to form the sulphates, of which there are two; one being produced as a proto, and the other as a per-salt. The proto-sulphate is obtained by dissolving iron in dilute sulphuric acid: during the process, water is decomposed, and abundance of hydrogen is evolved. The proto is easily converted into the per-sulphate by exposure to the air, or by the addition of nitrous or nitric acid, when the oxide enters into the state of protoxide. Green copperas is a proto-sulphate of iron in an impure state. It abounds in many kinds of coal, &c., after exposure to the air, by which means the sulphide is gradually converted into the sulphate. In the Isle of Sheppy, at the mouth of the Thames, the process goes on at an enormous rate; for the soil abounds in nodules of iron pyrites, which are continually undergoing decomposition. Sulphate of iron is also found in many mineral springs; and frequently the oxygen it absorbs from the atmosphere, is the cause of the reddish-brown powder observed when such waters are allowed to remain still for a time. A solution of proto-sulphate of iron, from its great attraction for oxygen, is often employed with the nitric oxides, in the estimation of the presence of oxygen in air, &c. The proto and per-sulphates are largely used as fixing agents in photographic processes. The nitrate of iron is also similarly employed, and is produced when iron is dissolved in nitric acid.

Copperas (the proto-sulphate) is much employed as a dyeing agent for producing, with logwood, gall-nuts, sumach, &c., various shades of slate and black colours; and, for the same reason, it is used by calico-printers: but for these purposes the acetate is generally preferred, as it is more readily decomposed by the gallic acid found in the vegetable substances we have named. The acetate may be prepared by decomposing the sulphate by means of acetate of lead, when sulphate of lead is precipitated, and acetate of iron held in solution. Acetic acid and common vinegar, or sour beer, &c., may be left to digest on old iron hoops or other articles; which is, of course, a more economical, although slower, process for forming the acetate. These, and many other salts of iron, such as the citrate, succinate, &c., to which we might direct attention, are not, from their comparative rarity, of general importance. The citrate is employed in medicine, and the succinate as a chemical test.

The combination of iron with cyanogen, however, deserves by no means a passing notice, as the prussiate salts are extensively used as a dyeing agent, and are constantly employed in the laboratory as tests, &c. The cyanide of iron is obtained by adding cyanide of potassium to a solution of proto-sulphate of iron; and is, at first, a white powder. As, however, every salt of iron, unless in a peroxidised state, has a great attraction for oxygen; the cyanide, if exposed freely to the air, becomes of a blue colour. The most important cyanide of iron is that generally known by the term of yellow prussiate of potass. It consists of lemon-coloured crystals, of a regular form. This substance is produced on a large scale in this country for the use of dyes. Animal matter, such as hoofs, horns, old leather, blood, &c., is projected into a large iron pot heated red-hot, potash and old iron being also added; a general decomposition ensues; and when perfect fusion is effected, the mass is raked out, and, after being cooled, is dissolved in water, and, subsequently, crystallisation produces the yellow prussiate. During the process, a combination is effected between the carbon, nitrogen, iron, and potass. A new substance is thereby generated, termed ferro-cyanogen, which, with hydrogen, forms hydro-ferrocyanic acid; analogous, in some respects, to hydrocyanic acid. But the actual composition of the yellow prussiate is that of 1 equivalent of ferro-cyanogen; 2 equivalents of cyanide of potassium; and 3 of water: hence it is termed ferrocyanide of potassium.

Many years ago we had occasion to notice the accidental production of Prussian blue—which we shall presently describe—whilst visiting some gas-works. Now coal contains carbon, nitrogen, iron, &c.; and at the present time, the prussiate is produced by the union of these elements, in a similar way to that which we named as accidentally occurring; charcoal, air, oxide of iron, and carbonate of potash being heated together in suitable arrangements.

The yellow prussiate of potass is converted into the red, or ferri-cyanide—so called in contradistinction to the ferro-cyanide, which we have just described—by passing chlorine gas through an aqueous solution of the latter salt, by which means one equivalent of chloride of potassium is removed. It is chiefly used as a test for iron, giving a distinctive character as to the oxidation of the iron in the solution tested. The uses of the yellow and red prussiates, in this respect, will be considered under the head of Chemical Analysis.

By adding solutions of the yellow prussiate of potass to per-sulphate of iron, or to a solution of the proto-sulphate, to which nitric acid has been added, a rich, blue precipitate is produced, termed Prussian blue. It is the formation of this substance in woollen and cotton stuffs, by the successive use of the salts named, that such are dyed of a blue colour for many purposes, instead of by indigo. The colour is, however, turned green, and eventually destroyed by alkalies, and is therefore not of a permanent nature. Prussian blue is largely used as a pigment. If a solution of sulpho-cyanide of potass, produced by heating sulphur, carbonate, and prussiate of potash together to redness, and dissolving the fused mass in water when cold, be added to those of the salts of iron, a deep, blood-red colour is produced. It is highly possible that some such combination as this is the cause of the colour of our blood, as all the necessary elements are present in that fluid.

We have hitherto confined our attention exclusively to the chemical character of the metal iron and its salts. We must now speak of others, which are of considerable interest, before concluding these remarks. It is well-known that iron has the power of becoming attractive by the loadstone, and that steel is subject to the magnetic influence to such an extent, as to be able to retain all the characteristics of the magnetic force. This fact adds to the value of iron as a metal; for by means of it we are enabled to construct the mariner's compass, so essential to navigation in the present day. This subject has already been fully discussed, however, in the section on Magnetism; to which we must, therefore, refer our readers for information on the matter.

LEAD.

THIS metal is one which has long been employed for various uses by mankind, and, like those previously described, it was well-known to the ancients. Its specific gravity, on an average, is 11·45; its symbol, Pb; and its equivalent = 103·7.

Lead is almost always found in nature in combination with sulphur; in this state it is called galena; and it is very generally distributed over most parts of Great Britain, especially in Derbyshire, Yorkshire, and Northumberland. It is obtained by roasting the ore with coal, and subsequently smelting it: the metal is then run into long bars called "pigs," in which state it is sent into the market.

It has a bluish colour, and may be polished; but soon tarnishes, owing to the formation of an oxide on its surface; its presence in solution may be detected by sulphuretted hydrogen gas. This gas, which is the cause of the offensive smell of drains, immediately turns any solution of lead to a dark-brown colour; hence the reason that white paint is so often discoloured, containing as it does the carbonate of lead.

Lead is easily melted, bent, and rolled, and is therefore valuable in forming piping, cisterns, and other vessels. It, however, produces a deadly poison when in a state of solution (as the carbonate in water), which frequently occurs from the use of lead cisterns, &c., if the water in them be nearly pure. This, at first sight, would seem paradoxical, but is easily explained. At the part where the lead, water, and air are in contact, the carbonate of lead is formed. The carbonic acid, gradually absorbed by the water, facilitates the solution of the salt, and so impregnates the liquid with lead. Now this does not happen if the water be hard, because the sulphates contained therein form an insoluble coating of the sulphate of lead, which prevents further action on the metal. Considerable opposition on this account was made, a few years ago, to the introduction of the water of Loch Katrine into Glasgow, as it was considered that the exceedingly soft nature of the water, owing to its freedom from salts, would induce the solution of the lead in the cisterns and pipes of the city. Whilst we fully admit the prudence and good feeling of those who raised the objections, we are equally glad to find that the denizens of that city still flourish, and that the purity of the water-supply has not been attended by any untoward danger. Numerous instances, however, have occurred in which serious injury, and even fatal results, have arisen from the water supplied to houses and stables having become impregnated with lead; and there is no doubt that many families have been afflicted by disease arising from this effect, which has not been discovered at the time, but assigned to other sources for an origin.

The combinations of lead are of considerable importance. An oxide is at once produced when lead is heated to the melting point. Its surface gradually becomes coated with a varied-coloured powder, through the oxidation of the metal; and thus red lead—a pigment in so much use—is produced. The protoxide of lead may be obtained by heating the carbonate of the metal to redness. Massicot and litharge are oxides of lead; the latter being boiled with linseed oil, produces the drying-oil used by painters. When dilute nitric acid is made to act on the ordinary red lead, a portion only is dissolved, and a nitrate of lead is produced. There, however, remains an insoluble brown powder at the bottom of the vessel, which resists the action of the acid, and is the peroxide. The sulphate of lead may be produced by adding sulphate of soda to the acetate of the metal, both being in solution. It will fall down as a white powder. The same result is obtained by digesting galena (the sulphide) with nitric acid, by which the sulphur is oxidised, and the sulphate, together with some unchanged sulphur, are produced. The chloride is also a white powder, and may be obtained by adding hydrochloric acid to the acetate of the metal. The acetate, or sugar of lead, which is employed in medicine, and for other purposes, is a crystalline substance, procured by digesting litharge in vinegar. It has a remarkably sweet taste; and hence its name. A basic acetate of lead exists, containing less acetic acid, and more base than the former. The chromate of lead is produced on adding a solution of chromate of potass to one of acetate of lead, when a yellow powder is precipitated, forming the yellow chrome used by painters. The orange chrome is formed when the yellow is boiled with lime.

The most important salt of this metal is the carbonate commonly known as white lead, and extensively used for painting purposes. It is easily obtained by mixing solutions of carbonate of soda with the acetate of lead; but for commercial purposes, a more economical plan is pursued. Thin plates of lead are exposed to the action of the fumes of spent tan, by means of which both acetic and carbonic acids are afforded. Gradually, the external surface becomes coated with a white powder, which is the carbonate. As this is formed, it is scraped off; and the surface left is again exposed to undergo the same process, until the entire plate is destroyed. It is found in commerce, both in the dry state and ground with oil. This manufacture is peculiarly dangerous to the health of the work-persons; and females are generally employed, as being less liable to the evil effects than males. Of late years, a compound of zinc has been substituted, to some extent, for white lead in house-painting.

There are other combinations of lead, such as the iodide, cyanide, &c.; which, however, are of minor importance, and are little beyond the range of chemical curiosities. If the tartrate of lead, produced by adding cream of tartar to acetate of lead in solution, be dried and introduced into a tube, which must then be exposed to a red heat and sealed, a finely divided form of lead will be produced. If the tube be broken, and its contents shaken out, they will catch fire, affording what is called pyrophorus.

ZINC.

DURING the last few years, the metal zinc has been applied to so many useful purposes, as to have superseded lead to a very large extent. It is produced from two kinds of ores, known as calamine and blende; the former being a carbonate, and the latter a sulphide of the metal. As is usual with most metallic ores, those of zinc are first roasted. By this the oxide is produced; and the reduction of the metal is effected by heating with pulverised coke. Being volatile, it distils over from the crucible in which it is reduced; these being placed in a peculiarly constructed furnace. It is afterwards cast into flat plates, and sold in that form as "spelter," in commerce. If one of these plates be broken, a beautiful crystalline appearance is presented. The facets, however, soon become oxidised, and lose their brilliancy. Zinc can be readily rolled out into sheets; but for that purpose it must be heated to 300° Fah., because both above and below that temperature the metal is brittle, and may even be pulverised.

Zinc may be drawn into a wire, which, however, has no great amount of tenacity. It is manufactured into nails, and has been extensively employed for coating iron instead of tin. This process is termed, most erroneously, "galvanising," for no voltaic current has anything to do with the process. The effect of covering iron with zinc is simply that of enclosing a readily oxidisable metal within one having less attraction for oxygen, and thus of protecting the interior plate from the action of air and moisture. The tenacity of the iron plate is diminished; and nails over-"galvanised" become so rotten as to be easily broken by the fingers, even when they are of considerable substance.—Whilst referring to the properties of lead, and its application to domestic purposes, we stated that considerable danger exists when it is employed to line water-cisterns for domestic use. Of late, zinc has been advantageously employed for that purpose. Its surface only becomes superficially coated with oxide, which is insoluble in water, in all ordinary cases; and hence no danger can arise from its use. Zinc is largely employed in the construction of voltaic batteries; and was most ingeniously proposed to be attached to the copper sheathing of vessels, so as to prevent the corrosive action of sea-water on them. The plan failed through a secondary result taking place; for shells of molluscs, &c., attached themselves to the copper, and so impeded the speed of the vessel.

Zinc, in combination with copper, produces brass: it forms alloys with other metals; and so its uses have been greatly extended.

Zinc, like iron, is a very combustible metal; for in the form of thin shavings it will burn with ease, affording a beautiful blue flame. If zinc be melted in a ladle, and, when red-hot, a portion be cast into the fire, the same result will be produced. A ladle may be filled with melted zinc, and small portions of nitre or chlorate of potass being cast on the surface, a splendid combustion results. A jet of oxygen directed on shavings of zinc which have been previously ignited, affords a beautiful spectacle. The combustion of zinc, as poles of the voltaic battery, produces a blue light fringed with red.

From the above experiments a white powder is always produced, which is the oxide of the metal. A hydrate of the oxide is obtained by adding a solution of potass to one of sulphate of zinc. The oxide has been extensively employed as a substitute for white lead; and it is to this substance we referred in our remarks on the carbonate of lead.

Chloride of zinc is obtained by dissolving the metal in hydrochloric acid. This compound has been recommended as an excellent disinfectant; and its value in this respect has been well tested by the medical boards of health throughout the kingdom. A small quantity of the solution is to be sprinkled over any offensive object; or in applying it in hospitals, &c., towels wetted with it may be hung in different parts of the wards or rooms. A remarkable instance of its efficacy came under our notice some years ago (1849). A metal manufacturer in the neighbourhood of London, employing a large number of hands, was in the habit of using chloride of zinc for soldering purposes; and considerable quantities of the waste material were allowed to run down a sewer. All his workmen escaped attacks of cholera, which was then prevalent to an alarming extent; as did the inhabitants, whose drains were connected with the sewer. About the same period, we observed a similar result from chloride of lime; the waste liquor, to the amount of hundreds of gallons, daily passing down a drain in the centre of a street situate in Lock's Fields, Walworth. Although on all sides deaths were occurring hourly from cholera, yet the whole of the inhabitants of the street escaped; as did the workmen in the adjacent factory.

The sulphide of zinc, or zinc blende, to which we have already alluded as an ore of the metal, may be obtained by heating the oxide with sulphur in an iron ladle. It has no specific uses as an artificial production.

The sulphate of zinc is readily procured by dissolving the metal in dilute sulphuric acid; and this is often done to procure hydrogen gas, which is afforded in abundance. It is largely produced by the use of Smee's batteries employed for electro-plating. A dilute solution, say twenty-grains, to a pint of distilled water, forms an excellent eye-wash. As an application in cases of burns and scalds, and in other cases, it may be used in place of "Goulard water," which is a weak solution of acetate of lead. The equivalent of zinc is 32·6; its symbol, Zn; and its specific gravity averages about 7·0.

TIN.

TIN has long been known as a metal capable of a variety of uses; and the British tin mines are spoken of by both Greek and Roman writers. The sources of this metal are the mines in Cornwall and Devonshire, in England; but considerable quantities are imported from Malacca. It has a whitish colour; produces a peculiar smell when rubbed, and a crackling noise on being bent. Alloyed with lead, it forms pewter, solder, &c.; with copper it affords bronze, gun and bell-metal, and bearings for shafting, &c. It is also largely employed for tinning iron and copper vessels for domestic use; preserving the former from oxidation, and preventing the poisonous effects of the latter through its protecting coat. Tin also forms a constituent of the material used in the manufacture of specula, for reflecting telescopes. Its equivalent is 59; the symbol, Sn; and its specific gravity, 7·25.

Tin combines with oxygen in two proportions. The protoxide is obtained by adding alkaline solutions to the proto-chloride, when a white powder is precipitated, which is a hydrate. The binoxide, or, as it is termed, stannic acid, is produced when tin is digested with nitric acid. It behaves as an acid, by forming salts with alkalies.

Sulphur and tin unite to form the sulphide and bisulphide. The latter may be produced by passing sulphuretted hydrogen through the per-chloride. Mosaic gold is a sulphide of tin, and is produced by heating an amalgam of tin with sulphur and sal ammoniac. The most important combinations of this metal are the chlorides, which are largely used in the arts. The proto-chloride is produced by dissolving tin in hydrochloric acid. This liquor is used as a mordaunt with vegetable colours, such as Sapan, Brazil wood, &c., in dyeing woollen and other yarns. The "purple of Cassius," which we have named in connection with gold, is produced by adding together the chlorides of the two metals; both being in solution. The per-chloride is produced when powdered tin is combined with chlorine gas, which should be passed over the metal in a porcelain tube. The product is a liquid which used to be known by the name of the fuming liquid of Libavius.

NICKEL.

NICKEL is chiefly known as a constituent of German silver, and is, comparatively speaking, a new metal. It has a greyish white colour; and, like iron, possesses magnetic properties. Its equivalent is 29·5; symbol, Ni; and it has a specific gravity of 8·7. It derives its name from the German term given to its ore. It is obtained from its combinations with arsenic, either by the moist or dry way. By dissolving the ore in nitric and sulphuric acids, the arsenic and nickel are separated; and by means of carbonate of potash, a double salt of nickel and potash is produced. From this the oxalate is procured, which affords nickel on being reduced by heat. The protoxide of nickel is obtained by adding an alkaline solution to the sulphate, and heating the precipitate, which is a hydrated oxide. The sulphate of nickel and other salts have a rich green colour. They have, however, no special application. We have already remarked that nickel forms a chief constituent of German silver, in which it is united with zinc and copper. The proportions of these metals vary according to the views or requirements of the manufacturer. Its use in connection with electro-metallurgy has already been fully dealt with, under that head, in the section on Electricity.

It is somewhat remarkable that nickel is generally found associated with iron in meteoric stones. The two metals have many qualities resembling each other. As we have seen, they are both subject to magnetic action; their equivalents and specific gravity do not greatly vary; and the colour and characteristics of several of their combinations are occasionally almost identical in some respects.

BISMUTH.

THIS metal is found in Cornwall, Germany, and other countries in Europe. It has a white crystalline appearance: it is obtained from an ore containing sulphur and arsenic; and is occasionally met with in an impure metallic state. It is distilled from its ores in iron tubes; and forms beautiful crystals on being melted and cooled. Its symbol is Bi; its equivalent, 213; and its specific gravity, 9·8. Alloyed with tin, it affords a material which has sonorous properties. Eight parts of bismuth, five of lead, and three of tin, form an alloy which fuses at a temperature a little less than that of boiling-water; and two parts of bismuth, one of lead, and one of tin, fuse at a temperature of 200°. For the amusement of young persons, such alloys are occasionally made into the form of spoons, which at once melt on being immersed in hot tea. It forms several oxides; the ter-oxide, $Bi\ O_3$, prepared from the ter-nitrate, $Bi\ O_3,\ 3NO_5$, is a yellow-coloured powder, which is used in the porcelain manufacture. The nitrates are the chief salts of the metal; and the one to which we have referred is prepared by digesting the metal with nitric acid, diluted with an equal quantity of water. The sub, or basic nitrate, forms the "pearl powder," used by ladies as a cosmetic; and is employed in medicine as an astringent and tonic. Bismuth, as a metal, is rarely used in the arts or manufactures. It is employed in constructing thermo-electric piles in combination with antimony.

ANTIMONY.

ANTIMONY, so valuable as a material in printing-types, and as a medicinal agent, is obtained both in a native state, and in combination with sulphur, as a ter-sulphide. Its ores are obtained from Singapore and Borneo. It is also found at Linz, in Germany. The sulphide is heated with charcoal, when it fuses: the mass so obtained is reduced to a coarse powder, and roasted with charcoal moistened with a solution of carbonate of soda. The mixture is then raised to a red heat, by which the metal is afforded in a tolerably pure state. It is very brittle and crystalline; has a bluish-white colour; and volatilises at a high temperature. It has the singular property of expanding as it cools. The latter circumstance renders it of great value for casting types, as the moulds in which they are formed are thereby more accurately filled, and the casting becomes more perfect. Its other alloys are also valuable. It forms a constituent of Britannia-metal; specula, for reflecting telescopes; bell-metal; and, associated with bismuth, it has the singular property of affording a current of electricity on the application of heat in the form of the thermo-electric pile. (See Bismuth). Its symbol is Sb; its equivalent = 129·; and the specific gravity, 6·8. Oxide of antimony is prepared by mixing the chloride with a hot solution of carbonate of soda, by which a grey powder is produced. This oxide, combined with tartaric acid and potash, forms the "tartar emetic," used as a medicinal agent. Antimonious acid, $Sb\ O_4$, is a constituent of James' powder, a well-known substance, employed in medicine. Antimonic acid, $Sb\ O_5$, is produced by acting on the metal by means of strong nitric acid; and forms salts with bases. Sulphur and antimony, united as sulphides, differ in appearance, in the native condition, from those prepared artificially. The natural sulphide is a dense black powder; and is much used by firework-makers to give coloured flames. Its peculiarity in this respect may be noticed by burning the following composition:—Nitre, four parts; sulphur, two parts; sulphide of antimony, one part.

Each ingredient is to be separately pounded, and the whole is then to be intimately mixed in a mortar. During combustion, the above produces an intense light, and is used for the manufacture of Bengal lights and sea-signals. We have frequently employed it for the purpose of taking photographs at night. The artificial sulphide has a rich red colour, and may be prepared by passing sulphuretted hydrogen through a solution of tartar emetic.

Kermes' mineral is produced by boiling the sulphide with potash or its carbonate, and has a reddish-brown colour. A sulphide of antimony, containing five equivalents of sulphur to one of the metal, is produced by passing sulphuretted hydrogen through a solution of the perchloride in hydrochloric acid, when it falls as a yellow powder.

If a little powdered antimony be heated and dropped into a jar of chlorine gas, it will take fire, producing a chloride of the metal. If the black sulphide be boiled with hydrochloric acid, the terchloride is formed ($Sb\ Cl_3$), which has been called the "butter of antimony." This, with olive oil, is used as a bronzing composition.

Antimony, like arsenic, has the singular property of combining with hydrogen, forming what is called antimoniuretted hydrogen. This may be procured by passing hydrogen through any solution of the metal. If a cold plate be held against the flame of this compound, metallic antimony will be deposited, which will not volatilise on the application of heat—a point of importance as distinguishing it from arseniuretted hydrogen, of which we shall have to speak hereafter.

ARSENIC.

The ores of this metal are very widely diffused in combination with those of other metals; and one of its oxides is the well-known poison, commonly called white arsenic. The importance of the subject of poisoning by this substance, will require, on our part, a somewhat extended notice of this element and its compounds; and we shall enter into a general description of the various methods which have been proposed for their detection.

The metal itself is obtainable from any of its combinations by first applying heat—say to the arseniurets of iron, copper, &c. White arsenic, or arsenious acid, sublimes; and if this be heated with charcoal in a test-tube, the metal can be readily procured as a crystalline substance having a metallic lustre. Its equivalent is 75; symbol, As; and the specific gravity about 6. When heated and in process of volatilisation, arsenic affords a smell resembling that of onions. We may here warn our readers to be exceedingly cautious with respect to inhaling the vapour, as it produces extreme inconvenience by acting on the membranes of the eyes and nose, even in minute quantities, and may prove highly dangerous if incautiously inhaled.

Arsenious acid is the most important combination of the metal. It contains one equivalent of arsenic to three of oxygen; and, as we have stated, is the poison known by the name of white arsenic. It is surprising that so deadly a poison should be so extensively employed. It has been stated, on good authority, that the inhabitants of some parts of Germany are accustomed to eat arsenious acid, and that it has the effect of making them plump in body, and sustains their power of ascending steep places, so far as the act of breathing is concerned. It has also been remarked, that so long as the habit is maintained, no inconvenience of any kind is produced; but if it be left off, then all the symptoms of arsenical poisoning present themselves. White arsenic is often given to horses, and produces a sleekness of skin not obtainable by any other means. As a sheep-wash, and to steep wheat, it is often employed; and it is no uncommon matter to find farmers purchasing the article extensively on market-days, for the purposes we have named. We need feel surprise, not that cases of arsenical poisoning so rarely occur, but rather that they are not more frequent, when we take into account the character of the individuals with whom it is entrusted. Arsenic with copper forms a pigment largely used for the purpose of printing a rich green colour on paper-hangings. This fact, in the opinion of many, has given rise to instances of poisoning in households, owing to the arsenical powder being brushed off from the walls, and being inhaled afterwards. Such an occurrence *may* take place, undoubtedly, in extraordinary circumstances; but we are strongly inclined to the opinion, that an accident only, and not a constant tendency, must exist to produce harm from such a source. Ladies' dresses have often their green tint heightened by means of Scheele's green; and persons employed in making such, and in the manufacture of wax and other artificial flowers, suffer seriously from the inhalation of arsenical powder thus used in their business. It is much to be regretted, that the personal charms of the *belle* must be enhanced by her poorer sisters paying the cost with the sacrifice of their health, and, possibly, their lives.

In the early attempts to manufacture candles from stearine, considerable difficulty was experienced in the tendency towards crystallising, which that substance always exhibited whilst cooling. White arsenic was used for the purpose of preventing this, and it was successful; but great danger arose to persons sitting in rooms wherein such candles were burned. Hence the use of arsenic was given up, and, by mechanical means, the difficulties we have named have been overcome. Another source of danger to a family, is the use of arsenic (by which term we shall in future call the arsenious acid, for the sake of simplicity) as a poison for rats and mice. To the ordinary observer, little difference exists between flour and that substance; and to prevent the danger arising from this source, an act of parliament has been passed, in which it is insisted that the white arsenic shall not be sold in its ordinary state, but mixed with other substances to give it a distinctive character. Another and extraordinary use of this poison is that of a cosmetic. For this purpose, a little of the white arsenic is put into the water in which the person is washing, and the face and hands are bathed with the solution. Independent of the certainty that the dissolved matter will be absorbed by the pores of the skin, small particles are likely to be introduced into the mouth, nose, and eyes; the least inconvenience from which would be that of causing inflammation of the membranes. But as the pores of the skin communicate with the blood-vessels, the poison is gradually absorbed by the system, and the consequences of its use soon become apparent by symptoms of arsenical poisoning, which supervene.

We may suggest a very simple way by means of which arsenic may readily be distinguished from any other substance usually found in our houses. Let a small pinch of the suspected powder be placed at the end of a penknife, and introduced into the flame of a candle. If it be white arsenic, it will soon pass off as vapour; or a small portion may be placed on some red-hot coal, when, if arsenic be present, it will volatilise, affording a smell exactly like that of onions.

We shall now proceed to familiarise the minds of our readers with the modes which may be adopted to detect the presence of arsenic—commencing with some comparatively rough experiments, and then giving those refined methods which the analytical chemist adopts in cases which form the subject of judicial investigation. If the student proceed successively with each, he may soon acquire skill in the detection of the poison.

(*a*.) Introduce a few grains of white arsenic into a test-tube, and apply the heat of a spirit-lamp to the powder. It will volatilise and condense on the cool part of the tube as a white crystalline substance.

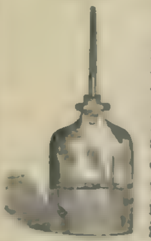
(*b*.) Mix a little white arsenic with some powdered charcoal, and apply heat as before. The oxide will be reduced, and *metallic* arsenic will be produced in dark metallic crystals.

(*c*.) Apply heat to these crystals, when they will immediately volatilise, recondensing in some cooler part of the tube.

The foregoing experiments will illustrate the characteristics of arsenic and arsenious acid, and the method by means of which the metal may be reduced.

Arsenic has, like antimony, the power of combining with hydrogen, and of forming what is termed arseniuretted hydrogen. This fact has been taken advantage of by Mr. Marsh, to afford a most ingenious and effective means for discovering the presence of arsenic in minute quantities. The best mode of illustrating this is as follows:—

Fig. 67.



Into the bottle represented in the following figure, introduce some water, zinc-shavings, and a little sulphuric acid; just as is done in making hydrogen gas. Refit the tube and cork; and after having allowed the mixture of air and hydrogen to escape, so as to prevent any danger of explosion, ignite the gas, and hold against the flame a white porcelain plate. If the zinc and acid be pure, nothing will be deposited on the plate except a little water. Dissolve a small portion of arsenious acid in hydrochloric acid, heating the mixture. When the solution is effected, pour a few drops into the bottle producing hydrogen, and instantly replace the tube. After a few seconds, apply a light to the jet; when it will be noticed that the flame has a blue colour; and if a cold plate be now held against it, crystals of arsenious acid will be at once produced; in the centre of which will be a black mark. Instead of the plate, let the flame play inside of a glass-tube. The arsenious acid and arsenic in a metallic state will both be deposited. These may then be again volatilised in the manner already referred to in experiments *a* and *c*, above. We need scarcely state, that in accurate analysis, it is of the utmost importance that all the materials employed should be pure. Zinc and sulphuric acid frequently contain arsenic; hence, if the commercial articles be used, they may possibly afford arsenic, which would render all results obtained valueless. This precaution, the experienced chemist, of course, always attends to; and is at all times scrupulously careful as to the purity of his tests.

If a piece of clean copper be dipped into the hot solution of arsenious acid just mentioned, and boiled therein, the copper will become coated, arsenic being reduced on its surface. The copper slip may then be removed; and after being washed in water and dried, the arsenic may be sublimed from its surface, if it be introduced into a test-tube, and heated by means of a spirit-lamp. This plan is Reinsch's test; but unless quite pure copper be employed, the results are vitiated and valueless; for copper frequently contains a portion of arsenic, obtained from its ores. In a late case of suspected poisoning, the accidental discovery of the impurity of the copper, after the analysis had been effected, alone saved the condemnation and execution of the suspected individual.

Arsenic in solution may also be detected by liquid tests. That of nitrate of silver with ammonia, when the alkali is not in excess, affords a yellow precipitate; of sulphate of copper and ammonia, a rich green one; and a stream of sulphuretted hydrogen produces a yellow sulphide. These tests, however, are never relied on in accurate investigations.

The foregoing remarks and experiments will give our readers a general idea of the various methods employed to detect arsenic. We shall now direct attention to the usual *modus operandi* in post-mortem investigations, when death has been suspected to have arisen from the use or administration of arsenic as a poison.

Arsenic is generally taken in comparatively small quantities—*i. e.*, of a few grains only. It is not very soluble in water of the temperature of the stomach; but it must be borne in mind that the acids in the stomach greatly facilitate its solution. When taken in large doses, it produces vomiting, which relieves the stomach of a portion of the poison. An instance of this kind occurred in Glasgow some years ago, when not less than a quarter of an ounce of the poison was discovered in the stomach of the deceased person, who had been violently sick for some time before death. The symptoms of arsenical poisoning are, violent retching, heat in the throat and palate, a metallic taste in the mouth, and great irritability of the whole system. The face wears an anxious appearance; and, indeed, most of the symptoms produced by irritant poisons are manifested. It is of the utmost importance that medical aid should at once be secured; but in the absence of this, the only antidote which can be readily obtained, is that of the peroxide of iron. To obtain it, add carbonate of ammonia to the ordinary tincture of steel, as long as a powder is thrown down; and to save time, pour off the clear liquid, adding water to the powder; again pour off the liquid, and add sufficient water to allow of the powder being readily suspended in it. Stir it up, and administer a wine-glass full continually, taking care that as much as possible of the oxide be swallowed at each dose. Meanwhile medical aid may have been obtained, and the patient so left in experienced hands. When the remedy cannot be arrived at, every attempt should be made to make the patient vomit, so as to clear the stomach as much as possible of the poison. If such precautions be taken before inflammation commences, considerable chance exists of preventing a fatal termination, because, as we before remarked, the comparative insolubility of arsenic favours the chance of recovery.

When any suspicion of foul play exists, every drop thrown up from the stomach should be carefully preserved, and be left untouched until placed in competent hands for analysis. Many a life has been preserved through the over-cunning of criminals. A small dose of arsenic is far more likely to prove fatal than a large one, for reasons we have already assigned; but the non-success of one attempt by no means leads to the supposition that it will not be repeated. Therefore, if symptoms such as we have described have been noticed, and any suspicion exist, then the analysis of the ejected matter may, if it do not lead to detection, terrify by the discovery it affords.

In post-mortem examinations, much chemical skill is required; and we cannot but remark on the careless and loose manner with which such delicate operations are entrusted to what we may safely call unpractised and ignorant persons. The ordinary medical practitioner is often one whose education has been generally on each branch of the knowledge of his profession; and those of us who are acquainted with the course of chemistry through which most students pass in our hospitals and colleges, previously to passing their examination, would not trust to such the analysis of a piece of chalk. Chemical analysis requires, for its successful pursuit, a most extensive knowledge, a careful manipulation, and a delicately-balanced judgment—which nothing but years of practice can afford—when detection, in cases of poisoning, is required. Other qualifications are also requisite; for if a chemist be merely informed that death has been caused by *an* irritant poison—unless he have sufficient medical knowledge to enable him to anticipate his progress by an acquaintance with the symptoms, &c.,

he may grope long enough, and not only not succeed, but waste the preserved matter, and so prevent discovery or accurate analysis impossible. Two instances of this nature came under our notice some few years ago. In one case, it was suspected that wine, which had been partaken of by several persons, had accidentally been kept in a bottle which had contained poison. The analyst was a clever chemist, but no physiologist; he never asked respecting the symptoms which had exhibited themselves. The quantity of poison, whatever it was, was exceedingly minute; and, for want of a proper clue, he examined for antimony, corrosive sublimate, &c. We happened to suggest, that as it was wine, it was highly possible arsenic might have been communicated to the bottle by its having been cleaned with shot. We also learned that the usual symptoms of arsenical poisoning had been noticed, but in a mild form. On using Marsh's test, a minute quantity of arsenic was actually discovered. In this case, had the chemist possessed medical knowledge, he might have gone straight to his work.

The second instance to which we shall refer, was that of suspected poisoning by arsenic; in which the medical man was utterly deficient of all the capabilities of a chemical analyst. In attempting the analysis, the entire contents of the stomach, with itself, were put into an evaporating dish; heat was applied; and, for want of proper care and experience, the dish was cracked, and the whole of its contents lost. The fragments, however, were collected; and, in more experienced hands, afforded the evidence sought for.

We may, therefore, be permitted to urge on coroners and juries the necessity, in every case in which suspicion lurks, that all available material should be placed in competent hands only, lest that which is of value may be lost, or crude and indigested opinions be expressed, to the damage of the suspected one, or the defeat of the ends of justice.

The directions we are about to give, are simply intended to enable the tyro in chemical science to apply his knowledge in gaining an insight into the *modus operandi*. If he be desirous of repeating the experiments on animals, he will do best to choose rabbits for his purpose, because he will not have to experience that disgust which the analysis of the stomach of cats, dogs, &c., is apt to excite. An anatomist soon gets over all squeamishness in the matter, whilst a tyro has to overcome many offensive details.

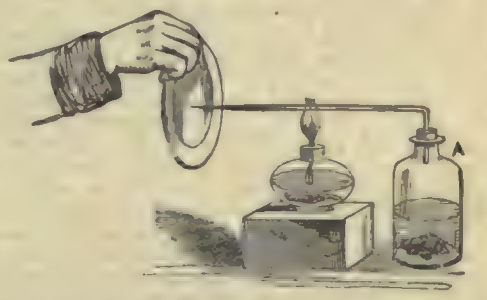
The belly of the poisoned animal is to be carefully opened by means of a sharp knife, or lancet; and the whole of the stomach, from the throat to the gut, removed, *unopened*—just as is done in trussing fowls. We shall be excused by our professional friends if we use unprofessional and plain language. The stomach, &c., is then to be placed in a clean dish, and divided lengthwise by means of the lancet, so as to expose the whole interior. This may be then carefully examined by means of a Stanhope lens, to detect the presence of any white powder. If such be present, remove all of it carefully with a *bone* spoon, or spatula, and wash it off carefully into a test-tube; put this aside for a time.

Supposing no powder to be seen—which will depend on the quantity of arsenic given, and the length of time which has elapsed from its administration—put the whole of the contents of the dish into an evaporating dish containing a little distilled water; and turning the stomach and gut inside out, wash them carefully for some time. This will wash or dissolve out all the unobserved powder, and that which has been dissolved or suspended in the food. Add to the mixture some strong acetic acid; and then cautiously evaporate the whole to dryness, taking care not to do so beyond what is absolutely necessary to ensure that all vapour is driven off. Allow the contents of the dish to cool, and then add some perfectly pure hydrochloric acid: great care must be taken in this respect; because all commercial acid contains arsenic, which would vitiate the result. Heat the acid with the contents of the dish for some time, adding more as evaporation proceeds; and after the lapse of half-an-hour, pour the liquid into a clean glass. By these means all the arsenious acid will have been converted into the chloride, and the solution will be free from organic matter. The rest of the process is very simple. Part of the solution may be boiled with some pure copper, according to Reinsch's plan, and the rest introduced into the hydrogen bottle, according to the method of Marsh. By means of either process, both of which we have already described, any arsenic present will be at once detected.

The powder previously removed into the test-tube may be heated with hydrochloric acid, and the same tests applied as above, when the results afforded will, of course, be the same, although more distinct in their indications.

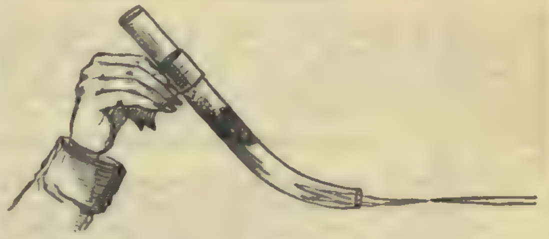
We have thus endeavoured to put the student into possession of what we consider to be the best mode of conducting an elementary analysis for the detection of arsenic. We need only add, that in this, as in all other chemical investigations, care and keen observation are eminently required. In analysing vomited matter, the practitioner must follow the same course; if, however, Marsh's test be adopted, care is required that the animal matter do not cause so much frothing up as to cast the contents out of the hydrogen bottle. This may partly be prevented by pouring a few drops of almond oil on the surface of the liquid in the bottle. It is an excellent plan to heat the tube through which the gas issues before reaching the jet. This is easily done by bending the tube in the manner shown in the following engraving

Fig. 68.



The flame may be received in a bent tube, as represented below, when the double layer of metallic arsenic, and the white arsenic, will be readily noticed.

Fig. 69.



We have already frequently noticed the combination of arsenic with copper, forming the pigment termed Scheele's green. The chloride has also been often mentioned in our previous remarks. The chief other compounds of the metal are those with sulphur; one of which, realgar, is an ore of the metal, of a red colour, containing one equivalent of arsenic to two of sulphur. The other is known by the name of orpiment; and it has a yellow colour.

Arsenic, like copper and iron, produces an acid bearing its name; its symbol is $As\,O_5$. It forms salts with bases; is a white substance; and yields a red precipitate with nitrate of silver alone; whilst arsenious acid requires the addition of ammonia. By this difference, with the colour of the precipitate, the two may be distinguished. Before concluding these remarks, we refer our readers to what we have stated in reference to the combination of antimony with hydrogen, as distinguishing it from that of arsenic with the same gas.*

MANGANESE.

This metal is never found in a native state; and its attraction for oxygen is so great, that it almost immediately passes into a state of oxide after its preparation. It may be obtained from its protoxide by fusing, on exposing the latter, with charcoal and borax, to an intense white heat. Metallic manganese, so produced, presents a similar appearance to cast-iron, which it resembles in many other respects. Its symbol is Mn; the equivalent, 27·5; and its specific gravity about 8.

As we have already stated, manganese readily unites with oxygen. The protoxide is to be obtained by passing hydrogen gas over the binoxide; and it forms salts with acids, of which the sulphate is best known. The most important oxide is that represented by the symbol MnO_2, which is largely employed in various ways. In the laboratory it is used as a source of oxygen gas; and chlorine is produced, by its aid, from common salt. In Glasgow, Manchester, and Newcastle, immense quantities of the peroxide are employed in the manufacture of chloride of lime; and it is also used in calico-printing, for the purpose of producing brown and black colours. Some of the salts of manganese have had similar applications to useful purposes; and common writing-inks have been made from them—the advantage in their use consisting in the fact, that the marks produced cannot be altered, but are rather deepened by age. Some proto-salts of manganese have been used as a test for the presence of ozone. For this purpose, manganic acid, MnO_3, in combination with potash, as the manganate of potash, has been proposed, which darkens to a purple tint as the permanganate, on exposure to air containing free ozone. The chief uses of the oxide of the metal, beyond those already indicated, are in connection with the manufacture of glass. If heated with oxide of iron during the melting of glass, it deoxidises that metal, and so diminishes the tint which oxide of iron affords. If heated in quantities, it affords a blue or amethyst tint; and for a similar object it has been employed in the production of artificial gems. The chief source of proto-salts is the carbonate, and in the manufacture of chloride of lime: this is produced by converting the sulphate into that salt, and thence obtaining the metal again in a state of peroxide. We have observed this process adopted only at the works of Messrs. Tennant, of Glasgow.

COBALT.

This metal is, except in its state of combination, little more than a chemical curiosity. It may be obtained by heating the oxalate with charcoal: when fused, it resembles a mass of cast-iron in many respects; and, amongst others, in that of being magnetic. Its equivalent is 29·5. Specific gravity a little over 8; and its symbol, Co.

In union with one equivalent of oxygen, it affords a protoxide, which may be obtained by adding a solution of potash to the sulphate, by which a hydrate is afforded of a fine blue colour. If chlorine be transmitted through water holding the protoxide in suspension, the sesqui-oxide (Co_2, O_3) is afforded.

The proto-chloride of cobalt has been employed as a sympathetic ink. Words written by means of a quill pen with this ink are almost invisible, or of a very slight pink colour, if the solution be dilute. If, however, the paper on which they are written be held near the fire, the writing will appear of a fine blue colour, which will vanish on the paper becoming cool. Landscapes painted with this and the chloride of copper, have a very pretty and interesting effect.

Nitrate of cobalt is chiefly used as a blow-pipe test, in which respect we shall have to speak hereafter of it.

The most important combination of cobalt, in a commercial point of view, is that with siliceous matter, which produces the substance called "smalts." This is obtained by roasting the ore, and subsequent fusion with sand and potash, by means of which a kind of glass is formed. This, in the state of fine powder, is employed by paper manufacturers, bleachers, and others, for the purpose of giving a blue tint to their goods. Oxide of cobalt is also employed in the manufacture of artificial gems, to which it imparts a blue colour.

CHROMIUM.

In its metallic state chromium is of no importance; but in combination with potash as a chromate, it is largely used for various purposes. The metal is obtained from its oxide by heating it with charcoal powder and oil, with which it is made into a paste. An intense heat reduces the metal, which then presents several of the characters of cast-iron. Its ore is a chromate of iron, found in many parts of Europe. Chromium has an equivalent of 26·7. Its specific gravity is nearly 6; and its symbol, Cr.

Chromium readily unites with oxygen, producing oxide and an acid. The protoxide is an unstable compound, passing rapidly to a greater state of oxidation. The sesqui-oxide has a green colour, and may be produced by heating chromate of potash, and passing over it a stream of chlorine gas, by which chloride of potassium and the sesqui-oxide are afforded. This substance, in union with iron, forms the chromate of that metal, and consequently the ore called chrome iron.

Chromic acid is the most important of the oxygen combinations. It results from a union of one equivalent of the metal with three of oxygen, CrO_3; and may be obtained from the bichromate of potash by means of sulphuric acid. The latter unites with the potash, setting the acid free, which has, like some of its salts, a rich ruby-red colour. Combined with potash it forms both the chromate and bichromate—the former affording lemon-coloured crystals, and the latter having a rich orange or red colour. The bichromate of potass is used by calico-printers and dyers; and chromate of lead affords the well-known pigment—chrome yellow. A higher state of oxidation affords perchromic acid, which, however, has not been completely investigated.

Chromium combines with chlorine, forming two chlorides. With oxygen and chlorine, chloro-chromic acid is produced. This may be obtained by fusing together common salt and bichromate of potash; and, by subsequently adding sulphuric acid, and distilling the mixture in a retort, the chloro-chromic acid passes over as a deep red-coloured liquid. The other compounds of chromium are of no special importance. The oxides are used to produce coloured glass; and by the union with potash and other alkalies, a kind of alum is produced, analogous to those afforded by alumina and the alkalies.

CADMIUM.

Our readers will have noticed that we have described each metal in the order of its comparative value and general importance; and this course has been chosen in the belief, that the general reader and the student would have their interest in our subject best maintained. We are now approaching that limit beyond which the natural history of the elements becomes a matter of importance to the experienced chemist only.

Cadmium generally occurs in combination with zinc, and, like that metal, is volatile. It is dissociated from zinc through its greater volatility; and its presence during the distillation of that metal, is evidenced by the production of a brown-coloured flame; that of zinc having a blue tint. Cadmium is somewhat like silver in appearance, but much softer, and is readily fusible. Its equivalent is 56: specific gravity, 8·6; and its symbol is Cd. Its combinations with oxygen and the salts produced from its protoxide, have no special application in the arts, &c., and are only of interest as the productions of an individual element.

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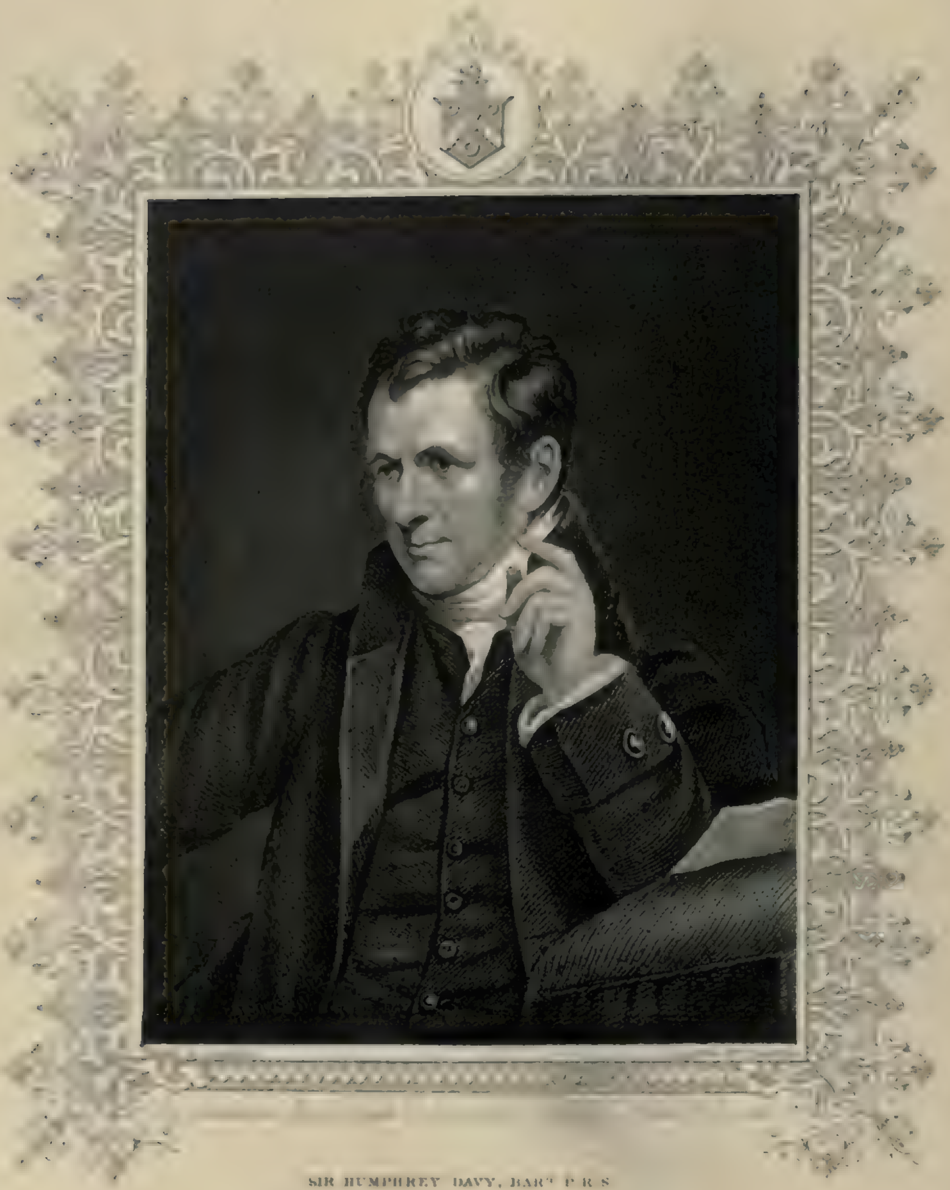
* See *ante*, p. 381.



SIR HUMPHREY DAVY, BART. P.R.S.

Humphry Davy

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OSMIUM, PALLADIUM, IRIDIUM, RHODIUM, AND RUTHENIUM.

THESE metals are found associated with platina. Osmium has a specific gravity of about 10: its equivalent is 99·6; and the symbol, Os. It somewhat resembles platina in appearance; with oxygen, it forms several compounds, amongst which is the osmic acid. This forms salts with bases, termed osmiates.

Palladium—symbol, Pd; equivalent, 53·3; specific gravity, 11·8—also resembles platina, but is soluble in nitric acid. Its other combinations are extremely analogous to those of platina. Its chief uses have been for purposes in which a freedom from ready oxidation is required; and it has lately been largely employed as a substitute for gold in fixing teeth, and in making artificial palates.

Iridium much resembles, in every respect, the metal platina. It combines with oxygen, chlorine, &c. Its chief use has been to form the points of steel pens, &c., for which purpose its extreme hardness renders it valuable. Rhodium and ruthenium much resemble platina in their properties. Rhodium has had a similar application to that we have named in respect to iridium.

VANADIUM, TUNGSTEN, MOLYBDENUM, URANIUM, ETC.

THESE metals are all so rare as to scarcely ever come under the observation of the chemist. Vanadium is found in combination with iron and lead, from which it is separated as vanadic acid. It combines, in various proportions, with oxygen, forming oxides; and with chlorine, two chlorides are producible.

Tungsten is obtained from a mineral called wolfram, and with oxygen produces a binoxide, and an acid called tungstic acid. A combination of this with soda, the tungstate of soda, has lately been highly recommended as a material by means of which ladies' cotton and other dresses may be rendered non-inflammable. We fear that its use in this respect is extremely limited, however valuable its properties may be.

Molybdenum is a white metal, forming oxides and the molybdic acid. This produces salts with bases. The metal is obtained from the native sulphide—a somewhat rare mineral.

Uranium is obtained from a mineral called pitchblende. It forms oxides, the chief use of which is that of colouring glass. The substance so formed has peculiar optical properties in reference to epipolic dispersion of light.

Titanium is obtained from a mineral called rutile. It combines with oxygen, producing oxides and the titanic acid. In some of its combinations it resembles the earth silica.

Tellurium may be termed a quasi metal. It resembles silver in colour, and with oxygen forms the telluric acid. Some analogy exists betwixt this element and selenium.

Our readers will find, in our list of the elements, many other substances belonging to the family of metals proper. Their properties are, however, so uncertain, and their interest so slight, as not to require any detailed notice. The two most interesting metallic elements lately discovered by means of the spectrum analysis of Messrs. Bunsen and Kirchoff, have been named cæsium and rubidium; but the quantities of these yet produced are so small as not to warrant any dependence on the characteristics they present.

KALIGENEOUS METALS, OR THOSE PRODUCING ALKALIES.—POTASSIUM.

FROM the barren waste of information presented by our limited knowledge of many of the elements we have just referred to, we turn with pleasure to the description of the alkaline metals, in which every interest of a scientific and social nature is involved. There are few substances more completely identified with our daily life, manufactures, domestic matters, &c., than the alkalies, potass and soda. All our cleansing, bleaching, and numerous other processes depend on their use. The production of soaps, candles, the purification of oils, the constitution of common salt, and many other products, are identified with them. And we therefore feel it necessary to give an extended notice both of their elements and their numerous combinations.

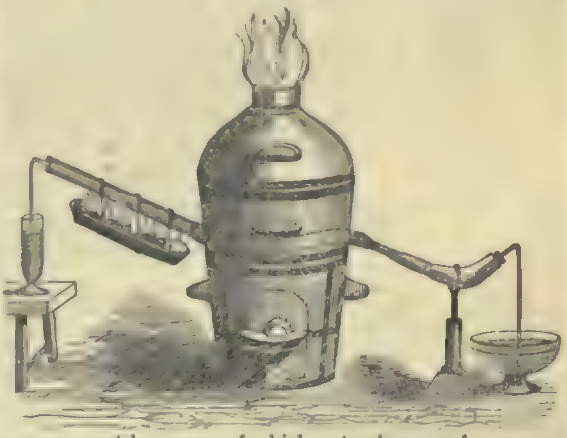
Perhaps no discovery ever made in chemical science, gave rise to such general excitement as that by which Sir Humphry Davy demonstrated that potash and soda contained metals as a constituent. We learn that all ranks crowded daily in the laboratory of the Royal Institution, to witness the new productions. Indeed, by means of these metals, our knowledge of the constitution of numerous other bodies has been largely extended. Each earth was successively examined; and as potassium and sodium have so great an affinity for oxygen, it is a matter of no surprise that this affinity was employed as a means of analysis, and that, too, with successful results. Perhaps the most surprising practical application of the discovery, has been that of an abundant production of a valuable and useful metal, in large quantities, from common clay, or other compounds of alumina. Chemistry is ever affording us surprising and unlooked-for results; and chemists, whilst constantly hoping for new discoveries, do not forget their duty to mankind at large, in applying for the benefit of mankind, in a useful form, that which, seen by the light of scientific research, is regarded by them with philosophical interest only. With these prefatory remarks, we shall now proceed to describe the character of the alkaline metals, commencing with potassium and its compounds.

POTASSIUM, as its name would imply, is obtained from potass. If a piece of fused potass be placed on some platina foil connected with a powerful voltaic battery, whilst a platina wire is brought into contact with the alkali, considerable effervescence will take place. Bright sparks will fly off; and if a little mercury be placed also in contact with the potass, it will form an amalgam with the potassium thus produced. The amalgam, if dropped into water, will decompose the liquid, owing to the associated potassium combining with the oxygen, and setting the hydrogen free.

By this mode of producing potassium, the voltaic current is employed to separate the oxygen from the metal; the oxide of potassium is thus decomposed, and the metal obtained in minute quantities.

Potassium, however, is readily procured in any quantity by means of the following process. The oxide of potassium, or potass, is first fused, and then allowed to run over iron-turnings heated to whiteness. These seize the oxygen of the potass, and the metal potassium is thus set free. The following engraving illustrates an arrangement by means of which potassium can be manufactured on the small scale. In the centre is a furnace,

Fig. 70.



containing an iron tube, filled with iron-turnings. On the left is another tube, holding the potass, which is fused by a small furnace hung beneath it. From this the fused potass runs on to the iron-turnings, when, being decomposed, the potassium is produced. This distils over, and falls into a basin holding naphtha free from oxygen. The tube, on the left-hand side, acts as a kind of safety-valve for the escape of vapour, and prevents any risk of accident in case the iron tube should become filled up.

There are other modes of making potassium on the large scale; but that we have described will acquaint our readers with the general principles of its manufacture.

Potassium has a specific gravity of ·86; its symbol is K (kalium); and its equivalent, 39·2. In its ordinary form, as sold at the shops, it is in the shape of peas, having, externally, a black appearance, owing to the decomposition of the naphtha in which it is preserved for the purpose of preventing its oxidation. If one of these globules be cut open, a shining metallic appearance is presented, which, however, is soon lost, owing to the action of the oxygen in the atmosphere. If a piece of potassium be cast on to water, that liquid is at once decomposed, and the liberated hydrogen combining with a portion of potassium in the form of potassiuretted hydrogen, catches fire. This fact was applied by Mr. Macintosh to the construction of a new kind of shell for warlike purposes. An ordinary bomb-shell was to be filled with any inflammable liquid, and at the opening a few small pieces of potassium were fixed. This shell, on being fired from a mortar and striking an object, would break, and its contents falling out, the potassium would inflame the liquid. A simple experiment, illustrating the principle, may be easily tried by filling a thin glass globe with ether, and dropping into it a pea of potassium. If the glass be then thrown into a vessel of water, so that it may be broken at its sides, the shell will break, and the potassium, coming in contact with the water, will at once inflame the ether. However ingenious this plan may be, the practical difficulties existing in its application have prevented its adoption. We need scarcely add that ether and potassium are articles scarcely fit to be entrusted in the hands of our sailors. Potassium fuses at a temperature of 150°, and may be distilled over like water, if out of contact with the atmosphere. It may also be, in one sense, welded together at common temperatures, as it has a kind of paste-like consistency.

Potass, potash, or pearlash, is an oxide of potassium (KO): the two latter names being applied to the article as found in commerce. It is produced as a hydrate when potassium is exposed to the action of the atmosphere. Commercially, it is obtained by burning various plants; hence its name of potash. The salts set free are then dissolved out by water, when the carbonate of potash, one of the products, is decomposed by boiling with lime. The latter seizes the carbonic acid, forming chalk, and the liquor is then concentrated in iron vessels, until a nearly solid substance is produced. The temperature being raised, the potass becomes fused, and is poured into moulds. In the ordinary state it is a rough mass; but as sold by the chemists, it is generally in the shape of small sticks, which, from their action on animals, are used as a caustic. Potash readily absorbs, in this state, both moisture and carbonic acid. As a liquid, it forms what is termed "oil of tartar."

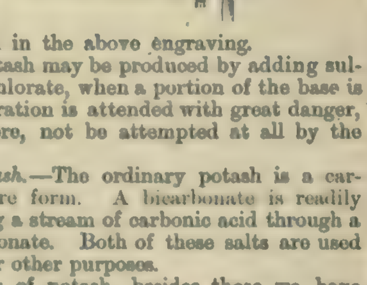
Potash is largely employed for various purposes. With oils it forms soft soaps. Its detergent properties make it of great value to the bleacher, the laundress, and others. A peroxide of potassium is produced when the metal is burned in oxygen gas.

Sulphur and potassium unite to form a sulphide, on potass being fused, together with sulphur. The various salts produced by the union of potass with acids require an individual notice, from their general importance.

There are two sulphates of potass. The sulphate is produced when sulphuric acid and potass, or almost any of its salts, are added together. It is a crystalline substance, and affords no acid reaction. The bisulphate is produced by heating the sulphate with sulphuric acid; by means of which an extra equivalent of acid is added to it. The nitrate of potash—common nitre, or saltpetre—is of great value; forming the chief ingredient of gunpowder. It is imported in large quantities from India; which, in fact, is the chief source of its supply. It is found on the surface of the earth; and results from the decomposition of animal matter. For the same reason it is always present in wells situated near a churchyard; and is the cause of the peculiar cooling taste water presents when obtained from such offensive and dangerous sources. Saltpetre requires refining before being employed in the gunpowder manufacture; and this is effected by means of repeated solution and crystallisation. In its purest form, nitre is found in beautiful six-sided crystals. When fused into balls, it affords the sal-prunella of the shops. Heated alone, it gives off oxygen and nitrogen; but in contact with carbonaceous matter, it affords its oxygen; and carbonic acid, and carbonate of potash, are the results of the combustion. Nitre is extensively used by firework-makers; it is an anti-septic, and may be used in curing meat; and for similar reasons it was employed by the Egyptians in the process of embalming.

Chlorate of Potash.—We shall describe this salt next to nitre, because in almost every respect it resembles that substance. It is produced by passing chlorine through a solution of potass, by which the chloric acid resulting unites with the base. The liquid is then crystallised, and the chlorate is afforded in small flat crystals. This salt has become of great importance in connection with the manufacture of lucifer-matches; and from being only a chemical curiosity, it is now an article of extensive manufacture. On being heated it affords pure oxygen gas. Carbonaceous matter mixed with it deflagrates with great violence. It was proposed to use it in place of nitre; but it so readily parts with its oxygen, and forms with sulphur so explosive a compound, as to present the greatest danger in such an attempt. A few grains of chlorate of potash in powder, mixed with sulphur, and rubbed in a mortar, or struck with a hammer, explode with great violence. If a little sugar and chlorate of potass be mixed together, they may be inflamed by means of a drop of sulphuric acid. And so readily does this salt promote combustion, that pieces of phosphorus in a glass of water, with a few of its crystals, may be inflamed under water. This experiment is best accomplished by pouring on to the crystals and phosphorus some strong sulphuric acid through a funnel, in the manner represented in the above engraving.

Fig. 71.



Perchlorate of potash may be produced by adding sulphuric acid to the chlorate, when a portion of the base is removed. The operation is attended with great danger, and should, therefore, not be attempted at all by the student.

Carbonate of Potash.—The ordinary potash is a carbonate in an impure form. A bicarbonate is readily produced by passing a stream of carbonic acid through a solution of the carbonate. Both of these salts are used in medicine, and for other purposes.

The combinations of potash, besides those we have named, are very numerous. The chloride of potassium is afforded when the chlorate of potass is heated until all its oxygen is driven off, when a white crystalline mass is left, which is very soluble in water—produces great cold during its solution, and is analogous to common salt. The iodide of potassium has been frequently named in our previous pages, especially in connection with photography; in which, also, the bromide is largely used. With tartaric acid, potash produces the ordinary bitartrate, or cream of tartar. With chromic acid, the chromate and bichromate are formed, of which we fully spoke under the head of Chromium. The acetate, oxalate, citrate, malate, prussiate, and numerous other salts, are produced by the union of potash with the respective acids. Were we to attempt to give a list of all the salts producible by the union of potass with acids, we should have to include the whole of the latter. All the combinations of potash are soluble in water; and hence they become of great value as tests in the laboratory. The carbonate is often employed as a flux, both for blow-pipe investigations, and for the analysis of siliceous matters. Potassium itself is largely employed as a deoxidising agent, owing to its powerful attraction for oxygen.

SODIUM.

In many respects sodium resembles the metal potassium. It is prepared in an exactly similar manner; the carbonate of soda, however, being used. Its specific gravity nearly equals that of water; its symbol is Na, and the equivalent 23. Sodium will not inflame if cast in cold water. If, however, hot water be used, or the metal be placed on a piece of wetted blotting or filtering-paper, it will decompose water, and afford a yellow flame. The production of this colour by salts of soda in flame, is a general characteristic of their presence.

Soda is the protoxide of sodium; and, as an article of manufacture, it is of great importance. The ordinary soda of commerce is a sub-carbonate, from which the hydrate may be obtained by boiling a solution of the salt with lime. The earth combines with the carbonic acid, and so leaves the soda itself in its caustic state.

Common salt is the source whence soda is manufactured. The salt is mixed with dilute sulphuric acid, which so forms the sulphate; the hydrochloric acid passing off in a gaseous state. The sulphate is then mixed with sawdust or small coal, and roasted for some time. By this the sulphur is deoxidised, and the sulphuric acid decomposed. An impure carbonate of soda is produced, which is purified by solution and crystallisation. Soda ash, as it is termed, is the same substance in the form of a powder. It, however, generally contains a less proportion of carbonic acid; approaching more nearly to a pure fused hydrate.

Soda is largely employed for numerous purposes. With oils, it forms ordinary hard soap—uniting with the stearine of the fat, and so producing a stearate of soda. The common yellow soap of commerce often contains resinous matter, which is used for the sake of cheapness. Mottled soap is produced by adding a small portion of copperas (sulphate of iron), by which the veins, noticed in that article, are obtained. The better kind of soaps, such as curd, &c., are made from mutton fat; and brown Windsor, and other scented preparations, are manufactured therefrom. In bleaching, soda is used to remove grease and resinous matters from the yarn and cloth; but of late years, lime has been substituted for it, as being a cheaper material. The domestic uses of soda we need not refer to.

Soda unites to form soluble salts with most of the acids; and the following are amongst the chief of its salts:—Sulphate of soda, formerly known as Glauber's salts, is easily prepared by adding sulphuric acid to any salt of soda. It is extremely soluble in water; and may be employed by the student in initiatory experiments on crystallisation. If this salt be dissolved in boiling-water to saturation, and the solution be put away to cool in a glass flask carefully corked up, it will not crystallise if kept quite still. The instant the cork is removed or a small piece of sand be dropped in, the whole liquid will shoot out into beautiful crystals, and be converted into a solid mass. If the external part of the vessel be held in the palm of the hand, a considerable evolution of heat will be also noticed.

Soda and sulphur unite to form sulphites and the hyposulphite. The latter salt has been extensively employed as a "fixing" agent by photographers; and is produced by heating the sulphate with sulphur. It is a crystalline body; and in a solution with water, readily dissolves oxide of silver—hence its employment in the way we have named. Nitrate of soda is prepared by adding nitric acid to any of the soda salts, with the exception of the sulphate. In its properties it greatly resembles nitre;* and, in fact, is sometimes employed as a substitute for that material, in the manufacture of gunpowder and fireworks, for which purpose it is largely imported into this country from India. Its natural production is due to the presence of decomposing animal matter, which affords the nitric acid of the salt.

Chloride of sodium is the well-known common salt. This substance abounds in nature; and is a constituent of sea and river water, in variable proportions. It is obtained from various sources—as the evaporation of sea-water. Our supplies for domestic use are drawn from the salt mines, of which the most notable are those of Cheshire, in England, and Cracow, on the continent. The rock-salt has a yellow or brownish colour, owing to the presence of oxide of iron. On being dissolved, however, this oxide falls down in an insoluble form. The liquor is then evaporated, and left to crystallise, when white salt, such as is used at our tables, is afforded. Pure chloride of sodium is obtained by adding hydrochloric acid to a solution of carbonate of soda.

Common salt seems to be essential to animal existence; for not only is it used as a condiment by man, but it is a great favourite with almost all animals, forming a constituent of their bodies. We may refer our readers for further information on this subject, to the remarks of Dr. Moleschott, at a previous page.†

We have already described the carbonate of soda, as obtained from common salt. In an impure form it is produced from the ashes of sea-plants; and, as such, is known as barilla in commerce. At one time, the only source of soda was that of burning sea-weed; and in many parts of the English and Scottish coasts, the names of small villages are derived from the "pans" in which the barilla, or kelp, was produced. If a stream of carbonic acid be passed through a solution of the carbonate, a bicarbonate is afforded, which is largely used as an antacid in medicine. A natural carbonate, containing two equivalents of soda to three of carbonic acid, is found in some places; but not in quantities to make it of much value as an article of commerce.

Borax, which is a biborate of soda, is largely imported from Tuscany, which affords a natural supply of boracic acid. This salt is much used by metallurgists and others, as a flux; and with many oxides it affords a glass. It is much used in the production of artificial gems.

With acetic, oxalic, and other vegetable acids, soda unites to form salts; but as these are of no special importance, we shall not further allude to them.

LITHIUM.

This metal, like potassium and sodium, may be obtained from its oxide by means of a powerful voltaic battery; and in its general characteristics it is much like them. Its equivalent is 6·5; the symbol, L; and its specific gravity much less than either of the alkaline metals. Its oxide, lithia, is analogous to soda and potass; and, like them, possesses alkaline properties. It is a constituent of the tourmaline and other minerals; and forms a link between the alkalies and earths. As, however, it is of great rarity, we shall not occupy any space in its consideration. We may observe, that Messrs. Bunsen and Kirchoff have discovered, by means of their spectrum analysis, a new element, which has been termed cæsium, in some of the compounds of lithia.

TERRIGENEOUS METALS, OR THOSE PRODUCING EARTHS.

Having described the properties of the metals proper, and of those producing alkalies, we shall now turn to a very interesting branch of chemical science, and proceed to describe those metals which, by oxidation, produce earths. Although the face of nature presents a most diversified appearance, from the barren rocks to the cultivated soil, still the number of earths is very limited; and taking the average of the soil, &c., of this kingdom, we do not find its essential ingredients to exceed three or four in number, of which silex, lime, and alumina are the chief. Of course, in minute quantities, many other substances are present, which are essential to the growth and existence of plants. Common clay is a mixture of alumina and oxide of iron. The sand of our hills and the sea-shore is composed of silicious matter, or pounded flint. Gravel is mostly composed of silex and oxide of iron, which gives it its colour. The chalk rocks, which abound in Kent and other places, are nearly pure carbonate of lime. Granite, felspar, quartz, &c., which form the primitive rocks, are chiefly composed of silex; whilst those of the secondary and other formations have alumina, silex, magnesia, and lime, in constantly varying proportions. The earths have been divided into classes; namely, earths proper, and alkaline earths; of the latter, are lime, magnesia, baryta, and strontia. To the general observer of nature, as well as to the geologist and chemist, the disintegration of the rocks, and the productions of soil, are matters of deep interest. The action of air and water gradually crumbling down the most solid mass; the first evidence of vegetable existence, as seen in the humble moss, which, whilst robbing the rock to supply itself with food, converts that rock from a barren waste to a fruitful soil; the liberation of the constituents of the rock, which afterwards afford nourishment to the grass and tree;—these and numerous other changes, gradually but constantly taking place, invest the study of the earths with the deepest interest. But we must reserve our remarks on this subject for that of Agricultural Chemistry, in which we may more properly dilate on the variety of changes which occur in the production of soil, plants, &c.; and we shall at once proceed to describe the earths, the metals, and compounds, together with their varied uses in commerce, medicine, and manufactures.

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* See *ante*, p. 386.

† *Ante*, p. 354.

ALUMINIUM.

ONE of the greatest curiosities of modern times, is the production of a valuable metal from common clay, which, in its pure form, is simply an oxide of the metal aluminium, or, as it is also called, aluminum. Its discovery has resulted from those of Sir Humphry Davy, who, as we have already remarked, was the first to prove that the alkalies, potass, and soda are the oxides of metals. Aluminium has a specific gravity of 2·5; and is therefore of about the same as that of glass; its symbol is Al; and the equivalent = 13·7.

It is best produced by the process of M. St. Claire Deville, of Paris, to whom we are indebted for the commercial importance of the metal. A mineral called "cryolite" is its most convenient source; but the following plan may be adopted to procure it from clay:—This material, with sugar and charcoal, is to be made into a paste, which is then to be placed in a platina tube. The tube is to be heated in a convenient furnace to a red heat; and chlorine gas is passed over the paste. By these means the chloride of aluminium is produced through the following chemical change which takes place. The oxygen leaves the aluminium of the clay, and the chlorine takes its place; so that we find the chloride formed in place of the oxide. Now, as the chloride of the metal is very easily sublimated or removed by heat, it speedily passes away from the platina tube, and collects in a receiver placed at the other end of the tube, in the form of a primrose-coloured powder. Great care must be taken that the atmosphere has no access to the receiver, because the moisture it naturally contains would be absorbed by the salt-like substance, and such would be thereby rendered unfit for the purpose of obtaining the metal. The chloride thus produced is then heated with the metal sodium, by means of which the chlorine is taken from the aluminium, and combines with the sodium, forming common salt. The metal aluminium is then found at the bottom of the crucible, of a white colour, and having the appearance of silver: and in many of its qualities it is similar to that metal.

The first bar of aluminium which was exhibited in this country, was sent by the present Emperor of the French to the Polytechnic Institution, Regent-street, London; and it excited, at the time, the deepest interest in scientific circles. On the occasion of its first exhibition, many of the leading scientific celebrities of the kingdom were present; and Messrs. Hunt and Roskell, of Bond-street (her majesty's jewellers), kindly lent numerous specimens of emeralds, and other precious stones, for the purpose of elucidating the *rationale* of the process. We may here remark, that emeralds, rubies, garnets, and the corundum, with other gems, are largely composed of aluminium in the state of oxide—such as the pure alumina, of which we have been speaking.

Aluminium is susceptible of a high polish. It has naturally a whiter colour than pure silver; and whilst that metal soon tarnishes, or loses its polish when exposed to air containing sulphur, which invariably occurs in places wherein coal-gas is burned, aluminium retains its polish, because its affinity for sulphur is very slight. It is very ductile, and may therefore be drawn into a wire. It may also be easily beaten out into sheets or plates, and in that form may be employed for a vast variety of purposes. It has been made into brooches, spectacle-frames, helmets, spoons, chemical apparatus, and various other utensils; and, so far as we can judge, it promises to be of extreme value, generally speaking, in its employment for all purposes where freedom from action by chemical agents is required.

The sulphuric, nitric, and hydrochloric acids have but a slight, if any action on it. Indeed, in our own experiments we have found that its chief enemy (if we may so call it) is any kind of alkali, such as potass and soda.

If a bar of the metal be immersed in either of these alkalies, the water in which they are dissolved is at once decomposed, and hydrogen gas is given off. A jelly-like substance forms on the surface of the metal; and on chemical examination, such is found to be the hydrate of alumina. This, however, only occurs when the alkaline solution is very strong—a circumstance rarely, if ever, occurring in daily life.

It will thus be observed, that modern chemistry, although it does not turn whatever it touches literally into gold, yet it has the power of producing from the very dust which we shake off our feet, a metal valuable to us in the highest degree. It may be safely said of experimental science—

"Whatever it touches it adorns."

Give the chemist the refuse of our manufactures, and he will, by the potent aid of his art, almost create that which is truly valuable from that which appeared next akin to nothing. He calls out of the chaos of nature its most valued elements; and by the intelligence which he exercises, he invests with interest that which had been a matter of offence and repugnance.

Alumina is the only oxide of the metal; and it may be produced by adding a solution of carbonate of ammonia to one of common alum. The precipitate, when dried, affords a white powder, which is the earth alumina.

This substance, as found in the form of clay, is of great importance; and as such, is the material from which China, pottery, bricks, &c., are produced. The colour of clay varies considerably; the purest, or "kaolin," being nearly white. London clay has a slate or blue colour; whilst that from which the lower kinds of pots, pans, bricks, and tiles are produced, is of a brown tint before being heated, and attains a red colour after undergoing the process of baking.

The quality of whiteness, comparative transparency, &c., found in the highest kinds of porcelain, are due to the nature of the clay employed in the manufacture. With the kaolin, a quantity of flint, reduced to an impalpable powder, is mixed, so that an homogeneous paste is produced. This then undergoes the process of fashioning into the shape of the articles required; but as, in the process of baking, the material contracts nearly one-third in bulk, allowance is made in the size of the article as it leaves the hands of the potter. Of all implements of manufacture, the potter's wheel seems to have undergone the least alteration by the lapse of time; for, on comparing the instrument in use at the present day with those employed three thousand years ago, as illustrated in ancient drawings and sculpture, their identity is almost complete.

The process of firing or baking is one requiring the greatest care, because, if the material have been improperly mixed, the different parts will contract unequally, and the vessel will be destroyed. Each article, as it is produced by the potter, is removed to a kiln fitted with shelves, on which the vessels to be baked are arranged. A fire is then kindled in the centre of the kiln, and by it the mass of the articles is aggregated and made into a compact body. The vessels so baked are allowed to cool gradually, and so to assume their proper shape and consistency. In this state, however, they are porous, and unfit for domestic use. The next process, therefore, is that of glazing, by means of which the external surface is converted into a kind of glass. This is effected by washing the surfaces of the vessels by means of a mixture of powdered felspar, borax, &c., for the better class of goods, and of common salt for common goods. The articles are again removed to a kiln, and by means of an intense heat the surface is fused, and a glaze formed thereon. On cooling, the pottery is then ready for sale. If patterns such as we observe in china and plates, &c., be required, such are painted on before glazing; and in that process they are "burnt in" on the surface of the ware.

The preceding remarks afford a general insight into the manufacture of all kinds of ceramic ware. Our limits proscribe us from entering into minute details. Those of our readers who may be desirous of full information on the subject, and yet have not the opportunity of visiting a pottery, may trace every part of the process in the brick-field, which is a rough but accurate type of every class of the manufacture.

Alum.—Alumina, in combination with an alkali and sulphuric acid, gives rise to a series of compounds known in commerce as alums. The manufacture of alum is always carried on in close proximity to the place where the slate or suitable clay is found; and at Campsie and Hurlet, near Glasgow, immense quantities of the article are produced. The slate is first roasted, by which it is reduced into powder. Exposed to the atmosphere, a chemical decomposition ensues. Sulphuric acid is produced by the oxidation of the sulphide of iron, and so a sulphate of alumina is formed. Potash being added thereto, a double salt results, forming a sulphate of potash and alumina, which is the ordinary alum of commerce. Instead of potash, soda or ammonia may be substituted with a similar result.

The chief uses of alum are those in dyeing; for the alumina which it contains has the power of attracting the colouring matter of vegetable bodies. For this reason it has been termed a mordaunt. The principle of its employment for this purpose may be readily understood by adding an infusion of many vegetable colouring substances to one of alum, and on precipitating the alumina by means of an alkaline carbonate, a coloured precipitate will be produced. In the process of dyeing, the yarn or cloth is first steeped in a solution of alum, and subsequently boiled in one of the colouring matter, by means of which the colour is precipitated in a solid form, and fills up the minute tubes of which the fibres of cotton and wool are composed. Alum is also employed in medicine as an astringent. The other combinations of alumina do not require special notice.

SILICON.

We have placed this substance amongst those producing earths, rather as a matter of convenience—silica, its oxide, being one of the most common constituents of rocks and soils. It may be obtained from pure silex, which is a combination of silicon with oxygen, by heating the silex with the potassium. It is a powder of a dark-brown colour, and, in some respects, is more analogous to carbon than to the metals. Its symbol is Si, and its equivalent 21·3.

Silica, silex, or silicic acid, is found in its purest state as rock-crystal; it is also the chief constituent of common flints, and of the sand of the sea-shore, &c. It is composed of one equivalent of silicon with three of oxygen, $Si\,O_3$. It may be prepared artificially by fusing sand with four times its weight of carbonate of soda: the product is afterwards to be dissolved in water and hydrochloric acid, and the solution is evaporated to dryness, when a white insoluble powder is afforded, which is the silica. It has a gritty feeling in the mouth, and is hard to the touch. Silica, in a crystallised state, is often to be noticed on the outside of some kinds of grasses. A chloride of silicon may be produced by similar means mentioned for obtaining chloride of aluminium, silicon being substituted for silica in the process. Silicic acid unites with soda, &c., to form silicates; and, by such means, a soluble glass is formed, which has been much used as a preparation for coating stone liable to decay by atmospheric influences. The application of silica promises to be of great importance; and it has been proposed to arrest the progress of decomposition now going on in the stone used in building the Houses of Parliament, which is rapidly being disintegrated by a variety of causes.

The most important use of silica, in a commercial point of view, is that of the manufacture of glass of all kinds, of which it is an essential ingredient. For this purpose, sand is fused with soda, together with a little oxide of lead, to give it fusibility. The "metal," as it is called, is kept in a state of fusion for some hours, until all the ingredients are completely mixed. The colour of glass varies, and has a greenish tint if oxide of iron be present. Coloured glass is produced by burning various metallic oxides into ordinary glass at a high temperature. The oxides used for this purpose have already been named in our previous pages, under the heads of their individual metals. (See Iron, Copper, Cobalt, Chromium, Manganese, &c.) If a proper proportion of alkali be employed, ordinary glass is unacted on by air and moisture. The lapse of time, however, indicates that a very slight action takes place on most kinds of glass, although the progress is very slow. Artificial gems are produced by melting together metallic oxides, silica, and alkalies; and to so great a perfection has this art arrived, that some productions of this kind almost equal the true stone in their beauty of colour and other qualities. Hydro-fluoric acid is the only one in which silica is soluble, and it is accordingly employed in etching on glass surfaces.

CALCIUM.

This metal, which is at present very rarely obtained, may be produced by heating lime with potassium or sodium. Its equivalent is 20, and the symbol Ca. It is highly possible that if the attention of chemists were turned to its production in quantities, another valuable addition would be made to our stock of useful metals. The general properties of calcium are but little known.

The well-known earth, lime, is an oxide of the metal, $Ca\,O$; and it may be obtained by heating chalk (its carbonate) to redness in an open vessel. By this, the carbonic acid is driven off: access of air is required, because the gas is liable to form a heavy superstratum if the operation be carried on in a closed vessel. On the large scale, lime is produced by burning chalk or limestone in a kiln. Soft and hard lime are respectively produced by the use of these two materials.

Lime is used for a great variety of purposes. With sand, it forms mortar, after being slaked; during which process, a great amount of heat is set free; sufficient, indeed, to set fire to wood. It frequently happens that wooden stores, sheds, carts, &c., are set fire to when a shower of rain falls on lime stored in them. The value of lime in mortar, consists in the fact that it gradually becomes a compact solid, by the absorption of carbonic acid from the air, for which it has a great attraction. It is used by bleachers as a cheap detergent; and candle-makers employ it to saponify tallow in the process of the stearine manufacture. It is an essential constituent of all soils; and is found in almost every vegetable and animal, in the form of either sulphate, carbonate, or phosphate. In the process of tanning, it is used to remove hair from the skins; and in solution as lime-water, is a useful depiliatory for removing superfluous hairs from the head and face.

Chloride of lime, so called, is produced by passing chlorine gas over slaked lime, by means of which the gas unites with the earth, to form what is termed a hypochlorite. The manufacture of this substance is carried on to a large extent for the use of bleachers; and perhaps no chemical product is of more value to us, in a social point of view, than this substance. By means of its solution in water, as much cotton yarn or cloth can be bleached in a few hours, as would have required weeks under the old system of bleaching in the air. The process is conducted as follows:—The yarn is first boiled in lime and water, to remove grease, resin, &c., and then immersed in dilute sulphuric acid, to dissolve out the oxide of iron usually present. It is then immersed in a tepid solution of chloride of lime; and having been taken out and drained, it is dipped into a trough containing acid and water, which sets free the chlorine. After being washed in abundance of water, the yarn is then dried; and will have acquired a perfectly white colour during the process. Chloride of lime is much used also as a disinfectant, for which it is extremely valuable.

Lime unites with chlorine, forming the chloride of calcium. This substance is much used in chemical laboratories to absorb moisture. It is prepared by saturating hydrochloric acid with lime; and after evaporating the water as far as possible, the crystals are fused in a crucible. The chloride must be preserved from contact with the atmosphere, or it will speedily become liquid from the absorption of water.

Fluorine and lime unite to form a fluoride, of which a well-known mineral, fluor spar, is composed. This is the chief source of fluorine and hydro-fluoric acid. The phosphide of lime has been already described, under the head of Phosphorus.*

Sulphate of lime forms the well-known plaster of Paris. It may be prepared by adding sulphuric acid to the carbonate; but, on the large scale, it is produced by burning the crystallised form of the salt, called gypsum. Its uses are numerous; amongst which, the most important are that of producing a cement, and for making casts, statues, &c. It has a great attraction for water, and so becomes solid on being mixed with that liquid; hence its employment for the purposes we have referred to. Crystals of the sulphate abound in London clay, wherein they may be readily distinguished by their glistening appearance.

Carbonic acid and lime unite to form the carbonate, which abounds in some parts of the world as chalk and limestone. Chalk is the source whence lime is obtained; and both are much used as a dressing for land. Carbonate of lime is found in both the vegetable and animal kingdoms in abundance. Although insoluble in pure water, it is readily dissolved by water holding carbonic acid in solution. It is the chief cause of the hardness of water; but it may be removed by adding lime-water, which precipitates the lime in solution as bicarbonate, by seizing its carbonic acid. Dr. Clarke, of Aberdeen, very ingeniously proposed to apply this process for rendering hard water soft; and it has been long in use at Charlton, near Woolwich, where, owing to the water being obtained from the chalk hills, the water is very hard. On boiling water containing chalk in solution, the latter is deposited; hence the cause of the fur in tea-kettles and steam-boilers. Stalactites are formed by the dripping of water charged with the bicarbonate of lime, through crevices in caves, &c., of which there are numerous instances in Derbyshire. The doubly refracting Iceland spar, and the beautiful mineral Arragonite, are both carbonates of lime. These may be generally found in the crevices of limestone rocks; and are produced by crystallisation of the soluble carbonate, as it trickles down their surface. Marble is a carbonate of lime which has undergone fusion. If chalk be rammed into a gun-barrel, and then exposed to an intense heat in a smith's forge, it will present, on cooling, the appearance of marble. When pure, that substance is of a white crystalline texture; and the black or coloured veins in it are produced by the presence of metallic oxides, especially that of iron.

Some of the most important salts of lime are the phosphates. They form, as we have frequently remarked, an essential constituent of bone. The super-phosphate, so called in commerce, is produced by adding sulphuric acid to calcined bones, when sulphate of lime is also afforded. This is a valuable manure, and as such, is highly prized by farmers. Guano, and all animal excrementitious substances contain it in considerable quantities. A phosphate may be readily produced by adding phosphate of soda to the chloride of calcium in solution.

In medicine, lime, as a carbonate, the creta preparata of the shops, is used as an antacid, which is also an excellent dentifrice. It is an essential ingredient in sea-water; hence the large quantities of chalk contained in sea-shells. The eggs of all birds are chiefly composed of it and albumen; and it is a singular fact, that pod-bearing vegetables bear no fruit in its absence; whilst in the animal kingdom, birds will lay eggs without shells if deprived of its use.

MAGNESIUM.

THIS metal forms the basis of the well-known earth magnesia. It is obtained by heating potassium and the chloride of magnesium together, in a test-tube or crucible. At the moment that decomposition takes place, both light and heat are evolved, and the metal magnesium is set free. It has an appearance similar to silver; its symbol is Mg, and its equivalent 12·2. Magnesia is an oxide of the metal; and is a somewhat abundant earth. It is a constituent of many rocks, especially of the magnesian limestone. Sea-shells also contain it; and as a chloride it is found in the water of the ocean. In its pure state it is a white powder, scarcely soluble in water, and belongs to the alkaline earths. Its chief salts are the sulphate and carbonate.

The sulphate is the well-known Epsom salts, so largely employed in medicine. It is a constituent of many mineral waters, and derives its name from the town of Epsom, in the wells near which it is found. It is obtained by digesting sulphuric acid with magnesian limestone; the solution being afterwards crystallised.

The carbonate of magnesia, also used in medicine as an antacid, is found abundantly in a native state, combined with lime. The rocks formed of these materials are termed dolomite; and they are extensively used for building purposes; the new Houses of Parliament, and many other public buildings in London, &c., being constructed of it. When first precipitated, the carbonate is a bulky white powder, which, on being heated, contracts considerably. The two kinds are sold in the shops as light and heavy magnesia. It is soluble in excess of carbonic acid; and in that way can be obtained in crystals. Murray's fluid magnesia is prepared by dissolving the carbonate in water containing carbonic acid in solution.

The chloride is produced by adding hydrochloric acid to the carbonate; or, still better, by the addition of sal-ammoniac to this solution; when, after evaporation, it is to be fused. It must be preserved from access to air, as it is highly deliquescent. As we before mentioned, the metal is obtained from this substance.

Magnesia combines with nitric acid, forming a nitrate; and with ammonia and phosphoric acid, it produces a double salt, scarcely soluble in water.

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* *See ante*, p. 371.

Neither magnesia nor its salts have any application to the arts or manufactures; and their uses are almost entirely confined to medicinal purposes.

BARIUM.

THIS metal is obtained by the action of potassium on baryta, its oxide, and presents a similar appearance to those obtained from other earths. Its symbol is Ba, and the equivalent equals 68·5.

Baryta derives its name from its great specific gravity, being the heaviest of the earths. It is the protoxide of the metal, and may be obtained by heating the carbonate to redness, when a grayish-white powder is produced, much resembling lime in its leading properties. In the state of a hydrate, baryta may be crystallised; it has an alkaline reaction on test-papers, and is highly poisonous.

The sulphate of baryta, or heavy spar, is found in many places. It may be prepared artificially, by adding sulphuric acid or a sulphate to any soluble salt of baryta; and the affinity between the base and acid is such, that baryta can separate sulphuric acid from any of its salts in solution. Baryta salts are, in fact, the usual tests employed in the laboratory for the presence of that acid. The nitrate is easily obtained by dissolving the native carbonate in nitric acid; and it is used as a reagent. If dissolved in alcohol, it gives a rich green flame on ignition of the spirit. It is largely employed by firework-makers as an ingredient in green fire. The chloride, which is also a useful test, is readily produced by the solution of the carbonate in hydrochloric acid, from which the salt, in a crystallised state, is readily obtained.

A peroxide of baryta is an interesting compound, from the fact that it enables us to procure a peroxide of hydrogen. It is produced by heating caustic baryta in oxygen gas, at a temperature a little short of a red heat.

STRONTIUM.

THIS metal, like barium, is, at present, only obtainable in minute quantities, either by voltaic action as an amalgam, or by the action of heat and potassium on strontia. It has similar properties to barium; its equivalent is 43·8, and the symbol, Sr.

Strontia is obtained by heating the carbonate, which is a somewhat abundant mineral, found at Strontian, in Scotland. It has an alkaline action on test-papers, and is soluble in water, from which it may be obtained as a crystalline hydrate. Native sulphate of strontia is termed celestine, and is found in some of the northern counties of England and Scotland in a crystalline state, some of the specimens presenting a beautiful appearance. The nitrate is produced by dissolving the carbonate in nitric acid. This salt affords a fine deep-red flame when burned with spirits of wine, and is used in the manufacture of fireworks to produce coloured fire, &c., which affords a splendid deep-red colour. The carbonate, as we have remarked, is found in a natural state. It may be produced artificially, for laboratory uses, by exposing the hydrate to the action of carbonic acid. The chloride and other salts are producible by means similar to those for making barytic salts.

It only remains for us to mention, that there are four earths very rarely met with, which complete the list of that class of bodies. They are Thorina, Yttria, Glucina, and Zirconia. Thorina is obtained from the mineral called Thorite. Yttria from Gadolonite, &c. Zirconia from Zircons; and Glûcina from a few minerals, amongst which the emerald is the chief. The properties of these earths generally agree with those of alumina or other earths already described; but as they have been scarcely investigated, any description of their individual properties is almost impossible. They respectively afford metallic bases analogous to those of earths generally.

TESTS FOR METALS, ETC.

IN our previous pages we have described the leading properties of metals proper, alkaline, and earth-producing metals; and we shall presume that our readers have become well acquainted with their individual characteristics. We shall now proceed to mention the usual mode of detecting the presence of the leading metals and their compounds. This will serve the double purpose of teaching the art of testing and of giving exercises, by means of which the student may become practically acquainted with the various states and conditions in which metals and their oxides are met with in nature, or in the course of laboratory practice. We shall resume the plan of experiments, and point out the various tests, and the mode of using them in the manner adopted by analytical chemists. Our readers, by trying the following experiments, will thus obtain an insight into the methods of inorganic and organic analysis, on which we shall have to dilate hereafter.

The object of a test is to discover, by the addition of one body to another, the presence of that which is being sought for; and our article on Arsenic illustrates one application of this fact. To apply the proper test in any case requires that we should be fully acquainted with the properties of the test, and also of the substance tested; and nearly every substance so employed has already been fully described. If our readers, therefore, have any difficulty in this respect, they need only refer to the substances named under the heads of the respective metals, when they will at once obtain the required information.

The state of solution, if not always necessary, is, at all events, the most convenient for the purposes of testing. The particles of a body are then in the most favourable condition to obey the laws of chemical affinity. Being suspended in a fluid, they are easily moved; and so, whether remaining in solution or precipitated as a powder, they are at once under our command. For the following experiments a number of test-tubes, Florence flasks, some filtering paper, a funnel, stirrers, &c., are required. We may also notice, that in many cases slips of window-glass, or white plates, will answer every purpose; for on either of these the peculiar and distinctive colour, &c., produced by a test, are readily observed.

To assist our readers, we shall give a list of a few tests which will be needed; and shall also refer to the page in which they have already been mentioned. With the exception of the acids, all the tests should be dissolved in *distilled* water, and kept in glass-stoppered bottles. Besides the sulphuric, nitric, and hydrochloric acids, the following are the chief tests:—

Hydro-sulphuric acid	See p.	370.
Hydro-sulphate of ammonia	,,	370.
Caustic potass	,,	386.
Carbonate of potass	,,	386.
Prussiate ,, ,,	,,	378.
Caustic soda	,,	387.
Carbonate of soda	,,	387.
Chloride of sodium	,,	387.
Sulphate of iron	,,	377.
Liquid ammonia	,,	324.
Carbonate of ammonia	,,	324.
Chloride of barium	,,	391.
Nitrate of baryta	,,	391.
,, ,, silver	,,	374.
Sulphate of soda	,,	387.
Chloride of platina	,,	372.
Chromate of potass	,,	384.
Nitrate of cobalt	,,	384.
Iodide of potassium	,,	386.

Besides the above, a few slips of copper, zinc, and iron, and some litmus and turmeric paper should be provided.

We shall now show how these tests are employed; first giving some introductory experiments to initiate our readers, before we proceed to give separate tests for each metal.

Experiment 68.—Dissolve a crystal of the proto-sulphate of iron in distilled water; and divide the solution into three test-tubes. Into one pour a few drops of a solution of the prussiate of potass; a blue precipitate will be produced.

Experiment 69.—Into another of the test-tubes pour a few drops of tincture of galls, which is sold by all chemists and druggists; a black colour will be afforded.

Experiment 70.—Into another test-tube drop some solution of carbonate of soda; a dark-green precipitate will fall down, which, on being exposed to the air, will turn to a red colour.

It follows, from the above experiments, that the presence of iron may be detected by three methods; each test employed affording a different result. We have chosen iron as the subject of our illustration, because the characteristics of its presence are so easily noticed.

But such is not the case with many other substances. For instance, white powders or precipitates are thrown down from many solutions; and to distinguish between them is by no means easy. Each precipitate, however, has a distinctive character, as the following experiments will prove.

Experiment 71.—Add together, in a test-tube, solutions of common salt and nitrate of silver. It will be noticed that the white precipitate has a peculiar appearance, like curdled milk.

Experiment 72.—Add a little solution of chloride of barium to one of sulphate of soda. A white precipitate will be produced in a pulverulent form.

Now these two precipitates, although both white, are readily distinguished by their physical appearance; and may be still more so by the action of light; for if that of the silver be exposed to the rays of the sun, it will turn black, whilst the precipitate of sulphate of baryta, in the last experiment, will maintain its white appearance.

In precipitates which are coloured, very little difficulty exists, as the colours vary so considerably. But in some cases great caution is required in the amount of test which is used; because an excess will not only re-dissolve a precipitate, but will even prevent its production. Of this the following are two instances:—

Experiment 73.—Add a *little* soda-water to some lime-water. Carbonate of lime will be thrown down. Afterwards add soda-water in abundance, when the *excess* of carbonic acid will at once re-dissolve the precipitate.*

Experiment 74.—To a solution of corrosive sublimate, add a few drops of one of iodide of potassium. A yellow precipitate will be afforded. Add still more of the iodide, when the biniodide of mercury, which has a red colour, will be produced. Afterwards add the iodide of potassium in excess, and the precipitate will be entirely dissolved. A few drops of the corrosive sublimate being added to the liquid, will first produce a red, and then a yellow iodide, which would be again dissolved by an excess of the iodide of potassium.

These experiments will impress on the mind of the student the necessity of caution and care in testing; and will, we trust, correct the almost invariable practice of the tyro in using *large* quantities of any test.

We may now show how accidental circumstances will often cause considerable difficulty to the careless or ignorant person; and even experienced chemists may, through the neglect of proper precautions, be led into error. Not only is it required that proper tests be employed, but that they should also be perfectly pure, and free from all foreign matter.

Experiment 75.—Dissolve a little nitrate of silver in distilled water, so as to obtain a perfectly transparent solution. Fill a test-tube with distilled, and another with river-water; and to each add a few drops of the solution of the silver salt. The distilled water will afford no change, but the river-water will give an opal-coloured precipitate, owing to the presence of common salt and other chlorides.

We may here remark on the necessity of great cleanliness in all vessels used for testing purposes. In long laboratory experience, we have noticed numerous failures occurring to young analysts through neglect in this respect. In very accurate experiments, test-glasses, &c., should invariably be washed by distilled water, lest some of the salts contained in common water should remain on the surface after washing.

Occasionally precipitates do not readily exhibit themselves, as may be seen in the following experiment:—

Experiment 76.—Add a little tartaric acid to a solution of carbonate of potass, in a test-tube perfectly free from scratches, &c., on the glass surface. No precipitate will be noticed. Now rub the surface of the glass by means of a stirrer, or rough substance of any kind. A precipitate, composed of minute crystals, will soon be produced, owing to an angular surface, which is favourable to crystallisation being carried on.

Occasionally chemical changes soon take place in substances kept as tests. An instance of this may be found in the solution of iron; for if the proto-sulphate, in a pure state, be tested by means of prussiate of potass, a light greyish-blue precipitate will be produced. If, however, the solution of the iron-salt has been kept for any length of time in a bottle *half filled* with it, the oxygen of the air in the bottle will have converted the protoxide into the peroxide; and the precipitate, by means of the prussiate of potash, will have a dark-blue colour.

Many metallic oxides, &c., may be readily distinguished by the colour which they afford to flame; of which the following are instances:—

Experiment 77.—Dissolve common salt in spirits of wine, and ignite the solution; a yellow flame will be afforded.

Experiment 78.—Dissolve in the same way, separately, the nitrates of baryta and strontia. These will afford, respectively, a green and red flame.

In using the blow-pipe, these indications are of extreme importance, and are relied on by the chemist when minute quantities of bodies alone can be obtained.

Hydro-sulphuric acid, and hydro-sulphate of ammonia, are much employed as tests for metals. Either may be produced by the methods described at a previous page;† and the study of their indications will be of the utmost importance to the student, and absolutely necessary before he commences a course of analytical chemistry.

The solution of many metals may be decomposed, and the metal set free by immersing slips of other metals; of which the following afford instances:—

Experiment 79.—Immerse a piece of silver foil in a solution of chloride of platina. The latter metal will be deposited on the silver as a black powder.

Experiment 80.—Into a solution of nitrate of silver, introduce a piece of polished copper, when the silver will be at once reduced to a metallic state.

Experiment 81.—Dip a piece of clean iron into a solution of sulphate of copper. The latter metal will be at once precipitated on the iron as a brown powder, which consists of pure copper.

The foregoing experiments will familiarise the minds of our readers with the general principles which must be followed in examining the characteristics of metals, &c., by testing. For the methods of bringing them into solution, our previous observations and instructions, under their separate heads, must be followed; and we strongly advise all who are desirous of acquiring and maintaining the knowledge of the properties of metals, to keep the precipitates produced in the experiments by them in a dry state. Such should be put into corked test-tubes, and properly labelled, with the name in full, together with the symbol showing their constitution. By such an arrangement, the results of previous investigations are kept ready for reference; and in the course of further investigations, many difficulties may thereby be avoided. In our earlier days, in chemical study, we adopted this plan; and in subsequent investigations of a more scientific nature, have often arrived at speedy conclusions; whilst our companions had to refer to half-a-dozen books for descriptions, in print, of what they ought to have had visibly before them. Of course, when a person becomes experienced, such aids are unnecessary, but never useless. Our remarks have still more force when applied to crystalline substances, in which the external form varies so greatly.

We shall now proceed to point out the various tests which may be applied for each metal; and which our readers should employ, if they would obtain an extended and practical knowledge of chemical science. Of all branches of science, that of chemistry is essentially experimental.

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* See *ante*, p. 390.

† See *ante*, p. 370.

Platina.—A full description of the leading properties of this metal has been given at page 372. The following are tests for its presence in solution:—

Solution Tested.	Test.	Results.
Bichloride	Silver plate	Black powder.
—	Sal-ammoniac..........	Yellow crystalline precipitate.
—	Hydro-sulphuric acid	Black sulphide.
—	Hydro-sulphate of ammonia	Do.

Gold.—See p. 372.

Chloride	Proto-sulphate of iron	Metallic gold.
—	Proto-chloride of tin..	Purple powder.
—	Hydro-sulphuric acid	Black sulphide.
—	Hydro-sulphate of ammonia	Do.

Silver.—See p. 373.

Nitrate............	Copper plate..........	Metallic silver.
—	Common salt.........	A white curdy precipitate, which turns black on exposure to light.
—	Iodide of potassium..	Yellow powder.
—	Hydro-sulphuric acid	Black sulphide.
—	Hydro-sulphate of ammonia	Do.
—	Potass, ammonia, soda, lime, &c. ...	Brown oxide.
—	Cyanide of potassium	White powder.

Mercury.—See p. 374.

Chloride, Corrosive sublimate, or other soluble salt.........	Iodide of potassium..	Yellow powder.
—	Do. in excess......	Red do., eventually re-dissolved.
—	Alkalies................	Black oxide.
—	Hydro-sulphuric acid, hydro-sulphate of ammonia	Black sulphide.
—	Plate of copper or zinc	Metallic mercury.

Copper.—See p. 376.

Sulphate, &c. ...	Potass	Blue powder.
—	Ammonia............	Do., which re-dissolves in excess of the test, affording a deep-blue colour.
—	Hydro-sulphuric acid and hydro-sulphate of ammonia	Brown-black sulphide.
—	Iron or zinc plate.....	Metallic copper.
—	Prussiate of potass....	Red-brown powder.

Iron.—See p. 376.

Proto-sulphate, &c.	Ferro-cyanide of potassium	Light-blue precipitate, which speedily darkens.
Per-salts	Do.	Dark-blue precipitate.
Proto-salts	Ferri-cyanide of potassium	Deep-blue precipitate.
Per-salts	Do.	No precipitate, but a red-coloured liquid.
Proto-salts	Alkalies................	Green powder, turning to red.
Per-salts	Do.	Red oxide.
—	Sulpho-cyanide of potassium...........	Blood-red liquid.

Lead.—See p. 378.

Solution Tested.	Test.	Results.
Soluble salts.....	Hydro-sulphuric acid, hydro-sulphate of ammonia	Blackish-brown sulphide.
—	Chromate of potass...	Yellow chromate.
—	Alkalies................	White precipitate.
—	Hydrochloric acid and chlorides	White chloride.
Soluble salts.....	Sulphuric acid and sulphates	White sulphate.
—	Iodide of potassium..	Yellow iodide.
Acetate...........	Metallic zinc..........	Metallic lead.

Zinc.—See p. 379.

Soluble salts	Hydro-sulphuric acid, and hydro-sulphate of ammonia	White sulphide.
—	Alkalies.................	White oxide.
—	Carbonates of alkalies	Do.—That in excess of ammonia is redissolved.

Tin.—See p. 380.

Soluble salts.....	Hydro-sulphuric acid, and hydro-sulphate of ammonia.........	Yellow or brown sulphide.
—	Alkalies.................	White oxide.
—	Chloride of gold......	Purple powder (See *Gold.*)
Per-salts	Metallic zinc..........	Metallic tin.

Nickel.—See p. 380.

Soluble salts ...	Hydro-sulphate of ammonia	Black sulphide.
—	Hydro-sulphuric acid	No precipitate.
—	Potass, &c............	Green oxide.
—	Cyanide of potassium	Do. cyanide.

Bismuth.—See p. 380.

Soluble salts ...	Hydro-sulphuric acid, and hydro-sulphate of ammonia	Black sulphide.
—	Alkalies	White oxide.
—	Chromate of potass...	Yellow chromate, soluble in nitric acid, by which it differs from chromate of lead.

Antimony.—See p. 380.

Soluble salts ...	Hydro-sulphuric acid	Red sulphide.
—	Hydro-sulphate of ammonia	Red sulphide soluble in excess.
—	Alkalies	White oxide.
—	Metallic zinc	Metallic antimony as a black powder.

(See remarks as to antimoniuretted hydrogen, at p. 381.)

Arsenic—Arsenious Acid.—The student is referred to our extended remarks on the detection of arsenic, at p. 381, *et seq.*; in which full information on the various tests usually employed is given.

Manganese.—See p. 384.

Solution Tested.	Test.	Results.
Soluble salts ...	Hydro-sulphate of ammonia	Flesh-coloured precipitate of the sulphide.
—	Alkalies	White oxide, turning to nearly black by exposure to the atmosphere.

Cobalt —See p. 384.

Soluble salts ...	Hydro-sulphate of ammonia	Black sulphide.
—	Hydro-sulphuric acid, with neutral solutions..................	Do.
—	Potass..................	Blue precipitate, turning green in the air.
	Ammonia..............	Blue precipitate, resoluble in excess, producing a red liquid.
—	Carbonates of alkalies	The pink carbonate of the metal.

With borax a blue glass is produced by the heat of the blow-pipe, which is highly characteristic of cobalt.

Chromium.—See p. 384.

Solution Tested.	Test.	Results.
Soluble salts ...	Potass	Green oxide, soluble in excess.
—	Acetate of lead	Chromate of lead.
Chromic acid and salts	Nitrate of silver	Purple chromate of silver.

Cadmium.—See p. 384.

Solution Tested.	Test.	Results.
Soluble salts	Hydro-sulphuric acid and hydro-sulphate of ammonia	Yellow sulphide.
—	Potass, &c.	White oxide.
—	Carbonates of alkalies	White carbonate.

Omitting potassium and sodium, and their compounds, which do not afford insoluble precipitates or other effects similar to those we have described in connection with the metals in the previous list, we pass on to consider the tests for the terrigeneous earths and their oxides.

Aluminum.—See p. 388

Solution Tested.	Test.	Results.
Alumina in solution, as alum, &c.	Potass	White hydrate.
—	Ammonia	Do.

With proto-nitrate of cobalt it produces a sky-blue mass under the blow-pipe, if heated to redness on charcoal.

Silicon.—See p. 389.

Silica.—Its insolubility and general characteristics distinguish silica from every other substance.

Calcium.—See p. 389.

Solution Tested.	Test.	Results.
Lime, solution of, in water	Carbonic acid	The carbonate, or chalk. This precipitate is soluble in excess of the acid.
Lime salts	Sulphuric acid and sulphates	The sulphate as a white powder.
—	Oxalic acid	White oxalate.
—	Oxalate of ammonia	Do.
—	Alkalies and their carbonates	White precipitate.
—	Phosphoric acid	Do.

Magnesium.—See p. 390.

Solution Tested.	Test.	Results.
Magnesia	Ammonia	White hydrate.
—	Potass	Do. do.
—	Phosphate of soda	Do. phosphate.
—	Oxalate of ammonia	Do. oxalate.

Barium.—See p. 391.

Baryta, salts of	Alkaline carbonate	The white carbonate.
—	Sulphuric acid and sulphates	The white sulphate.
—	Phosphate of soda	White phosphate.

Salts of Baryta.—Tinged flame of a rich green colour.

Strontium.—See p. 391.

Strontia.—The tests for baryta exhibit a similar reaction with salts of strontia. The sulphate, however, is partially soluble in water. The chloride and nitrate give a rich red colour to flame, which is highly characteristic of their presence.

The foregoing tables of tests for metals and their compounds, will afford the student an opportunity of making himself acquainted with their leading properties and characteristics; and will also be a valuable introduction to the principles on which chemical analysis of inorganic bodies is carried on. We should advise that he first prepare the salts of the chief metals, and retain the products in both a crystalline and dissolved state. He may afterwards apply the tests we have named to the solutions, and note the resulting products. The more extended the course of experiments which he may institute of this kind, the more perfect will be his acquaintance with the nature of substances he may afterwards meet with, or which are produced in analytical investigations. In fact, without such a preliminary study, his attempts at analysis will prove but guess-work, resulting in disappointment and loss of time.

CHAPTER V.

INORGANIC ANALYSIS.

HAVING described the chief properties of all the elementary bodies which the student is likely to meet with; and having also mentioned the different tests which may be employed to detect their presence, and that of their compounds, we shall proceed to describe the methods adopted by chemists in discovering the composition of bodies which may be casually presented for examination.

We need scarcely say that the object of an analysis is to find out either the proximate or ultimate elements of which a substance is compounded. In most cases we are content to find the proximate substance, rather than to separate the elementary bodies entirely from each other. Thus, for instance, in analysing a piece of limestone, we are content to find how much lime, magnesia, &c., it may contain; and do not trouble ourselves about the metals of which those earths are the oxides. In some cases, however, we must ascertain the quantity of the elementary body present. Thus, in examining the ores of gold and silver, we always desire to ascertain how much of the pure metal is present. But chemical analysis may be divided into two parts; that is, qualitative and quantitative analysis. By the former, we merely ascertain the fact that a body is present; but by the latter, we arrive at a knowledge of its quantity in any substance which may be analysed.

Qualitative analysis always precedes that of quantitative; because, of course, we must first detect the presence of a body before we can ascertain in what proportion it exists. We shall, therefore, first draw the attention of our readers to that branch of the science, and point out the various methods which may be adopted.

Generally speaking, substances presented for analysis contain several compounds; and it requires considerable skill to apply the proper test, so that only one compound at a time shall be removed or detected. For instance, supposing a solution contained both sulphuric and hydrochloric acids. If one of nitrate of silver were added to it, then both acids would be precipitated together as the sulphate and chloride of silver; and they would either have to be separated by troublesome processes, or be left as an unresolvable mass. But if we first precipitate the sulphuric acid, by means of nitrate of baryta, in the form of the sulphate of that earth, we shall only leave nitric and hydrochloric acid and their compounds in solution. We filter off the insoluble sulphate of baryta; and pouring the clear liquid into another vessel, we may then add nitrate of silver, when all the chlorine will combine with the metal as the chloride, which will then be precipitated.

But even in this simple process another precaution must be attended to; for if, in the last case, we added *chloride* of barium to the solution, for the purpose of precipitating the sulphuric acid, then we should introduce more chlorine into the liquid, which would be thrown down on the addition of the nitrate of silver: hence, such a careless analysis would afford far more chlorine than should really have been present. This instance will suffice to show that we must not try tests at random, but in an orderly and consecutive manner; which is a point of great importance to the young student; for many attempt tests at a venture, and so only succeed in confusing themselves—becoming frequently disgusted at their want of success. We have seen such take the first out of a row of test-bottles, and trying one thing after another, in the vain hope of discovering the substances contained in a solution; and after expending considerable time in a chemical steeple-chase, they have ended with the conviction that they could not undertake analysis; in which opinion their tutors have often most rationally acquiesced.

We shall first give some preliminary and practical directions, and then proceed to point out a systematic course of qualitative chemical analysis.

Before commencing, it is well to provide a sufficiency of glass vessels of various sorts, for the operations which have to be pursued. And, to the young beginner, test-tubes are of great value. He can, by means of them, boil solutions, test them, and perform many operations on the small scale. To commence with—a few test-tubes, Florence flasks, two or three evaporating dishes, a spirit-lamp, pestle and mortar, some glass tubing, a funnel, and some filtering-paper, are the chief requisites. A book should be kept, in which *every* experiment should be registered; and small pieces of paper, marked with the letters of the alphabet, and others with the numerals, must also be at hand. Such are employed to distinguish the results of the different tests. Thus, one *series* of experiments may be called A; whilst each separate addition of a test in the series may be marked 1, 2, &c., on the test-glasses. By such means the experiments are classified; and they should be registered after the same order in the note-book. To illustrate this, we may refer to our directions for arsenical analysis;[*] in which we supposed, that after the arsenious acid had been brought into solution, three different methods, viz., Marsh's, Reinsch's, and the liquid tests, had been employed. The original liquid and the results might have been marked as follows:—

A.—Solution of Arsenious Acid.	Results.
1. Marsh's test	Arsenic deposited.
2. Reinsch's do.	Metallic arsenic reduced.
3. Liquid	Precipitates afforded.

By such a plan the progress of the analyst is materially assisted, errors are prevented, and the results kept for reference. It is also of great importance that the notes of experiments should be made at the moment the results are obtained. Young chemists, and even old beginners, are so eager to arrive at the end of their examinations, that they neglect these intermediate precautions. After they have thus proceeded for some time, they become perfectly bewildered, and are quite at a loss to distinguish one result from another, simply because they will not take the trouble to note each step as they proceed. In most laboratories for young pupils, it would be an advantage if the old fable of the tortoise and the hare were printed in capitals around the room. We proceed to practical details.

SOLUTION.—The first step in an analysis is to bring the body into solution, if it be not already in that state; and this may be effected in various ways. Some bodies are readily soluble in water; and the substance, in that case, may be dissolved by that fluid alone. It must be remembered, that most substances are more soluble in hot liquids than at ordinary temperatures; and hence the application of heat is required. Sometimes long boiling is requisite; and for such purposes Florence flasks and test-tubes may be employed. No more liquid should be used than is absolutely necessary, because this increases the bulk of the solution, and also diminishes its strength. As the water evaporates, more should be added, or otherwise a portion of the solid will be deposited on cooling, and crystallisation may also take place. All bodies more soluble in hot than in cold water will deposit some portion on cooling, if a hot saturated solution be made—that is, when the *hot* liquid has dissolved as much as it can.

It is always best to reduce a solid substance into powder, because, by that means, more particles are brought into contact with the fluid in the same time. The operation is thus facilitated. For this purpose, the pestle and mortar are to be used; the mass being first broken up into fragments, a *little* of which at a time is to be reduced to powder in the mortar. If too much be employed at once, the pulverisation will be incomplete. In pulverising some substances great difficulty is experienced; but this can be overcome by various expedients. Thus it is impossible to reduce camphor to a powder by itself; but the addition of a few drops of spirits of wine renders the operation quite easy. Flints, again, are difficult to pound; but if they are heated red-hot, and then quenched suddenly in cold water, their pulverisation may be readily performed.

Some substances are quite insoluble in water; and we may here state that we are confining our remarks, for the present, entirely to inorganic or mineral bodies. With such, a different course of treatment must be adopted. Take, for instance, a piece of chalk, which is entirely insoluble in distilled water. If it be placed in dilute sulphuric acid, no solution would be effected for the purpose of analysis, because the insoluble sulphate would be afforded. If, however, nitric or hydrochloric acid were employed, a solution would be at once produced. But such a solution would differ considerably from an aqueous one; for, in the latter, the substance is merely suspended—as sugar dissolved in water; which may be restored, without chemical change, to its previous solid state; whilst a solution of chalk in nitric acid is attended with decomposition, the nitrate of lime being produced in place of the carbonate. Still, for all analytical purposes, the result is the same, so that the body be in a state of solution at last. Clay may be taken as another example; for all except its silicious portion may be dissolved by acids, with the aid of heat, and thus a solution is afforded which can afterwards be conveniently tested.

The mention of silicious matter—such as sand, flint, quartz, &c.—brings us to remark on a class of bodies which are insoluble in all menstrua, with the exception of hydro-fluoric acid; and which, from its nature, forbids its employment, except in very peculiar cases. Supposing it were required to analyse a piece of flint, it would be necessary to fuse it with a flux, by means of a red heat at least. The plan to be adopted in such cases is as follows:—The body having been reduced to a fine powder, is to be mixed in a mortar with three or four times its weight of carbonate of potass, or soda. The mixture is to be put into a Berlin porcelain, or, in rough experiments, a Hessian or English crucible. It is then to be exposed to a red heat for two or three hours. By this process the silicious matter will form a soluble glass, and so a solution will be afforded, which may be dealt with in a manner we shall presently describe.

The metals and their alloys are all more or less soluble in some acid, but their ores have sometimes to be disposed of differently. At times they must be roasted, to drive off the combined sulphur, &c. This is effected by placing the substance in a common crucible, and exposing the contents to a red heat for some time. The residual mass may then be generally dissolved in an acid or other solvent. Some means may be occasionally tried, which produce secondary but required results: thus, the lead ore called galena is decomposed into sulphate of lead by long boiling with nitric acid.

Supposing solution to have been effected by any of the foregoing means, or others, the next step is that of examining or testing the solution, which affords either a precipitate or a change of colour. In some cases, neither of these results may follow. In this case, our next resource is crystallisation. Each of these cases we shall proceed to examine.

PRECIPITATION.—When a solid body is removed by means of a test from a solution, the substance so separated, whether it fall down or not, is called a precipitate, of which so many examples have already been noticed. We shall not repeat here the numerous instructions already given on this point, but rather mention minutiæ of practice to which we have not yet alluded. Some precipitates will not fall down or separate, owing to the extreme fineness of their particles; in such cases, heat is to be applied to the vessel containing them, which will generally cause the particles to coalesce. During precipitation, the liquid ought to be constantly stirred to assist the process; and an abundance of fluid, if not otherwise objectionable, is, generally speaking, an advantage. If it be desired to preserve the precipitate, the contents of the vessel, after being well stirred, are to be poured into a funnel holding filtering-paper. The paper should be first moistened, so as to facilitate the process; and the precipitate will thus be separated from the liquid, which will pass through, and may be received in another vessel. The precipitate, whilst on the filter, should be carefully washed with the solvent, so that any soluble portion may be removed; and the washings, as they are termed, are to be added to the liquid already passed through.

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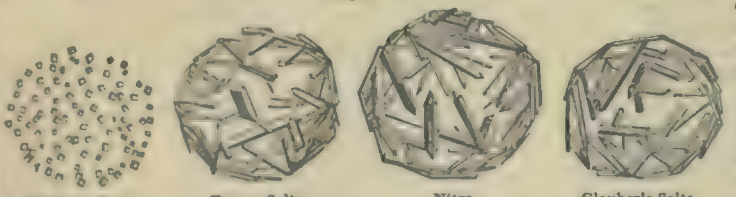
[*] See *ante*, p. 382, *et seq.*

If a precipitate be not formed, but a coloured liquid produced, great care should be taken as to the way in which the colour is examined. Some liquids afford a different colour when viewed by transmitted in place of reflected light. In such cases, the best plan is to pour a small portion of the liquid on to a clean white plate or dish, when the colour may then readily be noticed. Occasionally precipitates will change colour by exposure to the air; as, for instance, the cyanide of iron, the oxide of manganese, &c. Others will change colour by the application of heat—*e.g.*, the oxide of zinc—all of which, and many other circumstances, must be keenly and carefully noticed.

Crystallisation.—In many cases in which no precipitate is afforded, the process of crystallisation may be employed. By this we are enabled to give the solid body a definite form; for almost every salt crystallises in some peculiar shape so as to make it recognisable. The following engraving affords an illustration of the fact.

Fig. 72.



Common Salt. Epsom Salts. Nitre. Glauber's Salts.

Now here we observe four different forms of crystals, each of which is readily distinguished from the other. The student may readily produce these crystals by dissolving each of the salts separately in hot water to saturation, and then placing a drop of each on a piece of window-glass. As the liquid evaporates, the salt will reassume the solid form, and present its characteristic development.

The process of evaporation is generally carried on in evaporating dishes, heat being applied until a pellicle forms on the surface of the liquid. The vessel is then to be removed to a place where it will not be disturbed, when crystals will gradually be formed. The slower the evaporation is conducted, the more regular will the crystals produced be. By such means, highly soluble and less soluble salts may be separated. Thus chlorate of potass and chloride of potassium are simultaneously afforded in the production of the chlorate; but the latter is easily *crystallised away* from the former by its greater insolubility. In a similar manner, alum is purified from iron and other substances. The best vessels for crystallisation only, are rough glazed earthenware dishes, which, by their uneven surface, assist crystallisation; whilst for refined experiments, vessels of Berlin porcelain, silver, or platina, are usually employed. The two last, however, are only needed for special purposes. After a liquid has yielded as many crystals as it can, on fresh evaporation more will be afforded. Occasionally, however, a liquid will actually redissolve crystals which have been formed in it. This arises either from an increase of temperature, or from a difference of specific gravity in the upper and lower portions of the fluid.

Some crystals, when exposed to the air, *deliquesce*; that is, they absorb moisture from the atmosphere. This occurs with chloride of calcium, chloride of aluminum, and many other substances. Others lose water, or *effloresce*, as in the case of sub-carbonate of soda in crystals (the ordinary "soda" of commerce), which is thereby converted into a powder-like substance.

Large and perfect crystals may be obtained from a solution by immersing in it a crystal of the crystallising body. A perfect one should be chosen for this purpose; and, by such means, enormous crystals of the prussiate and bichromate of potass may be produced.

Distillation and Sublimation.—These operations differ more in name than in principle, their object being to separate volatile from solid substances. By distillation, water, alcohol, &c., may be separated from bodies which may have been dissolved in them. Mercury, in a similar manner, is distilled from its amalgams and ores, and obtained in a pure state. For such purposes, glass retorts are employed when a temperature less than a red heat is required; whilst, for other operations, vessels of porcelain, iron, or platina must be used, as in the preparation of potassium, sodium, the chloride of aluminium, &c.

Sublimation can generally be effected by means of flasks or test-tubes. Thus sulphur, arsenic, iodine, &c., may be volatilised in a manner already mentioned under their separate heads in our previous pages. This operation, however, is by no means common in the course of analysis.

The Blow-Pipe.—We have thus given an outline of the leading operations in analysis, and shall now make a few remarks on the blow-pipe, an instrument of great use in experimental chemistry, and by means of which we can get, on the small scale, all the results of the most powerful furnace. We can hardly recommend, as the *best*, any form of the instrument; but, for our own part, always employ either the common tinman's blow-pipe, or that invented by Dr. Black, which can be obtained at any instrument-maker's.

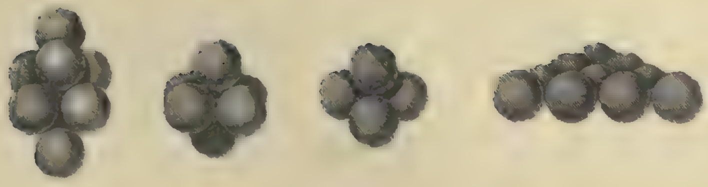
Much art is required in using the blow-pipe. The air should be blown from the cheeks only, and not from the chest. By that plan, a person may keep up a continuous stream of air for a long time without fatigue, and the mouth may be replenished with air from the lungs by an operation something like the reverse of swallowing. It is impossible further to describe the method, as practice alone is of use. The object the student should have in view, is that of keeping up the stream of air, free from puffing, and to thereby maintain a steady flame.

The flame consists of two parts. The inner one ends with a blue colour, at the point of which the most intense heat is produced. The outer flame—that is, that farthest removed from the jet—affords much less heat, being in direct contact with the external air. The uses of these two flames are accordingly different; the inner one is employed for reduction, and the outer one for oxidation. A wax or composite candle affords a suitable flame for the blow-pipe. Small rings of platina wire, some platina foil, and pieces of charcoal, are needed as supports for the substances to be examined. It must be borne in mind, that the charcoal is employed as a deoxidator—that is, for the purposes of reducing oxides to their metallic state; whilst the ring, or foil of platina, is used to produce beads, &c. If the latter were employed in reduction, the metals would form an alloy in many cases, which would destroy the platina support. The use of the charcoal may easily be seen by placing a little red oxide of lead in a small hole made in the charcoal support by means of a penknife. If the blue flame be then directed on it, metallic lead will be at once reduced.

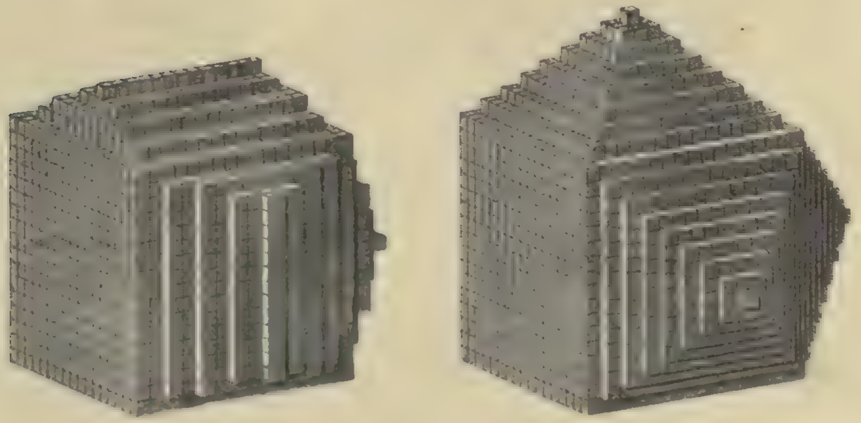
By means of the blow-pipe, a preliminary examination of many substances may be made, which will save a vast deal of time in subsequent processes. This remark especially applies to metallurgical chemistry, and to the examination of minerals generally. It should be the object of the young chemist, therefore, to familiarise himself, by constant practice, with the use of the blow-pipe. He may commence by reducing metals on charcoal supports, using only a minute portion of the oxides, &c., for the purpose. The use of the oxidating flame may similarly be tried on metals. Employing a little borax, carbonate of soda, &c., he may proceed to produce coloured buttons, by the fusion, at the reducing flame, of those substances, for

Crystallization.

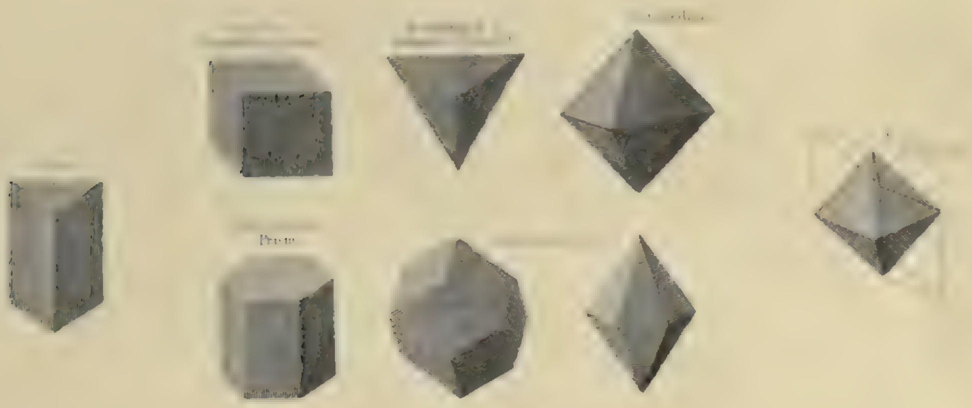
See Page 396



Arrangement of the Molecules of A Body having A Crystalline Form



Molecular Constitution of A Crystalline Body



Various Forms of Crystals, in Relation to their Geometric Elements.

Crystals Revealing their Elementary Forms during Solution

which rings of platina wire are to be employed. The flux is first melted so as to form a little bead, which will fill up the ring. On this the oxide is placed, and fused with the flux. The oxides of cobalt, manganese, and chromium are useful for initiatory experiments. In all cases the heat should be employed gently at first, until the required temperature be obtained. In the absence of this precaution, charcoal supports and refractory minerals will fly into pieces; and it is by no means unlikely that the first intimation of this result will be feelingly conveyed to the operator by the lodgment of a red-hot fragment in the eye, by which he will be forcibly impressed with the necessity of care for the future.

Presuming that the reader will have become acquainted with the various processes involved in general analysis, we shall proceed to give details of the methods adopted in laboratory practice.

QUALITATIVE ANALYSIS.

In the following pages we shall not follow the usual plan of giving directions for a systematic course, by means of which every body which can possibly be met with may be detected, but confine ourselves to instances which will afford types of the methods to be employed. To those of our readers who may be desirous of entering extensively into the subject, we recommend the works of Fresenius, Parnell, and Rose; in which may be found directions for almost every analysis which can be undertaken. We shall adapt our directions chiefly for such who have never attempted to analyse, and so fit them for a perusal of the works we have named, and for the subsequent prosecution of more elaborate researches.

The external qualities of a body will often give a general insight into the nature of its composition. The value of this, however, will depend on the extent of general information which the operator may possess. An experienced chemist can often give an approximate idea of the composition of bodies by a few preliminary experiments, or even by simple inspection. The physical characters of specific gravity, colour, form, whether crystalline or amorphous, suggest the connection of any substance with others of a similar class. Thus, no one having any acquaintance with mineralogy would mistake quartz for Iceland spar; for one is hard, crystallises in prisms having six sides, will scratch glass, &c.; whilst the latter is soft, splits into rhomboids, and may therefore be readily scratched by glass, iron, or almost any other body. Metallic alloys, oxides, &c., are generally to be distinguished by their specific gravity, lustre, and colour; and hence, for them, the usual tests for metals may be employed.

Almost every inorganic substance is composed of oxides of metals proper, or those of the alkaline and terrigeneous metals; and maintaining the distinction incident to the individual character of each class, the student may often make a rough and tolerably accurate guess of the substance under examination.

In illustrating the different methods of qualitative examination, we shall take instances of well-known substances, progressing from such as afford comparatively little difficulty, to others of a more complex character.

Our first attempt shall be a qualitative analysis of a piece of marble, by which we shall show that it mainly consists of carbonic acid and lime. The student will thus learn to detect the presence of a gas and an earth, and so take a first step in his analytical progress.

Experiment 82.—Take a small piece of marble, of the size of a pea; and having reduced it into powder, boil it for some time in distilled water. Pour some of the liquid on to a piece of clean window-glass, and hold it over the spirit-lamp flame until the glass be dry. No deposit will take place, because the marble is insoluble in water.

(*a.*) The next step is to find some acid which will dissolve the marble; and for this purpose dilute hydrochloric acid may be poured on the powder, which should be placed in a test-tube of considerable size. The acid should be added little by little, waiting till all effervescence has ceased before more is poured into the tube. It will be evident that a gas, which formed part of the marble, is escaping; but this we will consider hereafter.

(*b.*) Having effected the solution, we must next test it, and discover, if possible, what the solid substance which has dissolved consists of. It is evident that that has united with the hydrochloric acid, and the substance with which it was previously in combination has been removed. The very appearance and source of the marble indicate that it belongs to the earths, and it must therefore consist of one of these. But as all the earths are soluble in hydrochloric acid, we must add some test which shall detect that really present; and herein lies the art of the analyst. If we add alkalies or their carbonates, in solution, we may have *any* of the earths precipitated as a white powder; hence, for the present, they will not assist us. But we find that sulphuric acid will not precipitate *all* the earths, for the sulphate of magnesia and alumina are soluble in water. We accordingly add a little dilute sulphuric acid to the solution made in *a*, and find a copious white precipitate thrown down.

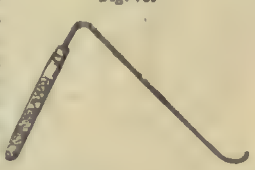
(*c.*) The precipitate obtained in *b* may either be the sulphate of baryta, strontia, or lime; and for all practical purposes we may consider one of these to be present. A portion of the precipitate may accordingly be taken, and having been washed in a test-tube with distilled water, the washings being thrown away, the powder is first to be agitated in abundance of distilled water, and a little of the liquid is then to be poured on a piece of clean window-glass, or platina foil. This is to be held over the spirit-lamp until quite dry. If any deposit be effected, it must either be sulphate of strontia or sulphate of lime, because the sulphate of baryta is insoluble in water. We therefore conclude that the solid substance dissolved in *b* must be one of the two former earths.

(*d.*) As we may infer that either the sulphates of strontia or lime alone are present, we add to the solution obtained in *c* one of sulphate of lime, which may be made by agitating that substance with plenty of cold water, and filtering off the clear solution. A portion of this being added to solution *c*, if a precipitate fall, we know that it cannot be sulphate of lime, for that would not be thrown down by a solution of itself. We therefore at once infer that strontia is present. For an equally good reason, we are assured that strontia is *not* present if *no* precipitate take place. In such a case we conclude that lime alone could be present, and therefore that earth must have been the solid matter dissolved in *a*, by the hydrochloric acid.

Having so far gained a knowledge of one of the constituents of the marble, we proceed to examine the nature of the gas given off whilst the solution was being effected; and this may easily be done as follows:—

(*e.*) Take a small piece of the marble, and put it into a test-tube fitted with a cork and bent tube, as represented in the margin. Add a little dilute hydrochloric acid; and having fitted in the tube, allow the gas which is given off to bubble through some lime-water. If no precipitate be produced, then the gas is not carbonic acid. If, however, a precipitate be thrown down, then carbonic acid is certainly present, which will be at once evident by the formation of chalk or carbonate of lime.

Fig. 73.



It will thus be shown that the marble is chiefly composed of two substances—lime and carbonic acid: and another step may be taken to verify the investigation.

(*f.*) Heat a small piece of the marble in an *open* crucible, or in an iron ladle, at a red heat for some time. It will be found that the colour and appearance of the remainder will have entirely changed—a yellowish-coloured substance will remain. If this be placed in a few drops of water it will heat and fall to powder; and, in fact, will present every characteristic of the earth, lime.

The marble having thus afforded carbonic acid and lime, will also, most likely, contain a small portion of iron ; hence another step is necessary to detect the oxide of that metal.

(*g.*) To a small quantity of the solution *a*, add a few drops of nitric acid, and heat it. Then pour in a few drops of a solution of prussiate of potass, when, if iron be present, a blue precipitate will at once be produced.

We may now consider the qualitative analysis of the marble to have been completed and verified ; and to make our progress more easy, we shall recapitulate each step we have taken.

1. We tried the solubility of the marble in water.
2. In *a* we effected a solution in hydrochloric acid.
3. In *b* we obtained a precipitate by means of sulphuric acid.
4. In *c* we showed that only the sulphates of strontia or lime could have been produced.
5. In *d* we found that the precipitate could be sulphate of lime only.
6. By the process named in *e* we proved the presence of carbonic acid.
7. We verified our previous discoveries by heating the substance as in *f*.
8. And, lastly, anticipating the presence of iron, we tested for its oxide, and obtained a precipitate of Prussian blue in *g*.

We have thus succeeded in obtaining, separately, each of the constituents of the marble by way of analysis. Each step, it will be observed, depends on that taken previously ; and from this easy example, the student may learn the general method of conducting the qualitative analysis of many substances.

In carrying out this initiatory experiment, we have chosen a substance which is so simple in its constitution as to leave no doubt of accurate results, because every specimen of marble and chalk has, as near as possible, an identical constitution. We must, however, inform the student, that baryta, strontia, lime, and magnesia, present, as near as can be, the same appearance when tested in solution by some reagents. For instance, the carbonate of the alkalies would precipitate the carbonate of each of these earths as a white powder. The same may be said of oxalic acid and oxalate of ammonia. Hence, had such tests only been employed, the student would scarcely have advanced a step, because the indications produced would have been undistinguishable. By the plan, however, which we have described, each source of error was successively eliminated, and so the true composition of the marble was at last arrived at. The progress in chemical analysis, when properly conducted, is just as logical as a problem in geometry or an equation in algebra. The value of lines or quantities is gradually estimated in those sciences from data ; and similarly in chemistry, we obtain, in a physical form, analogous results.

We must next examine more difficult instances ; and by suggesting mixtures of various substances, shall indicate the means to be employed in detecting their constituents.

Experiment 83.—We shall now proceed to analyse a piece of clay, which, generally speaking, contains four or more substances. Our attention, however, will be confined to the detection of the silica, alumina, lime, and iron ; and the following plan may be adopted.

(*a.*) Take a small piece of the brown-coloured clay, such as is used in brickmaking, and heat it for some time in a ladle, but not red-hot, for the purpose of driving off water in a free state, and to break up the crystals of sulphate of lime which may be present. Transfer the mass so obtained to a glass flask ; add abundance of distilled water, and agitate the contents of the vessel for some time. This will dissolve out a portion of sulphate of lime, which is more soluble in cold than in hot water. Let the whole stand for some time, and then pour off the clear liquid into another vessel. Evaporate a portion of the liquid to dryness in an evaporating dish ; if a powder be obtained which is barely soluble in much water, it may be assumed as sulphate of lime, but may also be tested as in Experiment 82.

(*b.*) A brown-coloured mass will be left in the flask, which is insoluble in water. To this add dilute hydrochloric acid, and boil for some time to dissolve all soluble matter out. By this the carbonate of lime, alumina, and iron, will be removed. The clear liquid is accordingly to be poured off into another vessel, and more hydrochloric acid added to the powder in the flask. The contents are to be boiled, and when cool, are to be added to the previous solution. The remaining mass is to be washed with distilled water, and the washings are also to be added to the solution already effected. By these means, all matter soluble in dilute hydrochloric acid will have been removed, and little else but silica, which is insoluble, will remain. Of this we shall speak hereafter.

(*c.*) The solution from *b* will contain lime, alumina, and iron ; and each of these is to be tested for separately. Were we to add alkalies, or an alkaline carbonate, we should obtain an abundant precipitate of *all* the bodies in solution ; and this would only perplex us, and compel a re-solution of some of them. Our object, however, may be readily attained by first removing the lime as sulphate ; secondly, the alumina ; and, lastly, the iron may be discovered by a separate process. Now, as alumina and iron are both soluble in sulphuric acid, and lime produces an almost insoluble precipitate with it, we add dilute sulphuric acid to the solution, and so obtain the sulphate of lime. By this we get rid of one of the three constituents of the clay—namely, the carbonate of lime ; the existing sulphate having already been disposed of.

(*d.*) The solution freed from the salts of lime most likely to be present, now contains alumina and oxide of iron ; and after adding a little nitric acid, boil for a short time in a flask ; then divide the liquid into two portions. To one add prussiate of potass, when the iron will fall down as a blue precipitate. The presence of iron is thus evidenced.

(*e.*) To the remaining portion of the solution in *d*, add a *dilute* solution of potass. A precipitate will at once be afforded of alumina : allow it to settle, and then add an *excess* of a solution of potass, and heat it. The alumina will be completely dissolved. To this add a solution of sal-ammoniac, when the alumina will be again thrown down. The alumina present in the clay is thus detected.

We have thus discovered that clay consists of alumina, lime, and oxide of iron ; but have yet to examine the insoluble matter left in process *b*, after digesting with hydrochloric acid. This leads us to another branch of analysis—namely, that in which matter containing silica is to be examined. The following course must be adopted.

(*f.*) As silicious matter is insoluble in all liquids except hydrofluoric acid,* we must bring it into a soluble state by means of a flux. The mass left in *b* is therefore to be removed, washed, and dried, and then to be mixed in the mortar with three or four times its weight of carbonate of soda, or potass. It is then to be transferred to a Berlin porcelain or English crucible ; and heat for two hours at a full red heat. A gas-furnace may be employed if a Berlin crucible be used ; or a common coke fire will answer equally as well, if common crucibles are adopted. The sand and alkali will unite, and form a silicate, which is easily soluble in water. As soon as the entire mass is completely fused, the crucible is to be removed from the fire, and allowed to cool. The whole may then be placed in a large evaporating dish, and dilute hydrochloric acid is to be poured into the crucible ; or its contents may be broken out, and put into the evaporating dish alone, as a little loss is no object in a qualitative analysis of this kind.

(*g.*) The resulting mixture from the last process is to be cautiously evaporated to dryness ; and when the dish is quite cool, dilute hydrochloric acid is to be poured in. This will dissolve out all but the silica, which will be left as a gritty white powder. It is thus also removed from the clay.

* See *ante*, p. 339.

(*h.*) But the solution effected in the last process by hydrochloric acid, after the silica is removed, may contain both alumina and oxide of iron. It may be tested in exactly the same manner as indicated under *d* and *e*, when these substances, which had been held in chemical union with the silica, will also be discovered.

By such means the presence of silica, alumina, lime, and iron may be detected in clay; and also the combination of the three last-named bodies in union with the silica as sand. In this analysis, we still further discover the necessity of a successive process in analytical investigations, if we would command reliable results. We may recapitulate our analysis as follows :—

1. We removed matter soluble in water.
2. We effected the solution of a portion of the clay in hydrochloric acid, leaving the insoluble part for further experiments (*b.*)
3. We next got rid of the carbonate of lime in the form of a sulphate (*c.*)
4. The iron was removed by the next process (*d.*)
5. The alumina was precipitated as a hydrate in *e.*
6. We afterwards effected the decomposition of the sand, and obtained its silica as a white powder (*f* and *g.*)
7. And lastly, the other constituents of the sand, in chemical union with the silica, were also discovered (*h.*)

The foregoing examples of qualitative analysis may serve the student as types of the methods generally to be pursued in inorganic qualitative analysis, when earths alone are sought for; and to the agriculturist such may prove a kind of general guide to a rough analysis of an average soil. We shall next give some instances, in which the detection of metals proper are more especially involved, and which may prove useful to those desirous of investigating the nature of veins of metallic ores, which may occur in their neighbourhood. Before proceeding to the following experiments, the reader is advised to peruse the tables of tests for the metals, given at a previous page;* by which he will be the better prepared to understand the operation of each reagent in any special case.

Experiment 84.—Dissolve together, in a test-tube, a few crystals of sulphate of copper, and proto-sulphate of iron. You will thus obtain a mixed solution of the two metals; the object being to detect the presence of each separately. Now, if the ordinary method of precipitating the oxide, carbonate or prussiate, were adopted, by means of those salts of the alkalies, the salts of the metals would be thrown down together. Other means must therefore be tried: and either of the two following methods may be adopted :—

1. Divide the solution into two parts, in separate test-tubes. Into one immerse a piece of clean iron plate, having first added a few drops of sulphuric acid. Chemical action will at once ensue, and the copper will be reduced to a metallic state, completely from the liquid.

2. Immerse two pieces of platina foil, attached to each wire proceeding from a single cell of a voltaic battery, in the other solution, and allow the current to pass for some time. Eventually the copper will be reduced at the negative pole in the metallic state.

3. The copper may be removed from an *acid* solution of the two metals, by means of sulphuretted hydrogen. For this purpose, some sulphuric acid is first to be added to the solution of the salts of the two metals, and then a current of sulphuretted hydrogen is to be passed through. The black sulphide of copper will be at once produced; whilst the iron still remains in solution. It can afterwards be precipitated by means of any of the alkalies; but the solution must first be boiled, to remove any hydro-sulphuric acid left after the precipitation of the copper. This experiment illustrates methods by which voltaic action may often be employed for the removal of platina, silver, mercury, and copper, from mixed metallic solutions.

Experiment 85.—We shall now offer a more complex experiment, in which two metals and an earth are found in solution; the object being to detect and remove each separately. Dissolve a few grains each of the nitrates of silver, lead, and baryta together in a test-tube; and from this solution precipitate each substance as follows :

(*a.*) Add common salt in solution to the above solution. The chlorides of silver and lead will be precipitated together. Filter these off by means of the funnel, &c., preserving the clear liquid, and washing the precipitate with distilled water, which must then be added to the original solution left after filtration. Put the precipitates into a test-tube, and add liquid ammonia. This will dissolve the chloride of silver, and leave the lead untouched, by which that metal will be removed entirely.

(*b.*) To the solution of silver by ammonia, produced in *a*, add an excess of dilute hydrochloric acid, when the chloride will be precipitated, and may be known by changing to a dark colour by the action of light.

(*c.*) The original solution will now only contain the salt of baryta, which is readily precipitated by adding a sulphate, or dilute sulphuric acid; and thus the baryta will also be removed.

Now, had we used sulphuric acid in the first instance in *a*, we should have precipitated together the sulphates of each of the bases, and would have lost almost all chance of dividing them. This, again, is another illustration of the necessity of rule in using chemical reagents. It is of the utmost importance, in adding a test, that we take care not to introduce any substance which may in any way invalidate or modify the different steps of an analysis. Thus the chloride of barium, or nitrate of baryta, may sometimes be used to precipitate sulphuric acid from solutions. At others, the introduction of chlorine would cause insoluble precipitates, and so render the result fallacious in that or other ways, as the following experiment will show :—

Experiment 86.—Add a few drops each of sulphuric and hydrochloric acid together. To the solution add some of *nitrate* of baryta, when an insoluble sulphate of baryta will be precipitated. Then add, after the removal of the sulphuric acid, a little nitrate of silver to the original solution: a precipitate of chloride of silver will be produced; and the hydrochloric acid will then be also removed. Now, had the chloride of barium been used instead of the nitrate, then it would of itself have produced a precipitate of the chloride of silver, independently of the hydrochloric acid already present; and hence if a quantitative analysis had been desired, the amount of chlorine would have been indicated far in excess of the truth. We may here mention another source of error, which occasionally arises in testing with either of the salts we have named. Both the chloride of barium and the nitrate of baryta are apt to be precipitated as very minute crystals, when used as tests in concentrated solutions. They, however, immediately disappear when water is added, being readily soluble in that fluid. They are, of course, thus at once removed from the insoluble sulphate.

We may make a practical application of some of these facts in undertaking a quantitative analysis of mineral waters. But as some of our readers may not have access to any, we shall suggest a way of imitating the composition of one. For this purpose dissolve a few grains of each of the following salts in a pint of distilled water; viz.—Sulphate of magnesia (Epsom salts); chloride of sodium (common salt); sulphate of lime (plaster of Paris); sulphate of iron (copperas).

Now, these substances are generally found in most mineral waters; and very often in common river-water. The salts are all to be well shaken up; and will dissolve, with the exception of the sulphate of lime, of which only a small portion will enter into solution. It is required that we should detect the bases, *i.e.*, the magnesia, lime, and iron, and the common salt. The solution must be filtered so as to be perfectly clear; and the following steps are to be taken :—

(*a.*) Pour a portion of the solution into an evaporating dish, capable of holding about two fluid ounces; and evaporate by the heat of a spirit-lamp; adding more of the fluid as it is lost in vapour. By this means the solution is to be concentrated, and at last evaporated to dryness. This should be effected at the conclusion of the process, by means of a water-bath, lest the basin exposed to the heat of the flame should crack. Such an arrangement is easily made, by placing the evaporating dish in another of larger size, filled with boiling-water, and kept at that heat; or the small dish may be placed on the top of a tin saucepan, the water in which is to be kept boiling. All the solid substances will thus be removed from solution; and if only a very small quantity of boiling-water be then poured on to the mass, all the substances will be again dissolved, with the exception of the sulphate of lime, which is thus removed from the mixture. The fluid is then to be poured into a clean test-tube; more water added to it; and the remaining sulphate of lime may be tested as directed in Experiment 82.

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* See *ante*, p. 393.

(*b*.) We shall now have left in the solution a very minute portion of sulphate of lime (which we may neglect), and the sulphates of magnesia and iron with the common salt.

The common salt, or rather its chlorine, is first to be removed by means of nitrate of silver, as in Experiment 85. The solution must then be cleared of the precipitate by means of filtration.

(*c*.) The clear solution now only contains sulphate of magnesia and iron; and we have to obtain these two separately. A little hydro-sulphate of ammonia is to be added, when the iron is precipitated as the black sulphide.

(*d*.) We have now nothing remaining but the salt of magnesia. The solution having been filtered from the sulphide of iron, is to have added to it some solution of potass, and to be boiled, when the magnesia will be precipitated as a white hydrate.

Thus the lime, common salt, iron, and sulphate of magnesia, may successively be detected in the manner we have pointed out.

We have thus given some experiments illustrating the methods by means of which inorganic substances containing earths and metallic oxides may be analysed. We shall now proceed to what will be more difficult to the tyro—namely, the means we have of detecting the presence of alkalies. These substances and their compounds are, with few exceptions, soluble in water; and they therefore cannot be generally detected in the manner, or with the ease, that the metals are susceptible of. Some of the salts are more easily detected than others. Thus the nitrates and chlorates of each of the alkalies, deflagrate with burning charcoal, producing carbonates, which may be distinguished from those of the earths by their great solubility in water. The salts of potass give a violet tint in the blow-pipe flame; whilst those of soda afford a yellow colour. Potass and soda may, however, be distinguished by means of the chloride of platina and tartaric acid. With the chloride of platina, potass combines and affords a yellow crystalline substance. With tartaric acid, it produces the comparatively insoluble bitartrate of potass, or cream of tartar. Now soda produces no precipitate with the platina salt, and none with tartaric acid, unless a very concentrated solution be employed. The chloride of platina also affords a yellow crystalline product with ammonia and its salts. But the latter alkali may always be distinguished by the smell, which may be produced by heating its salts with lime or potass. The ammonia is set free; and if a piece of moist turmeric paper be held over the fumes, it will turn red—a colour which is lost as the alkali evaporates. A glass rod moistened with hydrochloric acid, affords white fumes of sal-ammoniac. If a piece of horn or flesh be heated, and burnt in a test-tube, and either of these tests be applied, the presence of ammonia will at once be made evident. All the alkalies change the yellow colour of turmeric to red, and restore the colour to blue, of previously reddened litmus-paper. We advise the student to make repeated experiments with potass and soda in the blow-pipe flame, so that he may acquire the knack of distinguishing them. He may also watch the effect of their giving colour to the flame of burning spirits of wine. We may suggest the following experiments, as calculated to familiarise the mind with some of the appearances of the alkalies, as met with in analysis.

Experiment 87.—Fuse some sand in a crucible, with four times its weight of carbonate of potass. Separate the silica as directed in Experiment 83, (*f*); but add nitric instead of hydrochloric acid, to dissolve out the glassy matter. Having filtered the solution from the silica, evaporate to dryness, and take the residue away, throwing it on a piece of red-hot charcoal. By this means a deflagration will take place, and the charcoal and oxygen of the nitric acid forming carbonic acid, carbonate of potass will be produced. Put the piece of charcoal into distilled water, to dissolve away the alkaline carbonate; and having filtered the solution, add some chloride of platina.* The yellow crystalline double salts of chloride of platina and potassium will be produced. In this experiment, three points may be noticed—the production of the carbonate of potass; its easy solubility; and the production of a double salt of platina and potassium.

Experiment 88.—Ammonia is never met with in what we may term a mineral state, and is always the product of organic decomposition. Its reaction, however, may be noticed by adding the platina salt to a solution of sal-ammoniac, when a double salt, analogous to that of potass, will be formed. It has, however, a lighter colour than the potass precipitate. We shall have to give full details as to the detection of ammonia when we treat on organic analysis.

There is a class of substances often met with in inorganic analysis, which may, generally speaking, be obtained from their combinations by means of sublimation. Of these, arsenic and sulphur are the most likely to claim the attention of the student. Any ore containing, or suspected to contain, either of these substances, may be placed, being first reduced to powder, in a test-tube or Florence flask. Heat is then to be applied, when the arsenic present will sublime as the white arsenious acid, and the sulphur settle as a yellow crystalline substance; but, if ignited, producing sulphurous acid gas. The sulphides of some of the metals may be readily analysed by the following method.

Experiment 89.—Take a small piece of galena, the native sulphide of lead; and having reduced it to powder, digest it in a flask with nitric acid, until the whole is converted into a white powder. This is the sulphate of lead, produced by the oxidation of the sulphide at the expense of the nitric acid. Occasionally sulphur will be set free in the solid form, but this is easily recognised by its general properties.

Mercury is also a volatilisable body, but will only be met with in those conditions in which its presence will be easily recognised. The same may be remarked in reference to iodine and bromine.

Our previous remarks and experiments, with the tables of tests given at page 393, will thus enable the student to undertake a general qualitative analysis of most of the substances which he may ordinarily meet with in inorganic analysis; and we shall give a systematic arrangement which is generally applicable in all cases. In this we have interwoven the plan recommended by Fresenius with that which we have generally adopted ourselves. We shall suppose that a great variety of all sorts of compounds may be present, and that each is to be separately eliminated in the course of analysis. Such a state of combination cannot occur in nature; but the plan has the advantages of pointing out the various appearances and indications of most of the compounds which can possibly be met with in an extended and long practice in analytical investigation.

GENERAL PLAN FOR INORGANIC QUALITATIVE ANALYSIS.

1. Ascertain, as far as possible, the specific gravity and other physical characters of the body; and refer to the general properties of metals, alkalies, earths, and salts, for general ideas of its constitution.

2. Reduce, if requisite, to powder. Heat this, and

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* See *ante*, p. 372.

observe whether partial fusion, sublimation, or other notable change takes place. If, on heating, the mass turn black, organic matter must be present. If any smell be produced, test for ammonia, sulphur, &c.

3. If any particular change do not take place, proceed to dissolve in water, first employing the fluid in a cold state, then heating to ebullition.

4. Use any indication which the blow-pipe may afford. By this, oxides may be reduced by charcoal supports. Earths may produce glasses; and some oxides may afford coloured glasses with borax, carbonate of potash, &c., on platina foil or wire.

5. If insoluble in water, use dilute hydrochloric or nitric acid. In most cases these acids act as solvents; and, with the exception of silica, few substances withstand their separate or joint action. If solution be effected, proceed to test for earths, oxides of metals proper, and alkalies, as already pointed out.

6. Consulting the tables and experiments already given, employ such tests as shall not interfere with the general result. Do not introduce any substance which might vitiate the ultimate requirements of the analysis.

7. Leave all substances insoluble in the acids, &c., already named, for fusion with fluxes, as most likely such would consist chiefly of silica and silicates.

8. As a general rule, leave the detection of any alkalies supposed to be present to the end of the process. The possible presence of ammonia alone is to be excepted, as it is desirable that this alkali should be disposed of as soon as possible.

9. A careful use of hydro-sulphuric acid and hydrosulphate of ammonia, is one of the most valuable guides the chemist possesses, as by them the peculiar character of metals may be readily classified. The following table will be found useful in directing to a general knowledge of the sulphides formed by their use. We have also included the results afforded on adding the alkalies to the various solutions of the oxides, salts, &c.

TABLE *of Precipitates afforded by Hydro-sulphuric Acid (sulphuretted hydrogen) and the Hydro-sulphate of Ammonia, in Solutions of most of the Elementary Bodies and their Compounds, and those obtained by testing with Potass, Ammonia, &c.*

Elements, Oxides, &c.	Test. Hydro-sulphuric Acid.	Test. Hydro-sulphate of Ammonia.	Potass.	Ammonia.	Carbonate of
Alkalies. Potass Soda Ammonia	No precipitate	No precipitate	No precipitate	No precipitate	No precipitate.
Alkaline Earths. Baryta	Do.	Do.	Hydrate in concentrated solution	Do.	All alkalies. Carbonate.
Strontia	Do.	Do.	Do.	Do.	Do.
Lime	Do.	Do.	Do.	Do.	Do.
Magnesia	Do.	Do.	Hydrate	Partial precipitate of hydrate	Potass—The carbonate and hydrate precipitated. Ammonia—scarcely.
Alumina.	Do.	White precipitate of the hydrate	Hydrate, soluble in excess	Hydrate, soluble in excess in the absence of potass and soda	
Chromium, Oxide of	Do.	The black hydrate	Hydrate, green, soluble in excess	Hydrate, scarcely soluble in excess	
Zinc. Oxide	White sulphide in neutral solutions. In acid solutions—none	White sulphide from neutral solutions: precipitate not redissolved by excess	White hydrate, soluble in excess	Hydrate, soluble in excess	Potass — Carbonate insoluble in excess. Ammonia — Carbonate soluble in excess.
Manganese. Protoxide	No precipitate in acid or neutral solutions. A flesh-coloured precipitate from alkaline solutions	Flesh-coloured sulphide from neutral solutions, changing to dark-brown when exposed to the air. Insoluble in excess of the test	White hydrate, which changes to brown, and at last to black, on exposure to the air by both alkalies		
Nickel, Oxide of	No precipitate in acid or neutral; a black in alkaline solutions	Black sulphide in neutral solutions; partly soluble in excess	Green hydrate, insoluble in excess	Hydrate, soluble in excess, turning to blue	
Cobalt. Protoxide	Scarce precipitate in neutral solutions; none in acid; black in alkaline solutions	Black sulphide in neutral solutions	Blue, turning to green on exposure to air	Same as potass, but soluble, producing a brown-coloured fluid	

Elements, Oxides, &c.	Test Hydro-sulphuric acid	Test, Hydrosulphate of Ammonia.	Potass.	Ammonia.	Carbonate of
Iron.					
Protoxide	None in acid solutions; black in alkaline	Black sulphide	Green hydrate, turning to red on exposure to the air	Same as potass	
Peroxide	In neutral and acid solutions a precipitate of sulphur	Black sulphide	Red hydrate	Do.	
Silver.					
Oxide	Black sulphide	Do.	Brown oxide, insoluble in excess	Brown oxide, readily soluble in excess	
Mercury.					
Protoxide	Black sulphide	Do.	Black oxide, insoluble	Same as potass	
Peroxide	White, orange, or black, according to quantity of test	White, orange, or black, according to quantity of test	Red-brown oxide, turning to yellow by excess	White precipitate	
Lead.					
Oxide	Black sulphide	Black sulphide	White precipitate, slightly soluble in excess	White precipitate, insoluble in excess	
Copper.					
Oxide	Brown-black sulphide	Brown-black sulphide, slightly soluble in excess of the test.	Blue hydrate, becoming black on being boiled	Blue, soluble in excess, affording a deep-blue liquid	All alkalies—Carbonate.
Bismuth.					
Oxide	Black sulphide	Black sulphide	White hydrate, insoluble	Same as potass	
Cadmium.	Yellow sulphide	Yellow sulphide	White hydrate, insoluble	White hydrate, readily soluble	Alkalies—White carbonate insoluble in excess.
Gold.					
Peroxide	Black sulphide	Black sulphide, soluble in excess of the test	None.—In very concentrated solutions a yellow peroxide	Yellow fulminating compound	
Platina.					
Peroxide	From acid and neutral solutions, black-brown sulphide	Black-brown sulphide, soluble in excess of the test	Yellow crystalline double salt Both these precipitates may be redissolved in excess of precipitants, on being boiled therein	Same as potass	
Antimony.					
Oxide	Scarce from neutral; orange-red sulphide from acid solutions	Orange-red sulphide, soluble in excess	White hydrate, soluble in excess	White hydrate, insoluble in excess	Alkalies — Same as ammonia.
Tin.					
Protoxide	From neutral and acid solution, a brown sulphide	Brown sulphide, scarcely soluble in excess—Yellow bisulphide if the test contain free sulphur	White hydrate, soluble in excess	White hydrate, insoluble in excess	Do.
Peroxide	Yellow bisulphide, from acid and neutral solutions	Yellow bisulphide, soluble in excess of the test	Do.	Do.	Do.
Arsenic.					
White oxide, or arsenious acid	Yellow sulphide in acid, and none in alkaline solutions	None till the addition of an acid	Form with alkalies, &c., arsenites and arseniates. See article "Arsenic" for full tests (p. 381, *et seq.*)		
Arsenic acid	None in alkaline and neutral; yellow sulphide in acid solutions	Do.			
Uranium.					
Oxide	None	Brown sulphide	Yellow hydrate	Soda—Same as potass	

By means of the indications thus afforded by hydrosulphuric acid and hydro-sulphate of ammonia, the characteristics of the metals, in respect to their sulphides, are readily grouped; and hence a great assistance is afforded in analytical investigations. The results obtained by the addition of the alkalies and their carbonates are equally valuable; and the previous tables will thus be found of essential service to the student. As they are the result of direct experiment, he may place every reliance on them, although a repetition of the experiments will prove of great use, as tending to impress the facts on the mind.

We have thus afforded the student ample means for commencing a course of qualitative analysis. Commencing with bodies of which he knows the constitution, or can find it by referring to our previous pages, he may gradually pass on to examine others of a more complex character. From such he may proceed to analyse minerals of various kinds; and having acquired tact, skill, extended information, keenness of observation, and last, but by no means least, *patience*, he will be qualified to detect the presence of poisons, under circumstances in which, from the minuteness of their presence, all his chemical knowledge and qualifications will be called into exercise. We shall now give general directions for the prosecution of *quantitative* analysis, presuming that our readers have already acquired sufficient information in the previous department of analytical research to enable them to make further progress.

INORGANIC QUANTITATIVE ANALYSIS.

Whilst in the qualitative analysis of any substance we are content to simply ascertain the *presence* of the various constituents, the object of quantitative analysis is that of finding the *quantity* of each substance composing the body under examination. We need hardly state that great skill and care in manipulating is required. In former days 100 grains were generally taken for an analysis; a course which, whilst in appearance would afford a chance of obtaining reliable results, actually defeated that object, by the diffusion, of both the substance tested and the reagent, in large quantities of liquids. The operations, of course, were on the large scale. Furnaces, vessels, and every apparatus were of the largest size; and a laboratory for such purposes could only be commanded by persons possessed of great pecuniary resources. At the present time, a table, a gas-furnace, test-tubes, and other small apparatus, are considered quite sufficient for all purposes. Only a few grains of any substance are needed; and in some cases less than a grain satisfies the requirements of an experienced chemist. Former sources of error are entirely avoided; and an accuracy, which in the earlier days of chemical science was not dreamt of, is expected from even the pupil; much more so from the professor. It will be impossible for us to give detailed instructions for the quantitative analysis of every class of bodies which come under examination in the laboratory. For such information we must refer our readers to works exclusively devoted to the subject. We shall be content to give general outlines for the direction of the student, and must leave to his industry and skill the chance of his becoming proficient in this interesting branch of experimental science.

Of course, before attempting to ascertain the relative quantities of the constituents in any substance, their existence must be ascertained by means of a qualitative analysis. And this must not be cursorily performed; for if any one, however minute, of the bodies constituting the whole quantity be neglected in a preliminary examination, then an accurate quantitative analysis cannot be effected. Hence, as we have already remarked, considerable chemical experience is necessary to qualify any one for an analyst. A vast variety of minor circumstances must also be kept in mind. Thus, some substances almost entirely insoluble in distilled water, are readily dissolved if others are present; and this often occurs through the introduction of salts, &c., in the course of analysis. For instance, many oxides, &c., precipitated by an exact equivalent of potass and ammonia, are insoluble in water. If, however, an excess of the test be present, a portion of the precipitate will be redissolved, and a source of serious error introduced. Many cases of this kind may be noticed in the tables of tests, &c., in our previous pages. Occasionally, the presence of certain salts, or their absence, will interfere with precipitation completely. Thus, the sulphate of lead is readily precipitated from the acetate by means of sulphuric acid, or a sulphate; but if the sulphate of ammonia, or other salts of that alkali be present, precipitation may be entirely prevented.

In quantitative analysis, it is of the utmost importance that the whole of the precipitate be thrown down. This will not occur where a sufficiency of liquid is not used; but caution in this respect is requisite, for an excess may interfere injuriously—*e. g.*, the sulphate of lime, which is barely soluble in water, and may be almost entirely precipitated in strong solutions; whilst a portion will remain in solution in those which are soluble. Temperature, also, materially influences precipitation; for occasionally a precipitate will remain suspended in the fluid at ordinary temperatures; whilst, on boiling, it will be speedily thrown down. Such is the case with the ferro-cyanide of iron, and many other salts. At times the physical condition of the precipitating vessel will influence the result—*e. g.*, the bitartrate of potass will only be thrown down on a roughened surface.

Great care is required in collecting precipitates during filtration. In transferring them from the precipitating jar to the filter, a lipped vessel should always be used, which should be repeatedly washed out, and the washing added to the filtrate. The choice of filtering-paper is also of great importance. Two sorts are sold for chemical purposes; one being of a porous, open nature, suitable for common operations; and the other of a fine texture, which alone should be employed for analytical purposes. All paper contains a portion of earthy matter, the quantity of which varies according to the material employed, and the water used at the mill. In some parts of England we have detected both lead and iron in paper manufactured with water derived from proximity to mines of those metals. It is therefore a good plan to purchase a quantity of filtering-paper of one manufacture; and having ascertained the amount of ash it affords per 100 grains, by burning it—the next step is to make a qualitative analysis of the ash, so as to learn the possible nature of the impurity it may communicate. Some recommend to soak the paper in dilute nitric or hydrochloric acids; but this, whilst not affording any great advantage, is a troublesome process; and even, if not carefully employed, may introduce fresh sources of error. Thus, if the paper were soaked in hydrochloric acid, and every trace of the latter had not been removed, an analysis of any substance containing salts of silver or lead, would afford a deficiency of the base, by the conversion of a portion into a chloride of the metal, which would most likely remain in the pores of the paper itself.

In very delicate investigations account must be taken of the moisture which the paper contains. Persons unaccustomed to the delicate nature of some analyses, would scarcely credit how necessary it is to ensure accuracy in all respects. It is best to use the filtering-paper after completely drying it. Pieces are then to be cut off, as near as possible the same size, one of which is employed to weigh the substance before analysis, and the others in its course; for nothing should be weighed in the scale-pan itself. The exact weight of the dried filter must, of course, be ascertained, and carefully noted down. It is by no means a useless plan to make a nick on all the papers likely to be used for one analysis, as by that means a chance of error may be avoided; and no inconvenience can arise, because they can thereby be all identified as belonging to the same kind of paper, and to be used for the same purpose.

For quantitative analysis a most delicate balance is required. In the previous experiments, any pair of scales capable of weighing as low as grains, may be employed; but in estimating the quantities of constituents, a balance which will weigh accurately to the $\frac{1}{1000}$th part of a grain, is absolutely needed. Of course, we cannot recommend the instrument made by any particular manufacturer; but we may point out various particulars, to which the attention of our readers must be directed, if they would obtain reliable results in their investigations.

In the balance, we observe a lever whose fulcrum is midway between its two extremities, and which, when perfect, should lie in an exact horizontal line, or parallel to one drawn through the fulcrum. The operation of weighing consists in the addition of two weights, one at either extremity of the lever; which, if in excess of each other, shall cause the lever to be depressed beneath the horizontal, on the side on which the excess rests; but which, if equal to each other, shall allow the whole lever to be parallel to the horizontal line of rest. Now, at first sight it would seem to be an easy matter, according to the skill of the workman, to construct an accurate balance; but one or two points have to be attended to. That of the centre of gravity is an essential one; for if that be in the centre of the lever, then the scales will have a tendency to "set" when over-loaded—that is, the horizontal line will not be regained when even exactly the same weights are on each part of the scale. This difficulty is overcome by so arranging the centre of oscillation, or that on which the beam turns, with the centre of gravity, so that the latter may be above the former. In proportion as this point is attended to, then, within certain limits, does the value of the balance depend. The homogeneous character of the metal of which the balance is composed, is another matter of some importance; for although, theoretically, we *presume* the lever to be inflexible, it is really not so; and an excess of weight, or carelessness of weighing, may seriously damage the balance-beam—in many cases partly through a want of homogeneity in its parts.

In the operation of weighing, many precautions have to be attended to. As we have already said, no substance should be weighed except on paper or in a glass tube; for if the operator be neglectful of this, the scale-pan may become oxidated, and so prove heavier than its fellow. Besides, portions of the substance are sure to adhere; and the rough usage required to remove them is totally opposed to the delicate treatment a good balance should receive at the hands of its owner. Heated substances should not be weighed, because, as they would tend to expand the arm of the beam under which they are placed, they would seem to weigh heavier than they really did; and hence a serious source of error would be introduced.

The pans should never be loaded whilst suspended from the arms of the beam. This never need be done; for in all chemical balances, arrangements are made by which the pans, arms, &c., can all be suspended simultaneously for the operation of weighing, and be brought to rest again when it is concluded. By such means all undue or unnecessary strain on the arms of the balance is avoided, and a highly possible source of injury removed. The balance should always be kept away from the fumes of acids, &c. They are generally enclosed in glass cases, which, except when in use, prevent any great risk of injury.

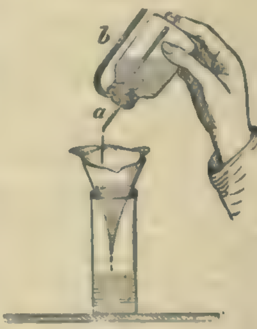
The analyst will require a considerable variety of weights, from the fraction of a grain upward; the smaller being made of platina. Small weights are readily made from fine platina wire by weighing a portion, and then dividing it for halves, quarters, &c., of the normal weights. The weights should not be touched by the finger, but put into, and removed from, the scale-pan by means of a pair of tongs. Small pieces of tin or lead-foil, or small shot, are useful in counterpoising crucibles, &c. They must be kept scrupulously clean. Glass beads are preferable, because they do not acquire an increase of weight by oxidation.

It must be kept in mind, that many substances, whilst being weighed, acquire a certain amount of moisture from the air, and so increase in weight; as, for instance, caustic potass, soda, barytes, lime, &c. These are termed hygrometric substances; and such should always be weighed in very thin test-tubes. This is a point of considerable importance in organic analysis, in which the substances operated on are generally minute in quantity, and the black oxide with which they are burned being hygrometric.

In very delicate balances, which will turn with at least the $\frac{1}{1000}$th part of the weight in the scale—say with a load of 500 to 800 grains—the indications of an index will assist in a further attainment of accurate weighing, and thereby another decimal figure may be added to the result. This, however, can only be done by an experienced person. Meanwhile, we direct the attention of the tyro to the fact, so that he may gradually acquire a knowledge of the peculiarities of the chemical balance.

Before proceeding to make remarks specially applicable to certain precipitates, we may give a few more general directions. The precipitate having been collected on the filter, must be carefully washed by means of a gentle stream of water, if insoluble in that liquid; and this is best done by means of a washing-bottle. This may be of any form or shape: a convenient one is represented in the margin. It consists of an ordinary glass bottle, fitted wth two glass tubes, having a bore equal to about the tenth of an inch. That marked *a*, is one from which the water flows; whilst the air passes through *b*, to take the place of the escaping fluid. It is easily filled by drawing air out by the mouth through the tube *b*, whilst *a* is immersed in a vessel of distilled water. When thus filled, the bottle is used for washing, as shown in the cut. The tube *a* should be drawn to a jet-like point, so that the stream of water may flow gently on to the precipitate, and not drive holes in the filter.

Fig. 74.



The removal of the filter from the funnel requires care, because the filtering-paper has little tenacity, and hence may readily break in pieces. The best plan is to tilt the funnel on one side, and blow between the paper and the glass surface. This will separate them, and a good hold may then be taken of the filter for its removal. The next step is that of drying the precipitate; and this requires care. In most cases a water-bath is all that is requisite; and one of these is readily constructed by means of two evaporating dishes. The filter being placed in one, it is floated in another containing water kept at a boiling state. This or any other similar plan may be adopted. Water-baths, however, made in Berlin porcelain, may be procured of the instrument-makers. If a higher temperature be required, as for the evaporation of acids, &c., then a sand-bath must be used; for which purpose an iron or copper bowl, filled with sand, and placed over the spirit-lamp or furnace, will answer. The evaporating dish containing the filter, &c., to be dried, is placed in the sand. In some cases it is desirable to burn the filter, and to weigh the residual precipitate. This is best done in a Berlin or platina crucible. We must here warn our readers never to burn precipitates containing oxides of metals proper, either with paper or in a platina crucible. The charcoal of the paper would most likely reduce the metal. And metals such as lead, &c., if coming in contact with a red-hot platina crucible, would render it useless by forming an alloy. The alkalies, earths, and their compounds alone, should find a place in a platina vessel at a high temperature. Some substances, during drying, decrepitate; that is, they cast off small portions, owing to the formation of steam, which, expanding between their particles, affords small explosions. Such occasion loss; and a cover of any kind, such as another evaporating dish, should be inverted over that containing the drying precipitate. In employing the sand-bath, care should be taken, if a filter is being dried, that it do not attain a temperature sufficient to char it, as that would occasion a loss of weight, owing to the decomposition of the paper.

When a filter is very dry it also becomes rigid; and the least touch may break it, or cause the dry powder it contains to jerk out. This is a point requiring attention and care. We have seen young students destroy all the hoped-for reward of their previous attempts, for want of delicacy of touch and manipulation in this respect.

In some instances, the only result obtained—which is, generally speaking, far from being satisfactory—is that of crystalline product. Under this head the alkalies must be placed; as, with little exception, no solid precipitate can be obtained from which they can be estimated. Potass, for instance, may be weighed as a sulphate, nitrate, &c. Soda may be similarly estimated. In such, and many other similar instances, when crystallisation is had recourse to, it must carefully be noticed whether the crystals are likely to possess water as a constituent. We do not here refer to water in a liquid state, but in chemical combination. In some cases crystals lose their water of crystallisation, and from this cause serious loss may occur in the estimation of the base. The water may not be driven off at the boiling temperature; hence a water-bath may be used. Generally speaking, all such salts, however, suffer loss when the temperature is raised beyond that point; and the sand-bath must therefore be employed with great caution.

Presuming that the student has mastered the details through which we have been proceeding, his last step will be that of finding the relative value in respect to the chemical equivalents of bases, &c., of the products which he has obtained. This, however, is not a question of very great difficulty; because the simplest rules of arithmetic alone are involved. In fact, the Rule of Three or Proportion is that which is to be employed, and of which we shall give some examples.

Suppose, for example, that in Experiment 82, p. 397, in our remarks on qualitative analysis, 100 grains of marble had been employed, and that after dissolving the solid in hydrochloric acid, of which a known weight had been added, it was found that 44 grains had been lost. Now, as by the subsequent part of the process it was discovered that the gas which had escaped was carbonic acid, and that the solid with which the acid was in combination was lime, we wish to know how much lime was actually present in 100 grains of marble. As that substance lost 44 grains of carbonic acid, and as 28 grains of lime unite with 22 grains of carbonic acid to form 50 grains of marble, we should have the following proportions :—

	One equivalent of CO_2.		The amount of CO_2 in 100 grains of marble.		One equivalent of lime, Ca O
As	22	:	44	: :	28

Now multiplying 44 by 28, and dividing by the first term as in ordinary Proportion, we obtain—

44
28

352
88

22) 1232

56

Hence there will be 56 grains of lime in 100 grains of marble. Of course this long sum was not required to prove the fact; as it is evident that there will be twice as much lime in 100 as in 50 grains of marble; but we give this easy illustration that our readers may understand the process.

We will give another illustration, which, although on the same principle, yet involves a little more apparent difficulty. We will suppose that an alloy has been dissolved in nitric acid, and that the silver has been precipitated as chloride of silver, of which there are 48 grains obtained; how much chlorine and silver are individually in that quantity?

Now the equivalent of chloride of silver is—

Chlorine	36
Silver	108
	Or 144

And we must accordingly make two proportions to ascertain the amount of each element present.

Commencing with the silver, we say—

	Ar Cl	Ar Cl		Ar		Ar
As	144	: 48	: :	108	:	36 as below.

108
48

864
432

144) 5184 (36 grains of silver.
432

864
864

In this the 48 is placed in the second term of the proportion, because the answer must necessarily be smaller than the third term.

With the chlorine we say—

	Ar Cl	Ar Cl		Cl		Cl
As	144	: 48	: :	36	:	12 grains as below.

36
48

288
144

144) 1728 (12 grains of chlorine.
144

288
288

Now, by the first proportion, we find of silver	36 grs.
,, by the second proportion, of chlorine	12 ,,
	48

Hence we find the total equal to that of the number of grains found in the precipitate.

We shall now take another case; in which, having ascertained, as above, the exact amount of each substance in a precipitate, the per-centage of each, or its proportion in one hundred grains, is desired. This is a very common question, especially in assaying and metallurgical processes generally.

The method to be adopted is precisely similar to that illustrated above; and we will take our last example to show the *modus operandi.*

Supposing that the 48 grains of chloride of silver were precipitated from 600 grains of a lead ore containing silver: how much *per cent.* of silver does the ore contain? We state it as—

600 : 36 : : 100 : 6 per cent. as below.
36

600) 3600 (6
3600

Hence such an ore would contain 6 per cent. of metallic silver; or, in other words, every 100 grains of the ore will afford six grains of pure silver.

Occasionally, the proposition is in the reverse form; that is, the per-centage being given—required the amount in any number of grains; say, for instance, in 600. Then we state it—

: 100 : 6 : 600
6

100 3600

36, or 36 grains in 600 of the ore.

Our readers not well conversant with arithmetic, must be cautious how they draw out their proportions. Some years ago, we entrusted to a pupil the analysis of some copper ore, by way of trying his abilities. We gave him fifty grains on which to try his skill. In the course of a day or so, he stated that he had obtained 200 grains of pure copper from it. The result was certainly astonishing, but its wonder immediately lessened on the arithmetical process being examined. The analysis was near enough; but in casting the proportion, he had misstated it, and brought out 200 grains instead of two per cent., by some mysterious process of calculation.

It will be seen, from the preceding remarks, that an extended knowledge of the equivalents and constitution of chemical compounds is essential to progress in quantitative analysis, and a free use of the symbols is equally requisite, as much time is saved thereby. It is an excellent plan for the student to copy out the equivalents of all the simple and compound bodies he can gain information of, and writing them, in a tabular form, on a piece of cardboard, he should hang it up in some conspicuous place in his laboratory. By this much time is saved, and reference to books is rendered unnecessary. The latter soon get spoilt in a laboratory. It is also advisable to have extended tables of specific gravities, and other particulars, for the purpose of constant reference, which should always be ready to hand.

The foregoing general hints will be sufficient to remind the student of many important matters requiring his attention whilst pursuing quantitative analysis. We shall now pass on to give an instance or two of the practical method, and to mention various particulars which have to be attended to in special cases. We must mention, however, the following rules as never in any case to be neglected.

1. A qualitative analysis should always first be made.
2. Care must be taken in quantitative analysis never to lose a drop of solution or a particle of a precipitate.
3. All washings must be added to the original solution from which the precipitate has been thrown down, and the precipitate itself must be washed (if insoluble in water) until every trace of soluble matter has been removed.
4. The precipitate must be thoroughly and carefully dried. It is not to be weighed till cool; and if hygrometric, must, for that purpose, be placed in a thin glass tube, or between two watch-glasses.
5. And, lastly, every step of the analysis must be carefully noted down as it occurs, and each vessel should be labelled with letters, &c., corresponding to those attached to entries in the note-book.

PRACTICAL ILLUSTRATIONS.—For the sake of more fully explaining the methods adopted in quantitative analysis, we shall select the following examples, which may be taken as types of the general plans to be followed in most cases. There will be sufficient variety to enable the student to comprehend numerous other instances with which he may meet.

Experiment 90.—Dissolve a three-penny piece in nitric acid. Now, as silver is always alloyed with copper to form the material from which our coin is manufactured, nitrates of silver and copper will be left in solution. It is required to make an analysis affording the actual weight of each metal composing the coin. The solution is to be diluted with distilled water, and to this a solution of common salt is to be added so long as a white precipitate is formed. The silver is thus removed as the chloride, whilst the copper remains in solution.

(*a.*) Pour solution and precipitate cautiously and gently on to a weighed filter,* supported in a funnel, and receive the liquid, which will run through, in another vessel. Wet the filter with distilled water before pouring in the solution, &c.† When all the liquid has passed through, wash the precipitate by means of the washing-bottle,‡ so as to remove every trace of the copper solution, &c. As the latter will spread by capillary attraction through every part of the paper, wash the filter as carefully as the precipitate itself. The washings, of course, must run through to the solution in the lower vessel, as they will contain a minute portion of copper. When the washing is completed, remove the filter carefully from the funnel,§ and place it on the water-bath, leaving it there till completely dry. Whilst the chloride of silver is being dried, the copper left in solution may be obtained.

(*b.*) If the solution left after removing the chloride have been largely increased in bulk by using too much water for washing, evaporate until not more than an ounce in measure is left. To this add a little hydrochloric acid, and then a few zinc shavings or some clean iron scraps. Heat the solution gently for some time, and then evaporate nearly to dryness. Add a little dilute sulphuric acid, to remove all excess of iron or zinc. The precipitate produced will be metallic copper, which is to be collected on a weighed filter, and well washed. It is then to be quickly dried by means of the water-bath.

The next step is to weigh each product of the analysis. We will take the chloride of silver first. If the operation have been properly conducted, and supposing the filter to weigh 20 grains, it and the precipitate together should weigh $46\frac{3}{4}$ grains nearly. Hence the precipitate will afford $26\frac{3}{4}$ grains of chloride of silver. The amount of silver is easily found from this;‖ for the proportion will be—

As an equivalent of chloride of silver is to that of the product of the analysis, so is the amount of silver contained in an equivalent to that contained in the precipitate; or, expressed in figures—

$$\text{As } 144 \;:\; 26\tfrac{3}{4} \;::\; 108 \;:\; 20\tfrac{1}{16} \text{ grains.}$$

$$\begin{array}{r@{\,}l}
 & 108 \\
 & 26\frac{3}{4} \\
\hline
 & 648 \\
 & 216 \\
 & 54 \\
 & 27 \\
\hline
144) & 2889\ (20\tfrac{9}{144} = 20\tfrac{1}{16} \text{ grains.} \\
 & 288 \\
\hline
 & 9
\end{array}$$

We thus find that the coin contained $20\frac{1}{16}$ grains of pure silver.

If the copper be then weighed after drying, it will afford about $1\frac{3}{4}$ grains. Thus the composition of the three-penny piece will be—

Of silver	$20\frac{1}{16}$ grains
Copper...........................	$1\frac{3}{4}$
Total	$21\frac{13}{16}$ grains.

Which is a little less than that of the new coin, as issued from the mint.

It is usual to express all figures in an analysis in whole numbers and decimals; the latter being used in place of fractions: the results of the analysis will then stand as under:—

Silver	20·0625
Copper...........................	1·7500
Total	21·8125 grains.

The above examples may be easily repeated with successful results, because both the products of the operations are insoluble in water; and therefore no error can arise except that proceeding from carelessness. In many instances, the slight solubility of a precipitate interferes with the accuracy of the result, and in such cases the total of the original weight is made up after analysis, by adding in to the amounts of the found bodies, figures representing the actual loss, and marking them as such.

In the next example we shall confine our attention to weighing only one of the substances present in the body analysed—namely, the sulphuric acid.

Experiment 91.—Dissolve twenty grains of the sulphate of potash in an ounce of distilled water, and add thereto so much solution of the chloride of barium as is required to precipitate the whole of the acid as sulphate of baryta. When the precipitate is complete, filter, wash, and dry as before; and after drying, weigh with the filter. With such a quantity of the sulphate of potash, a precipitate of twenty-nine grains of sulphate of baryta will be produced. Now the equivalent of sulphate of baryta = 116; and that of sulphuric acid in the anhydrous state = 40. And hence we have,

---

* See *ante*, p. 403.

† *Ante*, p. 396.

‡ *Ante*, p. 404.

§ See *ante*, p. 404.

‖ *Ante*, p. 405.

$$\begin{array}{lllllll} \text{As } 116 & : & 29 & :: & 40 & : & 10 \\ & & & & 29 & & \\ \hline & & & & 360 & & \\ & & & & 80 & & \\ \hline & & & 116\,) & 1160 & (\,10 \text{ grains.} & \\ & & & & 1160 & & \end{array}$$

Hence we find that twenty grains of sulphate of potass contain, according to the analysis, ten grains of sulphuric acid. The actual composition of the salt is

$$\text{Equivalent of} \left\{ \begin{array}{l} \text{Sulphate of} \\ \quad \text{potass} \ldots \end{array} \right\} = \begin{array}{ll} \text{Sulphuric acid} & 40 \\ \text{Potass} \;.\;.\;.\;. & 47{\cdot}2 \\ \hline & 87{\cdot}2 \end{array}$$

So that a little error has been made in such an analysis.

The preceding will afford initiative experiments for the beginner; and by practising simple examples, he will gradually acquire the dexterity and accuracy only obtained by constant practice and experience.

In quantitative analysis, there are some processes which are specially applicable for certain purposes. One of these, cupellation, we shall now describe, because it is extensively used both by analytical chemists and by metallurgists in refining and assaying. A peculiar form of furnace is requisite for this operation, and an instrument called a muffle. In the preceding engraving, a view of a furnace externally, and as arranged for the reception of a muffle, is shown. On the left hand view, an opening is seen in the body of the furnace, into which the muffle is introduced.

Fig. 75.

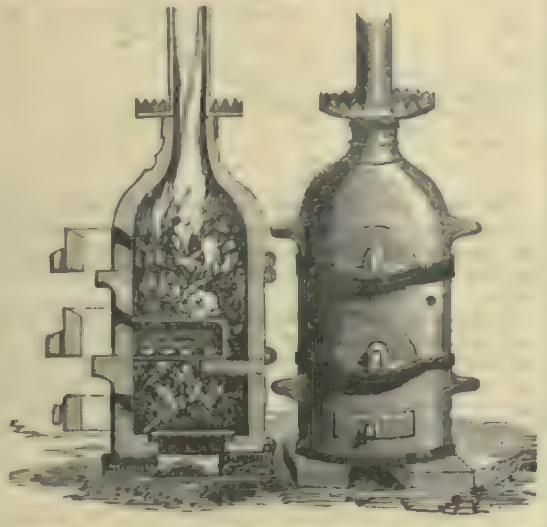
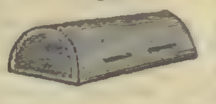


Fig. 76.



The muffle is made of earthenware, and in this the cupel is placed. A view of an ordinary form of the muffle is represented in the margin. The cupel itself is a small vessel, made of bone ashes, which are employed on account of their porosity.

The operation of cupellation depends on the fact, that, when gold and silver, which do not become oxidised at a red heat, are alloyed with metals that do, the latter separate from the noble metals, leaving them in an almost pure state. Supposing silver is to be assayed, a few grains are put into some pure lead foil. The two are then placed in the muffle, already red-hot in the furnace. By the peculiar arrangement of the whole apparatus, a current of air passes over the silver and lead; and so long as lead or any other readily oxidisable metal remains, the two present a dull appearance. The operator must carefully watch; and quite suddenly he will perceive the silver to present a most brilliant appearance, owing at that moment to the almost entire removal of all the baser metals. The process is then concluded; and the cupel having been allowed to cool, the button of pure metal may be removed and weighed. The cupellation of gold is conducted in a similar manner, with the exception that silver and lead are both employed, instead of lead by itself. By this an alloy of silver and gold is produced at the conclusion of the cupellation. The silver is afterwards removed by means of nitric acid; the gold being thus left in a pure state.

This process is one requiring considerable care; for at times the metals, at the instant of their becoming pure, swell up. This is especially the case with silver; for its bead occasionally bursts, by which a loss of metal is sustained.

With this we shall conclude our remarks on quantitative analysis, so far as inorganic bodies are concerned. The subject is of so extensive a character, that were we to attempt to give full details, we should have to far exceed the limits which our plan imposes on us. We have already named the best works which the student can consult.* In them almost every substance which can be met with, is treated on in full particulars. Analysis, from its numerous uses, has become a distinct branch of chemical science; and one in which, to be proficient, requires an undivided attention. Our object has been to give a general idea of the subject, so that the beginner may have an opportunity of learning leading principles and methods only.

CHAPTER VI.

THE CHEMISTRY OF VEGETABLE SUBSTANCES, AND THEIR PRODUCTS.

It has long been customary to arrange the description of the chemical properties of organic bodies under the head of Organic Chemistry; but owing to the extension of our knowledge during the last few years, the distinction which had existed between the animate and inanimate creation, in a chemical point of view, has been removed, or so greatly modified as to scarcely exist. We propose, therefore, to devote the following chapters to a description of such bodies as exist in, or are, the products of vegetable substances only. With respect to the chemistry of animal life, that has been so fully dilated on in the chapter on Food, &c., as to leave nothing to add to it.

During the last twenty years, the study of the chemistry of vegetable substances, &c., has made astonishing progress. To Dr. Liebig, of Giessen, we are preeminently indebted for masterly and accurate researches in this branch of experimental science. It has been chiefly owing to his discoveries that so deep an interest has been generally taken in the subject, and that agriculture has advanced to the rank of a science, in place of being a pursuit of chance and speculation, as it had formerly existed.

In other branches of industry, similar results have accrued, owing to the rapid applications which have been made of newly-developed principles. These have been so numerous as to defy enumeration. In the art of dyeing, for instance, the discovery and use of aniline has completed an entire revolution in the processes employed; and gas-tar, which previously was employed almost solely for the preservation of timber, is now the source of those beautiful colours, mauve, Magenta, &c., which afford such rich tints to cotton and silken fabrics.

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* See *ante*, p. 397.

We shall first give a general account of such substances, &c., which being directly or mediately produced from plants, have for their chief component parts, the elements of hydrogen, oxygen, and carbon, or any two of these. Having thus, as it were, given the elementary principles of this department of chemical science, we shall proceed to examine other and more complex bodies.

With scarcely an exception, vegetable bodies consist entirely of carbon and water, or the elements of water united with carbon in various proportions. To familiarise the mind of the student with the products of vegetable substances resulting from chemical action, we shall here give the formulæ of water, carbonic acid, and other compounds, in which these or their elements are involved. We shall use their symbols, to save space in indicating their composition.

Water . . .	HO
Carbonic acid . .	CO_2
Alcohol . . .	$C_4 H_6 O_2$
Ether . . .	$C_4 H_5 O$
Acetic acid . .	$C_4 H_3 O_3$
Oxalic acid . .	$2CO + O = C_2 O_3$
Tartaric acid . .	$C_8 H_4 O_{10}$
Sugar . . .	$C_{12} H_{11} O_{11}$
Starch . . .	$C_{12} H_{10} O_{10}$

Now, on inspecting the above list, we find that a union of the same elements, in various proportions, is capable of producing substances having entirely different properties. Thus, alcohol, ether, acetic acid, tartaric acid, and sugar, each contain precisely the same elements, but in different proportions; and yet there is not the least apparent similarity in their specific properties. These facts just point out the extensive variety of forms and substances which carbon, oxygen, and hydrogen can produce by their union, and the learner may thus have some idea of the wide range of that interesting field of investigation into which we are now entering.

We shall here point out two singular and instructive facts connected with the composition of four of the substances to which we have called attention—namely, sugar, alcohol, ether, and acetic acid. In producing alcohol from sugar, as in the ordinary process of fermentation, we find that carbonic acid is largely produced, which passes off in a gaseous form. This accounts for the difference of composition between the sugar and alcohol. In producing ether from alcohol, we lose no carbonic acid, but only an equivalent of water; so that alcohol may be regarded as composed of ether + water. Acetic acid (which is the cause of the acid flavour of ordinary vinegar) is readily produced from alcohol, and in that process we lose three equivalents of hydrogen and gain one of oxygen, by the oxidising effect of the atmospheric air employed; so that acetic acid may be regarded as a compound of four equivalents of carbon united with three of water. We can thus produce from sugar, so far as we have gone, three different bodies having exactly the same elements in various proportions, each different, and yet chemically analogous.

Referring now for a moment to oxalic acid, we find that really consists of a union of carbonic oxide (CO) with carbonic acid (CO_2), for the two would afford two equivalents of carbon and three of oxygen, which is precisely the constitution of oxalic acid in an anhydrous state. But if we speak of ordinary oxalic acid, we shall find that it is combined with three equivalents of water (3HO), which would make the combination as follows:—

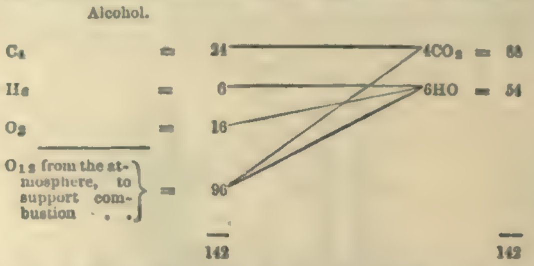
$$C_2 + O_3 + 3HO \text{ ; or, } C_2 + O_6 + H_3.$$

And in this formula we see how oxalic acid may be produced from starch or sugar (the usual source). Now, if we add concentrated sulphuric acid to oxalic acid, having the latter formula, decomposition ensues, its water is removed, and an equivalent, each of carbonic oxide and acid, is afforded.

As we proceed, we shall have to extend the number of elements in our formulæ, and include nitrogen. We shall, however, for the sake of simplicity, defer our remarks on this point till we have more fully described the simpler kinds of organic compounds.

The existence of carbon in vegetable bodies is constantly evidenced to our senses in the process of burning. Thus, in baking, toasting, roasting, and burning any vegetable body, a black mass is produced, which is owing to the carbon forming part of it being set free, as charcoal. In any of these processes, the hydrogen and part of the oxygen escape as water. The remaining portion of the oxygen combines with a portion of the carbon, and passes away as carbonic acid. Even the solid charcoal left, will gradually be converted into carbonic acid by combining with the atmospheric oxygen which surrounds it. The following formula shows the results of an entire combustion of a portion of absolute alcohol, which we shall take as a type of such results obtained during the combustion of other substances.

Results of the Combustion of an Equivalent of Alcohol.



We thus perceive, that by the combustion of one equivalent of alcohol with twelve of oxygen gained from the atmosphere, the oxidation of the spirit should produce four equivalents of carbonic acid and six of water.

In the early pages of this section, we have had frequent occasion to remark on the beautiful simplicity, and the value of the doctrine of chemical equivalents. But as we proceed in our investigation, we shall still further admire the sagacity of its discoverer. It is impossible to speak too highly of its use in this department of experimental science; in fact, organic chemistry has developed the nature of atomic values beyond the most sanguine expectations of the great Dalton.

The remarks and illustrations we have made, are, of course, only intended as introductory to our subject. We shall next speak of organic analysis, because it lays at the foundation of our investigations; and without its exercise it would be impossible to proceed a step in relating the natural history of the various substances we shall have to examine.

ORGANIC ANALYSIS.

In our previous pages* we gave extended directions for the qualitative and quantitative analysis of inorganic bodies. The principles and details we then explained, whilst incident to the analysis of organic bodies, are so, however, only to a limited extent, because our range over the nature of the elements with which we have to deal is also limited. In inorganic analysis we may happen to find any of about sixty different elements, and an almost interminable number of their compounds. In organic analysis we chiefly wish to detect the proportions of carbon, hydrogen, and oxygen present, and occasionally have to expect nitrogen. We do not trouble ourselves so much in reference to what we may call accidental substances, which are not absolutely essential to the typical constitution of

* See *ante*, pp. 394—407, inclusive.

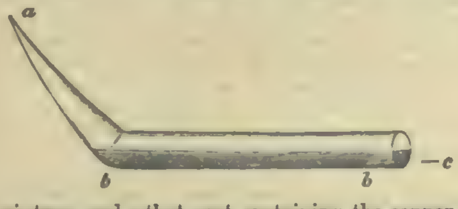
the bodies under examination, because we can almost always obtain vegetable products in a condition verging on absolute purity. Of course, if an analysis were required for commercial purposes we must be very exact in that respect also. Philosophically, we may consider flour, for instance, to be a compound of gluten and starch, and the analysis of it, and the detection of the elements present, would be restrained within the limits we have named. If, however, we have to conduct the analysis for agricultural purposes, we should pay far greater heed to what we have termed the "accidentals," such as silicious matter, phosphate of lime, &c., because the presence of such substances in the soil whence the wheat is produced is essential to its growth. If, therefore, we desired to analyse the wheat as a guide to the management of the soil, with respect to manure, &c., this matter then becomes of the highest importance. We shall, however, leave such till we enter upon the department of agricultural chemistry, to which they properly belong. Similar remarks are applicable to other technical applications of organic analysis.

We have already stated that the carbon, hydrogen, and oxygen existing in a vegetable product, are all resolvable into carbonic acid and water by the process of combustion, and the fact points out the course which must be adopted in conducting an analytical investigation. Many methods, both of a special and general nature, have been proposed; but the following is an outline of the plan usually adopted at the present time, and for which we are chiefly indebted to Dr. Liebig:—

(*a*). The substance to be analysed must, if solid, be reduced to as fine a powder as possible; this is best effected in a glazed Berlin porcelain mortar. After being powdered sufficiently fine, it may then be introduced into a tube, and exposed to a heat short of boiling water, if it will bear that without decomposition, which almost every substance will safely do.

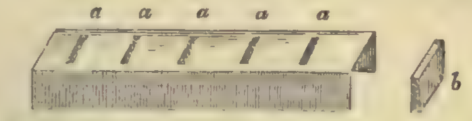
(*b*). It has then to be mixed with a considerable quantity of the black oxide of copper. This substance affords oxygen to any combustible mixed with it, when the two are exposed to a red heat. This is effected by introducing both into a German glass tube about eighteen inches long and an inch in diameter, which must have one end drawn out to a fine point. The substance to be analysed is to be mixed with the oxide whilst the latter is in a warm state, so as to prevent any danger of the absorption of moisture. A little clean oxide is first introduced into the tube, then the mixture of the substance and oxide; and some more oxide is then added, which, of course, will be at the open end of the tube. The following cut represents the tube as filled, *a* being the fine end, which is closed, *b b* the portion filled with the mixture, and *c* that part containing the copper turnings. When filled, the tube is to be laid horizontally, and gently tapped, by means of which the mixture will be spread evenly throughout the length. About three grains of the substance—which, of course, must be accurately weighed—is to be mixed with about an ounce to an ounce and a-half of the oxide.

Fig. 77.



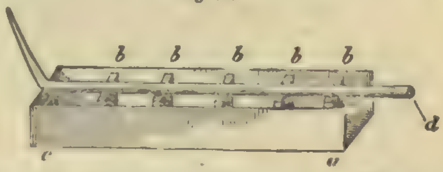
(*c*). This tube is then placed in a peculiar shaped furnace, so that the combustion may proceed in a gradual manner, commencing from the wide end of the tube, till it is completed at the narrow end, which is to be last heated. The wide end, however, projects beyond the furnace, for reasons we shall presently mention. The furnace is made of wrought iron plate, and should be about a foot long, and four inches wide and deep. At about two inches distance throughout its length, holes half an inch wide, and two inches long, are to be cut—*a*, *a*, *a*, *a*, *a*, *a*, as in the following figure, which represents the furnace, with the lower part uppermost (Fig. 78). Between each of these holes a small iron upright is placed (see *b* in Fig. 78), on which the tube is to rest during the process of combustion.

Fig. 78.



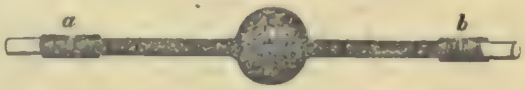
The fuel used is charcoal, which is first introduced in small quantities, at *a*, and gradually added, so as to carry on the combustion from *a* to *c* (Fig. 79). By these means a complete combination of the elements contained in the substance in the tube is effected, with the oxygen of the oxide of copper.

Fig. 79.



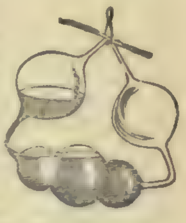
(*d*). At the end *d*, in the figure above, a tube is to be attached by means of an india-rubber connexion. This tube is to contain fused chloride of calcium, for the purpose of receiving the aqueous vapour generated during the combustion. By this, the amount of water produced is readily ascertained; and so the hydrogen and oxygen of the mixture are weighed away. A useful form of tube for this purpose is represented in the following cut (Fig. 80), in which is shown that part which is attached, by india rubber connexions, to the combustion tube, placed in the furnace; and *b*, that end attached to the potass apparatus, to be described immediately.

Fig. 80.



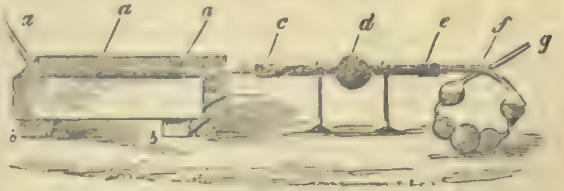
(*e*). The next piece of apparatus which is attached to the tube, shown above at *b*, is the potass bulb, represented in the annexed cut. The object of this is the absorption of carbonic acid, by means of pure liquid caustic potass, with which it is nearly filled; and by means of which the carbonic acid, produced during the combustion, becomes removed, and affords us the opportunity of ascertaining the amount of carbon in the body under examination. This potass bulb is attached, air-tight, to the end of the chloride of calcium tube, and so forms the end of the whole arrangement.

Fig. 81.



It will be perceived that we have only hitherto referred to the analyses of those substances containing carbon, hydrogen, and oxygen; and having described the different apparatus used, we subjoin a cut representing the whole in operation.

Fig. 82.



a a a, the combustion tube; *b b*, furnace resting on two bricks; *c*, connection of the combustion tube with the chloride of calcium tube, *d*; *e*, the connection between the chloride of calcium tube and, *f*, the potass apparatus; *g*, the open end of ditto.

The following particulars must be carefully attended to in a quantitative analysis conducted on the preceding plan.

If the article be a solid it must be most carefully weighed, and absence of hygrometric moisture should be carefully ensured. If, however, a liquid be examined, it should be inclosed in a glass tube, having a fine extremity. After the tube has been weighed, the liquid is to be introduced by expelling the air by heat, and holding the capillary opening beneath the surface of the liquid. A portion will thus be forced in by the pressure of the air. This tube is then to be let down to the closed end, marked *b*, in Fig. 77; and the remainder of the combustion tube is to be filled as before, with oxide of copper. The process is to be conducted as before, with the exception, that a gentle heat is to be applied to the end of the combustion tube holding the liquid. This will expel a portion of the liquid, which, passing as vapour to the heated portion of the oxide of copper, will be decomposed. Of course, this operation requires great care in manipulation; and but a little at a time of the liquid should be expelled, lest any portion might escape combustion.

The connections must all be as complete as possible, so as to be quite air-tight. The only open part will be the end of the potass apparatus. The latter should have its three lower bulbs *quite* full, and the two upper, *half* filled. At first a little air will pass off, which, of course, is owing to the expansion by heat of that contained in the combustion tube. The chloride of calcium tube must be accurately weighed when filled, each end being fitted with corks, which are to be weighed with it. These, of course, are to be removed on using the tube, but must be weighed again with it after the analysis is concluded, as their absence would of course introduce a source of error in the estimation of the increased weight due to the absorption of water, resulting from the combustion.

The same remarks hold good with the potass apparatus; for the amount of carbonic acid produced is known by the increase of weight, resulting from its absorption by the liquid. It must therefore be accurately weighed before and after the analysis, and the increase of weight indicates the amount of carbonic acid absorbed. At the moment of the analysis being concluded, which will be noticed when the liquid potass is not driven to and fro in the bulbs, the capillary end of the combustion tube is to be broken, and by the mouth—or, still better, the end of a syringe or air-pump attached by means of an india-rubber connection—air is to be drawn through the whole apparatus; and thus any water or carbonic acid unabsorbed is drawn through the calcium and potass tubes. It is true that a source of error may arise from the admission of air; but if this be not carried to any excess, the error will be but trifling.

When nitrogen is present in any substance to be analysed, a separate analysis must be effected to determine its presence. In place of filling the potass apparatus with that alkali, a solution of the chloride of platina is to be employed. The substance under examination is mixed with a portion of soda ash and lime, and heated as before. By this means the nitrogen produces ammonia, which, combining with the platina salt, forms the double chloride of ammonium and platina, in which form the nitrogen is estimated. The nature of the double salt has already been referred to under the head of Platina.[*]

The delicate nature of organic analysis requires long experience and great tact for its successful performance; and we should far exceed the limits imposed on us, were we to enter into every detail. Our object has been to point out general outlines; and we must refer our readers to works on the subject, in which full particulars are entered into, and every minutiæ of the processes which have been recommend are given in their proper order.

We may, however, just add, by way of illustration, the results of an analysis in which we shall suppose that the amount of carbon, hydrogen, and oxygen has been determined.

For instance, a combustible substance has been burned with oxide of copper, after the plans already recommended; and on weighing the chloride of calcium tube and contents, we find that its *increase* is 2·25 grains; and the *increase* in weight of the potass apparatus and contents, is 5·5 grains; how much hydrogen, carbon, and oxygen were present in the substance, supposing that it had weighed 3·75 grains?

1. We have 2·25 grains of water produced therefore. As the equiv. of water = 9 is to hydrogen = 1, so is 2·25 to ·25.

Or, As 9 : 1 : : 2·25 : $\frac{2{\cdot}25}{9}$ = ·25

Hence we have of hydrogen = ·25 grains.
And necessarily the rest is oxygen = 2·00 „
2·25 „

2. Now, we find that the contents of the potass apparatus sustained an increase of 5·5 grains. We therefore say—

As the equiv. of CO_2 = 22 is to C = 6, so is 5·5 to 1·5.

Or, As 22 : 6 : : 5·5 :
6

22) 33·0 (1·5
22

110
110

In other words, the substance analysed contained

Of hydrogen ·25 grains.
„ oxygen 2·00 „
„ carbon 1·50 „

Its original weight 3·75

If we wish to find the equivalents of each element present, we shall easily obtain them by multiplying each of the quantities in the above example by 4; thus—

H = ·25 × 4 = 1 = one equivalent.
O = 2·00 × 4 = 8 = „
C = 1·50 × 4 = 6 = „

Hence the supposititious substance is composed of one equivalent each, of hydrogen, oxygen, and carbon. We must here mention that we have invented a substance expressly for our purpose, so that it might contain only one equivalent of each element. This has been done only for the sake of simplicity; and our readers can easily extend our example to facts, by multiplying the proportion, as given above, by the formulæ for any chosen body, maintaining of course the relative proportions which exist therein.

Thus, if 3·75 grains of ether had been analysed, its formula being $C_4 + H_5 + O$, we should have had

Of hydrogen = ·507 grains.
„ oxygen = ·811 „
„ carbon = 2·432 „

3·750 „

Now, as the weight employed is nearly one-tenth of that of an equivalent of ether, which is 37, we shall nearly obtain the equivalent of each of the substances, by multiplying each of the above weights by ten. This result would be more exact if we used the factor $\frac{37}{3{\cdot}75}$; thus, the equivalent of the

Hydrogen $\frac{{\cdot}507 \times 37}{3{\cdot}75} = 5$

Oxygen $\frac{{\cdot}811 \times 37}{3{\cdot}75} = 8$

Carbon $\frac{2{\cdot}432 \times 37}{3{\cdot}75} = 24$

Or, ether = $C_4 + H_5 + O = 37$

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[*] See *ante*, p. 372.

These illustrations will be sufficient to give our readers a general idea of the calculations involved in organic analysis, with the exception of that of the percentage, which is precisely similar to that explained already.*—We shall now proceed to investigate the properties of numerous organic substances—commencing with starch, sugar, &c.; and subsequently shall examine their chemical products, which are of a more complex character.

CHAPTER VII.

STARCH, SUGAR, ETC., AND THEIR PRODUCTS.

IN commencing the chemical history of vegetable products, we may observe that such must be to a large extent very cursory. The advance which chemical science has made during the last fifteen years in respect to the branch of organic products, has been so extensive, that were we to attempt to enter into it fully, we should require more space than has been devoted to all the subjects yet treated on in this volume. We are, therefore, compelled to select and indicate, rather than to define, all the divisions of this interesting department of experimental science.

The first substance to which we shall direct the attention of our readers is starch in its various forms. Starch is to be found in almost every plant, in some form or other. The most familiar instances which we can adduce of its presence, is that of its being found in wheat, potatoes, and other such plants as are used for common articles of food. If flour be carefully washed with *cold* water, a milky liquid passes off, which contains the starch of the wheat. If the liquid be allowed to settle, the starch will be deposited as a white powder, being separated from the gluten, which is the other component of the flour. If potatoes are broken up and pounded, the starch of which they are chiefly composed may be separated in a similar manner. During the existence of the potato disease, the fact that much starch may be obtained from potatoes, even in a diseased state, was utilised; and large quantities of that substance were thus obtained for the use of calico-printers, by which a considerable amount of wheat, previously used for that purpose, was saved. Starch forms, also, the chief part of arrow-root, sago, tapioca, and most other farinaceous foods.

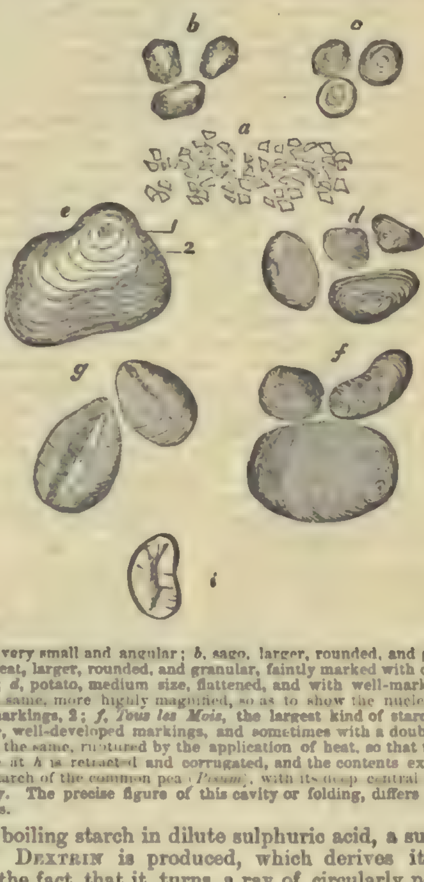
The physical characters of this substance vary with the sources whence it is obtained. To all external appearance, as viewed by the naked eye, it seems merely a granular powder; but if placed under a microscope, the various kinds of starch show cells, differing considerably in their external characteristics. The following engraving and description will assist our readers in this respect; and if they possess a microscope of ordinary power, they may readily make their own examinations of various forms of feculine products.

If starch be mixed with cold water, no notable change takes place; on mixing it with boiling water, however, the cells swell enormously, and eventually burst, as is familiarly known to be the case in cooking potatoes, making paste, and starch, for laundry use. By these means the real starch, which is enclosed in a membrane, is set free by the heat rupturing its covering.

We shall have eventually to notice the office which starch fulfils in reference to the propagation and growth of plants, and the simultaneous production of sugar. We shall now proceed to describe its chemical characters.

Starch is composed of carbon, hydrogen, and oxygen. Its symbol and proportions are $C_{12} H_{10} O_{10}$. It is precipitated by metallic oxides, and other substances in solution. Its proper test, however, is iodine, with which it forms a dark-blue colour, provided a cold solution of the test be employed. The simplest way of detecting its presence, is to add a solution of iodide of potassium to the substance supposed to contain starch; a few drops of sulphuric acid are then to be poured into the mixture; and if starch be present, a dark indigo colour is at once produced. A heated solution affords no such reaction. Starched paper dipped in a solution of iodide of potassium, is a test for the presence of free ozone, which sets the iodine free from the salt, and so produces a blue colour in proportion to its presence in a free state.

Fig. 83.—The more common forms of the Starch Cell.



a, rice, very small and angular; *b*, sago, larger, rounded, and granular; *c*, wheat, larger, rounded, and granular, faintly marked with concentric lines; *d*, potato, medium size, flattened, and with well-marked lines; *e*, the same, more highly magnified, so as to show the nucleus, 1, and the markings, 2; *f*, *Tous les Mois*, the largest kind of starch, of oval shape, well-developed markings, and sometimes with a double hilum, 1; *g*, the same, ruptured by the application of heat, so that the membrane at *h* is retracted and corrugated, and the contents exposed; *i*, the starch of the common pea (*Pisum*), with its deep central folding or cavity. The precise figure of this cavity or folding, differs in various grains.

By boiling starch in dilute sulphuric acid, a substance called DEXTRIN is produced, which derives its name from the fact that it turns a ray of circularly polarised light towards the right hand. LIGNIN is a substance forming a large proportion of cotton, and other fibrous materials; and these, by being boiled with diluted sulphuric acid, may be readily converted, by the change of their lignin, into grape-sugar. Of this singular fact we shall have to speak more fully hereafter.

GUMS proper, soluble in water, are a common product of plants. Soluble in cold water, they afford an adhesive solution, which is used for a variety of purposes. The ordinary gum-arabic may be taken as a type of the class; and from it a substance called *arabin* may be obtained, which is composed of $C_{12} H_{11} O_{11}$; and thus has a similar composition to sugar. The presence of gums in solution, is shown by their precipitation with the neutral or basic acetate of lead, also in solution.

INULIN is a substance analogous to starch, from which, however, it differs, in giving a brown colour when tested by iodine. It is insoluble in cold water, but dissolves in that liquid when heated.

PECTIN is somewhat analogous to starch. It is the jelly of fruits, such as is produced in the culinary process of preserving fruits. The red currant affords it in great abundance. It produces an acid called pectic acid, which combines with bases to form salts.

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* See *ante*, p. 405.

We shall now pass on to consider a very important class of bodies, of which cane-sugar is the type, and from which a vast variety of articles of food and medicine may be obtained. The following, as well as the precedingly-named substances, are, as we shall observe, composed of oxygen, hydrogen, and carbon only.

SUGAR.—This important article of food may be obtained from various sources. That used in this country is the produce of the sugar-cane, so largely grown in tropical countries. The canes, when ripe, are cut down and conveyed to rolling-mills, between the rollers of which they are passed, so as to squeeze out the juice. This, when obtained in quantity, is heated; and slaked lime is added to clarify it. The clear liquid is then quickly evaporated and crystallised, being repeatedly stirred, to prevent the formation of large crystals. It is then drained to remove the molasses. Loaf-sugar is made from the product of the above process, purified by the addition of blood, &c., with which it is heated. The albumen separates all foreign matter; and thus sugar in a pure form may be obtained. In France, the beet-root is largely cultivated for the purpose of producing sugar; and in North America, large quantities are obtained from the maple-tree. The sap of this tree is obtained by "tapping," the juice being afterwards purified in a similar manner to that already described. Sugar, as we shall presently see, is produced from many sources, both of a vegetable and animal nature. Amongst the latter we may mention the sugar of goat, asses, and human milk.

Sugar, however, differs essentially in its chemical characters, according to the source whence it is obtained. From fruits—such as the grape—from honey and urine, during the disease of diabetes, the kind called *grape-sugar* is producible. This sort has not the sweetness of cane-sugar. It differs also in the form of its crystals, and in its less solubility in water.

The following table exhibits the chemical composition of various kinds of sugar:—

Cane-sugar	$C_{12}\ H_{11}\ O_{11}$
Grape "	$C_{12}\ H_{14}\ O_{14}$
Milk "	$C_{12}\ H_{12}\ O_{12}$

Cane-sugar may be converted into grape-sugar by means of dilute sulphuric acid.

A peculiar sugar, termed mannite, may be obtained from some plants, as the ash; it is the ordinary manna of commerce, and is composed of $C_{12}\ H_{14}\ O_{12}$.

Owing to the great demand for milk sugar in the homœopathic system of medicine, an immense quantity of this substance is now produced. Of this, some finely crystallised specimens were exhibited in the International Exhibition of 1862; they were produced in Switzerland, from the milk of goats.

We shall now proceed to speak of the various substances which may be obtained from sugar, by chemical and manufacturing processes.

OXALIC ACID.—If sugar be heated with dilute nitric acid, the well known oxalic acid is produced. The nitric acid parts with its oxygen, and so oxidises the sugar: crystals of the oxalic acid may thus be obtained, having a composition of $C_2\ O_3 + 3\ HO$, or a combination of one equivalent of acid with three of water. In this curious metamorphosis we have an astonishing change occurring between the material employed and the resulting product. The former, an article of food in universal request amongst all nations, may readily be converted into a most virulent poison, by comparatively simple means, affording another instance of the surprising changes which chemistry continually affords us.

Oxalic acid, if heated with sulphuric acid, undergoes decomposition, carbonic acid and carbonic oxide being produced. With bases, it forms the salts called oxalates, of which the binoxalate of potass and the oxalate of ammonia are the most important. The salt of potass exists in the plant called sorrel, and also in the English kind of rhubarb used as food, being the cause of its sour taste. It is sold in the shops as salts of sorrel, and is used for taking out ink-marks from linen, &c., having the power of forming a soluble salt with protoxide of iron. Oxalate of ammonia is chiefly used as a chemical test for lime, with which it forms an oxalate of lime, as an insoluble white powder. Oxalic acid unites also with oxides of metals, such as those of iron, copper, nickel, &c.

Saccharic acid may also be produced by the action of dilute nitric acid on sugar. It is composed of $C_{12}\ H_8\ O_{14} + 2\ HO$. Mucic acid is obtained by the action of nitric acid on gum, and the sugar of milk. Its composition is $C_{12}\ H_8\ O_{14} + 2\,HO$, and is therefore analogous to saccharic acid.

ALCOHOL.—This well-known liquid is the product of the vinous fermentation of grape-sugar. It is sold in commerce as spirits of wine. And to its presence in brandy, gin, rum, whiskey, wines and beer, their peculiar effects of producing intoxication are due. Alcohol is readily produced by adding yeast to a solution of sugar in water, at a temperature varying between 70° and 80° Fahrenheit. Carbonic acid is largely disengaged; and when fermentation is complete, and the liquid has become clear, it must be distilled in a retort, the first portions only being retained. The product is alcohol largely diluted with water. By repeated distillation with subcarbonate of potass, or dry lime, the water may be separated, until nearly absolute or pure alcohol is obtained. In its pure state, alcohol has a composition of $C_4\ H_6\ O_2$; compared with water = 1000, its specific gravity is 0·794. It boils at 173° Fahrenheit; and is converted into a vapour which is readily condensed, and which, like the liquid itself, is highly inflammable. "Proof spirit," such as is referred to in the excise laws, contains half pure alcohol, the remainder being water. Its specific gravity is 0·92 nearly. The uses of alcohol are very numerous in arts and manufactures. From its solvent powers it is used to make varnishes; but for this purpose, methylated spirits are now employed, of which we shall have to speak hereafter.

The most important use of alcohol is that of a beverage; and to this point we shall now direct the attention of our readers.

From our preceding remarks, it will be gathered that the fermentation of saccharine matter leads to the production of alcohol in all cases. This is strictly true; but the commercial products differ according to the source of the sugar. Thus, whiskey is obtained by the fermentation of the sugar of malt, as in beer. Rum is produced when coarse cane-sugar is employed. Brandy and wines, generally, are the produce of the juice of the grape. The differences of these liquids, then, are owing to the admixture of the alcohol they contain, with other substances dissolved and held in solution by it. In corn spirit, or that obtained from potatoes, a coarse offensive oil is found, which gives the peculiar smoky flavour of whiskey. In wines, the peculiar and distinguishing taste is due to the presence of an ether, which, at the same time, differs in each wine, and gives it its characteristic flavour—and so on. Again, the per-centage of pure alcohol varies greatly. In London porter, it does not exceed 3 or 4 per cent.; whilst in Spanish wines, such as sherry and port, as much as 20 to 30 per cent. is found; and brandy often contains as much as 50 per cent. of pure spirit. Although, therefore, the same chemical principles are involved in the manufacture of each kind of spirituous or fermented beverage, the modes of manufacture differ greatly. Beer, the favourite beverage of our own country, is produced from malt. Malt is produced from barley by a process which produces germination, and the simultaneous conversion of the starch of the seed into sugar. This is effected by moistening the seed, which should be kept together in masses; it thus heats, but is prevented from gaining too high a temperature by being repeatedly stirred on the malting-floor. It is then kiln-dried, which stops any further change. During this process a substance called *diastase* is produced, which, acting on the starch, converts it, eventually, into sugar.

The malt is crushed, and added to a proper proportion of water, at a temperature of 170° Fahrenheit. In this it is infused for some hours, and produces a sweet liquid, called wort. This is drawn off quite clear, and, being pumped up into coppers, is boiled with hops, which impart a bitter flavour, improve the beer, and assist in keeping it from any change which might deteriorate its flavour. If a considerable quantity of hops be used, bitter ale is produced, in which the flavour of the hop is predominant. In some beer, such as the Burton strong ale, more saccharine matter is left, and less hops used. Such ales have a highly feeding effect; whilst those of the bitter kind are excellent tonics.

After the boiling with hops is completed, the liquor is rapidly cooled, and yeast added to it, by which fermentation is effected. The difference of beer produced at the various breweries depends much on the management of this part of the process, and all the skill of the brewer is thereby called into exercise. When the fermentation has been carried on to a certain extent, it is stopped by drawing off the beer from the yeast into barrels. By long keeping, ale loses its sweetness, and acquires a strong taste; hence the difference between mild, or new, and old ale.

It has long been considered that the different flavour of ales is due to the water employed. But that such is not the case may be proved by the fact, that whilst each ale brewed by the great brewers in London, differs in its flavour from the rest, all the breweries are supplied by water from the same source, either Artesian wells, or the Thames, or New River water companies. Again, it cannot be owing to the softness of the water, for the presence of a little lime is advantageous. In respect to this, we may notice, that the water used by the Scotch brewers is generally very soft, and yet the class of ales they produce is quite different to that brewed in London, and the porter is vastly inferior.

Porter owes its dark colour to the addition of high-dried malt, which contains a substance termed *caramel;* and this may also be procured by strongly heating sugar, which becomes of a dark-brown colour. The high character which London porter holds as a beverage, is partly due to the care used in, and the extent of, its production. If it be required for export, an entirely different article is brewed, as that intended for London use would soon turn sour. Even if sent to other parts of the kingdom the same point is attended to; hence a Londoner will often find that the "London porter" he may drink in a provincial town, is much stronger than that he partakes of at home. We cannot here stop to notice the numerous adulterations which this beverage undergoes in its transit from the brewery, through the cellars of the publican, to the consumer. Sugar, copperas, tobacco, Spanish liquorice, treacle, *coculus Indicus, cum multis aliis,* &c., all join with water to make every two barrels of *genuine* porter into three of the Londoner's drink; and thus, more "porter" by far is drunk in London annually, than is really made at the breweries which profess to supply it. Stout is merely porter brewed of a greater strength and body.

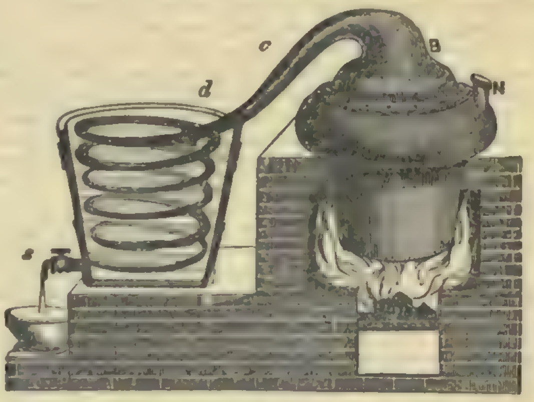
In this country, no wine, properly so called, is manufactured. The British wines owe their spirituous part to the addition of brandy to the fruits after which they are called. Foreign wines are the product of the grape, and vary greatly in their qualities. French and German wines contain from ten to eighteen per cent. of alcohol, whilst sherry and port have so much as twenty-five per cent.

In the south of France, Italy, Spain, Portugal, and many parts of Germany, the vine is extensively cultivated for the purpose of making wine; and the colour of the husks, or skins, of the grape, is the cause of the colour of the wine produced from them. The nature of the soil and the climate have a decided effect on the quality of wine produced; and hence a connoisseur will readily detect, not only the difference in the wines, but also of the vintage, some years being more favourable than others for the production of the grape.

When the grapes are ripe, they are carefully gathered, and the juice is pressed out and received in large vats. No addition of yeast is required for fermentation, as the grape itself contains a fermentable principle in its vegetable albumen, which absorbs oxygen, and so promotes the decomposition of the grape-sugar, and its subsequent conversion into alcohol. This is, however, a delicate and uncertain process, as the nature of the wine is materially influenced by it; and the result depends on the amount of vegetable fermentative matter the grapes may contain. After the wine has been bottled, a slight fermentation progresses in some kinds, and hence the production of carbonic acid, as in champagne and other sparkling wines. The gas escapes from the liquid in uncorking the bottle, just as a similar result takes place in opening a bottle of soda-water. Port, and some other wines, deposit a considerable quantity of tartrate of potass; and this, mixed with the colouring matter, forms the crust, so well known in old port wine, and which lines that side of the bottle which has been laid down. Many attempts have been made to deposit this substance rapidly by chemical means, but without success. During the last few years the production of wine has steadily increased, and the colonies of Great Britain, in Africa and Australia, are producing considerable quantities of this favourite beverage.

The production of spirituous liquors, commonly so called, depends on the processes we have already described; being supplemented by another—that of distillation. Brandy is procured from wine by distilling the spirit from that liquid. The following engraving will give a general idea of the form and uses of a still.

Fig. 84.



A represents the body of the still, containing the liquor to be distilled, which is introduced through an opening N. The still is built into a furnace, to the action of which it should expose as much surface as possible. B is the head of the still, which is connected by means of a pipe *c*, with a metallic worm *d*. This worm is placed in a vessel, which is kept filled with cold water. Heat being applied to the liquid in the still, the alcohol rises in vapour, and passing to the worm, it becomes condensed again into a liquid state, issuing from the cock *s* into any suitable receiver.

Now, by such means, alcohol may be readily distilled from any liquor containing it; and, as we have mentioned, brandy may thus be produced from wine.

Gin and whiskey are produced from malt in the following manner:—The malt is infused in hot water, and then immediately fermented as far as possible, so as to convert all the sugar into alcohol. Generally speaking, the malt is mixed with a considerable proportion of raw grain, and hence the cost of the spirit is lessened, and its quantity increased. The "wort," thus fermented, is transferred to the still, and thus whiskey is produced. For making gin, the whiskey, or raw spirit, has to be rectified;-that is, its fusel oil, which gives the flavour to it, must be removed, and other oils substituted for it. This is effected by a second distillation of the raw spirit, to which salts of potass are added. Gin *should* be a mixture of alcohol and the oil obtained from juniper berries; but, generally speaking, oil of turpentine is largely substituted, and hence arise some highly prejudicial effects on the system of the habitual gin-drinker.

Liqueurs are procured by dissolving various essential oils in alcohol of various strengths. Perfumes are manufactured in a similar manner; and occasionally both are subjected to another distillation, so as to ensure complete mixture.

We must now pass on to consider the effect produced on sugar when the fermentation is pushed on to what has been termed the acetous state, or that in which the spirit becomes converted into an acid.

If the wort of the brewer, or wash of the distiller be left, after fermentation, to the free action of the atmosphere, an acid, called the acetous, is eventually produced. Indeed, this is the method often adopted for the production of the common vinegar of commerce, and results from the atmospheric oxidation of the alcohol. On similar principles, but by different means, alcohol diluted with a little water may be converted into acetic acid. This is effected by dropping it on some finely divided platina,* when the smell of acid produced will be abundantly evident. Another arrangement, in which the oxidation and slow combustion of alcohol is produced, is described at the page to which we have just referred. Acetal ($C_{12} H_{14} O_4$) is a colourless liquid, and may be produced by the slow oxidation of alcohol through the action of finely divided platina.

Acetic acid combines with several bases forming salts, called acetates. The acetates of lead have already been referred to at a previous page;† the acetate of iron is employed as a mordaunt in dyeing, and for this purpose is largely manufactured in the North of England.

Acetate of alumina is similarly employed.

Of late years, acetic acid has been extensively used in photography, as a developing agent. That of the strongest kind is generally preferred for the purpose, and is sold under the name of "Glacial Acetic Acid." It is prepared from the acetate of soda, by distilling that salt with sulphuric acid; and from the liquid distilled over, crystals may be readily obtained at a temperature less than 60° Fahrenheit. A considerable quantity of vinegar, containing a portion of acetic acid, with many impurities, has lately been manufactured by the agency of the "vinegar plant." This is a species of fungus, which produces the acetous fermentation in solutions of saccharine matter. The chief source, however, of the vinegar of commerce is ale, which, from having been kept too long, has become "hard" and sour.

Various combinations analogous to acetic acid are producible from alcohol, but these are of special interest to the professional chemist only.

ETHERS.

An interesting group of substances, which has of late years received much attention, is found in the ethers which are producible from alcohol by distillation with acids. And we must here call attention to a singular phenomenon, in the fact that a substance called *Ethyl* ($C_4 H_5$), symbol Ae, can assume all the properties of a base, just as a metal might do with oxygen and acid. In fact, this substance, ethyl, may replace such bases; form haloid salts, with chlorine, iodine, &c.; analogously to sodium or other metals, it may be oxidised; and thus it opens out a very curious range of organic compounds to our investigation.

Adopting the "Ethyl theory," as it is termed, we shall view ethyl as the base of a system of compounds; and from its various combinations the following substances are obtained, which, with the symbols, we have arranged in a tabular form:—

Base—Ethyl	$C_4 H_5$
Oxide of base—Ether . .	$C_4 H_5 O$
Hydrate of Oxide . . .	$C_4 H_5 O + HO$

Confining our attention, for the present, to the above compounds, we may state that ethyl has been isolated from its iodide ($C_4 H_5 I$) by the action of zinc and heat. It is, when condensed, a colourless fluid. Its oxide ($C_4 H_5 O$) is the ordinary "sulphuric ether" of the shops, which is obtained by distilling rectified spirits of wine with sulphuric acid. The vapour should be received in a vessel surrounded with pounded ice. Its specific gravity is less than alcohol, being 0·720 (water = 1·000). It is a very volatile fluid; boils in the air at 96°, and under freezing-point in the vacuum of an air-pump; is highly combustible, and burns with a bright flame. This ether is employed in medicine as a stimulant. If its vapour be breathed, which may be done by pouring a little of the liquid into a bladder fitted with a mouthpiece, it produces very similar effects to those of the "laughing gas." And it was for some time used, before the discovery of chloroform, in surgical operations, to destroy the sense of pain.

On passing the vapour of ether through a red-hot tube it undergoes decomposition, and "aldehyde" is produced. This substance is a hydrated oxide of "acetyl" ($C_4 H_3$), and hence its composition is $C_4 H_3 O + HO$. A connection with the ethyl series, and that to which acetic acid belongs, is thus established. And the products of the decomposition of alcohol and ether are respectively homologated.

By using hydrochloric instead of sulphuric acid, in distillation with spirits of wine, as just described, the chloride of ethyl is produced ($C_4 H_5 Cl$), commonly known as chloric ether. Nitric ether ($C_4 H_5 NO_6$) is similarly produced, by the action of nitric acid on rectified spirits of wine.

We have confined our attention, chiefly, to those forms of the combinations of ethyl which are more generally known. A large number of others, however, has been discovered; and as they have no special interest we shall only give the symbols showing their constitution.

Ethyl Compounds, in addition to those already named.

Bromide of	Ethyl . .	$\overline{C_4 H_5} + Br$
Sulphide „	„ . .	$\overline{C_4 H_5} + S$
Cyanide „	„ . .	$\overline{C_4 H_5} + Cy$
Sulphate of Oxide of	„ .	$O + \overline{C_4 H_5} + SO_3$
Nitrate „	„	$O + \overline{C_4 H_5} + NO_5$
Carbonate of	„	$O + \overline{C_4 H_5} + CO_2$
Oxalate „	„	$O + \overline{C_4 H_5} + C_2 O_3$
Acetate „	„	$O + \overline{C_4 H_5} + C_4 H_3 O_3$

An inspection of the above table will at once convince the student how completely ethyl replaces a metallic base, in its haloids and salts. And this fact has formed the basis of an entirely new doctrine in chemical science, namely, that of substitution. We shall have again to refer to this point as we proceed.

With other substances than those we have referred to, ethyl forms interesting compounds. Wine, for instance, owes its *bouquet*, or rich flavour, to the presence of what is termed "*Œnanthic*" ether. Butyric, valerianic, magaric, and stearic ether are compounds of ethyl, with the acids bearing their names.

The other derivatives of alcohol possess, with one exception, less interest than those to which we have directed attention. And we shall, therefore, conclude with some account of chloroform, to which great attention has been directed, owing to its employment in connection with surgical operations.

Chloral is produced by passing dry chlorine through alcohol, free from water. If, to the product, soda or potass be added in solution, a decomposition ensues, and chloroform is produced. But the plan usually adopted is, to distil alcohol with chloride of lime. The liquid thus afforded is chloroform; it has a pleasant smell, and, if inhaled, has the peculiar property of so far influencing the nervous system as to produce almost total insensibility to pain. Its effects depend, however, very materially, on the temperament of the person inhaling it. In some instances, we have noticed great cerebral excitement, leading to violent acts. In a case which came under our notice, some years ago, a lad, 12 years of age, part of whose finger had to be amputated, was placed under the influence of the agent. To all appearance he was perfectly insensible to pain; he was yet able to describe each step of the operation, which he did as each was finished, with an amazing exactness. On being restored (if we may so say) to consciousness, he was perfectly unaware that the top joint had been removed, and would not believe the fact until he saw his finger, on the next day, when it was uncovered for the purpose of dressing it. In several cases, the administration of chloroform has been attended with fatal consequences, in which, however, its action could scarcely be properly assumed as the sole cause of death.

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* See ante, p. 372.

† See ante, p 379.

It may be as well for us to give a general glance over the subject to which attention has been drawn in this chapter, before proceeding further, inasmuch as it will, to a large extent, form a basis on which we shall hereafter have to build. Starch, sugar, and their products, were the subjects on which we were to treat. We have found that, from these vegetable products, a vast variety of substances, having no physical similarity to them, can be obtained. Alcohol, ethers, acids, each having their own peculiarity of constitution, are readily producible from the well-known articles of our daily food. Even a cursory glance over the different substances we have mentioned, or referred to, will be sufficient to lead every thoughtful mind to reflect on the astonishing multiplicity of compounds which may arise from the combination of three elements, in variable proportions. We have, hitherto, noticed that, from sugar alone, deadly poisons, favourite beverages of all classes, medicines, condiments, and interesting philosophical results, are easily obtained. Wonderful as may be such discoveries, they are but an index to the innumerable instances of a similar kind which organic chemistry presents to our view. Perhaps, in no branch of scientific inquiry can we find an instance in which so few agents are, by the fiat of the Creator, employed in so great a variety of ways and purposes. This fact has rendered the study of this portion of our subject a matter of the deepest interest to most living chemists. But, to the sagacity and researches of Liebig every civilised nation is indebted, in a commercial and social point of view; his labours, chiefly, have opened out a valuable and extensive field of inquiry, and have made popular a subject of great difficulty.

CHAPTER VIII.

METHYL, AMYL, ETC., AND THEIR PRODUCTS.

In the last chapter, we remarked, that the substance called ethyl, had the peculiar property of acting as a base, and of forming certain definite compounds with oxygen, chlorine, &c., and acids. We now shall show that other substances exist, having analogous properties, equally capable of acting as bases, and of producing compounds having very similar constitution to those produced by ethyl.

The base of the first of the series is called Methyl—symbol, Me—and it is composed of $C_2\ H_3$. It is contained in ordinary wood-spirit, which is a hydrated oxide of methyl. The latter is obtained by the distillation of wood. The acid liquor produced is neutralised by lime, and the spirit is produced therefrom. The product is a limpid liquid, having an unpleasant smell; it readily burns on being inflamed; and is largely employed for a variety of purposes in the arts; by cabinet-makers, as a solvent for varnishes; and to burn in lamps. Its specific gravity varies with its purity, but should be about 0·795 (water = 1·000).

Oxide of methyl is the analogue of ether, and may be produced by distilling wood-spirit, just described, with sulphuric acid. The chloride, iodide, bromide, sulphide, and cyanide of methyl, are similar in constitution to the compounds of the different substances with ethyl, and have the same formula, excepting that that of methyl is substituted. Similar observations apply to the combination of sulphuric and other acids with methyl.

During the slow combustion of wood-spirit a peculiar acid is produced, which has also been obtained from the well-known insect, the ant. It has, therefore, been called formic acid. Combined with one equivalent of water, its constitution is $C_2\ HO_3\ HO$. Formic acid readily unites with bases forming the salts called formates. It has a pungent smell, and readily blisters the skin if applied to it.

AMYL has a composition analogous to the substance we have already described; for, like ethyl and methyl, it contains carbon and hydrogen only. Its general symbol is Ayl, and it consists of $C_{10}\ H_{11}$. Common fused oil is a hydrate of its oxide. This liquid was long treated as a waste product in distilleries. It is readily produced in the distillation of spirit from potatoes; has a strong smell, and is combustible.

Amyl forms a similar series of compounds to ethyl; hence we have its oxide, or amyl-ether, the chloride, bromide, iodide, &c.

Benzoyl—symbol Bz—is, to some extent, analogous in its basic properties to ethyl, &c. Bitter-almond oil, which is a combination of benzoyl ($C_{14}\ H_5\ O_2$) with hydrogen, is a substance well known as an article in general use for the purpose of perfumery, &c.; and is a product of the distillation of water in which bitter-almonds have been steeped. This substance, which is a hydrate of benzoyl, becomes converted into benzoic acid by oxidation.

Benzoic acid is chiefly obtained, for ordinary use, by the sublimation of gum-benzoin. When the latter substance is heated, the acid rises in vapour, and condenses in fine crystals. During its volatilisation, the acid has a very powerful smell; it is hence used as an ingredient in pastiles, incense, &c. It readily forms salts with bases, called benzoates.

Benzoyl forms numerous compounds besides those we have mentioned, but they are not generally of interest. The constitution of some of them is given below, in tabular order, with which we shall content ourselves, without entering into any further description of them:—

Chloride of benzoyl . . .	Bz Cl
Iodide ,, ,, . . .	Bz I
Sulphide ,, ,, . . .	Bz S

A peculiar acid, having some connection with the benzoyl series, may here be named. It is found in the urine of domestic animals, and is termed hippuric acid. It is scarcely soluble in water; is decomposed into benzoic acid, &c., by heat, and forms salts with alkalies. This acid contains a portion of nitrogen as a constituent.

We shall include with the series two other bases, analogous to those we have described—namely, kakodyl and salicyl; although the latter appears to have some analogy to the vegetable alkalies, which we shall have to speak of presently.

Kakodyl—symbol Kd—is a peculiarly offensive, and fearfully poisonous compound, composed of $C_4\ H_6\ As$, thus including an equivalent of a metal, arsenic, in its composition. Its oxide is obtained by distillation, from a mixture of acetate of potass and arsenious acid; the product is to be condensed in a cold vessel, when a dense liquid will be obtained, which is the impure oxide. To obtain it in a pure state it must be re-distilled with potass, in a retort, filled with hydrogen. This operation is of the most dangerous character, from the poisonous nature of the substance produced.

Like ethyl, &c., kakodyl produces a chloride, iodide, &c. It is also susceptible of oxidation, producing the kakodylic acid, having a constitution of $C_4\ H_6\ O_3$.

The whole of the combinations of kakodyl are curious, from the intensely powerful effects they have on the animal system. And we should strongly urge on our readers the prudence of being content with their brief description, rather than to attempt their production.

Salicyl, to which we now direct attention, is obtained from a product of vegetable life, salicin, and derives its name from the willow tree, in the bark of which it may be found. The crystals of salicin have a beautifully silken appearance. It is prepared by infusing the bark of the willow, poplar, &c., in hot water, and heating with the protoxide of lead. Hydrosulphuric acid* is then to be passed through the solution, to remove the lead as sulphide. Salicyl is the base of this substance.

There are other compounds analogous to ethyl, &c., which we shall omit to notice altogether, or refer to them in a more convenient place.

In the perusal of this chapter, our readers will have perceived that we have entered into a novel branch of organic chemistry, in which we find fresh analogies, and a very interesting series of compounds. As we may have again to refer to the substances we have described, we shall, for the present, end our remarks by presenting the symbols of the constitution of each, in a tabular form, so as to be convenient for future reference; and shall then proceed to give a description of the alkaloids and vegetable acids.

Table showing the constitution of Ethyl, Methyl, Amyl, Benzoyl, and some of their Compounds.

Name.			Symbol.	Composition.
	ETHYL	....	Ae	$C_4 H_5$
Ether	,,	Oxide of	Ae O	$C_4 H_5 O$
Alcohol	,,	Hydrate of Oxide of	Ae O + HO	$C_4 H_5 O + HO$
	METHYL	....	Me	$C_2 H_3$
Wood-ether	,,	Oxide of....................	Me O	$C_2 H_3 O$
Wood-spirit	,,	Hydrate of Oxide of...	Me O + HO	$C_2 H_3 O + HO$
	AMYL	....	Ayl	$C_{10} H_{10}$
Ether	,,	Oxide of	Ayl O	$C_{10} H_{11} O$
Fusel oil	,,	Hydrate of Oxide of	Ayl O + HO	$C_{10} H_{11} O + HO$
	BENZOYL	....	Bz	$C_{14} H_5 O_2$
Bitter almond oil	,,	Hydride of	Bz H	$C_{14} H_5 O_2 H$
Benzoic acid	,,	Hydrate of Oxide of	Bz O + HO	$C_{14} H_5 O_2 O + HO$

CHAPTER IX.

VEGETABLE ALKALIES OR ALKALOIDS, AND ACIDS.

In our last chapter we included a description of salicyl, because its analogies are in accordance with those of ethyl, &c. Salicin, which contains it, would appear to belong to the alkaloids. We have accordingly chosen it as the last of the basic series, before commencing an account of the vegeto-alkalies.

These substances differ materially from all other organic matters with which we have yet been engaged, in the fact of their containing nitrogen as an element of their composition. They are, generally speaking, principles existing in minute quantities in the plants from which they are obtained. They are but slightly soluble in water, their proper menstrum of solution being alcohol. With acids they act as bases, and in their specific character resemble the alkalies; hence they have received the title of alkaloids. Some of them are extremely poisonous, whilst others exert a highly beneficial influence on the human system. After giving a general description of the alkaloids, we shall proceed to describe some of the leading vegetable acids.

STRYCHNINE.—This alkaloid is obtained from the *nux vomica*, which also contains another substance called BRUCINE: in fact, these two alkaloids are often mistaken for each other, in unskilful hands. Strychnine, in the pure state, has a crystalline form, an intensely bitter taste, is barely soluble in water, but dissolves easily in boiling spirits of wine. Brucine may be separated from strychnine by its solubility in *cold* alcohol. It is also more soluble by far, in water, than strychnine.

A sulphate of strychnine is readily produced, as is the acetate, by the action of the corresponding acids on the alkaloid. Brucine forms analogous salts with the same acids.

Of late years, strychnine, or, as it is often called, strychnia, has obtained a painful notoriety in connection with cases of poisoning; more especially in that of the celebrated Palmer's case. It is, perhaps, the most dreadful poison which can be administered to a human being. In quantities of a fraction of a grain to one grain, according to the constitution of the victim, its effects are fatal. Rigid contortions of the muscles of the body are its symptoms of poisoning. The sufferer retains his mental powers undiminished, although he is racked with pain; and, at last, he is put out of misery by convulsions, which are truly horrible to behold.

The tests for the presence of strychnine are twofold—chemical and physiological; the latter having been first introduced to notice by the late lamented Dr. Marshall Hall.

If a grain of strychnine be placed on a plate with a little powdered bichromate of potass near it, and a drop of concentrated sulphuric acid be let fall between them, a rich purple colour is instantly produced when all three are stirred together, but which speedily changes to a red tint, which is permanent. The same colour is produced by using the red ferri-cyanide of potass in place of the bi-chromate.

The physiological test is more satisfactory, and may be illustrated as follows:—Dissolve $\frac{1}{4}$th of a grain of the acetate of strychnine in a little boiling alcohol, and add thereto two ounces of cold water. Place in the solution two small frogs. For some time no effect will be apparent. This is best conducted in a tall glass jar, the top of which should be covered, for reasons which will be presently seen.

After a time the poison will be absorbed into the system of the animals. If the glass which contains them be then shaken, the poor creatures will instantly suffer from convulsions—stretching out each limb, which will become rigid; they will utter piercing shrieks, and show all the usual effects of strychnine poisoning. After a time the eyeballs will swell, and death ends their sufferings.

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* See *ante*, p. 376.

During the trial of Palmer, we were requested to undertake a series of experiments, in order to discover whether strychnine suffered decomposition simultaneously with that of an animal body. For the purpose of ascertaining this, we tried the following :—

Six young frogs were poisoned by means of a grain of strychnine dissolved in the manner already indicated. When quite dead, the bodies were carefully washed, to remove all traces of strychnine not absorbed into the animal textures. They were then cut to pieces, and exposed for a fortnight (in summer-time), till the mass was completely putrid. Copious washing with distilled water was then employed, and the mass boiled for some hours in alcohol. The liquid was filtered, and evaporated to dryness, sulphuric acid being then added to form the sulphate. The product was then dissolved, and administered to two frogs, who speedily passed into convulsions and died. Now these experiments were conducted with the full expectation and hope that the decomposition of the strychnine would have resulted, but it was shown that such did not take place. Every possible care was taken to insure accuracy of result, and therefore the experiments may be depended upon.

There seems to be no remedy for strychnine as a poison. We may, however, mention, that on attempting to put some of the numerous frogs we experimented on, during the time named, out of their misery, we gave them a very large dose of cyanide of potassium, in solution. This, instead of killing them, prolonged their lives. Other substances were also tried, which we cannot now name from memory, but as deadly poisons in general. This would seem to indicate, that some substances having, in the normal state of the animal, a fatal effect, are positively antidotal when it suffers from poisoning by strychnine. On this point, however, we can offer no decided opinion, as our experiments were only carried on in reference to the hopes of some, that their result might induce a reprieve of the criminal under sentence of death. The results we have named, however, deserve further and careful investigation. During the above course of experiments, we may further mention, that exceedingly small doses of strychnine seemed to produce a tonic effect. This was noticed through occasionally tasting the ends of the fingers, to ascertain that all accidentally adhering strychnine had been washed away, and so to prevent its introduction into any opening of the skin—a circumstance which sometimes occurs in investigations of bodies containing poisons, &c. The formula for strychnine has been stated as $C_{42}\ H_{22}\ N_2\ O_4$. And of Brucine, $C_{46}\ H_{26}\ N_2\ O_8$.

Quinine and **Cinchonine**.—These are obtained from the well-known Peruvian bark, which, from their presence, has been used so much in medicine, as tonic. Quinine, however, as a disulphate, has been generally substituted. Both these substances assume a crystallised form, and neither are readily soluble in water ; but in this respect quinine has the advantage of cinchonine. They are both very bitter. The sulphate of quinine is generally sold in minute crystals, and, for medicinal purposes, is dissolved in dilute sulphuric acid. An acid called Kinic acid may be obtained from the Peruvian bark, which unites with bases, and also affords, with other substances, a curious range of compounds.

Morphine.—This alkaloid is obtained from opium, by a somewhat tedious process. It is soluble in boiling alcohol, and but sparingly so in water. With sulphuric, nitric, hydrochloric, and phosphoric acids, it forms salts. The sulphate and acetate are used as medicines.

Narcotine and **Codeine** are also obtainable from opium ; and, in some respects, they resemble morphine. The sedative effects of opium are due to the presence of the alkaloids. And morphine salts are now chiefly employed in place of the vegetable substance, which always contains a host of impurities. A peculiar acid, the meconic, which forms salts with bases, is also found in the opium of commerce.

Nicotine, is found in tobacco, and is a deadly poison. Like **Conine**, which is produced from hemlock, it forms salts with acids, which may be obtained in a crystallised state. Similar remarks apply to **Hyoscyamine**, from the *hyoscyamus;* **Atropine**, from the *atropa belladonna;* and **Aconitine**, which is the active principle of the *aconitum napellus.* There are many other vegetable substances which afford similar principles, all of which possess a bitter taste, and are generally poisonous when taken internally.

Theine.—This principle, which is identical with that called **Caffeine**—the names varying according to the source of the substances—is found respectively in the tea and coffee so much used as beverages ; and it is to the presence of these alkaloids that is owing the refreshing and exhilarating effect of those drinks.—**Theobromine** is similarly contained in the cacao-nut. Caffeine, or Theine, is soluble in cold water, but much more so in the heated liquid ; hence the use of boiling-water in making the infusion of either tea or coffee. Salts of this substance have been obtained, and also a double one with the chloride of platina.

It will be noticed, generally, that each of the alkaloids is but slightly soluble in water, however they may be dissolved in boiling alcohol. They form salts with many of the acids, and are distinguished from other organic substances, having a vegetable origin, by containing nitrogen. They are mostly deadly poisons, and exist in minute quantities in the vegetable from which they are obtained.

The general method of obtaining them is to steep the plant or bark in hot water, or to make a strong decoction. Sulphuric or acetic acid is added, to form a salt; and, in some cases, these are decomposed by hydrate of lime. The alkaloid is thus set free, and may be dissolved out by boiling alcohol, from which it is afterwards obtained by evaporation and crystallisation. Generally speaking, the chemical constitution of their substances has not been satisfactorily obtained; and we shall not attempt to guide our readers by giving hypothetical formulæ, in which entire dependence cannot be placed, more especially when no particular necessity exists, or advantage can arise from such a course.

VEGETABLE ACIDS.

Having described the chemical character of the leading alkaloids, we shall now add some remarks on those acids which are obtained from some plants, omitting such as the oxalic, &c., to which allusion has already been made at page 412, in connection with sugar, &c.

Generally speaking, the vegetable acids are freely soluble in both hot and cold water ; gallic acid, however, forms an exception in the latter case. They have all a sour taste ; generally affect blue litmus paper ; form soluble salts with alkalies and metallic oxides ; and are decomposed into other acids by the aid of heat.

Tartaric Acid may be obtained from the ordinary cream of tartar. An impure tartrate of potass is found in wines,* which is the chief source of the acid. From this salt an insoluble tartrate of lime is produced, which is subsequently decomposed by sulphuric acid. From this the tartaric acid may be crystallised. The chief salts of this acid are those of potass and soda. It, however, forms salts with the earths. With antimony and potass it forms the tartar-emetic used in medicine. By being heated, tartaric acid becomes changed into a variety of other substances having an acid character: and an anhydrous acid may be obtained, freed from two equivalents of water, which the commercial product always possesses. Racemic acid has a definite relation to tartaric acid, and is obtained also from some kinds of wine. It is frequently termed Paratartaric acid. The formula for anhydrous tartaric acid is $C_8\ H_4\ O_{10}$: and that of the commercial substance is the same, with the addition of two equivalents of water, or $C_8\ H_4\ O_{10} + 2HO$.

Malic Acid—$C_8\ H_4\ O_8 + 2HO$—is found in apples and other fruit, and is also united with oxalic acid in the common rhubarb stalks. It is also present in the berries of the mountain-ash. This acid unites with bases forming salts called malates, of which the malate of lead is of the most interest, as being the salt from which the acid is obtained.

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* *See ante*, p. 413.

Citric Acid is found in lemons, in great abundance, from the juice of which it may be obtained by adding chalk, which affords a citrate of lime. Dilute sulphuric acid is to be added to the precipitate, which will then be decomposed. The citric acid may be crystallised from the remaining liquid. In the anhydrous state, the formula for citric acid is $C_{12}\ H_5\ O_{11}$, but it is in a crystallised state united with five equivalents of water, as produced above. Its proper constitution, however, may be represented by the addition of three equivalents (3HO) to the above formula. Citric acid unites with bases, forming citrates, of which the alkalies are soluble, whilst those of the earths are insoluble in water.

Gallic Acid.—The ordinary oak-bark, gall-nuts, catechu, sumac, &c., contain two acids, the gallic and tannic, which have considerable commercial importance, inasmuch as the tanning of leather, the manufacture of ink, and photographic processes, depend on them.

Gallic acid may be procured by allowing an infusion of powdered nut-galls to be exposed to the air, until the whole solution, &c., has become completely mouldy: from this, gallic acid may be dissolved out and crystallised. It is now prepared in large quantities as a photographic agent, being much used in developing pictures by various processes.

When gallic acid is heated, another acid, the pyrogallic, is produced, which is also largely used in photography. The formula for gallic acid is $C_7\ HO_3 + 2HO$; whilst that of the pyrogallic is $C_6\ H_3\ O_3$.

Tannic Acid—$C_{18}\ H_5\ O_9 + 3HO$—is the active agent in oak-bark, nut-galls, sumac, &c., which renders them so valuable in the manufacture of leather and ink. It has the property of precipitating gelatine, &c., as found in skins, as an insoluble substance. In the process of tanning, the gelatine of the skins is thus transformed, and they are thereby rendered not liable to further change. With the oxide of iron a black colour is produced; and hence the usual method of making ink, and of dyeing textile fabrics of a black colour, is that of employing solutions of iron, and infusions of substances containing tannic acid. The acid is generally obtained by means of the action of ether on nut-galls. The liquid is added to the powdered nut, and the solution of acid is allowed to evaporate spontaneously, when it will afford crystals of the acid.

With the exception of the tannates of iron, as in ink, and dye products, the tannates and gallates are of no importance.

We have thus presented descriptions of the chief vegetable acids to which the attention of the student need be directed. Some of them, as we have seen, are of considerable commercial importance. Their salts are all decomposible by the mineral acids, by means of which the organic acids themselves may generally be set free, and obtained in an isolated form. In almost every case they may be had in crystals.

CHAPTER X.

MISCELLANEOUS VEGETABLE SUBSTANCES.

OILS, RESINS, COAL AND ITS PRODUCTS, PARAFFINE, ROCK-OIL, PETROLEUM, VEGETABLE COLOURING MATTER, ANILINE, ETC.

We shall now briefly describe a variety of vegetable products, of considerable use for numerous purposes, and obtained from various sources, either during the growth or from the decay of plants.

Oils.—Vegetable oils much resemble those obtained from animals in many respects. They give a greasy stain; if of the "fixed" kind, are inflammable, of light specific gravity, and consist chiefly of carbon and hydrogen; hence their use for illuminating purposes.

The *fixed* oils—such as that of the olive or almond, &c.—are distinguished from the *volatile* by many qualities. They cannot be distilled except at a high temperature, and then they are generally decomposed. The volatile, such as oil of lavender, or otto of roses, readily evaporate, even at ordinary temperatures. If a drop of otto of roses and one of olive oil be placed on blotting-paper, the essential oil will speedily evaporate by a gentle heat, whilst the fixed oil stains the paper with an unctuous mark. It is customary to adulterate the otto with spermaceti; and the above plan affords a ready means of detecting the imposition; for, if any of the spermaceti be present in a specimen of the otto, the volatile oil will pass off, whilst the fatty matter remains on the paper.

Generally speaking, the oils derive their names from the plant producing them; and the following gives a description of the best known substances of each class.

Fixed Oils.—These oils consist chiefly of two distinct substances: one a fluid, to which the name *oleine* is applied; and the other a solid matter, termed *stearine* and *margarine*, when obtained from animal substances, or *palmitine* or *cocine*, when produced from vegetable, as the palm or cocoa-nut. Vegetable oils are generally obtained from the fruit or seeds of the plant. Thus, olive oil is expressed from the fruit; whilst rape, linseed, castor, &c., afford oil from the seed. Their chemical composition varies considerably. Generally speaking, they contain about equal proportions of carbon and hydrogen, a little oxygen, and an equivalent of water. Some are nearly solid, as palm oil, at ordinary temperatures; but even the most fluid become semi-solid on exposure to a temperature of 32°, as is familiarly known in the case of salad oil, the most fluid, which "freezes" in winter-time. Some oils, by the absorption of oxygen, become what are termed "drying oils," such as linseed, which, if boiled with litharge, affords the drying oil used by house-painters. All fixed oils are insoluble in water; most form soap with that fluid, or emulsions, by the agencies of alkalies; and some are soluble in alcohol and ether.

Vegetable wax is obtained from some plants belonging to the family *Myricaceæ*, &c., and large quantities of this substance are now imported from Japan, South America, and the Cape.

The following table gives the specific gravity of some vegetable oils, water = 1,000:—

Rape oil	913
Colza	914
Olive	916
Almond (sweet)	917
Poppy	922
Sunflower	926
Cocoa	926
Linseed	930 to 960
Castor	965

Although the specific gravity of the fixed oils is the method of distinguishing them, the action of sulphuric acid in a concentrated state has been proposed. One drop of the acid is added to a few of the oil to be examined, and the reaction may be carefully noticed. Each oil presents a distinct effect, at once characteristic of its nature. The subject, however, is one into which we cannot here enter in detail.*

Olive oil is the produce of the *Olea Europæa*, and is obtained from the fruit by pressure and fermentation.

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* See Article, *Artificial Illumination*, Sect. II.

Almond oil is afforded by the almond tree, the nuts of which give the oil, on the kernels being crushed or pressed.

Linseed oil is afforded by the seeds of the flax plant; and the seeds of the Indian hemp also contain abundance of oil. The cake left after the extraction of oil from linseed, is largely used as food for fattening cattle.

Rape and *Colza oil* are the produce of seeds of the *Brassica* family. The latter oil has been much employed of late for lamps. That of the rape-seed is much used for the same purpose, but both require refining from various substances with which they are mixed in the raw state.

Nut, Poppy, Cotton, Mustard, Sunflower, &c., oils, are obtained from the seeds of the plants bearing their names, and have different uses, depending on their specific qualities. They are employed in the manufacture of soap, for illuminating and lubricating purposes; and, generally speaking, the solid refuse left after extracting the oil, may be employed as food for cattle. There are numerous other vegetable fixed oils, to which we need not allude separately.

VOLATILE OILS.—These oils are the produce, in most cases, of the flowers of plants, which owe their fragrance to them. In some cases, however—as oil of turpentine—the oil is obtained from the wood of a tree. Volatile oils are generally soluble in alcohol, and hence are largely used as perfumes.

Perfumes have for ages been highly esteemed; and in the earliest records of biblical and general history, we read of various productions of that class. This will be a matter of no surprise, when we remark that the climate of Greece, Turkey, and Egypt, countries in which the early history of the world is involved, is such as to be most favourable for the production of flowers affording the best perfumes. At the present day, we are indebted to those countries for many of our best essential oils; and the otto or attar of roses is indigenous to Damascus and its neighbourhood.

In England many places are devoted to the cultivation of flowers intended for the use of the perfumer. Mitcham and its neighbourhood, and other places in Surrey, are especially noted for the cultivation of flowers for perfumery and medical uses. In Germany and on the banks of the Rhine, as at Vierlander, the cultivation of flowers is an object of importance to the farmer and landholder.

There are many ways of obtaining the perfumes contained in flowers. Some yield their essential oil readily; others part with it more slowly. The following methods may be tried, and they serve as illustrations of the various modes adopted according to circumstances:—

1st.—Dip some fine cotton lamp-wick in the purest olive oil, and lay the petals of the flowers on these threads. Place these over each other in layers, and after some time loosen the threads, and soak them in a glass of warm water. The oil will gradually ooze out, and, with the water, may then be transferred to a still. As distillation proceeds, the more volatile oil will separate from the fixed one, and, passing over, may be condensed in the worm, together with a portion of the vapour of water. As the oil and water condense, the two will separate, and the essential oil will float on the water surface, from which it may easily be removed. For this purpose it is better to receive the condensed liquids in a glass vessel, because in that they will be more readily distinguished.

2nd.—The flowers may be rubbed together with dry chalk or loaf sugar in a mortar, and water is then to be added. The mixture, so obtained, is to be transferred to the still, and the usual process of distillation being carried out, the oil is readily separated. The essential oil of lemons is obtained very easily by rubbing the outside of the peel with loaf sugar, which, on being dissolved in water in the body of the still, soon yields up the oil by distillation.

3rd.—The leaves of the flowers may be soaked for some time in a strong solution of common salt in water. This extracts the oil, and the liquid can then be distilled.

4th.—But the readiest way in most cases is to steep the flowers for some time in strong spirits of wine. The spirituous solution so obtained is then to be transferred to the body of the still, and the vapour being condensed in the worm, will be found to contain the essential oil in union with the spirit.

We have already pointed out other properties of essential oils. They consist chiefly of carbon, hydrogen, and oxygen; but in some nitrogen is found; and in the oil of the onion, &c., sulphur is also present. It will be unnecessary for us to describe such oils as those of roses, lavender, neroli, lemons, bergamot, &c., as their general qualities are so well known. We shall therefore confine our remarks to the following:—

Turpentine and *Camphor.*—These substances are both the produce of the wood of trees. Oil of turpentine is obtained from the pine, larch, and fir. The tree is "tapped;" that is, a hole is cut into it a little above the ground, and the resin exudes, and is collected in suitable vessels. The oil is obtained from the crude product by distillation. Its specific gravity is 870—water = 1000, and it is highly inflammable. Common turpentine is an impure form of the liquid.

Camphor is obtained from a plant, the *Laurus Camphora*, by distillation of the wood. This well-known substance is highly inflammable, can be sublimed by a moderate heat, is but slightly soluble in water, but readily in alcohol, and has numerous medicinal and other uses. A peculiar acid, the camphoric, is obtained by the action of nitric acid and heat on camphor. A substance resembling camphor may be produced by passing hydrochloric acid gas through oil of turpentine.

Cedar and sandal-wood, and valerian root, each afford an essential oil from the solid portion of their substance, having distinctive chemical and odoriferous properties. The preceding oils have carbon, hydrogen, and oxygen solely as their constituents. The oils obtained from mustard seed, onion, garlic, eschalots, horse-radish, and assafœtida, contain also sulphur and nitrogen. The presence of sulphur in mustard is readily noticed by its well-known action on the silver of mustard-spoons. These articles of domestic use should, therefore, be gilt, so as to prevent the production of the black sulphide of silver, which so frequently covers the unprotected silver surface. Mustard also affords a fixed oil, to which allusion has already been made.

RESINS.—Under this head some very interesting products range. They are generally obtained from the tree affording them, by the process of "tapping," and the juice which exudes soon acquires solidity. Ordinary resin is thus a product of the pine, fir, larch, &c.

Caoutchouc, or India-rubber, is one of the most valuable of these products. It is obtained by making a hole in the stem of the *Ficus Elastica*. A milky juice passes out, which gradually becomes solid. The natural colour of caoutchouc is nearly white; its dark tint is acquired by the bottles into which it is made being smoked over a fire. Caoutchouc is soluble in ether, some kinds of naptha, and petroleum; and, in this state, is used as a varnish for waterproofing purposes. It is composed of carbon and hydrogen, and is highly elastic and inflammable. Mixed or worked up with sulphur, it loses a portion of elasticity, but acquires other useful properties. Its varied applications are well known; and, with the exception of gutta-percha, few vegetable products have been so extensively employed in making articles of comfort and luxury.

Gutta-percha is also the product of a tree, but differs from India-rubber in many important respects. It is less elastic, but does not lose its elasticity by being heated to a high temperature, as is the case with India-rubber. In their chemical characteristics they also differ, and in their solubility in menstrua. Both are non-conductors of electricity, and have therefore been extensively used for insulating telegraph wires. Electrical machines have been made of each of these materials, and they present great advantages to the electrician, owing to the very slight deposition of moisture on their surface in damp weather. The non-conducting power of heat possessed by gutta-percha is of great value, when that substance is employed in making water-pipes, as the enclosed liquid rarely freezes.

Lac, amber, mastic, dragon's-blood, &c , are resins, some of which are soluble in alcohol, and thus afford varnishes of different kinds. We may include pitch under this head, as it is a product of vegetable life, being obtained from the fir tribe of trees, in a similar manner to that already described

COAL AND ITS PRODUCTS.

Under this head we should be tempted to enlarge very considerably if space permitted ; but as the subject has been fully entered into in Section II., under the head of *Artificial Illumination*, we need do no more than present, in some respects, a general outline of the subject.

Coal is unquestionably of vegetable origin, and seems to have resulted from the decomposition of plants nearly allied to the ferns of our time. Indeed, the fossils of the coal measures afford every indication of that kind. *Lignite* and jet seem to be wood in a state of transition towards coal. Several years ago, we picked up at Tynemouth, near Newcastle, on the coast, a nut, which contained decomposed or rotten wood, charcoal, coal, and iron pyrites, all in one solid piece ; and this specimen well illustrates the agencies of heat and chemical decomposition, which have been active in producing the valuable fuel to which we are so much indebted for our position as a manufacturing nation. It would seem that the plants were first converted into peat ; that, subsequently, the land bearing them subsided ; and, being covered by strata, the pressure on the surface, and terrestrial heat, changed these plants into the forms we now recognise as coal.

Other minerals afford products analogous to those of coal. We merely mention this for the present, as we shall shortly have to describe such at some length.

Coal is chiefly composed of carbon and hydrogen, with some oxygen, and a slight per-centage of nitrogen, in certain kinds. When exposed to the action of heat, a vast variety of compounds of these substances is produced, many of which can be resolved into other substances. Indeed, a mass of coal may produce so many compounds that it would be impossible to describe all in a volume of moderate size Supposing considerable heat to be applied, the residue consists of a black mass called coke, which is nearly pure carbon.

The volatile products consist of olefiant gas, carburetted hydrogen, ammonia, carbonic oxide, carbonic acid, bisulphide of carbon, tarry matter, &c., &c., some of which we shall now describe.

Light Carburetted Hydrogen, or *Marsh gas*, so called from its frequency in districts containing pools of stagnant water, is a compound of CH_2, and may be obtained from various sources. It is highly inflammable ; and hence is partly the cause of the illuminating power of coal gas. It escapes from its natural source, the coal mine, in large quantities, occasioning those fearful accidents in mines of that material.—*Olefiant gas*, so called from its producing an oily fluid, by the agency of chlorine, is composed of $C_4 H_4$, and may be obtained by the action of concentrated sulphuric acid on alcohol. This gas is also inflammable, and burns with a rich yellow flame. The other volatile products to which we have alluded, have already been mentioned, under the heads of Nitrogen, Carbon, and Sulphur, respectively.

The manufacture of gas from coal is carried on to an enormous extent in this country ; but however interesting the subject may be, we must pass it over for others of greater importance in a scientific point of view. Besides, the article on Artificial Illumination, in the section on Light, affords every theoretical and practical particular. But one product of the distillation of coal or wood demands some notice, on account of the curious chemical products it affords. We refer to what is usually called tar.

Tar may be obtained by distilling wood or coal in close vessels, and condensing the products ; some of them we have already considered at page 415, to which we must refer our readers.

Creosote is a product of the destructive distillation of resinous wood. Its smell may be recognised in cured fish, smoked hams, &c. ; and its name is derived from the antiseptic power it possesses. Its preparation is somewhat complex : it is separated from wood-tar by distillation, and is composed of $C_{14} H_8 O_2$.—*Eupione* is another product of wood-tar. It differs from creosote in being readily inflammable, of less density, and also in its chemical composition, its formula being $C_5 H_6$. From some kinds of wood, a kind of paraffine is produced, to which, as obtained from another source, we shall have to direct attention hereafter.

Carbolic acid is a product of coal-tar, and is, or is nearly, identical with creosote in most of its properties. Its formula is $C_{12} H_5 O + HO$. It has been much used of late years as a powerful disinfectant. This product of coal introduces us to an interesting series of substances, ranged under the generic term, phenyl—$C_{12} H_5$—of which carbolic acid is a hydrated oxide. Connected with this series is aniline, a product alike of coal and indigo. We here meet with one of those extraordinary discoveries in chemistry which for many years may lie hid, only noticed by the scientific chemist, and yet may suddenly expand into the most useful form of applied science. For a long time, aniline was recognised as a principle of the colouring matter of indigo ; but it is only lately that, by the teachings of chemical science, we have been able to produce precisely the same substance from coal. At the present time, aniline, as obtained from coal-tar, is exclusively used in many dyeing establishments, for affording the colours—blue, mauve, Magenta, Solferino, &c.—which far exceed, in brilliancy, those produced from vegetable sources. Aniline is manufactured to a large extent from tar, by a patent process, and the philosophy of the subject we shall investigate when speaking of indigo.

Phenyl is perfectly analogous to ethyl, methyl, amyl, &c., which we have described at page 414, and subsequently ; and affords an oxide (the hydrate of which is carbolic acid), a chloride, &c.

Coal Naphtha.—This liquid is also obtained from coal-tar, being one of the last products of its distillation. After being removed from the tar in a crude state, it is redistilled, and becomes what is termed *light naphtha* It is an extremely mobile fluid, has a strong smell, a density of 890, water = 1000, and has been much used as an illuminating agent in what are called naphtha lamps. Benzole, which has already been described, may be produced from this naphtha by careful distillation, and subsequent agitation with sulphuric and nitric acids. Benzole thus procured is a liquid of a density of 850, water = 1000 ; is highly inflammable, and has lately been much used for naphthalising gas, and as a detergent for articles of clothing.

Rock Oil, or Petroleum.—In many parts of the world, a large quantity of oil spontaneously exudes from the minerals of some rocks which are adjacent, either to the coal measures, or in rocks containing lignite, &c. The oil wells of Birma, Baku, Syria, and at Colebrookdale, are instances of the kind ; but they all sink into insignificance in comparison with the wells lately discovered in Canada. As these have excited the deepest interest in all circles, we shall give somewhat extended particulars, and in the words of eye witnesses.

The oil deposits at present known seem confined to districts in the counties of Enniskillen, Brooke, and Sombra, in which parts the oil wells were first discovered. One of the largest yields a supply of no less than seven thousand barrels of oil daily, which flows out spontaneously, just like water. Immense quantities of the oil are wasted for want of casks to store it up.

As first afforded, this oil is highly inflammable; so much so, indeed, that serious accidents have been caused through its ignition. It has, however, been stated that it may be so refined as not to ignite under a temperature of 130°. It can be used as a lubricant, and affords naphtha, benzine, paraffine, and many other useful products. The following remarks are from a paper by Professor Hind, of Toronto University :—

"The extraordinary cheapness of petroleum, as an illuminator, is too well known in Canada to require any special notice. Notwithstanding the comparative dearness, in this country and the United States, of the chemicals (sulphuric acid and alkalies) which are required to purify, deodorise, and fit it for burning in lamps, yet it is, at 45 cents a gallon, incomparably the cheapest illuminator which has yet been manufactured; and it threatens, for domestic purposes, to drive all other means of illumination out of the field. But in Britain, France, and Germany, where acid and alkalies are abundant and cheap, and where various products, such as benzole, tar, &c., can be utilised with profit, the preparation of purified petroleum can be effected at so much cheaper a rate than on this continent, as to nearly make up the difference in the cost of the raw material which freight and insurance would add to it. Petroleum has arrested the production of coal oil or keroline on this continent; it will soon arrest, if proper steps are taken, the production of shale oil in France and Germany."

The following Table shows the comparative advantage of this Petroleum, or Rock Oil, over all other burning oils :—

Description of Oil.	Price per Gallon.		Intensity of Light by the Photometer.	Amount of Light from equal Quantity.	Cost of an equal Quantity of Light in Decimals.
	s.	d.			
Petroleum, or Rock Oil.	2	0	13·70	2·60	2·00
Sperm	7	6	2·00	·95	20·00
Camphine	5	0	5·00	1·30	10·00
Rape or Colza	4	0	2·10	1·50	6·50
Lard	4	0	1·50	·70	14·50
Whale	2	9	2·40	·85	8·25

Paraffine.—We have already alluded to a substance called paraffine, as obtained from wood-tar, and which is so named on account of its weak affinity, or slight inclination to combine with other bodies. Of late years, a large quantity of paraffine and paraffine oil has been obtained from Boghead coal, or Torbane-hill mineral, under a patent taken out by Mr. James Young, of Bathgate, near Edinburgh. The substance whence it is produced has been the "chameleon" of chemistry; for almost all kinds of differing opinions have been pronounced respecting it: the point to decide being, as to whether it is a mineral proper, or a coal—a question yet unsatisfactorily determined. The following is an account of Mr. Young's process :—

The raw material is heated in an iron retort; then exposed to hot steam, by which the oil, tar, and paraffine are volatilised. After being condensed, the product is distilled, and subsequently purified by means of sulphuric acid and oxide of manganese, or bi-chromate of potash. The residue having been washed, is distilled a second time, and subsequently cooled, strained, and pressed. It is then heated to 400° with concentrated sulphuric acid; and having been washed in hot water and soda, it is afforded as a solid body, semi-transparent, and presenting an appearance of bleached wax, but of a more pearly hue. In this form it is made into candles, which burn brilliantly, and without smoke. The oil produced from the raw material is employed almost universally in this country in lamps, and also as a lubricant for machinery.

Paraffine and its oil are prepared from various other substances; and lately it has been proposed to employ bog-peat in Ireland for the purpose; but we believe that the process has not been commercially successful.

Bitumen or Asphaltum is a solid substance, obtained chiefly from the neighbourhood of places abounding in other hydro-carbon products. The Dead Sea has long been noted for its adjacent bitumen pits. This substance is also found in India, America, and other places. It is of a black colour, inflammable, and being easily melted and moulded, is much used as a paving material when mixed with sand. In this form it is employed for rendering arches and railway tunnels waterproof, also as a basement flooring in warehouses.

Many substances analogous to coal, lignite, bitumen, &c., are found in various countries; but such are more conveniently described in the sections on *Mineralogy* and *Geology*.

VEGETABLE COLOURING MATTER.

Until within a recent period, most colours used in calico-printing, paper-staining, dyeing, &c., were chiefly obtained from vegetable sources. Mineral dyes, however, have been lately much in favour, and have the advantage, where they can be used, of being lasting, and easily applied. Some of these thus used, and also as pigments, have been described in our previous pages; we shall, therefore, here chiefly direct attention to colours of a vegetable origin.

The natural colours of leaves and flowers are due to a peculiar principle which is subject to the action of heat, light, and chemical action, but which is lost on the death of the plant. It is there a vital principle only. Colours employed for dyeing, &c., are extracted from the plant after its vitality has ceased, and are resident in the leaves, stems, roots, and flowers. Red dyes are obtained from madder, or rubia tinctorum, safflower, or carthamus, Brazil wood, logwood, sapan wood, the cochineal insect, &c.; blue colours are afforded by indigo, orchil, litmus, &c.; and yellow dyes are produced from fustic, turmeric, saffron, &c.

But all these colours *per se* are fugitive, and require a *mordaunt* to fix them in the fabrics to which they are applied. The action of a mordaunt is readily illustrated by that of iron and an infusion of logwood used in dyeing black. If a piece of cotton were simply dipped in the infusion of logwood, it would only acquire a dirty red-brown colour; but if it be first soaked in a solution of sulphate of iron, the oxide of the metal attaches itself to the fibres of the material, and, on being introduced into the logwood infusion, a black and permanent colour is produced. The mordaunts usually employed are salts of iron, alumina, and tin, others being used only to a limited extent.

Madder is chiefly employed in dyeing red. It is the root of a plant, and is imported into this country from the continent in a state of powder, having a dark-red colour. From it a peculiar principle, called *garancine*, is produced by means of sulphuric acid. This preparation is superseding the use of the raw madder, because it is more economical, cleanly, and effective. Other principles may be extracted from madder, such as *purpurine*, *alizarine*, *xanthine*, &c., of which the alizarine is the most important, because it is really the colouring principle of the rest, and is the chief constituent of the garancine of commerce. The celebrated Turkey red dye, which withstands the action of most chemical substances, is obtained by means of madder. The process is long and troublesome; and we may refer our readers to a late edition of Berthollet's *Bleaching and Dyeing*, for a full account of all its details.

Cochineal is properly an animal dye, but its colouring powers are due to the cactus, on which it feeds. With alumina, a decoction of the insect affords a rich red colour, used in dyeing silk, and in producing "carmine." We have succeeded in producing some rich red precipitates from a cold infusion of the cactus-flower and solutions of carbonate of soda and citric acid, employed in the manner we are about to describe in connection with safflower.

Safflower is a kind of saffron, and affords two colouring principles—a yellow and a red—the former being abundant and useless, whilst the latter is obtained only in small quantities, and is very valuable as a dye for silks, producing reds of the purest colour and of every shade, from pink to deep poppy.

The safflower must be washed in *cold* water, until all the yellow colour is removed. The residue of solid matter is then to be steeped in a solution of carbonate of soda, also cold; and, after some time, cotton wool is to be introduced so as to absorb the colour. It will appear of a muddy red tint; but on the addition of a solution of citric acid, or lemon juice, a magnificent red colour is afforded. By a repetition of this process the cotton may be filled with colour, which can afterwards be removed by the same means as those just employed. The colour is thus obtained isolated. A large quantity of the colour is manufactured on the banks of the Lea, near London; and also near Paris. The winter months are chosen for the purpose, as the heat of summer spoils the tint. From the small quantity of colour produced from the raw material, it is very expensive, and its use is chiefly confined to dyeing silks, and making "carmine saucers."

Decoctions made by boiling the chips of Japan, Brazil, and logwood, afford a red colour, with tin and alum as a mordaunt; and a black with salts of iron. These substances are chiefly used for dyeing wool and cotton yarn. Each of them contains some tannic acid, as do sumac, gall-nuts, oak-bark, walnut-peels, &c.; and such, with iron, afford black dyes of various depths.

Yellow colours are produced by the action of alumina, as a mordaunt on infusions of turmeric, &c.; but these colours are generally fugitive. Intermediate tints of brown, maroon, &c., are produced by successively dyeing the stuffs a yellow and red colour, until the desired tint be arrived at.

Blue vegetable colours are obtained from some lichens, amongst which the *rocella tinctoria* is that most commonly used. All the lichens, however, afford colours of some kind, and even those of a yellow and red tint. Orchil, used in dyeing silk, is obtained from the *rocella*, by means of ammonia, or more economically, urine, which has so far undergone decomposition as to afford ammonia. From these plants, peculiar colouring principles, such as *orcine*, &c., may be obtained, which are analogous to garancine, alizarine, &c.

Indigo has, till the discovery of the production of aniline from coal-tar,* been the chief source of the permanent blue of the dyer. The colour from indigo is not obtained, as in most other cases, by infusion or decoction in water, but by means of the strongest sulphuric acid. The raw material, as imported, is in blue-coloured cakes; these are powdered, and to them the strongest sulphuric acid is added. By this, the real principle of indigo is dissolved out. The principle of dyeing with this substance is that of first deoxidising it, which is done by means of the proto-sulphate of iron. This renders the indigo in a state fitted for absorption by the fabric, which, after being dipped into the solution, becomes of a deep blue colour on exposure to the oxygen of the air, and the colour then becomes permanent. In the process, the alkaline earth, lime, is used with the proto-salt of iron. And this introduces us to the production of aniline from a vegetable, as we have shown its production already from coal-tar.

If indigo be acted on by a hot solution of potass, and then distilled, aniline is produced as a nearly colourless liquor. It is highly volatile, soluble in water, and on being oxidised by chromic acid, affords a rich purple tint. It matters not what it is produced from, for it is equally obtainable from indigo, nitro-benzole, and coal-tar. And this is a matter of not only deep philosophical interest, but, in the uses to which it is applied, has become a most important article of commerce.†

Into the varied treatment which this substance undergoes, we of course cannot enter; and we have made our remarks more extended on it than we should have done, solely because it affords an instance of a most remarkable practical application of purely scientific research, and which we are glad to say has been the means of enriching those to whose perseverance we have been indebted for its manufacture in quantities.

Aniline forms numerous combinations with other bodies; its chemical formula has been stated as $C_{12}H_7N$, and thus it contains one equivalent of nitrogen. Stating it in its relation to the phenyl series,‡ we may use the formula—Pyl H_2N, or, $C_{12}H_5 + H_2N$.

In this chapter we have selected some of the chief of the many miscellaneous products of vegetable life, and have endeavoured to arrange them in such connection as would most conduce to their being understood by our readers. With them we shall dismiss the subject of vegetable chemistry, advising those who desire to prosecute the subject still further, to peruse the works of Fownes, Graham, Miller, Odling, and others, to whom we have ourselves been indebted for some of the facts which have been adduced.

CHAPTER XI.

ANIMAL SUBSTANCES.

ALBUMEN—FIBRINE—CASEINE—GELATINE—PROTEINE—KREATINE—BLOOD—OIL AND FATS—URINE AND UREA—BONES, ETC.—MANUFACTURES FROM ANIMAL SUBSTANCES—MANURES, CANDLES, SOAP, TANNING, ETC.

THE department of chemistry devoted to the consideration of animal matter in general, has already been the subject of an elaborate paper in our previous pages; we shall, therefore, here confine ourselves to a mere outline of facts.

The elementary constituents of animal life are, like those of vegetable existence, but comparatively few in number. Carbon, hydrogen, oxygen, and nitrogen, are, if we may so call them, the essentials; whilst sulphur, phosphorus, lime, and metallic oxides, in exceedingly small proportions, are the accidentals of animal matter. And yet, without a due proportion of each, the animal would be incomplete; for phosphorus is essential to the brain, lime to the bones. And thus the entirety is made up of parts, having definite ratios, which cannot be interfered with, lessened, or increased, without producing great changes in the being which may be their subject.

We need not here enter into any description of the individual elements; that has already been done in our previous pages: we shall, therefore, call attention to a few substances which either are part of, are secreted, or excreted, by the animal system.

ALBUMEN.—This substance is best typified in the white of a hen's egg, which contains the yolk suspended in nearly pure albumen. At a temperature of 160°, it becomes solid, as is well known in the domestic operation of boiling eggs. It then ceases to be soluble in water; and if exposed to a high temperature, it affords ammonia, and impure charcoal. Albumen is found in the blood, and in many secretions of the animal. It also exists in the juice, fruit, and seeds of plants. When in solution, it is precipitated by metallic salts, those of mercury being eligible, and by some acids, as the tannic, &c. Its chemical composition is stated by Mulder, as—

Carbon	53·5
Hydrogen	7·0
Nitrogen	15·5
Oxygen	22·0
Phosphorus	·4
Sulphur	1·6
	100·0

FIBRINE, so named from its fibrous appearance, is a constituent of blood, from which it may be removed by beating up that liquid with twigs, to which the febrine adheres. In a solid state it exists in flesh, in the gluten of flour, and in grasses. The chemical composition of fibrine is—

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* See *ante*, p. 420.

† See *ante*, p. 420.

‡ See *ante*, p. 420.

Carbon	52·7
Hydrogen	6·9
Nitrogen	15·4
Oxygen	23·5
Phosphorus	·3
Sulphur	1·2
	100·0

From this we observe, that it scarcely differs from albumen, and in many of its characteristics it resembles that substance.

Caseine is found in milk; and, in fact, cheese is nearly pure caseine. A vegetable substance, legumine, much resembles caseine, and is found in cabbage and other plants. Pure caseine is insoluble in water and alcohol, but readily dissolves in alkaline solutions. In making cheese, the caseine is coagulated by means of an acid, obtained from the stomach of the calf. This is called rennet, and hydrochloric acid may be substituted for it, having, to some extent, a like effect. Caseine contains carbon, hydrogen, nitrogen, oxygen, and sulphur.

Gelatine and **Chondrine** contain the same elementary bodies. They are easily extracted from tissues, bones, &c., by means of hot water; and calf's-foot jelly is an instance of the former substance; whilst isinglass, glue, and size, represent it in a more solid state. In manufacturing bone manure, much chondrine is often produced. This substance differs from gelatine in not assuming what is generally termed a *gelatinous* state, and is not precipitable by tannic acid, which precipitates gelatine readily.

Proteine is obtainable from albumen, fibrine, &c., through their decomposition by an alkali, and subsequent precipitation by acetic acid. It seems to be a proximate principle of albumen, fibrine, and caseine.

Kreatine is a principle existing in flesh, from which it may be extracted by aqueous infusion. It is composed of carbon, hydrogen, oxygen, and nitrogen. It is soluble in hot water, and is extracted from heat in the process of stewing, &c.

Blood.—This liquid is essential to animal existence, and consists of water and organic matter. Its specific gravity varies, but may be stated at 1·05, water = 1·00. The red particles, which owe their colour to *hæmatosine*, are the heaviest portion of the liquid. The blood corpuscles vary in size in different animals; and lately M. Duboscq, of Paris, has rendered great service to physiology by the photographs he has produced of these globules. Their general appearance is accordingly characteristic of their origin. The following cuts will illustrate the fact: Fig. 85 representing the blood globules of man; Fig. 86, those of fishes.

Fig. 85.

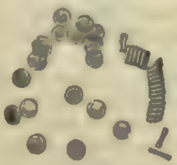
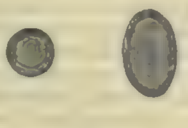


Fig. 86.



We need scarcely state, that by means of the blood every part of the body is nourished; hence it must contain every substance required in the animal from which it has been obtained for analysis. The following gives the proportions of proximate principles in human blood in a thousand parts:—

	Male.	Female.
Water	779·0	791·0
Red particles	141·0	127·0
Albumen	69·4	70·5
Earthy salts, &c.	6·8	7·4
Fatty particles	1·6	1·6

The above proportions differ from those given by other authors; but we need scarcely state that the composition of the blood incessantly varies in the same individual, owing to change of diet, disease, and other causes.

Oils and Fats.—At a previous page we have drawn attention to the oily matters produced by vegetables. To these the animal fats present much resemblance. Generally speaking, fat may be resolved by the action of alkalies and acids into oleine, stearine, margarine, and glycerine, each of which afford acids, theoleic, stearic, &c.; and such unite with alkalies and other bases. Thus soap is a stearate of soda; and by the addition to it of a dilute acid, the stearic acid is set free. Stearine is thus manufactured for making candles. The fat is made into an earth-soap by means of lime; to this dilute sulphuric acid is added, which sets the stearine, or stearic acid, free. By pressure and purification, it is eventually presented in a solid form. Margarine, oleine, and glycerine are simultaneously produced by the above process.

Spermaceti is a solid obtained from a species of whale, and is analogous to stearine in most of its characters. Butter is an animal fat, obtained from the milk of the female.

Amongst the secretions, such as bile, milk, urine, &c., we need only single out urea, which is a constituent of urine, and is interesting, as it may be produced artificially. For this purpose cyanate of potash is dissolved, together with sulphate of ammonia. The solution is evaporated to dryness; and the urea so formed, is extracted by means of alcohol. From this solution crystallised urea may be obtained, affording a rare instance of an animal product being imitable by the direct combination of its elements, apart from vital action. Its composition is $C_2\ H_4\ N_2\ O_2$. Uric acid is a component of urine, and is afforded in a solid state in guano, and the urine of serpents. Hippuric acid exists in the urine of gramnivorous animals.

Bones.—The framework or bones of animals consists of earthy matter, united by means of organic substances. The composition of bone varies, not only in different animals, but in the sexes of each. The following table affords an analysis of human bones:—

Animal matter	33·30
Phosphate of lime and fluoride of calcium	53·04
Carbonate of lime	11·30
Phosphate of magnesia	1·16
Soda and common salt	1·20
	100·00

In the crustacea, and marine animals generally, the carbonate of lime chiefly abounds; and thus the shells of mollusca are readily dissolved away, and engraved on, by means of hydrochloric acid.

Skin, nails, and horns may be considered as nearly identical in their constitution and production; the same may be remarked of hair and wool. In hair, both manganese and sulphur have been found.

Into the philosophy of nutrition, the production of bone, flesh, muscle, nerve, the secretions, &c., we cannot enter, as such will be fully considered in the section on *Comparative Physiology*, at the commencement of Vol. II. Our design has been to simply refer to those parts of the animal, the consideration of which is confined to the chemical details, and not to trench on subjects on which our space and arrangement forbid us to enlarge.

We may, however, add a few remarks on the manufactures connected with animal chemistry. Of late years, much attention has been paid to the production of animal manures. The bone, after being calcined, is drenched with dilute sulphuric acid, by which a superphosphate of lime is produced, which is of great value to the farmer. Blood, and other animal waste, have similarly been applied; and scarcely any substance of an animal nature exists, that is not now used for the purpose.

Hoofs, horns, blood, &c., are used in the manufacture of prussiate of potass, and in the process of cementation. Skins are, as is well known, converted into leather, by the process of tanning, in which the tannic acid* converts the soluble into insoluble substances.

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* See *ante*, p. 415.

Fat, oils, wax, &c., are largely used in the manufacture of soap and candles by means of the process described in connection with stearine. This branch of manufacture has enormously increased of late years, owing to the stimulus which chemical discoveries have afforded.

Bones and urine are employed in the manufacture of phosphorus, a substance which, a few years ago, was only a chemical curiosity, but which, at the present day, is an absolute necessity (in the lucifer-match) to, perhaps, every civilised home in the world. Glue, size, isinglass, patent gelatine, are each produced from skins or bone; and the latter substance has become of considerable commercial importance in the manufacture of portable soups. We decline, for the sake of our readers, to indicate its possible sources in many cases.

Space fails us to name all the uses to which animal substances are now applied. Chemistry helps us, not only to use up what would otherwise be waste, but actually converts, in some cases, substances, literally as "cheap as dirt," and real dirt, into others as valuable as gold. Indeed, its chief object, at the present day, is to turn to profit that which has been cast away as useless in former times. Aniline, aluminum, and a host of other products, afford us instances of the kind, and prove, alike to the philosopher and the artisan, that nothing was made in vain, or without its definite object and use.

Whilst it is the province of science to discover and indicate, the natural energy and instinct of man tend to utilise; and thus each succeeding age builds its monument on the *débris* of that which preceded it, only to afford materials for that which has to follow.

CHAPTER XII.

CHEMISTRY APPLIED TO AGRICULTURE.

WITHIN the last few years chemistry has been extensively employed in connection with agricultural pursuits. For centuries, agriculture had been little else than a system of conjecture and chance, in which the experience or opinion of each farmer was followed, without reference to principles or experiment. At the present time, on the contrary, science becomes the guide of every homestead —the aid of chemistry is everywhere sought, and agricultural societies and colleges are the means of disseminating its teachings amongst the masses.

At first sight it would seem that agricultural chemistry is a subject so extensive, that an acquaintance with it would involve almost a lifetime of research and experience; but such is not the case. We have simply to learn the constituents of any plant, and we shall at once perceive the nature of the soil required for its production. The question of climate, period of sowing and reaping, drainage, and similar matters, do not fall within the scope of chemistry. Such are purely matters of judgment and experience, for which no science can be substituted, except the knowledge acquired by repeated and long-continued observation.

The chemist has to decide on the question of the component parts of soils actually present, and the nature of such which, whilst absent, may be supplied in the form of manure. To do this, he carefully analyses the grain which is to be sown, and the soil in which it is intended to sow it; and whatever the latter is deficient in, his result should tend to point out, and to suggest a supply.

Let us first examine into the most important of all kinds of food, the cereals, such as wheat, rye, &c. The ear of wheat consists of seed, which is composed of gluten and starch. The latter substance is abundant in most plants, especially in cereals, potatoes, &c. It consists entirely of carbon, hydrogen, and oxygen,* whilst gluten has an additional element in the form of nitrogen. But besides the starch and gluten, earthy matter of various kinds is found in the seed and stem, such as silex, lime, &c.; and these, whilst they act as the bone to the plant during its vitality, become bone in the animal through the absorption of those substances in the system from the vegetable food on which it subsists. It matters not whether animal or vegetable food be used by man, exclusively, or mixed together. In either case he acquires the mineral matter of which his bones, teeth, &c., stand in need;—from the plant directly, in a crude state; or from the animal, as the ox and sheep, in a form more readily adapted for assimilation. Indeed, the domestic animals and plants used as food are simply channels, by which the elements of the soil are changed from inanimate dust to the constituents of sentient humanity. In this process, for a time only, inanimate nature is robbed of part of her own; but this occurs only for a brief space. Daily we give back, as excreta, portions of earthy and saline matter; and, at last, when we return "dust to dust, ashes to ashes," we repay the debt we have incurred during the brief period of our existence; and our mortal bodies, in their decay, literally and truly return to the earth from which they were first made. Nothing is lost; but, on the contrary, the particles of matter undergo an incessant change of combination—flitting at one moment as dust in the sunbeam, and anon forming the material of man, made in God's own image.

If we carefully examine the constituents of an average soil, we shall find that their number is comparatively few. The following is an analysis of one of the best soils found in the grazing pastures of Devonshire and Cheshire. It contains of

Silica	per cent.	70·2
Alumina	,,	19·2
Carbonate of lime	,,	0·4
Oxide of iron	,,	6·0
Water and organic matter . .	,,	4·1
Chloride of sodium (common salt)	,,	·1
		100.0

Now the whole of the above ingredients go to form the solid matter of a plant, but only in part; for the great proportion of flour, potatoes, &c., consists of carbon and water, derived, not from the soil, but from the atmosphere which surrounds the plant.

When a seed commences to germinate it produces two organs—the root, or radicle, which is to inhabit the soil; and the plumule, which, rising beyond the soil into the atmosphere, is to become the means by which the carbonic acid floating in the air is to be absorbed, and its carbon assimilated to the body of the plant. With this part of the growth the farmer has nothing to do; he must leave all to nature. With the root, however, he has everything to engage his attention; because that supplies the plant with the stamina which enables it to acquire a living and productive form.

Now, as soils differ in their nature, they are accordingly fit for the growth of corresponding plants, and every plant, as is well known, will not flourish equally in every soil. Clay consists chiefly of silica and alumina; and when it contains about forty per cent. of sand, it affords a clay-loam. The colour of clay is caused by the presence of oxide of iron. If a soil contain from ten to twenty per cent. of lime, it becomes a marl; and with a greater proportion, as in the chalky soils of Kent and Sussex, a calcareous, or lime soil is afforded.

When a soil is too close, inferior crops are produced; and some plants, as the potato, will not grow, or will produce inferior crops. Hence, in travelling over extensive ranges of country, we find great diversity of crops. In Kent, Essex, Suffolk, &c., we find cereal produce prevailing, with rotations of turnips and potatoes. In the Midland Counties grass is chiefly cultivated, for grazing purposes. In Lincolnshire and Cambridgeshire we see heavy corn crops, owing to the land containing the essential constituents in the soil.

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* See *ante*, p. 411.

Soils differ greatly, not only in their chemical constitution, but also in respect to their physical characteristics. A solid clay will not permit water to pass through it, whilst sand allows that fluid to percolate most readily. Now, generally speaking, the clay is rich in all the substances which are required to produce the plant, whilst sand is comparatively deficient; and yet a better crop shall be produced from sand soil than from heavy clay. This fact shows that we must not depend solely in the results of a chemical analysis to direct us in the choice of a soil. Indeed, we have seen in Invernesshire and Nairnshire, in the north of Scotland, soils (if they deserve the name) consisting of nothing but unmixed sea-sand, in their natural state, but which, by means of manures, afford excellent crops of wheat, barley, and oats. An excess of moisture in soils checks vitality equally with droughts; therefore, not only is a clay improved, in a chemical point of view, by admixture with sand and lime, but its physical character is altered, and it becomes more readily drained, "warmer," and therefore fitter for the production of any kind of crops.

After one kind of plant has been grown in a soil, it will be found to have abstracted such parts of that soil as have been required to afford it nourishment; and thus the soil has become deteriorated. Now, it is a remarkable fact, that the absence of an ingredient will entirely prevent the production of seeds and fruit. Lime is essentially necessary for this purpose; in fact, pea and bean plants will bear flowers but not seed in its absence. We have at this moment before us a luxuriant crop of beans, grown in a soil in which the same kind of seed was sown last year, but, in the absence of lime, was absolutely unproductive. At the early part of the present year, the soil was well dug up and limed, and now every stem is loaded with pods. On another soil, we tried precisely the same experiment some years ago; half was limed, and the other half was exhausted: the latter bore plenty of leaves, but not a pod; while the former afforded abundant crops, the same kind of seed being sown in each. It is true that such are extraordinary examples; but the farmer has plenty of them, although not so distinctly marked as these. Poor crops generally indicate that their producer has failed in his duty and practical knowledge. This is the great secret of the success which attends farming in the north of England and in Scotland. There the *climate* is everything that can be against the farmer, but his practical experience overcomes the natural deficiencies of the soil and climate; and thus he can produce crops on land which, in the south of England, farmers would only compare to those of a stone wall, as to its probabilities of producing power. We have seen oats growing on a limestone rock, on which the soil rarely exceeded four inches in depth. There the soil had to be brought to the rock; whilst in the southern counties, rich loam, extending many feet in depth, may be found. When a deep soil is superficially exhausted, it may be improved by being deeply ploughed. By this means the subsoil is brought to the surface, and so mixes with and replaces that which had been exhausted by the crops that had grown on it. In a similar manner, the rotation of crops benefits a soil; for if turnips, for instance, be grown where wheat has been reaped, they merely take up water; whilst the cereal has withdrawn silex, alumina, lime, &c. The atmosphere then has time to break up and decompose such elements into a soluble state, and so fit the soil, by a kind of reproductive process, for a repetition of the cereal growth after a lapse of time. Herein consists the advantage of allowing the fallowing of land. The soil is left to itself for a year or two; the atmosphere and moisture decompose the felspar and other minerals of which it is composed, and thus render it fit for growing various crops in succession.

Another point of great importance is the degree of pulverisation of land. As a plant grows it shoots out from the main root a vast number of radicles, by means of which it absorbs, from a thousand sources, its mineral food. If these radicles are confined by a close soil, the plant droops and dies; whilst if the soil be porous, and finely divided, these little roots can extend in all directions, and so become feeders for the main root and the plant. Ploughing, digging, harrowing, the burrowing of animals, as moles, worms, &c., the decay of vegetable fibres, all make such channels in the soil, and also assist in its drainage; and many manures, such as straw, and the like, thus effect a physical as well as a chemical improvement in land. This is a matter of great importance; although we feel sure that it is systematically neglected, and generally by pure accident only obtained. Burning clay has lately been much resorted to for this purpose; and, where coal is cheap, is an excellent practice. The retention and close character of the clay is destroyed, and the soil rendered into a finer state of division, and therefore more porous.

MANURES—ANALYSIS OF SOILS.—An experienced farmer will readily guess as to the nature of the substances abstracted from a soil by any crop; but chemical analysis more certainly points out this, and also suggests what kind of manure may be required to restore fertility. We shall not enter into minute details of the plan of analysis, as that has already been done at page 394, and succeeding pages. We shall only give directions as to the preliminary part of the process.

It will generally happen that more than one kind of soil is contained in a field, or many fields may consist of the same kind. An average of the soil should, therefore, be gathered, and the various samples mixed together. The first point to ascertain will be the amount of moisture which the soil contains. For this purpose all stones should be removed, and the earth may be weighed. It is then to be exposed for some hours to a temperature not exceeding 212° Fah. This will drive off all absorbed water, and the loss of weight will indicate the amount present.

The absorbing power of a soil may be learned by drying and weighing samples as above. They may then be well wetted, and left till all loose water has drained off. Their comparative weights being then ascertained, their absorptive power will also be learned.

After the amount of water in the soil has been ascertained, the next step will be to find out how much soluble matter it contains. This may be done by agitating the specimen for some time with cold water, when the soluble parts may be removed and tested, in the way directed at page 398. The insoluble portion may then be digested with hydrochloric acid, to remove iron, &c., and the remainder of the analysis carried out in the way suggested at the page referred to, *et seq.* By such a method, and following the instructions already given, any one of ordinary intelligence may gain sufficient knowledge of the constituents of a soil, and distinguish the earths and metallic oxides it contains. A quantitative analysis requires considerable knowledge and experience, and is better left to the professional chemist.

Having discovered what substance the soil is deficient of, the next step is to supply the required material. This is done by manures; and the choice of the proper one is decided by the results of the analysis. Such a multitude of these preparations is offered at the present day, and each is stated to be the "very best," that a farmer will, in many cases, be in a state of perplexity as to what he should choose. We shall not increase his difficulties by giving directions, which, at the best, would be but suggestive, if of use at all. So much depends on drainage, climate, and other physical circumstances, besides those of a chemical character, that it would be impossible for us to specify a universal panacea for all the ills that earth is heir to. We shall, therefore, simply name the proximate elements essential, and the manures which contain them, and leave to practical experience their choice and application. In purchasing any kind of manure, we may warn the farmer that he may be greatly deceived. The practice of adulterating guano with sand, &c., is very common; and, in every case, a certificate of analysis should be insisted on at the time of purchase. Many respectable houses always furnish this.

Substances required for Soils.

Silex	As in	Sand.
Alumina	„	Clay.
Lime	„	Chalk and limestone.
Oxide of iron . .	„	Sand and clay.
Phosphorus . . .	„	Phosphate of lime.
Sulphur	„	Plaster of Paris (sulphate of lime).
Nitrogen	„	Ammonia.
Carbon	„	Chalk, carbonic acid, vegetable matter, &c.

The following supply some or all of these substances :—

1. *Sewage.*—This abounds in every town, and has frequently been utilised to a great extent. Near Edinburgh there are fields which, from being comparatively worthless, now produce several crops of grass annually, owing to the sewage of the city being allowed to flow over them. Generally speaking, however, sewage matter is far too much diluted with water to be of much value, except in places where it can be readily applied by pumping, or the natural fall of the land. Of course it contains *every* substance required on the land, because it is the product of all that has been taken from the land in the shape of corn, grass, and the flesh of oxen, sheep, &c., fed on it. When it can be economically applied, it is highly valuable; but on clayey grounds, the immense amount of moisture deposited may prove hurtful. On sandy soils it becomes a most eligible manure.

2. *Guano.*—This substance is in high repute. It contains nitrogen (as ammonia), phosphorus (as phosphate of lime), silex, lime, &c. Indeed, we may consider it as the concentrated equivalent of town sewage. It is composed of the decomposed excreta, feathers, bones, &c., of birds.

3. *Artificial Manures.*—Next to the above come a legion of manures prepared artificially, all resembling sewage matter and guano, and containing the elements to which we have referred. To enumerate such would be impossible. Some are of great value; and in many we are inclined to believe that the only thing of value about them is the cask in which they are contained. Of such, we say to the farmer *Caveat Emptor.*

4. *Super-phosphates.*—These manures are made by calcining bones, and adding sulphuric acid thereto. A super-phosphate of lime with sulphate of lime is produced, affording phosphorus, sulphur, and lime. These are generally sold as bone manures, and are of great value to most soils.

5. *Lime, Chalk, Limestone.*—Some lands require the destruction of vegetable matter, which is effected by the addition of—

6. *Caustic Lime.*—This may be readily obtained by burning chalk, which drives off the carbonic acid. Chalk gives soft, and limestone hard, lime. Chalk or limestone is eligible for lands deficient of lime, but not requiring that substance for the purposes just named. They gradually become soluble, by the absorption of carbonic acid from the atmosphere.

7. *Fish, Seaweed, &c.*—These are readily procurable at the sea-side, and contain lime, common salt, and many other substances. They are each spread broad-cast on the land, or heaped up that they may putrefy. In Kent this manure is largely used; and sometimes sprats are so plentiful on the south coast of England, that they are employed to a very great extent by the farmers.

In the above epitome of manures, we have not included stable dung, human excreta, &c. The latter is used almost exclusively in China, and we need scarcely say is highly valuable. It is dried and made into cakes, in which form it is sold to the agriculturist in that country.

We have, in these observations on the subject of Agricultural Chemistry, confined ourselves solely to the results of personal experience in most parts of Great Britain and Ireland. We have met and conversed with farmers of every degree of intelligence and experience, and must say, that to depend on the dicta of any one or any class, would be to go counter to the rules of sound farming. What would be excellent as a plan in the south of England, would bring starvation to the doors of a northern farmer. In Ireland things spring up despite human intelligence; in Scotland, in despite of nature; and in England, these extremes meet. But the progress of science and railroads is effecting immense revolutions. Agricultural societies present to their members the results of extended experience and observation; the sons of farmers are now almost universally acquainted with, or are being instructed in chemistry; and the "dark ages" of agriculture are passing away into the things that were.

Of course we have only given a sketch of the application of chemical science; we shall conclude by recommending the perusal, by such of our readers as may need greater detail, of the works of Davy, Liebig, Johnston, and other eminent men, to whom humanity will ever owe a lasting debt of gratitude. The Proceedings of the Royal Agricultural Societies also afford much valuable information to the practical farmer on all the points we have named.

CHAPTER XIII.

THE SANITARY APPLICATIONS OF CHEMISTRY.

NOTHING is more gratifying to the true philosopher than to find that the facts and laws he has discovered may be applied for the social benefit of mankind; and just in proportion as this is successful, so he gains the reward of his labours.

Of late years, the subject of Social Science has been extensively ventilated; and, under the able and active superintendence of Lord Brougham, an association has been formed, with the object of discussing all questions relating to the moral, mental, and physical well-being of our nation. It is with the last department, in its relation to chemical science, that we shall briefly deal.

Health, individually and socially, depends chiefly on an abundant supply of fresh air, good (not "pure") water, and wholesome food. The latter is assimilated with the animal system; the water assists in diluting the food, in its assimilation, and in the formation of the blood, flesh, bones, &c., conveying the soluble matters to their proper destination; whilst the air, acting on the carbon of the food, causes its slow combustion, and this affords heat to the animal.

We, however, necessarily give off, in our most healthy condition, various excreta, consisting of gases, liquids, and solids; of which the chief are, carbonic acid from the breath and skin, other gases the product of digestion, &c., urine, and fæces; all of which, whilst decidedly poisonous to humanity, are extremely valuable as food for plants. In other words, what we have done with and must get rid of, is exactly that which the vegetable kingdom must have, and hence arises a compensating process between animal and vegetable life and their exigencies. The subject has already been entered into in the last chapter; we shall, therefore, not require to describe the uses of animal products. In dealing with this subject, we shall confine our remarks almost solely to the question of pure air; for the supply of water and the adulteration of food have already become subjects of government interference, and may be safely left to public management. Our supply of air, on the contrary, is purely a *personal* question, and so deserves the most careful consideration.

During the process of respiration, carbonic acid is abundantly produced in the lungs, and is, of course, exhaled as the breath is exspired. With this, a considerable amount of fœtid animal matter is also exhaled, which adds to the impurities of the surrounding air. If a person sit in a room destitute of arrangements for ventilation, he must, after a time, re-breathe these poisonous exhalations, and by so doing vitiate the blood, which depends on the supply of pure air for its oxygenation. The close smell of bedrooms in the morning, after one or more persons have slept in them, is caused by these exhalations; and the fœtor of alleys and courts, inhabited by the poorer classes, is due to the same cause, aided, however, by imperfect sewers, closets, &c.

Now the atmosphere, in its normal condition, contains only from four to six parts of carbonic acid in ten thousand of air. In unventilated rooms this proportion enormously increases; and an idea of this may be gained from the following example of a boys' school, extracted from a Parliamentary Report:—

Capacity of room . . .	22,140 cubic feet.
Number of boys present . .	164
Carbonic acid present in the air of the room after having been occupied 2½ hours .	$\frac{23 \cdot 71}{10000}$

Thus we see that the proportion of carbonic acid to that of air had increased from 5 to nearly 24 parts per 10,000, or nearly five times its normal quantity. How can we wonder, then, that fevers abound in crowded localities when such palpable facts are presented to us. It must be borne in mind, also, that besides the carbonic acid afforded, animal matter, other gases, and impure aqueous vapour are produced in such cases. The presence of these soon become evident from the languor into which the children fall; and then the physical evils resulting from breathing impure air, exert a visible and undeniable influence.

Defective drainage is another cause of impure air, and in many cases a most fatal one. The sewers, of course, carry off every species of excreta and waste from our houses. These substances all mix together, and mutually decompose, giving off many hydro-carbons, sulphuretted hydrogen, and other deleterious substances. When the connection between the house-drain and the main sewers is defective, these gases will enter the house, especially when the barometrical column is falling; hence arises the "smell of drains" just previous to wet weather. Defective drainage and typhoid diseases are always to be found concomitant; and the only effective remedy lies in removing the cause of the evil.

Disinfectants.—We may here conveniently refer to substances which have been used as disinfectants, omitting such as pastiles, perfumes, &c., which merely mask, but do not remove, pernicious exhalations. Chloride of lime, chloride of zinc, carbolic acid, caustic lime, and charcoal, are the most effective that can be used, consistent with abundance of supply, and consequent cheapness.

The two chlorides are of great value, because they decompose, sooner or later, most of the hydro-carbons in sewage gases; they are easily applied, and are very cheap. Carbolic acid and lime are not so fit for domestic use as those we have named, but can be employed on the large scale. The former may be cast down sewers; whilst the lime is suitable for covering graveyards, and any places whence effluvia are given off. Of late years, immense quantities of lime have been thrown into that great common sewer, the River Thames, during the summer months, and with the most beneficial effect.

Charcoal, whether as obtained from peat or other substances, is of great value. It is readily applied, and is of slight cost. Some years ago it was proposed, by Drs. Stenhouse and Watson, to purify the air supplied to hospitals, by passing it over trays of charcoal, or charcoal containing platina in a minute state of division. The following extract from Dr. Watson's remarks, gives the *rationale* of the method, and also affords some instances in which charcoal may be usefully applied:—

"Charcoal, as is well known, has the power of absorbing various gases in large quantities, and, perhaps, few more readily than those which arise during the decay of dead animal and vegetable substances. But charcoal does something more than simply absorb; for it is evident that, if it possessed no other property, a point would shortly be reached, when, having become quite saturated, it would cease to act. Charcoal, then, has another power in addition to that of absorption; and for the illustration of this I would refer to the following striking experiment:—Certain dead animals were placed in an open box, and covered with a layer of roughly-pounded wood charcoal, rather less than three inches in thickness, and all the decomposible portions disappeared more rapidly than if they had been buried in the ordinary way. Moreover, the boxes containing these dead animals were kept for many months in a room in which several persons were employed during the day, but still no disagreeable effluvia were detected, and health remained unaffected.

"All decaying animal and vegetable matters give off, during decomposition, fœtid and deleterious gases, which, in the end, tell as fatally upon the human constitution as does the bite of a viper, or the most insidious poison known to the chemist; and the animals referred to in these experiments formed no exception to this rule—a fact which could be readily ascertained by removing a portion of the charcoal so as to get nearer to the putrid mass below. From this, therefore, it follows, that not only does the charcoal hasten decomposition, but that those deadly gases that are constantly being given off, become, in their passage through the charcoal, converted into inodorous and comparatively harmless ones.

"The explanation of this peculiar result is simple and very beautiful. Charcoal is an extremely porous substance, presenting through its mass an almost incredible amount of surface; and upon this depends its power of absorbing various gases in large quantities. The oxygen of the air is the great vivifier of nature. The deadly emanations given off by decomposing matters are in what is called a 'low state of oxidation;' that is, they contain a comparatively small proportion of oxygen. Combine them with, or force them to take up, more of this purifying element, and the point is gained: that which, perhaps, an instant before would have proved most hurtful if breathed, becomes now almost entirely resolved into comparatively harmless combinations.

"Now charcoal contains within its pores a very large proportion of oxygen, amounting to rather more than eight times its bulk. As already shown, it absorbs the various putrid gases with avidity, and in this way they are brought into intimate contact with the condensed oxygen existing in the charcoal, and the result is as has been described.

"Such, then, is the action of common charcoal in dealing with the fœtid gases of decomposition. It not only absorbs but destroys them; that done, it gives out the resulting comparatively harmless products; room is made for more oxygen from the air, and more of these bad gases from whatever source, and thus the process ceaselessly goes on. In this manner the charcoal is, so to speak, constantly purifying itself; and, under ordinary circumstances, with occasional exposure in front of a large fire, or to the sun's rays, its powers remain intact for an almost indefinite period."

For domestic purposes, as the above extract will show, charcoal is of great use. It may be employed either in powder or in pieces of about the size of a pea, and, in that state, should be thrown over the object to be disinfected.

Ventilation.—This subject, although its end is of a purely chemical character, more properly appertains to the Laws of Heat;* we shall, therefore, confine our remarks to general principles only.

The gases given off from the lungs, are nitrogen and carbonic acid, in a heated state. The former has a specific gravity of 0·97, whilst the latter has that of 1·524. We should hence suppose, at first, that the nitrogen would rise in the air of a room, and the carbonic acid would fall to the lower parts; but this never happens, unless either gas is present in great excess. On the contrary, by the action of gaseous diffusion they intermingle, and so form a homogeneous combination in all ordinary circumstances. In a *closed* room, therefore, every part will soon arrive at a literally stagnant state. The only motion of the impure air will be produced by the animal heat, which causes it to rise, and the cooling effect of the windows and walls, which has a contrary influence. On opening a door, however, a fresh series of physical causes comes into operation. A draught of cool, and therefore heavy, air *sets into* the room at the lower part, whilst one of hot and light air *escapes outwards*, at the top. This may be readily noticed by holding a lighted candle successively at the bottom, middle, and top of an open doorway, when these currents will be noticed ; the middle part, however, affording no current. This simple experiment teaches us the philosophy of ventilation in the most effective manner ; and every ventilating arrangement which has yet been, or is likely to be proposed, rests for its basis on this fact.

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* In the section on *Heat*, this subject is treated on in considerable practical detail.

When open fires are found in a room, of course a constant draught is maintained ; and this, doubtless, diminishes the unhealthy character of apartments in which we often see men, women, children, cats, dogs, &c., &c., herding together. A fire must be kept for cooking purposes ; and *all* the animals we have named generally sleep on the floor, or but little removed above it. They then, to some extent, enjoy a constant change of air when at rest ; whilst to a person entering such a room the effluvia would be intolerable, because *his head* would be much above the chimney opening, through which the lower stratum of impure air passes away. On these vital and interesting subjects, however, we must not enlarge. We commend them to the study of all our readers, as a matter of duty on their part. A little sound practical advice to a poor man, earning his weekly wages and supporting a large family, is of much greater pecuniary value than a liberal donation of money. The last he will soon spend ; the former will save him, in medicine and loss of time (his money) by illness, some pounds perchance in the year. Indeed, we never see a broken pane of glass in the window of a working man's room, without being thankful that the brittle nature of the material most mercifully provides for him one of the greatest blessings of nature—free air, and also ventilation —both of which he wilfully denies himself, to his own loss of health and resources. Well would it be if the city missionaries, so usefully employed in our large towns, would, in such a manner of giving sanitary advice, *completely* follow the pattern of the Great Physician, who "went about doing good," not to the souls alone, but generally *first* to the bodies of men. However much we may endeavour to raise the moral condition of man, we must not neglect his physical state. Our relationship, as beings, is threefold—moral, mental, and physical. At no instant in this life do they exceed each other in their exigencies. They have been united by the Creator from the moment man was created, and they are not put asunder until He commands, and death becomes the means of their dissolution.

Distantly connected with the subject of ventilation is that of clothing, in so far as it concerns the removal of those gases which pass from the skin. It is a remarkable fact, that animals may be as readily poisoned by the absorption of poisonous gases through the skin as if such were inhaled by the lungs. Some years ago, when the Niger expedition was first proposed, an inquiry was made into the causes of the unhealthiness that prevails in regions surrounding the Niger and other tropical rivers. After much dispute on the subject, it was considered that the presence of free sulphuretted hydrogen was the proximate cause ; and that not only would danger arise from inspiring the tainted air, but that the entire surface of the body was also predisposed to absorb the fatal exhalations. To prove this, rabbits were enclosed in air-tight vessels, filled with sulphuretted hydrogen, in such a manner that the heads of the animals were left free in the open air ; so that the poisonous gases could not pass into the system by means of the lungs. It was found that a rapid absorption took place through the skin, which soon proved fatal. If, however, the bodies were well oiled, then the pores of the skin being filled up, absorption did not take place ; and so the animals, placed in the circumstances referred to, were comparatively uninjured. Applying, to some extent, such principles to the question of the human subject, we shall see that, unless the clothing permits a certain amount of ventilation of the body, either by its being porous, or only covering the skin loosely, the animal exhalations will be retained on the surface of the skin, and will be reabsorbed, to the manifest detriment of the individual. Hence the danger which arises in part from the use of what is termed "water-proof" clothing. The gaseous and liquid secretions are kept in contact with the skin, and are thus productive of serious consequences. The use of water-proof shoes hence frequently causes "tender feet;" for the ammonia or acetic acid secreted, according to the physical constitution of the person, acts as an irritant of the skin ; and thus, for the same reason, the nonporosity of the leather—new shoes always hurt the feet, preventing, as they do, the escape of the perspiration.

It is thus found that flannel worn next the skin, and woollen clothes generally, are the most healthy. They retain, through their non-conducting power, the heat of the body ; and yet, by their porosity, permit of a constant circulation of the air they inclose, into which the excreted gases, &c., pass off. They have also another advantage, which is that of acting as an irritant to the skin. They stimulate the nerves on its surface, and thus the pores are kept continually open.

CLEANLINESS.—The remarks just made bring us to another subject of great importance—that of personal cleanliness ; and in this, chemistry and chemical manufactures play a most important part. It is well known that the body of a human being contains some millions of outlets, through which water, &c., continually pass out. The liquid and gaseous substances escape, or else are absorbed by the clothing ; whilst some portion of the solid is retained in the skin and in the pores. This solid matter consists of animal fat, common salt, acids, and other substances, which, if not removed, will close the perspiratory pores, and prevent that relief to the system which those pores are intended to afford. In removing the matter, water alone is not sufficient. This may be readily noticed on any one taking a bath, either hot or cold, after violent exercise. The water will flow from the skin without really wetting it, simply because the fatty matter, deposited on the surface, intervenes between the skin and the liquid. If, however, an alkali, either free or in the form of soap, be used, then the fatty substances are easily removed. Soap, indeed, is simply a convenient method of applying an alkaline solution in such a way that, whilst its detergent properties may be employed, its caustic effects, which would destroy the skin, are avoided. Washing powders, and other substances of the class, are those generally containing either potass, or soda in a free state ; and from their known power of combining with the animal fatty acids, their value as detergents arises. Of course, what is required to cleanse the skin, is equally useful in the operation of washing articles of clothing. If such are not cleansed from the impurities which they have absorbed from the skin, they soon not only acquire a fœtid smell, but become positively dangerous. Kept near the absorbing pores, they afford a constant supply of impure matter, which, if reabsorbed, is productive of any kind of disorder of the typhoid class. We are, perhaps, not wrong in applying a similar observation to ragged clothes as we have done to broken windows ; for what one does to ventilate the room of the poor man, the other does to his body—each allowing an unwelcome, but necessary supply of fresh air.

WATER.—We have already stated that we do not intend to enter into the question of the supply of water, but we may make a few remarks respecting its chemical relations to sanitary questions. Pure water, in the chemical sense, never occurs in Nature. The nearest approach to it is rain ; which, however, is always contaminated with gaseous exhalations, if collected on land; and with saline matter if it fall near the sea-shore. Absorbed by the earth, and percolating the interstices of sand, rocks, &c., it acquires a variety of substances, which dissolve in it. When these are in excess, the water is termed "mineral water;" and usually contains lime, magnesia, oxide of iron, and alkaline salts. Such water can neither be used for washing, drinking, or culinary purposes, owing to its hardness. When such saline ingredients are in less proportion, we have a liquid resembling river water; and such partakes of the character of that supplied to most of the large cities and towns in the kingdom. Generally speaking, such water contains from fifteen to twenty-five grains of inorganic, and from one to four grains of organic matter to the imperial gallon; besides a small proportion of oxygen, carbonic acid, and other gases in a dissolved state.

Confining our attention solely to the organic matter (for the other questions have been frequently inquired into in our previous pages), we may observe that both vegetable organisms and animalcules are its constituents. It has been noticed that, in seasons when diarrhœa is most prevalent, the presence of putrefied or putrefying organic matter is always in excess in most sources of our water-supply; and so universally is this the case, that no competent authority doubts the direct connection between the disease and this circumstance.

If there be one subject in which we are most at fault with respect to chemical science, it is that of the operation of poison on the human system; especially that of vegetable and animal poisons. Their action is so subtle, and frequently the external physical effects of it so speedily pass away, that nothing but rough conjecture is left us in respect to that action.

The most careful chemical analysis has failed us in attempting to detect the details of poisoning by putrefied matter, whatever may be its origin. It has been ingeniously suggested that the result is due to a kind of inoculation; for it has been found, that if the matter of putrefied sausages be introduced into the system, in a manner similar to that adopted in vaccination, the blood soon becomes vitiated, and the death of the subject speedily ensues. However, to extend such a theory to all cases of death arising from the use of impure water, would hardly be permissible; we can only point out the evils which may arise, advise that they be carefully avoided, and hope that, eventually, we may gain some deeper insight into this painful, but deeply interesting subject.

It is a remarkable fact that some kinds of water really become purer after undergoing a kind of fermenting process. The animal and vegetable matter they contain, undergoes decomposition; and assuming, by the rearrangement of their elements, new forms of combination, the water itself is left in an almost pure state. The water supplied from the Thames to shipping, affords a notable example of this; and is, therefore, preferable to any other kind, when required for long voyages. Of late years, the general adoption of steam as a method of propulsion, and the invention of various forms of apparatus for distilling sea-water, have been of great advantage to persons undertaking long voyages, and have greatly conduced to the health and comfort of the passengers.

As remarked at page 426, water acts as a solvent and diluent to the human system. Whatever we take in the water we drink, must travel through the system, and be assimilated to its structure, or be rejected as urine, together with other substances which it has dissolved during its progress. This will show of what extreme importance it is that the water of which we partake should be of good quality. As we have before observed, it must not be chemically pure, for then it would be almost valueless; but it should only contain such substances as shall be beneficial, or, at all events, not prejudicial to health. Let us for a moment glance at what the office of water is to our system; and in the term we shall include such liquids as milk, broth, infusion of tea, &c., which are almost entirely composed of water.

First, as a diluent, water moistens and lubricates all the surfaces of our internal organs. The nose, eyes, ears, mouth, larynx, lungs, stomach, joints, &c., constantly require it; and when, in febrile attacks, we suffer from its comparative absence, the consequences are sufficiently well known to all. Then, as a solvent, it conveys albumen, fibrine, caseine, &c., to the surface of the stomach, in such a form as that organ can readily assimilate them; and it carries those proximate principles in the blood to each part of the human frame. In the body it produces the muscles, fat, fibre, nerves, &c., by this power of conveyance. On suitable parts it deposits its saline substances to form bone. Externally, it is essential to the production of hair, nails, &c.; and it is constantly employed in conveying to the skin the materials of which that substance is composed, and by which it is constantly renewed as it is worn away or injured. After having performed this, and many other functions, it returns to other parts of the system, bearing with it all the valueless and waste products of the entire system, till it arrives at those organs by means of which the body is relieved of the excretions.

From this hasty glance, our readers will at once perceive how essential water is to us as an active chemical agent in our system. But we may go still further, and remind them that a great proportion of our bodies consists of water. This is constantly passing off by the lungs and the skin; and the liquids we drink have, therefore, another office than those we have referred to, in that of maintaining the mass of the body of a uniform size.

We, however, cannot further enlarge by such details. Our object has been to show, in a general manner, the chemical relationship which water has on the health; and we must leave our readers to make such practical applications as the subject is capable of.

Chemical Manufactures and Sanitary Laws.—It is a most unfortunate circumstance, that many of our manufactories are not only the causes of disease to those engaged in them, but spread their evil influence for considerable distances around them as a centre. According to late legal definitions, such manufactories may be deemed "nuisances," if carried on to the prejudice of the health of the surrounding neighbourhood. It is equally unfortunate that such manufactories must necessarily be surrounded by workpeople, who, as they are engaged in the factories, must reside near their work.

It would be impossible for us here to enumerate the various occupations in which chemistry is involved, and which may be really dangerous in a sanitary point of view. The manufacture of chemical products, bone-boiling, artificial manure-making, and the like, come under the category; and, from the fact that they, generally speaking, require a great bulk of cheap material to afford but a comparatively small amount of commercial and valuable product, they cannot be carried on at a profit unless they are situated near the places from which the raw material is obtained. If even they were removed from the neighbourhood of large towns, still their exigencies would soon tend to produce almost a town of workpeople around them; and hence a temporary delay, but not a cure, of the evil would result.

It would thus seem, at first sight, that such manufactories must necessarily exist as a source of constant injury to health, and that either the people who subsist on them must die of disease, or, in their absence, of starvation. But such need not be the case, for, in almost every instance, an improved system of drainage, and the erection of tall chimneys for the purpose of carrying away smoke, vapours, &c., will mitigate all the evils which exist at the present time. The astonishing improvement which has taken place in the city of London, since the "Smoke Act" has been enforced, proves that where there is the will there is also the way; and, in the absence of the will, the strong arm of the law can soon effect every desirable result.

It is true that some manufactures must always be unhealthy; such as those of the production of white lead, arsenic, steel grinding, and the like. In such cases, however, much may be done by scientific means, urged by humane endeavours. At all events, we may rest quite assured that no manufacture can be legitimately carried on which must necessarily cause the death of its labourers; and we may equally be certain, that as, in manufacturing, we merely avail ourselves of the laws of nature applied for special purposes, there are such methods of conducting our processes as shall render them quite consistent with sanitary requirements, if our prejudices and ignorance do not stand in the way.

Much foolish prejudice, however, exists in the mind of the public against all places in which chemical substances are produced; and hence sulphuric acid and soda manufacturers have been the subjects of considerable persecution. In many cases, positive benefit may arise from the purification and disinfectation of the sewers, by the waste running from such factories. We have met with several instances of the kind in London, and other large cities, presenting cases in which whole streets have been protected from disease by this cause. The erection of tall chimneys for carrying off the fumes and vapours, and an abundant supply of water, will generally remove all sources of danger. It is, however, extremely desirable that competent inspectors should be appointed, whose duties should not be confined to the prevention of nuisance *outside* the factory, but who should also have a judicious control over the internal arrangements, and thus be enabled to keep a watchful eye on the health of the work-folks *inside* the gates. It will frequently be found that the ill-health of labourers in chemical works, does not so much arise from the inhalation of fumes, &c., as from their constant exposure to changes of temperature, and from the fact that most of the processes are carried on in open sheds, affording little or no protection from draughts of air, and wet weather. By these the strongest man must soon be reduced to a state of disease, through checked perspiration, &c.

Two singular instances, in which the production of disease seems at first sight unavoidable, are those of glass-silvering, and lucifer match-making. In both of these, the vapour of the substances used seems to be absorbed by the worker, until his entire frame is permeated. In silvering glasses, large quantities of quicksilver are used. The progress of disease is noted by the salivation produced; and at last the individual becomes almost a complete carious mass. In making lucifer-matches, the phosphorus is similarly absorbed by the system, affecting the bones of the face and palate until these are completely eat away. But in this extreme case a remedy has been discovered; for if amorphous phosphorus be employed,[*] all danger vanishes, and the work people suffer no inconvenience of any kind. On the continent, the use of the ordinary phosphorus is forbidden, for the foregoing reason, and thus a most painful and incurable disease is prevented.

Our space will not permit us to enlarge further on this interesting subject, or we might point out several instances in which the employment of poisonous substances is not only dangerous, but positively wicked. The manufacture of green artificial leaves, &c., is carried on by the use of the arseniate of copper. Scores of young females are hourly exposed to the pernicious dust of the compound in close rooms, for no other purpose than to administer to the vanity of others.[†] No one can hesitate to pronounce against such occupations, whether in respect to the evil resulting to the workers, or to the danger which most certainly arises from wearing the product of their labours. In many a ball-room, the ladies carry, in their ornaments and dress, as much arsenic as would suffice to poison all present; and, generally, the arsenical dust is wafted from their dresses and head-dresses, in quantities of which they have but little idea. This is one out of many similar employments in which females are engaged; and may thus serve as a type of manufactures, which less devotion to fashion would at once cause to be abolished; and in being carried on, serve no useful end.

The following Tables of WEIGHTS AND MEASURES will be found useful by the student. A more extended series will be given in connection with Applied Mechanics.

ENGLISH WEIGHTS.

	Grains.		Ounces.		Pound.
Troy weight . . .	480	=	1		—
	5760	=	12	=	1
Avoirdupois weight	437½	=	1		—
	7000	=	16	=	1

FRENCH WEIGHTS.

The standard is the gramme = 15·434 troy grains; and the decimal system is adopted. Thus the

Milligramme . . = ·015434
Centigramme . . = ·154340
Decigramme . . . = 1·543400
Gramme . . . = 15·434000
Decagramme . . = 154·340000
Hectogramme . . = 1543·400000
Kilogramme . . . = 15434·000000 = 2¼ pounds avoirdupois weight, English.

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[*] See *ante*, p. 370.

[†] See *ante*, p. 381.

ADDENDA.

THE NEW METAL, THALLIUM.

In the chapter treating on Metals,[*] we have mentioned the discovery of two, by means of the spectrum analysis; and an extended use of the same agency has resulted in the addition of a third to that class of elementary bodies. In the Great Exhibition of 1862, a few grains of this metal were exhibited, which had been obtained from the residue of a process for making sulphuric acid. Its discovery arose from a rich green spectral band being produced, when a portion of the material above named was submitted to the spectrum examination; and the name *Thallium* was applied to it, on account of the colour of the flame. Since that time Mr. Crookes has devoted his attention to the production of the metal on a larger scale; and has so far succeeded as to have prepared several ounces. In its physical character it resembles lead, being very soft, marking paper like that metal, and having a specific gravity of 11·4. Its chemical equivalent is 203. It forms two or three oxides; and unites with acids, affording salts, which crystallise in beautiful forms. The chief sources have hitherto been substances united with sulphur—such as native sulphur—ores of copper, &c. The small quantities, comparatively speaking, yet procured, have not enabled Mr. Crookes to give a probable estimate of its commercial value and application; but, as it seems pretty widely disseminated in mineral bodies, it may possibly become of some importance in the arts and manufactures.

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[*] See *ante*, p. 312.

SECTION VII.

MATHEMATICS.

CHAPTER I.

ARITHMETIC.

In the previous pages, the attention of our readers has been chiefly directed to the forces which operate on every kind of matter, and to the different qualities of those substances of which material bodies are formed. We have hitherto scarcely required more than the ordinary rules of arithmetic in the calculations of quantities, time, and space; but we shall now proceed to an entirely different range of subjects. These will afford a full exposition of the different branches of mathematical science, the rules of which enable us to express the relative value of both the forces acting on, and the quantities of, matter, together with every other particular involving the use of numbers, symbols, &c. As each subject will have a special introduction, it is not necessary that we should here occupy any considerable space in discussing their nature. We shall rather confine ourselves to advising our readers as to the best method of studying. We may first impress on their minds the necessity of proceeding *step by step*. If any person were to glance at a page in trigonometry, or the differential calculus, it would be a matter of no surprise if he were to hopelessly despair of ever mastering the subject. On the other hand, an acquaintance with the common rules of arithmetic and algebra, and the elements of geometry having been once acquired, little or no difficulty will be experienced in the investigation of the higher branches of those subjects, provided the student makes himself thoroughly acquainted with every rule, *successively*, as he proceeds. It is incumbent, therefore, that the old Greek motto, "Hasten slowly," should become the rule of every tyro in mathematical science, if success and proficiency are to be obtained. We must add, that the entire work, both in the preceding and succeeding pages, has been so arranged as to *gradually* inure the mind to close reasoning. When we have to investigate the laws of astronomical science, all the rules of each branch of mathematics will be involved; and it is therefore essential to those who would penetrate into the depths of that science, that they should have become tolerably proficient in each branch of mathematics. Without this, the study of astronomy is little better than "star-gazing;" which, while it may afford excitement to our imagination, has but little effect in increasing our knowledge and expanding our mental powers.

Many of our readers may have no desire to become deep mathematicians. On such we would urge that mathematics may not only be studied for the sake of their applications, but may be used as a mental gymnasium. In every other branch of science, truth is veiled under theories; whilst, in mathematics, she appears unclouded by a single doubt. Each proposition of geometry is absolutely demonstrable. No doubt can be left of their truth; and in such investigations the mind undergoes an exact although severe training, which fits it for exercise in the daily affairs of life, as much as for the accurate study of nature and nature's laws.

The articles on each of the following subjects have been written by different authors, whose names have been already mentioned in the table of contents. The editor has ventured on such alterations in the original papers as he deemed would assist the reader or student in cases of apparent difficulty, which often occur to those unused to the phraseology and technicalities of mathematical science. The arrangement of each paper has been made so that the preceding shall be introductory to those which follow; and thus the student may progress from one subject to another, until he has become fully acquainted with each branch of the science.

Introductory.—The present section is to be devoted to that branch of study implied in the term Mathematics—a term which comprehends one of the most extensive and important departments of human knowledge. By most people, it is considered also as one of the most *difficult* departments; and many, with time and talents for the task, are deterred from entering upon a study which would amply repay the expenditure of both, by this mistaken prejudice. Every science, no doubt, has its hard and knotty points; and in no intellectual pursuit can distinction be attained without labour, thought, and perseverance; yet if there be one subject of scientific inquiry which, more than any other, is distinguished by the simplicity, certainty, and obviousness of its fundamental principles—by the irresistible evidence by which position after position is established—and by the systematic gradations by which layer after layer of the intellectual structure is completed—that subject is Mathematics.

In other topics of research, there is generally more or less of hypothesis or conjecture: there are obscure recesses, into which the light of truth and demonstration cannot penetrate, and where fancy and imagination are sometimes permitted to guide our steps. But there are no perplexities of this kind in mathematics—no ingenious theories to mislead, and no conflicting opinions to bewilder; our progress here is exclusively under the unerring direction of Truth herself; and it is her torch alone that lights up the path.

Whatever, therefore, may be the difficulties connected with the study of mathematics, it is plain that they do not arise from our having to grope our way in darkness and uncertainty; the asperities of the road are as clearly revealed before us as the level and unobstructed track; and all that the earnest student requires, is some friendly hand to aid him in surmounting these in the earlier stages of his progress.

It is this sort of aid that we here propose to supply. We do not undertake to conduct the scientific inquirer through the entire regions of mathematical research—ours is a far less ambitious aim: we write for the young—for the self-dependent—the solitary—and, perchance, the unfriended student. The office we here take upon ourselves will be performed, if we succeed in the endeavour to assist *him*. This is the only object at which we now aim; and we think it right thus explicitly to declare it, in order to forewarn those who may desire information on the more recondite researches of science.

It may be proper to mention, however, that although we now propose to limit our labours to an exposition of the elementary principles of mathematical learning, and to economise space as much as possible; yet, within the bounds prescribed, we shall take care that every subject receive a full and fair elucidation, and that it be discussed to an extent amply sufficient for the purposes of general education. We hope, too, by avoiding all attempts at magisterial dignity of style, and addressing our readers in the familiar language of social intercourse, to secure their attention, and win their confidence. That we may be fortunate enough, by clearness and simplicity of explanation, to awaken in some a genuine love for science, and a desire to prosecute their researches in writings of wider scope and higher pretensions, is our earnest desire.

We have thought it advisable to commence our work on elementary Mathematics with a preliminary treatise on Arithmetic—the groundwork upon which the entire system, with the exception of pure geometry, ultimately rests. Books on arithmetic, however, are so numerous, and so easily accessible, that we might have been held excused from introducing so hackneyed a subject into a work which, though confessedly of an elementary character, is, nevertheless, intended to embrace a range of topics beyond the ordinary limits of a schoolboy course. But it unfortunately so happens, that books on arithmetic, with few exceptions, are little more than mere depositories of practical rules and mechanical operations; and are, therefore, but ill-suited to prepare the young for that higher kind of exertion—higher, because more intellectual—which science, properly so called, always demands.

A boy who has gone through his "Walkingame," and who, generally speaking, is then introduced to "Euclid," is naturally enough bewildered by the total dissimilarity of the two authors—not from difference of subject, but of manner of exposition. The former has abundantly supplied him with *rules*, but no reasons; the latter gives him *reasons*, but no rules: the one has loaded his memory, and employed his fingers; while the other appeals to his judgment, and exercises his understanding.

To make, in this way, the passage from arithmetic to geometry an abrupt transition from the mechanical to the intellectual, we conceive to be a serious defect in educational training. Arithmetic is as much a science as geometry: there is not a rule in the one, any more than there is a theorem in the other, that is not founded on reason, and demonstrably true. And even viewing arithmetic merely in reference to its practical utility in commercial affairs, to the demands upon it in the counting-house and the shop—we still contend that its principles should be rationally taught—not authoritatively declared, inasmuch as that which has engaged the understanding, and been received from a conviction of its truth, is more securely retained in the memory than what is committed to it by rote. Rules, unsupported by reasons, are hard to learn, and hard to remember; but, when the practical precept is associated in the mind with the theoretical principle on which it depends, we learn and remember with ease and satisfaction; for, instead of words, we get knowledge.

These considerations have prevailed with us; and have determined us to render this course of elementary mathematics complete, by commencing at the very foundation. It must be borne in mind, however, that our design is not to exhaust here any subject of which we treat: it is rather to excite an appetite for knowledge, than to satiate it. It will not be expected, therefore, that our treatise on arithmetic is to be co-extensive with what, under that name, is usually put into a schoolboy's hands; the bulk of such books arises, in a great measure, from the system of instruction condemned in the preceding observations; the object of which system seems to be to inculcate a knack of readily applying *rules*, by experimenting upon numerous examples, under the guidance of the prescribed directions. This, as the reader has already been made aware, is not our object; we propose to explain *principles*, and to furnish the reasons that justify the rules; persuaded, that if the former be thoroughly apprehended, there need be but little anxiety felt about the mere verbal memory of the latter.

It would extend these introductory remarks far beyond the space that can be allotted to them, to enter into any detailed account here of the several particulars to be introduced in this division of the work; but we cannot conclude them without a few words more especially addressed to those who have resolved to place themselves under our instructions.

It is a common thing with students in science to be frequently making inquiries as to the *use* of what they are learning. "What is the *use* of this?" is a question put at every turn; generally to the annoyance of the teacher, and often to the discredit of the learner.

The use of any intellectual pursuit—employing the term *use* in its higher and more honourable signification—is to be realised in the mental satisfaction and the mental elevation it communicates. You do wrong to estimate science solely and exclusively in proportion as it visibly contributes to our animal wants and enjoyments: there is an intellectual pleasure in the very process of acquiring knowledge; while the conscious possession of it raises the human being in the scale of creation, and thus enables him to contemplate its wonders from a more exalted position. It is in this way that knowledge, like virtue, to which indeed it is allied, is said to be its own reward; for the study of science is accompanied with gratifications of the purest and loftiest kind; and is productive of advantages to the student altogether distinct from the benefits conferred by its applications to the practical purposes of life. It invigorates and enlarges the faculties—refines and elevates the desires—and adorns and dignifies the entire character, withdrawing our thoughts from what is mean and degrading, and inclining them to the noblest and worthiest of objects—to the love and veneration, and therefore to the practice, of TRUTH.

These advantages, though unconnected with outward and tangible results, are surely too precious to be entirely overlooked in any correct estimate of the value of scientific pursuits. When, in the course of these, you find yourself engaged in an inquiry that may seem to have but little relation to the things of sense, and are, on this account, disposed to ask—what is the *use* of it? just reflect whether the intellectual exercise has not been combined with intellectual gratification; and whether there be not an abstract beauty in the result arrived at, that awakens pleasurable feelings, practically realised, though hard to be described. We think the reflection will, in general, suggest an answer to the inquiry in some measure satisfactory.

These tranquil and purely intellectual enjoyments have prompted and encouraged the efforts of the wisest of men—have cheered and sustained them amidst penury and neglect; and, under the persecutions of power, wielded by ignorance, have supplied a consolation second only to that which Divine revelation affords; and they have finally been regarded as no mean reward for a life thus tried, and toils thus endured. They were considered as compensation sufficient for the labours even of NEWTON—many of whose discoveries in science would, in all probability, never have been given to the world but for the urgent interference of private friends.

We are anxious that you should be influenced by considerations such as these; and that you should regard science as something more than a ministering agent to our animal comforts, or even to our social gratification and convenience. The practical benefits of science—and more especially of those departments of it connected with the subjects of the present volume—are in little danger of being overlooked or undervalued; they are spread profusely around us; and are felt and enjoyed by all. And there thus seemed to be all the more need for directing your attention to collateral advantages—less palpable and striking, and therefore less likely to be duly appreciated.

We shall now proceed to the business before us: we shall assume no knowledge at all on your part in reference to the topics to be discussed; and, in even so simple a subject as ARITHMETIC, we shall begin at the beginning.

Arithmetic.—Arithmetic is that branch of knowledge that teaches us how to perform calculations by means of numbers. The rules which direct the various operations constitute the *art* of arithmetic: the reasons and principles on which these are founded belong to the *theory* of arithmetic; and the theory and practice united, form the *science* of arithmetic. It is this that we are now going to explain. You are aware that the symbols, or marks, employed in this subject are called *figures*, and that they are as follow:—1, 2, 3, 4, 5, 6, 7, 8, 9, together with the mark 0, called *nought*, or *cipher*, or *zero*, and which stands for *nothing*. This 0 is also called a *figure*, so that there are *ten* figures in arithmetic: the number of *units*, or *ones*, which each stands for, is here written—

nothing,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.
0	1	2	3	4	5	6	7	8	9

Each of these figures is also called a *number:* but the word number has a wider meaning. Thus 26, 43, 57, &c., are all numbers, each of which consists of *two figures*; the first number is *twenty-six*, the second *forty-three*, the third *fifty-seven*, and so on; so, that you see, the 2 in the first stands for *two tens*, the 4 in the second for *four tens*, and the 5 in the third for *five tens*. In like manner, 368 is a number of *three figures*. The first figure, 3, stands for *three hundred;* the second, 6, for *six tens*, or *sixty;* and the third, 8, for *eight ones*, or *units:* the number itself standing for *three hundred and sixty-eight*.

You thus perceive that a figure which, when written singly, stands merely for so many *units*, changes its meaning, or value, according to the *place* it occupies in a number of several figures. If it occupy the last, or right-hand place, it still stands for units; but if it be in the next place, to the left, it stands for so many *tens;* if in the place next to that, for so many *hundreds;* and if it occupy the fourth place from the end, it stands for so many *thousands:* the number 7352, for instance, is seven thousand three hundred and fifty-two.

Figures thus have a *local* value; that is, a value depending upon the places they occupy in a number. The following is a number of *twelve* figures; and when the local values, of these figures are written against them, it supplies what is usually called the

NUMERATION TABLE.

2	4	6	8	7	3	1	5	9	4	3	8
hundreds of thousands of millions.	tens of thousands of millions.	thousands of millions.	hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.

And this number is read thus:—two hundred and forty-six thousand eight hundred and seventy-three MILLION, one hundred and fifty-nine THOUSAND, four hundred and thirty-eight.

A person beginning to learn arithmetic will be enabled, by means of the above table, to *read* any number—that is, to express its value *in words*. In a large number like that here given, the easiest way to proceed is this. Cut off the *last three* figures, then the next three, then the next, and so on; thus dividing the figures into sets of three as far as possible. A glance at the *table* shows that the leading figure of each set is *hundreds* of something; that of the first set, on the right, is *hundreds* of *units*, or simply *hundreds;* that of the next set is *hundreds* of thousands; that of the next, *hundreds* of millions; and so on. And by thus finding out the *local* value of the leading figure in each *period*, as it is called, you may read the number with ease. For example, the number 68547329, when divided into periods, as here proposed, is 68,547,329: pointing to the 3, you say *hundreds*, and passing to the 5, *hundreds* of thousands; the incomplete period, 68, must therefore be 68 *millions;* and the entire number 68 million, 547 thousand, 329; or, expressing the value wholly in words, it is sixty-eight million, five hundred and forty-seven thousand, three hundred and twenty-nine. In a similar way we find the number 42638572613, or 42,638,572,613, to be 42 thousand 638 million, 572 thousand 613. If you wished to put this wholly into words, all you would have to do would be to write forty-two for 42, six hundred and thirty-eight for 638, five hundred and seventy-two for 572, and six hundred and thirteen for 613. The leading figure of a complete period, you know, is always *hundreds*, according to the plan we have proposed; and when you have found by the table *what* these hundreds are, or from practice can recollect what they are, you can have no difficulty in reading the number. When any of the figures are *noughts*, a little extra care is, however, necessary. Thus, the number 460305007, which, divided into periods, is 460,305,007, is read four hundred and sixty million, three hundred and five thousand, and seven.

From what has now been said, you clearly see what is meant by the *local* value of a figure. If it occupy the place of *units*, its value is so many *ones;* if it be in the place of *tens*, its value is *ten times* as many ones—that is, it represents so many *tens;* if it be in the place of *hundreds*, its value is *ten times* as many *tens;* if in the place of *thousands*, it is *ten times* as many *hundreds;* and so on. So that, as you advance a figure, place after place, towards the left, you increase its value *tenfold* at every remove: thus, 8 is simply eight *ones;* 80, where the 8 is now in the *second* place, and *nothing*, or *nought*, in the first place, is *eighty*, or *ten times* eight; 800, where the 8 is in the *third* place, and *noughts* in the first and second places, is *eight hundred*, or *ten times* eighty; and so on. This tenfold increase in the value of a figure, when it is removed one place from right to left, explains why our system of numeration is called the *decimal system;* the word *decimal* being derived from a Latin word, meaning *ten*. It was a beautiful contrivance thus to give a *local*, as well as an absolute value, to the symbols, or figures, used in the *notation* of arithmetic.[*] You see that by this happy idea we are enabled to express all numbers whatever by the help of only *ten* different marks, or symbols: whether we owe it to the Arabs, to the Greeks, or Romans, is a question on which there is still some doubt.

It ought, perhaps, to be mentioned, that although the numbers considered above do not extend beyond twelve figures, numbers with more figures than these may occur, and that there are words to express the additional periods. If the number have *thirteen* figures, the leading figure on the left would stand for so many *billions;* and if a complete additional period were joined to a number of twelve figures, making a number of *fifteen* figures, then the leading figure on the left would, of course, be *hundreds* of billions. But *billions*, *trillions*, *quadrillions*, &c., are names so seldom employed or wanted, that the numeration table need not be encumbered with them. It may be worth a passing notice, too, that no distinct ideas are conveyed by any of these terms: beyond a very moderate extent our notions of the value of numbers become confused. The number of *ones* in a million, even, is hard to conceive: it is a thousand thousand, and would take more than twenty-three days to count, though you kept at it for twelve hours a-day, and counted one every second. Our ten figures, or *digits*, as they are often called (*digitus* being Latin for finger, and our ten fingers suggesting the word[†])—our ten figures thus enable us accurately to express on paper, without the error of a single unit, numbers too great to be even conceived or imagined.

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[*] The marks, or symbols, made use of in any science, constitute the *notation* of that science.

[†] *Names* frequently throw light on the origin of *things:* it is interesting to notice, that the name *digit* is plainly significant of the early rude method of counting on the *fingers;* and that the name *calculation* as plainly refers to the primitive practice of reckoning with *pebbles* (*calculus*, a pebble). In a similar way *electricity* derives its name from *electron*—amber; *magnetism*, from *magnesia*, &c.

Before concluding these remarks on numeration, it may be as well to show the beginner how a number expressed in *words* may be translated into *figures*. This is not quite so easy as to translate figures into words; the plan is as follows:—

Write down a row of *noughts*, or *ciphers*, and, as if these blanks were numbers, mark off the *periods:* then, commencing at the first cipher on the left, put under each the proper *figure* in the number proposed, taking care that it be in its proper *place:* if any *vacancies* appear under the corresponding ciphers, fill them up with *noughts*. Thus, let it be required to put into figures the number five hundred and six million, thirty-four thousand, and forty-eight. We know that the place of *millions* has *six* places to the right of it; we therefore put a nought for the millions, and write six noughts after it, and, as we see, from *hundreds* being the leading word in the written expression, that the first period will be a *complete* period, we *prefix* two noughts more. The requisite row of noughts, divided as proposed, is as in the margin; and under these we now have to write, in their proper places, the figures 5, 6, 3, 4, 4, 8, and then to fill up the gaps with noughts; we thus find the number, when written in figures, to be 506,034,048. The learner will be able to do without such helps as these after a little practice; he should accustom himself to express in words the numbers he uses, when these are of moderate extent, and not content himself with merely looking at them.

000 000 000
506 034 048

We shall now proceed to the four fundamental operations of arithmetic: these are *addition*, *subtraction*, *multiplication*, and *division*. There are no calculations, however long and intricate, that are not composed of one or more of these four.

Simple Addition.—Addition teaches us how to add numbers together, and so to find the *sum* of all. It is called *simple* addition, when the numbers to be added either have no reference to particular *things* or *objects*, or when the things referred to are *all of the same denomination:* thus, if 24 pounds, 37 pounds, 82 pounds, &c., were all to be added together, the operation would be that of simple addition; but if 24 pounds 7 shillings, 37 pounds 2 shillings, 82 pounds 12 shillings, &c., were to be added, then, as pounds and shillings are different things, the operation would not be *simple*, but *compound* addition; one of the first set of things being called a *simple* quantity, and one of the other set a *compound* quantity. The rule for performing simple addition is as follows:—

RULE.—Arrange the numbers to be added one under another, so that the first column of figures on the right may be *units*, the next column on its left *tens*, the next *hundreds*, and so on. This is nothing more than preserving each figure in its proper place. Add up the units' column: if it amount to a sum expressed by only *one* figure, put this figure down under the units' column. But if it be a number of more than one figure, the *last* figure only of that number—the *units'* figure—is to be put down, and the number expressed by what is left, after rubbing out the figure thus put down, is to be *carried* to the next, or tens' column, and added in with that column.

If the sum of the tens' column be a number of a single figure, it is to be put down under that column; but if it be a number of more than one figure, then, as before, only the last, or units' figure, of that number, is to be put down, and the number which is expressed, after the figure put down is rubbed out, is to be carried to, and added in with, the figures in the next column, and so on; observing, that when the last column is reached, the entire sum of that column is to be put down. Suppose, for example, the following numbers are to be added together—namely, 246, 357, 26, 148, and 6; then, writing the numbers, one under another, as in the margin, so that the *first* column on the right may be a column of *units*, the next a column of *tens*, and the next a column of *hundreds*, we proceed, under the direction of the rule, as follows: 6 and 8 are 14, and 6 are 20, and 7 are 27, and 6 are 33; there are, therefore, in the first column, 33 units; that is to say, 3 *tens*, and 3 *units:* the 3 units we put, of course, under the column of units, but we carry the 3 *tens* to the next, or tens' column, and say—3 and 4 are 7, and 2 are 9, and 5 are 14, and 4 are 18; that is 18 *tens:* the 8 we put down, but the number left, after rubbing this out, namely 1, we carry to the next column, as it is clear we ought to do; for this 1 is one more place to the left; it stands for *one hundred*, and therefore belongs to the hundreds' column: any figure next, on the left, to a figure that stands for *tens*, must, from the principles of numeration, stand for *hundreds*. Carrying, therefore, the 1 to the hundreds' column, we say, 1 and 1 are 2, and 3 are 5, and 2 are 7; that is, 7 *hundreds:* so that the sum of the proposed numbers is 783; that is, seven hundred and eighty-three. From this operation you see that the figures of the *sum* are all carefully put in their proper places, so that each has its own *local* value; the numbers *carried* from one column to the next, are so carried because they really *belong* to the place, one in advance, to the left. Arranging these numbers one under another, as before, taking care not to disturb their local positions, we proceed thus: 3 and 2 are 5, and 8 are 13, and 3 are 16, and 2 are 18; 8 and carry 1: 1 and 5 are 6, and 7 are 13, and 6 are 19; 9 and carry 1: 1 and 7 are 8, and 7 are 15, and 8 are 23, and 4 are 27; 7 and carry 2: 2 and 7 are 9, and 4 are 13, and 8 are 21; therefore the sum is 21798: that is, twenty-one thousand seven hundred and ninety-eight. It is plain from the foregoing illustrations, that the rule for addition is in strict accordance with the system of notation and numeration already explained, and that it must always lead to the correct result. There is no *figure* higher than 9: *ten*, of any denomination (*hundreds*, *thousands*, &c.), is *one* of the next higher denomination; so that in adding up any column of figures, all of the same denomination, for every *ten* in the sum, *one* must be carried to the next column; and, therefore, as many *ones* as *tens*.

246
357
26
148
6
——
783

8462
873
758
4702
7003
——
21798

You have already seen that the marks used in the *notation* of arithmetic are *figures:* besides these, other marks are frequently employed to *indicate operations* with these figures, and to express relations among them: thus, instead of saying 2 *and* 5 *are equal to* 7, the form $2+5=7$, is used to express the same thing: the mark $+$ being the *sign for addition*, and the mark $=$ the *sign for equality:* this must be borne in mind: $+$ is called *plus;* so that $2+5=7$, may be read 2 *plus* 5 *equals* 7, or 2 *plus* 5 *are equal to* 7. The following, therefore, are statements in symbols, instead of in words, which you will at once understand: $2+5+1=8$; $3+4+2=9$; $6+5+3+1=15$. A few examples in addition are here given under this form. You will have to arrange the numbers in them in columns, as in the two examples worked above; and, if the results of your addition be correct, they will be found to agree with the numbers to the right of the sign of equality:—

(1.) $324+643+201+46=1214$.
(2.) $36+320+708+17+3=1084$.
(3.) $5684+340+7006+309+824=14163$.
(4.) $20065+8473+751+92083+564+92=122028$.

Simple Subtraction.—Subtraction teaches us how to subtract the smaller of two numbers from the greater, or to find their difference, which is called the *remainder*. The operation is called *simple* subtraction when the numbers refer to things of the same denomination, as in *simple* addition. The rule for simple subtraction is as follows:—

RULE.—Put the smaller number under the greater, taking care, as in addition, that units shall be under units, tens under tens, and so on.

Then, beginning at the *units*, subtract each figure in the lower row from the figure above it, if the lower figure be not the *greater* of the two, and put the remainder underneath (see the operation in the margin, where 34572 is subtracted from 68594, and the remainder found to be 34022).

From 68594
Subt. 34572
Rem. 34022

But if you come to a lower figure, which is *greater* than the figure above it, add 10 to the upper figure, and then subtract, putting down the remainder as before, and taking care to *carry* 1 to the next figure of the lower row.

From 64927
Subt. 27385
Rem. 37542

For example: let it be required to subtract 27385 from 64927; then, placing the former number below the latter (as in the margin), we proceed thus: 5 from 7, and 2 remain: 8 from—not 2—but 12, and 4 remain; carry 1: 4 from 9, and 5 remain; 7 from 14, and 7 remain; carry 1: 3 from 6, and 3 remain.

All that requires explanation here is the *carrying*, as in the former rule. In the preceding example we see that the 8 cannot be taken from the figure above it, because *this* is only 2: we, therefore, add 10 to the 2, converting it into 12; but the adding 10 to any figure is simply putting 1 before it; that is, it is adding 1 to the *preceding* figure, which 1, by carrying it to the next lower or subtractive figure, is taken away again at the next step. In like manner the 4, in the upper row, is converted into 14, and the one thus prefixed to it is afterwards taken away, by 1 being carried to the next lower figure, and 3 subtracted instead of 2. It is plain that in *subtraction* the carrying can never amount to more than 1.

From 130741392
Subt. 86025704
Rem. 44715688

As another example, let 86025704 be subtracted from 130741392: then, having arranged the numbers as in the margin, we proceed thus: 4 from 12, 8; carry 1: 1 from 9, 8: 7 from 13, 6; carry 1: 6 from 11, 5; carry 1: 3 from 4, 1: 0 from 7, 7: 6 from 10, 4; carry 1: 9 from 13, 4: therefore the remainder is 44715688.

There is a *sign for subtraction* as well as one for addition: it is the little mark — placed before the number to be subtracted; it is called *minus:* 5—2 is therefore 5 *minus* 2, that is, 5 *diminished* by 2; the remainder, or difference, is of course 3: so that 5—2=3. By help of the plus and minus signs, we can easily connect together in a single row a set of numbers, of which some are to be added, and others to be subtracted; thus, 4+6—3—2 means that 4 and 6 are to be *added*, and 3 and 2 are to be *subtracted;* so that 4+6—3—2=5. Instead of subtracting first 3 and then 2, we may, of course, subtract 5 at once; so that the above is the same as 10—5=5; and whenever addition and subtraction operations are indicated in this way, it will always be best to find first the sum of the *additive* quantities, then the sum of the *subtractive* quantities, and then, as in the foregoing example, to find the *difference* of the two results: in this manner the result of 125+427—684+237—15, is computed as in the margin, and found to be 90. So that 125+427—684+237—15=90.

125 684
427 15
237 ——
—— 699
789
699
——
90 result.

We shall here add a few examples to be worked in a similar manner:—

(1.) 361+483—246—179=419.
(2.) 573—184+602—67=924.
(3.) 86243+721—649—70+13—9=86249.
(4.) 12064+700628—109641+637—2604=601084.
(5.) 23596—625+72311075—13758—3506285+6879=68820882.

In order to *prove* whether subtraction is correctly performed, *add* the remainder to the number which has been subtracted—that is, to the *lower* of the two proposed numbers; the *sum* will be the *upper* number, if the work be correct: thus, in each of the two examples above, we have—

Subtractive number	27385	86025704
Remainder . . .	37542	44715688
Upper number .	64927	130741392

Simple Multiplication.—Multiplication is the method of finding the *sum* of any number of *equal* quantities, without the trouble of repeating them, one under another, and adding them up; it is a short way of obtaining the *results of addition*, when the numbers or quantities to be added are all *equal.* When the quantities are not only equal, but all of one denomination, the operation is called *simple* multiplication.

To perform this operation readily, a *table*, called the *multiplication table*, must first be learnt; and the result which arises from multiplying one number by another, provided neither be greater than 12, must be committed to memory; it is one of the few operations in arithmetic where the memory of rules is indispensable.

The number by which another is to be multiplied, is called the *multiplier;* the number which is multiplied, the *multiplicand;* and the result obtained—which, as just stated, is the same as would be got by writing down the multiplicand as often as there are units in the multiplier, and adding all up, is called the *product.* The multiplication table shows what the product is in every case in which neither multiplicand nor multiplier exceeds 12; and, by knowing this table, the product may always be found, whatever numbers be proposed as multiplicand and multiplier. It may be as well to mention here, that the numbers called by these names, when spoken of together, are generally called *factors* of the product, as they *make* or produce it: thus, 2 and 3 are *factors* of 6, since 3 taken twice, or 2 taken three times, make or produce 6.

MULTIPLICATION TABLE.

Twice		3 times		4 times		5 times		6 times		7 times		8 times		9 times		10 times		11 times		12 times	
1 are	2	1 are	3	1 are	4	1 are	5	1 are	6	1 are	7	1 are	8	1 are	9	1 are	10	1 are	11	1 are	12
2	4	2	[illegible]	2	[illegible]	2	10	2	12	2	14	2	16	2	18	2	20	2	22	2	24
3	6	3	9	3	12	3	15	[illegible]	18	3	21	3	24	3	27	3	30	3	33	3	[illegible]
4	8	4	12	4	16	4	20	[illegible]	24	4	28	4	32	4	36	4	40	4	44	4	48
5	10	5	15	5	20	5	25	5	30	5	35	5	40	5	45	5	50	5	55	5	60
6	12	6	18	6	24	6	30	6	36	6	42	6	48	6	54	6	60	6	66	6	72
7	14	7	21	7	28	7	35	7	42	7	49	7	56	7	63	7	70	7	77	7	84
8	16	8	24	8	32	8	40	8	48	8	56	8	64	8	72	8	80	8	88	8	96
9	18	9	27	9	36	9	45	9	54	9	63	9	72	9	81	9	[illegible]	9	99	9	108
10	20	10	30	10	40	10	50	10	60	10	70	10	80	10	90	10	100	10	110	10	120
11	22	11	33	11	44	11	55	11	66	11	77	11	88	11	99	11	110	11	121	11	132
12	24	12	36	12	48	12	60	12	72	12	84	12	96	12	108	12	120	12	132	12	144

That the product in any case is really what the table states it to be, the learner can easily prove for himself; he has only to take the multiplicand as often as there are units in the multiplier, and, by addition, to find the sum of all; thus, the table states that 8 times 6 are 48, which is true, because 6, written eight times, and all added, produce 48, that is,

6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 = 48.

and so of any other pair of *factors* within the limits of the table.

I. *When the Multiplier is not greater than* 12.

RULE.—Put the multiplier under the multiplicand, units under units; and, by aid of the table, multiply each figure of the multiplicand, commencing at the units' figure, by the multiplier. Set down the right-hand figure only of the product when it is a number of more than one figure, and carry as in addition.

For example: multiply 6432 by 4. The multiplier 4 being placed under the multiplicand 6432, as in the margin, we proceed thus: 4 times 2 are 8: 4 times 3 are 12; 2 and carry 1: 4 times 4 are 16, and 1 are 17; 7 and carry 1: 4 times 6 are 24, and 1 are 25.

```
Multiplicand  6432
Multiplier       4
              ————
Product . .  25728
```

A beginner, with the table before him, can easily perform operations of this kind; but he must learn to work them without looking at the table. It is as well to show him the time and trouble saved, by actually exhibiting the work of such examples by addition, as here annexed. The following, worked like the example above, require no further explanation:—

```
 6432
 6432
 6432
 6432
 ————
25728
```

```
 78326    9073214    57387    264135
     8          9       10        12
——————   ————————   ——————   ———————
626608   81658926   573870   3169620
```

The multiplication by 10, as in the third of these examples, requires, in fact, no actual work, or reference to the table. You know that a number becomes ten times as great by simply putting a 0 after the figures; this causes each figure to advance a place to the *left*, so that its local value is increased tenfold. In like manner, a number becomes multiplied by 100 when *two* 0's are added to it; by 1000 when *three* are added, and so on, as is evident from *numeration*. The *cipher*, though in itself of no value, thus plays an important part in our notation; by filling up what would otherwise be gaps between figures, it keeps them in their proper places, and preserves their local values; and by being put after a number, it has the effect of multiplying that number by 10, 100, &c., according as it is written once, twice, &c.

The *sign* for multiplication is × placed between the *factors*, thus:—(1.) 346×7=2422. (2.) 6047×5=30235. (3.) 246053×6=1476318. (4.) 53274×800=42619200. In working this fourth example, the plan is to consider 8 only as the multiplier, and to put the ciphers to the right of it, as in the margin, annexing them afterwards to the product by 8. (5.) 470329×11=5173619. (6.) 375842×12=4510104.

```
   53274
     800
————————
42619200
```

II. *When the Multiplier is greater than* 12.

RULE.—Place the multiplier under the multiplicand, units under units, tens under tens, &c.

Commencing with the units' figure, multiply by *each* in succession, and arrange the several rows of results, so that the first figure on the right in each row may be *directly under* the multiplying figure that produced it. Add up all these products, and the sum will be the complete product.

For example, if we have to multiply 426 by 34, we place the 34 under the 26, and proceed thus:—4 times 6 are 24; 4 and carry 2: 4 times 2 are 8, and 2 are 10; 0 and carry 1: 4 times 4 are 16, and 1 are 17. The first row is now completed, and we begin anew, with the next figure, 3, as multiplier, taking care to put the first figure we get in the new row directly under this 3. 3 times 6 are 18; 8 and carry 1: 3 times 2 are 6, and 1 are 7: 3 times 4 are 12. The rows are now completed, so that, drawing a line and adding up, we find the product to be 14484.

```
  426
   34
 ————
 1704
1278
—————
14484
```

You see, from the local position of our second multiplier, 3, that it is in reality 30, and 426×30=12780: adding this product to the former product, that given by the 4, as in the margin, the whole product by 34 must necessarily be the result; and you see that it agrees with that in the example.

```
 1704
12780
—————
14484
```

If our multiplier had been a number of *three* figures, as 534, then, to the *partial* products obtained we must have added the product due to the 5; which, having regard to its local value, is 500; and if we retain the noughts, the whole operation would be as here annexed. And it is plain that we may always omit the noughts, provided we take care, as the rule directs, to put the first figure of each partial product directly under the multiplying figure, which supplies that product. It is worthy of notice, too, that the product will always be the same, whichever of the two numbers be regarded as the multiplier: you may easily satisfy yourself that 426 multiplied by 534, is the same as 534 multiplied by 426. To be convinced that this principle is perfectly general, you have only to assure yourself of the fact within the limits of the multiplication table, which you may do by replacing multiplication by addition, as shown in a previous example; that is, proving to yourself that 3 times 7 is the same as 7 times 3; that 5 times 8 is the same as 8 times 5, and so on, as the table declares: because, whatever be the two factors, the multiplication of one by the other is made up only of multiplications within the limits of the table. It is in general most convenient to take that for the multiplier which gives the fewer partial products, or rows of figures. (See the operations in the margin.)

```
   426
   534
  ————
  1704
 12780
213000
——————
227484
```

```
  2647      356
   356     2647
  ————     ————
 15882     2492
13235     1424
7941     2136
——————   712
942332   ——————
         942332
```

The learner may now exercise himself in the process, by showing that the following statements are true:—

(1.) 4214×24=101136.
(2.) 658×243=159894.
(3.) 3264×2300=7507200.
(4.) 15607×3094=48288058.
(5.) Show that 243×616=9×9×11×8×7×3.
(6.) Show that 2048×1936=64×121×32×16.

When the multiplier consists of two figures, forming a number greater than 12, there are two partial products, or rows of figures, to add up; but, with a little address, the product may be written down at once, whenever the multiplier does not exceed 20. Suppose, for instance, it were 16, then, if we multiply by the 6, and, as we go on, add in not only what we *carry* from any figure of the multiplicand, but also the immediately *preceding* figure of the multiplicand, the complete product will be obtained in one line, as in the margin; the operation being carried on thus: 6 times 8 are 48, 8 and carry 4: 6 times 7 are 42 and 4 are 46 and 8 are 54; 4 and carry 5: 6 times 3 are 18 and 5 are 23 and 7 are 30; 0 and carry 3: 6 times 2 are 12 and 3 are 15 and 3 are 18; 8 and carry 1: 1 and 2 are 3. It will be advisable for the learner to practise this short way with the multipliers, 13, 14, 15, 16, 17, 18, 19. Multiplication which is thus performed in one line, is called *short* multiplication: when there are more lines, it is *long* multiplication.

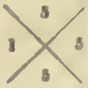
```
      2378
        16
      ————
     38048
This is the
   same as
      2378
        16
     —————
     14268
     2378
     —————
     38048
```

Method of proving multiplication by casting out nines.

We shall here mention a useful method of trying whether the product of two numbers is correct; but must postpone the explanation of the *principle* of the method till you arrive at *Algebra*. We can only mention here, that if any number be divided by 9, the remainder will be the same as would arise from dividing the *sum* of the figures in that number by 9: for 10 is equal to *once* 9+1; 100 is equal to 11 times 9+1; 1000 to 111 times 9+1; and so on: that is, the remainder arising from dividing 1, followed by any number of *noughts*, by 9, is always 1. Consequently the remainder arising from dividing 2, or 3, or 4, &c., followed by any number of noughts, is 2, or 3, or 4—the same as the figure preceding the noughts. It therefore follows, that whether we divide a number, such as 4326, which is of course 4000+300+20+6, by 9, or simply divide 4+3+2+6, that is 15—the sum of the figures—by 9, we must, in each case, get the same remainder. This property, taken in connection with the principle referred to above, and to be proved in Algebra (see the multiplication of compound quantities in Algebra), suggests the following rule:—

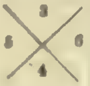
Rule.—Add together the figures of the multiplicand, not counting any 9 that may occur, rejecting also 9, whenever, in adding up, the sum amounts to 9 or more: when all the figures are added, the result will therefore be less than 9: note this result. Proceed in like manner with the figures of the multiplier; noting the result. Multiply the two results together; retaining, as before, only what is left after the rejection of all the *nines* the new result contains. Do the same thing with the figures of the product; and compare this third result with that just found: if the two be the same, the work may be presumed to be correct; if they differ, it is certainly wrong.

The usual way of noting the four results is to make a cross, to put the first in the left hand opening; the second in the opposite opening; the third above, and the fourth below. If the upper and lower results are the same, the work is *most likely* correct, but otherwise it is wrong.



Let us proceed in this way to test the accuracy of the work at page 436. Commencing at the right of the multiplicand, we say 7 and 4 are 11, therefore rejecting 9, 2 and 6 are 8 and 2 are 10: the first result, therefore, rejecting 9 from this 10, is 1, which we place in the opening of the cross to the left. Taking now the multiplier, we say 6 and 5, 11; 2 and 3, 5, the second result, which we place opposite the former. The product of the two is 5, with no 9 to reject: this is the third result, to be placed above. Lastly, taking the product, we say 2 and 3 are 5, and 3 are 8, and 2 are 10: 1 and 4 are 5; which is the fourth result, and, as it agrees with the preceding, we conclude the work to be correct.

It is plain, however, that if any of the figures in the product were made to exchange places, the agreement of the third and fourth results would remain, though the product would be wrong; as would also be the case if one figure of it were increased and another diminished, by the same number: all, therefore, that we can safely infer, is, that the agreement spoken of *must* have place if the work be correct; so that if it fail the work is wrong. Suppose, for instance, that we had made 73084163×7584=554270392192: then, applying the test, we get, from the first factor, the result 5; from the second, the result 6; and from the product of these, the result 3: but, from the above-stated product of the two numbers, the result is 4: this product, therefore, is incorrect; and, upon revising the multiplication, we find that the 3, after the nought, should have been a 2.



Simple Division.—The operation by which we find how many times one number or quantity is contained in another number or quantity of the same kind, is called *division.* It is also the operation by which we find the 4th part, the 5th part, &c., of a number or quantity. The number or quantity divided is called the *dividend;* that by which we divide it, the *divisor;* and the result obtained, the *quotient.*

You must not fall into the common mistake of considering the quotient to express always *how many times* the dividend contains the divisor: the 4th part of a mere number tells us how many times that number contains 4; but the fourth part of a *quantity*—a sum of money, for instance—is just the fourth part, and nothing else:—it is itself also a sum of money. The division is called *simple* when the quantities concerned are of but one denomination. On coming to the division of *compound* quantities, you will find some further remarks on the true nature of division in general; at present both dividend and divisor, and therefore the quotient, are to be regarded as mere *numbers.* They are, in fact, only abstract expressions for the idea of quantity.

I. *When the Divisor is not greater than* 12.

Rule.—Place the divisor to the *left* of the dividend, with a mark of separation, thus), between the two.

Draw a line beneath the dividend, and, by the multiplication table, find how many times the divisor is contained in the first figure of the dividend, or in the number expressed by the first *two* figures, or even in the number expressed by the first *three* figures, should the number given by the first, and even by the first two, be *smaller* than the divisor; and write the quotient under the line, taking care to observe what is *over*, as the divisor may be contained a certain number of times in the number expressed by the leading figure or figures, and leave something *over.*

Proceed to the next figure of the dividend; regard what was over, if anything, to be *prefixed* to it; and find how many times the divisor is contained in the number you thus get; putting the quotient down, and, as before, carrying what is over to the next figure of the dividend, to which you must regard it as *prefixed.* And in this way figure after figure of the complete quotient is to be found, till all the figures of the dividend have been used. Should there be anything over at the end, this is called the *remainder:* it is to be written beside the quotient figures, with the divisor placed under it, and a line of separation between them.

$$\begin{array}{r} 3)25602 \\ \hline 8534 \\ \hline \end{array}$$

Suppose, for example, we have to divide 25602 by 3, then placing dividend and divisor (3) as in the margin, we proceed thus: 3 is contained in 2, *no* times; so that nothing is to be placed under the 2: 3 is contained in 25, 8 times and 1 over; 8 and carry 1: this 1, regarded as *prefixed* to the 6, gives the number 16: we therefore say: 3 in 16, 5 times and 1 over: 3 in 10, 3 times and 1 over: 3 in 12, 4 times. Therefore, the quotient is 8534; and this is the *complete* quotient, as there is no *remainder.*

$$\begin{array}{r} 5)7804623 \\ \hline 1560924\frac{3}{5} \end{array}$$

Again, suppose it were proposed to divide 7804623 by 5, we should say 5 in 7, 1; and 2 over: 5 in 28, 5; and 3 over: 5 in 30, 6: 5 in 4, 0: 5 in 46, 9; and 1 over: 5 in 12, 2; and 2 over: 5 in 23, 4; and 3 over. As there is here a remainder 3, we annex it, with the divisor 5 under it, to the figures of the quotient, and call $1560924\frac{3}{5}$, the complete *quotient.*

The principle upon which the foregoing operation depends is pretty evident: the leading figure in the dividend above is 7000000: the fifth part of this is 1000000 and 2000000 over; that is, with the local value of the next figure 8, 2800000; the fifth part of this is 500000, and 300000 besides; the fifth part of which is 60000: the fifth part of the 4000—the local value of the next figure—is 0 thousands, and 4000 over; this, with the local value of the 6, is 4600; of which the fifth part is 900, and 100 over; this, with the 20, is 120; the fifth part of which is 20, and 20 over; and lastly, the fifth part of the remaining 23 is 4, and 3 over; and, to imply that this 3 still remains to be divided, it is put down with the 5 underneath; because one number, placed in this way under another, is a form frequently used to denote that the *upper* number is to be divided by the *lower.* Hence the fifth part of the proposed number is 1560924, and the fifth part of 3 besides: this quotient being made up of the several parts which arise from taking a fifth of each of the above-mentioned component portions of the number.

The *sign* for division is ÷, which stands for the words *divided by:* thus, $6 \div 2 = 3$ is a short way of stating that 6 divided by 2 is equal to 3. As noticed above, there is another way of indicating division, namely, by putting the dividend above and the divisor below, a short line separating the two: thus, $\frac{6}{2} = 3$ expresses the same thing as the notation above. The learner may exercise himself in the rule just explained by proving by it the truth of the following statements expressed in one or other of the forms of notation here adverted to:

(1.) $2185095 \div 3 = 728365.$
(2.) $2208184 \div 8 = 276023.$
(3.) $47051412 \div 6 = 7841902.$
(4.) $\frac{462508}{4} = 115627.$
(5.) $\frac{835465}{6} = 139244\frac{1}{6}.$
(6.) $\frac{13900424}{7} = 1985774\frac{6}{7}.$
(7.) $\frac{2375920}{11} = 215992\frac{8}{11}.$
(8.) $\frac{46285439}{12} = 3857119\frac{11}{12}.$
(9.) $\frac{65283029}{10} = 6528302\frac{9}{10}.$

It is plain that division by 10 requires no work: the quotient is always the dividend itself, wanting the last figure, which is the *remainder*, and which, therefore, written as in this last example, with the divisor underneath, completes the quotient. In a similar way, to divide by 100 we have simply to cut off *two* figures from the dividend for remainder; to divide by 1000, to cut off *three* figures; and so on: thus—

$$\frac{78546}{100} = 785\tfrac{46}{100},\ \frac{78546}{1000} = 78\tfrac{546}{1000},\ \&c.$$

All this is obvious, because

$$78546 = 78500 + 46 = 78000 + 546,\ \&c.$$

II. *When the Divisor is greater than* 12.

RULE.—Place the divisor to the left of the dividend as in the former case, and to the right mark off a place for the figures of the quotient.

Find how many times the *leading figure* of the divisor is contained in that of the dividend, or in the number expressed by the first *two* figures, if the leading figure of the dividend be smaller than that of the divisor; and put the figure expressing the number of times in the quotient's place.

Multiply the divisor by this first quotient-figure, and subtract the product from the number formed by the leading figures of the dividend, and to the remainder annex the next figure of the dividend. The number thus formed will be a new dividend, and the number of times it contains the divisor—to be found as before—will be the second quotient-figure, the product of which and the divisor, being subtracted from the new dividend, will give a second remainder, to which the *next* figure of the original dividend is to be joined, and the operation continued till all the figures of the dividend have been used.

An example worked at length will explain the operation better than any verbal rule.

Let it be required to divide 256438 by 346.

```
346)256438(741
    2422
    ————
     1423
     1384
     ————
      398
      346
      ———
       52
```

Placing the divisor on the left of the dividend, and marking off a place for the quotient on the right, we look at the leading figure of the divisor and also at that of the dividend, with the view of seeing whether the latter contains the former, which it does not, 3 being greater than 2: we therefore commence with the number 25, formed by the first *two* figures of the dividend, and seeing that 3 is contained in 25, 8 times, we should put 8 for the first quotient-figure; but bearing in mind, that when the *whole* divisor is multiplied by this 8, we must attend to the *carryings*, we perceive that 8 is too great, we therefore try 7, and find 7 times 346 to be 2422, a number *less* than 2564 above it, so that we can obey the direction of the rule and *subtract:* the remainder is 142, which, when the next figure of the dividend is brought down, becomes 1423. We now take *this* as a dividend; and, looking only at *leading figures* in this new dividend and in the divisor, we see that the latter *will go*, as it is called, 4 times; we therefore put 4 for the second quotient-figure; and multiplying and subtracting, we get 39 for the second remainder; and, by bringing down another figure, 398 for a new dividend: the divisor goes into this *once;* so that the quotient is 741, and the final remainder 52: this remainder, as in the former case, must be annexed, with the divisor underneath, to the quotient-figures; so that the complete quotient is $741\frac{52}{346}$, which is the 346th part of 256438. Of the truth of this you may convince yourself by observing that 256438 has been cut up into portions, and the 346th part of each portion found; for the work above is nothing else but that here annexed, with useless repetitions suppressed. According to this arrangement it is at once seen that 700 is the 346th part of 242200, that 40 is the 346th part of 13840, and that 1 is the 346th part of 346, and that of 52, the 346th part, still remains to be taken. Now, $242200 + 13840 + 346 + 52 = 256438$; consequently 741, together with $\frac{52}{346}$, is the 346th part of the number proposed.

```
346)256438(700
    242200
    ——————
346) 14238(40
     13840
     —————
346)   398(1
       346
       ———
        52
```

It must be noticed that if any *dividend*, formed by a remainder and a figure brought down, should be *less* than the divisor, that the divisor will go *no times* in that dividend; so that a 0 will be the corresponding quotient-figure; and that then a *second* figure must be brought down, as in the operation here annexed; where the complete quotient is $102\frac{21}{472}$.

```
472)48165(102
    472
    ———
      965
      944
      ———
       21
```

There is another thing also to be attended to. Sometimes the divisor ends with zeros or noughts: when such is the case, the best way is to cut the ciphers off, and entirely to disregard them in the division, cutting off, however, at the same time, as many figures from the end of the dividend, which latter figures help to form the final remainder: you will see by operating on the same example first with the ciphers retained, and then with the ciphers dismissed, that nothing is omitted but useless ciphers: the complete quotient being, by either way, $61\frac{126}{2700}$. In thus completing the quotient, by means of the final remainder, take care to restore the ciphers that were temporarily cut off from the divisor: in some books on arithmetic this has not been named. The following examples are subjoined for practice:—

```
2700)164826(61
     16200
     —————
      2826
      2700
      ————
  Rem. 126

27.00)1648.26(61
      162
      ———
       28
       27
      ———
  Rem. 126
```

(1.) $\frac{13301}{47} = 283$
(2.) $159894 \div 658 = 243.$
(3.) $\frac{278643}{35} = 7961\frac{8}{35}.$
(4.) $780967 \div 3700 = 211\frac{267}{3700}.$
(5.) $\frac{36326599}{1342} = 27069\frac{1}{1342}.$
(6.) $3939040647 \div 6889 = 571787\frac{4}{6889}.$

To *prove* whether the quotient, in any case, is correct, it is only necessary to multiply it and the divisor together: the product will be the dividend, if the operation be correct, as is obvious; for the object of division is to find a number which by its multiplication with the divisor shall equal the dividend. In thus proving division, if there be a *remainder*, this is to be added to the product of the divisor and quotient-figures. For example, to prove whether the work above is correct, we multiply and add as in the margin; and as the result is the same as the dividend, we may be sure that the quotient is right.

```
    472
    102
    ———
    944
   472
     21 Rem.
  —————
  48165
```

III. *When the Divisor is composed of Factors, none of which exceeds 12.*

The operation just explained is called *long* division, to distinguish it from the shorter process of the preceding case, where the divisor is not greater than 12. If any divisor be found to be the product of factors, each of which does not exceed this limit, the division may be performed by successive applications of the shorter rule: for we may divide first by one factor, then the quotient by another factor, the new quotient by a third factor; and so on, till all the factors have been used.

It is possible that the division by the first factor may leave a remainder; if so, it must, of course, be preserved: the division by the second factor may also leave a remainder; if so, multiply it by the *first* divisor, and add in the former remainder; the result will be the complete remainder as far as the operation has been carried: if there be a third division, and a third remainder, we must multiply it by both the *first and second* divisors, adding in the former complete remainder: and so on, till all the divisions are completed. For example, suppose we have to divide 38214 by 63: then since $63=7\times9$, we may operate as in the margin: the final quotient being $606\frac{36}{63}$. In like manner, if we have to divide 24611 by 126; then, since $126=3\times6\times7$, the operation by short division, is that here annexed: the remainder from the first division is 2; that from the second is 1; and this 1, multiplied by the first divisor 3, and the former remainder being taken in, gives 5 for the complete second remainder: the remainder from the third division is 2, which multiplied by 3, and by 6, *both* the former divisors, that is by 18, gives 36; which, with the preceding remainder 5, makes 41;—the final complete remainder: hence the complete quotient is $195\frac{41}{126}$.

```
7) 38214
   ------
9)  5459...1
   ------
     606...36rem.

3) 24611
   ------
6)  8203...2
   ------
7)  1367...5
   ------
     195...41rem.
```

The method here described of obtaining the final remainder, and thence completing the final quotient, cannot be clearly explained till some knowledge of *fractions* be acquired: parts of a whole, as one-half, one-third, two-fifths, &c., and which are denoted by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, &c., are called *fractions:* when you are a little acquainted with the management of these, the reason of the foregoing directions will be plainly seen.

Upon the principles now delivered depend all the operations of arithmetic. In what has preceded, they have been applied only to what are called *abstract* numbers, without any reference to particular objects or articles. It remains to show the application of the same principles to *concrete* quantities; that is, to real commodities, or things—as to money, weights, measures, &c. And, in order to this, a few *Tables* connected with these matters must first be given.

We may urge on the student the necessity of learning the whole of these tables by heart. He will find such to afford him great assistance; and it will prevent the necessity of frequent reference to them.—ED.

TABLES OF MONEY, TIME, WEIGHTS, AND MEASURES.

I.—MONEY.

Gold Coins.

Name.	Value. s.	d.	Weight troy. dwt.	gr.*
Sovereign (*one pound*)	20	0	5	$3\frac{171}{623}$
Half-sovereign	10	0	2	$13\frac{397}{623}$
Guinea†	21	0	5	$9\frac{39}{89}$
Half-guinea	10	6	2	$16\frac{64}{89}$

Silver Coins.

Name.	Value. s.	d.	Weight troy. dwt.	gr.
Crown	5	0	18	$4\frac{4}{11}$
Half-crown	2	6	9	$2\frac{2}{11}$
Shilling	0	12	3	$15\frac{3}{11}$
Sixpence	0	6	1	$19\frac{7}{11}$
Fourpenny-piece	0	4	1	$5\frac{1}{11}$
Threepenny-piece	0	3		$21\frac{9}{11}$

Pence Table.

	s.	d.		s.	d.
4 farthings make	0	1	80 pence make	6	8
12 pence ,,	1	0	84 ,, ,,	7	0
20 ,, ,,	1	8	90 ,, ,,	7	6
24 ,, ,,	2	0	96 ,, ,,	8	0
30 ,, ,,	2	6	100 ,, ,,	8	4
36 ,, ,,	3	0	108 ,, ,,	9	0
40 ,, ,,	3	4	110 ,, ,,	9	2
48 ,, ,,	4	0	120 ,, ,,	10	0
50 ,, ,,	4	2	130 ,, ,,	10	10
60 ,, ,,	5	0	132 ,, ,,	11	0
70 ,, ,,	5	10	140 ,, ,,	11	8
72 ,, ,,	6	0	144 ,, ,,	12	0

A farthing—that is, *one-fourth* of a penny—is represented thus, $\frac{1}{4}d.$; a halfpenny, thus, $\frac{1}{2}d.$; and three farthings, thus, $\frac{3}{4}d.$ To express a fraction of a farthing, the letter *f* is put against the fraction: thus, $\frac{1}{2}f.$ means half a farthing; $\frac{3}{5}f.$, *three-fifths* of a farthing, &c.

II.—TIME.

60 seconds	make 1 minute.
60 minutes	,, 1 hour.
24 hours	,, 1 day.
7 days	,, 1 week.
52 weeks 1 day, or 365 days	,, 1 year.
366 days	,, 1 *leap* year.

III.—AVOIRDUPOIS WEIGHT.

16 drams (*dr.*)	make 1 ounce.
16 ounces (*oz.*)	,, 1 pound.
28 pounds (*lb.*)	,, 1 quarter.
4 quarters (*qr.*), or 112 lb.	,, 1 hundredwt.
20 hundredweight (*cwt.*)	,, 1 ton.

IV.—TROY WEIGHT.

24 grains (*gr.*)	make 1 pennywt.
20 pennyweights (*dwt.*)	,, 1 ounce.
12 ounces (*oz.*)	,, 1 pound.

V.—APOTHECARIES' WEIGHT.

20 grains (*gr.*)	make 1 scruple.
3 scruples (℈)	,, 1 dram.
8 drams (ʒ)	,, 1 ounce.
12 ounces (*oz.*, or ℥)	,, 1 pound.

VI.—MEASURES OF LENGTH, OR LONG MEASURE.

12 inches	make 1 foot.
3 feet	,, 1 yard.
6 feet	,, 1 fathom.
5½ yards	,, 1 rod, pole, or perch.
4 perches, or 100 links	,, 1 chain (22 yards).
40 poles	,, 1 furlong.
8 furlongs	,, 1 mile.
3 miles	,, 1 league.

Cloth.

2¼ inches	make 1 nail.
4 nails	,, 1 qr. of a yard.
5 quarters	,, 1 English ell.
3 quarters	,, 1 Flemish ell.

VII.—MEASURES OF SURFACE, OR SQUARE MEASURE.

144 square inches	make 1 square foot.
9 square feet	,, 1 square yard.
30¼ square yards	,, 1 sq. rod, pole, or perch.
40 square perches	,, 1 rood.
4 roods, or 160 sq. perches	,, 1 acre.
10 sq. chains, or 100,000 sq. links	,, 1 acre.
640 acres	,, 1 square mile.
100 square feet	,, 1 square of flooring.
272¼ square feet	,, 1 sq. rod of brick-work.

---

* The mark *dwt.* stands for *pennyweights*, and *gr.* for *grains:* see the table of Troy weight. The reader can scarcely require to be informed that £ stands for *pounds*, *s.* for *shillings*, and *d.* for *pence*.

† The *coin* guinea has been long abolished, but the *name* is still retained for 21*s*. The name *pound* is given to 20*s*., because the quantity of silver in this sum originally weighed a pound troy.

VIII.—MEASURES OF SOLIDS, OR CUBIC MEASURE.*

1728 cubic inches	make	1 cubic foot.
27 cubic feet	,,	1 cubic yard.

IX.—MEASURES FOR LIQUID AND DRY GOODS.

4 gills	make	1 pint.
2 pints	,,	1 quart.
4 quarts	,,	1 gallon.
2 gallons	,,	1 peck.
4 pecks	,,	1 bushel.
8 bushels	,,	1 quarter.
2 cwt. of coals	,,	1 sack.
10 sacks	,,	1 ton.

It may be well to notice here, that the *avoirdupois* pound contains 7,000 grains, of which 5,760 make a pound *troy;* so that 144 pounds avoirdupois are equal to 175 pounds troy. The ounce troy *exceeds* the ounce avoirdupois by 42½ grains. The gallon contains 10 pounds avoirdupois of distilled water, and its solid measure is 277 cubic inches and 274 thousandths of an inch.

Reduction.—Arithmetic is now to be applied to *concrete* quantities, such as those named in the foregoing tables: hitherto its operations have been confined to *abstract* numbers. The name *reduction* is given to the methods by which quantities are changed to others of the same values, but of different denominations; as, for instance, the changing, or *reducing*, pounds to shillings, pence to farthings, yards to miles, minutes, hours, &c., to years—and so on. There are two rules for such reductions: the one applying when the quantity is to be converted from a higher to a lower denomination—as, for instance, from pounds to pence; and the other applying when the change is to be from a lower denomination to a higher, as from pence to pounds.

I. To reduce a Quantity to one of Lower Denomination.

RULE.—From the previous table see how many of the *next* lower denomination make 1 of the higher; multiply by this *number:* the product will be the number of quantities of the next lower denomination. If any of the lower denomination be connected with the proposed quantity, the number of these must be added in with the product.

Suppose, for example, we have to reduce £136 8*s.* 4¾*d.* to pence. Then, as 20*s.* make £1, we multiply the number 136 by the number 20, adding in the number 8: the product is 2728, the number of shillings. Again, since 12 pence make 1*s.*, we multiply the last number obtained by 12, taking in the 4: the product is 32740, the number of pence. And lastly, multiplying this by 4, because 4 farthings make 1*d.*, and taking in the 3, the number of farthings, we get 130963 for the number of farthings required.

£ s. d.
136 8 4¾
20
2728 numb. of shillings.
12
32740 numb. of pence.
4
130963 numb. of farthings.

Although we have been dealing here with *concrete* quantities, yet, after all, our operations are performed entirely with *abstract* numbers. We do not multiply £136 by 20, because we should then get £2728 for the product; much less do we multiply by 20 *shillings* (as some books direct us to do); for to attempt to *multiply* by *shillings* is to attempt an absurdity: "20 shillings times 136 pounds," is a mode of expression as ridiculous as it is meaningless.

As a second example, let it be required to reduce 217 days 14 hours and 36 minutes to minutes. Since 24 hours make 1 day, we multiply the number 217 by 24; and in adding in the 14, we include the *units* in the units' amount of the product—that is, in the first result of the first partial product—and the *tens* (1) in the first result of the second partial product. We thus get 5222, the number of hours; this number we multiply by 60, because 60 minutes make 1 hour, and we add in the 36—units with units, and tens with tens, as before: and we thus find the number of minutes to be 313356.

d. h. m.
217 14 36
24
872
435
5222
60
313356

Sometimes we have to multiply by a *fraction;* as, for instance, when perches of *length* are to be reduced to yards; for you see by the *table* that 5½ yards make 1 perch: also, in reducing *square perches* to *square yards*, we have to multiply by 30¼, the number of square yards in 1 square perch. Now, to *multiply* by ½ means simply to take *half* the multiplicand; that is, to *divide* it by 2; and to multiply by ¼, means to take a *fourth part*, or to divide the multiplicand by 4. This is certainly a departure from the primitive meaning of the word *multiply;* but it is sanctioned by common practice. It is customary to speak of two-and-a-half times this, or three-and-a-quarter times that, and so on: thus, two-and-a-half times 4 we know to mean 10; and two-and-a-quarter times, 9. The way to introduce such fractional parts in the arithmetical operation will be sufficiently seen from the two examples worked in the margin; the first being to reduce 248 *linear* perches to *linear* yards, and the second to reduce 248 *square* perches to *square* yards. If the number of perches had been 249, the multiplier ½ would have given 124½, and the multiplier ¼, 62¼. By aid of the tables, which ought, indeed, to be committed to memory, you will easily be able to show the truth of the following statements, namely:—

Linear perches.
2) 248
5½
1240
124 for ½
1364 yards.

Square perches.
4) 248
30¼
7440
62 for ¼.
7502 sq. yds.

(1.) 13*s.* 4*d.* = 160*d.*
(2.) £32 1*s.* 6*d.* = 7698*d.*
(3.) £5 12*s.* 4½*d.* = 5394 farthings.
(4.) 27 cwt. 2 qr. 22 lb. = 3102 lb.
(5.) 17 lb. 6 oz. 14 dwt. *troy* = 4214 dwt.
(6.) 131 miles 3 furlgs. 10 perches 3 yds. = 231278 yds.
(7.) 29 days 3 hours 21 minutes = 41961 minutes.
(8.) 37 acres 3 roods 12 perches = 183073 yards.
(9.) 239¼ gallons = 7664 gills.
(10.) 327 square perches = 9891¾ square yards.
(11.) 263 tons 18 cwt. 3 qr. 21 lb. = 591241 lb.

To reduce a Quantity to one of Higher Denomination.

RULE.—Find by the table how many of the given denomination make 1 of the next higher, and divide by this number; the quotient will express how many of the next higher denomination are in the proposed quantity. In like manner, divide by the number expressing how many of the new denomination make 1 of the next higher to it; and so on, till the required denomination is reached. Suppose, for instance, we had to find how many pounds there were in 2640397 farthings. Dividing the number of farthings by 4, we get the number of pence—namely, 660099, and one farthing over. Dividing the number of pence by 12, we get the number of shillings—namely, 55008, and three pence over; and lastly, dividing by 20, we get the number of pounds—namely, 2750, and 8*s.* over. Consequently, in the proposed number of farthings, there are £2750 8*s.* 3¼*d.*

4) 2640397
12) 660099...¼*d.*
2.0) 5500.8...3*d.*
£2750 8*s.* 3¼*d.*

Again: let it be required to convert 591241 lb. into tons, cwt., &c. As 28 lb. make 1 qr., the next higher denomination to pounds, we divide first by 28, or by 7 and by 4, the two *factors* of 28, as it is better to use *short* division: we thus get 21115, the number of quarters, with 21 lb. over. This number, divided by 4, gives the number of cwts.—namely, 5278 and 3 qrs. over: and lastly, dividing by 20, the number of cwt. in 1 ton, we get finally 263, the number of tons: so that there are 263 tons 18 cwt. 3 qr. 21 lb. in 591241 lb.

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* A *cube* is a solid of six equal square faces, like a common die. If the edge of the figure be 1 inch, the solid is a *cubic* inch; while each face is a square inch. In a similar solid of which the edge is 1 foot, there are 1728 of the smaller cubes, or cubic inches.

7) 591241
4) 84463
4) 21115...21 lb.
2.0) 527.8...3 qrs.
263 t. 18 cwt. 3 qr. 21 lb.

All this is so easy and obvious that we need not occupy space with any more worked-out examples. We shall merely give one cautionary direction—it is this: that when you have to *divide* by 5½, bring both this divisor and the dividend into *halves*; that is, double both; making the divisor 11, instead of 5½; but remember that the *remainder* will be so many *halves*. In like manner, when you have to divide by 30¼, bring all into *quarters*; that is, divide 4 times the dividend by 121, which is 4 times 30¼; remembering, however, that the *remainder* will be *quarters*; so that a fourth part of the number, which is the remainder, will be the number of *wholes*. See the operation in the margin, where the *factors* of 121, viz., 11, 11, are used to get the quotient by short division. This quotient shows that there are 81 square perches, and 51 *quarter-yards* over; that is, 12¾ square yards: the result would therefore be written, 81 square perches, 12¾ square yards.

Square yards.
2463
4
11) 9852
11) 895...7
81...51

The examples given at page 440, may be employed for exercise in this rule, by taking in each the quantity on the right of the sign of equality, and converting it into that on the left; but others are added here:—

(1.) 28635 seconds = 7 h. 57m. 15 sec.
(2.) 10085760 gr. = 1751 lb. troy.
(3.) 633600 inches = 10 miles.
(4.) 397024 yds. = 225 mi. 4 fur. 26 per. 1 yd.
(5.) 91476 sq. ft. = 2 ac. 0 rds. 16 per.
(6.) 100000 cub. in. = 2 cub. yds. 3 ft. 1504 in.

The four fundamental operations of arithmetic may now be applied, in order, to *compound* quantities.

Addition of Compound Quantities.—To add together a set of concrete quantities of different denominations, the rule is as follows:—

RULE.—Arrange the quantities to be added one under another, so that all in the same vertical column may be of the same denomination.

Add up the quantities of *lowest* denomination: find how many of the next denomination are contained in the sum: put the *remainder* under the column, and carry the quotient to the next column.

Proceed in this way, from column to column, till all have been added up.

The principle of this rule is too obvious to require any explanation; the carryings merely transfer the quantities of advanced denominations to the columns in which those denominations are arranged, just as in the addition of abstract numbers.

£	s.	d.
17	9	3¼
42	13	4¾
16	10	2½
7	2	9½
1	18	10½
85	14	6½.

Thus the sums of money in the margin are arranged so that the denomination *farthings* forms one column, the denomination *pence* the next, *shillings* the next, and *pounds* the next. The sum of the farthings' column is 10 farthings, in which are contained 2 pence, and there are 2 farthings, or ½ over; this ½ is therefore put down, and the 2 pence carried to the pence column; the sum of this column is 30 pence, that is, 2*s.* 6*d.*: the 6*d.* is put down, and the 2*s.* carried to the shillings' column, the sum of which is 54*s.*, that is, £2 14*s.*; we therefore say 14, and carry 2; and this 2 added in with the pounds' column, makes the amount of that column £85; therefore the sum of the whole is £85 14*s.* 6½*d.*

It may be noticed, that in adding up the shillings' column of an account, the best way is to disregard the *tens* in that column till all the *units* have been added; then, having reached the top unit-figure, to proceed downwards, taking in every *ten* that appears. Thus, in the present example, the sum of units' column of shillings is 24; so that proceeding downwards, taking in each *ten* as we meet with it, we say 34, 44, 54; so that the sum is 54*s.*, or £2 14*s.*

lb.	oz.	dr.
8	13	11
9	10	13
4	6	9
11	11	15
7	3	8
41	14	8

Two other examples are here annexed; the one in *Avoirdupois weight*, and the other in *Time*. In the former the sum of the drams is found to be 56 dr.; by reduction, we find that in these drams there are 3 oz. 8 dr.; we therefore put down 8 dr. and carry the 3 oz. to the next column, which gives 46 oz., or 2 lb. 14 oz.; writing down the 14 oz., and carrying the 2 lb. to the column of lbs., we get 41 lb. for the sum of this column: therefore the whole sum is 41 lb. 14 oz. 8 dr.

d.	h.	m.	s.
34	13	9	15
18	9	0	37
27	21	11	19
14	18	23	4
10	7	14	16
13	14	21	19
119	11	19	50

In the next example the column of seconds amount to 110 seconds; that is, to 1 minute 50 seconds: the 50 seconds is put down, and the 1 minute carried to the next column, the amount of which is 79 minutes; that is, 1 hour 19 minutes, 19 and carry 1: the hour column amounts to 83 hours, or 3 days 11 hours; 11 and carry 3 to the day's column, the amount of which is 119: therefore the whole amount is 119 days, 11 hours, 19 minutes, 50 seconds.

Subtraction of Compound Quantities.—The subtraction of concrete quantities, of different denominations, is effected by the following rule:—

RULE.—Place the less of the two quantities under the greater, arranging the denominations as in addition.

Commence with the *lowest* denomination, and subtract, if the upper number be sufficiently great; if not, increase it by as many as will make 1 of the *next* denomination, and *then* subtract, taking care afterwards to carry 1, as in the subtraction of abstract numbers: and proceed in like manner with each denomination till the subtraction is finished.

£	s.	d.
124	16	9½
75	19	3¾
48	17	5¾

In this way the difference between £124 16*s.* 9½*d.* and £75 19*s.* 3¾*d.* is found, as in the margin. Since 3 farthings cannot be taken from 2 farthings, we increase the 2 farthings by 4 farthings, or 1*d.*, and say 3 from 6 and 3 remain; that is, ¾*d.*: carry 1: 4 from 9 and 5 remain: 19 from 36 (increasing the 16*s.* by 20*s.**) and 17 remain; carry 1: 6 from 14 and 8 remain; carry 1: 8 from 12 and 4 remain; therefore the difference is £48 17*s.* 5¾*d.*

m.	fur.	per.	yd.
43	0	1	1
24	6	21	2
18	1	19	4½

Again, suppose we have to subtract 24 miles, 6 furlongs, 21 perches, 2 yards from 43 miles, 1 perch, 1 yard. Then, having placed the quantities as in the margin, and seeing that the 1 yard is too small, we increase it by 1 perch, that is by 5½ yards, and subtract 2 yards from 6½ yards; we thus get the remainder 4½ yards; and carry 1: and as 40 perches make 1 furlong, we subtract 22 from 41, and get 19 for remainder: carrying 1 to the 6 we subtract 7 from 8—the furlongs in 1 mile—and get 1 for remainder: and carrying 1 to the 4, it merely remains to subtract 25 from 43: the complete remainder is therefore 18 miles, 1 furlong, 19 perches, 4½ yards.

If the complete remainder in an operation of this kind be added to the compound quantity immediately above it—that is, to the subtractive quantity—the sum will be equal to the upper row; that is, to the quantity which has been diminished: so that we may prove in this way the correctness of the subtraction. The following examples, if thus worked and proved, will afford exercise both in subtraction and addition :—

---

* Instead of thus increasing the upper term by the unit of next higher denomination, the learner will find it a little easier to subtract at once from this unit, expressed in the lower denomination, and to *add* the remainder to the term above: thus we may say, 19 from 20, 1; and 16 make 17.

(1.) Subtract £374 11s. 8¼d. from £920 17s. 7½d.
(2.) Subtract £173 9s. 4¾d. from £200.
(3.) Subtract 8 lb. 4oz. 23 gr. from 23 lb. 11 oz. 21 gr.
(4.) Subtract 342 mls. 6 fur. 4 per. 4 yds. from 687 mls. 3 fur. 1 per.
(5.) Subtract 324 gall. 2 quarts 1 pint from 570 gallons 1 quart.
(6.) Subtract 3 roods 7 perches 23 yards from 2½ acres.
(7.) Subtract 121 sq. yds. 7 ft. 132 in. from 237 sq. yds. 3 ft. 101 in.
(8.) Subtract 18 cubic yds. 37 ft. 211 in. from 47 cubic yds. 13 ft. 75 in.

Multiplication of Compound Quantities.—From the nature of multiplication, it is plain that a concrete quantity can be multiplied only by *an abstract number;* indeed —whatever be the multiplicand—the multiplier, which simply denotes how many times the former is to be taken, must necessarily be a mere number. Strange to say, however, books on arithmetic, of the most recent date, are to be found, in which the multiplication together of *concrete* quantities is insisted upon, and pretended to be taught. People have disputed over and over again about the product of £19 19s. 11¾d., multiplied by itself! They might as well have disputed about the multiplication of Cheapside by Lombard Street; or, as Mr. Walker pithily expresses it, about multiplying "5lbs. of beef by 3 bars of music."* This last operation, palpably absurd as the thing is, the arithmeticians referred to would not for a moment hesitate to undertake, provided the beef and music occurred in a rule-of-three question, as indeed they very well might; for they refer to the rule-of-three in justification of such a process.† Is it not ridiculous to appeal to a *rule* instead of to reason and common sense, in a subject which professes to be founded on rational principles, and supported by demonstration? We wish to warn you earnestly against this: receive no rule in any department of mathematics, the truth of which is not evident to your own understanding; and, therefore, in strict accordance with common sense. Multiplication is merely a short way of doing addition, and *addition* may always replace it: you have only to write the multiplicand down the proposed number of times, and to add all up. The *sum* is what in multiplication is called the *product;* but how could a sum of money be written down £19 19s. 11¾d. *times?* Even in common reduction, similar absurdities are to be met with in the books. If you wish to convert pounds into shillings, you are told to multiply the *pounds* by 20, and sometimes, which is worse, to multiply them by 20s. But if you multiply pounds by 20, you get—not shillings —but 20 times as many *pounds,* as is obvious: what you really do, is to multiply the *number* denoting *how many* pounds by 20; because there must be 20 times that *number* of shillings.

The following is the rule for multiplying a compound quantity by a number :—

RULE I.—*When the Multiplier is not greater than* 12.

Put the multiplier under the quantity of least denomination: multiply that quantity by it, and divide the product by the *number* that expresses how many of such quantities make 1 of the next denomination: put down the *remainder,* and carry the quotient to the product arising from the multiplication of the next term—and so on till all the terms have been multiplied.

When the multiplier is greater than 12, and is yet such as to admit of being formed from *factors,* multiply by each factor in succession, as in short multiplication.

The table of factors at the end will be found very useful in enabling us to tell at a glance whether any number not exceeding 100,000, can be decomposed into factors, within the limits of the multiplication table; and if so, what the factors are.

Multiply £23 14s. 7¾d. by 7. Putting the multiplier 7 under the farthings, and multiplying them by the 7, the product is 21 farthings; and dividing 21 by 4, the number of farthings in a penny, we get 5 and 1 over; so that in 21 farthings there are 5 pence and 1 farthing: we put down the 1 under *farthings,* and carry the 5 pence to the 49, the pence product; which gives 54 pence, or 4s. 6d.: we put down the 6d., and carry the 4 to the shillings product, and thus get 102 shillings, or £5 2s; and putting down the 2s., we carry the 5 to the pounds' product. The complete product is thus £166 2s. 6¼d.

£	s.	d.
23	14	7¾
		7
166	2	6¼

Suppose the multiplier, had been 105, then seeing by the table that 105 = 7 × 5 × 3; after the multiplication by 7, as above, we should have again multiplied by 5, and then by 3, as in the margin; from which we see that 105 times £23 14s. 7¾d. is £2491 17s. 9¾d.

£	s.	d.
23	14	7¾
		7
166	2	6¼
		5
830	12	7¼
		3
2491	17	9¾

RULE II.—*When the Multiplier exceeds* 12, *and is not divisible into factors, each less than* 13.

Take that number in the table which is *nearest* to the proposed multiplier, whether greater or less, and use the factors of this number. To the final product *add,* if the number be *less,* and *subtract* from it if the number be greater, the product arising from multiplying the given quantity by the *difference* between the multiplier and number taken from the table; the result will obviously be the complete product required.

For example, if the multiplier of the sum above had been 107 instead of 105, we should still have taken 105, and have used the factors of it, as just shown; but to the product by these factors, we should have *added* twice the multiplicand; we should thus have got 105 times the sum and twice the sum, that is 107 times the sum as proposed. If the multiplier had been 109, then from 112 times, that is, from 8 × 7 × 2 times, we should have *subtracted* 3 times the original multiplicand as in the margin, and should thus have found £23 14s 7¾d. × 109 = £2586 16s. 4¾d.

£	s.	d.
23	14	7¾ × 3
		8
189	17	2
		7
1329	0	2
		2
2658	0	4
71	3	11¼
2586	16	4¾

The following statements are left for the learner to verify after the manner now shown :—

(1.) £8 18s. 6d. × 6 = £53 11s. 0d.
(2.) £148 7s. 0¼d. × 9 = £1335 3s. 2¼d.
(3.) £148 7s. 0¼d. × 63 = £9346 2s. 3¾d.
(4.) 6s. 10¼d. × 97 = £33 6s. 10½d.
(5.) 15 mil. 3 fur. 2 per. 4 yds. × 75 = 1153 mil. 6 fur. 4 per. 3 yds.

Division of Compound Quantities.—Division of concrete quantities may be viewed under two aspects, accordingly as the divisor is itself a concrete quantity or merely abstract number.

If you have to divide by a concrete quantity, your object is to find *how many times* the smaller quantity—the divisor—is contained in the larger—the dividend. But if you have to divide by an abstract number, you then seek to divide the proposed quantity into *as many equal parts* as there are units in the divisor. These, you see, are two different objects; and precision and accuracy of thought require that you should bear in mind the distinction. When you divide one concrete quantity by another, your quotient is, of course, an *abstract number:* but when you divide a concrete quantity by an abstract number, your quotient is also a *concrete quantity* of the same kind. You will remember that we are not writing

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* *Philosophy of Arithmetic,* p. 58.

† Paganini was a very wonderful performer on the violin. Many people would have given a good deal of beef for a few bars of his music. Suppose, in time of need, he had exchanged 11 bars for 5lb., how many lb. might have been exchanged, at the same rate, for 3 bars? This is a rule-of-three question, and there are plenty of books—Walkingame for instance —that would direct the following *stating* :—

Bars of music.		lb. of beef.		bars of music.		lb. of beef.
11	:	5	::	3	:	1 4/11

And to get this 1 4-11 lb. of beef, they would direct the beef and the music, in the second and third terms, to be multiplied together! The author of this would thus incorporate the beef and music.

a book on Arithmetic, exclusively with a view to mercantile practice. Our endeavour is to prepare you for a course of mathematical study; and we therefore wish you to cultivate habits of thought and reflection—to know what you are actually about, and not to feel contented by merely following a *rule*. We shall not insist upon any marked departure from the customary forms of expression, in the practical directions for working an example; but must insist upon accuracy of *thought*, whatever want of precision in *language* custom may authorise.

To divide a compound quantity by a *number*, the rule is this:—

RULE.—Commence with the *highest* denomination, and take the proposed part of *it*; reduce what is over to the next denomination, and carry the result to the next term of the dividend; take the proposed part of the sum, reducing what is over, and carrying as before; and so on, to the end.

Thus, if the 7th part of £22 15*s*. 9*d*. be required, we find it as in the margin: the 7th part of £22 is £3 and 20*s*. over: this, carried to the 15*s*., gives 35*s*.; the 7th part of which is 5*s*., and there is nothing to carry: the 7th part of 9*d*. is 1*d*.; and 2*d*., or 8 farthings, over; the 7th part of which is 1 farthing, and $\frac{1}{7}$ of a farthing; hence, the 7th part of the proposed sum is £3 5*s* $1\frac{1}{4}d. + \frac{1}{7}f.$

```
   £   s.  d.
 7)22  15  9
 ------------
    3   5  1¼+⅐f.
```

If the divisor exceed 12, we must proceed, upon the same principle, by long division, unless it can be decomposed into convenient factors; when the operation need to consist only of successive steps like that explained above.

When the divisor, instead of being an *abstract number*, is a *concrete quantity*, of the same kind as the dividend, the rule is as follows:—

RULE.—Reduce both dividend and divisor to the *lowest denomination* found in *either*, and then perform the division exactly as in the case of mere numbers: the quotient will denote the *number of times* the smaller quantity is contained in the greater.

For example, let it be required to divide £63 7*s*. by £13 2*s*. 3*d*. Then, as *pence* is the lowest denomination that occurs, we reduce *both* quantities to *pence*, and then divide as in the margin: the quotient shows that the smaller sum is contained in the larger between 4 and 5 times: it is contained in it 4 times and a *fractional part* of a time, represented by $\frac{2616}{3147}$, which is nearly another time, but not quite. You may shorten the work a little by reducing the two quantities—not to *pence*, but to *three-pences*—as shown below; observing, that as 4 three-pences make 1 shilling, we multiply the number of shillings by 4, and take in the odd threepence. From this mode of working, we should conclude that the dividend contains the divisor 4 times and a part of a time, denoted by the fraction $\frac{872}{1049}$, which differs from the former fraction only in appearance—not in value; for if we wish to express that one number is to be divided by another, we may, as you are aware, do so by writing the latter below the former, or by writing twice, three times, &c., the latter below twice, three times, &c. the former. This is pretty obvious, since the quotient of dividend and divisor is the same, whatever number *both* be multiplied by. It will be seen that the upper and lower numbers of the first fraction are only those of the second, each multiplied by 3.

```
  £  s. d.     £  s.
 13  2  3     63  7
 20           20
 ---          ----
 262          1267
  12            12
 ---          ----
3147)        15204(4
             12588
             -----
              2616
```

```
  £  s. d.     £  s.
 13  2  3     63  7
 20           20
 ---          ----
 262          1267
   4             4
 ---          ----
1049)         5068(4
              4196
              ----
               872
```

There is no room in this treatise for many examples. We shall, however, give two. The first is to show that £33 19*s*. 6*d*. divided by 13 gives £2 11*s*. 6*d*. for quotient; the second is to show that the same sum divided by £13 gives $2\frac{319}{520}$ for quotient. In working the second example, you had better reduce to *sixpences*, not to *pence*.

Fractions.—What has preceded suffices to convey a general, and, we hope, a pretty accurate notion of the arithmetic of *integral* quantities. We are now to show how the fundamental operations are to be applied to *fractions*. It has been impossible to avoid all allusion to fractions in the foregoing part of the subject, because they force themselves upon our notice even when operating upon *integers*; but the arithmetic of fractions remains to be explained, and, indeed, the formal definition of a fraction to be given. In strictness, a fraction is a part of a *whole*—that is, it is *less* than the quantity of which it is said to be a fraction. Thus, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{26}{43}$, are strictly fractions—*proper* fractions. The first denotes a third part of unit, or 1; the second a fifth part of 2; the third a forty-third part of 26; each part being *less* than one whole. But $\frac{4}{3}$, $\frac{7}{5}$, $\frac{64}{43}$, are also called fractions, though four-thirds, seven-fifths, sixty-four forty thirds, are all greater than one whole, as is plain. Fractions such as these, where the upper number, called the *numerator*, is not less than the lower, called the *denominator*, are said to be *improper fractions*. You will readily see why these terms, numerator and denominator, are so applied: the upper number *enumerates*, or states the number of parts of that particular *denomination* indicated by the lower number. Thus, $\frac{3}{4}$ means *three* of the parts called *fourths*: if it were $\frac{3}{4}$ of £1, then, since one-fourth is 5*s*., three-fourths, or $\frac{3}{4}$, would be 15*s*., and so on. Instead of reading this fraction *three-fourths*, we may, if we please, say *three divided by four*. Three pounds divided by the number 4, is evidently the same as three-fourths of one pound; and any fraction may be viewed in either of these two ways: thus it is matter of indifference whether we call $\frac{5}{7}$, *five-sevenths*, or 5 *divided by* 7. A moment's reflection will convince that five-sevenths of anything, is the same as a seventh part of *five* such things; for a seventh part of one of them added to a seventh part of another, then again this sum increased by a seventh part of another, and so on, till a seventh part of each of the five has been taken, and all these sevenths added, will be equal to all five divided by 7.

The fractional notation is perfectly general—*any number* may be expressed in it; a whole number, or an *integer*, as well as a fraction properly so called. Thus 6, 8, &c., may be written $\frac{6}{1}$, $\frac{8}{1}$, &c.; and it is sometimes convenient to write integers this way. *Here* the denominator is *unit*, or 1; but we may express an integer in the form of a fraction with *any* denominator. Thus, if 7 for denominator be chosen, the two numbers, 6, 8, may be written $\frac{42}{7}$, $\frac{56}{7}$, as is evident: you have only to multiply the number by the chosen denominator, and to place the factor, thus used as a multiplier, underneath—that is, as a divisor. The numerator and denominator are called the *terms* of the fraction; and when an integer is united to a fraction, the whole is called a *mixed* number. Thus, $2\frac{1}{3}$; $3\frac{5}{7}$, &c., are *mixed numbers*.

To reduce a Mixed Number to an Improper Fraction.

The rule is this: multiply the integer by the denominator of the fraction; add the product to the numerator, and put the denominator underneath. Thus, $2\frac{1}{3} = \frac{7}{3}$, $3\frac{5}{7} = \frac{26}{7}$; for 2 is evidently $\frac{6}{3}$, and $\frac{6}{3} + \frac{1}{3} = \frac{7}{3}$. In like manner 3 is $\frac{21}{7}$, and $\frac{21}{7} + \frac{5}{7} = \frac{26}{7}$; and so on. Here are other examples: $5\frac{3}{5} = \frac{28}{5}$; $4\frac{7}{8} = \frac{39}{8}$; $7\frac{13}{9} = \frac{76}{9}$; $12\frac{8}{11} = \frac{140}{11}$. To accomplish the contrary purpose—that is,

To reduce an Improper Fraction to a Mixed Number,

You have only to perform the division indicated by the denominator, and to annex to the quotient the fractional correction as in common division. Thus, $\frac{28}{5} = 5\frac{3}{5}$, $\frac{39}{8} = 4\frac{7}{8}$, $\frac{76}{9} = 8\frac{4}{9}$, and so on.

To reduce Fractions with Different Denominators, to others of the same Value, with Equal Denominators.

This is one of the most important operations in the arithmetic of fractions; for till fractions appear with a *common denominator*, they can neither be added to nor subtracted from one another: the reduction of fractions to a common denominator is thus a preliminary, indispensably necessary, for the application to them of the first rules of arithmetic. The operation is also useful in enabling us to discover at a glance which of two fractions, however nearly equal, is really the greater. Thus, of the two fractions, $\frac{33}{44}$, $\frac{32}{44}$, we see in a moment which is the greater, *because* their denominators are the same; but we could not so readily and confidently state which is the greater of $\frac{3}{4}$ and $\frac{8}{11}$: yet the former are only these reduced to a common denominator—the values are the same.

The rule for reducing fractions to a common denominator is as follows:—

RULE.—Multiply *each* numerator by the product of the denominators of all the *other fractions*; we shall thus get the *numerators* of the changed fractions.

Multiply *all* the denominators together; the product will be the *common denominator* belonging to each changed numerator.

For example: in order to reduce the fractions $\frac{2}{3}$, $\frac{1}{5}$, $\frac{3}{7}$, to others of the same value with the same denominator, we proceed as follows:—

$$\left.\begin{aligned} 2 \times 5 \times 7 &= 70 \\ 1 \times 3 \times 7 &= 21 \\ 3 \times 3 \times 5 &= 45 \end{aligned}\right\} \text{the new numerators;}$$
$$3 \times 5 \times 7 = 105 \quad \text{the common denominator;}$$

therefore the equivalent fractions, changed in form as required, are $\frac{70}{105}$, $\frac{21}{105}$, $\frac{45}{105}$.

If you compare these with the original fractions, you will see that they each arise from multiplying the numerator and denominator of the former by the same number. Thus, $\frac{70}{105}=\frac{2 \times 35}{3 \times 35}$; and it is obvious that one number divided by another (in this case 2 by 3), is the same as 35 times the former divided by 35 times the latter, or any number of times the former divided by the same number of times the latter. If you have any doubt of this, just consider, if you had to divide 2*s.* among 3 people, whether the *share* of each would not be the same as if you had to divide 35 times 2*s.*—that is, 70*s.*—among 35 times 3 people—that is, 105 people. It is plain that, in either case, each would get a third part of 2*s.*, or two-thirds of 1*s.*; or, to view the matter more generally, it is self-evident, that if you *multiply* any quantity by a number, and then *divide* by the same number, you virtually leave the quantity, as to *value*, untouched; for multiplication and division by the *same* number, are two operations which mutually neutralise one another. We may, therefore, always multiply numerator and denominator of a fraction by any number, without changing the value of the fraction.

The rule just given will *always* effect the object proposed by it; but not always in the shortest way. In particular cases it will be desirable to proceed differently. Thus, if the fractions $\frac{2}{3}$, $\frac{4}{6}$, $\frac{1}{2}$, are to be changed into equivalent ones with a common denominator, you see, by looking at the denominators, that the thing may be brought about without interfering with the middle fraction at all: you have only to multiply the terms of the first fraction by 2, and those of the third by 3, to get the desired result—the changed fractions being found in this way to be $\frac{4}{6}$, $\frac{4}{6}$, $\frac{3}{6}$. If you had applied the *rule*, the new fractions would have been $\frac{24}{36}$, $\frac{24}{36}$, $\frac{18}{36}$, forms far less simple than those above, although the same in *value*; they would be got by multiplying the terms of the simpler fractions, each by 6. In bringing fractions to a common denominator, you should always be on the look-out for the simplest multiplier of the terms of each that will accomplish the object, and use the *rule* only as matter of necessity—that is, only when simpler multipliers than the rule supplies, do not present themselves. Suppose you had $\frac{3}{8}$, $\frac{5}{6}$, $\frac{5}{3}$, do you not see, from a glance at the denominators, that if the first be multiplied by 3, the second by 4, and the third by 8, that the products will be all alike? Multiply, then, the terms of the first fraction by 3, those of the second by 4, and those of the third by 8, and you will get the following—viz., $\frac{9}{24}$, $\frac{20}{24}$, $\frac{40}{24}$, for equivalent fractions with a common denominator. The *rule* would have given you these—$\frac{54}{144}$, $\frac{120}{144}$, $\frac{240}{144}$, which, although equal to, are far less simple than the former.

The *smallest* number that can be a common denominator of a row of fractions is evidently the smallest number that is divisible by each of the given denominators: it is called the *least common multiple* of those denominators. There is a rule for finding the least common multiple of a set of numbers; but you see that it may often be discovered, without any rule, by a little reflection. We shall give you but one more instance here, since the reduction of fractions to a common denominator, as observed above, will form a necessary preparation for addition and subtraction. Let the fractions be $\frac{2}{3}$, $\frac{5}{9}$, $\frac{1}{6}$, $\frac{3}{2}$: here you see that the first two are brought to a common denominator by merely multiplying the terms of the first by 3; so that these two fractions are $\frac{6}{9}$, $\frac{5}{9}$. Again, the last two are brought to a common denominator by merely multiplying the terms of the fourth by 3; so that these two fractions are $\frac{1}{6}$, $\frac{9}{6}$. We have, therefore, now only to find the *least* number which will divide by 9 and 6; and it requires but little thought to discover that 18 is that number; so that we reach the desired result by the following steps:—

$$\begin{aligned} &\tfrac{2}{3},\ \tfrac{5}{9},\ \tfrac{1}{6},\ \tfrac{3}{2} \\ \text{or, }&\tfrac{6}{9},\ \tfrac{5}{9},\ \tfrac{1}{6},\ \tfrac{9}{6} \\ \text{or, }&\tfrac{12}{18},\ \tfrac{10}{18},\ \tfrac{3}{18},\ \tfrac{27}{18} \end{aligned}$$

where you see that a fraction in either of the lower rows is merely that above, with its *terms* multiplied by the same number. If you had applied the rule to the first row of fractions, you would have got $3 \times 9 \times 6 \times 2 = 324$ for a common denominator, instead of the more simple number 18.

Addition and Subtraction of Fractions.—RULE.—Reduce the fractions to equivalent ones, having a common denominator: then add or subtract the *numerators*, as proposed, and put the common denominator under the result.

For instance, let the fractions be $\frac{4}{9}$ and $\frac{2}{7}$: these, reduced to a common denominator, are $\frac{28}{63}$ and $\frac{18}{63}$, therefore $\frac{28}{63}+\frac{18}{63}=\frac{46}{63}$, their *sum*: $\frac{28}{63}-\frac{18}{63}=\frac{10}{63}$, their *difference*.

Again, let it be required to find the value of $\frac{3}{5}+\frac{2}{9}-\frac{1}{3}$. Here the second and third fractions will have a common denominator, if the terms of the third are multiplied by 3: the differing denominators will then be 5 and 9; that is, we shall have $\frac{3}{5}+\frac{2}{9}-\frac{3}{9}=\frac{3}{5}-\frac{1}{9}$; that is, by the rule for the common denominator, $\frac{27}{45}-\frac{5}{45}=\frac{22}{45}$.

Suppose the value of $1-\frac{8}{15}-\frac{9}{20}$ were required. It is easy to see that the denominators will be made alike if the first be multiplied by 4 and the second by 3; so that, multiplying numerators, as well as denominators, we have $1-\frac{32}{60}-\frac{27}{60}=1-\frac{59}{60}=\frac{60}{60}-\frac{59}{60}=\frac{1}{60}$.

And in like manner are the results following obtained: (1.) $\frac{1}{2}+\frac{2}{7}=\frac{11}{14}$. (2.) $\frac{1}{7}-\frac{1}{11}=\frac{4}{77}$. (3.) $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}=2\frac{1}{4}$. (4.) $\frac{16}{21}-\frac{8}{15}=\frac{8}{35}$. (5.) $6\frac{3}{4}-5\frac{1}{3}=1\frac{5}{12}$. (6.) $11\frac{2}{3}+8\frac{7}{9}-9\frac{19}{22}=10\frac{115}{198}$.

In the subtraction of mixed quantities, it sometimes happens that the fractional part of the subtracted quantity is greater than the fractional part of that from which it is to be taken: when this is the case, it is better to convert a unit of the latter into a fraction, and to incorporate it with the fractional part: we shall thus have an improper fraction, from which the subtractive fraction may be taken; thus, if we had to take $3\frac{5}{6}$ from $5\frac{3}{8}$, we see, when the fractions are brought to a common denominator, by multiplying the terms of the first by 4, and those of the second by 3, that the subtractive fraction $\frac{20}{24}$ is greater than $\frac{9}{24}$, the other fraction; we therefore *fractionise* a unit of the 5, considering 5 to be $4\frac{24}{24}$, and therefore $5\frac{3}{8}$, or $5\frac{9}{24}$, to be $4\frac{33}{24}$; so that the work stands thus: $5\frac{3}{8}-3\frac{5}{6}=5\frac{9}{24}-3\frac{20}{24}=4\frac{33}{24}-3\frac{20}{24}=1\frac{13}{24}$.

Multiplication and Division of Fractions.—Multiplication.—If we have to multiply a fraction by a whole number, the product will, of course, be as many times that fraction as there are units in the whole number: thus, $\frac{2}{5} \times 3 = \frac{6}{5}$; that is, 3 times *two-fifths*: the *denominator* is not operated upon, because *this* merely states the *denominations*, not the *number* of them. If, instead of 3, the multiplier had been only a fourth part of 3, that is $\frac{3}{4}$, then only a fourth part of the above product,

$\frac{6}{5}$, could have resulted. A fourth part of $\frac{6}{5}$ is $\frac{6}{20}$, because a number (in this instance 6) divided first by 5, and then again by 4, gives the same result as a single division of it by 20. We know, therefore, that $\frac{2}{5}\times\frac{3}{4}=\frac{6}{20}$; that is, it is the product of the numerators, divided by the product of the denominators.

The same reasoning evidently applies, whatever be the fractions to be multiplied together: the product of the numerators, divided by the product of the denominators, is the product of the fractions.

Division.—Let us now try to discover how fractions are to be divided; and first let us consider the case in which the divisor, like the multiplier, in the foregoing instance, is a whole number.

If we have to perform the operation $\frac{2}{5}\div3$, we see that the quotient of 2 by 5 is to be divided by 3: but this we know, from simple division, is the same as the quotient of 2 by 15; for, if anything be divided by 5 and then the quotient by 3, the result is the same as we should get by dividing that thing at once by 5×3 or 15; therefore, $\frac{2}{5}\div3=\frac{2}{15}$.

But if, instead of 3, the divisor were $\frac{3}{4}$, that is, only a *fourth part* of 3, it is plain that the quotient ought to be 4 *times as great*; namely, $\frac{8}{15}$; so that $\frac{2}{5}\div\frac{3}{4}=\frac{8}{15}$; the same result as we should get by *inverting* the divisor $\frac{3}{4}$, and *multiplying*, instead of *dividing*; for $\frac{2}{5}\times\frac{4}{3}=\frac{8}{15}$. And as the reasoning evidently applies, whatever be the fractions chosen, we infer that division of fractions may always be converted into multiplication by inverting the terms of the divisor; that is, in fact, by turning the divisor upside down. Thus, $\frac{2}{5}\div\frac{6}{7}=\frac{2}{5}\times\frac{7}{6}=\frac{14}{30}$. We are thus led to the following rules, viz.:—

RULE.—*For multiplication.*—Multiply the numerators together for the numerator of the product.

Multiply the denominators together for the denominator of the product.

RULE.—*For division.*—Invert the terms of the divisor, and then proceed as in multiplication.

It is desirable that fractions which appear in the results of any operations, should be reduced to their lowest terms; that is, that both numerator and denominator should be divided by whatever number *will* divide them. Thus, in the instance above, where the product of $\frac{2}{5}$ and $\frac{3}{4}$ was found to be $\frac{6}{20}$, the result should be freed from the factor 2, common to both numerator and denominator; that is, we should divide both terms by 2, and write the result in the more simple form, $\frac{3}{10}$. The division of both terms by the same number cannot alter the *value* of the fraction, otherwise the *multiplication* of the terms of a fraction by the same number would alter its value, which we know to be not the case.

In multiplication and division, we may often prevent the entrance of these superfluous factors in the result; and it is of course better to do so than to allow them to enter, and then to remove them; thus, in multiplying $\frac{2}{5}$ by $\frac{3}{4}$ we foresee that, as 2 occurs in a numerator and 4 in a denominator, 2 will also occur in both numerator and denominator of the product, unless we previously prevent its entrance: this we should do by regarding the proposed fractions as $\frac{1}{5}$, $\frac{3}{2}$; the product of which is $\frac{3}{10}$. All the factors that enter alike into numerators and denominators, should thus be removed, for you then get your product in the simplest form at once, without being at the trouble to reduce it to lower terms. We shall here give examples of this:

(1.) $\frac{7}{8}\times\frac{6}{11}=\frac{7}{4}\times\frac{3}{11}=\frac{21}{44}$. (2.) $\frac{8}{9}\times\frac{3}{5}=\frac{8}{3}\times\frac{1}{5}=\frac{8}{15}$. (3.) $\frac{4}{5}\div\frac{16}{15}=\frac{4}{5}\times\frac{15}{16}=\frac{1}{1}\times\frac{3}{4}=\frac{3}{4}$. (4.) $\frac{11}{4}\div\frac{5}{6}=\frac{11}{4}\times\frac{6}{5}=\frac{11}{2}\times\frac{3}{5}=\frac{33}{10}=3\frac{3}{10}$. (5.) $\frac{19}{20}\div\frac{3}{5}=\frac{19}{20}\times\frac{5}{3}=\frac{19}{4}\times\frac{1}{3}=\frac{19}{12}$.

In this way the entrance of common factors into the numerator and denominator of each result in the following examples is to be provided against:—

(1.) $\frac{7}{8}\times\frac{12}{13}=\frac{21}{26}$. (2.) $\frac{13}{60}\div\frac{26}{50}=\frac{5}{12}$. (3.) $\frac{3}{10}\div\frac{1}{30}=9$. (4.) $\frac{2}{3}\div\frac{3}{11}=2\frac{4}{9}$. (5.) $\frac{13}{14}\div\frac{13}{15}=1\frac{1}{14}$. (6.) $\frac{7}{8}\times\frac{8}{7}=1$.

In these examples, the rules are applied to pure fractions. If we have to deal with mixed quantities, then we must reduce them to improper fractions before using either rule: thus $3\frac{2}{5}\div2\frac{1}{7}=\frac{17}{5}\div\frac{15}{7}=\frac{17}{5}\times\frac{7}{15}=\frac{119}{75}=1\frac{44}{75}$.

Proportion.—Four quantities are said to be in *proportion* when the first divided by the second is the same *abstract number* as the third divided by the fourth: thus, the four numbers, 6, 3, 8, 4 are in proportion, because $\frac{6}{3}=\frac{8}{4}$; and of any two equal fractions, the four terms are in proportion.

The quotient which arises from dividing one quantity by another *of the same kind*, is called the *ratio* of the former to the latter: thus, the ratio of 6 to 3 is 2, and the ratio of 8 to 4 is 2; *ratio* being only another name for the *quotient* of two quantities of the same kind. A proportion is thus said to be an equality of ratios: *ratio* implies two terms; *proportion*, four. The first term of a ratio (the dividend) is called the *antecedent*, the second (the divisor) its *consequent*. Instead of writing the antecedent as the numerator and denominator of a fraction to express the ratio, the same thing is indicated by simply putting two dots between them; thus, 6 : 3 is the same as $\frac{6}{3}$; and 8 : 4 the same as $\frac{8}{4}$; so that the proportion above may be expressed in this way, 6 : 3 = 8 : 4. Instead of the sign of equality, it is more common to use four dots, and to write the proportion thus: 6 : 3 :: 8 : 4, which is read, 6 is to 3 as 8 is to 4; a form of expression intended to imply that 6 is related to 3, in point of magnitude, just as 8 is related to 4. This idea of relative or comparative magnitude, which is essential to the correct notion of proportion, forbids our considering the term *ratio* in the same unrestricted sense as the term *quotient*: the two terms are to be regarded as meaning the same thing only when dividend and divisor are quantities of the same kind: *ratio is always an abstract number*. Now, the name quotient, as we have seen, is applied not only to abstract numbers, but to the concrete quantities that arise from taking the third, fourth, fifth, &c. part of concrete quantities. Be careful to observe this distinction; and to remember that the ratio of an antecedent to its consequent *always* has reference to the *number of times*, and parts of a time, by which the former contains the latter: so that it would be absurd to speak of the ratio of one thing to another of a different kind; as, for instance, of the ratio of £6 to the *number* 3; or of 8 cwt. to £4, and so on.

It thus appears, that if four quantities, ranged in order as above, form a proportion, the first and second must necessarily be of the same kind; and also that the third and fourth must be of the same kind: thus the following are proportions; they express equal ratios, the ratio in each case being the abstract number 2—

£6 : £3 :: 8 cwt. : 4 cwt.; again, 6 yds. : 3 yds. :: 8 oz : 4 oz.

From what has now been said, you see that the essential condition, and the only condition which four quantities must fulfil, in order that they may form a proportion, is this, namely—

$$\frac{\text{First term}}{\text{Second term}}=\frac{\text{Third term}}{\text{Fourth term}}$$

each of these fractions being an abstract number. If two fractions are equal, we know that two equal fractions will also result from inverting their terms; so that from the above we may infer that

$$\frac{\text{Second term}}{\text{First term}}=\frac{\text{Fourth term}}{\text{Third term}}$$

These being abstract numbers, we may multiply *any* quantity by either: the results will, of course, be equal, whichever we take for the multiplier, since they themselves are equal. Let us multiply "Third term" by each: the results will be

$$\frac{\text{Second term}}{\text{First term}}\times\text{Third term}=\text{Fourth term}.$$

You can have no doubt about the second result, as you know that divisor, multiplied by quotient, gives dividend; and the fraction above, on the right, denotes the *quotient* of the division of "Fourth term" by "Third term."

It appears, then, that in order to find the fourth term of a proportion, when the first three terms are given, we have only to divide the second by the first, and to multiply the third by the abstract number furnished by their quotient. Now, if the first and second terms are concrete quantities, you know that you *cannot* divide one by the other, and thus get the abstract number here spoken of, till the quantities are reduced to the same denomination.* When, however, this is done, you may entirely disregard *denomination*, and consider the resulting numbers to be *abstract* numbers; for the *quotient* is the same in both cases: thus $\frac{£6}{£3}$ is the abstract number 2; and so is $\frac{6}{3}$, or $\frac{6 \text{ cwt.}}{3 \text{ cwt.}}$, or $\frac{6 \text{ yds.}}{3 \text{ yds.}}$

This, though a very obvious, is yet an important fact; as we learn from it that, when the first and second terms of a proportion are reduced to the same denomination, we may consider both numerator and denominator of the fraction $\frac{\text{Second term}}{\text{First term}}$ as abstract numbers, as well as the fraction itself; and therefore that, without violating any principle of arithmetic, we may write the above equality, namely—

$$\frac{\text{Second term}}{\text{First term}} \times \text{Third term} = \text{Fourth term}$$

in the form

$$\frac{\text{Second term} \times \text{Third term}}{\text{First term}} = \text{Fourth term}$$

which is often the more convenient in practice; that is, after the reduction to the same denomination, as spoken of above, we may multiply the third term by the second (regarded as a *number*), and divide the product by the *number* denoting the first term.

We have thus established the general principle of what is called the *Rule of Three*, the object of which is, when three terms of a proportion are given, to find the fourth.

Rule of Three.—1. Write the three given terms in a row, taking care that the *third* term is a quantity of the same kind as the required *fourth* term; and also that, according as this fourth term is to be greater or less than the third, so may the *second* of the given terms be greater or less than the first. This is called *stating* the question.

2. Having thus properly stated the question, bring the first and second terms to the same denomination; and regard the results as *abstract numbers*, the *denomination* being suppressed.

3. Multiply the third term by the second, which is now a *number*; and divide the product by the first, which is also a number: the quotient will be the required fourth term, in the same denomination, of course, as that in which the third term was used.

You may sometimes simplify this operation: for the first and second terms may each be divisible by the same number; in which case you may employ only the quotients instead of the quantities themselves, on the principle that a fraction is not altered in value by discarding factors common to numerator and denominator. You may, also, in like manner, divide the first and third terms, when possible, on the same principle; and we would recommend you, in rule-of-three operations, to be always on the look-out for these means of simplification.

We shall now show the application of the rule by some examples.

1. If 16 cwt. cost £42 8*s.*, how much will 26 cwt. cost?

```
             £  s.   £  s.
16 : 26 :: 42  8 : 68 18
 8 : 13     20
           ----
           848
            13
           ----
          2544
           848
           ----
        8)11024s.
        ---------
       2,0)137,8s.
       -----------
        £68 18s.
```

As the answer or fourth term of the proportion is to be *money*, the £42 8*s.* given in the question must be the third term.

As the greater the number of cwt. the greater will be the cost, we must arrange the first two terms of the proportion, so that the second may be the greater; consequently, the stating will be 16 cwt. : 26 cwt. :: £42 8*s.* : the required fourth term. But as the first and second terms are already in the same denomination, namely, cwt., no reduction of them is necessary; we therefore, in the stating, entirely disregard this common denomination, and insert merely the abstract numbers, 16 and 26, as in the margin. But, a glance at these two numbers shows that each is divisible by 2; we, therefore, replace them by the quotients 8 and 13; and it now only remains to multiply £42 8*s.* by the number 13, and to divide the product by 8, to get the sum of money required. For convenience, we reduce the given sum to shillings; thus bring out the answer or required fourth term in shillings, and then convert these into pounds. When the work is finished, as here annexed, we complete the proportion, in the stating, by putting the result, viz., £68 18*s.* for the fourth term.

You see that, by dividing the first and second terms in the stating by 2, we have effected a little saving of figures in the work: but we might have simplified further, and have shortened the operation still more, by dividing the third term and the reduced first term both by 8; so that, having regard to the utmost attainable simplicity, we should have proceeded with the preliminary statings as here annexed, and thus have reduced the subsequent work to the small amount of figures here shown; 13 times 6*s.* is 78*s.*; that is, £3 18*s.*: putting down the 18*s.*, and carrying the £3, we have 13 times £5 = £65; and the £3 carried makes £68. Here is another example.

```
              £  s.
16 : 26 :: 42  8
 8 : 13
 1 : 13 ::  5  6
              13
          -------
          £68 18s.
```

2. If 28 persons reap a harvest in 36 days, how many will be required to reap it in 21 days?

As the answer is to be number of persons, the 28 persons given in the question must be the third term of the proportion; and as the fewer the days the greater must be the number of workmen, we arrange the first two terms of the proportion so that the second may be the greater; the stating is, therefore, 21 days : 36 days :: 28 persons : the number of persons required.

```
          Persons. Persons.
21 : 36 ::  28   :  48
 7 : 12
 1 : 12 ::   4
            12
            --
            48 persons.
```

But, as the first two terms are in the same denomination, we suppress *denomination*, and use only the abstract numbers 21 : 36. These we see have a common divisor, 3; we therefore replace them by the quotients 7 : 12; but the 7, and the 28 in the *third* term will divide by 7; we thus get the stating in the simple form 1 : 12 :: 4 persons; and then proceed as in the margin.

One example more must suffice.

A mass of 106 lb. of Australian gold, sold at the rate of £3 6*s.* 8*d.* per ounce: how much did it fetch?

```
                   £  s.  d.   £
1 oz. : 106 lb. :: 3  6   8 : 4240
          12       20
        ----      ----
        1272       66
         800       12
        ----      ----
    12)1017600    800
    ----------    ----
     2,0)8480,0
      --------
       £4240
```

Stating the question as in the margin, putting the greater weight in the second term, because the greater sum must be in the fourth, we see that the first and second terms differ in denomination; we must, therefore, reduce the second to ounces, before we can regard the stating in the proper form for working with: it then becomes 1 oz. : 1272 oz. :: £3 6*s.* 8*d.*

The common denomination *ounces* is now suppressed, and, for convenience, the money is reduced to *pence*, the denomination in which the required fourth term must therefore appear: we have then merely to multiply 800 *pence* by the abstract number 1272; and the required value comes out 1017600 pence; for, as the first term is 1, there is no division.

Instead of making 1272 the multiplier of 800, we make 800 the multiplier of 1272, for convenience; as we know that the *number*, furnished for the product, is the same in one case as the other; this *number*, therefore, represents so many *pence*; which, when reduced to *pounds*, is £4240.

---

* See *ante*, p. 443.

The following are a few examples for exercise:—

(1.) If 57 cwt. cost £216, what will 95 cwt. cost?—Ans. £360.

(2.) If 148 gallons cost £119 10*s*., how many gallons will £89 12*s* 6*d*. buy?—Ans. 111.

(3.) What is the value of 2 qr. 24 lb. at £5 7*s*. 4*d*. per cwt.?—Ans. £3 16*s*. 8*d*.

(4.) What is the income of a person who pays £22 7*s*. 5*d*. for income tax, at the rate of 7*d*. in the pound?—Ans. £767.

(5.) $44\frac{1}{2}$ guineas used to be coined out of 1 lb. of standard gold: how many sovereigns are now coined out of this weight?—Ans. $46\frac{29}{40}$.

(6.) 66*s*. are coined out of 1 lb. of standard silver: what is gained in coining £100 of silver, if the price of the silver be 5*s* 2*d*. per oz.?—Ans. £6 $9\frac{1}{31}$*s*.

The Double Rule of Three.—The double rule of three is so called because there are at least *two* single rule-of-three statings implied in it. The following is an example, namely:—

If 12 horses plough 11 acres in 5 days, how many horses will plough 33 acres in 18 days?

This may be divided into two single rule-of-three questions; thus—1st. If 12 horses plough 11 acres, how many will plough 33 acres *in the same time?*

$$11 : 33 :: 12 \text{ horses} : \frac{12 \times 33}{11} \text{ horses} = 36 \text{ horses.}$$

2nd. If 36 horses can perform a work in 5 days, how many can perform *the same* in 18 days? By the former rule,

$$18 : 5 :: 36 \text{ horses} : \frac{36 \times 5}{18} \text{ horses} = 10 \text{ horses.}$$

It is plain that by these two single rule-of-three operations, the correct answer to the question is obtained; but it is more readily obtained by the following arrangement:—

$$\left.\begin{matrix} 11 : 33 \\ 18 : 5 \end{matrix}\right\} :: 12 \text{ horses} : \frac{12 \times 33 \times 5}{11 \times 18} \text{ horses} = 10 \text{ horses.}$$

The fourth term of this *compound proportion*, as it is called, being got by multiplying the third term by the product of the *consequents*, and then dividing by the product of the *antecedents*; and it is by the same multiplications and divisions that the final result is arrived at in the two distinct statings above. This more compact form of working is described in the following rule:—

RULE.—Put for the third term that one of the given quantities which is of the same kind as the quantity sought, just as in the single rule of three.

Then taking any pair of the remaining quantities like in kind, complete the stating, as if for the single rule of three, paying no regard to the other quantities, or rather considering them to remain *the same*.

Then take another pair, like in kind, as a new antecedent and consequent to be placed under the former pair; these, with the third term above, completing a second single rule-of-three stating. And proceed in this way till all the pairs are used.

Multiply the third term by the product of all the consequents, and divide the result by the product of all the antecedents, and the answer will be obtained.

Each given antecedent and consequent is of course to be regarded as an abstract number. It is convenient to *indicate* merely the several multiplications, at first, to place the divisor under the dividend, in the form of a fraction, as in the above example, and then, before performing the operations, to expunge factors common to numerator and denominator.

EXAMPLE.—If £15 12*s*. pay 16 labourers for 18 days, how many labourers will £35 2*s*. pay, at the same rate, for 24 days?

As the answer is to be a certain number of *labourers*, the given 16 labourers is to be the third term; then taking *days* for the first antecedent and consequent, and money for the second antecedent and consequent, attending to whether either consequent should be greater or less than its antecedent, as in the former rule, the operation is as follows:—

$$\left.\begin{matrix} 24 & : & 18 \\ \pounds 15\ 12s. & : & \pounds 35\ 2s. \end{matrix}\right\} :: 16 \text{ labourers} : 27 \text{ labourers.}$$

$$\begin{matrix} 20 & & 20 \\ \hline 312 & & 702 \end{matrix} \text{ therefore}$$

$$\frac{16 \times 18 \times 702}{24 \times 312} \text{ lab.} = \frac{4 \times 3 \times 351}{156} \text{ lab.} = \frac{351}{13} \text{ lab.} = 27 \text{ lab.}$$

The 18 is placed in the second term, because *fewer* labourers are required for 24 days, the *work* being the same, than for 18; and the £35 2*s*. is placed in the second term, because *more* labourers can be paid for that sum than for £15 12*s*., the *time* being the same. If the question had been worked by two single rule-of-three statings, we should have had

$$\text{1st. } 24 : 18 :: 16 \text{ lab.} : \frac{16 \times 18}{24} \text{ lab.} = 12.$$

$$\text{2nd. } 312 : 702 :: \frac{16 \times 18}{24} \text{ lab.} : \frac{16 \times 18 \times 702}{24 \times 312} \text{ lab.} = 27 \text{ lab.}$$

You see, therefore, that the double rule of three merely compounds the several single proportions into one; it is thus called *compound proportion*.

We add two examples for exercise in this rule:—

1. If 8 persons can be boarded for 16 weeks for £42, how long will £100 support 6 persons at the same rate?—Ans. $50\frac{50}{63}$ weeks.

2. If a family of 13 persons spend £64 in butcher's meat in 8 months, when meat is 6*d*. per lb., how much money, at the same rate of consumption, should a family of 12 persons spend in 9 months, when meat is $6\frac{1}{2}$*d*. per lb.?—Ans. £72.

In this example, there would be *three* separate statings, if the question were worked by the single rule of three; these are here to be compounded into *one*.

Decimals.—It was observed at the commencement of this chapter, that in our system of arithmetic, numbers are expressed in the *decimal* notation, and the reason for this designation was stated:—it is simply this—namely, that the unit of any figure in a number has a value of *ten times* the unit of the figure in the next place to the right. Thus, in a number consisting of unit-figures—as for instance, in the number 1111—the second unit, beginning with the right-hand one, is 10 times the first, the third 10 times the second, the fourth 10 times the third, and so on; or beginning with the first on the left, the second is the tenth part of the first, the third the tenth part of the second, and so on till we come down to the last unit, which is merely *one*. Now, we may evidently extend this principle still further; and, on the same plan, may represent one-tenth of *one*, one-tenth of *this*, or one-hundredth of *one*, one-thousandth of *one*, and so on, by simply putting some mark of separation between the *integers* and these *fractions*. The mark actually used is a *dot*, thus: 1111·1111. The unit next the dot, on the left, is 1; the unit one place from this on the left is 10; the next is 100; the next, 1000; and so on. In like manner, the unit one place from the 1 on the right, is $\frac{1}{10}$, the next $\frac{1}{100}$, the next $\frac{1}{1000}$, and so on. This being agreed upon, it is easy to interpret such a number as 36·427: it is $36 + \frac{4}{10} + \frac{2}{100} + \frac{7}{1000}$; each figure, to the right of the point, being a *fraction* of a known denominator; the denominator being 10 for the first figure, 100 for the second, 1000 for the third, and so on. The *sum* of the fractions represented by the *decimal* ·427, above, is obviously $\frac{427}{1000}$; in like manner, the fraction expressed by ·2643 is $\frac{2643}{10000}$; and in general the denominator of the equivalent fraction is always 1 followed by as many zeros as there are decimal places, the numerator being the number itself, when the prefixed dot, or *decimal point*, as it is called, is suppressed. You will thus easily see that the following are so many *identities*—namely:—

$24{\cdot}6 = 24\frac{6}{10}$; $136{\cdot}54 = 136\frac{54}{100}$; $73{\cdot}641 = 73\frac{641}{1000}$; $2{\cdot}07 = 2\frac{7}{100}$.

Any decimal may therefore be converted into its equivalent fraction at sight: it will be shown presently how any fraction may be converted into its equivalent decimal, though not with the same rapidity.

It is pretty evident that whatever whole number be prefixed to a decimal, the same may be prefixed to the numerator of the fraction which replaces that decimal: thus, taking the previous values, we have

$24·6=\frac{246}{10}$; $136·64=\frac{13664}{100}$; $73·641=\frac{73641}{1000}$; $2·07=\frac{207}{100}$.

for this is only reducing the foregoing *mixed quantities* to improper fractions.

To reduce a Proper Fraction to a Decimal.

RULE.—Annex a zero to the numerator, and then actually divide by the denominator: if there be a remainder, annex another zero, and continue the division, still annexing a zero, either till the division terminates without remainder, or till as many decimals as are considered necessary are obtained; the quotient, with the decimal point before it, will be the value of the fraction in decimals.

```
8)3·000
 ——————
  ·375 = 3/8
```

For example: let it be required to express $\frac{3}{8}$ in decimals; the operation is that in the margin. That $\frac{3}{8}$ = ·375 is easily proved; for $\frac{3}{8}=\frac{3000}{8000}$; consequently, dividing numerator and denominator by 8, we have $\frac{3}{8}=\frac{375}{1000}$ = ·375, from the very nature of decimals. If an *improper* fraction had been chosen, the operation would clearly have been just the same, only there would have been an integer prefixed to the decimal: thus, the operation for $\frac{19}{8}$ would have been as here annexed, showing that $\frac{19}{8}$ = 2·375. We need not take the trouble of actually annexing the zeros, as here: it is enough that we proceed as if they were inserted, as in the marginal example, for reducing $\frac{6}{11}$ to a decimal; where it is plain, from the remainders, that 54 would recur continually; so that $\frac{6}{11}$ is equal to a *recurring* decimal; the recurring *period* being 54. As a final example, let it be required to convert $\frac{8}{113}$ into a decimal. When one 0 is annexed to the 8, the divisor 113 will go *no times;* therefore, the first decimal place is to be occupied with a 0. Annexing now a second 0, the next decimal figure is 7, and the work proceeds as in the margin: the *noughts* being suppressed, though conceived to be annexed to the 8, and brought down one at a time, as in ordinary division. The quotient shows that $\frac{8}{113}$ = ·07079, &c.: the decimals may be carried out to any extent; but if we stop the work here, the error cannot be so great as ·00001; that is, it is less than $\frac{1}{100000}$: but it is obvious that, by continuing the work, we can make the error as small as we please.

```
8)19·000
 ———————
  2·375

11)6
  ————
  ·5454, &c.

113)8  (·07079, &c.
   791
   ———
     9
   791
   ———
   109
  1017
  ————
    73, &c.
```

The following are a few examples for exercise:—

(1.) $\frac{15}{8}$ = 1·875. (2.) $\frac{7}{16}$ = ·4375.
(3.) $\frac{11}{40}$ = ·275. (4.) $\frac{1}{32}$ = ·03125.
(5.) $\frac{1}{13}$ = ·076923, &c. (6.) $\frac{5}{1024}$ = ·00488, &c.

Addition and Subtraction of Decimals.—The rules for these fundamental operations are in reality the same as those for integers. We must here be careful not only to place units under units, tens under tens, and so on, but also place *tenths* under *tenths*, *hundredths* under *hundredths*, &c.: that is, the *decimal points* must all range under one another in the same vertical line. This attended to, the operations are just the same as those with integers. (See the operations in the margin.)

```
Addition.   Subtraction.
 23·628       14·705
  4·1056       3·9281
   ·0137     ———————
   ·0042      10·7769
   ·75       ———————
 ———————        1·34
 28·5015         ·0657
 ———————     ———————
                1·2743
```

Multiplication of Decimals.—Multiplication requires no special rule. The multiplier is to be placed under the multiplicand, just as if both were integers, no regard being paid to the decimal points. The only thing to be attended to is the marking off the proper number of decimal places in the product; and that is a very easy matter. We have seen that a number involving decimals is, in fact, a fraction with that number, the decimal point being suppressed, for numerator, and 1, followed by as many ciphers as there are decimal places, for denominator. Two such fractions multiplied together, being the product of the numerators divided by the product of the denominators, will therefore be a fraction of which the denominator is 1, followed by as many ciphers as there are in both factors. Consequently, in the multiplication of decimals, as many decimal places are to be marked off in the product as there are decimal places in *both* factors.

The example in the margin will suffice for illustration. As there are three decimals in the multiplicand, and two in the multiplier, *five* are marked off in the product.

```
   23·462
    17·31
   ——————
    23462
   70386
  164234
  23462
 —————————
 406·12722
```

Division of Decimals.—This operation, like that of multiplication, is the same for decimals as for integers; and the way to estimate the number of decimal places in the quotient is suggested by the plan adopted in multiplication.

All the decimals employed in the dividend, including, of course, whatever ciphers may have been added to it to carry on the division, are to be counted. We have then only to provide so many in the quotient, that when added to the number of them in the divisor, we may have just as many as in the dividend.

If the quotient figures, though all be considered as decimals, be too few in number to make up, with those in the divisor, the number in the dividend, then ciphers sufficient for this purpose are to be *prefixed* to the quotient figures, and the decimal point to be placed before them. (See the second example in the margin.) In the first of these examples, *six* have been used in the dividend, and as there are *two* in the divisor, there must be *four* in the quotient, which is therefore 10·0515. The last decimal, 5, is a little too great; but it is easy to see that if we had made it 4, the error in *defect* would have exceeded the present error in *excess;* and in limiting the number of decimals, we always make the *last* figure as near the truth as possible. In the second example, *five* decimals have been used in the dividend; and as there is but *one* in the divisor, *four* are required in the quotient; and to make up this number, a cipher is prefixed. The quotient is, therefore, ·0265, as far as the decimals have been carried: that is, to four places.

```
2·35)23·621(10·0515
     235
     ———
       121
      1175
      ————
        35
       235
       ———
       115
      1175

32·4)·86 (·0265
     648
     ———
     212
    1944
    ————
     176
    1620
    ————
     140
```

The following examples will serve for exercise in these two rules:—

(1.) ·321096 × ·2465 = ·079150164.
(2.) 464·3 × ·00521 = 2·419003.
(3.) 56·10833 ÷ 17·371 = 3·23.
(4.) 2·419003 ÷ 464·3 = ·00521.

Extraction of the Square Root.—If a number be multiplied by itself, the product is called *the second power*, or the *square*, of that number. If this also be multiplied by the same number, the product is called *the third* power, or the *cube* of that number: and so on for the *fourth power, fifth power, &c.* This raising of powers, which is called *involution*, is therefore nothing more than the multiplication together of *equal factors*, and is easy enough. But the reverse operation—that is, to find the factor which, involved in this manner, shall produce a *given number*—is a problem not so readily disposed of. The factor referred to is called the *root* of the power; so that the reverse problem spoken of is the problem of the *extraction of roots.* To extract the *square* root of a given number, is to find a number which, when squared, or raised to the second power, or, which is the same thing, when multiplied by itself, shall reproduce the given number. The rule for this operation is as follows:—

RULE.—Commencing at the units' figure, cut off *two* figures, then two more, and so on, thus dividing the number into *periods*, as they are called.

If the number of figures be *even*, we shall therefore have two figures in every period; but if *odd*, the first period, on the left, will have but a single figure.

Take the square root of this first period, or find the root of the nearest square to it, which nearest square is not, however, to be *greater* than the number forming the first period. The root thus found will be the leading figure of the root of the proposed number, and the square of this leading figure is to be subtracted from the first period.

To the remainder annex the second period: the resulting number will be the *first dividend*, and the divisor for it is to be found as follows:—

Put *twice* the root-figure, just found, in the divisor's place; the leading portion of the complete divisor will thus be obtained. This leading portion, as in common division, is enough to suggest the corresponding quotient-figure, which is now to be found, and annexed to the former root-figure, and *also* to the leading portion of the divisor employed in finding it: the divisor will then be *completed;* so that proceeding as in common division, a second remainder will be obtained, to which the *next* period being united, a *second dividend* will result.

To find the corresponding divisor, put *twice* the number, of *two figures*, now in the root's place, in the divisor's place: we shall thus have the leading portion of the complete divisor, by aid of which a *third* root-figure may be found; which, as before, is to be annexed to the former root-figures, and *also* to the incomplete divisor, in order to complete it; and the work is to be then carried another step, as in common division; and so on, till all the periods have been brought down, and annexed, step after step, to the successive remainders, thus supplying the successive *dividends*. If there be a remainder after all the periods have been brought down, the operation may be continued by annexing successive periods of *ciphers* to the successive remainders, or successive periods of the decimals in the given number, should any be there: in either case, the root-figures become *decimals* from the time that ciphers or decimals are annexed.

As the root multiplied by itself produces the original number, the decimal places in the root must, of course, always be just half as many as have been employed in determining it. And the root is only to be regarded as an *approximation* to the truth, whenever a remainder is left at the step where the work terminates.

Here follow three examples, worked by these precepts.

(1.) 45,96,84(678
36

127)996
889

1348)10784
10784

(2.) 31,64,06,25(5625
25

106)664
636

1122)2806
2244

11245)56225
56225

(3.) 3,·65)1·910497, &c.
1

29)265
261

381)400
381

38204)190000
152816

382089)3718400
3438801

3820987)27959900
26746909

1212991, &c.

The symbol for the operation of the square root is $\sqrt{\ }$; which is no doubt a degenerated form of *r*, the initial letter of the word *radix*, or *root;* it appears from the foregoing operations that

$\sqrt{459684}=678$; $\sqrt{31640625}=5625$; $\sqrt{3{\cdot}65}=1{\cdot}910497$, &c., the last root being true as far as the sixth decimal place.

By the same process it is found that
(1.) $\sqrt{473256}=687{\cdot}936$. (2.) $\sqrt{784{\cdot}375}=28{\cdot}0067$.
(3.) $\sqrt{11}=3{\cdot}316625$. (4.) $\sqrt{{\cdot}3236068}={\cdot}5688645$.
(5.) $\sqrt{794\frac{1}{5}}=\sqrt{794{\cdot}2}=28{\cdot}18155425$.

A TABLE *of those Factors of the Composite Numbers from* 75 *to* 10000, *which fall within the limits of the Multiplication Table.*

No.	Factors.	No.	Factors.	No.	Factors.
75	5 5 3	1078	11 7 7 2	3969	9 9 7 7
98	7 7 2	1089	11 11 9	3993	11 11 11 3
105	7 5 3	1125	9 5 5 5	4032	9 8 8 7
112	8 7 2	1134	9 9 7 2	4096	8 8 8 8
125	5 5 5	1152	12 12 8	4116	12 7 7 7
126	9 7 2	1155	11 7 5 3	4125	11 5 5 5 3
128	8 8 2	1176	8 7 7 3	4158	11 9 7 6
135	9 5 3	1188	12 11 9	4224	11 8 8 6
147	7 7 3	1215	9 9 5 3	4235	11 11 7 5
154	11 7 2	1225	7 7 5 5	4312	11 8 7 7
162	9 9 2	1232	11 8 7 2	4356	11 11 9 4
165	11 5 3	1296	12 12 9	4374	9 9 9 6
168	8 7 3	1323	9 7 7 3	4375	7 5 5 5 5
175	7 5 5	1331	11 11 11	4455	11 9 9 5
176	11 8 2	1344	8 8 7 3	4536	9 9 8 7
189	9 7 3	1372	7 7 7 4	4608	9 8 8 8
192	12 8 2	1375	11 5 5 5	4704	12 8 7 7
196	7 7 4	1386	11 9 7 2	4725	9 7 5 5 3
198	11 9 2	1408	11 8 8 2	4752	11 9 8 6
216	12 9 2	1452	12 11 11	4802	7 7 7 7 2
224	8 7 4	1458	9 9 9 2	4851	11 9 7 7
225	9 5 5	1485	11 9 5 3	4928	11 8 8 7
231	11 7 3	1512	9 8 7 3	5082	11 11 7 6
242	11 11 2	1536	8 8 8 3	5103	9 9 9 7
243	9 9 3	1568	8 7 7 4	5145	7 7 7 5 3
245	7 7 5	1575	9 7 5 5	5184	9 9 8 8
252	12 7 3	1584	12 12 11	5292	12 9 7 7
256	8 8 4	1617	11 7 7 3	5324	11 11 11 4
264	11 6 4	1694	11 11 7 2	5346	11 9 9 6
275	11 5 5	1701	9 9 7 3	5376	12 8 8 7
288	12 12 2	1715	7 7 7 5	5445	11 11 9 5
294	7 7 6	1728	12 12 12	5488	8 7 7 7 2
297	11 9 3	1764	9 7 7 4	5544	11 9 8 7
308	11 7 4	1782	11 9 9 2	5625	9 5 5 5 5
315	9 7 5	1792	8 8 7 4	5632	11 8 8 8
324	9 9 4	1815	11 11 5 3	5775	11 7 5 5 3
336	12 7 4	1848	11 8 7 3	5808	11 11 8 6
343	7 7 7	1875	5 5 5 5 3	5832	9 9 9 8
352	11 8 4	1925	11 7 5 5	5929	11 11 7 7
363	11 11 3	1936	11 11 4 4	6048	12 9 8 7
375	5 5 5 3	1944	9 9 8 3	6075	9 9 5 5 3
378	9 7 6	2016	9 8 7 4	6125	7 7 5 5 5
384	8 8 6	2025	9 9 5 5	6144	12 8 8 8
385	11 7 5	2048	8 8 8 4	6174	9 7 7 7 2
392	8 7 7	2058	7 7 7 6	6237	11 9 9 7
396	11 9 4	2079	11 9 7 3	6272	8 8 7 7 2
405	9 9 5	2112	11 8 8 3	6336	11 9 8 8
432	12 9 4	2156	11 7 7 4	6468	12 11 7 7
441	9 7 7	2178	11 11 9 2	6534	11 11 9 6
448	8 8 7	2187	9 9 9 3	6561	9 9 9 9
462	11 7 6	2205	9 7 7 5	6615	9 7 7 5 3
484	11 11 4	2268	9 9 7 4	6655	11 11 11 5
486	9 9 6	2304	9 8 8 4	6776	11 11 8 7
495	11 9 5	2352	8 7 7 6	6804	12 9 9 7
504	9 8 7	2376	11 9 8 3	6875	11 5 5 5 5
512	8 8 8	2401	7 7 7 7	6912	12 9 8 8
525	7 5 5 3	2464	11 8 7 4	7056	9 8 7 7 2
528	12 11 4	2475	11 9 5 5	7128	11 9 9 8
539	11 7 7	2541	11 11 7 3	7168	8 8 8 7 2
567	9 9 7	2592	9 9 8 4	7203	7 7 7 7 3
576	12 12 4	2625	7 5 5 5 3	7392	12 11 8 7
588	12 7 7	2646	9 7 7 6	7425	11 9 5 5 3
594	11 9 6	2662	11 11 11 2	7546	11 7 7 7 2
605	11 11 5	2673	11 9 9 3	7623	11 11 9 7
616	11 8 7	2688	8 8 7 6	7744	11 11 8 8
625	5 5 5 5	2695	11 7 7 5	7776	12 9 9 8
648	9 9 8	2744	8 7 7 7	7875	9 7 5 5 5
672	12 8 7	2772	11 9 7 4	7938	9 9 7 7 2
675	9 5 5 3	2816	11 8 8 4	7986	11 11 11 6
686	7 7 7 2	2835	9 9 7 5	8019	11 9 9 9
693	11 9 7	2904	11 11 8 3	8064	9 8 8 7 2
704	11 8 8	2916	9 9 9 4	8085	11 7 7 5 3
726	11 11 6	3024	9 8 7 6	8192	8 8 8 4 4
729	9 9 9	3025	11 11 5 5	8232	8 7 7 7 3
735	7 7 5 3	3072	8 8 8 6	8316	12 11 9 7
756	12 9 7	3087	9 7 7 7	8448	12 11 8 8
768	12 8 8	3125	5 5 5 5 5	8505	9 9 7 5 3
784	8 7 7 2	3136	8 8 7 7	8575	7 7 7 5 5
792	12 11 6	3168	11 9 8 4	8624	11 8 7 7 2
825	11 5 5 3	3234	11 7 7 6	8712	11 11 9 8
847	11 11 7	3267	11 11 9 3	8748	12 9 9 9
864	12 9 8	3375	9 5 5 5 3	9072	9 9 8 7 2
875	7 5 5 5	3388	11 11 7 4	9075	11 11 5 5 3
882	9 7 7 2	3402	9 9 7 6	9216	9 8 8 4 4
891	11 9 9	3456	9 8 8 6	9261	9 7 7 7 3
896	8 8 7 2	3465	11 9 7 5	9317	11 11 11 7
924	12 11 7	3528	9 8 7 7	9375	5 5 5 5 5 3
945	9 7 5 3	3564	11 9 9 4	9408	8 8 7 7 3
968	11 11 8	3584	8 8 8 7	9504	12 11 9 8
972	12 9 9	3645	9 9 9 5	9604	7 7 7 7 4
1008	12 12 7	3675	7 7 5 5 3	9625	11 7 5 5 5
1024	8 8 8 2	3696	11 8 7 6	9702	11 9 7 7 2
1029	7 7 7 3	3773	11 7 7 7	9801	11 11 9 9
1056	12 11 8	3872	11 11 8 4	9856	11 8 7 4 4
		3888	9 9 8 6		

ADDENDA.

The preceding able article, by Professor Young, may, perhaps, be rendered more practically useful, if we extend the principles therein laid down to some of the requirements of mercantile life. For this purpose we shall explain a very simple plan of calculating the cost of any number of articles, at any price, reckoned in pence. The ordinary "Rule of Three" may be employed for that purpose, or the old plan of "Practice" is equally eligible where accuracy of result alone is desired. In the hurry of business, however, a more rapid mode of calculation is desirable; and we have no hesitation in recommending that which we are about to describe.

It is first necessary that the table subjoined be learned carefully by heart, because the success of the system depends essentially thereon. In using the table so committed to memory, it is only required to multiply the value of the number of articles reckoned at a *penny* each, and reduced to pounds, shillings, and pence, by the price of each article. Suppose, for example, that 240 articles are sold at sixpence each, what would be their total value?

By referring to the table, we find that 240 articles, at one penny each, would amount to one pound. Now, multiplying this by the price in pence, we have the total of six pounds. If this sum were reckoned by the old method, it would stand as follows:—

240 articles
6 price each
12)1440 total value in pence
20)120
6 0 0 value in pounds.

Now it will be at once seen how much time and figuring is saved by the plan we propose. The only difficulty is that of learning off the table: this, however, is soon done by any person possessed of an average memory. It may be as well that we should give the rule in an explicit form; which is as follows:—

Rule.—Suppose the number of articles to be of the value of one penny each, and by memory reduce the amount to its value in pounds, shillings, and pence. Multiply the value so obtained by the price in pence, and the total will be the sum sought. If farthings constitute part of the price, then the fourth for a farthing, the half for a halfpenny, and the three-fourths for three farthings, of the value of the article at a penny, must be added to the sum obtained.

Pence Table reduced to Shillings and Pounds.

	£	s.	d.		£	s.	d.
60 pence =	0	5	0	1260 pence =	5	5	0
120 ,,	0	10	0	1320 ,,	5	10	0
180 ,,	0	15	0	1380 ,,	5	15	0
240 ,,	1	0	0	1440 ,,	6	0	0
300 ,,	1	5	0	1500 ,,	6	5	0
360 ,,	1	10	0	1560 ,,	6	10	0
420 ,,	1	15	0	1620 ,,	6	15	0
480 ,,	2	0	0	1680 ,,	7	0	0
540 ,,	2	5	0	1740 ,,	7	5	0
600 ,,	2	10	0	1800 ,,	7	10	0
660 ,,	2	15	0	1860 ,,	7	15	0
720 ,,	3	0	0	1920 ,,	8	0	0
780 ,,	3	5	0	1980 ,,	8	5	0
840 ,,	3	10	0	2040 ,,	8	10	0
900 ,,	3	15	0	2100 ,,	8	15	0
960 ,,	4	0	0	2160 ,,	9	0	0
1020 ,,	4	5	0	2220 ,,	9	5	0
1080 ,,	4	10	0	2280 ,,	9	10	0
1140 ,,	4	15	0	2340 ,,	9	15	0
1200 ,,	5	0	0	2400 ,,	10	0	0

It will be seen that the above table contains the value of so many articles at one penny each. We have restricted it to every five shillings of value; and if any intermediate quantities have to be calculated, their value at a penny each can easily be reckoned and added to that sum next lowest in amount, as found in the table. The following examples will assist our readers in acquiring the use of this method:—

How much will 550 articles at 7*d.* each come to?
550 at a penny = £2 5 10
Multiply by 7
£16 0 10 Answer.

How much will 768 articles at 8¾*d.* each amount to?
768 at a penny = £3 4 0
Multiply by 8
25 12 0
Halfpenny ½ . . 1 12 0
Farthing ¼ . . 0 16 0
¾*d.* £28 0 0 Answer.

From the above instances our readers will at once see how simple and easily managed this plan is; and we feel convinced, from our own experience, that its use will prove of great advantage to all engaged in commercial pursuits. If the price of the articles involves shillings, the values may be obtained by multiplying by the number of shillings, and reducing the product to pounds, &c., adding this to the amount of the cost of the article in the pence, as obtained above. Of course, if pounds are also involved, then the number of articles must be multiplied by the number representing pounds. The following is an instance of the kind:—

Required the value of 250 articles at £1 2*s.* 8*d.* each?

		£	s.	d.
250 at £1	=	250	0	0
250 ,, 2*s.* = $\frac{1}{10}$ of £1	=	25	0	0
250 ,, 8*d.* {= £1 0 10 8				
		8	6	8
		£283	6	8 Answer.

(Ed.)

The following table of aliquot parts may be frequently found useful:—

A Table of Aliquot Parts of Money and Weights.

A Penny.		
½*d*	=	$\frac{1}{2}$
¼	,,	$\frac{1}{4}$

A Shilling.		
1*d.*	=	$\frac{1}{12}$
1½	,,	$\frac{1}{8}$
2	,,	$\frac{1}{6}$
3	,,	$\frac{1}{4}$
4	,,	$\frac{1}{3}$
6	,,	$\frac{1}{2}$

A Pound.		
1*d.*	=	$\frac{1}{240}$
1½	,,	$\frac{1}{160}$
2	,,	$\frac{1}{120}$
3	,,	$\frac{1}{80}$
3¾	,,	$\frac{1}{64}$
4	,,	$\frac{1}{60}$
5	,,	$\frac{1}{48}$
6	,,	$\frac{1}{40}$
7½	,,	$\frac{1}{32}$
8	,,	$\frac{1}{30}$
10	,,	$\frac{1}{24}$

A Pound.		
1*s.*	=	$\frac{1}{20}$
1*s.* 3*d.*	,,	$\frac{1}{16}$
1*s.* 4*d.*	,,	$\frac{1}{15}$
1*s.* 8*d.*	,,	$\frac{1}{12}$
2*s.*	,,	$\frac{1}{10}$
2*s.* 6*d.*	,,	$\frac{1}{8}$
3*s.* 4*d.*	,,	$\frac{1}{6}$
4*s.*	,,	$\frac{1}{5}$
5*s.*	,,	$\frac{1}{4}$
6*s.* 8*d.*	,,	$\frac{1}{3}$
10*s.*	,,	$\frac{1}{2}$

A Pound Troy.		
6 oz.	=	$\frac{1}{2}$
4 oz.	,,	$\frac{1}{3}$
3 oz.	,,	$\frac{1}{4}$
2 oz.	,,	$\frac{1}{6}$
1 oz. 10 dwts.		$\frac{1}{8}$
1 oz.	=	$\frac{1}{12}$

An Ounce Troy.		
10 dwt.	=	$\frac{1}{2}$
6 dwt. 16 gr.		$\frac{1}{3}$
5 dwt.	,,	$\frac{1}{4}$
4 dwt.	,,	$\frac{1}{5}$
3 dwt. 8 gr.		$\frac{1}{6}$
2 dwt. 12 gr.		$\frac{1}{8}$
2 dwt.	=	$\frac{1}{10}$
1 dwt. 16 gr.		$\frac{1}{12}$

A Ton.		
10 cwt.	=	$\frac{1}{2}$
5 cwt.	,,	$\frac{1}{4}$
4 cwt.	,,	$\frac{1}{5}$
2 cwt.	,,	$\frac{1}{10}$
1 cwt.	,,	$\frac{1}{20}$

A Cwt.		
2 qrs.	=	$\frac{1}{2}$
1 qr.	,,	$\frac{1}{4}$
16 lb.	,,	$\frac{1}{7}$
14 lb.	,,	$\frac{1}{8}$
8 lb.	,,	$\frac{1}{14}$
7 lb.	,,	$\frac{1}{16}$
3½	,,	$\frac{1}{32}$
2 lb.	,,	$\frac{1}{56}$
1¾	,,	$\frac{1}{64}$

½ cwt. or 56 lb.		
28 lbs.	=	$\frac{1}{2}$
14 lbs.	,,	$\frac{1}{4}$
8 lbs.	,,	$\frac{1}{7}$
7 lbs.	,,	$\frac{1}{8}$

¼ cwt. or 28 lb.		
14 lb.	=	$\frac{1}{2}$
7 lb.	,,	$\frac{1}{4}$
4 lb.	,,	$\frac{1}{7}$
3½ lb.	,,	$\frac{1}{8}$

A lb. Avoir.		
8 oz.	=	$\frac{1}{2}$
4 oz.	,,	$\frac{1}{4}$
2 oz.	,,	$\frac{1}{8}$
1 oz.	,,	$\frac{1}{16}$

CHAPTER II.

ALGEBRA.

WE need scarcely remark, that the study of Algebra requires a previous acquaintance with the principles of common Arithmetic; it is upon these that its operations are based, and the processes of Algebra are, for the most part, only the processes of Arithmetic, extended and rendered more comprehensive by the aid of a new set of symbols, taken in combination with the well-known symbols of Arithmetic. The symbols of Arithmetic are the *figures* of Arithmetic 1, 2, 3, &c.; and the new symbols introduced by Algebra are the letters of the alphabet, *a*, *b*, *c*, &c. As noticed at page 433, the peculiar marks or symbols employed in any science constitute the *notation* of that science; the notation of Algebra is therefore of a mixed character, consisting of the figures of Arithmetic and of the letters of the alphabet. The *letters* may be regarded as standing for *figures*; and the reason why figures alone will not enable us to do all that Algebra will, is that a *figure* always has a fixed and invariable signification: thus a 3 or a 5, or a 7, conveys the same meaning to everybody—no one uses the mark or symbol 3 to denote anything but *three*. But a *letter*, as *a*, or *b*, may be used to denote *any figure whatever:* a 3, or a 4, or a 7, &c.; and it may therefore be made to stand even for a figure whose value may be unknown to us, and which value it may be the object of the inquiry to discover. This is one of the principal advantages of algebraical notation; that part of this notation, which is borrowed from Arithmetic, enables us to express the *known* figures in any mathematical investigation by marks intelligible to all; while the other part, the letters, enables us to represent the figures or quantities, at the outset *unknown*, but which it is the object of the problem to determine.

But besides the marks or symbols of *quantity*—the figures and letters—other marks are introduced into the notation, as *signs of operation*. Some of these are used alike, both in Arithmetic and Algebra—as the sign (+) for addition, the sign (—) for subtraction, and one or two others with which the reader of the Arithmetic must be already familiar. The signs of operation in most frequent request in Algebra we shall now explain, and shall give a few easy examples of their application before entering on the formal rules of the science. We wish the reader to understand, at the outset, that we are about to exhibit the first principles of Algebra to *mere beginners*—to persons who, as yet, know nothing at all of the subject. Some experience in elementary teaching has convinced us, that to succeed in an undertaking of this kind, the instructor must forego all self-importance, descend to the level of his pupil, and, as it were, sit familiarly down by his side, and address him in that simple and unadorned style that no intelligent individual can fail to understand. We shall endeavour, therefore, to be very plain and simple in the language employed.

Definitions and Explanations of First Principles.—You already know, from arithmetic, that *numbers* are of two kinds:—*abstract* numbers, and *concrete* numbers. An abstract number is simply a *figure*, or number formed by two or more figures: thus, 3, 23, 147, &c., are abstract numbers; but 3 *oz.*, 23 *ft.*, £147, are *concrete* numbers. Abstract numbers merely denote how many *times* or *repetitions*; and, accordingly, whenever you use a *multiplier*, you use an abstract number. Concrete numbers denote how many *things*; and some mark or symbol, to tell us *what* the things are, must always be joined to the figure or figures which tell us the *number* of them. The mark or symbol *oz.*, as you know, means *ounces*; the symbol *ft.*, *feet*; and £ stands for *pounds*, in money. In algebra, *letters* are used to stand for numbers, whether they be abstract or concrete. Instead of writing down the abstract number, 23, a letter, *a*, for instance, may be put to represent 23; in like manner, instead of writing down the concrete number 23 *ft.*, a single letter may be made to stand for it. You see, therefore, that a *letter* serves for a number of *either* kind, while a *figure* must have a particular symbol joined to it when a *concrete* quantity is to be represented.

It is the business of a teacher of algebra to show how the operations carried on in arithmetic, by the help of figures *alone*, may be conducted with figures and letters *both*. The letters, as just noticed, representing numbers, whether abstract or concrete, are called *symbols of quantity;* and the marks or signs which *indicate* operations performed with the letters, are called *signs of operation*. We are now to explain to you some of these signs of operation: most of them, however, have been used in the Arithmetic.

The mark + is the *sign of addition:* whenever you see it put before a number or letter, you are to understand that the *addition* of whatever that number or letter signifies is meant: thus, $7 + 3$, which is read 7 *plus* 3, denotes that the 3 is to be *added* to the 7; in like manner, $a + b$, that is, *a plus b*, signifies that the *b* is to be added to the *a:* but you will ask—how can a *b* be added to an *a*? Our reply to this is, that *a* and *b* both stand for *numbers*, abstract or concrete; and although we say, according to custom, that "*b* is to be added to *a*," what we really mean is, that the *number* represented by *b* is to be added to the *number* represented by *a*. In the case of 7 *plus* 3, that is, of $7 + 3$, we can obey the direction of the sign of operation, and we know that 10 is the result; but in the case of $a + b$, we *cannot* obey the direction of the sign till we know the interpretation of *a* and *b:* we can only *indicate* the addition—but cannot *perform* it.

The sign — is the sign of subtraction: whenever it is placed before a quantity, it indicates that the quantity is to be *subtracted:* thus, $7 - 3$, which is read 7 *minus* 3, means that 3 is to be subtracted from 7; and, in like manner, $a - b$ means that *b* is to be subtracted from *a*. In the case of $7 - 3$, the result of the subtraction is 4; the result of $a - b$ cannot be given so long as the numbers represented by *a* and *b* remain unknown.

The sign × placed between two quantities denotes the *multiplication* of those quantities together: it is called *the sign of multiplication;* thus, 7×3 means that 7 and 3 are to be multiplied together; and, in like manner, $a \times b$ means the product of *a* and *b*. In the case of 7×3, the result or product is 21; in the case of $a \times b$, the result remains unknown till the number represented by the multiplier is stated. Every multiplier, as you are aware, must be an abstract number; if *a* stand for the abstract number 5, then $a \times b$ is $5 \times b$; that is, 5 *times b*, whatever *b* may stand for. The sign just explained is not *always* used to indicate multiplication; instead of it a *dot* placed between the two quantities, is often employed to mean the same operation: thus, 7.3 and *a.b* mean the same as 7×3 and $a \times b$; and, *in the case of letters*, even this sign is usually omitted, and the letters simply written side by side, without any intervening mark for multiplication at all; thus *ab*, *a.b*, and $a \times b$, all equally mean *the product of a and b*. When *figures* are to be multiplied, *some* sign for multiplication *must* be put between them, to prevent misunderstanding; if, when we meant 7×3, or 7.3, we put merely 73, we should be thought to mean *seventy-three*, instead of *seven times three*.

The sign ÷, placed before a quantity, denotes *division by that quantity:* it is called the *sign of division;* thus, $12 \div 4$ means 12 divided by four, and $a \div b$ means *a* divided by *b*. Division is otherwise indicated by writing the dividend above, and the divisor below a short line, as in the notation for a fraction: thus, $12 \div 4$ and $\frac{12}{4}$ mean the same thing; as also do $a \div b$ and $\frac{a}{b}$. As in the former instances, so here, the operation can be only *indicated*, not actually *performed*, except in the case of *numbers*.

The sign = stands for the words *equal to;* it is called *the sign of equality:* thus, $5 + 2 = 7$ is a brief way of saying that 5 plus 2 are *equal to* 7; and $5 - 2 = 3$ states, in like manner, that 5 minus 2 is *equal to* 3. Also $4x + 2x = 6x$, states that four times x plus twice x are equal to six times x; and $4x - 2x = 2x$, expresses, in symbols, that four times x minus twice x is equal to twice x.

A *figure*, or number, prefixed to a letter, as a *multiplier*, is called the *coefficient* of that letter; thus, 4 is the coefficient of x in the quantity $4x$; and 6 is the coefficient of x in $6x$. In like manner, 8 is the coefficient of xy in the quantity $8xy$, and 23 is the coefficient of xyz in the quantity $23xyz$.

Every quantity which, like each of those just noticed, is *not separated into parts* by any plus or minus sign, is called *a simple quantity*, or *a simple expression*—or it is said to consist of but a single *term*. But when a quantity is made up of parts, linked together by plus or minus signs, the quantity is called *a compound quantity*, or *a compound expression*. The following are simple expressions: they each consist of but one *term*.

$$4ax,\ 7aby,\ 14xyz,\ \frac{2ab}{7},\ \frac{3abx}{4y},\ 37amnxz,\ \&c.$$

The following are compound expressions: the first consists of two terms, the second of three terms, and the third of four terms.

$$5a - 2b,\ 3ax + 5by - 4z,\ 16axy - \frac{4bcz}{3x} - 2dmx + 5en.$$

You see what in Algebra is meant by a *term* and an *expression;* the first row of quantities above, is formed by *six* distinct *expressions*, each expression consisting of only one *term:* the second row is formed by *three expressions*, of which the first consists of two terms, the second of three terms, and the third of four terms. The first row is a row of *simple* expressions, the second a row of *compound* expressions.

We shall now give you a few easy exercises by which you may prove to yourself whether you fully understand the meaning of the signs already explained. You must not forget that whenever two or more quantities are placed side by side, without any sign of operation between them, the *multiplication* of those quantities, or rather of the *numbers* they represent, is always meant.

EXERCISES.

NOTE.—In these exercises $a=4$, $b=2$, $c=3$, $d=5$, $m=8$, $n=1$.

All that you have to do is to give these interpretations to the letters, thus translating the algebra into arithmetic, and then to put down the numerical value of each expression. To prevent mistakes we will here show you how to deal with example 4; that is, with $21m - 9d$. As m is presumed to stand for 8, and d for 5, the translation of the expression is $21 \times 8 - 9 \times 5$; that is, $168 - 45 = 123$, which is the numerical value of the proposed expression, on the supposition that m and d stand for 8 and 5 respectively. And in a similar manner are all the other examples to be treated. Do not forget that when the multiplication of *numbers* is indicated, the *multiplication sign* must be interposed between them.

Find the values in numbers of the following expressions:—

1. $3a + 4c$[*]
2. $5c - 2a$
3. $13n + b$
4. $21m - 9d$
5. $7d + 4n - 2a$
6. $3a + 4b - 5c$
7. $6m - 5n - 3b$
8. $14 - 3c + m$
9. $11b + n - 13$
10. $-4d + 5m - 2n$
11. $\frac{3m}{4} + 6 - \frac{m}{2}$
12. $\frac{5a}{2} + \frac{4d}{5} - 2c$
13. $\frac{2a + 3b}{7} + m - n$
14. $\frac{3a - 4b + 6c - 2}{10}$
15. $3ab + dm - 5b + 6cn - 9 \times 2$
16. $2abm - 3cdm + \frac{3dm}{8} + ad$
17. $\frac{5am - 2bc + n - 49}{140} - n$
18. $\frac{24}{m} + \frac{5abc}{24} - \frac{60}{bcd} - \frac{m + n}{2b + d}$

The first ten of these exercises are free from fractions: the next four all contain *algebraic fractions:* the fourteenth is a *single* fraction; it is an algebraic expression consisting of but *one term*. The numerator of this expression is, however, a *compound* quantity, as it is made up of *simple* quantities, united together by the signs + and —. The leading term in each of the above expressions, with the exception of Exercise 10, has no *sign* prefixed to it. You must take notice that a term, without any prefixed sign, is always to be regarded as *plus:* the actual insertion of the +, before a leading term, is unnecessary; since, when it is *minus*, the sign — is always put before it. The *plus* quantities are all called *positive* quantities, and the *minus* quantities, *negative* quantities. There is another thing which you must also take note of. You know that the *numerical multiplier* prefixed to a letter, or to a group of letters, is called the *coefficient* of the letter or letters connected with it. Now, although no coefficient should appear before a letter, you are not to say that the coefficient is *nothing*, any more than you are to say, when the plus sign is absent, that the *sign* is nothing: the second term in Exercise 3, is $+ b$, that is, plus *once* b, and the coefficient 1, although not actually written, is to be understood. If the question were asked, therefore, you should say that the coefficient of the b is 1: this 1 is omitted from before the b on the same principle that the + is omitted from before the $13n$; the insertion of either would add to the number of symbols, without adding any clearness to the meaning of the expression; for $13n + b$ can never be mistaken for anything but $+ 13n + 1b$, that is, *thirteen times n plus b;* or, as an algebraist would read it, *thirteen n plus b;* the word *times* being suppressed. Exercise 15 would be read thus: *three a, b; plus d, m; minus five b; plus six c, n; minus twice nine, or 9 times 2.*

From your recollection of the terms employed in common arithmetic, you will know the meaning of the word *factor:* the numbers which, multiplied together, give a *product*, are called *factors* of that product: so here, in algebra, every *multiplier* is a *factor:* thus the factors of $5axy$ are 5, a, x, and y: the factors of $bcyz$ are b, c, y, and z. A product is not altered by changing the order of its factors: the following different arrangements of the factors all imply the same product: $bcyz$, $bycz$, $cyzb$, &c., &c.; but it is usual, when *letters* are the factors, to write them one after another in the order in which they follow in the alphabet; so that the first of the above arrangements would be that generally adopted. In a similar way, since quantities connected together, some by the *additive* or *positive* sign, and others by the *subtractive* or *negative* sign, furnish the same *result*, in whatever order they succeed one another, the order fixed upon is mere matter of accident or choice: Exercise 10 is the same expression whether written

$-4d + 5m - 2n$, or $5m - 4d - 2n$, or $-2n - 4d + 5m$; but the *second* of these ways would be preferred simply because a *sign* is thus saved. The leading term there being *positive*, the leading sign + is dispensed with.

From the explanations that have now been given, and from the practice afforded by the exercises already worked, you must perceive this fact—that although the sign + before an algebraic quantity is a direction to *add*, and the sign — a direction to *subtract*, yet you cannot, in general, obey these directions till the algebra is converted into arithmetic, by a translation of the

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[*] The Answers to all the Exercises and Questions in this Chapter will be found at the end.

letters into *numbers*. But in many cases you can actually add and subtract, though you may remain in utter ignorance as to the *meaning of the letters:* there is such a thing as addition of *algebra* as well as addition of common numbers; and subtraction of *algebra* as well as arithmetical subtraction. We shall now introduce you to these operations; you will at once see how it happens that you can actually *perform* the operations indicated by + and —, even upon quantities whose values are unknown, by examining a simple instance or two, before proceeding to RULES. Suppose we have such an expression as $5ax + 3ax - 7ax + 2ax - ax$; you would surely not require to know what ax stands for, before you could *perform* the operations indicated by the signs; you would say to yourself—whatever thing ax may represent, 5 *plus* 3 of them must make 8 of those things: 8 of them *minus* 7 of them leaves *one;* this, with the two following, make 3 of them, which diminished by *one*, gives 2 of the things, whatever they be, as the amount of the whole row; you would thus be sure that the result is $2ax$; you would thus state with confidence that

$$5ax + 3ax - 7ax + 2ax - ax = 2ax.$$

The *compound* expression on the left of the sign of equality, is thus reduced to the *simple* expression on the right, solely because the several quantities in the compound expression are *like quantities;* that is, they differ in nothing except in *coefficient* and in *sign;* the *letters* are the same in all. It is only when the terms of a compound expression are *unlike quantities* that the operations indicated by + and — cannot be performed till the values of the letters are stated. You will no doubt be able of yourself to put the proper simple value, on the right of the sign of equality in each of the following cases:

1. $2xy - 4xy + 13xy + 3xy =$
2. $-axz - 7axz + 11axz + axz =$
3. $9mnx + 5mnx - 13mnx =$
4. $6\frac{a}{b} + 2\frac{a}{b} - 5\frac{a}{b} - 2\frac{a}{b} + \frac{a}{b} =$

ADDITION.

Addition of algebra is the finding the amount of a set of quantities, of which some are *additive*, and the others *subtractive*. It differs, therefore, from addition in common arithmetic in this; that, in the latter operation all the numbers are *additive*. Addition of algebra therefore combines the two operations which, in arithmetic, are called addition and subtraction.

CASE I.—*When the quantities to be added are all* like quantities.

RULE 1. Find the sum of the *positive coefficients*.
2. Find the sum of the *negative coefficients*.
3. Take the *difference* of these two sums, and prefix to that difference the sign belonging to the *greater* sum.
4. Annex to the difference the *letters* common to all the quantities, and the correct sum will be obtained.

1.
$$\begin{array}{r} 3a \\ 5a \\ -2a \\ 7a \\ -4a \\ \hline 9a \\ \hline \end{array}$$

2.
$$\begin{array}{r} 7x \\ -3x \\ -2x \\ 13x \\ -x \\ \hline 14x \\ \hline \end{array}$$

3.
$$\begin{array}{r} 6ay \\ 2ay \\ -5ay \\ ay \\ -9ay \\ \hline -5ay \\ \hline \end{array}$$

4.
$$\begin{array}{r} 2ax + 3by \\ 3ax - 2by \\ -5ax + 4by \\ 4ax - by \\ 6ax + 7by \\ \hline 10ax + 11by \\ \hline \end{array}$$

5.
$$\begin{array}{r} 5axz - 3bcy \\ -6axz - 2bcy \\ 2axz + 5bcy \\ -3axz - bcy \\ -4axz + 4bcy \\ \hline -6axz + 3bcy \\ \hline \end{array}$$

The fourth and fifth of these examples each consists of two vertical rows of *like quantities;* the *first row is always that which is first computed;* so that in algebra we begin with the column at which, in arithmetic, we end. The reason why the columns are added up in this order is, that it is more convenient to write the results of the several columns, with the proper signs, from left to right, than from right to left.

From carefully looking over these examples, you will see that the only *work* performed is the adding up of all the *positive coefficients* in each column, and all the *negative coefficients* in separate totals, and then writing the *difference* of these sums for the coefficient of the result, prefixing to it the sign belonging to the greater of the two sums, and then writing against it the *letters* common to all the terms in the column. The work, therefore, is purely arithmetical; the *letters* in the finished result are simply *copied* from the terms above.

EXAMPLES FOR EXERCISE.

1.
$$\begin{array}{r} 2b \\ 7b \\ 9b \\ 11b \\ -3b \\ \hline \\ \hline \end{array}$$

2.
$$\begin{array}{r} 4ax \\ -2ax \\ 3ax \\ 7ax \\ 12ax \\ \hline \\ \hline \end{array}$$

3.
$$\begin{array}{r} 6by \\ 3by \\ -4by \\ -5by \\ 7by \\ \hline \\ \hline \end{array}$$

4.
$$\begin{array}{r} 3axy \\ -axy \\ -2axy \\ 7axy \\ -12axy \\ \hline \\ \hline \end{array}$$

5.
$$\begin{array}{r} 4a + 3x \\ 2a - 7x \\ 6a - 2x \\ 5a + x \\ -a - x \\ \hline \\ \hline \end{array}$$

6.
$$\begin{array}{r} 3ax - 2bz \\ 5ax + 3bz \\ -4ax - 8bz \\ -2ax + 6bz \\ 7ax - 4bz \\ \hline \\ \hline \end{array}$$

7.
$$\begin{array}{r} 8pqy + 3mx - n \\ 2pqy - 4mx + 2n \\ -3pqy - 2mx - 5n \\ -6pqy - 5mx - 3n \\ 4pqy + 6mx + 4n \\ \hline \\ \hline \end{array}$$

8.
$$\begin{array}{r} 4xyz - 2ab \phantom{{}-13} \\ 3xyz + 7ab - 6 \\ -2xyz - 3ab + 7 \\ 5xyz - ab - 13 \\ -8xyz + 4ab + 9 \\ \hline \\ \hline \end{array}$$

9.
$$\begin{array}{rrrr} -abz & -3x & +a & -3 \\ -2abz & +x & -2a & \\ -4abz & -7x & & +2 \\ 5abz & -13x & +4a & \\ 11abz & & +5a & \\ \hline \\ \hline \end{array}$$

10.
$$\begin{array}{rrr} 9acx & +2bey & -kg \\ 2acx & -7bey & +3kg \\ -4acx & -3bey & -2kg \\ acx & & +8kg \\ 7acx & -5bey & \\ & -4bey & -kg \\ \hline \\ \hline \end{array}$$

11.
$$\begin{array}{rrr} 3z & +abc & -17mp \\ & 5abc & +4mp \\ 7z & -2abc & -3mp \\ 5z & & +9mp \\ & 13abc & -2mp \\ & -4abc & +mp \\ \hline \\ \hline \end{array}$$

12.
$$\begin{array}{rrr} 34ky & -3mz & +2abc \\ -16ky & +9mz & -8abc \\ & -5mz & \\ 4ky & +6mz & -3abc \\ -5ky & & +7abc \\ 2ky & -11mz & -abc \\ \hline \\ \hline \end{array}$$

Add together the following quantities:—

13. $7abz-3cey+2mx$, $+3abz-7mx$, $4cey+mx$, $+9mx$.
14. $2gx+6pqy-13$, $+8pqy+11$, $+7gx-2$, $-3gx-pqy$, $+17$.
15. $3axy$, $7ab-c$, $3axy+2ab+5c$, $-axy-5ab-2c$.
16. $5xyz-2am+3bn$, $7am-7bn$, $6xyz-bn$, $4am-5bn$.
17. $6bcx+3ny$, $7mz$, $-8ny$, $2bcx-ny$, $5ny-3mz$, $-2mz$.
18. $9exz-13$, $7bcy+4$, $6exz-4bcy+1$, $13bcy-3$, $4exz-bcy$.
19. $-4mpy-2ax+3$, $-5ax-7$, $3mpy-2$, $mpy+8ax+6$.
20. $-cgx-4$, $-6ab+2$, $3cgx-ab-5$, 17, $4cgx+7ab$, -13.

CASE II.—*When the quantities to be added are* not all *like* quantities.

When rows of quantities are arranged one under another, so as to present a set of vertical columns, as in Examples 1 to 12 above, if the vertical rows are not rows of *like* quantities, then you will have carefully to examine *all* the rows, and to pick out from among them the different sets of *like* quantities, and add them together as before; those quantities that have none *like* them must be merely connected with their signs to the sums thus found; for we cannot *actually* add or subtract when the quantities are unlike. The following is the rule.

RULE.—Add together the different sets of *like* quantities; and put down each separate result, exactly as in the last case; then, to the sum thus found, connect, by the signs belonging to them, the remaining *unlike* quantities.

Although it matters not in what order a row of algebraical terms is written, yet *in addition*, it is usual to commence with the quantity at the *top* of the left-hand column, and to select from among all the columns the several quantities like *it*; then passing to the quantity at the top of the next column, to add together the quantities like *it*, and so on.

1.

$$\begin{array}{r} 3x - 7y + 2z \\ 4y + 6z - x \\ -3z - 2y + a \\ 4x + 3z - y \\ \hline 6x - 6y + 8z + a \\ \hline \end{array}$$

2.

$$\begin{array}{r} -5ax + 2by - 7 \\ 3by + 18 - 4z \\ 4ax - 9 - by \\ 26 + 3ax - 2by \\ \hline 2ax + 2by + 28 - 4z \\ \hline \end{array}$$

3.

$$\begin{array}{r} 7xy - 4az + 2bc \\ 6az + 5mn - 3p \\ 2bc - 3xy + 8az \\ -xy - bc - az \\ -4az + 3xy - 4bc \\ \hline 6xy + 5az - bc + 5mn - 3p \\ \hline \end{array}$$

4.

$$\begin{array}{r} -2yz - 3ab \\ -5cx + 2yz + 5ab \\ 7ab - 3cx + yz \\ 4ek - 6yz - cx \\ 2cx + yz - 8 \\ \hline -4yz + 9ab - 7cx + 4ek - 8 \\ \hline \end{array}$$

EXAMPLES FOR EXERCISE.

1.

$$\begin{array}{r} 2ax + 3by + 4c \\ 5by + 3c - ax \\ 7c + 4ax - 2by \\ 3cx - 5ax + 12 \\ \hline \\ \hline \end{array}$$

2.

$$\begin{array}{r} 5xy - 3az + 7 \\ 6xy + 4bx - 3 \\ 4az - 2xy + c \\ -19 + ax - xy \\ \hline \\ \hline \end{array}$$

3.

$$\begin{array}{r} 9yz + 8ax - 3bc \\ -2en - 4bc + 2yz \\ xyz + 3ax - 4yz \\ -2ax - yz + 3en \\ \hline \\ \hline \end{array}$$

4.

$$\begin{array}{r} 4ab - 3cd + 5ef \\ -7e - 4ab \\ 6cd + ef - 6e \\ 2ef - 3cd + 14 \\ 3e + ab - cd \\ \hline \\ \hline \end{array}$$

5.

$$\begin{array}{r} -7xy - 3bz + 4w \\ -5bz + 4xy - 3cd \\ 2w - 6bz + 9xy \\ -4xy + 3w - bz \\ -16 + 5xy + w \\ \hline \\ \hline \end{array}$$

6.

$$\begin{array}{r} 2amz + 2nx - 3bv \\ 6cw - 8bv - amz \\ 5bv + 4amz + 6 \\ -amz - 2bv - 7nx \\ -8nx - 14 \\ \hline \\ \hline \end{array}$$

From comparing the examples in this second case of addition with those given in the first, you will see that, although the placing of the several expressions to be added one under another was of considerable assistance *there*, because the *like* quantities all appeared in vertical rows, yet the arrangement is of no advantage *here*, since we have to pick out the *like* quantities after a careful search for them among the entire set of expressions. We could do this just as well if the expressions were all written side by side, as in the next examples, without taking the trouble of first arranging them one under another. If you think it easier, however, you may select first one set of like quantities out of the expressions in each of the following examples, and arrange them as in Case I.; then a second set, and so on. You may thus change this Case II. into Case I., taking care, however, that the *unlike* quantities be connected to the sum of the *like* quantities in the final result.

7. $2axy-3bz+4,\ 7bz-6,\ 5+2bz-3axy,\ 4bz-3c+2.$
8. $-6abc-3de-4f,\ 6f-2abc+de,\ 4de-3abc+8,\ abc-f.$
9. $4\frac{1}{2}z+2y-7x+9,\ 3x-\frac{1}{2}y+z,\ 8y-5z+2x,\ 2y-3x.$
10. $8\frac{x}{y}+ax-5b,\ 6b-4\frac{x}{y}-3ax,\ 2\frac{x}{z}-3\frac{x}{y},\ 4-7b.$
11. $-7xyz-3\frac{z}{a}-2bc,\ 5\frac{z}{a}-4mn+p,\ 12bc+3xyz,\ -xyz-7\frac{z}{a}.$
12. $\frac{2}{3}ax-5y+7z-8,\ -\frac{1}{2}y-2z,\ 4z-6ax+2y,\ 3y-9z+ax.$

SUBTRACTION.

Before giving you the rule for subtraction of algebra, we must explain the principle upon which the rule is founded; for whatever part of mathematics be studied, you must never confide in a "Rule" till convinced of its correctness.

Now, let us take any two numbers at random, say 9 and 4; and let us endeavour to *subtract* the latter from the former, when the algebraic signs +, —are, the one or the other, prefixed to the 9 and the 4, and try to put down what we are certain must be the correct *remainder*. You know that to *subtract* means to *take away;* if, therefore, we can *actually* take away the 4 with its sign, from the 9 with *its* sign, we shall be sure of the remainder sought. In order to this, let us write 9 in this form, namely, 9 + 4 — 4, which you see, although it takes up more room is only 9, for 4 — 4 is *nothing*. From the 9 thus expressed, *take away* the + 4, that is, fancy it actually taken up in your fingers and removed, or which will answer as well, take it away by rubbing it out; what remains is evidently 9 — 4 or 5.

Again, from the same expression for the 9, now take the — 4 away; that is, rub it out: what remains is evidently 9 + 4 or 13.

You thus see that if from a positive number, as 9, you have to subtract a positive number, as 4, the true remainder will be got by *changing the sign* of the number to be subtracted, and *then adding*, as in the margin. And that if, from a positive number, you have to subtract a *negative* number, the true remainder will be got, in like manner, by *changing the sign* of the number to be subtracted, and *then adding;* as in the margin.

$$\begin{array}{r} 9 \\ -4 \\ \hline 5 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ 4 \\ \hline 13 \\ \hline \end{array}$$

Hitherto, the 9 has been supposed *positive:* let it be negative, and, in imitation of the plan above, let us write this — 9 in the form — 9 + 4 — 4.

From the — 9, thus written, take away, or remove, the + 4; the remainder is — 9 — 4, or — 13.

From the same expression for the — 9, take away the — 4; the remainder is — 9 + 4, or — 5.

You see, therefore, that if from a negative number, as — 9, you have to subtract a positive number, as 4, the true remainder will be got by *changing the sign* of the number to be subtracted, and *then adding*, as in the margin. And that if from a negative number you have to subtract a negative number, the true remainder will, in like manner, be got by *changing the sign* of the number to be subtracted, and *then adding*, as in the margin.

$$\begin{array}{r} -9 \\ -4 \\ \hline -13 \\ \hline \end{array} \qquad \begin{array}{r} -9 \\ 4 \\ \hline -5 \\ \hline \end{array}$$

This reasoning, which of course applies to any pair of numbers, as well as to 9 and 4, at once suggests the following rule for subtraction of algebra.

RULE.—Change the signs of all the terms in the quantity to be *subtracted*, or *conceive them to be changed;* then proceed as if it were *addition* instead of subtraction: the result will be the *remainder*.

$$\begin{array}{lr} \text{1. From} & 9a + 6b - 5c \\ \text{Take} & 4a - 2b + 3c \\ \hline \text{Rem.} & 5a + 8b - 8c \\ \hline \end{array} \quad \left\{ \begin{array}{l} \text{or, which is the} \\ \text{same thing,} \end{array} \right\} \quad \begin{array}{lr} \text{To} & 9a + 6b - 5c \\ \text{Add} & -4a + 2b - 3c \\ \hline & 5a + 8b - 8c \\ \hline \end{array}$$

In the operation on the left, the signs of the subtractive terms are only *conceived* to be changed, and then *addition* performed. On the right the change is actually made, and the terms then added. We need scarcely say that the results of subtraction, like those of addition, are written from left to right.

$$\begin{array}{ll} \text{2. From} & 4ax - 3by - 7cz \\ \text{Take} & 5ax + 4by - 9cz - 2 \\ \hline \text{Rem.} & -ax - 7by + 2cz + 2 \\ \hline \end{array}$$

$$\begin{array}{ll} \text{3. From} & 8mz + bx - 6ay - 2 \\ \text{Take} & -2mz - 3bx + 4ay - 3 \\ \hline \text{Rem.} & 10mz + 4bx - 10ay + 1 \\ \hline \end{array}$$

As subtraction is thus converted into addition, all that has been said about addition applies of course here. Terms that have no *like* are to be brought down in the *remainder* with their *proper* signs; those that are in the subtractive row *with changed signs.*

EXAMPLES FOR EXERCISE.

1.
$$\begin{array}{l} 5x - 6y + 3z \\ 2x + 4y - 5z \\ \hline \\ \hline \end{array}$$

2.
$$\begin{array}{l} 2ax + 4by - 7cz \\ 3ax - 2by + 4cz \\ \hline \\ \hline \end{array}$$

3.
$$\begin{array}{l} -4xy - 6ax + 7 \\ -5xy + 9ax - 4 \\ \hline \\ \hline \end{array}$$

4.
$$\begin{array}{l} -23ay - 14bx + 6 \\ -18ay - 9bx \\ \hline \\ \hline \end{array}$$

5.
$$\begin{array}{l} 11abc + 3de - 6fg \\ -9abc - 7de + 5 \\ \hline \\ \hline \end{array}$$

6.
$$\begin{array}{l} 2bz - 3ax - m \\ -4bz + 2ax + en \\ \hline \\ \hline \end{array}$$

7.
$$\begin{array}{l} 8 - 6 + 2 - 4 - 7 \\ 3 + 5 - 9 + 2 - 4 \\ \hline \\ \hline \end{array}$$

8.
$$\begin{array}{l} 10dy - 11ex - 12fz - 3m \\ 9dy - 11ex + 12fz + 8n - 4 \\ \hline \\ \hline \end{array}$$

9.
$$\begin{array}{l} -7ayz - 9bx + 6c - 2e \\ 4ayz + 8bx + 2c - 4d \\ \hline \\ \hline \end{array}$$

10.
$$\begin{array}{l} 9cex - 7aby + 2dx + 11 \\ -3cex + aby - 2dx - 3z \\ \hline \\ \hline \end{array}$$

In the following examples the suitable *arrangement* of the terms, so as to bring like quantities under one another, is left for you to manage yourself.

11. From $5ax - 3by + 7cz$ take $2by - 3ax + 2cz$.
12. From $-4bz - 2cy - ax$ take $3ax + 4cy - 8bz$.
13. From $7ay + 5bx - 16$ take $-3bx + 4ay + 3cz$.
14. From $4\frac{1}{2}x - 2y + 13z - 4$ take $5z + 6y - \frac{1}{2}x$.
15. From $-8\frac{1}{2}axy - 3\frac{1}{2}bz + 4m$ take $-6\frac{1}{2}bz - 4axy + 2n$.
16. From $6\frac{1}{2}yz - 5\frac{1}{2}ax - 2\frac{1}{4}$ take $4ay - 3\frac{1}{2}ax + 4yz + \frac{3}{4}$.

If you have correctly worked the examples now given, you will have acquired a pretty good knowledge of Algebraical Addition and Subtraction, and have become familiar with the meaning and use of the plus and minus signs. We shall now explain a few further particulars, and shall then give a short specimen of the application of Algebra to what are called Simple Equations. These will throw some light upon the practical utility of the science; they will afford an insight into the value of algebraic symbols in matters of calculation, and enable you to see the great advantage in such matters of combining *letters* with *figures.*

You already know that the sign *minus* prefixed to a quantity indicates the *subtraction* of that quantity. Hitherto the sign has been prefixed to *simple* quantities only; but it may be prefixed to a *compound* quantity, so as to indicate that all the simple terms of which it is composed are to be subtracted. In order to this it is only necessary to unite all the terms by some link, so as to imply that when a sign is put before the compound quantity, that sign is to affect *the whole quantity,* or every individual term of which it is composed: whatever be the link employed, it is called a *vinculum.* Suppose we have the compound quantity $4a - 2b + 3c$ to subtract from $9a + 6b - 5c$, as in Exercise 1, page 454. We might *indicate* the subtraction thus:—

$$9a + 6b - 5c - (4a - 2b + 3c);\ \text{or}$$

$$9a + 6b - 5c - \left\{ 4a - 2b + 3c \right\};\ \text{or}$$

$$9a + 6b - 5c - [4a - 2b + 3c].$$

Such *vincula* as these are also called *brackets.* It is plain, from the rule of subtraction, that a *bracketed quantity* may be freed from the brackets, when a *minus sign* is prefixed to it, by simply *changing* the *signs* of all the terms which compose it. Thus the preceding expression, without brackets, is,

$$9a + 6b - 5c - 4a + 2b - 3c = 5a + 8b - 8c.$$

And the subtraction of a compound quantity may in general be performed in this way without taking the trouble of writing it under the quantity it is to be taken from. When the terms to be subtracted are all connected, with changed signs, as above, to the other quantity, we shall merely have a row of terms to be united, like with like, as in addition; just the same as in the example now given.

If the sign *plus* appear before a bracketed quantity, then, since *addition* implies no change of sign, the signs must remain undisturbed, though the brackets be removed: thus—

$$9a + 6b - 5c + (4a - 2b + 3c) = 9a + 6b - 5c + 4a - 2b + 3c$$
$$= 13a + 4b - 2c.$$

Now, that you may never find yourself puzzled, as to signs, when you have to free an expression from brackets, always be careful to notice the *sign,* whether + or —, which precedes the bracket; fancy this sign rubbed out along with the brackets; if it be +, the terms thus set free present their proper signs, without any change being necessary. Of course the *leading* term within the brackets, if itself a *plus* term, as the $4a$, above, will not have its *sign* actually inserted; so that when the + *before* the bracket is rubbed out with the brackets themselves, there will be a gap as between the $5c$ and the $4a$ here; you need scarcely be told that in this gap the + belonging to the $4a$ must be inserted, because $4a$ is *now* not a leading quantity.

But if the sign before the bracket be —, then having rubbed out this sign, with the brackets, or having fancied it rubbed out, write all the terms, thus set free, with *changed signs.* For example—

$$8ax + by + (-5ax - 3by) = 8ax + by - 5ax - 3by$$
$$= 3ax - 2by.$$

$$8ax + by - (-5ax - 3by) = 8ax + by + 5ax + 3by$$
$$= 13ax + 4by.$$

You see that in the first of these expressions you have only to fancy that, with the tip of your finger, you rub out the marks + (, between the by and the $-5ax$; and in the second expression, that you in like manner rub out — (, between the by and the $-5ax$, and *then* that the signs of the terms thus set free are *changed.* By attending to these hints, you will not be likely "*to stick*" at bracketed expressions.

Coefficients are frequently found before bracketed quantities: you are aware that coefficients are *multipliers* or *factors.* When the brackets are removed, the factor is to be introduced into each simple term, as in the instances following:—

$$3(4a - 2b + 3c) = 12a - 6b + 9c;\ 6(-5ax - 3by)$$
$$= -30ax - 18by$$

$$-4(-3x - 5ay + 2) = 12x + 20ay - 8;$$
$$2a(3xy - 2bz) = 6axy - 4abz.$$

As already stated, brackets are not the only kind of *vincula* used to bind a set of simple quantities into one compound whole; a *bar* or *line,* put over the row of quantities, is sometimes, though less frequently, employed; thus, $a + b + \overline{a - b}$, and $a + b - \overline{a - b}$, are the same as $a + b + (a - b)$, and $a + b - (a - b)$; but this kind of vinculum is getting out of use, though the bar or line, which separates the numerator from the denominator of a fraction, still performs the office of a vinculum when there are several terms in the *numerator;* thus, in the fraction$-\dfrac{ax - by + c}{m}$, the minus sign, *before the bar of separation,* operates as it would do if the compound numerator were enclosed in brackets; thus, $\dfrac{-(ax - by + c)}{m}$, that is, the fraction is the same as $\dfrac{-ax + by - c}{m}$

We shall not detain you further with these explanations, except to remark, that the two instances given before, to show the meaning of the *bar-vinculum*, suggest two arithmetical theorems, which, although very simple, you will do well to remember, as they are useful. The expressions referred to are—

$a + b + (a - b) = 2a$, and $a + b - (a - b) = 2b$.

Now, as a and b may stand for any two numbers whatever, we learn from these results that—

1. *The sum* $(a + b)$ *of any two numbers, increased by their difference* $(a - b)$, *is twice the greater number*, that is, $2a$.

2. *The sum of any two numbers, diminished by their difference, is twice the less*; viz., $2b$.

Take, for instance, the two numbers 7 and 3; their sum 10, increased by their difference 4, is 14—double of the *greater*, 7. And the same sum, 10, diminished by their difference, 4, is 6—double of the *less;* and the same of any pair of numbers whatever.

SIMPLE EQUATIONS.

We shall now show a few applications of the principles laid down in the foregoing articles to the *solution of simple equations*. What we here propose to give, must be regarded, however, as only an introduction to the complete consideration of the subject, which will be resumed hereafter.

An *equation* is merely a statement, in the characters of algebra or arithmetic, that two quantities are *equal:* thus, that *two and five are equal to seven*, is a statement of equality, which, when expressed in figures and signs, thus, $2 + 5 = 7$, is an equation. In like manner, all these are equations—namely: $6 + 2 - 3 = 5$, $7 - 4 = 6 - 3$, $2 + 8 - 1 = 9$, $3x - 2 = 10$, &c., &c. All but the last are purely arithmetical equations; the last is an *algebraical* equation, as it contains the algebraical character or symbol x. You see that an equation consists of two *members* or *sides:* one on the left of the sign of equality, and the other on the right. These members or sides, if equally increased, or equally diminished, equally multiplied, or equally divided, give results that must evidently be *new equations*, because *equal* quantities thus operated upon must give *equal* quantities for the results of the operations. This is a general axiom which you must pay attention to, as your success in solving the equations about to be given will depend chiefly upon your skill in applying it. But we must tell you what is meant by *solving* an equation.

You are said to have *solved* a question in arithmetic when you have worked it out and arrived at the *answer;* till you have completed the calculation, the answer remains *unknown*. It is the same in algebra; *the solution of an equation* is the finding the value or interpretation of some letter or letters, which at the outset is unknown. Unknown values are usually represented by letters towards the *end* of the alphabet, as z, y, x, &c.; while letters whose numerical value are already known, are chosen from the beginning, as a, b, c, &c. In the equation given above, namely, $3x - 2 = 10$, x represents a number, at present unknown, such that three times that number, diminished by 2, is equal to 10; the operation by which the number, or value of x, is discovered, is the solution of the equation. This is effected as follows :—

Add 2 to each *side;* the result is the new equation $3x = 12$; that is, 3 times x is equal to 12.

Divide each side of this by 3: the result is $x = 4$: so that x, at first an unknown value, turns out to be equal to 4; and you see that three times 4 diminished by 2 *is* equal to 10.

As here, so in the examples which follow, x will always be used to stand for the value at the outset unknown; so that the solution of the equation will be the *finding the value of* x; the operations for this purpose will be very easy: they are called *Transposition* and *Clearing Fractions*.

Transposition.—The operation called transposition consists simply in taking quantities from one side of an equation, and putting them on the other side, still, however, taking care to preserve the *equality* of the two sides; for whatever operations we perform, we must never disturb the equality of the two sides: the *result* of each operation must still be an *equation*. Now, if you remove a quantity from one side of an equation, and place it on the other side; that is, if you rub it out, sign and all, from one side, and then write it down on the other, the change you thus make will not disturb the equality of the two sides, provided only the quantity rubbed out be written on the other side with a *changed sign*. You will be convinced of this by an example or two. Let there be the equation $4x - 5 = 3x - 2$: then if we wish the 5 on the left to be removed to the right, all we have to do is to *add* 5 to *both sides* of the equation; the result is the equation $4x = 3x - 2 + 5$. Here you see that the 5 is *transposed;* it is taken from the left side and put on the right *with changed sign*. The new equation is therefore $4x = 3x + 3$. Again: if we now wish to remove the $3x$ from the right to the left, all we have to do is to *subtract* $3x$ from *both sides;* the result is the equation $4x - 3x = 3$. Here you see that the $3x$ is *transposed:* it is taken from the right and put on the left *with changed sign*. The new equation is therefore $x = 3$. We have thus actually *solved* the equation $4x - 5 = 3x - 2$; the x in this equation, at the outset unknown in value, is now found out to stand for 3. If you put this 3 for the x in the proposed equation, it becomes the numerical equation $12 - 5 = 9 - 2$; which you see is true, each side being 7.

Clearing Fractions.—When a fraction occurs in an equation, we may *clear* the equation of the fraction by multiplying both sides of it by the denominator of the fraction. You know from common arithmetic, that if you multiply a fraction by its denominator, the result is simply the numerator: thus, $\frac{2}{3} \times 3 = 2$; $\frac{4}{5} \times 5 = 4$; $\frac{2}{7} \times 7 = 2$, and so on. In like manner, $\frac{x}{3} \times 3 = x$; $\frac{4x}{5} \times 5 = 4x$; $\frac{2a}{7} \times 7 = 2a$, &c. If, therefore, we have such an equation as $2x + \frac{x}{3} + 2 = 22 - x$ it will become cleared of fractions by multiplying both sides by 3: the result of this multiplication is the new equation $6x + x + 6 = 66 - 3x$, which is free from fractions. To complete the solution of the equation, that is, to find the value of x, we must, by transposition, bring *all* the unknown quantities to one side of the equation, and *all* the known quantities to the other: we shall thus have $6x + x + 3x = 66 - 6$; that is, $10x = 60$: consequently, dividing each side by 10, there results finally $x = 6$: and if this be put for x in the original equation, you will find the two sides of it to be numerically equal, for each side will be 16.

To solve a simple equation containing only one unknown quantity.

RULE I.—If there be a fraction in the equation, clear it away by multiplying both sides by the *denominator* of the fraction.

2. If known and unknown quantities are linked together, separate them by *transposition;* so that *all* the unknown quantities may appear on one side of the equation, and all the known quantities on the other.

3. Collect the terms on each side into one sum, so that there may be only a single unknown term on one side of the equation, and a single known term on the other.

4. Lastly, divide each side by the coefficient or multiplier of x, the unknown quantity, the result will be x *alone* on one side, and its value or interpretation on the other side of the sign of equality.

EXAMPLE 1.—Given the equation $5x - 8 = 3x + 2$ to find the value of x. As it is usual to choose the *left* hand side of the equation for the unknown quantities, and the other side for the known numbers, we shall *transpose* the known number 8, and thus get the equation $5x = 3x + 2 + 8$; then transposing the unknown quantity $3x$, we shall obtain the equation $5x - 3x = 2 + 8$, the *left* side of which is occupied wholly with unknown, and the *right* side wholly with known quantities. Reducing now each side to a single term, the equation is $2x=10$; and dividing by 2, there results $x=5$. The steps of the solution, as you would be expected to solve the equation, would stand as follows:—

Given equation, $5x-8=3x+2$
By transposition, $5x-3x=2+8$
Collecting the terms, $2x=10$
Dividing by 2, $x=5$

2 Given the equation $2x-9=31-3x$ to find the value of x.

By transposing, $2x+3x=31+9$
Collecting, $5x=40$
Dividing by 5, $x=8$

3. Given the equation $\frac{2x}{3}+\frac{x}{4}-22=0$ to find the value of x.

Multiplying by 3, to clear the first fraction,

$$2x+\frac{3x}{4}-66=0$$

Multiplying by 4, to clear the remaining fraction,

$$8x+3x-264=0$$

Transposing, $8x+3x=264$
Collecting, $11x=264$
Dividing by 11, $x=24$

4. Given $\frac{x}{3}+\frac{x}{4}+\frac{x}{5}=1$, to find the value of x.

Multiplying by 3, $x+\frac{3x}{4}+\frac{3x}{5}=3$
" by 4, $4x+3x+\frac{12x}{5}=12$
" by 5, $20x+15x+12x=60$
* $\therefore$ $47x=60$, $\therefore x=\frac{60}{47}=1\frac{13}{47}$

EXAMPLES FOR EXERCISE.

Find the value of x in each of the following equations:—

NOTE.—Since any quantity may be removed from one side of an equation to the other, provided its *sign be changed*, it follows that if *all* the signs on one side, and *all* the signs on the other be changed, the *equality* of the two sides will not be disturbed; for this changing of *all* the signs is merely the same as *transposing* the two sides.

1. $2x-7=x+4$
2. $5x-3=3x+5$
3. $6x+2=4x+8$
4. $3x-1=x+7$.
5. $8x-13=4x-1$
6. $2(x+3)=20$
7. $3(x-4)=6$
8. $4(x-1)=x+5$
9. $5(x+2)=2(x+8)$
10. $3(2x-3)=4(x-2)$
11. $\frac{x}{3}+\frac{x}{4}=7$
12. $\frac{x}{3}-\frac{x}{4}=1$
13. $\frac{2x}{3}-\frac{x}{4}=5$
14. $\frac{x}{2}+\frac{x}{3}=31-\frac{x}{5}$
15. $\frac{x}{3}+\frac{x}{4}+\frac{x}{6}=1$
16. $2x-(48-x)=x+12$
17. $\frac{5(3x+2)}{2}=35$
18. $\frac{x+3}{5}-1=2-\frac{x}{7}$
19. $\frac{x}{2}+\frac{x}{3}-\frac{x}{5}-1=0$
20. $\frac{x}{2}+\frac{x}{3}-\frac{x}{4}-\frac{1}{2}=0$
21. $11-\frac{5x}{8}=\frac{x}{2}-\frac{2x}{3}$
22. $4x-2(3-x)=\frac{x+3}{2}$
23. $6x-\frac{4-3x}{5}=\frac{29}{5}$
24. $\frac{x-6}{2}+\frac{x}{3}=20-\frac{x-2}{2}$
25. $\frac{3x-13}{7}+\frac{11-4x}{3}=0$
26. $6-\frac{x-1}{2}-\frac{x-2}{3}=\frac{3-x}{4}$

QUESTIONS TO BE SOLVED BY SIMPLE EQUATIONS.

Every *question*, whether in arithmetic or algebra, has for its object the discovery of some unknown value by means of the conditions, stated in the question, which connect it with *known* values. In algebra, the first thing to be done is to express these conditions in the form of an *equation*, some letter, as x, being made to stand for the unknown quantity; the solution of the *equation* is the solution of the *question*. If there are several unknown quantities to be determined, as many distinct equations will in general be necessary; at present those questions only will be considered that furnish a single equation with one unknown quantity. The method of translating such questions into equations will be better learnt by the study of a few examples than by verbal directions. We shall therefore here give some instances of the modes of proceeding.

1. There are two numbers of which the difference is 8 and the sum 38: what are the numbers?

Let x stand for the smaller of the two numbers: then $x+8$ must represent the greater: and since by the question the sum of the two is 38, we have the equation,

$$x+x+8=38.$$

Or transposing the 8, $2x=30$, $\therefore x=15$, the smaller number; and as $x+8$ is the greater, $\therefore 15+8=23$, is the greater.

You see that these numbers satisfy the conditions of the question; for $23-15=8$, and $23+15=38$.

2. From two places, 160 miles apart, two persons, A and B, set out at the same time to meet each other. A travels 18 miles a-day, and B 22 miles: in how many days will they meet?

Suppose they meet in x days: then A will have travelled $18x$ miles, and B, $22x$ miles: the *sum* of these distances is, by the question, 160 miles: hence we have the equation,

$$18x+22x=160;$$

that is, $40x=160$, $\therefore x=4$, the number of days.

3. A father is three times as old as his son; but five years ago he was four times as old: what are their ages now?

Let the present age of the son be x years: then, by the question, that of the father is $3x$ years. Also five years ago the age of the son must have been $x-5$, and that of the father $3x-5$. The question tells us that this latter number is four times the former: that is,

$$3x-5=4(x-5);$$

or, removing the vinculum, $3x-5=4x-20$; $\therefore$ transposing the 20 to the left, and the $3x$ to the right,

$$20-5=4x-3x, \therefore 15=x;$$

that is, the son's age is 15 years; and $3x$, the age of the father, is 45.

4. A vessel holding 120 gallons is partly filled by a spout which delivers 14 gallons in a minute: this is then turned off, and a second spout, delivering 9 gallons in a minute, completes the filling of the vessel. How long did *each* spout run, the time occupied by *both* in filling the vessel being 10 minutes?

Suppose the first ran x minutes: then the second ran $10-x$ minutes. As the first delivers 14 gallons a minute, the quantity delivered by this spout must be $14x$ gallons; and the quantity delivered by the other, at 9 gallons a minute, must be $9(10-x)$ gallons; $\therefore$ the *whole* number of gallons delivered is,

$14x+9(10-x)=120$, by the question;
that is, removing vinculum, $14x+90-9x=120$;
$\therefore$ transposing, $14x-9x=120-90$;
collecting, $5x=30$, $\therefore x=6$.

$\therefore$ the first spout ran 6 minutes, and the second $10-6=4$ minutes.

5. What number is that of which the third part exceeds the fifth part by 48?

Let x be the number: then by the question

$$\frac{x}{3}-\frac{x}{5}=48$$

Multiplying by 3, $x-\frac{3x}{5}=144$
" by 5, $5x-3x=720$;
that is, $2x=720$, $\therefore x=360$, the number required.

The third part of this is 120; the fifth part is 72; and the difference between these parts is 48.

6. A vessel can be filled from a tap in 3 hours, and from a second tap in 5 hours: in what time will it be filled if both taps run together?

Let the number of hours be x: then the part of the vessel filled in *one* hour will be the $\frac{1}{x}$ part.

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* The sign $\therefore$, stands for the word *therefore*; on account of its convenience it will be frequently used hereafter.

But the part filled by the first tap in 1 hour is the $\frac{1}{3}$ part, and by the second tap the $\frac{1}{5}$ part;

$$\therefore \frac{1}{3}+\frac{1}{5}=\frac{1}{x}; \text{ that is, } \frac{8}{15}=\frac{1}{x}$$

Multiplying by 15, $8=\frac{15}{x}$

" by x, $8x=15$, $\therefore x=\frac{15}{8}=1\frac{7}{8}$

∴ the time is one hour and seven-eighths.

These specimens of the solution of questions by simple equations must suffice for the present. It will be seen, from an examination of them, that the only thing of any difficulty, in thus applying the first principles of Algebra, is the translation of the conditions of the question into algebraical language, under the form of *an equation;* the process by which this equation is to be solved, and the unknown quantity in it discovered, is in general sufficiently suggested by the appearance of the equation itself. As the aim is to isolate the x, so that it alone may occupy one side of the equation, while known numbers occupy the other, every step in the solution is made to contribute to this end. By transposition, clearing fractions, collecting like terms, &c., the equation is made to pass through one change after another, till at length a single unknown term appears on one side, and a known number, which is its interpretation, on the other. It then only remains to divide each side by the coefficient of x, if it have a coefficient other than unity, and x itself becomes known.

QUESTIONS FOR EXERCISE.

1. There are two numbers, of which the difference is 9, and the sum 43: what are the numbers?

2. From two places, 108 miles apart, two persons, A and B, set out at the same time to meet each other. A travels 17 miles a-day, and B travels 18: in how many days will they meet?

3. Find two numbers of which the difference is 13, and which are such that if 17 be added to their sum, the whole will amount to 62.

4. There are two numbers, of which the difference is 15, and which are such that if 7 times the less be subtracted from 5 times the greater, the difference is 19: what are the numbers?

5. A person starts from a certain place, and travels at the rate of 4 miles an hour. After he has been gone 10 hours, a horseman, riding 9 miles an hour, is dispatched after him: how many hours must the horseman ride to overtake him?

6. A person has 264 coins—sovereigns and florins; he has $4\frac{1}{2}$ times as many florins as sovereigns: how many of each coin has he?

7. A person spends $\frac{1}{4}$th of his yearly income in board and lodging, $\frac{1}{3}$th in clothes and other expenses, and he lays by £85 a-year: what is his income?

8. What number is that whose *third* part exceeds its *fifth* part by 72?

9. I have a certain number in my thoughts. I multiply it by 7, add 3 to the product, and divide the sum by 2. I then find that if I subtract 4 from the quotient, I get 15: what number am I thinking of?

10. A man 40 years old has a son 9 years old; the father is therefore more than 4 times as old as his son: in how many years will the father be only twice as old as his son?

11. Two persons, A and B, 120 miles apart, set out at the same time to meet each other. A goes 3 miles an hour, and B 5 miles: what distance will each have travelled when they meet?

12. Divide £250 among A, B, and C, so that B may have £23 more than A, and C £105 more than B.

13. A can execute a piece of work in 3 days which takes B 7 days to perform: in how many days can it be done if A and B work together?

14. A cistern can be filled by three pipes; by the first in 2 hours, by the second in 3, and by the third in 4: in what time can it be filled by all the pipes running together?

15. Solve the preceding question when the first pipe fills the cistern in 1 hour 20 minutes; the second in 3 hours 20 minutes; and the third in 5 hours.

16. After A has been working 4 days at a job which he can finish in 10 days, B is sent to help him; they finish it together in 2 days: in what time could B alone have done the whole?

17. Divide £143 among A, B, and C, so that A may receive twice as much as B, and B three times as much as C.

18. A person has 40 quarts of superior wine, worth 7*s.* a quart; he wishes, however, so to reduce its quality as that he may sell it at 4*s.* 6*d.* a quart: how much water must he add?

19. Divide 90 into four parts, such that if the first be increased by 2, the second diminished by 2, the third multiplied by 2, and the fourth divided by 2, the results may all be equal.

20. Divide 39 into four parts, such that if the first be increased by 1, the second diminished by 2, the third multiplied by 3, and the fourth divided by 4, the results may all be equal.

The preceding examples may serve to show how the first principles of Algebra may be applied to inquiries of a practical nature. It is time that we now proceed to the other two rules, multiplication and division; for, as in Arithmetic, the four rules comprehend all the operations of the science. But two or three particulars must be previously defined.

We know that when factors are multiplied together, the result is called *a product;* if the factors are all *equal*, the product is called *a power* of the factor whose repetition in the multiplication has produced it: thus, in the following instances, namely—
$5+5=25$; $3+3+3=27$; $2+2+2+2=16$, &c., 25 is *the second power*, or *the square* of 5; 27 is *the third power*, or *the cube* of 3; 16 is *the fourth power* of 2; 32 is *the fifth power* of 2; 64 *the sixth power*, and so on. The fourth power of a is $aaaa$; the fifth power of x is $xxxxx$, and so on. But as this repetition of the factors is tedious and cumbersome, it is agreed to represent a power by writing down the factor only *once*, and placing over the right hand upper corner the *number* which denotes the *repetitions;* thus, the fourth power of a is written a^4, the fifth power of x is written x^5, and so on. Suppose, for example, that x stands for 3; then $x=3$, $x^2=9$, $x^3=27$, $x^4=81$, $x^5=243$, $x^6=729$, $x^7=2187$, &c. The small figures, thus used in the notation for powers, are called *exponents* or *indices;* the exponent or index for the *cube*, or *third power*, is 3, that for the fourth power 4, and so on.

The number or quantity, which thus produces a *power*, is called a *root* of that power; thus, 3 is the *square root* of 9, the *cube root* of 27, the *fourth root* of 81, and so on. There is a convenient notation for *roots* as well as for *powers*.

The sign $\sqrt{\ }$, for a root, is called the radical sign; it is prefixed to the quantity whose root is meant, and a small figure, denoting *what* root is to be understood, is connected with it; thus, $\sqrt[2]{4}$ means the *second*, or *square* root of 4, that is, 2; because the *second power*, or *square* of 2, is 4; in like manner, $\sqrt[3]{8}$ means the *third*, or *cube* root of 8, namely, 2; because the *third power*, or *cube* of 2, is 8; $\sqrt[4]{x}$ means the *fourth root* of x; that is, it is a number such that the *fourth power* of that number is the number that x stands for. It must be observed, however, that in the case of the *second* or *square* root, the little 2 is always omitted; so that when there is no *index-figure* connected with the *radical sign* $\sqrt{\ }$, the *square root* is always to be understood. You will now easily make out the following statements or equations:—

$$\sqrt{9}=3,\ \sqrt[3]{125}=5,\ \sqrt[4]{a^4}=a,\ \sqrt[5]{x^5}=x,\ \sqrt{z^2}=z.$$

But besides the radical sign, there is another contrivance for indicating roots—a contrivance like that for denoting powers, namely, the attaching an *exponent* to the quantity whose root is meant; thus, the equations above, expressed in this other form of notation, are as follows:—

$$9^{\frac{1}{2}}=3,\ 125^{\frac{1}{3}}=5,\ (a^4)^{\frac{1}{4}}=a,\ (x^5)^{\frac{1}{5}}=x,\ (z^2)^{\frac{1}{2}}=z.$$

In the case of a *power* of a quantity, the exponent is a whole number, which indicates *what power* is meant; in the case of a root, the exponent is a fraction, the denominator of which indicates *what root* is meant. The *power* of a proposed quantity may be easily determined, as the *multiplication* may be actually executed with but little trouble; but to find a specified *root* of a proposed quantity—even of a common number—is often a difficult matter; some numbers, indeed, have no *exact* roots. But we shall have to speak about roots again.

MULTIPLICATION.

CASE I.—*When the factors are simple quantities.*

RULE 1. Observe whether the *signs* of the two factors are *like* or *unlike*; if they are *like*, that is both + or both —, write + for the sign of the product; but if they are *unlike*, that is, one + and the other —, write — for the sign of the product.

2. After the sign, write the product of the *coefficients*.

3. After the product of the coefficients, write that of the *letters*: that is, put down the letters in both factors, one after the other, without any sign between them, and the complete product will be exhibited.

Thus, if we have to multiply together the two factors, $-4ax$ and $3by$, we first observe the signs; these being *unlike*, we know that the sign of the product is *minus*; after this *minus* we write 12, the product of the coefficients, and finally we place against the 12 the quantity $abxy$, this being the product of the letters when arranged in alphabetical order, ∴ the product is $-12abxy$. Again, if we have to multiply $-7bxz$ by $-5acy$, then the signs being *like*, we write + for the sign of the product, 35 for the product of the coefficients, and $abcxyz$ for the product of the letters; ∴ $-7bxz \times -5acy = 35abcxyz$, the *plus* sign being omitted, as unnecessary. In a similar way we have—

$$3mx \times 6ay = 18amxy,\ 2bcy \times -4ax = -8abcxy.$$
$$-4a^2x \times 5a^3bx = -20a^5bx^2,\ -3bz^2 \times -8a^3bxz = 24a^3b^2xz^3.$$

You will observe that the *third* of these examples is the same as $-4aax \times 5aaabx = -20aaaaabxx$; and that this result, in the more brief notation for *powers*, is $-20a^5bx^2$. And you must perceive that, by always adopting this notation, the multiplication of *powers of the same quantity* is reduced simply to the *addition of the exponents* of the factors; thus, the factors in $x^2x^3x^5$, factors which are all powers of the same quantity x, are x^2, x^3, x^5; the *sum* of the exponents is $2+3+5=10$; so that $x^2x^3x^5=x^{10}$; nothing, therefore, can be more easy than the multiplication of powers of *the same* quantity. We need scarcely tell you that x is the same as x^1, so that $xx=x^2$, $x^2x=x^3$, &c.

The preceding direction, as to the *sign* of the product of two factors, is called the *Rule for the signs*. It is briefly expressed thus:—*Like signs give* PLUS; *unlike signs*, MINUS.

You may satisfy yourself that *the rule for the signs* is true as follows:—

Take any two factors whatever, say 7 and 3; then all the possible varieties, as to signs, will be these—namely,

$$7\times3,\ -7\times3,\ 7\times-3,\ -7\times-3.$$

The first is the case of common arithmetic, the product being 21; the second case requires no consideration; for -7, repeated three times, is necessarily -21. The third case is peculiar; but we may arrive at the true product thus: increase the multiplier -3 by 4; the product, whatever it be, will obviously be 4 times 7—that is, 28 *too great*; but the multiplier increased by 4 becomes 1; and *once* 7 is 7, and as this is 28 too great, the correct product must be $7-28$, that is -21, ∴ $7\times-3=-21$.

In like manner, in the fourth case, increase the multiplier -3 by 4; then, as before, the product will be four times -7, that is, -28 too great; in other words, -28 must be *subtracted* from the erroneous product to make it correct; but the multiplier increased by 4, is 1, and once -7 is -7; ∴ the correct product is $-7+28$, that is 21; because the sign of the -28 must be *changed* when subtracted; ∴ $-7\times-3=21$. You thus see that when the factors have *like* signs, the product is *plus*; and that when they have *unlike* signs, the product is *minus*; and, from the foregoing reasoning, it is plain that the same conclusion would have followed if any other two factors had been chosen; ∴ the *rule for the signs* in multiplication is general.

Ex. 1. $\begin{array}{l} 7ax \\ 3b \\ \hline 21abx \\ \hline \end{array}$ 2. $\begin{array}{l} -7ax^2 \\ 3bx \\ \hline -21abx^3 \\ \hline \end{array}$

3. $\begin{array}{l} -7x^2y^3 \\ -3ax^3 \\ \hline 21ax^5y^3 \\ \hline \end{array}$ 4. $\begin{array}{l} -8ayz^2 \\ -7bxy \\ \hline 56abxy^2z^2 \\ \hline \end{array}$

6. $\begin{array}{l} 13a^2b^3x \\ -9ab^2x^4 \\ \hline -117a^3b^5x^5 \\ \hline \end{array}$ 7. $\begin{array}{l} 16cy^3z^4 \\ -ab^2z^2 \\ \hline -16ab^2cy^3z^6 \\ \hline \end{array}$

8. $\begin{array}{l} -\frac{1}{2}a^3x^4y^5 \\ -6ax^2y^3 \\ \hline 3a^4x^6y^8 \\ \hline \end{array}$ 9. $\begin{array}{l} -\frac{1}{2}x^4y^6z^2 \\ \frac{1}{2}ax^3y^6z^2 \\ \hline -\frac{1}{4}ax^7y^6z^3 \\ \hline \end{array}$

10. $3a^4x^2z^3\times7a^2xy^3=21a^6x^3y^3z^3$
11. $-4x^5yz^3\times5x^2y^4z=-20x^7y^5z^4$
12. $-6b^2cx^6\times-3b^4c^2x=18b^6c^3x^7$
13. $-\frac{2}{3}a^2z^3\times\frac{3}{5}ax^2z^4=-\frac{2}{5}a^3x^2z^7$

EXAMPLES FOR EXERCISE.

[NOTE.—You will observe that the rule for the signs enables us to fix the sign of the product of *two* factors only; but it is unnecessary that it should do more than this; if there are *three* factors, the product of *two* becomes *a factor* to be combined with the *third*. It is plain that, however numerous the factors when the number of them preceded by the *minus* sign is *odd*, the sign of the product is *minus*; and that when the number of minus factors is *even*, the sign of the product is *plus*.]

1. $9a^3y^2\times4a^2y^3$ 2. $-7bx^2y^3\times8b^4y^5$
3. $-4a^2x^3z^4\times-6$ 4. $-5b^3c^2y\times-3c^4y^3z$
5. $11a^4xy^2z^3\times-9a^5x^7y^5z^2$ 6. $-13a^2x^3y^4\times14a^4x^3y^2$
7. $\frac{1}{2}b^4x^3y^2\times-4b^2xy^5$ 8. $\frac{1}{2}ax^4z^2\times\frac{1}{2}x^3yz^4$
9. $2ax^2y\times-3a^2y^3\times-4a^3x^4$
10. $-3b^2yz\times-2y^2z^2\times-4ab^3y$
11. $\frac{1}{2}axy^3\times\frac{2}{3}x^2y\times-3a^2y^2$
12. $\frac{2}{3}a^2z^3\times-\frac{1}{4}x^2y^3\times-\frac{3}{5}a^3y^2z$
13. $-\frac{5}{6}a^2y^3\times-3b^2z^3\times-\frac{2}{7}abx^2yz$
14. $\frac{7}{8}a^3x\times-\frac{4}{5}x^2y\times-2y^2z\times-10a^2xz^3$

CASE II.—*When the multiplicand is a compound quantity, and the multiplier a simple quantity.*

RULE.—Multiply each simple term in the multiplicand by the multiplier, beginning always at the *left* hand; connect the several products together by their proper signs, and the complete product will be exhibited.

1. $\begin{array}{l} 4a^2x^3-2by^2 \\ 3axy \\ \hline 12a^3x^4y-6abxy^3 \\ \hline \end{array}$ 2. — $\begin{array}{l} 3axy^2-4bz+2a^2 \\ 5axz^2 \\ \hline -15a^2x^2y^3z^2-20abxz^3+10a^3xz^2 \\ \hline \end{array}$

3. $\begin{array}{l} 6x^2yz-2ay^2-3b^2y \\ -2xz^2 \\ \hline -12x^3yz^3+4axy^2z^2+6b^2xyz^2 \\ \hline \end{array}$

4. $\begin{array}{l} -5ax^3y-by^2z+\frac{1}{2}ab^3 \\ -6a^2by^3 \\ \hline 30a^3bx^3y^4+6a^2b^2y^5z-3a^3b^4y^3 \\ \hline \end{array}$

5. $\begin{array}{l} 4a^2x+\frac{1}{2}by^2+az^3 \\ -abx \\ \hline -4a^3bx^2-\frac{1}{2}ab^2xy^2-a^2bxz^3 \\ \hline \end{array}$

EXAMPLES FOR EXERCISE.

1. $$\begin{array}{l} 3ax^2-2bxy\\ 4a^2x\\ \hline \\ \hline \end{array}$$

2. $$\begin{array}{l} 5a^2y^2-4x^2y^3\\ -3axy\\ \hline \\ \hline \end{array}$$

3. $$\begin{array}{l} -6b^2yz^2+4ab^2y^2\\ -2ayz\\ \hline \\ \hline \end{array}$$

4. $$\begin{array}{l} 7a^2b^4x^6-3x^2y^4+5\\ 4ax^3y\\ \hline \\ \hline \end{array}$$

5. $$\begin{array}{l} 9m^2nz^2-7a^2y^2-2b^2x^3y\\ -7an^2x^3\\ \hline \\ \hline \end{array}$$

In the following examples, the *vinculum* or *brackets* may be removed, and the simple factor (the multiplier) placed under the leading term of the compound one, as in the preceding instances; but in most cases you will be able to write down the result of the multiplication at once, without resorting to this arrangement.

6. $(2a^2bx^3-5xy^3)\ 3ax^2y$
7. $(2axy^3-3x^2z+2x)\times-4axy$
8. $(mxy^3+3ny^3-2y^2z)\times-6m^2x^2y$
9. $(4y^2z^3-3ax^5-5by^3)\times-5ab^2y$
10. $(5cx^2z^3-y^4z^2-3z^4)\ 12c^2x^3yz^3$
11. $(\frac{2}{3}ax^2-\frac{1}{2}by^3-2cz^2)\times-12a^2bc^3$
12. $\{4ay^2-(2b^2z^3-yz-2)3a^2yz^3$
13. $\{2bx^2y^3-(5cy^2z+4yz^3-3z^5)\}\times-3x^2y$
14. $\{5a^2x^3-(3b^2y^2+2a^2x^3-4ay^3)\}\times-2a^2b^2xy$

CASE III.—*When multiplicand and multiplier are both compound quantities.*

RULE 1.—Multiply *all* the terms of the multiplicand by *each* term of the multiplier, proceeding with each as in the last case.

2. Collect together the several products that are *like*, as in addition, and to the sums of these unite, by their proper signs, the other products; and the complete product will be obtained.

1. Multiply $x+y$ by $x-y$.

$$\begin{array}{r} x+y\\ x-y\\ \hline x^2+xy\\ -xy-y^2\\ \hline \end{array}$$

$$\therefore (x+y)(x-y)=x^2\quad-y^2$$

2. Multiply $x+y$ by $x+y$.

$$\begin{array}{r} x+y\\ x+y\\ \hline x^2+xy\\ xy+y^2\\ \hline \end{array}$$

$$\therefore (x+y)^2=x^2+2xy+y^2$$

3. Multiply $x-y$ by $x-y$.

$$\begin{array}{r} x-y\\ x-y\\ \hline x^2-xy\\ -xy+y^2\\ \hline \end{array}$$

$$\therefore (x-y)^2=x^2-2xy+y^2$$

We have a remark to make in reference to these three examples; you will do well to keep it in remembrance. It is this: since x and y stand for any two numbers whatever, we learn that—

1. The sum of two numbers $(x+y)$ multiplied by their difference $(x-y)$ is the difference of the squares of those numbers (x^2-y^2).

2. The square of the sum $(x+y)$ of two numbers (x and y) is equal to the sum of their squares (x^2+y^2), together with twice their product $(2xy)$.

3. The square of the difference $(x-y)$ of two numbers is equal to the sum of their squares (x^2+y^2), diminished by twice their product $(2xy)$.

EXAMPLE.—Let 7 be one number and 3 the other: then, 1st, their sum, 10, multiplied by their difference, 4, is 40; the *squares* of the numbers are 49 and 9; so that $7^2-3^2=(7+3)(7-3)=40$. 2nd. $(7+3)^2=10^2=100$, and $7^2+3^2=49+9=58$; also *twice* 7×3, that is, $2\times7\times3=42$; and $58+42=100$. 3rd. $(7-3)^2=4^2=16$; and $58-42=16$.

4. Multiply $(x+y)^2$ by $x+y$, that is, multiply out or *develope* $(x+y)^3$.

$$\begin{array}{r} (x+y)^2=x^2+2xy+y^2\\ x+y\\ \hline x^3+2x^2y+\ xy^2\\ x^2y+2xy^2+y^3\\ \hline \end{array}$$

$$\therefore (x+y)^3=x^3+3x^2y+3xy^2+y^3$$

5. *Develope*, or multiply out $(x-4)\ (x+3)$; $(x+5)\ (x-7)$; $(2x+3)\ (3x+2)$ and $(x+a)\ (x+b)$.

$$\begin{array}{rcr} x-4 & \qquad & x+5\\ x+3 & & x-7\\ \hline x^2-4x & & x^2+5x\\ 3x-12 & & -7x-35\\ \hline x^2-x-12 & & x^2-2x-35\\ \hline \end{array}$$

$$\begin{array}{rcr} 2x+3 & \qquad & x+a\\ 3x+2 & & x+b\\ \hline 6x^2+9x & & x^2+ax\\ 4x+6 & & bx+ab\\ \hline 6x^2+13x+6 & & x^2+(a+b)x+ab\\ \hline \end{array}$$

In the third of these results, the $13x$ is got by actually *adding* the coefficients 9 and 4: in the fourth example, the corresponding coefficients of x, not being *numbers*, cannot be actually *added;* but the addition may nevertheless be *indicated*, as above: and where, in the *third* example, the middle term of the product was written $13x$, that is, $(9+4)x$, the corresponding term *here* is written $(a+b)x$; this is better than making *two* terms of the same quantity, and writing it thus, $ax+bx$. *Here* the coefficients of x are a and b; and, being *letters*, they are called *literal coefficients:* the coefficient of x, in the result above, is $(a+b)$. Whenever a common factor enters several of the terms of a compound expression, this common factor may always be written outside a vinculum, and whatever multiplies it, *within;* repetitions of the same factor are thus avoided; and in the answers to questions, or the final results of operations, this more compact form of expression should be adopted. Thus, such a result as $abx+4cx-3mx$, would have an unfinished appearance; it should be changed into $(ab+4c-3m)x$. *This*, you will see, is more easily computed than the former, when the letters are replaced by the numbers they represent. Suppose, for instance, $a=12$, $b=6$, $c=5$, and $m=9$; and that $x=23$; you will find that the numerical value of the expression is more readily obtained from the second form than from the first. And we may as well tell you here, once for all, that algebraists always take care that their *results* are presented in a form that will give the *least* trouble to the arithmetical computer. This principle will in general be observed in what follows.

We shall now work an example which suggests a principle of some interest in arithmetic.

(6.) Multiply $n+r$ by $n'+r'$.

$$\begin{array}{r} n+r\\ n'+r'\\ \hline nn'+n'r\\ nr'+rr'\\ \hline nn'+n'r+nr'+rr'\\ \hline \end{array}$$

Now suppose a number to be divided by any other number, say 9, and to leave a remainder, which we may denote by r: then, if we express this number, *wanting the remainder*, by n, the entire number will be denoted by $n+r$. In like manner, a second number may be denoted by $n'+r'$, where n and n' represent numbers exactly divisible by 9; and r and r' numbers less than 9. You see by the above expression for the product of the two proposed numbers $n+r$ and $n'+r'$, that the whole of it is necessarily divisible by 9, except the part $r\,r'$, because n or n' is a factor of every other term: you may, therefore, at once conclude that if you divide a *multiplicand* and its *multiplier* each by 9 (or indeed by any other number), and note the remainders (r, r'), and then divide the product of these remainders by 9 (or the other number), the remainder arising from this last division must be the very same as the remainder arising from dividing the *product* of multiplier and multiplicand by 9 (or the other number). And thus you have the principle of *the method of proving multiplication by casting out nines.* (Arith., p. 437). The reason that 9 is chosen for divisor is, because that for the divisor 9, the remainder is the same whether the number itself, or only the sum of the figures composing it, be divided; and it is easier to sum up the figures, and reject the *nines*, than to perform the division on the number.

EXAMPLES FOR EXERCISE.

1. Multiply $2ax^2 - 3bx - 2$ by $3x^2 - 2x$
2. Multiply $4ay + 3bx - 4c$ by $3by + 2c$
3. Multiply out $(5x^3 - 4x^2 + 3x - 2)\,(2x^2 - x + 1)$
4. Multiply out $(x+2)\,(x-2)\,(x+3)\,(x-3)$
5. Multiply out $(x+a)\,(x-a)\,(x^2-a^2)$
6. Multiply out $(2x+3a)\,(4x-5a)\,(x+a)$
7. $(3ax-b)\,(4ax+c)\,(5ax-3)$
8. $(4ax+3by-1)\,(2ax-by+2)$
9. $(x-y)^2(x+y)$ 10. $\{x^2-(a+b)\,x+c\}\,(x-c)$
11. $3^2(x^2-a^2)^2$ 12. $(2x+3)\,(2x-3)\,(4x^2+9)$
13. $\{(x+2)\,(x-2)\}^2$
14. $(x^2-\frac{1}{2}x+1)\,(x^2+2x-1)$

INVOLUTION.

Involution is nothing more than *multiplication:* it is a term employed to signify that the factors multiplied together are all *equal*, the product or result being a *power*. Example 4, for instance, page 460, is a case of involution; for $x+y$ is *raised*, as it is called, or *involved*, to the third power. Involution is thus the operation of raising a proposed quantity to a proposed power, and this operation is *multiplication*. The following are examples of the involution of *simple quantities*.

$(2ax)^3 = 8a^3x^3$, $(2a^2x)^4 = 16a^8x^4$, $(-ax^2y^3)^5 = -a^5x^{10}y^{15}$, &c.

The RULE for obtaining the powers in such cases as these is pretty obvious.

To the power of the *coefficient* annex the letters, with their several exponents *multiplied by the exponent of the power.*

The *rule of signs* must of course be attended to. If the quantity to be raised or involved be *negative*, the sign of every *even* power must be *positive*—that of every *odd* power *negative*. See following examples:—

1. $(3a^2x^2)^2$ 2. $(7by^3)^4$
3. $(-4a^2b^3x^4)^3$ 4. $(2x^4y^2z)^5$
5. $(-3a^3y^2z^2)^6$ 6. $(-8b^2x^5y^3)^3$
7. $(5a^2b^3c^4)^4$ 8. $(-2^2a^3x^2y)^3$

When the quantity to be involved is a compound quantity, the proposed power of it is to be found by multiplying the quantity the requisite number of times by itself, as in the examples below:—

1. To find the cube of $a-2x$.

$$\begin{array}{rl}
 & a-2x \\
 & a-2x \\
\hline
 & a^2-2ax \\
 & \quad\;\; -2ax+4x^2 \\
\hline
(a-2x)^2 = & a^2-4ax+4x^2 \\
 & a-2x \\
\hline
 & a^3-4a^2x+4ax^2 \\
 & \quad -2a^2x+8ax^2-8x^3 \\
\hline
\therefore (a-2x^3) = & a^3-6a^2x+12ax^2-8x^3
\end{array}$$
[*]

2. To find the cube of $3ax-4y$.

$$\begin{array}{rl}
 & 3ax-4y \\
 & 3ax-4y \\
\hline
 & 9a^2x^2-12axy \\
 & \qquad\quad -12axy+16y^2 \\
\hline
(3ax-4y)^2 = & 9a^2x^2-24axy+16y^2 \\
 & 3ax-4y \\
\hline
 & 27a^3x^3-72a^2x^2y+48axy^2 \\
 & \qquad\quad -36a^2x^2y+96axy^2-64y^3 \\
\hline
\end{array}$$

$$\therefore (3ax-4y)^3 = 27a^3x^3-108a^2x^2y+144axy^2-64y^3$$

EXAMPLES FOR EXERCISE.

1. $(a+2x)^2$ 2. $(3ax-4y)^3$ 3. $(a+b+c)^2$
4. $(3x^2+2x+5)^2$ 5. $(a-2b+3c)^2$
6. $\{(a+x)\,(a-x)\}^2$ 7. $\{(x+2)\,(x-2)\}^3$
8. $\{(2x-3)\,(2x+3)\}^4$ 9. $\{(x-a)^2+2ax\}^4$
10. $\{(x+1)\,(x-1)\,(x^2+1)\}^2$ 11. $\{(x-1^2\}^3$

The following is a table of the powers of $(a+x)$ *developed* in order, from $(a+x)^1$ up to $(a+x)^8$: the result of the actual multiplication is called the *development*, and sometimes the *expansion* of a power. The table exhibits the developments of the powers of an expression, consisting of *two* simple terms, a and x. Every expression of two terms is called a *binomial.*

Table of the developments of the powers of a binomial.

$(a+x)^1 = a+x$
$(a+x)^2 = a^2+2ax+x^2$
$(a+x)^3 = a^3+3a^2x+3ax^2+x^3$
$(a+x)^4 = a^4+4a^3x+6a^2x^2+4ax^3+x^4$
$(a+x)^5 = a^5+5a^4x+10a^3x^2+10a^2x^3+5ax^4+x^5$
$(a+x)^6 = a^6+6a^5x+15a^4x^2+20a^3x^3+15a^2x^4+6ax^5+x^6$
$(a+x)^7 = a^7+7a^6x+21a^5x^2+35a^4x^3+35a^3x^4+x21a^2x^5+7ax^6+x^7$
$(a+x)^8 = a^8+8a^7x+28a^6x^2+56a^5x^3+70a^4x^4+56a^3x^5+28a^2x^6+8ax^7+x^8$

This table, which may be carried to any extent, shows that the coefficients of the terms in the development of a binomial follow one another according to a remarkable *law*, by observing which, they may be derived, each from that which precedes it, with very little trouble, so that the actual involution of the binomial may always be dispensed with. By examining the several rows of coefficients, you will discover the law to be this; namely—

If the coefficient of any term be multiplied by the exponent of a in that term, and the product be divided by the number which marks the *place* of the term in the row (as the 2nd place, 3rd place, &c.), the quotient will be the coefficient of the *next* term. Thus, look at the development of $(a+x)^7$. The *first* term in that development we might know to be a^7, without actual multiplication. The coefficient of this first term is 1; and $\frac{7\times1}{1}=7$ is the next coefficient, the complete term being $7a^6x$. This is the second term; and $\frac{6\times7}{2}=21$, is the next coefficient, the third term being $21a^5x^2$; and from this we get $\frac{5\times21}{3}=35$, the next coefficient, the fourth term being $35a^4x^3$. In like manner, from this we find $\frac{4\times35}{4}=35$, the next coefficient; and in this way all the coefficients may be found, one after another, without involution. As to the *letters* connected with these coefficients, the writing of *them* involves no trouble at all. The *first* term is the first term of the binomial, with the exponent of the power over it; the *last* term is the other term of the binomial, with the same exponent over *it;* this is the highest exponent in the row of terms; in each intermediate term, *both* the letters occur in conjunction; and, in proceeding from term to term, the exponents of a regularly *descend* in value, and the exponents of x as regularly *ascend;* that is, the powers of a are—

$$a^7 \quad a^6 \quad a^5 \quad a^4 \quad a^3 \quad a^2 \quad a^1$$

and those of x, $\quad x^1 \quad x^2 \quad x^3 \quad x^4 \quad x^5 \quad x^6 \quad x^7$

the *sum* of the exponents of *both* letters always making

---

[*] See Example 1, p. 460.

7, in every term. And, generally, the sum of the exponents, in each term, is always equal to the exponent of the power.

There are one or two other things which it is important to take note of in looking at the foregoing developments.

1. The number of terms is always *one more* than the number which marks the power of the binomial. Thus, when the exponent is 1, the number of terms is *two*; when the exponent is 2, the number of terms is *three*, and so on; ∴ when the exponent is *odd*, the number of terms is *even*; and when the exponent is *even*, the number of terms is *odd*.

You cannot, therefore, with propriety, speak of *a middle term*, except when the exponent of the power is *even*; when it is *odd*, there are *two middle terms*; and you see that the *coefficients* of these middle terms are always *the same*—another remarkable circumstance.

2. But perhaps the most remarkable thing of all is, that when, in the case of an *even* power, you have reached the middle coefficient, as above directed, the remaining coefficients are got by simply writing those which precede the middle one *in reverse order*; so that these remaining coefficients require no *computing*. When, in the case of an *odd* power, you have reached the first of the two middle coefficients, you have only to *repeat* this coefficient, and then to write all the coefficients, before the middle ones, in reverse order, as in the other case; so that no coefficient beyond the middle *one*, or the middle *pair*, need ever be *computed*; they have only to be *copied*.

The present examination affords a good opportunity to bring another particular for our attention. The foregoing *table* presents us with a set of *equations*, but they differ from the equations solved at pages 456, &c., in a very marked manner; you must take notice of this. The equations at the pages referred to fix certain *conditions*, which the value of x must be such as to satisfy—no *other* values of x would do; but in the equations in the preceding *table*, no *conditions* are implied—both a and x in each may stand for anything whatever; for the second side, or member, is only the first side *developed*, unfolded, or spread out; in other words, one side is nothing but the other side put in a different form; so that, write what we will for a and x, the two sides must of necessity remain equal. Equations such as these, in which one side is only the other in a changed form, are called *identical equations*, or simply *identities*. Now, as in identities anything may be substituted for the *letters*, you may in the table put $-y$ for x; you will thus get the developments of the different powers of $a-y$, or, to be uniform, of $a-x$; all you have to do is to change x into $-x$ on both sides; you will see that the only change in the development is that the *second*, and every *alternate* term, becomes *minus*; thus—

$$(a-x)^3 = a^3 - 3a^2x + 3ax^2 - x^3$$
$$(a-x)^4 = a^4 - 4a^3x + 6a^2x^2 - 4ax^3 + x^4,$$

and so on. With these changes, therefore, in the alternate signs, the table exhibits the developments of the power both of $a+x$ and $a-x$.

What has now been said in reference to the powers of $a+x$, that is, in reference to the powers of a *plus or minus* x, must be very carefully read over, and fully understood. We have been giving to you the leading particulars of the celebrated BINOMIAL THEOREM, and you must try to impress them on your mind. In order to this, you should write out the developments of $(a-x)^6$, $(a-x)^7$, &c., and to try $(a+x)^9$, $(a-x)^9$, &c., finding the coefficients according to the short and easy method explained above; that is, deriving them, one after another, from the *law* shown to prevail, as far at least as to the eighth power. It prevails universally, but the *general* demonstration of the *Binomial Theorem* requires more advanced principles of Algebra; you will do right to refuse assent to the law *at present*, for an exponent higher than 8, unless you like to put it to the test for 9, 10, &c. You will find the theorem in a more general form hereafter. As an application of the Binomial Theorem, let it be required to develope $(a-3y)^5$, which is the same as $(a+x)^5$, when $-3y$ is put for x.

From what is shown above, it appears that the terms, without the coefficient, are—

$$a^5,\ a^4\,(3y),\ a^3\,(3y)^2,\ a^2\,(3y)^3,\ a\,(3y)^4,\ (3y)^5.$$

And from the law of the coefficients, these are—

$$1,\ 5,\ \frac{4\times 5}{2}\ \text{(or 10)},\ 10,\ 5,\ 1;$$

the last three being those of the first three written in reverse order; therefore, remembering to write the terms alternately plus and minus,

$$\begin{aligned}(a-3y)^5 &= a^5 - 5a^4\,(3y) + 10a^3\,(3y)^2 - 10a^2\,(3y)^3 + 5a\,(3y)^4 - (3y)^5,\\ &= a^5 - 15a^4y + 90a^3y^2 - 270a^2y^3 + 405ay^4 - 243y^5\end{aligned}$$

Again, let Example 2, page 461, be taken; namely, $(3ax-4y)^3$. The terms, without the coefficients, are—

$$(3ax)^3,\ (3ax)^2 4y,\ 3ax(4y)^2,\ (4y)^3;$$

therefore, introducing the coefficients, 1, 3, 3, 1, we have—

$$\begin{aligned}(3ax-4y)^3 &= (3ax)^3 - 3\,(3ax)^2 4y + 3.3ax\,(4y)^2 - (4y)^3\\ &= 27a^3x^3 - 108a^2x^2\,y + 144axy^2 - 64y^3.\end{aligned}$$

EXAMPLES FOR EXERCISE.

1. $(a+y)^4$. 2. $(a-x)^5$. 3. $(a+2x)^3$.
4. $(1-x)^6$. 5. $(1+3x)^4$. 6. $(x+2y^2)^5$.
7. $(x^2+3y^2)^6$. 8. $(2a-x)^6$. 9. $(x-2y^2)^7$.

NOTE.—The preceding process may be extended to expressions of *three* terms, *four* terms, &c., in the manner following:—

$$\begin{aligned}[a+b+c]^2 = [\,(a+b)+c]^2 &= (a+b)^2 + 2\,(a+b)\,c + c^2\\ &= a^2 + 2ab + b^2 + 2\,(a+b)\,c + c^2\\ &= a^2 + b^2 + c^2 + 2\,(ab + ac + bc)\end{aligned}$$

$$\begin{aligned}[a+b+c+d]^2 = [\,(a+b)+(c+d)\,]^2 &= (a+b)^2 + 2\,(a+b)\,(c+d) + (c+d)^2\\ &= a^2 + 2\,ab + b^2 + 2\,(ac + ad + bc + bd) + c^2 + 2cd + d^2\\ &= a^2 + b^2 + c^2 + d^2 + 2\,(ab + ac + ad + bc + bd + cd).\end{aligned}$$

So that the square of the sum of *three* quantities, or of the sum of *four* quantities, is equal to the squares of the quantities themselves, together with twice the sum of the products of every possible pair of them; and the same is true for *five*, and for any number of quantities. An extended use of involution will be found in the calculation of logarithms, in a subsequent chapter.

DIVISION.

When one quantity (called the *dividend*) is to be divided by another (called the *divisor*), the object is to find a third quantity (called the *quotient*), such, that if the quotient and divisor be multiplied together, the product will be the dividend; and this is to be brought about whether the quantities concerned are the figures of arithmetic, or the symbols of algebra. The rule for division is thus suggested by that for multiplication; it is as follows:—

CASE I.—*When dividend and divisor are both simple quantities.*

RULE 1.—Determine the *sign* of the quotient, on the principle that if the signs of dividend and divisor be like, the quotient is *plus*: if unlike, *minus*.

2. Having found the sign, next find the coefficient by dividing the coefficient of the dividend by that of the divisor, as in common arithmetic.

3. To the coefficient, annex the *letters*, which must be such, that when they are united to the letters already in the divisor, they may exactly make up those in the dividend.

It is plain, that by following these precepts, you will get a quantity for quotient such, that when it and the divisor are multiplied together, the resulting sign will be that of the dividend, the resulting coefficient also that of the dividend, and finally the resulting letters, arising from combining those of divisor and quotient together, will be the same as the letters in the dividend.

If it should happen that there are letters in the divisor, that is to say *factors* in the divisor, which are not also in the dividend; then, like as in arithmetic, since actual division by these cannot be performed, they must be written as divisors in the quotient; the quotient will then appear as a *fraction* incapable of reduction to a simpler form—just as in division of numbers.

1. Divide $-12a^3x^2y$ by $3ax$. Here $\frac{-12a^3x^2y}{3ax} = -4a^2xy$; for the signs being unlike the sign of the quotient is *minus*; also 12 divided by 3 gives 4, so that -4 is the coefficient in the quotient; and since, from looking at the letters in dividend and divisor, we see that two *a*'s, one *x*, and a *y*, must be united to the divisor to make up the letters in the dividend, these wanting letters are those which the quotient must supply; the complete quotient is therefore $-4a^2xy$. If the divisor had been $3axz$, then the foregoing quotient must have been further divided by *z*; that is, $\frac{-12a^3x^2y}{3axz} = \frac{-4a^2xy}{z}$; for it is plain that to divide by $3axz$ is the same as first to divide by $3ax$ and *afterwards* by the other factor *z*. In like manner, to divide by $3axy^2$ is the same as first to divide by $3axy$, and then again to divide by the other factor *y*, $\therefore \frac{-12a^3x^2y}{3axy^2} = -\frac{4a^2x}{y}$. All this is the same as in arithmetic; thus, if we have to divide 48 by 36, we may proceed as follows: $\frac{48}{36} = \frac{12.4}{12.3} = \frac{4}{3}$; for $\frac{12.4}{12} = 4$, and this divided by the other factor 3, gives $\frac{4}{3}$*

2. $\frac{14a^4x^6y^2}{2ax^3y^2} = 7a^3x^3$ 3. $\frac{-8a^5x^2z}{4a^3x^3z^2} = \frac{-2a^2}{xz}$

4. $\frac{-12b^3y^7x^2}{-4b^4y^3x} = \frac{3y^4x}{b}$ 5. $\frac{7ax^3y^5}{2ax^2y^4} = \frac{7xy}{2}$

6. $\frac{-3x^4y^5z^6}{-4ax^2y^3} = \frac{3x^2y^2z^6}{4a}$ 7. $\frac{-9ay^3z^4}{-6a^2y^2z} = \frac{3yz^3}{2a}$

8. $\frac{-16a^2x^3\sqrt{y}}{-4ax^2} = 4ax\sqrt{y}$

9. $\frac{-4xy\sqrt{z}}{6x^3y^2} = \frac{-2\sqrt{z}}{3x^2y}$ or $-\frac{2\sqrt{z}}{3x^2y}$

NOTE.—When the numerator of a fraction is *minus*, and its denominator *plus*, it is matter of indifference whether we put the minus before the *numerator*, or before the entire fraction; because, from the rule of signs, a *minus* quantity is the result of the division indicated. Thus, in the last example it is indifferent whether we write $-\frac{2\sqrt{z}}{3x^2y}$ or $\frac{-2\sqrt{z}}{3x^2y}$. The same is true when the denominator is *minus*, and the numerator *plus*; for in division of one quantity by another, as in multiplication, whichever of the quantities be minus, provided only the other be plus, the result is minus. Thus, the following all express the same thing: $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$. For you see, that in either case, when *a* and *b* are interpreted, and the division performed, the quotient is *minus*. For instance—

$$\frac{-6}{2} = -3,\ \frac{6}{-2} = -3,\ -\frac{6}{2} = -3, \text{ \&c.}$$

Before proceeding to the following exercises, it will be well for you to look again at precept 3 of the Rule, from which you may yourself draw an inference of some importance in the general theory of exponents. The inference is, that when *the same letter* occurs in both dividend and divisor, and that the exponent of it in the former is greater than that in the latter; the *quotient*, as far as this letter is concerned, is got by simply subtracting the smaller exponent from the greater, and placing the difference over the letter; thus, in example 2, a^4 in the dividend, and a^1 in the divisor, give a^3 in the quotient, and x^6 in the dividend, and x^3 in the divisor, give x^3 in the quotient; the y^2 in *both* is *cancelled*. You thus see, that when the same letter or quantity is concerned, division becomes the *subtraction of exponents*: multiplication, as you are aware, being the *addition* of *exponents*. You will no doubt think, when the exponent in the divisor is *greater* than that in the dividend, that this view of division must be abandoned; but it is not so, as you will shortly see.

EXAMPLES FOR EXERCISE.

1. Divide $8ax^2y^2$ by $4axy$
2. $-6bx^3y^2z$ by $2bx^3y$
3. $-12x^3y^4z^5$ by $-3x^2y^3z^2$
4. $16ax^2y^3$ by $-4a^3xy^2$
5. $7ax^4yz^2$ by $5a^2x^2yz^3$
6. $-8b^6y^3z$ by $6b^3y^6z^2$
7. $24cy^3$ by $-21c^3y^2z$
8. $-36a^2x^2y^3$ by $2\times7a^2x^4y$
9. $5abx^2\sqrt{y}$ by $10ab^3x\sqrt{y}$
10. $-3a^2c^5x^4$ by $-12ac^2x^4\sqrt{z}$
11. $-2mx^4y^3$ by $8m^2x^4y^6$
12. $-13x^2y^6z^{\frac{1}{2}}$ by $-26y^4\sqrt{z}$†

CASE II.—*When the dividend is a compound quantity, and the divisor a simple quantity.*

RULE 1. Find the quotient of the divisor, and *each term* of the dividend by the former rule.

2. Connect the separate quotients together, by the signs which belong to them, and the complete quotient will be obtained.

1. $\frac{4x^3y^4 - 6x^2y^3 + 8xy^2}{2xy^2} = 2x^2y^2 - 3xy + 4$
2. $\frac{12ax^2z^3 - 8a^2xz^2 - 4xz}{4xz^2} = 3axz - 2a^2 - z^{\frac{1}{3}}$
3. $\frac{21a^3x^2 + 7a^2x^3 - (42a^4x - 28)}{7a^2x}$. Here the *compound* term in the numerator is to be *subtracted* from what precedes, *therefore* the signs of the subtractive quantities are to be changed, and they are then to be added.‡ Hence the fraction is—

$$\frac{21a^3x^2 + 7a^2x^3 - 42a^4x + 28}{7a^2x} = 3ax + x^2 - 6a^2 + \frac{4}{a^2x}$$

EXAMPLES FOR EXERCISE.

1. $\frac{9a^2x^3 - 3a^3x^3 + 6a^4x}{3a^2x}$
2. $\frac{12x^3yz^4 + 6x^3y^3z^3 - 3x^2y^2}{3x^2yz^2}$
3. $\frac{3ax\,(4xy - 8ay^2) + 12axy}{6axy}$
4. $\frac{8x^3y^4 - 4(4ax - 2y)x}{4x^2y}$
5. $\frac{8a^2x^2y^2 - 2(3ax^2y^2z^2 - 1)}{4axy}$
6. $\frac{16a^4b^2x^3 - (8 - 3a^2bx^4)}{4a^2bx^2}$

CASE III.—*When both dividend and divisor are compound quantities.*

RULE 1. Arrange the terms of dividend and divisor so that the exponents of the powers of some one letter in both of them may appear in decreasing or increasing order; that is, if *x*, for instance, be the letter chosen to govern the arrangement, place the terms in either of these two ways:—Let the term containing the highest power of *x* be put first; that containing the next power immediately after, and so on; or else let the term containing the lowest power of *x* be put first, that containing the next lower immediately after, and so on; but do not write the terms at random.

2. The dividend and divisor, thus arranged, being placed as dividend and divisor are placed in arithmetic, divide the *first* term of the dividend by the *first* term of the divisor; the result is the first term of the quotient.

3. Then, as in arithmetic, multiply the *whole divisor* by the part of the quotient thus found, and subtract the product from the dividend.

4. To the remainder annex another term, brought down from the dividend, or annex more of the terms if more are seen, from the extent of the divisor, to be required; the row of terms thus got is a new dividend, with which, and the divisor, proceed as at first, and a second quotient term will be obtained. And in this way

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* The learner will perceive, that having indicated the division by a fraction, whether the fraction be algebraical or purely numerical, all we have to do is to *cancel* all the *factors* common to numerator and denominator: the result is the value of the fraction, reduced to its simplest form.

† See *ante*, p. 458.

‡ See p. 455.

the operation is to proceed, like the corresponding operation in arithmetic, till all the terms of the dividend have been brought down. When the last term has been brought down, and annexed to the remainder—no term in the dividend having been overlooked or neglected—then the row of terms thus got is the *complete remainder.* Should it happen that the divisor will not *go* into this complete remainder, the operation is at an end, and you do just as you would in the similar case of arithmetic; you place this complete remainder, with the divisor underneath, in the quotient, as a fractional correction.

Divide $6x^2 - ax - a^2$ by $2x - a$. Here dividend and divisor are arranged according to the *descending* powers of x, the highest power of that letter occurring in the first term. If the arrangement be the reverse of this, the operation will be that of the second form below; *minus* the dividend is divided by *minus* the divisor for convenience.*

$$\begin{array}{l}
1.\quad 2x - a)6x^2 - ax - a^2(3x + a \\
\qquad\qquad \underline{6x^2 - 3ax} \\
\qquad\qquad\qquad 2ax - a^2 \\
\qquad\qquad\qquad \underline{2ax - a^2}
\end{array}
\qquad\qquad
\begin{array}{l}
2.\quad a - 2x)a^2 + ax - 6x^2(a + 3x \\
\qquad\qquad \underline{a^2 - 2ax} \\
\qquad\qquad\qquad 3ax - 6x^2 \\
\qquad\qquad\qquad \underline{3ax - 6x^2}
\end{array}$$

$$\begin{array}{l}
3.\quad x^2 + 2ax + a^2)x^5 + 5ax^4 + 10a^2x^3 + 10a^3x^2 + 5a^4x + a^5(x^3 + 3ax^2 + 3a^2x + a^3 \\
\qquad\qquad\qquad\qquad \underline{x^5 + 2ax^4 + a^2x^3} \\
\qquad\qquad\qquad\qquad\qquad 3ax^4 + 9a^2x^3 + 10a^3x^2 \\
\qquad\qquad\qquad\qquad\qquad \underline{3ax^4 + 6a^2x^3 + 3a^3x^2} \\
\qquad\qquad\qquad\qquad\qquad\qquad 3a^2x^3 + 7a^3x^2 + 5a^4x \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{3a^2x^3 + 6a^3x^2 + 3a^4x} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad a^3x^2 + 2a^4x + a^5 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underline{a^3x^2 + 2a^4x + a^5}
\end{array}$$

$$\begin{array}{l}
4.\quad x - 2)2x^4 - 32\,\dagger(2x^3 + 4x^2 + 8x + 16 \\
\qquad\qquad \underline{2x^4 - 4x^3} \\
\qquad\qquad\qquad 4x^3 \\
\qquad\qquad\qquad \underline{4x^3 - 8x^2} \\
\qquad\qquad\qquad\qquad 8x^2 \\
\qquad\qquad\qquad\qquad \underline{8x^2 - 16x} \\
\qquad\qquad\qquad\qquad\qquad 16x - 32 \\
\qquad\qquad\qquad\qquad\qquad \underline{16x - 32}
\end{array}$$

$$\begin{array}{l}
5.\quad 4x^2 - 7x)12x^5 - 13x^4 - 14x^3 + 3x^2 + 1(3x^3 + 2x^2 + 2 + \dfrac{14x + 1}{4x^2 - 7x} \\
\qquad\qquad\qquad \underline{12x^5 - 21x^4} \\
\qquad\qquad\qquad\qquad 8x^4 - 14x^3 \\
\qquad\qquad\qquad\qquad \underline{8x^4 - 14x^3} \\
\qquad\qquad\qquad\qquad\qquad 8x^2 + 1 \\
\qquad\qquad\qquad\qquad\qquad \underline{8x^2 - 14x} \\
\qquad\qquad\qquad \text{Remainder,} \qquad 14x + 1
\end{array}$$

EXAMPLES FOR EXERCISE.

1. Divide $x^2 - 2x - 35$ by $x - 7$
2. Divide $x^2 - x - 12$ by $x + 3$
3. Divide $6x^2 + 13x + 6$ by $3x + 2$
4. Divide $12x^3 + 29x^2 + 14x$ by $3x^2 + 2x$‡
5. Divide $18x^3 - 33x^2 + 44x - 35$ by $3x^2 - 2x + 5$
6. Divide $x^4 - y^4$ by $x - y$
7. Divide $x^4 - ax^3 - 7ax^2 + 8a^2x - a^3$ by $x - a$
8. Divide $x^2 + (a + b)\,x + ab$ by $x + a$
9. Divide $8x^4y + 2x^3y - 2x^2 - 3x^2y + x$ by $4x^2y + 3xy - 1$
10. Divide $x^6 - x^4 + x^3 - x^2 - 1$ by $x^2 + x - 1$
11. Divide $3\,(2x^2 + 3)\,x^2 - 20x$ by $3\,(x - 1)\,x$
12. Divide $x^2 + px + q$ by $x - a$
13. Divide $x^3 + px^2 + qx + r$ by $x - a$

[It is worthy of notice, that in each of the last two examples, the final *remainder* is the very same as the *dividend*, when the x in it is removed and a is written instead.]

ON EXPONENTS, ROOTS, SURDS.

We have already stated (page 458) that there are two ways of indicating a root of a quantity: one way by means of the *radical sign* $\surd$, placed before the quantity, and another by the introduction of a fraction, written in smaller character, over the right-hand corner of the quantity proposed—this corner fraction being called an *exponent* or *index.* The radical sign is used exclusively for *roots;* exponents are used alike for *roots* and for *powers*—thus, a^4 is the fourth *power* of a, and $a^{\frac{1}{4}}$ is the fourth *root* of a. But exponents or indices have a wider application still. As yet we have had to deal only with *positive* exponents; algebraists, however, have introduced *negative* exponents. We must give some account of these—showing how they have arisen, and what meaning is attached to them.

Your attention has already been drawn to the fact (page 458), that when a quantity with an exponent denoting a power, is to be divided by the same quantity with a smaller exponent, the subtraction of the latter exponent from the former is all that is necessary; thus

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* Whether one quantity be divided by another, or *minus* the former by *minus* the latter, the result is the same; for $\frac{a}{b}$ is the same as $\frac{-a}{-b}$ whatever a and b may stand for; it is of course the same with multiplication: the factors a, b, give the same product as the factors $-a$, $-b$.

† This is evidently the same as $2x^4 + 0x^3 + 0x^2 + 0x - 32$; so that the -32 need not be brought down till we arrive at the concluding step of the work.

‡ In exercises in division, always expunge, before you begin the operation, whatever factor is obviously common to both dividend and divisor, as you would do if you had to deal with a *fraction*, the dividend being the numerator, and the divisor the denominator. In the example above, the factor x obviously enters all the terms of both dividend and divisor; it is therefore a useless encumbrance, and should be expunged; the division will then be $12x^2 + 29x + 14$ by $3x + 2$. Example 11 admits of a like simplification.

$$\frac{a^5}{a^2}=a^{5-2}=a^3\ ;\ \ \frac{a^3}{a^2}=\underline{a}^{3-2}=\underline{a}^1=\underline{a},\ \&c.$$

Now, this mode of proceeding has suggested the extension of notation adverted to above, giving rise to such expressions as a^0, a^{-1}, a^{-2}, &c.: thus, carrying out the foregoing principle, we have

$$\frac{a^1}{a^1}=a^{1-1}=a^0\ ;\ \text{but}\ \frac{a^1}{a^1}\ \text{or}\ \frac{a}{a}=1,\ \therefore a^0=1,\ \text{a remarkabie result.}$$

$$\text{In like manner,}\ \frac{a^0}{a^1}a^{0-1}=a^{-1}\ ;\ \text{but}\ \frac{a^0}{a}=\frac{1}{a^1}\ \therefore a^{-1}=\frac{1}{a}$$

$$\text{Similarly,}\ \frac{a^{-1}}{a^1}=a^{-1-1}=a^{-2}\ ;\ \text{but}\ \frac{a^{-1}}{a}=\frac{1}{a^2},$$

$$\therefore a^{-2}=\frac{1}{a^2}$$

And in the same way, $a^{-3}=\frac{1}{a^3}$, $a^{-4}=\frac{1}{a^4}$; and *generally*, $\frac{1}{a^n}=a^{-n}$; whatever whole number n may stand for. We thus see that without making any inquiry as to the particular values of m and n, we may always write $\frac{a^m}{a^n}=a^{m-n}$; if $m=3$ and $n=5$, then $\frac{a^3}{a^5}=a^{-2}$, which, as seen above, is only another way of writing $\frac{1}{a^2}$. It may here be observed that *unity* or 1, divided by any quantity, is called the *reciprocal* of that quantity; thus, $\frac{1}{a^2}$ is the reciprocal of a^2; $\frac{1}{a}$ is the reciprocal of a; and so on. And any quantity with a *negative exponent* stands for the reciprocal of that quantity with a *positive exponent*; thus, $a^{-\frac{1}{2}}$ means $\frac{1}{a^{\frac{1}{2}}}$, $a^{-\frac{1}{3}}$ means $\frac{1}{a^{\frac{1}{3}}}$, &c.; so that not only powers and roots, but also the reciprocals of powers and roots, are represented by exponents.

In the instances hitherto given of *fractional* exponents, the numerator of the fraction has always been *unit* or 1, the notation implying a *root* of the quantity to which the exponent is attached. When a *power* of this root is to be indicated, or the *root* of a power, the somewhat cumbersome form used at page 458—namely, $(a^4)^{\frac{1}{3}}$ for the third root of the fourth power of a, or $(a^{\frac{1}{3}})^4$ for the fourth power of the third root of a, is not the notation usually employed; a single fraction is made to serve; the *numerator* of the fraction denoting the *power*, and the *denominator* the *root; thus* $a^{\frac{4}{3}}$ would stand indifferently for the third root of the fourth power of a, or for the fourth power of the third root of a. Whether you regard the power to be taken first and then the root, or the root first and then the power, is of no moment; the *result* is the same. Thus suppose you have $8^{\frac{2}{3}}$; if you regard this as the second power (or square) of the cube or third root of 8, then since the cube root of 8 is 2 (seeing that $2^3=8$), you have $8^{\frac{2}{3}}=2^2=4$; but if, on the contrary, you regard it as the cube root of the square of 8, then since the square of 8 is 64, you have $8^{\frac{2}{3}}=64^{\frac{1}{3}}=4$, as before (seeing that $4^3=64$). Or take the more general case noticed above; namely, $a^{\frac{4}{3}}$. If you regard this as $(a^4)^{\frac{1}{3}}$, you consider it the same as $(aaaa)^{\frac{1}{3}}$. And if you regard $a^{\frac{4}{3}}$ as $(a^{\frac{1}{3}})_4$, you consider it the same as $a^{\frac{1}{3}}a^{\frac{1}{3}}a^{\frac{1}{3}}a^{\frac{1}{3}}$. Now these two results differ only in appearance; for let $a^{\frac{1}{3}}$, that is, the cube root of a, whatever it be, be denoted by c; then, of course, a is c^3; and it is plain that $(c^3c^3c^3c^3)^{\frac{1}{3}}=cccc$; the first side of this *identical equation* being the former of the above expressions, and the second side the latter. It is therefore matter of choice whether you call $a^{\frac{m}{n}}$ the nth root of a^m, or the mth power of $a^{\frac{1}{n}}$; and, in actual numbers, whichever of these two views is most convenient for the purposes of arithmetic, may be taken.

Whenever an exponent in the form of a fraction is such that the numerator and denominator are *the same*, as in $a^{\frac{m}{m}}$, that exponent may be replaced by unit, or 1; because if the mth *power* of any quantity be taken, and then the mth *root* of the result, the original quantity (in this case a) is of course brought back again.

The second operation merely undoes what the first does; the two operations mutually destroy one another, and are $\therefore$ of no effect, so that $a^{\frac{m}{m}}=a$. In consequence of this, such an expression as $a^{\frac{p}{q}\cdot\frac{m}{m}}$ is the same as $a^{\frac{p}{q}}$, since the former means the mth root of the mth power of *this.* When a fractional exponent, therefore, has a factor common to both numerator and denominator, the common factor may be expunged. All that is said above applies as well to negative as to positive exponents, as each expression there considered may be equally regarded as the denominator of a fraction whose numerator is 1. The following is a view of the principal operations with exponents when they are attached to the same quantity:—

$a^{\frac{pm}{qm}}=a^{\frac{p}{q}}$; for example, $a^{\frac{4}{6}}=a^{\frac{2}{3}\cdot\frac{2}{2}}=a^{\frac{2}{3}}$; $a^{\frac{4}{2}}=a^{2\cdot\frac{2}{2}}=a^2$; &c.

$a^m\times a^n=a^{m+n}$; for example, $a^3\times a^2=a^5$; $a^{\frac{1}{3}}\times a^{\frac{1}{2}}=a^{\frac{1}{3}+\frac{1}{2}}=a^{\frac{5}{6}}$

$a^m\div a^n=a^{m-n}$; for example, $a^5\div a^2=a^3$; $a^{\frac{1}{2}}\div a^{\frac{1}{3}}=a^{\frac{1}{2}-\frac{1}{3}}=a^{\frac{1}{6}}$

$(a^m)^n=a^{mn}$; for example, $(a^3)^2=a$; $(a^{\frac{1}{3}})^{\frac{1}{2}}=a^{\frac{1}{6}}$; &c.

That is—

To MULTIPLY. *Add the exponents.*

To DIVIDE. *Subtract the exp. of divisor from exp. of dividend.*

To EXPRESS A POWER OR ROOT. *Multiply the exp. of the quantity, by the exp. of the power or root of it which is proposed.*

In *evolution*, or the *extraction of roots*, there are some particulars respecting *signs* which require to be especially mentioned. In *involution*, or the raising of powers, you have seen that the sign of the result is always fixed by the rule of signs; it is not so in the reverse operation of extraction. For instance, the square of 4 is 16, the 16 being *plus*, whether the 4 be *plus* or *minus;* but the square root of 16 is *ambiguous* as to sign; the numerical value of the root is, of course, 4; but we have as much right to prefix a *minus* to this 4 as a *plus;* since $(-4)^2$ and 4^2 are equally 16. Hence $\sqrt{16}=\pm 4$; that is *plus* or *minus* 4. And there is a like ambiguity, as to sign, in every *even root* of a positive quantity; because the corresponding *even power* of that root would be the same whether a + or a — be prefixed to it: $\sqrt{4}$ or $4^{\frac{1}{2}}$ is ± 2; $\sqrt[4]{16}$ or $16^{\frac{1}{4}}$—is ± 2, and so on. As to an *even root* of a *negative* quantity, the thing is *impossible.* Such an expression as $\sqrt{-4}$ implies an impossible operation; for the *square root* of a quantity is that which, when squared, reproduces the quantity. Now a quantity squared, whether its sign be + or —, is always +; it is impossible $\therefore$ that -4 can be the *square* of anything. Such expressions as $\sqrt{-4}$, $\sqrt{-9}$, $\sqrt{-1}$, &c., are $\therefore$ called *imaginary* or *impossible* quantities; and they answer this purpose, namely—whenever they occur in the solution of a problem, you may take it as a sure indication that the problem implies some impossibility or contradiction. You thus see that the numbers of arithmetic, when introduced into algebra, divide themselves not only into *positive* and *negative*, but also into *real* and *imaginary.* Real numbers, too, are separated into two classes; namely, *rational* and *irrational*, or *surd.* The following are examples of *surds;* namely, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{10}$, &c. Surds, you see, are *roots;* but roots of numbers that are not themselves the reverse *powers:* 3, 5, 7, 10, &c., are not *squares;* they have $\therefore$ no *exact square roots.* Nevertheless, either of these numbers being proposed, we can always find another number, such that its square shall approach as near to the proposed number as we please; we can thus *approximate*, as it is called, to the square root of 3, or 5, or 7, &c., to any degree of nearness, as was shown in the operation for the square root in decimal arithmetic. There is thus a marked difference between $\sqrt{3}$ and $\sqrt{-3}$; the value of the former can be approximated to, to any degree of nearness; the value of the latter cannot be approached to at all—it has no existence. It may be noticed, however, that both *surds* and *imaginary quantities* may sometimes be simplified in appearance. Thus, $\sqrt{8} = \sqrt{4\cdot2} = 2\sqrt{2}$; $\sqrt{27} = \sqrt{9\cdot3} = 3\sqrt{3}$; $\sqrt{-8} = \sqrt{-2\cdot4} = 2\sqrt{-2}$; $\sqrt{-27} = \sqrt{-3\cdot9} = 3\sqrt{-3}$; $\sqrt{-4} = 2\sqrt{-1}$, &c. This can always be done when the number under $\sqrt{\ }$ can be separated into two factors, one of which is a square; the operation implied in $\sqrt{\ }$ being actually *performed* on this square, it becomes freed from the radical, which then applies only to the other factor. You should never leave under the radical, in any final result, a factor upon which the operation indicated by the radical can be actually performed. Thus, such an expression as $\sqrt{a^2b}$ should be reduced to the simpler form $a\sqrt{b}$. In like manner, $\sqrt[3]{a^3x^3y} = ax\sqrt[3]{y}$; $\sqrt[3]{16ax^3} = \sqrt[3]{8\cdot2ax^3} = 2x\sqrt[3]{2a}$; and so on. It is customary to call such forms as $\sqrt{b}$, $\sqrt[3]{y}$, $\sqrt[3]{2a}$, &c., in which the operation under the radical cannot be performed, *algebraical surds*; although, if the letters were interpreted, it is quite possible that the algebraical surd might prove a *rational number*. Thus, if b were 4, then $\sqrt{b}$ would be ± 2; which is rational, though ambiguous as to sign.

We shall now give you a few examples on the *evolution* of simple quantities, or quantities consisting of but *one term*. It has been stated (p. 461) that quantities consisting of *two* terms are called *binomials*; you will be prepared to expect, therefore, that those of *one* term are often called *monomials*, those of *three* terms *trinomials*, those of four *quadrinomials*; a quantity consisting of more terms comes under the general name of *polynomial* or *multinomial*.

To extract a proposed root of a simple or monomial quantity.

RULE 1. Write the root of the *coefficient* with its proper sign, remembering that an *odd* root, like an *odd* power of any quantity, has the *same sign* as that quantity; but that an *even* root of a *positive* quantity takes the *double sign* $\pm$.

2. *Divide* the exponent of each letter by the exponent of the radical; or, which is the same thing, *multiply* by the fractional index, by which the radical may be replaced; and the proposed root will be obtained.

1. $\sqrt{16a^4x} = 4a^2\sqrt{x}$, or $4a^2x^{\frac12}$. Here the square root of the coefficient 16 is 4; the letters with their exponents are a^4x^1, the index or exp. of the radical is 2; ∴ dividing the exponents by this, the *letters* of the result are $a^{\frac42}x^{\frac12} = a^2x^{\frac12}$. Or replacing the radical by the equivalent exponent, the ex. is $(16a^4x)^{\frac12} = 4(a^4x^1)^{\frac12}$; and *multiplying* the exponents *within* the vinculum, by the exp. *without*, we have $4(a^4x^1)^{\frac12} = 4a^{\frac42}x^{\frac12} = 4a^2\sqrt{x}$. The double sign is not placed before the 4: the reason is, that the extraction is not completed; $\sqrt{x}$ is a still unperformed operation, and the double sign is regarded as still implied in $\sqrt{\ }$.

2. $\sqrt{9a^2x^6} = 3a^{\frac22}x^{\frac62} = \pm 3ax^3$

3. $\sqrt{2a^3x^4} = \sqrt{2a^2ax^4} = ax^2\sqrt{2a}$

4. $\sqrt[3]{-8a^6x^3} = -2a^{\frac63}x = -2a^2x$

5. $\sqrt[3]{27ab^2x^3} = 3a^{\frac13}b^{\frac23}x = 3x\sqrt[3]{ab^2}$

It is to be observed, that when an irrational or surd quantity occurs among the factors of a term, the *letters* composing that term are not arranged in alphabetical order except when the surd is expressed by a fractional exponent; when it is denoted by the sign $\sqrt{\ }$, it is always placed *last*, to prevent all mistake as to the extent of influence of this sign. If the result in the last ex. had been written $3\sqrt[3]{ab^2x}$, it would evidently have conveyed a wrong meaning; it is true this might have been remedied by marking the extent to which $\sqrt[3]{\ }$ reaches by a bar-vinculum, thus, $3\sqrt[3]{\overline{ab^2}}.\,x$; but superfluous marks and signs are always to be avoided, so that the form $3x\sqrt[3]{ab^2}$ is to be preferred; a bar over ab^2 is not required, when it is understood that $\sqrt{\ }$ covers, or extends to, *all* the factors which follow it. We have, indeed, put a bar over similar expressions at page 455, but did so for fear that *you* should limit the influence of the radical; it is best omitted when there is no danger of such mistake being made.

6. $\frac{1}{\sqrt{a^3b^4}} = \frac{1}{ab^2\sqrt{a}}$ or $= a^{-\frac32}b^{-2}$, or $= a^{-1}a^{-\frac12}b^{-2}$.

The first of these forms, namely $\frac{1}{ab^2\sqrt{a}}$, is that which would, in general, be employed; negative indices, though of much importance in certain general investigations, being seldom used in particular expressions. But we must tell you that algebraists do not like to leave surd quantities in the *denominators of fractions*; for as has been said before (page 460), they wish their final results to be in a form the best suited for arithmetical computation. Now, suppose you had to compute $\frac{1}{\sqrt{a}}$, a being interpreted to you;—say $a = 11$. You would have first to find the square root of 11, to several places of decimals; suppose *four* places were considered sufficient, you would then have $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{11}} = \frac{1}{3\cdot3166}$, and the *division* here indicated would have to be performed. But instead of taking this course, let numerator and denominator of $\frac{1}{\sqrt{a}}$ be multiplied by $\sqrt{a}$, which, from the first principles of arithmetical fractions, you know to be allowable, $\frac{1}{\sqrt{a}}$ would thus be changed into $\frac{\sqrt{a}}{a}$; and you would have to compute $\frac{\sqrt{a}}{a} = \frac{\sqrt{11}}{11} = \frac{3\cdot3166}{11}$. You can *now*, in a moment, pronounce the final result to be ·3015 . . . , which, of course, is the same as the result of $\frac{1}{3\cdot3166}$, but it would have taken you much longer to discover it. Ex. 6 therefore should not be left in the state in which we have left it above; the steps should be:—

$$\frac{1}{\sqrt{a^3b^4}} = \frac{1}{ab^2\sqrt{a}} = \frac{\sqrt{\ }}{ab^2a} = \frac{\sqrt{a}}{a^2b^2}.$$

EXAMPLES FOR EXERCISE.

1. $\sqrt{a^4x^6y}$ 2. $\sqrt[4]{2^4a^4b^8}$ 3. $\sqrt[3]{8a^3x^6z}$
4. $\sqrt[3]{-8a^4x^3y^2}$ 5. $\sqrt{-2a^4y^6z^8}$ 6. $\sqrt{3a^5y^4x}$
7. $(a^2b^3x^4)^{\frac12}$ 8. $\sqrt{x^{\frac13}y^2z^2}$ 9. $\sqrt{\frac{a^2}{bx^2}}$
10. $\sqrt{a^3x^{-4}}$ 11. $\sqrt{a^{-3}x^{-4}y^{-5}}$ 12. $\sqrt{\frac{8a^3}{27x^3y^4}}$
13. $(-8a^3x^2y^6)^{\frac13}$ 14. $(9b^4x^2y^3)^{-\frac12}$ 15. $x^2(81x^6y^7z^8)^{-\frac14}$
16. $\sqrt[3]{-16a^5x^6y^{-9}}$ 17. $(32a^{\frac12}x^2y^4)^{-\frac15}$ 18. $(32a^{-3}x^{-6}y^3)^{-\frac15}$

FRACTIONS.

Algebraic fractious are treated exactly the same as numerical fractions in common arithmetic; the only difference being in the symbols, and not in the operations performed.

To reduce a mixed quantity to an improper fraction.

A quantity is called a mixed quantity when it is partly integral and partly fractional; such a quantity may always be changed into an equivalent one, wholly fractional in form; the result is called an *improper* fraction, as well in arithmetic as in algebra, because an integral quantity is really absorbed into it. The rule for reducing a quantity partly integral and partly fractional to a form wholly fractional, is as follows:—

RULE.—Multiply the integral part by the denominator of the fractional part; connect the product with the numerator, by the sign of that numerator, and place the denominator underneath.

1. $a+\frac{x}{b}=\frac{ab+x}{b}$. Here nothing more is done than replacing a by $\frac{ab}{b}$, which is, of course, equivalent to it. The integral part a is converted into a fractional form, by multiplying it by the *given* denominator b, and dividing the result by the same letter, thus destroying the effect of the multiplication. And this is all that is done in operations of this kind.

2. $a+b-\frac{c}{x}=\frac{(a+b)x-c}{x}$ 3. $a-\frac{b}{x-y}=\frac{a(x-y)-b}{x-y}$

4. $a-\frac{x-y}{b}=\frac{ab-x+y}{b}$

5. $2+\frac{x^2+y^2}{xy}=\frac{2xy+x^2+y^2}{xy}=\frac{(x+y)^2}{xy}$*

6. $\frac{(x+y)^2}{xy}-2=\frac{(x-y)^2}{xy}$ 7. $a+b+\frac{b^2}{a-b}=\frac{a^2}{a-b}$

8. $\sqrt{x}-\frac{x-1}{\sqrt{x}}=\frac{1}{\sqrt{x}}=\frac{\sqrt{x}}{x}$†

To reduce an improper fraction to a mixed quantity.

RULE.—Divide the numerator by the denominator, as far as the division can be carried, and to the quotient annex the *remainder* with the denominator underneath. If there be no remainder, the quotient, which will then be wholly integral, will be the complete value; showing that the so-called improper fraction is an integral quantity disguised under a fractional form.

This operation, you see, is nothing but that of *division:* the examples at page 463, serve as well as any to illustrate the present rule; but a few others may be added here.

1. $\frac{12a^2-4a+3b}{4a}=3a-1+\frac{3b}{4a}$

2. $\frac{a^2-x^2+1}{a-x}=a+x+\frac{1}{a-x}$

3. $\frac{4x^3-25x^2+20x+25}{4x+3}=x^2-7x+10+\frac{x-5}{4x+3}$

4. $\frac{x^4-8x+7}{x^2-3x+2}=x^2+3x+7+\frac{7x-7}{x^2-3x+2}$

EXAMPLES FOR EXERCISE IN THE TWO PRECEDING RULES.

Prove that the following are IDENTICAL EQUATIONS:—

1. $a+x+\frac{x^2}{a-x}=\frac{a^2}{a-x}$

2. $3-y-\frac{2-y^2}{3+y}=\frac{7}{3+y}$

3. $\left(\frac{b+c}{b-c}\right)^2-1=\frac{4bc}{(b-c)^2}$

4. $\frac{a^3+x^3}{a-x}=a^2+ax+x^2+\frac{2x^3}{a-x}$

5. $\frac{a^3-x^3}{a^2+ax+x^2}=a-x$

6. $x^2-a^2+\frac{x^4+a^4}{x^2+a^2}=\frac{2x^4}{x^2+a^2}$

7. $\frac{(x+y+z)(x+y-z)}{2xy}-1=\frac{x^2+y^2-z^2}{2xy}$

8. $\frac{(x+y+z)(x+y-z)}{2xy}-2=\frac{(x-y)^2-z^2}{2xy}$

9. $\frac{x^2+y^2-z^2}{2xy}-1=\frac{(x-y-z)(x-y+z)}{2xy}$

10. $x^2-3x-\frac{3x(3-x)}{x-2}=\frac{x\{(x-1)^2-4\}}{x-2}$

To reduce fractions to a common denominator.

Before fractions can be either added or subtracted they must be reduced to a common denominator. No two quantities can be actually added together, or subtracted the one from the other, so long as the *denominations* of the quantities are different. We cannot *add* 4 shillings to 2 pounds; but can only connect them together as distinct quantities, till both are brought to the same denomination. It is only then that they can be actually incorporated in one sum; 40*s.* and 4*s.* make 44*s.*; the 2 and the 4 make 6; but these are neither pounds nor shillings, and have no meaning in reference to the things proposed. A fraction, like a concrete quantity, denotes a stated number of things of a stated denomination; the *number* is expressed by the *numerator*, the *denomination* by the *denominator;* and ∴ so long as the denominators are different, two fractions can no more be added or subtracted than pounds and shillings. It follows, therefore, that before fractions can be fitted for addition or subtraction, they must be prepared for these operations by a previous reduction of them to common denominators. The rule for this reduction is given below; it is founded upon the obvious principle that a fraction is not altered in value, though the numerator be multiplied by any quantity whatever, provided only that the denominator be multiplied by the same thing; since the new factor in the denominator just counteracts the influence of the same factor in the numerator: a multiplication, and then a division by the same thing, leaves the quantity operated upon, whatever it may be, virtually untouched.

RULE 1.—Multiply *each* numerator by the product of all the denominators *except its own;* the results will be the numerators of the changed fractions.

2. The product of *all* the denominators will be the denominator common to all the changed fractions.

This rule will effect the reduction of fractions to a common denominator in all cases; but sometimes the desired change may be brought about in an easier way. The common denominator, found by the rule, is evidently such that each of the proposed denominators is always *a factor* of it. It is the object of the rule to make sure of such a number in every case; but it often happens that a *smaller* number exists, such that each denominator is a factor of it; and whenever such smaller number can be discovered with little trouble, it is of course better to use *it*, than a larger number. The smallest number possible is called the *least common multiple* of the denominators; and a little examination of the denominators will often enable us to arrive at it very readily, as will be seen in some of the following examples:—

1. $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\frac{ayz}{xyz}+\frac{bxz}{xyz}+\frac{cxy}{xyz}$

Here the three fractions on the right of the sign of equality are respectively the same, in value, as those on the left; and the denominator xyz is common to all.

2. $\frac{x+a}{2}+\frac{b}{3}+\frac{1}{5}=\frac{15(x+a)}{30}+\frac{10b}{30}+\frac{6}{30}$

3. $\frac{2x+3}{x}-\frac{5x+1}{3}=\frac{6x+9}{3x}-\frac{5x^2+x}{3x}$

4. $\frac{2x+1}{6a}+\frac{5x}{4}$. Here it is plain that 12 is a number, and the *least* number, such that 6 and 4 are each a factor of it, ∴ $12a$ is the least common multiple of $6a$ and 4, so that the common denominator need be no higher than $12a$; this $12a$ is the first denominator above multiplied by 2, ∴ multiplying the numerator also by 2, the changed fraction is $\frac{2(2x+1)}{12a}$. Again, the same $12a$ is the second denominator multiplied by $3a$, ∴ multiplying the numerator also by $3a$, the second changed fraction is $\frac{15ax}{12a}$; so that $\frac{2x+1}{6a}+\frac{5x}{4}=\frac{2(2x+1)}{12a}+\frac{15ax}{12a}$ By the *rule* the common denominator would have been twice as great.

5. $\frac{2x}{x^2-a^2}+\frac{3}{x+a}$ Here, since we know that $(x+a)$

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* See *ante*, p. 460.   † See p. 465.

$(x-a)=x^2-a^2$, the second fraction will be converted into one with a denominator the same as the first, by merely multiplying numerator and denominator of the second by $x-a$, $\therefore$ the fractions are $\frac{2x}{x^2-a^2}+\frac{3(x-a)}{x^2-a^2}$.

6. $\frac{2x}{3ab^3}+\frac{x}{2a^2b^2}+\frac{5}{6ab}$. Here the *coefficient* of the common denominator is 6, the least number divisible by coefficients 3, 2, and 6; the letters are a^2b^3, the least quantity divisible by a^2b^3, a^2b^2, and ab, $\therefore$ the changed fractions are $\frac{4ax}{6a^2b^3}+\frac{3bx}{6a^2b^3}+\frac{5a^2b^2}{6a^2b^3}$; each fraction *here* differing from that which it replaces only in appearance —it is the original fraction with numerator and denominator multiplied by the same thing.

It will not be necessary that we should give any exercises expressly for practice in the preceding rule. The examples furnished in addition and subtraction— the rule for which we are now about to give, will equally afford practice in the foregoing operation; for, as you have already seen, all fractions must have a common denominator before they can be either added or subtracted. We shall therefore at once proceed to

ADDITION AND SUBTRACTION OF FRACTIONS.

Rule.—*Addition.*—Reduce the fractions to equivalent ones with a common denominator, which place under the *sum* of the changed numerators.

Subtraction.—Reduce the fractions to equivalent ones with a common denominator, which place under the *difference* of the changed numerators.

1. $\frac{x+a}{2}+\frac{b}{3}+\frac{1}{5}=\frac{15(x+a)+10b+6}{30}$

2. $\frac{2x+3}{x}-\frac{5x+1}{3}=\frac{9+5x-5x^2}{3x}=\frac{9+5x(1-x)}{3x}$

3. $\frac{a+b}{a-b}+\frac{a-b}{a+b}=\frac{(a+b)^2+(a-b)^2}{a^2-b^2}=\frac{2(a^2+b^2)}{a^2-b^2}$

4. $\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{(a+b)^2-(a-b)^2}{a^2-b^2}=\frac{4ab}{a^2-b^2}$

5. $\frac{2x-3}{3x}-\frac{4x+2}{3}=\frac{2x-3-(4x+2)x}{3x}=-\frac{4x^2+3}{3x}$

6. $\frac{1}{a^n}+\frac{1}{a^n+1}=\frac{a+1}{a^n+1}$

7. $1-\frac{x^2-y^2}{x^2+y^2}=\frac{x^2+y^2-(x^2-y^2}{x^2+y^2}=\frac{2y^2}{x^2+y^2}$

8. $\frac{x^2+y^2}{xy}-2=\frac{x^2+y^2-2xy}{xy}=\frac{(x-y)^2}{xy}$

In this example, as well as in the preceding, an *integer* occurs in connection with a fraction; and it may here be noticed, that an integer may always be put in the form of a fraction by simply giving to it 1 for denominator: thus, in the present example, the 2 may be regarded as $\frac{2}{1}$.

EXAMPLES FOR EXERCISE.

Note.—It sometimes happens that the sum or difference of fractions is a fraction such that numerator and denominator have a factor common to both. Such common factors should always be expunged from the final result, whenever they are seen to enter: they only serve to give to the fraction an appearance unnecessarily complicated. A fraction thus deprived of all common factors in numerator and denominator is said to be in its *lowest terms*, the numerator and denominator being called *the terms of the fraction*.

1. $\frac{2x-5}{3}+\frac{x-1}{2x}$
2. $\frac{x}{x-3}+\frac{x}{x+3}$
3. $\frac{10x-9}{8}-\frac{3x-5}{7}$
4. $\frac{2x-3}{7x}-\frac{x-5}{8x}$
5. $\frac{x-a}{x^2-ax+a^2}+\frac{1}{x+a}$
6. $1-\frac{x-a}{x+a}$
7. $\frac{a^2x}{xy}-\frac{b^2x}{yz}$
8. $\frac{1}{x}+\frac{1}{x^2}-\frac{1}{x^n}$
9. $\frac{1}{2x-a}+\frac{1}{2x+a}-\frac{4x^2}{4x^2-a^2}$
10. $\frac{1+x}{1+x+x^2}-\frac{1-x}{1-x+x^2}$
11. $\frac{a^2+b^2}{a^2-b^2}+\frac{a+b}{a-b}-\frac{a-b}{a+b}+2$
12. $\frac{y}{y-3}-\frac{y-3}{y}+\frac{y}{y+3}-\frac{y+3}{y}$
13. Show that $\frac{x}{x+y}+\frac{y}{x-y}=\frac{x}{x-y}-\frac{y}{x+y}$
14. $\frac{x}{x+z}-\frac{y}{y+z}=\frac{z}{y+z}-\frac{z}{x+z}$

Note.—The student may obtain considerable assistance in the study of the addition, &c., of algebraic fractions, by combining with it that of the arithmetical branch of the same subject.*—Ed.

MULTIPLICATION OF FRACTIONS.

If a fraction $\frac{a}{b}$ is to be multiplied by an integer c, we have to take a things, of denomination b, c times; the product is therefore $\frac{ac}{b}$; that is, ac of those things, just as if we had to multiply a pounds, or a ounces, by c, the product would be ac of those pounds or ounces. But, if instead of c, we are to multiply only by the dth part of c, then, of course, the product will be only the dth part of the former; that is, it will be $\frac{ac}{b}\div d$, or $\frac{ac}{bd}$, $\therefore \frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$. This suggests the rule, which is as follows:

Rule 1.—Multiply the numerators together, the result will be the *numerator of the product.*

2 Multiply the denominators together, and the result will be the *denominator of the product.*

Note.—Before performing the operation, see whether either of the fractions can be reduced to *lower terms;* if so, reduce the fraction. And after the operation, see whether a like reduction can be easily made in the result; if so, reduce accordingly. (See preceding Note).

It is of importance, in dealing with algebraic fractions, that we always keep in remembrance the factors of such expressions as a^2-b^2, a^4-b^4, a^6-b^6, &c.; as also of $a^2+2ab+b^2$, and $a^2-2ab+b^2$. You have seen at page 460, that the *difference* of the *squares* of two quantities is the *product of the sum and difference* of the quantities; so that $a^2-b^2=(a+b)(a-b)$; $a^4-b^4=(a^2-b^2)(a^2-b^2)$; $a^6-b^6=(a^3+b^3)(a^3-b^3)$, &c.; also that $a^2+2ab+b^2=(a+b)(a+b)$, or $(a+b)^2$; and that $a^2-2ab+b^2=(a-b)(a-b)$, or $(a-b)^2$. As there will be frequent occasion to apply these truths in operations with fractions, we repeat them here, that you may have full warning of what you will be expected to remember.

1. $\frac{3x}{x-1}\times\frac{2x}{x+1}=\frac{6x^2}{(x-1)(x+1)}=\frac{6x^2}{(x-1)^2}$

2. $\frac{3x^2-x}{5}\times\frac{10}{2x^2-3x}$. Here it is easy to see that the factor x will enter into both numerator and denominator of the product: it is useless to allow it thus to enter, and afterwards to expunge it; we, therefore, silently suppress or *cancel* this common factor, and so preclude its appearance altogether. In like manner, the factor 5, foreseen to enter numerator and denominator of the product, is, at the outset, *cancelled;* so that, before actually multiplying, we imagine the factors changed for $\frac{3x-1}{1}\times\frac{2}{2x-3}=\frac{2(3x-1)}{2x-3}$, the product in its lowest terms. You are aware that all this is exactly what we should do in common arithmetic: and have, in fact, nothing to learn in the management of algebraic fractions that is not equally necessary in the fractions of pure arithmetic. Suppose the example here commented upon had been $\frac{3x^2-x}{5}\times\frac{10}{2x^2-4x}$; then, seeing that the second fraction has the factor 2 common to numerator and denominator, you would here, as in arithmetic,

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* See *ante*, p. 443, *et seq.*

expunge or cancel that factor; and, to make each step perfectly plain, might write—

$$\frac{3x^2-x}{5}\times\frac{10}{2x^2-4x}=\frac{3x^2-x}{5}\times\frac{5}{x^2-2x}=\frac{3x-1}{1}\times\frac{1}{x-2}=\frac{3x-1}{x-2}$$

EXAMPLES FOR EXERCISE.

1. $\frac{4x}{7}\times\frac{5}{9x}$ 2. $\frac{4x+1}{3}\times\frac{6x}{7}$ 3. $\frac{4x+2}{3}\times\frac{5x}{2x+1}$

4. $\frac{2x-1}{x+2}\times\frac{3x+1}{x-3}$ 5. $\frac{a+b}{b}\times\frac{b^2x}{a^2-b^2}$

6. $\frac{x^3-a^2x}{a^2}\times\frac{x}{ax-a^2}$ 7. $\frac{x^n}{y^m}\times\frac{x^m}{y^n}$ 8. $\frac{x^n}{y^m}\times\frac{y^n}{x^m}$

9. $\frac{3\sqrt{x}}{y}\times\frac{2y^2}{5\sqrt{x}}$ 10. $\frac{x^2-4a^2}{x-a}\times\frac{x^2-a^2}{x+2a}$

11. $\frac{2x}{7}\times\frac{4x}{9}\times\frac{7}{4x}\times\frac{9x}{5}$ 12. $\frac{4x+2}{3}\times\frac{5x}{2x+1}\times\frac{9x-3}{x}$

13. $\frac{(x-y)^2}{x^2-y^2}\times\frac{x+y}{x-y}$ 14. $\frac{(x-y)^2}{x^4-y^4}\times\frac{x+y}{x-y}$

15. $\left(a-\frac{b^2}{a}\right)\left(\frac{a}{b}+\frac{b}{a}\right)$

DIVISION OF FRACTIONS.

If a fraction $\frac{a}{b}$ is to be divided by an integer c, the result is of course $\frac{a}{bc}$; but if it is to be divided, not by c, but by only the dth part of c, that is, by $\frac{c}{d}$, then the quotient will be d times the former quotient, as is obvious; that is to say, the quotient will be $\frac{a}{bc}\times d$, or $\frac{ad}{bc}$. Hence division is converted into multiplication, as directed in the following rule:—

RULE.—Invert the terms of the *divisor*, or turn it upside down, and proceed as in multiplication.

1. $\frac{3x-2}{2x}\div\frac{5x+1}{7x}=\frac{3x-2}{2x}\times\frac{7x}{5x+1}=\frac{7(3x-2)}{2(5x+1)}$

2. $\frac{x^4-y^4}{(x-y)^2}\div\frac{x^2+xy}{x-y}=\frac{x^4-y^4}{(x-y)^2}\times\frac{x-y}{x(x+y)}=\frac{(x^2+y^2)(x^2-y^2)}{x-y}\times\frac{1}{x(x+y)}=\frac{(x^2+y^2)(x^2-y^2)}{x^2-y^2}\times\frac{1}{x}=\frac{x^2+y^2}{x}$

3. $\left(\frac{a^2}{x}-\frac{x^3}{a^2}\right)\div\left(\frac{a}{x}+\frac{x}{a}\right)=\frac{a^4-x^4}{a^2x}\times\frac{ax}{x^2+x^2}=\frac{a^2-x^2}{a}$

EXAMPLES FOR EXERCISE.

NOTE.—*Before* inverting the divisor, you may always expunge factors common to either *both numerators*, or to *both denominators:* thus, the x in the denominators of example worked in the last page, might have been omitted, as also $x-y$ in each denominator of Example 1.

1. $\frac{x}{x-2}\div\frac{x-1}{x^2+2}$ 2. $\frac{a+1}{a}\div\frac{a^2-1}{a}$

3. $\frac{x^2-9}{5}\div\frac{x+3}{4}$ 4. $\frac{6x^2-2x}{4-x^2}\div\frac{x^2}{2-x}$

5. $\frac{9x^2-4y^2}{x+y}\div\frac{3x-2y}{x^2-y^2}$ 6. $\frac{x^3-y^3}{x^2-2xy+y^2}\div\frac{x^2+xy}{x-y}$

CHAPTER III.

ALGEBRA.—EQUATIONS.

AT page 456, a rule was given for the solution of a simple equation, and several examples were proposed to show its practical application. These examples, however, were selected with a view to your acquaintance with the operations of algebra at that stage of your progress. You may now attempt equations of a higher order of difficulty, and apply the following more general rule for that purpose.

GENERAL RULE *for the solution of a simple equation with one unknown quantity.*

1. Clear the equation of fractions, if there be any. This may be done by multiplying *each* numerator by *all* the denominators except its own; or by taking a common multiple of the denominators and then multiplying *each numerator* by this multiple, after suppressing that factor in it which is equal to the *denominator.*

2. Clear the equation of radical signs, if there be any. This is done by causing the quantity under the radical we wish to remove, to stand *alone* on one side of the equation, and then performing the operation the *reverse* of that indicated by the radical, on *both* sides of the equation. Thus, if the radical be $\sqrt{\ }$, we must *square* both sides; if it be $\sqrt[3]{\ }$, we must *cube* both sides; and so on. This reverse operation evidently disengages the quantity from the radical; if *two* radicals enter, the operation must be repeated in reference to the *second* radical. Transposing, collecting, &c., are sufficiently explained in the former rule. It need only be added here, that the example itself must suggest the order in which the precepts given in either rule should follow one another.

1. $\frac{x}{5}+\frac{x}{4}=\frac{x}{2}-\frac{x}{3}+17$. Here, if each numerator be multiplied by the product of the denominators of all the *other* fractions (regarding 17 as $\frac{17}{1}$), we shall have $24x+30x=60x-40x+2040$, $\therefore 24x+30x+40x-60x=2040$; that is, $34x=2040$, $\therefore x=\frac{2040}{34}=60$.

But if we take the least common multiple of 5, 4, 2, 3, which is easily seen to be 60, we shall have to multiply the first numerator by $\frac{60}{5}$ or 12, the second by $\frac{60}{4}$ or 15, the third by $\frac{60}{2}$ or 30, the fourth by $\frac{60}{3}$ or 20, and the last (17) by $\frac{60}{1}$ or 60. The equation cleared of fractions in this way will be—

$$12x+15x=30x-20x+1020,\ \therefore 12x+15x+20x-30x=1020.$$

$$\text{That is, } 17x=1020,\ \therefore x=\frac{1020}{17}=60.$$

It is plain, that, whichever method be used, the equation free from fractions always arises from multiplying both sides of the original equation by the same quantity; thus, in the first way, each side is multiplied by 120, the product of the denominators; in the second way, each side is multiplied by 60, the least common multiple of the denominators.

2. $x+\sqrt{6x+x^2}=3$. Transposing the x, agreeably to precept 2, of the rule, $\sqrt{6x+x^2}=3-x$; and squaring both sides, $6x+x^2=9-6x+x^2$

$$\therefore 6x=9-6x,\ \therefore 12x=9,\ \therefore x=\frac{9}{12}=\frac{3}{4}.$$

3. $\sqrt{x+7}-\sqrt{x}=1$. Transposing, $\sqrt{x+7}=\sqrt{x}+1$. Squaring both sides, $x+7=x+2\sqrt{x}+1$. Transposing again, to get the radical *alone* on one side, $x-x+7-1=6=2\sqrt{x}$, $\therefore 3=\sqrt{x}$, $\therefore$ (squaring) $9=x$.

4. $\frac{1}{a^2-x^2}-a=\frac{ax}{a-x}+\frac{a}{a+x}$. Here it is evident that $(a-x)(a+x)$, or a^2-x^2 is the least common multiple of the denominators.

$$\therefore 1-a(a^2-x^2)=ax(a+x)+a(a-x),$$

that is, $1-a^3+ax^2=a^2x+ax^2+a^2-ax$. Transposing the unknown terms to the left, $\therefore ax-a^2x=a^3+a^2-1$, that is, $(1-a)ax=a^3+a^2-1$.

$$\therefore x = \frac{a^3 + a^2 - 1}{(1-a)a}, \text{ or } \frac{(a+1)a^2 - 1}{(1-a)a}.$$

As already remarked at page 467, we may, if we please, always *change the signs* of numerator and denominator of a fraction, so that it would be equally correct to write the preceding expression for x thus: $x = \frac{1-(a+1)a^2}{(a-1)a}$.

5. $\sqrt{a+x} + \sqrt{x} + b = c$. Transposing, $\sqrt{a+x} = c - b - \sqrt{x}$.
Squaring, $a + x = (c-b)^2 - 2(c-b)\sqrt{x} + x$.
Transposing, $2(c-b)\sqrt{x} = (c-b)^2 - a$.
Squaring. $4(c-b)^2 x = \{(c-b)^2 - a\}^2$.

$$\therefore x = \left\{\frac{(c-b)^2 - a}{2(c-b)}\right\}^2.$$

This example is in appearance difficult, but in reality it is easy. If you look at the operations actually *performed*, you will see that they are very trifling; the work is for the most part merely *indicated*, not *executed*; and such, in general, is the case when we have to deal exclusively with *literal*, and not with *numerical* quantities. The work of an algebraic problem is usually the greater, as the merely numerical operations are greater; because these must be actually performed: it is the *arithmetic*, not the algebra, that occasions the labour. An example purely algebraical, like that above, requires only a little address in the deduction of one step from another; a little caution in the management of signs, and some care in *indicating* processes, which arithmetic must *perform* when the symbols are interpreted by numbers. It is not easy, however, to embody in a *rule* all the expedients and artifices which may with advantage be resorted to in the solution of a simple equation; these must be acquired by observation and experience, by the exercise of your own ingenuity, and the suggestions of common sense. A little thought and reflection will often do more than the most elaborate *rule*. The following example will serve to illustrate these remarks:—

6. $\frac{\sqrt{a^2+x^2}+x}{\sqrt{a^2+x^2}-x} = \frac{b}{c}$. In this example, the removal of the radicals is evidently the first thing we should try to effect; and it is easy to see that if we were to attempt this by implicitly following the precepts in the rule, we should soon have to deal with complicated expressions. Let us try to evade these by a little ingenuity. You see that if we were at liberty to subtract the denominator of the first member from the numerator, the upper radical would be removed at once: this suggests the subtracting 1 from each side; we therefore make the following the first step, namely—

$$\frac{\sqrt{a^2+x^2}+x}{\sqrt{a^2+x^2}-x} - 1 = \frac{b}{c} - 1\text{; that is, } \frac{2x}{\sqrt{a^2+x^2}-x} = \frac{b-c}{c}.$$

And as it is in general less inconvenient to have a radical in the numerator than in the denominator of a fraction, we shall take the *reciprocals* of these fractions; that is, shall simply turn them upside down, writing the equation thus—

$$\frac{\sqrt{a^2+x^2}-x}{2x} = \frac{c}{b-c} \therefore \sqrt{a^2+x^2} = \left(\frac{2c}{b-c}+1\right)x$$
$$= \frac{b+c}{b-c}x \ldots [\text{A}]$$

$$\therefore \text{squaring each side, } a^2 + x^2 = \left(\frac{b+c}{b-c}\right)^2 x^2 \therefore \frac{a^2}{x^2} + 1$$
$$= \left(\frac{b+c}{b-c}\right)^2$$

$$\therefore \frac{a^2}{x^2} = \left(\frac{b+c}{b-c}\right)^2 - 1 = \frac{4bc}{(b-c)^2} \therefore \frac{x^2}{a^2} = \frac{(b-c)^2}{4bc}$$
$$\therefore x = \frac{a(b-c)}{2\sqrt{bc}}, \text{ or } = \frac{a(b-c)\sqrt{bc}}{2bc}.$$

The step marked [A] may be reached a little differently by help of a property of *two equal fractions*, which you will do well to remember, and apply in examples like that above. Let $\frac{p}{q}, \frac{b}{c}$ be two equal fractions ($\frac{p}{q}$ may stand for the first member of the given equation above); then *subtracting* 1 from each, we have—

$$\frac{p-q}{q} = \frac{b-c}{c}.$$

Also, *adding* 1 to each, we have—

$$\frac{p+q}{q} = \frac{b+c}{c}.$$

Now divide these results by the former, and we get—

$$\frac{p+q}{p-q} = \frac{b+c}{b-c}.$$

By applying this property to the given equation, we get at once—

$$\frac{\sqrt{a^2+x^2}}{x} = \frac{b+c}{b-c}, \text{ and thence } \sqrt{a^2+x^2} = \frac{b+c}{b-c}x.$$

EXAMPLES FOR EXERCISE.

NOTE.—Examples 8, 9, 10, 11, and 16, have been selected with a view to the application of the principle just explained. Remember, in Ex. 15, that *sum.* × *diff.* = *diff. of squares.*

1. $\frac{2x}{5} + \frac{x}{2} = 22 + \frac{x}{6}$ 2. $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 6\tfrac{1}{2}$

3. $\frac{x}{2} - \frac{x}{4} = \frac{x}{3} - \frac{x}{6} + 1$ 4. $\frac{18+x}{x} = \frac{6}{x} + 5$

5. $21 + \frac{3x-11}{16} = \frac{5(x-1)}{8} + \frac{97-7x}{2}$

6. $\sqrt{x^2+7} + x = 7$ 7. $\sqrt{x-a} - b = 0$

8. $\frac{\sqrt{x}+2}{\sqrt{x}-2} = \frac{3}{2}$ 9. $\frac{\sqrt{x}+28}{\sqrt{x}+4} = \frac{\sqrt{x}+38}{\sqrt{x}+6}$

10. $\frac{\sqrt{x}+2a}{\sqrt{x}+b} = \frac{\sqrt{x}+4a}{\sqrt{x}+3b}$ 11. $\frac{\sqrt{ax}-a}{\sqrt{ax}+a} = \frac{3\sqrt{ax}-2a}{3\sqrt{ax}+5a}$

12. $\sqrt{(x-24)} = \sqrt{x} - 2$ 13. $\sqrt{(4x+21)} = 2\sqrt{x} + 1$

14. $\sqrt[3]{(2x+3)} = 3$ 15. $\frac{5x-9}{\sqrt{5x}+3} = \frac{\sqrt{5x}-1}{2}$

16. $\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}} = \frac{1}{2}$

Simple Equations with Two Unknown Quantities.

There are *three* methods of solving a pair of simple equations containing two unknown quantities. We shall illustrate them by an example:—

1. Find the values of x and y in the equations,

$$\begin{cases} 2x + 5y = 23, \\ 3x - 2y = 6. \end{cases}$$

From the first equa., by trans., $2x = 23 - 5y$ ∴

$$x = \frac{23-5y}{2}.$$

From the second equa., by trans., $3x = 6 + 2y$ ∴

$$x = \frac{6+2y}{3}.$$

Equating these two different expressions for x, we have $\frac{23-5y}{2} = \frac{6+2y}{3}$. ∴ (clearing) $69 - 15y = 12 + 4y$ ∴ trans. $-19y = -57$ ∴ $y = 3$. Substituting this value for y in either of the expressions for x, the second, for instance, we have $x = \frac{6+6}{3} = 4$ ∴ $x = 4$, $y = 3$. This method, you see, consists in equating the expressions for the *same* unknown quantity, as deduced from the two equations.

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2. Having found an expression for x from *one* of the equations, as, for instance, $x = \frac{6+2y}{3}$, substitute it for x in the *other* equation, and we get $\frac{2(6+2y)}{3} + 5y = 23$; ∴ clearing and removing brackets,

$$12 + 4y + 15y = 69 \therefore 19y = 57 \therefore y = 3,$$

so that x or $\frac{6+2y}{3}=\frac{6+6}{3}=4 \mathrel{.\cdot.} x=4,\ y=3.$

This is called the method of substitution.

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3. In the third method, the first object is to convert the proposed equations into forms such that one of the unknown quantities may have *the same coefficient* in each; thus:—multiplying the first equation by 3, and the second by 2, they become changed into

$$\begin{aligned} 6x+15y&=69\\ 6x-\ \ 4y&=12 \end{aligned}$$

By subtracting, $19y=57 \therefore y=3$

And this value put for y in one of the given equations, in the second for instance, there results, $3x-6=6 \therefore$

$$x=\frac{12}{3}=4.$$

Or without, in this way, borrowing from the method of substitution, multiply the first equation by 2, and the second by 5: they then change into

$$\begin{aligned} 4x+10y&=46\\ 15x-10y&=30 \end{aligned}$$

By adding, $19x=76 \therefore x=4$

This is the method of *equalising the coefficients* of the same unknown quantity in the two equations. When this equality is brought about, then addition or subtraction, according as the *signs* of the equal coefficients are unlike or like, will of course remove one of the unknown quantities altogether: when one of the unknown quantities is thus got rid of, it is said to be *eliminated*.

The operations by the three methods may be stated in a rule as follows:—

RULE 1. *By equating two expressions for the same unknown quantity.*—Find an expression for one of the unknown quantities from the first equation. Find an expression for the same from the second equation. Equate these two expressions, and you will then have but a single equation with one unknown quantity, the value of which may be found by the former rules. The value of the other is got by substituting the *value* just found in one of the expressions in place of its *symbols*.

2. *By substitution.*—Find an expression for one of the unknown quantities from either equation, and substitute this expression in place of that unknown in the other equation: an equation with but one unknown quantity will be the result.

3. *By equalising coefficients.*—Multiply the two equations by such numbers (or quantities), the smaller the better, as will cause the resulting coefficients of one of the unknown quantities to be *the same* in the two changed equations: then by addition or subtraction, according as the equal coefficients have unlike or like signs, an equation will arise having only one unknown quantity. *Both* may be *eliminated* one after the other in this way; or having eliminated one, and then found the value of the other from the resulting equation, the value of the former may then be got by substitution, as in the example above.

1. Find the values of x and y from the equations $\left\{\begin{aligned} \frac{x}{6}+\frac{y}{4}&=6\\ \frac{x}{4}+\frac{y}{6}&=5\tfrac{2}{3} \end{aligned}\right.$

Multiplying the first equation by 2 and the second by 3,

$$\begin{aligned} \frac{x}{3}+\frac{y}{2}&=12\\ \frac{3x}{4}+\frac{y}{2}&=17 \end{aligned}$$

Subtracting, $\frac{3x}{4}-\frac{x}{3}=5 \therefore 9x-4x=60 \therefore 5x=60 \therefore x=12$

$$\therefore \text{(equation 1) } 2+\frac{y}{4}=6 \therefore \frac{y}{4}=4 \therefore y=16.$$

2. $\left.\begin{aligned} \frac{x}{2}+\frac{y}{3}&=7\\ \frac{x}{3}+\frac{y}{2}&=8 \end{aligned}\right\} \therefore$ clearing, $\left.\begin{aligned} 3x+2y&=42\\ 2x+3y&=48 \end{aligned}\right\}$ . . . [A] $\therefore x=24-\frac{3y}{2}$

Substituting this value of x in the first equation,

$$72-\frac{9y}{2}+2y=42$$

$$\therefore 144-9y+4y=84 \therefore 60=5y \therefore 12=y$$

$$\text{And } x=24-\frac{3y}{2}=24-18=6.$$

Or, subtracting the upper from the lower of [A],

$$-x+y=6 \therefore y=6+x.$$

Substituting this in the former,

$3x+12+2x=42 \therefore 5x=30 \therefore x=6 \therefore y=6+x=12.$

This latter is the easier mode of proceeding: such slight departures from *rule*, as in equations with one unknown, may often be adopted with advantage.

EXAMPLES FOR EXERCISE.

1. $\left.\begin{aligned} 2x+3y&=23\\ 5x-2y&=10 \end{aligned}\right\}$ 2. $\left.\begin{aligned} 5x+4y&=58\\ 3x+7y&=67 \end{aligned}\right\}$

3. $\left.\begin{aligned} \frac{x}{8}+8y&=194\\ \frac{y}{8}+8x&=131 \end{aligned}\right\}$ 4. $\left.\begin{aligned} \frac{x+y}{3}&=5\\ \frac{y-x}{7}&=1 \end{aligned}\right\}$

5. $\left.\begin{aligned} \frac{3x-7y}{3}&=\frac{2x+y-1}{5}\\ 2-\frac{x-y}{5}&=0 \end{aligned}\right\}$ 6. $\left.\begin{aligned} \frac{x+2}{3}+8y&=31\\ \frac{y+5}{4}+10x&=192 \end{aligned}\right\}$

7. $\left.\begin{aligned} ax+by&=1\\ a'x-b'y&=1 \end{aligned}\right\}$ 8. $\left.\begin{aligned} ax+by&=c\\ a'x+b'y&=c \end{aligned}\right\}$

9. $\left.\begin{aligned} \frac{1}{x}-\frac{1}{y}&=12\\ \frac{3}{x}+\frac{5}{y}&=2\tfrac{4}{7} \end{aligned}\right\}$ Here the unknowns had better be regarded as $\frac{1}{x}, \frac{1}{y}$; when these are found, the reciprocals can be taken.

10. $\left.\begin{aligned} \frac{x-2}{5}-\frac{10-x}{3}-\frac{y-10}{4}&=0\\ \frac{2(y+2)}{3}-\frac{2x+y}{8}-\frac{x+13}{4}&=0 \end{aligned}\right\}$

11. $\left.\begin{aligned} \frac{x+6}{x+4}&=\frac{y+2}{y-2}\\ \frac{x+3}{x+1}&=\frac{y-7}{y-13} \end{aligned}\right\}$* 12. $\left.\begin{aligned} x+y&=19\\ x^2-y^2&=95 \end{aligned}\right\}$

Questions in Simple Equations with One or Two Unknown Quantities.

1. Find a number such that whether it be divided into two equal parts, or into three equal parts, the product of the parts shall be the same.

Let x be the number; then by the question

$$\frac{x}{2}\times\frac{x}{2}=\frac{x}{3}\times\frac{x}{3}\times\frac{x}{3}$$

that is $\frac{x^2}{4}=\frac{x^3}{27} \therefore \frac{1}{4}=\frac{x}{27} \therefore 27=4x \therefore \frac{27}{4}=6\tfrac{3}{4}=x.$

When the question implies *parts* of the unknown quantity, fractions may generally be avoided in the solution by representing the unknown not by x, but a multiple of x, such that the proposed parts may be integral: thus, in the present case—

Let $6x$ be the number; then by the question

$$3x\times 3x=2x\times 2x\times 2x$$

$$\text{that is } 9x^2=8x^3 \therefore 9=8x \therefore x=\frac{9}{8} \therefore 6x=\frac{27}{4}=6\tfrac{3}{4}.$$

2. A person after spending one-fifth of his income and £10 more, found that he had £35 more than half his income left; what was his income?

Let $10x$ represent his income; then by the question, the number of pounds he spent is $2x+10$, so that he had $10x-(2x+10)$ left; that is $8x-10$; but by the question he had $5x+35$ left,

$\therefore 8x-10=5x+35 \therefore 3x=45 \therefore x=15 \therefore 10x=150,$ so that his income was £150.

3. There is a fraction such that if 3 be added to the numerator, its value is $\frac{1}{3}$; and if 1 be subtracted from the denominator, its value is $\frac{1}{5}$: what is the fraction?

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* See *ante*, p. 470.

Let x be the numerator and y the denominator; then the fraction is $\frac{x}{y}$; and by the question

$$\left.\begin{aligned}\frac{x+3}{y}&=\frac{1}{3}\\ \frac{x}{y-1}&=\frac{1}{5}\end{aligned}\right\}\text{ or }\left\{\begin{aligned}&3x+9=y\\ &5x=y-1,\text{ or }y=5x+1.\end{aligned}\right.$$

Hence, equating these two expressions for y, we have $5x+1=3x+9\;\therefore\;2x=8\;\therefore\;x=4$, the numerator.

And $y=5x+1=21$, the denominator, $\therefore\;\frac{4}{21}$ is the fraction.

4. A man and his wife could drink a barrel of beer in 15 days; but after drinking together 6 days, the woman alone drank the remainder in 30 days. In what time could either alone drink the whole barrel?

Suppose the man could drink it in x days, and the woman in y days; then in *one* day the man's share is $\frac{1}{x}$ of the whole, and the woman's $\frac{1}{y}$; so that 15 times the sum of these shares is 1 whole barrel: that is—

$$\frac{15}{x}+\frac{15}{y}=1.$$

But by the question

$$\frac{6}{x}+\frac{6}{y}+\frac{30}{y}=1;\text{ that is, }\frac{6}{x}+\frac{36}{y}=1.$$

So that dividing the first equation by 15, and this last by 6, we have

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{15}.$$

$$\frac{1}{x}+\frac{6}{y}=\frac{1}{6}.$$

By subtracting

$$\frac{5}{y}=\frac{1}{6}-\frac{1}{15}=\frac{3}{30}=\frac{1}{10}\;\therefore\;y=50.$$

Also since, $\frac{5}{y}=\frac{1}{10}\;\therefore\;\frac{15}{y}=\frac{3}{10}\;\therefore\;\frac{15}{x}=1-\frac{15}{y}=1-\frac{3}{10}=\frac{7}{10}$

$$\therefore\;7x=150;\;\therefore\;x=\frac{150}{7}=21\tfrac{3}{7}.$$

Consequently, the man alone could drink it in $21\frac{3}{7}$ days, and the woman in 50 days.

In this example, $\frac{1}{x}$ and $\frac{1}{y}$ are regarded as the unknown quantities, as recommended at page 471, Example 9.

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1. Find a number such that if it be increased by one-half, one-third, and one-fourth of itself, the sum shall be 50.

2. There is a fraction such that if 4 be added to the denominator the value is a $\frac{1}{2}$; and if 3 be added to the numerator, the value is $\frac{3}{4}$: required the fraction.

3. What number is that such that if it be increased by 7, the square root of the sum shall be equal to the square root of the number itself and 1 more?

4. Fifty labourers are engaged to remove an obstruction on a railway: some of them are by agreement to receive ninepence each, and the others fifteen pence. Just £2 are paid to them: but, no memorandum having been made, it is required to find how many worked for 9*d.*, and how many for 15*d.*

5. A person ordered a quantity of rum and brandy, for which he paid £19 4*s.*: the brandy was 9*s.* a quart, and the rum 6*s.* He has, however, forgotten the exact quantity of each he has to receive; but he remembers that if his brandy had been rum, and his rum brandy, his outlay would have been £1 13*s.* less. How many quarts of each did he buy?

6. A person has spirits at 12*s.* a gallon, and at £1 a gallon; how much of each sort must he take to make a gallon worth 14*s.*?

7. A merchant has spirits at *a* shillings a gallon, and at *b* shillings a gallon; how much of each must he take to make a mixture of *d* gallons worth *c* shillings a gallon?

8. In the composition of a certain quantity of gunpowder, two-thirds of the whole + 10lbs. was nitre; one-sixth of the whole — 4½lbs. was sulphur; and the charcoal was one-seventh of the nitre, all but 2lbs. How many lbs. of gunpowder were there?

9. A farmer wishes to mix 28 bushels of barley at 2*s.* 4*d.* a bushel with rye at 3*s.* a bushel, and wheat at 4*s.* a bushel, so that the whole may make 100 bushels worth 3*s.* 4*d.* a bushel; how much rye and wheat must he use?

10. Two persons, A and B, are engaged on a work which they can finish in 16 days; but after working together 4 days, A is called off; and B alone finishes it in 36 days more. In how many days could each do it separately?

11. A composition of copper and tin, containing 100 cubic inches, weighed 505 ounces; how many ounces of each metal did it contain, supposing a cubic inch of copper to weigh 5¼ oz., and a cubic inch of tin to weigh 4¼ oz.?

12. A cask is supplied by three spouts, which can fill it in *a* minutes, *b* minutes, and *c* minutes respectively; in what time will it be filled if all flow together?

Simple equations with three unknown quantities may be solved by imitating the operations in equations with two unknown; that is, by first eliminating one of the unknown from *two* of the equations, and then eliminating the *same* unknown from one of those two and the third; the results of the eliminations will be a pair of equations, with only two unknown quantities: for example,

A and B can perform a piece of work in 8 days; A and C in 9 days; and B and C in 10 days. In how many days can each alone perform it?

Suppose A, B, and C can do the xth part, the yth part, and the zth part respectively in *one* day: then by the question,

$8x+8y=1$ (*the whole*), $9x+9z=1$, $10y+10z=1$.

To equalise the coefficients of y, multiply the first by 5, and the third by 4; and we have

$$40x+40y=5$$

$$40x+40y=4$$

$\therefore$ Subtracting, $40x-40z=1$. Mult. this by 9, and the second equa. $9x+9z=1$ by 40:

$$360x-360z=9$$

$$360x+360z=40$$

Adding and subtracting, $720x=49$ and $720z=31$,

$$\therefore\;x=\frac{49}{720},\text{ and }z=\frac{31}{720},$$

$$\therefore\;10y+10z=10y+\frac{31}{72}=1\;\therefore\;10y=\frac{41}{72}\;\therefore\;y=\frac{41}{720}.$$

Hence A can do

$\frac{49}{720}$ of the whole in 1 day; B can do $\frac{41}{720}$; and C, $\frac{31}{720}$.

$\therefore$ A can do the whole in

$\frac{720}{49}$ days, B in $\frac{720}{41}$ days, and C in $\frac{720}{31}$ days.

That is, in $14\frac{34}{49}$ days, $17\frac{23}{41}$ days, and $23\frac{7}{31}$ days, respectively.

Equations which, like those considered in the preceding pages, imply distinct conditions, all existing together, are called *simultaneous equations:* the conditions of the foregoing problem are implied in the three simultaneous equations at the commencement of the solution.

Quadratic Equations with One Unknown Quantity.

A quadratic is an equation in which the *square* of the unknown quantity enters, and enters in such a way as to require a peculiar process for its removal. It is possible that even a simple equation may contain the square of the unknown; but then, in order that it may deserve the name of a *simple* equation, this square must be removable by transposition or division, or by some other of the operations common to simple equations in general; whereas a *quadratic*, properly so called, requires for its solution an additional principle; what this is, is now to be explained.

And first, you are to observe that the square of a quantity, consisting of but *one* term; that is, the square of a *monomial* is itself a monomial: thus the square of x is x^2; the square of ax is a^2x^2, and so on. But the square of a *binomial* consists of *three* terms; that is, it is a *trinomial:* thus, $(x+a)^2=x^2+2ax+a^2$, $(x-a)^2=x^2-2ax+a^2$, and so on.

The next thing to observe is, that if only the first and second terms of the square of a binomial, $x+a$, or $x-a$, be given, the *third* term of it can always be found; for you see that *this third term is nothing more than the square of half the coefficient in the second term.* Thus, if the first and second terms, viz., x^2+2ax, or x^2-2ax, be written down, we can complete the square to which these two terms belong, by dividing the coefficient of x, namely $2a$ or $-2a$, by 2; and then *adding* the square of the result. Thus, half the coefficient of x, in x^2+2ax, is a; the square of this is a^2; adding this $\therefore$ to the two terms proposed, we have $x^2+2ax+a^2$, for the complete square of $x+a$. In like manner, knowing the two terms x^2-2ax, we have only to add to them the square of half $-2a$, that is, $(-a)^2$, or a^2, to get $x^2-2ax+a^2$, the complete square of $(x-a)^2$. It follows from this, that whenever we have an expression of this form—namely, x^2+px, whether p be positive or negative, we can always add to it a term, easily found, that will make the expression a complete square; this term being no other than the square of $\frac{1}{2}p$; that is $\frac{1}{4}p^2$; and moreover, that the root of the square may at once be pronounced to be $x+\frac{1}{2}p$. In fact, common multiplication shows that $x^2+px+\frac{1}{4}p^2=(x+\frac{1}{2}p)^2$, whether p be + or —.

We shall give you an example or two of *completing the square*, as it is called—

1. Having the two terms x^2+6x, complete the square to which these two terms belong, and express the root of that square. Here half the coefficient of x is 3, $\therefore$ the wanting term is $3^2=9$, $\therefore$ the square is x^2+6x+9, and the root being x, with the half coefficient taken with its own sign added, we have $x^2+6x+9=(x+3)^2$.

2. Given x^2-8x to complete the square. Here the half coefficient is -4, of which the square is 16, $\therefore$ the complete square is $x^2-8x+16$, the root of which being x, together with the half coefficient, we have $x^2-8x+16=(x-4)^2$.

3. Given x^2-3x to complete the square. Here the half coefficient is $-\frac{3}{2}$, $\therefore$ its square is $\frac{9}{4}$, $\therefore$ the complete square is $x^2-3x+\frac{9}{4}=(x-\frac{3}{2})^2$.

You can now find no difficulty in rendering an expression of the form x^2+px, in which the coefficient of x^2 is *unity*, and that of x *anything*, a complete square. It is the accomplishment of this single matter that constitutes the whole mystery of quadratic equations.

To solve a Quadratic Equation.

RULE 1.—As in a simple equation dispose all the unknown terms on one side of the equation, and all the known on the other; the unknown side, when the terms are collected, will then consist of but *two* terms—the first containing x^2, and the second x simply.

2. If x^2, in the first term, have a coefficient other than unity, divide both sides by that coefficient; the equation will thus be reduced to the form $x^2+px=a$, where a is the *known* side, and p a known coefficient.

+3. Add the square of half this coefficient to both sides; *the unknown side will then be a complete square*, viz., the square of $x+\frac{1}{2}p$, the *sign* of p being the same as that in the above reduced form; and the *known side* will be $a+\frac{1}{4}p^2$.

4. Extract the square root of each side, and the result will be a *simple equation*, viz., $x+\frac{1}{2}p=\surd(a+\frac{1}{4}p^2)$.

You will observe, that although we *extract the square root of each side*, yet as far as the unknown side is concerned, there is, in reality, *no actual extraction performed*; you have nothing to do but to write down x *together with* half the coefficient of x, in the reduced form x^2+px; but, as respects the *known* side, when this is a *number*, the square root is actually found by common arithmetic, unless it is seen to be a surd, when we may leave it under the sign $\surd$, the extraction being merely *indicated*. You must remember, however, that when the square root of a number is actually *determined*, the *sign* to be prefixed to that root is *ambiguous*;* it may be either + or —.

1. Find the values of x in the equation $x^2+6x=55$.
Completing the square, $x^2+6x+9=64$.
Extracting the square root, $x+3=\pm 8$.
$$\therefore x=-3\pm 8=5, \text{ or } -11.$$
If 5 be put for x, the proposed equation is $25+30=55$.
If -11 be put for x, the equation is $121-66=55$.

2. $3x^2+2x-9=76$. Transposing, and then dividing by 3, the coefficient of x^2, $x^2+\frac{2}{3}x=\frac{85}{3}$.
Completing the square, $x^2+\frac{2}{3}x+\frac{1}{9}=\frac{85}{3}+\frac{1}{9}=\frac{256}{9}$.
Extracting the root, $x+\frac{1}{3}=\surd\frac{256}{9}=\pm\frac{16}{3}$.
$$\therefore x=-\tfrac{1}{3}\pm\tfrac{16}{3}=5 \text{ or } -\tfrac{17}{3}.$$

3. $3x^2-9x-4=80$. Transposing and dividing by 3, $x^2-3x=28$.
Completing the square, $x^2-3x+\frac{9}{4}=28+\frac{9}{4}=\frac{121}{4}$.
Extracting the root, $x-\frac{3}{2}=\surd\frac{121}{4}=\pm\frac{11}{2}$.
$$\therefore x=\tfrac{3}{2}\pm\tfrac{11}{2}=7, \text{ or } -4.$$

4. $\frac{x}{5}+\frac{5}{x}=5\frac{1}{5}$. Multiplying by $5x$, to clear fractions,
$$x^2+25=26x, \therefore x^2-26x=-25.$$
Completing the square, $x^2-26x+13^2=169-25=144$.
Extracting the root, $x-13=\surd 144=\pm 12$.
$$\therefore x=13\pm 12=25, \text{ or } 1.$$

5. $\frac{7}{x+1}+\frac{2}{x}=5$. Clearing fractions,
$$7x+2x+2=5x^2+5x.$$
Transposing, $5x^2-4x=2 \therefore x^2-\frac{4}{5}x=\frac{2}{5}$.
Completing the square, $x^2-\frac{4}{5}x+\frac{4}{25}=\frac{2}{5}+\frac{4}{25}=\frac{14}{25}$.
Extracting the root, $x-\frac{2}{5}=\surd\frac{14}{25}=\frac{1}{5}\surd 14$.
$$\therefore x=\frac{2}{5}+\frac{1}{5}\surd 14.$$

The square root of 14 is found, by arithmetic, to be 3·742; and $\frac{1}{5}$ of this is ·748; also since $\frac{2}{5}=\cdot4$, we have $x=\cdot4\pm\cdot748=1\cdot148$, or $-\cdot348$.

6. $\frac{x+11}{x}+\frac{9+4x}{x^2}=7$. Multiplying by x^2, to clear fractions, $x^2+11x+9+4x=7x^2 \therefore 6x^2-15x=9$.
Or dividing by 6, $x^2-\frac{5}{2}x=\frac{3}{2}$.
Completing the square, $x^2-\frac{5}{2}x+\frac{25}{16}=\frac{3}{2}+\frac{25}{16}=\frac{49}{16}$.
Extracting the root, $x-\frac{5}{4}=\surd\frac{49}{16}=\pm\frac{7}{4}$.
$$\therefore x=\frac{5}{4}\pm\frac{7}{4}=3, \text{ or } -\frac{1}{2}.$$

7. $\frac{2\surd x+2}{4+\surd x}=\frac{4-\surd x}{\surd x}$. Clearing fractions, and remembering that sum × dif. = dif. sqrs.
$$2x+2\surd x=16-x \therefore 2\surd x=16-3x.$$
Squaring, to remove the radical,
$$4x=256-96x+9x^2 \therefore 9x^2-100x=-256.$$
Dividing by 9, $x^2-\frac{100}{9}x=-\frac{256}{9}$.
Completing the square,
$$x^2-\frac{100}{9}x+\frac{2500}{81}=\frac{2500}{81}-\frac{256}{9}=\frac{196}{81}$$
Extracting the root, $x-\frac{50}{9}=\surd\frac{196}{81}=\pm\frac{14}{9}$
$$\therefore x=\frac{50}{9}\pm\frac{14}{9}=\frac{64}{9}, \text{ or } 4.$$

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* See *ante*, p. 466.

We might have completed the solution without clearing the radical from the equation $2x + 2\sqrt{x} = 16 - x$, thus: by transposing,

$$3x + 2\sqrt{x} = 16 \therefore x + \frac{2}{3}\sqrt{x} = \frac{16}{3}.$$

Now, since x in the first term is the *square* of $\sqrt{x}$ in the second, it is plain that we may treat this as a quadratic equation, regarding $\sqrt{x}$ as the unknown quantity $\therefore$ completing the square,

$$x + \frac{2}{3}\sqrt{x} + \frac{1}{9} = \frac{16}{3} + \frac{1}{9} = \frac{49}{9}$$

Extracting the root,

$$\sqrt{x} + \frac{1}{3} = \sqrt{\frac{49}{9}} = \pm\frac{7}{3}$$

$$\therefore \sqrt{x} = -\frac{1}{3} \pm \frac{7}{3} = 2, \text{ or } -\frac{8}{3}; \quad \therefore x = 4, \text{ or } \frac{64}{9}.$$

NOTE.—And in a similar way may any equation of the form $x^{2n} + px^n + a$, in which the exponent of x in the first term is double that of x in the second, be solved as a quadratic; the equation just considered is $x + \frac{2}{3}x^{\frac{1}{2}} = \frac{16}{3}$, where the exp. 1 of x, in the first term, is double $\frac{1}{2}$, the exp. of x in the second.

EXAMPLES FOR EXERCISE.

1. $x^2 - 4x = 45.$ 2. $x^2 + 8x = 33$
3. $x^2 - 8x = 9.$ 4. $3x^2 + 2x = 161.$
5. $7x^2 - 20x = 32.$ 6. $x^2 - x - 170 = 40.$
7. $5x^2 + 4x = 273.$ 8. $\frac{4x^2 - x}{3} = 11.$
9. $x + \frac{7x - 8}{x} = 9.$ 10. $5x^2 + 3 = 4x + 159.$
11. $6x + \frac{35 - 3x}{x} - 44 = 0.$ 12. $\frac{x-2}{x+2} + \frac{x+2}{x-2} = 14.$
13. $\frac{48}{x^2} + \frac{32}{x} = 11$ or $48x^{-2} + 32x^{-1} = 11.$*
14. $x^4 - 40x^2 + 39 = 0.$ 15. $x^2 + ax + b = 0.$
16. $\frac{x+a}{x-a} + \frac{x-a}{x+a} = b.$ 17. $\frac{x}{2} - \frac{1}{5}\sqrt{x} = 22\frac{1}{2}.$
18.† $(3x - 2)(1 - x) = 4.$ 19. $15x^2 - 48x + 45 = 0.$
20. $\sqrt{\frac{x-2}{x+2}} + \sqrt{\frac{x+2}{x-2}} = 4.$

The preceding method of solving a quadratic equation is that which is usually employed for the purpose; in certain cases, however, it is attended with inconvenience; in those cases, namely, where the completion of the square introduces numerical *fractions* into the work. We shall now show you an improvement upon the common rule; by adopting which, in practice, you will prevent the introduction of fractions.

The most general form of the quadratic equation, after the proper preliminary reductions, is $ax^2 + bx = c$. Let each side be multiplied by $4a$; the equation will then be $4a^2x^2 + 4abx = 4ac$, or, which is the same thing,

$$(2ax)^2 + 2b(2ax) = 4ac.$$

The first member of this, you will observe, consists of two terms—namely, the *square* of $2ax$ and $2b$ times $2ax$; so that, regarding $2ax$ as the unknown quantity, we shall *complete the square* by adding b^2; and the root of that square will be $2ax + b$: we therefore proceed thus—

Completing the square, $(2ax)^2 + 2b(2ax) + b^2 = 4ac + b^2$.

Extracting the root, $2ax + b = \sqrt{(4ac + b^2)}$.

Hence any quadratic may be reduced to a simple equation, of which this is the general model, without the introduction of any fractions. The steps from the proposed quadratic $ax^2 + bx = c$, to the simple equation $2ax + b = \sqrt{(4ac + b^2)}$, need not be gone through in each particular example; the reduced equation may be derived at once as follows:—

Rule II.—Take twice the coefficient of x^2 in the proposed quadratic; this will be the coefficient of x in the derived simple equation.

To the first term thus found, connect with its own sign the coefficient of x in that proposed: and the first member of the derived simple equation will be obtained.

To form the second member, multiply the second member of the proposed, by four times the coefficient of x^2, add the square of the next coefficient (that of x) to the result, and cover the whole with the radical sign.

This is the *rule* for the solution of any quadratic; it is a mere translation in words of the *formula* for the solution; namely,

Proposed quadratic.	*Derived simple equation.*
$ax^2 + bx = c.$	$2ax + b = \sqrt{(4ac + b^2)}.$

You may either commit the *rule* to memory, or keep the *formula* before your mind's eye; remembering that, as a *square* is always *plus*, b^2 is to take the *positive* sign, whether b, in the quadratic, is + or —. Thus (see Ex. 6, p. 473),

Quadratic. *Simple equation.*

$6x^2 - 15x = 9$

or

$2x^2 - 5x = 3$ . . . . $4x - 5 = \sqrt{(24 + 25)} = \sqrt{49} = \pm 7.$

Again, Ex. 5, p. 473,

$5x^2 - 4x = 2$ . . . . $10x - 4 = \sqrt{(40 + 16)} = \sqrt{56} = 2\sqrt{14}.$

Also Ex. 3, p. 473,

$x^2 - 3x = 28$ . . . . $2x - 3 = \sqrt{(112 + 9)} = \sqrt{121} = \pm 11.$

And from each of these simple equations you may deduce the value of x, at a single step: thus—

$$\text{Ex. 3. } x = \frac{3 \pm 11}{2} = 7, \text{ or } -4;$$

$$\text{Ex. 5. } x = \frac{4 + 2\sqrt{14}}{10} = \frac{2}{5} + \frac{1}{5}\sqrt{14};$$

$$\text{Ex. 6. } x = \frac{5 \pm 7}{4} = 3, \text{ or } -\frac{1}{2},$$

as at the pages referred to. All the examples worked by the common method you may re-solve in this way, which will be much the shorter and easier, whenever fractions are unavoidable by the former rule. We refrain from giving additional examples here, solely because we expect you to act upon this recommendation.

To solve a pair of Equations with Two Unknown Quantities, when one is a Simple Equation and the other a Quadratic.

The mode of proceeding in this case scarcely requires any formal directions. From the simple equation an expression for one of the unknowns may be easily found in terms of the other and *known* quantities, as at page 471; and this expression substituted in the quadratic will, of course, give an equation with only one unknown quantity; the other being *eliminated* in consequence of the substitution. Thus—

1. $\left.\begin{array}{c} x + 2y = 7 \\ x^2 + 3xy - y^2 = 23 \end{array}\right\}$ From the first equation $x = 7 - 2y$. Substituting this in the second, it becomes—

$$(7 - 2y)^2 + 3(7 - 2y)y - y^2 = 23; \text{ that is, } 49 - 28y + 4y^2 + 21y - 6y^2 - y^2 = 23, \text{ or } 3y^2 + 7y = 26$$

$$\therefore \text{ Rule II., } 6y + 7 = \sqrt{(312 + 49)} = \sqrt{361} = \pm 19$$

$$\therefore y = \frac{-7 \pm 19}{6} = 2, \text{ or } -\frac{13}{3} \therefore x = 7 - 2y = 3, \text{ or } 15\frac{2}{3};$$

so that the two values of x and y which satisfy the proposed pair of simultaneous equations are either—

$$\left.\begin{array}{l} x = 3 \\ y = 2 \end{array}\right\} \text{ or } \left\{\begin{array}{l} x = 15\frac{2}{3} \\ y = -4\frac{1}{3} \end{array}\right.$$

2. $\left.\begin{array}{c} 2x - 3y = 1 \\ 2x^2 + xy - 5y^2 = 20 \end{array}\right\}$ From the first equation $x = \frac{1 + 3y}{2}$.

Substituting this in the second, it becomes—

$$2\left(\frac{1 + 3y}{2}\right)^2 + \frac{1 + 3y}{2}y - 5y^2 = 20; \text{ that is, } \frac{1 + 6y + 9y^2}{2} + \frac{y + 3y^2}{2} - 5y^2 = 20,$$ or, multiplying by 2, and transposing,

$2y^2 + 7y = 39$

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* See *ante*, p. 465.

† The roots of this equation, and also those of equation 19, are imaginary; that is, the values of x in each will be found to involve the square root of a negative number; we may conclude, therefore, that it is not possible to satisfy either of these equations by any real value of x (see p. 465).

$$\therefore \text{ Rule II., } 4y+7=\surd(312+49)=\surd 361=\pm 19$$

$$\therefore y=\frac{-7\pm 19}{4}=3, \text{ or } -\frac{13}{2} \therefore x=\frac{1+3y}{2}=5, \text{ or } -9\tfrac{1}{4},$$

so that the values of x and y are either—

$$\left.\begin{matrix} x=5 \\ y=3 \end{matrix}\right\} \text{ or } \left\{\begin{matrix} x=-9\tfrac{1}{4} \\ y=-6\tfrac{1}{2} \end{matrix}\right.$$

Sometimes pairs of equations, coming under the present head, may be most conveniently solved, independently of *rule*, by exercising a little ingenuity. Here is an example :—

3. $\left.\begin{matrix} x+y=12 \\ x^2+y^2=74 \end{matrix}\right\}$ By squaring the first equation, and then subtracting the second, $2xy=70 \therefore 4xy=140$.

$$\begin{array}{llcl} \text{From the sq. of the first, viz.} & x^2+2xy+y^2 & = & 144 \\ \text{Subtract} & 4xy & = & 140 \\ \hline & \therefore x^2-2xy+y^2 & = & 4 \\ \text{Extracting the square root,} & x-y & = & \pm 2 \\ \text{But,} & x+y & = & 12 \\ \hline \end{array}$$

$$\therefore \text{ adding and subtracting, } 2x=12\pm 2, \text{ and } 2y=12\mp 2$$

$$\therefore x=7, \text{ or } 5; \text{ and } y=5, \text{ or } 7$$

that is, the values of x and y are either—

$$\left.\begin{matrix} x=7 \\ y=5 \end{matrix}\right\} \text{ or } \left\{\begin{matrix} y=7 \\ x=5 \end{matrix}\right.$$

4. In a similar way you may treat the equations $x-y=6$, $x^2+y^2=50$; and you will find that the values of x and y are either $x=7$, $y=1$; or $x=-1$, $y=-7$.

Questions requiring the Solution of Quadratic Equations.

1. The fore-wheel of a carriage makes 6 revolutions more than the hind-wheel in going 120 yards; but it is found that if the circumference of each wheel be increased 1 yard, it will make only 4 revolutions more than the hind-wheel in the same distance; required the circumference of each wheel.

Let $x=$ No. of yards in circ. of larger wheel,
$y=$ . . . . smaller wheel,

then $\frac{120}{x}=$ No. of revolutions of the former, and $\frac{120}{y}=$ No. of revolutions of latter; and by the question, $\frac{120}{x}=\frac{120}{y}-6 \therefore 20y=20x-xy \therefore xy=20x-20y$. . [A].

Also by the question, $\frac{120}{x+1}=\frac{120}{y+1}-4 \therefore 30(y+1$ $=(x+1)(29-y)$, that is, $30y+30=29x+29-xy-y \therefore xy=29x-31y-1$.

Substituting this expression for xy in the equation [A], we have—

$$29x-31y-1=20x-20y \therefore 9x=11y+1 \text{ . . . . [B].}$$

This equation [B] is a simple equation, and [A] is a quadratic, or an equation of *two dimensions*, because of the term xy.

From [A] we have $x=\frac{11y+1}{9}$; this substituted in [B] gives—

$$\frac{11y^2+y}{9}=\frac{220y+20}{9}-20y \therefore 11y^2+y=220y+20-180y$$

$$=40y+20 \therefore 11y^2-39y=20$$

$$\therefore \text{ Rule II., } 22y-39=\surd(880+39^2)=\surd 2401=\pm 49.$$

$$\therefore y=\frac{39+49}{22}=4, \text{ or } -\frac{5}{11} \therefore x=\frac{11y+1}{9}=5, \text{ or } -\frac{1}{9}.$$

From the nature of the question, it is plain that the *negative* values of x and y are inadmissible; they fulfil the *algebraical* conditions [A] and [B], as well as the *positive* values, for there is nothing in those conditions to exclude them; and it will often be found that the algebraic translation is free from the particular restrictions embodied in the question itself. The algebra furnishes *all* the values of the symbols, whether positive or negative, real or imaginary; and those of them are afterwards to be rejected which the restrictions of the question necessarily exclude. In the present case the only answers to the question are $x=5$, and $y=4$

2. A company at a tavern had £8 15*s*. to pay; but before the bill was settled, two of them left; in consequence of which, those who remained had each 10*s*. more to pay. How many persons were in company at first?

Let x represent the number: then $\frac{175}{x}$ is the share of each in shillings, and $\frac{175}{x-2}$ the portion each paid, after two had left: the difference is $\frac{175}{x-2}-\frac{175}{x}=10$ by the question, $\therefore$ clearing fractions,

$$175x-175x+350=10x(x-2);$$

or, dividing by 10, $35=x^2-2x$, or $x^2-2x=35$,

$$\therefore 2x-2=\surd(140+4)=\pm 12 \therefore x=\frac{2\pm 12}{2}$$

$$=1\pm 6=7, \text{ or } -5.$$

Consequently they were 7 persons at first.

3. What number is that which, being divided by the product of its two digits, the quotient is 2; and if 27 be added to it, the digits will be inverted or transposed?

Let x and y be the digits, then the number is $10x+y$; and when the digits are inverted or transposed, the number is $10y+x$.

By the question, $\frac{10x+y}{xy}=2 \therefore 10x+y=2xy$ . . . . [A]

and $10x+y+27=10y+x \therefore 9x+27=9y \therefore x+3=y$.

Substituting this value of y in [A], we have—

$$11x+3=2x^2+6x \therefore 2x^2-5x=3$$

$$\therefore 4x-5=\surd(24+25)=\surd 49=\pm 7.$$

$$x=\frac{5\pm 7}{4}=3, \text{ or } -\tfrac{1}{2} \therefore y=x+3=6, \text{ or } 2\tfrac{1}{2}.$$

The only admissible values are $x=3$, and $y=6$, $\therefore$ the number is 36.

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1. Divide the number 33 into two such parts, that their product may be 162.

2. Find two numbers whose difference is 9, and which are also such that their sum multiplied by the greater gives 266 for the product.

3. A company at a tavern had £7 4*s*. to pay; but two of them having left, the others had each 1*s*. more to pay than his fair share; how many persons were there at first?

4. A purse contains 24 coins of silver and copper; each copper coin is worth as many pence as there are silver coins, and each silver coin is worth as many pence as there are copper coins: and the whole is worth 18*s*. How many are there of each?

5. Two messengers, A and B, were dispatched to the same place, 90 miles distant. A, by riding one mile an hour more than B, arrives at his destination an hour before him. How many miles an hour did each travel?

6. A grocer sold 80lbs. of mace and 100lbs. of cloves for £65; but he sold 60lbs. more of cloves for £20 than he did of mace for £10. What was the price of 1lb. of each?

7. The product of two numbers is 240, and they are such, that if one of them be increased by 4, and the other diminished by 3, the product of the results is still 240. Find the numbers.

8. A and B set out at the same time for a place 150 miles distant. A travels 3 miles an hour faster than B, and arrives at the place 8_3 hours before him. How many miles did each travel per hour?

9. What number is that the sum of whose digits is 15, and if 31 be added to their product, the digits will be transposed?

10. There is a certain number consisting of two digits. The left-hand digit is equal to three times the right-hand one; and if 12 be subtracted from the number, the remainder will be equal to the square of the left-hand digit. What is the number?

A pair of simultaneous equations, with two unknown quantities, cannot in general be solved without the aid of higher equations, when the proposed ones are *both* quadratics. There are only two particular classes of such simultaneous equations to which general methods of solution, capable of being explained here, apply. These classes are called respectively *homogeneous equations*, and *symmetrical equations*. We shall very briefly consider them in the present article.

Homogeneous Quadratics.—A pair of quadratics is said to be a pair of homogeneous equations when each unknown term in both is of *two dimensions*; that is, when no term occurs without either the *square* of one of the unknown quantities, or the *product* of both; the presence of *two* unknown factors—whether equal or unequal—in a term, being the circumstance that renders that term of *two* dimensions. The following is a pair of homogeneous quadratics, each unknown term being of two dimensions:

$$\left.\begin{array}{l} x^2+xy=77 \\ xy-y^2=12 \end{array}\right\}$$

The following is the general way of solving such equations:—Put zy for x, and the equations become

$$z^2y^2+zy^2=77, \text{ and } zy^2-y^2=12.$$

From the first, $y^2=\frac{77}{z^2+z}$; from the second, $y^2=\frac{12}{z-1}$ . . . . [A]

$$\therefore \frac{77}{z^2+z}=\frac{12}{z-1} \therefore 12z^2-65z=-77.$$

By means of this contrivance; viz., the putting z times one of the unknown quantities for the other, we easily obtain a quadratic equation involving z only: solving this, we have

$$24z-65=\surd(-77\times48+65^2)=\surd529=\pm23$$

$$z=\frac{65\pm23}{24}=\frac{11}{3}, \text{ or } \frac{7}{4}.$$

Substituting each of these in equation [A], we have

$$y^2=\frac{12}{2\frac{2}{3}}, \text{ or } \frac{12}{\frac{3}{4}}; \text{ that is, } y^2=\frac{9}{2}, \text{ or } 16 \therefore y=\frac{3}{\surd2}=\frac{3}{2}\surd2, \text{ or } \pm4$$

$$\therefore x=zy=\frac{11}{2}\surd2, \text{ or } \pm7.$$

You see that each of the unknown quantities has *four* values; for $\surd2$ is either + or −: such is usually the case when the proposed equations are each of *two* dimensions. From the first equation above $y=\frac{77-x^2}{x}$; and if this be put for y in the second, the result, cleared of fractions, would evidently be an equation of the *fourth* degree; and it is proved, in the *Theory of Equations*, that an equation always has as many roots as there are units in the number which marks its degree: this is the reason that the values of x and y above are four in number.

Symmetrical Quadratics.—An equation is said to be symmetrical, in reference to the unknown quantities which enter it, when they so enter that they may be interchanged without producing any alteration in the equation: thus $x+y=a$; $x^2+3xy+y^2=b$; $2x^3-3x^2y^2+2y^3=c$, &c., are all symmetrical equations; because for every x you may put y, and for every y, x, without altering either of the equations. The following is a pair of symmetrical quadratics; namely,

$$\left.\begin{array}{r} x^2+y^2-x-y=18 \\ xy+x+y=19 \end{array}\right\}$$

The *general* method of dealing with such equations is as follows:—Put $u+v$ for x, and $u-v$ for y, and the equations become

$$\left.\begin{array}{r} (u+v)^2+(u-v)^2-2u=18 \\ (u+v)(u-v)+2u=19 \end{array}\right\} \text{or} \left\{\begin{array}{l} u^2+v^2-u=9 \\ u^2-v^2+2u=19 \end{array}\right. \quad . \ . \ . \ [A]$$

By adding $2u^2+u=28 \therefore 4u+1=\surd(224+1)=\pm15 \therefore$

$$u=\frac{-1\pm15}{4}=\frac{7}{2}, \text{ or } -4$$

$$\therefore [A], \frac{49}{4}+v^2-\frac{7}{2}=9 \therefore v^2=\frac{1}{4} \therefore v=\pm\frac{1}{2}$$

$$16+v^2+4=9 \therefore v^2=-11 \therefore v=\surd-11$$

$$\therefore x=u+v=\frac{7}{2}\pm\frac{1}{2}, \text{ or } -4\pm\surd-11; \text{ that is, } x=4, 3, \text{ or, } -4\pm\surd-11$$

$$y=u-v=\frac{7}{2}\pm\frac{1}{2}, \text{ or } -4\pm\surd-11; \text{ that is } y=3, 4, \text{ or } -4\pm\surd-11$$

The two classes of equations here exemplified will always yield to the foregoing methods: but particular cases of these, as also of other kinds of simultaneous equations for the solution of which no general rules can be given, may often be solved more easily by the exercise of a little tact and ingenuity: for instance, you would not think of applying the first method to such a pair of equations as $x^2+xy=9$, $y^2+xy=16$; because the slightest inspection of them would suggest to you the better way of *adding* them together, which would give you $x^2+2xy+y^2=25$, whence $x+y=\pm5$; and since the first is $(x+y)x=9$, you would have at once $\pm5x=9$; and thence $x=\pm\frac{9}{5} \therefore y=\pm5-x=\pm5\pm\frac{9}{5}=\frac{\pm25\pm9}{5}=\pm\frac{16}{5}$. Or you may proceed thus: the two equations give $x+y=\frac{9}{x}$, and $x+y=\frac{16}{y} \therefore 9y=16x \therefore y=\frac{16}{9}x \therefore$ by substitution in the given equations, you would have

$$\frac{25}{9}x^2=9, \text{ and } \frac{400}{81}x^2=16.$$

These equations give no *different* values for x, since the second is nothing but $\frac{16}{9}$ times the first: hence the only values of x are $x=\pm\frac{9}{5} \therefore y=\frac{16}{9}x=\pm\frac{16}{5}$.

CHAPTER IV.

ALGEBRA.—RATIO AND PROPORTION.

THE abstract number which arises from dividing one quantity by another of the same kind, is called the *ratio* of the former to the latter. *Ratio*, therefore, is only another name for the *quotient* of one quantity by another of the same kind:—the first quantity (the dividend) is called the *antecedent* of the ratio, and the second (the divisor) the *consequent:* these are the *terms* of the ratio. Thus, of the two quantities a, b, the ratio is $\frac{a}{b}$; or, which is the same thing, $a\div b$; but the little mark between the two dots, in the symbol for division, is usually omitted in expressing a ratio; so that the *ratio of a to b* would be denoted by $a:b$. Thus, the ratio of 8 to 4, that is $8:4$ is $\frac{8}{4}$ or 2; this ratio is of course the same as the ratio $4:2$, or the ratio $2:1$.

When there are four quantities such that the ratio of the first to the second is equal to the ratio of the third to the fourth, the four quantities are said to be in proportion: thus, if the four quantities, a, b, c, d, be such that $\frac{a}{b}=\frac{c}{d}$, or, which is the same thing, $a:b=c:d$, then a, b, c, d are in proportion; and we express this by *saying* that a is to b as c is to d; or by *writing* $a:b::c:d$. Of these four *terms*, the first and last (a and d) are called the *extremes*; and the intermediate terms (b and c) the *means*.

From what is here said, you see that two equal fractions may always be converted into a proportion, and a proportion into two equal fractions. All the properties of proportional quantities may therefore be derived from those of equal fractions: the following are the most useful :—

1. In four proportional numbers, $a : b :: c : d$, the product of the extremes is equal to the product of the means: for since $\frac{a}{b} = \frac{c}{d}$ $\therefore ad = bc$.

Conversely, if the product of one pair of factors be equal to the product of another pair, the four factors are in proportion: the factors of one product being the means, and those of the other the extremes: thus, if $ad = bc$, then $a : b :: c : d$; because from $ad = bc$, we get, by dividing each side by db, $\frac{a}{b} = \frac{c}{d}$.

3. If four quantities are proportional, they are proportional when taken *inversely*; that is, the second is to the first as the fourth to the third: thus, if $a : b :: c : d$, then $b : a :: d : c$; because from $\frac{a}{b} = \frac{c}{d}$ we get, by taking the reciprocals, $\frac{b}{a} = \frac{d}{c}$.

4. They are also proportional if taken alternately, provided the four quantities are all of the same kind; that is, the first is to the third, as the second to the fourth: thus, if $a : b :: c : d$ then $a : c :: b : d$ if the quantities are alike in kind; because from $\frac{a}{b} = \frac{c}{d}$ we get, by mutiplying by $\frac{b}{c}$, $\frac{a}{c} = \frac{b}{d}$. If a and c were quantities of different kinds, as for instance, pounds and yards, they could of course have no ratio; *ratio* being always an *abstract* number expressing how many times the antecedent is contained in the consequent.

5. *Three* quantities a, b, c are in proportion when they supply the equality $\frac{a}{b} = \frac{b}{c}$; that is, when $a : b :: b : c$. It is plain that three such quantities must always be of the same kind; the first of them is to the third as the square of the first to the square of the second; that is, $a : c :: a^2 : b^2$; because from $\frac{a}{b} = \frac{b}{c}$, we get, by multiplying by $\frac{b}{c}$, $\frac{a}{c} = \frac{b^2}{c^2} = \frac{a^2}{b^2}$.

6. It is further obvious that you may multiply or divide the first and second terms of a proportion by any number, and the third and fourth by any number, without disturbing the proportion; for $\frac{a}{b} = \frac{c}{d}$ is the same as $\frac{ma}{mb} = \frac{nc}{nd}$. And that the first and third may be multiplied or divided by any number, as also the second and fourth; for from $\frac{a}{b} = \frac{c}{d}$ we get $\frac{ma}{nb} = \frac{mc}{nd}$. Moreover, since $\frac{a}{b} = \frac{c}{d}$ leads to $\frac{a^m}{b^m} = \frac{c^m}{d^m}$, whatever be m, it follows that the same powers or roots of four proportional numbers are also in proportion.

7. The terms of two proportions, when they are numbers, may also be multiplied together; that is if—

$$a : b :: c : d$$
and $$e : f :: g : h$$
then $$ae : bf :: cg : dh$$

for this is only multiplying the equations $\frac{a}{b} = \frac{c}{d}$, $\frac{e}{f} = \frac{g}{h}$, together.

8. If $\left.\begin{array}{l} a : b :: c : d \\ \text{and } b : e :: d : f \end{array}\right\}$ then $a : e :: c : f$. This follows from multiplying together the equations $\frac{a}{b} = \frac{c}{d}$, $\frac{b}{e} = \frac{d}{f}$ which give $\frac{ab}{be} = \frac{cd}{df}$, or $\frac{a}{e} = \frac{c}{f}$.

By thus deducing different equations from the equal fractions or ratios $\frac{a}{b} = \frac{c}{d}$ an endless variety of sets of proportionals may be obtained; but the most general of these deductions is derived from the following principle, namely :—

If two fractions, $\frac{a}{b}$, $\frac{c}{d}$ are equal, then we may replace the terms a, b of the first by any expressions involving a and b that are *homogeneous* in reference to these quantities, provided we also replace the terms c, d by expressions involving c and d in the same manner. For instance—

$2a^2 - 3ab + b^2$ is homogeneous as respects a and b: each term being of two dimensions; so also is $5ab + 4a^2 - 2b^2$; we may therefore substitute these for a and b, provided we put the similar expressions $2c^2 - 3cd + d^2$ and $5cd + 4c^2 - 2d^2$ for c and d; that is, we may infer that because—

$$\frac{a}{d} = \frac{c}{d} \therefore \frac{2a^2 - 3ab + b^2}{5ab + 4a^2 - 2b^2} = \frac{2c^2 - 3cd + d^2}{5cd + 4c^2 - 2d^2}.$$

The reason of this is pretty obvious; two fractions cannot be *equal* unless one is convertible into the other by multiplying num. and den. of the former by some factor (m); so that in the above, c must be $= ma$ and $d = mb$. Now, if in the *second* of the changed fractions above, you put ma, mb for their equals c and d, you will see at once that that fraction will be nothing but the *first* fraction with its num. and den. multiplied by m^2. If the homogeneous expressions chosen for the terms of the first fraction had been of *three* dimensions, then after the substitution of ma, mb, for c, d in the similar terms of the second fraction, the result would have differed from the first fraction only by the num. and den. being multiplied by m^3, and so on, as is obvious. The particular case of this general theorem which is most frequently employed is this, namely :—

$$\frac{a}{b} = \frac{c}{d} \therefore \frac{a \pm mb}{a \pm nb} = \frac{c \pm md}{c \pm nd}.$$

or, from $a : b :: c : d$

$$\therefore a \pm mb : a \pm nb :: c \pm md : c \pm nd$$

where the values of m and n are arbitrary. In most applications they are chosen each equal to 1, or one equal to 0, and the other equal to 1. We need scarcely mention, that when any of the conditions of a question are expressed by a *proportion*, the product of the extremes equated to the product of the means converts the proportion into an *equation*.

ARITHMETICAL AND GEOMETRICAL PROGRESSIONS.

Arithmetical Progression.

An arithmetical progression is a row, or *series* of quantities, such, that each quantity, after the first, is merely the *preceding* quantity with some invariable value added to it, or subtracted from it: thus, 1, 3, 5, 7, &c., and 1, −1, −3, −5, &c., are arithmetical progressions; the terms after the first in the former are obtained, each from the preceding, by adding 2; and in the latter by subtracting 2. The constant quantity thus added or subtracted is called the common difference: it is usually denoted by d; and as d may stand for a quantity either positive or negative, the following will be the *general way* of expressing an arithmetical progression—viz.:

$$a,\ a + d,\ a + 2d,\ a + 3d,\ a + 4d,\ \cdots\ a + (n - 1)d;$$

where n stands for the number of the terms: the last term being evidently derived from the first by adding to it as many times d as there are terms *after* the first; that is $n - 1$ times d; so that calling the last term l, we have $l = a + (n - 1)d$. You thus see that the problem—Given the first term, the common difference and the number of terms, to find the last term, is one of very easy solution. We shall therefore proceed to the next, which is this :—

Given the first term (a) the com. diff. (d), and the number of terms (n) to find the sum of the series (S).

$$S=a+(a+d)+(a+2d)+(a+3d)+\ldots\ldots l,$$

or writing the terms in reverse order,

$$S=l+(l-d)+(l-2d)+(l-3d)+\ldots\ldots a,$$

$$\therefore \text{adding, } 2S=(a+l)+(a+l)+(a+l)+(a+l)+\ldots\ldots a+l.$$

And as there are n terms in each series for S, there must be n *equal* terms in this last series; that is to say,

$$2S=n\,(a+l)\ \therefore S=\tfrac{1}{2}n\,(a+l);\ \text{but } l=a+(n-1)d$$

$$\therefore S=\tfrac{1}{2}n\,\{2a+(n-1)d\}$$

The foregoing expressions for l and S will enable you to determine any two of the five quantities concerned when the other three are given; as shown in the following examples.

1. The first term of an arithmetical progression is 5, and the ninth term 37; what is the common difference of the terms?

For the particular numbers here given, the formula

$$l=a+(n-1)d$$

$$\text{is } 37=5+8d\ \therefore d=\frac{37-5}{8}=4.$$

Therefore, the series is 5, 9, 13, 17, 21, 25, 29, 33, 37.

2. Required $1+3+5+7+9+$ &c., to twenty terms.

$$S=\tfrac{1}{2}n\,\{2a+(n-1)d\}$$

$$=10\ \{2+19\times 2\}=400.$$

3. How many terms of the series $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+$ &c. (where $d=-\frac{1}{6}$) must be taken so that the sum may be *nothing*?

$$S=\frac{1}{2}n\left\{2a+(n-1)d\right\}$$

$$\therefore 0=\frac{1}{2}n\ \left\{1-\frac{1}{6}(n-1)\right\}$$

$$=\frac{1}{2}n\left(\frac{7}{6}-\frac{n}{6}\right)\therefore 0=7n-n^2\ \therefore 0=7-n$$

$$\therefore n=7$$

therefore, the series is $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+0-\frac{1}{6}-\frac{1}{3}-\frac{1}{2}$.

4. Insert four *arithmetical means* between 2 and 17. Here 2 is the first, and 17 the *sixth* term of an arithmetical progression, and we have to find the common difference.

$$l=a+(n-1)d$$

$$17=2+5d\ \therefore d=\frac{15}{5}=3$$

∴ the means are, 5, 8, 11, 14; which evidently fill up the gap between the first term and the sixth; the series being 2, 5, 8, 11, 14, 17.

5. How many terms of the series $9+7+5$, &c., will give 24?

$$S=\tfrac{1}{2}n\ \{\ 2a+(n-1)d\ \}$$

$$\therefore 24=\tfrac{1}{2}n\ \{\ 18-2(n-1)\ \}=10n-n^2$$

$$\therefore n^2-10n=-24\ \therefore 2n-10=\sqrt{100-96}=\pm 2$$

$$\therefore n=4 \text{ or } 6.$$

Consequently, whether we take *four* terms of the proposed series or *six*, the sum in either case will be 24; thus—

four terms $9+7+5+3=24.$
six terms $9+7+5+3+1-1=24.$

EXAMPLES FOR EXERCISE.

1. Find the sum of sixteen terms of the series—

$$1+2+3+4+\text{\&c.}$$

2. Find the sum of fourteen terms of the series—

$$4+3+2+1+0-1-2-\text{\&c.}$$

3. Sum the series $\frac{1}{2}+1\frac{1}{3}+2\frac{1}{6}+$ &c., to twenty terms.

4. Insert three arithmetical means between 2 and 6.

5. Insert five arithmetical means between $\frac{1}{2}$ and $-\frac{1}{2}$.

6. The first term is 5, and the fifteenth 47; what is the common difference?

7. How many terms of the series $12+11\frac{1}{2}+11+10\frac{1}{2}$, &c., must be taken to make 55?

8. The first term of an arithmetical progression is 7, the common difference $-\frac{2}{3}$: required the ninth term.

9. The sum of eight terms of an arithmetical series is 2, and the common difference $-1\frac{1}{2}$: required the first term.

10. The first term of an arithmetical series is $-3\frac{1}{3}$, the common difference $\frac{1}{3}$: required the sum of twenty-one terms.

Geometrical Progression.

In an arithmetical progression, the several terms give equal *differences*; in a geometrical progression they give equal *quotients*; and this constant quotient, arising from dividing any term by that which immediately precedes it, is called the *common ratio*. The general form for a geometrical series, continued as far as n terms, is

$$a,\ ar,\ ar^2,\ ar^3,\ ar^4,\ \ .\ \ .\ \ .\ \ .\ \ .\ ar^{n-1}$$

where a is the first term, and r the common ratio; the last, or nth term, being $l=ar^{n-1}$; which formula enables us readily to find any remote term (the nth term) when the first term and the common ratio are given.

By glancing at the above general form for a geometrical series, you will at once see that the product of any two terms is always equal to the product of any other two equidistant from them; and that if the number of terms be *odd*, the product of the *extreme* terms is always equal to the square of the middle or *mean* term. Thus:—the product of the first and fifth terms is equal to that of the second and fourth, as also to the square of the third: that is—

$$ar^4\times a=ar^3\times ar=(ar^2)^2=a^2r^4.$$

When the first term of a geometrical progression is given, it is plain that we only require to know the common ratio in order to write down the succeeding terms to any extent; and from this it follows that when the leading term and any remote term are given, we may always find the intervening terms, and thus fill up the gap: for instance, if a and ar^4 be given, to find the *three* intervening terms, we should divide ar^4 by a; and knowing—from the fact that there are *five* terms altogether—that the quotient would be the *fourth* power of the ratio, we should obtain the ratio itself by taking the fourth root of that quotient: by help of the ratio, the wanting terms may be easily supplied, as noticed above. These intervening terms are called *geometrical means* between the given extremes; if only *one* mean is to be inserted between two extremes, it is found by taking the square root of the product of the extremes. Suppose, for example, it were required to find a geometrical mean between 2 and 8: then, since the square root of 8 × 2, or 16, is 4, we know the mean to be this number; the progression being 2, 4, 8: if the proposed extremes had been −2, −8, we should have taken the *minus* root of 16, and have written the progression thus, −2, −4, −8: nevertheless, whether the given extremes be both *plus* or both *minus*, the square root of their product with *either* sign may be truly regarded as a *mean*, for the following are geometrical progressions, as well as those above, 2, −4, 8; −2, 4, −8: the common ratio *here* being −2; in the sets above it was + 2.

Again, suppose we had to insert *two* geometrical means between 3 and 81: then, as there are *four* terms altogether, the exponent $n-1$ above, is here 3, $\therefore 81\div 3=27=r^3\ \therefore r=3$. Hence the progression is 3, 9, 27, 81; the required *means* being 9 and 27.

To find the sum (S) *of n terms of a geometrical progression.*

$$\text{Since } S=a+ar+ar^2+ar^3+ar^4+\ .\ .\ .\ ar^{n-2}+ar^{n-1}$$

$$\therefore Sr=ar+ar^2+ar^3+ar^4+\ .\ .\ .\ ar^{n-2}+ar^{n-1}+ar^n.$$

Subtracting the first expression from the second—

$$Sr-S+ar^n-a\ \therefore S=\frac{ar^n-a}{r-1},\ \text{or}=a\,\frac{r^n-1}{r-1}.$$

We have already seen that the last term l is ar^{n-1}, so that $rl = ar^n$, ∴ the first expression for S may be written

$$S = \frac{rl-a}{r-1} \quad . \quad . \quad . \quad . \quad (1),$$

and this formula, expressed in words, furnishes the following rule to obtain the sum.

Rule.—Multiply the last term by the ratio; from the product subtract the first term, and divide the remainder by the ratio minus 1.

This rule applies, of course, whether the ratio be whole or fractional, positive or negative. When it is a *proper fraction*, the series is a *decreasing* one; and if carried on to an unlimited extent, must supply terms approaching nearer and nearer to zero or nothing, the terms at length differing from 0 by quantities too small to be assigned. Although we cannot *write down* such an infinite number of terms, yet we may affirm, with confidence, that they are all comprehended between these extreme limits, namely, the first term a, and 0; with these limits included; for including the 0 in the series cannot affect the value of the *sum*. In finding this sum we may therefore regard 0 as the *last* term of the infinite decreasing series; so that, for all such series, the formula above, l being 0, will be—

$$S = \frac{a}{1-r} \quad . \quad . \quad . \quad . \quad (2);$$

and from this you see—strange as the thing may appear—that it is *easier* to find the entire sum of an *infinite* geometrical decreasing series, than to find the sum of three or four of its leading terms; because the formula (1) involves more calculation than the formula (2). For distinction sake, it is usual to replace the symbol S, for the sum of a *finite* series, Σ when the series is infinite; that is, to write the formulæ (1) and (2) thus:—

$$S = \frac{rl-a}{r-1}, \quad \Sigma = \frac{a}{1-r}.$$

1. Required the sixth term and the sum of six terms of the series $1+2+4+$ &c., in which $a = 1$, $r = 2$, and $n = 6$, $l = ar^{n-1} = 2^5 = 32$; $S = \frac{rl-a}{r-1} = 2 \times 32 - 1 = 63$.

2. Required the sum of five terms of the series $1 - 4 = 16 - 64 +$ &c., in which $r = -4$.

$$\text{Here } l = 256; \; S = \frac{rl-a}{r-1} = \frac{-1024-1}{-5} = 205.$$

3. Required the sum to seven terms, as also to infinity of each of the series, $1 + \frac{1}{2} + \frac{1}{4} +$ &c., and $1 - \frac{1}{2} + \frac{1}{4} -$ &c. In the first series $r = \frac{1}{2}$, $n = 7$ ∴ $l = ar^{n-1} = (\frac{1}{2})^6 = \frac{1}{64}$.

$$\therefore S = \frac{rl-a}{r-1} = \left(\frac{1}{128} - 1\right) \div -\frac{1}{2} = \frac{127}{128} \times 2 = 1\frac{63}{64}.$$

This is the sum of seven terms. When the number of terms is infinite, that is when $n = \infty$,* the sum is—

$$\Sigma = \frac{a}{1-r} = 1 \div \tfrac{1}{2} = 2;$$

so that the sum of the *infinite* series is just double of the first term; and you see how much more easily the sum of an infinite number of terms is found than the sum of seven terms.

In the second series $r = -\frac{1}{2}$, ∴ for the seventh term

$$\text{we have } l = ar^{n-1} = \left(-\frac{1}{2}\right)^6 = \frac{1}{64} \therefore S = \frac{rl-a}{r-1}$$

$$= \left(\frac{1}{128} - 1\right) \div -\frac{3}{2} = \frac{127}{128} \times \frac{2}{3} = \frac{127}{192}$$

$$\text{And when } n = \infty, \; \Sigma = \frac{a}{1-r} = 1 \div \frac{3}{2} = 1 \times \frac{2}{3} = \frac{2}{3}.$$

4. Insert five geometrical means between 2 and $\frac{1}{32}$.

Here, when the means are supplied, there will be *seven* terms ∴ $n = 7$; also $a = 2$, and $l = \frac{1}{32}$; but $l = ar^{n-1}$

$$= 2r^6 \therefore 2r^6 = \frac{1}{32} \therefore r^6 = \frac{1}{64} \therefore r = \pm\frac{1}{2}.$$

Consequently the five means are ± 1, $\frac{1}{2}$, $\pm \frac{1}{4}$, $\frac{1}{8}$ $\pm \frac{1}{16}$, so that the complete seven terms are either $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$, or $2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32}$.

---

[*] This symbol stands for the word *infinite*.

EXAMPLES FOR EXERCISE.

1. Required the sum of five terms of the series $1 + 2^2 + 2^4 + 2^6 +$ &c.
2. Eight terms of $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} +$ &c.
3. Ten terms of $1 + 2 + 4 + 8 +$ &c.
4. What is the geometrical mean between 6 and 54?
5. Insert two geometrical means between 2 and 54.
6. Sum the series $\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} +$ &c., to affinity; that is, find the true fractional value of the recurring decimal ·1111. . .
7. The first term of a geometrical series is 3, the common ratio 5, and the last term 375: find the sum.
8. The first term of a geometrical series is $\frac{5}{6}$, and the common ratio $-\frac{1}{5}$: find the sum to infinity.
9. Insert three geometrical means between $\frac{1}{3}$ and $\frac{2}{3}$.
10. Required the sum of the infinite series,

$$1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \text{\&c.}$$

and thence deduce the values of Σ in the particular cases of $x = 2$, and $x = -2$.

Besides the two classes of progressions just treated of, there is a third kind which may here be briefly adverted to, namely, *harmonical progression*. This name is given to every series of which the terms are the *reciprocals* of those of an arithmetical progression. Thus the reciprocals of the terms of the arithmetical series $1 + 2 + 3 + \;.\;.\;.\;.\; n$ give the harmonical series $1 + \frac{1}{2} + \frac{1}{3} + \;.\;.\;.\;.\; \frac{1}{n}$. It has been found that musical strings, of equal thickness and tension, when their lengths are as the numbers 1, $\frac{1}{2}$, $\frac{1}{3}$, &c., are in harmony when sounded together; and hence the term *harmonical progression*.

There is no general formula for the *sum* of an harmonical series, but any number of harmonical *means* may be inserted between two given extremes by help of the corresponding problem in arithmetical progression. Thus:—

Insert four harmonical means between 2 and 4.

First insert four arithmetical means between $\frac{1}{2}$ and $\frac{1}{4}$, from the formula $l = a + (n-1)d$, where $a = \frac{1}{2}$, $n = 6$, and $l = \frac{1}{4}$: we thus get $\frac{1}{4} = \frac{1}{2} + 5d$; ∴ $d = (\frac{1}{4} - \frac{1}{2}) \div 5 = -\frac{1}{20}$ ∴ since $\frac{1}{2} = \frac{10}{20}$, by applying the constant difference $-\frac{1}{20}$, we have, for the four *arithmetical* means, the quantities $\frac{9}{20}$, $\frac{8}{20}$, $\frac{7}{20}$, $\frac{6}{20}$; that is $\frac{9}{20}$, $\frac{2}{5}$, $\frac{7}{20}$, $\frac{3}{10}$; and, taking the reciprocals of these, the required *harmonical* means are

$$2\tfrac{2}{9},\; 2\tfrac{1}{2},\; 2\tfrac{6}{7},\; 3\tfrac{1}{3}.$$

And in this way you will find that two harmonical means, between 1 and 2, are $1\frac{1}{5}$ and $1\frac{1}{2}$; and that four, between 2 and 12, are $2\frac{2}{5}$, 3, 4 and 6.

Questions in which Proportion and Progression are concerned.

1. Divide 18 into two parts so that the squares of those parts may be the one to the other, as 25 to 16.

Let x be one part, then $18 - x$ is the other, and by the question,

$$\begin{aligned} & x^2 : (18-x)^2 :: 25 : 16 \\ \therefore \text{(Theo. 6, p. 477)}\; & x : 18 - x :: 5 : 4 \\ \text{or (p. 477)}\; & x : 18 :: 5 : 9 \\ \therefore \text{(Theo. 6, p. 477)}\; & x : 2 :: 5 : 1 \end{aligned}$$

∴ (Theo. 1, p. 477) $x = 10$ ∴ the parts are 10 and 8.

2. A merchant sold some brandy for £39, and gained as much per cent. as the brandy cost him. What was the original price?

Let £x = the cost price, then the gain was £x per £100,

$$\therefore 100 : x :: x : \frac{x^2}{100},$$ the number of pounds gained.

But the number of pounds gained was also $39 - x$

$$\therefore \frac{x^2}{100} = 39 - x$$

$$* \therefore x^2 + 100x = 3900 \therefore x^2 + 100x + 50^2$$
$$= 3900 + 2500 = 6400.$$
$$\therefore x + 50 = \pm 80 \therefore x = 30 \text{ or } -130.$$

∴ the cost price was £30; the negative value of x, though satisfying the algebraical conditions, being excluded by the nature of the question.

3. The sum of three numbers in arithmetical progression is 9, and the sum of their cubes 153. What are the numbers?

Let $x - y$, x, $x + y$ represent the numbers; then

$$x - y + x + x + y = 3x = 9 \therefore x = 3$$
$$(x-y)^3 + x^3 + (x+y)^3 = 3x^3 + 6xy^2 = 153 \therefore y^2 = \frac{153 - 3x^3}{6x} = 4 \therefore y = \pm 2,$$ ∴ the numbers are either 1, 3, 5; or 5, 3, 1; being the same set, whether the common difference y be taken positively or negatively.

4. A person travels a distance of 198 miles as follows: namely, 30 miles the first day, 28 the second, 26 the third, and so on: in how many days will he have finished the journey?

Here we have an arithmetical progression, in which $a = 30$, $d = -2$, and $S = 193$, to find n, the number of terms or days.

By the formula, p. 478, $S = \frac{1}{2} n \left\{2a + (n-1)d\right\}$, that is,

$$198 = \frac{1}{2} n \left\{60 - (n-1)2\right\} = 31n - n^2 \therefore n^2 - 31n = -198.$$
$$\dagger \therefore 2n - 31 = \sqrt{(31^2 - 792)} = \sqrt{169} = \pm 13.$$
$$\therefore n = \frac{31 \pm 13}{2} = 22 \text{ or } 9.$$

It would seem from this result, that he might arrive at his journey's end either in 9 days, or in 22 days. But you must always remember, in solving particular questions by algebra, that the equation which embodies the conditions of the question is not necessarily *restricted* to those conditions. The solution will furnish *all* the values which can satisfy the *equation;* whereas, from the limited nature of the question, some of these may be inadmissible. You perceive, in a moment, that the question solved above is of wider meaning than that proposed: it includes the proposed and something more; for it is this—namely, The sum of an arithmetical series is 198, the first term is 30, and the common difference —2: find the number of terms. The answer is perfectly general; for the series consists of either the *nine* terms $30 + 28 + 26 + 24 + 22 + 20 + 18 + 16 + 14 = 198$, or of the twenty-two terms furnished by extending the series to the thirteen additional terms $+ 12 + 10 + 8 + 6 + 4 + 2 + 0 - 2 - 4 - 6 - 8 - 10 - 12$; for you see that these additional terms amount to 0; and therefore that the sum of *nine* terms, or of *twenty-two*, equally amounts to 198. It is plain, therefore, that the algebra would have been imperfect if $n = 22$ had not been contained in the result, though the limitation of the question excludes it: this limitation evidently is, that each day's travel *adds* to the distance from the starting-point: the traveller is presumed not to go back, so that a *subtractive* day's journey is forbidden.

5. The sum of three numbers in geometrical progression is 13; and the product of the mean and sum of the extremes is 30; required the numbers.

Let the numbers be represented by $\frac{x}{y}$, x, and xy; the condition that they are to form a geometrical progression being thus secured.

Then the other two conditions are as follow—namely,

$$\frac{x}{y} + x + xy = 13$$
$$\text{and } \left(\frac{x}{y} + xy\right) x = 30$$

From the first of these equations

$$\frac{x}{y} + xy = 13 - x \ldots\ldots (A).$$

From the second $\frac{x}{y} + xy = \frac{30}{x}$, $\therefore 13 - x = \frac{30}{x}$,

$$\therefore 13x - x^2 = 30, \text{ or } x^2 - 13x = -30,$$

Consequently, by Rule II. of quadratic equations, page 474,

$$2x - 13 = \sqrt{(-120 + 13^2)} = \sqrt{49} = \pm 7,$$
$$\therefore x = \frac{13 \pm 7}{2} = 10, \text{ or } 3.$$

Therefore, substituting 3 for x in equation (A), we have $\frac{3}{y} + 3y = 13 - 3 = 10 \therefore 3 + 3y^2 = 10y$, or $3y^2 - 10y = -3$;

$$\ddagger\ 6y - 10 = \sqrt{(-36 + 100)} = \sqrt{64} = \pm 8.$$
$$\therefore y = \frac{10 \pm 8}{6} = 3, \text{ or } \frac{1}{3}$$

Hence, putting 3 for x, and 3 for y, in $\frac{x}{y}$, x, xy, the numbers are 1, 3, 9; or putting 3 for x, and $\frac{1}{3}$ for y, the numbers are 9, 3, 1; forming the same geometrical progression written in reverse order. If, instead of $x = 3$, the other value $x = 10$ had been taken, the corresponding values of y would have been imaginary, or impossible: thus, putting 10 for x in (A), the equation becomes

$$\frac{10}{y} + 10y = 13 - 10 = 3 \therefore 10 + 10y^2 = 3y,$$
$$\text{or } 10y^2 - 3y = -10,$$
$$\therefore 20y - 3 = \sqrt{(-400 + 9)} = \sqrt{-391},$$
$$\therefore y = \frac{3 \pm \sqrt{-391}}{20}$$

These two values of y are imaginary; so that the three numbers, 1, 3, 9, determined above, are the only *real* numbers that fulfil the conditions of the question.

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1. Divide 49 into two parts such, that the greater increased by 6 may be to the less diminished by 11 as 9 to 2.

2. Two hundred stones are placed in a straight line at intervals of 2 feet, the first stone being 20 yards in advance of a basket: suppose a person starting from the basket collects the stones, and returns them one by one to the basket. How much ground does he go over?

3. The sum of four numbers in geometrical progression is equal to 1 added to the common ratio; and $\frac{1}{16}$ is the first term: required the numbers.

4. From two towns 165 miles apart, A and B set out to meet each other. A travels 1 mile the first day, 2 the second, 3 the third, and so on: B travels 20 miles the first day, 18 the second, 16 the third, and so on: in how many days will they meet?

5. The sum of the first and second of four numbers in geometrical progression is 15, and the sum of the third and fourth 60: what are the numbers?

6. The sum of three numbers in geometrical progression is 35; and the mean term is to the difference of the extremes as 2 to 3: what are the numbers?

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* See Quadratic Equations; *ante*, p. 473.

† *Ibid.*, Rule II.; *ante*, p. 474.

‡ See Quadratic Equations, Rule II.; *ante*, p. 474.

CHAPTER V.

ALGEBRA.—MISCELLANEOUS INVESTIGATIONS.

A FEW particulars connected with the elements of algebra remain to be noticed. These might have been introduced earlier, and are so introduced in most books on the subject. We have thought it expedient, however, with a view to facilitate the progress of the student, to postpone them to this place.

To Extract the Square Root of a Polynomial

When we set about extracting the square root of a quantity, whether a number or an algebraical expression, we proceed on the supposition that the proposed quantity is really a *square;* that is, that it is capable of being produced from two *equal* factors. You know, that in *numbers*, this is not always the case; 2, 7, 8, 10, &c., are not *squares*, and therefore, in strictness, have no *square roots;* by the aid of decimals, however, we can *approximate*, as it is called, to the square root of any of these numbers; that is, we can find a number such that the square of it shall differ from the number proposed by as small a decimal as we please; so, that for all *practical* purposes, the root, thus determined, may be used for that of the proposed number; since it is the *exact* root of a number differing from the proposed one, only by a decimal too minute to be appreciated. The difference here spoken of would be actually exhibited by the *remainder* left at the close of the operation; for, by subtracting this remainder from the number proposed, we should convert that number into a complete square; we always take care, in such arithmetical operations, to push the approximative process sufficiently far to render the correctional remainder of no moment, in reference to the practical inquiry in hand.

It is the same with algebraical expressions or *polynomials;* we proceed (by the rule presently to be given) on the supposition that the square root actually exists, carrying on the process, as in arithmetic, till the terms of the polynomial have all been used up. If a remainder be left, we conclude that the polynomial is not a square, and can assign the correction necessary to make it a square; for this correction, as in arithmetic, is the *remainder.* In algebra, it is usual to stop as soon as the proposed polynomial is exhausted; the object being, in general, more to ascertain whether the expression submitted to the process is a square or not, than to seek algebraical approximations to imperfect squares. Indeed, the term *algebraical* approximation would be meaningless, as you must see; for, in the absence of all *numerical* interpretation of our symbols, how could we speak of a set of symbolical expressions *approximating*, as to value, to any other such expression? When numerical values are given to the letters, we may with propriety speak of such approximations; but, even then, what are *approximations* for some numerical values, will be *departures* for others.

Let us now take a complete algebraic square, and try to discover by what process its root may be evolved. And first, let the square be that of a binomial; namely, the square of $a + b$, which is $a^2 + 2\,ab + b^2$.

$$\begin{array}{rl} & a^2 + 2ab + b^2\,(a + b \\ & a^2 \\ \cline{2-2} 2a + b\,] & 2ab + b^2 \\ & 2ab + b^2 \\ \cline{2-2} \end{array}$$

Now write down this expression as in the margin, and mark off a place for the root, to the right. The square root of the first term is a, which we know to be the first term of the sought root; and subtracting a^2 from the given expression, we get $2ab + b^2$ for a remainder. Consider this remainder as a *dividend*, and let us see whether, by help of the term a, already in the root, we cannot discover the leading term of a *divisor* for this dividend, suited to give b, the other term of the root. A glance at the dividend shows that the proper leading term is just double the partial root a, already obtained; we therefore write $2a$ for the first term of the divisor we are seeking to form; and as, in algebra, the *first term* of a divisor is all we want, to get a term of the quotient, we at once pronounce b to be the second term of the quotient or root; and, to *complete* the divisor, we put this b, not only in the quotient, but *also* in the divisor; and we find that b times the complete divisor, $2a + b$, equals the dividend; and thus the operation terminates, and the required root is discovered.

Guided by this easy process, let us now endeavour to evolve the root from the square of a trinomial; namely, from the square of $a + b + c$, which is $(a + b)^2 + 2(a + b)c + c^2$; this being evidently the square of $(a + b) + c$. Writing the polynomial in the usual way, it would be $a^2 + 2ab + b^2 + 2(a + b)\,c + c^2$; and the following method, for discovering the terms a, b, c, of the root, one after another, is in exact imitation of the operation above.

$$\begin{array}{rll} & a^2 + 2\,ab + b^2 + 2\,(a + b)\,c + c^2\,[a + b + c & \\ & a^2 & \\ \cline{2-2} 2a + b\,] & 2\,ab + b^2 & \\ & 2\,ab + b^2 & \\ \cline{2-2} 2(a + b) + c\,] & & 2\,(a + b)\,c + c^2 \\ & & 2\,(a + b)\,c + c^2 \\ \cline{3-3} \end{array}$$

Here the first term a of the root is found as before: we then get the remainder or *dividend* $2ab + b^2$; the double of the known portion a of the root furnishes the *trial* or *incomplete* divisor $2a$, by help of which the second term b, of the root is found; after which the divisor is completed by joining this b to the incomplete form: we then get a second remainder or *dividend* $2(a + b)\,c + c^2$, and take the double of what is now in the root for the corresponding incomplete divisor: this gives for quotient c the third term of the root; and the addition of this c completes the divisor, the multiplication of which by the quotient-term c, finishes the process, as no remainder is left.

In like manner, by applying the same proceeding to the square of a quadrinomial $a + b + c + d$, we should obtain the several terms of the root, one after another; the polynomial whose root is sought being written in the form

$$a^2 + (2\,a + b)\,b + (2(a + b) + c)\,c + d^2,$$

as a mere contemplation of this form, in connection with the above process, makes sufficiently evident. And as the square of *any* polynomial $(a + b + c + d + .\ . + l)^2$, may be written

$$a^2 + (2a + b)\,b + (2(a + b) + c)\,c + (2(a + b + c) + d)\,d + \ldots. + l^2,$$

the process is general: it is expressed in words as follows:—

RULE 1.—Arrange the terms of the proposed polynomial as if for division, marking off a place as for quotient.

2.—Find the square root of the leading term, put it in the quotient's place, and the square of it under the leading term, which will of course be equal to it. Draw a line, as if for subtraction, and bring down, under it, the next *two* terms of the polynomial: these will form a *dividend*, and a place, to the left of it, is now to be marked off, for a *divisor.*

3.—Put *twice* the root-term, just found, in the divisor's place: see how often this *incomplete* divisor is contained in the leading term of the dividend, and connect the quotient, with its proper sign, both to the root-term, and to the incomplete divisor: the divisor will thus be *completed.*

4.—Multiply the complete divisor by the root-term just found, subtract the product from the dividend, and to the remainder unite the two next terms of the polynomial; and a *second* dividend will be obtained.

Proceed with this as with the former, marking off a place for a new divisor, and putting in that place *twice* the root-quantity already found, for an *incomplete* divisor, by aid of which the third root term may be found, which, added to the incomplete divisor, renders it *complete*. And this uniform process is to be continued till all the terms of the polynomial have been brought down, as in the following examples :—

$$\begin{array}{rl}
1. & 9x^4-12x^3+16x^2-8x+4\;(3x^2-2x+2 \\
 & 9x^4 \\
6x^2-2x] & -12x^3+16x^2 \\
 & -12x^3+\;\;4x^2 \\
6x^2-4x+2] & \qquad\qquad 12x^2-8x+4 \\
 & \qquad\qquad 12x^2-8x+4
\end{array}$$

$$\begin{array}{rl}
2. & 4x^6-24x^5+60x^4-80x^3+60x^2-24x+4\,(2x^3-6x^2+6x-2 \\
 & 4x^6 \\
4x^3-6x^2] & -24x^5+60x^4 \\
 & -24x^5+36x^4 \\
4x^3-12x^2+6x] & \qquad 24x^4-80x^3+60x^2 \\
 & \qquad 24x^4-72x^3+36x^2 \\
4x^3-12x^2+12x-2] & \qquad\qquad -8x^3+24x^2-24x+4 \\
 & \qquad\qquad -8x^3+24x^2-24x+4.
\end{array}$$

$$\begin{array}{rl}
3. & a^{4m}-4a^{3m+n}+6a^{2m+2n}-4a^{m+3n}+a^{4n}\;(a^{2m}-2a^{m+n}+a^{2n}. \\
 & a^{4m} \\
2a^{2m}-2a^{m+n}] & -4a^{3m+n}+6a^{2m+2n} \\
 & -4a^{3m+n}+4a^{2m+2n} \\
2a^{2m}-4a^{m+n}+a^{2n}] & \qquad 2a^{2m+2n}-4a^{m+3n}+a^{4n} \\
 & \qquad 2a^{2m+2n}-4a^{m+3n}+a^{4n}
\end{array}$$

$$\begin{array}{rl}
4. & 4x^4-4x^3-3x^2+2x+4\;(2x^2-x-1 \\
 & 4x^4 \\
4x^2-x] & -4x^3-3x^2 \\
 & -4x^3+x^2 \\
4x^2-2x-1] & \quad -4x^2+2x+4 \\
 & \quad -4x^2+2x+1 \\
 & \qquad\qquad\qquad 3
\end{array}$$

In this example there is a *remainder*, after all the terms of the proposed polynomial have been used: we infer therefore that the polynomial is not a complete square; but that it would be made one, by subtracting 3 (the remainder) from it: the expression $2x^2-x-1$ is the complete square root of $4x^4-4x^3-3x^2+2x+1$: the polynomial proposed cannot be produced from two *equal* factors: it is the product of the unequal factors $(2x^2-x-1+\surd-3)\,(2x^2-x-1-\surd-3)$.

EXAMPLES FOR EXERCISE.

Extract the square root of each of the following expressions :—

1. $x^2+4bx+4b^2$. *Root*, $x+2b$.
2. $9x^4+12x^3+10x^2+4x+1$. *Root*, $3x^2+2x+1$.
3. $9x^4+12x^3+34x^2+20x+25$. *Root*, $3x^2+2x+5$.
4. $x^6+4x^5+2x^4+9x^2-4x+4$. *Root*, x^3+2x^2-x+2.
5. $x^6-4x^5+10x^4-4x^3-7x^2+24x+16$. *Root*, x^3-2x^2+3x+4.
6. $4x^4+6x^3+\frac{89}{4}x^2+15x+25$. *Root*, $2x^2+\frac{3}{2}x+5$.

In this last example you may, if you please, multiply the polynomial by 4, in order to get rid of the fraction: the square root of the result will, of course, be the square root of the given expression, multiplied by the square root of 4: that is, by 2; you must, therefore, remember to divide the root by 2.

The above general rule, for the square root of an algebraical expression, applies, of course, to *numbers*; and suggests the arithmetical operation given at page 449, in the Arithmetic. The first step in this operation, is to separate the figures of the number into periods, by marking off two figures, commencing at the units' place, then two to the left of these, and so on. This enables us to determine the *local value* of the leading figure of the root, and thence the number of integer-places in the complete root. Thus, taking for example the number 76807696, we see, by help of the periods, that the first figure of the root must be in the place of *thousands*, the *square* of it being so many millions, like the 76 in the first period. The detailed operation, as suggested by the Algebra, is therefore as follows :—

$$\begin{array}{rll}
 & 76\;80\;76\;96(8000+700+60+4= & \\
 & 64\;00\;00\;00 & 8700+60+4= \\
16000+700=16700) & 12\;80\;76\;96 & 8760+4=8764 \\
 & 11\;69\;00\;00 & \\
17400+60=17460) & \;\;1\;11\;76\;96 & \\
 & \;\;1\;04\;76\;00 & \\
17520+4=17524) & \qquad 7\;00\;96 & \\
 & \qquad 7\;00\;96 &
\end{array}$$

The above operation is the same, in principle, as that at page 449 of the Arithmetic; by a reference to which you will see that the more compact form, which the work assumes in the place referred to, arises merely from the suppression of the useless *ciphers* or *noughts*, and the postponement of the successive periods till they are actually wanted in the several dividends.

To Extract the Square Root of a Binomial, one of whose terms is Rational, and the other a Quadratic Surd.

It sometimes happens that expressions of the form

$\sqrt{(a+\sqrt{b})}$ occur in the results of algebraical problems: if they be left in this form, and then when a and b are interpreted, the operations of arithmetic be applied to them, you see that if b be irrational, we shall have to find the square root of b to a certain number of decimals, and then to extract the square root of $a+\sqrt{b}$, a quantity *necessarily* involving decimals. The square root of b, may be taken out of a table of square roots to several decimal places, when b consists of not more than four or five figures; but the square root of a number consisting of three or four figures, followed by five or six decimals, cannot be found to any degree of nicety by existing tables; so that, in such a case, the arithmetical operation must be executed. As we have told you before, algebraists seek such a form for their results as will give arithmeticians the least trouble in the numerical computation of them; and this is one reason why they have sought to convert $\sqrt{(a+\sqrt{b})}$, into the simpler form $\sqrt{a'}+\sqrt{b'}$.

In order to show how this is done, a theorem or two respecting binomial surds must first be established.

1. The square root of a quantity cannot be *partly* rational and *partly* a quadratic surd: that is to say, the condition $\sqrt{p}=q+\sqrt{r}$ is impossible, provided p and r are not themselves squares.

For, assuming this equation to be possible, we should have, by squaring each side $p=q^2+2q\sqrt{r}+r$, so that $\sqrt{r}=\frac{p-q^2-r}{2q}$, a rational quantity, which is contrary to the supposition, as it shows that r must be the *square* of the second member of this equation.

2. In every equation of the form $a+\sqrt{b}=x+\sqrt{y}$, where a and x are *rational* quantities, and $\sqrt{b}$ *irrational* or *surd*, a must be equal to x, and therefore b will equal y. For if a and x had any difference, then by transposing the a, we should have the square root of a quantity, not itself a square, expressed either by a rational quantity, which is absurd, or by a quantity partly rational and partly irrational, which has been shown above to be impossible. It follows that $\sqrt{y}$ must be irrational as well as $\sqrt{b}$.

3. If $\sqrt{(a+\sqrt{b})}=x+y$: then must $\sqrt{(a-\sqrt{b})}=x-y$; where $\sqrt{b}$ is a quadratic surd, and x, y, one or both, also quadratic surds.

For by squaring the first equation $a+\sqrt{b}=x^2+2xy+y^2$, where x^2+y^2 is necessarily *rational*, and $\therefore 2xy$ irrational (by 2 above) being equal to $\sqrt{b}$: that is (by 2),

$$a=x^2+y^2,\ \sqrt{b}=2xy,\ \therefore a-\sqrt{b}=x^2-2xy+y^2,\ \therefore \sqrt{(a-\sqrt{b})}=x-y.$$

By help of this latter principle, combined with theo. 2 above, the extraction of the square root of a binomial surd consisting of a rational term, and of a quadratic irrational term, may be effected, as in the following examples:—

1. Extract the square root of $7+2\sqrt{10}$.

$$\text{Put } \sqrt{(7+2\sqrt{10})}=x+y,\ \therefore \sqrt{(7-2\sqrt{10})}=x-y;$$

$$\therefore \text{taking the product, } \sqrt{(49-40)}=x^2-y^2,\ \therefore 3=x^2-y^2.$$

$$\text{But } 7+2\sqrt{10}=x^2+2xy+y^2,\ \therefore 7=x^2+y^2,\ \therefore 10=2x^2,\ 4=2y^2$$

$$\therefore x=\sqrt{5},\ y=\sqrt{2},\ \therefore \sqrt{(7+2\sqrt{10})}=\sqrt{5}+\sqrt{2}.$$

You will at once see that the change of $\sqrt{(7+2\sqrt{10})}$ into $\sqrt{5}+\sqrt{2}$ facilitates the arithmetical computation: a table of square roots gives us $\sqrt{5}=2{\cdot}2360680$, and $\sqrt{2}=1{\cdot}4142136$; so that the root sought is $3{\cdot}6502816$. If the unchanged form be used, we should have, from the table, $\sqrt{10}=3{\cdot}1622777$; twice this increased by 7 is $13{\cdot}3245554$, and the square root of this we should have to find by actual extraction, on account of the necessarily limited extent of the tables.

2. Find the square root of $10-\sqrt{96}$.

$$\text{Put } \sqrt{(10+\sqrt{96})}=x+y,\ \therefore \sqrt{(10-\sqrt{96})}=x-y.$$

$$\left.\begin{aligned}\therefore \sqrt{(100-96)}=x^2-y^2=2.\\ \text{Also } x^2+2xy+y^2=10+\sqrt{96},\ \therefore x^2+y^2=10\end{aligned}\right\}$$

$$\because x^2=6,\ y^2=4,\ \therefore \sqrt{(10-\sqrt{96})}=\sqrt{6}-2.$$

3. Find general expressions for the square roots of $a+\sqrt{b}$, and $a-\sqrt{b}$.

Putting $x+y$, and $x-y$, for the required roots, we have—

$$\left.\begin{aligned}a+\sqrt{b}=x^2+2xy+y^2\\ a-\sqrt{b}=x^2-2xy+y^2\end{aligned}\right\}\ \therefore x^2+y^2=a$$

$$\text{Also } x^2-y^2=\sqrt{(a^2-b)}$$

By adding and subtracting

$$x^2=\frac{a+\sqrt{(a^2-b)}}{2},\ y^2=\frac{a-\sqrt{(a^2-b)}}{2}$$

therefore $x+y$, $x-y$, that is $\sqrt{(a+\sqrt{b})}$ and $\sqrt{(a-\sqrt{b})}$ are respectively

$$\sqrt{(a+\sqrt{b})}=\sqrt{\frac{a+\sqrt{(a^2-b)}}{2}}+\sqrt{\frac{a-\sqrt{(a^2-b)}}{2}}$$

$$\sqrt{(a-\sqrt{b})}=\sqrt{\frac{a+\sqrt{(a^2-b)}}{2}}-\sqrt{\frac{a-\sqrt{(a^2-b)}}{2}}$$

These general expressions show that, whenever a^2-b is not *a square*, nothing is gained by changing $\sqrt{(a+\sqrt{b})}$ into the new form, but in fact something lost in the way of simplification: the changed form is less complex than the original only when a^2-b is a square: it is prudent, therefore, to ascertain whether or not this be the case, before entering upon the proposed transformation; and to leave the form unchanged if a^2-b be not a square.

EXAMPLES FOR EXERCISE.

1. $\sqrt{(19+8\sqrt{3})}=4+\sqrt{3}$
2. $\sqrt{(12-\sqrt{140})}=\sqrt{7}-\sqrt{5}$
3. $\sqrt{(7-2\sqrt{10})}=\sqrt{5}-\sqrt{2}$
4. $\sqrt{(8+\sqrt{39})}=\frac{1}{2}(\sqrt{26}+\sqrt{6})$
5. $\sqrt{(76-32\sqrt{3})}=8-2\sqrt{3}$
6. $\sqrt{(31+12\sqrt{-5})}=6+\sqrt{-5}$.

On Multipliers which render Binomial Surds rational.

It has already been noticed (page 466), that it is in general inconvenient to have the result of an algebraic operation in the form of a fraction with a surd denominator; and when the surd so occurring is *monomial*, the means of removing it have been explained. We are now to show how, in like manner, a fraction having a binomial surd $\sqrt{a}+\sqrt{b}$ or $\sqrt{a}-\sqrt{b}$ for denominator, may be converted into an equivalent fraction with a rational denominator; in other words, to explain the method of *rationalising* binomial surds of the above form. This is very easy, the rationalising multiplier being at once suggested from the binomial surd itself: if it be $\sqrt{a}+\sqrt{b}$, the multiplier is obviously $\sqrt{a}-\sqrt{b}$; and if it be $\sqrt{a}-\sqrt{b}$, the multiplier is $\sqrt{a}+\sqrt{b}$; and you thus have a useful application of the principle that the *sum* multiplied by the *difference* of two quantities gives the difference of their squares. Suppose, for example, our fraction is $\frac{3}{\sqrt{5}+\sqrt{7}}$, a form which is inconvenient for computation, because after getting $\sqrt{5}+\sqrt{7}$ we should have the troublesome operation of dividing 3 by a number consisting of many figures. But by multiplying numerator and denominator by $\sqrt{5}-\sqrt{7}$, we change the fraction into $\frac{3(\sqrt{5}-\sqrt{7})}{5-7}=\frac{3}{2}(\sqrt{7}-\sqrt{5})$, by which change the long division spoken of is avoided.

As a second example, let $\frac{\sqrt{3}+1}{5-\sqrt{3}}$ be proposed. The rationalising multiplier here is $5+\sqrt{3}$, and the changed fraction is $\frac{(\sqrt{3}+1)(5+\sqrt{3})}{25-3}=\frac{4+3\sqrt{3}}{11}$.

Sometimes the binomial surd to be rationalised is of the form $\sqrt[3]{a}\pm\sqrt[3]{b}$: in this case the suitable multiplier will be *trinomial*: it will consist of the *squares* of both the irrational terms, and of their product with changed sign, as actual multiplication will show; for

$$(\sqrt[3]{a}\pm\sqrt[3]{b})(\sqrt[3]{a^2}\mp\sqrt[3]{ab}+\sqrt[3]{b^2})=a\pm b.$$

The rationalising multiplier is therefore easily remembered: the following is an example of its application:—

Convert the fraction $\frac{5}{\sqrt[3]{5}-\sqrt[3]{2}}$ into an equivalent one with a rational denominator.

$$\frac{5}{\sqrt[3]{5}-\sqrt[3]{2}}=\frac{5(\sqrt[3]{25}+\sqrt[3]{10}+\sqrt[3]{4})}{5-2}=\frac{5}{3}(\sqrt[3]{25}+\sqrt[3]{10}+\sqrt[3]{4}).$$

And this form, though involving *three* cube roots, is of easier calculation than the original, if tables of cube roots are used; because there is no long-division operation.

EXAMPLES FOR EXERCISE.

Prove that—

1. $\frac{3}{2\sqrt{7}-3\sqrt{2}}=\frac{3}{10}(2\sqrt{7}+3\sqrt{2}).$
2. $\frac{2+\sqrt{3}}{3+\sqrt{3}}=\frac{3+\sqrt{3}}{6}$
3. $\frac{\sqrt{6}}{\sqrt{8}+\sqrt{3}}=\frac{4\sqrt{3}-3\sqrt{2}}{5}.$
4. $\frac{2}{\sqrt[3]{3}-\sqrt[3]{2}}=2(\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}).$
5. $\frac{5}{\sqrt[3]{7}-\sqrt[3]{5}}=\frac{5}{2}(\sqrt[3]{49}+\sqrt[3]{35}+\sqrt[3]{25}).$
6. $\frac{\sqrt{12}-\sqrt{10}}{\sqrt{6}+\sqrt{5}}=11\sqrt{2}-4\sqrt{15}.$

To Extract the Cube Root of a Compound Quantity.

In order to discover the means of arriving at the cube root of a polynomial, we may proceed in imitation of the course adopted for the determination of the square root. Thus, to begin with the simplest case, let us take the cube of $a+b$; that is, the expression $a^3+3a^2b+3ab^2+b^3$; and, availing ourselves of our previous knowledge of the cube root of this expression, let us inquire by what steps it may be evolved.

We see that the first term a of the root is at once obtained from the leading term of the polynomial; we thus have the step

$$\begin{array}{l} a^3+3a^2b+3ab^2+b^3(a \\ \underline{a^3} \\ \quad 3a^2b+3ab^2+b^3; \end{array}$$

and regarding the remainder, here exhibited, as a dividend, it remains to find a divisor of it, such that the quotient may be b. It is plain that three times the square of the root term a, just found, taken as a *trial* or incomplete divisor, suffices to suggest the second term b of the root; and we see, moreover, that if three times the product of the *two* root terms, a and b, as also the square of b, be added to the trial divisor $3a^2$, that the complete divisor, corresponding to the quotient b, will be obtained; hence the finished process is as follows:—

$$\begin{array}{rl} & a^3+3a^2b+3ab^2+b^3(a+b \\ & \underline{a^3} \\ 3a^2+3ab+b^2) & 3a^2b+3ab^2+b^3 \\ & \underline{3a^2b+3ab^2+b^3.} \end{array}$$

If the root consist of three terms, $a+b+c$, that is, if the polynomial be $(a+b+c)^3$, or $(a+b)^3+3(a+b)^2c+3(a+b)c^2+c^3$, then the portion $(a+b)^3$ of this polynomial may be exhausted, as above, and the first two terms $a+b$ of the root thus found. And, just as in the former case, b was derived from a, so here it is plain that c may be derived from $a+b$. From these considerations the following rule is suggested:—

RULE.—Arrange the terms according to the powers of one of the letters, as in the operation for the square root. Put the cube root of the leading term in the quotient's place, subtract the cube of it from the polynomial, bringing down the next *three* terms for a dividend, to the left of which mark off a place for the divisor. In this place put *three times* the square of the root term just found; this will be the *trial divisor*, and the quotient it suggests will be the second term of the root.

To *three times* the product of this new term and the preceding, add the square of the new term, and connect the result to the trial divisor; the whole will be the *complete divisor.*

Multiply the complete divisor by the new term, subtract the product from the dividend, and annex three more terms of the polynomial to the remainder; the whole will be the second dividend.

For the corresponding trial divisor put *three times* the square of the root-quantity now found, in the divisor's place; and with this trial divisor find the third term of the root; and then complete the divisor, by adding to what is already in the divisor's place *three times* the product of the new term and the preceding part of the root, and also the square of the new term; proceed then as in division, adding three new terms of the polynomial to the remainder; and so on, till all the terms of the polynomial have been brought down.

You will perceive that the principal part of the work consists in forming the successive divisors, each of which is made up of *three* portions—namely, thrice the square of the part of the root previously obtained, thrice the product of this part and the new term, and the square of that new term. The first of these portions is what we have called the *trial divisor;* but you will not fail to notice, that, as in the operation for the square root, the *leading term* of the *first* divisor is always sufficient to make known any subsequent term of the root; so that after the first divisor is obtained, every new term of the root may be discovered at once, without the aid of any special trial divisor; and therefore every subsequent divisor may be inserted at once, in its proper place, and in its complete form.

The following is an example of the operation:—

$$\begin{array}{ll} & x^6-6x^5+15x^4-20x^3+15x^2-6x+1\,(x^2-2x+1 \\ & \underline{x^6} \\ 3x^4-6x^3+4x^2) & -6x^5+15x^4-20x^3 \\ & \underline{-6x^5+12x^4-8x^3} \\ 3x^4-12x^3+15x^2-6x+1) & 3x^4-12x^3+15x^2-6x+1 \\ \text{This is made up of} & \underline{3x^4-12x^3+15x^2-6x+1} \\ 3(x^2-2x)^2+3(x^2-2x)+1 & \end{array}$$

As a second example the following may be taken:—

$$\begin{array}{ll} & 27x^6-54x^5+63x^4-44x^3+21x^2-6x+1\,(3x^2-2x+1 \\ & \underline{27x^6} \\ 27x^4-18x^3+4x^2) & -54x^5+63x^4-44x^3 \\ & \underline{-54x^5+36x^4-8x^3} \\ 27x^4-36x^3+21x^2-6x+1) & 27x^4-36x^3+21x^2-6x+1 \\ \text{This is made up of} & \underline{27x^4-36x^3+21x^2-6x+1} \\ 3(3x^2-2x)^2+3(3x^2-2x)+1 & \end{array}$$

The operation, as just conducted, is tolerably short; but there is some work, in finding the several divisors, which does not appear to the eye in looking at the finished form. We shall explain another mode of proceeding, much more complicated in appearance, but very simple indeed in the performance. It presents to the eye three columns of work, as below; they are formed thus:—

Write down 0, 0, and the given polynomial, as the leading quantities of three vertical columns of work; and then proceed to fill up the column thus:—Put the first term of the root not only in the quotient's place, but also under the first 0; to which, for uniformity sake, you may consider it to be *added*; multiply the sum by the same root-term, and add the product to the next 0; multiply the sum still by the same root-term, and *subtract* the product from the polynomial, bringing down, however, only *three* terms for remainder.

Return to the first column, and add the same term anew, multiplying the sum and carrying the product to the second column as before, but not extending the work to the third column. Return *again* to the first column, repeating the former operation, and there stop. With the last result in the second column, taken as a trial divisor, find the second term of the root; and go over with *this* exactly the same process, extending the several columns, by successive additions, till the polynomial is exhausted. As an inspection of the operation will convey a clearer idea of the several steps than words will do, we here give the work of the last example in the form proposed.

$$
\begin{array}[t]{l}
0\\
3x^2\\ \hline
3x^2\\
3x^2\\ \hline
6x^2\\
3x^2\\ \hline
9x^2\\
\quad -2x\\ \hline
9x^2-2x\\
\quad -2x\\ \hline
9x^2-4x\\
\quad -2x\\ \hline
9x^2-6x\\
\qquad +1\\ \hline
9x^2-6x+1
\end{array}
\qquad\qquad
\begin{array}[t]{l}
0\\
9x^4\\ \hline
9x^4\\
18x^4\\ \hline
27x^4\\
\quad -18x^3+4x^2\\ \hline
27x^4-18x^3+4x^2\\
\qquad -18x^3+8x^2\\ \hline
27x^4-36x^3+12x^2\\
\qquad\qquad\qquad 9x^2-6x+1\\ \hline
27x^4-26x^3+21x^2-6x+1
\end{array}
\qquad
\begin{array}[t]{l}
27x^6-54x^5+63x^4-44x^3+21x^2-6x+1\,(3x^2-2x+1.\\
27x^6\\ \hline
\quad -54x^5+63x^4-44x^3\\
\quad -54x^5+36x^4-\ \,8x^3\\ \hline
\qquad 27x^4-36x^3+21x^2-6x+1\\
\qquad 27x^4-36x^3+21x^2-6x+1\\ \hline
\end{array}
$$

NOTE.—It would be very easy to contrive means of abridging the work here exhibited in the fullest detail: but a learner will more clearly see the entire texture of the process by having every item of it thus put before him.

By the very same process, the fifth root, the seventh root, and indeed any root whatever of a polynomial, may be extracted: we have only to form, as above, as many columns of work as are sufficient to mark the number of the root; that is, five columns for the fifth root, seven for the seventh root, and so on; to construct the first column by that number of additions of the first root-term, of the second root-term, and so on; and to carry on the formation of the other columns as above. Whether you adopt this method of arranging the work of extracting the cube root of a *polynomial* or not, it is the arrangement we would recommend you always to observe, in extracting the cube root of a *number*; as it precludes the necessity of retaining in the memory a somewhat intricate rule: nothing can be more easy to remember than the very simple series of recurring operations of which the several steps of the following process are made up: the close of each of these steps we have marked by an Egyptian figure, in order that you may the more distinctly see where one step of the operation ends, and another begins. We call that a *step* which comprises the work between the determination of one root-figure and that of the immediately succeeding figure.

1. Required the cube root of 189119224.

$$
\begin{array}[t]{r}
0\\ 5\\ \hline 5\\ 5\\ \hline 10\\ 5\\ \rule[0.5ex]{1.5em}{0.4pt}\;1\\ 15\\ 7\\ \hline 157\\ 7\\ \hline 164\\ 7\\ \rule[0.5ex]{1.5em}{0.4pt}\;2\\ 171\\ 4\\ \hline 1714
\end{array}
\qquad\qquad
\begin{array}[t]{r}
0\\ 25\\ \hline 25\\ 50\\ \rule[0.5ex]{1.5em}{0.4pt}\;1\\ 75\\ 1099\\ \hline 8599\\ 1148\\ \hline 9747\\ 6856\\ \hline 981556
\end{array}
\qquad\qquad
\begin{array}[t]{r}
189{,}119{,}224(574\\ 125\\ \rule[0.5ex]{4em}{0.4pt}\;1\\ 64119\\ 60193\\ \rule[0.5ex]{4em}{0.4pt}\;2\\ 3926224\\ 3926224\\ \hline
\end{array}
$$

2. Required the cube root of 469,640,998,917.

$$
\begin{array}[t]{r}
0\\ 7\\ \hline 7\\ 7\\ \hline 14\\ 7\\ \rule[0.5ex]{1.5em}{0.4pt}\;1\\ 21\\ 7\\ \hline 217\\ 7\\ \hline 224\\ 7\\ \rule[0.5ex]{1.5em}{0.4pt}\;2\\ 231\\ 7\\ \hline 2317\\ 7\\ \hline 2324\\ 7\\ \rule[0.5ex]{1.5em}{0.4pt}\;3\\ 2331\\ 3\\ \hline 23313
\end{array}
\qquad\qquad
\begin{array}[t]{r}
0\\ 49\\ \hline 49\\ 98\\ \rule[0.5ex]{1.5em}{0.4pt}\;1\\ 147\\ 1519\\ \hline 16219\\ 1568\\ \rule[0.5ex]{3em}{0.4pt}\;2\\ 17787\\ 16219\\ \hline 1794919\\ 16268\\ \rule[0.5ex]{4em}{0.4pt}\;3\\ 1811187\\ 69939\\ \hline 181188639
\end{array}
\qquad\qquad
\begin{array}[t]{r}
469{,}640{,}998{,}917(7773\\ 343\\ \rule[0.5ex]{4em}{0.4pt}\;1\\ 126640\\ 113533\\ \rule[0.5ex]{4em}{0.4pt}\;2\\ 13107998\\ 12564433\\ \rule[0.5ex]{4em}{0.4pt}\;3\\ 543565917\\ 543565917\\ \hline
\end{array}
$$

3. Required the cube root of 3 to five or six places of decimals.

0	0	3(1·442249 . . . .
1	1	1
—	—	— 1
1	1	2
1	2	1·744
—	— 1	—— 2
2	2	·256
1	1·36	·241984
— 1	——	—— 3
3	4·36	14016
·4	1·52	12458883
—	—— 2	—— 4
3·4	5·88	1557,112
·4	·1696	1248
—	——	——
3·8	6·0496	309
4	·1712	249
— 2	—— 3	——
4·2	6·2208	60
4	8644	56
—	——	—
4·24	6·229444	
4	8648	
——	—— 4	
4·28	6,·2,3,8092	
4		
—— 3		
4·32		
2		
——		
4·322		
2		
——		
4·324		
2		
—— 4		
4·326		

It is easy to foresee, that after the step marked 4 (Ex. 3) is reached, the work of the subsequent steps can have little or no influence upon the *three* leading figures 6·23 of the forthcoming *divisors;* so that regarding these three figures as constant, and recognising the others, 8092, only for the sake of what is *carried* from them, we may, as above, make sure of at least *three* true decimals of the root, beyond the three already found, *by common division*, provided we reject a figure of the constant divisor, 6·23, at each step, taking care to secure accuracy in the carryings from the rejected figures.

Ex. 1. The cube root of $x^6 + 9x^5 + 6x^4 - 99x^3 - 42x^2 + 441x - 343$ is $x^2 + 3x - 7$.

2. The cube root of $x^6 + 6x^5 + 40x^3 - 96x - 64$, that is of $x^6 + 6x^5 + 0x^4 - 40x^3 + 0x^2 + 96x - 64$, is $x^2 + 2x - 4$.

3. The cube root of 12994449551 is 2351.

4. The cube root of 2 is 1·25992104989.

5. The cube root of 959 is 9·8614218.

6. The cube root of $x^6 - 15x^5y + 69x^4y^2 - 138x^2y^4 - 60xy^5 - 8x^6$ is $x^2 - 5xy - 2y^2$.

We here conclude the treatise on Elementary Algebra. The subject, in its widest acceptation, is one of very considerable extent—we might almost say of unlimited extent; as there are no definite bounds to its operations. In the preceding treatise, our object has been to unfold to you, fully and perspicuously, the leading principles of the science; and thus to lay a sufficiently secure basis for future researches. There is one department of the subject—the general theory of Logarithms and Series—which we have not touched upon here. It is a part of Algebra which is marked by peculiar features, and is occupied with investigations different in kind and in object from those necessary for the solution of an algebraical equation, or for the reduction of an algebraical expression; and is, moreover, of sufficient importance to merit distinct consideration. We shall, therefore, first give solutions to the various exercises contained in the foregoing chapters; and in the succeeding one, the subject of Series and Logarithms will be fully entered into. An extended Table of Logarithms of natural numbers will be also included.

SOLUTIONS TO THE EXERCISES IN THE CHAPTERS ON ALGEBRA.

At page 452 of the Algebra, a promise was given to furnish the Answers to all the Examples proposed for exercise in that subject. Considering for whom the work was expressly written—for persons not merely unacquainted with the very alphabet of Algebraical Science, but also precluded from the advantages of academical instruction—we have thought that more acceptable service would be rendered by supplying sketches of the solutions themselves, rather than a mere register of the *results*. You will therefore regard what follows as furnishing a Key to the unworked examples; showing briefly—but, we hope, clearly—the processes by which the answers are to be obtained.

In Addition and Subtraction, however, all that can here be done is to put down the totals; for the result of an addition or subtraction example exhibits, in itself, the whole work.

Exercises. Page 452.

1. $3a + 4c = 3.4 + 4.3 = 12 + 12 = 24$.
2. $5c - 2a = 5.3 - 2.4 = 15 - 8 = 7$.
3. $13n + b = 13.1 + 2 = 13 + 2 = 15$.
4. $21m - 9d = 21.8 - 9.5 = 168 - 45 = 123$.
5. $7d + 4n - 2a = 7.5 + 4 - 2.4 = 35 + 4 - 8 = 31$.
6. $3a + 4b - 5c = 3.4 + 4.2 - 5.3 = 12 + 8 - 15 = 5$.
7. $6m - 5n - 3b = 6.8 - 5 - 3.2 = 48 - 5 - 6 = 37$.
8. $14 - 3c + m = 14 - 3.3 + 8 = 14 - 9 + 8 = 13$.
9. $11b + n - 13 = 11.2 + 1 - 13 = 10$.
10. $-4d + 5m - 2n = -4.5 + 5.8 - 2 = 18$.
11. $\frac{3m}{4} + 6 - \frac{m}{2} = \frac{3.8}{4} + 6 - \frac{8}{2} = 6 + 6 - 4 = 8$.
12. $\frac{5a}{2} + \frac{4d}{5} - 2c = \frac{5.4}{2} + \frac{4.5}{5} - 2.3 = 10 + 4 - 6 = 8$.
13. $\frac{2a + 3b}{7} + m - n = \frac{2.4 + 3.2}{7} + 8 - 1 = 2 + 8 - 1 = 9$.
14. $\frac{3a - 4b + 6c - 2}{10} = \frac{3.4 - 4.2 + 6.3 - 2}{10} = \frac{20}{10} = 2$.
15. $3ab + dm - 5b + 6cn - 18 = 3.4.2 + 5.8 - 5.2 + 6.3 - 18 = 54$.
16. $2abm - 3cdm + \frac{3dm}{8} + ad = 2.4.2.8 - 3.3.5.8 + \frac{3.5.8}{8} + 4.5 = -197$.
17. $\frac{5am - 2bc + n - 49}{140} - n = \frac{5.4.8 - 2.2.3 + 1 - 49}{140} - 1 = -\frac{2}{7}$.
18. $\frac{24}{m} + \frac{5abc}{24} - \frac{60}{bcd} - \frac{m + n}{2b + d} = \frac{24}{8} + \frac{5.4.2.3}{24} - \frac{60}{2.3.5} - \frac{8 + 1}{2.2 + 5} = 5$.

Page 453. Ex. 1. $2xy - 4xy + 13xy + 3xy = 14xy$.
2. $-axz - 7axz + 11axz + axz = 4axz$.
3. $9mnx + 5mnx - 13mnx = mnx$.
4. $6\frac{a}{b} + 2\frac{a}{b} - 5\frac{a}{b} - 2\frac{a}{b} + \frac{a}{b} = 2\frac{a}{b}$.

ADDITION. CASE I. PAGE 453.

1. $26b$. 2. $24ax$. 3. $7by$. 4. $-5axy$.
5. $16a-6x$. 6. $9ax-5bz$. 7. $5pqy-2mx-3n$.
8. $2xyz+5ab-3$. 9. $9abz-22x+8a-1$.
10. $15acz-17bey+4kg$. 11. $15z+18abc-8mp$.
12. $19ky-4mz-3abc$. 13. $10abz+cey+5mx$.
14. $6gx+13pqy+13$. 15. $5axy+4ab+2c$.
16. $11xyz+9am-10bn$. 17. $8bcx-ny+2mz$.
18. $19exz+15bcy-11$. 19. ax. 20. $6cgx-3$.

Page 454. Case II. 1. $6by+3ez+14c+12$.
2. $8xy+2az+c-15$. 3. $6yz+9ax-7bc+xyz+en$.
4. $ab-cd+8ef-10e+14$.
5. $7xy-15bz+10w-3cd-16$.
6. $4amz-13nx-8bv+6cw-8$.
7. $-axy+10bz-3c+5$. 8. $-10abc+2de+f+8$.
9. $-\frac{1}{2}z+11\frac{1}{3}y-4x+9$. 10. $\frac{x}{y}-2az+2\frac{z}{x}-6b+4$.
11. $-5xyz-5\frac{z}{a}+10bc-4mn+p$. 12. $4\frac{1}{3}ax-\frac{1}{2}y-8$.

SUBTRACTION. PAGE 455.

1. $3x-10y+8z$. 2. $-ax+6by-11cz$.
3. $xy-15ax+11$. 4. $-5ay-5bx+6$.
5. $20abc+10de-6fg-5$. 6. $6bz-5ax-m-en$.
7. $5-11+11-6-3=-4$.
8. $dy-24fz-3m-8n+4$.
9. $-11ayz-17bx+4c+4d-2e$.
10. $12cex-8aby+4dx+3z+11$.
11. $8ax-5by+5cz$. 12. $4bz-6cy-4ax$.
13. $3ay+8bx-3cz-16$. 14. $5x-8y+8z-4$.
15. $-4\frac{1}{3}axy+3bz+4m-2n$. 16. $2\frac{1}{2}yz-2ax-3$.

SIMPLE EQUATIONS. EXAMPLES FOR EXERCISE. PAGE 457.

1. $2x-7=x+4$. Transposing, $2x-x=4+7$, that is, $x=11$.
2. $5x-3=3x+5$. Transposing, $5x-3x=5+3$, $\therefore 2x=8$ $\therefore x=4$.
3. $6x+2=4x+8$. Transposing, $6x-4x=8-2$, $\therefore 2x=6$ $\therefore x=3$.
4. $3x-1=x+7$. Transposing, $3x-x=7+1$, $\therefore 2x=8$ $\therefore =4$.
5. $8x-13=4x-1$. Transposing, $8x-4x=13-1$, $\therefore 4x=12$ $\therefore x=3$.
6. $2(x+3)=20$, that is, $2x+6=20$. Transposing, $2x=20-6=14$, $\therefore x=7$.
7. $3(x-4)=6$, that is, $3x-12=6$. Transposing, $3x=18$, $\therefore x=6$.
8. $4(x-1)=x+5$, that is, $4x-4=x+5$. Transposing, $3x=9$, $\therefore x=3$.
9. $5(x+2)=2(x+8)$, that is, $5x+10=2x+16$. Transposing, $3=6$, $\therefore x=2$.
10. $3(2x-3)=4(x-2)$, that is, $6x-9=4x-8$. Transposing, $2x=1$, $\therefore x=\frac{1}{2}$.

11. $\frac{x}{3}+\frac{x}{4}=7$. Multiplying by 3, to clear the first fraction,

$$x+\frac{3x}{4}=21.$$

Multiplying by 4, $4x+3x=84$, $\therefore 7x=84$, $\therefore x=\frac{84}{7}=12$.

12. $\frac{x}{3}-\frac{x}{4}=1$. Multiplying by 3, $x-\frac{3x}{4}=3$.

Multiplying by 4, $4x-3x=12$, $\therefore x=12$.

13. $\frac{2x}{3}-\frac{x}{4}=5$. Multiplying by 3, $2x-\frac{3x}{4}=15$.

Multiplying by 4, $8x-3x=60$, $\therefore 5x=60$, $\therefore x=12$.

14. $\frac{x}{2}+\frac{x}{3}=31-\frac{x}{5}$. Multiplying by 2,

$$x+\frac{2x}{3}=62-\frac{2x}{5}.$$

Multiplying by 3, $3x+2x=186-\frac{6x}{5}$.

" 5, $15x+10x=930-6x$, $\therefore 15x+10x+6x=930$.

That is, $31x=930$, $\therefore x=\frac{930}{31}=30$.

15. $\frac{x}{3}+\frac{x}{4}+\frac{x}{6}=1$. Multiplying by 3,

$$x+\frac{3x}{4}+\frac{3x}{6}=3.$$

Multiplying by 4, $4x+3x+2x=12$, $\therefore 9x=12$, $\therefore x=\frac{12}{9}=1\frac{1}{3}$.

16. $2x-(48-x)=x+12$, that is, $2x-48+x=x+12$, or transposing, $2x=12+48=60$, $\therefore x=30$

17. $\frac{5(3x+2)}{2}=35$. Multiplying by 2, and removing vinculum,

$$15x+10=70,\ \therefore 15x=60,\ \therefore x=4.$$

18. $\frac{x+3}{5}-1=2-\frac{x}{7}$, or $\frac{x+3}{5}=3-\frac{x}{7}$. Multiplying by 5,

$$x+3=15-\frac{5x}{7},\ \text{or}\ x=12-\frac{5x}{7}.$$

Multiplying by 7, $7x=84-5x$, $\therefore 12x=84$, $\therefore x=7$.

19. $\frac{x}{2}+\frac{x}{3}-\frac{x}{5}-1=0$. Multiplying by 2,

$$x+\frac{2x}{3}-\frac{2x}{5}-2=0.$$

$$\text{Multiplying by 3, } 3x+2x-\frac{6x}{5}-6=0, \text{ or } 5x-\frac{6x}{5}=6.$$

$$\text{,, by 5, } 25x-6x=30, \therefore 10x=30, \therefore x=\frac{30}{19}=1\tfrac{11}{19}.$$

20. $\frac{x}{2}+\frac{x}{3}-\frac{x}{4}-\frac{1}{2}=0$. Multiplying by 2,

$$x+\frac{2x}{3}-\frac{x}{2}-1=0.$$

$$\text{Multiplying by 3, } 3x+2x-\frac{3x}{2}-3=0, \text{ or } 5x-\frac{3x}{2}-3=0.$$

$$\text{,, by 2, } 10x-3x-6=0, \therefore 7x=6, \therefore, x=\frac{6}{7}.$$

21. $11-\frac{5x}{8}=\frac{x}{2}-\frac{2x}{3}$. Multiplying by 8,

$$88-5x=4x-\frac{16x}{3}.$$

$$\text{Multiplying by 3, } 264-15x=12x-16x, \therefore 264=11x, \therefore x=\frac{264}{11}=24.$$

22. $4x-2(3-x)=\frac{x+3}{2}$, that is, $4x-6+2x=\frac{x+3}{2}$, or $6x-6=\frac{x+3}{2}$.

$$\text{Multiplying by 2, } 12x-12=x+3, \therefore 11x=15, \therefore x=\frac{15}{11}=1\tfrac{4}{11}.$$

23. $6x-\frac{4-3x}{5}=\frac{29}{5}$. Multiplying by 5,

$$30x-4+3x=29, \therefore 33x=33, \therefore x=1.$$

24. $\frac{x-6}{5}+\frac{x}{3}=20-\frac{x-2}{2}$, that is, $\frac{x}{2}-3+\frac{x}{3}=20-\frac{x}{2}+1$, or by transposing, $x+\frac{x}{3}=24$, $\therefore$ mult. by 3, $3x+x=72$, $\therefore 4x=72$, $\therefore x=18$.

25. $\frac{3x-13}{7}+\frac{11-4x}{3}=0$. Multiplying by 7,

$$3x-13+\frac{77-28x}{3}=0.$$

$$\text{Multiplying by 3, } 9x-39+77-28x=0.$$

$$\text{Transposing, } 38=19x, \therefore x=2.$$

26. $6-\frac{x-1}{2}-\frac{x-2}{3}=\frac{3-x}{4}$. Multiplying by 4, in order that both the first and last fractions may be cleared at the same time, we have

$$24-2x+2-\frac{4x-8}{3}=3-x,$$

$$\text{Multiplying by 3, } 72-6x+6-4x+8=9-3x.$$

$$\text{Transposing, } 72+6+8-9=6x+4x-3x.$$

$$\text{Collecting, } 77=7x, \therefore x=11.$$

QUESTIONS TO BE SOLVED BY SIMPLE EQUATIONS.
PAGE 458.

1. There are two numbers of which the difference is 9, and the sum 43. What are the numbers?

Let x represent the smaller of the two numbers, then by the question, the other must be $x+9$, and their sum

$$x+x+9=43, \text{ that is, } 2x=43-9,$$

$$\therefore x=\frac{34}{2}=17,$$

the smaller number, and $\therefore 17+9=26$, the larger number.

2. From two places, 108 miles apart, two persons, A and B, set out at the same time, to meet each other. A travels 17 miles a-day, and B travels 18; in how many days will they meet?

Suppose they meet in x days: then by the question, the first, A, will have travelled $17x$ miles, and the second, B, $18x$ miles; and since the sum of these distances is the distance between the two places, we have the equation

$$17x+18x=108, \therefore 35x=108, \therefore x=3\tfrac{3}{35} \text{ days.}$$

3. Find two numbers of which the difference is 13, and which are such, that if 17 be added to their sum, the whole will amount to 62.

Let x be the greater number; then, by the question, $x-13$ is the less; also

$$x+x-13+17=62, \therefore 2x=62+13-17=58,$$

$\therefore x=29$, the greater number, and $\therefore 29-13=16$, the less number.

4. There are two numbers of which the difference is 15, and which are such that if 7 times the less be subtracted from 5 times the greater, the difference will be 19: what are the numbers!

Let x be the greater number; then, by the question, $x-15$ is the less; also

$$5x-(7x-105)=19, \text{ that is, } -2x+105=19,$$

$$\therefore -2x=-86,$$

$$\therefore x=\frac{-86}{-2}=43, \text{ the greater number, and}$$

$$\therefore 43-15=28, \text{ the less number.}$$

5. A person starts from a certain place, and travels at the rate of 4 miles an hour. After he has gone 10 hours, a horseman, riding 9 miles an hour, is dispatched after him: how many hours must the horseman ride to overtake him?

Suppose the horseman overtakes him after riding x hours; then, by the question, the pedestrian will have walked $40+4x$ miles, and the horseman will have ridden $9x$ miles; and since the distance travelled by each is the same, we have the equation

$$9x=40+4x, \therefore 5x=40, \therefore x=8;$$

hence the horseman has ridden 8 hours, so that each person must have travelled 9 × 8 = 72 miles.

6. A person has 264 coins—sovereigns and florins; he has $4\frac{1}{2}$ times as many florins as sovereigns: how many of each coin has he?

Suppose he had x sovereigns; then, by the question, he must have had $4\frac{1}{2}x$ florins; so that

$$x + 4\tfrac{1}{2}x = 264, \text{ that is, } 5\tfrac{1}{2}x = 264,$$
$$\text{or } 11x = 528, \therefore x = 48.$$

Consequently he had 48 sovereigns, and $\therefore 48 \times 4\frac{1}{2} = 216$ florins.

7. A person spends $\frac{1}{4}$th of his yearly income in board and lodging, $\frac{1}{7}$th in clothes and other expenses, and he lays by £85 a-year: what is his income?

To avoid fractions, suppose $28x$ to be the number of pounds he receives yearly, 28 being chosen because it is divisible by both 4 and 7: then, by the question, he spends in board and lodging $7x$ pounds, and in clothes, &c., $4x$ pounds. Consequently, since he lays by £85, we have the equation

$$7x + 4x + 85 = 28x, \text{ that is, } 11x + 85 = 28x.$$
$$\text{Transposing, } 85 = 17x, \therefore x = 5, \therefore 28x = 140.$$

Hence his yearly income was £140.

8. What number is that whose third part exceeds its fifth part by 72?

To avoid fractions, let $15x$ represent the number; then, by the question,

$$5x - 3x = 72, \therefore 2x = 72, \therefore x = 36,$$
$$\therefore \text{the number required is } 36 \times 15 = 540.$$

9. I have a certain number in my thoughts. I multiply it by 7, add 3 to the product, and divide the sum by 2. I then find that if I subtract 4 from the quotient I get 15: what number am I thinking of?

Let x represent the number; then, by the question,

$$\frac{7x + 3}{2} - 4 = 15, \text{ or } \frac{7x + 3}{2} = 19.$$

Multiplying by 2, $7x + 3 = 38$, $\therefore 7x = 35$, $\therefore x = 5$, the number thought of.

10. A man, 40 years old, has a son 9 years old: the father is therefore more than four times as old as his son. In how many years will the father be only twice as old as his son?

Suppose in x years: the father will then be $40 + x$ years old, and the son $9 + x$ years: by the question, the former number is to be double of the latter: hence the equation

$$40 + x = 18 + 2x, \therefore 40 - 18 = x = 22.$$

Therefore the father will be twice as old as his son in 22 years; in which time the father will be 62, and the son 31.

11. Two persons, A and B, 120 miles apart, set out at the same time to meet each other. A goes 3 miles an hour, and B 5 miles: what distance will each have travelled when they meet?

Suppose that A has travelled x *hours*, then B also must have travelled x *hours*. A must therefore have gone $3x$ miles, and B $5x$ miles; and since together they must have travelled 120 miles, we have the equation

$$3x + 5x = 120, \text{ that is, } 8x = 120, \therefore x = 15.$$

The *time* occupied by each is therefore 15 hours; so that A must have travelled 15 × 3 = 45 miles, and B 15 × 5 = 75 miles.

Otherwise.—Suppose A goes x miles, then B goes $120 - x$ miles: the time occupied by A, at 3 miles an hour, is therefore $\frac{x}{3}$ hours; and the time occupied by B, at 5 miles an hour, is $\frac{120 - x}{5}$ hours; but the times are *equal*. Hence the equation

$$\frac{x}{3} = \frac{120 - x}{5} = 24 - \frac{x}{5}.$$

$$\text{Multiplying by 3, } x = 72 - \frac{3x}{5}.$$

$$\text{Multiplying by 5, } 5x = 360 - 3x, \therefore 8x, = 360,$$
$$\therefore x = 45, \text{ A's distance.}$$
$$\therefore 120 - 45 = 75, \text{ B's distance.}$$

12. Divide £250 among A, B, and C, so that B may have £23 more than A, and C £105 more than B.

Let A's share be x pounds, then B's is $x + 23$; and C's $x + 23 + 105$; and the sum of the shares is 250: hence the equation

$$x + x + 23 + x + 23 + 105 = 250.$$

Collecting, and transposing, $3x = 99$, $\therefore x = 33$: hence the shares are as follow:—

A's share £33; B's £56; C's £161; and their sum is £250.

13. A can execute a piece of work in 3 days, which takes B 7 days to perform: in how many days can it be done if A and B work together?

Suppose they can do it in x days; then since A can do one-third of it in 1 day, he can do $\frac{x}{3}$ in x days: in like manner B can do $\frac{x}{7}$ of the whole in x days: hence, when working together, they do $\frac{x}{3} + \frac{x}{7}$ in x days; but these *parts* make up the whole work since they do it *all* in x days;

$$\therefore \frac{x}{3} + \frac{x}{7} = 1 \textit{ whole}$$

$$\text{Multiply by 3,} \quad x + \frac{3x}{7} = 3$$

$$\text{,,} \quad \text{by 7,} \quad 7x + 3x = 21, \therefore 10x = 21, \therefore x = 2\tfrac{1}{10}.$$

Hence they complete the work in two days and one-tenth.

14. A cistern can be filled by three pipes; by the first in 2 hours, by the second in 3, and by the third in 4; in what time can it be filled by all the pipes running together?

Suppose it can be filled in x hours; then since the first can supply one-half in 1 hour, it can supply $\frac{x}{2}$ in x hours; in like manner the second can supply $\frac{x}{3}$ in x hours, and the third $\frac{x}{4}$; and by the question the sum of these parts is the *whole*.

$$\therefore \frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 1.$$

Multiplying by 4, in order that the first and third fractions may be removed at the same time, we have

$$2x + \frac{4x}{3} + x = 4, \text{ that is, } 3x + \frac{4x}{3} = 4.$$

Multiplying by 3, $9x + 4x = 12$, $\therefore 13x = 12$, $\therefore x = \frac{12}{13}$; hence, when they all run together, the pipes will fill the cistern in $\frac{12}{13}$h. = 55 min. $23\frac{1}{13}$ sec.

15. Solve the preceding question, when the first pipe fills the cistern in 1 hour 20 minutes; the second in 3 hours 20 minutes, and the third in 5 hours.

Imitating the foregoing solution, x being the required number of hours as before, the part of the whole supplied by the first pipe is $\frac{x}{1\frac{1}{3}}$, the part supplied by the second is $\frac{x}{3\frac{1}{3}}$, and the part supplied by the third is $\frac{x}{5}$, in x hours; hence, since these parts make up the whole, we have

$$\frac{x}{1\frac{1}{3}} + \frac{x}{3\frac{1}{3}} + \frac{x}{5} = 1.$$

Or, multiplying the terms of the first pair of fractions by 3, $\frac{3x}{4} + \frac{3x}{10} + \frac{x}{5} = 1$.

$$\text{Multiplying by 4, } 3x + \frac{6x}{5} + \frac{4x}{5} = 4, \text{ or } 3x + 2x = 4,$$

$$\text{that is, } 5x = 4, \therefore x = \frac{4}{5};$$

hence, the time is $\frac{4}{5}$ h. = 48 minutes.

16. After A has been working 4 days at a job, which he can finish in 10 days, B is sent to help him; they finish it together in 2 days: in what time could B alone have done the whole?

Suppose B can finish it in x days; then he can do $\frac{1}{x}$ of it in 1 day, so that in the 2 days he does $\frac{2}{x}$ of it. Now, as A does $\frac{1}{10}$ of it in a day, in the 4 days, working alone, he has done $\frac{4}{10}$ or $\frac{2}{5}$ of it; hence, when B commences there is only $\frac{3}{5}$ of it to be done, so that $\frac{2}{x} + \frac{2}{10} = \frac{3}{5}$, or $\frac{2}{x} + \frac{1}{5} = \frac{3}{5}$, $\therefore \frac{2}{x} = \frac{2}{5}$, $\therefore \frac{1}{x} = \frac{1}{5}$.

Multiplying by x, and then by 5, we have $5 = x$; hence, B can finish the work alone in 5 days.

17. Divide £143 among A, B, and C, so that A may receive twice as much as B, and B three times as much as C.

Suppose C's share to be x pounds
then B's ,, $3x$,,
and A's ,, $6x$,,

$\therefore$ the sum of the shares is $10x = 143$,

$$\therefore x = \frac{\overset{\pounds}{143}}{10} = \pounds 14\ 6s.$$

$\therefore$ C's share = £14 6*s.*; B's, £42 18*s.*; A's, £85 16*s.*; and the sum of which is the whole = £143.

18. A person has 40 quarts of superior wine worth 7*s.* a quart; he wishes, however, so to reduce its quality as that he may sell it at 4*s.* 6*d.* a quart; how much water must he add?

Suppose the water to be x quarts, then the entire number of quarts in the mixture will be $40 + x$, and by the question the worth of the pure wine is 280*s.*, and that of the reduced wine is $4\frac{1}{2}(40 + x)$ shillings. As the worth is to remain the same, we have the equation

$$4\tfrac{1}{2}(40 + x) = 280, \text{ or } 180 + 4\tfrac{1}{2}x = 280$$

$$\therefore 4\tfrac{1}{2}x = 100;\ \text{and} \therefore x = \frac{100}{4\frac{1}{2}} = \frac{200}{9} = 22\tfrac{2}{9};$$

hence, the quantity of water to be added is $22\frac{2}{9}$ quarts; so that the mixture will make $62\frac{2}{9}$ quarts. The value of this at $4\frac{1}{2}$*s.* a quart is found, as in the margin, to be 280*s.*, which is the value of the unreduced 40 quarts.

$$\begin{array}{r} 2)\,62\frac{2}{9} \\ 4\frac{1}{2} \\ \hline 248\frac{8}{9} \\ 31\frac{1}{9} \\ \hline 280s. \end{array}$$

19. Divide 90 into four parts, such, that if the first be increased by 2, the second diminished by 2, the third multiplied by 2, and the fourth divided by 2, the results may all be equal.

Let the first part be $x - 2$: this increased by 2, is x.
Then the second will be $x + 2$: ,, diminished by 2, is x.
,, third ,, $\frac{x}{2}$: ,, multiplied by 2, is x.
,, fourth ,, $\underline{2x}$: ,, divided by 2, is x.

$$\text{The sum of these is } 4x + \frac{x}{2} = 90.$$

Multiplying by 2, $8x + x = 180$, $\therefore 9x = 180$, $\therefore x = 20$; hence, the required parts are 18, 22, 10, and 40, which together make 90.

20. Divide 39 into four parts, such, that if the first be increased by 1, the second diminished by 2, the third multiplied by 3, and the fourth divided by 4, the results may all be equal.

Let the first part be $x - 1$: this increased by 1 is x.
Then the second will be $x + 2$: this diminished by 2 is x.
,, third ,, $\frac{x}{3}$: this multiplied by 3 is x.
,, fourth ,, $\underline{4x}$: this divided by 4 is x.

$$\text{The sum of these is } 6x + \frac{x}{3} + 1 = 39, \therefore 6x + \frac{x}{3} = 38.$$

Multiplying by 3, $18x + x = 114$, $\therefore 19x = 114$, $\therefore x = 6$; hence, the required parts are 5, 8, 2, and 24, the sum of which is 39.

From the last two examples you will perceive, that although, in general, the unknown quantity sought is best represented by a single symbol x, yet the conditions of the question may be such as to suggest a more convenient form for it: a judicious form of representation at the outset will often save several steps of work in the solution. When fractions are foreseen to enter the equation, when the symbol for the unknown quantity is x, it will always be better to use instead of x, such a multiple of x as will preclude their entrance, as in Examples 7 and 8, just given.

MULTIPLICATION. CASE I. PAGE 459.

EXAMPLES FOR EXERCISE.

1. $$\begin{array}{l} 9a^3y^2 \\ 4a^2y^3 \\ \hline 36a^5y^5 \\ \hline \end{array}$$

2. $$\begin{array}{l} -7bx^2y^3 \\ 8b^4y^5 \\ \hline -56b^5x^2y^8 \\ \hline \end{array}$$

3. $$\begin{array}{l} -4a^2x^3z^4 \\ -6 \\ \hline 24a^2x^3z^4 \\ \hline \end{array}$$

4. $$\begin{array}{l} -5b^3c^2y \\ -3c^4y^3z \\ \hline 15b^3c^6y^4z \\ \hline \end{array}$$

5. $$\begin{array}{l} 11a^4xy^2z^3 \\ -9a^5x^7y^5z^2 \\ \hline -99a^9x^8y^7z^5 \\ \hline \end{array}$$

6. $$\begin{array}{l} -13a^2x^3y^4 \\ 14a^4x^3y^2 \\ \hline -182a^6x^6y^6 \\ \hline \end{array}$$

7. $$\begin{array}{l} \frac{1}{2}b^4x^3y^2 \\ -4b^2xy^5 \\ \hline -2b^6x^4y^7 \\ \hline \end{array}$$

8. $$\begin{array}{l} \frac{1}{3}ax^4z^2 \\ \frac{3}{4}x^3yz^4 \\ \hline \frac{1}{4}ax^7yz^6 \\ \hline \end{array}$$

9. $$\begin{array}{l} 2ax^2y \\ -3a^2y^3 \\ \hline -6a^3x^2y^4 \\ -4a^3x^4 \\ \hline 24a^6x^6y^4 \end{array}$$

10. $$\begin{array}{l} -3b^2yz \\ -2y^2z^2 \\ \hline 6b^2y^3z^3 \\ -4ab^3y \\ \hline -24ab^5y^4z^3 \end{array}$$

11. $$\begin{array}{l} \frac{1}{2}axy^3 \\ \frac{2}{3}x^2y \\ \hline \frac{1}{3}ax^3y^4 \\ -3a^2y^2 \\ \hline -a^3x^3y^6 \end{array}$$

12. $$\begin{array}{l} \frac{2}{3}a^2z^3 \\ -\frac{3}{4}x^2y^3 \\ \hline -\frac{1}{2}a^2x^2y^3z^3 \\ -\frac{1}{5}a^3y^2z \\ \hline \frac{1}{10}a^5x^2y^5z^4 \end{array}$$

13. $$\begin{array}{l} -\frac{8}{9}a^2y^2 \\ -3b^2z^2 \\ \hline \frac{8}{3}a^2b^2y^2z^2 \\ -\frac{5}{8}abx^2yz \\ \hline -\frac{5}{3}a^3b^3x^2y^3z^3 \\ \hline \end{array}$$

14. $$\begin{array}{l} \frac{7}{8}a^3x \\ -\frac{4}{5}x^2y \\ \hline -\frac{7}{10}a^3x^3y \\ -2y^2z \\ \hline \frac{7}{5}a^3x^3y^3z \\ -10a^2xz^2 \\ \hline -14a^5x^4y^3z^3 \\ \hline \end{array}$$

MULTIPLICATION. CASE II. PAGE 460.

1. $$\begin{array}{l} 3ax^3 - 2bxy \\ 4a^2x \\ \hline 12a^3x^4 - 8a^2bx^2y \\ \hline \end{array}$$

2. $$\begin{array}{l} 5a^2y^2 - 4x^2y^3 \\ -3axy \\ \hline -15a^3xy^3 + 12ax^3y^4 \\ \hline \end{array}$$

3. $$\begin{array}{l} -6b^2yz^3 + 4ab^2y^2 \\ -2ayz \\ \hline 12ab^2y^2z^4 - 8a^2b^2y^3z \\ \hline \end{array}$$

4. $$\begin{array}{l} 7a^3b^4x^5 - 3x^2y^4 + 5 \\ 4ax^3y \\ \hline 28a^4b^4x^8y - 12ax^5y^5 + 20ax^3y \\ \hline \end{array}$$

5. $$\begin{array}{l} 9m^2nz^3 - 7a^2y^3 - 2b^3x^3y \\ -\ 7an^3x^2 \\ \hline -63am^2n^4x^2z^3 + 49a^3n^3x^2y^3 + 14ab^3n^3x^5y. \\ \hline \end{array}$$

6. $(2a^2bx^2 - 5xy^2) \times 3ax^2y = 6a^3bx^4y - 15ax^3y^3.$

7. $(2axy^2 - 3x^2z + 2x) \times -4axy = 8a^2x^2y^3 + 12ax^3yz - 8ax^2y.$

8. $(mxy^3 + 3ny^2 - 2y^3z) \times -6m^2x^2y = -6m^3x^3y^4 - 18m^2nx^2y^3 + 12m^2x^2y^4z.$

9. $(4y^3z^2 - 3az^5 - 5by^3) \times -5ab^2y = -20ab^2y^4z^2 + 15a^2b^2yz^5 + 25ab^3y^4.$

10. $(5cx^2z^3 - y^4z^2 - 3z^4)\,12c^2x^3yz^2 = 60c^3x^5yz^5 - 12c^2x^3y^5z^4 - 36c^2x^3yz^6.$

11. $(\frac{2}{3}ax^2 - \frac{1}{4}by^2 - 2cz^2) \times -12a^2bc^3 = -8a^3bc^3x^2 + 3a^2b^2c^3y^2 + 24a^2bc^4z^2.$

12. $\{4ay^2 - (2b^2z^2 - yz - 2)\}\,3a^2yz^3 = \{4ay^2 - 2b^2z^2 + yz + 2\}\,3a^2yz^3 = 12a^3y^3z^3 - 6a^2b^2yz^5 + 3a^2y^2z^4 + 6a^2yz^3.$

13. $\{2bx^2y^3 - (5cy^2z + 4yz^2 - 3z^3)\} \times -3x^2y = \{2bx^2y^3 - 5cy^2z - 4yz^2 + 3z^3\} \times -3x^2y = -6bx^4y^4 + 15cx^2y^3z + 12x^2y^2z^2 - 9x^2yz^3.$

14. $\{5a^2x^3 - (3b^2y^2 + 2a^2x^2 - 4ay^3)\} \times -2a^2b^2xy = \{5a^2x^3 - 3b^2y^2 - 2a^2x^2 + 4ay^3\} \times -2a^2b^2xy = -10a^4b^2x^4y + 6a^2b^4xy^3 + 4a^4b^2x^3y - 8a^3b^2xy^4.$

MULTIPLICATION. CASE III. PAGE 461.

1.
$$\begin{array}{l}
2ax^2 - 3bx - 2 \\
3x^2 - 2x \\ \hline
6ax^4 - 9bx^3 - 6x^2 \\
\qquad - 4ax^3 + 6bx^2 + 4x \\ \hline
6ax^4 - (4a + 9b)\,x^3 + 6(b - 1)x^2 + 4x. \\ \hline
\end{array}$$

2.
$$\begin{array}{l}
4ay + 3bx - 4c \\
3by + 2c \\ \hline
12aby^2 + 9b^2xy - 12bcy \\
\qquad\qquad 8acy + 6bcx - 8c^2 \\ \hline
12aby^2 + (9b^2x + 8ac)y + 6bc(x - 2y) - 8c^2. \\ \hline
\end{array}$$

3.
$$\begin{array}{l}
5x^3 - 4x^2 + 3x - 2 \\
2x^2 - x + 1 \\ \hline
10x^5 - 8x^4 + 6x^3 - 4x^2 \\
\qquad - 5x^4 + 4x^3 - 3x^2 + 2x \\
\qquad\qquad\qquad 5x^3 - 4x^2 + 3x - 2 \\ \hline
10x^5 - 13x^4 + 15x^3 - 11x^2 + 5x - 2. \\ \hline
\end{array}$$

4. $(x + 2)(x - 2) = x^2 - 4$; $(x + 3)(x - 3) = x^2 - 9$
$$\begin{array}{l}
x^2 - 4 \\
x^2 - 9 \\ \hline
x^4 - 4x^2 \\
\qquad - 9x^2 + 36 \\ \hline
x^4 - 13x^2 + 36.^* \\ \hline
\end{array}$$

5. $(x + a)(x - a) = x^2 - a^2$; $(x^2 - a^2)(a^2 - a^2) = x^4 - 2a^2x^2 + a^4.$

6.
$$\begin{array}{l}
2x + 3a \\
4x - 5a \\ \hline
8x^2 + 12ax \\
\qquad - 10ax - 15a^2 \\ \hline
8x^2 + 2ax - 15a^2 \\
\quad x + a \\ \hline
8x^3 + 2ax^2 - 15a^2x \\
\qquad\quad 8ax^2 + 2a^2x - 15a^3 \\ \hline
8x^3 + 10ax^2 - 13a^2x - 15a^3. \\ \hline
\end{array}$$

7.
$$\begin{array}{l}
3ax - b \\
4ax + c \\ \hline
12a^2x^2 - 4abx \\
\qquad\qquad 3acx - bc \\ \hline
12a^2x^2 + (3c - 4b)ax - bc \\
\quad 5ax - 3 \\ \hline
60a^3x^3 + 5(3c - 4b)a^2x^2 - 5abcx \\
\qquad\quad - 36a^2x^2 - 3(3c - 4b)ax + 3bc \\ \hline
60a^3x^3 + \{5(3c - 4b) - 36\}\,a^2x^2 - \{3(3c - 4b) + 5bc\}\,ax + 3bc. \\ \hline
\end{array}$$

8.
$$\begin{array}{l}
4ax + 3by - 1 \\
2ax - by + 2 \\ \hline
8a^2x^2 + 6abxy - 2ax \\
\qquad - 4abxy - 3b^2y^2 + by \\
\qquad\qquad\qquad 8ax + 6by - 2 \\ \hline
8a^2x^2 + 2abxy + 6ax - 3b^2y^2 + 7by - 2. \\ \hline
\end{array}$$

9. $(x - y)^3(x + y) = (x - y)^2(x - y)(x + y) = (x - y)^2(x^2 - y^2)$
$$\begin{array}{l}
x^2 - 2xy + y^2 \\
x^2 - y^2 \\ \hline
x^4 - 2x^3y + x^2y^2 \\
\qquad - x^2y^2 + 2xy^3 - y^4 \\ \hline
x^4 - 2x^3y + 2xy^3 - y^4. \\ \hline
\end{array}$$

10.
$$\begin{array}{l}
x^2 - (a + b)x + c \\
x - c \\ \hline
x^3 - (a + b)x^2 + cx \\
\qquad - cx^2 + (a + b)cx - c^2 \\ \hline
x^3 - (a + b + c)x^2 + (a + b + 1)cx - c^2 \\ \hline
\end{array}$$

11. $3^3(x^2 - a^2)^2 = 27(x^4 - 2a^2x^2 + a^4.$

12. $(2x + 3)(2x - 3)(4x^2 + 9) = 4x^2 - 9)(4x^2 + 9) = 16x^4 - 81.$

13. $\{(x + 2)(x - 2)\}^2 = (x^2 - 4)^2 = x^4 - 8x^2 + 16.$†

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* See Algebra, p. 460.

† See *ante*, p. 460.

14.
$$\begin{array}{l}
x^2 - \frac{1}{2}x + 1 \\
x^2 + 2x - 1 \\
\hline
x^4 - \frac{1}{2}x^3 + x^2 \\
\qquad 2x^3 - x^2 + 2x \\
\qquad\qquad - x^2 + \frac{1}{2}x - 1 \\
\hline
x^4 + 1\frac{1}{2}x^3 - x^2 + 2\frac{1}{2}x - 1.
\end{array}$$

INVOLUTION. PAGE 461.

1. $(3a^2x^2)^2 = 9a^4x^4.$
2. $(7by^3)^4 = 2401b^4y^{12}.$
3. $(-4a^2b^3x^4)^3 = -64a^6b^9x^{12}.$
4. $(2x^4y^2z)^5 = 32x^{20}y^{10}z^5.$
5. $(-3a^3y^3z^2)^5 = -243a^{15}y^{15}z^{10}.$
6. $(-8b^2x^6y^3)^3 = -512b^6x^{18}y^9.$
7. $(5a^2b^3c^4)^4 = 625a^8b^{12}c^{16}.$
8. $(-2^2a^3x^2y)^3 = -64a^9x^6y^3.$

INVOLUTION. EXAMPLES FOR EXERCISE. PAGE 461.

1. $(a + 2x)^3 = (a + 2x)^2 (a + 2x) = \begin{cases} a^2 + 4x + 4x^2.^* \\ a + 2x \end{cases}$
$$\begin{array}{l}
a^3 + 4ax + 4ax^2 \\
\qquad 2a^2x + 8x^2 + 8x^3 \\
\hline
a^3 + 2a(a + 2)x + 4(a + 2)x^2 + 8x^3. \\
\hline
\end{array}$$

2. $(3ax - 4y)^3 = (3ax - 4y)^2 (3ax - 4y) = \begin{cases} 9a^2x^2 - 24axy + 16y^2.^\dagger \\ 3ax - 4y \end{cases}$
$$\begin{array}{l}
27a^3x^3 - 72a^2x^2y + 48axy^2 \\
\qquad\qquad - 36a^2x^2y + 96axy^2 - 64y^3 \\
\hline
27a^3x^3 - 108a^2x^2y + 144axy^2 - 64y^3. \\
\hline
\end{array}$$

3.
$$\begin{array}{l}
a + b + c^{\ddagger} \\
a + b + c \\
\hline
a^2 + ab + ac \\
\qquad ab + b^2 + bc \\
\qquad\qquad ac + bc + c^2 \\
\hline
a^2 + 2ab + 2ac + b^2 + 2bc + c^2. \\
\hline
\end{array}$$

4.
$$\begin{array}{l}
3x^2 + 2x + 5 \\
3x^2 + 2x + 5 \\
\hline
9x^4 + 6x^3 + 15x^2 \\
\qquad 6x^3 + 4x^2 + 10x \\
\qquad\qquad 15x^2 + 10x + 25 \\
\hline
9x^4 + 12x^3 + 34x^2 + 20x + 25. \\
\hline
\end{array}$$

5.
$$\begin{array}{l}
a - 2b + 3c \\
a - 2b + 3c \\
\hline
a^2 - 2ab + 3ac \\
\quad - 2ab + 4b^2 - 6bc \\
\qquad\qquad 3ac - 6bc + 9c^2 \\
\hline
a^2 - 4ab + 6ac + 4b^2 - 12bc + 9c^2. \\
\hline
\end{array}$$

6. $\{(a + x)(a - x)\}^3 = (a^2 - x^2)^3$
$= (a^2 - x^2)^2 (a^2 - x^2) = \begin{cases} a^4 - 2a^2x^2 + x^4 \\ a^2 - x^2 \end{cases}$
$$\begin{array}{l}
a^6 - 2a^4x^2 + a^2x^4 \\
\qquad - a^4x^2 + 2a^2x^4 - x^6 \\
\hline
a^6 - 3a^4x^2 + 3a^2x^4 - x^6. \\
\hline
\end{array}$$

7. $\{(x + 2)(x - 2)\}^3 = (x^2 - 4)^3 = (x^2 - 4)^2 (x^2 - 4) = \begin{cases} x^4 - 8x^2 + 16 \\ x^2 - 4 \end{cases}$
$$\begin{array}{l}
x^6 - 8x^4 + 16x^2 \\
\qquad - 4x^4 + 32x^2 - 64 \\
\hline
x^6 - 12x^4 + 48x^2 - 64. \\
\hline
\end{array}$$

8. $\{(2x - 3)(2x + 3)\}^4 = (4x^2 - 9)^4 =$
$(4x^2 - 9)^2 (4x^2 - 9)^2 = \begin{cases} 16x^4 - 72x^2 + 81 \\ 16x^4 - 72x^2 + 81 \end{cases}$
$$\begin{array}{l}
256x^8 - 1152x^6 + 1296x^4 \\
\qquad - 1152x^6 + 5184x^4 - 5832x^2 \\
\qquad\qquad\qquad 1296x^4 - 5832x^2 + 6561 \\
\hline
256x^8 - 2304x^6 + 7776x^4 - 11664x^2 + 6561. \\
\hline
\end{array}$$

9. $(x - a)^2 + 2ax = x^2 - 2ax + a^2 + 2ax = x^2 + a^2$
And $(x^2 + a^2)^4 = \begin{cases} x^4 + 2a^2x^2 + a^4 \\ x^4 + 2a^2x^2 + a^4 \end{cases}$
$$\begin{array}{l}
x^8 + 2a^2x^6 + a^4x^4 \\
\qquad 2a^2x^6 + 4a^4x^4 + 2a^6x^2 \\
\qquad\qquad a^4x^4 + 2a^6x^2 + a^8 \\
\hline
\end{array}$$
$\therefore \{(x - a)^2 + 2ax\}^4 = x^8 + 4a^2x^6 + 6a^4x^4 + 4a^6x^2 + a^8$

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[*] See Example 2, p. 460.

[†] See Example 3, p. 460.

[‡] See Note, p. 462.

10. $(x+1)(x-1)(x^2+1)=(x^2-1)(x^2+1) = x^4-1$, and $(x^4-1)^2 = x^8-2x^4+1$.

11. $(x-1)^2 = x^2-2x+1 \quad \therefore$

$$\begin{array}{r}
x^2-2x+1 \\
x^2-2x+1 \\
\hline
x^4-2x^3+x^2 \qquad\qquad \\
-2x^3+4x^2-2x \qquad \\
x^2-2x+1 \\
\hline
x^4-4x^3+6x^2-4x+1 \\
x^2-2x+1 \\
\hline
x^6-4x^5+6x^4-4x^3+x^2 \qquad\qquad\qquad \\
-2x^5+8x^4-12x^3+8x^2-2x \qquad \\
x^4-4x^3+6x^2-4x+1 \\
\hline
\end{array}$$

$$\therefore \{(x-1)^2\}^3 = x^6-6x^5+15x^4-20x^3+15x^2-6x+1.$$

NOTE.—In working the foregoing examples, it will be seen that frequent application has been made of the principles marked 1, 2, 3, at page 460. They should always be used in like manner whenever opportunity occurs. The Binomial Theorem, in the next article, is, however, of still wider application, and you should make yourself well acquainted with its form: the eleventh example here worked out in full, is solved in a moment by aid of the Binomial Theorem, as you will see by a reference to the table of developments at the middle of page 461. The example in question is to find the sixth power of $x-1$; for $\{(x-1)^2\}^3$ is the same as $(x-1)^6$. If you suppose a in the table to be -1, you will see that the development of $(x-1)^6$ is the same as that found above by actual multiplication; the directions at page 462 show how this development might have been written down at once.

BINOMIAL THEOREM. EXAMPLES FOR EXERCISE. PAGE 462.

1. To develop $(a+y)^4$. The terms without the coefficients are

$$a^4,\ a^3y,\ a^2y^2,\ ay^3,\ y^4.$$

The coefficients, as far as the third, are

$$1,\ 4,\ \frac{3\times 4}{2} = 6$$

$$\therefore (a+y)^4 = a^4+4a^3y+6a^2y^2+4ay^3+y^4.$$

Instead of finding the terms without the coefficients, and then the coefficients in a separate step, it is just as easy to write down each term in succession in its complete form, attending to the directions at page 461.

2. $(a-x)^5 = a^5-5a^4x+10a^3x^2-10a^2x^3+5ax^4-x^5$.

3. $(a+2x)^3 = a^3+3a^2(2x)+3a(2x)^2+(2x)^3$
$= a^3+6a^2x+12ax^2+8x^3$.

4. $(1-x)^6 = 1-6x+15x^2-20x^3+15x^4-6x^5+x^6$.

In this example the powers of 1, the first term of the binomial $1-x$, are, of course, suppressed; but the exponents of those powers are mentally employed, as in other cases, to form the coefficients.

5. $(1+3x)^4 = 1+4(3x)+6(3x)^2+4(3x)^3+(3x)^4$
$= 1+12x+54x^2+108x^3+81x^4$.

6. $(x+2y^2)^5 = x^5+5x^4(2y^2)+10x^3(2y^2)^2+10x^2(2y^2)^3+5x(2y^2)^4+(2y^2)^5$
$= x^5+10x^4y^2+40x^3y^4+80x^2y^6+80xy^8+32y^{10}$.

7. $(x^2+3y^2)^5 = (x^2)^5+5(x^2)^4(3y^2)+10(x^2)^3(3y^2)^2+10(x^2)^2(3y^2)^3+5(x^2)(3y^2)^4+(3y^2)^5$
$= x^{10}+15x^8y^2+90x^6y^4+270x^4y^6+405x^2y^8+243y^{10}$.

8. $(2a-x)^6 = (2a)^6-6(2a)^5x+15(2a)^4x^2-20(2a)^3x^3+15(2a)^2x^4-6(2a)x^5+x^6$
$= 64a^6-192a^5x+240a^4x^2-160a^3x^3+60a^2x^4-12ax^5+x^6$.

9. $(x-2y^2)^7 = x^7-7x^6(2y^2)+21x^5(2y^2)^2-35x^4(2y^2)^3+35x^3(2y^2)^4-21x^2(2y^2)^5+7x(2y^2)^6-(2y^2)^7$
$= x^7-14x^6y^2+84x^5y^4-280x^4y^6+560x^3y^8-672x^2y^{10}+448xy^{12}-128y^{14}$.

DIVISION. CASE I. EXAMPLES FOR EXERCISE. PAGE 463.

1. $\frac{8ax^2y^2}{4axy} = 2xy$.
2. $\frac{-6bx^3y^2z}{2bx^3y} = -3yz$.
3. $\frac{-12x^3y^4z^5}{-3x^2y^3z^2} = 4xyz^3$.
4. $\frac{16ax^2y^3}{-4a^2xy^2} = -\frac{4xy}{a}$.
5. $\frac{7ax^4yz^3}{-5a^2x^2yz^3} = -\frac{7x^2}{5a}$.
6. $\frac{-8b^6y^3z}{6b^3y^6z^2} = -\frac{4b^3}{3y^3z}$.
7. $\frac{24cy^3}{-21c^2y^2z} = -\frac{8y}{7cz}$.
8. $\frac{-36a^2x^2y^3}{2\times 7a^2x^4y} = -\frac{18y^2}{7x^2}$.
9. $\frac{5abx^2\sqrt{y}}{10ab^2x\sqrt{y}} = \frac{x}{2b}$.
10. $\frac{-3a^2c^3x^4}{-12ac^2x^5\sqrt{z}} = \frac{ac}{4x\sqrt{z}}$.
11. $\frac{-2mx^4y^3}{8m^2x^4y^6} = -\frac{1}{4my^3}$.
12. $\frac{-13x^2y^5z^{\frac{1}{2}}}{-26y^4\sqrt{z}} = \frac{x^2y}{2}$.

DIVISION. CASE II. PAGE 463.

1. $\frac{9a^2x^2-3a^3x^3+6a^4x}{3a^2x} = 3x-ax^2+2a^2$.

2. $\frac{12x^3yz^4+6x^2y^3z^3-3x^2y^3}{3x^2yz^2} = 4xz^2+2y^2-\frac{y}{z^2}$.

3. $$\frac{3ax(4xy - 8ay^2) + 12axy}{6axy} = \frac{12ax^2y - 24a^2xy^2 + 12axy}{6axy} = 2x - 4ay + 2.$$

4. $$\frac{8x^3y^4 - 4(4ax - 2y)x}{4x^2y} = \frac{8x^3y^4 - 16ax^2 + 8xy}{4x^2y} = 2xy^3 - \frac{4a}{y} + \frac{2}{x}.$$

The last two examples might have been treated somewhat more simply; thus, cancelling $3ax$ from dividend and divisor in Example 3, we have

$$\frac{4xy - 8ay^2 + 4y}{2y} = 2x - 4ay + 2$$

It is easy to see that instead of $3ax$, $6ax$ might have been cancelled. In like manner, in Example 4, by cancelling $4x$, we have

$$\frac{2x^2y^4 - 4ax + 2y}{xy} = 2xy^3 - \frac{4a}{y} + \frac{2}{x}.$$

5. Cancelling 2, from dividend and divisor, we have

$$\frac{4a^2x^2y^2 - 3ax^2y^2z^2 + 1}{2axy} = 2axy - \frac{3}{2}xy^2z^2 + \frac{1}{2axy}.$$

6. $$\frac{16a^4b^2x^3 - 8 + 3a^2bx^4}{4a^2bx^3} = 4a^2bx + \frac{3x^2}{4} - \frac{2}{a^2bx^3}.$$

DIVISION. CASE III. PAGE 464.

1. $$\begin{array}{l} x - 7)x^2 - 2x - 35(x + 5. \\ \quad \underline{x^2 - 7x} \\ \qquad 5x - 35 \\ \qquad \underline{5x - 35} \end{array}$$

2. $$\begin{array}{l} x + 3)x^2 - x - 12(x - 4. \\ \quad \underline{x^2 + 3x} \\ \qquad - 4x - 12 \\ \qquad \underline{- 4x - 12} \end{array}$$

3. $$\begin{array}{l} 3x + 2)6x^2 + 13x + 6(2x + 3. \\ \quad \underline{6x^2 + 4x} \\ \qquad 9x + 6 \\ \qquad \underline{9x + 6} \end{array}$$

4. $$\begin{array}{l} 3x + 2)12x^2 + 29x + 14(4x + 7. \\ \quad \underline{12x^2 + 8x} \\ \qquad 21x + 14 \\ \qquad \underline{21x + 14} \end{array}$$

5. $$\begin{array}{l} 3x^2 - 2x + 5)18x^3 - 33x^2 + 44x - 35(6x - 7. \\ \quad \underline{18x^3 - 12x^2 + 30x} \\ \qquad - 21x^2 + 14x - 35 \\ \qquad \underline{- 21x^2 + 14x - 35} \end{array}$$

6. $$\begin{array}{l} x - y)x^4 - y^4(x^3 + x^2y + xy^2 + y^3. \\ \quad \underline{x^4 - x^3y} \\ \qquad x^3y - y_4 \\ \qquad \underline{x^3y - x^2y^2} \\ \qquad\quad x^2y^2 - y^4 \\ \qquad\quad \underline{x^2y^2 - xy^3} \\ \qquad\qquad xy^3 - y^4 \\ \qquad\qquad \underline{xy^3 - y_4} \end{array}$$

Example 6 may be worked differently, thus:

$$\frac{x^4 - y^4}{x - y} = \frac{(x^2 + y^2)(x^2 - y^2)}{x - y} = (x^2 + y^2)(x + y), \therefore \left.\begin{array}{l} x^2 + y^2 \\ x + y \end{array}\right\} \text{by multiplying}$$

$$\overline{\underline{x^3 + xy^2 + x^2y + y^3.}}$$

7. $$\begin{array}{l} x - a)x^4 - ax^3 - 7ax^2 + 8a^2x - a^3(x^3 - 7ax + a^2. \\ \quad \underline{x^4 - ax^3} \\ \qquad - 7ax^2 + 8a^2x \\ \qquad \underline{- 7ax^2 + 7a^2x} \\ \qquad\quad a^2x - a^3 \\ \qquad\quad \underline{a^2x - a^3} \end{array}$$

8. $$\begin{array}{l} x + a)x^2 + (a + b)x + ab(x + b \\ \quad \underline{x^2 + ax} \\ \qquad bx + ab \\ \qquad \underline{bx + ab} \end{array}$$

9. $$\begin{array}{l} 4x^2y + 3xy - 1)8x^4y + 2x^3y - 2x^2 - 3x^2y + x(2x^2 - x. \\ \quad \underline{8x^4y + 6x^3y - 2x^2} \\ \qquad - 4x^3y - 3x^2y + x \\ \qquad \underline{- 4x^3y - 3x^2y + x} \end{array}$$

10. $$\begin{array}{l} x^2 + x - 1)x^6 - x^4 + x^3 - x^2 - 1(x^4 - x^3 + x^2 - x + 1 - \dfrac{2x}{x^2 + x - 1} \\ \quad \underline{x^6 + x^5 - x} \\ \qquad - x^5 + x^3 - x^2 \\ \qquad \underline{- x^5 - x^4 + x^3} \\ \qquad\quad x^4 - x^2 - 1 \\ \qquad\quad \underline{x^4 + x^3 - x^2} \\ \qquad\qquad - x^3 - 1 \\ \qquad\qquad \underline{- x^3 - x^2 + x} \\ \qquad\qquad\quad x^2 - x - 1 \\ \qquad\qquad\quad \underline{x^2 + x - 1} \\ \qquad\qquad\qquad - 2x \quad \text{Remainder.} \end{array}$$

11. Cancelling x, from dividend and divisor, and removing the vincula, the operation is as follows:

$$3x-3)6x^3+9x^2-20(2x^2+5x+5-\frac{4}{3x-3}$$
$$\underline{6x^3-6x^2}$$
$$15x^2-20$$
$$\underline{15x^2-15x}$$
$$15x-20$$
$$\underline{15x-15}$$
$$\underline{-4} \text{ Rem.}$$

12. $$x-a)x^2+px+q(x+a+p+\frac{a^2+pa+q}{x-a}$$
$$\underline{x^2-ax}$$
$$(a+p)x+q$$
$$\underline{(a+p)x-a^2-pa}$$
$$\underline{a^2+pa+q} \text{ Rem.}$$

13. $$x-a)x^3+px^2+qx+r(x^2+(a+p)x+a^2+pa+q+\frac{a^3+pa^2+qa+r}{x-a}$$
$$\underline{x^3-ax^2}$$
$$(a+p)x^2+qx$$
$$\underline{(a+p)x^2-(a^2+pa)x}$$
$$(a^2+pa+q)x+r$$
$$\underline{(a^2+pa+q)x-a^3-pa^2-qa}$$
$$a^3+pa^2+qa+r \text{ Rem.}$$

The final remainder in Example 12 is a^2+pa+q, and that in Example 13 is a^3+pa^2+qa+r; in each case the remainder is the very same expression as we should get by rubbing out the x in the dividend and putting a instead, as observed at page 462. The property is perfectly general: if any polynomial proceeding according to the powers of x be divided by $x-a$, the remainder will always be the same as the polynomial itself when the x is replaced by a. Hence, when a is any given number, you will be able to ascertain, without actual division, whether a proposed polynomial is divisible by $x-a$ or not: thus, suppose it were asked—Is $x^2-2x-35$ divisible by $x-7$? Putting 7 for x in $x^2-2x-35$, we get $49-14-35=0$. Hence, the *remainder* is *nothing*, so that the expression *is* divisible by $x-7$. (See Ex. 1).

In like manner, referring to Ex. 2, and substituting -3 for x in x^2-x-12, it becomes $9+3-12=0$. Hence, x^2-x-12 is divisible by $x+3$. And a glance will serve to show that the division may be performed without remainder in Ex. 7.

SQUARE ROOT OF A SIMPLE QUANTITY. EXAMPLES FOR EXERCISE. PAGE 466.

1. $\sqrt{a^4x^6y}=a^2x^3\sqrt{y}$.
2. $\sqrt[4]{2^4a^4b^8}=\pm 2ab^2$.
3. $\sqrt[3]{8a^3x^6z}=2ax^2\sqrt[3]{z}$.
4. $\sqrt[3]{-8a^4x^3y^2}=-2ax\sqrt[3]{ay^2}$.
5. $\sqrt{-2a^4y^6z^8}=a^2y^3z^4\sqrt{-2}$.
6. $\sqrt{3a^5y^4x}=a^2y^2\sqrt{3ax}$.
7. $(a^2b^3x^4)^{\frac{1}{2}}=abx^2b^{\frac{1}{2}}$.
8. $\sqrt{x^{\frac{1}{2}}y^2z^2}=yz\sqrt[4]{x}$.
9. $\sqrt{\frac{a^2}{bx^2}}=\frac{a}{x\sqrt{b}}=\frac{a\sqrt{b}}{bx}$.
10. $\sqrt{a^3x^{-4}}=ax^{-2}\sqrt{a}$.
11. $\sqrt{a^{-3}x^{-4}y^{-5}}=a^{-\frac{3}{2}}x^{-2}y^{-\frac{5}{2}}$.
12. $\sqrt{\frac{8a^3}{27x^3y^4}}=\sqrt{\frac{4.2a^3}{9.3x^3y^4}}=\frac{2a}{3xy^2}\sqrt{\frac{2a}{3x}}=\frac{2a}{3xy^2}\frac{\sqrt{6ax}}{3x}$.
13. $(-8a^3x^2y^6)^{\frac{1}{3}}=-2ay^2\sqrt[3]{x^2}$.
14. $(9b^4x^2y^3)^{-\frac{1}{2}}=\frac{1}{\sqrt{9b^4x^2y^3}}=\frac{1}{3b^2xy\sqrt{y}}=\frac{\sqrt{y}}{3b^2xy^2}$.
15. $x^2(81x^6y^7z^8)^{-\frac{1}{3}}=\frac{x^2}{\sqrt[3]{81x^6y^7z^8}}=\frac{x^2}{3x^2y^2z^2\sqrt[3]{3yz^2}}=\frac{1}{3y^2z^2\sqrt[3]{3yz^2}}=\frac{(3yz^2)^{\frac{2}{3}}}{9y^3z^4}=\frac{\sqrt[3]{9y^2z}}{9y^3z^3}$.
16. $\sqrt[3]{-16a^5x^6y^{-9}}=-2ax^2y^{-3}\sqrt[3]{2a^2}$.
17. $(32a^{\frac{1}{2}}x^2y^4)^{-\frac{1}{2}}=32^{-\frac{1}{2}}a^{-\frac{1}{4}}x^{-1}y^{-2}=\frac{1}{4xy^2\sqrt{2}.\sqrt[4]{a}}=\frac{\sqrt{2}.\sqrt[4]{a}}{8axy^2}$.
18. $(32a^{-3}x^{-6}y^3)^{-\frac{1}{3}}=32^{-\frac{1}{3}}ax^2y^{-1}=\frac{ax^2}{2y\sqrt[3]{4}}=\frac{ax^2\sqrt[3]{16}}{8y}=\frac{ax^2\sqrt[3]{2}}{4y}$.

REDUCTION OF FRACTIONS. EXAMPLES FOR EXERCISE. PAGE 467.

1. $a+x+\frac{x^2}{a-x}=\frac{a^2-x^2+x^2}{a-x}=\frac{a^2}{a-x}$.
2. $3-y-\frac{2-y^2}{3+y}=\frac{9-y^2-2+y^2}{3+y}=\frac{7}{3+y}$.
3. $\left(\frac{b+c}{b-c}\right)^2-1=\frac{b^2+2bc+c^2-(b^2-2bc+c^2)}{(b-c)^2}=\frac{4bc}{(b-c)^2}$.
4. $\frac{a^3+x^3}{a-x}=$, by actual division, $a^2+ax+x^2+\frac{2x^3}{a-x}$.
5. By actual division, $\frac{a^3-x^3}{a^2+ax+x^2}=a-x$.
6. $x^2-a^2+\frac{x^4+a^4}{x^2+a^2}=\frac{x^4-a^4+x^4+a^4}{x^2+a^2}=\frac{2x^4}{x^2+a^2}$.
7. $\frac{(x+y+z)(x+y-z}{2xy}-1=\frac{[(x+y)+z][(x+y)-z]-2xy}{2xy}=\frac{(x+y)^2-z^2-2xy}{2xy}=\frac{x^2+y^2-z^2}{2xy}$.

8. $\frac{[(x+y)+z][(x+y)-z]}{2xy}-2=\frac{(x+y)^2-z^2-4xy}{2xy}=\frac{x^2-2xy+y^2-z^2}{2xy}=\frac{(x-y)^2-z^2}{2xy}.$

9. $\frac{x^2+y^2-z^2}{2xy}-1=\frac{x^2-2xy+y^2-z^2}{2xy}=\frac{(x-y)^2-z^2}{2xy}=\frac{(x-y-z)(x+y+z).}{2xy}$

10. $x^2-3x-\frac{3x\ (3-x}{x-2}=\frac{x\{(x-3)(x-2)-3(3-x)\}}{x-2}=$

$\frac{x\{(x-3)(x-2)+3(x-3)\}}{x-2}=\frac{x(x-3)(x-2+3)}{x-2}=\frac{x(x-3)(x+1)}{x-2}$

$=\frac{x(x^2-2x-3)}{x-2}=\frac{x(x^2-2x+1-4)}{x-2}=\frac{x\{(x-1)^2-4\}}{x-2}.$

ADDITION AND SUBTRACTION OF FRACTIONS. EXAMPLES FOR EXERCISE. PAGE 468.

1. $\frac{2x-5}{3}+\frac{x-1}{2x}=\frac{4x^2-10x+3x-3}{6x}=\frac{4x^2-7x-3}{6x}.$

2. $\frac{x}{x-3}+\frac{x}{x+3}=\frac{x^2+3x+x^2-3x}{x^2-9}=\frac{2x^2}{x^2-9}.$

3. $\frac{10x-9}{8}-\frac{3x-5}{7}=\frac{70x-63-24x+40}{56}=\frac{46x-23}{56}.$

4. $\frac{2x-3}{7x}-\frac{x-5}{8x}=\frac{16x^2-24x-7x^2+35x}{56x^2}=\frac{9x^2+11x}{56x^2}=\frac{9x+11}{56x}.$

5. $\frac{x-a}{x^2-ax+a^2}+\frac{1}{x+a}=\frac{x^2-a^2+x^2-ax+a^2}{x^3+a^3}=\frac{2x^2-ax}{x^3+a^3}.$

6. $1-\frac{x-a}{x+a}=\frac{x+a-x+a}{x+a}=\frac{2a}{x+a}$ 7. $\frac{a^2z}{xy}-\frac{b^2x}{yz}=\frac{a^2z^2-b^2x^2}{xyz}.$

In this seventh example, the terms of the first fraction are multiplied by z, and those of the second by x; the denominators thus become each equal to xyz.

8. $\frac{1}{x}+\frac{1}{x^2}-\frac{1}{x^n}$. Multiplying the terms of the first fraction by x^{n-1}, and those of the second by x^{n-2}, we have $\frac{x^{n-1}+x^{n-2}-1}{x^n}$

9. $\frac{1}{2x-a}+\frac{1}{2x+a}-\frac{4x^2}{4x^2-a^2}=\frac{2x+a+2x-a-4x^2}{4x^2-a^2}=\frac{4x-4x^2}{4x^2-a^2}.$

10. $\frac{1+x}{1+x+x^2}-\frac{1-x}{1-x+x^2}=\frac{(1+x)(1-x+x^2)-(1+x+x^2)(1-x)}{\{(1+x^2)+x\}\ \{(1+x^2)-x\}}=\frac{(1+x)x^2-x^2(1-x)}{(1+x^2)^2-x^2}=\frac{2x^3}{1+x^2+x^4}$

11. $\frac{a^2+b^2}{a^2-b^2}+\frac{a+b}{a-b}-\frac{a-b}{a+b}+2=\frac{a^2+b^2+a^2-b^2-(a^2-b^2)+2(a^2-b^2)}{a^2-b^2}=\frac{3a^2-b^2}{a^2-b^2}.$

12. $\frac{y}{y-3}-\frac{y-3}{y}+\frac{y}{y+3}-\frac{y+3}{y}.$

The second and fourth fractions united give $-\frac{2y}{y}=-2$, the first and third make $\frac{y^2+3y+y^2-3y}{y^2-9}=\frac{2y^2}{y^2-9}$. Hence the sum of all is $\frac{2y^2}{y^2-9}-2=\frac{2y^2-2y^2+18}{y^2-9}=\frac{18}{y^2-9}.$

13. $\frac{x}{x+y}+\frac{y}{x-y}=\frac{x^2-xy+xy+y^2}{(x+y)(x-y)}=\frac{x^2+y^2}{x^2-y^2}$

and $\frac{x}{x-y}-\frac{y}{x+y}=\frac{x^2+xy-xy+y^2}{(x-y)(x+y)}=\frac{x^2+y^2}{x^2-y^2}$

14. $\frac{x}{x+z}-\frac{y}{y+z}=\frac{xy+xz-xy-yz}{(x+z)(y+z)}=\frac{(x-y)z}{(x+z)(y+z)}$

and $\frac{z}{y+z}-\frac{z}{x+z}=\frac{xz+z^2-yz-z^2}{(y+z)(x+z)}=\frac{(x-y)z}{(x+z)(y+z)}.$

MULTIPLICATION OF FRACTIONS. EXAMPLES FOR EXERCISE. PAGE 469.

1. $\frac{4x}{7}\times\frac{5}{9x}=\frac{4}{7}\times\frac{5}{9}=\frac{20x}{63x}$ 2. $\frac{4x+1}{3}\times\frac{6x}{7}=(4x+1)\frac{2x}{7}=\frac{8x^2+2x}{7}.$

3. $\frac{4x+2}{3}\times\frac{5x}{2x+1}=\frac{2}{3}\times 5x=\frac{10x}{3}.$ 4. $\frac{2x-1}{x+2}\times\frac{3x+1}{x-3}=\frac{6x^2-x-1}{x^2-x-6}.$

5. $\frac{a+b}{b}\times\frac{b^2x}{a^2-b^2}=1\times\frac{bx}{a-b}=\frac{bx}{a-b}.$ 6. $\frac{x^3-a^2x}{a^2}\times\frac{x}{ax-a^2}=\frac{x^4-a^2x^2}{a^3x-a^3}.$

7. $\frac{x^n}{y^m}\times\frac{x^m}{y^n}=\frac{x^{m+n}}{y^{m+n}}.$ 8. $\frac{x^n}{y^m}\times\frac{y^n}{x^m}=\frac{xy^{2n}}{xy^{2m}}.$

9. $\frac{3\sqrt{x}}{y}\times\frac{2y^2}{5\sqrt{x}}=\frac{6y}{5}.$ 10. $\frac{x^2-4a^2}{x-a}\times\frac{x^3-a^3}{x+2a}=(x-2a)(x^2+ax+a^2)=x^3-ax^2-a^2x-2a^3.$

In this tenth example it is seen at once that the numerator of the second fraction is divisible by the denominator of the first, because if a be put for x in x^3-a^3, the result is 0.*

11. $\frac{2x}{7}\times\frac{4x}{9}\times\frac{7}{4x}\times\frac{9x}{5}=2x\times 1\times\frac{x}{5}=\frac{2x^2}{5}.$

12. $\frac{4x+2}{3}\times\frac{5x}{2x+1}\times\frac{9x-3}{x}=2\times 5\times(3x-1)=30x-10.$ 13. $\frac{(x-y)^2}{x^2-y^2}\times\frac{x+y}{x-y}=\frac{x-y}{x-y}=1.$

14. $\frac{(x-y)^2}{x^4-y^4}\times\frac{x+y}{x-y}=\frac{x-y}{(x^2-y^2)(x^2+y^2)}\times(x+y)=\frac{x-y}{(x-y)(x^2+y^2)}=\frac{1}{x^2+y^2}.$

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* See p. 495.

15. $(a-\frac{b^2}{a})(\frac{a}{b}+\frac{b}{a})=\frac{a^2-b^2}{a}\cdot\frac{a^2+b^2}{ab}=\frac{a^4-b^4}{a^2b}$; Otherwise,

$$(a-\frac{b^2}{a})(\frac{a}{b}+\frac{b}{a})=b(\frac{a}{b}-\frac{b}{a})(\frac{a}{b}+\frac{b}{a})=b(\frac{a^2}{b^2}-\frac{b^2}{a^2})=b\,\frac{a^4-b^4}{a^2b^2}=\frac{a^4-b^4}{a^2b}.$$

DIVISION OF FRACTIONS. EXAMPLES FOR EXERCISE. PAGE 469.

1. $\frac{x}{x-2}\times\frac{x^2+2}{x-1}=\frac{x^3+2x}{x^2-3x+2}$.

2. $\frac{a+1}{a}\times\frac{a}{a^2-1}=\frac{1}{a-1}$.

3. $\frac{x^2-9}{5}\div\frac{x+3}{4}=\frac{x-3}{5}\div\frac{1}{4}=\frac{x-3}{5}\times 4=\frac{4x-12}{5}$.

4. $\frac{6x^2-2x}{4-x^2}\div\frac{x^2}{2+x}=\frac{6x-2}{2-x}\div x=\frac{6x-2}{2x-x^2}$.

5. $\frac{9x^2-4y^2}{x+y}\div\frac{3x-2y}{x^2-y^2}=\frac{3x+2y}{1}\div\frac{1}{x-y}=(3x+2y)(x-y)=3x^2-xy-2y^2$.

6. $\frac{x^2-y^2}{x^2-2xy+y^2}\div\frac{x^2+xy}{x-y}=\frac{x+y}{x-y}\div\frac{x(x+y)}{x-y}=1\div x=\frac{1}{x}$.

SIMPLE EQUATIONS IN GENERAL. EXAMPLES IN EQUATIONS WITH ONLY ONE UNKNOWN QUANTITY. PAGE 470.

1. $\frac{2x}{5}+\frac{x}{2}=22+\frac{x}{5}$. Multiplying the terms of the second fraction by 3,

$$\frac{2x}{5}+\frac{3x}{6}=22+\frac{x}{6}.$$

Transposing, $\frac{2x}{5}+\frac{x}{3}=22$.

Clearing, $6x+5x=330$, that is, $11x=330$ $\therefore x=30$.

This might also have been readily solved by multiplying each side of the proposed equation by 30, the least number divisible by all the denominators; the equation thus cleared of fractions would have been

$$12x+15x=660+5x$$
$$\therefore 27x-5x=660,\ \therefore 22x=660,\ \therefore x=30.$$

2. $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=6\frac{1}{2}$. Multiplying by 12, the least common multiple of the denominators, we have

$$6x+4x+3x=78,$$
that is, $13x=78$, $\therefore x=6$.

3. $\frac{x}{2}-\frac{x}{4}=\frac{x}{3}-\frac{x}{6}+1$, or $\frac{2x}{4}-\frac{x}{4}=\frac{2x}{6}-\frac{x}{6}+1$; that is,

$$\frac{x}{4}=\frac{x}{6}+1.$$

Multiplying by 12, $3x=2x+12$, $\therefore x=12$.

4. $\frac{18+x}{x}=\frac{6}{x}+5$, that is, $\frac{18}{x}+1=\frac{6}{x}+5$, $\therefore \frac{12}{x}=4\ \therefore 12=4x\ \therefore x=3$.

5. $21+\frac{3x-11}{16}=\frac{5(x-1)}{8}+\frac{97-7x}{2}$. Multiplying by 16,

$$336+3x-11=10x-10+776-56x.$$

Transposing, $56x+3x-10x=776+11-10-336$

$$\therefore 49x=441,\ \therefore x=9.$$

6. $\sqrt{(x^2+7)}+x=7$, $\therefore \sqrt{(x^2+7)}=7-x$.

Squaring, $x^2+7=49-14x+x^2$, $\therefore 14x=42$, $\therefore x=3$.

7. $\sqrt{(x-a)}-b=0$, $\therefore \sqrt{(x-a)}=b$, $\therefore x-a=b^2$, $\therefore x=a+b^2$.

8. $\frac{\sqrt{x}+2}{\sqrt{x}-2}=\frac{3}{2}$. Applying the principle explained at page 468, we have $\frac{\sqrt{x}}{2}=\frac{5}{1}$, $\therefore \sqrt{x}=10$, $\therefore x=100$.

9. $\frac{\sqrt{x}+28}{\sqrt{x}+4}=\frac{\sqrt{x}+38}{\sqrt{x}+6}$. Applying the principle referred to in the last example, we have

$$\frac{\sqrt{x}+16}{12}=\frac{\sqrt{x}+22}{16}.$$ Multiplying by 3.4.4,

$$4\sqrt{x}+64=3\sqrt{x}+66,\ \therefore \sqrt{x}=2,\ \therefore x=4.$$

10. $\frac{\sqrt{x}+2a}{\sqrt{x}+b}=\frac{\sqrt{x}+4a}{\sqrt{x}+3b}$. By the principle at page 468,

$$\frac{2\sqrt{x}+2a+b}{2a-b}=\frac{2\sqrt{x}+4a+3b}{4a-3b};$$ that is,

$$\frac{2\sqrt{x}}{2a-b}+\frac{2a+b}{2a-b}=\frac{2\sqrt{x}}{4a-3b}+\frac{4a+3b}{4a-3b}.$$

Transposing, $\frac{2\sqrt{x}}{2a-b}-\frac{2\sqrt{x}}{4a-3b}=\frac{4a+3b}{4a-3b}-\frac{2a+b}{2a-b}$.

Multiplying by $(2a-b)(4a-3b)$,

$$8a\sqrt{x}-6b\sqrt{x}-4a\sqrt{x}+2b\sqrt{x}=(2a-b)(4a+3b)-(2a+b)(4a-3b)$$
$$\therefore (4a-4b)\sqrt{x}=-8ab+12ab=4ab,$$
$$\therefore \sqrt{x}=\frac{ab}{a-b},\ \therefore x=\left(\frac{ab}{a-b}\right)^2.$$

11. $\frac{\sqrt{ax}-a}{\sqrt{ax}+a}=\frac{3\sqrt{ax}-2a}{3\sqrt{ax}+5a}$. Applying the foregoing principle, $\frac{\sqrt{ax}}{a}=\frac{6\sqrt{ax}+3a}{7a}$,

$$\text{Clearing, } 7\sqrt{ax}=6\sqrt{ax}+3a,\ \therefore\ \sqrt{ax}=3a$$
$$\therefore\ ax=9a^2,\ \therefore\ x=9a.$$

12. $\sqrt{(x-24)}=\sqrt{x}-2$. Squaring each side,

$$x-24=x-4\sqrt{x}+4.$$
$$\text{Transposing, } 4\sqrt{x}=28,\ \therefore\ \sqrt{x}=7,\ \therefore\ x=49.$$

13. $\sqrt{(4x+21)}=2\sqrt{x}+1$. Squaring each side,

$$4x+21=4x+4\sqrt{x}+1.$$
$$\text{Transposing, } 20=4\sqrt{x},\ \therefore\ \sqrt{x}=5,\ \therefore\ x=25.$$

14. $\sqrt[3]{(2x+5)}=3$. Cubing each side,

$$2x+3=27,\ \therefore\ 2x=24,\ \therefore\ x=12.$$

15. $\frac{5x-9}{\sqrt{5x}+3}=\frac{\sqrt{5x}-1}{2}$. The numerator of the first fraction is the difference of the squares of the two quantities in the denominator; therefore, the numerator is actually divisible by the denominator, so that the equation is the same as

$$\sqrt{5x}-3=\frac{\sqrt{5x}-1}{2},\ \therefore\ 2\sqrt{5x}-6=\sqrt{5x}-1,$$
$$\therefore\ \sqrt{5x}=5,\ \therefore\ 5x=25,\ \therefore\ x=5.$$

16. $\frac{\sqrt{(x+1)}-\sqrt{(x-1)}}{\sqrt{(x+1)}+\sqrt{(x-1)}}=\frac{1}{2}$. By the principle at page 468,

$$\frac{\sqrt{(x+1)}}{\sqrt{(x-1)}}=\frac{3}{1},\ \therefore\ \frac{x+1}{x-1}=9$$
$$\therefore\ x+1=9x-9,\ \therefore\ 10=8x,\ \therefore\ x=\frac{10}{8}=1\tfrac{1}{4}.$$

SIMPLE EQUATIONS WITH TWO UNKNOWN QUANTITIES.
EXAMPLES FOR EXERCISE. PAGE 471.

1. $\left.\begin{array}{l}2x+3y=23\\5x-2y=10\end{array}\right\}$ 1st. Proceeding by Rule I., we have from the first equation $x=\frac{23-3y}{2}$; and from the second $x=\frac{10+2y}{5}$; hence equating these two expressions for x,

$$\frac{23-3y}{2}=\frac{10+2y}{5},$$
$$\text{Clearing fractions, } 115-15y=20+4y,$$
$$\text{Transposing, } 95=19y,\ \therefore\ y=5,$$
$$\therefore\ x=\frac{10+2y}{5}=\frac{10+10}{5}=\frac{20}{5}=4.$$

2nd. Proceeding by Rule II., we have from the second equation $x=\frac{10+2y}{5}=2+\frac{2y}{5}$.

Substituting this in the first we get

$$4+\frac{4y}{5}+3y=23,\ \therefore\ \frac{4y}{5}+3y=19,$$
$$\text{Clearing, } 4y+15y=95,\ \therefore\ 19y=95,\ \therefore\ y=5;$$
$$\text{And since } x=\frac{10+2y}{5},\ \therefore\ x=\frac{20}{5}=4.$$

3rd. Proceeding by the third method, multiplying the first equation by 2, the second by 3, and then adding the results, we have

$$19x=76,\ \therefore\ x=4.$$

Again, multiplying the first equation by 5, the second by 2, and subtracting the results, we get

$$19y=95,\ \therefore\ y=5.$$

2. $\left.\begin{array}{l}5x+4y=58\\3x+7y=67\end{array}\right\}$ Multiplying the first equation by 3, and the second by 5, there results

$$\begin{array}{r}15x+12y=174\\15x+35y=335\\\hline\end{array}$$
$$\text{Subtracting,}\quad 23y=161,\ \therefore\ y=\frac{161}{23}=7.$$

And since from the first equation $x=\frac{58-4y}{5}$,

$$\therefore\ x=\frac{58-28}{5}=6.$$

Otherwise, thus: multiplying the first equation by 2, we have

$$\begin{array}{lr}
 & 10x+8y=116\\
 & 3x+7y=\ \ 67\\\hline
\text{Subtracting,} & 7x+\ y=\ \ 49,\ \therefore\ y=49-7x\\
\text{Multiplying by 3,} & 21x+3y=147\\
\text{First equa.} & 5x+4y=\ \ 58\\\hline
\text{Subtracting,} & -16x+y=-89,\ \therefore\ y=16x-89
\end{array}$$
$$\therefore\ 16x-89=49-7x,\ \therefore\ 23x=138,\ \therefore\ x=6,$$
$$\therefore\ y=49-7x=49-42=7.$$

3. $\left.\begin{array}{l}\frac{x}{8}+8y=194\\\frac{y}{8}+8x=131\end{array}\right\}$ Clearing fractions, these equations become

$$x+64y=1552$$
$$y+64x=1048$$

By adding these $65x+65y=2600,\ \therefore\ y+x=\frac{2600}{65}=40$

And by subtracting, $63y-63x=504,\ \therefore\ y-x=\frac{504}{63}=8$

$\therefore$ adding and subtracting, $2y=48$, and $2x=32$,

$$\therefore\ y=24, \text{ and } x=16.$$

NOTE.—The rule has been departed from in the preceding solution; and you perceive that considerable advantage on the score of neatness and simplicity has been gained in consequence. By the rule we should have proceeded thus:

From the first equation $x=1552-64y$

Substituting in the second, $y+64(1552-64y)=1048$;

That is, $64\times1552-1048=(64^2-1)y$

$$\therefore\ y=\frac{64\times1552-1018}{64^2-1}=\frac{64\times1552-1048}{65\times63}$$
$$=\frac{98280}{5.9.7.13}=24$$

And $x=1552-64y=1552-64\times24=16$.

4. $\left.\begin{array}{l}\frac{x+y}{3}=5\\\frac{y-x}{7}=1\end{array}\right\}$ Clearing, these equations become

$$x+y=15$$
$$y-x=\ \ 7$$

Adding and subtracting, we get

$$2y=22, \text{ and } 2x=8,\ \therefore\ y=11,\ x=4.$$

5. $\left.\begin{array}{l}\frac{3x-7y}{3}=\frac{2x+y+1}{5}\\2-\frac{x-y}{5}=0\end{array}\right\}$ Clearing fractions, we have

$$15x-35y=6x+3y+3$$
$$10-x+y=0$$

Or, transposing,
$$9x - 38y = 3 \ldots (1)$$
$$x - y = 10 \ldots (2)$$
$$\therefore 9x - 9y = 90 \ldots (3)$$
Subtracting (1) from (3), $29y = 87, \therefore y = 3$
$$\therefore x = 10 + y = 13.$$

6. $\left.\begin{aligned}\frac{x+2}{3} + 8y &= 31\\ \frac{y+5}{4} + 10x &= 192\end{aligned}\right\}$ Clearing the fractions, we have
$$x + 2 + 24y = 93$$
$$y + 5 + 40x = 768$$
Or, $x + 24y = 91$
$$40x + y = 763.$$
Multiplying the first by 40,
$$40x + 960y = 3640$$
Subtracting, $959y = 2877, \therefore y = \frac{2877}{959} = 3$
$$\therefore x = \frac{763 - y}{40} = \frac{760}{40} = 19.$$

7. $\left.\begin{aligned}ax + by &= 1\\ a'x + b'y &= 1\end{aligned}\right\}$ Equalising the coefficients of x, we have
$$aa'x + a'by = a'$$
$$aa'x + ab'y = a$$
Subtracting, $(ab' - a'b)y = a - a'$
$$\therefore y = \frac{a - a'}{ab' - a'b}.$$
In like manner equalising the coefficients of y,
$$ab'x + b'by = b'$$
$$a'bx + b'by = b$$
Subtracting, $(a'b - ab')x = b - b'$
$$\therefore x = \frac{b - b'}{a'b - ab'}.$$

8. $\left.\begin{aligned}ax + by &= c\\ a'x + b'y &= c'\end{aligned}\right\}$ Equalising the coefficients of x, we have
$$aa'x + a'by = a'c$$
$$aa'x + ab'y = ac'$$
Subtracting, $(ab' - a'b)y = ac' - a'c$
$$\therefore y = \frac{ac' - a'c}{ab' - a'b}.$$
In like manner equalising the coefficients of y, we have
$$ab'x + b'by = b'c$$
$$a'bx + b'by = bc'$$
Subtracting, $(a'b - ab')x = bc' - b'c$
$$\therefore x = \frac{bc' - b'c}{a'b - ab'}.$$

9. $\left.\begin{aligned}\frac{1}{x} - \frac{1}{y} &= 12\\ \frac{3}{x} + \frac{5}{y} &= 2\tfrac{5}{7}\end{aligned}\right\}$ Multiplying the first equation by 3, and then subtracting the second, we have,
$$-\frac{8}{y} = 36 - 2\tfrac{5}{7} = 33\tfrac{2}{7}.$$
Multiplying by $7y$, $-56 = 233y, \therefore y = -\frac{56}{233}.$

Also, $\frac{1}{x} = 12 + \frac{1}{y} = 12 - \frac{233}{56} = \frac{439}{56}, \therefore x = \frac{56}{439}$

10. $\left.\begin{aligned}\frac{x-2}{5} - \frac{10-x}{3} - \frac{y-10}{4} &= 0\\ \frac{2(y+2)}{3} - \frac{2x+y}{8} - \frac{x+13}{4} &= 0\end{aligned}\right\}$ Clearing fractions, these equations become,
$$12x - 24 - 200 + 20x - 15y + 150 = 0$$
$$16y + 32 - 6x - 3y - 6x - 78 = 0.$$
Transposing, $32x - 15y = 74$
$$-12x + 13y = 46.$$
Multiplying the first by 3, and the second by 8, we have,
$$96x - 45y = 222$$
$$-96x + 104y = 368$$
Adding, $59y = 590, \therefore y = 10;$

and since $13y - 46 = 12x, \therefore 130 - 46 = 12x,$
$$\therefore x = \frac{84}{12} = 7.$$

11. $\left.\begin{aligned}\frac{x+6}{x+4} &= \frac{y+2}{y-2}\\ \frac{x+3}{x+1} &= \frac{y-7}{y-13}\end{aligned}\right\}$ Applying to these fractions the general principle at page 468, we have
$$\frac{x+5}{1} = \frac{y}{2}, \therefore 2x + 10 = y$$
$$\frac{x+3}{1} = \frac{y-10}{3}, \therefore 3x + 9 = y - 10.$$
Equating the two expressions for y, we have
$$3x + 19 = 2x + 10, \therefore x = -9$$
$$\therefore y = 2x + 10 = -18 + 10 = -8.$$

12. $\left.\begin{aligned}x + y &= 19\\ x^2 - y^2 &= 95\end{aligned}\right\}$ Dividing the second equation by the first, we have
$$x - y = 5$$
Also, $x + y = 19$

$\therefore$ adding and subtracting, $2x = 24$, and $2y = 14$,
$$\therefore x = 12, \text{ and } y = 7.$$

QUESTIONS IN SIMPLE EQUATIONS. PAGE 472.

1. Find a number such that if it be increased by one-half, one-third, and one-fourth of itself, the sum shall be 50.

Let x be the number; then by the question,
$$x + \frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 50.$$
Multiplying by 12, to clear the fractions,
$$12x + 6x + 4x + 3x = 600,$$
that is, $25x = 600, \therefore x = 24.$

Otherwise.—To avoid fractions, put $12x$ for the number; then by the question,
$$12x + 6x + 4x + 3x = 50,$$
that is, $25x = 50, \therefore x = 2, \therefore 12x = 24.$

2. There is a fraction such that if 4 be added to the denominator, the value is $\frac{1}{4}$; and if 3 be added to the numerator the value is $\frac{3}{4}$: required the fraction.

Let the fraction be $\frac{x}{y}$; then by the question, $\frac{x}{y+4} = \frac{1}{4}$, and $\frac{x+3}{y} = \frac{3}{4}$,

therefore clearing the fractions, $4x = y + 4$
$$4x + 12 = 3y$$
Subtracting $12 = 2y - 4,$
$$\therefore 2y = 16, \therefore y = 8,$$
Also $x = \frac{y}{4} + 1 = 3, \therefore$ the fraction is $\frac{3}{8}$.

3. What number is that, such, that if it be increased by 7, the square root of the sum shall be equal to the square root of the number itself and 1 more?

Let x be the number, then the condition is that
$$\sqrt{(x+7)} = \sqrt{x} + 1;$$
Squaring, $x + 7 = x + 2\sqrt{x} + 1;$

Transposing $6 = 2\sqrt{x}, \therefore 3 = \sqrt{x}, \therefore 9 = x;$

hence the number is 9.

4. Fifty labourers are engaged to remove an obstruction on a railway; some of them are, by agreement, to receive ninepence each, and the others fifteen pence. Just £2 are paid to them; but, no memorandum having been made, it is required to find how many worked for 9*d.*, and how many for 15*d.*

Suppose there are x workmen at 9*d.*, and y at 15*d.*, then the number of pence received by the former is $9x$, and the number received by the latter $15y$; and since by the question the number of pence paid altogether is $2 \times 20 \times 12 = 480$, we must have the equation
$$9x + 15y = 480$$
Moreover, $x + y = 50$ by the question.
$$\therefore 9x + 9y = 450$$
Subtracting, $6y = 30, \therefore y = 5, \therefore x = 45.$

Consequently, 5 of the labourers received 15*d.*, and 45 received 9*d.*

Otherwise.—Let x be the number of labourers at 9*d.*, then by the question $50 - x$ was the number at 15*d.*; consequently the former received $9x$ pence, and the latter $15(50 - x)$ pence; hence, as they received together 480 pence, we must have the equation

$$15(50 - x) + 9x = 480;$$

That is, $750 - 15x + 9x = 480$;

Transposing, $270 = 6x$, $\therefore x = 45$, and $50 - x = 5$;
Consequently, 45 received 9*d*. each, and 5 received 15*d*.

5. A person ordered a quantity of rum and brandy, for which he paid £19 4*s*.; the brandy was 9*s*. a quart, and the rum 6*s*. He has, however, forgotten the exact quantity of each which he has to receive; but he remembers that if his brandy had been rum, and his rum brandy, his outlay would have been £1 13*s*. less. How many quarts of each did he buy?

Suppose he bought x quarts of rum and y quarts of brandy; then, by the question, the number of shillings paid for the two was $6x + 9y$; but if the brandy had been 6*s*. a quart and the rum 9*s*., the number of shillings paid would have been $9x + 6y$. The actual price paid was 384*s*.; the sum that would have been paid in the latter case, we are told, is 33*s*. less than this, that is, 351*s*.; hence we have these two equations—viz.,

$$\begin{aligned} 6x + 9y &= 384 \\ 9x + 6y &= 351 \end{aligned} \quad \therefore \quad \begin{aligned} 2x + 3y &= 128 \\ 3x + 2y &= 117. \end{aligned}$$

By adding these two equations, we have

$$5x + 5y = 245 \;\therefore\; y + x = 49$$

By subtracting, $y - x = 11$

Therefore adding and subtracting, $2y = 60$, $2x = 38$, $\therefore x = 19$, and $y = 30$; so that he had 19 quarts of rum and 30 quarts of brandy.

Otherwise.—Suppose he had x quarts of rum; for this he paid $6x$ shillings by the question; therefore $384 - 6x$ is the number of shillings paid for the brandy; and as 1 quart of brandy cost 9*s*. the number of quarts must have been $\frac{384 - 6x}{9}$, or $\frac{128 - 2x}{3}$. Now, by the question, if these quarts had cost 6*s*. each, and the x quarts 9*s*. each, the number of shillings paid would have been only 351: hence we have the equation

$$\frac{128 - 2x}{3} \times 6 + 9x = 351,$$

or, $256 - 4x + 9x = 351$,

$$\therefore 5x = 95, \;\therefore x = 19, \text{ quarts of rum,}$$

and $\frac{128 - 2x}{3} = \frac{128 - 38}{3} = 30$, quarts of brandy.

It is plain that the former is the easier mode of solution; and it will usually be found that both thought and work are diminished when as many unknown symbols are employed in the solution of a question as there are distinct conditions embodied in that question. It is a mistake to suppose that it is always easier to solve a question with only one unknown quantity than with two, when two are implied in the conditions; the contrary is more frequently true. Some of the present examples will sufficiently show this.

6. A person has spirits at 12*s*. a gallon, and at £1 a gallon; how much of each must he take to make a gallon worth 14*s*.? Let the fractional part of a gallon at 12*s*. be represented by x, and that of a gallon at 20*s*. by y; then

$$x + y = 1,$$

and the price of the mixture $= 12x + 20y = 14$, by the question.

Multiplying the first of these equations by 12, and subtracting

$$8y = 2 \;\therefore\; y = \frac{1}{4}; \text{ and } x = 1 - y = \frac{3}{4}$$

Consequently the mixture must consist of $\frac{1}{4}$ gallon at 20*s*., and $\frac{3}{4}$ gallon at 12*s*.

Otherwise.—Suppose he take x gal. at 12*s*.; then he must take $1 - x$ at 20*s*.; the price of the former is $12x$ shillings, and that of the latter $20\,(1 - x)$ shillings; therefore the price of the whole gallon of the mixture is

$$12x + 20 - 20x = 14, \text{ by the question.}$$

Transposing, $6 = 8x$, $\therefore \frac{3}{4} = x$; $\therefore y = 1 - x = \frac{1}{4}$, so that there must be $\frac{3}{4}$ gal. at 12*s*., and $\frac{1}{4}$ gal. at 20*s*.

7. A merchant has spirits at a shillings a gallon, and at b shillings a gallon; how much of each must he take to make a mixture of d gallons worth c shillings a gallon? Suppose he take x gallons at a shillings, and y at b shillings; then the worth of the mixture of $x + y$ gallons—that is, of d gallons—is

$$ax + by = cd, \text{ by the question;}$$

$$\text{also } x + y = d.$$

Mult. by a, $ax + ay = ad$

Subtract, $(a - b)y = (a - c)d$, $\therefore y = \frac{(a - c)d}{a - b}$.

In like manner, multiplying by b, and subtracting,

$$(a - b)x = (c - b)d, \;\therefore x = \frac{(c - b)d}{a - b}.$$

Otherwise.—Suppose he take x gallons at a shillings, then, by the question, he must take $d - x$ gallons at b shillings: the price of the mixture is,

$$ax + b(d - x) = cd, \text{ by the question; that is,}$$

$(a - b)x = (c - b)d$, $\therefore x = \frac{(c - b)d}{a - b}$, gals. at a shillings,

and $d - x = d - \frac{(c - b)d}{a - b} = \frac{(a - c)d}{a - b}$, gals. at b shillings.

8. In a composition of a certain quantity of gunpowder, two-thirds of the whole + 10 lbs. was nitre; one-sixth of the whole — $4\frac{1}{2}$ lbs. was sulphur; and the charcoal was one-seventh of the nitre, all but 2 lbs. How many lbs. of gunpowder were there?

Suppose there were x lbs. of gunpowder;
then there were $\frac{2x}{3} + 10$ of nitre;
" $\frac{x}{6} - 4\frac{1}{2}$ of sulphur;
" $\frac{1}{7}(\frac{2x}{3} + 10) - 2$ of charcoal;

consequently, adding the ingredients, we have

$$\frac{8}{7}\left(\frac{2x}{3} + 10\right) + \frac{x}{6} + 6\tfrac{1}{2} = x;$$

or, multiplying by 6,

$$\frac{8}{7}(4x + 60) + x + 39 = 6x,$$

$$\therefore \frac{8}{7}(4x + 60) - 39 = 5x.$$

Multiplying by 7,

$$32x + 480 + 273 = 35x,$$

$$\therefore 207 = 3x, \;\therefore x = 69.$$

Consequently there were 69 lbs. of gunpowder.

9. A farmer wishes to mix 28 bushels of barley, at 2*s*. 4*d*. a bushel, with rye at 3*s*. a bushel, and wheat at 4*s*. a bushel, so that the whole may make 100 bushels worth 3*s*. 4*d*. a bushel: how much rye and wheat must he use?

Suppose x bushels of rye, and y of wheat; then $x + y + 28$ must make 100.

Now the price of x bushels of rye is $36x$ pence
" y " wheat is $48y$ "
" 28 " barley is 28×28 pence,

$\therefore$ the price of the $x + y + 28$ bushels is $36x + 48y + 28 \times 28$ pence; which, by the question, is equal to 100×40: hence we have

$$x + y + 28 = 100 \; . \; . \; . \; . \; (1)$$

$$36x + 48y + 28 \times 28 = 100 \times 40;$$

or, dividing the latter equation by 4,

$$9x + 12y + 28 \times 7 = 100 \times 10$$

that is, $9x + 12y + 196 = 1000 \; . \; . \; . \; . \; (2).$

Multiplying (1) by 9, $9x + 9y + 252 = 900$

Subtracting, $3y - 56 = 100$

$$\therefore y = \frac{156}{3} = 52$$

$$\therefore (1),\ x = 100 - 28 - y = 72 - 52 = 20.$$

Hence there must be 20 bushels of rye, and 52 bushels of wheat.

Otherwise.—Suppose there were x bushels of rye; then there must have been $100 - 28 - x$ bushels of wheat. The worth of the x bushels of rye is $36x$ pence, and the worth of the $72 - x$ bushels of wheat is $48(72 - x)$ pence; also the worth of the 28 bushels of barley is 28×28 pence. Hence, because the 100 bushels is worth 4000 pence, by the question, we have

$$36x + 48(72 - x) + 28 \times 28 = 4000$$

Dividing by 4, $\quad 9x + 12(72 - x) + 28 \times 7 = 1000$

that is, $\quad -3x + 864 + 196 = 1000$

$\therefore -3x = 1000 - 1060 = -60$, $\therefore x = 20$, bushels of rye

$\therefore 100 - 28 - x = 72 - 20 = 52$, bushels of wheat.

This latter solution is perhaps somewhat easier than the former: you may vary both by employing shillings throughout instead of pence; representing the 2*s.* 4*d.* and the 3*s.* 4*d.* by $2\frac{1}{3}$*s.*, and $3\frac{1}{3}$*s.* respectively.

10. Two persons, A and B, are engaged on a work which they can finish in 16 days; but after working together 4 days, A is called off; and B alone finishes it in 36 days more In how many days could each do it separately? Suppose A can do it in x days, and B in y days: then in one day they can do the $\frac{1}{x}$ and the $\frac{1}{y}$ part respectively, and consequently in 16 days they can do $\frac{16}{x} + \frac{16}{y}$, which by the question, is the whole.

$$\frac{16}{x} + \frac{16}{y} = 1.$$

Also in 4 days they do the $\frac{4}{x} + \frac{4}{y}$ part, that is, the $\frac{1}{4}$ part; so that after that time the $\frac{3}{4}$ part remains to be done by B. Now, in 1 day B does the $\frac{1}{36}$ part of this; that is, the $\frac{3}{144} = \frac{1}{48}$ of the whole; but he does the $\frac{1}{y}$ of the whole in 1 day; hence

$$\frac{1}{y} = \frac{1}{48}, \therefore y = 48.$$

Substituting this value of y in the former equation, we have

$$\frac{16}{x} + \frac{16}{48} = 1$$

$$\therefore \frac{16}{x} = 1 - \frac{1}{3} = \frac{2}{3}, \therefore 2x = 48, \therefore x = 24.$$

Hence A can finish the work in 24 days, and B in 48 days.

Otherwise, without Algebra.—Since A and B have worked together 4 days, they can complete what is left in 12 days; and, therefore, $\frac{1}{12}$ of it in 1 day, while B alone can do only the $\frac{1}{36}$ in 1 day; so that the part of it done by A in 1 day is

$$\frac{1}{12} - \frac{1}{36} = \frac{1}{18};$$

hence A is twice as good a workman as B, so that A and B together are as good as 3 B's; and, therefore, since 3 B's would do the whole in 16 days, *one* B would take 3 times as long, or 48 days; and, consequently, A would take 24 days—half the time.

Again, with one unknown quantity.—By the question A and B have done $\frac{1}{4}$ of the work before they separate, hence $\frac{3}{4}$ of it remains to be done. Suppose A can do the whole in x days, then in 1 day he can do $\frac{1}{x}$ of it; but A and B together do $\frac{1}{16}$ in a day; hence B's part in a day is $\frac{1}{16} - \frac{1}{x}$; but B's part is also $\frac{1}{36}$ of $\frac{3}{4}$; hence

$$\frac{1}{16} - \frac{1}{x} = \frac{1}{48}$$

$$\therefore \frac{1}{x} = \frac{1}{16} - \frac{1}{48} = \frac{1}{24}, \therefore x = 24, \text{ A's days.}$$

And since B does the $\frac{1}{48}$ part in 1 day, he can do the whole in 48 days.

11. A composition of copper and tin containing 100 cubic inches, weighed 505 ounces; how many ounces of each metal did it contain, supposing a cubic inch of copper to weigh $5\frac{1}{4}$ oz., and a cubic inch of tin to weigh $4\frac{1}{4}$ oz.?

Suppose there were x ounces of copper and y ounces of tin: then $x + y = 505$. Also by the question,

$$\frac{x}{5\frac{1}{4}} + \frac{y}{4\frac{1}{4}} = 100, \therefore \frac{x}{21} + \frac{y}{17} = 25$$

$$\therefore 17x + 21y = 8925$$

Mult. 1st equa. by 17, $\quad 17x + 17y = 8585$

Subtracting, $\quad 4y = 340, \therefore y = 85$

$$\therefore x = 505 - y = 505 - 85 = 420.$$

Hence there were 420 oz. of copper and 85 oz. of tin.

This is solved with only one unknown quantity, by putting x for the number of ounces of copper, and $505 - x$ for the number of ounces of tin.

12. A cask is supplied by three spouts, which can fill it in a minutes, b minutes, and c minutes, respectively; in what time will it be filled if all flow together?

Let x be the number of minutes, then the part of the whole supplied in 1 minute is

$$\frac{1}{x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}, \text{ by the question,}$$

Clearing, $abc = (ab + ac + bc)x$

$$\therefore x = \frac{abc}{ab + ac + bc}.$$

Questions of this kind may be treated rather differently, thus: since the first spout can fill the cask in a minutes, it can fill bc such casks in abc minutes. In like manner the second spout can fill ac such casks in abc minutes, and the third ab such casks in the same time; so that when all flow together, they can fill $ab + ac + bc$ casks in abc minutes, and, consequently, *one* cask in

$$\frac{abc}{ab + ac + bc} \text{ minutes.}$$

QUADRATIC EQUATIONS. EXAMPLES FOR EXERCISE.

PAGE 474.

1. $x^2 - 4x = 45$. Completing the square,

$$x^2 - 4x + 4 = 49.$$

Extracting the root, $\quad x - 2 = \pm 7,$

$$\therefore x = 2 \pm 7 = 9 \text{ or } -5.$$

2. $x^2 + 8x = 33$. Completing the square,

$$x^2 + 8x + 16 = 49.$$

Extracting the root, $\quad x + 4 = \pm 7,$

$$\therefore x = -4 \pm 7 = 3 \text{ or } -11.$$

3. $x^2 - 8x = 9$. Completing the square,

$$x^2 - 8x + 16 = 25.$$

Extracting the root, $\quad x - 4 = \pm 5,$

$$\therefore x = 4 \pm 5 = 9 \text{ or } -1.$$

4. $3x^2 + 2x = 161$. Dividing by 3, $x^2 + \frac{2}{3}x = \frac{161}{3}$.

Completing the square, $\quad x^2 + \frac{2}{3}x + \frac{1}{9} = \frac{161}{3} + \frac{1}{9} = \frac{484}{9}.$

Extracting the root, $\quad x + \frac{1}{3} = \pm \frac{22}{3},$

$$\therefore x = -\frac{1}{3} \pm \frac{22}{3} = 7 \text{ or } -\frac{23}{3}.$$

Or, by Rule II., page 474,

$$3x^2 + 2x = 161, \therefore 6x + 2 = \sqrt{(1932 + 4)} = \sqrt{1936}$$
$$= \pm 44,$$

$$\therefore x = \frac{-2 \pm 44}{6} = 7 \text{ or } -\frac{23}{3}.$$

5. $7x^2 - 20x = 32$. Dividing by 7, $x^2 - \frac{20}{7}x = \frac{32}{7}$.

Completing the square, $x^2 - \frac{20}{7}x + \frac{100}{49} = \frac{32}{7} + \frac{100}{49} = \frac{324}{49}.$

Extracting the root, $\quad x - \frac{10}{7} = \sqrt{\frac{324}{49}} = \pm \frac{18}{7}.$

$$\therefore x=\frac{10}{7}\pm\frac{18}{7}=4 \text{ or} -1\tfrac{1}{7}.$$

Or, by Rule II., page 474,

$7x^2-20x=32$, $\therefore 14x-20=\sqrt{(32\times 28+400)}=\pm 36$,

$$\therefore x=\frac{20\pm 36}{14}=4, \text{ or} -1\tfrac{1}{7}.$$

6. $x^2-x-170=40$, or $x^2-x=210$. Completing the square,

$$x^2-x+\frac{1}{4}=210+\frac{1}{4}=\frac{841}{4}.$$

Extracting the root, $x-\frac{1}{2}=\sqrt{\frac{841}{4}}=\pm\frac{29}{2}$,

$$\therefore x=\frac{1}{2}\pm\frac{29}{2}=15, \text{ or} -14.$$

Or by Rule II., page 474,

$x^2-x=210$, $\therefore 2x-1=\sqrt{(840+1)}=\pm 29$,

$$\therefore x=\frac{1\pm 29}{2}=15, \text{ or} -14.$$

7. $5x^2+4x=273$. Dividing by 5, $x^2+\frac{4}{5}x=\frac{273}{5}$

Completing the square, $x^2+\frac{4}{5}x+\frac{4}{25}=\frac{273}{5}+\frac{4}{25}=\frac{1369}{25}$.

Extracting the root, $x+\frac{2}{5}=\sqrt{\frac{1369}{25}}=\pm\frac{37}{5}$.

$$\therefore x=-\frac{2}{5}\pm\frac{37}{5}=7, \text{ or} -7\tfrac{4}{5}.$$

Or, by Rule II., page 474,

$5x^2+4x=273$, $\therefore 10x+4=\sqrt{(273\times 20+16)}=\sqrt{5476}=\pm 74$

$$\therefore x=\frac{-4\pm 74}{10}=7, \text{ or} -7\tfrac{4}{5}.$$

8. $\frac{4x^2-x}{3}=11$. Multiplying by 3, $4x^2-x=33$.

Dividing by 4, $x^2-\frac{1}{4}x=\frac{33}{4}$

Completing the square, $x^2-\frac{1}{4}x+\frac{1}{64}=\frac{33}{4}+\frac{1}{64}=\frac{529}{64}$

Extracting the root, $x-\frac{1}{8}=\sqrt{\frac{529}{64}}=\pm\frac{23}{8}$

$$\therefore x=\frac{1}{8}\pm\frac{23}{8}=3, \text{ or} -2\tfrac{3}{4}.$$

Or, by Rule II., page 474,

$4x^2-x=33$, $\therefore 8x-1=\sqrt{(33\times 16+1)}=\sqrt{529}=\pm 23$,

$$\therefore x=\frac{1\pm 23}{8}=3, \text{ or} -2\tfrac{3}{4}.$$

9. $x+\frac{7x-8}{x}=9$. Multiplying by x, $x^2+7x-8=9x$, $\therefore x^2-2x=8$.

Completing the square, $x^2-2x+1=9$.

Extracting the root, $x-1=\pm 3$, $\therefore x=4$, or -2.

10. $5x^2+3=4x+159$, $\therefore 5x^2-4x=156$. Dividing by 5,

$$x^2-\frac{4}{5}x=\frac{156}{5}.$$

Completing the square,

$$x^2-\frac{4}{5}x+\frac{4}{25}=\frac{156}{5}+\frac{4}{25}=\frac{784}{25}.$$

Extracting the root, $x-\frac{2}{5}=\sqrt{\frac{784}{25}}=\pm\frac{28}{5}$

$$\therefore x=\frac{2}{5}\pm\frac{28}{5}=6, \text{ or} -5\tfrac{1}{5}.$$

Or, by Rule II., page 474,

$5x^2-4x=156$, $\therefore 10x-4=\sqrt{(156\times 20+16)}=\sqrt{3136}=\pm 56$.

$$\therefore x=\frac{4\pm 56}{10}=6, \text{ or} -5\tfrac{1}{5}.$$

11. $6x+\frac{35-3x}{x}-44=0$, or $6x+\frac{35}{x}-3-44=0$;

that is, $6x+\frac{35}{x}-47=0$.

Multiplying by x, $6x^2+35-47x=0$.

$\therefore 6x^2-47x=-35$. Dividing by 6,

$$x^2-\frac{47}{6}x=-\frac{35}{6}.$$

Completing the square,

$$x^2-\frac{47}{6}x+\left(\frac{47}{12}\right)^2=\frac{2209}{144}-\frac{35}{6}=\frac{1369}{144}.$$

Extracting the root,

$$x-\frac{47}{12}=\sqrt{\frac{1369}{144}}=\pm\frac{37}{12},$$

$$\therefore x=\frac{47}{12}\pm\frac{37}{12}=7, \text{ or } \frac{5}{6}.$$

Or by Rule II., page 474,

$6x^2-47x=-35$, $\therefore 12x-47=\sqrt{(-35\times 24+47^2)}=\sqrt{1369}=\pm 37$, $\therefore x=\frac{47\pm 37}{12}=7$, or $\frac{5}{6}$.

12. $\frac{x-2}{x+2}+\frac{x+2}{x-2}=14$. Clearing the fractions,

$$x^2-4x+4+x^2+4x+4=14(x^2-4).$$

Collecting and transposing,

$$64=12x^2, \therefore x=\sqrt{\frac{64}{12}}=\frac{4}{\sqrt{3}}=\frac{4}{3}\sqrt{3}.$$

13. $48x^{-2}+32x^{-1}=11$. Dividing by 48,

$$x^{-2}+\frac{2}{3}x^{-1}=\frac{11}{48}.$$

Completing the square,

$$x^{-2}+\frac{2}{3}x^{-1}+\frac{1}{9}=\frac{11}{48}+\frac{1}{9}=\frac{49}{144}.$$

Extracting the root, $x^{-1}+\frac{1}{3}=\pm\frac{7}{12}$.

$$\therefore x^{-1}, \text{ that is } \frac{1}{x}=-\frac{1}{3}\pm\frac{7}{12}=\frac{1}{4}, \text{ or} -\frac{11}{12}.$$

$$\therefore x=4, \text{ or} -1\tfrac{1}{11}.$$

Or, by Rule II., page 474,

$48x^{-2}+32x^{-1}=11$, $\therefore 96x^{-1}+32=\sqrt{(48\times 44+32^2)}=\sqrt{3136}=\pm 56$,

$$\therefore x^{-1}=\frac{1}{x}=\frac{-32\pm 56}{96}=\frac{1}{4}, \text{ or} -\frac{11}{12},$$

$$\therefore x=4, \text{ or} -1\tfrac{1}{11}.$$

14. $x^4-40x^2+39=0$, or $(x^2)^2-40(x^2)=-39$.

Completing the square, $(x^2)^2-40(x^2)+400=361$.

Extracting the root, $x^2-20=\sqrt{361}=\pm 19$,

$\therefore x^2=20\pm 19=1$ or 39, $\therefore x=\pm 1$, or $\pm\sqrt{39}$.

15. $x^2+ax+b=0$, $\therefore x^2+ax=-b$.

Completing the square.

$$x^2+ax+\frac{1}{4}a^2=\frac{1}{4}a^2-b=\frac{a^2-4b}{4},$$

Extracting the root, $x+\frac{1}{2}a=\frac{\sqrt{(a^2-4b)}}{2}$

$$\therefore x=\frac{-a\pm\sqrt{(a^2-4b)}}{2}.$$

This may be regarded as a general formula for the solution of any quadratic equation whatever; for by putting particular values for a and b in this result, we shall have the proper values of x in the quadratic to which those particular values of the coefficients belong. We further learn from this formula a few general principles well worthy of notice: thus,

1. In every quadratic, put into the form $x^2+ax+b=0$, if $a^2=4b$, the two roots, must be *equal*; since, in this case the general form for the roots is $x=\frac{-a\pm 0}{2}$; so that each root is half the coefficient of x, taken with changed sign.

2. If a^2 be greater than $4b$, the two roots are unequal

and *real;* for a^2-4b, under the radical, is in this case positive.

3. If a^2 be less than $4b$, the two roots must be *imaginary;* for then a^2-4b, under the radical, is negative.

Hence, by examining the coefficients of a quadratic, we may ascertain the character of its roots without actually solving the equation.

16. $\frac{x+a}{x-a}+\frac{x-a}{x+a}=b$. Clearing fractions,

$$(x+a)^2+(x-a)^2=b(x^2-a^2)$$

that is, $2x^2+2a^2=bx^2-a^2b$

Transposing, $(b-2)x^2=a^2(2+b)$,

$$\therefore x^2=\frac{a^2(b+2)}{b-2},$$

$$\therefore x=\pm a\sqrt{\frac{b+2}{b-2}}.$$

17. Put y for $\sqrt{x}$, then multiplying by 6, we have

$$3y^2-2y=133,$$

Dividing by 3, $y^2-\frac{2}{3}y=\frac{133}{3}$.

Completing the square, $y^2-\frac{2}{3}y+\frac{1}{9}=\frac{133}{3}+\frac{1}{9}=\frac{400}{9}$,

Extracting the root, $y-\frac{1}{3}=\sqrt{\frac{400}{9}}=\pm\frac{20}{3}$.

$$\therefore y=\frac{1}{3}\pm\frac{20}{3}=7, \text{ or } -\frac{19}{3}=-6\tfrac{1}{3},$$

$$\therefore y^2=x=49, \text{ or } 40\tfrac{1}{9}.$$

Or, by Rule II., page 474,

$3y^2-2y=133$, $\therefore 6y-2=\sqrt{(1596+4)}=\sqrt{1600}=\pm 40$,

$$\therefore y=\frac{2\pm 40}{6}=7, \text{ or } -6\tfrac{1}{3},$$

$$\therefore y^2=x=49, \text{ or } 40\tfrac{1}{9}.$$

18. $(3x-2)(1-x)=4$; that is,

$$-3x^2+x-2=4, \therefore 3x^2-x=-2.$$

Dividing by 3, $x^2-\frac{1}{3}x=-\frac{2}{3}$,

Completing the square,

$$x^2-\frac{1}{3}x+\frac{1}{36}=\frac{1}{36}-\frac{2}{3}=-\frac{23}{36}.$$

Extracting the root, $x-\frac{1}{6}=\pm\frac{1}{6}\sqrt{-23}$,

$$\therefore x=\frac{1\pm\sqrt{-23}}{6}$$

19. $15x^2-48x+45=0$, or $15x^2-48x=-45$.

Dividing by 15,

$$x^2-\frac{16}{5}x=-3.$$

Completing the square,

$$x^2-\frac{16}{5}x+\frac{64}{25}=\frac{64}{25}-3=-\frac{11}{25}.$$

Extracting the root,

$$x-\frac{8}{5}=\frac{1}{5}\sqrt{-11}, \therefore x=\frac{8\pm\sqrt{-11}}{5}$$

We might have known, without actually solving either of the two equations in Examples 18 and 19, that the roots were imaginary, for these equations being $3x^2-x+2=0$, and $15x^2-48x+45=0$, we see at once that four times the product of the extreme terms exceeds the square of the middle term. (See the principle 3, above.)

20. $\sqrt{\frac{x-2}{x+2}}+\sqrt{\frac{x+2}{x-2}}=4$.

Squaring, $\frac{x-2}{x+2}+2+\frac{x+2}{x-2}=16$,

$$\therefore \frac{x-2}{x+2}+\frac{x+2}{x-2}=14,$$

and this equation has already been solved in Example 12.

QUESTIONS REQUIRING QUADRATIC EQUATIONS.
PAGE 475.

1. Divide the number 33 into two such parts, that their product may be 162.

Let x be one part, then $33-x$ is the other; and by the question,

$$33x-x^2=162; \text{ or } x^2-33x=-162,$$

$\therefore$ Rule II., $2x-33=\sqrt{(1089-648)}=\sqrt{441}=\pm 21$,

$$\therefore x=\frac{33\pm 21}{2}=27, \text{ or } 6.$$

$$\therefore 33-x=6, \text{ or } 27.$$

Hence the parts are 6, and 27.

Otherwise.—Let the two parts be x and y; then by the question

$$x+y=33, \text{ and } xy=162$$
$$\therefore x^2+2xy+y^2=1089$$
$$\text{And } \quad 4xy=648$$

Subtracting, $x^2-2xy+y^2=441$

That is,

$(x-y)^2=441$, $\therefore x-y=\pm 21$

And since $x+y=33$

$\therefore$ adding and subtracting, $2x=33\pm 21$, $\therefore x=27$ or 6

$2y=33\mp 21$, $\therefore y=6$ or 27.

2. Find two numbers whose difference is 9, and which are also such that their sum multiplied by the greater gives 266 for the product.

Let x be the greater number, then $x-9$ is the less, and by the question, $(2x-9)x=266$; that is,

$$2x^2-9x=266$$

$\therefore$ Rule II., $4x-9=\sqrt{(2128+81)}=\sqrt{2209}=\pm 47$

$$\therefore x=\frac{9\pm 47}{4}=14, \text{ or } -\frac{19}{2}=-9\tfrac{1}{2}$$

$$\therefore x-9=5, \text{ or } -18\tfrac{1}{2};$$

hence the numbers are either 5 and 14, or $-9\frac{1}{2}$ and $-18\frac{1}{2}$.

Otherwise.—Let x and y be the numbers; then by the question, $x-y=9$, and $(x+y)x=266$.

From the first of these equations, $x=y+9$, $\therefore$ by substitution in the second, $(2y+9)(y+9)=266$; or

$$2y^2+27y+81=266, \therefore 2y^2+27y=185$$

$\therefore$ Rule II., $4y+27=\sqrt{(1480+729)}=\sqrt{2209}=\pm 47$

$$\therefore y=\frac{-27\pm 47}{4}=5 \text{ or } -18\tfrac{1}{2}$$

$$\therefore x=y+9=14, \text{ or } -9\tfrac{1}{2}$$

Or thus: $x-y=9$

$x+y=\frac{266}{x}$

Adding, $2x=9+\frac{266}{x}$, $\therefore 2x^2-9x=266$,

and the remaining steps as in the first solution.

3. A company at a tavern had £7 4s. to pay; but two of them having left, the others had each 1s. more to pay than his fair share: how many persons were there at first?

Suppose there were x persons, then the fair share of each was $\frac{144}{x}$ shillings; but after two had left, the share to be paid by each of the others was $\frac{144}{x-2}$; and by the question,

$$\frac{144}{x-2}=\frac{144}{x}+1$$

Clearing fractions, $144x=144x-288+x^2-2x$

$\therefore x^2-2x=288$, $\therefore$ completing the square,

$x^2-2x+1=289$, $\therefore$ extracting, $x-1=\pm 17$,

$$\therefore x=18, \text{ or } -16.$$

Consequently there were 18 persons at first. The share of each was therefore 8s.; but when two had left, the share of each of the remaining 16 was 9s.

4. A purse contains 24 coins of silver and copper;

each copper coin is worth as many pence as there are silver coins, and each silver coin is worth as many pence as there are copper coins; and the whole is worth 18s.; how many are there of each?

Suppose there were x of one sort, and therefore $24 - x$ of the other: then the worth of the whole in pence is

$$(24 - x)x + (24 - x)x = 216, \text{ or } 2(24 - x)x = 216$$
$$\therefore 2x^2 - 48x = -216, \therefore x^2 - 24x = -108.$$

Completing the square, $x^2 - 24x + 144 = 36$, $\therefore x - 12 = \pm 6$, $\therefore x = 18$, or -6, $\therefore 24 - x = 6$: hence there were 18 silver coins and 6 copper coins, or 6 silver and 18 copper.

Otherwise.—Let the number of coins be x and y: then

$$x + y = 24, \text{ and } 2xy = 216$$
$$\therefore (x + y)^2 = 576, \text{ and } 4xy = 432.$$

Hence by subtraction, $(x - y)^2 = 144$, $\therefore x - y = \pm 12$.
$\therefore$ by adding to and subtracting from the first equation, we have $2x = 36$, $2y = 12$; $\therefore x = 18$, $y = 6$.

The *negative* values are suppressed, because they are inadmissible, from the nature of the question.

5. Two messengers, A and B, were dispatched to the same place, 90 miles distant. A, by riding one mile an hour more than B, arrives at his destination an hour before him. How many miles did each travel?

Suppose A travels x miles an hour, then he reaches his destination in $\frac{90}{x}$ hours; and B, by travelling 1 mile an hour more—that is, $x + 1$ miles an hour—reaches it in $\frac{90}{x+1}$ hours. And, by the question,

$$\frac{90}{x} = \frac{90}{x+1} + 1 \text{ or } \frac{90}{x} = \frac{91 + x}{x+1}$$

Clearing fractions, $90x + 90 = 91x + x^2$
Transposing, $x^2 + x = 90$

$$\therefore \text{Rule II., } 2x + 1 = \sqrt{(360 + 1)} = \sqrt{361} = \pm 19,$$
$$\therefore x = \frac{-1 + 19}{2} = 9, \therefore x + 1 = 10.$$

Hence A travels 9 miles an hour, and B 10 miles an hour. The negative value of x is suppressed, as it is excluded by the conditions of the question. You will observe that algebra gives *all* the values of the unknown quantity that can satisfy the *equation*: if there are any restrictions in the *question* embodied in that equation, the final results must, of course, conform to those restrictions.

6. A grocer sold 80lbs. of mace and 100lbs. of cloves for £65; but he sold 60lbs. more of cloves for £20, than he did of mace for £10. What was the price of 1lb. of each?

Suppose the price of the mace per lb. was x pounds, and that of the cloves y pounds; then by the question, $80x + 100y = 65$. Also, the number of lbs. of cloves for £20 is $\frac{20}{x}$, and of mace for £10 is $\frac{10}{x}$; and by the question, $\frac{20}{y} = \frac{10}{x} + 60$: hence, the two equations are

$$80x + 100y = 65, \therefore 16x + 20y = 13 \quad . \ . \ . \ (1)$$
$$\text{and } \frac{20}{y} = \frac{10}{x} + 60, \therefore \frac{2}{y} - \frac{1}{x} = 6 \quad . \ . \ . \ (2)$$

Clearing the last of fractions, $\therefore 2x - y = 6xy$ . . . (3)

Substituting in (3) the expression for x given by (1)—viz., $x = \frac{13 - 20y}{16}$, it becomes

$$\frac{13 - 20y}{8} - y = \frac{39y - 60y^2}{8}$$

Multiplying by 8, $\therefore 13 - 20y - 8y = 39y - 60y^2$
Transposing, $60y^2 - 67y = -13$

$$\therefore \text{Rule II., } 120y - 67 = \sqrt{(-13 \times 240 + 67^2)}$$
$$= \sqrt{1369} = \pm 37 \therefore y = \frac{67 \pm 37}{120} = \frac{1}{4}, \text{ or } \frac{13}{15}$$

$\therefore x = \frac{13 - 20y}{16} = \frac{13 - 5}{16} = \frac{1}{2}$; the other value of x is negative: hence the price of 1lb. of mace was £$\frac{1}{2}$, or 10s., and of 1 lb. of cloves £$\frac{1}{4}$, or 5s.

Otherwise.—Multiply (1) and (2) together; we then have

$$32\frac{x}{y} - 20\frac{y}{x} - 16 + 40 = 78, \therefore 32\frac{x}{y} - 20\frac{y}{x} = 54,$$
$$\therefore 16\frac{x}{y} - 10\frac{y}{x} = 27.$$

Multiplying (2) by $8x$, $16\frac{x}{y} - 8 = 48x$

Subtracting, $10\frac{y}{x} - 8 = 48x - 27$

Dividing (1) by $2x$, $8 + 10\frac{y}{x} = \frac{13}{2x}$, $\therefore 10\frac{y}{x} = \frac{13}{2x} - 8$.

Hence, by substitution in the preceding equation,

$$\frac{13}{2x} - 16 = 48x - 27, \text{ or } \frac{13}{2x} = 48x - 11.$$

Clearing the fraction,

$$96x^2 - 22x = 13, \therefore \text{Rule II., } 192x - 22 = \sqrt{(96 \times 52 + 22^2)} = \pm 74$$
$$\therefore x = \frac{22 \pm 74}{192} = \frac{1}{2}, \text{ or } -\frac{13}{48}$$

and, rejecting the negative value of x, we have from (1)

$$y = \frac{13 - 16x}{20} = \frac{13 - 8}{20} = \frac{1}{4}$$

Therefore, the price of 1lb. of mace was £$\frac{1}{2}$, or 10s.; and of 1lb. of cloves £$\frac{1}{4}$, or 5s., as before.

7. The product of two numbers is 240, and they are such that if one of them be increased by 4, and the other diminished by 3, the product of the results is still 240. Find the numbers.

Let x and y represent the numbers; then by the question,

$$xy = 240, \text{ and } (x + 4)(y - 3) = 240,$$
$$\text{That is, } xy - 3x + 4y - 12 = 240$$

Subtracting this from the first equation, $3x - 4y + 12 = 0$, $\therefore x = \frac{4y - 12}{3}$

$$\therefore xy = \frac{4y^2 - 12y}{3} = 240, \therefore 4y^2 - 12y = 720$$
$$\therefore y^2 - 3y = 180, \therefore \text{Rule II., } 2y - 3 = \sqrt{(720 + 9)}$$
$$= \pm 27 \therefore y = \frac{3 \pm 27}{2} = 15, \text{ or } -12$$
$$\therefore x = \frac{240}{y} = 16, \text{ or } -20.$$

Hence the numbers are either 16 and 15, or -20 and -12.

8. A and B set out at the same time for a place 150 miles distant. A travels 3 miles an hour faster than B, and arrives at the place $8\frac{1}{3}$ hours before him. How many miles did each travel per hour?

Suppose A travels x miles per hour, then B travels $x - 3$ miles an hour; and the number of hours occupied by A is $\frac{150}{x}$, the number occupied by B is $\frac{150}{x-3}$; but by the question, this latter number exceeds the former by $8\frac{1}{3}$,

$$\therefore \frac{150}{x} + 8\tfrac{1}{3} = \frac{150}{x-3}$$

Clearing fractions, $450x - 1350 + 25x^2 - 75x = 450x$
Transposing, $25x^2 - 75x = 1350$, $\therefore x^2 - 3x = 54$,

$$\therefore \text{Rule II., } 2x - 3 = \sqrt{(216 + 9)} = \sqrt{225} = \pm 15$$
$$\therefore x = \frac{3 \pm 15}{2} = 9, \text{ or } -6,$$
$$\therefore x - 3 = 6.$$

Hence, A travels at the rate of 9 miles an hour, and B at the rate of 6 miles an hour.

9. What number is that the sum of whose digits is 15, and if 31 be added to their product, the digits will be transposed?

Let x and y denote the two digits; then the number is $10x + y$; and by the question

$$x + y = 15, \text{ and } xy + 31 = 10y + x.$$

From the first, $x = 15 - y$, $\therefore (15 - y)y + 31 = 10y + 15 - y$.

That is, $15y - y^2 + 31 = 9y + 15$, $\therefore y^2 - 6y = 16$.

Completing the square,

$$y^2 - 6y + 9 = 25, \therefore y - 3 = \pm 5, \therefore y = 3 \pm 5 = 8,$$
$$\text{or} -2 \;\therefore x = 15 - y = 7.$$

Hence the digits are 7 and 8, and the number is 78.

10. There is a certain number consisting of two digits. The left-hand digit is equal to three times the right-hand one; and if 12 be subtracted from the number, the remainder will be equal to the square of the left-hand digit. What is the number?

Let x and y represent the two digits; then the number is $10x + y$. And by the question,

$$x = 3y, \text{ and } 10x + y - 12 = x^2$$
$$\therefore \text{ by substitution, } 30y + y - 12 = 9y^2$$
$$\text{Transposing, } 9y^2 - 31y = -12$$
$$\therefore \text{Rule II., } 18y - 31 = \sqrt{(-36 \times 12 + 31^2)}$$
$$= \sqrt{529} = \pm 23 \;\therefore y = \frac{31 \pm 23}{18} = 3, \text{ or } \frac{4}{9}.$$

The fractional value must be rejected, $\therefore x = 3y = 9$; hence the number is 93.

Otherwise.—As above,

$$10x + y - 12 = x^2, \text{ and } x - 3y = 0,$$
$$\text{Multiplying the first by 3, } 30x + 3y - 36 = 3x^2$$
$$\begin{array}{lrcl} \text{Adding the second,} & x - 3y & = & 0 \\ \hline & \therefore\; 31x - 36 & = & 3x^2 \end{array}$$

Transposing, $3x^2 - 31x = -36$, $\therefore$ Rule II., $6x - 31$

$$= \sqrt{(-36 \times 12 + 31^2)} = \pm 23 \therefore x = \frac{31 + 23}{6} = 9,$$
$$\therefore y = \frac{x}{3} = 3;$$

hence the digits are 9 and 3, and the number is 93.

Arithmetical Progression. Examples for Exercise.

Page 478.

1. Find the sum of sixteen terms of the series $1 + 2 + 3 + 4 +$ &c.

By the formula, $S = \frac{1}{2}n\{2a + (n - 1)d\}$, where $a = 1$, $d = 1$, and $n = 16$.

Consequently $S = 8\{2 + 15\} = 136$, the sum required.

2. Find the sum of fourteen terms of the series $4 + 3 + 2 + 1 + 0 - 1 - 2 -$ &c. Here $a = 4$, $d = -1$, and $n = 14$: hence

$$S = \tfrac{1}{2}n\{2a + (n - 1)d\}$$
$$= 7\{8 - 13\} = -35, \text{ the sum.}$$

3. Sum the series $\frac{1}{2} + 1\frac{1}{2} + 2\frac{1}{2} +$ &c. to twenty terms. Here $a = \frac{1}{2}$, $d = 1$, and $n = 20$: therefore

$$S = \tfrac{1}{2}n\{2a + (n - 1)d\}$$
$$= 10\{1 + 19\} = 200, \text{ the sum.}$$

4. Insert three arithmetical means between 2 and 0. By the formula, $l = a + (n - 1)d$, where $a = 2$, $n = 5$, $l = 0$. $\therefore 0 = 2 + 4d$, $\therefore d = -\frac{2}{4} = -\frac{1}{2}$.

Hence, when the means are inserted, the series is

$$2, 1\tfrac{1}{2}, 1, \tfrac{1}{2}, 0.$$

5. Insert five arithmetical means between $\frac{1}{2}$ and $-\frac{1}{2}$.

$$l = a + (n - 1)d,$$
$$\therefore -\tfrac{1}{2} = \tfrac{1}{2} + 6d, \therefore d = -\tfrac{1}{6}.$$

Hence, when the means are inserted, the series is

$$\tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{6}, 0, -\tfrac{1}{6}, -\tfrac{1}{3}, -\tfrac{1}{2}.$$

6. The first term is 5, and the fifteenth 47: what is the common difference?

$$l = a + (n - 1)d,$$
$$\therefore 47 = 5 + 14d, \therefore d = \frac{42}{14} = 3.$$

7. How many terms of the series $12 + 11\frac{1}{2} + 11 + 10\frac{1}{2} +$ &c., must be taken to make 55?

$$S = \tfrac{1}{2}n\{2a + (n - 1)d\}$$
$$\therefore 55 = \tfrac{1}{2}n\{24 - (n - 1)\tfrac{1}{2}\} = 12n - \frac{n^2 + n}{4}$$
$$\therefore 220 = 49n - n^2, \therefore n^2 - 49n = -220$$
$$\therefore \text{Rule II., p. 474, } 2n - 49 = \sqrt{(49^2 - 880)} = \sqrt{1521} = \pm 39,$$
$$\therefore n = \frac{49 \pm 39}{2} = 5, \text{ or } 44.$$

Hence, whether five terms, or forty-four terms of the series be taken, the sum will be 55.

8. The first term of an arithmetical progression is 7, and the common difference $-\frac{3}{2}$: required the ninth term.

$$l = a + (n - 1)d,$$
$$\therefore l = 7 - 8 \times \frac{3}{2} = 7 - 12 = -5, \text{ the ninth term.}$$

9. The sum of eight terms of an arithmetical series is 2, and the common difference $-1\frac{1}{2}$; required the first term.

$$S = \tfrac{1}{2}n\{2a + (n - 1)d\}$$
$$\therefore 2 = 4\{2a - 7 \times 1\tfrac{1}{2}\}, \therefore 1 = 4a - 21,$$
$$\therefore 4a = 22, \therefore a = 5\tfrac{1}{2}, \text{ the first term.}$$

10. The first term of an arithmetical series is $-3\frac{1}{3}$, and the common difference $\frac{1}{5}$: required the sum of twenty-one terms.

$$S = \tfrac{1}{2}n\{2a + (n - 1)d\}$$
$$\therefore S = \frac{21}{2}\{-6\tfrac{2}{3} + \frac{20}{5}\} = 21\{-3\tfrac{1}{3} + 2\}$$
$$= -70 + 42 = -28, \text{ the sum.}$$

Geometrical Progression. Examples for Exercise.

Page 479.

1. Required the sum of five terms of the series.

$$1 + 2^2 + 2^4 + 2^6 + \text{\&c.}$$
$$l = ar^{n-1}, S = \frac{rl - a}{r - 1}, \text{ where } a = 1, r = 4, n = 5,$$
$$\therefore l = 256, \therefore S = \frac{1024 - 1}{3} = 341, \text{ the sum.}$$

2. Required the sum of eight terms of

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \text{\&c.}$$
$$l = ar^{n-1}, S = \frac{rl - a}{r - 1}, \text{ where } a = 1, r = -\frac{1}{2}, n = 8,$$
$$\therefore l = -\frac{1}{128}, \therefore S = \frac{\frac{1}{256} - 1}{-\frac{3}{2}} = \frac{255}{256} \times \frac{2}{3} = \frac{85}{128}, \text{ the sum.}$$

3. Required the sum of ten terms of $1 + 2 + 4 + 8 +$ &c.

$$l = ar^{n-1}, S = \frac{rl - a}{r - 1}, \text{ where } a = 1, r = 2, n = 10,$$
$$\therefore l = 512, \therefore S = \frac{1024 - 1}{1} = 1023, \text{ the sum.}$$

4. What is the geometrical mean between 6 and 54?

$$54 \times 6 = 324, \text{ and } \sqrt{324} = \pm 18, \text{ mean.}$$

Hence when the mean is inserted, the progression is either 6, 18, 54 or 6, -18, 54, the ratio in the former case being 3, and in the latter -3.

5. Insert two geometrical means between 2 and 54. When the two means are inserted there will be four terms, therefore $n = 4$; also $a = 2$, and $l = 54$; and we have to find r. By the formula, $l = ar^{n-1}$, $\therefore 54 = 2r^3$,

$$\therefore r^3 = 27, \therefore r = \sqrt[3]{27} = 3.$$

Hence, supplying the means, the progression is 2, 6, 18, 54.

6. Sum the series $\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} +$ &c. to infinity.

By the formula,

$$\Sigma = \frac{a}{1 - r}, \therefore \Sigma = \frac{1}{10} \div \frac{9}{10} = \frac{1}{9}, \therefore \cdot 111 \ldots = \frac{1}{9}.$$

7. The first term of a geometrical series is 3, the

common ratio 5, and the last term 375: find the sum.

$$S = \frac{rl - a}{r - 1}, \therefore S = \frac{1875 - 3}{4} = 468, \text{ the sum.}$$

8. The first term of a geometrical series is $\frac{5}{2}$, and the common ratio $-\frac{1}{5}$: find the sum to infinity.

By the formula, $\Sigma = \frac{a}{1 - r}$, $\therefore \Sigma = \frac{5}{2} + \frac{6}{5} = 2\frac{1}{12}$, the sum.

9. **Insert three geometrical means between $\frac{1}{2}$ and $\frac{2}{9}$.**

When the means are inserted there will be five terms, therefore $n = 5$; also $a = \frac{1}{2}$, and $l = \frac{2}{9}$; and we have to find r.

By the formula, $l = ar^{n-1}$, $\therefore \frac{2}{9} = \frac{1}{2}r^4$,

$$\therefore r^4 = \frac{4}{9}, \therefore r^2 = \pm\frac{2}{3}, \therefore r = \sqrt{\pm\frac{2}{3}}.$$

Hence the progression is

$$\frac{1}{2}, \frac{1}{2}\sqrt{\pm\frac{2}{3}}, \pm\frac{1}{3}, \pm\frac{1}{3}\sqrt{\pm\frac{2}{3}}, \frac{2}{9}.$$

10. Required the sum of the infinite series

$$1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \&c.,$$

and then deduce Σ for $x = 2$, and $x = -2$,

$$\Sigma = \frac{a}{1 - r}, \therefore \Sigma = 1 \div \left(1 - \frac{1}{x}\right) = \frac{x}{x - 1}.$$

$$\text{When } x = 2, \Sigma = 2; \text{ when } x = -2, \Sigma = \frac{2}{3}.$$

QUESTIONS IN WHICH PROPORTION AND PROGRESSION ARE CONCERNED. PAGE 480.

1. Divide 49 into two parts, such, that the greater increased by 6, may be to the less diminished by 11, as 9 to 2.

Let x be the greater, then $49 - x$ is the less, and by the question

$$x + 6 : 38 - x :: 9 : 2.$$

Multiplying extremes and means, we have

$$2x + 12 = 342 - 9x, \therefore 11x = 330, \therefore x = 30,$$

$\therefore 49 - x = 19$: hence the two parts are 30 and 19.

2. Two hundred stones are placed in a straight line, at intervals of 2 feet, the first stone being 20 yards in advance of a basket. Suppose a person, starting from the basket, collects the stones, and returns them, one by one to the basket. How much ground does he go over?

Upon returning the first stone, he will have gone 40 yards; upon returning the second, 40 yards and 4 feet; upon returning the third, 40 yards and 8 feet, and so on; hence we have to find the sum of the arithmetical progression, following, each term being feet.

$$120 + 124 + 128 + 132 + \&c., \text{ to 200 terms.}$$

By the formula, $S = \frac{1}{2}n\{2a + (n - 1)d\}$

$$\therefore S = 100\{240 + 199 \times 4\} = 103,600 \text{ feet,}$$

and 103,600 feet = $34,533\frac{1}{3}$ yards = $19\frac{1}{2}$ miles $213\frac{1}{3}$ yards.

3. The sum of four numbers in geometrical progression is equal to 1 added to the common ratio; and $\frac{1}{17}$ is the first term: required the numbers.

Let r be the common ratio, then since

$$S = a\frac{r^n - 1}{r - 1}, \therefore r + 1 = \frac{r^4 - 1}{17(r - 1)} = \frac{(r^2 - 1)(r^2 + 1)}{17(r - 1)}$$

$$\text{That is, } r + 1 = \frac{(r + 1)(r^2 + 1)}{17}, \therefore 17 = r^2 + 1,$$

$$\therefore r^2 = 16, \therefore r = \pm 4;$$

Hence the progression is $\frac{1}{17}, \pm\frac{4}{17}, \frac{16}{17}, \pm\frac{64}{17}$; that is to say, either of the two progressions, following, will fulfil the conditions, namely,

$$\frac{1}{17}, \frac{4}{17}, \frac{16}{17}, \frac{64}{17};$$

$$\text{or, } \frac{1}{17}, -\frac{4}{17}, \frac{16}{17}, -\frac{64}{17}.$$

4. From two towns, 165 miles apart, A and B set out to meet each other. A travels 1 mile the first day, 2 the second, 3 the third, and so on; B travels 20 miles the first day, 18 the second, 16 the third, and so on: in how many days will they meet?

Suppose they meet in n days, then

$$1 + 2 + 3 + \&c., \text{ to } n \text{ terms} = \tfrac{1}{2}n\{2 + (n - 1)\}$$
$$= \text{A's distance}$$
$$\text{and } 20 + 18 + 16 + \&c., \text{ to } n \text{ terms} = \tfrac{1}{2}n\{40 - 2(n - 1)\}$$
$$= \text{B's distance}$$

$$\therefore \text{ the sum of the distances} = \tfrac{1}{2}n\{42 - n + 1\} = 165$$
$$\therefore 42n - n^2 + n = 330, \therefore n^2 - 43n = -330,$$
$$\therefore \text{Rule II., page 474, } 2n - 43 = \sqrt{(43^2 - 1320)}$$
$$= \sqrt{529} = \pm 23$$
$$\therefore n = \frac{43 \pm 23}{2} = 10, \text{ or } 33.$$

Hence they meet in 10 days. If it were a condition that B is to continue travelling after he meets and passes A, diminishing his distance by two miles daily, he will on the eleventh day travel 0 miles, and therefore on the twelfth day − 2 miles; that is, he will *return back* 2 miles, on the thirteenth day 4 miles, and so on. The second value of n above, shows that in 33 days B will in this way again come up with A, by *overtaking* him. In these 33 days A will have proceeded directly onwards to a distance of 561 miles, while B will have been *following* him for a distance of 396 miles, for B's whole distance is expressed by − 396; and 561 − 396 = 165: so that the *algebraic sum* of the two distances still makes 165 miles; and the only condition implied in the algebraical statement of the question is, that the sum of the two distances shall be 165 miles.

5. The sum of the first and second of four numbers in geometrical progression is 15, and the sum of the third and fourth 60; what are the numbers?

Let the geometrical progression be

$$\frac{x}{y^2}, \frac{x}{y}, x, xy,$$

then, by the question, the conditions are

$$\frac{x}{y^2} + \frac{x}{y} = 15 \quad . \quad . \quad . \quad (1),$$

$$\text{and} \quad x + xy = 60 \quad . \quad . \quad . \quad (2),$$

and since 4 times the first is equal to the second, we have, after dividing each by x,

$$\frac{4}{y^2} + \frac{4}{y} = 1 + y, \therefore \frac{4}{y^2} - 1 = y - \frac{4}{y},$$

or, which is the same thing, $\frac{4}{y^2} - 1 = -y\left(\frac{4}{y^2} - 1\right)$,

that is, transposing, $(y + 1)\left(\frac{4}{y^2} - 1\right) = 0 \quad . \quad . \quad . \quad (3).$

Now a product is 0, *whichever* of the factors is 0; hence we must have either

$$y + 1 = 0, \text{ or } \frac{4}{y^2} - 1 = 0.$$

The first of these conditions cannot be admitted, because it is plain, that for $y = -1$, the equation (2), becoming $0 = 60$, is impossible: hence the condition which is to satisfy (3) is

$$\frac{4}{y^2} - 1 = 0, \therefore 4 - y^2 = 0, \therefore y^2 = 4, \therefore y = \pm 2,$$

$$\therefore (2), x = \frac{60}{1 + y} = \frac{60}{1 \pm 2} = 20, \text{ or } -60.$$

Hence the progression is either

$$5, 10, 20, 60, \text{ or } -15, 30, -60, 120.$$

6. The sum of three numbers in geometrical progression is 35; and the mean term is to the difference of the extremes as 2 to 3; what are the numbers?

Let the three numbers be $\frac{x}{y}$, x, and xy; then by the question

$$\frac{x}{y} + x + xy = 35 \; . \; . \; . \; (1),$$

$$\text{and } x : xy - \frac{x}{y} : : 2 : 3, \; \therefore 3x = 2\left(xy - \frac{x}{y}\right),$$

$$\therefore 3 = 2y - \frac{2}{y} \; . \; . \; . \; (2),$$

$$\therefore 3y = 2y^2 - 2, \; \therefore 2y^2 - 3y = 2,$$

Rule II., page 474, $4y - 3 = \sqrt{(16 + 9)}$
$= \sqrt{25} = \pm 5,$

$$\therefore y = \frac{3 \pm 5}{4} = 2, \text{ or } -\tfrac{1}{2}.$$

Hence, (1) becomes

$$\frac{x}{2} + x + 2x = 35, \text{ or } -2x + x - \frac{x}{2} = 35,$$

that is, $3\frac{1}{2}x = 35$, or $-1\frac{1}{2}x = 35$,

$$\therefore x = 10, \text{ or, } -23\tfrac{1}{3}.$$

Hence the numbers are either

5, 10, and 20 ; or $46\frac{2}{3}$, $-23\frac{1}{3}$, and $11\frac{2}{3}$,

both of which sets satisfy the conditions of the question.

EXTRACTION OF THE SQUARE ROOT. EXAMPLES FOR EXERCISE. PAGE 482.

1.
$$\begin{array}{ll}
 & x^2 + 4bx + 4b^2(x + 2b \\
 & x^2 \\
2x + 2b) & 4bx + 4b^2 \\
 & 4bx + 4b^2.
\end{array}$$

2.
$$\begin{array}{ll}
 & 9x^4 + 12x^3 + 10x^2 + 4x + 1(3x^2 + 2x + 1 \\
 & 9x^4 \\
6x^2 + 2x) & 12x^3 + 10x^2 \\
 & 12x^3 + 4x^2 \\
6x^2 + 4x + 1) & 6x^2 + 4x + 1 \\
 & 6x^2 + 4x + 1
\end{array}$$

3.
$$\begin{array}{ll}
 & 9x^4 + 12x^3 + 34x^2 + 20x + 25)3x^2 + 2x + 5 \\
 & 9x^4 \\
6x^2 + 2x) & 12x^3 + 34x^2 \\
 & 12x^3 + 4x^2 \\
6x^2 + 4x + 5) & 30x^2 + 20x + 25 \\
 & 30x^2 + 20x + 25
\end{array}$$

4.
$$\begin{array}{ll}
 & x^6 + 4x^5 + 2x^4 + 0x^3{}^{*} + 9x^2 - 4x + 4(x^3 + 2x^2 - x + 2 \\
 & x^6 \\
2x^3 + 2x^2) & 4x^5 + 2x^4 \\
 & 4x^5 + 4x^4 \\
2x^3 + 4x^2 - x) & -2x^4 + 0x^3 + 9x^2 \\
 & -2x^4 - 4x^3 + x^2 \\
2x^3 + 4x^2 - 2x + 2 & 4x^3 + 8x^2 - 4x + 4 \\
 & 4x^3 + 8x^2 - 4x + 4
\end{array}$$

5.
$$\begin{array}{ll}
 & x^6 - 4x^5 + 10x^4 - 4x^3 - 7x^2 + 24x + 16(x^3 - 2x^2 + 3x + 4 \\
 & x^6 \\
2x^3 - 2x^2) & -4x^5 + 10x^4 \\
 & -4x^5 + 4x^4 \\
2x^3 - 4x^2 + 3x) & 6x^4 - 4x^3 - 7x^2 \\
 & 6x^4 - 12x^3 + 9x^2 \\
2x^3 - 4x^2 + 6x + 4) & 8x^3 - 16x^2 + 24x + 16 \\
 & 8x^3 - 16x^2 + 24x + 16
\end{array}$$

6.
$$\begin{array}{ll}
 & 4x^4 + 6x^3 + \frac{89}{4}x^2 + 15x + 25(2x^2 + \frac{3}{2}x + 5 \\
 & 4x^4 \\
4x^2 + \frac{3}{2}x) & 6x^3 + \frac{89}{4}x^2 \\
 & 6x^3 + \frac{9}{4}x^2 \\
4x^2 + 3x + 5) & \frac{80}{4}x^2 + 15x + 25 \\
 & 20x^2 + 15x + 25
\end{array}$$

---

[*] The $0x^3$ in the above square is of no value, and has been retained solely to indicate the successive diminution in the value of the exponents of x.—ED.

Otherwise.—$16x^4 + 24x^3 + 89x^2 + 60x + 100(4x^2 + 3x + 10$

$16x^4$ $\quad\therefore$ Root $= 2x^2 + \frac{3}{2}x + 5$

$8x^2 + 3x)$ $\quad 24x^3 + 89x^2$
$\quad 24x^3 + 9x^2$

$8x^2 + 6x + 10$ $\quad 80x^2 + 60x + 100$
$\quad 80x^2 + 60x + 100$

SQUARE ROOT OF A BINOMIAL SURD. EXAMPLES FOR EXERCISE. PAGE 483.

1. Put $\sqrt{(19 + 8\sqrt{3})} = x + y$, $\therefore \sqrt{(19 - 8\sqrt{3})} = x - y$. Taking the product,
$\sqrt{(361 - 192)} = x^2 - y^2$; that is, $13 = x^2 - y^2$. Also,
$19 + 8\sqrt{3} = x^2 + 2xy + y^2$, $\therefore 19 = x^2 + y^2$
$\therefore \overline{32} = 2x^2,\ 6 = 2y^2$
$\therefore x^2 = 16, \therefore x = 4;\ y^2 = 3, \therefore y = \sqrt{3}$
$\therefore \sqrt{(19 + 8\sqrt{3})} = 4 + \sqrt{3}$.

2. $\sqrt{(12 - \sqrt{140})} = x - y$, $\therefore \sqrt{(12 + \sqrt{140})} = x + y$. Taking the product,
$\sqrt{(144 - 140)} = x^2 - y^2$; that is, $2 = x^2 - y^2$.
Also, $12 - \sqrt{140} = x^2 - 2xy + y^2$, $\therefore 12 = x^2 + y^2$
$\therefore \overline{14} = 2x^2,\ 10 = 2y^2$
$\therefore x^2 = 7, \therefore x = \sqrt{7};\ y^2 = 5, \therefore y = \sqrt{5}$.
$\therefore \sqrt{(12 - \sqrt{140})} = \sqrt{7} - \sqrt{5}$.

3. $\sqrt{(7 - 2\sqrt{10})} = x - y$, $\therefore \sqrt{(7 + 2\sqrt{10})} = x + y$. Taking the product,
$\sqrt{(49 - 40)} = x^2 - y^2$; that is, $3 = x^2 - y^2$

Also, $7 - 2\sqrt{10} = x^2 - 2xy + y^2$, $\therefore 7 = x^2 + y^2$
$\therefore \overline{10} = 2x^2,\ 4 = 2y^2$
$\therefore x^2 = 5, \therefore x = \sqrt{5};\ y^2 = 2, \therefore y = \sqrt{2}$
$\therefore \sqrt{(7 - 2\sqrt{10})} = \sqrt{5} - \sqrt{2}$.

4. $\sqrt{(8 + \sqrt{39})} = x + y$, $\therefore \sqrt{(8 - \sqrt{39})} = x - y$. Taking the product,
$\sqrt{(64 - 39)} = x^2 - y^2$; that is, $5 = x^2 - y^2$

Also, $8 + \sqrt{39} = x^2 + 2xy + y^2$, $\therefore 8 = x^2 + y^2$
$\therefore \overline{13} = 2x^2,\ 3 = 2y^2$
$\therefore x^2 = \frac{13}{2}, \therefore x = \frac{1}{2}\sqrt{26};\ y^2 = \frac{3}{2}, \therefore y = \frac{1}{2}\sqrt{6}$
$\therefore \sqrt{(8 + \sqrt{39})} = \frac{1}{2}(\sqrt{26} + \sqrt{6})$.

5. $\sqrt{(76 - 32\sqrt{3})} = x - y$, $\therefore \sqrt{(76 + 32\sqrt{3})} = x + y$. Taking the product, $\sqrt{(5776 - 3072)} = x^2 - y^2$; that is, $52 = x^2 - y^2$.
Also, $76 - 32\sqrt{3} = x^2 - 2xy + y^2$, $\therefore 76 = x^2 + y^2$
$\therefore \overline{128} = 2x^2,\ 24 = 2y^2$;
$\therefore x^2 = 64, \therefore x = 8;\ y^2 = 12, \therefore y = 2\sqrt{3}$,
$\therefore \sqrt{(76 - 32\sqrt{3})} = 8 - 2\sqrt{3}$.

6. $\sqrt{(31 + 12\sqrt{-5})} = x + y$, $\therefore \sqrt{(31 - 12\sqrt{-5})} = x - y$. Taking the product, $\sqrt{(961 + 720)} = x^2 - y^2$; that is, $41 = x^2 - y^2$.

Also, $31 + 12\sqrt{-5} = x^2 + 2xy + y^2$, $\therefore 31 = x^2 + y^2$
$\therefore \overline{72} = 2x^2,\ 10 = -2y^2$;
$\therefore x^2 = 36, \therefore x = 6;\ y^2 = -5, \therefore y = \sqrt{-5}$,
$\therefore \sqrt{(31 + 12\sqrt{-5})} = 6 + \sqrt{-5}$.

TO RATIONALISE BINOMIAL SURDS. EXAMPLES FOR EXERCISE. PAGE 484.

1. Prove that $\frac{3}{2\sqrt{7} - 3\sqrt{2}} = \frac{3}{10}(2\sqrt{7} + 3\sqrt{2})$.

Multiplying numerator and denominator by $2\sqrt{7} + 3\sqrt{2}$,

$$\frac{3}{2\sqrt{7} - 3\sqrt{2}} = \frac{3(2\sqrt{7} + 3\sqrt{2})}{28 - 18} = \frac{3}{10}(2\sqrt{7} + 3\sqrt{2}).$$

2. Prove that $\frac{2 + \sqrt{3}}{3 + \sqrt{3}} = \frac{3 + \sqrt{3}}{6}$. Multiplying the terms by $3 - \sqrt{3}$,

$$\frac{2 + \sqrt{3}}{3 + \sqrt{3}} = \frac{(2 + \sqrt{3})(3 - \sqrt{3})}{6} = \frac{3 + \sqrt{3}}{6}.$$

3. Prove that $\frac{\sqrt{6}}{\sqrt{8}+\sqrt{3}}=\frac{4\sqrt{3}-3\sqrt{2}}{5}$

$$\frac{\sqrt{6}}{\sqrt{8}+\sqrt{3}}=\frac{\sqrt{6}\times(\sqrt{8}-\sqrt{3})}{8\sqrt{\ }+\sqrt{3})\,(\sqrt{8}-\sqrt{3})}=\frac{\sqrt{48}-\sqrt{18}}{5}=\frac{4\sqrt{3}-3\sqrt{2}}{5}.$$

4. Prove that $\frac{2}{\sqrt{3}-\sqrt[3]{2}}=2(\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4})$.

$$\frac{2}{\sqrt[3]{3}-\sqrt[3]{2}}=\frac{2(\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4})}{(\sqrt[3]{3}-\sqrt[3]{2})\,(\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4})}=\frac{2(\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4})}{3-2}$$
$$=2(\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}).$$

5. Prove that $\frac{5}{\sqrt[3]{7}-\sqrt[3]{5}}=\frac{5}{2}(\sqrt[3]{49}+\sqrt[3]{35}+\sqrt[3]{25})$.

Multiplying numerator and denominator by $\sqrt[3]{49}+\sqrt[3]{35}+\sqrt[3]{25}$, we have

$$\frac{5}{\sqrt[3]{7}-\sqrt[3]{5}}=\frac{5(\sqrt[3]{49}+\sqrt[3]{35}+\sqrt[3]{25})}{7-5}=\frac{5}{2}(\sqrt[3]{49}+\sqrt[3]{35}+\sqrt[3]{25}).$$

6. Prove that $\frac{\sqrt{12}-\sqrt{10}}{\sqrt{6}+\sqrt{5}}=11\sqrt{2}-4\sqrt{15}\sqrt{\ }$. Multiplying numerator and denominator by $\sqrt{6}-\sqrt{5}$,

$$\frac{\sqrt{12}-\sqrt{10}}{\sqrt{6}+\sqrt{5}}=\frac{(\sqrt{12}-\sqrt{10})\,(\sqrt{6}-\sqrt{5})}{6-5}=\sqrt{72}-2\sqrt{60}+\sqrt{50}$$
$$=6\sqrt{2}-4\sqrt{15}+5\sqrt{2}=11\sqrt{2}-4\sqrt{15}.$$

EXTRACTION OF THE CUBE ROOT PAGE 486.

1.

$$\begin{array}{lll}
0 & 0 & x^6+9x^5+6x^4-99x^3-42x^2+441x-343(x^2+3x-7^{*} \\
x^2 & x^4 & x^6 \\
x^2 & x^4 & 9x^5+6x^4-99x3 \\
x^2 & 2x^4 & 9x^5+27x^4+27x^3 \\
2x^2 & 3x^4 & -21x^4-126x^3-42x^2+441x-343 \\
x^2 & \quad 9x^3+9x^2 & -21x^4-126x^3-42x^2+441x-343 \\
3x^2 & 3x^4+9x^3+9x^2 & \\
\quad 3x & \quad 9x^3+18x^2 & \\
3x^2+3x & 3x^4+18x^3+27x^2 & \\
\quad 3x & \qquad -21x^2-63x+49 & \\
3x^2+6x & 3x^4+18x^3+6x^2-63x+49 & \\
\quad 3x & & \\
3x^2+9x & & \\
\quad -7 & & \\
3x^2+9x-7 & &
\end{array}$$

Hence the cube root of the proposed polynomial is x^2+3x-7.

2.

$$\begin{array}{lll}
0 & 0 & x^6+6x^5+0x^4\dagger-40x^3+0x^2+96x-64(x^2+2x-4 \\
x^2 & x^4 & x^6 \\
x^2 & x^4 & 6x^5+0x^4-40x^3 \\
x^2 & 2x^4 & 6x^5+12x^4+8x^3 \\
2x^2 & 3x^4 & -12x^4-48x^3+0x^2+96x-64 \\
x^2 & \quad 6x^3+4x^2 & -12x^4-48x^3+0x^2+96x-64 \\
3x^2 & 3x^4+6x^3+4x^2 & \\
\quad 2x & \quad 6x^3+8x^2 & \\
3x^2+2x & 3x^4+12x^3+12x^2 & \\
\quad 2x & \qquad -12x^2-24x+16 & \\
3x^2+4x & 3x^4+12x^3+0x^2-24x+16 & \\
\quad 2x & & \\
3x^2+6x & & \\
\quad -4 & & \\
3x^2+6x-4 & &
\end{array}$$

Hence the cube root is x^2+2x-4.

---

* See the Note appended to the last example in this Key.

† This is of no value, and merely completes the series of exponents of x.

```
3.  0          0           12,994,449,551(2351, the
    2          4           8                 cube root.
    —          —           ———— 1
    2          4           4994
    2          8           4167
    —          —— 1        ———— 2
    4          12           827449
    2           189         810875
    — 1        ———          ————— 3
    6          1389          16574551
     3          198          16574551
    —          ——— 2         —————
    63         1587
     3           3475
    —          ————
    66         162175
     3           3500
    — 2        ———— 3
    69         165675
     5              7051
    —          —————
    695        16574551
     5         —————
    —
    700
     5
    — 3
    705
      1
    —
    7051
```

In this method of extracting the cube root, all abridgment of the work has been disregarded, in order that the operation may be exhibited in the fullest detail.

```
4.  0          0           2 (1·25992104989 . . , the
    1          1           1                 cube root.
    —          —           — 1
    1          1           1
    1          2            ·728
    —          — 1         ——— 2
    2          3            ·272
    1           ·64         ·225125
    — 1        ———          ———— 3
    3          3·64          46875
     ·2         ·68          42491979
    —          ——— 2         ————— 4
    3·2        4·32           4383021
     ·2         ·1825         4282778799
    —          ————           —————— 5
    3·4        4·5025          100242201
     ·2         ·1850           95242392
    — 2        ———— 3          ————
    3·6        4·6875           4999809
      5          33831          4762199
    ——         —————           ———
    3·65       4·721331          237610
      5          33912           190488
    ——         ———— 4            ———
    3·70       4·755243           47122
      5          340011           42860
    —— 3       —————              ———
    3·75       4·75864311          4262
      9          340092            3810
    ——         ————— 5             ——
    3·759      4·76204403           452
      9            7599             429
    ——         —————                ——
    3·768      4·7621196,2           23
      9            756
    —— 4       ————
    3·777      4·7621952
      9            38
    ——         ————
    3·7779     4·762199,0
      9            4
    ——         ————
    3·7788     4·7,6,2,2,03
      9
    —— 5
    3·7,7,9,7
```

In the preceding example the decimals of the root are interminable, since 2 is not an exact cube; it is required to approximate to the cube root as far as eleven or twelve places of decimals only; so that there is no necessity, in carrying on the work of development, to make any provision for decimals beyond this prescribed limit; and, accordingly, in the operation here shown, the decimals in the several columns have been restrained from accumulating to an unnecessary extent. You will, of course, remember that in multiplying a number abridged of final decimals, the *carryings* from the rejected decimals are to be taken account of, as noticed at page 486, ALGEBRA.

Hence the cube root of 2, true as far as eleven places of decimals, is $\sqrt[3]{2} = 1{\cdot}25992104989$. In the third column of the work, the decimals have not been allowed to accumulate after the step marked 5, since more decimals, to the right of those furnished at that step, would have been superfluous, if only eleven or twelve decimals are required in the root; and, in order to provide against this unnecessary increase of decimals, the two preceding columns are curtailed of the decimals which are foreseen to contribute only to that portion of the third column thus rejected. Another decimal—viz., 4, or 5 very nearly—might obviously have been added to those already in the root; so that we may with safety affirm that $\sqrt[3]{2} = 1{\cdot}259921049895$ is true, to the nearest unit, in the twelfth place of decimals. Had the suppression of additional decimals been postponed till another step of the work had been reached, we should have found

$$\sqrt[3]{2} = 1{\cdot}259921049894873 \ldots .$$

which is true as far as fifteen places of decimals.

Such an extent of figures is very rarely, if ever, required in actual practice, and they have been given here merely to show the accuracy with which a cube root may be extracted by the foregoing contracted method.

```
5.  0          0           959(9·8614218 . . . .
    9          81          729
    —          —           ——— 1
    9          81          230
    9          162         212·192
    —          ——— 1       ———— 2
    18         243          17·808
     9          22·24       17·393256
    — 1        ————         ————— 3
    27         265·24         ·414744
      ·8        22·88         ·291688
    ——         ——— 2          ————
    27·8       288·12         ·123056
      ·8         1·7676       ·116692
    ——         —————          ————
    28·6       289·8876          6364
      ·8         1·7712          5835
    —— 2       ————— 3           ——
    29·4       291·6588           529
       6            296           292
    ——         ————               —
    29·46      291·688,4          237
       6             30           233
    ——         ————               —
    29·52      291·718              4
       6            12
    —— 3       ————
    2,9,·5,8   291·73,0
                      1
               ————
               29,1·,7,4
```

Hence, the cube root of 959 is 9·8614218, which is true to seven places of decimals.

6. To find the cube root of

$$x^6 - 15x^5y + 69x^4y^2 - 138x^2y^4 - 60xy^5 - 8x^6.$$

From this expression a term was accidentally omitted; but as it is instructive to deal with it as it is, the wanting term is purposely left unsupplied, and the work carried on as in next page.

$$\begin{array}{lll}
 & 0 & x^6 - 15x^5y + 69x^4y^2 - 138x^2y^4 - 60xy^5 - 8x^6(x^2 - 5xy - 2y^2 \\
x^2 & x^4 & x^6 \\
\hline
x^2 & x^4 & -15x^5y + 69x^4y^2 + 0x^3y^3 \\
x^2 & 2x^4 & -15x^5y + 75x^4y^2 - 125x^3y^3 \\
\hline
2x^3 & 3x^4 & -6x^4y^2 + 125x^3y^3 - 138x^2y^4 - 60xy^5 - 8x^6 \\
x^3 & -15x^3y + 25x^2y^2 & -6x^4y^2 + 60x^3y^3 - 138x^2y^4 - 60xy^5 - 8x^6 \\
\hline
3x^3 & 3x^4 - 15x^3y + 25x^2y^2 & 65x^3y^3 \\
-5xy & -15x^3y + 50x^2y^2 & \\
\hline
3x^2 - 5xy & 3x^4 - 30x^3y + 75x^2y^2 & \\
-5xy & -6x^2y^2 + 30xy^3 + 4y^4 & \\
\hline
3x^2 - 10xy & 3x^4 - 30x^3y + 69x^2y^2 + 30xy^3 + 4y^4 & \\
-5xy & & \\
\hline
3x^2 - 15xy & & \\
-2y^2 & & \\
\hline
3x^2 - 15xy - 2y^2 & &
\end{array}$$

Hence the cube root of the proposed polynomial is $x^2 - 5xy - 2y^2$, with $65x^3y^3$ for remainder; so that the expression proposed differs by $65x^3y^3$ from the complete cube of $x^2 - 5xy - 2y^2$. If the remainder be introduced, with changed sign, into the polynomial, it will then become a complete cube: that is,

$$x^6 - 15x^5y + 69x^4y^2 - 65x^3y^3 - 138x^2y^4 - 60xy^5 - 8x^6 = (x^2 - 5xy - 2y^2)^3.$$

And in this way we may always ascertain what expression must be introduced into a proposed polynomial, in order to render it a complete cube.

NOTE.—The process adopted above, in imitation of that at page 485, of the Algebra, occupies a good deal more space than the work by the old method; but it must be observed that *here* there are no bye-operations: the whole is placed fairly before the eye, and every step may be performed with great ease and rapidity, and may be readily revised in case of mistake—advantages not to be overlooked.

The solutions now completed will, we think, afford to the learner all the aid he can reasonably expect in his efforts to make himself acquainted with the PRINCIPLES OF ALGEBRA. Wherever his own attempts to accomplish the satisfactory solution of a question fail, he will find it of much service to him to have at his command the assistance here offered; and even when his endeavours are successful, he will often gain instruction by comparing his own work with the details thus given. In the expectation that the present collection of solutions will be consulted with a view to such instruction, we have frequently given different methods of working out the same example, and of thus exhibiting certain algebraical artifices and expedients—familiar enough to proficients, but which can be acquired by a learner only by practice, and the careful examination of illustrative models. No one can become an algebraist by *Rule:* all that rules can do is to dictate the mode of applying general principles; while, in many special cases, independent judgment and ingenuity must be exercised, in reference to the particular circumstances of the inquiry, in order to avoid an unnecessary accumulation of symbols, and a needless outlay of time and trouble—in fact, in order to give to the algebraic process that compactness of form, and neatness of finish, which constitute what is called an elegant solution. This mastery over his subject the student can acquire only by consulting the best models, and by cultivating the power, himself, of proceeding under the guidance of his own free judgment and penetration, unfettered by the shackles of *rules.* A learner should always be on the look-out for those facilitating expedients, which are often to be brought into operation in cases, in reference to which rules can afford but imperfect guidance. He would do well, however, to keep in remembrance that a *short* solution is not necessarily the *best* solution: what looks short upon paper may be the result of more mental exertion than what looks long; the art is so to manage that the expenditure of time and thought may be reduced to the smallest possible amount: a solution thus characterised is entitled to be called an elegant solution. To those merely mechanical operations—the performance of which it is the business of the early parts of algebra to teach—these remarks do not, of course, apply; they refer exclusively to the subject of problem-solving, and to the more advanced portions of algebraical research.

CHAPTER VI.

SERIES AND LOGARITHMS.

ON SERIES.

1. *The Principle of the Permanence of Equivalent Forms.*

IT was stated at page 451, that "the processes of Algebra are, for the most part, only processes of Arithmetic, extended and rendered more comprehensive by the aid of a new set of symbols, taken in combination with the well-known symbols of Arithmetic;" and in the explanations following, it is made to appear that Algebra is a generalisation of Arithmetic—that whereas 2, 5 . . . represent certain special numbers, *a*, *b* . . . represent *any* numbers. This is professedly an elementary view of the case; and, as an elementary view, is quite sufficient. But when the nature of the generalisation is more closely considered, it appears that, in what is commonly called Algebra, there are really two distinct, though closely connected sciences, which may be called respectively Arithmetical Algebra, and Symbolical Algebra. In Arithmetical Algebra, "the symbols represent numbers, whether abstract or concrete, whole or fractional, and the operations to which they are subject are assumed to be identical, in meaning and extent, with the operations of the same name in common arithmetic. The only distinction between the two sciences consists in the substitution of general symbols for digital numbers."

Thus, in arithmetic, it is impossible to subtract 7 from 5: so that $5 - 7$ is impossible; and hence in arithmetical Algebra, when we write $a - b$, we do so with the tacit assumption that $a > b$. If we generalise a step farther than this, and allow ourselves to write $a - b$ for all values of a and b, then it is clear that the negative sign has a more extended meaning than that of mere subtraction; and it remains for us to ascertain what this more extended meaning is. The science which concerns itself with this second generalisation is called *Symbolical Algebra*. Thus, then, we have, in all three sciences—

(1). Arithmetic, in which the symbols employed are *particular* in *form*, and *particular* in *value*.

(2). Arithmetical Algebra, in which the symbols employed are *general* in *form*, but *particular* in *value*.

(3). Symbolical Algebra, in which the symbols employed are *general* in *form*, and also *general* in *value*.

Thus, as the second of these sciences is a generalisation of the first, so the third is a generalisation of the second. The principle in accordance with which this second generalisation is conducted is called that of "The permanence of equivalent forms." The principle may be stated as follows:—

"Whatever algebraical forms are equivalent, when the symbols are general in form, but specific in value, will be equivalent likewise when the symbols are general in value, as well as in form."

For the full exposition of relations between these two sciences, the advanced reader is referred to a *Treatise on Algebra*, by George Peacock, D.D., to whom is due the detection of the co-existence of these two sciences in that which is generally treated as one science—Algebra. We shall have several occasions to make use of the principles above enunciated in the course of the following pages. As an example of their application, we will reconsider the Theory of Indices already treated in p. 465.

2. *On the Theory of Indices.*

We have already seen that a^m signifies $a \times a \times a \times a$, &c., to m factors, when, of course, m must be a whole number.

In like manner a^n signifies $a \times a \times a$, &c., to n factors. Hence—

$$a^m \times a^n = a \times a \times a \ldots \text{to} (m+n) \text{ factors,}$$

and therefore $a^m \times a^n = a^{m+n}$.

This is a result in *Arithmetical Algebra*. It is perfectly *general in form*, but it is *particular* in value; for m and n are, by the definition of a^m, limited to being positive whole numbers. If we suppose m and n to have negative or fractional values, this involves a generalisation of our original definition; and the question arises, what meaning we must assign to such expressions as a^{-n} $a^{\frac{p}{q}}$. To answer it, we proceed in the following manner:—By assuming m and n general in value as well as in form, we enter the domains of Symbolical Algebra; hence, by the principle of the permanence of equivalent forms, under all circumstances—

$$a^m \times a^n = a^{m+n},$$

$$\therefore a^m \times a^n \times a^r = a^{m+n} \times a^r = a^{m+n+r};$$

and so on for any number of terms. Hence—

$$a^{\frac{p}{q}} \times a^{\frac{p}{q}} \times a^{\frac{p}{q}} \times \ldots \text{to } q \text{ terms} = a^{\frac{p}{q}+\frac{p}{q}+\frac{p}{q}+\ldots \text{ to } q \text{ fraction}}$$

$$\left(a^{\frac{p}{q}}\right)^q = a^p.$$

$$\therefore a^{\frac{p}{q}} = \sqrt[q]{a^p}.$$

i.e., $a^{\frac{p}{q}}$ must (in accordance with our general principle) signify the q^{th} root of the p^{th} power of a.

Again, $a^m \times a^n \times a^{-n} = a^{m+n} = a^m$,

$$\therefore a^n \times a^{-n} = 1.$$

$$\therefore a^{-n} = \frac{1}{a^n}.$$

Hence, we see that in assigning the meaning $\sqrt{a}$ to $a^{\frac{1}{2}}$, we are doing so not arbitrarily, but in accordance with a principle which lies at the foundation of Algebra.

3. *On Impossible Expressions.*

Again, we know that $\sqrt{a^2} = a$. In like manner if we were to have $-a^2$, this is $= a^2 \times (-1)$, and $\therefore \sqrt{-a^2} = a\sqrt{-1}$. The expression $\sqrt{-1}$ is frequently spoken of as an "impossible quantity," an "imaginary expression," and so on, since -1 cannot be produced by multiplying either $+1$ by $+1$, or -1 by -1. In reality, however, $\sqrt{-1}$ is as possible or as impossible as -1; for in arithmetical Algebra a and $\sqrt{a}$ are only admissible on the supposition that a is positive. In symbolical Algebra this restriction is removed, and therefore in *that* both $-a$ and $\sqrt{-a}$ are admissible. Hence in future investigations we shall make use of $\sqrt{-1}$ just as freely as $\sqrt{a}$, whenever it may suit our purpose, quite undeterred by the circumstance of its so-called impossibility. Of course there are many differences between the symbols -1 and $\sqrt{-1}$; for instance, the interpretation of the former is a much simpler matter than the interpretation of the latter, and in some cases $a - b$ belongs to arithmetical Algebra; but $a + b\sqrt{-1}$ *never* does. We cannot enter further into the matter now; what we have said will be enough to explain that we are justified in introducing into our calculations expressions which are called "imaginary," or "impossible."

4. *To prove that if* $A_0 + A_1x + A_2x^2 + \cdots + Anx^n = a_0 + a_1x + a_2x^2 + \ldots + a_nx_n$, *for all values of* x. *Then* $A_0 = a_0$, $A_1 = a_1$, $A_2 = a_2$, $-A_n = a^n$.

For since $A_0 + A_1x + \ldots + A_nx^n = a_0 + a_1x + \ldots + a_nx^n$ are equivalent for ALL values of x; they are equivalent when $x = 0$. $\therefore Ao = a_0$, and $\therefore A_1x + A_2x^n + \ldots + A_nx^n = a_1x + a_2x^n + \ldots + a^nx^n$; $\therefore A_1 + A_2 x + \cdot\cdot + A_nx^{n-1} = a_1 + a_2x + \ldots + a_nx^{n-1}$ for all values of x: and hence when $x = o$, $\therefore A_1 = a_1$, and so on. Hence $A_2 = a_2 \ldots$ and $A_n = a_n$.

This is called the principle of Indeterminate Coefficients.

It will be seen, that in the case supposed, where the number of terms in each series is finite, the proof is quite rigid. If each series were infinite the proof would not then be conclusive; and, accordingly, we shall refrain from using this principle except in cases where no objection can be raised to its use. Such as the following:—

(1). To resolve $\frac{1-2x+3x^2}{(x-1)(x-2)(x-3}$ into partial fractions.

Assume $\frac{1-2x+3x^2}{(x-1)(x-2)(x-3)} = \frac{A_1}{x-1} + \frac{A_2}{x-2} + \frac{A_3}{x-3}$.

$\therefore 3x^2 - 2x + 1 = A_1(x-2)(x-3) + A_2(x-3)(x-1) + A_3(x-1)(x-2) = x^2(A_1 + A_2 + A_3) - x(5A_1 + 4A_2 + 3A_3) + 6A_1\ 3A_2 + 2A_3$.

This being true for all values of x, we have

$$\begin{aligned} A_1 + A_2 + A_3 &= 3. \\ 5A_1 + 4A_2 + 3A_3 &= 2 \\ 6A_1 + 3A_2 + 2A_3 &= 1. \\ \therefore 2A_1 + A_2 &= -7. \\ 4A_1 + A_2 &= -5 \\ \hline 2A_1 &= 2. \end{aligned}$$

$$\therefore A_1 = 1.$$
$$\therefore A_2 = -9.$$
$$A_2 = 11.$$

$$\therefore \frac{3x^2-2x+1}{(x-1)(x-2)(x-3)} = \frac{1}{x-1} - \frac{9}{x-2} + \frac{11}{x-3}.$$

N.B.—A fraction written in the above form is said to be resolved into its partial fractions.

(2). Resolve $\frac{x-1}{(x+1)(x^2+3)}$ into its partial fractions.

Assume $\frac{x-1}{(x+1)(x^2+3)} = \frac{A}{x-1} + \frac{Mx+N}{x^2+3}$.

$$\therefore x - 1 = x^2(A + M) + x(M + N) + 3A + N$$

$$\begin{aligned} \therefore A + M &= 0 \\ M + N &= 1 \\ 3A + N &= -1 \\ \therefore A - N &= -1 \\ \therefore 4A &= -2 \\ A &= -\tfrac{1}{2}. \\ N &= \tfrac{1}{2} \\ M &= \tfrac{1}{2}. \end{aligned}$$

$$\therefore \frac{x-1}{(x+1)(x^2+8)} = -\frac{1}{2(x+1)} + \frac{x+1}{2(x^2+3)}.$$

The student can prove the following:—

(1). $\frac{3x^2+4x-1}{(x+1)(x-1)(x-2)} = \frac{1}{x+1} + \frac{1}{x-1} + \frac{1}{x-2}$

(2). $$\frac{2x+3}{(x+2)(x-3)(x-5)} = -\frac{1}{35(x+2)} - \frac{11}{10(x-3} + \frac{79}{70(x-5)}$$

(3). $$\frac{x}{(x-a)(x-b)} = \frac{a}{a-b}\cdot\frac{1}{x-a} - \frac{b}{a-b}\cdot\frac{1}{x-b}.$$

(4). $$\frac{1}{(x-a)(x-b)(x-c)} = \frac{1}{(c-a)(b-a)}\cdot\frac{1}{x-a} + \frac{1}{(a-b)(c-b)}\cdot\frac{1}{x-b} + \frac{1}{(a-c)(b-c)}\cdot\frac{1}{x-c}$$

(5). A series is a number of algebraical expressions, each of which is connected with those which precede it in some determinate manner.

For example:—In the treatise on Elementary Algebra, we have had examples of series in the arithmetical and geometrical progressions.* In the former case, each term is derived from the one preceding it by adding a certain known number called the common difference. In the latter case, each term is derived from the one preceding it by multiplying that term by a certain known number called the common ratio. Hence,

$$a + (a+b) + (a+2b) + (a+3b) \cdots\cdots$$
$$a + ar + ar^2 + ar^3 + \cdots\cdots$$
$$\text{and } 1 + r + r^2 + r^3 + \cdots\cdots$$

are series.

Def.—A series is called a finite series when it has an assignable last term. It is called an infinite series when, if we fix on any term whatever, there are terms beyond it.

Thus, $1 + r + r^2 + \cdots\cdots + r^n$ is a finite series. But $1 + r + r^2 + \cdots\cdots$ *ad inf.* is an infinite series, because, if we take any term whatever—for instance, the 50th, or 500th, or 5000th—there are always terms beyond it.

6. *To explain what is meant by a Convergent and a Divergent Series.*

Def.—If the sum of the terms of a series has an arithmetical limit when the number of terms is infinite, that series is *convergent*; if otherwise, it is *divergent*.

If we divide 1 by $1 - r$, we shall produce $1 + r + r^2 + \cdots\cdots$ which series we can continue to produce to any number of terms whatever. Hence the fraction $\frac{1}{1-r}$, and the series, $1 + r + r^2 + \cdots\cdots$ *ad inf.* are equivalent to each other; or

$$\frac{1}{1-r} = 1 + r + r^2 + r^3 + \textit{ad infinitum}.$$

Now, it has been already proved, that if $r < 1$, by taking a sufficiently large number of terms, the numerical value of the series can be made to approach to the numerical value of $\frac{1}{1-r}$, to within any assignable limits. For instance, if $r = \frac{1}{2}$ then $\frac{1}{1-r} = 2$; and if we take four terms the series equals 1·875. If we take five terms it equals 1·9375; if six terms, it equals 1·96875; and hence in the extreme case, when we suppose the number of terms to be infinitely large, the series is actually equal to 2. And hence if r be less than 1,

$$\frac{1}{1-r} = 1 + r + r^2 + r^3 + \textit{ad infinitum},$$

where by the sign $=$ we mean that the fraction $\frac{1}{1-r}$ is arithmetically equal to the series. But if r be greater than 1, for instance, if equal to 2, the fraction equals -1; whereas if we take four terms, the series 15; if five terms, 31; if six terms, 63; and so on, where there is no trace of approximation towards arithmetical equality between the series and the fraction. In the former case the series is said to be *convergent*, in the latter *divergent*; and if we include both cases in the expression,

$$\frac{1}{1-r} = 1 + r + r^2 + \ldots \textit{ ad infinitum};$$

it must be understood that the sign $=$ signifies algebraically equivalent, not arithmetically equal. This explanation will be sufficient to enable the student to understand the meaning of the terms convergent and divergent, when applied to special series. The general questions that are suggested by series, and their convergency and divergency, belong to the higher parts of the science—and many of them are still doubtful.

7. *A test for ascertaining the Convergency of a given Series.*

We have already seen that—

$\frac{1}{1-r} = 1 + r + r^2 + r^3 + \ldots.$ is convergent when r is < 1. Hence if we have a series

$$\text{A} + \text{B} + \text{C} + \text{D} + \ldots.$$

and can show that $\text{B} < r\text{A}$, $\text{C} < r^2\text{A}$, $\text{D} < r^3\text{A}$, &c. Then—

$$\text{A} + \text{B} + \text{C} + \text{D} + \ldots < \text{A}\,(1 + r + r^2 + r^3 + \ldots.).$$

This latter is convergent if r be < 1. And if so, the former must plainly be convergent too. This gives us a test for ascertaining whether a given series is convergent, which we shall find useful hereafter. The student must remember that, though all series which submit to this test are convergent, many may be convergent which do not submit to it.

For instance, to ascertain whether the series

$\theta + \frac{\theta^3}{1\cdot2\cdot3} + \frac{\theta^5}{1\cdot2\cdot3\cdot4\cdot5} + \text{\&c.}$, *ad infinitum*, is convergent when $\theta < 2$.

The series may be written—

$$\theta\left\{1 + \frac{\theta^2}{2\,3\cdot} + \frac{\theta^4}{2\cdot3\cdot4\cdot5} + \ldots\right\}$$

$$\frac{\theta^2}{2\cdot3} = \frac{\theta}{2} \times \frac{\theta}{3} < \frac{\theta}{2}\cdot\frac{\theta}{2} \text{ or is } < \left(\frac{\theta}{2}\right)^2$$

Similarly, $\frac{\theta^4}{2\cdot3\cdot4\cdot5} < \left(\frac{\theta}{2}\right)^4$ and so on for the other terms; hence the given series is less than

$$\theta\left\{1 + \left(\frac{\theta}{2}\right)^2 + \left(\frac{\theta}{2}\right)^4 + \ldots \textit{ ad infin.}\right\}$$

and this is convergent if $\left(\frac{\theta}{2}\right)^2 < 1$, or if $\theta < 2$; and hence the given series is convergent if $\theta < 2$.

N.B. $\frac{1}{1+r} = 1 - r + r^2 - r^3 + \ldots.$ is true arithmetically when $r < 1$. Hence, the above test of convergency holds good when the terms are alternately positive and negative.

In the following pages we shall confine our attention to three series—the *Binomial*, the *Exponential*, and the *Logarithmic*.

8. *To state the Binomial Theorem.*†

If $a.\ b.\ n.$ are any numbers whatever, then—

$$(a+b)^n = a^n + n.a^{n-1}b + \frac{n(n-1)}{1\cdot2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1\cdot2\cdot3}a^{n-3}b^3 + \text{\&c.}$$

The reader will observe that the first term is a_n, that in each term the index of a diminishes by unity, while the power of b continually increases by unity, so that the sum of the indices of each term is n.

Again, the coefficient of each term has for each denominator the continued products $1.2.3\ldots.$ up to the index of b *inclusive*; and for numerator, the continued product $n\,(n-1)\,(n-2)\ldots.$ down to the index of a

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* See *ante*, p. 477, *et seq.*

† See *ante*, p. 462.

exclusive. Thus the coefficient of the term which involves $a^{n-r}b^{r}$ is

$$\frac{n.(n-1)\,(n-2)\,\ldots\ldots(n-r+1)}{1\cdot\ \ 2.\ \ \ 3.\ \ \ldots\ldots\ \ r}.$$

9. *To prove the Binomial Theorem when n is a positive Integer.*

(*a*) To show that $(a+b)^n = a^n + na^{n-1}b + \ldots.$

By actual multiplication,

$$\begin{aligned}(a+b)^2 &= a^2 + 2ab + \ldots.\\ (a+b)^3 &= a^3 + 3a^2b + \ldots.\\ (a+b)^4 &= a^4 + 4a^3b + \ldots.\end{aligned}$$

These results plainly suggest the assumption

$$(a+b)^m = a^m + ma^{m-1}b + \ldots.$$

Multiply both sides by $a+2$, and we have

$$(a+b)^{m+1} = a^{m+1} + (m+1)\,a^mb + \ldots.$$

which is clearly of the same form as the assumption, *i. e.*, this has $m+1$, wherever that has m. Hence, if the theorem be true for m, it must also be true for $m+1$. Now it is true for 4, ∴ it is true for 5, ∴ for 6, and so on; therefore it is always true for any positive whole number.

$$\therefore (a+b)^n = a^n + na^{n-1}b + \ldots. \qquad (a)$$

N.B.—If $a = 1$, and $b = x$, we of course have

$$(1+x)^n = 1 + nx + \ldots. \qquad (b)$$

(*β*) To show that—

$$(1+x)^n = 1 + nx + \frac{n.(n-1)}{1.2}x^2 + \frac{n\,(n-1)\,(n-2)}{1.2.3.}x^3 + \ldots$$

For suppose—

$$(1+x)^n = 1 + nx + A_2x^2 + A_3x^3 + A_4x^4 + \ldots \quad (c)$$

It is plain, since $(1+x)^n$ means $(1+x)$ multiplied into itself n times, that this series is finite, so that we may employ the principle of indeterminate coefficients.

In the series A_2, A_3, A_4, ….. do not at all depend on x, and will therefore continue the same for all values of x, so that, for instance—

$$(1+y)^n = 1 + ny + A_2y^2 + A_3y^3 + A_4y^4 + \ldots.$$

In equation (*a*), write $x = y + z$. Then—

$$\begin{aligned}(1+y+z)^n &= 1 + n\,(y+z) + A_2\,(y+z)^2 + A_3\,(y+z)^3 + \ldots.\\ &= 1 + ny + nz + A_2\,(y^2 + 2yz + \ldots) + A_3\,(y^3 + 3y^22 + \ldots.)\\ &= 1 + ny + A_2y^2 + A_3y^3 + A_4y^4 + \ldots.\\ &\quad + z\{n + 2A_2y + 3A_3y^2 + 4A_4y^3 + \ldots.\} + \&c. \ (d)\end{aligned}$$

which is true for all values of y and z.

Again, if in equation (*a*) we take $1+y = a$ and $z = b$, we have—

$$(\overline{1+y}+z)^n = (1+y)^n + n\,(1+y)^{n-1}z + \ldots.\ (E).$$

Now, (*d*) and (*e*) are the same for all values of z, ∴ the coefficient of z in each must be the same, ∴

$$n\,(1+y)^{n-1} = n + 2A_2y + 3A_3y^2 + 4A_4y^3 + \ldots.$$

for all values of y; multiply both sides by $1+y$,

$$\begin{aligned}\therefore n\,(1+y)^n = n &+ 2A_2y + 3A_3y^2 + 4A_4y^3 + \ldots.\\ &+ ny + 2A_2y^2 + 3A_3y^3 + \ldots.\end{aligned}$$

But by equation, (*c*) $n\,(1+y)^n = n + n^2y + A_2ny^2 + A_3ny^3 + A_4ny^4 + \ldots.$

$$\begin{aligned}&\therefore n + n^2y + nA_2y^2 + nA_3y^3 + nA_4y^4 + \ldots.\\ &= n + (2A_2 + n)y + (3A_3 + 2A_2)y^2 + (4A_4 + 3A_3)y^4 + \ldots.\end{aligned}$$

These expressions are true for all values of y.

$$\begin{aligned}\text{Hence } 2A_2 + n &= n^2\\ 3A_3 + 2A_2 &= nA_2\\ 4A_4 + 3A_3 &= nA_3\end{aligned}$$

$$\therefore 2A_2 = n^2 - n = n(n-1(, \ \therefore A_2 = \frac{n(n-1)}{1\cdot2}$$

$$3A_3 = A_2(n-2) \qquad A_3 = \frac{n(n-1)(n-2)}{1\cdot2\cdot3}$$

$$4A_4 = A_3(n-3) \qquad A_4 = \frac{n(n-1)(n-2)(n-3)}{1\cdot2\cdot3\cdot4}$$

Hence—

$$(1+x)_n = 1 + nx + \frac{n(n-1)}{1\cdot2}x^2 + \frac{n(n-1)(n-2)}{1\cdot2\cdot3}x^3 \ \ldots.\ (I).$$

The student will observe the manner in which each successive coefficient is derived from the one that goes before it. He will easily see that if we look in the $r-1^{th}$ and r^{th} terms—viz., $A_{r-1.}\,x_{r-1} + A_rx_r + \ldots.$ we should then have an equation—

$$\begin{aligned}rA_r - (r-1)\,A_{r-1} &= nA_{r-1},\\ \therefore rA_r &= A_{r-1}\,(n-r+1).\end{aligned}$$

so that the general term of the expansion will be—

$$\frac{n.(n-1)\,(n-2)\ldots(n-r+1)}{1\cdot\ \ 2\cdot\ \ \ 3\cdot\ \ \ldots\ \ r}x_r.$$

He will also observe that if n be a whole number when r is greater than n, there will be in the general term a factor $n - \overline{n+1} + 1 = o$; or x^n is the last term of the series, also the coef. of

$$x^n = \frac{n.n-1\ldots2\cdot1}{1\cdot2\ldots.n} = 1. \text{ Hence—}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\cdot2}x^2 + \frac{n(n-1)\,(n-1)}{1\cdot2\cdot3}x^3 + \ldots + x^n$$

(*γ*) To prove the series—

$$(a+b)^n = a^n + n.a^{n-1}b + \frac{n.(n-1)}{1\cdot2}a^{n-2}b^2 + \ldots + b^n$$

For in the last series write $x = \frac{b}{a}$. Then-

$$\left(1+\frac{b}{a}\right)^n = 1 + n\,\frac{b}{a} + \frac{n(n-1)}{1\cdot2}\frac{b^2}{a^2} + \ldots. + \frac{b^n}{a^n}.$$

$$\text{Now, } a\left(1+\frac{b}{a}\right) = a + b,$$

$$\therefore a^n\left(1+\frac{b}{a}\right)^n = (a+b)^n.$$

$$\text{But } a^n\left(1+\frac{b}{a}\right)^n = a^n + n.\frac{b}{a}\times a^n + \frac{n.(n-1)}{1\cdot2}\frac{b^2}{a^2}\times a^n + \ldots + \frac{b^n}{a^n}\times - a^n$$

$$\therefore (a+b)^n = a^n + na^{n-1}b + \frac{n.(n-1)}{1\cdot2}a^{n-2}b^2 + \ldots + b^n\ \ldots\ (II),$$

which is the Binomial Theorem when n is a positive integer.

In page 461, a table of the developments of powers of a binomial is given. These may be immediately deduced from the series we have just proved. Thus, to develop, or expand $(a+x)^8$, we have—

$$(a+x)^n = a^n + n\,a^{n-1}x + \frac{n.n-1}{1\cdot2}a^{n-2}x^2 + \ldots.$$

$$\begin{aligned}\therefore (a+x)^8 = a^8 &+ 8\,a^7x\frac{8\cdot7}{1\cdot2}a^6x^2 + \frac{8\cdot7\cdot6}{1\cdot2\cdot3}a^5x^3 + \frac{8\cdot7\cdot6\cdot5}{1\cdot2\cdot3\cdot4}a^4x^4,\\ &+ \frac{8\cdot7\cdot6\cdot5\cdot4}{1\cdot2\cdot3\cdot4\cdot5}a^3x^5 + \frac{8\cdot7\cdot6\cdot5\cdot4\cdot3}{1\cdot2\cdot3\cdot4\cdot5\cdot6}.a^2x^6 +\\ &+ \frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7}.\,ax^7 + \frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8}.\,x^8\\ = a^8 &+ 8a^7x + \frac{8\cdot7}{1\cdot2}a^6x^2 + \frac{8\cdot7\cdot6}{1\cdot2\cdot3}a^5x^3 + \frac{8\cdot7\cdot6\cdot5}{1\cdot2.3.4}.\,a^4x^4\\ &+ \frac{8\cdot7\cdot6}{1\cdot2.3}a^3x^5 + \frac{8\cdot7}{1\cdot2}a^2x^6 + 8ax^7 + x^8.\\ = a^8 &+ 8a^7x + 28a^6x^2 + 56a^5x^3 + 70a^4x^4 + 56a^3x^5\\ &+ 28a^2x^6 + 8ax^7 + x^8.\end{aligned}$$

The student will observe that the coefficients of a^7x and of ax^7 are the same, as also of a^6x^2 and a^2x^6 of a^5x^3 and a^3x^5.

And, in general, if we write the series, whether we begin from a or from x, we get the same coefficients. Thus—

$$x^9 + 8x^7a + 28x^6a^2 + 56x^5a^3 + 70x^4a^4 + 56x^3a^5 + 28x^2a^6 + 8xa^7 + a^9$$

$$a^9 + 8a^7x + 28a^6x^2 + 56a^5x^3 + 70a^4x^4 + 56a^3x^5 + 28a^2x^6 + 8ax^7 + x^5.$$

It can easily be seen that this must be the case, for the former of these two is $(x + a)^9$, and the latter $(a + x)^9$, which are clearly the same thing. This consideration greatly facilitates our expansion of a binomial. Thus—

$$(a + x)^{10} = a^{10} + 10\,a^9x + \frac{10\cdot9}{1\cdot2}a^8x^2 + \frac{10\cdot9\cdot8}{1\cdot2\cdot3}\,a^7x^3 + \frac{10\cdot9\cdot8\cdot7}{1\cdot2\cdot3\cdot4}\,a^6x^4 + \frac{10\cdot9\cdot8\cdot7\cdot6}{1\cdot2\cdot3\cdot4\cdot5}\,a^5x^5 + \ldots$$

$$= a^{10} + 10a^9x + 45a^8x^2 + 120a^7x^3 + 210a^6x^4 + 252a^5x^5 + 210a^4x^6 + 120a^3x^7 + 45a^2x^8 + 10ax^9 + x^{10}.$$

In like manner—

$$(a + x)^9 = a^9 + 9a^8x + 36a^7x^2 + 84a^6x^3 + 121a^5x^4 + 121a^4x^5 + 84a^3x^6 + 36a^2x^7 + 9ax^8 + x^9.$$

It will also be observed that, in the case of $(a + x)^{10}$ there is a middle term, $252a^5x^5$; but in the case of $(a + x)^9$ there is no middle term. In fact, it is plain that the expansion of $(a + b)^n$ contains $n + 1$ terms, for the expansion contains one term in which b does not appear, and also terms containing b, b^2, b^3 . . . . up to b^n. Hence, if n be even, $n + 1$ is odd, and the expansion has a middle term; if n be odd, $n + 1$ is even, and the expansion is without a middle term.

10. *To prove the Binomial Theorem for any value of n.*

Let us use the notation

$$f(n) = 1 + nx + \frac{n.(n-1)}{1\cdot2}x^2 + \ldots.$$

for all values of n.

$$\therefore f(m) = 1 + mx + \frac{m.(m-1)}{1\cdot2}x^2. + \ldots$$

and

$$f(m+n) = 1 + (m+n)x + \frac{(m+n)(m+n-1)}{1\cdot2}x^2 +,$$

&c.

Now, when m and n are positive integers, we obtain from equation (I)

$$(1+x)^m = 1 + mx + \frac{m.(m-1)}{1\cdot2}.\,x^2 + \ldots.$$

$$(1+x)^n = 1 + nx + \frac{n.(n-1)}{1\cdot2}x^2 + \ldots.$$

$$(1+x)^{m+n} = 1 + (m+n)x + \frac{(m+n)(m+n-1)}{1\cdot2}x^2 + \ldots.$$

$$\therefore (1 + mx + \frac{m.(m-1)}{1\cdot2}x^2 + \ldots) \times (1 + nx + \frac{n(n-1)}{1\cdot2}x^2 + \ldots)$$

$$= (1+x)^m(1+x)^n = (1+x)^{m+n} = 1 + (m+n)x + \frac{(m+n)(m+n-1)}{1\cdot2}x^2 + \text{\&c.}$$

Hence, when m and n are any positive integers whatever,

$$f(m)\,f(n) = f(m+n)$$

In accordance with the principle of the permanence of equivalent forms, we assume this to hold good, whatever m and n are, and then interpret the meaning we must assign to $f(m)$ consistently with this assumption.

It is plain that

$$f(m)\,f(n)\,f(p) = f(m+n)\,f(p) = f(m+n+p),$$

and so on for any number of terms.

Hence,

$$f(\tfrac{p}{q}) \times f(\tfrac{p}{q}) \times f(\tfrac{p}{q}) \text{ to } q \text{ factors} = f(\tfrac{p}{q} + \tfrac{p}{q} + \tfrac{p}{q} + \text{to } q \text{ fractions}) \therefore \left\{f(\tfrac{p}{q})\right\}^q = f(p).$$

Now p is a positive integer,

$$\therefore 1 + px + \frac{p.(p-1)}{1\cdot2}x^2 + \ldots. = (1+x)^p$$

$$\therefore \quad f(p) = (1+x)^p$$

$$\therefore \left\{f(\tfrac{p}{q})\right\}^q = (1+x)^p$$

$$\therefore f(\tfrac{p}{q}) = (1+x)^{\frac{p}{q}}$$

Let $\frac{p}{q} = n$

$$\therefore \quad (1+x)^n = f(n) = 1 + nx + \frac{n.(n-1)}{1\cdot2}x^2 + \ldots.$$

when n is a fraction.

Again, if n be negative we have

$$f(n)\,f(m)\,f(-m) = f(n+m-m) = f(n)$$

$$\therefore f(m)\,f(-m) = 1\cdot$$

$$f(-m) = \frac{1}{f(m)}.$$

Now, m is positive $\therefore f(m) = (1+x)^m$

$$\therefore \quad \frac{1}{f(m)} = (1+n)^{-m}$$

$$\therefore \quad (1+x)^{-m} = f(-m).$$

Let $n = -m$

$$\therefore (1+x)^n = f(n) = 1 + nx + \frac{n.(n-1)}{1\cdot2}x^2 + \ldots.$$

when n is negative.

And therefore, for all values of n positive or negative, integral or fractional,

$$(1+x)^n = 1 + nx + \frac{n.(n-1)}{1\cdot2}x^2 + \frac{n(n-1)(n-2)}{1\cdot2\cdot3}x^3 + \ldots. \text{ (III.)}$$

Let $x = \frac{b}{a}$

$$(1 - \frac{b}{a})^n = 1 + n.\frac{b}{a} + \frac{n.(n-1)}{1\cdot2}\cdot\,\frac{b^2}{a^2} + \ldots.$$

Now, $a^n(1 + \frac{b}{a})^n = (a+b)^n$

$$\therefore (a+b)^n = a^n + na^{n-1}b + \frac{n.(n-1)}{1\cdot2}a^{n-1}b^2 + \ldots. \text{(IV.)}$$

which is the Binomial Theorem, and we have proved it to hold good for all values of n positive or negative, fractional or integral.

N.B.—The Binomial Theorem was discovered by Newton; it is very important, and is constantly used in almost every part of mathematics; it is therefore very necessary that the student should be quite familiar with it. It will be well if he will carefully examine the following results.

(1.) To show that

$$(1-x)^n = 1 - n.x + \frac{n.(n-1)}{1\cdot2}x^2 - \frac{n(n-1)(n-2)}{1\cdot2\cdot3}x^3 + \text{\&c.}$$

We obtain this from equation (IV.) by writing $a = 1$, and $b = -x$.

(2). To show that

$$2^n = 1 + n + \frac{n\cdot(n-1)}{1\cdot2} + \frac{n\cdot(n-1)(n-2)}{1\cdot2\cdot3} + \ldots.$$

write in the series (IV.) $a = 1$ and $b = 1$, and we obtain this result.

(3). To show that

$$n + \frac{n(n-1)(n-2)}{1\cdot2\cdot3} + \text{\&c.} = 1 + \frac{n(n-1)}{1\cdot2} + \text{\&c.}$$

write in the series (IV.) $a = 1$ and $b = -1$. Then

$$0 = 1 - n + \frac{n(n-1)}{1\cdot2} - \frac{n.(n-1)(n-2)}{1\cdot2\cdot3} - \text{\&c.}$$

Whence the result.

(4). To show that

$$\frac{1}{\sqrt{1-x}} = 1 + \tfrac{1}{2}x + \frac{1\cdot3}{2\cdot4}x^2 + \frac{1\cdot3\cdot5}{2\cdot4\cdot6}x^3 + \&c.$$

For $\frac{1}{\sqrt{1-x}} = (1-x)^{-\frac{1}{2}}$.

Hence, by the general formula (IV.), this equals

$$1 + (-\tfrac{1}{2})(-x) + \frac{(-\frac{1}{2})(-\frac{1}{2}-1)}{1\cdot2}(-x)^2$$
$$+ \frac{(-\frac{1}{2})(-\frac{1}{2}-1)(-\frac{1}{2}-2)}{1\cdot2\cdot3\,(-2^3)}$$
$$= 1 + \tfrac{1}{2}x + \frac{\frac{1}{2}\cdot\frac{3}{2}}{1\cdot2}x^2 + \frac{\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}}{1\cdot2\cdot3}x^3 + \&c.$$

Whence the result.

(5). To ascertain whether the series $1 + nx + \frac{n(n-1)}{1\cdot2}x^2 + \ldots$ is convergent.

For the sake of brevity, let us write the series

$$1 + nx + \ldots\ldots + A_r x^r + A_{r+1}x^{r+1} + A_{r+2}x^{r+2} + \ldots\ldots$$

where $A_r = \frac{n.(n-1)\ldots\ldots(n-r+1)}{1\,2\quad\ldots\ldots\quad r}$

$$\therefore A_{r+1} = A_r\frac{n-r}{r+1}$$
$$A_{r+2} = A_{r+1}\frac{n-r-1}{r+2}$$

Now $\frac{n-r}{r+1} = \frac{n+1}{r+1} - 1.$

$$\frac{n-r-1}{r+2} = \frac{n+1}{r+2} - 1.$$

Now, r can be taken so large that $\frac{n+1}{r+2} < 1$; and therefore $\frac{n+1}{r+2} < 1$. So that if $\frac{n-r}{r+1} < 1$. $\frac{n-r-1}{r+2}$ is also < 1.

Also, these fractions are all negative, ∴ if A_r be positive, A_{r+1} is negative, A_{r+2} is positive, and so on. But if we neglect the consideration of the signs from what has been proved, it appears that $A_r, > A_{r+1}, A^{r+1} > A_{r+2}$, &c.

Hence, the part of the series beginning with $A_r x^r$; *i.e.*

$$A_r x^r + A_{r+1} + x^{r+1} + A_{r+2}x^{r+2} + \ldots\ldots$$

has its terms (if x be positive) alternately positive and negative, and is term by term less than

$$A_r x^r - A_r x^{r+1} + A_r x^{r+2} - \&c.\ldots\ldots$$

provided $A_r < 1$.

But this latter series equals—

$$A_r x^r (1 - x + x^2 -)\ \&c.$$

which is convergent of $x < 1$.

Hence, provided $x < 1$, the series, at all events, after a certain number of terms, converges.

(6). Hence, if $x < 1$.

$$(1+x)^n = 1 + nx + \frac{n\cdot(n-1)}{1\cdot2}x^2 + \ldots\ldots$$

is true arithmetically. We may therefore apply this formula to extracting roots of numbers. Thus to extract the 5th root of 1·1. The fifth root of 1·1, is $(1+\frac{1}{10})^{\frac{1}{5}}$.

Now,

$$(1+x)^{\frac{1}{5}} = 1 + \frac{1}{5}x + \frac{1\cdot4}{5\cdot10\cdot}x^2 + \frac{1\cdot4\cdot9}{5\cdot10\cdot15\cdot}x^3$$
$$- \frac{1\cdot4\cdot9\cdot14\cdot}{5\cdot10\cdot15\cdot20\cdot}x^4 + \frac{1\cdot4\cdot9\cdot14\cdot19}{5\cdot10\cdot15\cdot20\cdot25\cdot}x^5 - \&c.$$

$$1 \qquad = 1 \qquad\qquad \frac{4}{5\cdot10\cdot}\ \frac{1}{10^2} = \cdot0008$$
$$\frac{1}{5} + \frac{1}{10} \qquad = \cdot02$$
$$\frac{1\cdot4\cdot9}{5\cdot10\,15} + \frac{1}{10^3} \qquad = \cdot000048 \qquad\qquad \frac{1\cdot4\,9\,14}{5\,10\cdot15\,20\cdot} + \frac{1}{10^4} = \begin{array}{r}\cdot0008\\ \cdot00000336\\ \hline \cdot00080336\end{array}$$

$$\begin{array}{r} 1\cdot020048\\ 000803{,}36\\ \hline 1\cdot019245\end{array}$$

which is the 5th root of 1·1, true to 5 places of decimals.

11. *To show that* $a_x = 1 + Ax + \frac{A^2x^2}{1\cdot2} + \frac{A^3x^3}{1\cdot2\cdot3} + \ldots\ldots$ *ad infinitum.*

where $A = (a-1) - \frac{(a-1)^2}{2} + \frac{(a-1)^3}{3} - \&c.$

This is called the Exponential Theorem or series.

By the Binomial Theorem.

$$a^x = (1 + \overline{a-1})^x$$
$$= 1 + x\cdot(a-1) + \frac{x(x-1)}{1\cdot2}(a-1)^2 + \frac{x(x-1)(x-2)}{1\cdot2\cdot3}(a-1)^3 + \ldots\ldots \quad (a)$$

Now, it is plain that if we multiply the factors of the coefficients of $(a-1)^2$, $(a-1)^3$, $(a-1)^4$ . . . . together, we may re-arrange the series so that it shall become

$$a^x = 1 + Ax + Bx^2 + Cx^3 + Dx^4 + \ldots\ldots \qquad (b)$$

where A, B, C, D, ... contain $(a-1)$ and its powers in some determinate manner. For example, if we examine (a), we shall find that each term, after the first, contains the first power of x; viz., the second term contains $x(a-1)$, the third contains $-x.\frac{(a-1)^2}{2}$, the fourth contains $x.\frac{(a-1)^3}{3}$, the fifth $-x\cdot\frac{(a-1)^4}{4}$, and so on; hence the term involving the first power of x when (a) is re-arranged, must be

$$x\left\{(a-1) - \frac{(a-1)^2}{2} + \frac{(a-1)^3}{3} - \frac{(a-1)^4}{4} + \right\}\ \&c.$$

$$\therefore A = (a-1) - \frac{(a-1)^2}{2} + \frac{(a-1)^3}{3}$$
$$- \frac{(a-1)^4}{4} + \ldots\ldots \qquad (c)$$

In like manner, we might find B. C. D. . . . . in series (b). Instead of doing so, we proceed as follows: since (b) is true for all values of x, we must have—

$$a^y = 1 + Ay + By^2 + Cy^3 + Dy^4\ldots\ldots \qquad (d)$$

Now, $a^x \times a^y = a^{x+y}$.

$$\begin{aligned}\text{But, } a^x \times a^y = 1 + Ax + Bx^2 + Cx^3 + Dx^4 + \ldots\ldots\}\\ + y\,\{A + A^2x + ABx^2 + ACx^3 + \ldots\ldots\}\\ + y^2\,\{B + ABx + B^2x^2 + \ldots\ldots\}\\ + y^3\,\{C + ACx + \ldots\ldots\}\\ + y^4\,\{D + \ldots\ldots\}\end{aligned}$$

$$\begin{array}{rrrrrl} = 1 + Ax & + Bx^2 & + Cx^3 & + Dx^4 & + \ldots\ldots & \\ + Ay & + A^2xy & + ABx^2y & + ACx^3y & + \ldots\ldots & \\ & + By^2 & + ABxy^2 & + B^2x^2y^2 & + \ldots\ldots & (e)\\ & & + Cy^3 & + ACxy^3 & + \ldots\ldots & \\ & & & + Dy^4 & + \ldots\ldots & \end{array}$$

$$\begin{aligned}\text{And } a^{x+y} &= 1 + \text{A}(x+y) + \text{B}(x+y)^2 + \text{C}(x+y)^3 + \ldots\ldots \\ &= 1 + \text{A}x + \text{B}x^2 + \text{C}x^3 + \text{D}x^4 + \quad \ldots\ldots \\ &\quad + \text{A}y + 2\text{B}xy + 3\text{C}x^2y + 4\text{D}x^3y + \ldots\ldots \\ &\quad + \text{B}y^2 + 3\text{C}xy^2 + 6\text{D}x^2y^2 + \ldots\ldots \quad (f) \\ &\quad + \text{C}y^3 + 4\text{D}xy^3 + \quad \ldots\ldots \\ &\quad + \text{D}y^4 + \quad \ldots\ldots\end{aligned}$$

Now, from the early part of this article, it appears that there is *one definite expansion of* a^x, and $\therefore$ of a^{x+y}. Hence (*e*) and (*f*), which are each the expansion of a^{x+y}, are not merely equal, but are actually identical; therefore they must be, term by term, the same.

$$\therefore \text{A}^2 = 2\text{B} \qquad \therefore \text{B} = \frac{\text{A}^2}{1 \cdot 2}$$

$$\text{AB} = 3\text{C} \qquad \text{C} = \frac{\text{A}^3}{1 \cdot 2 \cdot 3}.$$

$$\text{AC} = 4\text{D} \qquad \text{D} = \frac{\text{A}^4}{1 \cdot 2 \cdot 3 \cdot 4}$$

Hence,

$$a^x = 1 + \text{A}x + \frac{\text{A}^2x^2}{1 \cdot 2} + \frac{\text{A}^3x^3}{1 \cdot 2 \cdot 3} + \frac{\text{A}^4x^4}{1 \cdot 2 \cdot 3 \cdot 4} + \ldots \text{(V.)},$$

where

$$\text{A} = a - 1 - \frac{(a-1)^2}{2} + \frac{(a-1)^3}{3} - \frac{(a-1)^4}{4} + \ldots \quad \text{(VI.)}$$

N.B.—Suppose *e* to be such a number that—

$$1 = (e-1) - \frac{(e-1)^2}{2} + \frac{(e-1)^3}{3} - \frac{(e-1)^4}{4} + \ldots\ldots$$

$$\text{Then } e^x = 1 + x + \frac{x^2}{1 \cdot 2} + \frac{x^3}{1 \cdot 2 \cdot 3} + \frac{x^4}{1 \cdot 2 \cdot 3 \cdot 4} + \ldots\ldots$$

This number *e* is very important, and is called the base of the Napierian Logarithms, for reasons to be explained hereafter. We can easily ascertain its value in the following manner:—In the above series let $x = 1$. Then—

$$e = 1 + 1 + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \ldots$$

This series is convergent, since it is clearly—

$$< 1 + \left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\right)$$

The series, therefore, gives the numerical value of *e*. This value may be calculated as follows:—

2	1·000000000
3	·50000000
4	·16666666
5	·041666666
6	·008333333
7	·001388888
8	·000198412
9	·000024801
10	·000002755
11	·000000275
12	·000000025
13	·000000002
	·000000000

These decimals are respectively $\frac{1}{1 \cdot 2}$, $\frac{1}{1 \cdot 2 \cdot 3}$, $\frac{1}{1 \cdot 2 \cdot 3 \cdot 4}$, $\frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}$, &c. Their sum is ·718281827; and hence $e =$ 2·7182818, which is quite accurate, so far as it goes.

The student will observe, that the reasoning in the above article is founded upon the assumptions (1), that a^x can be expanded in a series of ascending powers of x; (2), that it can be expanded in only one series of that kind. These assumptions, in the present case, may be considered as resting on the fact that the expansion of a^x is simply a transformation of the binomial theorem. The same remark applies to the following article:—If we make the assumption general—viz., that every function of x can be expanded in a single series of ascending powers of x, we enter upon a question which has given rise to many discussions, which cannot be further noticed here.

Definition.—If $a^y = x$, then y is called the logarithm of x to the base a, and is generally written $y = \log._a x$ where $\log._a$ means "logarithm to base *a*."

12. *To show that* $\text{A} \log._e (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$ + &c., *where* A *has the same value as in the last article.* This is called the logarithmic series.

$$\text{Let } y = \log._a (1+x).$$

$$\text{Then by definition } 1 + x = a^y.$$

$$\therefore (1+x)^n = a^{ny}.$$

$$\text{By the last article, if } k = x - \frac{x^2}{2} + \frac{x^3}{3} - \text{\&c.}$$

$$(1+x)^n = 1 + k.n + \frac{k^2n^2}{1 \cdot 2} + \text{\&c.}$$

$$\text{and } a^{ny} = 1 + \text{A}yn + \frac{\text{A}^2y^2n^2}{1 \cdot 2} + \text{\&c.}$$

Now, since $(1+x)^n$ is identically the same as a^{ny}, these two series must be identically the same.

$$\therefore \text{A}y = k.$$

$$\text{i.e., A} \log._a (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots \text{(VII).}$$

$$\text{where A} = (a-1) - \frac{(a-1)^2}{2} + \frac{(a-1)^3}{3} - \text{\&c.}$$

$$\text{N.B.—Now } 1 = (e-1) - \frac{(e-1)}{2} + \frac{(e-1)^3}{3} - \text{\&c.}$$

$$\therefore \log._e (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \text{\&c.}$$

$$\therefore \log._e a = (a-1) - \frac{(a-1)^2}{2} + \frac{(a-1)^3}{3} - \text{\&c.}$$

$$\therefore \text{A} = \log._e a.$$

Hence, equation (VII.) may be written—

$$\log._e a \log._a (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \text{\&c.}$$

It will be observed that if x be < 1, the series

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \text{\&c.}$$

is term by term less than $x - x^2 + x^3 - \text{\&c.}$ and is therefore convergent, provided x is < 1.

ON THE CALCULATION OF LOGARITHMS.

13. *On the Calculation of the Arithmetical Values of Quantities expressed by Infinite Series.*

In the Treatise on Elementary Algebra, the method has been explained of obtaining in numbers the value of an algebraical expression, when definite values are assigned to the letters composing the expression.* For instance, if $a = 2$ and $b = 5$, then $(2a + b)(b - a) = 27$. The student may ask, how can an infinite series be reduced? Although we have already given three instances of the manner of doing this, the question is well worth a distinct consideration. We have already seen that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots\ldots$$

Now, if $x = \frac{1}{2}$, the fraction is equal to 2; and we know that if we took the whole number of terms of the series we should get exactly 2. The first two terms are 1·5; the first three 1·75; the first four 1·875; the first five terms 1·9375; the first six 1·96875; each result being nearer to the truth than the one before. Thus, by taking a sufficiently large number of terms, we can get as near to the exact value as we like. The series, in fact, affords the means of *approximating* to the true value. Of course, in such a case as the above, we should not care for the approximation, since we can so readily get the real value. But in the large majority of cases we cannot get at the real value, or even the real value cannot be expressed by digits at all—*i.e.*, is not commensurable with unity; in such cases the approximate value is the only one we can get, *and the series is the means by which we get it.* For instance, we have called the base of the Napierian logarithm e. But what is e? It is such a number that

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* See *ante*, p. 451.

$$1 = (e - 1) - \tfrac{1}{2}(e - 1)^2 + \tfrac{1}{3}(e -)^3 - \&c.$$

This is an equation we have no method—no direct method—of solving. We have seen, however, that e is expressed by the series

$$e = 1 + 1 + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \&c.$$

From this, as we have already seen, we can find that $e = 2{\cdot}7182818$, *&c.* The student may think an approximation a very unsatisfactory result; but he must remember two things:—(1). That greater part of the quantities we have to deal with *cannot* be expressed in whole numbers, or in vulgar fractions—*e.g.*, so common and elementary an expression as $\sqrt{2}$ *cannot* be expressed as a vulgar fraction; and (2), that in practice no measurement is *accurate*, but is known to lie within certain limits. For instance, if a tailor measure a piece of cloth, he calls it a yard, though it may happen to be a quarter of an inch more or less. In like manner the most refined scientific measurements (the length of the second's pendulum, of an arc of the meridian, &c.), are generally the *means* of several results, and are accurate to within certain very small limits. Now, in approximating to a result by means of a series, we can always get to within any given limits that may be assigned. And thus approximations by means of series are as accurate as any, the most refined, measurements can be.

In practice, if we know a number to be true for the first six or seven places of decimals, it is generally known with sufficient or even more than sufficient accuracy. Thus, if we are certain that x. lies between 3·16754 and 3·16755, we may call it 3·16754; although if we calculated to a greater nicety we might obtain $x = 3{\cdot}1675438295$, for the error we commit is $< \frac{1}{10000000}$, *e.g.*, an error less than $\frac{1}{4}$ of an inch in one mile.

It is to be observed, that in calculating the value of a series, we must calculate each term to one or two more places of decimals than the result we wish to obtain, so as to be quite sure that we *carry* the right number to the seventh place. Thus, in finding the value of e (Art. 11), we calculated each term to nine places of decimals, to ensure that our result should be true to seven places.

It is also to be observed, that in cutting off the eight and subsequent decimal places, if the eighth place be 5, 6, 7, 8, or 9, we add 1 to the seventh place; but if 4, 3, 2, 1, or 0, we simply omit it. Thus we reckon

$2{\cdot}59716345827 = 2{\cdot}5971635$

But $2{\cdot}59716343254 = 2{\cdot}5971634$.

For it is plain that 2·59716345827 is nearer to 2·5971635 than to 2·5971634: whereas, as in the second instance, the contrary is the case.

We now proceed to consider the subject of logarithms in detail.

14. *To Explain the Principle on which Logarithms may be used to Facilitate Calculations.*

From the definition of a logarithm already given, it follows that if $M = a^x$ then x is the logarithm of M to the base a; and if $N = a^y$ then y is the logarithm of N to the base a, Now observe $M \times N = a^{x+y}$, whence it is plain that the *multiplication* of one number by another corresponds to the *addition* of their logarithms. In like manner, $M \div N = a^{x-y}$, or the division of one number by another corresponds to the subtraction of the logarithm of the dividend from that of the divisor. Again, $M^m = a^{mx}$, or the raising of a number to a given power corresponds to the multiplication of the logarithm by that power. In like manner $M^{\frac{1}{m}} = a^{\frac{x}{m}}$, or the extraction of the root of a given number corresponds to the division of the logarithm by that root. So that if we knew the logarithm which corresponds to any number whatever, and wished to find the product of two numbers, we should merely have to write down the logarithms of the numbers, add them, and then the number whose logarithm is that sum will be the product of the two given numbers; and similarly for the other rules.

Now, tables have been calculated which give us the logarithm corresponding to any number between 1 and 10,000,000. Hence, by using these tables properly, multiplication is performed by means of addition; and in like manner division by means of subtraction, involution by multiplication, and evolution by division.

In the following pages we shall first explain the method by which these tables are calculated, and then proceed to show how they are practically employed.*

15. *The following results follow manifestly from what has been said.*

(1). That if $P = Q$, then $\log._e P = \log._e Q$.

(2). Since $a^1 = a$. $\therefore \log._a a = 1$.

(3). Since $a^0 = 1$. $\therefore \log._a 1 = 0$.

(4). If $M = a^x$, $N = a^y$, then $x = \log._a M$, $y = \log._a N$.
Now, $MN = a^x \times a^y = a^{x+y}$ $\therefore x + y = \log._a MN$

$$\therefore \log._a M + \log._a N = \log._a MN.$$

(5). Similarly, $a^x \div a^y = a^{x-y}$, $\therefore \frac{M}{N} = a^{x-y}$.

$$\therefore \quad x - y = \log._a \frac{M}{N}.$$

$$\therefore \quad \log._a M - \log._a N = \log._a \frac{M}{N}.$$

(6). Again, if $a^x = M$. $\therefore \log._a M = x$.
Now, $M^m = a^{mx}$ $\therefore \log._a M^m = mx$.
$\therefore m \log._a M = \log._a M^m$.

(7). Similarly, $M^{\frac{1}{m}} = a^{\frac{x}{m}}$

$$\therefore \quad \frac{1}{m} \log._a M = \log._a M^{\frac{1}{m}}$$

(8). From (4) it is manifest that
$\log._a M + \log._a N + \log._a P + \ldots . = \log._a MNP \ldots .$

16. *To show that* $\log._a b, + \log._b x = \log._a x$.

Now, suppose $x = a^m$, and $x = b^n$
$\therefore \log._a x = m$, and $\log._b x = n$.

But since $a^m = b^n$ we must have $a = b^{\frac{n}{m}}$

$$\therefore \quad \log._a a = \log._a b^{\frac{n}{m}}$$

$$\therefore \quad 1 = \frac{n}{m} \log._a b.$$

$$\therefore \quad n . \log._a b = m$$

$$\text{or } \log._a b \log._b x = \log._a x.$$

From this it follows, that if we know the logarithm of a given number to a given base, we can find its logarithm to another base, by dividing the first logarithm by the logarithm of the new base; for instance, suppose our tables give the logarithms of numbers to the base 10, and suppose we wished to find the logarithm of a given number (N) to the base 19, we have

$$\log._{10} 19 \log._{19} N = \log._{10} N.$$

Then $\log._{10} 19$ and $\log._{10} N$ are given by the tables, and therefore we know $\log._{19} N$ by division.

17. *The practical advantage of Calculating Logarithms to the Base* 10.

The tables of Logarithms commonly printed, are logarithms to the base of 10. In all future articles, whenever we write $\log. x$, we mean logarithm to the base 10.

We might calculate tables to the base e; and the calculation is obviously rendered much easier when this base is employed by the circumstance that

$$\log._e x = x - \frac{x^2}{2} + \frac{x^3}{3} - \&c.$$

whereas, if we use the base 10. we have

$$\log._e 10. \log. x = x - \frac{x^2}{2} + \frac{x^3}{3} - \&c.$$

---

* An extended table of logarithms will be found at the end of this chapter.

And, in point of fact, the inventor of logarithms, Napier, actually calculated logarithms to this base e, which is hence called the base of the Napierian logarithms.

For the purposes of numerical calculation, however, the base 10 possesses the following decisive advantage over any other.

Suppose $10^x = N$

Then $10^{x+n} = N \times 10^n$.

Now, suppose n to be a whole number, then $N. \times 10^n$ has the same digits as N in the same order, and only differs from it in having its decimal point shifted n places to the right; and again—

$$10^{x-n} = \frac{N}{10^n}.$$

And when n is a whole number, $\frac{N}{10^n}$ only differs from N in having its decimal point shifted n places to the left. It follows, therefore, that *the decimal part* of the logarithm of a number is the same wherever the decimal point may be in the number, and that for every place that the decimal point in the number is shifted to the right, 1. is added to the logarithm; and for every place, it is shifted to the left, 1. is subtracted from the logarithm. Thus the table gives us

log. 7·5684 = ·8790041.
∴ log. 75·684 = 1·8790041.
log. 756·84 = 2·8790041.

So again—

log. ·75684 = —1 + ·8790041.

or, as it is more generally written—

log. ·75684 = $\bar{1}{\cdot}8790041$.

Similarly—

log. ·00075684 = $\bar{4}{\cdot}8790041$.

It is plain, then, that one calculation gives us the logarithm of the above five numbers, and in fact of as many numbers as can be made by shifting the decimal point to different positions in the combination 75684; but if we adopted any other base, we should require a separate calculation for each of them. This advantage, which the base 10 has over any other, was first seen and applied by Briggs, who was professor at Oxford about the year 1670; the logarithms are, therefore, sometimes called the "Briggian Logarithms."

The student will perceive that the base 10 has this advantage, in consequence of our system of notation being decimal. If our system were duodecimal, our logarithms would then have to be calculated to the base 12, to be possessed of a like advantage, and so on for any other system.

N.B.—The decimal part of a logarithm is called the *mantissa:* the whole number is called the *characteristic.*

18. *To show that every number has a Calculable Logarithm.*

It will be observed that—

$$\log._e a = (a-1) - \frac{(a-1)^2}{2} + \frac{(a-1)^3}{3} -, \&c.,$$

may not be (for aught we have yet shown) convergent, unless a < 2. Now, if a be any number, a root of it can always be found, say the n^{th}, which shall be less than 2. Now,

$$\log.^e a^{\frac{1}{n}} = (a^{\frac{1}{n}} - 1) - \frac{(a^{\frac{1}{n}} - 1)}{2} + \frac{(a^{\frac{1}{n}} - 1)^3}{3}, - \&c.$$

which, since $a^{\frac{1}{n}} - 1$ is less than 1, is convergent. Now, $\log.^e a^{\frac{1}{n}} = \frac{1}{n}\log._e a, \therefore \log._e a = n\{(a^{\frac{1}{n}} - 1) - \frac{1}{2}(a^{\frac{1}{n}} - 1)^2,$ $+ \frac{1}{3}(a^{\frac{1}{n}} - 1)^3 - \&c.,\}$ which series being convergent, if n be properly chosen, for every value of a, the value of $\log._e a$ might be found from it, for all values of a. To calculate a table from such a formula would be most laborious; but as the calculation of the common tables presupposes that we know $\log._e 10$, if we treat the subject in its logical order, it would be necessary to calculate $\log.^e 10$ from this series, before going further. We shall find, if we do so, that—

$$\log._e 10 = 2{\cdot}3025851.$$

Hence—

$$\frac{1}{\log._e 10} = {\cdot}434294481.$$

In future pages we shall denote this number by μ. It is called the modulus of the tabular logarithm.

The logarithmic series to the base 10 we have already seen to be—

$$\log._e 10 \log.(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} -, \&c.$$

$$\therefore \log.(1+x) = \mu\left\{x - \frac{x^2}{2} + \frac{x^3}{3} - \&c.\right\}.$$

19. *To derive from the Logarithmic series, others from which the numerical values of Logarithms may be calculated.*

We have seen that

$$\log.(1+x) = \mu\left\{x - \frac{x^2}{2} + \frac{x^3}{3} - \&c.\right\}$$

$$\therefore \log.(1-x) = \mu\left\{-x - \frac{x^2}{2} - \frac{x^3}{3} - \&c.\right\}$$

$$\text{Now } \log.(1+x) - \log.(1-x) = \log.\frac{1+x}{1-x}.$$

$$\therefore \log.\frac{1+x}{1-x} = 2\mu\left\{x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \ldots\right\}$$

$$\text{Now suppose } \frac{1+x}{1-x} = \frac{q}{p} \therefore x = \frac{q-p}{q+p}.$$

$$\therefore \log.\frac{q}{p} = 2\mu\left\{\frac{q-p}{q+p} + \frac{1}{3}\left(\frac{q-p}{q+p}\right)^3 + \ldots\right\}$$

Suppose $q = p + 1$.

$$\therefore \log.\frac{p+1}{p} = 2\mu\left\{\frac{1}{2p+1} + \frac{1}{3}\cdot\frac{1}{(2p+1)^3} + \ldots\right\}$$

$$\therefore \log. p+1) = \log. p. + 2\mu\left\{\frac{1}{2p+1} + \frac{1}{3}\cdot\frac{1}{(2p+1)^3} + \ldots\right\} \text{ VIII.}$$

From this we can calculate successively the values of log. 2, log. 3, log. 4 .... as far as we please.

Thus, remembering that log. 1 = 0, we have

$$\log. 2 = 2\mu\left\{\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{3^3} + \frac{1}{5}\cdot\frac{1}{3^5} + \ldots\right\}$$

$$\log. 3 = \log. 2 + 2\mu\left\{\frac{1}{5} + \frac{1}{3}\cdot\frac{1}{5^3} + \frac{1}{5}\cdot\frac{1}{5^5} + \ldots\right\}$$

$$\log. 4 = \log. 3 + 2\mu\left\{\frac{1}{7} + \frac{1}{3}\cdot\frac{1}{7^3} + \frac{1}{5}\cdot\frac{1}{7^5} \ldots\right\}$$

In the ordinary tables the logarithms are given calculated to 7 places of decimals. Hence, in making the calculation from the above series we must take in every term >. 00000001; or, since each term is greater than the one that comes after it, we must reduce each term to decimals until we find one < ·00000001, which we can omit, together with all that come after it; all that go before it being reduced to decimals and added together, give the required logarithm. Thus, to find log. 11:—here $p = 10 \therefore 2p + 1 = 21$.

$$\therefore \log. 11 = \log. 10 + 2\mu\left\{\frac{1}{21} + \frac{1}{3}\cdot\frac{1}{21^3} + \frac{1}{5}\cdot\frac{1}{21^5} + \ldots\right\}$$

Now,

$$\frac{1}{21} = ·047619047$$

$$\frac{1}{3}\cdot\frac{1}{21^3} = ·000035993.$$

$$\frac{1}{5}\cdot\frac{1}{21^5} = ·000000049. \qquad \frac{1}{7}\cdot\frac{1}{21^7} < ·0000000001$$

$$\overline{·047655089}$$

$$\text{and } 2\mu = ·868588962$$

$$\therefore 2\mu \times \left\{\frac{1}{21} + \frac{1}{3}\cdot\frac{1}{21^3} + \frac{1}{5}\cdot\frac{1}{21^3} + \cdots\right\} = \begin{matrix}·047655089 \times ·868588962 = \\ ·041392674\end{matrix}$$

But log. 10 = 1.

∴ log. 11 = 1·0413927. To seven places of decimals, which is the logarithm you will find registered in the table. From the formula

$$\log.\frac{q}{p} = 2\mu\left\{\frac{q-p}{q+p} + \frac{1}{3}\cdot\left(\frac{q-p^3}{q+p}\right) + \ldots\right\}.$$

Another very convenient formula can be derived.

$$\text{Thus let } \frac{q}{p} = \frac{x^2}{x^2-1}\cdot \quad \text{Then } \frac{q-p}{q+p} = \frac{1}{2x^2-1}\cdot$$

$$\therefore \log.\frac{x^2}{x^2-1} = 2\mu\left\{\frac{1}{2x^2-1} + \frac{1}{3}\cdot\frac{1}{(2x^2-1)^3} + \ldots\right\}.$$

$$\text{Now } \log.\frac{x^2}{x^2-1} = \log.\frac{x^2}{(x-1)(x+1)} =$$

$$2\log.x - \log.(x-1) - \log.(x+1).$$

$$\therefore \log.(x+1) = 2\log.x - \log.(x-1) - 2\mu\left\{\frac{1}{2x^2-1} + \&c.\right.$$

20. *To explain the use of this last Series.*

The use of this formula, which converges very rapidly, gives us any logarithm in terms of the two that precede it. It will be observed in this formula that $\frac{1}{3}\cdot\frac{1}{(2x^2-1)^3}$ and all the terms following it can be omitted, provided

$$\frac{1}{3}\cdot\frac{1}{(2x^2-1)^3} < .00000001$$

$$\text{and } \therefore \text{ if } \frac{1}{(2x^2-1)^3} < ·000000027.$$

$$\text{and } \therefore \text{ if } \frac{1}{2x^2-1} < ·003.$$

$$\text{Or if } 2x^2 - 1 > 334.$$

$$\therefore \text{ if } x^2 > 169.$$

$$\therefore \text{ if } x > 13.$$

Hence, if we employ this formula to calculate logarithms, we have, for all numbers greater than 13—

$$\log.(x+1) = 2\log.x - \log.(x-1) - \frac{2\mu}{2x^2-1}$$

Again, suppose $x > 10000$. Then

$$\frac{2\mu}{2x^2-1} = \frac{\mu}{x^2}\cdot \times \frac{1}{1-}\frac{1}{2x^2} = \frac{\mu}{x^2}\cdot\left\{1 + \frac{1}{2x^2} + \ldots\right\}.$$

and

$$\frac{\mu}{2x^4} < .00000001.$$

Hence, the formula finally reduces itself to

$$\log.(x+1) = 2\log.x - \log.(x-1) - \frac{\mu}{x^2} \ldots \text{(IX.)}$$

$$\text{Or } \log.(x+1) - \log.x = \log.x - \log.(x-1) - \frac{\mu}{x^2}.$$

$$\text{Now } \frac{\mu}{x^2} - \frac{\mu}{(x+n)^2} = \frac{\mu}{x^2}\cdot\left\{\frac{2n}{x} - \frac{3n^2}{x^2}\ \&c.\right\}.$$

$$\therefore \text{ if } x > 10000, \text{ in which case } \frac{\mu}{x^2} > .0000001.$$

Hence, if we only calculate to 7 places of decimals, n must be at least many hundreds for $\frac{\mu}{x^2}$ to differ sensibly from $\frac{\mu}{(x+n)^2}\cdot$ Now, the formula would give us

$$\log.(x+n+1) - \log.(x+n) = \log.(x+n) -$$

$$\log.(x+n-1) - \frac{\mu}{(x+n)^2}$$

which, unless n is several hundreds, we have seen is practically the same as

$$\log.(x+n) - \log.(x+n-1) = \log.(x+n-1)$$

$$-\log.(x+n-2) - \frac{\mu}{x}$$

$$\text{So that } \log.(x+1) - \log.x = \log.x - \log.(x-1) - \frac{\mu}{x^2}.$$

$$\log.(x+2) - \log.x = \log.(x+1) - \log.x - \frac{\mu}{x^2}.$$

$$\log.(x+3) - \log.(x+2) = \log.(x+2)$$

$$-\log.(x+1) - \frac{\mu}{x^2}.$$

$$\log.(x+n) - \log.(x+n-1) = \log.(x+n-1)$$

$$-\log.(x+n-2) - \frac{\mu}{x^2}$$

∴ adding together

$$\log.(x+n) - \log.x = \log.(x+n-1) - \log.(x-1)$$

$$-n\frac{\mu}{x^2}$$

$$\therefore \log.(x+n) - \log.(x+n-1) = \log.x$$

$$-\log.(x-1) - n.\frac{\mu}{x^2}$$

Now, if n be sufficiently small for $n\frac{\mu}{x^2}$ to be < ·0000001, it is clear that we may omit $n.\frac{\mu}{x^2}$ so long as this is the case, and hence the differences between the successive logarithms will continue the same within that limit.

For instance, we can show by formula (VIII.) that

$$\log.(10000) = 4·00000000.$$

$$\log.(10001) = 4·00004342945.$$

$$\therefore \log.(10001) - \log.10000 = ·00004343.$$

$$\text{and} \qquad \frac{\mu}{(10000)^2} = ·0000000042934.$$

$$\text{Hence } n\frac{\mu}{(100000)^2} < ·0000001.$$

until $n = 20$. So that the logarithms of 10001, 10002, 10003 . . . . 10020. can be found the one from the one before it by merely adding .0000434294.

We shall then have to calculate log. (10020) and log. (10021) from the original formula (VIII.) and find how far we can use the difference between these for deducing log. (10022) by log. (10023), &c. After some 20 or 30 logarithms are thus found by simple addition, a fresh calculation will become necessary: by proceeding in this manner, without any exorbitant labour, a table which gives the logarithm for every number from 10000 up to

99999 can be constructed; which is practically the same as from 1 to 100000.

It is to be observed, that with such a table, by means of a very simple subsidiary calculation, we can obtain the logarithm of any number from 1 to 10000000.

Thus, the tables give us log. 73894, log. 73895, *i.e.*, log. 7389400, log. 7389500, for these only differ from those in the characteristic.

The subsidiary calculation referred to enables us to fill up any one of the logarithms of 7389401, &c., up to 7389499.

21. *To explain the construction and use of the Table of Proportional Parts.*

Suppose N to be a number such as that above referred to, > 1000000; and suppose its log. to be given in the table; then log. $(N + 100)$ is also given in the tables. From these data we want to find log. $(N + \delta)$ where δ lies between 0 and 100.

Now $\log. (N + \delta) = \log. N \overline{\left(1 + \frac{\delta}{N}\right)}$

$$= \log. N + \log. \left(1 + \frac{\delta}{N}\right) = \log. N + \mu \left\{ \frac{\delta}{N} - \frac{1}{2}\left(\frac{\delta}{N}\right)^2 \right.$$

$$\left. \ldots \text{\&c.} \right\}$$

Now $\delta < 100 \quad > 1000000$

$$\therefore \left(\frac{\delta}{N}\right)^2 < \frac{1}{(10000)^2} < \cdot00000001$$

which can be omitted, since we only take in the first seven places of logarithms.

$$\therefore \log. (N + \delta) = \log. N + \frac{\mu}{N} \cdot \delta$$

$$\therefore \log. (N + 100) = \log. N + \frac{\mu}{N} . 100.$$

Now, log. $(N + 100) - \log. N = \Delta$ (suppose) is given by the tables, and we see that—

$$\Delta = \frac{\mu}{N} 100$$

$$\therefore \log. (N + \delta) = \log. N + \Delta \times \frac{\delta}{100}.$$

which is true for every value of δ from 1 to 99.

If a and b are the digits of δ, so that $\delta = 10.a + b$. This formula can be written—

$$\log. (N + \delta; = \log. N + \frac{\Delta a}{10} + \frac{1}{10} \cdot \frac{\Delta b}{10}.$$

We have already seen that the difference between two consecutive logarithms is the same for several logarithms together; accordingly, a small subsidiary table, giving $\frac{\Delta}{10}$. $\frac{\Delta \times 2}{10}$, $\frac{\Delta \times 3}{10}$ .... $\frac{\Delta \times 9}{10}$, is calculated for each different value of Δ, and is printed, as Δ occurs, in the margin of the table. For instance, Δ corresponding to log. 28568, is ·0000152; or, as it is written, 152; it being understood that the last figure, 2, falls under the seventh decimal of the logarithm, In this case the subsidiary table is the accompanying. It is called a Table of Proportional Parts. Since a and b are digits, this table gives at once $\frac{\Delta a}{10}$ and $\frac{\Delta b}{10}$, and therefore $\frac{1}{10} \cdot \frac{\Delta b}{10}$. Hence, by means of this table, we can determine log. $(N + \delta)$ from log. N by addition only.

	152.
1	15
2	30
3	46
4	61
5	76
6	91
7	106
8	122
9	137

Thus, log. 2856800 = 6·4558798.

Now, $\frac{\Delta \times 3}{10} = \cdot0000046$, and $\frac{1}{10} \cdot \frac{\Delta \times 7}{10} = \cdot00000106$ $= \cdot0000011$, as we only take in seven places of decimals. Hence, log. 2856837 = 6·4558798 + ·0000046 + ·0000011 = 6·4558855.

We have now given a full explanation of the principles on which Logarithmic Tables are calculated, so far as that explanation is possible in a purely elementary treatise, and have exemplified those principles in the case of the ordinary tables which (practically) give the logarithms of numbers from 1 up to 10,000,000, to seven places of decimals. The student may ask, what would be done if a case occur in which we have numbers exceeding 10,000,000? The answer to the question is the following:—If the number were, for instance, 97536982, and if the calculation demanded so much accuracy that we could not consider this as equal to 97536980, then a more refined set of tables would be necessary; in point of fact, however, for all ordinary calculations, the degree of accuracy which the common tables allow of is sufficient.

We now proceed to explain the practical method of using the tables:—

THE USE OF A TABLE OF LOGARITHMS.

For the purpose of explanation, the following is printed from p. 78 of Hülsse's edition of Vega's Logarithms.

N. 46000 * L. 662

N	O	1	2	3	4	5	6	7	8	9	P	P
4600	662 7578	7673	7767	7862	7956	8050	8145	8239	8334	8428		
4601	8522	8617	8711	8805	8900	8994	9089	9183	9277	9372		95
4602	9466	9561	9655	9749	9844	9938	*0032	*0127	*0221	*0315	1	10
4603	663 0410	0504	0598	0693	0787	0881	0976	1070	1164	1259	2	19
4604	1353	1447	1542	1636	1730	1825	1919	2013	2108	2282	3	29
											4	38
											5	48
											6	57
											7	67
											8	76
											9	86

(1). *To find the Logarithm, when the number is given in the Tables.*

(*a*). To find the mantissa.

Suppose we want to find the logarithm of 46017. The number N. 46000 at the top of the page will direct us to the page on which we shall find 46017; the number 4601, in the column marked N, will give us the line in which we shall find what we want. Pass your eye along the line, 4601, until it comes to column 7, there we find 9183. You will observe that there is 662 written in column O; this is to be written before every one of the numbers under the other columns, and is only written once to render the tables more compact. Write this in front of 9183, which we before found, and we obtain 6629183. This is the mantissa of the logarithm of 46017; it is, therefore, a decimal, and must be written ·6629183.

So again, to find mantissa of logarithm of 46035, look down col. N for 4603, look along the line 4603 till you come to col. 5, when you find 0881; before this prefix 663, and ·6630881 is the mantissa of logarithm of 46035.

Again, to find the mantissa of a logarithm of 46028, look for 4602 in col. N; along this line, in col. 8, you find *0221; the asterisk in front of this 0221 shows that we must add 1 to the 662, and thus we obtain for the mantissa of this logarithm ·6630221.

(3). To find the characteristic.

We have already explained the principle of doing this: if we apply that principle, we shall obtain*

log. 4.6017 = ·6629182
log. 4601·7 = 3·6629182
log. 4601700. = 6·6629182
log. ·00046017 = $\bar{4}$·6629182
log. ·46017 = $\bar{1}$·6629182

If you examine these cases, you will find that they suggest the following rule:—"*Place your pen between the first and second* FIGURE (NOT *cipher*), *and count one for each figure or cipher, until you come to the decimal point; the number this gives will be the characteristic: if you count to the right, the characteristic is positive; if to the left, the characteristic is negative.* Thus, in finding log. 4·6017, if you place your pen between the first figure (4), and second (6), it falls on the decimal point; in this case, therefore, there is no characteristic. Next, in the case of log. 4601·7. place your pen between (4) and (6), and count $\begin{smallmatrix}4,601\cdot7,\\ |123\end{smallmatrix}$ the characteristic is 3; and as you count to the right, it is *plus* 3. Next, in the case log. 4601700. here the decimal point falls behind the last cipher. Hence, counting as before, we have $\begin{smallmatrix}4\,601700\\ |\,123456,\end{smallmatrix}$ and the characteristic is *plus* 6, Again, in the case, log. ·00046017, the first *figure* is, as before, 4. Hence counting, we have $\begin{smallmatrix}\cdot0004,\ 6017;\\ 4321\end{smallmatrix}$ but here we count to the left, so that the characteristic is negative or $\bar{4}$. Again, in the case, log. ·4601. we have $\begin{smallmatrix}\cdot4\ 6017,\\ 1\,|\end{smallmatrix}$ and the characteristic is $\bar{1}$.

Instead of writing log ·00046017 = $\bar{4}$·6629182, this is frequently written 6·6629182. To explain this, observe that $\bar{4}$·6629182 means — 4 + ·6629182, which clearly equals 6 + ·6629182 — 10, or 6·6629182 — 10. It is usual to omit the — 10. and write 6·6629182: no experienced calculator would forget the — 10, although it is not written down; but as this tract is intended for beginners, we shall never omit the — 10, but as it may suit our purpose write log. ·00046017 = $\bar{4}$·6629182 or 6·6629182 — 10.

To find log. 46. Since 46 = 46000 and the table gives mantissa log. 46000 = ·6627576.

∴ log. 46 = 1·6627576.

Hence, to find the logarithm of any number given in the table, first find the mantissa, and then prefix to it the characteristic, in the manner above explained.

Find the logarithms of the following numbers from the tables at the end of this chapter, viz.:—

72·643	7264·3	8·5972	6315.9
84658	6397200	64532	6·5973.
·057234	·000058	762	·035872
·35872	63500	635	6·35
20000	200	·02	·029

(2). *To find the Logarithm of a Number not given in the Tables.*

The rule for the characteristic is the same as given above. For finding the mantissa we proceed as follows:

The student will observe that each logarithm on p. 521, differs from the one before it by 94 or 95. Call this 95, and construct a table of proportional parts as before explained; this is printed in the column marked PP. We then proceed as follows:—To find log. ·0460267.

N.	L
46026	6630032
P.P 7	67
460267	·6630099

N L

∴ log. ·0460267 = $\bar{2}$·6630099.

In practice this is arranged as follows:—

46026	·6530032
7	67
log. ·0460267	$\bar{2}$·6630099

Again, to find log. 460 3629.

46036		·6630976
2		19
9		8,6
log. 460·3629	=	2·6631004

To find log. 4604·508.

46045	·6631825
08	7.6
log. 460·4508	2·6631833

In like manner the student may find the logarithms of
75·84653 ·0927543. 13·02528.

(3). *To find the Number corresponding to a given Logarithm.*

It very rarely happens that the logarithm is exactly to be found in the tables. If it be, the only difficulty we have to contend with in such a case is that of fixing the decimal point. For instance, find the number corresponding to the logarithm 3·6629089. At the top of the page we have L662; this will direct us to the page on which the logarithm will be found; then, looking in the other part of the table we find 9089, in the column 6 of the line marked 4601 of column N. ∴ the number corresponding to the mantissa ·6629089 is 46016. To fix upon the position of the decimal point, we must modify the rule previously given: place the pen between the first and second figure, and count off as many figures as there are units in the characteristic—to the right if the characteristic be positive, to the left if negative; and if there are not figures enough, add or prefix as many ciphers as necessary; thus, in the present case, $\begin{smallmatrix}4|601\cdot6\\ 123\end{smallmatrix}$

∴ the number corresponding to logarithm 3·6629089 is 4601.6; similarly, that corresponding to $\bar{3}$·6629089 is ·0046016.

(4). *To find the Number corresponding to a given Logarithm, which does not exactly occur in the Tables. We proceed as follows:—*

Find the number corresponding to the logarithm 2·6629319.

The Logarithms 6629277 and 6629372 are in the table; the number, therefore, will be between 46018 and 46019. It will therefore be the former, with something added on. To find this "something" we proceed as follows:—

	6629319
Logarithm next less 46018	6629277
	42
Prop. part corresponding to	4 38
	03 .... 3·8

∴ logarithm of 4601843 is 6629319.
∴ logarithm of 460·1843 is 2·6629319.

(5). *To find the Arithmetical complement of the Logarithm of a Number.*

N.B.—If x be any number whatever, then the ar. comp. of $x = 10 - x$.

Now 10 — 3·7568274 = 6·2431726.
10 — $\bar{2}$·3907526 = 11·6092474.
10 — ·9328243 = 9·0671260.

If you examine these, you will find that the subtraction is performed by subtracting the *last figure* (to the right hand) from 10, and each of the rest from 9; in fact, to take the first case, we should proceed as follows: 4 from 10 leaves 6, and carry 1. Then 1 + 7 = 8; take 8 from 10, leaves 2, and carry 1; but taking 8 from 10 is of course the same thing as taking 7 from 9, and so on. The student may, perhaps, think this very obvious, but he will do well not to despise it.

Hence, to find the ar. comp. of the logarithm of a number, find the logarithm and subtract it from 10, in the manner already explained.

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* See Art. 17, p. 518.

e. g. Find ar. comp. of log. 46·028.
log. 46·028 = 1·6630221.
ar. . comp. log. 46·028 = 8·3369778.

(6). *To find the product of several numbers by means of a Table of Logarithms.*

We have seen that if N = xyz .... Then
log. N = log. x + log. y + log. z +, &c. ....

Hence, find the logarithm of each number, add them together—this gives the logarithm of the quotient—find the number corresponding to this logarithm, and we have the product itself.

Ex. Find the product of 52·731 × 6·0032 × ·0759
log. 52·731 = 1·7220660
log. 6·0032 = ·7783828
log. ·0759 = 8· 8802418 — 10.

1·3806906
24·026 ·3806815

91
5 91

24·0265 Ans.

N.B.—In any example of this kind, never use a negative characteristic, such as $\bar{2}$·8802418. but 8·8802418 — 10 as above; by doing so, there is nothing but straightforward addition to be performed until the end, when —10 can be easily struck off the characteristic of the sum.

(7). *To divide one number by another by means of a Table of Logarithms.*

We have seen that if $N = \frac{x}{y}$ ∴ log. N = log. x — log. y.
∴ log. N = log. x + 10 — log. y — 10
∴ log. N = log. x + Ar. Comp. log. y — 10.

Hence, "To log. of numerator, add ar. comp. logarithm of denominator, and subtract 10 from the sum—this gives logarithm of quotient.—Find the number corresponding to this logarithm, and the number is the quotient required."

Ex. Divide 37·052 by 6741·6.
log. 37·052 = 1·5688117
Ar. : C. : log. 6741·6 — 10 = 6·1712370 — 10

$\bar{3}$·7400487
54960 ·7400647

20
2 16

4
5 4,0

·005496025 Ans.

(8). *Similarly, if* $N = \frac{abc}{xyz}$. We have—

log. N = log. a + log. b + log. c + Ar. C. log. x — 10 + Ar. C. log. y — 10 + Ar. C. log. z — 10.

By this means log. N is found by a single addition sum. Thus, find the value of $\frac{3·0972 \times 56·035}{·002 \times 8746·5 \times 3·124}$.

log. 3·0972 ·4909693
log. 56·035 1·748594.
Ar. C. log. ·002 — 10 12·6989700 — 10
Ar. C. log. 8746·5 — 10 6·0581657 — 10
Ar. C. log. 3·124 — 10 9·5052890 — 10

·5018534
3 1758 ·5018531

3
02 2,7

3·175802 Ans.

(9). *To find any power of a Number, we have seen that if* $N = a^n$ *log.* N = n *log.* a.

Hence, if we multiply log. of the number by the index, we obtain the logarithm of the power of the given number; and finding the number corresponding, we obtain the power itself.

Thus, find the fifth power of 2·00573
2·0057 ·3022660
3 65
log. 2·00573 = ·3022725
Multiply by index 5

1·5113625
32 461 ·5113619

04 6
5,4

32·46104 Ans.

Again, find the third power of ·02751.
log. ·02751 8·4394906 — 10
3
25 3184718 — 30
or, $\bar{5}$·3184718.

We may in practice write this as as follows:—
log. ·02751 8·4394906 — 10
3

$\bar{5}$·3184718
20810 ·3184599

119
5 104

15
·7 14,6

·00002081957 Ans.

(10). *To find the Root of any Number.*

If $N = a^{\frac{1}{n}}$, then,

$$\text{log. } N = \frac{1}{n}.\ \text{log. } a$$

Hence, "Find the logarithm of the given number; divide it by the number indicating the root—this is the logarithm of the required root—the corresponding number is the root itself."

Thus, extract the 5th root of 72·095.
log. 72·095 5)1·8979051

·3795810
2·3965 ·3795774

36
2 36

2·39652 Ans.

Again: extract the 7th root of ·00972.

Log. ·00972 = $\bar{3}$ ·9876663 = 7·9876663 — 10.

All this has to be divided by 7. This will be effected most easily by adding and subtracting such a multiple of 10 as shall make the negative part 70.

i.e., take log. ·00972 = 67·9876663 — 70.
7)67·9876663 — 70

9 7125238 — 10.
51585 7125234

4
05 4.2

·5158505. Ans.

If we had to extract the 6th root of the above number, we must of course take log. ·00972 = 57·9876663 — 60. And again, to extract the cube root, we must take log. ·00972 = 57·9876663 — 30.

(11). The student must exercise himself in working several examples, like each of those above given. When he has done so, he may then, for practice, work some more complicated examples, such as the following:—

Find the value of $\dfrac{31^{\frac{2}{3}} \times (\cdot05796)^3}{15\sqrt{2}}$

(a)	log. $31^{\frac{2}{3}}$	1·4913617
		2
		3)2·9827234
		·9942411
(b)	log. $(\cdot05796)^3$	8·7631284 — 10
		3
		6·2893852 — 10
(c)	log. 15	1·1760913.
(d)	log. $\sqrt{2}$	2) ·3010300
		·1505150
Hence	log. $31^{\frac{2}{3}}$	·9942411
	log. $(\cdot05796)^3$	6·2893852 — 10
	Ar. C. log. 15 — 10.	8·8239087 — 10.
	Ar. C. log. $\sqrt{2}$ — 10	9·8494850 — 10
		$\bar{5}$.9570200
	90577	·9570179
	4	21
		19
	4	2

·00009057744 Ans.

N.B.—To find the logarithms of a mixed number, reduce it to an improper fraction.

To find the logarithm of a vulgar fraction, use the formula—

If $N = \frac{a}{b}$ then log. N = log. a + Ar. Com. log. b — 10,

precisely as in division, excepting that there is no occasion to find N itself.

Thus, in the following example—

Find log. of $\dfrac{37052}{67416}$

log. 37052 = 4·5688117
Ar. Com. log. 67416—10 = 5·1712370 — 10

9.7400487 — 10

$\therefore$ log. $\dfrac{37052}{67416} = \bar{1}\cdot7400487$

[Before entering into another department of mathematics, we shall make a few observations on those which have already been explained. Although few of our readers will have perused the preceding sections of this volume without possessing some knowledge of mathematics; still the chapters on Arithmetic and Algebra, by Professor Young, are admirably calculated to introduce, in an easy and consecutive method, not only the first elements of the sciences, but also to prepare the way for the more difficult studies involved in Trigonometry, and Algebra applied generally to Geometry and the Conic Sections. With the exception of the chapter on Series and Logarithms, we think that all our readers will have readily mastered what has been advanced. There is no doubt that the last subject presents many points of interest, and yet requires considerable attention so as to permit of its thorough acquaintance. Much of the success of the student, in this respect, will depend on his having well studied what has been taught in reference to the Binomial Theorem, and Arithmetical and Geometrical Progression. Before commencing the study of the later books in Euclid, we advise that he should re-read the chapters devoted to the consideration of the subjects we last named, together with that on Series, &c. The necessity of this he will find out when he arrives at the investigation of the values of sines, tangents, &c., in our future pages. The calculation of trigonometrical tables depends entirely on the application of the principles already explained, in combination with those involved in the elements of Geometry, both plane and spherical. Without such a preparation as we urge on our readers, the study of Trigonometry will be very perplexing. On the other hand, one who has thoroughly acquainted himself with all that has preceded it, will find his future progress comparatively easy. We have arranged each branch of this section on the plan of gradually unfolding the subject. The reader, therefore, cannot, with advantage to himself, omit *any* part. Our plan is progressive, as his course must also be; and following that method, we can promise that his difficulties will gradually melt away before perseverance and close attention. We may extend these remarks also to the study of Euclid. If a person will only dip here and there into that admirable compendium of reasoning, he will *never* understand it. The whole of each "book" depends on reasoning based on the premises enunciated at its beginning. Each proposition is not only derived from its predecessors, but prepares the way for that which follows. Hence, the study of geometry must be taken as a whole; and any one link in the chain being broken, is fatal to the success of those who attempt its study. If any of our readers intend pursuing mathematical science in its higher branches, nothing is more essential than a thorough knowledge of the principles taught in the preceding and subsequent chapters in this section.—ED.]

MATHEMATICAL TABLES.

A TABLE OF THE LOGARITHMS OF ALL NUMBERS,

FROM 1 TO 10,000.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
0		62	7923917	124	0934217	186	2695129	248	3944517	310	4913617	372	5705429
1	0000000	63	7993405	125	0969100	187	2718416	249	3961993	311	4927604	373	5717088
2	3010300	64	8061800	126	1003705	188	2741578	250	3979400	312	4941546	374	5728716
3	4771213	65	8129134	127	1038037	189	2764618	251	3996737	313	4955443	375	5740313
4	6020600	66	8195439	128	1072100	190	2787536	252	4014005	314	4969296	376	5751878
5	6989700	67	8260748	129	1105897	191	2810334	253	4031205	315	4983106	377	5763414
6	7781513	68	8325089	130	1139434	192	2833012	254	4048337	316	4996871	378	5774918
7	8450980	69	8388491	131	1172713	193	2855573	255	4065402	317	5010593	379	5786392
8	9030900	70	8450980	132	1205739	194	2878017	256	4082400	318	5024271	380	5797836
9	9542425	71	8512583	133	1238516	195	2900346	257	4099331	319	5037907	381	5809250
10	0000000	72	8573325	134	1271048	196	2922561	258	4116197	320	5051500	382	5820634
11	0413927	73	8633229	135	1303338	197	2944662	259	4132998	321	5065050	383	5831988
12	0791812	74	8692317	136	1335389	198	2966652	260	4149733	322	5078559	384	5843312
13	1139434	75	8750613	137	1367206	199	2988531	261	4166405	323	5092025	385	5854607
14	1461280	76	8808136	138	1398791	200	3010300	262	4183013	324	5105450	386	5865873
15	1760913	77	8864907	139	1430148	201	3031961	263	4199557	325	5118834	387	5877110
16	2041200	78	8920946	140	1461280	202	3053514	264	4216039	326	5132176	388	5888317
17	2304489	79	8976271	141	1492191	203	3074960	265	4232459	327	5145478	389	5899496
18	2552725	80	9030900	142	1522883	204	3096302	266	4248816	328	5158738	390	5910646
19	2787536	81	9084850	143	1553360	205	3117539	267	4265113	329	5171959	391	5921768
20	3010300	82	9138139	144	1583625	206	3138672	268	4281348	330	5185139	392	5932861
21	3222193	83	9190781	145	1613680	207	3159703	269	4297523	331	5198280	393	5943926
22	3424227	84	9242793	146	1643529	208	3180633	270	4313638	332	5211381	394	5954962
23	3617278	85	9294189	147	1673173	209	3201463	271	4329693	333	5224442	395	5965971
24	3802112	86	9344985	148	1702617	210	3222193	272	4345689	334	5237465	396	5976952
25	3979400	87	9395193	149	1731863	211	3242825	273	4361626	335	5250448	397	5987905
26	4149733	88	9444827	150	1760913	212	3263359	274	4377506	336	5263393	398	5998831
27	4313638	89	9493900	151	1789769	213	3283796	275	4393327	337	5276299	399	6009729
28	4471580	90	9542425	152	1818436	214	3304138	276	4409091	338	5289167	400	6020600
29	4623980	91	9590414	153	1846914	215	3324385	277	4424798	339	5301997	401	6031444
30	4771213	92	9637878	154	1875207	216	3344538	278	4440448	340	5314789	402	6042261
31	4913617	93	9684829	155	1903317	217	3364597	279	4456042	341	5327544	403	6053050
32	5051500	94	9731279	156	1931246	218	3384565	280	4471580	342	5340261	404	6063814
33	5185139	95	9777236	157	1958997	219	3404441	281	4487063	343	5352941	405	6074550
34	5314789	96	9822712	158	1986571	220	3424227	282	4502491	344	5365584	406	6085260
35	5440680	97	9867717	159	2013971	221	3443923	283	4517864	345	5378191	407	6095944
36	5563025	98	9912261	160	2041200	222	3463530	284	4533183	346	5390761	408	6106602
37	5682017	99	9956352	161	2068259	223	3483049	285	4548449	347	5403295	409	6117233
38	5797836	100	0000000	162	2095150	224	3502480	286	4563660	348	5415792	410	6127839
39	5910646	101	0043214	163	2121876	225	3521825	287	4578819	349	5428254	411	6138418
40	6020600	102	0086002	164	2148438	226	3541084	288	4593925	350	5440680	412	6148972
41	6127839	103	0128372	165	2174839	227	3560259	289	4608978	351	5453071	413	6159501
42	6232493	104	0170333	166	2201081	228	3579348	290	4623980	352	5465427	414	6170003
43	6334685	105	0211893	167	2227165	229	3598355	291	4638930	353	5477747	415	6180481
44	6434527	106	0253059	168	2253093	230	3617278	292	4653829	354	5490033	416	6190933
45	6532125	107	0293838	169	2278867	231	3636120	293	4668676	355	5502284	417	6201361
46	6627578	108	0334238	170	2304489	232	3654880	294	4683473	356	5514500	418	6211763
47	6720979	109	0374265	171	2329961	233	3673559	295	4698220	357	5526682	419	6222140
48	6812412	110	0413927	172	2355284	234	3692159	296	4712917	358	5538830	420	6232493
49	6901961	111	0453230	173	2380461	235	3710679	297	4727564	359	5550944	421	6242821
50	6989700	112	0492180	174	2405492	236	3729120	298	4742163	360	5563025	422	6253125
51	7075702	113	0530784	175	2430380	237	3747483	299	4756712	361	5575072	423	6263404
52	7160033	114	0569049	176	2455127	238	3765770	300	4771213	362	5587086	424	6273659
53	7242759	115	0606978	177	2479733	239	3783979	301	4785665	363	5599066	425	6283889
54	7323938	116	0644580	178	2504200	240	3802112	302	4800069	364	5611014	426	6294096
55	7403627	117	0681859	179	2528530	241	3802170	303	4814426	365	5622929	427	6304279
56	7481880	118	0718320	180	2552725	242	3838154	304	4828736	366	5634811	428	6314438
57	7558749	119	0755470	181	2576786	243	3856063	305	4842998	367	5646661	429	6324573
58	7634280	120	0791812	182	2600714	244	3873898	306	4857214	368	5658478	430	6334685
59	7708520	121	0827854	183	2624511	245	3891661	307	4871384	369	5670264	431	6344773
60	7781513	122	0863598	184	2648178	246	3909351	308	4885507	370	5682017	432	6354837
61	7853298	123	0899051	185	2671717	247	3926970	309	4899585	371	5693739	433	6364879

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
434	6374897	510	7075702	586	7678976	662	8208589	738	8680564	814	9106244	890	9493900
435	6384893	511	7084209	587	7686381	663	8215135	739	8686444	815	9111576	891	9498777
436	6394865	512	7092700	588	7693773	664	8221681	740	8692317	816	9116901	892	9503649
437	6404814	513	7101174	589	7701153	665	8228216	741	8698182	817	9122221	893	9508515
438	6414741	514	7109631	590	7708520	666	8234742	742	8704039	818	9127533	894	9513375
439	6424645	515	7118072	591	7715875	667	8241258	743	8709888	819	9132839	895	9518230
440	6434527	516	7126497	592	7723217	668	8247765	744	8715729	820	9138139	896	9523080
441	6444386	517	7134905	593	7730547	669	8254261	745	8721563	821	9143432	897	9527924
442	6454223	518	7143298	594	7737864	670	8260748	746	8727388	822	9148718	898	9532763
443	6464037	519	7151674	595	7745170	671	8267225	747	8733206	823	9153998	899	9537597
444	6473830	520	7160033	596	7752463	672	8273693	748	8739016	824	9159272	900	9542425
445	6483600	521	7168377	597	7759743	673	8280151	749	8744818	825	9164539	901	9547248
446	6493349	522	7176705	598	7767012	674	8286599	750	8750613	826	9169800	902	9552065
447	6503075	523	7185017	599	7774268	675	8293038	751	8756399	827	9175055	903	9556878
448	6512780	524	7193313	600	7781513	676	8299467	752	8762178	828	9180303	904	9561684
449	6522463	525	7201593	601	7788745	677	8305887	753	8767950	829	9185545	905	9566486
450	6532125	526	7209857	602	7795965	678	8312297	754	8773713	830	9190781	906	9571282
451	6541765	527	7218106	603	7803173	679	8318698	755	8779470	831	9196010	907	9576073
452	6551384	528	7226339	604	7810369	680	8325089	756	8785218	832	9201233	908	9580858
453	6560982	529	7234557	605	7817554	681	8331471	757	8790959	833	9206450	909	9585639
454	6570559	530	7242759	606	7824726	682	8337844	758	8796692	834	9211661	910	9590414
455	6580114	531	7250945	607	7831887	683	8344207	759	8802418	835	9216865	911	9595184
456	6589648	532	7259116	608	7839036	684	8350561	760	8808136	836	9222063	912	9599948
457	6599162	533	7267272	609	7846173	685	8356906	761	8813847	837	9227255	913	9604708
458	6608655	534	7275413	610	7853298	686	8363241	762	8819550	838	9232440	914	9609462
459	6618127	535	7283538	611	7860412	687	8369567	763	8825245	839	9237620	915	9614211
460	6627578	536	7291648	612	7867514	688	8375884	764	8830934	840	9242793	916	9618955
461	6637009	537	7299743	613	7874605	689	8382192	765	8836614	841	9247960	917	9623693
462	6646420	538	7307823	614	7881684	690	8388491	766	8842288	842	9253121	918	9628427
463	6655810	539	7315888	615	7888751	691	8394780	767	8847954	843	9258276	919	9633155
464	6665180	540	7323983	616	7895807	692	8401061	768	8853612	844	9263424	920	9637878
465	6674530	541	7331973	617	7902852	693	8407332	769	8859263	845	9268567	921	9642596
466	6683859	542	7339993	618	7909885	694	8413595	770	8864907	846	9273704	922	9647309
467	6693169	543	7347998	619	7916906	695	8419843	771	8870544	847	9278834	923	9652017
468	6702459	544	7355989	620	7923917	696	8426092	772	8876173	848	9283959	924	9656720
469	6711728	545	7363965	621	7930916	697	8432328	773	8881795	849	9289077	925	9661417
470	6720979	546	7371926	622	7937904	698	8438554	774	8887410	850	9294189	926	9666110
471	6730209	547	7379873	623	7944880	699	8444772	775	8893017	851	9299296	927	9670797
472	6739420	548	7387806	624	7951846	700	8450980	776	8898617	852	9304396	928	9675480
473	6748611	549	7395723	625	7958800	701	8457180	777	8904210	853	9309490	929	9680157
474	6757783	550	7403627	626	7965743	702	8463371	778	8909796	854	9314579	930	9684829
475	6766936	551	7411516	627	7972675	703	8469553	779	8915375	855	9319661	931	9689497
476	6776070	552	7419391	628	7979596	704	8475727	780	8920946	856	9324738	932	9694159
477	6785184	553	7427251	629	7986506	705	8481891	781	8926510	857	9329808	933	9698816
478	6794279	554	7435098	630	7993405	706	8488047	782	8932068	858	9334873	934	9703469
479	6803355	555	7442930	631	8000294	707	8494194	783	8937618	859	9339932	935	9708116
480	6812412	556	7450748	632	8007171	708	8500333	784	8943161	860	9344985	936	9712758
481	6821451	557	7458552	633	8014037	709	8506462	785	8948696	861	9350032	937	9717396
482	6830470	558	7466342	634	8020893	710	8512583	786	8954225	862	9355073	938	9722028
483	6839471	559	7474118	635	8027737	711	8518696	787	8959747	863	9360108	939	9726656
484	6848454	560	7481880	636	8034571	712	8524800	788	8965262	864	9365137	940	9731279
485	6857417	561	7489629	637	8041394	713	8530895	789	8970770	865	9370161	941	9735896
486	6866363	562	7497363	638	8048207	714	8536982	790	8976271	866	9375179	942	9740509
487	6875290	563	7505084	639	8055009	715	8543060	791	8981765	867	9380191	943	9745117
488	6884198	564	7512791	640	8061800	716	8549130	792	8987252	868	9385197	944	9749720
489	6893089	565	7520484	641	8068580	717	8555192	793	8992732	869	9390198	945	9754318
490	6901961	566	7528164	642	8075350	718	8561244	794	8998205	870	9395193	946	9758911
491	6910815	567	7535831	643	8082110	719	8567289	795	9003671	871	9400182	947	9763500
492	6919651	568	7543483	644	8088859	720	8573325	796	9009131	872	9405165	948	9768083
493	6928469	569	7551123	645	8095597	721	8579353	797	9014583	873	9410142	949	9772662
494	6937269	570	7558749	646	8102325	722	8585372	798	9020029	874	9415114	950	9777236
495	6946052	571	7566361	647	8109043	723	8591383	799	9025468	875	9420081	951	9781805
496	6954814	572	7573960	648	8115750	724	8597386	800	9030900	876	9425041	952	9786369
497	6963564	573	7581546	649	8122447	725	8603380	801	9036325	877	9429996	953	9790929
498	6972293	574	7589119	650	8129134	726	8609366	802	9041744	878	9434945	954	9795484
499	6981005	575	7596678	651	8135810	727	8615344	803	9047155	879	9439889	955	9800034
500	6989700	576	7604225	652	8142476	728	8621314	804	9052560	880	9444827	956	9804579
501	6998377	577	7611758	653	8149132	729	8627275	805	9057959	881	9449759	957	9809119
502	7007037	578	7619278	654	8155777	730	8633229	806	9063350	882	9454686	958	9813655
503	7015680	579	7626786	655	8162413	731	8639174	807	9068735	883	9459607	959	9818186
504	7024305	580	7634280	656	8169038	732	8645111	808	9074114	884	9464523	960	9822712
505	7032914	581	7641761	657	8175654	733	8651040	809	9079485	885	9469433	961	9827234
506	7041505	582	7649230	658	8182259	734	8656961	810	9084850	886	7474337	962	9831751
507	7050080	583	7656685	659	8188854	735	8662873	811	9090209	887	9479236	963	9836263
508	7058637	584	7664128	660	8195439	736	8668778	812	9095560	888	9484130	964	9840770
509	7067178	585	7671559	661	8202015	737	8674675	813	9100905	889	9489018	965	9845273

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
966	9849771	971	9872192	976	9894498	981	9916690	986	9938769	991	9960737	996	9982593
967	9854265	972	9876663	977	9898946	982	9921115	987	9943172	992	9965117	997	9986952
968	9858754	973	9881128	978	9903389	983	9925535	988	9947569	993	9969492	998	9991305
969	9863233	974	9885590	979	9907827	984	9929951	989	9951963	994	9973864	999	9995655
970	9867717	975	9890046	980	9912261	985	9934362	990	9956352	995	9978231		

[Nos. 1000—1609.] TABLE OF LOGARITHMS FROM 1,000 TO 10,000. [Log. 0000000—2065560.]

[Note.—This table is simply a continuation of the preceding one; but when we arrive at No. 434, we commence giving a Table of Differences, by means of which Nos. exceeding 10,000 may be easily calculated.]

No.	0	1	2	3	4	5	6	7	8	9
100	0000000	04341	08677	13009	17337	0021661	25980	30295	34605	38912
101	43214	47512	51805	56094	60380	64660	68937	73210	77478	81742
102	86002	90257	94509	98756	03000	0107239	11474	15704	19931	24154
103	0128372	32587	36797	41003	45205	49403	53598	57788	61974	66155
104	70333	74507	78677	82843	87005	91163	95317	99467	03613	07755
105	0211893	16027	20157	24284	28406	0232525	36639	40750	44857	48960
106	53059	57154	61245	65333	69416	73496	77572	81644	85713	89777
107	93838	97895	01948	05997	10043	0314085	18123	22157	26188	30214
108	0334238	38257	42273	46285	50293	54297	58298	62295	66289	70279
109	74265	78248	82226	86202	90173	94141	98106	02066	06023	09977
110	0413927	17873	21816	25755	29691	0433623	37551	41476	45398	49315
111	53230	57141	61048	64952	68852	72749	76642	80532	84418	88301
112	92180	96056	99929	03798	07663	0511525	15384	19239	23091	26939
113	0530784	34626	38464	42299	46131	49959	53783	57605	61423	65237
114	69049	72856	76661	80462	84260	88055	91846	95634	99419	03200
115	0606978	10753	14525	18293	22058	0625820	29578	33334	37086	40834
116	44580	48322	52061	55797	59530	63259	66986	70709	74428	78145
117	81859	85569	89276	92980	96681	0700379	04073	07765	11453	15138
118	0718820	22499	26175	29847	33517	37184	40847	44507	48164	51819
119	55470	59118	62763	66404	70043	73679	77312	80942	84568	88192
120	91812	95430	99045	02656	06265	0809870	13473	17073	20669	24263
121	0827854	41441	35026	38608	42187	45763	49336	52906	56473	60037
122	63598	67157	70712	74265	77814	81361	84905	88446	91984	95519
123	99051	02581	06107	09631	13152	0916670	20185	23697	27206	30713
124	0934217	37718	41216	44711	48204	51694	55180	58665	62146	65624
125	69100	72573	76043	79511	82975	86437	89896	93353	96806	00257
126	1003705	07151	10594	14034	17471	1020905	24337	27766	31193	34616
127	38037	41456	44871	48284	51694	55102	58507	61909	65309	68705
128	72100	75491	78880	82267	85650	89031	92410	95785	99159	02529
129	1105897	09262	12625	15985	19343	1122698	26050	29400	32747	36092
130	39434	42773	46110	49444	52776	56105	59432	62756	66077	69396
131	72713	76027	79338	82647	85954	89258	92559	95858	99154	02448
132	1205739	09028	12315	15598	18880	1222159	25435	28709	31981	35250
133	38516	41781	45042	48301	51558	54813	58065	61314	64561	67806
134	71048	74288	77525	80760	83993	87223	90451	93676	96899	00119
135	1303338	06553	09767	12978	16187	1319393	22597	25798	28998	32195
136	35389	38581	41771	44959	48144	51327	54507	57685	60861	64034
137	67206	70375	73541	76705	79867	83027	86184	89339	92492	95643
138	98791	01937	05080	08222	11361	1414498	17632	20765	23895	27022
139	1430148	33271	36392	39511	42628	45742	48854	51964	55072	58177
140	61280	64381	67480	70577	73671	76763	79853	82941	86027	89110
141	92191	95270	98347	01422	04494	1507564	10633	13699	16762	19824
142	1522883	25941	28996	32049	35100	38149	41195	44240	47282	50322
143	53360	56396	59430	62462	65492	68519	71544	74568	77589	80608
144	83625	86640	89653	92663	95672	98678	01683	04685	07686	10684
145	1613680	16674	19666	22656	25644	1628630	31614	34596	37575	40553
146	43529	46502	49474	52443	55411	58376	61340	64301	67261	70218
147	73173	76127	79078	82027	84975	87920	90864	93805	96744	99682
148	1702617	05551	08482	11412	14339	1717265	20188	23110	26029	28947
149	31863	34776	37688	40598	43506	46412	49316	52218	55118	58016
150	60913	63807	66699	69590	72478	75365	78250	81133	84013	86892
151	89769	92645	95518	98389	01259	1804126	06992	09856	12718	15578
152	1818436	21292	24147	26999	29850	32698	35545	38390	41234	44075
153	46914	49752	52588	55422	58254	61084	63912	66739	69563	72386
154	75207	78026	80844	83659	86473	89285	92095	94903	97710	00514
155	1903317	06118	08917	11715	14510	1917304	20096	22886	25675	28461
156	31246	34029	36810	39590	42367	45143	47918	50690	53461	56229
157	58997	61762	64525	67287	70047	72806	75562	78317	81070	83821
158	86571	89319	92065	94809	97552	2000293	03032	05769	08505	11239
159	2013971	16702	19431	22158	24883	27607	30329	33049	35768	38485
160	41200	43913	46625	49335	52044	54750	57455	60159	62860	65560

No.	0	1	2	3	4	5	6	7	8	9
161	68259	70955	73650	76344	79035	81725	84414	87100	89785	92468
162	95150	97830	00508	03185	05860	2108534	11205	13876	16544	19211
163	2121876	24540	27202	29862	32521	35178	37833	40487	43139	45790
164	48438	51086	53732	56376	59018	61659	64298	66936	69572	72207
165	74839	77471	80100	82729	85355	87980	90603	93225	95845	98464
166	2201081	03696	06310	08922	11533	2214142	16750	19356	21960	24563
167	27165	29764	32363	34959	37555	40148	42740	45331	47920	50507
168	53093	55677	58260	60841	63421	65999	68576	71151	73724	76296
169	78867	81436	84004	86570	89134	91697	94258	96818	99377	01934
170	2304489	07043	09596	12146	14696	2317244	19790	22335	24879	27421
171	29961	32500	35038	37574	40108	42641	45173	47703	50232	52759
172	55284	57809	60331	62853	65373	67891	70408	72923	75437	77950
173	80461	82971	85479	87986	90491	92995	95497	97998	00498	02996
174	2405492	07988	10482	12974	15465	2417954	20442	22929	25414	27898
175	30380	32861	35341	37819	40296	42771	45245	47718	50189	52658
176	2455127	57594	60059	62523	64986	2467447	69907	72365	74823	77278
177	79733	82186	84637	87087	89536	91984	94430	96874	99318	01759
178	2504200	06639	09077	11513	13949	2516382	18815	21246	23675	26103
179	28530	30956	33380	35803	38224	40645	43063	45481	47897	50312
180	52725	55137	57548	59957	62365	64772	67177	69582	71984	74386
181	76786	79185	81582	83978	86373	88766	91158	93549	95939	98327
182	2600714	03099	05484	07867	10248	2612629	15008	17385	19762	22137
183	24511	26883	29255	31625	33993	36361	38727	41092	43455	45817
184	48178	50538	52896	55253	57609	59964	62317	64669	67020	69369
185	71717	74064	76410	78754	81097	83439	85780	88119	90457	92794
186	95129	97464	99797	02129	04459	2706788	09116	11443	13769	16093
187	2718416	20738	23058	25378	27696	30013	32328	34643	36956	39268
188	41578	43888	46196	48503	50809	53114	55417	57719	60020	62320
189	64618	66915	69211	71506	73800	76092	78383	80673	82962	85250
190	87536	89821	92105	94388	96669	98950	01229	03507	05784	08059
191	2810334	12607	14879	17150	19419	2821688	23955	26221	28486	30750
192	33012	35274	37534	39793	42051	44307	46563	48817	51070	53322
193	55573	57823	60071	62319	64565	66810	69054	71296	73538	75778
194	78017	80255	82492	84728	86963	89196	91428	93660	95890	98118
195	2900346	02573	04798	07022	09246	2911468	13689	15908	18127	20344
196	22561	24776	26990	29203	31415	33626	35835	38044	40251	42457
197	44662	46866	49069	51271	53471	55671	57869	60067	62263	64458
198	66652	68845	71037	73227	75417	77605	79792	81979	84164	86348
199	88531	90713	92893	95073	97252	99429	01605	03781	05955	08128
200	3010300	12471	14641	16809	18977	3021144	23309	25474	27637	29799
201	31961	34121	36280	38438	40595	42751	44905	47059	49212	51363
202	53514	55663	57812	59959	62105	64250	66394	68537	70680	72820
203	74960	77099	79237	81374	83509	85644	87778	89910	92042	94172
204	96302	98430	00557	02684	04809	3106933	09056	11178	13300	15420
205	3117539	19657	21774	23889	26004	3128118	30231	32343	34454	36563
206	38672	40780	42887	44992	47097	49201	51303	53405	55505	57605
207	59703	61801	63898	65993	68088	70181	72273	74365	76455	78545
208	80633	82721	84807	86893	88977	91061	93143	95224	97305	99384
209	3201463	03540	05617	07692	09767	3211840	13913	15984	18055	20124
210	22193	24261	26327	28393	30457	32521	34584	36645	38706	40766
211	42825	44882	46939	48995	51050	53104	55157	57209	59260	61310
212	63359	65407	67454	69500	71545	73589	75633	77675	79716	81757
213	83796	85834	87872	89909	91944	93979	96012	98045	00077	02108
214	3304138	06167	08195	10222	12248	3314273	16297	18320	20343	22364
215	24385	26404	28423	30440	32457	34473	36488	38501	40514	42526
216	44538	46548	48557	50565	52573	54579	56585	58589	60593	62596
217	64597	66598	68598	70597	72595	74593	76589	78584	80579	82572
218	84565	86557	88547	90537	92526	94514	96502	98488	00473	02458
219	3404141	06424	08405	10386	12366	3414345	16323	18301	20277	22252
220	24227	26200	28173	30145	32116	34086	36055	38023	39991	41957
221	43923	45887	47851	49814	51776	53737	55698	57657	59615	61573
222	63530	65486	67441	69395	71348	73300	75252	77202	79152	81101
223	83049	84996	86942	88887	90832	92775	94718	96660	98601	00541
224	3502480	04419	06356	08293	10229	3512163	14098	16031	17963	19895
225	21825	23755	25684	27612	29539	31465	33391	35316	37239	39162
226	41684	43006	44926	46846	48764	50682	52599	54515	56431	58345
227	60259	62171	64083	65994	67905	69814	71723	73630	75537	77443
228	79348	81253	83156	85059	86961	88862	90762	92662	94560	96458
229	98355	00251	02146	04041	05934	3607827	09719	11610	13500	15390
230	3617278	19166	21053	22939	24825	26709	28593	30476	32358	34239
231	36120	37999	39878	41756	43634	45510	47386	49260	51134	53007
232	54880	56751	58622	60492	62361	64230	66097	67964	69830	71695
233	73559	75423	77285	79147	81009	82869	84728	86587	88445	90302
234	92159	94014	95869	97723	99576	3701428	03280	05131	06981	08830
235	3710679	12526	14373	16219	18065	19909	21753	23596	25438	27279
236	29120	30960	32799	34637	36475	38311	40147	41983	43817	45651

No.	0	1	2	3	4	5	6	7	8	9
237	3747483	49316	51147	52977	54807	3756636	58464	60292	62119	63944
238	65770	67594	69418	71240	73063	74884	76704	78524	80343	82161
239	83979	85796	37612	89427	91241	93055	94868	96680	98492	00302
240	3802112	03922	05730	07538	09345	3811151	12956	14761	16565	18368
241	20170	21972	23773	25573	27373	29171	30969	32767	34563	36359
242	38154	39948	41741	43534	45326	47117	48908	50698	52487	54275
243	56063	57850	59636	61421	63206	64990	66773	68555	70337	72118
244	73898	75678	77457	79235	81012	82789	84565	86340	88114	89888
245	91661	93433	95205	96975	98746	3900515	02284	04052	05819	07585
246	3909351	11116	12880	14644	16407	18169	19931	21691	23452	25211
247	26970	28727	30485	32241	33997	35752	37506	39260	41013	42765
248	44517	46268	48018	49767	51516	53264	55011	56758	58504	60249
249	61993	63737	65480	67223	68964	70705	72446	74185	75924	77663
250	79400	81137	82873	84608	86343	88077	89811	91543	93275	95007
251	96737	98467	00196	01925	03653	4005380	07106	08832	10557	12282
252	4014005	15728	17451	19173	20894	22614	24333	26052	27771	29488
253	31205	32921	34637	36352	38066	39780	41492	43205	44916	46627
254	48337	50047	51755	53464	55171	56878	58584	60289	61994	63698
255	65402	67105	68807	70508	72209	73909	75608	77307	79005	80703
256	82400	84096	85791	87486	89180	90874	92567	94259	95950	97641
257	99331	01021	02710	04398	06085	4107772	09459	11144	12829	14513
258	4116197	17880	19562	21244	22925	24605	26285	27964	29643	31321
259	32998	34674	36350	38025	39700	41374	43047	44719	46391	48063
260	49733	51404	53073	54742	56410	58077	59744	61410	63076	64741
261	66405	68069	69732	71394	73056	74717	76377	78037	79696	81355
262	83013	84670	86327	87983	89638	91293	92947	94601	96254	97906
263	99557	01208	02859	04509	06158	4207806	09454	11101	12748	14394
264	4216039	17684	19328	20972	22615	24257	25898	27539	29180	30820
265	32459	34097	35735	37372	39009	40645	42281	43916	45550	47183
266	48816	50449	52081	53712	55342	56972	58601	60230	61858	63486
267	65113	66739	68365	69990	71614	73238	74861	76484	78106	79727
268	81348	82968	84588	86207	87825	89443	91060	92677	94293	95908
269	97523	99137	00751	02364	03976	4305588	07199	08809	10419	12029
270	4313638	15246	16853	18460	20067	21672	23278	24883	26487	28090
271	29693	31295	32897	34498	36098	37698	39298	40896	42495	44092
272	45689	47285	48881	50476	52071	53665	55259	56851	58444	60035
273	61626	63217	64807	66396	67985	69573	71161	72748	74334	75920
274	77506	79090	80675	82258	83841	85423	87005	88587	90167	91747
275	93327	94906	96484	98062	99639	4401216	02792	84368	05943	07517
276	4409091	10664	12237	13809	15380	16951	18522	20092	21661	23230
277	24798	26305	27932	29499	31065	32630	34195	35759	37322	38885
278	40448	42010	43571	45132	46692	48251	49811	51370	52928	54485
279	56042	57598	59154	60709	62264	63818	65372	66925	68477	70029
280	71580	73131	74681	76231	77780	79329	80877	82424	83971	85517
281	87063	88608	90153	91697	93241	94784	96327	97868	99410	00951
282	4502491	04031	05570	07109	08647	4510185	11722	13258	14794	16329
283	17864	19399	20932	22466	23998	25531	27062	28593	30124	31654
284	33183	34712	36241	37769	39296	40823	42349	43875	45400	46924
285	48449	49972	51495	53018	54540	56061	57582	59102	60622	62142
286	63660	65179	66696	68213	69730	71246	72762	74277	75791	77305
287	78819	80332	81844	83356	84868	86378	87889	89399	90908	92417
288	93925	95433	96940	98446	99953	4601458	02963	04468	05972	07475
289	4608978	10481	11983	13484	14985	16486	17986	19485	20984	22482
290	23980	25477	26974	28470	29966	31461	32956	34450	35944	37437
291	38930	40422	41914	43405	44895	46386	47875	49364	50853	52341
292	53829	55316	56802	58288	59774	61259	62743	64227	65711	67194
293	68676	70158	71640	73121	74601	76081	77561	79039	80518	81996
294	83473	84950	86427	87903	89378	96853	92327	93801	95275	96748
295	98220	99692	01164	02634	04105	4705575	07044	08513	09982	11450
296	4712917	14384	15851	17317	18782	20247	21711	23175	24639	26102
297	27564	29027	30488	31949	33410	34870	36329	37788	39247	40705
298	42163	43620	45076	46533	47988	49443	50898	52352	53806	55259
299	56712	58164	59616	61067	62518	63968	65418	66867	68316	69765
300	71213	72660	74107	75553	76999	78445	79890	81334	82778	84222
301	85665	87108	88550	89991	91432	92873	94313	95753	97192	98631
302	4800069	01507	02945	04381	05818	4807254	08689	10124	11559	12993
303	14426	15859	17292	18724	20156	21587	23018	24448	25878	27307
304	28736	30164	31592	33020	34446	35873	37299	38725	40150	41574
305	42998	44422	45845	47268	48690	50112	51533	52954	54375	55795
306	57214	58633	60052	61470	62888	64305	65722	67138	68554	69969
307	71384	72798	74212	75626	77839	78451	79863	81275	82686	84097
308	85507	86917	88326	89735	91144	92552	93959	95366	96773	98179
309	99585	00990	02395	03799	05203	4906607	08010	09412	10814	12216
310	4913617	15018	16418	17818	19217	20616	22015	23413	24810	26207
311	27604	29000	30396	31791	33186	34581	35974	37368	38761	40154
312	41546	42938	44329	45720	47110	48500	49890	51279	52667	54050

No.	0	1	2	3	4	5	6	7	8	9
313	4955443	56831	58218	59604	60990	4962375	63761	65145	66529	67913
314	69296	70679	72062	73444	74825	76206	77587	78967	80347	81727
315	83106	84484	85862	87240	88617	89994	91370	92746	94121	95496
316	96871	98245	99619	00992	02365	5003737	05109	06481	07852	09222
317	5010593	11962	13332	14701	16069	17437	18805	20172	21539	22905
318	24271	25637	27002	28366	29731	31094	32458	33821	35183	36545
319	37907	39268	40629	41989	43349	44709	46068	47426	48785	50142
320	51500	52857	54213	55569	56925	58280	59635	60990	62344	63697
321	65050	66403	67755	69107	70459	71810	73160	74511	75860	77210
322	78559	79907	81255	82603	83950	85297	86644	87990	89335	90680
323	92025	93370	94714	96057	97400	98743	00085	01427	02768	04109
324	5105450	06790	08130	09469	10808	5112147	13485	14823	16160	17497
325	18834	20170	21505	22841	24175	25510	26844	28178	29511	30844
326	32176	33508	34840	36171	37502	38832	40162	41491	42820	44149
327	45478	46805	48133	49460	50787	52113	53439	54764	56089	57414
328	58738	60062	61386	62709	64031	65354	66676	67997	69318	70639
329	71959	73279	74598	75917	77236	78554	79872	81189	82507	83823
330	85139	86455	87771	89086	90400	91715	93028	94342	95655	96968
331	98280	99592	00903	02214	03525	5204835	06145	07455	08764	10073
332	5211381	12689	13996	15303	16610	17916	19222	20528	21833	23138
333	24442	25746	27050	28353	29656	30958	32260	33562	34863	36164
334	37465	38765	40064	41364	42663	43961	45259	46557	47854	49151
335	50448	51744	53040	54336	55631	56925	58220	59513	60807	62100
336	63393	64685	65977	67269	68560	69851	71141	72431	73721	75010
337	76299	77588	78876	80163	81451	82738	84024	85311	86596	87882
338	89167	90452	91736	93020	94304	95587	96870	98152	99434	00716
339	5301997	03278	04558	05839	07118	5308398	09677	10955	12234	13512
340	14789	16066	17343	18619	19896	21171	22446	23721	24996	26270
341	27544	28817	30090	31363	32635	33907	35179	36450	37721	38991
342	40261	41531	42800	44069	45338	46606	47874	49141	50408	51675
343	52941	54207	55473	56738	58003	59267	60532	61795	63059	64322
344	65584	66847	68109	69370	70631	71892	73153	74413	75673	76932
345	78191	79450	80708	81966	83223	84481	85737	86994	88250	89506
346	90761	92016	93271	94525	95779	97032	98286	99538	00791	02043
347	5403295	04546	05797	07048	08298	5409548	10798	12047	13296	14544
348	15792	17040	18288	19535	20781	22028	23274	24519	25765	27010
349	28254	29498	30742	31986	33229	34472	35714	36956	38198	39439
350	40680	41921	43161	44401	45641	46880	48119	49358	50596	51834
351	53071	54308	55545	56781	58018	59253	60489	61724	62958	64193
352	65427	66660	67894	69126	70359	71591	72823	74055	75286	76517
353	77747	78977	80207	81436	82665	83894	85123	86351	87578	88806
354	90033	91259	92486	93712	94937	96162	97387	98612	99836	01060
355	5502284	03507	04730	05952	07174	5508396	09618	10839	12059	13280
356	14500	15720	16939	18158	19377	20595	21813	23031	24248	25465
357	26682	27899	29115	30330	31545	32760	33975	35189	36403	37617
358	38830	40043	41256	42468	43680	44892	46103	47314	48524	49735
359	50944	52154	53363	54572	55781	56989	58197	59404	60612	61818
360	63025	64231	65437	66643	67848	69053	70257	71461	72665	73869
361	75072	76275	77477	78680	79881	81083	82284	83485	84686	85886
362	87086	88285	89484	90683	91882	93080	94278	95476	96673	97870
363	99066	00262	01458	02654	03849	5605044	06239	07433	08627	09821
364	5611014	12207	13399	14592	15784	16975	18167	19358	20548	21739
365	22929	24118	25308	26497	27685	28874	30062	31250	32437	33624
366	34811	35997	37183	38369	39555	40740	41925	43109	44293	45477
367	46661	47844	49027	50209	51392	52573	53755	54936	56117	57298
368	58478	59658	60838	62017	63196	64375	65553	66731	67909	69087
369	70264	71440	72617	73793	74969	76144	77320	78495	79669	80843
370	82017	83191	84364	85537	86710	87882	89054	90226	91397	92568
371	93739	94910	96080	97249	98419	99588	00757	01926	03094	04262
372	5705429	06597	07764	08930	10097	5711263	12429	13594	14759	15924
373	17088	18252	19416	20580	21743	22906	24069	25231	26393	27555
374	28716	29877	31038	32198	33358	34518	35678	36837	37996	39154
375	40313	41471	42628	43786	44943	46099	47256	48412	49568	50723
376	51878	53033	54188	55342	56496	57650	58803	59956	61109	62261
377	63414	64565	65717	66868	68019	69170	70320	71470	72620	73769
378	74918	76067	77215	78363	79511	80659	81806	82953	84100	85246
379	86392	87538	88683	89828	90973	92118	93262	94406	95550	96693
380	97836	98979	00121	01263	02405	5803547	04688	05829	06969	08110
381	5809250	10389	11529	12668	13807	14945	16084	17222	18359	19497
382	20634	21770	22907	24043	25179	26314	27450	28585	29719	30854
383	31988	33122	34255	35388	36521	37654	38786	39918	41050	42181
384	43312	44443	45574	46704	47834	48963	50093	51222	52351	53479
385	54607	55735	56863	57990	59117	60244	61370	62496	63622	64748
386	65873	66998	68123	69247	70371	71495	72618	73742	74865	75987
387	77110	78232	79353	80475	81596	82717	83838	84958	86078	87198
388	88317	89436	90555	91674	92792	93910	95028	96145	97263	98379

No.	0	1	2	3	4	5	6	7	8	9
389	5899496	00612	01728	02844	03959	5905075	06189	07304	08418	09532
390	5910646	11760	12873	13986	15093	16210	17322	18434	19546	20657
391	21768	22878	23988	25098	26208	27318	28427	29536	30644	31753
392	32861	33968	35076	36183	37290	38397	39503	40609	41715	42820
393	43926	45030	46135	47239	48344	49447	50551	51654	52757	53860
394	54962	56064	57166	58268	59369	60470	61571	62671	63771	64871
395	65971	67070	68169	69268	70367	71465	72563	73661	74758	75855
396	76952	78048	79145	80241	81336	82432	83527	84622	85717	86811
397	87905	88999	90092	91186	92279	93371	94464	95556	96648	97739
398	98831	99922	01013	02103	03193	6004283	05373	06462	07551	08640
399	6009729	10817	11905	12993	14081	15168	16255	17341	18428	19514
400	20600	21686	22771	23856	24941	26025	27109	28193	29277	30361
401	31444	32527	33609	34692	35774	36855	37937	39018	40099	41180
402	42261	43341	44421	45500	46580	47659	48738	49816	50895	51973
403	53050	54128	55205	56282	57359	58435	59512	60587	61663	62739
404	63814	64889	65963	67037	68111	69185	70259	71332	72405	73478
405	74550	75622	76694	77766	78837	79909	80979	82050	83120	84191
406	85260	86330	87399	88468	89537	90605	91674	92742	93809	94877
407	95944	97011	98078	99144	00210	6101276	02342	03407	04472	05537
408	6106602	07666	08730	09794	10857	11921	12984	14046	15109	16171
409	17233	18295	19356	20417	21478	22539	23599	24660	25720	26779
410	27839	28898	29957	31015	32074	33132	34189	35247	36304	37361
411	38418	39475	40531	41587	42643	43698	44754	45809	46863	47918
412	48972	50026	51080	52133	53187	54240	55292	56345	57397	58449
413	59501	60552	61603	62654	63705	64755	65805	66855	67905	68954
414	70003	71052	72101	73149	74197	75245	76293	77340	78387	79434
415	80481	81527	82573	83619	84665	85710	86755	87800	88845	89889
416	90933	91977	93021	94064	95107	96150	97193	98235	99277	00319
417	6201361	02402	03443	04484	05524	6206565	07605	08645	09684	10724
418	11763	12802	13840	14879	15917	16955	17992	19030	20067	21104
419	22140	23177	24213	25249	26284	27320	28355	29390	30424	31459
420	32493	33527	34560	35594	36627	37660	38693	39725	40757	41789
421	42821	43852	44884	45915	46945	47976	49006	50036	51066	52095
422	53125	54154	55182	56211	57239	58267	59295	60322	61350	62377
423	63404	64430	65457	66483	67509	68534	69560	70585	71610	72634
424	73659	74683	75707	76730	77754	78777	79800	80823	81845	82867
425	83889	84911	85933	86954	87975	88996	90016	91037	92057	93076
426	94096	95115	96134	97153	98172	99190	00209	01226	02244	03262
427	6304279	05296	06312	07329	08345	6309361	10377	11393	12408	13423
428	14438	15452	16467	17481	18495	19508	20522	21535	22548	23560
429	24573	25585	26597	27609	28620	29632	30643	31654	32664	33674
430	34685	35694	36704	37713	38723	39732	40740	41749	42757	43765
431	44773	45780	46788	47795	48801	49808	50814	51820	52826	53832
432	54837	55843	56848	57852	58857	59861	60865	61869	62873	63876
433	64879	65882	66884	67887	68889	69891	70893	71894	72895	73897
434	74897	75898	76898	77898	78898	79898	80897	81896	82895	83894

The Tabular Difference of Logarithms, from 4350 to 9999, are added in the remainder of the Table.

No.	0	1	2	3	4	5	6	7	8	9	Dif.
435	6384893	5891	6889	7887	8884	6389882	0879	1876	2872	3869	998
436	6394865	5861	6857	7852	8847	6399842	0837	1832	2826	3820	995
437	6404814	5808	6802	7795	8788	6409781	0773	1765	2758	3749	993
438	6414741	5733	6724	7715	8705	6419696	0686	1676	2666	3656	991
439	6424645	5634	6623	7612	8601	6429589	0577	1565	2552	3540	989
440	6434527	5514	6500	7487	8473	6439459	0445	1431	2416	3401	986
441	6444386	5371	6355	7339	8323	6449307	0291	1274	2257	3240	984
442	6454223	5205	6187	7169	8151	6459133	0114	1095	2076	3057	982
443	6464037	5018	5998	6977	7957	6468936	9915	0894	1873	2851	980
444	6473830	4808	5786	6763	7741	6478718	9695	0671	1648	2624	978
445	6483600	4576	5552	6527	7502	6488477	9452	0426	1401	2375	975
446	6493349	4322	5296	6269	7242	6498215	9187	0160	1132	2104	973
447	6503075	4047	5018	5989	6960	6507930	8901	9871	0841	1811	971
448	6512780	3749	4719	5687	6656	6517624	8593	9561	0528	1496	969
449	6522463	3431	4397	5364	6331	6527297	8263	9229	0195	1160	966
450	6532125	3090	4055	5019	5984	6536948	7912	8876	9839	0802	964
451	6541765	2728	3691	4653	5616	6546578	7539	8501	9462	0423	962
452	6551384	2345	3306	4266	5226	6556186	7145	8105	9064	0023	960
453	6560982	1941	2899	3857	4815	6565773	6730	7688	8645	9602	958
454	6570559	1515	2471	3427	4383	6575339	6294	7250	8205	9159	956
455	6580114	1068	2023	2977	3930	6584884	5837	6790	7743	8696	954
456	6589648	0601	1553	2505	3456	6594408	5359	6310	7261	8212	952
457	6599162	0112	1062	2012	2962	6603911	4860	5809	6758	7706	950
458	6608655	9603	0551	1499	2446	6613393	4341	5287	6234	7181	947

No.	0	1	2	3	4	5	6	7	8	9	Dif.
459	6618127	9073	0019	0964	1910	6622855	3800	4745	5690	6634	945
460	6627578	8522	9466	0410	1353	6632296	3239	4182	5125	6067	943
461	6637009	7951	8893	9835	0776	6641717	2658	3599	4539	5480	941
462	6646420	7360	8299	9239	0178	6651117	2056	2995	3934	4872	939
463	6655810	6748	7686	8623	9560	6660497	1434	2371	3307	4244	937
464	6665180	6116	7051	7987	8922	6669857	0792	1727	2661	3595	935
465	6674530	5463	6397	7331	8264	6679197	0130	1062	1995	2927	933
466	6683859	4791	5723	6654	7585	6688516	9447	0378	1308	2239	931
467	6693169	4099	5028	5958	6887	6697816	8745	9674	0602	1530	929
468	6702459	3386	4314	5242	6169	6707096	8023	8950	9876	0802	927
469	6711728	2654	3580	4506	5431	6716356	7281	8206	9130	0054	925
470	6720979	1903	2826	3750	4673	6725596	6519	7442	8365	9287	923
471	6730209	1131	2053	2974	3896	6734817	5738	6659	7579	8500	921
472	6739420	0340	1260	2179	3099	6744018	4937	5856	6775	7693	919
473	6748611	9529	0447	1365	2283	6753200	4117	5034	5951	6867	917
474	6757783	8700	9615	0531	1447	6762362	3277	4192	5107	6022	915
475	6766936	7850	8764	9678	0592	6771505	2418	3332	4244	5157	913
476	6776070	6982	7894	8806	9718	6780629	1540	2452	3362	4273	911
477	6785184	6094	7004	7914	8824	6789734	0643	1552	2461	3370	910
478	6794279	5187	6096	7004	7912	6798819	9727	0634	1541	2448	908
479	6803355	4262	5168	6074	6980	6807886	8792	9697	0602	1507	906
480	6812412	3317	4222	5126	6030	6816934	7838	8741	9645	0548	904
481	6821451	2354	3256	4159	5061	6825963	6865	7766	8668	9569	902
482	6830470	1371	2272	3173	4073	6834973	5873	6773	7673	8572	900
483	6839471	0370	1269	2168	3066	6843965	4863	5761	6659	7556	898
484	6848454	9351	0248	1145	2041	6852938	3834	4730	5626	6522	896
485	6857417	8313	9208	0103	0998	6861892	2787	3681	4575	5469	895
486	6866363	7256	8150	9043	9936	6870828	1721	2613	3506	4398	893
487	6875290	6181	7073	7964	8855	6879746	0637	1528	2418	3308	891
488	6884198	5088	5978	6867	7757	6888646	9535	0423	1312	2200	889
489	6893089	3977	4864	5752	6640	6897527	8414	9301	0188	1074	887
490	6901961	2847	3733	4619	5505	6906390	7275	8161	9046	9930	885
491	6910815	1699	2584	3468	4352	6915235	6119	7002	7885	8768	884
492	6919651	0534	1416	2298	3180	6924062	4944	5826	6707	7588	882
493	6928469	9350	0231	1111	1991	6932872	3752	4631	5511	6390	880
494	6937269	8149	9027	9906	0785	6941663	2541	3419	4297	5175	878
495	6946052	6929	7806	8683	9560	6950437	1312	2189	3065	3941	877
496	6954817	5692	6568	7443	8318	6959193	0067	0942	1816	2690	875
497	6963564	4438	5311	6185	7058	6967931	8804	9676	0549	1421	873
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499	6981005	1876	2746	3616	4485	6985355	6224	7093	7963	8831	870
500	6989700	0569	1437	2305	3173	6994041	4908	5776	6643	7510	868
501	6998377	9244	0111	0977	1843	7002709	3575	4441	5307	6172	866
502	7007037	7902	8767	9632	0496	7011361	2225	3089	3953	4816	864
503	7015680	6543	7406	8269	9132	7019995	0857	1720	2582	3444	863
504	7024305	5167	6028	6890	7751	7028612	9472	0333	1193	2054	861
505	7032914	3774	4633	5493	6352	7037212	8071	8930	9788	0647	860
506	7041505	2363	3221	4079	4937	7045794	6652	7509	8366	9223	858
507	7050080	0936	1792	2649	3505	7054360	5216	6072	6927	7782	856
508	7058637	9492	0347	1201	2055	7062910	3764	4617	5471	6325	854
509	7067178	8031	8884	9737	0589	7071442	2294	3146	3998	4850	852
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511	7084209	5059	5908	6758	7607	7088456	9305	0154	1003	1851	849
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513	7101174	2020	2866	3714	4559	7105404	6250	7096	7941	8788	846
514	7109631	0476	1321	2165	3010	7113854	4698	5542	6385	7229	844
515	7118072	8915	9759	0601	1444	7122287	3129	3971	4813	5655	843
516	7126497	7339	8180	9021	9862	7130703	1544	2385	3225	4065	841
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521	7168377	9211	0044	0877	1710	7172543	3376	4208	5041	5873	833
522	7176705	7537	8369	9200	0032	7180863	1694	2525	3356	4186	831
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527	7218106	8930	9754	0578	1401	7222225	3048	3871	4694	5517	823
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533	7267272	8087	8901	9716	0530	7271344	2158	2972	3786	4599	814
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537	7299743	0552	1360	2168	2977	7303785	4593	5400	6208	7015	808
538	7307823	8630	9437	0244	1051	7311857	2663	3470	4276	5082	806
539	7315888	6693	7499	8304	9109	7319914	0719	1524	2329	3133	805
540	7323938	4742	5546	6350	7153	7327957	8760	9564	0367	1170	804
541	7331973	2775	3578	4380	5183	7335985	6787	7588	8390	9192	802
542	7339993	0794	1595	2396	3197	7343997	4798	5598	6398	7198	801
543	7347998	8798	9598	0397	1196	7351995	2794	3593	4392	5191	799
544	7355989	6787	7585	8383	9181	7359979	0776	1574	2371	3168	798
545	7363965	4762	5558	6355	7151	7367948	8744	9540	0335	1131	796
546	7371926	2722	3517	4312	5107	7375902	6696	7491	8285	9079	795
547	7379873	0667	1461	2254	3048	7383841	4634	5427	6220	7013	793
548	7387806	8598	9390	0182	0974	7391766	2558	3350	4141	4932	792
549	7395723	6514	7305	8096	8887	7399677	0467	1257	2047	2837	791
550	7403627	4416	5206	5995	6784	7407573	8362	9151	9939	0728	789
551	7411516	2304	3092	3880	4668	7415455	6243	7030	7817	8604	788
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553	7427251	8037	8822	9607	0392	7431176	1961	2745	3530	4314	785
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555	7442930	3712	4495	5277	6059	7446841	7622	8404	9185	9967	782
556	7450748	1529	2310	3091	3871	7454652	5432	6212	6992	7772	781
557	7458552	9332	0111	0890	1670	7462449	3228	4006	4785	5564	779
558	7466342	7120	7898	8676	9454	7470232	1009	1787	2564	3341	778
559	7474118	4895	5672	6448	7225	7478001	8777	9553	0329	1105	776
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561	7489629	0403	1177	1950	2724	7493498	4271	5044	5817	6590	774
562	7497363	8136	8908	9681	0453	7501225	1997	2769	3541	4312	772
563	7505084	5855	6626	7398	8168	7508939	9710	0480	1251	2021	771
564	7512791	3561	4331	5101	5870	7516639	7409	8178	8947	9716	770
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576	7604225	4979	5733	6486	7240	7607993	8746	9500	0253	1005	753
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590	7708520	9256	9992	0728	1463	7712199	2934	3670	4405	5140	736
591	7715875	6610	7344	8079	8813	7719547	0282	1016	1750	2483	735
592	7723217	3951	4684	5417	6150	7726883	7616	8349	9082	9814	733
593	7730547	1279	2011	2743	3475	7734207	4939	5670	6402	7133	732
594	7737864	8595	9326	0057	0788	7741519	2249	2979	3710	4440	731
595	7745170	5899	6629	7359	8088	7748818	9547	0276	1005	1734	729
596	7752463	3191	3920	4648	5376	7756104	6832	7560	8288	9016	728
597	7759743	0471	1198	1925	2652	7763379	4106	4833	5559	6285	727
598	7767012	7738	8464	9190	9916	7770641	1367	2093	2818	3543	726
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600	7781512	2236	2960	3683	4407	7785130	5853	6576	7299	8022	723
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606	7824726	5443	6159	6876	7592	7828308	9024	9740	0456	1171	716
607	7831887	2602	3318	4033	4748	7835463	6178	6892	7607	8321	715
608	7839036	9750	0464	1178	1892	7842606	3319	4033	4746	5460	714
609	7846173	6886	7599	8312	9024	7849737	0450	1162	1874	2586	713
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613	7874605	5313	6021	6730	7438	7878146	8853	9561	0269	0976	708
614	7881684	2391	3098	3805	4512	7885219	5926	6632	7339	8045	707
615	7888751	9457	0163	0869	1575	7892281	2986	3691	4397	5102	706
616	7895807	6512	7217	7922	8626	7899331	0035	0739	1444	2148	705
617	7902853	3555	4259	4963	5666	7906370	7073	7776	8479	9182	703
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623	7944880	5577	6274	6971	7668	7948365	9061	9757	0454	1150	696
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626	7965743	6437	7131	7824	8517	7969211	9904	0597	1290	1983	693
627	7972675	3368	4060	1753	5445	7976137	6829	7521	8213	8905	692
628	7979596	0288	0970	1671	2362	7983053	3744	4435	5125	5816	691
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630	7993405	4095	4784	5473	6162	7996851	7540	8228	8917	9605	689
631	8000294	0982	1670	2358	3046	8003731	4421	5109	5796	6484	688
632	8007171	7858	8545	9232	9919	8010605	1292	1978	2665	3351	687
633	8014037	4723	5409	6095	6781	8017466	8152	8837	9522	0207	686
634	8020893	1577	2262	2947	3632	8024316	5001	5685	6369	7053	684
635	8027737	8421	9105	9789	0472	8031155	1839	2522	3205	3888	683
636	8034571	5254	5937	6619	7302	8037984	8666	9348	0030	0712	682
637	8041394	2076	2758	3439	4121	8044802	5483	6164	6845	7526	681
638	8048207	8887	9568	0248	0929	8051609	2289	2969	3649	4329	680
639	8055009	5688	6368	7047	7726	8058405	9085	9763	0442	1121	679
640	8061800	2478	3157	3835	4513	8065191	5869	6547	7225	7903	678
641	8068580	9258	9935	0612	1290	8071967	2644	3320	3997	4674	677
642	8075350	6027	6703	7379	8055	8078731	9407	0083	0759	1434	676
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645	8095597	6270	6944	7617	8290	8098962	9635	0308	0980	1653	673
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651	8135810	6477	7144	7811	8477	8139144	9811	0477	1144	1810	667
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654	8155777	6441	7105	7769	8433	8159096	9760	0423	1087	1750	664
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656	8169038	9700	0362	1024	1686	8172347	3009	3670	4331	4993	662
657	8175654	6315	6975	7636	8297	8178958	9618	0278	0939	1599	660
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659	8188854	9513	0172	0831	1489	8192148	2806	3465	4123	4781	658
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661	8202015	2672	3328	3985	4642	8205298	5955	6611	7268	7924	657
662	8208580	9236	9892	0548	1203	8211859	2514	3170	3825	4480	656
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666	8234742	5394	6046	6698	7350	8238002	8653	9305	9956	0607	652
667	8241258	1909	2560	3211	3862	8244513	5163	5814	6464	7114	651
668	8247765	8415	9065	9715	0364	8251014	1664	2313	2963	3612	650
669	8254261	4910	5559	6208	6857	8257506	8154	8803	9451	0100	649
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671	8267225	7872	8519	9166	9813	8270460	1107	1753	2400	3046	647
672	8273693	4339	4985	5631	6277	8276923	7569	8214	8860	9505	646
673	8280151	0796	1441	2086	2731	8283376	4021	4665	5310	5955	645
674	8286599	7243	7887	8532	9176	8289820	0463	1107	1751	2394	644
675	8293038	3681	4324	4967	5611	8296254	6896	7539	8182	8824	643
676	8299467	0109	0752	1394	2036	8302678	3320	3962	4604	5245	642
677	8305887	6528	7169	7811	8452	8309093	9734	0375	1016	1656	641
678	8312297	2937	3578	4218	4858	8315499	6139	6778	7418	8058	640
679	8318698	9337	9977	0616	1255	8321895	2534	3173	3812	4450	639
680	8325089	5728	6366	7005	7643	8328281	8919	9558	0195	0833	638
681	8331471	2109	2746	3384	4021	8334659	5296	5933	6570	7207	637
682	8337844	8480	9117	9754	0390	8341027	1663	2299	2935	3571	636
683	8344207	4843	5479	0114	6750	8347385	8021	8656	9291	9926	635
684	8350561	1196	1831	2465	3100	8353735	4369	5003	5638	6272	635
685	8356906	7540	8174	8807	9441	8360075	0708	1341	1975	2608	634
686	8363241	3874	4507	5140	5773	8366405	7038	7670	8303	8935	633

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689	8382192	2822	3453	4083	4713	8385343	5973	6602	7232	7861	630
690	8388491	9120	9750	0379	1008	8391637	2266	2895	3523	4152	629
691	8394780	5409	6037	6666	7294	8397922	8550	9178	9806	0433	628
692	8401061	1688	2316	2943	3571	8404198	4825	5452	6079	6706	627
693	8407332	7959	8586	9212	9838	8410465	1091	1717	2343	2969	626
694	8413595	4220	4846	5472	6097	8416723	7348	7973	8598	9223	625
695	8419848	0473	1098	1722	2347	8422971	3596	4220	4844	5468	624
696	8426092	6716	7340	7964	8588	8429211	9835	0458	1081	1705	624
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698	8438554	9176	9798	0420	1042	8441664	2286	2907	3529	4150	622
699	8444772	5393	6014	6635	7256	8447877	8498	9119	9739	0360	621
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775	8893017	3577	4138	4694	5258	8895818	6378	6938	7498	8058	560
776	8898617	9177	9736	0296	0855	8901415	1974	2533	3092	3651	559
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779	8915375	5932	6489	7057	7604	8918161	8718	9275	9832	0389	557
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881	9449759	0252	0745	1238	1730	9452223	2716	3208	3701	4193	493
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908	9580858	1337	1815	2293	2771	9583250	3727	4205	4683	5161	478
909	9585639	6117	6594	7072	7549	9588027	8505	8982	9459	9937	478
910	9590414	0891	1368	1845	2322	9592800	3276	3753	4230	4707	477
911	9595184	5660	6137	6614	7090	9597567	8043	8520	8996	9472	477
912	9599948	0425	0901	1377	1853	9602329	2803	3281	3756	4232	476
913	9604708	5183	5659	6135	6610	9607086	7561	8036	8512	8987	476
914	9609462	9937	0412	0887	1362	9611837	2312	2787	3262	3736	475

No.	0	1	2	3	4	5	6	7	8	9	Dif.
915	9614211	4686	5160	5635	6109	9616583	7058	7532	8006	8481	474
916	9618955	9429	9903	0377	0851	9621325	1799	2272	2746	3220	474
917	9623693	4167	4640	5114	5587	9626061	6534	7007	7481	7954	473
918	9628427	8900	9373	9846	0319	9630792	1264	1737	2210	2683	473
919	9633155	3628	4100	4573	5045	9635517	5990	6462	6934	7406	472
920	9637878	8350	8822	9294	9766	9640238	0710	1181	1653	2125	472
921	9642596	3068	3539	4011	4482	9644953	5425	5896	6367	6838	471
922	9647309	7780	8251	8722	9193	9649664	0135	0605	1076	1546	471
923	9652017	2488	2958	3428	3899	9654369	4839	5309	5780	6250	470
924	9656720	7190	7660	8130	8599	9659069	9539	0009	0478	0948	470
925	9661417	1887	2356	2826	3295	9663764	4233	4703	5172	5641	469
926	9666110	6579	7048	7517	7985	9668454	8923	9392	9860	0329	469
927	9670797	1266	1734	2203	2671	9673139	3607	4076	4544	5012	468
928	9675480	5948	6416	6884	7351	9677819	8287	8754	9222	9690	468
929	9680157	0625	1092	1559	2027	9682494	2961	3428	3895	4362	467
930	9684829	5296	5763	6230	6697	9687164	7630	8097	8564	9030	467
931	9689497	9963	0430	0896	1362	9691829	2295	2761	3227	3693	466
932	9694159	4625	5091	5557	6023	9696488	6954	7420	7885	8351	466
933	9698816	9282	9747	0213	0678	9701143	1608	2074	2539	3004	465
934	9703469	3934	4399	4863	5328	9705793	6258	6722	7187	7652	465
935	9708116	8581	9045	9509	9974	9710438	0902	1366	1830	2294	464
936	9712758	3222	3686	4150	4614	9715078	5542	6005	6469	6932	464
937	9717396	7859	8323	8786	9249	9719713	0176	0639	1102	1565	463
938	9722028	2491	2954	3417	3880	9724343	4805	5268	5731	6193	463
939	9726656	7118	7581	8043	8506	9728968	9430	9892	0354	0816	462
940	9731279	1741	2202	2664	3120	9733588	4050	4511	4973	5435	462
941	9735896	6358	6819	7281	7742	9738203	8664	9126	9587	0048	461
942	9740509	0970	1431	1892	2353	9742814	3274	3735	4196	4656	461
943	9745117	5577	6038	6498	6959	9747419	7879	8340	8800	9260	460
944	9749720	0180	0640	1100	1560	9752020	2479	2939	3399	3858	460
945	9754318	4778	5237	5697	6156	9756615	7075	7534	7993	8452	459
946	9758911	9370	9829	0288	0747	9761206	1665	2124	2582	3041	459
947	9763500	3958	4417	4875	5334	9765792	6251	6709	7167	7625	458
948	9768083	8541	9000	9458	9915	9770373	0831	1289	1747	2204	458
949	9772662	3120	3577	4035	4492	9774950	5407	5864	6322	6779	457
950	9777236	7693	8150	8607	9064	9779521	9978	0435	0892	1348	457
951	9781805	2262	2718	3175	3631	9784088	4544	5001	5457	5913	457
952	9786369	6826	7282	7738	8194	9788650	9106	9562	0017	0473	456
953	9790929	1385	1840	2296	2751	9793205	3662	4118	4573	5028	456
954	9795484	5939	6394	6849	7304	9797759	8214	8669	9124	9579	455
955	9800034	0488	0943	1398	1852	9802307	2761	3216	3670	4125	455
956	9804579	5033	5487	5942	6396	9806850	7304	7758	8212	8666	454
957	9809119	9573	0027	0481	0934	9811388	1841	2295	2748	3202	454
958	9813655	4108	4562	5015	5468	9815921	6374	6827	7280	7733	453
959	9818186	8639	9092	9544	9997	9820450	0902	1355	1807	2260	453
960	9822712	3165	3617	4069	4522	9824974	5426	5878	6330	6782	452
961	9827234	7686	8138	8589	9041	9839493	9945	0396	0848	1299	452
962	9831751	2202	2654	3105	3556	9834007	4459	4910	5361	5812	451
963	9836263	6714	7165	7616	8066	9838517	8968	9419	9869	0320	451
964	9840770	1221	1671	2122	2572	9843022	3473	3923	4373	4823	450
965	9845273	5723	6173	6623	7073	9847523	7973	8422	8872	9322	450
966	9849771	0221	0670	1120	1569	9852019	2468	2917	3366	3816	449
967	9854265	4714	5163	5612	6061	9856510	6959	7407	7856	8305	449
968	9858754	9202	9651	0099	0548	9860996	1445	1893	2341	2790	449
969	9863238	3686	4134	4582	5030	9865478	5926	6374	6822	7270	448
970	9867717	8165	8613	9060	9508	9869955	0403	0850	1298	1745	448
971	9872192	2640	3087	3534	3981	9874428	4875	5322	5769	6216	447
972	9876663	7109	7556	8003	8450	9878896	9343	9789	0236	0682	447
973	9881128	1575	2021	2467	2913	9883360	3806	4252	4698	5144	446
974	9885590	6035	6481	6927	7373	9887818	8264	8710	9155	9601	446
975	9890046	0492	0937	1382	1828	9892273	2718	3163	3608	4053	445
976	9894498	4943	5388	5833	6278	9896722	7167	7612	8057	8501	445
977	9898946	9390	9835	0279	0723	9901168	1612	2056	2500	2944	444
978	9903389	3833	4277	4721	5164	9905608	6052	6496	6940	7383	444
979	9907827	8271	8714	9158	9601	9910044	0488	0931	1374	1818	443
980	9912261	2704	3147	3590	4033	9914476	4919	5362	5805	6247	443
981	9916690	7133	7575	8018	8461	9918903	9345	9788	0230	0673	442
982	9921115	1557	1999	2441	2884	9923326	3768	4210	4651	5093	442
983	9925535	5977	6419	6860	7302	9927744	8185	8627	9068	9510	442
984	9929951	0392	0834	1275	1716	9932157	2598	3039	3480	3921	441
985	9934362	4803	5244	5685	6126	9936566	7007	7447	7888	8329	441
986	9938769	9209	9650	0090	0531	9940971	1411	1851	2291	2731	440
987	9943171	3611	4051	4491	4931	9945371	5811	6250	6690	7130	440
988	9947569	8009	8448	8888	9327	9949767	0206	0645	1085	1524	440
989	9951963	2402	2841	3280	3719	9954158	4597	5036	5474	5913	439
990	9956352	6791	7229	7668	8106	9958545	8983	9422	9860	0298	439

No.	0	1	2	3	4	5	6	7	8	9	Dif.
991	9960736	1175	1613	2051	2489	9962927	3365	3803	4241	4679	438
992	9965117	5554	5992	6430	6867	9967305	7743	8180	8618	9055	438
993	9969492	9930	0367	0804	1241	9971679	2116	2553	2990	3427	437
994	9973864	4301	4737	5174	5611	9976048	6484	6921	7358	7794	437
995	9978231	8667	9104	9540	9976	9980413	0849	1285	1721	2157	436
996	9982593	3029	3465	3901	4337	9984773	5209	5645	6080	6516	436
997	9986951	7387	7823	8258	8694	9989129	9564	0000	0435	0870	435
998	9991305	1740	2176	2611	3046	9993481	3916	4350	4785	5220	435
999	9995655	6089	6524	6959	7393	9997828	8262	8697	9131	9566	434

MISCELLANEOUS TABLES.

The following tables will be found useful in many calculations, especially those of the squares, cubes, and roots, which will be repeatedly needed in the sections on Mechanical Philosophy and Applied Mechanics. The table giving Napierian Logarithms will be found of interest in connection with the observations at page 517, *et seq.*

NAPIERIAN LOGARITHMS

OF NUMBERS, FROM 1 TO 100.

No.	Logarithm.	No.	Logarithm.
2	0·69314718	52	3·95124372
3	1·09861229	53	3·97029191
4	1·38629436	54	3·98898405
5	1·60943791	55	4·00733319
6	1·79175947	56	4·02535169
7	1·94591015	57	4·04305127
8	2·07944154	58	4·06044301
9	2·19722458	59	4·07753744
10	2·30258509	60	4·09434456
11	2·39789527	61	4·11087386
12	2·48490665	62	4·12713439
13	2·56494936	63	4·14313473
14	2·63905733	64	4·15888308
15	2·70805020	65	4·17438727
16	2·77258872	66	4·18965474
17	2·83321334	67	4·20469262
18	2·89037176	68	4·21950771
19	2·94443898	69	4·23410650
20	2·99573227	70	4·24849524
21	3·04452244	71	4·26267988
22	3·09104245	72	4·27666612
23	3·13549422	73	4·29045944
24	3·17805383	74	4·30406509
25	3·21887582	75	4·31748811
26	3·25809654	76	4·33073334
27	3·29583687	77	4·34380542
28	3·33220451	78	4·35670883
29	3·36729583	79	4·36944785
30	3·40119738	80	4·38202663
31	3·43398720	81	4·39444915
32	3·46573590	82	4·40671925
33	3·49650756	83	4·41884061
34	3·52636052	84	4·43081680
35	3·55534806	85	4·44265126
36	3·58351894	86	4·45434730
37	3·61091791	87	4·46590812
38	3·63758616	88	4·47733681
39	3·66356165	89	4·48863637
40	3·68887945	90	4·49980967
41	3·71357207	91	4·51085951
42	3·73766962	92	4·52178858
43	3·76120012	93	4·53259949
44	3·78418963	94	4·54329478
45	3·80666249	95	4·55387689
46	3·82864140	96	4·56434819
47	3·85014760	97	4·57471098
48	3·87120101	98	4·58496748
49	3·89182030	99	4·59511985
50	3·91202301	100	4·60517019
51	3·93182563		

SQUARES AND CUBES.

No.	Square.	Cube.	No.	Square.	Cube.
2	4	8	52	2704	140608
3	9	27	53	2809	148877
4	16	64	54	2916	157464
5	25	125	55	3025	166375
6	36	216	56	3136	175616
7	49	343	57	3249	185193
8	64	512	58	3364	195112
9	81	729	59	3481	205379
10	100	1000	60	3600	216000
11	121	1331	61	3721	226981
12	144	1728	62	3844	238328
13	169	2197	63	3969	250047
14	196	2744	64	4096	262144
15	225	3375	65	4225	274625
16	256	4096	66	4356	287496
17	289	4913	67	4489	300763
18	324	5832	68	4624	314432
19	361	6859	69	4761	328509
20	400	8000	70	4900	343000
21	441	9261	71	5041	357911
22	484	10648	72	5184	373248
23	529	12167	73	5329	389017
24	576	13824	74	5476	405224
25	625	15625	75	5625	421875
26	676	17576	76	5776	438976
27	729	19683	77	5929	456533
28	784	21952	78	6084	474552
29	841	24389	79	6241	493039
30	900	27000	80	6400	512000
31	961	29791	81	6561	531441
32	1024	32768	82	6724	551368
33	1089	35937	83	6889	571787
34	1156	39304	84	7056	592704
35	1225	42875	85	7225	614125
36	1296	46656	86	7396	636056
37	1369	50653	87	7569	658503
38	1444	54872	88	7744	681472
39	1521	59319	89	7921	704969
40	1600	64000	90	8100	729000
41	1681	68921	91	8281	753571
42	1764	74088	92	8464	778688
43	1849	79507	93	8649	804357
44	1936	85184	94	8836	830584
45	2025	91125	95	9025	857375
46	2116	97336	96	9216	884736
47	2209	103823	97	9409	912673
48	2304	110592	98	9604	941192
49	2401	117649	99	9801	970299
50	2500	125000	100	10000	1000000
51	2601	132651			

SQUARE ROOTS.

No.	Square root.	No.	Square root.	No.	Square root.
2	1·4142136	35	5·9160798	68	8·2462113
3	1·7320508	36	6·0000000	69	8·3066239
4	2·0000000	37	6·0827625	70	8·3666003
5	2·2360680	38	6·1644140	71	8·4261498
6	2·4494897	39	6·2449980	72	8·4852814
7	2·6457513	40	6·3245553	73	8·5440037
8	2·8284271	41	6·4031242	74	8·6023253
9	3·0000000	42	6·4807407	75	8·6602540
10	3·1622777	43	6·5574385	76	8·7177979
11	3·3166248	44	6·6332496	77	8·7749644
12	3·4641016	45	6·7082039	78	8·8317609
13	3·6055513	46	6·7823300	79	8·8881944
14	3·7416574	47	6·8556546	80	8·9442719
15	3·8729833	48	6·9282032	81	9·0000000
16	4·0000000	49	7·0000000	82	9·0553851
17	4·1231056	50	7·0710678	83	9·1104336
18	4·2426407	51	7·1414284	84	9·1651514
19	4·3588989	52	7·2111026	85	9·2195445
20	4·4721360	53	7·2801099	86	9·2736185
21	4·5825757	54	7·3484692	87	9·3273791
22	4·6904158	55	7·4161985	88	9·3808315
23	4·7958315	56	7·4833148	89	9·4339811
24	4·8989795	57	7·5498344	90	9·4868330
25	5·0000000	58	7·6157731	91	9·5393920
26	5·0990195	59	7·6811457	92	9·5916630
27	5·1961524	60	7·7459667	93	9·6436508
28	5·2915026	61	7·8102497	94	9·6953597
29	5·3851648	62	7·8740079	95	9·7467943
30	5·4772256	63	7·9372539	96	9·7979590
31	5·5677644	64	8·0000000	97	9·8488578
32	5·6568542	65	8·0622577	98	9·8994949
33	5·7445626	66	8·1240384	99	9·9498744
34	5·8309519	67	8·1853528	100	10·000000

CUBE ROOTS.

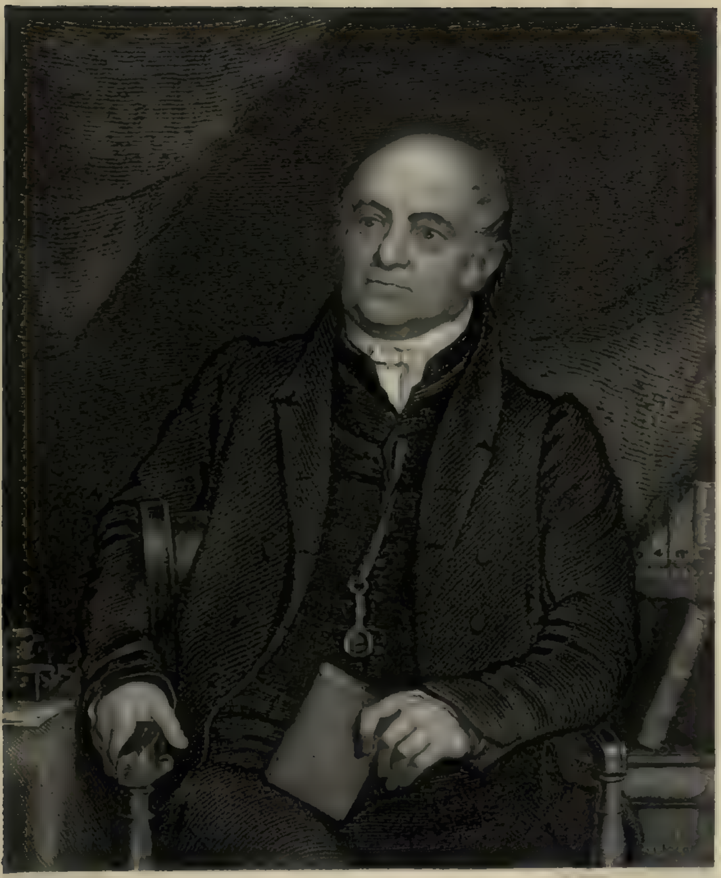
No.	Cube root.	No.	Cube root.	No.	Cube root.
2	1·2599210	35	3·2710663	68	4·0816551
3	1·4422496	36	3·3019272	69	4·1015661
4	1·5874011	37	3·3322218	70	4·1212853
5	1·7099759	38	3·3619754	71	4·1408178
6	1·8171206	39	3·3912114	72	4·1601676
7	1·9129312	40	3·4199519	73	4·1793390
8	2·0000000	41	3·4482172	74	4·1983364
9	2·0800837	42	3·4760266	75	4·2171633
10	2·1544347	43	3·5033981	76	4·2358236
11	2·2239801	44	3·5303483	77	4·2543210
12	2·2894286	45	3·5568933	78	4·2726586
13	2·3513347	46	3·5830479	79	4·2908404
14	2·4101422	47	3·6088261	80	4·3088695
15	2·4662121	48	3·6342411	81	4·3267487
16	2·5198421	49	3·6593057	82	4·3444815
17	2·5712816	50	3·6840314	83	4·3620707
18	2·6207414	51	3·7084298	84	4·3795191
19	2·6684016	52	3·7325111	85	4·3968296
20	2·7144177	53	3·7562858	86	4·4140049
21	2·7589243	54	3·7797631	87	4·4310476
22	2·8020393	55	3·8029525	88	4·4479602
23	2·8438670	56	3·8258624	89	4·4647451
24	2·8844991	57	3·8485011	90	4·4814047
25	2·9240177	58	3·8708766	91	4·4979414
26	2·9624960	59	3·8929965	92	4·5143574
27	3·0000000	60	3·9148676	93	4·5306549
28	3·0365889	61	3·9364972	94	4·5468359
29	3·0723168	62	3·9578915	95	4·5629026
30	3·1072325	63	3·9790571	96	4·5788570
31	3·1413806	64	4·0000000	97	4·5947009
32	3·1748021	65	4·0207256	98	4·6104363
33	3·2075343	66	4·0412401	99	4·6260650
34	3·2396118	67	4·0615480	100	4·6415888

LENGTHS OF CIRCULAR ARCS.

1°	·0174533	60°	1·0471976	1′	2909	1″	48
2	·0349066	70	1·2217305	2	5818	2	97
3	·0523599	80	1·3962634	3	8727	3	145
4	·0698132	90	1·5707963	4	11636	4	194
5	·0872665	100	1·7453293	5	14544	5	242
6	·1047198	110	1·9198622	6	17453	6	291
7	·1221730	120	2·0943951	7	20362	7	339
8	·1396263	130	2·2689280	8	23271	8	388
9	·1570796	140	2·4434610	9	26180	9	436
10	·1745329	150	2·6179939	10	29089	10	485
20	·3490659	160	2·7925268	20	58178	20	970
30	·5235988	170	2·9670597	30	87266	30	1454
40	·6981317	180	3·1415927	40	116355	40	1939
50	·8726646	360	6·2831853	50	145444	50	2424

LENGTHS OF CHORDS AND SAGITTAS.

D.	Chord.	Sagitta.	D.	Chord.	Sagitta.
5°	·087239	·000952	95°	1·474555	·324410
10	·174311	·003805	100	1·532089	·357212
15	·261052	·008555	105	1·586707	·391239
20	·347296	·015192	110	1·638304	·426424
25	·432879	·023704	115	1·686783	·462700
30	·517638	·034074	120	1·732051	·500000
35	·601412	·046283	125	1·774022	·538251
40	·684040	·060307	130	1·812616	·577382
45	·765367	·076120	135	1·847759	·617317
50	·845237	·093692	140	1·879385	·657980
55	·923497	·112989	145	1·907434	·699294
60	1·000000	·133975	150	1·931852	·741181
65	1·074599	·156609	155	1·952592	·783560
70	1·147153	·180848	160	1·969616	·826352
75	1·217523	·206647	165	1·982890	·869474
80	1·285575	·233956	170	1·992389	·912844
85	1·351180	·262723	175	1·998096	·956381
90	1·414214	·292893	180	2·000000	1·000000



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Olinthus Gregory.

MATHEMATICS—GEOMETRY.

CHAPTER I.

PLANE GEOMETRY.—THE ELEMENTS OF EUCLID.

INTRODUCTION.

In order to acquire clear conceptions of a *point*, a *line*, and a *surface*, with the definitions of which Euclid sets out, it will be best for the learner to consider them, at first, in connection with a *solid;* that is, with something that has *length*, *breadth*, and *thickness.* All external objects—things that can be seen and felt—are *solids;* and however small the solid may be, it must have some length, some breadth, and some thickness: using these terms in their ordinary acceptation—that is, in the sense in which they are used in common discourse.

The *boundaries* of a solid are called the *surface*, or the *superficies*, of the solid; and, as a boundary, cannot have *thickness;* because, if it had, it would be a *part* of the solid, and not a side, face, or boundary of it: it follows that a surface can have *length* and *breadth* only.

Again—the surface itself has its boundaries, or limits: *these* boundaries are called *lines:* and as a *boundary* of a surface cannot have either breadth or thickness, since it is no *part* of the surface, much less a part of the solid, it follows that a line has *length* only.

Lastly, a line has its limits—a beginning and a termination: we speak of these, in common language, as the *ends* of the line. Euclid calls them *points;* and it is plain that a point, being no *part* of the line, cannot have length; and as breadth and thickness are excluded even from the line itself, it follows that a point has no dimensions or *magnitude.* It merely indicates *position;* the position, namely, of the commencement or termination of a line.

Euclid frequently speaks of taking a point *in* a line, without meaning an *extremity* of the line; but we may conceive a line to be crossed, or cut by other lines, and thus to be divided into shorter portions. Each portion has its extremities; so that we may conceive as many points in the line as we please. We cannot represent *length only* to the eye; it is necessary, therefore, in contemplating the black marks on paper, by which lines are represented, entirely to disregard the *breadth* of them; and to fix the attention upon the *length alone:* the *eye* may see breadth and thickness; the *mind* takes note of length only. It is usual to commence the study of Geometry with definitions, which are as follows.

ELEMENTS OF EUCLID.—BOOK I.

DEFINITIONS.

I.

A *point* has no magnitude: it has *position* only.

II.

A *line* has *length* only.

III.

A *straight line* is that which lies *evenly* between its extreme points.

It is distinguished by uniformity of direction between its extremities.

IV.

A *surface*, or *superficies*, is that which has length and breadth only.

V.

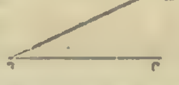
Therefore the boundaries of a surface are *lines.*

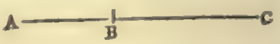
VI.

A *plane surface*, or simply a *plane*, is that in which, whatever two points be taken, the straight line, having these points for extremities, lies *wholly* in that surface.

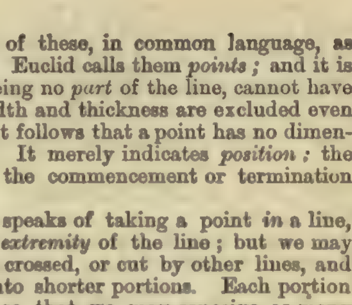
VII.

A *plane rectilineal angle* is the *opening* between two straight lines, which meet together, but which do not unite so as to form one continued straight line.





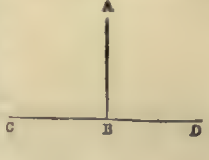
Thus two straight lines A B, C B, meet each other in the point B, forming an *opening*, of which B A and B C are the boundaries, or limits: this opening is called a *plane rectilineal angle*, or simply an *angle.* The two straight lines A B, C B, in the second representation, also meet each other; but they form no *opening or angle;* they unite in forming a continued straight line, A C.



An angle is sometimes referred to by simply naming the letter, placed at the point in which the lines forming the angle meet: in this way reference is made to the angle B, or the angle F, meaning the opening between B A, B C, or that between F D, F E. But if there be two or more openings, or angles, at the same point, this mode of reference will not do: the *sides*, or *boundaries*, of the particular one of those angles meant, must also be pointed out. Thus, if we wish to refer to the angle whose sides are B G, B H, attention must, in some way, be distinctly directed to these sides; because, as there are several angles at B, a reference to this point *alone* would be insufficient. The plan adopted is this: not only the letter (B) at the *vertex* of the angle, as this point is called, is used, but the other two letters (G, H) which mark the *sides* of the angle—the former letter being always placed *between* these two. Thus GBH, or HBG, means the angle whose *vertex* is B, and whose sides are B G, B H. In like manner A B H, or H B A, refers to the angle whose vertex is B, and whose sides are B A, B H: but there is no necessity to [illegible] three letters to denote an angle when there are no other angles at the same point or vertex.

VIII.

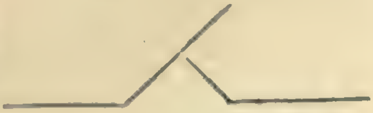
When a straight line (A B) standing on another straight line (C D) makes the adjacent angles (A B C, A B D) equal to one another, each of the angles is called a *right angle;* and the straight lines are said to be *perpendicular* to each other.



Thus the two lines, A B, C D, are perpendicular to each other.

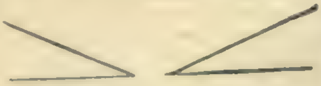
IX.

An *obtuse* angle is that which is greater than a right angle.



X.

An *acute* angle is that which is less than a right angle.

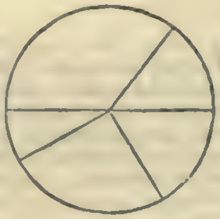


XI.

A *figure* is that which is enclosed by one or more boundaries.

XII.

A *circle* is a plane figure enclosed by one line, which is called the *circumference* of the circle: it is such, that all straight lines, drawn from a certain point within the figure to the circumference, are equal to one another.



XIII.

This point is called the *centre* of the circle.

And any line drawn from the centre to the circumference is called a *radius* of the circle.

XIV.

A *diameter* of a circle is a straight line drawn through the centre, and terminated both ways by the circumference.

A radius is therefore half the diameter, or a *semidiameter*.

XV.

A *semicircle* is the figure contained by a diameter and the part of the circumference cut off by the diameter.

XVI.

Rectilineal figures are those which are enclosed by straight lines only.

XVII.

Trilateral figures, or *triangles*, are enclosed by three straight lines.

XVIII.

Quadrilateral figures, by four straight lines.

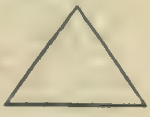
XIX.

Multilateral figures, or *polygons*, by more than four straight lines.

The term polygon, however, is often employed as a general name for rectilineal figures of all kinds, without regard to the number of the sides; so that the rectilineal figures defined above may, without impropriety, be called polygons of three and of four sides respectively.

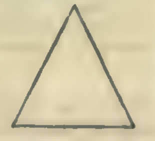
XX.

An *equilateral triangle* is that which has its three sides *equal*.



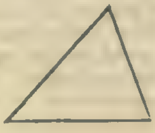
XXI.

An *isosceles triangle* has two of its sides equal.



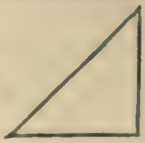
XXII.

A *scalene triangle* has its three sides unequal.



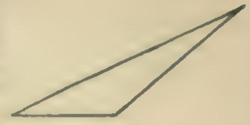
XXIII.

A *right-angled triangle* has one of its angles a right angle.



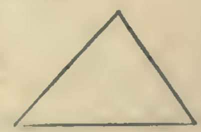
XXIV.

An *obtuse-angled triangle* has one of its angles *obtuse*, or greater than a right angle.



XXV.

An *acute-angled triangle* has each of its angles *acute*, or less than a right angle.



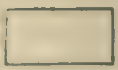
XXVI.

Of four-sided figures, a *square* is that which has all its sides equal, and all its angles right angles.

If the four sides are *equal*, and only *one* of the angles affirmed to be right, the other three angles must be right, of necessity, as will be hereafter proved; this definition is therefore *redundant*.

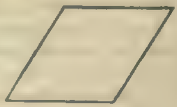
XXVII.

A *rectangle* is that which has all its four angles right angles, whether its sides be equal or not.



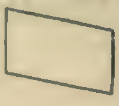
XXVIII.

A *rhombus* is that which has all its sides equal, but its angles are not right angles.



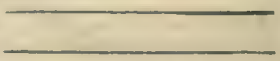
XXIX.

A *rhomboid* is that which has its opposite sides equal to one another, but all its sides are not equal, nor its angles right angles.



XXX.

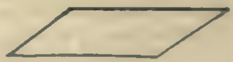
Parallel straight lines are such as are in the same plane, and which, being produced ever so far both ways, do not meet.



XXXI.

If each pair of opposite sides of a quadrilateral be parallel lines, the figure is called a *parallelogram*.

It will be seen hereafter that squares, rectangles, rhombuses, and rhomboids, are all *parallelograms*.

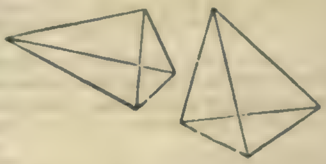


XXXII.

All other four-sided figures, besides those here defined, are called *trapeziums*.

XXXIII.

A line drawn across a figure, joining two opposite corners, or *vertices*, is called a *diagonal*.



POSTULATES.

I.

Let it be granted that a straight line may be drawn from any one point to any other point.

II.

That a terminated straight line may be prolonged to any length in a straight line.

III.

And that circle may be described from any centre, at any distance from that centre.

AXIOMS.

I.

Things which are equal to the same thing are equal to one another.

II.

If equals be added to equals, the wholes will be equal.

III.

If equals be taken from equals, the remainders will be equal.

IV.

If equals be added to unequals, the wholes will be unequal.

V.

If equals be taken from unequals, the remainders will be unequal.

VI.

Things which are double of the same are equal to one another.

VII.

Things which are halves of the same are equal to one another.

VIII.

Magnitudes which coincide, or which may be conceived to coincide, with one another, are *equal*.

IX.

The whole is greater than its part.

X.

Two straight lines cannot enclose a space.

XI.

All right angles are equal to one another.

MARKS AND SIGNS USED FOR ABBREVIATION.

= is the sign of *equality*, and signifies that the quantities between which it is placed are *equal*.

+ is the sign of addition, and signifies that the quantities between which it is placed are to be *added*.

— is the sign of subtraction, and implies that the quantity *after* it, is to be *subtracted* from the quantity *before* it.

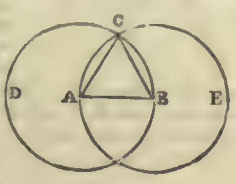
∴ stands for the word *therefore*, or *consequently*.

Besides these marks the following contractions are also frequently used; namely, *ax.* for *axiom*; *post.* for *postulate*; *prob.* for *problem*; *theo.* for *theorem*; *prop.* for *proposition*; *const.* for *construction*; and *hyp.* for *hypothesis*. Q. E. D. stands for "*Quod erat demonstrandum;*" that is, which was to be demonstrated.

PROPOSITION I.—PROBLEM.

To describe an equilateral triangle (A B C) *upon a given finite straight line* (A B).

With centre A, and radius A B, describe the circle * Post. 3. B C D;* and with centre B, and the same radius, B A, describe the circle A C E.



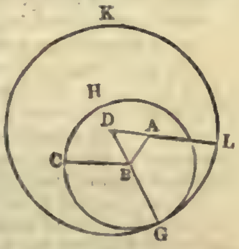
From C, the point in which the circles cut each other, draw * Post. 1. C A, C B,* *the triangle* A B C *shall be equilateral.*

† Def. 12. Because A is the * Ax. 1. centre of the circle B C D, A C = A B,† and because B is the centre of the circle A C E, B C = A B, ∴ A C = B C;* so that A C, B C, A B, are equal to one another; consequently *the triangle* A B C *is equilateral*, and it is described upon A B, *which was to be done.*

PROPOSITION II.—PROBLEM.

From a given point (A) *to draw a straight line* (A L) *equal to a given straight line* (B C).

From A to B draw AB,* and upon AB describe * Post. 1. * Pr. 1. the equilateral triangle B D A.* With centre B, and radius B C, describe the circle † Post. 3. C G H,† and produce DB to meet it in G. Again: with D as centre, and D G as radius, describe the circle G K L, and produce D A to * Post. 2. meet it in L;* A L *shall be equal to* B C.



Because B is the centre of the circle C G H, B C = † Def. 12. B G.†

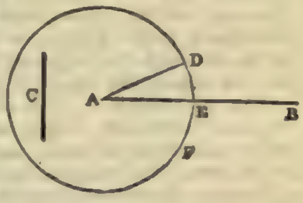
Because D is the centre of the circle G K L, D L = D G.

But D A, D B, parts of D L, D G, are also * Const. equal,* ∴ the remainders, A L, B G, are † Ax. 3. equal;† so that A L, B C are each equal to B G, and are, consequently, equal to each other.* * Ax. 1. Wherefore, *from the given point*, A, *a straight line*, A L, *has been drawn equal to* B C. *Which was to be done.*

PROPOSITION III.—PROBLEM.

From the greater (A B) *of two given straight lines* (A B and C), *to cut off a part equal to the less* (C).

* Pr. 2. From A draw A D = C,* and with centre A, and radius A D, describe the circle D E F,† † Post. 3. cutting off A E from A B: A E shall be = C.

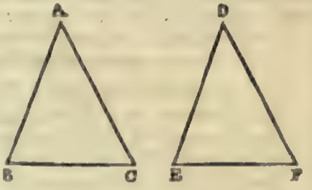


Because A is the centre of the circle DEF, A E = * Def. 12. A D.* But † Const. C = A D,† ∴ A E and C are each = * Ax. 1. A D, ∴ A E = C;* so that, *from the greater line*, A B, *a part* A E, *has been cut off, equal to the less*, C. *Which was to be done.*

PROPOSITION IV.—THEOREM.

If two triangles (A B C, D E F) *have two sides* (A B, A C) *of the one equal to two sides* (D E, D F) *of the other, each to each; that is,* A B = D E, *and* A C = D F, *and have likewise the angles contained by those sides equal, that is, the angle* A = *the angle* D, *then their bases or third sides* (B C, E F) *shall be equal, and also their other angles each to each, namely, those to which the equal sides are opposite, that is, the angle* B = *the angle* E, *and the angle* C = *the angle* F; *the surfaces of the triangles shall also be equal.*

For, conceive the triangle A B C to be laid upon, or applied to, the triangle D E F so that the point A may be on D, and the line A B on D E; then the point B shall fall on E, because A B = * Hyp. D E.* And A B thus coinciding with D E, A C must fall on D F, because the angle A = the angle † Hyp. D;† also the * Hyp. point C must fall on F, because A C = D F.* But B was proved to coincide with E; and since B coincides with E, and C with F, the base B C must coincide with the base E F, otherwise the two straight lines, B C, E F, thus made to coincide at their extremities, would † Ax. 10. inclose a space, which is impossible;† ∴ *the base* B C, *coinciding with the base* E F, *is equal to* * Ax. 8. E F.* Moreover, since the two lines, A B, B C coincide with the two D E, E F, the angle B coincides with the angle E; and since the two lines A C, C B coincide with the two D F, F E, the angle C coincides with the angle F; ∴ *the angle* B = *the angle* E, *and the*



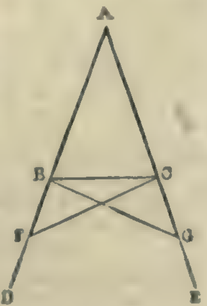
* Ax. 8. *angle* C = *the angle* F.* And as the triangle A B C thus coincides, in every respect, with the triangle D E F, *the surfaces of the triangles must be* † Ax. 8. *equal*; † ∴ *if two triangles, &c.* Q.E.D.

PROPOSITION V.—Theorem.

The angles (A B C, A C B) *at the base* (B C) *of an isosceles triangle* (A B C) *are equal; and if the equal sides* (A B, A C) *be prolonged, the angles* (D B C, E C B) *upon the other side of the base shall be equal.*

In B D take any point, F, and from A E, the greater, * Pr. 3. cut off A G = A F, the less;* and draw F C, G B.

† Const. Because F A = G A,† and A C = AB* ∴ F A, * Hyp. A C are equal to G A, AB, each to each; and the angle A is common to both the triangles A F C, A G B; † Pr. 4. ∴ F C = G B, † *the angle* A C F = *the angle* A B G,† and *the angle* A F C = *the angle* A G B,† these being the angles to which the equal sides are opposite.



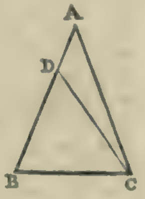
Again: because the whole A F = the whole A G, and that the parts A B, A C are equal, the remainders B F, C G are * Ax 3. equal;* and it was proved that F C = G B, ∴ the two sides B F, F C, are equal to the two C G, G B, each to each: it was also proved that the angle B F C = the angle C G B, ∴ *the angle* F B C † Pr. 4. = *the angle* G C B,† and the angle B C F = the angle C B G. And since it was demonstrated that the whole angle A B G, = the whole angle A C F, the parts of which, the angles C B G, B C F, are also equal, ∴ *the remaining angle* A B C = *the remaining angle* * Ax. 3. A C B.* These are the angles *at the base* of the isosceles triangle A B C, and the angles F B C, G C B, before proved to be equal, are the angles *on the other side of the base.* Therefore *the angles at the base, &c.* Q.E.D.

Corollary.—Every *equilateral* triangle is *equiangular.*

PROPOSITION VI.—Theorem.

If two angles (A B C, A C B) *of a triangle are equal, the sides* (A C, A B) *which subtend*, or are opposite to, *the equal angles shall be equal.*

For, if A B be not equal to A C, one of them, as A B, must be the greater. Let a part B D, equal to A C, the * Pr. 3. less, be cut off,* and draw D C. Then, because in the triangles D B C, A C B, D B = A C, and B C common to both, the two sides D B, B C are = the two sides A C, C B, each to each; and the angles D B C, * Hyp. A C B are equal;* ∴ the triangle D B C = the triangle A C B,† † Pr. 4. a part to the whole, which is absurd. Therefore, A B, A C *are not unequal—that is, they are equal*; ∴ *if two angles, &c.* Q. E. D.

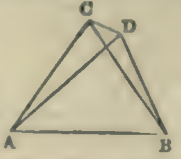


Cor.—Every *equiangular* triangle is *equilateral.*

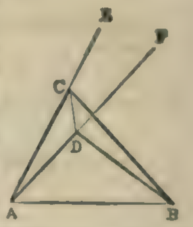
PROPOSITION VII.—Theorem.

Upon the same base (A B), *and on the same side of it, there cannot be two triangles* (A C B, A D B), *having their sides* (A C, A D), *which are terminated in one extremity of the base, equal to one another, and likewise* (B C, B D) *those which are terminated in the other extremity.*

Join C D: then, in the case in which the vertex of each triangle is *without* the other tri- * Hyp. angle, because A C = A D,* † Pr. 5. the angle A C D = A D C.† But the angle A C D is greater than the angle B C D, which is only a part of it; ∴ the angle A D C is greater also than B C D: much more, then, is the angle B D C * Hyp. greater than B C D. Again: because B C = B D,* † Pr. 5. the angle B D C = B C D;† but it has been demonstrated to be greater than B C D, which is impossible, ∴ *in this case it is impossible that* A C = A D, *and likewise* B C = B D.



But if one of the vertices, as D, be *within* the other triangle, prolong A C, A D to E, F. Then, because * Hyp. A C = A D* in the triangle A C D, the angles E C D, F D C, upon the other side of the † Pr. 5. base C D, are equal;† but the angle E C D is greater * Ax. 9. than B C D,* ∴ F D C is likewise greater than B C D: much more, then, is B D C greater than B C D. Again: because B C = † Hyp. B D,† the angle B D C = * Pr. 5. B C D;* but B D C was proved greater than B C D, which is impossible, ∴ *in this case also it is impossible that* A C = A D, *and likewise* B C = B D.

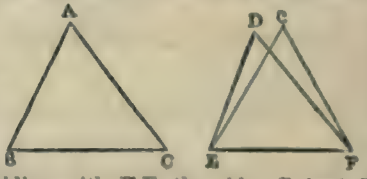


The case in which the vertex of one triangle is *upon* † Ax. 9. a *side* of the other, needs no demonstration.† Therefore, *upon the same base, &c.* Q. E. D.

PROPOSITION VIII.—Theorem.

If two triangles (A B C, D E F) *have two sides* (A B, A C) *of the one equal to two sides* (D E, D F) *of the other, each to each, and have likewise their bases or third sides* (B C, E F) *equal, then the angle* (A) *contained by the two sides of the one shall be equal to the angle* (D) *contained by the two sides equal to them of the other.*

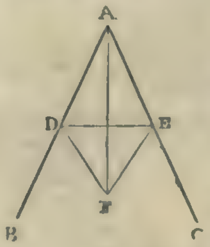
For if the triangle A B C be applied to D E F, so that the point B may be on E, and B C on E F, the point C shall coincide with F, because B C = * Hyp. E F.* And B C thus coinciding with E F, B A and A C must coincide with E D and D F; for if the base B C, coinciding with E F, the sides B A, A C could fall otherwise than on E D, D F, and have different situations, as E G, G F, then upon the same base E F, and on the same side of it, there could be two triangles having the sides E D, E G, terminated in one extremity of the base, equal to one another, and *likewise* the sides F D, F G terminated in the other extremity. But this † Pr. 7. is impossible;† ∴ if the bases coincide, the remaining sides cannot but coincide, and ∴ *the angle* A *must coincide with the angle* D, *and be equal to* * Ax. 8. *it.** Therefore, *if two triangles, &c.* Q.E.D.



PROPOSITION IX.—Problem.

To bisect a given rectilineal angle (B A C); *that is, to divide it into two equal angles.*

Take any point D in A B, and from A C cut off A E = * Pr. 3. A D:* draw D E, and upon it describe an equilateral † Pr. 1. triangle D E F, † so that the vertex F may be on the opposite side of D E to the vertex A. Draw A F, then A F shall bisect the angle B A C.



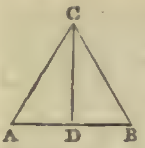
* Const. Because A D = A E,* and that A F is common to the two triangles D A F, E A F: the two sides D A, A F are = the two sides E A, A F, each to each, and the base D F = the base E F; ∴ the angle D A F * Pr. 8. = the angle E A F;* ∴ *the given angle* B A C *is bisected by the straight line* A F. *Which was to be done.*

PROPOSITION X.—Problem.

To bisect a given finite straight line (A B); *that is, to divide it into two equal parts.*

* Pr. 1. Upon AB describe an equilateral triangle ABC,* and bisect the angle A C B by the straight line

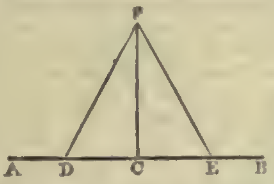
† Pr. 9. C D ;† A B shall be cut into two equal parts in the point D.



Because C A = C B, and C D common to the two triangles A C D, B C D, the two sides C A, C D are = the two C B, C D, each to each ; and the angle A C D = the angle B C D ; ∴ the base A D = * Pr. 4. the base B D;* ∴ *the given line* A B *is divided into two equal parts in the point* D. *Which was to be done.*

PROPOSITION XI.—Problem.

To draw a straight line at right angles to a given straight line (A B), *from a given point* (C) *in it.*



Take any point D in A C, and make C E = C D. Upon D E describe an equilateral triangle D F E,* * Pr. 1. and draw C F ; C F shall be at right angles to A B.

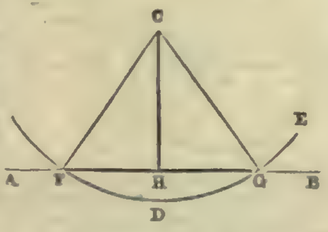
Because D C = E C, and that F C is common to the two triangles DCF, E C F ; the two sides D C, C F are = the two E C, C F, each to each, and the base D F = base E F ; ∴ the angle D C F = angle * Pr. 8. E C F,* and they are adjacent angles. But when the adjacent angles which one straight line makes with another are *equal*, each is called *a right angle* ;† † Def. 8. ∴ *from the given point* C *in the given straight line* A B, *a straight line* C F *has been drawn at right angles to* A B. *Which was to be done.*

PROPOSITION XII.—Problem.

To draw a straight line perpendicular to a given straight line (AB) *of unlimited length, from a given point* (C) *without it.*

Take any point D upon the other side of A B, and with centre C and radius C D describe the circle * Post. 3. E G F* meeting A B in F, G : bisect F G in † Pr. 10. H,† and join C, H.

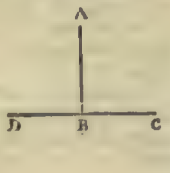
The straight line C H shall be perpendicular to A B. * Const. Draw CF, CG. Then because F H = G H,* and H C common to the two triangles F H C, G H C, the two sides F H, H C are = the two sides G H, HC, each to each, and the base C F = the † Def. 12. base CG;† ∴ the angle C H F = the adjacent angle * Pr. 8. C H G ;* each of them is ∴ a † Def. 8. right angle,† and consequently CH *is perpendicular to* A B ;† ∴ *from the given point* C *a perpendicular* C H *to the given line* A B *has been drawn. Which was to be done.*

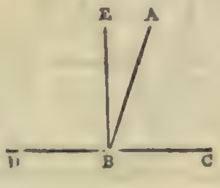


PROPOSITION XIII.—Theorem.

The angles which one straight line (AB) *makes with another* (D C) *upon one side of it, are either two right angles, or are together equal to two right angles.*

For the angle A B C is either equal to A B D, or it is not. * Def. 8. If A B C be = A B D, *each is a right angle.** But if they are not equal, from B draw B E at right † Pr. 11. angles to D C :† then the angles E B C, E B D are right angles, also E B C = E B A + A B C. To each of these equals add E B D, ∴ E B C + E B D = * Ax. 2. E B D + E B A + A B C.* Again : A B D = E B D + E B A. Add A B C to each of these equals, ∴ † Ax. 2. A B D + A B C = E B D + E B A + A B C ;† but it was demonstrated that E B C + E B D are equal to the latter three angles, ∴ A B D + A B C = E B D * Ax. 1. + E B C :* that is, *the two angles* A B D, A B C *are together equal to two right angles.*

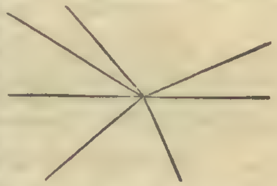




Therefore *the angles which one straight line*, &c. Q.E.D.

Cor. 1.—It is manifest from this, that if two straight lines cross one another, forming *four* angles at the point of intersection, these four angles are together equal to *four right angles.*

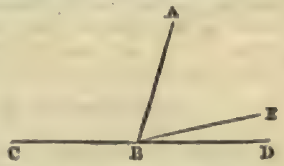
Cor. 2.—And moreover, that if in a straight line any point between its extremities be taken, from which any number of straight lines are drawn, some on one side and some on the other, all the angles thus made at the point will together be equal to four right angles ; those on one side of the proposed line being together equal to two right angles, and those on the other side equal also to two right angles.



PROPOSITION XIV.—Theorem.

If at a point (B) *in a straight line* (A B) *two other straight lines,* (B C, B D), *upon opposite sides of it, make the adjacent angles* (A B C, A B D) *together equal to two right angles, the two straight lines* (B C, B D) *shall be in one and the same straight line.*

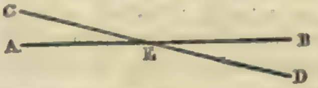
For, if B D be not in the same straight line with C B, let BE be in the same straight line with it. Then because A B makes, with the straight line C B E upon one side of it, the angles ABC, ABE, these are together = two * Pr. 13. right angles.* But A B C, A B D are likewise together = two † Hyp. right angles,† ∴ ABC + ABE = ABC * Ax. 1. + A B D.* Take away the common angle A B C, and there † Ax. 3. remains ABE = ABD, the less to the greater, which is impossible ; ∴ B E is *not* in the same straight line with BC. And in like manner may it be proved that no line, except BD, can be the prolongation of CB; ∴ CB, BD *are in one and the same straight line.* Wherefore, *if at a point,* &c. Q.E.D.



PROPOSITION XV.—Theorem.

If two straight lines (A B, C D) *cut one another, the vertical* (or opposite) *angles shall be equal ;* that is, A E C = D E B, and C E B = A E D.

The angles which A E makes with C D, on one side of * Pr. 13. it, are together equal to two right angles ;* that is, A E C + A E D = two right angles. Again : the angles which D E makes with A B are also together equal to two right an- † Pr. 13. gles ;* that is DEA + DEB = two right angles ; ∴ AEC + AED † Ax. 1. = D E A + D E B.† Take away the common * Ax. 3. angle AED, and there remains AEC = DEB.* And in a similar manner it may be proved that C E B = A E D. Therefore, *if two straight lines,* &c. Q.E.D.



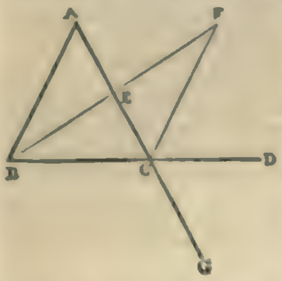
PROPOSITION XVI.—Theorem.

If one side (B C) *of a triangle be prolonged, the exterior angle* (A C D) *shall be greater than either of the interior opposite* (or more remote) *angles.* (B A C, A B C).

* Pr. 10. Bisect A C in E,* draw B E, which prolong, † Pr. 3. and make E F = equal B E :† join F C.

* Const. Because A E = C E,* and E B = E F ;* A E, E B are = C E, E F, each to each and the angle A E B

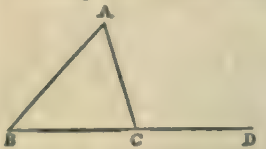
† Pr. 18. = C E F ;† ∴ A B = C F,‡ and the angle A = ‡ Pr. 4. E C F : but A C D is greater than E C F, ∴ A C D *is greater than* A ; ∴ if one side, B C, of a triangle be prolonged, the exterior angle A C D is greater than the interior angle A *opposite to the side prolonged* : but if the side prolonged be A C, the exterior angle will be B C G, and the interior angle opposite, A B C ; ∴ B C G is greater than A B C. But * Pr. 15. A C D = B C G,* ∴ A C D is greater than A B C ; and it was before proved that A C D is greater also than A, ∴ A C D *is greater than either of the interior opposite angles* B A C, A B C. Therefore, *if one side, &c.* Q.E.D.



PROPOSITION XVII.—THEOREM.

Any two angles of a triangle (A B C) *are together less than two right angles.*

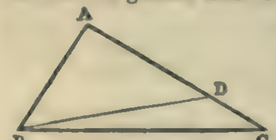
Prolong one of the sides, as B C to D ; then the exterior angle A C D is greater than the interior opposite * Pr. 16. angle B.* To each of these add A C B ; then A C D, A C B are together greater than B, † Ax. 4. A C B ;† but A C D, A C B are together equal to * Pr. 13. two right angles ;* ∴ B, A C B *are together less than two right angles.* And if B A be prolonged, it may be proved, in like manner, that A, B are together less than two right angles ; or if C A be prolonged, that A and A C B are together less than two right angles. Therefore, *any two angles, &c.* Q. E. D.



PROPOSITION XVIII.—THEOREM.

The greater side of every triangle (A B C) *is opposite to the greater angle.*

Let A C be greater than A B ; the angle A B C shall be greater than the angle C. From A C the greater cut off A D = A B the less, and draw B D. Then because A D B is an exterior angle of the triangle B D C, * Pr. 16. it is greater than C :* † Pr. 5. but A D B = A B D,† ∴ the angle A B D is likewise greater than C ; much more, then, is the angle A B C greater than C. Therefore, *the greater side, &c.* Q. E. D.

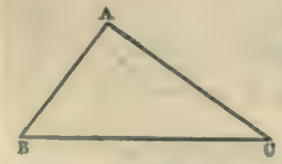


PROPOSITION XIX.—THEOREM.

The greater angle of every triangle (A B C) *is subtended by the greater side,* or has the greater side opposite to it.

Let the angle B be greater than C ; then A C shall be greater than A B.

For, if it be not greater, A C must either be equal to A B, or less than it. If it were *equal,* the angle B * Pr. 5. would = C ;* but it is not, ∴ A C *is not* = A B. If it were *less,* the angle B would be less than † Pr. 18. C ;† but it is not, ∴ A C *is not less than* A B ; and it was shown that it is not *equal* to A B, ∴ A C *is greater than* A B, ∴ *the greater angle, &c.* Q.E.D.

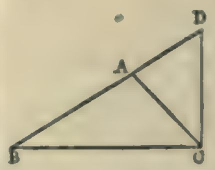


PROPOSITION XX.—THEOREM.

Any two sides (B A, A C) *of a triangle* (A B C) *are together greater than the third side* (B C).

Prolong one of the two sides, as B A, to D, and make * Pr. 3. A D = A C,* the other of the two sides : join D, C.

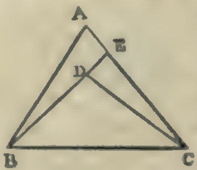
Because A D = A C, the angle † Pr. 5. A C D = D ;† but the angle B C D is greater than A C D, ∴ B C D is greater than D. And because the angle B C D of the triangle D B C is greater than the angle D, and that the greater angle is subtended by the greater * Pr. 19. side,* the side B D is greater than B C : but B D = B A + A D = B A + A C, ∴ B A + A C *is greater than* B C. Therefore, *any two sides, &c.* Q.E.D.



PROPOSITION XXI.—THEOREM.

If from the ends of a side of a triangle there be drawn two straight lines (B D, C D) *to a point within the triangle, these shall be together less than the other two sides* (A B, A C) *of the triangle, but shall contain a greater angle.*

Prolong B D to E. The two sides B A, A E of the triangle A B E are together greater * Pr. 20. than B E.* To each of these unequals add E C, ∴ B A, A C are greater than B E, E C. Again : the two sides C E, E D of the triangle C E D are together greater † Pr. 20. than C D.† To each of these add D B, ∴ C E, E B are greater than C D, D B. But it was shown that B A, A C are greater than B E, E C ; much more then are B A, A C greater than B D, D C. Again : the exterior angle B D C of the triangle C D E is greater * Pr. 16. than C E D ;* and the exterior angle C E B of the triangle A B E is greater than B A C :* *much more then is* B D C *greater than* B A C. Therefore, *if from the ends, &c.* Q.E.D.

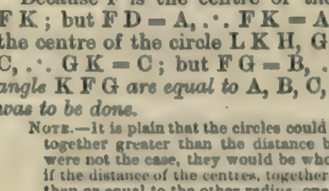


PROPOSITION XXII.—PROBLEM.

To make a triangle of which the sides shall be equal to three given straight lines (A, B, C), *each to each, but any* * Pr. 20. *two of these must be greater than the third.**

Take a straight line D E, terminated at the point D, but unlimited towards E, and make D F = A, F G = B, and G H † Pr. 3. = C.† With centre F and radius F D describe the circle D K L ; and with centre G and radius G H describe the circle H K L, cutting the former in K. Draw K F, K G : the triangle K F G has its three sides equal to the three lines A, B, C.

Because F is the centre of the circle D K L, F D = F K ; but F D = A, ∴ F K = A. Again : because G is the centre of the circle L K H, G H = G K ; but G H = C, ∴ G K = C ; but F G = B, ∴ *the sides of the triangle* K F G *are equal to* A, B, C, *each to each. Which was to be done.*



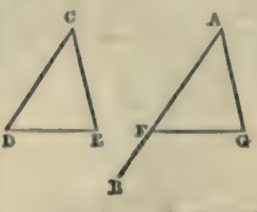
NOTE.—It is plain that the circles could not *cut* unless their radii were together greater than the distance between their centres : if such were not the case, they would be wholly *without* one another. And if the distance of the centres, together with one radius, were smaller than or equal to the other radius, one circle would be wholly *within* the other : the hypothesis precludes both of these circumstances.

PROPOSITION XXIII.—PROBLEM.

At a given point (A) *in a given straight line* (AB) *to make an angle equal to a given angle* (C).

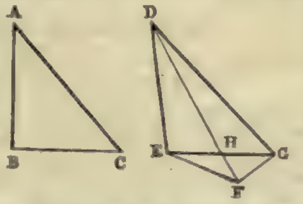
In C D, C E take any points D, E, one in each, and draw D E. Make the triangle A F G, the sides of which shall be equal to those of C D E—namely, A F = C D, A G = C E, * Pr. 22. and F G = D E ;* then the angle A shall be equal to C.

Because A F = C D, A G = C E, and F G = D E, ∴ * Pr. 8. *the angle* A = *the angle* C,* ∴ *at the given point, &c. Which was to be done.*



PROPOSITION XXIV.—THEOREM.

If two triangles (A B C, D E F) *have two sides* (A B, A C) *of the one equal to two sides* (D E, D F) *of the other, each to each, but the angle* (B A C) *contained by the two sides of one of them greater than the angle* (E D F) *contained by the two sides equal to them of the other; the base* (B C) *of that which has the greater angle shall be greater than the base* (E F) *of the other.*

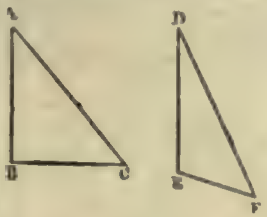


Of the two sides D E, D F, let D E be the side which is *not greater* than the other. Make the angle • Pr. 23. E D G = A,* and make D G = A C or † Pr. 3. D F.† Draw E G, G F, and let H be the point where E G is cut, either by D F or by D F prolonged.

Because D E is not greater than D G, the angle D G E is not greater than • Pr. 19. D E G;* but D H G *is* greater than D E G, ∴ D H G is greater than D G H, and ∴ D G is greater † Pr. 19. than D H;† but D F = D G, ∴ D F is greater than D H, ∴ H G F *is a part of* D G F. Again: the two sides B A, A C are = the two E D, D G, each to • Hyp. & Const. each, and the angle A = E D G,* ∴ B C = E G. And because D F = D G, the angle D F G † Pr. 5. = D G F:† but it was proved that D G F is greater than E G F, ∴ D F G is greater than E G F; much more then is E F G greater than E G F, ∴ in the triangle F E G the angle E F G is greater than E G F, • Pr. 19. ∴ E G is greater than E F;* but, as already proved, E G = B C, ∴ B C *is greater than* E F. Therefore, *if two triangles, &c.* Q.E.D.

PROPOSITION XXV.—THEOREM.

If two triangles (A B C, D E F) *have two sides* (A B, A C) *of the one equal to two sides* (D E, D F) *of the other, each to each, but the base* (B C) *of one greater than the base* (E F) *of the other, the angle* (A) *contained by the sides of that with the greater base shall be greater than the angle* (D) *contained by the sides equal to those of the other.*

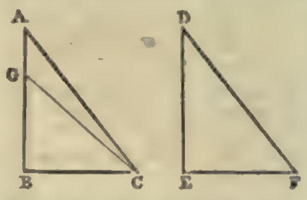


For if A be not greater than D, it must be either equal to it or less. A is not *equal* to D, for then B C would be = E F;* • Pr. 4. but it is not. A is not *less* than D, for then B C would be less than E F:* • Pr. 24. but it is not.† As † Hyp. therefore it is neither equal to nor less than D, A *must be greater than* D, ∴ *if two triangles, &c.* Q.E.D.

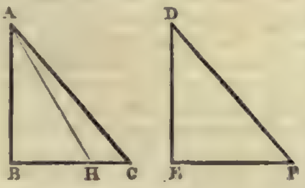
PROPOSITION XXVI.—THEOREM.

If two triangles (A B C, D E F) *have two angles* (B, C) *of the one equal to two* (E, F) *of the other, each to each; and one side equal to one side—viz., either the sides* (B C, E F) *adjacent to the equal angles, or the sides* (A B, D E) *opposite to equal angles in each; then shall the other sides be equal, each to each, and also the third angle of the one to the third angle of the other.*

First let B C = E F, the sides *adjacent* to the angles that are equal each to each. Then if A B, D E be *unequal*, one of them must be the greater. Let A B be the greater, and make B G = D E,* • Pr. 3. and draw G C: then because B G = E D, and B C = E F,† † Hyp. and that the angle B = E, ∴ the angle G C B = • Pr. 4. F:* but the angle A C B = F,† ∴ the angle † Hyp. G C B = A C B, the less to the greater; which is impossible; ∴ A B *is not unequal to* D E, *that is, it is equal to it*, ∴ in the two triangles A B C, D E F, the two sides A B, B C and the included angle B in the one, are respectively equal to the two sides D E, E F and the included angle E in the other, ∴ A C = D F, *and the* • Pr. 4. *angle* A = D.*

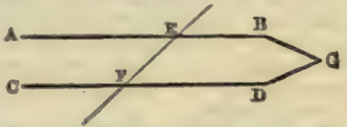


Next let A B = D E, the sides *opposite* to equal angles; in this case, likewise, the other sides shall be equal—namely, A C = D F, and B C = E F and also the angle B A C = D. For, if B C, E F be unequal, let B C be the greater, and make B H = E F.* • Pr. 3. Join A, H: then because B H = E F, and A B = D E, the two A B, B H = D E, E F, each to each; and they contain equal angles; • Pr. 4. ∴ A H = D F,* and the angle B H A = E F D: † Hyp. but E F D = B C A,† ∴ B H A = B C A, the exterior angle equal to the interior and opposite, which • Pr. 16. is impossible;* ∴ B C is not unequal to E F, ∴ B C = E F: and A B = D E, ∴ the two A B, B C = D E, E F, each to each; and they contain equal angles, ∴ A C = D F, and B A C = E D F, ∴ *if two triangles, &c.* Q.E.D.



PROPOSITION XXVII.—THEOREM.

If a straight line (E F) *falling upon two other straight lines* (A B, C D) *makes the alternate angles* (A E F, E F D) *equal, these two straight lines shall be parallel.*

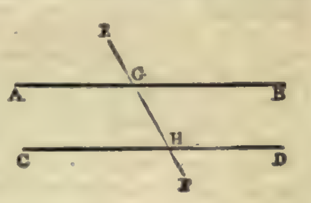


For if they be not parallel they will meet, when prolonged, either towards B, D, or towards A, C. Let them be prolonged and meet in the point G; then G E F will be a *triangle*, and its exterior angle A E F must be • Pr. 16. greater than the interior and opposite angle † Hyp. E F G;* but it is also equal to it,† which is impossible; ∴ A B, C D, when prolonged, do not meet towards B, D. In like manner it may be proved that they do not meet towards A, C. ∴ *they are parallel*, ∴ • Def. 3. *if a straight line, &c.* Q. E. D.

PROPOSITION XXVIII.—THEOREM.

If a straight line (E F) *falling upon two other straight lines* (A B, C D) *make the exterior angle* (E G B) *equal to the interior and opposite angle* (G H D) *upon the same side of the line; or make the interior angles* (B G H, G H D) *upon the same side together equal to two right angles; the two straight lines* (A B, C D) *shall be parallel.*

Because the angle E G B = the angle G H D,* and • Hyp. E G B = A G H,† ∴ A G H = G H D; and † Pr. 15. these are *alternate* angles, ∴ A B *is parallel to* C D.*



• Pr. 27. Again: because the angles B G H, G H D are together = two right angles,* and • Hyp. that A G H, B G H are together also = two right angles,† ∴ † Pr. 13. A G H + B G H = B G H + G H D. Take away B G H, then A G H = G H D; and these are • Pr. 27. *alternate* angles, ∴ A B *is parallel to* C D,* ∴ *if a straight line, &c.* Q. E. D.

NOTE.—Propositions XXVII. and XXVIII. clearly prove the *existence* of parallel lines, or of lines such that, however far they be prolonged, they can never meet. And by aid of the first of these propositions, if a straight line be given, one parallel to it may always be drawn (see Prop. XXXI.) But, to proceed further in the doctrine of parallel lines, requires assent to a principle which must now be formally stated, and unhesitatingly admitted as true: it is the principle affirmed in Euclid's twelfth axiom. In most editions of Euclid, this is placed, with the other axioms, at the commencement of the Book. It has been here kept out of view till it can no longer be dispensed with; and this has been done because, in the first place, we should not be called upon to give assent to what concerns anything, of the possible existence of which there may be reasonable doubt. Till he has reached Prop. XXVII. (which proposition is all-sufficient for the construction of Prop. XXXI.), the learner may fairly question whether it be *possible* for what are called parallel lines to exist: he now knows that parallel or *never-meeting* lines may be actually drawn.

In the next place, from the property demonstrated in Proposition XVII., namely, that any two angles of a *triangle* are together less than two right angles, he knows—what he could not know at an earlier stage of his progress—an important particular respecting a pair of *meeting-lines*, crossed by a third line; namely, that in a pair of *meeting-lines*, the two interior angles on one side of the crossing line, are together *less than two right angles;* and, as a consequence (Prop. XXVIII.), that when these interior angles are together *equal* to two right angles, the lines crossed must be *non-meeting*, or *parallel lines*. It is the *converse* of the first-mentioned property that he is now to be called upon to receive as true; viz., that if a straight line, crossing *a pair of lines*, make the two interior angles, on the same side of it, together *less than two right angles*, the pair crossed shall be *meeting-lines*. This is the twelfth axiom, and is thus expressed in Euclid:—

AXIOM XII.—If a straight line meet two straight lines, so as to make the two interior angles, on the same side of it, taken together, *less than two right angles*, these straight lines, being prolonged, shall at length *meet* upon that side on which are the angles that are less than two right angles.

The propositions already referred to enable us to see distinctly what it is that this axiom assumes, and moreover inform us that the assumption is, at least, perfectly *consistent with demonstrated truth*. That it is necessarily true, must be admitted, upon reflecting for a moment upon that peculiarity of a *straight* line, really implied in its designation, though not expressly adverted to in its definition—*its undeviating sameness of direction.*

It is obvious, from this uniformity of direction, that if two straight lines, however far prolonged, *never* meet, then, at no part of their course, can either make any approach towards the other; for if two straight lines *approach* one another, their continuance, in the same undeviating directions, necessitates their *meeting*, if indefinitely prolonged. We cannot doubt this, and yet have an accurate conception of an unlimited straight line; since *uniformity of direction* must enter that conception. It follows, therefore, that parallel lines must be, throughout, *equidistant lines.* But two *distinct* straight lines, through the same point, cannot be throughout equally distant from a third; so that two straight lines, through the same point, cannot *both* be parallel to the same straight line. It has been seen (Prop. XXVIII.) that *one* (C D) is parallel to another (A B), if the interior angles (B G H, D H G) be *equal* to two right angles; a *second* (H K), which would cause the interior angles (B G H, K H G) to be *less* than two right angles, being a distinct line from C D, must therefore *meet* A B if prolonged. And this is the assertion of the twelfth axiom.

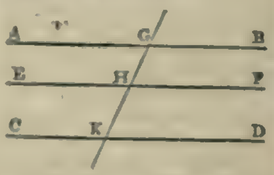
PROPOSITION XXIX.—THEOREM.

If a straight line (E F) *fall upon two parallel straight lines* (A B, C D) *it makes the alternate angles* (A G H, G H D) *equal; and the exterior angle* (E G B) = *the interior and opposite* (G H D) *upon the same side; and likewise the two interior angles* (B G H, G H D) *upon the same side together* = *two right angles.*—[See the preceding diagram.]

For if A G H be not = G H D, one of them, as A G H, must be the greater. Add the angle B G H to each of them, ∴ A G H + B G H are greater than B G H + G H D. But A G H + B G H = two right angles,* ∴
* Pr. 13. B G H + G H D are less than two right
† Ax. 12. angles, ∴ A B, C D, if prolonged, will meet,† which is impossible, since (by hypothesis) they are parallel; ∴ A G H *is not unequal to* G H D, *that is, it is*
* Pr. 15. *equal to it.* Again: A G H = E G B,* ∴ E G B = G H D. Add to each of these B G H, ∴ E G B + B G H = G H D + B G H; but E G B + B G H = two
† Pr. 13. right angles,† ∴ B G H + G H D = *two right angles,* ∴ *if a straight line, &c.* Q. E. D.

PROPOSITION XXX.—THEOREM.

Straight lines (AB, C D) *which are parallel to the same straight line* (E F) *are parallel to each other.*

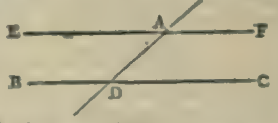


Let the straight line G H K cut A B, E F, C D: then, because A B is parallel to E F, the angle A G K
* Pr. 29. = G H F,* and because E F, C D are also parallel, the angle G H F
† Pr. 29. = G K D,† ∴
* Pr. 27. A G K = G K D, ∴ A B *is parallel to* C D.* ∴ *straight lines, &c.* Q. E D.

PROPOSITION XXXI.—PROBLEM.

To draw a straight line through a given point (A) *parallel to a given straight line* (B C).

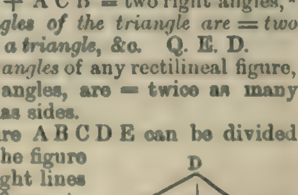
In B C take any point D: join A D; and at the point
* Pr. 23. A in A D make the angle D A E = A D C;* and prolong E A to F; then E F *shall be parallel to* B C.



Because A D, falling upon E F, B C, makes the alternate angles E A D, A D C equal, E F *is paral-*
* Pr. 27. *lel to* B C,* ∴ *through the given point A a line E F parallel to* B C *is drawn. Which was to be done.*

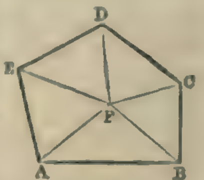
PROPOSITION XXXII.—THEOREM.

If a side (B C) *of a triangle* (A B C) *be prolonged, the exterior angle* (A C D) *is* = *the two interior and opposite angles* (A, B); *and the three interior angles of every triangle are together* = *two right angles.*



* Pr. 31. Through C draw C E parallel to A B;* then the alternate angles B A C, A C E are equal,†
† Pr. 29. and because B D falls upon the said parallels, the exterior angle E C D is = the interior and opposite angle
* Pr. 29. A B C.* It was proved that A C E = B A C, ∴ *the whole exterior angle* A C D = A + B, *both the interior and opposite angles.* To each of these equals add A C B, ∴ A C D + A C B = A + B + A C B: but A C D + A C B = two right angles,*
* Pr. 13. ∴ *the three angles of the triangle are* = *two right angles,* ∴ *if a side of a triangle, &c.* Q. E. D.

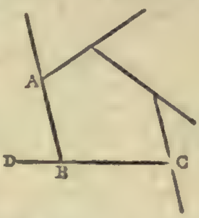
COR. 1.—All the *interior angles* of any rectilineal figure, together with four right angles, are = twice as many right angles as the figure has sides.



For, any rectilineal figure A B C D E can be divided into as many triangles as the figure has sides, by drawing straight lines from a point F within the figure to each of its vertices. And, by the preceding prop., all the angles of these triangles are = twice as many right angles as there are triangles; that is, as there are sides of the figure. But these same angles are equal to the angles of the figure together with the angles at F, the common vertex of the triangles; that is, to-
* Pr. 13. gether with four right angles;* ∴ *all the*
Cor. 2. *angles of the figure, together with four right*

angles, are = twice as many right angles as the figure has sides.

COR. 2.—All the *exterior angles* of any rectilineal figure are together = four right angles.

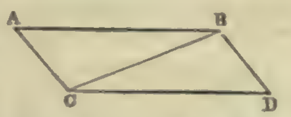


Because every interior angle A B C, with its adjacent exterior angle A B D, is = two
* Pr. 13. right angles,* .·. all the interior + all the exterior are = twice as many right angles as there are sides to the figure; that is, by the above corollary, to all the interior angles + four right angles, .·. *all the exterior angles are = four right angles.*

This remarkable property can scarcely fail to arrest the student's special attention; as, previously to its demonstration, he would be little likely to expect that if the sides of a rectilineal figure be prolonged, one after another, the exterior angles thus formed would have the *same amount* whether the figure had *three* sides, or *three thousand.*

PROPOSITION XXXIII.—THEOREM.

The straight lines (A C, B D) *which join the extremities of two equal and parallel straight lines* (A B, C D) *towards the same parts, are also themselves equal and parallel.*

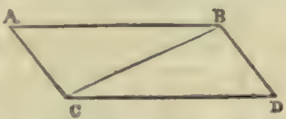


Draw B C, which joins the extremities of the parallels towards *opposite* parts; then the alternate angles A B C,
* Pr. 29. B C D are equal.* And because A B = C D, and B C common to the two triangles A B C, D C B, the two sides A B, B C and the included angle are respectively = the two D C, C B and the included angle; .·. A C =
† Pr. 4. B D,† and the angle A C B = D B C; and these
* Pr. 27. are *alternate* angles, .·. A C *is parallel to* B D;* *and it was shown that* A C = B D; .·.*the straight lines, &c.* Q. E. D.

PROPOSITION XXXIV.—THEOREM.

The opposite sides and angles of a parallelogram A C D B *are equal, and the diagonal* (B C) *bisects it; that is, divides it into two equal parts.*

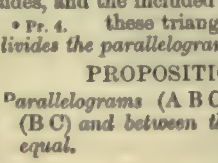
DEF.—A parallelogram is a four-sided figure, of which the opposite sides are parallel; and the *diagonal* is the straight line joining two of its opposite vertices.



Because A B is parallel to C D, and B C meets them, the angle A B C =
* Pr. 29. D C B;* and because A C is parallel to B D, and B C meets them,
† Pr. 29. the angle A C B = D B C,† .·. the two triangles A B C, D C B have two angles A B C, A C B, in the one = D C B, D B C, in the other, each to each, and the side B C, adjacent to the equal angles, common to the two triangles; .·. A B = C D, *and* A C = B D; *and the*
* Pr. 26. *angle* A = D.* Again: because the angle A B C = D C B, and the angle D B C = A C B, .·. *the whole angle* A B D = *the whole angle* A C D; and it was proved that A = D, .·. *the opposite sides and angles of a parallelogram are equal.* Also the diagonal bisects it: for it has been shown that the triangle A C B has two sides, and the included angle A = respectively to two sides, and the included angle D in the triangle D B C, .·.
* Pr. 4. these triangles are equal,* .·. *the diagonal* B C *divides the parallelogram into two equal parts.*

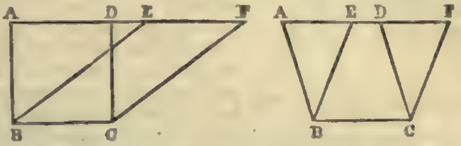
PROPOSITION XXXV.—THEOREM.

Parallelograms (A B C D, E B C F) *upon the same base* (B C) *and between the same parallels* (A F, B C) *are equal.*



Suppose, first, that the sides A D, E F, opposite to the base B C, terminate in the *same point* D; then, since each parallelogram is double of the triangle B D C,*
* Pr. 34. *the parallelograms are equal.*

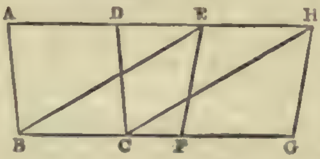
Next, let the sides A D, E F terminate in different points, D, E; then A D = B C,
† Pr. 34. and E F = B C,† .·. A D = E F, and D E is



common, .·. the whole, or remainder, A E = the whole, or
* Pr. 34. remainder, D F: also A B = D C,* .·. A E, A B
† Pr. 29. = D F, D C, each to each; also the angle A = F D C,† .·. the triangle E A B = triangle F D C. Take the triangle E A B from the trapezium A B C F, and from the *same* trapezium take the equal triangle F D C; the remainders must be equal; that is, *the parallelogram* E B C F = *the parallelogram* A B C D; .·. *parallelograms upon the same base, &c.* Q. E. D.

PROPOSITION XXXVI.—THEOREM.

Parallelograms (A B C D, E F G H) *upon equal bases* (B C, F G) *and between the same parallels* (A H, B G) *are equal.*



Draw B E, C H. Then because B C = F G, and F G = E H; .·. B C = E H; and these are parallels, and joined towards the same parts by B E, C H, .·. E B, H C are equal and
* Pr. 33. parallel,* .·. E B C H is a paral-
† Pr. 34. lelogram,†
* Def. and it is = A B C D,* also E B C H = E F G H,*
* Pr. 35. .·. *the parallelogram* A B C D = E F G H, .·. *parallelograms upon equal bases, &c.* Q. E. D.

PROPOSITION XXXVII.—THEOREM.

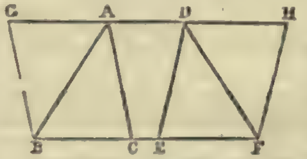
Triangles (A B C, D B C) *on the same base* (B C) *and between the same parallels* (E F, B C) *are equal.*

This proposition is only a particular case of that which follows; and as the particular is not made use of in the more general demonstration, it may be omitted, as quite superfluous. Prop. XXXVIII. proves that triangles between the same parallels are equal, provided only that their bases are *equal*, without any restriction as to whether the bases *coincide* or not. Whatever is proved as to *equal* things is, of course, proved when the things are not only *equal*, but *identical*. Prop. XXXV. is, like the present, only a particular case of that next in order, but the proof of the latter requires that the particular case be previously established; so that, although Prop. XXXVI. really includes Prop. XXXV., yet Prop. XXXV. must not on that account be suppressed.

The enunciation of the present useless proposition is retained here, solely in order that Euclid's subsequent propositions may not be disturbed.

PROPOSITION XXXVIII.—THEOREM.

Triangles (A B C, D E F) *upon equal bases* (B C, E F) *and between the same parallels* (B F, G H) *are equal.*



Through B draw B G
* Pr. 31. parallel to C A,* and through F draw F H parallel to E D,* the lines thus drawn terminating in G H. Then G B C A, D E F H are parallelograms,†
† Pr. 34. and, being on equal bases, and between the
Def. same parallels, they are *equal*,* .·. their halves
* Pr. 36. are equal; that is, *the triangle* A B C = *the triangle* D E F, .·. *triangles upon equal bases, &c.* Q. E. D.

PROPOSITION XXXIX.—THEOREM.

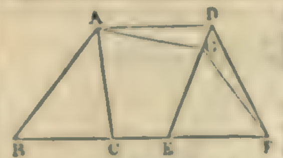
Equal triangles (A B C, D B C) *upon the same base* (B C) *and on the same side of it, are between the same parallels.*

This proposition, like Prop. XXXVII., is superfluous; it is included in the next.

PROPOSITION XL.—THEOREM.

Equal triangles (A B C, D E F) *on the same side of the same straight line, and having equal bases, are between the same parallels.*

Draw A D. Then if A D be not parallel to B F, let

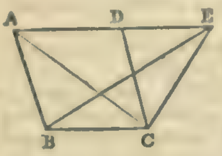


some other line A G be parallel to B F, and draw G E F. The triangle A B C = • Hyp. the triangle G E F,† as they are on equal bases and between the same parallels. But the triangle A B C • Pr. 38. = the triangle D E F,* ∴ triangle D E F = triangle G E F, though one is only a part of the other, which is impossible, ∴ any other line A G is *not* parallel to B F, ∴ A D *is the parallel to* B F *drawn from* A, ∴ *equal triangles*, &c. Q. E. D.

NOTE.—Whether the *supposed* parallel, A G, be considered to meet E D, or E D prolonged—that is, whether A G be imagined on the one side of A D, or on the other—the reasoning remains the same.

PROPOSITION XLI.—THEOREM.

If a parallelogram (A B C D) *and a triangle* (E B C) *be upon the same base and between the same parallels, the parallelogram shall be double the triangle.*



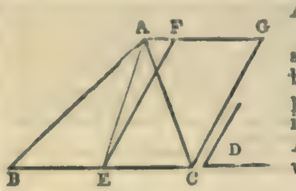
Draw A C, then triangle A B C = triangle E B C, because they are on the same base and are between the same parallels.* But • Pr. 37. A B C D is double the † Pr. 34. triangle A B C,† ∴ A B C D *is also double the triangle* E B C, ∴ *if a parallelogram, &c.* Q. E. D.

NOTE.—The learner will perceive that this thorem is unnecessarily restricted to the case in which the bases *coincide*; he may generalise it himself, proving that if a parallelogram and a triangle be upon *equal* bases, &c.

PROPOSITION XLII.—PROBLEM.

To describe a parallelogram that shall be equal to a given triangle (A B C), *and have one of its angles equal to a given angle* (D).

• Pr. 10. Bisect B C in E,* and make the angle CEF † Pr. 23. = D.† Also through A draw A F G parallel • Pr. 31. to B C, and through C, C G parallel to E F:* then F E C G is the parallelogram required. For draw A E: the triangle A B E = † Pr. 38. triangle A E C,† since they are on equal bases and between the same parallels, ∴ triangle A B C is double the triangle A E C. But F E C G is likewise double the triangle • Pr. 41. A E C,* ∴ *parallelogram* F E C G = *triangle* A B C, *and the angle* C E F † Const. = D,† ∴ a parallelogram has been described as required.

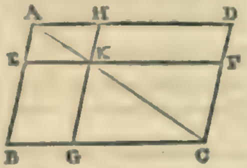


PROPOSITION XLIII.—THEOREM.

The complements (B K, K D) *of the parallelograms* (E H, G F) *which are about the diagonal* (A C) *of a parallelogram are equal.*

The parallelogram E H, G D, through which the diagonal A C passes. are said to be *about the diagonal*, and the remaining parallelograms B K, K D, which make up the whole figure, are called the *complements* of the former.

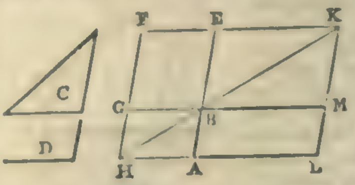
Because B D is a parallelogram, and A C its diagonal, • Pr. 34. ∴ triangle A B C = triangle A D C.* Again, because E H is a parallelogram, ∴ triangle A E K = † Pr. 34. triangle A H K;† and, for a like reason, triangle K G C = triangle K F C, ∴ A E K + K G C = A H K + K F C. But the whole A B C = the whole A D C, ∴ the remainders—that is, the *complements* B K, K D—*are equal*, ∴ *the complements, &c.* Q. E. D.



PROPOSITION XLIV.—PROBLEM.

To a given finite straight line (A B) *to apply a parallelogram which shall be equal to a given triangle* (C), *and have one of its angles equal to a given angle* (D).

Make the parallelogram B F = triangle C, and having • Pr. 42. the angle E B G = D,* so that B E may be in the same straight line with A B; and prolong F G to H. † Pr. 31. Through A draw A H, parallel to B G or E F,† meeting the prolongation of F G in H, and draw H B. Then because H F falls on the parallels A H, E F, the angles A H F, H F E are together = two right • Pr. 29. angles,* ∴ B H F + H F E are *less* than two right angles, ∴ H B, F E, if prolonged, must † Ax. 12. meet.† Let them meet in K; and draw K L • Pr. 31. parallel to E A, or F H;* and prolong H A, G B, to L, M. Then L F is a parallelogram, of which the diagonal is H K; and L B, B F are the *complements* of the parallelograms A G, M E, about the diagonal, † Pr. 43. • Const. ∴ L B = B F;† but B F = C,* ∴ L B = C. † Pr. 15. And because the angle G B E = A B M,† and • Const. G B E = D,* ∴ A B M = D, ∴ *to the straight line* A B *the parallelogram* L B *is applied* = C, *and having an angle* A B M = D. Which was to be done

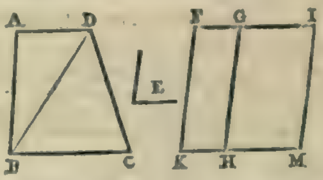


PROPOSITION XLV.—PROBLEM.

To describe a parallelogram equal to a given rectilineal figure, and having an angle equal to a given angle (E).

First, let the figure be a *four-sided* one, A B C D. Draw D B, and describe the parallelogram F H = the • Pr. 42. triangle A D B, and having the angle K = E;* and to G H apply the parallelogram G M = the triangle † Pr. 44. D B C, having the angle G H M = E.† *The figure* F M *shall be the parallelogram required.*

• Const. Because the angles K, G H M are each = E,* ∴ K = G H M. Add to each of these the angle K H G, ∴ K + K H G = G H M + K H G; but K + K H G = two right angles,† † Pr. 29. ∴ G H M + K H G = two right angles, and consequently K H, H M are in the • Pr. 14. same straight line.* And because H G meets the parallels K M, F G, the angle M H G = H G F;† † Pr. 29. add to each of these the angle H G L, ∴ M H G + H G L = H G F + H G L; but the former are • Pr. 29 = two right angles;* ∴ H G F + H G L = two right angles, ∴ F G, G L are in the same straight † Pr. 14. line.† And because K F, M L are each • Pr. 30. parallel to H G, ∴ K F is parallel to M L,* and K M, F L are by construction parallel; ∴ F M is a † Pr. 34. parallelogram.† And because the triangle Def. A B D = F H, and the triangle D B C = • Const. G M,* ∴ *the whole* A C = *the parallelogram* F M. Again, let the figure be *five-sided*. then, having drawn D C, there will be, besides the four-sided figure A C, a triangle upon D C. A parallelogram, equal to this triangle, may be applied to L M, just as G M = D B C was applied to G H; and thus, however numerous be the sides, *a parallelogram may be described equal to the given rectilineal figure, and having the angle* K = *the given angle* E.

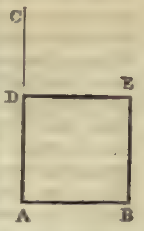


COR.—From this it is manifest how, to a given straight line, to apply a parallelogram which shall have an angle equal to a given rectilineal angle, and shall be equal to a given rectilineal figure.

PROPOSITION XLVI.—PROBLEM.

To describe a square upon a given finite straight line (A B).

• Pr. 11. Draw A C at right angles to A B.* and make † Pr. 3. A D = A B.† Through D draw D E parallel to • Pr. 31. A B, and through B draw B E parallel to A D,*



∴ A E is a parallelogram, ∴ A B † Pr. 34 = D E, and A D = B E;† but A B = A D, ∴ the four sides A B, A D, D E, E B are all equal, ∴ A E is *equilateral.* Likewise, all its angles are right angles; for since A D meets the parallels A B, D E, the angles A + A D E * Pr. 29. = two right angles;* but A † Const. is a right angle,† ∴ A D E is a right angle; but the opposite angles * Pr. 34. of a parallelogram are equal,* ∴ B, E are each right angles, ∴ the figure A E is *rectangular*, and it was proved to be *equilateral;* it is ∴ *a square, and it is described upon the given straight line* A B. Which was to be done.

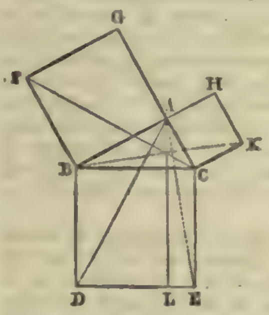
Cor.—Hence, every parallelogram that has *one* right angle, has all its angles right angles.

Note.—It may be easily proved that the figure A E is rectangular, provided it have *one* right angle A, and that it be equilateral; for if the diagonal D B be drawn, the figure will be divided into two equal triangles (Pr. VIII.), ∴ A = E, ∴ the triangles are right-angled isosceles triangles, and (Pr. V. and XXXII.) each base angle is half a right angle, ∴ the angles A, B, E, D are all right angles. Hence, *a square is a four-sided figure, which has all its sides equal, and one of its angles a right angle;* that the other three are also right angles is demonstrable, as above, and ought not to be assumed in a definition.

PROPOSITION XLVII.—Theorem.

In any right-angled triangle (B A C), *the square* (B E) *described upon the side* (B C) *subtending the right angle, is equal to the squares* (B G, C H) *described upon the sides containing the right angles.*

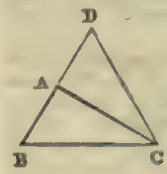
* Pr. 46. The squares being described,* through A draw † Pr. 31. A L parallel to B D or C E;† draw also A D, * Hyp. F C. Then, because B A C is a right angle,* and that B A G is also a right angle, C A, A G are in the † Pr. 14. same straight line.† For a like reason, A B, A H are in the same straight line. Now the angle D B C = F B A, each being a right angle; add to each the angle A B C, ∴ D B A = F B C; also the two sides A B, B D = the two F B, B C, each to each, ∴ *the triangle* A B D = *the triangle* F B C.* * Pr. 4. Now the parallelogram B L is double the triangle A B D, because they are on the same base B D, and between the same paral- † Pr. 41. lels B D, A L;† and the square B G is double the triangle F B C, because these also are on the same base, and are between the same parallels F B, G C; but the doubles of equals are themselves equal, ∴ B L = B G.



In like manner, by drawing A E, B K, it may be demonstrated that C L = C H, ∴ the whole square B E = the two squares B G, C H; that is, *the square described upon* B C *is equal to the squares described upon* A B, A C, ∴ *in any right-angled triangle, &c.* Q. E. D.

PROPOSITION XLVIII.—Theorem.

If the square described upon (B C) *one of the sides of a triangle* (A B C) *be equal to the squares described upon the other two sides, the angle* (A) *contained by these two sides is a right angle.*



From A draw A D at right angles * Pr. 11. to A C,* and make A D † Pr. 3. = A B,† and draw D C. Then, because A D = A B, the square of A D = the square of A B: to each of these add the square of A C, ∴ the squares of A D, A C = the squares of A B, A C. But the square of D C = the squares of A D, A C, because * Pr. 47. D A C is a right angle,* and the square of B C is by hyp. = the squares of A B, A C, ∴ the square of D C = the square of B C, ∴ D C = B C. Hence, in the two triangles A B C, A D C, there are two sides B A, A C in the one, equal to the two D A, A C in the other, each to each, and the base B C equal to the base * Pr. 8. D C, ∴ the angle B A C = D A C.* But D A C is a right angle, ∴ B A C is a right angle, ∴ *if a square, &c.* Q. E. D.

REMARKS AND COMMENTS ON THE FIRST BOOK OF EUCLID.

The following observations on the general character of geometrical reasoning, and on the First Book of Euclid in particular, are intended for the guidance and instruction of those whose acquaintance with the subject is limited to what has now been delivered. We think it very probable that, among such persons, there may be some who, however attentively they may have read the portion now completed, have failed to perceive, so clearly as is desirable, the main object and intention of a course of geometrical study. It is true that the demonstrations themselves are so free from obscurity, and so thoroughly convincing, that no doubt can remain on the mind of an attentive reader as to the truth of the several conclusions arrived at; so that anything added to these demonstrations, by way of elucidation of the steps, or as confirmatory of the results, would be felt by the merest beginner to be an incumbrance rather than an aid.

There is no doubt, however, that Geometry is sometimes taken up with erroneous expectations as to what it teaches; and is read with a pliant docility of mind—a passive acquiescence in the dicta of the teacher—which Euclid himself would be the first to condemn. It is chiefly for the purpose of guarding *you* against such mistakes that we append the following remarks to the first book of the elements. We should have *prefixed* them, could we have been quite certain that you would have been familiar with the geometrical terms which would have been employed. We offer them here, in the expectation that you will give the foregoing part a second reading, guided by the additional light here to be given, in reference to the objects and advantages of Geometry, and also as regards the true spirit in which its principles should be studied.

A youth, destined ultimately for some mechanical or scientific occupation, is told—and properly told—that, to excel in his calling, he must study Euclid; this study is usually associated in his mind with a case of mathematical instruments, scales, compasses, parallel-rulers, &c. Now, without expressly *forbidding* these things, it is of importance that he should be distinctly informed that, for aught that appears to the contrary, Euclid never handled, or even saw, compasses, parallel-rulers, &c., in his life. It is certain that he gives no countenance to the use of any such mechanical contrivances in his work. Had Euclid been asked, there is no doubt that he would have declared his inability to describe a circle, and even to draw a straight line. What *we* call *practical geometry* Euclid was entirely regardless of; indeed the application of geometry to the practical business of life was viewed by the ancient geometricians rather as a degradation of the purely intellectual science they cultivated, than as enhancing its value; and we accordingly find that but few of Euclid's *problems*, or practical constructions, are such as a skilful workman would follow.

Important and extensive as are the practical applications of geometry, it should nevertheless be borne in mind that the *Elements* would have existed, just as they now do, if these applications had never been thought of. The availability of geometry in practice is merely a contingent and accidental circumstance, uncontemplated by the geometricians, just as their theory of the conic sections was elaborated without any prospective regard to the future demands of physical astronomy, or to the discoveries of Newton.

And it is right to contemplate geometry under this purely intellectual aspect, and to study it as a strictly abstract science. Practical operations are all more or less imperfect. There are imperfections of vision, of the hand, of the instruments employed. Pure Euclidean geometry tolerates no such imperfections, however minute or unimportant they may be in a practical point of view. The circle of Euclid is a *perfect* circle—such a circle, in fact, as no human being ever formed. In like manner, the straight line of Euclid is rigorously what it is affirmed to be—*perfectly* straight, and *perfectly* breadthless. You see, therefore, that we were fully warranted in saying, that Euclid could not describe a circle, nor draw a straight line; he has not, indeed, attempted to do either; the marks and diagrams which he exhibits to the eye, in connection with his reasonings, are nothing more than the outward symbols, of what *actually exists in the mind alone.* And the truths of geometry become applicable to visible and tangible squares, circles, &c., only on the supposition or *assumption* that they are perfect copies of our intellectual conceptions of these things. A very able writer on Logic and mental Philosophy (Mr. Stuart Mill), denies to the lines and figures of geometry, the perfection here contended for. Assuming that all our conceptions of form originate in our contemplation of outward objects, which is no doubt true, he maintains that our ideas of squares, circles, &c., are only copies of the confessedly imperfect forms presented to our eyes. We would submit, however, that the mind can *conceive* what it may surpass the powers of the hand to *execute;* and that we can *imagine* a perfection which art cannot attain. A mere approximation to the perfect form—which is all that can be presented to the eye—will suggest the practically-unattainable perfection to the mind; and it may be safely asserted that the very infirmity of our visual organs contributes to this perfection; since defects, too minute to be visible in the outward object, cannot possibly accompany the mental impression of that object.

We have thought it right, in these introductory remarks, thus to state broadly, and unambiguously, what the subject-matter of geometry really is. The objects with which it deals, and to which its reasonings are applied, are our perfect mental conceptions of figure, and not the imperfect pictured forms which, to help these conceptions, are traced upon paper. These are merely the outward representations, or visible symbols of the purely intellectual forms which they very conveniently serve to suggest; though, from the physical and instrumental imperfections which we know to be attached to them, they are not accurately the things themselves; these latter, by an act of abstraction, being freed from all material encumbrances; so that, in fact, the forms and figures of geometry are exclusively in the *mind*, and not in *matter*.

This is no bar to the practical applications of the science. Whether a tangible square be perfect or imperfect is of no moment, practically speaking, so long as its imperfections are undiscoverable by the senses; inasmuch as the rigorous conclusions of geometry may be applied to it without practical or appreciable error.

We now proceed to consider the basis upon which the entire structure of geometry rests—a basis so simple that a child might lay the foundation-stones; yet supporting a fabric which, though so extensive, is, at the same time, so secure, that the most powerful intellect cannot disturb its stability. We need scarcely say that we allude to the axioms and postulates of the science; and, in connection with the consideration of these, shall take occasion to offer some suggestions as to the proper frame of mind in which Euclid should be studied.

Supposing, then, that you have your Euclid in your hands, we commence by first directing your attention to the fact, that the work is divided into distinct sections, or *Books;* and that each book commences with an explanation of the technical terms employed in it; with a concise but satisfactory description of the lines and figures to be reasoned about, and a statement of the elementary propositions to be *admitted as possible*, in the practical constructions, and of the elementary propositions to be *admitted as true* in the reasonings.

You will at once see, that in entering upon any doctrine which is to be established, not by the influence of authority, but by the force of reasoning and sound argument, it is of much importance that preliminaries such as these should be clearly and satisfactorily settled. If a person desire to communicate his own convictions to another, and, in undertaking to do so, make *reasoning* the only channel through which to convey them, there must be—first, a mutual concurrence as to the meaning of the terms employed; and secondly, a like concurrence as to the fundamental principles to be assumed by the one party, and admitted by the other. A good deal of what goes by the name of reasoning and argument, in the common affairs of life, is nothing but a sort of wrangling disputation, solely from the neglect to establish a clear understanding on these points at the outset. Euclid is careful to preclude this fertile source of ambiguity, confusion, and error. He commences the several portions of his subject with *Definitions* of the things to be discussed, and of the peculiar terms to be employed in the discussion; he then tells you what he expects you to admit as practically, or at least, as *conceivably* possible; and lastly, what he requires you to concede, without demonstration, as necessarily true.

You should not hurry over the definitions; they have been framed with great care. The character of a good definition is this: that it is just sufficiently descriptive of the thing defined to distinguish it from all other things, but not more than sufficient for this purpose. If anything more than what merely suffices to identify the object defined be declared in a definition, that definition is said to be *redundant:* it involves the *assumption* of some property or peculiarity of the object, which it is the province of *reasoning* to deduce from the properly-restricted definition of it. All the properties of geometrical figures are in this way deduced from, or as it were drawn out of, the definitions of those figures; for in the definitions they are all virtually implied, and lie concealed. If you were to define an equilateral triangle as that which has three equal sides and three equal angles, you would make a statement which is quite correct, as a *statement*, but very faulty as a *definition:* the equality of the three sides *necessitates* the equality of the three angles (Prop. V., Cor.), so that the equality of the angles is virtually implied in the equality of the sides—a truth which must be discovered to us by reasoning, not assumed in a so-called definition. You will observe that Euclid invariably constructs his figures solely in reference to the descriptions of those figures embodied in the definitions, quite regardless, at the time, of all other properties of them; and you will perceive that he has furnished particulars just sufficient for this purpose, without one superfluous item.

We need scarcely state, that in speaking of Euclid here, the emendations of Simson and other modern editors are uniformly kept in view. It is much to be regretted that, in the editions of Euclid most generally studied, acknowledged blemishes are allowed to remain in the text, while the proper emendations are given in the form of notes at the end. Our veneration for a writer on science should never be considered as ground sufficient for us to endorse his errors and defects, nor even to except to them only indirectly, and in the form of supplementary annotation; they ought, in justice to him, as well as to those for whom he wrote, to be expunged from the text of his instructions. In a work of taste or imagination, the case would be different; an editor would have no right to replace the author's views and peculiarities by his own; but a book of science, so extensively used in education as Euclid is, should be rendered as perfect as possible; and an editor of such a book could incur little blame for expunging every admitted blemish from the text of his author. We have generally acted under this impression.

Having thus previously established the *existence* of his geometrical forms, Euclid then proceeds, in his *Theorems*, to deduce, by reasoning, all those properties necessarily, though not obviously, implied in the definitions; but, as already noticed, before these existences can be proved, that is to say, before the constructions employed by Euclid in his *Problems* can be actually effected, assent must be given to the practicability, or, to be more explicit, to the conceivability of certain fundamental *operations:* these are enumerated in the *Postulates:* and that before the reasonings in his *Theorems* can be entered upon, assent must, in like manner, be given to certain fundamental and necessary *truths;* these are enumerated in the *Axioms.* You perceive, therefore, that the definitions furnish the raw material worked *upon*, and that the postulates and axioms furnish the implements worked *with*—the postulates supplying the elements of the constructions, the axioms the elements of the reasonings.

And here we must caution you against a very prevalent mistake. Do not for a moment imagine that Euclid required his postulates to be *granted*, because the fundamental operations, under that head, are so easy of performance; nor that his axioms are to be assented to because the truths so-called are so easy of proof. His reasons for these preliminary stipulations were of a directly opposite kind; he bargains with you to grant the possibility of his fundamental problems (the postulates), solely because he is unable, practically, to perform them; and he calls upon you to admit, without proof, his fundamental theorems (the axioms), solely because he is unable to demonstrate them. A postulate and an axiom should each have a twofold character: a postulate should be a conceivable, but, at the same time, a really impracticable *operation;* an axiom should be a self-evident, but, at the same time, an indemonstrable *truth.* Euclid asks us to "*grant* that a straight line may be drawn from any one point to any other," from sheer necessity; the apparent simplicity of the operation is in reality a cause of its difficulty. What operation, still more simple, could be made subsidiary to the drawing of a straight line? And how could he direct the performance of the latter, without some operation still more elementary? Besides, an isolated straight line, according to Euclid's strict definition, has no visible or external existence. Euclid's *line* is merely the abstraction *length;* and length, unaccompanied by other dimensions, cannot, of course, be actually exhibited. The finest line that you could draw upon paper would be a solid bar of ink; and the finest line an artist could engrave upon steel would be a sunken channel, with both breadth and depth. You may possibly think that, as the physical or material lines here adverted to are so very slender, it is not worth while to make any objection to them on the score of their width or thickness; but the "near enough," or the "that'll do" system, has no place whatever in *Euclid's* system, which is one of rigid, uncompromising accuracy. A line that you could draw and exhibit would no more be regarded as a line that Euclid had defined, than a beam of timber would be. Remember that with *him* "a miss is as good as a mile."

As with the postulates, so with the axioms; they are inserted from necessity, and solely because Euclid was unable to demonstrate them. That self-evidence alone was not considered by him as sufficient to justify the claim of a proposition to a place among the axioms, is plain, from his uniform practice of demonstrating whatever can be demonstrated, and assuming only what cannot; taking care, however, *in general*, that the truth assumed shall be, not only an ultimate truth, but also a thing perceived to be true as soon as enunciated. We say, *in general*, because there is one remarkable exception—the 12th axiom of the first book is indemonstrable, but not self-evident; it has one of the characteristics of an axiom, but not the other. In the preceding book, we have thought it prudent to keep this so-called axiom out of sight, till the 29th proposition was reached; because up to that point its aid is not required: and arguments, abundantly sufficient to produce full conviction of its truth, could be adduced *then*, though they could not have been employed at the opening of the subject.

Now you must not be surprised or disappointed that geometry—pre-eminently the science of demonstrated truth—should thus require to rest upon principles which must be gratuitously admitted. No reasoning process whatever can even be conceived to exist, unsupported by a like foundation. A proposition may be affirmed on the one side, and denied on the other; but the matter cannot be reasoned out—it cannot be *argued*, unless some common first principle or principles be at the commencement agreed to by both parties. If everything be denied, there may be assertion and contradiction, dispute and altercation, but certainly no *argument.* The noticeable thing in Euclid's first principles, or axioms, is, that, with the exception mentioned above, they are such as nobody in his senses would think of controverting, inasmuch as the truth of them is self-evident; that is, so immediately obvious, that nothing of the kind, anterior to them in obviousness and simplicity, can possibly be adduced; for if anything could, then *that* thing—being the more simple and elementary—would itself become the axiom, or first principle, by aid of which the former might be demonstrated. You see, therefore, that it is essential to the very nature of an axiom that it should be too simple and elementary to admit of demonstration by help of anything more simple and elementary. The axioms are, on this account, self-evident, indemonstrable truths. Proposition II., of Book III.—namely, that "If any two points be taken in the circumference of a circle, the straight line which joins them shall fall *within* the circle"—is a proposition as self-evident, to any one who has a clear conception of a circle, as that which affirms that "two straight lines cannot inclose a space;" but as it is demonstrable, it is very properly placed in the body of the work.

But it is time to give you a few words of advice as to the disposition of mind with which you should sit down to the study of geometry, and to notice some of the intellectual advantages which you have a right to expect from the time and attention devoted to the subject.

You have already seen what the preliminary conditions are, which Euclid makes with you: he considers these to have been conceded, without the slightest qualification or reserve. As far as he is committed, he will take care that they are faithfully adhered to, and he stipulates that they shall be equally binding upon you. They are fully and fairly placed before you in the Definitions, the Postulates, and the Axioms. In these matters he seems to assume a sort of magisterial authority, from which he allows no appeal. If you refuse to subscribe to the conditions which he himself lays down, he, on his part, refuses to be your guide: he can conduct you through the rich domain you wish to explore upon no other terms: he is inflexible as to his preliminary arrangements for the journey; and he as good as tells you, as he told King Ptolemy of old, that "there is no royal road to geometry."

If, therefore, you have any notion still lingering in your mind, about a line, or a triangle, or a circle, &c., not strictly in accordance with what he authoritatively declares to be *the* notion, you must dismiss either it or *him.* If you have any scruples or misgivings about his postulates or axioms, you must, in like manner, overcome them: you must examine them, and re-examine them, till you are fully convinced that the postulates are really conceivable operations, and that the axioms are really unquestionable truths; for, depend upon it, he will tie you down most rigidly to the *conditions*, and allow of no escape, evasion, or qualification. It matters nothing to him by what process of mind you satisfy yourself of the truth of his axioms, nor by what mechanical contrivances you seek to give an outward representation of lines, triangles, and circles; you may use pen, ruler, compasses, or whatever you please, for your own individual convenience or assistance; but remember, that he himself takes no cognizance of these things. If, without any instrumental aid at all, and by mere freedom of hand, you were to sketch the outline of an inclosed figure, and make *that* the representation, to yourself, of a circle, Euclid would find no fault with you, provided only you still, in imagination, endowed it with the characteristics of the true circle that he had defined. You will find a carpenter, or a mason, much harder to please, in a matter of this kind, than you would find Euclid. If you have carefully read what has preceded, you will see that we are quite justified in making this statement; for you will have learned that Euclid is not concerned with the *representations*, but with the *things* themselves, the purely intellectual conceptions.

Taking it, then, as a settled matter, that you receive

Euclid's definitions—that you *do* "grant" his postulates, and that you *do* fully acquiesce in the truth of his axioms, all preparatory ground is cleared; and you may proceed at once to the business proposed. And here you are to observe, that both you and your author occupy different positions. Euclid at once, from this point, drops his authority as a master; you withdraw your submission as a docile and obedient pupil. What he *says*, you are to regard no longer—only what he *proves*. You are to exercise a vigilant watchfulness over every statement he makes, receiving just so much of it as you cannot help receiving, and no more. *Faith*, in anything he advances, is not to be thought of. Be as sceptical as you please—nay, as sceptical as you can. Euclid would not thank you for any gratuitous concession whatever; all he demands is, that you will honestly respect the preliminary articles of agreement; and, in spite of all your opposition, and of all your scepticism, he will *compel* you —however much against your wish—to do unqualified homage—not to *him*—but to the truth he propounds.

And this is the attitude of mind you are to assume in entering upon the propositions of geometry. There must be no yielding to the dicta of a teacher—no intellectual obeisance to the authority of a great name. Every truth you acquire, you must *so* acquire as to feel and know it to be a truth, from your own perfect individual *conviction* that it is so. Your conviction must be so thoroughly inwrought and complete, that if a Newton, or even a greater than Newton, should attempt to controvert a truth thus secured, the effect of such an attempt upon your mind would be about the same as an endeavour to convince you that you are an inhabitant of the moon.

Now we think that, from these unqualified statements, you may fairly make two inferences, well worthy of consideration. The first is, that even in this, our frail and erring state, there is offered to our notice a system of unadulterated and incontrovertible TRUTH, built up by purely human effort, and consolidated and rendered imperishable by purely human reason. The second inference is, that the reasoning process, by which such an intellectual structure has been reared, must surely be of the most faultless kind—no logical error can have been committed—no conceivable objection unanticipated, and no case of exception unprovided for.

These considerations alone seem to us amply sufficient to incline all who have the time and opportunity, to a diligent study of geometry, apart from all regard to practical applications. Only reflect for a moment upon the *habits of mind* which such study must necessarily foster, where they in any way exist, or *create* where they are wanting. The frequent contemplation of *Truth* has a salutary and an ennobling influence. Next to Inspired Truth, the truths of pure science furnish the most exalted materials upon which the human mind can exercise its powers. He who is earnestly and *successfully* engaged in this exercise, comes, at length, to love truth for its own intrinsic excellence; to be fascinated with its unadorned beauty; and to entertain increased repugnance towards the deformities of falsehood. Habits of mind, whether good or bad, are the fruits of seeds usually sown in youth; they become formed and fixed from the natural effects of those trains of thought in which we most frequently indulge in early life; and hence the study of geometry, and of the sciences which carry out its pure principles, have an important influence, even in a *moral* point of view. To secure the operation of this influence is surely deserving an effort. The properties of geometrical figures may be matters of perfect indifference to us—we may take but little direct interest in what relates to triangles, parallelograms, and circles; but we cannot be indifferent to a truthful habit of mind; and though all the theorems of Euclid be forgotten, yet if *this* remain as an abiding result, how great will be the acquisition we shall have made!

But the *intellectual* advantages connected with the study of the "exact sciences," are even more certain and palpable than the *moral* advantages here alluded to. You cannot read a proposition of Euclid as it ought to be read—and indeed as it must be read, in order to be fully understood—without a concentration of attention more intense than most other subjects, out of mathematics, demand; and since, as just noticed, there must be no disposition to admit anything whatever, without the most complete conviction of its truth, a habit of scrutinising evidence, and of distinguishing between plausibilities and proofs, is insensibly but securely acquired. There is, perhaps, no faculty of the mind which in early life stands in more need of cultivation than the reasoning faculty; for that every one reasons, or at least engages in what goes by the name of reasoning, is so generally admitted as a distinguishing peculiarity, that man has been even defined to be "a reasoning animal." Locke says, "Would you have a man reason *well*—let him learn geometry;" that is to say, if to reason well be the only end in view—all the truths of Euclid being regarded as utterly valueless—still let geometry be studied. Yet geometry supplies no rules; it prescribes no directions for conducting a logical process; but, what is better, it places before us a collection of the most exquisite models. It teaches by example, not by precept; and no one, with proper attention, can fail to profit by its lessons.

You see, therefore, that Euclid is something more than a mere problem-book for the use of architects and surveyors: it is the most finished treatise on the "Art of Reasoning" that the world possesses; and it is chiefly as such that we are anxious to recommend to our young friends a careful study of its contents. It may fail to render you much direct professional service; but the mental discipline it furnishes will strengthen your judgment, improve your logic, give additional acuteness to your penetration, and, in fact, so enlarge and invigorate all the faculties of your mind, that you will be enabled to bring a higher degree of intellectual power to bear upon any pursuit in which you may earnestly engage. It is not the properties of geometrical figures that can do this: it is the reasonings by which they are established. The several stages at which you arrive, in your progress through Euclid, may present but few points of attraction; but you *must* be benefited by the invigorating influence of the journey.

And here it may perhaps be as well, in order to prevent misunderstanding, that we should offer a remark or two in reference to a direction given you above—namely, that you should approach the demonstrations of Euclid in a *sceptical* spirit. It has been foolishly, and most unjustly affirmed by some, that the study of pure science has a tendency to produce *general scepticism*. Now all truths are harmonious; they have common features and common attractions, recommending themselves to our homage by the same dignified aspect and bearing. How can scientific truth ever be out of keeping with inspired truth? A religious sceptic is generally something more than a mere *neutral* as respects divine things: he is usually a *denier*; that is, he embraces a *negative proposition*, and acts upon it without proof! How does geometry sanction this? In Euclid there are negative propositions as well as affirmative ones. We recommend you, anterior to proof, to be equally sceptical as to *both*. In things out of geometry, geometrical demonstration is, of course, not to be had; it would be folly to look for it; yet, if in such things an affirmative be declared on the one hand, and a negative on the other, do you not think that the logic of geometry, as well as the logic of common sense, would incline us to *that*, in support of which *some* evidence was offered, rather than to that which had no such support at all?

But there are sceptics of a different stamp from the class noticed above; men of literary and philosophical habits, who do not content themselves with a "cold negation." They address themselves to the task of undermining the existing evidences of the truth of Christianity.

Such a man was David Hume, a distinguished writer of the last century. He wrote an essay to prove that *a miracle* never could have been performed, or, at least, that we have no reliable evidence of its performance. He laid down certain preliminary principles, and dressed his performance in the garb of *fair reasoning.* But Hume knew nothing of mathematics; and, in invading its province, he trod upon slippery ground, and *fell.* Those who *did* know something of the "exact sciences" betook themselves to the investigation; they started upon precisely the same principles as Hume did, without the assumption of a single additional particular, and they arrived, by a train of reasoning which it is impossible to disturb, at a conclusion *directly opposite to his.* Had Hume known but a little of mathematics, and had he revised his so-called *argument,* in the same sceptical spirit which we recommend to you, a regard for his own literary reputation merely, would have precluded him from ever publishing to the world his *Essay on Miracles.**

You have already seen that the propositions of geometry are of two kinds—problems and theorems. A proposition is called a *problem,* when the thing proposed is an operation to be performed—a construction to be effected: its object is a practical result, to be brought about by a suitable disposal and combination of the elementary materials furnished by the *postulates.* A proposition is called a *theorem,* when the thing proposed is a truth to be demonstrated; and for this demonstration the elementary materials are furnished by the *axioms.* It usually happens, however, that the proof of a theorem requires the previous introduction of certain lines and constructions; and hence it is that Euclid commences his first book with problems instead of theorems. The only theorem in this book, that is quite independent of a problem, is Proposition IV. Euclid probably considered it more systematic to introduce the problems, which he foresaw would be indispensable as soon as the fourth proposition was disposed of, at the *beginning* of the book, than to interpose them between propositions so closely related as the fourth and fifth.

Of the three problems thus found to be necessary to meet the demands of the subsequent theorems, the second is the only one which seems to require any comment. Beginners in general find it difficult, and are apt to consider that there is an unnecessary parade of geometrical apparatus exhibited to effect so simple a matter as the drawing of one straight line equal to another. There is no doubt that a mere mechanic would pronounce the proceeding a very round-about one; he would accomplish the business at once, by drawing a straight line from A at random; and then, having taken the length B C in his compasses, he would apply one foot at A, fixing it there as a centre, and with the other foot would cut off the required length A L; thus dispensing with all Euclid's machinery—the circles and the equilateral triangle. Now this is all very well for the purposes of the practical workman, who neither seeks nor expects rigid accuracy in his constructions; but you must remember that Euclid ignores compasses, and that the instrumental transference of one line to another is not warranted by any postulate. No one can take *accurately* any stipulated length in a pair of compasses; the limitation of his vision precludes his pronouncing, with perfect certainty, that he has got *exactly* the proposed length, neither more nor less. If the minute error, whether in excess or in defect, be only so small as to escape his senses, he cannot take cognizance of it; and he not only practically, but from necessity disregards it. But without any additional postulate, Euclid shows you how the thing proposed may be done without the possibility of any error at all. In the ordinary editions of Euclid, we think justice is scarcely done to the process indicated. You are directed, first, to draw a line A E *longer* than B C, and are then shown how to cut off a part A L *equal* to B C. The ingenuity of Euclid's mode of proceeding would be more apparent, if no superfluity of length were at first introduced. It certainly seems a thing of much greater difficulty to draw a line from a point A, till a certain prescribed length be attained, and then, but not till then, to stop. We would therefore recommend you to leave the prolonging of D A to the very last; so that, having performed every other part of the construction, prolong D A, as a final step, till the prolongation reaches to the circumference of the outer circle. A line A L will thus have been drawn equal to B C, and there will be no excess of length to throw away. This is the mode of proceeding adopted in the present work.

As to the other two problems, the first and third, but little need be said; the directions given by Euclid for the construction of them are too clear and explicit to render further explanation necessary. It may be well, however, to invite your attention to two particulars in connection with Proposition I., which instructs us how to describe an equilateral triangle upon a given finite straight line. Some commentators object to the word *finite* as superfluous, considering the condition, "a *given* straight line," to imply that the *length* is fixed and determinate. But a line may be given in *position* only, without any limitation as to *length,* as in Proposition XI.; or it may be given in *length* merely, without any restriction as to *position:* thus the line to be constructed in Proposition II. is to have a given length, but is unrestricted as to position or direction. By "a given *finite* straight line," Euclid means a line of given length, and with given extremities; and if any objection at all be made, in reference to this word *finite,* we think it should be urged against the omission of it in Proposition II., rather than against the introduction of it in Proposition I. In the *first* proposition, we dare say Euclid thought it prudent to prevent all cavil, to state explicitly that the extremities, A, B, are given; and that he did not consider it necessary to repeat this in Proposition II. Whenever a line is given in position merely, and restriction as to its length forbidden, Euclid characterises it as "a given straight line of unlimited length," as in Proposition XII.; but when it is a matter of perfect indifference whereabouts the extremities are (position alone being all that we are concerned with), the terms "a given straight line" are those always employed, as in Proposition XXXI. A mere glance at this proposition will show you that the given straight line of Proposition I. cannot be so entirely free from restriction, as to length, as the given straight line of Proposition XXXI.; and hence the propriety of the restrictive term *finite* in the former. We should not have said so much about a mere word, had it not been for the hypercriticism of others. You must therefore regard these remarks, not as a comment upon *Euclid,* but as a comment upon his commentators.

The other matter we should wish you to notice, in connection with this first proposition, is, that what is called "the point C, in which the circles cut," is, in fact, either of *two* points, one on each side of A B, so that a second equilateral triangle may be described, on the opposite side of the given line; and it is this latter position which the equilateral triangle, introduced into the construction of Proposition IX., is to take. We shall only further notice, that it would have been somewhat more explicit if Euclid had referred to the point C as where the *circumferences* cut, rather than as where the *circles* cut; though it is quite true that the circles themselves interpenetrate there; but the distinction, very properly made by Euclid in his definitions, between circle and circumference, is in danger of being overlooked by a beginner, in consequence of Euclid's mode of expression, in reference to intersecting circumferences.

Proposition IV. is the first of Euclid's *theorems;* and, being the first, and involving no construction, its proof depends solely on the axioms. The demonstration hinges upon what has been called the method of *superposition;* that is, the imagining one figure to be placed upon another, with a view to their perfect adaptation and coincidence, and thence to the inference of their complete *equality* (Axiom 8). If you wish to try the experiment whether or not you have any taste or aptitude for geometrical reasoning, you may, if you please, *commence* with this theorem, and study the three preceding problems afterwards. It is a very beautiful specimen of Euclid's mode of argumentation; and is quite within range of the powers of the merest beginner. But before you address yourself to it, it may be as well to reflect, that the relative *positions* of two things can have nothing to do with their relative *magnitudes*; that the two triangles, affirmed under certain conditions to be *equal*, and placed side by side before you on the paper, must remain equal, however their relative positions be altered; whether one be turned upside down, or be made to overlap the other, can make no difference as to the equality or inequality of the two in *magnitude*. You will thus see that, for the purpose of instituting an inquiry as to the equality of the triangles, it is quite allowable to imagine one to be placed upon, or to be applied to the other, with the view of ascertaining whether, by a suitable adjustment, a complete adaptation may be brought about. In this manner Euclid directs you to apply the triangle A B C to the triangle D E F in a certain way—namely, so that the point A may be on D, and the straight line A B upon D E. He then affirms that B must coincide with E; and as he never affirms anything without immediately answering the inquiry *why?* he adds, *because* A B is *equal* to D E. The adjustment is thus brought about as far as the sides A B, D E are concerned. He then asserts that, this partial adjustment remaining undisturbed, A C must fall upon D F, *because* the angle or opening A is equal to the angle or opening D. If A C fell *beyond* D F, the angle A would be *greater* than the angle D; and if A C fell *short of* D F, the angle A would be *less* than the angle D; Euclid's conclusion, therefore, is irresistible. As A C then necessarily falls upon D F, the point C must as necessarily fall upon or coincide with the point F, *because* A C is *equal* to D F. And thus having proved, first, that B coincides with E, and then that C coincides with F, he infers, in virtue of the 10th axiom, that the base B C must coincide with the base E F. Hence the adaptation is complete; there is perfect coincidence, and therefore perfect equality in every respect.

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* See Babbage's *Bridgewater Treatise*—Appendix. Also Young's *Three Lectures on Mathematics.*

Proposition V. is always found to be more or less perplexing to a learner; and it is certainly one of the most, if not *the* most knotty of the propositions in the first book. We would recommend a beginner, after completing the construction as directed, to erase the base B C of the original triangle, in order that nothing may divert his attention from the two triangles A F C, A G B. The line thus expunged may be restored after these triangles have been proved to be in all respects equal: they are nothing more than the two triangles already considered in Proposition IV. in a different position, one triangle partially overlapping the other. Young students are sometimes deterred from prosecuting the study of Euclid by the length and difficulty of this proposition. They should be apprised that the propositions are not arranged in the order of their difficulty; that none more troublesome than this fifth will ever after be met with, and that the last theorem in the book is quite as easy as the first. A great point will be gained, if you master this fifth proposition; for you may then conclude with confidence that you will find yourself fully adequate to all that follows: but you must not come to this conclusion till, closing the book, you find yourself able to demonstrate the theorem step by step without a reference to it. This mode of testing your progress must be resorted to all along. It is not enough that you read and understand Euclid's demonstrations—you must acquire the ability of furnishing these demonstrations yourself; you may vary the language, but you must preserve the rigour of the argument, affirming nothing without a reason.

We do not clearly see why the fifth proposition should be called *Pons asinorum*, or *the asses' bridge*. They say it is because "asses" stick at it; but we believe it was the *twentieth* proposition that was so designated by some of the ancients; for Proclus informs us, in his *Commentary on Euclid*, that the Epicureans derided the twentieth proposition as being manifest "even to asses;" for if a bundle of hay were placed at one extremity of the base of a triangle, and an ass at the other, the animal would not be such an ass as to take the crooked path to the hay instead of the straight one; as he would know the direct course to be the shorter: *this* was therefore called the *asses' bridge*.

The sixth proposition you will find very easy after the fifth: it is what is called the *converse* of the first part of the former proposition. A theorem is said to be the converse of another, when the hypothesis and the consequence, in that other, change places. The hypothesis in Proposition V. is, that two *sides* of the triangle are equal; the inference or consequence is, that the two *angles* opposite to them are equal. The hypothesis in Proposition VI. is, that two *angles* of a triangle are equal; and the consequence is, that the two *sides* opposite to them are equal: the one proposition is therefore the converse of the other. In general, Euclid demonstrates the converse of a previous theorem *indirectly*, or by what is called the *reductio ad absurdum* method; that is, he commences with a denial of the truth stated; and, reasoning from the contradictory statement, as if it were true, shows that an absurdity, or impossibility, is the unavoidable consequence; thus proving that the thing contradicted cannot be otherwise than true. Throughout the whole of this book, the last proposition is the only converse theorem that is not demonstrated in this indirect manner. It is not every theorem that is true both directly and conversely. You should take note of those that Euclid proves to be *convertible*, and endeavour to discover for yourself which of his propositions hold conversely, though only proved directly. For example, Proposition XXXIV. proves that if the opposite sides of a quadrilateral are *parallel*, they are likewise *equal*. It is also true conversely, that if the opposite sides are *equal*, they are likewise *parallel*, as you may prove for yourself, after the direct proposition has been established.

Proposition VII. is merely subsidiary to the proposition next following; it is what in some geometrical writings would be called a *Lemma*. You see that the demonstration of it rests almost entirely on Proposition V. In some modern books on geometry this proposition is dispensed with, and the eighth established independently of it; but, as an intellectual exercise, Proposition VII. is as useful as any in the book. Besides, a proposition, though manifestly introduced as merely subsidiary to something else, may yet possess intrinsic excellence of its own sufficient to justify its retention in the system. For instance, the proposition before us teaches us this fact, which is certainly not without interest; namely, that a physical triangle, supposed to have its sides freely movable about joints at its vertices, cannot possibly be thrust out of shape by any force whatever. You may break the bars forming the framework, but you cannot make the frame itself assume another shape.

Proposition VIII. is the *second* proposition, in the geometry of triangles, which proves that two triangles are equal in every respect—that is, that each is but an exact copy of the other—provided *three* things in one are respectively equal to three corresponding things in the other. The three things may be two sides and the included angle, as we learn by Proposition IV., or the three sides, as the present proposition teaches. And we may as well observe here, that there is only one other proposition in the Elements where the like equality of two triangles is inferred from an equality of three things in one to three corresponding things in the other: it is Proposition XXVI. On these three propositions the practical part of *plane trigonometry* is founded. A triangle, in the language of trigonometry, is said to have six *parts*—the three sides and the three angles; and when certain sets of *three* of these are given—either of the sets, namely, mentioned in Propositions IV., VIII., and XXVI.—the remaining three, which we see by these propositions must be fixed and invariable, become determinable, and are matters of computation.

And here it may not be amiss to say a word or two about the form of expression continually employed by Euclid, when comparing figures together, for the purpose of establishing their equality. He always speaks of two sides or angles of the one being equal to two sides or angles of the other, *each to each*. Learners are apt to omit this qualifying condition, "each to each," as if the frequent repetition of these words were only so much useless tautology; but precision requires that they should always be retained. If you were to say that two sides of one triangle are equal to two sides of another, your meaning might be taken to be, that the *aggregate* or *sum* of the two sides of one triangle is equal to the aggregate or sum of the two sides of the other; but the addition of the words "each to each" would preclude the possibility of such a mistake, and would show that the sides, taken separately and individually in the one triangle, were affirmed to be equal to corresponding sides in the other.

The four propositions next following are *problems*. You may be pretty sure that Euclid has postponed them till they became indispensable. We do not think Euclid *liked* problems; at all events, there is less careful finish about them than in his theorems. Proposition IX., for instance, needs mending a little; it professes to teach how to bisect an angle, of whatever magnitude it may be. Now, suppose that the triangle A D E, in the book, is an *equilateral* triangle, and that we want to bisect the angle D A E. Euclid tells us to construct an equilateral triangle on D E; and without the diagram before our eyes, where the construction is exhibited in its completed state, we should naturally describe the equilateral triangle he directs, *above* D E, and not *below:* in which case we should get nothing; for our new equilateral triangle would simply cover the one already there, and the point, F, falling on A, would have no separate existence; so that there would be no guide to the drawing of A F, the bisecting line; it should have been distinctly stated, therefore, that the equilateral triangle, to be described on D E, should have its vertex, F, on the opposite of D E to the point A. This restriction is introduced in the present edition.

In going over Euclid's proposition without the book, as we have recommended above, always refrain from *copying the diagrams.* We know that such is the usual practice; but it should be condemned. The progress of the diagram should just keep pace with that of the text, and no line should be introduced till it is actually demanded by the text. It would be nearly as faulty as to write out the whole text, and then to supply the diagram (as the boy did who said he would tell the story first, and draw the picture afterwards), as to commence with the completed diagram, and then supply the text. In a printed book, the diagram must, of course, be presented *completed:* but in your own private practice you should make it grow to maturity along with the text. In the whole course of your geometrical studies, let us urge upon you never to allow your judgment or conviction to be in the slightest degree biassed by your visual impressions from the *diagram.* Let two lines look ever so like two equal lines, do not forestal the reasoning, and conclude them equal from their appearance; remember always that you are engaged in a purely *intellectual* process, and that you are not to be allured by the *matter* from the *mind.* Graphical accuracy, in the figured form, is of no moment; logical accuracy, in the abstract reasoning, is all that you have to attend to; and therefore we think it worse than waste of time to be over-scrupulous with scale and compasses, in reference to the lines introduced into Euclid's demonstrations; we have already given you some hints on this matter.

There is nothing that calls for special notice till we reach Proposition XVI. This is easy enough, as far as Euclid carries the demonstration; but when, at the close, he says, as in other editions of the Elements he is made to do, "in the same manner it may be demonstrated," a beginner is likely to feel a difficulty. There is really a good deal to do before the proof can be completed; and, when completed, "in the *same* manner," there is a needless amount of complication. We would advise you to finish the reasoning rather differently. By carefully looking at the argument, you will see that this truth is established, and nothing more; namely, that if one side of a triangle (*any* side, of course) be produced, the exterior angle is *greater than that interior angle which is opposite to the side thus produced;* the angle A C D is thus greater than A. Let now A C be produced to G; then, since the exterior angle is greater than that interior one which is opposite to the side produced, the angle B C G is greater than A B C; but B C G is equal to A C D (by the fifteenth), therefore A C D is greater than A B C; but A C D was shown to be greater also than B A C; *therefore* A C D is greater than *either* of the interior and opposite angles, B A C, A B C. It is this form of completing the demonstration that has been adopted in the present work.

Proposition XVII. would seem, at first sight, to have been introduced without any object. The truth of it is clearly implied in Proposition XXXII., and it is not required in any of the intervening propositions. But that Euclid had an object is not to be questioned; and it seems to have been this:—It was desirable that, at some convenient place, before the introduction of the theorems respecting parallel lines, something should be established by demonstration that would diminish the repugnance, very properly felt at the outset, to the *twelfth axiom.* We have recommended you (p. 540) to keep this axiom in the background till you arrive at Proposition XXIX., where a reference to it becomes indispensably necessary. The axiom is no other than the *converse* of this seventeenth proposition; *this* shows that if two meeting or non-parallel lines, B A, C A, be cut by a third line, B D, the two interior angles, C B A, B C A, on the same side of it, are together *less than two right angles;* and the twelfth axiom asserts, *conversely,* that if a straight line cutting two others make the two interior angles on the same side of it *less than two right angles,* those must be non-parallel or meeting lines.

The seventeenth proposition, therefore, enables us to see more clearly the exact amount of assent demanded of us by the twelfth axiom, and prevents our overrating that amount. If two lines cut by a third *meet,* the two interior angles are less than two right angles: this is *proved.* If two lines cut by a third make the two interior angles less than two right angles, *they meet:* this is *assumed.*

Passing over, for the present, the intermediate propositions, let us suppose Proposition XXIX. to be reached. The two propositions immediately preceding sufficiently show that the lines called parallel lines *exist;* the twenty-ninth demonstrates a property of them, admitting the truth of the axiom just mentioned. Geometers without number have tried, some to evade this axiom altogether, and others to prove it by establishing the converse of Proposition XVII.; but all have failed. What can be the cause of this failure? Is it not in the imperfect definition of a *straight line?* Our conception of a straight line, independently of all formal definition, necessarily involves two ideas; namely, that of *length,* and that of *uniformity of direction. Length* is implied in the word *line;* and *invariability of direction* in the term *straight.* A line which changes its direction is a *crooked* line or a *curved* line; a line that never changes its direction is a *straight* line. Now it necessarily follows, from this uniformity of direction, that if two straight lines, however far prolonged, can *never* meet, then at no part of their course can either make any *approach* towards the other; for if two lines, proceeding in any two directions, *approach* and continue undeviatingly to pursue those directions, they cannot fail eventually to *meet.* It follows, therefore, that parallels must *throughout* be equidistant; but *two* distinct straight lines, through the same point, cannot *throughout* be equidistant from a third; so that *two* straight lines through a point cannot *both* be parallel to the same straight line. Proposition XXVIII. shows that *one* (C D) will be parallel to another (A B), provided a line, cutting both, make the interior angles together *equal* to two right angles; a *second* line through H, which causes the interior angles to be *less* than two right angles, being distinct from C D, must therefore meet A B, if prolonged; and this is the assertion of the twelfth axiom.

What is here said, remember, is not a *demonstration* of this *axiom.* An *axiom,* you know, is an indemonstrable truth. All we wish to show is, that it *is* an axiom; that is, a truth necessarily implied in the correct conception of the thing to which it refers. If you steadily contemplate your conception of a straight line, giving due consideration to its distinguishing peculiarity —*uniformity of direction*—you must see that *two*, from the same point, cannot *both* be equidistant from a third; *one* may be parallel to this third, or everywhere equidistant from it, since, as we have seen, parallels are *possible*; but *one* must of necessity meet it.

Of the propositions passed over, the 22nd and 24th are the only ones requiring any special notice here. Both these, as given in the text of Dr. Simson, are open to objection. In the first of them, it is taken for granted that the two circles employed in the construction must *cut* one another; and in the second, it is assumed that the point F falls *below* the line E G. You will find these defects acknowledged in the notes at the end of Simson's *Euclid;* but they were first pointed out by Mr. Thomas Simpson, in his *Elements of Geometry*. The emendations of the latter were, however, but ill received by the "restorer of Euclid," who treated the "remarker," as he called him, with a good deal of contempt; the more to be reprobated, as the poor self-taught weaver (for such Simpson in early life was) was very superior, as a man of science, to his academical opponent, great as were the merits of the latter in the field of ancient geometry. The biography of Thomas Simpson is full of instruction and encouragement to the young and unaided student, who cannot fail to view with interest the steps by which a person in Simpson's position, without books, money, or friends, plying his humble calling among the lowest ranks of society, was conducted, by the force of perseverance, to the proud eminence which he eventually attained. In the annals of science he ranks among the most distinguished mathematicians of the last century; and yet, at the age of nineteen, he was ignorant of the first rudiments of common arithmetic.[*]

The defect above alluded to, in Dr. Simson's version of the 24th Proposition, is removed in the present edition; and the objection made to the reasoning in the 22nd may be disposed of as follows:—After having described the circles, as at page 538, reason thus: *One of these circles cannot be wholly without the other*, for then F G, the distance of their centres, would be either equal to, or *greater* than the sum of the radii; but, by hypothesis, it is *less*. *Neither can one of the circles—*as, for instance, that whose centre is G—*be wholly within the other;* for then the radius, F D, of the latter would be equal to, or *greater* than F H; but, by hypothesis, it is *less*; hence, since one circle can be neither *wholly* without the other, nor *wholly* within—they must be *partly without* and *partly within* one another, ∴ they must *cut* in some point K. This completion of the proof may be introduced in a second reading of the first book.

Proposition XXXII. is among the most interesting theorems of this first book; but an objection to the demonstration of it may be made, the occasion for which had better be removed. You are directed to draw, through the point C, a line C E parallel to A B, by Proposition XXXI.; and it is then inferred that the alternate angles B A C, A C E are equal, by an appeal to Proposition XXIX.; but to draw the parallel C E, surely everybody would proceed by making the angle A C E equal to B A C; that is to say, we should first make the alternate angles equal to get the parallels, and should then make use of the parallels to prove the alternate angles equal. You will at once see that we should avoid this circuitous method of proceeding, by making the angle A C E equal to B A C by Proposition XXIII.; and then inferring the parallelism of A B, C E from Proposition XXVII., so that Proposition XXXI. need not be called into operation at all.

The corollaries to this proposition are remarkably beautiful; and the second, especially, cannot fail to excite, in a person who reads it for the first time, a feeling of surprise. It would indeed be a feeling of *incredulity*, if this were possible in geometry. That the sum of the exterior angles, formed by prolonging the sides of a rectilineal figure, should always be *exactly* the same, whether the figure have three sides or as many thousand, is a truth so far beyond the reach of practical observation and experiment, and apparently so improbable, that, in the absence of geometry, its existence could scarcely have been suspected, much less established; and yet an argument of half-a-dozen lines produces in every mind the fullest conviction of the fact.

But the corollary that precedes this, though less striking, has, perhaps, the greater practical interest; among other things, we learn from it that—the sum of the angles of a four-sided figure is *twice* as great as the sum of the angles of a three-sided figure; the sum of the angles of a five-sided figure, *three* times as great; of a six-sided figure, *four* times as great, and so on; but the most noticeable practicable inference is, that only *three* regular figures,[†] namely, the equilateral triangle, the square, and the regular six-sided figure or *hexagon*, can, by repetition, completely cover a surface; in other words, that, without leaving any blanks or interstices, we may cover a surface with a mosaic work of equilateral triangles, or of squares, or of regular hexagons, but not with regular figures of any other kind. It would be impossible, for instance, to form a piece of tessellated pavement with slabs of any other regular figure but one of these three; because the uniting together of any other forms, by adjusting side to side, would not fill up the space about the corners—there would be either left an angular gap, or else the stones must overlap one another. You will readily see the truth of this from the following considerations:—

Let us first consider the equilateral triangle: as the three angles make two right angles, each must be $\frac{1}{3}$ of two right angles—that is, $\frac{1}{6}$ of *four* right angles; consequently, if *six* equilateral triangles were placed side by side, a corner or vertex of each being at the same common point, all the angular space about that point would be completely occupied; and no one triangle would overlap another, for the angles about a point amount to just four right angles (Prop. XIII., Cor. 2). Let us next consider the square; and, as each angle of a square is a right angle, it is plain that *four* squares, each with a vertex at the same point, when placed in contact, will exactly fill the space about the point.

The figure next in order is the regular *pentagon*, or five-sided figure. The corollary teaches us that the sum of its five equal angles amounts to six right angles; consequently each angle is *one* right angle and *a fifth*. Now you cannot multiply $1\frac{1}{5}$ by any whole number that will make the product 4—no such number exists; in other words, you cannot arrange the angles of pentagons round a point, as the common vertex of all, so as to fill up the *four* right angles about that point; the pentagon, therefore, must be rejected. The next figure is the hexagon. By the corollary we learn that its six equal angles amount to eight right angles: consequently each angle is equal to $\frac{1}{6}$ of eight right angles; or, which is the same thing, to $\frac{1}{3}$ of four right angles. It follows, therefore, that three regular hexagons, placed side by side, round a point, as a common vertex, will exactly fill the space about that point.

We need not extend the examination any further; for, as the sides of regular figures increase in number, the angles increase in magnitude; and, as it has just been seen that *three* angles of a six-sided regular figure, make four right angles, more than *two* of a seven, or eight, or nine-sided figure, could never be required; but if *two* equal angles of any figure could make *four* right angles, each angle would equal *two* right angles, which is absurd. It thus follows, that if space about a point is to be filled up by the juxtaposition of regular figures, these figures must be either equilateral triangles, squares, or regular hexagons.

Of these three classes of figures, it is demonstrable, by more advanced principles, that the hexagon will enclose a given amount of space, with less extent of outline or border than either of the others; so that if a given surface were to be divided into regular compartments, of equal area, by a network of costly materials, economy, guided by science, would suggest the hexagon as the figure to be chosen. And this is the figure selected by the bee in the construction of the honeycomb. You have seen that of all the figures of geometry, there are but *three* which can so cover a surface as to leave no waste of room—no interstices. The bee chooses one of *them*. Of these three, the hexagon is that which most economises material:—the bee chooses *it*. It is necessary, too, as well for compactness and strength, as for the safe lodgment of the grub, that the hexagonal cell should terminate in a solid angle. Her choice of angles, that *would do*, is to be made from an infinite variety; but among all these, mathematicians have discovered, by a profound analysis, that there is *one*, and but one, mode of formation by which the object would be attained with the least expenditure of materials; and this one the bee adopts. She closes her hexagonal tube with an angular termination, formed by three plane faces. Each plane cuts off or excludes a portion of the tubular space; but the space within the solid angle just makes up for what is thus rejected; and the faces of the solid angle are so shaped, and so inclined, as to fulfil all the mathematical conditions of a *minimum* of surface; so that while the angular space just compensates for the tubular space cut off, it effects the compensation not only with *less* amount of material, but with the *least* amount possible. The compensating space might be secured in an infinite number of ways. Of this endless variety there is *one* way more economical than any other, and that one is chosen by the bee.

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[*] See account of Simpson, in *The Pursuit of Knowledge under Difficulties;* also Hutton's *Mathematical Dictionary*.

[†] Right-lined figures are said to be *regular*, when they are both *equilateral* and *equiangular*.

You must regard this reference to the architecture of the bee as a digression, into which we have been allured by the preceding examination of Euclid's corollary. But the subject is full of instruction; and when your mathematical knowledge is sufficiently extensive to enable you fully to estimate the science of the honeycomb, you may, like the ancient geometrician Pappus, feel even additional reverence for the Creator's power, and additional gratitude that you have been endowed with faculties to comprehend and admire the exquisite geometry of the bee.

Proposition XXXVII. may be omitted. It is included in the more general theorem which follows, and which is demonstrated independently of the particular case. What is proved of triangles upon *equal* bases is, of course, proved of triangles on the *same* base.

Propositions XXXV. and XXXVI., about parallelograms, seem to be related to one another, just as these two propositions about triangles are related. But there is a difference; for, although the thirty-fifth is only a particular case of that which follows, yet it is not superfluous, since the particular case is required in the proof of the general theorem. The thirty-ninth proposition, however, like the thirty-seventh, *is* superfluous; as the general theorem which follows does not require its aid. The forty-first, too, might have been replaced by a proposition of wider generality. You can give it the extension here suggested by putting "equal bases" for "the same base" in the enunciation, and modifying the construction and demonstration accordingly.

Before passing on to the few remaining propositions, it may be worth while to notice that the term *trapezium* is used in the thirty-fifth proposition for the first time. This is, in fact, the *only* place in which it occurs in the *Elements*, though it is a term frequently employed in the applications of geometry to practical matters, such as mensuration, surveying, &c.

Proposition XLII. requires a word or two. The line A E is improperly introduced, in other editions, into the *construction*; you will see, by attending to the details, that the required parallelogram is completed without any aid from this line; it is in the *demonstration* alone that A E becomes necessary, and therefore its introduction should be deferred till the construction is completed.

Of the forty-seventh proposition but little need be said here. It is stated to have been discovered by Pythagoras, who is recorded to have sacrificed a hundred head of oxen to the gods on the occasion; but this is probably a fable. The multiplication table has also been generally attributed to the same philosopher; but the modern French geometrician, M. Chasles, in his historical researches, has shown this to be a mistake.

The proposition in question is one of the most important, in its practical applications, of all the theorems of geometry. Many forms of demonstration have been given; but that of Euclid is not to be surpassed in elegance and clearness.

The last proposition is remarkable only for the peculiarity of its demonstration; it is the converse of the preceding, and is demonstrated in the *direct* manner, contrary to Euclid's general practice of demonstrating converse propositions *indirectly*. As an exercise, you may supply an indirect demonstration yourself: other exercises on the first book are here subjoined.

EXERCISES ON BOOK I.

1. Prove that the two diagonals of a parallelogram bisect each other.

2. If in the sides of a square, at equal distances from the four vertices, four points be taken, one in each side, the figure formed by the four straight lines, joining the four points, will also be a square.

3. From two given points to draw two straight lines, to meet in a given straight line, and to make equal angles with it.

4. Prove that the two lines drawn, as in the last exercise, are, together, *less* than any other two lines from the points meeting in the given line.

5. To divide a right angle into three equal parts.

6. The diagonals of a rhombus intersect each other at right angles.

7. If the three sides of a triangle be bisected, the perpendiculars drawn to the sides from the three points of bisection will all meet in the same point.

8. The straight line which bisects two sides of a triangle will be parallel to the third side.

9. If the middle points in the sides of any quadrilateral figure are joined by four straight lines, the figure so formed will be a parallelogram.

10. The sum of the angles of a four-sided figure is twice as great as the sum of the angles of a three-sided figure; the sum of the angles of a five-sided figure, three times as great; of a six-sided figure, four times as great; and so on.

11. Through a given point, between two non-parallel straight lines, to draw a third straight line, terminating in the former, which shall be bisected at the given point.

12. From whatever point within an equilateral triangle perpendiculars be drawn to the sides, their sum shall always be the same.

13. If from any point within a parallelogram, lines be drawn to the four vertices, each pair of opposite triangles thus formed will be, together, equal to half the parallelogram.

14. If two triangles have two sides of the one equal to two sides of the other, each to each, and if the angle contained by the two sides of the one, *together with* that contained by the two sides of the other, make two right angles, the two triangles will be equal in surface, or area.

15. In the figure to Proposition XLVII., if F D, G H, E K, be joined, the triangles F B D, G A H, K C E, will all be equal to one another, and to A B C.

16. Prove the *converse* of Proposition XXXIV.; namely—if the opposite sides of a quadrilateral be equal, or if the opposite angles be equal, the figure will be a parallelogram.

17. If the points of bisection of the three sides of a triangle be joined, the triangle will be divided into four component triangles, all equal to one another.

CHAPTER II.

ELEMENTS OF EUCLID.—BOOK II.

DEFINITIONS.

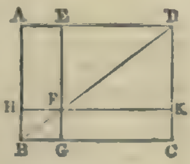
I.

Every right-angled parallelogram, or *rectangle*, is said to be *contained* by any two of the sides which contain one of its angles: that is, by any two *adjacent* sides.

Thus the rectangle A C is said to be *contained* by the adjacent sides A B, B C, or by A D, D C, and is often called, for brevity, "the rectangle A B·B C;" or "the rectangle A D·D C." And when the adjacent sides are made equal to two detached lines, each to each, it is common to refer to the rectangle as contained by the lines to which the adjacent sides have been made equal. Thus the rectangle B H, in Proposition I. following, is referred to as the rectangle A·B C, because B G = A.

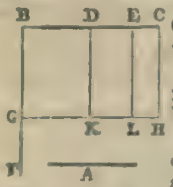
II.

In every parallelogram, either of the parallelograms about a diagonal, together with the two complements, is called a *gnomon*. Thus the parallelogram H G, together with the complements A F, F C, is a gnomon; it is expressed by the *three* letters A G K, or E H C, at the opposite vertices of the parallelograms, which make the gnomon.



PROPOSITION I.—THEOREM.

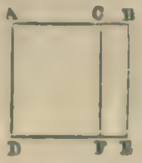
If there be two straight lines (A and B C), *one of which* (B C) *is divided into any number of parts, the rectangle contained by the two straight lines is equal to the rectangles contained by the undivided line and the several parts* (B D, D E, &c.) *of the divided line.*



* 11 I. From B draw B F at right angles to B C,* † 3 I. and make B G = A.† Though G draw G H parallel to B C, and through D, E, C, draw D K, E L, C H * 31 I. parallel to B G.* Then the rectangle B H = B K + D L + E H; but B H is contained by A and B C, † Const. for B G = A,† and B K is contained by A and B D, for B G = A; also, D L is contained by A and D E, * 34 I. for D K = B G* = A; and, in like manner, E H is contained by A and E C, ∴ *the rectangle* A·B C = A·B D + A·D E + A·E C, however many divisions there may be in B C; ∴ *if there be two straight lines, &c.* Q. E. D.

PROPOSITION II.—THEOREM.

If a straight line (A B) *be divided into any two parts* (A C, C B) *the rectangles contained by the whole and each of the parts, are together equal to the square of the whole line:* that is, A B·A C + A B·B C = A B^2.

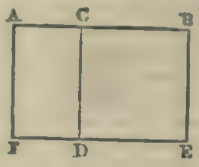


* 46 I. Upon A B describe the square A E,* and through C draw C F parallel to A D or † 31 I. B E.† Then A E = A F + * Const. C E; but A E is A B^2,* and A F = A D·A C = A B·A C, because A D = A B; also C E = A B·B C, for B E = A B, ∴ A B·A C + A B·B C = A B^2; ∴ *if a straight line, &c.*—Q. E. D.

NOTE.—This proposition might have been made a corollary to the preceding, since it is only that particular case of the former in which the two proposed lines (A, BC) are *equal*. It is also obvious that the restriction of the number of parts to *two* is unnecessary.

PROPOSITION III.—THEOREM.

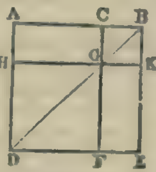
If a straight line (A B) *be divided into any two parts* (A C, C B), *the rectangle contained by the whole and one of the parts is equal to the rectangle contained by the two parts, together with the square of the aforesaid part:* that is, A B B C = A C C B + B C^2.



Upon B C describe the square * 46 I. C E;* prolong E D to F, and through A draw A F parallel † 31 I. to C D or B E.† Then A E = A D + C E; but A E is the rectangle A B·B C, for B C = B E; and A D is the rectangle A C·C B, for C D = C B; also C E is B C^2, ∴ A B B C = A C·C B + B C^2; ∴ *if a straight line, &c.* Q. E. D.

PROPOSITION IV.—THEOREM.

If a straight line (A B) *be divided into any two parts* (A C, C B) *the square of the whole line is equal to the squares of the two parts, together with twice the rectangle contained by the parts:* that is, A B^2 = A C^2 + C B^2 + twice A C·C B

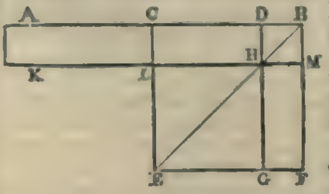


* 46 I. Upon A B describe the square A E:* draw B D; through C draw C G F parallel to A D or B E, and † 31 I. through G draw H K parallel to A B or D E;† then C F, A D being parallels, the angle B G C = * 29 I. A D B:* but A D B = A B D, because A B = † 5 I. A D,† A E being a square, ∴ C G B = C B G, * 6 I. ∴ C B = C G;* but C B = G K, and C G = B K, ∴ C K is *equilateral*. It is likewise *rectangular*, for C G, B K being parallels, the angles K B C, G C B are = two right an- † 29 I. gles:† but K B C is a right angle, ∴ G C B is a right angle, ∴ C G K, G K B, opposite to these, * 34 I. are right angles,* ∴ C K is *rectangular*; and since it is also *equilateral*, it is *a square*—the square of C B. For a similar reason H F is the square of H G, or of A C, ∴ H F, C K are A C^2, C B^2. And because the complement A G = † 43 I. the complement G E,† and A G is the rectangle A C·C B, for C G = C B, ∴ G E = A C C B, ∴ A G + G E = twice A C·C B; and H F + C K = A C^2 + C B^2, ∴ H F + C K + A G + G E = A C^2 + C B^2 + twice A C·C B; that is, A B^2 = A C^2 + C B^2 + *twice* A C·C B, ∴ *if a straight line, &c.* Q E D.

COR.—From this it is manifest, that parallelograms about the diagonal of a square are likewise squares.

PROPOSITION V.—THEOREM.

If a straight line (A B) *be divided into two equal parts* (in C), *and also into two unequal parts* (in D), *the rectangle contained by the unequal parts, together with the square of* (C D) *the line between the points of section, is equal to the square of half the line:* that is, A D·D B + C D^2 = C B^2.



* 46 I. Upon C B describe the square C F:* draw B E; through D draw D H G parallel to C E † 31 I. or B F,† and through A draw A K parallel to C L or B M. The complement C H = * 43 I. H F:* to each of these add D M, ∴ C M = D F: but C M = † 36 I. A L,† since A C = C B, ∴ A L = D F; to each of these add C H, ∴ A H = D F + C H: but A H is the * 4 II. Cor. rectangle A D·D B, for D H = D B;* and D F + C H is the gnomon C M G, ∴ C M G = A D·D B:

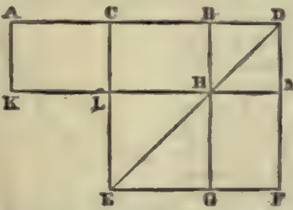
† 4 II. Cor. to each of these add LG or CD^2,† $\therefore CMG + LG = AD\cdot DB + CD^2$: but $CMG + LG = CF = CB^2$, $\therefore AD\cdot DB + CD^2 = CB^2$; $\therefore$ *if a straight line, &c.* Q. E. D.

Cor.—From this it is manifest, that the difference of the squares of two unequal lines, AC, CD, is equal to the rectangle contained by their sum and difference; that is, $AC^2 - CD^2 = AD\cdot DB$, where $AD = AC + CD$, and $DB = CB - CD = AC - CD$.

PROPOSITION VI.—Theorem.

If a straight line (AB) *be bisected* (in C), *and prolonged to any point* (D), *the rectangle contained by the whole line* (AD) *thus prolonged, and the prolonged part* (BD), *together with the square of* (CB) *half the line bisected, is equal to the square of* (CD), *the line made up of the half and the prolonged part:* that is, $AD\cdot DB + CB^2 = CD^2$.

• 46 I. Upon CD describe the square CF:* draw DE, and through B draw BHG parallel to CE or DF, and through H draw KLM parallel to † 31 I. AD, or EF;† also through A draw AK parallel to CL or DM. Then because $AC = CB$, • 36 I. $AL = CH$;* but $CH = HF$,† $\therefore AL$ † 43 I. $= HF$: to each of these add CM, $\therefore AM =$ gnomon CMG; but AM is the rectangle $AD\cdot DB$, • 4 II. Cor. for $DM = DB$,* $\therefore CMG = AD\cdot DB$: † 4 II. Cor. add LG or CB^2† to each of these, $\therefore CMG + LG = ADDB + CB^2$; but $CMG + LG = CF = CD,^2$ $\therefore AD\cdot DB + CB^2 = CD^2$; $\therefore$ *if a straight line, &c.* Q. E. D.



PROPOSITION VII.—Theorem.

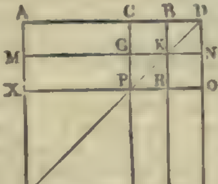
If a straight line (A B) *be divided into any two parts* (in C), *the squares of the whole line, and of one of the parts, are equal to twice the rectangle contained by the whole and that part, together with the square of the other part:* that is, $AB^2 + BC^2 =$ twice $AB\cdot BC + AC^2$. [See Diagram, Prop. IV.]

• 46 I. Upon AB describe the square AE,* and construct the figure as in the preceding propositions. Then † 43 I. because $AG = GE$,† if CK be added to each, $AK = CE$, $\therefore AK + CE =$ twice AK. But $AK + CE =$ gnomon $AKF + CK$, $\therefore AKF + CK =$ twice AK; but twice $AK =$ twice $AB\cdot BC$, for • 4 II. Cor. $BK = BC$,* $\therefore AKF + CK =$ twice $AB\ BC$. To each of these add HF or AC^2, $\therefore AKF + CK + HF =$ twice $AB\cdot BC + AC^2$: but $AKF + CK + HF = AB^2 + BC^2$, $\therefore AB^2 + BC^2 =$ twice $AB\cdot BC + AC^2$; $\therefore$ *if a straight line, &c.* Q. E. D.

PROPOSITION VIII.—Theorem.

If a straight line (A B) *be divided into any two parts* (in C), *four times the rectangle contained by the whole and one of the parts* (BC), *together with the square of the other part* (A C), *is equal to the square of the line* ($AD = AB + BC$), *made up of the whole and that part:* that is, four times $AB\cdot BC + AC^2 = AD^2$.

• 3 I. Prolong AB to D, so that $BD = CB$;* † 46 I. and upon AD describe the square AF;† and construct two figures such as in the preceding. Then because $CB = BD$, and $CB = GK$, and $BD = KN$,* • 34 I. $\therefore GK = KN$. For a like reason, $PR = RO$; and because $CB = BD$, and $GK = KN$, $\therefore CK = BN$, † 36 I. and $GR = RN$.† But $CK = BN$, being com- • 43 I. plements,* $\therefore BN = GR$, $\therefore$ the four rectangles, BN, CK, GR, RN, are *equal*, and are therefore quadruple of CK. Again, because $CB = BD$, and $BD =$ † 4 II. Cor. BK† $= CG$, • 34 I. and $CB = GK$* $=$ † 4 II. Cor. GP,† $\therefore CG = GP$, and because $CG = GP$, and $PR = RO$, $\therefore AG =$ • 36 I. MP,* and $PL = RF$. But $MP = PL$, being † 43 I. complements,† $\therefore AG = RF$ $\therefore$ the four, AG, MP, PL, RF, are equal, and are together quadruple AG. And it was proved that the four, CK, BN, GR, RN, are quadruple CK, $\therefore$ the eight rectangles, which compose the gnomon AOH, are quadruple of AK; and because $AK = AB\cdot BC$, for $BK = BC$, $\therefore$ four times $AB\cdot BC =$ four times AK: but it was proved that $AOH =$ four times AK, $\therefore$ four times $AB\cdot BC = AOH$. To each of • 4 II. Cor. these add XH or AC^2,* $\therefore$ four times $AB\cdot BC + AC^2 = AOH + XH$; but $AOH + XH = AD^2$, $\therefore$ *four times* $AB\cdot BC + AC^2 = AD^2$; $\therefore$ *if a straight line, &c.* Q. E. D.

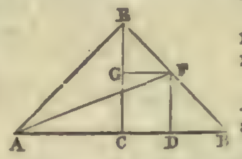


Note.—For another and shorter demonstration of this proposition, see the remarks at the end of Book II.

PROPOSITION IX.—Theorem.

If a straight line (A B) *be divided into two equal, and also into two unequal parts* (in C and D), *the squares of the two unequal parts are together double of the square of half the line* (A C), *and of the square of the line* (C D), *between the points of section:* that is, $AD^2 + DB^2 =$ twice $AC^2 +$ twice CD^2.

• 11 I. From C draw CE at right angles to AB;* † 3 I. and make it $= AC$ or CB.† Draw EA, EB; and through D draw DF parallel to CE, and through F • 31 I. draw FG parallel to BA:* draw also AF. Then because $AC = CE$, the angle $EAC = AEC$;† † 5 I. and because ACE is a right angle, the two others, AEC, EAC, must together make one right • 32 I. angle;* and as they are equal, each must be half a right angle. For a like reason, each of the angles, CEB, EBC, is half a right angle, $\therefore AEB$ is a right angle. And because GEF is half a right angle, and EGF a right angle, for it is $=$ † 29 I. ECB,† $\therefore$ the remaining angle EFG is half a right angle, $\therefore GEF = EFG$, • 6 I. and $\therefore EG = GF$.* Again, because B is half a right angle, and FDB a right angle, † 29 I. for it is $= ECB$,† $\therefore$ the remaining angle, BFD, is half a right angle, $\therefore$ • 6 I. $B = BFD$, and $\therefore DF = DB$.* And because $AC = CE$, $\therefore AC^2 = CE^2$, $\therefore AC^2 + CE^2 =$ twice AC^2; but $AE^2 = AC^2 + CE^2$, because ACE is a right † 47 I. angle,† $\therefore AE^2 =$ twice AC^2. Again, because $EG = GF$, $\therefore EG^2 = GF^2$, $\therefore EG^2 + GF^2 =$ twice • 47 I. GF^2: but $EF^2 = EG^2 + GF^2$,* $\therefore EF^2 =$ † 34 I. twice GF^2; and $GF = CD$,† $\therefore EF^2 =$ twice CD^2: but $AE^2 =$ twice AC^2, $\therefore AE^2 + EF^2 =$ twice • 47 I. $AC^2 +$ twice CD^2: but $AF^2 = AE^2 + EF^2$,* because AEF is a right angle, $\therefore AF^2 =$ twice $AC^2 +$ twice CD^2: but $AD^2 + DF^2 = AF^2$, because ADF is † 47 I. a right angle,† $\therefore AD^2 + DF^2 =$ twice $AC^2 +$ twice CD^2; and $DF = DB$, $\therefore AD^2 + DB^2 =$ *twice* $AC^2 +$ *twice* CD^2; $\therefore$ *if a straight line, &c.* Q. E. D.

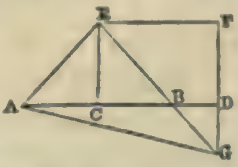


PROPOSITION X.—Theorem.

If a straight line (A B) *be bisected* (in C), *and prolonged to any point* (D), *the square of the whole line thus prolonged, and the square of the part of it prolonged* (D B), *are together double the square of* (A C) *half the line bisected, and of the square of* (C D) *the line made up of the half and the prolonged part:* that is, $AD^2 + DB^2 =$ twice $AC^2 +$ twice CD^2.

• 11 I. From C draw CE at right angles to AB,* † 3 I. and make it $= AC$ or CB;† and draw AE, EB. Through E draw EF parallel to AB, and through • 31 I. D draw DF parallel to CE.* Then because EC, FD are parallels, the angles $CEF + EFD =$ two † 29 I. right angles,† $\therefore BEF + EFD$ are *less* than two right angles, $\therefore EB$, FD, if prolonged, will meet • Ax. 12. towards B, D.* Let them meet in G; and draw AG. Then because $AC = CE$, the angle CEA † 5 I. $= EAC$,† and ACE is a right angle, $\therefore CEA$, • 32 I. EAC are each half a right angle.* For a like reason, CEB, EBC are each half a right angle, $\therefore$ AEB is a right angle. And because EBC is half a right angle, DBG also is half a right angle, for † 15 I. they are vertically opposite :† but BDG is a right angle, being $= DCE$, $\therefore$ the remaining angle DGB is half a right angle, and is $\therefore = DBG$, $\therefore BD =$ * 6 I. DG.*

Again, because EGF is half a right angle, and that F is a right angle, being $=$ the opposite angle ECD,† $\therefore$ † 34 I. the remaining angle FEG is half a right * 6 I. angle, $\therefore FEG = EGF$, $\therefore GF = FE$.*



And because $EC = CA$, $\therefore EC^2 = CA^2$, $\therefore EC^2 + CA^2 =$ twice CA^2: but $EA^2 = EC^2 +$ † 47 I. CA^2,† $\therefore EA^2 =$ twice AC^2. Again, because $GF = FE$, $\therefore GF^2 = FE^2$, $\therefore GF^2 + FE^2 =$ twice FE^2; but * 47 I. $EG^2 = GF^2 + FE^2$,* † 34 I. $\therefore EG^2 =$ twice FE^2, and $FE = CD$,† $\therefore EG^2 =$ twice CD^2. But it was demonstrated that $EA^2 =$ twice CA^2, $\therefore AE^2 + EG^2 =$ twice $AC^2 +$ twice CD^2; * 47 I. but $AG^2 = AE^2 + EG^2$,* $\therefore AG^2 =$ twice † 47 I. $AC^2 +$ twice CD^2; but $AD^2 + DG^2 = AG^2$,† $\therefore AD^2 + DG^2 =$ twice $AC^2 +$ twice CD^2; but $DG = DB$, $\therefore AD^2 + DB^2 =$ *twice* $AC^2 +$ *twice* CD^2; $\therefore$ *if a straight line, &c.*

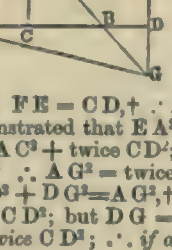
NOTE.—For other demonstrations of the last two propositions, see the remarks at the end of this book.

PROPOSITION XI.—PROBLEM.

To divide a given straight line (AB) *into two parts, so that the rectangle contained by the whole, and one of the parts, shall be equal to the square of the other part.*

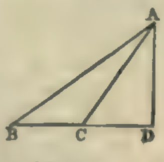
* 11 & 3 I. From A draw AC perpendicular and equal † 10 I. to AB;* bisect AC in E,† and draw BE. * 3 I. † 3 I. Prolong CA to F, making $EF = EB$,* and from AB cut off $AH = AF$:† AB *shall be divided in* H, *so that* $AB \cdot BH = AH^2$.

* 46 I. Complete the square BC, as also the square FH;* and prolong GH to K. Then because AC is bisected in E and prolonged to F, the rectangle $CF \cdot FA +$ † 6 II. $AE^2 = EF^2$;† but $EF =$ * Const. EB,* $\therefore CF \cdot FA + AE^2 = EB^2$; but $BA^2 + AE^2 = EB^2$, because EAB is a right angle, $\therefore CF \cdot FA + AE^2 = BA^2 + AE^2$: take away AE^2, which is common to both, $\therefore CF \cdot FA = BA^2$. But $FK = CF \cdot FA$, for $AF = FG$; and $AD = BA^2$, $\therefore FK = AD$. Take away AK, common to both, $\therefore FH = HD$: but $HD = AB \cdot BH$, for $AB = BD$, and FH is AH^2, $\therefore AB \cdot BH = AH^2$; $\therefore AB$ *is divided in* H, *so that* $AB \cdot BH = AH^2$. Which was to be done.



PROPOSITION XII.—THEOREM.

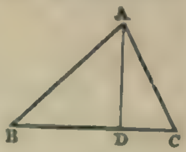
In an obtuse-angled triangle (ABC), *if a perpendicular* (AD) *be drawn from either of the acute angles to the opposite side prolonged, the square of the side* (AB) *subtending the obtuse angle is greater than the squares of the sides* (BC, CA) *containing the obtuse angle, by twice the rectangle contained by the side upon which, when prolonged, the perpendicular falls, and the line, without the triangle, between the perpendicular and the obtuse angle;* that is, $AB^2 = BC^2 + CA^2 +$ twice $BC \cdot CD$.



Because BD is divided into two parts in C, $BD^2 =$ * 4 II. $BC^2 + CD^2 +$ twice $BC \cdot CD$.* To each of these add DA^2, $\therefore BD^2 + DA^2 = BC^2 + CD^2 + DA^2 +$ twice $BC \cdot CD$; but $AB^2 = BD^2 + DA^2$, and $CA^2 =$ † 47 I. $CD^2 + DA^2$,† $\therefore AB^2 = BC^2 + CA^2 +$ *twice* $BC \cdot CD$; that is, *the square of* AB *is greater than the squares of* BC, CA *by twice* BC·CD; $\therefore$ *in an obtuse-angled triangle, &c.* Q. E. D.

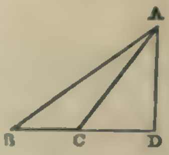
PROPOSITION XIII.—THEOREM.

In every triangle (ABC), *the square of a side* (AC) *subtending an acute angle, is less than the squares of the sides* (CB, BA) *containing that angle, by twice the rectangle* (CB·BD) *contained by either of these sides, and the straight line intercepted between the perpendicular* (AD) *and the acute angle* (B): that is, $AC^2 = CB^2 + BA^2 -$ twice $CB \cdot BD$.



First, let AD fall *within* the triangle; then, because CB is divided into two parts in D, $CB^2 + BD^2 =$ twice * 7 II. $CB \cdot BD + DC^2$.* To each of these add AD^2, $\therefore CB^2 + BD^2 + AD^2 =$ twice $CB \cdot BD + DC^2 + AD^2$: but $BA^2 = BD^2 + AD^2$, and $AC^2 = DC^2$ † 47 I. $+ AD^2$,† $\therefore CB^2 + BA^2 = AC^2 +$ twice $CB \cdot BD$. From each of these take twice $CB \cdot BD$, $\therefore AC^2 = CB^2 + BA^2 -$ twice $CB \cdot BD$.

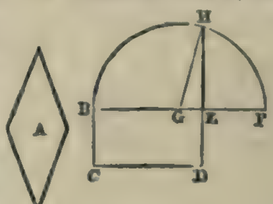
Secondly, let AD fall without the triangle; then, because D is a right * 16 I. angle, ACB is greater than † 12 II. a right angle,* $\therefore AB^2 = AC^2 + CB^2 +$ twice $BC \cdot CD$.† To each of these add BC^2, $\therefore AB^2 + BC^2 = AC^2 +$ twice $BC^2 +$ twice $BC \cdot CD$. But because BD is divided into two parts in C, $DB \cdot BC = BC \cdot CD + BC^2$,* * 3 II. and the doubles of these being equal, $\therefore AB^2 + BC^2 = AC^2 +$ twice $DB \cdot BC$. From each of these take twice $DB \cdot BC$, $\therefore AC^2 = AB^2 + BC^2 -$ *twice* $DB \cdot BC$; $\therefore$ *in every triangle, &c.* Q. E. D.



PROPOSITION XIV.—PROBLEM.

To describe a square that shall be equal to a given rectilineal figure (A).

* 45 I. Describe the rectangle $BD = A$.* Then, if the sides of it, BE, ED, are *equal*, it is a *square*, and what was required is now done; but if they are not equal, prolong one of them, BE to F, and make $EF = ED$. † 10 I. Bisect BF in G,† and with centre G and radius GB, or GF, describe the semicircle BHF, and prolong DE to H. The square described upon EH shall be equal to the given figure A. Draw GH. Then because BF is divided into two equal parts in G, and into two unequal parts in E, $BE \cdot EF + EG^2$ * 5 II. $= GF^2$;* but $GF = GH$, $\therefore BE \cdot EF + EG^2 = GH^2$; but HE^2 † 47 I. $+ EG^2 = GH^2$† $\therefore BE \cdot EF + EG_2 = HE^2 + EG^2$. Take away EG^2, which is common to both, $\therefore BEEF = EH^2$; but $BE \cdot EF = BD$, because $EF = ED$, $\therefore BD = EH^2$; but $BD = A$, $\therefore EH^2 = A$; $\therefore$ *a square described upon* EH *is* $=$ *to the given rectilineal figure.* Which was to be done.



REMARKS ON BOOK II.

All the propositions in this book, except the last three, concern the divisions of straight lines into parts, and the equality of rectangles having certain of these parts for their sides. The first proposition is little more than an axiom, as its object is to show that any rectangle BH is equal to the sum of the partial rectangles BK, DL, &c., into which it is divided: but, as already observed (p. 545), Euclid does not regard as an axiom any proposition that admits of demonstration.

Proposition II. might, without impropriety, have been appended as a corollary to Proposition I., as already stated in the text; since, if the line A, in Proposition I., be considered to be *equal* to the line BC, this proposition will virtually include the next following.

Proposition V. authorises us to affirm, as an inference, that the rectangle contained by the two *halves* of a straight line, is *greater* than the rectangle contained by any two *unequal* parts of it, since it shows that to this latter rectangle something must be *added*, to make up the square of half the line.

In attempting the propositions in this book, without reference to the text, you will find it useful to keep in remembrance, that in all the constructions, up to Proposition VIII. inclusive, the *greatest* square mentioned in the enunciation is always to be described *first;* and that in each proposition after the fourth, the lines employed in that fourth are always to be introduced. After this partial construction, the completion of the diagram, in each case, will readily present itself.

It may, too, be deserving of notice, that the demonstrations in Propositions V. and VI. may, with advantage, be conducted rather differently from the method of Euclid. Thus, in Proposition V., having proved, as in the text, that the *projecting* rectangle A L is equal to the *marginal* rectangle D F, we may proceed as follows:—Take each of these rectangles from the entire figure; then the remainder C F will be equal to the remainder $AH + LG$; that is, to the rectangle $AD \cdot DB$ (for $DB = DH$), and the square of C D, since $LH = CD$: and this completes the demonstration.

In like manner, in Proposition VI., having proved, as in the text, that the projecting rectangle A L is equal to the marginal rectangle H F, take each rectangle from the entire figure; then the remainder C F is equal to the remainder $AM + LG$; that is, $CD^2 = AD \cdot DB + CB^2$, which completes the proof.

Proposition VIII. is usually found, by a beginner, to be a little perplexing; and Propositions IX. and X., though very elegantly demonstrated by Euclid, are felt to be somewhat lengthy. We shall, therefore, here show how these three propositions may be otherwise demonstrated.

Prop. VIII.—Having made

$BD = CB$, the reasoning may be as follows:

By Prop. IV.,

$$AD^2 = AB^2 + BD^2 + 2AB \cdot BD;$$

that is, $AD^2 = AB^2 + CB^2 + 2AB \cdot CB$.

Also, Prop. VII.,

$$AB^2 + CB^2 = 2AB \cdot CB + AC^2;$$

$$\therefore AD^2 = 2ABCB + AC^2 + 2AB \cdot CB;$$

that is, $AD^2 = 4AB \cdot CB + AC^2$.

Which was to be demonstrated.

Prop. IX.—By Prop. IV.,

$$AD^2 = AC^2 + CD^2 + 2AC \cdot CD;$$

$$\therefore AD^2 + DB^2 = AC^2 + CD^2 + 2AC \cdot CD + DB^2;$$

that is, $AD^2 + DB^2 = BC^2 + CD^2 + 2BC \cdot CD + DB^2$.

But, Prop. VII.,

$$BC^2 + CD^2 = 2BC \cdot CD + DB^2;$$

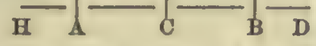
$$AD^2 + DB^2 = BC^2 + CD^2 + BC^2 + CD^2;$$

that is, $AD^2 + DB^2 = 2BC^2 + 2CD^2 = 2AC^2 + 2CD^2$.

Which was to be demonstrated.

Proposition X.—The demonstration of this may be made to depend on the preceding proposition, as follows:—

Prolong C A to H,
making $AH = BD$;
then $HC = CD$, and
$HB = DA$.



And since H D is divided equally in C, and unequally in B, ∴ by Proposition IX.,

$$HB^2 + BD^2 = 2HC^2 + 2CB^2;$$

that is, $AD^2 + BD^2 = 2CD^2 + 2AC^2$.

Which was to be demonstrated.

Although the foregoing demonstrations occupy less space than those of Euclid, we would by no means recommend them to your preference. Those of Euclid are among the finest of his specimens of clear and consecutive reasoning, and you will observe that they are altogether independent of second-book propositions. They are thus simpler, though longer than those given above. There is, too, a beauty in the reiterated appeals to the forty-seventh of the first book, that more than compensates for the length of the argument. The demonstrations above, however, may be useful as exercises on the application of second-book propositions.

There are but two *problems* in this second book; and each of these, as given in the editions of Simson and others, exemplifies the remark made in the commentary on the preceding book, in reference to Euclid's apparent indifference as to the neatness and finish of his *constructions*.

In the editions alluded to, the whole of the diagram in Proposition XI., with the exception of the prolongation of G H to K, is made to appear as essential to the construction of the problem; that is, to the determination of the point H. You have seen, however, that to the discovery of this point, the actual construction of the two squares A D, A G is not necessary; they are wanted only as a part of the machinery in the *demonstration* that the point H, previously found, divides the proposed line as required.

In Proposition XIV., too, the drawing of G H is made to enter into the construction of the problem, instead of being postponed for use in the demonstration, as is done in this edition.

You will not fail to notice the connection between Propositions XII. and XIII., and Proposition XLVII. of the first book. The three together furnish certain corresponding relations between one side of a triangle and the other two, whether the angle opposite that one be right, obtuse, or acute.

It may not, perhaps, be out of place here to notice, that the word *equal*, employed so frequently in this and in the preceding book, in reference to rectangles, triangles, &c., is used by Euclid in two somewhat different senses. In the earlier propositions of the first book—the fourth, eighth, and twenty-sixth, for instance—the test of equality is perfect *coincidence*, as the result of superposition; but in Propositions XXXV., XXXVI., &c., as also in most of the propositions of this second book, the condition of coincidence is excluded, and the figures are declared to be equal if they are proved to inclose the same extent of surface. Legendre, a distinguished French geometer, has proposed to discriminate between these two kinds of equality: figures which, though equal in surface, do not admit of coincidence, he prefers to call—not *equal* figures, but *equivalent* figures. We think the distinction an appropriate one; and while upon these minor matters of mere phraseology, we would venture further to suggest, that instead of speaking of lines as *greater* and *less*, the more explicit and restrictive terms *longer* and *shorter* would be preferable; and that *part* of a line, or *portion* of a line, are, either of them, designations that might appropriately supply the place of *segment* of a line.

EXERCISES ON BOOKS I. AND II.

1. In any triangle, the squares of the two sides are together equal to the square of half the base, and of the straight line from the vertex to the middle of the base.

2. The squares of the four sides of a parallelogram, are together equal to the squares of the two diagonals.

3. If from any point lines be drawn to the four vertices of a rectangle, the squares of those drawn to opposite vertices will, together, be equal to the squares of the other two.

4. In every parallelogram, the squares of the four sides are, together, equal to the squares of the two diagonals.

5. If from any point whatever, lines be drawn to the four vertices of a parallelogram, twice the sum of their squares will be equal to the squares of the diagonals, together with eight times the square of the line drawn from the given point to the intersection of the diagonals.

6. From the last exercise prove that the following curious property exists; namely:—If from the point of intersection of the diagonals of a parallelogram, a circle be described with any radius, the squares of the lines drawn from any point in the circumference to the four corners of the parallelogram, will always amount to the same sum.

7. In any quadrilateral, the sum of the squares of the four sides is equal to the sum of the squares of the diagonals, together with four times the square of the line joining the middle points of the diagonals.

8. Prove the *converse* of Example 2; namely:—If the squares of the sides of a quadrilateral are together equal to the squares of the diagonals, the figure must be a parallelogram.

9. Let a straight line be divided according to Proposition XI., Book II., and from the greater segment cut off a part equal to the less: prove that this greater segment will thus be divided also according to the same proposition.

In Book III., the properties of circles are investigated.

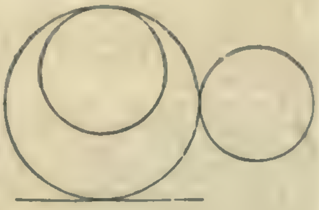
CHAPTER III.

ELEMENTS OF EUCLID.—BOOK III.

DEFINITIONS.

I.

A straight line is said to *touch* a circle when it *meets* the circle, and being prolonged, does not *cut* it.



II.

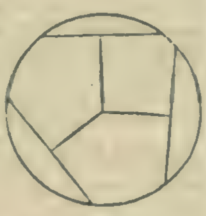
Circles are said to *touch* one another, which *meet*, but do not cut one another.

III.

Straight lines are said to be equally distant from the centre of a circle, when the perpendiculars drawn to them from the centre are equal.

IV.

And the straight line on which the greater perpendicular falls is said to be farther from the centre.

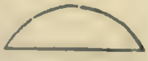


V.

An *arc* of a circle is any part of the circumference.

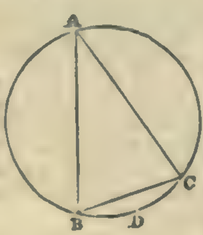
VI.

A *segment* of a circle is the figure contained by an arc of a circle and the straight line joining its extremities, which straight line is called the *chord of the arc*, or the *base of the segment.*



VII.

An *angle in a segment* is the angle contained by two straight lines drawn from any point in the arc of the segment, to the extremities of the chord of the arc, or base of the segment. Thus A is an angle in the segment B C D.

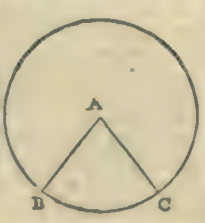


VIII.

And an angle is said to insist or *stand* upon the arc intercepted between the straight lines that contain the angle. Thus A stands upon the arc B D C.

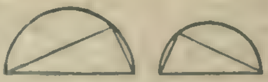
IX.

A *sector* of a circle is the figure contained by two straight lines drawn from the centre, and the arc between them. A B C is a *sector*, so is the remaining part of the circle.



X.

Similar segments of circles are those in which the angles are equal, or which contain equal angles.

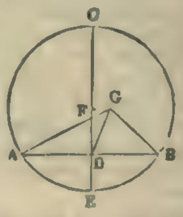


PROPOSITION I.—PROBLEM.

To find the centre of a given circle (A B C).

Draw any straight line A B, terminating in the •10 I. circumference, and bisect it in D:* draw D C at †11 I. right angles to A B,† prolong it to E, and bisect •10 I. C E in F:* the point F shall be the centre of the circle.

For if it be not, let G, a point *out of the line* C E, be the centre: and draw G A, G D, G B. Then because †Const. D A = D B,† and D G common to the two triangles A D G, B D G, the two sides A D, D G are = the two B D, D G, each to each, and the base G A = the base G B, •Def. 12 I. because G is the centre,* ∴ the angle G D A = G D B:† †8 I. but when a straight line standing upon another makes the adjacent angles *equal*, each angle is •Def. 8 I. a right angle:* ∴ G D B is a right angle: but †Const. F D B is likewise a right angle,† ∴ G D B = F D B, the less to the greater, which is impossible, ∴ the centre cann to be at any point *out of the line* C E: it must ∴ be at a point *in* C E, and ∴ at the *middle* of C E, ∴ F *is the centre,* which was to be found.

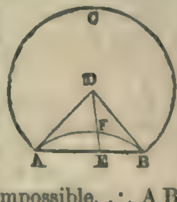


Cor.—From this it is manifest that if *a chord* of a circle (as A B) be bisected by a perpendicular, that perpendicular will pass through the centre.

PROPOSITION II.—THEOREM.

If two points (A B) *be taken in the circumference of a circle* (A B C), *the straight line or chord* (A B) *which joins them shall fall within the circle.*

For if not, let it fall without, as A E B. Find D, •1 III. the centre of the circle,* and draw D A, D B. In the arc A B take any point F, draw D F, and produce †Def. 12 I. it to E: then because D A = D B,† the angle •5 I. D A B = D B A:* and because A E, a side of the triangle D A E, is prolonged to B, the exterior angle †16 I. D E B is greater than D A E:† but D A E = D B E, as just proved: ∴ D E B is greater than D B E; but to the greater angle the greater side is •19 I. opposite,* ∴ D B is greater †Def. 12 I. than D E: but D B = D F,† ∴ D F is greater than D E, which is impossible, ∴ A B does *not* fall without the circle. And since D B, the radius of the circle, is greater than D E, and that E is *any point* on the chord, between A and B, ∴ no point between A and B can be *on* the circumference, ∴ *the chord* A B *is wholly within the circle,* ∴ *if any two points,* &c. Q. E. D.

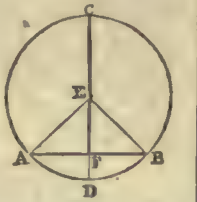


PROPOSITION III.—THEOREM.

If a straight line (C D) *through the centre of a circle* (A B C) *bisect a chord* (A B), *which does not pass through the centre, it shall cut it at right angles; and if it cut the chord at right angles, it shall bisect it.*

•1 III. Take E, the centre of the circle,* and draw

† Hyp. E A, E B. Then because A F = F B,† and F E common to the two triangles A F E, B F E, and also * 8 I. E A = E B, ∴ the angle A F E = B F E,* † Def. 8 I. ∴ *each is a right angle*,† ∴ C D, *drawn through the centre, bisecting the chord* A B, *that does not pass through the centre, cuts it at right angles.*

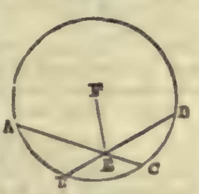


Now let C D cut A B at right angles: C D shall bisect A B. For, the same construction being made, because E A = E B, the angle E A F * 5 I. = E B F;* and the right angle A F E = B F E, ∴ in the two triangles, E A F, E B F, there are two angles in the one = two in the other, each to each; and E F, opposite to equal angles, one in each, is com- † 26 I. mon to both, ∴ A F = F B;† that is, *the chord* A B *is bisected by the diameter* C D, *which is perpendicular to it*; ∴ *if a straight line, &c.* Q. E. D.

PROPOSITION IV.—THEOREM.

If in a circle (A B C D) *two chords* (A C, B D) *cut one another, and do not pass through the centre, they do not bisect each other.*

For let them cut in E; and suppose A E = E C, *and* B E = E D. If *one* of the chords pass through the centre, it is plain it cannot be bisected by the other which does not pass through the centre, because the middle of the former is the centre.

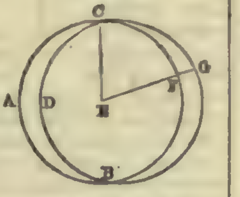


But if neither pass through the centre, take F, the centre;* * 1 III. and draw F E. Then, because F E, through the centre, bisects A C *not* through the centre, † 3 III. it cuts it at right angles,† ∴ F E A is a right angle. Again, * Hyp. because F E bisects B D,* a chord not through the centre, it cuts it also at right † 3 III. angles,† ∴ F E B is a right angle; but it was proved that F E A is a right angle, ∴ F E A = F E B, the less to the greater, which is impossible, ∴ A C, B D *do not bisect one another*; ∴ *if in a circle, &c.* Q. E. D.

PROPOSITION V.—THEOREM.

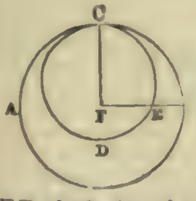
If two circles (A B C, C D G) *cut one another* (as in the point C), *they shall not have the same centre.*

For, if it be supposed possible, let E be the common centre. Draw E C, and draw E F G, meeting the circumferences in F and G. Then, because E is the centre of the circle A B C, * Def. 13 I. E C = E F.* Again, because E is the centre of the circle C D G, E C = E G. But it was shown that E C = E F, ∴ E F = E G, the less to the greater, which is impossible, ∴ *the two circles have not a common centre*; ∴ *if two circles, &c.* Q. E. D.



PROPOSITION VI.—THEOREM.

If two circles (A B C, C D E) *touch one another, they shall not have the same centre.*

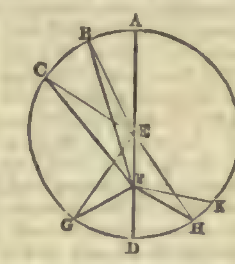


Let them touch at C, then if they have a common centre, let it be F. Draw F C, and F E B, meeting the circumferences in E and B. Then, because F is the centre of A B C, * Def. 13 I. F C = F B;* also, because F is the centre of C D E, F C = F E:* but it was shown that F C = F B, ∴ F E = F B, the less to the greater, which is impossible, ∴ *the two circles have not a common centre*; ∴ *if two circles, &c.* Q. E. D.

PROPOSITION VII.—THEOREM.

If any point (F), *not the centre, be taken in the diameter* (A D) *of a circle* (A B C D), *then of all lines* (F A, F B, F C, &c.) *which can be drawn from it to the circumference, the greatest is that* (F A) *which passes through the centre* (E), *and* (F D), *the other part of the diameter, is the least. And of the others, that which is nearer to the line through the centre is always greater than one more remote. And from the same point* (F) *there can be drawn two, and only two, straight lines that are equal, one upon each side of the diameter.*

Draw E B; then because any two sides of a triangle are together greater than the third side, F E, E B are * 20 I. greater than F B;* but E A = E B, ∴ F E + E A; that is, F A is greater than F B, and F B is any other line, ∴ *the line through the centre is the greatest.* Again, draw E C: then, because E B = E C, and F E common to the triangles B E F, C E F, ∴ the two sides F E, E B are = the two F E, E C, each to each; but the angle B E F is greater than C E F, ∴ F B is greater than † 24 I. F C,† ∴ *the line nearer to that through the centre is greater than one more remote.*



Again, draw E G: then because E F + F G are greater * 20 I. than E G,* and that E G = E D, ∴ E F + F G are greater than E D. Take away the common part E F, ∴ F G is greater than F D, and F G is any other line, ∴ F D *is the least of all the lines from* F *to the circumference.*

Also *two*, but only two, equal straight lines can be drawn from F to the circumference, one upon each side of the diameter A D. For F G being any straight line † 23 I. from F, at E make the angle F E H = F E G,† and draw F H. Then, because E G = E H, and E F common to the triangles G E F, H E F, the two sides E G, E F are = the two E H, E F, each to each, and the * Const. angle G E F = H E F,* ∴ F G = F H.† But, † 4 I. besides F H, no other line equal to F G can be drawn from F to the circumference. For, if there can, let it be F K: then, because F K = F G and F G = F H, ∴ F K = F H; but one of these must be nearer to the line through the centre than the other, so that a line nearer to that through the centre is *equal* to one more remote, which has been proved to be impossible, ∴ *besides* F H *no other line* = F G *can be drawn*; ∴ *if any point, &c.* Q. E. D.

PROPOSITION VIII.—THEOREM.

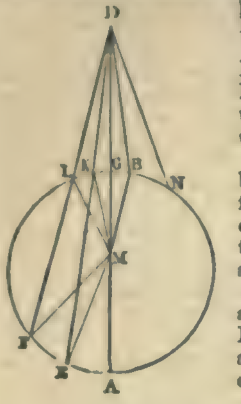
If any point (D) *be taken without a circle* (A B F), *and straight lines* (D A, D E, D F, &c.) *be drawn from it to the circumference, of which one* (D A) *passes through the centre* (M), *of those which fall upon the concave part of the circumference, the greatest is that* (D A) *which passes through the centre; and of the rest, that which is nearer to the one through the centre is always greater than one more remote. But of those which fall upon the convex part of the circumference, the least is that* (D G) *which, when prolonged, passes through the centre; and of the rest, that which is nearer to the least is always less than one more remote. Also two, and only two, equal straight lines can be drawn from the same point* (D) *to the circumference, one upon each side of the line through the centre.*

* 1 III. Take M the centre of the circle,* and draw M E. Then because A M = E M, add M D to each, ∴ A D = E M + M D; but E M + M D are greater than † 20 I. E D,† ∴ A D is greater than E D, and E D is any other line, ∴ A D, *the line through the centre, is the longest.*

Again, draw M F. Then, because E M = F M, the two E M, M D are = the two F M, M D, each to each; but the angle E M D is greater than F M D, ∴ E D is * 24 I. greater than F D;* that is, *a line* D E, *nearer to that through the centre, is greater than* D F, *one more remote.*

Now draw M K. Then because M K + K D are greater

† 20 I. than M D,† and that M K = M G, ∴ the remainder K D is greater than the remainder G D; that is, G D is less than K D, any other line from D to the convex part of the circumference, ∴ D G *is the least line.* Draw M L: then because M L D is a triangle, and that M K, D K are drawn from the extremities of a side to a point K, within the triangle, ∴ M K + D K are less than M L * 21 I. + D L:* but M K = M L, ∴ the remainder D K is less than the remainder D L; that is, *a line nearer to the least is less than one more remote.*



Also, there can be drawn *two*, but only two, equal straight lines from D to the circumference, one on each side of D A, the line through the centre. For draw any straight line D K, and at M make † 23 I. the angle D M B = D M K;† and draw D B. Then because M K = M B, the two M K, M D are = the two M B, M D, each to each; and the angle K M D = * Const. B M D,* ∴ D K = † 4 I. D B,† *so that two equal lines can be drawn*, and the demonstration is the same whether D K be drawn to the convex or to the concave part of the circumference. But, besides D B, no straight line drawn from D to the circumference, can be = D K. For, if there can, let it be D N. Then because D K = D N, and D K = D B, ∴ D N = D B; but one of these must be nearer to that through the centre than the other, ∴ a line nearer to that through the centre is = one more remote, which has been proved to be impossible; ∴ *if any point, &c.* Q. E. D.

PROPOSITION IX.—THEOREM.

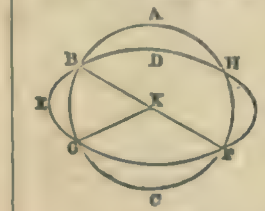
If any point be taken, from which there may be drawn more than two equal straight lines to the circumference of a circle, that point is the centre of the circle.

For if the point were *not* the centre, only *two* equal straight lines could be drawn from it to the circumfer- * 7 & 8 III. ence;* ∴ *if any point, &c.*

PROPOSITION X.—THEOREM.

One circumference of a circle cannot cut another in more points than two.

For suppose the circumference F A B C to cut the circumference D E G F in more than two points, viz., in B, G, F. Take the centre K, of the circle F A B C,* * 1 III. and draw K B, K G, K F. Then because K is the centre of the circle F A B, these lines are all *equal;* so that from a point K, to the other circumference D E G F, *more than two* equal straight lines are drawn, viz., the *three* K B, K G, K F, ∴ K is the centre † 9 III. of the circle D E G F,† ∴ the same point is the centre of two circles that cut one * 5 III. another, which is impossible;* ∴ *one circumference, &c.* Q. E. D.

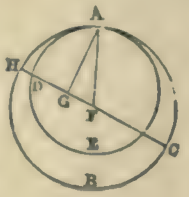


PROPOSITION XI.—THEOREM.

If one circle (A D E) *touch another* (A B C) *internally in a point* (A), *the straight line which joins their centres being prolonged, shall pass through the point of contact* (A).

Let F be the centre of A B C, and G, the centre of A D E; F G, when prolonged, shall pass through A.

For if not, let F G, prolonged, cut the circumferences in D and H. Draw A F, A G: then, because two sides of a triangle are together greater than the third side, * 20 I. F G + G A are greater than F A:* but F A = F H, ∴ F G + G A are greater than F H. Take away the common part F G, ∴ G A is greater than G H: but G A = G D,† ∴ † Def. 12 I. G D is greater than G H, which is impossible, even could D, H coincide, ∴ F G *being prolonged, cannot but pass through the point* A; ∴ *if one circle, &c.* Q. E. D.

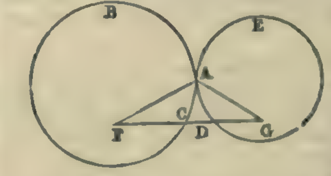


PROPOSITION XII.—THEOREM.

If two circles (A B C, A D E) *touch each other externally in a point* (A), *the straight line which joins their centres shall pass through the point of contact* (A).

Let F be the centre of A B C, and G that of A D E: F G shall pass through A.

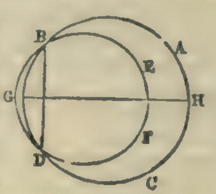
For if not, let F G cut the circumferences in C, D, and draw F A, G A. Then because F is the centre of A B C, F A = F C: and because G is the centre of A D E, G A = G D, ∴ F A + G A = F C + G D ∴ the whole F G *is not less than* F A + G A, even though C, D be supposed to coincide; but a side F G, of the triangle A F G, is less than the other two * 20 I. sides,* which is impossible, ∴ F G *cannot pass otherwise than through the point of contact* A; ∴ *if two circles, &c.* Q. E. D.

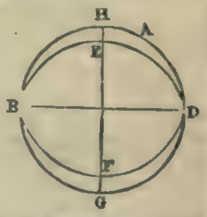


PROPOSITION XIII.—THEOREM.

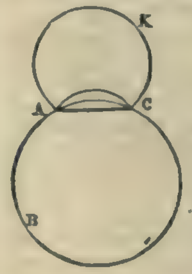
One circle cannot touch another in more than one point, whether it touch it on the inside or on the outside.

For, if it be supposed possible, let the circle E B F touch the circle A B C in *two* points; and first on the *inside*, in the points B, D. Join B, D, and draw G H, * 10 & 11 I. bisecting B D at right angles.* Then because the points B, D are in the circumference of each of the circles, B D falls within each of them, † 2 III. ∴ their centres are in G H, * 1 III. Cor. which bisects B D at right angles,* ∴ G H passes through the point of contact B, and through the † 11 III. point of contact D:† but the points B, D are *without* the straight line G H, which is absurd; ∴ *one circle cannot touch another on the inside in more than one point.*





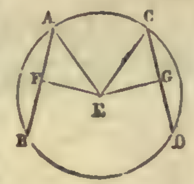
Nor can two circles touch on the *outside* in more than one point. For suppose the circles A C K, A B C to touch in *two* points, A, C. Join A, C: then because A, C are in the circumference A C K, A C, which joins them, falls within the circle A C K.* * 2 III. But the circle A C K is † Hyp. *without* the circle A B C,† ∴ A C is without this last circle; but, because A, C are in the circumference A B C, A C must be * 2 III. within the same circle,* which is absurd, ∴ *one circle cannot touch another on the outside in more than one point;* and it has been shown that they cannot touch on the *inside* in more than one point, ∴ *one circle, &c.* Q. E. D.



PROPOSITION XIV.—THEOREM.

Equal chords (A B, C D) *in a circle are equally distant* *from the centre; and chords equally distant from the centre are equal.*

• 1 III. Take E the centre of the circle A B D C,* and from E draw E F, E G perpen- † 12 I. diculars to A B, C D;† *then if* A B = C D, E F *must be* = E G. For draw E A, E C. Then because E F, from the centre, cuts A B, a chord not through the centre, at right angles, it bisects it,* ∴ • 3 III. A F = F B, ∴ A B is double of A F. For a like reason C D is double of C G. But A B † Hyp. = C D,† ∴ A F = C G. And because E A = E C, ∴ $EA^2 = EC^2$: but $AF^2 + FE^2 = EA^2$, • 47 I. because A F E is a right angle;* and, for a like reason, $CG^2 + GE^2 = EC^2$ ∴ $AF^2 + FE^2 = CG^2 + GE^2$: but $AF^2 = CG^2$, because A F = C G, ∴ $FE^2 = GE^2$, ∴ F E = G E; but chords of a circle are said to be equally distant from the centre when the perpendiculars drawn to them from the centre are equal,† † Def. 3 III. ∴ A B, C D *are equally distant from the centre.*



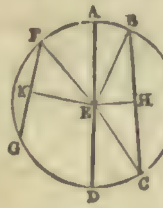
Next, let the chords A B, C D be equally distant from the centre: that is, let the perpendiculars E F, E G be equal: *then shall* A B = C D. For, the same construction being made, it may, as before, be demonstrated that A B is double of A F, and C D double of C G, and that $AF^2 + FE^2 = CG^2 + GE^2$: but $FE^2 = GE^2$, because F E • Hyp. = G E,* ∴ $AF^2 = CG^2$, ∴ A F = C G: but A B, C D are the doubles of A F, C G, as already proved, ∴ A B = C D; ∴ *equal chords, &c.* Q. E. D.

PROPOSITION XV.—THEOREM.

The diameter is the greatest chord in a circle: and of all others, that which is nearer to the centre is always greater than one more remote: and the greater is nearer to the centre than the less.

Let E be the centre of the circle A B C D, and let B C be nearer to it than F G: *the diameter* A D *is the greatest chord, and* B C *is greater than* F G.

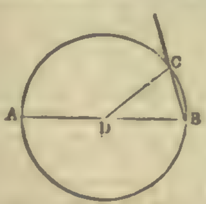
Draw E B, E C. Then because E A = E B and E D = E C, ∴ A D = E B + E C: but E B + E C are greater • 20 I. than B C,* ∴ A D *is greater than* B C. Again, draw E H, E K perpendiculars to B C, F G; then because B C is nearer to the centre than F G,† † Hyp. E H is less than E K; and, as was demonstrated in the preceding proposition, $EH^2 + HB^2 = EK^2 + KF^2$: but EH^2 is *less* than EK^2, because E H is less than E K, ∴ HB^2 is *greater* than KF^2; but B C is the double of H B,* • 3 III. and F G the double of K F, ∴ B C *is greater than* F G.



Next, let B C be greater than F G, then B C *shall be nearer to the centre than* F G; that is, the same construction being made, E H shall be less than E K. Because B C is greater than F G, B H is greater than F K, ∴ BH^2 is greater than FK^2. And $BH^2 + HE^2 = FK^2 + KE^2$, of which BH^2 is greater than FK^2, ∴ EH^2 is *less* than EK^2, ∴ E H is less than EK; that is, B C is nearer to the centre than F G; ∴ *the diameter, &c.* Q. E. D.

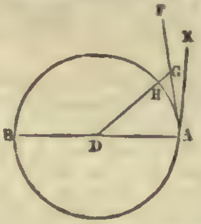
PROPOSITION XVI.—THEOREM.

The straight line drawn at right angles to the diameter (A B) *of a circle* (A B C), *from the extremity of it* (A) *falls without the circle; and no straight line can be drawn from that extremity between the straight line and the circumference, so as not to cut the circle.*



For if A E do not fall without the circle, let a part of it, as A C, fall within the circle; and from the centre D draw D C, to the point C where A E meets the circumference. Then because D A = D C, the angle † 5 I. D A C = D C A;† but D A C is a right angle,* • Hyp. ∴ D C A is a right angle, ∴ D A C + D C A † 17 I. = two right angles, which is impossible,† ∴ a part of A E, *at right angles to* B A, *does not fall within the circle.* And it may be demonstrated, in the same manner, that A E does not fall upon the circumference; ∴ A E *falls without the circle.*

Moreover, between A E and the circumference no straight line can be drawn from A which does not cut the circle. For let A F be supposed to be drawn between them; from D draw D G perpendicular to A F,* • 12 I. and let it meet the circumference in H. Because A G D is a right angle, and D A G less † 17 I. than a right angle,† D A • 19 I. is greater than D G;* but D A = D H, ∴ D H is greater than D G, a part than the whole, which is impossible, ∴ *no straight line can be drawn from* A *between* A E *and the circumference;* ∴ *the straight line, &c.* Q. E. D.



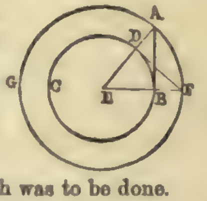
COR.—From this it is manifest that the straight line, at right angles to a diameter of a circle, from an extremity of that diameter, touches the circle; and that it touches it in one point only, because if it met the † 2 III. circle in two points, it would fall within it.† Also it is evident that there can be but one straight line which touches a circle in the same point.

PROPOSITION XVII.—PROBLEM.

To draw a straight line from a given point, either without or in the circumference of a circle (B C D), *which shall touch the circle.*

First let the given point be without the circle as A. • 1 III. Find E the centre,* and draw A E, cutting the circle in D; and with centre E, and radius E A, describe the circle A F G; from D draw D F at right angles to † 11 I. E A,† and draw E B F, A B. A B *shall touch the circle* B C D.

Because E is the centre of the circles B C D, A F G, E A = E F and E D = E B, ∴ the two sides A E, E B are = the two F E, E D, each to each, and the angle E is common to the two triangles A E B, F E D, ∴ the angle E B A • 4 I. = E D F;* but E D F is a right angle, ∴ E B A is a right angle, and E B is a radius; but a straight line from an extremity of a diameter, at right angles to it, † 16 III. Cor. touches the circle,† ∴ A B *touches the circle, and it is drawn from the given point* A; which was to be done.



But if the given point be *in* the circumference, as at D, draw D E to the centre E, and D F at right angles to • 16 III. Cor. D E; D F *touches the circle.**

PROPOSITION XVIII.—THEOREM.

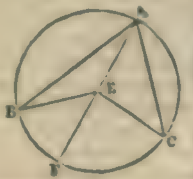
If a straight line touch a circle, the straight line drawn from the centre to the point of contact, shall be perpendicular to the line touching the circle.

This proposition is really included in Prop. XVI.: it affirms that the diameter is perpendicular to the touching line at its extremity: and Prop. XVI. shows that the perpendicular to the diameter at its extremity is the touching line, and the *only* touching line at that extremity. The next proposition is equally superfluous: the only thing proved by either is this—viz., that *a line which is perpendicular to the diameter must have the diameter perpendicular to it.* The two enunciations are retained here only to avoid interruption in the numbering of the propositions.

PROPOSITION XIX.—THEOREM.

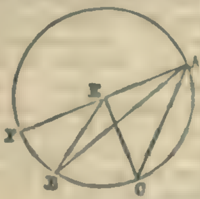
If a straight line touch a circle, and from the point of contact a straight line be drawn at right angles to the touching line, the centre of the circle shall be in that line.

PROPOSITION XX.—THEOREM.



The angle (B E C) *at the centre* (E) *of a circle* (A B C) *is double of the angle* (B A C) *at the circumference upon the same arc* (B C).

Draw A E, and prolong it to F. And first let the centre E be *within* B A C. Because E A = E B, the * 5 I. angle E A B = E B A,* ∴ E A B + E B A is double of E A B; † 32 I. but B E F = E A B + E B A,† ∴ B E F is double of E A B. For a like reason F E C = double of E A C, ∴ B E C *is double of* B A C.

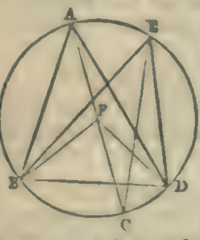


Next let the centre E be *without* B A C. It may be demonstrated, as in the first case, that F E C is double of F A C, and that F E B, a part of the first, is double of F A B, a part of the other, ∴ *the remaining angle* B E C *is double of the remaining angle* B A C; ∴ *the angle at the centre, &c.* Q. E. D.

PROPOSITION XXI.—THEOREM.

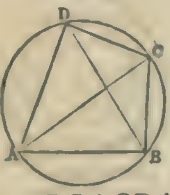
The angles (B A D, B E D) *in the same segment* (B A E D) *of a circle are equal.*

* 1 III. Take F the centre of a circle:* draw F B, F D, as also the diameter A C; and join E, C. Because B F C, at the centre, is upon the same arc B C, as B A C at the circumference, ∴ † 20 III. B F C is double of B A C.† Again, because B F C, at the centre, is upon the same arc B C, as B E C at the circumference, ∴ B F C is double of B E C. But it was shown that B F C is also double of B A C, ∴ B A C = B E C. In like manner it may be proved that C A D = C E D, ∴ *the whole angle* B A D = *the whole angle* B E D; ∴ *the angles, &c.* Q. E. D.



PROPOSITION XXII.—THEOREM.

The opposite angles of any quadrilateral figure (A B C D) *inscribed in a circle* (A B C D), *are together equal to two right angles.*



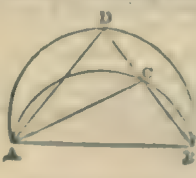
Draw the diagonals A C, B D. Then because the three angles of every triangle are together = two right * 32 I. angles,* ∴ C A B + A B C + B C A = two right angles: but † 21 III. C A B = C D B,† and B C A = B D A, ∴ C A B + B C A = C D A. To each of these add A B C, ∴ A B C + C A B + B C A = A B C + C D A, ∴ A B C + C D A = *two right angles:* and as the *four* * 32 I. angles of a quadrilateral = *four* right angles,* Cor. I. ∴ *the other two opposite angles are also = two right angles;* ∴ *the opposite angles, &c.* Q. E. D.

NOTE.—It appears from the foregoing property, that if one side (as A B) of a quadrilateral in a circle be prolonged, the exterior angle will be equal to the interior and opposite angle (D).

PROPOSITION XXIII.—THEOREM.

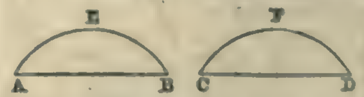
Upon the same straight line (A B), *and on the same side of it, there cannot be two similar segments of circles not coinciding with one another.*

For if it be supposed possible, let A C B, A D B be two similar segments, *not* coinciding with one another. Then, because the circles cut in the two points A, B, they can- * 10 III. not cut in any other point,* ∴ one segment must be within the other. Let A C B be within A D B: draw B C D, as also C A, D A. Then, because the segments are simi- † Hyp. lar,† and that similar segments of circles contain equal * Def. 10 III. angles,* ∴ the angle A C B = A D B, the exterior to † 16 I. the interior and opposite, which is impossible,† ∴ *there cannot be two similar segments of circles upon the same side of the same straight line, which do not coincide.* Q. E. D.



PROPOSITION XXIV.—THEOREM.

Similar segments of circles (A E B, C F D) *upon equal chords* (A B, C D) *are equal to one another.*

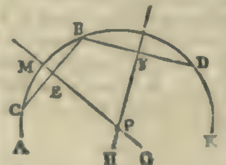


For if the segment A E B be applied to C F D, so that A may be on C, and A B on C D, B shall coincide with D, because A B = C D, ∴ A B coinciding with C D, *the segment* A E B *must coincide with the seg-* * 23 III. *ment* C F D,* *and is* ∴ *equal to it;* ∴ *similar segments, &c.* Q. E. D.

NOTE.—It obviously follows from this, that similar segments of circles upon equal chords have equal arcs.

PROPOSITION XXV.—PROBLEM.

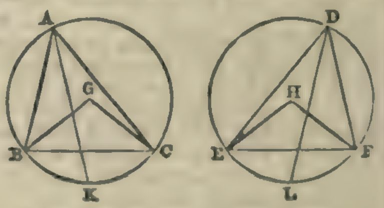
An arc (A B K) *of a circle being given, to describe the circle of which it is an arc.*



Take any point B in the arc, from which draw any two chords B C, B D. Bisect * 10 I. them in E and F,* and † 11 I. draw E G, F H at right angles to B C, B D. These perpendiculars will cut each other in a point P, which will be the centre of the circle. For the centre of the circle to which the arc B C belongs, is in the line E G, bisect- * 1 III. Cor. ing the chord C B at right angles:* it is also in the line F H, bisecting the chord B D at right angles: the centre being thus in *both* the lines E G, F H, must be at P, where they intersect, ∴ *with* P *as centre, and* P M *as radius, if a circle be described, it will be that of which the given arc is a part.* Which was to be done.

PROPOSITION XXVI.—THEOREM.

In equal circles (A B C, D E F) *equal angles stand upon equal arcs, whether they be at the centres or at the circumferences.*



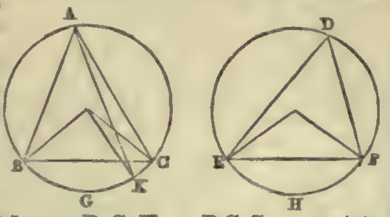
First, let the angles B G C, E H F at the *centres* be equal. Take any point, A, in the arc B A C, and any point, D, in the arc E D F: draw A B, A C, D E, D F, B C, E F. Then, because the circles are equal, the radii G B, G C are equal to the radii H E, H F, each to * Hyp. each; and the angle G = H,* ∴ B C = † 4 I. E F:† and because B A C = E D F,* the seg- * 20 III. ment B A C is similar to the segment E D F;† † Def. 10 III. and they have equal chords, ∴ the seg- * 24 III. ment B A C = segment E D F;* but the whole circle A B K C = the whole circle D E L F, ∴ the remaining segment B K C = E L F, ∴ *the arc* B K C = *arc* E L F.

Next let the angles B A C, E D F at the *circumferences* † 1 III. be equal. Find the centres G, H;† and draw G B, G C, B C; H E, H F, E F: then if B A C be less than a right angle, the angle B G C, at the centre, will stand on the same arc B K C as B A C, and will ∴ * 20 III. be double of B A C.* In like manner E H F will be double of E D F, ∴ the angles G, H at the centres are equal, and ∴, as before proved, the *arcs* B K C, E L F *must be equal.* But if B A C be not less † 9 I. than a right angle, bisect it,† as also the equal angle E D F, by the straight lines A K, D L, then the angles B A K, E D L being equal, and each less than a right angle, the arc B K is, by the first case, = the arc

E L. In like manner K C = L F, ∴ *the arcs* B K C, E L F *are equal,* ∴ *in equal circles, &c.* Q. E. D.

PROPOSITION XXVII.—Theorem.

In equal circles (A B C, D E F) *the angles* (B A C, E D F), *which stand upon equal arcs* (B G C, E H F), *are equal, whether they be at the centres or at the circumferences.*



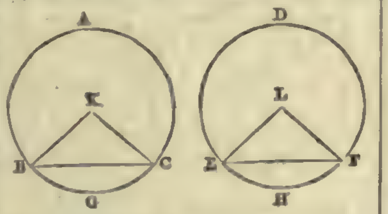
For if not, let one of them, as B A C, be the greater; and make the angle B A K = E D F;* * 23 I. then because in equal circles equal angles stand on equal † 26 III. arcs,† ∴ the arcs B G K, E H F are equal: but B G C = * Hyp. E H F,* ∴ B G K = B G C, a part to the whole, which is impossible, ∴ the angles B A C, E D F cannot be the one greater than the other; that is, *the angles are equal;* ∴ *in equal circles, &c.* Q. E. D.

Note.—The demonstrations of the last two propositions differ considerably from those of Euclid, which, upon examination, will be found to be incomplete, since they exclude the case in which the angles B A C, E D F, at the circumferences, are each not less than a right angle.

It will be observed that the demonstration of the second of these propositions equally applies whether the angles B A C, E D F be at the circumferences, as in the diagrams, or at the centres. These points are left unmarked by letters, in order that, in going over the reasoning, the letters A, D may be mentally transferred to the centres, and thus both cases be included in the the same demonstration.

PROPOSITION XXVIII.—Theorem.

In equal circles (A B C, D E F) *equal chords* (B C, E F) *cut off equal arcs* (B A C, E D F, *and* B G C, E H F) *the greater equal to the greater, and the less to the less.*



* 1 III. Take K, L, the centres of the circles,* and draw K B, K C, L E, L F. Then, because the circles are equal, K B, K C are = L E, L F, each to each, and B C = E F,† † Hyp. ∴ the angle B K C = * 8 I. E L F:\* but equal angles at the centres stand on equal arcs,† ∴ † 26 III. *the arc* B G C = E H F: but the whole cir- * Hyp. cumference A B C = the whole E D F,* ∴ *the remaining part* B A C = *the remaining part* E D F; ∴ *in equal circles, &c.* Q. E. D.

PROPOSITION XXIX.—Theorem.

In equal circles (A B C, D E F) *equal arcs* (B G C, E H F) *are subtended by equal chords* (B C, E F).

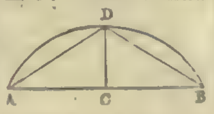
[See preceding diagrams.]

If the arcs are *semi-circumferences,* then B C, E F are *diameters;* and as the circles are equal, these diameters must be equal. But if the arcs be not semi-circumferences, take K, L, the centres of the circles,* and * 1 III. draw K B, K C, L E, L F. Then, because the arc B G C = the arc E H F, the angle B K C = † 27 III. E L F;† and because the circles are equal, K B, K C are equal to L E, L F, each to each, and the * 4 I. angles K, L are also equal, ∴ B C = E F;* ∴ *in equal circles, &c.* Q. E. D.

Note.—It is plain that what is proved in the preceding propositions, as to *equal circles,* necessarily holds in reference to *the same circle.*

PROPOSITION XXX.—Problem.

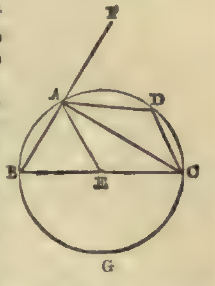
To bisect a given arc (A D B); *that is, to divide it into two equal parts.*



* 10 I. Draw A B and bisect it in C:\* from C draw C D at right angles to A B:† the † 11 I. arc A D B shall be bisected in D. Draw A D, B D. Then, because A C = C B, and C D common to the triangles A C D, B C D, the two sides A C, C D are = the two B C, C D, each to each; and the angle A C D = B C D, each being * 4 I. a right angle, ∴ A D = B D.* But equal chords cut off equal arcs, the greater are equal to the † 28 III. greater, and the less to the less,† ∴ *arc* A D = *arc* B D, each arc being less than a semicircle, because D C, or D C prolonged, passes through the centre; ∴ *the given arc is bisected in* D. Which was to be done.

PROPOSITION XXXI.—Theorem.

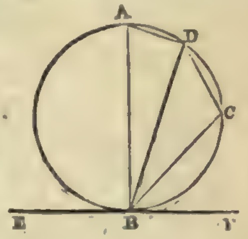
The angle (B A C) *in a semicircle* (B A D C) *is a right angle: the angle in a segment* (A B G C) *greater than a semicircle, is less than a right angle: and the angle* (D) *in a segment* (A D C) *less than a semicircle, is greater than a right angle.*



From the centre, E, draw E A, and produce B A to F. Then, because E B = E A, the * 5 I. angle E A B = E B A;* also because E A = E C, the angle E A C = E C A, ∴ the whole angle B A C = A B C + A C B: but F A C, the exterior angle of the triangle † 32 I. A B C, is = A B C + A C B,† ∴ B A C = F A C, ∴ each is a right * Def. 8 I. angle,* ∴ *the angle* B A C *in a semicircle is a right angle.* Again, because the two angles B, C, of the triangle A B C are together † 17 I. less than two right angles,† and that B A C has been proved to be a right angle, ∴ A B C must be *less* than a right angle, ∴ *the angle* B *in a segment* A B G C, *greater than a semicircle, is less than a right angle.* Lastly, because A B C D is a quadrilateral in a circle, the opposite angles * 22 III. B, D are together = two right angles:* but B has been proved to be *less* than a right angle, ∴ D, *in a segment* A D C *less than a semicircle, is greater than a right angle;* ∴ *the angle, &c.* Q. E. D.

PROPOSITION XXXII.—Theorem.

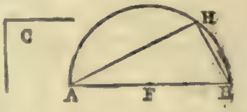
If a straight line (E F) *touch a circle, and from the point of contact* (B) *a straight line* (B D) *be drawn cutting the circle, the angles* (D B F, D B E) *which this line makes with the touching line shall be equal to the angles* (A, C) *in the alternate segments of the circle, each to each.*



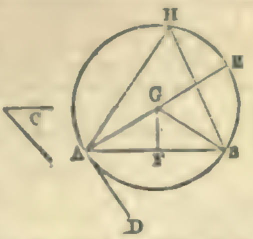
* 11 I. From B draw B A at right angles to E F:* take any point C in the arc D C B, and draw A D, D C, C B. Then, because E F touches the circle in B, and that B A is drawn at right angles to E F, the centre of the circle † 19 III. is in B A,† ∴ A D B, being an angle in a semicircle, is a right angle,* * 31 III. ∴ B A D + A B D † 32 I. = a right angle;† but A B F likewise is a right * Const. angle,* ∴ A B F = B A D + A B D. Take away the common angle A B D, ∴ D B F = A, *the angle in the alternate segment.* Again: because A C is a quadrilateral in a circle A + † 22 III. C = two right angles:† but D B F + D B E * 13 I. = two right angles,* ∴ D B F + D B E = A + C; and it has been proved that D B F = A, ∴ D B E = C, *the angle in the alternate segment;* ∴ *if a straight line, &c.* Q. E. D.

PROPOSITION XXXIII.—Problem.

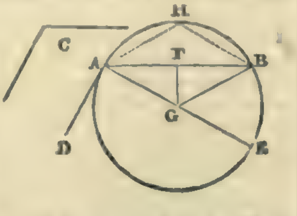
Upon a given straight line (A B) *to describe a segment of a circle capable of containing an angle equal to a given angle* (C).



* 10 I. First let C be a right angle: bisect A B in F,* and with F as centre, and radius F B, describe the semicircle A H B: then *the angle* H, formed by H A, H B, drawn from any point H in the arc, being in a semicircle, † 31 III. *is equal to the right angle* C.†



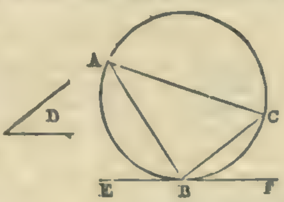
But if C be not a right angle, make B A D = C,* * 23 I. and draw A E at † 11 I. right angles to A D.† * 10 I. Bisect A B in F,* and draw F G at right angles † 11 I. to A B :† F G will meet A E in some point G, because G A F is less than a * Ax 12. right angle ;* and if from G as centre, with radius G A, a circle be described, it will pass through B, and the segment A H B will be that required. For draw G B : then because A F = F B, the two sides A F, F G are = the two B F, F G, each to each ; and the angle † Const. A F G = B F G,† ∴ G A = G B ;* ∴ the * 4 I. circle, with centre G, and radius G A, must pass through B. Again : because from A, an extremity of the diameter A E, A D is drawn at right angles to A E, ∴ A D touches † 16 III. Cor. the circle :† and because A B, drawn from the point of contact, cuts the circle, the angle D A B = H, in the alter- * 32 III. nate segment :* but D A B = C, ∴ H = C ; ∴ *upon* A B *the segment* A H B *of a circle is described, which contains an angle* H = C. Which was to be done.



PROPOSITION XXXIV.—PROBLEM.

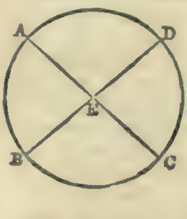
From a given circle (A B C) *to cut off a segment, which shall contain an angle equal to a given angle* (D).

Take any point B in the circumference, and draw E F touching the circle in B,* * 17 III. and make the † 23 I. angle F B C = D.† The segment B A C shall contain an angle A = D. Because E F touches the circle at B, the angle F B C = B A C, in the alternate * 32 III. segment :* but † Const. F B C = D,† ∴ the angle A, in the segment B A C, is = D ; ∴ *from the given circle the segment* B A C *is cut off containing an angle* = D. Which was to be done.



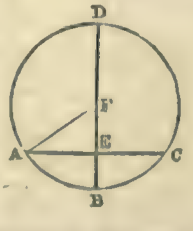
PROPOSITION XXXV.—THEOREM.

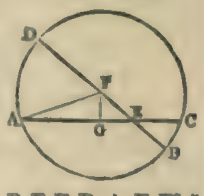
If two chords (A C, B D) *in a circle, cut one another, the rectangle* (A E·E C) *contained by the segments of one of them, is equal to the rectangle* (B E·E D) *contained by the segments of the other.*



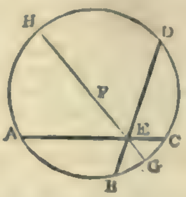
If the chords intersect in the centre E, it is evident that A E, E C, B E, E D, being all equal, *the rectangle* A E·E C = B E·E D.

But if one of them, B D, pass through the centre, and cut the other A C, which does *not* pass through the centre, *at right angles*, in E, then, if B D be bisected in F, F is the centre of the circle ; join A, F. Then, because B D, through the centre cuts A C, not through the centre, at right * 3 III. angles in E, A E = E C.* And because B D is cut into two equal parts in F, and into two unequal parts in E, $B E \cdot E D + E F^2$ † 5 II. $= F B^2$, or $F A^2$;† but * 47 I. $A E^2 + E F^2 = F A^2$,* ∴ $B E \cdot E D + E F^2 = A E^2 + E F^2$. Take away the common square $E F^2$, ∴ $B E \cdot E D = A E^2 = A E \cdot E C$. Again, let B D, which passes through the centre, cut A C, which does not pass through the centre in E, *but not at right angles* ; then, as before, if B D be bisected in F, F is the centre of the circle. Draw A F, and from F draw F G * 12 I. perpendicular to A C,* ∴ † 3 III. A G = G C,† ∴ $A E \cdot E C +$ * 5 II. $E G^2 = A G^2$.* To each of these add $G F^2$, ∴ $A E \cdot E C + E G^2 + G F^2 = A G^2 + G F^2$; but $E G^2$ † 47 I. $+ G F^2 = E F^2$,† and $A G^2 + G F^2 = A F^2$, ∴ $A E \cdot E C + E F^2 =$ * 5 II. $A F^2 = F B^2$; but $F B^2 = B E \cdot E D + E F^2$,* ∴ $A E \cdot E C + E F^2 = B E \cdot E D + E F^2$. Take away $E F^2$, ∴ A E·E C = B E·E D.



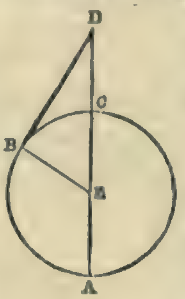


Lastly, let neither of the chords A C, B D pass through the centre. Take the centre F,† † 1 III. and through E, the intersection of the chords, draw the diameter G E F H. Then, because A E·E C = G E·E H, as also B E·E D = G E·E H, as already proved, ∴ A E·E C = B E·E D ; ∴ *if two chords, &c.* Q. E. D.

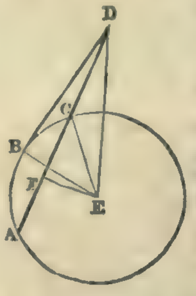


PROPOSITION XXXVI.—THEOREM.

If from any point (D) *without a circle* (A B C) *two straight lines be drawn, one of which* (D A) *cuts the circle, and the other* (D B) *touches it, the rectangle* (A D·D C) *contained by the whole line cutting the circle, and the part without the circle, shall be equal to* ($D B^2$) *the square of the line which touches it* ; that is, $A D \cdot D C = D B^2$.



The line D C A either passes through the centre, or it does not ; first let it pass through the centre E ; and draw E B ; then † 18 III. B is a right angle ;† and because A C is bisected in E, and prolonged to D, $A D \cdot D C + E C^2 =$ * 6 III. $E D^2$;* but E C = E B, ∴ $A D \cdot D C + E B^2 = E D^2$; but $E D^2 = E B^2 + B D^2$, because B is a † 47 I. right angle,† ∴ $A D \cdot D C + E B^2 = E B^2 + B D^2$. Take away $E B^2$, ∴ $A D \cdot D C = B D^2$. But if D C A do not pass through the centre, * 1 III. take the centre E,* and draw E F perpendicular to A C ;† † 12 I. draw also E B, E C, E D. Then because E F, passing through the centre, cuts A C, not through the centre at right * 3 III. angles, it also bisects it,* ∴ A F = F C ; and because A C is bisected in F, and prolonged to D, † 6 II. $A D \cdot D C + F C^2 = F D^2$.† To each of these add $F E^2$, ∴ $A D \cdot D C + F C^2 + F E^2 = F D^2 + F E^2$; but $E D^2 = F D^2 + F E^2$, because E F D is a right angle ;* * 47 I. and, for the same reason, $E C^2 = F C^2 + F E^2$, ∴ $A D \cdot D C + E C^2 = E D^2$; but E C = E B, ∴ $A D \cdot D C + E B^2 = E D^2$; but $E B^2 + B D^2 = E D^2$, because B is a right angle,† † 47 I. ∴ $A D \cdot D C + E B^2 = E B^2 + B D^2$. Take away $E B^2$, ∴ $A D \cdot D C = B D^2$.

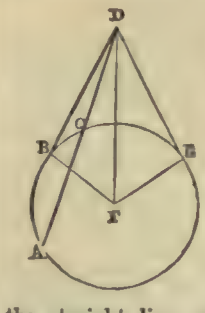


COR.—If from any point without a circle, two straight lines cutting it be drawn, the rectangles of the whole lines and the parts without the circle are equal ; for each rectangle is equal to the square of the line drawn from he point to touch the circle.

PROPOSITION XXXVII.—THEOREM.

If from a point (D) *without a circle* (A B C) *there be drawn two straight lines, one* (D C A) *to cut the circle, and the other* (D B) *to meet it ; and if the rectangle* (A D D C) *of the whole cutting line and the part without the circle be equal to* ($D B^2$) *the square of the line which meets it, the line* (D B) *which meets shall touch the circle.*

* 17 III. Draw D E touching the circle in E :* find



† 3 III. the centre F,† and draw F E, F B, F D: then E is a right * 18 III. angle.* And because D E touches, and D C A cuts the † 36 III. circle, $AD \cdot DC = DE^2$:† * Hyp. but $AD \cdot DC = DB^2$,* ∴ $DE = DB$; also $FE = FB$, ∴ D E, E F are = D B, B F, each to each; and the base F D is common to the two triangles D E F, D B F, † 8 I. ∴ the angle E = B:† but E was shown to be a right angle, ∴ B is a right angle: and B F prolonged is a diameter: but the straight line which is drawn at right angles to a diameter, from an extremity of it, touches the circle,* * 16 III. ∴ D B *touches the circle* A B C; ∴ *if from a point*, &c. Q. E. D.

REMARKS ON BOOK III.

The third book is wholly occupied with propositions concerning the circle. In the definitions we have introduced two terms, the want of which is very much felt in the Elements—the terms *arc* and *chord*. The absence of this latter term has been the occasion of much looseness of expression in certain propositions of Euclid's Third Book: in Proposition I., for instance, as Thomas Simpson has justly noticed, we are directed to "draw within the circle any straight line A B," without the limiting condition that its extremities are to be in the circumference; and the same want of precision is observable in Propositions III. and IV. By "a straight line in a circle," Euclid certainly means exclusively "a straight line in a circle, and terminated both ways by the circumference;" but this should have been expressly stated, and not left to be inferred from observation. Had Definition VII., Book IV. (Simson's Euclid), been placed among the definitions of the third book, the objection here advanced would have been removed; since, according to that definition, "A straight line is said to be placed in a circle, when the extremities of it are in the circumference of the circle;" but it is better to call such a line a *chord*, the name given to it in all other geometrical inquiries.

The demonstration in Proposition I. of Simson's Euclid is imperfect, since all that it proves is, that the centre of the circle must be in the line C E, without determining the particular *point* in that line; the omission is supplied in the present edition.

The enunciation of Proposition VI. in Simson is, "If two circles touch one another *internally*, they shall not have the same centre;" but as it is plain that each circle cannot be *within* the other, the word *internally* should be omitted. If two circles touch *externally*, then, since each must be wholly *without* the other, it is obvious that they cannot have the same centre. In Proposition VII. certain lines are introduced by Euclid—namely, the lines E C, E G—long before they are wanted; and the diagram is thus unnecessarily complicated at the outset of the demonstration. We have already stated (page 549), with a view to an orderly and systematic arrangement of the steps of the argument, that the diagram should not proceed, in advance of the text: the more you depart from this plan, the more you depart from simplicity; since you thus encumber the diagram with lines that serve no purpose but to distract your attention. In demonstrating this seventh proposition without the book, you should commence by exhibiting in your diagram only the lines F B, F C, in addition to the diameter A D: you should then draw E B, for the purpose of proving that F A is greater than F B, whatever line, other than F A, F B may be: one part of the proposition is thus disposed of. The next step is to draw E C, the aid of which is now required—but not till now—to prove that F B, a line nearer to that passing through the centre, is greater than any other line F C more remote: another part of the theorem is thus demonstrated. You may now, if you please, introduce F G, and then draw E G, for the purpose of proving that F D is shorter than any other line drawn from F. We say you may introduce F G *if you please;* for, in strictness, there is no absolute occasion for it; F C, or F B, already drawn, would answer the purpose equally well: of the remaining part of the construction nothing need be said here.

In Proposition VIII. we have actually omitted Euclid's superfluous line—the line corresponding to that which, as just noticed, may be dispensed with in the preceding proposition. As the diagram here is somewhat more complicated, it was thought that the suppression of an unnecessary line from D, and *consequently* of two lines in connection with it from M, would be a desirable relief to it. You will perceive it to be so if you compare the diagram here, with that in other editions of Euclid.

The demonstration of Proposition IX., given in the present edition, is not that of Euclid, though substituted for it by De Chales, and some other editors. In the earlier copies of Euclid, two different demonstrations of this simple theorem are given. They have both been preserved by Gregory, Stone, Williamson, and Bonnycastle. It is the second of these two only that Simson retains; but, as remarked by Williamson, there is a defect in the reasoning, which, however, it is scarcely worth while to remove. The demonstration given in the text is so easy and obvious, that, as Austin justly observes, "we cannot suppose Euclid would have overlooked it." It is probable, therefore, that the demonstrations here discarded are not Euclid's own, but the interpolations of some other writer.

Although it is proved in Proposition XX. that an angle at the centre of a circle is double that at the circumference subtended by the same arc, yet it must not be inferred that to *every* angle at the circumference there corresponds an angle at the centre, upon the same arc, which is double of the former. If the angle at the circumference stand upon a semi-circumference, there can evidently be *no* corresponding angle at the centre upon the same arc; and if the angle at the circumference stand upon an arc greater than a semi-circumference, the angle at the centre, the sides of which terminate in the extremities of the same arc, will stand—not upon *that* arc—but upon the opposite part of the circumference.

The construction of Proposition XXV. is different in the present edition from that given in Euclid, which requires three diagrams: moreover, the "*segment* of a circle," as Euclid has it, need not be given, only the *arc*. Propositions XXVI. and XXVII. differ also in matters of construction from those of Euclid; for, as they at present stand in Simson, they are insufficiently demonstrated; as noticed in the text.

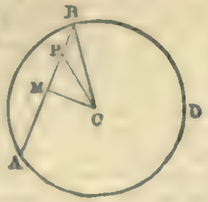
Proposition XXXIII.—In all the editions of Euclid that we have seen, there is a superfluous line (the line B E) introduced into the second of the three diagrams; and in some editions the line B E is also introduced into the third diagram, for what purpose we are unable to conceive; the tendency, though of course not the intention, is to mislead.

The demonstration applies equally to *both* diagrams. In the first, the line B E, which goes to form the angle E, is quite in keeping with the text, as this is really an angle in the segment spoken of. But in the second of these diagrams, the line is not merely superfluous—it is *wrong;* since the angle E, of which it is a side, is an angle in a segment different from that to which the text refers. In order that the proper angle should be exhibited in both diagrams, the vertex of it should be at H, as shown by the dotted lines in this edition, but there is no absolute occasion to introduce this angle at all.

Propositions XXXV. and XXXVI. may be otherwise and more easily established, as follows:—

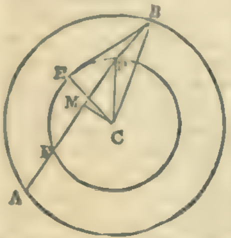
Proposition XXXV.—Let P be any point *within* a circle A B D. It is required to prove that whatever chord, A B, be drawn through P, the rectangle $AP \cdot PB$ will be always the same.

* 1 III. Find the centre C;* and draw C M at right † 12 I. angles to A B;† draw also C P, C B. Then,



* 3 III. because A B is divided equally in M,* and unequally in P, $A P \cdot P B + P M^2$ † 3 II. $= M B^2$.† Add $C M^2$ to each, ∴ $A P \cdot P B + P M^2 + C M^2 = M B^2 + C M^2$; that is, $A P \cdot$ * 47 I. $P B + P C^2 = B C^2$.* Take $P C^2$ from each, ∴ $A P \cdot P B = B C^2 - P C^2$. But $B C^2$ is always the same wherever in the circumference the point B may be, and $P C^2$ is always the same whatever chord be drawn through P, ∴ $B C^2 - P C^2$ is always the same, ∴ $A P \cdot P B$ is always the same, whatever chord (A B) be drawn through P; ∴ if two chords cut one another in any point P, the rectangle contained by the segments of one of them is equal to that contained by the segments of the other.

Proposition XXXVI.—Let B be any point *without* a circle P E F. It is required to prove that whatever line B P F be drawn, *cutting* the circle, the rectangle $B F \cdot B P$ shall be equal to the square of B E, a line drawn *touching* the circle.



* 1 III. Find the centre C,* and draw C B, C P, C E. With this same centre C, and radius C B, describe a circle, and produce B F to meet its circumference in A. Then (last Proposition) $A P \cdot B P = B C^2$ † 47 I. $- P C^2 = B C^2 - E C^2 = B E^2$.† Draw the perpendicular C M to A B; then C M bisects both A B * 3 III. and F P,* ∴ $A F = P B$, ∴ $A P = B F$. But it was proved that $A P \cdot P B = B E^2$; ∴ $B F \cdot B P = B E^2$, which was to be demonstrated.

An obvious corollary follows from this proposition, viz.—

Cor. 1.—Since from any point without a circle *two* † 17 I. lines may be drawn to touch the circle,† these two touching lines must be *equal*.

We shall only further observe that, besides the particulars mentioned above, in which slight departures from the authorised text of Euclid have been made, other modifications, chiefly in the diagrams, have been introduced in certain propositions; but we have nowhere ventured upon any change, where the circumstances of the case did not fully justify and require it.

EXERCISES ON BOOKS I., II., III.

1. Prove the *converse* of Proposition XXII.—Book III.; namely, if the opposite angles of a quadrilateral be together equal to two right angles, a circle may be described about it.

2. A trapezium may be inscribed in a circle, provided two of the opposite sides are parallel, and that the two non-parallel sides are equal.

3. If a quadrilateral be described about a circle, that is, if the four sides touch the circle, one pair of opposite sides will always be equal to the other pair.

4. If from the vertices of the three angles of a triangle, perpendiculars be drawn to the opposite sides, they will intersect in the same point.

5. If two circles cut one another, the line joining the intersections of the circumferences, shall be perpendicular to the line joining the centres.

6. If two circles cut one another, and from one of the points of intersection diameters be drawn, the extremities of these diameters shall be in the same straight line as the other point of intersection.

7. If any two chords of a circle intersect at right angles, the squares upon their four segments will, together, be equal to the square upon the diameter.

8. If two circles touch each other, any straight line through the point of contact will cut off similar segments.

9. If a quadrilateral be described about a circle, the angles subtended at the centre by one pair of opposite sides, will, together, be equal to those subtended by the other pair; that is, to two right angles.

10. Find a point in the prolongation of a diameter of a circle, from which, if a line be drawn to touch the circle, it shall be equal to a given straight line.

11. Two chords A D, B C, are drawn in a semicircle from the extremities of the diameter A B; the chords intersect in P. Prove that the rectangles $A D \cdot A P$, $B C \cdot B P$ are together equal to the square of the diameter.

12. If from any point in the diameter of a semicircle two straight lines be drawn to the circumference, one to the middle of the arc, the other at right angles to the diameter, the squares upon these two lines always amount to the same sum, wherever the point be taken.

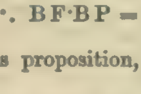
13. From a given point without a circle to draw a straight line to cut it, and to terminate in the circumference, such that the intercepted chord may have a given length, not greater than the diameter of the circle.

CHAPTER IV.

ELEMENTS OF EUCLID.—BOOK IV.

DEFINITIONS.

I.

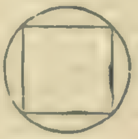


A rectilineal figure is said to be *inscribed* in another rectilineal figure, when all the *vertices* of the former are upon the *sides* of the latter, each upon each.

II.

A rectilineal figure is said to be *described* about another, when all the *sides* of the former pass through the *vertices* of the latter, each through each.

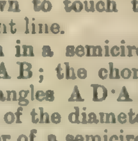
III.



A rectilineal figure is said to be inscribed in a circle, when all the vertices of the former are upon the circumference of the circle.

IV.

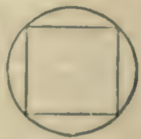
A rectilineal figure is said to be described about a circle when each side of the former *touches* the circumference of the circle.



V.

A circle is said to be inscribed in a rectilineal figure, when the circumference of the circle *touches* each side of the figure.

VI.



A circle is said to be described about a rectilineal figure, when its circumference passes through all the vertices of that figure.

VII.

A straight line is said to be *placed in a circle*, when the extremities of it are in the circumference of that circle.*

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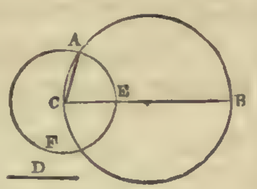
* See *ante*, Book III., Prop. I., III., and IV.

PROPOSITION I.—Problem.

In a given circle (A B C), *to place a straight line equal to a given straight line* (D), *which is not greater than the diameter of the circle.*



Draw B C, any diameter of the circle. Then if C B = D, the thing required is done; but if B C be not = D, • Hyp. it is greater;* make C E = D; and with centre C and radius C E, describe the circle A E F; join C A; C A shall be = D.

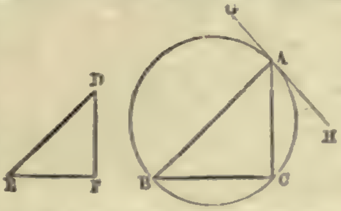
Because C is the centre of the circle A E F, C A = • Const. C E; but D = C E,* ∴ D = C A; *therefore in the circle* A B C, *a straight line, equal to* D, *is placed as required.*

Note.—The enunciation of this proposition might have been as follows:—From a given point, in the circumference of a given circle, to draw a *chord* equal to a given straight line, which is not greater than the diameter.

PROPOSITION II.—Problem.

In a given circle (A B C) *to inscribe a triangle equiangular to a given triangle* (D E F).

Take any point A in the circumference, and draw † 17 III. G A H to touch the circle in that point.† Make • 23 I. the angle H A C = E, and G A B = F,* and draw B C: A B C shall be the triangle required.



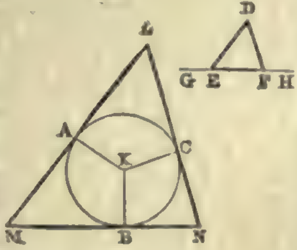
Because H A G touches the circle, and A C is drawn from the point of contact, the angle H A C = B, in the alternate seg- † 22 III. ment:† but H A C = E, ∴ B = E. For a like reason, C = F, ∴ the remaining angle B A C = the • 32 I. remaining angle D;* ∴ *the triangle* A B C, *inscribed in the circle* A B C, *is equiangular to* D E F. Which was to be done.

Note.—If the angle H A C has been made equal to F, instead of E, the resulting triangle would still have been equiangular to D E F: but the construction above furnishes a triangle not only equiangular, but similar in *position* to D E F. Like remarks apply to the next proposition.

PROPOSITION III.—Problem.

About a given circle (A B C) *to describe a triangle equiangular to a given triangle* (D E F).

Prolong E F both ways to G and H: find the • 1 III. centre K of the circle,* and draw any radius K B. Make the angle B K A = D E G, † 23 I. and the angle B K C = D F H:† and through A, B, C, draw L M, M N, N L to touch the • 17 I. circle:* these lines will meet and form a triangle L M N equiangular to D E F.

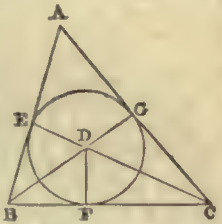


Because L M, M N, N L touch the circle at A, B, C, the lines K A, K B, K C from the centre make the angles at A, B, C *right* † 18 III. angles,† ∴ A M, B M would make with a line joining A, B, angles which are together *less* than two right angles, ∴ A M, • Ax. 12. B M, must *meet.** In like manner A L, C L, meet; as also B N, C N: and because the four angles of the quadrilateral A M B K, are = four right angles, for it can be divided into two triangles, and that two of the angles K A M, K B M, are right angles, ∴ the other two, A K B, A M B make two right angles. But D E G + † 13 I. D E F = two right angles,† ∴ A K B + A M B = D E G + D E F; but A K B = D E G, ∴ A M B = D E F. In like manner it may be proved that L N M = D F E, ∴ the remaining angle L = the remaining angle • 32 I. D;* ∴ the triangle L M N is equiangular to the triangle D E F; *and it is described about the circle.* Which was to be done.

PROPOSITION IV.—Problem.

To inscribe a circle in a given triangle (A B C).

• 9 I. Bisect the angles A B C, B C A* by the straight lines B D, C D, meeting one another in D, from which draw D E perpendicular to one of the sides.† † 12 I. The circle described with centre D, and radius D E, will be inscribed in the triangle.

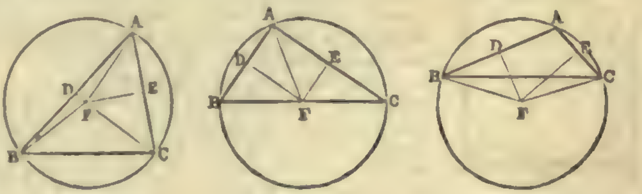


For draw D F, D G, perpendiculars to the other sides:* • 12 I. then, because the angle E B D = F B D, for A B C is † Const. bisected by B D,† and • Ax. 11. that B E D = B F D,* ∴ the triangles E B D, F B D have two angles of the one = two of the other, each to each; and the side B D, opposite to one of the equal angles in each, † 26 I. common to both, ∴ the other sides are equal;† that is, D E = D F. For a like reason, D G = D F, ∴ D E = D G, ∴ D E, D F, D G are all equal; and the circle described with centre D, and *either* of these for a radius, will pass through the extremities of *all;* and will *touch* A B, B C, C A, because the angles at E, F, G • 16 III. are right angles;* ∴ *the circle* E F G *is inscribed in the triangle* A B C. Which was to be done.

PROPOSITION V.—Problem.

To describe a circle about a given triangle (A B C).

• 10 I. Bisect A B, A C in D, E;* and from these points draw D F, E F, at right angles to A B, A C.† † 11 I. Then D F, E F must meet one another, because they make with a line joining D, E, angles on the same • Ax. 12. side, together *less* than two right angles.* Let them meet in F, and draw F A, then a circle described with centre F, and radius F A, will circumscribe the triangle A B C.

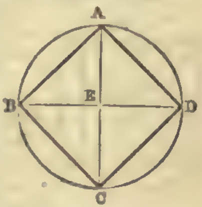


For if the point F be not in B C, draw B F, C F. Then, because A D = D B, and D F is common, and at † 4 I. right angles to A B, A F = F B.† In like manner, it may be shown that C F = A F, ∴ F B = F C, ∴ F A, F B, F C are all equal; ∴ the circle described with centre F, and *either* of these lines for radius, will pass through the extremities of *all, and be circumscribed about the triangle* A B C. Which was to be done.

PROPOSITION VI.—Problem.

To inscribe a square in a given circle (A B C D).

Draw any two diameters A C, B D at right angles to one another: draw A B, B C, C D, D A: the figure A B C D shall be the square required.



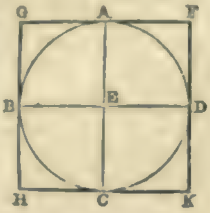
Because E is the centre, E B = E D; also E A is common, and at right angles to B D, ∴ • 4 I. A B = A D.* For a like reason C B, C D are respectively = A B, A D, ∴ the figure A B C D is *equilateral.* It is also *rectangular;* for B D being a diameter of the circle, B A D is a semicircle, ∴ B A D is a right angle.† † 31 III. For the same reason each of the angles A B C, B C D, C D A, like D A B, is a right angle, ∴

the figure is *rectangular*, ∴ it is a *square ; and it is inscribed in the circle* A B C D. Which was to be done.

PROPOSITION VII.—PROBLEM.

To describe a square about a given circle (A B C D).

Draw two diameters, A C, B D, at right angles to one another, and through A, B, C, D, draw F G, G H, H K, K F, touching the circle ;* the * 17 III. figure G H K F shall be the square required. Because F G touches the circle, and E A is drawn from the centre to the point of contact A, the angles at † 18 III. A are right angles.† For a like reason, the angles at B, C, D, are right angles. And because the angle A E B is a right angle, as likewise E B G, G H is * 28 I. parallel to A C ;* for a like reason A C is parallel to F K. In a similar manner may it be proved that G F, H K, are each parallel to B D, ∴ the figures G K, G C, A K, F B, B K, are parallelograms, ∴ G F † 34 I. = H K, and G H = F K ;† and because A C = B D, and that A C = G H = F K ; and B D = G F = H K, ∴ G H, F K, are each = G F or H K, ∴ the figure F G H K is *equilateral*. It is also *rectangular ;* for G E being a parallelogram, and A E B a right angle, * 34 I. G is a right angle ;* and G K, being a parallelogram with a right angle at G, is *rectangular*,† † Cor. 46 I. ∴ it is a *square ; and it is described about the circle* A B C D. Which was to be done.

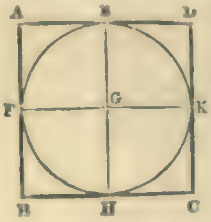


PROPOSITION VIII.—PROBLEM.

To inscribe a circle in a given square (A B C D).

* 10 I. Bisect each of the sides A B, A D in F, E ;* draw E H parallel to A B or D C, and F K parallel to † 31 I. A D or B C ;† then, if with their point of intersection G, as centre, and G F, or G E, as a radius, a circle be described, it will be that required.

Each of the figures A K, K B, A H, H D, A G, G C, B G, G D, being a parallelogram, their opposite sides are * 34 I. equal :* and because A D = A B, and A E = half A D, and A F = half A B, A E = A F, ∴ the sides opposite to these are equal—viz., F G = G E. In like manner, it may be demonstrated that G H = G K, and G H = G F, ∴ the four G E, G F, G H, G K are equal to one another ; and the circle described from centre G, with any one of them for a radius, will pass through the extremities of all, and will *touch* A B, B C, C D, D A, because the angles E, F, H, K are right angles ;† † 16 III. Cor. *the circle is* ∴ *inscribed in the square* A B C D. Which was to be done.

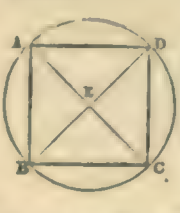


PROPOSITION IX.—PROBLEM.

To describe a circle about a given square (A B C D).

Draw A C, B D, intersecting in E : with E as centre, and E A as radius, describe a circle :—it will be that required.

Because D A = A B, and A C is common to the triangles D A C, B A C, the two sides D A, A C = the two B A, A C, each to each ; and D C = B C, ∴ the * 8 I. angle D A C = B A C ;* that is, the angle D A B is bisected by A C. In like manner, it may be proved that the angles A B C, B C D, C D A, are severally bisected by B D, C A. Again : the angle D A B = A B C, also E A B = half D A B, and E B A = half A B C, ∴ E A B = E B A, † 6 I. ∴ E A = E B.† In like manner, it may be demonstrated that E C, E D = E B, E A, each to each, ∴ the four E A, E B, E C, E D are equal to one another ; and the circle whose centre is E, and radius E A, passes through the extremities of all, *and is* ∴ *described about the square* A B C D. Which was to be done.

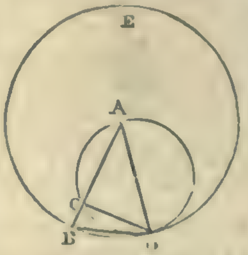


PROPOSITION X.—PROBLEM.

To describe an isosceles triangle having each of the angles at the base double of the third angle.

Take any straight line A B, and divide it in C, so * 11 II. that $A B B C = A C^2$,* and with centre A and radius A B, describe the circle B D E, in which place † 1 IV. B D = A C ;† and draw A D ; the triangle A B D shall be such that each of the angles A B D, A D B shall be double of B A D.

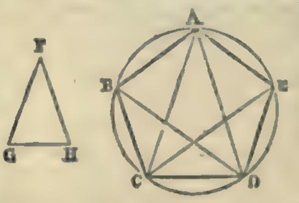
Draw D C ; and about the triangle A C D describe the circle A C D ; then because * Const. $A B \cdot B C = A C^2$,* and A C = B D, ∴ $A B \cdot B C = B D^2$, ∴ B D *touches* the cir- † 37 III. cle† A C D in D, ∴ the angle B D C = D A C ;* * 32 III. to each of these add C D A, ∴ B D A = D A C + C D A ; but the exterior † 32 I. angle B C D = D A C + C D A,† ∴ B D A = * 5 I. B C D ; but B D A = B, because A B = A D,* ∴ B = B C D, ∴ the three angles D B A, B D A, B C D, are equal to one another. Again, because the angle * 6 I. D B C = D C B, D C = D B ;* but D B was made = C A, ∴ C A = C D, ∴ the angle C D A = † 5 I. C A D,† ∴ C D A + C A D = twice C A D ; * 32 I. but B C D = C D A + C A D,* ∴ B C D = twice C A D ; but B C D = B D A = B, as before proved, ∴ each of the angles B D A, D B A is double of B A D, ∴ *an isosceles triangle* A B D *has been described, having each of the angles at the base double of the third angle.* Which was to be done.



PROPOSITION XI.—PROBLEM.

To inscribe an equilateral and equiangular pentagon in a given circle (A B C D).

Describe an isosceles triangle F G H, having each of * 10 IV. the angles G, H, double of F ;* and in the circle inscribe a triangle A C D, equiangular to the triangle F G H, so that the angle C A D may be † 2 IV. = F ;† then each of the angles A C D, A D C is double of C A D. Bisect A C D, A D C, by the straight lines C E, * 9 I. D B ;* and draw A B, B C, D E, E A. A B C D E shall be the pentagon required. Because each of the angles A C D, A D C is double of C A D, and that they are bisected by C E, D B, ∴ the five angles D A C, A C E, E C D, C D B, B D A are equal to one another ; ∴ the five arcs A B, † 26 III. B C, C D, D E, E A are equal to one another,† ∴ the lines A B, B C, C D, D E, E A are equal to one * 29 III. another,* ∴ the pentagon A B C D E is *equilateral*. It is also *equiangular*, for since the arc A B = the arc D E, if to each B C D be added, the whole A B C D = E D C B ; but these arcs subtend the angles † 27 III. A E D, B A E, ∴ A E D = B A E.† For a like reason each of the angles A B C, B C D, C D E = B A E or A E D ; ∴ the pentagon is *equiangular*, ∴ *in the given circle an equilateral and equiangular pentagon has been described.* Which was to be done.



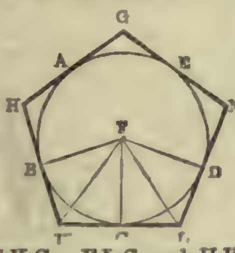
PROPOSITION XII.—PROBLEM.

To describe an equilateral and equiangular pentagon about a given circle (A B C D).

Let the vertices of a pentagon, inscribed in the circle, by last proposition, be at A, B, C, D, E ; so that the arcs A B, B C, C D, D E, E A are equal ; and through these points draw G H, H K, K L, L M, M G, touching

* 17 III. the circle :* the figure GHKLM shall be the pentagon required.

From the centre F draw FB, FK, FC, FL, FD. Then because KL touches the circle in C, FC is perpendicular † 18 III. to KL,† ∴ the angles at C are right angles : for a like reason the angles B, D are right angles. And because FCK is a right angle, $FK^2 =$ * 47 I. $FC^2 + CK^2$:* for a like reason $FK^2 = FB^2 + BK^2$, ∴ $FC^2 + CK^2 = FB^2 + BK^2$; but $FB^2 = FC^2$, ∴ $BK^2 = CK^2$, ∴ BK = CK. Again : because FB = FC, the two sides FB, FK = the two FC, FK ; and BK = CK, as just proved, ∴ the angle BFK = † 8 I. CFK,† and BKF = CKF, ∴ BFC = *twice* CFK, and BKC = *twice* CKF. For a similar reason CFD = twice CFL, and CLD = twice CLF : and because the arc BC = arc CD, the angle BFC = * 27 III. CFD :* and BFC = twice CFK, and CFD = twice CFL, ∴ CFK = CFL ; ∴ in the two triangles FKC, FLC, there are two angles of the one equal to two of the other, each to each, and the side FC, adjacent to equal angles in each, is common to both, * 26 I. ∴ KC = LC,* and the angle FKC = FLC, and KL is twice KC. In a similar manner it may be shown that HK is twice BK, and consequently, since it was proved that BK = KC, and that KL = twice KC, and HK = twice BK, HK = KL. In like manner it may be shown that GH, GM, ML are each = HK or KL : ∴ the pentagon GHKLM is *equilateral.* It is also *equiangular* ; for, since the angle FKC = FLC, and HKL = twice FKC, and KLM = twice FLC, as already proved, ∴ HKL = KLM. And in like manner it may be shown that each of the angles KHG, HGM, GML is = HKL or KLM ; ∴ the five angles GHK, HKL, KLM, LMG, MGH, being equal to one another, the pentagon is *equiangular, and it is described about the circle* A B C D. Which was to be done.

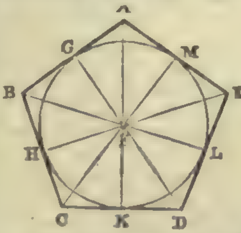


PROPOSITION XIII.—Problem.

To inscribe a circle in a given equilateral and equiangular pentagon (A B C D E).

Bisect the angles BCD, CDE by the straight lines, * 9 I. CF, DF ;* and from the point F, in which they meet, draw FK perpendicular to one of the sides ; then if with F as centre and FK as radius a circle be described, it will touch every side of the pentagon.

† Hyp. Draw FB, FA, FE ; then since BC = CD,† and CF is common to the triangles BCF, DCF, the two sides BC, CF = DC, CF each to each ; and the * 4 I. angle BCF = DCF, ∴ BF = FD,* and the angle CBF = CDF. And because CDE = twice † Const. CDF,† and that CDE = CBA, and CDF = CBF, as just proved, ∴ CBA = twice CBF, ∴ ABF = CBF, ∴ ABC is *bisected* by BF. In a similar manner it may be demonstrated that the angles BAE, AED are bisected by FA, FE. From F draw FG, FH, FL, FM perpendicular to AB, BC, DE, EA ; then because the angle HCF = KCF, and FHC = FKC, being right angles, ∴ in the triangles FHC, FKC, two angles of the one are equal to two of the other, each to each ; and the side FC opposite to one of the equal angles in each is common to both, * 26 I. ∴ FH = FK.* In like manner it may be proved that FL, FM, FG are each = FK, or FH, ∴ the five FG, FH, FK, FL, FM, are equal to one another, ∴ the circle described from centre F with either of them for radius, will pass through the extremities of all ; it will moreover *touch* AB, BC, CD, DE, EA ; since the angles at G, H, K, L, M, are right angles ;† † 16 III. ∴ *the circle is inscribed in the pentagon* ABCDE. Which was to be done.

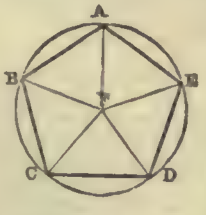


PROPOSITION XIV.—Problem.

To describe a circle about a given equilateral and equiangular pentagon (A B C D E).

Bisect the angles BCD, CDE by the straight lines * 9 I. CF, DF ;* and with the point F, in which they meet, as centre, and either of them as radius, if a circle be described, it will be that required.

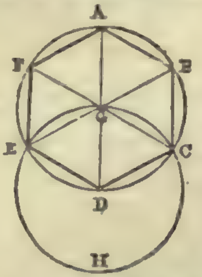
Draw FB, FA, FE. It may be demonstrated, as in the preceding proposition, that the angles CBA, BAE, AED are bisected by FB, FA, FE ; and because the angle BCD = CDE, and that FCD is half BCD, and CDF half CDE, ∴ FCD = † 6 I. FDC, ∴ CF = FD.† In like manner it may be demonstrated that FB, FA, FE are each = FC or FD, ∴ the five FA, FB, FC, FD, FE are equal to one another ; and ∴ the circle described from centre F, with either of them for radius, will pass through the extremities of all, and will ∴ *be described about the pentagon* ABCDE. Which was to be done.



PROPOSITION XV.—Problem.

To inscribe an equilateral and equiangular hexagon in a given circle (A C D F).

* 1 III. Find the centre G of the circle,* and draw the diameter AGD ; from D as centre with radius DG, describe the circle EGCH ; draw EG, CG, and prolong them to B, F ; and draw AB, BC, CD, DE, EF, FA ; ABCDEF shall be the equilateral and equiangular hexagon required. Because G is the centre of the circle ACDF, GE = GD ; and because D is the centre of the circle EGCH, DE = DG, ∴ GE = ED, and the triangle EGD is equilateral, ∴ the angles EGD, GDE, DEG, † 5 I. C. are equal to one another ;† and as they are * 32 I. together equal to two right angles,* ∴ EGD is the third part of two right angles. In a similar manner it may be demonstrated that DGC is the third part of two right angles ; and because GC makes with EB the adjacent angles EGC, CGB, equal to two right † 13 I. angles,† the remaining angle CGB is also the third part of two right angles, ∴ EGD, DGC, CGB, are equal to one another ; and to these are equal the vertical * 15 I. or opposite angles BGA, AGF, FGE ;* ∴ the six angles at G are equal to one another ; consequently the six arcs which subtend them are † 26 III. equal,† and ∴ the six * 29 III. chords of these arcs,* ∴ the hexagon ABCDEF is *equilateral.* It is also *equiangular;* for since the arc AF = ED, add ABCD to each ; ∴ the whole arc FABCD = EDCBA ; and the angle FED stands upon FABCD, and AFE upon EDCBA, ∴ AFE = FED.† † 27 III. In a similar manner it may be demonstrated that the other angles of the hexagon are each of them = AFE, or FED ; ∴ the hexagon is *equiangular ; and it is inscribed in the given circle* ACDF. Which was to be done.

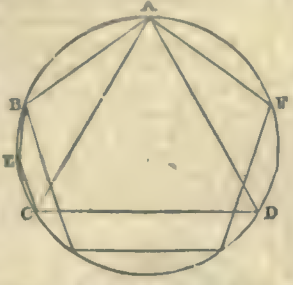


Cor.—From this it is manifest that a side of the hexagon is equal to the radius of the circle. And if through A, B, C, D, E, F lines be drawn touching the circle, an equilateral and equiangular hexagon will be described about it, as may be demonstrated from what has been said of the pentagon : and likewise a circle may be inscribed in a given equilateral and equiangular hexagon, and circumscribed about it, by a method like to that used for the pentagon.

PROPOSITION XVI.—Problem.

To inscribe an equilateral and equiangular quindecagon, or fifteen-sided figure, in a given circle (A B C D).

Let A C be a side of an equilateral triangle inscribed * 2 IV. in the circle,* and A B a side of an equilateral and equiangular pentagon inscribed in the same:† † 11 IV. then of such parts as the whole circumference contains *fifteen*, the arc A B C, being the third part of the circumference, contains *five*; and the arc A B, which is the fifth part of the circumference, contains *three*; ∴ B C, their difference, contains *two* of the same parts. Bisect * 30 III. B C in E,* ∴ B E, E C are each the fifteenth part of the whole circumference; ∴ if the straight lines B E, E C, be drawn, and straight lines equal to them be placed round, in the whole circle, an *equilateral* quindecagon will be inscribed in it; and that it is *equiangular* is plain, because each angle stands upon an arc, equal to the whole circumference diminished by the two arcs which its sides subtend, and which arcs are by construction equal, ∴ *an equilateral and equiangular quindecagon is inscribed in the circle.* Which was to be done.



And in the same manner as was done in the pentagon, if through the points of division made by inscribing the quindecagon, straight lines be drawn touching the circle, an equilateral and equiangular quindecagon will be described about it. And likewise, as in the pentagon, a circle may be inscribed in a given equilateral and equiangular quindecagon, and circumscribed about it.

REMARKS ON BOOK IV.

The propositions in this fourth book are all *problems*, relating chiefly to the construction of regular polygons in and about the circle. From the fourth we learn this theoretical truth; namely, that the straight lines bisecting the three *angles* of a triangle all meet in a point—the centre of the inscribed circle; and from the fifth, that the straight lines bisecting the three *sides* of a triangle also meet in a point—the centre of the circumscribing circle.

As the second proposition shows that a triangle equiangular to any proposed triangle may be inscribed in a circle, we know that an equilateral triangle may be inscribed; and we thus infer that the circumference of a circle may be geometrically divided into *three* equal parts. By bisecting each of these three equal arcs (Proposition XXX., Book III.), we may divide the circumference into *six* equal parts, as otherwise shown in Proposition XV.; and by another series of bisections, into *twelve* equal parts, and so on. The division of the *whole* circumference into *six* equal parts is obviously the division of the semi-circumference into *three;* the division of the whole into *twelve* equal parts is the division of the fourth of it—that is, of a *quadrant*, into *three* equal parts, and so on. We thus see that an arc equal to a fourth part, or to an eighth part, a sixteenth part, &c., of the entire circumference, may be divided geometrically into *three* equal parts. But the general problem to divide *any* arc of a circle into three equal parts, or, which amounts to the same thing, to *trisect an angle*, is a problem that all the geometry in Euclid has been found hitherto inadequate to accomplish. It is one of the great outstanding problems of antiquity; and more thought and labour have been expended upon it, during the last two thousand years, than perhaps any other problem has ever called into exercise. If you now learn this fact for the first time, you will be surprised that a thing apparently so simple as to divide an angle or an arc of a circle into *three* equal parts, should have been found to be a matter of such surpassing difficulty as to have baffled the efforts of the greatest geometers from the time of Euclid to the present day. We draw attention to it here mainly for the purpose of discouraging any fresh attempts by students in geometry to trisect an angle. Novices are very apt to enter upon such attempts from a vague notion that a successful solution of the problem would, in some way or other, advance science. But geometricians know better; and by *them* the inquiry has long been abandoned, not only as a hopeless one, but also as a comparatively useless one. The theorist has no need of this solution, because he is never stopped in any mathematical investigation from the want of it; the practical man has no need of it, for he can avail himself of mechanical methods which effect the trisection of an angle that departs from strict geometrical accuracy, by an amount of error too minute to be detected by his senses, though aided by the most finished instruments of measurement; and these methods he would continue to employ even were the problem ever to be brought within the scope of elementary geometry. You see, therefore, that the inquiry is one of pure curiosity, and nothing more; since the successful issue of it could supply no want either in theory or in practice.

By aid of certain curves beyond the limits of Euclidean geometry, the trisection of an angle may be readily effected, as will be hereafter shown; but what geometricians have been in quest of, is the accomplishment of this trisection by aid merely of the straight line and the circle—the only lines recognised by Euclid. Some have ventured to *say* that, with this limitation as to materials, the problem is impossible; but who can *prove* this? The third part of an angle of course *exists;* and there is therefore no just grounds for affirming that it cannot be found, except by the aid of machinery external to geometry. Till the beginning of the present century, there were the very same grounds for affirming that the circumference of a circle could not be divided into *seventeen* equal parts by common geometry; but in 1801, Gauss, a distinguished German mathematician, showed how this could be effected without going beyond the limits of elementary geometry. The discovery, though of no theoretical or practical value, made a good deal of noise at the time. The work containing this, and several analogous problems, was translated into French by Delisle, under the title of *Recherches Arithmétique;* but the division of the circumference into *seven* equal parts; or, which is the same thing, the problem to inscribe a regular *heptagon*, or seven-sided figure, in a circle, like the trisection of an arc, is not yet accomplished.

From what has now been said, you perceive that Euclid enables us (Proposition II.) to divide the circumference of a circle into 3, 6, 12, &c., equal parts; as also (Proposition VI.) into 4, 8, 16, &c., equal parts; and again (Proposition XI.) into 5, 10, 20, &c., equal parts; and finally (Proposition XVI.), into 15, 30, 60, &c., equal parts;—all these subdivisions of the circumference being obtained by the propositions here referred to, and the repeated application of Proposition XXX., Book III. As just observed, Gauss has extended Euclid's instructions, and taught us how to divide the circumference into 17, 34, 68, &c., equal parts; as also into other parts, in like manner excluded from Euclid's divisions—namely, into 257, 65537, &c.; each part, by repeated bisections, giving rise to a new series of subdivisions as above. And by combining some of these with the subdivisions of Euclid—as Euclid himself has combined the divisions into three parts and into five parts, in Proposition XVI., to get the division into fifteen parts—other sections of the circumference may be obtained. But the actual constructions, even in the simplest of these additional cases—the division, namely, into seventeen equal parts, are so very complicated as to be of no avail in actual practice; they are interesting merely as showing that this part of the Euclidean geometry is really susceptible of extension; and we have occupied your attention in this brief account of the speculations of geometers in reference to the division of the circumference, chiefly that you might see sufficient reason why so few of these divisions are accomplished by Euclid, and why he should pass at once from the six-sided polygon (Proposition XV.) to that of fifteen sides (Proposition XVI.), without anything being said as to the intermediate polygons of 7, 9, 11, 13, and 14 sides respectively. You now know that the omission arose from his inability to inscribe any of these polygons in a circle.

ON THE QUADRATURE OF THE CIRCLE.

Intimately connected with the researches just adverted to, is the problem of the quadrature of the circle, which, like that of the trisection of an angle, has for ages occasioned the fruitless expenditure of much valuable time and thought. We shall endeavour to give you here some notion of the meaning and object of this celebrated problem, not only because it is a matter of such historical interest, that you ought to know something about it; but because, moreover, in certain elementary writings on the subject, the thing is put before the student in an erroneous form; and, consequently, a wrong impression as to the real character of the problem is conveyed.

The problem of *squaring the circle*, as it is popularly called, has a twofold meaning—namely, the *geometrical* quadrature, and the *numerical* quadrature. In the first of these senses the problem is to construct a square that shall be equal in surface to a given circle; in the second, the problem is to express the numerical measure of the surface of a circle when the measure or length of its diameter is given in numbers. The former of these is the more ancient form of the problem; and all that can be fairly said of it is—as was said of the trisection of an angle—that the solution has never been effected; a square equal to a circle has never yet been constructed. We have no grounds for affirming that this construction is *impossible*, for the equivalent square *exists*. You may readily satisfy yourself of this by the following reflections:—The square on the *diameter* of the circle would be too great, and the square on the chord from an extremity of the diameter, to cut off a fourth part of the circumference, would be too small, since the former square would be circumscribed about the circle, and the latter inscribed in it; the circle therefore is in magnitude somewhere *between* the two. Conceive, now, the smaller of these two squares to expand continuously, still retaining its character as a square, till it arrives at the larger square in magnitude; then, as all intermediate magnitudes are thus reached and passed through, and as the circle is *one* of these intermediate magnitudes, it necessarily follows that our expanding square must, at a particular stage of its progress, have exactly attained the magnitude of the circle; so that if its progress could be arrested at that stage, or—to drop this idea of progression—if the individual square could be isolated and exhibited, the problem of the geometrical quadrature of the circle would be solved. It is plain, therefore, that there is nothing visionary or absurd in the search after this square, as if it were a thing that had no existence; although some very able geometers have, strangely enough, condemned the inquiry on these grounds. The only sound reasons for abandoning the investigation are these two; namely—first, that the problem has been earnestly and laboriously attempted, by the profoundest geometers, for thousands of years, and *they* have been obliged to abandon it in despair; and secondly, that the successful solution of it would be of no theoretical or practical value if furnished. As far as *utility* is concerned, the other form of the problem of the quadrature of the circle is by far the more important; that is, to discover the numerical measure of the surface of a circle from the measured length of its diameter being given. But, under this aspect of it, the accurate solution of the problem is really impracticable; it can be *proved* to be so; and the proof will be given in a subsequent part of the present mathematical course. It is just as impracticable as it is to assign accurately the square-root of 2; and, in fact, this square-root does repeatedly enter into the approximative numerical process. You will require to know something of *Proportion*, in the sense in which the term is employed by Euclid, before that process can be fully explained to you. This subject, together with the sixth book of Euclid, forms the object of the chapter next following; and at the end of it you will find the principles upon which the approximative quadrature of the circle depends, clearly exhibited; and the mode of computation pointed out. The plan is to compute first the surface of the inscribed four-sided equilateral figure or square; then the inscribed eight-sided figure; then the sixteen-sided figure; and so on, till the inscribed polygon differs insensibly from the circle. *How* the surfaces of these successive polygons are computed one after another, must be deferred for the present; but some of the *results* are exhibited in the margin, where the radius of the circle, whose surface is approximated to, is regarded as 1; that is, 1 inch, 1 foot, 1 yard, or *one* any measure.

No. of sides.	Surface of pol.
4	2
8	2·8284271
16	3·0614674
32	3·1214451
64	3·1365485
128	3·1403311
256	3·1412772
512	3·1415138
1024	3·1415729
2048	3·1415877
4096	3·1415914
8192	3·1415923
16384	3·1415925
32768	3·1415926

It appears, from the above table, that the surface of an inscribed regular polygon of 32768 sides is 3·1415926, which is correct as far as the decimals extend; this number, therefore, may be taken for the numerical measure of the circle itself; for it is plain that a regular polygon of so many thousand sides would be undistinguishable from, and therefore *practically* identical with, the circle in which it is inscribed. But, by continuing to double the number of sides of the polygon, the approximation to the circle may be pushed to any extent. De Lagny computed the decimals true to 128 places; and eighty more have been recently added on by Dr. Rutherford. It is observed by Montucla, that if we suppose a circle whose diameter is a thousand million times the distance of the sun from the earth, the *approximative* measure of the circumference, as computed by De Lagny, would differ from the *true* measure by a length less than the thousand millionth part of the thickness of a hair.

There is obviously no use, as far as practical purposes are concerned, in extending the approximation to anything like this extreme degree of nearness. The earlier computers were no doubt induced to carry on the decimals in the expectation that they would at length terminate; but, as already observed, and as will be hereafter proved, the decimals will go on for ever. It is right to infer, therefore, that the *exact* numerical measure of the surface of a circle does not exist, though a geometrical square equal to it does exist. The exact numerical value of $\sqrt{2}$ does not exist; yet $\sqrt{2}$ represents the diagonal of a square whose side is 1, which diagonal is of course an existent geometrical line.

EXERCISES ON BOOKS I.—IV.

1. Prove that if a pair of opposite angles of a quadrilateral be equal to two right angles, a circle may be described about it.—(Converse of Proposition XXII., Book III.)

2. If the diagonals of a quadrilateral divide one another, so that the rectangle contained by the parts of the one is equal to the rectangle contained by the parts of the other, then a circle may be described about the quadrilateral: required the proof.

3. If two opposite sides of a quadrilateral be prolonged to meet, and if it be found that the rectangle contained by one of the lines thus produced, and the part produced, are equal to the rectangle contained by the other line and the part produced, then a circle may be described about the quadrilateral: required the demonstration.

These two theorems are virtually the converses of Propositions XXXV. and XXXVI. of Book III.

4. If a circle be described about the square B E (see the Diagram to Proposition XLVII., Book I.), its circumference shall pass through the point where A D, F C intersect, and also through the point where A E, K B intersect: required the proof.

5. If an equilateral triangle be constructed on one side

of a given line, and on the other side two equilateral triangles, one on each half of the line, then the two lines drawn from the vertex of the larger triangle to the vertices of the smaller triangles will trisect the given line: required the proof.

6. Divide a given triangle into three equal parts by lines drawn from the vertices of the triangle to a point within it.

7. Inscribe a circle in a rhombus.

8. Prove that the square circumscribing a circle is double of the square inscribed in the same.

9. If a circle be inscribed in a right-angled triangle, and another be circumscribed about it, prove that the sum of the sides containing the right angle will be equal to the sum of the diameters.

10. From a given point in the arc of a circle, to draw a tangent thereto without first finding the centre of the circle.

11. The straight line bisecting any angle of a triangle cuts the circumference of the circumscribing circle in a point which is equidistant from the extremities of the opposite side, and from the centre of the inscribed circle: required the proof.

12. If from any point within an equilateral and equiangular polygon perpendiculars be drawn to the several sides, the sum of these perpendiculars will always be the same, wherever the point from which they are drawn be taken.

13. Through a given point within a circle it is required to draw the shortest chord possible.

CHAPTER V.

PROPORTION.

A TREATISE INTENDED AS A SUBSTITUTE FOR EUCLID'S BOOK V.

INTRODUCTORY.—In the foregoing portion of elementary geometry, we have given you the "Elements of Euclid," substantially, in all their integrity: the modifications we have introduced are for the most part merely of a verbal character; but while condensing the language we have been careful to preserve the spirit and rigour of the original. In the few instances in which we have thought an improvement might be introduced, or a defect supplied, we have not hesitated to offer the suggestion, and to propose the emendation: what little is done in this way is sufficiently detailed in the Remarks appended to the several books. You will, of course, submit these to your own judgment—always remembering that in matters connected with geometery, nothing is to be taken upon trust; mere opinion, unsupported by reasonings—reasonings which elevate it into proof—must be regarded, *in this subject*, as of but little worth.

We are now going to depart altogether from Euclid's method of exposition, and to place before you a treatise on Proportion, constructed on a different plan. We have come to this determination only after mature deliberation. It would, of course, be a much easier task to transfer Euclid's fifth book into these pages. We could find very little to remark upon in it, as the ancient Geometer has displayed so much sagacity and penetration in this, the most elaborate of all his writings, that he has left to the moderns little or no room for improvement: it must be studied just as it is (in Simson's restoration), or else be superseded in instruction by a treatise of equal generality, but of geater simplicity. You will understand, therefore, that we do not displace Euclid's fifth book because of its imperfections, or because of its inadequacy to completely accomplish its objects; but solely because of its great *difficulty* to a beginner. We will endeavour to give you here some notion of the cause of this difficulty. The subject of Euclid's fifth book is PROPORTION—*universal* proportion: that is, not *numerical* proportion merely, but proportion in reference to all magnitudes and quantities whatever, whether numbers, lines, surfaces, solids, or concrete quantities of any kind. With proportion *in numbers* you are already familiar:—this will be a help. You are also somewhat acquainted with proportion in *Algebra:* this will be a greater help; for proportion in Geometry really accomplishes no more for things in general, than the same doctrine in arithmetic and algebra accomplishes for what the notation of those sciences specially represents; and if this kind of proportion would do for geometry, the fifth book of Euclid would become a very easy matter indeed. But the obstacle to this is, that geometrical magnitudes, when compared together, are in many cases found to be *incommensurable;*—that is to say, two such magnitudes may be quite incapable of a common measurement—they may be of a nature not to admit of being *both* measured by one and the same unit of measurement, however minute the measuring unit be taken; and, consequently, *both* cannot be represented by *numbers.* We have already adverted to an instance of this kind* in the side and diagonal of a square, and to another in the diameter and circumference of a circle. You may divide the side of a square into as many equal parts as you please—from two parts to as many millions. In every case each part is, of course, a *measure* of the side; so that by applying such part, progressively, from one extremity of the side onwards towards the other extremity, that other extremity would at last be accurately reached.

But if the same measure, however small it be taken, be applied in like manner to the diagonal, the remote extremity of *it* can never be accurately reached; either an unmeasured *smaller* portion will still be left, or else the applied measure will overlap and project *beyond* that extremity. It is thus that these two lines are *incommensurable.* That they are so, could not have been found out by such practical or experimental tests as those here adverted to for illustration; they must be proved to be so by geometrical reasoning.

It may be as well to caution you here that you must not speak of a line or quantity, *by itself*, as being incommensurable; this would be absurd. The diagonal of a square is not itself incommensurable, since it has, of course, its third part, fourth part, hundredth part, &c., and is therefore *measurable* by each of those parts; but as none of them will *also* measure the *side*, the two, considered together, are incommensurable; there exists no measure common to *both.* In the same way in reference to the circle—the circumference itself is not incommensurable any more than the diameter; for each has its fourth part, sixth part, &c.; but it is incommensurable with its diameter; no length whatever can measure *both.* The circumference of a circle may be ten feet and some fraction of a foot; the diameter will necessarily be more than three feet; but the exact fraction of a foot, besides, it is not in the power of numbers to express.

Now although Euclid makes no mention of incommensurable quantities in his fifth book, he was well aware of their existence; and therefore, to render his theorems on proportion general, he had to take care that this class of quantities should be comprehended in his reasonings. But proportion limited to numbers, or to the symbols for numbers, would necessarily exclude incommensurables; he therefore had to proceed quite independently of arithmetic, and to secure to his propositions such a universality that each theorem should rigorously apply, whether the quantities or magnitudes spoken of be measurable, or beyond the powers of numerical representation. He has executed his difficult task with consummate ability; for, as Dr. Barrow remarks, "there is nothing, in the whole body of the Elements, of a more subtile invention—nothing more solidly established, and more accurately handled, than the doctrine of proportionals."

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* See *ante*, p. 569.

It is on account of the subtleties here adverted to, and which are of too refined a character for the generality of young students to comprehend, that we have resolved to replace the fifth book by the following treatise. We cannot promise that you will find the study of it *easy;* but it will certainly be much *less difficult* than the corresponding portion of Euclid's work; and you will enter upon it with considerable advantage, if you have read Arithmetical and Geometrical Progression in the chapter on Algebra.

DEFINITIONS.

I. Of two unequal magnitudes, of the same kind, the greater is said to *contain* the less as many times as there are parts in the greater equal to the less.

This definition is intended to convey the sense in which the word *contain* is to be understood in what follows. You will carefully observe that the term is not restricted by any condition as to whether or not the greater quantity leave any *remainder* after taking the less from it as often as possible. Thus 8 is said to contain 2 as often as 9 contains it, though there is a remainder in the latter case, and no remainder in the former.

II. One magnitude is said to be a *multiple* of another when it is equal to a certain number of times that other *exactly.* And the less of the two is in this case said to be a *sub-multiple* of the greater, or a *measure* of the greater.

Thus 8 is a multiple of 2, because 8 is equal to a certain number of times 2 *exactly;* namely, *four* times 2. And 2 is a sub-multiple of 8, or, as we should say in arithmetic, a *factor* of 8. Remember that a sub-multiple, or *measure* of any magnitude, is a smaller magnitude of the same kind which measures the former (the *multiple*) *exactly*, without leaving any remainder. 2 is not a sub-multiple or measure of 9; which is the same as saying, that 9 is not a multiple of 2.

III. Magnitudes—two or more—which have a *common measure*—that is, which are multiples of some other magnitude—are said to be *commensurable.* But if it be *impossible* that any such common measure can exist, then the magnitudes are said to be *incommensurable.*

All sets of abstract numbers, and of concrete quantities, like in kind, that can be accurately denoted by numbers, are commensurable; that is, they have a common measure. The expression, "common measure," as employed in arithmetic and algebra, is not synonymous with the geometrical meaning. In arithmetic we should say that the pairs of numbers 3, 7; 4, 11; 5, 12, &c., have no common measure; but even here a qualifying exception is always tacitly made: it is this—namely, *except unit.* All whole numbers contain 1 an exact number of times; though it is agreed in arithmetic that 1 shall not be recognised as a common measure. Geometry makes no such exception: whatever quantity is contained in another, an exact number of times, is a measure of that other. In like manner, 3½, 7¼, would not be regarded as having a common measure in arithmetic; yet as the first number contains ¼ exactly 14 times, and the second contains ¼ exactly 29 times, ¼ is a common measure of the two numbers according to the above definition of the term. And whatever numbers be compared together, whether they be whole or fractional, it will be found that there always exists some smaller number—either whole or fractional—that will exactly measure both. Suppose, for instance, the proposed numbers, when brought to a common denominator, have (say) 12 for the common denominator, then each denotes so many *twelfths:* that is, $\frac{1}{12}$ is a common measure of both. We do not say anything here about such expressions as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{7}$, &c.; geometrical rigour forbids our calling what these symbols stand for, definite numbers, as they involve an endless series of fractions or decimals, and can only be valued *approximately;* yet they may be accurately represented by *lines*, as Euclid has himself shown in a Book not now read (Book X.) We could not speak of any number being contained in $\sqrt{2}$ a certain number of times *exactly*, because $\sqrt{2}$ itself is not determinable exactly.

IV. *Equimultiples*, or *like multiples*, of two or more magnitudes, are those larger magnitudes which contain those of which they are multiples—each of each—*the same number of times.*

For instance, the numbers 8 and 12 are equimultiples of 2 and 3; for the former contain these, respectively, the same number of times—namely, *four* times. In like manner, 7½ and 10 are equimultiples of 2½ and 3⅓; for the former numbers contain these, respectively, *three* times.

V. And *like sub-multiples* are those which are contained in their respective multiples the same number of times.

Thus, in the instances just adduced, 2 and 3 are like sub-multiples or like measures of 8 and 12; 2½ and 3⅓ are like sub-multiples or measures of 7½ and 10.

VI. Four magnitudes are said to be *proportionals*, or to form a *proportion*, when the first cannot be contained in any multiple of the second, oftener than the third is contained in a *like multiple* of the fourth, nor the third in any multiple of the fourth, oftener than the first in a *like multiple* of the second.

The first and third of four such magnitudes are called *antecedents*, and the second and fourth their *consequents.* The definition affirms that an antecedent must be contained in each multiple of its consequent as often as the other antecedent is contained in a like multiple of its consequent, but not oftener.

VII. When four magnitudes are in proportion, the first antecedent is said to have the same *ratio* to its consequent that the second antecedent has to *its* consequent.

This term *ratio* has been the source of very considerable embarrassment to Geometers since the time of Euclid, and has been productive of much metaphysical disquisition and controversy. We think that on this subject, as well as in reference to the theory of parallel lines, mathematicians too often overlook the fact that the fundamental notions of geometry really exist in the mind anterior to, and independently of, the definitions of the science. These, for the most part, do not originate those notions, but only give to them the necessary degree of clearness and precision. If one pair of magnitudes be submitted to our contemplation, and then another pair be brought into comparison with them, as well as with each other, the mind is at once capable of forming a notion as to whether the *relative magnitudes* of the individuals of the first pair be the same or not as the relative magnitudes of those of the second pair: the *absolute* magnitudes of the individuals of one pair may be very different from the absolute magnitudes of those of the other pair; yet the former two may have the same *relation to one another*, as to magnitude, as the latter two; and the mind is quite capable of recognising and understanding this sameness of relation, or of *ratio*, as it is called above, before any *name* is given to the conception. It is this *equality of ratios* of two magnitudes brought into comparison with other two, that renders the *four* proportionals.

That the foregoing definition of proportion (Def. VI.) includes numerical proportion, in Arithmetic, will be obvious upon a little consideration. Proportion limited to numbers may be defined thus:—

Four numbers are proportionals when the first is contained *exactly*, as often in some multiple (any *one* multiple being sufficient) of the second, as the third is contained in a *like* multiple of the fourth.

It is plain that if this condition have place, the four numbers must be proportionals according to the common arithmetical notion; for it follows of necessity, that the quotient of the second by the first must then be the same as that of the fourth by the third, and consequently that the quotient of the first by the second must be the same as that of the third by the fourth.*

Definition VI., above, is, however, free from the restriction implied in the term *exactly*, which is introduced here into the particular case of it, applying exclusively to *numerical* proportion; since the general form of the definition admits of such restrictive qualification when numbers only are concerned; incommensurables being then excluded.

Four magnitudes not fulfilling the conditions of Definition VI. would evidently violate even this arithmetical condition of proportion. There can be no such things as proportionals out of the restrictions of the former definition; so that all proportionals, whether among commensurables or incommensurables, must be included in the general Definition VI.

VIII. The first and last of four proportionals are called the *extremes*, and the two intermediate ones the *means.*

IX. The magnitudes themselves are called the *terms* of the proportion; and those are called *homologous* or *like* terms which have the same *name;*—the *antecedents* forming one pair of homologous terms, and the *consequents* another pair.

X. Magnitudes, more than two, are said to form a *continued proportion* when each *consequent* in succession is taken for the *antecedent* of the term next following.

Thus, if A is to B, as B is to C, as C is to D, &c.; then A, B, C, &c., are in continued proportion.

XI. If the continued proportionals be but *three* in number, the middle one is called the *mean* term, and the others the *extremes.*

AXIOMS.

I. Equimultiples of the same magnitude, or of equal magnitudes, are equal to one another; so also are equi-sub-multiples.

II. A multiple of a greater magnitude exceeds a like multiple of a less; and a sub-multiple of a greater exceeds a like sub-multiple of a less.

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* See Arithmetic, p. 416.

III. The magnitude of which a multiple or sub-multiple is greater than a like multiple or sub-multiple of another, is greater than that other.

IV. Of any two magnitudes of the same kind, a multiple of one may be taken so great as to exceed the other.

Marks and Signs used for Abbreviation.

1. To express that four magnitudes, A, B, C, D, are proportionals, they are arranged thus: $A : B :: C : D$; and this expression is read, "as A is to B, so is C to D;" or, "A is to B, as C is to D."

2. And to express that magnitudes A, B, C, D, E, &c., form a series of continued proportionals, they are arranged thus: $A : B :: B : C :: C : D :: D : E$, &c., which is read, "as A is to B, so is B to C, and C to D, and D to E," &c.

3. Besides the marks $+$ and $-$ for addition and subtraction, the mark $\backsim$ is sometimes employed, to denote the *difference* of the two quantities between which it is placed; it is useful for this purpose when it is not stated *which* of the two quantities is the greater of the two; thus $A \backsim B$ means simply the difference between A and B, or rather between the things denoted by these letters, without any assertion as to which is the greater. The double mark $\overset{+}{\backsim}$ between two quantities signifies "the sum or difference" of those quantities. Thus by $6 \overset{+}{\backsim} 2$ we should understand "8 or 4."

4. The terms "greater than" and "less than" being of frequent use in what follows, convenient symbols for them are introduced: when $>$ is placed between two quantities it implies that the first of them is *greater than* the second; and when $<$ is placed between them, it denotes that the first of them is *less than* the second. Thus $A > B$ asserts that A is *greater than* B, or that A *exceeds* B; and $A < B$ affirms that A is *less than* B, or that B *exceeds* A.

5. In the following propositions, magnitudes (whether lines, surfaces, or solids) will be represented by the capital letters, A, B, C, &c. They may, indeed, be taken to represent any quantities whatever, whether abstract or concrete; as the reasonings will be found to apply without restriction as to the nature of the things represented by the letters. But when *multiples* of these quantities are taken (that is, when they are multiplied by *numbers*), these numbers will be represented by the small letters; and, in general, by m, n, p, q.

PROPOSITION I.—Theorem.

If any number of magnitudes be equimultiples of as many others, each of each, whatever multiple any one of the former is of the corresponding one of the latter, the *same multiple* is the *sum* of all the former, and of the *sum* of all the latter.

First, let there be but *two* magnitudes mA, mB, any equimultiples whatever of two others, A, B; the *sum* of the former shall be *the same* multiple of the *sum* of the latter.

For the sum of the former is $mA + mB$; that is, $m(A + B)$; and the sum of the latter is $A + B$; and $m(A + B)$ is *the same multiple* of $A + B$, that mA is of A, or mB of B.

Next, let there be *three* magnitudes mA, mB, mC, equimultiples of the three A, B, C.

The *sum* of the former three is $mA + mB + mC$; that is $m(A + B + C)$, and the *sum* of the latter three is $A + B + C$; and $m(A + B + C)$ is *the same multiple* of $A + B + C$, that mA is of A, or mB of B, or mC of C. And in the same manner is the proposition proved, when there are four magnitudes equimultiples of other four; when there are five magnitudes, six magnitudes, or any number of magnitudes, $\therefore$ *if any number of magnitudes*, &c. Q. E. D.

PROPOSITION II.—Theorem.

If, in any proportion, an antecedent and its consequent be respectively the same as an antecedent and its consequent in *another* proportion, the remaining antecedent and consequent in the former, together with the remaining antecedent and consequent in the latter, will form a proportion.

Let the two proportions be—

$$\begin{matrix} A : B :: C : D \\ A : B :: E : F, \end{matrix} \text{ then } C : D :: E : F.$$

For (Def. VI.) m being any whole number whatever, C is contained in mD as often as A is contained in mB, but not oftener.

In like manner E is contained in mF as often as A is contained in mB, but not oftener.

Therefore C cannot be contained oftener in mD than E is contained in mF; nor can E be contained oftener in mF, than C is contained in mD; and m is any whole number whatever, $\therefore$ (Def. VI.) *the four magnitudes*, C, D, E, F, *are proportionals*, $\therefore$ *if in any proportion, &c.* Q. E. D.

PROPOSITION III.—Theorem.

If, in any proportion, equimultiples of the antecedents and equimultiples of the consequents be taken; if the multiple of one of the antecedents be greater than that of its consequent, the multiple of the other antecedent will be greater than that of *its* consequent.

Let the proportion be—

$$A : B :: C : D.$$

If $mA > nB$, then $mC > nD$, and conversely; m and n being any whole numbers whatever.

For A is contained in mA exactly m times; but, by hypotheses, nB is *less* than mA, $\therefore$ A is contained in nB *less* than m times. But (Def. VI.) C is contained no oftener in nD, than A is contained in nB, $\therefore$ C is contained in nD, less than m times.

But C is contained in mC exactly m times, $\therefore$ C is contained in mC *oftener* than it is contained in nD, $\therefore$ $mC \quad nD$.

In like manner, if the hypothesis be that $mC > nD$, may it be shown that $mA > nB$, $\therefore$ *if in a proportion*, &c. Q. E. D.

Cor.—Since m and n may be any whole numbers whatever, let each be $= 1$: then it follows that:—

In a proportion, if one antecedent be greater than its consequent, the other antecedent will be greater than its consequent.

PROPOSITION IV.—Theorem.

In any proportion, according as one antecedent is greater than, less than, or equal to its consequent, so will the other antecedent be greater than, less than, or equal to *its* consequent.

It has already been proved (Prop. III., Cor.) that if one antecedent be *greater* than *its* consequent, the other antecedent will be *greater* than *its* consequent. Let the proportion be:—

$$A : B :: C : D.$$

1st. If $A < B$, then $C < D$, and conversely. Let $B - A = P$; then a number m exists such that $mP > A$ (Ax. 4), $\therefore$ mB must contain A *oftener* than it contains B. But mD contains C *as often* as mB contains A (Def. VI.), $\therefore$ mD contains C *oftener* than mB contains B; that is, oftener than m times, $\therefore$ $C < D$. Consequently, if $A < B$ then $C < D$. And in like manner may it be shown that if $C < D$ then $A < B$.

2nd. If $A = B$ then $C = D$, and conversely.

For when $A = B$, if it were possible that $C > D$, or $C < D$, or, when $C = D$, if it were possible that $A > B$, or $A < B$, the foregoing conclusions would be contradicted, $\therefore$ *in any proportion, &c.* Q. E. D.

PROPOSITION V.—Theorem

If four magnitudes be such, that whatever equimultiples of the antecedents, and whatever equimultiples of the consequents be taken, the multiple of one antecedent cannot be greater than that of its consequent, without the multiple of the other antecedent being greater also than that of its consequent, the four magnitudes are proportionals.

Let the four magnitudes be A, B, C, D. If they are *not* proportional, one of the antecedents, as A, must be contained in some multiple mB of its consequent *oftener* than C is contained in mD (Def. VI.) Therefore, p being any whole number, A must be contained oftener in pmB than C is contained in pmD.

Let nA be the *greatest* multiple of A that does not *exceed* pmB; then nA is not $> pmB$, and A is contained in nA, and in pmB the same number of times, namely, n times. Therefore C is contained in pmD less than n times. But C is contained in nC exactly n times, $\therefore$ $nC > pmD$. Consequently, of the four magnitudes, equimultiples of the antecedents, and equimultiples of the consequents, may be taken, as below :—

$$\begin{array}{cccc} A : & B : : & C : & D \\ nA & pmB & nC & pmD \end{array}$$

such that $nC > pmD$, and yet nA *not* $> pmB$: but, by hypothesis, this is impossible: therefore the magnitudes cannot be other than proportionals; $\therefore$ *if four magnitudes*, &c. Q. E. D.

PROPOSITION VI.—THEOREM.

In any proportion, if like multiples of the antecedents, and like multiples of the consequents, be taken, the results will form a proportion.

Of the four proportionals, let equimultiples of antecedents and of consequents be taken as below, m, n, p, and q being *any* whole numbers :—

$$\begin{array}{cccc} A : & B : : & C : & D \\ mA & nB & mC & nD \\ pmA & qnB & pmC & qnD \end{array}$$

Then (Prop. III.) if $pmA > qnB$, it must follow that $pmC > qnD$; or if $pmC > qnD$, that $pmA > qnB$. But pmA, pmC, are *any* equimultiples of mA, mC; and qnB, qnD, are *any* equimultiples of nB, nD. Consequently (Prop. V.),

$$mA : nB : : mC : nD.$$

Therefore, *in any proposition*, &c. Q. E. D

NOTE.—Either m or n may of course be *unit*; as also in every case where the multiples are unrestricted.

In Proposition V. it was demonstrated that—"If four magnitudes be such that whatever equimultiples of the antecedents, and whatever equimultiples of the consequents be taken, the multiple of one antecedent cannot be greater than that of its consequent, without the multiple of the other antecedent being greater also than *its* consequent, the four magnitudes are proportionals." It is now proved that these multiples themselves are also proportionals. Consequently (Prop. IV.), according as the first multiple is greater than, less than, or equal to the second, so will the third multiple be greater than, less than, or equal to the fourth multiple. And this is the condition which constitutes Euclid's criterion of proportional magnitudes, as embodied in his celebrated *fifth* definition, which, in the version of Playfair, is expressed as follows :—

Euclid's Definition of Proportionals (Def. V.)

If there be four magnitudes, and if any equimultiples whatsoever be taken of the first and third, and any equimultiples whatsoever of the second and fourth, and if according as the multiple of the first be greater than the multiple of the second, equal to it, or less, the multiple of the third also is greater than the multiple of the fourth, equal to it, or less; then the four magnitudes are proportionals. This definition of Euclid is deduced here as a *theorem*; but in what follows it will be referred to as "Def. V., page 573."

PROPOSITION VII.—THEOREM.

The terms of any proportion form also a proportion when they are taken INVERSELY; that is, the second term is to the first as the fourth is to the third, or the second has the same ratio to the first that the fourth has to the third.

Let the proportion be $A : B : : C : D$; then also $B : A : : D : C$.

For (Prop. VI.) $mA : nB : : mC : nD$.

And (Prop. IV.) according as nB is greater than, less than, or equal to mA, so is nD greater than, less than, or equal to mC; and m, n are any whole numbers whatever. But when this is the case in reference to four magnitudes B, A, D, C, they are proportionals. (Def. V., page 573).

$$\therefore B : A : : D : C.$$

Hence *the terms of any proportion*, &c. Q. E. D.

COR.—Therefore a consequent is contained as often in a multiple of its antecedent, as the other consequent is contained in a like multiple of its antecedent, but not oftener. (Def. VI.)

PROPOSITION VIII.—THEOREM.

In any proportion, if equimultiples of the first two terms be taken, and also equimultiples of the last two, the results will form a proportion.

Let the proportion be $A : B : : C : D$; then also

$$mA : mB : : nC : nD$$

By Def. VI., $A : B : : mA : mB$; and $C : D : : nC : nD$

But $A : B : : C : D$; $\therefore$ (Prop. II.) $C : D : : mA : mB$

and therefore, by the same prop., $mA : mB : : nC : nD$; $\therefore$ *in any proportion*, &c. Q. E. D.

PROPOSITION IX.—THEOREM.

In a proportion consisting of *homogeneous* magnitudes—that is, magnitudes all of the same kind—if one antecedent be greater than the other, the consequent of the former will be greater than the consequent of the latter.

Let the magnitudes forming the proportion $A : B : : C : D$ be all of the same kind, and let $A > C$; then also $B > D$. For let $A - C = P$; then (Ax. 4) there exists some number m, such that $mP > D$, and consequently such that mA contains D oftener than mC contains D.

But by taking the terms of the proportion inversely (Prop. VII.)

$$\begin{array}{cc} B : A : : & D : C \\ mA & mC \end{array}$$

$\therefore$ mA does not contain B oftener than mC contains D (Def. VI.), $\therefore$ mA contains D oftener than it contains B, therefore $B > D$ $\therefore$ *in a proportion*, &c. Q. E. D.

COR. I.—In a proportion consisting of *homogeneous* magnitudes, if one *consequent* be greater than the other, the *antecedent* of the former will be greater than that of the latter.

This follows from the present proposition by *inversion*; and, consequently, in a proportion whose terms are all homogeneous, if one antecedent be greater than, less than, or equal to the other antecedent, the consequent of the former will be greater than, less than, or equal to the consequent of the latter, and conversely.

COR. II.—Therefore (Prop. II.) if two proportions have three corresponding terms in each equal, each to each, the fourth terms will be equal.

NOTE.—It must be carefully observed that Proposition IX., as also X. and XI. following, apply only when the magnitudes are all four of the same kind. There can be no such relation as that implied in the word *ratio* between things of different kinds. In the other propositions hitherto discussed, it is necessary only that the first and second of the magnitudes be of the same kind, and that the third and fourth be also of the same kind. The latter pair, however, may differ in kind from the former pair: one pair may be lines, or numbers, and the other pair surfaces, or solids: but no *ratio* can exist between *heterogeneous* quantities, or quantities unlike in kind.

PROPOSITION X.—THEOREM.

If any number of *homogeneous* magnitudes be proportionals, then as one antecedent is to its consequent, so is the sum of all the antecedents to the sum of all the consequents.

First, let there be *four* proportionals, and let any equimultiples of the antecedents, and any equal multiples of the consequents, be taken thus :—

$$\begin{array}{cccc} A : & B : : & C : & D \\ mA & nB & mC & nD \end{array}$$

It is to be proved that

$$A : B : : A + C : B + D$$

By Prop. III. if $mA > nB$, then $mC > nD$; consequently, if $mA > nB$, then also $(mA + mC) > (nB + nD)$.

And conversely, if $(mA + mC) > (nB + nD)$, then $mA > nB$. For, from the first proportion, if this were not the case, neither could $mC > nD$; and, consequently, neither could $(mA + mC) > (nB + nD)$; $\therefore$ if this latter condition have place, so, of necessity, must the condition $mA > nB$.

$\therefore$ (Prop. V.) $A : B :: A + C : B + D$.

Next let there be six proportionals,

$$A : B :: C : D :: E : F.$$

It has already been proved, in reference to the last four, that

$$C : D :: C + E : D + F$$
$$\text{But } C : D :: A : B.$$

$\therefore$, (Prop. II.) $A : B :: C + E : D + F$.

Consequently, by the first case above,

$$A : B :: A + C + E : B + D + F.$$

And in a similar way as the proof for six proportionals has been derived from that for four, so may the proof for eight be derived from that for six; and so on for any number of proportionals; $\therefore$ *if any number of homogeneous magnitudes, &c.* Q. E. D.

PROPOSITION XI.—Theorem.

If the terms of a proportion are all of the *same kind*, they also form a proportion when taken ALTERNATELY; that is, the first is to the third as the second is to the fourth. Let the homogeneous proportionals be

$$A : B :: C : D; \text{ then also } A : C :: B : D.$$

For let any equimultiples of A, B, and any equimultiples of C, D, be taken and arranged as below,

$$\begin{array}{ccccccccccccccc} A & : & B & :: & C & : & D; & & A & : & C & :: & B & : & D \\ mA & & mB & & nC & & nD; & & mA & & nC & & mB & & nD \end{array}$$

Then (Prop. VI.) $mA : mB :: nC : nD$; $\therefore$ (Prop. IX.) if $mA > nC$, then $mB > nD$; and if $mB > nD$, then $mA > nC$.

$$\therefore \text{(Prop V.) } A : C :: B : D.$$

Hence, *if the terms of a proportion, &c.* Q. E. D.

PROPOSITION XII.—Theorem.

If in any proportion an antecedent be a multiple or submultiple of its consequent, the other antecedent will be a like multiple or submultiple of *its* consequent.

Let A, B, C, D be four proportionals, such that $A = mB$, then will $C = mD$. For since

$$A : B :: C : D$$

$\therefore$ (Prop. VI.) $A : mB :: C : mD$; but $A = mB$, $\therefore$ $C = mD$ (Prop. IV.)

Again, let the proportionals be such that $B = mA$; then will $D = mC$.

For by inversion (Prop. VII.) $B : A :: D : C$ and (Prop. VI.) $B : mA :: D : mC$, but $B = mA$ $\therefore$ $D = mC$.

$\therefore$ *if in any proportion, &c.* Q. E. D.

COR.—When the proportionals are *homogeneous*, if one antecedent be a multiple of the other, the consequent of the former will be a like multiple of the consequent of the latter.

PROPOSITION XIII.—Theorem.

In any proportion, the sum of an antecedent and its consequent is to either term, as the sum of the other antecedent and consequent is to the like term.

$$\begin{array}{rlllll} \text{Let the proportion be} & A & : B & :: & C & : D \\ \text{then} & A+B & : A & :: & C+D & : C \\ \text{also} & A+B & : B & :: & C+D & : D \end{array}$$

For A cannot be contained oftener in mB than C is contained in mD (Def. VI), $\therefore$ A cannot be contained oftener in $m(A + B)$ than C is contained in $m(C + D)$. Nor can C be contained oftener in $m(C + D)$ than A is contained in $m(A + B)$; for if it could, C would be contained in mD oftener than A in mB, which is impossible (Def. VI.) $\therefore$ $A : A + B :: C : C + D$.

Again, by inversion, $B : A :: D : C$; therefore, as just proved, $B : A + B :: D : C + D$; consequently inverting the two proportions now deduced,

$A + B : A :: C + D : C$, and $A + B : B :: C + D : D$.

$\therefore$ *in any proportion, &c.* Q. E. D.

COR.—If the proportionals are *homogeneous*, then, by alternation (Prop. XI.),

$A + B : C + D :: A : C$; and $A + B : C + D :: B : D$.

PROPOSITION XIV.—Theorem.

In any proportion, the difference between an antecedent and its consequent is to either term, as the difference between the other antecedent and consequent is to the like term.

Let $A : B :: C : D$ be any proportion; then taking any antecedent and consequent, as A, B, suppose first that $B > A$, and consequently (Prop. IV.) that $D > C$.

Take any equimultiples of B, D, and the same of $B-A$, $D-C$; and arrange the terms as usual—

$$\begin{array}{ccccccc} A & : & B & :: & C & : & D \\ & & mB & & & & mD \\ A & : & B-A & :: & C & : & D-C \\ & & mB-mA & & & & mD-mC \end{array}$$

Then (Def. VI.) A cannot be contained oftener in mB than C is contained in mD; but A is contained in mA just as often as C is contained in mC, namely, m times, without remainder. Therefore, A cannot be contained in $mB-mA$ oftener than C is contained in $mD-mC$.

In like manner may it be shown that C cannot be contained in $mD-mC$ oftener than A is contained in $mB-mA$; $\therefore$ (Def. VI.)

$$A : B-A :: C : D-C; \therefore \text{(Prop. XIII.)}$$
$$B : B-A :: D : D-C$$

And inverting these two proportions (Prop. VII.),

$B-A : A :: D-C : C$, and $B-A : B :: D-C : D$ which proves the theorem when $A > B$.

Next let $A > B$, and consequently (Prop. IV.) $C > D$.

By inversion, $B : A :: D : C$: $\therefore$ as proved above, $A-B : B :: C-D : D$, and $A-B : A :: C-D : C$, which proves the theorem when $A > B$; $\therefore$ $A \backsim B : A :: C \backsim D : C$; and $A \backsim B : B :: C \backsim D : C$ $\therefore$ *in any proportion, &c.* Q. E. D.

PROPOSITION XV.—Theorem.

If there be three magnitudes and other three, such that, whichever set be taken, the first in that set is to the second, as the second in the other set is to the third; then if the first in one set be greater than the third, the first in the other set also will be greater than the third.

Let the two sets of magnitudes be—

$$\left.\begin{array}{l} A, B, C \\ D, E, F \end{array}\right\} \text{ such that } \left\{\begin{array}{l} A : B :: E : F \\ D : E :: B : C \end{array}\right.$$

If $A > C$ then must $D > F$; and if $D > F$, then $A > C$. Let $A > C$: and take mA, mC, such equimultiples of A, C that mA may contain B *oftener* than mC contains B (Ax. 4); take also mE, the same multiple of E. Then since by the second proportion mE contains D only *as often* as mC contains B, $\therefore$ mA contains B *oftener* than mE contains D.

But mE contains F as often as mA contains B (Prop. VII., Cor.), therefore mE contains F *oftener* than mE contains D, $\therefore$ $D > F$. So that if $A > C$, then must $D > F$.

And in like manner may it be demonstrated that if $D > F$ then must $A > C$; $\therefore$ *if there be three magnitudes, &c.* Q. E. D.

PROPOSITION XVI.—Theorem.

If there be three magnitudes and other three such that whichever set be taken, the first in that set is to the second, as the second in the other set is to the third; then the first in the one set will be to the third, as the first in the other set is to the third.

Let the two sets of magnitudes be—

$$\left.\begin{matrix}A, B, C\\ D, E, F\end{matrix}\right\}\text{ such that }\left\{\begin{matrix}A : B :: E : F\\ D : E :: B : C\end{matrix}\right.$$

it is to be proved that A : C :: D : F.

Of A, B, D take any equimultiples, mA, mB, mD, and of C, E, F, any equimultiples, nC, nE, nF; then

$$\left.\begin{matrix}\text{(Prop. VIII.) } mA : mB :: nE : nF,\\ \text{and (Prop. VI.) } mD : nE :: mB : nC\end{matrix}\right\}\quad\left\{\begin{matrix}mA, mB, nC\\ mD, nE, nF.\end{matrix}\right.$$

Hence the first of the three magnitudes, on the right, are related to the other three, as in Proposition XV.; $\therefore$ if $mA > nC$, then $mD > nF$; or if $mD > nF$, then $mA > nC$. Consequently, (Prop. V.) A : C :: D : F; $\therefore$ *if there be three magnitudes*, &c. Q. E. D.

Cor.—If to one of the above sets a *fourth* magnitude P be *annexed*, and to the other set, a *fourth* magnitude Q be *prefixed*, such as to furnish an additional proportion, Q : D :: C : P; then, from the hypotheses, and the foregoing conclusion, namely, that—

$$\left.\begin{matrix}A, B, C, P\\ Q, D, E, F\end{matrix}\right\}\text{ are such that }\left\{\begin{matrix}A : C :: D : F\\ Q : D :: C : P,\end{matrix}\right.$$

it follows that A, C, P are related to Q, D, F, as in the proposition; and therefore that A : P :: Q : F. And in this way may the proposition be extended to any number of magnitudes.

PROPOSITION XVII.—Theorem.

If there be two sets of magnitudes, the number being the same in each set, such that the first is to the second in the one set, as the first to the second in the other set, the second to the third in the former set, as the second to the third in the latter set, and so on: then as the first magnitude in the one set is to the last, so is the first magnitude in the other set to the last.

First, let there be *three* magnitudes in each set, namely—

$$\left.\begin{matrix}A, B, C\\ D, E, F\end{matrix}\right\}\text{ such that }\left\{\begin{matrix}A : B :: D : E\\ B : C :: E : F,\end{matrix}\right.$$

then it is to be proved that A : C :: D : F.

Take any equimultiples, mA, mD of A, D; and any equimultiples nC, nF, of C, F; then (Props. VI. and VII.),

$$\left.\begin{matrix}B : mA :: E : mD\\ \text{and } B : nC :: E : nF\end{matrix}\right\}\quad\left\{\begin{matrix}A,\ C,\ D,\ F\\ mA, nC, mD, nF.\end{matrix}\right.$$

Let $mA > nC$, then (Ax. 4) a multiple pmA of mA, may be taken so great that pmA will contain B *oftener* than pnC will contain B. But pmD contains E as often as pmA contains B (Def. V.); while pnF contains E *only as often* as pmC contains B, $\therefore$ pmD contains E oftener than pnF contains E, $\therefore$ $mD > nF$; so that if $mA > nC$ then $mD > nF$. In like manner it may be proved, by interchanging A, C, with D, F, and also interchanging B with E, that if $mD > nF$, then $mA > nC$; $\therefore$ A : C :: D : F.

Next let there be *four* magnitudes in each set—namely,

$$\left.\begin{matrix}A, B, C, P\\ D, E, F, Q\end{matrix}\right\}\text{ such that }\left\{\begin{matrix}A : C :: D : F\\ C : P :: F : Q,\end{matrix}\right.$$

then it is to be proved that A : P :: D : Q. And this is done as in the first case, since the three magnitudes A, C, P are related, by the foregoing proportions, to the three D, F, Q, as that case supposes; $\therefore$ A : P :: D : Q. And in like manner may the case for *five* magnitudes be deduced from this for *four;* and so on for any number of magnitudes in each set; $\therefore$ *if there be two sets of magnitudes*, &c. Q. E. D.

Cor.—If the consequents in one proportion be the antecedents in another, a third proportion may be formed, having the same antecedents as the first, and the same consequents as the second; thus if—

$$\left.\begin{matrix}A : B :: C : D\\ \text{and } B : K :: D : L\end{matrix}\right\}\text{ then } A : K :: C : L.$$

Note.—It will be observed that, in the last three propositions, the magnitudes in each set are *all of the same kind;* but that those in one set, need not be of the same kind as those in the other sets.

PROPOSITION XVIII.—Theorem.

In any proportion, the sum of the first two terms is to their difference, as the sum of the other two is to their difference.

Let A : B :: C : D; then $A + B : A \backsim B :: C + D : C \backsim D$.

For (Prop. XIII.) $A + B : A :: C + D : C$, and (inverting Prop. XIV.) $A : A \backsim B :: C : C \backsim D$.

Consequently (Prop. XVII., Cor.) $A + B : A \backsim B :: C + D : C \backsim D$, $\therefore$ *in any proportion*, &c. Q. E. D.

PROPOSITION XIX.—Theorem.

If the antecedents in one proportion be the same as those in another, then the first antecedent is to the sum or difference of the first consequents, as the second antecedent is to the sum or difference of the second consequents.

Let the proportions be A : B :: C : D; and A : E :: C : F; it is to be proved that $A : B \underset{\backsim}{+} E :: C : D \underset{\backsim}{+} F$.

Inverting the first of the given proportions (Prop. VII.)

$$\left.\begin{matrix}B : A :: D : C\\ \text{but } A : E :: C : F\end{matrix}\right\}\begin{matrix}\therefore \text{ (Prop. XVII., Cor.)}\\ B : E :: D : F.\end{matrix}$$

Consequently (Props. XIII., XIV.) $B : B \underset{\backsim}{+} E :: D : D \underset{\backsim}{+} F$; and, comparing this with the first of the given proportions,

$A : B \underset{\backsim}{+} E :: C : D \underset{\backsim}{+} F$; $\therefore$ *if the antecedents*, &c. Q. E. D.

Cor.—If the terms are all *homogeneous*, then by alternation, $A : C :: B \underset{\backsim}{+} E : D \underset{\backsim}{+} F$; and $B : D :: B \underset{\backsim}{+} E : D \underset{\backsim}{+} F$.

PROPOSITION XX.—Theorem.

In a proportion whose terms are homogeneous, the sum of the greatest and least terms exceeds the sum of the other two.

Let A : B :: C : D; and first let an antecedent, A, be the greatest term; then will D be the least (Prop. IX.); and it is to be proved that $(A + D) > (B + C.)$

By Prop. XIV., and inversion, $A : A - B :: C : C - D$: but by hyp. $A > C$, $\therefore$ (Prop. IV.) $(A - B) > (C - D)$. To each of these unequals add $B + D$, then $(A + D) > (B + C)$.

Next, let a consequent, B, be the greatest: then by inverting the proportion, B : A :: D : C; and since by hyp. B is the greatest of these, C is the least, and $\therefore$ as just proved $(B + C) > (A + D)$; $\therefore$ *in a proportion*, &c. Q. E. D.

Cor.—If the proportion be A : B :: B : C, then $(A + C) > 2\,B$; that is, the sum of *the extremes*, in three proportionals, exceeds twice the *mean*.

The three propositions following, though of no application in the Sixth Book of Euclid, will be found useful in the consideration of *incommensurable quantities*.

PROPOSITION XXI.—Theorem.

If a magnitude measure each of two others, it will also measure their sum and difference.

Let A, B be any two magnitudes, and let C be a third magnitude which measures each; that is, such that $A = mC$ and $B = nC$, m and n being whole numbers. Then $A + B = mC + nC = (m + n)\,C$; also $A \backsim B = mC \backsim nC = (m \backsim n)\,C$: but C is contained in the former $m + n$ times *exactly*, and in the latter $m \backsim n$ times exactly, $\therefore$ C measures both $A + B$, and $A \backsim B$; $\therefore$ *if a magnitude*, &c. Q. E. D.

Cor.—If C measure B, and also $A + B$ and $A - B$, it must likewise measure A: for the sum of $A - B$ and B is A, and the difference of $A + B$ and B, is A; and as shown above, C measures both this sum and difference.

PROPOSITION XXII.—Problem.

Two magnitudes of the same kind being given to find their greatest common measure.

Let the two given magnitudes be A, B: it is required to find the greatest magnitude that will measure both. Let A be the greater of the two magnitudes, and from it take the greatest possible multiple of B (viz., *m*B), leaving a remainder C, less, of course, than B. In like manner take from B the greatest possible multiple of C (viz., *n*C), leaving a remainder D, less than C. In like manner take from C the greatest possible multiple of D (viz., *p*D), leaving a remainder E less than D; and so on, as in the margin: the greatest common measure will be that remainder which exactly measures the preceding one: for instance, if E measure D so that *q*E = D, then E is the greatest common measure of A and B.

B)A(M
*m*B
——
C)B(*n*
*n*C
——
D)C(*p*
*p*D
——
E)D(*q*
*q*E
——
&c.

For every common measure of A and B, as it measures B, will measure *m*B; consequently, as it at the same time measures A, it will measure A — *m*B, by last proposition; that is, it will measure C. This being the case, it must also measure *n*C, and therefore, last proposition, it must measure B — *n*C; that is, it must measure D: for similar reasons it must measure E; and so on. Consequently every measure of A and B *also* measures each of the remainders C, D, E, &c.; and as these remainders become less and less, it follows that that must be the *greatest* common measure which is exactly *equal* to the last of these remainders, and at which the operation terminates. But if the operation never terminate, the diminishing series of remainders being continued without end, then, as there is no *last* remainder, there can be no common measure at all—in other words, the magnitudes A, B will be *incommensurable*.

PROPOSITION XXIII.—THEOREM.

If one magnitude contain another, and leave a remainder, such that the greater of the two magnitudes is to the less, as the less is to that remainder, then the two magnitudes will be incommensurable. Let A, B be the two magnitudes, such that the greater A contains the less B, *m* times, leaving a remainder C; that is, such that A — *m*B = C; then if A : B : : B : C, the magnitudes A, B will be *incommensurable*.

For let C, D, E, &c., be the successive remainders in the operation for finding the common measure (Prop. XXII.) Then C cannot measure B, for then B would measure A, so that there would not be any *remainder* (Prop. XII.); but C is contained as often in B as B is contained in A (Def. VI.) Let *m*B be the greatest multiple of B which is contained in A, and take *m*C, an equimultiple of C: then (Prop. VIII.) A : B : : *m*B : *m*C, and (Props. XIV. and XI.) A : B : : A — *m*B : B — *m*C: but by hypothesis A — *m*B = C, and B — *m*C = D: therefore A : B : : C : D; and since A : B : : B : C, ∴ (Prop. II.) B : C : : C : D; hence D cannot measure C, inasmuch as C cannot measure B, (Prop. XII.)

Let now *n*C be the greatest multiple of C in B, and take *n*D, an equimultiple of D: then, from what is proved above, C : D : : D : E; hence E cannot measure D, inasmuch as D, as just proved, cannot measure C. And the reasoning is the same for every successive remainder; so that no remainder can ever measure the preceding remainder; and therefore the operation for the common measure can never terminate; that is, the two magnitudes A, B are *incommensurable*, ∴ *if one magnitude* &c. Q. E. D.

The operation explained in Prop. XXII. is that actually performed on a pair of *numbers*, when the object is to ascertain whether those numbers have a common measure, and to discover the *greatest* common measure.

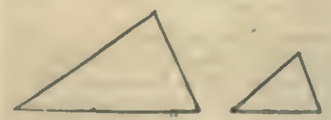
In this *arithmetical* process, should a remainder ever become 1, we conclude that no common measure of the two numbers exists; because *in Arithmetic*, as remarked at page 571, 1 is not regarded as an arithmetical common measure. When the two numbers have no factor in common, a unit-remainder must always occur to apprise us of the fact, after a finite number of steps of the work; since the remainders—all whole numbers—go on continually *diminishing*. But in magnitudes not susceptible of numerical representation, the operation referred to could not be practically applied—as long as *any* remainder occurred, so long must the work be continued; and therefore, in the case of incommensurable magnitudes, it would be endless, even if we could practically carry forward the steps. But the proposition just established, furnishes a geometrical test of incommensurability that may be readily appealed to, as will be seen in the proposition respecting the side and diagonal of a square, at the end of the Sixth Book. It will also be shown hereafter, by aid of the present theorem, that if a line be divided, as in Prop. XI., Book II., the two parts of that line will be incommensurable; and therefore that it would be quite impossible to express *both* by numerical values of their lengths.

CHAPTER VI.

ELEMENTS OF EUCLID.—BOOK VI.

DEFINITIONS.

I.



Similar rectilineal figures are those which have the several angles in one equal to those in the other, each to each, and the sides *about* the equal angles—that is, which *include* the equal angles, proportionals.

II.

Two sides of one figure are said to be *reciprocally proportional* to two sides of another, when one of the sides of the first is to one of the sides of the second as the remaining side of the second is to the remaining side of the first.

III.

A straight line is said to be cut in *extreme and mean ratio*, when the whole is to the greater segment as the greater segment is to the less.

IV.

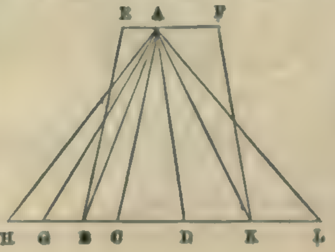
The *altitude* of any figure is the straight line drawn from its vertex perpendicular to the base, or opposite side, and terminating in that side, or the side prolonged.

Thus the perpendicular A D, drawn from the vertex A to the base B C, is the altitude of the triangle A B C, in Proposition VIII. in this book.

PROPOSITION I.—THEOREM.

Triangles and parallelograms of the same altitude are to one another as their bases.

Let the triangles A B C, A D K, and the parallelograms E C, F D, have the same altitude: then as the base B C is to the base DK, so is the triangle A B C to the triangle A D K, and the parallelogram E C to the parallelogram F D.



Produce B K both ways to H and L, and make B G, G H., &c., any number of

3 I. them, each equal to B C; and K L, &c., any number of them, each equal to D K; and draw A G, A H, &c., and A L, &c.; then the triangles A B C, A G B, †38 I. A H G, &c., are all equal;† and if the base of the triangle which is the sum of all these be *m* times B C, the triangle itself will be *m* times the triangle *38 I. A B C.* In like manner if D L be *n* times D K, the triangle A D L will be *n* times A D K.

base B C : base D K : : triangle A B C ; triangle A D K,
m. B C : *n*. D K : : *m*. A B C : *n*. A D K

Hence if *m*. B C = *n*. D K, then *m*. A B C = *n*. A D K; if *m*. B C > *n*. D K, then *m*. A B C > *n*. A D K; and if *m*. B C = *n*. D K, then *m*. A B C = *n*. A D K, and *m*, *n*. are any whole numbers whatever, ∴ (Def. A., page 573).

base B C : base D K : : triangle A B C : triangle A D K.

And because the parallelogram E C is double of the triangle A B C, and the parallelogram F D double of the triangle A D K, ∴ triangle A B C : triangle A D K : : †8 V. parallelogram E C : parallelogram F D;† and, comparing this proportion with the last, base B C : base *2 V. D K : : parallelogram E C : parallelogram F D,* ∴ *triangles and parallelograms*, &c. Q. E. D.

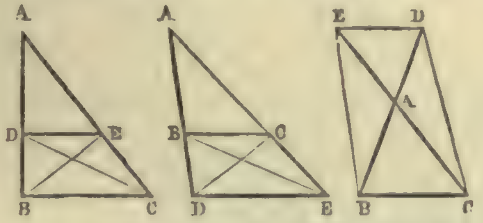
COR.—If triangles or parallelograms have *equal* altitudes, they are to one another as their bases.

For if the figures be so placed as to have their bases in the same straight line, and perpendiculars be drawn from the vertices to the bases, the straight line joining the vertices will be parallel to that in which are the *23 I. bases,* since the perpendiculars are both equal †28 I. and parallel;† and ∴ if the above construction be made, the demonstration will be the same.

PROPOSITION II.—THEOREM.

If a straight line (D E) *be drawn parallel to one of the sides* (B C) *of a triangle, it shall cut the other sides, or these produced, proportionally; and if the sides, or the sides produced, be cut proportionally, the straight line which joins the points of section shall be parallel to the remaining side of the triangle.*

For draw B E, C D. Then the triangle B D E = C D E,



because they are on the same base D E, and are between *37 I. the same parallels D E, B C;* and A D E is another triangle, ∴ B D E : A D E : : C D E : A D E. †1 VI. But B D E : A D E : : B D : D A.† For a similar reason C D E : A D E : : C E : E A,

2 V. ∴ B D : D A : : C E : E A.

Again, let the sides A B, A C, or these produced, be cut proportionally in the points D, E; that is, so that B D : D A : : C E : E A; and draw D E; *D E shall be parallel to B C.*

The same construction being made, because

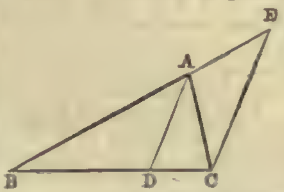
B D : D A : : C E : E A, and B D : †1 VI. D A : : B D E : A D E,† and C D E : A D E : : C E : *1 VI. E A.* } ∴ (2 V.) B D E : A D E : : C D E : A D E.

But as the consequents are equal the antecedents are †9 V. Cor. 1. equal,† ∴ triangle B D E = triangle C D E; and they are on the same base D E, and are between the same parallels, ∴ *D E is parallel to B C*; ∴ *if a straight line*, &c. Q. E. D.

PROPOSITION III.—THEOREM.

If an angle (B A C) *of a triangle, be divided into two equal angles, by a straight line which cuts the opposite base, the segments of the base* (B D, D C) *shall be to each other as the remaining sides of the triangle; and if the segments of the base are to each other as the remaining sides of the triangle, the straight line drawn from the vertex to the point of section, shall divide the vertical angle into two equal angles.*

31 I. Draw C E parallel to D A; and let B A produced meet C E in E. Because A C meets the parallels A D, E C, the angle A C E = C A D;† †29 I. but by hypothesis, C A D = B A D; ∴ B A D = A C E. Again, because B E meets the parallels A D, E C, the angle B A D *29 I. = A E C;* but it was proved that A C E = B A D, ∴ A C E = A E C, and ∴ A E = A C; and since A D is parallel to E C, a side of the triangle B C E; †2 VI. ∴ B D : D C : : B A : A E;† but A E = A C,



∴ B D : D C : : B A : A C.

Next let B D : D C : : B A : A C; and draw A D; the angle B A C shall be divided into two equal angles by A D.

The same construction being made, A D is parallel to *Const. E C,* and because

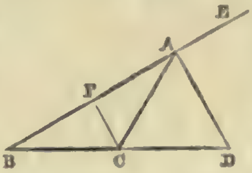
B D : D C : : B A : A C, and †2 VI. B D : D C : : B A : *9 V. Cor. A E,† } ∴ (2 V.) B A : A C : : B A : A E, ∴ A C = A E,*

and consequently the angle A E C = A C E; but A E C †29 I. = B A D,† and A C E = C A D,* ∴ B A D = *29 I. C A D, that is, *the angle B A C is divided into two equal angles by A D*; ∴ *if an angle*, &c. Q. E. D.

PROPOSITION A.—THEOREM.

If the outward angle (C A E) *of a triangle* (A B C) *made by producing one of its sides, be divided in two equal angles, by a straight line* (A D) *which cuts the base produced, the segments* (B D, D C) *between the dividing line and the extremities of the base, are to each other as the remaining sides of the triangle; and if the segments of the base produced are to each other as the remaining sides of the triangle, the straight line drawn from the vertex to the point of section, divides the outward angle of the triangle into two equal angles.*

31 I. Through C draw C F parallel to A D. Then because A C meets the parallels A D, F C, the angle A C F †29 I. = C A D;† but (hyp.) C A D = D A E, ∴ D A E = A C F. Again, because F E meets the parallels A D, F C, the angle D A E = C F A;* *29 I. but as just proved, D A E = A C F ∴ C F A = †6 I. A C F; ∴ A C = A F.† And because A D is parallel to F C, a side of the triangle B C F, ∴ B D : D C : : B A : A F; but A C = A F, ∴



B D : D C : : B A : A C.

Next, let B D : D C : : B A : A C; and draw A D; the angle C A D shall be equal to D A E.

The same construction being made, because

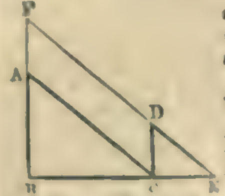
B D : D C : : B A : A C, and *2 VI. B D : D C : : B A : A F,* } ∴ (2 V.) B A : A C : : B A : A F, ∴ A C = A F,

†5 I. ∴ the angle A F C = A C F;† but A F C = *29 I. E A D, and A F C = C A D,* ∴ E A D = C A D; ∴ *if the outward angle*, &c. Q. E. D.

PROPOSITION IV.—THEOREM.

The sides about the equal angles of equiangular triangles (A B C, D C E) *are proportionals; and those which are opposite to the equal angles are* homologous *sides; that is, are the antecedents, or the consequents of the ratio.*

Let the angle A B C = D C E; A C B = D E C; and consequently B A C = C D E; and let the triangle D C E be placed so that its side C E may be contiguous to B C, and in the same straight line with it; then the angle

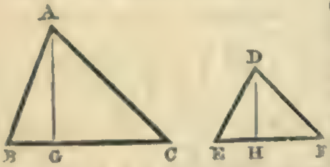


† Hyp. B C A = C E D.† Add to each the angle B; then B C A + B = C E D + B; but the former two are less than two right angles,* * 17 I. ∴ C E D + B, are less than two right angles, ∴ B A, E D, † Ax. 12 if produced will meet;† let them be produced and meet in F. Then because the angle B = * 28 I. DCE, BF is parallel to CD;* and because A C B = D E C, A C is parallel to F E; ∴ † 34 I. F C is a parallelogram,† ∴ A F = C D, and A C = F D; and because A C, C D are respectively parallel to F E, B F, sides of the triangle F B E, ∴ (1 VI.)

B C : C E : : B A : A F and B C : C E : : F D : D E } or since A F = C D, and F D = A C { B C : C E : : B A : C D; B C : C E : : A C : D E

* 2 V. ∴ B A : C D : : A C : D E.* Alternating these three proportions, B C : B A : : C E : C D; B C : A C : : C E : D E; B A : A C : : C D : D E. ∴ *the sides about the equal angles, &c.* Q. E. D.

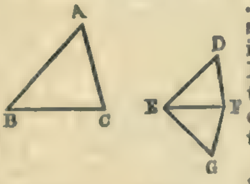
Cor.—In similar triangles (A B C, D E F), the bases (B C, E F) are to one another as the altitudes (A G, D H). For since the angles B, E are equal, as also the right angles G, H, the triangles A B G, D E H are equiangular, ∴ A B : A G : : D E : D H; but B C : A B : : E F : E D;* * Def. 1 VI. ∴ B C : A G : : E F : D H;† hence, alter- † 15 V. Cor. nately, the bases are as the altitudes.



PROPOSITION V.—Theorem.

If the sides of two triangles (A B C, D E F), *about two angles* (B, C) *of one, and two angles* (E, F) *of the other, be proportionals, the triangles shall be equiangular; and the equal angles shall be those which are opposite to homologous sides.*

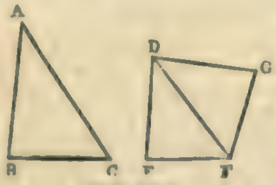
At the points E, F, in the straight line E F, make the angle F E G = B, and E F G = C; then the remaining angle G = A, and ∴ the triangles A B C, G E F, are equi- * 4 VI. angular, ∴ A B : B C : : G E : E F;* but (hyp.) † 2 VI. A B : B C : : D E : E F; ∴ D E : E F : : G E : E F;† ∴ D E = G E. For a like reason D F = F G. And because in the triangles D E F, G E F, D E = G E, and E F common, the two sides D E, E F are equal to the two G E, E F, each to each, and D F = G F; ∴ * 8 I. the angle D E F = G E F,* and the angle D F E = G F E, and E D F = E G F. And because D E F = G E F, and † Const. G E F = B,† ∴ D E F = B. For a like reason D F E = C, and ∴ D = A; ∴ *the triangle* D E F *is equiangular to* A B C; ∴ *if the sides, &c.* Q. E. D.



PROPOSITION VI.—Theorem.

If two triangles (A B C, D E F) *have an angle* (A) *of one, equal to an angle* (E D F) *of the other, and the sides about them proportionals, the triangles shall be equiangular, and shall have those angles equal which are opposite to the homologous sides.*

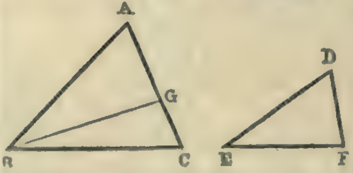
At the points D, F, in the straight line D F, make the angle F D G equal to either of the angles A, or E D F; and the angle D F G equal to C; then the remaining angle G = B, and ∴ the triangles A B C, D G F are equiangular; ∴ B A : A C : : G D * 4 VI. : D F;* but (hyp) B A : A C : : E D : : D F; ∴ E D : D F : : G D : D F;† ∴ † 2 V. E D = G D; ∴ in the triangles E D F, G D F, the two sides E D, D F = * Const. G D, D F, each to each, and the angle E D F = G D F;* ∴ E F = F G†, and the angle D F G = D F E, † 4 I. and G = E;* but D F G = C (const.) D F E = * 4 I. C; but E D F = A (const.), ∴ E = B; ∴ *the triangles* A B C, D E F *are equiangular;* ∴ *if two triangles, &c.* Q. E. D.



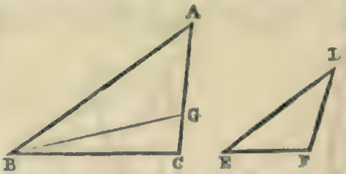
PROPOSITION VII.—Theorem.

If two triangles (A B C, D E F) *have an angle* (A) *of the one, equal to an angle* (D) *of the other, and the sides about two* (A B C, D E F) *proportionals; then if each of the remaining angles* (C, F) *be either less, or not less, than a right angle, the triangles shall be equiangular, and shall have those angles equal, about which the sides are proportionals.*

First let each of the angles C, F be *less* than a right angle: the angles A B C and E shall be equal, and ∴ the angle C to the angle F. For if A B C be not equal to E, one of them, as A B C, must be the greater. At the point B, in A B, make the angle A B G = E; then because A = D (hyp.) the remaining angle A G B must be * 32 I. = F;* ∴ the triangles A B G, D E F are equiangular, ∴ A B : B G : : D E : E F. But (hyp.) A B : B C : : D E : E F, ∴ A B : B C : : A B : B G, ∴ B C = B G,† and ∴ † 9 V. Cor. 1. B G C = B C G; but (hyp.) B C G is less than a right angle, ∴ B G C is less than a right angle; ∴ A G B must be *greater* than a right angle: but it was proved that A G B = F; ∴ F is greater than a right angle; but (hyp.) it is *less* than a right angle; which is absurd; ∴ the angles A B C, E are not unequal, that is, they are equal; and, as A = D (hyp.) ∴ C = F ∴ *the triangles* ABC, DEF *are equiangular.*



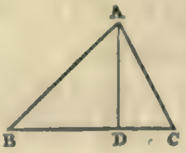
Next, let C, F be each *not less* than a right angle: the triangles shall also in this case be equiangular. For if it be denied, then, the same construction being made, it may be proved, as above, that B C = B G; and that ∴ the angle B G C = C: but (hyp.) C is not less than a right angle: ∴ B G C is not less than a right angle ∴ two angles of the triangle B G C are together not less than two right angles: which is impossible; ∴ *the triangle* A B C, *as in the first case, is equiangular to* D E F. ∴ *if two triangles, &c.* Q. E. D.



PROPOSITION VIII.—Theorem.

In a right-angled triangle (A B C), *if a perpendicular* (A D) *be drawn from the vertex of the right angle to the base, the triangles* (A B D, A C D) *on each side of it are similar to the whole triangle* (A B C) *and to one another.*

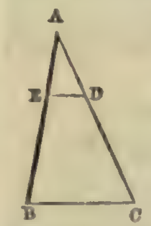
Because the angle B A C = A D B, each being a right angle, and that B is common to the two triangles A B C, A B D, ∴ the * 32 I. angle A C B = B A D;* ∴ the triangles A B C, A B D are equi- † 4 VI. and Def. 1. angular, ∴ they are similar.† In like manner it may be demonstrated that the triangles A B C, A C D are similar; ∴ *the triangles* A B D, A C D *being both similar to* A B C, *are similar to each other;* ∴ *in a right-angled triangle, &c.* Q. E. D.



Cor.—From this it is manifest that the perpendicular from the vertex of the right angle of a right-angled triangle to the base, is *a mean proportional* between the segments of the base; and also that each side about the right angle is a mean proportional between the base, and the segment of it adjacent to that side: for in the tri- * 4 VI. angles B D A, C D A, B D : D A : : D A : D C;* in the triangles A B C, D B A, B C : B A : : B A : B D; and in the triangles A B C, A C D, B C : C A : : C A : C D.

PROPOSITION IX.—PROBLEM.

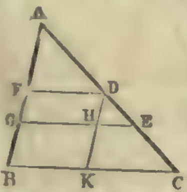
From a given straight line (A B) *to cut off any part required.*



From A draw a straight line A C, making any angle with A B; and in it take any point D; and take A C, the same multiple of A D, that A B is of the part to be cut off. Draw B C, and D E * 31 I. parallel to it,* then A E shall be the part required to be cut off. Because E D is parallel to B C, a side of the triangle A B C, ∴ C D : D A : : B E : E A; † 13 V. ∴ A C ; A D : : A B : A E ;† but * Const. A C is a multiple of A D,* ∴ A B is the same multiple of A E; that is, whatever part of A D is of A C, the same † 7 V. Cor. part is A E of A B,† ∴ *from* A B, *the part required,* A E *is cut off.* Which was to be done.

PROPOSITION X.—PROBLEM.

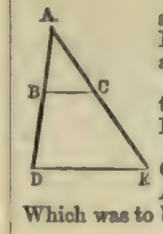
To divide a given straight line (A B) *similarly to a given divided straight line* (A C).



Let A C, placed so as to make any angle with A B, be divided in the points D, E. Draw B C; and through the points D, E draw D F, E G parallels to it; A B shall be divided in F, G similarly to A C. Through D draw D H K, parallel to A B: then each of the figures F H H B is a parallelogram, ∴ D H = * 34 I. F G,* and H K = G B; and because H E is parallel to K C, a side of the triangle DKC, ∴ C E : † 2 VI. E D : : K H : H D ;† but K H = B G, and H D = G F; ∴ C E : E D : : B G : G F. Again, because F D is parallel to G E, a side of the triangle A G E, ∴ E D : D A : : G F : F A : and it was before proved that C E : E D : : B G : G F; ∴ *the given straight line is divided similarly to* A C. Which was to be done.

PROPOSITION XI.—PROBLEM.

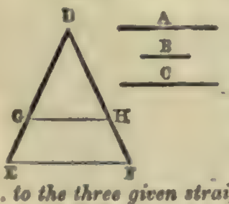
To find a third proportional to two given straight lines (A B, A C).



Let the lines be placed so as to make any angle A, and produce them to D, E. Make B D = A C; draw B C, and through D draw D E parallel to it:* * 31 I. C E shall be a third proportional to A B, A C. Because B C is parallel to D E, ∴ A B : B D : : A C : C E; but B D † Const. = A C ;† ∴ A B : A C : : A C : C E; ∴ *to the two given straight lines* A B, A C, *a third proportional* C E *is found.* Which was to be done. Q. E. D.

PROPOSITION XII.—PROBLEM.

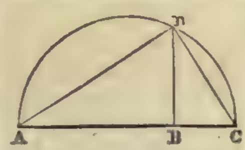
To find a fourth proportional to three given straight lines (A, B, C).



Take two straight lines D E, D F, making any angle D; and upon these take D G = A, G E = B, and D H = C. Draw G H, and E F parallel to it: F H shall be a fourth proportional to A, B, C. Because G H is parallel to E F, ∴ D G : G E : : D H : H F; but D G = A, G E = B, and D H = C,* * Const. ∴ A : B : : C : H F, ∴ *to the three given straight lines* A, B, C, *a fourth proportional* H F *is found.* Which was to be done.

PROPOSITION XIII.—PROBLEM.

To find a mean proportional between two given straight lines.

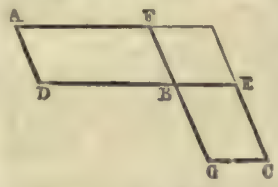


Let the parts A B, B C be equal to the two given straight lines: it is required to find a mean proportional between them. Upon A C describe the semicircle A D C, and from B draw B D perpendicular * 11 I. to A C;* B D shall be a mean proportional between A B, B C.

Draw A D, C D. Then because † 31 I. the angle A D C, in a semicircle is a right angle,† and that in the right-angled triangle A D C, D B is drawn from the vertex of the right angle perpendicular to the base, D B is a mean proportional between A B, B C,* * 8 VI. Cor. ∴ *between* A B, B C, *a mean proportional* D B, *is found.* Which was to be done.

PROPOSITION XIV.—THEOREM.

Equal parallelograms (A B, B C) *which have an angle* (B) *of the one equal to an angle of the other, have their sides about the equal angles reciprocally proportional: and parallelograms that have an angle of the one equal to an angle of the other, and the sides about those angles reciprocally proportional, are equal to one another.*



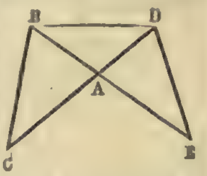
Let the sides D B, B E, be placed in the same straight line; then because F B D + F B E = two right angles, and that F B D = G B E (hyp.), ∴ G B E + F B E = two right angles; ∴ F B, B G are in one straight line. It is to be proved that DB : BE : : GB : BF. Complete the parallelogram F E. Then because A B = B C, AB : FE : : BC : FE; but AB : FE : : DB : BE,* * 1 VI. and B C : F E : : † 2 V. G B : B F, ∴ D B : B E : : G B : B F ;† ∴ *the sides of the equal parallelograms* A B, B C, *about their equal angles are reciprocally proportional.*

Next, let the sides about the equal angles be reciprocally proportional; namely: D B : B E : : G B : B F; then is A B = B C. Because D B : B E : : G B : B F, and DB : BE : : AB : F E, and GB : BF : : BC : FE,* * 1 VI. ∴ A B : F E : : B C : FE,† ∴ A B = B C, ∴ † 2 V. *equal parallelograms, &c.* Q. E. D.

PROPOSITION XV.—THEOREM.

Equal triangles (A B C, A D E), *which have an angle* (A) *of the one equal to an angle of the other, have their sides about the equal angles reciprocally proportional; and triangles which have an angle of the one equal to an angle of the other, and their sides about those angles reciprocally proportional, are equal to one another.*

Let the triangles be placed so that their sides, C A, A D may be in one straight line; then it may be proved, as in last proposition, that E A, A B are in one straight line. It is to be proved that C A : A D : : E A : A B.



Draw B D. Then because the triangles A B C, A D E are equal, ∴ triangle A B C : triangle A B D : : triangle A D E : triangle A B D; but triangle ABC : triangle ABD : : C A * 1 VI. : AD,* and triangle ADE : triangle A B D : : E A : A B, con- † 2 V. sequently† C A : A D : : E A : A B; ∴ *the sides of the equal triangles* A B C, A D E, *about the equal angles are reciprocally proportional.*

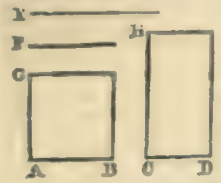
Next, let the sides of the triangles, A B C, A D E, about the equal angles be reciprocally proportional; namely, C A : A D : : E A : A B; then is triangle A B C = triangle A D E.

Draw BD as before. Then, because C A : A D : : E A : A B, and C A : A D : : triangle A B C : triangle ABD,* * 1 VI. and E A : A B : : triangle A D E : triangle A B D; ∴ triangle A B C : triangle A B D : : triangle † 2 V. A D E : triangle A B D ;† and the consequents * 9 V. Cor. being equal, the antecedents are equal,* ∴ *triangle* A B C = *triangle* A D E; ∴ *equal triangles, &c.* Q. E. D.

PROPOSITION XVI.—THEOREM.

If four straight lines (AB, CD, E, F) *are proportionals, the rectangle contained by the extremes* (A B, F) *is equal to that contained by the means* (CD, E): *and if the rectangle contained by the extremes be equal to that contained by the means, the four straight lines are proportionals.*

From A, C draw A G, C H, perpendiculars to A B, • 11 I. C D;* make A G = F, and C H = E, and † 31 I. complete the parallelograms B G, D H.† It is to be proved that these rectangles are equal.



Because A B : C D :: C H : A G, ∴ the sides of the parallelograms about the equal angles A, C, are reciprocally proportional; they are therefore equal to one • 14 VI. another;* and B G is the rectangle contained by A B, F, and D H the rectangle contained by C D, E; ∴ AB·F = C D·E. And if AB·F = CD E, then shall A B : C D :: E : F. The same construction being made, the rectangle B G = D H; and as they are equi- † 14 VI. angular also, ∴ A B : C D :: C H : A G;† that is A B : C D :: E : F; ∴ *if four straight lines, &c.* Q. E. D.

NOTE.—The next proposition is merely a corollary to this: it is that particular case of it in which the *means* are equal.

PROPOSITION XVII.—THEOREM.

If three straight lines be proportionals, the rectangle contained by the extremes is equal to the square of the mean, and if the rectangle contained by the extremes is equal to the square of the mean, the three straight lines are proportionals.

COR.—The first of the three proportionals (A, B, C) is to the third, as the square of the first to the square of the second.

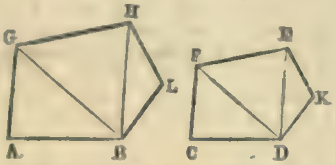
• 1 VI. For* A : C :: rectangle A·A : rectangle A·C; but A·C = B^2, ∴ A : C :: A^2 : B^2.

PROPOSITION XVIII.—PROBLEM.

Upon a given straight line (A B) *to describe a rectilineal figure similar, and similarly situated, to a given rectilineal figure.*

First, let the given rectilineal figure, C, D, E, F have *four* sides: it is required to describe on A B a figure similar, and similarly situated to C, D, E, F.

Draw D F; and at the points A, B, in A B, make the angle A = C, and ABG = CDF; then AGB = CFD;† † 32 I. ∴ the triangles F C D, G A B, are equiangular. At the points G, B in G B, make the angle B G H = D F E, • 32 I. and G B H = F D E; then G H B = F E D;* ∴ the triangles F D E, G B H are equiangular.



Because the angle A G B = C F D, and B G H = D F E; ∴ A G H = C F E. For like reasons A B H = C D E, ∴ since A = C, and G H B = F E D, *the figures* A B H G, C D E F, *are equiangular.* Moreover these figures have their sides about the equal angles proportionals; G A B, F C D being equiangular triangles, ∴ B A : A G † 4 VI. :: D C : C F; and A G : G B :: C F : F D;† also by reason of the equiangular triangles B G H, D F E, G B : G H :: F D : F E; ∴ A G : G H :: C F : F E. In like manner it may be proved that A B : B H :: C D : D E, and G H : H B :: F E : E D; ∴ *the equiangular figures* A B H G, C D E F *have their sides about the equal* • Def. 1 VI. *angles proportionals,* ∴ *they are similar.**

Next, let the given rectilinear figure C D K E F have *five* sides. Draw D E; and upon A B describe the figure A B H G, similar and similarly situated, to the quadrilateral C D E F, by the former case: and at the points B, H, in B H, make the angle H B L = E D K, and • 32 I. B H L = D E K, then L = K.*

Because the figures A B H G, C D E F are similar, the angle G H B = F E D; and B H L was made = D E K; ∴ G H L = F E K. For like reasons A B L = C D K; ∴ *the figures* A B L H G, C D K E F, *are equiangular.* And because the figures A B H G, C D E F are similar, ∴ G H : H B :: F E : E D; but H B : H L :: E D : E K, ∴ G H : H L :: F E : E K. For like reasons A B : B L :: C D : D K, and B L : L H :: D K : K E; ∴ *the equiangular figures* A B L H G, C D K E F *have their sides about the equal angles proportionals,* ∴ *they are similar.* And in like manner may a rectilineal figure of *six* sides, similar to a given one, be described upon A B; and so on. Which was to be done.

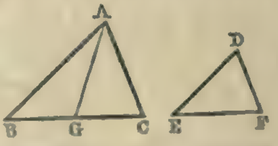
PROPOSITION XIX.—THEOREM.

Similar triangles (A B C, D E F) *are to one another, as the squares of their homologous sides.*

The triangles being similar, having the angle B = E, • Def 9 V. and the side B C homologous to E F,* that is such that A B : B C :: D E : E F, it is to be proved that tri·A B C : tri·D E F :: BC^2 : EF^2.

† 11 VI. Take B G, a third proportional to B C, E F;† so that B C : E F :: E F : B G; and draw G A.

Then because A B : B C :: D E : E F, alternately,* • 11 V. A B : D E :: B C : E F, but B C : E F :: E F † 2 V. : B G (Const.), ∴ A B : D E :: E F : B G;†



but triangles which have the sides about two equal angles reciprocally proportional are • 15 VI. *equal,** ∴ ABG = D E F. And because B C : E F :: E F : B G, ∴ B C : B G :: BC^2 : EF^2;† † 17 VI. Cor. but B C : B G :: triangle A B C : triangle • 1 VI. A B G;* ∴ triangle A B C : triangle A B G :: BC^2 : EF^2, but A B G = D E F, ∴ triangle A B C : triangle D E F :: BC^2 : EF^2; ∴ *similar triangles, &c.* Q. E. D.

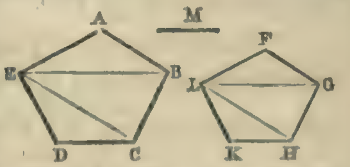
COR.—If three straight lines be proportionals, then as the first is to the third, so is any triangle upon the first, to a similar and similarly situated triangle upon the second.

PROPOSITION XX.—THEOREM.

Similar polygons may be divided into the same number of similar triangles, which are to one another as the polygons themselves: and the polygons are to one another as the squares of their homologous sides.

Let A B C D E, F G H K L be similar polygons, and let A B be the side homologous to F G.

From E, L draw the diagonals. Because the polygons are similar, the angle B A E = G F L, and B A : A E :: G F : F L; ∴ the triangles A B E, F G L have an angle in one equal to an angle in the other, and the sides about those equal angles proportionals, ∴ the triangles are • 6 & 4 VI. equiangular and similar,* ∴ the angle A B E = F G L: but since the polygons are similar, the whole angle A B C = the whole angle F G H, ∴ the remaining angle E B C = L G H.



Also E B : B A :: L G : G F, and from the similar polygons, A B : B C :: F G : G H; ∴ E B : B C :: L G : G H,† that † 17 V. Cor. is, the sides about the equal angles E B C, L G H are proportionals; • 6 VI. ∴ the triangles E B C, L G H are similar.* For like reasons the triangle E C D, L H K are similar, ∴ *the polygons are divided into the same number of similar triangles.* It is now to be proved that these triangles are, each to each, as the polygons themselves, and that these are to each other as the squares of their like sides.

Because the triangles A B E, F G L are similar, as • 19 VI. also B E C, G L H, ∴

$$\left.\begin{array}{l}\text{triangle } ABE : \text{triangle } FGL :: BE^2 : GL^{2*}\\ \text{triangle } BEC : \text{triangle } GLH :: BE^2 : GL^2\end{array}\right\}\therefore ABE$$

† 2 V. : F G L :: B E C : G L H.†

In like manner, because the similar triangles B E C, G L H, as also the similar triangles E C D, L H K, are as EC^2 to LH^2, ∴ B E C : G L H :: E C D : L H K, ∴ A B E : F G L :: B E C : G L H :: E C D : L H K. Con-

* 10 V. sequently,* A B E : F G L : : polygon A B C D E : polygon F G H K L ; but A B E : F G L : : A B^2 : FG^2,† † 19 VI. .·. (2 V.) *the polygons are to one another as the squares of their homologous sides* A B, F G ; *similar polygons, &c.* Q. E. D.

Cor.—It follows from this and the corollary to Prop. XIX., that if three straight lines be proportionals, the first is to the third as any rectilineal figure upon the first, to a similar and similarly described figure upon the second.

PROPOSITION XXI.—Theorem.

Rectilineal figures (A B) *which are similar to the same rectilineal figure* (C) *are also similar to one another.*

Because A is similar to C, they are equiangular, and * Def 1 VI. have also their sides about the equal angles proportionals.*

Again, because B is similar to C, they also are equiangular, and have their sides about the equal angles proportionals, .·. A, B are each of them equiangular to C, and have their sides about the equal angles of each of them and of C proportionals ; .·. A, C are equiangular, and have their sides about the equal angles propor- † 2 V. tionals ;† .·. A, B *are similar figures* ; .·. *rectilineal figures, &c.* Q. E. D.

PROPOSITION XXII.—Theorem.

If four straight lines (A B, C D, E F, G H) *are proportionals, the similar rectilineal figures, similarly described upon them, are also proportionals: and if the similar and similarly described rectilineal figures upon four straight lines are proportionals, the straight lines themselves also are proportionals.*

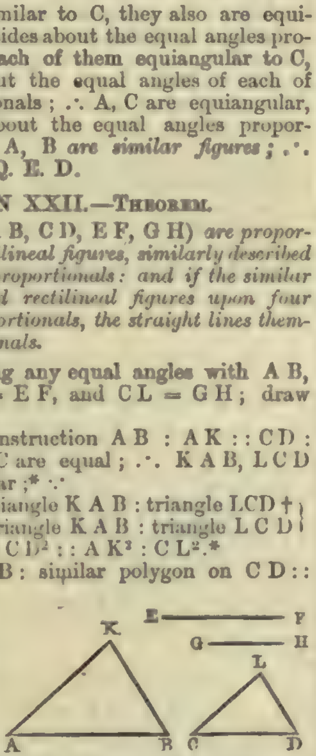
Draw A K, C L, making any equal angles with A B, C D ; make also A K = E F, and C L = G H ; draw K B, L D.

By hypothesis and construction A B : A K : : C D : C L ; and the angles A, C are equal ; .·. K A B, L C D * 6 VI. are equiangular ;* .·.

† 19 VI. A B^2 : C D^2 : : triangle K A B : triangle L C D †
A K^2 : C L^2 : : triangle K A B : triangle L C D

* 2 V. .·. A B^2 : C D^2 : : A K^2 : C L^2.*

But a polygon on A B : similar polygon on C D : : A B^2 : C D^2 ; and polygon on A K : similar polygon on C L : : A K^2 : C L^2 ; .·. polygon on A B : sim. polygon on C D : : polygon on E F : sim. polygon on G H.



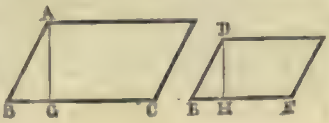
Again, let the last proportion have place, then A B : C D : : E F : G H.

Make equal angles at A and C, as before, as also A K = E F ; and as A B is to C D, so make A K to C L ;† † 12 VI. then the triangles K A B, L C D are equian- * 6 VI. gular ;* and .·., as proved above, A B^2 : C D^2 : : A K^2 : C L^2 ; but A K = E F, .·. A B^2 : C D^2 : : E F^2 : C L^2. But by hyp., the polygons on A B, C D, E F, G H are proportionals, and therefore, since similar poly- † 20 VI. gons are as the squares of their like sides† A B^2 : * 9 V. Cor. 2. C D^2 : : E F^2 : G H^2 ; .·. C L = G H.* But by construction, the four lines A B, C D, A K, C L, are proportionals ; .·. *the four lines* A B, C D, E F, G H *are proportionals* ; .·. *if four straight lines, &c.* Q. E. D.

PROPOSITION XXIII.—Theorem.

Equiangular parallelograms (A C, D F) *are to each other as the rectangles of their containing sides* (A B·B C *and* D E·E F).

Draw A G, D H, perpendiculars to B C, E F. Then because parallelograms on the same base and between the same parallels are equal, A C = A G·B C, and D F = D H·E F. Also the rectangles A G B C, A B·B C having the same altitude B C, are to each other as their bases, A G, A B. In like manner, the rectangles D H·E F, D E· * 1 VI. E F are to each other as their bases D H, D E.* But the triangles A B G, D E H having the angles at B and G respectively equal to those at E and H, are equi- † 4 VI. angular ; .·. A G : A B : : D H : D E.† .·. A G·B C : A B·B C : : D H·E F : D E·E F, and alternately* * 11 V. A G·B C : D H·E F : : A B·B C : D E·E F ; that is, *the parallelograms* A C, D F *are to each other as the rectangles* A B·B C, D E·E F *of their containing sides* ; .·. *equiangular parallelograms, &c.* Q. E. D.



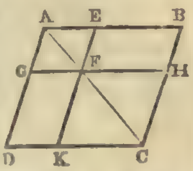
PROPOSITION XXIV.—Theorem.

Parallelograms (E G, H K) *about the diagonal* (A C) *of a parallelogram* (D B) *are similar to the whole, and to one another.*

Because D C, G F are parallels, the angle A D C = † 29 I. A G F ;† and because B C, E F are parallels, A B C = A E F ; and each of the angles B C D, E F G is equal to the opposite angle D A B ; .·. B C D = E F G ; .·. the parallelograms B D, E G are *equiangular.* And because the angle A B C = A E F, and that B A C is common to the two triangles B A C, E A F, .·. they are equiangular ; .·. A B : B C : : A E : * 4 VI. E F ;* that is, the opposite sides of parallelograms being equal,

A B : A D : : A E : A G
D C : C B : : G F : F E
C D : D A : : F G : G A

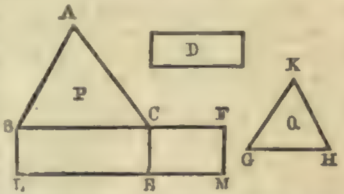
.·. the sides of the equiangular parallelograms B D, E G, about the equal angles, are proportionals : the parallelograms are therefore similar. For like reasons, the parallelograms B D, H K are similar ; .·. *each of the parallelograms* E G, H K *is similar to* B D : *they are therefore similar to each other* ;* * 21 VI. .·. *parallelograms, &c.* Q. E. D.



PROPOSITION XXV.—Problem.

To describe a rectilineal figure which shall be similar to one (P) *and equal to another given rectilineal figure* (D).

Upon B C, a side of the given figure, describe the † 45 I. Cor. parallelogram B E = P ;† and upon C E describe the parallelogram C M = D, and having the angle F C E = C B L ; then L B C + E C B = two right angles, .·. F C E + E C B = two right angles, .·. B C, C F are in a straight * 14 I. line ;* so also for like reasons are L E, E M. Between B C, C F find † 13 VI. a mean proportional G H ;† and upon it describe the rectilineal figure Q, similar and similarly * 18 VI. situated to the figure P,* Q shall be the figure required.

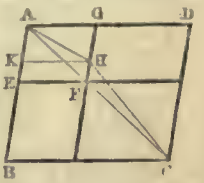


Because B C : G H : : G H : C F, .·. B C : C F : : P : † 20 VI. Cor. Q ;† but B C : C F : : B E : E F ; .·. P : Q : : B E : E F ; and P = B E, .·. Q = E F ; but E F = D, .·. Q = D, *and it is similar to* P. Which was to be done.

PROPOSITION XXVI.—Theorem.

If two similar parallelograms (B D, E G) *have a common angle* (at A) *and be similarly situated, they are about the same diagonal.*

For if not, let, if possible, the parallelogram B D have its diagonal A H C in a different straight line from A F, the diagonal of E G ; and let G F, or G F prolonged, meet A H C in H, and draw H K parallel to D A, or C B. Then B D, K G being about the same diagonal are similar, .·. D A : A B : : G A : A K ; but B D, E G * Hyp. are similar ;* .·. D A : A B : : G A : A E ; .·. G A : A E : : G A : A K ; .·. A K = A E which is impossible ; .·. B D, K G are *not* about the same diagonal, that is, the diagonal of B D



cannot be in a different straight line from that of E G, .·. *the parallelograms are about the same diagonal ; .·. if two similar parallelograms, &c.* Q. E. D.

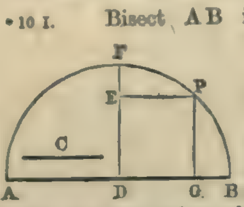
PROPOSITION XXVII.—THEOREM.

Of all the rectangles contained by the segments of a given straight line, the greatest is the rectangle of the two EQUAL *parts of the line ; that is, the square of half the line.*

This is an obvious corollary from Proposition V., Book II., as already noticed at page 553. The corresponding Proposition of Euclid is long and complicated, and Propositions XXVIII., XXIX., are still more so. Playfair, in his edition of the Elements, has therefore replaced these three Propositions by others of a simpler character; and his example is followed here; but the demonstrations of the next two Propositions will be found still simpler than those of Playfair.

PROPOSITION XXVIII.—PROBLEM.

To divide a given straight line (A B), *so that the rectangle contained by its segments may be equal to a given square* (C^2), *not greater than the square of half the line.*



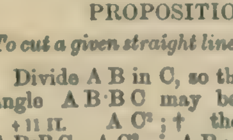
• 10 I. Bisect A B in D ;* then must A D be either = C, or ⁊ C. If A D = C, the thing required is done ; but if A D ⁊ C, draw D F perpendicular to A B, † † 11 I. and make D E = C, as also D F = A D. With centre D and radius D F describe a circle ; the circumference must pass through the points A, B, because D A, D F, D B are (by Const.) equal. Draw E P, parallel to A B, meeting the circumference in P ; and draw P G parallel to E D ; then G will be the point of division required.

• 34 I. For (Const.) E G is a rectangle, .·. P G = E D ;* but E D = C (Const.) .·. P G = C : but A G·G B = † 13 VI. P G^2,† .·. A G·G B = C,² .·. *A B is divided in* G, *so that* A G·G B = C^2 ; which was to be done.

PROPOSITION XXIX.—PROBLEM.

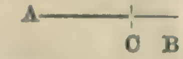
To prolong a given straight line (A B), *so that the rectangle contained by the segments between the extremities of the given line, and the point to which it is prolonged, may be equal to a given square* (C^2).

Bisect A B in D, draw B E perpendicular to it, and make B E = C ; draw D E, and with centre D and radius D E, describe a circle meeting A B prolonged in G, F ; then will A G·G B = C^2. Because D is the centre of the circle F E G, D F = D G ; and (Const.) D A = D B, .·. A F = B G. To each of these add A B, then • 13 VI. F B = A G. But F B·B G = B E^2,* .·. A G·G B = B E^2 = C^2, .·. A B *is prolonged to* G, *so that* A G·G B = C^2 ; which was to be done.



PROPOSITION XXX.—PROBLEM.

To cut a given straight line (AB) *in extreme and mean ratio.*

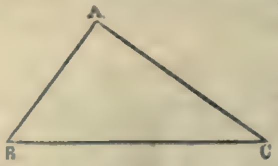


Divide A B in C, so that the rectangle A B·B C may be equal to † 11 II. A C^2 ; † then because A B·B C = A C^2, .·. A B : A C : : A C : • 17 VI. B C ;* .·. A B is cut in extreme and mean † Def. 3 VI. ratio ; † which was to be done.

NOTE.—An example is here furnished of the way in which pairs of *incommensurable* lines may be found at pleasure. It was proved (Prop. XXIII., Book V.), that if two lines A B, A C, are such that A B : A C : : A C : B C (= A B − A C), then A B, A C, are incommensurable. It would be impossible, therefore, to express the two lines A B, A C, accurately by *numbers* ; and it is thus that a theory of proportion, sufficiently comprehensive for the demands of geometry, could never be rigorously established by aid of numbers only : the bases of the proposed triangles in Proposition I. of this book, might, for aught we know to the contrary, be related as the lines A B, A C above ; and therefore could not be numerically expressed.

PROPOSITION XXXI.—THEOREM.

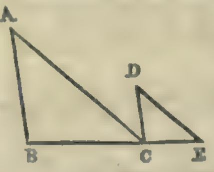
In a right-angled triangle (A B C) *the rectilineal figure described upon the side* (B C) *opposite to the right angle, is equal to the similar and similarly situated figures described upon the sides* (A B, A C) *containing the right angle.*



• 20 VI. For figure on AB : figure on AC : : AB^2 : AC^2 ;* therefore fig. on A B + fig. on A C : fig. on A C : : A B^2 † 13 V. + A C^2 : A C^2, † that is, fig. on A B + fig. on A C : fig. on A C : : B C^2 : A C^2. But fig. on B C : fig. on A C : : B C^2 : A C^2 ; • 2 V. consequently,* fig. on A B + fig. on A C : fig. on A C : : fig. on B C : fig. on A C, .·. the consequents being equal, the antecedents † 9 V. Cor. are equal, † .·. *fig. on* A B + *fig. on* A C = *fig. on* B C ; .·. *in a right-angled triangle, &c.* Q. E. D.

PROPOSITION XXXII.—THEOREM.

If two triangles (A B C, D C E) *which have two sides* (B A, A C) *of the one proportional to two sides* (C D, D E) *of the other be joined at an angle, so as to have their homologous sides* (A B, D C, and A C, D E) *parallel to one another, the remaining sides* (B C, C E) *shall be in a straight line.*



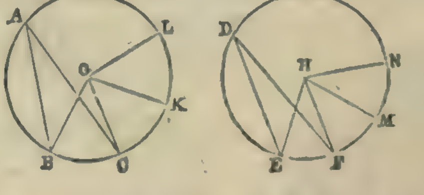
Because A B is parallel to D C, and A C meets them, the angle B A C = A C D ; for a similar reason C D E = A C D ; .·. B A C = C D E. And because the triangles A B C, D C E have an angle A = D, and the sides about those angles proportional, namely, B A : A C : : • Hyp. C D : D E,* the triangles † 6 VI. A B C, D C E, are equiangular ; † .·. angle A B C = D C E ; and it was proved that B A C = A C D, .·. A C E = A B C + B A C. Add A C B to each, .·. A C E + A C B = A B C + B A C + A C B = two right angles ; that is, at the point C, in A C, the two straight lines C B, C E, on opposite sides of it, make the adjacent angles A C E, A C B, together equal to two right angles ; .·. C B, C E *are in a straight line ; .·. if two triangles, &c.* Q. E. D.

PROPOSITION XXXIII.—THEOREM.

In equal circles (A B L, D E N), *angles* (B G C, E H F) *at the centres or* (B A C, E D F) *at the circumferences are to one another as the arcs* (B C, E F) *upon which they stand ; and so also are the sectors* (B G C, E H F).

NOTE.—In the following demonstration, by "the angle B G L" must be understood "the sum of the angles B G C, C G K, &c. ;" and by "the angle E H N," "the sum of the angles E H F, F H M, &c."

Take any number of arcs, C K, K L, each equal to B C ; and any number F M, M N, each equal to E F ; and draw G K, G L ; H M, H N ; then the angles at G • 27 III. are all equal,* .·. whatever multiple B L is of B C, the same multiple is the angle B G L of B G C. For a similar reason, whatever multiple E N is of E F, the same multiple is the angle E H N of E H F ; also if



B L = E N, then B G L = E H N ; if B L ⁊ E N, then B G L ⁊ E H N ; and if B L ∠ E N, then B G L ∠ E H N. Consequently (Def. V., p. 573) B C : E F : : B G C : E H F ; and since the angles A, D, are the halves of B G C, E H F, each of each, .·. B C : E F : : A : D ; .·. *the angles are as the arcs on which they stand.*

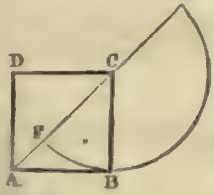
Again, the sectors B G C, C G K, K G L are equal; for it is manifest that, if applied to one another, they would coincide; in like manner, the sectors E H F, F H M, M H N, are also equal; consequently, if in the preceding demonstration "sectors" are substituted for "angles," the conclusion will be that B C : E F : : sector B G C : sector E H F. $\therefore$ *in equal circles, &c.* Q. E. D.

We shall conclude this chapter with two supplementary propositions of interest.

1. *The diagonal and side of a square are incommensurable.*

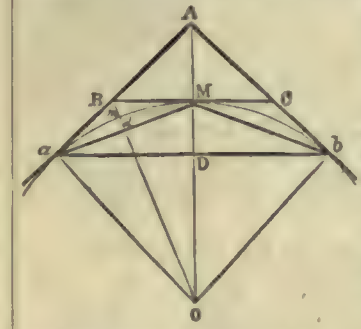
Let B D be a square: the diagonal A C is incommensurable with its side A B.

With C as centre, and C B as radius, describe the semicircle F B E: then A B C being a right angle, A B touches the circle (16 III.); $\therefore$ (36 III.) $AE \cdot AF = AB^2$ $\therefore$ (17 VI.) A E : A B : : A B : A F; $\therefore$ (23 V.) the lines A E, A B are incommensurable, $\therefore$ A C, A B, are also incommensurable; for if these had a common measure, that measure would likewise measure A C + AB, that is AE; so that B A E would have a common measure, which is shown to be impossible: $\therefore$ the diagonal and side of a square are incommensurable.



2. *The surface of a regular inscribed polygon, and that of a similar circumscribed polygon being given, to find the surfaces of regular inscribed and circumscribed polygons of double the number of sides.*

Let *ab* be a side of an *inscribed* polygon: the touching line *a*A, *b*C will each be half a side of the similar *circumscribed* polygon, as it is evident from what has been shown in reference to the inscribed and circumscribed pentagon in Propositions XI. and XII. of Book IV., and from what is said at the close of Prop. XVI. Let the surfaces of these two polygons be given. The chords *a*M, *b*M, drawn to the middle of the arc *a*M*b*, will be sides of an inscribed polygon of double the number of sides, and the touching line, B M C, will be a side of a similar circumscribing polygon, as is evident.



For brevity, let the surface of the inscribed polygon, whose side is *ab*, be represented by *p*, and that of the corresponding circumscribed polygon by P; also let the inscribed and circumscribed polygons of double the number of sides be denoted by p' and P'.

It is evident that the space O*a*D, is the same part of *p*, that O*a*A is of P,—that O*a*M is of p',—and that O*a*BM is of P'; for each of these spaces must be repeated exactly the same number of times to complete the polygon to which it belongs: consequently, since magnitudes are as their like multiples or sub-multiples, whatever proportions exist among these spaces, must also exist among the polygons of which they are sub-multiples. Now the right angled triangles OD*a*, OA*a* are similar, $\therefore$ O D : O*a* : : O*a* : O A; that is, O D : O M : : O M : O A; and since triangles of the same altitude are to one another as their bases, and that the altitude *a*D is the same for the triangles OD*a*, OM*a*, OA*a*, it follows, from the proportion just deduced, that O D*a* : O M*a* : : O M*a* : O A*a*; that is, the numerical measure of the surface of the triangle OM*a* is a mean proportional between the measures of OD*a*, OA*a*; consequently *the surface of the polygon p' is a mean proportional between the surfaces of p and P.*

Again, the right-angled triangles O D*a*, B M A, are also similar, $\therefore$ O D : O*a* : : B M : B A; that is, O D : O M : : *a*B : B A; consequently, since the altitude *a*D is the same for the triangles O D*a*, O M*a*, and the altitude O*a*, the same for the triangles O*a*B, O B A, and that triangles of the same altitude are to each other as their bases, it follows that O D*a* : O M*a* : : O*a*B : O B A; $\therefore$ (13 and 6 V.) OD*a* + O M*a* : 2O D*a* : : O*a*B + O B A : 2O*a*B; $\therefore p + p' : 2p :: P : P'$

The two conclusions now obtained are sufficient to enable us to compute the surfaces of inscribed and circumscribed regular polygons of 8, 16, 32, &c. sides, from having the surfaces of the inscribed and circumscribed squares already given. Thus, let the radius of the circle be numerically represented by 1, then if the given inscribed and circumscribed polygons (*p*, P) be *squares*, the side of the former will be $\sqrt{2}$, and that of the latter 2; and their surfaces will be 2 and 4 respectively; and, from what is proved above, the surface of the regular eight-sided *inscribed* polygon (p') will be a mean between the two squares *p*, P, $\therefore p' = \sqrt{8} = 2{\cdot}8284271$. Again, for the surface of the eight-sided *circumscribed* polygon (P') the proportion $p + p' : 2p :: P : P'$, gives,

$$= \frac{2p.\ P}{p + p'} = \frac{16}{2 + \sqrt{8}} = \frac{16}{4{\cdot}8284271} = 3{\cdot}3137085$$

And from these numerical expressions for the surfaces of inscribed and circumscribed polygons of eight sides, we may evidently, by repeating the operation, and substituting the values just obtained for *p* and P, deduce the numerical expressions for the surfaces of the sixteen-sided polygons, and so on to any extent: the results are as in the following table, which was in part given at page 569.

No. of sides.	Surf. of ins. pol.	Surf. of circ. pol.
4	2·0000000	4·0000000
8	2·8284271	3·3137085
16	3·0614674	3·1825979
32	3·1214451	3·1517249
64	3·1365485	3·1441184
128	3·1403311	3·1422236
256	3·1412772	3·1417504
512	3·1415138	3·1416321
1024	3·1415729	3·1416025
2048	3·1415877	3·1415951
4096	3·1415914	3·1415933
8192	3·1415923	3·1415928
16384	3·1415925	3·1415927
32768	3·1415926	3·1415926

From these numerical values for the surfaces of the inscribed and circumscribed polygons, it appears that when the number of the sides is so great as 32768, the two polygons differ so little from one another, that their numerical measures, as far as seven places of decimals, are absolutely the same. Now the circle, with which these polygons are connected, is manifestly *between the two*, as to amount of surface; being greater than the inscribed polygon, and less than the circumscribed one; consequently the surface of the circle must differ *less* from that of either polygon than the polygons differ from each other; and as the polygons themselves differ in numerical measure only, after the seventh decimal of the number 3·1415926, &c., it follows that this number, as far as the decimals extend, is the numerical expression for the surface of a circle whose radius is 1. By carrying on the foregoing process, the expression for the surface is found to be 3·141592653589793, &c.

CHAPTER VII.

PLANES:

BEING THE FIRST TWENTY-ONE PROPOSITIONS OF THE ELEVENTH BOOK OF EUCLID'S GEOMETRY.

INTRODUCTION.—The figures, lines, angles, &c., the properties of which form the subject of the First Six Books of Euclid's Geometry, are supposed to lie in one plane, or to be in space of two dimensions.* The following pages contain a few elementary propositions on the relations between lines, angles, &c., which do not lie in one plane, but are in solid space,—or space of *three dimensions.*† The student will find the following propositions very easy, *when once he has distinctly conceived the meaning of their enunciations.* The figures which are given to each proposition cannot represent the proposition to the eye so perfectly as in the former books, in consequence of their having to be drawn in perspective. It is hoped, however, that the shading introduced into the diagrams will aid the student in conceiving the proposition they belong to.

* i.e., of length and breadth.

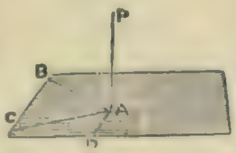
† i.e., of length, breadth, and thickness.

It is to be added, that, as in Plane Geometry, we are allowed to draw lines in any direction, and to produce them to any extent, so in solid Geometry we are allowed to draw planes in any direction, and to produce them to any extent. Moreover, two lines intersect in a point; in like manner it will be shown that two planes intersect in a line. Also, as we may suppose, a line to revolve round a point till it comes to a point on its plane, so we may suppose a plane to revolve round a given line until it comes to a given point situated anywhere in space.

DEFINITIONS.

I.

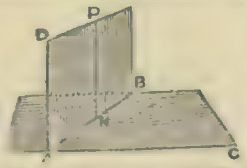
A straight line is perpendicular, or at right angles to a plane, when it makes right angles with every straight line in that plane which meets it.



Thus, if B D be a plane, P A a line perpendicular to it. Through A draw any lines A B, A C, A D . . . . in that plane, then will P A B, P A C, P A D, &c., be right angles.

II.

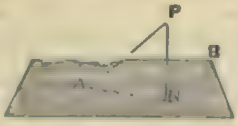
A plane is perpendicular to a plane, when a straight line drawn in one plane perpendicular to the intersection of the planes is at right angles to the other plane.



Thus, let A B D, A B C, be two planes, let the former be perpendicular to the latter, and let A B be the line of intersection of the planes in the plane A B D, draw P N at right angles to A B. Then is P N at right angles to the plane A B C.

III.

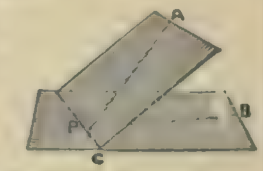
The inclination of a straight line to a plane, is the acute angle contained by that straight line, and another drawn from the point in which the first line meets the plane, to the point in which a perpendicular to the plane drawn from any point of the first line above the plane meets the same plane.



Thus, let A N B be a plane, A P a line meeting the plane in A; from P draw P N perpendicular to the plane, and meeting the plane in N. Join A N, then the angle P A N is the inclination of the line P A to the plane A N B.

IV.

The inclination of a plane to a plane, is the acute angle contained by two straight lines drawn from any one point of their common section at right angles to it, one upon one plane, the other upon the other.



Let P A C, P B C be two planes intersecting in the line P C. From P in the former plane, draw P A at right angles to P C; and from the same point P on the latter plane, draw P B at right angles to P C. Then if B P A be an acute angle, this is the inclination of the planes to each other.

V.

Two planes have the same inclination to one another which two other planes have, when the said angles of inclination are equal to one another.

VI.

Parallel planes are such as do not intersect, though produced ever so far in all directions.

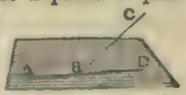
VII.

A solid angle is that which is made by the meeting of more than two plane angles, which are not in the same plane, in one point.

PROPOSITION I.—THEOREM.

One part (A B) *of a straight line* (A B C) *cannot be in a plane, and another part* (B C) *be above it.*

For let us suppose this possible, then since the straight line A B is in the plane, it can be produced in that plane: let it be produced to D. Now, suppose a plane to pass through the straight line A D, and let it be turned round that line, till it comes to the point C. Then because B and C are in the plane, the straight line B C is in it:* ∴ there are two straight lines A B C, A B D in the same plane, having a common segment A B, which is impossible.† Q. E. D.



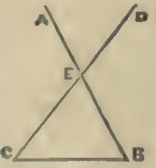
* 6 Def. I.

† Cor. 11 I.

PROPOSITION II.—THEOREM.

Two straight lines (A B, C D) *which cut one another (in the point* E) *are in one plane. And three straight lines* (B C, C E, E B) *which meet one another, are in one plane.*

Let any plane pass through E B, and let the plane be turned about E B produced if necessary, until it pass through the point C. Then because the points C and E are in this plane, the line C E is in it.* For the same reason the straight line B C is in the same plane, and by the hypothesis E B is in it; ∴ the three straight lines B C, C E, E B are in one plane. But A B is in the same plane as E B, and D C as E C.† Also A B and D C are in the same plane. Q. E. D.



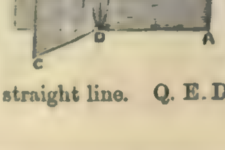
* 6 Def. I.

† 1 XI.

PROPOSITION III.—THEOREM.

If two planes (A B, B C) *cut one another, their common section* (D B) *is a straight line.*

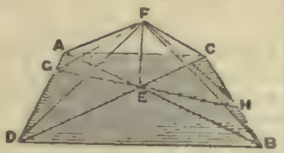
For, if not, since D, B are points in the plane A B, draw the straight line D F B in that plane, and similarly draw the line D E B in the plane B C. Then because these two straight lines have the same extremities, they enclose a space, which is absurd;* ∴ the common section B D cannot but be a straight line. Q. E. D.



* 3 I. Ax. I.

PROPOSITION IV.—THEOREM.

If a straight line (E F) *be at right angles to each of two straight lines* (A B, C D) *at their point of intersection* (E), *it shall also be at right angles to the plane* (A D B C), *which passes through them, i.e., to the plane in which they are.*

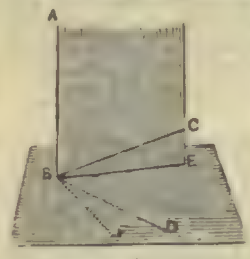


Take the straight lines E A, E B, E C, E D equal to each other. Join A D and B C. In the plane A D B C draw through E any line G E H, meeting A D and B C in G and H; we are to show that F E is perpendicular to G H. For join F A, F B, F C, F D, F G, F H. Now, because in the triangles A E D, B E C, the sides A E, E D = the sides C E, E B, and the angle A E D
• 15 I. = angle C E B,* ∴ A D = B C, and angle D A E
† 4 I. = angle E B C.† Again, in triangles A G E,
• 15 I. E B H we have the angle A E G = angle B E H;* and by what we have proved, angle G A E = E B H, and the side A E = side E B; ∴ G E = E H, and A G =
† 26 I. B H.† Again, since the sides A E, E F are equal to the sides D E, E F each to each, and the right
• Ax. 11 I. angle A E F = the right angle D E F;* ∴
† 4 I. A F = D F.† Similarly A F = F B, and E = F C. Hence in the triangles D A F, B C F we have the sides D A, A F = to the sides C B, B F, each to each, and the base D F = the base F C; ∴ the angle D A F =
• 8 I. the angle F B C.* Again, in the triangles G A F, H B F we have (by what we have already proved) the sides G A, A F = the sides H B, B F, each to each, and the angle G A F = angle H B F; ∴ base G F = base
† 4 I. F H.†

Hence (by what we have now proved), in the triangles G E F, H E F we have the sides G E, E F = the sides H E, E F, each to each, and the base F G = the base
• 8 I. F H, ∴ the angle G E F = the angle H E F,*
† 8 Def. I. which are therefore each right angles.† The same proof applies to any other line drawn through E, in the plane A B C. Hence E F is perpendicular to the
• 1 Def. XI. plane.* Q. E. D.

PROPOSITION V.—THEOREM.

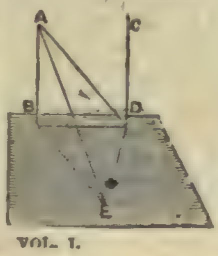
If three straight lines (B C, B D, B F) *meet all in one point* (B), *and a straight line* (B A) *stands at right angles to each of them at that point, these three straight lines are in one and the same plane.*



For if not, suppose the plane which passes through B F and B D not to pass through B C, and suppose the plane passing through A B and B C to cut the former plane in B E, then the straight lines B F, B D, B E are in the same plane, viz., the one passing through B D, B F, and A B is at right angles to B D, B E, and ∴ is also at
• 4 XI. right angles to B E.* Now, the angle A B C is a
† 11 Ax. I. right angle, ∴ angle A B C = angle A B F,† and they are both in the same plane, which is impos-
• 9 Ax. I. sible.* ∴ B C, B D, B F must be in the same plane. Q. E. D.

PROPOSITION VI.—THEOREM.

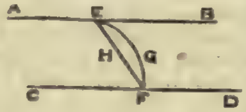
If two straight lines (A B, C D) *are at right angles to the same plane* (B D E), *they shall be parallel to one another.*



B, D are the points in which the lines meet the plane. Join B D, and at D draw D E in the plane perpendicular to B D. Make D E = A B. Join A D, B E, A E. Now, since A B is perpendicular to plane, A B D and A B E are right angles, then in triangles A B D, B D E the sides A B, B D = sides D E, D B, each to each, and angle A B D = angle B D E, each being a right angle, ∴ B E
• 4 I. = A D.* Then in triangles A D E, A B E, the sides A D, D E = the sides E B, B A, each to each, and the base A E common, ∴ angle A B E = angle A D E.†
† 8 I. But angle A B E is a right angle, ∴ A D E is a right angle. Now, because C D is perpendicular to the
• 3 Def. XI. plane, C D E is a right angle;* so that E D is at right angles to the three lines B D, A D, C D,
† 5 XI. which are therefore in the same plane;† but the plane which contains A D, D B contains A B,* ∴
• 2 XI. A B, B D, D C are in the same plane; now, angles A B D and B D C are right angles, and are ∴ together equal to two right angles, ∴ A B is parallel to
† 28 I. C D.† Q. E. D.

PROPOSITION VII.—THEOREM.

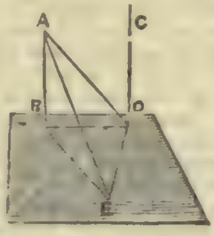
If two straight lines (A B, C D) *are parallel, the straight line drawn from any point* (E) *in the one, to any point* (F) *in the other, is in the same plane with the parallels.*



For if not, suppose E G F to be the straight line joining them, and suppose it do not fall in the plane. Since E and F are points in the plane, we can join them by a straight line, which lies wholly in the plane. Let this line be E H F. Then E H F and E G F are two straight lines, inclosing a space, which is impossible. Q. E. D.

PROPOSITION VIII.—THEOREM.

If two straight lines (A B, C D) *are parallel, and one of them* (A B) *is at right angles to a given plane* (B D E), *the other shall also be at right angles to the same plane.*



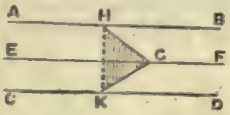
Let the lines meet the plane in B and D. Join B D. Then A B, B D, D C are in one
• 7 XI. plane.* Draw D E at right angles to B D, and in the
† 11 I. plane B D E.† Take A B = D E. Join A D, A E, B E. Now, A B being perpendicular to the plane, is perpendicular to B D
Def. XI. and B E.* Now, B D meets the parallel lines A B, C D, ∴ the angles A B D, B D C are together equal to two right an-
† 29 I. gles.† But A B D is a right angle, ∴ B D C is a right angle.

Again, in triangles A B D, B D E, the sides A B, B D = the sides E D, D B, and the right angle A B D = the
• 4 I. right angle B D E; ∴ A D = B E.* Hence in triangles A B E, A D E we have the sides A B, B E = the sides E D, D A, and the base A E is common, ∴
† 8 I. angle A B E = angle A D E.† But A B E is a right angle, ∴ A D E is a right angle; ∴ E D is at right angles to the lines B D and D A, and ∴ is at right angles
• 4 XI. to the plane passing through them,* and ∴ since C D is in the same plane with A D, B D, angle C D E is
† Def. XI. a right angle.† But we have already seen that B D C is a right angle, ∴ C D is at right angles to the
• 4 XI. plane passing through B D, D E,* i.e., is at right angles in the plane B D E. Q. E. D.

PROPOSITION IX.—THEOREM.

If two straight lines (A B, C D) *are each parallel to the same straight line* (E F), *but not both in the same plane with it, they are parallel to one another.*

For, take any point G in E F, and from G in the
• 11 I. plane AB, EF draw GH perpendicular to EF;*



and likewise from G in plane E F, C D, draw G K perpendicular to E F.

Then because G F is at right angles to G H and G K, it is at right angles to the plane
† 4 XI. H G K;† and since G F is at right angles to the plane H G K, and H B is parallel to G F, ∴ H B is
• 8 XI. at right angles to H G K.* Similarly K D is at right angles to H G K, ∴ H B and K D are parallel,†

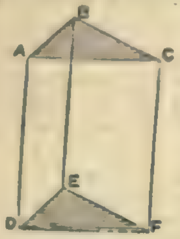
† 6 XI. or A B and C D are parallel to each other. Q. E. D.

PROPOSITION X.—THEOREM.

If two straight lines (A B, B C) *meeting one another be parallel to two others* (D E, E F) *also meeting one another, the latter two not being in the same plane with the former two, then the former two contain an angle* (A B C) *equal to the angle* (D E F) *contained by the latter two.*

We suppose that A B is parallel to D E, and B C to E F. Take B A = E D, and B C = E F, and join B E, A D, C F, A C, D F.

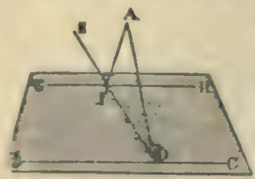
Then because A B is equal and parallel to D E, and their extremities are joined towards the same points B E and A D; ∴ B E is equal and
* 33 I. parallel to A D.* Similarly B E is equal and parallel to C F, and
† Ax. 1 I. ∴ A D is equal† and paral-
* 9 XI. lel* to C F. But the extremities of A D, C F are joined towards the same points by A C, and
† 33 I. D F, and therefore A C = D F.† Hence in triangles A B C, D E F we have the sides A B, B C = the side D E, E F, each to each, and the base A C = the base D F, ∴ the angle
* 8 I. A B C = the angle D E F.* Q. E. D.



PROPOSITION XI.—PROBLEM.

From a given point (A) *above a plane* (B C H) *to draw a line perpendicular to that plane.*

In the plane draw any line B C. From A draw A D
* 12 I. perpendicular to B C.* In the given plane from
† 11 I. D draw E D perpendicular to B C.† From A
* 12 I. draw A F perpendicular to E D,* then A F is the line required.

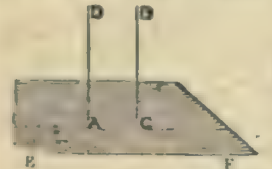


From through F draw G H,
† 31 I. parallel to BC.† Now, since B C is perpendicular to E D and A D, it is perpendicular to the plane passing
* 4 XI. through them,* ∴ G H is also perpendicular to the plane passing through E D
† 8 XI. and A D,† and ∴ is perpendicular to F A ;* ∴
* Def. XI. the angle A F G is a right angle. But, by construction A F D is a right angle, then since A F is perpendicular to the lines F G, F D it is also perpendicular
† 4 XI. to the plane passing through them,† *i. e.*, is perpendicular to the plane B C H. Which was to be done.

PROPOSITION XII.—PROBLEM.

From a given point (A) *in a given plane* (E F) *to draw a line perpendicular to that plane.*

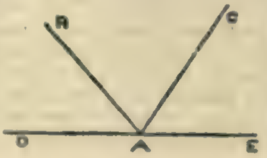
Take any point B above the plane, and from B draw B C perpendicular to the
* 11 XI. plane.* From A draw A D parallel to
† 33 I. B C ;† then because A D and B C are parallel, and B C is perpendicular to the plane, ∴ A D is also perpendicular to the
* 8 XI. plane.* Which was to be done.



PROPOSITION XIII.—THEOREM.

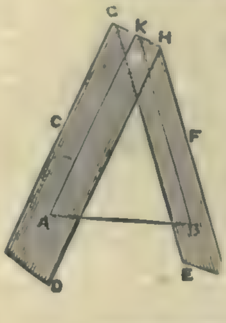
From the same point (A) *of a given plane, there cannot be two straight lines* (A B, A C) *at right angles to the plane, upon the same side of it ; and there can be but one perpendicular drawn to a plane from a given point above it.*

For, if possible, suppose A B, A C to be at right angles to a given plane. Suppose the plane which contains A B and A C to intersect the given plane in D E. Then C A, B A, D A, are in the same plane. Now C A is perpendicular to every line in the
* Def. XI. plane,* and ∴ to A D, ∴ C A D is a right angle. For the same reason B A D is a right angle, ∴
† Ax. 11 I. C A D = B A D,† which is absurd. Also from the same point above a given plane, two perpendiculars cannot be drawn to it, for if they could they would be parallel to one another,*
* 6 XI. which is absurd. Q. E. D.



PROPOSITION XIV.—THEOREM.

Planes (D C, E F) *to which the same straight line* (A B) *is perpendicular are parallel to each other.*

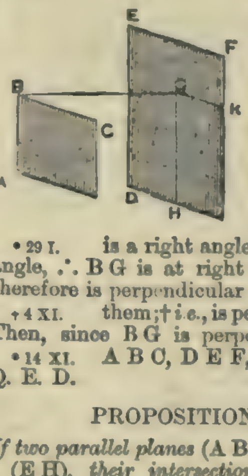


For if not, the planes must intersect ; let them intersect in the straight line H G, in which take any point K. Join K A, K B. Then K A is in the plane D C, ∴ K A B is a right angle.*
* Def. XI. Similarly A B K is a right angle, ∴ the two angles K A B, A B K of the triangle A B K, are equal to two right angles, which is absurd,† ∴
† 17 I. the two planes cannot intersect, and therefore are pa-
* Def. XI. rallel.* Q. E. D.

PROPOSITION XV.—THEOREM.

If two straight lines (A B, B C) *meeting one another be parallel to two other straight lines* (D E, E F) *which meet one another but are not in the same plane with the former two, then the plane passing through the former two* (A B, B C) *is parallel to the plane passing through the latter two* (D E, E F).

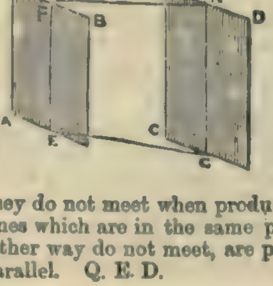
For, from B draw B G perpendicular to the plane
* 11 XI. A B C,* meeting the plane D E F in G. In this plane and through G, draw G H, G K, parallel to
† 31 I. D E, E F.† Then, since E D is parallel both to A B and G H, ∴ A B is
* 9 XI. parallel to G H.* Similarly G K is parallel to B C. Again, because B G is perpendicular to the plane A B C, it is perpendicular to
† Def. XI. A B,† ∴ A B G is a right angle ; but because A B is parallel to G H, A B G and B G H are together equal to two right angles,* ∴ B G H
* 29 I. is a right angle, Similarly B G K is a right angle, ∴ B G is at right angles to G H and G K, and therefore is perpendicular to the plane passing through
† 4 XI. them;† *i.e.*, is perpendicular to the plane D E F. Then, since B G is perpendicular to both the planes
* 14 XI. A B C, D E F, these planes are parallel.* Q. E. D.



PROPOSITION XVI.—THEOREM.

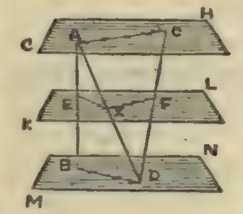
If two parallel planes (A B, C D) *are cut by a third plane* (E H), *their intersections with it* (E F, G H) *are parallels.*

For if not, they will meet when produced either on the side F H or E G ; let them be produced on the side F H, and meet in K. Then, since G H is in the plane C D, G H when produced is in
* 1 XI. that plane,* ∴ G H K is in the plane C D. Similarly E F K is in the plane A B, ∴ the plane A B meets the plane C D, for they have a common point K, and therefore is not parallel to it, which is contrary to the hypothesis, ∴ E F and G H do not meet when produced on the side F H. Similarly they do not meet when produced on the side E G. But lines which are in the same planes, and being produced either way do not meet, are parallel, ∴ E F and G H are parallel. Q. E. D.



PROPOSITION XVII.—Theorem.

If two straight lines (A B, C D) *are cut by parallel planes* (G H, K L, M N, *in points* A C, E F, B D), *they are cut in the same ratio,* (*i. e.,* A E : E B :: C F : F D).



For, join A D meeting K L in X, and join A C, E X, X F, and B D. Then, because the plane A B D cuts the parallel planes K L, M N in the lines E X and B D, these are parallels.* • 16 XI. Similarly A C is parallel to X F; now because E X is parallel to B D, and A C to X F, we have,

† 2 VI. A E : E B :: A X : X D†

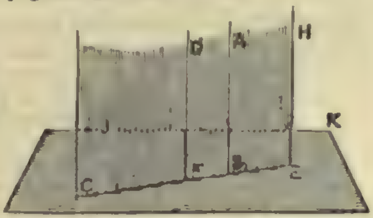
and A X : X D :: C F : F D

∴ A E : F B :: C F : F D. Q. E. D.

PROPOSITION XVIII.—Theorem.

If a straight line (A B) *be at right angles to a plane* (C K), *every plane that passes through the line is at right angles to the plane.*

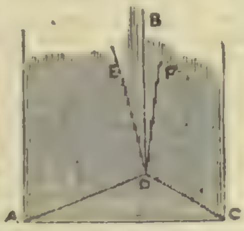
Let C H be any plane passing through A B, and let it intersect C K in the line C E. From any point F in C E, and in the plane C H, draw F G at right angles • 11 I. to C B.* Then, because A B is perpendicular to the † Def. XI. plane it is perpendicular to C E,† ∴ A B F is a right angle. But G B F is also a right angle, ∴ G F B and F B A are together equal to two right angles; • 28 I. ∴ G F is parallel to B A,* and A B is perpendicular to the plane C K, ∴ G F is perpendicular to † 8 XI. the plane C K;† similarly any other line in C H drawn perpendicular to C E is perpendicular to the plane C K, ∴ plane C H is perpendicular to plane C K. In like manner it can be proved that any other plane passing through A B is perpendicular to C K. Q. E. D.



PROPOSITION XIX.—Theorem.

If two planes (A B, B C) *which cut one another* (*in the line* B D) *be each perpendicular to a third plane* (A D C), *the common section* (B D) *is also perpendicular to the same plane.*

Let plane A B intersect plane A D C in A D, and let plane B C intersect plane A D C in D C. Then, if B D is not perpendicular to A D C, from point D in plane A B, draw D E perpendicular to A D,* • 11 I. and similarly in B C draw D F perpendicular to D C. Then, because plane A B is perpendicular to A D C, we have E D perpendicular to † Def. XI. A D C,† similarly F D is perpendicular to A D C; ∴ from the point A D in the plane A D C, two lines D E, D F are drawn perpendicular to that plane, • 13 XI. which is absurd :* ∴ B D is perpendicular to the third plane A D C. Q. E. D.



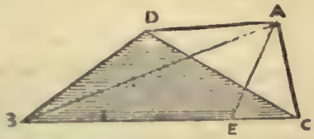
PROPOSITION XX.—Theorem.

If a solid angle (*at* A) *be contained by three plane angles* (B A C, C A D, D A B), *any two of them are greater than the third.*

If the three plane angles are equal, any one of them is clearly less than the other two.

If the three angles are not equal, let B A C be that which is not less than either of the other two.

At A in B A, and in plane B A C, make the angle B A E = angle • 23 I. B A D,* make A E = A D, through E draw B E C, meeting A B and A C in B and C. Join D B, D C.

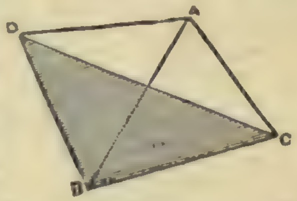


Then in triangles B A D, B A E, the sides B A, A D = the sides B A, A E each to each, and the angle B A D = † 4 I. the angle B A E, ∴ the base B D = base B E.† Now, the sides B D, D C are together greater than B C;* • 20 I. *i. e.,* greater than B E, C E; taking away the equals B D, B E, we have left D C greater than E C.† † 5 Ax I. Then in the triangles E A C, D A C, we have the sides D A, A C = the sides E A, A C, each to each. But the base D C is greater than the base E C, ∴ the • 25 I. angle D A C is greater than the angle E A C.* To each of these add the equals D A B, B A E. Then the two B A D, D A C are together greater than B A C.* † 4 Ax. I. Q. E. D.

PROPOSITION XXI.—Theorem.

Every solid angle (A) *is contained by plane angles, which, together, are less than four right angles.*

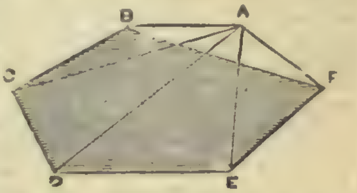
First, let the angle (A) be contained by three plane angles B A C, C A D, D A B. Join B C, C D, D B. Now, since the angles of any triangle are together equal to two right • 32 I. angles,* we have the angles of any three triangles together equal to six right angles, ∴ B A C + C A D + D A B + A B C + B C A + A C D + C D A + A D B + D B A = six right angles.



Now, because D is a solid angle, C D A + A D B are † 20 XI. greater than C D B.* Similarly D B A + A B C are greater than D B C, and B C A + A C D are greater than B C D.

∴ B A C + C A D + D A B + C D B + D B C + B C D are less than six right angles. Now, B C D is a triangle, and ∴ C D B + D B C + B C D = 2 right angles, ∴ B A C + C A D + D A B are together less than four right angles. But these contain the solid angle A, ∴ when three angles contain a solid angle, they are together less than four right angles. Q. E. D.

Next, suppose the angle A to be contained by any number of plane angles. Then all these plane angles are together less than four right angles; for supposing the lines containing the angle to be cut by a plane, so that we obtain a polygon B C D E F instead of the triangle in the first case. As before, we can easily prove that the angles containing the solid angle, together with the angles of the polygon, are less than twice as many right angles as there are sides of the polygon; but twice as many right angles as there are sides of the polygon are equal to the angles† • Cor. 33 I. of the polygon, together with four right angles; ∴ the plane angles forming solid angles at A, together with angles of polygon, are less than angles of polygon, together with four right angles, ∴ angles forming solid angles are together less than four right angles. Q. E. D.



The study of the last two propositions will be of value in connection with the section on Crystallography.—Ed.

CHAPTER VIII.

SPHERICAL GEOMETRY.

Introduction.—This chapter on Spherical Geometry, is intended to be strictly introductory to the cognate science of Spherical Trigonometry. To explain their relation, the following, for our present purposes, will suffice. The Science of Geometry, as given in the first six books of Euclid's Geometry, contains, along with others, a variety of propositions concerning the relations between the sides and angles of plane triangles. And these propositions may be *directly* applied to solve a variety of problems by *construction*—*e. g.*, if we are asked to construct an equilateral and equiangular pentagon on a given straight line, we can do this, with rule and compasses, by skilfully availing ourselves of certain properties of lines and angles which Euclid has proved. But if the question were asked—given that one side of a triangle is so many feet long, and that the angles adjacent to that side are respectively certain parts of a right angle, how many feet long are the remaining sides? The question is one, not of *construction*, but of *calculation*, and we cannot solve it directly, but only by the intervention of a science which shall give algebraical expressions for the relations between the sides and angles of triangles. Such a science has been invented, and is called *plane trigonometry*. It clearly *presupposes* a knowledge of the relations which Euclid has established, and assumes them as its basis. Now, suppose the triangles to be described, not on a plane, but on the surface of a sphere, spherical trigonometry is the science which gives us the means of *calculating* from given data the sides and angles of *such* triangles. This science, therefore, stands to the *spherical* triangle in the same relation that plane trigonometry stands to the *plane* triangle. And as the latter science rests on that part of the science of Geometry which treats of plane triangles as its basis, so the former science must rest on another portion of the science of Geometry, which shall treat of triangles described on the surface of a sphere as its basis.

The need of such a science as Spherical Trigonometry will be apparent to any one who reflects on the circumstance that the surface of the globe is (very nearly) spherical; consequently, all the triangles calculated in the course of a survey on a large scale are, when reduced to the surface of the earth, spherical triangles. Hence, surveying on a large scale (Geodesy) cannot be carried on without the investigations of spherical trigonometry. Again, in practical astronomy, the positions of all the heavenly bodies are referred to the surface of the great sphere—that, namely, which has the centre of the earth, supposed to be fixed, for its centre; and thus the triangles recognised in practical astronomy are spherical triangles, and the requisite calculations cannot be carried on except by means of the science of spherical trigonometry.

Having thus explained that this so needful science demands as its basis the investigation of certain properties of the spherical triangle, we will proceed to investigate those properties. As already stated, we shall confine ourselves strictly to such propositions as we shall hereafter need in treating of spherical trigonometry.

We have already, in general terms, said that a spherical triangle is one described on the surface of a sphere; we must, however, define this and other points more accurately, which we shall do as we proceed.

It is to be observed, that we suppose that we can draw any plane through any three given points; or, which is the same thing, through any straight line, and through a point not on that same straight line; also that we can cut any given solid, by a plane, in any direction whatever.

N.B.—From Euclid VI., 33, it appears, that in a given circle, any angle at the centre is proportional to the arc on which it stands. The arc is therefore said to measure the angle.

DEFINITIONS.

I.

A *solid* is a space which has three dimensions—namely, length, breadth, and thickness.

II.

A *sphere* is a solid bounded by a surface, of which every point is equally distant from a point within it called the *centre*.

III.

The *radius* of a sphere is a straight line drawn from the centre, to any point in the surface of the sphere.

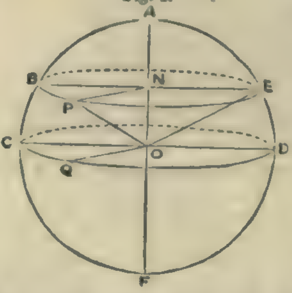
IV.

A straight line drawn through the centre, and terminated both ways by the surface of the sphere, is called the *diameter*.

PROPOSITION I.

Every section of a sphere made by a plane is a circle.

Fig. 1.



Let A B C D be the sphere; draw O A any radius whatever; let B P E N be a plane cutting the sphere's surface in the line B P E. It is supposed that the plane of the paper passes through the centre of the sphere perpendicularly to this cutting plane, which also cuts the radius O A in the point N; and suppose O A to be perpendicular to the plane; take P, any point in the line B P E, join P N, P O, O E. Then because P N is in the plane B P E, and O N is perpendicular to the plane, ∴ P N O is a right angle. For the same reason O N E is a right angle, ∴ in the triangles P N O, O N E, we have $PN^2 + NO^2 = PO^2$ and $EN^2 + NO^2 = OE^2$. Now, O P = O E, because each are radii of the sphere, ∴ $PN^2 + NO^2 = EN^2 + NO^2$, ∴ P N = E N. Similarly of any other point in the line B P E, ∴ B P E is a circle, the centre of which is N.

If the plane pass through the centre of the sphere, as plane C Q D, take Q any point in the line in which the plane cuts the surface of the sphere. Join O Q, O D. Then O Q, O D are radii of the sphere, ∴ O Q = O D. Similarly of any other point in the line C Q D, ∴ the section C Q D is a circle. Hence every section of a sphere is a circle. Q. E. D.

N.B.—The figure, as thus drawn, will, it is hoped, be quite comprehensible; it may be observed, however, that the circle A C D is the section of the sphere made by the plane of the paper. The other sections of the sphere are made by planes which intersect the plane of the paper in straight lines—*e.g.*, in B E, and in C D, and these sections are seen in perspective, as C Q D, B P E.

Def. V.—The section of a sphere made by a plane which does not pass through the centre is called a *small* circle. Thus, in Fig. 1, B P E is a small circle.

Def. VI.—The section of a sphere made by a plane which passes through the centre is called a *great* circle. Thus, in Fig. 1, C Q D is a great circle.

PROPOSITION II.—Theorem.

A great circle may be drawn through any two points on the surface of a sphere, but in general not through more than two points.

For, taking any two points on the surface of the sphere, we can draw a plane through them, and this plane can be made to pass through any third point, *viz.*, through the centre of a sphere. The section of the sphere made by this plane is a great circle, and the two points clearly lie on it.

These three points determine the plane, and ∴ we cannot be sure of its passing through any other point, whether on the surface of the sphere or not. Q. E. D.

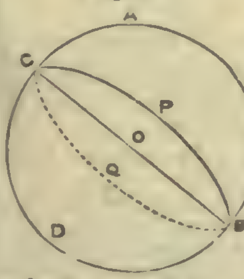
Cor.—It is plain that a small circle may be made to pass through any three points. For a plane being drawn through two points can be made to pass through a third; and if these three points are on the surface of a sphere, the plane cuts the sphere in a circle on which these three points lie.

N.B.—We shall henceforth assume that we can draw arcs of great circles in any possible direction; for instance, through any two points. For this is merely equivalent to drawing a plane through the centre of the sphere, and those two points, which of course cuts the sphere in the required great circle.

PROPOSITION III.

Two great circles bisect one another.

Fig. 2.

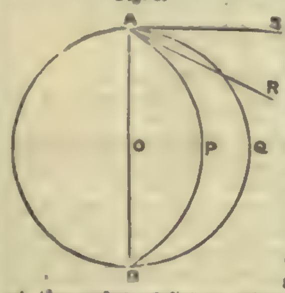


For suppose A B C D, the section made by the plane of the paper, to be one circle, and B P C, a section made by any other plane B P C Q, to be the other, then these planes intersect in the straight line B C, which passing through O, the centre of the sphere B C is a diameter of each of the circles, and ∴ bisects each of the circles, ∴ B P C = C Q B, and B A C = C D B, or the two great circles bisect each other. Q. E. D.

PROPOSITION IV.

The inclination of two great circles is the angle between the tangents drawn to those circles at their point of intersection.

Fig. 3.



Let one great circle be that made by the plane of the paper A Q B. Let A P B, the other circle, be that made by any other plane, then the diameter A O B is the line in which the planes intersect; in plane of paper draw A S perpendicular to A B; in the plane A P B draw A R perpendicular to A B. Then (*Planes, ante*, p. 590, Def. 4) R A S is the angle between the planes. But A R touches the circle A P B at point A, and A S touches the circle A Q B at point A (Euclid III., 16), ∴ the inclination of two great circles is the angle between their tangents at the point of intersection. Q. E. D.

N.B.—The angle R A S, between the tangents R A, S A, is generally supposed to be measured on the sphere, and is called the angle P A Q.

Def. VII.—If from the centre of a sphere a line be drawn perpendicular to the plane of any circle, whether great or small, and be produced both ways to meet the surface of the sphere, the points in which that line meets the surface of the sphere, are called the *poles* of the circle.

Thus, in Fig. 1, let C Q D be the plane of a great circle, O being the centre of the sphere. Through O draw O A perpendicular to the plane C Q D, and produce it to meet the surface of the sphere in F and A. Then F and A are the *poles* of the great circle C Q D.

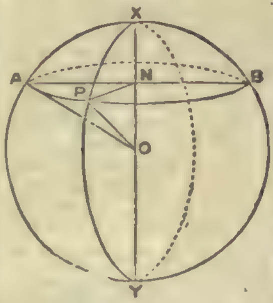
Again, if B P E be the plane of a small circle B P E, from O draw O N perpendicular to that plane, produce O N both ways to meet the surface of the sphere in A and F. These are the poles of the small circle B P E. It is usual to call A (the pole nearest to the small circle), *the* pole of the circle.

From the demonstration of Proposition I., it is plain that N is the centre of the circle B P E.

PROPOSITION V.

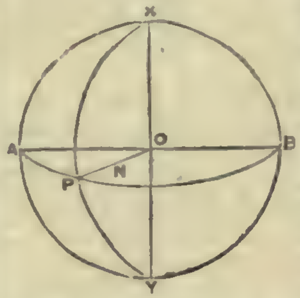
If from the pole of a circle great circles be drawn to any two points of that circle, the intercepted arcs are equal.

Fig. 4.



(1). In the case of a small circle, let A P B be the plane of the small circle, and X O Y perpendicular to that plane, and meeting it in N, then X Y are the poles of the circle, and N is its centre. Let X A Y, the section made by the plane of the paper, be one great circle, X P Y the section made by the plane passing through any other point P be the other great circle. We have to prove that the arc X P = arc X A. For, since N is the centre of the circle A P B, we have A N = P N; also, since O is the centre of the sphere O A = O P, ∴ in the triangle A O N, P O N, we have the sides A O, O N, = the sides P O, O N, each to each, and the base A N = the base P N, ∴ the angle A O N = angle P O N. But in equal circles, equal angles stand on equal circumferences, ∴ the arc A X = arc P X.

Fig. 5.



(2). In the case of a great circle, let A P B be the great circle, O its centre, and X O Y perpendicular to the plane A P B. Then, X Y are the poles of the circle. Let the section made by the plane of the paper X A Y be one of the great circles, and let X P Y be the other; we have to show that arc A X = arc P X—join O A, O P. Then, because X O is perpendicular to the plane, X O A and X O P are right angles, and ∴ are equal to each other. Hence as before, arc A X = arc P X. Q. E. D.

Cor. 1.—Hence the pole of a circle is equally distant from every point of that circle. *The distance being measured along a great circle.*

Cor. 2.—In the case of the pole of the great circle, it is plain, since A O X is a right angle, that A X is the fourth part (or *quadrant*) of the great circle A X B Y.

Cor. 3.—Also any plane passing through the poles of a great circle is clearly perpendicular to the plane of that circle; since the line joining the poles X O Y (Fig. 5) is perpendicular to the plane A P B. (See *Planes*, p. 590, Def. 2).

Cor. 3 (Fig. 5).—The inclination of the two great circles X A Y, X P Y is clearly measured by the arc A P. For P O and A O are each perpendicular to X Y, the line of intersection of the great circle, ∴ P O A is the inclination of the great circles, and P O A is measured by the arc A P.

Cor. 4.—We have already seen that the angle A X P, *i. e.*, the angle between the tangents to the circles at the point X, is the inclination between the planes, ∴ A P measures the angle A X P. (See Prop. IV.)

Def. VIII.—A spherical triangle is the portion of a surface of a sphere contained by the arcs of three great circles.

Thus, let A X B, C Z D, E Y F be three great circles which intersect in the points Z Y X, then the space

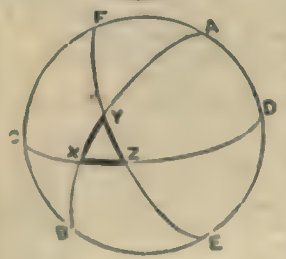


Fig. 6

Z X Y inclosed by the arcs Z Y, Y X, X Z is called a spherical triangle.

It will be observed, that the great circle in a spherical triangle is analogous to the straight line in the case of a plane triangle; but there is this difference to be observed, that two straight lines, when produced, never meet, whereas two great circles, when produced, always meet, *viz.*, in a point distanced from the other by a whole semicircle. To consider the result of this circumstance, we will suppose the section made by the plane of the paper (A B *ab*) Fig. 7, to be one great circle, and the two others to be A C*ac* and B C*bc*, O the centre of the sphere, A O*a*, B O*b*, C O*c*, are diameters. It will be seen that A B C is a spherical triangle, as in the case of figure 6. But the three circles, in addition to A B C, make seven other spherical triangles, *viz.*, *ab*C, B C*a*, *b*C A, on the one hemisphere, ABC*a*, and AB*c*, *abc*, *bc*A, B*ca*, on the hemisphere A*cba*.

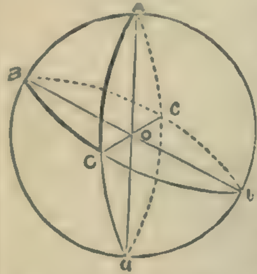


Fig. 7.

Also, it is plain that the triangles on the one hemisphere are equal to those on the other, each to each; thus, the triangle A B C is equal to the triangle *abc*. For A C*a*, and C*ac* are each halves of the same circle, and ∴ are equal to each other; take away the common part C*a*, and we have left A C = *ac*. Similarly A B = *ab*, and B C = *bc*. Again, the angle of the triangle *acb*, which is the angle between the planes, is equal to the angle A C B, which is also the angle between the planes; similarly the other angles are equal, and they are described on the surface of the same sphere; if, therefore, the triangles were superimposed, they would coincide, and are ∴ equal.

N.B.—The student will do well to consider very carefully the above observations: he must also take notice of the assumption, that triangles taken off the surface of the same sphere will coincide, provided their sides and angles are equal. This is merely assuming that the *curvature* of the same sphere is the same at all parts; which is obviously true, as the following consideration will assure us:—Suppose we have two spheres of equal radii—suppose these centres to coincide—then, since every point in each sphere is equally distant from their common centre, their surfaces coincide, and will continue to coincide however we may move either of them, provided their centres continue to coincide.

It is plain that the side A B measures the angle AOB. Hence the side of a spherical triangle is spoken of as an angle, *viz.*, the plane angle it subtends at the centre of the sphere.

PROPOSITION VI.

Any two sides of a spherical triangle are together greater than the third, and the three sides of the triangle are together less than four right angles.

For (*Planes*, see Prop. 20), if a solid angle be contained by three plane angles, any two are greater than a third; but (Fig. 7) the solid angle at O is contained by A O B, B O C, C O A, ∴ any two of these are greater than a third; and hence any two of the three sides of A B C (which sides measure these angles respectively) must be greater than the third.

Again (*Planes*, Prop. 21), the three angles, A O B, B O C, C O A, are together less than four right angles; and ∴ the three sides, A B, B C, C A, which measures these angles must be less than four right angles. Q.E.D.

Def. IX.—A *lune* is the portion of the surface of a sphere inclosed by the arcs of two great circles.

Thus (Fig. 7), A B*a* is a lune.

Def. X.—The angle of a *lune* is the angle between the two great circles which bound it.

Thus (Fig. 7), B A*a* is the angle of the lune.

PROPOSITION VII.

On equal spheres, if the angles of two lunes are equal, the lunes themselves are equal.

Let A C B D, A E B F, be two lunes described on equal and coincident spheres, having the angle C A D = angle E A F, these lines shall be equal; for suppose the one sphere to revolve till the circle A E B coincides with A C B, then because angle C A D = angle E A F, we must have A F B coinciding with A D B. The lunes, therefore, coincide and are equal.

N.B.—In the above demonstration we have assumed that the lunes have the same extremities, A B: we are obviously entitled to do this, since by shifting the spheres these extremities can be brought to coincide. Also, the proposition is plainly true when the lunes are on the *same* sphere. Also, it is manifest that the *greater* lune has the *greater* angle, and *vice versâ*.

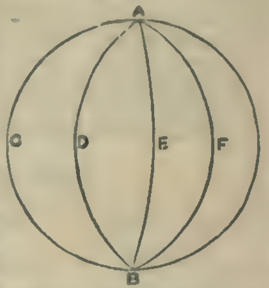


Fig. 8.

PROPOSITION VIII.

In the same or equal spheres, lunes are to each other in the ratio of their angles.

For let l and m be two lunes, the angles of which are a and b; let A be any multiple of a, and L the lune corresponding to A; then it is plain that L is the same multiple of l that A is of a. Similarly let B be any multiple of b, and let M be the corresponding lune; then it is plain that M is the same multiple of m that B is of b. We have then four magnitudes, l, m, a, b; and of the first and third we have taken any equimultiples, L and A; and of the second and fourth we have taken any equimultiples, M and B. Now, by last proposition, if $L > M$, A is $> B$; if equal, equal; if less, less ∴ (Def. V., p. 571), $l : m :: a : b$. Q. E. D.

Cor.—It is plain that the area of half a hemisphere is a lune whose angle is a right angle, ∴ if X be the area of a sphere, and if A be any lune whose angle is B,

$$A : \frac{X}{4} :: B : \text{one right angle.}$$

$$\therefore A : X :: B : \text{four right angles.}$$

Or area of lune : area of sphere : : anglé of lune : four right angles.

Def. XI.—The spherical excess of a spherical triangle is the excess of the sum of its three angles over two right angles.

It will be seen by the next proposition that the sum of the three angles of a spherical triangle are really greater than two right angles—the excess of the angles above two right angles is clearly due to the *sphericity* of the triangle; hence the term "spherical excess."

PROPOSITION IX.

To prove that the angles of a spherical triangle are together greater than two right angles, and that the area of a spherical triangle has to the area of half the sphere on which it is described the same ratio that the spherical excess has to two right angles.

For (Fig. 7) let A B C be the triangle, then the lunes corresponding to each of the angles are B*a*A corresponding to A, B*b*A corresponding to B, C*abc* corresponding to C. For the sake of brevity, call these lunes respectively L_1, L_2, L_3.

Then by Corol. to Prop. VIII.,

$$L_1 : \text{area of sphere} :: A : 4 \text{ right angles.}$$

$$\text{or, } L_1 : \text{area of hemisphere} :: A : 2 \text{ right angles.}$$

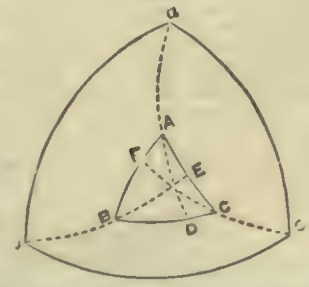
Similarly—

L_2 : area hemisphere : : B : 2 right angles.
L_3 : area hemisphere : : C : 2 right angles.

∴ $L_1 + L_2 + L_3$: area hemisphere : : $A + B + C$: two right angles. Now, the three lunes clearly make up the hemisphere B A C*ab*, together with the triangle *abc*. Hence the three lunes are in all cases greater than a hemisphere, and ∴ the three angles of the triangle are together greater than two right angles.

Again, triangle *abc* is equal to triangle A B C (Remarks on Def. VIII.), ∴ $L_1 + L_2 + L_3$ = area of hemisphere + area of triangle A B C. Also, $A + B + C$ = two right angles + spherical excess ; ∴ area hemisphere + triangle A B C : area hemisphere : : two right angles + spherical excess : two right angles, ∴ area triangle A B C : area hemisphere : : spherical excess : two right angles. Q. E. D.

Fig. 9.



DEF. XII.—The triangle formed by the great circles which join the poles at the sides of a given triangle, is called the polar or supplemental triangle.

Thus, let A B C be a given triangle ; then if *a*, *b*, *c*, be respectively the poles of the sides B C, C A and A B, *abc* is the polar triangle. And clearly, if *a*A be joined by the arc of a great circle, and this be produced to meet B C in D, then *a*D is a quadrant of a circle, and A D is perpendicular to B C, and similarly of the other poles.

The relation between the given triangle and its polar triangle is very important, as will be seen when we come to employ its properties in Spherical Trigonometry ; the properties on which its importance depends are proved in the following propositions.

PROPOSITION X.

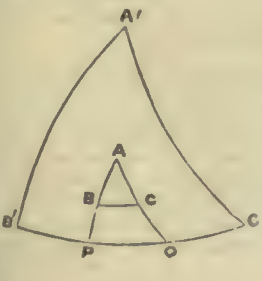
If two great circles intersect, their points of intersection will be the poles of the great circle which passes through their poles.

For (in Fig. 5), take the plane of the paper for the plane of one circle, and Y P X for any other, so that X and Y are the points of intersection of the two circles ; through O draw a plane A P B perpendicular to X Y, and ∴ perpendicular to both the planes X A B, and X P Y, and ∴ the plane A P B will contain the lines drawn perpendicular to those planes, and therefore will contain the poles of the two given circles. Hence A P B is the great circle joining the poles of the circles X A Y and X P Y ; but Y O X is perpendicular to the plane A P B, ∴ X and Y are the poles of the circle A P B ; *i. e.*, are the poles of the great circle which joins the poles of the two given circles.

PROPOSITION XI.

If A B C *be a given triangle, and* A′B′C′, *is its polar triangle, then is* A B C *the polar triangle of* A′B′C′.

Fig 10.



For since C′ is the pole of A B, and B′ is the pole of A C, ∴ (by last Prop.) the point of intersection A of A B and A C is the pole of the great circle joining B′C′, *i. e.*, A is the pole of B′C′, similarly B is the pole of C′A′, and C the pole of A′B′. Q. E. D.

PROPOSITION XII.

If A B C *be a triangle, and* A′B′C′ *its polar triangle, then the arc on the sphere, which measures the angle* A, *together with the side* B′C′, *equals the semi-circumference of a great circle.* (See Fig. 10).

For, produce A B, A C to meet B′C′ in P and Q. Then because A P and A Q are quadrants, P Q is the arc that measures the angle A (Cor. 4, Prop. V) ; now B′C′ + P Q = B′P + C′Q + Q P = B′Q + C′P. But since B′ is the pole of A C, ∴ B′Q is a quadrant. Similarly C′P is a quadrant, and the two together are a semicircle, ∴ B′C′, together with the arc on the great circle which measures A, equals the semi-circumference of a great circle.

COR. 1.—If for these arcs we substitute the angles they measure, we may state the proposition as follows :—

$$A + B'C' = 2 \text{ right angles.}$$

Similarly—

$$B + C'A' = 2 \text{ right angles,}$$
$$C + A'B' = 2 \text{ right angles.}$$

COR. 2.—And since ABC is the polar triangle of A′B′C′, we have—

$$A' + BC = 2 \text{ right angles.}$$
$$B' + CA = 2 \text{ right angles.}$$
$$C' + AB = 2 \text{ right angles.}$$

COR. 3.—Hence, the sum of the angles of any triangle, together with the sides of the polar triangle, = six right angles. But the sides of the polar triangle must have some magnitude, and must be less than four right angles (Prop. VII.) Hence the three angles of a triangle must be less than six, and greater than two right angles.

It is plain, since the three angles of a spherical triangle are greater than two right angles, and less than six right angles, that a spherical triangle may have one, two, or even three of its angles right angles.

DEF. XIII.—A right-angled spherical triangle is one which has *one* or *more* right angles.

DEF. XIV.—A quadrantal triangle is one which has at the least one side a right angle, *i. e.*, the quadrant of a great circle.

PROPOSITION XIII.

If A B C *be a right-angled triangle, having a right angle* C, *and* A′B′C′ *be its polar triangle, then* A′B′C′ *is a quadrantal triangle, having the side* A′B′ *a quadrant.*

For by the last proposition (Cor. 1)—

$$C + A'B' = \text{two right angles.}$$

Now, C is a right angle, ∴ A′B′ is a right angle, *i. e.*, is a quadrant. Q. E. D.

COR.—Hence, if all three angles, A′B′C, are right angles, the sides of the polar triangle are all right angles. For if two sides of a triangle are right angles, the third side measures the opposite angle ; ∴ if the third side be also a right angle, all the angles are right angles. Hence in the polar triangle the sides and angles are *all* right angles. And since the angles of the polar triangle are each right angles, the sides of A B C will be right angles (last Prop., Cor. 2). Hence, if all the angles of any triangle are right angles, the sides are right angles (*i. e.*, quadrants) also.

With this chapter we conclude the Elements of Geometry, both plane and spherical. In the succeeding one, the subject of Practical Geometry will be treated on, as well as Conic Sections. In the section on Astronomy, the student will require a considerable knowledge of the properties of the ellipse, parabola, &c. ; which are the most important of the conic sections, properly so called ; excluding, however, the circle, which has already been discussed in the Elements of Euclid. The subject of Trigonometry, plane and spherical, will follow Practical Geometry. An extended series of the logarithms of sines, tangents, &c., will be given at the end of this section.—ED.

CHAPTER IX.

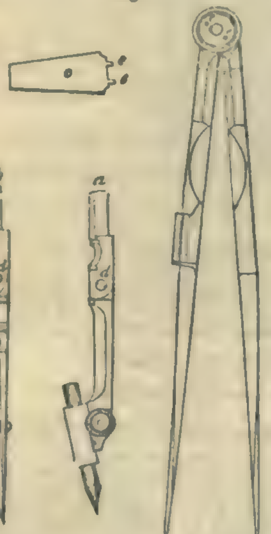
PRACTICAL GEOMETRY.

THE preceding portions of this section having given, at the commencement of the several Books of Euclid, the general definitions of *a point, a line*, &c., also the *Postulates* and *Axioms*, it is unnecessary to repeat them, it being sufficient for the student to refer to them when requisite, in order to give him a clear understanding or conception of the Problem he may at the time have under discussion. It is not here intended to give all the Problems contained in a complete treatise on Practical Geometry, but merely a selection of those which may be considered most useful in assisting the mechanical draughtsman, workman, or others who may be engaged in like pursuits.

INSTRUMENTS.—For the purpose of performing the construction of the different Geometrical figures, the only instruments absolutely required are a pair of compasses, a ruler, a lead pencil, and a drawing pen.

Compasses.—The best form of compasses, or dividers (Fig. 1), are made of metal, such as brass or silver, from five to six inches in length, having steel points, and formed with one of the points or legs movable, which at any time, as occasion may require, can be replaced by another containing a pencil leg (*a*), or a pen leg (*b*), the pen being constructed in the same manner as the drawing pen, afterwards described. The various uses to which the compasses may be applied are well known; the principal, however, being to measure or transfer distances, and when fitted with a pencil or pen, to describe circles, the one with black-lead pencil, the other with ink.* The small figure (*c*) is an instrument used for tightening the joint of the compasses when the legs work too easily, or the reverse; the two points (*e e*) at the one end fitting into two small holes (*e e*) at the head of the compasses—the other end being used for screwing up the nails (*d d*) in the pencil or pen leg, so as to make the joint work easily.

Fig. 1.



Ruler.—The ruler in general use is merely a bar of wood or metal, the edges being formed straight, and, for convenience, should be from six to twelve inches in length, about an inch in breadth, and is for the purpose of guiding the motion of the pencil or pen in a straight line or direction.

Pencil.—It may perhaps be considered unnecessary to give a description of a common drawing pencil; but as there are a few who, in commencing to draw mathematical figures, can form a proper point to their pencils, a few words may suffice to show the best method of doing it.

A pencil for drawing (Fig. 2) is generally prepared by sharpening the wood and lead, so as to form a fine point, similar to (No. 1) in the cut; but the point (No. 2), as shown in the figure, is the best form used for mechanical drawing, and is made by cutting two sides of the wood and lead flat, and leaving but a small flat edge on the other two sides, the one side showing a broad point, as at (*a*), the other a fine point, as at (*b*). In this way, the lead of the pencil may be kept close to the ruler, and at the same time draw a fine line. A very simple and beautiful little instrument, not much known, called the "pencil Cutter and Sharpener" (Fig. 3), has been invented, which forms the pencil point (No. 1), and is used by placing the pencil through the guides (*a*) into the hole or cone (*b*); and by turning it round with the hand against the knife edge (*c*), the point of the pencil is gradually formed.

Fig. 2.

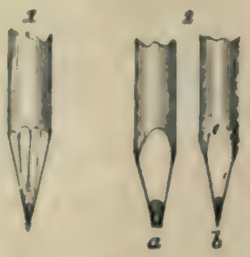
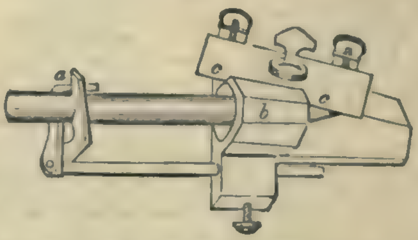
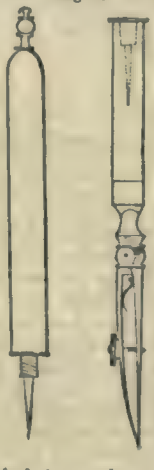


Fig. 3.



Drawing Pen.—The drawing pen (Fig. 4), like a pencil, is used for drawing straight lines, guided along the edge of the ruler; it is usually made in two parts—*viz.*, the pen and the handle. The pen part consists of two blades, with steel points, so bent that the ends or points meet, but leaving a space or cavity for the ink; and in order to draw lines of different thickness, those blades can be opened more or less by a small screw. The best pens are also made with a joint to one of the blades, to admit of the pen being more easily cleaned by separating them. The other, or upper part, forming the handle of the pen, screws into the lower portion, having attached to it a short piece of steel with a very fine point, generally called the protecting pin, and is used for setting off points in a line, or marking their intersections.

Fig. 4.

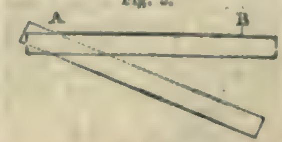


Having briefly described the few instruments requisite, it will be necessary to show their application, by performing the three following simple problems, generally given as Postulates (Euclid, Definitions, page 541), which, however, by constructing, the student will the more readily become familiar with the instruments and their use.

PROBLEM I.—*To draw a straight line from any one point* (A) *to any other point* (B).

Lay one edge of the ruler upon the point A, as shown in dotted lines in the figure, and move the ruler round until it coincides with the point B; draw the point of the pencil or pen from A to B, keeping it always close to the edge of the ruler: the trace or mark left upon the surface of the paper is the straight line required.

Fig. 5.



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* The ink commonly used in drawing mathematical figures, &c., is that known as China Ink, which, being rubbed on a plate or palette, with a little water, runs more freely from the pen, and dries more quickly than common black writing ink; it also has the advantage of not running or blotting so much on the paper.

PROBLEM II.—*To prolong or produce the straight line* (A B) *to any length towards* (C), *in a straight line.*

Place one edge of the ruler upon the point B at the extremity of the line A B; take any point in A B, such as *a*, so far from B that the distance may be less than the length of the ruler; make the ruler coincide with that point *a*, and draw as before with the pencil or pen from B towards C, which will leave the line produced as required. This operation may be repeated as often as wished, so as to prolong a line to any extent.

Fig. 6.

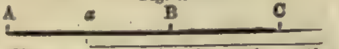
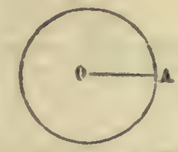


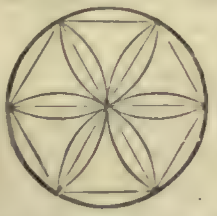
Fig. 7.



PROBLEM III.—*To describe a circle from a centre* (O), *and at any distance* (O A) *from the centre* (O).

Place the fixed leg or point of the compasses in the centre O, opening them until they take in the given distance O A or radius of the circle to be drawn; then move them round, describing the circumference of the required circle.

Fig. 8.



After performing the three preceding problems, it will be useful, in order to acquire a free use of the instruments in describing circles, to draw lines through points of intersection, join points by straight lines, and thus to produce the accompanying figure several times. The student may thereafter proceed to the construction of the following problems.

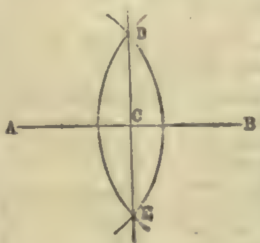
PROBLEM I.

To bisect or divide into two equal parts a given straight line (A B). (Euclid, Book I., Prop. X.)*

1ST METHOD.—*When there is not sufficient space in the drawing on either side of the given line.* By means of the compasses, place a leg at one of the extremities of the line, open them until the other leg is at any other point near the middle of the line; mark this point; transfer the leg of the compasses to the other extremity of the line, and mark the distance of the other leg from this point; if the two points so marked do not coincide, open the compasses to as near the middle of these two points as the eye will direct: proceed as before; the distance between the new points will be smaller than before, and a trial or two will enable us to bisect the line accurately.

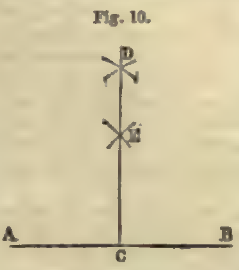
2ND METHOD.†—*When the given line* (A B) *occurs in or near the centre of the drawing, and there is sufficient room on both sides of the given line* (A B). With the centre A (Fig. 9), and any radius greater than the half of A B, describe an arc; with the centre B, and the same radius, describe another arc, cutting the former in D and E; join the points of intersection D and E, cutting the line A B in C, which will bisect A B in the point C.

Fig. 9.



3RD METHOD.—*When the given line* (A B) *occurs near the bottom of the drawing, and sufficient room is only left on one side of the given line* (A B). With centres A and B (Fig. 10), and any equal radii greater than the half of A B (this radius should be as long as can be conveniently got), describe arcs cutting each other in the point D. With the same centres, and less equal radii, also greater than the half of A B, describe arcs cutting each other in the point E; join D E, and produce the line until it cuts A B in C, which will be the point of bisection of A B.

Fig. 10.

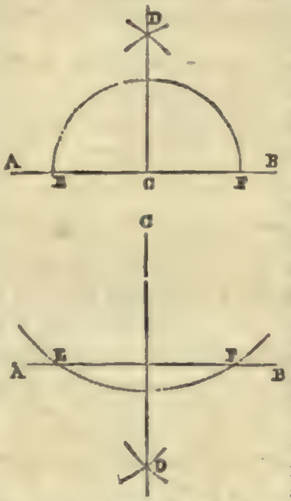


PROBLEM II.

To draw a perpendicular or a straight line at right angles to a given straight line (A B) *from a given point* (C). (Euclid, Book I., Prop. XI. and XII.)

1ST METHOD.—*When the given point* (C) *is within or without the given line* (A B) *and near the middle of it.* With the centre C (Fig. 11), and any radius, describe an arc cutting the line A B in the points E and F. With the centres E F, and the same, or any other equal radii, describe arcs cutting each other in the point D; join D C, then the line D C is perpendicular to the line A B as required.

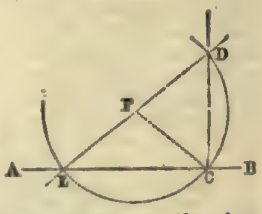
Fig. 11.



2ND METHOD.—*When the given point* (C) *is within and at or near the extremity of the given line* (A B). Take any point F (Fig. 12) above the line A B, and with the radius F C describe the arc E C D, cutting A B in the point E; join E F, and produce it until it cuts the arc E C D in the point D, and join D C, which will be perpendicular to A B, as required.

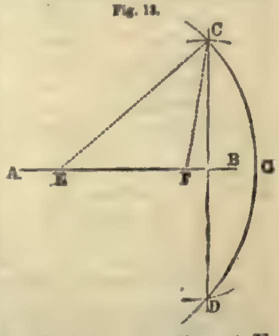
3RD METHOD.—*When the given point* (C) *is without and opposite the extremity of the given line* (A B). In the line A B (Fig 13) take any two convenient points E, F. With the centre E, and radius E C, describe the arc C G D, and with the centre F, and radius F C, describe an arc cutting the former arc C G D in the point C and D; join C D; the line C D will be perpendicular to the given line A B, as required.

Fig. 12.



A very simple method of setting off a perpendicular on the ground may be performed with a tape line in the following manner:—Suppose it were required to set out a straight line of road (C D) (Fig. 12) at right angles, or perpendicular to another straight line of road (A B) from any point (C) in A B. Put in a pin at the point C, and also one at E, along one side of the line of road A B, and three feet distant from C; put the end of the tape line at E,

Fig. 13.



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* References are made to the propositions in Euclid, where several of the Problems are demonstrated, in order that the student, should he wish to investigate the Problems further, may do so by examining the methods of construction there adopted.

† This is the method most frequently used to bisect a line; but in practice it is not necessary to draw the whole arcs, but merely the intersections, and by means of the ruler placed on those points, to mark the centre point C. The line D C or D E, in this and the following Problem, when drawn, will also be found to be perpendicular to A B.

pass it round the pin at C, carrying it up in the direction of the line of new road to D. Bring the line back to the pin at E, and holding the end of the tape line fast at E, stretch the line tight. When stretched, this will give a triangle, having sides three (E C), four (C D), and five (E D) feet respectively in length, of which D C will be perpendicular to E C, and form one side of the new road required.

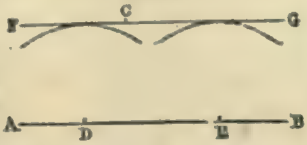
This method of setting off a perpendicular is taken from that beautiful theorem in Euclid (Book I., Prop. XLVII.), where in this case, E D, the hypothenuse of the triangle, is equal to 5 feet, and the two sides E C and C D, 3 and 4 feet respectively; therefore $5^2 = 3^2 + 4^2$, or $25 = 9 + 16$. The sum of the three sides being 12, or the whole length of line used; but any numbers may be used (consistent with the length of the tape line), provided they fulfil the required value. It may be easily remembered that any multiple of the numbers used, *viz.*, 3, 4, 5, will suffice.

PROBLEM III.

To draw a straight line parallel to a given straight line (A B), *and through any given point* (C). (Euclid, Book I., Prop. XXXI.)

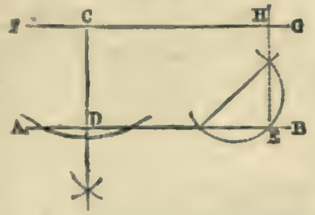
1st Method.—Take any two points, D and E (Fig. 14), in the given line A B, the further apart from each other the better; with the centres D and E and radii equal to the distance between the point C and the line A B* describe two arcs, and draw a straight line to touch or be a tangent to both arcs, then the line G F is parallel to A B.

Fig. 14.



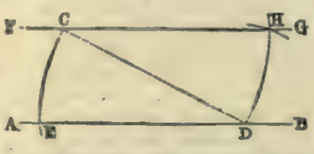
2nd Method.—From the point C (Fig. 15) draw the line C D perpendicular to A B; at another point anywhere in the line A B, but as far from D as convenient, draw E H perpendicular to A B. Make E H equal to C D, and draw through the points C and H the straight line F G, which will be parallel to A B.

Fig. 15.



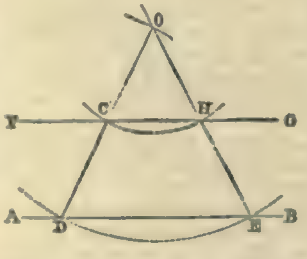
3rd Method.—Take any point D (Fig. 16), the further from C the better, in the line A B; join D C, and with the centre D, and radius D C (or any other convenient radius which will cut D C), describe an arc cutting A B in E; also with the centre C, and the same radius, describe an arc D H; again with the centre D, and a radius equal to the distance between C and E, describe an arc cutting the latter arc in H; draw the line F G passing through the points C and H; the line F G will be parallel to A B.

Fig. 16.



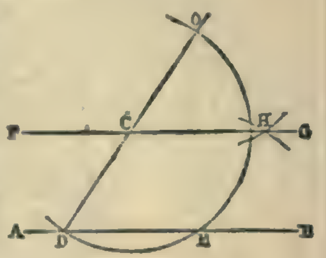
4th Method.—Take any point D (Fig. 17) in the line A B; join D C and produce it, making C O equal to C D; with the centre O and radii O C and O D, describe two arcs, the latter cutting A B in E; join O E, cutting the lesser arc in the point H; draw the line F G passing through the points C and H, then F G will be parallel to A B.

Fig. 17.



5th Method.—Take any point D (Fig. 18) in the line A B; join D C and produce it; with the centre C, and radius C D, describe a circle, cutting A B in E, and, D C produced, in O; with the centres E and O, and the same radius, describe arcs intersecting in the point H; draw F G passing through the points C and H; then F G will be parallel to A B.

Fig. 18.

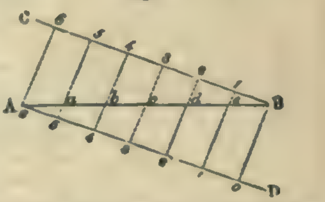


PROBLEM IV.

To divide a given straight line (A B) *into any given number* (*say* 6) *of equal parts.*

Through A (Fig. 19) draw a line A D at any angle with A B, and through B draw B C parallel to A D; from A set off six equal parts on the line A D (*those equal parts should be as nearly the length of the required equal parts of* A B *as can be estimated*), and from B on the line B C set off the same number (six) equal parts of the same length as before; join B and *o*, 1 and 1, 2 and 2, 3 and 3, 4 and 4, 5 and 5, A and 6; then the points where those lines cut A B, *viz.*, in *a*, *b*, *c*, *d*, *e*, will divide the line A B into the required number (six) of equal parts.

Fig. 19.

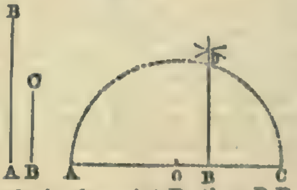


PROBLEM V.

To find a mean or mean proportional between two given lines (A B *and* B C).

Draw a straight line A B C (Fig. 20) equal in length to AB and BC together; bisect AC in O, and with the centre O and radius O A, or O C, describe a semicircle; from the point B = B C, the small line measured from C on A O C, draw a line perpendicular to A C, and meeting the semicircle in the point D, then B D is a mean or mean proportional between A B and B C.

Fig. 20.

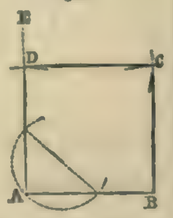


PROBLEM VI.

To describe a square on a given straight line (A B). (Euclid, Book I., Prop. XLVI.)

From the point A (Fig. 21) draw A E perpendicular to A B (Problem II.), and from A E cut off A D equal to A B; draw D C and C B parallel to A B and A D respectively, intersecting at the point C; or, with B and D as centres, and radii equal to A B, describe arcs intersecting at the point C, and join D C and B C, then A B C D is the square required.†

Fig. 21.



PROBLEM VII.

To describe a square which shall be equal to any number of given squares (1, 2, 3, 4, 5).

Draw any two lines, A B and A C (Fig. 22), at right angles to each other; in A B make A D equal to the side of the square 1, and in A C make A E equal to the side of the square 2, and join D E; then in A B make A F equal to D E, and in A C make A G equal to the side of the square 3, and join F G; again, in A B make A H equal to F G, and in A C, A K equal to the side of the square 4, and join H K; lastly, in A B make A L equal to H K, and in A C make A M equal to the side of the square 5, and join L M; then the square L M N O, described on the line L M, is equal to the sum of the squares 1, 2, 3, 4, 5.

---

* The distance between C and the line A B is easily got by placing one point of the compasses at C, and opening them until the other point just touches. F G may be drawn parallel to A B at any distance apart, by merely using that distance as the radii of the arcs, as in Fig. 15.

† Of course, any rectangle whose length and breadth are given, can be constructed in a similar manner, by making the perpendicular equal to the breadth, and either drawing parallels or using the length of the sides as radii.

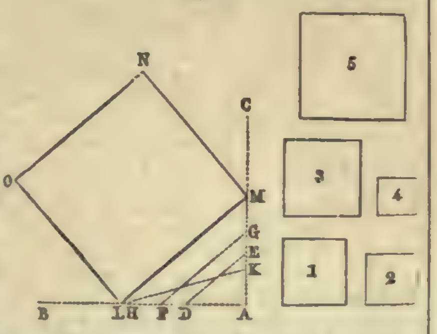


Fig. 22.

By proceeding in a similar manner till all the sides of any number of given squares are employed, a square may be constructed equal to the sum of the given number of squares; this, as well as the next problem, depending throughout their whole construction on Euclid, Book I., Prop. XLVII.

PROBLEM VIII.

To describe a square which shall be equal to the difference of any two given squares (F *and* G).

Draw any two lines, A B and A C (Fig. 23), at right angles to each other; in A C make A D equal to the side of the less of the two given squares, F; with the centre D, and a radius equal to a side of the other given square G, describe an arc cutting A B in E; the square A E H K, described on the line A E, is the square equal to the difference of the two squares F and G.

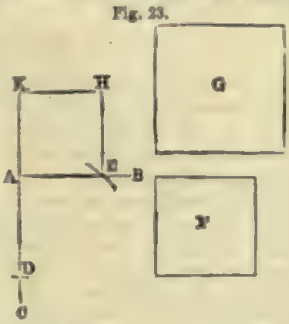


Fig. 23.

PROBLEM IX.

To describe a square which shall be equal to a given parallelogram (A B C D).

Draw any line G K, which shall be a mean proportional between the two sides, A B and B C of the given parallelogram (*Problem* V.), and upon the line G K describe the square H G K L, which will be equal to the given parallelogram, A B C D. (Fig. 24).

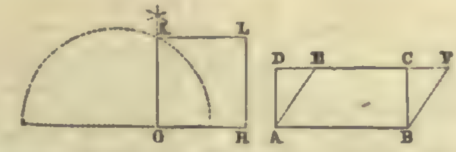


Fig. 24.

When the given parallelogram is not a rectangle, as A E F B, *then draw a rectangle* A B C D, *standing on the same base, and between the same parallels* (Euclid, Book I., Prop. XXXV.), *equal to it, and proceed as before.*

PROBLEM X.

To draw through a given point (E) *a straight line, which shall tend to the intersection of two given straight lines* (A B, C D), *but whose point of intersection* (O) *falls beyond the limits of the drawing* (L M N).

1st Method.—Through the given point E (Fig. 25), draw any line E C, meeting A B in A, and C D in C, draw any other line H K parallel to E C, meeting A B and C D in G and K; join C G, and through K draw K F parallel to it, meeting A B in F; join E G, and through F draw F G parallel to it, meeting K G produced in H; join E H, which, if produced, would pass through the point of intersection O.

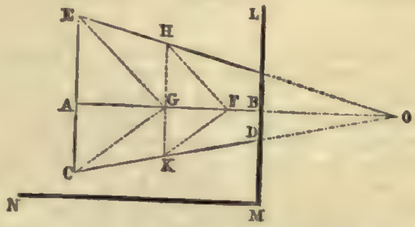


Fig. 25.

2nd Method.—Draw any two parallels E C and G K (Fig. 26), as before; join A K and C G, intersecting in F, and through F draw P Q parallel to E C or G K. Join E K, cutting P Q in R, and through the point R draw C H, meeting the line G K produced in H; join E H, which, if produced, would pass through the point of intersection O.

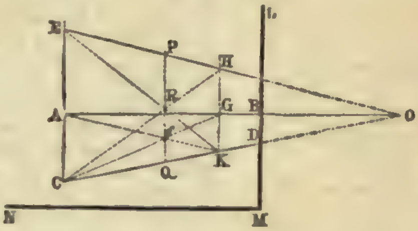


Fig. 26.

PROBLEM XI.

At a given point (E) *in a given straight line* (E D), *to make an angle equal to a given angle* (A B C). (Euclid, Book I., Prop. XXIII.)

With the centre B (Fig. 27) and any convenient radius, describe an arc, cutting A B in H, and B C in G; with the centre E and the same radius, describe an arc cutting E D in the point L; with the centre L and a radius equal to the distance H G, describe another arc, intersecting in the point K; join E K, and produce it if necessary; then the angle D E F will be equal to the angle A B C, as required.

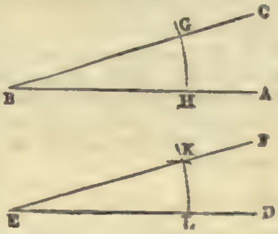


Fig. 27.

PROBLEM XII.

To bisect any given angle (A B C). (Euclid, Book I., Prop. IX.)

With the centre B (Fig. 28) and any convenient radius, describe an arc cutting B A in D, and B C in E, with D and E as centres, and any equal radii, describe arcs intersecting in the point F; join B F; then the angle A B C will be bisected by B F, or divided into two equal angles, A B F, and C B F.

By again bisecting the angles A B F, C B F, the angle A B C will be divided into four equal angles, and so on; by proceeding in a similar manner, the angle A B C may be divided into any *even* number of equal angles, continuing in geometrical progression—*viz.*, 2, 4, 8, 16, 32, 64, &c.

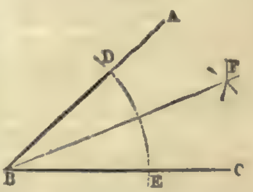
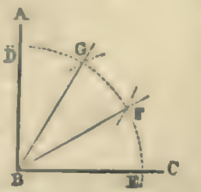


Fig. 28.

PROBLEM XIII.

To trisect or divide into three equal angles a given right angle (A B C).

Fig. 29.

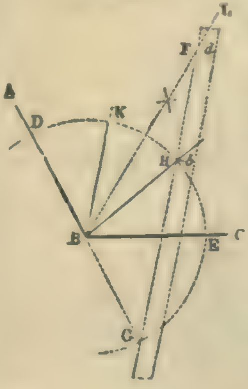


With the centre B (Fig. 29) and any radius, describe an arc, cutting B A in D, and B C in E, and from the points D and E as centres, and the same radius as before, describe [illegible] cutting the former arc in F and G; join B and the points of intersection F and G; then the angle A B C is divided into three equal angles, A B G, G B F, F B C.

PROBLEM XIV.

To trisect or divide into three equal angles any given angle (A B C).

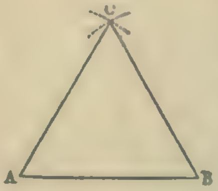
Fig. 30.



With the centre B (Fig. 30) and any radius, describe a circle cutting B A in D, and B C in E; bisect the angle A B C by the line B L (Problem XII.), and produce A B till it meets the circle in G; on the edge of the ruler mark off a distance (*bd*) equal to the length of the radius B D or B E, and lay the ruler at the point G, moving it until the point *b* cuts the circle, and the point *d* intersects the line B L, in H and F respectively; join B H, and set off H K equal to E H, leaving K D also equal to E H, and join B K; then the angle A B C will be divided into three equal angles, A B K, K B H, and H B C.

PROBLEM XV.

Fig. 31.



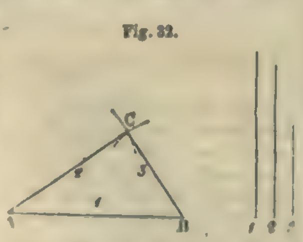
To describe an equilateral triangle upon a given straight line (AB). (Euclid, Book I., Prop. I.)

With the centres A and B (Fig. 31) and radii equal to A B, describe arcs intersecting in the point C; join A C and B C; the triangle A B C will be equilateral.

PROBLEM XVI.

To construct a triangle, whose sides shall be respectively equal to three given straight lines (1, 2, 3), *any two of which are greater than the third.* (Euclid, Book I., Prop. XXII.)

Fig. 32.



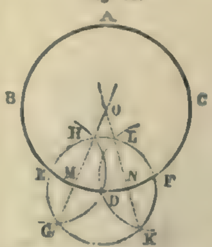
Draw any straight line, A B, (Fig. 32) equal to the given line 1, with the centre A and a radius equal to the given line 2 describe an arc, and with the centre B, and a radius equal to the given line 3, describe another arc, cutting the former at the point C; join A C and B C; then A B C will be the triangle required.

PROBLEM XVII.

*To find the centre and radius of a given circle.** (Euclid, Book III., Prop. I.)

1st Method.—In the circumference of the given circle A B C (Fig. 33), take any point D, and from it as a centre, with any radius, describe a circle cutting the circumference of the given circle in the points E and F, with which as centres, and the same radius, describe arcs intersecting the circle E F in the points G H and K L. Draw lines passing through the points H and G, L and K, and produce them until they intersect each other in the point O, which is the centre of the circle, and O M or O N the radius.

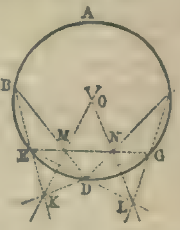
Fig. 33.



By this method the arcs D E *and* D F *are bisected, and the problem may be performed by merely drawing any two chords, such as* D E *and* D F, *bisecting those chords by perpendiculars, which being produced, until they intersect each other, will give the centre.*

2nd Method.—Take any five points, B, E, D, G, C (Fig. 34), in the circumference of the given circle A B C, equidistant from each other; draw lines passing through B E and G D, meeting in the point K; also through C G and E D, meeting in the point L; join E G, and draw B D and C D, cutting the line E G in M and N respectively; and through the points of intersection K M, and L N, draw lines, and produce them until they intersect each other in the point O, which is the centre of the given circle A B C.

Fig. 34.

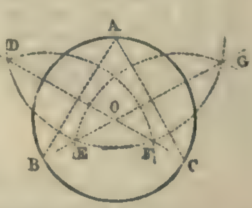


PROBLEM XVIII.

To describe a circle that shall pass through any three given points (A B C), *which are not in a straight line.*

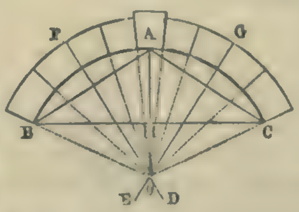
From the point A (Fig. 35), as a centre, with any convenient radius greater than half the distance A B or A C, describe an arc; with B and C as centres, and the same radii, describe arcs cutting the former in the points D, F and E G; draw lines passing through those points intersecting each other in the point O, and with the centre O, and a radius equal to the distance O A, O B, or O C, describe a circle which will necessarily pass through the three given points A, B, and C.

Fig. 35.



This problem will be found most applicable in the case of a circular arch, whose span (B C) and rise (A H) (Fig. 36) are given, and it is required to describe it; join A B and A C, bisect them by the perpendiculars D F and E G; then, as before, the point of intersection O is the centre with which to describe the circle.†

Fig. 36.



The joints of the stones or *voussoirs* forming the arch are drawn from the centre, and form continuations of the radii.

PROBLEM XIX.

To find the point in a given straight line (A B), *drawn from the one extremity of a given arc* (A E C), *through which the other extremity* (C) *of the given arc, if continued, would pass, and without using the centre.*

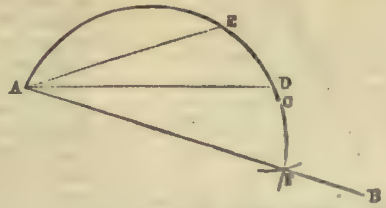
From A (Fig. 37) draw any chord A D, and also any other chord A E, making the angle D A E equal to the angle B A D; with the centre D and radius D E describe an arc cutting A B in F; then F is the point in A B, through which the arc A E C, if continued, would pass.

---

* This Problem is performed in Euclid by drawing any chord, bisecting it by a perpendicular meeting the given circle at its two extremities, which gives the diameter of the circle, by bisecting which, both the centre and radius of the circle are found.

† See *Euclid*, Book III., Prop. XXV.

Fig. 37.



When the centre is either not known, or at too great a distance to be conveniently used (as very frequently occurs in practice), this problem will be found exceedingly useful.

PROBLEM XX.

To draw a tangent to a given circle (A B C) *through a given point* D *in the circumference.* (Euclid, Book III., Prop. XVII.)

1st Method.—If the centre O of the given circle A B C (Fig. 38) be not given, find it by Problem XVII.; join O D, and through the given point D draw E F perpendicular to O D; then E F is the tangent required.

Fig. 38.

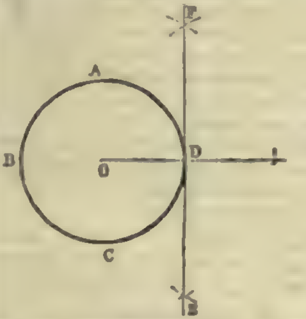
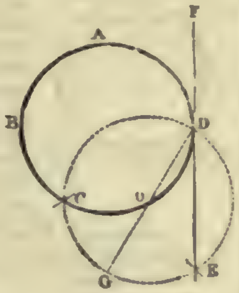


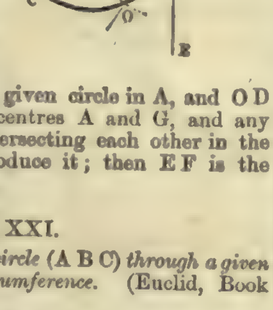
Fig. 39.



2nd Method.—Take any other point O (Fig. 39) in the circumference of the given circle A B C; join D O and produce it; with the centre O, and radius O D describe a circle cutting the given circle in C, and D O produced in G; make the arc G E equal to the arc G C; join E D and produce it; then E F is the tangent required.

3rd Method.—Take any other point O (Fig. 40) in the circumference of the given circle A B C; join O D and produce it; with the centre D, and radius D O, describe a circle cutting the given circle in A, and O D produced in G; with the centres A and G, and any equal radii, describe arcs intersecting each other in the point F; join F D and produce it; then E F is the tangent required.

Fig. 40.



PROBLEM XXI.

To draw a tangent to a given circle (A B C) *through a given point* (D) *without the circumference.* (Euclid, Book III., Prop. XVII.)

1st Method.—Find the centre O (Fig. 41) of the given circle A B C; join O D, and bisect it in G; with the centre G, and radius G O or G D, describe a semicircle, cutting the given circle A B C in the point C, which is the point of contact; join D C and produce it; then E D is the tangent required, touching the circle at the point C, and drawn from the point D.

Fig. 41.

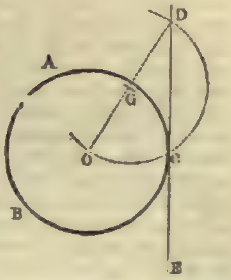
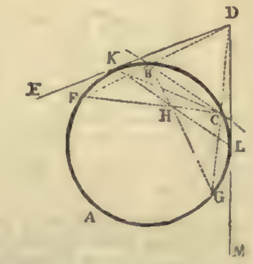


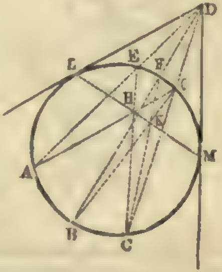
Fig. 42.



2nd Method.—With the given point D (Fig. 42) as a centre, and any radius, describe an arc cutting the given circle A B C in B and C; join D B, D C, and produce them till they meet the circumference of the given circle A B C in F and G; join B G and C F, cutting each other in the point H; join B C, and through H draw a line parallel to B C, and meeting the circle in the points K L; draw from the point D lines D E and D M, passing through K and L, which will be tangents to the circle A B C.

3rd Method.—From the given point D (Fig. 43) draw any three lines cutting the given circle in E, A in F, B and in G, C respectively; join G B, F C, intersecting in K, and G A, E C, intersecting in H; draw through H and K a line meeting the circle in the points L, M which will be the points of contact; and lines drawn from D through those points L, M will be tangents to the circle A B C.

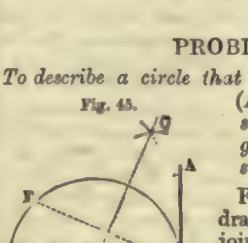
Fig. 43.



PROBLEM XXII.

To describe a circle that shall touch two given straight lines (A B *and* C D) *not in the same straight line, and touching at a given point* (E) *in one of them* (A B).

Fig. 44.

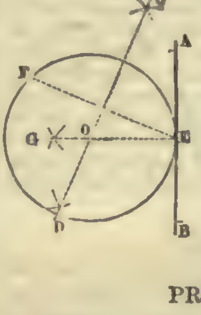


Produce the given lines, AB and C D (Fig. 44), till they meet in the point G; make G F equal to G E: from E and F draw E O and F O perpendicular to A B and C D, respectively intersecting in the point O; and with the centre O, and radius O E or O F, describe the circle HEF, which will touch the given lines A B, C D, and also A B in the point E, as required.

PROBLEM XXIII.

To describe a circle that shall touch a given straight line (A B) *in a given point* (E), *and shall also pass through another given point* (F) *not in the same straight line* (A B).

Fig. 45.

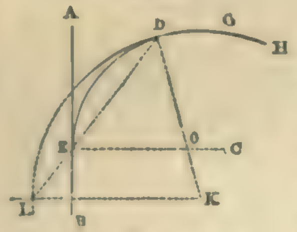


From the given point E (Fig. 45) draw E G perpendicular to A B; join E F, and bisect it by a perpendicular C D, cutting E G in the point O, with which as a centre, and the radius O E, describe the circle E D F, which will touch the line A B in the given point E, and also pass through the given point F.

PROBLEM XXIV.

To describe a part of a circle that shall touch a given *straight line* (A B) *in a given point* (E), *and shall also touch a given arc* (D G H).

Fig. 46.

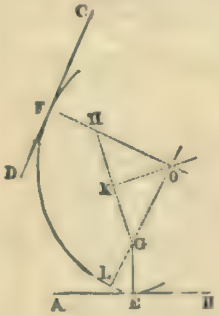


Draw E C (Fig. 46) perpendicular to A B; and through the centre K of the given arc D G H, draw K L parallel to E C, and meeting the arc D G H continued in the point L; join L E, and produce it to meet the given arc in the point D, and join D K, cutting E C in O; then with the centre O, and radius O E, describe a part of a circle which will touch the given line A B in the given point E, and also the given arc D G H in the point D.

PROBLEM XXV.

To describe two arcs that shall meet each other in the line of their centres, and shall touch two given straight lines (A B, C D) *at the given points* (E, F) *in those lines, the radius* (E G) *of the lesser arc being also given.*

Fig. 47.

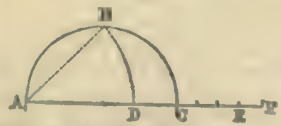


From the point E (Fig. 47) draw E G perpendicular to A B, and equal to the given radius, and from the point F draw F H perpendicular to C D, and make it equal to E G; join G H; bisect G H by a perpendicular K O, cutting F H produced, in the point O; join O G, and produce it, then with O as a centre, and radius O F, describe an arc meeting O G produced in L. With the centre G, and radius G E or G L, describe an arc meeting the former arc in L; then the arcs F L and E L are those required, meeting each other at the point L in the line L O, passing through both their centres G and O, and also touching A B and C D in E and F.

PROBLEM XXVI.

To find a straight line nearly equal to the length of a given arc (A B C).

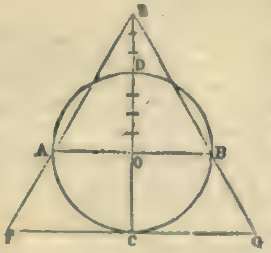
Fig. 48.



Bisect the given arc in B (Fig. 48), and draw the lines A B, A C; with the centre A, and radius A B, describe an arc cutting A C in D; make D E in A C produced, equal to A D, divide C E into three equal parts, and make E F equal to one of the parts; then A F will be nearly equal to the arc A B C.

PROBLEM XXVII.

Fig. 49.



To draw a straight line nearly equal to the semi-circumference of a given circle (A D B C).

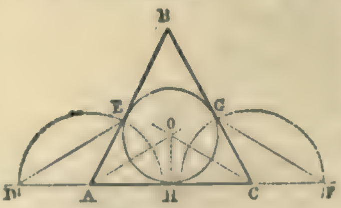
Draw the diameters A B and C D (Fig. 49) at right angles to each other; produce C D until D E is equal to three-fourths of D O; through C draw F G parallel to A B; join E A, E B, and produce them until they meet F G in F and G; then F G will be nearly equal to the semi-circumference of the circle A D B C.

PROBLEM XXVIII.

To inscribe a circle in a given triangle (A B C) *that shall touch all its sides.* (Euclid, Book IV., Prop. IV.)

Produce A C (Fig. 50) both ways; with the centre A, and any radius, describe an arc cutting A C produced in D, and A B in E; also with the centre C, and the same radius, describe an arc cutting AC produced in F, and B C in G; join D E and F G, and draw through A and C lines parallel to DE and F G respectively, and cutting each other in the point O; then O is the centre of the required circle (*the lines* A O *and* O C *will bisect the angles* B A C *and* A C B, *which might be done by Problem* XII.) From O draw O H perpendicular to A C; then with the centre O, and radius O H, describe a circle which will touch all the sides of the given triangle A B C.

Fig. 50.



PROBLEM XXIX.

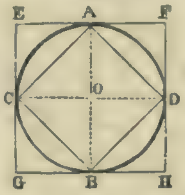
To describe a circle about a given triangle (A B C). (Euclid, Book IV., Prop. V.)

This Problem is simply to describe a circle passing through three given points, A B C, as given before in Problem XVIII.

PROBLEM XXX.

To draw a square within or about a given circle (A C B D). (Euclid, Book IV., Props. VI. and VII.)

Fig. 51.

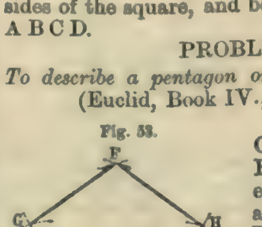


Draw through the centre O (Fig. 51) any two diameters, A B and C D, at right angles to each other, meeting the circle in the points A C B D. To *inscribe* a square in the circle, join those four points; then A C B D is the square required; and to *describe* a square about the circle, draw tangents or lines parallel to the diameters at those four points, then EFHG is the square required.

PROBLEM XXXI.

To circumscribe and inscribe circles to a given square (A B C D). (Euclid, Book IV., Props. VIII. and IX.)

Fig. 52.

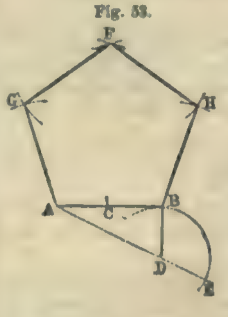


Draw the diagonals, A C, B D (Fig. 52), intersecting in the point O, the centre of the required circles. With the centre O, and radius O A, describe a circle which will pass through A, B, C and D, and be *circumscribed* about the given square A B C D. From O, draw O E perpendicular to D C; and again with the centre O, and radius O E, describe a circle which will touch all the four sides of the square, and be *inscribed* in the given square A B C D.

PROBLEM XXXII.

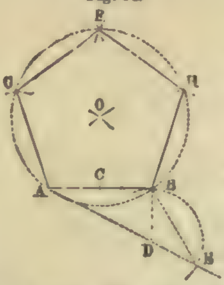
To describe a pentagon on a given straight line (A B) (Euclid, Book IV., Props. XI. and XII.)

Fig. 53.



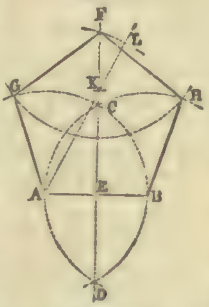
1st Method.—Bisect A B in C (Fig. 53), and from B draw B D perpendicular to A B, and equal to A C or B C; join A D and produce it; with the centre D and radius D B describe a circle cutting A D produced, in E; with the centres A and B and radii, each equal to A E, describe arcs intersecting in F; lastly, with the centres A and F, and radii, each equal to A B, describe arcs intersecting in G, and with B and F as centres, and the same radii, describe arcs intersecting in H; join A G, F G, F H, and B H; then A B H F G is the pentagon required.

Fig. 54.



2nd Method.—Bisect A B in C (Fig. 54), and from B draw BD perpendicular to AB, and equal to A C or B C; join A D and produce it; with the centre D, and radius D B, describe a circle cutting A D produced, in E; join B E, and with the centres A and B and radii, each equal to BE, describe arcs intersecting in O, with which as a centre and radius O A or O B, describe a circle. With the centres A and B and radii, each equal to AB, describe arcs cutting the circle in G and H; lastly, with the centre G or H, and the same radius, describe an arc cutting the circle in F, and join A G, G F, F H, H B; then A G F H B is the pentagon required.

Fig. 55.

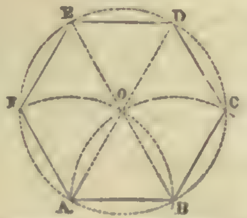


3rd Method.—With the centres A and B (Fig. 55), and radii, each equal to AB, describe circles intersecting in C and D; join C D, cutting A B in E, and produce it. Make E K equal to A B; join A K, and produce it, till K L is equal to A E or E B; with the centre A, and radius A L, describe an arc cutting D C produced in F; and with the centre F and radius A B describe a circle cutting the circles D A C and D B C in H and G respectively; join A G, F G, F H, and B H; then A B H F G is the pentagon required.

PROBLEM XXXIII.

To describe a regular hexagon upon a given straight line. (Euclid, Book IV., Prop. XV.)

Fig. 56.



With the centres A and B (Fig. 56), and radii, each equal to A B, describe circles intersecting in O; with the centre O, and the same radius, describe another circle, cutting the two former in F and C respectively; join A O and B O, and produce them till they meet the latter circle in D and E respectively; join BC, CD, D E, E F, and F A; then A B C D E F is the hexagon required. From this it will be seen that the side of a hexagon is equal to the radius of the circumscribed circle; and by merely finding the centre O as above, and setting off the length of the given line *six* times round the circumference, the hexagon will be constructed.

PROBLEM XXXIV.

To describe a regular octagon on a given straight line (AB).

With the centres A and B (Fig. 57), and radii equal to A B, describe circles intersecting in K; with the centre K, and the same radius, describe a circle cutting the two former in L and M; with the centres L and M, and still the same radius, describe arcs cutting the last circle in N and O; join A N and B O, by lines cutting the two first circles in P and Q, and produce them; join A Q and B P, and produce them till P G and Q D are each equal to A B; and lastly, with the centres G and D, and the same radii, equal to A B, describe circles cutting the two first circles in H and C, and the lines A N and B O produced, in F and E. Join B C, C D, D E, E F, F G, G H and H A; then A B C D E F G H is the octagon required.

Fig. 57.

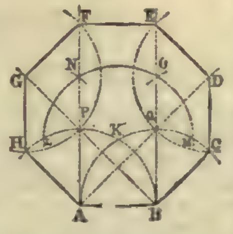
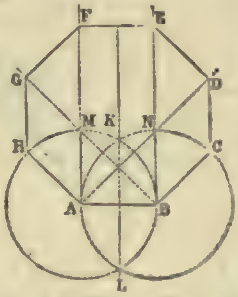


Fig. 58.

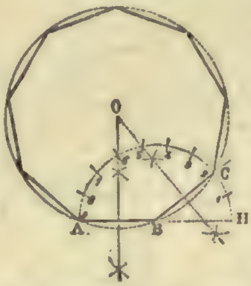


2nd Method.—With the centres A and B (Fig. 58), and radii, each equal to A B, describe circles cutting each other in K and L; join K L, and through A and B draw A F and B E parallel to K L, cutting the circles in M and N; join A N and B M, and parallel to them through A and B draw A H and B C, meeting the circles in H and C; through H and C draw H G and C D parallel to K L, and meeting B M and A N produced in G and D; and lastly, through G and D draw G F and D E parallel to A D and B G, cutting A M and B N produced in F and E; join FE; then ABCDEFGH is the octagon required.

PROBLEM XXXV.

To describe a regular polygon (containing any number of given sides) upon a given straight line (A B).

Fig. 59.



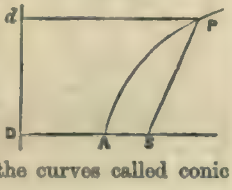
With the centre B (Fig. 59), and radius A B, describe a semicircle, meeting A B produced in H; divide the semicircle into as many equal parts as in the proposed polygon; join B and the second point of division C; bisect each of the sides A B and B C by perpendiculars intersecting in O; then with the centre O, and radius O A, O B, or O C, describe a circle, which will pass through the points A, B, and C; this circle will be that described about the required polygon; and therefore, by setting off A B as many times as required round the circle thus formed, the polygon will be described. The example given in the figure is a nonagon, or nine-sided figure.

CHAPTER X.

ON THE CONSTRUCTION OF THE CONIC SECTIONS.

THE preceding propositions depend entirely on the properties of straight lines and the circle, and therefore admit of construction by rule and compasses. Now, beside the circle there are several curves which are used more or less, by draughtsmen; these are for the most part drawn, when required, by determining accurately several points in them, which are then neatly joined by the hand. Of these curves the *ellipse* is the one oftenest needed, and is used, in fact, almost as much as the circle, in consequence of the perspective representation of a circle being an ellipse. Besides the *ellipse*, the *hyperbola* and *parabola* are often needed in the delineation of shadows. These three curves are generally called conic sections, for this reason—if a right cone is cut by a plane, the curve which bounds the section is one of these three, except in the special cases when the elliptic section degenerates into a circle, and the hyperbolic into two straight lines. The object of the few following pages is to prove such properties of these curves as shall enable us to give rules for their construction. The complete investigation of their properties, which forms a distinct and very interesting branch of mathematics, is not here intended.

Fig. 60.

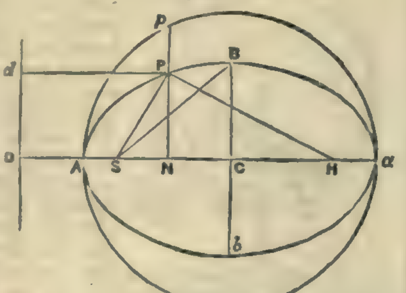


DEFINITION 1.—Let dD (Fig. 60) be a given fixed straight line, called the directrix; S a given fixed point, called the focus; P a movable point. Let Pd be perpendicular to the given straight line, then if P move in such a manner that S P bears a constant ratio to Pd, it traces out one of the curves called conic sections.

DEFINITION 2.—Suppose $SP : Pd :: e : 1$, then if $e \angle 1$, the curve is an *ellipse;* if $e = 1$, the curve is a *parabola;* if $e \not> 1$, the curve is a *hyperbola.* (e = the eccentricity of the denoted curve).

THE ELLIPSE.

Fig. 61.



Let Dd, S and P (Fig. 61) be the same as in the foregoing definitions; through S draw D Aa perpendicular to Dd. Then if A and a are so taken that $SA : AD :: e : 1$, and $Sa : aD :: e : 1$, then A and a are points in the ellipse; also the line A a is the *transverse* diameter or *major* axis; bisect A a in C, then C is the *centre* of the ellipse; through C draw B C b perpendicular to A a, then B b is called to *conjugate* to the transverse diameter, or the *minor* axis; take H, so that $a\text{H} = \text{AS}$, or so that $\text{CS} = \text{CH}$, then H and S are called the *foci* (singular *focus*).

(1.) *To show that in the ellipse* $\text{CS} = e, \text{AS}$.

We have already seen that $\text{AS} = e. \text{AD}$, also that $\text{S}a = e. a\text{D}$ $\therefore a\text{S} - \text{SA} = e\,(a\text{D} - \text{DA})$, and $a\text{H} = \text{AS}$. $\therefore \text{SH} = e. \text{A}a$, or $\text{SC} = e. \text{AC}$. Q. E. D.

N.B.—e is called the eccentricity of the ellipse.

(2.) *To show that* $\text{SB} = \text{AC}$.

Manifestly $\text{SB} = e. \text{DC} = e\,\text{AD} + e\,\text{AC} = \text{AS} + \text{SC} = \text{AC}$. Q. E. D.

COR.—Hence, $\text{BC}^2 = \text{SB}^2 - \text{SC}^2 = \text{AC}^2 - \text{SC}^2 = \text{AC}^2 (1 - e^2)$.

(3.) *To show that* $\text{SP} = \text{AC} - e. \text{CN}$.

Manifestly,

$$\begin{aligned}\text{SP} &= e\,\text{DN} = e\,(\text{D}c - \text{CN}) = e\,(\text{DA} + \text{AC}) - e\,\text{CN}\\ &= e\,\text{DA} + e\,\text{AC} - e\,\text{CN} = \text{AS} + \text{SC} - e\,\text{CN}\\ &= \text{AC} - e\,\text{CN}.\end{aligned}$$

(4.) *To show that* $\dfrac{\text{PN}^2}{\text{CB}^2} + \dfrac{\text{CN}^2}{\text{CA}^2} = 1$.

For $\text{SP}^2 = \text{SN}^2 + \text{PN}^2$ (Eucl. I., 47). But $\text{SP} = \text{AC} - e\,\text{CN}$ and $\text{SN} = \text{SC} - \text{CN}$ $\therefore (\text{AC} - e\,\text{CN})^2 = (\text{SC} - \text{CN})^2 + \text{PN}^2$ and $\text{SC} = e\,\text{AC}$.

$$\therefore \text{AC}^2 - 2\,e\,\text{AC}, \text{CN} + e^2\,\text{CN}^2 = e^2\,\text{AC}^2 - 2\,e\,\text{AC}\,\text{CN} + \text{CN}^2 + \text{PN}^2.$$

$$\therefore \text{AC}^2 (1 - e^2) = \text{CN}^2 (1 - e^2) + \text{PN}^2.$$

$$\therefore 1 = \frac{\text{CN}^2 (1 - e^2)}{\text{AC}^2 (1 - e^2)} + \frac{\text{PN}^2}{\text{AC}^2 (1 - e^2)} \qquad \text{Now } \text{AC}^2 (1 - e) = \text{CB}^2.$$

$$\therefore 1 = \frac{\text{CN}^2}{\text{AC}^2} + \frac{\text{PN}^2}{\text{BC}^2}$$

$$\therefore \frac{\text{PN}^2}{\text{BC}^2} + \frac{\text{CN}^2}{\text{AC}^2} = 1. \qquad \text{Q. E. D.}$$

COR.—With centre C and radius C A, describe a circle Apa; produce N P to meet the circumference in p; join Cp, which is plainly equal to A C.

Now $\quad p\text{N}^2 + \text{CN}^2 = \text{C}p^2 = \text{CA}^2$.

$$\therefore \frac{\text{PN}^2}{\text{CA}^2} + \frac{\text{CN}^2}{\text{CA}^2} = 1.$$

$$\therefore \frac{\text{PN}^2}{\text{CB}^2} = \frac{p\text{N}^2}{\text{CA}^2}$$

Or, $\quad \text{PN} : p\text{N} :: \text{CB} : \text{CA}$.

This result, and the previous one, we shall have occasion to use in the article on Mensuration in a future chapter.

(5.) *To show that* $\text{HP} = \text{AC} + e\,\text{CN}$.

For $\text{HP}^2 = \text{PN}^2 + \text{NH}^2 = \text{PN}^2 + (\text{CN} + \text{CS})^2 = \text{PN}^2 + (\text{CN} + e\,\text{CA})^2$.

Now from art. 4. $\text{PN}^2 = \text{CB}^2 - \dfrac{\text{CB}^2}{\text{CA}^2}\text{CN}^2 = (1 - e)^2\,\text{CA}^2 - (1 - e^2)\,\text{CN}^2$.

$$\therefore \text{HP}^2 = \text{CA}^2 - e^2\,\text{CA}^2 - \text{CN}^2 + e^2\,\text{CN}^2 + \text{CN}^2 + 2\,e\,\text{CA}, \text{CN} + e^2\,\text{CA}^2.$$

$$= \text{CA}^2 + 2\,e\,\text{CA}, \text{CN} + e^2\,\text{CA}^2 = (\text{CA} + e\,\text{CN})^2.$$

$$\therefore \text{HP} = \text{CA} + e\,\text{CN}.$$

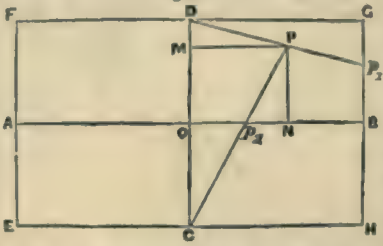
COR.—Hence, $\text{SP} + \text{HP} = \text{CA} - e\,\text{CN} + \text{CA} + e\,\text{CN} = 2\,\text{CA} = \text{A}a$.

Or, $\text{SP} + \text{PH}$ = the major axis of the Ellipse.

On this property of the Ellipse the first and third practical methods of construction depend.

(6.) *If* EFGH (Fig. 62) *be a rectangular parallelogram, and* D C *and* A B *are the lines joining the bisections of the opposite sides, divide* G B *into* n, *equal parts, and also* O B *into* N, *equal parts. Let* Gp_1 *contain* p *of the parts into which* G P *is divided, and let* Op_2 *contain* p *of the equal parts into which* O B *is divided, join* Dp_1 *and* Cp_2 *and produce it to meet* Dp_1 *in* P; *then* P *is a point in the Ellipse, whose major and minor axes are* A B *and* O D.

Fig. 62.



For, drawn P N parallel to D O and $\therefore$ perpendicular to AB.

Now, $Op_2 = \frac{p}{n} OB$. and $Gp_1 = \frac{p}{n} OD$.

Also, $\frac{OD - PN}{ON} = \frac{OD - p_1B}{OB} = \frac{Gp}{OB} = \frac{p}{n} \cdot \frac{OD}{OB}$.

Again, draw P M perpendicular D O. Then

$$\frac{Op_2}{OC} = \frac{MP}{MC}, \text{ or } \frac{ON}{OC + OM} = \frac{p}{n} \cdot \frac{OB}{OC}.$$

$$\text{Now } OC = OD \therefore \frac{OD + PN}{ON} = \frac{n}{p} \cdot \frac{OD}{OB}.$$

But we have already seen that $\frac{OD - PN}{ON} = \frac{p}{n} \cdot \frac{OD}{OB}$.

$$\therefore \frac{OD^2 - PN^2}{ON^2} = \frac{OD^2}{OB^2}.$$

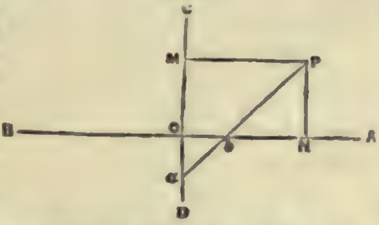
$$\therefore \frac{PN^2}{OD^2} + \frac{ON^2}{OB^2} = 1.$$

Hence by article (4), P is a point in an ellipse, whose major and minor axes are O B and O D respectively.

On this principle the second practical method is founded.

(7.) *If* A B, C D (Fig 63) *are two lines at right angles to each other, and bisecting each other in the point* O, *then if* P*ab be a straight line, so placed that* P*b* = O C *and* P*a* = O A, *then the point* P *is in an ellipse, whose axes are* A B *and* C D.

Fig. 63.



For draw P N, P M at right angles to O C and O A. Now by similar triangles,

$$\frac{Ob}{ab} = \frac{PM}{Pa} = \frac{ON}{OA}.$$

$$\text{and } \frac{Oa}{ab} = \frac{PN}{Pb} = \frac{PN}{OC}.$$

$\therefore \frac{PN^2}{OC^2} + \frac{ON^2}{OA^2} = \frac{Ob^2 + Oa^2}{ab^2} = \frac{ab^2}{ab^2} = 1$. Since bOa is a right angle.

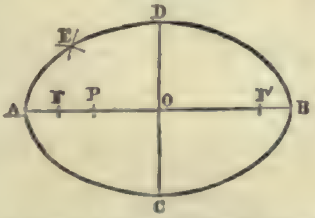
Hence (art. 4) P is a point in the specified ellipse. Q. E. D.

This is the principle of the trammel described in the fourth practical method.

To describe an Ellipse, the transverse (A B) *and conjugate* (C D) *diameters being given.*

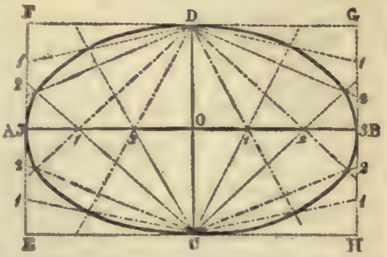
1st Method.—Draw A B and C D, bisecting each other at right angles, in the centre O (Fig. 64); with the centre D or C and radius O A or O B, describe arcs cutting A B in F and F′, which points will be the foci of the ellipse; take any other point P in the transverse diameter A B, and with the centres F, F′ and radii equal to A P and P B, respectively, describe arcs intersecting in the point E, which will be a point in the curve. By taking several points in the transverse diameter, and proceeding in a similar manner, as many points in the curve may be found as may be required, through which the ellipse may be drawn.

Fig. 64.



2nd Method.—Through the extremities of the given diameters draw the rectangle E F G H (Fig. 65); divide A E, A F and B H, B G into any number of equal parts (say 3); also divide A O and O B into the same number of equal parts; join D and the points of division in A F and B G, also C and those in A E, and B H. Again, from D draw lines through the points of division in A O and O B, intersecting the lines drawn from C to the points in A E and B H; and in the same manner from C draw lines through the points of division in A O and O B, intersecting the lines drawn from D to the points in A F and B G; the intersections thus derived are points in the curve through which the required ellipse may be drawn.

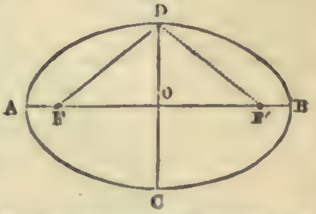
Fig. 65.



3rd Method.—An ellipse may easily be drawn by means of a thread, in the following manner:—

Find the foci F and F′ (Fig. 66) as in the first method; take a piece of thread equal in length to the distance between the foci added to the transverse diameter, and fasten the two ends; having fixed two pins in the foci, pass the thread round them and stretch it, place a pencil in the angle made by the thread when stretched, and commencing at one end (D) of the conjugate diameter, the thread being in the position F D F (as shown in dotted lines), move the pencil-point round, keeping the thread always stretched until the point again meets at D; the ellipse will be drawn.

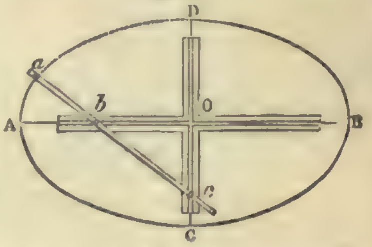
Fig. 66.



4th Method.—An instrument called a *trammel* is sometimes used for describing an ellipse, and is generally formed of two pieces of wood fixed at right angles to each other, and having a groove running through the centre of each, the groove being made a little wider at the bottom than at the top, in order to keep the guides *b*, *c*, (Fig. 67), attached to the bar, *abc*, in the groove; the bar, *abc*, should have a pencil fixed at the end *a*, the two guides, *b* and *c*, being movable or sliding along the bar, but when set ready for use, secured by small screws in the proper positions. The method of using the trammel is as follows:—

1st. Set the grooved pieces of wood on the given diameters A B and C D of the ellipse (as in the figure); the centre, where the two pieces cross, being on the centre O of the ellipse.

Fig. 67.



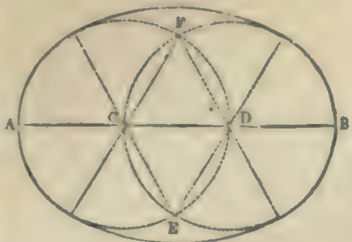
2nd. Make the distance from the pencil at *a* to the first sliding-guide at *b* equal to half the conjugate diameter, (or *ab* equal to O D or O C); and from the same point *a* to the second sliding-guide at *c* equal to half the transverse diameter (or *ac* equal to O A or O B).

3rd. Move the end *a* round, commencing at the point D, allowing the guides to move freely in the grooves, and the pencil will be found to describe the ellipse required.

The oval, although not an ellipse, so nearly resembles it, that very frequently, from the simplicity of drawing an *oval*, it is used in place of an *ellipse*. It is formed of parts of circles by the following methods:—

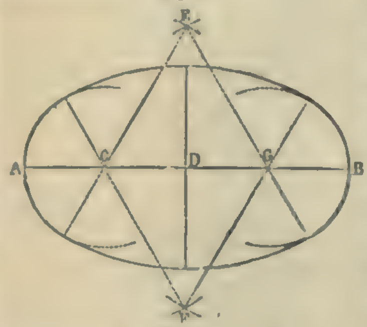
1st Method.—Divide any line A B (Fig. 68) representing the transverse diameter, into three equal parts by the points C D. With the centres C and D and the equal radii C A and D B, describe two circles. At the points E and F, where they intersect, as centres, and the equal radii A D or B C, describe parts of other two circles joining the circumferences of the two first circles (*the exact points of junction of the two circles may be got by joining the centres of both circles, as shown by the dotted lines*), which will complete the oval required.

Fig. 68.



2nd Method.—It is often required to draw a *flat or long-shaped oval*, in which case divide the line A B (Fig. 69) into four equal parts by the points C, D, G, and with the centres C and G and radii equal to C A and G B, describe two circles; also with the same centres and radii equal to any two parts of the line A B, such as A D, describe arcs intersecting at E and F, with which, as centres and radii equal to three parts, of A B, such as A G, describe portions of other two circles joining the circumferences of the first circles, which will complete the oval required.

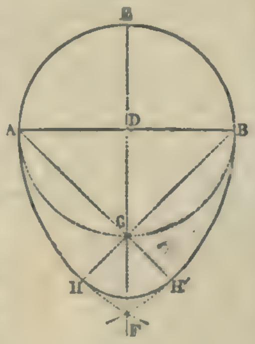
Fig. 69.



3rd Method.—*To draw an egg-shaped oval.*

Bisect the diameter A B in the point D (Fig. 70), with which, as a centre, and D A or D B as a radius, describe a circle cutting the diameter E F in the point G: with the centres A and B and radii equal to A B describe two arcs, A H and B H′; join A G and B G, and produce them until they cut the arcs A H and B H′ in the points H and H′, and with the centre G and radius G H, describe a part of a circle touching the two latter drawn arcs, which will complete the oval.

Fig. 70.

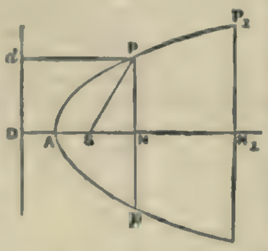


THE PARABOLA.

(1.) *To show that* $PN^2 = 4AS, AN$.

Let D*d* (Fig. 71) be the directrix, S the focus; draw D A N perpendicular to D*d* through S; this line is called the *axis* of the parabola. Draw P N perpendicular to D N, and P*d* perpendicular to P N. Now S P = P*d*. Hence if A S = A D, A is a point in the parabola, and is called its vertex.

Fig. 71.



Now $PS = Pd$ (by Def. 2) $= DN = DA + AN = AS + AN$.

Also $PS^2 = PN^2 + SN^2 = PN^2 + (AN - AS)^2$ $\therefore$ $(AS + AN)^2 = PN^2 + (AN - AS)^2$.

$$\therefore PN^2 = 4AS, AN.$$

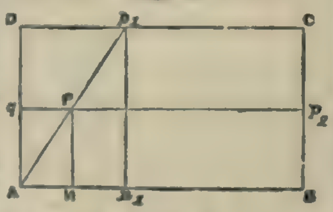
Cor. (1.) Hence A S. is a third proportional to A N, and the half of P N.

Cor. (2.) Also if P_1 be any other point, and $P_1 N_1$ be parallel to P N, we have $P_1N_1^2 : PN^2 :: 4AS.AN_1 : 4AS.AN :: AN_1 : AN$.

N.B.—If $P_1 N_1$ bound the curve it is sometimes called the base, and AN_1 the height of the curve. Also it is plain that if P N be produced, so that $pN = NP$, then $pN^2 = 4AS.AN$, and therefore p is a point in the parabola; which is therefore *symmetrical* about AN_1; the line Pp is called a double ordinate.

(2.) If A B C D (Fig. 72) be any rectangle, and if D C and C B are each divided into n equal parts, and let $Dp_1 = p$ of the equal parts into which C D is divided, and $Bp_2 = p$ of the parts into which C B is divided; join Ap_1 meeting p_2q (parallel to A B) in P; then P is a point in the parabola which has for its base C B and height B A.

Fig. 72.



For, draw P N perpendicular to A B, and $p_1 n_1$ parallel to P N.

$$\text{Now, } Bp_2 = \frac{p_1}{n_1}.CB.,\ An_1 = \frac{p_1}{n_1}.\ AB \text{ and } PN = Bp_2.$$

$$\text{Also, } \frac{PN}{AN} = \frac{p_1n_1}{An_1} = \frac{CBn_1}{ABp_1}$$

$$\text{And } PN = \frac{p_1}{n_1}\ CB.$$

$$\because \frac{PN^2}{AN} = \frac{CB^2}{AB}$$

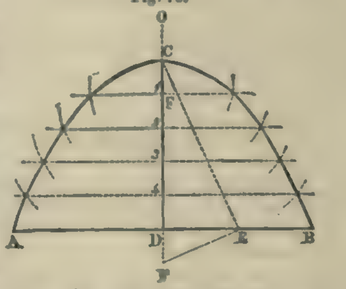
$\therefore PN^2 : CB^2 :: AN : AB$.

$\therefore$ P is a point in the specified parabola by corollary to last article. The second of the following practical methods is founded on the above principles. The first and third depend entirely on the definition of the parabola.

To describe a parabola, any absciss (that is, a portion of, as C1 C2, &c.) *of the axis and the corresponding ordinate being given; or, the half width of the base* (A D *or* D B), *and the height of the curve* (C D) *being given.*

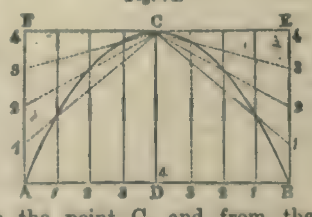
1st Method.—Bisect D B (Fig. 73) in the point E; join C E, and from E draw E F perpendicular to C E, and meeting C D produced in F; make C O in D C produced and C F_1 each equal to D F, F_1 will be the focus of the required parabola. Take any number of points, 1, 2, 3, 4 (&c.), in C D; through them draw double ordinates, or lines perpendicular to the axis C D; then with the centre F_1, and radii O F_1, O1, O2, O3, O4 (O &c.), describe arcs cutting the ordinates; the curve drawn through the points of intersection will be the parabola required.

Fig. 73.



2nd Method.—A B (Fig. 74) being the width at the base, and C D the height of the curve, as before, construct the parallelogram A B E F, divide D A, D B and A F, B E respectively into the same number of equal parts in the points 1, 2, 3, 4; from the points of division in A F and B E draw lines to the point C, and from the points of division in D A and D B draw lines perpen-

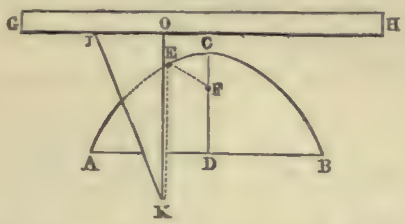
Fig. 74.



dicular to A B, or parallel to C D, cutting the former lines; then the curve drawn through the points of intersection will be the parabola required.

3rd Method.—Place a ruler G H (Fig. 75) at any convenient distance from C, parallel to the base A B, and take a piece of wood (called a square or set square), made in the form of a right-angled triangle I O K, placing the base I O against the ruler, and the other edge O K to coincide with the line C D; having found the focus F (as in the 1st method), fasten one end of a string at F, place a pencil at the point C, passing the string round it, and bringing the string back to K, fasten it to the end or point of the triangle; move the triangle or square along the ruler, keeping the pencil always against the edge of the square (as at E), and with the string stretched the pencil will describe one-half of the curve. By reversing the square and proceeding in a similar manner, the other half may be drawn, and the parabola required completed.

Fig. 75.



THE HYPERBOLA.

If the reasoning employed in the case of the ellipse be carefully gone over, attending only to the difference that will result from the circumstance that $e > 1$, the student will readily deduce that the following expressions will be found to hold good of the hyperbola, which are entirely analogous to the corresponding expressions in the case of the ellipse.

(1.) $SC = e\,AC$.

(2.) BC must be so taken that $BC^2 = CS^2 - CA^2 = (e^2 - 1)\,CA^2$.

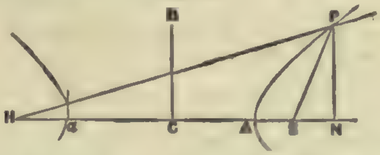
(3.) $SP = e.\,CN - AC$.

(4.) $\dfrac{PN^2}{CB^2} - \dfrac{CN^2}{CA^2} = -1$.

(5.) $HP = e.\,CN + AC$.

(6.) $HP - SP = 2\,AC$.

Fig. 76.



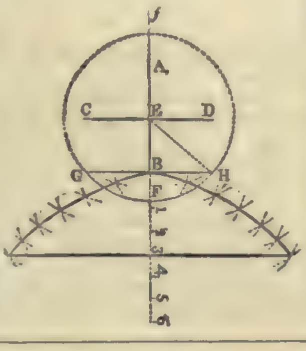
It is on this (6th) property that the following practical construction is founded.

The terms major axis, transverse axis, &c., in the hyperbola, are entirely analogous to the same terms in the ellipse.

To describe a hyperbola, the transverse (A B) *and conjugate* (C D) *axes being given.*

Through B (Fig. 77), at one end of the transverse axis A B, draw G H parallel to the conjugate axis C D, and make G H equal to C D; with the centre E and radius, equal to E H, describe a circle cutting A B, produced both ways, in the points F and f, which will be the foci of opposite hyperbolas; take any number of points, 1, 2, 3, 4 (&c.), in A B produced, and with the centres F and f, and radii B1, B2, B3, B4, (B &c.), and A1, A2, A3, A4 (A &c.), describe arcs cutting each other; the curve drawn through the points of intersection will be the hyperbola required.

Fig. 77.



The following consideration materially simplifies the construction of a hyperbola.

Through A (Fig. 78) draw A D parallel and equal to B C, join C D and produce it indefinitely, take P any point in the curve, draw pPN, meeting CD produced in p. Then

$$\frac{DA}{CA} = \frac{pN}{CN}$$

$$\therefore \frac{pN^2}{CB^2} - \frac{CN^2}{CA^2} = 0$$

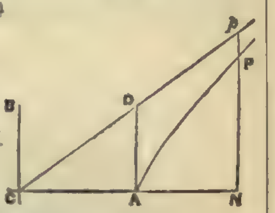
$$\text{But (4)} \quad \frac{PN^2}{CB^2} - \frac{CN^2}{CA^2} = -1$$

$$\therefore \text{subtracting } \frac{pN^2 - PN^2}{CB^2} = 1$$

$$\therefore (pN - PN)(pN + PN) = CB^2$$

$$\therefore Pp = \frac{CB^2}{pN + PN}.$$

Fig. 78.



Now it is plain that PN and pN increase as P is farther from A, or N from A, and therefore, the farther P is from the vertex the nearer it approaches C D produced. C D is called an asymptote—after a short distance the curve *sensibly* coincides with the asymptote. Hence, if in the practical construction above given E H be produced, it will serve as a *guide* to the curve, which can be drawn very accurately after a very few points have been determined by the construction.

THE CYCLOID.

Besides the conic sections, which we have briefly discussed above, there are several curves possessing curious or useful properties. Amongst the chief of these is the cycloid. The construction of this curve is sometimes useful to the artist. The following will suffice to explain the nature of the curve, and the method of its construction.

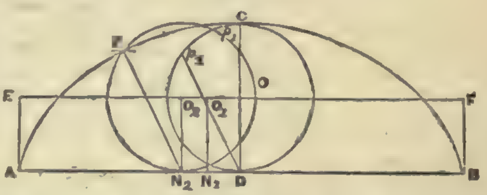
The cycloid is a curve formed by a point in the circumference of a circle (called the generating circle), revolving on a straight or level line; it may be best described as the curve traced out by a point in the wheel of a carriage when in motion along a level road. When the generating circle revolves round another circle, the curve described by a point in the circle is then called an epicycloid, and is constructed in a similar manner to the cycloid.

To describe a cycloid, the diameter or width at the base (AB), *and height* (C D), *of the curve being given.*

The most common method of describing the cycloid is by placing a ruler along the line A B, and taking a circle, such as a shilling, &c., according to the size or height of the curve required, and having fixed a point in that circle, to move it slowly along the ruler, marking different points in the curve, or by keeping a pencil fixed at the point chosen, and thus describing the curve. This, however, is liable to error, as the circle used very often slips, and cannot then revolve accurately; by construction, however, the curve may be correctly formed thus:—

Let A C B (Fig. 79) represent the curve in question, A B its base, C D its height. Bisect C D in O, and through O draw E F parallel to A B; with centre O and radius O C describe a circle, and divide the circumference into any number of equal parts, as Cp_1, Cp_2, . . . join

Fig. 79.



Dp_1 Dp_2 . . . Again, divide A D into the same number of equal parts, D N_1 D N_2 . . . . and draw O_1 N_1 O_2 N_2 . . . . parallel to C D; with centre O_2 and radius C D describe a circle, and with centre N_2 and radius Dp_2 describe an arc cutting the circle in the point P. This is a point in the curve, similarly with the centre O_1 and radius C D. Another circle may be described and intersected by the arc of a circle described with centre N_1 and radius Dp_1, and thus another point be determined; and so on for any number of points, which, being joined carefully, will give the curve in question.

We thus conclude our treatise on Practical Geometry. It has been our endeavour to confine it in extent to that which it is absolutely necessary the scientific draughtsman should be familiar with; and in the foregoing pages will be found all that is required for practical purposes—no really essential propositions being omitted.

CHAPTER XI.

PLANE TRIGONOMETRY.

1. *On representing Lines and Angles by Numbers.*

If we have a line of any length, we can represent it numerically by the number of times it contains a given line, which we take to represent unity. Thus, if we take a line a foot long to be the unit of length, a line seven feet long can be represented by 7. Of course, the same holds good of any other line. And so when we speak of a line 8, 5, or whatever the number may be, we mean that the line in question contains 8 or 5 of the given unit, as 8 feet, or 5 feet. And of course, if we can represent lines by numbers, we can generalise the numbers by letters, and thus we can represent lines by algebraical symbols; so that $a\,b\,c$, $x\,y\,z$, &c., may be understood to represent lines. In the same manner as before, if we speak of a line a, we mean a line containing as many units of length (*e.g.*, feet) as a contains units of number.

On the same principle we may express angles by numbers or by letters. This is done by dividing the right angle into 90 equal parts, each of which is called a degree, and dividing the degree into 60 equal parts, each called a minute, and the minute into 60 equal parts, each called a second. An angle is then expressed as being so many degrees, with odd minutes and seconds, *e.g.*, 36 degrees, 57 minutes, 31 seconds (which is usually written $36° \, 57' \, 31''$), in the same manner as a line is expressed by so many yards, with odd feet and inches.

Of course, as we can thus represent angles by numbers, we may also represent them by letters, and may have angles A B C; where the angle A (for instance) means that the angle contains as many degrees and parts of a degree as A contains units and parts of a unit. It is usual to denote angles either by Roman capital letters, A B C; or else by Greek small letters, $\alpha, \beta, \gamma \ldots \theta, \phi, \psi \ldots$ while generally the small Roman $a\,b\,c$ denote lines. This is, of course, only a conventional arrangement.

In the same manner as we may measure lines either by feet or yards, or miles, so we might take, as the unit of angular measure, any other part of the right angle than the $\frac{1}{90}$th; and, in fact, at the end of last century, when the decimal notation was introduced into France, it was proposed by certain French mathematicians, to make the degree the $\frac{1}{100}$th part of the right angle. The proposition was at no time extensively accepted, and is now quite abandoned.

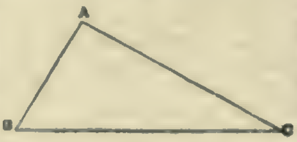
2. *Definition of the Science of Trigonometry.*

We are thus enabled to express lines and angles by numbers; and this is the first step towards making calculations in which lines and angles are the data. However, before these calculations can be performed, it is necessary that the relations which exist between straight lines and angles should be investigated. It is the object of the science of Trigonometry to make these investigations.

The object of the science will, perhaps, be more clearly stated, if we limit the definition so as to make it correspond more closely to its derivational meaning, by saying that *the science of Trigonometry has for its object the investigation of the relations which exist between the sides and angles of triangles and the algebraical expression of those relations.*

The immediate application of the science is to the calculation of certain parts of a triangle from certain given parts; *e. g.*, having given the sides B A, A C, and the angle B A C of the triangle A B C, we can calculate the magnitude of the side B C.

Fig. 1.



The science has, however, very many other uses besides the one from which its name is taken—viz., the measurement of triangles.

3. *The Circular Measure of an Angle.*

The measures above given, enable us to compare arithmetically one straight line with another, and one angle with another. But it is to be observed, that an angle and a line are heterogeneous magnitudes; and therefore, if we would perform algebraical operations into which lines and angles enter, we must devise some plan of measuring angles that shall express them by means of lines, or of the ratios of lines.

In fact, when we speak of an angle (of 57° suppose) it tells us what the angle is, but does not at once give us the means of comparing that angle with given lines.

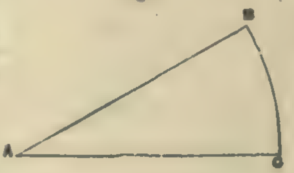
The measure of the angle adopted for the purpose of such calculations, is called the circular measure.

It is founded on the two well-known geometrical propositions.

(*a*) That in circles of the same radius, the angle is proportional to the arch which subtends it.

(*b*) And that for the same angle, in circles of different radii, the arc varies as the radius.

Fig. 2.



If a = the arc B C,
θ = the angle B A C, subtended by the arc B C,
r = the radius A C.

we may express these propositions by the two variations

$$a \backsim \theta \text{ when } r \text{ is constant.}$$
$$a \backsim r \text{ when } \theta \text{ is constant.}$$
$$\therefore a \backsim r\theta \text{ when both vary.}$$
$$\therefore \theta \backsim \frac{a}{r}$$

or the angle is measured by the ratio of the arc to the radius.

If we take the unit of angle to be the angle which is subtended by an arc of the same length as the radius, then

$$\theta = \frac{a}{r}.$$

In this case, the angle being measured by the ratio of two lines, it can enter a calculation in which we are dealing with lines.

N.B.—We can easily find the number of degrees in the angle which is the unit of circular measure.

Let x = the number of degrees required. Then x is subtended by an arc of the length of the radius = r.

Now an angle of 180° is subtended by a semicircle; *i. e.*, by an arc = πr, where $\pi = 3{\cdot}14159$.

$$\therefore \frac{x^\circ}{180^\circ} = \frac{r}{\pi r} = \frac{1}{\pi}$$

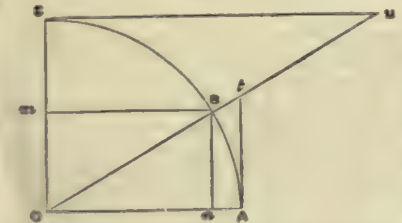
$$\therefore x = \frac{180^\circ}{3{\cdot}14159} = 57^\circ{\cdot}29577.$$

4. *The Definitions of the Trigonometrical Lines and Ratios.*

It is, however, generally more convenient, and for our present purposes necessary, to determine an angle not by an arc, or by the ratio of an arc to its radius, but by certain straight lines, or by the ratio of certain straight lines to each other. These lines, or, as they are now more commonly regarded, these ratios, are called respectively the sine, tangent, secant, cosine, cotangent, or cosecant of the angle. We proceed to define these terms.

Let A O B be an angle O A. Draw O C perpendicular to O A, and with the centre O and any radius O A describe an arc of a circle, meeting O C in C.

Fig. 3.



Draw B*n*, B*m*, perpendicular to O A, to O C; at A and C draw A*t*, C*u*, perpendicular to O A, O C. Then B*n* is defined to be the sine of the angle A O B, to the radius O A.

A*t* is defined as the tangent of A O B.
O*t* . . . . secant of A O B.
A*n* . . versed sine of A O B.
Hence, also B*m* (or O*n*) is the sine of B O C.
C*u* . . tangent of B O C.
O*u* . . secant of B O C.

Now B O C is 90° – A.

And B*m*, C*u*, O*u*, are defined as being the cosine, cotangent, cosecant respectively of A O B.

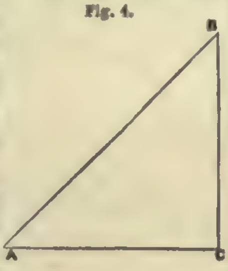
Hence—

$$\left.\begin{array}{r} Bn = \text{sine } A \\ Bm \text{ or } On = \text{cosine } A \\ At = \text{tan. } A \\ Cu = \text{cotan. } A \\ Ot = \text{sec. } A \\ Ou = \text{cosec. } A \end{array}\right\} \text{to the radius } OA.$$

To this method there is the obvious objection that the sines, &c., of a given angle have different values, according as they are referred to different radii; accordingly, instead of defining the sines, &c., as lines, it is, as above stated, now more usual to define them as ratios; by which means all consideration of the radius to which the sines are referred is avoided. According to this method the definitions are given as follow :—

C A B any right-angled triangle having the right angle at C.

Fig. 4.



Then The sine of A is $\frac{BC}{AB}$

The tangent of A is $\frac{BC}{AC}$

The secant of A is $\frac{AB}{AC}$

Hence— The sine of B is $\frac{AC}{AB}$

The tangent of B is $\frac{AC}{BC}$

The secant of B is $\frac{AB}{BC}$

But because A = 90 — B
The sine of B is the cosine A.
The tangent of B, cotangent of A.
The secant of A is the cosecant of A.

Hence— Sine $A = \frac{BC}{AB}$ Cosine $A = \frac{AC}{AB}$

Tan. $A = \frac{BC}{AC}$ Cotan. $A = \frac{AC}{BC}$

Sec. $A = \frac{AB}{AC}$ Cose. $A = \frac{AB}{BC}$

N.B.—The angle which with another makes up 90° is called the complement of that angle. Hence B is the complement of A; and the cosine, cotangent, and cosecant of an angle are evidently the sine, tangent, and secant of its complement.

It is plain (Euc. VI. 4) that the values of these ratios depend solely on the angle, and are quite independent of the magnitude of the sides of the triangles. If, then, we can by any means calculate the value of these ratios, which correspond to any angle, these values can be arranged in a table; and it is plain that, having such tables, if we have given any one of the ratios defined above, we know the angle; and *vice versâ*, if we have the angle given, we know the ratio.

Such tables have been calculated on principles to be hereafter explained;—for our present purpose it is sufficient for us distinctly to understand, that if we have given to us the numerical value of any one of the ratios, we know the angle that corresponds to it, and *vice versâ.*

5. *On the relations existing between the Trigonometric Ratios of the same Angle.*

Let A B C be a right-angled triangle,
A any given angle.
C the right angle.

Then $AC^2 + BC^2 = AB^2$ (Eucl. 47, Book I.)

$$\therefore \frac{AC^2}{AB} + \frac{BC}{AB} = 1.$$

$$\therefore \text{cos. }^2A + \text{sin. }^2A = 1 \ldots\ldots (1).$$

Again $$\text{tan. } A = \frac{BC}{AC}$$

$$\text{cotan. } A = \frac{AC}{BC}.$$

$$\therefore \text{tan. } A,\ \text{cotan. } B\ A = 1. \ldots\ldots (2).$$

Again, $$\text{tan. } A = \frac{BC}{AC} = \frac{\frac{BC}{AB}}{\frac{AC}{AB}}$$

$$\therefore \text{tan. } A = \frac{\text{sin. } A}{\text{cos. } A} \ldots\ \ldots (3).$$

Again, $$\text{secant } A = \frac{AB}{AC} = \frac{1}{\frac{AC}{AB}}$$

$$\therefore \text{sec. } A = \frac{1}{\text{cos. } A} \ldots\ldots (4).$$

In the same manner it may be easily proved that

$$\text{Cotan. } A = \frac{\text{cos. } A}{\text{sin. } A} \ldots\ldots (5).$$

$$\text{Cosec. } A = \frac{1}{\text{sin. } A} \ldots\ldots (6).$$

$$\frac{\text{Sec. } A}{\text{Cosec. } A} = \text{tan. } A. \ldots\ldots (7).$$

It is of very great importance that the student be familiar with the relations we have just established. He will therefore do well to perform the following exercises :—

Show that

(1). $\text{sin. } A = \sqrt{1 - \text{cos.}^2 A}$.

(2). $\text{tan. } A = \sqrt{\frac{1}{\text{cos. }^2A} - 1}$.

(3). $\text{tan. } A \text{ cosec. } A = \text{sec. } A$.

(4). $\text{tan. } A + \text{cotan. } A = \frac{1}{\text{sin. } A.\ \text{cos. } A}$.

(5). $\text{cosec. } A - \text{sin. } A = \text{cos. } A \text{ cotan. } A$.

(6). $\frac{1 + \text{cos. } A}{\text{sin. }^2A} = \frac{1}{1 - \text{cos. } A}$

(7). $\frac{1}{\sin.^2 A} = 1 + \text{cotan.}^2 A.$

(8). Express each of the trigonometrical ratios of an angle A in terms of the sine of A.

There are two or three angles, the numerical values of the trigonometrical ratios of which can be easily determined. These angles are 45°, 60°, and 30°.

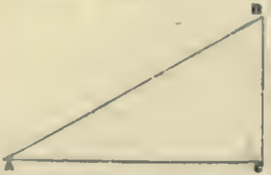
6. *To find the Trigonometrical Ratios of an Angle of 45°.*

A B C, a right-angled triangle. C the right angle.
If $A = 45°$, then $A = B$, and $AC = BC$.
Now, $AC^2 + BC^2 = AB^2$. (Eucl. I. 47).

$\therefore\ 2AC^2 = AB^2,$

or $2BC^2 = AB^2.$

Fig. 5.



$\therefore\ \sin. 45° = \frac{BC}{AB} = \frac{1}{\sqrt{2}}$

$\tan. 45° = \frac{BC}{AC} = 1.$

$\sec. 45° = \frac{AB}{AC} = \sqrt{2},$

and $\text{cosin.}\ 45° = \sin. (90 - 45) = \sin. 45°.$

$\therefore\ \cos. 45° = \frac{1}{\sqrt{2}}$

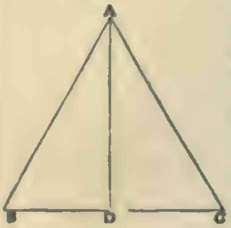
Similarly, $\cot. 45° = 1.$

$\text{cosec.}\ 45° = \sqrt{2}.$

7. *To find the Trigonometrical Ratios of an Angle of 60°.*

A B C, an equilateral triangle. The angle A B C is one of 60°. Draw A D perpendicular to B C. Now $BD = \frac{1}{2}.\ BC = \frac{1}{2}.\ AB$ and $AD^2 = AB^2 - BD^2 = \frac{3}{4}.\ AB^2.$

Fig. 6.



$\therefore AD = \frac{\sqrt{3}}{2}\cdot AB.$

$\therefore \sin. 60° = \frac{AD}{AB} = \frac{\sqrt{3}}{2}.$

$\tan. 60° = \frac{AD}{BD} = \frac{\frac{\sqrt{3}}{2}\cdot AB}{\frac{1}{2}\ AB} = \sqrt{3}.$

$\sec. 60° = \frac{BD}{AB} = 2.$

$\cos. 60° = \frac{BD}{AB} = \frac{1}{2}.$

$\text{cotan.}\ 60° = \frac{BD}{AD} = \frac{1}{\sqrt{3}}.$

$\text{cosec.}\ 60° = \frac{AB}{AD} = \frac{2}{\sqrt{3}}.$

Since $30° = 90° - 60°$
we shall have $\sin. 30° = \cos. 60° = \frac{1}{2}.$

And similarly $\tan. 30° = \frac{1}{\sqrt{3}}$ $\sec. 30° = \frac{2}{\sqrt{3}}.$

$\cos. 30° = \frac{\sqrt{3}}{2}.$ $\text{Cotan.}\ 30° = \sqrt{3}.$

$\text{Cosec.}\ 60° = \frac{1}{2}.$

8. *Generalisation and Extension of the Principles and Definitions previously laid down.*

The definitions above given hold good for angles that are less than ninety degrees; the definition, both of an angle and of the ratios which determine it, admit of and require extension; the nature of which extension and the principle on which it is made we will now proceed to explain.

9. *The use of the Negative Sign to denote position.*

Fig. 7.

Let A B be a line, the length of which is a. Let B C be a line, the length of which is b. Then it is plain that A C is $a - b$. This distance, A C, is arrived at by measuring a distance (a) to the right from A, and then measuring another distance (b) to the left from B, the $+a$ and the $-b$ being measured in opposite directions.

It appears, therefore, that when a stands for a line measured from a given point in one direction, $-a$ will stand for a line of the same length measured in the opposite direction. In other words, the magnitude of the line is determined by the number of units in a, while the direction is determined by its sign.

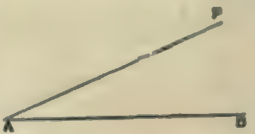
Fig. 8.

It is generally understood that $+a$ signifies a line measured to the right of a given point, as A B, and therefore that $-a$ signifies a line measure to the left of the fixed point, as AB′.

10. *Extension of the Definition of an Angle.*

We now proceed to extend the definition of an angle. An angle, as defined by Euclid—*i. e.*, as the inclination of one line to another—must be less than two right angles. But if we regard an angle as the space swept out by a right line revolving in one plane, about a fixed point in a given straight line, we clearly remove the limit imposed by Euclid's definition on the magnitude of the angle.

Fig. 9.

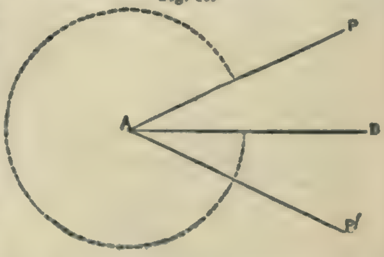


Thus if A be the fixed point in the fixed line A B, A P the movable line, let the angle B A P, according to Euclid's definition, be A.

Now it is plain that in one revolution AP passes through an angle equal to four right angles, or 360°. Moreover, A P will always come to its present position after one, two, or any number of revolutions; and therefore, according to our extended definition, B A P may be either A or 360° + A, or 2 × 360° + A, or, generally, $360°n + A$, where n is any integer.

11. *Negative Angles.*

Fig. 10.



In the same manner as we have shown that $+a$ and $-a$ mean equal lines measured in a contrary direction, so + A and — A will mean angles measured in contrary directions. Thus, AB and AP have the same meaning as before. Let A P′ be so placed that $P'AB = PAB$; then if $PAB = A$, $P'AB = -A$.

It is plain that if A P′ comes into the position A P, it must revolve through an angular space of $-360° + A$, as denoted by the portion of a dotted circle in figure, or through a certain number of total revolutions besides $-360° + A$. Hence B A P may also be represented by $-360° + A$, $-360° \times 2 + A$, or generally by $-360° \times n + A$, where n is any positive integer.

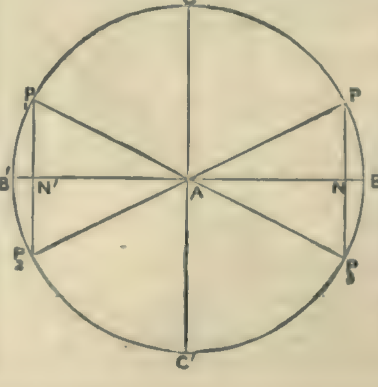
Hence we conclude that if A be any geometrical angle, its most general trigonometrical form will be

$$n \times 360° + A,$$

where n is any positive or negative integer whatever.

12. *Extension of Definitions of Trigonometrical Ratios.*

Fig. 11.



We now proceed to consider the trigonometrical ratios of angles greater than a right angle. We shall, in the first instance, confine our attention to the sines and cosines of angles, less than four right angles.

We have already explained that

$$\text{Sin. } B\,A\,P = \frac{P\,N}{A\,P}$$

$$\text{Cos. } B\,A\,P = \frac{A\,N}{A\,P}$$

Now as A N is measured along A B to the right, A N is positive. And if we reckon lines measured upward, from A towards C positive, it is plain that P N, being measured parallel to that direction, is positive.

The signs of the sine and cosine of an angle less than ninety degrees are then, by this way of reckoning the signs of the measurements, positive, as they should be.

Now if we consider an angle B A P_1, it is clear that P_1 N′ stands in the same relation to B A P_1 that P N does to B A P.

Hence we define

$$\text{Sin. } B\,A\,P_1 = \frac{P_1 N'}{A\,P_1}$$

$$\text{Cos. } B\,A\,P_1 = \frac{A\,N'}{A\,P_1}$$

It is plain that P_1 N′ is positive, and A N′ is negative. Hence

The sine of an angle ⁊ 90° ∠ 180° is positive, and the cosine of an angle ⁊ 90° ∠ 180° is negative.

In like manner, if B A P_2 be the angle subtended by the circumference B P_1 P_2,

$$\text{Sin. } B\,A\,P_2 = \frac{P_2 N'}{A\,P_2}$$

$$\text{Cos. } B\,A\,P_2 = \frac{A\,N'}{A\,P_2}$$

Now P_2 N′ is negative, and A N′ is negative. Hence the sine of an angle ⁊ 180° ∠ 270° is negative, and the cosine of an angle ⁊ 180° ∠ 270° is negative. In like manner if P_3 A B signify the angle subtended by the circumference B P_1 P_2 P_3,

i. e., ⁊ 270° but ∠ 360°.

$$\text{Sin. } B\,A\,P_3 = \frac{P_3 N}{A\,P_3}$$

$$\text{Cos. } B\,A\,P_3 = \frac{A\,N}{A\,P_3},$$

And P_3 N is negative, and A N is positive. Hence, sine of an angle ⁊ 270° ∠ 360° is negative: and the cosine of an angle ⁊ 270° ∠ 360° is positive. These four angles which we have considered are said to be in the first, second, third, and fourth quadrants respectively.

By means of the above, if we have given the signs both of sine and cosine of an angle, we can tell in what quadrant it must lie.

Thus if sine $\theta = +m$ and cos. $\theta = -n$, θ must lie in the second quadrant; *i.e.*, must be greater than 90° and less than 180°.

13. *To express the Trigonometrical Ratios of any angle in terms of those of an angle less than* 90°.

Again, the trigonometrical ratios of any angle can be expressed by means of the ratios of an angle less than 90°.

For if, in the same figure, B A P, B′A P_1, B′A P_2, B A P_3, are equal to one another, and therefore the lines P N, P_1 N′, P_2 N′, P_3 N, are equal in magnitude; as also are A N and A N.

If then we take account both of sign and magnitude,

$$\text{Sin. } B\,A\,P_1 = \frac{P_1 N'}{A\,P_1} = \frac{P\,N}{A\,P} = \text{sin. } B\,A\,P.$$

$$\therefore \ \text{sin. } (180° - A) = \text{sin. } A.$$

$$\text{Sin. } B\,A\,P_2 = \frac{P_2 N'}{A\,P_2} = \frac{-P\,N}{A\,P} = -\text{sin. } B\,A\,P.$$

$$\therefore \text{ sin. } (180° + A) = -\text{sin. } A.$$

$$\text{Sin. } B\,A\,P_3 = \frac{P_3 N}{A\,P_3} = \frac{-P\,N}{A\,P} = -\text{sin. } B\,A\,P.$$

$$\text{Sin. } (360° - A) = -\text{sin. } A.$$

In the same manner it is easy to show that

$$\text{Cos. } (180° - A) = -\text{cos. } A.$$

$$\text{Cos. } (180° + A) = -\text{cos. } A.$$

$$\text{Cos. } (360° - A) = \text{cos. } A.$$

For example, we have seen that cos. $60° = \frac{1}{2}$.

Cos. $120° = \text{cos. } (180° - 60°) = -\text{ cos. } 60° = -\frac{1}{2}$.

Similarly—

Sin. $315° = \text{sin. } (360° - 45°) = -\text{ sin. } 45° = -\frac{1}{2}$.

If we consider the case of the tangent of an angle, ∠A being an angle, $<90°$.

$$\text{Tan. } (180° - A) = \frac{\text{sin. } (180° - A)}{\text{cos. } (180° - A)} = \frac{+\text{sin. } A}{-\text{cos. } A} = -\text{tan. } A.$$

$$\text{Tan. } (180° + A) = \frac{\text{sin. } (180° + A)}{\text{cos. } (180° + A)} = \frac{-\text{sin. } A}{-\text{cos. } A} = \text{tan. } A.$$

$$\text{Tan. } (360° - A) = \frac{\text{sin. } (360° - A)}{\text{cos. } (360° - A)} = \frac{-\text{sin. } A}{\text{cos. } A} = -\text{tan. } A.$$

In the same manner we may express the other trigonometrical ratios of angles greater than 90° by means of those angles less than 90.

It is to be observed, that if we suppose A P to make one complete revolution from A P, it returns to its present position. So that A N and N P are the same both in magnitude and direction for the angle 360° + A as for A.

The same is true of any number of complete revolutions.

Hence, if f denote any trigonometrical ratio whatever,

$$f\,(n\,360° + A) = f\,(A),$$

where n is any positive integer whatever. Thus,

$$\text{Sin. } (n\,360° + A) = \text{sin. } (A)$$

$$\text{Cos. } (n\,360° + A) = \text{cos. } (A)$$

And so on.

There are a great variety of relation similar to those above deduced. The following are worth notice:—

We have before stated that if B A P = A, then P_3 A B = − A.

$$\therefore \text{sin. } (-A) = \text{sin. } B\,A\,P_3 = \frac{NP_3}{AP_3}$$

$$= -\frac{PN}{AP} = -\text{ sin. } A.$$

$$\therefore \text{ sin. } (-A) = -\text{ sin. } A.$$

Similarly,

$$\text{Cos. } (-A) = \text{cos. } A,$$

$$\text{and } \therefore \text{ tan. } (-A) = \text{tan. } A.$$

$$\text{and } \quad \text{cot. } (-A) = -\text{ cot. } A.$$

This result can be arrived at by reference to formulas previously proved.

We have seen that under all circumstances

$$f\,(n.360° + A) = f\,(A).$$

$$\therefore \text{ sin. } (360° - A) = \text{sin. } (-A).$$

But we have also seen that

$$\text{sin. } (360° - A) = -\text{ sin. } A;$$

$$\therefore \text{sin. } (-A) = -\text{ sin. } A$$

14. *On the Magnitudes of the Trigonometrical Functions of Angles*, 0°, 90°, 180°, 270°.

The definition of sine B A P tells us that

$$\text{sin. } A = \frac{PN}{AP}.$$

Now if A P coincide with A B, A = 0 and P N = 0, ∴ sin. 0° = 0.

If A P revolve round A, P N increases until A P coincides with A C, when A = 90°, and P N = A P.

$$\therefore \text{ sin. } 90° = 1.$$

After which, as A P revolves towards A B′, P N decreases until at A B coincides with A B^1, when P N = 0, A = 180°, and therefore

$$\text{Sin. } 180° = 0.$$

As A P revolves from A B′ towards A C, P N increases negatively until A P coincides with A C, when A = 270° and P N = − A P.

$$\therefore \text{ sin. } 270° = -1.$$

As A P revolves from A C′ towards its original position A B, P N decreases negatively, until when A P coincides with A B, we have A = 360°, and P N vanishes.

$$\therefore \text{ sin. } 360° = 0.$$

In like manner, if we take the cosine we have

$$\text{Cos. } A = \frac{AN}{AP}.$$

Now when A = 0 A N = A P ∴ cos. 0° = 1.
A = 90° A N = 0 ∴ cos. 90° = 0.
A = 180° A N = − A P cos. 180° = − 1.
A = 270° A N = 0 cos. 270° = 0.
A = 360 A N = A P cos. 360° = 1.

Now $\tan. A = \dfrac{\sin. A}{\cos. A}$

$$\therefore \tan. 0^\circ = \frac{\sin. 0^\circ}{\cos. 0^\circ} = \frac{0}{1} = 0$$

$$\tan. 90^\circ = \frac{\sin. 90^\circ}{\cos. 90^\circ} = \frac{1}{0} = \infty$$

$$\tan. 180^\circ = \frac{\sin. 180^\circ}{\cos. 180^\circ} = \frac{0}{1} = 0$$

$$\tan. 270^\circ = \frac{\sin. 270^\circ}{\cos. 270^\circ} = \frac{1}{0} = \infty$$

$$\tan. 360^\circ = \frac{\sin. 360^\circ}{\cos. 360^\circ} = \frac{0}{1} = 0$$

If we apply similar reasoning to the various trigonometric functions to that employed in discussing the variations of the sine of A, we obtain results which may be arranged in a tabular form, as follows :—

QUADRANT.

Line.	0°	< 90° > 0	90°	< 180° > 90°	180°	< 270° > 180°	270°	< 360° > 270°	360°
Sine	0	+	1	+	0	—	—1	—	0
Cosine	1	+	0	—	—1	—	0	+	1
Tangent	0	+	∞	—	0	+	∞	—	0
Cotangent . . .	∞	+	0	—	∞	+	0	—	∞
Secant	1	+	∞	—	—1	—	∞	+	1
Cosecant	∞	+	1	+	∞	—	—1	—	∞

The student will do well to verify carefully all the results given in this table ; he will also observe that the trigonometrical ratios illustrate the principle that if a function of a variable changes its sign, it must pass through the values of either zero (.0) or infinity (∞).

As the values of the ratios are continuous, the ratios increase gradually to their greatest value, and then decrease to their least. Thus, to take the case of the sine of an angle, which we call θ.

Sine θ increases from 0, when $\theta = 0^\circ$ up to 1, when $\theta = 90^\circ$. It then decreases to 0, when $\theta = 180^\circ$; after which it still further decreases till it equals -1, when $\theta = 270^\circ$, and finally increases up to 0, when $\theta = 360^\circ$.

15. *To determine all the angles which have the same sine, or cosine.*

There is another class of questions presented to us by this extension of our definition of an angle, *viz.*, having given a trigonometrical ratio of an angle, to find all the angles corresponding to it.

For example, $\tan. \theta = p$.

Now, if we did not reckon any angles but those less than 180°, as is the case in geometry, we could only have one value of θ corresponding to a given value of $\tan. \theta$. Suppose this value $= a$.

Then, if we take the trigonometrical or generalised conception of an angle, we shall have another $= 180^\circ + a$.

And since no trigonometrical ratio changes either its value or its sign when its angle is increased by any multiple of 360°, it is plain that in addition to a we shall have a series of values, $360 + a$, $2 \times 360^\circ + a$, $3 \times 360^\circ + a + \ldots.$, $n\,360^\circ + a$; and in addition to the value $180^\circ + a$, we shall have a series of values, $360^\circ + 180^\circ + a$, $2 \times 360^\circ + 180^\circ + a$, $3 \times 360^\circ + 180^\circ + a$, $\ldots.\ n\,360^\circ + 180^\circ + a$. Both these series may be included in one formula,

$$\theta = m\,180^\circ + a,$$

where m is any integer number whatever.

In the same manner, if

$$\text{Cos. } \theta = q,$$

and a be the value of θ less than 180°, which has for its cosine q, then all the values of θ which have a cosine q are included in the formula,

$$\theta = m.\ 360^\circ \pm a,$$

where m is any integer.

And similarly if

$$\text{Sin. } \theta = q,$$

the value of θ is included in the formulas—

$$\theta = 2m\ 180^\circ + a,$$

$$\text{and } \theta = (2m + 1)\ 180^\circ - a,$$

m being any integer whatever.

These may be included in one formula, as follows :—

$$\theta = k.180^\circ + (-1)^k a,$$

where k is any integer whatever ; for when k is even, $(-1)^k$ is positive ; and when k is uneven, $(-1)^k$ is negative.

As practice in the preceding articles, the student may verify the following results :

1. $\text{Sin. } 180^\circ = 0.$
2. $\text{Sin. } 135 = \dfrac{1}{\sqrt{2}}$
3. $\text{Cos. } 135^\circ = \dfrac{1}{\sqrt{2}} = \cos.\ 225^\circ.$
4. $\text{Cos. } 315^\circ = \dfrac{1}{\sqrt{2}} = \sin.\ 405^\circ.$
5. $\text{Sin. } 120^\circ = \dfrac{1}{2} = -\sin.\ 300^\circ.$
6. $\text{Cos. } 300^\circ = \dfrac{\sqrt{3}}{2} = -\cos.\ 150^\circ.$
7. $\text{Sin. } 150^\circ = \dfrac{\sqrt{3}}{2}.$
8. $\text{Cos. } 300^\circ = \dfrac{1}{2}.$

Given that sine $\theta = \dfrac{1}{2}$, show that the following are the values of θ which satisfy that equation, 30°, 150°, 390°, 510°, 750°, 870°, &c.

Given that cos. $\theta = \dfrac{1}{\sqrt{2}}$, show that the following are the values of θ which satisfy this equation, 45°, 315°, 405°, 675°, 765°, &c.

Given that tan. $\theta = 1$, show that the values of θ are 45°, 225°, 405°, 585°, 765°, &c.

THE RELATIONS BETWEEN THE TRIGONOMETRICAL FUNCTIONS OF DIFFERENT ANGLES.

The formulas we have already proved hold good of the ratios of the same angle ; we now proceed to investigate the formulas which express the relations between two or more different angles. There is a very great variety of formulas of this kind, and they admit of an almost infinite number of combinations and modifications. They are, however, all derived mediately or immediately from the following four :—

$\text{Sin. } (A + B) = \sin. A, \cos. B + \sin. B, \cos. A. \ldots.$ (8).
$\text{Sin. } (A - B) = \sin. A, \cos. B - \sin. B, \cos. A. \ldots.$ (9).
$\text{Cos. } (A + B) = \cos. A, \cos. B - \sin. A, \sin. B. \ldots.$ (10).
$\text{Cos. } (A - B) = \cos. A, \cos. B + \sin. A, \sin. B. \ldots.$ (11).

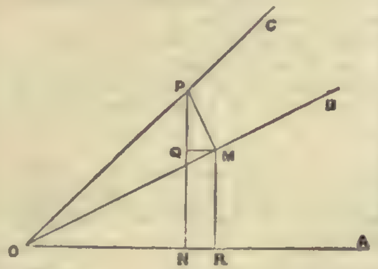
These four formulas can be easily remembered—and it is of great importance that they should be remembered

—by observing that the sine of the sum of two angles is the sum of the product of the sine of the first angle and the cosine of the second, and of the product of the sine of the second angle and the cosine of the first; while the sine of the difference of the angles is the former product minus the latter. The cosine of the sum of two angles is the product of the cosines of those angles minus the product of their sines; and the cosine of the difference of two angles is the sum of the product of the cosines and of the product of the sines.

16. *To prove the Formula*

Sin. (A + B) = sin. A cos. B + sin. B cos. A.

Fig. 12.



Let AOB be the angle A, BOC the angle B; ∴ A O C is the angle A+B. In O C take any point P, and from P let fall PN, PM, perpendiculars on O A and O B, and from M let fall M Q and M R, perpendiculars on PN and O R.

Then—

$$\text{sin. }(A+B)=\frac{PN}{QP}=\frac{QN+PQ}{OP}=\frac{QN}{OP}+\frac{PQ}{OP}$$

$$=\frac{MR}{OP}+\frac{PQ}{OP}=\frac{MR}{OM}\cdot\frac{OM}{OP}+\frac{PQ}{PM}\cdot\frac{PM}{OP}$$

Now $\frac{MR}{ON}=$ sin. A and $\frac{OM}{OP}=$ cos. B

And since PMO is a right angle, QPM = QMO = MOR,

$$\therefore\ \frac{PQ}{PM}=\frac{OR}{OM}=\text{cos. }A=\frac{PM}{OP}=\text{sin. }B\ ;$$

∴ sin. (A + B) = sin. A. cos. B + sin. B, cos. A.

17. *To prove the Formula*

Cos. (A + B) = cos. A. cos. B − sin. A, sin. B.

For as before,

$$\text{Cos. }(A+B)=\frac{ON}{OP}=\frac{OR-QM}{OP}=\frac{OR}{OP}-\frac{QM}{OP}$$

$$=\frac{OR}{OM}\cdot\frac{OM}{OP}-\frac{QM}{PM}\cdot\frac{PM}{OP}$$

Now $\frac{OR}{OM}=$ cos. A. $\frac{OM}{OP}=$ cos. B,

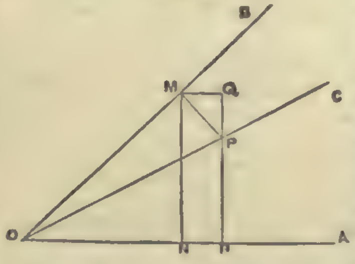
And $\frac{QM}{PM}=\frac{MR}{OM}=$ sin. A. $\frac{PM}{OP}=$ sin. B;

∴ cos. (A + B) = cos. A. cos. B − sin. A. sin. B.

18. *To prove the Formula*

Sin. (A − B) = sin. A. cos. B − sin. B. cos. A.

Fig. 13.



Let A O B = A. B O C = B. Take in O C any point P; from P draw P M and P R perpendicular to O B and O A. Draw M N and M Q perpendicular to O A and M N. Then A O C = A − B.

$$\therefore\ \text{sin. }(A-B)=\frac{PR}{OP}=\frac{QN-QP}{OP}=\frac{MR}{OP}-\frac{QP}{OP}$$

$$=\frac{MR}{OM}\cdot\frac{OM}{OP}-\frac{QP}{PM}\cdot\frac{PM}{OP}.$$

Now $\frac{MR}{OM}=$ sin. A. $\frac{OM}{OP}=$ cos. B.

Again ∠ M P Q = Q M B = B O A = A.

$$\therefore\ \frac{PQ}{MP}=\frac{QR}{OM}=\text{cos. }A.\ \frac{PM}{OP}=\text{sine }B.$$

∴ sin. (A − B) = sin. A. cos. B − sin. B. cos A.

19. *To prove the Formula*

Cos. (A − B) = cos. A. cos. B + sin. A. sin. B.

$$\text{cos. }(A-B)=\frac{ON}{OP}=\frac{OR}{OP}+\frac{MQ}{OP}=\frac{OR}{OM}\cdot\frac{OM}{OP}+\frac{MQ}{MP}\cdot\frac{MP}{OP}$$

But $\frac{OR}{OM}=$ cos. A. $\frac{OM}{OP}=$ cos. B. $\frac{MQ}{MP}=$ sin. A and $\frac{MP}{OP}=$ sin. B.

∴ cos. (A − B) = cos. A. cos. B + sin. A. sin. B.

20. *To extend the above proofs in special cases.*

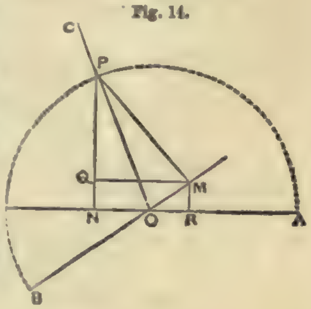
The proofs above given are clearly limited to the cases in which A, B, and A + B or A − B, are each less than 90°. They admit of extension to any case whatever. Thus—

If A is $> 180^\circ < 270^\circ$, B $> 90^\circ < 180^\circ$, and A − B $> 90^\circ < 180^\circ$.

To show that

Sin. (A − B) = sin. A cos. B − sin. B cos. A.

Fig. 14.



In this case the figure will be the following:—

A O B is the angle A, measured as indicated by the dotted circle. B O C is the angle B. ∴ C O A is the angle A − B.

From any point P in O C draw P N, P M perpendicular to O A and O B produced, and from M draw M Q, M R perpendicular P N and O A.

Then

$$\text{Sin. }(A-B)=\frac{PN}{PO}=\frac{PQ}{PO}+\frac{QN}{PO}=\frac{PQ}{PO}$$

$$+\frac{MR}{PO}=\frac{PQ}{PM}\cdot\frac{PM}{PO}+\frac{MR}{MO}\cdot\frac{POM}{PO}$$

Now $\frac{PQ}{PM}$ cos. M P Q = cos. A O M = cos. (A − 180°)

= cos. (180° − A) = − cos. A.

$\frac{PM}{PO}=$ sin. P O M = sin. (180° − B) = sin. B.

Similarly $\frac{MR}{MO}=$ sin. M O A = − sin. A

$\frac{OM}{PM}=$ cos. P O M = − cos. B;

∴ Sin. (A − B) = sin. A. cos. B − sin. B. cos. A.

In the same case,

$$\text{cos. }(A-B)=-\frac{ON}{OP}=-\frac{NR-OR}{OP}$$

$$=-\frac{QM}{OP}+\frac{OR}{OP}=-\frac{MQ}{MP}\cdot\frac{MP}{OP}+\frac{OR}{OM}\cdot\frac{OM}{OP}.$$

Now $\frac{MQ}{MP}=$ sin. MPQ = sin. MOA = − sin. A,

$\frac{MP}{OP}=$ sin. POM = sin. POB = sin. B.

$\frac{OR}{OM}=$ cos. MOA = − cos. A.

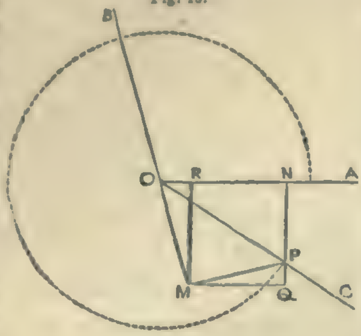
$\frac{OM}{OP}=$ cos. POM = − cos. B.

∴ cos. (A − B) = cos. A cos. B + sin. A sin. B.

Again, to show that sin. (A + B) = sin. A cos. B + sin. B cos. A.

When A + B > 270° ∠ 360°   A > 90° ∠ 180°
B > 180° ∠ 270°.

Fig. 15.



Let A O B be the angle A; B O C be the angle B. Then A O C is the angle A + B, B O C, and therefore A O C, being measured as indicated by the dotted circles. In C take any point P and draw P N, P M perpendiculars to O A and O B produced, and from M draw M R and M Q perpendiculars to O A and N P produced.

Then

$$\text{Sine }(A+B)=-\frac{PN}{OP}=\frac{PQ-QN}{OP}$$

$$=\frac{PQ}{OP}-\frac{RM}{OP}=\frac{PQ}{MP}\cdot\frac{MP}{OP}+\frac{RM}{OM}\cdot\frac{OM}{OP}$$

$$\text{Now }\frac{PQ}{OM}=\cos.\ MPQ=\cos POA=-\cos A.$$

$$\frac{MP}{OP}=\sin.\ POM=-\sin.\ B.$$

$$\frac{RM}{OM}=\sin.\ AOM=\sin.\ A.$$

$$\frac{OM}{OP}=\cos.\ POM=-\cos.\ B.$$

$$\therefore \sin.\ (A+B)=\sin.\ A.\cos.\ B+\sin.\ B.\cos.\ A.$$

The student may verify for practice the formulas in the following cases :—

Sin. (A + B) A ⁊ 90° ∠ 180° A + B ⁊ 180° B ⁊ 90°
Cos. (A − B) A ⁊ 90° ∠ 180° A + B ∠ 90° B ⁊ 90°
Cos. (A + B) A ⁊ 180° ∠ 270° A + B ⁊ 360° ∠ 450°
B ⁊ 90° ∠ 180°

21. *The principle on which the proof may be considered as established generally.*

The above examples will be sufficient to satisfy the reader that the four formulas above given hold good for all angles whatever. It is worth while to observe, however, that, independently of these examples, this follows from the circumstance that the extension given above to the definitions of angular magnitude, and of the trigonometric ratios, are made in strict accordance with the extension given to the meaning of the negative sign in algebra. Thus, $a-b$ primarily signifies that the number b is to be subtracted from the number a; if therefore b be greater than a, $a-b$ is impossible, unless we generalise the definition of the negative sign. If we do this so as to render $a-b$ susceptible of meaning for all values of a and b, then whatever theorem we prove to be true of $a-b$, and its combinations with the restriction, will be equally true of $a-b$, and its combinations without the restriction.

In like manner, if we prove a trigonometrical formula to hold good for all geometrical angles, these will equally hold good of the angles when defined according to the trigonometrical conception of an angle explained above. The principle which we have to guide us in all these generalisations, is called "The Principle of the Permanence of Equivalent Forms," and is that which lies at the root of all extensions of merely Arithmetical Algebra, as explained in the treatise on Logarithms and Series. (See Chapter VI., p. 512, in this section). The reader who wishes to see a full account of the application of this principle to Trigonometry, will do well to consult Dr. Peacock's *Algebra*, vol. ii., p. 144, &c., 2nd edition.

22. *Relation between the Four Fundamental Formulas.*

It is to be observed that the last three of the four formulas given above can be derived from the first of them.

Thus, $\sin.\ (A+B)=\sin.\ A\cos.\ B+\sin.\ B\cos.\ A.$
For B write $-B$. Now $\sin.\ (-B)=-\sin.\ B.$
And $\cos.\ (-B)=-\cos.\ B.$

$$\therefore \sin.\ (A-B)=\sin.\ A\cos.\ B-\sin.\ B\cos.\ A.$$

Again

$$\begin{aligned}\cos.\ (A+B)&=\sin.\ (90^\circ-A-B)\\&=\sin.\ (90^\circ-A)\cos.\ B-\sin.\ B\cos.\ (90^\circ-A)\\&=\cos.\ A\cos.\ B-\sin.\ A\sin.\ B.\end{aligned}$$

Again

$$\begin{aligned}(\text{Cos. }(A-B)&=\sin.\ (\overline{90^\circ-A}+B)\\&=\sin.\ (90^\circ-A)\cos.\ B+\cos.\ (90^\circ-A)\\&=\cos.\ A\cos.\ B+\sin.\ A\sin.\ B.\end{aligned}$$

23. *Formulas derived from the Fundamental ones.*

From these four the following formulas of frequent occurrence can easily be derived :—

$$\text{Sin. }A\cos.\ B+\sin.\ B\cos.\ A=\sin.\ (A+B)$$

$$\text{Sin. }A\cos.\ B-\sin.\ B\cos.\ A=\sin.\ (A-B).$$

∴ adding $2\sin.\ A\cos.\ B=\sin.\ (A+B)+\sin.\ (A-B)$; and subtracting $2\cos.\ A\sin.\ B=\sin.\ (A+B)-\sin.\ (A-B)$.

Similarly,

$$2\sin.\ A\sin.\ B=\cos.\ (A-B)-\cos.\ (A+B)\ldots.\quad(12)$$

$$2\cos.\ A\cos.\ B=\cos.\ (A-B)+\cos.\ (A+B)\ldots.\quad(13)$$

The same formulas are of frequent occurrence in a different form. Evidently

$$\theta=\frac{\theta+\phi}{2}+\frac{\theta-\phi}{2}$$

$$\phi=\frac{\theta+\phi}{2}-\frac{\theta-\phi}{2}$$

$$\therefore \sin.\ \theta=\sin.\frac{\theta+\phi}{2}\cos.\frac{\theta-\phi}{2}+\sin.\frac{\theta-\phi}{2}\cos.\frac{\theta+\phi}{2}$$

$$\sin.\ \phi=\sin.\frac{\theta+\phi}{2}\cos.\frac{\theta-\phi}{2}-\sin.\frac{\theta-\phi}{2}\cos.\frac{\theta+\phi}{2}$$

$$\therefore \sin.\ \theta+\sin.\ \phi=2\sin.\frac{\theta+\phi}{2}\cdot\cos.\frac{\theta-\phi}{2}\quad\ldots.(14).$$

$$\sin.\ \theta-\sin.\ \phi=2\cos.\frac{\theta+\phi}{2}\cdot\sin.\frac{\theta-\phi}{2}\quad\ldots.(15).$$

Similarly,

$$\text{Cos. }\theta+\cos.\ \phi=2\cos.\frac{\theta+\phi}{2}\cdot\cos.\frac{\theta-\phi}{2}\quad\ldots.(16).$$

$$\text{Cos. }\phi-\cos.\ \theta=2\sin.\frac{\theta+\phi}{2}\cdot\cos.\frac{\theta-\phi}{2}\quad\ldots.(17).$$

24. *Formula for the Tangent of the Sum of Two Angles.*

Again, we can easily derive from the formulas for the sines and cosines of A + B and A — B, expressions for the tangents of A + B and A − B. Thus,

$$\text{Tan. }(A+B)=\frac{\sin.\ (A+B)}{\cos.\ (A+B)}$$

$$=\frac{\sin.\ A\cos.\ B+\sin.\ B\cos.\ A}{\cos.\ A\cos.\ B-\sin.\ A\sin.\ B}$$

Divide both numerator of this fraction by cos. A cos. B.

$$\therefore \tan.\ (A+B)=\frac{\dfrac{\sin.\ A\cos.\ B}{\cos.\ A\cos.\ B}+\dfrac{\sin.\ B\cos.\ A}{\cos.\ B\cos.\ A}}{\dfrac{\cos.\ A\cos.\ B}{\cos.\ A\cos.\ B}-\dfrac{\sin.\ A\sin.\ B}{\cos.\ A\cos.\ B}}$$

$$=\frac{\dfrac{\sin.\ A}{\cos.\ A}+\dfrac{\sin.\ B}{\cos.\ B}}{1-\dfrac{\sin.\ A\sin.\ B}{\cos.\ A\cos.\ B}}=\frac{\tan.\ A+\tan.\ B}{1-\tan.\ A\tan.\ B}\quad\ldots.(18).$$

Similarly,

$$\text{Tan. }(A-B)=\frac{\tan.\ A-\tan.\ B}{1-\tan.\ A\tan.\ D}\quad\ldots.(19).$$

25. *Expressions in which the Sum of Three Angles occur.*

We can easily derive from the above, expressions for the sines, cosines, &c., of the sum of three or more angles. Thus,

$\text{Sin. }(A+B+C)=\sin.\ (A+B)\cos.\ C+\cos.\ (A+B)\sin.\ C=(\sin.\ A\cos.\ B+\sin.\ B\cos.\ A)\cos.\ C+(\cos.\ A\cos.\ B-\sin.\ A\sin.\ B),\sin.\ C=\sin.\ A\cos.\ B\cos.\ C+\sin.\ B\cos.\ C\cos.\ A+\sin.\ C\cos.\ A\cos.\ B-\sin.\ A\sin.\ B\sin.\ C.$

In the same manner,

Cos. (A + B + C) = cos. A cos. B cos. C — cos. A sin. B. sin. C — cos. B sin. C sin. A — cos. C sin. A sin. B.

And hence,

$$\text{Tan. }(A+B+C) = \frac{\text{tan. }A + \text{tan. }B + \text{tan }C - \text{tan. }A\text{ tan. }B\text{ tan. }C}{1 - \text{tan. }B\text{ tan. }C - \text{tan. }C\text{ tan. }A - \text{tan. }A\text{ tan. }B.}$$

26. *Certain other Formulas.*

Again, since—

$$\text{Sin. }\theta + \text{sin. }\phi = 2\text{ sin.}\frac{\theta+\phi}{2}\cdot\text{ cos. }\frac{\theta-\phi}{2}.$$

$$\text{Sin. }\theta - \text{sin. }\phi = 2\text{ cos.}\frac{\theta+\phi}{2}\cdot\text{sin.}\frac{\theta-\phi}{2}.$$

$$\therefore\ \frac{\text{Sin. }\theta + \text{sin. }\phi}{\text{Sin. }\theta + \text{sin. }\phi} = \frac{\text{sin. }\frac{\theta+\phi}{2}}{\text{cos. }\frac{\theta+\phi}{2}}\cdot\frac{\text{cos. }\frac{\theta-\phi}{2}.}{\text{sin. }\frac{\theta-\phi}{2}.}$$

$$= \text{tan. }\frac{\theta+\phi}{2}\cdot\text{ cotan. }\frac{\theta-\phi}{2}\ldots\ldots(20).$$

In like manner,

$$\frac{\text{Cos. }\phi - \text{cos. }\theta}{\text{Cos. }\phi + \text{cos. }\theta} = \text{tan. }\frac{\theta+\phi}{2}\text{tan.}\frac{\theta-\phi}{2}\ldots\ldots(21).$$

There are many similar combinations of the trigonometric ratios besides those above given. These are of very frequent occurrence, and the student who has thoroughly mastered the above will be at no loss in investigating other combinations that may occur in his subsequent reading.

27. *The Sines, Cosines, &c., of Multiples of given Angles.*

We have already seen that

Sin. (A + B) = sin. A, cos. B + sin. B, cos. A.

This being true of all values of A and B is true when A = B, and ∴ when A + B = 2A.

$$\therefore\ \text{Sin. }2A = 2\text{ sin. }A,\text{ cos. }A\ \ldots\ (22).$$

Similarly, since

$$\text{Cos. }(A+B) = \text{cos. }A,\text{ cos. }B - \text{sin. }A,\text{ sin. }B.$$

$$\therefore\ \text{Cos. }2A = \text{cos.}^2A - \text{sin.}^2A.\ \ldots\ldots\ (23).$$

$$\text{Now, }1 = \text{cos.}^2A + \text{sin.}^2A.$$

Add this equation to (23), and we obtain

$$\text{Cos. }2A = 2\text{ cos.}^2A - 1.$$

$$\text{And Cos. }2A = 1 - 2\text{ sin.}^2A.$$

Similarly,

$$\text{Tan. }2A = \frac{2\text{ tan. }A}{1-\text{tan.}^2A}\ldots\ldots\ (24)$$

Again,

$$\begin{aligned}\text{Sin. }3A &= \text{sin. }(2A + A).\\ &= \text{sin. }2A,\text{ cos. }A + \text{cos. }2A,\text{ sin. }A\\ &= 2\text{ sin }A,\text{ cos. }A,\text{cos.}A + (\text{Cos.}^2A - \text{sin.}^2A)\text{sin. }A.\\ &+ 3\text{ sin. }A,\text{ cos}^2A - \text{sin.}^3A.\\ &= 3\text{ sin. }A - 4\text{ sin. }^3A.\ \ldots\ldots\ (25).\end{aligned}$$

Similarly,

$$\text{Cos. }3A = 3\text{ Cos. }A + 4\text{ cos.}^3A.\ \ldots\ldots\ (26),$$

and

$$\text{Tan. }3A = \frac{3\text{ tan. }A - \text{tan.}^3A}{1-3\text{ tan.}^2A}\ldots\ldots\ (27).$$

28. *Determination of Sine, &c., of an Angle in terms of the Sine, &c., of the Sub-multiples of that Angle.*

From these expressions we may derive others expressing the sines, &c., of an angle in terms of the sines, &c. of the submultiples of that angle. Thus, writing $\frac{\theta}{2}$ for A, we have

$$\text{From (22)}\quad \text{Sin. }\theta = 2\text{ sin.}\frac{\theta}{2}\text{ cos. }\frac{\theta}{2}.$$

$$\text{From (23)}\quad \text{Cos. }\theta = \text{cos.}^2\frac{\theta}{2} - \text{sin.}^2\frac{\theta}{2}$$

$$= 1 - 2\text{ sin.}^2\frac{\theta}{2}$$

$$= 2\text{ cos.}^2\frac{\theta}{2} - 1.$$

$$\text{From (24)}\quad \text{Tan. }\theta = \frac{2\text{ tan.}\frac{\theta}{2}}{1-\text{tan.}^2\frac{\theta}{2}}.$$

And writing ϕ for $\frac{A}{3}$ we have

$$\text{From (25)}\quad \text{Sin. }\phi = 3\text{ sin. }\frac{\phi}{3} - 4\text{ sin.}^3\frac{\phi}{3}.$$

$$\text{From (26)}\ \text{Cos. }\phi = -3\text{ cos. }\frac{\phi}{3} + 4\text{ cos.}^3\frac{\phi}{3}$$

$$\text{From (27)}\quad \text{Tan. }\phi = \frac{3\text{ tan. }\frac{\phi}{3} - \text{tan.}^3\frac{\phi}{3}}{1-3\text{ tan.}^2\frac{\phi}{3}}$$

29. *On the "Ambiguities" resulting from the use of the above Formulas.*

These formulas enable us to solve the following question :—Having given the sine, &c., of an angle, we can find from it the sine, &c., of double that angle ; and conversely having given the sine, &c., of an angle, we can find the sine, &c., of half that angle.

(*a*). Thus, having given sin. A = p, to find cos. 2 A, we have

$$\text{Cos. }2A = 1 - 2\text{ sin.}^2A = 1 - 2p^2$$

and so, having given sin. A = p to find sin. 2 A.

Since sin. A = p cos. A = $\sqrt{1-p^2}$

$\therefore$ sin. 2 A = $\pm 2p\sqrt{1-p^2}$.

It will be seen, from the above formulas, that for one given value (p) of sin. A, there is one value of cos. 2 A, while there are two of sine 2 A equal in magnitude, but of different signs. This is sometimes spoken of as an ambiguity. It will be observed, however, that the ambiguity in the determination of sin. 2 A arises necessarily from the data, since it appears by considering the values of A which satisfy the equation.

$$\text{Sin. }A = p,$$

that there will be one value of cos. 2 A and two values of sin. 2 A resulting from the data. Thus, if A^1 be one angle which satisties the equation, then all the values of A are included in the two formulas.

$$A = 2m\,180^\circ + A^1$$
$$\text{and }A = (2m+1)\,180^\circ + A^1$$

m being any integer whatever ;

$$\therefore\ \text{cos. }2A = \text{cos. }(2m\,360^1 + 2A') = \text{cos. }2A^1,$$
$$\text{or} = \text{cos. }(2m+1)\,360^\circ - 2A^1) = \text{cos. }2A^1$$

Under all circumstances, therefore,

$$\text{Cos. }2A = \text{cos. }2A^1,$$

and therefore has but one value. Whereas

$$\text{Sin. }2A = \text{sin. }(2m\,360^1 + 2A^1) = \text{sin. }2A^1.$$
$$\text{or} = \text{sin. }\{(\overline{2m+1})\,360^\circ - 2A^1)\} = -\text{sin. }2A^1.$$

$\therefore$ sin. 2 A has the two values + sin. 2 A^1 and −1 sin. 2 A^1.

If, however, we know A, or even the limits between which A lies, as well as that sin. A = p, then all indeterminateness vanishes from the expression for sin. 2 A. Thus, if A be less than 90°, then 2 A is $\angle$ 180° and sin° 2 A must be positive. And, again, if A $\not>$ 90° $\angle$ 180°, then 2 A $\not>$ 180° $\angle$ 360°, and the sin. 2 A is negative in the former case ; therefore,

$$\text{Sin. }2A = 2p\sqrt{1-p^2}.$$

In the latter,

$$\text{Sin. }2A = -2p\sqrt{-1p^2}.$$

(*b*). Again, we have

$$\text{Cos. } \theta = 1 - 2 \text{ sin. } \frac{{}^2\theta}{2} = 2 \text{ cos. } \frac{{}^2\theta}{2} - 1.$$

$$\therefore \text{sin. } \frac{\theta}{2} = \sqrt{\frac{1 - \text{cos. } \theta}{2}}$$

$$\text{Cos. } \frac{\theta}{2} = \sqrt{\frac{1 + \text{cos. } \theta}{2}}$$

Hence it appears that for a given value of cos. θ there will be two values of sin. $\frac{\theta}{2}$, likewise two values of cos. $\frac{\theta}{2}$ equal in magnitude, but with different signs. This is, as before, necessarily the case, if we only know the value of cos. θ. For if we have a given value p of cos. θ, so that

$$\text{Cos. } \theta = p,$$

and if θ^1 be a value of θ which satisfies this equation, then all the values of θ, which satisfy this equation, are expressed by,

$$\theta = m\,(360^\circ \pm \theta^1).$$

$$\therefore \text{sin. } \frac{\theta}{2} = \text{sin. } (m\ 180^\circ \pm \frac{\theta^1}{2})$$

$$\text{which} = \pm \text{ sin. } \frac{\theta^1}{2} \text{ if } m \text{ be even,}$$

$$\text{or, } \mp \text{ sin. } \frac{\theta^1}{2} \text{ if } m \text{ be odd,}$$

under any circumstances; therefore there must be two values of sin. $\frac{\theta}{2}$ equal in magnitude, but with different signs; the same result as that we obtained from the equations for sin. $\frac{\theta}{2}$ and cos. $\frac{\theta}{2}$

(*c*). Again, we have given sin. θ to find sin. $\frac{\theta}{2}$ and cos. $\frac{\theta}{2}$

We have—

$$\text{sin. }^2 \frac{\theta}{2} + \text{cos. }^2 \frac{\theta}{2} = 1$$

$$2 \text{ sin. } \frac{\theta}{2} \text{ cos. } \frac{\theta}{2} = \text{sin. } \theta$$

Adding these equations we obtain,

$$\text{sin. }^2 \frac{\theta}{2} + 2 \text{ sin. } \frac{\theta}{2} \text{ cos. } \frac{\theta}{2} + \text{cos. }^2 \frac{\theta}{2} = 1 + \text{sin. } \theta$$

And, subtracting the second from the first,

$$\text{sin. }^2 \frac{\theta}{2} - 2 \text{ sin. } \frac{\theta}{2} \text{ cos. } \frac{\theta}{2} + \text{cos. }^2 \frac{\theta}{2} = 1 - \text{sin. } \theta$$

Extracting the square root of each of these equations, and we have—

$$\text{sin. } \frac{\theta}{2} + \text{cos. } \frac{\theta}{2} = \sqrt{1 + \text{sin.}}$$

$$\text{sin. } \frac{\theta}{2} - \text{cos. } \frac{\theta}{2} = \sqrt{1 - \text{sin. } \theta}$$

$\therefore$ Adding—

$$2 \text{ sin. } \frac{\theta}{2} = \sqrt{1 + \text{sin. } \theta} + \sqrt{1 - \text{sin. } \theta}$$

and subtracting—

$$2 \text{ cos. } \frac{\theta}{2} = \sqrt{1 + \text{sin. } \theta} - \sqrt{1 - \text{sin. } \theta}$$

$$\therefore \text{sin. } \frac{\theta}{2} = \tfrac{1}{2} \left\{ \sqrt{1 + \text{sin. } \theta} + \sqrt{1 - \text{sin. } \theta} \right\}$$

$$\text{and cos. } \frac{\theta}{2} = \tfrac{1}{2} \left\{ \sqrt{1 + \text{sin. } \theta} - \sqrt{1 - \text{sin. } \theta} \right\}$$

And since each square root has two sines, it follows that if we have given merely the value of sin. θ, *i. e.*, sin. $\theta = p$. we have *four* different values of sin. $\frac{\theta}{2}$ viz. $\tfrac{1}{2}(\sqrt{1+p.} + \sqrt{1-p.})$, $\tfrac{1}{2}(\sqrt{1+p.} - \sqrt{1-p.})$ $\tfrac{1}{2}(-\sqrt{1+p.} + \sqrt{1-p.})$, and $\tfrac{1}{2}(-\sqrt{1+p.} - \sqrt{1-p.})$

We may prove, as before, that this amount of indeterminateness is involved in the data: for if

$$\text{sin. } \theta = p,$$

and if θ' be a value of θ. which satisfies this equation, then all the values of θ which satisfy the equation, are given by the formulas.

$$\theta = 2\,m.\ 180^\circ + \theta$$
$$\text{and } \theta = (2\,m. + 1)\ 180^\circ - \theta'$$

where m is any whole number whatever.

$\therefore$ sin. $\frac{\theta}{2}$ may be either one of the two forms, as follows:—

Sin. $(m.180^\circ + \frac{\theta^\circ}{2})$ or sin. $(m.\ 180^\circ + 90 - \frac{\theta'}{2})$ or according as m is odd or even, *i. e.*, according as m is $2\,n$ or $2\,n + 1$.

One of the four—

$$\text{Sin. } (2\,n.\ 180^\circ + \frac{\theta'}{2}) \text{ sin. } (\overline{2\,n + 1.}\ 180^\circ + \frac{\theta'}{2})$$

$$\text{Sin. } (2\,n.\ 180^\circ + 90 - \frac{\theta'}{2}) \text{ or sin. } \overline{(2\,n + 1.}\ 180^\circ$$
$$+\ 90^\circ - \frac{\theta'}{2})$$

And these are respectively equal to

$$\text{Sin. } \frac{\theta'}{2},\ -\text{sin. } \frac{\theta'}{2},\ \text{cos. } \frac{\theta'}{2}, \text{ and } -\text{cos. } \frac{\theta'}{2}.$$

i. e., it may have one of the four different values—

$$\text{Sin. } \frac{\theta}{2},\ -\text{sin. } \frac{\theta'}{2}, \text{cos. } \frac{\theta'}{2},\ -\text{cos. } \frac{\theta'}{2}.$$

If, however, we know the limits within which the value of θ lies, this indeterminateness vanishes. Thus, if $\theta \urcorner 90^\circ \angle 180^\circ$, then $\frac{\theta}{2} \urcorner 45^\circ \angle 90^\circ$, and therefore sin. $\frac{\theta}{2}$ must be $\urcorner \frac{1}{\sqrt{2}} \angle 1$. and be positive. Hence, in this case,

$$\text{Sin. } \frac{\theta}{2} = \frac{1}{2} \{+\sqrt{1 + \text{sin. } \theta} + \sqrt{1 - \text{sin. } \theta}\}.$$

$$\text{For when } \theta = 90 \text{ this gives sin. } \frac{\theta}{2} = \frac{1}{\sqrt{2}}$$

$$\theta = 180 - \text{sin. } \frac{\theta}{2} = 1.$$

and between these limits the value of the formula continually increases.

If, however, $\theta \urcorner 270^\circ \angle 360^\circ$, we have

$$\text{Sin. } \frac{\theta}{2} = \frac{1}{2} \{-\sqrt{1 + \text{sin. } \theta} + \sqrt{1 - \text{sin. } \theta}\}.$$

The student will do well to verify this for himself.

The same kind of reasoning applies to the formula for cos. $\frac{\theta}{2}$.

(*d*). Again the formula

$$\text{Tan. } \theta = \frac{2 \text{ tan. } \frac{\theta}{2}}{1 - \text{tan}^2 \frac{\theta}{2}}$$

enables us to determine tan. $\frac{\theta}{2}$ when tan. θ is given. For we can easily put the equation into the form

$$\text{Tan. }^2 \frac{\theta}{2} + \frac{2}{\text{tan. } \theta}.\ \text{tan. } \frac{\theta}{2} = 1.$$

$$\therefore \text{ tan. } \frac{\theta}{2} + \frac{1}{\text{tan. } \theta} = \pm \frac{\sqrt{1 + \text{tan.}^2}}{\text{tan. } \theta}.\ \text{tan. } \frac{\theta}{2}$$

$$= \frac{1 \pm \sqrt{1 + \text{tan.}^2 \theta}}{\text{tan. } \theta}$$

It will be observed that for one value of tan. θ we have two values of tan. $\frac{\theta}{2}$. The ambiguity is involved in the data in this case as in others. For if

$$\tan.\ \theta = p.$$

then θ' being one value of θ, all the values of θ are included in the formula

$$\theta = m.\ 180^\circ + \theta'.$$

$\therefore$ according as m is odd or even, *i. e.*, according as m is equal to $2n$ or $2n+1$,

$$\frac{\theta}{2} = n\,180^\circ + \frac{\theta'}{2}$$

$$\text{or} \frac{\theta}{2} = n.\ 180^\circ + 90^\circ + \frac{\theta'}{2}$$

That is,

$$\text{Tan.}\ \frac{\theta}{2} = \tan.\ n\,180^\circ + \frac{\theta'}{2}$$

$$\text{or,} \qquad \tan.\ (n\,180^\circ + 90^\circ + \frac{\theta'}{2})$$

according as m is odd or even; and therefore

$$\tan.\ \frac{\theta'}{2} = \tan.\ \frac{\theta'}{2} \text{ or } \tan.\ (90^\circ + \frac{\theta'}{2})$$

$$= \tan.\ \frac{\theta'}{2} \text{ or} - \text{cotan.}\ \frac{\theta'}{2}$$

Two different values, as also appeared from the formula.

30. *On the Numerical Value of the Sines, Cosines, &c., of the Angles* 15°, 30°, 45°, 60°, 75°.

We have already investigated the value of the trigonometrical ratios of 45°, 30°, and 60°. By the aid of the above formulas we can investigate the values of many others. For example, of 15° and of 18°.

Sin. 15° = sin. (45° − 30°) = sin. 45° cos. 30° − sin. 30°, cos. 45°.

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}} \text{ which also equals cos. } 75^\circ,$$

since 75° = 90° − 15°.

Cos. 15° = cos. (45° − 30°) = cos. 45° cos. 30° + sin. 45° sin. 30°

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin.\ 75^\circ$$

Hence—

$$\text{Tan.}\ 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \text{cotan.}\ 75^\circ$$

$$\text{Cotan.}\ 15^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1}. = \tan.\ 75^\circ$$

$$\text{Sec.}\ 15^\circ = \frac{2\sqrt{2}}{\sqrt{3}+1}. = \text{cosec.}\ 75^\circ$$

$$\text{Cosec.}\ 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}. = \text{sec.}\ 75^\circ$$

The student will observe that this investigation of the ratios of an angle of 15°, together with those previously investigated, gives the ratios of the series of angles 15°, 30°, 45°, 60°, 75°, 90°.

31. *To investigate the Trigonometrical Ratios of an Angle of* 18°, 36°, 54°, 72°.

Since $\qquad 54^\circ = 90^\circ - 36^\circ,$

if we write θ for 18°, we have

$$3\theta = 90^\circ - 2\theta,$$

and therefore

$$\text{Cos.}\ 3\theta = \sin.\ 2\theta.$$

$$\therefore\ 4\,\text{Cos.}^3\theta - 3\cos.\ \theta = 2\sin.\ \theta,\ \cos.\ \theta.$$

$$\therefore\ 4\,\text{Cos.}^2\theta - 3\cos.\ \theta = 2\sin.\ \theta.$$

$$\therefore 1 - 4\sin.^2\theta = 2\sin.\ \theta.$$

$$4\,\text{Sin.}^2\theta + 2\sin.\ \theta = 1.$$

$$\therefore 4\,\text{Sin.}^2\theta + 2\sin.\ \theta + \frac{1}{4} = \frac{5}{4}.$$

$$\therefore 2\,\text{Sin.}\ \theta = \frac{-1 \pm \sqrt{5}}{2}.$$

$$\text{Sin.}\ \theta = \frac{-1 \pm \sqrt{5}}{4}.$$

This is a case of ambiguity similar to those above explained, and if we only had the equation

$$4\,\text{Sin.}^2\theta + 2\sin.\ \theta = 1,$$

we should have the two values of sin. θ just given, viz.,

$$\frac{-1+\sqrt{5}}{4} \text{ and } \frac{-1-\sqrt{5}}{4}.$$

But as we not only have the equation, but also know that $\theta = 18^\circ$, this enables us to choose the only admissible value.

$$\text{Sin.}\ 18^\circ = \frac{-1+\sqrt{5}}{4}.$$

For the other value of sin. θ being negative cannot be the sine of 18°. Hence,

$$\text{Cos.}^2 18^\circ = 1 - \sin.^2 18^\circ =$$

$$1 - \frac{3-1\sqrt{5}}{8} = \frac{5+\sqrt{5}}{8}$$

$$\therefore \cos.\ 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}.$$

$$\text{Hence, } \sin.\ 36^\circ = \sin.\ 2 \times 18^\circ = \frac{(\sqrt{5}-1)\sqrt{2\sqrt{5}+10}}{8}$$

$$\text{and } \cos.\ 36^\circ = \frac{\sqrt{5}+1}{4}.$$

and sin. 36° = cos. 54°, and sin. 18° = cos. 72°.

Hence we evidently can obtain the trig. ratios of the angles 18°, 36°, 54°, 72°.

Again

$$\text{Sin.}\ 3^\circ = (18^\circ - 15^\circ.)$$

$$= \sin.\ 18^\circ \cos.\ 15^\circ - \cos.\ 18^\circ \sin.\ 15^\circ.$$

$$= \frac{\sqrt{5}-1}{4} \times \frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{10+2\sqrt{5}}}{4}\,\frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$= \frac{(\sqrt{15}-1)(\sqrt{3}+1) - (\sqrt{3}-1)\sqrt{10+2\sqrt{5}}}{8\sqrt{2}}.$$

Hence we may clearly obtain numerical values for the trigonometrical ratios of the series of degrees, 3°, 6°, 9°, 12°, 15°, &c.

N.B.—In reading the preceding pages the student will have observed, that in many instances, when a method of reasoning has been applied to one case, it has been merely indicated that the same method is applicable to a similar case. In all these instances he will do well to *write out at full length* the reasoning in these similar cases. By this means he will ensure a thorough comprehension of this part of the subject, and become familiar with the various combinations that trigonometric ratios can form. In regard to this very subject, Dr. Peacock observes:—"It should be the first lesson of a student, in every branch of science, not to form his own estimate of the importance of elementary views and propositions, which are very frequently repulsive or uninteresting, and such as cannot be thoroughly mastered and remembered without a great sacrifice of time and labour." To assist in obtaining this familiarity he may perform the following exercises:—

(1). $\frac{1}{2} + \cos.\ 2\,A = 2\sin.\ (60^\circ - A)\sin.\ (60^\circ + A.)$
(Remember that cos. 120° = $-\frac{1}{2}$.)

(2). $4\cos.\ m\,\theta\cos.\ n\,\theta\cos.\ r\,\theta = \cos.\ (m+n+r)\,\theta + \cos.\ (m+n-r)\,\theta + \cos.\ (m-n+r)\,\theta + \cos.\ (m-n+r)\,\theta$
(Remember that cos. (A + B) + cos. (A − B) = 2 cos. A cos. B

(3). $4\sin.^2\theta\cos.\ \theta = \cos.\ \theta\cos.\ 3\,\theta.$

(4). $\text{Tan.}\ A + \tan.\ B = \frac{\sin.\ (A+B)}{\cos.\ A\cos.\ B}$

$$\left(\text{Remember that tan. A} = \frac{\text{sin. A}}{\text{cos. A}}\right).$$

(5). $\text{Tan. }^2\text{A} - \text{tan. }^2\text{B} = \dfrac{\text{sin. (A + B) sin. (A − B)}}{\text{cos. }^2\text{A cos. }^2\text{B.}}$

(Remember that $\text{tan. }^2\text{A} - \text{tan. }^2\text{B} = (\text{tan. A} + \text{tan. B})(\text{tan. A} - \text{tan. B.})$)

(6). If $\dfrac{m \text{ sin. } \beta}{n \text{ sin. } a} = \dfrac{\text{cos. } (\text{B} + x)}{\text{cos. } (a - x)}$ then

$$\text{Tan. } x = \frac{n \text{ cotan. } \beta - m \text{ cotan. } a}{n + m.}$$

(7). If $\theta + \phi + \psi = 90^\circ$ show that

(a) $\text{Tan. } \phi \text{ tan. } \psi + \text{tan. } \psi \text{ tan. } \theta + \text{tan. } \theta \text{ tan. } \phi = 1.$

(b) $\text{Tan. } \theta + \text{tan. } \phi + \text{tan. } \psi = \text{tan. } \theta + \text{tan. } \phi \text{ tan. } \psi + \text{sec. } \theta \text{ sec. } \phi \text{ sec. } \psi$

(Remember that $\text{sin. } (\theta + \phi + \psi) = 1.$
and $\text{cos. } (\theta + \phi + \psi) = 0.$

(8). If $\dfrac{m \text{ tan. } (a - x)}{\text{cos. }^2 x} = \dfrac{n \text{ tan. } x}{\text{cos. }^2 (a - x)}$

Then, $\dfrac{\text{sin. } (2a - 2x)}{\text{sin. } 2x} = \dfrac{n}{m}.$

$$\text{and tan. } (a - 2x) = \frac{n - m}{n + m} \text{ tan. } a.$$

(9). If $\text{tan. A} + 3 \text{ cos. A} = 4$, show that A has two series of values, one of which is 45°, 225°, 405°, 585°, √. . . (i. e., tan. A = 1).

(10). If $\text{cos. } n\text{A} + \text{cos. } (n - 2) \text{ A} = \text{cos. A}$, show that A must have the values

$$\frac{30^\circ}{n - 1} \cdot \frac{330^\circ}{n - 1} \cdot \frac{390^\circ}{n - 1} \cdot \frac{690^\circ}{n - 1} \cdot \frac{750^\circ}{n - 1} \cdots$$

(11). If $\text{sin. } (x + a) + \text{cos. } (x + a) = \text{sin. } (x - a) + \text{cos. } (x - a)$, then x must be 45°, 225°, 405°. . . . .

(12). If $\text{sin. A} + \text{sin. } (\theta - \text{A}) + \text{sin. } (2\theta + \text{A}) = \text{sin. } (\theta + \text{A}) + \text{sin. } (2\theta - \text{A.})$

(13). If $2 \text{ sin. } (\theta - \phi) = 1$, and $\text{sin. } (\theta - \phi) = \text{cos. } (\theta + \phi)$, then we shall have $\theta = 45^\circ$, $\phi = 15^\circ$.

(14). If $\text{Tan. } \theta = \text{tan.}^3 \dfrac{\phi}{2}$ and $\text{cos. }^2\phi = \dfrac{m^2 - 1}{3}.$

$$\text{Then } \frac{m^2 - 1}{3} = \left(\frac{\text{cos. } \frac{2}{3}\theta - \text{sin. } \frac{2}{3}\theta}{\text{cos. } \frac{2}{3}\theta + \text{sin. } \frac{2}{3}\theta}\right)^2.$$

(15). If $\text{sin.}^2 2\theta - \text{sin.}^2\theta = \frac{1}{4}.$

Show that $\text{sin. } \theta = \pm \dfrac{1 + \sqrt{5}}{4}$, and show that the values of θ are given by the series,

$$\begin{array}{llll} 18^\circ & 162^\circ & 378^\circ & 528^\circ \ldots\ldots \\ 54^\circ & 126^\circ & 416^\circ & 486^\circ \ldots\ldots \\ 198^\circ & 342^\circ & 558^\circ & 702^\circ \ldots\ldots \\ 234^\circ & 306^\circ & 594^\circ & 666^\circ \ldots\ldots \end{array}$$

(16). Show that the series of angles in No. 9 can be expressed by the formula $m.\ 180^\circ + 45^\circ$.

In No. 10, by the formula, $\dfrac{m.\ 360^\circ \pm 30^\circ}{n - 1}.$

In No. 11, by the formula, $m.\ 180^\circ + 45^\circ$.
In No. 15, by the formulas, $m.\ 180^\circ \pm 18^\circ$.
$m.\ 180^\circ \pm 54^\circ$.

31. *On Inverse Trigonometrical Ratios Explanatory.*

The following notation, which is part of a general system of notation originally proposed by Sir J. Herschel, is very generally adopted, and is very convenient:—

If $\text{tan. } \theta = p$.

Then $\theta = \text{tan. }^{-1} p$.

i e., $\text{tan. }^{-1} p$. means the angle whose tangent is p. In like manner $\text{sin. }^{-1} p$ means the angle whose sine is p, and $\text{cos. }^{-1} p$ the angle whose cosine is p.

The system of notation originally proposed was the following :—If $\text{sin. } \theta = p$ and θ an angle, the arc subtending which, divided by radius, is equal to p; then, sin. θ is the sin. of p, and therefore $\text{sin. } \theta, = \text{sin. } (\text{sin. } \theta)$, and Sir J. Herschel proposed to write

$$\text{Sin. } (\text{sin. } \theta) = (\text{sin. }^2\theta),$$

reserving the notation $(\text{sin. A})^2$ for the squares of the sin. of θ. Upon this principle,

$$\text{Sin. } (\text{sin. } (\text{sin.. } (\ldots \text{ sin. } \theta))) = \text{sin}^n \theta,$$

and evidently

$$\text{sin.}^n (\text{sin.}^m \theta) = \text{sin.}^{n+m} \theta$$

or the notation follows the law of indices,* and the interpretation that $\text{sin.}^{-1} p$ must obtain, is that it is the angle whose sine is p.

In like manner

$$\text{log. (log. (log. } a) \text{ is written log. }^3 a$$

and $\text{log. }^{-1} a$, signifies the number whose logarithm is a.

Of this system the only part that has obtained any extensive currency is that given above in the case of the inverse trigonometric ratios.

32. *Formulas connecting inverse Trigonometric Ratios.*

There are some formulas in which these inverse ratios occur that are worthy of notice.

$$(a).\ \text{Tan.}^{-1} m + \text{tan.}^{-1} n = \text{tan.}^{-1} \frac{m + n}{1 - mn}.$$

For if $\text{tan. } \theta = m$ and $\text{tan. } \phi = n$

$$\text{tan. } (\theta + \phi) = \frac{\text{tan. } \theta + \text{tan. } \phi}{1 - \text{tan. } \theta \text{ tan. } \phi} = \frac{m + n}{1 - mn}.$$

$$\text{Now } \theta = \text{tan.}^{-1} m \text{ and } \phi = \text{tan.}^{-1} n.$$

$$\therefore \theta + \phi = \text{tan.}^{-1} m + \text{tan.}^{-1} n$$

$$\text{But } \theta + \phi = \text{tan.}^{-1} \frac{m + n}{1 - mn}.$$

$$\therefore \text{tan.}^{-1} m + \text{tan.}^{-1} n = \text{tan.}^{-1} \frac{m + n}{1 - mn}. \ldots (28).$$

Hence,

$$2 \text{ tan.}^{-1} m = \text{tan.}^{-1} \frac{2m}{1 - m^2}.$$

and

$$\text{Tan.}^{-1} m - \text{tan.}^{-1} n = \text{tan.}^{-1} \frac{m - n}{1 - mn.}$$

Again,

$$\text{sin.}^{-1} m + \text{sin.}^{-1} n = \text{sin.}^{-1} \{m \sqrt{1 - n^2} + n \sqrt{1 - m^2}\}.$$

For let $\text{sin.}^{-1} m = \theta$, $\text{sin.}^{-1} n = \phi$.

$$\therefore m = \text{sin. } \theta \ n = \text{sin. } \phi.\ \sqrt{1 - m^2}$$
$$= \text{cos. } \theta \ \sqrt{1 - n^2} = \text{cos. } \phi.$$

$$\text{Now sin. } (\theta + \phi) = \text{sin. } \theta.\ \text{cos. } \phi + \text{cos. } \theta.\ \text{sin. } \phi$$
$$= m \sqrt{1 - n^2} + n \sqrt{1 - m^2}$$

$$\theta + \phi = \text{sin. }^{-1} \{m \sqrt{1 - n^2} + n \sqrt{1 - m^2}\}$$

Or,

$$\text{Sin. }^{-1} m + \text{sin. }^{-1} n = \text{sin. }^{-1} \{m \sqrt{1 - n^2} + n \sqrt{1 - m^2}\} \ldots . (29).$$

Similarly,

$$\text{Cos. }^{-1} m - \text{cos. }^{-1} n = \text{cos. }^{-1} \{mn - \sqrt{1 - m^2} \sqrt{1 - n^2}\}.$$
$$\text{Cos. }^{-1} m + \text{sin. }^{-1} n = \text{sin. }^{-1} \{mn + \sqrt{1 - n^2} \sqrt{1 - m^2}\}.$$

33. *Examples.*

Show that

$$\text{Tan.}^{-1} \tfrac{1}{2} + \text{tan.}^{-1} \tfrac{1}{3} = 45^\circ$$

$$\text{For tan.}^{-1} \frac{1}{2} + \text{tan.}^{-1} \frac{1}{3} = \text{tan.}^{-1} \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}$$

$$= \text{tan.}^{-1} \frac{3 + 2}{6 - 1} = \text{tan.}^{-1} 1. = 45^\circ$$

It is to be observed that when we say $\text{tan.}^{-1} = 45^\circ$ we mean that this is *one* value. All the values of $\text{tan.}^{-1} 1$. are, of course, given by the formula $m 180^\circ + 45^\circ$.

Show that

(1). $\text{Sin.}^{-1} \dfrac{2}{\sqrt{5}} + \text{tan.}^{-1} \dfrac{1}{3} = 45^\circ.$

(2). $\text{Sin.}^{-1} m = \text{tan.}^{-1} \dfrac{m}{\sqrt{1 - m^2}}$

(3). $\text{Cos.}^{-1} m + \text{tan.}^{-1} m = \text{tan.}^{-1} \dfrac{\sqrt{1 - m^2} + m^2}{m - m \sqrt{1 \ m^2}}$

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* See *ante*, p. 465.

(4). $4 \tan.^{-1} \frac{1}{5} = \tan.^{-1} \frac{120}{119}$.

(5). $4 \tan.^{-1} \frac{1}{5} - \tan.^{-1} \frac{1}{239} = 45^\circ$

(6). $\tan.^{-1} m + \tan.^{-1} n = \sin.^{-1} \left\{ \frac{m+n}{\sqrt{(1+m^2)(1+n^2)}} \right\}$

(7). If $\theta = \tan.^{-1} \frac{1}{\sqrt{3}}$ $\phi = \tan.^{-1} \frac{1}{\sqrt{15}}$; then

$$\sin. (\theta + \phi) = \sin. 60^\circ \cos. 30^\circ$$

ON THE USE OF THE SUBSIDIARY ANGLES.

34. *Explanatory.*

In making trigonometrical calculations it is nearly always necessary to conduct them by means of logarithms. For the purpose of preparing a formula for logarithmic calculation, it is often necessary or convenient to express the sum or difference of two or more magnitudes by means of a product: this can generally be performed by introducing the sine, tangent, or some other ratio of an angle chosen for that purpose, which is called a subsidiary angle. One or two examples will explain the means employed for this purpose.

35. *Examples.*

(*a*) Thus, let $x^2 = a^2 + b^2$ to find x

we have $x^2 = a^2 \left(1 + \frac{b^2}{a^2}\right)$

$$\text{Assume } \frac{b}{a} = \tan. \theta$$

$$x^2 = a^2 (1 + \tan.^2 \theta) = \frac{a}{\cos.^2 \theta}.$$

$$\therefore x = \frac{a}{\cos. \theta}.$$

so that if a and b are any two numbers whatever, we are entitled to assume that $\frac{a}{b} = \tan. \theta$, since $\tan. \theta$ may have any value whatever from o to ∞, whether the value be positive or negative. But if we assume $\frac{a}{b} = \sin. \theta$, we must be sure that $a \angle b$, for otherwise $\frac{a}{b} > 1$, or $\sin. \theta > 1$, which is impossible.

(*b*) The following case is one that frequently occurs—

$$x = a \sin. A + b \cos. A.$$

$$\text{Assume } \frac{b}{a} = \tan. \theta$$

$$\text{then } x = a \left(\sin. A + \frac{b}{a} \cos. A\right)$$

$$= a (\sin. A + \tan. \theta \cos. A)$$

$$= a \frac{\sin. A \cos. \theta + \sin. \theta \cos. A}{\cos. \theta}$$

$$= a . \frac{\sin. (A + \theta)}{\cos. \theta}$$

(*c*) Again, if we have

$$x = \frac{a^2 - b^2}{a^2 + b^2}$$

$$\text{Assume } \frac{b}{a} = \tan. \theta$$

$$\therefore x = \frac{1 - \frac{b^2}{a^2}}{1 + \frac{b^2}{a^2}}$$

$$= \frac{1 - \tan.^2 \theta}{1 + \tan.^2 \theta}$$

$$= \frac{\cos.^2 \theta - \sin.^2 \theta}{\cos.^2 \theta + \sin.^2 \theta}$$

$$= \cos.^2 \theta - \sin.^2 \theta = \cos. 2\theta.$$

N.B.—It will frequently happen in calculations that we have previously used certain logarithms, and when this is the case the calculation is very materially shortened. Thus, in the above example, suppose we already know log. a and log. b. Then

$$\text{L. } \tan. \theta = \log. b - \log. a + 10.$$

which immediately enables us to find L. cos. 2θ; and therefore x, by only using the tables twice.

(*d*) Sin. A = cos. B cos. C, cos. a + sin. B sin. C.

$$\text{Assume } \frac{\cos. C \cos. a}{\sin. C.} = \tan. \phi$$

We obtain

$$\sin. A = \frac{\sin. C. \sin. (\phi + B.)}{\cos. \phi}$$

(*e*) If $x = \frac{p}{2} \left\{ 1 + \sqrt{1 - \frac{4q}{p^2}} \right\}$ where $p^2 > 4q$.

$$\text{Assume } \frac{4q}{p^2} = \sin.^2 \theta. \quad \text{Then}$$

$$x = p . \sin.^2 \frac{\theta}{2}.$$

(*f*) If $x = \frac{1 - \sqrt{1 - e^2}}{1 + \sqrt{1 - e^2}}$ $\quad e \angle 1.$

$$\text{Assume } e = \sin. \theta. \quad \text{Then}$$

$$x = \tan.^2 \frac{\theta}{2}.$$

(*g*) Such an example as the following frequently occurs in Astronomy:

If $x = m \cos. \theta + n \cos. (\theta + a)$. Express x in the form A. cos. $(\theta + B)$.

$$x = m \cos. \theta + n \cos. a \cos. \theta - n \sin. a \sin. \theta$$

$$= (m + n \cos. a) \cos. \theta - n \sin. a \sin. \theta$$

$$= (m + n \cos. a) \left(\frac{\cos. \phi \cos. \theta - \sin. \phi \sin. \theta}{\cos. \phi} \right)$$

$$= \frac{m + n \cos. a}{\cos. \phi} . \cos. (\theta + \phi).$$

$$\text{where } \tan. \phi = \frac{n \sin. a}{m + n \cos. a}$$

It will be observed that the expression for tan. ϕ, is not expressed in products and quotients only; to effect this we must introduce another angle, ϕ'. Thus:—

$$\text{Assume} \quad \tan \phi' = \frac{n}{m} \cos. a$$

$$\text{Then} \quad m + n \cos. a = m (1 + \tan. \phi')$$

$$= \frac{m (\cos. \phi' + \sin. \phi')}{\cos. \phi'}$$

$$= \frac{m (\sin. \phi' \cos. 45^\circ + \cos. \phi' \sin. 45) \sqrt{2}}{\cos. \phi'}$$

$$= \frac{m \sin. (\phi' + 45^\circ) \sqrt{2}}{\cos. \phi'}$$

$$\therefore \tan. \phi = \frac{n \sin. a \cos. \phi'}{m \sin. (\phi' + 45^\circ) \sqrt{2}}$$

$$\text{and} \quad x = \frac{m \sqrt{2} . \sin. (\phi' + 45^\circ) \cos. (\theta + \phi}{n . \sin. a \cos. \phi \cos. \phi'}$$

Then x is in the form required.

THE RELATION BETWEEN THE SIDES AND ANGLES OF TRIANGLES.

In the following articles, abc represent the sides of a triangle, and A B C the angles which they subtend.

(36). *To show that in every triangle* $\frac{a}{b} = \frac{\sin. A}{\sin. B}$

Let A B C be the triangle, from C draw C N perpendicular to A B. Then whether A be acute or obtuse,

Fig. 16.

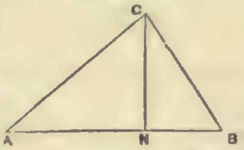
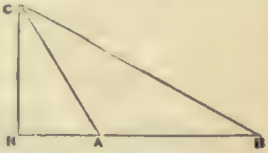


Fig. 17.



$$\text{Sin. } A = \frac{CN}{AC} \text{ and } \sin. B = \frac{CN}{CB}.$$

$$\therefore \frac{\text{Sin. A}}{\text{Sin. B}} = \frac{\text{CN}}{\text{AC}} \div \frac{\text{CN}}{\text{CB}} = \frac{\text{CN}}{\text{AC}} \times \frac{\text{CB}}{\text{CN}} = \frac{\text{CB}}{\text{CA}}$$

$$\therefore \frac{\text{Sin. A}}{\text{Sin. B}} = \frac{a}{b} \ldots\ldots (30)$$

$$\text{Similarly } \frac{\text{Sin. B}}{\text{Sin. C}} = \frac{b}{c}.$$

N.B.—These relations manifestly can be written in the form

$$\frac{\text{Sin. A}}{a} = \frac{\text{Sin. B}}{b} = \frac{\text{Sin. C}}{c} \ldots\ldots (31)$$

(37). *To prove the Formula.*

$$a^2 = b^2 + c^2 - 2\,bc.\cos.\text{A}.$$

If A be an acute angle. Let A B C be the triangle, draw C N perpendicular to the base of the triangle A B. Then (Euclid ii. 12);

$$\text{BC}^2 = \text{AC}^2 + \text{AB}^2 - 2\,\text{BA}\cdot\text{AN}.$$
$$\text{Now, AN} = \text{AC}.\cos.\text{CAB} = b\cos.\text{A}.$$
$$\therefore a^2 = b^2 + c^2 - 2bc.\cos.\text{A}.$$

Again, if A be an obtuse angle. Then drawing C N perpendicular to AB produced, we have by Euclid, ii. 13,

$$\text{BC}^2 = \text{AB}^2 + \text{AC}^2 + 2\,\text{BA}.\text{AN}$$
$$\text{and AN} = \text{CA}\cos.\text{CAN} = b\cos.(180^\circ - \text{A})$$
$$= -\,b\cos.\text{A}.$$
$$\therefore a^2 = b^2 + c^2 - 2\,ab\cos.\text{A}.$$

Hence whether A be acute or obtuse

$$a^2 = b^2 + c^2 - 2bc\cos.\text{A}\ldots\ldots (32).$$

Similarly,

$$b^2 = c^2 + a^2 - 2ca.\cos.\text{B}.$$
$$c^2 = a^2 + b^2 - 2ab.\cos.\text{C}.$$

(38). *To deduce the Formulas of Articles* 37 *from those of* 36.

These formulas can be immediately deduced from the formula—

$$\frac{\sin.\text{A}}{a} = \frac{\sin.\text{B}}{b} = \frac{\sin.\text{C}}{c}$$

without reference to Euclid's demonstration. Thus, since the three angles of a triangle are together equal to two right angles, we have

$$\text{A} + \text{B} + \text{C} = 180.^\circ$$
$$\therefore \sin.(\text{A} + \text{B}) = \sin.(180^\circ - \text{C}) = \sin.\text{C}.$$
$$\therefore \sin.\text{A}\cos.\text{B} + \sin.\text{B}\cos.\text{A} = \sin\text{C}.$$
$$\therefore \frac{\sin.\text{A}}{\sin.\text{C}}\cos.\text{B} + \frac{\sin.\text{B}}{\sin.\text{C}}.\cos.\text{A} = 1;\text{ or}$$
$$\frac{a}{c}\cos.\text{B} + \frac{b}{c}\cos.\text{A} = 1.$$
$$\therefore a\cos.\text{B} + b\cos.\text{A} = \text{C}.$$
$$\therefore a^2\cos.^2\text{B} + b^2\cos.^2\text{C} + 2\,a\,b\cos.\text{A}\cos.\text{B} = c^2.$$
$$\text{Now } \frac{a}{b} = \frac{\sin.\text{A}}{\sin.\text{B}}$$
$$\therefore a\sin.\text{B} - b\sin.\text{A} = 0$$
$$\therefore a^2\sin.^2\text{B} + b^2\sin.^2\text{A} - 2\,a\,b\sin.\text{A}\sin.\text{B} = 0$$

adding these two equations together, and remembering that $\sin.^2\text{A} + \cos.^2\text{A} = 1$, we have

$$a^2 + b^2 + 2\,ab\,(\cos.\text{A}\cos.\text{B} - \sin.\text{A}\sin.\text{B}) = c^2$$
$$\text{But }\cos.\text{A}\cos.\text{B} - \sin.\text{A}\sin.\text{B} = \cos.(\text{A} + \text{B}).$$
$$\text{Now, }\cos.(\text{A} + \text{B}) = \cos.(180^\circ - \text{C}) = -\cos.\text{C}.$$
$$\therefore a^2 + b^2 - 2\,ab\cos.\text{C} = c^2.$$

The other formulas can be derived in the same manner.

(39). *Certain Derived Formulas.*

(*a*) Again, since

$$a^2 = b^2 + c^2 - 2bc\cos.\text{A}.$$
$$\therefore \cos.\text{A} = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\therefore 1 + \cos.\text{A} = 1 + \frac{b^2 + c^2 - a^2}{2bc}$$
$$\text{And } 1 - \cos.\text{A} = 1 - \frac{b^2 + c^2 - a^2}{2bc}$$
$$\therefore 1 + \cos.\text{A} = \frac{b^2 + 2bc + c^2 - a^2}{2bc}$$
$$1 - \cos.\text{A} = \frac{a^2 - b^2 + 2bc - c^2}{2bc}$$
$$\therefore 2\cos.^2\frac{\text{A}}{2} = \frac{(b + c)^2 - a^2}{2bc.}$$
$$2\sin.^2\frac{\text{A}}{1} = \frac{a^2 - (b - c)^2}{2bc.}$$
$$\therefore \cos.^2\frac{\text{A}}{2} = \frac{(b + c + a)(b + c - a)}{4bc}$$
$$\sin.^2\frac{\text{A}}{2} = \frac{(a - b + c)(a + b - c)}{4bc}$$

$$\text{Now suppose}\quad 2s = a + b + c$$
$$\therefore 2(s - a) = b + c - a$$
$$2(s - b) = a - b + c$$
$$2(s - c) = a + b - c.$$

$$\therefore \cos.\frac{\text{A}}{2} = \sqrt{\frac{s.(s - a)}{bc}}\ldots\ldots (33)$$
$$\sin.\frac{\text{A}}{2} = \sqrt{\frac{(s - b)(s - c)}{bc}}\ldots\ldots (34)$$
$$\text{Now, }\sin.\text{A} = 2\sin.\frac{\text{A}}{2}\cos.\frac{\text{A}}{2}.$$
$$\therefore \sin.\text{A} = \frac{2}{bc}\sqrt{s.(s - a)(s - b)(s - c)}\ldots\ldots (35)$$

In regard to these formulas it will be observed that the angle and the denominators are always the three letters, *i.e.*, if the angle is A the denominator is *bc*; if B, the denominator is *ca*, and so on. Again, in (33) it will be seen that the sides in the numerator correspond to the angle, and in (34) the sides in the numerator correspond to those in the denominator. Hence we can always adapt these formulas to any angle, thus :—

$$\sin.\text{B} = \frac{2}{ac}\sqrt{s.(s - a)(s - b)(s - c)}$$
$$\cos.\frac{\text{C}}{2} = \sqrt{\frac{s.(s - c)}{ab}}$$

(*b*) The following relations are important :—

$$\text{Since } \frac{a}{b} = \frac{\sin.\text{A}}{\sin.\text{B}}.$$
$$\therefore \frac{a + b}{b} = \frac{\sin.\text{A}. + \sin.\text{B}}{\sin.\text{B}}$$
$$\frac{a - b}{b} = \frac{\sin.\text{A} - \sin.\text{B}}{\sin.\text{B}}$$
$$\therefore \frac{a - b}{a + b} = \frac{\sin.\text{A} - \sin.\text{B}}{\sin.\text{A} + \sin.\text{B}}$$
$$= \frac{2\sin.\frac{\text{A} - \text{B}}{2}\cos.\frac{\text{A} + \text{B}}{2}}{2\cos.\frac{\text{A} - \text{B}}{2}\sin.\frac{\text{A} + \text{B}}{2}}$$
$$= \tan.\frac{\text{A} - \text{B}}{2}\cot.\frac{\text{A} + \text{B}}{2}.$$
$$\text{Now}\quad \text{A} + \text{B} = 180^\circ - \text{C}.$$
$$\therefore \cot.\frac{\text{A} - \text{B}}{2} = \cot.\left(90^\circ - \frac{\text{C}}{2}\right) = \tan.\frac{\text{C}}{2}$$
$$\therefore \tan.\frac{\text{A} - \text{B}}{2}\tan.\frac{\text{C}}{2} = \frac{a - b}{a + b}.\ldots (36)$$

(40). *To obtain Formula* (32) *in a form adapted for Logarithmic calculation.*

The above formulas, expressing the relation between the sides and angles of a triangle, are in a form adapted for logarithmic computation, except

$$c^2 = a^2 + b^2 - 2ab \text{ cos. C.}$$

This can easily be written in such a form, by introducing a subsidiary angle. Thus,

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab \text{ cos. C.} \\ &= a^2 + b^2 + 2ab - 2ab - 2ab \text{ cos. C.} \\ &= a^2 + b^2 + 2ab - 2ab\,(1 + \text{cos. C}). \\ &= (a+b)^2 - 4ab \text{ cos.}^2 \frac{C}{2} \\ &= (a+b)^2 \left\{ 1 - \frac{4\,ab \text{ cos.}^2 \frac{C}{2}}{(a+b)^2} \right\} \end{aligned}$$

$$\text{Now } a + b > \sqrt{2ab}. \quad \therefore \frac{2\sqrt{ab.} \text{ cos. } \frac{C}{2}}{a+b} < 1.$$

We may therefore assume

$$\frac{2\sqrt{ab.} \text{ cos. } \frac{C}{2}}{a+b} = \text{cos. } \theta.$$

$$\therefore c^2 = (a+b)^2\,(1 - \text{cos.}\ ^2\theta) = (a+b)^2 \text{ sin.}\ ^2\theta$$

$$\therefore c = (a+b) \text{ sin. } \theta \ldots\ldots (37)$$

Or we may proceed as follows,

$$\text{Since } 1 = \text{cos.}^2 \frac{C}{2} + \text{sin.}^2 \frac{C}{2}$$

$$\text{And cos. C} = \text{cos.}^2 \frac{C}{2} - \text{sin.}^2 \frac{C}{2}$$

$$\begin{aligned} \therefore c^2 &= (a^2 + b^2)\left(\text{cos.}^2 \frac{C}{2} + \text{sin.}^2 \frac{C}{2}\right) \\ &\quad - 2\,ab\left(\text{cos.}^2 \frac{C}{2} - \text{sin.}^2 \frac{C}{2}\right) \\ &= (a^2 + 2\,ab + b^2) \text{ sin.}^2 \frac{C}{2} + (a^2 - 2\,ab + b^2) \text{ cos.}^2 \frac{C}{2} \\ &= (a+b)^2 \text{ sin.}^2 \frac{C}{2} + (a-b)^2 \text{ cos.}^2 \frac{C}{2}. \\ &= (a+b)^2 \text{ sin.}^2 \frac{C}{2} \left\{ 1 + \left(\frac{a-b}{a+b}\right)^2 \text{ cotan.}^2 \frac{C}{2} \right\}. \end{aligned}$$

$$\text{Assume tan. } \theta = \frac{a-b}{a+b} \text{ cotan. } \frac{C}{2}.$$

$$\therefore c^2 = (a+b)^2 \text{ sin.}^2 \frac{C}{2}. \ (1 + \text{tan.}^2\ \theta)$$

$$\text{or } c = \frac{(a+b) \text{ sin. } \frac{C}{2}}{\text{cos. } \theta} \ldots\ldots (38).$$

Also the calculation of $\text{tan.} \frac{A-B}{2} = \frac{a-b}{a+b} \text{ cotan. } \frac{C}{2}$ can be simplified, if we have already log. a and log. b; by introducing a subsidiary angle. Thus,

$$\frac{a-b}{a+b} = \frac{1 - \frac{b}{a}}{1 + \frac{b}{a} + \frac{b}{a}}$$

$$\text{Assume} \qquad \text{tan. } \theta = \frac{b}{a}.$$

$$\therefore \frac{a-b}{a+b} = \frac{1 - \text{tan. } \theta}{1 + \text{tan. } \theta} = \frac{\text{tan. } 45° - \text{tan. } \theta}{1 + \text{tan. } 45° \text{ tan. } \theta}$$

$$\text{for tan. } 45° = 1.$$

$$\therefore \frac{a-b}{a+b} = \text{tan. } (45° - \theta)$$

$$\text{and tan. } \frac{A-B}{2} = \text{tan. } (45° - \theta) \text{ cotan. } \frac{C}{2} \ldots\ldots (39)$$

There are several Theorems which can be deduced by means of the relations proved above. The following are a few.

(41). *To find the Area of a Triangle in terms of its sides.*

In figure (16) we evidently have

$$\text{Area triangle} = \tfrac{1}{2}.\ \text{C N. A B.}$$

$$\text{Now C N.} = \text{A C sin. A} = b \text{ sin. A.}$$

$$\begin{aligned} \therefore \text{area} &= \frac{bc}{2} \text{ sin. A} \\ &= \frac{bc}{2} \times \frac{2}{bc} \sqrt{s.\ (s-a)\ (s-b)\ (s-c)}. \ \ldots (35). \\ &= \sqrt{s.\ (s-a)\ (s-b)\ (s-c)} \end{aligned}$$

N.B.—If $a = b = c$. or triangle equilateral, then $s = \frac{3a}{2}$

$$\therefore \text{area} = \frac{a^2\sqrt{3}}{4}$$

If $a = b$. or triangle isosceles

$$s = a + \frac{c}{2}\ s - a + s - b + \frac{c}{2}$$

$$s - c = a - \frac{c}{2}.$$

$$\therefore \text{area} = \frac{c}{2}\sqrt{\frac{a^2 - c^2}{4}}$$

(42). *To find the Radius of Inscribed Circle on terms of sides.*

Let A B C be the triangle.
O the centre of inscribed circle
Join O A, O B, OC.
Now area A B C = area B O C + area C O A + area A O B.
Let r = required radius.

$$\text{Then, area B O C} = \frac{ra}{2}$$

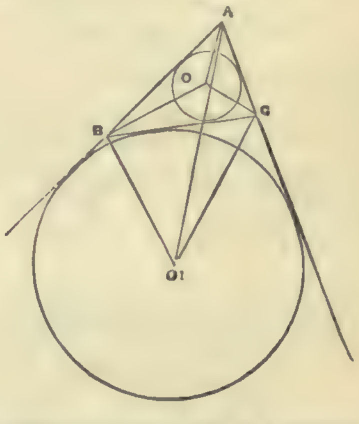
$$\text{area C O A} = \frac{rb}{2}.$$

$$\text{area } \frac{\text{A O B}}{2} = \frac{rc}{2}.$$

$$\therefore r\frac{a+b+c}{2} = \text{area.}$$

$$\therefore r = \frac{\sqrt{s\,(s-a)\,(s-b)\,(s-c)}.}{s}$$

Fig. 18.



The circles which touch one side of a triangle, and the two other sides produced, are sometimes called the *susscribed* circles. Let r_a r_b r_c be the radii of those circles which touch the sides a b c respectively.

Then if O_1 be the centre of the circle which touches the side B C, join O_1A, O_1B, O_1C. We clearly have

$$- \text{Area B } O_1,\ \text{C} + \text{area C } O_1,\ \text{A} + \text{area A } O_1,\ \text{C} = \text{area A B C.}$$

$$\therefore -\frac{r_a\ a}{2} + \frac{r_a\ b}{2} + \frac{r_a\ c}{2} = \text{area.}$$

$$\therefore r_a\ (s-a) = \sqrt{s.\ (s-a)\ (s-b)\ (s-c)}$$

$$r_a = \frac{\sqrt{s.\ (s-a)(s-b)(s-c)}}{(s-a)}$$

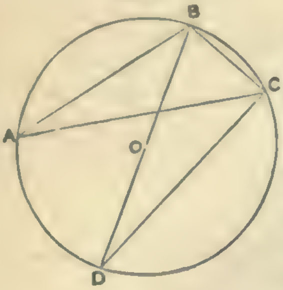
Similarly

$$r_b = \frac{\sqrt{s.\ (s-a)\ (s-b)\ (s-c)}}{(s-b)}$$

$$r_c = \frac{\sqrt{s.\ (s-a\ (s-b)\ (s-c)}}{(s-c)}$$

(43). *To find the Area of the Circumscribed Circle.*

Fig. 19.



Let A B C be the triangle. Circumscribe a circle about it, the centre of which is O. Join B O and produce it to meet the circumference in D. Join D C. Then (Euclid III.—21) B D C = B A C, and B C D is a right angle. (Euclid III.—31.) Now B D sin. B D C = B C.

∴ If R = radius of circumscribed circle, 2 R. sin. A = a.

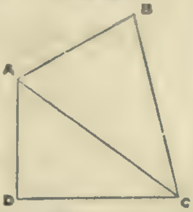
$$\therefore \frac{\mathrm{R}}{bc}\sqrt{s.\ (s-a)\ (s-b)\ (s-c)} = a$$

$$\mathrm{R} = \frac{abc}{\sqrt{s.\ (s-a)\ (s-b)\ (s-c)}}$$

(44). *To find the Area of a Quadrilateral inscribed in a Circle in terms of the sides.*

A B C D a quadrilateral inscriptible in a circle, let the four sides A B, B C, C D, D A, be respectively $a\ b\ c\ d$. Join A C.

Fig. 20.



Now, if A B C = θ A D C = 180° — θ (Euclid III.—22.)

Hence, $\mathrm{A\ C}^2 = a^2 + b^2 - 2ab \cos.\ \theta$

and $\mathrm{A\ C}^2 = c^2 + d^2 - 2cd \cos.\ (180° - \theta)$

$\therefore a^2 + b^2 - 2ab \cos.\ \theta = c^2 + d^2 + 2cd \cos.\ \theta$

$$\therefore a^2 + b^2 - 2ab \cos.\ \theta = c^2 + d^2 + 2cd \cos.\ \theta$$

$$\therefore \cos.\ \theta = \frac{a^2 + b^2 - c^2 - d^2}{2ab + 2cd}.$$

$$\text{Now, } 2 \cos.^2 \frac{\theta}{2} = 1 + \cos.\ \theta$$

$$\therefore 2 \cos.^2 \frac{\theta}{2} = 1 + \frac{a^2 + b^2 - c^2 - d^2}{2ab + 2cd}$$

$$= \frac{(a+b)^2 - (c-d)^2}{2ab + 2cd} = \frac{(a+b+c-d)\ (a+b-c+d)}{2ab + 2cd}$$

$$= \frac{2.\ (s-d)\ (s-c)}{ab + cd} \qquad \text{if } 2s = a + b + c + d.$$

Similarly,

$$2 \sin.^2 \frac{\theta}{2} = 1 - \frac{a^2 + b^2 - c^2 - d^2}{2ab + 2cd}$$

$$= \frac{2.\ (s-b)\ (s-c)}{ab + cd}$$

$$\mathrm{Sin.}^2 \theta = \frac{4\ (s-a)\ (s-b)\ (s-c)\ (s-d)}{(ab + cd)^2}$$

But area of triangle A D C

$$= \frac{\mathrm{AD,\ DC}}{2} \sin.\ \mathrm{A\ D\ C} = \frac{dc}{2} \sin.\ (180° - \theta)$$

$$= \frac{dc}{2} \sin.\ \theta$$

$$\text{Area of triangle A B C} = \frac{\mathrm{AB,\ BC}}{2} \sin.\ \mathrm{BC}.$$

$$= \frac{ab}{2} \sin.\ \theta.$$

$$\therefore \text{The whole area} \frac{\sin.\ \theta}{2} (ab + dc).$$

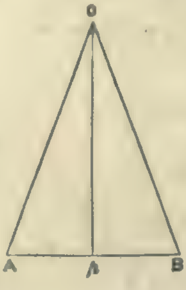
Hence area of quadrilateral

$$= \sqrt{(s-a)\ (s-b)\ (s-c)\ (s-d)}.$$

(45). *To find the Area of a Polygon of (n) sides inscribed in a given Circle.*

If A B be one side of the polygon, O the centre of the circle, let r be the radius, and n the number of sides. Then area of polygon = n × (area triangle O A B).

Fig. 21.



Now angle $\mathrm{A\,O\,B} = \frac{360°}{n}$.

Draw Op perpendicular to A B, Then

$$\mathrm{A\,O}p = \tfrac{1}{2}\,\mathrm{A\,O\,B} \therefore \mathrm{A\,O}p = \frac{180°}{n}.$$

Then area of triangle

$$= \frac{\mathrm{A\,B} \times \mathrm{O}p}{2} = \mathrm{A}p \times \mathrm{O}p.$$

$$\mathrm{A}p. = r \sin.\ \frac{180°}{n}.\quad \mathrm{O}p. = r \cos.\ \frac{180°}{n}.$$

$$\therefore \text{Area triangle} = r^2 \sin.\ \frac{180°}{n} \cos.\ \frac{180°}{n}.$$

$$= \tfrac{1}{2}\, r^2 \sin.\ \frac{360°}{n}.$$

If a be one side of the polygon,

$$\text{Then } \frac{a}{2} = r \sin.\ \frac{180°}{n}.$$

$$\therefore r = \frac{a}{2 \sin.\ \frac{180°}{n}}$$

$$\therefore \text{area of triangle} = \frac{a^2}{4 \sin.^2 \frac{180}{n}} \times \sin.\ \frac{180°}{n} \cos.\ \frac{180°}{n}$$

$$= \frac{a^2}{4} \text{cotan.}\ \frac{180°}{n}.$$

Hence area of polygon $= \frac{n}{2} r^2 \sin.\ \frac{360°}{n}$. when radius is given,

And area of polygon $= \frac{na^2}{4} \text{cotan.} \frac{180°}{n}$ when side is given.

The area of circumscribed polygon can in like manner be proved to equal

$$n r^2 \tan.\ \frac{180°}{n}.$$

CHAPTER XII.

TRIGONOMETRICAL SERIES AND TABLES.

(46) GENERAL EXPLANATIONS.

THE preceding pages give the theory of plane trigonometry. To complete this branch of mathematical science it remains to explain the mode of deducing numerical results from the above theory, and to perform the actual calculations of the parts of triangles from which the science derives its name. We have already seen that the trigonometrical ratios of certain angles are known; for example, we know that $\sin. 45^\circ = \frac{1}{\sqrt{2}}$, $\sin. 60^\circ = \frac{\sqrt{3}}{2}$, and some others, and knowing these values we can determine log. sin. 45°, log. sin. 60°, and so on. Now instead of knowing only the trigonometrical functions of certain angles, we want to know the trigonometrical functions of every angle from 0° up to 90°. And as the calculations are for the most part carried on by means of logarithms, we require to know also the logarithms of these functions. These values have been calculated and arranged in tables in a similar manner to the tables of logarithms as before explained; we purpose in the following pages to explain the principles on which this calculation has been effected.*

In our article on series and logarithms, we found it necessary to investigate certain algebraical series before proceeding to discuss the nature of logarithms; in like manner we shall find it now necessary to investigate certain trigonometrical series before explaining the construction of trigonometrical tables. In the following article on trigonometrical series we shall always keep this object strictly in view, a circumstance that will account for the absence of certain series that are often given in treatises of trigonometry. There is a very large number of such series; we treat the series here simply as the *means of arriving at the numerical results*, and on this principle make the following selection from that large number. The student will do well to observe, in studying any branch of mathematics, in which algebraical formulas occur, that unless he understands the method of obtaining arithmetical results from his formulas, he has not as yet mastered the subject.

The first article is an example of a limiting value. We would recommend the student to consider it very carefully. The proposition to be proved is that the limiting value of $\frac{\sin. \theta}{\theta} = 1$, where, of course, θ is in circular measure. The following will sufficiently explain the meaning of the statement. When θ becomes very small, $\frac{\sin. \theta}{\theta}$ becomes very nearly equal to 1, and the smaller θ becomes the more nearly $\frac{\sin. \theta}{\theta}$ becomes equal to 1; but so long as θ has any value, however small, $\frac{\sin. \theta}{\theta}$ can never actually equal 1. Then the value which limits the value of $\frac{\sin. \theta}{\theta}$ is 1, while the value which limits the values of θ is 0, and we *assume* that if θ actually equals 0, $\frac{\sin. \theta}{\theta}$ actually equals 1. The assumption is in point of fact *an axiom*. We cannot, however, discuss the questions here that this statement gives rise to. There are several methods by which the proposition can be proved. The following is a modification of Newton's sixth Lemma.

(47). *To show that when $\theta = 0$ we must have $\frac{\sin. \theta}{\theta} = 1$*

Let AP_1P be the arc of a circle, the centre of which is O. AT a tangent, AP a chord. Produce AP to p, and AT to t, draw PT and pt parallel to AO, draw po parallel to PO. Then angle $Apo = APO = PAO$. Since $AO = PO$, and therefore $Ao = op$, with centre o and radius op describe a circular arc, Ap. Then the angle AOP being equal to Aop, we have—

$$\frac{\text{arc } AP}{AO} = \frac{\text{arc } Ap}{Ao},$$

since each measures the equal angles.

Again, draw PN and pn parallel to At.

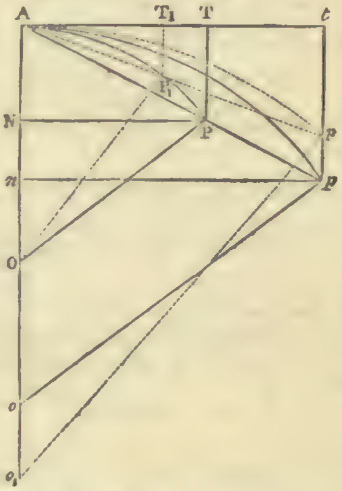
$$\text{Then } \frac{AT}{AO} = \frac{PN}{PO} = \sin. AOP.$$

$$\frac{At}{Ao} = \frac{pn}{po} = \sin. Aop.$$

$$\therefore \frac{AT}{AO} = \frac{At}{Ao}$$

$$\therefore \frac{\text{arc } AP}{AT} = \frac{\text{arc } Ap}{At}. \qquad (a)$$

Fig. 22.



Now, suppose P to move along the arc to P_1, and suppose pt to remain fixed, produce the chord AP_1, to meet pt in p, make the chord AP_1 to meet pt in p_1, make the angle $AP_1O = Ap_1o_1$. Then as before—

$$\frac{\text{arc } AP_1}{AT_1} = \frac{\text{arc } Ap_1}{At}.$$

so that in all cases the equation (a) holds good. Now when P moves up to A, p moves up to t, and when P coincides with A, p coincides with t, and then p and t coinciding, the changing arc Ap coincides with At, and therefore in the extreme or limiting case.

$$\frac{\text{arc } Ap}{At} = 1.$$

and hence on the limiting case, when AP vanishes,

$$\frac{\text{arc } AP}{AT} = 1.$$

Now let $AOP = \theta$, where θ is in circular measure, then—

$$\frac{\text{arc } AP}{AO} = \theta.$$

$$\text{and} \qquad \frac{AT}{AO} = \frac{PN}{PO} = \sin. \theta.$$

$$\text{Now} \qquad \frac{\text{arc } AP}{AT} = \frac{\frac{\text{arc } AP}{PO}}{\frac{AT}{PO}} = \frac{\sin. \theta}{\theta}.$$

---

* Tables of natural sines and cosines, and logarithmic sines, cosines, tangents, &c., will be found at the end of this section.

Hence, in the limit when θ equals zero,

$$\frac{\sin.\ \theta}{\theta} = 1. \qquad \text{Q. E. D.}$$

Cor. Under all circumstances—

$$\frac{\tan.\ \theta}{\theta} = \frac{\sin.\ \theta}{\theta} \times \frac{1}{\cos.\ \theta}.$$

Now when $\theta = 0$ $\quad \dfrac{\sin.\ \theta}{\theta} \times \dfrac{1}{\cos.\ \theta} = 1.$

Hence, in the limiting case, when $\theta = 0$.

$$\frac{\tan.\ \theta}{\theta} = 1.$$

(48). *De Moivre's Theorem.*

(*a*). To prove that $(\cos.\ \theta + \sqrt{-1}\sin.\ \theta) \times (\cos.\ \phi + \sqrt{-1}\sin.\ \phi) = \cos.\ (\theta+\phi) + \sqrt{-1}\sin.\ (\theta+\phi)$.

For by actual multiplication—

$(\cos.\ \theta + \sqrt{-1}\sin.\ \theta)(\cos.\ \phi + \sqrt{-1}\sin.\ \phi) = \cos.\ \theta\cos.\ \phi + \sqrt{-1}\sin.\ \theta\cos.\ \phi + \sqrt{-1}\cos.\ \theta\sin.\ \phi - \sin.\ \theta\sin.\ \phi = \cos.\ \theta\cos.\ \phi - \sin.\ \theta\sin.\ \phi + \sqrt{-1}\left\{\sin.\ \theta\cos.\ \phi + \cos.\ \theta\sin.\ \phi\right\} = \cos.\ (\theta+\phi) + \sqrt{-1}\sin.\ (\theta+\phi)$.

Hence—

$(\cos.\ \theta + \sqrt{-1}\sin.\ \theta)(\cos.\ \phi + \sqrt{-1}\sin.\ \phi)(\cos.\ \psi + \sqrt{-1}\sin.\ \psi) = \left\{\cos.\ (\theta+\phi) + \sqrt{-1}\sin.\ (\theta+\phi)\right\}(\cos.\ \psi + \sqrt{-1}\sin.\ \psi) = \cos.\ (\theta+\phi+\psi) + \sqrt{-1}\sin.\ (\theta+\phi+\psi)$.

And generally if we had n angles $\theta_1 \ldots \theta_2 \ldots \theta_n$, we should have—

$$\left\{\cos.\ \theta_1 + \sqrt{-1}\sin.\ \theta_1\right\}\left\{\cos.\ \theta_2 + \sqrt{-1}\sin.\ \theta_2\right\} \ldots \left\{\cos.\ \theta_n + \sqrt{-1}\sin.\ \theta_n\right\} = \cos.\ (\theta_1 + \theta_2 + \ldots\ldots \theta_n) + \sqrt{-1}\sin.\ (\theta_1 + \theta_2 + -\ldots + \theta_n) \ldots\ldots (40).$$

(*b*) To show that $(\cos.\ \theta + \sqrt{-1}\sin.\ \theta)^n = \cos.\ n\theta + \sqrt{-1}\sin.\ n\theta$ for all integral and positive values of n.

In equation (40), suppose $\theta_1\ \theta_2 \ldots \theta_n$ to be each equal to one another and to θ.

Then— $\quad \theta_1 + \theta_2 - \ldots + \theta_n = n\theta.$

And—

$(\cos.\ \theta_1 + \sqrt{-1}\sin.\ \theta_1)(\cos.\ \theta_2 + \sqrt{-1}\sin.\ \theta_2) \ldots (\cos.\ \theta_n + \sqrt{-1}\sin.\ \theta_n) = (\cos.\ \theta + \sqrt{-1}\sin.\ \theta)(\cos.\ \theta + \sqrt{-1}\sin.\ \theta) \ldots (\cos.\ \theta + \sqrt{-1}\sin.\ \theta) = (\cos.\ \theta + \sqrt{-1}\sin.\ \theta)_n.$

And $\cos.\ (\theta_1 + \theta_2 + \ldots + \theta_n) + \sqrt{-1}\sin.\ (\theta_1 + \theta_2 + \ldots + \theta_n)$

$= \cos.\ n\theta + \sqrt{-1}\sin.\ n\theta$

Hence

$(\cos.\ \theta + \sqrt{-1}\sin.\ \theta)^n = \cos.\ n\theta + \sqrt{-1}\sin.\ n\theta.$

(*c*). To prove the same theorem when n is a negative whole number.

For by multiplication, $(\cos.\ n\theta + \sqrt{-1}\sin.\ n\theta)$, $(\cos.\ n\theta - \sqrt{-1}\sin.\ n\theta) = \cos.^2 n\theta + \sin.^2 n\theta = 1$.

$$\text{now } 1 = \frac{(\cos.\ \theta + \sqrt{-1}\sin.\ \theta)^n}{(\cos.\ \theta + \sqrt{-1}\sin.\ \theta)^n}$$

$= (\cos.\ \theta + \sqrt{-1}\sin.\ \theta),^n (\cos.\ \theta + \sqrt{-1}\sin.\ \theta)^{-n}$

$= (\cos.\ n\theta + \sqrt{-1}\sin.\ n\theta), (\cos.\ \theta + \sqrt{-1}\sin.\ \theta)^{-n}$

$\therefore (\cos.\ n\theta - \sqrt{-1}\sin.\ n\theta), (\cos.\ \theta + \sqrt{-1}\sin.\ \theta)^{-n} =$

$(\cos.\ n\theta + \sqrt{-1}\sin.\ n\theta), (\cos.\ n\theta - \sqrt{-1}\sin.\ n\theta)$;

$\therefore (\cos.\ \theta + \sqrt{-1}\sin.\ \theta)^{-n} = \cos.\ n\theta - \sqrt{-1}\sin.\ n\theta)$

$= \cos.\ (-n\theta) + \sqrt{-1}\sin.\ (-n\theta).$

Since $\quad \cos.\ (-n\theta) = \cos.\ n\theta$
$\qquad\quad \sin.\ (-n\theta) = -\sin.\ n.\ \theta.$

Hence the theorem is true when n is negative.

(*d*). To prove the same theorem when n is fractional.

Let $n = \dfrac{p}{q}$

$$\text{Now } \left(\cos.\ \frac{p\theta}{q} + \sqrt{-1}\sin.\ \frac{p\theta}{q}\right)^q = \cos.\ p\theta + \sqrt{-1}\sin.\ p\theta.$$

$$= (\cos.\ \theta + \sqrt{-1}\sin.\ \theta)^p$$

$$\therefore \cos.\ \frac{p\theta}{q} + \sqrt{-1}\sin.\ \frac{p\theta}{q} = \sqrt[q]{(\cos.\ \theta + \sqrt{-1}\sin.\ \theta)^p}$$

$$= (\cos.\ \theta + \sqrt{-1}\sin.\ \theta)^{\frac{p}{q}}$$

and hence, in all cases—

$$(\cos.\ \theta + \sqrt{-1}\sin.\ \theta)^n = \cos.\ n\theta + \sqrt{-1}\sin.\ n\theta \ldots\ldots (41).$$

Whether n be positive or negative, integral or fraction; which theorem is called De Moivre's theorem.

(49). *To express sin. m θ and cos. m θ in terms of powers of sin. θ and cos. θ.*

Since

$$(\cos.\ m\theta + \sqrt{-1}\sin.\ m\theta) = (\cos\ \theta + \sqrt{-1}\sin.\ \theta)^m$$

$$= \cos.^m\theta - \frac{m.\ \overline{m-1}}{1 \cdot 2}\cos.^{m-2}\theta\sin.^2\theta$$

$$+ \frac{m.\ \overline{m-1}.\ \overline{m-2}.\ \overline{m-3}}{1.2.3.4} \times \cos^{m-4}\theta\sin.^4\theta - \&c.$$

$$+ \sqrt{-1}\left\{m.\ \cos.^{m-1}\theta\sin\theta - \frac{m.\ \overline{m-1}\ \overline{m.-2}}{1.2.3}\cos.^{m-3}\theta\sin.^3\theta + \right\}$$

Hence equating possible and impossible quantities,

$$\cos.\ m\theta = \cos.^m\theta - \frac{\overline{m\ m-1}}{1.2}\cos.^{m-2}\theta\sin.^2\theta$$

$$+ \frac{m.\ \overline{m-1}.\ \overline{m-2}.\ \overline{m-3}}{1.2.3.4}\cos.^{m-4}\sin.^4\theta - \&c. \ldots (42).$$

and

$$\text{Sin. } m\theta = m\cos.^{m-1}\theta\sin.\ \theta - \frac{m.\ \overline{m-1}.\ \overline{m-2}}{1.2.3}\cos.^{m-3}\theta\sin.^3\theta + \ldots (43)^*$$

(50). *To express sin. φ, and cos. φ, in terms of φ.*

Let $m\theta = \phi$, and $\therefore \theta = \dfrac{\phi}{m}$.

Hence in the expression of the last article for cos. $m\theta$, we have

$$\cos.\ \phi = \left(\cos.\ \frac{\phi}{m}\right)^m - \frac{m.\ (m-1)}{1.2}\left(\cos.\ \frac{\phi}{m}\right)^{m-2}\sin.^2\frac{\phi}{m}$$

$$+ \frac{m.\ (m-1)(m-2)(m-3)}{1.2.3.4}\left(\cos.\ \frac{\phi}{m}\right)^{m-4}\sin.^4\frac{\phi}{m}\ \&c.$$

$$= \left(\cos.\ \frac{\phi}{m}\right)^m - \frac{\left(1-\frac{1}{m}\right)}{1.2}\left(\cos.\ \frac{\phi}{m}\right)^{m-2}\left(\sin.^2\frac{\phi}{m}\right)m^2$$

$$+ \frac{\left(1-\frac{1}{m}\right)\left(1-\frac{2}{m}\right)\left(1-\frac{3}{m}\right)}{1.2.3.4}\left(\cos.\ \frac{\phi}{m}\right)^{m-4}\left(\sin.^4\frac{\phi}{m}\right)m^4$$

$$- \&c.$$

* We assume in this article that if $a + \sqrt{-1}.\ b = A + \sqrt{-1}\ B$. where $a.\ b.\ A.\ B.$ are *real*. Then $a = A$ and $B = b$. To prove this, suppose A to be unequal to a and let $A = a + x$. and $B = b + y$.

$$\therefore a + b\sqrt{-1} = a + x + (b+y)\sqrt{-1}$$
$$\therefore o = x + y\sqrt{-1}$$
$$\therefore -x = y\sqrt{-1}$$
$$\therefore x^2 = -y^2$$
$$\therefore x^2 + y^2 = o.$$

Now x^2 and y^2 must each be positive.

$$\therefore x = o \text{ and } y = o$$
$$\therefore A = a.\ \text{and } B = b.$$

$$= \left(\cos.\ \frac{\phi}{m}\right)^m - \frac{1-\frac{1}{m}}{1\cdot2}\left(\cos.\ \frac{\phi}{m}\right)^{m-2}\left(\frac{\sin.\ \frac{\phi}{m}}{\frac{\phi}{m}}\right)^2\phi^2$$

$$+\frac{\left(1-\frac{1}{m}\right)\left(1-\frac{2}{m}\right)\left(1-\frac{3}{m}\right)}{1.2.3.4}$$

$$\left(\cos.\ \frac{\phi}{m}\right)^{m-4}\left(\frac{\sin.\ \frac{\phi}{m}}{\frac{\phi}{m}}\right)^4\phi^4 - \&c.$$

This formula is true for every value of m, however large m may be; but if we suppose m to become very large, then in the limiting or extreme case when m is infinity $\frac{\phi}{m}$ is zero, and hence in this limiting case

$$\frac{\sin.\ \frac{\phi}{m}}{\frac{\phi}{m}} = 1.$$

Also $1-\frac{1}{m}=1.\quad 1-\frac{2}{m}=1.\qquad \&c.$

Also $\quad \left(\cos.\ \frac{\phi}{m}\right)^m = 1.$

And therefore

$$\cos.\ \phi = 1-\frac{\phi^2}{1.2}+\frac{\phi^4}{1.2.3.4}-\&c.\ \ldots (44).$$

By reasoning in precisely the same way from the formula for sin. $m\,\theta$ we shall obtain

$$\sin.\ \phi = \phi - \frac{\phi^3}{1.2.3} + \frac{\phi^5}{1.2.3.4.5} - \&c.\ \ldots (45).$$

Cor.—If ϕ be so small that we can omit ϕ^3, we shall clearly have

$$\sin.\ \phi = \phi \text{ and } \cos.\ \phi = 1 - \frac{\phi^2}{2}$$

(51). *To obtain sin. θ. cos. θ. and tan. θ in terms of Exponentials.*

Since

$$\sin.\ \theta = \theta - \frac{\theta^3}{1.2.3} + \frac{\theta^5}{1.2.3.4.5} - \&c.$$

$$\&\ \cos.\ \theta = 1 - \frac{\theta^2}{1.2} + \frac{\theta^4}{1.2.3.4} - \&c.$$

$$\therefore \sqrt{-1}.\ \sin.\ \theta = \theta\sqrt{-1} - \frac{\theta^3\sqrt{-1}}{1.2.3}$$

$$+\frac{\theta^5\sqrt{-1}}{1.2.3.4.5} - \&c.$$

$$\therefore \cos.\ \theta + \sqrt{-1}.\ \sin.\ \theta = 1 + \theta\sqrt{-1} - \frac{\theta^2}{1.2}$$

$$+\frac{\theta^3\sqrt{-1}}{1.2.3} - \frac{\theta^4}{1.2.3.4} + \frac{\theta^5\sqrt{-1}}{1.2.3.4.5}$$

$$\&\ \cos.\ \theta - \sqrt{-1}.\ \sin.\ \theta = 1 - \theta\sqrt{-1} + \frac{\theta^2}{1.2}$$

$$-\frac{\theta^3\sqrt{-1}}{1.2.3} + \frac{\theta^4}{1.2.3.4} - \frac{\theta^5\sqrt{-1}}{1.2.3.4.5}$$

Now

$(\sqrt{-1})^2 = -1.$
$(\sqrt{-1})^3 = (\sqrt{-1})^2\sqrt{-1} = -\sqrt{-1}.$
$(\sqrt{-1})^4 = (\sqrt{-1})^2(\sqrt{-1})^2 = (-1)(-1) = 1.$

$$\therefore \cos.\ \theta + \sqrt{-1}\sin.\ \theta = 1 + \theta\sqrt{-1} + \frac{(\theta\sqrt{-1})^2}{1.2}$$

$$+\frac{(\theta\sqrt{-1})^3}{1.2.3} + \cdots$$

and

$$\cos.\ \theta - \sqrt{-1}\sin.\ \theta = 1 - \theta\sqrt{-1} + \frac{(\theta\sqrt{-1})^2}{1.2}$$

$$-\frac{(\theta\sqrt{-1})^3}{1.2.3} + \cdots\ \theta\sqrt{-1}.$$

$$\therefore \cos.\ \theta + \sqrt{-1}\sin.\ \theta = e\ \ldots (46).$$

And $\cos.\ \theta - \sqrt{-1}\sin.\ \theta = e^{-\theta\sqrt{-1}}\ \ldots (47).$

by the exponential theorem (Art. on Series)*

$\therefore$ dividing the former by the latter

$$\frac{\cos.\ \theta + \sqrt{-1}\sin.\ \theta}{\cos.\ \theta - \sqrt{-1}\sin.\ \theta} = e^{\theta\sqrt{-1}} + e^{-\theta\sqrt{-1}}$$

$$\therefore \frac{1+\sqrt{-1}\tan.\ \theta}{1-\sqrt{-1}\tan.\ \theta} = e^{2\theta\sqrt{-1}}\ \ldots (48).$$

Also adding (46) and (47) we obtain

$$2\cos.\ \theta = e^{\theta\sqrt{-1}} + e^{-\theta\sqrt{-1}}\ \ldots (49).$$

and subtracting (47) from (46) we obtain

$$2\sqrt{-1}\sin.\ \theta = e^{\theta\sqrt{-1}} - e.^{-\theta\sqrt{-1}}\ \ldots (50)$$

(52). *To obtain an expression for θ in terms of tan. θ and its powers.*

Since from equation (48)

$$e^{2\theta\sqrt{-1}} = \frac{1+\sqrt{-1}\tan.\ \theta}{1-\sqrt{-1}\tan.\ \theta}$$

Taking logarithms on both sides.

$$2\,\theta\sqrt{-1} = \log.\ (1+\sqrt{-1}\tan.\ \theta) - \log.\ (1-\sqrt{-1}\tan.\ \theta)$$

$$= \frac{\sqrt{-1}\tan.\ \theta}{1} - \frac{(\sqrt{-1}\tan.\ \theta)^2}{2} + \frac{(\sqrt{-1}\tan.\ \theta)^3}{3}. - \&c.$$

$$-\left\{-\frac{\sqrt{-1}\tan.\ \theta}{1} - \frac{(\sqrt{-1}\tan.\ \theta}{1.2} - \frac{(\sqrt{-1}\tan.\ \theta)^3}{3}\ \ldots\right\}$$

$$= 2\left\{\frac{\sqrt{-1}\tan.\ \theta}{1} + \frac{(\sqrt{-1}\tan.\ \theta)^3}{3} + \frac{(\sqrt{-1}\tan.\ \theta)^5}{5} + \cdots\right\}$$

$$= 2\sqrt{-1}\left\{\tan.\ \theta - \frac{\tan.^3\theta}{3} + \frac{\tan.^5\theta}{5} - \&c.\right\}$$

$$\therefore \theta = \tan.\ \theta - \frac{\tan.^3\theta}{3} + \frac{\tan.^5\theta}{5} - \&c.$$

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* See *ante*, p. 516.

or if, $\theta = \tan.^{-1} x$. Then

$$\tan.^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \&c. \qquad \ldots (51).$$

If x be positive, this value of $\tan.^{-1} x$ is, of course, that value which is less than $\frac{\pi}{2}$. Hence, if we wish to express *all* the values of $\tan.^{-1} x$ in terms of x, we ought to use the formula

$$\tan.^{-1} x = m\,\pi + \left\{ x - \frac{x^3}{3} + \frac{x^5}{5} - \&c. \right\}$$

Where m is any positive or negative integer

(53). *To calculate the value of π. i. e., the ratio which the Arc of a Semicircle bears to its Radius.*

Let a be the arc to radius r subtending an angle of 45°. Then since the semicircle subtends an angle of 180°, $\frac{\pi}{4} = \frac{a}{r}$. Hence $\frac{\pi}{4}$ is the circular measure of an angle of 45°.

$$\therefore \tan.\frac{\pi}{4} = 1$$

and

$$\frac{\pi}{4} = \tan.^{-1} 1$$

Hence, from equation (51).

$$\frac{\pi}{4} = \tan.^{-1} 1 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

and from this we might obtain the numerical value of π. But it is much more expeditiously determined in the following manner,

$$2\tan.^{-1}\frac{1}{5} = \tan.^{-1}\frac{\frac{2}{5}}{1-\frac{1}{25}} = \tan.^{-1}\frac{5}{12}$$

$$4\tan.^{-1}\frac{1}{5} = 2\tan.^{-1}\frac{5}{12} = \tan.^{-1}\frac{\frac{5}{6}}{1-\frac{25}{144}} = \tan.^{-1}\frac{120}{119}$$

$$\mathrm{Tan.}^{-1}\frac{120}{119} - \tan.^{-1} 1 = \tan.^{-1}\frac{\frac{120}{119}-1}{1+\frac{120}{119}} = \tan.^{-1}\frac{1}{239}$$

$$\therefore \tan.^{-1} 1 = \tan.^{-1}\frac{120}{119} - \tan.^{-1}\frac{1}{239}$$

$$= 4\tan.^{-1}\frac{1}{5} - \tan.^{-1}\frac{1}{239}$$

$$\therefore \frac{\pi}{4} = 4\tan.^{-1}\frac{1}{5} - \tan.^{-1}\frac{1}{239}$$

$$= 4\left\{\frac{1}{5} - \frac{1}{3}\cdot\frac{1}{5^3} + \frac{1}{5}\cdot\frac{1}{5^5} - \&c.\right\}$$

$$- \left\{\frac{1}{239} - \frac{1}{3}\cdot\frac{1}{239^3} + \frac{1}{5}\cdot\frac{1}{239^5} - \&c.\right\}$$

The expression, it will be observed, consists of the difference between two series. Their values can be calculated separately, and the value of π. be found from thence in the following manner :—

(1). We obtain the value of $\frac{1}{5} - \frac{1}{3}\cdot\frac{1}{5^3} + \frac{1}{5}\cdot\frac{1}{5^5} -$ &c., to the 12th place of decimals in the following manner.

$$\frac{1}{5} = \cdot2$$

$$\frac{1}{5^3} = \cdot008$$

$$\frac{1}{5^5} = \cdot00032$$

$$\frac{1}{5^7} = \cdot0000128$$

$$\frac{1}{5^9} = \cdot000000512$$

$$\frac{1}{5^{11}} = \cdot00000002048$$

$$\frac{1}{5^{13}} = \cdot000000000819$$

$$\frac{1}{5^{15}} = \cdot000000000032$$

$$\frac{1}{5^{17}} = \cdot000000000001.$$

From these we can easily obtain, first, the negative, and then the positive part of the expression ; and subtracting we obtain the whole value required.

$$\frac{1}{3}\cdot\frac{1}{5^3} = \cdot\,002666666666$$

$$\frac{1}{7}\cdot\frac{1}{5^7} = \cdot\,000001828571$$

$$\frac{1}{11}\cdot\frac{1}{5^{11}} = \cdot\,000000001861$$

$$\frac{1}{15}\cdot\frac{1}{5^{15}} = \cdot\,000000000002$$

$$.\,002668497100$$

Again

$$\frac{1}{5} = \cdot\,2.$$

$$\frac{1}{5}\cdot\frac{1}{5^5} = \cdot\,000064$$

$$\frac{1}{9}\cdot\frac{1}{5^9} = \cdot\,000000056888$$

$$\frac{1}{13}\cdot\frac{1}{5^{13}} = \cdot\,000000000063$$

$$.\,200064056951$$

$$.\,002668497100$$

$$.\,197395559851$$

$$4$$

$$.\,789582239404$$

Which is the value of decimals

$$4\left\{\frac{1}{5} - \frac{1}{3}\cdot\frac{1}{5^3} + \&c.\right\} \text{ to 12 places.}$$

(2). To calculate the value of $\left\{\frac{1}{239} - \frac{1}{3}\cdot\frac{1}{239^3} + \ldots\right\}$

$$\frac{1}{239} = \cdot004184100418$$

$$\frac{1}{239^3} = \cdot000000073249$$

$$\frac{1}{239^5} = \cdot000000000001$$

$$\therefore \frac{1}{239} = \cdot004184100418$$

$$\frac{1}{3}\cdot\frac{1}{239^3} = \cdot000000024416$$

$$\cdot004184076002$$

Which is the value of $\left\{\frac{1}{239} - \frac{1}{3}\cdot\frac{1}{239^3} + -\right\}$ to 12 places of decimals.

If we subtract this from the result last found we shall obtain $\frac{\pi}{4}$. and ∴ multiplying that difference by 4, we shall obtain the value of π. required. Thus,

$$\begin{array}{r} {\cdot}789582239404 \\ {\cdot}004184076002 \\ \hline {\cdot}785398163402 \\ 4 \\ \hline 3{\cdot}141592653608 \end{array}$$

The calculation is carried to 12 places of decimals, but is not trustworthy beyond the first 10.

Hence (to 10 places of decimals)

$$\pi = 3{\cdot}1415926536$$

If we had wanted to obtain the value of π to a larger number of places of decimals, we should have had to carry the calculations throughout to a correspondingly greater extent.

Thus, we can show that

$$\pi = 3{\cdot}141592653589794^{*}$$

We can very conveniently apply De Moivre's formula to the solution of Binomial Equations, *i.e.*, equations of the form $x^n + a = 0$

(54). *To explain what we mean by the roots of a Binomial Equation.*

If we take the case $x^n + a = 0$, and if a be b^n, then the equation becomes $0^n + b^n = 0$, and if $x = by$. $x^n = b^n y^n$, and then the equation becomes

$$y^n + 1 = 0$$

where the 1 may be either positive or negative. It might seem at first sight that this has only one *root*, *viz.*, unity, but to conclude so would be an error, as a little consideration will make quite plain; for take the case

$$x^2 - 1 = 0$$

Then x is either $+1$, or -1; *i.e.*, has two values. And if we take the case

$$x^3 - 1 = 0$$

Then because

$$x^3 - 1 = (x - 1)(x^2 + x + 1)$$

we shall have

$$(x - 1)(x^2 + x + 1) = 0$$

whence x will have values corresponding to

$$x - 1 = 0$$
$$x^2 + x + 1 = 0$$

From the former of them we find that $x = 1$, and from the second that

$$x^2 + x + \tfrac{1}{4} = -\tfrac{3}{4}$$
$$\therefore x = \frac{-1 \pm \sqrt{-3}}{4}$$

And hence x has the three values.

$$1. \quad \frac{-1 + \sqrt{-3}}{2} \text{ and } \frac{-1 - \sqrt{-3}}{2}$$

which are three roots of the binomial equation

$$x^3 - 1 = 0$$

And hence we manifestly have that the roots of

$$x^3 - a^3 = 0$$

are

$$a. \quad \frac{-1 + \sqrt{-3}}{2}\, a \text{ and } \frac{-1 - \sqrt{-3}}{2}\, a.$$

In like manner

$$x^n + 1 = 0$$

has n different roots, which indeed follows from the general principle, that every equation of the n^{th} degree has n different roots, real or imaginary.

(55). *To find the roots of the Equation,* $x^n + 1 = 0$.

Since, $\cos. (2p + 1)\pi + \sqrt{-1} \sin. 2p + 1)\pi = -1$ whatever be the value of p. provided it be an integer. And since

$$x^n = -1$$

we must have

$$x^n = \cos. (2p + 1)\pi + \sqrt{-1} \sin. (2p + 1)\pi.$$

$$\therefore \quad x = (\cos. \overline{2p + 1}\,\pi + \sqrt{-1} \sin. \overline{2p + 1}\,\pi)^{\frac{1}{n}}$$

$$\text{or } x = \cos. \frac{2p + 1}{n}\pi + \sqrt{-1} \sin. \frac{2p + 1}{n}\pi.$$

Hence $\cos. \frac{2p + 1}{n}. \pi + \sqrt{-1} \sin. \frac{2p + 1}{n} \pi$. is a root of the binomial equation whatever integral value we may give to p.

We shall prove that there are n different values of this formula, corresponding to the different values of p, viz. : 0, 1, 2, 3. . . . $\overline{n - 1}$, and that there are *no more* than n values.

(*a*). Let p and q be two values, each $< n$; and if possible let

$$\cos. \frac{2p + 1}{n}. \pi + \sqrt{-1} \sin. \frac{2p + 1}{n}. \pi$$
$$= \cos. \frac{2q + 1}{n} \pi + \sqrt{-1} \sin. \frac{2p + 1}{n} \pi.$$

Hence equating possible and impossible terms,

$$\cos. \frac{2p + 1}{n}. \pi = \cos. \frac{2q + 1}{n} \pi.$$
$$\sin. \frac{2p + 1}{n} \pi = \sin. \frac{2q + 1}{n}. \pi.$$

Hence the angles must differ by some multiple of 2π, say by $k \times 2\pi$.

$$\therefore \frac{2p+1}{n}. \pi = \frac{2q+1}{n}. \pi \pm 2k\pi.$$

$$\therefore p - q = k.n. \text{ a multiple of } n,$$

and therefore either p or q must be greater than n. Hence all the values of x which correspond to the values of $p \angle n$, viz., 0, 1, 2, 3, ... $n - 1$, are different from each other.

(*b*). Again, if we give p a value r greater than n, then the corresponding value of x will be the same as one of those which is given by some value of p, that is less than n, as for instance q.

For suppose $p = kn + q$.

where k is a whole number, and q is less than n.

Then, $\cos. \frac{2p+1}{n}\pi + \sqrt{-1} \sin. \frac{2p+1}{n}\pi =$

$$\cos. \frac{2kn + 2q + 1}{n}\pi + \sqrt{-1} \sin. \frac{2kn + 2q + 1}{n}\pi =$$
$$\cos. \left(2k\pi + \frac{2q+1}{n}\pi\right) + \sqrt{-1} \sin. \left(2k\pi + \frac{2q+1}{n}\pi\right) =$$
$$\cos. \frac{2q+1}{n}\pi + \sqrt{-1} \sin. \frac{2q+1}{n}\pi.$$

Hence by giving to p. successively the values of 0.1.2.3 . . . . . $n - 1$. in the formula

$$x = \cos. \frac{2p+1}{n}\pi + \sqrt{-1} \sin. \frac{2p+1}{n}\pi.$$

we obtain all the roots of the Equation

$$x^n + 1 = 0.$$

Thus. To find the roots of the Equation

$$x^6 + 1 = 0.$$

the roots are given by the formula

$$x = \cos. \frac{2p+1}{6}\pi + \sqrt{-1} \sin. \frac{2p+1}{6}\pi.$$

and are therefore respectively

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* In connection with this, the student may advantageously peruse the remarks of Professor Young, in Chapter IV. on Geometry, in which he refers to the quadrature of the circle. (See p. 577).

$$\cos.\frac{\pi}{6}+\sqrt{-1}\sin.\frac{\pi}{6}.=\cos.30^\circ+\sqrt{-1}\sin.30^\circ=\frac{\sqrt{3}+\sqrt{-1}}{2}$$

$$\cos.\frac{3\pi}{6}+\sqrt{-1}\sin.\frac{3\pi}{6}.=\cos.90^\circ+\sqrt{-1}\sin.90^\circ=\sqrt{-1}$$

$$\cos.\frac{5\pi}{6}+\sqrt{-1}\sin.\frac{5\pi}{6}.=\cos.150^\circ+\sqrt{-1}\sin.150^\circ=\frac{-\sqrt{3}+\sqrt{-1}}{2}$$

$$\cos.\frac{7\pi}{6}+\sqrt{-1}\sin.\frac{7\pi}{6}.=\cos.210^\circ+\sqrt{-1}\sin.210^\circ=\frac{-\sqrt{3}-\sqrt{-1}}{2}$$

$$\cos.\frac{9\pi}{6}+\sqrt{-1}\sin.\frac{9\pi}{6}.=\cos.270^\circ+\sqrt{-1}\sin.270^\circ=-\sqrt{-1}$$

$$\cos.\frac{11\pi}{6}+\sqrt{-1}\sin.\frac{11\pi}{6}.=\cos.330^\circ+\sqrt{-1}\sin.330^\circ=\frac{\sqrt{3}-\sqrt{-1}}{2}$$

Similarly if we have to find the roots of the Equation $x^n-1=0$. Since $\cos.2p\pi+\sqrt{-1}\sin.2p\pi=1$ we shall have all the roots given in the formula $x=\cos.\frac{2p\pi}{n}+\sqrt{-1}\sin.\frac{2p\pi}{n}$.

(56). *To resolve x^{2n-1} into factors.*[*]

If $a_0\ a_1\ a_2\ \ldots\ldots\ a^{2n-1}$ are the roots of the Equation $x^{2n}-1=0$. Then $x^{2n}-1=(x-a_0)(x-a_1)(x-a_2)\ldots\ldots(x-a^{2n-1})$.

Now all the roots of the Equation are included in the expression $\cos.\frac{2p\pi}{2n}+\sqrt{-1}\sin.\frac{2p\pi}{2n}$.

If then we suppose $p\angle n$. and a_p be the corresponding root $a_p=\cos.\frac{p\pi}{n}+\sqrt{-1}\sin.\frac{p\pi}{n}$.

Now observe that there will be another root a^{2n-p}. which will equal $\cos.\frac{\overline{2n-p}\,\pi}{n}+\sqrt{-1}\sin.\frac{\overline{2n-p}\,\pi}{n}$

$$=\cos.\left(2\pi-\frac{p\pi}{n}\right)+\sqrt{-1}\sin.\left(2\pi-\frac{p\pi}{n}\right).$$

Hence $a^{2n-p}=\cos.\frac{p\pi}{n}-\sqrt{-1}\sin.\frac{p\pi}{n}$. where $p\angle n$, and so we have a pair of factors $x-a^p$ and $x-a^{2n-p}$

$$\text{which are equal to}\left(x-\cos.\frac{p\pi}{n}-\sqrt{-1}\sin.\frac{p\pi}{n}\right)\left(x-\cos.\frac{p\pi}{n}+\sqrt{-1}\sin.\frac{p\pi}{n}\right)$$

$$=x^2-2x\cos.\frac{p\pi}{n}+1,\text{ by multiplication.}$$

Where p may take any value, $1\cdot2\cdot3\ldots\ldots n-1$. It will be observed that $\left\{\begin{array}{l}x-a_0=x-1.\\x-a_n=x+1.\end{array}\right.$ $\therefore(x-a_0)(x-a_n)=x^2-1$

$$\text{and}\ \therefore x^{2n}-1=(x^2-1)\left(x^2-2x\cos.\frac{\pi}{n}+1\right)\left(x^2-2x\cos.\frac{2\pi}{n}+1\right)\ldots\left(x^2-2x\cos.\frac{\overline{n-1}\,\pi}{n}+1\right)\ (52)$$

$$\text{Corol. Hence, } x^{2n}-a^{2n}=(x^2-a^2)\left(x^2-2ax\cos.\frac{\pi}{n}+a^2\right)\ldots\ldots+\left(x-2ax\cos.\frac{\overline{n-1}\,\pi}{n}+a^2\right)\ldots(53)$$

$$\text{For since } y^{2n-1}=(y^2-1)\left(y^2-2y\cos.\frac{\pi}{n}+1\right)\left(y^2-2y\cos.\frac{2\pi}{n}+1\right)\ldots\ldots\left(y^2-2y\cos.\frac{\overline{n-1}\,\pi}{n}+1\right)$$

Let $y=\frac{x}{a}$.

$$\therefore\frac{x^{2n}}{a^{2n}}-1=\left(\frac{x^2}{a^2}-1\right)\left(\frac{x^2}{a^2}-2\frac{x}{a}\cos.\frac{\pi}{n}+1\right)\ldots\left(\frac{x^2}{a^2}-2\frac{x}{a}\cos.\frac{\overline{n-1}\,\pi}{n}+1\right)$$

Now as there are n factors, if we clear the equation of fractions, multiplying the right-hand side by a^{2n} will be the same thing as multiplying each factor by a^2.

Hence :—

$$x^{2n}-a^{2n}=\left(x^2-a^2\right)\left(x^2-2ax\cos.\frac{\pi}{n}+a^2\right)\ldots\left(x^2-2ax\cos.\frac{\overline{n-1}\,\pi}{n}+a^2\right)$$

By a precisely similar process we shall be able to prove that

$$x^{2n}+1=\left(x^2-2x\cos.\frac{\pi}{2n}+1\right)\left(x^2-2x\cos.\frac{3\pi}{2n}+1\right)\ldots\ldots\left(x^2-2x\cos.\frac{\overline{2n-1}\,\pi}{2n}+1\right)\ldots.(54).$$

And hence

$$x^{2n}+a^{2n}=\left(x^2-2ax\cos.\frac{\pi}{2n}+a^2\right)\left(x^2-2ax\cos.\frac{3\pi}{n}+a^2\right)\ldots..\left(x^2-2ax\cos.\frac{\overline{2n-1}\,\pi}{2n}+a^2\right)\ldots..(55).$$

---

[*] If $a_1, a_2, a_3\ldots\ldots a_n$ are the roots of the equation, $x^{n-1}=0$ Then $x^{n-1}=(x-a_1)(x-a_2)(x-a_3)\ldots\ldots(x-a_n)$ for all values of x.

For if we actually divide x^{n-1} by $x-a_1$, we shall obtain a remainder $a_1{}^{n-1}$; now, since a_1 is a root of the equation $x^{n-1}=0$, we have $a_1{}^{n-1}=0$, *i. e.*, x^{n-1} is divisible by $x-a_1$, or $x-a_1$, is a factor of x^{n-1}. Similarly $x-a_2$. $x-a_3\ldots x-a_n$ are each factors of x^{n-1}, and since x^{n-1} cannot have more than n factors, it is plain that

$$x^{n-1}=(x-a_1)(x-a_2)\ldots\ldots(x-a_n).$$

(57). *To prove that* $2^{2n-1} \sin.^2 \frac{\pi}{2n} \sin.^2 \frac{2\pi}{2n} \sin.^2 \frac{3\pi}{2n} \ldots \sin.^2 \frac{\overline{n-1}\,\pi}{2n} = 2n.$

We have by actual division—

$$\frac{x^{2n}-1}{x-1} = x^{2n-1} + x^{2n-2} + x^{3n-3} + \ldots x + 1.$$

Hence, when $x = 1$. The limiting value of $\frac{x^{2n}-1}{x-1}$ will equal $1+1+1+\ldots+1+1$ to $2n$ terms $= 2n$.

Again by the last article—

$$\frac{x^{2n}-1}{x-1} = (x+1.)\left(x^2 - 2x\cos.\frac{\pi}{n}+1\right)\left(x^2-2x\cos.\frac{2\pi}{n}+1\right)\ldots\left(x^2-2x\cos.\frac{\overline{n-1}\,\pi}{n}+1\right)$$

and hence when $x = 1$, the limiting value of $\frac{x^{2n}-1}{x-1}$.

$$\text{is } 2\left(2-2\cos.\frac{\pi}{n}\right)\left(2-2\cos.\frac{2\pi}{n}\right)\ldots\left(2-2\cos.\frac{\overline{n-1}\,\pi}{n}\right)$$

and therefore these two limiting values are equal, and

$$2n = 2^n\left(1-\cos.\frac{\pi}{n}\right)\left(1-\cos.\frac{2\pi}{n}\right)\ldots\left(1-\cos.\frac{\overline{n-1}\,\pi}{n}\right)$$

$$= 2^n \times 2\sin.\frac{2\pi}{2n}\,2\sin.^2\frac{2\pi}{2n}\,2\sin.^2\frac{3\pi}{2n}\ldots 2\sin.^2\frac{\overline{n-1}\,\pi}{2n} = 2^n \times 2^{n-1}\sin.\frac{2\pi}{2n}.\sin.^2\frac{2\pi}{2n}.\sin.^2\frac{3\pi}{2n}\ldots\sin.^2\frac{\overline{n-1}\,\pi}{2n}$$

$$= 2^{2n-1}\sin.^2\frac{\pi}{2n}\sin.^2\frac{2\pi}{2n}\sin.^2\frac{3\pi}{2n}\ldots\sin.^2\frac{\overline{n-1}\,\pi}{2n}. \quad \ldots (56).$$

In the same manner we may prove that

$$2 = 2^{2n}\sin.^2\frac{\pi}{4n}\sin.^2\frac{3\pi}{4n}\sin.^2\frac{5\pi}{4n}\ldots\sin.^2\frac{\overline{2n-1}\,\pi.}{4n}. \quad \ldots (57).$$

For

$$x^{2n}+1 = \left(x^2-2x\cos.\frac{\pi}{2n}+1\right)\left(x^2-2x\cos.\frac{3\pi}{2n}+1\right)\ldots\left(x^2-2x\cos.\frac{\overline{2n-1}\,\pi.}{2n}+1\right)$$

Let $x = 1$, and then observing that

$$x^2 - 2x\cos.\frac{p\pi}{2n}+1 = 2\left(1-\cos.\frac{p\pi.}{2n.}\right)$$

$$= 2^2\sin.^2\frac{p\pi.}{4n.}$$

We shall obtain the required formula.

(58). *To express* sin. x *in a series of factors.*

The above formula (53) for the factors of $x^{2n} - a^{2n}$ is true for all values of x and a

Suppose $x = 1 + \frac{z}{2n}$ and $a = 1 - \frac{z}{2n}$.

$$\text{Then } x^2 - 2ax\cos.\frac{p\pi}{n} + a^2 = 1 + \frac{z}{n} + \frac{z^2}{4n^2} - 2\left(1-\frac{z^2}{4n^2}\right)\cos.\frac{p\pi}{n} + 1 - \frac{z}{n} + \frac{z^2}{4n^2}$$

$$= 2\left(1+\frac{z^2}{4n^2}\right) - 2\left(1-\frac{z^2}{4n^2}\right)\cos.\frac{p\pi}{n}.$$

$$= 2\left(1-\cos.\frac{p\pi}{n}\right) + 2\frac{z^2}{4n^2}\left(1+\cos.\frac{p\pi}{n}.\right)$$

$$= 4\sin.^2\frac{p\pi}{2n} + 4.\frac{z^2}{4n^2}.\cos.^2\frac{p\pi.}{2n}$$

$$= 4\sin.^2\frac{p\pi}{2n}.\left(1+\frac{z^2}{4n^2}\cot an.^2\frac{p\pi}{2n}\right)$$

Hence the factors on the right-hand side of Equation (53) will be equal to

$$\frac{2z}{n}.4\sin.^2\frac{\pi}{2n}\left(1+\frac{z^2}{4n^2}.\text{cotan.}^2\frac{\pi}{2n}\right)4\sin.^2\frac{2\pi}{2n}\times$$

$$\left(1+\frac{z^2}{4n^2}\text{cotan.}^2\frac{\pi}{2n}\right)\ldots 4\sin.^2\frac{(n-1)\pi}{2n}\left(1+\frac{z^2}{4n^2}\text{cotan.}^2\frac{\overline{n-1}\,\pi}{2n}\right)$$

$$= \frac{2z}{n}.4\sin.^2\frac{\pi}{2n}4\sin.^2\frac{2\pi}{2n}4\sin.^2\frac{3\pi}{2n},\ldots 4\sin.^2\frac{\overline{n-1}\,\pi}{n}$$

$$\times\left(1+\frac{z^2}{4n^2}.\text{cotan.}^2\frac{\pi}{2n}\right)\left(1+\frac{z^2}{4n^2}\text{cotan.}^2\frac{2\pi}{2n}\right)\cdot\left(1+\frac{z^2}{4n^2}\text{cotan.}^2\frac{\overline{n-1}\,\pi}{2n}\right)$$

$$= \frac{z}{n}.2^{2n-1}\sin.^2\frac{\pi}{2n}.\sin.^2\frac{2\pi}{2n}\ldots\sin.^2\frac{\overline{n-1}\,\pi}{2n}\times$$

$$\left(1+\frac{z^2}{4n^2}.\text{cotan.}^2\frac{\pi}{2n}\right)\left(1+\frac{z^2}{4n^2}.\text{cotan.}^2\frac{2\pi}{2n}\right)\ldots\left(1+\frac{z^2}{4n^2}\text{cotan.}^2\frac{\overline{n-1}\,\pi}{2n}\right)$$

$$= 2z\left(1+\frac{z^2}{4n^2}\text{cotan.}^2\frac{\pi}{2n}\right)\left(1+\frac{z^2}{4n^2}\text{cotan.}^2\frac{2\pi}{2n}\right)\ldots\left(1+\frac{z^2}{4n^2}\text{cotan.}^2\frac{\overline{n-1}\,\pi}{2n}\right)\ldots (a).$$

Since by the last article

$$2^{2n-1} \sin.^2 \frac{\pi}{2n} \ldots . \sin.^2 \frac{\overline{n-1}\,\pi}{2n} = 2n.$$

Again

$$x^{2n} - a^{2n} = \left(1 + \frac{z}{2n}\right)^{2n} - \left(1 - \frac{z}{2n}\right)^{2n}.$$

and expanding each by the Binomial Theorem.

$$x^{2n} - a^{2n} = \left(1 + 2n.\frac{z}{2n} + \frac{2n.\overline{2n-1}}{1.2}\cdot\frac{z^2}{4n^2} + \frac{2n.\overline{2n-1}\,\overline{2n-2}}{1.2.3}\cdot\frac{z^3}{8n^3} + \right)$$

$$- \left(1 - 2n.\frac{z}{2n} + \frac{2n.\overline{2n-1}}{1.2}\cdot\frac{z^2}{4n^2} - \frac{2n.\overline{2n-1}\,\overline{2n-2}}{1.2.3}\cdot\frac{z^3}{8n^3} + \right)$$

$$= 2\left\{2n.\frac{z}{2n} + \frac{2n.\overline{2n-1}\,\overline{2n-2}}{1.2.3}\cdot\frac{z^3}{8n^3} + \ldots . \right\} \ldots . (b).$$

Now (*a*) and (*b*) are equal under all circumstances, and therefore their limiting values when $2n$ becomes very large are equal, *i. e.*, when $\frac{1}{2n}\,\frac{2}{2n} \ldots . \frac{p}{2n}$ are each equal to zero.

$$\text{Now since } \frac{z^2}{4n^2}\cdot \text{cotan.}^2 \frac{p\pi}{2n} = \frac{z^2}{4n^2}\cdot\frac{1}{\tan.^2 \frac{p\pi}{2n}}$$

$$= \frac{z^2}{p^2\pi^2}\cdot\frac{\frac{p^2\pi^2}{4n^2}}{\tan.^2 \frac{p\pi}{2n}} = \frac{z^2}{p^2\pi^2}\cdot\left\{\frac{\frac{p\pi}{2n}}{\tan. \frac{p\pi}{2n}}\right\}^2.$$

But in the limit when $2n$ is very large $\frac{p\pi}{2n} = 0.$

And the limit of $\frac{\tan.\,\theta}{\theta}$ when $\theta = 0$ is unity.

Hence the limiting value of $\dfrac{\frac{p\pi}{2n}}{\tan.\,\frac{p\pi}{2n}} = 1.$

And therefore the limiting value of $\frac{z^2}{4n^3}$ cotan.$^2 \frac{p\pi}{2n} = \frac{z^2}{p^2\pi^2}$

Hence the limiting value of the expression (*a*) is

$$2z\left\{1 + \frac{z^2}{\pi^2}\right\}\ \left\{1 + \frac{z^2}{4\pi^2}\right\}\ \left\{1 + \frac{z^2}{9\pi^2}\right\} \ldots . -$$

Again since (*b*) can be written

$$2z\left\{1 + \frac{\left(1 - \frac{1}{2n}\right)\ \left(1 - \frac{2}{2n}\right)}{1.2.3}\,z^2 + \frac{\left(1 - \frac{1}{2n}\right)\ldots\left(1 - \frac{4}{2n}\right)}{1.2.3.4.5}\,z^4 \ldots . \right\}$$

The limiting value of (*b*) is $\quad 2z\left(1 + \frac{z^2}{1.2.3} + \frac{z^4}{1.2.3.4.5} + -\right)$

Hence,

$$1 + \frac{z^2}{1\cdot 2\cdot 3} + \frac{z^4}{1\cdot 2\cdot 3\cdot 4\cdot 5} + \ldots = \left(1 + \frac{z^2}{\pi^2}\right)\ \left(1 + \frac{z^2}{2^2\pi^2}\right) \ldots .$$

or writing $-z^2$ for z^2, we have—

$$1 - \frac{z^2}{1\cdot 2\cdot 3} + \frac{z^4}{1\cdot 2\cdot 3\cdot 4\cdot 5} = \left(1 - \frac{z^2}{\pi^2}\right)\ \left(1 - \frac{z^2}{2^2\pi^2}\right)\ \left(1 - \frac{z^2}{3^2\pi^2}\right) \ldots .$$

Multiply both sides by z, and we obtain

$$z - \frac{z^3}{1\cdot 2\cdot 3} + \frac{z^5}{1\cdot 2\cdot 3\cdot 4\cdot 5} = z\left(1 - \frac{z^2}{\pi^2}\right)\ \left(1 - \frac{z^2}{2^2\pi^2}\right) \ldots .$$

But, $\sin.\, z = z - \frac{z^3}{1\cdot 2\cdot 3} + \frac{z^5}{1\cdot 2\cdot 3\cdot 4\cdot 5} - \&c.$

$$\therefore \text{Sin. } z = z\left(1 - \frac{z^2}{\pi^2}\right)\ \left(1 - \frac{z^2}{2^2\pi^2}\right)\ \left(1 - \frac{z^2}{3^2\pi^2}\right) \ldots . - (58).$$

which is the expression required.

In the same manner, by taking $x^{2n} + a^{2n}$ and its factors from formula (55), we can obtain cos. x in factors.

$$\text{Thus, let } x = 1 + \frac{z}{2n}\cdot a = 1 - \frac{z}{2n}\cdot$$

$$\therefore x^{2n} + a^{2n} = 2\left\{1 + \frac{2n.\overline{2n-1}}{1.2}\cdot\frac{{}^2}{4n^2} + \frac{2n.\overline{2n-1}\ \overline{2n-2}\ \overline{2n-3}}{1.2.3.4}\,\frac{2^4}{16n^4} + \ldots \right\}$$

And as before the limiting value of

$x^{2n} + a^{2n}$ is $2\left(1 + \frac{z^2}{1.2} + \frac{z^4}{1.2.3.4} + \dots\right)$. (a)

Again since

$$x^2 - 2ax \cos. \frac{\overline{2p+1}n}{2n} + a^2 = 2\left(1 + \frac{z^2}{4n^2}\right) - 2\left(1 - \frac{z^2}{4n^2}\right) \frac{\cos. \overline{2p+1}\,\pi}{2n}$$

$$= 4 \sin.^2 \frac{\overline{2p+1}\pi}{4n} \left\{ 1 + \frac{z^2}{4n^2} \cot an.^2 \frac{\overline{2p+1}\pi}{4n} \right\}$$

The factors of $x^{2n} + a^{2n}$ become

$$2^{2n} \sin.^2 \frac{\pi}{4n} \sin.^2 \frac{3\pi}{4n} \sin.^2 \frac{5\pi}{4n} \dots \sin.^2 \frac{\overline{2n-1}\pi}{4n} \times$$

$$\left(1 + \frac{z^2}{4n^2} \text{cotan.}^2 \frac{\pi}{4n}\right)\left(1 + \frac{z^2}{4n^2}.\text{cotan.}^2 \frac{3\pi}{4n}\right)\left(1 + \frac{z^2}{4n^2} \text{cotan.}^2 \frac{5\pi}{4n}\right) \dots$$

$$= 2\left(1 + \frac{z^2}{4n^2} \text{cotan.}^2 \frac{\pi}{4n}\right)\left(1 + \frac{z^2}{4n^2}.\text{cotan.}^2 \frac{3\pi}{4n}\right)\left(1 + \frac{z^2}{4n^2} \text{cotan.}^2 \frac{5\pi}{4n}\right) \dots,$$

$$\text{Now, } \frac{z^2}{4n^2} \cdot \text{cotan.}^2 \frac{\overline{2p+1}\,\pi}{4n} = \frac{4z^2}{(\overline{2p+1}\,\pi)^2} \cdot \left\{ \frac{\frac{\overline{2p+1}\,\pi}{4n}}{\tan. \frac{\overline{2p+1}\,\pi}{4n}} \right\}$$

The limiting value of which is $\frac{4z^2}{\overline{2p+1}^2\pi^2}$

Hence the limiting value of $x^{2n} + a^{2n}$ derived from the expression for its factors will be

$$2\left(1 + \frac{4z^2}{\pi^2}\right)\left(1 + \frac{4z^2}{3^2\pi^2}\right)\left(1 + \frac{4z^2}{5^2\pi^2}\right) \dots (b)$$

And the two limiting values (a) and (b) are equal. Hence

$$1 + \frac{z^2}{1.2} + \frac{z^4}{1.2.3.4} + \dots = \left(1 + \frac{4z^2}{\pi^2}\right)\left(1 + \frac{4z^2}{3^2\pi_2}\right)\left(1 + \frac{4z2}{5^2\pi^2}\right) \dots$$

Write $-z^2$ for $+z^2$ (as before) and we obtain

$$1 - \frac{z^2}{1.2} + \frac{z^4}{1.2.3.4} - \&c. = \left(1 - \frac{4z^2}{\pi^2}\right)\left(1 - \frac{4z^2}{3^2\pi^2}\right)\left(1 - \frac{4z^2}{5^2\pi^2}\right)$$

$$\therefore \cos. z = \left(1 - \frac{4z^2}{\pi^2}\right)\left(1 - \frac{4z^2}{3^2\pi^2}\right)\left(1 - \frac{4z^2}{5^2\pi^2}\right) \dots (59)$$

The series given in the preceding articles are sufficient for our purpose. We now proceed to explain the method of calculating trigonometrical tables.

The Tables of the trigonometrical functions are of two kinds. (1). Those which give the numerical values of the sines, cosines of the angles, &c. (2). Those which give the values of the logarithms of the sines, cosines, &c., of the angles. The former are called Tables of natural sines; the latter, Tables of logarithmic sines.

We shall explain the method by which each of these tables is calculated.

(59). *To show that* sin. θ *is less than* θ *and greater than*

$$\theta - \frac{\theta^3}{4}$$

For, $\sin. \theta = 2 \sin. \frac{\theta}{2} \cos. \frac{\theta}{2} = 2 \sin. \frac{\theta}{2}\left(1 - \sin.^2 \frac{\theta}{2}\right)$

Now $1 - \sin.^2 \frac{\theta}{2}$ must be less than 1.

$\therefore \sin. \theta \angle 2 \sin. \frac{\theta}{2}$.

$\therefore \sin. \frac{\theta}{2} \angle 2 \sin. \frac{\theta}{2^2}$.

$\therefore \sin. \frac{\theta}{2^2} \angle 2 \sin. \frac{\theta}{2^3}$.

$\therefore \text{Sin.} \frac{\theta}{2^{n-1}} \angle 2 \sin. \frac{\theta}{2^n}$.

Hence $\sin. \theta \angle 2^n \sin. \frac{\theta}{2^n}$. whatever n may be.

$\frac{\text{Sin. } \theta}{\theta} \angle \frac{\sin. \frac{\theta}{2^n}}{\frac{\theta}{2^n}}$ for all values of n, however large n may be; and therefore, in the limiting case, when n is infinitely large, & $\therefore 2^n$ infinitely large, or when $\frac{\theta}{2^n} = 0$.

But in this case $\frac{\sin. \frac{\theta}{2^n}}{\frac{\theta}{2^n}} = 1$,

$\therefore \frac{\sin. \theta}{\theta} < 1$.

or $\sin. \theta < \theta$

In like manner we can easily prove that $\tan. \theta > \theta$

For $\tan. \theta = \frac{2 \tan. \frac{\theta}{2}}{1 - \tan.^2 \frac{\theta}{2}}$

$\therefore \tan. \theta > 2 \tan. \frac{\theta}{2}$

Hence by same reasoning as before, $\tan. \theta > 2^n \tan. \frac{\theta}{2^n}$

$\therefore \frac{\tan. \theta}{\theta} > \frac{\tan. \frac{\theta}{2^n}}{\frac{\theta}{2^n}}$

Now, when n is infinitely large $\frac{\theta}{2^n} = 0$

and $\frac{\tan. \frac{\theta}{2^n}}{\frac{\theta}{2^n}}$ in this case $= 1$.

$$\therefore \frac{\tan.\,\theta}{\theta} > 1$$

$$\text{or } \tan.\,\theta > \theta$$

(2). To show that $\sin.\,\theta > \theta - \frac{\theta^3}{4}$.

$$\text{For } \sin.\,\theta = 2\sin.\,\frac{\theta}{2}\ \cos.\,\frac{\theta}{2}$$

$$= 2\tan.\,\frac{\theta}{2}\cos.^2\frac{\theta}{2}$$

$$= 2\tan.\,\frac{\theta}{2}\left(1-\sin.^2\frac{\theta}{2}\right)$$

$$=\theta.\,\frac{\tan.\,\frac{\theta}{2}}{\frac{\theta}{2}}\left\{1-\frac{\theta^2}{4}\left(\frac{\sin.\,\frac{\theta}{2}}{\frac{\theta}{2}}\right)^2\right\}$$

Now $\dfrac{\tan.\,\frac{\theta}{2}}{\frac{\theta}{2}}$ being > 1 and $\left\{\dfrac{\sin.\,\frac{\theta}{2}}{\frac{\theta}{2}}\right\}^2$ being $\angle 1$.

$$1-\frac{\theta^2}{4}\ \text{must be} \angle\ 1-\frac{\theta^2}{4}\left(\frac{\sin.\,\frac{\theta}{2}}{\frac{\theta}{2}}\right)^2$$

$$\therefore \sin.\,\theta \text{ must be } > \theta\left(1-\frac{\theta^2}{4}\right)$$

$$\therefore \sin.\,\theta > \theta-\frac{\theta^3}{4}$$

(60). *To calculate the value of* sin. 1′ *and* cos. 1′.

Let θ be an angle of 1′ measured by the circular measure. Then,—

$$\theta = \pi - \frac{1'}{180^\circ} = \frac{\pi}{180 \times 60}.$$

$\therefore$ Taking the value of π previously given,

$$\theta = \cdot 00029088820.$$

$$\frac{\theta^3}{4} = \cdot 00000000006$$

Hence, if we only take in the first *ten* decimal places,

$$\sin.\,1' = \cdot\,0002908882.$$

And since

$$\cos.\,1' = \sqrt{1-\sin.^2 1'} = 1-\frac{1}{2}\sin.^2 1 - \frac{1}{8}\sin.^4 1'\ldots$$

$$= .\ 9999999577.$$

Cor.—Hence, it is plain that if our approximation do not extend beyond ten places of decimals,

Circular measure of angle of 1′ = sin. 1′.
Similarly, Circular measure of angle of 1″ = sin. 1″.
$\therefore$ Circular measure of angle of n'' = n sin. 1″, provided $n \angle 60$.

(61). *To calculate the Sines of* 2′ 3′ 4′. . . . .

Since $\sin.\,(A + B) + \sin.\,(A - B) = 2\cos.\,B, \sin.\,A.$
we have $\sin.\,(n + 1)' + \sin.\,(n - 1)' = 2\cos.\,1' \sin.\,n'.$
Now $\cos.\,1' = 1 - \cdot\,0000000423\ . = 1 - k.$
$\therefore \sin.\,(n + 1)' + \sin.\,(n - 1)' = 2\sin\,n' - 2k \cdot \sin.\,n'.$
$\therefore \sin.\,(n + 1)' - \sin.\,n' = \sin.\,n' - \sin.\,(n - 1)' - 2k\sin\,n'.$

This formula is very convenient for calculating the sines and cosines of successive angles. Thus,

$\text{Sin.}\,2' - \sin.\,1' = \sin.\,1' - 2k\sin.\,1'.$
$\text{Sin.}\,3' - \sin.\,2' = \sin.\,2' - \sin.\,1' - 2k\sin.\,2'$
$\text{Sin.}\,4' - \sin.\,3' = \sin.\,3' - \sin.\,2' - 2k\sin.\,3'.$

and so on. It will be observed that the first member of the right-hand side of each equation is given by the former equation. So that the only term requiring multiplication is $2k\sin.\,1'$, $2k\sin.\,2'$, $2k\sin.\,3'$ in each equation. This multiplication can be greatly facilitated by forming a table in which $2k$ is multiplied by each digit, thus :—

$2k =$	846
2	1692
3	2538
4	3384
5	4230
6	5076
7	5922
8	6768
9	7614

where the seven zeros in front of the significant digits of $2k$ are suppressed. By means of this table, which resembles that of the proportional parts in the table of logarithms, the multiplication can be performed by means of addition only. It will also be observed that in the case of the sines of the first few minutes the products of k will have more than ten zeros, and therefore can be omitted, and that under all circumstances they will have at the least seven, so that the multiplication is soon performed. Thus, suppose

$$\begin{array}{rl} \text{Sin.}\,p' = & \cdot\,3759264827 \\ 2k.\,\sin.\,p' = & \cdot\,0000000253{,}8 \\ & 59{,}3 \\ & 4{,}2 \\ & 7 \\ \hline & .0000000318 \end{array}$$

By this means we can successively obtain the sines 1′ 2′ 3′ 4′, and so on for every minute up to 45°.

(62). *To obtain the Cosines of the Angles* 2′ 3′ 4′, *&c.*

Since $\cos.\,(A + B) + \cos.\,(A - B) = 2\cos.\,A\cos.\,B.$
$\therefore \text{Cos.}\,(n + 1)' + \cos.\,(n - 1)' = 2\cos.\,1\cos.\,n'.$
Hence as before

$$\text{Cos.}\ (n+1)' - \cos.\,n' = \cos.\,n' - \cos.\,\overline{n-1}' - 2k\cos.\,n'.$$

or, since the cosine continually decreases,

$$\text{Cos.}\,n' - \cos.\,\overline{n+1}' = \cos.\,\overline{n-1}' + \cos.\,n' + 2k\cos.\,n'.$$

Hence

$\text{Cos.}\,1' - \cos.\,2' = \cos.\,0' - \cos.\,1' + 2k\cos.\,1'.$
$\text{Cos.}\,2' - \cos.\,3' = \cos.\,1' - \cos.\,2' + 2k\cos.\,2'.$
$\text{Cos.}\,3' - \cos.\,4' = \cos.\,2' - \cos.\,3' + 2k\cos.\,3'.$

The method of calculation is precisely similar to that of the sines. It will be further observed, that since sin. (90°— A) = cos. A, that we need not to continue the calculations of the sines and cosines of the angles beyond 45°. For example, if we know sin. 23° 15′ and cosin. 23° 15′, these are respectively cosin. 66° 45′ and sin. 66° 45′.

(63). *Simplification of calculation in case of certain Angles.*

Again, the calculation of sines and cosines of angles greater than 30° can be very much simplified ; for

$$\text{Sin.}\,(30^\circ + \theta) + \sin.\,(30 - \theta) = 2\sin.\,30^\circ\cos.\,\theta = \cos.\,\theta$$

$$\text{Since } \sin.\,30^\circ = \frac{1}{2}$$

$$\therefore \sin.\,(30^\circ + \theta) = \cos.\,\theta - \sin.\,(30^\circ - \theta)\,;$$

now if $30^\circ + \theta$ be less than 60°, then θ, and $30^\circ - \theta$ are each less than 30°. Hence by our previous calculations we know both cos. θ and sin. $(30^\circ - \theta)$, and therefore obtain sin. $(30^\circ + \theta)$ by subtraction.

Thus sin. (41°. 15′) = cos. (11°, 15′) — sin. (18°. 45′).

It is plain that by this formula we can calculate the sines of angles from 30° to 60°.

And since cos. $(60^\circ - \theta)$ = sin. $(30^\circ + \theta)$ this calculation of the sines from 30° to 60° gives the sines from 30° to 45°, and the cosines from 45° to 30° ; which is what we want to complete the tables from 0 to 45°. Thus if we calculate by the preceding formula, sin. (51°. 33′), this is the same thing as cos. (38°. 27′).

(64). *Method of checking the calculation.*

It will be observed, that according to the method above given, the sine of a given number of degrees or minutes is inferred from the sine of the number preceding. Hence, if an error be made at any one point, say in sin. 3° 15′, it will be propagated into the sines of every succeeding angle ; to arrest the progress of any such error, as well as to verify the correctness of the calculations, at different points of their progress, when no such errors exist, it is usual to interpose the values of any such terms as can be calculated by independent methods. Thus we have already seen that the sines, &c., of the angles 15°. 30°. 45° ... can be readily expressed, as well as those of 18°. 36°. 54°...* And that hence we can obtain the sines, &c., of an angle of 3°. These and others will act as *stops* in the series, and also serve to verify the accuracy of the calculation up to the point where they are inserted.

(65). *The calculation of Tangents and Cotangents.*

By the methods now explained we can calculate the natural sines and cosines of all angles from 0° to 90° for every minute of a degree. The natural tangents of angles between 0° and 45° can be obtained by simple division, since tan. $\theta = \frac{\sin.\ \theta}{\cos.\ \theta}$ From 45° up to 90° we can obtain the tangent by the formula,

$$\text{Tan. } (45° + \theta) = 2 \tan.\ 2\,\theta + \tan.\ (45° - \theta.)$$

To prove this formula.

$$\text{Tan. } (45° + \theta) - \tan.\ (45° - \theta) = \frac{\sin.\ (45° + \theta)}{\cos.\ (45° + \theta)} - \frac{\sin.\ (45° - \theta)}{\cos.\ (45° - \theta)}$$

$$= \frac{\sin.\ (45° + \theta) \cos.\ (45° - \theta) - \sin.\ (45° - \theta) \cos.\ (45° + \theta)}{\cos.\ (45° + \theta) \cos.\ (45° - \theta)}$$

$$= \frac{\sin.\ 2\,\theta}{\cos.\ (45° - \theta) \cos.\ (90° - 45° - \theta)}$$

$$= \frac{2 \sin.\ 2\,\theta}{2 \cos.\ (45° - \theta) \sin.\ (45° - \theta)}$$

$$= \frac{2 \sin.\ 2\,\theta}{\cos.\ (90° - 2\,\theta)} = \frac{2 \sin.\ 2\,\theta}{\cos.\ 2\,\theta}$$

$$= 2 \tan.\ 2\,\theta$$

$$\therefore \tan.\ (45° + \theta) = 2 \tan.\ 2\,\theta + \tan.\ (45° - \theta)$$

Since θ is less than 45°. $2\,\theta$ is less than $45° + \theta$. and hence, whatever be the value of θ, we shall have already calculated tan. $2\,\theta$. before we need tan. $2\,\theta$. for the determination of tan. $(45° + \theta)$.

$$\text{Thus} \quad \tan.\ 81° = 2 \tan.\ 72° + \tan.\ 9°$$

where before we calculate tan. 81° we shall already have calculated tan. 72°.

It is plain that if we know the tangents of angles from 0° to 90°, we also know the cotangents of the angles from 90° to 0°.

(66). *Formulas of verification.*

There are many formulas by which the accuracy of the tables, when calculated, can be tested. The following are some of them :

(1). $\text{Cos. } \theta = \sin.\ (30° + \theta) + \sin.\ (30° - \theta)$.

(2). $\text{Sin. } \theta + \sin.\ (72° + \theta) - \sin.\ (72° - \theta) = \sin.\ (36° + \theta) - \sin.\ (36° - \theta)$.

(3). $\text{Cos. } (90° - \theta) + \cos.\ (18° - \theta) - \cos.\ (18° + \theta) = \cos.\ (54° - \theta) - \cos.\ (54° + \theta)$.

The student will readily verify these formulas if he remember the numerical values found in previous articles for the sines, &c., of 18°.†

(67). *Another method of calculating Tables of natural Sines and Cosines.*

Besides the method already given for calculating the natural sines and cosines of angles, there is another more convenient than that. The following is an account of this second method.

We have $$\text{Sin. } x = x - \frac{x^3}{1.2.3} + \frac{x^5}{1.2.3.4.5} - \&c. \ldots\ldots$$

Now suppose x to be an angle $= \frac{m}{n} \cdot \frac{\pi}{2}$. Then

$$\text{Sin. } \frac{m}{n} \cdot \frac{\pi}{2} = \frac{m}{n} \cdot \frac{\pi}{2} + \left(\frac{m}{n}\right)\left(\frac{\pi}{2}\right)^5 \times \frac{1}{120} + \left(\frac{m}{n}\right)^9 \left(\frac{\pi}{2}\right)^9 \frac{1}{362880} + \ldots\ldots - \left\{ \left(\frac{m}{n}\right)^3 \left(\frac{\pi}{2}\right)^3 \frac{1}{6} + \left(\frac{m}{n}\right)^7 \left(\frac{\pi}{2}\right)^7 \frac{1}{5040} + \cdots\cdots \right\}$$

Now we have already seen that $\frac{\pi}{2} = 1{\cdot}570796326794897$.

Hence if we reduce the coefficients of $\frac{m}{n} \cdot \left(\frac{m}{n}\right)^3 \ldots\ldots$ to numbers, we have

$$\text{Sin. } \frac{m}{n} \cdot 90°$$

$$= \frac{m}{n} \times 1.570796326794897 - \left(\frac{m}{n}\right)^3 \times 0.645964097506246$$

$$+ \left(\frac{m}{n}\right)^5 \times 0.079692626246167 - \left(\frac{m}{n}\right)^7 \times 0.004681754135319$$

$$+ \left(\frac{m}{n}\right)^9 \times 0.000160441184787 - \left(\frac{m}{n}\right)^{11} \times 0.000003598843235$$

$$+ \left(\frac{m}{n}\right)^{13} \times 0.000000056921729 - \left(\frac{m}{n}\right)^{15} \times 0.000000000668804$$

$$+ \left(\frac{m}{n}\right)^{17} \times 0.000000000006067 - \left(\frac{m}{n}\right)^{19} \times 0.000000000000044$$

$$+ \&c. \ldots$$

A similar formula can be calculated for cos. $\frac{m}{n} \cdot 90°$ It will be observed that $\frac{m}{n}$ is always a fraction less than $\frac{1}{2}$, since we only require the sines and cosines of angles less than 45°. Hence these series converge very rapidly, and from them we can easily calculate the sines and cosines for each degree from 1° up to 45°. When these are known, the sines and cosines of the angles for intervals of 1′, or if necessary 10″ or of 1″, can be found by the "method of interpolations."

The method of interpolations involves mathematics of a higher order than is admissible in an elementary course. The advanced reader will find an account of the application of this method to the calculation of Tables of natural sines and cosines in Airy's *Treatise on Trigonometry*, in the *Encyclopædia Metropolitana.*

Calculation of Logarithmic Sines, &c.

We now proceed to give an account of the method of constructing Tables of the logarithms of sines, cosines, &c., of angles.

N. B.—The sines and cosines of angles are never greater than 1. Consequently their logarithms are negative. The numbers printed in the tables are always the logarithms of the sines, &c., with 10 added to them ; thus if L denote the tabular logarithm, and *log.* the ordinary logarithm

$$\text{L. sin. } \theta = \log.\ \sin.\ \theta + 10.$$

This notation will be observed throughout the following pages.

The Tables may be constructed by calculating the logarithms of the natural sines and cosines. And then as we have already found the natural sines and cosines of angles for intervals of 1′ from 0° to 45°, this table will give us the log. sines, and cosines for intervals of 1′ from 0° to 45°.

They are, however, more generally calculated by an independent process, of which the following is an account :

(68). *To obtain an expression for* log. sin. $\frac{m}{n} \cdot 90°$ *in a form adapted for calculation.*

We have already seen (page 634) that

$$\text{Sin. } x = x \left(1 - \frac{x^2}{\pi^2}\right)\left(1 - \frac{x^2}{2^2\,\pi^2}\right)\left(1 - \frac{x^2}{3^2\,\pi^2}\right)\left(1 - \frac{x^2}{4^2\,\pi^2}\right)$$

---

* See *ante*, p. 621.

† See *ante*, p. 621.

hence

$$\text{Sin.}\,\frac{m}{n}.\frac{\pi}{2}=\frac{m}{n}.\frac{\pi}{2}.\left(1-\frac{m^2}{4n^2}\right)\left(1-\frac{m^2}{4^2n^2}\right)\left(1-\frac{m^2}{6^2n^2}\right)\left(1-\frac{m^2}{8^2n^2}\right)$$

taking the logarithms on both sides.

$$\text{Log. sin.}\frac{m}{n}.\frac{\pi}{2}=\text{log.}\,\frac{m}{n}+\text{log.}\,\frac{\pi}{2}+\text{log.}\,\frac{4n^2-m^2}{4n^2}+\text{log.}\left(1-\frac{1}{4^2}.\frac{m^2}{n^2}\right)+\text{log.}\left(1-\frac{1}{6^2}.\frac{m^2}{n^2}\right)+\text{log.}\left(1-\frac{1}{8^2}.\frac{m^2}{n^2}\right)+\ldots$$

$$=\text{log. } m-\text{log. } n+\text{log. }\pi-\text{log. }2+\text{log. }(4m^2-m^2)-\text{log. }4n^2$$

$$-M\left(\frac{1}{4^2}\frac{m^2}{n^2}+\frac{1}{2}.\frac{1}{4^4}.\frac{m^4}{n^4}+\frac{1}{3}\frac{1}{4^6}.\frac{m^6}{n^6}+\ldots\right)$$

$$-M\left(\frac{1}{6^2}.\frac{m^2}{n^2}+\frac{1}{2}.\frac{1}{6^4}.\frac{m^4}{n^4}+\frac{1}{3}.\frac{1}{6^6}.\frac{m^6}{n^6}+\ldots\right)$$

$$-M\left(\frac{1}{8^2}.\frac{m^2}{n^2}+\frac{1}{2}.\frac{1}{8^4}.\frac{m^4}{n^4}+\frac{1}{3}.\frac{1}{8^6}.\frac{m^6}{n^6}+\ldots\right)$$

$$-\&c.$$

Where M is the modulus of the common logarithms.

$$\therefore\ \text{Log. sin.}\,\frac{m}{n}.\frac{\pi}{2}=$$

$$\text{log. } m+\text{log. }(2n-m)+\text{log. }(2n+3)-3\text{ log. } n+\text{log. }\pi-\text{log. }8.$$

$$-M\left(\frac{1}{4^2}+\frac{1}{6^2}+\frac{1}{8^2}+\frac{1}{10^2}+\ldots\ldots\right)\frac{m^2}{n^2}$$

$$-\frac{M}{2}\left(\frac{1}{4^4}+\frac{1}{6^4}+\frac{1}{8^4}+\frac{1}{10^4}+\ldots\ldots\right)\frac{m^4}{n^4}$$

$$-\frac{M}{3}\left(\frac{1}{4^6}+\frac{1}{6^6}+\frac{1}{8^6}+\frac{1}{10^6}+\ldots\ldots\right)\frac{m^6}{n^6}-\&c.$$

Now log. π — log. 8 = 9·594059885702190 — 10
And M = ·434294481903252

Hence reducing the coefficients of $\frac{m^2}{n^2}, \frac{m^4}{n^4}\cdots\cdot$ to numbers (in the same manner as we calculated the value of π in article 53), and remembering that L. sin. $\frac{m}{n}$ 90° = log. sin. $\frac{m}{n}$ 90° + 10 we obtain

$$\text{L. sin.}\frac{m}{n}\cdot 90^\circ=\text{log. } m+\text{log. }(2n-m)+\text{log. }(2n+m)-3\text{ log. } n.+9{\cdot}594059885702190.$$

$$-\frac{m_2}{n^2}\times 0{\cdot}070022826605902-\frac{m^4}{n^4}\times 0{\cdot}001117266441662$$

$$-\frac{m^6}{n^6}\times 0{\cdot}000039229146454-\frac{m^8}{n^8}\times 0{\cdot}000001729270798$$

$$-\frac{m^{10}}{n^{10}}\times 0{\cdot}000000084362986-\frac{m^{12}}{n^{12}}\times 0{\cdot}000000004348716$$

$$-\frac{m^{14}}{n^{14}}\times 0{\cdot}000000000231931-\frac{m^{16}}{n^{16}}\times 0{\cdot}000000000012659$$

$$-\frac{m^{18}}{n^{18}}\times 0{\cdot}000000000000703-\frac{m^{20}}{n^{20}}\times 0{\cdot}000000000000040$$

$$-\&c.$$

It can be proved in the same manner, by means of the expression

$$\text{Cos. } x=\left(1-\frac{4x^2}{\pi^2}\right)\left(1-\frac{4x^2}{9\pi^2}\right)\ldots\ldots\text{ that}$$

L cos. $\frac{m}{n}$90° = 10 + log. $(n-m)$ + log. $(n+m)$ — 2 log. n + terms involving $\frac{m^2}{n^2}, \frac{m^4}{n^4}$ &c., similar to those in L. sin. $\frac{m}{n}$90°

By means of the former of these expressions we may calculate logarithmic sines of angles from 45° to 90°, or by means of the latter logarithmic cosines of angles from 0° to 45°, which clearly comes to the same thing. We can then calculate remaining logarithms by means of the formula

L. sin. A = L. sin. 2A — L. cos. A + 9·698970004336019

To prove this formula

Since sin. 2A = 2 sin. A cos. A
log. sin. 2A = log. 2 + log. sin. A + log. cos. A
∴ L. sin. 2A = L. sin. A + L. cos. A + log. 2 — 10.
And 10 — log. 2 = 9·698970004336019.

Hence the formula.

We may also employ the formula for L sin. $\frac{m}{n}$ 90° to calculate the L. sines for angles of large intervals, as for 1° 2° 3° . . . . and then apply the method of interpolations referred to in article 67.

(69). *Explanations.*

If we take a table of logarithms of numbers we shall observe that the numbers run quite regularly, viz., 10000, 10001, 10002, &c., and the corresponding logarithms are entered in the table. But if we take a table of logarithms of sines, &c., we shall find that the logarithms of the sines and tangents of small angles are given for much smaller divisions of the angle than of the larger angles. Thus, in Hülsse's edition of *Vega's Tables*, we find the logarithmic sines, cosines, tangents, and cotangents for every minute from 6° 0′ up to 45° 0′, while they are given for every 10″ from 0° 0′ 0″, up to 6° 0′ 0″; and to this general table are prefixed two others, the former giving the sines and tangents for every tenth part of a second from 0° 0′ 0″ up to 0° 0′ 59″.9, the latter giving the sines and tangents for intervals of 1″ from 0° 0′ 0″ up to 1° 29′ 59″; the need for these minute calculations appears from the following considerations:—

To show that

$$\text{L. sin.}(\theta+n'')-\text{L.sin.}\,\theta\doteq Mn\,\text{sin. }1''\text{cotan.}\theta\left(1-\frac{n\,\text{sin. }1''}{\text{sin. }2\,\theta}\right)$$

where n is a small number,
we have

$$\text{Log. sin. }(\theta+\delta)-\text{log. sin. }\theta=\text{log. }\frac{\text{sin. }(\theta+\delta)}{\text{sin. }\theta}$$

$$=\text{log. }(\text{cos. }\delta+\text{cotan. }\theta\text{ sin. }\delta).$$

Now if δ be small, so that we may omit δ^3 . . . . *

---

* In this and the following articles are several instances of approximation which must be carefully attended to. We say if δ be so small that we may omit δ^3 and all the higher powers of δ, then cos. $\delta = 1-\frac{\delta^2}{2}$ and sin. $\delta=\delta$. For this, see cor. to article (50).

We then obtain

$$\text{log. sin. }(\theta+\delta)-\text{log.sin. }\theta=M\left\{\left(\text{cotan. }\theta.\,\delta-\frac{1}{2}\delta^2\right)-\frac{1}{2}\left(\text{cotan. }\theta.\delta-\frac{\delta^2}{2}\right)^2\right\}$$

$$\text{which is of course}=M\left(\text{cotan. }\theta.\delta-\frac{\delta^2}{2}-\frac{1}{2}\delta^2\text{ cotan.}^2\,\theta+\frac{1}{2}\text{cotan. }\theta.\,\delta^3-\frac{\delta^4}{8}+\&c.\right)$$

$$\text{which }=M\left\{\text{cotan. }\theta.\delta-\frac{1}{2}.\delta^2\,(1+\text{cotan.}^2\,\theta)\right\}$$

if we omit δ^3, δ^4, &c., and hence the result of the text.

So again in another article. If we omit θ^4, and all higher powers of θ, we have

$$\text{sin. }\theta=\theta-\frac{\theta^3}{6}\qquad\therefore\quad\frac{\text{sin. }\theta}{\theta}=1-\frac{\theta^2}{6}$$

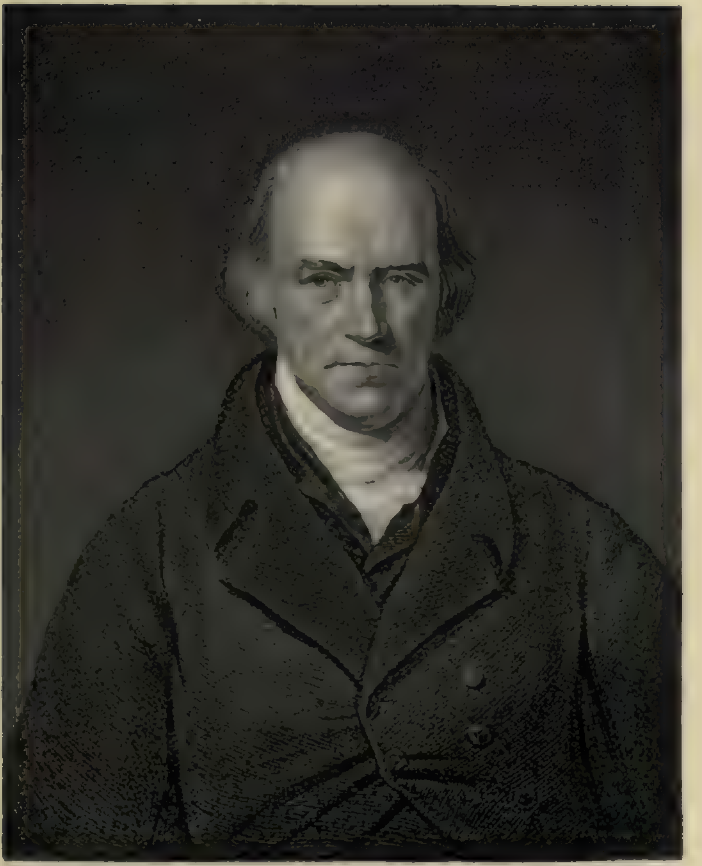
$$\text{Now }\left(1-\frac{\theta^2}{6}\right)^{\frac{1}{3}}=1-\frac{1}{3}.\frac{\theta^2}{2}-\frac{1}{9}.\left(\frac{\theta^2}{2}\right)^2\&c.$$

$$=1-\frac{\theta^2}{6}-\frac{\theta^4}{36}\&c.$$

by the Binomial Theorem

$$\therefore\left(1-\frac{\theta^2}{2}\right)^{\frac{1}{3}}=1-\frac{\theta^2}{6}\text{ if we omit }\theta^4\,\&c.$$

$$\therefore\frac{\text{sin. }\theta}{\theta}=\left(1-\frac{\theta^2}{2}\right)^{\frac{1}{3}}=\left(\text{cos. }\theta\right)^{\frac{1}{3}}$$



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$$\text{we have cos. } \delta = 1 - \frac{\delta^2}{2} \text{ and sin. } \delta = \delta.$$

$$\therefore \log.\sin.(\theta+\delta) - \log.\sin.\theta = \log.\left(1 + \text{cotan.}.\theta\delta - \frac{\delta^2}{2}\right)$$

$$\text{Now } \log.(1+x) = \text{M.}\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \&\text{c.}\right)$$

$$\therefore \log.\sin.(\theta+\delta) - \log.\sin.\theta.$$

$$\text{M.}\left\{\left(\text{cotan.}\theta.\delta - \frac{\delta^2}{2}\right) - \frac{1}{2}\left(\delta\,\text{cotan.}\,\theta - \frac{\delta^2}{2}\right)^2 \cdots\right\}$$

$$= \text{M.}\left(\delta\,\text{cotan.}\,\theta - \frac{1}{2\sin.^2\theta}\cdot\delta^2\right) \text{ omitting } \delta^3 \cdots$$

$$= \text{M. } \delta\,\text{cotan.}\,\theta\left(1 - \frac{\delta}{\sin.\,2\,\theta}\cdot\right).$$

Now if δ be an angle of n'' then δ the circular measure of this angle, equals $n.\sin.1''$.

$$\therefore \text{L. sin. }(\theta+n'') - \text{L. sin. }\theta = \text{M}\,n\,\sin.1''\ \text{cotan.}\,\theta \times \left(1 - \frac{n\sin.1''}{\sin 2\ \theta}\right). \ (60).$$

In like manner we may prove that

$$\text{L. tan. }(\theta+n'') - \text{L. tan. }\theta = \frac{\text{M}\,n\,\sin.\,1''}{\cos.^2\theta} \times \left(1 - n\sin.\ 1''\tan.\ \theta\right)$$

Now suppose that our tables are calculated for angles that differ by so small an angle δ'', that we can neglect the second term of Equation (60) so that

$$\text{L. sin. }(\theta+\delta'') - \text{L. sin. }\theta = \text{M}\,\delta\,\sin.\,1''\,\text{cotan. }\theta.$$

Also, if $n < \delta$, *à fortiori*,

$$\text{L. sin. }(\theta+n'') - \text{L. sin. }\theta = \text{M}\,n\,\sin.\,1''\,\text{cotan. }\theta.$$

Now, for any value of θ the Tables give us

$\text{L. sin. }(\theta+\delta'') - \text{L. sin. }\theta$ call this $\triangle$

$$\therefore \qquad \triangle = \text{M}\,\delta\,\sin\ 1''\,\text{cotan. }\theta.$$

$$\therefore \text{L. sin. }(\theta+n'') - \text{L. sin. }\theta = \frac{n}{\delta}\triangle$$

i.e., when two angles are nearly equal, difference between log. sines is proportional to the difference of angles.

Now $\frac{\triangle}{\delta}$ is the difference corresponding to $1''$. This can easily be multiplied by n. Also L. sin. θ is given by the Tables, and hence we can find L. sin. $(\theta + n'')$ and conversely having given L. sin. $(\theta + n'')$, where only L. sin. θ is given in tables, we can easily find the n seconds.

If the numerical values of formula (60) are taken for different angles, it will be found that in order to make the difference between two consecutive L. sines, given in the Tables proportional to the number of seconds, we must have $\delta = 1''$ from 0° up to 1°. 30′., and $\delta = 10''$ from 1°. 30′ up to 5°, and $\delta = 60''$ from 5° upward—the log. sines being calculated to 7 places of decimals.*

The practical mode of employing the Tables will be readily understood from the following examples :—

Thus, : (1). Find L. sin. (15° 11′ 16″ · 5).

By tables . . . . . . L. sin. 15° 11′ = 9·4181495

Do. diff. 1″ = 77, 55 × 10″ = 775, 5

6″ = 465, 3

·5 = 38, 7

L. sin. (15° 11′ 16″ · 5) = 9·4182775

(2). Find L. sin. (3° 19′ 37″ · 4).

By tables . . . . . . L. sin. (3° 19′ 30″) = 8·7634252

Do. diff, 1″ = 362.3 × 7″ = 2534, 1

4 = 144, 9

L. sin. (3° 19′ 37″ · 4) = 8·7636931

(3). Find θ. If L. sin. θ = 9·7645827

L. sin. θ = 9·7645827

L. sin. (35° 33′) = 9·7644849

978

Diff. 1″ = 29·45 × 30 893·5

84·5

2 58·9

25·6

·8 23·56

$\therefore \theta = 35°.\ 33'.\ 32''.8.$

We proceed in precisely the same manner with the logarithms of the other trigonometric functions, with this exception, that what is true of the L. sines and L. tangents of small angles, is true of the L. cosines and L. cotangents of angles which are nearly equal to 90°. For, since sin. θ = cos. $(90 - \theta)$ and tan. θ = cotan. $(90 - \theta)$, then if θ be small, $90° - \theta$ is nearly equal to 90°.

(71). *Another way of treating the L. sines of small angles.*

If the tables give the log. sines, *&c.*, for intervals of 1′ throughout, and not the refined tables for small angles before spoken of, we can find the accurate value of a small angle whose L. sine is given, and *vice versâ*, by the following process.

If θ be small, so that we can omit θ^4 θ^5, *&c.*, we have (Art. 50)

$$\sin.\ \theta = \theta - \frac{\theta^3}{6}.$$

$$\&\ \cos.\ \theta = 1 - \frac{\theta^2}{2}$$

$$\text{Hence} \qquad \frac{\sin.\ \theta}{\theta} = 1 - \frac{\theta^2}{6} = \left(1 - \frac{\theta^2}{2}\right)^{\frac{1}{3}} = \cos.^{\frac{1}{3}}\theta$$

Now suppose θ to contain n'' $\therefore \theta = n\sin.\ 1''$

$$\therefore \frac{\sin.\ \theta}{n\sin.\ 1''} = \cos.^{\frac{1}{3}}\theta$$

$$\therefore \log.\sin.\theta - \log.n. - \log.\sin.1'' = \frac{1}{3}\log.\cos.\theta$$

$$\therefore \text{L. sin. }\theta - \log.n - \text{L. sin. }1'' = \frac{1}{3}.\text{L.cos. }\theta - \frac{10}{3}$$

$$\therefore \text{L. sin. }\theta = \log.n + \frac{1}{3}\text{L. cos. }\theta + \text{L. sin. }1'' - \frac{10}{3}$$

$$\&\ \ \text{L. sin. }1'' = 4{\cdot}6855749.$$

$$\therefore \text{L. sin.} = \log.n + \frac{1}{3}\text{L. cos }\theta + 1{\cdot}3522416$$

$$\text{and } \log.n = \text{L. sin. }\theta - \frac{1}{3}.\text{L. cos. }\theta - 1{\cdot}3522416$$

It is to be observed, that when θ is very small, cos. θ changes very slowly. Hence in the term L. cos. θ we may use the number of minutes in θ, omitting the odd seconds, and then, the former formula gives us L. sin. θ, when θ or n'' is given, and the latter gives us the n'' when L. sin. θ is given.

---

* See Tables at page 666, *et seq.*

For instance,

Find θ, having given that L. sin. = 7.2777613. On looking into Tables we find that θ is between 6′ and 7′, hence in this case cos. θ = cos. 6′

$$\begin{array}{lrl} & \text{L. sin.}\,\theta & = 7{\cdot}2777613 \\ & -\tfrac{1}{3}\,\text{L. cos.}\,6' & = 6{\cdot}6666668-10. \\ & -+1{\cdot}3522416 & = \underline{8{\cdot}6477584}-10. \\ & & \ 2{\cdot}5921865 \\ & & \ \underline{\ {\cdot}5921768} \\ 391.00 & & \qquad\quad 97 \\ 8 & & \qquad\quad 90 \\ \underline{6} & & \qquad\quad \overline{7.} \\ 391.0086 = n. & & \therefore \theta = 6',\ 31''\ .0086 \end{array}$$

If we find n'' by more refined tables

$$\begin{array}{rl} \text{L. sin.}\,\theta & = 7{\cdot}2777613 \\ \text{L. sin.}\,6'.31'' & = \underline{7{\cdot}2777514} \\ & \quad 99{,}00 \\ & \quad \underline{75{,}21} \qquad 1 \\ & \quad 23{,}790 \\ & \quad \underline{22{,}563} \qquad 3 \end{array}$$

$\therefore \theta = 6'.31''.013.$

$\therefore$ by this method $\quad n = 391{\cdot}013$

which does not differ materially from the result we obtained by the less refined table and the formula.

Find L. sin. θ when $\theta = 7'.31''.37$.

$$\begin{array}{ll} \text{Here} \quad n = 451.37 & \text{L. cos.}\,\theta = \text{L. cos.}\,7' \\ \text{L. sin.}\,\theta = \log.\,n & = 2{\cdot}6545327 \\ +\dfrac{1}{3}\,\text{L. cos.}\,7' & = 3{\cdot}3333330 \\ +1{\cdot}3522416 & = \underline{1{\cdot}3522416} \\ & \ 7{\cdot}3401073 \end{array}$$

The more refined tables give us at once

$$\begin{array}{lrcr} & \text{L. sin.}\,7'31'' & = & 7{\cdot}3397511 \\ \text{Diff. } 1'' = 9619 \times & {\cdot}3 & = & 2885{,}7 \\ & {\cdot}07 & = & \underline{673{,}33} \\ & & & 7{\cdot}3401070 \end{array}$$

In like manner if we have θ so small an angle that $\theta^4\,\theta^6$ . . . may be omitted, and if θ be an angle of n seconds,

$$\tan.\,\theta = \frac{\sin.\,\theta}{\cos.\,\theta} = \frac{\theta - \dfrac{\theta^3}{6}}{1 - \dfrac{\theta^2}{2}} = \left(\theta - \frac{\theta^3}{6}\right)\left(1 + \frac{\theta^2}{2}\right)$$

$$\therefore \frac{\tan.\,\theta}{\theta} = 1 + \frac{1}{3}\,\theta^2 = \left(1 - \frac{\theta^2}{2}\right)^{-\frac{2}{3}} = (\cos.\,\theta)^{-\frac{2}{3}}.$$

$$\therefore \log.\,\tan.\,\theta - \log.\,n\,\sin.\,1'' = -\frac{2}{3}.\,\log.\,\cos.\,\theta$$

$$\text{L. } \tan.\,\theta = \log.\,n + \text{L. sin. } 1'' + \frac{2}{3}.\,(10 - \text{L. cos. } \theta).$$

$$\text{L. } \tan.\,\theta = \log.\,n + 11{\cdot}3322416 - \frac{2}{3}\,\text{L. cos. } \theta.$$

This formula can be used in the same manner as the one for L. sin. θ.

(72). *Delambre's method for L. sines of small Angles.*

There is another method of treating the L sines of small angles, of which the following is an account.

$$\text{Sin. } \theta = \theta - \frac{\theta^3}{6} + \frac{\theta^5}{120} - \text{\&c.}$$

$$\therefore \log.\,\frac{\sin.\,\theta}{\theta} = \log.\,\left\{1 - \left(\frac{\theta^3}{6} - \frac{\theta^4}{120}\right)\right\} \cdot \text{Omitting } \theta^5$$

$$= -\,\text{M}\left\{\left(\frac{\theta^2}{6} - \frac{\theta^4}{120}\right) + \frac{1}{2}\left(\frac{\theta^2}{6} - \frac{\theta^4}{120}\right)^2 + \ldots .\right\}$$

$$= -\,\text{M}\left(\frac{\theta^2}{6} + \frac{\theta^4}{180}.\right)$$

From this a table for $\log.\,\dfrac{\sin.\,\theta.\,\sin.\,1''}{\theta}$ can easily be calculated. Now suppose θ to contain n''

Then $\quad n\,\sin.\,1'' = \theta$

$$\therefore \quad \frac{\sin.\,n''}{n} = \frac{\sin.\,\theta.\,\sin.\,1''}{\theta}$$

$$\therefore \quad \text{L. sin. } n'' = \log.\,n + \text{L.}\,\frac{\sin.\,\theta.\,\sin.\,1''}{\theta}$$

and since $\text{L.}\,\dfrac{\sin.\,\theta.\,\sin.\,1''}{\theta}$ is given by the Tables, if we know n we can at once find L. sin. n''. And conversely if we have given L. sin. θ, we can find n; for sin. $\theta = \theta = n$ sin. 1″ nearly. $\therefore$ log. n = L. sin. θ — L. sin. 1″ nearly, whence we know n approximately. This will enable us to obtain the value of $\log.\,\dfrac{\sin.\,\theta.\,\sin.\,1''}{\theta}$ from the Tables with sufficient accuracy; which being known, we have $\log.\,n = \text{L. sin. } \theta - \log.\,\dfrac{\sin.\,\theta\,\sin.\,1''}{\theta}$, which gives us n accurately.

It can easily be shown, that in calculating tables for $\log.\,\dfrac{\sin.\,\theta}{\theta}$ from the formula,

$$\log.\,\frac{\sin.\,\theta}{\theta} = -\,\text{M}\left(\frac{\theta^2}{6} + \frac{\theta^4}{180}.\right)$$

that for tables of seven places of decimals up to 5°, θ^4 can be omitted.

For when θ is an angle of 5°, $\theta = \dfrac{5\,\pi}{180}$

$$\therefore \text{M}\,\frac{\theta^4}{180} = \text{M}\,\frac{5^4\,\pi^4}{(180)^5} = .0000001{,}39927 \ldots$$

Let us now look back and consider what we have done in the previous pages. (1). We have investigated the various general relations existing between the trigonometrical functions. (2). We have investigated the general relations existing between the sides and angles of triangles. (3). We have fully discussed the mode of constructing trigonometrical tables, *i. e.*, the means of obtaining numerical results from our trigonometrical formulas.

We now proceed to the actual numerical calculation of the sides and angles of triangles, which we shall find will invariably consist in reducing formulas to numbers. For instance,

Given sin. $(m + x)$ — sin. $(m - x)$ = cos. $(m - n)$ — cos. $(m + n)$

Find x when $m = 12°\cdot 13'$ and $n = 7°\cdot 37'$.

$$\begin{array}{ll} \text{Since} & \sin.\,(m + x) - \sin.\,(m - x) = 2\,\sin.\,x\,\cos.\,m \\ \text{and} & \cos.\,(m - n) - \cos.\,(m + n) = 2\,\sin.\,m\,\sin.\,n. \\ & \therefore 2\,\sin.\,x\,\cos.\,m = 2\,\sin.\,m.\,\sin.\,n. \\ & \therefore \sin.\,x = \tan.\,m.\,\sin.\,n. \\ & \therefore \log.\,\sin.\,x = \log.\,\tan.\,m + \log.\,\sin.\,n. \\ & \therefore \log.\,\sin.\,x + 10 = \log.\,\tan.\,m. + 10 + \log. \\ & \quad \sin.\,n + 10 - 10 \\ \text{and} & \therefore \text{L. sin. } x = \text{L. tan. } m. + \text{L. sin. } n - 10. \\ & \text{L. tan. } m = 9{\cdot}3354823 \\ & \text{L. sin. } n = \underline{9{\cdot}1223624} - 10 \\ & \qquad\qquad 8{\cdot}4578447 \\ \text{L. sin. } 1°\ 38'\cdot 40'' & \qquad\qquad \underline{8{\cdot}4578369} \\ & \qquad\qquad\quad 78 \\ \text{diff. } 1'' = 732{,}8 \times {\cdot}1 & \qquad\qquad\quad \underline{73{\cdot}28} \end{array}$$

Ans. $x = 1°.\,38'.\,40''.\,1.$

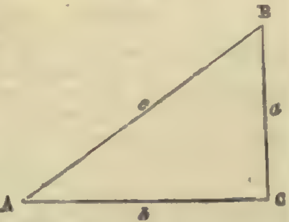
CHAPTER XIII.

THE NUMERICAL SOLUTION OF RIGHT-ANGLED TRIANGLES.

THE parts of a right-angled triangle are the three sides, two angles and the right angle; if any two of the former five, one of the two being a side, are given, we can calculate the remaining three. The following are the methods employed in the various cases, which are these.

(1). Given the base and perpendicular.
(2). Given the hypothenuse and another side.
(3). Given the base or perpendicular and an angle.
(4). Given the hypothenuse and an angle.

Fig. 23.



(1). *Given the Base and Perpendicular; i. e., given a. b. find A. B. c.*

$\tan. A = \frac{a}{b}$ gives A.

$B = 90° - A$. Gives B.

$\text{Cos. } A = \frac{b}{c}$.

or $c = \frac{b}{\cos. A}$. Gives c.

These can be put into forms adapted for logarithmic calculation, as follows :—

$$\log. \tan. A = \log. a - \log. b.$$
$$\therefore 10 + \log. \tan. A = \log. a + 10 - \log. b.$$
$$\text{or, L. } \tan. A = \log. a + \text{ar.: comp.: } \log. b. \quad (a).$$

Similarly,

$$\text{Log. } c = \log. b. - \log. \cos. A.$$
$$= \log. b. + 10 - (10 + \log. \cos. A)$$
$$= \log. b. + 10 - \text{L. } \cos. A.$$
$$\therefore \log. c = \log. b. + \text{ar.: comp.: L. } \cos. A. \quad (b).$$

(*a*) and (*b*) are the formulas actually used in calculation. Thus,

Given $a = 7564 \cdot 5$ yds. $b = 3987 \cdot 4$ yds.
Find A. B. and *c*.

To find A. from (*a*) we have,

$\text{Log. } \tan. A = \log. a + \text{ar.: comp.: } \log. b.$
$= \log. 7564 \cdot 5 = 3 \cdot 8787802$
$+ \text{ar.: comp.: } \log. 3987 \cdot 4 \quad 6 \cdot 3993102$

$10 \cdot 2780904$
$\text{L. } \tan. 62° 12' \quad 10 \cdot 2779915$

diff. for 1″ $= 51 \cdot 05) \; 989 \cdot 00 \; (19$
5105

47850
$\therefore A = 62° 12' 19'' \quad 45945$
90

and $B = 27° 47' 41''$

To find *c* we have,

$\text{Log. } c = \log. b + \text{ar.: comp.: L. } \cos. A$
$\log. b = 3 \cdot 6006898$
$\text{ar.: comp.: L } \cos. 62° 12' 19'' = \text{ar.: comp.: } 9 \cdot 6686623 = \cdot 3313377$

$3 \cdot 9320275$
$8551 \cdot 2 \quad \cdot 9320271$
$08 \text{ diff.} = 4$

$\therefore c = 8551 \cdot 208$

(2). *Given the Hypothenuse and another side.*

e. g., given *a c* to find A B and *b*.

We have. $\sin. A = \frac{a}{c}$.

$$\therefore \log. \sin. A = \log. a - \log. c.$$
$$\therefore \text{L. } \sin. A = \log. a + \text{ar.: comp.. } \log. c \quad (a)$$

which gives A and ∴ B which equals 90° — A.

Again $\cos. A = \frac{b}{c}$.

$$\therefore \quad b = c \cos. A.$$
$$\therefore \quad \log. b = \log. c + \text{L. } \cos. A - 10. \quad (b)$$

Ex. Given $a = 724 \cdot 5$. $c = 1005 \cdot 4$.
We shall have $A = 46° 6' 17''$. $B = 43° 53' 43''$.
$b = 697 \cdot 086$.

(3). *Given the Base or Perpendicular and an Angle.*

e. g., given *a*. A., find B. *b*, *c*.

We have $B = 90° - A$.

Again $c = \frac{a}{\sin. A}$

$$\therefore \log. c = \log. a - \log. \sin. A$$
$$\therefore \log. c = \log. a + \text{ar. : comp. : L. } \sin. A. \quad (a)$$

Again $c = \frac{b}{\cos. A}$

$$\therefore \log. b = \log. c + \text{L. } \cos. A - 10. \quad (b)$$

In using formula (*b*), it must be remembered that in the calculation of formula (*a*) we have already found log. *c*.

Example. Given $a = 7643 \cdot 5$ $A = 37°.18'$.
We shall find $B = 52°.42'$
$c = 12613 \cdot 4$
$b = 10033 \cdot 53$

(4). *Given the Hypothenuse and an Angle.*

e. g., given *c*. A, find B. *b*. *a*.

We have $B = 90° - A$.
$b = c. \cos. A$.

$$\therefore \log. b = \log. c + \text{L. } \cos. A - 10. \quad (a)$$

Again $a = c \sin. A$.

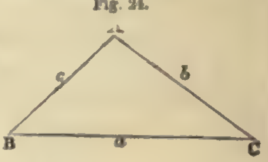
$$\therefore \log. a = \log. c + \text{L. } \sin. A - 10. \quad (b)$$

Example. Given $c = 7234 \cdot 5$ $A = 33°.19'$.
We have $B = 56°.41'$.
$b = 6045 \cdot 493$.
$a = 3973 \cdot 665$.

The cases of oblique triangles are in like manner reducible to four, viz.—

(1). Given three sides to find the angles.
(2). Given two sides and the included angle.
(3). Given two sides and the angle opposite to them.
(4). Given one side and two angles.

Fig. 24.



(1). *Given three sides to find the angles, i. e.,* given *a b. c.* to find A. B. C.

We have $2s = a + b + c$.

$$\therefore \tan.^2 \frac{A}{2} = \frac{(s-b)(s-c)}{s. (s-a)}$$

$$\therefore 2 \log. \tan. \frac{A}{2} = \log. (s - b) + \log. (s - c) - \log. s - \log. (s - a)$$

$$\therefore 2 \log. \tan. \frac{A}{2} + 20 = \log. (s-b) + \log. (s-c) + 10 - \log. s + 10 - \log. (s-a)$$

$$\therefore 2 \text{ L. } \tan. \frac{A}{2} = \log. (s - b) + \log. (s - c) + \text{ar. : comp. : } \log. s + \text{ar. : comp. } \log. (s-a)$$

This formula gives us $\frac{A}{2}$. We shall have to find B from the formula

$$2 \text{ L. } \tan. \frac{B}{2} = \log. (s-c) + \log. (s-a) + \text{ar. : comp. : } \log. s + \text{ar. : comp. : } \log. (s-b)$$

It will be observed that all the logarithms needed for finding B and C have been already used in finding A, whence, when A is known, B and C are found with very little trouble.

Instead of using the formula which gives us tan. $\frac{A}{2}$, we may use those which give sin. $\frac{A}{2}$ or cos. $\frac{A}{2}$.

Example given, $a = 490$ $b = 960$ $c = 660$ we shall find that—

$$\begin{aligned} A &= 28° \,.\, 10' \,.\, 12'' \\ B &= 112 \,.\, 20 \,.\, 40 \\ C &= 39 \,.\, 29 \,.\, 8 \end{aligned}$$

or $$A + B + C = 180 \,.\, 0 \,.\, 0$$
which circumstance is a test of the accuracy of the calculation.

(2). *Given two sides and the included Angle.*
e. g., given $a\ b\ C$, find $A\ B\ c$.

We have
$$\tan. \frac{A - B}{2} = \frac{a - b}{a + b} \tan. \frac{A + B}{2}.$$

$$\therefore \text{L} \tan. \frac{A - B}{2} = \log. (a - b) + \text{ar.: comp.: } \log. (a + b).$$
$$+ \text{L} \tan. \frac{A + B}{2} - 10. \qquad (a)$$

It must be observed that $\frac{A + B}{2} = 90° - \frac{C}{2}$ and is therefore known. Wherefore (a) gives us $\frac{A - B}{2} = a$ a known angle. Then,

$$\frac{A + B}{2} = 90 - \frac{C}{2}$$
$$\frac{A - B}{2} = a$$
$$\therefore \qquad A = 90° + a - \frac{C}{2}$$
$$\text{and } B = 90° - a - \frac{C}{2}.$$

whence A and B are known.

To find c we have
$$c = a \frac{\sin. C}{\sin. A} \qquad (b)$$

$\therefore \log. c = \log. a + \text{L} . \sin. C + \text{ar. : comp. : L} . \sin. A - 10$, which will give us c.

We may employ a subsidiary angle θ in the following manner (see Art. 40).

$$\text{Assume} \qquad \tan. \theta = \frac{a - b}{a + b} \cdot \cot\text{an}. \frac{C}{2}$$
$$\therefore c = \frac{(a + b) \sin. \frac{C}{2}.}{\cos. \theta}$$

$$\log. \tan. \theta = \log. (a - b) + \text{ar. : comp. : } \log. (a + b)$$
$$+ \log. \cot\text{an}. \frac{C}{2} - 10 \qquad (c)$$

$$\log. c = \log. (a + b) + \log. \sin. \frac{C}{2} + \text{ar. : comp. :}$$
$$\log. \cos. \theta - 10. \qquad (d)$$

By using (c) and (d) we can obtain c without first finding A and B.

If we solve by the former methods we require six logarithms, viz. : those of $(a - b)$, $(a + b)$, tan. $\frac{A + B}{2}$ sin. C., and sin A.

If by the latter we require five logarithms $(a - b)$, $(a + b)$, cotan. $\frac{C}{2}$, sin. $\frac{C}{2}$, and cos. θ; therefore the latter method possesses a slight advantage over the former.

The θ in formula (c) is evidently the same as $\frac{A - B}{2}$ in formula (a).

If $b \mathrel{\not<} a$ the formulas become
$$\tan. \frac{B - A}{2} = \frac{b - a}{b + a} \cdot \tan. \frac{A + B}{2}$$
$$\text{and} \qquad \tan. \theta = \frac{b - a}{b + a} \cdot \cot\text{an}. \frac{C}{2}.$$

Example:—Given $a = 562.$ $b = 320.$ $C = 128° 4'$

We shall find
$$\begin{aligned} A &= 33° 34' 40'' \\ B &= 18° 21' 20'' \\ c &= 800 \cdot 008. \end{aligned}$$

If we take the second method we shall find
$$\log. \tan. \theta = 9 \cdot 1258960$$
$$\therefore \log. \cos. \theta = 9 \cdot 9961568$$

And, as before, we shall find,
$$c = 800 \cdot 008.$$

(3). *Given two sides, and an Angle opposite to one of them;*
e. g., given B. b. c. find A. C. a.

$$\text{We have} \qquad \sin. C = \frac{c}{b} \cdot \sin. B.$$

$$\therefore \log. \sin. C = \log. c + \log. \sin. B + \text{ar. : comp.}$$
$$\log. b - 10 \ldots\ldots\ldots (a)$$

which gives C. Then
$$A = 180° - (B + C)$$
which gives A. Then
$$a = b \frac{\sin. A}{\sin. B}$$
or, $\log. a = \log. b + \log. \sin. A + \text{ar. : comp. :}$
$$\log. \sin. B - 10 \ \ldots\ldots (b)$$
which gives a.

N.B.—This is sometimes called the ambiguous case of a triangle; for we have sin. C given us, whence we know $C = a$, where $a \angle 90°$. Now, sin. $a = \sin. (180 - a)$ $\therefore$ C may also equal $180 - a$. Hence this equation may give us *two* triangles, one which has $C = a$, the other having $C = 180° - a$. In many cases, however, this ambiguity does not exist.

(1). $B =$ or $\mathrel{\not<} 90°$. Then $C = 180° - a$ is inadmissible, since C would then be $\mathrel{\not<} 90°$, and $B + C > 180°$, which is impossible.

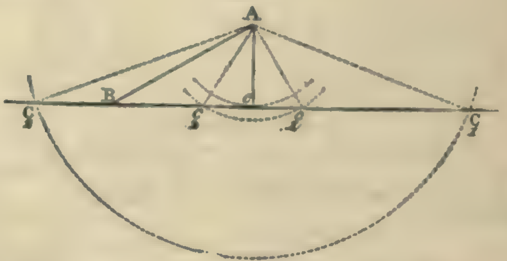
(2). If $B < 90°$. Then

- (a). If $c \sin. B = b$ then $\sin. C = 1$ or $C = 90°$, and the other value of $C = 180° - 90° = 90°$, or in this case there is no ambiguity.
- (β). If $c < b$. Then $C < B$, hence if $C = a$. $a < B$, and $180° - a + B > 180°$, or in this case the angle $180° - a$ is inadmissible, and there is no ambiguity.
- (γ). But if $c > b$. Then both $c = a$, and $c = 180° - a$ are admissible, and there are two triangles determinable from the given values, b. c. B.

This can be illustrated geometrically as follows:—

$AB = c$
$ABc = B.$

Fig. 25.



with centre A and radius b describe a circle. Then,

- (a). If $b = c \sin. B$, the circle will touch Bc in c, and the triangle will be A Bc.
- (β). If $c < b$ the circle will cut Bc, in two points $C_2\,C_1$, on *different* sides B, and the triangle is ABC_1.
- (γ). If $c > b$ the circle cuts Bc in two points c_1 c_2 on the *same* side of B, and the triangle may be either ABc_2 or ABc_1; which is the reason of this case being called ambiguous.

Example:—Given $B = 45°$ $b = 305$ $c = 219 \cdot 5$

$$\begin{aligned} \text{Then} \quad C &= 30° . 36' . 22'' \\ A &= 104° . 23' . 38'' \\ a &= 416.8344. \end{aligned}$$

(4). *Given, one side and two Angles; e.g., given, a B C, to find A. c, b.*

We have $A = 180° - (B + C)$. which gives A.

$$\text{Now,} \qquad \frac{b}{a} = \frac{\sin. B}{\sin. A} \qquad \frac{c}{a} = \frac{\sin. C}{\sin. A}.$$

$\therefore$ log. b = log. a + log. sin. B + ar.: comp.: log. sin. A — 10. and log. c = log. a + log. sin. C + ar. comp.: log. sin. A — 10. Whence we can immediately calculate b, &c.

Example:—

Given B = 49°. C = 29°. 19′ and a = 95·4, we shall find

$A = 101°. 41'$
$b = 73·52255$
$c = 47·69952$

Several practical applications of the science of Trigonometry will be found in the following chapter on Mensuration.

CHAPTER XIV.

MENSURATION.

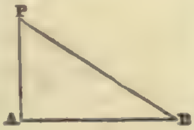
THE object of the following pages is to show the practical application of the truths investigated in previous chapters on Geometry and Trigonometry to solve a variety of questions about measurements such as may frequently occur in practical life. For instance, the determination of the area of a field, the distance between two inaccessible points, the solid content of a pyramid, and others. We shall endeavour in each instance to refer these questions to the principles on which they rest. By doing so, the attentive reader will be able to apply the same principles to cases that may occur practically, but which our limits forbid us to treat of. We shall take in order a variety of questions concerning the determination—(1), of Heights and Distances; (2), of Areas of Surfaces; (3), of Contents of Solids.

I. ON HEIGHTS AND DISTANCES.

In the following articles we are supposed to have the means of measuring the angle subtended at the eye of the observer by the line joining two points. The instrument by which this can be done is called a sextant. For measuring the angle subtended by two objects on a horizontal plane, and for determining the vertical elevation of one point above another, an instrument called a theodolite can be used.

(1). *To determine the height of a Tower standing on a Horizontal Plane, the base of which is accessible.*

Fig. 1.



Let A P be the tower. B any convenient place for the observer. At B measure the angle A B P; also measure the line A B. Then, if A P = x,

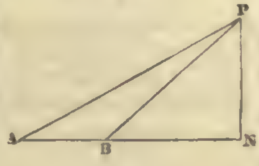
$x = a$ tan. B.

$\therefore$ log. x = log. a + L. tan. B − 10. which gives x.

N.B.—If P be the top of a steeple, half the breadth of the tower must be added to the measurement, which is made to the outside of the tower. It will be observed, moreover, that B is the place of the observer's eye, and $\therefore$ A P, or x, is the height of P, above the horizontal plane passing through the observer's eye. Hence, if h be the height of the eye above the ground, $x + h$ is the height of the summit of the tower above the ground.

(2). *To determine the height of a Tower, the base of which is inaccessible.*

Fig. 2.



Let A B N be the horizontal plane; P N the height required. At B measure angle P B N; move backward to another point, A, taking care that A and B are both on the same vertical plane passing through P, and measure A B, and the angle P A N.

Let P B N = β. P A B = a. A B = a.
B P = x. P N = y.

Now angle A P B = $(\beta - a)$

$$\therefore y = a. \frac{\sin\ a}{\sin. (\beta - a)}$$

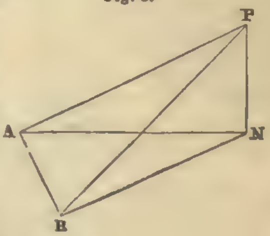
and $x = y \sin. \beta$.

$$\therefore x = a\,\frac{\sin.\ a.\ \sin.\ \beta}{\sin. (\beta - a)}$$

$\therefore$ log. x = log. a + L. sin. a + L. sin. β + ar. comp. L. sin. $(\beta - a) - 20$.

(3). *To determine the height of the Tower in the last article, when the nature of the ground does not admit of the observer moving backward.*

Fig. 3.



Let P N be the vertical height required; place pickets at two points, A and B, APN and B P N being two different vertical planes.

Measure A B = a. Let P N = x.

Measure angle P A N = a.
P A B = A P B A = B.

Then x = A P sin. a.

Now A P B = 180° − (A + B)

$$\therefore A P = a. \frac{\sin. B}{\sin. (A + B)}$$

$$\therefore x = a. \frac{\sin.\ a.\ \sin.\ B}{\sin. (A + B)}.$$

$\therefore$ log. x = log. a + L. sin. a + L. sin. B + ar. comp. L. sin. (A + B) − 20.

In the above case such an angle as P A N being the vertical elevation of P above the horizon, is measured by a theodolite. While the angle P A B, which is not in a horizontal plane, being the angle subtended by the distance between two objects, is measured by a sextant.

If the object be a long way off, we may measure the angle subtended at the eye by the line joining the object and its image in still water, by a sextant; half that angle will be the vertical elevation of the object.

Example:—We observe that the altitude of a hill is 3° 15′. On proceeding $1\frac{3}{4}$ miles towards it, its altitude is 15° 37′. Find the height of that hill, neglecting the sphericity of the earth.

Here (Fig. 2) A B = $1\frac{3}{4}$ miles.
P A B = 3° 15′.
P B N = 15° 37′.
$\therefore$ P B N − P A B = 12° 22′.

Then by the formula proved in Article (2)

$$P N = a. \frac{\sin. PBN \sin. PAB}{\sin. (PBN - PAB)}$$

$\therefore$ log. P N = log. a + L. sin. P B N + L. sin. P A B + ar. comp. L. sin. (P B N − P A B) − 20.

log. 1·75	·2430380
L. sin. 15° 37′	9·4300750
L. sin. 3° 15′	8·7535278
Ar. comp. L. sin. 12° 22′	·6692473 − 20
	1·2958881
19764	2958748
6	133
·197646 of a mile 6 =	132

or $\frac{1}{5}$ of a mile very nearly. (*Answer.*)

(4). *To find the distance between two points, one of which is accessible and the other inaccessible.*

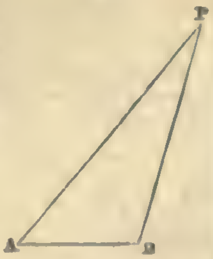


Fig. 4.

Let P be the inaccessible point.

A the accessible.

Drive a picket in at A, and drive in another at any convenient point, B.

Measure the angles P A B and P B A, also measure the line A B.

Then by the fourth case of the solution of triangles

$$\text{A P} = \text{A B}.\frac{\sin.\ \text{APB}}{\sin.\ \text{ABP}}$$

$$= \text{A B}.\frac{\sin.\ (\text{PAB} + \text{ABP})}{\sin.\ \text{ABP}}$$

$\therefore$ calling A B = a. P A B = A. A B P = B. we have

$$\log.\ \text{A P} = \log.\ a + \text{L. sin. } (\text{A} + \text{B}) + \text{ar. comp. L. sin. B} - 10.$$

(5). *To find the distance between two points neither of which is accessible.*

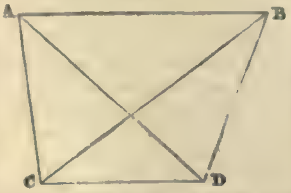


Fig. 5.

Let A, B, be the two points; place on the ground two pickets, C and D, such that the distance between C and D can be measured, and that from each of C and D, the two inaccessible points and the other picket may be visible.

Measure C D = p A C B = C B C D = C′ A D B = D A D C = D′

Then in triangle A C D we know one side and two angles, and therefore can calculate A C.

In triangle B C D we know one side and two angles, and therefore can calculate B C.

And finally in triangle A C B we have already calculated A C, B C, and have measured the angle C, and therefore can determine A B.

Of course, from the first two triangles we can also determine A D and B D, and then in triangle A B D we know two sides, and the included angle, and hence can determine A B, and can use these two calculations for checks upon each other.

The calculation is performed as follows :—

Call the sides and angles of A B C — a. b. c. A. B. C. and observe that C A D = 180° — (C + C′ + D′) and C B D=180° (C′ + D + D′). Then from triangle A C D

$$\text{A C} = \text{C D}\frac{\sin.\ \text{CAD}}{\sin.\ \text{CDA}}$$

$$\therefore \log.\ \text{A C} = \log.\ \text{C D} + \text{L. sin. C A D} + \text{ar. comp. log. sin. C D A} - 10.$$

or

$$\log.\ b = \log.\ p + \text{L. sin. } (\text{C} + \text{C}' + \text{D}') + \text{ar.: comp.: log. sin. D}' - 10 \tag{1}$$

Similarly

$$\log.\ a = \log.\ p. + \text{L. sin. } (\text{C}' + \text{D} + \text{D}') + \text{ar.: comp.: log. sin. C}' - 10 \tag{2}$$

From triangle A C B we have $\frac{\text{A} + \text{B}}{2} = 90° - \frac{\text{C}}{2}$

$$\tan.\ \frac{\text{A} - \text{B}}{2} = \frac{a - b}{a + b}\ \tan.\ \frac{\text{A} + \text{B}}{2}$$

$$\text{assume } \tan.\ \phi = \frac{b}{a}\ \therefore \frac{a - b}{a + b} = \frac{1 - \tan.\ \phi}{1 + \tan.\ \phi} = \tan.\ (45° - \phi)$$

$$\therefore \text{L. tan. } \phi = \log.\ b - \log.\ a + 10 \tag{3}$$

$$= \log.\ b + \text{ar.: comp.: log. } a$$

$$\text{and L. tan.} \frac{\text{A} - \text{B}}{2} = \text{L. tan. } (45° - \phi)$$

$$+ \text{L. tan.} \frac{\text{A} + \text{B}}{2} - 10 \tag{4}$$

whence we obtain $\frac{\text{A} - \text{B}}{2}$ and $\therefore$ A and B since we know $\frac{\text{A} + \text{B}}{2}$

Finally from triangle A B C

$$c = a\frac{\sin.\ \text{C}}{\sin.\ \text{A}}.$$

$$\therefore \log.\ c = \log.\ a + \text{L. sin. C} + \text{ar.: comp.: L. sin. A} - 10 \tag{5}$$

It will be observed that in the above calculation we do not require a and b, but only log. a and log. b, which are given by equations (1) and (2). (Compare Plane Trig., Art. 40).

Example:—Given C D = 372.5 yds. C = 123°.15′. C′ = 13°.42′. D = 129°.11′. D′ = 19°.13′.

(1) Log. b = log. 372.5 = 2.5711263
+ L. sin. 156°.10′ 9.6064647
+ ar. comp. L. sin. 19°.13′ — 10 .4826176 — 10.
2.6602086

(2) Log. a = log. 372.5 2.5711263
+ L. sin. 162°.6′ 9.4876426
+ ar. comp. L. sin. 13°.42′ — 10 .6255483 — 10.
2.6843172

(3) L. tan. ϕ = log. b = 2.6602086
+ ar. comp. log. a 7.3156828
9.9758914
ϕ = 43° 24′ 9.9757318
4218)159600(37.8
12654
ϕ = 43° 24′ 37″ 8 33060
45 29526
45 — ϕ = 1° 35′ 22″ 2 35340

(4) $\text{L. tan. } \frac{\text{A} - \text{B}}{2} = \text{L. tan. } (45° - \phi)$
$+ \text{L. tan. } \frac{\text{A} + \text{B}}{2} - 10.$
= L. tan. 1° 35′ 22″.2 = 8·4432492
L. tan. 28° 22′ 30″ 9·7326024 — 10
8·1758516
0° 51′ 30″ 8·1755658
2 1408)2858(2
$\frac{\text{A} - \text{B}}{2}$ = 0° 51′ 32″ 2816
42

$\frac{\text{A} + \text{B}}{2}$ = 28° 22′ 30″
A = 29° 14′ 2″
B = 27° 30′ 58″

(5). log. c = log. a + L. sin. C + ar. comp. L. sin. A — 10.
= log. a. = 2·6843172
L. sin. 56° 45′ 9·9223549
ar. comp. L. sin. 29° 14′ 2″ ·3112458 — 10
2·9179179
827.78 = ·9179149
30
$\therefore$ c = 827.786 yds. 6 = 32
827.786 *Answer.*

II. THE MENSURATION OF AREAS.

(1). *To find the Area of a Rectangle.*

Let A B C D *abcd* be two rectangles. Then (Euclid, VI., 23) the areas of these rectangles are to each other in the ratio compounded of the ratios of the sides, *i. e.*, in the ratio compounded of the two, *ab* : A B and *bc* : B C; and if we suppose these lines to be represented by numbers, this compounded ratio is *ab* × *bc*:

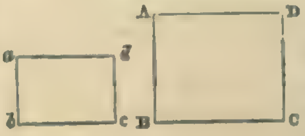


Fig. 6.

A B × B C. Hence rectangle ac : rectangle B C : : $ab \times bc$: A B × B C.

Now suppose $ab = 1$, and $cb = 1$. Then the area ac is the unit of area, *i. e.*, is a square inch, or a square foot, or a square yard, according as ab is an inch, foot, or a yard. In this case

$$\text{Rectangle BC} = \text{AB} \times \text{BC}.$$

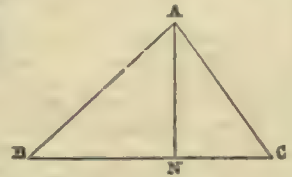
Hence, if a and b be the sides of a rectangle, its area is ab, *i. e.*, it contains as many *units of area* as the product of the number of units of length in one side, by the number of units of length in the other.

Cor.—If a be the side of a square, its area is a^2.

(2). *To find the Area of a Triangle.*

Let A B C be the triangle; from A draw A N perpendicular to B C. Then, since the area of A B C is half that of a rectangle, whose base is B C and height A N, the area of triangle $= \dfrac{\text{BC} \times \text{AN}}{2}$

Fig. 7.



Cor.—(1). If we have given AB. BC and angle B.

$$\text{Area triangle} = \frac{\text{BC} \times \text{AB sin. B}}{2} = \frac{ac \text{ sin. B}}{2}$$

$$\text{Since} \quad \text{AN} = \text{AB sin. B}.$$

(2). If we have given B C and the angles of triangle,

$$\text{Then, since } \frac{c}{a} = \frac{\text{sin. C}}{\text{sin. A}}$$

$$\text{Area triangle} = \frac{a^2 \text{ sin. B sin. C}}{2 \text{ sin. A}}$$

(3). If we have all the sides given (by Trigon., Art. 41)

$$\text{Area triangle} = \sqrt{s.\,(s-a)\,(s-b)\,(s-c)}$$
$$\text{where } 2s = a + b + c.$$

(3). *To find the Area of a Parallelogram.*

If $a.b$ be the sides of the parallelogram, and A the angle contained by those sides, then by the last article the area of half the parallelogram

$$= \frac{ab \text{ sin. A}}{2}$$

∴ Area of parallelogram $= ab$ sin. A.

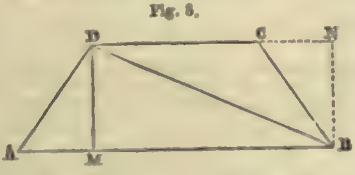
Or, if p is the perpendicular distance between two parallel sides, each of which $= a$, then

$$\text{Area of parallelogram} = ap.$$

(4). *To find the Area of a Trapezoid.*

Let A B C D be the trapezoid of which the side A B is parallel to the side C D. Join B D, draw D M perpendicular to A B, and B N perpendicular to D C, or D C produced, and let $a =$ A B. $b =$ D C and $p =$ D M or B N.

Fig. 8.



Then
$$\text{Area triangle ABD} = \tfrac{1}{2}\,ap$$
$$\text{Area triangle BCD} = \tfrac{1}{2}\,bp$$
$$\therefore \text{Area ABCD} = \tfrac{1}{2}\,p\,(a + b)$$

Or the area of a trapezoid $= \frac{1}{2}$ sum of parallel sides *plus* the perpendicular distance between them.

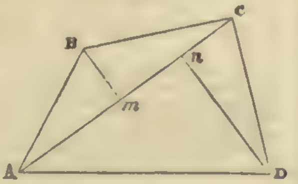
(5). *To find the Area of a Trapezium.*

Let A B C D (Fig. 9), be the trapezium; join AC, from B and D let fall perpendiculars B m, D n, upon AC.

The area of triangle A B C $= \frac{1}{2}$ Bm. A C and area of triangle A D C $= \frac{1}{2}$ Dn. A C.

∴ Area of trapezium $= \frac{1}{2}\,\text{AC}\left(\text{B}m + \text{D}n.\right)$

Fig. 9.

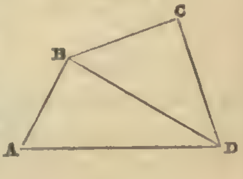


If instead of having the perpendiculars Bm and Dn given, we have the sides A B. B C. C D. D A. and a diagonal, we must find the area of each triangle separately by the formula.

$$\text{Area triangle} = \sqrt{s.\,(s-a)\,(s-b)\,(s-c)}$$

Or, again, if we have the sides and one angle, as A, given, we can proceed as follows.—Let A B. $= a$. B C $= b$. C D $= c$. DA $= d$. Join BD (Fig. 10), then we can calculate B D from triangle A B D by second case of oblique-angled triangles, and having calculated B D we can determine the areas of the triangles as before.

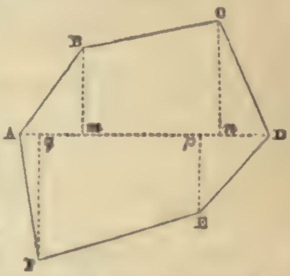
Fig. 10.



(6). *To determine the Area of an irregular Polygon.*

Suppose A B C D E F to be an irregular polygon, its area can be determined by effecting the following measurements; join A D, the longest distance across the figure from B C E F, draw perpendiculars to A D., viz. Bm, Cn, Ep, Fq, respectively; measure Am, mn, nD, Cn, Bm, Aq, qp, pD, Ep, Fq.

Fig. 11.



Then area of polygon $=$ ABm + B$mn$C + CnD + DpE + E$pq$F + FqA.

$+ \frac{1}{2}\,\text{A}m \times \text{B}m + \frac{1}{2}\,mn \times (\text{B}m + \text{C}n) + \frac{1}{2}\,\text{C}n \times n\text{D} + \frac{1}{2}.\,\text{D}p + p\text{E} + \frac{1}{2}\,pq\,(\text{E}p + \text{F}q) + \frac{1}{2}\text{A}q \times q\text{F}.$

(7). *To determine the Area of a regular Polygon.*

Various formulas have been already given for this in Chapter XI., on Trigonometry (Art. 45). It is there shown that if n be the number of sides of the polygon, and a the length of one of the sides,

$$\text{Area} = \frac{na^2}{4} \text{ cotan. } \frac{180^\circ}{n}.$$

Hence in case of pentagon,

$$\text{Area} = \frac{5a^2}{4} \text{ cotan. } 36^\circ = \frac{5\,(\sqrt{5} + 1)\,a^2}{4\sqrt{10 - 2\sqrt{5}}}$$

In case of hexagon,

$$\text{Area} = \frac{3a^2}{2} \text{ cotan. } 30^\circ = \frac{3\sqrt{3} \cdot a^2}{2}$$

And so on in other cases.

(8). *To determine the Area of a Circle.*

If n is the number of sides of a regular polygon inscribed in a circle whose radius is r, then, as we have already seen (Trig., Art. 45),

$$\text{Area polygon} = \frac{nr^2}{2} \text{ sin. } \frac{2\pi}{n}.$$

$$= r^2\pi \, \frac{\text{sin. } \frac{2\pi}{n}}{\frac{2\pi}{n}}$$

Now if we increase the number of sides of the polygon, it becomes more and more nearly equal to the circle, and in the limiting case when the number of sides is infinitely great, it becomes the circle;* but when $n = \infty$ $\therefore \frac{2\pi}{n} = 0$ and (Trig., Art. 47) in this case

$$\frac{\text{sin. } \frac{2\pi}{n}}{\frac{2\pi}{n}} = 1$$

$$\therefore \text{Area of circle} = r^2\pi.$$

Cor.—Hence area of quadrant $= \dfrac{r^2\pi}{4}. = \dfrac{1}{2}\,r\,\dfrac{r\pi}{2}.$

---

* Compare with the statement in the text what is said on p. 577.

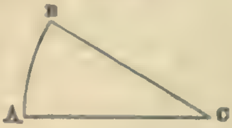
Now $\frac{r\pi}{2}$ = the length of arc on which quadrant stands.

$\therefore$ Area of quadrant $= \frac{r}{2} \times$ arc on which quadrant stands.

(9). *To find Area of a Sector of a Circle.*

Let A O B be any sector of a circle,
O A = r. A O B = θ (in circular measure),
$\therefore$ A B = $r\theta$.

Fig. 12.



Now if A O B were a quadrant, A B would equal $\frac{r\pi}{2}$ and the area of the quadrant = $\frac{r^2\pi}{4}$.

But in equal circles sectors are to each other as the arcs on which they stand.

$$\therefore \text{Area sector} : \text{area quadrant} :: r\theta : \frac{r\pi}{2}.$$

$$\therefore \text{Area sector} : \frac{r^2\pi}{4} :: \theta : \frac{\pi}{2}.$$

$$\therefore \text{Area sector} = \frac{r^2\theta}{2}.$$

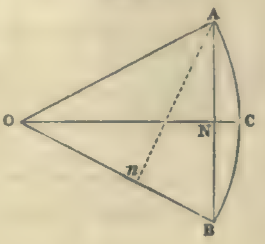
$$= \frac{1}{2} r \times r\theta.$$

$$= \frac{1}{2} r \times (\text{arc on which sector stands}).$$

(10). *To find the Area of a Segment of a Circle.*

Let O A C B be a sector of a circle. Join A B. We are required to find the area of the segment A B C. Draw O N C perpendicular to A B. Let A O B = θ. O B = r.

Fig. 13.



$\therefore$ ON $= r \cos. \frac{\theta}{2}$ AN $= r \sin. \frac{\theta}{2}$.

$\therefore$ area A B O $= r^2 \sin. \frac{\theta}{2} \times \cos. \frac{\theta}{2} = \frac{1}{2} . r^2 \sin. \theta$.

Now area A C B O $= \frac{1}{2} r^2 \theta$.

$\therefore$ area of segment $= \frac{r^2}{2} (\theta - \sin. \theta)$.

Cor.—From A draw A n perpendicular to O B. Then A $n = r \sin. \theta$, also arc A C B $= r\theta$.

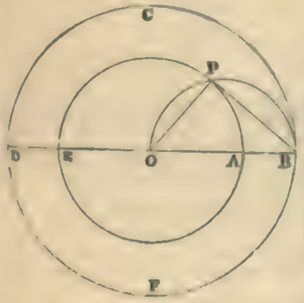
Hence the area of segment

$$= \frac{r}{2} (r\theta - r \sin. \theta)$$

$$= \frac{r}{2} \times (\text{A C B} - \text{A}n)$$

(11). *To find the space between two Concentric Circles, viz.,* A B C E D F.

Fig. 14.



Let O A = r O B = r_1.

Then area of interior circle = πr^2, and area of exterior circle = πr_1^2

$\therefore$ Area of the space between the circles

$$= \pi (r_1^2 - r^2)$$

On O B as a diameter, describe a circle O P B cutting the interior circle in P; join O P. P B, then O P B is a right angle,

BP^2 = OB^2 — OP^2 $= r_1^2 - r^2$

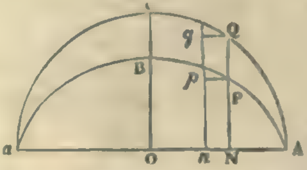
$\therefore \pi$ B P$^2 = \pi (r_1^2 - r^2)$

also since O B P is a right angle, B P touches the circle, or the area of the space in question is equal to the area of a circle whose radius is the length of the line drawn from any point in the exterior circle to touch the interior circle.

The cited cases of areas of plane figures are the chief of those which belong to Elementary Mathematics. The determination of areas bounded by curved lines belongs, of right, to the Integral Calculus; the following propositions, however, are best given here, though the reasoning is not of a strictly elementary character.

(12). *To find the Area of an Ellipse.*[*]

Fig. 15.



Let aB A be a semi-ellipse.

O A its semi-major axis = a.

O B its semi-minor axis = b.

On aA as a diameter describe a semicircle aC A. In A B take any point P and draw an ordinate P N perpendicular to O A, and produce it to meet the circle in Q, and draw another ordinate qpn parallel and near to Q P N, and complete the parallelograms nP, nQ

Now, by a property of the ellipse,

$$\text{NQ} : \text{NP} :: a : b$$
$$\therefore n\text{Q} : n\text{P} :: a : b$$

and hence, if we suppose a series of parallelograms to be described in circle and ellipse, the same proportion will hold good between each of these, and therefore

Sum of parallelograms in semicircle : sum of parallelograms in semi-ellipse :: a : b.; and this being true, however great the number may be, is true in the limit.

Now the semi-ellipse is the limit of the parallelograms inscribed in it, and the semicircle is the limit of parallelograms inscribed in it.

$$\therefore \text{semicircle} : \text{semi-ellipse} :: a : b$$
$$\therefore \text{circle} : \text{ellipse} :: \pi a^2 : \pi a b$$

But area of circle = πa^2

$\therefore$ area of ellipse = πab.

(13). *Let q* P Q *be a portion of a Parabola. It is required to find its Area.* (Fig. 17).

Bisect Qq in V, draw the diameter P V. Through P draw r P R parallel to Q V q, and draw Q R and qr parallel to P V. Then the area q P Q is two-thirds of qr R Q.

Take p and p_1, two points in P Q, and through p_1 draw $p_1 m_1$, $p_1 n_1$ parallel to P V and Q V, and through p draw pm pn also parallel to P V, Q V, and produce them to meet $p_1 n_1$ in q_1 and $p_1 m_1$ in q respectively. Then

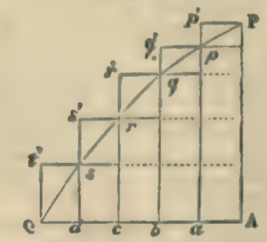
Parallelogram pm_1 : parallelogram pn_1 :: $pn \times nn_1$: $pm \times mm_1$.

$$:: pn \times (\text{P}n_1 - \text{P}n) : \text{P}n \times (p_1 n_1 - pn)$$

---

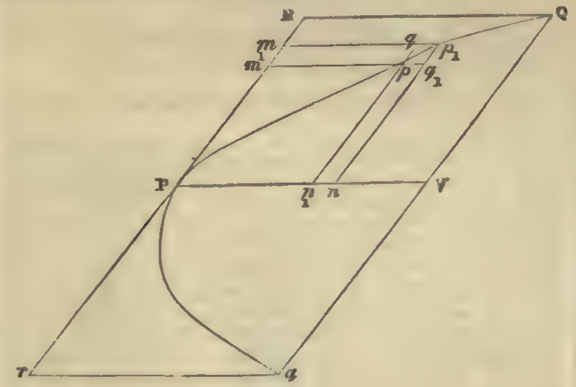
[*] Let A P Q be an area included by two straight lines A B. A P, and the curve P Q; divide A Q into equal parts Aa, ab, bc cd, dQ, and on these lines draw the rectangles Ap, aq, br, cs *within* the curved areas, and complete the parallelograms Ap', aq', br', cs', dt. Then it is plain that the difference between the interior parallelograms (Ap, aq, br, cs) and the exterior parallelograms (Ap', aq', br', cs', dt') will equal Ap'. Now, by making the number of parallelograms very large, Aa, and $\therefore$ Ap' will become very small, and may be made less than any magnitude that may be assigned. Now the curvilinear area is clearly greater than the interior and less than the exterior parallelograms, and therefore differs from the interior parallelogram by a quantity less than Ap'; *i.e.*, a quantity that can be made less than any that can be assigned: and therefore the curvilinear area is the limit to which the sum of the interior parallelograms continually approaches when their number is increased.

Fig. 16.



Now, by a property of the parabola

Fig. 17.



$Pn : P_1 n_1 :: pn^2 : p_1 n_1^2$

$\therefore Pn : Pn_1 - Pn :: pn^2 : p_1 n_1^2 - pn^2$

or $Pn : Pn_1 - Pn :: pn^2 : (p_1 n_1 - pn)(p_1 n_1 + pn)$

$\therefore Pn \times (p_1 n_1 - pn) : (Pn_1 - Pn)\, pn :: pn : p_1 n_1 + pn$

$\therefore$ parallelogram pm_1 : parallelogram $pn_1 :: pn : p_1 n + pn$.

Now in the limit $p_1 n_1$ differs from pn by a quantity less than any that can be assigned.

$\therefore$ In the limit $pn : p_1 n_1 + pn :: 1 : 2$

$\therefore$ In the limit parallelogram pm_1 : parallelogram $pn_1 :: 1 : 2$

which proportion is true for *each* pair of parallelograms inscribed in P Q R and P Q V, and $\therefore$ is true of *all*.

$\therefore$ In the limit sum of parallelograms in P Q R : sum of parallelograms in P Q V :: 1 : 2

But the area P Q R is the limit of the sum of the parallelograms inscribed in it; and P Q V is the limit of the sum of the parallelograms inscribed in it.

$\therefore PQR : PQV :: 1 : 2$

$\therefore PQVR : PQV :: 3 : 2$

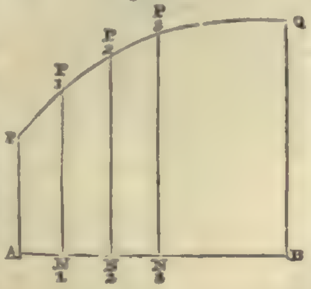
Now P Q V R is the half of qr Q R, and P Q V is the half of P Qq.

$\therefore PQq = \frac{2}{3} rq.\ QR.$ Q.E.D.

(14). *To find approximately the Area of a Plane Figure bounded by a Curve.*

Let A B P Q be the figure; divide A B into equal parts, AN_1, $N_1 N_2$, $N_2 N_3$, &c., and draw ordinates $P_1 N_1$, $P_2 N_2$, $P_3 N_3$, &c., parallel to A P or B Q, and perpendicular to A B.

Fig. 18.



(1). As a first approximation the curved area may be considered as identical with the polygonal area inclosed by P A B Q and the chords PP_1, P_1P_2, P_2P_3, &c. In this case, the area required $= AN_1P_1P + N_1N_2P_2P_1 + N_2N_3P_3P_2$, &c.

$$= AN_1 \times \frac{AP + P_1N_1}{2} + N_1N_2 \times \frac{P_1N_1 + P_2N_2}{2}$$

$$+ N_2N_3 \times \frac{P_2N_2 + P_3N_3}{2}, \text{ \&c.}$$

$$= AN_1\left(\frac{AP}{2} + P_1N_1 + P_2N_2 + P_3 \quad + \dots + \frac{QB}{2}\right)$$

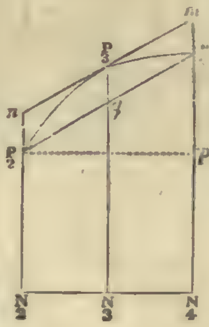
Or the area = (the distance between any two consecutive ordinates) × (half the sum of the extreme ordinates, together with the sum of all the intermediate ordinates)

Or if $AN_1 = \triangle$ $AP = a$ $QB = b$
$P_1N_1 = y_1$ $P_2N_2 = y_2$ $P_3N_3 = y_3$ &c.

$$ABQP = \triangle\left(\frac{a+b}{2} + y_1 + y_2 + y_3 + \dots + y_n\right)$$

(2). The above is a good approximation, but a much better may be found in the following manner. Consider the portion of the curve between any three consecutive P's such as $P_2 P_3 P_4$, through P_3 draw a line nP_3m touching the curve, and let it meet N_2P_2, N_4P_4 produced in n and m. Draw P_2p perpendicular to P_4N_4, join P_2P_4 meeting P_3N_3 in q. Now we may consider $P_2P_3P_4$ a portion of a parabola,* and $\therefore$ the curvilinear area $P_2P_3P_4 = \frac{2}{3}$ of nP_2P_4m.

Fig. 19.



$$= \frac{2}{3} mP_4 \times P_2 p.$$

$$= \frac{2}{3} P_3 q \times P_2 p.$$

Now $N_3 q = \frac{1}{2}(y_2 + y^4)$ and $P_2 p = N_2 N_4 = 2\triangle$

$$\therefore P_3 q = P_3 N_3 - q N_3 = y_3 - \frac{1}{2}(y_2 + y_4)$$

$$\therefore P_2 P_3 P_4 = \frac{4\triangle}{3}\left\{y_3 - \frac{1}{2}(y_2 + y_4)\right\}$$

Now the area $P_2 P_4 N_4 N_2 = \triangle(y_2 + y_4)$

$\therefore$ the whole area $P_2 P_3 P_4 N_4 N_2 = \frac{\triangle}{3}(4y_3 + y_2 + y_4)$

Hence the area of the figure P A B Q (Fig. 18), which equals $PAN_2P_2 + P_2NP_4N_4 + P_4N_4P_6N_6 + \dots$

$$= \frac{\triangle}{3}.\ (a + 4y_1 + y) + \frac{\triangle}{3}(y_2 + 4y_3 + y_4) +$$

$$\frac{\triangle}{3}(y_4 + 4y_5 + y_6) + \dots \quad + \frac{\triangle}{3}.\ (y_{2n} + 4y_{2n-1} + b)$$

reckoning that there are $2n + 1$ (an odd number) ordinates drawn between the two extreme ones P A, Q B.

$$\therefore PABQ = \frac{\triangle}{3}.\left\{a + b + 4(y_1 + y_3 + y_5 + \dots + y_{2n+1} \atop + 2(y_2 + y_4 + y_6 + \dots + y_{2n})\right\}$$

or the area

$= \frac{\triangle}{3} \times$ (sum of extreme sides + 4 sum of odd ordinates + 2 sum of even ordinates).

Hence the Rule :—At equal distances along the base A B draw an odd number of ordinates parallel to the parallel sides of the figure; then take the sum of the parallel sides, of four times the odd ordinates, and twice the even ordinates, and multiply the third part of this sum by the common distance between the ordinates, and this product is the area of the figure.

Besides the above areas of plane surfaces, there are areas of certain solids which will be best given here, viz., the areas of a prism, a cylinder, a pyramid, a cone, and a sphere.

Def. 1.—A prism is a solid bounded by two equal and similar rectilinear figures in parallel planes and by parallelograms.

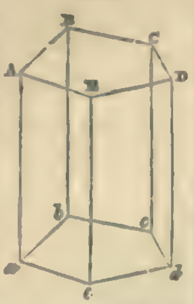
Thus ABCDE, *abcde*, is a prism bounded by two equal and similar pentagons whose planes are parallel (ABCDE, and *abcde*), and by parallelograms A*a*B*b*,

---

* It appears from Newton, (Lemma XI., Lect. 1,) that in any curve of finite curvature, $P_3 q$ is in the limit as the square of qP_4. Now it is the characteristic property of the parabola that $P_3 q$, is as the square of qP_4; hence every curve of finite curvature tends to a parabola as its limit, and so if we take a small arc $P_2 P_3 P_4$ we may consider it to be a parabolic arc, without making any appreciable error.

BbcC, &c. A prism is called a right prism when the planes of the parallelograms are perpendicular to the planes of the base (*i.e.*, *abcde* in the accompanying figure).

Fig. 20.

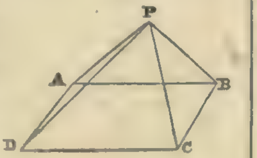


DEF. 2.—A pyramid is a solid bounded by any plane rectilinear figure, and by triangles having a common vertex, and for bases the sides of the rectilinear figure respectively.

Thus (Fig. 21) P A B C D is a pyramid, on a quadrilateral base, A B C D.

DEF. 3.—A cylinder is a solid whose surface is traced out by a straight line which always moves parallel to its first position, and whose extremity is guided by a given curve.

Fig. 21.



Thus (Fig. 22) if *acb* is a circle, and A*a* a straight line perpendicular to the plane of the circle, then ABC *abc* is a cylinder which is traced out by a line C*c*, that is always parallel and equal to A*a*. This cylinder is strictly defined as a "right cylinder with a circular base;" it is, however, in elementary treatises generally called "a cylinder."

DEF. 4.—A cone is a solid, the surface of which is traced out by a straight line, one end of which passes through a fixed point, and the other end through a given plane curve, called its base.

Fig. 22.

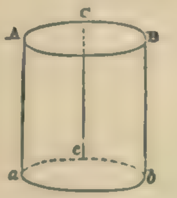
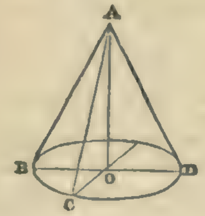


Fig. 23.



If B C D (Fig. 23) is a circle, and A a fixed point, the surface A B C D is called a cone on a circular base. If O is the centre of the circle, join A; then draw straight lines from A to different points in circumference of circle, making equal angles with A O; or if (which is the same thing) A O is perpendicular to plane of circle, A B C D is strictly defined as "a right cone with a circular base;" or, as it is more generally called in elementary treatises, a "right cone."

N. B.—It is manifest that if a regular polygon be inscribed in the circle B C D, and its angular points are joined with A, that the resulting solid will be a pyramid inscribed in the cone; and also, that if we increase the number of sides in the polygon, the inscribed pyramid will approach more nearly to the cone; and, since the circle is the limit of the inscribed polygon, the cone will be the limit of the inscribed pyramid.

Similarly, the cylinder will be the limit of the inscribed prism.

(15). *To find the Area of a right Prism.*

In figure (20) AE*ea* is a rectangle. Since the planes A*b*, A*e* are perpendicular to the base, their cone of intersection A*a* is also perpendicular to the base, and therefore angle A*ae* is a right angle. Hence the area of AE*ea* is A*a* × A E, similarly of all the other parallelograms. Hence the area of the parallelograms is A*a* (A E + E D + D C + ...) = A E × (the perimeter of the base). Hence the whole area = height × perimeter of base + areas of the two ends.

(16). *To find the Area of the surface of a right Cylinder.*

Since the surface of the cylinder is the limit of the surface of the inscribed prism, and the area of the prism = height × perimeter of base + 2 × base, whatever be the number of sides to the base, this will be true in the limit when base is a circle.

∴ Area of a cylinder = height × perimeter of base + 2 × base.

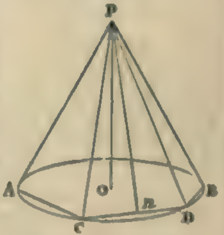
Hence, if h = height, and a = radius of base.

Area of cylinder $= 2\pi ah + 2\pi a^2 = 2\pi a(a+h)$.

(17). *To find the Area of the curved surface of a right Cone.*

Let P A C D B be the cone, A C D B its circular base, O the centre of the circle, then P O is at right angles to plane of the circle. In the circle inscribe any regular polygon, A C D, &c., and join P A, P C, P D, &c. Bisect C D in *n*, join P*n*. Then P*n* is perpendicular to C D, and ∴ the area of triangle P C D = ½ C D, P*n*. Now the line joining P with the bisection of any other side of the polygon is equal to P*n*.

Fig. 24.



Hence area of pyramid inscribed in cone

$$= \tfrac{1}{2}\,\mathrm{PN}\left(\mathrm{AC}+\mathrm{CD}+\mathrm{DB}+\ldots\right)$$

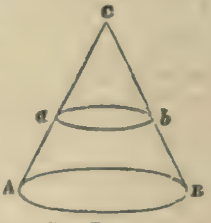
$$= \tfrac{1}{2}\,\mathrm{PN} \times \text{perimeter of polygon.}$$

Now, this is true whatever be the number of sides the polygon may have, and hence is true in the limit; when perimeter of polygon = circumference of circle, and P*n* is drawn to a point in circumference, or is = slant side of cone.

∴ Area of curved surface of cone = ½ slant side × circumference of base.

COR.—If we suppose that the surface of a cone is capable of being unwrapt, it is plain that its surface will be a sector of a circle whose radius is the slant side of cone, and base of the same length as the circumference. It is plain (Mensuration, Art. 9), that the area of this sector is, as it should be, the same as that of the surface of cone.

Fig. 25.

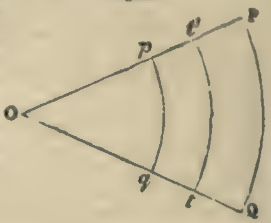


DEF.—The frustum of a cone or pyramid is the portion cut off by a plane parallel to the base; thus, A B *b a* is a frustum of the cone C A B.

(18). *To find the Area of the Frustum of a Cone.*

Suppose a sector, O P Q, (Fig. 26) of a circle is described with radius O P = C A (Fig. 25), and if its base P Q = circumference of A B, we have seen that the area of P O Q is the same as that of the cone C A B. Take O*q* = C*a*, and describe the arc O*qp*. Then as before, area of *opq* = area of C*ab*, and ∴ the area of frustum + *q*QP*p*.

Fig 26.



Let angle POQ $= \theta$. $Oq = r'$ $OQ = r$

$\therefore$ Area $Opq = \tfrac{1}{2} r'^2 \theta$

$\therefore$ Area $OPQ = \tfrac{1}{2} r^2 \theta$

$\therefore$ Area $qOPp = \tfrac{1}{2} \theta (r^2 - r'^2)$

$$= \theta \frac{r+r'}{2} \times (r-r').$$

Bisect qQ in t and draw arc tt'

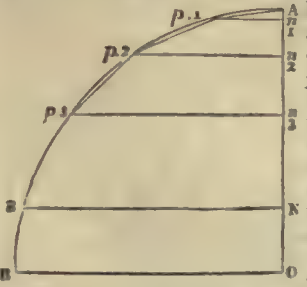
Then $\dfrac{r+r'}{2}\theta = tt'$ and $r - r' = qQ$.

∴ Area of frustum = rectangle between Q*q* and *tt'* or between slant side of cone and circumference of mean section of frustum.

(19). *To find the Area of a portion of the surface of a Sphere.*

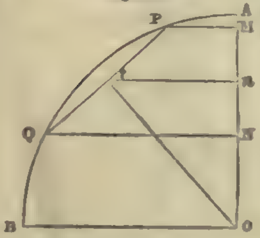
Let A B be a quadrant of a cirle whose radius is A B or O B. If we suppose the quadrant to revolve round A O it will describe a hemisphere; and if we suppose a number of equal chords, Ap_1, $p_1{}'$ p_2, p_2, p_3, &c., to be drawn from point to point of A B, these chords in the revolution will describe frustums of cones: now the arc A R is the limit of the chords $Ap_1 + p_1 p_2 + p_2 p_3 + \ldots$ and hence the area of the portion of sphere described by A R will be the limit of the sum of the frustums of cones described by $Ap_1 \, p_1 p_2 \, p_2 p_3 \ldots$

Fig. 27.



Let P Q (Fig. 28) be one of these chords: draw P M, Q N perpendicular to A O. Draw O perpendicular to P Q, and tn perpendicular to A O. Now tn is the radius of the mean section of the frustum of cone described by P Q, and therefore area of that frustum

Fig. 28.



$$= 2\pi tn \times PQ$$

Now $tn = Ot \sin. tOA.$

and $PQ \sin. PQN = NM.$

or since $PQN = tOA.$

$PQ \sin. tOA = NM$

$\therefore tn. PQ = Ot. NM.$

And area of frustum

$$= 2\pi Ot. NM.$$

In Fig. 27, draw $p_1 n_1 \, p_2 n_2 \, p_3 n_3$ perpendicularly to A O. Then, since, $Ap_1 \, p_1 p_2 \, p_2 p_3$ are all equals, the perpendiculars on them are equal, and therefore the sum of areas of frustum of cone

$$= 2\pi Ot \times (An_1 + n_1 n_2 + n_2 n_3 + \ldots.)$$
$$= 2\pi Ot \times AN.$$

if we only consider the portion of sphere described by A R. This is true, however great the number of chords, and is therefore true in the limit; but in the limit Ot = radius of sphere.

$\therefore$ Area of portion of sphere, whose height is A N.

$$= 2\pi OA. AN.$$

If we take the whole sphere. $AN = 2OA.$

$\therefore$ Area of sphere $= 4\pi. (OA)^2.$

COR.—In the sphere $2\pi \times OA$ = circumference of a great circle.

$\therefore$ Area of portion of a sphere = height of portion × circumference of a great circle.

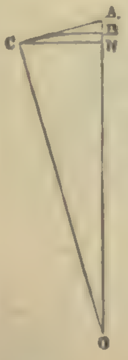
If we imagine a right cylinder to be described about a sphere, its curved area = circumference of a great circle × diameter of sphere = area of sphere.

Hence, "area of sphere = area of circumscribing cylinder."

Exam.:—How many square miles of sea are visible from the top of a mast 80 feet above the surface?

Let O be the centre of the earth; draw O A. O C., so that A C is perpendicular to O C.; draw C N perpendicular to O A. Then, if $BN = x$. $OB = r$. The area visible from A will be the area of the part of the sphere whose depth is B N

Fig. 29.



$$i.e., \text{ will } = 2\pi r x.$$

Now, let $AB = p$. Then, by similar triangles

$$r + p : r :: r : r - x.$$
$$\therefore r - x = \frac{r^2}{r+p}.$$
$$\therefore \quad x = \frac{rp}{r+p}.$$
$$\text{and visible area} = \frac{2\pi r^2 p}{r+p}.$$
$$= 2\pi rp \text{ very nearly.}$$

Now, $\pi = 3{\cdot}14159.$

$r = 3958$ miles

$$p = 80 \text{ feet} = \frac{80}{5250} = \frac{1}{66}$$

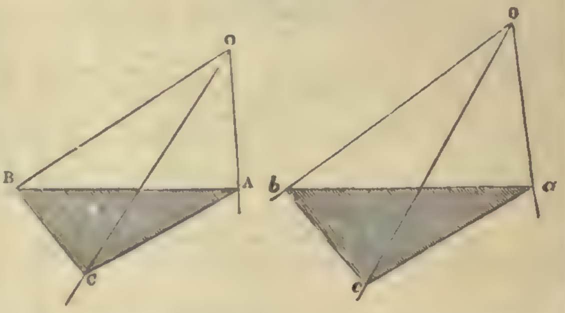
$$\therefore \text{visible area} = 2 \times 3{\cdot}14159 \times 3958 \times \frac{1}{66}$$
$$= 378 \text{ nearly.}$$

III. THE MENSURATION OF SOLIDS.

(1). *If two solid Angles are each contained by three Plane Angles, that are equal each to each, then the inclination of the Planes will be equal, each to each.*

Let O, o, be two solid angles contained by the plane angles A O B, B O C, C O A, and aob, boc, coa, respectively. Then the plane A O B is inclined to plane C O A at the same angle that aob is inclined to aoc. For, take $OA = oa$, and let the plane C A B be perpendicular to O A, and cab perpendicular to oa. Then angles C A O, B A O, cao, bao, are right angles; and C A B, cab, are the inclination of the planes in question.

Fig. 30.



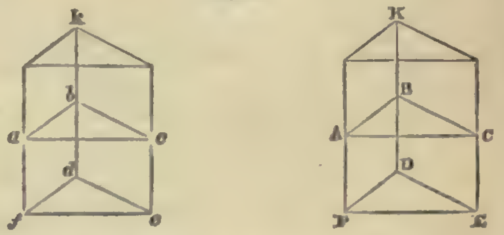
Now in triangles A O B, aob. we have $OA = oa$ and angles B O A, O A B = angles boa, oab, each to each, $\therefore OB = ob$ and $AB = ab$ (Euclid, I., 26); similarly in triangles C O A, coa, we have $OC = oc$ and $AC = ac$.

Then in triangles B O C, boc, we have the sides B O, O C = sides bo, oc, each to each, the included angle $BOC = boc$. $\therefore$ (Euclid, I., 4) the base CB = base cb.

Hence, finally, in triangle A B C, abc, we have the sides B A, A C = the sides ba, ac each to each, and the base BC = base bc. $\therefore$ (Euclid, I., 8) $CAB = cab$. Q. E. D.

COR. 1.—Hence (Fig. 30), if the solid angle O be superimposed on the solid angle o, so that A O coincides with O oa, and the plane A O B with plane aob, then, because angle AOB = angle aob the line O B coincides with ob; and because inclination of plane C O A to A O B equals that of coa to cob, the plane C O A will coincide with coa, and hence O C with oc.

Fig. 31.

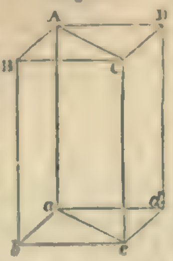


COR. 2.—If A B C D E F and $abcdef$ are two prisms, the edges of which are equal each to each, and the plane angles at B equal those at b, the prisms are equal in all respects. (See Figs. 31 and 32).

For, if the triangle A B C be applied to abc so that A B coincides with ab, and B C with bc, it is plain by the last corollary that B D coincides with bd. And hence the prisms will coincide throughout.

Cor. 3.—If A B C D *abcd* be a parallelepiped[*] (*i. e.*, a prism on a parallelogram for a base), it can be divided into two equal prisms by a plane A C*ca* passing through the diagonals of its bases; for it is obvious that the edges of these prisms are equal, each to each, and also that the plane angles containing the solid angles at D and B are equal. (See Fig. 32).

Fig. 32.



Cor. 4.—Again (in Fig. 31), if we suppose the base D E F of the one prism to be in all respects equal to that of the other *def*, and if the angles at D are equal to the angles at *d*, each to each, then it is plain that if D B is > *db* the prism B D F E is > prism *bdfe*, and if D B < *db*, the prism B D F E < prism *bdfe*. Also if B D is double of *bd*, the prism B D F E is double of the prism *bdfe*; and generally if B D is any multiple of *bd*, then B D F E is the same multiple of *bdfe*.

Cor. 5.—If we suppose (Fig. 31) B D produced to K so that D K is any multiple of D B, the prism K D F E is the same multiple of B D F E; and if we suppose *db* produced to *k* so that *kd* is any multiple of *bd*, then *kdfe* is the same multiple of *bdfe*. But if K D > *kd*, K D F E > *kdfe*; if equal, equal; if less, less ∴ (Euclid, V., p. 136).

B D F E : *bdfe* : : B D : *bd*.

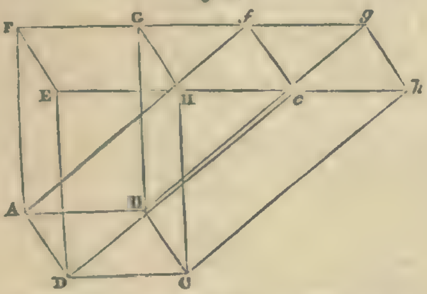
Cor. 6.—The results proved in Cors. 4 and 5 to be true of prisms, are manifestly true of paral^{pps}.

(2). *Parallelepipeds on the same Base and of the same Altitude are equal to one another.*

If the paral^{pps} are on the same base and of the same altitude, it is manifest that the ends opposite the common base are in the same plane.

(*a*) Suppose two of the edges of the ends opposite to the base of each figure to be in the same straight line.

Fig. 33.



Let D B E G, D B*eg*, be the parallelepipeds on the same base B D, and of the same altitudes, and suppose that E H, *eh*, are in the same straight line, and also F G, *fg*, in the same straight line. Then it is obvious that E*e* = H*h*, H C = E D, and angle *h*H C = angle *e*E D.

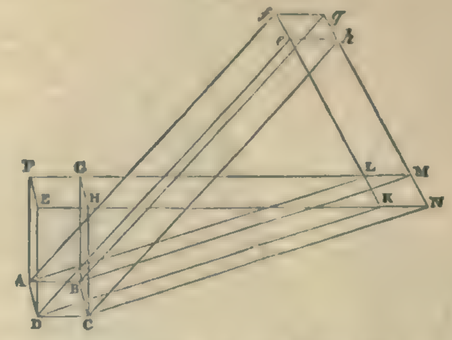
∴ the base *e*E D = base *h*H C, also H G = F E, and the plane angles that form the solid angles at H and E are equal each to each, ∴ the prism *h*C H G = the prism *e*D E F. Hence if we suppose the former prism taken from the whole figure *h*C D E F, and the latter to be taken from the same figure, the remainders will be equal ∴ D B E G = D B*eg*.

(*b*) Suppose that no two of the edges of the side opposite to the base are in the same straight line.

Let D B E G, D B*eg* (Fig. 34), be the paral^{ppd} on the same base B D, and being of the same altitude, their ends E G, *eg*, are in the same plane. Produce E H, F G, *fe*, *gh*, to meet, they will evidently form a parallelogram; let this parallelogram be K L M N, then L K, M N, and F E, G H = A D, B C, each to each, and K N, L M = *eh*, *fg* = D C, A B, each to each, and angle M L K = angle G F E = angle B A D.

∴ L K M N is equal to A B C D. Join A L, B M, C N, D K. Then D B M K is a parallelepiped; and by the first part of this proposition, B D M K is equal to each of B D G E, and B D*ge*. ∴ B D G E = B D*ge*. Q. E. D.

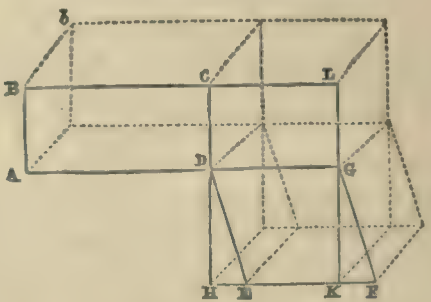
Fig. 34.



(3). *Solid Parallelepipeds on equal Bases and of the same Altitudes are equal to each other.*

(*a*) Suppose the edges to be perpendicular to the bases.

Fig. 35.



To avoid a complicated figure we will only letter the bases, and will call the edges of figure that are perpendicular to the bases by the letters at the angles of the base. Thus the edge B means the edge perpendicular to the base at the point B—*i. e.*, B*b*. Let A C, D F be the bases where we suppose the solids to be so placed as to have a common edge D, and the sides A D, D E in the same straight line; produce C D, and F E to meet in H; through G draw L G K parallel to C D, and produce B C to meet K G in L

Now since D H = G K, and H E is evidently = K F and the angle D H E = G K F ∴ the base D H E = base G K F; and since edges H and K are perpendicular to base, the solid angle at H is contained by plane angles respectively equal to those containing the angle K; and hence the prisms whose bases are D H E and G K F are equal. Add the solid on base D E K G to both, ∴ paral^{ppd} on D K = paral^{ppd} D F.

Now paral^{ppd} B D : paral^{ppd} D L : : A D : : D G : : B D : D L

and paral^{ppd} D L : paral^{ppd} D K : : C D : D H : : D L : D K

∴ (Ex equali).

paral^{ppd} B D : paral^{ppd} D K : : B D : D K

But B D = D K, since (Eucl. I., 35) D K = D F

∴ paral^{ppd} B D = paral^{ppd} D K = paral^{ppd} D F.

(*b*) If the edges are not perpendicular to the bases, call the paral^{ppd} P and Q. If on P's base a paral^{ppd} *p* be described, whose edges are perpendicular to base, and whose altitude is same as P, then *p* = P by last Proposition; and if *q* be in like manner described on same base as Q, but having its edges perpendicular to the base, then (last Proposition) *q* = Q, and by the former part of this Proposition, *q* = *p*, ∴ Q = P. Q. E. D.

Cor. 1.—Hence if there be two paral^{ppd} of the same altitude, but the base of the one be double of the base of the other, the former is double of the latter; and generally if the base of the one be any multiple of the base of the other, the former paral^{ppd} is the same multiple of the latter; and hence, by reasoning similar to that employed in Cor. 5, Prop. I. of the present article, it appears that paral$^{\text{ppds}}$ of equal altitudes are as their bases. And again, if they have equal bases, they are to one another as their altitudes.

---

[*] We shall use throughout the abbreviation paral^{ppd} for parallelepiped.

Cor. 2.—Let there be two paral$^{\text{ppds}}$ P, P′, the base and height of one of which are A and h, and one of the other A′ and h', where Ah A′h' are in numbers. And suppose S to be a third paral$^{\text{ppd}}$ on the base A′ and the height h.

Then
$$P : S :: A : A'$$
$$S : P' :: h : h'$$
$$\therefore P : P' :: Ah : A'h'$$

Cor. 3.—If we suppose P′ to be a cube which has one of its edges equal to unity, then A′ = 1. and h' = 1.

$$\therefore P : P' :: Ah : 1.$$

hence if we consider P′ to be the unit of solid measure (a cubic inch, or foot, for instance) then

$$P = Ah.$$

Hence the volume of a paral$^{\text{ppd}}$ is found by multiplying the area of one face by the distance between that face and the opposite one.

Cor. 4.—We have seen that a prism on a triangular base is half of a paral$^{\text{ppd}}$ of the same altitude, and on a base which is double of the triangle. Hence if A be the area of the triangle, and h the height of the prism, the volume of this paral$^{\text{ppd}}$ = 2A × h, and ∴ the volume of the prism = A × h; or the volume of a prism on a triangular base is found by multiplying the area of the base by the altitude.

Cor. 5.—It is plain that a prism on a polygonal base can be divided by planes passing through one edge of the prism into a number of prisms on triangular bases, each having the same altitude as the original prism.

Let A_1 A_2 A_3 . . . be the areas of these triangles, and h the common altitude, the volumes of all these prisms

$$= (A_1 + A_2 + A_3 + \ldots).\ h$$

But the polygon is equal to all the triangles; hence if A be the area of the polygon,

$$A = A_1 + A_2 + A_3 + \ldots\ \ldots$$

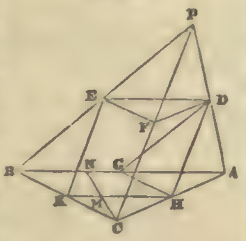
∴ The volume of the prism on a polygonal base = Ah, or is equal to the base multiplied by the altitude.

Cor. 6.—It is plain that Cor. 5 is true of a regular polygon of any number of sides, and therefore is true in limit; now when the number of sides of the polygon is increased, its limit is the circumscribing circle, and the limit of the corresponding prism is the circumscribing cylinder. Hence the volume of a cylinder = Ah, or base × height.

N.B.—When we speak of the volume of a solid, a prism for instance, being equal to the base multiplied by the height, it is of course understood that all the measurements are referred to the same unit. Thus, if we were asked what is the volume of a prism whose base is 2 square feet, and height 18 inches?—the answer is, not 2 × 18, but $2 \times \frac{18}{12} = 3$, and the 3 is in CUBIC FEET. The cubic foot and the square foot being the unit of content and of area corresponding to the linear unit *one foot*.

(4). *If* P, A B C *be any pyramid on a triangular base, and if through the middle point* (D) *of one of the sides* (A P) *we draw planes* (D G H *and* D E F) *parallel to one of the faces* (P B C) *and to the base* (A B C) *of the pyramid, then if we suppose a plane to be drawn through* D E *and* D H, *cutting off a prism* E F D, H K C, *the figure* E D G H C B *is double of the prism* E D F, H K C.

Fig. 36.

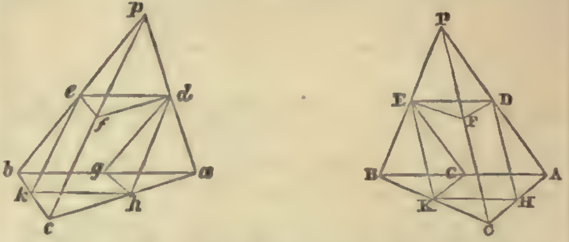


Since the plane E D F is parallel to A B C, E D is parallel to A B. Now A P is bisected in D. ∴ P B is bisected in E. Similarly A C is bisected in H. Also since plane D E F is parallel to A B C. ∴ E D is parallel to K H, and ∴ K H is parallel to A B. Draw C N perpendicular to A B, meeting K H in M; then since C H : H A : : C M : M N (Euclid VI., 2), and C H = H A ∴ C M = M N. Hence, if from N and C perpendiculars are drawn on the plane, D E K H, they are equal. Hence paral$^{\text{ppds}}$ on the base D E K H, with these perpendiculars respectively for altitudes, are equal. Now the prism B E K H D G is half the former, and the prism K C H D F E is half the latter. Hence the prisms are equal, and they are together double of one of them. But the two prisms make up the figure E D G H C B, which is, therefore, double of the prism E D F H K C.

(5). *Pyramids on triangular Bases and of equal Altitudes are to one another as their Bases.*

Let P, A B C, p, abc be the two pyramids, and let them be divided by planes as in the last proposition.

Fig. 37.



Now efd : bca in the duplicate ratio of fd : ca; but fd : ca : : 1 : 2 ∴ efd : bca : : 1 : 4. Similarly E F D : B C A : : 1 : 4 ∴ efd : bca : : E F D : B C A or efd : E F D : : bca : B C A. But the prism $efdkch$: prism E F D K C H : : efd : E F D : : abc : A B C. ∴ (by last proposition)

The double prism $edghcb$: E D G H C B : : abc : A B C.

Now if we suppose pd and P D, and also da and D A to be equal, figures will be formed p,dfe, and P, D F E of the same kind as in p,abc and P, A B C, and also in d,agh and D, A G H, and these figures will have to each other the ratios efd : E F D and agh : A G H respectively; which ratios are each equal to the ratio abc : A B C. And the same will be true, however often we continue to bisect the bisections of the sides, and hence

All the double prisms in
p, abc : all those in P, A B C : : abc : A B C.

And the number of successive bisections being as great as we please, this is true in the limit. Now the pyramids are the limits of the sum of these double prisms. Hence

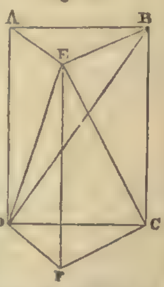
p,abc : P, A B C : : abc : A B C. Q. E. D.

Cor.: Hence pyramids on equal bases and of the same altitude are equal.

(6). *Every Prism on a triangular Base can be divided into three equal Pyramids.*

Let A B E, D F C be the prism; draw a plane through D E and C, cutting off the pyramid E, D F C, and through E D and B draw a plane dividing the remainder into two pyramids E, A B D and E, B C D. Now since these two latter have their vertices coincident at E, and since their bases A B D, B D E are manifestly equal, being halves of the parallelogram A C, the two pyramids E, A B D and E, B D E are equal. Now the pyramid E, A B D is clearly the same as the pyramid D, A B E. But the base ABE = base D E F, and the perpendicular from E on D F E, is equal to that from D on A B E since these planes are parallel; hence the pyramid D, A B E = pyramid E, D F C, and therefore the three pyramids are equal.

Fig. 38.



COR. 1.—Hence if a pyramid and a prism have equal triangular bases and are of equal altitudes, the pyramid is one-third part of the prism.

COR. 2.—Let A = area of base, h = height of pyramid. Then the volume of the prism on base A and of height $h = Ah$. And ∴ the volume of pyramid $= \frac{Ah}{3}$.

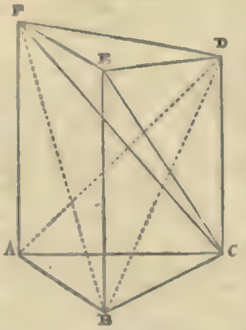
COR. 3.—If we have a pyramid on a polygonal base of which the area is A, it can be divided into triangles whose areas we will suppose to be A_1 A_2 A_3 . . . . so that $A = A_1 + A_2 + A_3 + \ldots$ Now if h be the height of the pyramid, then its volume being clearly the sum of the pyramids whose bases are A_1 A_2 A_3 . . . . and height h, will equal

$$\frac{1}{3}h\left(A_1 + A_2 + A_3 + \cdots\right) = \frac{1}{3}Ah.$$

COR. 4.—This is true, however great the number of sides there are to the polygon, and hence is true in the limit. Now if we suppose the polygon to be regular, its limit is the circumscribing circle, and the limit of the pyramid is the circumscribing cone ∴ volume of cone $= \frac{Ah}{3}$ where A is the area of the circular base.

(7). *To determine the Volume of the Frustum of a right Prism on a triangular Base.*

Fig. 39.

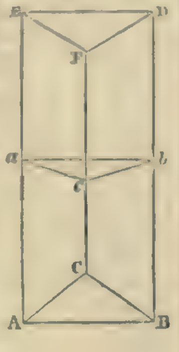


Let A B C, D E F be the frustum, where A B C is perpendicular to the edges. Let A = area of A B C, and let h_1 h_2 h_3 be the edges A F, B E, C D respectively, and V the required volume. Join F C and suppose a plane to pass through F C E, cutting off the pyramid C, FED, and another through E C A cutting off the pyramids E, A B C, E, A C F, these three make up the volume V. Join F B, D B, and D A. Now volume of E, A B C $= A\frac{h_2}{3}$; volume of E, F A C = volume of B, F A C; since perpendiculars from E and B on the plane A C F are clearly equal, and B, F A C is the same pyramid as F, A B C, the volume of which is $A\frac{h_1}{3}$.

Again, since F A is parallel to D C, the triangle A C D is = the triangle F C D; and we have already seen that the perpendiculars from E and B on the plane A C D F are equal, ∴ the pyramid E, C D F = the pyramid B, A C D, *i. e.* = D A B C, the volume of which is $\frac{Ah_3}{3}$.

$$\therefore\ V = A\frac{h_1}{3} + A\frac{h_2}{3} + \frac{h_3}{3}. = A\frac{h^1 + h_2 + h_3}{3}$$

Fig 40.



COR.—If the prism be A B C, F E D, in which neither of the ends is perpendicular to the edges, take *abc* an area whose plane is perpendicular to the edges, and let A = area of *abc*.

Let $AE = h_1$, $BD = h_2$ $CF = h_3$
$aE = x_1$ $bD = x_2$ $cF = x_3$
$aA = y_1$ $bB = y_2$ $cC = y_3$
Then if V = volume required
V = *abc*. D E F + *abc* A B C

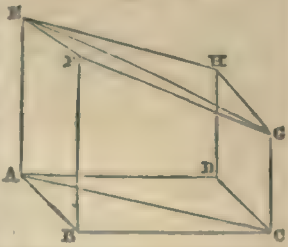
$$= A\frac{x_1 + x_2 + x_3}{3} + A\frac{y_1 + y_2 + y_3}{3}$$

$$= A\frac{x_1 + y_1 + x_2 + y_2 + x_3 + y_3}{3}$$

$$= A\frac{h_1 + h_2 + h_3}{3}$$

(8). *To determine the Volume of a Frustum of a Right Prism on a base which is a Parallelogram.*

Fig. 41.



Let A B C D E F G H be the frustum in question; let h_1 h_2 h_3 h_4 be the edges at A. B. C. D. respectively; draw A C, and if a plane pass through A C and E it divides A B . . . . H into two frustums similar to that in the last proposition. Let V be the required volume, and A_1 the area A B C which is = area ADC (Euclid, I., 34). Now the volumes of the two triangular prisms are respectively

$$A_1\frac{h_1 + h_2 + h_3}{3},\ \text{and}\ A_1\frac{h_3 + h_4 + h_1}{3}$$

$$\therefore V = \frac{1}{3}A_1(h_1 + h_2 + h_3) + \frac{1}{3}A_1(h_3 + h_4 + h_1)$$

If we had divided the figure by a plane passing through B D, we should have had

$$V = \frac{1}{3}A_1(h_2 + h_3 + h_4) + \frac{1}{3}A_1\ (h_4 + h_1 + h_2$$

Similarly if we had divided at C and D, we should have had respectively

$$V = \frac{1}{3}A_1(h_3 + h_4 + h_1) + \frac{1}{3}A_1(h_1 + h_2 + h_3)$$

$$V = \frac{1}{3}A_1(h_4 + h_1 + h_2) + \frac{1}{3}A_1(h_2 + h_3 + h_4)$$

Now let $S = h_1 + h_2 + h_3 + h_4$. Then adding the four values of V we have

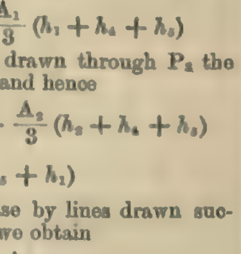
$$4V = \frac{1}{3}A_1\left(S + S + S\right) + \frac{1}{3}A_1\left(S + S + S\right)$$
$$= 2A_1.\ S$$

∴ If A be the whole area A B C D. Since $A = 2A_1$

$$V = A\frac{S}{4} = A\ \frac{h_1 + h_2 + h_3 + h_4}{4}$$

(9). *To find the Volume of a Frustum of a right Prism on a regular Pentagonal Base.*

Fig. 42.



Let P_1 P_2 P_3 P_4 P_5 be the base, and let h_1 h_2 h_3 h_4 h_5 be the edges corresponding to these angles. Join P_1 P_3 and P_1 P_4, and let the areas of P_1 P_2 P_3 and P_1 P_4 P_5, which are equal, be each A_1, and the area of P_1 P_3 P_4 be A_2. Let V be the required volume, then this volume consists of three portions of prisms of the same kind as in article (7) and hence

$$V = \frac{A_1}{3}(h_1 + h_2 + h_3)$$
$$+ \frac{A_2}{3}(h_1 + h_3 + h_4) + \frac{A_1}{3}(h_1 + h_4 + h_5)$$

Now divide the base by lines drawn through P_2 the areas will be the same as before, and hence

$$V = \frac{A_1}{3}(h_2 + h_3 + h_4) + \frac{A_2}{3}(h_2 + h_4 + h_5)$$
$$+ \frac{A_1}{3}(h_2 + h_5 + h_1)$$

and similarly by dividing the base by lines drawn successively through P_3 P_4 and P_5 we obtain

$$V = \frac{A_1}{3}(h_3 + h_4 + h_5) + \frac{A_2}{3}(h_3 + h_5 + h_1)$$
$$+ \frac{A_1}{3}(h_3 + h_1 + h_2)$$

$$V = \frac{A_1}{3}(h_4 + h_5 + h_1) + \frac{A_2}{3}(h_4 + h_1 + h_2)$$
$$+ \frac{A_1}{3}(h_4 + h_2 + h_3)$$

$$V = \frac{A_1}{3}(h_5 + h_1 + h_2) + \frac{A_2}{3}(h_5 + h_2 + h_3)$$
$$+ \frac{A_1}{3}(h_5 + h_3 + h_4)$$

Then adding these together, and writing

$$h_1 + h_2 + h_3 + h_4 + h_5 = S$$

we have $5\,V = \frac{A_1}{3}(S + S + S)$

$$+ \frac{A_2}{3}(S + S + S) + \frac{A_1}{3}(S + S + S)$$
$$\therefore 5\,V = (2A_1 + A_2)\,S = A.\,S.$$

If A = area of base.

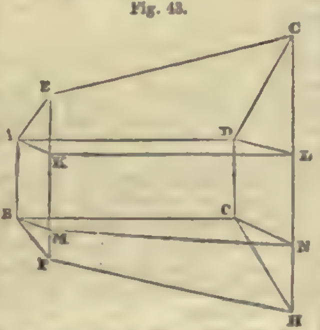
$$\therefore V = A.\,\frac{h_1 + h_2 + h_3 + h_4 + h_5}{5}$$

The student can easily prove that a similar formula is true in the case of a prism on a hexagonal base, or indeed on any base which is a regular polygon.

(10). *To find the Volume of a Prismoid.*

Def.—A prismoid is a solid of the form represented in the accompanying figure. A B C D is a rectangle, and A B F E, D E H G, are planes perpendicular to the planes of the rectangle, E D and C F are planes inclined at given angles to the plane of the rectangle, the lines E F and G H being parallel to A B and C D.

Fig. 43.



Through A D and B C draw planes A D L K and B C N M perpendicular to the plane of the rectangle; then divide the given figure into three—viz., A E K D L G, B D N, and B M F C H N.

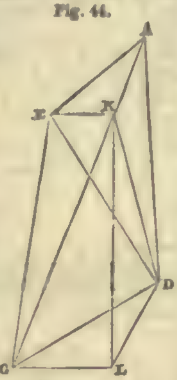
Let A B = a A D = h A K = x D L = x_1 E K = b G L = b_1 M F = c N H = c_1

Then (a) the figure B D N is of the same kind as in Article 8. Now the area D B = ah; if therefore V_1 is its volume,

$$V_1 = \frac{ah}{4}(x + x + x_1 + x_2) = \frac{ah}{2}(x + x_1)$$

(b). Let V_2 be the volume of the figure A E K D L G, (Fig. 44), draw planes through E D,K and G D,K, dividing the figure into three pyramids, D G L,K, A E K,D, and D E G,K.

Fig. 44.



The volume of G L D,K

$$= GLD \times \frac{KL}{3} = \frac{x_1 b_1}{2} \times \frac{h}{3} = \frac{h}{6} x_1 b_1$$

Similarly, volume of A E K,D $= \frac{h}{6} xb$.

Now the volume of G E D,K is to that of A D E,K as G E D is to A D E, or as A E to G D; or since triangles A E K, G D L are similar, these volumes are as A K to D L, or as E K to G L.

$\therefore$ Volume G E D,K

$$= \frac{h}{6} x\,b \times \frac{x_1}{x} \text{ or } = \frac{h}{6} \cdot xb \times \frac{b_1}{b}$$
$$= \frac{h}{6} x_1\,b \text{ or } = \frac{h}{6} \,.\, xb$$
$$= \frac{h}{6} \cdot \frac{x_1 b + x b_1}{2}$$
$$\therefore V_2 = \frac{h}{6}\left(x_1 b_1 + xb + \frac{x_1 b + x b_1}{2}\right)$$

(c). Similarly, if V_3 is the volume of M B F C N H (Fig. 43)

$$V_3 = \frac{h}{6}\left(x_1 c_1 + xc + \frac{xc_1 + xc_1}{2}\right)$$

Now if V be the required volume,

$$V = V_1 + V_2 + V_3.$$
$$\therefore V = \frac{h}{6}\left\{3\,a\,(x + x_1) + x_1\,(b_1 + c_1) + x\,(b + c)\right.$$
$$\left. + x_1\,\frac{b + c}{2} + x\,\frac{b_1 + c_1}{2}\right\}$$

or rearranging the right-hand side of the equation,

$$V = \frac{h}{6}\left\{x\left(a + \frac{b + c}{2}\right) + x_1\left(a + \frac{b_1 + c_1}{2}\right)\right.$$
$$\left. + (x + x_1)\left(2\,a + \frac{b + c}{2} + \frac{b_1 + c_1}{2}\right)\right\}$$

Now in this expression,

$x\left(a + \frac{b + c}{2}\right)$ is the area of A B F E. (Fig. 43).

$x_1\left(a + \frac{b_1 + c_1}{2}\right)$ is the area of D C H G.

and $(x + x_1)\left(2a + \frac{b + c}{2} + \frac{b_1 + c_1}{2}\right)$, which equals

$$4 \times \frac{x + x_1}{2}\left\{\frac{1}{2}\left(a + \frac{b + c}{2}\right) + \frac{1}{2}\left(a + \frac{b_1 + c_1}{2}\right)\right\}.$$

is clearly four times the area of a section made by a plane parallel to A B F E and D E H G, and half way between them.

Hence the solid content of a prismoid is found by the following rule: "To the areas of the ends, add four times the area of the mean section; multiply this sum by one-sixth of the height of the figure; this product is the volume required."

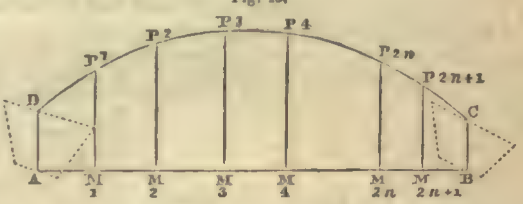
In practical cases it will generally happen that $b = c$, and $b_1 = c_1$. This does not affect the enunciation of *the rule*, but simplifies the formula, which becomes

$$V = \frac{h}{6}\left\{3a\,(x + x_1) + x_1\,(2\,b_1 + b) + x\,(2\,b + b_1)\right\}.$$

(11). *To find the Solid Content of a Railway Cutting.*

In the last article, the figure (Fig. 43) is very nearly that of a portion of a railway cutting, in which A B C D is the road, A G and B H, the sloping sides of the embankment, and hence the solid content required can be found by means of the rule given in the last article. It is to be observed that the rule requires E G and F H to be straight lines, or, as an approximation, to be very nearly straight lines, or E H to be a plane, which is not true if the cutting be a long one. For this case we derive the following rule from the above formula.

Fig. 45.



Let A B C represent a section made by a vertical plane of the hill to be cut through, A B the level of the road, and suppose sections of the cutting to be made by planes perpendicular to A B, at equal distances along that line, viz., at M_1 M_2 M_3 . . . M_{2n+1}, let the terminal sections at A and B be a and b, and the sections at P_1 M_1 P_2 M_2 . . $p_1\,p_2\,p_3 \ldots p_{2n}\,p_{2n+1}$, the number of sections being odd, and let the common distance between the sections be h. Now by last article the volume of portion

$$\text{A M}_2\text{ P}_2\text{ D is }\frac{\text{A M}_2}{6}.\ (a+4p_1+p_2)\text{ or}$$

$$\frac{h}{3}(a+4p_1+p_2)$$

and the volume of the portion $P_2\ M_2\ M_4\ P_4$ is $\frac{h}{3}\ (p_2+4p_3+p_4)$, and so on;

and the volume of $P_{2n}\,M_{2n}$ B C is $\frac{h}{3}.(p_{2n}+4p_{2+1}+b)$. Hence, by addition, the whole volume required is

$$\frac{h}{3}.\left\{(a+b+4(p_1+p_3+\ldots+p_{2n}+_1)+2(p_2+p_4\ldots p_{2n})\right\}$$

which may be expressed as a rule as follows.

Between the first and last sections make an odd number of sections at equal distances along the road, the planes of the section being perpendicular to the road; then one-third part of the common distance multiplied by the sum of the first and last sections, with four times the sum of the odd sections, and twice the sum of the even sections, gives the volume between the first and last section.

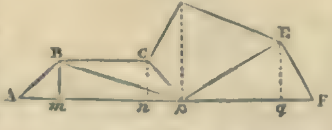
The student will observe that this rule is the same as that for finding the area of a figure bounded by a curve, which has been already given (p. 647), excepting that the ordinates in the former rule are replaced by trapezoidal sections in the latter. The formula given in article 10 is called the prismoidal formula. It will be observed that the material on each side of a cutting being generally the same, is the reason the inclination of the planes A G and B H to B D being generally the same as stated in the last article. Mr. Macneill, to whom the prismoidal formula is due, has constructed tables founded on that formula, by which the volume of a cutting is very readily calculated. The volume of an embankment is to be found by the same formula, since, as a question of mere figure, an embankment is only an inverted cutting. If the calculation be made directly from the formula, it is very tedious; the value of the tables above referred to is therefore very great, and is enhanced by the following circumstance:—In constructing a long line of railway, the earth taken out of the cutting should be sufficient to form the embankment, otherwise land must be purchased for the mere purpose of obtaining earth; to effect this end of making the volume of the embankments equal that of cuttings, the ascents and descents (gradients) of the line of road have to be properly chosen, and this can only be done by trial, so that the calculation may have to be performed two or three times before a right adjustment can be hit upon. It is worth adding that, as a general rule, a cutting is followed not by a long level, but by an embankment; if possible, these two are adjusted to each other, to prevent the need of carrying earth from long distances.

12. *To find the Solid Content of a Military Earth-work.*

The form of a military earth-work will be understood from the following explanations:—

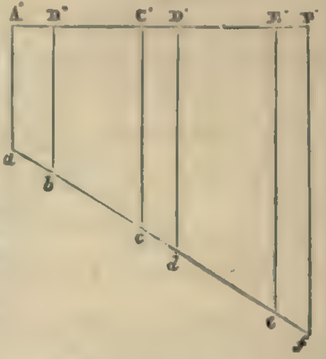
The form of a section of the work made by a vertical plane perpendicular to the face of the work is such as A B C D E F: from B, C, D, E draw perpendiculars to A F, viz., B*m*, C*n*, D*p*, E*q*, then A*m*, *mn*, *np*, *pq*, *q*F, are of known magnitudes, as also are B*m*, C*n*, D*p*, E*q*. The plan of the work will be of the accompanying kind, viz., F′*f* (Fig. 47) is the line corresponding to F (Fig. 46), E′*e* the line corresponding to E, and so on for the others; the lengths of all these lines are known; we will call them *a*, *b*, *c*, *d*, *e*, *f*, respectively.

Fig. 46.



In Fig. 46, join *p*E, *p*C, *p*B, dividing the section of the work into triangles A*p*B, B*p*C, C*p*D, D*p*E, and E*p*F; call these areas respectively $A_1\ A_2\ A_3\ A_4\ A_5$, then the contents of the work are clearly equivalent to the frustums of five prisms (similar to that in the Corol. to article 7), which have the area of perpendicular section A_5, and edges *f*, *e*, *d*, section A_4 and edges *e*, *d*, *d*, section A_3, and edges *d*, *d*, *c*, section A_2 and edges *c*, *d*, *b*, and section A_1 and edges *b*, *d*, *a*, it being evident that the edge through $p=d$. Hence if the whole volume equal V, we have

Fig. 47.



$$V=\frac{A_5}{3}(f+e+d)+\frac{A_4}{3}(e+d+d)+$$

$$\frac{A_3}{3}(d+d+c)$$

$$+\frac{A_2}{3}(c+d+b)+\frac{A_1}{3}(b+d+a)$$

$$\begin{aligned}\therefore 3\,V=a\,A_1+b\,(A_1+A_2)+c\,(A_2+A_3)\\+d(A_3+A_4)+c\,(A_4+A_5)+f\,A_5)\\+d\,(A_1+A_2+A_3+A_4+A_5)\end{aligned}$$

In which formula it will be observed that each line in the plan is multiplied by the triangle, or by the sum of the triangles, which have an angular point in that line, and that the line *d*D′ is also multiplied by the whole area of the section. Hence if these products are formed and added together, the required volume is one-third of the sum.

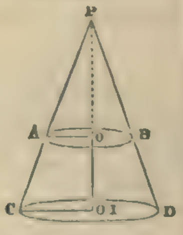
13. *To find the Volume of the Frustum of a right Cone made by a Plane parallel to the Base.*

Let A B C D be the frustum of the cone P C D. Join P O O_1 where O O_1 are the centres of the ends of the frustums. Let A O = *r*, C $O_1 = r_1$, $OO_2 = h$. P O = *x*.

Then volume A B P = $\frac{\pi r^2 x}{3}$, volume

P C D = $\frac{\pi r_1{}^2\,(x+h)}{3}$

Fig. 48.



∴ Volume of frustum =

$$\frac{\pi}{3}\ r_1{}^2 h+\frac{\pi\,(r_1{}^2-r_2{}^2)}{3}x$$

Now $x+h:r_1::x:r$.
$\therefore h:x::r_1-r:r$.

$$\therefore\text{ Volume of frustum}=\frac{\pi}{3}r_1{}^2h+\frac{\pi\,(r_1{}^2-r^2)}{3}\,\frac{hr}{r_1-r}$$

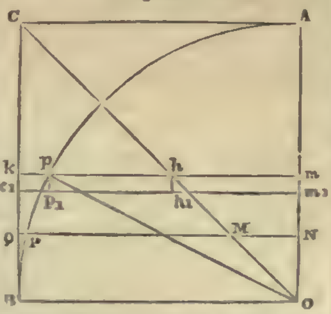
$$=\frac{\pi h}{3}.\ (r_1{}^2+r_1r+r^2)$$

which is the same as the volume of three cones whose common height is *h*, and which have the radii of their bases respectively *r*, r_1 and a mean proportional between *r* and r_1.

14. *To find the Volume of a Portion of a Sphere.*

Let A B O be a quadrant of a circle, draw B C, A C, tangents at A, and B, join O C, and take any point P in A B and draw Q P N parallel to B O; then if the whole figure revolve round A O, the quadrant A B O will clearly describe a hemisphere, the square A O B C will describe a cylinder, on a base whose radius is O B, and height A O, and the triangle A C O will describe a cone on a base whose radius is A C and height A O. Also A P N will describe a portion of a sphere, A C Q N a cylinder whose height is A N and radius A C, and A C M N a frustum of a cone, the radii of whose ends are A C and M N, and height A N. Divide A N into any number of equal parts, of which let $m\,m_1$ be one, through m, draw mk parallel to B O, meeting A B in p, and Q C in k, and through m_1 draw a line $m_1\ k_1$ parallel to mk, and from p and h draw pp_1 and hh_1 perpendicular to $m_1\ k_1$. Join Op. Now $Op^2 = mp^2 + mO^2$ since Omp is a right angle. But since triangle Omh is similar to O A C and A C = A O $\therefore Om = mh$ also $km = BO = Op \therefore mk^2 = mp^2 + mh^2 \therefore \pi \times mk^2 = \pi \times mp^2 + \pi \times mh^2$ or (Mensuration of Areas, Art. 8) the circle, the radius of which is mk, is equal to the sum of those whose radii are mp, and mh; also since $kk_1 = pp_1 = hh_1$ we have

Fig. 49.



$$\pi\ mk^2 \times kk_1 = \pi.\ mp^2 \times pp_1 + \pi\ mh^2 \times hh_1$$

or the cylinder, the radius of whose base is mk and height kk_1, is equal to the cylinder, the radius of whose base is mp, and height pp_1, together with that the radius of whose base is mk, and height hh_1; but these cylinders are those which will be described by mk_1 mp_1 and mh_1, when the whole figure revolves round A O. And the same is true of the cylinders corresponding to any other one of the equal parts into which A N is divided, and therefore is true of all of them. Now all the cylinders corresponding to km_1 make up the cylinder described by A Q, and all those described by m_1p, make up the series of cylinders inside the sphere, and those described by $m_1\,h$ make up those described about the cone. Hence,

Cylinder A Q = sum of cylinders inside portion of sphere + sum of cylinders outside portion of cone.

This being true, however small $m\,m_1$ may be, is true in the limit, but the portion of the sphere is the limit of the inscribed cylinders, and the frustum of cone is the limit of the cylinders outside the cone (compare Note, p. 646). Hence,

Cylinder described by A Q = portion of sphere described by A P N + portion of cone described by A C M N.

Now let r = radius of sphere h = height of A N, and V_1 = volume of portion of sphere N M = O N = $r - h$.

Now the column of cylinder described by A Q = $\pi r^2 h$. And volume of frustum of cone described by A C M N

$$= \frac{\pi h}{3} \times \left\{ r^2 + (r-h)\,r + (r-h)^2 \right\}$$

$$\therefore \pi r^2 h = V_1 + \frac{\pi h}{3} \left\{ r^2 + (r-h)\,r + (r-h)^2 \right\}$$

$$\therefore 3V_1 = 3\pi\,r^2 h - \pi h \left\{ r^2 + (r-h)\,r + (r-h)^2 \right\}$$

$$= \pi h \left(3r\,h - h^2 \right)$$

$$\therefore V_1 = \frac{\pi h^2}{3} \cdot \left(3r - h \right).$$

Hence if V = volume of the whole sphere, in which case $h = 2r$.

$$V = \frac{4\pi\,r^3}{3}$$

Now the volume of cylinder circumscribing sphere $= \pi r^2 \times 2r$.

$$\therefore V = \frac{2}{3}.\ \text{(circumscribing cylinder).}$$

Cor. 1.—The above proposition may be demonstrated in the following manner. Suppose a solid having any number of plane faces to be described in the sphere, and let $A_1\ A_2\ A_3$ &c., be the areas of these faces, and $p_1\ p_2\ p_3$ &c., be the perpendicular distances of these faces from the centre of the sphere; now this inscribed solid may be conceived to be made up of pyramids, the bases of which are the faces of the solids, having the centre of the sphere for their common vertex. Hence, if the volume of the solids is V_1

$$V_1 = \frac{1}{3} A_1\,p_1 + \frac{1}{3} A_2\,p_2 + \frac{1}{3} A_3\,p_3 + \ldots$$

Now this is true, however great the number of faces may be, and $\therefore$ is true in the limit, but in the limit, $p_1\ p_2\ p_3 \ldots$ become equal to one another, and to r the radius of sphere. Hence,

Limit of $A_1\,p_1 + A_2\,p_2 + A_3\,p_3 + \ldots = r \times$ (limit of $A_1 + A_2 \times A_3 \ldots$)

Now the limit of $A_1 + A_2 + A_3 + \ldots$ = surface of sphere = $4\pi\,r^2$ (Mensuration of Areas, Art. 19). Also the limit of $V_1 = V$ the volume of sphere.

$$\therefore V = \frac{1}{3} r \times 4\pi\,r^2 = \frac{4\pi\,r^3}{3}.$$

Cor. 2.—To find the volume of the portion of the sphere corresponding to B P N O. Let the volume be called V, and let h = O N which is = M N. Then

V = cylinder B N — cone O N M.

Now volume of cylinder B N $= \pi r^2 h$

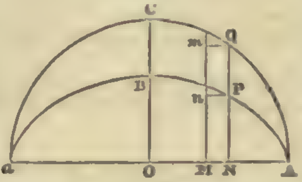
$$\text{Volume of cone M N O} = \frac{1}{3}\ \pi h^3$$

$$\therefore V = \pi r^2 h - \frac{1}{3}\ \pi h^3 = \pi h \left(r^2 - \frac{1}{3}\,h^2 \right)$$

15. *To find the Volume of a Spheroid.*

Def.—A spheroid is a figure formed by the revolution of an ellipse about one of its axes; if about the major axis it is called a prolate spheroid, if about its minor axis it is called an oblate spheroid.

Fig. 50.



Let A Ba be a semi-ellipse, O A its semi-major axis, O B its semi-minor axis. With centre O and radius O A describe a semicircle A Ca, draw Q P N through any point in the ellipse parallel to O C, draw Mnm parallel to N P Q and complete the rectangles M P, M Q (Mensuration of Areas, Art. 12).

Now if the figure revolve round Aa the semi-ellipse will describe a prolate spheroid and the circle a sphere, also M P and M Q will describe cylinders with altitudes M N, and having the radii of the bases N P and N Q respectively, and $\therefore$ having volumes π M N $\times$ P N^2 and π. M N $\times$ N Q^2. Now by a property of the ellipse,

$$QN : PN :: a : b$$
$$\therefore QN^2 : PN^2 :: a^2 : b^2$$

$$\text{But}\quad \text{Cylind. } MQ : \text{cylind. } MP :: MQ^2 : MP^2$$
$$\therefore \text{cylind. } MQ : \text{cylind. } MP :: a^2 : b^2$$

and the same is true of any other cylinders described in like manner within the sphere and spheroid, and $\therefore$ is true of their sum

$\therefore$ all cylinders within sphere : all within spheroid :: $a^2 : b^2$

and this being true, however many cylinders there may be, *i.e.*, however small we suppose M N to be, is true in the limit, but the sphere is the limit of its inscribed cylinders, and the spheroid the limit of its inscribed cylinder,

$$\therefore \text{Sphere : spheroid} :: a^2 : b^2 :: \frac{4\pi a^3}{3} : \frac{4\pi ab^2}{3}.$$

$$\text{But volume of sphere} = \frac{4\pi a^3}{3}.$$

$$\therefore \text{Volume of spheroid} = \frac{4\pi ab^2}{3}.$$

Cor.—In like manner the volume of an oblate spheroid is $\dfrac{4\pi a^2 b}{3}$

16. *To find the Volume of a portion of a Spheroid cut off by a plane Perpendicular to the Axis of Revolution.*

In figure 50, suppose we wish to find the volume of the portion of the spheroid corresponding to the portion A P N of the generating ellipse; then, as in Art. 15, we shall have

Portion of Spheroid : portion of sphere : : $b^2 : a^2$

Now (Mensuration of Solids, Art. 14), if $AN = h$ the volume of the portion of the sphere is equal to $\frac{\pi h^2}{3}(3a - h.)$

$\therefore$ If V_1 is the volume required,

$$V_1 = \frac{\pi h^2 b^2}{3a^2}(3a - h).$$

Similarly, if we wish to find the volume of the portion corresponding to O N P B, and if V_2 is this volume, and $h = ON$,

$$V_2 = \pi h b^2 \left(1 - \frac{1}{3} \cdot \frac{h^2}{a^2}\right)$$

as is evident from Corol. 2, Art. 14, Mensuration of Solids.

17. *To find the Volume of a Cask.*

We may consider a cask to be either the middle portion of a spheroid, or two frustums of equal cones joined together at their bases, though it will not coincide with either of these forms exactly. It is to be observed that excise officers generally consider casks to be of the first form.

The measurements that are most easily made in practice are the diameters of the end, and of the middle section, and the distance between the ends; we will call these d, D, and k, respectively, and investigate the rule in each of the above cases.

(*a*). Suppose the cask to be a portion of a spheroid. Its volume V will be double that of V_2 in the last article

$$\therefore V = 2\pi h b^2 \left(1 - \frac{1}{3}\frac{h^2}{a^2}\right)$$

Also by a property of the ellipse, (See Fig. 50),

$$\frac{PN^2}{OB^2} + \frac{ON^2}{OA^2} = 1.$$

$$\therefore \frac{PN^2}{b^2} + \frac{h^2}{a^2} = 1.$$

$$\therefore \frac{1}{3} \cdot \frac{PN^2}{b^2} + \frac{1}{3} \cdot \frac{h^2}{a^2} = \frac{1}{3}.$$

$$\therefore \frac{2}{3} + \frac{1}{3} \cdot \frac{PN^2}{b} = 1 - \frac{1}{3}\frac{h^2}{a^2}$$

$$\therefore V = 2\pi h b^2 \left(\frac{2}{3} + \frac{1}{3}\frac{PN^2}{b^2}\right)$$

$$\therefore V = \frac{2\pi h}{12}\left(8b^2 + 4\,PN^2\right)$$

Now $2h = k$. $2b = D$ $2.\,PN = d$

$$\therefore V = \frac{\pi}{12} k \left(2.\,D^2 + d^2\right).$$

Also $\frac{\pi}{12} = .2618$ very nearly, hence the rule. "To twice the square of the middle diameter add the square of the end diameter, and multiply the sum by the length of the cask; this product multiplied by .2618 gives the contents of the cask."

(*b*). Suppose the cask to have the form of a double frustum of a cone.

If v be the volume required, v is clearly double of the volume A B C D, Fig. 48. And hence by Article 13, Mensuration of Solids,

$$v = \frac{2\pi h}{3}\left(r_1^2 + r r_1 + r^2\right)$$

Now $D = 2r_1$, $d = 2r$ and $k = 2h$

$$\therefore v = \frac{\pi}{12} \cdot k \left(D^2 + Dd + d^2\right).$$

Hence the rule, "To the product of the diameters add the sum of their squares; multiply this by the length of the cask, then the whole product multiplied by .2618 gives the contents of the cask."

N. B.—If the measurements are made in inches, the above rules give the required contents in cubic inches; to obtain the contents in gallons we must divide by 277.274, since 277.274 cubic inches go to one gallon.

COR.—It is evident that v, the value given by the second rule, is less than the true contents of the cask: it is to be observed, also, that V, the value given by the former rule, is generally greater than the true value; so that the true value will lie somewhere between these two results. Hence we can easily estimate the amount of accuracy in each of the above determinations. Thus, $V - v$ is clearly greater than the difference between the true result, and either of those given by rule, and v is less than the true value of the contents; hence the error committed by either way of making the calculation cannot be so great as the $\left(\frac{V - v}{v}\right)^{\text{th}}$ of the whole. Now

$$V - v = \frac{\pi k}{12}\left(2D^2 + d^2\right) - \frac{\pi k}{12}\left(D^2 + Dd + d^2\right)$$

$$= \frac{\pi k}{12}\left(D^2 - Dd\right) = \frac{\pi k}{12} D(D - d.)$$

$$\therefore \frac{V - v}{v} = \frac{\frac{\pi k}{12} D(D - d)}{\frac{\pi k}{12}\left(D^2 + Dd + d^2\right)} = \frac{D(D - d)}{D^2 + Dd + d^2}.$$

If $D = d + n$. Then $Dd = D^2 - Dn$, and $d^2 = D^2 - 2Dn + n^2$.

$$\therefore \frac{V - v}{v} = \frac{Dn}{3D^2 - 3Dn + n^2} = \frac{\frac{n}{3D}}{1 - \frac{n}{D} + \frac{n^2}{3D^2}}.$$

which is an expression for a limit of the part of the whole, by the approximate, which differs from the true value. *e.g.*, Suppose the diameters to be 18 and 20 inches respectively,

then $n = 2$ inches and $\frac{n}{D} = \frac{1}{10}$, and there the error committed by calculating according to either rule cannot be so much as $\frac{\frac{1}{30}}{1 - \frac{1}{10} + \frac{1}{300}}$ of the whole, or so much as $\frac{10}{271}^{\text{th}}$ or $\frac{1}{27}^{\text{th}}$ (very nearly) of the whole. If the cask in question had an interior length of two feet, then, by the first rule, its contents are 25·47 gallons, and, by the second, its contents are 24.56 gallons. So that the error committed by either way of gauging must be less than one gallon.

CHAPTER XV.

SPHERICAL TRIGONOMETRY.

Before reading the following Chapter, the student will do well to reperuse that on Spherical Geometry already given (p. 596, &c.) He will there find the definitions enunciated, and the chief properties of spherical triangles proved, which are employed as the premises, and whence the formulas of the following pages are deduced. It has been stated that the chief applications of this science are found in practical astronomy and geodesy; also it is observed, on p. 598, that the side of a spherical triangle measures the angle it subtends at the centre of the sphere, and hence is spoken of as an angle. Now it is to be remarked, that in practical astronomy, the measurements made by the various instruments are invariably the angles subtended at the eye of the observer, by arcs of the great sphere; for instance, the altitude of a star is measured directly as an angle,—so that in these cases the radius of the sphere never enters into consideration. But in the case of measurements on the earth's surface, if we have a distance measured along a great circle in miles or yards, which is to enter into our calculations, we must determine the angle these yards or miles subtend at the earth's centre: thus if a be the length in question, r the radius of the earth, θ the angle, then $\theta = \frac{a}{r}$ where θ is in circular measure. If θ contain $n°$, then $n° = \frac{a\,180°}{\pi r}$. Further it will be observed, that in case the sides of a spherical triangle are small compared with the radius of the sphere, the triangle does not differ sensibly from a plane triangle: *e. g.*, a triangle on the earth's surface, the sides of which are each about a mile long, will not differ sensibly from a plane triangle, unless the measurements are made with very refined instruments. Hence it is manifest that the plane triangle is the limit of a spherical triangle, and accordingly we shall find that the formulas for the solution of spherical triangles are quite analogous to those that have been already deduced for the solution of plane triangles (p. 616, *et seq.*), and we shall see that the latter can be deduced from the former by considering the plane triangle as the limit of the spherical triangle.

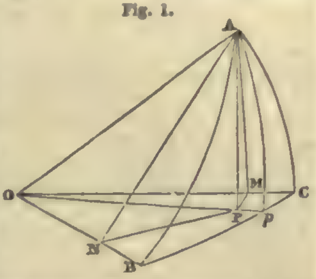
N.B.—The following results already proved on p. 599 are very important. Let A, B, C, a, b, c, be the angles and sides of any spherical triangle, and A′, B′, C′, a', b', c', the angles and sides of the corresponding polar triangle.

$$\text{Then}\quad A + a' = B + b' = C + c' = 180°$$
$$\text{And}\quad A' + a = B' + b = C' + c = 180°$$

We shall employ this notation for the angles and sides of a spherical triangle, and of its polar triangle, throughout the following pages.

(1). *To show that the Sines of the Angles of a Spherical Triangle are proportioned to the Sines of the opposite sides.*

Fig. 1.



Let A B C be the triangle, O the centre of the sphere; join O A, O B, O C; through A draw a plane A N P perpendicular to O B, cutting the plane A O B in A N and B O C in N P, these lines are perpendicular to O B, and the angle A N P measures the inclination of the planes, and is ∴ equal to the angle B of the triangle. Through A draw another plane A M P perpendicular to O C, cutting A O C in A M, C O B in M P, and A M P in A P, then A M and M P are perpendicular to O C, the angle A M P is equal to the angle C of the triangle, and A P is perpendicular to the plane B O A, and ∴ A P N and A P M are each right angles. Hence

$$\text{Sin. B} = \frac{AP}{AN}, \text{ and sin. C} = \frac{AP}{AM}. \quad \therefore \frac{\text{sin. B}}{\text{sin. C}} = \frac{AM}{AN}.$$

Also since c is the angle A O B and b the angle A O C

$$\text{sin. } c = \frac{AN}{OA}, \text{ and sin. } b = \frac{AM}{OA}. \quad \therefore \frac{\text{sin. } b}{\text{sin. } c} = \frac{AM}{AN}$$

$$\therefore \frac{\text{sin. B}}{\text{sin. } c} = \frac{\text{sin. } b}{\text{sin. } c}, \text{ or } \frac{\text{sin. B}}{\text{sin. } b} = \frac{\text{sin. C}}{\text{sin. } c}$$

The same proof holds good of the other sides and angles.

$$\text{Hence}\quad \frac{\text{sin. A}}{\text{sin. } a} = \frac{\text{sin. B}}{\text{sin. } b} = \frac{\text{sin. C}}{\text{sin. } c} \ldots \quad (1). \quad \text{Q.E.D.}$$

Cor. 1.—Suppose a, b, c, to be the lengths of the sides BC, CA, AB, then the angles denoted in formula (1) by a, b, c, are $\frac{a}{r}$, $\frac{b}{r}$, $\frac{c}{r}$ in circular measure, r being the radius of sphere. Hence

$$\frac{\text{sin. B}}{\text{sin. C}} = \frac{\text{sin. }\frac{b}{r}}{\text{sin. }\frac{c}{r}} = \frac{\frac{b}{r}\cdot\frac{\text{sin. }\frac{b}{r}}{\frac{b}{r}}}{\frac{c}{r}\cdot\frac{\text{sin. }\frac{c}{r}}{\frac{c}{r}}} = \frac{b}{c}\cdot\frac{\frac{\text{sin. }\frac{b}{r}}{\frac{b}{r}}}{\frac{\text{sin. }\frac{c}{r}}{\frac{c}{r}}}$$

Now in the limiting case when r is infinite, $\frac{a}{r} = 0$ and $\frac{b}{r} = 0$ and ∴ (Plane Trig., Art. 47)

$$\frac{\text{sin. }\frac{b}{r}}{\frac{b}{r}} = 1, \text{ and } \frac{\text{sin. }\frac{c}{r}}{\frac{c}{r}} = 1. \quad \therefore \text{ in the limit}$$

$$\frac{\text{sin. B}}{\text{sin. C}} = \frac{b}{c}\cdot$$ the formula for plane triangles (p. 627).

Cor. 2.—If through O and A P a plane be drawn cutting the surface of the sphere in Ap, then Ap is perpendicular to B C, and $\frac{AP}{OA}$ equals the sine of A O P: *i. e.*, is equal to the sine of Ap, which we will call p.

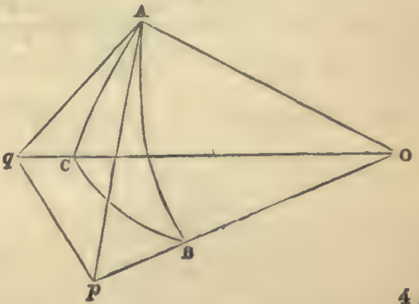
$$\text{Now sin. B sin. } c = \frac{AP}{AN} + \frac{AN}{OA} = \frac{AP}{AO} = \text{sin. A}p.$$

$$\therefore \text{sin. B sin } c = \text{sin. } p. \ldots \quad (2).$$

(2). *To prove the Formula* $\cos. A = \frac{\cos. a - \cos. b \cos. c}{\sin. b \sin. c}$

As before, let O be the centre of the sphere, and A B C

Fig. 2.



the triangle, join O A. O B. O C, and produce the planes A O B, B O C, C O A indefinitely; at A draw a plane Apq perpendicular to O A, cutting the planes A O B, B O C, C O A in Ap, pq, qA respectively, then since pA is on the plane A O B, and perpendicular to O A, and qA is on the plane C O A, and perpendicular to O A, pAq is the angle between the planes, and ∴ is equal to the angle A of the triangle; also the angle pOq is the angle subtended by B C: *i. e.*, is the angle a.

Hence (Plane Trig., Art. 37)

$$Ap^2 + Aq^2 - 2\,Ap.\,Aq\cos.\,A = pq^2 = Op^2 + Oq^2 - 2\,Op.\,Oq\cos.\,a.$$

$$\text{Now}\ \frac{Ap}{OA} = \tan.\,c = \frac{\sin.\,c}{\cos.\,c},\qquad \frac{Aq}{OA} = \tan.\,b = \frac{\sin.\,b}{\cos.\,b}.$$

$$\frac{OP}{OA} = \sec.\,c = \frac{1}{\cos.c}$$

$$\frac{Oq}{OA} = \sec.\,b = \frac{1}{\cos.b}$$

$$\therefore \frac{\sin.^2 c}{\cos.^2 c} + \frac{\sin.^2 b}{\cos.^2 b} - 2\frac{\sin.\,c\sin.\,b}{\cos.\,c\cos.\,b}\cos.\,A = \frac{1}{\cos.^2 c} + \frac{1}{\cos.^2 b} - \frac{2\cos.\,a}{\cos.\,c\cos.\,b}.$$

$$\therefore \frac{2\sin.\,c\sin.\,b}{\cos.\,c\cos.\,b}\cos.\,A = \frac{2\cos.\,a}{\cos.\,c\cos.\,b} - \frac{1}{\cos.^2 c} + \frac{\sin.^2 c}{\cos.^2 c} - \frac{1}{\cos.^2 b} + \frac{\sin.^2 b}{\cos.^2 b}.$$

$$\text{and}\qquad \frac{1}{\cos.^2 c} - \frac{\sin.^2 c}{\cos.^2 c} = \frac{\cos.^2 c}{\cos.^2 c} = 1.$$

$$\text{Similarly}\qquad \frac{1}{\cos.^2 b} - \frac{\sin.^2 b}{\cos.^2 c} = 1.$$

$$\therefore \frac{2\sin.\,c\sin.\,b}{\cos.\,c\cos.\,b}\cos.\,A = \frac{2\cos.\,a}{\cos.\,c\cos.\,b} - 2$$

$$\therefore \sin.\,c\sin.\,b\cos.\,A = \cos.\,a - \cos.\,c\cos.\,b$$

$$\therefore \cos.\,A = \frac{\cos.\,a - \cos.\,b\cos\,c}{\sin.\,b\sin.\,c}\cdot\cdot\cdot\cdot \qquad (3) \qquad \text{Q. E. D.}$$

Cor. 1.—If $a.b.c.$ represent the lengths of the sides of the triangle, and r the radius,

Then $\frac{a}{r}\cdot\ \frac{b}{r}\cdot\ \frac{c}{r}\cdot$ are the angles of formula (3) in circular measure.

Hence, remembering that $\cos.\,\theta = 1 - \frac{\theta^2}{1\cdot 2} + \frac{\theta^4}{1.2.3.4}\cdot\cdot$

and $\sin.\,\theta = \theta - \frac{\theta^3}{1.2.3} + \cdot\cdot\cdot$ we have

$$\text{Cos. A} = \frac{\left(1 - \frac{a^2}{2r^2} + \cdots\right) - \left(1 - \frac{b^2}{2r^2} + \cdots\right)\left(1 - \frac{c^2}{2r^2} + \cdots\right)}{\left(\frac{b}{r} - \frac{b^3}{cr^3} + \cdots\right)\left(\frac{c}{r} - \frac{c^3}{cr^3} + \cdot\cdot\right)}$$

$$= \frac{\left(1 - \frac{a^2}{2r^2}\right) - \left(1 - \frac{b^2}{2r^2} - \frac{c^2}{2r^2}\right) + \text{terms involving}\ \frac{1}{r^4}\cdots -}{\frac{bc}{r^2} + \cdots\ \text{terms involving}\ \frac{1}{r^3} + \cdots} = \frac{b^2 + c^2 - a^2 + \text{terms involving}\ \frac{1}{r}\cdot\ \frac{1}{r^2} + \cdots}{2bc + \text{terms involving}\ \frac{1}{r}\cdot\ \frac{1}{r^2} + \cdots}.$$

Now in the limit when r is infinite, the terms involving $\frac{1}{r}\cdot\ \frac{1}{r^2}\cdot\cdot\cdot\cdot$ will all disappear; hence in the limit,

$$\text{Cos. A} = \frac{b^2 + c^2 - a^2}{2\,bc}$$

$$\text{or}\qquad a^2 = b^2 + c^2 - 2\,bc\cos.\,A.$$

as in the case of the plane triangle. (Page 624).

Cor. 2.—In formula (3) substitute 180° — a' for A. 180° — A′ for a. 180° — B′ for b, and 180° — C′ for C, and we shall have

$$\text{Cos.}\ a' = \frac{\cos.\,A' + \cos.\,B'\cos.\,C'}{\sin.\,B'\sin.\,C'}.$$

This is true of the sides and angles of every polar triangle. Now it appears from Prop. XI., p. 599, that every triangle may be regarded as the polar triangle of some other; hence the above formula is perfectly general, and is true of every triangle, and we have

$$\text{Cos.}\ a = \frac{\cos.\,A + \cos\,B\cos.\,C}{\sin.\,B\sin.\,C}\cdot\cdot\cdot\cdot \qquad (4)$$

Cor. 3.—Formulas similar to (3) and (4) are, of course, true of Cos. B, and Cos. C, and of Cos. b and Cos. c.

(3). *To express the Formulas of the last Article in a Form adapted for Logarithmic Calculation.*

$$\text{Since}\qquad \text{Cos. A} = \frac{\cos.\,a - \cos.\,b\cos.\,c}{\sin.\,b\sin.\,c}$$

$$\therefore 1 + \cos.\,A = 1 + \frac{\cos.\,a - \cos.\,b\cos.\,c}{\sin.\,b\sin.\,c}$$

$$= \frac{\cos.\,a - \cos.\,b\cos.\,c + \sin.\,b\sin.\,c}{\sin.\,b\sin.\,c}$$

$$\text{And}\ 1 - \cos.\,A = 1 - \frac{\cos.\,a - \cos.\,b\cos.\,c}{\sin.\,b\sin.\,c}$$

$$= \frac{\cos.\,b\cos.\,c + \sin.\,b\sin.\,c - \cos.\,a}{\sin.\,b\sin.\,c}$$

$$\therefore 2\cos.^2\frac{A}{2} = \frac{\cos.\,a - (\cos.\,b\cos.\,c - \sin.\,b\sin.\,c)}{\sin.\,b\sin.\,c}$$

$$= \frac{\cos.\,a - \cos.\,(b + c)}{\sin.\,b\sin.\,c}$$

$$\text{And}\qquad 2\sin.^2\frac{A}{2} = \frac{\cos.\,(b - c) - \cos.\,a}{\sin.\,b\sin.\,c}$$

$$\therefore 2\cos.^2\frac{A}{2} = \frac{2\sin.\,\tfrac{1}{2}(a + b + c)\sin.\,\tfrac{1}{2}(b + c - a)}{\sin.\,b\sin.\,c}$$

$$\text{And}\ 2\sin.^2\frac{A}{2} = \frac{2\sin.\,\tfrac{1}{2}(a - b + c)\sin.\,\tfrac{1}{2}(a + b - c)}{\sin.\,b\sin.\,c}$$

Now if $a + b + c = 2\,s$, then $b + c - a = 2\,(s - a)$ $a - b + c = 2\,(s - b)$, and $a + b - c = 2\,(s - c)$.

$$\therefore \cos.^2\frac{A}{2} = \frac{\sin.\,s\sin.\,(s - a)}{\sin.\,b\sin.\,c} \qquad (5)$$

$$\sin.^2\frac{A}{2} = \frac{\sin.\,(s - b)\sin.\,(s - c)}{\sin.\,b\sin\,c} \qquad (6)$$

$$\therefore \tan.^2\frac{A}{2} = \frac{\sin.\,(s - b)\sin.\,(s - c)}{\sin.\,s\sin.\,(s - a)} \qquad (7)$$

$$\text{And since,}\qquad \sin.\,A = 2\sin.\,\frac{A}{2}\cos.\,\frac{A}{2}$$

$$\sin.^2 A = \frac{4}{\sin.^2 b\sin.^2 c}\ \sin,\,s.\ \sin.\,(s - a)\sin.\,(s - b)\sin.\,(s - c) \qquad (8)$$

these formulas are analogous to the formulas on p. 624, of Plane Trigonometry, which can be shown to be the limits of these in the same manner as in Cor. (1), Art. (1), and Cor. (1), Art. (2).

It is to be observed that, since any two sides of a triangle are greater than the third, $s - a$, $s - b$, $s - c$, are positive; and since all the sides of a triangle less than four right angles, s is less than two right angles, and *à fortiori*, $s - a$, $s - b$, $s - c$, are each less than two right angles: so that, sin. $s.$, sin. $(s - a)$, sin. $(s - b)$, sin. $(s - c)$, are each positive, also b and c are each less than 180°; so that sin. b and sin. c are always positive.

Hence, (5), (6), (7), (8), are always positive, and ∴ the values of $\cos. \frac{A}{2}$ $\sin.' \frac{A}{2}$ $\tan. \frac{A}{2}$ and $\sin. A$ derived from them are always real.

COR.—If we consider the case of the polar triangle $A = 180° - a'$ ∴ $\frac{A}{2} = 90° - \frac{a'}{2}$, $a = 180° - A'$, $b = 180° - B'$, $c = 180° - c'$.

$$\therefore s = 270° - \frac{1}{2}(A' + B' + C') = 270° - S'$$

$$\text{if } S' = \frac{1}{2}(A' + B' + C')$$

$$\therefore s - a = 90° - (S' - A')., \; s - b = 90° - (S' - B'),$$
$$s - c = 90° - (S' - C').$$

$$\text{Hence} \quad \sin.^2 \frac{a'}{2} = - \frac{\cos. S' \cos. (S' - A')}{\sin. B' \sin. C'}$$

with similar expressions for $\cos.^2 \frac{a'}{2}$. $\text{cotan.}^2 \frac{a'}{2}$. and $\sin.^2 a'$; and since these are true for all triangles, we shall have

$$\sin.^2 \frac{a}{2} = - \frac{\cos. S \cos. (S - A)}{\sin. B \sin. C} \tag{9}$$

$$\text{Similarly} \quad \cos.^2 \frac{a}{2} = \frac{\cos. (S - B) \cos. (S - C)}{\sin. B \sin. C} \tag{10}$$

$$\therefore \tan.^2 \frac{a}{2} = - \frac{\cos. S \cos. (S - A)}{\cos. (S - B) \cos. (S - C)} \tag{11}$$

$$\text{and } \sin.^2 a = \frac{4}{\sin.^2 B \sin.^2 C} \Big\{ - \cos. S \cos. (S - A) \cos. (S - B) \cos. (S - C) \Big\} \tag{12}$$

where $2S = A + B + C$. It is to be observed that $A + B + C$ must be less than six right angles, and greater than two right angles; ∴ $S > 90° < 270°$. ∴ cos. is *always negative*.[*]

Also $S - A$ being equal to $\frac{1}{2} . \Big(B + C - A\Big) = \frac{1}{2} \Big\{ (180° - (b' + c' - a') \Big\}$ must be $\angle$ 90°, since $b' + c' > a'$ ∴ $\cos. (S - A)$ is always positive, and similarly $\cos. (S - B)$ and $\cos. (S - C)$ are always positive, and ∴ (9), (10), (11), (12), though in appearance negative, are really always positive, and ∴ give us real values for $\sin. \frac{a}{2}$, $\cos. \frac{a}{2}$, $\tan. \frac{a}{2}$, and $\sin. a$.

Similar formulas, of course, exist for $\sin. \frac{b}{2}$. $\sin. \frac{c}{2}$ &c.

(4). *To prove the Formulas,*

$$\text{Tan.} \frac{A + B}{2} = \frac{\cos. \frac{a - b}{2}}{\cos. \frac{a + b}{2}} \text{cotan.} \frac{C}{2} \textit{ and } \text{Tan.} \frac{A - B}{2}$$

$$= \frac{\sin. \frac{a - b}{2}}{\sin. \frac{a + b}{2}} \text{cotan.} \frac{C}{2}$$

$$\text{Since} \quad \tan.^2 \frac{A}{2} = \frac{\sin. (s - b) \sin. (s - c)}{\sin. s. \sin. (s - a)}.$$

$$\tan.^2 \frac{B}{2} = \frac{\sin. (s - c) \sin. (s - a)}{\sin. s. \sin. (s - b)}$$

$$\tan.^2 \frac{C}{2} = \frac{\sin. (s - a) \sin. (s - b)}{\sin. s. \sin. (s - c)}$$

$$\therefore \tan.^2 \frac{B}{2} \tan.^2 \frac{C}{2} = \frac{\sin (s - c) \sin. (s - a)}{\sin. s \sin. (s - b)}$$
$$\times \frac{\sin. (s - a) \sin. (s - b)}{\sin. s. \sin. (s - c)} = \frac{\sin.^2 (s - a)}{\sin.^2 s}$$

$$\therefore \tan. \frac{B}{2} \tan. \frac{C}{2} = \frac{\sin. (s - a)}{\sin. s}$$

$$\text{Similarly,} \quad \tan. \frac{C}{2} \tan. \frac{A}{2} = \frac{\sin. (s - b)}{\sin. s} \text{ and}$$

$$\tan. \frac{A}{2} \tan. \frac{B}{2} = \frac{\sin. (s - c)}{\sin. s.}$$

$$\text{But } \tan. \frac{A + B}{2} \tan. \frac{C}{2} = \frac{\tan. \frac{B}{2} \tan. \frac{C}{2} + \tan. \frac{C}{2} \tan. \frac{A}{2}}{1 - \tan. \frac{A}{2} \tan. \frac{B}{2}}$$

$$= \frac{\frac{\sin. (s - a)}{\sin. s} + \frac{\sin. (s - b)}{\sin. s}}{1 - \frac{\sin. (s - c)}{\sin. s}}$$

$$\therefore \tan. \frac{A + B}{2} \tan. \frac{C}{2} = \frac{\sin. (s - a) + \sin. (s - b)}{\sin. s - \sin. (s - c)}$$

$$= \frac{2 \sin. \frac{1}{2}\Big(2s - a - b\Big) \cos. \frac{1}{2}\Big(b - a\Big)}{2 \sin. \frac{1}{2} c \cos. \frac{1}{2}\Big(2s - c\Big)}$$

$$= \frac{\sin. \frac{1}{2} c \cos. \frac{1}{2}\Big(a - b\Big)}{\sin. \frac{1}{2} c \cos. \frac{1}{2}\Big(a + b\Big)}.$$

$$\therefore \tan. \frac{A + B}{2} = \frac{\cos. \frac{1}{2}(a - b)}{\cos. \frac{1}{2}(a + b)} \text{cotan.} \frac{C}{2}. \tag{13}$$

$$\text{Similarly,} \quad \tan. \frac{A - B}{2} \tan. \frac{C}{2} = \frac{\sin. (s - b) - \sin. (s - a)}{\sin. s + \sin. (s - c)}$$

$$\frac{2 \sin. \frac{1}{2}(a - b) \cos. \frac{1}{2}\Big(2s - a - b\Big)}{2 \sin. \frac{1}{2}(s - c) \cos. \frac{1}{2} c,}$$

$$\therefore \tan. \frac{A - B}{2} = \frac{\sin. \frac{1}{2} (a - b)}{\sin. \frac{1}{2} (a + b)} \text{cotan.} \frac{C}{2} \tag{14}$$

These formulas are clearly analogous to (39) on p. 624, Plane Trigonometry.

COR.—If we take the polar triangle, since $\frac{A + B}{2} = 180° - \frac{a' + b'}{2}$, $\frac{A - B}{2} = \frac{b' - a'}{2}$, $\frac{a + b}{2} = 180° = \frac{A' + B'}{2}$; $\frac{a - b}{2} = \frac{B' - A'}{2}$, and $\frac{C}{2} = 90° - \frac{c'}{2}$.

We have from (13) and (14)

$$- \tan. \frac{a' + b'}{2} = \frac{\cos. \frac{B' - A'}{2}}{- \cos. \frac{B' + A'}{2}}, \tan. \frac{c'}{2},$$

$$\text{and } \tan. \frac{b' - a}{2} = \frac{\sin. \frac{B' - A'}{2}}{\sin. \frac{A' + B'}{2}} \tan. \frac{c'}{2}$$

Hence, remembering that the formulas for the polar triangle are perfectly general, we have

$$\tan. \frac{a + b}{2} = \frac{\cos. \frac{A - B}{2}}{\cos. \frac{A + B}{2}} \tan. \frac{c}{2} \tag{15}$$

$$\tan. \frac{a - b}{2} = \frac{\sin. \frac{A - B}{2}}{\sin. \frac{A + B}{2}} \tan. \frac{c}{2} \tag{16}$$

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[*] See *ante*, p. 616.

(5). *To prove the Formula* $\text{Cotan. } A \text{ sin. } C = \text{Cotan. } a \text{ sin. } b - \text{Cos. } C \text{ cos. } b$.

This formula is used in certain propositions : *e.g.*, it is employed in the astronomical problem of finding the aberration in declination.

From formula (2) we have

$$\text{Cos. } A \text{ sin. } b \text{ sin. } c = \text{cos. } a - \text{cos. } b \text{ cos. } c.$$

And from formula (3) we have

$$\text{Cos. } c = \text{cos. } a \text{ cos. } b + \text{sin. } a \text{ sin. } b \text{ cos. } C.$$

$$\therefore \text{Cos. } A \text{ sin. } b \text{ sin. } c = \text{cos. } a - \text{cos. } a \text{ cos.}^2 b - \text{sin. } a \text{ sin. } b \text{ cos. } b \text{ cos. } C.$$

$$\therefore \text{Cos. } A \text{ sin. } b \text{ sin. } c = \text{cos. } a \text{ sin.}^2 b - \text{sin. } a \text{ sin. } b \text{ cos. } b \text{ cos. } C.$$

$$\therefore \text{Cos. } a \text{ sin. } c = \text{cos. } a \text{ sin. } b - \text{sin. } a \text{ cos. } b \text{ cos. } C.$$

Also from formula (1) we have

$$\text{Sin. } c = \frac{\text{sin. } C \text{ sin. } a}{\text{sin. } A}$$

$$\therefore \text{Cos } A \frac{\text{sin. } C \text{ sin. } a}{\text{sin. } A} = \text{cos. } a \text{ sin. } b - \text{sin. } a \text{ cos. } b \text{ cos. } C.$$

$$\therefore \text{Cotan. } A \text{ sin. } C \text{ sin. } a = \text{cos. } a \text{ sin. } b - \text{sin. } a \text{ cos. } b \text{ cos. } C.$$

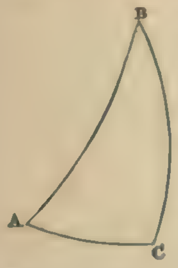
$$\therefore \text{Cotan. } A \text{ sin. } C = \text{cotan. } a \text{ sin. } b - \text{cos. } C \text{ cos. } b.$$

Q. E. D.

THE SOLUTION OF RIGHT-ANGLED SPHERICAL TRIANGLES.

There are as many as six different cases of right-angled spherical triangles, as will appear from the following considerations :—

Fig. 3.



Let A B C be the triangle, having a right angle at C. Then using the ordinary notation, all possible cases are the following :

(1). Given the base and perpendicular, *i.e.*, given a and b.

(2). Given the hypothenuse and another side, *i.e.*, given c and a, or c and b.

(3). Given the base or perpendicular, and an adjacent angle, *i.e.*, given a and B, or b and A.

(4). Given the base or perpendicular and an opposite angle, *i.e.*, given a and A, or b and B.

(5). Given the hypothenuse and an angle, *i.e.*, given c and A, or c and B.

(6). Given the two angles, *i.e.*, A and B.

If these cases be compared with those on p. 641, for plane triangles, it will be seen that the third case of plane triangles diverges into two cases, *viz.*, the third and fourth of spherical triangles, while the sixth case is peculiar to spherical triangles. Both of these differences are due to the circumstance, that in the spherical triangle, $A + B + C$ is not known, whereas in the plane triangle $A + B + C = 180°$

(6). *To investigate the Formulas on which the Solution of Right-angled Spherical Triangles depends.*

From the general formula $\dfrac{\text{sin. } c}{\text{sin. } C} = \dfrac{\text{sin. } b}{\text{sin. } B} = \dfrac{\text{sin. } a}{\text{sin. } A}$

Since $C = 90°$ and $\therefore \text{sin. } C = 1$, we have

$$\text{sin. } c = \frac{\text{sin. } b}{\text{sin. } B} = \frac{\text{sin. } a}{\text{sin. } A}$$

$$\therefore \text{sin. } b = \text{sin. } c \text{ sin. } B \qquad (17)$$

$$\text{Similarly} \quad \text{sin. } a = \text{sin. } c \text{ sin. } A \qquad (18)$$

Again, since $\text{cos. } C = \dfrac{\text{cos. } c - \text{cos. } a \text{ cos. } b}{\text{sin. } a \text{ sin. } b}$ and $\text{cos. } C = 0$, we have

$$\therefore \text{cos. } c = \text{cos. } a \text{ cos. } b \qquad (19)$$

Again, since $\text{cos. } C = \dfrac{\text{cos. } C + \text{cos. } A \text{ cos. } B}{\text{sin. } A \text{ sin } B}$ and $\text{cos. } C = 0$

$$\therefore \text{cos. } c = \text{cotan. } A \text{ cotan. } B \qquad (20)$$

Again, since $\text{cos. } a = \dfrac{\text{cos. } A + \text{cos. } B \text{ cos. } C}{\text{sin. } B \text{ sin. } C}$ and $\text{cos. } C = 0$ and $\text{sin. } C = 1$.

$$\therefore \text{cos. } A = \text{cos. } a \text{ sin. } B \qquad (21)$$

$$\text{Similarly} \quad \text{cos. } B = \text{cos. } b \text{ sin. } A \qquad (22)$$

Again, since $\text{cos. } A = \text{cos. } a \text{ sin. } B$ and $\text{sin. } B = \dfrac{\text{sin. } b}{\text{sin. } c}$

$\therefore \text{cos. } A = \dfrac{\text{sin. } b}{\text{sin. } c} \cdot \text{cos. } a$ and $\text{cos. } a = \dfrac{\text{cos. } c}{\text{cos. } b}$. by (2)

$$\therefore \text{cos. } A = \frac{\text{sin. } b}{\text{sin. } c} \cdot \frac{\text{cos. } c}{\text{cos. } b}.$$

$$\therefore \text{cos. } A = \text{tan. } b \text{ cotan. } c \qquad (23)$$

$$\text{Similarly} \quad \text{cos. } B = \text{tan. } a \text{ cotan. } c \qquad (24)$$

Again from (1) $\text{sin. } a = \text{sin. } c \text{ sin. } A = \dfrac{\text{sin. } b}{\text{sin. } B} \cdot \text{sin. } A$

from (2) and this $= \dfrac{\text{sin. } b \text{ cos. } B}{\text{cos. } b \text{ sin. } B} \times \dfrac{\text{sin. } A \text{ cos. } b}{\text{cos. } B}$

But by (4) $\text{cos. } B = \text{cos. } b \text{ sin. } A$.

$$\therefore \text{sin. } a = \text{tan. } b \text{ cotan. } B \qquad (25)$$

$$\text{Similarly} \quad \text{sin. } b = \text{tan. } a \text{ cotan. } A \qquad (26)$$

(7). *Napier's Rule for the Solution of Right-angled Spherical Triangles.*

The formulas given in the last article can be included in a single rule, which is very easily enunciated and remembered. It is generally called Napier's Rule, having been invented by Napier, who, as we have already stated, was the inventor of logarithms. Leaving out C, which is 90°, there are three sides and two angles in the triangle—*viz.*, a. b. c. A. B. We will call the base, the perpendicular, the complements of the hypothenuse and the angles, circular parts ; if we fix on any of these and call it the middle part, then of the remaining four, two will be adjacent, and the other two opposite: then it will be found that all the formulas of the last article are included in the following rule. "The sine of the middle part equals the product of the tangents of the adjacent parts, and also equals the product of the cosines of the opposite parts ;"

Or, Sin. mid. = tan. ad. = cos. op.

Thus, if $90° - A$ be the middle part, then $90° - c$ and b are the adjacent, and $90° - B$ and a are the opposite part.

The rule gives us $\text{sin. } (90° - A) = \text{tan. } (90° - c) \text{ tan. } b = \text{cos. } (90° - B) \text{ cos. } a$

$$\text{Or} \quad \text{Cos. } A = \text{tan. } b \text{ cotan. } c = \text{cos. } a \text{ sin. } B \qquad (a)$$

Similarly, if $90° - B$ is the middle part, then $90° - c$ and a are the adjacent, and $90° - A$ and b the opposite parts. The rule gives us

$$\text{Cos. } B = \text{tan. } a \text{ cotan. } c = \text{cos. } b \text{ sin. } A \qquad (b)$$

If a be the middle part then $90° - B$ and b are the adjacent, and $90° - c$ and $90° - A$ the opposite part, then the rule gives us

$$\text{Sin. } a = \text{tan. } b \text{ cotan. } B = \text{sin. } c \text{ sin. } A \qquad (c)$$

If b be the middle part then $90° - A$ and a are the adjacent parts, and $90° - c$ and $90° - A$ the opposite parts, then the rule gives us

$$\text{Sin. } b = \text{tan. } a \text{ cotan. } A = \text{sin. } c \text{ sin. } B \qquad (d)$$

Finally, if $90° - c$ be the middle part, then $90° - A$ and $90° - B$ are the adjacent, and a and b the opposite parts, then the rule gives us

$$\text{Cos. } c = \text{cotan. } A \text{ cotan. } B = \text{cos. } a \text{ cos. } b \qquad (e)$$

If the five formulas of the present article be compared with those of Article (6) they will be found identical. Hence Napier's Rule, as was stated, comprises all the formulas of Art. 6. It is a question whether, as a matter of practice, Napier's Rule is really more convenient than the disconnected formulas of Article (6).

(8). *To explain the Method of Solution in the cases of Right-angled Spherical Triangles.*

All the formulas of Art. 6, to which we refer in the present article, are expressed as products, and consequently are adapted for logarithmic calculations. The cases, as we have already seen, are the following:—

(1) Given a and b, find c, A and B
Cos. c is given by (19), then Cos. A is given by (23), and cos. B by (24)

(2) Given c and a, find b, A and B
Cos. b is given by (19), then cos. A is given by (23), and cos. B by (24)

(3) Given a and b, find b, c, and A
Cos. A is given by (21), then cos. b is given by (22), and tan. c by (23)

(4) Given a and A, find b, c, and B
Sin. B is given by (21), then cos. b is given by (22), and cos. c by (19)

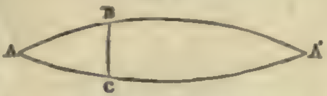
(5) Given c and A, find a, b, and B
Sin. b is given by (17), then cos. a is given by (19), and cos. B by (22)

(6) Given A and B, find a, b, and c
Cos. c is given by (20), cos. a by (21), and cos. b by (22).

It will be observed that each of the above determinations is clearly unambiguous, except the determination of B in (4), and b in (5), for these are the only two determinations made by means of sines; for which reason, if B′ and b' are the values less than 90° which satisfy (4) and (5), then 180° — B′, and 180° — b' also satisfy (4) and (5); and hence it would seem that in the former case there would in general be two values of b and two of c, corresponding to B′ and 180° — B′ respectively; and in the latter case, that there would be two values of a given by (3), and therefore two values of B given by (6). If more closely considered, however, it will appear that there is really no ambiguity in case (5). We will consider the cases separately.

In case (4) we have given A and a. Now AB and AC (Fig. 4), being produced, meet at A′, where A B A′ and A C A′ are each of arc 180° (see Spherical Geometry, pp. 598, 599) and the angle at A′ is equal to the angle at A. Hence the angle A and the side a belong equally to triangle A B C and to A′B C. And if we take the value of B less than 90° to be A B C, then C B A′ is 180° — B′, the second value indicated by the solution.

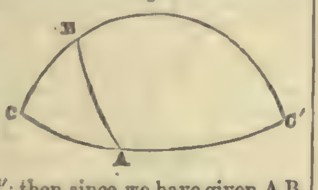
Fig. 4.



In case (5) — from (10) it appears that tan. a = sin. b tan. A.

Now sin. b is always positive, and hence the sign of tan. a must be the same as that of tan A. Hence if A > 90° a must be > 90°, and if a > 90° A must be < 90°, and A is given, hence only one of the two values of a is admissible. This also follows from geometrical considerations. Let C be the right angle, then C A and C B, when produced, meet in C′; then since we have given A B (c) and B A C (A) we determine b, *i.e.*, cA from the equation or 180° — b, *i.e.*, A C′; but A C′ belongs to a triangle on which the angle B A C′ is not A, but 180° — A, and ∴ the value A C′ is inadmissible.

Fig. 5.



THE SOLUTION OF OBLIQUE-ANGLED TRIANGLES.

(9). *To enumerate the Cases of Oblique-Angled Triangles.*

There are six cases of oblique-angled triangles, viz.:—

(1). Given three sides, *e. g.*, a. b. c

(2). Given two sides and the included angle, *e. g.*, a. b. and C.

(3). Given two sides, an angle opposite to one of them, *e. g.*, a. b. A

(4). Given one side and the two adjacent angles, *e. g.*, A. B. c

(5). Given one side, the opposite angle and another angle, *e. g.*, A. C. c

(6). Given the three angles, *e. g.*, A. B. C

As in the case of right-angled triangles, these six cases are analogous to the four cases of plane oblique-angled triangles (p. 641). But the fourth case of a plane triangle diverges on to the fourth and fifth of the spherical triangle, owing to the circumstance that A + B + C is not known in the case of the spherical triangle, whereas in the plane triangle A + B + C = 180°. For the same reason case (6) is peculiar to the spherical triangle.

(10). *To Solve the First Case of Oblique-Angled Triangles.*

We can obtain A from either of the formulas (5), (6), or (7), and then can obtain B and C from similar formulas. Of these formulas (7), which gives tan. $\frac{A}{2}$, is the most convenient if we wish to find both of the other angles. Compare the analogous case of Plane Triangles, p. 641.

(11). *To Solve the Second Case of Oblique-Angled Triangles.*

In this case we will suppose that we have given a b and C. Then from formulas (13) and (14) we can determine $\frac{A+B}{2}$ and $\frac{A-B}{2}$ and hence A and B; and then, knowing A and B, we can determine c from formula (1).

If, however, we wish to determine c directly, *i. e.*, independently of A and B, we can effect our object by introducing a subsidiary angle in a manner analogous to the corresponding case of plane triangles. (See pp. 623 and 642). Thus, from formula (3) we have

$$\text{Cos. C.} = \frac{\text{cos. } c - \text{cos. } a \text{ cos. } b}{\text{sin. } a \text{ sin. } b}$$

$$\begin{aligned} \therefore \quad \text{Cos. } c &= \text{cos. } a \text{ cos. } b + \text{sin. } a \text{ sin. } b \text{ cos. C} \\ \therefore 1 - \text{Cos. } c &= - \text{cos. } a \text{ cos. } b - \text{sin. } a \text{ sin. } b \text{ cos. C} \\ &= 1 - \text{cos. } a \text{ cos. } b + \text{sin. } a \text{ sin. } b \\ &\qquad - \text{sin. } a \text{ sin. } b\,(1 + \text{cos. C}) \\ &= 1 - (\text{cos. } a \text{ cos. } b - \text{sin. } a \text{ sin. } b) \\ &\qquad - \text{sin. } a \text{ sin. } b\,(1 + \text{cos. C}) \\ &= 1 - \text{cos. } (a + b) - \text{sin. } a \text{ sin. } b \\ &\qquad (1 + \text{cos. C}) \end{aligned}$$

Now $1 - \text{cos. A} = 2 \text{ sin.}^2 \frac{A}{2}$ and $1 + \text{cos. A} = 2 \text{ cos.}^2 \frac{A}{2}$

$$\therefore \quad \text{sin.}^2 \frac{c}{2} = \text{sin.}^2 \frac{a+b}{2} - \text{sin. } a \text{ sin. } b \text{ cos.}^2 \frac{C}{2}$$

$$\text{Assume} \quad \text{sin.}^2\,\theta = \text{sin. } a \text{ sin. } b \text{ cos.}^2 \frac{C}{2}$$

$$\begin{aligned} \therefore \quad \text{sin.}^2 \frac{c}{2} &= \text{sin.}^2 \frac{a+b}{2} - \text{sin.}^2\,\theta \\ &= \left(\text{sin. } \frac{a+b}{2} - \text{sin. } \theta\right)\left(\text{sin. } \frac{a+b}{2} + \text{sin. } \theta\right) \\ &= 2 \text{ sin. } \frac{1}{2}\left(\frac{a+b}{2} - \theta\right) \text{cos. } \frac{1}{2}\left(\frac{a+b}{2} + \theta\right) \\ &\times 2 \text{ sin. } \frac{1}{2}\left(\frac{a+b}{2} + \theta\right) \text{cos. } \frac{1}{2}\left(\frac{a+b}{2} - \theta\right) \\ &= 2 \text{ sin. } \frac{1}{2}\left(\frac{a+b}{2} - \theta\right) \text{cos. } \frac{1}{2}\left(\frac{a+b}{2} - \theta\right) \\ &\times 2 \text{ sin. } \frac{1}{2}\left(\frac{a+b}{2} + \theta\right) \text{cos. } \frac{1}{2}\left(\frac{a+b}{2} + \theta\right) \end{aligned}$$

$$\therefore \text{sin.}^2 \frac{c}{2} = \text{sin. } \left(\frac{a+b}{2} - \theta\right) \text{sin. } \left(\frac{a+b}{2} + \theta\right)$$

This latter method is very much easier than the former; for by this we only require five logarithms, whereas by that we require eight, for the determination of c.

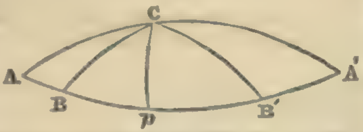
(12). *To Solve the Third Case of Oblique-Angled Triangles.*

In this case we will suppose that we have given a, b, A Then we obtain sin. B by formula (1); and knowing a, b, and A, B, we can determine C, by formula (13); and finally we can determine c by formula (1).

It will be observed that in this case, since the results depend on our determining B from a given value of sin. B, they will be ambiguous, as in the analogous case of plane triangles (p. 619); for if B′ be the value of $B < 90°$, which we derive from formula (1): then $180° - B'$ also satisfies formula (1).

This amount of ambiguity depends on the data. For let A B C be a triangle, having the angle $BAC = A$, $AC = b$ and $BC = c$. Produce A B and A C to meet in A′, draw $CB' = CB$. Then the given data belongs as much to the triangle A C B. as to A C B′. Moreover it is plain that $CBB' = CB'B$, and hence if $CB'A = B'$, we shall have $CBA = 180° - B'$, the same conclusion that we derived from the formula.

Fig. 6.



(13). *To solve the Fourth Case of Oblique-Angled Spherical Triangles.*

In this we suppose that we have given A. B. and C. Then $\frac{a+b}{2}$ is given from formula (15), and $\frac{a-b}{2}$ from formula (16); hence we obtain a and b, and then we obtain C. by formula (1). If we wish to obtain C without the previous calculation of a and b, we must introduce a subsidiary angle θ and proceed as in article (11). From formula (4) we have

$$\text{Cos. } C = -\cos. A \cos. B + \sin. A \sin. B \cos. c,$$

$$\text{Assume} \quad \sin.^2 \theta = \sin. A \sin. B \sin.^2 \frac{c}{2}.$$

And we shall obtain

$$\text{Cos.}^2 \frac{C}{2} = \sin.\left(\frac{A+B}{2} + \theta\right) \sin.\left(\frac{A+B}{2} - \theta\right)$$

(14). *To solve the Fifth Case of Oblique-Angled Triangles.*

In this case we will suppose that A. C. and c, are given. We shall obtain sin. a from formula (1). Now, formula (13) gives us

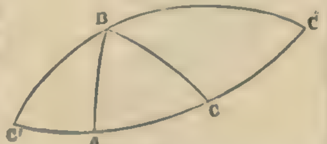
$$\text{Tan. } \frac{A+C}{2} = \frac{\cos. \frac{1}{2}(a-c)}{\cos. \frac{1}{2}(a+c)} \text{ cotan. } \frac{B}{2}$$

Whence we obtain B, and a similar modification of formula (15) will give us b; or, having B, we may obtain b from formula (1).

In this case a is determined from its sine, and therefore has two values, viz. a' and $180° - a'$; and if both these values are admissible, the case is ambiguous.

In the triangle A B C, let $AB = c$, $BAC = A$ and $BCA = C$, and suppose $BC = a$, draw $BC' = a'$; then if a' be greater than c, it is plain that A falls between C and C′; in this case produce C′ B and C′ C to meet in C″. Then the angle $BCC' = BC'C = BC''C$. Hence the data belong equally to the triangle B A C. and B A C″; and the case is ambiguous, provided $c < a$; also it will be observed that $BC'' = 180° - BC' = 180° - BC = 180° - a'$, as previously appeared from the calculation. If B C be less than B A, then C′ would fall between B and A, and the above construction would be no longer possible. Hence if $c > a$, the case is not ambiguous.

Fig. 7.



(15). *To solve the Sixth Case of Oblique-Angled Triangles.*

In this case we have given A, B, C. We shall find a from formula (11), and b and c from similar formulas. This case, however, never occurs in any of the practical applications of spherical trigonometry.

On the Solution of Quadrantal Triangles.

A quadrantal triangle (Spherical Geometry, Def. 14, p. 599) has one *side* of 90°, and ∴ the corresponding polar triangle is a right-angled triangle. From this consideration it would be easy to modify Napier's Rule to suit the case of the quadrantal triangle. In practice, however, it is better to treat them as oblique triangles: on doing so it will be found practically that the circumstance of one side being equal to 90° will introduce important simplifications.

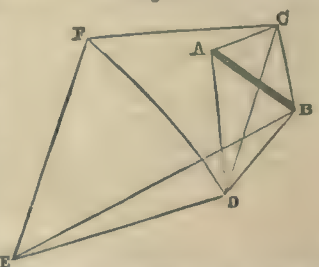
ON THE FORMULAS PECULIAR TO GEODETICAL OPERATIONS.

We have already stated, in general terms, that the science of spherical trigonometry finds one of its applications in Geodesy. It is to be observed that this application possesses some peculiarities in consequence of the sides of the triangles employed in a survey, on even the largest scale, being small compared with the radius of the earth, and consequently small when estimated in degrees or minutes; whereas, in astronomy, there is no limitation imposed on the magnitudes of the sides of the triangles employed. Our object in the present article is to explain concisely the results of this limitation, and to deduce certain formulas depending on it.

(16). *To state the Object of a Trigonometrical Survey of a Country.*

The object of the survey is, (1), to fix accurately the relative positions of certain chief points in the country, so as to lay them down on a map; and, (2), having fixed these chief points, then by means of subsidiary operations, to lay down in detail all the minor features of the country, its roads, rivers, towns, hamlets, &c. The accompanying figure will be sufficient to illustrate this matter for our present purpose, which is from an actual survey. A is a place called Ruckinge, B, High Nook, C, Allington, D, Lydd, E, Fairlight Down, and F, Tenterden. The line A B is measured very accurately, and is called the *base line*; and then the angles C A B, A B C, are measured; from these data, A C, and C B, can be calculated; then C B being known, the angles D C B, and C B D, can be measured, and thus C D, and D B, be determined. This operation is to be continued for any number of triangles whatever. It is usual, in the larger triangles, to measure all the three angles of any triangle, and not merely the two at the base; this is done with a view of keeping a check upon the various errors to which all observations are liable.

Fig. 8.

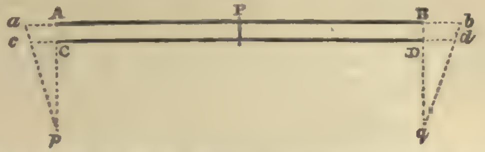


When the triangulating has been continued for some distance, it is necessary to compare the calculated length of a line that has been fixed upon, and then measure it; the coincidence of the two results is a verification of all previous measurements and calculations; hence such a line is called a *base of verification*. It is usual to choose stations that are from ten to twenty miles apart; also it is usual to choose for a *base line* a line of about four or five miles long. In late French surveys only two bases of verification have been used. The accuracy attainable in practice will be appreciated when the fact is stated that, in some English bases of verification, of four or five miles long, the computed and measured lengths have differed only by one or two inches. The operations of a trigonometrical survey are then two,—(1), the measurement of base lines; (2), the measurements of angles. We will proceed to consider each of these.

(17). *The Measurement of a Base Line.*

A space of open ground which is nearly level must be chosen, the line to be measured being indicated by stations, and stages erected, if necessary, to secure the horizontality of the base; the measure may be made by rods of glass, or steel, proper corrections being applied for temperature. A more convenient contrivance for securing accuracy in the measures has been devised of late for the Irish Survey: it is of the following kind:—A B, C D, are rods of platina and iron riveted together at the middle point P, and are exactly the same length for a given temperature: Ap, Bq, are marks affixed to the ends of each. Now the metals have different expansions for the same temperature; suppose then that A B, for a change in temperature, becomes ab, while C D becomes cd, then Ap will assume the position ap, and let $\mathrm{A}a = \delta a$ $\mathrm{C}c = \delta b$, then if Cp be taken so long, that $\frac{\mathrm{A}p}{\mathrm{C}p} = \frac{\delta a}{\delta b}$ then p will not be changed by the change of temperature; since, within very large limits, the expansion of a metal is proportional to the increase of temperature, and $\therefore$ $\frac{\delta a}{\delta b}$ is constant; the same arrangement being made at the end, B D, the distance pq, will not be affected by change of temperature.

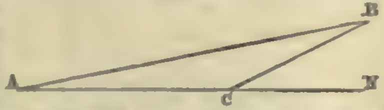
Fig. 9.



(18). *To Correct for want of Straightness in the Base Line.*

The nature of the ground may be such as to render it impossible to measure a perfectly straight base line of sufficient length. This was the case in some of the French surveys, where the actual measurement was of *two straight lines* inclined to each other at an angle of very nearly 180°.

Fig. 10.



Let A B be the line the length of which is required, the measurements are A C = b. C B = a. and B C N = θ in circular measure = n'' suppose. Then it is plain that A B $\angle$ $(a + b)$. Let us suppose A B = $a + b - x$; our object is to find x.

$$\mathrm{A\,B}^2 = a^2 + b^2 - 2\,ab \cos. (\pi - \theta) = a^2 + b^2 + 2\,ab \cos. \theta$$

Now (Plane Trig., Art. 50, cor.) $\cos. \theta = 1 - \frac{\theta^2}{2}$ · if we omit θ'' . . .

$$\therefore \mathrm{A\,B}^2 = a^2 + 2\,ab + b^2 - ab\theta^2 = (a+b)^2 \left\{1 - \frac{ab\theta^2}{(a+b)^2}\right\}$$

$$\therefore \mathrm{A\,B} = (a+b) \left\{1 - \frac{ab\theta^2}{(a+b)^2}\right\}^{\frac{1}{2}} = (a+b) \left\{1 - \frac{ab\theta^2}{2\,(a+b)^2}\right\} \text{ if we omit } \theta^4 \ldots$$

$$\therefore \mathrm{A\,B} = a + b - \frac{ab\theta^2}{2\,(a+b)}$$

$$\therefore x = \frac{ab\theta^2}{2\,(a+b)}$$

$$\text{Now} \quad \frac{\theta}{n} = \frac{\pi}{180 \times 60 \times 60} = \cdot000004848$$

$$\therefore x = \frac{ab}{a+b} \times \cdot0000000001175$$

The correction to be applied to the sum of a and b, to obtain the true distance, A B.

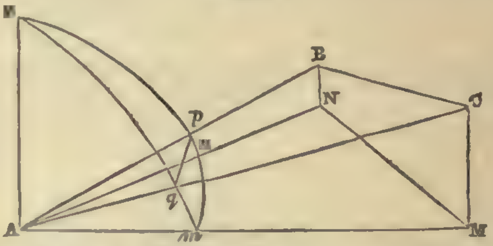
(19). *To measure the Angles of the Triangles of a Survey.*

Any two of the three angular points of one of these triangles are rarely in a horizontal plane passing through the third; the angle required is, of course, such a horizontal angle. Now the angles are measured either by a theodolite, or by a repeating circle—in the case of the former instrument, the vertical elevation of each object is observed, and the horizontal angle between them—that is to say, if A B C (Fig. 11) are the stations, A N M the horizontal plane through A, B N and C M perpendiculars from B and C on A M N, then by the theodolite we observe the angles B A N, C A M, and M A N; the last is the horizontal angle required. The theodolite is the instrument that has been used in the English surveys. But if a repeating circle be employed, the angle B A C is the one observed; and it is necessary to deduce from this the horizontal angle M A N. The repeating circle is the instrument used in the French surveys.

(20). *To determine the Correction for reducing an Angle to the Horizon.*

Let A B C M N be the same as in last article; with centre A and any radius describe a sphere, which meets the lines A B, A C, A N, A M in p, q, n, m, respectively. pn and mq, if joined by great circles, clearly meet in Z vertically over A, since the circles must be perpendicular to the horizontal plane. Then mn, or the angle mZn (Spherical Geometry, Prop. V., Cor. 4) is the angle required, and for its determination we have given pq = A, $pn = h$, $qm = h'$. Suppose $mZn = \mathrm{A} + \delta\mathrm{A}$, then our object is to determine δA.

Fig. 11.



Now in triangle pZq we have, by formula 3,

$$\cos. pZq = \frac{\cos. pq - \cos. pZ \cos. qZ}{\sin. pZ \cos. qZ}$$

Now $pZ = 90° - h$, and $qZ = 90° - h'$

$$\therefore \cos. (\mathrm{A} + \delta\mathrm{A}) = \frac{\cos. \mathrm{A} - \sin. h \sin. h}{\cos. h \cos. h'}$$

Now we will suppose δA so small that we can omit $\delta\mathrm{A}^2$ ... and h and h' so small that we can omit every power and product higher than h^2, hh', and h'^2.

$\therefore$ (Cor. Plane Trig., Art. 50) $\sin. \delta\mathrm{A} = \delta\mathrm{A}$, $\cos. \delta\mathrm{A} = 1$, $\sin. h = h$, $\sin. h' = h'$, $\cos. h = 1 - \frac{h^2}{2}$, and $\cos. h' = 1 - \frac{h'^2}{2}$

Now $\cos. (\mathrm{A} + \delta\mathrm{A}) = \cos. \mathrm{A} \cos. \delta\mathrm{A} - \sin. \mathrm{A} \sin. \delta\mathrm{A}$.

$$\text{Hence } \cos. \mathrm{A} - \delta\mathrm{A} \sin. \mathrm{A} = \frac{\cos. \mathrm{A} - hh'}{1 - \frac{1}{2}(h^2 + h'^2)} = \cos. \mathrm{A}$$

$$\cos. \mathrm{A} - hh' + \frac{h^2 + h'^2}{2} \cos. \mathrm{A}.$$

$$\therefore \delta\mathrm{A} \sin. \mathrm{A} = hh' - \frac{h^2 + h'^2}{2} \cos. \mathrm{A}$$

Now let $h + h' = p$, and $h - h' = q$. $\therefore 2h = p + q$, and $2h' = p - q$

$$\therefore 4hh' = p^2 - q^2, \text{ and } 2\,(h^2 + h'^2) = p^2 + q^2$$

$$\therefore 4\,\delta\mathrm{A} \sin. \mathrm{A} = p^2 - q^2 - (p^2 + q^2) \cos. \mathrm{A} = p^2\,(1 - \cos. \mathrm{A}) - q^2\,(1 + \cos. \mathrm{A})$$

$$\therefore 4\,\delta\mathrm{A}\ 2 \sin. \frac{\mathrm{A}}{2} \cos. \frac{\mathrm{A}}{2} = p^2 \times 2 \sin.^2 \frac{\mathrm{A}}{2} - q^2 \times 2 \cos.^2 \frac{\mathrm{A}}{2}$$

$$\therefore 4\,\delta\mathrm{A} = p^2 \frac{\sin. \frac{\mathrm{A}}{2}}{\cos. \frac{\mathrm{A}}{2}} - q^2 \frac{\cos. \frac{\mathrm{A}}{2}}{\sin. \frac{\mathrm{A}}{2}}$$

$$\therefore \delta A = \frac{1}{4}\left\{ p^2 \tan. \frac{A}{2} - q^2 \text{ cotan. } \frac{A}{2} \right\}$$

Hence, whether we use the theodolite, or the repeating circle, we obtain the same end; namely, the determination of the horizontal angle. It is plain that the horizontal angle being determined at each of the stations, these are the true angles, supposing the triangle spherical; and the triangle so determined is *spherical.*

We may proceed with the triangle thus obtained in either of the following ways:—

(21). *To Explain the Methods of Treating the Triangles thus obtained.*

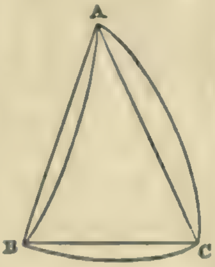
(a). *Delambre's Method.*

It is supposed that we have tables of the kind described in Art. 72, Plane Trigonometry. Now we know a, the length of a side in feet, hence $\frac{a}{r}$ the angle subtended at the centre by a is known, and hence log. $\frac{\sin. a}{a}$ is known; we will suppose the table to give us this for every value of a in feet. Then by simply adding log. a we obtain log sin. a. Now from formula (1). $\frac{\sin. b}{\sin. a} = \frac{\sin. B}{\sin. A}$; hence L. sin. b = L. sin. a + L. sin. B + ar : c : L. sin. A − 10, from whence b can be obtained.

(b). *By the Method of the Chordal Triangle.*

Let A B C be a given triangle, the sides of which are a, b, c, and angles A, B, C, join A B, B C, C D, by straight lines, then the plane triangle formed by these chords is called the chordal triangle. Let C − δC be the angle of the chordal triangle corresponding to the angle C of the spherical triangle. Now, by Plane Trigon., Art. 37,

Fig. 12.



2 (chord B C) (chord C A) cos. (C − δC) = (chord B C)² + (chord CA)² − (chord A B)²

$$\therefore 2 \sin. \frac{a}{2} \sin. \frac{b}{2} \cos. (C - \delta C)$$

$$= \sin.^2 \frac{a}{2} + \sin.^2 \frac{b}{2} - \sin.^2 \frac{c}{2}, \text{ since chord } \theta = 2 \sin. \frac{\theta}{2}$$

Again

$$\text{Sin. } a \sin. b \cos. C = \cos. c - \cos. a \cos. b =$$

$$1 - 2 \sin.^2 \frac{c}{2} - \left(1 - 2 \sin.^2 \frac{b}{2}\right) \left(1 - 2 \sin.^2 \frac{a}{2}\right)$$

$$\therefore 2 \text{ Sin.} \frac{a}{2} \sin. \frac{b}{2} \cos. \frac{a}{2} \cos. \frac{b}{2} \cos. C$$

$$= \sin.^2 \frac{a}{2} + \sin.^2 \frac{b}{2} - \sin.^2 \frac{c}{2} - 2 \sin.^2 \frac{a}{2} \sin.^2 \frac{b}{2}$$

$$= 2 \sin. \frac{a}{2} \sin. \frac{b}{2} \cos. (C - \delta C) - 2 \sin.^2 \frac{a}{2} \sin.^2 \frac{b}{2}$$

$$\therefore \text{Cos. } (C - \delta C) = \cos. \frac{a}{2} \cos. \frac{b}{2} \cos. C + \sin. \frac{a}{2} \sin. \frac{b}{2}$$

Now omitting $(\delta C)^2$ . . . we have cos. C + δ C sin. C

$$\therefore \delta C \sin. C = \sin. \frac{a}{2} \sin. \frac{b}{2} - \left(1 - \cos. \frac{a}{2} \cos. \frac{b}{2}\right) \cos. C.$$

Also $\sin. \frac{a}{2} = \frac{a}{2}$ and $\cos. \frac{a}{2} = 1 - \frac{a^2}{8}$, omitting a^3....

(Plane Trig., Art. 50, Cor.) ∴ δ C sin. C =

$$\frac{ab}{4} - \frac{a^3 + b^2}{8} \cos. C.$$

Let $a + b = e$, and $a - b = f$ ∴ 4 $ab = e^2 - f^2$, and 2 $(a^2 + b^2) = e^2 + f^2$ ∴ 16 δ C sin. C = $(e^2 - f^2) - (e^2 + f^2)$ cos. C = e^2 (1 − cos. C) − f^2 (1 + cos. C)

$$\therefore 32 \delta C \sin. \frac{C}{2} \cos. \frac{C}{2} = 2e^2 \sin.^2 \frac{C}{2} - 2f^2 \cos.^2 \frac{C}{2}$$

$$\therefore \delta C = \frac{1}{16}\left\{ e^2 \tan. \frac{C}{2} - f^2 \text{ cotan. } \frac{C}{2} \right\}$$

In this we suppose the radius of the sphere to be unity; and so if, for instance, e is in feet and equals m feet, we must have $e = \frac{m}{r}$ where r is the earth's radius in feet. If we wish for δC in seconds, let δC = n'' then $\frac{\delta C}{n} = \frac{\pi}{180 \times 60 \times 60}$ = 0·000004848, as we have already seen.

$$\therefore n = \frac{\delta C}{\cdot 000004848}$$

The above is the method used in the English surveys.

(c). *By the Method of Correcting the Angles, and Treating the Triangle as Plane.*

Let a, b, c, be the sides to radius r, then the circular measures of the sides are respectively $\frac{a}{r}$, $\frac{b}{r}$, $\frac{c}{r}$. Now

$$\text{Cos. } C = \frac{\cos. \frac{c}{r} - \cos. \frac{a}{r} \cos. \frac{b}{r}}{\sin. \frac{a}{r} \sin. \frac{b}{r}}$$

But if we suppose that we retain all terms up to the fourth order, *i.e.,* $\left(\frac{a^4}{r}\right) \left(\frac{b^4}{r}\right) \frac{a^2 b^2}{r^4}$ . . . . Then

$$\frac{1}{\text{Sin. } \frac{a}{r} \sin. \frac{b}{r}} = \frac{1}{\left(\frac{a}{r} - \frac{a^3}{6r^3}\right)\left(\frac{b}{r} - \frac{b^3}{6r^3}\right)}$$

$$= \frac{1}{\frac{ab}{r^2}\left(1 - \frac{a^2 + b^2}{6r^2}\right)} = \frac{1}{\frac{ab}{r^2}}\left(1 + \frac{a^2 + b^2}{6r^2}\right)$$

$$\text{and } \cos. \frac{c}{r} - \cos. \frac{a}{r} \cos. \frac{b}{r} = 1 - \frac{c^2}{2r^2} + \frac{c^4}{24r^4}$$

$$- \left(1 - \frac{a^2}{2r^2} + \frac{a^4}{24r^4}\right) \left(1 - \frac{b^2}{2r^2} + \frac{b^4}{24r^4}\right)$$

$$= 1 - \frac{c^2}{2r^2} + \frac{c^4}{24r^4} - \left\{1 - \frac{a^2 + b^2}{2r^2} + \frac{a^4 + b^4 + 6a^2 b^2}{24r^4}\right\}$$

$$= \frac{a^2 + b^2 - c^2}{2r^2} - \frac{a^4 + b^4 - c^4 + 6a^2 b^2}{24r^4}$$

$$\therefore \text{Cos.C} = \frac{1}{2\,ab}\left(1 + \frac{a^2 + b^2}{6r^2}\right)$$

$$\left(a^2 + b^2 - c^2 - \frac{a^4 + b^4 - c^4 + 6a^2 b^2}{12r^2}\right)$$

Taking in every term involving $\frac{1}{r^2}$

$$\therefore \text{Cos. } C = \frac{1}{2ab}\left\{ a^2 + b^2 - c^2 + \frac{(a^2 + b^2)(a^2 + b^2 - c^2)}{6r^2} - \frac{a^4 + b^4 - c^4 + 6a^2 b^2}{12r^2} \right\}$$

$$= \frac{a^2 + b^2 - c^2}{2ab} - \frac{1}{24abr^2}$$

$$\left(2a^2b^2 + 2b^2c^2 + 2c^2a^2 - c^4 - b^4 - a^4\right)$$

Now $2a^2b^2 + 2b^2c^2 + 2c^2a^2 - c^4 - b^4 - a^4 = 2b^2c^2 + 2c^2a^2 - c^4 - (b^2 - a^2)^2 = 4c^2b^2 - 2c^2 (b^2 - a^2) - c^4 - (b^2 - a^2)^2 = 4c^2b^2 - (c^2 + b^2 - a^2)^2 = (2cb + c^2 + b^2 - a^2)(2cb - c^2 - b^2 + a^2) =$

$$\left\{(b + c)^2 - a^2\right\} \left\{a^2 - (b - c)^2\right\}$$

$= (b + c + a)(b + c - a)(a + b - c)(a - b + c) = 16 \text{ A}^2$, if A be the area of a plane triangle whose sides are a, b, c. (Plane Trigon., Art. 41).

$$\therefore \text{Cos. } C = \frac{a^2 + b^2 - c^2}{2\,ab} - \frac{16}{24} \frac{\text{A}^2}{abr^2}$$

Now if δC be the correction to be applied to the triangle, in order that we may be enabled to reckon the triangle plane, so that $C - \delta C$ is the angle of a plane triangle whose sides are a, b, c, then

$$\text{Cos.}\,(C - \delta C) = \frac{a^2 + b^2 - c^2}{2ab} \text{ (Plane Trigon., Art. 37).}$$

$$\text{And} \qquad \text{Cos.}\,(C - \delta C) = \text{Cos. } C + \delta C.\ \text{sin. } C$$

$$\therefore \text{Cos. } C = \text{Cos. } C + \delta C,\ \text{sin. } C - \frac{2}{3}.\ \frac{A^2}{abr^2}$$

$$\therefore \delta C = \frac{2}{3}.\ \frac{A^2}{ab \text{ sin. } C.\ r^2} = \frac{1}{3}.\ \frac{A}{r^2}$$

Since the area of the triangle is $\frac{1}{2}\,ab \text{ sin. } C = A$.

Now if E be the spherical excess, *i.e.*, the excess of the sum of the three observed angles over two right angles, we have (Spherical Geometry, Prop. IX.)

$$A \ : \text{Area of hemisphere} : : E : 2\pi$$

E being in circular measure.

$$\therefore 2\pi A = E + 2\pi r^2$$

$$\therefore E = \frac{A}{r^2}$$

$$\therefore \delta C = \frac{1}{3} E.$$

The same clearly holds good of either of the other angles; hence the rule determines the spherical excess into three equal parts; subtract one part from each of the angles, and the triangle can then be considered plane. If δC be equal to n'' then, as before,

$$n'' = \frac{A}{3r^2 \times \cdot 000004848}.$$

It is to be observed that n is very small, *e.g.*, rarely more than 5 or 6; hence a small error in the area will produce no appreciable error. Hence A can be found on the supposition that the original triangle is plane.

[The remaining pages of this section will be occupied with tables giving the logarithmic sines, cosines, tangents, and cotangents for each minute in the quadrant. We add, however, a small table of natural sines and cosines, which will be useful in connection with other subjects that we shall subsequently have to enter on].

NATURAL SINES AND COSINES FOR EVERY DEGREE IN THE QUADRANT.

Sin.	Sin. 1°—30°	Cos.	Sin.	Sin. 31°—60°	Cos.	Sin.	Sin. 61°—90°	Cos.
1	0174524	89	31	5150381	59	61	8746197	29
2	0348995	88	32	5299193	58	62	8829476	28
3	0523360	87	33	5446390	57	63	8910065	27
4	0697565	86	34	5591929	56	64	8987940	26
5	0871557	85	35	5735764	55	65	9063078	25
6	1045285	84	36	5877853	54	66	9135455	24
7	1218693	83	37	6018150	53	67	9205049	23
8	1391731	82	38	6156615	52	68	9271839	22
9	1564345	81	39	6293204	51	69	9335804	21
10	1736482	80	40	6427876	50	70	9396926	20
11	1908090	79	41	6560590	49	71	9455186	19
12	2079117	78	42	6691306	48	72	9510565	18
13	2249511	77	43	6819984	47	73	9563048	17
14	2419219	76	44	6946584	46	74	9612617	16
15	2588190	75	45	7071068	45	75	9659258	15
16	2756374	74	46	7193398	44	76	9702957	14
17	2923717	73	47	7313537	43	77	9743701	13
18	3090170	72	48	7431448	42	78	9781476	12
19	3255682	71	49	7547096	41	79	9816272	11
20	3420201	70	50	7660444	40	80	9848078	10
21	3583679	69	51	7771460	39	81	9876883	9
22	3746066	68	52	7880108	38	82	9902681	8
23	3907311	67	53	7986355	37	83	9925462	7
24	4067366	66	54	8090170	36	84	9945219	6
25	4226183	65	55	8191520	35	85	9961947	5
26	4383711	64	56	8290376	34	86	9975641	4
27	4539905	63	57	8386706	33	87	9986295	3
28	4694716	62	58	8480481	32	88	9993908	2
29	4848096	61	59	8571673	31	89	9998477	1
30	5000000	60	60	8660254	30	90	1·0000000	0
Sin.	Cos. 60°—89°	Cos.	Sin.	Cos. 30°—59°	Cos.	Sin.	Cos. 0°—29°	Cos.

TABLE

OF

SINES, COSINES, TANGENTS, AND COTANGENTS;

From 0° to 90°.

The following Table gives the Logarithmic Sines, &c., for every Degree and Minute of a Quadrant, calculated to a Radius of 10.000.000, or

$$\text{Log. } \frac{\pi}{2} = 10.000.000.$$

Min.	0 Degrees.				Min.	Min.	0 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	0,0000000	10,0000000	0,0000000	Infinite.	60	30	7,9408419	9,9999835	7,9408584	12,0591416	30
1	6,4637261	9,9999999	6,4637261	13,5362739	59	31	7,9550819	9,9999823	7,9550996	12,0449004	29
2	6,7647561	9,9999999	6,7647562	13,2352438	58	32	7,9688698	9,9999812	7,9688886	12,0311114	28
3	6,9408475	9,9999998	6,9408475	13,0591525	57	33	7,9822334	9,9999800	7,9822534	12,0177406	27
4	7,0657860	9,9999997	7,0657863	12,9342137	56	34	7,9951980	9,9999788	7,9952192	12,0047808	26
5	7,1626960	9,9999995	7,1626964	12,8373036	55	35	8,0077867	9,9999775	8,0078092	11,9921908	25
6	7,2418771	9,9999993	7,2418778	12,7581222	54	36	8,0200207	9,9999762	8,0208445	11,9799555	24
7	7,3088239	9,9999991	7,3088248	12,6911752	53	37	8,0319195	9,9999748	8,0319446	11,9680554	23
8	7,3668157	9,9999988	7,3668169	12,6331831	52	38	8,0435009	9,9999735	8,0435274	11,9564726	22
9	7,4179681	9,9999985	7,4179696	12,5820304	51	39	8,0547814	9,9999721	8,0548094	11,9451906	21
10	7,4637255	9,9999982	7,4637273	12,5362727	50	40	8,0657763	9,9999706	8,0658057	11,9341943	20
11	7,5051181	9,9999978	7,5051203	12,4948797	49	41	8,0764997	9,9999691	8,0765306	11,9234694	19
12	7,5429065	9,9999974	7,5429091	12,4570909	48	42	8,0869646	9,9999676	8,0869970	11,9130030	18
13	7,5776684	9,9999969	7,5776715	12,4223285	47	43	8,0971832	9,9999660	8,0972172	11,9027828	17
14	7,6098530	9,9999964	7,6098566	12,3901434	46	44	8,1071669	9,9999644	8,1072025	11,8927975	16
15	7,6398160	9,9999959	7,6398201	12,3601799	45	45	8,1169262	9,9999628	8,1169634	11,8830366	15
16	7,6678445	9,9999953	7,6678492	12,3321508	44	46	8,1264710	9,9999611	8,1265099	11,8734901	14
17	7,6941733	9,9999947	7,6941786	12,3058214	43	47	8,1358104	9,9999594	8,1358510	11,8641490	13
18	7,7189966	9,9999940	7,7190026	12,2809974	42	48	8,1449532	9,9999577	8,1449956	11,8550044	12
19	7,7424775	9,9999934	7,7424841	12,2575159	41	49	8,1539075	9,9999559	8,1539516	11,8460484	11
20	7,7647537	9,9999927	7,7647610	12,2352390	40	50	8,1626808	9,9999541	8,1627267	11,8372733	10
21	7,7859427	9,9999919	7,7859508	12,2140492	39	51	8,1712804	9,9999522	8,1713282	11,8286718	9
22	7,8061458	9,9999911	7,8061547	12,1938453	38	52	8,1797129	9,9999503	8,1797626	11,8202374	8
23	7,8254507	9,9999903	7,8254604	12,1745396	37	53	8,1879848	9,9999484	8,1880364	11,8119636	7
24	7,8439338	9,9999894	7,8439444	12,1560556	36	54	8,1961020	9,9999464	8,1961556	11,8038444	6
25	7,8616623	9,9999885	7,8616738	12,1383262	35	55	8,2040703	9,9999444	8,2041259	11,7958741	5
26	7,8786953	9,9999876	7,8787077	12,1212923	34	56	8,2118949	9,9999424	8,2119526	11,7880474	4
27	7,8950854	9,9999866	7,8950988	12,1049012	33	57	8,2195811	9,9999403	8,2196408	11,7803592	3
28	7,9108793	9,9999856	7,9108938	12,0891062	32	58	8,2271335	9,9999382	8,2271953	11,7728047	2
29	7,9261190	9,9999845	7,9261344	12,0738656	31	59	8,2345568	9,9999360	8,2346208	11,7653792	1
30	7,9408419	9,9999835	7,9408584	12,0591416	30	60	8,2418553	9,9999338	8,2419215	11,7580785	0
Min.	Sine Com.	Sine.	Tan. Com.	Tang.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	89 Degrees.						89 Degrees.				

Min.	1 Degree.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	8,2418553	9,9999338	8,2419215	11,7580785	60
1	8,2490332	9,9999316	8,2491015	11,7508985	59
2	8,2560943	9,9999294	8,2561649	11,7438351	58
3	8,2630424	9,9999271	8,2631153	11,7368847	57
4	8,2698810	9,9999247	8,2699563	11,7300437	56
5	8,2766136	9,9999224	8,2766912	11,7233088	55
6	8,2832434	9,9999200	8,2833234	11,7166766	54
7	8,2897734	9,9999175	8,2898559	11,7101441	53
8	8,2962067	9,9999150	8,2962917	11,7037083	52
9	8,3025460	9,9999125	8,3026335	11,6973665	51
10	8,3087941	9,9999100	8,3088842	11,6911158	50
11	8,3149536	9,9999074	8,3150462	11,6849538	49
12	8,3210269	9,9999047	8,3211221	11,6788779	48
13	8,3270163	9,9999021	8,3271143	11,6728857	47
14	8,3329243	9,9998994	8,3330249	11,6669751	46
15	8,3387529	9,9998966	8,3388563	11,6611437	45
16	8,3445043	9,9998939	8,3446105	11,6553895	44
17	8,3501805	9,9998911	8,3502895	11,6497105	43
18	8,3557835	9,9998882	8,3558953	11,6441047	42
19	8,3613150	9,9998853	8,3614297	11,6385703	41
20	8,3667769	9,9998794	8,3668945	11,6331055	40
21	8,3721710	9,9998824	8,3722915	11,6277085	39
22	8,3774988	9,9998764	8,3776223	11,6223777	38
23	8,3827620	9,9998734	8,3828886	11,6171114	37
24	8,3879622	9,9998703	8,3880918	11,6119082	36
25	8,3931008	9,9998672	8,3932336	11,6067664	35
26	8,3981793	9,9998641	8,3983152	11,6016848	34
27	8,4031990	9,9998609	8,4033381	11,5966619	33
28	8,4081614	9,9998577	8,4083037	11,5916963	32
29	8,4130676	9,9998544	8,4132132	11,5867868	31
30	8,4179190	9,9998512	8,4180679	11,5819321	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	88 Degrees.				

Min.	1 Degree.				Min.
	Sine.	Sine. Com.	Tangent.	Tan. Comp.	
30	8,4179190	9,9998512	8,4180679	11,5819321	30
31	8,4227168	9,9998478	8,4228690	11,5771310	29
32	8,4274621	9,9998445	8,4276176	11,5723824	28
33	8,4321561	9,9998411	8,4323150	11,5676850	27
34	8,4367999	9,9998367	8,4369622	11,5630378	26
35	8,4413944	9,9998342	8,4415603	11,5584397	25
36	8,4459409	9,9998306	8,4461103	11,5538897	24
37	8,4504402	9,9998271	8,4506131	11,5493869	23
38	8,4548934	9,9998235	8,4550699	11,5449301	22
39	8,4593013	9,9998199	8,4594814	11,5405186	21
40	8,4636649	9,9998162	8,4638486	11,5361514	20
41	8,4679850	9,9998125	8,4681725	11,5318275	19
42	8,4722626	9,9998088	8,4724538	11,5275462	18
43	8,4764984	9,9998050	8,4766933	11,5233067	17
44	8,4806932	9,9998012	8,4808920	11,5191080	16
45	8,4848479	9,9997974	8,4850505	11,5149495	15
46	8,4889632	9,9997935	8,4891696	11,5108304	14
47	8,4930398	9,9997896	8,4932502	11,5067498	13
48	8,4970784	9,9997856	8,4972928	11,5027072	12
49	8,5010798	9,9997817	8,5012982	11,4987018	11
50	8,5050447	9,9997776	8,5052671	11,4947329	10
51	8,5089736	9,9997736	8,5092001	11,4907999	9
52	8,5128673	9,9997695	8,5130978	11,4869022	8
53	8,5167264	9,9997653	8,5169610	11,4830390	7
54	8,5205514	9,9997612	8,5207902	11,4792098	6
55	8,5243430	9,9997570	8,5245860	11,4754140	5
56	8,5281017	9,9997527	8,5283490	11,4716510	4
57	8,5318281	9,9997484	8,5320797	11,4679203	3
58	8,5355228	9,9997441	8,5357787	11,4642213	2
59	8,5391863	9,9997398	8,5394466	11,4605534	1
60	8,5428192	9,9997354	8,5430338	11,4569162	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	88 Degrees.				

Min.	2 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	8,5428192	9,9997354	8,5430838	11,4569162	60
1	8,5464218	9,9997309	8,5466909	11,4533091	59
2	8,5499948	9,9997265	8,5502683	11,4497317	58
3	8,5535386	9,9997220	8,5538166	11,4461834	57
4	8,5570536	9,9997174	8,5573362	11,4426638	56
5	8,5605404	9,9997128	8,5608276	11,4391724	55
6	8,5639994	9,9997082	8,5642912	11,4357088	54
7	8,5674310	9,9997036	8,5677275	11,4322725	53
8	8,5708357	9,9996989	8,5711368	11,4288632	52
9	8,5742139	9,9996942	8,5745197	11,4254803	51
10	8,5775660	9,9996894	8,5778766	11,4221234	50
11	8,5808923	9,9996846	8,5812077	11,4187923	49
12	8,5841933	9,9996798	8,5845136	11,4154864	48
13	8,5874694	9,9996749	8,5877945	11,4122055	47
14	8,5907209	9,9996700	8,5910509	11,4089491	46
15	8,5939483	9,9996650	8,5942832	11,4057168	45
16	8,5971517	9,9996601	8,5974917	11,4025083	44
17	8,6003317	9,9996550	8,6006767	11,3993233	43
18	8,6034886	9,9996500	8,6038386	11,3901614	42
19	8,6066226	9,9996449	8,6069777	11,3930223	41
20	8,6097341	9,9996398	8,6100943	11,3899057	40
21	8,6128233	9,9996346	8,6131889	11,3868111	39
22	8,6158910	9,9996294	8,6162616	11,3837384	38
23	8,6189369	9,9996242	8,6193127	11,3806873	37
24	8,6219616	9,9996189	8,6223427	11,3776573	36
25	8,6249653	9,9996136	8,6253518	11,3746482	35
26	8,6279484	9,9996082	8,6283402	11,3716598	34
27	8,6309111	9,9996028	8,6313083	11,3686917	33
28	8,6338537	9,9995974	8,6342563	11,3657437	32
29	8,6367764	9,9995919	8,6371845	11,3628155	31
30	8,6396796	9,9995865	8,6400931	11,3599069	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	87 Degrees.				

Min.	2 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	8,6396796	9,9995865	8,6400934	11,3599069	30
31	8,6425634	9,9995809	8,6429825	11,3570175	29
32	8,6454282	9,9995753	8,6458528	11,3541472	28
33	8,6482742	9,9995697	8,6487044	11,3512956	27
34	8,6511016	9,9995641	8,6515375	11,3484625	26
35	8,6539107	9,9995584	8,6543522	11,3456478	25
36	8,6567017	9,9995527	8,6571490	11,3428510	24
37	8,6594748	9,9995469	8,6599279	11,3400721	23
38	8,6622303	9,9995411	8,6626891	11,3373109	22
39	8,6649684	9,9995353	8,6654331	11,3345669	21
40	8,6676893	9,9995295	8,6681598	11,3318402	20
41	8,6703932	9,9995236	8,6708697	11,3291303	19
42	8,6730804	9,9995176	8,6735628	11,3264372	18
43	8,6757510	9,9995116	8,6762393	11,3237607	17
44	8,6784052	9,9995056	8,6788996	11,3211004	16
45	8,6810433	9,9994996	8,6815437	11,3184563	15
46	8,6836654	9,9994935	8,6841719	11,3158281	14
47	8,6862718	9,9994874	8,6867844	11,3132156	13
48	8,6888625	9,9994812	8,6893813	11,3106187	12
49	8,6914379	9,9994750	8,6919629	11,3080371	11
50	8,6939980	9,9994688	8,6945292	11,3054708	10
51	8,6965431	9,9994625	8,6970806	11,3029194	9
52	8,6990734	9,9994562	8,6996172	11,3003828	8
53	8,7015889	9,9994498	8,7021390	11,2978610	7
54	8,7040899	9,9994435	8,7046465	11,2953535	6
55	8,7065766	9,9994370	8,7071395	11,2928605	5
56	8,7090490	9,9994306	8,7096185	11,2903815	4
57	8,7115075	9,9994241	8,7120834	11,2879166	3
58	8,7139520	9,9994176	8,7145345	11,2854655	2
59	8,7163829	9,9994110	8,7169719	11,2830281	1
60	8,7188002	9,9994044	8,7193958	11,2806042	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	87 Degrees.				

Min.	3 Degrees.				Min.	Min.	3 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	8,7188002	9,9994044	8,7193958	11,2806042	60	30	8,7856753	9,9991892	8,7864861	11,2135139	30
1	8,7212040	9,9993978	8,7218063	11,2781937	59	31	8,7877359	9,9991815	8,7885544	11,2114456	29
2	8,7235946	9,9993911	8,7242035	11,2757965	58	32	8,7897867	9,9991737	8,7906130	11,2093870	28
3	8,7259721	9,9993844	8,7265877	11,2734123	57	33	8,7918278	9,9991659	8,7926620	11,2073380	27
4	8,7283366	9,9993776	8,7289589	11,2710411	56	34	8,7938594	9,9991580	8,7947014	11,2052986	26
5	8,7306882	9,9993708	8,7313174	11,2686826	55	35	8,7958814	9,9991501	8,7967313	11,2032687	25
6	8,7330272	9,9993640	8,7336631	11,2663369	54	36	8,7978941	9,9991422	8,7987519	11,2012481	24
7	8,7353535	9,9993572	8,7359964	11,2640036	53	37	8,7998974	9,9991342	8,8007632	11,1992368	23
8	8,7376675	9,9993503	8,7383172	11,2616828	52	38	8,8018915	9,9991262	8,8927653	11,1972347	22
9	8,7399691	9,9993433	8,7406258	11,2593742	51	39	8,8038764	9,9991182	8,8047583	11,1952417	21
10	8,7422586	9,9993364	8,7429222	11,2570778	50	40	8,8058523	9,9991101	8,8067422	11,1932578	20
11	8,7445360	9,9993293	8,7452067	11,2547933	49	41	8,8078192	9,9991020	8,8087172	11,1912828	19
12	8,7468015	9,9993223	8,7474792	11,2525208	48	42	8,8097772	9,9990938	8,8106834	11,1893166	18
13	8,7490553	9,9993152	8,7497400	11,2502600	47	43	8,8117264	9,9990856	8,8126407	11,1873593	17
14	8,7512973	9,9993081	8,7519892	11,2480108	46	44	8,8136668	9,9990774	8,8145894	11,1854106	16
15	8,7535278	9,9993009	8,7542269	11,2457731	45	45	8,8155985	9,9990691	8,8165294	11,1834706	15
16	8,7557469	9,9992938	8,7564531	11,2435469	44	46	8,8175217	9,9990608	8,8184608	11,1815392	14
17	8,7579546	9,9992865	8,7586681	11,2413319	43	47	8,8194363	9,9990525	8,8203838	11,1796162	13
18	8,7601512	9,9992793	8,7608719	11,2391281	42	48	8,8213425	9,9990441	8,8222984	11,1777016	12
19	8,7623366	9,9992720	8,7630647	11,2369353	41	49	8,8232404	9,9990357	8,8242046	11,1757954	11
20	8,7645111	9,9992646	8,7652465	11,2347535	40	50	8,8251299	9,9990273	8,8261026	11,1738974	10
21	8,7666747	9,9992572	8,7674175	11,2325825	39	51	8,8270112	9,9990188	8,8279924	11,1720076	9
22	8,7688275	9,9992498	8,7695777	11,2304223	38	52	8,8288844	9,9990103	8,8298741	11,1701259	8
23	8,7709697	9,9992424	8,7717274	11,2282726	37	53	8,8307495	9,9990017	8,8317478	11,1682522	7
24	8,7731014	9,9992349	8,7738665	11,2261335	36	54	8,8326066	9,9989931	8,8336134	11,1663866	6
25	8,7752226	9,9992274	8,7759952	11,2240048	35	55	8,8344557	9,9989845	8,8354712	11,1645288	5
26	8,7773334	9,9992198	8,7781136	11,2218864	34	56	8,8362969	9,9989758	8,8373211	11,1626789	4
27	8,7794340	9,9992122	8,7802218	11,2197782	33	57	8,8381304	9,9989671	8,8391633	11,1608367	3
28	8,7815244	9,9992046	8,7823199	11,2176801	32	58	8,8399561	9,9989584	8,8409977	11,1590023	2
29	8,7836048	9,9991969	8,7844079	11,2155921	31	59	8,8417741	9,9989496	8,8428245	11,1571755	1
30	8,7856753	9,9991892	8,7864861	11,2135139	30	60	8,8435845	9,9989408	8,8446437	11,1553563	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	86 Degrees.						86 Degrees.				

Min.	4 Degrees.				Min.	Min.	4 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	8,8435845	9,9989408	8,8446437	11,1553563	60	30	8,8946433	9,9986591	8,8959842	11,1040158	30
1	8,8453874	9,9989319	8,8464554	11,1535446	59	31	8,8962455	9,9986492	8,8975963	11,1024037	29
2	8,8471827	9,9989230	8,8482597	11,1517403	58	32	8,8978418	9,9986392	8,8992026	11,1007974	28
3	8,8489707	9,9989141	8,8500566	11,1499434	57	33	8,8994322	9,9986292	8,9008030	11,0991970	27
4	8,8507512	9,9989052	8,8518461	11,1481539	56	34	8,9010168	9,9986191	8,9023977	11,0970023	26
5	8,8525245	9,9988962	8,8536283	11,1463717	55	35	8,9025955	9,9986090	8,9039866	11,0960134	25
6	8,8542905	9,9988871	8,8554034	11,1445966	54	36	8,9041685	9,9985988	8,9055697	11,0944303	24
7	8,8560493	9,9988780	8,8571713	11,1428287	53	37	8,9057358	9,9985886	8,9071472	11,0928528	23
8	8,8578010	9,9988689	8,8589321	11,1410679	52	38	8,9072975	9,9985784	8,9087190	11,0912810	22
9	8,8595457	9,9988598	8,8606859	11,1393141	51	39	8,9088535	9,9985682	8,9102853	11,0897147	21
10	8,8612833	9,9988506	8,8624327	11,1375673	50	40	8,9104039	9,9985579	8,9118460	11,0881540	20
11	8,8630139	9,9988414	8,8641725	11,1358275	49	41	8,9119487	9,9985475	8,9134012	11,0865988	19
12	8,8647376	9,9988321	8,8659055	11,1340945	48	42	8,9134881	9,9985372	8,9149509	11,0850491	18
13	8,8664545	9,9988228	8,8676317	11,1323683	47	43	8,9150219	9,9985268	8,9164952	11,0835048	17
14	8,8681646	9,9988135	8,8693511	11,1306489	46	44	8,9165504	9,9985163	8,9180340	11,0819660	16
15	8,8698630	9,9988041	8,8710638	11,1289362	45	45	8,9180734	9,9985058	8,9195675	11,0894325	15
16	8,8715646	9,9987947	8,8727699	11,1272301	44	46	8,9195911	9,9984953	8,9210957	11,0789043	14
17	8,8732546	9,9987853	8,8744694	11,1255306	43	47	8,9211034	9,9984848	8,9226186	11,0773814	13
18	8,8749381	9,9987758	8,8761623	11,1238377	42	48	8,9226105	9,9984742	8,9241363	11,0758637	12
19	8,8766150	9,9987663	8,8778487	11,1221513	41	49	8,9241123	9,9984636	8,9256487	11,0743513	11
20	8,8782854	9,9987567	8,8795286	11,1204714	40	50	8,9256089	9,9984529	8,9271560	11,0728440	10
21	8,8799493	9,9987471	8,8812022	11,1187978	39	51	8,9271003	9,9984422	8,9286581	11,0713419	9
22	8,8816069	9,9987375	8,8828694	11,1171306	38	52	8,9285866	9,9984315	8,9301552	11,0698448	8
23	8,8832581	9,9987278	8,8845303	11,1154697	37	53	8,9300678	9,9984207	8,9316471	11,0683529	7
24	8,8849031	9,9987181	8,8861850	11,1138150	36	54	8,9315439	9,9984099	8,9331340	11,0668660	6
25	8,8865418	9,9987084	8,8878334	11,1121666	35	55	8,9330150	9,9983990	8,9346160	11,0653840	5
26	8,8881743	9,9986986	8,8894757	11,1105243	34	56	8,9344811	9,9983881	8,9360929	11,0639071	4
27	8,8898007	9,9986888	8,8911119	11,1088881	33	57	8,9359422	9,9983772	8,9375650	11,0624350	3
28	8,8914209	9,9986790	8,8927420	11,1072580	32	58	8,9373983	9,9983663	8,9390321	11,0609679	2
29	8,8930351	9,9986691	8,8943660	11,1056340	31	59	8,9388496	9,9983553	8,9404944	11,0595056	1
30	8,8946433	9,9986591	8,8959842	11,1040158	30	60	8,9402960	9,9983442	8,9419518	11,0580482	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	85 Degrees.						85 Degrees.				

Min.	5 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	8,9402960	9,9983442	8,9419518	11,0580482	60
1	8,9417376	9,9983332	8,9434044	11,0565956	59
2	8,9431743	9,9983220	8,9448523	11,0551477	58
3	8,9446063	9,9983109	8,9462954	11,0537046	57
4	8,9460335	9,9982997	8,9477338	11,0522662	56
5	8,9474561	9,9982885	8,9491676	11,0508324	55
6	8,9488739	9,9982772	8,9505967	11,0494033	54
7	8,9502871	9,9982660	8,9520211	11,0479789	53
8	8,9516957	9,9982546	8,9534410	11,0465590	52
9	8,9530996	9,9982433	8,9548564	11,0451436	51
10	8,9544991	9,9982318	8,9562672	11,0437328	50
11	8,9558940	9,9982204	8,9576735	11,0423265	49
12	8,9572843	9,9982089	8,9590754	11,0409246	48
13	8,9586703	9,9981974	8,9604728	11,0395272	47
14	8,9600517	9,9981859	8,9618659	11,0381341	46
15	8,9614288	9,9981743	8,9632545	11,0367455	45
16	8,9628014	9,9981629	8,9646388	11,0353612	44
17	8,9641697	9,9981510	8,9660188	11,0339812	43
18	8,9655337	9,9981393	8,9673944	11,0326056	42
19	8,9668934	9,9981275	8,9687658	11,0312342	41
20	8,9682487	9,9981158	8,9701330	11,0298670	40
21	8,9695999	9,9981040	8,9714959	11,0285041	39
22	8,9709468	9,9980921	8,9728547	11,0271453	38
23	8,9722895	9,9980802	8,9742092	11.0257908	37
24	8,9736280	9,9980683	8,9755597	11,0244403	36
25	8,9749624	9,9980563	8,9769060	11,0230940	35
26	8,9762926	9,9980443	8,9782483	11,0217517	34
27	8,9776188	9,9980323	8,9795865	11,0204135	33
28	8,9789498	9,9980202	8,9809206	11,0190794	32
29	8,9802589	9,9980081	8,9822507	11,0177493	31
30	8,9815729	9,9979960	8,9835769	11,0164231	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	84 Degrees.				

Min.	5 Degrees.				Min.
	Sine.	Sine. Com.	Tangent.	Tan. Comp.	
30	8,9815729	9,9979960	8,9835769	11,0164231	30
31	8,9828829	9,9979838	8,9848991	11,0151009	29
32	8,9841889	9,9979716	8,9862173	11,0137827	28
33	8,9854910	9,9979593	8,9875317	11,0124683	27
34	8,9867891	9,9979470	8,9888421	11,0111579	26
35	8,9880834	9,9979347	8,9901487	11,0098513	25
36	8,9893737	9,9979223	8,9914514	11,0085486	24
37	8,9906602	9,9979099	8,9927503	11,0072497	23
38	8,9919429	9,9978975	8,9940454	11,0059546	22
39	8,9932217	9,9978850	8,9953367	11,0046633	21
40	8,9944968	9,9978725	8,9966243	11,0033757	20
41	8,9957681	9,9978599	8,9979081	11,0020919	19
42	8,9970356	9,9978473	8,9991883	11,0008117	18
43	8,9982994	9,9978347	9,0004647	10,9995353	17
44	8,9995595	9,9978220	9,0017375	10,9982625	16
45	9,0008160	9,9978093	9,0030066	10,9969934	15
46	9,0020687	9,9977966	9,0042721	10,9957279	14
47	9,0033179	9,9977838	9,0055340	10,9944660	13
48	9,0045634	9,9977710	9,0067924	10,9932076	12
49	9,0058053	9,9977582	9,0080471	10,9919529	11
50	9,0070436	9,9977453	9,0092984	10,9907016	10
51	9,0082784	9,9977323	9,0105461	10,9894539	9
52	9,0095096	9,9977194	9,0117903	10,9882097	8
53	9,0107374	9,9977064	9,0130310	10,9869690	7
54	9,0119616	9,9976933	9,0142682	10,9857318	6
55	9,0131823	9,9976803	9,0155021	10,9844979	5
56	9,0143996	9,9976672	9,0167325	10,9832675	4
57	9,0156135	9,9976540	9,0179594	10,9820406	3
58	9,0168239	9,9976408	9,0191831	10,9808169	2
59	9,0180309	9,9976276	9,0204033	10,9795967	1
60	9,0192346	9,9976143	9,0216202	10,9783798	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	84 Degrees.				

Min.	6 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,0192346	9,9976143	9,0216202	10,9783798	60
1	9,0204348	9,9976011	9,0228338	10,9771662	59
2	9,0216318	9,9975877	9,0240441	10.9759559	58
3	9,0228254	9,9975743	9,0252510	10,9747490	57
4	9,0240157	9,9975609	9,0264548	10,9735432	56
5	9,0252027	9,9975475	9,0276552	10,9723448	55
6	9,0263865	9,9975340	9,0288524	10,9711476	54
7	9,0275669	9,9975205	9,0300464	10,9699536	53
8	9,0287442	9,9975069	9,0312373	10,9687627	52
9	9,0299182	9,9974933	9,0324249	10,9675751	51
10	9,0310890	9,9974797	9,0336093	10,9663907	50
11	9,0322567	9,9974660	9,0347906	10,9652094	49
12	9,0334212	9,9974523	9,0359688	10,9640312	48
13	9,0345825	9,9974386	9,0371439	10,9628561	47
14	9,0357407	9,9974248	9,0383159	10,9616841	46
15	9,0368958	9,9974110	9,0394848	10,9605152	45
16	9,0380477	9,9973971	9,0406506	10,9593494	44
17	9,0391966	9,9973833	9,0418134	10,9581866	43
18	9,0403424	9,9973693	9,0429731	10,9570269	42
19	9,0414852	9,9973554	9,0441299	10,9558701	41
20	9,0426249	9,9973414	9,0452836	10,9547164	40
21	9,0437617	9,9973273	9,0464343	10,9535657	39
22	9,0448954	9,9973132	9,0475821	10,9524179	38
23	9,0460261	9,9972991	9,0487270	10,9512730	37
24	9,0471538	9,9972850	9,0498689	10,9501311	36
25	9,0482786	9,9972708	9,0510078	10,9489922	35
26	9,0494005	9,9972566	9,0521439	10,9478561	34
27	9,0505194	9,9972423	9,0532771	10,9467229	33
28	9,0516354	9,9972280	9,0544074	10,9455926	32
29	9,0527485	9,9972137	9,0555349	10,9444651	31
30	9,0538588	9,9971993	9,0566595	10,9433405	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	83 Degrees.				

Min.	6 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,0538588	9,9971993	9,0566595	10,9433405	30
31	9,0549661	9,9971849	9,0577813	10,9422187	29
32	9,0560706	9,9971704	9,0589002	10,9410998	28
33	9,0571723	9,9971559	9,0600164	10,9399836	27
34	9,0582711	9,9971414	9,0611297	10,9388703	26
35	9,0593672	9,9971268	9,0622403	10,9377597	25
36	9,0604604	9,9971122	9,0633482	10,9366518	24
37	9,0615509	9,9970976	9,0644533	10,9355467	23
38	9,0626386	9,9970829	9,0655556	10,9344444	22
39	9,0637235	9,9970682	9,0666553	10,9333447	21
40	9,0648057	9,9970535	9,0677522	10,9322478	20
41	9,0658852	9,9970387	9,0688465	10,9311535	19
42	9,0669619	9,9970239	9,0699381	10,9300619	18
43	9,0680360	9,9970090	9,0710270	10,9289730	17
44	9,0691074	9,9969941	9,0721133	10,9278867	16
45	9,0701761	9,9969792	9,0731969	10,9268031	15
46	9,0712421	9,9969642	9,0742779	10,9257221	14
47	9,0723055	9,9969492	9,0753563	10,9246437	13
48	9,0733663	9,9969342	9,0764321	10,9235679	12
49	9,0744244	9,9969191	9,0775053	10,9224947	11
50	9,0754799	9,9969040	9,0785760	10,9214240	10
51	9,0765329	9,9968838	9,0796441	10,9203559	9
52	9,0775832	9,9968736	9,0807096	10,9192904	8
53	9,0786310	9,9968584	9,0817726	10,9182274	7
54	9,0796762	9,9968431	9,0828331	10,9171669	6
55	9,0807189	9,9968278	9,0838911	10,9161089	5
56	9,0817590	9,9968125	9,0849466	10,9150534	4
57	9,0827966	9,9967971	9,0859996	10,9140004	3
58	9,0838317	9,9967817	9,0870501	10,9129499	2
59	9,0848643	9,9967662	9,0880981	10,9119019	1
60	9,0858945	9,9967507	9,0891438	10,9108562	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	83 Degrees.				

	7 Degrees.						7 Degrees.				
Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	Min.	Min.	Sine.	Sine. Com.	Tangent.	Tan. Comp.	Min.
0	9,0858945	9,9967507	9,0891438	10,9108562	60	30	9,1156977	9,9962686	9,1194291	10,8805709	30
1	9,0869221	9,9967352	9,0901869	10,9098131	59	31	9,1166562	9,9962519	9,1204043	10,8795957	29
2	9,0879473	9,9967196	9,0912277	10,9087723	58	32	9,1176125	9,9962352	9,1213773	10,8786227	28
3	9,0889700	9,9967040	9,0922660	10,9077340	57	33	9,1185667	9,9962185	9,1223482	10,8776518	27
4	9,0899903	9,9966884	9,0933020	10,9066980	56	34	9,1195188	9,9962017	9,1233171	10,8766829	26
5	9,0910082	9,9966727	9,0943355	10,9056645	55	35	9,1204688	9,9961849	9,1242839	10,8757161	25
6	9,0920237	9,9966570	9,0953667	10,9046333	54	36	9,1214167	9,9961681	9,1252486	10,8747514	24
7	9,0930367	9,9966412	9,0963955	10,9036045	53	37	9,1223624	9,9961512	9,1262112	10,8737888	23
8	9,0940474	9,9966254	9,0974219	10,9025781	52	38	9,1233061	9,9961343	9,1271718	10,8728282	22
9	9,0950556	9,9966096	9,0984460	10,9015540	51	39	9,1242477	9,9961174	9,1281303	10,8718697	21
10	9,0960615	9,9965937	9,0994678	10,9005322	50	40	9,1251872	9,9961004	9,1290868	10,8709132	20
11	9,0970651	9,9965778	9,1004872	10,8995128	49	41	9,1261246	9,9960834	9,1300413	10,8699587	19
12	9,0980662	9,9965619	9,1015044	10,8984956	48	42	9,1270600	9,9960663	9,1309937	10,8690063	18
13	9,0990651	9,9965459	9,1025192	10,8974808	47	43	9,1279934	9,9960492	9,1319442	10,8680558	17
14	9,1000616	9,9965299	9,1035317	10,8964683	46	44	9,1289247	9,9960321	9,1328926	10,8671074	16
15	9,1010558	9,9965138	9,1045420	10,8954580	45	45	9,1298539	9,9960149	9,1338391	10,8661609	15
16	9,1020477	9,9964977	9,1055500	10,8944500	44	46	9,1307812	9,9959977	9,1347835	10,8652165	14
17	9,1030373	9,9964816	9,1065557	10,8934443	43	47	9,1317064	9,9959804	9,1357260	10,8642740	13
18	9,1040246	9,9964655	9,1075591	10,8924409	42	48	9,1326297	9,9959631	9,1366665	10,8633335	12
19	9,1050096	9,9964493	9,1085004	10,8914396	41	49	9,1335509	9,9959458	9,1376051	10,8623949	11
20	9,1059924	9,9964330	9,1095594	10,8904406	40	50	9,1344702	9,9959284	9,1385417	10,8614583	10
21	9,1069729	9,9964167	9,1105562	10,8894438	39	51	9,1353875	9,9959111	9,1394764	10,8605236	9
22	9,1079512	9,9964004	9,1115508	10,8884492	38	52	9,1363028	9,9958936	9,1404092	10,8595908	8
23	9,1089272	9,9963841	9,1125431	10,8874569	37	53	9,1372161	9,9958761	9,1413400	10,8586600	7
24	9,1099010	9,9963677	9,1135333	10,8864667	36	54	9,1381275	9,9958586	9,1422689	10,8577311	6
25	9,1108726	9,9963513	9,1145213	10,8854787	35	55	9,1390370	9,9958411	9,1431959	10,8568041	5
26	9,1118420	9,9963348	9,1155072	10,8844928	34	56	9,1399445	9,9958235	9,1441210	10,8558790	4
27	9,1128092	9,9963183	9,1164909	10,8835091	33	57	9,1408501	9,9958059	9,1450442	10,8549558	3
28	9,1137742	9,9963018	9,1174724	10,8825276	32	58	9,1417537	9,9957882	9,1459655	10,8540345	2
29	9,1147370	9,9962852	9,1184518	10,8815482	31	59	9,1426555	9,9957705	9,1468849	10,8531151	1
30	9,1156977	9,9962686	9,1194291	10,8805709	30	60	9,1435553	9,9957528	9,1478025	10,8521975	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	82 Degrees.						82 Degrees.				

	8 Degrees.						8 Degrees.				
Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	Min.	Min.	Sine.	Sine Com.	Tangent.	Tan. Comp.	Min.
0	9,1435553	9,9957528	9,1478025	10,8521975	60	30	9,1697021	9,9952033	9,1744988	10,8255012	30
1	9,1444532	9,9957350	9,1487182	10,8512818	59	31	9,1705465	9,9951844	9,1753622	10,8246378	29
2	9,1453493	9,9957172	9,1496321	10,8503679	58	32	9,1713893	9,9951654	9,1762239	10,8237761	28
3	9,1462435	9,9956993	9,1505441	10,8494559	57	33	9,1722305	9,9951464	9,1770840	10,8229160	27
4	9,1471358	9,9996815	9,1514543	10,8485457	56	34	9,1730699	9,9951274	9,1779425	10,8220575	26
5	9,1480262	9,9956635	9,1523627	10,8476373	55	35	9,1739077	9,9951084	9,1787993	10,8212007	25
6	9,1489148	9,9956456	9,1532692	10,8467308	54	36	9,1747439	9,9950893	9,1796546	10,8203454	24
7	9,1498015	9,9956276	9,1541739	10,8458261	53	37	9,1755784	9,9950702	9,1805082	10,8194918	23
8	9,1506864	9,9996095	9,1550769	10,8449231	52	38	9,1764112	9,9950510	9,1813602	10,8186398	22
9	9,1515694	9,9955915	9,1559780	10,8440220	51	39	9,1772425	9,9950318	9,1822106	10,8177894	21
10	9,1524507	9,9955734	9,1568773	10,8431227	50	40	9,1780721	9,9950126	9,1830595	10,8169405	20
11	9,1533301	9,9955552	9,1577748	10,8422252	49	41	9,1789001	9,9949933	9,1839068	10,8160932	19
12	9,1542076	9,9955370	9,1586706	10,8413294	48	42	9,1797265	9,9949740	9,1847525	10,8152475	18
13	9,1550834	9,9955188	9,1595646	10,8404354	47	43	9,1805512	9,9949546	9,1855966	10,8144034	17
14	9,1559574	9,9955505	9,1604569	10,8395431	46	44	9,1813744	9,9949352	9,1864392	10,8135608	16
15	9,1568296	9,9954822	9,1613473	10,8386527	45	45	9,1821960	9,9949158	9,1872802	10,8127198	15
16	9,1577000	9,9954639	9,1622361	10,8377639	44	46	9,1830160	9,9948964	9,1881196	10,8118804	14
17	9,1585686	9,9954455	9,1631231	10,8368769	43	47	9,1838344	9,9948769	9,1889575	10,8110425	13
18	9,1594354	9,9954271	9,1640083	10,8359917	42	48	9,1848512	9,9948573	9,1897939	10,8102061	12
19	9,1603005	9,9954087	9,1648919	10,8351081	41	49	9,1854665	9,9948377	9,1906287	10,8093713	11
20	9,1611639	9,9953902	9,1657737	10,8342263	40	50	9,1862802	9,9948181	9,1914621	10,8085379	10
21	9,1620254	9,9953717	9,1666538	10,8333462	39	51	9,1870923	9,9947985	9,1922939	10,8077061	9
22	9,1628853	9,9953531	9,1675322	10,8324678	38	52	9,1879029	9,9947788	9,1931241	10,8068759	8
23	9,1637434	9,9953345	9,1684089	10,8315911	37	53	9,1887120	9,9947591	9,1939529	10,8060471	7
24	9,1645998	9,9953159	9,1692839	10,8307161	36	54	9,1895195	9,9947393	9,1947802	10,8052198	6
25	9,1654544	9,9952972	9,1701572	10,8298428	35	55	9,1903254	9,9947195	9,1956059	10,8043941	5
26	9,1663074	9,9952785	9,1710289	10,8289711	34	56	9,1911299	9,9946997	9,1964302	10,8035698	4
27	9,1671586	9,9952597	9,1718989	10,8281011	33	57	9,1919328	9,9946798	9,1972530	10,8027470	3
28	9,1680081	9,9952409	9,1727672	10,8272328	32	58	9,1927342	9,9946599	9,1980743	10,8019257	2
29	9,1688559	9,9952221	9,1736338	10,8263662	31	59	9,1935341	9,9946399	9,1988941	10,8011059	1
30	9,1697021	9,9952033	9,1744988	10,8255012	30	60	9,1943324	9,9946199	9,1997125	10,8002875	0
Min.	Sine Com.	Sine	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	81 Degrees.						81 Degrees.				

Min.	9 Degrees.				Min.	Min.	9 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,1943324	9,9946199	9,1997125	10,8002875	60	30	9,2176092	9,9940027	9,2236065	10,7763935	30
1	9,1951293	9,9945999	9,2005294	10,7994706	59	31	9,2183635	9,9939815	9,2243819	10,7756181	29
2	9,1959247	9,9945798	9,2013449	10,7986551	58	32	9,2191164	9,9939603	9,2251561	10,7748439	28
3	9,1967186	9,9945597	9,2021588	10,7978412	57	33	9,2198680	9,9939391	9,2259289	10,7740711	27
4	9,1975110	9,9945396	9,2029714	10,7970286	56	34	9,2206182	9,9939178	9,2267004	10,7732996	26
5	9,1983019	9,9945194	9,2037825	10,7962175	55	35	9,2213671	9,9938965	9,2274706	10,7725294	25
6	9,1990913	9,9944992	9,2045922	10,7954078	54	36	9,2221147	9,9938752	9,2282395	10,7717605	24
7	9,1998793	9,9944789	9,2054004	10,7945996	53	37	9,2228609	9,9938538	9,2290071	10,7709929	23
8	9,2006658	9,9944587	9,2062072	10,7937928	52	38	9,2236059	9,9938324	9,2297735	10,7702265	22
9	9,2014509	9,9944383	9,2070126	10,7929874	51	39	9,2243495	9,9938109	9,2305386	10,7694614	21
10	9,2022345	9,9944180	9,2078165	10,7921835	50	40	9,2250918	9,9937894	9,2313024	10,7686976	20
11	9,2030167	9,9943975	9,2086191	10,7918809	49	41	9,2258328	9,9937679	9,2320650	10,7679350	19
12	9,2037974	9,9943771	9,2094203	10,7905797	48	42	9,2265725	9,9937463	9,2328262	10,7671738	18
13	9,2045766	9,9943566	9,2102200	10,7897800	47	43	9,2273110	9,9937247	9,2335863	10,7664137	17
14	9,2053545	9,9943361	9,2110184	10,7889816	46	44	9,2280481	9,9937030	9,2343451	10,7656549	16
15	9,2061309	9,9943156	9,2118153	10,7881847	45	45	9,2287839	9,9936813	9,2351026	10,7648974	15
16	9,2069059	9,9942950	9,2126109	10,7873891	44	46	9,2295185	9,9936596	9,2358589	10,7641411	14
17	9,2076795	9,9942743	9,2134051	10,7865949	43	47	9,2302518	9,9936378	9,2366139	10,7633861	13
18	9,2084516	9,9942537	9,2141980	10,7858020	42	48	9,2309838	9,9936160	9,2373678	10,7626322	12
19	9,2092224	9,9942330	9,2149894	10,7850106	41	49	9,2317145	9,9935942	9,2381203	10,7618797	11
20	9,2099917	9,9942122	9,2157795	10,7842205	40	50	9,2324440	9,9935723	9,2388717	10,7611283	10
21	9,2107597	9,9941914	9,2165683	10,7834317	39	51	9,2331722	9,9935504	9,2396218	10,7603782	9
22	9,2115263	9,9941706	9,2173556	10,7826444	38	52	9,2338992	9,9935285	9,2403708	10,7596292	8
23	9,2122914	9,9941498	9,2181417	10,7818583	37	53	9,2346249	9,9935065	9,2411185	10,7588815	7
24	9,2130552	9,9941289	9,2189264	10,7810736	36	54	9,2353494	9,9934844	9,2418650	10,7581350	6
25	9,2138176	9,9941079	9,2197097	10,7802903	35	55	9,2360726	9,9934624	9,2426103	10,7573897	5
26	9,2145787	9,9940870	9,2204917	10,7795083	34	56	9,2367946	9,9934403	9,2433543	10,7566457	4
27	9,2153384	9,9940659	9,2212724	10,7787276	33	57	9,2375153	9,9934181	9,2440972	10,7559028	3
28	9,2160967	9,9940449	9,2220518	10,7779482	32	58	9,2382349	9,9933959	9,2448389	10,7551611	2
29	9,2168536	9,9940238	9,2228298	10,7771702	31	59	9,2389532	9,9933737	9,2455794	10,7544206	1
30	9,2176092	9,9940027	9,2236065	10,7763935	30	60	9,2396702	9,9933515	9,2463188	10,7536812	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	80 Degrees.						80 Degrees.				

Min.	10 Degrees.				Min.	Min.	10 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,2396702	9,9933515	9,2463188	10,7536812	60	30	9,2606330	9,9926661	9,2679669	10,7320331	30
1	9,2403861	9,9933292	9,2470569	10,7529431	59	31	9,2613141	9,9926427	9,2686714	10,7313286	29
2	9,2411007	9,9933068	9,2477939	10,7522061	58	32	9,2619941	9,9926192	9,2693749	10,7306251	28
3	9,2418141	9,9932845	9,2485297	10,7514703	57	33	9,2626729	9,9925957	9,2700772	10,7299228	27
4	9,2425264	9,9932621	9,2492643	10,7507357	56	34	9,2633507	9,9925722	9,2707786	10,7292214	26
5	9,2432374	9,9932396	9,2499978	10,7500022	55	35	9,2640274	9,9925486	9,2714788	10,7285212	25
6	9,2439472	9,9932171	9,2507301	10,7492699	54	36	9,2647030	9,9925250	9,2721780	10,7278220	24
7	9,2446558	9,9931946	9,2514612	10,7485388	53	37	9,2653775	9,9925013	9,2728762	10,7271238	23
8	9,2453632	9,9931720	9,2521912	10,7478088	52	38	9,2660509	9,9924776	9,2735733	10,7264267	22
9	9,2460695	9,9931494	9,2529200	10,7470800	51	39	9,2667232	9,9924539	9,2742694	10,7257306	21
10	9,2467746	9,9931268	9,2536477	10,7463523	50	40	9,2673945	9,9924301	9,2749644	10,7250356	20
11	9,2474784	9,9931041	9,2543743	10,7456257	49	41	9,2680647	9,9924063	9,2756584	10,7243416	19
12	9,2481811	9,9930814	9,2550997	10,7449003	48	42	9,2687338	9,9923824	9,2763514	10,7236486	18
13	9,2488827	9,9930587	9,2558240	10,7441760	47	43	9,2694019	9,9923585	9,2770434	10,7229566	17
14	9,2495830	9,9930359	9,2565472	10,7434528	46	44	9,2700689	9,9923346	9,2777343	10,7222657	16
15	9,2502822	9,9930131	9,2572692	10,7427308	45	45	9,2707348	9,9923106	9,2784242	10,7215758	15
16	9,2509803	9,9929902	9,2579901	10,7420099	44	46	9,2713997	9,9922866	9,2791131	10,7208869	14
17	9,2516772	9,9929673	9,2587099	10,7412901	43	47	9,2720635	9,9922626	9,2798009	10,7201991	13
18	9,2523729	9,9929444	9,2594285	10,7405715	42	48	9,2727263	9,9922385	9,2804878	10,7195122	12
19	9,2530675	9,9929214	9,2601461	10,7398539	41	49	9,2733880	9,9922144	9,2811736	10,7188264	11
20	9,2537609	9,9928984	9,2608625	10,7391375	40	50	9,2740487	9,9921902	9,2818585	10,7181415	10
21	9,2544532	9,9928753	9,2615779	10,7384221	39	51	9,2747083	9,9921660	9,2825423	10,7174577	9
22	9,2551444	9,9928522	9,2622921	10,7377079	38	52	9,2753669	9,9921418	9,2832251	10,7167749	8
23	9,2558344	9,9928291	9,2630053	10,7369947	37	53	9,2760245	9,9921175	9,2839070	10,7160930	7
24	9,2565233	9,9928059	9,2637173	10,7362827	36	54	9,2766811	9,9920932	9,2845878	10,7154122	6
25	9,2572110	9,9927827	9,2644283	10,7355717	35	55	9,2773366	9,9920689	9,2852677	10,7147323	5
26	9,2578977	9,9927595	9,2651382	10,7348618	34	56	9,2779911	9,9920445	9,2859466	10,7140534	4
27	9,2585832	9,9927362	9,2658470	10,7341530	33	57	9,2786445	9,9920201	9,2866245	10,7133755	3
28	9,2592676	9,9927129	9,2665547	10,7334453	32	58	9,2792970	9,9919956	9,2873014	10,7126986	2
29	9,2599509	9,9926895	9,2672613	10,7327387	31	59	9,2799484	9,9919711	9,2879773	10,7120227	1
30	9,2606330	9,9926661	9,2679669	10,7320331	30	60	9,2805988	9,9919466	9,2886523	10,7113477	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	79 Degrees.						79 Degrees.				

Min.	11 Degrees.				Min.	Min.	11 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,2805988	9,9919466	9,2886523	10,7113477	60	30	9,2996553	9,9911927	9,3084626	10,6915374	30
1	9,2812483	9,9919220	9,2893263	10,7106737	59	31	9,3002758	9,9911670	9,3091088	10,6908912	29
2	9,2818967	9,9918974	9,2899993	10,7100007	58	32	9,3008953	9,9911412	9,3097541	10,6902459	28
3	9,2825441	9,9918727	9,2906713	10,7093287	57	33	9,3015140	9,9911154	9,3103985	10,6896015	27
4	9,2831905	9,9918480	9,2913424	10,7086576	56	34	9,3021317	9,9910896	9,3110421	10,6889579	26
5	9,2838359	9,9918233	9,2920126	10,7079874	55	35	9,3027485	9,9910637	9,3116848	10,6883152	25
6	9,2844803	9,9917986	9,2926817	10,7073183	54	36	9,3033644	9,9910378	9,3123266	10,6876734	24
7	9,2851237	9,9917737	9,2933500	10,7066500	53	37	9,3039794	9,9910119	9,3129675	10,6870325	23
8	9,2857661	9,9917489	9,2940172	10,7059828	52	38	9,3045934	9,9909859	9,3136076	10,6863924	22
9	9,2864076	9,9917240	9,2946836	10,7053164	51	39	9,3052066	9,9909598	9,3142468	10,6857532	21
10	9,2870480	9,9916991	9,2953489	10,7046511	50	40	9,3058189	9,9909338	9,3148851	10,6851149	20
11	9,2876875	9,9916741	9,2960134	10,7039866	49	41	9,3064303	9,9909077	9,3155226	10,6844774	19
12	9,2883260	9,9916492	9,2966769	10,7033231	48	42	9,3070407	9,9908815	9,3161592	10,6838408	18
13	9,2889636	9,9916241	9,2973395	10,7026605	47	43	9,3076503	9,9908553	9,3167950	10,6832050	17
14	9,2896001	9,9915990	9,2980011	10,7019989	46	44	9,3082590	9,9908291	9,3174299	10,6825701	16
15	9,2902357	9,9915739	9,2986618	10,7013382	45	45	9,3088668	9,9908029	9,3180640	10,6819360	15
16	9,2908704	9,9915488	9,2993216	10,7006784	44	46	9,3094737	9,9907766	9,3186972	10,6813028	14
17	9,2915040	9,9915236	9,2999804	10,7000196	43	47	9,3100798	9,9907502	9,3193295	10,6806705	13
18	9,2921367	9,9914984	9,3006383	10,6993617	42	48	9,3106849	9,9907239	9,3199611	10,6800389	12
19	9,2927685	9,9914731	9,3012954	10,6987046	41	49	9,3112892	9,9906974	9,3205918	10,6794082	11
20	9,2933993	9,9914478	9,3019514	10,6980486	40	50	9,3118926	9,9906710	9,3212216	10,6787784	10
21	9,2940291	9,9914225	9,3026066	10,6973934	39	51	9,3124951	9,9906445	9,3218506	10,6781494	9
22	9,2946580	9,9913971	9,3032609	10,6967391	38	52	9,3130968	9,9906180	9,3224788	10,6775212	8
23	9,2952859	9,9913717	9,3039143	10,6960857	37	53	9,3136976	9,9905914	9,3231061	10,6768939	7
24	9,2959129	9,9913462	9,3045667	10,6954333	36	54	9,3142975	9,9905648	9,3237327	10,6762673	6
25	9,2965390	9,9913207	9,3052183	10,6947817	35	55	9,3148965	9,9905382	9,3243584	10,6756416	5
26	9,2971641	9,9912952	9,3058689	10,6941311	34	56	9,3154947	9,9905115	9,3249832	10,6750168	4
27	9,2977883	9,9912696	9,3065187	10,6934813	33	57	9,3160921	9,9904848	9,3256073	10,6743927	3
28	9,2984116	9,9912440	9,3071675	10,6928325	32	58	9,3166885	9,9904580	9,3262305	10,6737695	2
29	9,2990339	9,9912184	9,3078155	10,6921845	31	59	9,3172841	9,9904312	9,3268629	10,6731471	1
30	9,2996553	9,9911927	9,3084626	10,6915374	30	60	9,3178789	9,9904044	9,3274745	10,6725255	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	78 Degrees.						78 Degrees.				

Min.	12 Degrees.				Min.	Min.	12 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,3178789	9,9904044	9,3274745	10,6725255	60	30	9,3353368	9,9895815	9,3457552	10,6542448	30
1	9,3184728	9,9903775	9,3280953	10,6719047	59	31	9,3359062	9,9895535	9,3463527	10,6536473	29
2	9,3190659	9,9903506	9,3287153	10,6712847	58	32	9,3364749	9,9895254	9,3469494	10,6530506	28
3	9,3196581	9,9903237	9,3293345	10,6706655	57	33	9,3370428	9,9894973	9,3475454	10,6524546	27
4	9,3202495	9,9902967	9,3299528	10,6700472	56	34	9,3376099	9,9894692	9,3481407	10,6518593	26
5	9,3208400	9,9902697	9,3305704	10,6694296	55	35	9,3381762	9,9894410	9,3487352	10,6512648	25
6	9,3214297	9,9902426	9,3311872	10,6688128	54	36	9,3387418	9,9894128	9,3493290	10,6506710	24
7	9,3220186	9,9902155	9,3318031	10,6681969	53	37	9,3393056	9,9893845	9,3499220	10,6500780	23
8	9,3226066	9,9901883	9,3324183	10,6675817	52	38	9,3398706	9,9893562	9,3504143	10,6494857	22
9	9,3231938	9,9901612	9,3330327	10,6669673	51	39	9,3404338	9,9893279	9,3511059	10,6488941	21
10	9,3237802	9,9901339	9,3336463	10,6663537	50	40	9,3409963	9,9892995	9,3516968	10,6483032	20
11	9,3243657	9,9901067	9,3342591	10,6657409	49	41	9,3415580	9,9892711	9,3522869	10,6477131	19
12	9,3249505	9,9900794	9,3348711	10,6651289	48	42	9,3421190	9,9892427	9,3528763	10,6471237	18
13	9,3255344	9,9900521	9,3354823	10,6645177	47	43	9,3426792	9,9892142	9,3534650	10,6465350	17
14	9,3261174	9,9900247	9,3360927	10,6639073	46	44	9,3432386	9,9891856	9,3540530	10,6459470	16
15	9,3266997	9,9899973	9,3367024	10,6632976	45	45	9,3437973	9,9891571	9,3546402	10,6453598	15
16	9,3272811	9,9899698	9,3373113	10,6626887	44	46	9,3443552	9,9891285	9,3552267	10,6447733	14
17	9,3278617	9,9899423	9,3379194	10,6620806	43	47	9,3449124	9,9890998	9,3558126	10,6441874	13
18	9,3284416	9,9899148	9,3385267	10,6614733	42	48	9,3454688	9,9890711	9,3563977	10,6436023	12
19	9,3290206	9,9898873	9,3391333	10,6608667	41	49	9,3460245	9,9890424	9,3569821	10,6430179	11
20	9,3295988	9,9898597	9,3397391	10,6602609	40	50	9,3465794	9,9890137	9,3575658	10,6424342	10
21	9,3301761	9,9898320	9,3403441	10,6596559	39	51	9,3471336	9,9889849	9,3581487	10,6418513	9
22	9,3307527	9,9898043	9,3409484	10,6590516	38	52	9,3476870	9,9889560	9,3587310	10,6412690	8
23	9,3313285	9,9897766	9,3415519	10,6584481	37	53	9,3482397	9,9889271	9,3593126	10,6406874	7
24	9,3319035	9,9897489	9,3421546	10,6578454	36	54	9,3487917	9,9888982	9,3598935	10,6401065	6
25	9,3324777	9,9897211	9,3427566	10,6572434	35	55	9,3493429	9,9888693	9,3604736	10,6395264	5
26	9,3330511	9,9896932	9,3433578	10,6566422	34	56	9,3498934	9,9888403	9,3610531	10,6389469	4
27	9,3336237	9,9896654	9,3439583	10,6560417	33	57	9,3504432	9,9888113	9,3616319	10,6383681	3
28	9,3341955	9,9896374	9,3445580	10,6554420	32	58	9,3509922	9,9887822	9,3622100	10,6377900	2
29	9,3347665	9,9896095	9,3451570	10,6548430	31	59	9,3515405	9,9887531	9,3627874	10,6372126	1
30	9,3353368	9,9895815	9,3457552	10,6542448	30	60	9,3520880	9,9887239	9,3633641	10,6366359	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	77 Degrees.						77 Degrees.				

Min.	13 Degrees.				Min.	Min.	13 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,3520880	9,9887239	9,3633641	10,6366359	60	30	9,3681853	9,9878315	9,3803537	10,6196463	30
1	9,3526349	9,9886947	9,3639401	10,6360599	59	31	9,3687111	9,9878012	9,3809100	10,6190900	29
2	9,3531810	9,9886655	9,3645155	10,6354845	58	32	9,3692363	9,9877708	9,3814655	10,6185345	28
3	9,3537264	9,9886363	9,3650901	10,6349099	57	33	9,3697608	9,9877404	9,3820205	10,6179795	27
4	9,3542710	9,9886070	9,3656641	10,6343359	56	34	9,3702847	9,9877099	9,3825748	10,6174252	26
5	9,3548150	9,9885776	9,3662374	10,6337626	55	35	9,3708079	9,9876794	9,3831285	10,6168715	25
6	9,3553582	9,9885482	9,3668100	10,6331900	54	36	9,3713304	9,9876488	9,3836816	10,6163184	24
7	9,3559007	9,9885188	9,3673819	10,6326181	53	37	9,3718523	9,9876183	9,3842340	10,6157660	23
8	9,3564426	9,9884894	9,3679532	10,6320468	52	38	9,3723735	9,9875876	9,3847858	10,6152142	22
9	9,3569836	9,9884599	9,3685238	10,6314762	51	39	9,3728940	9,9875570	9,3853370	10,6146630	21
10	9,3575240	9,9884303	9,3690937	10,6309063	50	40	9,3734139	9,9875263	9,3858876	10,6141124	20
11	9,3580637	9,9884008	9,3696629	10,6303371	49	41	9,3739331	9,9874955	9,3864376	10,6135624	19
12	9,3586027	9,9883712	9,3702315	10,6297685	48	42	9,3744517	9,9874648	9,3869869	10,6130131	18
13	9,3591409	9,9883415	9,3707994	10,6292006	47	43	9,3749696	9,9874339	9,3875356	10,6124644	17
14	9,3596785	9,9883118	9,3713667	10,6286333	46	44	9,3754868	9,9874031	9,3880837	10,6119163	16
15	9,3602154	9,9882821	9,3719333	10,6289667	45	45	9,3760034	9,9873722	9,3886312	10,6113688	15
16	9,3607515	9,9882523	9,3724992	10,6275008	44	46	9,3765194	9,9873413	9,3891781	10,6108219	14
17	9,3612870	9,9882225	9,3730645	10,6269355	43	47	9,3770347	9,9873103	9,3897244	10,6102756	13
18	9,3618217	9,9881927	9,3736291	10,6263709	42	48	9,3775493	9,9872793	9,3902700	10,6097300	12
19	9,3623558	9,9881628	9,3741930	10,6258070	41	49	9,3780633	9,9872482	9,3908151	10,6091849	11
20	9,3628892	9,9881329	9,3747563	10,6252437	40	50	9,3785767	9,9872171	9,3913595	10,6086405	10
21	9,3634219	9,9881029	9,3753190	10,6246810	39	51	9,3790894	9,9871860	9,3919034	10,6080966	9
22	9,3639539	9,9880729	9,3758810	10,6241190	38	52	9,3796015	9,9871549	9,3924466	10,6075534	8
23	9,3644852	9,9880429	9,3764423	10,6235577	37	53	9,3801129	9,9871236	9,3929893	10,6070107	7
24	9,3650158	9,9880128	9,3770030	10,6229970	36	54	9,3806237	9,9870924	9,3935313	10,6064687	6
25	9,3655458	9,9879827	9,3775631	10,6224369	35	55	9,3811339	9,9870611	9,3940727	10,6059275	5
26	9,3660750	9,9879525	9,3781225	10,6218775	34	56	9,3816434	9,9870298	9,3946136	10,6055864	4
27	9,3666036	9,9879223	9,3786813	10,6213187	33	57	9,3821523	9,9869984	9,3951538	10,6048462	3
28	9,3671315	9,9878921	9,3792394	10,6207006	32	58	9,3826605	9,9869670	9,3956935	10,6043065	2
29	9,3676587	9,9878618	9,3797969	10,6202031	31	59	9,3831682	9,9869356	9,3962326	10,6037674	1
30	9,3681853	9,9878315	9,3803537	10,6196463	30	60	9,3836752	9,9869041	9,3967711	10,6032289	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	76 Degrees.						76 Degrees.				

Min.	14 Degrees.				Min.	Min.	14 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,3836752	9,9869041	9,3967711	10,6032289	60	30	9,3985996	9,9859416	9,4126581	10,5873419	30
1	9,3841815	9,9868726	9,3973089	10,6026911	59	31	9,3990878	9,9859089	9,4131789	10,5868211	29
2	9,3846873	9,9868410	9,3978463	10,6021537	58	32	9,3995754	9,9858762	9,4136993	10,5863007	28
3	9,3851924	9,9868094	9,3983830	10,6016170	57	33	9,4000625	9,9858434	9,4142191	10,5857809	27
4	9,3856969	9,9867778	9,3989191	10,6010809	56	34	9,4005489	9,9858106	9,4147383	10,5852617	26
5	9,3862008	9,9867461	9,3994547	10,6005453	55	35	9,4010348	9,9857777	9,4152570	10,5847430	25
6	9,3867040	9,9867144	9,3999896	10,6000104	54	36	9,4015201	9,9857449	9,4157752	10,5842248	24
7	9,3872067	9,9866827	9,4005240	10,5994760	53	37	9,4020048	9,9857119	9,4162928	10,5837072	23
8	9,3877087	9,9866509	9,4010578	10,5989422	52	38	9,4024889	9,9856790	9,4168099	10,5831901	22
9	9,3882101	9,9866191	9,4015910	10,5984090	51	39	9,4029724	9,9856460	9,4173265	10,5826735	21
10	9,3887109	9,9865872	9,4021237	10,5978763	50	40	9,4034554	9,9856129	9,4178425	10,5821575	20
11	9,3892111	9,9865553	9,4026558	10,5973442	49	41	9,4039378	9,9855798	9,4183580	10,5816420	19
12	9,3897106	9,9865233	9,4031873	10,5968127	48	42	9,4044196	9,9855467	9,4188729	10,5811271	18
13	9,3902096	9,9864913	9,4037182	10,5962818	47	43	9,4049009	9,9855135	9,4193874	10,5806126	17
14	9,3907079	9,9864593	9,4042486	10,5957514	46	44	9,4053816	9,9854803	9,4199013	10,5800987	16
15	9,3912057	9,9864273	9,4047784	10,5952216	45	45	9,4058617	9,9854471	9,4204146	10,5795854	15
16	9,3917028	9,9863952	9,4053076	10,5946924	44	46	9,4063413	9,9854138	9,4209275	10,5790725	14
17	9,3921993	9,9863630	9,4058363	10,5941637	43	47	9,4068203	9,9853805	9,4214398	10,5785602	13
18	9,3926952	9,9863308	9,4063644	10,5936356	42	48	9,4072987	9,9853471	9,4219515	10,5780485	12
19	9,3931905	9,9862986	9,4068919	10,5931081	41	49	9,4077766	9,9853138	9,4224628	10,5775372	11
20	9,3936852	9,9862663	9,4074189	10,5925811	40	50	9,4082539	9,9852803	9,4229735	10,5770265	10
21	9,3941794	9,9862340	9,4079453	10,5920547	39	51	9,4087306	9,9852468	9,4234838	10,5765162	9
22	9,3946729	9,9862017	9,4084712	10,5915288	38	52	9,4092068	9,9852133	9,4239935	10,5760065	8
23	9,3951658	9,9861693	9,4089965	10,5910035	37	53	9,4096824	9,9851798	9,4245026	10,5754974	7
24	9,3956581	9,9861369	9,4095212	10,5904788	36	54	9,4101575	9,9851462	9,4250113	10,5749887	6
25	9,3961499	9,9861045	9,4100454	10,5899546	35	55	9,4106320	9,9851125	9,4255194	10,5744806	5
26	9,3966410	9,9860720	9,4105690	10,5894310	34	56	9,4111059	9,9850789	9,4260271	10,5739729	4
27	9,3971315	9,9860394	9,4110921	10,5889079	33	57	9,4115793	9,9850452	9,4265342	10,5734658	3
28	9,3976215	9,9860069	9,4116146	10,5883854	32	58	9,4120522	9,9850114	9,4270408	10,5729592	2
29	9,3981109	9,9859742	9,4121366	10,5878634	31	59	9,4125245	9,9849776	9,4275469	10,5724531	1
30	9,3985996	9,9859416	9,4126581	10,5873419	30	60	9,4129962	9,9849438	9,4280525	10,5719475	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	75 Degrees.						75 Degrees.				

Min.	15 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,4129962	9,9849438	9,4280525	10,5719475	60
1	9,4134674	9,9849099	9,4285575	10,5714425	59
2	9,4139381	9,9848760	9,4290621	10,5709379	58
3	9,4144082	9,9848420	9,4295661	10,5704339	57
4	9,4148778	9,9848081	9,4300697	10,5699303	56
5	9,4153468	9,9847740	9,4305727	10,5694273	55
6	9,4158152	9,9847400	9,4310753	10,5689247	54
7	9,4162832	9,9847059	9,4315773	10,5684227	53
8	9,4167506	9,9846717	9,4320789	10,5679211	52
9	9,4172174	9,9846375	9,4325799	10,5674201	51
10	9,4176837	9,9846033	9,4330804	10,5669196	50
11	9,4181495	9,9845690	9,4335805	10,5664195	49
12	9,4186148	9,9845347	9,4340800	10,5659200	48
13	9,4190795	9,9845004	9,4345791	10,5654209	47
14	9,4195436	9,9844660	9,4350776	10,5649224	46
15	9,4200073	9,9844316	9,4355757	10,5644243	45
16	9,4204704	9,9843971	9,4360733	10,5639267	44
17	9,4209330	9,9843626	9,4365704	10,5634296	43
18	9,4213950	9,9843281	9,4370670	10,5629330	42
19	9,4218566	9,9842935	9,4375631	10,5624369	41
20	9,4223176	9,9842589	9,4380587	10,5619413	40
21	9,4227780	9,9842242	9,4385538	10,5614462	39
22	9,4232380	9,9841895	9,4390485	10,5609515	38
23	9,4236974	9,9841548	9,4395426	10,5604574	37
24	9,4241563	9,9841200	9,4400363	10,5599637	36
25	9,4246147	9,9840852	9,4405295	10,5594705	35
26	9,4250726	9,9840503	9,4410222	10,5589778	34
27	9,4255299	9,9840154	9,4415145	10,5584855	33
28	9,4259867	9,9839805	9,4420062	10,5579938	32
29	9,4264430	9,9839455	9,4424975	10,5575025	31
30	9,4268988	9,9839105	9,4429883	10,5570117	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	74 Degrees.				

Min.	15 Degrees.				Min.
	Sine.	Sine. Com.	Tangent.	Tan. Comp.	
30	9,4268988	9,9839105	9,4429883	10,5570117	30
31	9,4273541	9,9838755	9,4434786	10,5565214	29
32	9,4278089	9,9838404	9,4439685	10,5560315	28
33	9,4282631	9,9838052	9,4444579	10,5555421	27
34	9,4287169	9,9837701	9,4449468	10,5550532	26
35	9,4291701	9,9837348	9,4454352	10,5545648	25
36	9,4296228	9,9836996	9,4459232	10,5540768	24
37	9,4300750	9,9836643	9,4464107	10,5535893	23
38	9,4305267	9,9836290	9,4468978	10,5531022	22
39	9,4309779	9,9835936	9,4473843	10,5526157	21
40	9,4314286	9,9835582	9,4478704	10,5521296	20
41	9,4318788	9,9835227	9,4483561	10,5516439	19
42	9,4323285	9,9834872	9,4488413	10,5511587	18
43	9,4327777	9,9834517	9,4493260	10,5506740	17
44	9,4332264	9,9834161	9,4498102	10,5501898	16
45	9,4336746	9,9833805	9,4502940	10,5497060	15
46	9,4341223	9,9833449	9,4507774	10,5492226	14
47	9,4345694	9,9833092	9,4512602	10,5487398	13
48	9,4350161	9,9832735	9,4517427	10,5482573	12
49	9,4354623	9,9832377	9,4522246	10,5477754	11
50	9,4359080	9,9832019	9,4527061	10,5472939	10
51	9,4363532	9,9831661	9,4531872	10,5468128	9
52	9,4367980	9,9831302	9,4536678	10,5463322	8
53	9,4372422	9,9830942	9,4541479	10,5458521	7
54	9,4376859	9,9830583	9,4546276	10,5453724	6
55	9,4381292	9,9830223	9,4551069	10,5448931	5
56	9,4385719	9,9829862	9,4555857	10,5444143	4
57	9,4390142	9,9829501	9,4560641	10,5439359	3
58	9,4394560	9,9829140	9,4565420	10,5434580	2
59	9,4398973	9,9828778	9,4570194	10,5429806	1
60	9,4403381	9,9828416	9,4574964	10,5425036	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	74 Degrees.				

Min.	16 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,4403381	9,9828416	9,4574964	10,5425036	60
1	9,4407784	9,9828054	9,4579730	10,5420270	59
2	9,4412182	9,9827691	9,4584491	10,5415509	58
3	9,4416576	9,9827328	9,4589248	10,5410752	57
4	9,4420965	9,9826964	9,4594001	10,5405999	56
5	9,4425249	9,9826600	9,4598749	10,5401251	55
6	9,4429728	9,9826236	9,4603492	10,5396508	54
7	9,4434103	9,9825871	9,4608232	10,5391768	53
8	9,4438472	9,9825506	9,4612967	10,5387033	52
9	9,4442837	9,9825140	9,4617697	10,5382303	51
10	9,4447197	9,9824774	9,4622423	10,5377577	50
11	9,4451553	9,9824408	9,4627145	10,5372855	49
12	9,4455903	9,9824041	9,4631863	10,5368137	48
13	9,4460250	9,9823674	9,4636576	10,5363424	47
14	9,4464591	9,9823306	9,4641285	10,5358715	46
15	9,4468927	9,9822938	9,4645990	10,5354010	45
16	9,4473259	9,9822569	9,4650690	10,5349310	44
17	9,4477586	9,9822201	9,4655386	10,5344614	43
18	9,4481909	9,9821831	9,4660078	10,5339922	42
19	9,4486227	9,9821462	9,4664765	10,5335235	41
20	9,4490540	9,9821092	9,4669448	10,5330552	40
21	9,4494849	9,9820721	9,4674127	10,5325873	39
22	9,4499153	9,9820351	9,4678802	10,5321198	38
23	9,4503452	9,9819979	9,4683473	10,5316527	37
24	9,4507747	9,9819608	9,4688139	10,5311861	36
25	9,4512037	9,9819236	9,4692801	10,5307199	35
26	9,4516322	9,9818863	9,4697459	10,5302541	34
27	9,4520603	9,9818490	9,4702112	10,5297888	33
28	9,4524879	9,9818117	9,4706762	10,5293238	32
29	9,4529151	9,9811744	9,4711407	10,5288593	31
30	9,4533418	9,9817370	9,4716048	10,5283952	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	73 Degrees.				

Min.	16 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,4533418	9,9817370	9,4716048	10,5283952	30
31	9,4537681	9,9816995	9,4720685	10,5279315	29
32	9,4541939	9,9816620	9,4725318	10,5274682	28
33	9,4546192	9,9816245	9,4729947	10,5270053	27
34	9,4550441	9,9815870	9,4734572	10,5265428	26
35	9,4554686	9,9815494	9,4739192	10,5260808	25
36	9,4558926	9,9815117	9,4743808	10,5256192	24
37	9,4563161	9,9814740	9,4748421	10,5251579	23
38	9,4567392	9,9814363	9,4753029	10,5246971	22
39	9,4571618	9,9813986	9,4757633	10,5242367	21
40	9,4575840	9,9813608	9,4762233	10,5237767	20
41	9,4580058	9,9813229	9,4766829	10,5233171	19
42	9,4584271	9,9812850	9,4771421	10,5228579	18
43	9,4588480	9,9812471	9,4776009	10,5223991	17
44	9,4592684	9,9812091	9,4780592	10,5219408	16
45	9,4596884	9,9811711	9,4785172	10,5214828	15
46	9,4601079	9,9811331	9,4789748	10,5210252	14
47	9,4605270	9,9810950	9,4794319	10,5205681	13
48	9,4609456	9,9810569	9,4798887	10,5201113	12
49	9,4613638	9,9810187	9,4803451	10,5196549	11
50	9,4617816	9,9809805	9,4808011	10,5191989	10
51	9,4621989	9,9809423	9,4812566	10,5187434	9
52	9,4626158	9,9809040	9,4817118	10,5182882	8
53	9,4630323	9,9808657	9,4821666	10,5178334	7
54	9,4634483	9,9808273	9,4826210	10,5173790	6
55	9,4638639	9,9807889	9,4830750	10,5169250	5
56	9,4642790	9,9807505	9,4835286	10,5164714	4
57	9,4646938	9,9807120	9,4839818	10,5160182	3
58	9,4651081	9,9806735	9,4844346	10,5155654	2
59	9,4655219	9,9806349	9,4848870	10,5151130	1
60	9,4659353	9,9805963	9,4853390	10,5146610	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	73 Degrees.				

Min.	17 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,4659353	9,9805963	9,4853390	10.5146610	60
1	9,4663483	9,9805577	9,4857907	10,5142093	59
2	9,4667609	9,9805190	9,4862419	10,5137581	58
3	9,4671730	9,9804803	9,4866928	10,5133072	57
4	9,4675848	9,9804415	9,4871433	10,5128567	56
5	9,4679960	9,9804027	9,4875933	10,5124067	55
6	9,4684069	9,9803639	9,4880430	10,5119570	54
7	9,4688173	9,9803250	9,4884924	10,5115076	53
8	9,4692273	9,9802860	9,4889413	10,5110587	52
9	9,4696369	9,9802471	9,4893898	10,5106102	51
10	9,4700461	9,9802081	9,4898380	10,5101620	50
11	9,4704548	9,9801690	9,4902858	10,5097142	49
12	9,4708631	9,9801299	9,4907332	10,5092668	48
13	9,4712710	9,9800908	9,4911802	10,5088198	47
14	9,4716785	9,9800516	9,4916269	10,5083731	46
15	9,4720856	9,9800124	9,4920731	10,5079269	45
16	9,4724922	9,9799732	9,4925190	10,5074810	44
17	9,4728985	9,9799339	9,4929646	10,5070354	43
18	9,4733043	9,9798946	9,4934097	10,5065903	42
19	9.4737097	9,9798552	9,4938545	10,5061455	41
20	9,4741146	9,9798158	9,4942988	10.5057012	40
21	9,4745192	9,9797764	9,4947429	10,5052571	39
22	9,4749234	9,9797369	9,4951865	10,5048135	38
23	9,4753271	9,9796973	9.4956298	10,5043702	37
24	9,4757304	9,9796578	9,4960727	10,5039273	36
25	9,4761334	9,9796182	9,4965152	10,5034848	35
26	9,4765359	9,9795785	9,4969574	10,5030426	34
27	9,4769380	9,9795388	9,4973991	10,5026009	33
28	9,4773396	9,9794991	9,4978406	10,5021594	32
29	9,4777409	9,9794593	9,4982816	10,5017184	31
30	9,4781418	9,9794195	9,4987223	10,5012777	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	72 Degrees.				

Min.	17 Degrees.				Min.
	Sine.	Sine. Com.	Tangent.	Tan. Comp.	
30	9,4781418	9,9794195	9,4987223	10,5012777	30
31	9,4785423	9,9793796	9,4991626	10,5008374	29
32	9,4789423	9,9793398	9,4996026	10,5003974	28
33	9,4793420	9,9792998	9,5000422	10,4999578	27
34	9,4797412	9,9792599	9,5004814	10,4995186	26
35	9,4801401	9,9792198	9,5009203	10,4990797	25
36	9,4805385	9,9791798	9,5013588	10,4986412	24
37	9,4809366	9,9791397	9,5017969	10,4982031	23
38	9,4813342	9,9790996	9,5022347	10,4977653	22
39	9,4817315	9,9790594	9,5026721	10,4973279	21
40	9,4821283	9,9790192	9,5031092	10,4968908	20
41	9,4825248	9,9789789	9,5035459	10,4964541	19
42	9,4829208	9,9789386	9,5039822	10,4960178	18
43	9,4833165	9,9788983	9,5044182	10,4955818	17
44	9,4837117	9,9788579	9,5048538	10,4951462	16
45	9,4841066	9,9788175	9,5052891	10,4947109	15
46	9,4845010	9,9787770	9,5057240	10,4942760	14
47	9,4848951	9,9787365	9,5061586	10,4938414	13
48	9,4852888	9,9786960	9,5065928	10,4934072	12
49	9,4856820	9,9786554	9,5070267	10,4929733	11
50	9,4860749	9,9786148	9,5074602	10,4925398	10
51	9,4864674	9,9785741	9,5078933	10,4921067	9
52	9,4868595	9,9785334	9,5083261	10,4916739	8
53	9,4872512	9,9784927	9,5087586	10,4912414	7
54	9,4876426	9,9784519	9,5091907	10,4908093	6
55	9,4880335	9,9784111	9,5096224	10,4903776	5
56	9,4884240	9,9783702	9,5100539	10,4899461	4
57	9,4888142	9,9783293	9,5104849	10,4895151	3
58	9,4892040	9,9782883	9,5109156	10,4890844	2
59	9,4895934	9,9782474	9,5113460	10,4886540	1
60	9,4899824	9,9782063	9,5117760	10,4882240	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	72 Degrees.				

Min.	18 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,4899824	9,9782063	9,5117760	10,4882240	60
1	9,4903710	9,9781653	9,5122057	10,4877943	59
2	9,4907592	9,9781241	9,5126351	10,4873649	58
3	9,4911471	9,9780830	9,5130641	10,4869359	57
4	9,4915345	9,9780418	9,5134927	10,4865073	56
5	9,4919216	9,9780006	9,5139210	10,4860790	55
6	9,4923083	9,9779593	9,5143490	10,4856510	54
7	9,4926946	9,9779180	9,5147766	10,4852234	53
8	9,4930806	9,9778766	9,5152039	10,4847961	52
9	9,4934661	9,9778353	9,5156309	10,4843691	51
10	9,4938513	9,9777938	9,5160575	10,4839425	50
11	9,4942361	9,9777523	9,5164838	10,4835162	49
12	9,4946205	9,9777108	9,5169097	10,4830903	48
13	9,4950046	9,9776693	9,5173353	10,4826647	47
14	9,4953883	9,9776277	9,5177606	10,4822394	46
15	9,4957716	9,9775860	9,5181855	10,4818145	45
16	9,4961545	9,9775444	9,5186101	10,4813899	44
17	9,4965370	9,9775026	9,5190344	10,4809656	43
18	9,4969192	9,9774609	9,5194583	10,4805417	42
19	9,4973010	9,9774191	9,5198819	10,4801181	41
20	9,4976824	9,9773772	9,5203052	10,4796948	40
21	9,4980635	9,9773354	9,5207282	10,4792718	39
22	9,4984442	9,9772934	9,5211508	10,4788492	38
23	9,4988245	9,9772515	9,5215730	10,4784270	37
24	9,4992045	9,9772095	9,5219950	10,4780050	36
25	9,4995840	9,9771674	9,5224166	10,4775834	35
26	9,4999633	9,9771253	9,5228379	10,4771621	34
27	9,5003421	9,9770832	9,5232589	10,4767411	33
28	9,5007206	9,9770410	9,5236795	10,4763205	32
29	9,5010987	9,9769988	9,5240999	10,4759001	31
30	9,5014764	9,9769566	9,5245199	10,4754801	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	71 Degrees.				

Min.	18 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,5014764	9,9769566	9,5245199	10,4754801	30
31	9,5018538	9,9769143	9,5249395	10,4750605	29
32	9,5022308	9,9768720	9,5253589	10,4746411	28
33	9,5026075	9,9768296	9,5257779	10,4742221	27
34	9,5029838	9,9767872	9,5261966	10,4738034	26
35	9,5033597	9,9767447	9,5266150	10,4733850	25
36	9,5037353	9,9767022	9,5270331	10,4729669	24
37	9,5041105	9,9766597	9,5274508	10,4725492	23
38	9,5044853	9,9766171	9,5278682	10,4721318	22
39	9,5048598	9,9765745	9,5282853	10,4717147	21
40	9,5052339	9,9765318	9,5287021	10,4712979	20
41	9,5056077	9,9764891	9,5291186	10,4708814	19
42	9,5059811	9,9764464	9,5295347	10,4704653	18
43	9,5063542	9,9764036	9,5299505	10,4700495	17
44	9,5067268	9,9763608	9,5303661	10,4696339	16
45	9,5070992	9,9763179	9,5307813	10,4692187	15
46	9,5074712	9,9762750	9,5311961	10,4688039	14
47	9,5078428	9,9762321	9,5316107	10,4683893	13
48	9,5082141	9,9761891	9,5320250	10,4679750	12
49	9,5085850	9,9761461	9,5324389	10,4675611	11
50	9,5089556	9,9761030	9,5328526	10,4671474	10
51	9,5093258	9,9760599	9,5332659	10,4667341	9
52	9,5096956	9,9760167	9,5336789	10,4663211	8
53	9,5100651	9,9759736	9,5340916	10,4659084	7
54	9,5104343	9,9759303	9,5345040	10,4654960	6
55	9,5108031	9,9758870	9,5349161	10,4650839	5
56	9,5111716	9,9758437	9,5353278	10,4646722	4
57	9,5115397	9,9758004	9,5357393	10,4642607	3
58	9,5119074	9,9757570	9,5361505	10,4638495	2
59	9,5122749	9,9757135	9,5365613	10,4634387	1
60	9,5126419	9,9756701	9,5369719	10,4630281	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	71 Degrees.				

Min.	19 Degrees.				Min.	Min.	19 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,5126419	9,9756701	9,5369718	10,4630281	60	30	9,5234953	9,9743466	9,5491487	10,4508513	30
1	9,5130086	9,9756265	9,5373821	10,4626179	59	31	9,5238518	9,9743018	9,5495500	10,4504500	29
2	9,5133750	9,9755830	9,5377920	10,4622080	58	32	9,5242081	9,9742570	9,5499511	10,4500489	28
3	9,5137410	9,9755394	9,5382017	10,4617983	57	33	9,5245640	9,9742122	9,5503519	10,4496481	27
4	9,5141067	9,9754957	9,5386110	10,4613890	56	34	9,5249196	9,9741673	9,5507523	10,4492477	26
5	9,5144721	9,9754521	9,5390200	10,4609800	55	35	9,5252749	9,9741224	9,5511525	10,4488475	25
6	9,5148371	9,9754083	9,5394287	10,4605713	54	36	9,5256298	9,9740774	9,5515524	10,4484476	24
7	9,5152017	9,9753646	9,5398371	10,4601629	53	37	9,5259844	9,9740324	9,5519521	10,4480479	23
8	9,5155660	9,9753208	9,5402453	10,4597547	52	38	9,5263387	9,9739873	9,5523514	10,4476486	22
9	9,5159300	9,9752769	9,5406531	10,4593469	51	39	9,5266927	9,9739422	9,5527504	10,4472496	21
10	9,5162936	9,9752330	9,5410606	10,4589394	50	40	9,5270463	9,9738971	9,5531492	10,4468508	20
11	9,5166569	9,9751891	9,5414678	10,4585322	49	41	9,5273997	9,9738519	9,5535477	10,4464523	19
12	9,5170198	9,9751451	9,5418747	10,4581253	48	42	9,5277526	9,9738067	9,5539459	10,4460541	18
13	9,5173824	9,9751011	9,5422813	10,4577187	47	43	9,5281053	9,9737615	9,5543438	10,4456562	17
14	9,5177447	9,9750570	9,5426877	10,4573123	46	44	9,5284577	9,9737162	9,5547415	10,4452585	16
15	9,5181066	9,9750129	9,5430937	10,4569063	45	45	9,5288097	9,9736709	9,5551388	10,4448612	15
16	9,5184682	9,9749688	9,5434994	10,4565006	44	46	9,5291614	9,9736255	9,5555359	10,4444641	14
17	9,5188295	9,9749246	9,5439048	10,4560952	43	47	9,5295128	9,9735801	9,5559327	10,4440673	13
18	9,5191904	9,9748804	9,5443100	10,4556900	42	48	9,5298638	9,9735346	9,5563292	10,4436708	12
19	9,5195510	9,9748361	9,5447148	10,4552852	41	49	9,5302146	9,9734891	9,5567255	10,4432745	11
20	9,5199112	9,9747918	9,5451193	10,4548807	40	50	9,5305650	9,9734435	9,5571214	10,4428786	10
21	9,5202711	9,9747475	9,5455236	10,4544764	39	51	9,5309151	9,9733980	9,5575171	10,4424829	9
22	9,5206307	9,9747031	9,5459276	10,4540724	38	52	9,5312649	9,9733523	9,5579125	10,4420875	8
23	9,5209899	9,9746587	9,5463312	10,4536688	37	53	9,5316143	9,9733067	9,5583077	10,4416923	7
24	9,5213488	9,9746142	9,5467346	10,4532654	36	54	9,5319635	9,9732610	9,5587025	10,4412975	6
25	9,5217074	9,9745697	9,5471377	10,4528623	35	55	9,5323123	9,9732152	9,5590971	10,4409029	5
26	9,5220656	9,9745252	9,5475405	10,4524595	34	56	9,5326608	9,9731694	9,5594914	10,4405086	4
27	9,5224235	9,9744806	9,5479430	10,4520570	33	57	9,5330090	9,9731236	9,5598854	10,4401146	3
28	9,5227811	9,9744359	9,5483452	10,4516548	32	58	9,5333569	9,9730777	9,5602792	10,4397208	2
29	9,5231383	9,9743913	9,5487471	10,4512529	31	59	9,5337044	9,9730318	9,5606727	10,4393273	1
30	9,5234953	9,9743466	9,5491487	10,4508513	30	60	9,5340517	9,9729858	9,5610659	10,4389341	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	70 Degrees.						70 Degrees.				

Min.	20 Degrees.				Min.	Min.	20 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,5340517	9,9729858	9,5610659	10,4389341	60	30	9,5443253	9,9715876	9,5727377	10,4272623	30
1	9,5343986	9,9729398	9,5614588	10,4385412	59	31	9,5446630	9,9715404	9,5731227	10,4268773	29
2	9,5347452	9,9728938	9,5618515	10,4381485	58	32	9,5450005	9,9714931	9,5735074	10,4264926	28
3	9,5350915	9,9728477	9,5622439	10,4377561	57	33	9,5453376	9,9714457	9,5738919	10,4261081	27
4	9,5354375	9,9728016	9,5626360	10,4373640	56	34	9,5456745	9,9713984	9,5742761	10,4257239	26
5	9,5357832	9,9727554	9,5630278	10,4369722	55	35	9,5460110	9,9713509	9,5746601	10,4253399	25
6	9,5361286	9,9727092	9,5634194	10,4365806	54	36	9,5463472	9,9713035	9,5750438	10,4249562	24
7	9,5364737	9,9726629	9,5638107	10,4361893	53	37	9,5466832	9,9712560	9,5754272	10,4245728	23
8	9,5368184	9,9726166	9,5642018	10,4357982	52	38	9,5470189	9,9712084	9,5758104	10,4241896	22
9	9,5371629	9,9725703	9,5645925	10,4354075	51	39	9,5473542	9,9711608	9,5761934	10,4238066	21
10	9,5375070	9,9725239	9,5649831	10,4350169	50	40	9,5476893	9,9711132	9,5765761	10,4234239	20
11	9,5378508	9,9724775	9,5653733	10,4346267	49	41	9,5480240	9,9710655	9,5769585	10,4230415	19
12	9,5381943	9,9724310	9,5657633	10,4342367	48	42	9,5483585	9,9710178	9,5773407	10,4226593	18
13	9,5385375	9,9723845	9,5661530	10,4338470	47	43	9,5486927	9,9709701	9,5777226	10,4222774	17
14	9,5388804	9,9723380	9,5665424	10,4334576	46	44	9,5490266	9,9709223	9,5781043	10,4218957	16
15	9,5392230	9,9722914	9,5669316	10,4330684	45	45	9,5493602	9,9708744	9,5784858	10,4215142	15
16	9,5395653	9,9722448	9,5673205	10,4326795	44	46	9,5496935	9,9708265	9,5788669	10,4211331	14
17	9,5399073	9,9721981	9,5677091	10,4322909	43	47	9,5500265	9,9707786	9,5792479	10,4207521	13
18	9,5402489	9,9721514	9,5680975	10,4319025	42	48	9,5503592	9,9707306	9,5796286	10,4203714	12
19	9,5405903	9,9721047	9,5684856	10,4315144	41	49	9,5506916	9,9706826	9,5800090	10,4199910	11
20	9,5409314	9,9720579	9,5688735	10,4311265	40	50	9,5510237	9,9706346	9,5803892	10,4196108	10
21	9,5412721	9,9720110	9,5692611	10,4307389	39	51	9,5513556	9,9705865	9,5807691	10,4192309	9
22	9,5416126	9,9719642	9,5696484	10,4303516	38	52	9,5516871	9,9705383	9,5811488	10,4188512	8
23	9,5419527	9,9719172	9,5700355	10,4299645	37	53	9,5520184	9,9704902	9,5815282	10,4184718	7
24	9,5422926	9,9718703	9,5704223	10,4295777	36	54	9,5523494	9,9704419	9,5819074	10,4180926	6
25	9,5426321	9,9718233	9,5708088	10,4291912	35	55	9,5526801	9,9703937	9,5822864	10,4177136	5
26	9,5429713	9,9717762	9,5711951	10,4288049	34	56	9,5530105	9,9703454	9,5826651	10,4173349	4
27	9,5433103	9,9717291	9,5715811	10,4284189	33	57	9,5533406	9,9702970	9,5830435	10,4169565	3
28	9,5436489	9,9716820	9,5719669	10,4280331	32	58	9,5536704	9,9702486	9,5834217	10,4165783	2
29	9,5439873	9,9716348	9,5723524	10,4276476	31	59	9,5539999	9,9702002	9,5837997	10,4162003	1
30	9,5443253	9,9715876	9,5727377	10,4272623	30	60	9,5543292	9,9701517	9,5841774	10,4158226	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	69 Degrees.						69 Degrees.				

Min.	21 Degrees.				Min.	Min.	21 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,5543292	9,9701517	9,5841774	10,4158226	60	30	9,5640754	9,9686779	9,5953975	10,4046025	30
1	9,5546581	9,9701032	9,5845549	10,4154451	59	31	9,5643960	9,9686281	9,5957679	10,4042321	29
2	9,5549868	9,9700547	9,5849321	10,4150679	58	32	9,5647163	9,9685783	9,5961380	10,4038620	28
3	9,5553152	9,9700061	9,5853091	10,4146909	57	33	9,5650363	9,9685284	9,5965079	10,4034921	27
4	9,5556433	9,9699574	9,5856859	10,4143141	56	34	9,5653561	9,9684785	9,5968776	10,4031224	26
5	9,5559711	9,9699087	9,5860624	10,4139376	55	35	9,5656756	9,9684286	9,5972470	10,4027530	25
6	9,5562987	9,9698600	9,5864386	10,4135614	54	36	9,5659948	9,9683786	9,5976162	10,4023838	24
7	9,5566259	9,9698112	9,5868147	10,4131853	53	37	9,5663137	9,9683285	9,5979852	10,4020148	23
8	9,5569529	9,9697624	9,5871904	10,4128096	52	38	9,5666324	9,9682784	9,5983540	10,4016460	22
9	9,5572796	9,9697136	9,5875660	10,4124340	51	39	9,5669508	9,9682283	9,5987225	10,4012775	21
10	9,5576060	9,9696647	9,5879413	10,4120587	50	40	9,5672689	9,9681781	9,5990908	10,4009092	20
11	9,5579321	9,9696158	9,5883163	10,4116837	49	41	9,5675868	9,9681279	9,5994588	10,4005412	19
12	9,5582579	9,9695668	9,5886912	10,4113088	48	42	9,5679044	9,9680777	9,5998267	10,4001733	18
13	9,5585835	9,9695177	9,5890657	10,4109343	47	43	9,5682217	9,9680274	9,6001943	10,3998057	17
14	9,5589088	9,9694687	9,5894401	10,4105599	46	44	9,5685387	9,9679771	9,6005617	10,3994383	16
15	9,5592338	9,9694196	9,5898142	10,4101858	45	45	9,5688555	9,9679267	9,6009289	10,3990711	15
16	9,5595585	9,9693704	9,5901881	10,4098119	44	46	9,5691721	9,9678763	9,6012958	10,3987042	14
17	9,5598829	9,9693212	9,5905617	10,4094383	43	47	9,5694883	9,9678258	9,6016625	10,3983375	13
18	9,5602071	9,9692720	9,5909351	10,4090649	42	48	9,5698043	9,9577753	9,6020290	10,3979710	12
19	9,5605310	9,9692227	9,5913082	10,4086918	41	49	9,5701200	9,9677247	9,6023953	10,3976047	11
20	9,5608546	9,9691734	9,5916812	10,4083188	40	50	9,5704355	9,9676741	9,6027613	10,3972387	10
21	9,5611779	9,9691241	9,5920539	10,4079461	39	51	9,5707506	9,9676235	9,6031271	10,3968729	9
22	9,5615010	9,9690746	9,5924263	10,4075737	38	52	9.5710656	9,9675728	9,6034927	10,3965073	8
23	9,5618237	9,9690252	9,5927985	10,4072015	37	53	9,5713802	9,9675221	9,6038581	10,3961419	7
24	9,5621462	9,9689757	9,5931705	10,4068295	36	54	9,5716946	9,9674713	9,6042233	10,3957767	6
25	9,5624685	9,9689262	9,5935423	10,4064577	35	55	9,5720087	9,9674205	9,6045882	10,3954118	5
26	9,5627904	9,9688760	9,5939138	10,4060862	34	56	9,5723226	9,9673697	9,6049529	10,3950471	4
27	9,5631121	9,9688270	9,5942851	10,4057149	33	57	9,5726362	9,9673188	9,6053174	10,3946826	3
28	9,5634335	9,9687773	9,5946561	10,4053439	32	58	9,5729495	9,9672679	9,6056817	10,3943183	2
29	9,5637546	9,9687276	9,5950269	10,4049731	31	59	9,5732626	9,9672169	9,6060457	10,3939543	1
30	9,5640754	9,9686779	9,5953975	10,4046025	30	60	9,5735754	9,9671659	9,6064096	10,3935904	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	68 Degrees.						68 Degrees.				

Min.	22 Degrees.				Min.	Min.	22 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,5735754	9,9671659	9,6064096	10,3935904	60	30	9,5828397	9,9656153	9,6172243	10,3827757	30
1	9,5738880	9,9671148	9,6067732	10,3932268	59	31	9,5831445	9,9655630	9,6175815	10,3824185	29
2	9,5742003	9,9670637	9,6071366	10,3928634	58	32	9,5834491	9,9655106	9,6179385	10,3820615	28
3	9,5745123	9,9670125	9,6074997	10,3925003	57	33	9,5837535	9,9654582	9,6182953	10,3817047	27
4	9,5748240	9,9669614	9,6078627	10,3921373	56	34	9,5840576	9,9654057	9,6186519	10,3813481	26
5	9,5751356	9,9669101	9,6082254	10,3917746	55	35	9,5843615	9,9653532	9,6190083	10,3809917	25
6	9,5754468	9,9668588	9,6085880	10,3914120	54	36	9,5846651	9,9653006	9,6193645	10,3806355	24
7	9,5757578	9,9668075	9,6089503	10,3910497	53	37	9,5849685	9,9652480	9,6197205	10,3802795	23
8	9,5760685	9,9667562	9,6093124	10,3906876	52	38	9,5852716	9,9651953	9,6200762	10,3799238	22
9	9,5763790	9,9667048	9,6096742	10,3903258	51	39	9,5855745	9,9651426	9,6204318	10,3795682	21
10	9,5766892	9,9666533	9,6100359	10,3899641	50	40	9,5858771	9,9650899	9,6207872	10,3792128	20
11	9,5769991	9,9666018	9,6103973	10,3896027	49	41	9,5861795	9,9650371	9,6211423	10,3788577	19
12	9,5773088	9,9665503	9,6107586	10,3892414	48	42	9,5864816	9,9649843	9,6214973	10,3785027	18
13	9,5776183	9,9664986	9,6111196	10,3888804	47	43	9,5867835	9,9649314	9,6218520	10,3781480	17
14	9,5779275	9,9664471	9,6114804	10,3885196	46	44	9,5870851	9,9648785	9,6222066	10,3777934	16
15	9.5782364	9,9663954	9,6118409	10,3881591	45	45	9,5873865	9,9648256	9,6225609	10,3774391	15
16	9,5785450	9,9663437	9,6122013	10,3877987	44	46	9,5876876	9,9647726	9,6229150	10,3770850	14
17	9,5788535	9,9662920	9,6125615	10,3874385	43	47	9,5879885	9,9647195	9,6232690	10,3767310	13
18	9,5791616	9,9662402	9,6129214	10,3870786	42	48	9,5882892	9,9646665	9,6236227	10,3763773	12
19	9,5794695	9,9661884	9,6132812	10,3867188	41	49	9,5885896	9,9646133	9,6239763	10,3760237	11
20	9,5797772	9,9661365	9,6136407	10,3863593	40	50	9,5888897	9,9645602	9,6243296	10,3756704	10
21	9,5800845	9,9660846	9,6140000	10,3860000	39	51	9,5891897	9,9645069	9,6246827	10,3753173	9
22	9,5803917	9,9660326	9,6143591	10,3856409	38	52	9,5894893	9,9644537	9,6250356	10,3749644	8
23	9,5806986	9,9659806	9,6147180	10,3852820	37	53	9,5897888	9,9644004	9,6253884	10,3746116	7
24	9,5810052	9,9659285	9,6150766	10,3849234	36	54	9,5900880	9,9643470	9,6257409	10,3742591	6
25	9,5813116	9,9658764	9,6154351	10,3845649	35	55	9,5903869	9,9642937	9,6260932	10,3739068	5
26	9,5816177	9,9658243	9,6157934	10,3842066	34	56	9,5906856	9,9642402	9,6264454	10,3735546	4
27	9,5819236	9,9657721	9,6161514	10,3838486	33	57	9,5909841	9,9641868	9,6267973	10,3732027	3
28	9,5822292	9,9657199	9,6165093	10,3834907	32	58	9,5912823	9,9641332	9,6271491	10,3728509	2
29	9,5825345	9,9656677	9,6168669	10,3831331	31	59	9,5915803	9,9640797	9,6275006	10,3724994	1
30	9,5828397	9,9656153	9,6172243	10,3827757	30	60	9,5918780	9,9640261	9,6278519	10,3721481	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	67 Degrees.						67 Degrees.				

Min.	23 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,5918780	9,9640261	9,6278519	10,3721481	60
1	9,5921755	9,9639724	9,6282031	10,3717969	59
2	9,5924728	9,9639187	9,6285540	10,3714460	58
3	9,5927698	9,9638650	9,6289048	10,3710952	57
4	9,5930666	9,9638112	9,6292553	10,3707447	56
5	9,5933631	9,9637574	9,6296057	10,3703943	55
6	9,5936594	9,9637036	9,6299558	10,3700442	54
7	9,5939555	9,9636496	9,6303058	10,3696942	53
8	9,5942513	9,9635957	9,6306556	10,3693444	52
9	9,5945469	9,9635417	9,6310052	10,3689948	51
10	9,5948422	9,9634877	9,6313545	10,3686455	50
11	9,5951373	9,9634336	9,6317037	10,3682963	49
12	9,5954322	9,9633795	9,6320527	10,3679473	48
13	9,5957268	9,9633253	9,6324015	10,3675985	47
14	9,5960212	9,9632711	9,6327501	10,3672499	46
15	9,5963154	9,9632168	9,6330985	10,3669015	45
16	9,5966093	9,9631625	9,6334468	10,3665532	44
17	9,5969030	9,9631082	9,6337948	10,3662052	43
18	9,5971965	9,9630538	9,6341426	10,3658574	42
19	9,5974897	9,9629994	9,6344903	10.3655097	41
20	9,5977827	9,9629449	9.6348378	10,3651622	40
21	9,5980754	9,9628904	9,6351850	10,3648150	39
22	9,5983679	9,9628358	9,6355321	10,3644679	38
23	9,5986602	9,9627812	9,6358790	10,3641210	37
24	9,5989523	9,9627266	9,6362257	10,3637743	36
25	9,5992441	9,9626719	9,6365722	10,3634278	35
26	9,5995357	9,9626172	9,6369185	10,3630815	34
27	9,5998270	9,9625624	9,6372646	10,3627354	33
28	9,6001181	9,9625076	9,6376106	10,3623894	32
29	9,6004090	9,9624527	9,6379563	10,3620437	31
30	9,6006997	9,9623978	9,6383019	10,3616981	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	66 Degrees.				

Min.	23 Degrees.				Min.
	Sine.	Sine. Com.	Tangent.	Tan. Comp.	
30	9,6006997	9,9623978	9,6383019	10,3616981	30
31	9,6009901	9,9623428	9,6386473	10,3613527	29
32	9,6012803	9,9622878	9,6389925	10,3610075	28
33	9,6015703	9,9622328	9,6393375	10,3606625	27
34	9,6018600	9,9621777	9,6396823	10,3603177	26
35	9,6021495	9,9621226	9,6400269	10,3599731	25
36	9,6024388	9,9620674	9,6403714	10,3596286	24
37	9,6027278	9,9620122	9,6407156	10,3592844	23
38	9,6030166	9,9619569	9,6410597	10,3589403	22
39	9,6033052	9,9619016	9,6414036	10,3585964	21
40	9,6035936	9,9618463	9,6417473	10,3582527	20
41	9,6038817	9,9617909	9,6420908	10,3579092	19
42	9,6041696	9,9617355	9,6424342	10,3575658	18
43	9,6044573	9,9616800	9,6427773	10,3572227	17
44	9,6047448	9,9616245	9,6431203	10,3568797	16
45	9,6050320	9,9615689	9,6434631	10,3565369	15
46	9,6053190	9,9615133	9,6438057	10,3561943	14
47	9,6056057	9,9614576	9,6441481	10,3558519	13
48	9,6058923	9,9614020	9,6444903	10,3555097	12
49	9,6061786	9,9613462	9,6448324	10,3551676	11
50	9,6064647	9,9612904	9,6451743	10,3548257	10
51	9,6067506	9,9612346	9,6455160	10,3544840	9
52	9,6070362	9,9611787	9,6458575	10,3541425	8
53	9,6073216	9,9611228	9,6461988	10,3538012	7
54	9,6076068	9,9610668	9,6465400	10,3534600	6
55	9,6078918	9,9610108	9,6468810	10,3531190	5
56	9,6081765	9,9609548	9,6472217	10,3527783	4
57	9,6084611	9,9608987	9,6475624	10,3524376	3
58	9,6087454	9,9608426	9,6479028	10,3520972	2
59	9,6090294	9,9607864	9,6482431	10,3517569	1
60	9,6093133	9,9607302	9,6485831	10,3514169	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	66 Degrees.				

Min.	24 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,6093133	9,9607302	9,6485831	10,3514169	60
1	9,6095969	9,9606739	9,6489230	10,3510770	59
2	9,6098803	9,9606176	9,6492628	10,3507372	58
3	9,6101635	9,9605612	9,6496023	10,3503977	57
4	9,6104465	9,9605048	9,6499417	10,3500583	56
5	9,6107293	9,9604484	9,6502809	10,3497191	55
6	9,6110118	9,9603919	9,6506199	10,3493801	54
7	9,6112941	9,9603354	9,6509587	10,3490413	53
8	9,6115762	9,9602788	9,6512974	10,3487026	52
9	9,6118580	9,9602222	9,6516359	10.3483641	51
10	9,6121397	9,9601655	9,6519742	10,3480258	50
11	9,6124211	9,9601088	9,6523123	10,3476877	49
12	9,6127023	9,9600520	9,6526503	10,3473497	48
13	9,6129833	9,9599952	9,6529881	10,3470119	47
14	9,6132641	9,9599384	9,6533257	10,3466743	46
15	9,6135446	9,9598815	9,6536631	10,3463369	45
16	9,6138250	9,9598246	9,6540004	10,3459996	44
17	9,6141051	9,9597676	9,6543375	10,3456625	43
18	9,6143850	9,9597106	9,6546744	10,3453256	42
19	9,6146647	9,9596535	9,6550112	10,3449888	41
20	9,6149441	9,9595964	9,6553477	10,3446523	40
21	9,6152234	9,9595393	9,6556841	10,3443159	39
22	9,6155024	9,9594821	9,6560204	10,3439796	38
23	9,6157812	9,9594248	9,6563564	10,3436436	37
24	9,6160599	9,9593675	9,6566923	10,3433077	36
25	9,6163382	9,9593102	9,6570280	10,3429720	35
26	9,6166164	9,9592528	9,6573636	10,3426364	34
27	9,6168944	9,9591954	9,6576989	10,3423011	33
28	9,6171741	9,9591380	9,6580341	10,3419659	32
29	9,6174496	9,9590805	9,6583692	10,3416308	31
30	9,6177270	9,9590229	9,6587041	10,3412959	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	65 Degrees.				

Min.	24 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,6177270	9,9590229	9,6587041	10,3412959	30
31	9,6180041	9,9589653	9,6590387	10,3409613	29
32	9,6182809	9,9589077	9,6593733	10,3406267	28
33	9,6185576	9,9588500	9,6597076	10.3402924	27
34	9,6188341	9,9587923	9,6600418	10,3399582	26
35	9,6191103	9,9587345	9,6603758	10,3396242	25
36	9,6193864	9,9586767	9,6607097	10,3392903	24
37	9,6196622	9,9586188	9,6610434	10,3389566	23
38	9,6199378	9,9585609	9,6613769	10,3386231	22
39	9,6202132	9,9585030	9,6617103	10,3382897	21
40	9,6204884	9,9584450	9,6620434	10,3379566	20
41	9,6207634	9,9583869	9,6623765	10,3376235	19
42	9,6210382	9,9583288	9,6627093	10,3372907	18
43	9,6213127	9,9582707	9,6630420	10,3369580	17
44	9,6215871	9,9582125	9,6633745	10,3366255	16
45	9,6218612	9,9581543	9,6637069	10,3362931	15
46	9,6221351	9,9580961	9,6640391	10,3359609	14
47	9,6224088	9,9580378	9,6643711	10,3356289	13
48	9,6226824	9,9579794	9,6647030	10,3352970	12
49	9,6229557	9,9579210	9,6650346	10,3349654	11
50	9,6232287	9,9578626	9,6653662	10,3346338	10
51	9,6235016	9,9578041	9,6656975	10,3343025	9
52	9,6237743	9,9577456	9,6660288	10,3339712	8
53	9,6240468	9,9576870	9,6663598	10,3336402	7
54	9,6243190	9,9576284	9,6666907	10,3333093	6
55	9,6245911	9,9575697	9,6670214	10.3329786	5
56	9,6248629	9,9575110	9,6673519	10,3326481	4
57	9,6251346	9,9574522	9,6676823	10,3323177	3
58	9,6254060	9,9573934	9,6680126	10,3319874	2
59	9,6256772	9,9573346	9,6683426	10,3316574	1
60	9,6259483	9,9572757	9,6686725	10,3313275	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	65 Degrees.				

Min.	25 Degrees.				Min.	Min.	25 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,6259483	9,9572757	9,6686725	10,3313275	60	30	9,6339844	9,9554882	9,6784961	10,3215039	30
1	9,6262191	9,9572168	9,6690023	10,3309977	59	31	9,6342491	9,9554280	9,6788211	10,3211789	29
2	9,6264897	9,9571578	9,6693319	10,3306681	58	32	9,6345137	9,9553676	9,6791460	10,3208540	28
3	9,6267601	9,9570988	9,6696613	10,3303387	57	33	9,6347780	9,9553073	9,6794708	10,3205292	27
4	9,6270303	9,9570397	9,6699906	10,3300094	56	34	9,6350422	9,9552469	9,6797953	10,3202047	26
5	9,6273003	9,9569806	9,6703197	10,3296803	55	35	9,6353062	9,9551864	9,6801198	10,3198802	25
6	9,6275701	9,9569215	9,6706486	10,3293514	54	36	9,6355699	9,9551259	9,6804440	10,3195560	24
7	9,6278397	9,9568623	9,6709774	10,3290226	53	37	9,6358335	9,9550653	9,6807682	10,3192318	23
8	9,6281090	9,9568030	9,6713060	10,3286940	52	38	9,6360969	9,9550047	9,6810921	10,3189079	22
9	9,6283782	9,9567437	9,6716345	10,3283655	51	39	9,6363601	9,9549441	9,6814160	10,3185840	21
10	9,6286472	9,9566844	9,6719628	10,3280372	50	40	9,6366231	9,9548834	9,6817396	10,3182604	20
11	9,6289160	9,9566250	9,6722910	10,3277090	49	41	9,6368859	9,9548227	9,6820632	10,3179368	19
12	9,6291845	9,9565656	9,6726190	10,3273810	48	42	9,6371484	9,9547619	9,6823865	10,3176135	18
13	9,6294529	9,9565061	9,6729468	10,3270532	47	43	9,6374108	9,9547011	9,6827098	10,3172902	17
14	9,6297211	9,9564466	9,6732745	10,3267255	46	44	9,6376731	9,9546402	9,6830328	10,3169672	16
15	9,6299890	9,9563870	9,6736020	10,3263980	45	45	9,6379351	9,9545793	9,6833557	10,3166443	15
16	9,6302568	9,9563274	9,6739294	10,3260706	44	46	9,6381969	9,9545184	9,6836785	10,3163215	14
17	9,6305243	9,9562678	9,6742566	10,3257434	43	47	9,6384585	9,9544574	9,6840011	10,3159989	13
18	9,6307917	9,9562081	9,6745836	10,3254164	42	48	9,6387199	9,9543963	9,6843236	10,3156764	12
19	9,6300589	9,9561483	9,6749105	10,3250895	41	49	9,6389812	9,9543352	9,6846459	10,3153541	11
20	9,6313258	9,9560886	9,6752372	10,3247628	40	50	9,6392422	9,9542741	9,6849681	10,3150319	10
21	9,6315926	9,9560287	9,6755638	10,3244362	39	51	9,6395030	9,9542129	9,6852901	10,3147099	9
22	9,6318591	9,9559689	9,6758903	10,3241097	38	52	9,6397637	9,9541517	9,6856120	10,3143880	8
23	9,6311255	9,9559089	9,6762165	10,3237835	37	53	9,6400241	9,9540904	9,6859338	10,3140662	7
24	9,6323916	9,9558490	9,6765426	10,3234574	36	54	9,6402844	9,9540291	9,6862553	10,3137447	6
25	9,6326576	9,9557890	9,6768686	10,3231314	35	55	9,6405445	9,9539677	9,6865768	10,3134232	5
26	9,6329233	9,9557289	9,6771944	10,3228056	34	56	9,6408044	9,9539063	9,6868981	10,3131019	4
27	9,6331889	9,9556688	9,6775201	10,3224799	33	57	9,6410640	9,9538448	9,6872192	10,3127808	3
28	9,6334542	9,9556087	9,6778456	10,3221544	32	58	9,6413235	9,9537833	9,6875402	10,3124598	2
29	9,6337194	9,9555485	9,6781709	10,3218291	31	59	9,6415828	9,9537218	9,6878611	10,3121389	1
30	9,6339844	9,9554882	9,6784961	10,3215039	30	60	9,6418420	9,9536602	9,6881818	10,3118182	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	64 Degrees.						64 Degrees.				

Min.	26 Degrees.				Min.	Min.	26 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,6418420	9,9536602	9,6881818	10,3118182	60	30	9,6495274	9,9517912	9,6977363	10,3022637	30
1	9,6421009	9,9535985	9,6885023	10,3114977	59	31	9,6497807	9,9517282	9,6980526	10,3019474	29
2	9,6423596	9,9535369	9,6888227	10,3111773	58	32	9,6500338	9,9516651	9,6983687	10,3016313	28
3	9,6426182	9,9534751	9,6891430	10,3108570	57	33	9,6502868	9,9516020	9,6986847	10,3013153	27
4	9,6428765	9,9534134	9,6894631	10,3105369	56	34	9,6505395	9,9515389	9,6990006	10,3009994	26
5	9,6431347	9,9533515	9,6897831	10,3102169	55	35	9,6507920	9,9514757	9,6993164	10,3006836	25
6	9,6433926	9,9532897	9,6901030	10,3098970	54	36	9,6510444	9,9514124	9,6996320	10,3003680	24
7	9,6436504	9,9532278	9,6904226	10,3095774	53	37	9,6512966	9,9513492	9,6999474	10,3000526	23
8	9,6439080	9,9531658	9,6907422	10,3092578	52	38	9,6515486	9,9512858	9,7002628	10,2997372	22
9	9,6441654	9,9531038	9,6910616	10,3089384	51	39	9,6518004	9,9512224	9,7005780	10,2994220	21
10	9,6444226	9,9530418	9,6913809	10,3086191	50	40	9,6520521	9,9511590	9,7008930	10,2991070	20
11	9,6446796	9,9529797	9,6917000	10,3083000	49	41	9,6523035	9,9510956	9,7012080	10,2987920	19
12	9,6449365	9,9529175	9,6920189	10,3079811	48	42	9,6525548	9,9510320	9,7015227	10,2984773	18
13	9,6451931	9,9528553	9,6923378	10,3076622	47	43	9,6528059	9,9509685	9,7018374	10,2981626	17
14	9,6454496	9,9527931	9,6926565	10,3073435	46	44	9,6530568	9,9509049	9,7021519	10,2978481	16
15	9,6457058	9,9527308	9,6939750	10,3070250	45	45	9,6533075	9,9508412	9,7024663	10,2975337	15
16	9,6459619	9,9526685	9,6932934	10,3067066	44	46	9,6535581	9,9507775	9,7027805	10,2972195	14
17	9,6462178	9,9526061	9,6936117	10,3063883	43	47	9,6538084	9,9507138	9,7030946	10,2969054	13
18	9,6464735	9,9525437	9,6939298	10,3060702	42	48	9,6540586	9,9506500	9,7034086	10,2965914	12
19	9,6467290	9,9524813	9,6942478	10,3057522	41	49	9,6543086	9,9505861	9,7037225	10,2962775	11
20	9,6469844	9,9524188	9,6945656	10,3054344	40	50	9,6545584	9,9505223	9,7040362	10,2959638	10
21	9,6472395	9,9523562	9,6948833	10,3051167	39	51	9,6548081	9,9504583	9,7043497	10,2956503	9
22	9,6474945	9,9522936	9,6952009	10,3047991	38	52	9,6550575	9,9503944	9,7046632	10,2953368	8
23	9,6477492	9,9522310	9,6955183	10,3044817	37	53	9,6553068	9,9503303	9,7049765	10,2950235	7
24	9,6480038	9,9521683	9,6958355	10,3041645	36	54	9,6555559	9,9502663	9,7052897	10,2947103	6
25	9,6482582	9,9521055	9,6961527	10,3038473	35	55	9,6558048	9,9502022	9,7056027	10,2943973	5
26	9,6485124	9,9520428	9,6964697	10,3035303	34	56	9,6560536	9,9501380	9,7059150	10,2940844	4
27	9,6487665	9,9519799	9,6967865	10,3032135	33	57	9,6563021	9,9500738	9,7062284	10,2937716	3
28	9,6490203	9,9519171	9,6971032	10,3028968	32	58	9,6565505	9,9500095	9,7065410	10,2934590	2
29	9,6492740	9,9518541	9,6974198	10,3025802	31	59	9,6567987	9,9499452	9,7068535	10,2931465	1
30	9,6495274	9,9517912	9,6977363	10,3022637	30	60	9,6570468	9,9498809	9,7071659	10,2928341	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	63 Degrees.						63 Degrees.				

Min.	27 Degrees.				Min.	Min.	27 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,6570468	9,9498809	9,7071659	10,2928341	60	30	9,6644056	9,9479287	9,7164767	10,2835233	30
1	9,6572946	9,9498165	9,7074781	10,2925219	59	31	9,6646482	9,9478631	9,7167851	10,2832149	29
2	9,6575423	9,9497521	9,7077902	10,2922098	58	32	9,6648906	9,9477973	9,7170933	10,2829067	28
3	9,6577898	9,9496876	9,7081022	10,2918978	57	33	9,6651329	9,9477314	9,7174014	10,2825986	27
4	9,6580371	9,9496230	9,7084141	10,2915859	56	34	9,6653749	9,9476655	9,7177095	10,2822906	26
5	9,6582842	9,9495585	9,7087258	10,2912742	55	35	9,6656168	9,9475995	9,7189173	10,2819827	25
6	9,6585312	9,9494938	9,7090374	10,2909626	54	36	9,6658586	9,9475335	9,7183251	10,2816749	24
7	9,6587780	9,9494292	9,7093488	10,2906512	53	37	9,6661001	9,9474674	9,7186327	10,2813673	23
8	9,6590246	9,9493645	9,7096601	10,2903399	52	38	9,6663415	9,9474013	9,7189402	10,2810598	22
9	9,6592710	9,9492997	9,7099713	10,2905287	51	39	9,6665828	9,9473352	9,7192476	10,2807524	21
10	9,6595173	9,9492349	9,7102824	10,2897176	50	40	9,6668238	9,9472689	9,7195549	10,2804451	20
11	9,6597634	9,9491700	9,7105933	10,2894067	49	41	9,6670647	9,9472027	9,7198620	10,2801380	19
12	9,6600093	9,9491051	9,7109041	10,2890959	48	42	9,6673054	9,9471364	9,7201690	10,2798310	18
13	9,6602530	9,9490402	9,7112148	10,2887852	47	43	9,6675459	9,9470700	9,7204759	10,2795241	17
14	9,6605005	9,9489752	9,7115254	10,2884746	46	44	9,6677863	9,9470036	9,7207827	10,2792173	16
15	9,6607459	9,9489101	9,7118358	10,2881642	45	45	9,6680265	9,9469372	9,7210893	10,2789107	15
16	9,6609911	9,9488450	9,7121461	10,2878539	44	46	9,6682665	9,9468707	9,7213958	10,2786042	14
17	9,6612361	9,9487799	9,7124562	10,2875438	43	47	9,6685064	9,9468042	9,7217022	10,2782978	13
18	9,6614810	9,9487147	9,7127662	10,2872338	42	48	9,6687461	9,9467376	9,7220085	10,2779915	12
19	9,6617257	9,9486495	9,7130761	10,2869239	41	49	9,6689856	9,9466710	9,7223147	10,2776853	11
20	9,6619702	9,9485842	9,7133859	10,2866141	40	50	9,6692252	9,9466043	9,7226207	10,2773793	10
21	9,6622145	9,9485189	9,7136956	10,2863044	39	51	9,6694642	9,9465376	9,7229266	10,2770734	9
22	9,6624586	9,9484535	9,7140051	10,2859949	38	52	9,6697032	9,9464708	9,7232324	10,2767676	8
23	9,6627026	9,9483881	9,7143145	10,2856855	37	53	9,6699420	9,9464040	9,7235381	10,2764619	7
24	9,6629464	9,9483227	9,7146237	10,2853763	36	54	9,6701807	9,9463371	9,7238436	10,2761564	6
25	9,6631900	9,9482572	9,7149329	10,2850671	35	55	9,6794192	9,9462702	9,7241490	10,2758510	5
26	9,6634335	9,9481916	9,7152419	10,2847581	34	56	9,6706576	9,9462032	9,7244543	10,2755457	4
27	9,6636768	9,9481260	9,7155508	10,2844492	33	57	9,6708958	9,9461362	9,7247595	10,2752405	3
28	9,6639199	9,9480604	9,7158595	10,2841405	32	58	9,6711338	9,9460692	9,7250646	10,2749354	2
29	9,6641628	9,9479947	9,7161682	10,2838318	31	59	9,6713716	9,9460021	9,7253695	10,2746305	1
30	9,6644056	9,9479289	9,7164767.	10,2835233	30	60	9,6716093	9,9459349	9,7256774	10,2743256	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	62 Degrees.						62 Degrees.				

Min.	28 Degrees.				Min.	Min.	28 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,6716093	9,9459349	9,7256744	10,2743256	60	30	9,6786629	9,9438985	9,7347644	10,2652356	30
1	9,6718468	9,9458677	9,7259791	10,2740209	59	31	9,6788955	9,9438299	9,7350656	10,2649344	29
2	9,6720841	9,9458005	9,7262837	10,2737163	58	32	9,6791279	9,9437612	9,7353667	10,2646333	28
3	9,6723213	9,9457332	9,7265881	10,2734119	57	33	9,6793602	9,9436925	9,7356677	10,2643323	27
4	9,6725583	9,9456659	9,7268925	10,2731075	56	34	9,6795923	9,9436238	9,7359685	10,2640315	26
5	9,6727952	9,9455985	9,7271967	10,2728033	55	35	9,6798243	9,9435549	9,7362693	10,2637307	25
6	9,6730319	9,9455310	9,7275008	10,2724992	54	36	9,6800560	9,9434861	9,7365699	10,2634301	24
7	9,6732684	9,9454636	9,7278048	10,2721952	53	37	9,6802877	9,9434172	9,7368705	10,2631295	23
8	9,6735047	9,9453960	9,7281087	10,2718913	52	38	9,6805191	9,9433482	9,7371709	10,2628291	22
9	9,6737409	9,9453285	9,7284124	10,2715876	51	39	9,6807504	9,9432792	9,7374712	10,2625288	21
10	9,6739769	9,9452609	9,7287161	10,2712839	50	40	9,6809816	9,9432102	9,7377714	10,2622286	20
11	9,6742128	9,9451932	9,7290196	10,2709804	49	41	9,6812126	9,9431411	9,7380715	10,2619285	19
12	9,6744485	9,9451255	9,7293230	10,2706770	48	42	9,6814434	9,9430720	9,7383714	10,2616286	18
13	9,6746840	9,9450577	9,7296263	10,2703737	47	43	9,6816741	9,9430028	9,7386713	10,2613287	17
14	9,6749194	9,9449899	9,7299295	10,2700705	46	44	9,6819046	9,9429335	9,7389710	10,2610290	16
15	9,6751546	9,9449220	9,7302325	10,2697675	45	45	9,6821349	9,9428643	9,7392707	10,2607293	15
16	9,6753896	9,9448541	9,7305354	10,2694646	44	46	9,6823651	9,9427949	9,7395702	10,2604298	14
17	9,6756245	9,9447862	9,7308383	10,2691617	43	47	9,6825952	9,9427255	9,7398696	10,2601304	13
18	9,6758592	9,9447182	9,7311410	10,2688590	42	48	9,6828250	9,9426561	9,7401689	10,2598311	12
19	9,6760937	9,9446501	9,7314436	10,2685564	41	49	9,6830548	9,9425866	9,7404681	10,2595319	11
20	9,6763281	9,9445821	9,7317460	10,2682540	40	50	9,6832843	9,9425171	9,7407672	10,2592328	10
21	9,6765623	9,9445139	9,7320484	10,2679516	39	51	9,6835137	9,9424476	9,7410662	10,2589338	9
22	9,6767963	9,9444457	9,7323506	10,2676494	38	52	9,6837430	9,9423779	9,7413650	10,2586350	8
23	9,6770302	9,9443775	9,7326527	10,2673473	37	53	9,6839720	9,9423083	9,7416638	10,2583362	7
24	9,6772640	9,9443092	9,7329547	10,2670453	36	54	9,6842010	9,9422386	9,7419624	10,2580376	6
25	9,6774975	9,9442409	9,7332566	10,2667434	35	55	9,6844297	9,9421688	9,7422609	10,2577391	5
26	9,6777309	9,9441725	9,7335584	10,2664416	34	56	9,6846583	9,9420990	9,7425594	10,2574406	4
27	9,6779642	9,9441041	9,7338601	10,2661399	33	57	9,6848868	9,9420291	9,7428577	10,2571423	3
28	9,6781972	9,9440356	9,7341616	10,2658384	32	58	9,6851151	9,9419592	9,7431559	10,2568441	2
29	9,6784301	9,9439671	9,7344631	10,2655369	31	59	9,6853432	9,9418893	9,7434540	10,2565460	1
30	9,6786629	9,9438985	9,7347644	10,2652356	30	60	9,6855712	9,9418193	9,7437520	10,2562480	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	61 Degrees.						61 Degrees.				

Min.	29 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,6855712	9,9418193	9,7437520	10,2562480	60
1	9,6857991	9,9417492	9,7440499	10,2559501	59
2	9,6860267	9,9416791	9,7443476	10,2556524	58
3	9,6862542	9,9416090	9,7446453	10,2553547	57
4	9,6864816	9,9415388	9,7449428	10,2550572	56
5	9,6867088	9,9414685	9,7452403	10,2547597	55
6	9,6869359	9,9413982	9,7455376	10,2544624	54
7	9,6871628	9,9413279	9,7458349	10,2541651	53
8	9,6873895	9,9412575	9,7461320	10,2538680	52
9	9,6876161	9,9411871	9,7464290	10,2535710	51
10	9,6878425	9,9411166	9,7467259	10,2532741	50
11	9,6880688	9,9410461	9,7470227	10,2529773	49
12	9,6882949	9,9409755	9,7473194	10,2526806	48
13	9,6885209	9,9409048	9,7476160	10,2523840	47
14	9,6887467	9,9408342	9,7479125	10,2520875	46
15	9,6889723	9,9407634	9,7482089	10,2517911	45
16	9,6891978	9,9406927	9,7485052	10,2514948	44
17	9,6894232	9,9406219	9,7488013	10,2511987	43
18	9,6896484	9,9405510	9,7490974	10,2509026	42
19	9,6898734	9,9404801	9,7493934	10,2506066	41
20	9,6900983	9,9404091	9,7496892	10,2503108	40
21	9,6903231	9,9403381	9,7499850	10,2500150	39
22	9,6905476	9,9402670	9,7502806	10,2497194	38
23	9,6907721	9,9401959	9,7505762	10,2494238	37
24	9,6909964	9,9401248	9,7508716	10,2491284	36
25	9,6912205	9,9400535	9,7511669	10,2488331	35
26	9,6914445	9,9399823	9,7514622	10,2485378	34
27	9,6916683	9,9399110	9,7517573	10,2482427	33
28	9,6918919	9,9398396	9,7520523	10,2479477	32
29	9,6921155	9,9397682	9,7523472	10,2476528	31
30	9,6923388	9,9396968	9,7526420	10,2473580	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	60 Degrees.				

Min.	29 Degrees.				Min.
	Sine.	Sine. Com.	Tangent.	Tan. Comp.	
30	9,6923388	9,9396968	9,7526420	10,2473580	30
31	9,6925620	9,9396253	9,7529368	10,2470632	29
32	9,6927851	9,9395537	9,7532314	10,2467686	28
33	9,6930080	9,9394821	9,7535259	10,2464741	27
34	9,6932308	9,9394105	9,7538203	10,2461797	26
35	9,6934534	9,9393388	9,7541146	10,2458854	25
36	9,6936758	9,9392671	9,7544088	10,2455912	24
37	9,6938981	9,9391953	9,7547029	10,2452971	23
38	9,6941203	9,9391234	9,7549969	10,2450031	22
39	9,6943423	9,9390515	9,7552908	10,2447092	21
40	9,6945642	9,9389796	9,7555846	10,2444154	20
41	9,6947859	9,9389076	9,7558783	10,2441217	19
42	9,6950074	9,9388356	9,7561718	10,2438282	18
43	9,6952288	9,9387635	9,7564653	10,2435347	17
44	9,6954501	9,9386914	9,7567587	10,2432413	16
45	9,6956712	9,9386192	9,7570520	10,2429480	15
46	9,6958922	9,9385470	9,7573452	10,2426548	14
47	9,6961130	9,9384747	9,7576383	10,2423617	13
48	9,6963336	9,9384024	9,7579313	10,2420689	12
49	9,6965541	9,9383300	9,7582242	10,2417758	11
50	9,6967745	9,9382576	9,7585170	10,2414830	10
51	9,6969947	9,9381851	9,7588096	10,2411904	9
52	9,6972148	9,9381126	9,7591022	10,2408978	8
53	9,6974347	9,9380400	9,7593947	10,2406053	7
54	9,6976545	9,9379674	9,7596871	10,2403129	6
55	9,6978741	9,9378947	9,7599794	10,2400206	5
56	9,6980936	9,9378220	9,7602716	10,2397284	4
57	9,6983129	9,9377492	9,7605637	10,2394363	3
58	9,6985321	9,9376764	9,7608557	10,2391443	2
59	9,6987511	9,9376035	9,7611476	10,2388524	1
60	9,6989700	9,9375306	9,7614394	10,2385606	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	60 Degrees.				

Min.	30 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,6989700	9,9375306	9,7614394	10,2385606	60
1	9,6991887	9,9374577	9,7617311	10,2382689	59
2	9,6994073	9,9373847	9,7620227	10,2379773	58
3	9,6996258	9,9373116	9,7623142	10,2376858	57
4	9,6998441	9,9372385	9,7626056	10,2373944	56
5	9,7000622	9,9371653	9,7628969	10,2371031	55
6	9,7002802	9,9370921	9,7631881	10,2368119	54
7	9,7004981	9,9370189	9,7634792	10,2365208	53
8	9,7007158	9,9369456	9,7637702	10,2362298	52
9	9,7009334	9,9368722	9,7640612	10,2359388	51
10	9,7011508	9,9367988	9,7643520	10,2356480	50
11	9,7013681	9,9367254	9,7646427	10,2353573	49
12	9,7015852	9,9366519	9,7649334	10,2350666	48
13	9,7018022	9,9365783	9,7652239	10,2347761	47
14	9,7020190	9,9365047	9,7655143	10,2344857	46
15	9,7022357	9,9364311	9,7658047	10,2341953	45
16	9,7024523	9,9363574	9,7660949	10,2339051	44
17	9,7026687	9,9362836	9,7663851	10,2336149	43
18	9,7028849	9,9362098	9,7666751	10,2333249	42
19	9,7031011	9,9361360	9,7669651	10,2330349	41
20	9,7033170	9,9360621	9,7672550	10,2327450	40
21	9,7035329	9,9359881	9,7675448	10,2324552	39
22	9,7037486	9,9359141	9,7678344	10,2321656	38
23	9,7039641	9,9358401	9,7681240	10,2318760	37
24	9,7041795	9,9357660	9,7684135	10,2315865	36
25	9,7043947	9,9356918	9,7687029	10,2312971	35
26	9,7046099	9,9356177	9,7689922	10,2310078	34
27	9,7048248	9,9355434	9,7692814	10,2307186	33
28	9,7050397	9,9354691	9,7695705	10,2304295	32
29	9,7052543	9,9353948	9,7698596	10,2301404	31
30	9,7054689	9,9353204	9,7701485	10,2298515	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	59 Degrees.				

Min.	30 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan Comp.	
30	9,7054689	9,9353204	9,7701485	10,2298515	30
31	9,7056833	9,9352459	9,7704373	10,2295627	29
32	9,7058975	9,9351715	9,7707261	10,2292739	28
33	9,7061116	9,9350969	9,7710147	10,2289853	27
34	9,7063256	9,9350223	9,7713033	10,2286967	26
35	9,7065394	9,9349477	9,7715917	10,2284083	25
36	9,7067131	9,9348730	9,7718801	10,2281199	24
37	9,7069667	9,9347983	9,7721684	10,2278316	23
38	9,7071801	9,9347235	9,7724566	10,2275434	22
39	9,7073933	9,9346486	9,7727447	10,2272553	21
40	9,7076064	9,9345738	9,7730327	10,2269673	20
41	9,7078194	9,9344988	9,7733206	10,2266794	19
42	9,7080323	9,9344238	9,7736084	10,2263916	18
43	9,7082450	9,9343488	9,7738961	10,2261039	17
44	9,7084575	9,9342737	9,7741838	10,2258162	16
45	9,7086699	9,9341986	9,7744713	10,2255287	15
46	9,7088822	9,9341234	9,7747588	10,2252412	14
47	9,7090943	9,9340482	9,7750462	10,2249538	13
48	9,7093063	9,9339729	9,7753334	10,2246666	12
49	9,7095182	9,9338976	9,7756206	10,2243794	11
50	9,7097299	9,9338222	9,7759077	10,2240923	10
51	9,7099415	9,9337467	9,7761947	10,2238053	9
52	9,7101529	9,9336713	9,7764816	10,2235184	8
53	9,7103642	9,9335957	9,7767685	10,2232315	7
54	9,7105753	9,9335201	9,7770552	10,2229448	6
55	9,7107863	9,9334445	9,7773418	10,2226582	5
56	9,7109972	9,9333688	9,7776284	10,2223716	4
57	9,7112080	9,9332931	9,7779149	10,2220851	3
58	9,7114186	9,9332173	9,7782012	10,2217988	2
59	9,7116290	9,9331415	9,7784875	10,2215125	1
60	9,7118393	9,9330656	9,7787737	10,2212263	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	59 Degrees				

Min.	31 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7118393	9,9330656	9,7787737	10,2212263	60
1	9,7120495	9,9329897	9,7790599	10,2209401	59
2	9,7122596	9,9329137	9,7793459	10,2206541	58
3	9,7124695	9,9328376	9,7796318	10,2203682	57
4	9,7126792	9,9327616	9,7799177	10,2200823	56
5	9,7128889	9,9326854	9,7802034	10,2197966	55
6	9,7130983	9,9326092	9,7804891	10,2195109	54
7	9,7133077	9,9325330	9,7807747	10,2192253	53
8	9,7135169	9,9324567	9,7810602	10,2189398	52
9	9,7137260	9,9323804	9,7813456	10,2186544	51
10	9,7139349	9,9323040	9,7816309	10,2183691	50
11	9,7141437	9,9322276	9,7819162	10,2180838	49
12	9,7143524	9,9321511	9,7822013	10,2177987	48
13	9,7145609	9,9320746	9,7824864	10,2175136	47
14	9,7147693	9,9319980	9,7827713	10,2172287	46
15	9,7149776	9,9319213	9,7830562	10,2169430	45
16	9,7151857	9,9318447	9,7833410	10,2166598	44
17	9,7153937	9,9317679	6,7836258	10,2163742	43
18	9,7156015	9,9316911	9,7839104	10,2160896	42
19	9,7158092	9,9316143	9,7841949	10,2158051	41
20	9,7160168	9,9315374	9,7844794	10,2155206	40
21	9,7162243	9,9314605	9,7847638	10,2152362	39
22	9,7164316	9,9313835	9,7850481	10,2149519	38
23	9,7166387	9,9313065	9,7853323	10,2146677	37
24	9,7168458	9,9312294	9,7856164	10,2143836	36
25	9,7170526	9,9311522	9,7859004	10,2140996	35
26	9,7172594	9,9310750	9,7861844	10,2138156	34
27	9,7174660	9,9309978	9,7864682	10,2135318	33
28	9,7176725	9,9309205	9,7867520	10,2132480	32
29	9,7178789	9,9308432	9,7870357	10,2129643	31
30	9,7180851	9,9307658	9,7873193	10,2126807	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	58 Degrees.				

Min.	31 Degrees.				Min.
	Sine.	Sine. Com.	Tangent.	Tan. Comp.	
30	9,7180851	9,9307658	9,7873193	10,2126807	30
31	9,7182912	9,9306883	9,7876028	10,2123972	29
32	9,7184971	9,9306109	9,7878863	10,2121137	28
33	9,7187030	9,9305333	9,7881696	10,2118304	27
34	9,7189086	9,9304557	9,7884529	10,2115471	26
35	9,7191142	9,9303781	9,7887361	10,2112639	25
36	9,7193196	9,9303004	9,7890192	10,2109808	24
37	9,7195249	9,9302226	9,7893023	10,2106977	23
38	9,7197300	9,9301448	9,7895852	10,2104148	22
39	9,7199350	9,9300670	9,7898681	10,2101319	21
40	9,7201399	9,9299891	9,7901508	10,2098492	20
41	9,7203447	9,9299112	9,7904335	10,2095665	19
42	9,7205493	9,9298332	9,7907161	10,2092839	18
43	9,7207538	9,9297551	9,7909987	10,2090013	17
44	9,7209581	9,9296770	9,7912811	10,2087189	16
45	9,7211623	9,9295989	9,7915635	10,2084365	15
46	9,7213664	9,9295207	9,7918458	10,2081542	14
47	9,7215704	9,9294424	9,7921280	10,2078720	13
48	9,7217742	9,9293641	9,7924101	10,2075899	12
49	9,7219779	9,9292857	9,7926921	10,2073079	11
50	9,7221814	9,9292073	9,7929741	10,2070259	10
51	9,7223848	9,9291289	9,7932560	10,2067440	9
52	9,7225881	9,9290504	9,7935378	10,2064622	8
53	9,7227913	9,9289718	9,7938195	10,2061805	7
54	9,7229943	9,9288932	9,7941011	10,2058989	6
55	9,7231972	9,9288145	9,7943827	10,2056173	5
56	9,7234000	9,9287358	9,7946641	10,2053359	4
57	9,7236026	9,9286571	9,7949455	10,2050545	3
58	9,7238051	9,9285783	9,7952268	10,2047732	2
59	9,7240075	9,9284994	9,7955081	10,2044919	1
60	9,7242097	9,9284205	9,7957892	10,2042108	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	58 Degrees.				

Min.	32 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7242097	9,9284205	9,7957892	10,2042108	60
1	9,7244118	9,9283415	9,7960703	10,2039297	59
2	9,7246138	9,9282625	9,7963513	10,2036487	58
3	9,7248156	9,9281834	9,7966322	10,2033678	57
4	9,7250174	9,9281043	9,7969130	10,2030870	56
5	9,7252189	9,9280251	9,7971938	10,2028062	55
6	9,7254204	9,9279459	9,7974745	10,2025255	54
7	9,7256217	9,9278666	9,7977551	10,2022449	53
8	9,7258229	9,9277873	9,7980356	10,2019644	52
9	9,7260240	9,9277079	9,7983160	10,2016840	51
10	9,7262249	9,9276285	9,7985964	10,2014036	50
11	9,7264257	9,9275490	9,7988767	10,2011233	49
12	9,7266264	9,9274695	9,7991569	10,2008431	48
13	9,7268269	9,9273899	9,7994370	10,2005630	47
14	9,7270273	9,9273103	9,7997170	10,2002830	46
15	9,7272276	9,9272306	9,7999970	10,2000030	45
16	9,7274278	9,9271509	9,8002769	10,1997231	44
17	9,7276278	9,9270711	9,8005567	10,1994433	43
18	9,7278277	9,9269913	9,8008365	10,1991635	42
19	9,7280275	9,9269114	9,8011161	10,1988839	41
20	9,7282271	9,9268314	9,8013957	10,1986043	40
21	9,7284267	9,9267514	9,8016752	10,1983248	39
22	9,7286260	9,9266714	9,8019546	10,1980454	38
23	9,7288253	9,9265913	9,8022340	10,1977660	37
24	9,7290244	9,9265112	9,8025133	10,1974867	36
25	9,7292234	9,9264310	9,8027925	10,1972075	35
26	9,7294223	9,9263507	9,8030716	10,1969284	34
27	9,7296211	9,9262704	9,8033506	10,1966494	33
28	9,7298197	9,9261901	9,8036296	10,1963704	32
29	9,7300182	9,9261096	9,8039085	10,1960915	31
30	9,7302165	9,9260292	9,8041873	10,1958127	30
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	57 Degrees.				

Min.	32 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.	
30	9,7302165	9,9260292	9,8041873	10,1958127	30
31	9,7304148	9,9259487	9,8044661	10,1955339	29
32	9,7306129	9,9258681	9,8047447	10,1952553	28
33	9,7308109	9,9257875	9,8050233	10,1949767	27
34	9,7310087	9,9257069	9,8053019	10,1946981	26
35	9,7312064	9,9256261	9,8055803	10,1944197	25
36	9,7314040	9,9255454	9,8058587	10,1941413	24
37	9,7316015	9,9254646	9,8061370	10,1938630	23
38	9,7317989	9,9253837	9,8064152	10,1935848	22
39	9,7319961	9,9253028	9,8066933	10,1933067	21
40	9,7321932	9,9252218	9,8069714	10,1930286	20
41	9,7323902	9,9251408	9,8072494	10,1927506	19
42	9,7325870	9,9250597	9,8075273	10,1924727	18
43	9,7327837	9,9249786	9,8078052	10,1921948	17
44	9,7329803	9,9248974	9,8080829	10,1919171	16
45	9,7331768	9,9248161	9,8083606	10,1916394	15
46	9,7333731	9,9247349	9,8086383	10,1913617	14
47	9,7335693	9,9246535	9,8089158	10,1910842	13
48	9,7337654	9,9245721	9,8091933	10,1908067	12
49	9,7339614	9,9244907	9,8094707	10,1905293	11
50	9,7341572	9,9244092	9,8097480	10,1902520	10
51	9,7343529	9,9243277	9,8100253	10,1899747	9
52	9,7345485	9,9242461	9,8103025	10,1896975	8
53	9,7347440	9,9241644	9,8105796	10,1894204	7
54	9,7349393	9,9240827	9,8108566	10,1891434	6
55	9,7351345	9,9240010	9,8111336	10,1888664	5
56	9,7353296	9,9239191	9,8114105	10,1885895	4
57	9,7355246	9,9238373	9,8116873	10,1883127	3
58	9,7357195	9,9237554	9,8119641	10,1880359	2
59	9,7359142	9,9236734	9,8122404	10,1877592	1
60	9,7361088	9,9235914	9,8125174	10,1874826	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	57 Degrees.				

Min.	33 Degrees.				Min.	Min.	33 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,7361088	9,9235914	9,8125174	10,1874826	60	30	9,7418895	9,9211066	9,8207829	10,1792171	30
1	9,7363032	9,9235093	9,8127939	10,1872061	59	31	9,7420803	9,9210229	9,8210574	10,1789426	29
2	9,7364976	9,9234272	9,8130704	10,1869296	58	32	9,7422710	9,9209393	9,8213317	10,1786683	28
3	9,7366918	9,9233450	9,8133468	10,1866532	57	33	9,7424616	9,9208555	9,8216060	10,1783940	27
4	9,7368859	9,9232628	9,8136231	10,1863769	56	34	9,7426520	9,9207717	9,8218803	10,1781197	26
5	9,7370799	9,9231805	9,8138993	10,1861007	55	35	9,7428423	9,9206878	9,8221545	10,1778455	25
6	9,7372737	9,9230982	9,8141755	10,1858245	54	36	9,7430325	9,9206039	9,8224286	10,1775714	24
7	9,7374675	9,9230158	9,8144516	10,1855484	53	37	9,7432226	9,9205200	9,8227026	10,1772974	23
8	9,7376611	9,9229334	9,8147277	10,1852723	52	38	9,7434126	9,9204360	9,8229766	10,1770234	22
9	9,7378546	9,9228509	9,8150036	10,1849964	51	39	9,7436024	9,9203519	9,8232505	10,1767495	21
10	9,7380479	9,9227684	9,8152795	10,1847205	50	40	9,7437921	9,9202678	9,8235244	10,1764756	20
11	9,7382412	9,9226858	9,8155554	10,1844446	49	41	9,7439817	9,9201836	9,8237981	10,1762019	19
12	9,7384343	9,9226032	9,8158311	10,1841689	48	42	9,7441712	9,9200994	9,8240719	10,1759281	18
13	9,7386273	9,9225205	9,8161068	10,1838932	47	43	9,7443606	9,9200151	9,8243455	10,1756545	17
14	9,7388201	9,9224377	9,8163824	10,1836176	46	44	9,7445498	9,9199308	9,8246191	10,1753809	16
15	9,7390129	9,9223549	9,8166580	10,1833420	45	45	9,7447390	9,9198464	9,8248926	10,1751074	15
16	9,7392055	9,9222721	9,8169335	10,1830665	44	46	9,7449280	9,9197619	9,8251660	10,1748340	14
17	9,7393980	9,9221891	9,8172089	10,1827911	43	47	9,7451169	9,9196775	9,8254394	10,1745606	13
18	9,7395904	9,9221062	9,8174842	10,1825158	42	48	9,7453056	9,9195929	9,8257127	10,1742873	12
19	9,7397827	9,9220232	9,8177595	10,1822405	41	49	9,7454943	9,9195083	9,8259860	10,1740140	11
20	9,7399748	9,9219401	9,8180347	10,1819653	40	50	9,7456828	9,9194237	9,8262592	10,1737408	10
21	9,7401668	9,9218570	9,8183098	10,1816902	39	51	9,7458712	9,9193390	9,8265323	10,1734677	9
22	9,7403587	9,9217738	9,8185849	10,1814151	38	52	9,7460595	9,9192542	9,8268053	10,1731947	8
23	9,7405505	9,9216906	9,8188599	10,1811401	37	53	9,7462477	9,9191694	9,8270783	10,1729217	7
24	9,7407421	9,9216073	9,8191348	10,1808652	36	54	9,7464358	9,9190845	9,8273513	10,1726487	6
25	9,7409337	9,9215240	9,8194096	10,1805904	35	55	9,7466237	9,9189996	9,8276241	10,1723759	5
26	9,7411251	9,9214406	9,8196844	10,1803156	34	56	9,7468115	9,9189146	9,8278969	10,1721031	4
27	9,7413164	9,9213572	9,8199592	10,1800408	33	57	9,7469992	9,9188296	9,8281696	10,1718304	3
28	9,7415075	9,9212737	9,8202338	10,1797662	32	58	9,7471868	9,9187445	9,8284423	10,1715577	2
29	9,7416986	9,9211902	9,8205084	10,1794916	31	59	9,7473743	9,9186594	9,8287149	10,1712851	1
30	9,7418895	9,9211066	9,8207829	10,1792171	30	60	9,7475617	9,9185742	9,8289874	10,1710126	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	56 Degrees.						56 Degrees.				

Min.	34 Degrees.				Min.	Min.	34 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7475617	9,9185742	9,8289874	10,1710126	60	30	9,7531230	9,9159937	9,8371343	10,1628657	30
1	9,7477489	9,9184890	9,8292599	10,1707401	59	31	9,7533118	9,9159069	9,8374049	10,1625951	29
2	9,7479360	9,9184037	9,8295323	10,1704677	58	32	9,7534954	9,9158200	9,8376755	10,1623245	28
3	9,7481230	9,9183183	9,8298047	10,1701953	57	33	9,7536790	9,9157330	9,8379460	10,1620540	27
4	9,7483099	9,9182329	9,8300769	10,1699231	56	34	9,7538624	9,9156460	9,8382164	10,1617836	26
5	9,7484967	9,9181475	9,8303492	10,1696508	55	35	9,7540457	9,9155589	9,8384867	10,1615133	25
6	9,7486833	9,9180620	9,8306213	10,1693787	54	36	9,7542288	9,9154718	9,8387571	10,1612429	24
7	9,7488698	9,9179764	9,8308934	10,1691066	53	37	9,7544119	9,9153846	9,8390273	10,1609727	23
8	9,7490562	9,9178908	9,8311654	10,1688346	52	38	9,7545948	9,9152974	9,8392975	10,1607025	22
9	9,7492425	9,9178051	9,8314374	10,1685626	51	39	9,7547777	9,9152101	9,8395676	10,1604324	21
10	9,7494287	9,9177194	9,8317093	10,1682907	50	40	9,7549604	9,9151228	9,8398377	10,1601623	20
11	9,7496148	9,9176336	9,8319811	10,1680189	49	41	9,7551431	9,9150354	9,8401077	10,1598923	19
12	9,7498007	9,9175478	9,8322529	10,1677471	48	42	9,7553256	9,9149479	9,8403776	10,1596224	18
13	9,7499866	9,9174619	9,8325246	10,1674754	47	43	9,7555080	9,9148604	9,8406475	10,1593525	17
14	9,7501723	9,9173760	9,8327963	10,1672037	46	44	9,7556902	9,9147729	9,8409174	10,1590826	16
15	9,7503579	9,9172900	9,8330679	10,1669321	45	45	9,7558724	9,9146852	9,8411871	10,1588129	15
16	9,7505434	9,9172040	9,8333394	10,1666606	44	46	9,7560544	9,9145976	9,8414569	10,1585431	14
17	9,7507287	9,9171179	9,8336109	10,1663891	43	47	9,7562364	9,9145099	9,8417265	10,1582735	13
18	9,7509140	9,9170317	9,8338823	10,1661177	42	48	9,7564182	9,9144221	9,8419961	10,1580039	12
19	9,7510991	9,9169455	9,8341536	10,1658464	41	49	9,7565999	9,9143342	9,8422657	10,1577343	11
20	9,7512842	9,9168593	9,8344249	10,1655751	40	50	9,7567815	9,9142464	9,8425351	10,1574649	10
21	9,7514691	9,9167730	9,8346961	10,1653039	39	51	9,7569630	9,9141584	9,8428046	10,1571954	9
22	9,7516538	9,9166866	9,8349673	10,1650327	38	52	9,7571444	9,9140704	9,8430739	10,1569261	8
23	9,7518385	9,9166002	9,8352384	10,1647616	37	53	9,7573256	9,9139824	9,8433432	10,1566568	7
24	9,7520231	9,9165137	9,8355094	10,1644906	36	54	9,7575068	9,9138943	9,8436125	10,1563875	6
25	9,7522075	9,9164272	9,8357804	10,1642196	35	55	9,7576878	9,9138061	9,8438817	10,1561183	5
26	9,7523919	9,9163406	9,8360513	10,1639487	34	56	9,7578687	9,9137179	9,8441508	10,1558492	4
27	9,7525761	9,9162539	9,8363221	10,1636779	33	57	9,7580495	9,9136296	9,8444199	10,1555801	3
28	9,7527602	9,9161673	9,8365929	10,1634071	32	58	9,7582302	9,9135413	9,8446889	10,1553111	2
29	9,7529442	9,9160805	9,8368636	10,1631364	31	59	9,7584108	9,9134530	9,8449579	10,1550421	1
30	9,7531280	9,9159937	9,8371343	10,1628657	30	60	9,7585913	9,9133645	9,8452268	10,1547732	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	55 Degrees.						55 Degrees.				

Min.	35 Degrees.				Min.	Min.	35 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,7585913	9,9133645	9,8452268	10,1547732	60	30	9,7639540	9,9106860	9,8532680	10,1467320	30
1	9,7587717	9,9132760	9,8454956	10,1545044	59	31	9,7641311	9,9105959	9,8535352	10,1464648	29
2	9,7589519	9,9131875	9,8457644	10,1542356	58	32	9,7643080	9,9105057	9,8538023	10,1461977	28
3	9,7591321	9,9130989	9,8460332	10,1539668	57	33	9,7644849	9,9104155	9,8540694	10,1459306	27
4	9,7593121	9,9130102	9,8463018	10,1536982	56	34	9,7646616	9,9103251	9,8543365	10,1456635	26
5	9,7594920	9,9129215	9,8465705	10,1534295	55	35	9,7648382	9,9102348	9,8546034	10,1453966	25
6	9,7596718	9,9128328	9,8468390	10,1531610	54	36	9,7650147	9,9101444	9,8548704	10,1451296	24
7	9,7598515	9,9127440	9,8471075	10,1528925	53	37	9,7651911	9,9100539	9,8551372	10,1448628	23
8	9,7600311	9,9129551	9,8473760	10,1526240	52	38	9,7653674	9,9099634	9,8554041	10,1445959	22
9	9,7602106	9,9125662	9,8476444	10,1523556	51	39	9,7655436	9,9098728	9,8556708	10,1443292	21
10	9,7603899	9,9124772	9,8479127	10,1520873	50	40	9,7657197	9,9097821	9,8559376	10,1440624	20
11	9,7605692	9,9123882	9,8481810	10,1518190	49	41	9,7658957	9,9096915	9,8562042	10,1437958	19
12	9,7607483	9,9122991	9,8484492	10,1515508	48	42	9,7660715	9,9096007	9,8564708	10,1435292	18
13	9,7609274	9,9122099	9,8487174	10,1512826	47	43	9,7662473	9,9095090	9,8567374	10,1432626	17
14	9,7611063	9,9121207	9,8489855	10,1510145	46	44	9,7664229	9,9094190	9,8570039	10,1429961	16
15	9,7612851	9,9120315	9,8492536	10,1507464	45	45	9,7665985	9,9093281	9,8572704	10,1427296	15
16	9,7614638	9,9119422	9,8495216	10,1504784	44	46	9,7667739	9,9092371	9,8575368	10,1424632	14
17	9,7616424	9,9118528	9,8497896	10,1502104	43	47	9,7669492	9,9091461	9,8578031	10,1421969	13
18	9,7618208	9,9117634	9,8500575	10,1499425	42	48	9,7671244	9,9090550	9,8580694	10,1419306	12
19	9,7619992	9,9116739	9,8503253	10,1496747	41	49	9,7672996	9,9089639	9,8583357	10,1416643	11
20	9,7621775	9,9115844	9,8505931	10,1494069	40	50	9,7674746	9,9088727	9,8586019	10,1413981	10
21	9,7623556	9,9114948	9,8508608	10,1491392	39	51	9,7676494	9,9087814	9,8588680	10,1411320	9
22	9,7625337	9,9114051	9,8511285	10,1488715	38	52	9,7678242	9,9086901	9,8591341	10,1408659	8
23	9,7627116	9,9113155	9,8513961	10,1486039	37	53	9,7679989	9,9085988	9,8594002	10,1405998	7
24	9,7628894	9,9112257	9,8516637	10,1483363	36	54	9,7681735	9,9085073	9,8596661	10,1403339	6
25	9,7630671	9,9111359	9,8519312	10,1480688	35	55	9,7683480	9,9084159	9,8599321	10,1400679	5
26	9,7632447	9,9110460	9,8521987	10,1478013	34	56	9,7685223	9,9083243	9,8601980	10,1398020	4
27	9,7634222	9,9109561	9,8524661	10,1475339	33	57	9,7686966	9,9082327	9,8604638	10,1395362	3
28	9,7635996	9,9108661	9,8527335	10,1472665	32	58	9,7688707	9,9081411	9,8607296	10,1392704	2
29	9,7637769	9,9107761	9,8530008	10,1469992	31	59	9,7690448	9,9080494	9,8609954	10,1390046	1
30	9,7639540	9,9106860	9,8532680	10,1467320	30	60	9,7692187	9,9079576	9,8612610	10,1387390	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	54 Degrees.						54 Degrees.				

Min.	36 Degrees.				Min.	Min.	36 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7692187	9,9079576	9,8612610	10,1387390	60	30	9,7743876	9,9051787	9,8692089	10,1307911	30
1	9,7693925	9,9078658	9,8615267	10,1384733	59	31	9,7745583	9,9050852	9,8694731	10,1305269	29
2	9,7695662	9,9077740	9,8617923	10,1382077	58	32	9,7747288	9,9049916	9,8697372	10,1302628	28
3	9,7697398	9,9076820	9,8620578	10,1379422	57	33	9,7748993	9,9048980	9,8700013	10,1299987	27
4	9,7699134	9,9075901	9,8623233	10,1376767	56	34	9,7750697	9,9048043	9,8702653	10,1297347	26
5	9,7700808	9,9074980	9,8625887	10,1374113	55	35	9,7752399	9,9047106	9,8705293	10,1294707	25
6	9,7702601	9,9074059	9,8628541	10,1371459	54	36	9,7754101	9,9046168	9,8707933	10,1292067	24
7	9,7704332	9,9073138	9,8631195	10,1368805	53	37	9,7755801	9,9045230	9,8710572	10,1289428	23
8	9,7706063	9,9072216	9,8633848	10,1366152	52	38	9,7757501	9,9044291	9,8713210	10,1286790	22
9	9,7707793	9,9071293	9,8636500	10,1363500	51	39	9,7759199	9,9043351	9,8715848	10,1284152	21
10	9,7709522	9,9070370	9,8639152	10,1360848	50	40	9,7760897	9,9042411	9,8718486	10,1281514	20
11	9,7711249	9,9069446	9,8641803	10,1358197	49	41	9,7762593	9,9041470	9,8721123	10,1278877	19
12	9,7712976	9,9068522	9,8644454	10,1355546	48	42	9,7764289	9,9040529	9,8723760	10,1276240	18
13	9,7714702	9,9067597	9,8647105	10,1352895	47	43	9,7765983	9,9039587	9,8726396	10,1273604	17
14	9,7716426	9,9066671	9,8649755	10,1350245	46	44	9,7767676	9,9038644	9,8729032	10,1270968	16
15	9,7718150	9,9065745	9,8652404	10,1347596	45	45	9,7769369	9,9037701	9,8731668	10,1268332	15
16	9,7719872	9,9064819	9,8655053	10,1344947	44	46	9,7771060	9,9036757	9,8734302	10,1265698	14
17	9,7721593	9,9063892	9,8657702	10,1342298	43	47	9,7772750	9,9035813	9,8736937	10,1263063	13
18	9,7723314	9,9062964	9,8660350	10,1339650	42	48	9,7774439	9,9034868	9,8739571	10,1260429	12
19	9,7725033	9,9062036	9,8662997	10,1337003	41	49	9,7776128	9,9033923	9,8742204	10,1257796	11
20	9,7726751	9,9061107	9,8665644	10,1334356	40	50	9,7777815	9,9032977	9,8744838	10,1255162	10
21	9,7728468	9,9060177	9,8668291	10,1331709	39	51	9,7779501	9,9032031	9,8747470	10,1252530	9
22	9.7730185	9.9059247	9,8670937	10,1329063	38	52	9,7781186	9,9031084	9,8750102	10,1249898	8
23	9,7731900	9,9058317	9,8673583	10,1326417	37	53	9,7782870	9,9030136	9,8752734	10,1247266	7
24	9,7733614	9,9057386	9,8676228	10,1323772	36	54	9,7784553	9,9029188	9,8755365	10,1244635	6
25	9,7735327	9,9056454	9,8678873	10,1321127	35	55	9,7786235	9,9028239	9,8757996	10,1242004	5
26	9,7737039	9,9055522	9,8681517	10,1318483	34	56	9,7787916	9,9027289	9,8760627	10,1239373	4
27	9,7738749	9,9054589	9,8684160	10,1315840	33	57	9,7789596	9,9026339	9,8763257	10,1236743	3
28	9,7740459	9,9053656	9,8686804	10,1313196	32	58	9,7791275	9,9025389	9,8765886	10,1234114	2
29	9,7742168	9,9052722	9,8689446	10,1310554	31	59	9,7792953	9,9024438	9,8768515	10,1231485	1
30	9,7743876	9,9051787	9,8692089	10,1307911	30	60	9,7794630	9,9023486	9,8771144	10,1228856	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	53 Degrees.						53 Degrees.				

Min.	37 Degrees.				Min.	Min.	37 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,7794630	9,9023486	9,8771144	10,1228856	60	30	9,7844471	9,8994667	9,8849805	10,1150195	30
1	9,7796306	9,9022534	9,8773772	10,1226228	59	31	9,7846117	9,8993697	9,8852420	10,1147580	29
2	9,7797981	9,9022581	9,8776400	10,1223600	58	32	9,7847762	9,8992727	9,8855035	10,1144965	28
3	9,7799655	9,9020628	9,8779027	10,1220973	57	33	9,7849406	9,8991756	9,8857650	10,1142350	27
4	9,7801328	9,9019674	9,8781654	10,1218346	56	34	9,7851049	9,8990784	9,8860264	10,1139736	26
5	9,7803000	9,9018719	9,8784281	10,1215719	55	35	9,7852691	9,8989812	9,8862878	10,1137122	25
6	9,7804671	9,9017764	9,8786907	10,1213093	54	36	9,7854332	9,8988840	9,8865492	10,1134508	24
7	9,7806341	9,9016808	9,8789533	10,1210467	53	37	9,7855972	9,8987867	9,8868105	10,1131895	23
8	9,7808010	9,9015852	9,8792158	10,1207842	52	38	9,7857611	9,8986893	9,8870718	10,1129282	22
9	9,7809677	9,9014895	9,8794782	10,1205218	51	39	9,7859249	9,8985919	9,8873330	10,1126670	21
10	9,7811344	9,9013938	9,8797407	10,1202593	50	40	9,7860886	9,8984944	9,8875942	10,1124058	20
11	9,7813010	9,9012980	9,8800031	10,1199969	49	41	9,7862522	9,8983968	9,8878554	10,1121446	19
12	9,7814675	9,9012021	9,8802654	10,1197346	48	42	9,7864157	9,8982992	9,8881165	10,1118835	18
13	9,7816339	9,9011062	9,8805277	10,1194723	47	43	9,7865791	9,8982015	9,8883775	10,1116225	17
14	9,7818002	9,9010102	9,8807900	10,1192100	46	44	9,7867424	9,8981038	9,8886386	10,1113614	16
15	9,7819664	9,9009142	9,8810522	10,1189478	45	45	9,7869056	9,8980060	9,8888996	10,1111004	15
16	9,7821324	9,9008181	9,8813144	10,1186856	44	46	9,7870687	9,8979082	9,8891605	10,1108395	14
17	9,7822984	9,9007219	9,8815765	10,1184235	43	47	9,7872317	9,8978103	9,8894214	10,1105786	13
18	9,7824643	9,9006257	9,8818386	10,1181614	42	48	9,7873946	9,8977123	9,8896823	10,1103177	12
19	9,7826301	9,9005294	9,8821007	10,1178993	41	49	9,7875574	9,8976143	9,8899432	10,1100568	11
20	9,7827958	9,9004331	9,8823627	10,1176373	40	50	9,7877202	9,8975162	9,8902040	10,1097960	10
21	9,7829614	9,9003367	9,8826246	10,1173754	39	51	9,7878828	9,8974181	9,8904647	10,1095353	9
22	9,7831268	9,9002403	9,8828866	10,1171134	38	52	9,7880453	9,8973199	9,8907254	10,1092746	8
23	9,7832922	9,9001438	9,8831484	10,1168516	37	53	9,7882077	9,8972216	9,8909861	10,1090139	7
24	9,7834575	9,9000472	9,8834103	10,1165897	36	54	9,7883701	9,8971233	9,8912468	10,1087532	6
25	9,7836227	9,8999506	9,8836721	10,1163279	35	55	9,7885323	9,8970249	9,8915074	10,1084926	5
26	9,7837878	9,8998539	9,8839338	10,1160662	34	56	9,7886944	9,8969265	9,8917679	10,1082321	4
27	9,7839528	9,8997572	9,8841956	10,1158044	33	57	9,7888565	9,8968280	9,8920285	10,1079715	3
28	9,7841177	9,8996604	9,8844572	10,1155428	32	58	9,7890184	9,8967294	9,8922890	10,1077110	2
29	9,7842824	9,8995636	9,8847189	10,1152811	31	59	9,7891802	9,8966308	9,8925494	10,1074506	1
30	9,7844471	9,8994667	9,8849805	10,1150195	30	60	9,7893420	9,8965321	9,8928098	10,1071902	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	52 Degrees.						52 Degrees.				

Min.	38 Degrees.				Min.	Min.	38 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,7893420	9,8965321	9,8928098	10,1071902	60	30	9,7941496	9,8935444	9,9006052	10,0993948	30
1	9,7895036	9,8964334	9,8930702	10,1069298	59	31	9,7943083	9,8934439	9,9008645	10,0991355	29
2	9,7896652	9,8963346	9,8933306	10,1066694	58	32	9,7944670	9,8933433	9,9011237	10,0988763	28
3	9,7898266	9,8962358	9,8935909	10,1064091	57	33	9,7946256	9,8932426	9,9013830	10,0986170	27
4	9,7899880	9,8961369	9,8938511	10,1061489	56	34	9,7947841	9,8931419	9,9016422	10,0983578	26
5	9,7901493	9,8960379	9,8941114	10,1058886	55	35	9,7949425	9,8930412	9,9019013	10,0980987	25
6	9,7903104	9,8959389	9,8943715	10,1056285	54	36	9,7951008	9,8929404	9,9021604	10,0978396	24
7	9,7904715	9,8958398	9,8946317	10,1053683	53	37	9,7952590	9,8928395	9,9024195	10,0975805	23
8	9,7906325	9,8957406	9,8948918	10,1051082	52	38	9,7954171	9,8927385	9,9026786	10,0973214	22
9	9,7907933	9,8956414	9,8951519	10,1048481	51	39	9,7955751	9,8926375	9,9029376	10,0970624	21
10	9,7909541	9,8955422	9,8954119	10,1045881	50	40	9,7957330	9,8925305	9,9031966	10,0968034	20
11	9,7911148	9,8954429	9,8956719	10,1043281	49	41	9,7958909	9,8924354	9,9034555	10,0965445	19
12	9,7912754	9,8953435	9,8959319	10,1040681	48	42	9,7960486	9,8923342	9,9037144	10,0962856	18
13	9,7914359	9,8952440	9,8961918	10,1038082	47	43	9,7962062	9,8922329	9,9039733	10,0960267	17
14	9,7915963	9,8951445	9.8964517	10,1035483	46	44	9,7963638	9,8921316	9,9042321	10,0957679	16
15	9,7917566	9,8950450	9,8967116	10,1032884	45	45	9,7965212	9,8920303	9,9044910	10,0955090	15
16	9,7919168	9,8949453	9,8969714	10,1030286	44	46	9,7966786	9,8919289	9,9047497	10,0952503	14
17	9,7920769	9,8948457	9,8972312	10,1027688	43	47	9,7968359	9,8918274	9,9050085	10,0949915	13
18	9,7922369	9,8947459	9,8974910	10,1025090	42	48	9,7969930	9,8917258	9,9052672	10,0947328	12
19	9,7923968	9,8946461	9,8977507	10,1022493	41	49	9,7971501	9,8916242	9,9055259	10,0944741	11
20	9,7925566	9,8945463	9,8980104	10,1019896	40	50	9,7973071	9,8915226	9,9057845	10,0942155	10
21	9,7927163	9,8944463	9,8982700	10,1017300	39	51	9,7974640	9,8914208	9,9060431	10,0939569	9
22	9.7928760	9,8943464	9,8985296	10,1014704	38	52	9,7976208	9,8913191	9,9063017	10,0936983	8
23	9,7930355	9,8942463	9,8987892	10,1012108	37	53	9,7977775	9,8912172	9,9065603	10,0934397	7
24	9,7931949	9,8941462	9,8990487	10,1009513	36	54	9,7979341	9,8911153	9,9068188	10,0931812	6
25	9,7933543	9,8940461	9,8993082	10,1006918	35	55	9,7980906	9,8910133	9,9070773	10,0929227	5
26	9,7935135	9,8939458	9,8995677	10,1004323	34	56	9,7982470	9,8909113	9,9073357	10,0926643	4
27	9,7936727	9,8938456	9,8998271	10,1001729	33	57	9,7984034	9,8908092	9,9075941	10,0924059	3
28	9,7938317	9,8937452	9,9000865	10,0999135	32	58	9,7985596	9,8907071	9,9078525	10,0921475	2
29	9,7939907	9,8936448	9,9003459	10,0996541	31	59	9,7987158	9,8906049	9,9081109	10,0918891	1
30	9,7941496	9,8935444	9,9006052	10,0993948	30	60	9,7988718	9,8905026	9,9083692	10,0916308	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com,	Sine.	Tan. Com.	Tangent.	Min.
	51 Degrees.						51 Degrees.				

Min.	39 Degrees.				Min.	Min.	39 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,7988718	9,8905026	9,9083692	10,0916308	60	30	9,8035105	9,8874061	9,9161045	10,0838955	30
1	9,7990278	9,8904003	9,9086275	10,0913725	59	31	9,8036637	9,8873019	9,9163618	10,0836382	29
2	9,7991836	9,8902979	9,9088858	10,0911142	58	32	9,8038168	9,8871977	9,9166192	10,0833808	28
3	9,7993394	9,8901954	9,9091440	10,0908560	57	33	9,8039699	9,8870934	9,9168765	10,0831235	27
4	9,7994951	9,8900929	9,9094022	10,0905978	56	34	9,8041228	9,8869890	9,9171338	10,0828662	26
5	9,7996507	9,8899903	9,9096603	10,0903397	55	35	9,8042757	9,8868846	9,9173911	10,0826089	25
6	9,7998062	9,8898877	9,9099185	10,0900815	54	36	9,8044284	9,8867801	9,9176483	10,0823517	24
7	9,7999616	9,8897850	9,9101766	10,0898234	53	37	9,8045811	9,8866756	9,9179055	10,0820945	23
8	9,8001169	9,8896822	9,9104347	10,0895653	52	38	9,8047336	9,8865710	9,9181627	10,0818373	22
9	9,8002721	9,8895794	9,9106927	10,0893073	51	39	9,8048861	9,8864663	9,9184198	10,0815802	21
10	9,8004272	9,8894765	9,9109507	10,0890493	50	40	9,8050385	9,8863616	9,9186769	10,0813231	20
11	9,8005823	9,8893736	9,9112087	10,0887913	49	41	9,8051908	9,8862568	9,9189340	10,0810660	19
12	9,8007372	9,8892706	9,9114666	10,0885334	48	42	9,8053430	9,8861519	9,9191911	10,0808089	18
13	9,8008921	9,8891675	9,9117245	10,0882755	47	43	9,8054951	9,8860470	9,9194481	10,0805519	17
14	9,8010468	9,8890644	9,9119824	10,0880176	46	44	9,8056472	9,8859420	9,9197051	10,0802949	16
15	9,8012015	9,8889612	9,9122403	10,0877597	45	45	9,8057991	9,8858370	9,9199621	10,0800379	15
16	9,8013561	9,8888580	9,9124981	10,0875019	44	46	9,8059510	9,8857319	9,9202191	10,0797809	14
17	9,8015106	9,8887547	9,9127559	10,0872441	43	47	9,8061027	9,8856267	9,9204760	10,0795240	13
18	9,8016649	9,8886513	9,9130137	10,0869863	42	48	9,8062544	9,8855215	9,9207329	10,0792671	12
19	9,8018192	9,8885479	9,9132714	10,0867286	41	49	9,8064060	9,8854162	9,9209898	10,0790102	11
20	9,8019735	9,8884444	9,9135291	10,0864709	40	50	9,8065575	9,8853109	9,9212466	10,0787534	10
21	9,8021276	9,8883408	9,9137868	10.0862132	39	51	9,8067089	9,8852055	9,9215034	10,0784966	9
22	9,8022816	9,8882372	9,9140444	10,0859556	38	52	9,8068602	9,8851000	9,9217602	10,0782398	8
23	9,8024355	9,8881335	9,9143020	10,0856980	37	53	9,8070114	9,8849945	9,9220170	10,0779830	7
24	9,8025894	9,8880298	9,9145596	10,0854404	36	54	9,8071626	9,8848889	9,9222737	10,0777263	6
25	9,8027431	9,8879260	9,9148171	10,0851829	35	55	9,8073136	9,8847832	9,9225304	10,0774696	5
26	9,8028968	9,8878221	9,9150747	10,0849253	34	56	9,8074646	9,8846775	9,9227871	10,0772129	4
27	9,8030504	9,8877182	9,9153322	10,0846678	33	57	9,8076154	9,8845717	9,9230437	10,0769563	3
28	9,8032038	9,8876142	9,9155896	10,0844104	32	58	9,8077662	9,8844659	9,9233004	10,0766996	2
29	9,8033572	9,8875102	9,9158471	10,0841529	31	59	9,8079169	9,8843599	9,9235570	10,0764430	1
30	9,8035105	9,8874061	9,9161045	10,0838955	30	60	9,8080675	9,8842540	9,9238135	10,0761865	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	50 Degrees.						50 Degrees.				

Min.	40 Degrees.				Min.	Min.	40 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,8080675	9,8842540	9,9238135	10,0761865	60	30	9,8125444	9,8810455	9,9314989	10,0685011	30
1	9,8082180	9,8841479	9,9240701	10,0759299	59	31	9,8126923	9,8809376	9,9317547	10,0682453	29
2	9,8083684	9,8840418	9,9243266	10,0756734	58	32	9,8128401	9,8808296	9,9320105	10,0679895	28
3	9,8085188	9,8839357	9,9245831	10,0754169	57	33	9,8129878	9,8807215	9,9322662	10,0677338	27
4	9,8086690	9,8838294	9,9248396	10,0751604	56	34	9,8131354	9,8806134	9,9325220	10,0674780	26
5	9,8088192	9,8837232	9,9250960	10,0749040	55	35	9,8132829	9,8805052	9,9327777	10,0672223	25
6	9,8089692	9,8836168	9,9253524	10,0746476	54	36	9,8134303	9,8803970	9,9330334	10,0669666	24
7	9,8091192	9,8835104	9,9256088	10,0743912	53	37	9,8135777	9,8802887	9,9332890	10.0667110	23
8	9,8092691	9,8834039	9,9258652	10,0741348	52	38	9,8137250	9,8801803	9,9335446	10.0664554	22
9	9,8094189	9,8832974	9,9261215	10,0738785	51	39	9,8138721	9,8800719	9,9338003	10,0661997	21
10	9,8095686	9,8831908	9,9263778	10,0736222	50	40	9,8140192	9,8799634	9,9340559	10,0659441	20
11	9,8097182	9,8830841	9,9266341	10,0733659	49	41	9,8141662	9,8798548	9,9343114	10,0656886	19
12	9,8098678	9,8829774	9,9268904	10,0731096	48	42	9,8143131	9,8797462	9,9345670	10,0654330	18
13	9,8100172	9,8828706	9,9271466	10,0728534	47	43	9,8144600	9,8796375	9,9348225	10,0651775	17
14	9,8101666	9,8827638	9,9274028	10,0725972	46	44	9,8146067	9,8795287	9,9350780	10,0649220	16
15	9,8103159	9,8826568	9,9276590	10,0723410	45	45	9,8147534	9,8794199	9,9353335	10,0646665	15
16	9,8104650	9,8825499	9,9279152	10,0720848	44	46	9,8148999	9,8793110	9,9355889	10,0644111	14
17	9,8106141	9,8824428	9,9281713	10,0718287	43	47	9,8150464	9,8792021	9,9358444	10,0641556	13
18	9,8107631	9,8823357	9,9284274	10,0715726	42	48	9,8151928	9,8790930	9,9360998	10,0639002	12
19	9,8109121	9,8822285	9,9286835	10,0713165	41	49	9,8153391	9,8789840	9,9363552	10,0636448	11
20	9,8110609	9,8821213	9,9289396	10,0710604	40	50	9,8154854	9,8788748	9,9366105	10,0633895	10
21	9,8112096	9,8820140	9,9291956	10,0708044	39	51	9,8156315	9,8787656	9,9368659	10,0631341	9
22	9,8113583	9,8819067	9,9294516	10,0705484	38	52	9,8157776	9,8786563	9,9371212	10,0628788	8
23	9,8115069	9,8817992	9,9297076	10,0702924	37	53	9,8159235	9,8785470	9,9373765	10,0626235	7
24	9,8116554	9,8816918	9,9299636	10,0700364	36	54	9,8160694	9,8784376	9,9376318	10,0623682	6
25	9,8118038	9,8815842	9,9302195	10,0697805	35	55	9,8162152	9,8783281	9,9378871	10,0621129	5
26	9,8119521	9,8814766	9,9304755	10,0695245	34	56	9,8163609	9,8782186	9,9381423	10,0618577	4
27	9,8121003	9,8813689	9,9307314	10,0692686	33	57	9,8165066	9,8781090	9,9383975	10,0616025	3
28	9,8122484	9,8812612	9,9309872	10,0690128	32	58	9,8166521	9,8779994	9,9386527	10,0613473	2
29	9,8123965	9,8811534	9,9312431	10,0687569	31	59	9,8167975	9,8778896	9,9389079	10,0610921	1
30	9,8125444	9,8810445	9,9314989	10,0685011	30	60	9,8169429	9,8777799	9,9391631	10,0608369	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	49 Degrees.						49 Degrees.				

Min.	41 Degrees.				Min.	Min.	41 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,8169429	9,8777799	9,9391631	10,0608369	60	30	9,8212646	9,8744561	9,9468084	10,0531916	30
1	9,8170882	9,8776700	9,9394182	10,0605818	59	31	9,8214073	9,8743443	9,9470630	10,0529370	29
2	9,8172334	9,8775601	9,9396733	10,0603267	58	32	9,8215500	9,8742325	9,9473175	10,0526825	28
3	9,8173785	9,8774501	9,9399284	10,0600716	57	33	9,8216926	9,8741205	9,9475720	10,0524280	27
4	9,8175235	9,8773401	9,9401835	10,0598165	56	34	9,8218351	9,8740085	9,9478265	10,0521735	26
5	9,8176685	9,8772300	9,9404385	10,0595615	55	35	9,8219775	9,8738965	9,9480810	10,0519190	25
6	9,8178133	9,8771198	9,9406936	10,0593064	54	36	9,8221198	9,8737844	9,9483355	10,0516645	24
7	9,8179581	9,8770096	9,9409486	10,0590514	53	37	9,8222621	9,8736722	9,9485899	10,0514101	23
8	9,8181028	9,8768993	9,9412036	10,0587964	52	38	9,8224042	9,8735599	9,9488443	10,0511557	22
9	9,8182474	9,8767889	9,9414585	10,0585415	51	39	9,8225463	9,8734476	9,9490987	10,0509013	21
10	9,8183919	9,8766785	9,9417135	10,0582865	50	40	9,8226883	9,8733352	9,9493531	10,0506469	20
11	9,8185364	9,8765680	9,9419684	10,0580316	49	41	9,8228302	9,8732227	9,9496075	10,0503925	19
12	9,8186807	9,8764574	9,9422233	10,0577767	48	42	9,8229721	9,8731102	9,9498619	10,0501381	18
13	9,8188250	9,8763468	9,9424782	10,0575218	47	43	9,8231138	9,8729976	9,9501162	10,0498838	17
14	9,8189692	9,8762361	9,9427331	10,0572669	46	44	9,8232555	9,8728849	9,9503705	10,0496295	16
15	9,8191133	9,8761253	9,9429879	10,0570121	45	45	9,8233971	9,8727722	9,9506248	10,0493752	15
16	9,8192573	9,8760145	9,9432428	10,0567572	44	46	9,8235386	9,8726594	9,9508791	10,0491209	14
17	9,8194012	9,8759036	9,9434976	10,0565024	43	47	9,8236800	9,8725466	9,9511334	10,0488666	13
18	9,8195450	9,8757927	9,9437524	10,0562476	42	48	9,8238213	9,8724337	9,9513876	10,0486124	12
19	9.8196888	9,8756816	9,9440072	10,0559928	41	49	9,8239626	9,8723207	9,9516419	10,0483581	11
20	9,8198325	9,8755706	9,9442619	10,0557381	40	50	9,8241037	9,8722076	9,9518961	10,0481039	10
21	9,8199761	9,8754594	9,9445166	10,0554834	39	51	9,8242448	9,8720945	9,9521503	10,0478497	9
22	9,8201196	9,8753482	9,9447714	10,0552286	38	52	9,8243858	9,8719813	9,9524045	10,0475955	8
23	9,8202630	9,8752369	9,9450261	10,0549739	37	53	9,8245267	9,8718681	9,9526587	10,0473413	7
24	9,8204063	9,8751256	9,9452807	10,0547193	36	54	9,8246676	9,8717548	9,9529128	10,0470872	6
25	9,8205496	9,8750142	9,9455354	10,0544646	35	55	9,8248083	9.8716414	9,9531670	10,0468330	5
26	9,8206927	9,8749027	9,9457900	10,0542100	34	56	9,8249490	9,8715279	9,9534211	10,0465789	4
27	9,8208358	9,8747912	9,9460447	10,0539553	33	57	9,8250896	9,8714144	9,9536752	10,0463248	3
28	9,8209788	9,8746795	9,9462993	10,0537007	32	58	9,8252301	9,8713008	9,9539293	10,0460707	2
29	9,8211217	9,8745679	9,9465539	10,0534461	31	59	9,8253705	9,8711872	9,9541834	10,0458166	1
30	9,8212646	9,8744561	9,9468084	10,0531916	30	60	9,8255109	9,8710735	9,9544374	10,0455626	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	48 Degrees.						48 Degrees.				

Min.	42 Degrees.				Min.	Min.	42 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,8255109	9,8710735	9,9544374	10,0455626	60	30	9,8296833	9,8676309	9,9620525	10,0379475	30
1	9,8256512	9,8709597	9,9546915	10,0453085	59	31	9,8298212	9,8675151	9,9623061	10,0376939	29
2	9,8257913	9,8708458	9,9549455	10,0450545	58	32	9,8299589	9,8673992	9,9625597	10,0374403	28
3	9,8259314	9,8707319	9,9551995	10,0448005	57	33	9,8300966	9,8672833	9,9628133	10,0371867	27
4	9,8260715	9.8706179	9,9554535	10,0445465	56	34	9,8302342	9,8671673	9,9630669	10,0369331	26
5	9,8262114	9,8705039	9,9557075	10,0442925	55	35	9,8303717	9,8670512	9,9633204	10,0366796	25
6	9,8263512	9,8703898	9,9559615	10,0440385	54	36	9,8305091	9,8669351	9,9635740	10,0364260	24
7	9,8264910	9,8702750	9,9562154	10,0437846	53	37	9,8306464	9,8668189	9,9638275	10,0361725	23
8	9,8266307	9,8701613	9,9564694	10,0435306	52	38	9,8307837	9,8667026	9,9640811	10,0359186	22
9	9,8267703	9,8700470	9,9567233	10,0432767	51	39	9,8309209	9,8665863	9,9643346	10,0356654	21
10	9,8269098	9,8699326	9,9569772	10,0430228	50	40	9,8310580	9,8664699	9,9645881	10,0354119	20
11	9,8270493	9,8698182	9,9572311	10,0427689	49	41	9,8311950	9,8663534	9,9648416	10,0351584	19
12	9,8271887	9,8697037	9,9574850	10,0425150	48	42	9,8313320	9,8662369	9,9650951	10,0349049	18
13	9,8273279	9,8695891	9,9577389	10,0422611	47	43	9,8314688	9,8661203	9,9653486	10,0346514	17
14	9,8274671	9,8694744	9,9579927	10,0420073	46	44	9,8316056	9,8660036	9,9656020	10,0343980	16
15	9,8276063	9,8693597	9,9582465	10,0417535	45	45	9,8317423	9,8658868	9,9658555	10,0341445	15
16	9,8277453	9,8692449	9,9585004	10,0414996	44	46	9,8318789	9,8657700	9,9661089	10,0338911	14
17	9,8278843	9,8691301	9,9587542	10,0412458	43	47	9,8320155	9,8656531	9,9663623	10,0336377	13
18	9,8280231	9,8690152	9,9590080	10,0409920	42	48	9,8321519	9,8655362	9,9666157	10,0333843	12
19	9,8281619	9,8689002	9,9592618	10,0407382	41	49	9,8322883	9,8654192	9,9668692	10,0331308	11
20	9,8283006	9,8687851	9,9595155	10,0404845	40	50	9,8324246	9,8653021	9,9671225	10,0328775	10
21	9,8284393	9,8686700	9,9597693	10,0402307	39	51	9,8325609	9,8651849	9,9673759	10,0326241	9
22	9,8285778	9,8685548	9,9600230	10,0399770	38	52	9,8326970	9,8650677	9,9676293	10,0323707	8
23	9,8287163	9,8684396	9,9602767	10,0397233	37	53	9,8328331	9,8649504	9,9678827	10,0321173	7
24	9,8288547	9,8683242	9,9605305	10,0394695	36	54	9,8329691	9,8648331	9,9681360	10,0318640	6
25	9,8289930	9,8682088	9,9607842	10,0392158	35	55	9,8331050	9,8647156	9,9683893	10,0316107	5
26	9,8291312	9,8680934	9,9610378	10,0389622	34	56	9,8332408	9,8645981	9,9686427	10,0313573	4
27	9,8292694	9,8679779	9,9612915	10,0387085	33	57	9,8333766	9,8644806	9,9688960	10,0311040	3
28	9,8294075	9,8678623	9,9615452	10,0384548	32	58	9,8335122	9,8643629	9,9691493	10,0308507	2
29	9,8295454	9,8677466	9,9617988	10,0382012	31	59	9,8336478	9,8642452	9,9694026	10,0305974	1
30	9,8296833	9,8676309	9,9620525	10,0379475	30	60	9,8337833	9,8641275	9,9696559	10,0303441	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	47 Degrees.						47 Degrees.				

Min.	43 Degrees.				Min.	Min.	43 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine. Com.	Tangent.	Tan. Comp.	
0	9,8337833	9,8641275	9,9696559	10,0303441	60	30	9,8378122	9,8605622	9,9772500	10,0227500	30
1	9,8339188	9,8640096	9,9699091	10,0300909	59	31	9,8379453	9,8604423	9,9775030	10,0224970	29
2	9,8340541	9,8638917	9,9701624	10,0298376	58	32	9,8380783	9,8603223	9,9777560	10,0222440	28
3	9,8341894	9,8637737	9,9704157	10,0295843	57	33	9,8382112	9,8602022	9,9780090	10,0219910	27
4	9,8343246	9,8636557	9,9706689	10,0293311	56	34	9,8383441	9,8600821	9,9782620	10,0217380	26
5	9,8344597	9,8635376	9,9709221	10,0290779	55	35	9,8384769	9,8599619	9,9785149	10,0214851	25
6	9,8345948	9,8634194	9,9711754	10,0288246	54	36	9,8386096	9,8598416	9,9787679	10,0212321	24
7	9,8347297	9,8633011	9,9714286	10,0285714	53	37	9,8387422	9,8597213	9,9790209	10,0209791	23
8	9,8348646	9,8631828	9,9716818	10,0283182	52	38	9,8388747	9,8596009	9,9792738	10,0207262	22
9	9,8349994	9,8630644	9,9719350	10,0280650	51	39	9,8390072	9,8594804	9,9795268	10,0204732	21
10	9,8351341	9,8629460	9,9721882	10,0278118	50	40	9,8391396	9,8593599	9,9797797	10,0202203	20
11	9,8352688	9,8628274	9,9724413	10,0275587	49	41	9,8392719	9,8592393	9,9800326	10,0199674	19
12	9,8354033	9,8627088	9,9726945	10,0273055	48	42	9,8394041	9,8591186	9,9802856	10,0197144	18
13	9,8355378	9,8625902	9,9729477	10,0270523	47	43	9,8395363	9,8589978	9,9805385	10,0194615	17
14	9,8356722	9,8624714	9,9732008	10,0267992	46	44	9,8396684	9,8588770	9,9807914	10,0192086	16
15	9,8358066	9,8623526	9,9734539	10,0265461	45	45	9,8398004	9,8587561	9,9810443	10,0189557	15
16	9,8359408	9,8622338	9,9737071	10,0262929	44	46	9,8399323	9,8586351	9,9812972	10,0187028	14
17	9,8360750	9,8621148	9,9739602	10,0260398	43	47	9,8400642	9,8585141	9,9815501	10,0184499	13
18	9,8362091	9,8619958	9,9742133	10,0257867	42	48	9,8401959	9,8583929	9,9818030	10,0181970	12
19	9,8363431	9,8618767	9,9744664	10,0255336	41	49	9,8403276	9,8582718	9,9820559	10,0179441	11
20	9,8364771	9,8617576	9,9747195	10,0252805	40	50	9,8404593	9,8581505	9,9823087	10,0176913	10
21	9,8366109	9,8616383	9,9749726	10,0250274	39	51	9,8405908	9,8580202	9,9825616	10,0174384	9
22	9,8367447	9,8615190	9,9752257	10,0247743	38	52	9,8407223	9,8579078	9,9828145	10,0171855	8
23	9,8368784	9,8613997	9,9754787	10,0245213	37	53	9,8408537	9,8577863	9,9830673	10,0169327	7
24	9,8370121	9,8612803	9,9757318	10,0242682	36	54	9,8409850	9,8576648	9,9833202	10,0166798	6
25	9,8371456	9,8611608	9,9759849	10,0240151	35	55	9,8411162	9,8575432	9,9835730	10,0164270	5
26	9,8372791	9,8610412	9,9762379	10,0237621	34	56	9,8412474	9,8574215	9,9838259	10,0161741	4
27	9,8374125	9,8609215	9,9764909	10,0235091	33	57	9,8413785	9,8572998	9,9840787	10,0159213	3
28	9,8375458	9,8608018	9,9767440	10,0232560	32	58	9,8415095	9,8571779	9,9843315	10,0156685	2
29	9,8376790	9,8606821	9,9769970	10,0230030	31	59	9,8416404	9,8570561	9,9845844	10,0154156	1
30	9,8378122	9,8605622	9,9772500	10,0227500	30	60	9,8417713	9,8569341	9,9848372	10,0151628	0
Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	46 Degrees.						46 Degrees.				

Min.	44 Degrees.				Min.	Min.	44 Degrees.				Min.
	Sine.	Sine Com.	Tangent.	Tan. Comp.			Sine.	Sine Com.	Tangent.	Tan. Comp.	
0	9,8417713	9,8569341	9,9848372	10,0151628	60	30	9,8456618	9,8532421	9,9924197	10,0075803	30
1	9,8419021	9,8568121	9,9850900	10,0149100	59	31	9,8457903	9,8531179	9,9926724	10,0073276	29
2	9,8420328	9,8566900	9,9853428	10,0146572	58	32	9,8459188	9,8529936	9,9929251	10,0070749	28
3	9,8424634	9,8565678	9,9855956	10,0144044	57	33	9,8460471	9,8528693	9,9931778	10,0068222	27
4	9,8422939	9,8564455	9,9858484	10,0141516	56	34	9,8461754	9,8527449	9,9934305	10,0065695	26
5	9,8424244	9,8563232	9,9861012	10,0138988	55	35	9,8463036	9,8526204	9,9936832	10,0063168	25
6	9,8425548	9,8562008	9,9863540	10,0136460	54	36	9,8464318	9,8524959	9,9939359	10,0060641	24
7	9,8426851	9,8560784	9,9866068	10,0133932	53	37	9,8465599	9,8523713	9,9941886	10,0058114	23
8	9,8428154	9,8559558	9,9868596	10,0131404	52	38	9,8466879	9,8522466	9,9944413	10,0055587	22
9	9,8429456	9,8558332	9,9871123	10,0128877	51	39	9,8468158	9,8521218	9,9946940	10,0053060	21
10	9,8430757	9,8557106	9,9873651	10,0126349	50	40	9,8469436	9,8519970	9,9949466	10,0050534	20
11	9,8432057	9,8555878	9,9876179	10,0123821	49	41	9,8470714	9,8518721	9,9951993	10,0048007	19
12	9,8433356	9,8554650	9,9878706	10,0121294	48	42	9,8471991	9,8517471	9,9954520	10,0045480	18
13	9,8434655	9,8553421	9,9881234	10,0118766	47	43	9,8473267	9,8516220	9,9957047	10,0042953	17
14	9,8435953	9,8552192	9,9883761	10,0116239	46	44	9,8474543	9,8514969	9,9959573	10,0040427	16
15	9,8437250	9,8550961	9,9886289	10,0113711	45	45	9,8475817	9,8513717	9,9962100	10,0037900	15
16	9,8438547	9,8549730	9,9888816	10,0111184	44	46	9,8477091	9,8512465	9,9964627	10,0035373	14
17	9,8439842	9,8548499	9,9891344	10,0108656	43	47	9,8478365	9,8511211	9,9967154	10,0032846	13
18	9,8441137	9,8547266	9,9893871	10,0106129	42	48	9,8479637	9,8509957	9,9969680	10,0030320	12
19	9,8442432	9,8546033	9,9896399	10,0103601	41	49	9,8480909	9,8508702	9,9972207	10,0027793	11
20	9,8443725	9,8544799	9,9898926	10,0101074	40	50	9,8482180	9,8507446	9,9974734	10,0025206	10
21	9,8445018	9,8543564	9,9901453	10,0098547	39	51	9,8483450	9,8506190	9,9977260	10,0022740	9
22	9,8446310	9,8542329	9,9903981	10,0096019	38	52	9,8484720	9,8504933	9,9979787	10,0020213	8
23	9,8447601	9,8541093	9,9906508	10,0093492	37	53	9,8485989	9,8503675	9,9982314	10,0017686	7
24	9,8448891	9,8539856	9,9909035	10,0090965	36	54	9,8487257	9,8502417	9,9984840	10,0015160	6
25	9,8450181	9,8538619	9,9911562	10,0088438	35	55	9,8488524	9,8501157	9,9987367	10,0012633	5
26	9,8451470	9,8537381	9,9914089	10,0085911	34	56	9,8489791	9,8499897	9,9989893	10,0010107	4
27	9,8452758	9,8536142	9,9916616	10,0083384	33	57	9,8491057	9,8498637	9,9992420	10,0007580	3
28	9,8454045	9,8534902	9,9919143	10,0080857	32	58	9,8492322	9,8497375	9,9994947	10,0005053	2
29	9,8455332	9,8533662	9,9921670	10,0078330	31	59	9,8493586	9,8496113	9,9997473	10,0002527	1
30	9,8456618	9,8532421	9,9924197	10,0075803	30	60	9,8494850	9,8494850	10,0000000	10,0000000	0
Min.	Sime Com.	Sine.	Tan. Com.	Tangent.	Min.	Min.	Sine Com.	Sine.	Tan. Com.	Tangent.	Min.
	5 Degrees.						45 Degrees.				

SECTION VIII.

MECHANICAL PHILOSOPHY.

CHAPTER I.

STATICS.

Mechanical Philosophy is that branch of Natural Philosophy which investigates the laws that govern the action of force on matter. By the application of the purely mathematical sciences to a few general results, suggested by experiment and universal experience, we demonstrate these laws in their highest degree of generalisation, and under all the various conditions, in which force can produce an effect on matter.

Matter may exist in three states—the solid, liquid, and aëriform. Force is that which either produces, or tends to produce, the motion of matter. Force may therefore be considered in relation to its effects on these three conditions of matter.

When forces produce the rest or equilibrium of the body, or system of bodies to which they are applied, we have the three following distinct subjects :—

1. **Statics**,—the rest, or equilibrium of *solid bodies* under the influence of forces.
2. **Hydrostatics**,—the rest, or equilibrium of *fluid bodies* under the influence of forces.
3. **Aerostatics**,—the rest, or equilibrium of *aëriform bodies* under the influence of forces.

When motion is the result of the application of forces to these conditions of matter, we have then these three subjects :—

1. **Dynamics**,—the motion of *solid bodies* produced by the influence of forces.
2. **Hydrodynamics**,—the motion of *fluid bodies* produced by the influence of forces.
3. **Aerodynamics, or Pneumatics**,—the motion of *aëriform bodies* produced by the influence of forces.

That branch of mechanical philosophy which treats of the rest, or equilibrium of solid bodies, when under the action of any forces, is called Statics. Statics is derived from a Greek word, στατική, the science of weight.

MATERIAL PARTICLE.—The smallest portion into which any solid body can be conceived to be divided by any means whatever, we call a Material Particle. This material particle is an abstract mental conception, altogether independent of any theories of the ultimate division of matter, and does not therefore correspond with the "atom" of the properties of matter. The material particle is a mere geometrical point, which we conceive destitute of every property save one—that of being set in motion, or brought to rest under the influence of forces which are supposed to act upon it. It is conceived to be destitute of form or sensible magnitude.

RIGID BODY.—A rigid body is a collection or assemblage of a number of material particles, held together in an invariable form, by forces of such intensity, that no conceivable force is supposed to be capable of altering the form of the body, or disturbing the position of any of its particles.

This rigid body is also a mental abstraction, having no representative in nature. No material solid, with which we are acquainted, is perfectly rigid, though the unknown molecular forces, which hold the particles of most solid bodies together, are so great, that for many practical purposes we may regard them, within certain limits, as rigid. The various solid bodies which occur in nature, though differing from one another by many properties, seem to possess one property in common—that of being able to transmit any forces applied to any one of their particles, unimpaired in their intensity, through all the particles of their substance, which are in the same straight line with the direction of the application of the force. Experiment shows that most solid bodies possess this property more or less; it is this which distinguishes, in a great measure, solids from fluids,—any force applied to a particle of a fluid being conveyed through every particle throughout its substance in every direction.

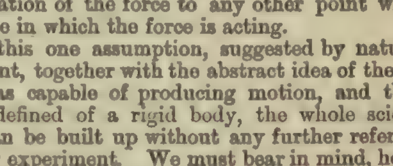
This property, which natural solid bodies possess more or less, we shall assume our rigid body to possess *perfectly*; and this will enable us to enunciate a principle upon which the whole science of Statics may be said to depend.

PRINCIPLE OF THE TRANSMISSION OF FORCE.—When a force, acting in combination with others, holds a rigid body in equilibrium, the equilibrium of the body will not be disturbed if we transfer the point of application of the force to any other point whatever in the line in which the force is acting.

Upon this one assumption, suggested by nature and experiment, together with the abstract idea of the nature of force as capable of producing motion, and the idea already defined of a rigid body, the whole science of Statics can be built up without any further reference to nature or experiment. We must bear in mind, however, when we apply the conclusions to which the science of Statics leads us, that these conclusions will only hold true, for the solid bodies of nature, so far as they possess the properties of perfect rigidity, and the perfect transmission of force. Within these limits we may apply our science to the action of forces on natural bodies with accuracy.

STATICS AN ABSTRACT SCIENCE.—Statics is an abstract mathematical science, and many of its results cannot be directly confirmed by experiment. Thus, one of the simplest propositions, which we shall hereafter prove, is, that if two weights, P and Q (Fig. 1) be suspended from the extremities of a rigid rod, A B, resting on the sharp edge of a support at F, the weights P and Q will balance one another, and the rod A B will be in a state of equilibrium, provided A F be as many inches in length as Q is ounces in weight, and F B as many inches in length as P is ounces in weight. This proposition cannot be proved by experiment, since it is only true on the supposition that A B is destitute of weight, and no bar,

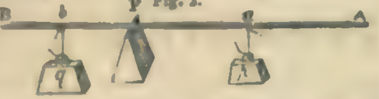
Fig. 1.



whose weight is inappreciable, can be found sufficiently rigid to support the weights P and Q, however small, and still remain perfectly straight under their pressure, and that of the sharp edge at F. It is true that another proposition of Statics would enable us to take into account the weight of the bar A B.

If the bar A B (Fig. 2) were uniform throughout in thickness and material, the bar would balance itself on the edge F, provided a weight q, equal the weight of the portion of the bar A F, were suspended from the point b, F b being equal the half of A F: and a weight, p, equal the weight of the portion of the bar F B, suspended from a, F a being equal the half of B F.

Fig. 2.



Supposing the difficulty of the weight of the bar thus obviated, there would still remain another difficulty in the way of a perfect experimental proof of our simple proposition. According to theory, the slightest additional weight, added to either of the weights P or Q, ought to destroy the equilibrium of the bar A B, and alter its perfectly horizontal position. But this supposes that the edge of the support F will exercise no friction on the surface of the rod A B. By a proper choice of materials, for the rod and the support, and also of the angle of the edge of the support, this friction may be considerably diminished, but it can never be absolutely destroyed. If we could succeed in fulfilling all the conditions we have indicated as necessary for the experimental proof of the very simple proposition of our science, which we have enunciated, we should be able to construct a perfect balance. That these difficulties are real, and not merely imaginary, is proved by the fact that a balance, sufficiently sensitive and accurate to supply the wants of the modern analytical chemist, is a very expensive instrument, and requires great skill and accuracy, as well as scientific knowledge, for its construction.

Though we cannot appeal to experiment for a rigid proof of our propositions, if we make the necessary allowances for the difference between the bodies on which our experiments are conducted, and the imaginary bodies which our science supposes perfectly rigid, and so forth, careful experiments will serve not only to give us clearer views of our science, but also to confirm our confidence in our abstract reasonings, by the approximate coincidence of our experimental results with our theoretical conclusions.

Thus, if in the experiment before alluded to, we make the necessary allowance for the weight of the bar, by applying the small weights p and q to the points a and b, our experiments will approximate to the theoretical proposition in proportion as the friction of F on the bar is diminished, and the bar A B is perfectly straight and accurately divided at the points a and b (Fig. 2).

REST OR EQUILIBRIUM.—Having defined a material particle and a rigid solid body, and their theoretical properties and condition, we must next consider what we mean by their rest or motion. We say that a body or a particle is in motion when we perceive the body or particle to change its position, and that it is at rest when it does not change its position. This clearly implies, however, that we have some means of measuring this alteration or change of position, by reference to other bodies which we conceive to be at rest. Hence it happens that what we suppose to be rest and motion, are generally only apparent rest and motion, and that there is no subject on which our senses more frequently lead us to form erroneous conclusions than when we attempt to discriminate the real or absolute rest and motion of objects from their apparent rest or motion. When we see a body change its position relatively to other bodies, we cannot tell whether the body which seems to move really does so or not, till we know whether the bodies relatively to which it appears to change its position, are themselves in a state of rest, or whether they are all moving together. It may so happen that the bodies may be moving, though we are ourselves unconscious of their motion; and in this case the body which apparently moves may be really at rest. Instances of this kind are of daily occurrence. A person on board ship, where his own body, as well as all the surrounding parts of the ship are apparently at rest, sees all the objects which the ship passes, carried, as it were, past him with great velocity; and were he not conscious, from other considerations, that the ship on which he stands were really in motion, he would have some difficulty in conceiving that the apparent motion of the stationary objects the ship passes by, was not real instead of imaginary. This is strikingly illustrated by looking out of a railway carriage when moving quickly; the objects, such as trees and houses, near the carriage, are apparently carried by the window with great rapidity, while those at a greater distance seem to pass more slowly. If, while waiting at a station, a train pass slowly by, it is often very difficult to tell whether the train in which we are seated be at rest or in motion, unless we correct our impression by looking out of the opposite window, where we can compare the position of the train with the fixed objects of the station.

APPARENT REST OR MOTION OF HEAVENLY BODIES.—A popular knowledge of astronomy is now so common, that we have no difficulty in persuading persons that the apparent motion of the sun and stars in the heavens is not real. Yet this is a fact which can only be demonstrated by a long course of intricate reasoning, founded on an immense number of careful observations.

To an ordinary observer, the vault of heaven, studded with stars, appears to revolve round an imaginary axis, while the earth seems perfectly at rest; each of the fixed stars describes a circle, which brings it back to the position in which it was first observed in an invariable interval of about 23 hours 56 minutes. What are called the fixed stars seem all to move together with the vault of heaven, and never to change their relative positions with respect to one another. The planets and comets, as well as the sun and moon, have an apparent motion among the fixed stars. In the case of the planets, comets, and moon, this apparent motion is exceedingly complex. Sometimes a planet will appear to move forward with great rapidity among the stars of a constellation; then it will come, as it were, to a stand-still, remain some time at rest, and then move backward. Ancient astronomers were well acquainted with the phenomena of real and apparent motion; yet after centuries of painful research and the invention of most complicated motions, to account for the apparent paths of the planets on the vault of heaven without avail, it was reserved for the genius of Copernicus and Kepler to resolve these motions into simpler ones, on the simple hypothesis that all the planets move round the sun in orbits nearly elliptical, and are never at rest.

Astronomy teaches us that every object on the earth's surface which appears to us at rest, is really subject, along with the earth on which it is placed, to two motions; one by which it is carried round the earth's axis by its diurnal rotation, and another by which it describes the earth's orbit round the sun in a year.

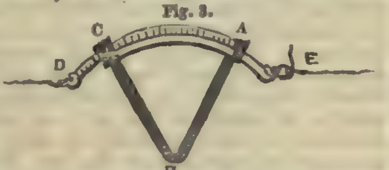
From these considerations it appears that there can be no absolute rest for any particle of matter by which we are surrounded; that perpetual motion is the real condition of all material objects; and that when we apply the reasonings and principles of the science of statics to nature, it is the apparent and not real rest of bodies to which we must have regard.

In the application of the principles and conclusions of statics to nature, we may consider the earth as absolutely fixed and at rest, and neglect its real motion without introducing any sensible error into our experiments. According to the law of inertia, all the bodies on the earth will partake of the earth's motion, and we know of no force which will deprive them of this motion; if, therefore, any forces produce the relative motion or rest of a terrestrial body, compared with bodies fixed with regard to the earth, that relative rest or motion will be the same whether the earth be really at rest or in motion. A familiar instance may make this assertion clear. If a ship under the influence of steam and tide, be moving rapidly through smooth water, all mechanical powers, such as blocks and pulleys, the windlass, and all the parts of the most complicated steam-engine will act under precisely the same circumstances, exert the same forces, be brought to a state of apparent rest or motion compared with the ship as a fixed object, whether the ship be at rest or in motion. A game of billiards, which requires considerable practical skill in mechanical science, may be played in the cabin of a ship without the players being aware whether the ship is at rest or in motion. Though, with respect to the earth, the balls, which were apparently at rest while the ship was moving, would be in reality not at rest, but moving with the ship.

MEASURE OF FORCE—DYNAMOMETERS.—We regard all forces as equal which produce the same mechanical effects; and in statics we consider forces as equal, which produce the same effects, when applied in a similar manner, in bringing a body into a state of equilibrium. Instruments, by means of which these effects are estimated, are called dynamometers, or force measurers.

A very simple dynamometer consists of a thin flat bar of steel, A B C, bent into an angle at B, and properly tempered, so that if any force be applied at A and C, to bring the extremities nearer together, the force of elasticity of the steel will cause the extremities, A and C, to resume their position as soon as the forces are removed. The greater the forces applied to A and C, the nearer these extremities of the bar will be brought to one another. To measure this effect, a circular arc of metal, A C D, is fixed perpendicularly to the surface of the steel bar at A, passing freely through an opening in the other extremity of the bar at C. Another arc, C A E, is similarly fixed at C, passing through an opening at A. Rings are fixed at D and E for the convenient application of forces whose effects are to be estimated; and one of the arcs, A D, is graduated by a number of equal divisions.

Fig. 3.



If the extremity E (Fig. 4) of the dynamometer be fixed to a beam, and a weight, P, hung from the ring at D, the graduated arc will show the statical effects of the weight P in bringing the dynamometer to a state of rest, with the extremities of the bent bar nearer to each other than they were before the application of the weight P. If, instead of applying a weight P, we pull with our hand a string attached to D (Fig. 5), till we bring the graduated part of the arc at A to the same division to which the weight P brought it, we produce with our hand a similar statical effect to that produced by the weight. We therefore conclude that the forces exerted by the weight and by the hand are equal to one another, though they are evidently forces of a very different character, the force of the weight being produced by the earth's attraction for the particles of matter of which it is composed, and the force exerted by the arm being derived from the contractions of the muscular fibres of the muscles of the arm, due to some force which we call the muscular force, of the nature of which we are perfectly ignorant.

Fig. 4.

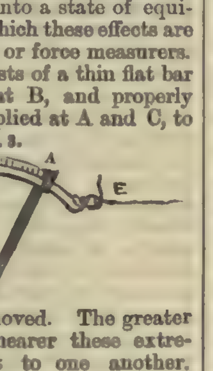
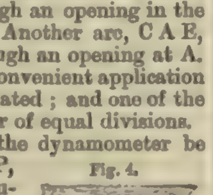
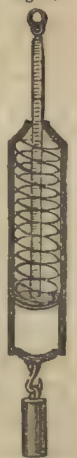


Fig. 5.



There are many other dynamometers besides that just described, such as one in which the statical effects of a force causing a spiral spring of wire contained in a metal cylinder to be compressed, is measured on a graduated rod, as in the accompanying figure (Fig. 6). The various kinds of balances and steel yards, which will be described hereafter, are all dynamometers.

Fig. 6.



UNIT OF FORCE.—The experiment on the dynamometer, just described, would lead us to infer, what is found to be true in practice, that weight is the most convenient measure of force which we can adopt in statics. We say that a force of ten or twelve pounds is exerted on a body, if the force produce the same statical effect on the body which a weight of ten or twelve pounds applied in a similar way would produce. The unit of force generally adopted in this country is that called a pound weight. But here we may be met with the inquiry, what is a pound weight? This is a very important question, and has been determined by legislative enactment, after careful deliberation, and the report of a scientific commission appointed for the consideration of the difficult subject of standard weights and measures.

POUNDS TROY AND AVOIRDUPOIS.—By the Act of Parliament (5th George IV., c. 74, passed in the year 1824, sections 4 and 5), it is enacted—"That from and after the first day of May, 1825, the standard brass weight of one pound troy weight, made in the year of 1758, now in the custody of the clerk of the House of Commons, shall be, and the same is hereby declared to be, the original and genuine standard of measure of weight, and that such brass weight shall be, and is hereby declared to be, the original and genuine standard measure of weight, and that such brass weight shall be and is hereby denominated the Imperial Standard Troy Pound, and shall be, and is hereby declared to be, the unit or only standard measure of weight from which all other weights shall be derived, computed, and ascertained, and that one twelfth part of the said troy pound shall be an ounce, and that one twentieth part of such ounce shall be a pennyweight, and that one twenty-fourth part of such pennyweight shall be a grain; so that 5,760 such grains shall be a troy pound, and that 7,000 such grains shall be, and they are hereby declared to be, a pound avoirdupois, and that one sixteenth part of the said pound avoirdupois shall be an ounce avoirdupois, and that one sixteenth part of such ounce shall be a dram.

"And whereas it is expedient that the said standard troy pound, if lost, destroyed, defaced, or otherwise injured, should be restored of the same weight by reference to some invariable natural standard; and whereas it has been ascertained by the commissioners appointed by his Majesty to inquire into the subject of weights and measures, that a cubic inch of distilled water, weighed in air by brass weights at the temperature of 62 degrees of Fahrenheit's thermometer, the barometer being at 30 inches, is equal to 252 grains, and 456 thousandth parts of a grain, of which, as aforesaid, the Imperial Standard Troy Pound contains 5760: Be it therefore enacted, that if at any time hereafter the said Imperial Troy Pound shall be lost, or shall in any manner be destroyed, defaced, or otherwise injured, it shall and may be restored by making, under the direction of the Lord High Treasurer, or the Commissioners of His Majesty's Treasury of the United Kingdom of Great Britain and Ireland, or any three of them for the time being, a new standard troy pound, bearing the same proportion to the weight of a cubic inch of distilled water, as the said standard pound, hereby established, bears to such cubic inch of water."

From this Act of Parliament it appears that the troy pound is a certain arbitrary piece of brass, which, weighed in air at a temperature of 62° Fahrenheit, the mercury in the barometer standing at a height of 30 inches, is equal to the weight of 22 cubic inches, and 815 thousand parts of a cubic inch of distilled water. Distilled water is therefore the natural standard to which the unit of weight is ultimately referred. This standard, however, is referred to a cubic inch, or a volume of water an inch in height, an inch in breadth, and an inch in depth.

We must, therefore, have some standard measure of length.

UNIT OF LENGTH.—The Act of Parliament just cited has also determined the standard unit of the measure of length. By the first section of that act, it is enacted—"That from and after the 1st day of May, 1825, the straight line, or distance between the centres of the two points in the gold studs in the straight brass rod, now in the custody of the clerk of the House of Commons, whereon the words and figures, 'Standard yard, 1760,' are engraved, shall be, and the same is hereby declared to be, the original and genuine standard of that measure of length or lineal extension called a yard, and that the same straight line or distance between the centres of the said two points in the said gold studs in the said brass rod, the brass being at the temperature of 62 degrees by Fahrenheit's thermometer, shall be and is hereby denominated the 'Imperial Standard Yard,' and shall be, and is hereby declared to be, the unit or only standard measure of extension, wherefrom or whereby all other measures of extension whatsoever, whether the same be lineal, superficial, or solid, shall be derived, computed, or ascertained; and that all measures of length shall be taken in parts or multiples, or certain proportions of the said standard yard; and that one-third part of the said standard yard shall be a foot, and the twelfth part of such foot shall be an inch, and that the pole or perch in length shall contain 5 such yards and a half, the furlong 220 such yards, and the mile 1,760 such yards.

"And whereas the said standard yard, if lost, destroyed, defaced, or otherwise injured, should be restored of the same length by reference to some invariable natural standard; and whereas it has been ascertained, by the commissioners appointed by his Majesty to inquire into the subject of weights and measures, that the said yard hereby declared to be the Imperial Standard Yard, when compared with a pendulum vibrating seconds of mean time in the latitude of London, in a vacuum, at the level of the sea, is in the proportion of 36 to 39 inches, and 1,393 ten thousandths part of an inch: Be it therefore enacted and declared, That if at any time hereafter the said imperial standard yard shall be in any manner destroyed, defaced, or otherwise injured, it shall and may be restored by making, under the direction of the Lord High Treasurer, or the Commissioners of His Majesty's Treasury of the United Kingdom of Great Britain and Ireland, or any three of them, for the time being, a new standard yard bearing the same proportion to such pendulum as aforesaid, as the said imperial standard yard bears to such pendulum."

UNIT OF TIME.—Time is the ultimate natural standard to which all measures of length, weight, and capacity are referred in this kingdom. Time certainly is one of the most convenient, and perhaps the only natural standard to which we can have recourse, to fix with scientific accuracy our units of measurement. The value of time in this respect depends upon the fact deduced by astronomers, from observations collected and registered for many centuries, that the mean length of day and night is invariable; in other words, that the earth always completes its rotation on its axis in the same period of time. This portion of time is obtained by observing the period which elapses between the passage of a fixed star over a certain imaginary line in the heavens, called the meridian of the place of observation, and its next appearance on this line, and is called a sidereal day. The sidereal day is divided into twenty-four hours, each hour into sixty minutes, and each minute into sixty seconds. A clock is an instrument for measuring time, and is set in motion by a weight; its motion is regulated by the vibrations of a pendulum, and the number of its vibrations are registered by means of a series of wheels, and indicated on the graduated face of the clock by the motion of hands or pointers. The length of the pendulum is so regulated that its time of vibration may be as nearly as possible one second; and these vibrations being registered by the clock, if the clock indicate that 24 hours, or 24 × 60 × 60, or 86,400 seconds have elapsed between one transit of the star over the meridian and another, the clock is said to be correct, and will afford us an accurate measure of sidereal time. Such a clock is called a sidereal clock, and is one of the most valuable and useful instruments in an observatory. Sidereal time is not, however, the time used for the ordinary transactions of life. The true solar day is the interval which elapses between one passage of the sun over the meridian and another, as shown by a dial or other means of astronomical observation; this day is not invariable in length, but changes from day to day, being sometimes longer and sometimes shorter. To avoid this inconvenience, a mean solar day is chosen by the supposed revolution of a fictitious sun, which shall be invariable in length; and this time is divided into 24 hours, and these hours into minutes and seconds, as in the case of the sidereal day. This mean solar day is the time used for the ordinary or civil reckoning of time. What is called the equation of time, is an astronomical calculation, which gives the difference between the time shown by the dial or the true solar time, and that indicated by the ordinary clocks, which show the mean solar or civil measure of time. The sidereal day is 23 hours, 56 minutes, 4 seconds, and nine hundredths of a second of a mean solar day; and the mean solar day is 24 hours, 3 minutes, 56 seconds, and 55 hundredth parts of a second of a sidereal day. The pendulum vibrating seconds, from which, by Act of Parliament, the standard measure of length, if lost, is to be obtained, is to be a pendulum vibrating a second of mean solar time under the conditions specified in the act. So great, however, are the scientific and mechanical difficulties to be overcome in determining accurately the length of this second's pendulum, that though the standard yard was so injured by the fire when the houses of Parliament were burnt down in 1834, and the standard pound troy altogether lost, no attempt has been made to restore the lost standards. The last scientific commission seems to have considered the attempt to do so altogether hopeless. Fortunately, the Royal Astronomical Society had a very beautiful scale constructed about the year 1832, and three feet of this scale were compared many hundreds of times with the Parliamentary standard; and this scale must now be considered the scientific English standard, and the best evidence of the parliamentary standard which exists. There is also great doubt as to the accuracy of Captain Kater's determination of the length of the second's pendulum which is adopted by the Act of Parliament; so that if the standard were to be restored according to that act, it would probably differ from the lost standard (Bailey's Report on a Standard Scale, Astronom. Soc. Mem. vol. ix.) Two Acts of Parliament, 5 and 6 Wm. IV., c. 63, and 16 Vic., c. 29, June, 1853, have been enacted since the fire, on the subject of weights and measures; but neither of them notice the loss of the standards, speaking of them as if they were still in existence.

FRENCH STANDARDS.—The French standards are derived from actual measurements of the earth's surface. From these measurements the length of a line drawn from the pole of the earth to the equator is deduced. The ten-millionth part of this line is called a metre; and this is the French standard of length. The decametre is 10 metres, the hecto-metre is 100 metres, the kilo-metre 1,000 metres, and the myria-metre is 10,000; while the deci-metre is the 10th part of a metre, the centi-metre the 100th part of a metre, and the millimetre the 1,000th part of a metre—Greek prefixes being used for the multiples of the metre, and Latin for its parts. The gramme, which is the standard of weight, is derived from the standard of length, and is equal to the weight of a cubic centi-metre of distilled water, weighed at the freezing point. The same prefixes are used before the gramme, which are added to the metre to express its multiples and parts. Thus a kilo-gramme is a 1,000 grammes, and a milli-gramme the 1,000th part of a gramme.

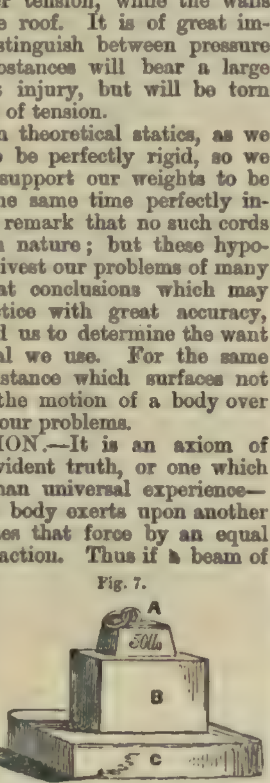
The metre is equal to 3·280899 English feet, and the kilo-gramme is equal to 3·204597 pounds avoirdupois.

PRESSURE AND TENSION.—When a material body is in a state of equilibrium under the influence of forces, the forces applied to the body may either have a tendency to press the particles of the body or to crush it, in which case the forces are called pressures, or they may have a tendency to separate the particles of the body or tear it, in which case they are denominated tensions. A weight placed on a body exerts a pressure on it. Two men pulling a rope, or weights suspended from a rope, exert a tension on the rope throughout its substance; and if one part of the rope be weaker than another, and the weight or force be sufficiently great, the rope will break or be torn asunder at that part. The tie-beams of a roof, which prevent the weight of the roof from thrusting the walls of a building out of the perpendicular, are under tension, while the walls support the pressure of the roof. It is of great importance, practically, to distinguish between pressure and tension; for some substances will bear a large amount of pressure without injury, but will be torn asunder by a far less amount of tension.

FLEXIBLE CORDS.—In theoretical statics, as we conceive our solid bodies to be perfectly rigid, so we conceive the strings which support our weights to be perfectly flexible, and at the same time perfectly inextensible. It is needless to remark that no such cords or strings are to be found in nature; but these hypothetical bodies enable us to divest our problems of many difficulties, and to arrive at conclusions which may afterwards be used in practice with great accuracy, when experiment has enabled us to determine the want of flexibility in the material we use. For the same reason, friction, or the resistance which surfaces not perfectly smooth oppose to the motion of a body over them, is at first neglected in our problems.

ACTION AND REACTION.—It is an axiom of statics—that is, it is a self-evident truth, or one which admits of no other proof than universal experience—that whatever force one rigid body exerts upon another rigid body, the latter opposes that force by an equal force, which is called its reaction. Thus if a beam of wood, B (Fig. 7) standing upright on a floor, C, support on one of its extremities a 50lb. weight, A, the weight A will exert a pressure of 50 lbs., acting downwards, on the beam, and the beam will convey this pressure to the floor. But the rigidity of the beam opposes a force to the weight, which prevents the weight from falling, or crushing the beam; and thus a reaction equal to 50 lbs. is exerted upwards by the beam upon the weight. Again, the beam presses on the floor with its own weight, in addition to that of the 50 lb. weight; and if the floor be strong enough, and of a material sufficiently rigid to prevent B sinking into it, the floor will sustain the pressure of A and B, which acts downwards; but it will react upwards on B with a force equal to the pressure B exerts upon it.

Fig. 7.



EQUILIBRIUM OF A MATERIAL PARTICLE.—If a material particle be acted upon by two forces which are equal to one another, but acting in opposite directions in the same straight line, they will neutralise one another, and the particle will be at rest. This is self-evident, and depends upon our fundamental idea of the equality of forces. Supposing, however, that the two forces do not act on the particle in the same straight line, but in the direction of straight lines inclined to one another at some angle, in what direction must a force be applied to the particle, and of what magnitude must it be, to neutralise the effect of these two forces, both in the case where the two forces are equal, and also where they are unequal? This is one of the most important, and, indeed, the fundamental proposition of statics. Before we can discover it we must adopt some means of representing forces and their directions.

GEOMETRICAL REPRESENTATION OF FORCES.—If we represent the material point by a geometrical point A (Fig. 8), we may draw a line, A P, to represent the direction in which a force, say of P pounds, is acting on the particle,, and A Q to represent the direction in which a force of Q pounds is acting on A; then if we take A P, P inches in length, and A Q, Q, inches in length, the lines A P and A Q will represent the forces P and Q acting on A, both in magnitude and direction. Thus if two forces, one of 4 lbs. and another of 3 lbs., are supposed to act on a particle at right angles, or perpendicular to one another, we should represent them by the lines A B, 3 inches in length, and A C, 4 inches in length, drawn perpendicularly to one another. (Fig. 8).

Fig. 8.

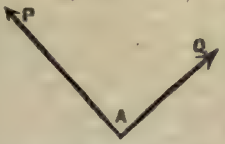
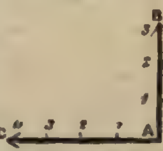


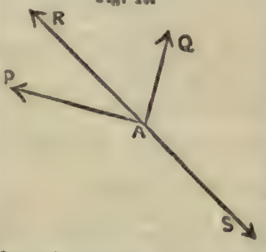
Fig. 9.



It is not necessary to use an inch as the representative of a pound, or the unit of weight. Any other convenient measure, such as the eighth or tenth of an inch, may be used, if we keep to the same scale throughout. It is usual to indicate the direction in which the force acts by the addition of an arrow-head to the line which represents it.

RESULTANT.—If two forces, P and Q (Fig. 10), represented in magnitude and direction by the lines A P and A Q, act upon the point A, so as to cause it to move, it must begin to move in some direction. Let A R represent that direction. Now we can conceive that some force, S, could be applied to A in the opposite direction to A R, in which A would begin to move, which would stop its motion and keep it at rest. Let A S represent this force in magnitude and direction. Then the particle A would be kept at rest by the three forces, P, Q, and S, acting in the directions A P, A Q, and A S. If, now, the line S A be produced to R, and A R be taken equal to A S, A R will represent in magnitude and direction a force, R, equal to S, which, if P and Q were removed, would counteract the effect of the force represented by A S on A. A R, therefore, represents, in magnitude and direction, a force which would have the same effect on A, when it acted upon it by itself, that the forces represented by A P and A Q would have, if they both acted on A.

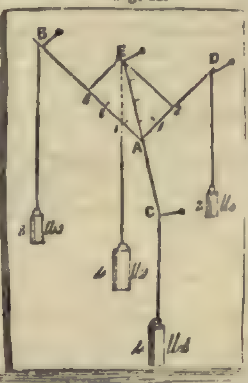
Fig. 10.



The force R, represented by A R, is called the *resultant* of the forces P and Q, represented by A P and A Q; and P and Q are called the *components* of R.

Thus, for example, if three perfectly smooth pegs, B, C, and D (Fig. 11), be inserted in a board standing in a vertical position, and three strings be attached to a point, or simply knotted together at A, and weights of 2 lb., 3 lb., and 4 lb. fixed to their extremities, as in the figure, the weights will balance each other; and, neglecting the friction on the pegs, B, C, and D, and supposing the strings perfectly flexible, the whole will come to rest in the position represented by the diagram. The point or knot A will be kept at rest by the tensions of the weights—that is, by a force of 3 lbs. acting in the direction of A B, 2 lbs. in the direction of A D, and 4 lbs. in that of A C. Let another peg, E, be inserted in the board, somewhere in the straight line C A produced, attach another string to A, and pass it over A E, with a weight of 4 lbs. at its other

Fig. 11.

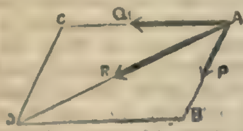


extremity, care being taken to support this latter weight so as not to allow it to act on A till we require it. If, now, the weights of 2 lb. and 3 lb. be supported so as to take off their tensions from A at the same instant that the second 4 lb. weight is allowed to exert its tension on A in the direction A E, the equilibrium of the point A will not be disturbed, since the tension of 4 lbs. acting in the direction A E, will exactly balance the tension of the 4 lb. acting in the direction A C. The tension of 4 lb. acting in the direction A E, produces, therefore, the same effect on the point A that the joint tensions of 3 lb. acting in the direction A B, and of 2 lb. in the direction A D, both together have upon it. The 4 lb. tension, acting in the direction A E, is called the *resultant* of the 3 lb. tension in the direction A B, and the 2 lb. tension in that of A D.

PROPOSITION I.

PARALLELOGRAM OF FORCES.—The proposition which enables us to represent the resultant of any two forces which act upon a material particle in magnitude and direction, when the magnitude and direction of the two forces are given, is called the parallelogram of forces, and is as follows :—

Fig. 12.



Let P and Q be two forces acting upon material point A. (Fig. 12).

Let the line A B represent the force P in magnitude and direction, and A C the force Q in magnitude and direction.

Through B draw B D parallel to A C, and through C, C D parallel to A B.

Let D be the point where the lines B D and C D meet.

Then, by construction, the figure A B D C is a parallelogram.

Join A D, A D is the diagonal of the parallelogram.

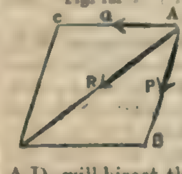
This diagonal A D will represent the resultant R of the two forces P and Q acting in the directions A B and A C, in magnitude and direction.

The parallelogram of forces may therefore be thus enunciated. *If two forces acting upon a material particle be represented in magnitude and direction by two adjacent sides of a parallelogram, the diagonal of the parallelogram, drawn through the point where these sides meet, will represent their resultant both in magnitude and direction.*

We shall first show that this proposition is true for the *direction* of the resultant, and then that it is also true for its *magnitude.*

1st. To prove that the parallelogram of forces is true for the *direction* of the resultant.

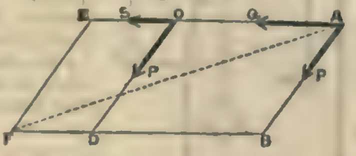
Fig. 13.



When the forces P and Q are equal, the direction of the resultant will manifestly bisect the angle B A C (Fig. 13), since no reason can be alleged why the resultant force R should incline more to one force, P, than to the other, Q. Since A B C D is a parallelogram whose sides A C and A B are equal, its diagonal, A D, will bisect the angle B A C, and therefore A D will represent the *direction* of the resultant when P and Q are equal.

Let us now assume the proposition to be true for two unequal forces P and Q, and also for two unequal forces P and S (Fig. 14), we can then prove that it will be true for the forces P and Q + S.

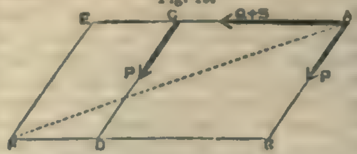
Fig. 14.



Draw a parallelogram A B D C, having one side A B proportional to the force P, and the adjacent side A C to that of Q ; produce A C to E, and make C E in the same proportion to S that the other lines bear to P and Q. Complete the parallelogram E D.

According to our assumption, the resultant of P and Q will act in the direction A D, and that of P and S in the direction C F. Now if all the points in the two parallelograms be supposed to be rigidly connected with one another, a force may be transferred from any point to another, provided the latter be in the same straight line in which the force is acting, without disturbing the equilibrium.

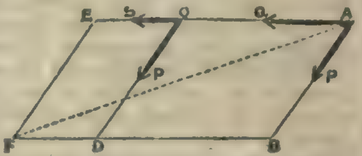
Fig. 15.



Hence the force S acting at C may be transferred to A (Fig. 15), and we shall then have a force Q + S acting on A in the direction A C, a force P acting on A in the direction A B, and a force P acting on C in the direction C D.

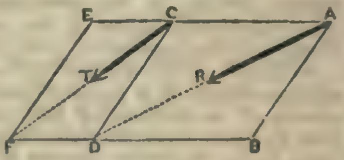
If the proposition be true for P and Q + S, their resultant will act in the direction A F, and these forces may be transferred from A to F. What we have therefore to show is, that the forces P and Q acting at A, and P and S acting at C, may be so transferred, by the principle of transmission of force, without altering the conditions of equilibrium, that we may have the forces Q + S and P acting at F and P at C (Fig. 16).

Fig. 16.



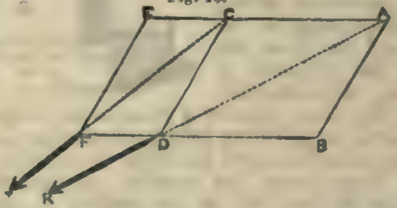
By our assumption, P and Q acting at A may be replaced by their resultant R (Fig. 17) acting in the diagonal A D of the parallelogram C B, and the forces P and S by their resultant T acting in the diagonal C F of the parallelogram E D.

Fig. 17.



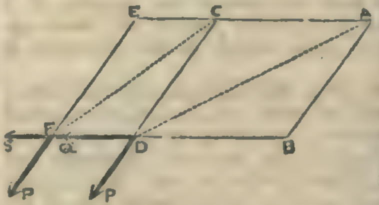
Now, according to the principle of the transmission of force, the force R may be transferred from A to D in the direction A D, and the force T from C to F in the direction C F, as represented in the accompanying diagram (Fig. 18).

Fig. 18.



Upon the same principle that we replaced the forces by their resultants, we may replace these resultants again by the forces from which they were obtained ; and we shall then have R replaced by the force P acting in the direction of C D produced, and Q in that of D F, and T replaced by P acting in E F produced, and S acting in D F produced (Fig. 19).

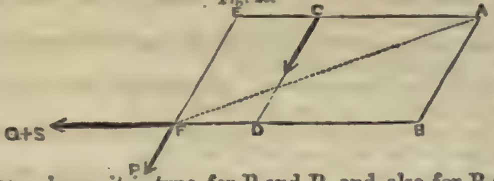
Fig. 19.



Lastly, the force Q may be transferred from D to F in the direction of D F produced (Fig. 20), and the force P from D to C in the direction C D ; so that we ultimately have the forces Q + S and P acting at F, and P at C, without having altered the conditions of equilibrium of any of the points of the parallelograms. Hence, if our proposition be true for two forces P and Q, and also for P and S, as regards the *direction* of the resultant, it is also true for the forces P and Q + S.

Now, we have shown that it is true for two equal forces; hence it is true for P and P, and also for P and P, it must therefore be true for P and P + P, or for P and 2 P. Being true for P and 2 P, and also for P and P, it is true for P and P + 2 P, or for P and 3 P, and so on; it may be shown to be true for P and *m* P, where *m* represents any whole number.

Fig. 20.



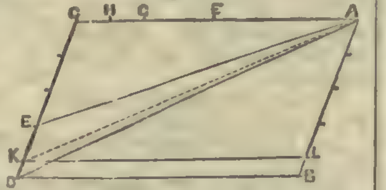
By similar reasoning the proposition may be extended to the forces *n* P and *m* P, where *m* and *n* represent any whole numbers whatever.

Hence the proposition is true, as to the *direction* of the result, for any two forces which are commensurable; or in other words, for any two forces which have a common measure, or can be expressed in terms of a common unit.

The proposition can be extended to the case of incommensurable forces, or forces which have no common measure.

Let the lines A B and A C (Fig. 21) represent two incommensurable forces, in magnitude and direction, of which A C is the greater. Complete the parallelogram A B D C, by drawing B D parallel to A C, and C D parallel to A B. If the resultant do not act in the direction of the diagonal A D of the parallelogram A B D C, let A E represent the direction in which it acts.

Fig. 21.



Divide the line A C in two equal parts in the point F; similarly divide F C in two equal parts in G, and G C in H; continue this subdivision until a part, such as H C, is obtained, which is less than D E.

A C may, therefore, be divided into a number of equal parts, each of which are equal to H C.

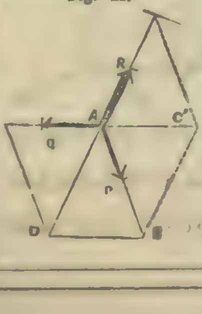
Set off distances each equal to H C along the line C D, commencing from the point C; then one of these divisions, such as K, must fall between E and D, since A C and C D have no common measure, and H C is less than E D.

From A B cut off A L = C K, and join K L and A K.

A C and A L will therefore have a common measure H C, and will consequently represent two commensurable forces; and A K, the diagonal of the parallelogram A L K C, will, by what we have previously proved, represent the direction of their resultant. The resultant A K, therefore, of the forces A L and A C, is further from A C than A E, the resultant of the forces A C and A B; but this cannot be true, since A B, being greater than A L, A E ought to be further from A C, or nearer to A B than A K. Consequently, the supposition that the resultant of A C and A B acts in the *direction* of the line A E, leads to an absurdity; and similarly it may be shown, that if it be supposed to act in any other direction than the line A D, the diagonal of the parallelogram A B D C, we shall be led to a like absurd conclusion.

Hence we infer, that if two forces, acting on a point, whether commensurable or incommensurable, be represented in magnitude and direction by two adjacent sides of a parallelogram, the diagonal of the parallelogram will represent the *direction* in which the resultant of these forces will act. We have next to show, that this diagonal will also represent the *magnitude* as well as the *direction* of the resultant.

Fig. 22.



Let A B, one side of the parallelogram A B C D (Fig. 22), represent a force P, in magnitude and direction, and the side A C, another force Q, in magnitude and direction, these two forces both acting on the point A.

Let R represent the force which is the resultant of the forces P and Q; this force will act, according to what we have already proved, in the *direction* A D, A D being the diagonal of the parallelogram A B C D.

Produce A D to D′, and take A D′ in the same proportion to R that A C bears to Q, or A B to P.

Draw D′ C′ parallel to A B, and B C′ parallel to A D′, meeting in C′, and join A C′, A B C′ D′ will be a parallelogram, A C′ its diagonal, and B C′ will be = A D′.

Since R is the resultant of the forces P and Q, a force R acting on A in the direction A D′, in the same straight line, but in the opposite direction to that in which the resultant of P and Q acts, will keep the point A at rest, when acted on by the forces P and Q.

Hence the forces P Q and R, represented in magnitude and direction by A B, A C and A D′ acting on A, will keep it at rest.

Any one of these three forces will be equal in magnitude to the resultant of the other two, but it will act in an opposite direction to it in the same straight line.

Now A C′, the diagonal of the parallelogram A D′ C′ B is the direction in which the resultant of the forces R and P represented by A D′ and A B acts. Therefore A C′ and A C must be in the same straight line; and since A C is parallel to B D, A C′ will also be parallel to B D; and since B C′ was drawn parallel to A D, A C′ B D must be a parallelogram, and B C′ will = A D.

But B C′ = A D′. Therefore A D = A D′; and since A D′ represents R in *magnitude*, A D will represent R both in *magnitude and direction*.

We are indebted to M. Duchayla, a French mathematician, for this very simple and beautiful demonstration of the parallelogram of forces. It may be proved by other methods, but they either require a knowledge of the higher branches of mathematical analysis, or else assume the principles of Dynamics.

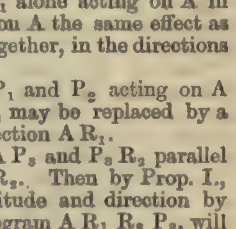
Some writers first demonstrate the properties of the lever, and from these deduce the parallelogram of forces.

PROPOSITION II.

To find the Resultant of any number of Forces acting on a Material Point in the same Plane.

Let P_1 P_2 P_3 and P_4, four forces acting on a point A in the same plane, be represented in *magnitude and direction* by the lines A P_1, A P_2, A P_3, and A P_4, (Fig. 23).

Fig. 23.



Draw P_1 R_1 parallel to A P_2, and P_2 R_1 parallel to A P_1, meeting in R_1; join A R_1. Then A R_1 the diagonal of the parallelogram A P_1 R_1 P_2 will by Prop. I. represent the resultant of the forces P_1 and P_2 acting on A in the directions A P_1 and A P_2.

Let R_1 be this force. Then R_1 alone acting on A in the direction A R_1 will produce on A the same effect as the two forces P_1 and P_2 acting together, in the directions A P_1 and A P_2.

Consequently the two forces P_1 and P_2 acting on A in the directions A P_1 and A P_2 may be replaced by a single force R_1, acting in the direction A R_1.

Again, draw R_1 R_2 parallel to A P_3 and P_3 R_2 parallel to A R_1 meeting in R_2; join A R_2. Then by Prop. I., a force R_2 represented in magnitude and direction by A R_2 the diagonal of the parallelogram A R_1 R_2 P_3, will have the same effect on A as the two forces R_1 and P_3 acting in the direction A R_1 and A P_3.

But R_1 acting on A in the directions A R_1 produces on A the same effect as P_1 and P_2 acting in the directions A P_1 and AP_2.

Hence the force R_2 acting on A in the direction A R_2 produces the same effect as the three forces P_1 P_2 and P_3 acting in the directions A P_1, A P_2, and A P_3.

Lastly, draw $P_4 R_3$ parallel to $A R_2$ and $R_2 R_3$ parallel to $A P_4$, meeting in R_3. Join $A R_3$. Then by Prop. I. a force R_3, represented in magnitude and direction by $A R_3$, will have the same effect on A as the two forces R_2 and P_4 have, acting in the directions $A R_2$ and $A P_4$.

But R_2, acting on A in direction $A R_2$, produces the same effect as P_1 P_2 and P_3 acting in the directions $A P_1$, $A P_2$, and $A P_3$.

Consequently a force R_3, acting in the direction $A R_3$, produces on A the same effect as the four forces P_1, P_2, P_3, and P_4, acting in the directions $A P_1$, $A P_2$, $A P_3$, and $A P_4$; or, in other words, $A R_3$ represents the resultant of these forces *in magnitude and direction.*

The same method may be extended to any number of forces, and affords an easy geometrical construction for finding the single resultant of any number of forces acting upon a material particle.

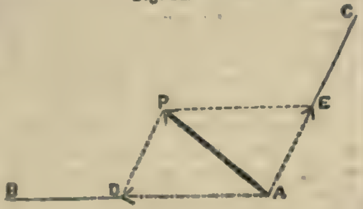
PROPOSITION III.

Resolution of Forces.

By means of the parallelogram of forces we can generally replace a single force by two others acting in any directions we please in the same plane; this is called *resolving* a force, and the forces by which it is replaced are termed its *resolved* parts.

Thus if a force P (Fig. 24) acting on a point A, be represented in magnitude and direction by the straight line A P; and A B, A C drawn through A be the arbitrary directions in which we wish to resolve the force P.

Fig. 24.



Through P draw P D parallel to A C meeting A B in D, and also PE parallel to A D meeting A C in E.

Then, by Prop. I., A E and A D will represent two forces in magnitude acting in the directions A C and A B, which produce on A the same effect as the simple force P acting in the direction A P, and therefore may replace that force without altering the conditions of equilibrium.

A E represents the *resolved* part of P along A C, and A D its *resolved* part in the direction A B.

In resolving forces, it is generally found more convenient to choose the direction A C perpendicular to A B.

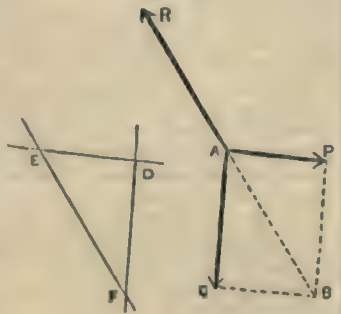
PROPOSITION IV.

Triangle of Forces.

If a material point be kept in equilibrium by the action of three forces acting upon it, in the same plane, the sides of any triangle drawn parallel to the directions of these three forces will be proportional to them; and conversely, three forces, acting on a material particle, will keep it at rest, if these forces be proportional to the sides of a triangle formed by drawing lines parallel to their directions.

Let P and Q (Fig. 25), two forces acting on A, be represented in magnitude and direction by the lines A P and A Q.

Fig. 25.



Through P draw P B parallel to A Q, and through Q, Q B parallel to A P meeting in B; join A B.

Produce A B to R, and make A R equal to A B.

Then Prop. I., a force R acting on A, represented in magnitude and direction by A R, will counteract the forces P and Q acting in the directions A P and A Q.

And A will be kept in equilibrium by the action of the three forces P, Q, and R acting in the directions A P, A Q, and A R.

Take any point E; through E draw E D parallel to A P, and E F parallel to A R.

In E D take any point D, and through D draw D F parallel to A Q, meeting E F in F.

Then by construction the triangle E D F will be equiangular to the triangles A P B or A Q B.

And, by Euc. B. VI., Prop. 4, E D : E F : F D : : A P : A B : B P : : P : R : Q.

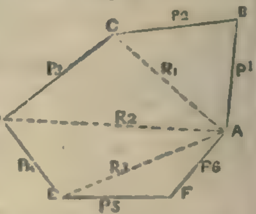
PROPOSITION V.

Polygon of Forces.

If a particle be acted on by any number of forces which are represented in magnitude, or are proportional to the sides of a polygon, it will be at rest, provided each force acts in a direction parallel to the side of the polygon to which it is proportional.

Let A B C D E F (Fig. 26) be a polygon, whose side A B, is proportional to the force P_1, B C to P_2, C D to P_3, D E to P_4, E F to P_5, and F A to P_6.

Fig. 26.



Join A C, A D, and A E.

Let $A C = R_1$, $A D = R_2$, and $A E = R_3$.

Then, by Prop. IV., R_1 will represent the resultant of the forces P_1 and P_2 in magnitude.

R_2 the resultant of R_1 and P_3, or of P_1, P_2, and P_3.

And R_3 the resultant of R_2, and P_4, or of P_1, P_2, P_3, and P_4.

But, by Prop. IV., since R_3, P_5, and P_6 are sides of the triangle A E F, R_3 will represent the resultant of P_5 and P_6.

Hence P_5 and P_6 will counteract the effect of the forces P_1, P_2, P_3, and P_4, provided the conditions of Prop. IV. be fulfilled; *i. e.*, provided these forces act upon a particle, in directions parallel to the sides of the polygon.

Since, in Prop. IV., any side of the triangle may be considered as the resultant of the other two, P_6 may be regarded as the resultant of R_3, and P_5, or of P_1, P_2, P_3, P_4, and P_5. Any one side, therefore, of the polygon may be taken as the resultant of all the others.

In the same manner the proposition may be extended to a polygon of seven, eight, or more sides.

The polygon of forces need not have all its sides in the same plane.

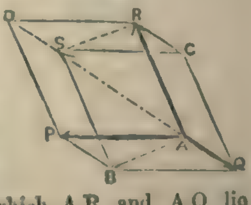
PROPOSITION VI.

Parallelopiped of Forces.

If three forces, whose directions are not in the same plane, act upon a point, and if they be represented in magnitude and direction by three adjacent edges of a parallelopiped, which meet in the point on which the forces act, the resultant of the forces will be represented in magnitude and direction by the diagonal drawn from this point to the opposite solid angle of the parallelopiped.

Let A be the point (Fig. 27), acted on by three forces, represented in magnitude and direction by A P, A Q, and A R, which are not in the same plane.

Fig. 27.



Then, Euc. B. xi. Prop. 2, A Q and A P are in the same plane; complete the parallelogram P A Q B in this plane, by drawing P B parallel to A Q, and Q B parallel to A P.

Similarly in the plane in which A R and A Q lie complete the parallelogram A R C Q.

And in the plane in which A R and A P lie, complete the parallelogram A R D P.

Also complete the parallelograms Q C B S in the plane

QCB, and the parallelogram PBSD in the plane DPB.

We shall then have constructed a parallelopiped, three of whose adjacent edges meeting at A are AP, AQ, and AR.

Join AB, AS, and RS. AB will be the diagonal of the parallelogram APBQ, and AS the diagonal of the parallelogram ARSB.

Hence AB will represent the resultant of the forces represented by AP and AQ in magnitude and direction.

And AS the resultant of the forces represented by AR and AB.

Therefore AS represents the resultants of the forces represented by AR, AP, and AQ, in magnitude and direction.

By the aid of this proposition, any single force acting on a particle may be resolved into three other forces not acting in the same plane. When a force is so resolved, it is generally found convenient to resolve it into three forces acting at right angles or perpendicularly to one another.

PROPOSITION VII.

Condition of equilibrium when a material particle is acted on by any number of forces, whose directions are all in the same straight line.

The condition of equilibrium in this case must evidently be, that the sum of all the forces acting on the particle in one direction must be equal to the sum of all the forces acting in the opposite direction.

It is usual to consider the forces acting in one direction as positive, and those in the opposite as negative. The condition of equilibrium, then, may be thus stated: the algebraical sum of all the forces is zero, or nothing.

Thus, if A be acted on by the positive forces, 4 lbs., 3 lbs., and 5 lbs., in one direction, and the negative forces, 6 lbs, 5 lbs, and 1 lb., in the opposite,

The algebraical sum of these forces will be $4 + 3 + 5 - 6 - 5 - 1 = 12 - 12 = 0$; and A will be at rest under the influence of these forces.

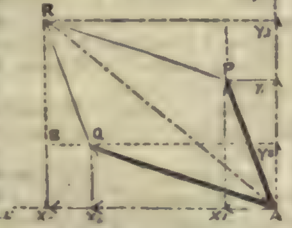
PROPOSITION VIII.

If two straight lines be drawn at right angles to one another through any point, and if two forces act on this point in any direction whatever in the same plane in which these lines are drawn, then the algebraical sum of the resolved parts of the two forces in the direction of each of the lines will be equal to the resolved part of their resultant in the same direction.

Let Ax and Ay (Fig. 29) be the straight lines drawn through the point A perpendicular to one another.

Lines so drawn are called *rectangular axes.*

Fig. 29.



Let two forces, P and Q, acting on A, be represented in magnitude and direction by AP and AQ.

Draw PR parallel to AQ, and QR to AP, meeting in R, and join AR.

Then, Prop. I., AR = R represents the resultant of P and Q in magnitude and direction.

Through the points P, Q, and R, draw PX_1, QX_2, and RX_3, perpendicular to Ax, and through the same points PY_1, QY_2, and RY_3, perpendicular to Ay.

By Prop. III., AX_1 and AY_1 will be the resolved parts of the force P in the directions Ax and Ay.

Similarly, AX_2 and AY_2 will be the resolved parts of Q; and AX_3 and AY_3 the resolved parts of R in the same directions.

Let $AX_1 = X_1$, $AX_2 = X_2$, $AX_3 = X_3$, $AY_1 = Y_1$, $AY_2 = Y_2$, and $AY_3 = Y_3$.

Produce the line Y_2Q to meet the line RX_3 in the point B.

Since APRQ is a parallelogram, AP = RQ; and also because BQ is parallel to Ax, and AP to RQ, the angle RQB = angle PAX.

But the angles RBQ and PX_1A are right angles by construction.

Hence, the triangles RQB and PAX_1 have the side AP = RQ, and the angles $PAX_1 = RQB$, and $PX_1A = RQB$.

Therefore, Euc., B. I., Prop. XXVI., $AX_1 = BQ$, and $PX = RB$.

And $X_1 + X_2 = AX_1 + AX_2 = BQ + AX_2 = X_3X_2 + AX_2 = AX_3 = X_3$, since $BQ = X_3X_2$, because by construction of figure BQX_2X_3 is a parallelogram.

And again $Y_1 + Y_2 = AY_1 + AY_2 = PX_1 + BX_3 = RB + BX_3 = RX_3 = AY_3 = Y_3$.

Since $AY_1 = PX_1$, $AY_2 = BX_3$, and $RX_3 = AY_3$, because AX_1PY_1, AX_3BY_2, and AX_3RY_3, are parallelograms by construction of figure.

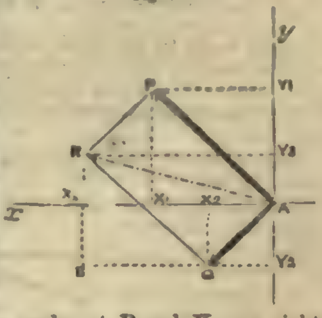
If, therefore, X_1Y_1 be the resolved parts of a force, P, along the rectangular axes, Ax and Ay, X_2Y_2 the resolved parts of Q, and X_3Y_3 the resolved parts of R, the resultant of P and Q in the direction of the same axes

$$X_3 = X_1 + X_2, \text{ and } Y_3 = Y_1 + Y_2.$$

The last figure has been so drawn that the forces P and Q both fall within the axes Ax and Ay; this is not always the case; it may sometimes happen, as in the following figure, that one of the axes may fall between the two forces; in this case it will be seen that the resolved part of the force Q, along the axis Ay, will fall on the opposite side of A, from the resolved forces of P and R along the same line.

The resolved forces which lie on one side of A are called positive, and those on the opposite side negative.

Fig. 30.



Consequently, the resolved parts of P (Fig. 30) will be X_1 and Y_1, of Q, X_2 and $-Y_2$, and of R, X_3 and Y_3. The same construction being made as in the last case, since APRQ is a parallelogram, AP = RQ, and since AX_1 is parallel to BQ, and AP to QR, the angle PAX_1 = angle RQB, also the angles at B and X_1 are right angles.

Hence, in triangles APX_1 and QBR (Euc., B. I., Prop. XXVI.), $RB = PX_1$, and $BQ = AX_1$.

$X_1 + X_2 = AX_1 + AX_2 = BQ + QY_2 = BY_2 = X_3A = X_3$.

$Y_1 - Y_2 = AY_1 - AY_2 = PX_1 - X_3B = RB - X_3B - RX_3 = AY_3 = Y_3$.

And in this case $X_3 = X_1 + X_2$, and $Y_3 = Y_1 - Y_2$.

A similar construction and demonstration will apply to every other position in which the rectangular axes may be placed with respect to the forces, and we may say, generally, that the algebraical sum of the resolved parts of any two forces acting upon a material point in the directions of any two rectangular axes, passing through that point, will be equal to the resolved parts of their resultant, along the same axes.

PROPOSITION IX.

To find the conditions of equilibrium of any number of forces acting upon a material point, the directions of the forces being all in the same plane, but not in the same straight line.

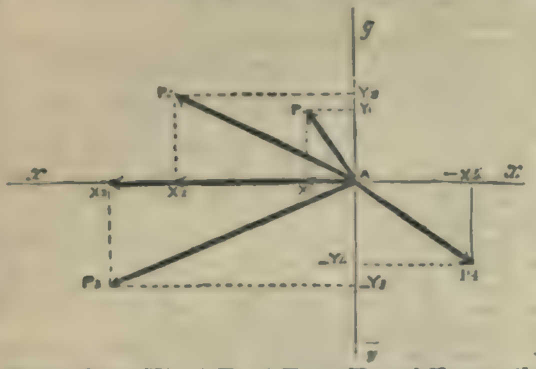
Let P_1, P_2, P_3, and P_4 (Fig. 31), be four forces acting upon A, represented in magnitude and direction by the lines AP_1, AP_2, AP_3, and AP_4.

Through A draw the rectangular axes Ax and Ay.

Through the points P_1, P_2, P_3, and P_4, draw P_1Y_1,

$P_2 Y_2$, $P_3 Y_3$, and $P_4 Y_4$, perpendicular to Ay, and $P_1 X_1$, $P_2 X_2$, $P_3 X_3$, and $P_4 X_4$, perpendicular to Ax.

Fig. 31.



Then, Prop. III., AX_1, AY_1, $= X_1$ and Y_1, are the resolved parts of P_1 in the direction of the axes Ax and Ay.

AX_2 and $AY_2 = X_2$ and Y_2 those of P_2, AX_3 and $-AY_3 = X_3$ and $-Y_3$ those of P_3 and $-AX_4$ and $-AY_4 - X_4$ and $-Y_4$ those of P_4 along the same axes.

Let R_1 be the resultant of the forces P_1 and P_2.

R_2 the resultant of the forces R_1 and P_2, or of P_1, P_2, and P_3.

And R_3 the resultant of the forces R_2 and P_3, or of P_1, P_2, P_3, and P_4.

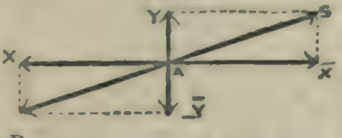
Then Prop. VIII., the resolved part of R_1 along the axis of $Ax = X_1 + X_2$, that of $R_2 = X_1 + X_2 + X_3$, and that of $R_3 = X_1 + X_2 + X_3 - X_4$.

Similarly the resolved part of R_1 along the axis $Ay = Y_1 + Y_2$, that of $R_2 = Y_1 + Y_2 + Y_3$, and that of $R_3 = Y_1 + Y_2 - Y_3 - Y_4$.

Let $X = X_1 + X_2 + X_3 - X_4$, the algebraical sum of the resolved parts of the four forces along the axis Ax, and $-Y = Y_1 + Y_2 - Y_3 - Y_4$, the algebraical sum of the resolved parts of these forces along the axis Ay.

Take $AX = X$ along the axis AX, and $A\bar{Y} = -Y$ along the axis AY.

Fig. 32.



Through X draw XR_3 (Fig. 32) perpendicular to AX, and through $-Y$, $-YR_3$ perpendicular to AY meeting in the point R_3. Join AR_3.

Then, Prop. III., AR_3 will represent the resultant of the forces P_1, P_2, P_3, and P_4, in magnitude and direction.

This single force AR_3, acting alone, will produce on A the same effect as the forces P_1, P_2, P_3, and P_4.

Produce AR_3 to AS, make $AS = AR_3$.

Then, Prop. VII., A force represented in magnitude and direction by AS will exactly counteract the effect of the force represented by AR_1.

Consequently, a force represented in magnitude and direction by AS, will keep the point A in equilibrium when acted on by the forces P_1, P_2, P_3, and P_4.

From S let fall SX perpendicular to AX, and SY perpendicular to AY.

Then, Prop. III., $A\bar{X}$ and AY are the resolved parts of AS along the rectangular axes. Now in the triangles AXR_3 and $AS\bar{X}$, the angle XAR_3 = angle $SA\bar{X}$, and the angles at X and X are right angles, and also $AR_3 = AS$.

Therefore $A\bar{X} = AX = X$, and $S\bar{X} = XR_3$.

$AXR_3\bar{Y}$ and $A\bar{X}SY$ are parallelograms.

Consequently, $S\bar{X} = AY$, and $XR_3 = A\bar{Y}$.

Therefore $AY = A\bar{Y} = -Y$.

The resolved parts of AS are therefore $-X$ and Y.

But $X = X_1 + X_2 + X_3 - X_4$, and $-Y = Y_1 + Y_2 - Y_3 - Y_4$.

Therefore $X_1 + X_2 + X_3 - X_4 - X = 0$, and $Y_1 + Y_2 - Y_3 - Y_4 + Y = 0$.

Or if five forces represented in magnitude and direction by AP_1, AP_2, AP_3, AP_4, and AS keep a point in equilibrium, the algebraical sum of the resolved parts of these forces along the rectangular axes passing through this point, will each be equal to zero.

The same demonstration may be applied to any number of forces. It may also be observed that the position of the rectangular axes is perfectly arbitrary, provided only that they are in the same plane in which the forces are supposed to act.

The conditions of equilibrium when all the forces are not in the same plane, involve the discussion of solid geometry. We shall reserve the consideration of this subject to a more advanced portion of the section.

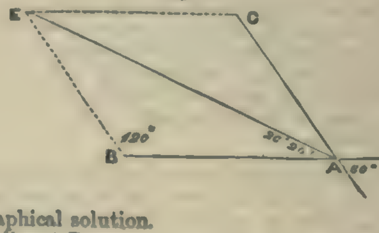
The student will observe that all the conditions of equilibrium, for a material point which we have considered, are geometrical deductions from the parallelogram of forces, and involve no new mechanical principles; and that the parallelogram of forces depends upon one mechanical principle, that of the transmission of a force from any one point to another rigidly connected with it, and in the direction of its action, without altering its effect.

We subjoin two or three examples, to show the method of applying the principles we have determined. Mechanical problems may be solved in two ways—graphically, *i. e.*, by accurately drawing the geometrial figures by mathematical instruments and scales, and obtaining a result, which can be measured by these instruments; or trigonometrically, *i.e.*, by computing the relations of the figures according to the principles of trigonometry, and thus arriving by means of trigonometrical tables at a more accurate result than the former will afford.

We would recommend the student to use both of these methods, as he will by this means obtain a clearer view of the subject than by accustoming himself only to one of them.

PROBLEM I.—*Two forces, represented by 12lbs. and 15lbs., are inclined to each other at an angle of 60°; required the magnitude of the resultant; and its inclination to the greater.*

Fig. 33.



1st, Graphical solution.

Draw a line AB.

Make AB equal to 15 parts of any convenient scale, as the 8th or the 10th of an inch.

Draw AC inclined to AB at an angle of 60°.

Take AC equal to 12 parts of the same scale, that AB is equal to 15 parts.

Through C draw CE parallel to AB, and through B, BE parallel to AC, meeting in the point E.

Join AE.

Then, Prop. I., AE will represent the resultant of the forces represented by AC and AB in magnitude and direction; and if AE be measured by the scale used in drawing AB and AC, it will be found to be $23\frac{4}{10}$ parts of that scale, and the angle EAB will measure very nearly 26° 20′.

The resulting force will therefore be represented by $23\frac{4}{10}$ lbs., and its inclination to the greater force will be 26° 20′ nearly.

The process might have been shortened by the use of Prop. IV., in which case we should have drawn AB = 15 parts of the scale, BE making an angle 180° − 60° or 120° with AB, and BE = 12 parts of the scale, and lastly joined A and E.

2nd. Trigonometrical solution.

$$
\begin{aligned}
AE^2 &= AB^2 + BE^2 - 2\,AB\cdot BE\ \cos.\ 120^\circ.\\
&= AB^2 + BE^2 + 2\,AB\cdot BE\ \cos.\ 60^\circ.\\
&= 12^2 + 15^2 + 2 \times 12 \times 15 \times \cdot 5.\\
&= 144 + 225 + 180 = 549.
\end{aligned}
$$

Hence, $AE = \sqrt{549} = 23\cdot 43$.

Again, $\frac{\sin.\ EAB}{\sin.\ EBA} = \frac{EB}{EA}$, or $\frac{\sin.\ EAB}{\sin.\ 120^\circ} = \frac{12}{23\cdot 43}$.

Hence, $\sin.\ EAB = \frac{12}{23\cdot 43}\sin.\ 120^\circ = \frac{12}{23\cdot 43}\sin.\ 60^\circ$.

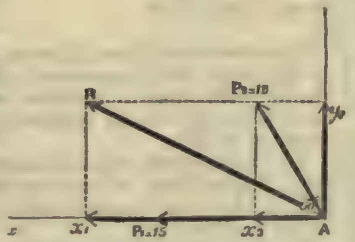
And Log. sin. E A B = Log. 12 + Log. sin. 60° — Log. 23·43.

$$
\begin{aligned}
&= 1\cdot 0791812 + 9\cdot 9375306 - 1\cdot 3697723\\
&= 9\cdot 6469395.\\
&= \text{Log. sin. } 26^\circ\ 20'\ \text{nearly.}
\end{aligned}
$$

Angle E A B = 26° 20′.

When the resultant of more than two forces is to be found, it will be more convenient to resolve the forces as in Prop. IX. We shall therefore work out the above example by that method.

Fig. 34.



Draw two straight lines, A x and A y at right angles to each other (Fig. 34).

Take $AP_1 = 15 =$ force P_1 along A x.

Draw $AP_2 = 12 =$ force P_2, making an angle 60° with AP_1.

Through P_2 draw $P_2\,y_2$ perpendicular to Ay, and $P_2\,x_2$ perpendicular to Ax.

Then Ax_2 Ay_2 are the resolved parts of P_2 along Ax and Ay; AP_1 is the resolved part of P_1 along the axis Ax, and its resolved part along Ay is zero.

Produce AP_1 to x', and make $P_1\,x' = Ax_2$.

Then $Ax' = AP_1 + Ax_2$ is the resolved part of the resultant of P_1 and P_2 along Ax and Ay_2 is the resolved part along Ay.

Through x' draw x' R perpendicular to Ax, and produce $y_2\,P_2$ to meet x' R in R. Join A R.

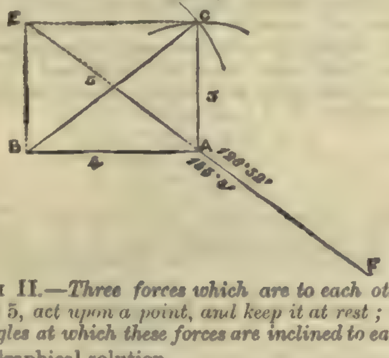
Then A R represents the resultant of P_1 and P_2 in magnitude and direction, $Ay_2 = AP_2 \cos.\ 60^\circ = 12 \times \cdot 5 = 6$, and $Ay_2 = P_2\,X_2 = AP_2 \sin.\ 60^\circ = 12 \times \cdot 866 = 10\cdot 392$.

But $AR^2 = Ay_2{}^2 + Ry_2{}^2 = Ay_2{}^2 + Ax_1{}^2 = (10\cdot 392)^2 + (15+6)^2 = (10\cdot 392)^2 + (21)^2. = 107\cdot 993664 + 441 = 548.993664$.

Therefore $AR = \sqrt{548.993664} = 23\cdot 430$.

Again, $\cos.\ RAx^1 = \frac{Ax^1}{AR} = \frac{21}{23\cdot 43} = \cdot 89628 = \cos.\ 26^\circ\ 20'$ nearly. And therefore angle R A x' = 26° 20′. And this is the inclination of the resultant to the greater force.

Fig. 35.



Problem II.—*Three forces which are to each other as 3, 4, and 5, act upon a point, and keep it at rest; required the angles at which these forces are inclined to each other.*

1st. Graphical solution.

Draw a line A B = 4 parts of some scale, with A as a centre and radius A C = 3 parts of this scale, describe an arc.

Also, with B as centre, and radius B C = 5 parts of the scale, describe another arc.

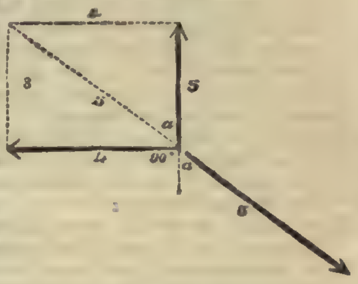
Let C be the point where the two arcs intersect.

Join A C and B C. Through C draw C E parallel to A B and through B, B E parallel to A C, meeting in E. Join A E and produce A E to F, making A F = A E. Then the forces which are to each other as 3, 4 and 5, acting upon the point A, and keeping it at rest, will be represented in magnitude and direction by the lines A C, A B, and A F; and the angles being measured by a protractor, or any other means used for measuring angles, it will be found that the angle B A C = 90°, the angle B A F = 143° 8′, and the angle C A F = 126° 52′, nearly.

2nd. Trigonometrical solution.

Since $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

Fig. 36.



The forces represented by 3 and 4 must be at right angles to each other, and their resultant 5, will be the diagonal of the rectangular parallelogram whose sides are as 3 and 4.

Let a be the angle between the resultant and the force represented by 3.

Then $\tan.\ a = \frac{4}{3} = 1\cdot 33333 = \tan.\ 53^\circ\ 8'$, nearly.

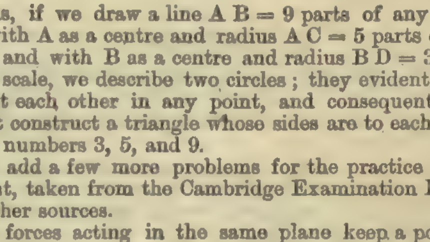
Hence, by a reference to the diagram it will be readily seen that the angle between the forces 3 and 5 is $180^\circ - a = 180^\circ - 53^\circ\ 8' = 126^\circ\ 52'$, while that between the forces 4 and 5 is $90^\circ + a = 90^\circ + 53^\circ\ 8' = 143^\circ\ 8'$.

Problem III.—*Three forces represented by the numbers 3, 5, and 9, cannot under circumstances produce equilibrium on a point.*

This is evidently true, since by Prop. IV., three forces can only produce equilibrium on a point, when a triangle can be described whose sides are respectively proportional to the magnitudes of the forces. Now, the sides of a triangle never can bear to each other the proportion of the numbers 3, 5, and 9, since in every triangle the sum of any two sides must always be greater than the third, and 3 + 5 = 8, a number less than 9.

If we attempt to solve the problem graphically, we shall soon perceive its impossibility.

Fig. 37.



Thus, if we draw a line A B = 9 parts of any scale, and with A as a centre and radius A C = 5 parts of the scale, and with B as a centre and radius B D = 3 parts of the scale, we describe two circles; they evidently will not cut each other in any point, and consequently we cannot construct a triangle whose sides are to each other as the numbers 3, 5, and 9.

We add a few more problems for the practice of the student, taken from the Cambridge Examination Papers and other sources.

Three forces acting in the same plane keep a point at rest; the angles between the directions of the forces are 135°, 120°, and 105°; compare their magnitudes.

Four forces represented by 1, 2, 3 and 4, act in the same plane on a point. The directions of the first and third are at right angles to each other; and so are the directions of the second and fourth; and the second is inclined at an angle of 60° to the first. Find the magnitude and direction of the resultant.

A circular hoop is supported in a horizontal position, and three weights of 4, 5 and 6 lbs. respectively are suspended over its circumference by three strings knotted together at the centre of the hoop. Neglecting the friction of the edge of the hoop, find the angles between the strings when there is an equilibrium.

Three equal forces, each equivalent to 6 lbs., act on a point; the first two are inclined to each other at an angle of 7[illegible]°, and the third is inclined at an angle of 15° to the first. Find the magnitude and direction of the resultant.

The resultant of two forces is 50 lbs., and the angles which it makes with their directions is 20° and 30°; find the component forces.

A boat, fastened to a fixed point by a rope, is acted on at the same time by the wind and the current. Suppose that the wind was S.E., the direction of the current S., and the direction of the boat from the fixed point S. 20° W., and also that the pressure on the point was 150 lbs., it is required to find the forces of the wind and the current.

In pulling a weight along the ground by a rope, inclined to the horizon, at an angle of 45°, a power of 40 lbs. was exerted; required the force with which the body was dragged horizontally.

Four forces are in the same plane, which are to each other as 6, 8, 10, and 12, act upon a given point, and are inclined to a given line at the angles 20°, 40°, 80°, and 150° respectively; find the magnitude and direction of a fifth force which shall balance the others.

EQUILIBRIUM OF A RIGID BODY.—Having considered the conditions of equilibrium of a material particle acted on by any number of forces acting in the same plane, we next proceed to consider the conditions of equilibrium of a rigid body under the same circumstances. Here it may be as well to repeat what has been stated before, that by the term *rigid body*, we understand a body composed of material particles held together by unknown molecular forces of such intensity that the body cannot be altered in shape, or its particles in any way displaced by any forces which can act upon it. This rigid body also possesses the property, that if any force be applied to it, its particles will transmit that force, unimpaired in intensity, to any point in the body, which lies in the line of direction in which the force is acting; consequently, the effect of a force on a rigid body will be the same, if we transfer its point of application from any one point in the rigid body to any other, provided these two points are in the line in which the force is supposed to act.

Unless it is otherwise stated, this *hypothetical rigid body* is also considered as being destitute of weight.

If two or more forces acting on a rigid body are applied to the same point of the body, the conditions of equilibrium will be the same as those for a material point under the same circumstances, for the same force which, when applied to the material point, would counteract the effect of these forces, would also keep the body in equilibrium, when applied to the point in the body upon which the forces act.

PROPOSITION X.

To find the magnitude and direction of the resultant of two forces acting on different points of a rigid body, the directions of the forces being in the same plane, but not parallel to each other.

Let a force P (Fig. 38), represented in magnitude and direction by the line A P, act upon the point A of a rigid body, and another force Q, represented by B Q, act upon the point B of the same body. B Q and A P both being the same plane.

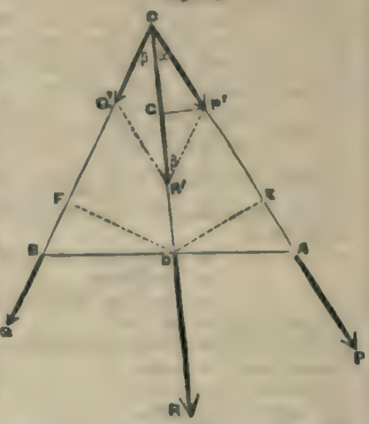
Join A B. Produce P A and Q B to meet in the point C.

Along C B take C Q′ = B Q, and along C A, C P′ = A P.

Draw Q′ R′ parallel to C P′, and P′ R′ parallel to C Q′, meeting in R′.

Join C R′, and produce C R′ to R, cutting A B in D.

Fig. 38.



Make D R = C R′. Then D R will represent the resultant of the forces P and Q in magnitude and direction.

By the principle of the transmission of forces, the points A B and C are supposed to be rigidly connected with each other.

The point of application of the force P may be transferred from A to C, and the force Q from B to C.

Then the forces P and Q acting at C may be replaced by the single force represented in magnitude and direction by C R′, Prop. I., and this force may be transferred from C to D, and be represented in magnitude and direction by D R.

This construction will enable us to represent, graphically, the resultant of any two forces acting on a rigid body in the same plane, but in directions not parallel to one another.

For the purpose of calculation, it will be convenient, however, to determine the geometrical relation of the point D to the forces P and Q, and their directions.

Through D draw D E (Fig. 38), perpendicular to C A, and D F perpendicular to C B; also through P′ draw P′ G perpendicular to C D.

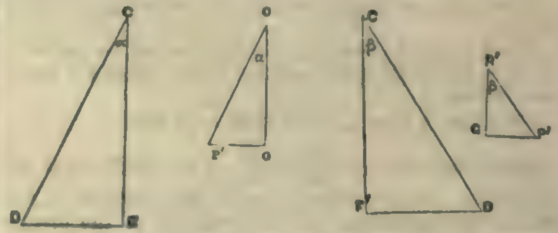
Let α represent the angle A C D, and β the angle B C D.

Then since by construction P′ R′ is parallel to C Q′, therefore (Euc., B. I., Prop. XXVII.), angle C R′ P′ = angle Q′ C R′ = β.

The angles at E, F, and G, are right angles by construction.

Hence the figure will afford us two pairs of similar triangles. (See Fig. 39).

Fig. 39.



Since in the triangles C D E, C G P′ the angles D C E and P′ C G are both = α, and those at E and G′ are right angles (as will probably be more readily perceived in the above figures than in the more complicated figure), it follows that the triangles C D E and C P′ G are equiangular triangles and similar to one another, and therefore (Euc., B. VI., Prop. IV.),

$$CD : DE :: CP' : GP'$$

$$\text{or, } \frac{CD}{DE} = \frac{CP'}{GP'}$$

Similarly, since the angles F C D and G R′ P′ are both equal to β, and the angles at F and G right angles, the triangles C F D and R′ G P′ are similar triangles, and

$$DF : CD :: GP' : P'R'$$

$$\text{or, } \frac{DF}{CD} = \frac{GP'}{P'R'}$$

Multiplying equals together, we have

$$\frac{CD}{DE} \cdot \frac{DF}{CD} = \frac{CP'}{GP'} \cdot \frac{GP'}{P'R'}$$

$$\text{and therefore } \frac{DF}{DE} = \frac{CP'}{P'R'}$$

But C P′ R′ Q′ (Fig. 38) is a parallelogram, and therefore P′ R′ = C Q′.

$$\text{Hence } \frac{DF}{DE} = \frac{CP'}{CQ'} = \frac{P}{Q}.$$

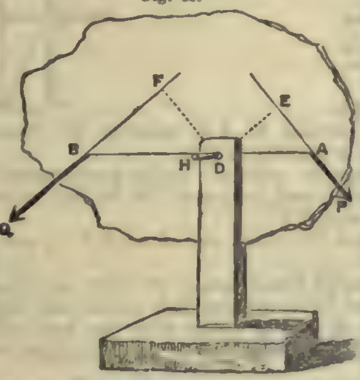
or, The perpendicular

$$\frac{\text{from the point D on the direction of the force Q}}{\text{from the point D on the direction of the force P}} = \frac{P}{Q}$$

Consequently, the perpendicular from the point D on the direction of the force Q multiplied by Q, equals the perpendicular from the point D on the direction of the force P multiplied by P.

MOMENT OF FORCES AND FULCRUM.—If we suppose an axis, H D (Fig. 40), passing through the plane in which the forces act, fixed with perfect rigidity, and perpendicular to that plane at the point D, and then placed on a stand, as in the accompanying figure, so as to allow the plane to move with perfect freedom about the axis H D, then, since the resultant of the forces P and Q acting at A and B passes through the point D, their joint effect will be counteracted by the reaction of the stand on the axis H D. Neglecting the weight of the plane and the friction of the axis, the plane will be in a state of equilibrium, under the influence of the two forces P and Q, acting at A and B in the directions A P and B Q, together with the reaction of the stand acting on the plane at the point D in a direction opposite to the resultant of the forces P and Q. If the rigid body be reduced to the rigid line A B, A B is called a lever, and the point D its fulcrum.

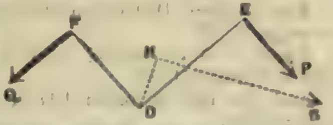
Fig. 40.



Since the conditions of equilibrium, or that the resultant of the forces B and Q should pass through D, are as above stated, that,

The perpendicular from D *on the direction of the force* Q *multiplied by* Q, *equals the perpendicular from* D *on the direction of the force* P, *multiplied by* P; it follows that the force P may be replaced by a force S without altering these conditions, provided that the perpendicular from D, D K on the direction K S of the force S multiplied by S, be equal to the perpendicular from D on the direction of the force P multiplied by P (Fig. 40).

Fig. 41.



The writers on Mechanics have applied the term moment or momentum to this product of the linear units in the length of the perpendicular from a given point on the direction of a force, by the units of force in the given force.

Thus, if the forces P, Q and S be represented in magnitude and direction by E P, F Q, and S K, and D E, D F and D K be the perpendiculars on those directions, P E × D E, Q F × D F, and S K × D K will be the respective moments or moments of the forces P Q and S, about the point D.

These products are not only termed the moments of the forces with respect to the point D, but also the moments of the forces with respect to axis H D, perpendicular to the plane in which they act.

The student must be careful not to confound these *statical moments, or momenta,* with the momenta of dynamics; the momentum of dynamics being the product of the mass of a body by its velocity.

If the force P alone were to act on the plane at the point A, it would evidently twist the plane about the fixed axis H D, and Q acting alone at B, would twist the plane about H D in a direction opposite to that in which P would do so. The tendency of P therefore to twist the plane about the axis H D, or point D, is counteracted by the tendency of Q to twist the plane in the opposite direction, in the case where equilibrium exists.

We have seen, however, that the tendency of Q to twist the body about the axis H D, will be counteracted by any other force whose statical moment, with respect to H D or D, is equal that of the force P.

Hence we may consider the statical moment of a force about a given point or axis, as a measure of its tendency to twist the plane to which it is applied, about that point or axis.

POSITIVE AND NEGATIVE MOMENTS.—The moments of those forces whose tendency is to twist a body about the axis or fulcrum, in the direction in which the hands of a watch move, are termed positive; those whose tendency is in the opposite direction, are called negative.

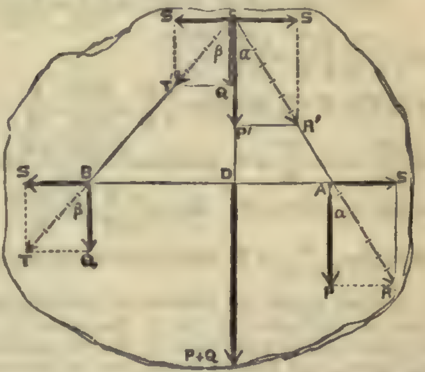
It will readily be seen, that if the directions of the forces P and Q in Proposition X. had been parallel to one another, the demonstration there used could not be applied; as, in that case, the directions of the forces could not be produced till they met.

PROPOSITION XI.

To find the magnitude and direction of the resultant of two forces acting on different points of a rigid body, the directions of the forces being in the same plane, but parallel to each other.

Let P and Q (Fig. 42) be the two forces, A and B the two points in the plane.

Fig. 42.



The force P being represented in magnitude and direction by the line A P, and Q by the line B Q.

Then A P and B Q are parallel to each other.

Join A B.

The conditions of equilibrium will not be altered if any arbitrary force S be applied to A in the direction A S, and to B in the opposite direction B S.

Let these equal and opposite forces be represented in magnitude and direction by A S and B S.

Consequently A S = B S.

Complete the parallelogram S B Q T and the parallelogram S A P R.

Join A R and B T.

Then A R = R represents the resultant of the forces P and S in magnitude and direction, and may be substituted for these forces.

Also B T = T represents the resultant of Q and S in magnitude and direction, and may be substituted for them.

Produce A R and B T to meet in the point C.

Through C draw C D parallel to A P or B Q, meeting A B in D.

The force R acting at A in direction A R may be transferred from R to C in the line R A C, and represented in magnitude and direction by C R′, C R′ being = A R.

Similarly T may be transferred from B to C, and represented in magnitude and direction by C T′, C T′ being equal to B T.

Through C draw the line S C S parallel to A B.

Also through T′ draw T′ Q′ parallel to B A, meeting C D in Q′, and T′ S parallel to D C, meeting S C in S.

Similarly through R′ draw R′ P′ parallel to A B, meeting C D in P′, and R′ S parallel to D C, meeting S C in S.

The parallelogram S C Q′ T′ is similar and equal in all respects to the parallelogram S B Q T, and the parallelogram C P′ R′ S to the parallelogram A P R S.

Hence, Prop. III., the force R acting at C, in the direction C R′, may be replaced by the forces S and P, represented in magnitude and direction by C S and C P′; also the force T acting at C in direction C T′, may be replaced by the forces S and Q′ acting in the directions C S and C Q′.

The equal and opposite forces S and S, acting in the line S C S, will destroy each other.

And the parallel forces P and Q acting at A and B are replaced by the forces P and Q acting at C in the direction C D, and these forces again may be transferred from C to D.

Hence the resultant of the forces P and Q acting at A and B, will be a force P + Q acting at D in a direction parallel to the directions of P and Q.

To determine the position of the point D.

Since P′ R′ is parallel to A D, the triangles C P′ R′, C D A are equiangular, and therefore, Euc., B. VI., P. 4—

$$CD : AD :: CP' : R'P', \text{ or } \frac{CD}{AD} = \frac{CP'}{R'P'}$$

Similarly because Q′ T′ is parallel to D B, the triangles C T′ Q′, C B D are equiangular, and

$$BD : CD :: T'Q' : CQ', \text{ or } \frac{BD}{CD} = \frac{T'Q'}{CQ'}$$

$$\text{Hence } \frac{CD}{AD} \cdot \frac{BD}{CD} = \frac{CP'}{R'P'} \cdot \frac{T'Q'}{CQ'}$$

$$\text{But } R'P' = T'Q', \text{ therefore } \frac{BD}{AD} = \frac{CP'}{CQ'} = \frac{P}{Q}$$

$$\text{And } \frac{BD}{AD} + 1 = \frac{P}{Q} + 1, \text{ therefore } \frac{BD + AD}{AD} = \frac{P + Q}{Q}$$

$$\text{But } BD + AD = AB. \quad \text{Hence } \frac{AB}{AD} = \frac{P + Q}{Q}$$

$$\text{And } AD = \frac{Q}{P + Q} AB, \text{ which determines the point D.}$$

Or, in other words, the distance of the fulcrum from the point of application of one of the forces P, is equal to the other force Q multiplied by the distance between the two forces, and divided by the sum of the two forces.

When A B is perpendicular to A P or B Q, we have, since—

$$\frac{BD}{AD} = \frac{P}{Q}$$

$$Q \times BD = P \times AD;$$

but in this case Q × B D is the moment of Q with respect to D, and P × A D the moment of P with respect to D. Hence in this case, as in the last proposition, the moments of P and Q with respect to D, are equal to each other.

In the case we have just considered, we supposed the two forces P and Q to be acting in the same direction; when they act in the opposite directions, our construction will be modified as represented by the accompanying diagram. (Fig. 43).

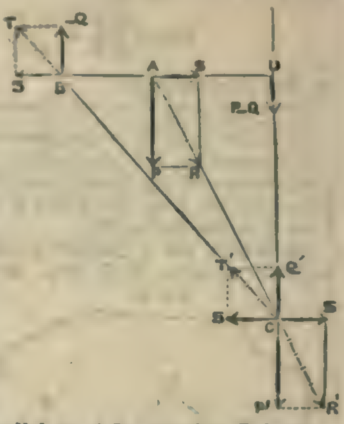
To A and B, as before, apply two equal and opposite forces represented in magnitude and direction by A S and B S.

Complete the parallelograms A P R S and B Q T S.

Join B T and A R, and produce them till they meet at C.

Through C draw C D parallel to A P or B Q, meeting B A produced in D.

Fig. 43.



The resultant of the forces S and Q acting at B will be represented in magnitude and direction by B T, and this force may be transferred from B to C, and represented by C T′ = B T.

The resultant of the forces P and S acting at A will also be represented by A R, and this may be transferred from A to C, and represented by C R′ = A R.

Through C draw S C S parallel to B A, through T′, T′ Q′ parallel to A B, T′ S parallel to D C, and through R′, R′ P′ parallel to A B, meeting D C produced in P′, and R′ S parallel to D C.

The force C T′ may be replaced by the forces represented by C Q′ and C S, and the force C R′ by the forces represented by C S and C P′.

But C S = B S, and C S = A S, and A S = B S by construction.

Hence the two equal forces acting on C in opposite directions in the line S C S destroy each other, and the forces P acting at A, and Q acting at B, are replaced by the two forces C Q′ and C P′ acting on C in opposite directions.

But C Q′ = B Q = Q, and C P′ = A P = P.

Hence these two forces may be represented by a single force P − Q acting at C in the direction C P′, and this force may be transferred from C to D in the line C D.

The resultant of the two parallel forces, P and Q, acting in opposite directions at the points A and B, will be a force P − Q acting at D in B A produced.

To determine the position of the point D.

Since A P is parallel to D C, the angle A R S = angle A C D, and the angles at C and D are common to the two triangles A R S, A C D.

Therefore the triangles A R S, A C D are equiangular and similar.

$$\text{And } AS : SR :: AD : DC, \text{ or } \frac{AS}{SR} = \frac{AD}{DC}$$

Also because B Q is parallel to C D, the angle T B Q = angle B C D.

Again, T Q is parallel to B D, therefore the angle Q T B = angle D B C.

And the two triangles T Q B, B D C are equiangular and similar.

$$\text{And } BQ : QT :: DC : BD, \text{ or } \frac{BQ}{QT} = \frac{DC}{BD}.$$

Multiplying the equal fractions together, we have

$$\frac{AS}{SR} \cdot \frac{BQ}{QT} = \frac{AD}{DC} \cdot \frac{DC}{BD}.$$

$$\text{But } QT = BS = AS, \text{ and } SR = AP$$

$$\text{Hence } \frac{BQ}{AP} = \frac{AD}{BD}, \text{ or } \frac{Q}{P} = \frac{AD}{BD}.$$

$$\text{Now } BD = AB + AD, \text{ therefore } \frac{Q}{P} = \frac{AD}{AB + AD}.$$

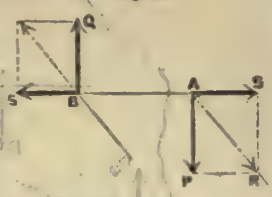
And Q. A B + Q. A D = P. A D, or Q. A B = (P − Q) A D.

$$\text{Therefore } AD = \frac{Q.\,AB}{P - Q}.$$

This result might have been obtained from the previous case by substituting − Q for Q, which would give P − Q for the resultant, but the expression for A D would be $-\frac{Q.\,AB}{P - Q}$, the negative sign signifying that the point D in this case would lie on a different side of A from what it would when the force Q was positive, or when both forces acted in the same direction.

In the last case, that is, when P and Q act in opposite directions, if P and Q are equal, we cannot represent their resultant by any single force.

Fig. 44.



For if we pursue the mode of construction adopted before, we shall find the line B T (Fig. 44), in this case parallel to the line A R; and consequently these lines, if produced, will never meet.

This admits of an easy proof, because B Q is parallel and equal to A P, and B S and A S are equal to each other, and are in the same line.

Therefore the parallelogram A P R S is similar and equal to the parallelogram B Q T S, and the angle P A R = the angle T B Q.

Also because A P is parallel to B Q, the angle P A B = angle Q B A.

And by addition, angle P A R + angle P A B = angle T B Q + angle Q B A.

Or the angle R A B = the angle T B A.

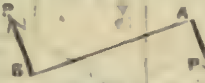
And, by Euc., B. I., P. XXVII., R A must be parallel to T B.

Our method in this case fails to discover a point D anywhere in the line A B produced, to which a single resultant can be applied.

This is indicated by the formulæ which we have already obtained of P − Q for the magnitude of the resultant, and $\frac{Q.\ AB}{P-Q}$ for A D, the distance of its point of application from A, since when Q = P, they give us P − P or 0 for the magnitude of the resultant, and $\frac{P.\ AB}{P-P}$ or $\frac{P\ \ AB}{0}$ the algebraical sign of infinity for A D; a result which shows that our problem in this case is impossible.

COUPLE.—When two equal and opposite parallel forces act at different points of a rigid body in the same plane, their effect, as we have seen, cannot be counteracted by any single force applied to the body, and their tendency will evidently be to twist the body in the direction of the plane in which they act. Thus P (Fig. 45) acting on A in the direction A P, and P acting on B in the direction B P will twist the body round in the direction B A P or A B P.

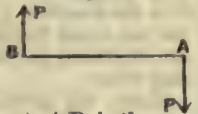
Fig. 45.



The term couple is applied to such a system of forces.

ARM OF A COUPLE.—The perpendicular distance between the directions in which the forces producing a couple act, is called the arm of that couple. Thus if the line A B (Fig. 46) is perpendicular to A P and B P, which represent the directions in which the forces producing a couple act, A B is the arm of that couple.

Fig. 46.



MOMENT OF A COUPLE.—The product of the arm of a couple and one of the forces producing it, is called the moment of that couple.

Thus if P be the force acting in the direction A P or B P, then P × A B is the *moment* of the couple produced by the couple whose *arm* is A B.

This moment is a measure of the tendency of the couple to twist the body on which it acts, and it is customary to indicate a couple by its moment.

EQUILIBRIUM OF A COUPLE.—Though no single force can be found which can counteract the effect of a couple on a rigid body, yet a couple may be found to neutralise the influence of another. Thus if a force Q (Fig. 47—1), equal and opposite to the force P represented by A Q be applied to A in the direction A Q of the line P A produced, and a similar and equal force at B in the line B P produced to Q, then the force Q at A being equal and opposite to that of P at A, but in the same straight line, will neutralise it.

Fig. 47—(1).

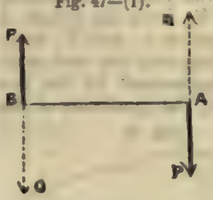
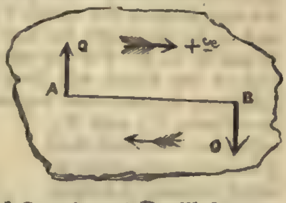


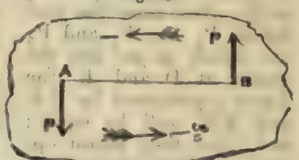
Fig. 47—(2).



Similarly, the forces P and Q acting at B will destroy each other, and the body will be in a state of equilibrium under the influence of two couples whose moments are P × A B and Q × A B, but which tend to twist the body in opposite directions. (Fig. 47—2).

A couple whose tendency is to twist the body in the direction in which the hands of a watch move is called a positive couple, such as Q × A B in the accompanying diagram (Fig. 47—2), while the couple which would cause the body to move in the opposite direction, such as P × A B, is called a negative couple. (Fig. 48).

Fig. 48.



It is also convenient to designate a couple by its moments; thus, when we speak of the couple P × A B, we mean the couple whose moment is P × A B, A B representing its arm, and P one of the equal forces acting at its extremity.

It is more convenient generally to represent the product of P and A B, by the symbol P · A B instead of P × A B.

AXIS OF A COUPLE.—The *axis* of a couple is a straight line, which is supposed to be drawn perpendicular to its plane, and proportional in length to its *moment.*

Thus, if the arm of a couple be 4 inches in length, and the forces acting at its extremities be both 5 pounds, and the arm of another couple be 6 inches, and the forces at its extremities be both 8 pounds, the *moments* of these couples will be represented by the numbers 20 and 48; and a line 20 inches in length, perpendicular to the plane of the first, and another of 48 inches, perpendicular to that of the second, will represent their respective axes.

PROPOSITION XII.

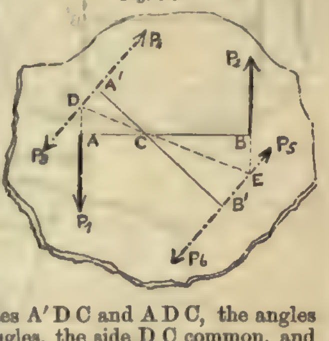
The arm of a couple may be turned round any point in that arm, in the plane of the couple, without altering the conditions of equilibrium.

Let A B represent the arm of the couple, P_1 and P_2, the forces acting at A and B. (Fig. 49).

In A B take any point C, turn A B round C into the new position A′ C B′.

Now at the points A′ and B′ we may apply equal and opposite forces, P_3, P_4, P_5, and P_6, perpendicular to the line A B′, and each equal to the force P_1 or P_2, without disturbing the conditions of equilibrium of the body on which the couple P·A B is supposed to act.

Fig. 49.



Produce the line A P_1 to meet A′P_3 in the point D, and P_2B to meet B′P_5 in E. Join C D and C E.

Because in the triangles A′ D C and A D C, the angles at A and A′ are right angles, the side D C common, and A′ C = A C.

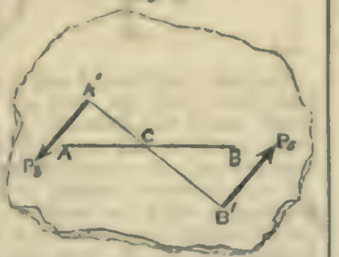
Therefore A′ D = A D, and the angles at D and C are bisected by C D.

Similarly, it may be shown that the angles at C and E are bisected by C E.

Hence C D and C E are both in the same straight line.

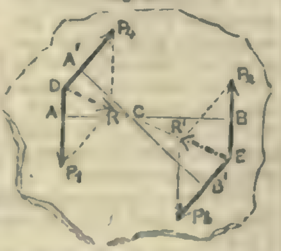
Since any force may be transferred from its point of application to any other point in the line of its action, we may remove the forces P_1 and P_4 from the points A and A′ to the point D, and the forces P_2 and P_6 from B and B′ to E. For the sake of clearness, the new position of the forces will then be represented by the two following figures—one representing the positions of the two forces P_3 and P_5, which have not had their points of application altered, forming a couple, and acting at the extremities of the arm A′ B′ (Fig. 50); and the other, the positions of the forces whose points of application have been changed. (Fig. 51).

Fig. 50.



Now, completing the parallelograms P_1 D P_4 R and E P_2 R′ P_6; D R and E R′ their diagonals will represent the resultants of the forces P_1 and P_3 acting at D, and P_2 and P_6 acting at E. And since the four forces P_1 P_4, P_2 and P_6 are equal, as are also the angles at D and E, the forces represented by D R and E R′ will be equal; and since they attain opposite directions in the same straight line D C E, they will counteract each other.

Fig. 51.



Hence on the whole we have replaced the couple P · A B by the couple P · A′ B′, without altering the conditions of equilibrium.

PROPOSITION XIII.

The effect of a couple will not be altered if its arm be removed to any position parallel to itself, either in its own plane or else in one parallel to it.

Firstly, let the arm A B of the couple P · A B be removed to the new position A′ B′ (Fig. 52—1), A′ B′ being parallel and equal to A B and in the same plane.

The conditions of equilibrium will not be altered if we apply to A′ two forces P_3 and P_4, acting in opposite directions perpendicularly to A′ B′, and both equal to P_1 or P_2.

Similarly we may apply at B′ the forces P_5 and P_6, each equal to the former, and perpendicular to A′ B′.

Fig. 52—(1).

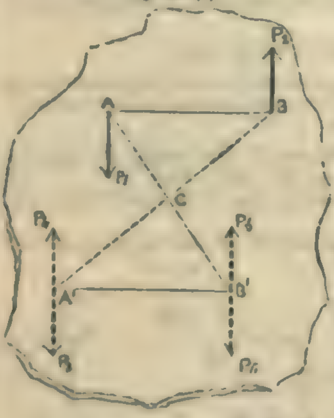
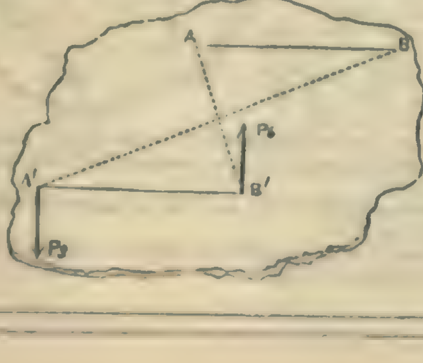
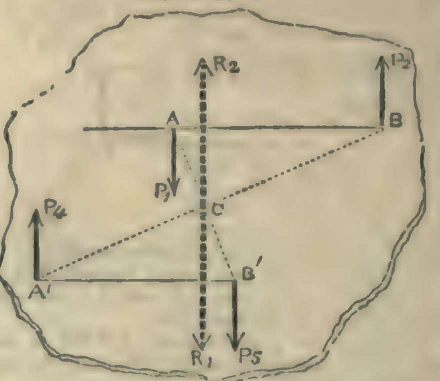


Fig. 52—(2).



Join A′ B, A B′ meeting in C. Then since A B is parallel and equal to A′ B′, we have in the triangles A B C, A′ B′ C, the angles at A and B equal to the angles at B′ and A′, and also A B = A′ B′. Hence A C = B′ C and B C = A′ C.

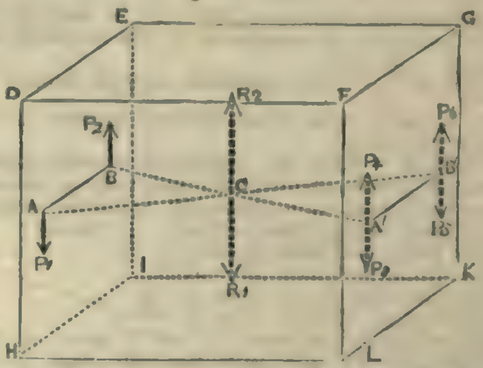
Fig. 52—(3).



Now for the sake of clearness we may suppose the six forces represented above divided into two groups, as in Nos. 2 and 3, one (2) consisting of the arm A′ B′ acted on by the equal and opposite forces P_3 and P_6, forming a couple P · A′ B′; and the other (3) of the four equal and parallel forces; P_1 and P_5 acting at A and B′ in one direction, and P_4 and P_2 at A′ and B in the opposite direction.

Now by Prop. XI., since A C = B′ C and $P_1 = P_5$, P_1 and P_5 acting at A and B′ will be equivalent to a single force C R_1 = 2 P acting at C in the direction C R_1 parallel to A P_1 or B′ P_5. Similarly P_4 and P_2 acting at A′ and B will be equivalent to C R_2 = 2 P acting at C in the direction C R_2 parallel to A′ P_4 or B P_2. But the two equal forces C R_1 and C R_2 acting on C in opposite directions in the same straight line will counteract each other.

Fig. 52—(4).



Hence the group of forces P_1, P_2, P_4, and P_5 will neutralise each other, and we shall only have the effect of the couple P · A′ B′ left, as represented in the other group (4).

On the whole, therefore, we have replaced P · A B by the couple P · A′ B′, the arm A′ B′ being parallel and equal to the arm A B and in the same plane with it.

Secondly, let the arm A B be removed to the position A′ B′ parallel to A B, in a plane parallel to the plane of the couple P · A B (4).

Let D E H I represent the plane of the couple P · A B F G K L a plane parallel to D E H I, the plane to which the arm A B is supposed to be removed.

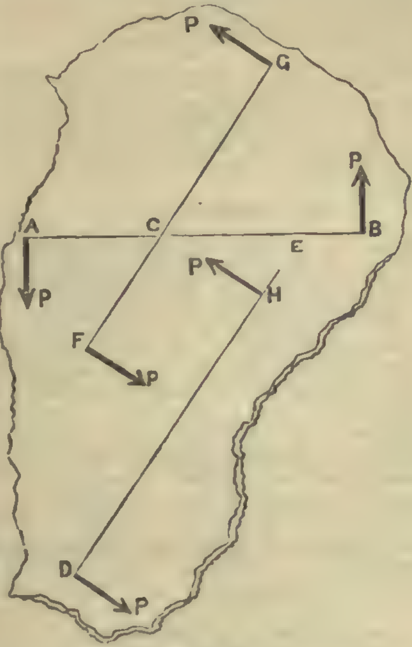
For the sake of clearness we may consider these planes as opposite faces of the parallelopiped D E F G H I K L.

Then A′ B′, the new position of the arm, will be drawn in the plane F G K L, parallel and equal to A B in the plane D E I H.

Join A B′, B A′ meeting in C, and apply, as in the preceding case, equal and opposite forces P_3, P_4, P_5, P_6

at A′ and B′, each equal and parallel to P_1 or P_2. Then Euc., B. XI., Prop. II., the triangles A B C, A′ B′ C are in the same plane, and as in the preceding case A C = B′ C′ and A′ C = B C. (Fig. 52—5).

Fig. 52—(5).



The parallel forces P_1 and P_5 acting at A and B′ are equivalent to C R_1 = 2 P acting at C parallel to P_1 or P_5; and the parallel forces P_2 and P_4 acting at B and A′ are equivalent to C R_2 = 2 P acting at C parallel to P_2 or P_4. The forces C R_1 and C R_2 neutralise each other, leaving, as in the previous case, the forces P_3 and P_6 acting at A′ and B′ as a couple P·A′ B′.

Combining the two last proportions, we see that the arm of a couple may be transferred from any position in its own plane, to any other position in that plane, or to any plane parallel to it.

Thus, if we wish to transfer the couple P· A B (5) to the position, in which one extremity of its arm shall correspond to the point D, and the arm itself be in the direction D E, all we have to do is to take any point C in A B, and through C draw F G parallel to D E. Then by Prop. XII., P · A B may be transferred to the position P · F G, and from that, by Prop. XIII., to the position P · D H. F G and D H being both equal to A B.

PROPOSITION XIV.

Couples acting in the same plane or in planes parallel to each other, will be equal if their moments be equal.

Let P· A B (Fig. 53—1) be a positive couple, and Q · D E (2) a negative one, either acting in the same plane with P · A B or else in one parallel to it. P being greater than Q, and D E greater than A B.

Then the couple Q · D E can be transferred so as to have its arm in such a position that one extremity D shall coincide with A (1), and its arm be the same line as A B. This is represented in (3), A F being equal to D E. Now, the opposite forces P and Q acting at A, are equivalent to a single force P—Q acting at A in the direction A P, as shown in (4).

Hence the effect of the two opposite couples P · A B and Q · D E on the rigid body, on which they are supposed to act, will be equivalent to the three parallel forces P—Q, P and Q acting at the points A, B and F in the same straight line A B F.

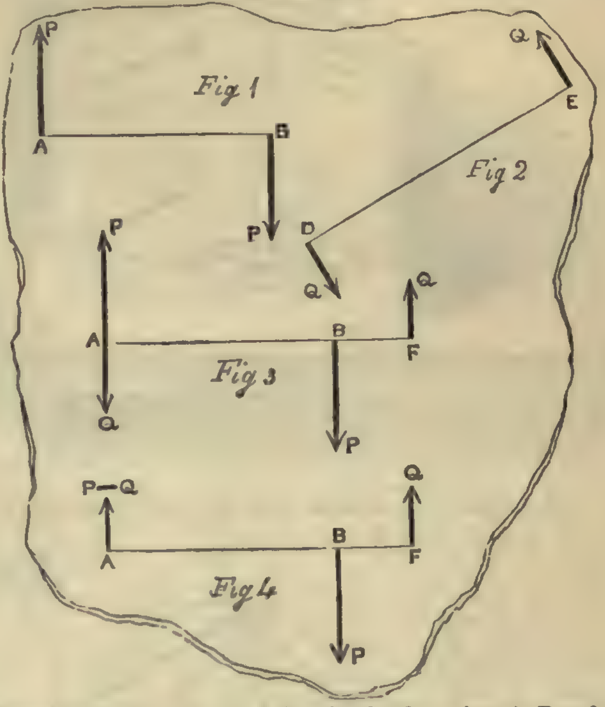
Now, if these couples P·A B and Q·D E, are such as to neutralise each other or produce equilibrium, the equilibrium will not be disturbed by the alteration we have made in the position of their arms. Hence, the parallel forces P — Q, P and Q acting in the points A, B and F in the line A B F, must in this case produce equilibrium, and by Prop. XI. we shall have

$$(P - Q)\ A B = Q \cdot B F$$

$$\text{or } P \cdot A B = Q \cdot A B + Q \cdot B F = Q\ (A B + B F)$$
$$= Q \cdot A\ F = Q \cdot D\ E\ ;$$

which shows that the moments of the two couples are equal. But a negative couple P A B would counteract the positive couple P·A B if their arms were in the same position.

Fig. 53.



Hence, two negative couples will be equivalent to each other, if their moments be equal, provided only that they act in the same plane or in planes which are parallel to each other.

The same reasoning will apply to positive couples.

PROPOSITION XV.

To find the resultant couple of two couples, which do not act in the same plane on a rigid body.

For the sake of clearness, we shall suppose the planes in which the two couples act, represented by the open pages of a book E F·G H K L, standing upon a table (Fig. 54—1), and inclined to each other at an angle θ. E H will be the intersection of the two pages or planes, which are both supposed to be perpendicular to the surface plane of the table.

Let P·A B be the couple acting in the plane L E H K, Q·C D that acting in the plane E F G H.

By the previous proposition the couple P·A B may be replaced by the couple P′· E H acting in the same plane L E H K, provided P′· E H = P·A B,

$$\text{or, } P' = P \cdot \frac{A B}{E H}$$

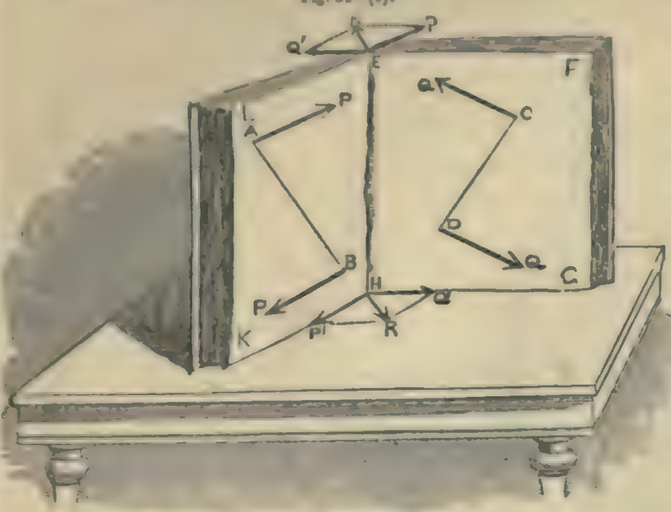
$$\text{In H K take H } P' = P \cdot \frac{A B.}{E H}$$

Produce L E to P′ and make E P′ = H P′.

Then P′ E H P′ will represent in magnitude and direction the couple P′· E H, which replaces P·A B

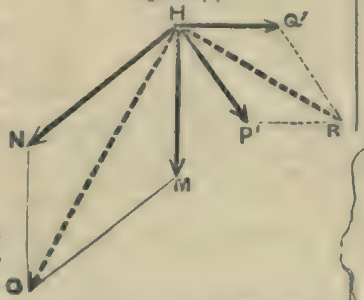
In a similar manner Q′ E H Q′ will represent in magnitude and direction the couple $Q' \cdot E \cdot H$, which will replace $Q \cdot DC$ provided $Q'H$ or $Q' = Q \cdot \frac{CD}{EH}$.

Fig. 54—(1).



Completing the parallelogram P′ H Q′ R on the plane of the table, and joining H R, H R its diagonal will represent a force R in magnitude and direction, which will replace the forces P′ and Q′.

Fig. 54—(2).



Since the plane of the table is at right angles to the planes E F G H, and L E H K, and consequently Euc., B. XI., Prop. XIX., to their intersection E H. Therefore H R in the plane of the table will be at right angles to E H.

Similarly, P′ and Q′ acting at E will be replaced by a force R, represented by E R acting at right angles to E H.

Hence, on the whole, we have replaced the couples P·A B and Q·C D, by a single couple R·E H, whose arm lies in the intersection of the planes in which the couples P·A B and Q·C D act.

Let us now suppose the parallelogram H Q′ R P′, which we have previously drawn on the plane of the table, to be drawn as in (2), on the plane of the paper.

Draw H M perpendicular to H Q′, H O perpendicular to H R, and H N perpendicular to H P′.

Then if θ = angle Q H P′, M H N will = θ, the angle M H O will = Q′ H R, and N H O = P′ H R.

Take H M = Q′·H E, H O = R·H E, and H N = P′·H E. Join M O and N O.

Then H M, H O, and H N will be the axes of the couples Q′. H E, R·H E, and P′·H E.

Because H P′ R Q′ is a parallelogram; therefore, the angle H P′ R = 180° $-\theta$ and by Trigonometry,

$$HR^2 = HP'^2 + P'R^2 - 2\,HP' \cdot P'R \cos.\ HP'R = HP'^2 + P'R^2 + 2\,HP' \cdot P'R \cdot \cos\theta$$

or $R^2 = P'^2 + Q'^2 + 2\,P' \cdot Q' \cdot \cos.\ \theta$

Multiplying both sides of the above equation by HE^2, we have

$$R^2\,HE^2 = P^2 \cdot HE^2 + Q^2 \cdot HE^2 + 2 \cdot P \cdot HE \cdot Q \cdot HE \cos.\ \theta$$

but H M = Q·H E, H O = R·H E, and H N = P′ H E by construction.

Hence, substituting these values in the equation, we have

$$HO^2 = HN^2 + HM^2 + 2\,HN \cdot HM \cdot \cos.\ \theta = HN^2 + HM^2 - 2\,HN \cdot HM \cos.\ NHM.$$

This is an equation identically the same as that we should arrive at if we suppose H M O N a parallelogram whose diagonal is H O.

Hence, if two sides of a parallelogram represent the axes of two component couples, its diagonal represents the axes of the resultant couple.

Similarly it may be shown that if the three edges of a parallelopiped represent the axes of three component couples, the diagonal of the parallelopiped will give the magnitude and direction of the axis of the resultant couple.

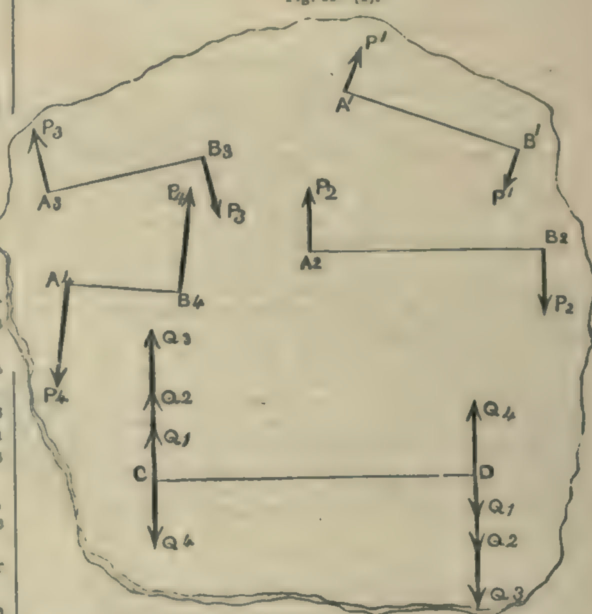
PROPOSITION XVI.

When any number of couples act in the same, or in parallel planes; the moment of the resultant couple is the algebraical sum of the moments of the component couples.

Let $P_1 \cdot A_1 B_1$, $P_2 \cdot A_2 B_2$, $P_3 \cdot A_3 B_3$ be three positive, and $P_4 \cdot A_4 B_4$ a negative couple acting in the same plane (Fig. 55—1).

Draw any arbitrary line C D, and apply at opposite extremities of it, two equal and opposite forces Q_1, such that $Q_1 \cdot CD = P_1 \cdot A_1 B_1$ or $Q_1 = P_1 \cdot \frac{A_1 B_1}{CD}$. Then Prop. XIV., the couple $P_1 \cdot A_1 B_1$ may be replaced by the couple $Q_1 \cdot CD$.

Fig. 55—(1).



Similarly apply at C and D two additional equal and opposite forces Q_2 such that $Q_2 = P_2 \cdot \frac{A_2 B^2}{CD}$

The couple $P_2 \cdot A_2 B_2$ may be replaced by the couple $Q_2 \cdot CD$.

Also the couple $P_3 \cdot A_3 B_3$ may be replaced by the couple $Q_3 \cdot CD$, provided $Q_3 = P_3 \cdot \frac{A_3 B_2}{CD}$

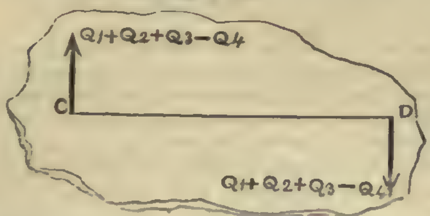
If one of the couples as $P_4 \cdot A_4 B_4$ act in the opposite direction to the others, it will be replaced by the couple $Q_4 \cdot CD$, where $Q_4 = P_4 \cdot \frac{A_4 B_4}{CD}$

The force Q_4 acting in the opposite direction to the forces Q_1, Q_2 and Q_3.

Hence, on the whole, we have replaced the four couples $P_1 \cdot A_1 B_1$, $P_2 \cdot A_2 B_2$, $P_3 \cdot A_3 B_3$, $P_4 \cdot A_4 B_4$, acting in any direction on a rigid body in the same plane by four couples, acting at the extremities of the same arm.

This is evidently equal to a single couple whose arm is C D, with equal and opposite forces $Q_1 + Q_2 + Q_3 - Q_4$ acting at each extremity. (Fig. 55—2).

Fig. 55—(2).



Hence we have
$(Q_1 + Q_2 + Q_3 - Q_4) \cdot C\,D = Q_1 \cdot C\,D + Q_2 \cdot C\,D + Q_3 \cdot C\,D - Q_4 \cdot C\,D$
$= P_1 \cdot A_1\,B_1 + P_2 \cdot A_2\,B_2 + P_3 \cdot A_3\,B_3 - P_4 \cdot A_4\,B_4$, or the resultant couple is the algebraical sum of the moments of the component couples.

The same reasoning may be extended from four to any number of couples, and, by Prop. XIII., to couples acting in parallel planes.

THEORY OF COUPLES.—The propositions from 12 to 16 contain the fundamental principles of what has been called the *theory of couples.* For this beautiful theory, which was introduced into the science of Mechanics about forty years since, we are indebted to the distinguished mathematician, M. Poinsot. After we have extended our 11th proposition from two to any number of parallel forces, we shall again return to the theory of couples, and determine, by its aid, the conditions of equilibrium for any number of forces acting on a rigid body.

PROPOSITION XVII.

To find the magnitude and direction of the resultant of any number of parallel forces acting on a rigid body in the same plane.

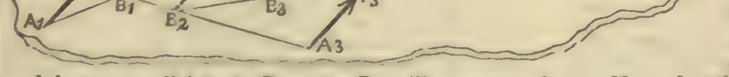
Let four parallel forces, P_1, P_2, P_3, and P_4 (Fig. 56), acting in the same plane on the points A_1, A_2, A_3, and A_4, be represented in magnitude and direction by $A_1\,P_1$, $A_2\,P_2$, $A_3\,P_3$, and $A_4\,P_4$.

Join $A_1\,A_2$; then by Prop. XI., $B_1\,Q_1 = P_1 + P_2$ applied at a point B_1 in $A_1\,A_2$ such that

$$A_1\,B_1 = \frac{P_2}{P_1 + P_2} A_1\,A_2$$

and drawn parallel to $A_1\,P_1$ or $A_2\,P_2$ will represent Q_1 the resultant of the parallel forces P_1 and P_2 in magnitude and direction.

Fig. 56.



Then join $B_1\,A_3$, in $B_1\,A_3$ take a point B_2 such that

$$B_1\,B_2 = \frac{P_3}{Q_1 + P_3} . B_1\,A_3 = \frac{P_3}{P_1 + P_2 + P_3} B_1\,A_3$$

Through B_2 draw $B_2\,Q_2$ parallel to $A_3\,P_3$ and $= P_1 + P_2 + P_3$.

$B_2\,Q_2$ will represent Q_2 the resultant of P_3 and Q_1, or of P_3, P_2, and P_1 in magnitude and direction.

Again, join $B_2\,A_4$ in $B_2\,A_4$, take a point B_3 such that

$$B_2\,B_3 = \frac{P_4}{Q_2 + P_4} B^2\,A_4 = \frac{P_4}{P_1 + P_2 + P_3 + P_4} B_2 A_4$$

Through B_3 draw $P_3\,Q_3 = Q_2 + P_4 = P_1 + P_2 + P_3 + P_4$ parallel to $A_4\,P_4$.

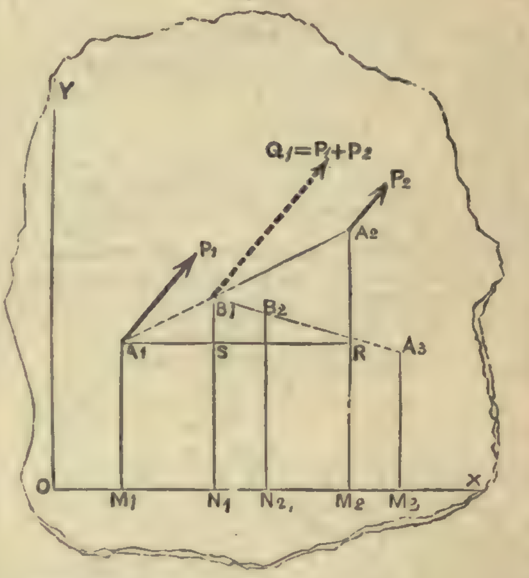
Then $B_3\,Q_3$ will represent Q_3 the resultant of the forces Q_2 and P_4, or of the four parallel forces P_1, P_2, P_3, and P_4 in magnitude and direction.

The same reasoning may be extended to any number of parallel forces.

It is sometimes far more convenient to refer these various points, A_1, A_2, A_3, A_4, B_1, B_2, B_3, &c., to two fixed arbitrary lines or axes drawn at right angles to each other, as in Props. VIII. and IX.

Let O X and O Y (Fig. 57), drawn through the point O at right angles to each other, be chosen as arbitrary *rectangular axes,* to which the points A_1, A_2, &c., B_1, B_2, &c., are to be referred.

Fig. 57.



Through A_1, B_1, and A_2 draw $A_1\,M_1$, $B_1\,N_1$, and $A_2\,M_2$ perpendicular to O X.

Then the lines $O\,M_1$ and $M_1\,A_1$, which determine the position of the point A_1 with respect to the axes O X and O Y, are called the *rectangular co-ordinates of the point* A_1.

Similarly, $O\,N_1$ and $N_1\,B_1$ are the *rectangular co-ordinates* of B_1; and $O\,M_2$ and $M_2\,A_2$ are those of the point A_2.

Let $O\,M_1$ be represented by the symbol x_1, $O\,M_2$ by x_2, $A_1\,M_1$ by y_1, and $A_2\,M_2$ by y_2.

Through A_1 draw $A_1\,R$ parallel to O X, cutting $B_1 N_1$ in S and $A_2\,M_2$ in R.

Then from the construction of the figure, it is evident that the angles at S, R, M_1, N_1 and M_2 are right angles; consequently $A_1\,S = M_1\,N_1$, $A_1\,R = M_1\,M_2$, and also $A_1\,M_1$, $S\,N_1$, and $R\,M_2$ are equal to each other.

Now, by the previous part of the proposition, the point B_1, was so taken in $A_1\,A_2$ that

$$A_1\,B_1 = \frac{P_2}{P_1 + P_2} A_1\,A_2, \text{ or } \frac{A_1\,B_1}{A_1\,A_2} = \frac{P_2}{P_1 + P_2}$$

Again, because $A_2\,R$ is parallel to $B_1\,S$, therefore Euc., B. VI., P. II.

$$\frac{A_1\,B_1}{A_1\,A_2} = \frac{A_1\,S}{A_1\,R} = \frac{B_1\,S}{A_2\,R}.$$

$$\text{But } \frac{A_1\,S}{A_1\,R} = \frac{M_1\,N_1}{M_1\,M_2} = \frac{O\,N_1 - O\,M_1}{O\,M_2 - O\,M_1} = \frac{O\,N_1 - x_1}{x_2 - x_1}$$

Hence since $\frac{A_1 S}{A_1 R} = \frac{A_1 B_1}{A_1 A_2} = \frac{P_2}{P_1 + P_2}$,

we have $\frac{O N_1 - x_1}{x_2 - x_1} = \frac{P_2}{P_1 + P_2}$.

Or $(O N_1 - x_1)(P_1 + P_2) = P_2 x_2 - P_2 x_1$.
And $O N_1 (P_1 + P_2) - P_1 x_1 - P_2 x_1 = P_2 x_2 - P_2 x_1$.
Hence $O N_1 (P_1 + P_2) = P_1 x_1 + P_2 x_2$.

And $O N_1 = \frac{P_1 x_1 + P_2 x_2}{P_1 + P_2}$.

Again, $\frac{B_1 S}{A_2 R} = \frac{B_1 N_1 - S N_1}{A_2 M_2 - R M_2} = \frac{B_1 N_1 - A_1 M_1}{A_2 M_2 - A_1 M_1}$
$= \frac{B_1 N_1 - y_1}{y_2 - y_1}$.

But $\frac{B_1 S}{A_1 R} = \frac{A_1 B_1}{A_1 A_2} = \frac{P_2}{P_1 + P_2}$.

Hence $\frac{B_1 N_1 - y_1}{y_2 - y_1} = \frac{P_2}{P_1 + P_2}$.

And $(B_1 N_1 - y_1)(P_1 + P_2) = P_2 y_2 - P_2 y_1$.
Or $B_1 N (P_1 + P_2) - P_1 y_1 - P_2 y_1 = P_2 y_2$ $P_2 y_1$.

And $B_1 N_1 = \frac{P_1 y_1 + P_2 y_2}{P_1 + P_2}$.

If, now, from B_2 and A_3 we draw $B_2 N_2$ and $A_3 M_3$ perpendicular to O X, and represent $O M_3$ by x_3 and $A_3 M_3$ by y_3.

Then, by a similar construction and demonstration to that used for the points A_1, B_1 and B_2, we can show that

$\frac{B_1 B_2}{B_1 A_3} = \frac{B_1 N_1 - B_2 N_2}{B_1 N_1 - A_3 M_3} = \frac{B_1 N_1 - B_2 N_2}{B_1 N_1 - y_3}$.

But it has been shown that $\frac{B_1 B_2}{B_1 A_3} = \frac{P_3}{P_1 + P_2 + P_3}$

Hence $\frac{P_3}{P_1 + P_2 + P_3} = \frac{B_1 N_1 - B_2 N_2}{B_1 N_1 - y_3}$.

Or $P_3 \cdot B_1 N_1 - P_3 y_3 = B_1 N_1 (P_1 + P_2) + P_3 \cdot B_1 N_1 - B_2 N_2 (P_1 + P_2 + P_3)$.

But $B_1 N_1 = \frac{P_1 y_1 + P_2 y_2}{P_1 + P_2}$, or $B_1 N_1 (P_1 + P_2) = P_1 y_1 + P_2 y_2$, and substituting this value in the above equation, and transposing, we have $B_2 N_2 (P_1 + P_2 + P_3) = P_1 y_1 + P_2 y_2 + P_3 y_3$.

Or $B_2 N_2 = \frac{P_1 y_1 + P_2 y_2 + P_3 y_3}{P_1 + P_2 + P_3}$.

In a similar manner it may be shown that

$$O N_2 = \frac{P_1 x_1 + P_2 x_2 + P_3 x_3}{P_1 + P_2 + P_3}.$$

If, now, perpendiculars $B_3 N_3$ and $A_4 M_4$ are supposed to be drawn from B_3 and A_4 to O X,

And $O M_4 = x_4$, and $M_4 A_4 = y_4$, we shall have by the carrying out the same method of demonstration

$$B_3 N_3 = \frac{P_1 y_1 + P_2 y_2 + P_3 y_3 + P_4 y_4}{P_1 + P_2 + P_3 + P_4}.$$

$$\text{And } O N_3 = \frac{P_1 x_1 + P_2 x_2 + P_3 x_3 + P_4 x_4}{P_1 + P_2 + P_3 + P_4}.$$

We might proceed from the case of four forces to five, from five to six, and so on; so that, assuming the symbols $\bar{x}$ and $\bar{y}$ to represent the rectangular co-ordinates of the point of application of the resultant of n parallel forces P_1, P_2, P_3, &c., P_n referred to the arbitrary axes O X and O Y; we shall have

$$\bar{x} = \frac{P_1 x_1 + P_2 x_2 + P_3 x_3 + \&c., + P_n x_n}{P_1 + P_2 + P_3 + \&c., + P_n}.$$

$$\text{And } \bar{y} = \frac{P_1 y_1 + P_2 y_2 + P_3 y_3 + \&c., + P_n y_n}{P_1 + P_2 + P_3 + \&c., + P_n}.$$

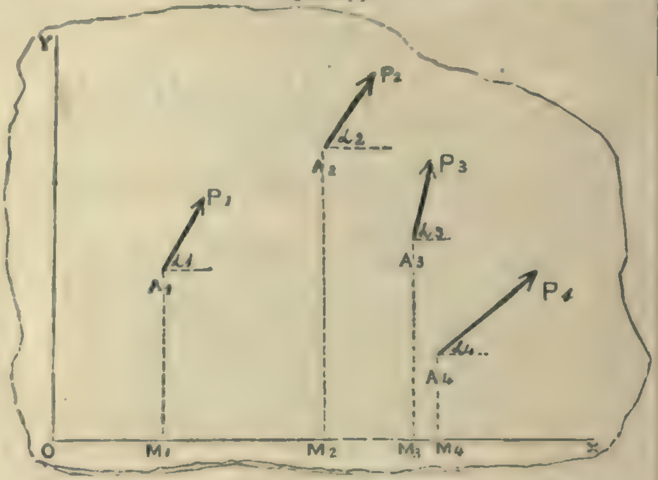
If any of the forces act in an opposite direction to the others, they must be taken with the negative sign; the co-ordinates of the various points of application of the forces must also be taken with the proper signs, determined by their position with regard to the axes.

PROPOSITION XVIII.

To find the magnitude and direction of the resultant force and resultant couple of any number of forces acting on a rigid body in the same plane, and the conditions under which there will be equilibrium.

Let P_1, P_2, P_3, and P_4 (Fig. 58—1), be four forces represented in magnitude and direction by $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$; acting on a rigid body in the same plane. A_1, A_2, A_3, and A_4, being the points of application of these forces to the rigid body. In the plane in which the forces act, take any two straight lines O X, O Y, at right angles to each other as rectangular axes.

Fig. 58—(1).



Let $O M_1 = x_1$, $A_1 M_1 = y_1$, be the rectangular co-ordinates of A_1, referred to the axes O X and O Y.

$O M_2 = x_2$, $A_2 M_2 = y_2$, those of A_2; $O M_3 = x_3$, $A_3 M_3 = y_3$ those of A_3; and $O M_4 = x_4$, $A_4 M_4 = y_4$, those of A_4.

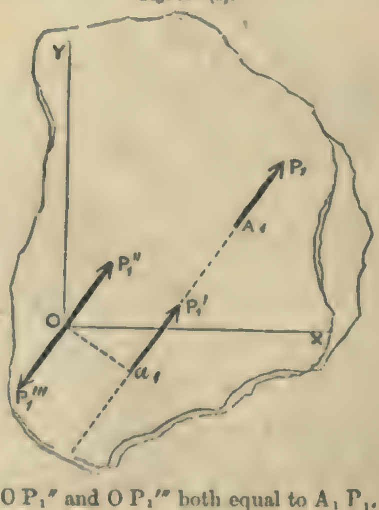
Also let a_1, a_2, a_3, and a_4 be the angles that $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$, make with lines drawn through A_1, A_2, A_3, and A_4 parallel to O X.

For the sake of clearness we will first confine our attention to the force P_1, acting at the point A_1.

Through O (2) draw $O a_1$ at right angles to $P_1 A_1$ produced.

Also through O draw $P_1'' P_1'''$ parallel to $A_1 P_1$.

Fig. 58—(2).



Make $O P_1''$ and $O P_1'''$ both equal to $A_1 P_1$.

Then without disturbing the equilibrium of the body, we may introduce two equal and opposite forces P_1, represented in magnitude and direction by $O P_1''$ and $O P_1'''$, acting at O, and also transfer the point of application of P_1 from A_1 to a_1.

Hence, on the whole, the effect of the force P_1 acting at A_1, is equivalent to a force P_1 acting at O in the direction $O P_1''$ parallel to $A_1 P_1$, and two equal and opposite forces represented in magnitude and direction by $O P_1'''$, $a_1 P_1'$ acting at right angles to $O a_1$ and forming a couple $P_1\ O a_1$, or a couple whose moment is P_1 multiplied by the perpendicular from O on the line $A_1 P_1$ produced.

In a similar manner the force P_2 (1) acting at A_2 may be replaced by a force P_2 acting at O, parallel in direction to $A_2 P_2$ and a couple whose moment is P_2 multiplied by the perpendicular from O on $A_2 P_2$ produced.

The force P_3 acting at A_3 (1), may be replaced by a force P_3 acting at O parallel to $A_3 P_3$, and a couple whose moment is P_3 multiplied by the perpendicular from O on $A_3 P_3$ produced.

And, lastly, P_4 acting at A_4 (1) may be replaced by a force P_4 acting at O parallel to $A_4 P_4$, and a couple whose moment is P_4 multiplied by the perpendicular from O on $A_4 P_4$ produced.

On the whole, therefore, the forces P_1, P_2, P_3, and P_4 acting at the points A_1, A_2, A_3, and A_4, may be replaced by forces P_1, P_2, P_3, and P_4, all acting at O in directions parallel to $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$, whose resultant will be a single force whose magnitude and direction may be found by Prop. II.; and four couples whose arms will have a common extremity O, and whose moments will be equal to P^1, P_2, P_3, and P_4, each respectively multiplied by the perpendicular from O on the original direction of the force; or on $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$ produced.

By Prop. XVI., these four couples will be equivalent to a single couple, whose moment is equal to the algebraical sum of their moments.

The single resultant force acting at O shows the tendency of the four forces P_1, P_2, P_3, and P_4, acting at O to move O in some rectilinear direction, while the resultant couple gives their tendency to twist the body in some direction round the point O.

In order, therefore, that the rigid body should be in equilibrium, when acted on by these forces, these resultants must each be equal to nothing, since if the resultant force alone were equal to nothing, the resultant couple would twist the body round O, as a fixed point; or if the moment of the resultant couple alone were equal to nothing, the body would move so as to keep O in a straight line.

Hence the *conditions of equilibrium* of four forces P_1, P_2, P_3, and P_4, acting on a rigid body in the same plane at the points A_1, A_2, A_3, and A_4 in the directions $P_1 A_1$, $P_2 A_2$, $P_3 A_3$, and $P_4 A_4$ are two.

First.—The resultant of four forces, respectively equal to P_1, P_2, P_3, and P_4, acting on any point O of the body, in directions respectively parallel to $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$, must be equal to zero.

Second.—The algebraical sum of the moments of the four couples, whose arms are the perpendiculars drawn from O on $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$ produced, and whose forces are respectively equal to P_1, P_2, P_3, and P_4, must likewise be equal to zero.

This latter condition is technically called taking the moments about the point O.

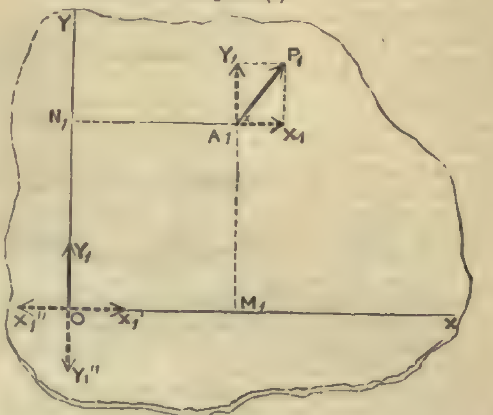
The reasoning above used for four forces may readily be extended to any number; it must also be observed that the position of the point O is perfectly arbitrary in the solution of problems. It is generally so chosen as to facilitate the solution, and to avoid unnecessary labour.

Instead of pursuing the preceding method, it is frequently advisable to resolve each of the forces P_1, P_2, P_3, and P_4 into two, acting in directions parallel to the arbitrary axes O X and O Y (1), as in Props. VIII. and XI.

Confining, as before, our attention first to the force P_1, acting at A_1 in the direction $A_1 P_1$.

Through A_1 (3) draw $A_1 X_1$ and $A_1 Y_1$ parallel to O X and O Y respectively; and through P_1, $P_1 X_1$, and $P_1 Y_1$ perpendicular to $A_1 X_1$, and $A_1 Y_1$. Again through A_1 draw $A_1 N_1$ perpendicular to O Y, and $A_1 M_1$ perpendicular to O X.

Fig. 58—(3).

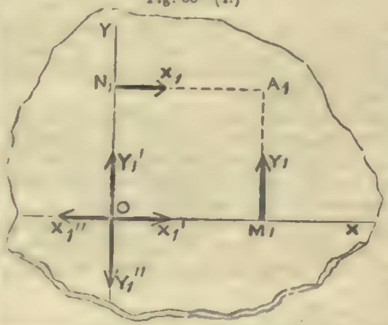


The force P_1 acting at A_1 may be replaced by two forces X_1 and Y_1 at right angles to each other, represented in magnitude and direction by $A_1 X_1$ and $A_1 Y_1$ (Prop. III.)

Without disturbing the conditions of equilibrium, two equal and opposite forces represented in magnitude and direction by $O X_1'$ and $O X_1''$, each equal to $A_1 X_1$, may be applied to O in the direction O X.

And two equal and opposite forces $O Y_1'$, $O Y_1''$, each equal to $A_1 Y_1$ may be applied to O in the direction O Y.

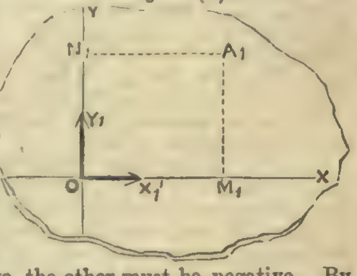
Fig. 58—(4.)



Also (4) the force X_1 may be transferred from A_1 to N_1, and the force Y_1 from A_1 to M_1.

The forces now acting on the body as in (4), may be divided into two groups, one as in (5), consisting of the forces X_1 and Y_1 acting at O, and represented in magnitude and direction by $O X_1'$ and $O Y_1'$; and the other as in (6), of the couples whose moments are X_1 multiplied by $O N_1$, and Y_1 multiplied by $O M_1$.

Fig. 58—(5.)



The tendency of the couple $X_1 \cdot O N_1$ is evidently to twist the body in the opposite direction to that in which the couple $Y_1 \cdot O N_1$ has a tendency to twist it. Hence, if one be considered positive, the other must be negative. By construction $A_1 M_1 = y_1$, and $A_1 N_1 = M_1 O = x_1$.

Hence the resultant moment of these two couples, Prop. XVI., will be

$$Y_1 \cdot x_1 - X_1 \cdot y_1.$$

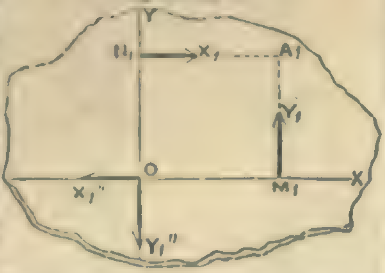
On the whole, therefore, we have replaced the force P_1 acting at A_1 by the forces X_1 and Y_1 acting on O in the direction of the axes O X and O Y, and a couple whose moment is equal to $Y_1 \cdot x_1 - X_1\, y_1$.

Referring to (3), we see that $A_1 X_1 = A_1 P_1 \cos. a_1$, and $A_1 Y_1 = A_1 P_1 \sin. a_1$.

Hence $X_1 = P_1$ cos. a_1 and $Y_1 = P_1$ sin. a_1.

In a similar manner it may be shown the force P_2 acting at A_2 may be replaced by the forces X_2 and Y_2 acting at O in the directions O X and O Y, and the couple whose moment is $Y_2 x_2 - X_2 y_2$, where $X^2 = P_2$ cos. a_2 and $Y_2 = P_2$ sin. a_2.

Fig. 58 b.



Likewise P_3 acting at A_3 may be replaced by X_3 and Y_3 acting at O in the directions O X and O Y and the couple whose moment is $Y_3 x_3 - X_3 y_3$, where $X_3 = P_3$ cos. a_3 and $Y_3 = P_3$ sin. a_3.

Lastly, P_4 acting at A_4 may be replaced by X_4 and Y_4 acting at O in the directions O X and O Y and the couple whose moment is $Y_4 x_4 - X_4 y_4$, where $X_4 = P_4$ cos. a_4 and $Y_4 = P_4$ sin. a_4.

On the whole, therefore, the four forces P_1, P_2, P_3, and P_4, acting at A_1, A_2, A_3, and A_4, in the directions $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$, have been replaced by four forces X_1, X_2, X_3, and X_4, acting at O in the direction O X, whose resultant is a single force equal to $X_1 + X_2 + X_3 + X_4$, acting at O in the direction O X; by four forces Y_1, Y_2, Y_3, and Y_4, acting at O in the direction O Y, whose resultant is a single force equal to $Y_1 + Y_2 + Y_3 + Y_4$ acting at O in the direction O Y; together with four couples, whose resultant couple, Prop. XVI., will be one whose moment is equal to

$$(Y_1 x_1 - X_1 y_1) + (Y_2 x_2 - X_2 y_2)$$
$$+ (Y_3 x_3 - X_3 y_3) + (Y_4 x_4 - X_4 y_4).$$

The two forces $(X_1 + X_2 + X_3 + X_4)$ acting at O in the direction O X, and $(Y_1 + Y_2 + Y_3 + Y_4)$ acting at O in the direction O Y, will have a single resultant, which may be found by Prop. I.

In order that the body may be in a position of equilibrium when acted on by the forces P_1, P_2, P_3, and P_4, applied at A_1, A_2, A_3, and A_4, in the directions $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_4 P_4$, this resultant must be zero, and so must the moment of the resultant couple.

In this case, excluding those conditions which belong to the couples, referring to Prop. IX., we shall have two conditions for the forces acting at O in the directions O X and O Y.

$$X_1 + X_2 + X_3 + X_4 = o$$
$$\text{And } Y_1 + Y_2 + Y_3 + Y_4 = o$$

The moment of the resulting couple being also zero, gives us a third condition.

$$(Y_1 x_1 + X_1 y_1) + (Y_2 x_2 - X_2 y_2) + (Y_3 x_3 - X_3 x_3) + (Y_4 x_4 - X_4 y_4) = 0.$$

Where $X_1 = P_1$ cos. a_1, $X_2 = P_2$ cos. a_2, $X_3 = P_3$ cos. a_3, $X_4 = P_4$ cos. a_4 · $Y_1 = P_1$ sin. a_1, $Y = P_2$ sin. a_2, $Y_3 = P_3$ sin. a_3, and $Y_4 = P_4$ sin. a_4.

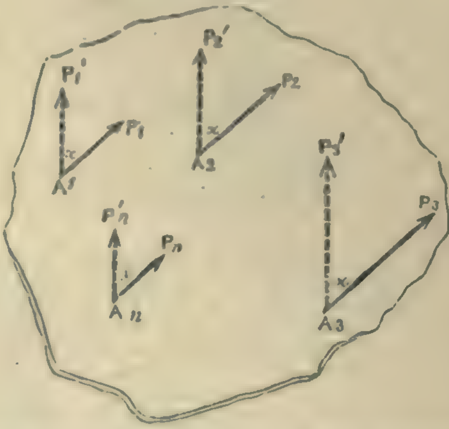
The above reasoning may readily be extended from four to any number of forces.

We have, for the sake of simplicity, drawn the directions of the forces P_1, P_2, P_3, and P_4, in such a manner that their co-ordinates and resolved portions should be positive. In other cases we must remember that if the resolved part of any force act in an opposite direction to that we have drawn, it must be considered negative;* and if one or both of the co-ordinates of the points of application of the force be negative, we have only to use the negative sign. Substituting these signs carefully in the above formulæ, we may extend them so as to include every possible case of any number of forces acting on a rigid body in the same place.

CENTRE OF GRAVITY.—By Prop. XVII. we found that if any number of parallel forces represented in magnitude and direction by P_1, P_2, P_3, &c., P_n (Fig. 59), acting on a rigid body in the same plane, at points A, A^2, A , &c., A_n, be referred to rectangular axes, and x_1, y_1; x_2, y_2; x_3, y_3; &c., x_n, y_n, be the rectangular co-ordinates of A_1, A_2, A_3, &c., A_n referred to these axes, the resultant force will be equal to $P_1' + P_2' + P_3' + P_n'$, acting in a direction parallel to P_1, P_2, &c., and applied at a point whose co-ordinates $\bar{x}$ and $\bar{y}$ may be found by the formulæ.

$$\bar{x} = \frac{P_1 x_1 + P_2 x_2 + P_3 x_3 + \&c., + P_n x_n}{P_1 + P_2 + P_3 + \&c., + P_n}$$
$$\bar{y} = \frac{P_1 y_1 + P_2 y + P_3 y_3 + \&c., + P_n y_n}{P_1 + P_2 + P_3 + \&c., + P_n}.$$

Fig. 50.



From these formulæ it is obvious that the position of the point of application of the resultant of the parallel forces is independent of their direction. Hence, if each of the forces represented in magnitude and direction by $A_1 P_1$, $A_2 P_2$, $A_3 P_3$, and $A_n P_n$, be turned through the same angle a, about the points A_1, A_2, A_3, &c., A_n, into the position shown by the dotted lines, the point of application of the resultant will not move.

For this reason this point is called the *centre of the parallel forces.*

We have seen that the weight of a heavy body is produced by the earth's attraction on each of the material particles of which it is composed, and that this attraction is the same for all kinds of matter. Hence, a cubic inch of iron weighs more than a cubic inch of wood; not because the earth's attraction is greater for particles of iron than for those of wood, but because a cubic inch of the former contains a greater number of gravitating particles than the latter.

The attraction which the earth exerts on all masses near its surface, on account of its larger relative mass, is so great, that for all masses which are not very large, we may neglect the attraction which the particles of these masses or those in their neighbourhood exert on one another. We can, on this account, rapidly determine, with considerable accuracy, the direction of the earth's attraction, or *gravity.*

Fig. 60.

Let a small weight of lead or brass P (Fig. 60), be fixed to one extremity of a thin flexible string A P, and suspended from a fixed point A. The weight P, after oscillating for some little time, will, if undisturbed, rest in such a position that the string A P shall point to the earth's centre. Such an instrument is called a *plummet* or a *plumb-line,* and the line in which it rests, the *vertical.* If two plumb-lines be suspended from points situated at different parts of the earth's surface, they cannot be exactly parallel, but must make a certain angle with each other, unless the one point lie in the antipodes of the other, in which case the directions will be in the same straight line.

For all practical purposes, on account of the comparative greatness of the earth's radius, we may say that two plumb-lines of any ordinary length, suspended within any apartment, however large, will be parallel to each other; since if in still water, at a distance of 200 yards from each other, we were to sink two plummets a mile and a-half in length, they would not deviate further than 3 inches from perfect parallelism. It is true that a large mass of matter, such as a mountain, will slightly deflect, as according to the theory of gravitation it ought, the plumb-line from the true vertical; but in ordinary cases this source of error may be neglected. Again, the attraction of the earth will vary according to the distance from its centre of the particle or body attracted; but this variation may also be neglected for bodies which are small when compared with the magnitude of the earth.

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* See ante, p. 708

Hence, in most cases, even such, for instance, as the largest line-of-battle ship, without introducing any sensible error, we may regard the attraction of the earth, on any mass of matter, as acting on every material particle composing that mass, in directions parallel to the plumb-line suspended near it.

The *centre of the parallel forces*, produced by the weights of the material particles of which any heavy body is composed, is called the *centre of gravity of that body.*

Hence, Prop. XVII. enables us to find the position of the centre of gravity of any number of heavy particles whose weights and positions are known.

From the properties of the *centre of parallel forces* we have already demonstrated, it follows that the *centre of gravity* of a heavy body *is that point within or without the body at which the whole of its weight may be conceived to act; and the body will produce the same mechanical effect, as if we were to suppose the whole of its weight concentrated in that point.* This enables us to extend Prop. XVII., to find the common centre of gravity of several bodies, whose weights and the position of whose respective centres of gravity are known.

Again, because the parallel forces exerted by the weights of the material particles of a body will have the same direction in whatever position it is placed, its centre of gravity will not change its position with respect to the body for any change in the position of the body. Hence, if the centre of gravity be fixed, the body will balance about it in every position, because the resultant of the weights of every one of its elementary particles will pass through the fixed point (the centre of gravity) in every position in which the body can be placed.

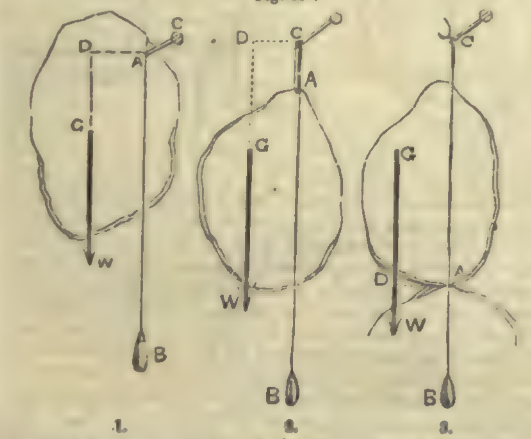
PROPOSITION XIX.

If a heavy body be in equilibrium when suspended from a point, or when resting on a point in contact with another body, its centre of gravity will be in the vertical line passing through the point of suspension or contact.

Let (Fig. 60*,) 1, 2, and 3 represent a section of the heavy body, taken through the plane passing through its centre of gravity G, and its point of suspension or contact A.

In (1) we suppose the body supported by a pin A C, passing through a hole at A, about which it can move freely; in (2) it is suspended from a point at A, by a string fixed at C; and in (3) it is supported by another body with which it is in contact at A.

Fig. 60*.



In all three cases let the vertical be represented by the plumb-line AB; the centre of gravity must lie in that line. If it do not, let it have some other position, as G.

From our definition of the centre of gravity, the weight of the body will produce the same effect upon the body as if, being destitute of weight, a force equal to its weight, acting vertically downwards, were applied to its centre of gravity.

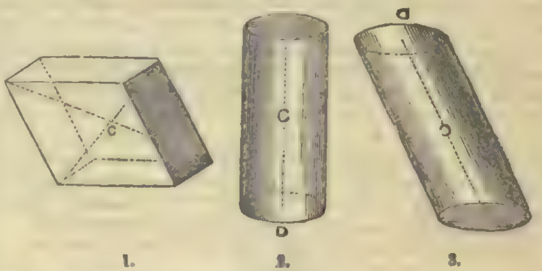
Let G W = W represent this force, the weight of the body, in magnitude and direction, and we then have the case of a rigid body without weight in equilibrium when acted on by the force W acting at G, and the reaction or tension at the point A.

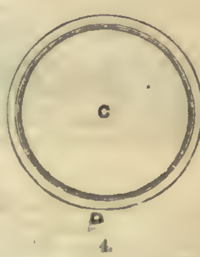
Through A draw A D perpendicular to G W or G W produced. Then by the principle of the transmission of force, W may be transferred from G to D, and W · A D will be the moment of a force tending to twist the body round the point A, which is counteracted by no other force. Consequently, equilibrium can only exist if W or A D be equal to nothing. But by the terms of the proposition, W cannot be equal to nothing. Hence, there can only be equilibrium where A D is nothing, in which case G must lie in the line A B.

In the same manner it can be shown that if a heavy body balance on a given straight line—as, for instance, the sharp edge of another body which is a straight line—its centre of gravity will lie in a straight line.

If any homogeneous heavy body be of a form which is symmetrical with respect to a certain point or line, the centre of gravity will be in that point or line; for the very idea of symmetry requires, that if any point be taken in the body, there must be another point in that body equidistant from the point or line about which it is symmetrical. Hence, since the centre of gravity is the centre of the parallel forces produced by the weights of the material particles composing the body; if it be homogeneous, that is, composed of particles of the same weight, and distributed uniformly throughout its substance, any one particle in the body will be balanced about the point or line around which its form is symmetrical, by another particle equal to it in weight, and equidistant from that point or line.

Fig. 61.





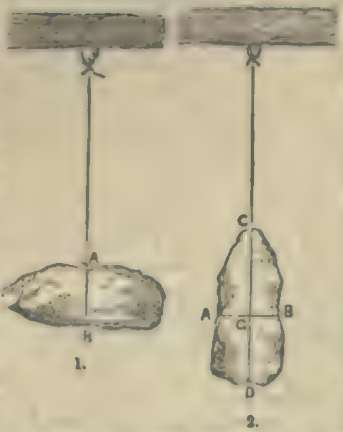
The centre of gravity, therefore, of a sphere, will be the centre of the sphere; that of a cube or oblique parallelopiped (Fig. 61—1), the point where two diagonals intersect each other. The centre of gravity of a right (2) or oblique (3) cylinder will be in the middle of its axis; and that of a ring (4) will be the centre of the ring.

The centre of gravity of a hollow sphere or cylinder will be the same as if it were solid, provided it be of the same thickness throughout. From this it follows that the centre of gravity of a body need not necessarily be a point within it.

EXPERIMENTAL DETERMINATION OF THE CENTRE OF GRAVITY.—The preceding proposition affords a means, in many instances, of determining the position of the centre of gravity of a body experimentally.

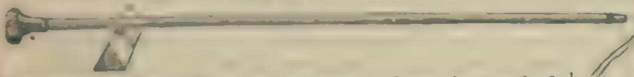
Thus, if a body be suspended by a string, attached to the point A (Fig. 62—1), the centre of gravity will lie in the vertical line A B passing through the body when it has attained a position in which it is perfectly at rest. We now suspend it (2) from another point C; let C D be the position of the vertical when it is at rest, this line will intersect the previous line A B in the point G, which will be the centre of gravity of the body. In many cases we shall thus be enabled to estimate the position of the centre of gravity. If the body A C B D (2) be bounded by plane surfaces, and be of uniform thickness throughout, we know that if the lines A B and C D be traced on both parallel surfaces, the centre of gravity will lie in the middle of the straight line joining the two points of intersection of these lines.

Fig. 62.



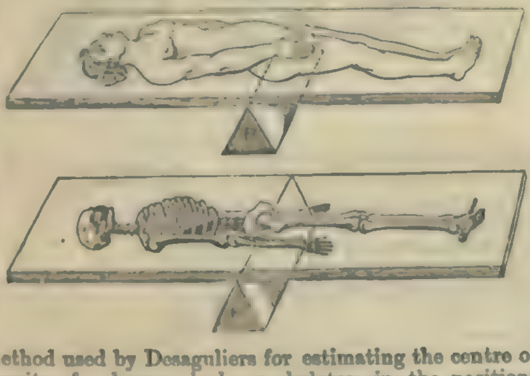
To find the centre of gravity of a walking-stick, which is supposed to be symmetrical with respect to an axis passing through its centre, and has a heavy head or handle, we have only to balance it on the sharp edge of a body, as indicated in Fig. 63, and we know the centre of gravity will lie in the point where the vertical plane passing through the edge intersects the axis of the stick.

Fig. 63.



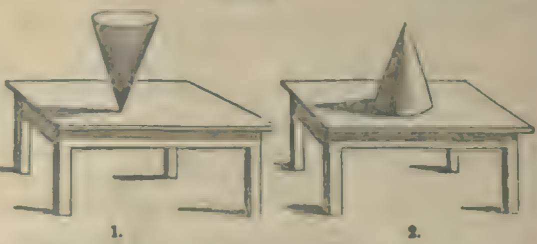
In these experimental determinations of the centre of gravity it does not signify whether the body be homogeneous or not: thus, the adjacent figures show the method used by Desaguliers for estimating the centre of gravity of a human body or skeleton, in the positions there indicated. (Fig. 64).

Fig. 64.



STABLE AND UNSTABLE EQUILIBRIUM.—Theoretically we say that a heavy body under the action of gravity, may be in equilibrium in a certain position, though we may not be able practically or experimentally to demonstrate it, because the slightest disturbance of the position of the body may destroy the conditions of its equilibrium. Thus, theoretically, a cone will be in a position of equilibrium whether it rest on a table (Fig. 65—1) on its apex, or (2) on its base. In the former case, however, the slightest possible movement to the one side or the other will destroy the equilibrium. The equilibrium which is theoretically but not practically possible, is called *unstable*, while that which is practicable is called *stable*. An egg will rest on its side in a position of *stable equilibrium*, while the attempt to balance it on one of its extremities will only be successful by imitating the well-known method Columbus used, to destroy its *unstable equilibrium*. A body may be said to be in *stable equilibrium* if after a slight disturbance it recovers its position of equilibrium; and in *unstable equilibrium* if after a slight disturbance it does not recover it. When a body is supported by its centre of gravity, it will rest in every position about that point; this has been called *indifferent equilibrium*.* Thus, a circle or any plane figure, of uniform thickness, will rest in every position, on an axis passing through its centre of gravity.

Fig. 65.

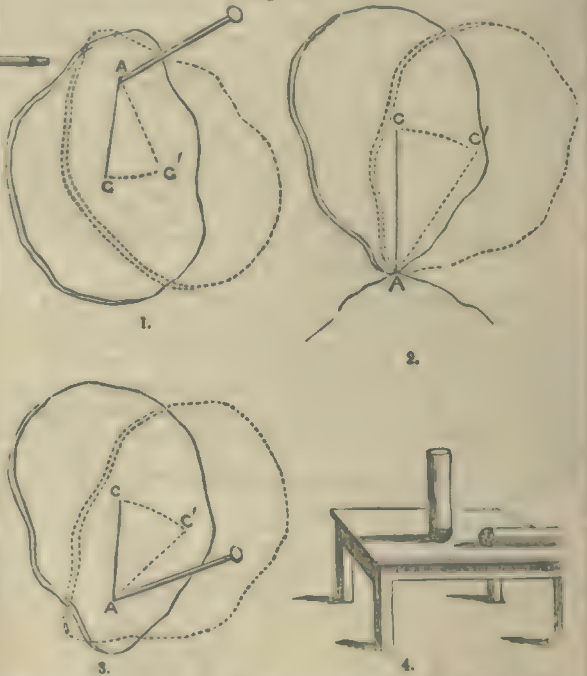


PROPOSITION XX.

The equilibrium of a body will be stable or unstable according as its centre of gravity is in the lowest or highest position possible.

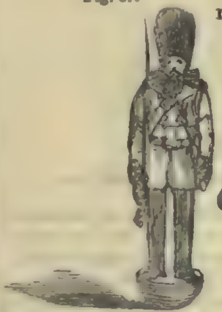
If a body be in a state of equilibrium, suspended by an axis at A (Fig. 66—1), about which it is capable of moving freely, or resting on a point A (2), any slight disturbance will cause the body to move to the right or left, as shown by the dotted lines, and the centre of gravity, to describe a small arc G G′; but the centre of gravity, having moved, however slightly, below the point G, the tendency of the weight of the body will evidently be to remove the centre of gravity further from the original position G. But if the centre of gravity be in its lowest position G (3) after any disturbance which should remove the point G to G′, the tendency of the weight will be to restore the body to its original position, and the point G′ to G.

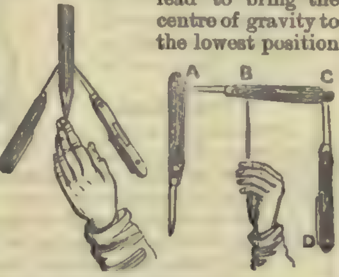
Fig. 66.



The tendency of the centre of gravity to recover its place if removed from its lowest to a higher position is well illustrated by fixing the half of a bullet (4), or any hemispherical heavy metallic body, to a cylinder of elder-pith, cork, or any other light substance. The centre of gravity of the whole, will lie somewhere in the heavy hemisphere; and if the cylinder be removed to any position, except it be laid almost absolutely on its side, it will speedily recover its upright position, as shown in the figure. Toys are constructed on this principle, the cylinder being cut into the form of a soldier; a regiment of such mimic troops being pressed nearly to the ground by passing a stick over them, seem immediately to spring up and recover their position as if by magic. (Fig. 67). The toys called *tumblers*, made of plaster of Paris, with a hemispherical bottom loaded with iron or lead to bring the centre of gravity to the lowest position possible, also owe their amusing properties to the same principle. Screens have been invented which, by this contrivance, right themselves after being pressed down. The preceding illustrations show how a pointed stick may be easily balanced on the tip of the finger by fixing two pen-knives in its side, thus converting an unstable into stable equilibrium; and how three pen-knives, placed in the position A, B, C, and D, may be kept in equilibrium on the point of a needle held in the hand. In both cases the centre of gravity must fall below the point on which the bodies are balanced.

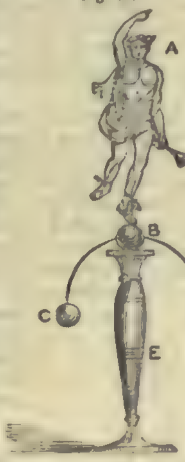
Fig. 67.





A small figure A (Fig. 68) with its foot fixed to a sphere B, through which passes a bent wire, having two leaden balls C and D attached to its extremities, if placed loosely on a stand E, will speedily recover its upright position after being moved from it; the figure being so constructed, that the centre of gravity of the three bodies A, C, and D, falls below the point of support; that is, where the sphere B rests on the stand E.

Fig. 68.

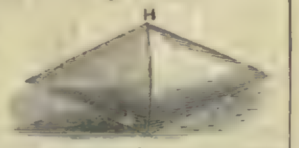


The apparent paradox of a double cone ascending an inclined plane by its own weight, is produced by the construction of the cone and plane being such that the centre of gravity of the cone really descends, and by its descent causes the cone to ascend the plane.

Construct an inclined plane of two equal pieces of straight wire A C and B C (Fig. 69—2), fixed at their extremities to three upright pieces A E, B F, and C D, standing perpendicular to the horizontal stand D E F: A E and B F being equal to each other, and C D less than A E or B F.

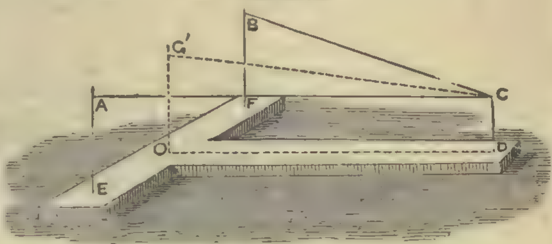
Let H be a double cone consisting of two right cones united together by their circular bases.

Fig. 69—(1).



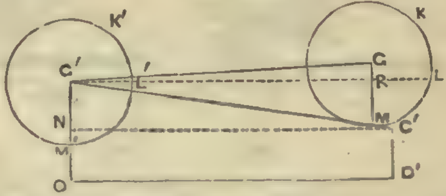
If the distance A B or E F be equal to or less than that between the two vertices of the cone, and the difference between A E and C D less than the radius of the circular base of the double cone, upon placing the cone near C with its circular base between the wires A C and B C, it will roll up the inclined plane till its extremities are stopped by the upright supports at A and B.

Fig. 69—(2).



That the centre of gravity really descends in this case will be readily seen by the diagram (3), which represents an imaginary section of the inclined plane through C D, perpendicular to E F, represented by the lines G′ O, O D, C D, G′ C (2).

Fig. 69—(3).



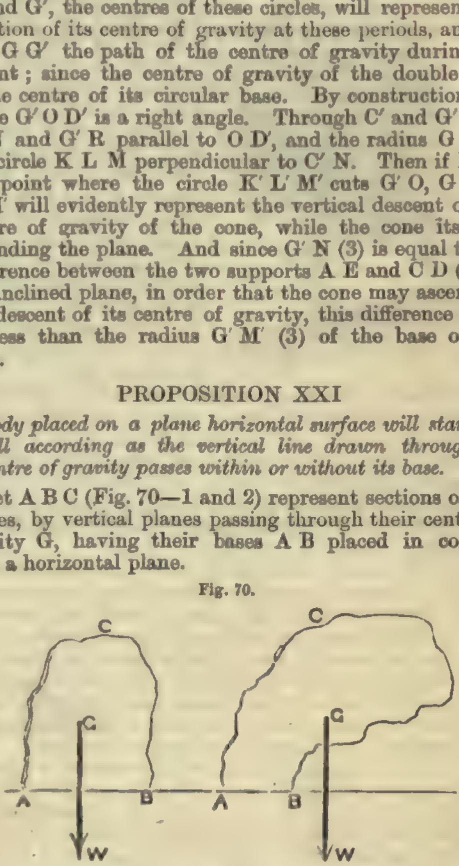
Let the circles K L M, K′ L′ M′ (3) represent the two positions of the circular base of the cone at the commencement and end of its ascent up the inclined plane; G and G′, the centres of these circles, will represent the position of its centre of gravity at these periods, and the line G G′ the path of the centre of gravity during the ascent; since the centre of gravity of the double cone is the centre of its circular base. By construction, the angle G′ O D′ is a right angle. Through C′ and G′ draw C′ N and G′ R parallel to O D′, and the radius G M of the circle K L M perpendicular to C′ N. Then if M′ be the point where the circle K′ L′ M′ cuts G′ O, G R or N M′ will evidently represent the vertical descent of the centre of gravity of the cone, while the cone itself is ascending the plane. And since G′ N (3) is equal to the difference between the two supports A E and C D (2) of the inclined plane, in order that the cone may ascend by the descent of its centre of gravity, this difference must be less than the radius G′ M′ (3) of the base of the cone.

PROPOSITION XXI

A body placed on a plane horizontal surface will stand or fall according as the vertical line drawn through its centre of gravity passes within or without its base.

Let A B C (Fig. 70—1 and 2) represent sections of two bodies, by vertical planes passing through their centre of gravity G, having their bases A B placed in contact with a horizontal plane.

Fig. 70.



1. 2.

In (1) the vertical G W passing through G falls within A B, and in (2) without A B.

The whole effect of the weight of the body is equivalent in both cases to a single force equal to that weight applied at G in the direction G W.

In (1) this force is evidently destroyed by the reaction of the plane, and the body will stand. In (2) this force has a moment about the point B, which is not counteracted by the resistance of the plane; and consequently the body will turn round B till it falls in such a position that G W will lie within the base on which it rests. This property is readily shown, experimentally, by taking two oblique cylinders (Fig. 71—1 and 2) of such lengths that the vertical passing through the centre of gravity of the one, when placed on a table, shall fall within its circular base, and the other without. The former will stand and the latter will fall when placed with their circular bases in contact with the table.

If the surface on which the body rests be inclined instead of horizontal, the same effect will be produced, provided the body be prevented from sliding down the inclined plane by friction or some other force.

Fig. 71.

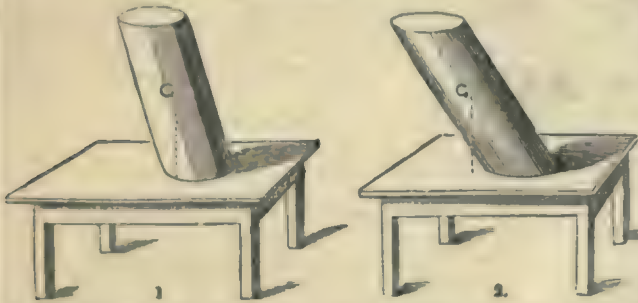
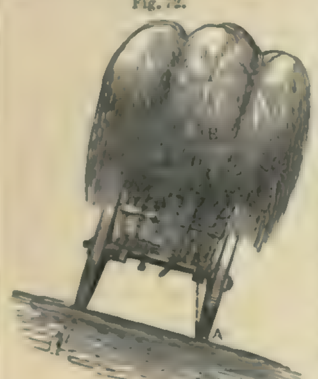


Fig. 72.



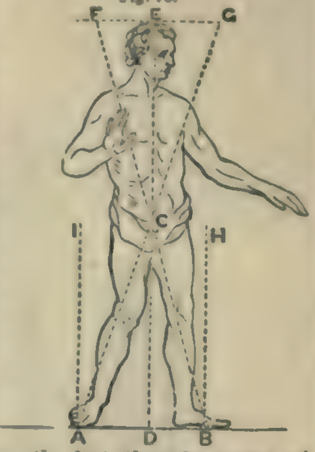
Thus, a loaded waggon passing along an inclined road will not be overturned so long as the vertical G A (Fig. 72) passing through the centre of gravity G of the load and waggon falls within the points where the wheels touch the road. As soon as the inclination of the road becomes so great, or the position of the centre of gravity of the load be such, that the vertical falls without these points, the waggon will be overturned.

The broader the base, and the lower the centre of gravity of the body, the firmer will be its stability. A waggon carrying a load of straw is more likely to be overturned than if it were laden with an equal weight of iron or some other heavy material; because, in the first instance, the distance of the centre of gravity of the load and waggon from the ground is much higher than in the latter.

The round tower which is the belfry of the cathedral at Pisa, is 190 feet high, and deviates from the perpendicular about 14 feet. At Bologna there is a square tower called Garisenda, 134 feet high, and deviating 9 feet from the perpendicular. Both these buildings are supposed to owe their inclination from the depression of the ground under their foundations; and they have not been overthrown, because the vertical, passing through their centre of gravity, still falls within their base.

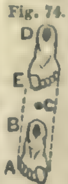
Desaguliers, in his *Course of Experimental Philosophy*, has the following interesting observations on the position of the centre of gravity in animal bodies when at rest, or in motion, in various positions. "When we stand upright, with our feet as represented in Fig. 73, the line of direction (*i. e., the vertical passing through the centre of gravity*) goes through the point C, and passes between our feet to D, and we may move our heads from E to F or G, and our bodies forwards, backwards, or sideways, as far as I or H, without danger of falling, or stirring our feet, as long as the line of direction traverses no farther than I A or H B, and falls anywhere within the space A B, which in this situation of our feet makes a pretty large base. But if we set one foot before the other, as in Fig. 74, a little push sideways will make the line of direction (which went through C) fall out of the base to the right or left towards E or B; in which case a man must fall if he do not quickly remove his feet to the position of Fig. 73.

Fig. 73.



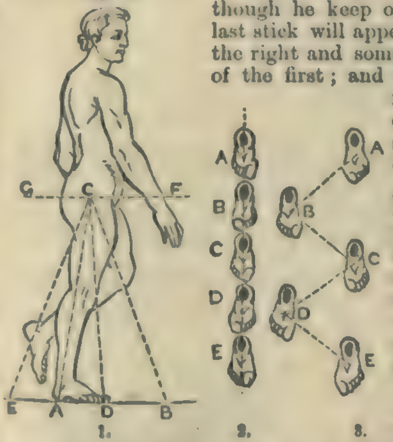
"When we stand upon either leg, we must bring our body so much over the foot, that the centre of gravity being directly over it, the line of direction may go through the sole of it; and in walking, the line of direction must travel through every place where each foot is set down, going successively through the points E, A, D, B (Fig. 75), while the centre of gravity goes through the points G, C, F, &c.; so that unless a man in walking straight forward, sets one foot directly before the other, the line of direction will not describe a straight line upon the plane where the man walks, but an indented line; that is, angles to the right and left, whilst the body of the man goes on in a waddling motion. This we see in the walking of fat people, and all others that straddle in their gait. The line of direction going through the points A, B, C, D, E, describes a straight line in Fig. 75 (2), where the feet are set before one another; but when the motion of one foot is in a parallel line with the motion of the other, an indented line is described by the motion of the centre of gravity above, and the line of direction as it cuts the ground at A, B, C, D, E (3).

Fig. 74.



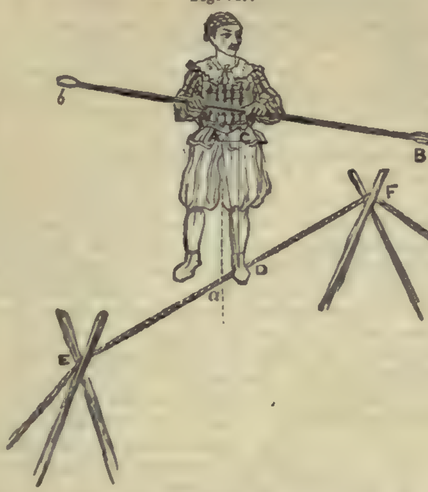
"It is not strictly true that any man in his common walk sets one foot so exactly before the other as to carry on the bottom of his line of direction in a straight line, as represented in (3); because if a straight line be drawn with chalk, it is difficult to walk straight along it: but the plainest proof is the observation of two upright sticks, of about the height of a man, the one painted white and the other black, and set up about ten yards beyond one another, in the same line that a man walks towards them; for in such a case, though he keep one eye shut, the last stick will appear sometimes on the right and sometimes on the left of the first; and the more so the nearer the man comes to the sticks. Rope-dancers, indeed, go in a straight line; but it is what they have learned by art, and inured themselves to by long practice; yet they must, even after all, have helps to keep their centre of gravity over the rope. They generally keep their eyes on some distant point in the same plane as the rope. They have commonly a long pole loaded at the ends with the balls of lead B *b* (Fig. 76), by the motion of which they can alter the position

Fig. 75.



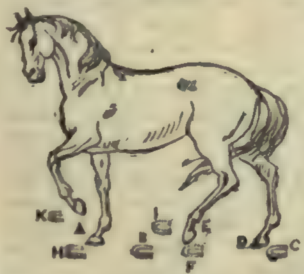
of the common centre of gravity of their body and the pole; as, for example, the centre of gravity of the rope-dancer C A being at A, his line of direction should go through *a*, off from the rope; but by moving the pole towards B, the common centre of gravity of the man and pole is brought to C, and the line of direction C D goes through the rope. Those that are well skilled in this art will sometimes use their arms only instead of a pole; and it is very common for several of them to dance with a flag, with which they strike the air the same way that the centre of gravity goes when the line of direction does not go through the rope; and by the reaction of the air the centre of gravity is brought back to its proper place.

Fig. 76.



"The ancients observing that horses and other quadrupeds, in galloping, lift up their two fore feet, and then their hind feet as soon as the fore feet are set down, did imagine that in walking, as well as pacing and trotting, a horse has two feet off of the ground at one time; and accordingly, in their brass or marble statues, they have represented their horses with two legs off of the ground diagonally opposite—as the right before and the left behind, or left before and right behind. The modern statuaries have also fallen into the same error, because in the quick walking of a horse, the eye cannot well distinguish; and therefore Borelli (*De motu animalium*) has shown from mechanical principles, that the motion of raising two feet at once in walking cannot be consistent with the wisdom and simplicity of nature.

Fig. 77.



"Let us consider a horse as an oblong machine sustained by the four legs, as four props or columns, resting on the points A, B, C, D, (Fig. 77), which make a rectangular quadrilateral figure; then the line of direction will fall perpendicularly on E, a point in or near the centre of the quadrilateral figure, which will make the station or standing of the horse the most firm. The progressive motion begins by one of the hind feet—as, for example, the left hind foot C, which, by strongly pressing back the ground, moves forward the centre of gravity, and consequently carries on the line of direction from E to G as itself moves from C to F. This done, the foot B is raised and carried forward as far as H, which motion of the foot is easy, because the line of direction first falls within the triangle A B D; secondly, within the trapezium A B F D—that is, the body of the horse is sustained by three or by four columns. Lastly, the three feet A, D, F, remaining firm, and taking the line of direction at G, immediately the left fore foot B is carried forward to H; and by the impulse already made, the centre of gravity is also carried over I—namely, the central point of the rhomb A H F D. The motion of the two left feet being completed, the impulse and motion of the right hind foot D begins, and then that of the right fore foot, and so on in the manner above described, as the animal moves forward.

"Ducks, geese, and the greatest part of the water-fowl, whose legs are set wide asunder for the convenience of their swimming, and turning quick in the water, have always a waddling motion upon land; but a cock, a stork, an ostrich, and most other birds that are not web-footed, walk almost directly forward without waddling (especially when they walk slowly), having their legs so placed as to put one foot before the other with greater ease. Thus quadrupeds seldom or never waddle, because they have commonly three feet upon the ground at a time; so that, however the base receiving the line of direction alters from a quadrangular to a triangular figure, that part of it, in which the line of direction falls, is always in or near the same line.

"When a man stands in a firm posture (Fig. 73) A B, the distance of his feet is the length of a quadrilateral figure, whose breadth is nearly the length of the feet, and D is the point under the centre of gravity C, where the line of direction falls. Let the lines A C and B C be drawn, then let these two lines and D C be continued to the points E F G so as to make the triangles E C G and A C B equal and similar; as long as the line F D (or a plane going through it) cuts the whole body of the man into two equal parts, the centre of gravity will be at C, and C D will be the line of direction. But if the body be inclined towards the left hand H, the centre of gravity will move from C to H, the line of direction will become H B, and the right foot being easily removed from A, may be carried on beyond B, by which means the man will go on towards the left. In like manner, by inclining towards I, the line of direction will be removed to I A, and the man go to the right. When a man stands upon one foot it is with some difficulty. For example, let the line of direction be C D (Fig. 75—1); by the motion of the blood and lungs, and other animal motions, the centre of gravity will be apt to vacillate or totter towards F or G on either side about the centre of motion D, where now the base is but small. If the line of direction come to B, the man must fall forwards, backwards if to E; and though A be under the heel of the foot, yet in the motion of the said line of direction from D to A, the body will be apt to go towards E, and so bring the line of direction beyond the base. This will more probably happen in the side motion of the body; so that the body will be in danger of falling, unless the right foot be put down towards that side where the body inclines. Birds stand upon one foot much more easily than men, because their line of direction being much shorter, and the base of one foot a large rhomboidal figure made by the four claws, the line of direction cannot go out of that base, unless the centre of gravity rises, which is impossible without a violent motion."

When a porter carries a burden upon his shoulders, he must stoop, because, if he should stand upright, the common centre of gravity of the man and burden would be so far brought back, that the line of direction would fall behind the feet.

PROPOSITION XXII.

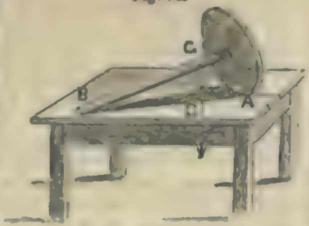
A heavy body rests upon a horizontal plane, to find the pressures produced by its weight upon the points of contact by which it is supported.

When a heavy body rests upon a horizontal plane, the pressures produced by its weight on the points on which it is in contact with the plane can be determined, if these points do not exceed three in number. When the points of contact are more than three, the pressure upon each is indeterminate.

If the body rest upon a single point, it is evident that the pressure on this point is equal to the weight of the body.

When the body rests upon two points A and B, as in Fig. 78, where the body is supposed to consist of a circular disc and a cylindrical stick, fixed perpendicularly to its centre, the pressures on these points may readily be determined.

Fig. 78.



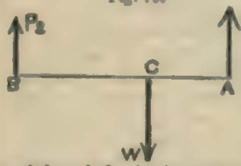
Let G be the centre of gravity of the body resting on the plane.

This body is evidently kept in equilibrium by its weight, and the equal and opposite reactions produced by the two pressures on the points A and B.

The vertical passing through G must therefore pass through the line adjoining A and B.

Let C be the point where the vertical cuts A B; the weight of the body acting at G may be transferred from G to C.

Fig. 79.



Then (Fig. 79) we have two reactions, P_1 and P_2, acting upwards, perpendicular to the plane at A and B; and a weight W equal to the weight of the body acting downwards, and also perpendicular to the plane.

Hence, Prop. XI., $P_1 + P_2 = W$, and $P_2 \cdot BC = P_1 \cdot AC$.

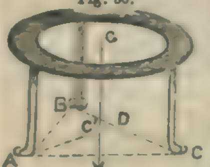
Two equations are thus obtained from which P_1 and P_2 may readily be determined, when the position of the centre of gravity G of the body is known.

The reactions P_1 and P_2 will be equal and opposite to the pressures produced by the body at A and B.

When the body rests upon three points A, B and C (Fig. 80), we may still find the pressures produced by its weight on these points.

Join the points A, B and C. In order that there may be equilibrium, the vertical passing through G, the centre of gravity of the body, must fall within the triangle A B C. (Prop. XXI.)

Fig. 80.



Let C' be the point where the vertical passing through C cuts the triangle A B C.

Join A C; and produce A C' to meet B C in D.

If W be the weight of the heavy body, we may transfer the force W from G to C. This force, acting perpendicular downwards at C, may be resolved into two forces, W_1 acting at A, and W_2 acting at D, both perpendicular to the plane of the triangle A B C.

Where $W_1 + W_2 = W$, and $W_1 \cdot AC = W_2 \cdot CD$.

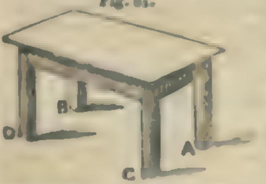
The force W_2 acting at D may again be resolved into two others, W_3 acting at B and W_4 at C, both perpendicular to the triangle A B C.

Where $W_3 + W_4 = W_2$, and $W_3 \cdot BD = W_4 \cdot CD$.

From these equations the forces W_1, W_3, and W_4, which will be the pressures exerted by the heavy body on the points A, B and C, may be readily determined.

If the heavy body rest on the plane by four points of support, as a table on its four legs, A, B, C, D, (Fig. 81), the pressures upon these points will be indeterminate if we consider the body perfectly rigid.

Fig. 81.



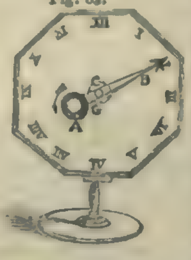
For since it may be supported by any three of the points of contact, the pressure on the fourth may be either nothing or some finite proportion of the weight of the body. The method used in the two preceding cases to determine the pressures will fail in this. The only condition we have is, that the sum of the pressures on the points A, B, C and D must be equal to the weight of the body; and if we consider them as weights acting perpendicularly to the plane on which the table rests, the centre of gravity of these weights will lie in the vertical, passing through the centre of gravity of the table.

The same reasoning may be extended to the cases where the points of contact are more than four.

THE MAGIC CLOCK.—The influence of the centre of gravity on the position of equilibrium of a heavy body, which can only turn round a fixed horizontal axis, is well illustrated by an ingenious contrivance called the *magic clock*.

A transparent glass dial-face has a hole pierced through its centre at C (Fig. 82); in this is a fixed horizontal axis; on this axis is placed the hour-hand of the clock, which can move freely round it to the right or left. The extremity of the hand opposite the pointer B is terminated by a hollow ring; in this ring there is a heavy spherical ball A, capable of moving freely round it. This ball is made to move uniformly round the interior of the ring, by a watch-movement concealed in the hand, once in twelve hours, in the direction indicated by the arrow in Fig. 82. The weight of the ball A is so proportioned to that of the hand with its concealed watch-work, that as A moves round the interior of the ring G, the common centre of gravity of the ball and hand would describe a circle round the centre of the axis C, if the hand were fixed in one position. Since the hand can move freely round C, for every new position of A the hand will assume that position which shall make C G vertical (Prop. XIX.) As A, therefore, is carried round the ring uniformly once in twelve hours, the pointer of the hand B moves with the same uniformity round the dial in the opposite direction, and indicates the hour.

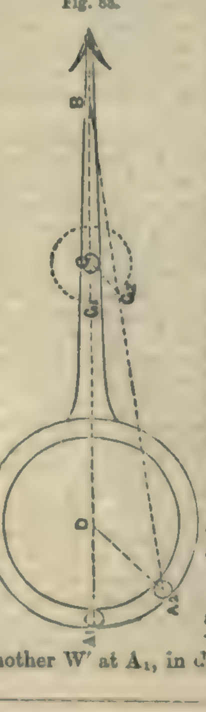
Fig. 82.



The ball A, and the watch-movement which causes it to turn round the ring, being both concealed, the hand seems to move by itself, as if by magic. The remarkable property that the hour-hand being made to move by your finger backwards or forwards, when left to itself, will, after a few oscillations, resume its position, and point to the correct hour, adds considerably to the illusion.

Let D (Fig. 83) represent the centre of the ring round which the spherical body circulates; A_2, the centre of the spherical body; C, the centre of the axis round which the hand moves; B, the centre of gravity of the hand and its concealed watch-work, exclusive of the movable spherical body; G_2, the common centre of gravity of the hand and spherical body. Also let W represent the weight of the hand and works exclusive of the sphere, whose weight is represented by W'.

Fig. 83.



Then, if W and W' be so chosen that $W \cdot BC = W' \cdot CD$, the common centre of gravity G_2 of the hand and movable sphere will describe a circle round C, if the hand be supposed to be fixed while A_2 describes a circle round D.

Let A_1 represent the position of the centre of the sphere, when it lies in B C D produced; G_1 the centre of gravity of the hand and sphere for this position.

Join D A_2, C G_2, and B G_2 A_2.

When the centre of the movable sphere is in the position A_1, we have a weight W acting at B, and another W' at A_1, in directions parallel to each other;

and since G_1 is the position of their resultant, we have (Prop. XI.)

$$W \cdot BG = W' \cdot A_1 G_1;$$

but by the construction of the instrument,

$$W \cdot BC = W' \cdot CD.$$

Hence $\frac{BG_1}{BC} = \frac{A_1 G_1}{CD}$, or $\frac{BC + CG_1}{BC} = \frac{A_1 C - CG_1}{CD}$.

Therefore $CD \cdot BC + CD \cdot CG_1 = A_1 C \cdot BC - BC \cdot CG_1$.

Or $CG_1 \cdot (CD + BC) = (A_1 C - CD) BC \cdot$

And $CG_1 \cdot BD = A_1 D \cdot BC$.

Or $CG_1 = \frac{A_1 D \cdot BC}{BD}$.

When the centre of the sphere is in the position A_2, we have a weight W acting at B, and a weight W′ at A_2, in parallel directions. Hence (Prop. XI.) he have

$$W \cdot BG_2 = W' \cdot A_2 G_2.$$

But $W \cdot BC = W' \cdot CD$.

Hence $\frac{BG_2}{BC} = \frac{A_2 G_2}{CD}$.

Therefore (Euc., B. VI., P. II.), the line CG_2 is parallel to $A_2 D$, and BCG_2 and BDA_2 are similar triangles; and consequently

$$\frac{CG_2}{A_2 D} = \frac{BC}{BD}, \text{ or } CG_2 = \frac{BC}{BD} \cdot A_2 D.$$

But $A_2 D = A_1 D$. Hence $CG = \frac{A_1 D \cdot BC}{BD}$. The same value which we have before obtained for CG_1.

Hence for every position A_2 of the centre of the movable sphere out of the line BCD, we shall have CG_2 parallel to DA_2, and equal to CG_1.

Fig. 84.

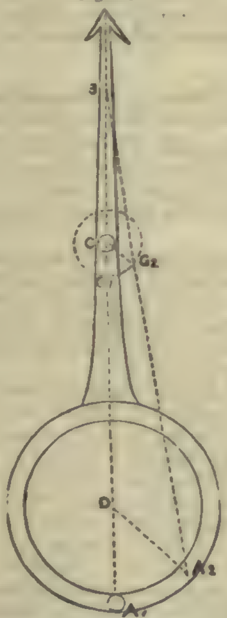
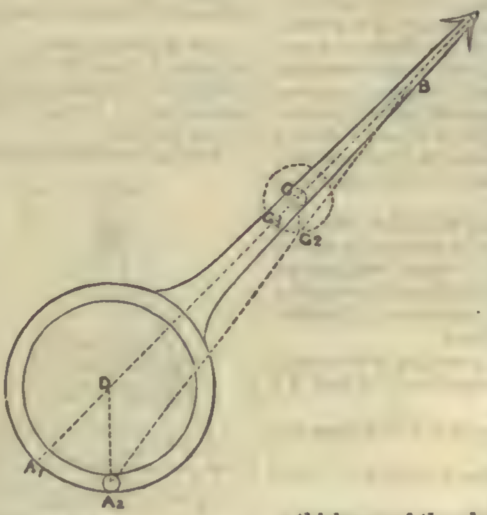


Fig. 85.

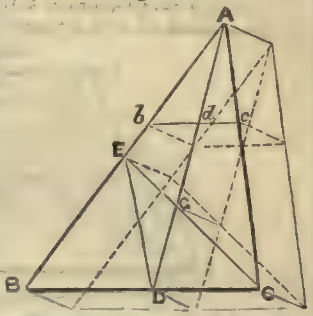


As A_1 therefore describes a circle round D, G_1 will describe a circle round C in the same direction, and uniformly with it; the radius of this circle being equal to the product of the radius of the circle described by the centre of the sphere, and the distance of B from C divided by BD.

When the centre of the sphere is in the position A_1, the hand will rest in such a position that CG_1 shall be vertical (Prop. XIX.), as shown in Fig. 84. When A_2 represents the position of the centre of the sphere, CG_2 will be vertical, and the hand will rest in the position shown by Fig. 85. Hence it is evident that, as the watch-movement concealed in the hand causes the centre of the sphere to move from A_1 to A_2, the extremity of the pointer of the hand will move through a similar arc in the opposite direction round C as its centre.

MATHEMATICAL DETERMINATION OF THE CENTRE OF GRAVITY OF A HEAVY BODY.—The application of Prop. XVII. to find the centre of gravity of a body of uniform or variable density, in most instances requires the aid of the differential and integral calculus before we can arrive at a solution. As this would lead us beyond the limits proposed for the mathematical department of our work, we give some instances of the determination of the centre of gravity of a few geometrical forms and solids of uniform density, depending upon the properties of the centre of gravity demonstrated in Prop. XIX.

Fig. 86.



EXAMPLE I.—*To find the centre of gravity of a triangular plate of uniform thickness and density.*

Let ABC (Fig. 86) be the surface of the triangular plate, bounded by two parallel triangular faces, and by parallelograms perpendicular to these faces.

Bisect BC in D. Join AD.

Through *b* any point in AB draw *bdc* parallel to BC, and cutting AD in *d*.

Then the triangle A*bd*, by construction, is similar to the triangle ABD, and the triangle A*dc* to the triangle ADC.

Hence (Euc., B. VI., P. IV.), $\frac{Ad}{AD} = \frac{bd}{BD}$, and $\frac{Ad}{AD} = \frac{dc}{DC}$.

Therefore $\frac{db}{BD} = \frac{dc}{DC}$, but $BD = DC$.

Hence $bd = dc$; and so it may be shown that every line drawn through any point in AB parallel to BC will be bisected by AD.

Now we may conceive the triangular plate, whose thickness is uniform, to be made up of a number of extremely thin slices, cut perpendicular to the surface, and parallel to BC.

Each of these slices, being symmetrical, will balance about its centre, and therefore every one of them will balance about a line drawn parallel to AD, and passing through the centre of the thickness of the plate.

The whole triangular plate will therefore balance about this line.

Next bisect AB in E, join EC cutting CD in G.

It may be shown, as before, that the triangular plate ABC will balance about a line parallel to EC, passing through the centre of the thickness of the plate.

Since the centre of gravity of the plate lies in both the lines passing through the centre of the thickness parallel to CE and AD, the intersection of these two lines must be the centre of gravity.

Hence a point in the middle of the thickness of the plate below G will be the centre of gravity of the plate, provided its thickness and density be uniform throughout.

To find G, join DE.

Then because AB is bisected in E, and BC in D,

Therefore (Euc., B. VI., Props. II. and IV.), ED is parallel to AC, and $ED = \frac{1}{2} AC$.

Hence the angle EDG = angle GCA, the angle at G is common, and therefore the triangle EGD is similar to the triangle AGC.

Therefore (Euc., B. VI., P. IV.), $\frac{DG}{DE} = \frac{AG}{AC}$.

Or $DG = \frac{AG \cdot DE}{AC} = \frac{1}{2} \cdot \frac{AG \cdot AC}{AC} = \frac{1}{2} AG$.

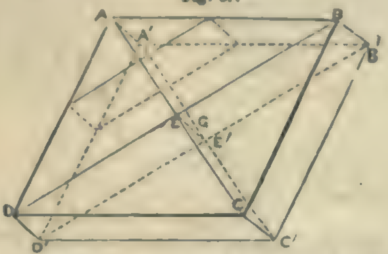
And $A D = A G + D G = 2 D G + D G = 3 D G$.

Or $D G = \frac{1}{3} A D$.

EXAMPLE II.—*To find the centre of gravity of a plate of uniform thickness and density bounded by parallel planes, whose upper and lower surfaces are parallelograms.*

Let the parallelogram A B C D (Fig. 87) represent the upper surface of the plate, A′ B′ C′ D′ the lower; the other boundary planes, such as B C C′ B′, being perpendicular to A B C D.

Fig. 87.



Join B D and A C meeting in E, B′ D and A′ C′ meeting in E′. A C will bisect all lines drawn through the parallelogram parallel to B D, and B D will bisect all similar lines drawn parallel to A C.

Then we may conceive, as in the last example, the plate as made up of a number of parallel plates, each parallel to the plane B D D′ B′, each of which will balance about a line drawn through the centres of A A and C C′.

Again, we may conceive the plate as composed of a number of parallel plates parallel to A A′ C′ C, which will each balance about a line drawn through the centre of D D and B B′.

G, the intersection of these lines, or the centre of E E′, will therefore be the centre of gravity of the plate.

To find the centre of gravity of the surface of any rectilineal figure, we have only to divide it into triangles, find the centre of gravity of each triangle, suppose weights acting at each of these centres of gravity proportional to the surface of the triangle at whose centre of gravity it is supposed to act, and then find the centre of gravity of these weights, which will be the centre of gravity of the figure.

It will be found, on calculation, that the centre of gravity of any triangular surface is the same as that of three spheres, or any other body, whose weights are equal, placed with their centres of gravity in the angular points of the triangle. This also applies to a parallelogram, but not to all four-sided figures.

EXAMPLE III.—*To find the centre of gravity of a homogeneous solid of uniform density, whose form is that of a pyramid on a triangular base.*

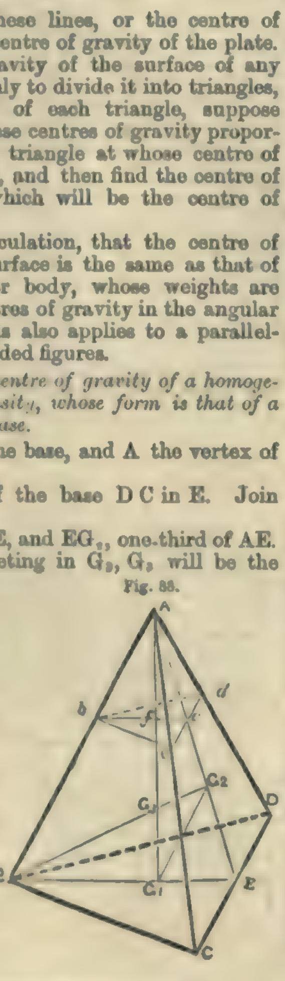
Let B C D (Fig. 88) be the base, and A the vertex of the pyramid.

Bisect one of the sides of the base D C in E. Join A E, B E.

Take EG_1, one-third of BE, and EG_2, one-third of AE.

Join $A G_1$ and $B G_2$ meeting in G_3, G_3 will be the centre of gravity of the pyramid.

Fig. 88.



In A C take any point *c*, through *c* draw *c d* parallel to C D cutting A E in *e*, and *b c* parallel to B C. Join *b d* and *b e* cutting $A G_1$ in *f*.

Then the plane *b c d* is evidently parallel to the plane B C D, and the line *b e* parallel to the line B E.

Hence by similar triangles $A E G_1$, $A e f$, $\frac{e f}{E G_1} = \frac{A e}{A E}$ and by similar triangles $A e b$ $A E B$, $\frac{b e}{B E} = \frac{A e}{A E}$

Therefore $\frac{e f}{E G_1} = \frac{b e}{B E}$ but $E G_1 = \frac{1}{3} B E$

Hence $e f = \frac{1}{3}$ of $b e$.

Again, because *c d* is parallel to C D, and C D is bisected in E, *c d* is also bisected by A E.

f therefore is the centre of gravity of the triangle *b c d*.

In a similar manner it may be shown that the centre of gravity of any triangular slice of the pyramid parallel to B C D must lie in the line $A G_1$; and since we may conceive the pyramid made up of an infinite number of such parallel slices, the centre of gravity of the whole pyramid must lie in $A G_1$.

By similar reasoning it may be shown that the centre of gravity of the pyramid lies in the line $B G_2$.

Consequently, G_3, the intersection of the lines $A G_1$ and $B G_2$, will be the centre of gravity of the pyramid.

Join $G_1 G_2$, then since $E G_2 = \frac{1}{3} A E$, and $E G_1 = \frac{1}{3} B E$, $G_1 G_2$ is parallel to A B. Consequently the triangle $G_3 G_1 G_2$ is similar to the triangle $G_3 A B$, and the triangle $E G_1 G_2$ to the triangle E B A.

Hence $\frac{G_1 G_3}{A G_3} = \frac{G_1 G_2}{A B}$ and $\frac{G_1 G_2}{A B} = \frac{E G_1}{B E} = \frac{1}{3}$.

Therefore $\frac{G_1 G_3}{A G} = \frac{1}{3}$ and $G_1 G_3 = \frac{1}{3} A G_3$

Or $G_1 G_3 = \frac{1}{4} A G_1$.

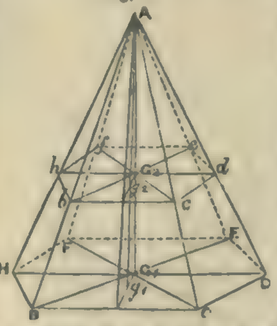
Hence the centre of gravity of the pyramid is found to be in the line joining the vertex and the centre of gravity of the base, at a distance of a fourth of its length.

EXAMPLE IV.—*To find the Centre of Gravity of any Pyramid of uniform density, whose base is any Polygon.*

Let A be the vertex of the pyramid, B C D E F H its polygonal base (Fig. 89). Let G_1 be the centre of gravity of this base; join $A G_1$, and in $A G_1$ take G_2 so that $G_1 G_2 = \frac{1}{4} A G_1$. G_2 will be the centre of gravity of the pyramid.

Join G_1 with each of the angular points of the base, through G draw the plane *b c d e f h* parallel to the base, and cutting A B in *b*, A C in *c*, &c., and join G_2 *c*, G_2 *d*, &c.

Fig. 89.



Let g_1 be the centre of gravity of the triangle $B G_1 C$, join $A g_1$ cutting the triangle $b G_2 c$ in g_2.

Then since the triangle $b G_2 c$ is parallel to the triangle B G C and $G_1 G_2 = \frac{1}{4} A G_1$; hence $g_1 g_2 = \frac{1}{4} A g_1$ and g_2, is the centre of gravity of the pyramid $A B C G_1$ on a triangular base G_1 B C, and it lies in the plane *b c d e f h*. In a similar manner it may be shown that the centres of gravity of every one of the triangular pyramids $A G_1$ C D, $A G_1$ D E. &c., into which the whole pyramid may be divided, all lie in the plane *b c d e f h*.

It follows, therefore, that the centre of gravity of the whole pyramid must lie in this plane.

Again the plane *b c d e f h* is in every respect similar to the base, and G_2 its centre of gravity will therefore lie in the line $A G_1$. In like manner it may be shown that the centre of gravity of every section of the pyramid parallel to the base will lie in $A G_1$; hence the centre of gravity of the whole pyramid must lie in $A G_1$.

We have shown, therefore, that the centre of gravity of the whole pyramid must be in the plane *b c d e f h*, and also in the line $A G_1$. Hence the centre of gravity must be their intersection G_2.

The centre of gravity of any pyramid of uniform density on a polygonal base, will lie in the line joining its vertex and the centre of gravity of the base, at a distance equal to three-fourths of its length from the former, or one-fourth of its length from the latter.

The above reasoning is altogether independent of the number of the sides of the polygon, and since we may conceive a curve to be made up of an infinite number of straight lines, or a polygon bounded by an infinite number of small sides, the above solution enables us to determine the centre of gravity of a cone with a curvilinear base.

Hence the centre of gravity of a right or oblique cone on any curvilinear base is found by joining the centre of gravity of the base with the apex of the cone, and taking a point in this line, equal to one-fourth of its length, measuring this point from the centre of gravity of the base.

CHAPTER II.

THE MECHANICAL POWERS.

Any instrument by means of which a force is communicated from one point to another, so as to keep at rest or set in motion a body acted on by another force, is called a machine. The simplest of these instruments are *cords*, *rods*, and *hard planes*; and these, by their combinations, form all complex machines, however various their forms and actions.

In statics, we only consider machines when the forces acting upon them are in a state of equilibrium; motion will be produced by an addition to some of the forces which produce equilibrium. But the discussion of this branch of the subject belongs to Dynamics, which will be investigated in the following chapter.

For the sake of simplicity, and to enable us to apply the theoretical principles we have already proved, we consider *cords* as being destitute of weight and perfectly flexible, *rods* and *planes* as perfectly rigid, inflexible, and without weight. When necessary, we can take the weights of the elementary parts of our machines into consideration, by considering their weights as a force proportional to their weight applied to the centre of gravity, in a vertical direction.

We also neglect the friction of all surfaces in contact with each other. It is evident, therefore, that our machines will be theoretical ones, which cannot exist in nature, and whose properties cannot be strictly proved by experiment. But if we determine the rigidity, flexibility, and friction of the substances composing our machine by experiment, and compare the force exerted by a real machine with the force it ought to exert by theory, we may arrive at a knowledge of the retarding forces produced by friction or want of flexibility; and thus by our theoretical knowledge of the combination of machines, estimate the forces produced by any actual complex machine, the friction or flexibility of whose elements we have determined.[*]

MECHANICAL POWERS.—The simplest combinations of these machines are called the *mechanical powers.* They are usually regarded as seven in number:—1, the lever; 2, the wheel and axle; 3, the toothed wheel; 4, the pulley; 5, the inclined plane; 6, the wedge; 7, the screw.

The wheel and axle, toothed wheel and pulley, may be regarded as modifications of the lever, and the wedge and screw as particular cases of the inclined plane.

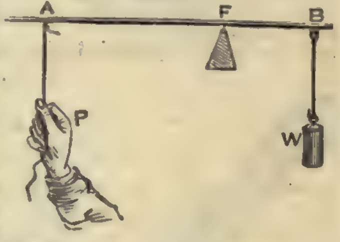
THE LEVER.—The simplest form of a lever is a straight rod, supposed to be inflexible and without weight, resting on a fixed point, somewhere in its length, about which it can turn freely, and having two forces applied at two other points of the rod.

The fixed point on which it rests, and about which it can turn, is called the *fulcrum;* one of the forces applied to it is called the *power*, and the other the *weight.* The distances of the points of application of the power and weight, from the fulcrum, are called the *arms* of the lever.

There are three kinds of levers, distinguished by the relative position of the *power*, *weight*, and *fulcrum*.

Lever of the first kind. — In the lever of the first kind the power P (Fig. 90) and the weight W act in the same direction on opposite sides of the fulcrum F. A crow-bar (Fig. 91), by means of which a man raises a heavy body W by placing one extremity B under W, and resting it on any body C while he presses the extremity B′, is an instance of a lever of the first kind.

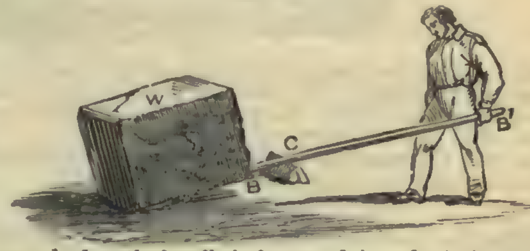
Fig. 90.



A poker is another. In this case the coals form the weight, the bar of the grate the fulcrum, and the hand the power.

The spade is a lever; the ground against which it is pressed when the handle is depressed, in order to turn up the earth in front of it, being the fulcrum.

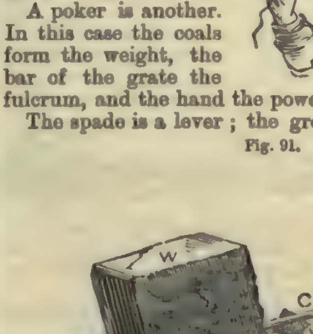
Fig. 91.



Scissors and carpenters' pincers are examples of double levers of the first kind.

Lever of the second kind.—In the lever of the second kind the power P (Fig. 92) and the weight W act in opposite directions on the same side of the fulcrum F, the weight being nearer to the fulcrum than the power.

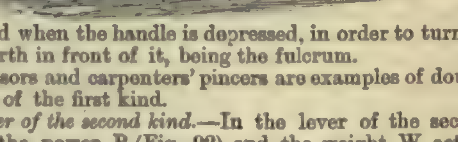
Fig. 92.



A cutting-knife (Fig. 93) and an oar are instances of levers of the second kind. In the case of the oar, the arm of the rower is the power, the pressure of the oar on the side of the boat is the weight, and the point of the blade of the oar, which is for a moment stationary in the water, forms the fulcrum.

Nutcrackers give a good illustration of a double lever of the second kind.

Fig. 93.



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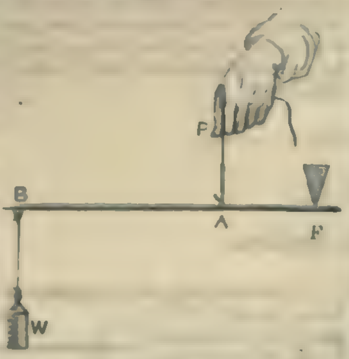
[*] See *Pressure, Tension,* and *Flexible Cords,* p. 693.

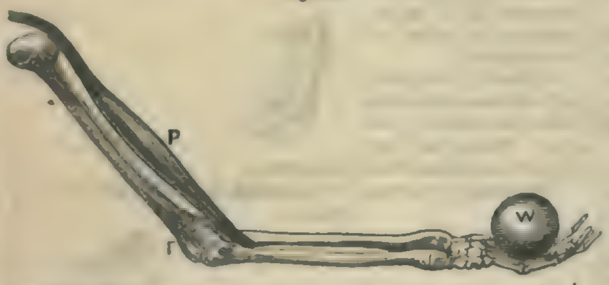
Lever of the third kind.—In the lever of the third kind the power P (Fig. 94) and the weight W act as in the second kind, in opposite directions on the same side of the fulcrum; but in the third species of lever, the power is nearer the fulcrum than the weight.

Fig. 94.



Most muscles in animals are generally inserted near the joint, as in Fig. 95, and act as levers of the third kind; the joint forms the fulcrum; the limb which the muscle moves, together with any resistance opposed to its motion, represents the weight; while the force exerted by the contraction of the muscle is the power. The treadle of a turning-lathe, or razor-grinder's machine, where the foot of the operator acts as the power, is a lever of the third kind.

Fig. 95.



A pair of tongs, and a pair of sheep-shears, afford familiar instances of double levers of the third kind.

PROPOSITION XXIII.

To find the Conditions of Equilibrium and the Pressure on the Fulcrum when two Parallel Forces act in the same plane on a straight lever.

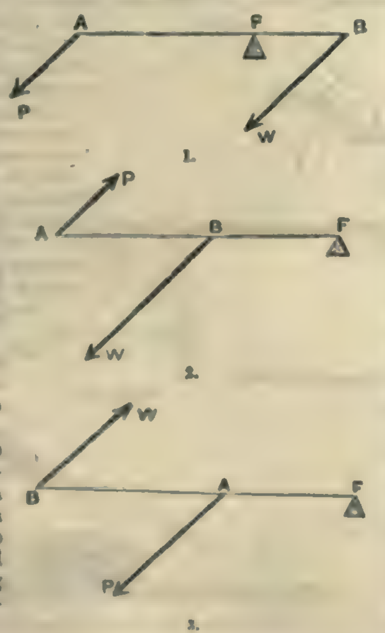
Let P and W be two parallel forces acting at A and B, on the straight lever A B whose fulcrum is F in each of the three kinds of levers.

And let A P and W B (Fig. 96—1, 2, and 3) represent these forces in magnitude and direction.

In order that there may be equilibrium, the resultant of the two parallel forces must pass through F, and the pressure on that point will be equal to their resultant, and this will be destroyed by the resistance of the fulcrum.

The proposition is therefore reduced to the case of Prop. XI., which we have already demonstrated; and from this we may learn that the pressure on F will be equal to P + W acting in a direction parallel to P or W.

Fig. 96.



And also that $P \cdot AF = W \cdot BF$ or $\frac{P}{W} = \frac{BF}{AF}$ in every case.

From this we find that in the lever of the first kind, the power may be less than, equal to, or greater than, the weight; in the second kind the power is always less; and in the third kind always greater than the weight.

In the preceding cases we have supposed the lever to be without weight; we shall show how to take the weight of lever into consideration, in the cases of the balance and steelyard.

PROPOSITION XXIV.

To find the Conditions of Equilibrium of two Forces acting in the same plane on a Bent Lever, when the directions of the Forces are not Parallel.

Let A C B (Fig. 97) be a bent lever consisting of a portion of a plane rigid body, supposed to be destitute of weight. C the fulcrum about which the lever turns. A and B the points of application of the two forces represented in magnitude and direction by A P and B W.

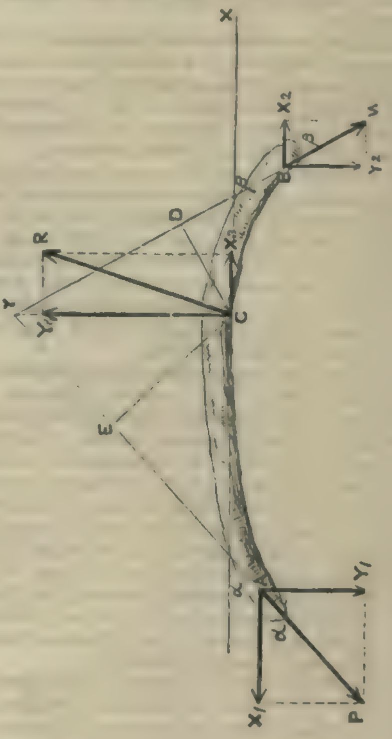


Fig. 97.

The effect of these forces will be to produce a pressure on C; and when there is equilibrium, this pressure must be destroyed by the reaction R of the fulcrum C, represented in magnitude and direction by C R.

Let α and β be the angles which A P and B W produced, make with a line C X drawn through C at right angles to the vertical C Y passing through C, and θ the angle C R makes with C X.

R and θ are two unknown quantities which we have to determine: through C draw C E and C D perpendicular to A P and B W produced.

We may regard our lever as a rigid body kept in equilibrium by the three forces P, W, and R. Resolving these forces into forces parallel to C X and C Y, X_1 and Y_1 being the resolved parts of P, X_2 and Y_2, those of W, and X_3 and Y_3 those of R; and taking moments about C, we have by Prop. XVIII., the following conditions of equilibrium.

(1.) $X_3 + X_2 - X_1 = 0$, or $R \cos. \theta + W \cos. \beta - P \cos. \alpha = 0$.

(2). $Y_2 - Y_3 - Y_1 = 0$, or $R \sin. \theta - W \sin. \beta - P \sin. a = 0$.

And (3). $P \cdot CE - W \cdot CD = 0$.

From this last equation we obtain $P \cdot CE = W \cdot CD$, or

$$\frac{P}{W} = \frac{CD}{CE} = \frac{\text{Perpendicular on direction of W.}}{\text{Perpendicular on direction of P.}}$$

From equations (1) and (2) we can determine R and θ, and consequently the magnitude and direction of the pressure on the fulcrum, which will be equal to R, and act in the direction opposite to C R.

Transposing equation

$$(1).\ R \cos. \theta = P \cos. a - W \cos. \beta$$
$$(2).\ R \sin. \theta = P \sin. a + W \sin. \beta$$

Dividing one of these equations by the other, we have

$$\tan. \theta = \frac{P \sin. a + W \sin. \beta}{P \cos. a - W \cos. \beta}, \text{ which determines } \theta.$$

Or, adding their squares, we have

$$R^2 (\cos.^2 \theta + \sin.^2 \theta) = P^2 (\cos.^2 a + \sin.^2 a) + W^2 (\cos.^2 \beta + \sin.^2 \beta) - 2\,P\,W (\cos. a \cos. \beta - \sin. a \sin. \beta).$$

Hence, by Trigonometry,

$$R^2 = P^2 + W^2 - 2\,P\,W \cos. (a + \beta),$$

which determines the magnitude of R.

If we had supposed B instead of C as the point of application of the fulcrum, and taken our moments about B, we should have had

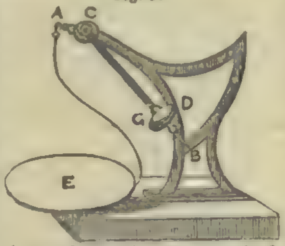
$$\frac{P}{R} = \frac{\text{Perpendicular on direction of R}}{\text{Perpendicular on direction of P}}.$$

Hence the condition of equilibrium in a lever of any kind, *is that the power must be to the weight, inversely as the perpendiculars drawn from the fulcrum on their directions.*

The bent lever evidently includes the straight one, as in the latter case A, B, and C are in the same straight line.

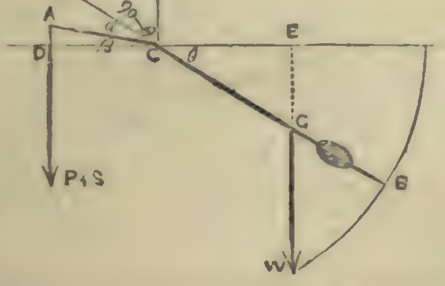
THE BENT-LEVER BALANCE.—The bent-lever balance (Fig. 98) is a machine which, within certain limits, enables us to weigh substances without the use of weights; it consists of a bent lever whose two arms are A C and B C, movable about a fulcrum C. The fulcrum C is fixed to a stand which carries a graduated arc; over this arc the extremity B of the lever moves as C B turns round the fulcrum C. From the other extremity A, a scale-pan E is so suspended as to have its centre in all positions of the lever in the vertical passing through A.

Fig. 98.



A weight D, is fixed to the arm C D so as to bring the centre of gravity of the whole lever and scale-pan to some point below the fulcrum C: the magnitude of this weight is so arranged that the extremity B of the lever shall point to zero on the graduated arc, when the scale-pan is empty, and the lever is in a state of equilibrium.

Fig. 99.



To graduate the arc, weights of 1, 2, 3, &c., pounds or ounces, or whatever denomination of weight the instrument is intended to indicate, are placed successively in the scale-pan; and the corresponding points of the arc over which B rests, are marked on the scale as 1, 2, 3, &c.

This balance is of great use for determining quickly the weights of bodies where extreme nicety is not essential.

To explain the graduation of this balance mathematically,

Let C (Fig. 99) be the fulcrum of the lever, A C and B C its arms; let G in B C be the centre of gravity of the whole lever, exclusive of the scale-pan and wire by which it is suspended. Let W represent this weight, and let $CA = a$ and $CG = b$. Also let S represent the weight of the scale-pan and wire by which it is suspended, P that of a body placed in the scale-pan.

Let θ be the angle C B makes with a line D C E drawn through C perpendicular to C F, the vertical passing through C; β the angle A C makes with D C E.

Also let $a = BCA$ be the angle the arms of the lever make with each other, and let $\acute{a} = 180 - a$, and let D and E be the points where the vertical lines passing through A and B cut the line D C E.

Then Prop. XXIV. $(P+S) \cdot CD = W \cdot CE$,

or $(P+S)\, a \cos. \beta = W \cdot b \cos. \theta$.

but $\beta = 90 - \theta + \acute{a} = 90 - (\theta - \acute{a})$; hence $\cos. \beta = \sin. (\theta - \acute{a})$.

Therefore $(P+S) a \sin. (\theta - \acute{a}) = Wb \cos. \theta$,

or $(P+S) a \{\sin. \theta \cos. \acute{a} - \cos. \theta \sin \acute{a}\} = Wb \cos. \theta$,

$$\text{and } \tan. \theta \cos. \acute{a} - \sin. \acute{a} = \frac{Wb}{(P+S)a}$$

$$\text{or } \tan. \theta = \frac{W \cdot b}{(P+S) a \cos. \acute{a}} + \tan. \acute{a}.$$

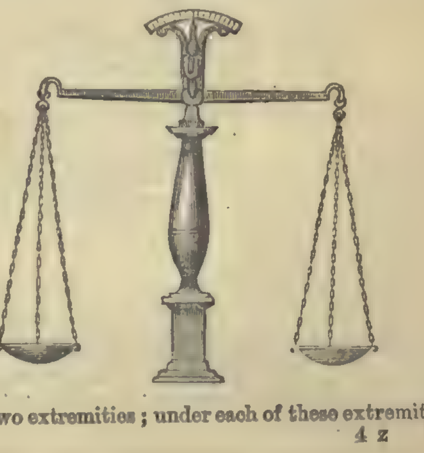
Hence if we wish to graduate the arc for pounds, we must take P successively equal to 0, 1, 2, 3, &c., pounds, in which case tan. θ will be successively equal to

$$\frac{W \cdot b}{Sa. \cos. \acute{a}} + \tan \acute{a},\ \frac{Wb}{(S+1)a. \cos. \acute{a}} + \tan. \acute{a},$$
$$\frac{Wb}{(S+2)a. \cos. \acute{a}} + \tan \acute{a},\ \frac{Wb}{(S+3)a. \cos. \acute{a}} + \tan a, \text{ \&c.},$$

where W and S represent the weight of the lever and scale-pan in pounds.

THE COMMON BALANCE.—This instrument is popularly called the *scales*, or a *pair of scales*, and is perhaps more frequently used than any other for determining the weights of substances or goods. It consists of a lever supported on an axis or fulcrum equally distant from its two extremities; under each of these extremities

Fig. 100.



a dish is suspended, in one of which the substance to be weighed is placed, and in the other the weights by which its value is determined.

The lever is so constructed as to be capable of moving on its axis in a vertical plane; and when a given weight is placed in one dish, and a substance equal in weight to it in the other, after vibrating some little time, it assumes a horizontal position. When a slight excess of weight is added to either dish, the lever again vibrates and assumes such a position of rest, that the extremity above the dish containing an excess of weight over the other, lies below the horizontal line. The lever is called the *beam*; and the two dishes, *scale-pans*.

There are three requisites in a balance:—

First.—When equal weights are placed in the scale-pans, the two extremities of the beam should rest in a perfectly horizontal line. This is called its *horizontality*.

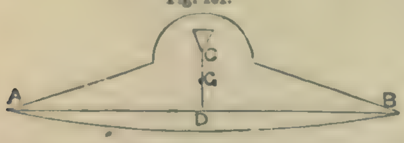
Second.—On the slightest addition of weight to either scale, the beam should lose its horizontal position. This is called its *sensibility*, and is measured by the smallest weight which causes the beam to depart from its *horizontality*.

Third.—After any disturbance the beam should assume a state of rest as speedily as possible: this is called its *stability*.

We shall now proceed to consider the mathematical conditions which must be satisfied, in order to obtain these requisites; for this purpose we must first confine our attention to the construction of the *beam*.

Let A B (Fig. 101) represent the beam, G its centre of gravity, and C the point on which its axis or fulcrum is supported.

Fig. 101.



Join A B, and produce C G to meet A B in the point D.

To secure the *horizontality* of the beam, the points C and G must not coincide, for if they did, from the properties of the centre of gravity, we should have indifferent equilibrium, and the beam, either by itself or when loaded at its extremities with equal weights, would rest in every position in which the line A B might be placed.

When C does not coincide with G, by Prop. XIX., and the beam is at rest, C G will assume a vertical position; hence in order that A B may be horizontal, C D must be perpendicular to A B. In order that this horizontal position may be that of equilibrum when equal weights are suspended from A and B, we must also have A D = B D.

The axis C is generally a prismatic piece of metal, which pierces and also is firmly fixed at right angles to the beam. This prism rests on one of its edges, technically called the *knife-edge*, on a plane or curved surface, so placed as a support on each side of the beam, that the edge or line of support about which the beam oscillates is horizontal and perpendicular to the plane in which it oscillates. The axis might be a cylinder or cone working in a socket, but the *knife-edge* is generally preferred, in order to avoid friction as much as possible.

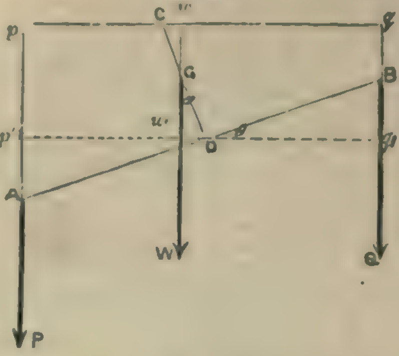
The conditions of *horizontality* given above require, therefore, that the plane passing through the line of support and the centre of gravity of the beam shall be at right angles to, and also bisect, the line passing through its extremities, or the points from which the scale-pans are suspended.

To determine the other two conditions, retaining the same letters as before, let two unequal weights P and Q (Fig 102), represented in magnitude and direction by A P and B Q, be suspended from A and B.

Let θ be the angle A B makes with the horizontal line p' q' drawn through the point D, and let p q cut A P in p and B Q in q.

Through C draw p q parallel to p' q', and cutting A P

Fig. 102.



in p, and B Q in q. Let W, represented in magnitude and direction by G W, be the weight of the beam and its axis, and let the line G W cut p q in w, p' q' in w'.

Also let A D = D B = a, C G = b, and C D = c.

Since C D is at right angles to A B, and θ is the angle A B makes with the horizontal line p' q', it will also make the angle θ with the vertical line w G w'.

Also because p q is parallel to p' q' and P p', w w' and q q' are parallel to each other, p w = p' w', and q w = q' w'.

Taking the moments of the three forces P, Q, and W, about the point C, we shall have, as a condition of equilibrium,

$$W \cdot Cw + Q \cdot Cq = P \cdot Cp.$$

$$\begin{aligned}
\text{But } Cq &= Cw + wq = Cw + w'q' \\
&= Cw + w'D + Dq' \\
&= CG \sin.\theta + GD \sin.\theta + DB \cos.\theta \\
&= b \sin.\theta + (c - b) \sin.\theta + a \cos.\theta \\
&= c \sin.\theta + a \cos.\theta.
\end{aligned}$$

$$\begin{aligned}
\text{Again } Cp &= pw - Cw = p'w' - Cw \\
&= p'D - w'D - Cw \\
&= a \cos.\theta - (c - b) \sin.\theta - b \sin.\theta \\
&= a \cos.\theta - c \sin.\theta.
\end{aligned}$$

Also $Cw = b \sin.\theta$.

Substituting these values of Cq, Cp, and Cw in the equation $W \cdot Cw + Q \cdot Cq = P \cdot Cp$, we have

$$W \cdot b \sin.\theta + Q(c \sin.\theta + a \cos.\theta) = P(a \cos.\theta - c \sin.\theta).$$

Dividing both sides by $\cos.\theta$

$$W \cdot b \tan.\theta + Q(c \tan.\theta + a) = P(a - c \tan.\theta);$$

$$\text{Or} \quad (Wb + Qc + Pc) \tan.\theta = (P - Q)a;$$

$$\text{And} \quad \frac{\tan.\theta}{P - Q} = \frac{a}{W \cdot b + (P + Q)c}.$$

Now the *sensibility* of the balance for a given difference of P and Q will be greater, the greater the angle θ; also for a given value of θ, the sensibility will be greater the smaller the difference between P and Q. Since $\tan.\theta$ is greater, the greater the angle θ, it follows that $\frac{\tan.\theta}{P - Q}$ is a measure of the sensibility of the balance.

Hence the greatest sensibility will be attained when $\frac{a}{Wb + (P + Q)c}$ is as great, or $\frac{Wb + (P + Q)c}{a}$ as small as possible.

In making this calculation we have neglected the friction of the edge of the fulcrum on its supports, which will have a tendency to diminish the sensibility.

In instruments where great accuracy is required, such as chemical, philosophical, and assay balances, this friction is diminished as much as possible by making the knife-edge of the fulcrum of hard polished steel, and the support on which it rests a plane of polished agate. Having thus obviated the diminution of sensibility due to the friction, it appears from the above expression for the sensibility of the balance, that it will be greater, the greater a is, and the less W, b, and c are.

We must, therefore, make the beam as light and as long as we possibly can, and the distances of G and D from C as small as may be consistent with other conditions.

The scale-pans and the cords, or apparatus by which they are suspended, must also be made as light as can be, compatible with the uses of the balance, as they increase the values of P and Q, whose sum is an element which diminishes the sensibility.

When equal weights act on A and B, the extremities of the beam, its stability is measured by its tendency to resume a state of rest, after its equilibrium has been disturbed. This tendency will depend upon the magnitude of the sum of the moments of the forces, about the point C, for any given position of the beam; *i.e.*, for any particular value of θ.

The sum of the moments of the three forces P, Q, and W about C, when A B is inclined at an angle θ to the horizontal, or C D at the same angle to the vertical, will be

$$W \cdot Cw + Q \cdot Cq - P \cdot Cp;$$

$$\text{or } Wb \sin.\theta + Q(c \sin.\theta + a \cos.\theta) - P(a \cos.\theta - c \sin.\theta).$$

When P and Q are equal, this expression becomes

$$Wb \sin.\theta + Pc \sin.\theta + Pc \sin.\theta;$$

$$\text{or } (W \cdot b + 2P \cdot c) \sin.\theta.$$

When θ is zero, sin. θ is also zero, and this expression becomes the same, or the sum of the moments is zero, which it must be, in order that there should be equilibrium. But for any other finite value of θ, the magnitude of the *stability* will be greater, as $W \cdot b + 2P \cdot c$ is greater; that is, as W, P, b and c are greater.

These conditions are contrary to those required for *sensibility*, which for any given value of a, demand that W, P, b and c shall be as small as possible.

Hence, while the arm of the balance remains of the same length, any increase of *sensibility* is made at the expense of its *stability*; but the quantities b, c, W and P remaining the same, we may increase the *sensibility* without injury to the *stability* by increasing a, or the length of the arm of the balance.

Where minute differences of weight are not matters of importance, and quickness of determination essential, as in weighing substances rapidly, which are not of great value in proportion to their weight, *stability* is of more importance than *sensibility*. The reverse, however, is the case where the substance is of great value in proportion to its weight, or where extreme accuracy is required.

In all that has been said, it must be remembered that the weight P includes that of the scale-pan, together with the apparatus by which it is suspended.

The scale-pan must be so suspended, that its centre of gravity, as well as that of the weight or substance placed in it, may be exactly in the vertical line passing through the extremity of the beam, from which it is suspended. From the mathematical expressions given above for *sensibility* and *stability*, both depending upon P, it follows that the same balance will have different degrees of these qualities according as it is used for determining greater or less weights. The quantities of substances which are used in philosophical or chemical balances being generally very small, a balance which is very *sensible* when no weight is placed in the scales, will be of almost equal *sensibility* for every weight with which it is intended to be used.

A needle is usually fixed to the beam in the direction of the line C G, which points vertically upwards or downwards when the beam is in a horizontal position. A graduated arc or scale attached to the support of the balance to indicate the arcs described by the extremity of this needle, as the beam oscillates, is a very convenient addition. When G is very near C, the oscillations will be very slow; in this case, the equality of the weights in the scale-pans may be ascertained by means of the index-needle, before the balance comes to a state of rest. When the weights are equal, the extremity of the needle will describe equal arcs on both sides of the vertical line; when they are unequal, the scale which preponderates will be indicated by a greater arc being described on that side than the other. This method renders *stability* of much less importance than sensibility.

Where great accuracy is required, the disuse of this long index-needle, almost in contact with the graduated arc, and the substitution in its place of a graduated arc attached to one end of the beam has been recommended. This arc is viewed through a fixed compound microscope, having a horizontal wire in the focus of the eye-piece; or by a mirror attached to the beam, in which the reflected image of a scale is viewed through a telescope.

In very accurate instruments the scale-pans are suspended from steel knife-edges, resting on agate planes fixed to the extremities of the beam. The balance should be constructed as much as possible of brass, as steel and iron are apt to acquire magnetic properties. Palladium has been used for the construction of the beam, and platinum is to be preferred for the scale-pans. When glass scale-pans are used, care must be taken not to excite their electrical properties; a difference of half a grain may be produced by merely cleaning one of the glass scales with a dry silk handkerchief. In constructing the beam, care must be taken by making it hollow, or if solid, by cutting out portions of it, to secure the greatest degree of lightness with the greatest length of arm which is compatible with its rigidity, or the retention of its form, under all the weights to which it is intended to be subjected.

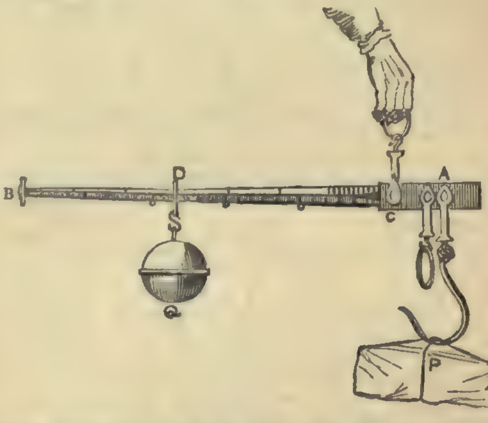
All very sensitive balances have a contrivance by which the knife-edges, which support the beam and scale-pans are lifted from the planes on which they play, whenever the instrument is not in use. The knife-edges are thus preserved from becoming blunt, or wearing the agate planes, and the beam is freed as much as possible from every strain, which would tend to alter its shape.

In addition to all these precautions, the balance should be enclosed in a glass case, for its preservation from dust and injury: to prevent error from the action of currents of air, a window at the side affords the means of introducing weights and the substance to be weighed, without removing the case. A cup containing quick-lime, or some other powerful absorbent of moisture, should also be kept within the case.

Ramsden constructed a balance for the Royal Society of such extreme *sensibility*, that when weighted with 10 pounds, it turned with about the thousandth part of a grain.

From the above description, it is manifest, that the construction of a perfect balance may be regarded as impossible, though one may be nearer than another to perfection. Borda invented a very simple method, by means of which very accurate results may be obtained by a balance sufficiently *sensitive* and well-constructed on its knife-edges, though the points of suspension of the scale-pans may not be equidistant from its fulcrum.

Fig. 103.



His ingenious device is to weigh the article whose weight is to be determined by weights or any other substance, such as sand, placed in the other scale: when equilibrium has been thus obtained, the article is removed and replaced by weights until equilibrium has been again restored. These latter weights determine that of the article; and thus any error arising from inequality of the arms of the balance is eliminated.

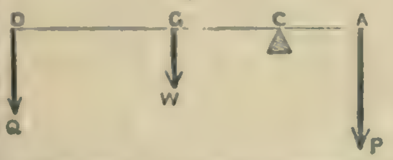
THE ROMAN STEELYARD OR BALANCE.—This balance consists of an iron or steel lever A B, (Fig. 103) with unequal arms A C and B C resting on a fulcrum with a knife-edge at C, which plays on a pivot passing through a support held or suspended by a ring. The substance P whose weight is to be determined, is suspended from the point A by a hook, while a constant weight Q, called the *counterpoise*, is attached to a ring D, which can be slipped along the longer arm C B. The edge of the arm C B is so graduated that the point D at which the counterpoise Q suspended from it balances P, determines the weight of the latter in pounds and ounces, or any other convenient denomination of weight for which the steelyard may have been graduated.

The steelyard is sometimes furnished with a second fulcrum and apparatus for its suspension, nearer to A than C; the position of the lever is then inverted, and another graduation on what was before the under edge of the arm B C, gives the weight of P for the new position of the fulcrum. The same instrument has thus two different ranges of weights for the same counterpoise, the fulcrum nearer to A being used for substances lying within a range of weights greater than that for which the other is graduated.

To Graduate the Roman Steelyard.—The Roman steelyard may be regarded as a heavy lever with unequal arms resting on a fulcrum C (Fig. 104).

Let W be the weight of the lever and the hook or scale-pan by which P is suspended, G the position of their joint centre of gravity, Q the weight of the counterpoise, together with the ring and chain by which it is suspended.

Fig. 104.



The steelyard may now be considered as a lever without weight, whose fulcrum is C, acted on by three parallel and vertical forces, P, W and Q, at the points A, G, and D, represented in magnitude and direction by the lines A P, G W, and D Q.

When these forces produce equilibrium, taking moments about the point C, we have

$$P \cdot A C = W \cdot G C + Q \cdot C D.$$

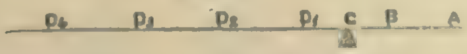
Hence, transposing, we have

$$Q \cdot C D = P \cdot A C - W \cdot G C$$

$$\text{or } C D = \frac{P}{Q} A C - \frac{W}{Q} G C.$$

Now let D_1, D_2, D_3, D_4, &c. (Fig. 105), represent the distances from C at which weights of one, two, three, four, &c., pounds, or any other unit of weight for which the steelyard is to be graduated, suspended from A are respectively balanced by the counterpoise.

Fig. 105.



Substituting, therefore, for P the numbers 1, 2, 3, 4, &c., and D_1, D_2, D_3, D_4, &c., for D in the above equation, we have

$$C D_1 = \frac{A C}{Q} - \frac{W}{Q} G C$$

$$C D_2 = 2 \frac{A C}{Q} - \frac{W}{Q} G C$$

$$C D_3 = 3 \frac{A C}{Q} - \frac{W}{Q} G C$$

$$C D_4 = 4 \frac{A C}{Q} - \frac{W}{Q} G C, \text{ \&c.}$$

In A C take a point B such that $B C = \frac{W}{Q} G C$, and make $B D_1 = \frac{A C}{Q}$, where Q represents the weight of the counterpoise, in the same unit which is chosen for the graduation, then make $D_1 D_2 = B D_1$, $D_2 D_3 = B D_1$, $D_3 D_4 = B D_1$, &c. D_1, D_2, D_3, D_4, &c., will mark the points at which the counterpoise Q will balance weights of 1, 2, 3, 4, &c., pounds suspended from A.

For

$$C D_1 = B D_1 - B C = \frac{A C}{Q} - \frac{W}{Q} G C.$$

$$C D_2 = B D_2 - B C = 2 B D_1 - B C = 2 \frac{A C}{Q} - \frac{W}{Q} G C.$$

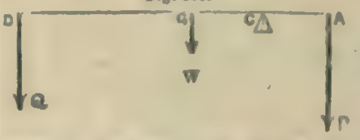
$$C D_3 = B D_3 - B C = 3 B D_1 - B C = 3 \frac{A C}{Q} - \frac{W}{Q} G C.$$

$$C D_4 = B D_4 - B C = 4 B D_1 - B C = 4 \frac{A C}{Q} - \frac{W}{Q} G C.$$

When the instrument is so constructed that the centre of gravity G of the steelyard and scale, or hook, lies in the vertical line passing through C, the quantity B C or $\frac{W}{Q} G C$ becomes zero, and the points of graduation are taken at equal intervals from C.

THE DANISH STEELYARD OR BALANCE.—This instrument differs from the Roman steelyard in these respects, that the counterpoise is fixed at one of the extremities, while the fulcrum is movable. The edge of the steelyard is graduated, and the point at which the fulcrum is placed to cause the fixed counterpoise to balance the substance whose weight is to be determined, marks its value.

Fig. 106.



To Graduate the Danish Steelyard.—Let P, W, Q, and the straight lines P A, G W, and D Q (Fig. 106), represent the same forces in this case, as in that of the Roman steelyard; in this instance, however, it is C and not D which is the movable point.

Then taking moments of the forces as before, about the point C, we have

$$\begin{aligned} P \cdot A C &= W \cdot G C + Q \cdot D C \\ &= W (A G - A C) + Q (A D - A C). \end{aligned}$$

By transposition $(P + W + Q) A C = W \cdot A G + Q \cdot A D$;

$$\text{or } A C = \frac{W \cdot A G + Q \cdot A D}{P + W + Q}.$$

Let C_1, C_2, C_3, C_4, &c., represent the points at which 1, 2, 3, 4, &c., pounds will be balanced by the counterpoise when the fulcrum is placed under these points. Hence substituting C_1, C_2, C_3, C_4, for C, and 1, 2, 3, 4, &c., for P in the preceding expression, we have

$$A C_1 = \frac{W \cdot A G + Q \cdot A D}{1 + W + G}$$

$$A C_2 = \frac{W \cdot A G + Q \cdot A D}{2 + W + G}$$

$$A C_3 = \frac{W \cdot A G + Q \cdot A D}{3 + W + G}$$

$$A C_4 = \frac{W \cdot A G + Q \cdot A D}{4 + W + G}.$$

The reciprocals of these quantities are in arithmetical progression, and therefore the distances of the points C_1, C_2, C_3, C_4, &c., from A will form an harmonical progression.

THE BALANCE OF QUINTENZ.—This balance, named after its inventor Quintenz, is frequently employed on railways for determining the weight of luggage, and affords a good example of a combination of levers. It is represented in Fig. 108. Fig. 107 is added for the purpose of showing its mechanism more clearly. A platform A B on which the body Q, which we wish to be weighed, is placed, is fixed firmly to an upright piece of framework B C. B C again is attached rigidly to an oblique piece D C fixed in D, as shown in Fig. 107, so that the whole portion A B C D forms one rigid and inflexible body. This rigid piece A B C D is supported at E by a knife-edge fulcrum, resting on the bar or lever F G, and at D by a rod H K, which is firmly fixed to D, and suspended by a hook at K from the lever L M N.

Fig. 107.

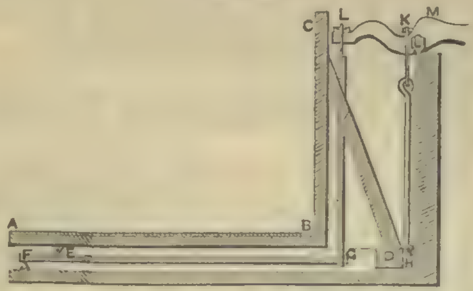
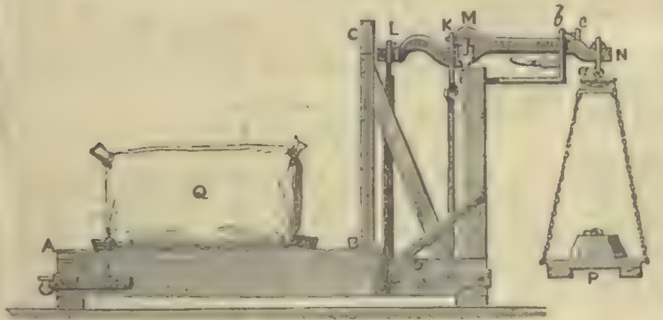


Fig. 108.



The whole weight, therefore, of Q and the framework A B C D rests on the points E and K.

The lever F G is supported by a fulcrum at one extremity F, while the other extremity G is suspended by a rod L G, hanging at L from the extremity of the lever L M N.

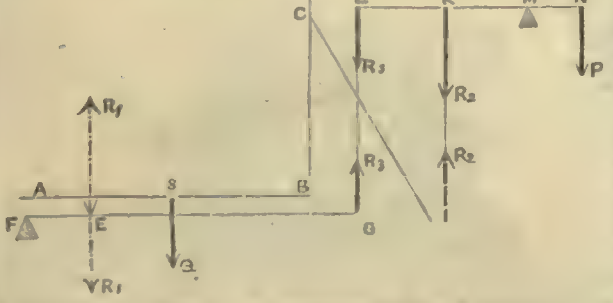
Lastly, the lever L M N is supported by a fulcrum M, which is fixed to the framework of the machine; the extremity N of this lever has a scale-pan suspended from it, in which weights may be placed.

The points E, L, and K are so chosen that $\frac{ML}{MK} = \frac{FG}{FE}$.

To avoid the consideration of the weights of the various parts of this machine, we will suppose that the weight of the scale-pan and the length of the arm M N of the lever L M N have been so chosen, as to produce an equilibrium of all the parts of the machine, and to keep L M N in a perfectly horizontal position, when no weight is placed on the platform A B.

To determine the weight P, which must be placed in the scale-pan suspended from N to balance a substance of a given weight placed on the platform A B, we will examine the conditions of equilibrium for the several parts of the machine.

Fig. 109.



Let the weights P and Q be represented in magnitude and direction by N P and S Q (Fig. 109.)

The weight of the substance placed on the platform A B being equal to a single force S Q passing through the vertical which passes through its centre of gravity, will be balanced by the reaction of the pressure R_1 which it produces on the fulcrum E, represented in magnitude and direction by ER_1, and the reaction R_2 of the tension it produces on the rod H K.

The pressure R_1 of the fulcrum E on the lever F G is kept in equilibrium by the reaction of the pressure it exerts on the fulcrum F, and the reaction R_3 of the tension it produces on the rod G L.

Finally, the lever L M N, resting on the fulcrum M, is acted on by three vertical forces P, R_2 and R_3, acting on the points N, K and L.

For the equilibrium of the rigid body A B C H we have as a condition

$$Q = R_1 + R_2.$$

For the lever F E G taking moments about E we have

$$R_1 \cdot EF = R_3 \cdot FG.$$

$$\text{Or } R_1 = R_3 \cdot \frac{FG}{EF}.$$

Lastly, for the lever L M N taking moments about M we have

$$P \cdot MN = R_2 \cdot MK + R_3 \cdot LM.$$

$$\text{Or } P \cdot \frac{MN}{MK} = R_2 + R_3 \cdot \frac{LM}{MK}.$$

But $\frac{LM}{MK} = \frac{FG}{EF}$ by the construction of the machine.

$$\text{Hence } P \cdot \frac{MN}{MK} = R_2 + R_3 \cdot \frac{FG}{EF} =$$

$$R_2 + R_1 = Q.$$

$$\text{And } \frac{P}{Q} = \frac{MK}{MN}.$$

If M N be taken ten times as long as M K, we have M N = 10 M K and Q = 10 P; and in this case a weight the tenth part of Q, when placed in the scale-pan, will produce equilibrium.

Instead of placing weights in the scale-pan P (Fig. 108), the arm of the lever M N may be graduated, and used as a Roman steelyard.

Before placing a body on the platform A B (Fig. 108) whose weight is to be determined, it is necessary to observe whether the arm of the lever M N is in a perfectly horizontal position. This is indicated by a horizontal pin *b* fixed to the framework of the instrument, being in the same line with a similar pin fixed to the arm of the lever. To bring these points into this position, small weights are added to or taken from a little cup *a*, placed under the point from which the scale-pan is suspended.

ROBERVAL'S BALANCE.—Many of the balances used in shops are constructed on the principle of this machine, which is interesting for its paradoxical character. It is apparently a lever, on the arms of which if two weights balance each other, they will still continue to do so from whatever points in those arms they may be suspended. The accompanying diagram will give an idea of the construction of this machine.

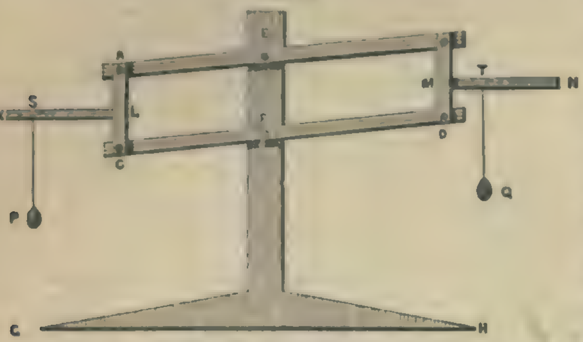
It consists of four bars, A B, C D, A C, and B D (Fig. 110); A B being equal to C D, and A C to B D. These four bars are united together by four pivots, A, B, C, and D; they also rest upon two pivots or axes E and F fixed to an upright bar E F, resting on a firm base G H.

The holes for the pivots E and F are so placed in A B and C D that B E is equal to D F.

Lastly, two bars K L and M N are fixed firmly at right angles to A C and B D, so that K L and A C form one rigid piece and M N and B D another.

The six pivots A, B, C, D, E, and F, are so constructed that the bars may move freely about them, with as little friction as possible, in the vertical plane while no lateral motion is permitted. From this construction it is evident that whatever position the framework A B C D may assume, when weights are suspended from the arms K L and M N, the four bars A B, C D, A C, and B D, will always form a parallelogram, and the arms K L and M N retain a horizontal position.

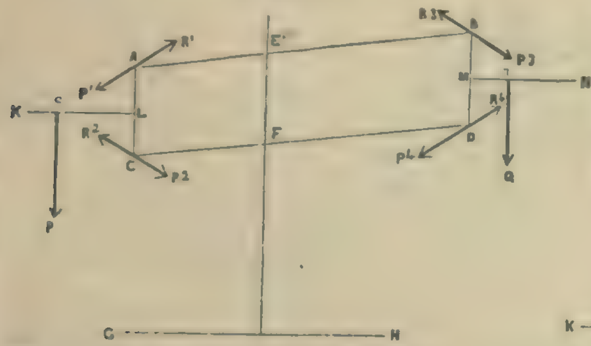
Fig. 110.



It is a peculiar property of this machine, that if two weights P and Q balance each other when suspended from two points S and T in K L and M N, they will also balance from whatever points in K L and M N they are suspended.

To show the properties on which this peculiar statical paradox depends, we shall neglect the weights of the various parts of the machine: or, which will come to the same thing, suppose the weights of its various parts so chosen that it shall be in equilibrium for every position in which the framework A B C D can be placed, when no weights are suspended from the arms K L and M N.

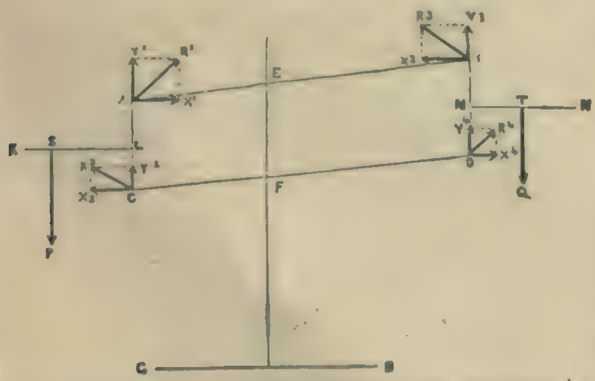
Fig. 111.



We also neglect the friction of the six pivots A, B, C, D, E, and F.

Let two weights P and Q, represented in magnitude and direction by S P and T Q (Fig. 111), be suspended from the arms K L and M N at the points S and T, and let us suppose that they balance each other

Fig. 112.



Also let the distances A E and C F be represented by a, E B and F D by b. Since K L and A C are rigidly connected, the pressure of P on S will be transmitted by the bars K L and A C to the pivots A and C, where it will produce two pressures P_1 and P_2.

These pressures are indeterminate, both in magnitude and direction.

Let P_1, the unknown pressure on A, be represented in magnitude and direction by AP_1; and P_2 that on C by CP_2.

As the machine is supposed to be in a state of equilibrium, P_1 will be counteracted by the reaction R_1 acting in the direction AR_1 (Fig. 112), AR_1 and AP_1 being in the same line, and $R_1 = P_1$.

Similarly P_2 will be counteracted by the equal and opposite reaction R^2, represented in magnitude and direction by CR_2.

Again, because the bars B D and M N are rigidly connected, the pressure of Q on T will be transmitted, and produce two pressures, P_3 and P_4, at B and D, which will be counteracted by equal and opposite reactions R_3 and R_4.

The bars K L and A C are kept in equilibrium by the three forces P, R_1, and R_2, acting at S, A and C in the directions S P, AR_1 and CR_2.

Resolving the forces represented by AR_1 and CR_2 into the vertical and horizontal forces X_1, Y_1, and X_2, Y_2, represented by AX_1, AY_1, CY_2, and CX_2 (Fig. 112).

We shall have by Prop. XVIII.

$$P = Y_1 + Y_2 \text{ and } X_1 = X_2.$$

The couple $X_1 \cdot AC$, whose tendency is to twist the rod A C, is entirely destroyed by the reaction of the pivots at A and C, and the forces Y_1 and Y_2 will be equal and opposite to two forces which will exert a pressure on the extremities A and C of the levers A B and C D.

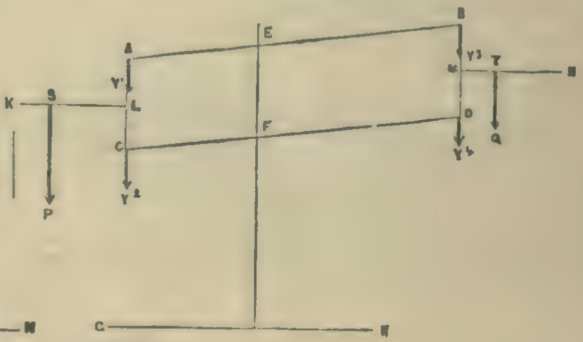
In a similar manner, by resolving the forces R_3 and R_4 into the horizontal and vertical forces X_3, Y_3, X_4, and Y_4, we shall have

$$Q = Y_3 + Y_4 \text{ and } X_3 = X_4.$$

The forces Y_3 and Y_4 being equal and opposite to two vertical pressures acting on the extremities B and D of the levers A B and C D.

Finally, we have the lever A B resting on the fulcrum E, kept in equilibrium by the forces Y_1 and Y_3 (Fig. 113).

Fig. 113.



Hence $Y_1 \cdot a = Y_3 \cdot b$.

Also for the lever C D we have

$$Y_2 \cdot a = Y_4 \cdot b.$$

Hence $(Y_1 + Y_2)a = (Y_3 + Y_4)b$;

But $Y_1 + Y_2 = P$, and $Y_3 + Y_4 = Q$;

Therefore $P \cdot a = Q \cdot b$.

Provided therefore that the pivots are strong enough to resist the lateral strains acting on them, the condition of equilibrium for the machine is, that P multiplied by a shall be equal to Q multiplied by b: a result entirely independent of the quantities c and d, or of the distances S L and M T.

In constructing balances on this principle, a and b are taken equal to each other, in which case E and F bisect A B and C D, and P and Q are equal.

WHEEL AND AXLE.—The second mechanical power is the wheel and axle; this machine, in its simplest form, consists of a circular wheel A B (Fig. 114) firmly fixed at right angles to a cylinder C D E F, so that both revolve together round a common axis G H.

The forces P and W are supposed to be applied by weights suspended from one extremity of the cord wrapped round the wheel or cylinder, to which the other extremity is attached. The forces, by this contrivance, always act at right angles to the circumferences of the wheel and cylinder.

The cords are supposed to be perfectly flexible, inextensible, and destitute of weight. Their friction on the surface of the wheel and axle, as well as the magnitude of their diameters, is also neglected.

Fig. 114.

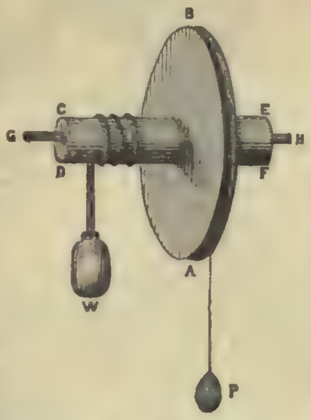


Fig. 115 represents a section of the wheel and cylinder perpendicular to their common axis.

Let A B be the radius of the wheel, and A C that of the cylinder or axle. Since the forces P and Q always act at right angles to the circumference of the wheel and axle, the force P may be represented in magnitude and direction by B P at right angles to A B, the force W by C W at right angles to A C.

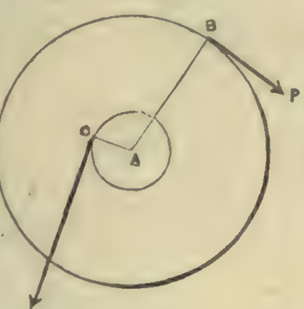
When these forces produce equilibrium, taking moments about A, we have

$$P \cdot AB = W \cdot AC \text{ or } \frac{P}{W} = \frac{AC}{AB},$$

that is

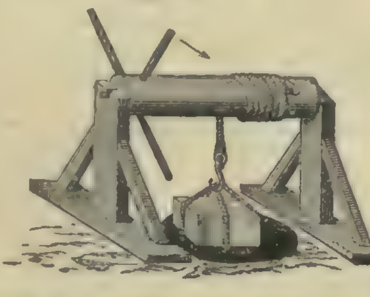
$$\frac{\text{The force acting on the surface of the wheel}}{\text{The force acting on the surface of the axle}} = \frac{\text{radius of axle}}{\text{radius of wheel}}.$$

Fig. 115.



This machine is only a modification of the lever; for referring to Fig. 115, we may consider it in its position of equilibrium as a bent lever, whose arms are A B and A C, and whose fulcrum is A.

Fig. 116.

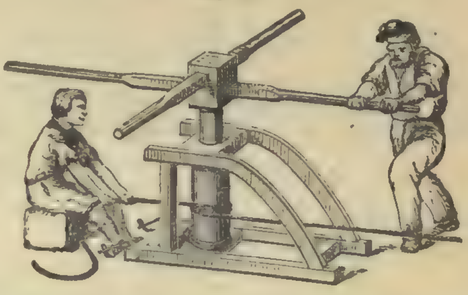


When it is in motion, we may regard the wheel and axle as made up of a number of spokes; these spokes come successively into action as levers, and thus the advantage of an endless leverage is produced, and the power and weight each act constantly in a straight line, instead of describing circular arcs as in the common lever.

Instead of the wheel, one or more bars are sometimes fixed at right angles to the axle, and these are often so disposed as to allow several men to act at once on the machine. When the axis of the axle is horizontal, a bar fixed at right angles to the extremity of the bar, forming what is called a winch, affords a convenient means for applying the force.

When the axis is horizontal, as in Fig. 116, the machine is called a *windlass;* when vertical, as in Fig. 117, a *capstan.*

Fig. 117.

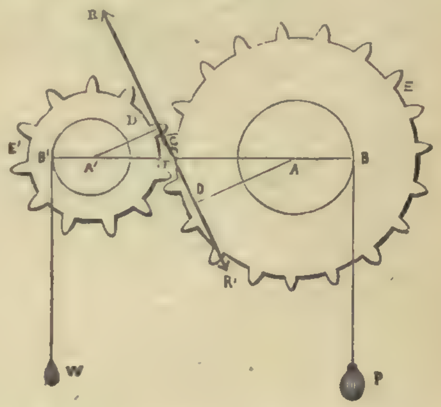


TOOTHED WHEELS.—The third mechanical power is the *toothed wheel;* it is extensively used in the construction of cranes, clock and watch-work, and almost every variety of machinery.

Toothed wheels consist of thin cylinders, having their circumferences indented or covered with projections called *teeth* or *cogs,* set at equal distances from each other.

If two such wheels have their axes placed in such a position that the surfaces of the wheels may be in the same plane, and the edges of their teeth touch each other, as in Fig. 118, and one of the wheels be made to revolve round its axis, its teeth will act in succession on the teeth of the other, and cause it to revolve round its axis in an opposite direction.

Fig. 118.



To determine the conditions of equilibrium for this machine, we will suppose A and A′ (Fig. 118) to be the axes about which the wheels revolve, and the weights P and W, which produce equilibrium, to be attached to the extremities of cords wrapped round, and having their other extremities fastened to, cylinders fixed perpendicularly to the wheels, and having a common axis with them.

Let A B and A′ B′ be the radii of these cylinders.

The weight P will communicate a tension to the rope B P, which will produce a pressure on the cylinder A B; this pressure will be communicated from the cylinder to the point C, where the tooth of the wheel C E is in contact with the tooth of the wheel C E′. In a similar manner, the pressure produced by W on the cylinder A′ B′ will be communicated to the same point.

These two pressures at C will act perpendicularly to the surfaces in contact, and when there is equilibrium they will destroy each other; consequently, they will be equal and opposite to each other. Let these pressures be represented by R, and their magnitudes and direction by C R and C R′.

From A and A′ draw A D and A D′ at right angles to C R, and join A A′ cutting C R in F. The wheel A E is kept in equilibrium by the force P acting in the direction B P, and R in the directions C R; and the wheel C E′ by the force W in the direction B′ W and R in the direction C R′.

Hence taking moments about A, we have $P \cdot AB = R' \cdot AD$
And taking moments about A′, we have $W \cdot A'B' = R \cdot A'D'$

Dividing one equation by the other $\frac{P \cdot AB}{W \cdot A'B'} = \frac{AD}{A'D'}$ or $\frac{P}{W}$

$$= \frac{AD}{A'D'} \cdot \frac{A'B'}{AB}$$

In the two triangles A D F, A′D′F, the angles at D and D′ are right angles, and the angle D F A = angle D F A′; hence the triangles are similar, and therefore

$$\frac{AD}{A'D'} = \frac{AF}{A'F}$$

and substituting this value we have

$$\frac{P}{W} = \frac{AF}{A'F} \cdot \frac{A'B'}{AB}$$

If we make A B = A′B′, the effect of the combination will depend upon the teeth of the wheels and their radii only, and then

$$\frac{P}{W} = \frac{AF}{A'F}$$

When the teeth are small in comparison with the radii of the wheels, A F and A′F will be nearly equal to these radii. And since the intervals between the teeth must be equal, in order that the wheels may work through a whole revolution, the number of teeth in each wheel will be proportional to their respective circumferences. Hence in this case

$$\frac{P}{W} = \frac{\text{radius of wheel } CE}{\text{radius of wheel } CE'} = \frac{\text{circumference of } CE}{\text{circumference of } CE'}$$
$$= \frac{\text{number of teeth in } CE}{\text{number of teeth in } CE'}$$

The edges of the teeth which come in contact with each other are sometimes formed of curves, which are portions of the curve called the involute of a circle. In this case, the point C retains the same position throughout the whole revolution of the wheel. The discussion of this property, as well as the best form of the teeth, requires a greater knowledge of the higher branches of geometry than can be assumed in an elementary work.

When the number of teeth in a wheel is small, the wheel is called a *pinion*, and the teeth *leaves*.

The axles about which the wheels revolve may be placed at right angles to each other, as in Figs. 119 and 120, or inclined at any angle to each other, as in Fig. 121.

Fig. 119.

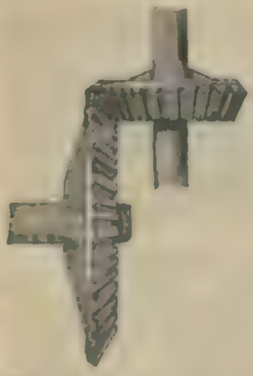
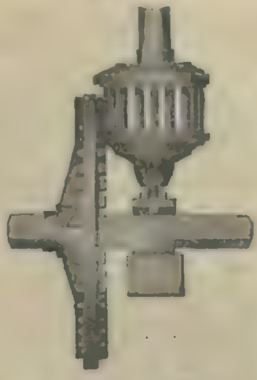


Fig. 120.



In Figs. 119 and 121 the pinions are placed on the surfaces of frustrums of cones, whose axes coincide with those of the wheels: the wheels are then called *bevelled wheels.* When the teeth project from the edges of the wheel, it is called a *spur* wheel; when they project from the surface of the wheel, as in Fig. 120, it is called a *crown* wheel.

Fig. 121.

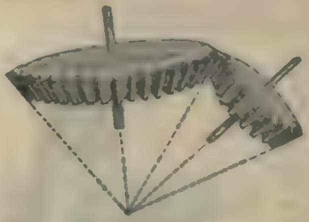
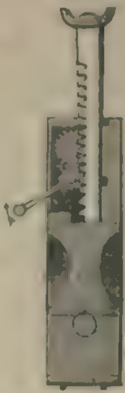


Fig. 122.



The teeth in which those of one of the wheels work, may be placed along the edge of a straight bar instead of the surface of a cylinder, as in Figs. 122 and 123; the bar being so confined as to allow it only to move in the direction of its length.

Fig. 123.

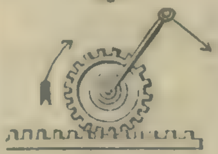
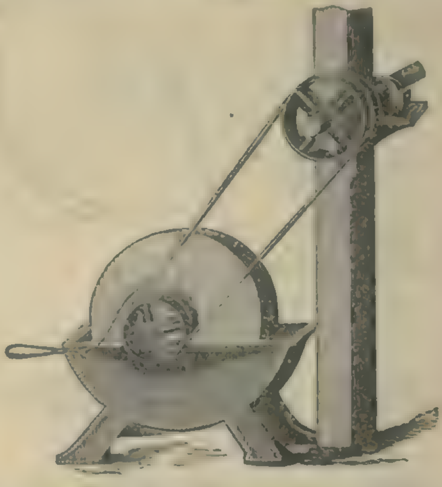
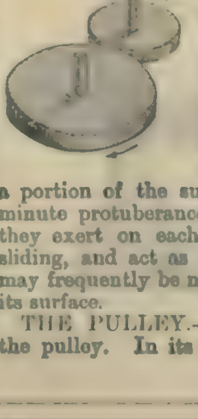


Fig. 124.



The arrangement in Fig. 122, where the wheel is made to revolve by a winch, is called a *jack*, and is often employed for raising heavy weights a small height.

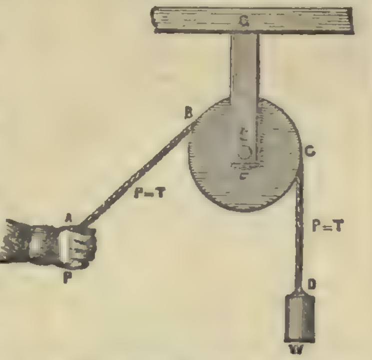
Fig. 125.



Wheels are sometimes turned by simple contact with each other, as in Fig. 125; and when they are at a distance from one another, as in Fig. 124, they may be made to act on each other by a band, strap, or chain passing over and in close contact with a portion of the surfaces of both. In these cases the minute protuberances of the surfaces, and the friction they exert on each other, prevent the surfaces from sliding, and act as minute teeth. A band which slips, may frequently be made to act by chalking or rosining its surface.

THE PULLEY.—The fourth mechanical power is the pulley. In its simplest form it consists of a solid circular wheel B C, Fig. 126, having a smooth groove cut in its circumference. This wheel revolves freely round an axis or pivot passing through its centre at right angles to its surface, and having its extremities fixed to a wooden or metallic frame F G. This frame may be either fixed or movable. The wheel is called the *sheaf*, *shiver*, *or rundle;* the axis the *gudgeon;* and the frame in which the axis is fixed, the *block* of the pulley.

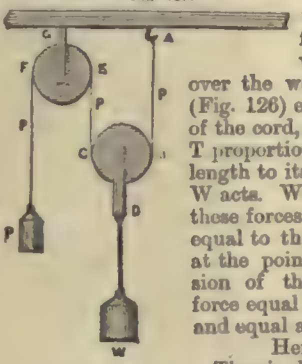
Fig. 126.



The force P is applied to one extremity of a cord passing freely over the grooved circumference of the sheaf, which moves with it round its axis, thus diminishing the friction of the cord; and the weight W, which is to be overcome, is attached to the other extremity of the cord

Several pulleys may be combined together, forming what is called a *system.* The cords which pass over the sheaves are supposed to be perfectly flexible and inextensible; the friction of the cords, as well as that of the axis, and when not specially mentioned, the weights of the blocks and sheaves, are not considered in our calculations.

Fig. 127.



The single pulley, with its block fixed, affords no mechanical advantage of the power over the weight, for the power P (Fig. 126) exerted on the extremity of the cord, communicates a tension T proportional to P throughout its length to its extremity D, at which W acts. When there is equilibrium, these forces P and W must be both equal to the tension T of the cord at the points A and D, or the tension of the cord will produce a force equal and opposite to P at A, and equal and opposite to W at D.

Hence P = W = T.

The single fixed pulley affords a convenient means for altering the direction of the application of a force to a machine. Thus a weight which acts vertically downwards, may be made to exert an equal force in any direction we require, by a proper position of the pulley and the support to which its block is fixed.

Single Movable Pulley.—Let a weight P be attached to one extremity of a cord passing over the fixed pulley F E (Fig. 127) round the movable pulley C B, and having its other extremity fastened to a fixed beam at A, the block of the pulley F E being fixed to the beam at G in such a manner that E C and A B shall be parallel. The weight W is attached to the block D of the pulley C B. Neglecting the weight of the pulley C B, P will communicate a tension throughout the cord, and this tension will produce a force P acting in the direction C E at E, and another force P acting in the direction B A at B (Fig. 127).

Hence when there is equilibrium, we shall have

$$W = P + P = 2P.$$

When the weight of the pulley is taken into account, this weight must be added to W. When the cords are not parallel, resolving the forces P acting at C, and P acting at B, into vertical and horizontal forces, as shown in Fig. 128, and equating the horizontal and vertical forces acting on B C, we have

$$P \sin. \phi = P \sin. \theta, \text{ hence } \phi = \theta$$

and W = P cos. θ + P cos φ = P cos. θ + P cos. θ = 2P cos. θ.

Where θ equals half the angle A B produced, makes with E C produced.

Fig. 128.

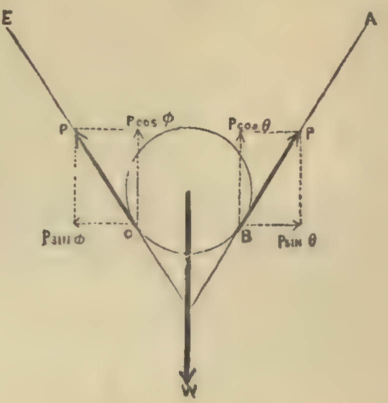
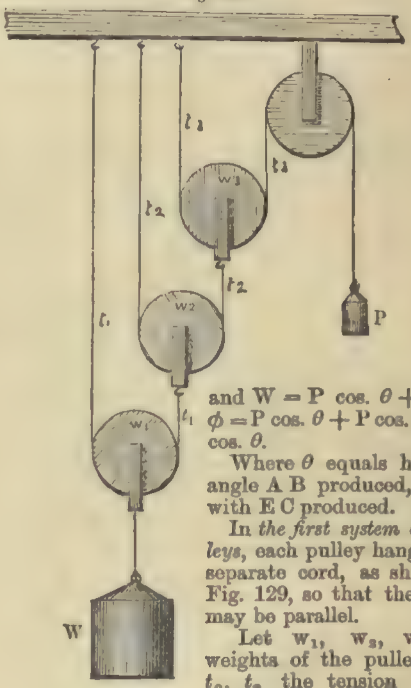


Fig. 129.



In *the first system of pulleys*, each pulley hangs by a separate cord, as shown in Fig. 129, so that the cords may be parallel.

Let w_1, w_2, w_3, be weights of the pulleys, t_1, t_2, t_3 the tension of the cords.

$W + w_1 = 2t_1$, $t_1 + w_2 = 2t_2$, $t_2 + w_3 = 2t_3$, and $t_3 = P$.

$$\text{Hence } P = \frac{w_3}{2} + \frac{t_2}{2} = \frac{w_3}{2} + \frac{w_2}{2^2} + \frac{t_1}{2^2} = \frac{w_3}{2} + \frac{w_2}{2^2} + \frac{w_1}{2^3} - \frac{W}{2^3}$$

If there had been four pulleys we should have had

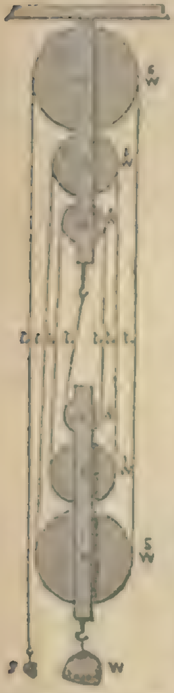
$$P = \frac{w_4}{2} + \frac{w_3}{2^4} + \frac{w_2}{2^3} + \frac{w_1}{2} = \frac{W}{2^4}$$

Neglecting the weights of the pulleys, w_1, w_2, &c., for three pulleys we shall have $P = \frac{W}{2^3}$, for four $P = \frac{W}{2^4}$ and generally for n pulleys $P = \frac{W}{2^n}$.

When the cords are not parallel, we may see, from the case of the single pulley, that each tension, t_1, t_2, t_3, &c., must be multiplied by the cosine of the angle which its cord makes with a line drawn parallel to the direction in which W acts.

This multiplication will necessarily diminish the tension, as the cosine will always be a fraction less than unity.

Fig. 130.



In *the second system of pulleys*, the cord to which P is attached passes over all the pulleys, as shown in Fig. 130. The pulleys are divided into two groups, the upper working in a common block which is fixed, and the lower in a common movable block. The other extremity of the cord to which P is attached, after passing over all the sheaves, is fixed to the upper block. W is attached to the lower block.

The weight or force P will communicate the same tension t_1 throughout the rope.

It is evident, therefore, that if we consider the ropes as parallel, and neglect the weight of the lower block and its sheaves when there is equilibrium, we shall have

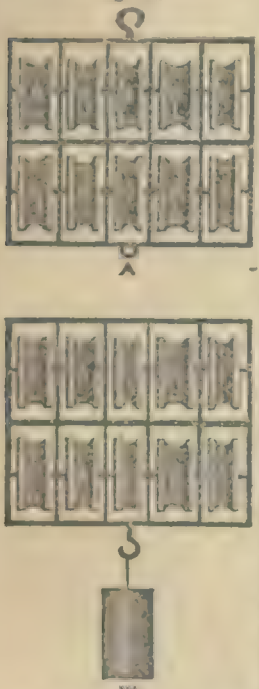
$$W = 6P$$

and generally we shall have $W = nP$ where n is the number of the portions into which the cord is divided by the two blocks.

When the weights of the sheaves and blocks are taken into consideration, it is evident that the weight of the lower block and its sheaves must be added to W.

If the positions of the cords be not parallel, each portion must have its tension multiplied by the cosine of the angle to which it is inclined to the direction of W, as in the first system of pulleys.

Fig. 131.



In Fig. 131 we have represented a very powerful arrangement, having the pulleys in each block arranged as shown in the figure. The sheaves of the upper row in the upper block, as well as those of the lower row of the lower block, are all of the same diameter; those of the lower row of the upper, and upper row of the lower, are equal to each other in diameter, though less than the former. The cord is omitted for the sake of clearness, but one extremity is fixed to the hook A below the lower block; it then passes in succession under each of the sheaves in the lower block, and over those in the upper in the order shown by the numbers attached to the sheaves in the figure: the power is applied to the extremity of the cord which passes over the pulley marked w_{20}.

As there are twenty portions of the cord between the two blocks, in this case W = 20P.

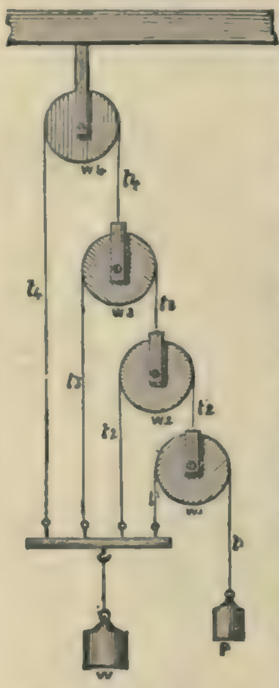
This arrangement permits the portions of the cords to be nearly parallel, and has the additional advantage that the power P acts immediately over the weight W, so that the parallelism of the cords will not be deranged, as it would otherwise be liable to be; it has, however, serious practical defects.

If the weight W be raised by the power P a given height, say for instance one foot, each portion of the cord between the two blocks must be raised the same height, which will necessarily cause P to descend 20 feet. From the arrangement of the cord, it is evident that one foot of the cord will pass over the pulley w_1, two feet over w_2, three feet over w_3, &c., and 20 over w_{20}.

Now since the sheaves w_1, w_2, w_3, &c., are of the same diameter, and revolve each round its own axis, and twice as much cord passes over w_2 as w, three times as much over w_3 as w_1, and so on; it follows that w_2 will revolve twice as fast, w_3 three times as fast, as w_1, and so on for the other sheaves.

This inequality of motion leads to great inequality in the wear of the various parts of the machine. If we attempt to remedy this defect by fixing the sheaves in the same row to a common axis, parts of the cord must necessarily scrape or slide over the grooves of the sheaves instead of moving with them; and this will lead to great increase of friction. To remedy these defects, Mr. White contrived the pulley known by his name. The sheaves in each block were cut out of one piece of wood or metal, being formed of a series of parallel and circular grooves, each increasing in diameter. The diameters of the grooves were made to bear to each other the same proportions as the portions of rope which were to pass over them—those for the lower block being as the odd numbers 1, 3, 5, 7, &c., and those in the upper as the even numbers 2, 4, 6, &c.

Fig. 132.



By this arrangement all inequality of wear was supposed to be obviated, and the friction was reduced nearly to that of the two axles of the blocks. But this ingenious contrivance is found practically to fail, as the elasticity of the cord, which is supposed by the arrangement to be inelastic or inextensible, is subject to continual change by the moisture or dryness of the atmosphere, and by the friction of the cord upon the grooves.

In *the third system of pulleys* each cord passes over a pulley, and has one extremity attached to the weight W, as shown in Fig. 132, and the cords are supposed to be parallel to each other.

Let w_1, w_2, w_3, w_4, represent the weights of the pulleys, t_1, t_2, t_3, t_4, the tension of the cords passing over them.

Then $t^1 = P.$

$t_2 = 2t_1 + w_1 = 2P + w_1.$

$t_3 = 2t_2 + w_2 = 2^2 P + 2w_1 + w_2.$

$t_4 = 2t_3 + w_3 = 2P^3 + 2^2 w_1 + 2w_2 + w_3.$

Therefore $W = t_1 + t_2 + t_3 + t_4.$

$= (1 + 2 + 2^2 + 2^3) P + (1 + 2 + 2^2) w_1 + (1 + 2) w_2 + w_3.$

$= (2^4 - 1) P + (2^3 - 1) w_1 + (2^2 - 1) w_2 + (2 - 1) w_3.$

The same reasoning may be extended to any number of pulleys n, in which case

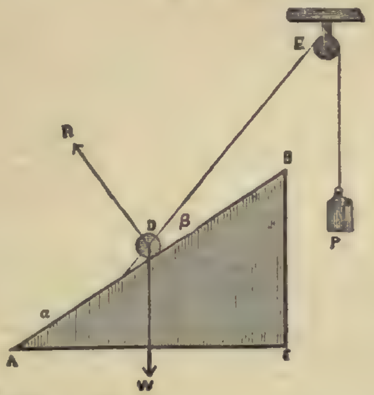
$$W = (2^n - 1) P + (2^{n-1} - 1) w_1 + (2^{n-2} - 1) w_2 + \&c. + (2 - 1) w_n.$$

When the weights of the pulleys are neglected, w_1, w_2, &c., w_n are equal to zero, and $W = (2^n - 1) P$.

When the strings are not parallel, their respective tensions must be multiplied by the cosine of the angle they make with the direction of W, as in the preceding systems.

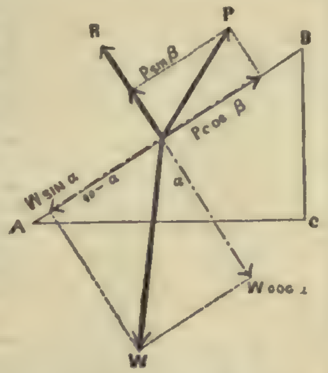
THE INCLINED PLANE.—The inclined plane is the fifth mechanical power. It consists of a plane surface A B, which is supposed to be perfect in hardness and smoothness, and inclined at some angle α to the horizontal line. A heavy body, whose weight is W (Fig. 133—1), resting on the plane, and supported by a force P acting in some direction D E constitutes the weight in this machine.

Fig. 133—(1).



Let β be the angle D E makes with B A, and B C be drawn at right angles to A C; α is called the inclination of the plane, B C its height, A B its length, and A C its base. When there is equilibrium, D will be acted on by three forces, the force P acting in the direction D E, the weight W acting vertically downwards, and the unknown reaction R produced by the pressure of the body D on the plane AB, which acts perpendicularly to the surface of the plane.

Fig. 133—(2).



Let R, P, and W be represented in magnitude and direction by D R, D P, and D W (2).

Resolving these forces parallel and perpendicular to the surface of the plane A B, as shown in Fig. 2, we have the following conditions of equilibrium :—

$$\text{W sin. } \alpha = \text{P cos. } \beta, \text{ or } \frac{\text{W}}{\text{P}} = \frac{\text{cos. } \beta}{\text{sin. } \alpha}.$$

The magnitude of R may be found from the equation

$$\text{R} + \text{P sin. } \beta = \text{W cos. } \alpha;$$

$$\text{or R} = \text{W cos. } \alpha - \text{P sin. } \beta = \text{W cos. } \alpha - \text{W} \frac{\text{sin. } \alpha \text{ sin. } \beta}{\text{cos. } \beta}$$

$$= \text{W} \left(\frac{\text{cos. } \alpha \text{ cos. } \beta - \text{sin. } \alpha \text{ sin. } \beta}{\text{cos. } \beta} \right) = \frac{\text{W cos. } (\alpha + \beta)}{\text{cos. } \beta}.$$

When P acts along the inclined plane in the direction D B, $\beta = 0$, and

$$\frac{\text{W}}{\text{P}} = \frac{1}{\text{sin. } \alpha} = \frac{\text{A B}}{\text{B C}} = \frac{\text{length of the plane}}{\text{height of the plane}};$$

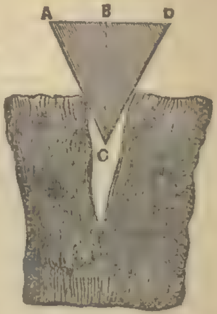
$$\text{also, R} = \text{W cos. } \alpha = \text{W} \frac{\text{A C}}{\text{A B}}$$

$$= \text{W} \times \frac{\text{base of the plane}}{\text{length of the plane}}.$$

Correctly speaking, the angles, &c., of (2) do not properly belong to the inclined plane of (1), unless the centre of gravity of the body D touches the plane A B, but to an imaginary plane passing through the point A and the centre of gravity of D.

THE WEDGE.—The sixth mechanical power is the wedge, which may be regarded as two equal and similar inclined planes A B C, B D C (Fig. 134), with their bases fixed together. It is used for cleaving substances, in which case the edge C is introduced into a cleft, and the surface A D struck by a hammer or mallet, so as to cause the wedge to enlarge the cleft and split the substance.

Fig. 134.

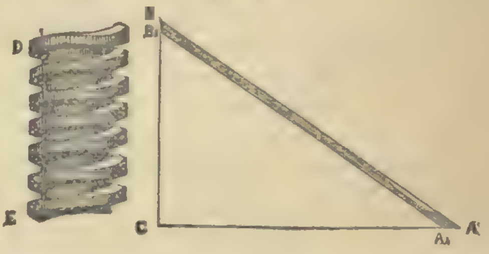


Considered as a statical machine, the power is a weight applied to A D sufficient to balance the pressures exerted by the substance into which the wedge is thrust on the sides A C and D C of the wedge, considering these surfaces as perfectly smooth.

The consideration of this machine is now omitted in many treatises, since "in the theory of the wedge, there are introduced so many conditions, which are perfectly inapplicable in practice, so many gratuitous assumptions and suppositions so inconsistent with practical truth, that the whole doctrine has little or no value. Nothing can more plainly demonstrate the inutility of the theory of the wedge, than that, in this theory, the power is supposed to be a pressure exerted on the back of the wedge, which is supposed to be capable of balancing the effect of the resistance in producing the recoil of the wedge. In all cases where the wedge is practically used, the friction of its faces with the resisting substance is sufficient to prevent the recoil; so that, strictly speaking, no force whatever is necessary to sustain the machine in equilibrium; and to move it, pressure is never resorted to—inasmuch as the slightest percussion is far more effective. The only general theoretical principle respecting the wedge, which obtains always in practice is, that its power is increased by diminishing the angle C D."*

THE SCREW.—The seventh mechanical power is the *screw*, and may be regarded as a modification of the inclined plane. If we take a triangular piece of paper A B C (Fig. 135), the angle at C being a right angle, in A C take a point A′, and through A′ draw A′ B′ parallel to A B. Then let the portion A A′ B′ B be blackened, and wrap the triangle A B C round a cylinder D E. Let now a groove be cut perpendicular to the surface of the cylinder, and following the direction of the dark spiral band, formed by the triangle A B C. The projecting portion between the groove is called the *thread* of the screw, and is evidently an inclined plane passing round the cylinder. The cylinder in which the groove is cut is called the *screw*. A hollow cylinder is now cut through a beam A B, and a spiral cavity, cut in the surface of the hollow cylinder, corresponding in magnitude to the thread of the screw, so that one may be regarded as the cast or mould of the other. This is called the *nut* of the screw.

Fig. 135



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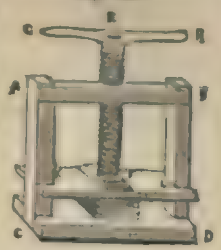
* "*Mechanics;*" *Society for the Diffusion of Useful Knowledge.*

Fig. 136.



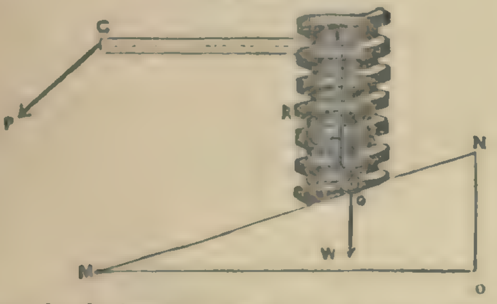
If A B (Fig. 136) be firmly fixed to the upright beams A C, B D, and these again to a plane base C D, the screw E F can be made to pass through the nut by causing it to revolve round its axis E F by means of the arms E H or E G, and as it moves through the nut its axis will always be vertical, and its extremity F will press upon a plane K L, so confined by the bars A C and B D in the frame A D C B, as to have only a vertical motion. The pressure of the screw will thus be communicated by K L to any substance placed between K L and C D.

To obtain the conditions of equilibrium of the screw, we neglect the weight of the screw itself, and the friction of the surface of the thread on that of the nut.

To simplify, as much as possible, the problem, we first suppose the portion of the thread which, by the action of a force P_1 at right angles to the extremity G of the lever G E (Fig. 136), communicates a pressure R_1 to the surface of the nut with which it is in contact, reduced to a single point Q (Fig. 137), and the surface of the nut unfolded into the inclined plane M N O.

The resistance R_1 which the surface of the nut opposes to the motion of Q, will be perpendicular to the surface M N of the inclined plane M N O, and the resolved portion of this force in a vertical position will give the pressure W_1, which the screw will communicate to the substance placed between K L and C D.

Fig. 137.



Let a be the length of the arm G E, b the radius of the cylinder of the screw, α the angle of the plane M N O.

Resolving R_1 vertically and horizontally, we have the following condition of equilibrium :—

$$W_1 = R_1 \cos. \alpha;$$

and taking moments about the axis of the screw,

$$a \cdot P_1 = b\, R_1 \sin. \alpha.$$

Now we may conceive the portion of the thread of the screw divided into a number of portions Q_1 Q_2, &c., Q_n, kept in equilibrium by forces P_1, P_2, &c., P_n, W_1, W_2, &c., W_n, and producing reactions R_1, R_2, &c., R_n on the surface of the nut.

$P_1 + P_2 +$ &c. $+ P_n$ will be the whole power P applied to the extremity of the arm G E, and $W_1 + W_2 +$ &c. $+ W_n$ will be the whole vertical pressure W which the screw will produce.

Then, as before, we shall have

$$\begin{array}{lll} W_2 = R_2 \cos. \alpha & \text{and} & aP_2 = bR_2 \sin. \alpha \\ W_3 = R_3 \cos. \alpha & & aP_3 = bR_3 \sin. \alpha \\ \text{\&c.} = \text{\&c.} & & \text{\&c.} = \text{\&c.} \\ W_n = R_n \cos. \alpha & & aP_n = bR_n \sin. \alpha \end{array}$$

Hence, adding these equations, we have

$$(W_1 + W_2 + W_3 + \text{\&c. } W_n) = (R_1 + R_2 + R_3 + \text{\&c. } + R_n) \cos. \alpha$$
$$a(P_1 + P_2 + P_3 + \text{\&c. } + P_n) = b(R_1 + R_2 + R_3 + \text{\&c. } + R_n) \sin \alpha$$

and dividing these equations, we have

$$\frac{W_1 + W_2 + W_3 + \text{\&c. } + W_n}{a(P_1 + P_2 + P_3 + \text{\&c. } + P_n)} = \frac{\cos. \alpha}{b \sin. \alpha}$$

$$\text{or } \frac{W}{aP} = \frac{\cos. \alpha}{b \sin \alpha}$$

$$\text{and } \frac{W}{P} = \frac{a}{b \tan. \alpha} = \frac{2 \pi a}{2 \pi b \tan. \alpha}$$

$$\text{or } \frac{W}{P} = \frac{\text{circumference of circle described by the extremity G of arm G E}}{\text{vertical distance between two threads of the screw}}$$

For if N G (Fig. 137) be the vertical distance between two threads of the screw, M O will be equal to $2\pi b$ and $N O = M O \tan. \alpha = 2\pi b \tan. \alpha$.

FRICTION.—In our previous investigations we have supposed all our surfaces in contact with each other to be perfectly smooth. (This perfect smoothness can never be attained in practice; the roughness, or want of smoothness, of two surfaces in contact with each other, opposes a resistance to their motion, over or along each other, which is called *friction*. From certain experiments it appears that friction may be due, in some measure, to the nature of the surfaces in contact with each other, and influenced by the molecular forces which the particles in contact may have with each other.

Experiments made with a number of different substances have led to the following laws :—

That the force of friction is proportional to the pressure acting on the surfaces in contact, so long as the materials of the surfaces in contact remain the same, and act at right angles to the direction of the pressure.

That for the same pressure, the friction is the same, whatever may be the magnitude of the surfaces in contact.

These laws are not strictly true, but are subject to considerable variation in certain extreme cases, as when the pressures are very great, and the surfaces in contact very small.

The friction of moving surfaces is also much less than that of the same surfaces in a state just bordering on motion.

For the state just bordering on motion, the friction of two smoothly planed pieces of wood, the grain being in the same direction, is half the pressure; for the same, the grain being in opposite directions, is one-fourth the pressure. The friction of two metallic surfaces is one-fourth the pressure; and of one surface metallic, and the other wood, one-fifth the pressure.

This friction may be greatly diminished by the use of lubricants, such as oil, tallow, black-lead, &c.

If the points in contact be mere lines, as in the case of the knife edge of the fulcrum of a lever, this friction may be considerably reduced. Thus, the friction of wooden surfaces is diminished in this case from one-fourth to one-twelfth the pressure, exerted by the surfaces in contact.

Having thus explained the leading laws of Statics, we shall proceed to investigate those of Dynamics.

Mechanics.

See Page 733.

LEVERS

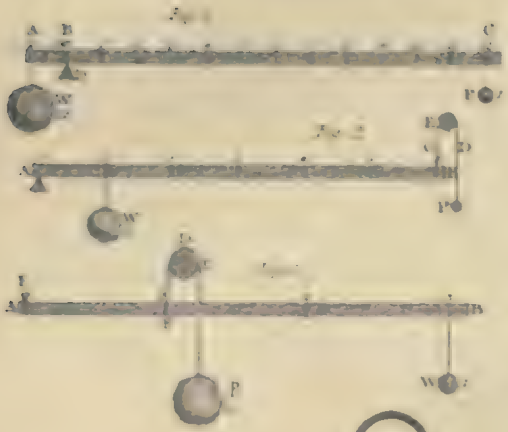
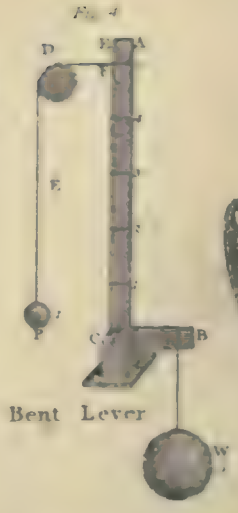


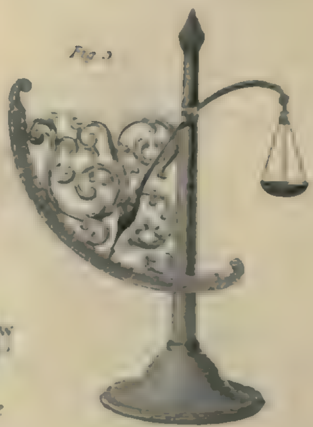
Fig. 4



Bent Lever

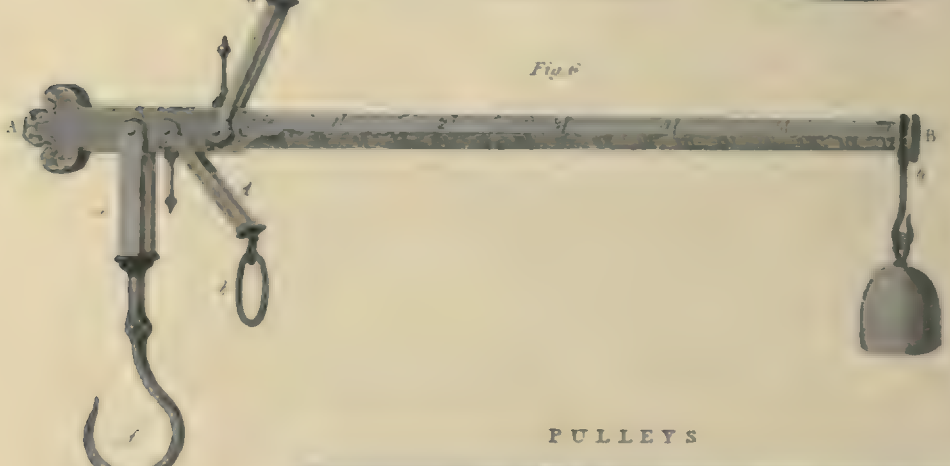
Bentlever Balance

Fig. 5



Roman Balance

Fig. 6



PULLEYS

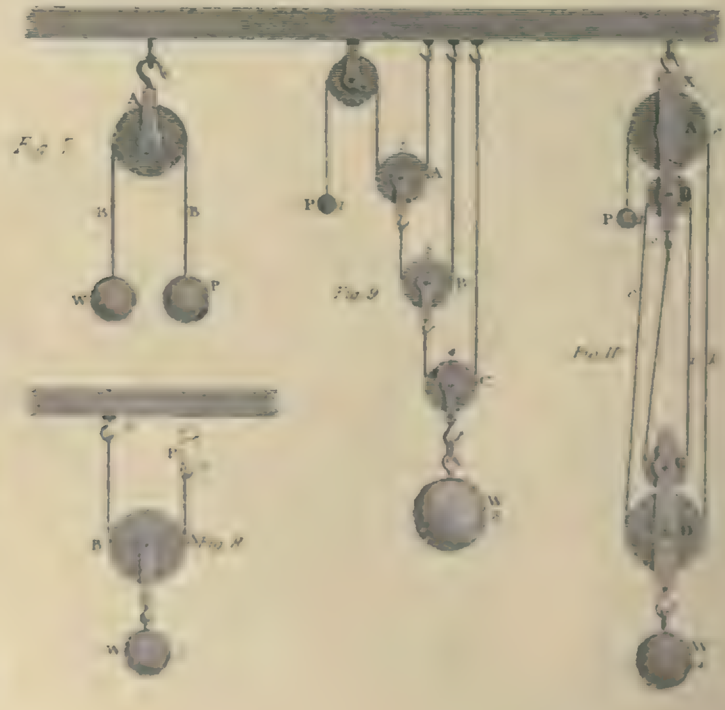
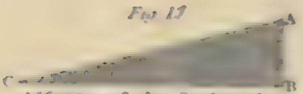


Fig. 11

Fig. 12



Affinity of the Inclined Plane to the Screw

Thread of the Screw
Fig. 10

ADDENDA.

[In the foregoing pages the student has had a full exposition of the theory of various mechanical arrangements, technically termed "powers." It has been thought desirable, however, to give further illustrations of the most commonly used machines; and accordingly such will be found in the annexed plate, respecting which the following observations may be made.

Figs. 1, 2, 3, represent the three orders of levers in their proper succession; the numerals on each lever and on the weights, and powers of application of force, will materially assist the student in understanding Propositions XXIII. and XXIV., as they show the position in which two unequal weights must be placed to neutralise each other's effect. The same observations apply to Fig. 4, which illustrates the bent lever. Figs. 5 and 6 are equally extended illustrations of the balance already described under the head of Levers. Figs. 7 to 10 illustrate various arrangements of the pulley; and Figs. 11, 12, and 13, refer to the affinity between the inclined plane and the screw. The following is a special description of each figure.

Fig. 1.—A lever of the first order. A, C, the lever; D, the fulcrum; W, the weight; P, the "power;" A, B, distance of weight from fulcrum; B, C, distance of "power" from fulcrum. In this cut, a "power" of one pound balances a weight of twelve pounds.

Fig. 2.—A lever of the second order. A, B, the lever; C, a cord (runs over pulley D), to which is attached the power, P, balancing the weight, W; E is the fulcrum.

Fig. 3.—A lever of the third order. Here the letters have the same signification as in Fig. 2, except that F is the point at which the power is applied.

Fig. 4.—The bent lever. A, B, the lever; C, the axis; D, a pulley over which runs a cord, E, supporting the power P, which thus balances the weight, W.

Fig. 5 is an application of the form of lever as a balance, described at page 721.

Fig. 6.—The Roman balance. A, B, is the lever; C, the fulcrum; *a*, *k*, an arm and ring by means of which the lever is suspended. The article to be weighed is suspended from the arm *e* by the hook *f*. If the lever be inverted, it is then suspended by *b*, *d*, as already explained at page 724.

Fig. 7 is a simple pulley, A; over it is stretched the cord B B, to which the "power" and weight P W, in this case equal, are attached. In Fig. 8, the power P is represented by a hand drawing upward the weight W, suspended to the pulley A, B. In Fig. 9, the power P balances the weight W, after the cord has been successively passed over pulleys A, B, C. The cord, however, is not continuous.* Fig. 10 is a further illustration of the system of pulleys described at page 730. A, D, and B, C, are two sets of pulleys, the lesser pair being smaller than the former. They are suspended at X; the cord stretches necessarily from the power P in the direction *e*, *k*, *o*, *u*, terminating at *s*, and balances the weight W.

Fig. 11 illustrates the inclined plane, of which A, B, is the base; C, D, the inclination; E, a descending weight; and F, G, the altitude. The theory of the inclined plane has been explained at page 731.

Fig. 12 shows an inclined plane, A, B, C, which if wrapped round a cylinder, as in Fig. 13, produces a screw whose threads are respectively *a*, *b*, *c*, *d*. C is the end of the plane, forming the terminal of the thread.—Ed.]

CHAPTER III.

DYNAMICS.

Introduction.—Every *body*, or material particle, is necessarily in a state of either rest or motion. A body strictly at rest is regarded as acted upon either by no force at all, or else by forces which oppose and neutralise one another. Investigations respecting forces which, acting upon a body or upon a system of bodies, thus keep the whole at rest, belong to the first division of Mechanical Philosophy—*Statics*. In this branch of the subject, we investigate the result of the several applied forces, which is *pressure* or *tension*, but no movement. In the second division, now to be treated of, the result of the applied forces, *motion*, falls under review: there may be pressure as well, but it is the motion or change of place of the body acted upon, that is the exclusive subject of *Dynamics*.

Of physical causes we know little or nothing: we observe motion or pressure, and we infer *force*—that is, some cause for the phenomena: but it is with the effects or phenomena alone that our observations and reasonings are concerned.

Ignorant, however, as we are of the essence of force, we cannot be under any doubt that it is the same in kind, when operating in the production of motion, as when operating in the production of pressure; for whenever, in the latter case, the resistance or obstacle is removed, motion necessarily ensues. But there is one consideration inseparably connected with that of motion, which has no place in statics: it is the consideration of *time*, an element which necessarily enters into the very idea of motion.

Nor can we conceive of a force which transmits its influence to a distant body—as the force of magnetism, or the force of gravity—as doing so independently of time. It is inconceivable that some interval does not elapse, however minute, between the cause and the effect: the transmission of a force through space must take time. A few years ago, the following question was proposed to the author of the present treatise:—A beam or bar uniformly strong—that is, resisting fracture in every part alike—is immovably fixed at one extremity, from which it hangs vertically; to the other, or lower extremity, is then attached a weight indefinitely great: where will the beam break? The condition is, that the beam has no tendency to break at one place more than at another, and yet the weight suspended to it is to be immeasurably great. The answer is, that time being required to transmit the force through the particles of the beam, the fracture takes place at the lowest part, the falling weight bringing with it only the lamina of material in immediate contact with it.

As the dynamical effect of force is motion, and as motion implies space passed through, and the time of passage, it is clear that, as in all physical inquiries, measuring causes by their effects, the measure of force must in some way be compounded of space and time.

The simplest kind of motion is *uniform* motion, or that in which the moving body passes through equal spaces in equal intervals of time; and the simplest path the moving body can describe, is a straight path. We can fully examine motion of *this* kind without any reference to force at all; we can take the motion as we find it, without any inquiry as to its origination; and ascertain all we wish to know respecting it. In fact, a body so moving is not, during the motion, acted upon by any force at all. Conceive an isolated body at rest, all other bodies and forces in nature being removed: it is plain that that body will remain at rest, for there is nothing to disturb its condition. In like manner, conceive such isolated body to be moving uniformly in a straight line: it must continue to move uniformly, for there is nothing to disturb its condition: to suppose a force to act upon it at any point of its path, and yet for the uniform onward motion to remain the same, would be to suppose a cause without an effect. A body, therefore, thus moving uniformly onward, is not actuated by any force whatever. It is true it could not pass from rest to uniform motion in a straight line without a cause—an impulse, for instance—but nothing acts upon it afterwards, or during its uniform motion; otherwise that uniformity would be interfered with.

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* See Fig. 129, *ante*, p. 729.

It is this simplest kind of motion that we shall first consider, and shall then proceed to that which is due, not to instantaneous impulse, but to *force* continuously acting upon the moving body. The proper method of measuring this force will shortly be explained.

UNIFORM RECTILINEAR MOTION.—In applied, as well as in pure mathematics, certain fundamental positions or postulates must at the outset be assented to. In Dynamics there are three such postulates: they are known as the three laws of motion.[*] The first of these is as follows:—

A material particle, if at rest, and unacted upon by any external force, will remain at rest. A material particle, if in motion, and unacted upon by any external force, will continue its motion uniformly, and in a straight line. This law is only a formal enunciation of the *inertia* of matter, by which is meant, its incapability of altering, *of itself*, the state into which it is put by any external cause, whether that be a state of rest or a state of motion.

It is plain that the motion spoken of, being neither unchecked nor expedited, nor its direction any way interfered with, must be uniform in its *rate*, and rectilinear in its course—the course originally impressed.

The *rate* of a body's uniform motion is estimated by the extent passed over in some determinate portion of time—a second, a minute, an hour, &c. In dynamical investigations, the *second* is generally taken for the unit of time; and what in popular language is called *rate* of motion, is here called *velocity*; hence if a body move uniformly over ten feet every second of time, we say that it moves with a velocity of ten feet, or that its velocity is ten feet "per second" being understood. Putting the symbol v for velocity, we should therefore have $v = 10$ feet. As a second is taken for the unit of time, so a foot is taken for the unit of length. It is common, however, to call the length of track described by a moving body, the *space* passed through, and to represent this length by s; but it will be borne in mind that by s, length only is meant.

The symbol employed for *number of seconds* is t: it is to be carefully observed that t does not stand for the concrete quantity *time*, but only for the *number* of seconds in that time; so that *length*, viz., *feet*, is the only concrete quantity represented by the symbols v, s, t, which are related to one another as follows; namely:—

$$v = \frac{s}{t},\ s = vt,\ t = \frac{s}{v}$$

So that any two of the three quantities concerned being known, the third may be immediately found. If, however, t be not reckoned from the commencement of the uniform motion, but only after a certain space s' has been passed over by the moving body, then s being the whole length of path from the commencement, the three equations will be

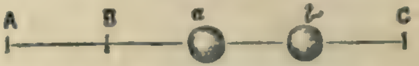
$$v = \frac{s-s'}{t},\ s = s' + vt,\ t = \frac{s-s'}{v}$$

Any one of these equations is sufficient for the entire theory of the motion of a body impelled by a single impulse, or influenced originally by any cause producing uniform motion.

It may, perhaps, be as well to remark here, that we speak of the path of a particle and of a body indiscriminately. In most works on Dynamics, all mention of *body* is avoided in this part of dynamics, and the motion of a single particle only considered. But this exclusion of body as an assemblage of particles is, we think, as injudicious as it is unnecessary. There *seems*, it is true, more mathematical precision of language in speaking of the line (s) traced out by a moving particle, than of what is described by a bulky body; but let it be only understood that by the path of a body we mean the line traced out by its centre of gravity, and all grounds for depriving the mass of volume, and reducing it to an indivisible isolated particle—of which indeed we can have no clear idea—will be removed. Throughout the present portion of dynamics, our investigations are entirely independent of mass or volume; and, consequently, to make a single physical particle the subject of those investigations, to the studied and systematic exclusion of body, is to perplex and mislead the learner—in fact, to convey to him the notion that what he is learning is applicable only to geometrical abstractions—to physical nonentities, and not to the actual material substances around us.

We shall now give an example of the application of the formulæ just established.

Fig. 138.



Two bodies (a, b, Fig. 138) animated by the uniform velocities, v, v', set out simultaneously from the points A B, and move in the direction of the straight line A B continued: required the time of their coming together.

Suppose a overtakes b at the point C, then

$$\text{A C} = vt,\ \text{B C} = v't;$$

that is, calling A C, s, and A B, s',

$$s = vt,\ s - s' = v't.$$

$$\therefore vt - s' = v't \therefore t = \frac{s'}{v - v'}$$

So that the abstract number denoting the units of time—that is, the number of seconds—will be found by dividing the space between the points of starting by the difference of the spaces denoting the velocities.

If uniform motion be the result of an impulse communicated to a body originally at rest, we may reasonably conclude that the velocity produced will be proportional to the intensity of the impulsive energy. For if a body receive a certain velocity in consequence of a certain impulse, it ought to acquire double that velocity if at any point of its path that impulse be repeated in the same direction; but if the second impulse, like the first, take place at the starting-point, it must unite with the first impulse, so that the consequence of a double intensity of impulse, will be a double velocity in the body impelled; and in like manner a triple intensity will produce a triple velocity; and so on.

VARIABLE RECTILINEAR MOTION.—When a moving body does not pass over equal spaces in equal times, the motion is not uniform, but variable. The rate at which the body moves, or its velocity at any instant, is the space it *would* describe in a second of time, measuring from that instant, provided all force were then withdrawn, and the body left to itself. Its motion during this second—as no force acts upon it—would of course be uniform.

Hence velocity in general is measured as follows:—

If the motion be uniform, the velocity is measured by the space actually passed over in a second. If the motion be variable, the velocity at any instant is measured by the space that *would be passed over* in a second, if all force were withdrawn at that instant, and the body left to proceed with the motion it then has.

ACCELERATING FORCE.—A body may so move as to acquire equal accessions of velocity in equal times—that is, it may move so that its velocity at any time t being v, we may have

at the times	t,	$t+1$,	$t+2$,	$t+3$,	$t+4$,	&c.
the velocities	v,	$v+v'$,	$v+2v'$,	$v+3v'$,	$v+4v'$,	&c.

Or the velocity may, in like manner, be retarded, v' being negative. Under such circumstances the velocity is said to be uniformly *accelerated*, or uniformly *retarded*.

The cause of the uniform acceleration or retardation of a body's velocity, we call *force*; and measuring causes by their effects, we take the amount of this constant acceleration or retardation, as the measure or representative of a constant force, continuously acting on the moving body.

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[*] The three laws of motion are formally enunciated at the end of this chapter.

The symbol for this force is f, which, viewed only in its effects, denotes merely the velocity *generated* (or destroyed) in a second of time. In the illustration above, for instance, $f = v'$.

It is of importance that the student do not attach any other meaning to this symbol f, than that here assigned to it. The nature of the occult influence called dynamical force—or, as it is more frequently named, *accelerative* force—nobody can explain. In the present inquiry, we have to do only with its effect; and this we see is merely augmentation (or diminution) of velocity; and therefore, like velocity itself, it is expressed by *space*—that is, by linear measure: it is not the *force* that is accelerated, but the *velocity*.

If the force be constant or uniform, the acceleration of the velocity is also constant or uniform, as supposed above; but if the force be variable, the acceleration of the velocity is also variable. The only office of accelerative force is to produce accelerative velocity; and it is only by this latter phenomenon that we become cognizant of its activity, and can estimate its intensity.

Although, as remarked at the outset, we are altogether unacquainted with the essence of the thing called force, yet we may have abundant means of ascertaining whether its effects are constant or variable; and it must be carefully borne in mind that, in speaking of a constant force, or of a variable force, we have reference solely to the constant effect, or the variable effect. The force itself, whatever it may be, may in reality remain entirely unchanged; and yet, if it affect a body differently at different distances from its supposed seat of motion, we should call it a *variable* force. For example, there is a force familiar to everybody called *gravity;* and there can be no doubt that this force, like the magnitude of the earth itself, remains unchanged: yet as we find that the higher above the surface a body be taken, the less it will weigh, and the less will its velocity be accelerated, we say—exclusively in reference to these variable effects—that gravity is a variable force. It may be observed, however, that as this variation of weight and acceleration is insensible at moderate distances from the earth's surface, in all the ordinary applications of dynamics to terrestrial matters, gravity may without error be regarded as a constant force.

It is of importance that the student have correct conceptions of the sense in which the terms velocity and force are employed in these inquiries, and also that he should clearly perceive that *space* is the only concrete quantity concerned in the present portion of dynamics. The two propositions following, contain the whole theory of the rectilinear motion of a body under the influence of a constant accelerating force.

1. If a constant accelerating force f act on a body now at rest, during the time t, producing in it at the end of that time a velocity v, then $v = ft$. For since f expresses the velocity generated in each second, it follows that in the t seconds, the velocity v will amount to ft, $\therefore v = ft$.

2. If s be the space through which the body is moved from rest by the constant accelerating force f, in the time t, then $s = \frac{1}{2} ft^2$.

Suppose the time t to be divided into n equal parts; then since equal velocities are generated in equal times, the velocity generated in the time $\frac{t}{n}$ is the nth part of that generated in the time t; that is, by last proposition, it is $\frac{ft}{n}$. Consequently,

at the end of the times $\frac{t}{n}, \frac{2t}{n}, \frac{3t}{n}, \ldots \frac{nt}{n}$ or t

the velocities will be $\frac{ft}{n}, \frac{2ft}{n}, \frac{3ft}{n}, \ldots \frac{nft}{n}$ or ft..(A)

and at the *commencement* of the same intervals, the velocities will be

$$0, \frac{ft}{n}, \frac{2ft}{n}, \ldots \frac{(n-1)ft}{n} \ldots \text{(B)}$$

If the several velocities (A) were uniform during the several equal intervals of time, then the whole space S described would be

$$S = \frac{ft}{n} \cdot \frac{t}{n} + \frac{2ft}{n} \cdot \frac{t}{n} + \frac{3ft}{n} \cdot \frac{t}{n} + \cdots + ft \cdot \frac{t}{n}.$$

And if, in like manner, the several velocities (B) were uniform during those intervals, the whole space S′ described would be

$$S' = 0 \cdot \frac{t}{n} + \frac{ft}{n} \cdot \frac{t}{n} + \frac{2ft}{n} \cdot \frac{t}{n} + \cdots + \frac{(n-1)ft}{n} \cdot \frac{n}{t}.$$

Now, it is obvious that the space s actually described, is intermediate between the two spaces S, S′; being less than the former, and greater than the latter. And this is evidently true, however short the several equal intervals; that is, however large the number n may be.

But the shorter the intervals be made, the closer do the values S and S′ approach to each other, and therefore to the intermediate value s, till at length, when the intervals are diminished down to zero, by n becoming infinite, all three values S, s, S′ unite and become identical.

The determination of s is therefore reduced to the following algebraical problem; namely—to determine the value of S or of S′ when n becomes infinite.

Now the general expression for the value of S is

$$(1 + 2 + 3 + \ldots + n)\frac{ft^2}{n^2}$$
$$= \frac{n(n+1)}{2} \cdot \frac{ft^2}{n^2} = \left(1 + \frac{1}{n}\right)\frac{ft^2}{2}$$

In like manner, the general expression for the value of S′ is

$$\{0 + 1 + 2 + \ldots + (n-1)\}\frac{ft^2}{n^2}$$
$$= \frac{n(n-1)}{2} \cdot \frac{ft^2}{n^2} = \left(1 - \frac{1}{n}\right)\frac{ft^2}{2}$$

When n is infinite, since in that case $\frac{1}{n} = 0$, each of these expressions becomes $\frac{1}{2}ft^2$: consequently $s = \frac{1}{2}ft^2$. It appears then, from these two propositions, that if a body at rest be acted upon by a constant and uniform accelerating force f, during t seconds, the velocity acquired and the space described will be expressed by the equations

$$v = ft \text{ and } s = \tfrac{1}{2}ft^2 \ldots. (1)$$

From these two important equations we may proceed to deduce some inferences.

1. The space described in any time, reckoning from the commencement of the motion, is half the space that *would be* described in the same time, if the body were to move from rest with a uniform velocity equal to the last acquired velocity.

For since by (1) $s = \frac{1}{2}(ft)\,t$, and $v = ft \therefore s = \frac{1}{2}vt$; but vt is the space described in t seconds with the uniform velocity v: hence the space s, actually described under the influence of a uniform accelerating force, is half the space that would be described in the same time, if the body were to commence with the final velocity, and move uniformly.

2. The spaces described in equal successive portions of time, from the commencement of the motion, are to one another as the odd numbers 1, 3, 5, 7, &c.

For let t in (1) be successively 1, 2, 3, 4, &c., then the spaces from beginning are $\frac{1}{2}f \cdot 1$, $\frac{1}{2}f \cdot 4$, $\frac{1}{2}f \cdot 9$, $\frac{1}{2}f \cdot 16$, &c.; therefore the spaces for the successive portions of time given by subtracting these spaces from the beginning, each from the following, are $\frac{1}{2}f \cdot 1$, $\frac{1}{2}f \cdot 3$, $\frac{1}{2}f \cdot 5$, $\frac{1}{2}f \cdot 7$, &c.; so that the spaces described from the beginning are as the squares of the times; and the spaces described in the successive equal portions of time are as the numbers 1, 3, 5, 7, &c.

3. By means of the two equations (1), any one of the four quantities f, t, v, s, may be eliminated, and an equation deduced, involving only the other three; so that any one of the four things—force, time, velocity, and space—may be expressed in terms of any two of the others: thus, as the simplest algebraic substitutions show, we have the following equivalent expressions:

Expressions for v, ft, $\sqrt{2fs}$, $\frac{2s}{t}$

" " s, $\frac{1}{2}ft^2$, $\frac{1}{2}vt$, $\frac{1}{2}\frac{v^2}{f}$

" " t, $\frac{v}{f}$, $\sqrt{\frac{2s}{f}}$, $\frac{2s}{v}$

" " f, $\frac{v}{t}$, $\frac{2s}{t^2}$, $\frac{1}{2}\frac{v^2}{s}$

4. A uniform accelerating force is measured by twice the space described from rest in one second.

For putting $t = 1$, we have $f = \frac{2s}{t^2} = 2s$; so that if we can only measure the space through which a body, acted upon by a uniform accelerating force, moves from a state of rest, in one second, we can correctly determine the *uniform effect* of that force on the body: it will be double the space thus passed through. In other words, this double space will be the constant increment of the velocity at each succeeding second; that is, the constant *acceleration* of the velocity.

It is found by experiment, that the attraction of the earth upon bodies near its surface—that is, the force of gravity—causes a body to fall from rest a distance of 16·1 feet in the first second of time. Consequently, the force of terrestrial gravity—which force is usually represented by g—is $g = 32·2$ feet: that is, this force, continuously soliciting a falling body, will accelerate its velocity 32·2 feet every second.

It may not be amiss to repeat here, before proceeding to practical illustrations, that where there is uniform velocity, in a straight line, there is no force. We should be compelled to admit this, as a necessary consequence of our valuation of force in dynamics:—where there is no acceleration of velocity, there can be no force urging the body onwards in its path.

The student must all along remember, that when we speak of a force acting on a body, we always refer to the mechanical condition of the body at the particular instant that has brought it to where we find it. The impulse that, acting on a body at rest, puts it in a state of uniform motion, acts only during the instant of passing from rest to rectilinear motion: it expires, as it were, in the act; so that at whatever point in the path described, the moving body comes under our examination, we are compelled to say that no force—not even the force of impulsion—is acting upon it *then*.

We shall now give a few examples connected with accelerating force.

EXAMPLES OF ACCELERATING FORCE.—1. A body moves from rest with a uniformly accelerated velocity, and after the lapse of 3 minutes 5 seconds, is found to have passed over 400 feet: required the accelerating force.

Here the time, namely, t secs. = 3 mins. 5 secs. = 185 secs., and the space described, namely, $s = 400$ feet, are given to determine f. From the formula $s = \frac{1}{2}ft^2$ we have

$$f = \frac{2s}{t^2} = \frac{800}{185^2} = \frac{32}{37^2} = ·0234 \text{ feet.}$$

Consequently the accelerating force is such as to increase the velocity ·0234 feet every second.

2. How far will a heavy body fall in four seconds?

Using g for f, we have $s = \frac{1}{2}gt^2$, where $g = 32·2$, and $t = 4$, $\therefore s = 16·1 \times 4^2 = 257·6$. Hence the distance is 257·6 feet.

3. In what time will a heavy body descend 400 feet?

$$s = \frac{1}{2}gt^2 \therefore 400 = 16·1t^2 \therefore t = \sqrt{\frac{400}{16·1}} = 5 \text{ seconds.}$$

4. If a body be projected vertically upwards with a velocity of 400 feet per second, how high will it ascend?

It will ascend to that height from which, if it were let fall, it would acquire a velocity of 400 feet upon reaching the earth; therefore since

$$s = \frac{1}{2}\frac{v^2}{g} \therefore s = \frac{1}{2}\frac{400^2}{32·2} = 2484 \text{ feet.}$$

5. Through what height must a body fall so that the velocity acquired may be equal to that height?

$$s = \frac{1}{2}\frac{v^2}{g}, \text{ but } s = v \therefore 1 = \frac{1}{2}\frac{v}{g} \therefore v = s = 2g = 64·4 \text{ feet.}$$

6. Through what height must a body fall to acquire a velocity of 100 yards a second?—Ans. 139·75 feet.

7. A body is projected upwards with a velocity of 1500 feet per second: how high will it ascend?—Ans. 34938 feet.

8. What time will elapse, after projecting the body in the last example, before it again reaches the earth?—Ans. 93 seconds.

In the foregoing inquiries respecting the rectilinear motion of bodies under the influence of an accelerating force, we have supposed the force to move the body from *rest*. We are now to consider the case in which the force begins to act upon a body already moving in the direction of that force, or receiving an impulse in that direction. Suppose a body to begin its motion with a velocity v', and, from the commencement, to be acted upon by a uniform accelerating force f; then, as before, v being the velocity acquired in t seconds, and s the space passed through in that time, the acquired velocity will evidently be made up of the original velocity v' and the velocity ft communicated by the force f; for this force adds to the velocity f feet every second, or ft feet in t seconds, so that in t seconds the acquired velocity is $v = v' + ft$.

Again, in virtue of the initial velocity v' alone, the body would pass over the space $v't$ in t seconds; and in the same time f alone would cause it to pass over the space $\frac{1}{2}ft^2$: hence the combination of these is the whole space passed over; that is, it is $s = v't + \frac{1}{2}ft^2$; so that the formulæ for finding the velocity and the space described are

$$v = v' + ft, \; s = v't + \frac{1}{2}ft^2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

If the initial velocity v' oppose that communicated by f, then v' is to be taken with a sign opposite to that of f.

From these two equations t may be easily eliminated, and a third equation obtained involving only the remaining quantities; thus, squaring the first, we have

$$v^2 = v^2 + 2v'ft + f^2t^2 = v'^2 + 2f(v't + \tfrac{1}{2}ft^2).$$

Hence, by the second equation,

$$v^2 = v'^2 + 2fs \quad . \; . \; . \quad (3)$$

As before, when gravity is the force, f is replaced by g.

Ex. 1. A body is projected vertically upwards, with a velocity of 480 feet per second; at what height will it be at the end of three seconds?

Here v', the velocity of projection, being 480 feet, this velocity continued uniform for 3 seconds would carry it to the height of 1440 feet; but gravity so counteracts its upward motion as to draw it back through a space of $\frac{1}{2}gt^2 = \frac{1}{2} \times 32.2 \times 3^2 = 144·9$ feet: hence the height to which the body is suffered to ascend is only 1440 — 144·9 = 1295·1 feet; that is, $s = v't - \frac{1}{2}gt^2 = 1295·1$ feet.

2. From an elevated position a body is projected vertically upwards with a velocity of 80 feet: required its place at the end of 6 seconds.

$$s = v't - \frac{1}{2}gt^2 = 80 \times 6 - \frac{1}{2}\,32·2 \times 6^2 = 480 - 579·6 = -99·6.$$

Hence, the place of the body is 99·6 feet *below* the place of projection.

3. With what velocity must a body be projected downwards from the top of a tower 150 feet high, to arrive at the bottom in two seconds?

$$s = v't + \tfrac{1}{2}gt^2 \therefore v' = \frac{s}{t} - \tfrac{1}{2}gt$$

$$= \frac{150}{2} - 16{\cdot}1 \times 2 = 42{\cdot}8 \text{ feet.}$$

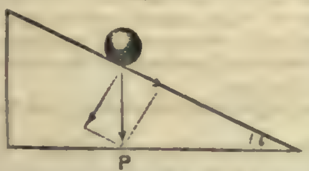
4. If a body be projected vertically downwards with a velocity of 20 feet, how far will it descend in 5 seconds?—Ans. 505·5 feet.

5. A body is projected upwards with a velocity of 100 feet: what will be its velocity when it has ascended 100 feet? (See Form 3.)—Ans. 51·6 feet.

6. In the preceding example, find at what time the body is 100 feet from the earth, as well in its descent as in its ascent.—Ans. 1·2 sec. and 4·7 secs. after projection.

MOTIONS OF BODIES DOWN INCLINED PLANES.—The fall of a body down an inclined plane is another instance of rectilinear motion under the influence of gravity. When a material substance is placed on an inclined plane, the force of gravity produces a certain vertical pressure P (Fig. 139): if we resolve this pressure in two directions, the one along the plane and the other perpendicular to it, the former component will be P sin. i, taking i for the inclination of the plane to the horizon; and to prevent the body from moving down, this is the force or pressure that must be counterbalanced.

Fig. 139.



As, therefore, P represents the pressure-force of gravity on the body in a vertical direction, and P sin. i the pressure in the direction of the plane, and as, when motion takes the place of these statical pressures, the accelerating force of gravity is proportional to them, we shall have for the acceleration down the plane,

$$P : P \sin. i :: g : g \sin. i.$$

Hence the body is urged down the inclined plane by the constant force,

$$g' = g \sin. i \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

And, therefore, substituting this value of g' for f in the formulæ at page 736, those formulæ will then comprise the entire theory of motion down an inclined plane.

If l represent the length of the plane, and h its height, then $\sin. i = \dfrac{h}{l}$: consequently the accelerating force down the plane is

$$g' = g\frac{h}{l} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

And, therefore, the velocity acquired in descending down the whole length l, that is, in descending through the space $s = l$, by the action of this force, must be (page 736)

$$v = \sqrt{2g'l} = \sqrt{2gh} \quad . \; . \; . \; . \; . \; . \quad (2)$$

This expression we see involves only the height h of the plane, and is independent of its length l: hence the velocity acquired in descending down all planes of the same height is the same, and equal to the velocity acquired by falling vertically through that height.

But the velocities of two bodies, one falling through the perpendicular height, and one falling down the length of the plane, are respectively

$$v = gt \text{ and } v' = gt' \sin. i \quad . \; . \; . \; . \quad (4)$$

where t and t' are the respective times occupied in falling: these expressions are therefore equal; that is,

$$gt = gt' \sin. i \therefore \frac{t}{t'} = \frac{\sin. i}{1}$$

So that the time of falling through the height is to the time of falling down the plane, as the sine of the plane's inclination to unity.

If we wish to know what extent of length down the plane a body will pass through, while another falls through the whole height, then, referring to the expression for the space (page 736), we have

Vertical fall, $h = \frac{1}{2}gt^2$; inclined fall in time t, $s = \frac{1}{2}g \sin. i\, t^2$;

$$\therefore h : s :: 1 : \sin. i, \text{ or } s = h \sin. i.$$

If, therefore, from B we draw the perpendicular B D, the length A D will be that fallen through by one body moving down the plane A C, while another body falls through the height A B, because

$$AD = AB \cos. A = AB \sin. C.$$

If we draw the vertical D B′, and B B′ perpendicular to D B, then, by this theorem, the time of falling down the oblique line D B would equal the time of falling down the vertical D B′, for D B = D B′ cos. D; but D B′ = A B, since A B is a parallelogram, therefore the time of falling down the vertical A B is the same as the time of falling down either of the oblique lines A D, D B at right angles to one another: hence this remarkable property of the circle, namely: If from the extremities A, B, (Fig. 141) of the vertical diameter A B, chords be drawn, as in the annexed diagram, a body would fall down either of them in the same time that it would fall through the vertical diameter A B.

Fig. 140.

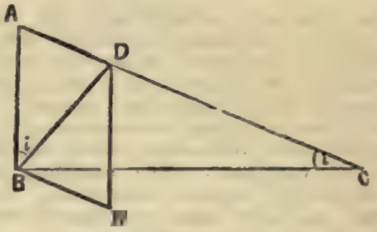
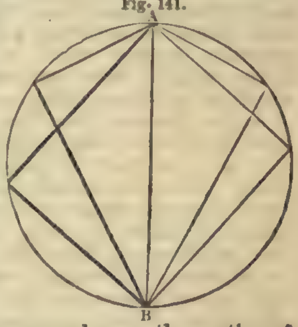


Fig. 141.



The following are a few examples on the motion of bodies down inclined planes:—

1. If the length of an inclined plane be 60 feet, and its inclination to the horizon 30°, what velocity would a body acquire in falling down it for two seconds?

By the formula (4), since $\sin. 30° = \frac{1}{2}$, we have $v = gt \sin. i = 32{\cdot}2 \times 2 \times \frac{1}{2} = 32{\cdot}2$ feet per sec. Hence the velocity is the same as would be acquired by a vertical fall in one second.

2. How long would a body be in falling down an inclined plane whose length is 100 feet, and inclination 60°?

Substituting the acceleration $g \sin. i$ for f, and l for s, in the expression for t, at page 736, and remembering that $\sin. 60° = \frac{1}{2}\sqrt{3}$, we have

$$t = \sqrt{\frac{2l}{g \sin. i}} = \sqrt{\frac{200}{32{\cdot}2 \times \frac{1}{2}\sqrt{3}}} = \sqrt{\frac{400}{32{\cdot}2 \times \sqrt{3}}}$$

$$= \frac{20}{\sqrt{(32{\cdot}2 \times \sqrt{3})}} = 2\tfrac{2}{3} \text{ seconds.}$$

3. A body is projected up an inclined plane whose height is $\frac{1}{6}$th of its length, with a velocity of 50 feet. Find its place and velocity after six seconds have elapsed.

In this case the force $g\dfrac{h}{l}$ (equation 2) *retards* the motion of the body, and must therefore be considered as negative: hence, from equation (2) page 736, we have

$$s = v't - \tfrac{1}{2}g\frac{h}{l}t^2$$

$$= 50 \times 6 - 16{\cdot}1 \times \tfrac{1}{6} \times 36 = 6\,(50 - 16{\cdot}1) = 203{\cdot}4 \text{ feet.}$$

$$\text{Also } v = v' - g \cdot \frac{h}{l}\,t$$

$$= 50 - 32{\cdot}2 = 17{\cdot}8 \text{ feet, the velocity required.}$$

4. If a body be projected up an inclined plane whose length is ten times its height, with a velocity of 30 feet, in what time will the velocity be destroyed, and the body begin to roll down?

The time is obviously the same as would be required to produce a velocity of 30 feet in a body falling from rest down the same plane: hence, substituting the acceleration $g\frac{h}{l}$ for f in the expression for t (page 736), we have

$$t = \frac{vl}{gh} = \frac{30 \times 10}{32{\cdot}2 \times 1} = \frac{300}{32{\cdot}2} = 9{\cdot}3 \text{ seconds.}$$

In the foregoing investigations, the student will observe that no allowance is made for the friction of bodies rolling down inclined planes; nor, whether bodies fall vertically, or obliquely as in this article, is the resistance of the air taken into account. These hindrances to free motion can be estimated only by practical and experimental researches, under various circumstances and conditions. The resistance of the air cannot be provided against, as is obvious; but in pieces of delicate machinery, many ingenious contrivances are resorted to, to diminish friction, and to render the departure from rigid mathematical theory as trifling as possible. The subject of friction will come under consideration in the section on Applied Mechanics.

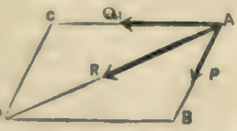
THE PARALLELOGRAM OF VELOCITIES.—The forces considered in Dynamics are influences of the same kind as those considered in Statics; they merely exhibit their effects in a different manner. What in Statics produces pressure, would in Dynamics—that is, if the thing pressed were removed—produce motion: the effects in both cases being proportional to the causes:—a double pressure from the same body, like a double acceleration of velocity, implies a double force.

It may, however, be proper here to anticipate a difficulty which the student may possibly feel: he may perhaps reason thus:—"A weight of ten pounds produces a pressure ten times that of one pound, yet the one pound weight, if let fall, is accelerated just as much as the ten-pound weight; whereas, from what is here said, it would seem that the heavier weight ought to be accelerated ten times as much as the lighter, seeing that its pressure is ten times as great."

But this apparent difficulty can arise only from a wrong conception of what is stated above: we are not comparing the pressure of *different* bodies under the same force acting upon all their particles, but the pressure of the *same* body under different acting forces: for example, if the same body were acted upon, now by the force of gravity, and hereafter if the same body, or an equal body, were acted upon by a force only half that of gravity, then, as stated above, the effect in the former case, whether that effect be pressure or acceleration, would be double the like effect in the latter case. If it were otherwise, than the statical measure of force (pressure) and the dynamical measure (acceleration of velocity) could not each of them be a correct measure or expression of the intensity of what we call force.

It has been fully proved in Statics (page 694), that if two forces P and Q act at A, and A B, A C represent their directions and intensities respectively, then will A D represent in direction and intensity their united effect.

Fig. 142.



The statical effect remains the same however long the forces or pressures act, for *time* is no element of consideration in Statics. But the dynamical effect is that at the instant the body acted on by the forces is at A, the continuous action of the forces, *afterwards*, not entering into consideration.

The parallelogram of *velocities* is independent of the parallelogram of forces. It will be remembered that the velocity of a body at any point of its path, is the space it *would* pass over in a unit of time, provided its rate or speed at that point were uniformly continued during the unit of time.

Suppose the body, when at A, were animated with a velocity that would alone carry it uniformly along A B to B, and with another velocity that would alone carry it uniformly along A C to C in the same time: these velocities may be considered as communicated by two simultaneous impulses in the directions A B, A C: the thing to be shown is, that the body would be carried uniformly along A D, and would arrive at D in the time spoken of.

For while the body is moving uniformly from A to B, conceive that the line A B, with the moving body on it, is carried parallel to itself, and with the second uniform velocity, up to C D: by the hypothesis, it will have arrived at C D in the same time that the body will have arrived at the extremity of the moving line: consequently, at the end of that time the body will be found at D. And that it will have arrived there with a uniform motion along the diagonal A D, will appear from considering, that if any point of its path were out of that diagonal, the uniform relation of the component velocities, namely A B : A C, would there be destroyed. That the diagonal is described with a uniform velocity—or that equal portions of it are passed over in equal portions of the time, is plain, because the uniformly moving line A B passes over equal portions of the diagonal in equal portions of the time.

Since, in the preceding figure, trigonometry gives for A D the expression

$$AD^2 = AB^2 + BD^2 - 2\,AB \cdot BD \cos. ABD,$$

and since $-\cos. ABD = +\cos. BAC$, if the directions of the component velocities v, v' make an angle a with each other, then the resultant velocity V will be

$$V^2 = v^2 + v'^2 + 2vv' \cos. a$$

From attending to the former part of the preceding examination, it will be seen that, however irregular the motion that would carry A to B, and however irregular the motion that would carry it to C, in the same time, the resultant of the two motions would necessarily carry it to D in that time, although not by the path A D, except the two component motions, during simultaneous portions of the time, are always as A B to A C. This truth will be of considerable importance in the next article on projectiles.

ON THE MOTION OF PROJECTILES.—If a body be projected obliquely upward or downward, the attractive force of gravity will cause it to take a curvilinear path. If the velocity of projection be considerable, and the body projected present much surface to the atmosphere, the resistance of the air will greatly modify the form of the curved path, and the range of the projectile; and it is no easy matter to determine one or the other. But if the body be supposed to move free from such obstruction (as in vacuum), all the circumstances connected with the flight of a projectile can be readily ascertained, as also, with a close degree of approximation, even in actual practice, if the velocity of projection be but small.

Supposing atmospheric resistance to be removed, the path of the projectile will always be the conic section called a *parabola* (see Practical Geometry, page 610), as we now proceed to show.

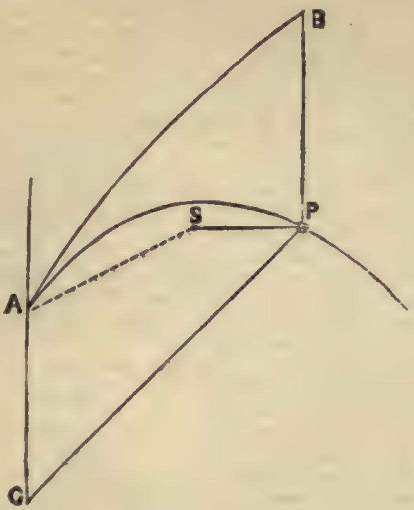
PATH OF PROJECTILE A PARABOLA.—Let A B (Fig. 143) be the direction in which a body is projected from the point A, with a velocity v. If no other motion were impressed upon the body, it would move uniformly along A B with the original velocity v, and in t seconds would arrive at B, supposing $AB = vt$. But as gravity acts on the body from the commencement, this force alone, in t seconds, would draw the body down, along A C, to C, supposing $AC = \frac{1}{2}gt^2$ (page 736).

Consequently, completing the parallelogram B C, the body in t seconds is found at P, as proved at the close of last article. Now

$$CP = AB = vt, \text{ and } AC = \tfrac{1}{2}gt^2$$

$$\therefore \frac{CP^2}{AC} = \frac{2v^2}{g} \therefore CP^2 = \frac{2v^2}{g} \cdot AC \quad . \quad . \quad . \quad (1)$$

Fig. 143.



This is what is called the *equation to a parabola*, A B parallel to C P being a tangent to the curve at the point A, and A C being parallel to the axis. The equation is usually written thus: $y^2 = ax$, where y is the ordinate C P of any P, and x the abscissa A C, of that point, while a is any constant multiplier of the abscissa. The equation when A is at the *vertex* of the curve, is investigated at page 610 of the PRACTICAL GEOMETRY. It follows, therefore, that the projectile in its flight always traces out a parabola: the constant multiplier in its equation is $\frac{2v^2}{g}$. It is known, from the theory of the parabola, that this constant multiplier is also 4 S A, S being the focus of the curve: it is further known, and is proved at the page just referred to, that the distance A S of any point A in the curve from the focus, is always equal to the distance of the same point from the *directrix*. Let this distance be called h, then

$$4\,\mathrm{SA} = \frac{2v^2}{g} = 4h \therefore v^2 = 2gh \quad . \quad . \quad . \quad . \quad (2)$$

But this value expresses the square of the velocity which a body would require, from the force of gravity g, by falling from rest, from the height h; consequently, the velocity of projection is equal to that which the body would acquire by falling from the directrix of the parabola, which it traces, down to the point of projection.

VELOCITY AT ANY POINT OF THE PATH.—It has just been shown that A being the point of projection, and v the velocity of projection, that velocity will be $v = \sqrt{(2g \cdot \mathrm{SA})}$. But any point P of the path may be regarded as the point of projection, and the corresponding velocity as the velocity of projection; so that the expression (2) applies equally to any point in the curve traced, h being the distance of that point below the directrix, and v the velocity at the same point. Hence, the velocity of the projectile at any point of its course is the same as the velocity it would acquire by falling vertically from the directrix down to that point. And it further follows, that at equal heights, in ascending and descending, the projectile will have equal velocity.

GREATEST HEIGHT.—In order to ascertain the greatest height to which the projectile will ascend, we must of course know the velocity of projection, and the direction of that velocity. It will also be convenient to replace this velocity by two component velocities equivalent to it in effect:—the one a horizontal velocity, and the other a vertical velocity (see page 738). If a be the angle of projection, that is, the angle of elevation of the initial direction above the horizontal line, then the velocity v of projection, will be equivalent to the horizontal velocity $v.\cos.a$, combined with the vertical velocity $v.\sin.a$.

Now, it is plain that gravity, which acts only in a *vertical* direction, cannot in any way disturb the *horizontal* velocity $v.\cos.a$, so that this velocity is the same at every point of the path. But the vertical velocity $v.\sin.a$, having the whole influence of gravity to check and oppose it, will be utterly destroyed and reduced to 0, when gravity has acted sufficiently long to impress on the body (supposing it left free to obey its solicitations) a downward velocity, equal to the upward velocity $v.\sin.a$. The spaces through which a body must fall to acquire this velocity, or the height to which it must ascend to lose this velocity, is

$$s = \tfrac{1}{2}\frac{(\text{velocity})^2}{g} = \tfrac{1}{2}\frac{v^2\sin.^2 a}{g} = h\sin.^2 a \text{ (by equa. 2)}$$

Hence, the greatest height to which the projectile discharged in a given direction can ascend, will be found by multiplying the height (h) through which a body must fall, to acquire the velocity (v) of projection, by the square of the sine of the angle (a) of elevation.

With the same projectile, velocity v, the highest point to which the projectile can ascend under different elevations, will of course be that due to the elevation of 90°; and we see accordingly, that the multiplier $\sin.^2 a$ is greater for this value of a, than for any other.

TIME OF FLIGHT.—The time occupied in the flight of the projectile, that is, the time from discharging it till it falls to the horizontal plane passing through the point of departure, will of course be double the time in which the vertical velocity is destroyed, as the body must fall to the horizontal plane with the same vertical velocity with which it left it. The time in which, by gravity, the velocity $v\sin.a$ would be generated, is

$$t = \frac{\text{velocity}}{g} = \frac{v.\sin.a}{g}$$

$$\therefore 2t = 2\frac{v.\sin.a}{g}, \text{ the time of flight.}$$

The time of flight, therefore, as might be expected, is the greater (with the same velocity of projection), the nearer the direction of projection approaches to the vertical; since sin. a increase if a increase and does not exceed 90°.

RANGE OF THE PROJECTILE.—The range is the distance on the horizontal plane, through the point of departure, at which the projectile falls. It has just been seen that the time of flight is $2\frac{v.\sin.a}{g}$, and since the horizontal velocity $v.\cos.a$ is uniform all this time, we have only to find the space due to the velocity $v.\cos.a$ in the time $2\frac{v.\sin.a}{g}$, that is, we have

$$\text{Range} = v\cos.a \times 2\frac{v\sin.a}{g} = \frac{v^2 \times 2\sin.a\cos.a}{g} = \frac{v^2}{g}\sin.2a$$

Hence, with the same velocity v of projection, the range increases as sin. $2a$ increases, and this it does as a increases from $a = 0$ up to $a = 45°$: therefore, with the same velocity of projection, the range at an elevation of 45° will be greater than at any other elevation whatever. And since the sine of an angle is the same as the sine of its supplement, that is, since sin. $2a$ is the same as sin. $(180 - 2a)$, it follows that the elevation $90° - a$ gives the same range as the elevation a: hence the ranges of projectiles, at any elevations above 45°, are the same as the ranges at elevations as much below 45°.

The length of the *maximum* range, that is, when $a = 45°$, is shown above to be $\frac{v^2}{g}$; v being the initial velocity, and this, as proved above (equation 2), is the same as $2h$: so that the length of the maximum range is equal to twice the height due to the velocity of projection.

If the range of a shot with a known elevation of the piece be ascertained, it will be easy to determine what the range would be with the same charge of powder at any other elevation. For calling the maximum range, or that due to the elevation $a=45°$, R, and the range due to any other elevation (a') r', we have, from the result just deduced, $r'=R \sin. 2a'$; so that by help of the maximum range, any other range, with the same charge, may be easily found. Or if we know any range r, corresponding to the elevation a, then to determine the range r', corresponding to another elevation a', we have the two equations,

$$r'=R \sin. 2a' \text{ and } r=R \sin. 2a$$

$$\therefore \frac{r'}{r}=\frac{\sin. 2a'}{\sin. 2a} \therefore r'=\frac{\sin. 2a'}{\sin. 2a}.r.$$

The following are the principal results in the foregoing theory of projectiles in a non-resisting medium :—

$$\text{Time of flight, } t=2\frac{v}{g}\sin. a=2\sin. a\sqrt{\frac{2h}{g}}$$

$$\text{Range, } r=\frac{v^2}{g}\sin. 2a=2h\sin. 2a',$$

$$\text{Greatest range, } R=\frac{v^2}{g}=2h,$$

$$\text{Greatest height, } H=h\sin.^2 a.$$

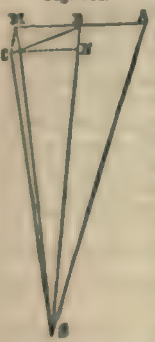
Where g is 32·2 feet, a the angle of elevation, v the velocity of projection, and h the height from which a body must fall from rest to acquire that velocity.

These results, however, ought to be regarded as but purely theoretical. In practice, the resistance of the air is so great in high velocities, as to render them almost useless : the mathematical doctrine of projectiles, through a resisting atmosphere, is full of difficulties ; and as the laws of resistance are as yet but insufficiently established, practical men must be guided chiefly by the results of actual experiments. The experiments in gunnery, by Dr. Hutton, as given in his *Mathematical Tracts*, may be consulted with advantage.*

CIRCULAR MOTION—CENTRIFUGAL FORCE.—If a body move uniformly in the circumference of a circle, the force to which that motion is due, must reside in the centre of that circle. This is only a particular case of the following more general proposition : namely, that if a body describe any curve, in virtue of a force continually diverting it from its wonted rectilinear path, that force must reside in a point such that, conceiving a line to join that point with the moving body, this line, moving with the body, must sweep over equal sectional areas in equal times.

This may be readily proved by first establishing the direct proposition, namely—that if a body describe a curve about any centre of force, and a line be supposed to unite that centre with the body, equal areas will be described in equal times.

Fig. 144.

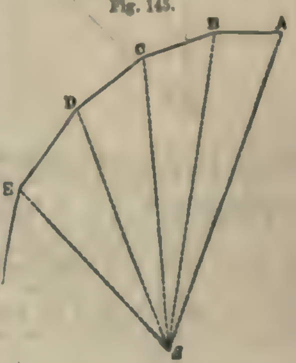


Let an attractive force at S (Fig. 144), act upon a body at the distance S B ; and to simplify the inquiry, imagine at first, that instead of acting continuously, the force exerts itself only at short regular intervals ; that, in fact, it consists of only equal successive impulses.

Let A B be the rectilinear path described by the body during any one interval between two successive impulses. At B the impulse is repeated, and the course of the body is diverted into the new path B C, which is described during the next interval ; at the end of which another impulse, in the direction C S, bends its path again, and so on. In the two intervals here considered, the triangular areas S A B, S B C, will have been described ; and it will be easy to show that these areas, thus described in equal times, are equal. For if, when the body was at B, no fresh impulse had been given to it, it would have moved forward uniformly to M, describing B M, the continuation of A B, equal to A B ; in which case the triangular areas S A B, S B M, would, of course, have been equal : it remains to show that the latter of these is also equal to the triangular area S B C.

The impulse from S, if it had acted alone on the body when at B, would have brought it to some point N in B S, in the prescribed interval of time ; and its uniform motion along A B, if undisturbed, would in the same interval have brought it to M. In virtue of these two uniform motions, the one from B to N, and the other from B to M, in the same time, the actual motion of the body is along the diagonal B C of the parallelogram M N. Hence M C being parallel to B S, the two triangles B M S, B C S, upon the same base B S, are between the same parallels, and are therefore equal ; therefore the triangle B C S is equal to the triangle A B S. It follows, therefore, that in the equal times, equal triangular spaces have been described.

Fig. 145.



In the same manner as it has now been shown, that the triangular area S A B, described in the first interval of time, is equal to that S B C described in the second equal interval, so may it be shown that the triangular area described in the third interval, is equal to that described in the second, and so on. Hence these triangular areas, in the margin (Fig. 145), being all equal, it follows, uniting any number of them in one area, that equal areas are thus described in equal times.

As this is true, however small the individual intervals of time between the impulses may be, it is true when these intervals are insensibly small, or when the successive impulses follow in one continuous force, and the lines A B, B C, &c., unite in one continuous curve. Hence, when a body is constrained to move in a curve, by the continuous action of a central force, equal areas are described about the centre of force in equal times.

Conversely, if equal areas are described about a point in equal times, the centre of force governing the motion of the body must be at that point.

Suppose the two equal areas S A B, S B C, to be described about S in equal times (Fig. 144) : then if the force acting on B were not in the direction B S, M C, parallel to the direction of the force, would not be parallel to B S ; that is, two triangles S B C, S B M on the same base B S, though *not* between the same parallels, are equal, which is impossible (Euc. 40 of I).

It follows from this, that, if a body under the influence of a continuous force, move uniformly in the circumference of a circle, the force must be at the centre of that circle, because equal sectional areas are described in equal times, since equal arcs are.

The force by which a body moving in a curve is drawn at any instant towards the centre of attraction S, is called the *centripetal* force at the distance the body is at that instant : the opposing force with which the body tends to fly off from the centre, and proceed in a rectilinear path, is called the *centrifugal* force. In circular motion these forces are everywhere exactly equal ; for neither, at any point of the circular path, prevails over the other, the distance from the centre of force S being everywhere the same. Let the arc A B (Fig. 146) be described in one second of time : draw B E perpendicular to A S ; then in one second, the body, originally at A, will have fallen from its wonted straight path A M, a distance = A E towards the attractive force at S : hence, 2 A E expresses the intensity of that force (page 736) acting on A. Join BA′; then since the arc A B differs insensibly from its chord (for the time of describing it may be regarded as minute as we please), we may regard A B A′ as a right-angled plane triangle, since the angle B is in a semicircle; therefore (Euc. 8, VI).

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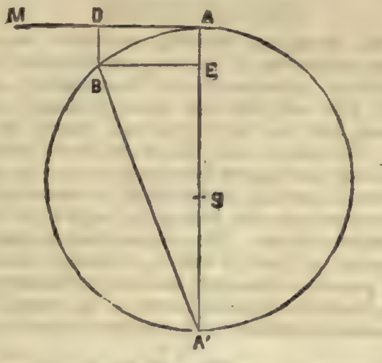
* These, however, have become of little use if applied in calculating the velocity of bullets and balls of a conical shape, as used in the rifle and rifled ordnance, by which the elements of resistance and "windage" are so materially lessened.—ED.

Fig. 146.



$$\text{AE} : \text{AE} :: \text{AB} : \text{AA}'$$

$$\therefore \text{AE} = \frac{\text{AB}^2}{\text{AA}'} = \frac{\text{AB}^2}{2\,\text{AS}} \therefore 2\,\text{AE} = \frac{\text{AB}^2}{\text{AS}}.$$

Now 2 A E represents the accelerating force at S, or, taken in an opposite direction, it represents the centrifugal force f, and A B represents the velocity v in the curve; consequently the centrifugal force $f = \frac{v^2}{r}$, where r = radius.

If as usual π be made to stand for the number 3·14159, &c., the whole circumference of the circle will be $2\,\pi\,r$; therefore, calling the whole time of describing the circumference—that is, the periodic time—t, and remembering that the uniform velocity v is equal to the whole space divided by the whole time, we have

$$v = \frac{2\,\pi\,r}{t},\ f = \frac{4\pi^2 r}{t^2}.$$

In the foregoing reasoning we are required to take t so many *seconds;* but the unit of time may be regarded as much smaller than this as we please; and thus the error of confounding the arc A B with its chord may be entirely removed, so that we may conclude rigorously that—

The centrifugal force in a circle varies as the radius divided by the square of the time of describing it: thus, for another circle, we should have $\text{F} = \frac{4\pi^2 \text{R}}{\text{T}^2}$;

$$\therefore f : \text{F} :: \frac{r}{t^2} : \frac{\text{R}}{\text{T}^2};$$

where f, F stand for the centrifugal or centripetal forces at the respective centres of circles of radii r, R, and in which the periodic times are t, T. If $\text{R} = r$, that is, if the circles are equal, the centrifugal forces F, f are inversely as the squares of the times T, t.

An interesting application of these results is to the determination of the centrifugal force at different places on the surface of the earth, from knowing the time of one rotation on its axis. But in studying this application, the student will not fail to observe that the circular path of a body, on the surface of the rotating earth, is not described under the same circumstances as the circular path is considered to be described in the present article. *Here* there is no solid matter interposed between the circulating body and the central force; there is nothing to prevent its falling to the centre at any point of its orbit; it is kept always at the same distance from the centre, simply because the force pulling it towards that centre is exactly balanced by that driving it from it: the centrifugal force is just sufficient to deprive the body of all weight or pressure towards the centre. With the earth it is different: if a perforation be bored in the earth, beneath a body on its surface, the body will fall down it; because the centrifugal force, driving it from the centre, is less than the attracting, or centripetal, force pulling it the other way.

CENTRIFUGAL FORCE AT THE EARTH'S SURFACE.—The earth, by its diurnal rotation, carries round, with a uniform velocity, every point on its surface in 86164 seconds. At the equator, the radius R is about 20922000 feet; therefore the centrifugal force F at the equator is

$$\text{F} = \frac{4\,\pi\,\text{R}}{\text{T}^2} \times \frac{4\,\pi \times 20922000}{86164^2} = \cdot 111245 \text{ feet.}$$

As this force opposes the force of gravity, it follows that if the earth had no rotation on its axis—that is, if no centrifugal force existed—gravity at the equator, instead of being what it really is, namely, $g = 32\cdot088$ feet, would be $\text{G} = g + \cdot111245$, and thus the weight of a body there would be a $\frac{\cdot111245}{32\cdot088}$ part more than it actually is.

The ratio of G to F being

$$32\cdot119 : \cdot1112 \text{ or } 289 : 1 \text{ nearly};$$

$$\therefore \text{F} = \frac{\text{G}}{219} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

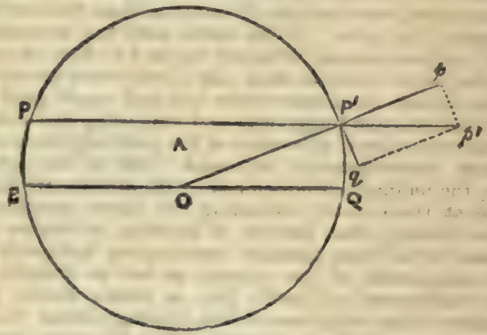
Now every parallel to the equator being carried round in the same time T, as the equator itself, by representing the centrifugal force in the parallel whose latitude is l, and radius r, by f, we shall have

$$\frac{f}{\text{F}} = \frac{r}{\text{R}} \therefore f = \text{F}\frac{r}{\text{R}} = \frac{\text{G}}{289} \cos l \quad . \quad . \quad . \quad (2)$$

Since, as is evident, $r = \text{R} \cos l$.

The force of gravity is not diminished by the whole of the centrifugal force, except at the equator; because in any parallel P A P′ (Fig. 147) this force acts, not in the direction P′ p wholly opposed to the force of gravity, but in the direction P′ p'. If, therefore, we decompose the force P′ p in the perpendicular directions P′ p, P′ q, the former component, being directly opposed to the force of gravity, and the latter component being a force acting tangentially, and therefore urging the particle P towards the equator, we shall have

Fig. 147.



Force opposing gravity, $\text{P}'\,p = \text{P}'\,p' \cos.\ p\ \text{P}'\,p' = f \cos.\ l$.

Hence (2) the expression for the diminution of gravity, in consequence of centrifugal force, at latitude l, is

$$\text{Diminution of equatorial gravity G in lat. } l = \frac{\text{G}}{289} \cos.^2 l.$$

Consequently the amount of diminution varies as the square of the cosine of the latitude.

The other component P′ q of the centrifugal force at P, being tangential, tends to drive the particles of the revolving body from the region of the poles to that of the equator, and to cause the body to assume the figure of an oblate spheroid, which is the figure that the earth has assumed: the expression for the tangential force is

$$\text{Tangential force, P}'\,q = f \sin.\ l = \frac{\text{G}}{289} \sin.\ l \cos.\ l$$

$$= \frac{\text{G}}{578} \sin.\ 2l,$$

which therefore varies as the sine of twice the latitude. The force thus tending to accumulate matter about the equatorial regions, is obviously greatest at lat. 45°, for there $2\,l = 1$.

From the foregoing principles we may readily determine the time in which the earth must perform its diurnal rotation, in order that the centrifugal force at the equator may be exactly equal to the force of gravity there; that is, in order that a body at the equator may have no weight.

Let T' represent the time of rotation corresponding to which the centrifugal force would be equal to that of gravity G; then the time at present being T, since the centrifugal forces in the same circle are inversely as the squares of the times of describing that circle (page 741), we have equation (1)

$$G : \frac{G}{289} :: T^2 : T'^2;$$

$$\therefore T' = \frac{T}{\sqrt{289}} = \frac{T}{17}.$$

Hence, if the diurnal rotation of the earth were performed in the 17th part of the time really occupied—that is, if it were to turn round seventeen times as rapidly—bodies at the equator would lose all their weight; and would, therefore, if placed at a small distance above the surface, remain suspended without any support. If the rotation were more rapid than this, no body could remain on the surface: everything would be repelled from it by the centrifugal force.

It has been demonstrated, however, that a spheroid, if fluid, and of the same density as the earth, could not remain in equilibrium if it were to rotate in a shorter time than 2h. 25m. 36s. (See *Airy's Tracts*, second edition, p. 150).

From what has now been shown, it will be observed that the superior accumulation of matter about the equatorial regions of the earth arises from two circumstances, or rather it is a twofold effect of the same cause. The centrifugal force generated by the diurnal rotation, acting in planes perpendicular to the axis, and on all the particles in each plane, and on the remoter particles of the plane more intensely than on the nearer, necessarily occasions the surface of the earth about the equator to recede more from the axis of rotation than the surface nearer to the poles. If the earth were not a rotating body, and an external influence opposed to that of gravitation were to diffuse itself like a belt round the equatorial regions, the matter under this belt would yield to the influence, and bulge out; and supposing the entire surface to be fluid, the waters of the polar regions would become proportionately depressed. But in the actual case, the bulging out occasioned by the direct centrifugal force at the equator, exceeding the direct centrifugal force at every other parallel of latitude, is further increased by all these parallels contributing indirectly to the enlargement of the equator, as well as directly to the enlargement of itself; for, as shown above, the equator is the only one of the rotating circles which expends none of the force upon it in a tangential direction. Every other circle—every parallel of latitude on either side, contributes tangentially its own supply of matter towards the equatorial regions; this supply, however, becoming less and less as the parallels recede from that of 45°.

On account of the solid materials of which a portion of the earth is composed, and the particles of which resist separation by their cohesion, the effects of centrifugal force are in some degree counteracted; but it is the general opinion of philosophers, that the present solid parts of the earth were once in a fluid or semi-fluid state —in fact, in a state of fusion; and that the outer crust has become solidified by cooling.

But those portions of the earth's surface which still remain fluid, observably yield to the tangential influence of the centrifugal force. There is a tendency, as observation shows, in the waters north and south of the equator to flow towards that circle; and as they flow from parts where the velocity of the diurnal rotation is less than the velocity at the equator, the streams from high latitudes are gradually left more and more behind in their lateral approach to the equator, where the diurnal velocity is greatest. As this velocity is towards the east, we may therefore expect to meet with great westerly currents in the open seas north and south of the equator which is conformable to experience.

MOVING FORCES.

In the foregoing remarks we have confined our attention exclusively to motion, unconnected with any considerations in reference to the quantity of matter moved. And although we have repeatedly used the term *body*, yet we have paid no regard to bulk or weight. The forces with which we have been dealing have, indeed, throughout been supposed to be analogous to the force of gravity, which gives the same velocity, in the same circumstances, to a single particle of matter as to the largest volume. We propose now to enter upon a few inquiries in which it will be necessary to take into account, *quantity of matter;* and as preliminary to these, some explanation must be given of the sense in which certain terms, hereafter to be employed, are to be understood.

Every person has a tolerably clear notion of *weight:* if two substances, however differing in bulk or volume, have the same weight at the same place, we infer that they contain the same quantities of matter; this is briefly expressed by saying that they are equal in *mass.* Yet mass and weight are not to be regarded as expressing the same thing; if any mass, or quantity of matter, were weighed at the pole, and then the same mass weighed at the equator, these weights would be different; though of course the transportation of the matter, from the pole to the equator, could not produce any change in its constitution.

A solid body may be considered as made up of particles or atoms, the aggregate of which constitute its volume: these particles may be packed (so to say) more closely in one body than in another of the same volume; so that the former may contain a greater quantity of matter than the latter; that is, a greater number of particles. Referring to this more compact constitution of the body, we say that it is more *dense* than the other body. *Density*, therefore, has reference to the quantity of matter in a given bulk; and if of two bodies of equal bulk, one be found to weigh, at the same place, *m* times as much as the other, the former is said to have *m* times the density of the other; it contains *m* times the quantity of matter, and is therefore *m* times the mass of the other.

The proper way, therefore, to represent the mass of any volume V, with a view to a correct comparison with other volumes, is this: Mass = Density × Volume; or, in symbols, $M = D\,V$. And it is to be observed that density is employed only comparatively, not absolutely; as when we say that lead has about eleven times the density of water, we mean that about eleven times as much matter is packed into a given volume of lead, as into an equal volume of water; so that the mass of a cubic inch of lead is eleven times the mass of a cubic inch of water.

As already observed, the mass of a body is proportional to its weight at the same place: the dynamical effect of this weight (W), or rather its dynamical measure, is Mg, where M is the mass, and g the force of gravity acting on it. And, in like manner, if a force f analogous to, but different in intensity from gravity, were to act on the mass M, the dynamical measure of the effect would be Mf, which is called the *moving force;* so that in the equation $F = Mf$, M is the mass, or quantity of matter moved; f is the accelerating force acting on it, and F is the moving force, the dynamical measure of the effect. In statics this would be pressure or tension.

The product of the mass, by the velocity with which it moves, is called the *momentum* of the body; as the velocity increases, therefore, so does the momentum. In uniform velocity, the momentum is constant: in uniformity accelerated velocity, additional momentum is generated every second; the increments of momentum being constant, like the increments of velocity. In the preceding expression for F, namely, $F = Mf$, f is no other than the constant increment of velocity generated in one second; consequently the momentum generated in one second, expresses, in fact, the moving force F; just as the velocity generated in one second represents the accelerating force f.

This latter force, as sufficiently seen in the preceding remarks, is all we have to attend to in investigating the motion of any single body subjected to its influence. As it acts upon all the particles of the body alike, the consideration of the number of these particles, or the mass of the body, would be superfluous. But when, instead of a single isolated body, we have to examine into the motions of a system of bodies, connected together by any ties, as cords, rods, &c., or mutually acting upon one another in any way; then, as it is easy to perceive, the quantity of matter in each body becomes an important item of consideration.

From what has now been said, the student will be prepared to give his assent to the following expressions, where V represents volume, and D density :—

The mass $M = D\,V$;

The moving force $F = M f \therefore f = \frac{F}{M}$ the accelerating force;

The momentum $= M v$.

But in order that he may attach an intelligible meaning to these symbols and relations, and may not mistake their proper signification, we shall here exemplify their interpretation in a particular case.

Suppose the volume of a body, that is, its magnitude, to be 10 cubic feet, and that in each cubic foot there is three times the quantity of matter that there is in a cubic foot of some standard substance—say pure water. Then, representing the density of water by unity or 1, the quantity of matter in the body before us, that is, the mass M will be

$M = D\,V = 3 \times 10 =$ the mass of 30 cubic feet of pure water;

that is to say, there is as much matter in the proposed 10 cubic feet as there is in 30 cubic feet of pure water.

If this mass move with a uniform velocity of 12 feet per second, the momentum of it, or the force of the blow with which it would strike an obstacle, would be

$M v =$ the momentum of 30 cubic feet of pure water, or of 30 cubic feet of a substance of the same density as water, moving at the same rate,—viz., 12 feet per second.

If the velocity of the moving mass, instead of being uniform, is constantly accelerated, the constant acceleration being $f = 6$ feet per second, then there is an acceleration or accumulation of momentum; and as, when uniform, the momentum is $M v$, so the constant accumulation of it, called the moving force, is

$M f =$ the moving force (or constant accumulation of momentum) of 30 cubic feet of a substance of the same density as water accelerated 6 feet per second.

When the velocity of the moving body is uniform, there is no moving force; that is, there is no accumulation of momentum, because there is no accumulation of velocity. If motion be not the result of the force f acting on the body, the effect must be pressure or weight.

The preceding explanation of the sense in which the symbols are to be understood, may be varied a little as follows:—Let the quantity of matter in a cubic foot of pure water be taken for the unit of mass, and the density of pure water for the unit of density.

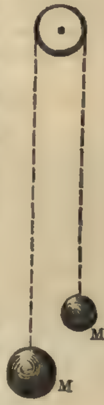
Let the momentum of a cubic foot of pure water, or of a substance of equal density, moving with a uniform velocity of one foot per second, be taken for the unit of momentum. Let the accumulation of momentum, or the constant quantity of momentum generated in one second in a cubic foot of pure water, or in the same volume of a substance of equal density, moving with an accelerating velocity of one foot per second, be taken for the unit of moving force. Then V representing the *number* of cubic feet in the volume of any body B, M the *number* of units of mass, v the *number* of feet per second in the velocity, and f the *number* of feet per second of acceleration, we shall have

$M = D\,V$, the number of units of mass in the body B;
$M v$, the number of units of momentum;
$M f$, the number of units of moving force.

We shall now proceed to some applications of the theory of moving forces.

APPLICATIONS OF MOVING FORCES.—1. Two heavy bodies whose masses are M M′, are connected together by a string which passes over a fixed pulley: required the circumstances of the motion.

Fig. 148.



Let M be the greater of the two masses; then M — M′ is the mass which, acted upon by gravity, causes M to descend and M′ to ascend. The moving force F, therefore, to which the motion is due, is $F = (M - M')\,g$; and, therefore, the accelerating force f, which, as above, is equal to F divided by the entire mass moved, is

$$f = \frac{M - M'}{M + M'}\,g.$$

And since the velocity generated in t seconds is ft, and the space passed over $\frac{1}{2}ft^2$ (page 736), we shall have, for the velocity of M downwards, or of M′ upwards in t seconds,

$$v = \frac{M - M'}{M + M'}\,gt, \text{ and for space, } s = \frac{1}{2}\,\frac{M - M'}{M + M'}\,gt^2.$$

These expressions determine the velocity with which M descends at the end of t seconds, and the space through which it will have fallen in that time.

TENSION OF THE STRING.—If the string were fastened to the pulley, so as to prevent motion, M, acted upon by the accelerating force of gravity g, would exert upon it a pressure or tension Mg; but this is diminished in consequence of a part of the accelerating force, namely f, acting upon M, and wholly expended in producing motion: hence the pressure or tension suffered by the string is only the difference between the two pressures, namely:

$$T = M\,(g - f) = M\left(g - \frac{M - M'}{M + M'}\,g\right) = \frac{2\,M\,M'}{M + M'}\,g.$$

This, therefore, is the expression for the tension of the string. With regard to the other mass, M′ not only is the accelerating force of gravity g, acting downwards upon M′, wholly exerted in stretching the string, but the upward accelerating force f in addition: hence the pressure or tension due to M′ is

$$T = M'\,(g + f) = M'\left(g + \frac{M - M'}{M + M'}\,g\right) = \frac{2\,M\,M'}{M + M'}\,g,$$

the same as before; as of course it ought to be.

PRESSURE ON THE AXIS OF THE PULLEY.—What is tension as respects the string, becomes, of course, pressure when acting on the pulley. As the tension of the string is on each side the same, namely, T, as determined above, therefore the pressure on the pulley is twice this—namely:

$$\text{Pressure on axis of pulley} = \frac{4\,M\,M'}{M + M'}\,g.$$

It is scarcely necessary to remind the student, that any mass multiplied by g, in other words, the moving force due to gravity, is mere weight when exerted statically.

Thus, calling the weights of the masses M and M′, W and W′ respectively, we shall find that the foregoing expression for T is the same as $T = \frac{2\,W\,W'}{W + W'}$.

For the expression at first given may evidently be written thus, namely:—

$$T = \frac{2\,Mg \cdot M'g}{Mg + M'g}.$$

Suppose, for instance, the weights hanging to the string were 7 lbs. and 5 lbs. respectively; then the tension of the string would be $\frac{2 \times 7 \times 5}{7 \times 5}$ lbs. = 5⅚ lbs.; that is, a weight of 5⅚ lbs. would, if suspended to a fixed string, produce the same tension as is actually produced in the moving string. In like manner, the axis of the pulley sustains a pressure equal to 11⅔ lbs. As the masses are proportional to the weights, we may, in a similar way, get a numerical expression for the accelerative force *f*—that is, by substituting the weights for the masses in the above value of *f*. Thus, taking the weights as here assumed, we have

$$\text{Acceleration, } f = \frac{7-5}{7+5}\,32{\cdot}2 = 5{\cdot}36 \text{ feet per second,}$$

the accelerating force that gives motion to the system being ⅙th of the accelerative force of gravity.

As already noticed, the moving force M*g*, which combines quantity of matter with the force acting upon it, is *pressure* or *weight* when motion is not the result of the action of the force on the mass. The symbol *g*, therefore, which stands for acceleration in Dynamics, should stand for pressure or weight in Statics, as in the expression for T given above; namely,

$$T = \frac{2\,Mg \cdot M'g}{Mg + M'g} = \frac{2\,W\,W'}{W + W'}$$

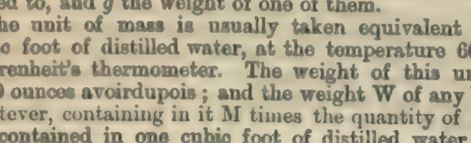
The learner, therefore, will not fall into the mistake of regarding such an expression as W = M*g* equivalent to the statement, that "weight is equal to mass multiplied by 32·2 feet." This would, of course, be absurd. The symbol *g* here stands for "the weight impressed by gravity on the unit of mass," whatever that unit may be selected to be. The unit of mass being assumed or fixed upon by common consent, then, in the above expression, M is the *number* of these units in the body referred to, and *g* the weight of one of them.

The unit of mass is usually taken equivalent to a cubic foot of distilled water, at the temperature 60° of Fahrenheit's thermometer. The weight of this unit is 1000 ounces avoirdupois; and the weight W of any body whatever, containing in it M times the quantity of matter contained in one cubic foot of distilled water, will always be W = M × 1000 ounces; and the same would obviously be the case if the unit of mass were anything else, *g* representing always the weight of that unit.

Whenever, in any mechanical inquiry, the investigation involves the mixed consideration of motion and statical pressure, although the motion and the pressure may be equally due to the same thing, and the same symbol (*g* for instance) be employed indifferently for the one or other of these different effects, yet as the *effect* is all that we intend the symbol to represent, it is plain that its signification in the one case must be different from its signification in the other. And it is of importance that the student always bear this in remembrance: he will find some further reference to the matter under the head of HYDROSTATICS.

2. Two weights W, W′ (Fig. 149), connected by a string passing over a small pulley C, are placed upon the two inclined planes C A, C B: it is required to determine the circumstances of their motion.

Fig. 149.



Let, as before, M, M′ represent the two masses. The moving force of W, in the direction W A, is M*g* sin. *i*; and the moving force of W′ in the direction W′B is M′*g* sin. *i*′: these oppose one another; so that the moving force F, to which the motion is due, is the difference of them; namely:—

$$F = (M \sin. i \backsim M' \sin. i)\,g.$$

And consequently the accelerating force *f*, which is equal to F divided by the sum of the masses moved, is

$$f = \frac{M \sin i \backsim M' \sin i'}{M + M'}\,gt.$$

This therefore is the expression for the accelerating force which urges W down the plane C A, and which, consequently, draws W′ up the plane C B.

And since the velocity generated in *t* seconds is *ft*, and the space passed over $\frac{1}{2}ft^2$, the velocity acquired by W downwards, or by W′ upwards, in *t* seconds, is

$$v = \frac{M \sin i \backsim M' \sin i'}{M + M'}\,gt;$$

and the space described is

$$s = \frac{M \sin i \backsim M' \sin i'}{2\,(M + M')}\,gt^2.$$

If the two planes were vertical, the problem would become identical with the last; we should then have sin. *i* and sin. *i*′ each equal to 1; and we see that the expressions for *v*, *s* would be those already deduced.

If only one of the planes were vertical, the problem would be converted into this—namely, to determine the circumstances of the motion when one body W, hanging vertically, draws another body W′ up an inclined plane. In this case, sin. *i* = 1; and we have only to put this in the preceding expressions, in order to obtain the necessary particulars of the motion.

Again, if one of the planes were vertical and the other horizontal, the problem would be to determine the motion when W, hanging vertically, draws W′ along a horizontal plane. In this case, sin. *i* = 1, and sin. *i*′ = 0; so that the accelerating force would be

$$f = \frac{M}{M + M'}\,g.$$

TENSION OF THE STRING.—For the tension of the string, under the original conditions, we have, if W be the descending weight,

$$T = M\left(g \sin. i - \frac{M \sin. i - M' \sin. i'}{M + M'}\,g\right)$$
$$= \frac{M\,M'}{M + M'}\,(\sin. i + \sin. i')\,g$$

which becomes the same as the expression for T in the last problem, when sin. *i* and sin. *i*′ are each of them 1. By substituting W for M*g*, and W′ for M′*g*, in each of the foregoing formulæ, numerical expressions will be obtained, as in the former problem.

3. Two weights W, W′ are attached, the latter to a wheel, and the former to its axle: to determine their motions.

Let M be the mass of W acting at the axle, and M′ the mass of W′, acting at the rim of the wheel: let also R be the radius of the wheel, and *r* that of the axle.

Then the moving forces M′*g*, M*g* acting at the distances R, *r* from the centre of motion, if free, would have the respective effects M′*g* · R, and M*g* · *r*, which, connected as they are, oppose each other. And if *f*′, *f* be the actual accelerations of W′, W, the moving forces, at the distances R, *r* from the centre of motion, are actually M′*f*′ · R, and M*f* · *r*; so that

$$M'g \cdot R - Mg \cdot r = M'f' \cdot R + Mf \cdot r \quad . \quad . \quad . \quad . \quad (1)$$

The accelerations *f*′, *f* must be to each other as the radii R, *r*, the velocities themselves being always in this constant ratio, since the wheel and axle both turn in the same time; therefore $f' = \frac{R}{r} f$, therefore

$$(M' \cdot R - Mr)\,g = \left(M\,\frac{R^2}{r} + Mr\right) f$$

$$\therefore f = \frac{M'\,Rr - Mr^2}{M'\,R^2 + Mr^2}\,g$$

the accelerating force of the ascending weight W.

And since $f' = \frac{R}{r} f$, we have

$$f = \frac{M'\,R^2 - M\,Rr}{M'\,R^2 + Mr^2}\,g$$

for the accelerating force of the descending weight W′.

The velocity, in t seconds, of the former, is

$$v = ft = \frac{M'\,Rr - Mr^2}{M'\,Rr + Mr^2}\,gt$$

and the space, $s = \frac{1}{g}\,ft^2 = \frac{1}{2}\,\frac{M'\,Rr - Mr^2}{M'\,R^2 + Mr^2}\,gt^2$

And for the latter we have

$$v' = \frac{M'\,R^2 - M\,Rr}{M'\,R + Mr^2}\,gt$$

$$s' = \frac{1}{2}\,\frac{M'\,R^2 - M\,Rr}{M'\,R + Mr^2}\,gt^2$$

The student will perceive that the first member of the equation (1), expressing the difference between the moving forces that *would* act if the weights were free, is the whole amount of moving force in actual operation; which whole amount is obviously expressed by the second member of the equation.

IMPACT OR IMPULSE.—In this article we propose to consider the circumstances of the motions of bodies moving in certain straight lines from the effects of impulse, and impinging against one another.

Let any specified quantity of matter be considered as the unit of mass: if it be projected by any given impulse, it will move with a constant velocity proportional to the intensity of that impulse; this velocity may therefore be taken to represent the magnitude of the impulsion. If we take two such units of mass, and two such impulses act on them simultaneously, and in the same direction, the relative positions of the moving masses will be always preserved; so that if the two units be blended into one mass, and the two impulses be thus made to unite and form a double impulse, the same velocity will be impressed on the compounded mass. And it is plain that if M such units be blended into one mass, which receives an impulse M times the intensity we have supposed to be applied to the single unit, the same velocity would still be impressed.

Consequently, when any body whose mass is M moves from the effect of an impulse, the correct expression for the intensity of that impulse must be Mv = the mass multiplied by the velocity, and which, as before stated (p. 743), is the *momentum* of the mass: hence the momentum measures the intensity of the impulse.

COLLISION OF BODIES: DIRECT IMPACT.—When two bodies moving by impulsion in the same straight line come into collision, the shock is called direct impact. Suppose the bodies which thus impinge to be entirely inelastic, or of such materials that they are blended by the impact into one mass, our object will be, from knowing the intensity of each separate impulse—that is, the momentum of each body—to determine the momentum of the blended mass; and thence the circumstances of its motion. The following example will illustrate how the necessary particulars are to be ascertained:—

Example.—Two inelastic bodies, whose masses are M, M′, move in the same straight line, with velocities v, v': required the velocity of their united masses after impact.

The momentum of M is Mv, and that of M′, $M'v'$, the combination of these—namely, $Mv + M'v'$, is the momentum of the united mass $M + M'$.

But if v be the velocity of this mass, its momentum must be $(M + M')\,v_1$: hence, to determine the velocity v_1 after the collision, we have the equation

$$(M + M')\,v_1 = Mv + M'v'.$$

$$\therefore v_1 = \frac{Mv + M'v'}{M + M} \quad \ldots . \ (1),$$

the velocity required.

This result is on the supposition that the bodies are moving in the same direction, or that one of them, M′, overtakes the other; but if they are moving in opposite directions so that the two meet, then the expression for v_1 will be

$$v_1 = \frac{Mv \backsim M'v'}{M + M'} \quad \ldots . \ (2).$$

If Mv exceed $M'v'$, then, after the collision, the mass will move in the direction in which M was proceeding; but if $M'v'$ exceed Mv, it will move in the direction in which M′ was proceeding, as is obvious.

If $Mv = M'v'$, then the collision, at meeting, will destroy the motion of each, and bring both bodies to a stand-still, since v_1 will then be 0.

If one of the masses M′ be at rest, then since $v_1 = 0$, the velocity after impact will be

$$v' = \frac{Mv}{M + M'} \quad \ldots . \ (3).$$

In all cases, the momentum after impact must be the sum of the momenta before impact, regarding those which act in opposite directions as having opposite signs; so that the momentum lost by one body, by the collision, is exactly equal to the momentum gained by the other. In the case (3) above, the momentum of the whole mass, after impact, being $(M + M')\,v_1 = Mv$, and the momentum gained by M′, which was at rest, being $M'v_1$, this must be the momentum lost by M. This loss of momentum in M shows that the mass M′ opposes a resistance to the communication of motion, and $M'v_1$ expresses the value of that resistance, which, as the mass was at rest, can be due only to the inertia of the body.

Let us now suppose that the bodies, instead of being perfectly inelastic, are perfectly elastic; and that, as in the former case, they move so as to impinge at some point in the common line described by their centres of gravity.

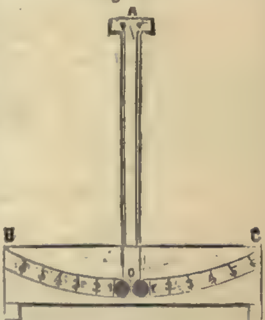
Elastic bodies are such as yield to the force of impact, and undergo a compression, and therefore a change of figure: the elasticity is that inherent force which the body exerts to recover its original form.

If the restoring force which the body exerts to recover its original figure be equal to the impressing force at the point of impact, so that the original form of the body is perfectly restored, and in the same length of time that it took to alter that form, the elasticity is said to be perfect.

In the case of perfect elasticity, whatever velocity one body loses during the action of the compressing force, it afterwards loses just as much more during the action of the restoring force; and whatever velocity the other body gains during the compression, it gains as much more during the restitution. For, at the end of the time occupied in the compression, there is the same communication of momentum as in the case of inelastic bodies: at that instant the bodies, then in the closest union, must have equal velocities. The force of restitution, exactly equal to that of compression, then acts; and another effect, the opposite to that of collision, takes place: the collision brings the bodies together; the force of elasticity drives them asunder.

No bodies in nature are perfectly elastic, or completely inelastic. The most elastic substance at present known is glass. Newton, by a contrivance similar to that represented in Fig. 150, determined a very close approximation to the defect from perfect elasticity of several substances. If two balls of the same substance, and in every other respect equal, were suspended from A by slender threads, and each let fall from equal distances, measured from the vertical, on the graduated arc B C, they would, after collision at o, each return to the point in the arc from which it started, if the elasticity were perfect. With ivory balls, the elasticity bore to

Fig. 150.



perfect elasticity the ratio of 8 to 9; with glass balls, the ratio was 15 to 16. From recent experiments, of a different kind, Mr. Eaton Hodgkinson has determined the ratios to be, for ivory ·81, and for glass ·94; the latter ratio differing but very little from that deduced by Newton. The fractional or decimal part that the elasticity is of perfect elasticity in any substance, is called the *modulus of elasticity* of that substance.

Example.—Two perfectly elastic bodies, whose masses are M, M′, moving with velocities v, v', strike with direct impact: it is required to determine their velocities afterwards. While the compression continues, the bodies move as one mass; and therefore with the velocity

$$v_1 = \frac{Mv + M'v'}{M + M'} \ldots\ldots (1)$$

So that M will lose the velocity $v - v_1$, and M′ will lose the velocity $v' - v_1$; and these express the velocities communicated, but in opposite directions, by the force of restitution. Hence the velocities, after impact, will be

$$\text{of the mass M, } v - 2(v - v_1) \ldots\ldots (2)$$
$$\text{and of the mass M}', v' - 2(v' - v_1) \ldots\ldots (3)$$

Or, substituting for v_1 its value (1), we have

$$\text{velocity of M, } v - 2\,\frac{M'(v - v')}{M + M'} \ldots\ldots (4)$$

$$\text{velocity of M}', v - 2\,\frac{M(v' - v)}{M + M'} \ldots\ldots (5)$$

If each mass be multiplied by the velocity it has after impact, the sum of the products will be $Mv + M'v'$; which is also the sum of the momenta before impact: hence the sum of the momenta after impact is the same as the sum of the momenta before impact. If we subtract (3) from (2), the remainder will be $v' - v$: hence the difference of the velocities is the same both before and after impact.

If $M = M'$, that is, if the bodies are perfectly equal in mass, as well as perfectly elastic, the expressions (4) and (5) show that, after impact, the bodies will exchange their velocities, M′ moving with the velocity v, and M with the velocity v'. Hence, if one of the bodies be perfectly at rest, the other which strikes it will impart to it its entire velocity, and will itself rest in its place.

If the elasticity be imperfect, the foregoing formulæ will require modification. Let e be the modulus of elasticity of the impinging bodies; that is, let the force of restitution after compression be only the eth part of that of compression: then the *additional* velocity lost by M from the action of this force, instead of being equal to that lost directly—namely, $v - v_1$, will be only $e(v - v_1)$; and the additional velocity lost by M′, only $e(v' - v_1)$; so that the velocities after the impact will be

$$\text{of the mass M, } v - (1 + e)(v - v_1)$$
$$= v - (1 + e)\frac{M'(v - v')}{M + M'}$$

$$\text{and of the mass M}', v' - (1 + e)(v - v_1)$$
$$= v_1 - (1 + e)\frac{M(v' - v)}{M + M'}$$

The momenta after impact will be, for M, the first of these expressions multiplied by M; and for M′, the second multiplied by M′: the original momenta are for the former, Mv, and for the latter $M'v'$. Hence, the momentum lost by one of the bodies, by the impact, is gained by the other: for, as the above expressions show, this momentum is

$$\text{for M, } (1 + e)\frac{MM'(v - v')}{M + M'}$$

$$\text{and for M}', (1 + e)\frac{MM'(v' - v)}{M + M'}$$

which are the same in value, but opposite in signs.

When the bodies are perfectly elastic, it has been seen above, as an immediate consequence of equations (2) and (3), that the sum of the momenta before impact is the same as the sum of the momenta after impact; and also that the difference of the velocities of the two bodies must be the same after impact as before; that is, calling the velocities after impact V_1 and V', we must have

$$MV + M'V' = Mv + M'v'.$$

$$\text{Also} \quad V - V' = v' - v.$$

$$\therefore M(V - v) = M'(v' - V'),$$

$$\text{and} \quad V + v = v' + V'.$$

Now, if we multiply these two last results together, we shall have the equation

$$M(V^2 - v^2) = M'(v'^2 - V'^2).$$

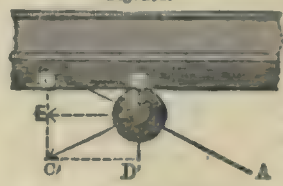
$$\text{Consequently} \quad MV^2 + M'V'^2 = Mv^2 + M'v'^2.$$

We may therefore conclude that when the bodies are perfectly elastic, the sum of the products of each body into the square of its velocity, is the same both before and after impact.

A particular name is given to the product of a mass into the square of the velocity with which it moves—it is called the *vis viva*, or the *living force*; so that in the collision of perfectly elastic bodies there is no loss of *vis viva* occasioned by the impact. The consideration of the force, thus called the *vis viva*, enters largely into certain inquiries connected with the motion of fluids.

1. A ball whose elasticity is e, strikes a perfectly hard plane—the raised edge of a billiard-table, for example: required the motion of the ball after the impact. Let A B (Fig. 151) be the straight line described by the centre of the ball before impact, and call the angle A B D′, a; then, v being the velocity along A B C, let it be represented by B C. The components of this velocity—the one perpendicular to the plane, and the other parallel to it—are B D, B E. The component B E is not modified by the impact; but B D, by the force of restitution, is converted into B D′: hence, compounding the velocities B E, B D′, the path, and velocity v_1, after impact, will be denoted by B C′.

Fig. 151.



The velocity B D′ is $e \times BD' = ev\cos. a$.

The velocity B E is $v\sin. a$;

because the velocity parallel to the plane is unaltered by the impact,

$$\therefore v_1{}^2 = (ev\cos. a)^2 + (v\sin. a)^2 = v^2(e^2\cos.^2 a + \sin.^2 a)$$
$$= v^2(e^2\cos.^2 a + 1 - \cos.^2 a) = v^2\{1 - (1 - e^2)\cos.^2 a\} \quad . \quad . \quad . \quad . \quad (1)$$

For the direction B C′, after impact, we have

$$\cot. a' = \frac{BD'}{DC'} = \frac{ev\cos. a}{v\sin. a} = e\cot. a \ldots\ldots (2)$$

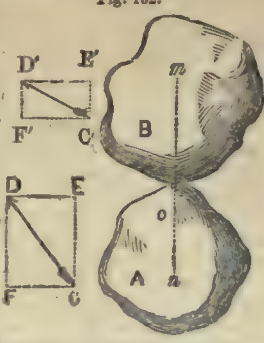
If the elasticity be perfect, that is, if $e = 1$, then, from (1) and (2), $v_1 = v$; and $a' = a$: hence, in the case of perfect elasticity, the ball will rebound with the velocity with which it struck; and the path it takes will make an angle of reflection equal to the angle of incidence.

But if the elasticity be imperfect, that is, if e be less than 1, then, from (1), v_1 will be less than v; and, from (2), cot. a' will be less than cot. a; and consequently a' will be greater than a: hence, in the case of imperfect elasticity, the velocity after impact will be less than the velocity before impact; but the direction after impact will make a greater angle with the perpendicular than the direction before impact.

2. The following diagram (Fig. 152) represents two bodies, A and B, impinging upon each other at the point o; C D, representing in magnitude and direction the velocity of the former, and C′D′, in magnitude and direction,

the velocity of the latter. Let *mn* be the line perpendicular to their surfaces at *o*. By the resolution of velocities, A may be regarded as animated by two velocities, of which one, C E, is parallel to *nm*, and the other, C F, perpendicular to the same line. In like manner, B may be regarded as animated by the velocity C′ E′, parallel to *nm*, and the velocity C′ F′ perpendicular to *nm*.

Fig. 152.



If A and B, at the instant of contact, were animated solely by the velocities C F, C′ F′, they would merely slide one past the other, and would experience no impact or shock: the shock they suffer is therefore due solely to the velocities C E, C′ E′; and even then, that any shock may take place, the former velocity must exceed the latter.

The intensity of the impact is the same, therefore, as if the two velocities C E, C′ E′, alone existed; and the two bodies will move as in the case of direct impact, only that these motions will be combined with the unchanged velocities C F, C′ F′: we shall therefore only have to compound the velocity of A in the direction *nm*, after the direct impact spoken of, with the velocity C F, perpendicular to *nm*, in order to obtain the magnitude and direction of A after the shock; and in a similar way are the magnitude and direction of the velocity of B to be found.

NOTE.—It must be noticed by the student, that what has here been taught respecting the collision of bodies, concerns only their rectilinear motions—their actual advance in space. Unless the line of direction in which two bodies strike, pass through the centre of gravity of each, rotation, as well as translation in space, will invariably be the result: the motion of translation is all that is sought to be determined in discussions on the collision of bodies; and it can be proved that this progressive motion is not in the slightest degree modified by the rotation of the impelled body. More advanced principles have fully established the following general proposition, viz.—When a body is acted upon by any impulsive forces, of which the resultant does not pass through the centre of gravity, the body will have in consequence a double motion:—1, the centre of gravity will move as if the forces were immediately applied to it; and 2, the body will rotate as if this centre were absolutely fixed. But want of space compels us to bring the subject to a close: our object has been merely to present to the young student a clear and perspicuous development of the fundamental principles of Dynamics, and not to carry him forward into those higher and more imposing applications of those principles, which necessarily demand a knowledge of much more recondite mathematical theories.

The student who has carefully mastered what is here delivered, will, we hope, find his study of the more advanced dynamical researches somewhat facilitated by the previous perusal of this elementary tract. Before concluding it, however, it is proper to give a formal enunciation of what have been called Newton's Three Laws of Motion: these, as already observed at page 734, are certain dynamical axioms, or postulates, assuming principles of too fundamental a character to admit of being rigorously established either by abstract reasoning or by experimental proof. In the foregoing pages we have, in general, tacitly taken these for granted, in several special topics of inquiry: we have preferred this course, to the usual custom of stating the three laws of motion in all their generality at the outset of the subject, because we think that their meaning and applicability cannot be clearly understood and perceived, till some familiarity with the language of Dynamics, and with a few of its more elementary problems, has been acquired.

THE THREE LAWS OF MOTION.—There are various forms of expression in which Newton's three laws are delivered by different writers on Dynamics: the following enunciation of them will perhaps be found as intelligible as any.

First Law.—A body once at rest, will remain at rest, unless acted upon by some external force; and if moving in any direction, will continue to move in that direction, unless acted on by some external force.

Second Law.—When a force acts upon a body in motion, the effect of this action is the same, in magnitude and direction, as if it acted on the body at rest.

Third Law.—This was stated by Newton as follows:—"Action and reaction are equal and contrary;" that is to say, A cannot act mechanically upon B, without A itself being reacted upon equally, but in an opposite direction.

The conjoined effect of velocity, and the moving mass, is *momentum*, as defined at page 743; and this momentum is the dynamical evidence of the "action" referred to in the law, which therefore merely affirms, that whatever momentum a body communicates in any direction, *that* momentum it loses in that direction; or, which is the same thing, it receives an equal momentum in a contrary direction.

A great deal of abstract argument, and of mechanical contrivance and experiment, have been employed to demonstrate the truth of these positions; we have neither space nor inclination to enumerate them: some things *must* be taken for granted, as fundamental or primitive principles, in every department of science. Reflection and common sense, exercised in the examination of such first principles, will usually produce a stronger amount of conviction of their truth than persuasive arguments or approximative experiments. Their verisimilitude, which renders it hard even to imagine a contravention of them, must be accepted instead of rigid proof: and when it is known that the results of the remotest investigations, all primarily resting on the truth of these assumptions, are in every instance verified by actual experience, the original *primâ facie* probability of their correctness becomes elevated into absolute certainty.

We may remark, in conclusion, that the fundamental principles of Dynamics, and the strict mathematical theories and deductions founded on them, find their fullest verification only in the movements of the heavenly bodies. Terrestrial mechanics is encumbered with many considerations operating as hindrances and drawbacks to the rigid application of those theories. Machines of human contrivance perform their functions through the intervention of rods and bars, wheels and pinions; and thus the consideration of *friction*, an obstacle that sometimes largely modifies the purely mathematical results, becomes imperatively necessary in practical mechanics. But the machinery of the heavens, without any physical ties to hold it together, goes on in obedience to an Almighty Immaterial agency; and thus the phenomena exhibited are in the completest harmony with the accurate deductions of mathematical science.

The modifying influence of friction, in the ordinary mechanical arrangements, will be examined into in the treatise on APPLIED MECHANICS.

The following particulars will often be found of service in dynamical inquiries:—

Force of gravity in the latitude of London, 51° 31′ 8″ N, 32·1908 feet.

Force of gravity in the latitude of Paris, 48° 50′ 14″ N, 32·1820 feet.

Force of gravity near the Equator, latitude 0° 1′ 34″ S, 32·0881 feet.

Force of gravity at Spitzbergen, latitude 79° 49′ 58″ N, 38·2526.

Length of the pendulum beating seconds in the latitude of London, 39·139 inches.

Equatorial diameter of the earth, 7925·465 geographical miles.

Polar diameter of the earth, 7898·972 geographical miles.

A French metre = 39·3708 English inches.

A French gramme = 15·434 grains.

CHAPTER IV.

HYDROSTATICS.

Hydrostatics is the science which treats of fluids, when maintaining a state of rest or equilibrium under the action of mechanical forces or pressures. Fluids are distinguished from solid bodies by obvious and remarkable peculiarities; and, in consequence of these differences, the statics and dynamics of the latter become only partially applicable to the former; so that the statics of water, and the dynamics of water, require a distinct and separate consideration. These two divisions of the general science of mechanics, thus applied exclusively to water, and such-like fluids, are called respectively *Hydrostatics* and *Hydrodynamics.*

To define a *fluid* would be to describe exactly what it is—its internal constitution, as well as the phenomena peculiar to that constitution, which it presents to our senses. But this is beyond our power: we know that, in common with all material existences connected with this earth, it has weight; but that, unlike solid bodies, it is without that entire cohesion of parts by which a solid preserves its shape in whatever position it be placed, and in virtue of which it is moved, as a whole, by a force applied only to a part.

It is this comparative absence of cohesion from among the constituent particles of a fluid, that renders those particles so freely movable among themselves, that causes them so readily to yield to any impression, and to obey the slightest effort to separate and detach them, as also to admit of the passage of solid bodies through them. It is the want of cohesion, too, that causes a fluid to change the figure it is made to assume, when supported in a vessel, as soon as any of that support is removed. A fluid presses laterally as well as vertically; and, in consequence of the lateral pressure, tends to spread itself horizontally when left unconstrained.

In all these particulars it differs entirely from a solid body: in *this* all the particles mutually cohere, and, in the absence of violence, maintain their relative positions. In a fluid, on the contrary, they are entirely free to interchange situations, and to move among themselves.

In this general statement of the characteristic differences between a solid and a fluid, it is to be borne in mind that we refer only to fluids such as we find them on this earth—fluids acted upon by the force of gravitation, and therefore having weight. If we conceive a vessel of water to be suspended in space, and gravity and every other force to be removed, then the fluid having no weight, there would be no pressure on the vessel in any direction; so that the water would preserve the shape of the vessel even though the latter were removed. In the absence of all force, there would be nothing to disturb the original arrangement of the component parts of the fluid mass.

TWO KINDS OF FLUIDS.—Fluids are divided into compressible fluids and incompressible fluids: the former are those which by pressure may be forced to contract into smaller space—such a fluid is the air we breathe, and every kind of gas. The incompressible fluids are those which cannot by pressure be reduced to smaller bulk. Whether, in strictness, any such fluid actually exists, is more than we can say; because, however it might resist compression from the mechanical forces at our command, it would be presumptuous to conclude that compression was impossible. This, there is reason to believe, is not the case with any fluid: water was formerly thought to be incompressible; but, by great mechanical force, it has in a slight degree been actually reduced to smaller compass. Still this, and the other common liquids, are so little compressible, that no practical error can arise from treating them as incompressible fluids.*

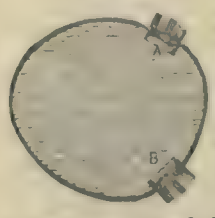
They are also classed under the heads of Elastic Fluids and Inelastic Fluids: the former are such as yield to compression; and upon the pressure being removed, expand to their former bulk. Air and gases are elastic fluids. Water, and liquids generally, are of course regarded as inelastic, as they do not admit of compression (in this sense) in any degree worth notice.

FLUID ELASTICITY.—The term elasticity, as applied to fluids, is not precisely the same in meaning as when applied to solids. A solid body is elastic when, having yielded to the force of a blow, an equal force, called the force of restitution, is exerted in the opposite direction, restoring the body to its original figure: there is no condition as to whether or not what is called the force of compression, actually causes the body to contract into smaller compass. But as regards fluids, elasticity refers to their contraction by pressure into smaller bulk, and their subsequent expansion to the original volume when the pressure is withdrawn. It is in this sense that the elasticity of fluids is to be understood. But taking the term with the signification attached to it in the mechanics of solid bodies, fluids may certainly be considered as elastic: a quantity of water poured down from any height on a hard substance, will to a certain extent rebound, and disperse itself in spray; and so likewise will it do if poured upon water itself. A flat stone, or an oyster-shell, thrown very obliquely on the surface of a pool of water will also rebound; and even a cannon-ball will do the same; but in none of these instances is the fluid compressed into smaller bulk.

TRANSMISSION OF PRESSURE.—The fundamental property of water and all other fluids is, that a pressure applied to any part of their surface is transmitted equally in all directions throughout the entire volume of the fluid: this is called the principle of equal pressure, and may be proved in various ways: for instance, if a vessel be perforated all round at any depth, and glass tubes, however bent, be inserted in the apertures, and then water or any other liquid be poured into the vessel, we find that when the level of the perforations is reached, the water presses into the tubes; and however high we raise the level of the water in the vessel, to the same level it always rises in each tube—thus showing, that on the same extent of surface at the same depth, there must be the same pressure.

Again: let a vessel of any shape be filled with water, (Fig. 153), and to any two equal perforations in its sides, let pistons A, B, be fitted; let the proper amount of pressure be applied to the piston A to keep the fluid in its place, and also the proper amount of pressure to the piston B. Then it will be found that any additional pressure applied to one of the pistons is transmitted to the other; so that equal additional pressures must be applied to both pistons to keep the fluid in its place, or to prevent one of the pistons from being forced outwards.

Fig. 153.



And if the vessel were covered with such pistons, the same pressure would be transmitted to each of the others that is applied to any *one* of them.

This very remarkable property of fluids is perfectly general, having place whatever be the contour or external form of the vessel: if, for instance, the form be that in Fig. 154—namely, A B C E F, then the downward pressure at B, arising from the weight of the water in the tube A B, produces an equal upward pressure upon every portion of surface, equal to the base of the tube, between B and C, and between B and F. Also the pressure of the fluid upon the base D E of the vessel is the same as it would be if the vessel were enlarged to the form H E, and the compartments H B, G B filled up with the fluid, the sides or partitions B C, B F being removed; for before the removal of these sides, the upward pressure on B C is the same as the downward pressure on it, from the weight of the fluid afterwards introduced into H B; the two pressures therefore neutralise one another, and thus there is no additional pressure sustained by the bottom of the vessel D E: in like manner is there no additional pressure from the additional fluid in the compartment G B.

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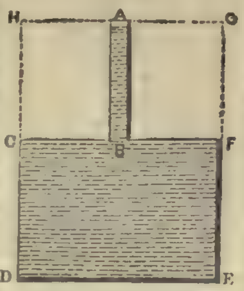
* By late experiments, it has been shown that fresh water contained in a vessel, and sunk to great depths in the ocean, undergoes evident compression.—Ed.

Fig. 154.



This is a property of fluids so extraordinary, that it has been called the *Hydrostatic Paradox*; namely, that the quantity, and therefore the weight of the fluid, may be indefinitely increased, and no increase of pressure be sustained by the bottom of the vessel, the pressure on the bottom being due entirely to the *height* of the fluid, and quite independent of its other dimensions.

Suppose in any vessel C B (Fig. 155), a solid body M, as a mass of lead, were suspended; then if a fluid be poured in to fill up the empty space about M, the pressure on the bottom of the vessel will be the same as if the mass M were removed, and its place supplied by additional fluid: the pressures on the sides, too, of the vessel must remain unaltered, whether the mass M be of lead or of the fluid; for the mass, whatever it be, so that it be incompressible by the surrounding fluid, cannot in any way modify the pressures exerted on the bottom and sides of the vessel when filled wholly with the fluid. It may be observed here, that not only is the *pressure* on the bottom of the vessel the same when the fluid in it is only the border of fluid surrounding M, as when, M being removed, it is quite full; but the *weight* of the entire vessel and contents is the same in both cases, though M be firmly supported by the beam E—for only so much of M is thus supported as is equal to the excess of its weight above the weight of the fluid which fills up the space occupied by M upon the removal of that body. The pressure of a fluid on the base of a vessel is no indication or measure of the weight of the fluid: the pressure on the base A B is the same whether the vessel filled with the fluid be C A B D, or E A B F (Fig. 156), though the weights are of course different.

Fig. 155.

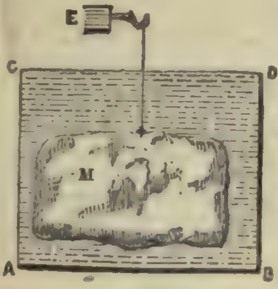
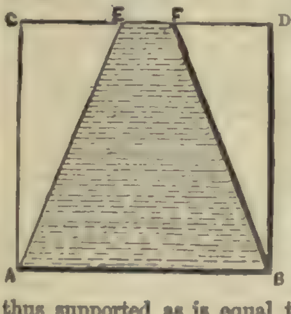


Fig. 156.

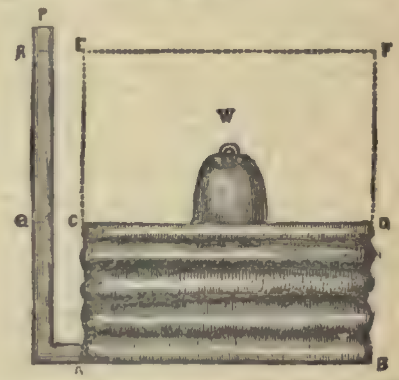


It thus happens, that by enlarging the base of a vessel, and narrowing the upper part, or by narrowing the upper part only, we may cause the pressure of the contained fluid on the base to *exceed* its weight in any given ratio: for instance, if the vessel be of the form of a cone, standing on its base, the pressure on the base will be three times the weight of the fluid itself. For the pressure on the base will be the same as if that base supported a cylinder of fluid instead of a cone of the same height; and we know that the content of the cylinder is three times the content of the inscribed cone.

Paradoxical as the statement may seem, that the pressure on the base is three times the whole weight of the fluid, the fact may be readily explained on the principle of the equal transmission of fluid-pressure. If the hollow cone be of heavy metal, and of sufficient weight to be completely water-tight when merely placed on the base, then, however great this weight of metal, if only the cone be sufficiently high, and water be poured into it through an orifice at the top, the upward pressure of the fluid will act with greater and greater intensity against the interior surface, till at length the cone will be seen to rise, forced upwards by the superior pressure, and the water will escape. This will be the subject of a problem hereafter. Equal pressures upward and downward have no effect on the weight.

HYDROSTATIC PARADOX.—But what is more emphatically called the hydrostatic paradox is this:—A B, C D (Fig. 157) are two stout boards connected together, like the boards of a pair of bellows, by a water-tight leathern band: if water be introduced into the enclosure through the pipe P or otherwise, the upward pressure of the fluid will separate the boards; and upon stopping the further supply of water, the fluid will stand at the same level both in the tube and in the receptacle C B; so that the small portion of water in the tube up to Q, balances or keeps in equilibrium the large body of water C B. If now a heavy weight be placed on the board C D—a weight equal to that of a mass of water that would fill up the space E D above the board—then the additional small portion of water filling the tube up to R would keep that weight supported.

Fig. 157.

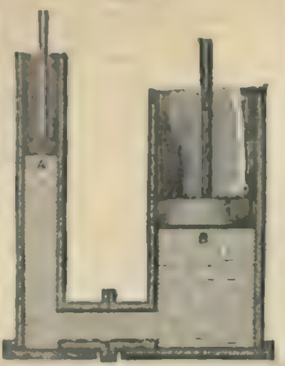


If, instead of water, the cavity C B be inflated with air, by a person standing on C D, and blowing into the tube P Q with his mouth, the same effect will take place; the person may thus easily raise himself higher and higher, and may therefore literally be said to be "blowing himself up." Such a contrivance is called the *hydrostatic bellows*.

All the phenomena hitherto enumerated are necessary consequences of the transmission of fluid-pressure in all directions. In the illustration just given, the downward pressure of the slender column of water P Q is transmitted as an upward pressure upon every area equal to the section Q of the entire surface C D; and thus the downward pressure of the body of water E D, or of the equivalent weight W, is counterbalanced.

BRAMAH'S HYDROSTATIC PRESS.—The hydrostatic bellows is little more than a philosophical toy for illustrating, in a popular and striking manner, that peculiar and important property of fluids by which pressure on a small surface is communicated undiminished to every portion, equal in area, of a large surface. Without the aid of levers, or pulleys, or wheels, or other such mechanical contrivances for accumulating and concentrating force, this fundamental property of water enables us to command as great a pressing force as we please at the expense of as little applied power as we please. One of the most interesting and useful exemplifications of this is furnished by Bramah's hydrostatic press, a machine the principle of which will be sufficiently understood from the following diagram (Fig. 158),

Fig. 158.



without entering into minute details of its construction. Water communicates with two cylinders of metal, as in the figure: one of these cylinders is of considerably larger section than the other. Water-tight pistons A, B being fitted to both, any force or pressure applied downwards to the smaller A, acts upwards upon the larger B with an intensity as many times the applied pressure as the area of it is contained in the area of B; so that by diminishing the diameter of A, or by increasing the diameter of B, we may make the upward pressure upon B as many times the downward pressure upon A as we please. Suppose, for instance, the diameter of the cylinder A is half an inch, and that of the cylinder B one foot; then as the areas of circles are proportional to the squares of their diameters, we shall have

$$(\tfrac{1}{2})^2 : 12^2 :: \text{applied pressure} :: \text{resulting pressure} = 576 \text{ times the applied pressure};$$

so that a pressure of 1 cwt. applied to the piston A, will produce a pressure of 576 cwt., or nearly 29 tons upon the piston B. It is plain that, by means of a lever, applied to the piston-rod A, the downward pressure may be increased to any required amount; and, in fact, by increasing the disparity in the diameters of the cylinders, and using only a moderate leverage on A, the transmitted pressure may be made as great as the metal cylinders can sustain. Something of this kind, no doubt, takes place in nature: water finds its way down the chinks and crevices of rocks and mountains, settling in whatever cavity within there may be to receive it. This cavity in time becomes filled, and a number of slender and irregular columns of water reaching from the reservoir to the surface, the upward and lateral pressure of the reservoir becomes at length greater than the rocky receptacle can sustain, a rupture takes place at the weakest part, and devastation is spread around. On a small scale, this effect is actually produced artificially in mining; where water-pressure is sometimes thus introduced for the purpose of blasting rocks.

It may not be altogether unworthy of notice here, that the Creator has provided a remarkable preventive for these destructive effects of fluid-pressure when exerted through a high column. The sap of trees, extending from the roots to the height of 80 or 100 feet, if it gravitated like the column of water in a Bramah's press, would rupture the trunks of the largest trees; but when fluid is introduced into *very narrow* tubes, an upward force, called capillary attraction, acts on the fluid in opposition to its downward pressure; and it is this force which sustains the sap in trees, and neutralises the downward pressure of the fluid.

EXPLANATION OF TERMS.—In the treatise on Dynamics (page 742), the meaning of the term *mass* was explained, and the distinction shown between it and weight. The weight or pressure produced by a mass M under the influence of gravity, may be denoted thus; namely, $W = Mg$. But in applying this notation, care should be taken to preserve consistency of meaning between the two members of the equation. M should be regarded as so many units of mass, just as, in Dynamics, t is regarded as so many units of time: the unit of mass may be arbitrarily chosen; and whatever it be,

$$\text{one unit of mass} \times g = \text{weight of the unit};$$

so that here g is not to be regarded as the symbol for the accelerating force of gravity, but for the *weight-force*—the force which gives to the mass M its weight or pressure; and employing M as an abstract number, namely, the number of units of mass, we have

$$1 \times g = \text{weight of the unit of mass};$$

that is, in the present inquiry, g stands for the following effect—namely, the weight impressed by gravity on the unit of mass. This is a definite and perfectly intelligible measure of the effect of gravity as a statical force; with the acceleration produced by it we have here no concern; and when, as in the current works on this subject, it is said, "let M represent mass, and g the accelerating force of gravity, and we shall have $W = Mg$," the language is calculated to mislead the student into the supposition, that the mass of a body is the 32nd part of its weight, which is of course an absurdity. The term "accelerating force" should never be employed in any statical inquiry, as it is unintelligible without reference to motion: the effect of the influence so called is, in statics, continued but stationary pressure or weight, and nothing else.

In order to estimate the mass or quantity of matter in any body, it is necessary—as in all other cases of measurement—to have reference to some conventional standard, as the unit of measure: accordingly, by general consent, the mass-unit is the quantity of matter contained in the volume unit (a cubic foot, or cubic inch) of distilled water at a certain temperature. Consequently, g stands for the weight of a cubic foot or inch, as may be agreed upon, of distilled water: as the cubic foot weighs just 1000 ounces, this is the unit to be preferred; so that the expression $W = Mg$ implies, that if we take the number of cubic feet in a body of distilled water containing the same quantity of matter as (and therefore of the weight of) W, then 1000 oz. multiplied by that number will give W, the multiplying *number* being all that is represented by M.

If a body contain D times as much matter as a body of distilled water of equal volume, the former is said to have D times the density of water; so that the density of water being taken = 1, that is, being taken for the unit of density, D will denote the density of the body: hence, if V be the volume, or rather the *number* of cubic feet in the body, DV will be the number of cubic feet in its equivalent, as to quantity of matter, of water; and therefore

$$W = Mg = DVg \quad . \quad . \quad . \quad . \quad (1)$$

Also if a body weigh S times as much as a body of distilled water of equal volume, the *specific gravity* of the former is said to be S. By the specific gravity, therefore, of any substance, is simply meant the ratio of the weight of any volume of it, to the weight of an equal volume of distilled water.

Both the density and specific gravity of any substance are expressed by the same abstract number: thus the density of mercury, as compared with distilled water, is 14; its specific gravity also is 14; but the density refers entirely to the mass, the specific gravity to the weight. As there is 14 times as much matter in a cubic foot of mercury as in a cubic foot of distilled water, the density of the former is 14 times that of the latter; and since, as a consequence of this superior density, a cubic foot of mercury is 14 times as heavy as a cubic foot of distilled water, the specific gravity of mercury is 14 also.

Hence, W and V denoting weight and volume as before, we have

$$W = S\,Vg \ (g = \text{one thousand ounces}) \quad . \quad . \quad . \quad . \quad (2)$$

INTERPRETATION OF SYMBOLS.—From the explanations now given, it will be perceived that the symbols to be hereafter employed have the following significations; namely:—

M = the *number* of cubic feet in a body of water containing the same quantity of matter (and therefore of the same weight) as the body proposed.
V = the *number* of cubic feet in the body proposed.
D = the *number* by which a volume of water must be multiplied, to give the same quantity of matter as is contained in an *equal* volume of the proposed body.
S = the *number* by which a volume of water must be multiplied to give the same weight as an *equal* volume of the proposed body. Hence, D and S

are the same abstract numbers; but the former refers to quantity of matter, and the latter to weight.

$g = 1000$ oz. avoirdupois: the weight or pressure communicated by gravity to a cubic foot of distilled water, at a temperature of 60° Fahrenheit.

All these symbols, except the pressure or weight g produced by gravity, are abstract numbers: the symbol W is of course the concrete quantity *weight*, as in common language. And weight—including under this term both pressure and tension—is the only concrete magnitude with which we have to do in Statics.

FLUID PRESSURES.—1. When a fluid is at rest, its upper surface is horizontal.

Let P Q (Fig. 159) be any two points in the upper surface of a fluid at rest, and the vertical lines P A, Q B be drawn, A B being a horizontal line of particles of the fluid. Then as A B is in equilibrio, the pressures on the extremities A, B in the horizontal direction must be equal; but the pressure on A, in the direction A B, is the same as the vertical pressure of the column of particles P A; and the pressure on B, in the direction of B A, is the same as the vertical pressure of the column of particles Q B. But these pressures are equal; therefore the column of particles P A is equal to the column of particles Q B; that is, the points P, Q are equally distant from the horizontal line A B, and are therefore themselves in a horizontal line; and in the same way is it shown that *any* two points on the upper surface are in a horizontal line, and therefore that the entire surface is a horizontal plane. It is evident, too, that if A B be the upper surface of a fluid sustaining a lighter fluid that does not mix with it, the separating surface A B will be horizontal; for, from what is shown above, every point of A B is pressed alike.

Fig. 159.

It must be observed, that what are called vertical lines, are lines perpendicular to the surface of the earth; and the upper surface of a fluid, here shown to be perpendicular to these verticals, will therefore be a surface parallel to that of the earth—a spherical surface; but it is customary to call but a small portion of such a large surface, a plane.

It is upon this property of fluids that the value of levelling instruments depends; that is, instruments which serve to show whether or not any two points are at the same horizontal level. The common level consists of a bent tube (see Fig. 160), open at the ends, which are turned up. The tube is nearly filled with a fluid, generally mercury, which supports two floats bearing sights, with a slender wire across, the wires being at equal distances from the floats. When held in the hand, the two surfaces bearing the floats are necessarily horizontal, however the tube itself may be inclined; and consequently the two wires are always on the same horizontal plane; also, whatever other objects, seen through the sights, may be on the same level as the wires, must likewise be in the same horizontal plane, or on the same level.

Fig. 160.

2. The pressure perpendicular to a surface immersed in a fluid, is equal to the weight of a column of the fluid whose base is the area A of the surface, and whose altitude is the perpendicular depth of the centre of gravity of the surface.

For let the vertical length of any linear column of particles pressing on the surface be a_1, and the point of the surface pressed be P_1. Regarding this point as a small area, we have for the pressure or weight of the column, SgP_1a_1, S being the specific gravity of the fluid. In like manner, for another vertical column of length a_2, pressing on another point P_2, we have SgP_2a_2; and so on. Hence, the whole pressure perpendicular to the surface is

$$Sg\,(P_1a_1+P_2a_2+P_3a_3+\ .\ \ .\ \ .\ \ .)$$

But if G be the depth of the centre of gravity of the assemblage of points P_1, P_2, P_3, &c., that is, of the proposed surface A, then by Statics,

$$P_1a_1+P_2a_2+P_3a_3+\ .\ .\ .\ .=(P_1+P_2+P_3+\ .\ .\ .\ .)\,G$$

$$\therefore \text{ Pressure perp. to the surface}=SAG\,.\,g$$

where AG is the volume V of a column of fluid of base equal to the area A and height G; so that $W=SVg$. Hence, if a given area A, immersed in a fluid, revolve round its centre of gravity, the pressure perpendicular to its surface must be the same in every position. Also if the area be a rectangle, the pressure upon it, when it forms the bottom of a vessel, will be double the pressure upon it when it forms one of the vertical sides; so that the pressure upon the four sides of a cubical vessel filled with liquid, is equal to twice the pressure on the base, that is, to twice the weight of the fluid.

If the sides of a vessel filled with fluid are all vertical, the entire pressure on the sides is equal to the weight of a column of the fluid whose base is the rectangle formed by developing the sides into a plane, and whose height is half that of the fluid.

By means of the preceding proposition, it is easy to find the amount of pressure sustained by a rectangular dam, or by a pair of flood-gates. If we multiply the area of the dam, or flood-gate, by half the depth of the water, we shall have the volume of water the weight of which will be the pressure. For example: let the water be 8 feet deep, and the breadth of the flood-gate 6 feet; then the area of the surface pressed is 48 feet: hence, $48\times4=192$ cubic feet of water $=12000$ pounds $=5\frac{5}{14}$ tons.

Since the centre of gravity of a straight line is at its middle point, if two straight lines a_1, a_2 be placed vertically in a fluid, the upper extremities of each being on the surface, then pressure on a_1 : pressure on

$$a_2 :: a_1\frac{a_1}{2} : a_2\frac{a_2}{2} :: a_1^2 : a_2^2$$

that is, the pressures are as the squares of the lengths. But if the lines are inclined to the surface of the fluid at the angles α_1, α_2 respectively, the perpendicular depths of the centre of gravity are

$$\tfrac{1}{2}a_1 \sin.\ \alpha_1, \text{ and } \tfrac{1}{2}a_2 \sin.\ \alpha_2$$

The pressures are therefore as

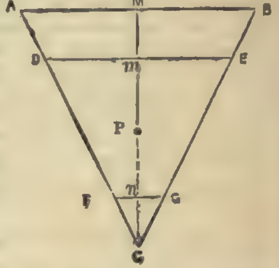
$$a_1\times\tfrac{1}{2}a_1 \sin.\ \alpha_1 : a_2\times\tfrac{1}{2}a_2 \sin.\ \alpha_2,$$

or as $$a_1^2 \sin.\ \alpha_1 : a_2^2 \sin.\ \alpha_2 ;$$

that is, as the squares of the lines into the sines of the angles of inclination, or as the squares of the lines themselves if the inclinations are equal.

If a triangle be immersed at any inclination in a fluid, with its vertex downwards and its base horizontal, and at the surface of the fluid, then the pressures on any two lines DE, FG (Fig. 161), the one as distant from the base as the other is from the vertex, will be equal.

Fig. 161.



For draw C M bisecting A B, and therefore bisecting the parallels D E, F G in m and n.

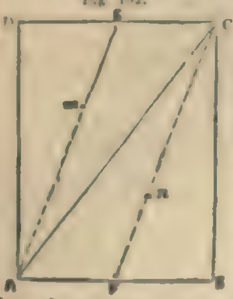
Now the pressures on D E, F F are as D E . Mm : F G . Mn; that is, as D E . Cn : F G . Cm. But Cn : Cm :: F G : D E; hence the pressures are as D E . F G : F G . D E; that is, they are equal.

If the triangle be reversed, C being at the surface of the fluid, the student may easily prove that the pressures on any two parallels are as the squares of those parallels, or as the squares of their depths.

3. If one of the sides of a vessel filled with fluid be a

rectangle, and if this rectangle be divided by a diagonal two triangles, then the pressure on one triangle will be double that on the other.

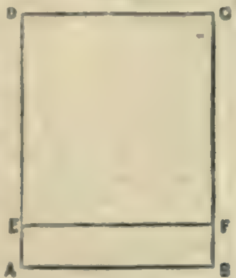
Fig. 162.



For let A B C D (Fig. 162) be the rectangle, and A C a diagonal. Let A E bisect D C, and C F, A B; and take $Em = \frac{1}{3}EA$, and $Fn = \frac{1}{3}FC$: then m and n are the centres of gravity of the two triangles. The depths of these centres are to one another as $Em : Cn$; that is, as $\frac{1}{3} : \frac{2}{3}$, or as $1 : 2$; and as the areas of the two triangles are equal, therefore the pressures upon them are as $1 : 2$.

4. Problem.—To divide the preceding rectangle, by lines parallel to the base, into n rectangles, so that the pressure on each may be the same.

Fig. 163.



Let E B (Fig. 163) be the lowest rectangle; then, since the centre of gravity of a rectangle is its middle point, the depth of the centre of gravity of A C is $\frac{1}{2}DA$, and of that of E C, $\frac{1}{2}DE$. The pressures, therefore, on these rectangles are as

$$DA \times \tfrac{1}{2}DA : DE \times \tfrac{1}{2}DE,$$
$$\text{or as } DA^2 : DE^2.$$

But the pressure on A C is divided into n equal pressures, and the pressure on A C into $n-1$ equal pressures: hence

$$DA^2 : DE^2 :: n : n-1.$$

$$\therefore DE = \sqrt{\frac{n-1}{n}}\,.\,DA.$$

$$\therefore EA = DA - DE = DA\left\{1 - \sqrt{\frac{n-1}{n}}\right\}$$

If, for instance, the rectangle is to be divided into two rectangles, so that the pressure on each may be the same—that is, half the whole pressure—then since $n=2$, $DE = \frac{1}{\sqrt{2}}DA = \frac{1}{2}DA\sqrt{2}$.

It appears from this proposition, that in constructing a flood-gate, or an upright embankment, to resist the pressure of water against it, it is unnecessary to make the material equally strong throughout; the proper degree of strength being secured at the bottom, where the pressure is greatest, the strength upwards may be diminished in the proportion of DA^2 to DE^2; that is, the strength may decrease as the square of the depth decreases.

Problem.—To determine the pressure upon the internal surface of a hollow sphere filled with fluid.

The centre of gravity of the surface pressed, is at the centre of the sphere, so that the distance of the centre of gravity from the surface of the fluid is the radius r. The area of the surface of the sphere is $4\pi r^2$, where π stands for 3·1416: hence, the pressure is the same as that of a cylindrical column of the fluid of base $4\pi r^2$ and altitude r.

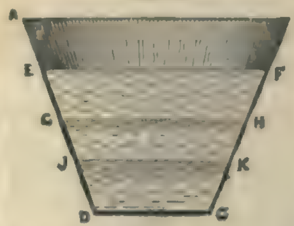
If S be the specific gravity, the weight of this column of the fluid is $4\pi r^3Sg$; but the volume of the sphere is $\frac{4}{3}\pi r^3$, and therefore the weight of the contained fluid is $\frac{4}{3}\pi r^3Sg$. Hence the pressure on the internal surface of the sphere is three times the weight of the contained fluid.

If a cone have its base equal to the surface of the sphere, and its altitude equal to the radius of the sphere, the pressure on the base of the cone will be the same as that on the surface of the sphere, when both are filled with the same fluid (page 749).

Problem.—To determine the pressure on the horizontal base of a vessel containing different fluids.

Let E F, G H, J K (Fig. 164), be the surfaces of the different fluids; these surfaces are all horizontal (p. 751). Let p, p_1, p_2, p_3, be the perpendicular depths of the several layers of fluid J C, G K, E H, A F, and S, S_1, S_2, S_3 their respective specific gravities: then the pressures of these several layers on the base D C will be,

Fig. 164.



$$DC\,.\,pSg,\ DC\,.\,p_1S_1g,\ DC\,.\,p_2S_2g,\ CD\,.\,p_3S_3g$$

and consequently the whole pressure on the base will be

$$DC\,(pS + p_1S_1 + p_2S_2 + p_3S_3)g$$

that is, the pressure on the base is found thus: multiply the area of the base by the sum of the products of the perpendicular thickness of each fluid into its specific gravity: the number of cubic feet, in the result, will be the number of 1,000 ounces of pressure.

If the depths of the several layers of fluid are all equal, then the pressure on the base will be

$$DC\,.\,p\,(S + S_1 + S_2 + S_3)\,g$$

so that if we multiply the area of the base by the perpendicular height of one of the layers of the fluid, and the product by the sum of the specific gravities, the result will be the number of 1,000 ounces of pressure.

Example.—A cylindrical vessel is filled with mercury to half its height, and the remainder is then filled with water. Supposing the specific gravity of mercury to be 14, required the pressure on the base, and on the concave perpendicular surface.

Let a be the height of the cylinder, and r the radius of its base: then the area of the base of the cylinder will be πr^2; and since $S = 14$, and $S_1 = 1$, we shall have for the pressure on the base

$$\pi r^2\,.\,\tfrac{1}{2}a\,(14+1)\,1000 \text{ oz.} = \frac{15}{2}\pi r^2a \times 1000 \text{ oz.}$$

The pressure on the concave surface of the lower half of the vessel is the same as if the water in the upper half were removed, and a column of mercury $\frac{1}{14}$th as high substituted in its stead, making the height of the entire column of mercury $(\frac{1}{2} + \frac{1}{28})a$. Hence, the area of the lower half of the concave surface being $2\pi r\frac{1}{2}a$, and the depth of the centre of gravity of it $\frac{1}{4}a + \frac{1}{28}a = \frac{2}{7}a$, we have for the pressure on that surface

$$2\pi r\tfrac{1}{2}a\,.\,\tfrac{2}{7}a\,.\,14g = 4\pi ra^2 \times 1000 \text{ oz. } (=g)$$

The pressure on the upper half of the surface is $2\pi r\frac{1}{2}a\,.\,\frac{1}{4}ag = \frac{1}{4}\pi ra^2g$: therefore the whole pressure on the concave surface is $4\pi ra^2g + \frac{1}{4}\pi ra^2g = \frac{17}{4}\pi ra^2 \times 1000$ oz. $(=g)$.

Or the pressure on the concave surface may be found thus:—remembering the fundamental principle, that a pressure exerted on the surface of a fluid is transmitted in all directions;

Pressure on lower half of cylinder by the mercury alone,

$$2\pi r\tfrac{1}{2}a\,.\,\tfrac{1}{4}a \times 14g$$

Pressure of the water on the surface of the mercury,

$$2\pi r\tfrac{1}{2}a\,.\,\tfrac{1}{2}a \times 1g$$

Pressure of the water on the upper half of the cylinder,

$$2\pi r\tfrac{1}{2}a\,.\,\tfrac{1}{4}a \times 1g$$

Sum of the pressures,

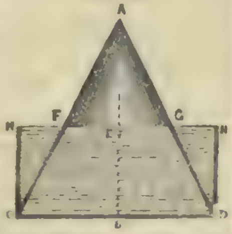
$$\frac{17}{4}\pi ra^2g = \frac{17}{4}\pi ra^2 \times 1000 \text{ oz. } (=g).$$

In a similar manner may the united pressures of layers, equal or unequal, of different fluids be always ascertained: the bottom layer exercises its own pressure, and this is increased by the weight of the whole superincumbent mass: the second layer in like manner exercises its own pressure, increased by the weight of the mass above *it*; and so on.

PROBLEM.—A hollow cone rests in a vessel with its base on a smooth horizontal plane, and water is poured in at the top of the cone. How high will the water rise before it lifts the cone off its support and escapes? (See Fig. 165).

As the water rises in the cone, it exercises an upward pressure on its slant sides, which increases as the perpendicular height of the fluid increases; and as soon as this upward pressure becomes equal to the downward pressure or weight of the conical shell, the equilibrium is just maintained, and no more water; that is, no more upward pressure can be sustained by the cone, which will therefore be lifted up, and give egress to the water: we have therefore to find the height of water, the upward pressure of which is just equal to the weight of the cone.

Fig. 165.



The upward pressure, thus just balanced by the weight of the cone, would be equally balanced if a cylinder on the same base, and of the same altitude as the water, were to surround the cone, and the vacant space to be filled with the fluid. Thus if B E (Fig. 165) were the height of the water, and if a cylinder H D were to surround the cone, and the space H F C to be filled with water, the downward pressure or weight of the water in this space would replace the weight of the cone, so that the equilibrium would remain undisturbed if the cone were withdrawn; we have therefore to determine the height B E of the cylinder, so that the weight of water contained between it and the cone may be exactly equal to the weight of the cone.

Let A B = a, B C = b, A E = x: then the volume of the cone A C D is $\frac{1}{3}\pi b^2 a$; and the volume of the cone A F G, being to that of A C D as x^3 to a^3, is $\frac{1}{3}\pi b^2 a \times \frac{x^3}{a^3}$: hence the volume of water in the frustum F D is

$$\text{volume of water in F D} = \tfrac{1}{3}\pi b^2 a\left(1-\frac{x^3}{a^3}\right)$$

$$\text{weight of the same} = \tfrac{1}{3}\pi b^2 a\left(1-\frac{x^3}{a^3}\right)g \;.\;.\;.\;(1)$$

the specific gravity of the fluid being 1.

Also, if the cone were removed, the volume of water in the cylinder H D would be

$$\text{volume of water in cylinder} = \pi b^2(a-x)$$
$$\text{weight of the same} = \pi b^2(a-x)\,g \;.\;.\;.\;(2)$$

Hence, subtracting (1) from (2), we have for the weight of water between the cylinder and cone—that is, for the upward pressure of the water in the cone on its sides,

$$\text{upward pressure on cone} = \pi b^2 g\left\{(a-x)-\tfrac{1}{3}a\left(1-\frac{x^3}{a^3}\right)\right\}$$

If, therefore, the weight of the cone be w, we shall have to solve the cubic equation

$$\pi b^2 g\left\{(a-x)-\tfrac{1}{3}a\left(1-\frac{x^3}{a^3}\right)\right\} = w$$

$$\text{or}\qquad x^3-3a^2x+2a^3=\frac{3a^2w}{\pi b^2 g}$$

$$\text{or}\qquad \left(\frac{x}{a}\right)^3-3\left(\frac{x}{a}\right)+2=\frac{3w}{\pi b^2 ag}$$

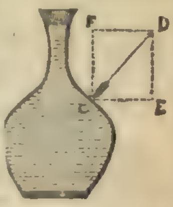
The upward pressure of the fluid compels an equal pressure downwards on the base; the water in the cavity between the cone and cylinder is just sufficient to balance the upward pressure, or to replace the resistance of the sides, the pressure on the base remaining undisturbed; and it is thus that the base of the cone supports not only the water in it, but also an amount of pressure equal to the weight of the additional water between the cone and cylinder.

In any vessel containing fluid, where all the *vertical* pressures are downwards—that is, where the sides do not any of them incline inwards—the sum of the vertical pressures must be equal to the weight of the fluid. For every vertical line of particles presses downwards with the weight of those particles; so that the whole vertical pressure is the whole weight of the fluid.

The pressure is, therefore, the same as it would be if the fluid were to become solid, and the sides of the vessel to be removed; and the effect is the same as if the entire pressure or weight were concentrated in the centre of gravity.

The horizontal pressure at any depth, is of course the same all round the vessel at that depth, the pressure of every horizontal line of particles being equal to the pressure of a vertical line of particles reaching from the surface to the horizontal line. The downward vertical pressure on the bottom of a vessel can exceed the weight of the fluid contained in the vessel, only when one or more of its sides, or a portion of a side, inclines inwards, occasioning an upward pressure, which reacts downwards on the base, the excess of the pressure on which, above the weight of the fluid, is just equal to this additional pressure; and the vertical pressure on the bottom can fall short of the weight of the fluid in the vessel, only when a side, or portion of a side, inclines outward, occasioning a downward pressure on that side. The whole downward pressure on the sides, together with that on the base, makes up the weight of the fluid.

Fig. 166.



Take, for example, the case of a common decanter: the pressure at any point C (Fig. 166) is in the direction of the straight line perpendicular to the surface. This may be decomposed into two pressures, of which one C E is horizontal, and the other C F vertical: this last pressure being directed upwards in the figure, if the point C had been near to the bottom, it would have been directed downward, on account of the curvature there being in a contrary direction. If we conceive the pressure at every point of the interior surface of the decanter to be decomposed in like manner into a horizontal and vertical pressure, there will be a series of horizontal pressures like C E, and a series of vertical pressures like C F. The horizontal components mutually destroy one another; otherwise the decanter would tend to move horizontally, which is not the case. Of the vertical components, some are upward pressures and the others downward pressures: they may, therefore, be replaced by a single vertical force, which will act upward or downward, according as the component vertical forces upward or downward prevail.* If this simple resultant pressure act upward, the pressure on the bottom of the decanter will exceed the weight of the liquid, because the downward pressure on the bottom, *minus* this upward pressure, must be equal to the weight: on the contrary, if the resultant be a downward pressure, then because the downward pressure on the bottom, *plus* this other downward pressure, is equal to the weight, the pressure on the bottom will be less than the weight.

PROBLEM.—To determine the resultant of all the pressures of a fluid upon the surface of a body immersed in it.

Instead of the immersed body C (Fig. 167), conceive the fluid it displaces to become solidified: the surrounding pressures will keep the solidified fluid at rest, just as if it were in its original state. But for these pressures, the mass would fall downwards in virtue of its weight: the resultant of the pressures, therefore, just balances the weight, and acts in a direction opposite to that of gravity; that is, vertically upwards through the centre of gravity of the mass. And as the surrounding fluid exerts the same pressures, whatever be the body whose surface is pressed, it follows that the resultant of the pressures on the surface of any solid body, is equal in intensity to the weight of the fluid displaced by it, and is directed vertically upwards through the centre of gravity of the fluid displaced.

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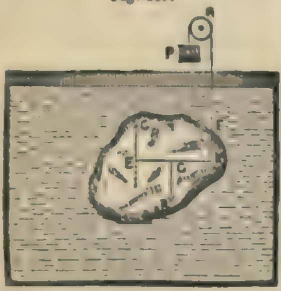
* See *Statics; ante*, p. 726.

When the solid floats at rest, the weight of the fluid displaced is equal to the weight of the solid; and the centre of gravity of the solid and that of the displaced fluid are in the same vertical line.

For the pressure of the fluid on the portion of surface immersed, and the weight of the solid, are the only forces acting: hence, as the body is at rest, the resultant of the pressures on it must be equal and opposite to the downward pressure, or weight, of the body, and act in the same vertical straight line. But since, as just shown, the resultant of the pressures (equal to the weight of the displaced fluid) is directed upwards through the centre of gravity of the fluid displaced, and the weight of the solid is directed downwards through the centre of gravity of the solid, it follows that the resultant of the pressures, that is, the weight of the fluid displaced, is equal to the weight of the solid, and that the centres of gravity of both are in the same vertical line. If, instead of floating on the surface, the body be kept from sinking by a force, acting along a string, just sufficient to keep the body at rest, the conditions of equilibrium may be found as follows:—

Let G be the centre of gravity of the body suspended in the fluid by the string F A: let E C, G B be verticals through the centre of gravity of the displaced fluid and through the centre of gravity of the body. Let also

Fig. 167.



W = weight of the body, and W′ = weight of fluid displaced;

T = tension of the string;

V = volume of the fluid displaced;

and D = density of the fluid.

The body is kept at rest by the forces $W' = D\,Vg$, acting in the direction E C; W, acting in the direction G B; and T, acting in the direction F A. These three forces must therefore be all in one plane; and T must be $= W - W'$. Through G draw E G perpendicular to E C, F A; then as the body is at rest, the moments of the forces W, W′ to turn the body about K in opposite directions, are equal (STATICS, p. 716).

$$\therefore W \,.\, GK = W' \,.\, EK = D\,Vg \,.\, EK.$$

Hence the weight acting over the pulley A upon the body at F, just sufficient to keep the body from sinking, is $W - W'$; and in order that it may be kept from turning, there must be the condition $W \,.\, GK = W' \,.\, EK$. Should the body be lighter than the fluid, and tend to float instead of to sink, then the force on F to prevent its rising will of course be $W' - W$; the other condition to prevent turning remaining the same.

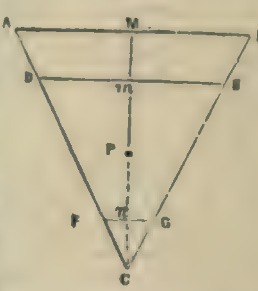
As noticed at page 749, if the vessel be full before plunging the body into the fluid, the quantity of the fluid which the immersion of the body causes to run over will occasion no diminution of the weight of the vessel and contents, nor yet any modification of the pressures on the bottom and sides: for the body merely fills the place of the bulk of fluid which its immersion drives out of the vessel. The circumstances as to the weight and pressures are the same as if the fluid, that originally occupied the space now filled by the body, had become solidified while at rest in the vessel. The weight P in the above diagram, and which measures the tension of the string, measures the excess of weight in the body above the weight of the water it displaces.

THE CENTRE OF PRESSURE.—The pressures of a fluid against the different points of a plane surface may be regarded as a system of parallel forces, acting perpendicular to the plane: the resultant of these forces is therefore perpendicular to the plane; and the magnitude or intensity of the resultant has already been shown (page 751) to be equal to a column of the fluid whose base is the surface pressed, and whose altitude is equal to the depth of the centre of gravity of the surface below the level of the fluid. The centre of pressure is that point of the surface to which, if a single force equal and opposite to the resultant of the pressures were applied, the plane would be kept at rest.

PROBLEM.—To find the centre of pressure of a fluid on a triangle whose base is horizontal and at the surface of the fluid.

Let A B C (Fig. 168) be the triangle, and draw C M to bisect the base A B; also let D E, F G be two lines parallel to A B, and drawn so that the distances M m, n C may be equal. These lines being horizontal are uniformly pressed throughout, so that the centre of pressure on each is at its middle point; and, as already proved at page 751, the pressure on one, is the same as the pressure on the other. Consequently we may regard the extremities m, n of the line m n as pressed by equal forces: the resultant of these is therefore equal to the sum of both applied to the middle point P, which point is evidently the middle point of C M.

Fig. 168.

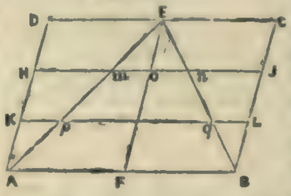


Whatever two lines be taken equidistant from M and C, the point of application P of the resultant remains the same; and as the whole pressure on the triangle may be regarded as made up of all these linear pressures, it follows that the resultant pressure must pass through P, which is therefore the centre of pressure on the triangle.

PROBLEM.—To find the centre of pressure of a fluid on a parallelogram, one of whose sides coincides with the surface of the fluid.

Let A C (Fig. 169) be the parallelogram, and draw E F bisecting the opposite sides D C, A B. The centre of pressure is necessarily in E F, as the pressures on each side of it are equal. Draw E A, E B, as also horizontal lines H J, K L, &c. Then the pressure on one of these lines, as H J, is to the pressure on A B as E o to E F; that is, as m n to A B. Hence, representing the pressure on A B by the line A B, the lines m, n, p, q, &c., will correctly represent the pressures on H J, K L, &c. Consequently if the fluid were all removed, and a pressure equal to that originally on A B be applied to that line, and also a pressure to every line p q, m n, &c., in the triangle, the pressures being always proportional to the lengths of these parallels, the parallelogram, in which the pressed triangle is inscribed, will still be at rest. But the resultant of all the pressures, thus uniformly diffused over the triangle, must pass through the centre of gravity of the triangle. Hence the centre of gravity of the triangle is the centre of pressure of the fluid on the parallelogram; and consequently the centre of pressure on the parallelogram, one of whose sides is at the surface of the fluid, is on the bisecting line E F, and at a depth equal to two-thirds the depth of the opposite or lowest side of the parallelogram. Again: suppose the upper side of the parallelogram to be below the surface of the fluid, but parallel to it; let H J, for instance, be the upper side of the parallelogram, and D C the surface of the fluid. Then, as shown above, the pressure on the parallelogram H C may be replaced by a pressure uniformly diffused over the triangle E m n; and the pressure on the parallelogram A C, by the extension of the pressure on E m n uniformly over the triangle E A B: hence the pressure on the parallelogram A J may be replaced by a pressure uniformly spread over the trapezium A m n B. Con-

Fig. 169.



sequently the centre of pressure of the parallelogram A J is the centre of gravity of the trapezium A $m n$ B.

As, in the first case, when the upper side of the parallelogram is at the surface of the fluid, the centre of the pressure is always on the same line E F, and at the same depth, however close the middle point E is to the extremities D, C—that is, however slender the parallelogram may be—it follows, that the centre of pressure of a straight line E F, having one extremity at the surface of the fluid, is at $\frac{2}{3}$rds the length of E F below E. The centre of pressure being the point about which all the pressing forces balance, it is, evidently, the point most in need of support, or where the opposing force should be more especially applied, to resist the pressure of the fluid on the surface, and thus to prevent rupture. The staves of vats and casks, which may be regarded as so many rectangles, should be each more especially strengthened at one-third of their lengths from the bottom.

SPECIFIC GRAVITIES OF BODIES.—When bodies are compared together, having the same specific gravity, any volume of one must of course have the same weight as an equal volume of each of the others; so that the weights of such bodies are to one another as their volumes. The volume of any irregular body may be ascertained by the bulk of water it displaces by being immersed (though more conveniently from its weight and specific gravity); or, for the purpose of comparing different volumes, we may merely observe the height to which the water rises in the two cases in a cylindrical vessel, upon the immersion of the bodies. The following is an example.

Example.—A mass of gold immersed in a cylinder containing water, caused the surface to rise a inches; a mass of silver of the same weight caused it to rise b inches; and a mass still of the same weight, but composed of gold and silver, caused it to rise c inches. What was the proportion of gold and silver in the compound mass?

Let x be the volume of the gold, and y that of the silver: then as the volumes immersed are as the elevations of the surface caused by the immersions, we have

$c : a :: x + y : \frac{a}{c}(x + y)$ the volume of the mass of gold.

$c : b :: x + y : \frac{b}{c}(x + y)$ „ „ „ silver.

The weights of these masses being equal, let W be the weight of each; then the weights being as the volumes when the specific gravities are the same,

$\frac{a}{c}(x + y) : x :: \mathrm{W} : \frac{c}{a} \cdot \frac{\mathrm{W}x}{x + y}$ the weight of gold in the compound,

$\frac{b}{c}(x + y) : y :: \mathrm{W} : \frac{c}{b} \cdot \frac{\mathrm{W}y}{x + y}$ the weight of silver in the compound.

$$\therefore \frac{c}{a} \cdot \frac{\mathrm{W}x}{x + y} + \frac{c}{b} \cdot \frac{\mathrm{W}y}{x + y} = \mathrm{W}$$
$$\therefore bcx + acy = ab\,(x + y)$$
$$\therefore (bc - ab)\,x = (ab - ac)y$$
$$\therefore x : y :: a\,(b - c) : b\,(c - a)$$

It is probable that in some such way as this Archimedes solved the problem proposed to him by Hiero, King of Syracuse, who having ordered a crown of gold to be made, suspected that the crown furnished to him was a mixture of gold and silver, and wished the truth to be ascertained without injuring the workmanship.

As the weight of one volume is to that of another of different specific gravity as the product of volume and specific gravity of the former is to the product of volume and specific gravity of the latter; therefore, if S, S′ represent the specific gravity of gold and silver respectively, the weights of x and y will be the one to the other as Sx to S′y; therefore, by the foregoing proportion,

wt. of the gold (x) : wt. of the silver (y)

$$:: a\,(b - c)\,\mathrm{S} : b\,(c - a)\,\mathrm{S}'$$
$$\text{or} :: \frac{a\,(b - c)}{b\,(c - a)} \cdot \frac{\mathrm{S}}{\mathrm{S}'} : 1$$

Consequently the weight of the compound is

$$\left\{\frac{a\,(b - c)}{b\,(c - a)} \cdot \frac{\mathrm{S}}{\mathrm{S}'} + 1\right\} \times \text{wt. of the silver} = \mathrm{W}$$

∴ the weight of the silver in the compound is

$$\mathrm{W} \div \left\{\frac{a\,(b - c)}{b\,(c - a)} \cdot \frac{\mathrm{S}}{\mathrm{S}'} + 1\right\}$$

PROBLEM.—Having given the volumes and specific gravities, or the weights and specific gravities of several bodies, to find the specific gravity of the compound.

1. Let the volumes V_1, V_2, V_3, &c., be given with the specific gravities S_1, S_2, S_3, &c.; and let S be the specific gravity of the compound; then since $\mathrm{W} = \mathrm{SV}g$, we have for the weight of $\mathrm{V}_1 + \mathrm{V}_2 + \mathrm{V}_3 + \&c.$

$$(\mathrm{S}_1\mathrm{V}_1 + \mathrm{S}_2\mathrm{V}_2 + \mathrm{S}_3\mathrm{V}_3 + \&c.)\,g$$
$$= \mathrm{S}\,(\mathrm{V}_1 + \mathrm{V}_2 + \mathrm{V}_3 + \&c.)\,g$$
$$\therefore \mathrm{S} = \frac{\mathrm{S}_1\mathrm{V}_1 + \mathrm{S}_2\mathrm{V}_2 + \mathrm{S}_3\mathrm{V}_3 + \&c.}{\mathrm{V}_1 + \mathrm{V}_2 + \mathrm{V}_3 + \&c.}$$

2. Let the weights W_1, W_2, W_3, &c., with the specific gravities S_1, S_2, S_3, &c. be given, S being the specific gravity of the compound as before: then since $\mathrm{V} = \frac{\mathrm{W}}{\mathrm{S}g}$, we have for the volume of $\mathrm{W}_1 + \mathrm{W}_2 + \mathrm{W}_3 + \&c.$

$$\frac{\mathrm{W}_1}{\mathrm{S}_1 g} + \frac{\mathrm{W}_2}{\mathrm{S}_2 g} + \frac{\mathrm{W}_3}{\mathrm{S}_3 g} + \&c.$$
$$= \frac{\mathrm{W}_1 + \mathrm{W}_2 + \mathrm{W}_3 + \&c.}{\mathrm{S}g}$$
$$\therefore \mathrm{S} = \frac{(\mathrm{W}_1 + \mathrm{W}_2 + \mathrm{W}_3 + \&c.)\,\mathrm{S}_1\,\mathrm{S}_2\,\mathrm{S}_3\,\&c.}{\mathrm{W}_1\,\mathrm{S}_1\,\mathrm{S}_2\,\&c. + \mathrm{W}_2\,\mathrm{S}_2\,\mathrm{S}_3\,\&c. + \mathrm{W}_3\,\mathrm{S}_3\,\mathrm{S}_4\,\&c.}$$

If m equal volumes are mixed, the specific gravity of the compound is

$$\mathrm{S} = \frac{\mathrm{S}_1 + \mathrm{S}_2 + \mathrm{S}_3 + \&c.}{m}$$

If m equal weights are mixed, the specific gravity of the compound is

$$\mathrm{S} = \frac{m\mathrm{S}_1\,\mathrm{S}_2\,\mathrm{S}_3 \ldots\ldots \mathrm{S}_m}{\mathrm{S}_1\,\mathrm{S}_2 \ldots \mathrm{S}_{m-1} + \mathrm{S}_2\,\mathrm{S}_3 \ldots \mathrm{S}_m + \&c.}$$

When there are only two bodies to be compounded, then for equal volumes, $\mathrm{S} = \frac{\mathrm{S}_1 + \mathrm{S}_2}{2}$; and for equal weights, $\frac{2\mathrm{S}_1\mathrm{S}_2}{\mathrm{S}_1 + \mathrm{S}_2}$, the former value being an arithmetic, and the latter an harmonic mean, between the specific gravities of the two substances.

PROBLEM.—To determine the volume of any substance, however irregular, of known specific gravity.

Let S be its specific gravity, and W its weight in ounces; then, since

$$\mathrm{W} = \mathrm{SV}g. \therefore \mathrm{V} = \frac{\mathrm{W}}{1000\,\mathrm{S}\ \text{oz.}}\ \text{cubic feet.}$$

In a similar way may the capacity of an irregular vessel be ascertained. Let the weight of the water that will fill the vessel be w ounces, then the capacity or volume of the vessel will be $\frac{w}{1000}$ cubic feet. If the result is to be in cubic inches instead of in cubic feet, the divisor 1000 must be replaced by the multiplier 1·728, because $\frac{1728}{1000} = 1{\cdot}728$: so that w being the number of the avoirdupois ounces in the weight, the volume or capacity will be $\mathrm{V} = 1{\cdot}728\,\frac{w}{\mathrm{S}}$ cubic inches.

Should w be the number of troy, instead of the number of avoirdupois ounces, then since

1 oz. troy : 1 oz. avoirdupois :: 480 : 437·5

$$\therefore 1 \text{ oz. avoirdupois} = \frac{437{\cdot}5}{480} \text{ oz. troy} = {\cdot}911458 \text{ oz. troy}$$

$$\therefore V = \frac{1728}{{\cdot}911458}\,\frac{w}{S} \text{ cubic inches} = \frac{1}{{\cdot}52746}\,\frac{w}{S} \text{ cubic inches.}$$

The divisor ·52746 is the troy weight in oz. of a cubic inch of water, as is obvious:—it is the value of w when $V=1$ and $S=1$. The troy weight of a cubic inch of water in grains is 253·1808 grains.

It is in general easier to ascertain the weight of a minute body than to measure accurately its dimensions; and thus the following is the method usually recommended for finding the diameter of a small sphere of known specific gravity and ascertained weight in grains troy. Let d be the diameter and w the weight in grains troy of the sphere, and S the specific gravity of the substance of which it is formed; then the volume of a sphere whose diameter is 1 inch being ·523598 inches, we have for the troy weight of an equal sphere of water

$$1 : {\cdot}523598 :: 253{\cdot}1808 \text{ grains} : 132{\cdot}5648 \text{ grains},$$

the weight of a sphere of water of 1 inch in diameter.

Now the volumes, and therefore the weights, of spheres of the same substance being as the cubes of their diameters, we have

$$1^3 : d^3 :: 132{\cdot}5648 \text{ grains} : 132{\cdot}5648d^3 \text{ grains},$$

the weight of the small sphere of water of diameter d; therefore the weight w of the proposed sphere is

$$w = 132{\cdot}5648d^3.\,S \text{ grains} \therefore d = {\cdot}19612\sqrt{\frac{w}{S}}$$

the *number* of grains being put for w.

When a body is wholly immersed in a fluid, the weight lost is to the whole weight as the specific gravity (S) of the fluid to the specific gravity (S′) of the solid.

The weight lost is the weight of the displaced fluid (page 754); and, from the general relation $W = SVg$, the weight of bodies of the same volume are as their specific gravities; therefore,

$$\text{wt. of displaced fluid} : \text{wt. of solid} :: S : S'$$
$$\text{that is, wt. lost} : \text{whole wt.} :: S : S' \ldots (1)$$

If the same body be immersed in another fluid whose specific gravity is S_1, then

$$\text{wt. lost} : \text{whole wt.} :: S_1 : S' \ldots (2)$$

The second and fourth terms being the same in the proportions (1) (2), it follows that the weights lost in the two fluids are as the specific gravities of those fluids; and hence may be ascertained the specific gravity of any fluid (obtainable in sufficient quantity for the experiment), when the specific gravity of any other fluid is known.

Also, the true weight of a body may be readily determined, from knowing what it weighs in each of two fluids of known specific gravities; for let W be the true weight and w, w' the weights in two fluids whose specific gravities are S, S′: then the weights lost are $W - w$ and $W - w'$ respectively.

$$\therefore W - w : W - w' :: S : S'.$$
$$\therefore S'W - S'w = SW - Sw',$$
$$\therefore (S' - S)W = S'w - Sw'.$$
$$\therefore W = \frac{S'w - Sw'}{S' - S}, \text{ true weight of}$$

the body.

And thus, from having the true weight of the body, and the weight lost in one of the fluids, we may, by the proportion (1) above, find also the specific gravity of the body.

By the true or absolute weight of a body is to be understood the weight of it in a vacuum, that is, free from the presence of even the air: the absolute weight, therefore, is equal to the weight of the body in air, increased by the absolute weight of a volume of air equal to that of the body; but for bodies of small volume this minute increase is inappreciable.

Example 1.—A piece of uniform substance, whose true weight is 64 grains, is found to weigh only 48 grains when immersed in distilled water: required the specific gravity (S) of the substance.

The weight lost is $64 - 48 = 16$ grains,

$$\therefore 16 : 64 :: 1 \text{ (specific gravity of water)} : 4.$$

Hence the specific gravity of the body is $S = 4$.

2. A body weighs 130 grains in one fluid, and 68 in another; but the true weight of the body is 240 grains: required the relative specific gravities of the two fluids.

The weights lost are 110 grains and 172 grains, and, as before, these are as the specific gravities of the fluids.

$$\therefore \frac{S}{S'} = \frac{110}{172}, \text{ or } S = \frac{110}{172}S'.$$

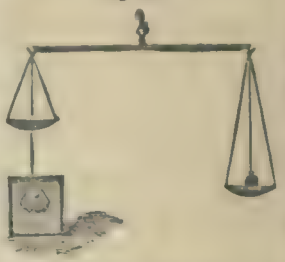
If one of the fluids be air in which the body is weighed, and the other water of specific gravity 1, then knowing the specific gravity S of the air, we have for the real weight W of the body, that is, for its weight in vacuo,

$$W = \frac{w - Sw'}{1 - S} = w + (w - w')S + (w - w')S^2 + \&c.$$
$$= w + (w - w')S \text{ very nearly,}$$

because S being a small fraction, S^2 is an equally small fraction of *it*.

The scales used in weighing bodies for the purpose of ascertaining specific gravities are called the *hydrostatic balance* (Fig. 169*). The body, after being weighed in the air, is suspended by a fine thread from one of the scale-pans, and immersed in distilled water, as in the margin; and when the weights in the other scale-pan bring the scale-beam into a horizontal position, the weight of the body in the fluid will be determined.

Fig. 169*



But in what has preceded, the solid is supposed to be heavier than an equal volume of the fluid in which it is immersed; when it is lighter than the fluid, the method of proceeding is this:—

Take a body sufficiently heavy to sink the lighter body with it, when both are united and immersed together in the fluid. Find the weight, in the fluid, of the heavier body by itself; then the weight, in the fluid, of the united mass: this latter weight will obviously be *less* than the former; as the lighter body displaces more than its own weight of water, the difference will show how *much* more. Call this difference between the two weights w', and the true weight of the lighter body w, then the weight of the fluid displaced by the lighter body will be $w + w'$. Consequently, $w + w' : w :: S$ (specific gravity of fluid) : S′ (specific gravity of the solid).

$$\therefore S' = S\frac{w}{w + w'}$$

The weight w', by which the weight of the displaced fluid exceeds that of the body, expresses the *buoyancy* of the body, or the upward pressure upon it, in virtue of which it would begin to ascend if left to itself.

If the specific gravity of a body in the form of powder is to be found, the preceding method may still be employed: the powder may be imbedded in wax, or some such yielding material, sufficiently heavy to cause the compound to sink in the fluid; the weight in the fluid, of the wax by itself, being found, and then the weight in the fluid, of the compound, the specific gravity of the powder, previously weighed in vacuo, will be given by the foregoing formula.

PROBLEM.—If in two fluids which do not mix, a solid be immersed, and rest partly in one fluid and partly in the other, to find the ratio of the two parts when the specific gravities of the solid and the fluids are known.

Let V′ be the volume immersed in the lower or heavier fluid, and V the volume immersed in the upper or lighter fluid; then from the condition $W = VSg$ we necessarily have

$$VS + V'S' = (V + V')S'',$$

where S′, S are the specific gravities of the two fluids, and S″ the specific gravity of the solid.

$\therefore V(S - S'') = V'(S'' - S')$ $\therefore \frac{V}{V'} = \frac{S'' - S'}{S - S''}$, the ratio.

Hence, adding 1 to each side,

$$\frac{V + V'}{V'} = \frac{S - S'}{S - S''}$$

If the solid float on *one* fluid, then in these results $S = 0$, that is, the upper portion V of the body is in vacuo; therefore $\frac{V}{V'} = \frac{S' - S''}{S''}$. Also $\frac{V + V'}{V'} = \frac{S'}{S''}$.

Consequently, if a body float on a fluid in vacuo, and then the air or any other lighter fluid be admitted, the body will rise, and leave a less portion of it immersed in the original fluid. Also, if a body float on a fluid, the part immersed V' is to the whole body $V + V'$ as the specific gravity S'' of the body to the specific gravity S' of the fluid.

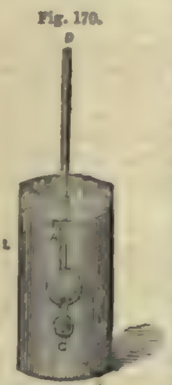
Consequently if the body float upon a second fluid of specific gravity S_1, and if V_1 be the part of the body immersed, then we have the two proportions

$$\left.\begin{array}{rl} V' : V + V' :: S'' : S' \\ \text{and } V_1 : V + V' :: S'' : S_1 \end{array}\right\} \therefore V'S' = V_1 S_1$$
$$\therefore V' : V_1 :: S_1 : S'$$

That is, the parts of the body immersed, or the volumes of the fluids displaced, are inversely as the specific gravities of those fluids. Upon this principle the instrument called the hydrometer, for measuring the specific gravities of different liquids, is constructed.

THE HYDROMETER.—There are several modifications of this useful instrument; in its simplest form, the hydrometer consists of two hollow spheres, usually of glass—a larger, and then smaller one below it, communicating with each other. Into the lower sphere mercury or small shot is introduced, in order that the spheres may sink in the liquid. From the upper sphere rises a cylindrical stem, the axis of which is in the straight line through the centres of gravity of the spheres; so that when the instrument rests in the fluid, the stem may be vertical. Let the instrument be placed in water, the stem will sink to some point A; let it then be placed in a liquid L that will cause the stem to sink to B; then, by the above-mentioned proposition, the specific gravity of L is to that of water (1), as the volume or magnitude of the body A C, to the volume or magnitude of B C.

Fig. 170.



$$\therefore \text{specific gravity of L} = \frac{\text{volume of A C}}{\text{volume of B C}}$$
$$= \frac{\text{volume of (D C} - \text{D A)}}{\text{volume of (D C} - \text{D B)}}$$

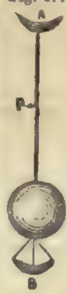
The whole volume of the instrument—that is, the volume of water it would displace if wholly immersed in it—is regarded as consisting of 4,000 parts, and the stem is divided so that each division is one of those parts. Suppose the stem to contain 50 of the parts, numbered from D downwards, and that it sinks to 30 in one liquid L_1, and to 20 in another liquid L_2: then

$$\frac{\text{Specific gravity of } L_1}{\text{Specific gravity of } L_2} = \frac{4000 - 20}{4000 - 30} = \frac{3980}{3970}$$

And in this way are the specific gravities of different liquids compared.

NICHOLSON'S HYDROMETER.—This differs from the common hydrometer chiefly in having a dish at each end, and in serving for measuring the specific gravities of solids as well as fluids. The stem in this instrument is a slender wire of hardened steel; and the adjustment is such that when a given weight—1000 grains for instance—is placed in the upper dish A (Fig. 171), the instrument will sink, in distilled water at the temperature of 60° Fah., to the point P in the middle of the stem. It is used as follows :—

Fig. 171.



To Compare the Specific Gravities of Two Liquids L, L′.—Let W be the weight of the hydrometer, w the weight which must be placed in the dish A to sink the instrument to the point P in the liquid L, w_1 the weight which must be placed in A to sink the instrument to P in the liquid L′: then

weight of the volume of L displaced $= W + w$
weight of same volume of L′ displaced $= W + w_1$

$$\frac{\text{specific gravity of L}}{\text{specific gravity of L}'} = \frac{W + w}{W + w_1}.$$

To find the Specific Gravity of a Solid.—Let w be the weight which, placed in the dish A, causes the instrument to sink to P in the water. Place the solid to be examined, or a fragment of it, in A, and let w' be the weight to be added to sink the instrument to P. Then remove the solid to the lower dish B, and let w'' be the additional weight to be added to the upper dish to sink the instrument to P.

It is plain that the weight of the solid in air, in $A = w - w'$
and that its weight in water, in $B = w - w''$

Hence the weight of water displaced by the solid $= w'' - w'$

$\therefore \frac{\text{specific gravity of the solid}}{\text{specific gravity of water}} = \frac{w - w'}{w'' - w'} =$ specific gravity of the solid, the specific gravity of water being 1.

Or, without placing any additional weight in the upper dish, observe to what depth the solid alone sinks the instrument in water; then remove the solid to the lower dish, and some additional weight w will be necessary to sink the instrument to *the same* depth; so that, calling the weight of the solid W, we shall have

$$\text{specific gravity of the solid} = \frac{W}{w}.$$

The weight W of the solid under examination is readily ascertained by finding what weight supplying its place in the upper dish will sink the instrument to the same depth.

It is evident that the instrument becomes more sensitive the slenderer the wire stem be made. If an additional small weight—a grain for instance—be placed in the dish A, an additional grain weight of the liquid must be displaced by the sinking of a suitable additional length of the stem; so that the finer this stem is, the greater must the additional length be. The diameter of the stem is in general $\frac{1}{40}$th of an inch; and the instrument will rise or fall nearly one inch by the abstraction or addition of $\frac{1}{10}$th of a grain. For the weight of a cubic inch of water is 253·1808 grains troy; and therefore the volume of $\frac{1}{10}$th of a grain is $\frac{1}{2531{\cdot}808}$ of a cubic inch. Now the area of a transverse section of the wire stem is $\frac{1}{40^2} \times {\cdot}7854 = \frac{7854}{16000000}$, and if the volume due to $\frac{1}{10}$th of a grain, namely $\frac{1}{2531{\cdot}808}$, be divided by this area, the quotient will be about $\frac{4}{5}$ths of an inch, for the length of wire that will rise or fall in water when the weight differs by $\frac{1}{10}$th of a grain. In a liquid lighter than water the length will of course be greater.

The hydrometer is the instrument used by officers of the Excise to ascertain the strength of spirituous liquors, or in what degree they are above or below proof. Proof-spirits consist of half alcohol, or pure spirit, and half water: when placed in this mixture, the exciseman's hydrometer sinks to the point marked *proof* upon the scale; if the liquor be below proof, or have more than half water, the instrument will not sink so far as proof; and if it be above proof, it will sink further, and the surface of the liquor will stand *above* proof.

But in all determinations of specific gravity, account must be taken of the temperature of the substance at the time of the experiment: all bodies expand by heat, and therefore become specifically lighter. Spirits more especially are susceptible of this increase of volume by increase of temperature: a cubic inch of brandy, for instance, will weigh about ten grains more in winter than in summer, unless the change of temperature is obviated by artificial means. It is necessary, therefore, in measuring the specific gravity of a liquid, to notice to what extent the temperature of it differs from the standard temperature of 60° Fahrenheit; and then, from knowing, by previous experiment, the amount of expansion or contraction of the liquid for different degrees of temperature, to reduce the specific gravity to what it would be at the temperature of 60°.

We shall now give a few examples in practical illustration of the preceding articles on specific gravity. The following examples will be useful in connection with Applied Mechanics.

Examples.—1. If a piece of stone weigh 10 lb. in air, and only 6¾ lb. in water, required the specific gravity of the stone.—Ans. 3·077.

2. A piece of elm weighs 15 lb. in air, and a piece of copper weighing 18 lb. in air, and 16 lb. in water, is affixed to it; the compound weighs 6 lb. in water: required the specific gravity of the elm.—Ans. 6.

3. A piece of cast-iron, of specific gravity 7·425, weighed 40 ounces in air, and 35·61 ounces in a fluid: required the specific gravity of the fluid.—Ans. 1.

4. Required the number of cubic inches in a block of stone of specific gravity 2·52, which weighs 1 cwt.—Ans. 1228⅘ nearly.

5. A composition of 1 cwt. consists of tin of specific gravity 7·32, and of copper of specific gravity 9; the specific gravity of the composition is 8·784: required the quantities of tin and copper in the mixture.—Ans. 100 lb. of copper and 12 lb. of tin.

6. The dimensions of one of the marble stones in the walls of Baibeck are, length 63 feet, and breadth and thickness each 12 feet; the specific gravity of the stone is 2·7: required the weight of the block.—Ans. 683⅖ tons nearly.

7. A solid, weighing 250 grains in air, weighs 147 grains in one fluid, and 120 in another: what is the ratio of the specific gravity of the fluids?—Ans. As 103 to 130.

8. A cubical iceberg is 100 feet above the level of the sea, the sides of it being vertical: given the specific gravity of sea-water 1·0263, and of the ice ·9214: required the number of cubic feet in the mass.—Ans. 936302452 cubic feet.

9. The weight of a common hydrometer is equal to that of a volume v of water; the part to which the stem is attached weighs a volume v' of water; the radius of the stem is r; and the specific gravity of a liquid, in which the instrument is immersed, is s: required the length l of stem immersed when in equilibrio.

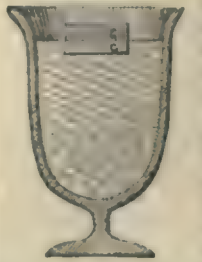
$$\text{Ans. } l = \frac{v - v'}{\pi r^2 s}.$$

EQUILIBRIUM OF A FLOATING BODY.—In order that a body may float on a liquid and remain at rest, two conditions are necessary:

1. The weight of the body must be less than the weight of a volume of the fluid of the same bulk as the body; it must be equal to the weight of the volume displaced by that portion of the bulk which is immersed in it.

2. The centre of gravity of the body, and the centre of gravity of the fluid displaced by the partial immersion, must both be situated in the same vertical straight line (p. 753).

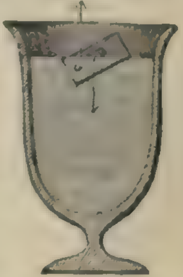
Fig. 172.



These are the only conditions necessary to secure the equilibrium of the floating body. If either of them be wanting, the body (though it may not sink) will move, and can come to a state of rest only when these two conditions are secured.

Suppose, for instance, a body floating at rest on the water, as in Fig. 172, the centres of gravity of the body and of the displaced fluid being in the same vertical line, to be disturbed from its position, as in Fig. 173. The fluid displaced then becomes changed in form and situation, though not in volume; its centre of gravity has shifted, and is no longer in the same vertical with the centre of gravity of the body. The body, therefore, is now acted upon by two forces tending to turn it round into the position from which it has been disturbed; namely, the force due to the weight of the body acting downwards through G, and the force due to the pressure of the displaced fluid acting upwards through G′.

Fig. 173.



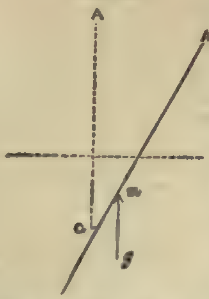
When a body thus slightly disturbed tends, as in this case, to return to its original position of equilibrium, that position is said to be one of *stable equilibrium*, in reference to the disturbance in question. If the disturbance were to act vertically on the body through its centre of gravity, and therefore tending either to depress the body lower in the fluid or to raise it further out; in the former case the additional upward pressure, and in the latter the additional downward weight, would bring the body into its original state of rest, the equilibrium being always stable in reference to disturbance in a vertical direction.

Unstable equilibrium is that position of the floating body, from which, if disturbed, it yields to the disturbance; and instead of recovering its original condition of equilibrium, departs from it more and more, till it finds either some new position of equilibrium, or upsets and sinks.

When a floating body is slightly disturbed from rest by being turned about its centre of gravity, the line through that centre, originally vertical, becomes slightly oblique; the downward pressure or weight is then in the direction of a new vertical line through the same centre; the upward pressure, too, in consequence of the change of figure and position of the displaced fluid causing a change in the centre of gravity of that fluid, is also in a new vertical line. The point, where this new vertical meets the slightly oblique line through the centre of gravity of the body, is called the *metacentre;* and the vertical itself is called the *line of support.* We have already seen that it is necessary for the line of support to pass through the centre of gravity of the body when the body is at rest (p. 753).

The position of the metacentre, in reference to the centre of gravity of the body when slightly disturbed, determines whether or not the disturbance will be counteracted, and the body right itself by returning to its former position of equilibrium; that is to say, whether or not this position of equilibrium is a *stable* position. If the metacentre be at a point on the oblique line, slightly turned out of its vertical position by the disturbance, which is *above* the centre of gravity on that line, then it is plain the upward pressure, acting along the vertical (line of support) through the metacentre, must re-establish the oblique line, or *axis* referred to, in its vertical position. For the centre of gravity is, as it were, a fixed point, round which the body turns; and

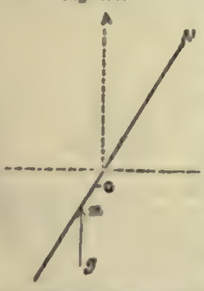
Fig. 174.



the upward force at the metacentre, *above* this centre, acting with a leverage, and unopposed, must restore to the axis its verticality. But if the metacentre be *below* the centre of gravity of the body, then the upward pressure has a contrary effect, and the oblique axis, on which it acts, is turned still more out of the vertical direction.

For instance, G (Fig. 174) being the centre of gravity of the floating body, if G A′ be the axis disturbed from its original vertical position G A, by any force which no longer acts, and if m be the metacentre, or point where the line of support gm, passing through the centre of gravity g of the displaced fluid, meets G A′, it is plain that the progress of the disturbance is arrested, and G A′ restored to its vertical position. If, on the other hand, the metacentre be situated, in reference to the centre of gravity G of the body, as in Fig. 175,—that is, below G, then the upward pressure upon it assists the deviation from the vertical; and thus G A′ goes on inclining more and more.

Fig. 175.



It is thus obvious, that when the metacentre is *above* the centre of gravity of the disturbed body, the disturbance will be rectified, and the equilibrium which has been disturbed is *stable.*

But when the metacentre is *below* the centre of gravity, the disturbance becomes aggravated; and the equilibrium thus disturbed is *unstable.* It is possible that the line of support gm may pass through G, the centre of gravity of the body: in this case, the body remains at rest in its new position, for the conditions of equilibrium are then as rigorously fulfilled as at first: the equilibrium in such circumstances is said to be *indifferent;* it is also sometimes called *neutral* equilibrium. This kind of equilibrium evidently has a place in a floating sphere and cylinder, and also in a body, of whatever shape, floating in a fluid of equal specific gravity as itself—that is, if it be allowed to call the equilibrium in this case *floating,* no part of the body being above the surface of the fluid.

From what has now been said, it is obvious how important it is, in the construction and loading of ships, to make the centre of gravity of the whole body so low that the metacentre may always be above that point, even for a considerable disturbance. The centre of gravity is sometimes elevated above its original position, in a gale of wind, by the shifting of the cargo: if the elevation be sufficient to bring it above the metacentre, the vessel must capsize.

Example.—A rectangular beam, the transverse vertical section of which is a square, and the specific gravity of which is one-half that of the water on which it floats, rests with one of its faces parallel to the surface of the water: it is required to determine whether the equilibrium is stable or unstable in reference to a slight transverse disturbance.

As the transverse vertical sections are all equal squares, it will be sufficient to consider merely the middle section A B C D (Fig. 176).

The centre of gravity of this is at G, the middle of the square; that is, G H = $\frac{1}{2}$ A B; and if g H = $\frac{1}{4}$ A B, then g will be the centre of gravity of the displaced fluid; for the body floats half in and half out of the water, as by hypothesis it is only half the weight of its bulk of water. E F is the plane of flotation, and H K the vertical axis through the centre of gravity G.

Let now the body be turned round its centre of gravity, through a small angle (Fig. 177), F G f = θ, causing the line of flotation to be ef, and the former vertical axis to become the oblique line H K; we have to find g', the centre of gravity of the displaced fluid $e\,f$ C B in this new position of the body, and thence the point O where the vertical g'O cuts the axis H K: this point O is the metacentre.

Fig. 176.

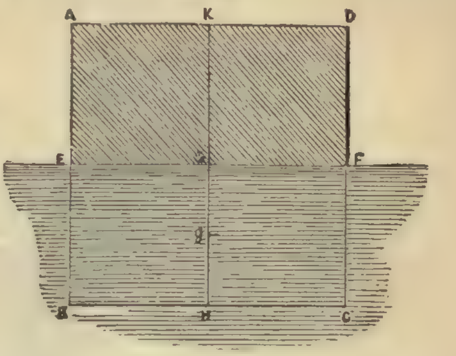
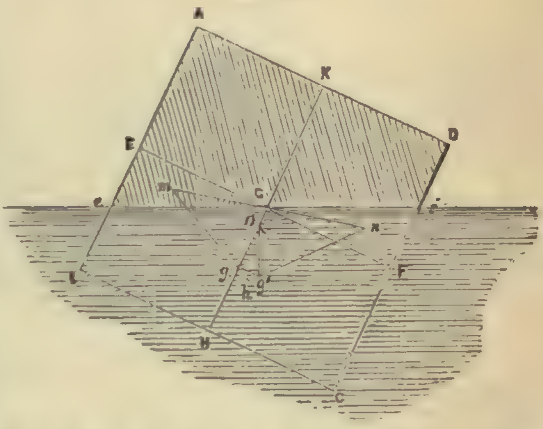


Fig. 177.



Let m, n be the centres of gravity of the portions E G e, F G f, and let h be the centre of gravity of the portion e G F C B.

Then by Statics, $hg : mg :: \text{EG}e : e\text{GFCB}$ and $hg' : ng' :: \text{FG}f : e\text{GFCB}$.

But E G e = F G f; consequently, $hg : mg :: hg' : ng'$ therefore gg' is parallel to mn

$$\therefore \frac{gg'}{mn} = \frac{hg}{gm} = \frac{\text{EG}e}{e\text{GFCB}}, \text{ by the first proportion,}$$

$$= \frac{\text{EG}e}{\text{EFCB} - \text{EG}e}$$

Now, as the angle of disturbance θ is considered as very small, the portion E G e must be insignificant in comparison with E F C B: hence

$$gg' = mn\frac{\text{EG}e}{\text{EFCB}} \quad . \quad . \quad . \quad . \quad (1)$$

Moreover, on account of the smallness of θ, sin. θ and θ may be regarded as equal; and we shall then have for the three quantities $m\,n$, E G e, E F C B, the values $m\,n = 2\,\text{G}\,m = \frac{4}{3}\,\text{G E}$, G E being the distance of the vertex G from the base, or from the *middle* of the base, the base being small in consequence of the minuteness of θ.

$$\text{EG}e = \tfrac{1}{2}\text{GE}^2 \times \theta,\ \text{EFCB} = 2\text{GE}^2$$

$$\therefore (1),\ gg' = \tfrac{4}{3}\,\text{GE} \times \tfrac{1}{4}\,\theta = \tfrac{1}{3}\,\text{GE} \times \theta = \tfrac{1}{3}\,\text{GE} \times \text{angle } gOg'$$

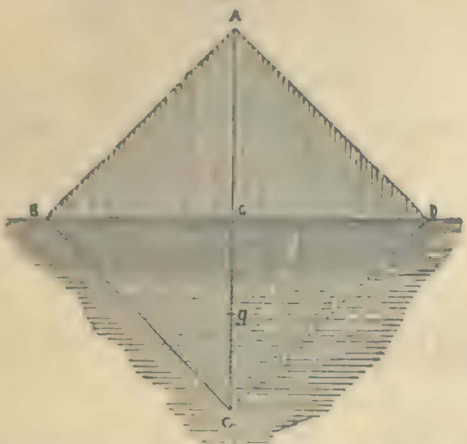
But $\quad Og \times \text{angle } gOg' = gg' \;\therefore \tfrac{1}{3}\,\text{GE} = Og$, that is

$$Og = \tfrac{1}{3}\,\text{GH}$$

$$\therefore \text{OH} = (\tfrac{1}{3} + \tfrac{1}{2})\,\text{GH} = \tfrac{5}{6}\,\text{GH} \;\therefore \text{GO} = \tfrac{1}{6}\,\text{GH}$$

Hence, the equilibrium is unstable, and therefore the beam will roll partly over, though displaced through only a very small angle. When such a position is reached that the diagonal A C becomes vertical, the body will slightly oscillate till the resistance of the fluid destroys its motion, when it will firmly settle in a position of stable equilibrium with the diagonal A C vertical. (Fig. 178). For it is plain, that when this position is arrived at, the centre of gravity G of the body, and the centre of gravity *g* of the displaced fluid, are again in the same vertical; but the inertia of the rolling mass will cause the axis C A to incline a little beyond the vertical position; the centre of gravity *g* of the displacement will thus move a little towards the right, as in the former case; and *g* being now nearer to G, the vertical form *g*′ will meet the slightly inclined axis above G, so that the new position of equilibrium into which the body finally settles will be stable.

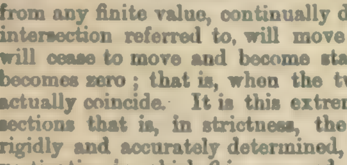
Fig. 178.



In reference to the foregoing investigation, it may be proper here to remark, that the student is not to infer that the resulting determination of the metacentre is merely a close approximation to its true position from our having regarded θ as deprived of appreciable value. The metacentre has been defined to be a point where certain two lines intersect: these two lines approach nearer and nearer to coincidence as θ diminishes. Now if θ, from any finite value, continually diminish, the point of intersection referred to, will move along the axis, and will cease to move and become stationary only when θ becomes zero; that is, when the two intersecting lines actually coincide. It is this extreme limit of the intersections that is, in strictness, the metacentre. It is rigidly and accurately determined, from any general investigation in which θ is assumed to be a small value, only in the extreme case of that hypothesis; that is, only when θ has diminished down to zero. It will be seen, by re-examining the operations above, that the final result strictly follows when θ becomes zero; or when O is the extreme limit of the intersections of the line of support with the axis H K. The metacentre, in reference to a displacement in a given direction, is thus a fixed point on the vertical, through the centre of gravity; and has nothing to do with extent of disturbance. When the body is at rest, the vertical through the centre of gravity of the body, and that through the centre of gravity of the fluid displaced, become one and the same line: if the equilibrium be disturbed, these verticals separate, and the point about which, in this separation, they *begin* to turn, is the metacentre.

FLUIDS COMMUNICATING THROUGH BENT TUBES.—In speaking of the Levelling Instrument at page 751, we have regarded it as a vessel of fluid; and from the proposition which suggested a reference to it, have inferred that the two surfaces of the bent tube were horizontal or level; but the following direct proof that the surfaces of the fluid in any bent receptacle are necessarily horizontal, is deserving of notice for its simplicity. We take it from the *Cours Elémentaire de Mécanique* of De Launay.

Fig. 179.



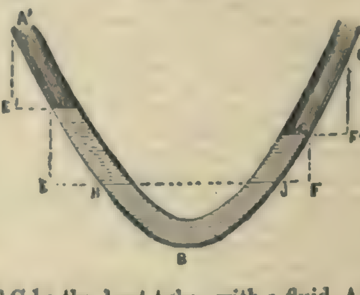
Let A B (Fig. 179) be two points taken in the interior of the fluid, and on the horizontal line wholly in the channel of communication A B, between the two ascending portions of the bent tube; these two portions being of any relative bulk whatever.

In order that the two points A and B may be at rest, the pressures at A and B must be equal, and must be the same as the pressure upon every point of the line A B. Now the pressure on A is equal to the weight of the vertical column of particles C A: the pressure on B, on account of the obliquity of the arm B M, is not so easily found. It may be obtained as follows:—The pressure on B is equal to the pressure on D, augmented by the weight of the column of particles D B. The pressure on D is the same as that on E; but the pressure on E is equal to the pressure on F, augmented by the weight of the column of particles F E: consequently the pressure on B is equal to the pressure on F, augmented by the weight of the two columns of particles D B, F E. Proceeding in this way, and observing that the pressure on M is nothing, we see that the pressure on B is equal to the weight of the five columns of particles D B, F E, H G, K I, M L. And since the pressures on A and B are equal, it follows that the five vertical lines D B, F E, H G, K I, M L, are together equal to the vertical line C A. Consequently every point in the surfaces at C and M is at the same vertical distance from the horizontal plane passing through the two points A, B: these surfaces are therefore horizontal.

In order that the foregoing phenomenon may always be exhibited, it is of course necessary that the fluid introduced into the tube be of uniform density, or that equal vertical columns of it press with equal weights. When two distinct fluids, which do not intermix, balance in a bent tube, or in any two vessels having a channel of free communication with each other, the proportion becomes modified, as follows:—

If two fluids which do not intermix are in equilibrio in a bent tube, the heights of their surfaces above the horizontal plane, where the one fluid rests upon the other, are inversely as their special gravities.

Fig. 180.



Let A B C be the bent tube, with a fluid A H of specific gravity S resting upon another fluid H B C of specific gravity S′, the horizontal plane of their separation being at H.

The pressure of the fluid A H on the horizontal plane H is H × A E × S*g*; the pressure of the fluid C J, upward on the same plane, is H × C F × S′*g*; and the plane H sustains only the upward pressure communicated by C J, since, H J being horizontal, if C J were removed, there would be no upward pressure on H.

Hence, as the fluids are at rest, the pressures on H must be equal.

$$\therefore \text{H} \times \text{A E} \times \text{S}g = \text{H} \times \text{C F} \times \text{S}'g$$

$$\therefore \text{A E} \times \text{S} = \text{C F} \times \text{S}', \text{ or } \frac{\text{A E}}{\text{C F}} = \frac{\text{S}'}{\text{S}}$$

that is, the heights of the surfaces A, C, above the horizontal plane of separation H J, are inversely as the specific gravities of the fluids A H, C J.

If S′ = S, we then have the case of a single uniform fluid before considered, and the result gives A E = C F;

that is, the surfaces are at the same height above a horizontal plane, or are level. Suppose above A and C, two other fluids of specific gravities S″, S‴, to be introduced into the tube, and that the equilibrium remains undisturbed; then the new fluid in C′ C communicates to the horizontal plane A the only upward pressure it receives, and this by hypothesis is balanced by the downward pressure of the new fluid in A′ A. Hence, as before,

$$\mathrm{A'E'} \times \mathrm{S''} = \mathrm{C'F'} \times \mathrm{S'''} \quad \therefore \mathrm{A\,E} \,.\, \mathrm{S} + \mathrm{A\,E''} \,.\, \mathrm{S''} = \mathrm{C\,F} \,.\, \mathrm{S'} + \mathrm{C'F'} \,.\, \mathrm{S'''}$$

and so on for any number of superposed fluids in equilibrio. Hence, if any number of fluids be in equilibrio in a bent tube, the sum of the products of the vertical heights into the specific gravities of the fluids in one branch of the tube, will be equal to the sum of the corresponding products in the other branch. The vertical height of each fluid is estimated from the horizontal plane on which it rests; the height of the lowest fluid H B C is estimated from the lowest point B; the vertical distance of B from H is the height in one branch of the tube; and the vertical distance of B from C, is the height in the other branch.

In illustrations such as the above, the tubes are always represented as circular; but, as the reasoning shows, the shape of the channel which connects the two upper surfaces of a fluid together does not enter into consideration. A pool of water connected by underground open passages, however tortuous, with an adjacent river, will, on the above principle, have its surface on the same level as that of the river. Also, water conveyed by pipes from a reservoir can never deliver a supply to any place, on a higher level, than the surface of the source, without the application of mechanical force or pressure.

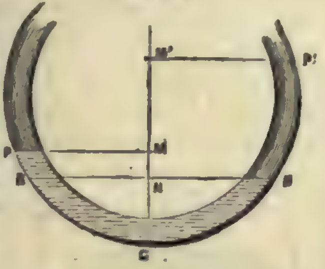
The student must be apprised, that in discussing the circumstances of the equilibrium of fluids in tubes, though the shape of the tube is of no moment, nor are the sectional area of the branches at any height, provided only that these exceed a certain amount of smallness; yet, if the section at any part be so small as to bring into operation what is called capillary attraction, the fluid surfaces in the two branches will not stand at the same level. If the section be circular, the diameter of it should exceed $\frac{1}{16}$th of an inch, in order that capillary attraction may not oppose that of gravity, and thus lessen downward pressure. We shall devote a short article to this curious subject presently.

Problem.—Equal lengths of two fluids which do not intermix, and whose specific gravities are as m to 1, are poured into a uniform circular tube; required the position in which they will rest.

Let P C H (Fig. 181) be the heavier fluid, and P′ H the lighter, the lengths of the arcs P H, P′ H being equal; let a represent each of these lengths; and draw P M, P′ M′ each perpendicular to the vertical diameter, as also H N R.

Then the lighter fluid P′ H is upheld entirely by the pressure of the portion P R of the heavier fluid; hence the downward pressure of the lighter fluid on the horizontal plane H must be equal to the upward pressure of the portion P R of the heavier fluid, transmitted to that plane. The heights of these equilibrating portions of fluid are respectively M′ N, and M N; and as these are inversely as their specific gravities, from what is shown above, we have

Fig. 181.



$$\mathrm{M'N : MN} :: m : 1 \;\therefore\; \mathrm{M'N} = m \,.\, \mathrm{MN} \;\ldots\ldots\ldots (1)$$

Let the arc C P be represented by x ∴ C H $= a - x$, and C P′ $= a - x + a = 2a - x$, then M′ N and M N may be found in terms of a and x as follows, observing that

$$\mathrm{M'\,N = M'\,C - N\,C}$$
$$\text{and } \mathrm{M\,N = M\,C - N\,C.}$$

$$\mathrm{M'\,C} = \text{ver. sin. } (2a - x),\ \mathrm{N\,C} = \text{ver. sin. } (a - x),\ \mathrm{M\,C} = \text{ver. sin. } x;$$

$$\therefore \mathrm{M'N} = \text{ver. sin. } (2a - x) - \text{ver. sin. } (a - x),$$
$$\mathrm{M\,N} = \text{ver. sin. } x - \text{ver. sin. } (a - x);$$
$$\therefore (1) \text{ ver. sin. } (2a - x) - \text{ver. sin. } (a - x) = m \text{ ver. sin. } x - m \text{ ver. sin. } (a - x) \;.\,.\; (2).$$

Or, since ver. sin. = R — cos., where R is the radius of the circle, we may use cosines instead of versed sines; and reduce the geometrical to the ordinary trigonometrical cosines; that is, to the scale R = 1; we shall only have to regard a and x as standing, not for lengths of arc, but for the *degrees* in those lengths; we shall thus have from (2)

$$1 - \cos.\ (2a - x) - 1 + \cos.\ (a - x) = m - m \cos.\ x - m + m \cos.\ (a - x);$$

that is, $\cos.\ (a - x) - \cos.\ (2a - x) = m \cos.\ (a - x) - m \cos.\ x$;

$$\therefore \cos.\ (2a - x) + (m - 1) \cos.\ (a - x) = m \cos.\ x;$$

that is, $\cos.\ 2a \cos.\ x + \sin.\ 2a \sin.\ x + (m - 1)(\cos.\ a \cos.\ x + \sin.\ a \sin.\ x) = m \cos.\ x$;

$$\therefore \cos.\ 2a + \sin.\ 2a \tan.\ x + (m - 1)(\cos.\ a + \sin.\, a \tan.\ x) = m;$$

$$\therefore \tan.\ x = \frac{m - \cos.\ 2a - (m - 1) \cos.\ a}{\sin.\ 2a + (m - 1) \sin.\ a} \;\ldots\ldots\ldots (3)$$

From this expression the trigonometrical tangent of the arc x may be found from the tables; and thence the number of degrees in that arc, or the angle which C P subtends at the centre of the circle.

Let $a = \frac{\pi}{2}$, then $\cos.\ a = 0$, $\cos.\ 2a = -1$, $\sin.\ a = 1$, $\sin.\ 2a = 0$, and the expression in these circumstances becomes

$$\tan.\ x = \frac{m + 1}{m - 1} \;\ldots\ldots\; (4)$$

If $m = 1$, that is, if the two fluids be the same, then (3) gives

$$\tan.\ x = \frac{1 - \cos.\ 2a}{\sin.\ 2a} = \frac{1 - (1 - 2 \sin.^2 a)}{2 \sin.\ a \cos.\ a}$$
$$= \frac{\sin.\ a}{\cos.\ a} = \tan.\ a.$$

that is, the arcs a and x are equal, as we otherwise know they must be.

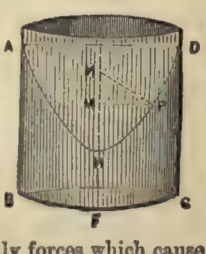
EQUILIBRIUM OF A ROTATING FLUID.—In all the preceding investigations, the only force supposed to act on the fluid is the force of gravity; we shall give an illustration of a case in which the equilibrium is due to the united effects of gravity and centrifugal force.

Problem.—A cylindrical vessel, containing water or any other liquid, revolves round its axis, perpendicular to the horizon, with a given angular velocity; to determine the form assumed by the internal surface of the fluid.

The rotation of the vessel necessarily puts the contained fluid in motion, and this motion continues till all the forces balance and the fluid attains a fixed form and position; it is in this state of equilibrium that we have to consider it.

Suppose A H D (Fig. 182) to be the form assumed by the surface of the fluid when the state of equilibrium is attained, and P any particle on that surface. From P draw P M perpendicular to the axis of the cylinder, and P N perpendicular to the curve at P. The forces which keep P in equilibrio are—1. Gravity, acting in the vertical direction N M; 2. The centrifugal force, acting in the horizontal direction M P; and 3. The pressure or reaction of the particle in the direction P N perpendicular to the surface; for it is plain that the vertical and horizontal forces acting on P are the only forces which cause

Fig. 182.



the normal pressure of P on the surface H P D; so that this normal pressure is the resultant of the other two, and therefore, taken in the opposite direction P N, neutralises the former, and keeps P at rest.

Let a be the angular velocity of rotation, that is, the angle turned through by M P or F C in a second of time; then the circular arc turned through by P in a second will be ay, putting y for P M; that is to say, the velocity v, of the point P, is $v = ay$.

Now the centrifugal force acting on P is

$$\frac{v^2}{y} = \frac{a^2y^2}{y} = a^2y. \quad \text{(Dynamics, page 741).}$$

Also (STATICS, Prop. IV., page 696),

N M : M P : : gravity : centrifugal force . . . . (1)

$$\text{that is, N M} : y :: g : a^2y \therefore \text{N M} = \frac{g}{a^2}.$$

The line N M, thus determined, is called the subnormal to the curve A H D at the point P; and we see that the curve is such that the subnormal at any point is a constant quantity. This property belongs exclusively to the *parabola*. Hence the surface of equilibrium is that generated by the rotation of a parabola about its axis; that is, it is a *paraboloid*.

In reference to the foregoing investigation, it may be useful to the student to attend to the following remarks:—

It will have been perceived that the problem proposed is of a mixed character—partly dynamical and partly statical. The forces first generate motion in P, which point continues to move till these forces become balanced, and P takes a position of rest. The force of gravity g, and the centrifugal force a^2y, both act dynamically, and are therefore to be expressed in *feet*. The symbol a, representing the angular velocity of rotation, is an abstract number—it is not an *angle*. Here, as in all dynamical inquiries, angular velocity is estimated thus:—A circle, whose radius is represented by 1, is imagined to be described about the centre of motion (about M in the present case); the semicircumference of this circle is 3·1416; and the extent of arc of the same circle, turned through in a second, is a, the angular velocity of the rotation, which is therefore 3·1416 multiplied by some number, whole or fractional. If, therefore, y or M P be measured in inches, the velocity of P is ay inches; if in feet, it is ay feet. As g is measured in feet—viz. $g = 32{\cdot}2$ feet, it is necessary that M P or y should also be measured in feet; otherwise the proportion (1) would be incongruous. The final result, namely, N M $= \frac{g}{a^2}$, gives the length of N M in feet: it is 32·2 feet divided by the abstract number a^2. Suppose the vessel performed 10 rotations in a second; then the angular velocity would be

$$a = 3{\cdot}1416 \times 20 = 62{\cdot}832 \therefore a^2 = 62{\cdot}832^2 = 3947{\cdot}86$$

$$\therefore \text{N M} = \frac{32{\cdot}2}{3947{\cdot}86} \text{ feet} = \frac{386{\cdot}4}{3947{\cdot}86} \text{ inches.}$$

We see that, with the same angular velocity of rotation, the curve is the same whatever be the magnitude of the vessel: it is further obvious that, as the cylindrical shape of the vessel is no item of consideration in the above solution, the conclusion remains the same whatever the shape may be, provided that the horizontal sections be circles, and the vertical axis of revolution pass through their centres.

Suppose the vessel to be itself in the form of a paraboloid: then the velocity of rotation, necessary to cause all the fluid to run over, may be easily determined. For the vessel will be emptied as soon as every point P has, through the intensity of the centrifugal force, reached the side of the vessel; that is, when the subnormal N M at any point P is identical with the subnormal at the point where the horizontal line M P meets the side of the vessel.

Calling, then, the constant subnormal of the given parabola l, we have $l = \frac{g}{a^2} \therefore a = \sqrt{\frac{g}{l}}$, the angular velocity per second at which the vessel must rotate in order that the empty paraboloid, formed by the rotation, may coincide with the concavity of the vessel itself.

From this expression for the angular velocity, the time in which each complete revolution must be performed may be ascertained thus:—

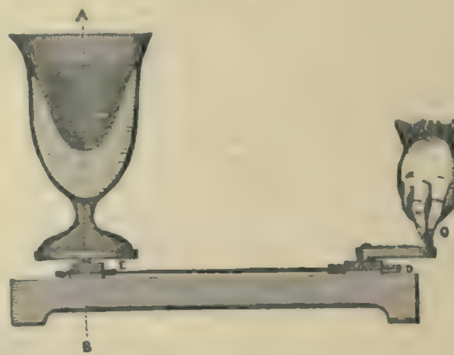
$$a : 3{\cdot}1416 :: 1 \text{ sec.} : \frac{3{\cdot}1416}{a} \text{ sec.} = \text{time of a semi-rotation}$$

$$\therefore \text{time of a rotation} = 2\frac{3{\cdot}1416}{a} = 3{\cdot}1416 \times 2\sqrt{\frac{l}{g}} \text{ seconds}$$

the subnormal l being measured in feet, and g being 32·2 feet. It appears, therefore, that with this rate of rotation, the fluid will rise and run over the vessel, till only what covers and adheres to the sides is left.

The following figure will illustrate the manner of communicating rapid rotatory motion to a vessel of water. D and E are two pulleys, round which passes an endless cord; by means of a winch C, the vessel may be made to rotate round its vertical axis A B as rapidly as we please.

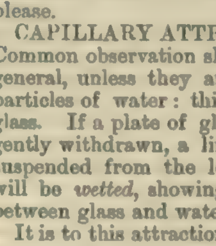
Fig. 183



CAPILLARY ATTRACTION AND REPULSION.—Common observation shows that the surfaces of solids in general, unless they are unctuous surfaces, attract the particles of water: this is more especially noticeable in glass. If a plate of glass be dipped in water, and then gently withdrawn, a line of fluid particles will be seen suspended from the lower edge, and the whole surface will be *wetted*, showing that there exists an attraction between glass and water.

It is to this attraction that the rise of water in a capillary tube is attributed: the bore of such a tube being very minute, from $\frac{1}{30}$th to $\frac{1}{50}$th of an inch in diameter, the thread of water is so slender that the surface of glass, above its upper extremity, may exercise attractive force sufficient to raise the upper particle, when the contiguous particles must follow to fill up the vacuum: this lengthening of the thread increasing till gravity, or the weight of the little column, counterbalances this capillary attraction. It is observed that the height to which the water is raised in a capillary tube, is inversely as the diameter of the tube. And this fact is sufficient to teach us the form of the curve which the upper surface of water assumes, when acted upon by the two glass plates which embrace it.

Fig. 184.



For the distance between the plates at B (Fig. 184), is to the distance between them at C, as the height of the liquid at C, to the height at B. Hence, putting A B $= x$, and A C $= x'$, and the corresponding heights y and y', and remembering, from Euclid, Prop. II., Book VI., that the distances between the plates at B and C are as A B to A C, we have

$$x : x' :: y' : y \therefore xy = x'y';$$

that is, the rectangle xy is a constant quantity, wherever B may be within the limits of capillarity. As this is the distinguishing property of the hyperbola, the axes of reference being the asymptotes, we infer that the curve presented by the upper surface of the fluid interposed between the plates, is an hyperbola. The curve ceases, of course, to be that of the hyperbola beyond the limits of capillarity. At a distance from the junction of the plates, at which the interval between them exceeds about $\frac{1}{50}$th of an inch, the curve will terminate: there will still be an elevation of the liquid up the side of each plate, but the line traced by its surface will become a straight line.

Where the distance between the plates is $\frac{1}{200}$th of an inch, the fluid, if water, is observed to rise 5 inches; and therefore where the distance between the plates is $\frac{1}{100}$th of an inch, the rise of the water is $2\frac{1}{2}$ inches. The elevation of the liquid is found to be quite independent of the thickness of the glass: if the bore of the tube, or the distance between the interior surfaces of the plate, be the same, the elevation is unaffected by the thickness, so that the quantity of matter in the glass does not contribute to the ascent of the fluid; and if the interior surfaces of the plates, or the bore of the tube, be coated with a film of oil, the capillary attraction is destroyed. On this account, it has been inferred that the attraction is exerted entirely by the surface of the glass, and that the liquid must be in complete contact with that surface, for the phenomenon to take place: the attractive energy, it would seem, has no penetrating power, it being stifled and rendered inoperative by the interposition of even the thinnest lamina of oil or grease.

It may be remarked, too, that the elevation of the fluid is not near so great between plates, as in a tube whose diameter is equal to the distance of the plates: the rise is something less than half as great in the former case as in the latter. But this might be expected, as in the tube, the thread of fluid is completely surrounded by the glass surface; the same law, however, namely, that the height of the fluid is inversely proportional to the distance between the plates, is observed to have place.

The fluids which exhibit the phenomena here noticed are exclusively the watery fluids, all of which rise to different heights in the same tube; and some of the heavier rise higher than the lighter: spirits of wine, for instance, rises only about $\frac{2}{5}$ths as high as water, which of all liquids rises to the greatest height. Mercury, however, does not rise at all; on the contrary, the tube and the plates seem to exercise a repellant energy, and cause the mercury to descend: this is called *capillary repulsion.*

The diameter of a capillary tube is not found by actual measurement; the method recommended is this:—Put a known weight w of grains of mercury into the tube, and let the length of tube it occupies be l inches; then representing the diameter of a transverse section of the tube by d, we have for the volume of the mercury $d^2l \times \cdot 7854$; and as a cubic inch of mercury, when pure, weighs 3443 grains, we have

$$1 : d^2l \times \cdot 7854 :: 3443 \text{ grains} : w \text{ grains};$$

$$\therefore d = \sqrt{\left(\frac{w}{l} \cdot \frac{1}{3443 \times \cdot 7854}\right)} = \cdot 09123 \sqrt{\frac{w}{l}} \text{ inches.}$$

Whatever the force called capillary attraction and repulsion may be, it is plain that it cannot be analogous to that so universally diffused throughout the material universe, and called the force of gravitation; because the intensity of *this* influence increases with the mass of the body exercising it: whereas the capillary influence is more like that of *electricity*, which, when in a state of equilibrium, seems to be confined to mere surfaces. It appears not improbable, therefore, that capillarity may be a peculiar manifestation of electric force exerted by the surface of the glass on the particles of liquid in its immediate neighbourhood.

CHAPTER V.

HYDRODYNAMICS.

Hydrodynamics treats of the motion of incompressible fluids, and, as the name implies, more especially of *water.* The consideration of compressible or elastic fluids, whether in a state of equilibrium or of motion, comes under the head of *Pneumatics*, a subject to which the next chapter will be devoted. But, from our limited space, we are precluded from discussing these remaining topics at any great length. This, however, is scarcely to be regretted, for the physical laws which regulate the motions of fluids are so imperfectly ascertained, that, except in those few particulars which constitute the mere elements of the subject, the investigations of hydrodynamics, founded on hypothetical data, are of little practical value; and sometimes even lead to results clearly contradicted by experiment. We shall, therefore, confine ourselves, in this brief summary, to those elementary portions of the mathematical theory which actual experiment confirms, or where the practical obstacles to such confirmation are clearly ascertained, and may be estimated and allowed for.

If water run through a pipe or other channel, always filling it, the velocity of the fluid at any part will be inversely as the area of the transverse section of the channel at that part. For let A, A′ represent the area of any two transverse sections, the water being supposed to run from A towards A′; then whatever quantity passes through A in a given time, the same quantity must pass through A′ in that time; since if a less quantity passed through, then the fluid between the sections, always remaining full, would be condensed; and if a greater quantity passed through, the fluid between the sections would be expanded, as the channel is always full. As each consequence is incompatible with an incompressible fluid, it follows that equal quantities of fluid must pass through A and A′ in the same time.

If, therefore, the velocity of the stream through A be v feet per second, and the velocity through A′, v' feet per second, we shall have

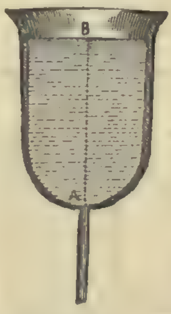
$$Av = A'v' \;\therefore\; A : A' :: v' : v;$$

that is, the velocities are inversely as the areas of the sections.

SPOUTING OF FLUIDS.—The velocity with which a fluid issues from a small orifice in the bottom or side of a vessel, kept constantly full, is equal to the velocity that would be acquired by a heavy body falling freely through the height of the surface of the fluid above the hole.

Suppose the orifice A (Fig. 185) to be at the bottom of the vessel, then the thin lamina of fluid covering the orifice A is forced out by the action of gravity upon it, and by the superincumbent pressure of the column of fluid whose base is A and altitude A B. Conceive the column A B to consist of n such laminæ, thus supposed to be acted upon at A; that is, calling the thickness of the lamina 1, let A B $= n$; then before motion, the statical forces or pressures on A are its own weight, and the weight or pressure of the column A B, and these are as 1 to n; consequently the dynamical effect of these forces, when motion takes place, must be also as 1 to n. Now the dynamical effect, that is, the velocity generated in the lamina A, in the time of falling through its own thickness 1, regarding this time as the unit of time, is 2 (Dynamics, page 736): hence

Fig. 185.



$$1 : n :: 2 : 2n;$$

therefore $2n$ is the velocity generated in the lamina A by the column of fluid in the assumed unit of time; and as this time is indefinitely small, it must be the velocity which the fluid has at the orifice itself, or that with which it actually spouts out. As the vessel is kept constantly full, the initial circumstances continue invariable, so that the fluid is discharged with the same uniform velocity.

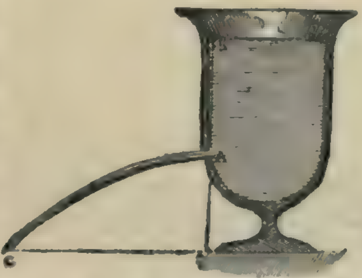
Now, if a heavy body fall freely through a height h by the action of gravity it acquires during the time of fall, a velocity $v = 2h$; that is, a velocity that would carry it through $2h$ in the same time (DYNAMICS, page 736). Consequently the fluid issues from the orifice A with a constant velocity equal to that which a heavy body would acquire in falling through the height B A.

If this height be h feet, the accelerating force of gravity being $g = 32{\cdot}2$ feet, we have, for the constant velocity of the spouting fluid, $v = \sqrt{2gh}$, the velocity per second.

The fluid spouts out with the same velocity whether the small orifice be in the side or in the bottom of the vessel, provided only that in both cases the hole be at the same depth below the surface of the fluid, since the pressure of a fluid is the same in all directions at the same depth.

The stream being, as it were, thus projected with a uniform velocity v, the curve it will describe in issuing from the side of a vessel (as in the margin) will be a *parabola*, of which the directrix is the horizontal line through B (Fig. 186), the surface of the fluid (DYNAMICS, page 739). The hole through which the fluid spouts must be so small as to render the pressures at different parts of it insensibly different from that at the centre of the orifice.

Fig. 186.



QUANTITY OF FLUID DISCHARGED PER SECOND.—As the velocity is constant, the quantity of fluid discharged per second is found by multiplying the area of the orifice by the length $\sqrt{2gh}$; thus, if a be the area of the orifice, the discharge per second is $a\sqrt{2gh}$ cubic feet. Let $h = 20$ inches, and $a = 1$ square inch, then $a\sqrt{2gh} = \sqrt{(2 \times 32{\cdot}2 \times 12 \times 20)} = \sqrt{15456} = 124{\cdot}3$ cubic inches.

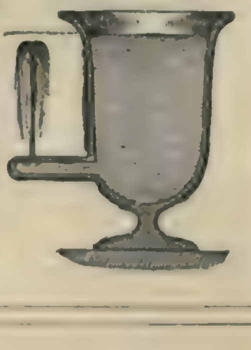
Since $a\sqrt{2gh}$ is the discharge per second, in t seconds a quantity Q will be discharged, such that

$$Q = at\sqrt{2gh} \;\therefore\; t = \frac{Q}{a\sqrt{2gh}} \quad . \quad . \quad . \quad . \quad (1)$$

This, therefore, expresses the time in which a proposed quantity Q of the fluid will be delivered, Q being expressed in cubic feet.

THE VENA CONTRACTA.—The foregoing conclusions respecting the velocity with which a fluid spouts from a small hole in the vessel containing it, and the quantity of the fluid delivered in a given time, have been compared with many carefully conducted experiments. The most satisfactory way of ascertaining practically the velocity of the discharge, is by observing the altitude to which the fluid spouts when forced to take an upward direction, as in the accompanying figure. If the uniform velocity of the issuing fluid be that due to the fall of a heavy body from the surface to the orifice, the fluid ought to spout up as high as that surface; and such is found to be very nearly the case. We might expect that the theoretical result would not be rigidly fulfilled, but that the height reached would fall a little short of the surface of the fluid in the vessel; because friction and the resistance of the air would act as opposing forces. But there is another cause in operation which must now be noticed

Fig. 187.



When a passage is made in a vessel for the exit of the fluid contained in it, the equilibrium of the entire mass is disturbed, and motion takes place among all the particles. The instant that the plug is removed, the velocity is due to the direct pressure upon it; but as lateral and oblique pressures all round the hole, cause the neighbouring fluid to flow sideways towards the main stream, when this surrounding fluid reaches the aperture, it does so with a certain velocity transverse to the direction of the main stream, which, in consequence, becomes slightly contracted a little beyond the opening. This contraction of the fluid vein is called by Newton the *vena contracta*; and it is through the section of the vena contracta that the fluid flows, as we have supposed it to flow through the orifice itself. The distance of this section from the orifice is nearly equal to the radius of the orifice, and its area about $\frac{5}{8}$ths of that of the orifice.

If, therefore, the orifice be small—as, for instance, a gimlet-hole in a cask—the confounding the orifice with the vena contracta can occasion but very little error in the height or distance to which the fluid spouts; but a considerable departure from the truth will result from calculating the quantity of fluid delivered in any time, on the supposition that the sectional area of the spouting stream is A instead of $\frac{5}{8}$ A; for instance, in the example above, the quantity of fluid discharged per second, instead of being 124·3 cubic inches, is only 77·7 cubic inches.

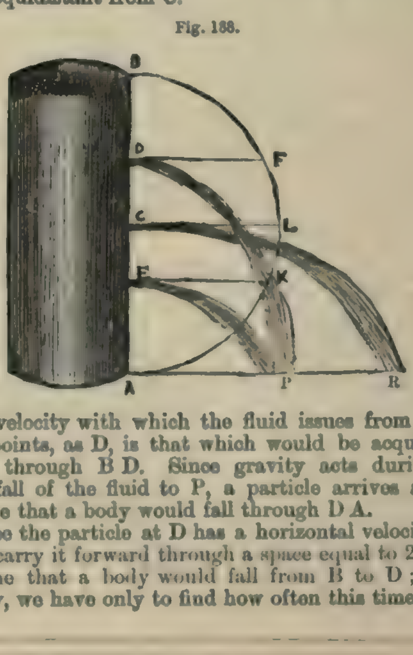
If the vessel be not kept full, and the surface be therefore allowed to descend, the velocity with which the descent commences will be, to the velocity with which it passes through the vena contracta, as the area of the section at the latter to the area of the surface (page 763).

Since the curve, which the fluid spouting from a small hole in the side of the vessel assumes, is that of a parabola—the same curve, in fact, that is described by a projectile impelled in the same direction and with the same velocity as the fluid at the vena contracta—everything connected with the extent of range, time of describing it, greatest altitude, &c., may be determined as in the theory of projectiles. The following are a few illustrations:—

If the fluid spout from a point in the middle of the upright side of a full vessel, the horizontal range will be the greatest; and from points at equal distances above and below the middle point, the ranges will be equal.

Let A B (Fig. 188) be the upright side of the vessel perforated at the middle point C, and also at two points D, E, equidistant from C.

Fig. 188.



The velocity with which the fluid issues from one of these points, as D, is that which would be acquired in falling through B D. Since gravity acts during the whole fall of the fluid to P, a particle arrives at P in the time that a body would fall through D A.

Hence the particle at D has a horizontal velocity that would carry it forward through a space equal to 2 B D in the time that a body would fall from B to D; consequently, we have only to find how often this time is contained in the time of falling from D to A, and to multiply 2 B D by the resulting number, in order to get the horizontal range A P. Now the times of falling through any spaces are as the square roots of those spaces (DYNAMICS, page 736); therefore the required multiplying number is $\sqrt{\frac{\mathrm{DA}}{\mathrm{BD}}}$: consequently,

$$\text{Range AP} = 2\mathrm{BD} \times \sqrt{\frac{\mathrm{DA}}{\mathrm{BD}}} = 2\sqrt{\mathrm{BD} \cdot \mathrm{DA}}.$$

If, therefore, with centre C a semicircle be described upon A B, and the perpendicular D F be drawn, the range of A P will be twice the length of this perpendicular. (Euc., XIII., Book VI.)

In like manner, the range of the fluid spouting from E will be twice the length of the perpendicular E K; and as these perpendiculars are equal, being equidistant from C, the ranges from D and E are equal.

As the perpendicular C L from the centre C is the longest that can be drawn from A B to the arc of the semicircle, the range A R = 2 C L, of the fluid spouting from C, is the greatest of all ranges.

These theoretical results are fully confirmed by experiment, when the orifices in the vessel are small, so that the vena contracta may be sufficiently near the vessel to be regarded, without sensible error, as at the orifice itself.

As the perpendicular D F is the sine of the arc B F, we see that the horizontal range from any orifice, is twice the sine of the arc of a circle whose diameter is the depth of the fluid, and whose versed sine is the depth of the orifice. By the depth of the fluid may be understood the depth of the horizontal plane on which the range is measured below the upper surface of the fluid; for whether the bottom of the vessel be in this plane, or be raised above it, makes no difference. The range from any orifice, measured on any horizontal plane, is the same, provided the distance between that orifice and the surface be the same, whether the fluid extend down to that plane or not.

PROBLEM.—A fluid issues from the side of a vessel through an oblique jet: to determine the horizontal range.

Let a be the angle of direction of the jet above the horizontal line, and h the height of the surface of the fluid above the orifice: then (DYNAMICS, page 739) the range r on the horizontal plane through the orifice will be $r = 2h \sin. 2a$. This is the greatest when $\sin. 2a$ is greatest; that is, when $a = 45°$, the range then being $\mathrm{R} = 2h$.

The velocity with which a fluid spouts from a small orifice, as the surface of the fluid descends, varies as the square root of the height of the surface above the hole.

For the velocity, at any instant, is always equal to that acquired by falling from the surface to the orifice; and this velocity varies as the square root of the space fallen through: hence the velocity varies as the square root of the height of the surface above the hole. The velocity, therefore, is retarded as the surface descends, but not uniformly retarded, unless the horizontal sections of the vessel are all equal in area; for the more extensive the section—or the surface of the fluid—at any instant of the descent, the more slowly will it increase its distance from the top of the vessel, and the less, therefore, will the velocity of the issuing fluid be retarded.

But if the horizontal sections of the vessel be all equal, then the velocity of the issuing fluid, as likewise the velocity of the descending surface, will be uniformly retarded.

For let V be the velocity of the descending surface at any instant, and v that at the orifice: let also the areas of the surface and orifice be A and a: then (page 763)

$$\mathrm{V} : v :: a : \mathrm{A} \therefore \mathrm{V} = \frac{a}{\mathrm{A}} v.$$

Now, as shown above, v varies as the square root of the space s between the descending surface and the orifice: we may put, therefore, $c\sqrt{s}$ for v, and write

$$\mathrm{V} = \frac{ac}{\mathrm{A}}\sqrt{s} = \sqrt{\frac{a^2c^2}{\mathrm{A}^2} s} = \sqrt{2fs}$$

by putting $2f$ for $\left(\frac{ac}{\mathrm{A}}\right)^2$. As this agrees with the general expressions for the velocity in uniformly accelerating or retarding forces (DYNAMICS, page 736), we infer that the velocity V is uniformly retarded; and since v is V multiplied by an invariable number, we conclude that v, the velocity of the spouting fluid, is also uniformly retarded. Consequently, the volumes or quantities of fluid discharged in equal times, decrease as the numbers 1, 3, 5, 7, &c., taken in reverse order. (DYNAMICS, page 736).

If a vessel of uniform horizontal section be kept constantly full, then twice the quantity which the vessel holds, will run out from a hole in the bottom, in the same time that the vessel would empty itself.

For the surface of the descending fluid is uniformly retarded as the fluid runs out; and when it arrives at the bottom, the velocity is destroyed: the space which the surface *would* describe in the same time, with the first velocity of it uniformly continued, would be twice the actual space described; and the quantity discharged in any time when the vessel is kept constantly full, is the same as what *would* be discharged if the surface descended uniformly with the first velocity: hence the quantity discharged in the one case is double that discharged in the other.

PROBLEM.—To find the time in which a vessel of uniform horizontal section will empty itself through a small orifice in the bottom.

Let A be the area of any horizontal section of the fluid, h the height of the vessel, and a the area of the orifice. Then the quantity of fluid that would be discharged in the time t of emptying, if the first velocity were to continue uniform, would be twice the contents of the vessel; that is, the quantity would be $2\mathrm{A}h$; so that if v be the velocity with which the fluid would then uniformly issue, we should have

$$atv = 2\mathrm{A}h \therefore t = \frac{2\mathrm{A}h}{av}$$

$$\text{But (page 764) } v = \sqrt{2gh} \therefore t = \frac{\mathrm{A}}{a} \times 2 \frac{h}{\sqrt{2gh}} = \frac{\mathrm{A}}{a}\sqrt{\frac{2h}{g}}$$

which therefore expresses the number of seconds occupied in emptying the vessel through the orifice.

It follows from this expression for t, that the times of emptying cylinders or prisms through equal orifices in their bases, vary as $\mathrm{A}\sqrt{h}$. If the bases of the vessels be equal in area, then the times vary as $\sqrt{h}$. If the altitudes are equal, the times vary as the bases. The time of emptying any altitude $h - h'$ of the vessel, is found by substracting from the time of emptying the whole height h, the time of emptying the height h': thus, since

$$\left.\begin{array}{r} t = \frac{\mathrm{A}}{a}\sqrt{\frac{2h}{g}} \\ \text{and } t' = \frac{\mathrm{A}}{a}\sqrt{\frac{2h'}{g}} \end{array}\right\} \therefore t - t' = \frac{\mathrm{A}}{a}(\sqrt{h} - \sqrt{h'})\sqrt{\frac{2}{g}} \quad \ldots (1)$$

the number of seconds elapsed during the descent of the surface of the fluid from the height h to the height h'.

The time in which the surface would descend from the height h to the height h', or the time in which the fluid occupying this interval would run out, if the vessel were kept constantly full, by equation (1), page 764, is

$$\frac{\mathrm{Q}}{a\sqrt{2gh}} = \frac{\mathrm{A}(h - h')}{a\sqrt{2gh}} \quad \ldots \ldots (2).$$

Hence the time of discharging the proposed quantity Q, when there is no supply from without, is to the time of discharging the same quantity when the vessel is kept constantly full, as

$$\frac{A}{a}(\sqrt{h}-\sqrt{h'})\sqrt{\frac{2}{g}} : \frac{A}{a}\times\frac{h-h'}{\sqrt{2gh}}, \text{ or as}$$

$$2(\sqrt{h}-\sqrt{h'}) : \frac{h-h'}{\sqrt{h}},$$

$$\text{or as } 2 : \frac{\sqrt{h}-\sqrt{h'}}{\sqrt{h}}, \text{ or } 2 : 1-\sqrt{\frac{h'}{h}}.$$

If the quantity Q be the whole contents of the vessel—that is, if h' be zero—then the times of discharging the vesselful of fluid in the two cases are as 2 to 1, as already inferred at page 765.

In order to test the accuracy of the theory by experiment, (1) is the most suitable formula for the purpose, since the time occupied by the surface in descending from one level to another, can be correctly observed; but the instant when the last portion of fluid escapes cannot be accurately noted; for when the vessel is near exhaustion, the fluid ceases to fill the orifice, and is discharged in drops from the edges of the hole. The time occupied by the surface descending through a certain space, is found to agree very closely with the theoretical expression for it.

THE CLEPSYDRA.—The clepsydra, or water-clock, is a contrivance for measuring time by the descent of the surface of water in a vessel, the water flowing through an orifice at the bottom. The most convenient form for the vessel is that in which the surface descends through equal vertical spaces in equal times. This form may be determined as follows:—

Let the altitude of the surface at any instant be x, and the radius of the surface—that is, the corresponding ordinate of the generating curve—y. Then the velocity at the orifice at that instant is $\sqrt{2gx}$ (page 764), and the area of the descending surface πy^2, where $\pi=3{\cdot}1416$. The velocity V of the descending surface is found by the theorem at page 763, which gives

$$\pi y^2 : a :: \sqrt{2gx} : V = \frac{a}{\pi y^2}\sqrt{2gx}$$

where a is the area of the orifice.

Since the surface is to descend uniformly, V must be a constant quantity, which can be suitably assumed in reference to the whole time of emptying: putting c for this constant, we have

$$c=\frac{a}{\pi y^2}\sqrt{2gx} \therefore y^4=\frac{2a^2g}{\pi^2c^2}x.$$

This, therefore, is the equation of the curve, the rotation of which about its vertical axis x will generate the surface of the vessel. This generating curve is a parabola of the fourth order.

But the surface need not be a surface of revolution, or one having all its horizontal sections circles. They may all be rectangles. Let the sides of the rectangular section, at the altitude x from the base, be y and p, then the area of the section is py, and we have

$$c=\frac{a}{py}\sqrt{2gx} \therefore y^2=\frac{2a^2g}{p^2c^2}x,$$

so that if p be the same for every section—that is, if the rectangular sections have all the same breadth—the curve bounding the vessel towards the lengths of these rectangles will be the common parabola.

THE RESISTANCE OF FLUIDS.—All bodies moving in a fluid are impeded in their progress by the resistance of the fluid, which must itself be moved, in order that motion may take place in the immersed body. The resistance to motion in the fluid is of course mainly due to its inertia; friction and the tenacity of the fluid, add somewhat to the resistance; but the retarding influence of these is too inconsiderable to render the consideration of them of much practical consequence.

It may safely be assumed that the resistance on a plane surface, moving with a given velocity through stagnant water, is the same as when, the plane being at rest instead of the water, a stream moving with the same velocity acts against *it*.

If a stream act perpendicularly against a plane surface, or if the surface act against the quiescent fluid by moving perpendicular to its surface through the fluid, the resistance will be as the area of the plane, the density of the fluid, and the square of the velocity conjointly.

For the velocity v of the stream being regarded as uniform throughout the whole depth of the plane, the resistance is the same at every point of the plane; and therefore, other circumstances being the same, it is proportional to the area A of the plane.

But the quantity of fluid matter striking against the plane, in a given time, is proportional to the density and velocity of the fluid conjointly; and this quantity of matter strikes with a velocity v, and therefore with a momentum equal to

quantity of matter × v.

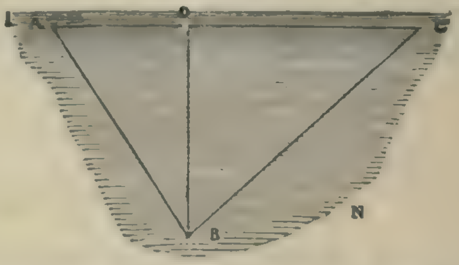
Hence, D being the density of the fluid, the resistance of the area A varies as A Dv^2.

If a body were to fall by the force of gravity till it acquired the velocity v of the stream, the space fallen through would be $s=\frac{1}{2}\frac{v}{g}$ (DYNAMICS, page 736); this, therefore, would vary as the square of the velocity; so does the resistance A Dv^2; hence the resistance varies as the weight of a column of the fluid, whose base is the area of the plane, and the altitude equals the space through which a heavy body must fall to acquire the velocity of the stream or of the moving plane.

The force with which a stream acts perpendicularly upon a plane opposed to it obliquely, varies as the square of the sine of the inclination of the plane to the stream.

Let A B (Fig. 189) and L N be sections of the plane and stream, moving in the direction of the latter; draw A C in that direction, meeting B C perpendicular to A B in C; draw also B D perpendicular to A C.

Fig. 189.

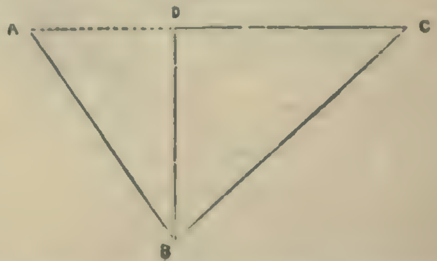


Now the quantity of fluid acting against A B is the same as that which acts perpendicularly against D B; it varies, therefore, as the length of B D; that is, as the sine of the angle A. If the velocity with which this quantity moves in the direction of the stream be represented by A C, then B C will represent the velocity with which it moves perpendicular to A B; and just as B D varies as the sine of the angle A, so does B C vary as the sine of the angle A; hence the force with which the stream acts perpendicularly upon the plane varies as sin.² A.

The force impelling the plane in the direction of the stream, varies as the cube of the sine of the inclination of the plane to that direction.

Let B C (Fig. 190), perpendicular to A B, represent the force on A B perpendicular to it; this may be resolved into two forces B D, D C, the latter D C in the direction of the stream, and the former B D perpendi-

Fig. 190.



cular to that direction. Now D C, the force in the direction of the stream, varies as B C sin. D B C; that is, as B C sin. A: but, as shown above, B C itself varies as $\sin.^2 A$; therefore, the force in the direction of the stream varies as $\sin.^3 A$.

This, of course, is on the assumption that the other component of the force of the stream in its own direction —namely, the component represented by B A, in the direction of the plane itself—is of no effect.

The force B D, perpendicular to the stream, is as B C cos. D B C = B C cos. A; hence the force with which the stream impels an oblique plane upwards, in a direction perpendicular to the stream, varies as $\sin.^2 A$ cos. A.

It must be observed, that in the foregoing propositions the plane is regarded as fixed while receiving the force of the stream, or else the fluid is regarded as quiescent, and the plane as moving through it. If both be regarded as moving in the same or in opposite directions, then the velocity spoken of above must be considered as the relative velocity; that is, it is the difference of the velocities if the plane and stream move in the same direction, and the sum of the velocities if they move in the opposite direction. Thus, if V be the velocity of the stream, and v the velocity of a float-board of an undershot water-wheel, then the perpendicular pressure on the float-board will vary as $(V-v)^2$.

The student has already been apprised, at the commencement of the present brief chapter on the motion of incompressible fluids, that the results of the mathematical theory often require considerable modification to render them accordant with actual experiment. This arises from the difficulty of assigning correct values to all the circumstances attending that motion. We have seen, in treating of fluids spouting from an orifice, that observation has discovered the orifice not to be the narrowest channel through which the stream uniformly flows, and consequently that the velocity there, is less than at the vena contracta, which ought therefore to be regarded as the true orifice.

It was found by Newton that the velocity at the vena contracta is, to the velocity of the orifice, as $\sqrt{2}$ to 1; and since, in falling bodies, the spaces passed through are as the squares of the velocities acquired, it follows that the space to be fallen through to give a body the velocity with which the fluid spouts from the *orifice*, is only half the space necessary to give the velocity at the vena contracta; that is, the velocity at the orifice would be acquired by a body falling through only half the altitude of the surface above the orifice.

In like manner, in treating of fluid resistances, the commonest observation shows that a stream impinging upon a plane surface cannot have the full effect which theory assigns. The velocity of the stream is, to a certain degree, impeded by what may be termed the backwater; moreover, the velocity is not in general the same at all depths. Yet the results of theory may usually be applied with safety in the *comparison* of different similar cases: for instance, the comparative results as to quantity of fluid discharged, time of emptying, &c., from equal orifices in two different vessels, disregarding the vena contracta, would accord very nearly with experiment. And, in like manner, fluid-pressures may be relatively estimated and computed from theory: for instance, though theory may assign an erroneous value for the effect of a stream upon the rudder of a ship, yet the position of *greatest* effect, as deduced from theory, may very well accord with actual observation; so the effect of a stream of water upon the floats of a water-wheel may be incorrectly assigned by theory, and yet the degree in which the velocity of the wheel should fall short of that of the stream, in order that the *greatest* effect may be produced, is found to be verified pretty closely by practical experience.

To determine particular values for the unknown, or arbitrary quantities, which enter into a general expression, so that for those values the expression may be a maximum or a minimum, requires the aid of the differential calculus. But in the two cases mentioned above, the principles to be employed are so elementary, and the knowledge required of that science so trifling, that although in general we are interdicted from using the calculus, we shall give the two problems adverted to as a conclusion to the present portion of our subject. We may premise, however, that in order to determine x so that any expression of the form

$$A + Bx + Cx^2 + Dx^3 + \&c. \quad . \quad . \quad . \quad . \quad (1)$$

may be a maximum, all we have to do is to disregard the term independent of x—that is, the term A—to write down all the other terms, first converting each *exponent* of x into a factor, and writing each exponent a unit smaller; thus

$$B + 2Cx + 3Dx^2 + \&c.,$$

and then to equate this expression to 0: that is, to solve the equation $B + 2Cx + 3Dx^2 + \&c. = 0$. The value of x, necessary to render (1) the greatest possible—if it *can* be made the greatest possible for any value of x—will be found by solving this equation.

Problem.—To find the angle at which the rudder of a ship must be inclined to the stream, so that the effect, in the direction perpendicular to the stream, may be the greatest possible.

It has been seen above that the effect varies as

$$\sin.^2 A \cos. A, \text{ or as } (1 - \cos.^2 A) \cos. A = \cos. A - \cos.^3 A = x - x^3$$

where x is put for the cosine of the required inclination.

$$\text{Since } x - x^3 = \text{a maximum}$$
$$\therefore 1 - 3x^2 = 0 \quad \therefore x = \sqrt{\tfrac{1}{3}}$$

hence the angle whose cosine is $\sqrt{\frac{1}{3}}$ will be the inclination necessary.

Problem.—The velocity V of the stream being given, to determine the velocity v of an undershot-wheel, so that the greatest possible effect may be produced.

The pressure upon the same area of float-board will vary as $(V - v)^2$. As this pressure moves with the velocity v, the effect will be greatest when

$$(V - v)^2 v = V^2 v - 2Vv^2 + v^3 = \text{a maximum};$$

in order to which, the unknown quantity v must satisfy the condition

$$V^2 - 4Vv + 3v^2 = 0$$

The solution of this quadratic gives

$$v = V, \text{ and } v = \tfrac{1}{3}V$$

The first value of v renders the proposed expression a minimum: that is, the least quantity of work is performed by the wheel when it moves with the same velocity as the stream—which is obvious: the other value of v is that which renders the expression a maximum; and shows that the greatest amount of work is performed when the wheel moves with a velocity equal to one-third the velocity of the stream. But, for the practical effects of water-power, acting through the medium of water-wheels, and other hydraulic machines, the student is referred to the section on Applied Mechanics, in the present volume.

[In the preceding chapters, the theories of momentum in dynamics, and the equal pressure of liquids in all directions as a fact in hydrostatics, have been investigated. Presuming the student to have become acquainted with the principles involved in each of these subjects, we shall now illustrate their application in pile-driving and pile-removing. In the construction of bridges, &c., it is often desirable to obtain a more solid foundation than the earth at the surface affords. This is especially the case when a bridge has to be built over a river, and the arches of which must be divided so as to be supported by their base on the bed of the stream. By means of the *momentum* of a mass of metal, piles are driven into the earth, forming the bed, and for this purpose the pile-driving machine is used. The following is a description of one of these machines, as represented in the annexed plate, Figs. 1 and 2.

Fig. 1 gives an end view of the machine. A A are two upright supports, generally about eighteen feet high. B is the base on which they are fixed. C is a fly-wheel for equalising the motion; and *k k* are winches turning G, a cylinder on which a rope is wound. By means of the rope, the "monkey," E, a heavy mass of metal attached to it by the head F at *f*, is raised by a pulley to the top of the uprights at *m*. The mode of detaching the monkey, after its being raised to the top, is thus managed:—In *e* (Fig. 1) we notice an arrangement like a pair of pincers. The same may be seen in Fig. 3, in which F, a cross-bar, receives the axis of these clamps. In Fig. 4, *n n* represents two rings, from each of which a cord is hung, so that they may be held in the hands of the workmen beneath. Now, supposing the "monkey" to have been raised to the top of the machine, and that its lower part is vertically over a pile, the men in attendance pull the cords stretched from *n n* (Fig. 4), and the clamps at once open and let fall the monkey. The separated clamp is represented in Fig. 5, with the cord as stretched from the hand of the person holding it. The "monkey," falling a height of some sixteen or twenty feet, acquires a velocity of nearly forty feet per second;* and being of great weight, its momentum† is sufficient to drive the pile receiving it with great force into the earth. The "monkey" is then again attached to the clamps, *e*, and raised to the top of the machine, again to fall. The operation is repeated until the pile has been driven sufficiently into the soil. Fig. 2 is a side view of the same machine, with the exception that an arrangement of teethed wheels is employed, as shown at G *g*. C is a block fixed to the uprights A for the sake of giving it additional support. Sometimes the flooring is extended, as at *a a*, and these joists are bolted firmly into a platform on which the machine stands.

Fig. 6 illustrates an application of hydrostatic laws. The principle of the hydrostatic power has already been explained at page 750. In this case, it is applied for the purpose of removing piles, which have been driven into the earth by the pile-driving machine. A is a bed of piles requiring to be removed, B being one of them. C D is a block of timber, which acts as a lever, whose fulcrum is E. This lever belongs to the second order,‡ the resistant or weight being in the lines *l*, *m*, *a*. The latter letter shows the chain by means of which the pile is attached to the lever: *d* is the head of the piston, and moves through a collar *b*. The piston is inside the cylinder F. The pump which works the press may now be described:—*h* is its handle, working on a pivot *g*; *f* is the plunger, which works in the cylinder *k*. Water is thus driven through the pipe *e* into F; and the power thus produced effects the removal of the sunken pile B by the action of the lever, &c. It will be seen that the principles of the lever, and the pressure of liquids, are both employed in this ingenious arrangement. A similar arrangement is adapted to hoists, cranes, &c.—Ed.]

CHAPTER VI.

PNEUMATICS.

THE fluids treated of in the foregoing chapter are all regarded as inelastic or incompressible: it remains for us to devote a few pages to the consideration of those of an elastic or compressible nature. The most important of these is the atmosphere with which we are surrounded: and it is in especial reference to this, that the examination of the mechanical properties of elastic fluids is said to belong to the science of *Pneumatics*—derived from a Greek term signifying, the air we breathe.

TRANSMISSION OF PRESSURE: COMPRESSION.—The mechanical properties of elastic and inelastic fluids are in many important particulars the same: the fundamental principle in Hydrostatics, for instance—namely, that a pressure applied to the surface of a fluid, is transmitted undiminished in all directions throughout the entire volume of the fluid—holds equally whether the fluid be elastic or inelastic; and in both cases may be put to the test of experiment in the same way. If the piston A, shown at page 748, play in a metal cylinder projecting into the interior of the vessel, and be forced inwards along the tube by a pressure or weight of 1 lb. say, it will advance under this pressure along the tube, and then stop: and it will be found that an additional pressure of 1 lb. must be applied to the piston B to prevent its being forced out.

In the case of a liquid, the only effect of the pressure on A is the transmission of it to every portion of the surface of equal area. In the case of the air, there is another and a distinct effect observably produced—namely, the compression of the fluid into smaller volume; and it is found by experiment that the volume diminishes just as the pressure increases: in other words, that the space into which any quantity of air is forced to compress itself, is inversely as the pressure applied. How this truth was ascertained we shall explain presently: it is necessary first to establish the fact that air has weight. It is possible to conceive that a substance like air may have the property of transmitting pressure, yet that it may not itself be a pressing or weighty substance.

EXPERIMENTAL PROOF OF ATMOSPHERIC PRESSURE.—But that the air exercises pressure, may be proved by many easy experiments: the boys' sucker is familiar to everybody. This toy consists of a piece of leather with a string passing through a perforation in the middle: upon being wetted so as to exclude the air, when it is pressed close to a smooth surface, it is found that considerable force must be applied to the string to detach the sucker; and if the surface of it be large, a stone of considerable weight may be thus lifted. What is it that thus presses the sucker and stone so firmly together? We are compelled to answer, the *air*, which surrounds them both as if they were one body. Before the sucker was applied, the air surrounded *it*: the pressure, if any, was alike on its upper and under surface: but when affixed to the stone, the pressure on the under surface of it was excluded, while that on the upper surface remained the same; and as an equal pressure is exerted on the corresponding area of the under surface of the stone, the two are, as it were, thus pinched or pressed together.

Again: take a glass tube, a foot or two long, open at both ends, and therefore full of air; plunge one end into a vessel of water: then we know from the first principles of Hydrostatics, that the level of the water will be the same both outside and within the tube. Let, now, the air in the tube be drawn out by the mouth, or pumped out by a machine fitted for the operation:—the water will be seen to rise within, and at length to fill the tube; and if the thumb be quickly applied to the top, so as to prevent the readmission of the air, the water thus filling the tube will remain suspended, just as it would do if an additional column of water of the same height as the unimmersed part of the tube pressed upon the surface of the water in the vessel. In the absence of this additional column of water, what is it that sustains the water in the tube? We are again compelled to answer, the pressure of the air which supplies its place.

We have here supposed the tube to be only a foot or two long, for convenience; but careful experiment has proved, that with a tube of sufficient extent, exhausted of air, the water would not cease to rise till it had attained the elevation of about 32 feet; which limit attained, it would remain stationary. On this large scale

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* See ante, p. 736.

† See ante p. 743.

‡ See ante, p. 719.

Hydrostatics.
See Page 768.

Fig. 1.

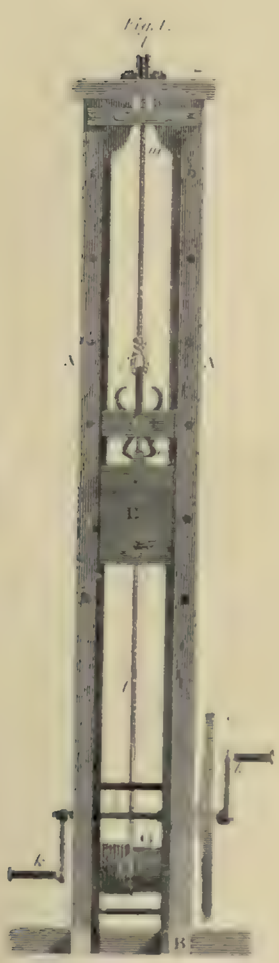


Fig. 3.

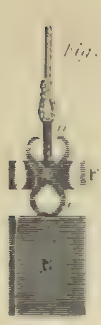
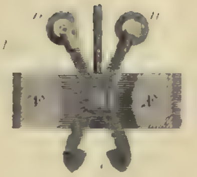


Fig. 4



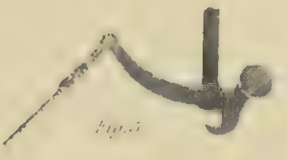
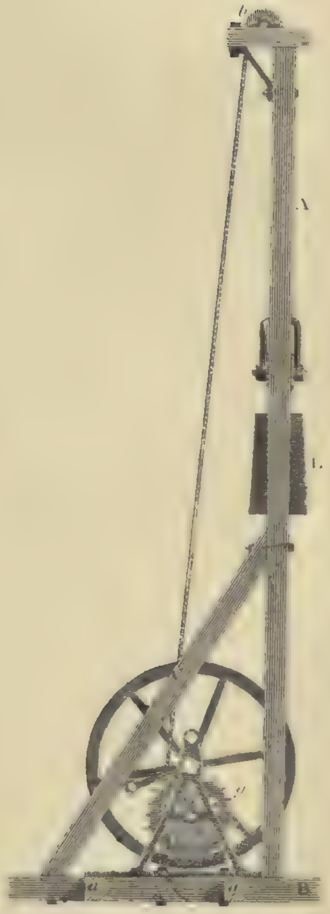


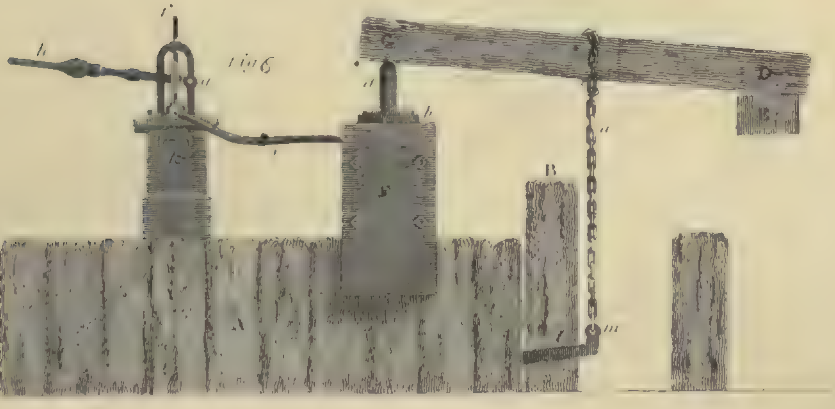
Fig. 5

Fig. 2.



Mr. Bramah's Machine for Drawing Piles out of the Ground.

Fig. 6



the experiment was actually performed, in 1647, by the celebrated Pascal: he procured glass tubes closed at one end, of forty feet long, and found, that when filled with water in a deep river, and then raised vertically with the open end downwards, the fluid ceased to fall when water in the tube stood about 32 English feet 2½ inches from the surface of the river. This experiment clearly proved that the pressure of the whole atmosphere on any surface was equal to the pressure of a column of water 32¼ feet high on the same surface. And thus was satisfactorily shown not only that air has weight, but what amount of weight was sustained by a given area of the surface of the earth pressed upon by the atmospheric column resting upon it: the weight sustained would be equal to that of a column of water, on the same area, about 32¼ feet high.

The same conclusion is obtained with a more manageable length of tube by using mercury instead of water. And in this way the experiment had been tried previously by Torricelli, and it is hence called the Torricellian experiment: it led to the invention of the barometer. When a tube about three feet in length, and closed at one end, is filled with mercury, and then inverted in a basin of that fluid, the column of mercury held suspended in the tube is found to be about 29 inches above the surface; and as mercury is about 13½ times the weight of the same volume of water, we arrive at the same result—namely, that the pressure of the atmosphere on any area, is the same as the pressure of a column of water on that area of about the height

$$29 \times 13\tfrac{1}{2} \text{ inches} = \text{about } 32\tfrac{1}{4} \text{ feet.}$$

An exact correspondence between the results of such experiments made at wide intervals of time must not be expected, because the weight of the atmosphere fluctuates, like the other changes in its condition. The average height of the mercurial column is about 30 inches, the ordinary range being between 28 inches and 31 inches; and as 30 cubic inches of mercury weigh about 15 lbs., every square inch of the surface of the earth, at the level of the sea, sustains, on the average, 15 lbs. of atmospheric pressure.

WEIGHT OF A VOLUME OF AIR.—Besides the foregoing methods of proving that the whole atmosphere exerts a pressure, upon the globe of the earth, equal to that which would be exerted by a sea of mercury 30 inches deep, covering its entire surface, a definite portion of the air about us may be taken, and, like any other material substance, be actually weighed in a balance.

Let a vessel containing a cubic foot, that is, 1000 ounces, of water be provided with a stop-cock screwed upon its neck. This vessel may be exhausted of its air by aid of the air-pump, a machine which will be hereafter described. Thus emptied, and the stop-cock closed so as to prevent the readmission of air, let the vessel be accurately weighed in a very sensible balance. (See STATICS, page 723).

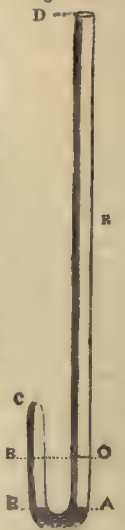
When the exact counterpoise is thus ascertained, let the stop-cock be opened and the air admitted into the vessel; the vessel thus filled, with the stop-cock again closed, to cut off all external pressure, will be seen at once to preponderate, and we shall find it necessary to put about 523 grains of additional weight in the other scale-pan to restore the equilibrium; thus showing that a cubic foot of air, at the surface of the earth, weighs about 523 grains.

We cannot, of course, speak accurately as to a few grains more or less, but it will be always found that at least an ounce more weight will be required to counterpoise the vessel when full of air, than was required before the air was admitted.

It is in this way ascertained, that when the barometer stands at 30 inches, and the temperature of the air is 52° of Fahrenheit's thermometer, the weight of water is to the weight of an equal volume of the air around us as 840 to 1.

LAW OF MARRIOTTE.—Having thus established the truth of the proposition that air has weight, we may now proceed to explain the way in which the law of Marriotte, namely, that the volumes into which air may be compressed are inversely as the pressures applied, is arrived at.

Fig. 191.



Let DAC (Fig. 191) be a bent tube of equal bore throughout, or at least equally wide throughout, the shorter vertical leg. Conceive this tube to be at first open at both ends C and D, and let a little mercury be poured in, so as to fill the bend of the tube; the level of the mercury at A and B will be at the same height AB in each tube, however dissimilar the two portions of the tube may be (HYDROSTATICS, page 751). Now let the end C be closed, and let such a quantity of mercury be poured in at D as will cause the surface, originally at C, to rise to B′, half-way between B and C. Upon measuring the additional column of mercury thus introduced into the tube—that is, having previously marked the level O of B′—if we now mark the level E when the surface B has risen to B′, we shall find that the length of the mercurial column O E is exactly equal to that of the mercurial column (as shown by the barometer) which represents the weight of the atmosphere. The inference is this: when the surface A sustained the pressure of only the column of air above it, the air in the shorter leg occupied the space B C; the air in B C was thus compressed into the volume it then had, simply by the weight of the atmosphere applied upwards to the surface B.

But when the weight of *two* atmospheric columns pressed on the surface B, that is, the weight of the barometric column of mercury in addition to the atmosphere, then the volume of air in B C occupied only half the space—namely, B′ C; that is, the original volume was compressed by the double pressure into *half* its former bulk. Again, if another column of mercury be poured into the tube at D, till the air in the other leg occupies only one-third of its original volume, we shall find that the height of the entire column of mercury, measuring the difference of the levels in two legs, is twice that of the barometric column; so that the air in B C, when pressed upwards by the weight of *three* atmospheres—the atmosphere itself pressing on the upper surface of the mercury and the double barometric column—is compressed into *one-third* of its original volume. Results in harmony with these are always found to follow whatever fractional portion of the whole atmospheric pressure be applied; and thus the law is experimentally established, that the volumes into which air is compressed by pressures are inversely as the intensities of those pressures. And this is only saying that the densities are directly proportional to the pressures.

As it is the elastic force of the condensed air that thus balances the additional pressures, we further infer that the elastic force is proportional to the density or to the force of compression.

A form somewhat more mathematical may be given to the foregoing account as follows:—

The mercury, at first poured into the tube at D, standing at the same level A B in both legs, and the end C of the tube being then closed, the space B C is occupied by the air in its existing state of atmospheric pressure at the time of the experiment, which pressure is indicated by the column of mercury in the barometric tube at that time. Fresh mercury is now poured in at D; its surface reaches the level E in one leg, and some lower level B′ O in the other, showing that the pressure of the air originally occupying C B, by now being forced to contract itself into C B′, exercises an increased pressure; so that now it not only balances the pressure of the atmosphere, in its original state at D, but also the column of mercury O E.

Let H be the height of the mercury in the barometer; then the pressure on the original volume of air B C is that of a column of mercury of base B and height H, while the pressure on the condensed volume, B′ C, is that of a column of mercury of the same base and of the height H + E O. Now, wherever B′ may be, it is found by actual measurement that

$$\mathrm{H + E\,O : H :: B\,C : B'C;}$$

and the pressures and volumes being as these linear dimensions, it follows that

Pressure on B′ C : Pressure on B C : : Volume B C : Volume B′ C.

And the same proportion is found to hold whatever be the original density of the air experimented upon ; and it equally has place for all elastic fluids.

The foregoing law, although generally called the law of Marriotte, is equally entitled to be called Boyle's law ; as it was announced, independently, by the latter philosopher, at nearly the same time. It was discovered by Boyle in the year 1662.

Since the force of compression on the air near the surface of the earth is that due to the weight of the superincumbent atmosphere, it follows that the air must be rarer the higher we ascend ; rarer, for instance, at the top of a mountain than at its base—a truth that has been turned to practical account in the way of measuring the height of a mountain by observing, with the barometer, the difference of atmospheric pressure at top and bottom.

With the general theoretical principles and the practical construction of this useful instrument, we here presume the reader to be already acquainted ; for although some account of the theory of the barometer might reasonably be expected in a treatise on Pneumatics, yet, as the subject appertains chiefly to METEOROLOGY, and as the space now at our disposal is limited, we must refer for the requisite information, upon what concerns the barometer and thermometer, to that section.

DENSITY OF THE ATMOSPHERE AT DIFFERENT HEIGHTS.—Since the density of air varies directly as the pressure sustained by it, the higher regions of the atmosphere having a less superincumbent load than the lower, must be proportionally less dense. This weight or pressure, as already noticed, will be affected by temperature : and moreover, the same particle, or volume of air, will weigh less, the higher it is situated above the surface of the earth, on account of the diminution in the force of gravity. But, leaving these comparatively unimportant modifying influences out of consideration—unimportant, at least, for moderate altitudes—and regarding both the temperature and the force of gravity as uniform, we may prove that—The density of the atmosphere at different heights above the surface of the earth, varies thus—namely, when the heights increase in arithmetical progression, the densities decrease in geometrical progression.

Conceive a uniform slender column of the atmosphere to be divided by horizontal sections into n equal parts ; the number n being so great, that each thin stratum of air may be regarded as of uniform density : then commencing at the bottom, or first stratum a_1, the several strata may be denoted by

$$a_1,\ a_2,\ a_3,\ a_4,\ \ldots\ a_n\ \ldots \quad (1)$$

and their densities by

$$d_1,\ d_2,\ d_3,\ d_4,\ \ldots\ d_n\ \ldots \quad (2)$$

It has been sufficiently explained (HYDROSTATICS, page 750) that, in speaking of the density of any substance, we always have reference to some assumed unit of density, and that the weight of the unit of volume (the unit of weight) of the standard substance, multiplied by the numerical expression (D) for the density of any other substance, gives the weight of a unit of volume of that other substance. The abstract numbers (2) therefore will equally express the number of units of weight of the volumes (1), provided we take the magnitude of a_1 or a_2, &c., for that of the unit of volume : and this we are of course at liberty to do, for in all investigations our unit of measure may be chosen at our convenience.

It may be well, however, in order to prevent confusion, to write the numbers (2), when regarded as so many units of weight, thus, namely :—

$$w_1,\ w_2,\ w_3,\ w_4,\ \ldots\ w_n.$$

Then, as the density of any stratum of air varies with the weight or pressure it sustains, we have

$$\left.\begin{array}{l} d_1 : d_2 :: w_2 + w_3 + w_4 + \ldots + w_n : w_3 + w_4 + \ldots + w_n \\ d_2 : d_3 :: w_3 + w_4 + w_5 + \ldots + w_n : w_4 + w_5 + \ldots + w_n \\ d_3 : d_4 :: w_4 + w_5 + w_6 + \ldots + w_n : w_5 + w_6 + \ldots + w_n \\ \qquad \&c. \qquad\qquad \&c. \qquad\qquad \&c. \end{array}\right\} (3).$$

But

$$\left.\begin{array}{l} d_1 : d \ :: w_1 : w_2 \\ d_2 : d_3 :: w_2 : w_3 \\ d_3 : d_4 :: w_3 : w_4 \end{array}\right\} (4).$$

Hence, putting w for d, in the foregoing proportions, alternating and compounding the ratios (ALGEBRA, page 477), we have

$$\begin{array}{l} w_1 : w_1 + w_2 + w_3 + w_4 + \ldots + w_n :: w_2 : w_2 + w_3 + w_4 + \ldots + w_n \\ w_2 : w_2 + w_3 + w_4 + w_5 + \ldots + w_n :: w_3 : w_3 + w_4 + w_5 + \ldots + w_n \\ w_3 : w_3 + w_4 + w_5 + w_6 + \ldots + w_n :: w_4 : w_4 + w_5 + w_6 + \ldots + w_n \end{array}$$

Consequently, alternating, and having regard to the proportions (3) and (4), we have

$$d_1 : d_2 :: d_2 : d_3 :: d_3 : d_4,\ \&c. ;$$

therefore the densities of the strata a_1, a_2, a_3, a_4, &c., of which the heights are in arithmetical progression, are themselves in geometrical progression ; and consequently the pressures sustained at different altitudes are also in geometrical progression.

DETERMINATION OF ALTITUDES BY THE BAROMETER.—Let A be the altitude in feet of any spot above the surface of the sea ; then measuring downwards from that spot, the distances 0, 1, 2, 3, &c., feet, the pressures upon each foot will form a geometrical increasing progression, by what is proved above. The relation, therefore, between the numbers expressing this progression, and the numbers 0, 1, 2, 3, &c , is the same as that between any numbers in geometrical progression, and their logarithms. The numbers 0, 1, 2, 3, expressing the distances downwards from the highest point, must therefore be the common logarithms of the numbers expressing the corresponding pressures, multiplied by some constant factor or modulus, which constant factor we may call K.

Hence, a expressing any lower altitude in feet, and P, p the pressures at the altitudes A, a, we shall have

$$\mathrm{A} - a = \mathrm{K} \log. \mathrm{P} - \log. p = \mathrm{K} \log. \frac{\mathrm{P}}{p}$$

If the lower station be at the level of the sea, then $a = 0$; and since the pressures P, p are indicated by the heights of the mercurial column in the barometer at the two stations, we have for A, the number of feet in the altitude of the upper station above the lower,

$$\mathrm{A} = \mathrm{K} \log. \frac{\mathrm{P}}{p} = \mathrm{K} \log. \frac{\text{height of barometer at upper station}}{\text{height of barometer at lower station,}}$$

But before this formula can be turned to practical account, we must be able to assign the numerical value of the constant multiplier K.

Let H be the height of the barometer at the level of the sea, and H′ the height at some known altitude A′; then

$$\mathrm{A'} = \mathrm{K} \log. \frac{\mathrm{H}}{\mathrm{H'}} = \mathrm{K} \log. \frac{\mathrm{P}}{p}$$

Let A′ = 1 foot, then

$$K = 1 \div \text{log. } \frac{H}{H'}, \text{ or } K = 1 \div \text{log. } \frac{P}{p}.$$

Now when the barometer stands at 30°, and the temperature of the air is 55° Fah., the weight of a volume of air is to that of an equal volume of mercury as 1·22 is to 13568 : hence the pressure of the 30 inches of mercury is equal to the pressure of a column of $\frac{13568 \times 30}{1{\cdot}22}$ inches of air of the same uniform density as that in which the barometer is placed—that is, of the air at the level of the sea. The height of this equiponderant column of homogeneous air is therefore

$$\frac{13568 \times 30}{1{\cdot}22 \times 12} \text{ feet} = 27803 \text{ feet ;}$$

and, therefore, the height of the column measured from a foot above the level of the sea, is 27802 feet.

$$\therefore K = 1 \div \text{log. } \frac{27803}{27802} = 1 \div \text{log. } (1 + \frac{1}{27802}).$$

But log.

$$(1 + \frac{1}{27802}) = {\cdot}43429448 \left\{ \frac{1}{27802} - \frac{1}{2} \frac{1}{27802^2} + \&c. \right\}$$

(MATHEMATICAL SCIENCES, page 519), therefore, neglecting the powers of so small a fraction,

$$K = 1 \div \frac{43429448}{27802} = \frac{27802}{{\cdot}43429448} = 64020 \text{ feet ;}$$

or dividing by 6, K = 10670 fathoms. Consequently the formula for the computation of any altitude A above the level of the sea, is

$$A = 10670 \text{ log. } \frac{\text{height of bar. at alt. A}}{\text{height of bar. at the sea-level.}} \text{ fathoms.}$$

The constant factor 10670 has been determined on the supposition that the temperature of the air is 55° Fah. The more convenient multiplier 10000 may be employed instead, by a suitable alteration of the temperature ; that is, by assuming the air to be in a different state. Experiment has shown that the altitude of a place, as deduced from the foregoing formula, will vary by $\frac{1}{435}$th of its whole value, for every degree by which the mean of the temperatures at the two stations differs from 55° ; the variation to be added when the mean exceeds 55°, and subtracted when the contrary is the case.

Now, the difference between 10670 and 10000 is 670, and the difference between 10670 and the value it would have for one degree less of temperature is $\frac{10670}{435} = 24{\cdot}5$: hence, to find for how many degrees less of temperature the difference is 670, we have the proportion

24·5 : 670 : : 1° : 27°.

Consequently 55° — 27° = 28° is the temperature of the air for which the altitude A in fathoms is

$$A = 10000 \text{ log. } \frac{\text{height of bar. at alt. A}}{\text{height of bar. at the sea-level.}}$$

And this value must be increased or diminished by the $\frac{1}{435}$th part of itself for every degree shown by the mean temperature of the two stations above or below 28°.

In taking the logarithms, we may employ only five figures for each, including the index 1. Regarding these as whole numbers, we take their difference ; then if E be the excess of the mean temperature of the two stations above 28°, we must multiply this difference by $\frac{E}{435}$, and add the result to the said difference to obtain the altitude in fathoms. This fraction of the difference must be subtracted if E be negative. The following is an example :—

Required the height of a mountain when the barometer at the bottom stands at 29·68 inches, and at the top at 25·28 inches, the mean temperature, that is, half the sum of the temperatures at the two stations, being 47°. The excess of the mean temperature above 28° is 47° — 28° = 19°

$$\begin{array}{lr} 10000 \times \begin{cases} \text{log. } 29{\cdot}68 \;.\;.\;.\;. \\ \text{log. } 25{\cdot}28 \;.\;.\;.\;. \end{cases} & \begin{array}{r} 14725 \\ 14028 \\ \hline 697 \end{array} \qquad \frac{E}{435} = \frac{19}{435} \\ \qquad\qquad 697 \times \frac{19}{435} = & 30 \\ \cline{2-2} \therefore \text{the height is} \;.\;.\;.\;.\;. & 727 \text{ fathoms} = 4362 \end{array}$$

feet.

Where minute accuracy is required, certain particulars, here disregarded, must be taken into consideration ; as, for instance, the latitude of the place of observation, and the dilatation of the mercurial column, which lengthens to the extent of $\frac{1}{10000}$th of the whole for every additional degree of temperature : but the determination of altitudes by the foregoing formula will, in general, differ from the truth only by a fathom or two, and will therefore be sufficiently accurate for most practical purposes. (See Chapter V.—Section, Meteorology.)

The higher we ascend into the regions of the atmosphere, the colder does it become. There are two reasons for this : in the first place, the direct rays of the sun pass through the air without being arrested, as it were, as in the case of a solid body, which absorbs a great portion of the heat poured upon it ; and, in the next place, experiment proves that air by rarefaction loses part of its original heat, so that when air of a certain temperature is relieved of part of the pressure upon it, and allowed to expand (as the air in the receiver of an air-pump), it is found that the temperature becomes less and less as the pressure diminishes ; so that if the original temperature is to be maintained, fresh heat must be communicated. As there is no external source for the supply of such heat in the more elevated regions of the atmosphere, the superior coldness of those regions becomes sufficiently accounted for.

WEIGHT OF THE WHOLE ATMOSPHERE.—We have already seen that the weight of the atmosphere surrounding the earth is equal to the weight of a surrounding coating of mercury, on the average 30 inches thick : the amount of this weight may be found by multiplying the volume of the mercurial shell by the specific gravity of the mercury, and the product by g = 1000 ounces (HYDROSTATICS, page 752). Let R = radius of the earth in feet, r = height of the mercury in feet, and s its specific gravity : then subtracting the volume of a sphere of radius R from that of a sphere of radius R + r, in order to get the volume of the spherical shell of mercury, we shall have for the weight W of that shell

$$W = \left\{ \frac{4\pi(R + r)^3}{3} - \frac{4\pi R^3}{3} \right\} sg$$

$$= \frac{4\pi}{3} \left\{ 3R^2r + 3Rr^2 + r^3 \right\} sg$$

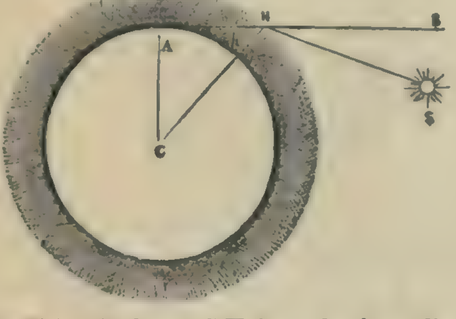
$$= \pi r \left\{ R^2 + Rr + \tfrac{1}{3}r^2 \right\} s + 1000 \text{ oz.}$$

This weight the celebrated Cotes calculated to be equal to that of a globe of lead of 60 miles in diameter, or upwards of 77,670,000,000,000,000 tons.

The weight of the atmosphere can be much more accurately estimated than its height, since the former can be submitted to experimental examination, while the latter is beyond our reach. But there are many cogent reasons which preclude the notion that the atmosphere is illimitable. Thus, we have seen that it has weight ; that is, that it is a material substance, like all other material substances, operated upon by the attraction of gravitation. There must be an elevation, therefore, at which its elasticity, or its tendency to expand further upwards, is just balanced by its gravitating tendency downwards, and which must therefore mark the limit of its altitude. Again, if its extent were boundless, the moon and all the other planets would each, by its attraction, appropriate a share of it ; and, as in the case of our earth, the density of it would increase towards the surface of each planet; and, more especially as regards the moon, its presence would become manifest by astronomical observation. But astronomers find the moon to be quite destitute of an atmosphere: it has no clouds, no rain; and some of the planets seem to be in the same predicament. That the moon has no atmosphere, and consequently can have no breathing inhabitants, is a fact interesting in itself; and it involves this other fact, equally interesting, that our atmosphere has its limits. Accurately to assign these limits is beyond the power of science; but the phenomenon of the refraction and reflection of light leads to the conclusion that it extends to the height of about forty or fifty miles. The average between those, namely, forty-five miles, is usually regarded by philosophers to be the height of our atmosphere. But that this conclusion may rest upon something better than mere conjecture, we shall give the reasonings by which it has been arrived at.

Atmospherical refraction is caused by the bending of the rays of light from a luminous body upon entering our atmosphere: till the outer boundary of the atmosphere is reached, nothing diverts the direction of a luminous ray. This refraction of the rays of the sun adds intensity to the twilight, or the light we enjoy after sunset. The twilight is found to continue till the sun is 18° below the horizon. Let, therefore, A H (Fig. 192) be the horizon of an observer at A, who will continue to have twilight till the ray S H from the descending sun makes with B H the angle S H B = 18°, and therefore the angle A H S is 162°.

Fig. 192.



The last glimmer of twilight is due, no doubt, mainly to the reflection of the ray S H from the particles of the atmosphere at H; and the radius C H being perpendicular to the reflecting surface at H, the angles A H C, S H C must be very nearly equal, and therefore each equal to about 81°. Now taking the radius C A of the earth at 4,000 miles, we have by Trigonometry,

$$CH = CA\,\text{cosec.}\,H = \frac{CA}{\sin.\,H} = \frac{4000}{\sin.\,81^\circ} = 4050 \text{ nearly.}$$

So that, according to this calculation, the height h H of the atmosphere is 4,050 — 4,000 = 50 miles.

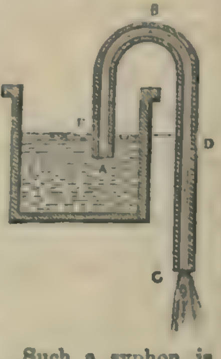
The height of an homogeneous atmosphere that would press, as the actual atmosphere is found to do, may be readily ascertained. Thus, if we take the specific gravity of air to that of water as 1 to 850, when the barometer stands at 30 inches, and the specific gravity of water to that of mercury as 1 to 14, we shall have the specific gravity of air to mercury as 1 to 11,900.

$\therefore$ 1 : 11,900 : : 30 inches : 357,000 inches = 5·63 miles. This, therefore, would be the height of an atmosphere pressing with the same weight as ours does, and of which the density is uniformly the same as the air at the surface of the earth. If the specific gravity of the air at the surface to that of mercury, when the barometer stands at 30 inches, be taken as 1 to 12,000, the height of the homogeneous atmosphere will be 12,000 × 30 inches = 10,000 yards = 5⅔ miles nearly. Hence the height of the equiponderant homogeneous atmosphere would be about 5⅔ miles.

THE SYPHON.—The weight of the atmosphere, like all the other forces spontaneously offered to us by nature, has, by the ingenuity of man, been made subservient to his wants and conveniences in a great variety of ways. Next to the force of gravitation, of which, indeed, the weight of the air is only a particular manifestation, atmospheric pressure is perhaps the most important of terrestrial phenomena; and it is not easy to estimate the amount of extra toil and privation to which mankind would be subjected, if the air we breathe, like the light we see, had no appreciable weight. It is our business now to give a short account of some of the contrivances by which atmospheric pressure has been turned to advantage in the practical affairs of life, and to explain the principles upon which these contrivances accomplish the purposes intended by them. The least complicated of these is the Syphon.

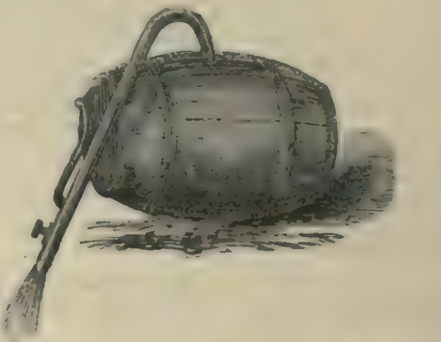
This instrument is simply a bent tube A B C (Fig. 193), employed for the purpose of exhausting a vessel of the liquid it may contain, or of transferring it to another vessel without the practical inconvenience—often the practical impossibility—of actually pouring the liquid from the one vessel into the other.

Fig. 193.



There are two ways of bringing the instrument into operation: the end of the shorter leg A may be inserted in the liquid, and then the air in the tube withdrawn by the mouth through a small pipe communicating with the tube near the extremity C of the longer leg. In this case there must be a stopcock between the pipe and C to cut off communication with the atmosphere pressing through C. Such a syphon is called the distiller's syphon: it is exhibited in operation in Fig. 194. In the second way the tube is inverted and filled with the fluid, and the ends A, C closed; the shorter end is then immersed, and both ends are opened.

Fig. 194.



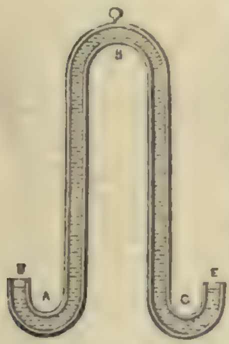
Suppose, as in the first way, that the tube, with the end A (Fig. 193) in the liquid and the end C stopped, has been exhausted of air: the pressure of the atmosphere on the exposed surface of the fluid in the vessel acts upwards at A, and forces the fluid into the vacant space with an energy sufficient, if it were water, and the tube were straight, to carry it to the height of thirty-two or thirty three feet; and to a greater height if it were a fluid lighter than water. As, however, the tube bends at a moderate height B, the ascending column is forced to accommodate itself to the course of the tube, and descends into the leg B D C. Arrived at B, the highest point, its descent down B C is expedited by the direct influence of gravity upon it; and C being opened, the liquid flows out.

Now it must be observed, that when the column has attained the height E B, it is not forced forward by the whole of the atmospheric pressure at E, but only by that pressure diminished by the weight of the column E B; so that when the column had extended itself to D, if the atmospheric air were admitted, the upward pressure on D, like that on E, would be equal to the whole atmospheric pressure diminished by the weight of the column D B; the two pressures, therefore, balance. But as soon as D is passed, and the column in B C lengthens, the equilibrium is destroyed, the downward pressure prevails, and the liquid falls through C upon finding a passage there: and the longer the part D C of the syphon below the surface D of the liquid, the greater will be the velocity with which it will issue, since the greater will be the preponderating pressure downwards.

In the second way of preparing the syphon, that is, by first filling it with the fluid, it is brought, in another manner, into the same circumstances as when it is filled, as above, through the pressure of the air on the surface of the liquid. The liquid will evidently cease to flow as soon as the surface of it descends to a level with the rising surface in the vessel to which it is transferred; at this stage, however, the syphon will remain full, as there will be a complete equilibrium.

In the syphon just explained, it is necessary that the legs be of unequal length; but there is another kind of syphon called the Wurtemburg syphon, in which the two legs are of the same length. In this instrument (see Fig. 195), the extremities of the equal legs are turned upwards so that the two open ends D, E may be on the horizontal level when the syphon is held upright. The instrument is kept constantly filled with water, which remains suspended in the two legs D A B, E C B, because the equal atmospheric pressures at D and E place them in a state of equilibrium; but if the leg terminating in D be immersed in water, the end D will sustain the additional pressure of the water reaching from D to the surface, and consequently E being free from such additional pressure, the fluid is forced out at that end. When the surface of the water in the vessel has descended to D, the stream from E stops, but the syphon remains full, and, thus filled, is taken out and hung up by a loop at B till again wanted.

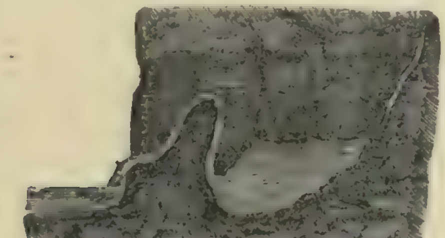
Fig. 195.



INTERMITTING SPRINGS.—The curious phenomena of intermitting springs, are referable to the foregoing principles. These springs, issuing from a fissure in a mountain side, flow for a certain period and then stop; after awhile the water flows again, and so on. These effects are produced through the operation of one of Nature's syphons. Fig. 196 exhibits a section of the mountain and the stream. A cavity exists in the former, as here represented; this is filled by gradual infiltrations, or by slender fissures communicating with the upper surface.

Suppose, now, that there is a syphon-like communication between the reservoir of water, supplied by these channels from above, and the mountain side; as this syphon is not artificially exhausted of air, it will not deliver the water in the reservoir till the surface of that water rises as high as the bend of the syphon B; after which it will begin to flow, pouring out a continued stream till the reservoir is emptied, or at least till the level reaches the immersed end of the syphon; the stream will then stop, and will commence to flow again only when, by a fresh accumulation of water from above, the level of B is again reached.

Fig. 196.



It is, of course, a condition necessary to the production of these results that the surface of the reservoir be subjected to the pressure of the atmosphere; but through the fissures which supply the cavity with water, air must have previously found its way; and, indeed, from the principle of the equal transmission of fluid-pressure, the weight of the atmosphere acting on the upper surface of the water in a single downward column, as on the water in the slender perforation to the right in the foregoing figure, acts equally on every part of the surface of the water in the reservoir.

THE HOUSEHOLD PUMP.—The common suction-pump is another important contrivance, which owes its usefulness entirely to the pressure of the atmosphere, although its dependence upon this agency was little suspected for many ages after its invention. As already noticed (page 769), Torricelli, a pupil of Galileo, early in the seventeenth century, was the first who fully recognised the influence of atmospheric pressure. He accounted for the ascent of water in the pump, and of mercury in an exhausted tube; and the invention of the barometer naturally followed.

Fig. 197.

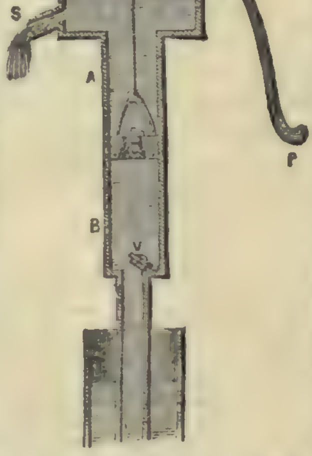


Fig. 197 represents a section of the suction-pump; A B is the tube, or working pump-barrel, communicating with the water in the well. In this barrel, an air-tight piston C, moved by the lever or pump-handle P, freely plays. At the lower extremity of the working barrel there is a valve V opening upwards: this valve separates the barrel from, and completely covers, what is called the suction or feeding-pipe, which is usually of smaller bore. In the piston C there is also a valve similarly opening upwards. The piston C is called the *sucker*.

Now, imagine the piston C to be at first at the bottom V of the barrel, and then to be raised by the action of the pump-handle. As the piston is air-tight, the pressure of the air in the barrel upon the ascending valve C keeps it closed, so that that air, having no escape below, is forced up and pumped out at the spout S. The vacuum thus produced in the barrel is immediately filled up by the ascent of the water through the feeding-pipe, for the water in the well sustains the pressure of the atmosphere on its surface—all except that portion of the surface which the feeding-pipe covers; and from *this* portion, as just explained, the air has been withdrawn. The barrel is thus filled with the water forced up by the pressure of the atmosphere on the exposed surface of that in the well; but no water escapes through the spout S, since all that has been raised is below the closed valve C; but it is retained suspended in the barrel, though the valve V still remains open, the upper surface of the water, immediately below the piston, being, of course, not more than 32 or 33 feet above the surface of the water in the well (page 769).

Upon now lowering the piston, the valve C opens by the resistance of the water confined in the barrel; for the downward pressure communicated to the confined water, and exercised through the lever P, closes the valve V, and prevents the water from being, by this extra pressure, forced back again into the well. Hence, upon again raising the piston, the upward resistance of the water above it closes the valve C, and the fluid is forced out of the spout S, while another vacuum is forming below the piston, and fresh water rising in the barrel. Thus every time the piston is raised, the lower valve opens and the upper one closes; while, on the contrary, every time the piston descends, the lower valve closes and the upper one opens; and in a well-made pump the barrel is kept constantly full.

It will of course be understood, in all that is said above, that the pressure of the atmosphere on the surface of the water in the well, or reservoir, need not be sufficiently great to cause the water in the pump to ascend as high as the spout, when, by the action of the handle, the air is withdrawn from the tube: if the length of the suction-pipe be 28 or 29 feet, or rather, if the valve at the top of it be at this height above the water in the well, the exhaustion of the air will be followed by the ascent of the water, through the suction-pipe, to some distance up the barrel of the pump. The subsequent raising of the water, thus introduced into the barrel, to the spout, is the sole result of the mechanical force applied to the pump-handle.

QUANTITY OF WATER DISCHARGED, AND FORCE APPLIED TO RAISE IT.—The quantity of water discharged at each stroke of the handle, supposing the barrel to be constantly full—that is, supposing the spout not too high—is a column of water whose base is the horizontal section of the piston, and of the height to which the piston is raised, called the length of the stroke: thus, if r be the radius of the section in feet, and l the length of the stroke in feet, we shall have

$$\text{Quantity discharged} = 3{\cdot}1416r^2l \text{ cubic feet};$$

or, since a cubic foot of water weighs 1000 ounces, or about 62½ pounds avoirdupois, and since an imperial gallon contains 10 lb. of water, we have

$$\text{Quantity discharged} = 3{\cdot}1416r^2l \times 6{\cdot}25 \text{ gallons}.$$

But the force necessary to raise this quantity, will be that required to raise the entire column of water extending from the surface of the reservoir to the surface of the water in the pump, the base of the column being the section of the piston. For let a be the number of linear feet of water above the piston, and b the number of feet below it, to the surface of the reservoir; let also p be the length of the column of water equal to the atmospheric pressure: then since, in raising the piston, the downward pressure of the atmosphere has to be overcome, the height of the equivalent column of water lifted is $a + p$; but the upward pressure against this, is equal to the weight of a column of water of height $p - b$: hence

$$(a + p) - (p - b) = a + b$$

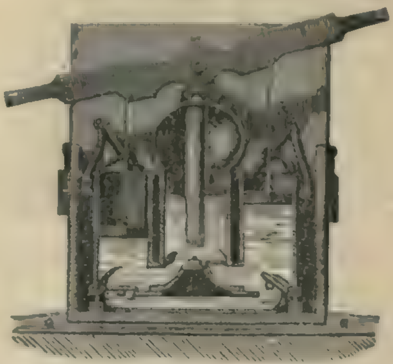
Consequently, the force necessary to lift the piston is that necessary to lift a column of water having the same section as the piston, and the height $(a + b)$, of the water in the pump, from the surface of that in the reservoir; and, in fact, additional force must be applied to pump out the water, on account of friction, and the weight of the piston and rod.

In the returning stroke of the pump-handle, the piston descends by its own weight, which is sufficient to overcome the friction, and the slight resistance of the water.

FORCING-PUMP AND FIRE-ENGINE.—As the pressure of the atmosphere varies, pumps are not constructed to raise water to a height above 28 or 29 feet; but the water thus raised may, by an additional contrivance, be forced upwards as much higher than this as we please. A pump for this purpose is called a *forcing pump*. In this pump there is no valve C in the sucker or piston; so that, after the exhaustion of the pump-barrel of air, and the consequent filling of it with water, no downward pressure on the piston could cause its descent, since the water is incompressible. A pipe, therefore, is inserted in the side of the barrel near the bottom, with a valve at the insertion opening outwards; the downward pressure on the solid piston forces open this valve, and drives the water into the tube, which may be carried upwards to any height, and be made to deliver the water there, provided only sufficient downward pressure act on the piston.

The fire-engine (Fig. 198) is essentially the combination of two forcing-pumps, the pistons of which are worked by a lever whose fulcrum is at its middle, and the power is applied at each end alternately; the water from the fire-plug is forced into the central receptacle, called the air-vessel, the air in which is thus condensed the more, the more water is forced in; the elastic force of this compressed air drives the water up the leathern hose, from which it issues through the delivery-pipe with a velocity proportionate to the pressure applied, or the condensation produced in the confined air. If the pistons be worked with sufficient energy to supply the air-vessel with water as rapidly as it is thus delivered, the stream will be invariable; if more water be forced into the vessel than can escape out of the delivery-pipe, the air-vessel will be in danger of bursting, by the increased pressure of the condensed air.

Fig. 198.

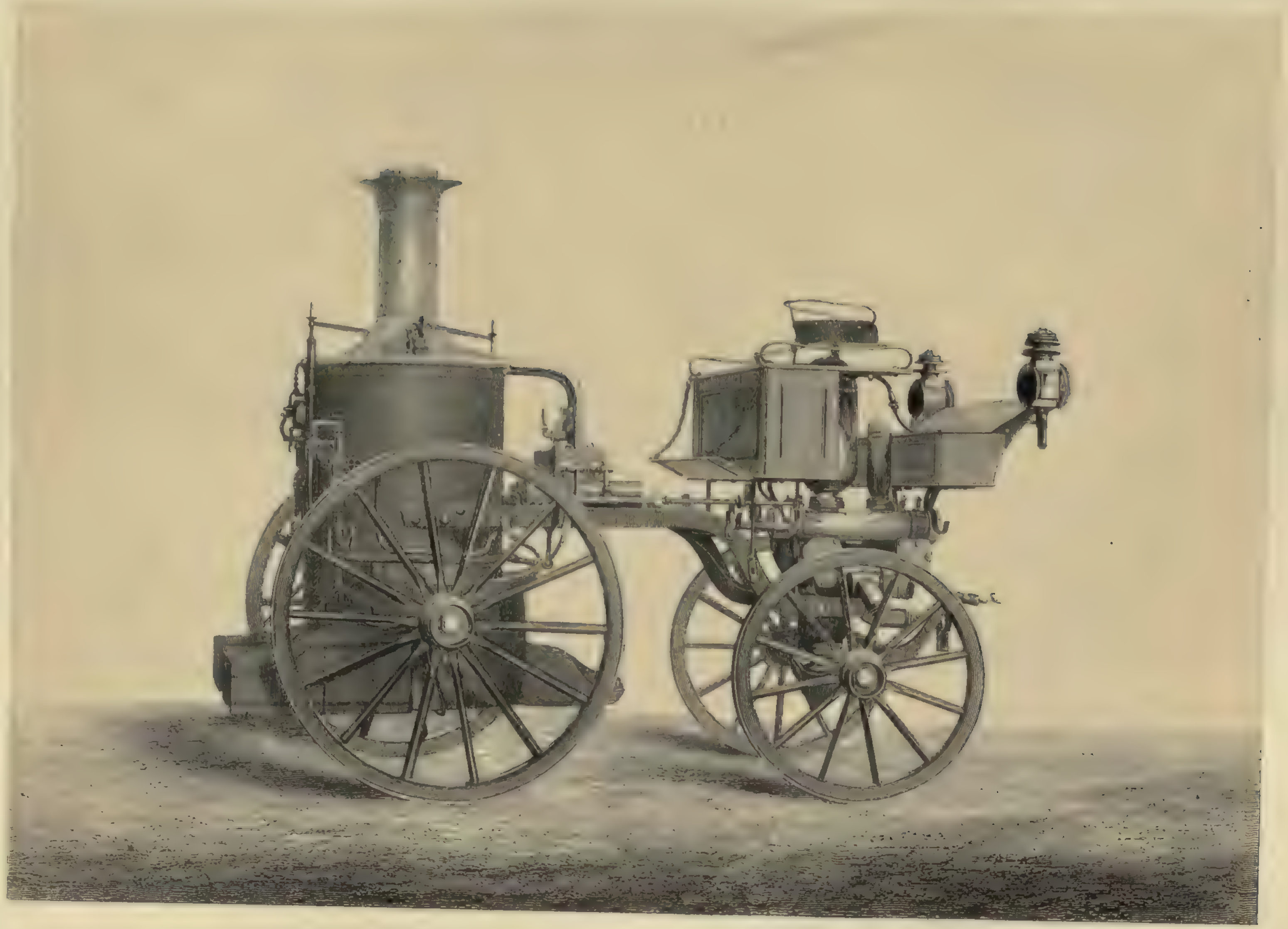


The intention of the air-vessel will be readily perceived by the student: the force applied to the two ends of the lever, which works the double pump, is necessarily an intermitting force; and without some contrivance to render the effect continuous, the water would issue from the delivery-pipe in jerks, by the separate impulsions given to it; but the continuous pressure of the condensed air in the air-vessel, causes a continuous flow of water along the hose, up to the point of issue. If the supply of water to the air-vessel vary, the velocity of the issuing stream will, of course, vary likewise; but there will be, nevertheless, no interruption in the continuousness of the discharge.

In fact, the air-vessel performs an office a good deal like that performed by the fly-wheel in ordinary machinery: it gives continuity to intermittent action, tends to equalise the effects of it, and prevents that strain on the structure which irregularities of action are apt to produce.

The metal pipe, from which the stream of water issues, is considerably smaller in bore than the leathern hose along which the fluid flows to it. A double purpose is accomplished by this disparity. In the first place, the friction in the hose is diminished, as a comparatively smaller portion of water comes in contact with the interior surface; and in the next place, the contraction of the stream, at the outlet, gives to it a proportional increase of velocity, so that the water is driven out of the narrow pipe with much greater force (page 764). It may be observed here, that the common syringe, or boys' squirt, is a miniature forcing-pump, or embryo fire-engine; the great contraction of the tube near the outlet, is the cause of the velocity with which the water issues, being so much greater than that with which the piston moves. In the syringe there is certainly no air-vessel, as in the fire-engine; but the pressure on the water is continuous, so long as the tube contains any.

THE DIVING-BELL.—The diver's bell is a heavy iron chest open at the bottom; it is called a *bell*, because, in its original construction, a bell-form figure was given to it. The interior is furnished with seats, sufficiently



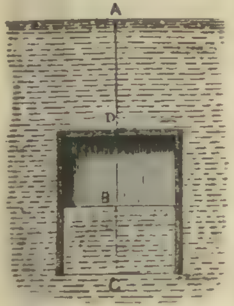
"THE SUTHERLAND" STEAM FIRE ENGINE.

BY MERRYWEATHER & SONS

high above the mouth to enable the persons sitting on them to keep their heads free from the water, which rises in the bell, in virtue of the upward pressure, as it descends into the sea, and to breathe the air which is thus forced up and condensed, above the rising water. By means of a flexible tube, communicating with the upper atmosphere through the top of the bell, fresh air can be pumped in, and the air unfit for respiration let out.

It is easy to find the space into which the air, originally in the bell, will be compressed when the chest is sunk to any depth below the surface.

Fig. 199.



Let A B (Fig. 199) be the distance between the surface of the water above, and the surface of the water in the bell. Let also h be the height of a column of the water, the pressure of which is equal to that of the atmosphere: then the original air in the bell sustained a pressure equal to that of a column of water, of the same horizontal section and height h; but when condensed by being forced up to B, it sustains a pressure equal to the same column h, and the additional column of water that is above B; that is, the pressure now sustained is equal to that of a column of water of height $h + \mathrm{A\,B}$.

$$\therefore \frac{\text{vol. of condensed air}}{\text{vol. of original air}} = \frac{\mathrm{D\,B}}{\mathrm{D\,C}} = \frac{h}{h + \mathrm{A\,B}} \text{ (page 769).}$$

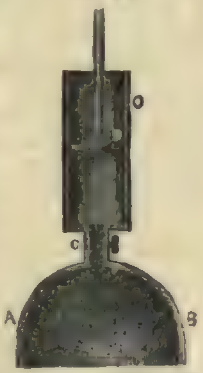
Putting, therefore, x for D B, the depth of the compressed air between the roof of the bell and the surface of the water within it, we have

$$\frac{x}{\mathrm{D\,C}} = \frac{h}{h + \mathrm{A\,D} + x} \quad \therefore x^2 + (h + \mathrm{A\,D})x = h\,\mathrm{D\,C}.$$

Hence, by solving this quadratic equation, which has but one positive root, the depth D B of the compressed air may readily be ascertained when the height D C of the bell, and the depth A D, to which its upper surface is sunk, are given.

THE CONDENSER.—The condenser consists of a strong vessel A B (Fig. 200), called the *receiver*, into which atmospheric air is forced and condensed by means of the following apparatus:—

Fig. 200.



A cylindrical barrel, opening into the receiver, and having a valve opening downwards at C, is furnished with a piston D, having a valve also opening downwards. As the piston descends, the valve D closes by the resistance of the air in C D, and the pressure opens the valve C, the compressed air passing into the receiver A B.

This piston having thus forced all the air originally in the barrel into the receiver, is raised up; and the superior pressure of the condensed air in A B immediately closes the valve C, while the downward pressure of the external atmosphere opens the valve D; so that the barrel becomes again filled with common air, and the operation is repeated.

In some condensers the piston D is made solid, and a small orifice is made at O near the top of the barrel, through which, upon raising the piston above it, the barrel is supplied with fresh air from without.

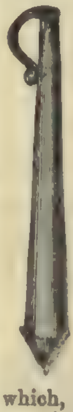
The density of the compressed air in the receiver, after n descents of the piston, may be ascertained thus:—Let R and B be the respective capacities of the receiver and the barrel, and let D be the density of the atmospheric air: then, R being the volume of this air in the receiver at first, and R + B the volume of it in the receiver after the first descent of the piston, the entire mass of air then in A B will be D . R + D . B; as the additional mass D . B is forced in at the second descent, A B will then contain the mass D . R + 2D . B; and a fresh mass D . B being thus forced in at every descent, we have for the whole quantity of air, after n descents,

$$\text{Quantity of air} = \mathrm{D}\,(\mathrm{R} + n\,\mathrm{B}) \;\therefore \text{ its density}$$
$$= \mathrm{D}\left(1 + n\frac{\mathrm{B}}{\mathrm{R}}\right)$$

which increases in arithmetical progression.

THE WINE-TASTER.—We may here notice, as among the minor contrivances by which the pressure of the air has been made available for practical purposes, the convenient little instrument called the wine-taster, which is much used in wine and ale cellars to draw out, through the bung-hole of a cask, a specimen of its contents. The marginal representation (Fig. 201) will give a sufficiently clear idea of the form of this contrivance, which is hollow, and has a small perforation at each end. When dipped into any liquid, the upper orifice being kept open to the atmosphere, the fluid rises through the lower orifice, till the level is the same inside and outside; the thumb is then pressed on the upper orifice, and the vessel withdrawn. The air previously within—the common atmospheric air, reaching from the surface of the liquid to the upper hole—expands and fills the enlarged space which the descent of the liquid leaves as the vessel is raised. The pressure within is thus diminished, while the external pressure upwards, on the lower orifice, is that of the unrarefied atmosphere; so that a portion of the liquid remains suspended in the tube, under which, if a glass be held, and the thumb removed, the sample will run out by its own weight.

Fig. 201.



THE AIR-PUMP.—The office of the air-pump is the opposite of that of the condenser, the purpose of it being to exhaust a receiver of the air contained in it. There are several forms of this machine; one of the best known is represented in Fig. 202, and is called Hawksbee's air-pump. The receiver, containing the air to be withdrawn, communicates, by means of a pipe, with two barrels, generally of polished brass, in which two closely-fitting pistons move by rack-work, as in the sectional outline exhibited in Fig. 203, where it will be observed that the descent of one piston compels the ascent of the other. The four valves, marked a, b, e, f, all open upwards. As one of the pistons descends, its valve opens, and the air in the barrel passes through, and rests above the piston when it has arrived at the bottom. During this operation the other piston ascends, with its valve closed, emptying the barrel in which it is fitted, of its air, the vacuum being supplied from the air in the receiver with which the barrel is in communication. The first piston is now raised, and *its* barrel emptied in

Fig. 202.

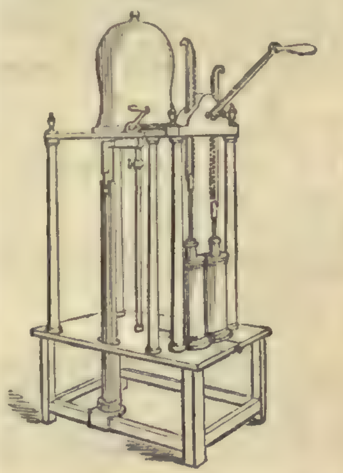
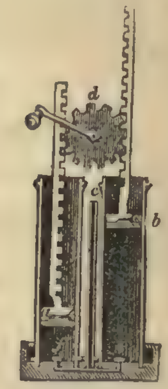


Fig. 203.



like manner, fresh air rushing in from the air in the receiver communicating with it. And these alternate ascents and descents of the piston render the air in the receiver so rarefied, that at length its pressure is inadequate to open the valves and pass into the barrels. It is plain that each turn of the toothed wheel, driving one piston to the bottom of the barrel and raising the other to the top, withdraws one barrel full of air; so that the air in the receiver, previously to this turn of the wheel, now occupies both the receiver and one of the barrels.

Let R be the capacity of the receiver, and B that of each barrel; and the density of the atmospheric air being D, let D_1, D_2, D_3, &c., be the densities of the air in the receiver after 1, 2, 3, &c., strokes or turns of the wheel: then since, at every turn, the volume of air R is dilated into R + B, we have

$$D_1\ (R + B) = D \, . \, R \ \therefore D_1 = D \frac{R}{R + B}$$

$$D_2\ (R + B) = D_1 \, . \, R \ \therefore D_2 = D \left(\frac{R}{R + B}\right)^2$$

$$D_3\ (R + B) = D_2 \, . \, R \ \therefore D_3 = D \left(\frac{R}{R + B}\right)^3$$

And generally $D_n = D \left(\frac{R}{R + B}\right)^n$, the density after n turns. Hence the density decreases in geometrical progression. As the mass, or quantity of air, is equal to its volume multiplied by its density (page 742), therefore after n strokes, the quantity of air in the receiver is

$$D_n \, R = \left(\frac{R}{R + B}\right)^n R = D \frac{R^{n+1}}{(R + B)^n}$$

The quantity of air in the receiver decreases, therefore, in geometrical progression; but as a decreasing geometrical progression may be continued indefinitely, it follows that the quantity can never be actually exhausted in any number of strokes of the air-pump.

As the pressure of the atmosphere on the receiver becomes very great after the pump has long worked, it is necessary that it possess considerable strength; and the bell-form, as exhibited in Fig. 202, is given to it to secure this condition, the glass of which it is formed being also very thick. The valves are usually of strong oil-silk, in little brass frames, traversed by a grating.

PROBLEM.—To find the number of strokes necessary to reduce the air in the receiver to a given density.

Let d be the given density: then D being the original density,

$$d = D \left(\frac{R}{R + B}\right)^n$$

$$\therefore \log. d = \log. D + n \ \{ \log. R - \log. (R + B) \}$$

$$\therefore n = \frac{\log. d - \log\ D}{\log. R - \log. (R + B)}$$

And if, as is usual, the original density, or that of the atmosphere in its ordinary state, be represented by 1, we shall have

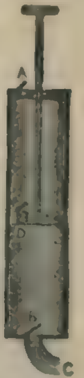
$$n = \frac{\log. d}{\log. R - \log. (R + B)}$$

The density d is here, of course, a proper fraction, and therefore log. d is negative, but the denominator being evidently negative also, the expression is positive; and here, as inferred above, if $d = 0$, that is, if the receiver be absolutely exhausted of air, log. d, and therefore n, is infinite.

Although two barrels are connected with the receiver in Hawksbee's air-pump, but only one barrel is exhausted at each stroke; yet the working of the two pistons simultaneously, at each turn of the wheel, considerably diminishes the labour of the operation; for the atmospheric pressures on the upper surfaces of the two pistons being the same, the force required to work the pump is only that necessary to overcome the difference of the pressures on the under surfaces of the pistons, and the friction of the pistons themselves. If only one barrel were employed, the ascent of its piston would be opposed by the difference, constantly augmenting, between the pressures on its upper and under surfaces, in addition to the friction. If, however, the ascending single piston could be relieved from the pressure of the atmosphere, a single exhausting barrel would answer every purpose, and the machine would be simplified.

SMEATON'S AIR-PUMP.—Such a simplification is given to the pump constructed by Smeaton, and called after his name. A B (Fig. 204) is the barrel or cylinder communicating with the receiver by means of the pipe B C. In this construction the barrel is closed at top, but furnished with a valve A opening upwards; the piston also has a valve D opening upwards; and a third valve B, likewise opening upwards, covers the pipe B C.

Fig. 204.



As the piston ascends from the bottom of the barrel, forcing up the air above it, and leaving a vacuum below, D is the only valve that closes; the atmospheric air originally in the barrel is forced out at A, and the exhausted cylinder is instantly supplied with air from the receiver through the pipe B C. Upon the descent of the piston, the valve A closes, the pressure from without exceeding that from within the cylinder, so that in its next, and in every succeeding ascent, the piston is relieved from the pressure of the atmosphere upon it.

If, as before, R, B represent the capacities of the receiver and barrel respectively, D the density of the atmospheric air, and D_n the density of the air in the receiver after n strokes of the piston, we shall have, as in the former case,

$$D_n = D \left(\frac{R}{R + B}\right)^n, \text{ the density after } n \text{ strokes.}$$

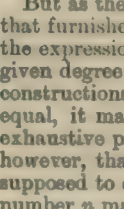
In this pump the exhaustion may be carried on to a much greater extent than in the common pump before described, because the valve D, being relieved from the downward pressure of the atmosphere, will open for a very slight pressure upwards, and consequently will allow of the passage of air in a more rarefied state.

But as the foregoing expression for D_n is the same as that furnished by Hawksbee's machine, and consequently the expression for n, the number of strokes producing a given degree of exhaustion, must be the same in both constructions, provided only that the barrels are all equal, it may seem that there is no difference in the exhaustive powers of the two. It must be remembered, however, that in the former investigation the valves are supposed to open at even the nth stroke, however high a number n may be; but as, in the former construction, the valve of the descending piston sustains the pressure of the atmosphere, which in the present contrivance is removed, a greater elasticity in the air below the valve is necessary to open it; so that, in Smeaton's pump, a greater degree of rarefraction will be ultimately attained.

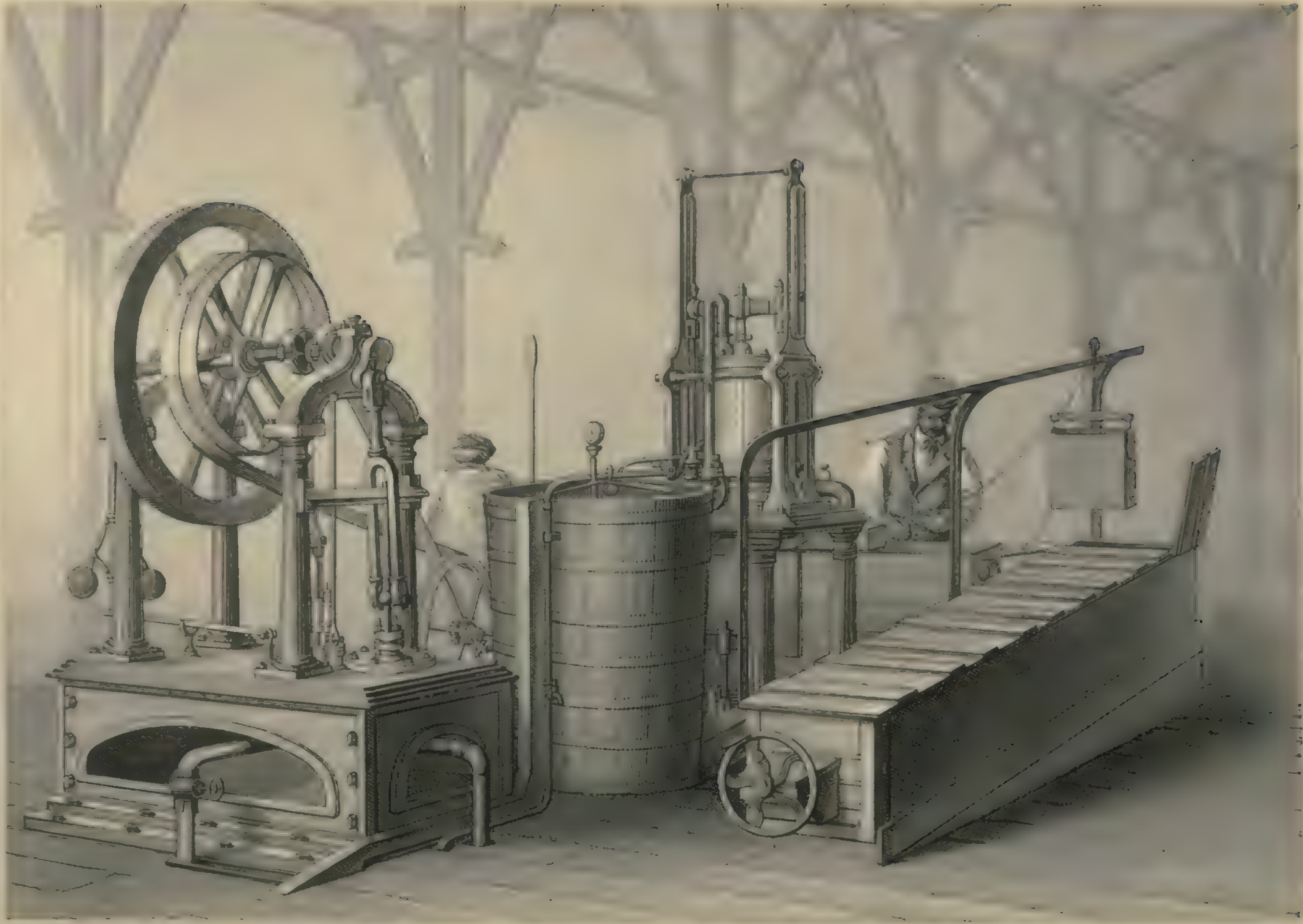
TATE'S AIR-PUMP.—This pump was first described by the inventor in the *Philosophical Magazine* for April, 1856. Its chief peculiarity is, that while it has, like Smeaton's pump, only a single cylinder or barrel, it has a double piston. This double piston performs the work of the two pistons in the ordinary double-barrelled air-pump, and that with only half the motion.

In the annexed diagram (Fig. 205), C D is the cylinder or barrel; A and B are solid pistons, rigidly connected by a rod K, and moved by the piston-rod A H, passing through a stuffing-box S. V and v are valves opening outwards; and R an open pipe, at the middle of the cylinder, leading to the receiver from which the air is to be exhausted.

Fig 205.



The distance between the extreme faces of the pistons is about ⅜ths of an inch less than one-half the length of the cylinder; this ⅜ths of an inch being the space requisite for clearing the exhausting-pipe R. The pistons are each about 1⅛ inch in thickness, and the rod K connecting them may be of any section consistent with strength. The effective length of the stroke is equal to the space between one side of the pipe



PATENT ICE MAKING MACHINE

[illegible]

[illegible]

R and the corresponding end-face of the cylinder, or it is very nearly equal to one-half the length of the cylinder.

In an upward stroke, the air above the piston A is propelled through the valve V into the atmosphere, while a vacuum is being formed beneath the piston B. When the piston A strikes against the top of the cylinder, the air from the receiver rushes through the pipe R, and diffuses itself through the lower half of the cylinder. In a downward stroke, the air beneath the piston B is propelled through the valve *v* into the atmosphere, while a vacuum is being formed above the piston, A, and so on. It will be observed that the double piston performs a double duty at every single stroke; for while a vacuum is being formed in one half of the cylinder by one piston, the other piston is propelling the air from the opposite half, into the atmosphere; hence much time and labour are saved.

ADDENDA.

THE PATENT ICE-MAKING MACHINE.—In temperate climes, sufficient ice is produced for summer consumption during the winter season; but in tropical countries, where it is unknown as a natural production, and its value is often at a high price, a method of producing it at pleasure has long been a desideratum.

In our Section on Heat we have given a list of freezing mixtures: these, however, can only be employed on the small scale.[*] It has, therefore, been frequently attempted to apply the air-pump to refrigerating purposes; and, perhaps, the most successful machine of the kind, is one which was shown in the Exhibition of 1862, and of which we shall give a short description.

It has been already stated, that ether boils at a very low temperature in a vacuum, and that if a little be placed in a watch-glass, resting in another containing water, the latter may easily be frozen in the receiver of an air-pump, whilst the air is being exhausted in the usual manner.[†] On the large scale, however, other arrangements for applying this principle must be adopted, both on account of speed and economy; for the attenuated atmosphere, being but a poor conductor of heat, would freeze but slowly any quantity of water exposed to its action; and the ether, which is an expensive material, must be recovered if possible, so as to be used repeatedly for the same purpose. The latter difficulty is overcome by permitting the restoration of the atmospheric pressure, when the ether regains its liquid form, which it had lost in the vacuum. To effect the rapid congelation of the water, strong brine is employed, being first cooled by the evaporation of ether in a receiver, from which the air is pumped by a steam-engine. The brine is a good conductor of heat; and, when thus cooled, it is caused to run round square cells containing the water to be frozen. The cells are placed at a little distance from each other, in a long trough, as represented in the folio engraving; and one of them is seen suspended over the trough on the right hand of the plate. The circulation of the brine is constantly kept up; fresh cold liquid being driven in by a pump, as that which has cooled the water is withdrawn. In a short space of time the water in the square cells is frozen, through the abstraction of its latent heat by the brine, and it may then be removed in solid flat cakes of ice. Fresh water is then introduced into the emptied cells; and thus the freezing process can be kept up without intermission, so long as the steam-engine is kept at work.

Comparatively speaking, in this climate the above is an expensive process; but when employed in hot countries, it produces ice in large quantities, at a price which is much lower than that hitherto charged. The machine has the advantage of requiring no chemical salts, as in the old method: in fact, the whole process is one of a purely mechanical nature.

BALLOONS.—In connection with the subject of Pneumatics, a few remarks on balloons may prove interesting to many of our readers. The ascent of a balloon in the air, is owing to the fact, that the contents, bag, car, &c., weigh less than the bulk of air they occupy. In other words, the balloon and its appendages, taken together, have less specific gravity than the atmosphere in which they float. This would, at first sight, appear paradoxical, but it is easily explained. Balloons may be raised by either of two means. If a large bag of paper, closed in all parts, except at a small orifice in its lower end, have the air it contains rarefied, by holding it over a gas flame, so that the air inside shall become heated; then, a less *weight* of hot air having the same *bulk* as cool air, will fill it. If the difference of weight between the heated air and that of the external atmosphere, bulk for bulk, equal that of the bag containing it, the bag will float in the air. On this principle the common fire-balloon is made to ascend. A piece of cotton wool, wetted with spirits of wine, is hung at the orifice of a large bag of tissue paper; and the spirits being inflamed, the internal air becomes rarefied, and the balloon ascends. The earlier attempts at aerostation, made by Montgolfier and others, were carried on in a similar manner. We need scarcely state that such a plan was highly dangerous. Large quantities of straw were burnt beneath the opening of a silk bag of considerable size, and thus the air it contained became rarefied. Occasionally, however, the machine caught fire, and fatal results occurred to those who had made the ascent.

The use of hydrogen was subsequently suggested; and it is a familiar lecture-table experiment to fill small balloons with that gas, when they immediately ascend in the air.[‡] Now, as pure hydrogen has a specific gravity of only, in round numbers, 0·07, whilst air (the standard of gases) = 1·00, it follows, that any bulk of hydrogen will only weigh one-fourteenth part of the same of atmospheric air. Hence, if a light and air-tight material be shaped into any hollow form, so that when filled with hydrogen, its weight, and that of the contained gas, shall be less than that of an equal bulk of air, of course it will at once fly upwards on being set free.

On the large scale, coal gas is always employed in filling balloons, because it is much cheaper, and almost always readily obtained. The external skin forming the balloon, consists of fine silk, which has been well varnished, to render it air-tight. A car is attached beneath it, by means of netting and cords. And such arrangements have been made of sufficient size to permit of the ascent of several persons.

Many attempts have been made to steer balloons, but all such have proved failures. At the present time, their use is confined to occasional ascents for scientific and military purposes, and as an attraction at places of public amusement. Balloons were frequently employed during the civil war in America, in 1862-'63: the British government have instituted experiments to test their value for reconnoiting purposes. Some account of the ascents of Mr. Glaisher, for meteorological observations, will be found in Chapter V., in the Section on Meteorology, p. 1174, *t. seq.*—EDITOR.

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[*] See *ante*, p. 37.

[†] See *ante*, p. 24

[‡] See *ante*, p. 320.

SECTION IX.

PRACTICAL OR APPLIED MECHANICS.

CHAPTER I.

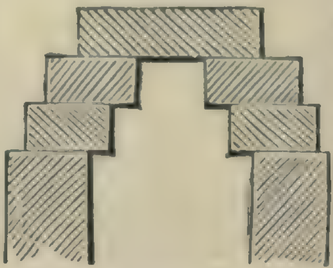
INTRODUCTORY.

Contents.—MECHANICS OF ANTIQUITY—MODERN MACHINERY—MECHANICS AND CHEMISTRY—STATICAL AND DYNAMICAL MECHANICS—STRENGTH OF MATERIALS—POWER—STANDARD OF POWER—SOURCES OF POWER—APPLICATION OF POWER—FRICTION—GOVERNING POWER—NATURE OF MACHINES—MECHANICAL EDUCATION.

PRACTICAL MECHANICS is the art of applying the theoretical principles of mechanical philosophy to materials, so that they may be fashioned, arranged, and combined in the various forms required by man. It is difficult in any case to draw an exact line of separation between the theoretical and the practical: for all true theory is founded on practice, and all skilful practice is the application of true theory. In the case of Mechanics, the separating line between the theory and the practice is eminently difficult to be traced; for the development of principles depends at every step upon practical experiment, and the improvements in mechanical arts are generally the results of extended research into the principles. It is true, indeed, that a great range of mechanical theory can be investigated by abstract reasoning upon a few simple principles, just as mathematical science is built upon a few definitions and truths; but it is remarkable that little progress was ever made in mechanical science until men began to translate the actual results of experiment. In Mathematics, which is certainly the science of all others requiring the least amount of experimental proof, the ancients made considerable progress, because the minds of men in former times were quite as well fitted for the investigation of abstract truths as those of modern mathematicians; but in the sciences which demand experimental proof—such as Chemistry, Optics, Astronomy, and Mechanics—little progress was ever made until philosophers began to observe facts, and thence reasoned to their causes. For many ages of the world's history, it was the custom of men pretending to science to start in their own minds a hypothesis or conjecture about some law of nature; to build upon this slippery foundation a vast scheme of nature; and then to wonder that natural phenomena did not suit their arrangement. In some such way as this the astronomers of old, with a few illustrious exceptions, took it for granted that an enormous, in their notions a boundless, mass, such as the earth, could not move; and, therefore, that the sun, moon, planets, and stars, revolved in daily cycle round it. And it was not until observations had been carried on for many centuries—until the records of these observations had been compared and digested, and until instruments had become so far perfected as to enable astronomers to observe with accuracy—that the true theory of the celestial phenomena was developed.

MECHANICS OF ANTIQUITY.—In respect of mechanical science, the ancients appear to have been very deficient. They had, indeed, tolerably true notions respecting the construction of buildings in such a manner as to ensure permanence and stability; but for these qualities they seem to have depended almost entirely on rude strength and massiveness, instead of skilful arrangement. Among the gigantic remains of Egyptian architecture, there is not found a single arch; the only approach to it is the vault of a passage hollowed like an an arch, but made of two or three immense blocks of stone cut to the vaulted form. Again, among the ruins of Central America, remarkable for their enormous extent and elaborate decoration, the passages of such buildings as remain tolerably entire are found to be covered by blocks of stone, arranged as in Fig. 1. The beautiful and stable arrangement of materials in the form of an arch does not seem to have occurred to the architects of antiquity, until the comparatively recent era of the Roman Empire. It is difficult to ascertain whether the ancients were acquainted with any of the forms of roofs now in use, on account of the perishable nature of the materials: but it is probable that they knew no such method of covering large spaces; and for covering apartments of more limited extent, they had no resource but the use of large masses of stone.

Fig. 1.



In respect of machinery, the ancients appear to have been extremely deficient, for we have records of no apparatus except certain implements of war and of the chase, and a few rude contrivances for irrigating and cultivating the ground. But, indeed, even in modern times, the mechanical arts had made little advance until the genius of Watt had given to man a power applicable alike to manipulation of the most delicate character, and to labour demanding the most gigantic strength. Could a man who lived in England but a hundred years ago now revisit his country, he would in mechanical art find a change far greater, an advance more astonishing, than the whole progress made from the creation of the world up to the time at which he lived.

MODERN MACHINERY.—That a man seeing all the wonders of modern art, as they have been seen of late years at the great Exhibitions, should endeavour to attain a knowledge of their nature and construction, may at first sight appear presumptuous, because of the vast extent and variety of the objects which we would have to study; and, indeed, unless one were prepared to devote his lifetime to mechanical art, he could scarcely hope to know accurately the details of modern machinery, much less to attain the skill of a master in all its varied departments. But, with a fair knowledge of some simple first principles, and an introductory glance at a few of the leading details that are common to all mechanical arrangements, we believe it would not be difficult to attain a good general knowledge of mechanical art, and a readiness at comprehending any mechanical arrangements that may be brought under our notice. Few will be inclined to dispute the advantages of possessing even such a moderate knowledge of Mechanics as this implies, for in modern times but little advance can be made in any useful pursuit without that aid. Practical Mechanics may indeed be called the handmaid of the other arts. The sculptor, the architect, the musician, the chemist, the astronomer, the surgeon, the merchant, the manufacturer, the builder, the military man, the civilian, the traveller, and the emigrant, all require her services—all reap the benefit of her labours.

It is generally supposed that inventors in the arts, as well as discoverers in the sciences, owe their success to some fortunate accident. We think it may, in most cases, be shown that such is not the fact. Almost every famous discovery, or useful invention, has been the result of long, often painful and laborious, thought and research; and success has often followed repeated failures; accuracy has succeeded repeated errors. That there is a natural genius for invention peculiar to some minds, cannot be doubted; but that genius alone, without labour and study, has ever led to brilliant results, may as firmly be denied. Mechanics, as an art, in a peculiar manner demands concentrated thought; there should be no waiting for some inspiration to bridge over a mechanical difficulty. Let a man gird himself to his task, and determine thoroughly to think out the subject he may have on hand, and we venture to predict that he will find few difficulties insurmountable.

MECHANICS AND CHEMISTRY.—The two sciences that deal with matter in its various affections and modifications are Chemical and Mechanical Philosophy. The former treats of the influences affecting its minute particles, and the combination of these particles; the latter has reference to masses of matter, and the forces and motions of masses, or aggregations of particles. Each of these two sciences frequently encroaches upon the other's domain, for there are many similar phenomena developed by the action of natural laws on masses as well as on particles, and thus it is often difficult to draw a line of separation between the subjects of the two sciences. Questions relating to heat, light, electricity, and magnetism, form part of both sciences alike, and can scarcely be said to belong to the one more than to the other. There are questions, however, which are purely chemical, and others as distinctly mechanical.

The chemist, examining a piece of matter, tries its specific gravity, its solubility or fusibility; analyses it, and finds the different elementary substances which compose it, and the precise quantities in which they are combined.

The practical mechanic, examining a structure or a machine, sees, feels, and measures the parts of which it consists, traces the laws that govern its equilibrium or its movements, and determines the principles which have been adopted in its construction.

STATICAL AND DYNAMICAL MECHANICS.—The objects of Practical Mechanics may be divided into Statical and Dynamical. Statical mechanics has reference to the formation and arrangement of materials intended to remain in a state of rest; its principal objects are permanence and stability. It has to consider the strength, elasticity, flexure, weight, and durability of the materials with which it deals. It has to employ the proper substances for its purposes, put them in their right places, make them of suitable forms, and unite them firmly together, or so arrange them, that the tendency to change their relative position when affected by external forces shall be the least possible. The civil and the naval architect, or the builder, whose business it is to make large fabrics by putting together numerous small pieces, is indebted for the durability and strength of his structure to the application of statical mechanics. So, also, the miner or the civil and military engineer, who has to form extensive excavations, erect embankments, bridges, tide-works, or fortifications, requires, an intimate knowledge of statical mechanics.

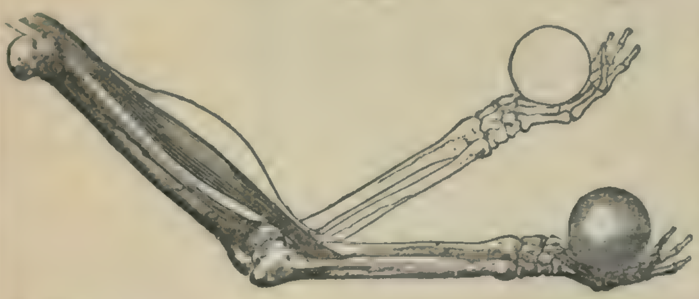
Dynamical mechanics, on the other hand, has reference to the forms and combinations of materials with a view to motion. Its principal objects are the generation, communication, and application of power and force, in order to change the forms or arrangements of materials. Its products are tools, implements, machines, engines, or apparatus; it teaches us how to choose the most suitable materials, produce the strongest and most serviceable forms, arrange their respective motions so as to secure regularity and diminish resistance, with a view to economy of labour and durability of structure. The farmer, brickmaker, and stone-worker, the carpenter and the smith, the spinner and weaver, the worker in pottery and metals, are all employed in changing the forms and arrangement of the materials submitted to them; and they have all to use the apparatus provided by the machinist, and the machinist practises the art of dynamical mechanics in devising and executing the implements suitable for their and his own use.

In statical and dynamical mechanics, the chief study is economy of material, labour, and cost. In any structure, whether for stability or movement, by an unsparing use of materials, it may generally be possible to secure the necessary strength. But it frequently happens that the usefulness of a construction would be seriously diminished by a rude accession of mere strength; and, in all cases, elegance of construction is manifested by the careful and ingenious arrangement of materials, and not by their absolute mass. Accordingly, we find, that as mechanical art advances, lightness and neatness of structure take the place of magnitude and weight; and improvements in existing constructions or arrangements are made to depend for their merits on the saving of material and labour which they effect. Perhaps the most stable structures that mankind have ever formed are the pyramids; and as works of enormous magnitude, of great age, and mysterious purpose, they excite admiration and wonder. But when one considers their form and the arrangement of their parts, their durability is not to be wondered at. A hill of mere sand, not blown upon by strong winds or washed by heavy rains, would retain for ever its pyramidal form, and would be an object of as great beauty and utility as the pyramid itself. The sovereigns of Egypt had probably at their command an enormous amount of human power, and knew no better object on which to employ it, than the erection of some immense pile, to gratify their pride while living, and contain their bodies when dead. We, living thousands of years after them, do not see the pain and labour of their slaves, and scarcely know how much misery might have been spared mankind, had their labour been applied to better uses. In the mere paving of the metropolis, we have more material and labour applied to a useful purpose than the amount required for the largest of the pyramids. In our harbours and docks, our canals and railways, we have works far more stupendous; and in our steam vessels and manufactories, we have power at work exceeding the capabilities of the whole ancient world. But with all this, it is the object of the engineer and machinist to study the greatest possible economy of material and of labour.

STRENGTH OF MATERIALS.—Whatever be the elements which constitute beauty, in works intended for ornament, we see beauty in things made for use, just in proportion to their fitness for their purpose. Parts out of place, deficiency or redundance of material, elaboration not called for, and deviation from just proportion, become glaring defects in any structure or apparatus formed for useful purposes; and no amount of decoration or finish can reconcile us to a disproportioned or unskilfully designed fabric. It is, therefore, most important that the practical mechanic should form an intimate acquaintance with the strength of the various materials with which he deals; their powers to resist strains in various directions, and the arrangements suitable for making use of these qualities to the greatest advantage. In the theoretical portion of this subject, there is much that is abstract and mathematical, yet nothing so difficult, but that the fair exercise of judgment may lead to very sound conclusions respecting it. All the main facts, however, respecting the strength of materials, are the result of experiment and observation; and the true use of theory in a subject like this, is to analyse and classify these results. If it be found, for instance, that several beams of timber, of known dimensions, are capable of sustaining certain strains applied to them in certain directions, it is not difficult to form general rules by which the strength of other beams differing in dimensions, or their strength to resist strains in certain other directions, may be computed. The careful mechanical designer either makes experiments for himself, or accepts the observations of others, and devises his structures in such a manner as to take advantage of these results. Practice in design, and frequent observation of successful works, do much to form the eye of a designer, so that the mere appearance of a thing, whether drawn or executed, satisfies his sense of just proportion, or the reverse. It is indeed a wonderful, as it is a most valuable property of the mind, that it can readily recognise and prefer the useful and suitable; and, as the just and the true recommend themselves to the mental and moral sense, so does the fitting in design recommend itself to the eye. Some individuals are gifted by nature with a keener sense and readier power of discriminating these qualities, and can therefore criticise more truly, or design more skilfully than others. But where there is a love for mechanical pursuits, or a desire to study mechanical works, we believe much of this power may be readily acquired.

POWER.—In the dynamical branch of Mechanics there are subjects of great interest and extensive application which demand the utmost attention; we allude to the means of deriving and communicating power. If we ask ourselves what power means, we can scarcely define it by any simple term; we may say that it is the capability of doing work or producing change. We start with the idea, which is a true one, that all matter is inert, incapable of receiving motion, of being brought to rest, or of undergoing any kind of change whatever of its own accord; some extraneous force or power has to be impressed upon it, and the quantity of change effected is the work done. When we lift a weight from the ground, the hand exerts a certain amount of power, and produces a certain amount of work; an act of mental volition communicates through the nerves, in some way unknown to us, an influence to the muscles of our arm, which causes them to contract. The muscles are attached at both ends to the bones, which are rigid levers jointed together. One end of such a bone or lever being fixed, as at the elbow-joint, and the muscle being contracted, the other end of the lever, to which is attached the hand holding the weight, is caused to move through a certain space, and thus to lift the weight a certain height (Fig. 2).

Fig. 2.



This is a case of power employed in producing motion; but, after the weight is lifted, power must still be exercised in retaining it in its position, for were the muscles for one instant relaxed, the hand and weight would drop downwards. If such be the case, we may at first sight imagine that our notion respecting the inertness of matter is incorrect, otherwise the hand and weight would of themselves remain where our muscular power had placed them. But, looking a little more closely into the facts, we find that while the matter with which we deal is really inert, there is a constant power acting upon it in opposition to our muscular power—the power of gravitating attraction, by which all bodies near the earth's surface are drawn down to it.

This power has been found, by extended experiment and observation, to act equally on all bodies and all parts of every body; and the measure of the force with which it acts on any body is the weight of that body. In estimating power, therefore, weight is the measure of one important element; and if we know the weight which any given power can lift from the earth, we have correct data for calculating its amount: in fact, we know the quantity of matter on which the power has effected a change. But there is another element of power quite as important as the quantity of matter acted on,—we mean the quantity of change produced upon it. If we find a certain amount of exertion necessary to lift a weight one foot from the ground, we shall find greater exertion necessary to lift it two feet. The change in both cases effected by the power applied, is a change of position; but the amount of change—that is, the distance through which the weight is moved—is in the one case double that in the other. Experiment and observation, as well as reasoning, prove distinctly, that the force required to lift a given weight, is exactly proportional to the distance through which it is lifted. We thus acquire the means of computing another element of power; and can combine this element, expressive of the amount of change effected, with the former, which expresses the quantity of matter upon which the change is produced.

In comparing numerically one power with another, we are therefore warranted in multiplying the weight lifted in each case, by the distance through which it is lifted, and comparing the results. Thus, if one man lift 20 lbs. 1 foot high, and another lift 30 lbs 1 foot high, we say that the powers exerted by the two men are as 20 to 30, or as 2 to 3; or if one lift 20 lbs. 2 feet high, and the other lift 20 lbs. 3 feet high, the powers they exert are as 20 multiplied by 2 (that is, 40), to 20 multiplied by 3 (that is, 60); or as 2 to 3. Again, let the one lift 20 lbs. 2 feet high, and the other 30 lbs. 3 feet high, their powers are as 20 multiplied by 2 (that is, 40), to 30 multiplied by 3 (that is, 90); or, more simply, as 4 to 9.

But, recurring to our definition of power as being the capability of effecting change, we must see that there is yet another element of calculation necessary for estimating different capabilities; as yet we have reckoned only the amount of work done, and have paid no regard to the time required for doing it. A few coral insects, labouring continuously for centuries, may form an island, which it would require thousands of workmen to raise in a short period; but yet the power of each workman far exceeds that of the insect, and the two powers can only be compared by supposing them to be exerted during equal times. Now, it is clear that a certain work being done, the shorter the period required for doing it, the greater is the power exerted; and conversely. In comparing powers numerically, we must therefore divide the work done by the time occupied in performing it. Suppose one man lifts 20 lbs. 12 feet high in 3 minutes, and another lifts 30 lbs. 10 feet high in 2 minutes, the work done by each is 20 lbs. × 12 feet, or 240, and 30 lbs. × 10 feet, or 300, respectively; but 240 divided by 3 minutes give 80, and 300 divided by 2 minutes give 150: the powers of the two are therefore in the proportion of these numbers, 80 to 150; or, more simply, 8 to 15.

STANDARD OF POWER.—We have now found means of measuring all the elements of power, and so determining its amount. For the sake of convenience, we fix upon some standard power with which we can compare others, just as we fix on a standard weight, such as a pound, or a standard length, as a foot; and knowing the value of several powers as compared with this standard, we can estimate their values as compared with one another. The standard power usually adopted in practical mechanics is a horse-power, which has been defined to be 33,000 lbs. lifted 1 foot high in 1 minute. The weight lifted, 33,000 lbs., was determined from the average of numerous experiments made with horses at work. Whether it be an accurate expression of the power of an average horse or not, is of little consequence, provided it be generally accepted and understood as a standard measure of power. Were there no such standard, each mechanic might make an estimate of his own: one might compute the work of a very strong horse, the other of a very weak one. Thus, steam-engines or other apparatus might be supplied of all different strengths and sizes, and yet purporting to be of equal powers; or apparatus of like strengths and dimensions might be stated to be of very different powers. But, this standard once fixed, it is the duty of a mechanic to estimate, by experiment or calculation, the weight which the engine he makes can lift through a given height in a given time, or the weight which moving through a given height in a given time, can work efficiently some apparatus which he may have fabricated; and he can then state the power which his engine furnishes, or which his apparatus requires, in terms intelligible to all the world. It is clear from what has preceded, that a horse-power does not mean precisely 33,000 lbs. lifted one foot in one minute, but a power equivalent to that; as, for instance, 330 lbs. lifted 100 feet in one minute, 3,300 lbs. lifted one foot in one-tenth of a minute; or, in fact, any weight such that, on multiplying it by the distance moved and dividing the product by the time occupied in the motion, the result shall be 33,000.

These considerations may be somewhat simplified by combining the distance and time into one term, which we call velocity or speed. Velocity is directly proportional to the distance passed over, and inversely proportional to the time occupied in the transit. A railway train that passes over 50 miles in an hour, has double the velocity of one that travels 25 miles an hour, because 50 is double 25. Again, a train that travels 25 miles in one hour has double the velocity of a train that passes over the same distance in two hours. To compare the velocities numerically, we divide the distances by the times of each.

In estimating powers, since our standard is given in feet for distance, and minutes for time, we divide the distance in feet by the time in minutes, and thus get the velocity. The weight in pounds lifted, multiplied by the velocity thus reckoned, gives a product which, being divided by 33,000, shows the number of horses'-power. Suppose an engineer were required to furnish a steam-engine capable of pumping 165,000 gallons of water every hour, to a height of 120 feet, he would reckon thus:—An hour contains 60 minutes; and 120 divided by 60, gives two feet per minute as the velocity with which the required volume of water must be lifted. A gallon of water weighs 10 lbs.; therefore, 165,000 gallons weigh 1,650,000 lbs.; this weight moved at the velocity of 2 feet per minute is equivalent to 1,650,000 × 2, that is 3,300,000 lbs. lifted 1 foot in 1 minute. Dividing this by 33,000, the quotient is 100 horse-power as the actual force required to do the work in the time given. He would, therefore, proceed to make an engine which, after providing for all mechanical losses in the operation, should be capable of producing this effect.

SOURCES OF POWER.—The principal sources of power are, the muscular forces of men and animals, the natural motions of air and water, the weight and elasticity of materials, and the changes effected in bodies by the action of heat, and by electrical and chemical action. It is the business of the mechanic to utilise these forces, to regulate and control them, to change or modify their directions, velocities, or intensities, so that they may be made to do certain work in the best, most economical, and expeditious manner. In addition, therefore, to his knowledge of the materials on which, or through which, these forces have to act, he must have an intimate acquaintance with the nature and laws of the forces themselves, as discovered by experiment, or investigated abstractly. It is a fortunate circumstance for man, considering the brief period allotted for individual research, that the laws of nature are of the most simple character, and that he is gifted with faculties that enable him to communicate the results of his investigations through great distances and over lengthened periods. Every successive discovery in natural science throws additional light over all that has preceded, opens up new fields for research, and suggests new modes of practical action. In the arts, likewise, an ingenious invention simplifies much that formerly was rude and cumbrous, facilitates operations formerly deemed impracticable, and furnishes the means of practising new and unheard-of branches of art. Let one but compare the state of mechanical art as it was before the time of Watt, and as it is now; and the more deeply he investigates the question, the more will he be surprised at the enormous change effected by a few simple but most ingenious modifications of a machine for utilising the power developed by subjecting water to the action of heat.

The steam-engine has indeed exerted so vast an influence on the arts, that we shall think it necessary to devote considerable space in what follows to its description in detail. We are the more strongly urged to this, because we believe that the student who can master the details of the construction and application of this apparatus, will be prepared to find his way through any branch of Practical Mechanics.

APPLICATION OF POWER.—The mere supply of forces for our various purposes would be of little benefit to us unless we had ingenuity sufficient to find modes of applying them. All work, as we have already hinted, means effecting changes of some kind or other; and all changes of matter imply motions either of masses or of parts. A most interesting and extensive branch of Practical Mechanics consists, therefore, in the discussion of the various modes of communicating and modifying motion. The two elements of which power consists—viz., the weight or mass acting, and the velocity with which it acts—are convertible terms. Having a certain power to work with, we can give up a certain amount of weight, and thereby gain in velocity, or sacrifice velocity in order to gain in weight. We cannot create any addition to the power either in weight or in velocity, nor can we annihilate any portion of it. We can, doubtless, apply the power with better effect to the work, and thereby save a loss; or we can misapply a portion of the power, and thereby render it ineffective for the purpose intended. A power applied to put in motion any part of a train of machinery, will be found acting at any other part of the same train.

If we could make our workmanship absolutely perfect, could have materials absolutely rigid and frictionless, and could remove our machinery from the influence of all extraneous existence, we should find in every part of it an equal amount of power. Some parts may move more slowly than others, but then they act with greater pressure or weight; some parts may act with less pressure, but then they move with greater velocity: in short, in every part of such a perfect machine we should find the pressure multiplied by the velocity—that is, the power or momentum, exactly the same. Even with our comparatively imperfect workmanship, we are quite safe in estimating according to this rule, without making much allowance for extraneous resistance. In the works of a clock, the wheel fixed to the barrel which carries the weight revolves so slowly, that its motion is quite imperceptible; and through the teeth of this wheel is conveyed the force which puts the whole clock-train in motion, a weight of many pounds. Again, the escapement-wheel, which sustains the motion of the pendulum, revolves with comparative rapidity, but with so little apparent power, that an opposing pressure of a few ounces might completely stop it; yet, on stopping the escapement-wheel, we stop also the barrel-wheel, and the few ounces applied to the one thus effectually oppose the many pounds acting on the other: in fact, the weight which acts on the barrel, multiplied by the velocity with which it moves, is equal to the weight necessary to stop the escapement, multiplied by the velocity with which it moves.

FRICTION.—We have alluded to resistances in machinery, the principal of which is friction, or the retardation caused by the rubbing together of imperfectly-smoothed surfaces. This is a subject about which little is known, or probably ever can be known. Some of its general laws have been successfully investigated, and a few general principles have been carried into practice; but its effects vary so much with every change of material—of speed, pressure, workmanship, and even temperature and other circumstances—that, after all, experience is the only real guide in all matters where it is to be considered. Except in a few mechanical arrangements where friction is employed as a useful resistance, as in the case of the friction-break of a locomotive or of a crane, it is generally the mechanic's object to diminish it as much as possible. The overcoming of friction is, in fact, the wasting of so much power; and as all machines are devised with a view to economy in the application of power, it becomes most important to reduce the waste to its minimum. In all devices for communicating and modifying motion, the question of friction becomes nearly as important as the question of strength; and many arrangements which are ingenious, and would be profitable, did friction not interfere, become comparatively useless, in consequence of its influence.

If one who had never made mechanics his study were introduced into a large manufactory, and had pointed out to him the steam-engines, the cranks, levers, wheels, pinions, straps, and other contrivances for communicating power throughout the building, and setting in motion each machine,—he would at first be bewildered, and inclined to believe that the art of communicating motion was intricate and complicated; but were he really to analyse the process carefully, and trace exactly the progress of the power or motion from the prime mover to the last machine, he would find the whole effected by the combination of a few simple mechanical elements; and wonder that through means so simple, results apparently so complicated could be attained.

Students of astronomy cannot help being struck by the circumstance, that the various motions of planets, satellites, and comets, and probably also of the stars and nebulæ, of the farthest and greatest, as well as of the nearest and smallest, of the heavenly bodies, all result from two simple forces, acting in various directions and with different intensities. The movements of the vast machine of the universe present, to a superficial observer, inextricable confusion and inexplicable irregularity; but to a mind like that of Newton all is skill, order, and beauty. The very disturbances to which the different parts of the grand machine are subjected are elements of stability; the irregularities are sources of permanence. In a system of machinery devised by human ingenuity and executed by human hands, there can, indeed, be no approach to this simplicity, order, and harmony; everything must be imperfect in proportion as man is imperfect, when compared with the Divine Mechanic of the Universe.

In every human work there must be elements of decay, sources of irregularity, and causes of instability; and these can only be reduced by diminishing the extent and complication of the work itself. Simplicity is, therefore, the great aim of the mechanic, especially in arrangements for communicating motion. Every wheel, every lever, every pulley that can be saved, is saved—not so much to avoid the cost of its introduction, as with a view to simplify the machinery, and thereby diminish the amount of friction and wear, and increase the permanence of the whole. To the eye of a practised mechanic, therefore, the beauty of a piece of machinery seems to depend more on its simplicity than on any other principle. The fewer the parts required in any apparatus to render it effective, the more ingenious is its contrivance, and the less is it liable to derangement, irregularity, and decay.

GOVERNING POWER.—Next in order to contrivances for communicating power or motion, may be studied those for regulating and governing it, so as to secure uniformity of action. All the forces we employ, with the exception of gravity, are subject to continual variations of intensity; and even the most uniformly regulated forces, when transmitted through trains of machinery necessarily imperfect, are subjected to considerable variations. Further, the forces we employ are chiefly used for effecting changes on materials; and as the qualities and conditions of the materials vary, so the quantities of force required to do the work upon them differ. It becomes, therefore, most important that arrangements should be devised for compensating all these variations of force; and accordingly, great ingenuity has been exhibited in contrivances for that purpose. We may quote a very beautiful specimen of mechanical skill applied for a purpose of this kind, in order to illustrate the great use of such arrangements. Wind-power is used to a great extent for putting machinery in motion; and yet, as we all know, nothing is more variable than the force of wind, both in direction and intensity. In former times, wind-mills were made so that the miller, watching the direction of the wind, could turn round the sails of his mill to face it, and furl or unfurl those sails as he found the breeze too strong or too light to give the velocity of movement he might require. But in modern wind-mills, the wind itself is made to regulate the machinery by simple but ingenious arrangements. In the first place, if the wind change in direction, it blows upon a small subsidiary set of sails placed so that they can be acted on by a wind that does not directly blow upon the main arms; by a little simple machinery, this side action of the wind is made to turn round the head of the mill until it brings the great sails into the proper position to receive its direct impulse—just as the wind acting on a weathercock brings the arrow-point round to face it, by its pressure on the broad feather at the other end. Again, should the force of the wind increase, this very increased force is made to act upon a regulating apparatus so as to furl the different divisions of the sails, or to turn the surfaces of which they are made up edgeways to its impulse, and thereby to diminish the surface on which it acts, proportionally to its increase of intensity. The consequence of these contrivances is, that from whatever quarter the wind may blow, the sails of the mill are always directly opposed to it so as to receive its full action; and whatever be its force, whether a gentle gale or a stiff breeze, the power communicated to the machinery does not greatly vary. Perhaps the most ingenious devices for regulating power and velocity are those employed in apparatus for the measurement of time—clocks, watches, and the like, which form one most interesting branch of mechanical art, Horology.

NATURE OF MACHINES.—Having obtained the power necessary for our purposes, and having found the means of communicating it to our machines, and of regulating its intensity so as to suit the work to be done, we have next to inquire into the nature of the machines themselves, or the contrivances through which the power is made to act on the inert material subjected to it.

We can scarcely venture to offer a very distinct classification of machines; nor, indeed, in a work of limited compass like the present, could we pretend to discuss in detail, all the different classes of machinery now in use. The subject is so extensive, and demands so minute a knowledge of what has been done by thousands of ingenious mechanics, that even the professed machinist cannot pretend to an intimate acquaintance with every branch of it. But the qualities of mind and the experimental training which render a man skilful and adept in one or two of these branches, make it easy for him to take up intelligently, any other branch that may be brought under his notice. The same general mechanical principles pervade all kinds of machinery, the modes of applying them being varied according to the nature of the material subjected to their operations, or to the kind of work to be done. With the exception of certain special apparatus, we believe almost all the machinery used in modern times may be classed under some of the following heads:—

I. *Machinery for raising Weights and giving Pressure.*—Among these we may notice especially the simple mechanical powers, the lever, wheel and axle, pulley, inclined plane, wedge, and screw.* Next, in order of complication, may be mentioned such machines as cranes, crabs, capstans and windlasses, slips, tackle and travellers, jacks, screw and lever presses, printing presses, and the like.

II. *Machinery for effecting Transit and Communication.*—This branch has of late years attained immense importance by the extended use of railways and steam vessels; and under the head of transit, it embraces the consideration of vehicles for land transport, and vessels for water carriage, and the various modes of putting them in motion. For the communication of intelligence, we now practise an art unknown in former times, and one deserving of detailed consideration from its marvellous nature and the important influences it is likely to exercise upon human civilisation. We allude to the electric-telegraph, to which perhaps greater ingenuity has been devoted within a few years than has been displayed in any other branch of mechanical art during preceding ages.

III. *Machinery for Moving Fluids.*—This branch naturally includes hydraulic apparatus, such as pumps, fire-engines, hydraulic presses and lifts; and also machinery for moving air, such as bellows, blowing-cylinders, fanners, and the like.

IV. *Machinery for changing the forms of Solid Materials.*—This is perhaps the most extensive branch of mechanical art, as it includes all apparatus for cutting, piercing, moulding, bending, crushing, and such like operations. It may be subdivided, according to the materials on which the operations are to be effected, as follows:—

1. Machinery for preparing animal and vegetable products; as oil, tallow, leather, flour, sugar, vegetable oils and extracts, caoutchouc, and gutta-percha.
2. Machinery for preparing mineral products; as stones, bricks, cements, ores, pottery-ware, glass, pigments, and the like.
3. Machinery for sawing, planing, moulding, bending, and carving timber.
4. Machinery for working metals; as in the operations of moulding and casting, forging, rolling, and wire-drawing; bending, shearing, punching, and rivetting; drilling, turning, and boring; planing, shaping, and the like.
5. Agricultural implements; or machinery for opening and triturating the soil, for sowing and reaping, for thrashing, grinding, and otherwise preparing produce.

V. *Machinery used in the manufacture of Textile Fabrics.*—This is likewise a most extensive and interesting branch of mechanical art, and one of the highest importance to us as a nation, as well as to the world at large. It includes directly the apparatus and processes employed in preparing the crude materials, such as wool, flax, cotton, and silk; the various operations of dressing, carding, spinning, weaving, dyeing, bleaching, calico-printing, and the like. Under the same head may also be classed the machinery for making ropes and cordage, and for the manufacture of paper.

VI. *Machinery for Measuring and Calculating.*—The apparatus included in this class are mostly of that exact character required for philosophical experiment and observation, such as indicators, dynamometers, gauges, balances, and mathematical and optical instruments; but there is one extensive and interesting branch, horology, devoted to apparatus for the measurement and division of time, which deserves especial consideration, as well from its usefulness as from the great ingenuity displayed in it. In this class we may include some other apparatus, which, though not distinctly falling within the scope of its title, yet present in some respects considerable similarity to some of those included; we mean automata and musical instruments.

VII. *Implements of War.*—It is lamentable to think how much mechanical skill has, in all ages, been devoted to the cruel purposes of destruction. In the present age, perhaps no branch of applied mechanics has called forth so much ingenuity as the contrivance of means of offence and defence. And so vast and costly have these contrivances become, that we may almost hope to see wars cease upon the earth because of the very perfection to which its implements have attained.

These seven classes, we believe, include the greater part of the machinery used in modern times. In what follows, we will not pretend to give detailed descriptions of the different machines included in these classes; we shall endeavour rather to select a few of a general character, involving in their construction the principles which are applied, in a modified form, to most of the others.

MECHANICAL EDUCATION.—While improved education and extended acquaintance with principles, have placed those who direct mechanical labour in a better position as to knowledge of their art, the division of labour and the extensive use of mechanical contrivances, instead of manual labour, have certainly lowered the position of the workmen as to general knowledge of their trade. Formerly, the millwright knew all about the machinery he made, and could turn his hand to all the operations required in its construction; now, there are mechanics who can only turn, others who can only file, and only very few who, to skill of hand, unite a knowledge of the machinery of which they execute portions. We are convinced that this state of things is injurious both to workman and to master; for a man can never labour with hearty good-will at work which he does not understand, and in which he, therefore, takes no intelligent interest; he can only act as the machine at which he works, and he is thus morally and intellectually degraded. We would have every workman understand the character and tendency of his operations, and take an interest in them. He would thus be relieved from much of that monotonous drudgery which his ignorance forces him to undergo; he would be prepared for emergencies; he would see modes of economising labour, and of improving the work on which he might be engaged; and he would be provided with a source of rational amusement in his leisure hours.

But not to the mechanical workman alone do we think that a general knowledge of practical mechanics should be supplied; we think that it should form part of the education of every one, whatever be his position and his prospects. We have seen of late years deplorable instances of want of contrivance in military affairs, and of sacrifices of blood and treasure in consequence. We are eminently a commercial, manufacturing, agricultural, and colonising nation; our commerce is conducted by means of railways, steam and sailing vessels, and requires warehouses, docks, and quays; our manufactures are all the product of mechanical contrivances; our agriculture is now rising to its just position as a mechanical and chemical art; and our colonies have been successful from the energy displayed by our emigrants, in giving scope to their natural genius for mechanical adaptation of the means to the end. Of late, we have assumed the position of a warlike nation, and we have sadly asked ourselves why there has not been employed in war any of that mechanical skill and ingenuity which characterise our peaceful arts.

Notwithstanding all this, practical mechanics have never yet been cultivated as a branch of general education; and consequently, every man, whatever be the walk of life in which he chooses to tread, has to begin his real education after he leaves school. We think this evil should be remedied—that every man should have instilled into his mind in his early years, a thorough knowledge of common things, so that when he advances in life he may enter more readily on any of the professions or trades practised at home, and be the better prepared for the emergencies of commerce, colonisation, or warlike expeditions abroad. Among the population of the land there lies dormant a vast amount of talent and ingenuity, which at present is so much loss of capital to our country and to the world. Let some opening be made for its cultivation, and we doubt not that a few years would bring about a more astonishing development of our resources, extension of our commerce, and improvement in our arts, than all the marvellous advance of the last thirty years. While we fully admit the high importance of other branches of mental and moral training, we would also strongly urge the advantages of insight into the mechanical arts, because through their extended cultivation we expect to secure a great increase of our material prosperity and comforts, a great diminution of labour, improvement of health and strength, and wide diffusion of intelligence among all classes.

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* See *ante*, p. 719, *et seq.* Section, *Mechanical Philosophy.*

CHAPTER II.

MECHANICAL DRAWING.

Contents.—USE OF DRAWING—PLANE SURFACES—DRAWING CUBES—DRAWING CYLINDERS—PROJECTION—SECTIONS—PERSPECTIVE—PROJECTION OF CURVES—UNFOLDED SURFACES—INTERSECTIONS—HATCHING—CENTRAL LINES—SHADOWS AND SHADES—INSTRUMENTS—SCALE OF DRAWINGS.

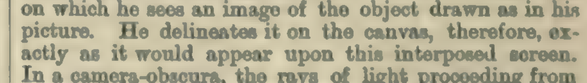
USE OF DRAWING.—In the actual practice of mechanical art, drawings are invaluable; they show the true forms, dimensions, and arrangements of machinery, to those accustomed to their use, with greater clearness than models or even the full-sized works themselves. The draughtsman devises the arrangement of his machinery; sketches it on paper; calculates the strength and proportion of the parts; draws them out full-size or to some suitable scale; studies their combinations on paper; improves this part, strengthens that part; modifies the forms so as to save complication, material, or cost of workmanship; traces the action of the whole; provides safeguards against accident or undue wear and strain; and having at length fully embodied his ideas on his plans, sections, and elevations, places them before the workman for execution. The workman has, in general, no need to study any of those details which fall within the province of the draughtsman; he has only to put his rule to the drawing, and measuring every dimension there indicated, shape the material with which he deals in exact accordance with it. Without drawings it would be impossible to make any advance in mechanical art. The paper, pencil, ink, and colours, are cheap materials; and the time and labour occupied in making a drawing are as nothing compared with the work in fashioning the solid material. To scheme on paper is, therefore, most advantageous in every point of view, as by that process only can be secured that economy of material and labour, harmony of action and justness of proportion, which constitute the beauty of a mechanical device. To scheme in the solid materials, is a most expensive as well as unsatisfactory process, and one that seldom leads to successful results. We know of no circumstance that can warrant its adoption, except in cases where there is no practical experience to guide the mechanic as to the power required, or the mode of operation suited to the work he has in hand. Even in such cases it is generally possible, by the exercise of a little judgment and ingenuity, to arrange some simple and inexpensive experiment, which may give to a practical man, a tolerable notion of the kind and scale of machinery that will be required.

A system of mechanical notation was proposed some years ago by the ingenious Mr. Babbage. In devising his calculating machinery, which consisted of a great number of parts, many of them merely repetitions, he found it difficult to imagine all their simultaneous movements without an excessive and painful exercise of that mental power which has to deal with such matters. He therefore attempted, and with considerable success, to trace by written symbols the flow of motion through a train of machinery. We are not aware that his system of notation has been adopted by practical men. Indeed, most of the complex machinery with which the engineer has to deal, is of a kind similar in many respects to works previously executed, embodying improvements that may have been from time to time effected in them; his mind is, therefore, in a manner prepared for the conception of the various motions and connections by his practice in watching those in machines in action, and he has comparatively little difficulty in fully imagining the intended action of the work on which he may be engaged. It will, therefore, be our first duty to offer a few practical suggestions on drawing, as applied to machines.

PLANE SURFACES.—When we have to draw a triangle, square, circle, or any other superficial figure, upon paper, we find no difficulty in giving a full and accurate delineation of it in respect of form and dimensions. The paper on which we draw is itself a plane surface; and so long as we are not limited in length and breadth, we can delineate any plane form whatever upon it. In the same manner, we can describe any of the flat faces of any solid object presented to us, such as a circle, pyramid, or prism; and give an accurate representation of any two of its dimensions, whether we call these by the names of length and breadth, or height and width. But when we have to delineate a solid body which has three dimensions, length, breadth, and thickness, we must either cut our paper to pieces, shape it so as to correspond with the different faces of the body, and put them together in similar order, so as to form a model of it; or we must have recourse to some device that shall enable us to comprehend for ourselves, and to communicate to others, an accurate notion of the solid body we propose to delineate on a flat surface. When the painter draws a portrait, or executes a landscape, he has to imagine, that between his eye and the object which he draws, there is interposed a surface or sheet of some kind on which he sees an image of the object drawn as in his picture. He delineates it on the canvas, therefore, exactly as it would appear upon this interposed screen. In a camera-obscura, the rays of light proceeding from every point of objects presented to it, are concentrated or gathered together by a lens, into a particular place or focus behind it. A piece of frosted glass in this focus offers a surface for these rays to illuminate, with their respective lights, shades, and colours; and an observer looking on the frosted glass, sees a perfect miniature picture of the objects. A plate of prepared metal or glass placed in the focus, receives exactly the same picture; and certain substances spread over its surface, rendered sensitive to chemical properties of rays of light, which the eye cannot appreciate, undergo changes in their constitution, which can be rendered permanently visible in photography.

DRAWING CUBES.—Pictures are thus produced by hand and by photography, which present, upon flat surfaces, delineations of solid bodies as they appear to the eye. But, if we were to apply a compass or a rule to the measurement of the bodies represented in these pictures, and also measure the objects themselves, we should find the angles, dimensions, and proportions of the picture, totally different from those of the real bodies.

Fig. 3.



Thus, a cube, such as a die, which has six equal square faces, marked by dots, as in Fig. 3, when represented on a picture, might present such an appearance as in Fig. 4, where only three sides are visible, none of them squares, and no two alike: not a single angle or dimension equal to those in he die itself. And yet the picture may be a perfectly true delineation of the die as it appears to the eye of an observer. Now, if a workman were furnished with such a picture, and required to make a die according to it, he could have no conception of its form and dimensions. For instance, he might cut a piece of plate to the shape of an irregular six-sided figure, with three lines engraved on it, meeting at a point near its middle, and certain dots marked at the distances indicated on the picture. He would then have made a solid body, which, viewed in one direction, would certainly present the appearance of the picture. Even if the picture were shaded, and marked by dotted lines, indicating the invisible edges of the die, as in Fig. 5, he could not, without further information as to its meaning, construct a solid body such as would answer to it. He would have to be told that it was a solid, having six equal flat sides, which, viewed in a certain direction, and at a certain distance from the eye, under light falling upon it at a certain angle, presented the appearance of the picture. He would then either model all conceivable kinds of six equal-sided bodies, and try each in the given aspect, or he would work out, geometrically, some other properties of the body, such as might enable him to construct it. If, on the other hand, instead of having such a perspective picture presented to him, he had a drawing like Fig. 3 placed before him, and were told to make a piece of ivory of such a shape that each of its six sides should be exactly of the form drawn, and either of its dimensions, or of some given multiple or fraction of these dimensions—as for instance, for every inch on the drawing, three inches in the die itself—he would at once comprehend and execute the work required of him.

Fig. 4.

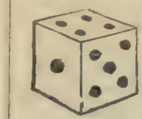
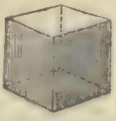
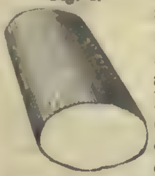


Fig. 5.



DRAWING CYLINDERS.—In like manner, if a cylinder, or roller, were pictured as it appears to the eye, with its light and shadow, as in Fig. 6, the workman might guess at its form, but could ascertain nothing as to its dimensions. But if he were told to make a thing of which Fig. 7 is the drawing, one-quarter of the full size, he could at once form his material to its proper diameter and length, by measuring the drawing, and allowing an inch in his work for every quarter of an inch on the drawing.

Fig. 6.



PROJECTION.—The object of mechanical drawing is, therefore, not to present a delineation of any object as it appears to the eye, but to furnish the exact figures and dimensions of its parts, in such an intelligible manner, that solid materials may be fashioned into shapes, whose parts shall have similar forms and proportions. The principle on which mechanical drawing is founded, is that of projection on plane surfaces, which we shall now endeavour to explain.

Fig. 7.

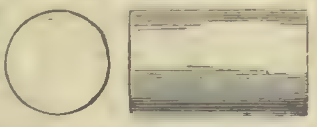
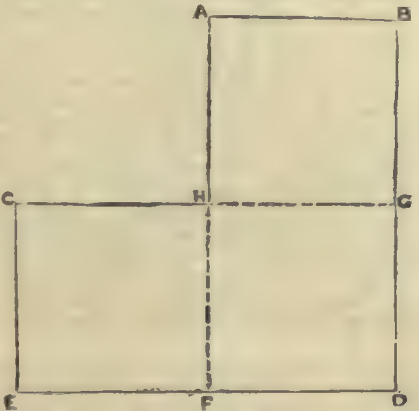


Fig. 8.



Suppose a sheet of paper were cut to the form A B D E C H (Fig. 8), and folded across where marked by the dotted lines H G and H F, the two parts A B G H and C H F E being turned up so as to stand at right angles or square to H G D F, and to one another as in Fig. 9, forming, as it were, three sides of a square box; then each of these three portions of the paper becomes what is called a "plane of projection," for receiving the representation of one of the sides of a solid body placed somewhere within the imaginary box, of which they form the sides. Let us suppose, for instance, that a die were suspended within the box (Fig. 10), and that from every point in each of the surfaces of the die exposed to the three sides of the box, lines were supposed to be drawn perpendicular or square to those sides as indicated by the dotted lines;—then the lines joining the intersections of the dotted lines, with the upright and horizontal surfaces, would enclose figures accurately representing, in form and dimensions, the respective sides of the die presented to them. The paper being then unfolded, would contain a mechanical drawing of the die (Fig. 11), exhibiting accurately the forms and dimensions of three of its sides. In order to get a drawing of the other three sides, the other sides of the imaginary box might have similar figures projected or thrown upon them. It is evident that whatever be the situation of the die, as to distance from either of three planes of projection, there will be no difference in the drawing; the form delineated will be the same, though it may appear on a different part of the paper.

Fig. 9.

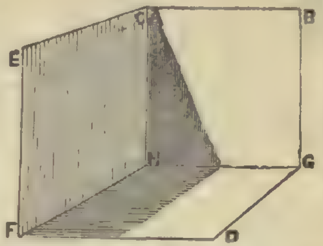
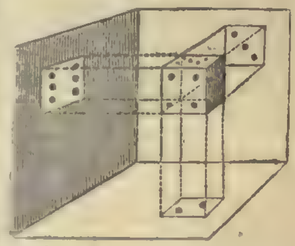
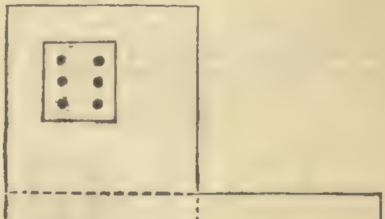


Fig. 10.



The names usually given to the three drawings of an object are these:—The projection on the horizontal plane or bottom of the box, is called the *plan:* and the projections on the vertical planes or upright sides, are called *elevations*. One of the elevations may be called the side elevation, or side view; and the other the end elevation, or end view; while the plan, when projected upwards, or on the top of the box, is sometimes called the bird's-

Fig. 11.



eye view, having nearly the form which an object would present to the eye of a bird soaring above it.

SECTIONS.—For drawing the exteriors of all solid bodies, bounded by straight lines and flat surfaces, these projections are generally sufficient. But it is often necessary to make such drawings as shall give a correct notion of the interior construction of bodies.

Suppose, for instance, we wished to have a cubical box constructed of wood, having a certain thickness, and fitted with a vertical partition of a certain height, and at a certain distance from the ends. In order to indicate this by a drawing, we must show its internal construction. To effect this, we may still use the three planes of projection; but as these planes are imaginary surfaces, we may easily conceive some of them to pass directly through the substance of the box: in other words, we may conceive the box to be sawn or cut across in any direction, and the form presented by the parts so cut, to be projected on the planes (Fig. 12). Thus, if a horizontal and a vertical cut were made, as indicated by the dotted lines, A A and B B, we should have a plan and a side elevation of the cut surface, in which the thickness of the wood, and the position and height of the partition, would be clearly shown. These views would be called *sections*, or the drawings of cuttings. The horizontal projection would be a sectional plan, and one of the vertical projections would be a longitudinal or transverse section, while the other would still remain as an end or side elevation. The dotted lines A A and D D, on the end elevation and section, correspond with the horizontal plane of section; and the lines B B and C C, on the end elevation and plan, correspond with the vertical plane of section. In a drawing, then, of a box, such as in Fig. 13, which would be seen on unfolding the paper of projection, enough of the form, thickness, position, and height of the partition would be represented to enable a workman to construct the box.

Fig. 12.

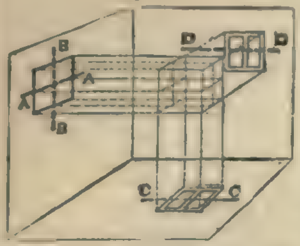


Fig. 13.

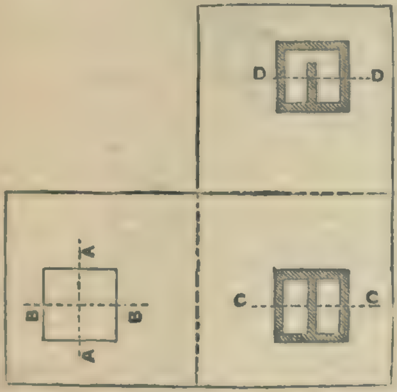
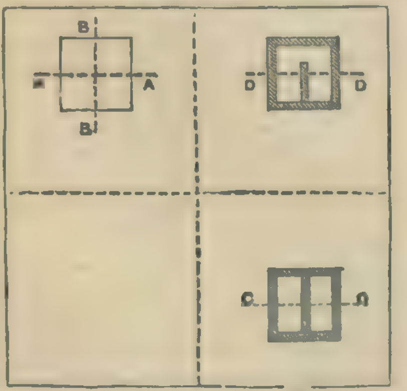


Fig. 14.

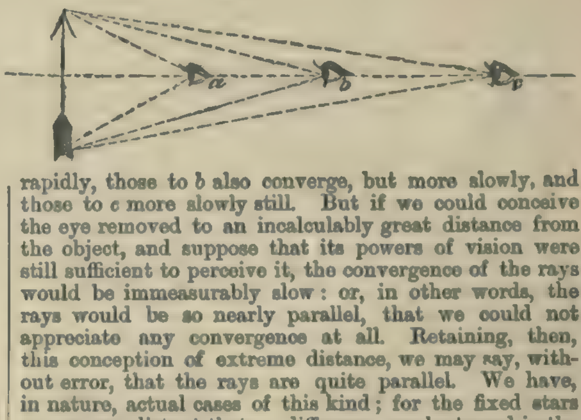


Now, if the part of the paper containing the end elevation were cut away from the rest, and turned round, so as to bring the letters on it into a more convenient position for reading them, as in Fig. 14, the accuracy of the drawing would in no respect be altered, provided always, that it were clearly understood what elevations, plans, or sections, the different parts of the drawing were intended to represent.

PERSPECTIVE.—There is another mode of considering this question. The rays of light by which the eye is enabled to perceive the form of an object, proceed from every point of the object, in straight lines to the eye. Suppose, then, one face of a die were presented directly to the eye, as in Fig. 15, the rays of reflected light, proceeding from every point in the surface of the die to the eye, would all converge or draw together towards the small opening which the pupil presents; and entering there, would produce the image which enables the spectator to see the object. This would be the case however far off, or however near, the eye were to the die; but the greater the distance, the less the convergence of the rays, or the more nearly are they parallel to each other, as may be clearly seen by the diagram, Fig. 16, where an arrow is supposed to be the object, and the rays of light proceeding from its extremities are indicated by the dotted lines, to an eye at the various positions, *a*, *b*, and *c*. While those to the eye, at *a*, converge very rapidly, those to *b* also converge, but more slowly, and those to *c* more slowly still. But if we could conceive the eye removed to an incalculably great distance from the object, and suppose that its powers of vision were still sufficient to perceive it, the convergence of the rays would be immeasurably slow: or, in other words, the rays would be so nearly parallel, that we could not appreciate any convergence at all. Retaining, then, this conception of extreme distance, we may say, without error, that the rays are quite parallel. We have, in nature, actual cases of this kind; for the fixed stars are so very distant that no difference can be traced in the direction of rays of light coming from them to the earth, at different parts of her orbit, although the distance across that orbit is nearly two hundred millions of miles.

Fig. 15.

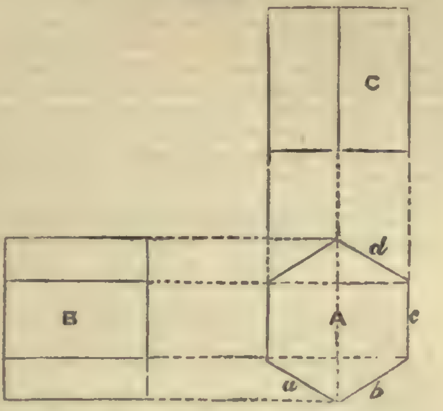
Fig. 16.



Now, we have already shown that the different projections of an object are formed by supposing parallel lines to be drawn from the different points of it to certain planes; and if we suppose these parallel lines to be rays of light, such as we have described, proceeding to an immeasurably distant eye, we attain a mode of understanding what these projections mean. According to this notion, the plan or horizontal projection is the view which an object would present when we look directly down on it, or directly up to it, from a great distance; the side and end elevations are the direct views of the side and end respectively; and sections are the direct views of the object when cut or sliced across in any direction. Bearing this in mind, we may then be prepared to make mechanical drawings of most objects with facility.

Suppose, for example, we had to draw an object such as mathematicians would call a hexagonal prism—that is to say, a solid figure having six equal oblong sides and two six-sided ends. Supposing it to stand on one of these ends, and that we looked directly down upon it, we should see its plan, which is a hexagon (A, Fig. 17).

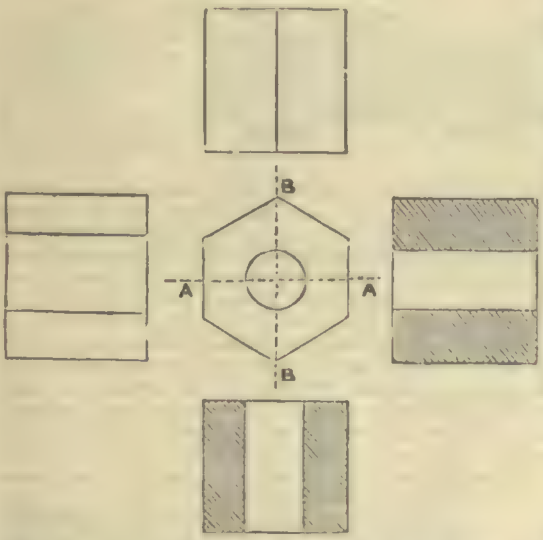
Fig. 17.



Again, looking directly on the angle formed by the two sides *a* and *b*, we should see the elevation C, showing the two retreating sides *a* and *b*; and looking direct on the side *c*, we should see the elevation B, showing the full front view of *c*, and views also of the two retreating sides *b* and *d*. The dotted lines represent portions of the supposed parallel rays of light proceeding from the angles of the object, and prolonged to form lines, projections, or drawings of the two views, the position of the top and bottom lines of these projections being determined by the height of the prism, which we suppose to be given.

Let us now suppose that the same prism is to be made with a circular hole passing through its centre from end to end, we may show the same views as before, and in addition to them two sections, as in Fig. 18.

Fig. 18.

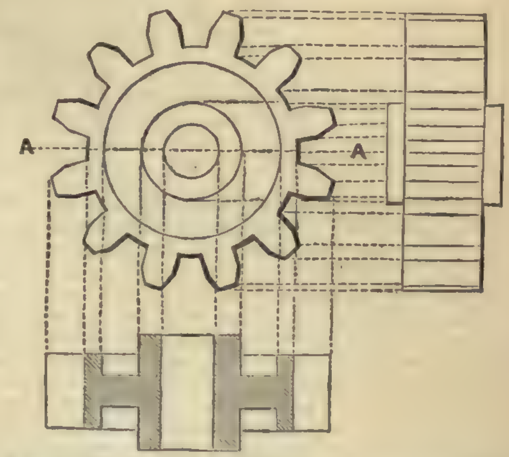


The lines A A and B B, through which the sections or cuttings are supposed to be made, are marked on the plan, and the sections are formed exactly like the elevations, by tracing the different points of the object where the section lines cut it. We have hatched or drawn diagonal lines across the solid parts which are supposed to have been cut asunder, in order to distinguish them, by a sort of shading, from the open circular part where no solid material is supposed to have been cut through.

We will now take a somewhat more complicated drawing as an illustration—for instance, a toothed wheel (Fig. 19), in front elevation, sectional plan on the line A A, and elevation as seen edgeways, which we shall call side elevation.

In this case, as in the former, the dotted lines are drawn parallel to each other from the different visible points or edges, and furnish us with the positions of these in the side elevation and plan; the width of the teeth and thickness of the material being supposed to be known independently.

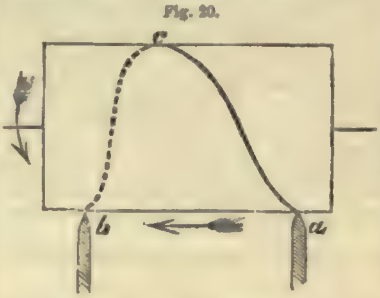
Fig. 19.



PROJECTION OF CURVES.—By the same method, drawings of curved surfaces may be made, some consideration being given in each case to the mode most suitable. Let us take for an example a drawing of a screw. In the first place, we must understand what a screw is. Suppose a cylinder or roller were revolving round its axis, and that while it was so revolving a pencil or sharp point were held against it. If the point were at rest while the roller revolved, a simple circle would be described on the surface of the latter. But suppose that while the roller revolved, the point were made to traverse lengthways along it, then a screw-line would be drawn on the surface (Fig. 20).

The pencil having begun to mark the roller at *a*, and having advanced to *b*, while the roller has made one complete turn, the screw line *a c b* would be marked on its surface. As we could not see both sides of the roller at once, we can only draw the part *a c* of the screw as visible, the other part *c b* being marked by a dotted line to show that it exists, and would be seen were the roller transparent. The whole line *a c b* would be called a thread or complete turn of the screw; and the distance *a b* would be called the pitch, or distance between two adjacent threads. It is clear that were the roller to continue to turn, and the pencil to advance, the thread would continue to be marked as far as the surface of the roller might extend. The screw marked in the figure would be technically called a "drunken thread," or be said to be of irregular pitch, as the relation between the velocities with which the roller revolved, and the pencil advanced respectively, is not of a fixed or constant character.

Fig. 20.



But if, while the roller revolved uniformly or with regular and equal velocity, the pencil also advanced uniformly, a thread would be traced perfectly regular and equal in all its parts, or a true screw, such as is used in mechanics, would be formed. We will now draw such a screw. Let A (Fig. 21) be the end-view or section of the roller, which is merely a circle, and let B be a side elevation; *a* being the position of the pencil when it began to mark the thread, *b* its position after one complete turn, and *c* its position after two turns; it is our object to find the shape of the line, or at least its drawing or projection on the elevation B, connecting the points a and b, and b and c, respectively. Now recollecting that a regular screw implies that while the roller makes any part of a turn, the pencil must make an advance through the same part of its pitch, we will divide the circumference of A into four equal parts by the points d, e, f, g, and also the two pitches $a\,b$ and $b\,c$ into four equal parts, marked by the points h, k, l, m, n, p. Then we know that while the roller has turned one quarter round, so as to bring the part of its surface e to the pencil, the pencil itself has moved one quarter of its pitch along, so as to be at the point h. If then we trace a line of projection from e along the elevation, and a line square to it from h, the point e_1, where these lines cross, must be a point in the projection of the thread. In the same manner, tracing lines from f and g and from k and l crossing them, we get other points, f_1 and g_1, in the thread; and repeating the process for the next thread, we get the points e_2, f_2, and g_2, similarly situated in it. But though we might, between the points d_1, e_1, f_1, &c., draw any sort of lines straight or crooked, we should not be warranted in assuming them to be the proper representations of the screw. We can, however, again subdivide the circumference of A and the pitch on B, and thus get a number of other points, as indicated in part on Fig. 22, which must be in the projection of half the screw. A curved line traced through these points, will then be a correct representation of half the thread; and as the thread is perfectly regular, the same form of curve will be repeated at the equal intervals of pitch, from d_1 to f_1, and d_2 to f_2. The intermediate parts of the thread from f_1 to d_2, and from f_2 to d_3, being traced on the side of the roller where g is, cannot be visible at the same time as the other portions: but if they could be seen, as in the case of a transparent roller, they would be symmetrical with the visible parts, but inverted, as indicated by the dotted lines in Fig. 23.

Fig. 21.

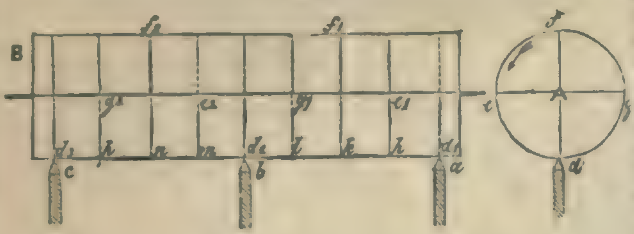
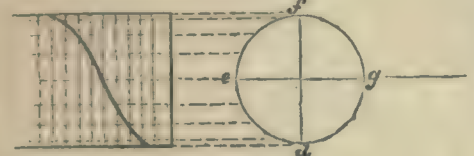
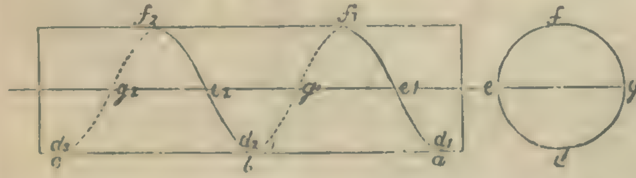


Fig. 22.



Next let us suppose that two pencils were held against the roller a little distance apart, so as to trace two screw-threads parallel with each other; the side elevation of this, as in Fig. 24, presents merely a repetition of the proper curved lines at the proper distance apart—that is to say, if any number of straight lines were drawn parallel to the axis or sides of the roller, the portions of those straight lines intercepted by the screw-curves, viz., $a\,b$, $a_1\,b_1$, $a_2\,b_2$, &c., also $c\,d$, $c_1\,d_1$, $c_2\,d_2$, &c., &c., would be all equal to one another. The ordinary practical mode of making such a drawing would be to set out the points of one portion of the curve, such as $a\ a_4$, by the method indicated on Fig. 22, to shape a piece of card-board or thin wood to fit it, and then marking out the proper distances $a\,b$, $a_1\,b_1$, $a_2\,b_2$, $b\,c$, $b_1\,c_1$, $b_2\,c_2$, $c\,d$, $e_1\,d_1$, $e_2\,d_2$, &c., and applying the shaped card to these points successively, trace by a pen or pencil as many repetitions of the curve as might be required. By a similar process, inverting the card, the elevation of the curves as they would appear through the roller were it transparent, as indicated by the dotted line in Fig. 23, could be repeated.

Fig. 23.



Recurring to Fig. 24, let us now suppose that all that part of the surface of the roller contained between the double threads were cut down to a certain depth, so as to leave a real solid screw-thread prominent, such as would be produced by winding a square wire obliquely round a cylinder. Our method of drawing this would be as follows (Fig. 25):—The transverse section shows two circles, one the boundary of the outside of the prominent thread, and the other the roller on which it is wound, or the bottom of the cutting made in the solid roller. Let half of both these circles be divided into any number of equal parts, and let the length $a\,b$, which the thread traverses during half a turn, be divided into the same number of equal parts; then, by drawing the vertical lines from the points where $a\,b$ is divided, representing the distances traversed by the describing point along the roller in equal portions of a half revolution; and by drawing horizontal lines from the points into which the half circumferences of the inner and outer circles are divided, representing the equal portions of the half revolution; the points, at which these vertical and horizontal lines cross each other, are points in the two screw curves respectively. A card shaped to each of these can be used to repeat them at equal intervals along the roller, so as to represent the continuation of the thread. A little consideration will show what portions of these lines would be visible in a side-view, and what parts would be concealed. It is usual to show, by full lines only, the visible portions, the invisible parts being marked, when necessary, by dotted lines.

Fig. 24.

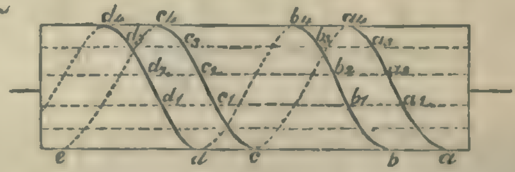
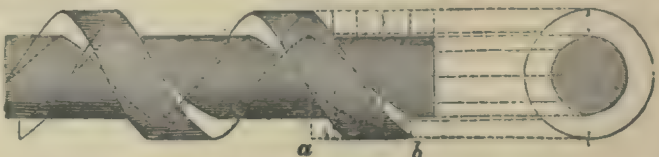


Fig. 25.



Other curves can be treated in the same way as screw-curves, the general principle being the same to all—namely, finding the projection of a number of points through which the curve must pass, and tracing through those points, a continuous curved line as nearly as the eye can judge or the hand execute. The greater the number of points projected, the more nearly accurate will be the drawing of the curve. But a little practice in drawing, and careful observation, soon enable a draughtsman to trace projections of curves with sufficient accuracy without requiring very many points for their determination. The drawing of a screw-curve, such as we have described, is as difficult as any of the projections that ordinarily occur in mechanical drawing. We have, therefore, dwelt upon it at some length, being convinced that the student who sees his way clearly through this example, will master most others without great difficulty.

UNFOLDED SURFACES.—Besides the method of drawing objects by projection, it is sometimes useful to represent surfaces developed or unfolded. This may be clearly understood by conceiving that a pattern of some kind—say a series of squares, as *a* in Fig. 26, is wrapped or folded round the roller *b*. We shall suppose that the piece of a flat paper, or fabric, containing the pattern, will fold exactly round the roller so that the edges meet. The projection of this pattern, as it would appear in a side elevation of the roller, may be drawn as in Fig. 27. Observing that the pattern is divided into three parts in height and five in width, we draw an elevation and a plan of the roller, as in Fig. 27, dividing the elevation into three equal parts in height, and dividing the circumference of the circular plan into five equal parts by the points *a*, *b*, *c*, *d*, *e*. Tracing up lines from such of these points as would be visible, looking on one side of the roller, viz., *a* and *b*, so as to cross the horizontal lines on the elevation, we get at once the elevation of the pattern as it would appear in projection when folded round the roller. This would really be a process of envelopment, for we have first drawn the pattern on a flat surface and then enveloped the roller in it. The process of development is just the converse; for it consists in developing or unfolding a surface already on the body, and then spreading it out flat. Suppose, for instance, that we had the elevation of a roller, such as Fig. 28, and we desired to develop the pattern which is represented in projection on its surface. We draw any required number of horizontal lines, at *a*, *b*, *c*, *d*, *e*, through the marked points of the pattern, and trace perpendicular lines from the same points till they meet the circumference of the circular plan in the points *f*, *g*, *h*, *k*, *l*. We then draw a straight line of the length of half the circumference of the circle, and make it the base of an oblong of the same height as the roller. Dividing the base by the points $f_3 g_3 h_3$, into parts of the same lengths respectively as the parts of the half circumference separated by *f g h* in the plan, and dividing the height, *a e*, by the points b_1 c_1 d_1, into parts corresponding with the divisions of the roller in height; and through all these points, drawing parallels and perpendiculars to the base; we get the positions of the different points f_1 g_1 h_1, &c., in the elevation, as they should be in the development, and can trace, through these points, the lines of the pattern. Should the pattern be complex, or made up of curved lines, we should have to trace the development of a greater number of points in order to get a correct drawing of it as it would appear when unfolded.

Fig. 26.

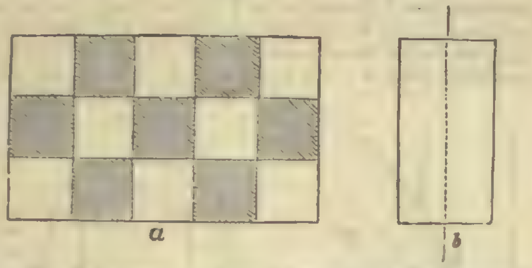


Fig. 27.

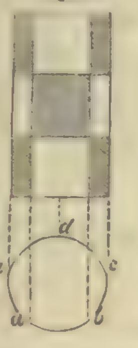
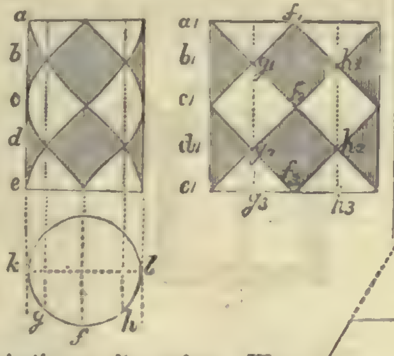
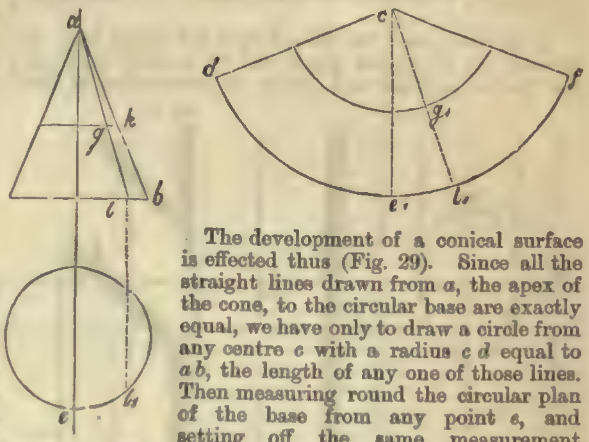


Fig. 28.



We may here observe that a conical surface is developable as well as that of a cylinder or roller; and the mechanical draughtsman has seldom occasion to deal with the development of any surfaces except these.

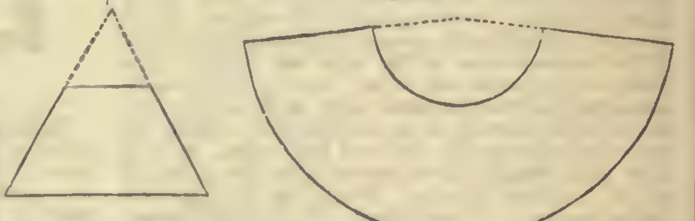
Fig. 29.



The development of a conical surface is effected thus (Fig. 29). Since all the straight lines drawn from *a*, the apex of the cone, to the circular base are exactly equal, we have only to draw a circle from any centre *c* with a radius *c d* equal to *a b*, the length of any one of those lines. Then measuring round the circular plan of the base from any point *e*, and setting off the same measurement from a point *d* to *f* on the circle we have described from the centre *c*, and drawing the straight lines *c d* and *c f*, we get the development of the conical surface. A piece of paper cut to the shape of the circular sector *c d f* would exactly fold round the cone, the edges *c d* and *c f* meeting. To develop any pattern, we should proceed, as in the case of the cylindrical surface, to find the development of a number of points. We shall show the method of proceeding with one point marked *g* in the elevation. Through the point *g* draw a parallel to the base *g k*, and a line *a g* from the apex meeting the base in *l*. Project *l* down to l_1 on the plan, and measuring round the circumference from *e* to l_1, set off the same measurement from e_1 to l_1 on the development, and draw the line $c\, l_1$, which will be the development of the line *a l*. Then from the centre *c* with a radius *a k*, draw a portion of the circle cutting $c\, l_1$ in g_1, and g_1 will be the development of the point *g*, for it is in the developed line $c\, l_1$, and at the proper distance from the apex. A similar process may be adopted with respect to any other points, and thus a pattern of any kind might be developed.

To develop the frustrum of a cone—that is, a cone with a portion of it towards the apex cut off, as in Fig. 30—we imagine the cone completed, as indicated by the dotted lines in the elevation, and have only to develop the complete cone and cut off the wanting part, leaving the development of the actual surface of the frustrum.

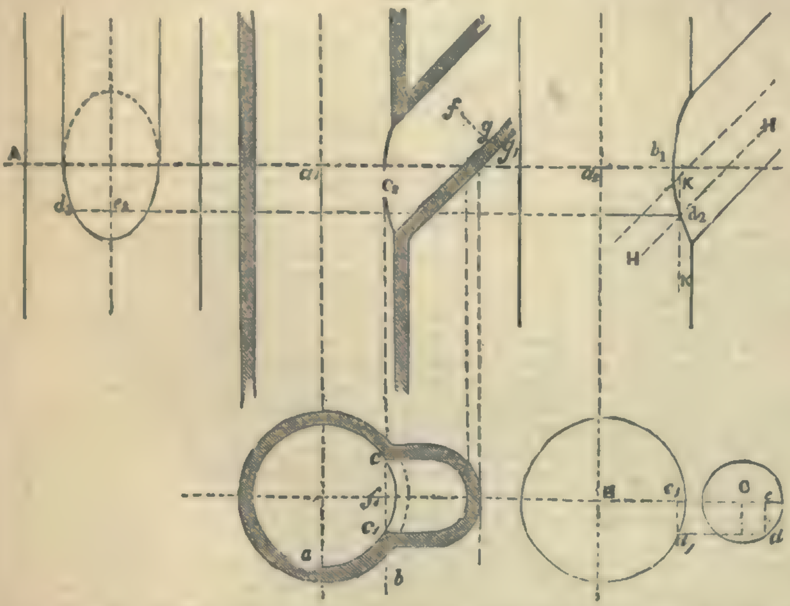
Fig. 30.



INTERSECTIONS.—In consequence of the facility with which materials can be shaped into cones, cylinders, or any forms which have circular sections, and for other reasons connected with strength and fitness, these forms are generally used in practical mechanics. It is therefore most important for the draughtsman to be expert in the projections and developments of such forms. Among the numerous problems in drawing them, there is an interesting, and often useful, class depending on the various intersections of one curved surface with another. We shall give one example of this to illustrate the method that may be very generally applied in most of them. Let us suppose that a pipe of a certain diameter branches out from another of a larger diameter at a certain angle, as indicated on the longitudinal section in Fig. 31; and that we have to draw two elevations and a transverse section of this, so as to show the form of the joint, or intersection of the branch with the main-pipe.

In the first place, we should draw the centre lines or axes of the cylindrical pipes, and then easily mark off the general outlines and points of intersection in the two elevations. The points c, c_1 in the transverse section, would be determined by setting off from f_1 on the centre line, the inside radius of the small pipe, as determined by the line $f\,g$ in the longitudinal section. These points would determine c_2 in the longitudinal section, by tracing the vertical line upwards till it meets the oblique centre line $f\,c_2$ in the longitudinal section. And in a similar manner, by tracing up a line from b on the transverse section, we should find a point immediately below b_1 on the side elevation, where the outsides of the two pipes intersect on the oblique centre line of the branch. There still remains for us to determine the particular curves of the intersection lying between the points so fixed; and this we must do by finding a number of other points in the different projections, which shall be the proper representations of points in the circumferences of the large and smaller pipes where they intersect each other. Let us, for instance, endeavour to determine some point in the curve of intersection represented on the side elevation; and as this must be a point on the outside of both pipes, we shall draw their outside circumferences from the centres B and C, connected by a line. Taking any point d in the circumference of C, and tracing on to d_1 on the circumference of B, and drawing the perpendiculars $d\,e$ and $d_1\,e_1$, we draw on the side elevation H H parallel to the axis of the small pipe at the distance C e from that axis, and K K parallel to the axis of the large pipe, at the distance B e_1. The point d_2, where these lines H H and K K intersect, is the projection of a point in the intersection of the circles on the side elevation. Tracing this across by a horizontal line to the front elevation, cutting its centre line at e_2, and setting off e_2 d_3 equal to e d_1, we get d_3 as the projection of the same point on the front elevation. And thus, by projecting a number of points, we should be enabled to trace through them a correct projection of the curves of intersection in their different aspects.

Fig. 31.



Hitherto we have only treated of the mode of drawing outlines; we may now proceed to discuss the various modes of rendering mechanical drawings distinct and more easily intelligible. These may be said to consist chiefly in hatching or section-lining, colouring, shading, and shadowing.

HATCHING.—Hatching is merely drawing parallel lines across the parts of any section, and are supposed to be cut through the solid material. Fig. 32 is the mere outline of the section of a hollow hexagonal prism; the lines convey no particular meaning, and they would apply as well to the end elevation of a prism with another projection from it; but if the section through the solid matter be hatched, as in Fig. 33, as if to indicate the appearance of the fibres of the cut material, it is understood that the drawing represents a section.

Fig. 32.

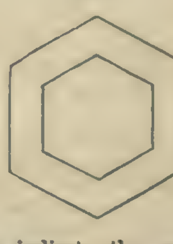
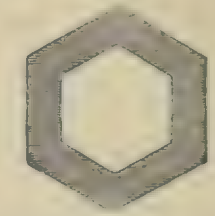
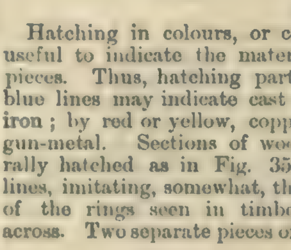


Fig. 33.



Hatching is also useful in some cases for distinguishing the different pieces of which the body drawn in section consists. This is effected by hatching one of the pieces by oblique lines lying in one direction, and an adjacent piece by lines lying in another direction. Thus, Fig. 34 may represent a section of a vessel with a separate cover fitted to it, and one of the bolts and nuts used for holding the cover in its place. The hatching lines of the vessel and its cover are oblique in opposite directions, and those of the bolt and its nut are vertical and horizontal respectively, so that all the separate pieces are distinguished.

Fig. 34.



Hatching in colours, or coloured lines, is sometimes useful to indicate the material as well as the separate pieces. Thus, hatching parts of a section by black or blue lines may indicate cast or wrought-iron; by red or yellow, copper, brass, or gun-metal. Sections of wood are generally hatched as in Fig. 35, by curved lines, imitating, somewhat, the appearance of the rings seen in timber when cut across. Two separate pieces of timber seen in cross section, side by side, may be hatched by these lines lying in opposite directions (Fig. 36); while timber in longitudinal section, may be hatched by crooked discontinuous lines, lying generally lengthways.

Fig. 35.

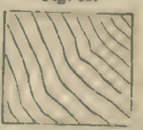
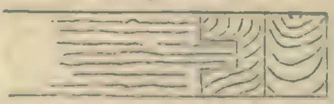


Fig. 36.



In working-drawings—that is to say, projections showing the parts in their proper dimensions, so that a workman can measure and transfer them to the solid material—colours are usefully employed to distinguish the different materials. Thus, parts tinted dark blue might represent cast-iron; while a pale blue might indicate wrought-iron or steel; yellow would mark brass or gun-metal; brownish-yellow, wood; brownish-red, copper; and so on. When any special material is to be used, its name is generally written on such part of the drawing as represents it.

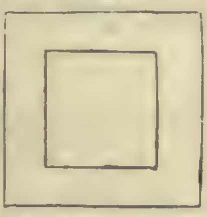
CENTRAL LINES.—In almost all mechanical works, or at least in their details, there is necessarily symmetry, or repetition of equal parts on different sides of some central points. For instance, a cylinder, cone, column, and such-like bodies, are symmetrical round their axes. The teeth of a wheel are symmetrical round its centre; and so with almost all the forms suited to machinery.

It is, therefore, most important in mechanical drawings, to mark the central or axial lines of the different parts. As these lines do not necessarily appear on the material, they are generally distinguished from those that mark outlines of actual form, by the use of a different-coloured ink; for instance, if the lines of the drawing generally be in black, as they usually are, the central lines may be ruled in red or blue. They should always be fine but distinct lines, because they serve as a starting-point for numerous dimensions. As an illustration, we may refer to Fig. 34, which we may suppose to be the section of a round or cylindrical box, the centre or axis of which is marked by the dotted line *a*. One of the bolts for fixing the cover is also shown; and as this bolt is also cylindrical, it has an axis or central line *b*, and the position of the bolt is sufficiently indicated on the section by figuring the distance of its axis from that of the cylinder, as it is marked, 10, on the figure. We have indicated the central lines *a* and *b* by dotted lines, to distinguish them from the outlines and hatchings; on a drawing they would probably be marked in blue or red, not dotted, but plain, fine, and distinct.

SHADOWS AND SHADES.—In order to render mechanical drawings more intelligible than they otherwise would be, recourse is had to shadowing and shading. The projections which constitute mechanical drawings are not representations of the actual appearances of the objects to the eye; they are merely the traces of their outlines formed by parallel lines drawn from all parts of them, to certain imaginary planes or flat surfaces, which are supposed to be transferred to the paper. In like manner, in the shadowing and shading of mechanical drawings, we must not attempt to give the natural appearance of the lights and shades visible on the objects. We must concert some system of illuminating their surfaces, in accordance with our system of projecting their outlines. The object of such light and shade is to make up for the deficiency of one of the dimensions of solid objects, depth, thickness, or distance, when we represent them on paper or any flat surface.

These outlines, however accurate, can never show more than two dimensions—length and breadth; and can convey no idea of parts projecting or receding from any surface represented. Nor can light and shade, applied to these outlines, give the other dimensions, so that they can be measured; but they can be applied with very good effect to indicate which parts are supposed to project or recede from the general surface, and the comparative amount of such superficial variations. If we have, for instance, the outline elevation of a square block of some material (Fig. 37), we cannot judge whether the outline is intended to represent a prominence or a hollow in the general surface of the block; but by a little shadowing we can give the effect of either, as in Fig. 38. In *a* we see that the inner part must project beyond the outer, in order to cast a shadow upon it; while in *b* the outer part of the surface must project beyond the inner for the same reason.

Fig. 37.



Again, shading may be very serviceable in giving a just idea of the form of an object seen in elevation. Thus *a* (Fig. 39) is the bare outline elevation of a roller standing on one end; but it gives no notion of the roundness of the object. *b* is the elevation shaded, and at once conveying the idea of roundness.

Fig. 38.

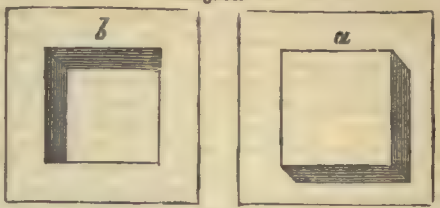


Fig. 39.

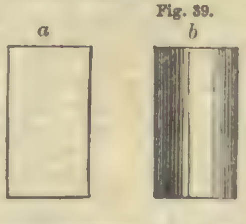
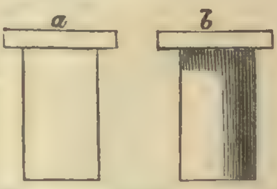


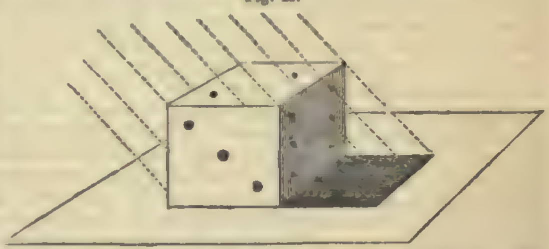
Fig. 40.



When the lights, shades, and shadows are all introduced in their proper places, as in Fig. 40, the notion of solid form is rendered very distinct, and a very clear conception of the object represented is conveyed to the mind.

The mode in which shadows are drawn is very similar to the system of projecting the outlines themselves. If we suppose a body, such as a die, to stand on a table, and numerous parallel rays of light, indicated by the dotted lines, to strike on it at an angle, some of them would be intercepted by the die, and thus all the surface of the table and the die would be illuminated, with the exception of the portions from which the light is kept by the interposition of the solid body. The dark surface thus left on the table, is a shadow of the die; and the unilluminated side of the die is a shade on the die. It appears, then, that the shadow is merely a projection of the figure made on a plane, by tracing oblique parallel lines from all the points in the outline of the body, to that plane.

Fig. 41.



But as this projection may, like the projection of the object itself, be effected on more than one plane, and as we are at liberty to suppose the object placed in light coming in any convenient direction, we can select such an obliquity of the rays as may at the same time furnish us with most distinct shadows, and with those most easily drawn. In selecting, then, the angle at which the rays of light shall strike, we can assume any that is not coincident with the planes of projection on which our drawing is made. But as these planes are at right angles, or square to one another, it seems natural to select half a right angle, 45°, or what is generally termed by workmen a *mitre-angle*, as the direction of the rays. Again, as a line drawn at such an angle may extend either from below upwards, or from the right towards the left, or from a distant point towards us, we must select the most suitable of these courses for giving us distinct shadows. As we are generally accustomed to see objects illuminated by light coming from above, it is natural that we should determine on employing rays coming from above downwards. Further, as our drawings are intended to represent the visible surfaces of objects, we must suppose those sides that are exposed to be illuminated; and therefore select rays, coming from behind us, and striking on and past the objects. As to whether these rays shall come in a direction from left towards right, or from right towards left, is a matter of indifference. The only circumstance which can affect our choice in this last respect is, that draughtsmen generally sit with their left side towards the light, and therefore are more likely to select rays coming in the direction to which they are accustomed: that is, from left towards right. Now, in order to see clearly the effect of rays supposed to come in the chosen direction, in illuminating certain surfaces, and in producing shades and shadows, we shall suppose a piece of paper, cut to the form of Fig. 42, consisting of three equal squares, and that portions of the diagonals of these squares are drawn, lying at mitre-angles, or angles of 45°, with the sides of the squares. Now, let us suppose that the paper is folded along the two lines $a\,b$ and $a\,c$, so as to turn up the two squares perpendicular to the third, and to each other, as in Fig. 43, forming three sides of a square box, with the diagonal lines drawn upon them. If the lines of projection upwards, towards the right and towards the front from the extremities of these three diagonal lines respectively, be drawn till they meet, as indicated by the dotted lines, we get a line $d\,e$ lying obliquely to each of three sides, of which line the diagonal lines are correct projections. Now, this line $d\,e$ would represent a portion of one of the rays of light which, as we suppose, illuminate the objects, and all the other rays would be parallel to it. The shades and shadows, therefore, of plans, elevations, and sections, projected on the three planes, will all be determined by lines lying at angles of 45° to the boundaries of those planes. As an example of the mode of correctly delineating shadows according to this law, we may explain the mode of defining the shadows in Fig. 44.

Fig. 42.

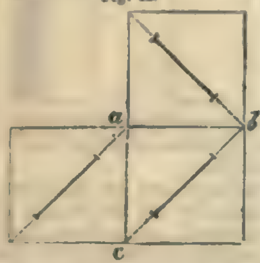
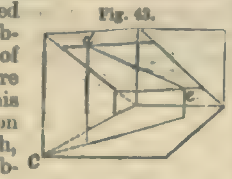
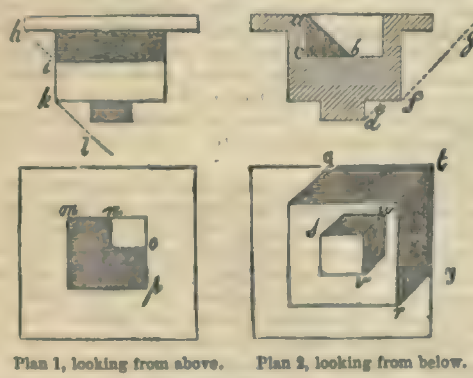


Fig. 43.



1. The line $a\,b$ drawn at 45° from the edge of the section, gives the breadth $c\,b$ of the bottom, over which the shadow of the side extends; we therefore set off $m\,n$ and $p\,o$ in the Plan 1, each equal to $c\,b$; and drawing lines from n and o parallel to the edges, we have the outline of that shadow.

Fig. 44.



Plan 1, looking from above. Plan 2, looking from below.

2. Drawing $d\,e$ and $f\,g$ at 45° from two lower edges of the section, we get the extent of the shadows observable in Plan 2, the inner shadow $s\,w\,v$ of the projecting part of the bottom being limited, because the line $d\,e$ falls within the edge of the bottom in section, while the shadow $q\,t\,y$ extends as far as the edges of the Plan 2, and would extend further if there were any surface to catch the shadow, because the line $f\,g$ in section extends beyond the edge of the body.

3. Again, in the elevation, $h\,i$ gives the point i as the limit of the shadow of the upper projection, while $k\,l$, extending beyond the projecting part of the bottom, shows that all its surface must be within the shadow.

For drawings of bodies, whose boundaries are plane surfaces, and whose edges are straight lines, the shadowing is very simple; and the proper inclinations and boundaries of the shadows almost suggest themselves, without the necessity for their actual projection. But when the bodies are bounded by curved surfaces, such as cylinders, cones, spheres, and the like, the projection of the shadows is somewhat more difficult. A little consideration, however, will in most of these cases enable the draughtsman to give a sufficiently faithful representation of the desired effect. It must not be supposed that the shadowing of mechanical drawings is intended to give them any merit in an artistic point of view; for, as they are not real perspective representations of objects, but only imaginary projections, so the shadows tinted on them are not the representations of shadows actually seen on the objects themselves, but geometrical projections of shadows, that would occur on certain suppositions as to the direction of illuminating rays. The only purpose of the shadowing is to give a clearer conception of the solid form intended to be shown; and the draughtsman should, therefore, take care to make these shadows geometrically accurate. After having solved a few problems by actual projection, he will find it easy to give tolerably faithful views of shadows by the eye. We will work out two problems in circular shadowing as examples of projection; and we have selected two kinds of shadows most commonly occurring in mechanical drawing for this purpose.

Fig. 45.

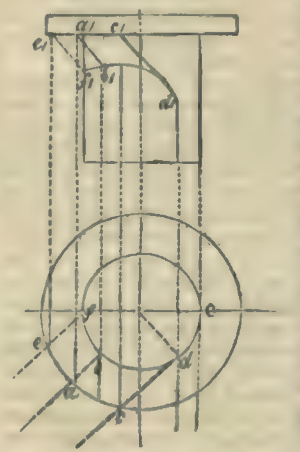
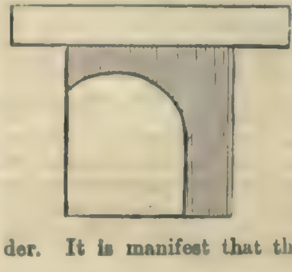


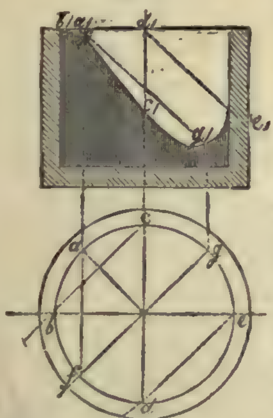
Fig. 46.



First, in Fig. 45, we have an elevation, and a plan looking from below, of a cylindrical body, with a flange or projecting rim round its upper end. The plan and elevation have a common centre line, corresponding with the axis. It is evident that the shadow of the lower edge of the flange is the boundary of the shadow of the flange on the elevation. If, then, we take any point, such as a on the plan, tracing it up to a_1 on the elevation, and drawing $a\,b$ at 45°, tracing up from b a line in elevation till it meets a line drawn at 45° from a_1, we get b_1 in the elevation as the shadow of a_1. Taking a number of points, such as a, and finding their shadows in a similar manner, we should be able to trace through them a curved line $f_1\,b_1\,d_1$ as the boundary of the shadow of the flange. The extreme point f_1 is found at once by taking f on the plan, drawing $f\,e$, tracing e up to e_1 on the elevation, and drawing $e_1\,f_1$. The line $c\,d$ on the plan is supposed to touch the inner circle in d; therefore, $d\,g$, the remainder of the visible part of that circle, must be all in shadow, as the body itself intercepts the rays. The line traced up from d to the elevation, and meeting the proper shadow line $c_1\,d_1$ in d_1, is therefore the boundary of the illuminated part of the elevation; and if the shadow part be filled in with a black tint, we have the appearance presented in Fig. 46 of the oblique projection or shadow of the flange on the cylinder. It is manifest that this shadow does not convey a proper notion of the cylindrical surfaces intended to be shown. The drawing requires shading to give the full effect; and after another example of simple shadowing, we will endeavour to show how the shading may be effected.

Referring now to Fig. 47, showing a plan and section of a cylinder open at top, we have to inquire as to the form of shadow in the section cast on the inside by the cylindrical side. Taking any points $b\ f\ d$ on the inner circle in plan, and drawing lines from them at 45° till they meet the opposite limb of the circle in the points $c\ g\ e$; tracing up these points by lines to the elevation, and also the points $b\ f\ d$ to $b_1\ f_1\ d_1$, and from the latter drawing lines at 45° to meet the others, we get the points $c_1\ g_1\ e_1$, in the boundary of the shadow. The point a, where a line at 45° touches the circle, being traced up to a_1 in the section, gives the commencement of the shadow; and the outline being filled in by a dark tint, presents the appearance of an interior cylindrical shadow. Like the exterior shadow shown in Fig. 46, shading is required in order to give the notion of a curved surface.

Fig. 47.



We shall, therefore, now proceed to discuss the question of shading; which consists in placing on a drawing, tints of various degrees of lightness or darkness, so as to represent the comparative amounts of reflected light from the different portions of the surface represented. We are not aware that the question of shading mechanical drawings has ever been discussed on geometrical grounds, like that of shadowing; it is, therefore, with considerable diffidence that we venture to offer some considerations which may furnish a clue to the proper variations of light and shade on projections. A skilful draughtsman has little difficulty in bring up a very correct, and even an artistic effect of light and shade on mechanical drawings; but we believe that it is to a practised eye and an expert hand that he owes his success. In shadowing, he trusts to the same elements of success; but as the outline of every shadow can be determined with mathematical accuracy, on the supposition of parallel rays of light proceeding at certain angles to the planes of projection, on the same supposition we think the variations of light and shade can also be determined. It must be confessed that, even if the lights and shades of a projection were determined in quantity, so that we could say such a part of a surface must have twice or thrice the darkness or the light of such another part, we should still labour under the difficulty of carrying out these dimensions of light and shade. We might, however, approach them by applying repeated dark tints to the shaded parts; making the number of equal tints laid on above one another, correspond with the degree of darkness determined on. But even without attempting any mathematically accurate mode of carrying out the theory in practice, we may at least derive useful hints from its investigation.

If we suppose the circle in Fig. 48 to be the plan of a cylinder, of which we wish to represent a shaded elevation, one half A D B would not be shown; the other half has its circumference divided into eight equal parts. We will suppose parallel rays of light R, R_1, R_2, R_3, R_4, R_5, R_6, to come at the proper angle (45°), and to be reflected from the surface of the cylinder in the directions r, r_1, r_2, r_3, r_4, r_5, r_6, respectively. These directions of the reflected rays are of course determined by drawing them so as to make the same angles with the radii C A, C 1, C 2, &c., as the incident rays make with those radii, but on opposite sides of them, according to the well-known optical principle that rays of light are reflected from a surface at the same angle as that with which they strike it. Now, of all those rays, that striking the point 3 is reflected most directly to an eye situated in the line C E, at a great distance from C, $3r_3$ being parallel to C E, and therefore the elevation of the point 3 should be the brightest. Again, the points 2 and 4 would appear equally illuminated, because the reflected rays $2\,r_2$ and $4\,r_4$ lie at equal obliquities to C E; but each of those points would appear less illuminated than the point 3, because of this obliquity. Farther, the points 1 and 5, whence the rays are reflected parallel to A B, mark the places where the illumination of the surface ceases; and were it perfectly smooth and polished, all beyond 1 towards A, and beyond 5 towards 6, would be perfectly dark. The portion 6 to B, receiving no light at all, would be represented in shadow. In the elevation then, if the points in the circumference of the plan be projected, and the surface darkened by lines or tints in accordance with the deficiency of reflected illumination at its different parts, we get a geometrically shaded representation of a cylinder.

Fig. 48.

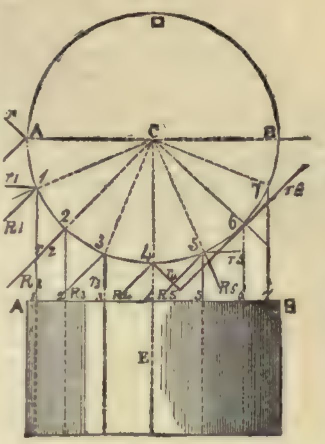
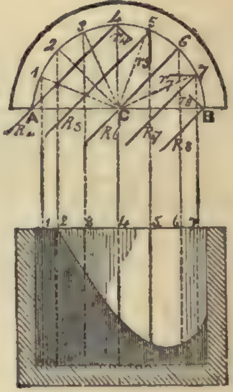


Fig. 49.



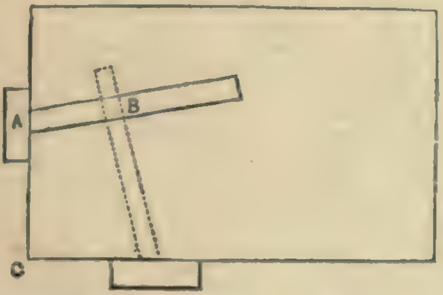
For the interior surface of a cylinder shown in section, the light and shade of the different parts may be found in a similar way, as marked in Fig. 49, where the point 5 giving the direct reflection will be the brightest, the points 4 and 6 having equal intensity of light, but each less than 5, because of the obliquity of the reflected rays; the part A 1 2 3 4 quite dark, because the light is intercepted by the edge A; the point 7, whence the ray is reflected parallel to A B, the last part of the illuminated surface; and 7 B dark, because the rays are reflected backwards. The section shows the shading in accordance with this variation of reflected lights, the lower part of the interior being completely shadowed in the form marked in Fig. 47.

The shading of other curved surfaces might be determined in a similar way; but as cylindrical surfaces are those which most commonly occur in drawings of machinery, we consider it sufficient to have pointed out the principles upon which the shading of them may be determined. A little practice will soon enable a draughtsman to give a rounded effect to drawings of such surfaces; and a little attention paid to this will often render plain and explicit a drawing, which would otherwise be comparatively obscure. The lights, shades, and shadows to complete drawings need not be determined by geometrical projections, which would often involve great labour without adequate results. A draughtsman accustomed to make drawings of machinery, forms, in his own mind, a very accurate conception of the solidity, projections, and hollows of the various parts, and throws in the shades and shadows by eye, in such a manner as to give a tolerable notion of these variations of surface. It is well, however, that the beginner should know the principles on which shading must depend, and he will then be better prepared for that success in their application, which practice of eye and hand can only give.

INSTRUMENTS.—Before concluding this part of our subject, we may say a few words respecting the instruments required by the mechanical draughtsman. The board, on which the paper is fixed, should be of well-seasoned wood, accurately right-angled, or, as it is commonly called, *square* at the angles. As wood is often apt to warp and shrink, according to changes in temperature and moisture, it is often necessary to have the edges of the board adjusted by the carpenter's plane, especially if the wood be new. Where great accuracy of drawing is required, this is a matter which should always be carefully looked to. A sheet of drawing-paper gently wetted by a sponge and clean water, will, if stretched and fastened on the drawing-board whilst moist, afford a surface as level as that of the board, and quite free from creases, when dry.

The drawing square, or ⊤ square, consists of a stock, with a projecting lip, which can slide along the edge of the drawing-board, and a blade, having the edges accurately straight and parallel. The edges of the blade should be at right angles to the face of the stock; but this is not of very great importance, for even if the blade made any other than a right angle with the stock, provided the angle C (Fig. 50) of the drawing-board be a right angle, the lines drawn by the square applied to both edges of the board would be at right angles to one another. Indeed, the blade and stock are often made so that the angle can be varied at pleasure, a screw being fitted in the stock in such a manner as to hold the blade fast at the angle required. This form of instrument is called the bevel ⊤ square, and is useful when there are to be drawn numerous lines parallel and at right angles to one another, but oblique to the edges of the board.

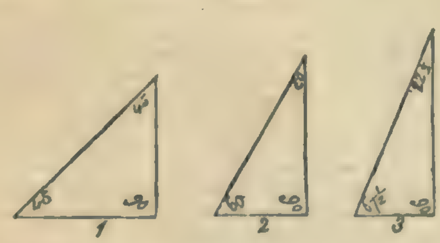
Fig. 50.



Set-squares are triangular pieces of wood having their sides accurately straight, and making particular angles with each other.

The most useful set-squares are the three shown in Fig. 51.

Fig. 51.



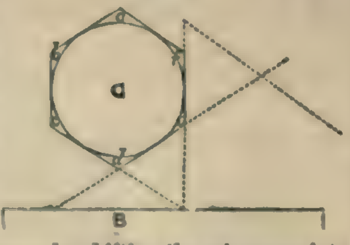
No. 1 is a triangle, having one angle a right angle, or 90°; and the other two each 45°, or half a right angle.

No. 2 has one angle 90°; another 30°, or one-third of a right angle; and the third 60°, or two-thirds of a right angle.

No. 3 has an angle 90°; another 22½°, or one-fourth of a right angle; and the third 67½°, or three-fourths of a right angle.

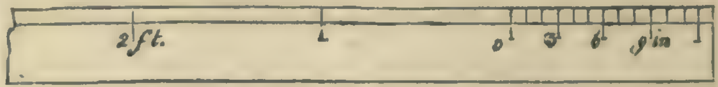
As an example of the use of these set-squares, we will suppose that we have to describe a hexagon round a given circle (Fig. 52). The circle being described round the centre G, the blade B of the ⊤ square being brought into any convenient position, the set-square No. 2 is applied to its edge in the different positions marked by the dotted lines, and the sides *f e*, *e d*, and *d c* of the hexagon drawn, touching the circle; the other sides, *c b*, *b a*, *a f*, being similarly drawn by shifting the set-square into suitable positions, which are not marked in the figure, to avoid confusion.

Fig. 52.



Scales are blades of ivory, boxwood, metal, or cardboard, having straight edges divided into equal parts, which are generally fractions of a foot or inch. In making drawings of machinery it is generally necessary to represent the parts, not of the real sizes to which they are made, but to some known scale, so that all the proportions of the parts are accurately maintained. For instance, had we to make drawings of a steam-engine, we should select some convenient size for the drawings; we might, for example, represent the engine one-eighth of its real size, and should form a scale every inch of which should stand for eight inches of the real work, every foot for eight feet, and every inch and a-half for one foot. This would be called a scale of one-eighth real size, or a scale of an inch and a-half to a foot, because an inch and a-half is the eighth part of a foot. Now, in making the drawing, or measuring from it when made, it would be troublesome and tedious to calculate what should be the size drawn of each dimension of the work; but having first formed a scale, such as Fig. 53, every inch and a half of which represents a foot, one of these feet being divided into twelve inches, we can at once apply it to the drawing, and set off any required dimension in feet and inches.

Fig. 53.



The most convenient scales for mechanical drawings are those of

6 in. to 1 foot, or $\frac{1}{2}$ real size.	1 in. to 1 foot, or $\frac{1}{12}$ real size.
3 „ 1 „ $\frac{1}{4}$ „	$\frac{1}{2}$ „ 1 „ $\frac{1}{24}$ „
2 „ 1 „ $\frac{1}{6}$ „	$\frac{1}{4}$ „ 1 „ $\frac{1}{48}$ „
1½ „ 1 „ $\frac{1}{8}$ „	$\frac{1}{8}$ „ 1 „ $\frac{1}{96}$ „

For architectural drawings, and drawings of large works generally, the following scales will be found convenient, in addition to the above:—

1 in. to 10 ft., or $\frac{1}{120}$ real size.	1 in. to 50 ft., or $\frac{1}{600}$ real size.
1 „ 20 „ $\frac{1}{240}$ „	1 „ 80 „ $\frac{1}{960}$ „
1 „ 40 „ $\frac{1}{480}$ „	1 „ 100 „ $\frac{1}{1200}$ „

For drawings of land-surveys, it is usual to employ chains as units of measurement; and scales are therefore made in terms of them, such as ten chains to one inch, and the like.

In plans of railway or canal works it is often necessary to measure various curves; and as the curves are generally made portions of circles for ease of setting out and execution, circular scales, divided equally round their circumference, are sometimes employed.

Offsets are short scales which act as set-squares as well as scales.

The annexed plate represents some of the most useful instruments required for mechanical drawing.

Fig. 1 is the Protractor, for setting off any desired angle. The instrument is placed on the paper, with the central point C at the point or vertex of the angle, and with its edge A B coinciding with one of the lines which has to contain the angle. If the angle were 40°, for

Mechanical Drawing

See Pages 794-5.

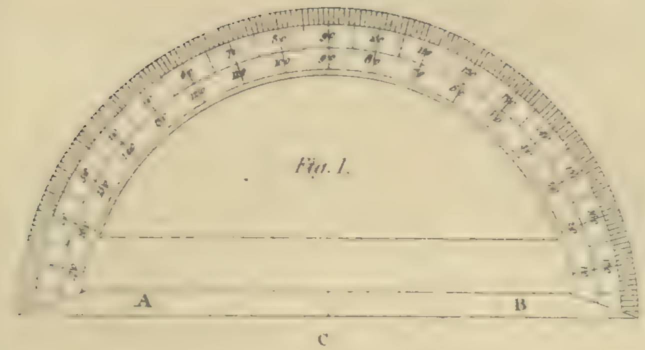


Fig. 1.

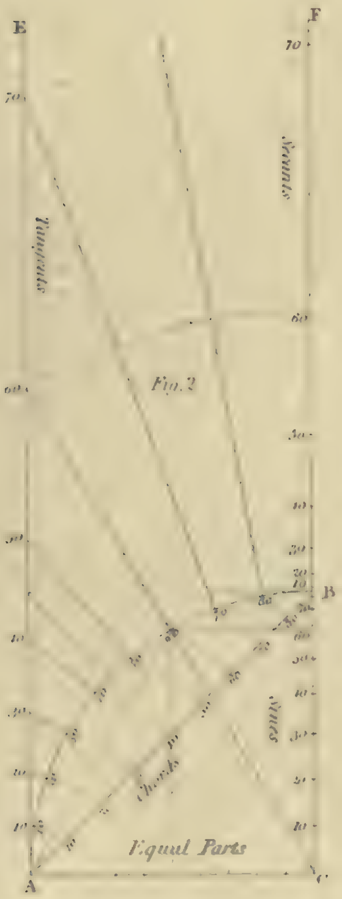


Fig. 2

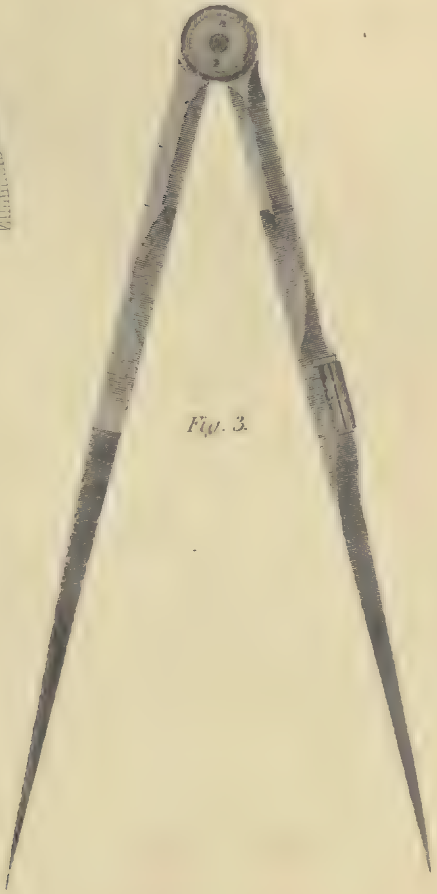


Fig. 3.

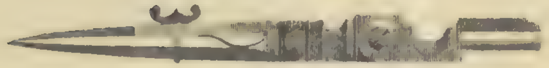
Fig. 4.


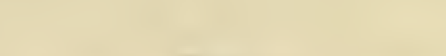
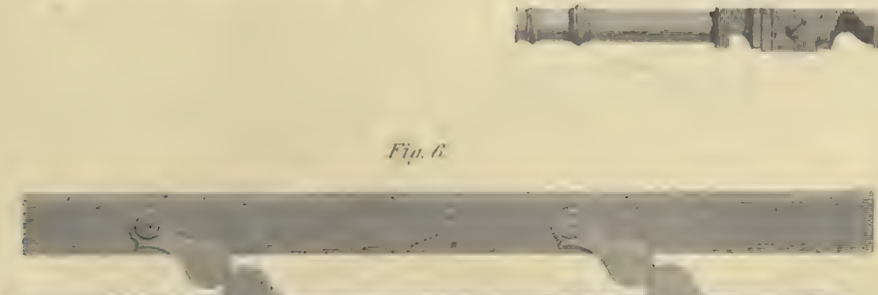
Fig. 5.


Fig. 6.


instance, the point D, corresponding to the 40° division of the Protractor, would be marked on the paper; and the instrument being removed, a straight line drawn through C and D would lie at 40° to the line C B, or at 140° to the line C A. The Protractor, being semicircular, contains two quadrants, each of 90°, making up 180°, and the arcs, or angles, are numbered, from 0° up to 180°, from each of its extremities.

Fig. 2 represents a Trigonometrical Scale, which is easily made, and is very useful in mapping or spherical projection. A quadrant, or arc of 90°, A B, is divided equally into arcs, each of 10°, or into smaller divisions, if required. The chords of those arcs are struck by circles drawn from the centre A, cutting the line A B. The base A C, or horizontal radius, is equally divided. The vertical radius C B is divided by lines, squared across from the divisions of the quadrant, and thus forms a scale of sines. The vertical line A E being cut by lines, or radii, drawn from C, and produced through the divisions of the quadrant, forms a scale of tangents; and the length of those extended radii, being laid down by describing circles from the centre C, upon the vertical line C F, forms a scale of secants. For drawing the meridians and parallels upon the projections of a sphere, either for geographical or for astronomical problems, these divisions are essential.

Fig. 3 represents the ordinary Compasses, or Dividers; and Fig. 4 and 5 show the Pen and Pencil-holders, which are substituted for one of the legs of the Compasses when circles are to be drawn on paper.

Fig. 6 represents the Parallel Ruler, consisting of two straight rulers, connected together by two equal and equally inclined radii. The edges of the two rulers being parallel to one another in any one position, maintain their parallelism when they are opened or closed; and thus, the one edge being laid to any line already drawn, another line can be drawn along the other edge, parallel to the first line at any required distance from it.

For remarks as to the use of those instruments, the reader may be referred to the Chapter on Practical Geometry. It may be farther observed here, that in laying down dimensions on paper, whether full size or to any scale, they should be always pointed off directly from the rule or scale on the drawing. The Compasses should be used only for dividing circular curves, to which straight scales cannot be applied. When they are used for straight dimensions, errors are introduced which do not occur when the draughtsman restricts himself to the scale. First is the chance of error in setting the points of the Compasses to the divisions of the scale. Secondly, the spring of the Compasses themselves. Thirdly, the uncertainty in the pricking of the paper. And lastly, the inaccuracy in drawing through the prick marks; to say nothing of the tear and wear of the scales, arising from the application of the sharp points to them, and the unseemly appearance of the drawing when full of prick marks.

SCALE OF DRAWING.—For the draughtsman employed in devising, on paper, work to be executed in solid materials, we strongly recommend that he should, as much as possible, endeavour to draw all details of their full size. Scale-drawings, especially of large works, are absolutely required to show the combination of all the parts in one machine or engine, full-sized drawings of which would be of most inconvenient size. But, in addition to the scale-drawings of the whole, it is the practice of the best engineers to execute full-sized drawings of details. One great advantage of this is, that the draughtsman sees better on the full size what should be the best forms, dimensions, and combinations of the parts in respect of strength, efficiency, and economy of material and labour; and another advantage consists in the facility with which the workman can read off the drawings, or transfer to the solid materials he labours on, the dimensions and forms marked out: he has only to apply his rule to the drawing and to his work, and make them agree. Farther, as it is customary for workmen to use rules divided into inches, and these again into halves, quarters, eighths, and sixteenths, the draughtsman should make his dimensions such as can be measured in those fractions. We have known numerous workmen who perfectly comprehended the eighth or sixteenth part of an inch, but had not the remotest conception of what was meant by the tenth or the twentieth part. Were the dimensions even figured by tenths, the workman would read it in quarters, eighths, or sixteenths, and would most probably err in reducing it from the one denomination to the other. It is, therefore, preferable, in the first instance, to draw or figure the dimensions in a denomination which the workman will employ; and thus a fruitful source of error will be avoided.

CHAPTER III.

STRENGTH OF MATERIALS.

Contents.—MATERIALS AND STRAINS—TENSION AND INTERNAL PRESSURE—COMPRESSION—TRANSVERSE STRAIN—DEFLECTION AND DISPOSAL OF MATERIALS—RULES FOR CALCULATING—TORSION—SHAFTS—CLIPPING OR SHEARING STRAIN.

MATERIALS AND STRAINS.—The materials with which the practical mechanic has principally to deal are metals, such as steel, wrought-iron, cast-iron, copper, brass and gun-metal, lead, tin, and zinc; timber, such as oak, teak, ash, beech, and other hard woods; fir, pine, cedar, and other soft woods; stones, bricks, mortars, and cements; cordage, straps and bands, and the like. The strains to which these materials may be subjected, may be classed under five heads:—

1. Tension, or a strain applied in the direction of the fibres of the body, so as to pull it asunder, as in the case of ropes, bands, tie-rods, and chains. The strength of a body to resist a strain of this kind, is called its cohesion, or the force with which its parts are held or knitted together.

2. Compression, or a strain applied also in the direction of the fibres so as to crush a body, as in the case of a column. Compressive strain is exactly opposite to tensive strain; and the power of a body to resist it seems to depend chiefly on its elasticity.

3. Transverse strain, or force applied perpendicularly to the fibres so as to break the body across, as in a beam loaded in the middle. In resisting a strain of this kind, the cohesion and elasticity of a body are both exercised.

4. Torsion or twisting, or a force applied at the end of a lever so as to turn one end of the body round its axis, while the other end is fixed, as in a shaft or axle.

5. Clipping or shearing, or a power applied to divide or cut a body across its fibres.

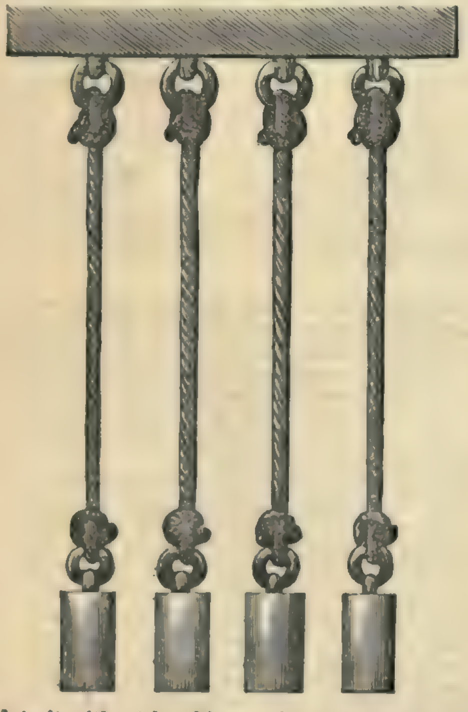
All the possible strains to which materials, whatever be their forms and arrangements, can be subjected, are of one or more of these classes. We therefore propose to consider them separately, and endeavour to apply the results of experiment and observation to their discussion, endeavouring to avoid, as far as possible, mere abstract mathematical investigations. We cannot, however, altogether dispense with mathematical aid, but shall endeavour to place the reasoning as much as possible on such a footing, that a student having a moderate acquaintance with the general principles of mathematics, may have little difficulty in following it. We must premise that the whole subject is as yet in an unsettled condition in some respects, for it has happened that very few writers have discussed it as a whole. Of those who have discussed portions of it, some may be said to have done so, with too little regard to its practical application; while others have erred in the opposite direction, giving mere empirical rules without theoretical reasons for their establishment. We shall endeavour to steer a middle course between these extremes, trying to show, as clearly as possible, the reasons for conclusions that have been arrived at, and quoting from the best authorities the results of experiments in a form that may render them conveniently applicable in practice.

1. TENSION.—We may consider that every body subjected to tension consists of numerous fibres laid side by side, and extending over the whole length of the body. Even if the body be not of a fibrous constitution, we may conceive its particles to be arranged in longitudinal rows, each particle being held to the next by some force which we call attraction of cohesion. A row of particles so held together, exactly corresponds with our notion of a fibre; and we are therefore warranted in treating all bodies as fibrous while we discuss their strength to resist tension.

Let us suppose that we have a rope capable of sustaining a hundredweight, and no more, without breaking. We should call the absolute cohesive strength of that rope one hundredweight, or 112 lbs. The length of the rope by which the weight is suspended has evidently no influence on its strength; for if any one part of its length be capable of sustaining the strain, and if it be uniform, every other part will be equally capable of sustaining it. It is true that were it hanging vertically, additions to its length would increase the weight, stretching its upper portion, because each portion of added length becomes an added weight. But considering the weight of the rope itself as part of the stretching force, we say that the given rope has an absolute cohesive strength of 112 lbs., whatever be its length.

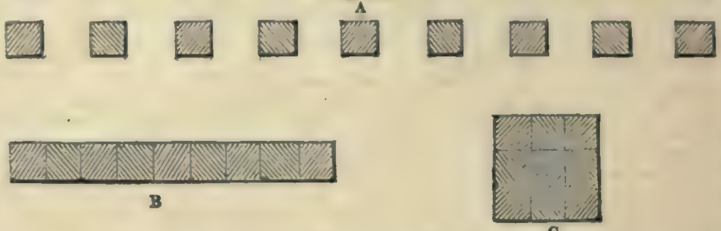
Now, if any number of such ropes were suspended in a row (Fig. 54), each might have a hundredweight attached to it without breaking, and the absolute cohesive strength of all together would be expressed by the number of hundredweights sustained; that is, by the number of ropes, reckoning each as having the strength of a hundredweight. Whether we deal with ropes, however, of hemp, or rods of wood or of iron, or of any other material, if we know the strength of one such rod, and the number of them sustaining equal weights, we know the total weight sustained by them all; that is to say, their combined strength. Let us assume, instead of ropes, that we experiment with square bars of iron measuring an inch each way, or having a square inch of sectional area. Whether a number of these bars, such as nine of them, be suspended in a row, of which A (Fig. 55) is the plan, with intervals between them; or in a close row with no intervals as in B, and forming a bar nine inches wide by one thick; or in a square, C, measuring three inches each way; there can be no difference in their combined strength, or in the total weight carried by the whole number of bars, however they may be arranged.

Fig. 54.



If, then, we know the weight sustained by a bar or rod of any material having a sectional area of one square inch, we can estimate the weight that can be sustained by any other bar of the same material, by calculating the number of square inches in its sectional area, and allowing the known strength for each square inch. For example: let us suppose that we find by experiment the average strength of a bar of wrought-iron, 1 square inch in section, to be 30 tons, or 67,200 lbs.; let us ascertain the strength of a bar 10 inches wide and 10 inches thick. Since the sectional area is 10 in. × 10 in., or 100 square inches, the strength is 30 tons × 100 sq. in., or 3,000 tons; or 67,200 lbs. × 100 sq. in., viz., 6,720,000 lbs. The strength of a round bar 1 inch in diameter would be estimated in the same way; for as the area of a circle 1 inch in diameter is 0·7854 sq. in., the strength of the round bar 1 inch in diameter would be 30 tons × 0·7854 sq. in., about 23½ tons; or 67,200 lbs. × 0·7854 sq. in., about 52,780 lbs.

Fig. 55.



Farther, as the areas of circles are in proportion to the squares of their respective diameters, the strength of a round bar of iron of any diameter is 23½ tons, or 52,780 lbs., multiplied by the square of the diameter in inches. Thus, a round bar of 10 inches in diameter has a strength of 23½ tons × 100 = 2,350 tons; for the area of a circle 10 inches in diameter is 100 times that of a circle 1 inch in diameter. All this mode of calculation manifestly proceeds on the simple principle, that every equal fibre or row of particles is supposed to act equally in resisting strain; and that as the number of such fibres or rows is proportional to the area of section, the strength of different bars or rods is therefore proportional to their areas of section.

So far the theoretical principle of cohesive strength appears very simple, and easily applicable in practice. It now becomes our business to inquire how far practical results agree with the theoretical principle. Many engineers have devoted great care to experiments on this subject; and although the results present considerable differences, yet all of them seem to bear out the principle we have laid down. As an example of the near approach which practical results make to the theoretical law, we may select some experiments made on round copper bars, as shown in the following table. The first column contains the diameters of the bars in inches; the second the sectional areas in square inches, or decimal parts of a square inch; the third contains the weights in tons, and decimal parts of tons, which were required to break the bars; and the fourth column contains the weights in tons per square inch of sectional area required to break or tear the bars asunder. The numbers in the last column are calculated by a simple proportion; thus, taking the bar 1 inch in diameter, the area of which is 0·7854 square inches, and the breaking-weight 17 tons, we say—

Sq. in.		Sq. in.		Tons.		Tons.
0·7854	:	1	: :	17	:	21·645;

the weight that would be required to break the bar were it one inch square.

TABLE I.—*Experiments on Cohesive Strength of Cylindrical Copper Bars.*

Diameter of bar: inches.	Area: sq. inches.	Breaking-weight: tons.	Breaking-weight reduced to 1 sq. inch: tons.
1⅛	0·9940	22	22·133
1	0 7854	17	21·645
⅞	0·6013	12·8	21·287
¾	0·4417	9	20·376
			Mean 21·36 tons.

It will be seen that, although the sectional areas of the bars differ very considerably, the breaking-weights, reduced to one square inch of area, do not vary greatly from their mean value.

Table II. contains the results of experiments on best hempen bower cables used in the navy. The first column contains the circumferences, or girths, of the cables in inches (the dimensions of cables being generally stated in terms of their girths); the second column contains their sectional areas in square inches; the third expresses the number of threads in each; the fourth column marks the actual breaking-weights; the fifth the breaking-weights per square inch of sectional area; and the sixth the breaking-weights per thread.

TABLE II.—*Experiments on the Cohesive Strengths of Best Hempen Bower Cables.*

Circ.: inchs.	Sect. area: sq. inchs.	No. of threads.	Breaking-weight: tons.	Break. wght. per sq. inch: tons.	Break. wght per thread: lbs.
23	42	2736	114	2·714	93⅓
18	26	1656	63	2 423	85¼
14½	17	1080	40	2·353	83
				Mean 2·5 nearly	87

The strengths in these cases also are very nearly as the sectional areas or numbers of threads. We shall finally quote the results of some experiments on the strength of wrought-iron bars of various qualities, as marked in Table III., where the breaking-weights per square inch are given as calculated from the experiments. The first column contains the description of bar, the second contains the dimensions, the third gives the amounts in inches which each foot of the bars stretched, and the fourth gives the breaking-weights per square inch of sectional area.

TABLE III.—*Experiments on the Cohesive Strengths and Extension of Wrought Iron Bars.*

Description and form.	Dimension before extension: inches.	Extension per foot: inches.	Breaking-weight per sq. inch of section: tons.
South Wales, cylindrcl.	1·38 diam.	1·81	29·3
Ditto, ditto.	1·50 „	1·86	29·8
Stafford, square	0 75 side.	1·22	27·2
Ditto, ditto	1·06 „	1·24	27·5
Welsh, square	1·00 „	2·38	29·0
Scrap. square	1·00 „	2·50	29·0
Common, cylindrical..	2·00 diam.	1·50	31·8
Stafford, square	1·00 side.	1·10	31·0
			Mean 29·3

By numerous experiments, such as those quoted, data have been determined from which we can readily calculate the dimensions of a bar of any material required to carry a given weight, or conversely, the weight which a given bar of material will carry. These data have been determined by the actual breakage of the bars tested; but in practice, where durability and adequate strength to meet contingencies of strain are required, it is necessary to give the materials we employ, considerably greater dimensions than such as would merely preserve them from breakage, or to load them with considerably less weights than those which would tear them asunder. Farther, it is found in practice, that materials subjected to considerable strains for long periods of time gradually lose their tenacity; and this degradation of quality must be provided against when permanence and stability are required.

In the case of metal bars, it is advisable not to load them beyond one-third of their actual breaking strain; or, in other words, to provide them of three times the strength sufficient to resist tearing asunder. In the case of ropes, timber, and such other materials as are of less uniform consistency than metals generally are, or are more liable to degradation, it is advisable to provide at least four times the breaking strength.

Table IV. contains data for the strength of a number of materials on which experiments have been made; and the numbers given may be safely used in calculation, as they are reduced to one-third or one-fourth of the breaking strength.

TABLE IV.—*Data for Calculating the Cohesive Strength of Bars of different Materials, determined from the Averages of numerous experiments. The numbers are given roundly, as no material error will arise in practice from their use.*

Col. 1.	Col. 2.			Col. 3.			Col. 4.	Col. 5.
Name of Material.	Strength per square inch of sectional area.			Strength per circular inch of sectional area.			Area in sq. inches to bear 1 ton.	Area in circ. inch. to bear 1 ton.
	Tons.		lbs.	Tons.		lbs.	sq. ins.	circ. ins.
Ash	2·2	or	5,000	1·73	or	3,900	0·45	0·57
Beech	1·5	„	3,400	1·18	„	2,700	0·66	0·84
Box..........................	2·7	„	6,000	2·12	„	4,700	0 37	0·47
Fir	1·6	„	3,600	1·26	„	2,800	0 62	0·79
Mahogany................	1·1	„	2,400	0 86	„	1,900	0·93	1·18
Oak	1·4	„	3,100	1·10	„	2,400	0·72	0 92
Pear-tree	1·3	„	3,000	1·02	„	2,360	0·75	0·96
Teak	2 0	„	4,500	1·71	„	3,500	0·50	0·64
Brass	2·7	„	6,000	2·12	„	4,700	0·373	0·476
Copper (cast)	2·8	„	6,300	2·20	„	4,900	0·356	0·454
Copper (wrought) ..	5.0	„	11,200	3·93	„	8,800	0 200	0·255
Iron (cast)	2·7	„	6,000	2 12	„	4,700	0·373	0·476
Iron (wrought), English	8·3	„	18,600	6·52	„	14,600	0·120	0·153
Ditto (Swedish)	10·7	„	24,000	8·41	„	19,000	0·093	0·118
Lead (cast............	0·3	„	650	0·24	„	500	3·446	4·390
Steel (blister or cast)	20·0	„	45,000	15·7	„	35,000	0 050	0·064
Steel (shear)..........	18.5	„	41,500	14·54	„	32.500	0·054	0·069
Tin (cast)	0·7	„	1,600	0·55	„	1,260	1·400	1·783

The strengths are given in square inches, and also in circular inches for convenience of calculation; and the following are the rules for calculating the strength of a given bar, or the dimensions required to bear a given load.

I. Given, the dimensions of a bar of any material to find its cohesive strength, or the constant load it will safely bear in the direction of its length.

1. When the bar is square.

Rule.—Multiply the number of inches in the side by itself: that is, square the number, and then multiply by the number opposite the given material in column 2 of Table IV.

Example 1.—Required the cohesive strength of a square bar of English wrought-iron of which each side measures 2¼ inches.

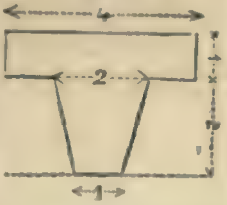
2¼ expressed decimally .	2·25
Multiplied by itself .	2·25
	1125
	450
	450
	5·0625
Multiply by tons in table .	8·3
	151875
	405000
	42·01875

Neglecting the decimal fraction, we find that we may safely load the bar with a weight of 42 tons.

2. When the bar is oblong, or of some other form having straight sides.

Rule.—Multiply the width by the thickness, or generally, find the sectional area in square inches, and multiply by the number in column 2 of the table.

Fig. 56.



Example 2.—Required the cohesive strength of a bar of wrought copper of the form and dimensions given in Fig. 56.

The upper part of the figure is an oblong or rectangle, of which the width is 4 inches and the thickness is 1 inch; its area is therefore 4 × 1, or 4 square inches.

The lower part of the figure is a trapezoid, of which the thickness at one edge is 2 inches, and at the other 1; the mean thickness is therefore 1½ inch, and the width being 2 inches, its area is 1½ × 2, or 3 square inches. The total area of the section is therefore 7 square inches: and 7 multiplied by 5 tons, the strength of wrought copper in column 2 of table, gives the strength of the bar 35 tons.

3. When the bar is cylindrical.

Rule.—Multiply the diameter in inches by itself, and by the number in column 3.

Example 3.—Required the cohesive strength of a fir pole 4 inches in diameter. 4 × 4 × 1·26 give 20·16, say 20 tons.

4. When the bar is of elliptical section.

Rule.—Multiply the greater diameter by the less, and by the number in column 3.

Example 4.—Required the cohesive strength of an elliptical lead bar, of which the greater diameter is 2 inches and the less 1½ inch: 2 × 1½ × 500 give 1500 lbs.

5. When the bar is cylindrical and hollow like a pipe.

Rule.—Multiply the outer diameter by itself, and also the inner by itself; subtract the one product from the other, and multiply the remainder by the number in column 3.

Example 5.—Required the cohesive strength of a brass tube 2 inches diameter outside, and 1½ diameter inside.

2 × 2 = 4
1½ × 1½ = 2¼

1¾ × 2·12 = 3·7 tons.

6. As a general rule, whatever be the form of section, find the area either in square inches or circular inches, and use as a multiplier the number in column 2 in the one case, and that in column 3 in the other. Or, find the strengths due to the several parts of the section, and add them together for the total strength.

Example 5.—Required the cohesive strength of a cast-iron bar of the form and dimensions given in Fig. 57.

Taking first the hollow semicircular part,

Outer dia. 5 in. × 5 = 25
Inner dia. 3 in. × 3 = 9

Take ½ for semicir.) 16

8 circ. in. × 2·12 tns. (col. 3) = 16·96 tns.

Two upright sides each
2 in. by 1 in. have an
area 2 × 1 × 2 . = 4 sq. ins.
Two lower flanges each
2½ in. by 1 in. have an
area 2 × 2½ × 1 . = 5

Square inches 9 × 2·7 tons (col. 2) = 24·3 tns.

Total cohesive strength 41·26 tns.
say 41 tons.

II. The converse operations are for calculating the dimensions of a bar required to carry a given load.

7. When the bar is square or oblong.

Rule.—Multiply the number in column 4, by the given number of tons, and the product gives the number of square inches of sectional area.

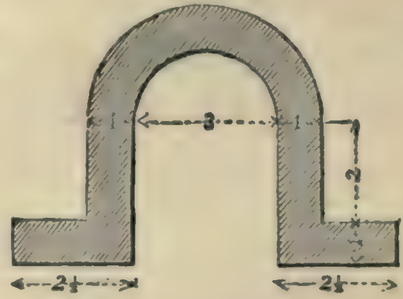
Example 7.—Required the area of a bar of wrought-iron capable of sustaining 10 tons.

Number from col. 4 . . . 0·120
Multiply by 10

1·2 square inch.

This sectional area may be attained by making the bar square about $1\frac{1}{10}$ inch of a side; or making it oblong, such as 2 inches by ⅝ inch thick, or any dimensions such that their product is at least 1·2 square inch.

Fig. 57.



8. When the bar is cylindrical.

Rule.—Multiply the number in col. 5 by the weight in tons, and take the square root of the product; it will be the diameter in inches.

Example 8.—Required the diameter of a cylindrical bar of wrought copper to carry 35 tons.

Number from col. 5 . . . 0·255
Multiply by 35

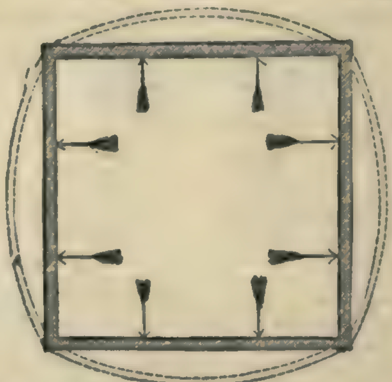
9· nearly.

The square root of 9 is 3 inches, the diameter of the bar.

9. For bars having sections of various forms and proportions, the calculations must be trials to a certain extent. The number of square inches required is found as in Example 7, and these may be disposed in any suitable form that may be required.

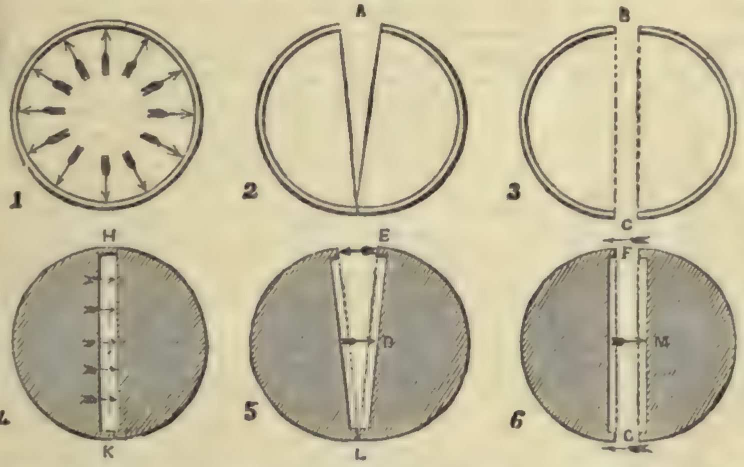
Before we leave the subject of tensive strength, we will discuss a case which frequently occurs in practice, and where the cohesive force of materials is employed in a vessel to resist a bursting or exploding strain arising from the pressure of a fluid within it. Were a boiler, pipe, or other vessel, intended to contain a fluid exerting considerable pressure, made of any form except circular in section, the internal pressure would change the form of section. Thus, suppose the section were of a square form, as in Fig. 58, the internal pressure acting equally on every part of the casing, as marked by the arrows, would bulge the flat sides, as indicated by the dotted lines; and, finally, were the material sufficiently pliable, would extend it into a circular form. After having attained a circular section, no farther change of form would be effected, and the pressure of the internal fluid

Fig. 58.



might be increased until it attained sufficient magnitude to burst or rend asunder the material. Vessels destined to sustain considerable internal pressure, are generally made circular in section; and a few simple considerations will enable us to ascertain the amount of strain, which the pressure in such vessels throws upon the material of the casing, and hence to compute the strength of material that should be employed in their construction in order that they may sustain a given pressure.

Fig. 59.



Let us suppose that a steam-boiler is of cylindrical form as in section 1 (Fig. 59), having pressure exerted on it equally, on every part of its surface, as indicated by the arrows. One effect of this pressure, if it exceeded the strength of the material containing it, would be to rend it asunder at some point A, section 2; or it might rend it at two opposite points, B and C, section 3, so as to force one-half of the casing away from the other. Now, if the material were of perfectly uniform consistency and strength throughout, there is no reason why one point A, or any two points B and C, should be selected for breakage more than any other points; but as, in practice, materials are never perfectly uniform, so, in theory, we may suppose the casing to be somewhat weaker at A, or at B and C, than elsewhere, and trace the influence of the pressure to effect a breakage at such points. Were we to suppose the whole of the vessel to be filled up with solid matter, except a thin film of fluid in the middle, as shown in section 4, exerting a pressure on every point of the flat side of the solid portion exposed to it, we should involve no change in the conditions, for the two points of the circumference, H and K, terminating the film of fluid, would have to resist exactly the same bursting-strain as if the rest of the casing were filled with fluid instead of solid matter. Farther, as this section would apply at any part of the length of the cylinder, we may take a belt or portion of its length, one inch in breadth, and trace the effect on it—an effect which would be repeated equally on every such belt one inch wide throughout the whole length of the cylinder, if the material were homogeneous throughout. Knowing the pressure which the internal fluid exerts on every square inch of the casing, we see, by section 4, that the bursting pressure on a belt of casing one inch wide, is to be measured by the pressure on a surface represented by the line H K—that is, on a surface having for length the diameter, and one inch in width. If this pressure burst the casing at one point E, section 5, so as to cause one-half to turn round the opposite point L as a fulcrum or centre; then, by the well-known principle of the lever, that a uniform pressure, acting at every point of the arm of a lever, has the same effect to turn it round its fulcrum, as if it were all collected into one force acting at the middle point of the arm, we see that the bursting pressure is equivalent to a force acting at D (as marked by the arrow), while the cohesive force of the casing acts at E, double the distance from the fulcrum L. Hence we conclude that the bursting force, as resisted at one point of the circumference, is half the pressure on the diameter, or the pressure on half the diameter, or on the radius. Again, if the vessel open at two points F and G, section 6, we see that the whole bursting force, marked by the central arrow, being resisted by two equal forces at F and G, each of those forces need be only half the bursting force; so that, in this case, we find that the effect to open the circumference is equal to half the pressure on the diameter as before.

To apply these considerations in practice, let us suppose that a boiler of English wrought-iron, 6 feet in diameter, has to sustain an internal pressure of 100 lbs. per square inch, and ascertain the necessary thickness of the plate to resist this force. Taking a belt of it 1 inch wide,

Half the diameter, 3 feet or	36 ins.
With a width of	1 in.
	—
Has a surface of	36 sq. ins.
Multiplying by pressure per sq. in. .	100 lbs.
	——
The pressure on half the diameter is .	3600 lbs.

which is the bursting strain on any belt of the circumference 1 inch wide.

By Table IV., English wrought-iron having 1 sq. in. area of section, sustains 18,600 lbs.; and by a simple proportion, 18,600 lbs. : 3,600 lbs. : : 1 sq. in : 0·2 sq. in. nearly. We find therefore, that the sectional area of the belt of circumference must be 0·2 sq. in.; or that, as it is 1 inch wide, its thickness must be 0·2 in., or $\frac{2}{10}$ or $\frac{1}{5}$ of an inch. In practice, boilers are made of numerous plates riveted together; and as the material is cut away by the rivet-holes, it is weaker at these places than elsewhere. It would, therefore, be advisable to make the strength of the plate at least double that calculated, which would give a thickness of about $\frac{2}{5}$ of an inch.

In the case of hydraulic presses which are made of cast-iron, and have to sustain enormous pressures, it is necessary to make the material of great thickness. There is, however, a limit to the pressure which such cylinders can sustain; for it is found that after a certain thickness of iron has been attained, the actual substance of the iron becomes compressed, so that the inner surface is considerably extended, while the outer surface sustains scarcely any bursting force. Additional thickness of metal does not therefore contribute proportional increase of strength. Indeed, the practical difficulty of casting thick masses of iron sound and solid in texture, renders it almost impossible to construct hydraulic presses of any very great force. It is doubtful whether, in any case, it is advisable to exceed a thickness of 6 or 7 inches when cast-iron is the material employed. The same remarks apply to a certain extent in the case of ordnance: but in making large guns or mortars, a considerable advantage results from the process of manufacture. The gun is cast solid, and afterwards the heart is bored out. Most of the unsound and spongy portions of the metal are thus removed, and the shell left is much sounder than it would be, were it cast hollow, as hydraulic presses generally are.

Vessels for containing fluids exerting pressure, are likewise subject to a bursting strain in the direction of their length, which is resisted by the cohesion of the casing. Taking the case of the boiler 6 feet in diam., and 0·2 in. thick, we may easily calculate what pressure it would bear before bursting in the direction of its length. The whole boiler may be taken as a hollow or tubular bar of wrought-iron, capable of sustaining a certain weight hung to it, equivalent to the pressure on one of its circular ends, tending to force or blow it to a distance from the other end.

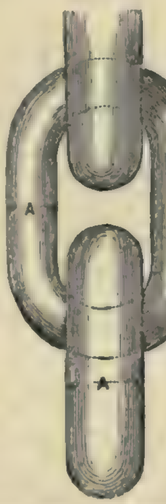
Outer diameter . . . 72·4 ins. × 72·4 =	5242 nearly
Inner diameter . . . 72·0 ins. × 72·0 =	5184
Area of ring in circ. ins.	58
Cohesive strength of 1 circ. in. wrought-iron	14600
Total strength of circumference	846800 lbs.

As the area of the end is 4,072 sq. ins., the boiler would bear a pressure of more than 200 lbs. per square inch, before parting lengthwise. Indeed, it will be found that when uniform thin material is employed, a cylindrical vessel subjected to internal fluid-pressure is almost exactly twice as strong lengthwise as it is circumferentially.[*] When the thickness is considerable, this proportion does not hold good.

It is a singular fact, that many metals, when drawn into wire, become stronger in respect of tensive strain. This may probably be owing to the wire-drawing process causing the particles to arrange themselves in continuous fibres. From numerous experiments made on iron wire less than ¼ inch in diameter, it appears that the strength of wire is about one-fourth, or 25 per cent., greater than that of iron bar; so that while the cohesive strength of wrought-iron bars may be taken at 8 tons per square inch of sectional area, that of iron wire may be taken at 10 tons; or taking bar at 6½ tons per circular inch, wire will have a strength of 8 tons per circular inch.

The following are the rules for computing the strengths of hempen ropes and chains. The dimensions of the ropes are stated in terms of girth or circumference, and those of the chains in terms of the diameters of the metal constituting the links. Thus the chain which has a diameter of ¾ inch at A (Fig. 60), is called a ¾-inch chain.

Fig. 60.



We have formed the rules so as to compute the strains at a lower rate than they are generally stated. It may be true that a new chain or rope will bear a much greater strain than that which we have allowed to it; but the chain loses strength by use—not so much from wear as from the particles of iron assuming a crystalline instead of a fibrous texture; and the rope loses strength by wear as well as by the gradual decay of its fibres. It is preferable, therefore, to err rather on the safe side, especially when it is considered that great damage may often result from the breaking of a chain or rope, and, what is still more carefully to be guarded against, serious injury to life and limb.

1. To find the diameter of a chain to carry a given weight.

Rule.—Multiply the weight (tons) to be carried by 30; the product will be the square of the diameter of the chain reckoned in 16ths of an inch.

Example.—Required the diameter of a chain to carry 10 tons. 10 × 30 = 300, which is near 324, the square of 18; therefore the diameter of the chain must be $\frac{18}{16}$ inch, or 1⅛ inch.

2. To find the weight which a chain of given diameter will carry.

Rule.—Divide the square of the diameter (reckoned in 16ths of an inch) by 30; the quotient will be the number of tons carried.

Example.—Required the strength of an inch chain: 1 inch is $\frac{16}{16}$ of an inch, and 16 × 16 = 256; dividing by 30 we have about 8½ tons.

3. To find the circumference of a rope to carry a given weight.

Rule.—Multiply the weight (tons) by 11; the product will be the square of the circumference in inches.

Example.—Required the size of a rope to carry 18 tons: 18 × 11 = 198, which is near 196 the square of 14 inches, the circumference required.

4. To find the weight which a given rope will carry.

Rule.—Divide the square of the circumference in inches by 11; the quotient will be the weight carried in tons.

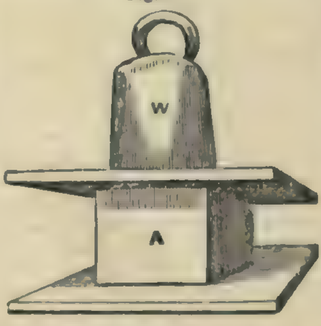
Example.—Required the strength of a 4-inch rope: 4 × 4 = 16, and 16 divided by 11 gives 1½ tons.

2. COMPRESSION.—The strength of materials to resist compressive strains, appears to depend chiefly upon some forces among their particles acting in the opposite direction to cohesive attraction. Indeed, the particles of every solid body appear to be ranged in such positions with respect to one another, and so balanced by cohesive attraction keeping them together on the one side, and by some repulsion resisting their nearer approach on the other, that considerable force is generally required to alter their relative distances. The intensity with which these forces act in any body, appears to be measured by their hardness, or their strength to resist external forces. But while cohesive attraction seems to follow a simple and regular law in any material—the amount of attraction or the strength to resist a tensive strain being proportional to the sectional area, or, in other words, to the number of particles upon which the attraction is exerted—experiments have as yet shown no very regular law as to strength of materials to resist compression. Fortunately, in practice, the other strains to which materials are subjected are generally so much more likely to affect them than mere compressive strain, that when we make the parts of our work sufficiently strong to resist the former, we are tolerably safe in respect of the latter.

Some valuable experiments have been made upon the strengths of building materials—wood and iron—to resist compression; and attempts to deduce laws from these experiments have not been wanting. We are not aware, however, that any satisfactory results have attended those efforts; and we fear that any rules founded on our present scanty information in respect of this subject, would tend rather to mislead than to assist the practical mechanic.

If we suppose that a cubical piece of any material A (Fig. 61) be loaded by a certain weight W, which it is just able to bear without being crushed, we may readily imagine that a number of such cubes—say nine, for instance—would bear nine times the weight which the one bears; and if these cubes were arranged either in a row (Fig. 55) at intervals, or close together, or in a square, they should still all together bear nine times the weight that one of them can bear. When the material we deal with is of uniform consistency, we are therefore warranted in reckoning, that its strength to resist compression, precisely as that to resist tension, is proportional to the area over which the compressive force is spread. Thus, a block of stone 4 feet long and 3 feet broad, should be able to sustain six times the compressing force which a block 2 feet long and 1 foot broad can sustain; because 4 × 3 = 12, the area of the one, is six times 2 × 1 = 2, the area of the other.

Fig. 61.



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[*] Let r = radius, t = thickness, a = cohesive strength of 1 square inch of casing, p = pressure to burst at circumference, and P = pressure to divide the casing lengthwise.

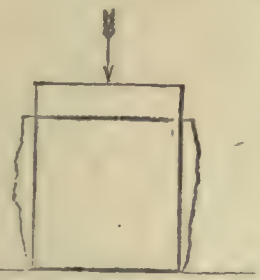
$$pr = at \therefore p = \frac{at}{r}$$

When t is small compared with r, the sectional area of casing is nearly $2\pi rt$; and as πr^2 = area of end,

$$\pi r^2 P = 2\pi rat \therefore P = 2\frac{at}{r} = 2p.$$

But if we endeavour to trace the practical effect of crushing any material, we shall find it difficult to establish the truth of this simple law, without making great allowances for irregularities in the consistency of the material, and in the application of the compressive force. Some materials of a soft consistence, such as clay or lead, on being subjected to vertical compression, merely spread out sideways. Thus, a piece of lead compressed by a heavy weight bulges in width, and thickness, to the form marked by the dotted lines (Fig. 62). Other materials of a fibrous texture, such as wood, become splintered by compression, owing to some of the fibres being driven or wedged into the interstices between others, and thus bursting them asunder. Hard and brittle materials, such as cast-iron, glass, stones, or bricks, are riven and splintered in most irregular and unexpected ways, some portions being actually pulverised in the process, or cracking away and occasionally flying off with considerable force from the rest.

Fig. 62.



These results, however, occur when the compressive strain is carried beyond what the material is capable of sustaining without disintegration; for all practical purposes, where we make the strength more than adequate to meet the load, we may very safely follow the simple law, and reckon the strength to be very nearly as the area of section at the weakest part. This, however, only holds good when we pay no regard to the height of the mass pressed upon. As we increase the height we must increase the strength, for several reasons, which we shall endeavour to illustrate. Suppose we take a cylindrical piece of cast-iron one inch in diameter and six inches high, and placing it upright, load it on the end until it will bear no more. As we place more and more weight upon it, the particles become pressed closer and closer together, and the mass yields or subsides a little in the direction of its length. After being compressed to a certain extent, the particles cannot get into closer contact; but some of them must insinuate or wedge themselves into the interstices between others, and thus the mass will tend to bulge sideways; or, being of a hard unpliable consistency, the sides will become fractured, and burst away. The mass thus weakened will finally all crumble under the pressure. Now let us take a cylindrical piece of the same material, and of the same diameter; but, instead of being six inches high, let it be merely a thin disc one-eighth or one-sixteenth of an inch thick. Here, although the area of section pressed upon be the same as before, yet the height being so much less, the number of particles that can be forced to one side or the other is much less, the number of interstices to receive pressed particles is less, the amount of bulging is diminished, and the whole resistance is enormously greater. As an example of the effect of increased height in diminishing strength to resist vertical pressure, we may quote some experiments made on pieces of cast-iron, all having square bases, a quarter of an inch each way, and various heights; whence it appears that as the height was increased, the strength to resist compression was diminished, so much, that the force to crush the piece $\frac{8}{8}$ inch, or 1 inch high, was little more than two-thirds of that required to crush the piece $\frac{3}{8}$ inch high.

Height. in.	Crushing Force. lbs.
$\frac{3}{8}$	9006
$\frac{4}{8}$	8845
$\frac{6}{8}$	8362
$\frac{7}{8}$	6430
$\frac{8}{8}$	6321

Farther, when the height is considerable, as in the case of columns, a slight want of uniformity in the material, or a slight want of equality in the distribution of the load, will cause one side to be affected more than the other, and thus produce flexure in the case of pliable materials, such as wood or wrought iron, or oblique separation in the case of stone. An extreme load pressing on a column just tottering on the verge of ruin, will certainly find out the weakest place, and there begin the demolition; and having once begun it, will very rapidly complete it. Thus, a knot in a wooden post, or a vein in a marble pillar, may become important elements in determining the mode of their fracture, and the amount of compression required to effect it. It appears, from some experiments made with wooden columns, that when the height is more than eight times the diameter, there is considerable risk of the column bending under an excessive load. With timber of uniform strength, and a load well balanced upon the column, when its height does not exceed this proportion, the wood will generally splinter under a crushing force. In columns of stone or brickwork, destined to sustain a heavy load, the ratio of height to diameter may be increased considerably above that of eight times, without the danger of their breaking by a sloping fracture. In such cases, however, the eye seems instinctively to judge of proportion; for a very tall slender column conveys a notion of instability, and is consequently wanting in that grace which depends, mainly, on the fitness of the object for the work it has to perform.

Cast-iron is found to be capable of supporting a very considerable vertical load without flexure or crushing; and columns of this material are, therefore, with safety made of tall and slender proportions. Even here, however, there is a limit beyond which the eye becomes dissatisfied; and accordingly, when the column is required to be of considerable height, it is generally made of greater diameter, the inside being hollowed out to save weight and material, without sensibly affecting the strength. Where, from inequalities of the load, there is any risk of unequal pressure on the summit of a cast-iron column, and of consequent tendency to bend, the material used in constructing the column is much more advantageously applied in the shell of a hollow column, or in the ribs of one, the section of which is a cross, than in one solid. We only advert to this now, as we shall have an opportunity of recurring to it when we discuss the question of transverse strain.

Some very valuable experiments have been made upon the crushing of building materials, such as stones and bricks. The compressive forces in most of them have been applied to cubical pieces of 1 inch or 2 inches. In some, where the pieces had a square base 1 inch broad, and a height of 2 inches, the force required to crush, has been found less than two-thirds of that required for the pieces only 1 inch in height. The increase of height, therefore, considerably diminishes the strength to resist compressive force. In connection with this subject, it may be interesting to inquire how high a brick building might be carried without becoming crushed by its own weight. We may suppose a massive brick pier or column divided into numerous separate columns, each $1\frac{1}{2}$ inch square in section, and each supported along its whole height by those around it, against lateral pressure or flexure. The lowest part of each of these columns would thus have to support the vertical weight of all the column above it; and if that weight exceeded the load required to crush a brick cube of $1\frac{1}{2}$ inch, the lowest part of the column would of course give way. Now, it has been found, by experiment, that the load necessary to crush a $1\frac{1}{2}$ inch cube of brick, varies from 1,200 lbs. up to 3,000 lbs., according to the different qualities of the brick. It would perhaps be safest to take the lower number, 1,200 lbs. The weight of 1 foot of brickwork having a $1\frac{1}{2}$ inch square base, is about 2 lbs.; therefore it would require a height of 600 feet to crush the base. An architect, wishing to secure the permanence of his building, would certainly not venture to raise it one-third of this height without having recourse to the ordinary expedient of spreading the base or foundation by extensive footings, so as to diffuse the crushing pressure over a much larger surface of material.

Table V. contains the average results of experiments made on the resistance of various materials to crushing force. We would, however, caution the mechanic against placing too much reliance on these results, as we think the number of experiments made, scarcely warrants their establishment as practical data.

TABLE V.—*Resisting Power of Materials to Crushing Force.*

Name of Material.	Dimension of cubical piece.	Crushing pressure.
Elm	1 inch	1300 lbs.
American pine	,,	1600 ,,
White deal	,,	1900 ,,
English oak	,,	3800 ,,
African oak	,,	5000 ,,
Chalk	1½ inch	1100 ,,
Soft brick	,,	1200 ,,
Red brick	,,	1800 ,,
Hard-burnt brick	,,	3000 ,,
Fire-brick	,,	3800 ,,
Grindstone grit	,,	8000 ,,
Portland stone	,,	10300 ,,
York paving	,,	12800 ,,
White marble	,,	13600 ,,
Cornish granite	,,	14300 ,,
Compact limestone	,,	17300 ,,
Peterhead granite (red)	,,	18600 ,,
Purbeck granite (red)	,,	20600 ,,
Hard freestone	,,	21200 ,,
White Italian marble	,,	21800 ,,
Aberdeen granite (blue)	,,	24500 ,,
Cast lead	¼ inch	480 ,,
Cast tin	,,	960 ,,
Wrought copper	,,	6400 ,,
Cast copper	,,	7300 ,,
Wrought iron	,,	8000 ,,
Brass	,,	10000 ,,
Cast iron	,,	10000 ,,

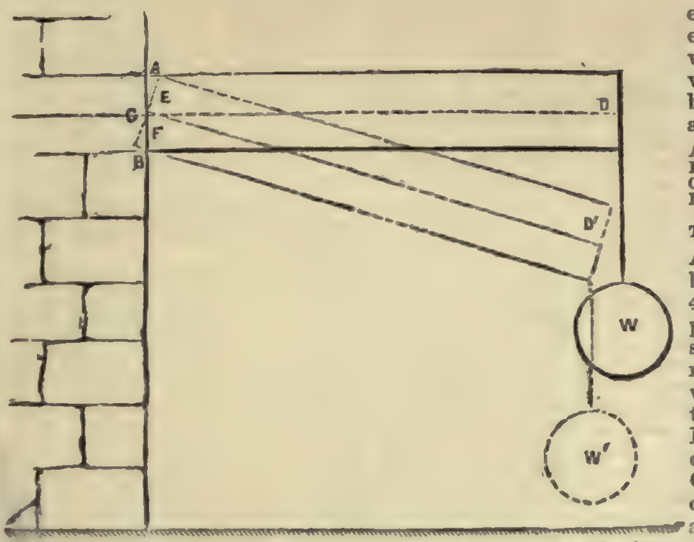
3. TRANSVERSE STRAIN.—The effect of a load on the end of a beam projecting from a wall, or on the middle of a beam supported at both ends, is to throw a transverse strain on the beam; and if the load be excessive, to break the beam transversely, or across its fibres. Of all questions respecting the strength of materials, this is certainly the most important. In architectural structures, the stability of roofs, floors, and walls supported on beams or girders; in civil engineering, the safety of girder-bridges and rails; in mechanical engineering, the strength of beams, levers, framing, and the like—all depend mainly upon the correct solution of questions relating to transverse strain. It is to this branch of the subject, therefore, that the consideration of engineers has been chiefly directed; and a vast number of experiments have been made with a view to ascertain practical data, from which the transverse strength of all useful materials may be calculated.

The theoretical investigation of transverse strain is by no means a difficult one; but the application of data in computation involves some very complicated questions, which the practical mechanic generally solves more by the eye, as he sees proportions in his drawing or model, than by any very accurate calculation. Such a mode of meeting the difficulty implies experience and observation; and without these, we fear no amount of theoretical knowledge, and no expertness in calculation, will serve for determining the strength of any structure. In machinery, especially, the strains to which parts may be subjected are so various in magnitude, owing to their respective movements, and the local qualities of materials differ so greatly, that the mere calculated strain and strength offer very little aid in guiding to the most suitable proportions. When the architect determines on the dimensions of an iron girder destined to carry a heavy wall, he can calculate, with tolerable precision, the weight to be supported, and the strength of girder required to carry it without danger. The girder is placed, the wall is built, and twenty years after, the load is the same, and the strength of the girder but little diminished.

But when the mechanic makes a pattern of a beam for a steam-engine, although he may readily calculate the strain which the pressure of steam on the piston throws upon the beam while the engine works steadily, he has little notion of the sudden though transient strain which may be thrown upon it by the sudden stoppage of some part of the machinery, or the occurrence of some slight obstacle to the movement. It not unfrequently occurs that a little water in the cylinder of a steam-engine, causes the fracture of some of the strongest parts. The machinery being all in motion, the piston descends upon a film of water which has no ready means of escape, and which being almost totally incompressible, offers as determined an obstacle as solid iron would do. In such a case, the momentum of all the moving parts must be suddenly destroyed; some of the beams, rods, or levers, through which the piston is connected with the rest of the engine, must give way; or the cylinder itself, which holds the water, must yield to the strain, and break under it. That we may form some idea of what damage such a strain as we have described may effect, we have only to reckon that in a large steam-engine there are 15 or 20 tons of iron, moving probably at an average velocity of 100 feet per minute, to be suddenly arrested. In destroying the momentum, as much force is expended as would be measured by the blow of a 68 lb. cannon-ball striking its mark at a very near range. Nor do these sudden and unexpected strains constitute the only difficulty under which the mechanic labours when he computes the strength of his work. He cannot always depend upon the internal soundness of the material with which he deals. Cast-iron is especially treacherous in this respect; and it often happens that a casting, externally sound, has some sponginess or air-bubbles under the skin, which are discovered only in the event of fracture. It is, therefore, his business not only to contrive devices for regulating the movements of his machinery, and for affording relief in cases of undue pressure, and to use every precaution against unsoundness in his materials; but also to provide such strength as shall meet the contingencies which a slight derangement may frequently bring about. It may be asked then, why, if the mechanic have to apply such excess of strength to meet contingencies, he should take any trouble in calculating or ascertaining practical data from which he may compute? The answer is plain. While he makes every part strong to excess, yet he has to maintain a proportionate strength in all. To make one part of a beam strong enough to sustain ten times its usual strain might be very proper; but to make another part of it capable of sustaining twenty times its load would be absurd, for then either the weaker part is only half as strong as it should be, or the stronger is twice as strong as required. The mechanic, then, must have a clear conception of how strains affect materials of different forms, and how far the change of one dimension or another may affect the strength, before he can venture to design or execute a machine justly proportioned in all its parts, and sufficiently strong throughout to meet the contingencies of its action, without undue waste of material and labour in its construction.

The most simple case of transverse strain is that to which a beam projecting from a wall is subjected, when a weight is suspended from its outer end. It is clear that the longer the beam, the greater the strain; and that if the beam give way anywhere, presuming it to be of uniform strength throughout, it will be at a point close to the wall where it is fixed, because at that point the weight acts with the greatest leverage. On tracing the breaking effect of the weight, we see that in bringing the end of the beam down to the position marked by the dotted lines, some action must take place among the fibres of the material at A B, where the weight acts with greatest power. This action must be that of tension among the upper fibres, and of compression among the lower ones; and there must be some point C in the beam where the extension and the compression of the fibres meet, and where there is neither of these actions. Such a point marks the position of what is called the *neutral axis*; that is, a line extending along the beam horizontally, and separating that part of the material which is extended or torn asunder, from that part which is compressed or squeezed together. It is manifest that, when the tearing asunder of the fibres commences, it must continue while the load continues to act, because the number of fibres resisting it is diminished, and those that remain, have therefore more load to bear.

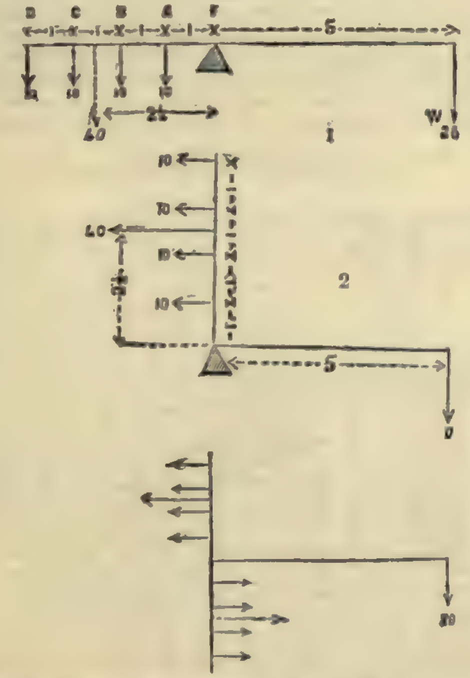
Fig. 63.



What we have to discover, however, is the strength of the beam while it is entire, or how much load W, at a certain distance from the wall, will bear without the destruction of the fibres. Supposing a horizontal line C D, drawn through the neutral axis, to form the long arm of a bent lever of which the fulcrum is C, one short arm C A, and the other C B, we have a certain power or weight W acting at the extremity of the long arm, and a number of resistances, viz., those of the fibres to extension, acting along the arm C A; also a number of resistances, viz., those of the fibres to compression, acting along the arm C B.

Now, when on one arm of a lever, a number of equal forces, equally distributed over the arm, acts against a given weight at the other end of the lever, the effect of all these forces is the same as if they were all collected into one force at the middle point of the arm on which they act (Fig. 64). This is true whether the lever be straight as 1, or bent with one arm as 2, or bent with two arms as 3. As a numerical example, let us suppose that the short arm of 1 is 4 feet long, and at each foot there hang 4 weights A, B, C, D, each of 10 lbs., as marked by the arrows; while the length of the other arm is 5 feet. A weight of 20 lbs. hung to the longer arm balances the four weights of 10 lbs. distributed along the shorter, because

Fig. 64.



A = 10 lbs. at 1 ft.	leverage is balanced by	2 lbs.	at 5 ft. lev.
B = 10 lbs. at 2 ft.	" "	4	"
C = 10 lbs. at 3 ft.	" "	6	"
D = 10 lbs. at 4 ft.	" "	8	"
Total A, B, C, D distributed . . .		20	"

And 20 lbs. at 5 feet leverage would be balanced by 40 lbs. at 2½ feet; that is, the total of the 4 weights A, B, C, D hung at their middle point, as marked by the dotted arrow. The same law will be found true, whatever the number of equally-distributed weights, and whether the lever be straight or bent. Applying this to the question of the beam, Fig. 63, let E be the middle point of A C, and F the middle of B C; then the load W acting with leverage C D has to resist the united tensive strength of the fibres along C A acting at the leverage C E, and the united compressive strength of those along C B acting with the leverage C F. Those resistances will necessarily be equal to one another, because the neutral axis C is the fulcrum, on each side of which they equalise themselves; and the tensive strength of C A, multiplied by its leverage C E, is therefore equal to the compressive strength of C B multiplied by the leverage C F; or, as C A is double of C E, and C B double of C F, and as doubles of equal things are themselves equal, the tensive strength of C A multiplied by the length C A is equal to the compressive strength of C B multiplied by the length C B; and each of those products is equal to the weight W multiplied by its leverage C D, the length of the beam, because each is double of half the effect of W to break the beam.

To show how this reasoning may be applied numerically, let us suppose that a beam of oak 1 inch thick and 6 inches deep, fixed in a wall so as to project 10 feet, is broken by a load hung at the end; and that while in the act of breaking, the neutral axis is observed to be 2 inches from the under side, and therefore 4 inches from the upper side of the beam, we may, from knowing the tensive strength of oak, estimate the weight required to break the beam. Taking this tensive force per square inch (or force required to tear asunder a square inch) at 12,000 lbs., we have in the case before us 4 square inches having a tensive force of 4 × 12,000 = 48,000 lbs., acting with leverage of 4 inches against leverage of 10 feet or 120 inches; and as 4 inches is the $\frac{1}{30}$th of 120 inches, the weight at D must be the $\frac{1}{30}$th of 48,000, viz., 1,600 lbs.

By such calculations as this we could compute the transverse strength of materials, from knowing their tensive strength, provided we knew the position of the neutral axis of fracture. The determination of this, however, is a point of very great difficulty; for it must, in the first place, depend upon the relative proportions of compressive and tensive strength: and as we are much in the dark as to the former, we cannot institute a comparison between it and the latter. In the second place, no material to which we can apply transverse strain resists completely, and then instantaneously gives way. While we add weight after weight, the extension of fibres at one side and the compression of those at the other goes on; the beam bends or becomes deflected; greater and greater strain is thrown on the outer fibres; the tensive and compressive strength of each varies as it is more and more extended or compressed; the neutral axis changes its position; fibres, which were, during part of the process, compressed, begin to be extended; and the condition of the material at and near the point of fracture becomes generally so altered as it approaches destruction, that we cannot clearly estimate its resistance, even if we were possessed of the most accurate knowledge of its strength to resist compression or extension. Owing to these circumstances, it becomes necessary to make distinct experiments on the transverse strength of bodies, and, by means of them, to establish certain data which we may apply in calculation. But while experiment and observation furnish the data or facts on which to calculate, we must still trust to reasoning for our methods of applying these facts; and fortunately the reasoning is of a simple character.

As to the neutral axis of fracture, without requiring to determine its position in any beam, we assume that, for beams of the same material, its distance from the upper or lower sides is proportional to the depth of the beam. Thus, if in a beam of ash 6 inches deep, it is 2 inches from the lower side and 4 inches from the upper; in a beam of ash 12 inches deep it will be 4 inches from the lower and 8 inches from the upper side; in one of 1 inch deep, it will be ⅓rd of an inch from the lower, and ⅔rds of an inch from the upper; and so on in regular proportion. We can see no reason why this should not be the case; and, so far as we can trace the circumstances which determine the position of the neutral axis, we see every reason for believing that it is so. This assumption greatly simplifies the rest of our reasoning; for it enables us to get rid of all calculation as to the actual position of the neutral axis, and to proceed with the comparison of beams as to transverse strength independently of it. We will suppose that we have three beams of the same material, all of equal lengths and breadths, but of the following depths: A depth 2 inches, B depth 4 inches, C depth 6 inches, and that we desire to compare their transverse strengths. Let us assume, for the sake of simplicity, that the neutral axis is midway in the depth; then in A we have, above the neutral axis, a set of fibres extending over 1 inch in depth, and acting with a leverage of 1 inch to resist the breaking load. In B we have fibres extending over 2 inches, and therefore double the number of those acting in A, acting with 2 inches leverage, or double the leverage of those in A. We see, then, that the strength of B must be twice 2, that is, 4 times the strength of A. Again, in C we have 3 inches depth of fibres and a leverage of 3 inches; therefore C has a strength of 3 times 3, or 9 times that of A. And so with any other depths, the strength being always as the depth multiplied by itself, or as the square of the depth. If we had assumed the neutral axis to be in any other position, such as ⅓rd of the depth from the lower side, we should still have found the same law to obtain; for so long as the beams we are considering are of the same material, and of similar form, the neutral axis must proportionally divide their depth, and leave proportional numbers of fibres to act with leverage proportional also to the depth.

We, therefore, have established the simple law, that in beams of the same form, length, breadth, and material, the transverse strengths are as the squares of the depths; and knowing the strength of a beam 1 inch deep, we can estimate the strength due to any other depth by multiplying that of the 1-inch beam twice, by the depth of the other in inches; that is, by the square of the depth.

Example.—An iron beam 1 inch deep breaks with a load of 2 tons: required the load that will break a similar beam 4 inches deep. The square of 4 is 16, and 2 × 16 = 32 tons, or 2 tons × 4 × 4 = 32 tons.

Now, let us ascertain what relation the transverse strengths of beams bear to their breadths. Suppose that a beam 1 inch broad carried a load of 1 ton, then another exactly like it will also carry 1 ton; a third will also carry 1 ton; and the three placed side by side, or united into one beam 3 inches broad, will of course carry 3 tons. Beams, then, of a certain material of equal lengths and depths, have transverse strengths which are proportional to their breadths; and if we know the strength of a beam 1 inch broad, we compute that of a beam having any other breadth, by multiplying the strength due to 1 inch of breadth, by the number of inches in the given breadth.

Example.—A fir beam 1 inch broad carries 10 cwt.: required the load carried by a fir beam 6 inches broad, of like length and depth. 10 cwt. × 6 ins. = 60 cwt., or 3 tons.

We may now combine the laws as to breadth and depth into one, and thus compute the transverse strength of a beam as depending on both dimensions. This law is, that the transverse strengths of beams of the same material and similar in form, are as the breadths multiplied by the squares of their depths; and if we know the strength of a beam 1 inch broad and 1 inch deep, we compute the strength of a beam having any other breadth and depth, by multiplying the strength of the 1-inch beam by the breadth in inches, and twice by the depth in inches of the other.

Example.—Suppose a beam 1 inch broad and 1 inch deep bears 5 cwt.: required the strength of a beam 4 inches broad and 6 inches deep.

5 cwt. × 4 ins. × 6 ins. × 6 ins. = 720 cwt. or 36 tons.

The strength of a beam is irrespective of its length; but the actual weight which it can carry depends upon the length, inasmuch as the leverage with which the weight acts, so as to fracture the beam, is greater, the greater the length. A weight of 12 tons hanging at the end of a beam 1 foot long, is exactly equivalent to a weight of 6 tons at 2 feet distance, of 4 tons at 3 feet, of 3 tons at 4 feet, of 2 tons at 6 feet, of 1 ton at 12 feet, of ½ ton at 24 feet, and so on—the weight multiplied by its distance being always a constant quantity. If, then, we know the load which a beam 1 foot long will carry at the end, we estimate the weight carried at any other distance by dividing that carried at 1 foot by the distance in feet; and combining this element with those of breadth and depth, we have the general rule embracing all the dimensions of a beam; *i. e.*, multiply the weight carried by a beam 1 inch broad, 1 inch deep, and 1 foot long, by the breadth in inches, and twice by the depth in inches, and divide the product by the length in feet.

Example.—A beam 1 inch square and 1 foot long, carries 5 cwt. at its end: required the load carried by a beam 4 inches broad, 6 inches deep, and 18 feet long.

$$\frac{5\text{ cwt.} \times 4\text{ ins.} \times 6\text{ ins.} \times 6\text{ ins.}}{18\text{ ft.}} = 40\text{ cwt., or 2 tons.}$$

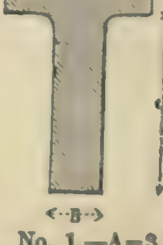
So far we have discussed the strength of beams of square or rectangular sections; but similar reasoning will apply to those of other forms. When the section is circular, the breadth being equal to the depth, we have to multiply the strength of a cylindrical beam 1 inch in diameter and 1 foot long, three times by the diameter in inches, and divide by the length in feet.

Example.—A cylindrical beam 1 inch diameter and 1 foot long, bears 3 cwt.; required the load supported by a cylindrical beam 4 inches in diameter and 8 feet long.

$$\frac{3\text{ cwt.} \times 4\text{ ins.} \times 4\text{ ins.} \times 4\text{ ins.}}{8\text{ ft.}} = 24\text{ cwt.}$$

When the section is of any form, such as the ⊤ shaped iron girder (Fig. 65), from our ignorance as to the position of the neutral axis of fracture, we are at a loss how to reckon the effect of the fibres at different parts of the section, and cannot therefore estimate the strength from any data obtained by experiments upon beams of square or rectangular section. But if we know the strength of a beam of certain dimensions, and of the form in question, we may pretty nearly estimate that of a beam of the same kind whose dimensions, as to breadth and depth respectively, are proportionally greater or less. For instance, to compare two ⊤ shaped beams of which the dimensions are as follows:—

Fig. 65.

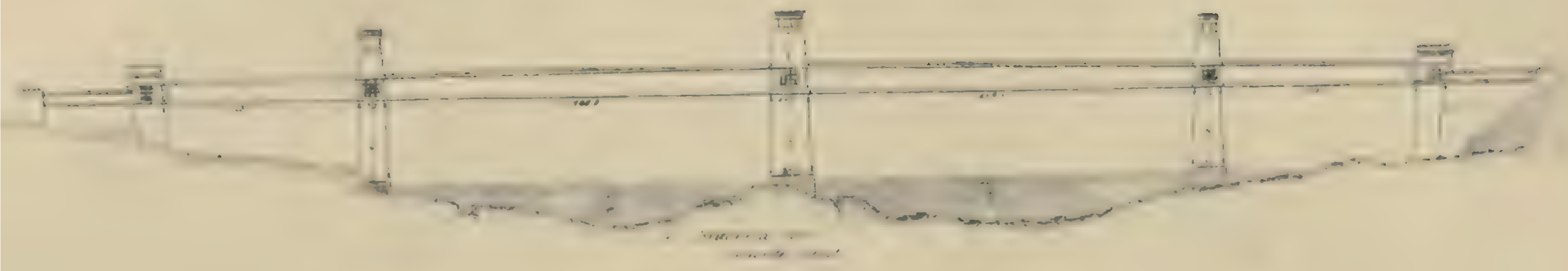


No. 1.—A=3 ins., B=1 in., C=4 ins., D=1 in., length 10 feet, is found to bear 2 tons: required the strength of No. 2, where A=6 ins., B=2 ins., C=6 ins., D=1½ in., length 15 feet.

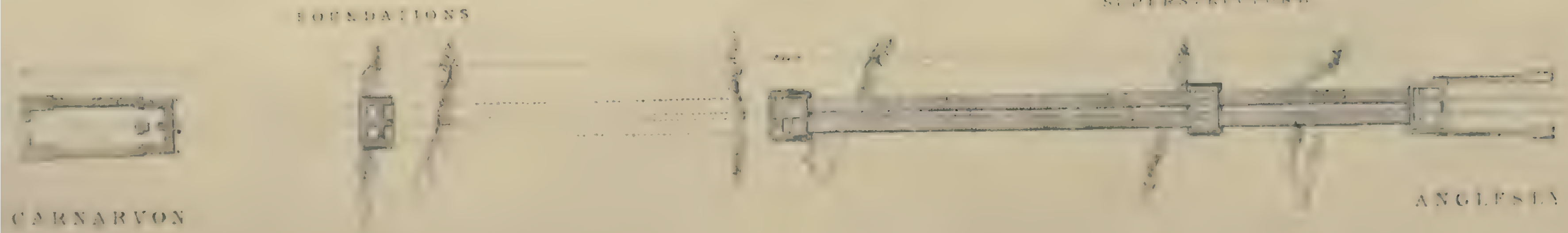
See pages 905 909

BRITANNIA TUBULAR BRIDGE.

ELEVATION



PLAN



Strength of No. 1 is as $\frac{3 \text{ ins.} \times 4 \times 4}{10} = 4{\cdot}8$

Strength of No. 2 is as $\frac{6 \text{ ins.} \times 6 \times 6}{15} = 14{\cdot}4.$

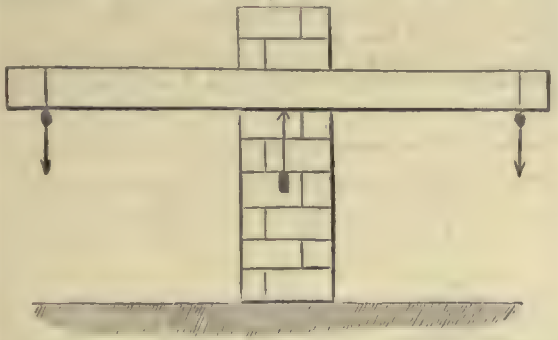
And by the simple proportion,

Strength No. 1. Strength No. 2. Tons. Tons.
4·8 : 14·4 :: 2 : 6, load carried by beam No. 2.

In the absence, then, of sufficient knowledge of the effect of material at different parts of the section in contributing to strength, for every different form of section we are under the necessity of making such experiments as shall fix data for calculation, to be applied to cases where the section is similar, but the dimensions different. A number of such experiments have been tried from time to time, and attempts have been made to settle, by theoretical reasoning as well as by practical results, the best form of section to be used; that is to say, the form which gives the greatest strength with the least material. We are not aware, however, that for any material, the subject has been so far investigated as to warrant us in laying down any absolute proportions; and the form of section must therefore be determined, in a great measure, by balancing a number of circumstances, and adopting such arrangements as shall combine the results most advantageously.

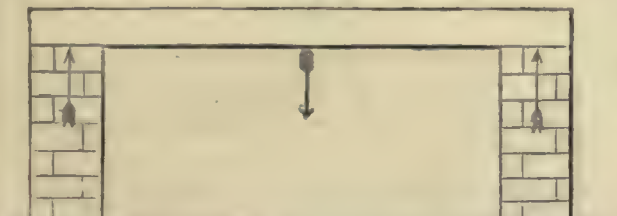
In a recent engineering work, a triumph of skill and perseverance over difficulties at first sight apparently insurmountable—we mean the Britannia Tubular Bridge across the Menai Straits—an enormous amount of preliminary investigation was conducted before the form and dimensions of the structure were finally determined. Models were made of all convenient forms, and tested against each other: one was found too weak in one place, another too weak in another place. Fresh models were made, with the weak parts strengthened by additional materials, and the mass of material in strong parts removed: the most suitable and convenient form was thus decided. A much larger model was made and tested as to deflection and fracture; and after a large comparison of results, the actual dimensions and details of construction of the full-sized structure were determined, and carried out with merited success.

Fig. 66.



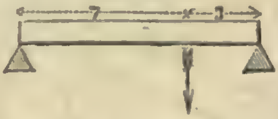
Hitherto we have referred only to the circumstance of a beam projecting and loaded at one end. This, however, is by no means the most ordinary condition under which materials are exposed to transverse strain. Beams are usually supported at both ends, and carry their load in the middle. We have, therefore, now to ascertain what relation exists between the strength of such a beam and that of a beam projecting and loaded at one end. If we suppose a beam built into a wall and projecting equally on both sides (Fig. 66), each end being loaded with an equal weight, it is clear that the wall supports double the weight suspended at each end. If we conceive the forces acting on such a beam inverted or turned upside down, we establish the conditions of a beam supported at both ends, and loaded in the middle (Fig. 67) with double the weight which each of the end supports has to bear. Now, if, in the first case, where the beam projected both ways from the wall, each end were loaded with the weight capable of breaking the beam, that is, up to its transverse strength; in the second case, the beam, which is twice the length of each projecting arm of the first, may be loaded with double the weight which hung from each end of the first, and this weight will measure its strength. We, therefore, come to the conclusion that the transverse strength of a beam, supported at both ends and loaded in the middle, is double that of a beam half the length at one end and loaded at the other. We have already shown that if we double the length of a projecting beam, we can load it with only half the weight; therefore, if each projecting arm of the beam were made equal in length to the beam supported at both ends, the former would bear only one-fourth of the weight which the latter can bear. For equal lengths, therefore, the strength of the beam supported at both ends and loaded in the middle, is just four times that of the beam fixed at one end and loaded at the other; or, conversely, the strength of the beam projecting is only one-fourth of that of the beam supported at both ends. Experiments upon transverse strain have been generally made upon beams supported on both ends; and tables of practical data founded on these experiments have been formed. Such tables generally contain the transverse strength of beams, having square and also circular sections one inch in diameter, and having a length of one foot between the supports, their load being supposed to be placed in the middle of their length. When the load is placed out of the middle it may be increased, because its leverage to break the beam, is diminished the farther it is from the middle point.

Fig. 67.



The mode of reckoning this diminution of strain may be best illustrated by an example. Suppose we found that a beam, 10 feet long, bore 42 cwt. at its middle point, and desired to ascertain how much it would bear suspended 2 feet from the middle—that is, 7 feet from one end and 3 feet from the other (Fig. 68)—we should proceed as follows:—Square half the length of the beam, or multiply 5 by itself, giving 25, and this by 42 cwt., the load sustained at the middle, product 1,050; now multiply 7 by 3 (the two portions into which the beam is divided), product 21, and divide the former product 1,050 by this, that gives a quotient 50 cwt., the load which the beam would carry 2 feet from the centre.

Fig. 68.

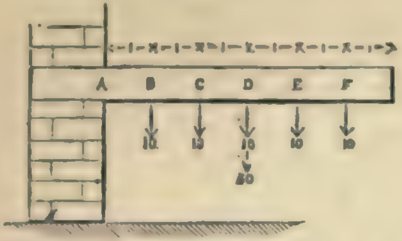


The simple principle of this computation is, that the load hung from any point of a beam, is to the load which may be hung from any other, as the product of the two lengths into which the second point divides the beam, is to the product of the two lengths into which the first point divides it. Thus, in the case we have given above, knowing that the beam bears 42 cwt. at the middle, or when it is divided into two lengths, each 5 feet, we say,

$$7 \times 3 = 21 : 5 \times 5 = 25 :: 42 : 50$$

It frequently happens, that beams have to bear a load not hung at any one point, but distributed uniformly over their length; as in the case of roofs, floors, and girder-bridges. Here, it may be readily seen, that the strain is only half that which it would be if the whole load were collected at the middle. If we suppose a beam projecting 6 feet from a wall, loaded at intervals of a foot by weights each 10 cwt. (Fig. 69), then, by the principle of the lever, the effect of each of the weights, to break the beam at A, may be reckoned as follows :—

Fig. 69.

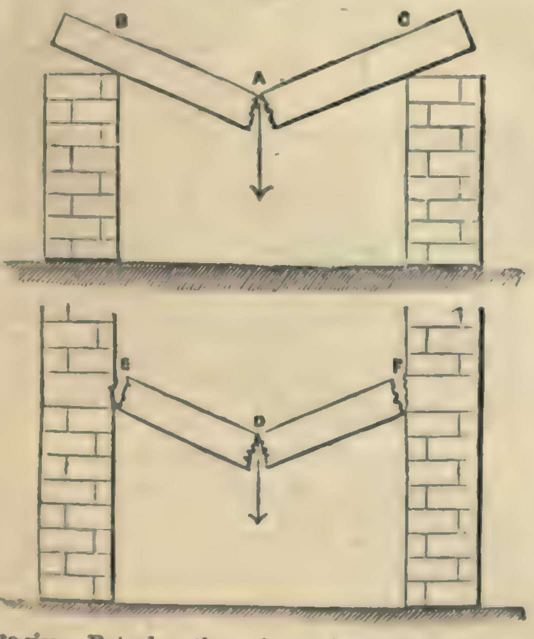


B=10 cwt.	acting	at 1	ft. from A	has the effect of	10	cwt. at	1	ft. leverage.
C=10	„	„ 2	„	„	20	„	1	„
D=10	„	„ 3	„	„	30	„	1	„
E=10	„	„ 4	„	„	40	„	1	„
F=10	„	„ 5	„	„	50	„	1	„
Total 50 cwt. distributed equally.					150	cwt. at	1	ft. leverage.

Or, as 150 cwt. at 1 foot leverage are equivalent to 50 cwt. at 3 feet, we find that the total strain is the same as if the total weight were collected at D, the middle point. Were we to assume a greater number of weights at smaller intervals, we should still find the same result; and the more numerous the weights, and smaller the intervals we assume, the more nearly do we approach to the case of a beam uniformly loaded over its whole length; whence we conclude that the effect of the distributed load is the same as if it were collected at the middle of the beam, and therefore just half of what it would be if hung at the end. Or, conversely, if the beam bear a certain load at its extreme end, it will bear double that weight distributed over its whole length. The same law applies in the case of a beam supported at both ends, and loaded uniformly throughout its length; the strain of the load is reduced to half what it would be if collected at the centre; or the beam will bear twice as much distributed weight, as it can bear at its middle point.

When a beam is not merely supported at each end, but fixed firmly there, its strength is increased by one-half. It would appear at first sight, that by fixing the ends of a beam we should double its strength, for the following reasons: when the beam is merely supported, an extreme load in the middle has only to effect one fracture at A (Fig. 70), the two ends B and C being free to rise. But when the ends are fixed, the load has not only to break the beam at the middle point D, but also at E and F; and to do this it must be double what it would be if the ends were free, as may be very simply computed thus. Suppose, in the first case, it required 12 cwt. to effect the single fracture at A, then in the second it would require likewise 12 cwt. in the middle to effect the fracture at D, and 6 cwt. in the middle to effect each of the fractures at E and F; making a total of 24 cwt. in the case of the beam with fixed ends.

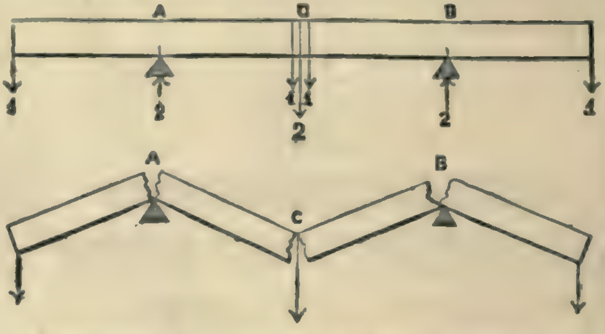
Fig. 70.



Many writers have taken this view, neglecting a circumstance which must in such cases occur, and which greatly modifies the distribution of strain on the middle and ends. A single glance at the figure shows that the amount of tension and compression on the fibres at the middle fracture must be double that at either of the end fractures; and hence each half of the weight required to produce either of the end fractures, or the total weight due to the middle fracture, must be 4 times either of those due to the end fractures. If then we supposed the total breaking-weight divided into 6 equal parts, 4 of those parts would act to break the beam in the middle, and 2 to break it at the ends. But the 4 parts required to effect the middle fracture must make up the breaking-weight due to a beam merely supported without being fixed at the ends; and the other 2 parts—that is, half as much more—make up the additional weight required when the ends are fixed. Taking the numerical example as before, if it required 12 cwt. to effect the single fracture at A, it would require as much to break the beam at D, and one-fourth, or 3 cwt., to break it at each of the points E and F, making a total of 18 cwt., which is the sum of 12 and 6, the ordinary breaking-weight increased by its half.

As this principle of computation accords better with experiment than the former, it is important that its demonstration should be clear, inasmuch as numerous theorists have adhered to the principle, that by fixing the ends of a beam its strength is doubled. According to them, the circumstances correspond with those of a beam (Fig. 71) resting on supports A and B, and projecting each way one half of its length beyond. A load of 1 at each end, balanced by two loads, each 1, in the middle, throws a breaking strain of 2 upon the beam at each prop; and an additional load of 2 in the middle will measure the breaking strain there, so that the total middle load is 4, or double the ordinary breaking strain. This is no doubt true, because if we suppose the fracture effected, the amount of compression and extension of the fibres at each of the points A, C, and B is the same; and therefore each fracture requires the same load to effect it. But in the case of the beam with ends not balanced but fixed, as we have already explained, the end fractures demand only half the amount of extension and compression due to the middle fracture.

Fig. 71.



When a load acts on a beam not perpendicularly or square to its length, but at some other angle, it throws less strain upon it, because the actual leverage of the weight is not the length of the beam as measured from A to B (Fig. 72), but the length of a line measured from A to C perpendicularly to the vertical line in which the weight acts. So when a beam is supported at both ends, but lies obliquely, the transverse strength is to be reckoned as that due to a beam of the length indicated by the horizontal line D E (Fig. 73) measured between the supports.

Fig. 72.

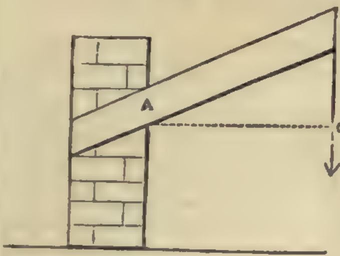
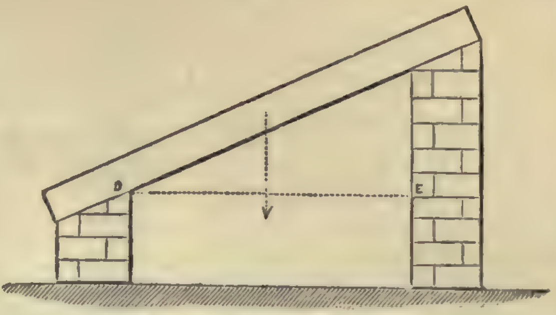


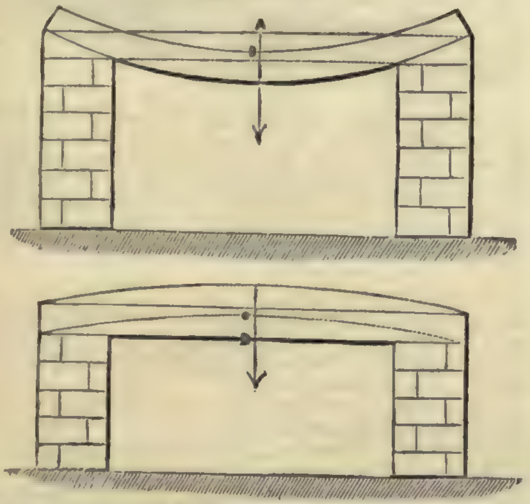
Fig. 73



DEFLECTION.—Hitherto, for the sake of simplicity, we have discussed the question of transverse strain as applied to materials perfectly inflexible, such as break, but cannot bend. We have, however, no practical experience of materials of this character, although stones, slates, or even cast-iron, approach very nearly to it. Timber beams, and those made of wrought-iron, bend considerably before they become fractured by transverse strain; and as, in such beams, deflection from their straight or horizontal condition may be inconvenient and unsuitable, it becomes important to estimate the amount of deflection which they will exhibit under certain loads, so that they be made slightly curved in the opposite direction before the load is placed upon them. If it were found, for instance, that a beam loaded with a certain weight deflected so far from the straight line that its middle point B (Fig. 74) sunk a certain distance—say 6 inches—below the horizontal line; then if the beam, instead of being made straight, were made somewhat arched, or *cambered*, as it is technically called, the load being placed upon it would still deflect it, and thus bring its surface to a horizontal line, if the camber or amount of arching C D were properly estimated.

The complete investigation of the question of deflection would involve us in mathematical reasoning of rather a complex character, which would scarcely be in place here; and, indeed, practical results as to deflection present so many irregularities, and so many deviations from any apparent law, that it is questionable whether theory would prove a very safe guide. Some writers on this subject have determined theoretically, that the amount of deflection of a beam of certain length, increases in the same proportion as the load, and that the deflection under a certain weight varies as the square of the length. If this law were true, a beam 20 feet long, with a certain weight on it, would be deflected four times as far as one 10 feet long, because the one has twice the length of the other, and the square of 2, or 2 multiplied into itself, is 4.

Fig. 74.



This law, however, has not by any means been found to be true in practice. More accurate investigators have furnished a law which, while it appears to be theoretically correct, presents results very nearly according with those of experiment. This law is, that the deflection of a beam (having a rectangular section) varies directly as the weight and as the cube of the length (or the length multiplied 3 times into itself), and inversely as the breadth and the cube of the depth. If, then, we knew the deflection of a certain beam, we might estimate that of another of the same material, but varying in all its dimensions. Suppose, for example, that a fir batten 1 inch broad and 2 inches deep, with a load of 1 cwt., deflected $\frac{1}{16}$th of an inch in a length of 3 feet, and we desired to know the deflection of a fir beam 3 inches broad, 10 inches deep, and 15 feet long, under a weight of 1 ton, or 20 cwt. In the case of the batten, which is 2 inches deep, since $2 \times 2 \times 2 = 8$, the deflection is only $\frac{1}{8}$th of what it would be were the depth 1 inch, because it is inversely as the cube of the depth. The deflection, then, of a batten 1 inch deep, under a load of 1 cwt., would be $8 \times \frac{1}{16} = \frac{1}{2}$ inch, the length being 3 feet. Further, the cube of 3, or $3 \times 3 \times 3$, is 27; and were the batten only 1 foot long instead of 3 feet, the deflection would be $\frac{1}{27}$th; the deflection then of a batten 1 inch broad, 1 inch deep, and 1 foot long, would be $\frac{1}{27}$th of $\frac{1}{2}$ an inch, or $\frac{1}{54}$th of an inch, with a load of 1 cwt. Having thus got an estimate for a beam with all the dimensions reduced to unity, we can apply it to any other beam, according to the law we have stated. This law is arithmetically applied by multiplying the deflection found above, $\frac{1}{54}$th of an inch, by the weight 20 cwt., by the cube of the length 15 feet (or 3 times by 15 feet), and dividing the product by the breadth 3 inches and the cube of the depth 10 inches; or the deflection is

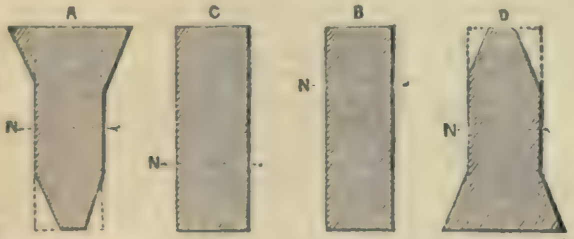
$$\frac{\frac{1}{54}\text{th in.} \times 20 \times 15 \times 15 \times 15}{3 \times 10 \times 10 \times 10} = \frac{5}{12}\text{ths of an inch,}$$

or nearly $\frac{1}{2}$ an inch.

As to the deflection of beams of various forms and materials, and subjected to strains under various conditions, although numerous experiments have been made, yet they have been conducted with too little reference to each other for us to develop any law of general application. In Barlow's *Treatise on the Strength of Materials*, the question of the deflection of wrought-iron, as applied in the construction of rails, is treated at considerable length; but the conditions of strength in malleable iron must differ very considerably from those in other materials, because their comparative tensive and compressive strengths differ very widely. Mr. Fairbairn, of Manchester, has lately conducted some extended experiments on this and allied subjects, which are well worthy of the attention of practical men. To show how cautious we ought to be in applying to any particular material the results deduced from experiments on other materials, we may instance the peculiar difference between cast and wrought-iron in respect of tensive and compressive strength. The direct cohesive strength of wrought-iron is about 3 times that of cast-iron; or, to bear a certain tensive strain, the area of section in cast, should be about 3 times that of wrought-iron. As to compressive strain, on the other hand, although it may be true that wrought-iron will sustain much greater compression than cast-iron before it becomes entirely crushed and disintegrated, yet the greater softness of the wrought-iron permits it to yield under pressure, and to become compressed to a considerable extent; while cast-iron scarcely yields perceptibly until it entirely gives way.

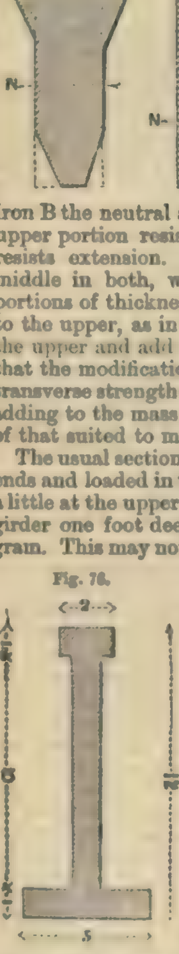
DISPOSAL OF MATERIAL.—If we apply these considerations to a beam subjected to a transverse strain, as when it is supported at both ends and loaded in the middle, we see that at the middle of the beam, just before fracture takes place, there must be a portion of the material above the neutral axis compressed, and a portion below it extended by the action of the load. If the resistance of the material to extension and to compression under those conditions be equal, the neutral axis would be in the middle of the beam; and its resistance to fracture would then be the greatest possible, because the total leverage of the compressed and extended portions on each side of it, would be greater when it is in the middle than when it is elsewhere. But if it happened that the material was more easily compressed than extended, it would be nearer the lower side; while if it were more easily fractured by extension than compression, it would be nearer the upper side. Desiring, however, in either case to secure the advantage of having it in the middle, and thereby giving both the compressed and the extended portions the greatest possible leverage, we should somewhat alter the form of transverse section, so as to increase the area of the weaker part, or add to it some fibres, diminishing the area of the stronger, and, upon the whole, not altering the quantity of material or total area of section, but only modifying it in such a way as to bring the neutral axis to the middle of the depth. In a rectangular beam of malleable iron C (Fig. 75), the neutral axis N is below the middle of the depth, because the upper portion yields more readily to compression than the lower to extension. In a beam of cast-iron B the neutral axis N is above the middle, because the upper portion resists compression more than the lower resists extension. To bring the neutral axis to the middle in both, we should for malleable iron remove portions of thickness from the lower edge, and add them to the upper, as in A; for cast-iron we should take from the upper and add to the lower, as in D. We thus find that the modification of form requisite for increasing the transverse strength of cast-iron of certain depth, without adding to the mass of material, is exactly the opposite of that suited to malleable iron.

Fig. 75.



The usual section of cast-iron girders, supported at both ends and loaded in the middle, is the inverted ⊤, swelled a little at the upper edge (Fig. 76). The dimensions of a girder one foot deep are nearly those marked in the diagram. This may not be precisely the best form for combining the greatest strength with economy of materials, but it approaches it; and, besides, it possesses practical advantages which deserve consideration. The flange, or increased width at the lower side, not only affords the additional strength required there in order to meet the tensive strain, but presents a projecting ledge for receiving the beams or arches which the girder may have to bear. The increased thickness at the upper side not only provides additional material to resist compressive strain at the greatest possible distance from the neutral axis, but serves to stiffen the girder so as to prevent it from bending or buckling sideways, either from contraction in cooling from its molten state, or from excessive strain in its place. The thickness of the metal in the middle part and in the lower flange is made as nearly equal as possible, because it is found practically that, in making iron castings, unequal thicknesses of metal cause unequal contractions or shrinkings in the metal as it cools, and thus tend to distort the work.

Fig. 76.



The best form of section for malleable iron rails is nearly the opposite of that for cast-iron girders, a ⊤ not inverted (Fig. 77). As, however, the upper part of the rail gets worn and uneven by the friction of the traffic, it is sometimes thought desirable to have the opportunity of inverting it, and thereby wearing both the upper and lower sides before the rail is rejected as worn out. The section is, therefore, made symmetrical above and below, as well as on both sides (Fig. 78).

Fig. 77.

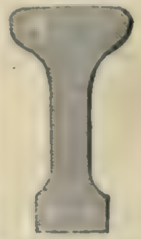
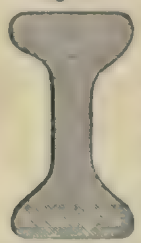


Fig. 78.



As we have shown above, the strength of any beam to resist transverse strain increases very greatly as the depth is increased. It is, therefore, of great importance in all cases of transverse strain to give as much depth as possible. This is often effected, not by increasing the total depth of the material, and thus adding greatly to its weight, but by introducing ribs or flanges, and thus dispersing a given quantity of material in a better form. If, for instance, we had to provide a square cast-iron plate, of sufficient strength to resist a great weight pressing on its middle (Fig. 79), while it is supported on two piers at the sides: instead of making the plate of solid iron, sufficiently thick to sustain the strain, we should probably make the upper part a thin flat plate (Fig. 80); and round the edges, as well as across the diagonals, provide projecting ribs of the greatest depth in the middle (Fig. 81); and thus attain sufficient strength and stiffness, while we should save a considerable quantity of material. This mode of attaining strength is particularly useful with such a material as cast-iron; for when it is formed in thick masses, the cooling of the outer crust, while the inner part of the mass remains fluid, sets or fixes the outside, and the subsequent contraction of the inside causes a sponginess or looseness of texture, which greatly diminishes strength. It is, therefore, of the utmost importance to attain the required strength without great thickness, as well for the sake of securing solidity and firmness of material, as for

Fig. 79.

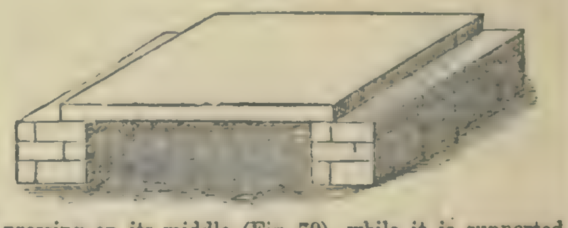


Fig. 80.

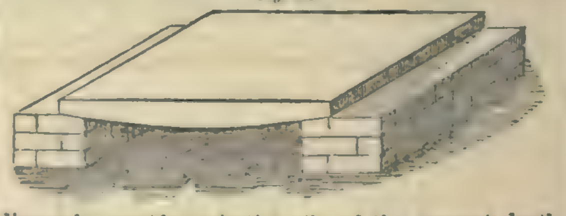
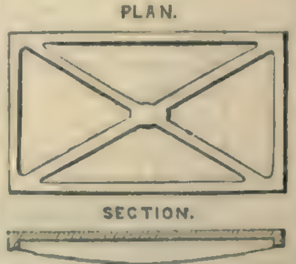


Fig. 81.



avoiding air-bubbles and flaws, which are apt to occur in thick castings; and which, if they do occur in thin castings, do not take so much from the strength, or, at all events, are more likely to be visible, and can be allowed for. In malleable iron bars, when they are required for strength and stiffness, the ⊤ form is usually employed. In plates, corrugating or wrinkling adds greatly to the stiffness, because it provides depth of material transversely.

The question of how to dispose material in order to secure the greatest strength with the least weight and cost, is indeed the main subject of mechanical contrivance as to form. We have already instanced the contrivance of the Britannia Tubular Bridge, as an example of skill in device going hand-in-hand with experiment. Were the material employed in one of the great tubes of this bridge all compressed into one solid bar, having a section of dimensions proportional to those of the tube, we question if it could support its own weight without breaking, even if the span were reduced to half that of the tube; while with one-fourth of the span, the deflection, from its own weight, would be enormous. In this tube, the material is disposed in such a manner as to attain the greatest strength with the least weight, and the most suitable form for the purposes of the traffic. The additional material required in the upper part of the section (as in the case of other wrought-iron girders), is arranged in the sides of cells or subsidiary tubes (Fig. 82); and due regard has been paid to the securing of lateral stiffness, to resist the pressure of strong winds against its immense side-surface, as well as to the attainment of vertical stiffness to resist the strain and shake of a heavy passing load. The annexed plate represents an elevation and plan of this great work, as much admired for its grandeur and simplicity as for its strength and durability.

Fig. 82.

In some other cases, where malleable iron is used in the construction of bridges, a girder has been formed of plates, disposed in a manner somewhat different. The upper part is a tube bent over to an arch-form (Fig. 83), and the lower portion forms a flange or ledge, as well for strength as for receiving the ends of the beams that carry the roadway. The upper and lower portions are connected by a longitudinal fin, with several transverse ribs for stiffening, and more firmly connecting the whole together. This ingenious arrangement of parts is due, we believe, to Mr. Brunel, the engineer.

Fig. 83.

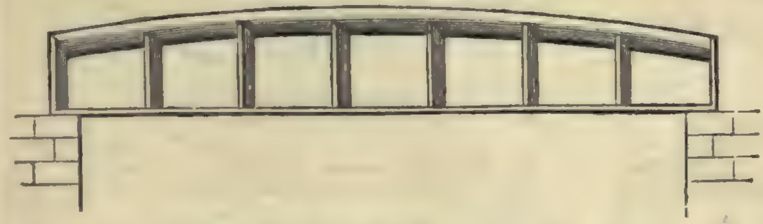
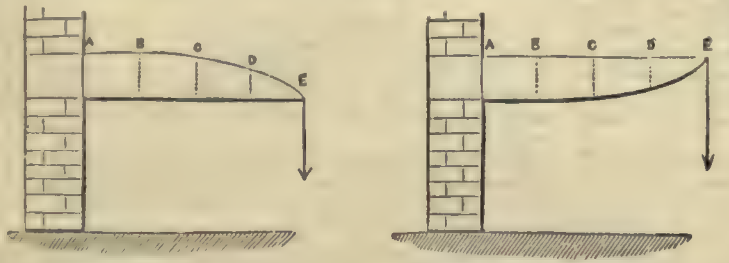


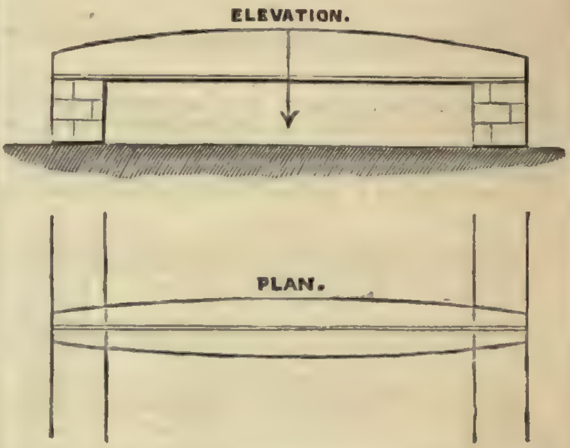
Fig. 84.



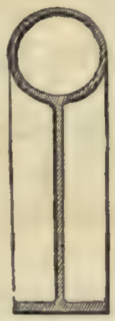
Not only, however, is there scope for modification of form of the transverse section so as to secure strength with economy of material, but the longitudinal section and a plan of a beam are also susceptible of modifications.

In the case of a beam projecting from a wall (Fig. 84), with a weight suspended at its extreme end, the strain is greatest at A, close to the support, and diminishes towards the end, because the leverage with which the weight acts to fracture the beam, diminishes. Thus, at C, midway, the strain is only half that at A; at B it is $\frac{3}{4}$ths; and at D it is $\frac{1}{4}$th. If, then, the breadth of the beam be uniform throughout its length, its depth may be with safety diminished towards the extremity. As the strength is proportioned to the square of the depth, the depths at B C D may be made such that their squares are respectively $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$ of the square of A. This may be done by removing material from the upper or lower side of the beam, so as to give it a curved outline above or below; and still the strength of the beam is maintained. The curve for such a beam is what is called a parabola, the peculiar property of which is, that a square of the length of each of the vertical lines or ordinates, A, B, C, D, is proportional to its distance from the apex or extremity E. Let us take, as an example, a beam of cast-iron projecting 12 feet, and 12 inches deep at A by 3 inches broad. The weight of such a beam of uniform depth throughout would be about 12 cwt. But by tapering it off, as we have indicated, its weight would be reduced to 8 cwt., $\frac{2}{3}$rds of what it was. Thus, not only is a saving effected in the cost of the beam, but as its own weight is an important part of the strain at A, this element of strain is considerably diminished, and the weight hanging from the end may be proportionately increased. The depths at the different points would be as follow :—

Fig. 85.



Section.

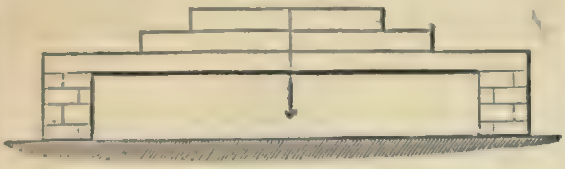


At A, close to the support, 12 inches.
" B, $\frac{3}{4}$ths of the length from A, about $10\frac{1}{2}$, because $10\frac{1}{2}$ squared is about $\frac{3}{4}$ths of 12 squared. $10\frac{1}{2} \times 10\frac{1}{2} = 110\frac{1}{4}$, and $12 \times 12 = 144$, $\frac{3}{4}$ths of which is 108.
" C, $\frac{1}{2}$ the length, or 6 feet from A, about $8\frac{1}{2}$ inches.
" D, $\frac{1}{4}$th the length, or 9 feet from A, 6 inches.

If the depth of the beam cannot conveniently be varied, the breadth may be diminished at a distance from the point of support; because the strength of a beam being at its breadth, and the leverage of the weight being lessened as we recede from the breaking-point, the breadth may be lessened in like proportion. Sometimes it is convenient to vary the depth and breadth also, and thus maintain a similar section throughout the whole length; that is, a section of like figure, but of varying dimensions.

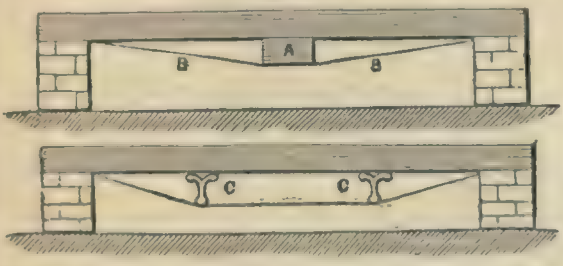
In the case of beams supported at both ends, and loaded in the middle (Fig. 85), the same principle is applied; the dimensions, in respect of depth and breadth, being made greatest in the middle, and diminished gradually towards the ends. This economy of material is easily effected in cast-iron girders, by making the pattern from which the casting is moulded of the requisite form.

Fig. 86.



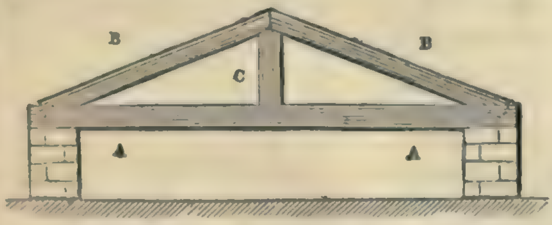
When timber beams are used, the increased depth towards the middle is attained by piling several beams on one another (Fig. 86). As the lower side of a beam resting at both ends is subject to a tensive strain, great additional transverse strength may be secured by straining a rod or chain B B (Fig. 87) from end to end of the beam, and blocking it off from the lower surface by means of one or more wooden or iron struts A or C C. By this arrangement the wooden beam, when loaded between the supports, is subjected to a compressive strain only, the whole of the tensive strain being thrown upon the rod or chain.

Fig. 87.



Again, by fixing a strut, or king-post, C (Fig. 88), upon the upper side of the beam A A, and connecting it by pieces B B to the ends, the compressive strain is thrown on B B, and the tensive strain only on A A. Indeed, there is no limit to the contrivances by means of which the direction and amount of the strain can be varied so as to economise material and secure stability. We have only instanced a few as examples of arrangements, a complete account of which would fill many volumes.

Fig. 88.



CALCULATION OF TRANSVERSE STRENGTH. The table of transverse strengths gives the load which may be safely placed on the middle of beams of different materials supported at both ends. The table applies to beams 1 foot long between the points of support. The weights are those for beams the transverse section of which in the middle is 1 inch square, or 1 inch broad and 1 inch deep; and these apply to all beams of rectangular section. The weights for beams of circular section, may be taken at two-thirds of those for square section. The following are the rules for computing the strengths of beams of various dimensions, according to the data furnished by the table:—

Table of transverse strengths of beams, 1 foot long between supports, lying loose at both ends, loaded in the middle, having a section 1 inch square.

Name of material.	Permanent load.	Name of material.	Permanent load.
Ash	250 lbs.	Oak (English)	190 lbs.
Beech	170 „	Pitch pine............	190 „
Deal	130 „	Teak	270 „
Elm	120 „	Iron (cast)	850 „
Oak (African)	230 „	Iron (wrought)......	1300 „

I. For beams supported at both ends and loaded in the middle. Given the length, breadth, and depth, to find the load.

Rule.—Multiply the number in the table by the breadth (in inches), twice by the depth (in inches), and divide by the length (in feet).

Example 1.—Required the transverse strength of a beam of teak 8 inches broad, 12 inches deep, and 14 feet between bearings.

Number from table opposite teak . .	270 lbs.
Multiply by breadth	8 ins.
	2160
Multiply by depth	12 ins.
	25920
Again by depth	12 ins.
Divide by length	14)311040
	22217 lbs.

Very nearly 10 tons, or 22,400 lbs.

Example 2.—Required the transverse strength of a wrought-iron bar $1\frac{1}{2}$ inch broad, 4 inches deep, and 6 feet long.

$$\frac{1300 \times 1\frac{1}{2} \times 4 \times 4}{6} = 5200 \text{ lbs., nearly } 46\tfrac{1}{2} \text{ cwt.}$$

For bars of circular section.

Rule.—Multiply the number in the table three times by the diameter (in inches), divide by the length (in feet), and deduct one-third of the result from itself.

Example 3.—Required the transverse strength of a beech roller, 2 inches diameter, and 4 feet long.

$$\frac{170 \times 2 \times 2 \times 2}{4} = 340$$

Deduct one-third = 113

227 lbs., about 2 cwt.

II. When the beam is fixed down at its ends.

Rule.—Find the strength as before (I.), and add to it its half.

Example 4.—Required the strength of a cast-iron bar 2 inches broad, 6 inches deep, 9 feet long, fixed at both ends.

$$\frac{850 \times 2 \times 6 \times 6}{9} = 6800 \text{ lbs., about 3 tons.}$$

And one-half 3400 „ $1\frac{1}{2}$ ton.

10200 „ $4\frac{1}{2}$ tons.

III. When the beam is supported loosely at both ends, and has the load uniformly distributed over its length.

Rule.—Find the strength as before (I.), and double it.

Example 5.—Required the strength of a deal rafter 3 inches broad, 11 inches deep, 10 feet long, loaded uniformly throughout its length.

$$\frac{130 \times 3 \times 11 \times 11}{10} \times 2 = 9438 \text{ lbs., about } 4\tfrac{1}{4} \text{ tons.}$$

IV. When the beam is fixed at both ends, and has a load uniformly distributed over its length.

Rule.—Find the strength as in I., and triple it.

Example 6.—Required the strength of a round wrought-iron bar 2 inches diameter, 10 feet long, fixed at ends, the load being uniformly distributed.

$$\frac{1300 \times 2 \times 2 \times 2}{10} = 1040$$

$$\text{Deduct one-third} = \underline{347}$$

$$693 \times 3 = 2079 \text{ lbs.}$$

V. When the beam is loaded by a weight not in the middle.

Rule.—Find the strength as before (I.), multiply twice by half the length, and divide successively by the length of each part into which the beam is divided by the point of suspension.

Example 7.—Required the strength of an ash beam 6 inches broad, 8 inches deep, 14 feet long, to carry a weight 3 feet from one end. Half the length is 7 feet, and the two parts are 11 and 3.

$$\frac{250 \times 6 \times 8 \times 8}{14} \times \frac{7 \times 7}{11 \times 3} = 10{,}182 \text{ lbs., about } 4\tfrac{1}{2} \text{ tons.}$$

VI. When the beam is fixed at one end and loaded at the other.

Rule.—Find the strength as in I., and divide it by 4.

Example 8.—Required the strength of an English oak beam 13 inches square, projecting 10 feet from a wall, to bear a load at the end.

$$\frac{190 \times 13 \times 13 \times 13}{10} = \underline{41743 \text{ lbs.}} \text{ strength by I.}$$

$$\text{Divide by 4, } \underline{10436 \text{ lbs.}}$$

These cases include most of the circumstances which occur in practice. The converse operations for finding the dimensions of a beam when we know the weight it has to bear, do not furnish us with the breadth and depth separately (except in the case of cylindrical and square beams when the depth and breadth are equal), but give us a result which is the product of the breadth by the square of the depth. We must, therefore, be guided by other circumstances in determining one of these dimensions; and having determined the one, we easily find the other. For instance, if we were required to provide a cast-iron girder of rectangular section, such that when placed on two walls 10 feet apart it should carry a load of 2 tons in the middle, we should reason thus:—By the table, a cast-iron rectangular beam 1 inch broad and 1 inch deep, and 1 foot between supports, bears 850 lbs.; one 10 feet between supports must be made ten times as strong to bear 850 lbs.; and to bear 2 tons, or 4480 lbs., it must be about five-and-a-quarter times as strong, because 4480 lbs. is about five-and-a-quarter times 850 lbs.

The strength, then, for the given length and weight must be $5\frac{1}{4} \times 10 = 52\frac{1}{2}$; or, more correctly,

$\frac{10 \times 4480}{850} = 52{\cdot}7$ times that of a beam 1 inch broad and 1 inch deep. But as the strength is as the breadth multiplied by the square of the depth, the number 52·7 must be the product of the breadth in inches, multiplied by the square of the depth, or twice by the depth; and if we determine one of these dimensions, we can easily ascertain the other: thus—

Assume breadth,	then depth,	because	nearly
1 in. . .	$7\frac{1}{4}$ in. . .	$1 \times 7\frac{1}{4} \times 7\frac{1}{4}$	$= 52{\cdot}7$
2 in. . .	$5\frac{1}{8}$ in. . .	$2 \times 5\frac{1}{8} \times 5\frac{1}{8}$	$= 52{\cdot}7$
3 in. . .	$4\frac{1}{8}$ in. . .	$3 \times 4\frac{1}{8} \times 4\frac{1}{8}$	$= 52{\cdot}7$
4 in. . .	$3\frac{5}{8}$ in. . .	$4 \times 3\frac{5}{8} \times 3\frac{5}{8}$	$= 52{\cdot}7$
5 in. . .	$3\frac{1}{4}$ in. . .	$5 \times 3\frac{1}{4} \times 3\frac{1}{4}$	$= 52{\cdot}7$

And so on, the one dimension being assumed according to circumstances of convenience, or for other reasons. When it is determined that the section of the beam shall be square, the product found as above is the cube of the breadth or depth, or it is the breadth multiplied 3 times by itself; and therefore the breadth or depth is found by taking the cube root of this product. In the case given, the cube root of 52·7 is about $3\frac{3}{4}$; and we therefore conclude that if the beam be of square section, its breadth or depth must be $3\frac{3}{4}$ inches, because $3\frac{3}{4} \times 3\frac{3}{4} \times 3\frac{3}{4} = 52{\cdot}7$ When the beam is cylindrical, the diameter is also determined. Since the strength of a cylindrical beam is about $\frac{2}{3}$ of a square one of the same material, we must make up the other $\frac{1}{3}$ by adding to its dimensions; and as $\frac{1}{3}$ is one-half of $\frac{2}{3}$, we must add to the product its half, and then take the cube root as for a square beam. In the case given, adding to 52·7 its half, or 26·35, we have 79·05, which is nearly the cube of $4\frac{1}{4}$; and we must therefore make the diameter of a cylindrical beam $4\frac{1}{4}$ inches, or more correctly 4·3 inches.

As a proof of the accuracy of our calculation, we have only to reverse the operation, and calculate the load which the beam of the dimensions we have thus determined will bear. If the result correspond with the given weight, we know that we are right. In the example we have taken, we have to find the strength of a cylindrical cast-iron beam 4·3 inches diameter and 10 feet long. By the rule for bars of circular section, case I.,

$$\frac{850 \times 4{\cdot}3 \times 4{\cdot}3 \times 4{\cdot}3}{10} = 6758 \text{ lbs.}$$

$$\text{Deduct } \tfrac{1}{3} = \underline{2253}$$

$$4505$$

Nearly 2 tons, or 4480 lbs., the load which the beam was intended to carry.

The following rules give the mode of calculating the *strength product* under various circumstances of strain, and then the method of dealing with the *strength product* when the form, proportions, or one of the sectional dimensions of the beam is given. The cases are numbered in the same order as those in the calculation of the load.

I. Beams supported at both ends and loaded in the middle: given the length and load to find the *strength product.*

Rule.—Multiply the load (in lbs.) by the length (in feet), and divide by the number in the table.

Example 9.—Required the *strength product* for a beam of teak 14 feet between bearings, to carry 10 tons in the middle.

$$10 \text{ tons} = 22{,}400 \text{ lbs.}$$

$$\frac{22{,}400 \times 14}{270} = 1161, \text{ strength product.}$$

8 ins. broad and 12 ins. deep would suit this case, because $8 \times 12 \times 12 = 1152$, nearly 1161.

II. When the beam is fixed down at the ends.

Rule.—Find the strength produced as in I., and deduct from it its $\frac{1}{3}$rd part.

Example 10.—A cast-iron bar fixed at both ends 9 feet apart has to carry 10,200 lbs.

$$\frac{10{,}200 \times 9}{850} = 108$$

$$\text{Deduct } \tfrac{1}{3} = \underline{36}$$

$$72 = \text{strength product.}$$

2 ins. broad and 6 ins. deep, for $2 \times 6 \times 6 = 72$.

III. When the beam is loosely supported, and the load uniformly distributed.

Rule.—Halve the strength product.

Example 11.—A deal rafter 10 feet long is loaded uniformly with 9438 lbs.

$$\frac{9438 \times 10}{130} \times \tfrac{1}{2} = 363$$

3 ins. broad and 11 ins. deep, for $3 \times 11 \times 11 = 363$.

IV. When the beam is fixed at both ends, and has the load uniformly distributed.

Rule.—Divide the strength product by 3.

Example 12.—A wrought-iron bar 10 feet long, fixed at its ends, is loaded with 2080 lbs., uniformly distributed.

$$\frac{2080 \times 10}{1300 \times 3} = 5.333, \text{ or } 5\tfrac{1}{3} \text{ the strength product.}$$

If the bar be round, we must increase this by one-half, making it 8; and as 8 is the cube of 2, or 2 × 2 × 2, the round bar must be 2 ins. diameter.

V. When the beam is loaded not in the middle.

Rule.—Multiply the strength product successively by the two lengths into which the weight divides the beam, and divide twice by half the total length.

Example 13.—An ash-beam 14 feet long, has 10,182 lbs. suspended from it 3 feet from one end (consequently 11 feet from the other, half the length being 7 feet).

$$\frac{10,182 \times 14}{250} \times \frac{11 \times 3}{7 \times 7} = 384.$$

6 inches broad and 8 inches deep, for 6 × 8 × 8 = 384.

VI. When the beam is fixed at one end and loaded at the other.

Rule.—Multiply the strength product by 4.

Example 14.—An English oak beam projects 10 feet, and carries 10,433 lbs. at the end.

$$\frac{10,433 \times 10}{190} \times 4 = 2196.$$

13 inches square, for 13 × 13 × 13 = 2197.

The following rules embody the methods of dealing with the strength product when found as above.

1. Given the depth to find the breadth.

Rule.—Divide the strength product twice by the depth (in inches), the quotient is the breadth (in inches).

Example 15.—Beam 6 inches deep to give strength product 72.

$$\frac{72}{6 \times 6} = 2 \text{ inches, the breadth}$$

2. Given the breadth to find the depth.

Rule.—Divide by the breadth, and take the square root of the quotient.

Example 16.—Beam 8 inches broad strength product 1161.

$\frac{1161}{8} = 145$, the square root of which is nearly 12, for 12 × 12 = 144.

3. When the beam is square in section.

Rule.—Take the cube root of the strength product.

Example 17.—Square beam, strength product 2196. Cube root of 2196 = 13 nearly, for 13 × 13 × 13 = 2197.

4. When the beam is cylindrical.

Rule.—Increase the strength product by its half, and take the cube root.

Example 18.—Cylindrical beam, strength product $5\frac{1}{3}$
Add half of $5\frac{1}{3}$ = $2\frac{2}{3}$

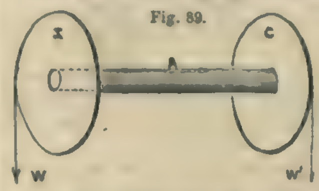
$$\overline{8}$$

Cube root of 8 = 2.

For beams of other forms than those which have square, rectangular, or circular sections, we cannot furnish any rule. Practical men, from long experience, and from a habit of observing the proportions suitable to certain strains, can form a tolerably accurate conception of the strength due to particular forms under various conditions. Before any mechanical work is executed, the appearance of the parts on the drawing recommends itself to a practised eye, or otherwise, according as the details are studied in consonance with the just proportions of strength or otherwise. In providing not only against absolute fracture, but also against deflection, vibration, and all such elements of degradation and weakness, the strength must be made very much in excess of that which the mere calculation of breaking-strain would indicate; and the different modes of providing adequate firmness and permanence without extravagant use of materials, are matters of ingenious contrivance for which no rule can be furnished.

4. TORSION.—As, in machinery, motion is generally conveyed from one part to another by means of shafts or spindles rotating on their axes, it becomes a matter of considerable importance to determine the strength of materials to resist a twisting or wrenching force. If we suppose an iron shaft fitted with a wheel at each end, one of which is driven by some prime mover, such as a steam-engine, and the other conveys the power or motion to some machinery, it is clear that the whole power so conveyed has to pass through the shaft; and the resistance of the machinery at the one end to the prime force applied at the other, subjects the shaft to a torsive or twisting strain.

Fig. 89.

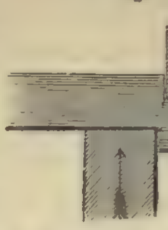


That we may simplify the view which we should take of this kind of strain, we shall suppose a shaft A (Fig. 89) fitted with a wheel at each end Z and C, round the circumference of each of which, in opposite directions, a rope passes, suspending weights W W′. The wheels and weights being equal and balanced, the shaft is not caused to turn in either direction; but both weights tend to twist the shaft in itself. If instead of one of the weights W′ we were to substitute a fixed pin or hook to which the rope might be attached, the rope would still be subjected to the same strain as if it had the weight suspended from it; and as it would react on its wheel with a force precisely equal to its tension, the twisting effect on the shaft would not be altered. We may, therefore, suppose the wheel and weight entirely removed from one end of the shaft, and that end fixed firmly in such a manner that it cannot turn or revolve on its axis, while it is throughout its whole length subjected to a twisting strain from the action of the weight at the other end. The power with which this weight tends to twist the shaft, depends both on its magnitude and on the size of the wheel round which its rope passes. The radius, or half-diameter, of the wheel is the leverage of the weight; and by increasing it we increase the torsive strain in like proportion. For ease of calculation, we will suppose the radius, or half-diameter, of the wheel to be 1 foot; and if we can find the torsion of a certain weight acting at a radius of 1 foot, we can reckon that of the weight at any other radius; or having found the weight to produce a certain torsion at 1 foot radius, we can easily reckon the radius at which the same weight would produce some other amount of torsion.

Thus, by doubling or trebling the radius we double or treble the torsive strain; by doubling or trebling the weight, we have to place it at one-half or one-third the radial distance from the centre of the shaft, in order to subject it to the same torsion; and so generally in simple proportion. The torsive strain, therefore, is as the weight multiplied by its radial distance. One ton at a radial distance of 3 feet, is equivalent to 3 tons at a radial distance of 1 foot, because 1 ton × 3 feet = 3 tons × 1 foot.

We have now to inquire what effect the dimensions of the shaft, or the quantity of material in it, has in resisting a certain torsive strain; how large the shaft must be to resist a given strain, or what strain a shaft of given dimensions can resist. The replies to these questions must depend upon the form of section and the nature of the material employed; but if we can ascertain, by direct experiment, the torsive strength of a shaft of certain material, form, and dimensions, we must endeavour to find a principle on which to reckon the strength of a shaft of the same form and material, but of different dimensions. We may imagine two plates of metal or other material, placed face to face, having such corresponding projections and hollows in their surfaces, that the projections of the one fit into the hollows of the other. We may suppose these plates pressed together, and some force applied at their edges, so as to push the one along the face of the other. It is manifest, that the force for this purpose must depend upon the amount of roughened surface in contact, upon the pressure squeezing the plates together, and the peculiarities of the roughness which their surfaces present. Instead

Fig. 90.



of two separate plates put artificially together, we may conceive, that at any place where a solid body might be shorn across, the particles of the body, or the grains of which it is composed, fit into the interstices of each other; and the two parts into which the body may be divided or shorn are held firmly together by the cohesion of the particles over the whole surface where the division takes place (Fig. 90). In order to effect a separation by pressing the body in opposite directions, as with shears, we should have to overcome the cohesion, and the resistance which the asperities present to the slip across the rough surface of separation. The actual amount of cohesive attraction and the resistance from asperity, must mainly depend upon the peculiar nature of the material acted on; but the relative amount of resistance which two pieces of the same material of different dimensions present, seems to depend simply upon the number of particles separated; that is to say, on the area of the section or quantity of surface separated. Thus, to separate crosswise, a piece of iron 2 inches square, we should expect to apply four times as much force as would be required to separate a piece of the same iron 1 inch square, because the piece 2 inches square has an area of section, or a surface of separation, four times as great as the piece 1 inch square. But, farther, if we suppose the separation to be effected by means of shears (Fig. 91), so arranged that the 2-inch piece is placed twice as far from the pivot or fulcrum of the lever, of which the shears form the arms, as the 1-inch piece, we should have to apply double the force to divide the 2-inch piece, on account of the disadvantage of leverage; and, on the whole, in such a case we should require eight times the force to divide the 2-inch piece that we require for the 1-inch piece. If now, instead of shearing across a square bar, we apply this principle to the fracture of a cylindrical bar or shaft, by torsion, the centre of the shaft becomes the fulcrum of the lever, which we may assume to be one foot long; and each portion of the sectional area of the shaft resists separation with a force proportional to its area, and to its leverage or distance from the centre C (Fig. 92). If we take any small square of the section, such as A, and suppose a ring traced inclosing it, it is clear that the resistance of every part of that ring of particles, of the same magnitude as A, offers the same resistance, because it has the same leverage, or is equally distant from the centre; and that, therefore, the total resistance of the ring is as its area multiplied by its leverage A C. But if the ring be very narrow, its area is nearly its circumference multiplied by its breadth, and its circumference is as its radius A C; therefore its resistance is as its breadth multiplied by its radius twice, or the square of its radius.

Fig. 91.

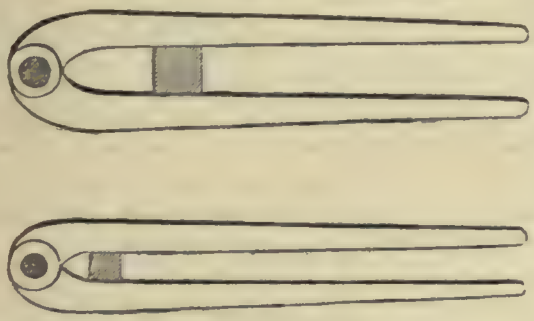
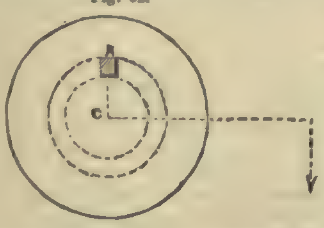
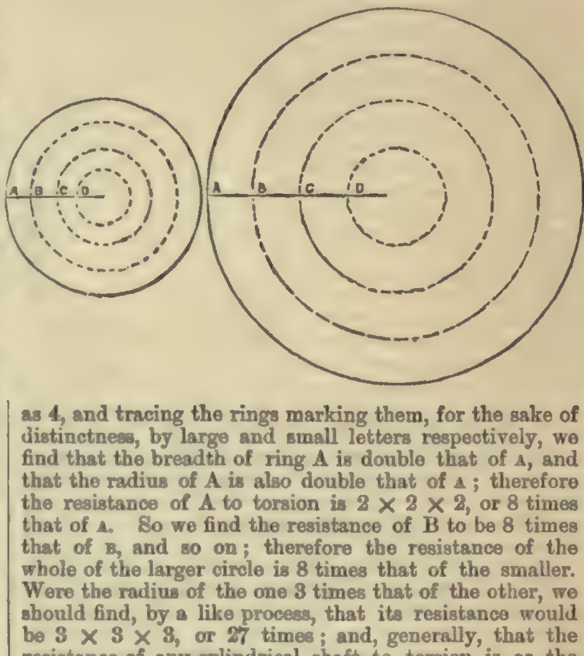


Fig. 92.



Were we to suppose, then, the whole circle divided into numerous narrow rings of equal breadth, the resistance of the whole to separation by torsion would be the sum of the resistances of all the rings, the resistance of each being as its breadth multiplied by the square of its radius. Now, if we compare a circle of one radius, such as 12 ins., with another, such as 6 ins. (Fig. 93), dividing each radius into an equal number of parts, such as 4, and tracing the rings marking them, for the sake of distinctness, by large and small letters respectively, we find that the breadth of ring A is double that of a, and that the radius of A is also double that of a; therefore the resistance of A to torsion is 2 × 2 × 2, or 8 times that of a. So we find the resistance of B to be 8 times that of b, and so on; therefore the resistance of the whole of the larger circle is 8 times that of the smaller. Were the radius of the one 3 times that of the other, we should find, by a like process, that its resistance would be 3 × 3 × 3, or 27 times; and, generally, that the resistance of any cylindrical shaft to torsion is as the cube of its radius, or of its diameter (which is the double of its radius).

Fig. 93.



This general proposition is almost self-evident; for if we admit that the resistance of any part of the circular section is as its area multiplied by the leverage at which it acts, the resistance of the whole circular section must be as its area multiplied by the mean or average leverage, of all its parts. The areas of circles are as the squares of their diameters, and in every circle, the mean leverage is proportional to the diameter; therefore the resistance to torsion is as the area or square of the diameter multiplied by the diameter—that is to say, as the cube of the diameter.

The reasoning we have used is founded on the assumption that the effect of excessive torsion is to divide or shear a shaft across at some place where the material is accidentally weaker than elsewhere. But in shafts of uniform strength throughout their length, and of fibrous texture, this is by no means the effect of destructive torsion. In a wrought-iron shaft, for instance, the fibres are made to twist, so that, in some cases, the outer surface presents the screw-form of a cord or rope; and ultimately they separate and present a fracture where parts seem to have been dragged out of their place by this twisting action. In producing such a twist of fibres, there must be a considerable elongation, particularly of the outer fibres; for it is longer round the thread of a screw than in a direct line from one end of a bar to the other; and the greater the diameter of the thread, or the farther it is from the centre, the longer is its course. If, then, we confine our attention to any set of fibres similarly situated, in two bars of different diameters subjected to torsion, we see, in the first place, that in the larger bar the number of fibres in the portion of the section referred to, is greater than the number in a similar portion of the smaller bar, in proportion as the square of the one diameter, or total area of the larger, exceeds that of the smaller. Again, in the larger bar the leverage of the set of fibres to resist the screw elongation, or the amount of the elongation which they have to undergo, exceeds that of the smaller, in proportion as the one diameter exceeds the other. Therefore, upon the whole, we must conclude, as before, that the resistance of the larger to torsion, is to that of the smaller, as the cube of the one diameter is to that of the other.

The same principles of reasoning apply equally to other than circular forms of section. The torsive strengths of square bars follow the same law; as do those of any other forms, provided they be similar, or have all their corresponding parts proportional. In comparing bars of circular with those of square section of equal diameter (Fig. 94), we readily see that the torsive strength of the square must exceed the other, not only on account of the square having the larger area, and, therefore, the greater number of fibres, or points of cohesion, but, also, because the additional area is situated at places farther from the centre, and, therefore, resisting with more leverage than any parts of the circular section. On the other hand, we observe, that while all the fibres of the circular section are supported against being forced aside by those around them, the fibres in the angles of the square section are not so supported, and may be expected to yield more readily to the twisting force. Upon the whole, it has been found that the torsive strength of the square section is about one-fourth greater than that of the circular. As the distance of any fibre from the centre contributes much to its resisting power, it may be advantageous to remove some of the material of a shaft from the inside, where it is of little value to resist torsion, and place it on the outside, where it acts with greatly increased leverage. On this principle, hollow or tubular shafts present the advantage of economising material, and securing strength with lightness; for each of the fibres in the ring section of a tube has so much greater leverage than those towards the centre of a shaft, that their number, and consequently the sectional area of the ring, may be considerably diminished without lessening the torsive strength.

Fig. 94.

From a number of experiments, made with a view of ascertaining the torsive strength of different materials, we are enabled to compile a table, from which the strength of a shaft of given materials and dimensions may be calculated; or, conversely, the dimensions may be computed of a shaft destined to sustain a given torsive strain.

Table of torsive strengths of cylindrical shafts 1 *inch in diameter; weights acting at* 1 *foot leverage.*

Name of material.	Permanent load.	Name of material.	Permanent load.
Brass	150 lbs.	Lead	34 lbs.
Copper	135 ,,	Steel (blister)	560 ,,
Gun-metal	170 ,,	Steel (cast)	590 ,,
Iron (wrought, English)	335 ,,	Steel (shear)	570 ,,
Iron (wrought, Swedish)	390 ,,	Tin	47 ,,
Iron (cast)	330 ,,		

The table, col. 2, gives the weight in pounds placed at the end of a lever 1 foot long, which a cylindrical shaft, 1 inch in diameter, will permanently sustain without injury; and the following rules embody the methods of computing the torsive strengths of other shafts, according to their diameter.

I. Given the diameter of a cylindrical shaft, and the length of lever at which the twisting weight acts, to find what weight it will permanently sustain.

Rule.—Multiply the number in the table three times by the diameter (in inches), and divide by the length of lever (in feet).

Example 1.—Required the strain that can be applied at the end of a lever 2 feet long to turn a cast-iron shaft 4 inches in diameter.

$$\frac{330 \times 4 \times 4 \times 4}{2} = 10{,}560 \text{ lbs., nearly } 4\tfrac{3}{4} \text{ tons.}$$

Example 2.—A rope passes round a pulley 3 feet in diameter, fixed to a gun-metal spindle 2½ inches in diameter; required the weight that may be attached to the rope.

Since the diameter of the pulley is 3 feet, the leverage of the weight is 1½ ft.; and

$$\frac{170 \times 2\tfrac{1}{2} \times 2\tfrac{1}{2} \times 2\tfrac{1}{2}}{1\tfrac{1}{2}} = 1{,}771 \text{ lbs., or nearly 16 cwt.}$$

II. When the shaft has a square section.

Rule.—Add ¼th to the weight.

Example.—Required the weight that may be hung to a lever 5 feet long, fixed to a wrought-iron (English) square shaft 2 inches broad.

$$\frac{335 \times 2 \times 2 \times 2}{5} = 536 \text{ lbs.}$$

$$\text{Add } \tfrac{1}{4} = 134 \text{ ,,}$$

$$670 \text{ lbs., nearly 6 cwt.}$$

III. Given the weight and lever, to find the diameter of a cylindrical shaft.

Rule.—Multiply the weight (in lbs.) by the length of lever (in feet), divide by the number in the table, and extract the cube root for the diameter (in inches).

Example 4.—A cast-steel spindle has to withstand the torsion of 5 tons at the end of a lever 8 feet long: required its diameter.

$$5 \text{ tons} = 11{,}200 \text{ lbs., and } \frac{11{,}200 \times 8}{590} = 152 \text{ nearly.}$$

The cube root of 152 is about 5⅜ inches.

IV. When the shaft is square.

Rule.—Multiply the weight by the lever, divide by the tabular number, deduct ⅕th of the result, and extract the cube root.

Example 5.—A square shaft of English wrought-iron has to sustain 6 cwt. at the end of a lever 5 feet long: required the breadth.

$$\frac{6 \text{ cwt.} = 672 \text{ lbs.} \times 5}{335} = 10$$

$$\text{Deduct } \tfrac{1}{5}\text{th} = \ 2$$

$$8, \text{ cube root 2 inches.}$$

The table and rules apply only to cases where a simple regular strain is applied. When shafts or spindles are intended to convey motion to machinery, they are generally subjected to great irregularities of torsive strain; and though they may only convey upon the whole a certain power, yet at different periods of their revolution they may be subjected to strains ranging from nothing up to many times the average strain due to the power they convey. If we consider for a moment the conditions under which the crank-shaft of a steam-engine rotates, we shall see the great variation of strain to which it is exposed. At two points of each revolution, what are technically called the *dead-centres*, the steam power has no effect to turn the shaft round; but, on the contrary, the momentum stored up in the fly-wheel turns the shaft, and through it gives movement to the parts of the engine. At two other points, when the connecting-rod of the engine is acting in the most favourable position, the shaft receives a torsive strain through the crank, greater than the total pressure of the steam on the piston; and the amount of this strain depends on the total steam-pressure on the piston, the obliquity of the connecting-rod, and the length of the crank. But all machinery is besides subject to accidental irregularities of strain. For instance, in the engines of a steam-vessel propelled by paddle-wheels, sometimes in a heavy sea, while the engines and paddles are moving at full speed, a wave strikes one paddle and suddenly immerses it to a great depth in water, so as at once to retard its rotation. The shock thus communicated to the engine, through the shaft which drives the paddle, is enormous, and occasionally more than 100 times the average force passing through the shaft. It is, therefore, essential to make all shafts intended to communicate power, of much greater strength than what is due to the mere average strain passing through them. The power that is communicated through any shaft is generally reckoned in horse-power; and as power consists of two elements—pressure or weight moved, and the velocity or speed with which it is moved—it is necessary to ascertain not only the assumed power passing through a shaft, but also the speed with which that shaft rotates, or the number of revolutions it makes in a given time, such as a minute, before we can compute the strain to which it is subjected, and the dimensions of which it should be made. The more quickly a shaft rotates in communicating a certain power, the less is the torsive strain to which it is subjected, for power is pressure or strain multiplied by velocity; and if, to produce a certain power, the velocity be increased, the strain must be proportionally diminished. The shafts of engines, and machinery connected with them, are generally made of wrought or cast-iron. As wrought-iron is more flexible and tenacious than cast-iron, it is less subject to fracture by sudden variations of strain, and is therefore preferable to cast-iron for shafts, and may be made considerably lighter. Again, the shafts which communicate the first effort of the power from the steam-pressure to the fly-wheel, are subjected to much greater variations of strain, than those which communicate the power afterwards from the fly-wheel shaft to other machinery. It is, therefore, advisable to give the first shafts, or prime movers as they are called, greater strength than is necessary for second movers. Practical men generally make the prime movers about twice as strong as the second movers; and the following rules embody the modes of computing dimensions of shafts for conveying given power at given velocities.

I. For wrought-iron prime movers, given the horse-power, and the number of revolutions per minute, to ascertain the diameter.

Rule.—Divide the power by the velocity, extract the cube root of the quotient, and multiply by 7, for the diameter in inches.

Example 1.—Required the diameter of a wrought-iron prime-moving shaft for 100 horse-power, making 20 revolutions per minute.

$\frac{100}{20}=5$, cube root 1·7 inches, and $7 \times 1·7=11·9$, nearly 12 inches.

II. For cast-iron prime movers.

Rule.—Divide the power by the velocity, and take 7½ times the cube root of the quotient.

Example 2.—Required the diameter of a cast-iron prime-moving shaft for 50 horse-power at 25 revolutions per minute.

$\frac{50}{25}=2$, cube root $1·26 \times 7\frac{1}{2}=9·45$ or 9½ inches.

III. For wrought-iron second movers.

Rule.—Divide the power by the velocity, and take 5½ times the cube root of the quotient.

Example 3.—Required the diameter of a wrought-iron second mover for 40 horse-power at 15 revolutions per minute.

$\frac{40}{15}=2·6$, cube root $1·4 \times 5\frac{1}{2}=7·7$ or 7¾ inches.

IV. For cast-iron second movers.

Rule.—Divide power by velocity, and take 6 times the cube root of the quotient.

Example 4.—Required the diameter of a cast-iron second mover for 22 horse-power at 36 revolutions per minute.

$\frac{22}{36}=0·6$, cube root $0·87 \times 6=5·22$ or 5¼ inches.

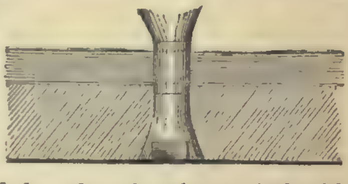
For shafts under very regular strain, and distantly connected with the prime mover, the multipliers may be for wrought-iron 5, and for cast-iron 5½.

The dimensions computed by the above rules are those of the weakest part of the shafts. When shafts are of great length, or have great weights, such as fly-wheels, and the like, fixed upon them, they are subject to the deflection due to transverse strain. This deflection in a revolving shaft is an important element of weakness, for it continually changes in direction as the one side or other of the shaft is uppermost, and thus subjects the material to all the degradation of alternate strains in opposite directions. In breaking a piece of wire, it is not unusual to bend it backwards and forwards until its tenacity is quite destroyed. The same kind of action occurs in a shaft too light for its length, or for the weight it bears, and gradually lessens its tenacity, and deprives the fibres of their continuous texture. The very best material employed in shafting undergoes a peculiar change in its molecular constitution, after having been long kept in revolution. The iron becomes quite short and crystalline, although at first fibrous and tenacious, and will often break off as short as cast-iron in a shaft which has been working for some years. This result is very common in railway axles, which are exposed to great vibration; and extreme cold has a similar tendency. For such cases the strengths should be made considerably in excess of what they are computed to be. In this, as in numerous other instances, experience must be the guide of the practical mechanic in the absence of set rules.

5. CLIPPING OR SHEARING STRAIN.—The strain which a body undergoes when it is divided across by means of shears, or any instrument consisting of two blades that pass one another like those of a scissors or shears, is of a kind very distinct from any of the other strains which we have discussed. Under *tensive* strain, the fibres are torn asunder by a force in the direction of their length; under *compressive* strain they are forced out of their place, and have their lateral cohesion destroyed; under *transverse* strain they are subjected partly to compression and partly to extension; under *torsive* strain they are extended in a screw or spiral direction; but under the *clipping* strain the particles are forced across each other, as we have described in endeavouring to investigate torsive strain on the principle of shearing.

Besides actual clipping by instruments for the purpose, there are various circumstances under which materials may be exposed to a strain of a similar character. In punching holes in a plate of metal, a similar action takes place. The punch (Fig. 95) is a piece of steel, the end of which is made of the size and shape of the hole to be punched, the edges being keen and not rounded off; the matrix is a block of steel, or iron faced with steel, having a hole such as is to be punched, with edges also keen. The punch and the matrix are so adjusted that the former shall pass exactly into the latter; and a plate of metal being placed on the matrix while the punch is up, has a hole pierced in it by its descent—the piece of metal being punched or pressed down into the hole of the matrix. When the edges of the punch and matrix are keen, and they accurately fit each other,

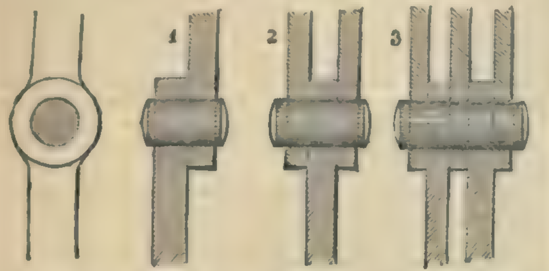
Fig. 95.



the metal of the plate is shorn cleanly round the circumference of the hole; and the force required to effect the operation, appears to be the same as would be required to shear a plate of the same material and thickness as that punched, over a length equal to that of the circumference of the hole.

Another example of shearing strain occurring in practice, is that to which the pins uniting the limbs of a chain are subject (Fig. 96). 1. If the joint consist of single limbs united by a pin, a strain applied to the limb longitudinally tends to shear the pin across at one place.

Fig. 96.



2. If the links be double and single alternately, the pin is exposed to be shorn at two places. 3. Where the links are three and two alternately, the shearing must take place at four places; and, generally, whatever be the number of links embracing each other at both sides, the number of places at which the pin must be shorn is double the smaller number of links. We naturally conclude that, if a certain power be required to shear a pin at one place, it would require double the force to shear it at two, triple at three, and so on, an additional force being required for each additional separation of the substance made. Practically, this is found to be true. Carrying the same reasoning to the consideration of the strains required for shearing pins of different sizes, we conclude that if we double the area of section—that is, the surface where the separation is effected or the number of fibres shorn—we have to double the force necessary to shear it. We, therefore, conclude that the strength to resist shearing, or the force required to shear, is proportional to the area of the body shorn at the place where the separation is effected: and here, also, our reasoning is borne out by experiment. A pin of iron 2 inches in diameter, requires 4 times the force to shear it that a pin 1 inch in diameter requires, because the area of the circle 2 inches is 4 times, or 2 × 2 times, the area of a circle 1 inch in diameter.

It is not uncommon, in machinery, to couple two rods A and B in the manner indicated in Fig. 97. The ends of the rods being turned truly cylindrical, and a socket C bored to fit on them, the ends are inserted in the socket, and keys D and E are driven into slits cut through the socket and the rods. On a great longitudinal strain being applied to the rods, so as to pull them out of the socket, fracture may occur in one of the following ways:—

1. Either of the rods, or the socket itself, may give way under the effort of direct tension; and if so, they must yield at the weakest place, which is manifestly where the keys pass through them, part of their sectional area being occupied by the keys, which add nothing to the cohesive strength.

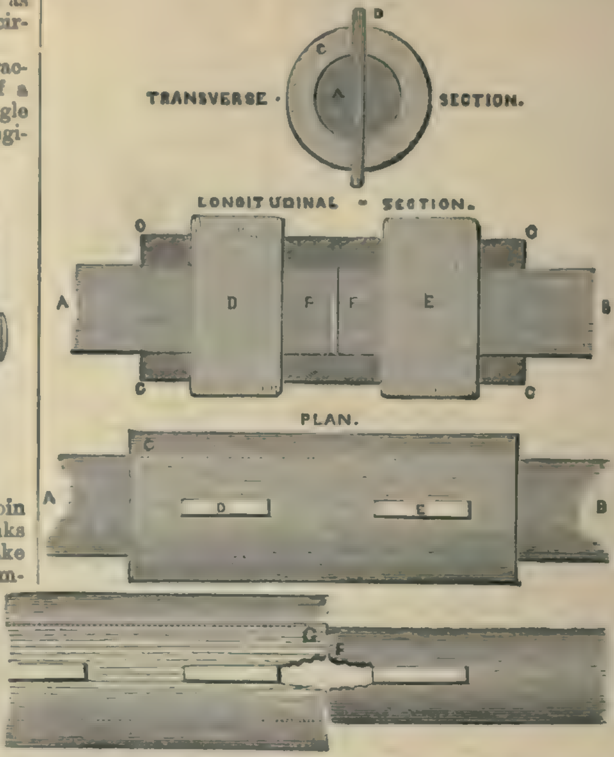
2. Either of the keys may be shorn across at the two places where they leave the socket, and enter the rods.

3. Either of the rods may have the material between the key and its end drawn out, so as to leave an opening F, the material forced out being punched or shorn, at both sides by the strain on the key.

4. Either end of the socket may have the material at G drawn out in a similar way.

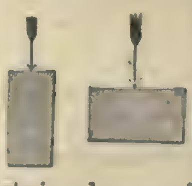
Now, if the material be of the same quality throughout, the strength to resist fracture in the 2nd, 3rd, and 4th ways may be equalised, because they are all shearing strains of the same kind, and we have only to make the several areas of the parts that would be shorn, all equal; that is to say, the sectional area of each key D and E should be equal to that of the end part F of each rod, because the shearing would separate two surfaces in either case; and the area of the part G of the socket need be only half that, because four surfaces would be separated. But we have as yet established no relation between the strength to resist tensive strain and that to resist shearing, and cannot, therefore, without further information, compute the proportional strength of the effective transverse sections of the rods and socket where the keys pass through them. The truth is, that very few experiments have been made upon the strength of materials to resist a shearing strain. Such as have been made with malleable iron, seem to show that it is precisely equal to the strength to resist tension. We believe that with other materials this law may be very safely assumed as correct. As the tensive strength is also proportional to the area of section, the areas to resist equal tensive and shearing strains should be equal. Accordingly, we should make the transverse sections of the rods and socket such, that the area of each part, on either side of the key, shall be equal to the transverse section of the key; for the key would be shorn at two places, and the rod or socket would be pulled asunder at the two places on either side of the key. On examining the figure, it will at once suggest itself that the keys should be made narrow and broad (measuring the breadth in the direction of the rods lengthways), so as to trench as little as possible upon the sectional area of the socket and rods.

Fig. 97.



Although for round or square pins, or bars, the law that the strength to resist shearing, is very nearly the same as the cohesive strength, yet there is little doubt that by increasing the depth of a bar exposed to be shorn, the breadth being diminished so as to retain the same area, the strength is materially increased. If we placed a bar of iron having 2 ins. × 1 in. of sectional area under a shearing strain edgeways, it would certainly resist more than when placed sideways. We are not aware, however, that experiments have been made to a sufficient extent to warrant us in laying down any law as to this.

Fig. 98.



In practice, no material error will arise from the assumption of the simple law we have already stated; and if strengths to resist shearing be calculated according to it, let the mechanic, as far as he can, contrive to bring the shearing strain against the greatest depth, and he will certainly be safe against fracture from this cause.

In punching a hole in metal, the surface of the separation produced, is the circumference of the hole, multiplied by the thickness or depth; and the resisting force should be equivalent to the tensive strength of a bar of the same material, having a sectional area equal to the surface of separation. Thus, in estimating the force required to punch a circular hole, 1 inch in diameter, through a plate of malleable iron ¾ths of an inch thick, we should reckon the circumference of a circle 1 inch in diameter is 3·1416 inches; multiplying by the thickness, ¾ths of an inch, we have the area of the surface of separation, 2·3562; and therefore the strength to resist the punching, is that of a bar of iron having a sectional area of 2·3562 square inches to resist fracture by tensive strain. The weight required to tear asunder a bar of iron 1 square inch in sectional area, is found, by experiment, to be about 26 tons; and the weight to tear asunder 2·3562 square inches would, therefore, be about 61 tons, which we reckon as the force required for punching the hole such as we have described.

The subject of shearing strain is, upon the whole, somewhat obscure. The experiments have been few, and made rather with a view to the solution of other questions than the determination of a law as to this strain. We would therefore caution the practical man against placing too much dependence on any computations made in respect of it. Where he is in doubt, and has no case practically carried out to which he may refer, it is far better that he should make experiments for himself; or, in the absence of data which he might deduce from them, at least he should make his work rather in excess of strength than otherwise. Great evils may often result from the execution of work, where the dimensions are too closely approximated to the results of calculation as to strains and strength. These evils are of an uncertain character—they may occur unexpectedly, under circumstances that may aggravate the mischief they cause; great expense may be incurred, limbs may be broken, life may be endangered or destroyed, and all because material has been grudgingly employed or disproportionally applied. But when strength is given in excess, to any structure intended to be permanent, the first evil, that of additional cost, is the only one—i tis known as to amount, is undergone, and appears not again—and the artificer has the satisfaction of thinking, that if difficulties or dangers arise during the use of the work he has contrived and formed, it must be from adventitious causes for which he cannot be held blameable.

Indeed, the whole subject of strength of materials is in an unsatisfactory state. Every new experiment made, every new work completed, adds somewhat to our practical knowledge in this department of mechanics. But so long as the materials we use vary in quality, are wanting in uniformity, and are subject to the changes which moisture, temperature, and other circumstances impose upon them, so long must we remain incapable of estimating correctly their strengths for any particular purposes, and so long must we be prepared to make large allowances for all such variations, and for contingencies which do not always readily suggest themselves.

CHAPTER IV.

SOURCES OF MECHANICAL POWER.

Contents.—MUSCULAR FORCE—WIND, DIRECT, OBLIQUE—WINDMILLS, SAILS, GOVERNORS—WATERWHEELS, UNDERSHOT, OVERSHOT, BREAST, AND COMBINED—DAMS AND LEADS—TURBINES—HYDRAULIC RAM—WEIGHT—FALLING BODIES—SPRINGS—HYDRAULIC LIFT AND CRANE—HEAT AND ELECTRICITY.

THE principal sources of power may be classed under five heads.

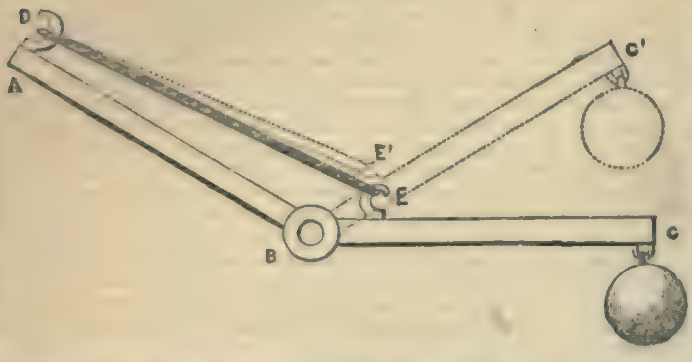
1. Muscular force of men and animals.
2. Natural movements of air.
3. Natural movement of water.
4. Weight and elasticity of bodies.
5. Heat, electricity, magnetism, and chemical action.

1. MUSCULAR POWER OF MEN AND ANIMALS.—It is not our province to discuss the mode in which the muscles of men and animals are put in action. This is a question which belongs to the physiologist. Suffice it to say, that the mechanism of animal structure is of that most perfect kind, which characterises all the works of Him who designed and executed it. The simple effort of *will*, or whatever that particular mental faculty be called, appears to communicate itself through the nerves which pervade the whole animal frame like a network of electric wires, and to operate immediately on the muscular tissue. The muscles, so far as we understand their nature, appear to have no innate force of their own, but are merely organised instruments put in action by the influence transmitted to them by the nerves, just as the index of an electric telegraph is perfectly inert until it is deflected, one way or other, by the subtile magnetic influence transmitted to it through the electric wire. Notwithstanding the immense variety of movements of which the different parts of the animal frame are capable, the action of the muscles, which effect those movements, is of one kind only—contraction in length, accompanied by expansion in diameter.

The eyes are caused to rotate in their sockets, and their focus as optical instruments can be varied; the lips, the tongue, the throat, and the jaws are moved so as to give voice to musical notes and articulate language; the shoulders, arms, wrists, hands, and fingers are put in motion so as to enable man to lift heavy weights, or to manipulate with consummate delicacy; the back, body, the legs and feet, are moved so that man may walk, run, or leap, row, or perform feats of strength and agility. And yet every one of these motions is effected by the longitudinal contractions of muscles under the nervous influence of volition. The frame-work of this exquisite machinery is the skeleton, consisting of numerous bones joined together. The form, length, strength, and situation of every bone have been devised with most minute care to suit its particular object. Each bone plays the part of a beam, and has attached to it a tendon or strong cord near its joint or fulcrum. This tendon is the end or continuation of a bundle of fleshy fibres, which constitute a muscle, fixed at the opposite end to some other part of the frame-work; and as these fibres contract in length under the nervous influence, the tendon or cord is pulled, and the bone or lever caused to move round its joint or fulcrum.

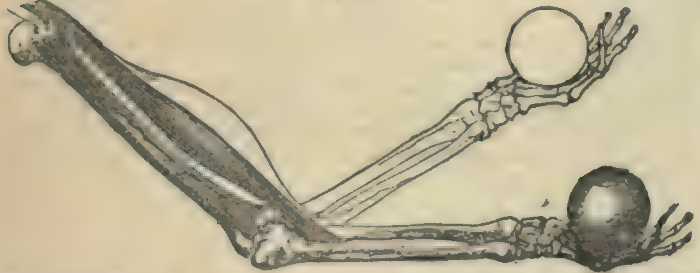
We may suppose A B (Fig. 99) to represent a fixed bar, having a joint at B fitted with a lever B C, from the extremity of which a weight is suspended; and D E a cord fixed at D to the bar A B, and at E to the lever B C. By shortening the cord D E we cause the lever to turn about its fulcrum B, to some such position as that marked by the dotted lines E′ C′. So in the human arm (Fig. 100), the upper bone is the bar jointed at the elbow; the radius bone terminates in the hand, in which a weight may be placed; the cord is the biceps muscle, fixed at the upper end, and terminating at the lower end in a tendon, which is fixed to the radius. The longitudinal contraction of this muscle pulls the tendon, and thus causes the radius to turn upon the elbow-joint, and raise the weight at its extremity.

Fig. 99.



Besides the direct movement of the bones as levers, such as we have described, there are numerous modified motions, sideways or rotatory, which are all effected by the contraction of muscles specially arranged for their respective purposes. For instance, in order to turn the hand upside down, it is necessary to make the arm rotate or turn upon its axis. This is effected by muscles placed spirally round the bones of the arm. By diminishing the length of one of these, the bones are caused to twist round. And so for every movement of every part of the frame, a special muscle is provided to effect the required motion by simple contraction.

Fig. 100.



The force or contracting energy of some of these muscles is enormous, as may readily be believed, when it is remembered that they are attached to the short end of a losing lever, and yet are capable of sustaining great weights at the extreme end, where the weight acts with immense leverage.

Some of the muscles are not placed exclusively under the control of the will, but act regularly without any effort of volition. Such are the muscles which effect the action of the heart, and organs of respiration. Some muscles, again, seem to be put in action by an effort of will of which we are not conscious. Such are those of the eye to direct and focus it, and those of the tongue, mouth, and throat in speaking. Although we *will* to speak or articulate certain sounds, we never think of the particular muscles that we must put in action, in order to effect the articulation desired. So in walking and other actions which have become familiar by habit, no thought is given to the special muscular movements required. It seems to be enough to *will* to walk, and the muscles obey. It is only when fatigue and exhaustion render their exercise painful, that we have to employ special acts of volition in order to effect our purpose.

In making use of muscular power as a moving force, it is important to call into exercise those muscles which require the least effort of will for their movement, and to apply them in the way least calculated to fatigue. The animals chiefly employed—the horse, the ox, and the camel—can only be used in particular ways, as for bearing and dragging loads. The horse and the ox are best adapted for dragging, as neither of them is capable of sustaining great weights. The camel, on the other hand, moves with facility under a very considerable load placed upon its back. The elephant is employed in tropical countries both for carrying and dragging, and appears to have enormous strength for either purpose. When human power is employed, it may be exercised in various ways. The operation in which man seems capable of exerting the greatest power, for the longest period, is that of rowing. In moving the oar against the resistance of the water, almost every muscle of the body is called into operation. The spine, bent forwards in the back-stroke, is thrown back as the pull is taken; the shoulders and arms are also thrown back, the legs are stiffened against the foot-board, and thus immense power is applied to the oar, without excessively straining any one or two particular muscles. In excavating earth or clay, conveying loads up steep inclinations by a wheelbarrow, and generally in the operations for which *navigators* are employed, a large amount of power is developed. The muscular frames of these men, and their long practice in particular operations requiring great strength, enable them to continue for long periods at work without much fatigue; and to one watching minutely their mode of working, while there is little appearance of effort, yet there is manifest a regulated play of the muscles, and a sort of contrivance for doing the work with the least possible movement, which indicate that every one of these muscular movements is the result of habit, without the need for a special effort of will.

In pumping, as in rowing, numerous muscles are exercised, and accordingly this kind of labour can be continued for a long period. There is, however, a monotony about the operation, and a want of visible effect to satisfy the mind of the workman, which tend to make him weary of his work. In rowing a boat, excavating, lifting or moving a load, there is a visible progress made, and the workman has a satisfaction in observing the change effected by his exertion, and can reckon the advance he has made, and the amount still remaining to be done; but in the continuous discharge of a body of water from a pump, drawn probably from an unseen source, and flowing into an unseen reservoir, or into one so large that additions to its contents produce no visible effect, there is nothing to reckon by, no progress apparent; and the workman accordingly becomes fatigued, more from want of mental satisfaction, than from actual weariness of body. It is true that cases frequently occur where seamen and passengers on board a leaky vessel, work for many days continuously at the pumps. But in such a case, life is at stake; and the excitement of danger, fear of death, and hope of safety, are impulses to exertion capable of overcoming much bodily fatigue and mental lassitude. In all operations demanding manual labour, it is good economy to let the workman see the effect of his efforts: let his mind be satisfied, and his body will the longer withstand fatigue. Where labour is used as a punishment, it is certainly an aggravation of the penalty to make the labour of a kind that manifests no result. The perpetual walking up the rounds of a treadmill, or perpetual turning a winch to drive unseen machinery, is therefore a penal labour of a most severe character, although the actual muscular exertion which either demands is not great. Prisoners working at the treadmill always prefer grinding corn to "grinding air," as they express the act of simply turning the wheel.

As rotatory motion is generally that kind of movement best applicable to machinery, being most easily modified in direction and velocity, labour at the winch is the kind of manual exertion chiefly used. Before steam machinery became common, the number of operations effected by manual labour was very much greater than it is now.

At present, except for special purposes—such as driving a printing-machine, or a turning-lathe, where the quantity of work to be done would not warrant the application of costly motive power—manual labour at the winch is little used. Engineers of the last generation, with whom human and animal labour was an important consideration as a source of motive power, devoted considerable care to experiments as to the best modes of applying it, and the quantity of power developed. The main result of their deductions was, that in moving machinery, the power of a horse was on the average equivalent to a weight of 33,000 lbs., lifted 1 foot high in 1 minute; and this has accordingly been adopted by engineers as a standard by which all powers, whether of steam-engines, water or wind-mills, are measured.

The power of a man is about one-fifth of that of a horse; that is to say, on the average, a man can exert a power equivalent to the lifting of a weight 33,000 lbs. one foot high in 5 minutes, or 6,600 lbs. one foot high in one minute. These estimates are not applicable to a single effort for a short period, but to labour such as could be sustained for hours daily, and continued on successive days. When the power of a man is continuously applied to a winch, it should not be estimated at more than one-half of that given above; that is to say, 3,300 lbs. raised one foot high in one minute. It is found in practice, that a man working a winch can, for a considerable period, move his hand through 120 feet per minute, exerting an average pressure of 30 lbs. At some parts of the revolution of the winch he exerts more than double this pressure, at other parts less than half; but, on the whole, the mean pressure may be taken at 30 lbs. Now, 30 × 120, gives a power of 3,600 lbs. raised one foot per minute, which is rather above the estimate of 3,300, given above. If the radius of the winch be 1 foot, the diameter of the circle in which the hand revolves is 2 feet, and its circumference $6\frac{2}{7}$ feet; therefore he makes about 19 revolutions per minute, for $19 \times 6\frac{2}{7} = 120$ nearly. For a short period, he could turn the winch at the rate of 25 or 30 revolutions per minute, and exert an average pressure at the handle of 40 lbs. to 50 lbs. But this high estimate should only be used in calculating when a weight is to be lifted during some minutes, and not when the work is constant. For driving machinery, affording a constant resistance during several hours, the lower estimate of the man's power, viz., 3,300 lbs., raised one foot high in one minute, should be taken.

In carrying a load on his back, over level ground, a man can develop a power of about 8,000 lbs. moved over one foot in one minute.

In carrying a load up a staircase, his power is equivalent to 2,000 lbs. lifted one foot high in one minute.

In dragging a hand cart on level ground, his power is about 20,000 lbs. moved one foot in one minute.

In rowing a boat, the power developed is about 25,000 lbs. moved one foot in one minute.

For temporary efforts, the following estimates have been made:—

A man can bear, for a short time, standing still, about 300 lbs. weight.

He can lift vertically about 280 lbs.

He can grasp by his hand with a pressure of about 100 lbs.

The principal applications of manual labour are not those in which great force is required to be developed. Dexterity and skill of manipulation are the qualities chiefly desired in a workman. As more and more progress is made in mechanics, so does the necessity for employing the bodily forces of men diminish. Indeed, a large proportion of the skilled workmen now employed are engaged in merely watching the operation of self-acting machines. Machines are more rigorously true in the work they perform than the hand; but they are limited to the constant repetition of similar operations. They cannot set their work, adjust their own movements, or vary their efforts, as man can do; but in all these matters they may be controlled by man. It is not our province to inquire into the moral effect produced on workmen, by the extended use of self-acting machinery. It is true that much of their corporal labour is saved, and that a vast deal more work is done more economically; and necessaries and luxuries are thus brought within the reach of millions who would else be deprived of them. At the same time, it must not be forgotten, that the workman is, to a certain extent, degraded into a mere machine, only a little elevated above that, the operations of which he watches and controls. He has no longer to acquire skill and dexterity in his craft—machinery does the work better than he could ever hope to do; he has not to think out the easiest and most effective methods of performing it—for the machine does a certain kind of work; the material is placed on it, the power is applied, and he has only to look on. Farther, the kind of work about which he is employed almost exclusively, is of the same unvarying character; and he is thus subjected to a monotony of occupation, which leaves many intellectual faculties dormant. In former generations, the millwright was a man conversant with machinery in general, and with the various operations required in its manufacture and application. He could make patterns, mould, forge, turn and fashion timber and metal to his uses; and generally—we may say necessarily—was a man of intelligence and ingenuity. Now we have patternmakers, moulders, smiths, turners, fitters, and workmen for attending planing, screwing, slotting, punching, and other machines; all, in a manner, different trades, requiring no versatility of mechanical talent, and not even dexterity of hand.

No doubt the vastly increased quantity of work is now done in all the better style for this subdivision of labour; but we fear the moral and intellectual character of the workman is considerably lowered.

2. NATURAL MOVEMENTS OF AIR.—History affords us no information as to the period when, or the race by whom, the movements of the winds and tides were converted into useful mechanical forces. Many savage nations, who for ages could have had no intercourse with people more civilised than themselves, possess vessels that float on the surface of the waters, impelled by the wind or the tidal currents of the sea and rivers, as well as by oars; and it is probable that the idea of utilising the motion of the wind, or the current of the water, suggested itself to many separate individuals, without any communication among each other. The most simple mode of applying the force of wind is that which is of most general use, the propulsion of vessels on the water. The most casual observation of the natural effects of wind, shows its greater power on larger surfaces than on small ones; and a little reflection would offer a reason for this, and suggest the extension of surface to receive the pressure of aërial currents, when greater force is sought to be derived from them. A savage floating in his canoe, finds that, by holding up the blade of his oar to catch the favourable breeze, he makes progress without effort; by holding up two oars, the velocity of his progress is increased; and by stretching between the oars a mat or skin, so as to increase largely the surface on which the wind can press, the speed of his canoe is proportionally augmented. From the first rude notion of a sail, the steps of transition to the complex rigging and arrangements of canvas in a first-rate man-of-war are easy and obvious; indeed, in making use of wind-power for propelling vessels, there are only two points requiring consideration: those are, first, the quantity of the force developed or required; and, secondly, the direction in which it is to be made to act. The amount of wind-force depends upon the velocity of its movements, and the quantity of surface on which it acts. At first sight it might appear, that by doubling the velocity of the wind, we should double its pressure on a certain area: but this is by no means the case, for on doubling the velocity, the pressure is quadrupled; on tripling the velocity, the pressure is increased nine times; and so on, the pressure being always proportioned to the square of the velocity, or the velocity multiplied by itself. On a little consideration, this law becomes obvious, as we will endeavour to show. Let us suppose the air to consist of a number of particles or

molecules; even if it do not consist of actual solid particles, we may, at all events, suppose it to consist of a number of separate masses, as small and light as we please; and though rare and attenuated as compared with solid bodies, yet certainly as much as they, subject to the general laws which govern all matter. Now, if we double the speed with which any one of these masses moves, we double the force with which it strikes on any body placed in its way; if we triple its speed, we triple the force of its blow; and so on, the force being always exactly proportional to the velocity, when any one mass or particle is taken into account. But when we extend our conception to the motion of an unlimited number of such particles, which constitute a continuous fluid, we observe that, by doubling the speed of the fluid's motion, we double the number of masses striking any body opposed to it in a given time; by tripling the speed we triple the number striking; and so on, the number of particles striking, as well as the force of each, being proportional to the velocity of movement.

Upon the whole, then, the pressure on any surface, arising from a continuous series of blows from masses of air, must be as the force of each, multiplied by the number in a given time; and as, on doubling the velocity, the number is doubled, the force or pressure is 2 × 2, or 4 times, on tripling the velocity the pressure is 3 × 3, or 9 times, and so on, the pressures being always, as we have stated above, proportional to the velocities multiplied by themselves, or the squares of the velocities. It is convenient to estimate the pressures of wind currents on some unit of surface, such as one square foot, while the velocities are generally stated as being at a certain number of miles per hour. The following table gives the pressures per square foot of surface produced by winds of different degrees of strength, as distinguished by their respective names, with the corresponding velocities in miles per hour.

Table of the Velocities of Winds in miles per hour, and their pressure in lbs. on a surface of one square foot.

	Miles per hour.	Pressure. lbs.
Hardly perceptible	1	0·005
Gentle breeze	5	0·123
Brisk gale	10	0·492
Very brisk gale	20	1·968
High wind	30	4·429
Very high	40	7·873
Storm	50	12·300
Great storm	60	17·715
Hurricane	80	31·490

The following is a simple rule for determining very nearly the pressure per square foot, when the velocity in miles per hour is known:—Multiply the velocity by itself, halve the product, and point off two figures to the right.

Example 1.—Required the pressure of a wind blowing at 40 miles per hour. 40 × 40 = 1600; half is 800; pointing off 2 figures, we have 8 lbs., which nearly agrees with 7·873, the pressure in the table.

Example 2.—Required the pressure of a tempest at 70 miles per hour.

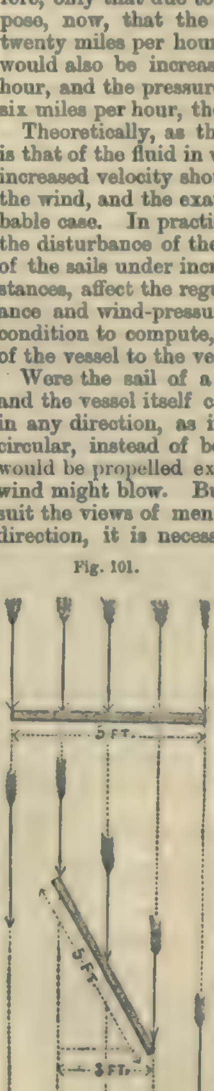
$$\begin{array}{r} 70 \\ 70 \\ \hline 2)4900 \\ \hline 24{\cdot}50 \text{ lbs.} \end{array}$$

The pressure of a wind on any surface, such as a square foot, being known, it is easy to estimate that on any other surface; for if, instead of 1 square foot, 2 square feet were opposed to the wind, we should find the pressure doubled in amount; on 3 square feet it would be tripled; and so on in regular proportion. Knowing, then, the velocity of the wind, we compute its pressure on any known surface by multiplying the pressure on 1 square foot by the surface in square feet. Thus, if we wished to ascertain the pressure of a gale at 20 miles per hour on a ship's sail 20 feet wide by 30 feet high, since 20 × 30 = 600 square feet of surface, and the pressure of the gale on 1 square foot being nearly 2 lbs., we have 600 sq. ft. × 2 lbs. = 1200 lbs., the total pressure on the sail.

When the wind is made use of as a motive force, the surface on which it blows must move with it, for if it simply rested opposed to the air, it would sustain pressure, but develop no motive force. The sail of a ship at anchor is certainly pressed on by the wind, and tends to force the vessel from its moorings; but until it does force it, no motive power is developed, for no motion is produced. But when a ship is in motion, the sail is, to a certain extent, going with the wind, and therefore receives a pressure due only to the excess of the velocity of the wind over that of the ship. Suppose the wind were blowing at the rate of ten miles per hour, and the ship sailing before it at the rate of seven miles per hour, the actual velocity with which the wind strikes the sail is only three miles per hour, the difference between its rate and that of the ship. The pressure on the sail is, therefore, only that due to three miles per hour. If we suppose, now, that the velocity of the wind increased to twenty miles per hour, the speed of the vessel propelled would also be increased probably to fourteen miles per hour, and the pressure on the sail would be that due to six miles per hour, the excess of twenty over fourteen.

Theoretically, as the resistance to the vessel's motion is that of the fluid in which it moves, the pressure due to increased velocity should follow the same law as that of the wind, and the example given above would be a probable case. In practice, however, the form of the vessel, the disturbance of the surface of the water, the bagging of the sails under increased pressure, and other circumstances, affect the regularity of the laws of water-resistance and wind-pressure so much, that we are not in a condition to compute, accurately, the ratio of the speed of the vessel to the velocity of the wind.

Fig. 101.



Were the sail of a vessel fixed in a middle position, and the vessel itself capable of moving with equal ease in any direction, as it would be if it were made quite circular, instead of being long and narrow, the vessel would be propelled exactly in the direction in which the wind might blow. But as this result would by no means suit the views of men desiring to sail in some particular direction, it is necessary to contrive the form of the vessel, and the position of the sails, so that the pressure of the wind in one direction may be converted into a pressure in another direction. The vessel is, therefore, made long and narrow, so that it may be subject to much less resistance from the water, when moving in the direction of its length, than if it were to move sideways; the sails are made capable of being turned about to different angles with the length of the vessel, so as to receive the wind obliquely; and, to direct the vessel and overcome any tendency of the wind to blow the front or back part of the vessel round out of its proper course, a rudder or helm is mounted at the stern, to balance, by its resistance in the water on one side or the other, the force tending to make the vessel deviate. When the wind strikes on a surface presented to it obliquely, its pressure is diminished. Suppose a certain surface five feet wide (Fig. 101) exposed directly to the wind, or at right angles to the direction of the wind, the pressure on it may be considered to be made up of five pressures each on one foot of the width. But if the same surface be turned obliquely to the wind, so that while the surface itself still remains five feet wide, the quantity of air intercepted is to be measured by three feet in width, the length of the line drawn at right angles to the direction of the wind intercepted between parallels to it from the extremities of the surface, then the pressure is only that on three feet of surface.

But besides the loss of surface from obliquity, there is also a loss in intensity of pressure. A column of air A D (Fig. 102) striking a surface exposed obliquely to it, would be reflected in D B, and press upon the surface in the line C D, which is perpendicular to the surface, and bisects the angle contained between A D, the line of impact, and D B, the line of reflection. The quantity of pressure compared with what it would be if the surface were not oblique, is found by taking A D any length, to represent the direct pressure, and drawing A C and D C respectively parallel and perpendicular to the oblique surface. Then, while A D measures the direct impulse, C D measures its effective impulse to move the surface in the direction D E, and A C measures the loss from obliquity, or the amount of force expended parallel to the oblique surface. and therefore not effective upon it.

Fig. 102.

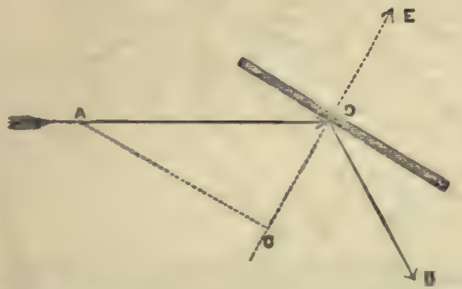
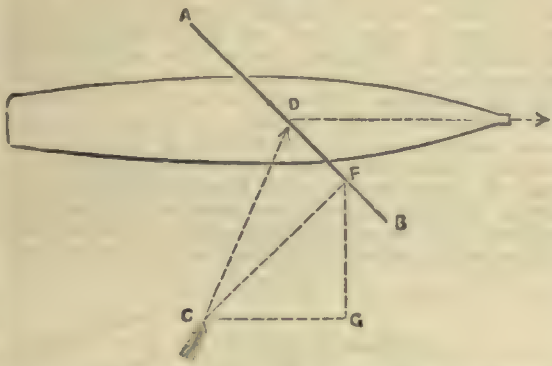


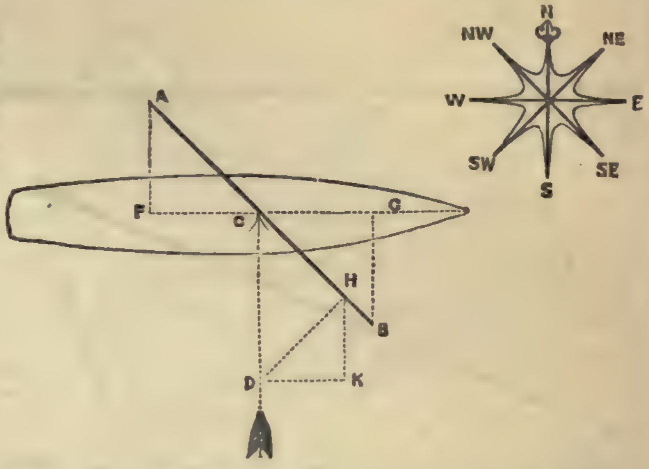
Fig. 103.



But farther, it is seldom the case that the effective pressure of the wind on the surface of a sail set obliquely to it, is employed directly. If A B (Fig. 103) represent the surface of the sail of a vessel, C D the direction of the wind, D E the direction of the vessel's course, we may take C D any length, representing the force of the wind; then C F, perpendicular to the sail, measures its effective force on the sail, and the length of C G, parallel to D E, and intercepted by F G, which is perpendicular to D E, measures the effective force in the direction of the ship's course. The line F G measures the effort of the wind to propel the vessel sideways, or to make what is technically called *leeway*.

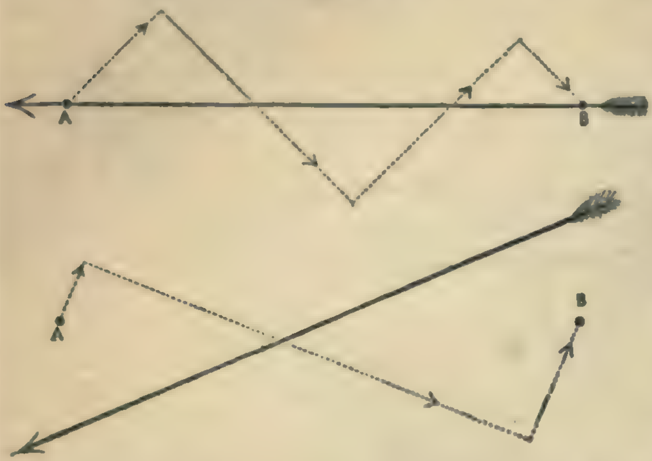
We may take a practical example as an illustration of how the effective impulse of the wind may be computed:—A sail 20 feet wide and 20 feet high is set with its edges directed to N.W. and S.E.; the wind blows at 30 miles an hour from the south, and the vessel sails due east with a speed of 10 miles per hour. Having drawn a plan of the vessel (Fig. 104), showing D C the direction of the wind, and A B that of the sail, in the first place from A and B we draw parallels to D C, cutting off F G, the effective width of sail to receive the wind. A B measures 20 feet, and F G would be found to measure about 14 feet; therefore the effective surface of the sail is 14 width × 20 height = 280 square feet. The velocity of the wind 30 miles per hour being represented by D C, its velocity, resolved perpendicularly to the sail, is measured by D H, about 21 miles per hour. This velocity, resolved into D K, in the direction of the vessel's course, becomes 15 miles per hour, which is to the actual speed of the vessel, 10 miles, as 3 to 2. The pressure on the surface of the sail is not that due to 21 miles per hour, the measurement of D H, because the vessel and sail are continually escaping from the wind; but to the excess of D H over the speed of escape, or the same part of 21 as 5 (the excess of the wind's velocity in D K over 10, the velocity of the vessel) is of 15, the wind's velocity in D K. As 5 is $\frac{1}{3}$rd of 15, and 7 is $\frac{1}{3}$rd of 21, the effective pressure on the sail is that due to 7 miles per hour—about 0·245 lbs. per square foot. This multiplied by 280 square feet, the effective surface of the sail, gives $68\frac{1}{2}$ lbs. for the pressure tending to blow off the sail from the mast while the vessel is moving at 10 miles per hour. Were the motion of the vessel entirely obstructed, the pressure on the sail would be that due to 21 miles per hour, about $2\frac{1}{4}$ lbs. per square foot, or $280 \times 2\frac{1}{4} =$ 630 lbs. The tendency of the wind to produce leeway, or to force the vessel sideways from its course, happens in this case to be the same as its direct force, for K H is equal to D K. If the vessel be six times as long as it is broad, the resistance to its movement sideways should be 6 times that to its direct movement, without taking into consideration the form of the part immersed. The shape, however, is made so as to offer as much resistance as possible sideways; so that, under equal impulses, the resistance to side-movement or leeway may probably be at least 10 times that to direct movement. It might be possible to compute the angle at which a sail should be set when the direction of the wind and the course of the vessel are given, so as to obtain the greatest possible effect. But as, in practice, the sails of a vessel are numerous and various in position and direction of action, it would be difficult to apply such a computation. A sailor, after a few trials of his vessel under sail, is able to estimate very correctly the angles of his sail and course with the direction of the wind, so as to get the best effect. If the wind blow almost directly against the ship, it is necessary to tack. If, for instance, in sailing from A to B (Fig. 105), the wind blew from B towards A, or nearly so, the vessel would be steered in various tacks or directions oblique to that of the wind, making progress on the whole. The choice of those tacks, so as to get over the given distance with as little deviation as possible, or, at all events, to do it with the least possible loss of time, is an important part of the sailor's art.

Fig. 104.



The force of wind has occasionally been applied to moving bodies on land, but not in a manner that can be generally used. Some of the Arctic voyagers having to travel long distances in sledges over ice, have taken advantage of a favourable wind to propel their vehicles by means of kites attached to them. Chariots have also been made with sails, so as to be propelled by wind along a road, as ships are propelled at sea. But the uncertainty and variations of the wind render such applications of its power matters of curious experiment rather than of practical utility.

Fig. 105.



Windmills.—For giving motion to machinery, windmills have been and still are very extensively used. Engineers of the last generation devoted great attention to the construction of windmills, and brought them to great perfection. The introduction of steam-power—a power economical, manageable, and always to be depended on—has, in a great measure, superseded that of wind as a mover of machinery. It is true that after the first cost of a windmill, the power is comparatively inexpensive; but it is so variable in intensity—sometimes, when it is not required, exerting great force, and sometimes, when it may be most wanted, totally ineffective—that it is generally preferable to apply a force, perhaps considerably more expensive in its production, but constant, steady, and completely under control.

Windmills are of two kinds, horizontal and vertical. The former have been very little used, for it is found in practice that they are by no means so effective as the latter. The mode of constructing a horizontal windmill is like that represented on the plan (Fig. 106), or some modification of the same principle of construction. A wheel, A, mounted on a vertical axis or shaft, and having flat vanes or boards fitted round its circumference, is inclosed in a circular casing, which is fitted with boards fixed obliquely, or in such lines as, if produced inwards, would touch the circumference of the wind-wheel. By this arrangement, the wind, from whatever point it may blow, causes the wheel to revolve in the same direction. Part of the breeze passes between the oblique boards of the casing, and acts on the blades of the wheel; while part is intercepted by the boards, and either reflected inwards so as to propel the blades in the same direction, or reflected outwards so as not to act upon them in the opposite direction. Sometimes horizontal windmills have been made with a casing partially surrounding the wind-wheel (Fig. 107), and capable of being turned round by means of a vane, so as to permit the wind only to act on one side of the wheel, while the other is completely sheltered.

Fig. 106.

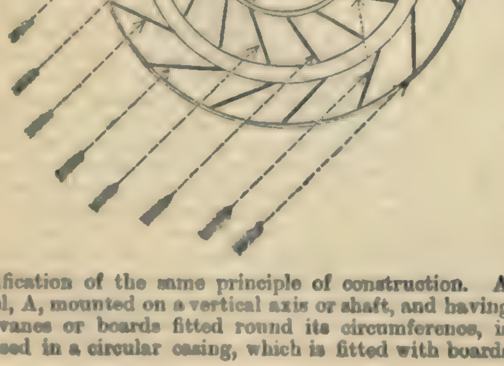
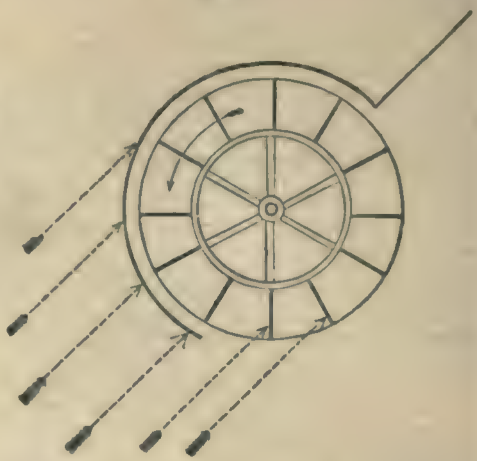
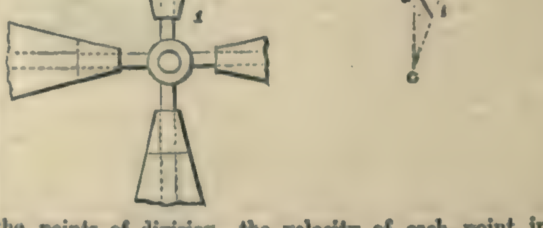


Fig. 107.



The vertical windmill, as is well known, consists of an axle or shaft, nearly horizontal, mounted in bearings at the summit of a tower, with four or more blades or sails attached to it. These sails are set at an angle with the axis, so that when the wind blows directly on the face of the mill, its oblique action on the sails is resolved into two forces—one in the direction of the axis, and the other perpendicular to it, the direction in which the sails revolve. Numerous experiments and computations were made to determine the most advantageous angles for setting the sails, and their most effective forms and proportions. If we suppose the radius of a sail divided into six equal parts (Fig. 108), and circles traced through the points of division, the velocity of each point in revolving, is proportional to the part of its circle intercepted between two radii, or proportional to its own radius. If, then, we make a series of plans of the sail at these different parts, we see that as we approach the centre we should increase the obliquity of the sail to its

Fig. 108.



plane of motion, so as to allow for its more slow escape sideways from the impulse of the wind. The sails accordingly are not made flat surfaces, inclined equally to the plane of their revolution, but surfaces of varying inclination, somewhat like portions of screw blades, twisting as it were from a certain obliquity at their extremes to a greater obliquity at the centre. The angles found most advantageous in practice are given by the celebrated engineer Smeaton as follow, as well as those used by some other engineers :—

Distance from centre	1	2	3	4	5	6
Inclination to plane of motion (Smeaton)	18°	19°	18°	16°	12½°	7°
Ditto (otherwise).........	24°	21°	18°	14°	9°	3°

In the angles given by Smeaton, an irregularity is observed in the first, which should, by theoretical reasoning, be greater than the second; whereas Smeaton makes it less. The following rule may be adopted as a very near approximation. To find the angle at which the sail should be inclined to the plane of revolution at any distance from the centre :—

Rule.—Multiply 18 twice by the distance from the centre; divide the product twice by the total radius, and subtract the quotient from 23; the remainder is the inclination in degrees.

Example.—In a windmill 60 feet in diameter, required the inclination of the sail 20 feet from the centre.

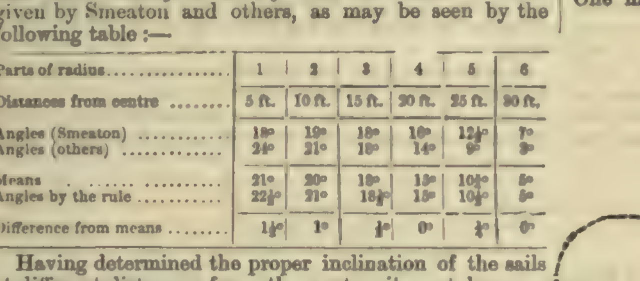
Here 30 feet is the total radius, and $\frac{18 \times 20 \times 20}{30 \times 30} = 8$, which, subtracted from 23, gives 15°, the angle of that point.

Were we to divide the radius 30 feet into 6 equal parts, and calculate the angles at each point, we should find them correspond nearly with the means of those given by Smeaton and others, as may be seen by the following table :—

Parts of radius................	1	2	3	4	5	6
Distances from centre	5 ft.	10 ft.	15 ft.	20 ft.	25 ft.	30 ft.
Angles (Smeaton)	18°	19°	18°	16°	12½°	7°
Angles (others)	24°	21°	18°	14°	9°	3°
Means	21°	20°	18°	15°	10¾°	5°
Angles by the rule	22½°	21°	18½°	15°	10½°	5°
Difference from means	1½°	1°	½°	0°	¼°	0°

Having determined the proper inclination of the sails at different distances from the centre, it next becomes important to inquire how much of the surface of the whole circle should be filled with sails. Mills are generally made with four strong wooden arms, or radii, fixed firmly in a central socket, and steadied and stiffened by tie-rods, connecting their extremities together, and with a projecting strut on the central boss. The width of each sail at the extreme, should be about half of the radius; so that in a mill 60 feet diameter, or 30 feet radius, each sail would be 15 feet wide at the extreme. The part of the arm next the centre, for about ⅙th of the radius, that is, 5 feet in the case supposed, is not fitted with sails, because the surface there is so little effective, as well from its short leverage as from its obstructing the wind reflected from the head of the turret behind it. The width at the inner end should be ⅓rd of the radius, or 10 feet. The surface of each sail is therefore 312½ square feet, and the total of the four is 312½ × 4 = 1,250 square feet.

Fig. 109.

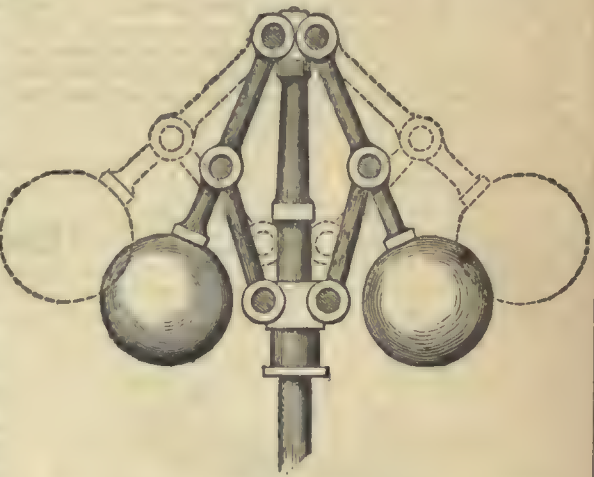


If A C be	. . .	30 feet.
Then A E	. . .	25 "
A D or E F	. .	10 "
A B	. . . .	5 "

The total area of a circle 60 feet in diameter is somewhat above 2,800 square feet, so that not half the surface of the circle is clothed with sails. There would be no disadvantage in extending the surface by making the sails broader, or more numerous, until it became ¾ths of the whole surface. Beyond this, additional sail-surface is disadvantageous, for it appears to obstruct the free passage of the currents reflected from the sails, and thus clogs their motions. It is found advantageous to arrange the surface of a sail somewhat in the proportions of the diagram (Fig. 109), which represents the front view of one sail.

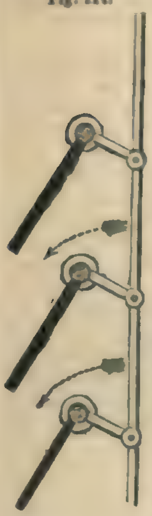
The covering of the surface, so as to catch the impulse of wind, formerly consisted of canvas fixed on a roller at one side of the arm, on which it could be rolled like a window-blind; or from which it could be unrolled so as to cover the whole sail, which was filled in with wooden framing to support the canvas pressed against it by the wind. Sometimes the canvas, instead of being in one sheet, was subdivided into numerous separate sheets mounted on rollers; and apparatus was provided so that the canvas might be wound on the rollers or unwound at pleasure, while the mill was in motion. As the wind is exceedingly variable, and as the quantity of work required of the mill also varies to a considerable extent, it was found necessary to provide some apparatus by which the mill might regulate itself, so that its velocity should not be excessive at one time and too small at another. One mode of effecting this object was to apply to the machinery of a mill a governor, like that of a steam-engine (which we shall have occasion to describe in detail hereafter). This governor consists of two heavy balls suspended from the summit of a vertical revolving spindle by jointed rods (Fig. 110). The spindle being at rest, the balls hang close to it on each side; but on the spindle being caused to revolve rapidly, the balls, impelled by centrifugal force, fly away from the central axis. as marked by the dotted lines. A system of levers and rods connected this apparatus with the sail-rollers, so that when the balls flew outwards, from increased velocity, the sails were furled; and when they fell inwards, from diminished speed of revolution, the sails were unfurled. The quantity of surface thus presented to the wind was adjusted to its force, and a tolerably equable velocity of the machinery was attained. In some more recent mills, an ingenious contrivance for regulating the surface of sail accord-

Fig. 110.



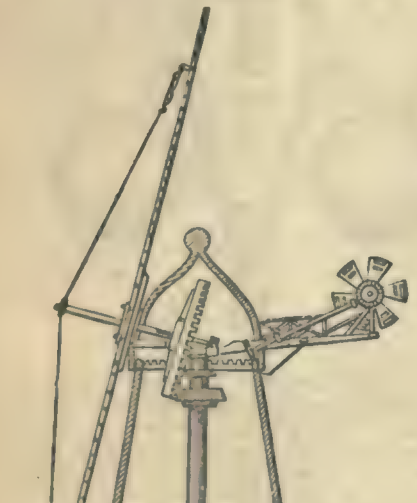
ing to the force of the wind, has been successfully adopted. The sails consist of a framework filled in with louvre-boards hinged on pivot-pins near one of their edges, and all connected by levers and rods, with a sliding-boss on the central axis of the windmill (Fig. 111). When the wind blows strongly against the louvre-boards, it forces them out of their vertical position, and passes freely through the openings between them. The surface of the sails is thus diminished by the pressure of the wind itself. To prevent its being too much diminished, the sliding-boss connected with the louvre-boards is pressed upon by a lever loaded by a certain weight sufficient to balance, as far as may be desirable, the pressure tending to force aside the louvres, and thus to keep them, to a certain extent, up to their work. When the load on the mill—that is to say, the quantity of work effected by it—is varied, the weight may be varied accordingly; and thus the effective amount of surface in the sails may be adjusted to the average force of the wind, and the work to be done by it. When the wind-force exceeds or falls short of its average, the greater or less inclination of the louvres very nearly compensates for the variation.

Fig. 111.



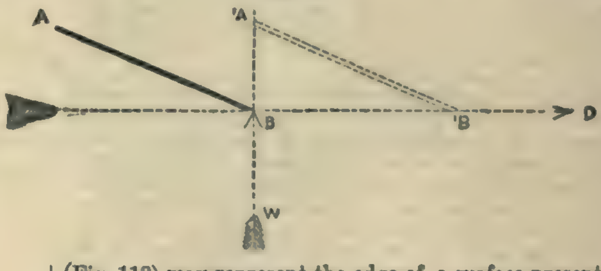
The sails of a windmill should directly face the wind in order to receive its most advantageous action; but, as the direction of the wind often changes, it is necessary to adopt some arrangement for varying that of the mill-shaft accordingly. The summit of the mill-tower, in which the mill-shaft is mounted, is therefore made to revolve, so that at any time the direction of the shaft may be varied, and the sails presented to the wind. In old mills, and indeed in many small mills still existing, this change of direction is effected by hand. A long lever is fixed to the movable cap or summit of the tower, and extends obliquely to the ground. The miller watches the direction of the wind, and by moving this lever, turns the cap round to its proper position. But in large mills this would require considerable power; and, moreover, constant attention would have to be paid to the changes of the wind. Were a single change neglected, the mill might be destroyed; for as the sails are made and strengthened by tie-rods to receive the wind's pressure on their face, a change of the wind to the opposite direction might throw a great strain on their back, for meeting which no provision is made. A simple mode of making the change of direction self-acting, is to fit the back of the cap with a large vane, which, like that of a weathercock, would cause the sails to be presented to the wind from whatever quarter it might blow. But when mills are of considerable size, the vane would require to be very large and cumbrous. The contrivance generally employed is neat and ingenious. Behind the cap (Fig. 112), on the side opposite that through which the wind-shaft passes, a framing is made to project outwards. On this framing there is mounted a small windmill, on an axis transverse to that of the main arms. The cap rests on rollers fitted in the circular top of the tower, so that it may move freely round; and a toothed circular rack is also fixed on the summit of the tower. A spindle, fitted with bevil-gearing so that it may be caused to revolve by the revolution of the small mill, conveys motion to a toothed pinion, which gears into the circular rack. When the main mill has its face presented to the wind, the small one stands edgeways to it, and therefore remains at rest; but as soon as the wind veers, it begins to act on one side or the other of the small mill, and thus causes it to revolve. The pinion is thus made to travel along the fixed rack, and turn the cap of the mill round, until the main mill is again brought to face the wind in its new direction. This arrangement is found to be very effective; and when it is properly applied, the mill requires no attention in respect of the direction of its sails to the wind.

Fig. 112.



In estimating the velocity with which the sails of a windmill revolve, we have to consider not only the force of the wind upon them, but also the resistance to their motion occasioned by the work done by the mill. A B (Fig. 113) may represent the edge of a surface presented obliquely to the wind, and capable of moving in the direction C D at right angles to that of the wind. If the surface be free and unresisted in its motion, and the wind be considered to produce its full effect upon it, the proportion of its velocity to that of the wind would be estimated by that of the line B B′ to the line B A′; for it is clear that while the wind travels over the distance B A′ the surface moves to the position dotted, that is, over B B′. But if the motion of the surface be resisted, its velocity in relation to that of the wind is diminished. In the case of windmill sails, we may suppose such a load of work on the mill that the velocity of the sails is not more than half what it would be were there no resistance. We may, therefore, assume that the velocity of the sail relatively to the wind would be expressed by the ratio of half the length of the line B B′ to the length of A B. Taking the wind as a gentle breeze, the velocity of which, in the table, is about 5 miles per hour, and the inclination of the sail or angle A′ B B′ half way from the centre 18°, we should find the half of B B′ to be about 1½ times A′ B, or the velocity of the sail 1½ × 5 = 7½ miles per hour—about 660 feet per minute. If the windmill be about 60 feet in diameter, the diameter of the middle point of the arm is 30 feet; the circumference of the

Fig. 113.



circle in which that point revolves is 94 feet, and the number of revolutions made per minute is therefore $\frac{660}{94}$ — about 7.

If, now, we calculate the speed of the extremities of the arms, we find that it is 1,320 feet per minute, or about 15 miles per hour; three times that of the wind, which we have assumed as 5 miles per hour. Did we assume a wind of greater velocity, we should have to take into account the self-regulating arrangement, which diminishes the amount of surface exposed, and therefore prevents the mill from attaining so much increase of speed as it would without regulation. Under ordinary circumstances, the speed of the outer extremities of the arms ranges from 20 to 30 miles per hour. We may assume 30 miles per hour when the wind blows at 10 miles, with a pressure of about ½ lb. on the square foot. The total surface of the sails unfurled, in a mill 60 feet diameter, is 1,250 square feet; we may suppose half lost by furling; leaving 625 effective. As the surface is set obliquely to the wind, the pressure in the direction of motion would be reduced from ½ lb. to about ⅐ lb., as a mean over the whole of the arms, giving a total pressure in the direction of motion of about 90 lbs. The mean velocity of the arms is half that of the extreme, 15 miles per hour, or 1,320 feet per minute. We have, therefore, 90 lbs. moving at 1,320 feet per minute, which is equivalent to a force of 90 × 1,320 = 118,800 lbs., moving at 1 foot per minute. A horse-power is reckoned as equivalent to 33,000 lbs., moved 1 foot per minute; therefore, the power of the mill we have reckoned is about 3½ horse-power.

When we double the diameter of a mill, we quadruple its power, for we quadruple its effective surface. The areas of circles are proportional to the squares of their diameters; and as the similar parts of the areas are occupied by sails, they are also as the squares of the diameters.

It is not at all an easy matter to estimate the powers of windmills. The proper guide as to power, velocity, and construction, is experience. Some of the works of Smeaton contain much valuable information respecting this branch of Practical Mechanics; and to these we must refer such of our readers as require a more full discussion of the subject than our limits permit us to offer.

As a force applied to the movement of machinery, wind has few advantages, except its little cost after the first outlay for a windmill has been made. It is chiefly available in flat countries, where there is no opportunity of obtaining the preferable power of water, and where there is little interruption to the aërial currents. In hilly countries windmills are often subject to derangement, from the excessive force of the gusts of wind that occur in such regions. In tropical countries, particularly islands and places near the sea-shore, the daily occurrence of the land and sea-breezes, occasioned by the action of the solar heat on the land, provides a certain amount of wind-power, which may be almost always depended on. But in these countries, on the other hand, there often occur tornadoes, or hurricanes of extreme violence, that sweep away almost everything that may oppose their progress; and thus frequently destroy windmills, and occasion renewed outlay in their re-construction. The principal use to which windmills are devoted in temperate climates, is for grinding corn; in tropical climates, such as the West Indian Islands, they are employed for driving sugar-cane mills. In fenny and marshy countries, such as Holland, or some of the eastern counties of England, they are used for drainage, either by working pumps or turning a wheel contrived for lifting the drainage water from the surface of the ground into canals at a higher level, by which it is carried off into the sea. In all situations, however, where the cost of fuel is not extravagantly great, steam-power is gradually superseding that of wind, because its certainty of action more than repays the cost of its production. Whole districts, the drainage of which is dependent on wind-power, may frequently remain many weeks under water from the prevalence of calm weather, and the agricultural operations of the season may be so seriously interfered with, that whole crops are lost, or become immensely deteriorated. In sugar-growing countries, again, the derangement of wind-machinery by a hurricane or tempest, may occur at the season when the sugar-canes have to be crushed; and the loss of a few days in crushing the canes, may seriously damage the sugar in respect of quantity as well as quality. Upon the whole, then, whenever the cost of fuel is not excessive, it is not advisable to incur the outlay of extensive works for securing wind-power. A very small steam-engine, kept constantly in operation, is far more effective than a windmill of much greater power, because the latter is so variable and uncertain in its action. The only operations suited to wind-power, are such as need not necessarily be completed at certain periods, but may be conducted occasionally as the wind may serve. Nor should the machinery driven by wind require very nice regularity in its action; for, notwithstanding all the ingenious arrangements for equalising the wind-force, it is still unsteady at the best.

Every part exposed to the wind should be greatly in excess of the strength required to resist the average strain to which it may be exposed. The tempest of an hour—nay, a momentary gust—may frequently destroy a windmill that has stood under ordinary winds for years. As a safeguard against too much strain, the windmill should always be left free to revolve, even if the machinery which it drives be thrown out of gear. The shaft or axis of the mill generally carries a large wheel, to which is fitted a strap of iron loaded so as to press on its circumference, and act as a friction-break either to hold the mill fast for purposes of repair during light winds, or to check its velocity when the winds are too strong for the work required.

WATER POWER.—The movements of water are much more serviceable for the purposes of power, and steady in their operation, than those of air. In level countries, where the streams are slow and languid in their flow, this power is not attainable; but in hilly countries, where the rivers and streams fall frequently from a high level to a lower, water-power is easily obtained, and is most advantageous as a steady, inexpensive prime mover. The most common way of employing water-power is to cause the current to act on the circumference of a large wheel, so as to give it a rotatory motion, which is communicated, by means of shafts and wheel-work, to the machinery required to be driven. Such water-mills are generally used for grinding or thrashing corn, crushing bones for manure, raising water to irrigate land; in mining districts, for crushing or otherwise operating on the ores; and in manufacturing districts, for working cotton, woollen, or flax machinery. Water-wheels are of three kinds, named according to the mode in which the water-current is made to act upon them:—

1. Undershot, when the wheel is fixed over a stream with inconsiderable fall, but considerable velocity.

2. Overshot, when the fall of the water is so great that the stream may be directed upon the upper part of the wheel.

3. Breast-wheels, when the stream can be directed on or near the middle or breast of the wheel.

1. *The Undershot-wheel* may be best understood by conceiving the action of the paddles of a steam-vessel reversed; that is to say, while in a steam-vessel the paddles are caused to revolve, and were the vessel fixed would produce a current in the water by their revolution, in the case of the undershot-wheel, the natural current of the water pressing on the floats immersed in it, causes the wheel to revolve. It is sufficiently clear that the power derived from this arrangement depends upon the intensity of the pressure which the water exerts on the floats, and the amount of surface pressed upon. If we suppose the wheel at rest, and its float standing vertically in the water, we may easily compute the pressure on every square foot of its surface by ascertaining the speed at which the water flows against it. This pressure, like that of the wind, is proportional to the square of the velocity of current; for by doubling the velocity, we

bring double the number of particles in contact with the float, and we also double the force of each particle in striking it; so that, upon the whole, we quadruple the pressure. So, also, by taking 3 times the velocity, we have 9 times the pressure; and, generally, if we know the pressure due to one velocity, such as 1 foot per second, we can compute that due to another velocity, such as 10 feet per second, by multiplying the velocity by itself, and taking the pressures in those proportions.

By an investigation of the mechanical laws which govern the motion of fluids, we ascertain that the velocity with which a fluid flows from an orifice in any vessel, is the same as that which a heavy body would acquire by falling through a height equal to that of the fluid column above the orifice.* This is found by the following rule:—Multiply the square root of the height (in feet) by 8; the product is the velocity (in feet per second). Thus, if a stone were dropped from a precipice 100 feet high (neglecting the resistance of the air to its fall), its velocity when it strikes the bottom would be about 80 feet per second; for 10 is the square root of 100, and 8 × 10 = 80 feet per second. If, on the other hand, we knew the velocity, we should be enabled to calculate the height of fall by reversing the rule; that is to say, divide the velocity (in feet per second) by 8, and square the result for the height (in feet). Thus, if we ascertained that a stone had in its fall acquired a velocity of 80 feet per second, we should reckon that it had fallen 100 feet, for $\frac{80}{8} = 10$, and 10 squared or $10 \times 10 = 100$.

These rules do not take into account the resistance offered by the air to the motion of the body falling through it. In falling through great heights, this resistance has very great effect on the velocity; but for small falls, it may be neglected without material error. If now, we apply these computations to the flow of currents of water, we can calculate the height due to a certain velocity of current; that is, the head of water whose pressure on its lower particles gives them the velocity in question. Thus, if on measuring the velocity of a stream at a rapid, we found it to be 16 feet per second, since $\frac{16}{8} = 2$, and $2 \times 2 = 4$, we should reckon that, in order to have this velocity, it must be pressed on by a column or head of water 4 feet high. It must, therefore, press upon any body immersed in it with the same force as it is itself subjected to; for it is the peculiar property of fluids to convey pressures equally in every direction through them.† It does not necessarily follow that, at the point where the velocity is measured, there is an actual fall of 4 feet: it is sufficient that the water has somewhere fallen enough to acquire the velocity measured, or that any forces whatever have combined to give it that velocity, or to reduce its velocity to that degree. The velocity is, in fact, the expression of the result of all the forces and resistances, that have acted on the water up to the point where it is measured. Having computed the head of water pressing, it is easy to compute the amount of pressure exerted on a square foot. A cubic foot of water weighs 62½ lbs.; therefore the bottom of a cubical box, measuring 1 foot every way, is pressed on by a force of 62½ lbs. when the box is filled with water; in other words, the pressure of a column of water 1 foot high, is 62½ lbs. on every square foot of bottom surface. Were the height of the column increased, the pressure would be increased in like proportion; every additional foot of height would throw an additional pressure of 62½ lbs. on every square foot. The intensity of pressure (in lbs. per square foot), then, is 62½ times the height in feet. Thus, the pressure due to a head of 4 feet, is 62½ × 4 = 250 lbs. per square foot. We may now combine the two computations—that for head due to given velocity, and that for pressure due to given head—into one rule, for determining the pressure per square foot due to a given velocity, as follows:—As the square of 8 is 64, which does not much differ from 62½, we may, without material inaccuracy, avoid dividing the velocity by 8 before squaring the quotient, and again multiplying by 62½, and thus have the very simple rule.

Square the velocity (in feet per second), and the result is nearly the pressure (in lbs. per square foot). Thus, when the stream is moving with a velocity of 16 feet per second, its pressure per square foot is 16 × 16 = 256 lbs. nearly. By the former computation, we found 250 lbs., less by 6; that is, less than $\frac{1}{41}$st part of the whole. So far we have found the means of computing the pressure on a float-board *at rest*, in a stream flowing with a given speed; but in the case of an undershot water-wheel, the float is in motion in the same direction with the current, and therefore the relative velocity of the latter, in acting upon it, is so much diminished. The relative velocities of the float and of the current may, of course, be varied by applying more or less resistance to the motion of the wheel. It is necessary to know the relation of the two velocities, so as to derive the best possible effect from the current. If we take a particular case, and try various relations, we may find the most advantageous—bearing in mind that the result to be obtained is the maximum of power or useful effect; that is to say, the pressure on any float multiplied by the velocity of its motion, which product gives the power of that float as a mover of machinery. If we take the velocity of a current as 6 feet per second, and form a table of velocities of float from 0 feet per second up to 6 feet per second, of pressures due to the excess of the stream's velocity over that of the float, and of the powers or products of those pressures multiplied by the corresponding velocities of float, we shall find the power greatest when the velocity of the float is exactly 2 feet per second, or ⅓rd of that of the stream. Thus—

Velocity of stream.		Velocity of float.		Excess of stream over float.		Pressure or excess squared.		Power or press. × veloc.
6	....	0	....	6	....	[illegible]	....	0
”	....	1	....	5	....	25	....	25
”	....	2	....	4	....	16	....	32
”	....	3	....	3	....	9	....	27
”	....	4	....	2	....	4	....	16
”	....	5	....	1	....	1	....	5
”	....	6	....	0	....	0	....	0

Were we to take any other velocity of stream we should find the same result, that the velocity of the float should be ⅓rd of that of the stream, in order to attain the maximum effect.‡ If this rule be adhered to, the excess of the stream's velocity over that of the float is ⅔rds of itself; and the pressure per square foot would be the square of ⅔rds of the stream's velocity, or $\frac{4}{9}$ths of the square of the velocity, since $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$. The power would depend on the number of square feet pressed upon, and the velocity of the float, and would therefore be found by multiplying the surface of the float by $\frac{4}{9}$ths of the square of the stream's velocity, and the product by the float's velocity, or ⅓rd of the stream's velocity. As there are generally several floats immersed in the water, it may appear that the surface acted on is considerably larger than that of one float; but when it is remembered that the volume of water contained between any two of the floats, if it press the one forward by its direct action, must equally press the other backward by its reaction, we cannot safely estimate more than the surface of one float as really effective. The velocities of currents are generally reckoned in feet per second, while the velocities of moving parts of machinery, in estimating

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* See *ante*, *Hydrodynamics*, p. 761.

† See *ante*, p. 751.

‡ If $V =$ velocity of stream, $v =$ velocity of float, $p =$ pressure per square foot $= (V - v)^2$ and $P =$ power $= pv = \overline{V - v}|^2 v$, to find v, so that P may be a maximum.

$$P = \overline{V - v}|^2 v = V^2 v - 2Vv^2 + v^3.$$

$$\frac{dP}{dv} = V^2 - 4Vv + 3v^2 = 0.$$

$$\therefore v^2 - \frac{4}{3}Vv = -\frac{V^2}{3} \text{ and } v^2 - \frac{4}{3}Vv + \frac{4}{9}V^2 = \frac{1}{9}V^2.$$

$$\text{Hence } v - \tfrac{2}{3}V = \pm \tfrac{1}{3}V, \text{ or } v = V, \text{ or } \tfrac{1}{3}V.$$

$\frac{d^2P}{dv^2} = -4V + 6v$, and substituting for v each of its values.

$-4V + 6V = 2V$ positive, gives P a minimum when $v = V$.

$-4V + 2V = -2V$ negative, gives P a maximum when $v = \frac{1}{3}V$.

power, are reckoned in feet per minute. Taking a horse-power as 33,000 lbs. lifted one foot high in one minute, and assuming the circumferential speed of an undershot-wheel as one-third of that of the current, we may estimate its power as a mover of machinery by the following rule :—

Multiply the surface of the float (in square feet) three times by the velocity of the stream (in feet per second), and divide the product by 3,800 ; the quotient expresses the horse-power.

Example.—An undershot-wheel, having floats 2 feet deep and 10 feet wide, is moved by a stream running at $7\frac{1}{2}$ miles per hour : required its power.

Here the surface of a float is $10 \times 2 = 20$ square feet.

The velocity is $7\frac{1}{2}$ miles per hour : or (as a mile is 5,280 feet, and an hour 3,600 seconds) the velocity is $\frac{5280 \times 7\frac{1}{2}}{3600} = 11$ feet per second.

The power therefore is $\frac{20 \times 11 \times 11 \times 11}{3800} = 7$ horse-power.

The speed of the floats being $\frac{1}{3}$rd of that of the current, is $\frac{11}{3}$ feet per second, or $\frac{11 \times 60}{3} = 220$ feet per minute. We may take this as the speed of the middle part of the float ; and if the wheel be 23 feet in extreme diameter, its diameter at the middle of the floats would be 21 feet, and circumference there 66 feet ; if this move at the rate of 220 feet per minute, the wheel must make $\frac{220}{66} = 3\frac{1}{3}$ revolutions per minute.

The speed of a stream may be generally estimated by throwing on it a body that floats, but is immersed some depth—as a bar of wood loaded at one end to float vertically—and watching the time occupied by its passage over a certain known distance. Care should be taken that the body fairly attain the speed of the current before its motion is reckoned in the time.

It is advantageous to make the inner edges of the floats to stand somewhat above the general level of the water, which becomes heaped up behind them, and would otherwise pour over the edges (Fig. 114). Indeed, the difference of level caused by this heaping up of the water behind, and its hollowing in front of a float, almost measures the head of water pressing on it. When practicable, the stream should be narrowed to the width of the wheel (Fig. 115), as by this means not only is its velocity augmented by the necessity of a certain body of water passing more quickly through a diminished channel ; but also the water, acting on the floats, is prevented from escaping sideways without giving its full effect.

Fig. 114.

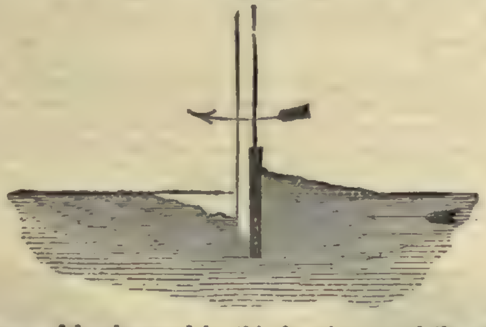
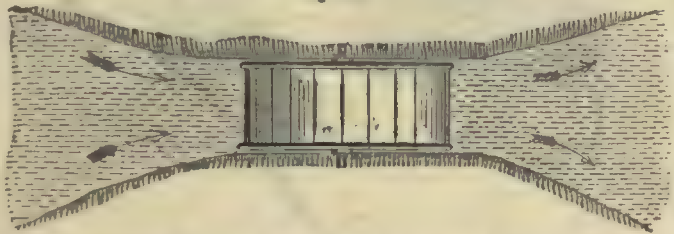


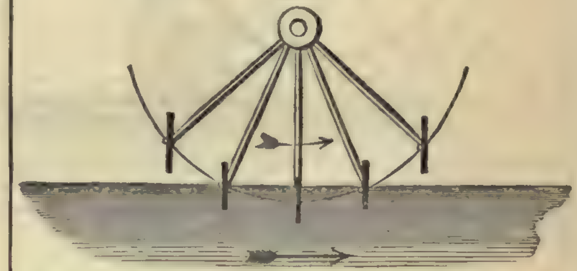
Fig. 115.



As the velocity of streams to which undershot-wheels are applicable is never very great, and as the velocity of the floats should not much exceed one-third of that of the stream, such wheels are necessarily slow in their revolution, and can therefore be applied with most advantage, in driving machinery where quick speeds are not required. When it is necessary to convert the slow revolution of the wheel to rapid motions in the machinery, there are considerable losses from the friction of the gearing. For such purposes as that of working pumps or fulling-mills, and generally for slow, heavy work, these wheels are very serviceable. The principal objection to their use arises from the circumstance, that with a stream of average rapidity, very little power is obtained without a very large and cumbrous wheel, involving considerable outlay, and extending over a great breadth of the stream. By making the diameter of the wheel large, no greater power is obtained, except what may be attributable to the more direct action of the water on the floats, which enter and leave the water more vertically when the wheel is large. The circumference of a large wheel should move with the same speed as that of a small one ; and, therefore, the greater the wheel, the smaller number of revolutions does it make in a given time. The only way of increasing the power is to extend the surface of the floats. This may be done by making them deeper or wider. Additional depth of the float, even where the depth of the stream permits it, is by no means so effective as additional width ; for a wide shallow float enters and leaves the water with ease, while a deeper one presses the surface of the water down in entering, and lifts it up in leaving, and thereby encounters considerable resistance to its motion.

Occasionally undershot-wheels have been made like the feathering paddles of steam-vessels, where the floats are capable of being turned on pivots (Fig. 116) so as to maintain a vertical position while immersed in the water, and thus receive its most direct impulse, while they enter and leave it with the least possible resistance.

Fig. 116.



When it is considered that twice in every day a great tidal stream flows and ebbs along our coast and in our estuaries, it is surprising that advantage has not more frequently been taken of this enormous power by the erection of undershot-wheels along the course of the tidal currents. In this country tide-mills are rare ; and neither their number nor magnitude render them important as sources of power. Occasionally, however, they have been employed with advantage. Where there is a great tidal stream, and consequently a considerable difference between the levels of high and low-water, fixed wheels would be almost useless ; for at high-water they would be too much immersed, and at low-water too little. It is in such cases necessary to mount them on a floating stage or barge, so that the whole mill may rise and fall with the tide, the amount of immersion remaining constant. Again, as the tide flows alternately in opposite directions, when it is required that the machinery move only in one direction, it is necessary that tide-mills, in such cases, should be fitted with apparatus by means of which the direction of movement may be reversed; or, where the mill is floated on a barge, the swinging of the barge by the tide effects the required change of position to suit the change in direction of the current.

If it were practicable to make use of the tidal stream in such a river as the Thames, without interfering with its navigation, the power derived from it would be enormous. If we suppose the breadth, 1,200 feet, occupied by tidal mills side by side, with floats immersed 3 feet deep, the total float-surface would be 3,600 square feet in one section of the river. The velocity of current we may take, on the average, as 3 miles per hour, nearly $4\frac{1}{2}$ feet per second; and the power, according to our rule, would be

$$\frac{3600 \times 4\frac{1}{2} \times 4\frac{1}{2} \times 4\frac{1}{2}}{3800} = 86 \text{ horse-power.}$$

Were such mills repeated at intervals of 220 feet along a mile of the river, there would be 24 of them, and the total power would be 86 × 24 = 2064 horse-power in a mile of the river's length. It is not, of course, presumed that such an arrangement is feasible: it is only offered as an illustration of the great mechanical power that might be derived from the natural movements of the water in tidal estuaries. In some rivers, such as the Rhine and the Seine, barges are moored carrying tidal mills of this kind. In such streams the level does not greatly vary, and the current sets continuously in one direction, so that the power is applied with constancy and facility.

Fig. 117.

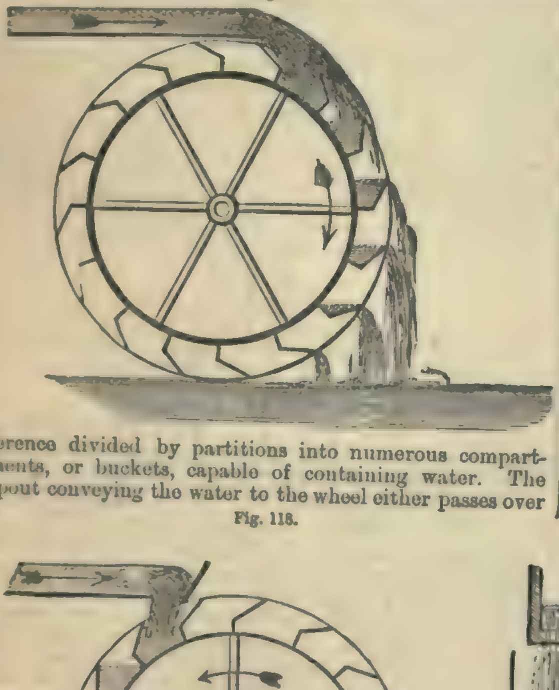
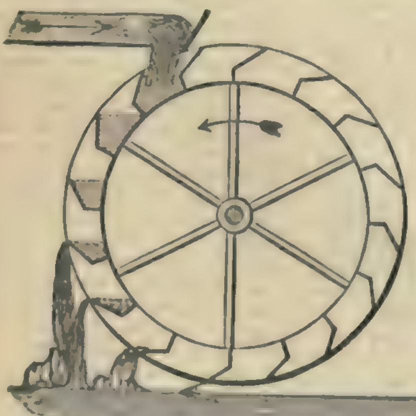


Fig. 118.



2. *The Overshot Water-wheel* (Fig. 117) has its circumference divided by partitions into numerous compartments, or buckets, capable of containing water. The spout conveying the water to the wheel either passes over its summit, or has a check at its end (Fig. 118), so as to discharge the water into the buckets a little beyond the summit of the wheel. As the wheel revolves, each successive bucket is brought under the spout and becomes filled with water, the weight of which, acting on one side of the wheel, is the moving force. The buckets, as they descend, become gradually emptied, and return up the unloaded side of the wheel, to be again filled, and descend.

Such wheels are only applicable where there is a considerable fall of water; for the height of the head, above the stream as it flows away from the wheel, technically called the *tail-water*, must be equal to the diameter of the wheel, or nearly so. In overshot-wheels the velocity of the water is no element of power, except in so far as the quantity conveyed by the spout to the wheel depends upon the velocity with which it flows. If the velocity of discharge be considerable, a positive disadvantage results from the too rapid dash of water into the buckets, causing it to overflow, while the bucket remains only partially filled. It is easy to see that the quantity of water issuing from the spout during the time which a bucket occupies in passing under it, should barely exceed that which the bucket will hold; if it fall short of that quantity, the bucket is only partially filled in its passage; and if it much exceed that quantity, the force of its flow causes it to dash over and become wasted without effectually filling the bucket. The diameter of the wheel being limited by the height of the fall, when it is desirable to take advantage of a large quantity of water discharged from the spout, the breadth of the wheel must be increased, and the water in the spout caused to spread itself out to a wide sheet, so as nearly to cover the whole breadth of the wheel. The sheet of water should always be a few inches narrower than the face of the wheel, to save the water from dashing ineffectively over the edges (Fig. 119). Another point of great importance in the construction of the buckets, is to leave a passage for air at the inner upper angle of each bucket, *a*, *a*, *a* (Fig. 119), otherwise the bucket can become only partially filled, in consequence of the elasticity of the air confined in it compelling the supply-water to dash over the edges instead of filling the bucket.

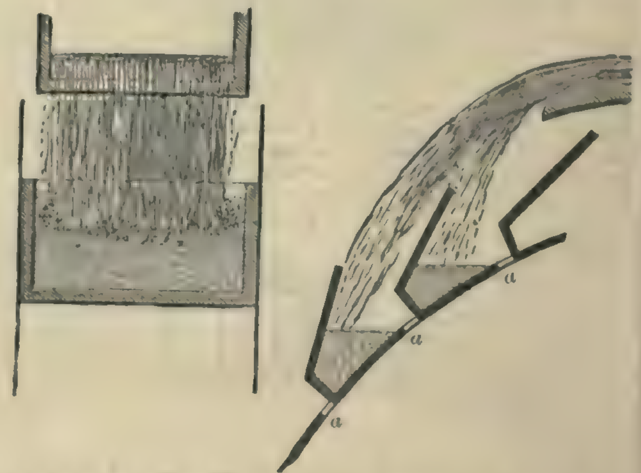
In constructing an overshot-wheel it is necessary to give consideration to the following points:—

1. The point of the circumference at which the spout should discharge so as best to fill the buckets.

2. The best form of bucket for receiving the water, and for retaining it, through a considerable part of its descent.

3. The best speed at which the circumference of the wheel should travel so as to obtain the greatest effect from the moving load of water which its buckets contain.

Fig. 119.



If we suppose that a fall of water about twenty-two feet in height is to act upon an overshot-wheel (Fig. 120), we may make the wheel about twenty-four feet in diameter, and the depth of the buckets, measured towards the centre, two feet. Dividing this depth into two equal parts, each one foot, marked by the dotted circle (Fig. 121), dividing the circumference of this circle into a convenient number of equal parts B B equal to the numder of buckets (40 in the diagram), and drawing lines C B A towards the centre of the wheel through the points of division, we are enabled to determine the form of the buckets. The casing extending between A A is called the sole of a bucket, and is left with a narrow slit open at A for the escape of air from the bucket when the water pours into it. The board A B is called the start, and the inclined part B C the arm of the bucket. Sometimes this inclined part is made in two parts at different degrees of obliquity, like the lines B D, D E; of which B D would be called the arm, and D E the wrist, the start A B being called the shoulder. These names are doubtless given from the resemblance of the section to the form of a bent arm. The whole circumference of buckets and soles is called the shrouding.

Fig. 120.

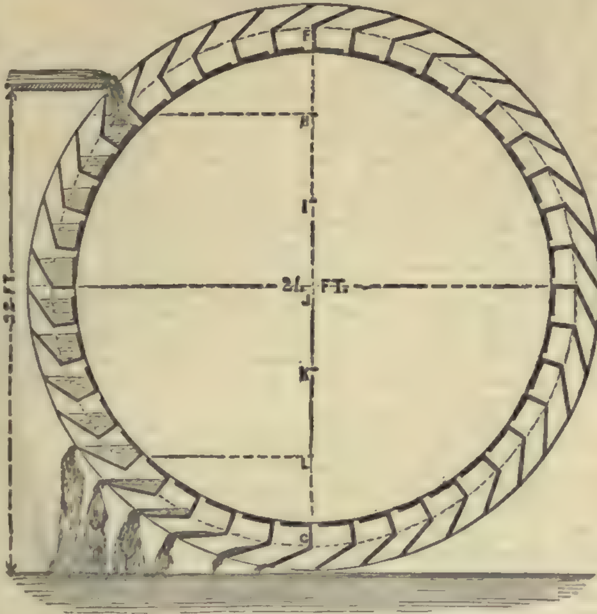
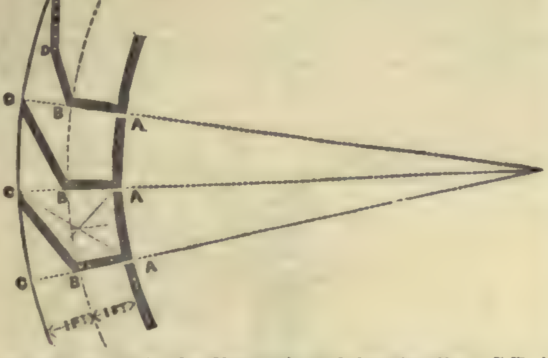


Fig. 121.



On dividing the vertical diameter F G of the mean circle of the shrouding into six equal parts at the points H, I, J, K, L (Fig. 120), and drawing horizontal lines through H and L to meet the circumference, we observe that at the upper line the bucket is filled, and therefore, the weight of its contents begins to act in causing the wheel to revolve, while at the lower line it begins to empty itself, and its action may there be considered to cease; or whatever effect the water may have beyond this point is so small that it may be neglected as an element of power. The total effect of the water, then, in causing the wheel to revolve, may be reckoned to be that of the weight of water contained in ten buckets descending through a height equal to two-thirds of the diameter of the mean circle, viz., $22 \times \frac{2}{3} = 15$ feet nearly. The diameter of the mean circle is equal to the height of fall; and we may, therefore, by taking a wheel of just proportions, generally obtain a descending weight acting through a vertical height two-thirds of the height of fall; and the weight itself, consisting of the contents of one-fourth of the total number of buckets. The capacity of those buckets for containing water depends manifestly on the breadth of the wheel or the length of the buckets, as well as their sectional area. The area of those in the diagram, reckoning up to the level line bounded by the air-slit at their filling-point, and by the lip of the bucket where the discharge begins, may be taken at little above $1\frac{1}{8}$ square foot; and we may suppose, for facility of calculation, that their length or the breadth of the wheel is 1 foot, giving each bucket a capacity for containing $1\frac{1}{8}$ cubic foot of water, weighing about 70 lbs. The contents of the 10 buckets, therefore, weigh 700 lbs.; and this weight is constantly moving with the velocity of the wheel.

In determining the absolute power to drive machinery, we must ascertain the velocity in relation to that with which the water flows from the spout. The circumference of the wheel must move at such a rate that no bucket shall pass the spout without being filled from it. The total circumference of the wheel being $75\frac{1}{2}$ feet, divided into 40 equal parts, we have, for the distance from lip to lip of each bucket, $\frac{75\frac{1}{2}}{40} = 1{\cdot}888$ nearly, or about $1\frac{7}{8}$ foot. If the wheel make one revolution per minute, each bucket passes any fixed point in $\frac{1}{40}$th of a minute, or $1\frac{1}{2}$ second; and the velocity of any point in the circumference is $\frac{75\frac{1}{2}}{60}$, about $1\frac{1}{4}$ foot per second. It has been stated, that the most advantageous circumferential velocity of an overshot-wheel is at the rate of 2 to 3 feet per second. Taking $2\frac{1}{2}$ feet per second for the case we are discussing, the wheel would make 2 revolutions per minute, and each bucket would pass a fixed point in $\frac{3}{4}$ths of a second. As the water issuing from the spout has a certain depth or thickness, some time of the bucket's passage must be deducted in order to ascertain the time allowed for influx of the water. Deducting $\frac{1}{3}$rd of the time, that is, $\frac{1}{4}''$ from $\frac{3}{4}''$, we have $\frac{1}{2}''$ as the time during which the bucket remains under the spout to be filled; and in this time $1\frac{1}{8}$ cubic foot, the contents of the bucket, must flow from the spout—that is, $2 \times 1\frac{1}{8} = 2\frac{1}{4}$ cubic feet in 1 second. As the spout is 1 foot broad, and we must not reckon the depth of water in it above 6 inches or $\frac{1}{2}$ a foot, the sectional area of the water-channel is $\frac{1}{2}$ a square foot, through which $2\frac{1}{4}$ cubic feet must flow per second. The velocity of the water must therefore be $2 \times 2\frac{1}{4} = 4\frac{1}{2}$ feet per second. Should the velocity of the stream be less than this, either the wheel must move more slowly or the spout must be inclined to meet it at a lower level, so that the water may attain greater velocity from additional fall. Should the velocity of the stream exceed this, either the wheel must be permitted to move more quickly, or it and the spout must be widened, so as to present greater capacity of bucket and diminish the speed of influx.

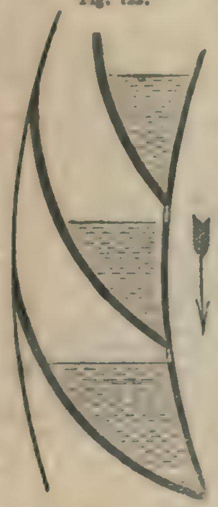
Recurring to the power of the wheel, which we suppose to revolve at the speed of $2\frac{1}{2}$ feet per second, or 150 feet per minute, with a force of 700 lbs. at its circumference, we find the effect to be equivalent to $700 \times 150 = 105000$ lbs. moved 1 foot per minute, $\frac{105000}{33000} =$ about $3\frac{1}{8}$ horse-power.

Had we estimated it in another way by taking the quantity of water issuing from the spout, viz., $2\frac{1}{4}$ cubic feet per second, or 135 cubic feet per minute, weighing about 8,440 lbs., and reckoning its fall or effective movement 15 feet, the distance descended while it remains in the buckets, we should have found the power to be $\frac{8440 \times 15}{33000}$ = about 3⅚ horse-power. But it must be remembered that all the buckets do not act with their full leverage to turn the wheel in descending through 15 feet, being nearer the centre of the wheel above and below the middle level than when they pass that point. The former estimate, therefore, of 3¼ horse-power, is to be taken as the more correct one.

In order to ascertain what fraction it is of the power actually developed by the descent of the water—that is, of the force necessary to raise the water up again to the level whence it flowed—we have to consider that 2¼ cubic feet of water issue from the spout every second, and descend 22 feet, or that a weight of 2¼ × 62½ = to about 140 lbs., moves 22 feet per second, or 22 × 60 = 1320 feet per minute; or that 140 × 1320 = 184,800 lbs. moved through 1 foot per minute. This is equivalent to $\frac{184800}{33000}$ = rather more than 5½ horse-power. Of this the mill has been found to render 3¼ horse-power, or about 57 per cent. available for driving machinery.

It has been stated by some engineers, that as much as 70 to 80 per cent. of the power expended by the fall of water has been made available by means of overshot-wheels; but we are inclined to think that, with the best known construction and proportions, the useful effect does not certainly exceed 70 per cent. of the water-power.

Fig. 122.



Of late years, many of these wheels have been made of iron: the partitions of the buckets are constructed of iron plates, bent to a curved form, and the obliquity is made considerably more than in the wooden shrouding of former times (Fig. 122). The diameter of such wheels is made somewhat greater than the height of fall, so that the water enters the buckets some distance below the summit, where the inclination of the bucket is suited for the reception of the stream. Even if the buckets were filled at the summit of a wheel, and did not empty themselves till they reached the lowest point, the additional effect of their contents would be of little advantage, as it would act more to press the wheel down on its bearings than to turn it round. It will be found advantageous in practice to reckon the diameter of the wheel as ⅛th more than the fall. Thus, for a fall of 24 feet, we should make the wheel 24 and ⅛th of 24, 3; altogether 27 feet in diameter.

It has been recommended that the velocity of the wheel should be made dependent on the height of the fall; that is to say, that it should be ¼th of the velocity which the water would acquire in reaching the bottom by free descent. We can see no reason why such a rule should be observed; for, as we have formerly stated, the velocity of the circumference should be so proportioned to that of the water flowing from the spout, that the buckets may be properly filled during their passage. It is true that, by inclining the spout, we may increase the speed of the stream flowing from it, and thus render a greater velocity of wheel practicable; but, being limited to a certain fall, whatever inclination we give to the spout, we take so much from the height after the water is delivered on the wheel, and consequently reduce the moving weight on the descending side of the wheel. We are, therefore, inclined to adhere to the maxim formerly received among millwrights, that the velocity of the wheel's circumference should not exceed 3 or 4 feet per second, and that, perhaps, it would most advantageously be fixed at 2 to 3 feet per second.

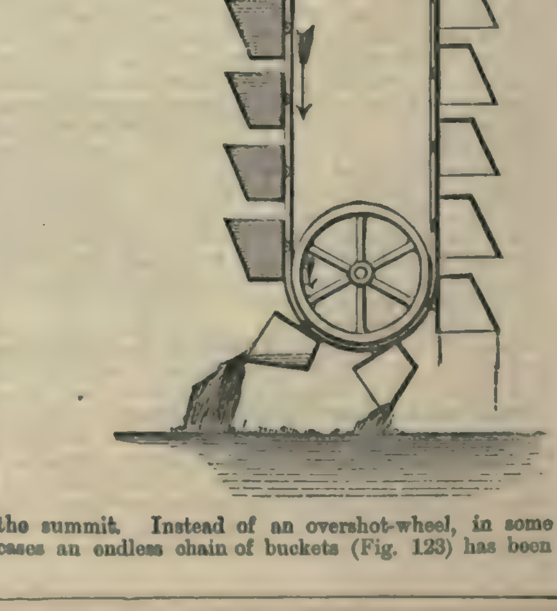
The number of buckets may be determined by making it double the number of feet in the wheel's diameter: thus, in a wheel 24 feet in diameter, the number of buckets would be 48. According to this rule, the space from lip to lip of buckets would always be about 1½ feet. Where the stream in the spout is wide and shallow, it may be made less; and where the stream is deep, it should be greater. But, practically, its size within a few inches is of no great importance; and we should recommend that a division of the circumference by 6, 8, 4, or such numbers and their multiples, should be made, so as to bring each division nearly to 18 inches.

In order to provide for the escape of air from the buckets, it is better to make their width exceed, by several inches at each side, that of the stream, than to provide air-slits in the sole; for, by this arrangement, each of the buckets may be made to hold a considerably greater quantity of water than when the air-slits limit its depth.

Every precaution should be taken to secure a free flow for the tail-water, as the resistance arising from the immersion of the lower part of the wheel in a languid stream, takes considerably from its effective force. It is better to sacrifice a few inches of head by inclining the tail-course, so as to give the water some velocity (at least that of the wheel) in its escape, than to let it act as a drag on the wheel, by making the tail-course too nearly level.

All descriptions of wheels where the water is received on their circumferences, fall under the denomination of overshot-wheels, even if the water be not shot over their summits; indeed, according to the systems now pursued in rendering water-power available, there is no case where a really *overshot*-wheel should be adopted. Instead of making the diameter of the wheel less than the height of fall, so that the spout could be carried over it, the diameter should always be greater, as we have described, so that the water may be delivered at some point below the summit. Instead of an overshot-wheel, in some cases an endless chain of buckets (Fig. 123) has been

Fig. 123.



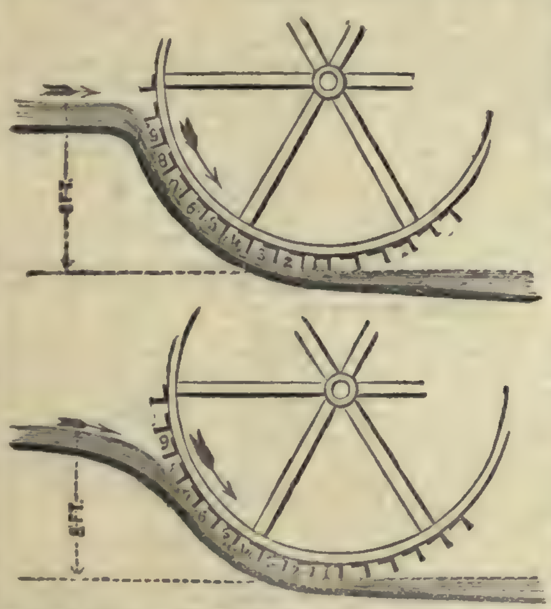
employed for obtaining power from a fall of water. In theory, this arrangement appears one likely to prove more effective than that of the wheel, for the weight of water is retained in the buckets, and acts with constant force throughout the whole descent. Practically, however, the apparatus is not of so substantial and permanent a character as the wheel; the chain has numerous joints, all subject to wear and decay from rust; and when they become deranged, the increased friction and inequality of action considerably diminish the efficiency of the apparatus.

3. *The breast-wheel* is an arrangement intermediate between the undershot and overshot-wheels. It consists of a wheel fitted with floats or paddle-boards round its circumference, revolving with its lower part in a channel which nearly fits it. Each float has a back-plate or sole, so that the wheel is somewhat like an overshot-wheel, with buckets open on their outer sides. When a considerable stream of water falls over a height not sufficient to render an overshot-wheel applicable, and yet greater than would be required for an undershot-wheel, the breast-wheel is applied with great advantage. The floats fit as nearly as possible, without rubbing-friction, to the bottom and sides of the channel, or sweep, in which they revolve; and thus, after passing the point where the water is delivered upon them, they act almost as close buckets, containing a load of water which urges them onwards.

At the point where the floats receive the water, some force arises from the impulse or velocity with which the water strikes them, as well as from the mere weight of their contents. Some millwrights have thought this impulse a most essential element of power, and have therefore contrived the spout so as to throw the water on the floats with great velocity. Others, and among them Smeaton, whose opinions on such matters are always to be received with respect, have arranged the spouts so as to deliver the water on the wheel at as high a level as possible. By this arrangement the impulse from velocity is lessened, but the height through which the water afterwards acts by weight is increased.

If we suppose that a certain stream, flowing with a velocity of 8 feet per second, and having a fall of 8 feet, is applied to driving a breast-wheel 20 feet in diameter, having 40 floats (Fig. 124), we may inquire whether it be more advantageous to deliver the stream at once on the wheel, or to slope its course downwards 3 feet before it meets the floats. In the one case we have the impulse due to a speed of 8 feet per second on one float marked 9, and the weight of the water contained in eight others, marked 1 to 8 inclusive. In the other case we have the impulse due to the increased velocity of stream upon one float marked 7, and the weight of water acting on six others marked 1 to 6 inclusive. Farther, as in the second case the velocity of the delivered water is greater, its stream must be shallower, and therefore it must strike on a less area of float; and if the wheels move at rates respectively proportional to those of their streams, the same quantity of water being supposed to be delivered in each case, each of the buckets in the second wheel must contain less water than each of those in the first. Let us assume that in each case the wheel revolves at a rate which makes its circumference travel at one-third of the velocity of the stream, which we found to be the most advantageous speed for receiving impulse in the case of undershot-wheels. In the first case the velocity would be $\frac{8}{3}=2\frac{2}{3}$ feet per second. In the second case we must calculate the velocity of stream due to increased fall. The fall to produce 8 feet per second is 1 foot; and adding to this the 3 feet of additional fall, we have a fall of 4 feet; the velocity due to which is 8 times its square root, or 16 feet per second. The circumference of the second wheel, then, travels at the rate of $\frac{16}{3}=5\frac{1}{3}$ feet per second, twice the velocity of the first; and if in the first the buckets be exactly filled, in the second they can only be half filled, or need have only half the capacity. If we take the area of float in the first case to be 1 square foot, and in the second ½ square foot, the float marked 9 in the first sustains a pressure due to 8 feet per second, the velocity of the water, less by 2⅔ feet per second its own velocity; that is, to 5⅓ feet per second, equivalent to a column ⁴⁄₉ths of a foot high on 1 square foot of area, about ⁴⁄₉ths × 62½ = 28 lbs. moving at the rate of 2⅔ feet per second, or 2⅔ × 60 = 160 feet per minute, which gives a power of 28 × 160 = 4,480 lbs. moving at 1 foot per minute. In the second case the float 7 is pressed on by a column sufficient to give 16 less by 5⅓, that is, 10⅔ feet per second, which implies a height of 1⁷⁄₉ foot; and this pressing on ½ square foot gives 56 lbs. moving at 5⅓ feet per second, equivalent to 17,920 lbs. moving at 1 foot per minute, 4 times the effect of float 9 in the first case, as might have been surmised, because the velocity is doubled.

Fig. 124.

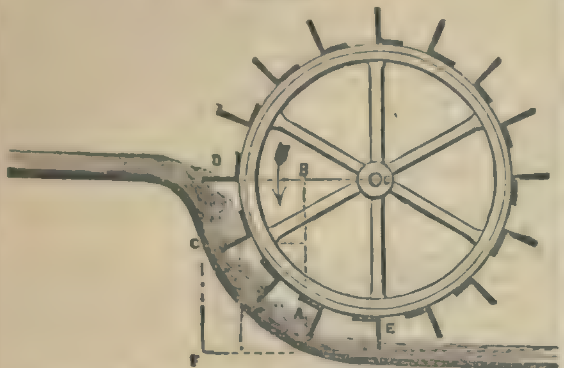


It remains now to compute the effect of the remaining floats in producing power. The total quantity of water issuing is 8 cubic feet per second, or 8 × 62½ × 60 = 30,000 lbs. per minute. In the first case this keeps 8 buckets continually full, and moves them at 2⅔ feet per second, or 160 feet per minute; in the second case it keeps 6 buckets half filled, or 3 buckets quite full, and moves them at 5⅓ feet per second, or 320 feet per minute. As each bucket holds 1 cubic foot, or 62½ lbs., the power of those in the first case is 8 × 160 × 62½ = 80,000 lbs. moving 1 foot per minute; and of those in the second, 3 × 320 × 62½ = 60,000 lbs. Adding to each of these results the power derived from the impulse of the water, we have in the first case 84,480 lbs. moved through 1 foot per minute = 2·54 horse-power; in the second case 77,920 lbs., equivalent to 2·36 horse-power. The result is, therefore, in favour of the first case; and thus Smeaton's view of the circumstances is borne out.

If the floats be tolerably well fitted to the sweep, so that there is little loss of water by escape past their edges, the circumferential speed of the wheel should be considerably more than one-third of that of the stream. A rate as high as two-thirds or three-fourths is practically attained with advantage. When this is the case, the impulse from excess of the stream's velocity over that of the float is much diminished, and the principal element of power is the load of the water contained in the buckets. If, then, the fall of the spout be made just sufficient to deliver the water supplied by the stream or reservoir, all the rest of the fall is most advantageously applied in the sweep, care being taken that sufficient fall is left to carry off the tail-water with full velocity, so that it do not become heaped up, and retard the ascending floats.

In estimating the power of a breast-wheel, we may suppose, for the sake of simplicity, that the water is delivered on the horizontal line of the centre, and keeps all the buckets, from that line to the bottom, full (Fig. 125). Now the effect of the weight of any bucket, such as A, to turn the wheel, depends upon the leverage with which it acts, which would be measured by the length C B of the horizontal line intercepted between the centre of the wheel and the middle of that bucket. Were we to divide the circumference from D to E into a great number of equal parts, and calculate their combined effect as dependent on the leverage with which they respectively act, we should find it to be the same as if one weight—bearing the same proportion to the total weight in the circumference D E as the length of C D, the radius, bears to the portion of the circumference D E—acted on at D. In other words, the effect of all the weight of water in D E to turn the wheel, is the same as that of a column D F of the same width and thickness hanging at D. The same principle is true if the water do not deliver at the level of the centre of the wheel; for if it be delivered at G, the effect of the weight of water between G E would be the same as that of a column of equal area and of the height G F acting at D.

Fig. 125.



If, now, we take the particular case of a wheel 25 feet in diameter, with buckets 1 foot broad and 1 foot deep, receiving the water at the level of the centre, and making 3 revolutions per minute, we may compute its power, and the proportion which its useful effect bears to the expended power of the water. The buckets being 1 foot deep, the circle passing through their middle points would have a diameter of 24 feet, and therefore a radius of 12 feet and a circumference of 75½ feet, making 3 revolutions per minute. The water in the buckets, therefore, moves at the rate of $75\frac{1}{2} \times 3 = 226\frac{1}{2}$ feet per minute; and the weight of the column, having an area of 1 square foot, and being 12 feet high, is $12 \times 62\frac{1}{2} = 750$ lbs. The power then is $\frac{750 \times 226\frac{1}{2}}{33000} =$ about $5\frac{1}{6}$ horse-power.

The quantity of water required to fill the buckets is 226½ cubic feet per minute, for it must three times fill the whole circumference every minute; and as there must be considerable waste from the inaccuracy with which the floats fit the bottom and sides of the sweep in which they revolve, we may reckon 20 per cent. more, or altogether 270 cubic feet per minute, to cover this waste; that is, 4½ cubic feet per second. If we take the stream at the spout 1 foot wide, and 9 inches or ¾ths of a foot deep, its area must be ¾ths of a square foot, through which 4½ cubic feet have to flow per second. The velocity of the water must, therefore, be 6 feet per second, or that due to a fall of nearly 7 inches. The water in working the wheel has to descend 12 feet, and we must allow at least 5 inches more of depth at the bottom of the wheel to clear the floats of back-water, and the total descent is therefore 13 feet: in other words, in order to raise the water up to the proper level to work the wheel, we should have to lift 270 cubic feet 13 feet high every minute. The power required for this would be $\frac{270 \times 62\frac{1}{2} \times 13}{33000}$ = about $6\frac{5}{8}$ horse-power. We found the effective power of the wheel about $5\frac{1}{6}$ horse-power; that is, 77 per cent. of the power expended. We believe that, practically, this estimate would be found too high, and that we could not depend on obtaining, in useful effect, more than 60 to 70 per cent. of the water-power expended.

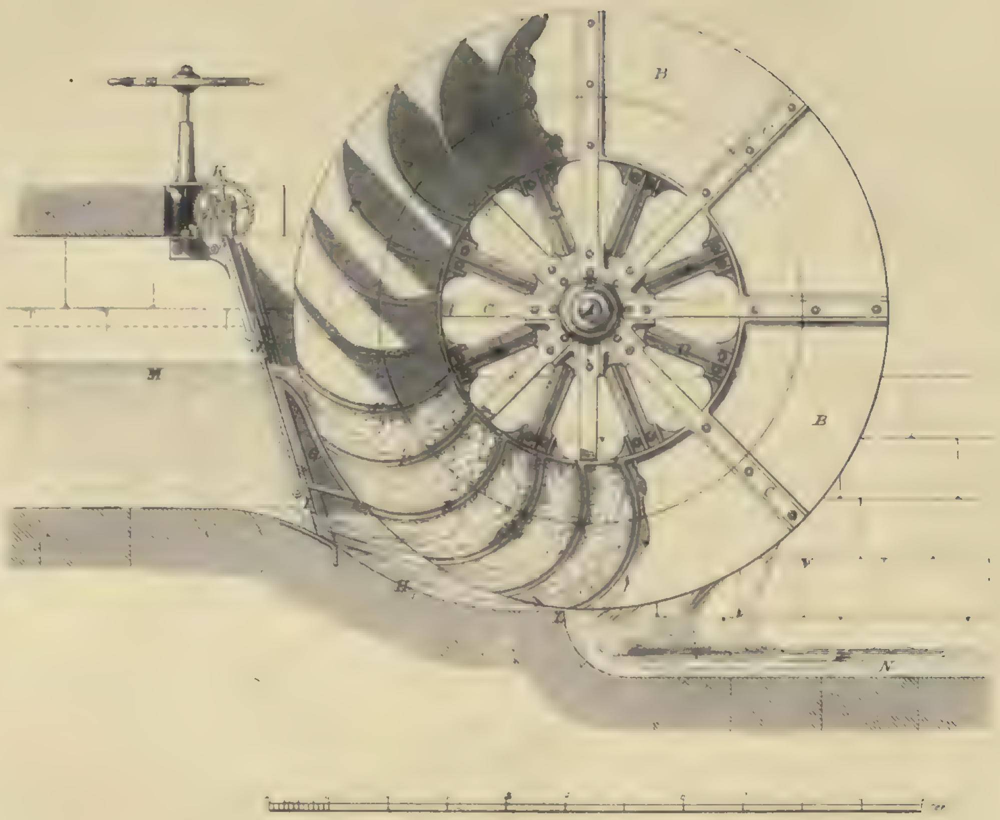
The annexed plate represents an ingenious and, we believe, a very effective construction of water-wheel, combining in one the principles of a breast and undershot-wheel. A strong boss, E, is fitted on to the main shaft or axle, A, on which the wheel turns. Two sets of arms, C C and D D, extend as radii from this boss, some of them, C, C, supporting the sides, B, of the buckets, and the others, D, D, supporting their bottoms or soles. The buckets or floats, F, F, are bent plates extending across the outer part of the wheel, and terminated at both ends by the casing, B, part of which is supposed to be removed in order to show the form of the buckets and the action of the water on them. The head of water, M, is fronted by a double sluice presenting two openings, I and J, at the lower and upper levels respectively, these openings being separated by a casting, G, formed in front to suit the sweep of the wheel. A hand-wheel and suitable gearing, K, are arranged above the sluice for causing it to slide up or down, and thereby regulating the amount of opening at I and J. Under the wheel the masonry, H, is formed in a curve suited to the motion of the water from the lower opening I, and is made to approach close to the edge of the wheel at L, so that no water may pass without acting on the floats. N is the tail-water, which is kept quite under the level of the wheel, so as to give no resistance to its rotation. By this construction, the water which issues from the higher opening, J, fills the buckets, and acts by weight to cause them to descend; while the water which issues from the lower opening I, with the increased velocity due to the head of water in M pressing upon it, rushes into the buckets, and presses the floats onwards, as in an undershot-wheel.

The terms *undershot*, *overshot*, and *breast-wheels*, have been applied in a somewhat different way from that in which we have used them. The term *undershot* has been used when the water is delivered on the wheel anywhere below the level of its centre, and thus the wheels which we have called *breast*-wheels would be among the *undershot;* the term *overshot* has been used in those cases only where the spout is actually carried over the summit of the wheel; and the term *breast* has been applied to wheels where the water is delivered somewhere above the central level. We think, however, that the classification we have adopted here is more distinct, as it refers not only to the different points where the water is delivered, but also to differences in the construction of the wheels. Thus, the *undershot*-wheel is that which receives the water-pressure on simple paddles or floats immersed in the current, and is acted on by its force only; the *overshot*-wheel receives the water at a high level in buckets formed in its circumference, and is moved simply by the weight of water contained in them; the *breast*-wheel receives the water on paddles or floats nearly fitting a sweep in which they revolve, and is thus put in motion partly by the weight of water lodged on and between the floats, and partly by the pressure on the floats arising from the velocity of the current. The peculiar construction of each kind of wheel is adapted to different conditions of the fall of water.

The undershot-wheel is to be used when there is a volume of water moving with considerable velocity, but with very little local fall, as in the case of river streams and tidal currents. The velocity of the current of a river arises from numerous little falls, or from a continuous inclination, without any considerable difference of level within a limited space. The velocity of a tidal current, again, arises from the pressure of the tidal wave, or body of ocean water, elevated above its average level

See page 832

COMPOUND BREAST & UNDERSHOT
WATER-WHEEL.



by the gravitating influence of the moon; but this wave appears only as a gentle and almost imperceptible inclination of the water surface, except in some estuaries, such as the Solway Frith, where it presents itself as an elevated body of water rushing with considerable velocity towards the land.

The overshot-wheel is applicable when the water has a considerable local fall, nearly equal to the diameter of the wheel; and the breast-wheel when the local fall is not great—less, for instance, than half the diameter of the wheel—but when it is of considerable volume, and moves with considerable velocity. In order to apply either an overshot or a breast-wheel, it is generally necessary to make extensive arrangements for conducting the water from an elevated level to the wheel, instead of permitting it to follow its natural channel. When a stream has a considerable fall—such as 40 or 50 feet in each mile of its length—a dam or weir is built across it at some convenient position, so as to check its progress there, and a new channel is formed for conveying its waters to the mill, and thence back to the bed of the stream at some point below the dam. As the artificial channel is made with only sufficient declivity to secure the flow of the water in such quantities as may be required, it is thus possible to obtain at the wheel, nearly the total fall which the channel of stream has, estimated from the point where the dam is built, to that where the tail-water of the mill re-enters. If, for instance, the stream in its natural channel is found to have a fall of 60 feet in a mile—this difference of level being made up either of numerous small local falls or of a continuous declivity, or both—an artificial channel is formed by its side, or as near it as the levels of the ground permit, having a constant declivity for half a mile, amounting to 5 feet of difference of level; the water acts on a wheel with a fall of 20 feet, and a declivity of 5 feet is allowed in the length of the tail-course. The difference of level in the channel for half a mile—that is to say, 30 feet—is thus made up, and the power due to $\frac{2}{3}$rds of that fall is thus secured for driving machinery. The current of the stream itself would probably not have so great a velocity at any place as to make it practically available for an undershot-wheel, on account of the irregularities of its channel, and the numerous resistances opposed to its progress.

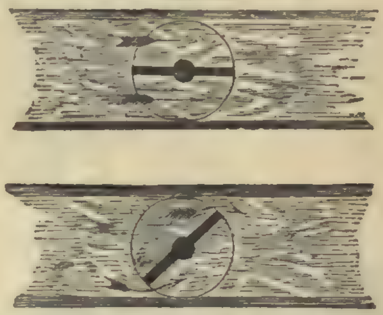
DAMS, OR RESERVOIRS.—Where there is no stream of sufficient magnitude to give the necessary power, and the power is not required to be constantly in operation, it is usual to form a large dam or reservoir for collecting the constant small tribute of the stream, so that the volume of water thus accumulated may be employed to drive the mill as occasion requires. For moving agricultural machinery, such as thrashing-machines, this plan is very commonly resorted to. The corn of a farm is generally thrashed in the winter season, when there is the best supply of water in the streams, and from the drainage of the soil. A small mill-dam or reservoir collects, during the night, sufficient water to drive the mill through the following day. And it is thus possible, even in localities where apparently no water-power can be obtained, to secure enough for the work to be done, by executing a properly contrived dam, and turning into it the drainage of the surrounding fields.

When water is applied to manufacturing purposes, requiring the constant supply of large volumes of water, reservoirs or dams are sometimes executed on a gigantic scale, in order to store up the superfluous supply of rainy weather against a season of drought. In some of the hilly districts of England and Scotland, these works are of a most important character, and the interests of large local populations are dependent on their efficiency and permanence. When the differences of level in the district are considerable, numerous mills are worked successively by the same water, that which has driven the higher, flowing along an artificial channel till it arrives at the next lower; and so on, in constant succession for great distances. In such cases, the mill-owners frequently combine to execute works for the benefit of all, and of much greater magnitude than a single capitalist could undertake. By such arrangements, they are enabled to throw an immense dam or retaining-wall across some valley, and thus collect in a vast reservoir the drainage of an extensive range of hills, which would otherwise flow along its natural course to the sea without being turned to useful account as a mover of machinery. While the rains or melting of the snows contribute much more water than is required, the reservoir is filled; and when the water in it attains the highest level required, it is permitted to overflow into its natural channel. When the season of drought arrives, the mills, that would otherwise be at a stand-still, derive an ample supply from the reservoir, extending, in some cases, over many square miles of valley. When the depth of this reservoir is considerable, great strength is required in the dam; and, notwithstanding the ingenuity and labour expended on some of these structures, they sometimes give way; and the enormous volume of water thus suddenly set free rushes impetuously onward to the sea, devastating whole districts in its course, destroying crops and buildings, and too frequently causing a great sacrifice of life.

The sluices or valves for opening and closing the water-channel of a mill are generally of very simple construction; they consist of a plate of wood or metal, made to fit against a framework fixed in the channel, and pressed against it by the water. This plate is made to slide upwards in grooves fitting it at each side; and when it is of large dimensions, it is raised or depressed by racks and pinions, or screws fitted with appropriate gearing, as represented at K, I, and J in the engraving of the compound wheel. By opening or shutting the sluice, the wheel is put in motion or stopped at pleasure. A channel is always provided to carry off the surplus water to the tail-course of the wheel, when it rises in the spout or lead, above the sill of the waste channel.

REGULATORS.—Various contrivances have been applied to regulate the speed of water-wheels. The most effectual is the steam-engine governor, or conical pendulum. A throttle-valve, or plate, moving on an axis or pivots at its middle, is fitted into the lead (Fig. 126). When it presents its edge to the current, it offers very little obstacle to its course; but when it is turned into an oblique position across the current, it arrests all the water except what can pass through the openings left between its edges and the sides of the lead. As it is poised on pivots in its middle, the pressure of water on each limb is the same; and the only force required to move it in either direction is that required for overcoming the friction of its pivots, and the resistance of the water to its movement through it, a force very inconsiderable. The governor is connected, by rods and levers, to a valve of this description in such a manner, that when, from too great velocity of the water-wheel, the governor-balls fly out from their axis, the valve is closed, or partly so, and the supply of water to the wheel diminished. When the wheel moves too slowly, the balls fall down to the axis, and cause the valve to open for the passage of a greater volume of water to the wheel. By this arrangement, the movement of the wheel is regulated with great nicety, and the quantity of water supplied to the wheel is suited exactly to the work which it has to do. The consequence is, a great saving of tear and wear to the

Fig. 126.



machinery, and a regularity of movement better suited to all those mechanical operations to which motive power is applied.

POWER OF STREAMS.—The power which can be derived from a given stream of water may be computed with tolerable accuracy. When, by levelling the ground, it is ascertained how much fall may be secured, making ample allowance for the declivity of channel to and from the intended wheel, the volume of water delivered in a certain time is to be computed by measuring the area of the existing channel, and the velocity with which the water flows through it. The area of the channel may be found by dropping a plumb-line, at numerous equal-measured intervals, across some part of the current where the water moves with tolerably equable velocity, and tracing out the section according to the measurements so taken. The area can then be calculated by the ordinary rule for mensuration of superficies: for example, if the total width of the surface of the stream be 9 feet, and the soundings taken at every foot be those marked in Fig. 127 (in feet and fractions of a foot), the area is the sum of all those depths, viz., 8 square feet. The velocity of the current may then be ascertained by throwing into it a floating body at some distance above a marked length of channel, so that before it floats within the range of the marked distance, it may have attained the speed of the current. The time of its passage over the marked distance may be then observed by a stop-watch. We may assume, for instance, that the marked distance is 20 feet, and that the floating body occupies 5 seconds in passing this; we conclude that its velocity is $\frac{20}{5} = 4$ feet per second. We must not, however, assume this to be the velocity of the whole stream, for at the bottom, and particularly at the shallow sides of the channel, the friction of the water on the rough surface considerably retards it. The effect of this retardation may be often observed upon a streak of foam spreading across the channel, the middle part advancing rapidly and breaking away from the side portions, which sometimes are even caused to move backwards by the eddying currents. In a shallow stream, like that in the case we have supposed, we should not, perhaps, be safe in assuming more than half the middle surface speed as the average of the whole stream, say 2 feet per second. Multiplying this by the area, we find that $8 \times 2 = 16$ cubic feet of water per second is delivered by the stream.

Fig. 127.

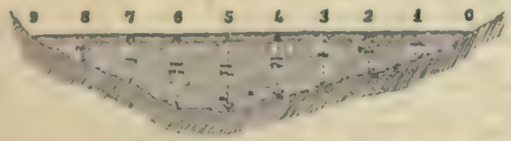
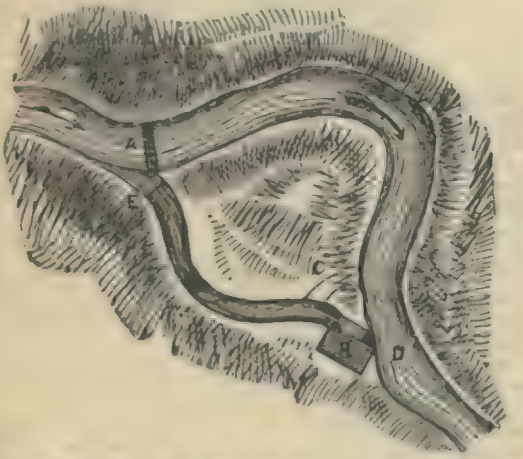


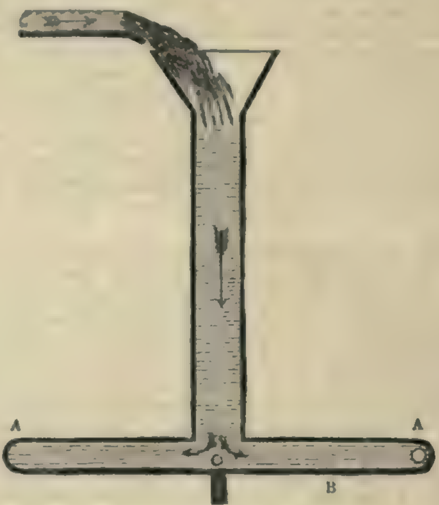
Fig. 128.



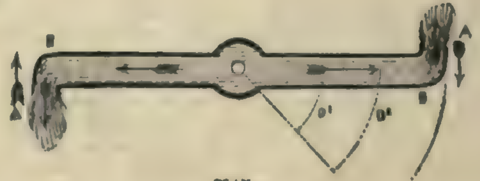
Having levelled the ground in the neighbourhood of the stream, we find that by building a dam 4 feet high at A (Fig. 128), and excavating a channel with a gentle declivity to B, a convenient site for a mill, and thence into the bed of the stream, we can get a fall of 15 feet at the mill-wheel, allowing for declivity of channel from A to B, and a proper fall for the tail-water from B to the stream at D. We should, therefore, have 16 cubic feet of water per second falling down 15 feet of vertical height as our moving power. To lift this quantity up the height of its fall, we should require $\frac{16 \times 60 \times 62\frac{1}{2} \times 15}{33000} = 27$ horse-power. By a proper arrangement of wheel, we may expect to secure about $\frac{2}{3}$rds of this for effective working power, which we should therefore estimate at $27 \times \frac{2}{3} = 18$ horse-power. A sluice would be fitted across the lead at E, so that the water might be excluded from it for purposes of cleaning or repairing the channel; and the proper sluice and regulating valve would be fitted at the mill with a sluice and waste water-course C for emptying the lead when the entering sluice at E is closed.

TURBINES.—When the volume of water is small, but the fall considerable, an apparatus called a turbine is frequently applied with great advantage. The principle of its action is similar to that of the well-known firework called the Catherine-wheel, or of the revolving jet sometimes applied to fountains. For a considerable period it has been known as a philosophical toy, called Barker's mill. This consists of a vertical tube, with two horizontal branches closed at the end, mounted on a vertical axis on which it can freely revolve (Fig. 129). Near the extremity of the horizontal arms, holes A A are made on opposite sides; and when water is poured into the upper part of the tube, it flows through these holes, and makes the arms revolve in the opposite direction. The cause of their motion may be very simply explained. If we suppose the apparatus at rest, and the holes closed, when the tube and arms are filled with water, every square inch of the inner surface of those arms is equally pressed on by the column of water in the vertical tube;

Fig. 129.



SECTION.

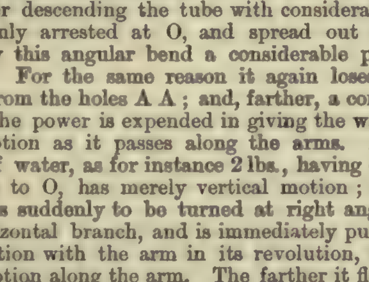


PLAN.

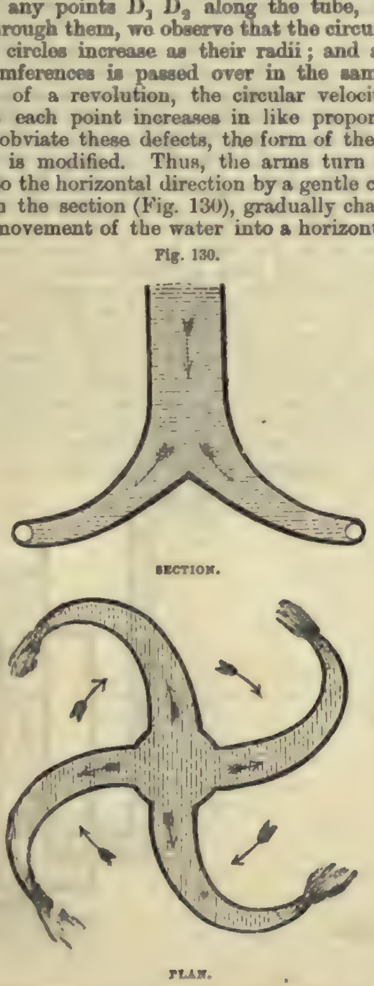
for it is the property of fluids to communicate pressure equally in all directions.* Under these circumstances, there is no tendency to produce motion in any direction; but if the holes A A be opened, then, while the pressure on one side of the tube B remains the same as before, that on the other is lessened by as much as its surface is diminished. If we suppose each hole to have an area of one square inch, then each side of the tube B sustains a pressure on one square inch more than the other side; in other words, there is a pressure on B exceeding that on A by that due to the area of the hole in A. This excess of pressure causes motion in the direction in which it acts—that is, opposite to the flow of the water issuing from the holes; and the force of the movement depends upon the amount of unbalanced area in each arm, and the intensity of pressure upon it.

In the simple Barker's mill there is considerable loss of power from impediments to the flow of the water. The water descending the tube with considerable speed, is suddenly arrested at O, and spread out laterally; losing by this angular bend a considerable part of its velocity. For the same reason it again loses speed in issuing from the holes A A; and, farther, a considerable part of the power is expended in giving the water a circular motion as it passes along the arms. A certain weight of water, as for instance 2 lbs., having descended the tube to O, has merely vertical motion; half of it (1 lb.) has suddenly to be turned at right angles along each horizontal branch, and is immediately put into circular motion with the arm in its revolution, as well as direct motion along the arm. The farther it flows along the tube, the more rapid is its circular motion; for if we take any points D_1 D_2 along the tube, and trace circles through them, we observe that the circumferences of these circles increase as their radii; and as each of the circumferences is passed over in the same period, the time of a revolution, the circular velocity of the water at each point increases in like proportion. In order to obviate these defects, the form of the tube and its arms is modified. Thus, the arms turn from the vertical to the horizontal direction by a gentle curvature, as seen in the section (Fig. 130), gradually changing the vertical movement of the water into a horizontal movement with little loss of force. And again, the arms, of which there may be any convenient number, bend also horizontally, as seen on the plan; so that while they revolve, the water contained in them is really moving almost in a straight line, instead of being swept round in a circle.

Fig. 130.



SECTION.



PLAN.

As the mouths of the arms must be made of such an area as to permit the issue of the water at the proper velocity due to its fall, if they are made too small, less water passes them than can be supplied, and the machine is not so powerful as it might be with the given supply of water. If, on the other hand, the mouths are made too large, the velocity of the issuing water is diminished; and the pressure on the opposite sides of the arms tending to drive them round, is diminished with it. The area of the mouths being decided according to the quantity of water, and its velocity from vertical fall, the arms are made to taper gradually to that area, so that the velocity of the water may gradually increase to suit the gradually diminished area of its channel, as it would naturally do during its vertical descent. Due consideration having been given to these points, as well as to the best mechanical arrangements for strength, durability, and economy of execution, the machine becomes a turbine, practically applicable in many cases with great advantage. M. Fourneyron in France, and Messrs. Whitelaw and Stirrat in Scotland, have executed many of these machines, and made interesting experiments on their power and the best modes of constructing them. Their simplicity and efficiency, and the small space they occupy, give them an advantage over water-wheels; and it is said that they are capable of deriving from a fall of water, quite as much effective power as wheels of the best construction, even if the volume of water be large. Experiments conducted by Morin in France, lead to the conclusion that turbines are actually more effective than wheels under similar circumstances, the useful effect averaging from 70 to 78 per cent. of the power of the water. It has been found that even the immersion of the arms to a depth of several feet in water does not materially affect their action; so that even greater height than that of the fall, measured to the level of the tail-water, can be taken advantage of.

In estimating the power that may be derived from a given fall by means of a turbine, $\frac{3}{4}$ths of the power required to raise the water up again, may be reckoned as the usual effect. Thus if the volume of water be 16 cubic feet per second, and the fall 15 feet, the power required to raise it would be $\frac{16 \times 60 \times 62\frac{1}{2} \times 15}{33000} = 27$ horse-power; and the power of the turbine to drive machinery may be taken at $\frac{3}{4}$ths of 27, about 20 horse-power.

It is probable that turbines will in many cases take the place of water-wheels. They are as yet comparatively novel, and not widely known, or looked on with prejudice; but as improvements are gradually made in their construction and adaptation, and as they become more common in their application, these prejudices will doubtless give way, and no longer interfere with the extended use of a simple and elegant apparatus, instead of the large and cumbrous wheels now generally used. For regulating the motion of turbines, arrangements may be made similar to those used for governing water-wheels; the quantity of water supplied to the turbine being regulated according to the speed required, and the work to which it is applied. The power of the turbine is found to be very nearly proportional to the quantity of water passing through it; so that having found its maximum power, or the greatest quantity of water that it will use, we can employ $\frac{1}{2}$ or $\frac{3}{4}$ths of that power by reducing the supply of water to $\frac{1}{2}$ or $\frac{3}{4}$ths of the maximum.

CONTRIVANCES FOR RAISING WATER.—Before quitting the subject of water-power, we may notice some contrivances by which a volume of water is made to raise a smaller volume to a greater height for purposes of irrigation or the like. The simplest of these is the Persian wheel. A breast or undershot-wheel of the ordinary kind, has a number of buckets hung on pivots to its circumference (Fig. 131). These buckets dip into

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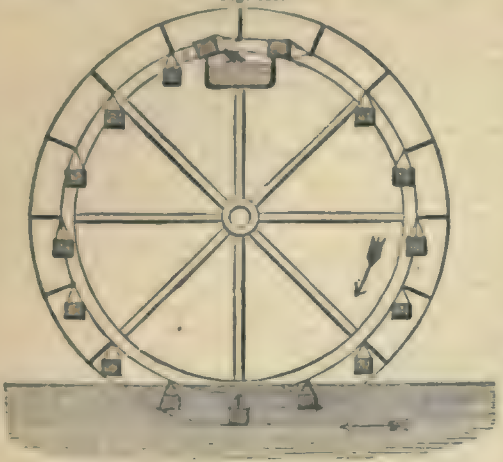
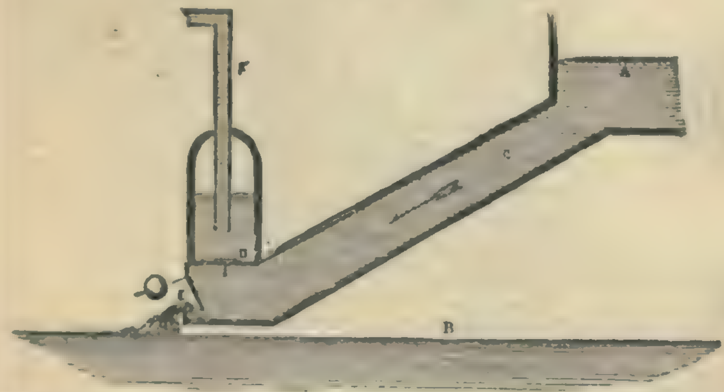
* See *ante*, *Hydrostatics*, p. 751.

Fig. 131.



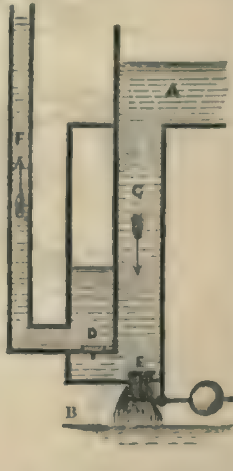
the water at their lowest level, and being filled, are carried up one side till they come in contact with the side of a spout fitted near the summit of the wheel. They are canted over by this spout, and discharge their contents into it, to be conveyed away for their required purposes. The empty buckets descend on the other side, to be again filled and lifted as before. Thus the force of a stream having inconsiderable fall is made to lift a certain quantity of water nearly the whole height of the wheel. The power of the wheel is expended on lifting a continuous weight of filled buckets up one side; and the quantity of water contained in them, as well as the height to which it is lifted, depends upon the size of the floats of the wheel and the pressure of the stream upon them. We may suppose a wheel 20 feet in diameter, with floats 8 feet broad and 1 ft. 6 ins. deep, worked at the circumferential velocity of 4 feet per second by a stream flowing at the rate of 12 feet per second. Subtracting 4 from 12, we get 8 feet per second as the excess of velocity of the stream over that of the float of the wheel; and the pressure due to that excess is ($\frac{8^2}{64}$ or 1 square) that of a column of water 1 foot high. The area of the float is $8 \times 1\frac{1}{2} = 12$ square feet; therefore the total pressure on the float is equivalent to 12 cubic feet of water, and it moves at the velocity of 4 feet per second, or 240 feet per minute, giving the same force as $240 \times 12 = 2880$ cubic feet of water moved 1 foot per minute. We may deduct $\frac{1}{3}$rd of this to allow for various losses, thus leaving an effective power equivalent to 1920 cubic feet of water lifted 1 foot high per minute, or $\frac{1920}{20} = 96$ cubic feet lifted 20 feet (the height of the wheel) per minute. As the wheel's circumference may be taken at 60 feet moving at the rate of 120 feet per minute, the wheel makes 2 revolutions per minute, and therefore twice in every minute lifts the contents of all its buckets, amounting to 96 cubic feet as we found above. The buckets may therefore altogether contain 48 cubic feet; and if their number be 24, each may have a capacity of 2 cubic feet. If we estimate the quantity of water acting on the wheel as the area of the float multiplied by the velocity of the stream, it appears that 8 ft. $\times 1\frac{1}{2}$ ft. $\times 12$ ft. $= 144$ cubic feet passes per second, or $144 \times 60 = 8640$ cubic feet per minute, of which 96 or $\frac{1}{90}$th part is lifted 20 feet high by the wheel which it moves.

Fig. 132.



The hydraulic ram is an ingenious contrivance, by which a small fall of a considerable body of water is made to raise a much smaller volume of water to a considerable height. From a reservoir A (Fig. 132), at the height of a few feet above the lower level of the stream at B, a large pipe conducts the water; this pipe has an aperture D, on its upper side near to its lower end, and the aperture is closed by a valve opening upwards, into an air-vessel, from which a small pipe F leads to a cistern at a level considerably above that of A. At the lower end of the inclined pipe there is a hinged valve E, opening inwards, and kept open by a weight fixed on a lever projecting from the valve. This weight is adjusted nicely, so as to counterbalance the pressure of the water on the surface of the valve E, but not greatly to exceed it. When the weight opens the valve, the whole of the water in the inclined pipe C begins to flow downwards, and issue at the opening made by the valve at E. Having acquired a certain velocity, it presses with greater force on the valve, and closes it in opposition to the weight, thereby completely arresting its own flow; but the momentum of the large body of water flowing along the pipe C cannot be suddenly destroyed, but must expend itself somewhere. It therefore lifts the small valve D with considerable force, and part of it flows into the air-vessel and up the pipe F. The momentum being thus absorbed, and the water in the pipe C having become still, the valve E again is opened by the weight, and the operation is repeated. Thus, by the alternate opening and closing of the valve E, under the quiescent and moving pressures of the water, a certain portion of the water is forced up the pipe F, and is prevented from returning by the closing of the valve D. The object of the air-vessel is to provide an elastic spring for the water propelled upwards: every time that the water is injected into it, the air in its upper part is compressed into a smaller space; and being perfectly elastic, tends to resume its former volume. It therefore exerts a pressure on the water, and continues its flow along the pipe F during the intervals that elapse between the successive discharges through the valve D. In estimating the power of this apparatus to raise water, we may suppose it arranged with the flow-pipe vertical instead of inclined, as it is usually made for convenience, the principle not being altered, but the details of calculation simplified by the vertical arrangement (Fig. 133). We may suppose that the height from the valve to the surface of the water in A is 4 feet, and that the area of the valve E is 1 square

Fig. 133.



foot. The pressure on the valve is, therefore, the weight of 4 cubic feet of water; namely, 4 × 62½ = 250 lbs. The effect of the weight to lift the valve by means of its lever must be somewhat greater than this. If the valve could be suddenly lifted so as to leave an opening to the full extent of its area, 1 square foot, the water would descend in C at the rate of 16 feet per second, the velocity due to 4 feet head; but as the valve opens gradually, and is only open for a short time, and that not to its full extent, we may take the velocity of the descending column at not more than ¼th of this rate; that is to say, at 4 feet per second. When this movement ensues, the valve is pressed on not only by the weight of the column above it, as before, but also by the weight of such a column as is due to a velocity of 4 feet per second, which will be found by calculation to be ¼th of a foot high, adding 62½ × ¼, about 15½ lbs. to the load on the valve. This additional load overcomes the leverage of the weight, and closes the valve; but the column of water, 1 foot in area and 4 feet high, contained in C, amounting to 4 cubic feet, is thus arrested whilst moving at the rate of 4 feet per second; and its momentum, which is equivalent to that of 4 × 4 = 16 cubic feet, moving at 1 foot per second, or 1 cubic foot at 16 feet per second, must be given out as a force propelling the water along the small pipe at the side. If we suppose that this pipe communicates with a cistern 36 feet high, the velocity due to that height is 48 feet per second; and the momentum of 1 cubic foot, moving at 16 feet per second, being equivalent to ⅓rd of a cubic foot moving at 48 feet per second, we should expect that this quantity—⅓rd of a cubic foot—would be propelled upwards to the high cistern at each closing of the valve. We must, however, recollect, that the momentum of the larger volume has not only to balance that of the smaller—it must considerably exceed it, because it has to lift the valve, to give the motion upwards to the column, to overcome friction in the pipes and other impediments, and, upon the whole, may be reckoned effective to the extent of not more than ¼th of its estimated force. We may estimate, then, that $\frac{1}{12}$th of a cubic foot is propelled up to the high cistern by the descent of 4 cubic feet through 4 feet from the lower cistern. Should the action not last so long as a second, a smaller volume will descend, and a proportionally smaller quantity will be sent upwards, and conversely; but the number of times the descent and ascent take place in a given period will be greater or less accordingly.

We have presented this calculation only as a rough approximation to the practical results. We are not aware that any carefully-conducted experiments with hydraulic rams have been given to the world, and we therefore do not venture to offer any estimate as a guide for practice; but have merely discussed a case with the view of opening the questions that have to be considered in dealing with such apparatus. In many situations where it may be desirable to raise water for the purpose of ornamental fountains, or of domestic supply, the hydraulic ram is applicable with great advantage. A neighbouring stream may be dammed so as to provide a fall of a few feet, if it have not sufficient local fall naturally; and the apparatus once fixed and properly adjusted, will continue effective for a long period. It is exceedingly simple, entirely self-acting, and seldom liable to derangement, if care be taken to fix gratings on the pipe, so that dirt or extraneous matter of any kind may be prevented from interfering with the action of the valves.

3. WEIGHT AND ELASTICITY OF BODIES.—The gravitating attraction which the earth exerts on bodies near its surface, and the force with which elastic bodies tend to resume the condition from which they have been withdrawn, are frequently employed for giving motion to machinery, chiefly when the power required is small, but exerted for considerable periods. Weight and elasticity are not really sources of power; they rather afford the means of storing up efforts of short duration for subsequent use during longer periods. Before a body can act by its weight, it must be lifted to the height whence it has to descend; and, in the same way, before a body can act by its elasticity, its condition must be changed to the extent through which it has afterwards to return. Thus, in winding up a clock or watch, as much power is exerted as is afterwards given out by the weight or spring in moving the machinery with which either is connected. The weight of the clock, and the spring of the watch, are merely instruments for absorbing, at the moment, a certain amount of power, and giving it out by degrees afterwards. The simplest, and, it may be said, the only general mode of employing the weight of a solid body to give impulse to machinery, is to attach it by a chain or string to a cylindrical barrel mounted on bearings at some height from the ground, and connected by gearing with the machinery which it is intended to move. The barrel being caused to revolve by hand, or any other convenient power applied to it, the string is wound round it, and the weight raised from the ground. When left to itself, the weight, attracted again towards the earth, descends, unwinding the string from the barrel, causing it to revolve, and thus giving the necessary impulse to the machinery. Did the machinery offer no resistance, the weight would always descend with a speed accelerated at every moment of its descent by the continued action of gravity, which exerts as much influence on a body in motion as on one at rest. In dropping a stone from a considerable height, whatever be its weight, we find that, during the first second of its descent, it acquires a velocity of 32 feet per second. Its velocity at the commencement was nothing, for it began to move from a state of rest; at every one of the instants into which we may conceive a second of time divided, it acquired more and more velocity, until it attained the final velocity of 32 feet per second. All these acquisitions in speed are equal in equal times, because the force of gravity is constant, and therefore exerts equal influences in equal times. Had the body descended during the whole second at the final velocity of 32 feet per second, it would of course have passed through 32 feet of space; had its velocity remained the initial velocity, which was nothing, it would have descended through 0 feet; but as the velocity began with 0 and ended with 32, its average throughout the second was 16 feet per second; and therefore the body descends in the first second through 16 feet. During the next second, the body, starting with a velocity of 32, acquires an additional velocity of 32, and therefore ends with a velocity of 64 feet per second; the average being 48 feet per second, and therefore the descent being 48 feet of height. Adding this to the space descended during the first second, 16 feet, we find that in the first 2 seconds the total descent is 64. Were we to pursue the investigation farther, we should find the velocity at the end of the third second 96 feet per second, and the total descent 144 feet, and so on according to the following law:—The velocity (in feet per second) acquired by a falling body is 32 times the time (in seconds).

The space (in feet) passed through by a falling body is 16 times the square of the time, or the square of 4 times the time. Hence it follows that the time (in seconds) occupied by the descent of a falling body through a given height (in feet), is ⅛th of the square root of the height; that the velocity (in feet per second) is 8 times the square root of the height; and that the height is the square of ⅛th of the velocity.

Example 1.—Required the velocity acquired by a falling body in 5 seconds.

32 × 5 = 160 feet per second.

Example 2.—Required the height fallen by a body in 5 seconds.

16 × 5 × 5 = 400 feet, or (4 × 5 =) 20 × 20 = 400.

Example 3.—Required the time occupied by a body falling through 400 feet.

Sq. root of 400 is 20, and ⅛th of 20 is 5 seconds.

Example 4.—Required the velocity acquired in falling 400 feet.

Sq. root of 400 is 20, and 8 times 20 is 160 feet per second.

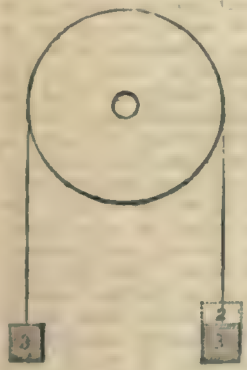
Example 5.—Required the distance fallen by a body when it has acquired a velocity of 160 feet per second.

$$\frac{100}{8}=20, \text{ and } 20 \times 20=400 \text{ feet.}$$

These rules only apply correctly when a body falls in vacuo, for the resistance of the air materially retards the velocities, especially when they become considerable, and when the body has considerable bulk in relation to its weight. Were it not for this resistance, every raindrop, descending as it does from a height of many hundred feet, would strike with a force as fatal as that of a rifle-bullet.

When part of the weight of a falling body is balanced by the ascent of some weight connected with it, its velocity is materially diminished. When two equal weights, each 3 lbs., are hung by a string over a pulley (Fig. 134), they exactly balance each other—that is to say, the gravitating attraction exerted on one is equal to that exerted on the other; and as the one cannot descend, in accordance with this force, without causing the other to ascend in opposition to it, no motion takes place; but if we add to the one 2 lbs., we apply to that side the gravitating force of 2 lbs., and the motion takes place in obedience to it. The force of 2 lbs. has, however, not only to put in motion its own mass, but also that of the two weights, in all 8 lbs., or 4 times its own weight; and therefore the velocity which they all acquire in 1 second can only be the ¼th of that which the 2 lbs. alone would acquire—that is to say, 8 feet per second instead of 32. The distance passed over must therefore be only ¼th of what it would be were the moving weight left to itself without having to put the others in motion. In all such cases, then, of weights partly balanced, if we know the total of the weights put in motion, and the excess of the one over the other, we find the velocity or the space in a given time by the following rule:—

Fig. 134.



Find the space or velocity as before, multiply by the excess of one weight, and divide by the sum of all.

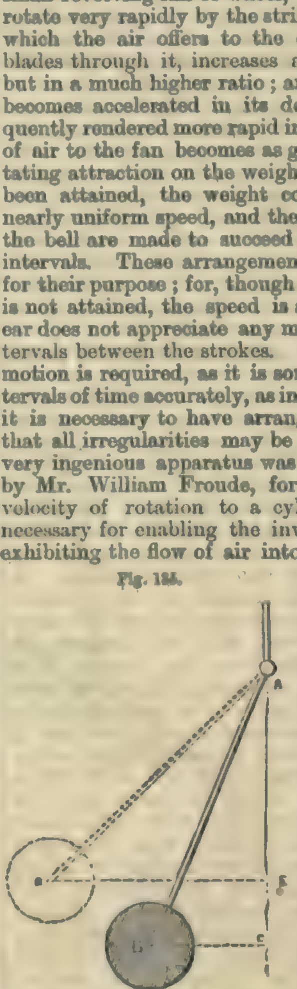
Example 6.—Required the space passed through in 4 seconds by a weight of 19 lbs. connected by a string over a pulley with one of 13 lbs.

The excess of the one is 6 lbs.; the sum of both is 32 lbs. The space would be for a single weight, $16 \times 4 \times 4 = 256$ feet, and

$\frac{256 \times 6}{32} = 48$ feet is the space descended in 4 seconds under the circumstances given.

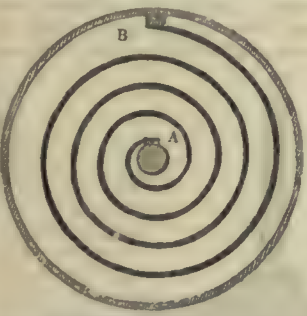
It does not often happen that, in practice, two different weights are hung over a pulley as we have just described. The effect, however, is just the same if there be one weight acting on a barrel, and if the movement of the barrel be resisted by some known force. If, for instance, we had a weight of 19 lbs. attached to a barrel connected by machinery with some other load, and we found that 13 lbs. hung to the barrel, instead of 19 lbs., would exactly balance the load upon the machinery, so that no movement of the barrel took place, we should estimate, as we have done above, that 19 lbs. had to set in motion itself and 13 lbs., the drag of the machinery, and that the space passed through in four seconds would be 48 feet, as we have computed. All such estimates, however, are made without regard to the resistance caused by friction. The rubbing surfaces of machinery are so irregular, and subject to such changes of condition by wear, temperature, deficiency of lubrication, and other causes, that friction cannot be estimated as a regular resistance. No machinery moved by a weight, without some special regulating apparatus, can be expected to move with uniform velocity, or with velocity varying according to any uniform law on account of these irregularities of resistance. The law of motion which a falling body obeys is, therefore, scarcely ever practically applicable. In all machinery moved by weights, some contrivances are introduced for providing a resistance so much greater than that of the mere friction, that the weights upon the whole may be made to descend uniformly, and not with the accelerated velocity due to gravitating force alone. Thus, in the time-keeping part of a clock, the weight which puts the whole in motion, is arrested at every moment of its descent by the pendulum and escapement. The motion of the weight downwards becomes in this case a series of extremely short descents, recurring at equal intervals, whenever the pallets of the escapement permit the train of wheels to move. The weight employed is so much in excess of the resistance from friction, that the irregularity in the time of each of its partial descents is exceedingly small. The regularity of the clock's motion, therefore, depends upon the uniformity of the times which the pendulum occupies in each of its beats, the interval during which the weight descends and the train moves, being too small for any irregularity to manifest itself. In the striking part of a clock, there is no necessity for extreme regularity. The whole striking train is kept at rest until the time-keeping part comes to such a point, that it removes the obstacle to the motion of the striking-weight. This, being relieved, begins to descend, and would go on descending with accelerated velocity, moving its train of wheels, and causing the hammer to strike the bell faster and faster, were not a resistance provided, sufficient to prevent the acceleration from becoming too great. The apparatus generally used for giving this resistance, consists of a small revolving fan or wheel, with flat blades, caused to rotate very rapidly by the striking train. The resistance which the air offers to the quick passage of the fan-blades through it, increases as their velocity increases, but in a much higher ratio; and gradually, as the weight becomes accelerated in its descent, and the fan consequently rendered more rapid in its rotation, the resistance of air to the fan becomes as great as the effort of gravitating attraction on the weight. When this velocity has been attained, the weight continues to descend with nearly uniform speed, and the strokes of the hammer on the bell are made to succeed each other at nearly equal intervals. These arrangements are sufficiently complete for their purpose; for, though a perfectly uniform motion is not attained, the speed is so nearly regular that the ear does not appreciate any marked difference in the intervals between the strokes. When a perfectly uniform motion is required, as it is sometimes for measuring intervals of time accurately, as in astronomical observations, it is necessary to have arrangements more delicate, so that all irregularities may be properly compensated. A very ingenious apparatus was contrived some years ago by Mr. William Froude, for giving perfectly uniform velocity of rotation to a cylinder. This motion was necessary for enabling the inventor to obtain diagrams exhibiting the flow of air into a vacuum, the propelling power of a screw on a boat fitted with it, and other interesting mechanical phenomena. In one of his apparatus he employed a flat disc B (Fig. 135) at the end of an arm suspended by a joint from a vertical axis A, caused to rotate by a train of wheels connected with a weight and with the cylinder, which was required to be uniformly moved. The disc revolving through the air offered a certain resistance, depending on its velocity; and if at any instant the velocity increased, the disc immediately moved outwards from the vertical line, owing to

Fig. 135.



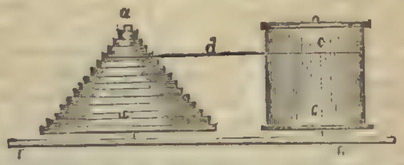
its increased centrifugal force, to some such position as that indicated by the dotted lines. When at this distance, the radius of its orbit D E was greater than the former radius B C; and therefore the speed with which the disc was driven through the air was proportionately greater, and the resistance of the air to its motion increased accordingly. The momentary increase of velocity, therefore, produced a considerable increase of resistance, and thus reduced itself to its average rate; and, conversely, a decrease of velocity diminished the resistance, and thus restored itself. By a proper adjustment of the disc as to area, the length of the arm by which it swung, and a weight on that arm, the apparatus served to move the cylinder for many minutes with a velocity in which there was not the slightest appreciable irregularity. A piece of paper was fixed on the cylinder, and a pencil, connected with a spring-balance acted on by the pressure to be measured, traced on the paper as it revolved a diagram, indicating the intensity of the pressure at every instant of the motion.

Fig. 136.



When the elasticity of a solid body is employed as a force for moving machinery, the body is called a spring, and generally consists of a strip of steel wound into a spiral form round an axis A, as shown in Fig. 136; one end of the strip being fixed to the axis, and the other at B to the side of the cylindrical box in which the spring is contained. When the box is fixed, as in a Geneva watch, the axis is turned round by a key, and the spring is thus drawn into closer convolutions. A wheel is fixed on the axis, and is connected by gearing with the escapement, and hands of the watch. The effort of the spring to unwind itself causes the axis to revolve in the direction opposite to that in which it was wound, and thus puts the train of wheels in motion. It is the property of all elastic bodies to exert a force which is nearly proportional to the amount of strain to which the body has been subjected. As we wind up the spiral spring of a watch, we apply greater and greater strain the farther we wind; and so when the spring unwinds itself it exerts the greatest force at first, and gradually decreases in power, the farther it is unwound. This decrease of force is an irregularity which no modification of the form of spring can obviate, but which may be considerably diminished by making the spring with a great number of convolutions. Thus, we may suppose that a spring of 20 convolutions, or turns, is wound up from its neutral condition to the full power required, by 10 revolutions of the axis. In unwinding itself it would lose by the first revolution $\frac{1}{10}$th of its force, by the second revolution another $\frac{1}{10}$th, and so on, until by five revolutions backwards it would lose $\frac{5}{10}$ths or $\frac{1}{2}$ of its full force. But if we suppose the spring had forty convolutions, and that we wound it up by 20 revolutions on the axis to the same force as the former spring, in unwinding itself it would lose by the first revolution $\frac{1}{20}$th, by the second another $\frac{1}{20}$th, and so on till by the fifth it would lose $\frac{5}{20}$ths or $\frac{1}{4}$th of its full force. The decrease of power in the second instance is only half that in the first, the axis performing in each case an equal number of revolutions. It is, therefore, important to give all springs which are required to exert a tolerably equal force throughout their recoil as great a number of convolutions as possible, and to employ as few revolutions of the axis as possible for moving the machinery. In other watches, where smallness of bulk is not so much studied, and in clocks moved by springs, the axis of the spring is fixed, and the box *c* (Fig. 137) containing it is caused to revolve by the recoil of the spring, and to wind upon its outer surface a small chain *d* attached to a fusee *a*. This fusee is a conical barrel, with a screw-groove cut in its surface to receive the coils of the chain. When the spring is beginning to recoil, and therefore acting with its greatest force, the chain pulls the fusee round at its upper part where the diameter is smallest, and therefore acts with least leverage on the train. As the spring loses force, the chain is pulled from a larger diameter of the fusee, and acts with more and more leverage on the train as its tension becomes less. By properly proportioning the increase in the diameter of the fusee to the diminution in the force of the spring, the power applied to the train can thus be made perfectly regular throughout the whole range of the spring's recoil. The fusee is not a regular cone, the outline of its section being part of a hyperbolic curve, that being the form which furnishes the proper proportion of increase of diameter to diminution of spring force.

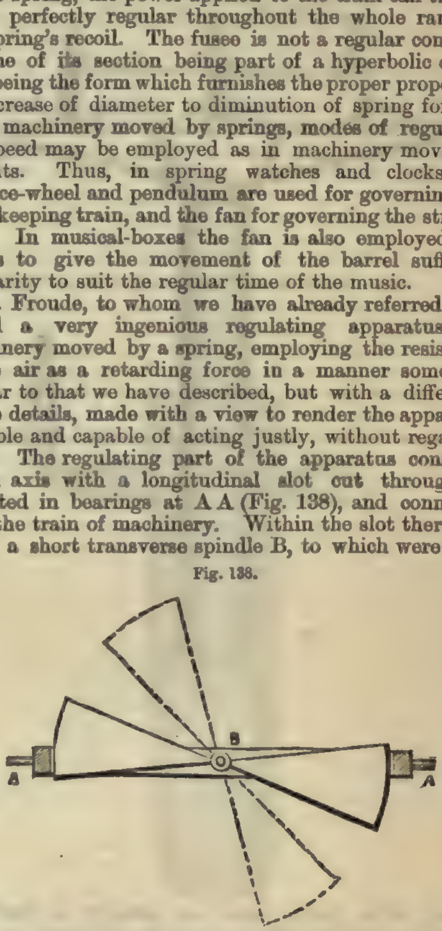
Fig. 137.



In machinery moved by springs, modes of regulating the speed may be employed as in machinery moved by weights. Thus, in spring watches and clocks, the balance-wheel and pendulum are used for governing the time-keeping train, and the fan for governing the striking part. In musical-boxes the fan is also employed, and serves to give the movement of the barrel sufficient regularity to suit the regular time of the music.

Mr. Froude, to whom we have already referred, contrived a very ingenious regulating apparatus for machinery moved by a spring, employing the resistance of the air as a retarding force in a manner somewhat similar to that we have described, but with a difference in the details, made with a view to render the apparatus portable and capable of acting justly, without regard to level. The regulating part of the apparatus consisted of an axis with a longitudinal slot cut through it, mounted in bearings at A A (Fig. 138), and connected with the train of machinery. Within the slot there was fitted a short transverse spindle B, to which were fixed two thin blades, kept in the position indicated in the figure by means of a small spring made to act on their spindle. When this apparatus was made to rotate with considerable velocity in the bearings A A, the blades tended, by their centrifugal force, to fly outwards to some such position as that marked by the dotted lines, and thus encountered more resistance from the air. By the proper adjustment of the area and weight of these blades and of the spring which acted on their spindle in opposition to their centrifugal force, extreme regularity in the motion of the train was maintained.

Fig. 138.

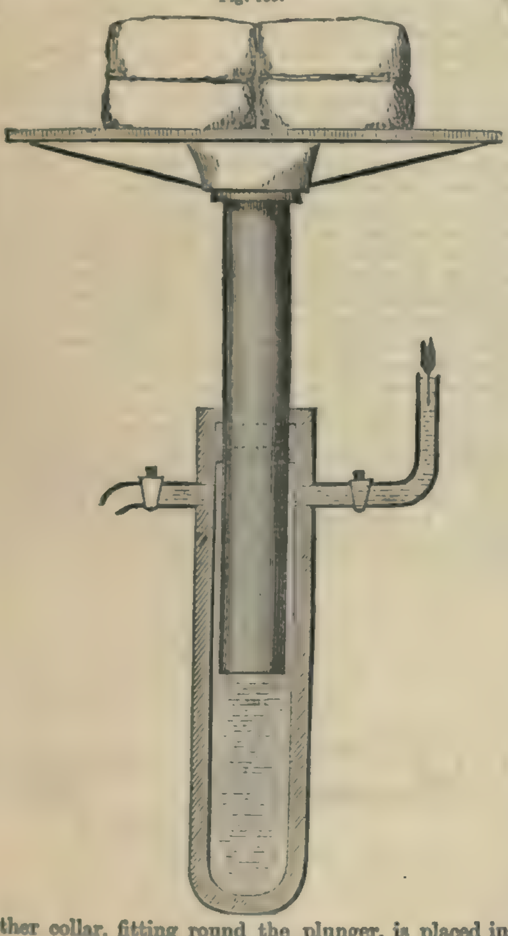


In general, for moving machinery with regular speed, weights are preferable to springs, because the force of a weight is constant, while that of a spring varies according to its tension. But weights act only vertically, while springs act in any direction. For all portable apparatus which cannot be maintained in a constant level so as to make the direction in which the weight acts constant, with relation to that on which it acts, springs must be employed. And so for the regulation of portable apparatus, escapements in which the elasticity of a spring is used for recoil, instead of the weight of a pendulum, must be likewise employed. The details of all such apparatus, and the many ingenious contrivances applied to them, form the subject of a distinct art, that of Horology.

PRESSURE AND ELASTICITY.—Besides contrivances for employing the weight and elasticity of solid bodies in giving motion to machinery, there are others by which the weight or pressure of liquids and the elasticity of gases are also applied for such purposes.

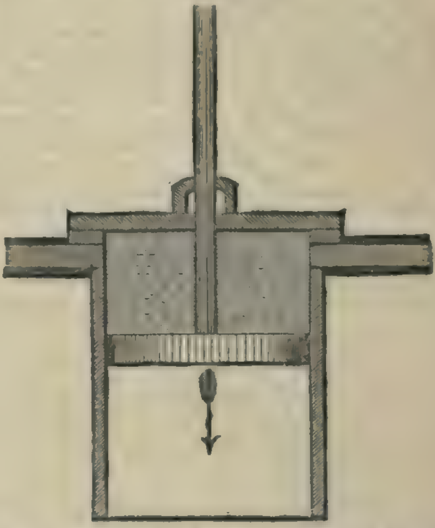
Among these the hydraulic lift and the hydraulic crane may be particularly noticed. The hydraulic lift, in its most simple form, consists of a cylinder closed at bottom, and fitted with a plunger or ram, which passes through the top and supports a stage (Fig. 139). A leather collar, fitting round the plunger, is placed in a recess provided in the neck of the cylinder, so as to prevent the escape of water around the plunger. A pipe from a high cistern conducts water into the cylinder, and another pipe permits the water to issue from it. Each of these pipes is provided with a stop-cock, so that the water may be admitted to the cylinder, or allowed to flow from it at pleasure. Since liquids communicate pressure equally in all directions, every part of the cylinder and the plunger is pressed upon, by a force proportioned to the height of the cistern which supplies the apparatus. Every 27 inches of height produces a pressure of 1 lb. on every square inch of surface exposed to it; and by making the area of the bottom of the plunger sufficiently large, a considerable weight can be raised on the stage which it carries. Thus if the cistern at the top of a house be 54 feet above the ground-floor, the pressure on the internal surface of any vessel on the ground-floor connected with it by a pipe, is 24 lbs. per square inch. If this vessel be a cylinder, such as we have described, fitted with a plunger 10 inches diameter, having therefore a sectional area of 78½ square inches, the pressure forcing this plunger upwards amounts to 78½, × 24 = 1884 lbs., and if from this we deduct 764 lbs. as a weight including that of the plunger and platform, there remains a pressure of 1120 lbs. forcing the plunger upwards. A load then of this amount, half a ton, may be placed on the platform when it is in its lowest position, and the cock being opened to the cistern, the whole will be raised to a height equal to that of the plunger, which may be made sufficient for lifting goods from one floor to another. By closing the cistern-cock and opening the other, the water is permitted to leave the cylinder; and the plunger, no longer subjected to upward pressure, descends to its former position.

Fig. 139.



The water-pressure may be made to act in a way somewhat different in detail, but similar in principle, by fitting the cylinder with a piston, connected by a rod passing tightly through the cylinder cover with a rope or chain, which may be led by pulleys in any convenient direction for lifting weights attached to it. The water-pressure acting on the piston, forces it down the cylinder, and thus pulls the chain and lifts the weight. Such an arrangement constitutes the hydraulic crane. The advantage of employing water for lifting weights in this manner consists in the circumstance that a small steam-engine, or other power, can be constantly employed in raising the water to a high cistern, and that great lifting power can be obtained for a short time by the expenditure of a portion of the water thus raised. The water lifted to a height, becomes, in fact, a reservoir of power, ready to be used when required, and accumulates the efforts of a small power acting through a considerable period, ready to be expended as a great power acting through a short time. Farther, this power is completely under control; for by the mere turning of a stop-cock, or the opening or closing of a valve, it can be put in action or arrested at pleasure; and the speed with which a weight is moved, and the height to which it is raised, can be regulated with the greatest nicety.

Fig. 140.



Apparatus has sometimes been applied in which water flowing from a high level, or compressed air, may be made to act as steam in a steam-engine, for giving motion to machinery. The construction of such apparatus is very similar to that of the steam-engine, and may be easily understood by one who has studied the arrangement of the latter.

The pressure of the atmosphere has also been employed as a moving force in the case of the atmospheric railway.

4. HEAT AND ELECTRICITY.—In treating of heat, electricity, magnetism, and chemical action, as sources of power, we embrace a very wide range of

matter, which may with advantage be sub-divided. We may here merely notice the branches which this division of the subject embraces.

Both heat and electricity appear to be universally the result of chemical action. They are not *new* forces created by the combination or separation of materials, but are merely the manifestations of force already stored up in bodies, and elicited by changes in their constitution. Practically speaking, however, heat may be said to be the force produced by the chemical action of combustion, or the union of oxygen with inflammable matter—a force which universally manifests itself as a repulsion, acting upon the particles of matter exposed to it. A pound of coal or carbon, or any other combustible, uniting with its proper equivalent of oxygen, produces a certain amount of heat, which is capable of expanding the bulk of any solid, liquid, or gaseous body exposed to it; of changing a solid into a liquid, or a liquid into a vapour, apparently by forcing the particles to a greater distance from each other. Combustible bodies are universally found throughout the earth; oxygen abounds in the atmosphere; and man has only to bring these two kinds of matter together, under proper conditions, in order to obtain the force developed by their union, which he can apply in many different ways as a motive power.

The various modes of thus applying heat may be distinguished under three heads.

1. The combustion of explosive substances, or substances like gunpowder, which very rapidly change from a condition in which they occupy small space, to one in which they occupy a very large space. The chief uses made of such forces are for blasting, and for giving motion to projectiles. The enormous force applied to these purposes may be best understood by observing, that any particular quantity of gunpowder, when ignited, expands to about 2,000 times its volume, in an interval of time so inappreciably small, that its expansion may almost be said to be instantaneous. Let us suppose, for instance, that a cannon of six inches bore is charged with a quantity of powder occupying about six inches of its length, and, on the powder, an iron ball is placed, weighing 32 lbs. Before the ignition of the powder, the surface of the ball is pressed on equally in all directions by the pressure of the atmosphere, about 15 lbs. on every square inch, or, its sectional area being 28 square inches, the total atmospheric pressure tending to push it in any direction is 28 × 15 = 420 lbs.; and as this pressure is exerted equally in every direction, no motion ensues. On the ignition of the powder, the pressure on one side is suddenly increased from that of one atmosphere, 420 lbs., to that of 2,000 atmospheres, or 840,000 lbs.; and thus a force is applied, at one side of the ball, exceeding 26,000 times its weight. Were this force continued for 1″ of time, the ball would acquire, in that time, a velocity 26,000 times that which gravity would give it, because the force is 26,000 times the weight or force of gravity; and as gravity gives in 1″ a velocity of 32 feet per second, the force of the gunpowder would in the same time give it a velocity of 26,000 × 32 = 832,000 feet per second, or nearly 570,000 miles per hour.

But, as the gun is comparatively short, seldom exceeding 9 or 10 feet in length, the pressure acts only during the short time which the ball occupies in traversing that distance. Also the elastic gases generated from the explosion of the gunpowder, instead of maintaining their enormous initial pressure during even this short period, lose more and more of it as they are permitted to expand by the forward motion of the ball; so that by the time it reaches the muzzle of the gun, their pressure would probably be reduced to less than 100 atmospheres. Moreover, as the ball does not precisely fit the gun, a considerable leakage is permitted round it, so that the pressure is still farther reduced, and even a certain back-pressure is caused by the gas which thus gets in front of the ball. From all these causes there results a diminished velocity of the ball, which seldom exceeds 2,000 feet per second, or about 1,360 miles per hour.

The whole subject of projectiles, and of the apparatus used for applying the combustion of explosive substances to give them motion, may be said to be in a transition state; and although of late years very great ingenuity has been displayed in contriving implements for this purpose, it would scarcely suit the limits of a work like the present to enter upon their discussion.

2. The combustion of fuel, without explosive violence, has been applied to cause the expansion of air for the purpose of producing motive power. The principle of this action is of a very simple character. Any volume of any gas, such as a cubic foot of air at a temperature of 32° (Fahr.), on receiving accessions of heat, increases in temperature and expands in bulk, and the amount of expansion depends on the increase of temperature, according to the following law:—For every added degree (Fahr.) of temperature, the volume expands $\frac{1}{480}$th part of itself. Thus, if the cubic foot were heated from 32° up to 72°, the increase of temperature being 40° (the difference between 32° and 72°), the increase of volume would be $\frac{40}{480}$ths, or $\frac{4}{48}$ths, or $\frac{1}{12}$th of a cubic foot; that is to say, 144 cubic inches. Or the volume at 72° would be 1 and $\frac{40}{480}$ths, or $\frac{520}{480}$ths of a cubic foot. If, now, this volume were heated up to 73°, or 1° more, its expansion would be $\frac{1}{480}$th, and its total volume would be $\frac{520}{480}$ths and $\frac{1}{480}$th; in all, $\frac{521}{480}$ths. Or its volume at 72° would be increased by $\frac{1}{520}$th of itself, on the accession of another degree of temperature. Now, if we observe that 480, the divisor which gives the expansion per degree of a volume at 32°, is 448 added to 32, and that 520, the corresponding divisor for 72°, is 448 added to 72, we readily see that the expansion per degree of any volume at any temperature is found by adding the constant number 448 to the temperature, and using the same as a divisor.* This leads to a rule by which, knowing the volume at any temperature, we can readily find the volume at any other temperature: thus—

Rule.—Add the constant number 448 to each of the temperatures, and state the proportion: as 448 + the temperature of the given volume is to 448 + the temperature of the required volume, so is the given volume to the required volume.

Example.—Required the expansion of 10 cubic feet, when heated from 60° to 212°.

			Given volume.		Required volume.
60°		212°			
448		448			
508	:	660 : :	10 cubic feet	:	13 cubic feet nearly.
		10			
		508)6600			
		13 nearly.			

Hence the expansion is nearly 3 cubic feet.

It may now be understood that, having the means of largely expanding any volume of air by adding heat, and of again contracting it by taking heat away, we have a force within our reach, capable of being applied as a motive power. Several contrivances for utilising this force have been made from time to time, but they all present so strong a similarity to the steam-engine, and they have, as yet, met with so little success in their

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* Let V be the volume at 32°
v_1 ,, ,, at t_1 of temperature
v_2 ,, ,, at t_2 of temperature

$$\text{Then since } v_1 = \text{V} + \text{V}\,\frac{t_1 - 32}{480} = \text{V}\left(1 + \frac{t_1 - 32}{480}\right) = \text{V}\,\frac{448 + t_1}{480}$$
$$\text{and since } v_2 = \text{V} + \text{V}\,\frac{t_1 - 32}{480} = \text{V}\left(1 + \frac{t_1 - 32}{480}\right) = \text{V}\,\frac{448 + t_1}{480}$$
$$\text{eliminating V, we have } \frac{480\,v_1}{448 + t_1} = \frac{480\,v_2}{448 + t_2},\ \text{or } v_2 = v_1\,\frac{448 + t_2}{448 + t_1}$$

practical application, that we do not think it necessary to occupy space by separately discussing them.

3. By far the most important application of heat to produce motive power, is that which is made in the steam-engine. Of all apparatus by which man has succeeded in rendering natural forces serviceable to himself, this is, perhaps, the most ingenious, as it is the most widely employed. It therefore forms a branch of Applied Mechanics, so important, and at the same time so extensive, that we propose to devote considerable space in what follows to its discussion.

Electricity and magnetism are the forces which have been applied so successfully in the electric telegraph—an apparatus which has now attained so great importance, and embraces so many ingenious contrivances, that it has been thought advisable to give it special consideration in another section. (SECTION IV.)

The efforts which have, from time to time, been made to render these forces available as prime movers of machinery, have not yet been crowned with success. We may, therefore, in a practical work like the present, dismiss the contrivances for this purpose with a few brief remarks as to their general character, referring the reader to those parts of this work which treat more especially of electro-magnetism, and of the machinery in which it has been applied.

One of the effects of electrical action being to decompose bodies subjected to its influence in one way, and to reproduce them from their elements in another way, an engine was contrived to act by the alternate decomposition of water into its elements—the elastic gases, oxygen and hydrogen—and its recomposition by their combustion. The one operation, decomposition, changing the small bulk of water into a large volume of the mixed gases, afforded a pressure; while the converse operation of recomposition removed the pressure by re-condensing the large volume of the gases to the small bulk of water.

The application of electro-magnetism as a prime mover has been attempted in various ways, all of which depend more or less on two special properties, by which electricity and magnetism appear to be connected. One of these is, that a current of electricity moving at right angles to a bar of iron renders it magnetic, each end of the bar becoming a pole. When the current is reversed in direction, the poles are immediately reversed, that which was South becoming North, and conversely; and when the current ceases, the magnetism disappears. By coiling the electric-wire many times round the bar, the power of the magnet is rendered very great; and as by reversing or arresting the current each end may be made to attract, repel, or to become entirely neutral, it might be expected that the force so developed should be readily applicable to the production of motive power. It has been found, however, that the attractive and repulsive influences act with extremely little force at any distance from the poles of the magnet, and the consequent limitation of range has rendered their application to machinery very difficult.

The other property by which electricity and magnetism are connected is, that a conducting wire wound round a permanent magnet, has a current of electricity induced in it, and the cells of an electrical battery surrounding a magnet are put in rotation round it as an axis. The motion produced by the mutual action of these forces has not, so far as we know, been as yet applied to any useful purpose; and, although many neat contrivances of the kind have been exhibited on the lecture-table, we fear that in practice the cost of producing power in this manner, is such as to prevent it from coming into competition with the simple combustion of fuel.

CHAPTER V.

THE STEAM-ENGINE.

Contents.—EXPANSIVE POWER OF HEAT—ELASTICITY OF GASES—TEMPERATURE AND PRESSURE—VOLUME OF STEAM—CONDENSATION—EXPANSION—THE BOILER—CORNISH BOILER—MARINE AND LOCOMOTIVE BOILERS—CONSTRUCTION OF BOILERS—INCRUSTATION—BOILER FITTINGS—FEED—SAFETY VALVE—PRESSURE GAUGE—CYLINDER—VALVES—SLIDE—CYLINDER AND PISTON—SHUT-OFF AND THROTTLE VALVES—FEED-PUMP—CRANK AND CONNECTING ROD—ECCENTRIC CAMS—GOVERNOR—DIRECT-ACTING ENGINES—VERTICAL, HORIZONTAL, LOCOMOTIVE, AND OSCILLATING ENGINES—ESTIMATION OF POWER—INDICATOR—CALCULATION OF POWER.

IT is usual for writers on the steam-engine to give an outline of its history, tracing up from the earliest times, successive contrivances and suggestions for rendering the elasticity of vapours applicable as a motive power. This mode of introducing the student to the consideration of the mechanical arrangements, which now constitute a complete steam-engine, is not without its advantages. It is calculated to open his mind to the defects of successive inventions, and to the modes in which they have been gradually removed, until what was at one time looked on as an ingenious and interesting philosophical toy, has become at last an apparatus that has changed the whole face of civilised society, and almost brought the elements within the control of man. By watching the steps of inventors in this progress, the student becomes well prepared to comprehend the details of the perfected apparatus, and the reasons why certain modifications have been introduced as improvements, while others have been rejected as practically inapplicable. But it happens with the steam-engine as with almost all other mechanical contrivances, that its earliest forms are by no means the most simple; and that improvements have often consisted more in discarding useless complications than in adding new parts. We think, upon the whole, therefore, that the steam-engine may be best studied in its most simple forms, regardless of their historical sequence, especially when a practical knowledge of its construction and applications is desired, rather than a merely theoretical acquaintance with its principles. We propose, therefore, to discuss, in the first place, the general properties of steam, and the modes adopted for generating it in a manner suitable for rendering it available as a prime mover. And we will, secondly, consider the special forms of apparatus, or engines, through which the steam is made to act for producing the movement of machinery. By this course we shall hope to give the reader a clear view of the whole subject; and we shall take such opportunities as offer themselves, of referring to the various inventions that have, from time to time, been brought into use.

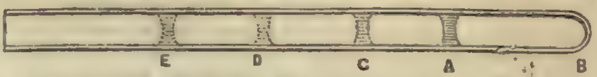
To the advantages arising from the use of steam as a motive power, to the changes that have been effected by its introduction, and the still greater changes that may be confidently expected from its more extended application, we need scarcely refer. These are patent to all; for in the present condition of society, few operations either of transit or manufacture are conducted, in which the steam-engine does not play the most prominent part.

EXPANSIVE POWER OF HEAT.—When a liquid is exposed to heat, it expands in bulk; and when the heat is carried to a certain point, part of the liquid rises from the rest in the form of vapour, occupying a much greater volume than it did in its liquid condition. It is probable that all substances, solid as well as liquid, are capable of being thus changed into the vaporous form. Some bodies, such as ether and alcohol among liquids, and iodine among solids, rise in vapour at comparatively low temperatures. Others, again, such as many of the metals, even after they have become liquid under the influence of heat, require very considerable accessions of temperature before they pass into vapour. Water, one of the substances most widely diffused in nature, and therefore most cheaply obtained, holds a middle course in this respect between these extremes, and attains, by no great accessions of temperature, properties which render it especially serviceable for human use. It is, indeed, among the most obvious of the beneficent arrangements of Providence, that this fluid—almost everywhere accessible, possessed of no corroding power like acids or alkalies, of no intoxicating qualities like spirits, not of inconvenient gravity like metals—should be at the same time capable of absorbing vast quantities of heat, and of giving it out in the shape of active motive power. It has been proposed to employ the vapours of ether, alcohol, sulphide of carbon, and mercury, for moving engines; but no advantages from their use in this way have presented themselves, such as could bring them into competition with water. We will, therefore, confine ourselves to it, as the only substance turned practically to account.

The atmosphere which envelops the earth, though invisible, is nevertheless possessed of weight, and presses on the surface of all bodies in it exactly as the water contained in a cistern presses on bodies immersed in it. The pressure of the atmosphere at the surface of the earth is about 15 lbs. on every square inch. This pressure varies with changes in the density of the atmosphere, but not to a great extent. The average is generally stated at somewhat less than 15 lbs. per square inch; but, for all practical calculations, 15 lbs. may be assumed without involving material error. Water contained in any vessel exposed to the air is thus pressed upon, and retains its liquid form at ordinary temperatures. But if the temperature of the water be raised to 212°, as measured by Fahrenheit's thermometer, its vapour at that temperature has an elastic force which exactly balances the atmospheric pressure, and part of the water consequently rises in the form of vapour, and mingles with the air, displacing as much of the latter as is equivalent to its own bulk. Were the pressure of the air removed, vapour would rise from the water at much lower temperatures than 212°, and occupy the vacant space in the vessel containing it. Thus, when a saucer containing water is placed under the receiver of an air-pump, as the air is extracted from the receiver, vapour rises from the water; and however low may be the temperature of the water, vapour continues to rise from it as the pressure of air on its surface is diminished. Even under the full atmospheric pressure, water is constantly evaporating at ordinary temperatures, for the air seems to have a power of dissolving water or taking it up in a vaporous form, just as water dissolves salt and retains it diffused through it in a liquid form.

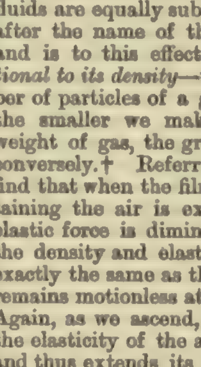
ELASTICITY OF FLUIDS.—At every different temperature at which vapour may exist, its elastic force is different, being greater the greater the temperature, but not proportionally so. When we inquire what is meant by the elastic force of a vapour, we can only say that it is a property common to all aëriform bodies, by which they react on any surface compressing them, or tend to expand into a larger volume than that in which they are confined. There seems, indeed, to exist, a repulsive force among the particles which constitute a gaseous body, pushing them asunder with equal power in every direction, and requiring some contrary compressing force to retain them in their places. We have no example of a gaseous body existing without being contained in an envelope of some kind, or being pressed upon by some surrounding medium. The air at the surface of the earth is pressed upon by the weight of air above it, and exerts an elastic force exactly balancing the pressure of the superincumbent fluid.* If its elastic force were greater than the force compressing it, it would expand and lift the air above it; or if less, it would collapse under a pressure greater than it could resist. As we ascend higher, the weight pressing on the air is diminished, because there being a certain amount of the atmosphere left below, there remains a less amount above. We find accordingly, that at any height above the earth's surface, the elastic force of the air is diminished in proportion as the pressure upon it is lessened. At the top of a high mountain, like Mont Blanc, the elastic force of the air is not more than half that of the air at the surface. If we were to introduce a drop of oil into a small glass tube closed at one end (Fig. 141), so that the oil should form a film, or diaphragm, across the tube at some point A, we should enclose between A and B a portion of air in the same condition as that surrounding us. If, now, we ascended a mountain, we should find the air-film at points C, D, E, successively, as we attained greater height. Before our ascent commenced, the elastic force of the air in A B pressed the oil-film outwards with exactly the same force as that of the weight of superincumbent air pressing it inwards, and accordingly the film remained at rest. As we ascended, we subjected the film to less external pressure, because we had less weight of air above us; and accordingly the elasticity of the air in A B overbalanced the pressure, and pushed the film to some such point as E; not by fits and starts, but gradually as we ascended.

Fig. 141.



We have now to inquire why there should be any point such as C, where the film could rest; in other words, why the elastic force of air in the tube, having once overbalanced the pressure of the external air, should again become equal to it, and no longer force the film outwards. The reply to this question involves the consideration of a most important law to which all elastic fluids are equally subjected. It is called Marriotte's law, after the name of the person who gave it to the world, and is to this effect:—*The elasticity of a gas is proportional to its density*—that is to say, the greater the number of particles of a gas we force into a certain space, or the smaller we make the space containing a certain weight of gas, the greater we make its elastic force, and conversely.† Referring now to the air in the tube, we find that when the film has arrived at C, the space containing the air is extended from A B to C B, and the elastic force is diminished in like proportion. In fact, the density and elasticity of the air within the film is exactly the same as that of the air without, and the film remains motionless at C as long as this equality subsists. Again, as we ascend, the external pressure diminishes; the elasticity of the air within pushes the film outwards, and thus extends its space until its elasticity is reduced to an equality with the pressure of the external air; and so on without limit. In descending, we should find the film moving inwards, according as it became subjected to a greater external pressure, to positions, such that the air confined within it became of a density sufficient to balance the pressure. Notwithstanding the extreme simplicity of Marriotte's law, it is one of the greatest importance, and should be thoroughly understood by any one desirous of an acquaintance with the steam-engine, for it applies to steam as closely as to any gas. From this law, it follows, that if we forced air or steam occupying a volume of 2 cubic feet into a vessel of the capacity of 1 cubic foot, we should find the elasticity doubled, because the density would be doubled, or the volume reduced to half. Conversely, if we permitted 1 cubic foot of air or steam to occupy a volume of 2 cubic feet, we should reduce its elastic force to half of what it was before expansion, because we should have doubled its volume, or reduced its density to half of what it was. In order to illustrate these facts, let us suppose that a cylindrical vessel is fitted with a piston that can

Fig. 142.



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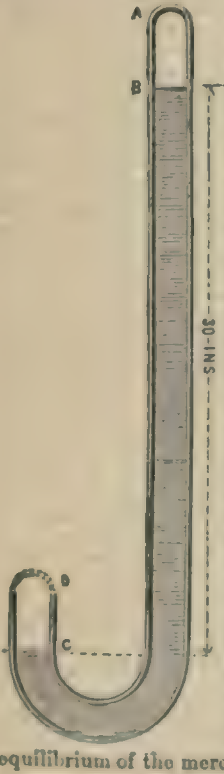
* See *ante*, p. 770, *Pneumatics*.

† See *ante*, p. 770, *Pneumatics*.

slide in it (Fig. 142), having an area of 1 square inch, and that a load of 20 lbs. placed on the piston forced it down to a position C D, an inch from A B, compressing below it the air or steam occupying the part A B D C of the cylinder. We should then say that the elasticity or pressure of the air or steam is 20 lbs. per square inch, because it balances or supports a load of 20 lbs. placed on a piston having an area of a square inch, exposed to its elastic force. If we suppose another inch added to the length of the cylinder, the added space A E F B being totally empty, and the former bottom A B suddenly withdrawn, so that the air or steam had its space or volume doubled, we should find 20 lbs. on the piston twice as much as it should be to retain the piston in its place, and we should have to replace it by a load of 10 lbs., because the elasticity of the air, or steam, would have become half of what it was before the doubling of its volume. We have already stated that the pressure or elasticity of the air, near the surface of the earth, is about 15 lbs. on every square inch. If then a cylinder like that which we have just described were filled with air under the piston, and placed in a vacuum, there would be required on the piston a load of 15 lbs. to retain it in its place. Were the load less than 15 lbs., the air under the piston would expand in volume and raise it; or were the load greater, the piston would be pressed downwards, increasing the density of the air below it, and proportionally its elasticity, until the load became exactly balanced. But a cylinder of the kind described, filled with air, and not placed in a vacuum, requires no actual weight on the piston, because the surrounding atmosphere affords a load exactly equivalent to the weight that would be required in a vacuum. If the cylinder were filled with steam instead of air, and with no load on the piston, the steam would be said to be at atmospheric pressure, because its elasticity tending to force the piston upwards, is exactly balanced by the pressure of the atmosphere tending to force it downwards. If the piston required a load of 15 lbs. upon it, the steam would be said to exert a pressure of two atmospheres, or of 15 lbs. above atmospheric pressure. So, if the steam sustained loads of 30 lbs., 45 lbs., 60 lbs., &c., placed on the piston, its pressure would be called that of 3, 4, 5, &c., atmospheres; or of 30 lbs., 45 lbs., 60 lbs., &c., above atmospheric pressure.

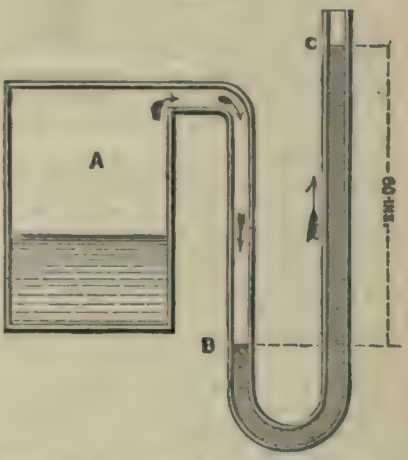
Fig. 143.



The pressure or elasticity of fluids, such as air or steam, is often expressed in terms of inches of mercury, or of the height of column of mercury which they can sustain. It happens that 2 cubic inches of mercury weigh very nearly 1 lb., and that 30 cubic inches weigh, consequently, about 15 lbs. Now, if we suppose a tube, having 1 square inch of sectional area, bent as in Fig. 143, and closed at both ends A and D, were filled with mercury to a height of 30 inches in one limb above the level in the other, the part A B being a perfect vacuum, and the part D C filled with air, since the weight of the column 30 inches high is 15 lbs., this pessure of 15 lbs. is communicated through the mercury in the bend to the air in C D, which consequently reacts with an elastic force equivalent to 15 lbs. on the surface of the mercury exposed to it. That is to say, the elasticity or pressure of the air in C D is 15 lbs., or 1 atmosphere; and the tube might be opened at D to the ordinary pressure of the atmosphere without effecting any change in the equilibrium of the mercurial column. The instrument in this form would become the ordinary barometer, which measures the pressure or density of the atmosphere by the height of a mercurial column sustained in a tube, every 2 inches of height of mercury corresponding to 1 lb. of pressure per square inch.

If a bent tube (Fig. 144) were connected with a vessel A containing water, on heat being applied to the water, steam would be generated in A, and press, by its elastic force, the mercury downwards in one limb of the tube and upwards in the other, until it attained such a position that the excess of weight of mercury in the one limb, and the pressure of the atmosphere on its upper surface C, should exactly balance the elastic force of the steam in A. If the height of C above B were 60 inches, the steam would be said to exert a pressure of 60 inches of mercury, or 30 lbs. per square inch above atmospheric pressure, or to have a total elasticity of 3 atmospheres.

Fig. 144.



TEMPERATURE AND PRESSURE.—We have already said that the elasticity of steam is greater the greater its temperature. There is no simple rule for calculating the pressure due to any given temperature, as the law which governs its variations is of rather an abstruse character. The following table of the pressures and corresponding temperatures of steam, or the vapour of water, is compiled from the results of numerous experiments made with a view to establish some law on the subject.

In a treatise like this we must abstain from the discussion of this law, on account of the advanced analysis required for its investigation. Nor need we offer a formula for calculating the pressure corresponding to a given temperature, as the table contains results sufficiently accurate for all practical purposes. The table only applies to the case of steam in a boiler, or vessel, in contact with the water from which it is generated. Were we to remove any portion of steam into another vessel, and then subject it to heat without water being in contact with it, we should simply expand its volume, as we should air or any other gas, by an increase of temperature; or confining its volume, elevate its pressure according to a totally different law, already discussed in reference to the heated air engine. (See p. 841). Or, were we to remove the steam to a separate vessel, and there cool it, a portion would be condensed, and the remainder would expand to fill the void, at a pressure reduced according to Marriotte's law.

In the first column of the table, the temperatures are marked in degrees of Fahrenheit's thermometer.

The second contains the pressures in atmospheres corresponding to the temperatures.

The third gives the pressure in inches of mercury, or the heights in inches of mercurial columns capable of balancing the elasticities.

The fourth column gives the pressures in pounds per square inch above that of the atmosphere; or the loads in pounds required, in addition to that of the atmosphere, to keep down a piston having an area of 1 square inch pressed upwards by the steam.

TABLE I.

Temperature, Fahrenheit.	Pressure in Atmospheres.	Pressure in inches of mercury.	Pressure in lbs. above atmospheric.
60°	0·017	0·5	14¾ below.
120°	0·120	3·6	13¼ "
180°	0·500	15	7½ "
212°	1·000	30	0 above.
250°	2·000	60	15 "
275°	3·000	90	30 "
290°	4·000	120	45 "
305°	5·000	150	60 "
320°	6·000	180	75 "
344°	8·000	240	105 "
372°	12·000	360	165 "
432°	20·000	600	285 "

We would remark, that for temperatures below 180° and above 344° the results are somewhat uncertain. It is within these limits, however, that any practical application of the table can be required, for we seldom have to do with steam pressing with elastic force of less than half an atmosphere on the one hand, or with pressures above eight atmospheres on the other.

VOLUME OF STEAM.—The great secret of the power derivable from steam, lies in the fact that a small volume of water is expanded by heat into a large volume of elastic vapour, tending to occupy a greatly increased space, and forcing any obstacle presented to its expansion, with the pressure due to its elasticity. The volume of steam produced from a given quantity of water has been variously stated; but we believe it may be very correctly estimated at 1600 times that of the water when the pressure is equivalent to 1 atmosphere. That is to say, a cubic inch of water contained in a vessel open to the air, when boiled off into steam, occupies 1600 cubic inches of bulk, and forces the air contained in the vessel away to the extent of that expanded volume; or expands with a force of 15 lbs. on every square inch, which is the measure of the atmospheric pressure, and therefore of the force resisting the expansion of the steam. But had the vessel containing the steam a volume of only 800 cubic inches, or half that to which the steam would expand at atmospheric pressure, then the density of the steam being doubled—or, in other words, the number of particles crowded into the space being doubled—the pressure on every part of the vessel would be doubled also. If the cover of the vessel were a movable piston having one square inch area, it would in this case require a load of 15 lbs., in addition to the atmospheric pressure upon it, to keep down the elastic steam within.

Looking at the question generally, we see that the volume occupied by steam from a certain quantity of water is inversely as the pressure, because the pressure is as the density, and the density is inversely as the volume. The following is the rule for calculating the volume of steam at any pressure, produced from a given volume of water.

Rule.—Multiply the volume of the water by 1600, and divide by the pressure in atmospheres.

Example 1.—Required the volume of steam at 4 atmospheres (or having a pressure of 45 lbs. per square inch above atmospheric pressure) generated from 3 cubic feet of water.

$$\frac{3\times1600}{4} = 1200 \text{ cubic feet of steam.}$$

Conversely, to find the quantity of water necessary to generate a given volume of steam at a given pressure.

Rule.—Multiply the volume of steam by the pressure in atmospheres, and divide by 1600.

Example 2.—Required the water necessary to generate 1200 cubic feet of steam at 4 atmospheres.

$$\frac{1200\times4}{1600} = 3 \text{ cubic feet of water.}$$

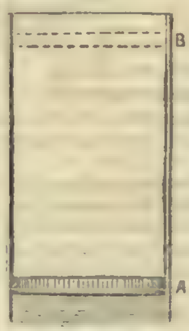
Example 3.—A cylinder 12 inches diameter and 20 inches long is filled 120 times per minute with steam, having a pressure of 30 lbs. above that of the atmosphere; required the quantity of water necessary to generate the steam.

The area of a circle 12 inches diameter is 113 square inches, and the capacity of the cylinder is 113 × 20 = 2260 cubic inches. This capacity filled 120 times gives a volume of steam = 2260 × 120 = 271,200 cubic inches. As the steam presses with 30 lbs. above that of the atmosphere, or altogether with 45 lbs., that is with 3 atmospheres, the volume of water is $\frac{271200\times3}{1600} = 508\frac{1}{2}$ cubic inches.

From these examples it is evident that a great force can be obtained by subjecting water to the action of heat. Just as a few grains of gunpowder on being ignited become suddenly transformed into a large volume of elastic gases, which by their expansive force propel a ball with great velocity, or burst asunder the solid rock; so a small quantity of water heated above 212° is changed into a perfectly elastic vapour, pressing upon the envelope containing it, and forcing any body, opposing its expansion, through a space sufficiently great to permit its enormous increase of bulk.

CONDENSATION.—But not only to the expansion of its volume does vaporised water owe its excellence as a moving force, for its increased volume can be suddenly reduced to a small bulk by the application of cold, or the removal of the heat which is necessary to its vaporous condition; and whatever force the steam exerted in expansion, is returned again by its condensation.

Fig. 145.



Thus if a vessel containing 1 cubic inch of water, and 799 cubic inches of air, were heated so as to turn the water into steam at twice the atmospheric pressure, the steam would force out the air and occupy its place; acting as it expanded with a pressure of 15 lbs. on every square inch above that of the atmosphere. Were the vessel now cooled so as to reduce the 800 cubic inches of steam to 1 cubic inch of water, the vacuum left by the condensed steam would be immediately filled by air pressed into it by the surrounding atmosphere with a force of 15 lbs. on every square inch. Were the vessel fitted with a piston or partition capable of sliding upwards or downwards in it, without permitting the passage of fluid round its edges, the effect would be the same; for if the piston at A (Fig. 145) were in contact with the water before boiling, and raised to B by its expansion into steam on heat being applied, it must, in rising, have been subjected to a pressure of 30 lbs. on every square inch of its under surface, so as to overbalance the atmospheric pressure on its upper surface by 15 lbs. On cold being applied so as to reduce the steam to its original volume of water, the pressure on the under surface of the piston being removed, that of the air on its upper surface again forces it downwards from B to A, its original position. Thus, both in the ascent and in the descent there is developed a force, applicable to the movement of machinery properly connected with the moving piston. In order that we may attain some notion of the amount of this force, let us suppose that the piston has an area of 1 square foot, and can rise and fall 2 feet, and that we have the means of generating and condensing the steam in the vessel 50 times in every minute.

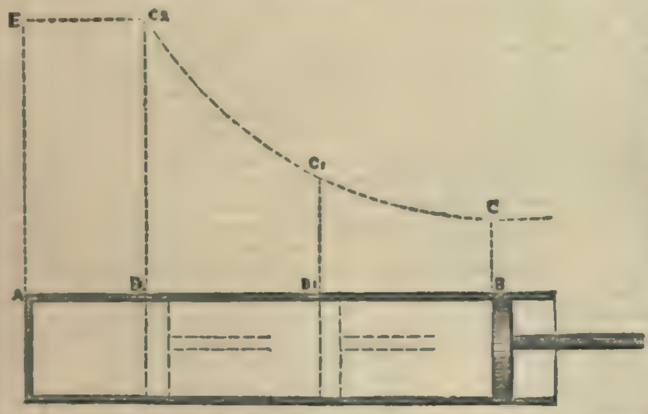
Since 1 square foot = ... 144 sq. in.,
And on every square inch the pressure is . 15 lbs.

The total pressure on the piston is . . . 2160 lbs.

This force is moved 2 feet up and 2 feet down 50 times per minute, or through a space of 200 feet per minute, and is therefore equivalent to 2160 × 200 = 432,000 lbs. moved through 1 foot per minute. A horse-power being reckoned at 33,000 lbs. moved 1 foot per minute, the force of the piston, as we have estimated it, is equivalent to $\frac{432000}{33000} = 13$ horse-power. From this example it will appear that by increasing the size of the piston, the distance through which it is moved, the rapidity of its alternations, and consequently the means of generating and condensing the steam, almost unlimited power can be attained.

In many steam-engines advantage is not taken of the power derivable from the condensation of the steam—its mere expansive power is employed; and, after having done its work, the expanded steam is allowed to escape into the atmosphere. This system is adopted for the sake of economy, lightness, and simplicity in the construction of the engine; and such engines are called *high-pressure* or *non-condensing*: *high-pressure*, because the steam must exert a pressure considerably higher than that of the atmosphere against which it has to act; or *non-condensing*, because the steam is not condensed after having done its work. In other steam-engines, called *low-pressure* or *condensing* engines, although greater power is derived from the steam, yet the machinery is rather more complex and heavy, and more liable to derangement; also a large supply of cold water is necessary to effect the condensation of the steam. Of late years many engines have been advantageously employed, in which the steam is first caused to act as it does in a non-condensing engine; but instead of being blown off into the air, it is afterwards made to do duty as in a condensing engine. Such are called *combined* engines, because the principles of expansion and condensation are combined in their action to a greater extent than in most others.

Fig. 146.



EXPANSION.—Before entering upon questions connected with the practical construction of the steam-engine, it will be advisable, in the first place, to discuss theoretically some of the principal facts connected with the expansion and pressure of steam, or generally of elastic fluids.

If we suppose a cylindrical vessel fitted with an air-tight piston, and containing within it a certain volume of air, steam, or any gaseous fluid perfectly elastic, we may represent the amount of pressure which the fluid exerts on the piston at B (Fig. 146) by the length of a vertical line B C. For example, if the area of the piston be 1 square inch, and the pressure on it at B be 15 lbs., we may draw a line or ordinate B C 15 inches long, taking 1 inch for each pound, or we might make it 15 half inches, or 15 tenths of an inch, using a half or a tenth of an inch to represent 1 lb. of pressure, or any other proportion that we may find convenient. If now we apply a force to the piston so as to push it along the cylinder to some place B_1, and there draw an ordinate B_1 C_1 having a length bearing to the pressure on the piston at B_1, the same proportion as the length of B C bears to the pressure at B; and if, farther, we drew an ordinate at B_2, and others at any number of intermediate points, we might, by tracing a curve C_2 C_1 C through all the points so determined, exhibit graphically the rate of variation of pressure according to that of volume or density. The law, to which we have already alluded, called Marriotte's law, is very simple, viz., that the volume multiplied by the pressure is always constant—that is to say, the length of A B (which represents the volume of gas when the piston is at B), multiplied by the height of B C (which represents the pressure at B), gives the same product as that of the length of A B_1 multiplied by B_1 C_1, or of A B_2 multiplied by B_2 C_2, the temperature remaining the same.[*]

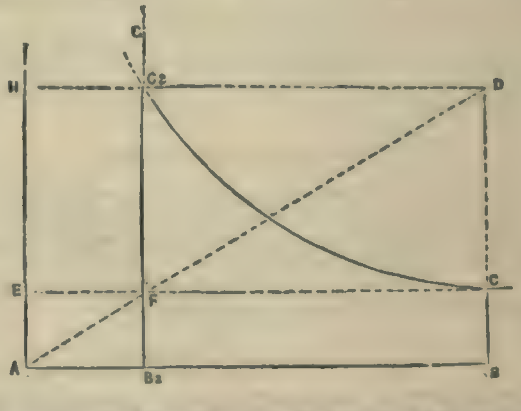
If, instead of filling the length A B of the cylinder with fluid, and then forcing in the piston so as to compress it, increasing the pressure in proportion to the increase of density, we supposed the piston to be at B_2, and the part A B_2 of the cylinder filled with elastic fluid, it would force the piston towards B with pressure gradually diminishing with the density according to the same law, and the curve C_2 C_1 C would terminate the ordinates representing pressures along the stroke or distance passed over by the piston. This is precisely the condition under which the piston of a steam-engine is ordinarily worked; and by taking a practical example we may ascertain what power is developed by the expansion of the steam. We shall suppose that in a steam cylinder of 1 square inch area the piston is close to the end A; and that by an opening there, steam, having a pressure of 4 atmospheres, 60 lbs. per square inch, is admitted so as to force the piston away from A to the position B_2, 1 inch from A. The steam-opening then being closed, the cylinder contains 1 cubic inch of steam 4 times the density of steam at atmospheric pressure, and pressing on the piston with a force of 60 lbs. When the piston has reached B_1, 2 inches from A, the steam has expanded to 2 cubic inches, and is therefore half its former density, or twice that of steam at atmospheric pressure, acting, therefore, on the piston with the force of 30 lbs. At B, 4 inches from A, the density is one-fourth of that at B_2, and the pressure on the piston is 15 lbs. By reckoning the pressures at a number of intermediate points, we might ascertain an average pressure of nearly 35 lbs. throughout the stroke, and thence calculate the power developed by the movement of the piston to be 35 lbs. moved through 4 inches, or 140 lbs. moved through 1 inch by the action of 1 cubic inch of steam at a pressure of 4 atmospheres—a quantity that would be generated from $\frac{1}{400}$th cubic inch of water.

Now, if we take another case, using the same quantity or weight of steam, but at a different pressure, we shall find a marked difference in the power developed. Let us suppose that steam at a pres-

---

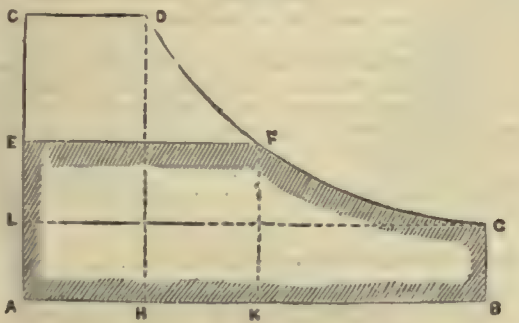
[*] The curve C_2 C_1 C (Fig. 146) is called the *hyperbola*, and it can readily be traced geometrically thus: If A B (Fig. 147) be the given volume or length of the cylinder when the pressure is B C, take any other point B_2 and draw an ordinate B_2 G cutting in F a line C E parallel to A B; join A F, and prolong it to meet B C in D, and through D draw D C_2 parallel to A B, cutting B_2 G in C_2; then C_2 is the point of the curve corresponding to B_2, for A B_2 × B_2 C_2, or the area of the rectangle A B_2 C_2 H, is equal to A B × B C, or the area of A B C E, as might readily be proved. Any number of points, such as C_2, being found, the curve can be traced through them. (See *ante*, p. 611, *Conic Sections*.)

Fig. 147.



sure of 2 atmospheres, or 30 lbs. per square inch, is admitted so as to force the piston through 2 inches of its stroke; we have 2 cubic inches of steam at 2 atmospheres, which are equivalent to 1 cubic inch at 4 atmospheres, and would be generated from the same quantity of water, $\frac{1}{400}$th cubic inch, at nearly the same expenditure of heating power. We should in this case find the average pressure on the piston, throughout the stroke of 4 inches, to be about 24 lbs., or equivalent to a weight of 96 lbs. moved through 1 inch. At first sight it appears startling, that by an expenditure of no more heating power in the first case, we should have obtained 45 per cent. more power than in the second; but such is the fact, nevertheless: and that it is so may be clearly seen by the diagram (Fig. 148). Let A B represent the stroke 4 inches long, A C the pressure, 4 atmospheres (represented by 4 inches in height), continued through 1 inch of the stroke, as shown by the line C D; then from D let the pressure-curve be drawn till it terminates at G, B G being 1 inch high, representing the terminal pressure 1 atmosphere. Then the area of the whole compound figure A C D F G B represents the total power developed in the first case by the expenditure of a quantity of steam represented by the area of A H D C. Again, let the pressure in the second case of 2 atmospheres, represented by A E, 2 inches high, be continued through 2 inches of the stroke to F; a portion of the same pressure-curve F G will enclose the pressures during the rest of the stroke. In this case the power is represented by the area of the shaded figure A B G F E, the quantity of steam expended being as the area of A K F E. Now the area of A K F E, or $2 \times 2 = 4$, is precisely the same as that of A H D C, or $1 \times 4 = 4$; while the area of A B G D C exceeds that of the shaded figure by the portion E F D C—so much clear gain of power by the higher initial pressure, and greater range of subsequent expansion. We have here supposed that no resistance is offered to the movement of the piston. If it be conceived to move in opposition to the pressure of the atmosphere, the gain by expansion of the steam is even more marked. A line G L represents the constant atmospheric resistance, and cuts off from both figures an area A B G L, which leaves the effective overplus of force impressed on the piston so as to move machinery, represented in the one case by the area of L G F E, and in the other by L G D C. Numerically, since we found the total force in the two cases to be equivalent to 96 lbs. and 140 lbs. respectively moved through 1 inch, and as the atmospheric resistance is 15 lbs. through 4 inches, or 60 lbs. through 1 inch in both cases, the effective forces are as 140 — 60, or 80, to 96 — 60, or 36; that is to say, the one is more than double the other.

Fig. 148.

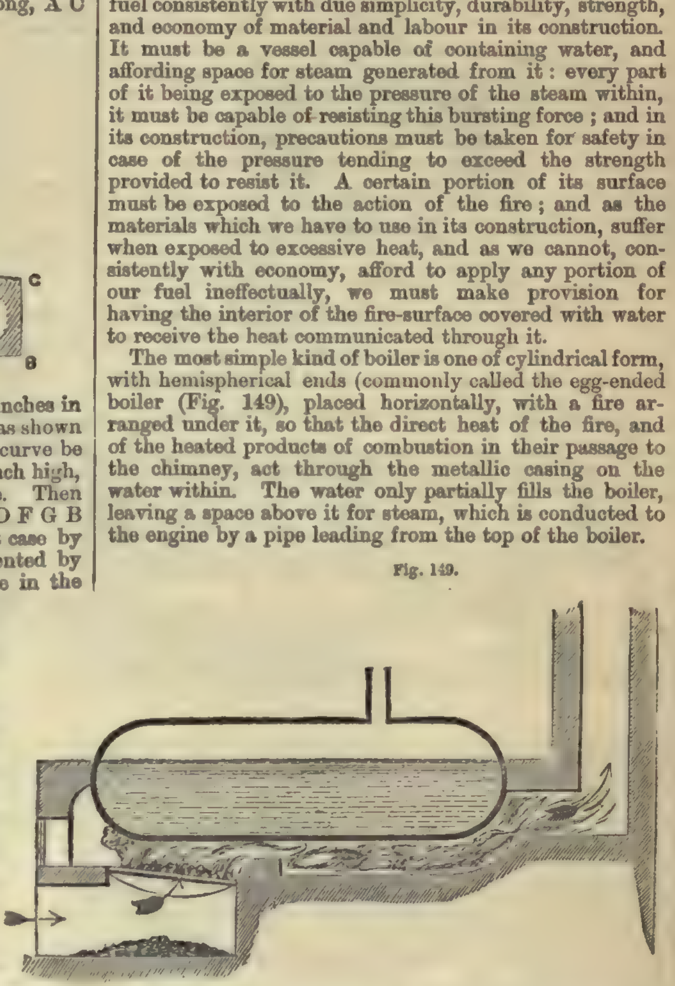


This mode of graphically representing forces is not only of an interesting, it is also of a most useful character. Watt invented an instrument which, being applied to a steam-engine, could draw upon a card a figure such as we have employed, representing the power developed by the action of the steam in the engine. He, however, seemed to consider it only as an interesting toy; but of late years this instrument has been found to be of the greatest utility. It is called the *indicator*, and is capable of not only representing with great accuracy the power developed in the engine to which it is applied, but also of exhibiting defects in design and construction, and of suggesting improvements. We shall have occasion hereafter to enter more particularly into the details of its construction and application.

THE BOILER.—In all steam-engines, the boiler or apparatus for generating the steam is a part of prime importance. In devising a good boiler, the problem is to obtain the greatest quantity of steam, or to boil off the greatest weight of water with the least weight of fuel consistently with due simplicity, durability, strength, and economy of material and labour in its construction. It must be a vessel capable of containing water, and affording space for steam generated from it: every part of it being exposed to the pressure of the steam within, it must be capable of resisting this bursting force; and in its construction, precautions must be taken for safety in case of the pressure tending to exceed the strength provided to resist it. A certain portion of its surface must be exposed to the action of the fire; and as the materials which we have to use in its construction, suffer when exposed to excessive heat, and as we cannot, consistently with economy, afford to apply any portion of our fuel ineffectually, we must make provision for having the interior of the fire-surface covered with water to receive the heat communicated through it.

The most simple kind of boiler is one of cylindrical form, with hemispherical ends (commonly called the egg-ended boiler (Fig. 149), placed horizontally, with a fire arranged under it, so that the direct heat of the fire, and of the heated products of combustion in their passage to the chimney, act through the metallic casing on the water within. The water only partially fills the boiler, leaving a space above it for steam, which is conducted to the engine by a pipe leading from the top of the boiler.

Fig. 149.

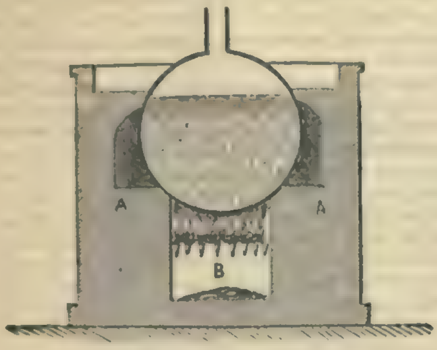


It is found by experiment, and indeed it seems to be a reasonable conclusion, that, within proper limits, having a certain quantity of fuel to dispose of, the larger the surface of water over which the heat developed from combustion is spread, the greater will be the heating effect produced, or the larger will be the quantity of water turned into steam. In fact, it becomes the object of the engineer to allow as little as possible of the heat to escape by the chimney, and consequently to arrange his boiler in such a manner as to make the water absorb the greatest possible quantity of heat, by exposing the largest possible surface to the combustion, and disposing that surface in the best manner.

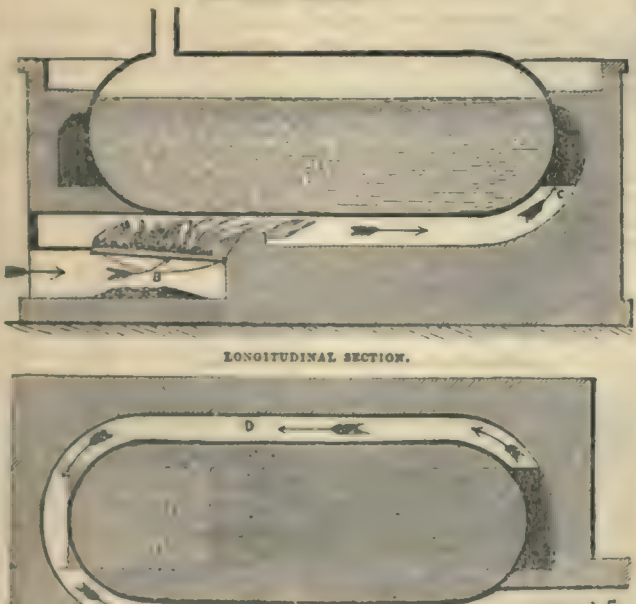
The strongest form in which a vessel can be made, when it is intended that it shall resist internal pressure, is that of a spherical shell. If, then, in the construction of steam-boilers strength alone were studied, the spherical form would be generally adopted. But of all forms of vessels, the spherical is that which has the smallest surface in proportion to its capacity, and it is consequently ill-adapted for the purpose of a boiler where a large amount of heating surface is important. Next to the spherical in point of strength, and superior to it in respect of superficial area, is the cylindrical form, with hemispherical or rounded ends (Fig. 149); and accordingly this form is very generally adopted for steam-boilers.

In order to render available as much as possible of the surface for receiving the heat, the heated products of combustion are not permitted to escape directly from the fire into the chimney, but are carried round the boiler by flues generally in the manner indicated in Fig. 150.

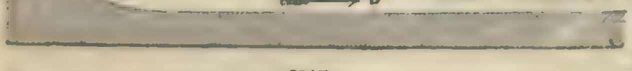
Fig. 150.



TRANSVERSE SECTION.



LONGITUDINAL SECTION.



PLAN.

The boiler is placed on two banks of brick-work A A, between which are fixed the fire-bars B, so that the flame may play on the bottom of the boiler. The products of combustion, heated to a high temperature, pass along under the bottom of the boiler, upwards at the far end C, thence along the side flues D D formed by brick-work, till they finally proceed by any convenient flue or channel E to the chimney. By this arrangement a large portion of the heat contained in the products of combustion is absorbed during their passage along the bottom and side surfaces of the boiler, and is given to the water contained in it; while the brick-work, being a very imperfect conductor of heat, permits very little to escape ineffectively. The power of a boiler arranged in this manner depends upon the extent of fire-grate, or quantity of combustible matter consumed in a given time, and the superficial area of boiler on which the products of combustion play in their course towards the chimney. That there should be some relation between those quantities will be evident from the following considerations. If the quantity of fuel consumed be very great while the flue-surface is small, the products of combustion will not have sufficient opportunity for parting with their heat, and will, therefore, carry up the chimney, and waste, a large amount of heating power, which, by better arrangements, might be made to tell upon the water. If, on the other hand, the fire-grate be too small, while the flue-surface is large, the products of combustion will have parted with their heat before reaching the chimney—a large portion of the flue-surface will thus be rendered useless, the draught (caused by the ascent of heated air in the chimney) will be sluggish, and the combustion slow. We believe that, practically, it will be found advantageous to adopt the following rules as to size of fire-grate and quantity of flue-surface in a boiler, such as we have described—the cylindrical, egg-ended boiler.

To the diameter of the boiler, multiplied by its length, add one-half: the result may be taken as the flue-surface; and this product (in square feet) should be 10 times the horse-power. Thus in a boiler 4 feet 6 inches diameter, and 15 feet long:—

Since 15 feet × 4½ feet = . . . 67½ square feet.
Add one-half . . . . 33¾ „ „

The flue-surface may be taken at 101¼

Or equivalent to 10 horse-power. Again, for every horse-power there should be ¾ths of a square foot of fire-grate. For 10 horse-power there should therefore be 7½ square feet of fire-grate, a surface that might be made up by taking the length of the fire, 3 feet 9 inches, and the width 2 feet, since 3 feet 9 inches × 2 feet = 7½ square feet.

The converse rule for finding the dimensions of a boiler suitable to a given power is the following:—From 10 times the horse-power subtract its ⅓rd part, and the result will be the product of the diameter by the length. Thus, to make a boiler of 10 horse-power:—

Since 10 × 10 = ... 100
Subtract ⅓rd = ... 33⅓

The product of diameter × length = . 66⅔

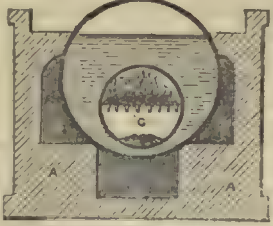
We are at liberty to take any convenient diameter and length that might make up this product within proper limits. Thus, making the

	ft.		ft. in.		
Diam.	3,	the length must be	22 3,	product	66¾ sq. ft.
„	4,	„ „	16 8,	„	66⅔ „
„	5,	„ „	13 4,	„	66⅔ „
„	6,	„ „	11 2,	„	67 „
„	7,	„ „	9 6,	„	66½ „

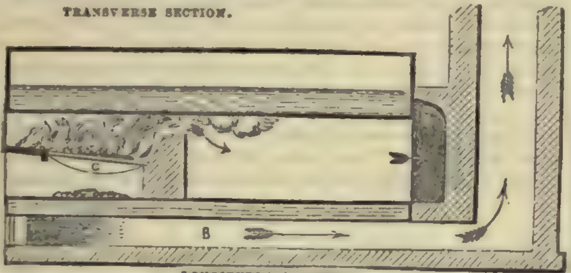
Any of these dimensions may be chosen according to the particular circumstances of the case. Were we to take a diameter smaller than 3 feet, with a greater length than 22 feet, or a diameter greater than 7 feet, with a length less than 9 feet 6 inches, we should find practical difficulties in fixing and working, and lose useful effect from the extreme lengthening or shortening of the flues.

CORNISH BOILER.—When it is desired to obtain greater heating surface within a smaller space, it is found very advantageous to construct the boiler of cylindrical form, with a cylindrical flue or tube passing through the water; so that not only may the exterior surface exposed to the flues receive heat from the products of combustion, but also the interior surface of the tube. When this tube is made of sufficient size to admit the fire within it, as in Fig. 151, the boiler is called a Cornish boiler, from the circumstance of its being first extensively applied with excellent effect in Cornwall. The arrangement of flues for a Cornish boiler is generally similar to that represented in the figure. The boiler rests on two banks of brick-work A A, with a space for the bottom flue B, left between them. The fire-grate is fixed at C, in the front portion of the tube; and the products of combustion pass along the tube, spread at the end E into the two side flues F F, descend at G G to the bottom flue B, and pass thence to the chimney. The quantity of flue-surface in a boiler of this kind exceeds that in the simple cylindrical boiler by nearly the internal surface of the tube. But as the hot products of combustion pass chiefly along the upper side of the tube, and leave its lower side generally covered with a coat of non-conducting soot and ashes, we cannot safely reckon much more than half the surface of the tube as effective heating surface; that is, 1½ time the tube's diameter multiplied by its length.

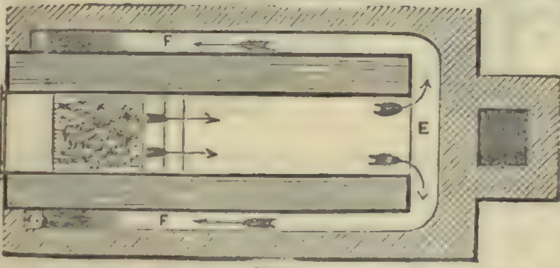
Fig. 151.



TRANSVERSE SECTION.



LONGITUDINAL SECTION.



PLAN.

In order, then, to estimate the power of a Cornish boiler, we should calculate the external surface as before, and add to it the product of 1½ time the diameter of the tube by the length for the total effective surface; allowing 10 square feet for every horse-power.

Or, more simply:—To the external diameter add the diameter of the tube (in feet), multiply by the length, add one-half to the product, and divide by 10 for the horse-power of the boiler.

Example.—A Cornish boiler, 5 feet diameter, and 12 feet long, has a flue-tube 3 feet diameter: required its power.

Diameter of boiler . . . . . 5 feet.
Add diameter of tube . . . . 3 „
8 „
Multiply by length . . . . . 12 „
96 square feet.
Add one-half of 96 = ... 48
Divide by . . . . . . 10) 144
Horse-power of boiler . . . . 14½ nearly.

The converse operation for finding the dimensions of the boiler when the power is given, would be:—From 10 times the power subtract its ⅓rd part; and the remainder gives the product of the length by the sum of the external and internal diameters.

Example.—Thus, for a boiler 14½ horse-power—
Since 10 × 14½ = ... 145
Subtract ⅓rd of 145 . . . . . . 48⅓
97 nearly.

Thus, 97 is the product of the length by the sum of the external and internal diameters.

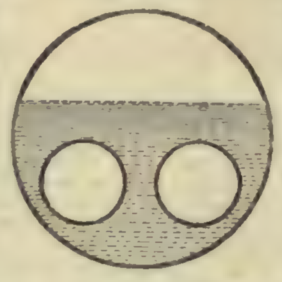
In this case, again, we must consult the circumstances of position for determining the suitable length and diameters. The diameter of the tube should not greatly exceed half that of the boiler, because there should be an ample covering of water over all the heated surfaces. And again, it should not be less than 1 or 2 feet, because it must admit sufficient area of fire-grate without excessive length. The fire-grate in the case given, reckoning $\frac{3}{4}$ths of a square foot per horse-power, should be about 11 square feet; and as a length of fire exceeding 5 or 6 feet would become inconvenient, we must take it at least 2 feet in breadth—that is to say, the diameter of the tube must be 2 feet. The diameter of the boiler might then be 4 feet, and the length would be for these diameters $\frac{97}{2+4} = 16$ feet. Were we to take the diameters as 3 feet for the tube and 5 feet for the boiler, the length would be $\frac{97}{3+5} = 12$ feet.

We believe it will be found practically advantageous to make the length a little more than three times the diameter of the boiler, the diameter of the tube being rather more than half that of the boiler. For 14½ horse-power, according to this proportion, we should have

Diameter of boiler	4 feet 6 inches
Diameter of tube	2 „ 6 „
Length of boiler	14 „ 0 „

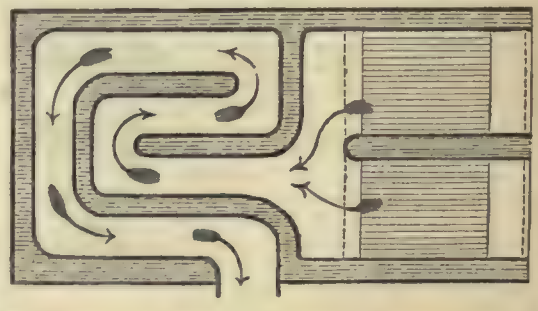
In cases where the dimensions of the boiler are considerable, two or more tubes are introduced, as in Fig. 152. The tubes are always placed as low as possible, allowing 4 to 6 inches between them and the outer casing, in order that their upper and hottest surfaces may be well covered with water, without interfering inconveniently with the steam space above.

Fig. 152.



MARINE AND LOCOMOTIVE BOILERS.—For marine steam-boilers, where brick-work setting would be inconvenient, it is usual to arrange the whole heating-surface within the boiler, by means of flues pervading it in all directions (Fig 153). The water is thus divided into sheets about 6 inches thick; heated on one or both sides by the products of combustion as they pass along the flues. In all such boilers a heating-surface of at least 10 square feet per horse-power, and fire-grate from ½ to $\frac{3}{4}$ths of a square foot per horse-power, should be provided. Nor should the flues be too small in sectional area, nor too much broken up or prolonged, lest the draught or rapidity of movement of air, and consequently of combustion, be interfered with. Of late years, tubular boilers have been very extensively adopted, both for marine and for land engines. These were first employed principally in locomotive engines, as they afforded a means of securing very large flue-surface within a limited

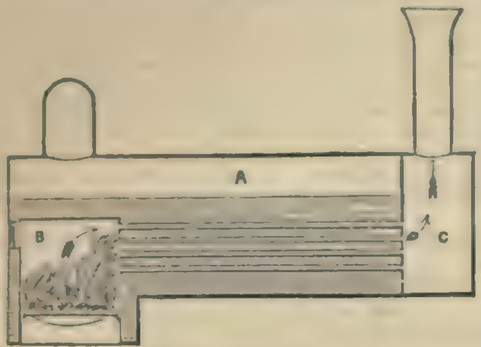
Fig. 153.

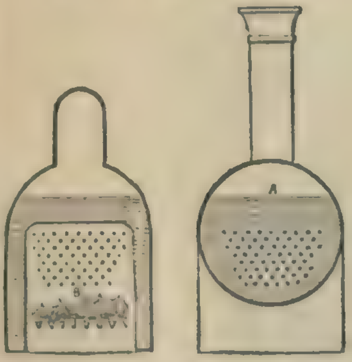


space, and without excessive weight of material. Fig. 154 represents the arrangement of tubes in a locomotive boiler.

The body of the boiler A is cylindrical: at one end B is the fire-box, surrounded by water space; at the other end C is the smoke-box, surmounted by the chimney. In the body are arranged numerous small tubes completely surrounded by water, through which the products of combustion pass in their progress from the fire to the chimney, delivering the greater portion of their heat to the surrounding water. The evaporating power of a boiler of this kind is very great, as we may readily believe on calculating the amount of heating-surface in a boiler of the following dimensions:—

Fig. 154.





Fire-box inside, 3 feet 6 inches × 3 feet 6 inches × 3 feet 6 inches, has 60 square feet actually exposed to the fire, and therefore most valuable as heating-surface. 120 tubes 2½ inches diameter and 10 feet long, give 800 square feet of effective flue surface; the whole, including smoke-box, being contained in a space about 13 feet long, 4 feet 6 inches wide, and 4 feet 6 inches high, and yet being equivalent to 80 or 90 horses' power.

CONSTRUCTION OF BOILERS.—We have now described, generally, the different kinds of boilers used for generating steam. They are, of course, subject to numerous modifications in their details and proportions, according to local circumstances and peculiarities of use and situation. They are generally made of wrought-iron plates, from ¼th inch to ¾ths inch in thickness, according to the magnitude of the work, and the amount of pressure which they are intended to sustain. These plates are made to overlap each other at the edges, and are fastened together by rivets or round pins of iron, passed red-hot through holes provided in the plates, and riveted over so as to form a head. A heavy piece of iron is held against the under side of the rivet-head, and the other end of the rivet is struck repeatedly by heavy hammers, and frequently finished by applying a tool hollowed to the shape of the intended head, and striking the tool by the hammers. The rivet, when the operation is complete, is of the form A (Fig. 155) when finished by smart hammering, called *staff-riveting;* and like B when finished by the tool, being then said to be *button-headed*. As the operation of riveting is performed when the rivet is red-hot, not only is the quality of the rivet not impaired by the hammering, as it would be if hammered when cold, but also the contraction or shrinking of the rivet in its length when it cools draws the edges of the two plates together with great force, and renders the joint impervious to fluid. When it is found by trial that the joints of the plates, or the edges of the rivet-heads, are not quite tight, as manifested by the leakage of water or steam through any of them, a blunt steel chisel or caulking tool is applied to the leaking edge, and struck smartly by a hammer, so as to caulk the joints, or force part of the iron into the crevice.

Fig. 155.

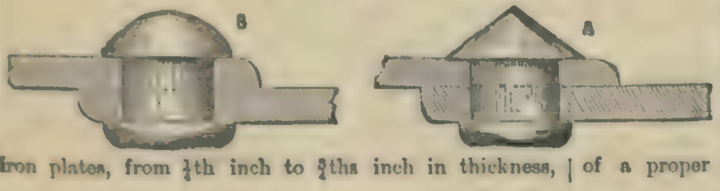
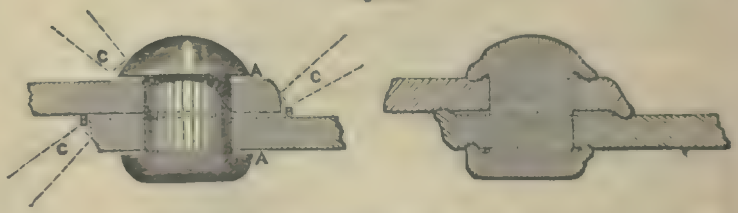
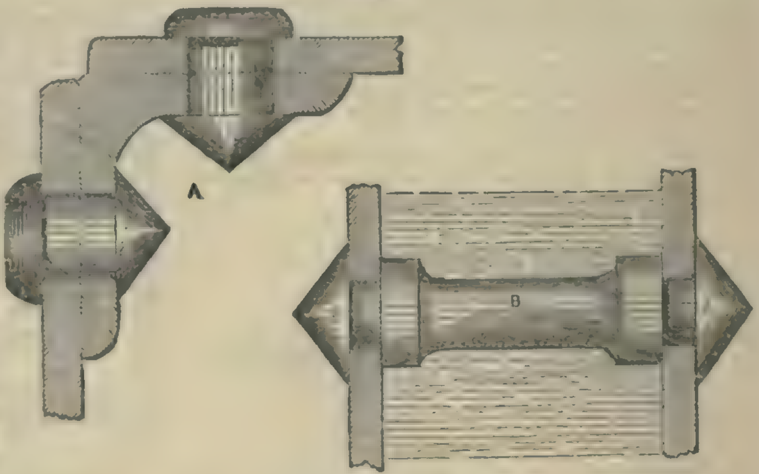


Fig. 156.



If we suppose, for instance, that an opening exists between the plates at B, and round the rivet-head at A (shown greatly exaggerated in Fig. 156), the caulking tool applied at the points marked C, forces the iron of the plate edges and of the rivet-heads into the interstices, and thus renders the jointing tight. For jointing the plates at the angles, a peculiar kind of iron, called *angle-iron*, indicated in section at A, Fig. 157, is employed; and where there are considerable flat surfaces of plate exposed to bursting, pressure stays, B, are introduced at proper intervals to prevent the plates from being forced asunder.

Fig. 157.



For fixing the tubes of tubular boilers in the plates through which they pass, holes are first bored in the plates of a proper size to fit the tubes tightly; and the tubes being cut of the proper lengths and put in their places, the ends are forced open by means of a conical tool driven by hammering into their mouths. Other methods of fixing tubes and stays are employed; and there are numerous other details of boiler-making of a technical character, upon which we need not enter. The furnace or fire-grate of a boiler is generally made of numerous bars of wrought or cast-iron, laid side by side so as to form a grating, on which the fuel is placed, leaving spaces about ½ inch wide between the bars for the passage of air upwards to support the combustion, and of the ashes or incombustible refuse downwards into the ash-pit. In front of the bars, there is generally placed a dead-plate or surface without openings, to receive the fresh fuel, which, lying there for some time exposed to the radiation of the fire beyond, parts with a portion of its gases, and is partially coked before it is pushed onwards to the fire-grate. The gases are ignited in their passage over the hot fire, and produce flame, which plays on the surfaces of the flues. When the supply of air is deficient, large volumes of these liberated gases, having numerous particles of carbon suspended in them, pass through the flues without ignition, and thence through the chimney as black smoke. When this happens, not only is a large and valuable part of the fuel wasted, but the air is inconveniently polluted. The object of smoke-consuming apparatus is to prevent this evil; and the general principle on which all such apparatus is constructed, is either to manage the production of these gases in such a continuous regular manner, as that sufficient air may be supplied for their combustion when they are sufficiently heated to ignite, or to supply heated air in some part of the flues, so as to turn into flame there, the gases that would otherwise escape unconsumed; or, finally, when the production of the gases is irregular, to supply air to burn them only at the proper times and in suitable proportions.

The chimney of a steam-boiler should be of sufficient area and height to produce a good draught or quick current of the heated products of combustion. When the draught is insufficient, the fresh air supplied to the fire is too small in volume, the combustion is retarded, and the flues are filled with smoke instead of flame. A chimney of 30 or 40 feet in height, and having an area of 1 square foot for 10 horse-power, generally gives a sufficient draught. Where height cannot be obtained, as in locomotive engines, the waste steam is made to rush up the chimney with considerable force, and thus to create an artificial draught. The chimney or flue leading to it is provided with a damper or slide, by which the area of passage may be diminished, and the draught reduced at pleasure. At convenient places in the flues, soot-doors are fixed for giving access to clean them from soot or ashes deposited there.

INCRUSTATION.—As all water supplied to boilers is more or less impure, or contains ingredients that become deposited while the water itself is driven off in the form of vapour, it is essential to provide for the frequent cleansing of boilers. In marine boilers particularly, where sea-water is necessarily employed, the deposit of saline ingredients, none of which are volatile, or pass off with the steam, is very large, and of a very troublesome character. It generally settles in the form of a hard, stony crust upon the interior surface of the boiler; and as this crust is a very bad conductor of heat, not only is there a waste of fuel where it exists, owing to its arresting the passage of the heat from the flues to the water, but there is a positive danger from the circumstance that the interior surfaces of the flues are thus over-heated, and the iron of which they are made becomes rapidly oxidised, scaling off in flakes, losing thickness and strength, and becoming liable to disruption from the pressure within exceeding the strength left to sustain it. As salt-water does not deposit rapidly until it becomes super-saturated with salt, it is usual to permit, frequently, the escape of a considerable quantity of the excessively salt-water in the boiler, and to replace it with new water from the sea, less salt and less liable to deposit. For effecting this object and also for emptying the boiler when required, a pipe and cock, called technically the *blow-off*, is fitted to the lower part of the boiler. This cock should be frequently opened, especially where the water is very dirty or of a saline character, and the worst part of the contents of the boiler, which, being heaviest, lie near the bottom, thus blown out. In order to save the loss of heat occasioned by frequently blowing off the hot impure water from marine boilers, and replacing it by cold, but purer water, an apparatus called the *change-water apparatus* is sometimes employed. It consists of a casing, with numerous small tubes arranged in it as in a tubular boiler. A portion of the contents of the boiler being always permitted to flow through the casing, and thence into the sea, a corresponding quantity of purer water is made to pass through the tubes to the boiler; and the latter is thus made to absorb in its passage through the tubes, a considerable portion of the heat given out by the former, in its passage round them.

The large amount of incrustation in marine boilers, and the consequent ineffective expenditure of fuel, have for many years proved a source of great cost and inconvenience in this application of steam-power. When it is recollected that a steam-vessel has to carry its fuel in a place which might otherwise be available for the stowage of cargo, or that fuel has often to be purchased at distant depôts, where its price is enhanced by the cost and difficulty of transport, it is a wonder that owners should have submitted so long to a large waste of fuel, which might have been avoided. A good many years ago, Mr. Hall introduced a surface-condenser, by which the steam, after working the engine, was recondensed into pure distilled water, and in that condition returned into the boiler to be again converted into steam; and so on, in continual circulation. Notwithstanding the great advantage of this arrangement in saving incrustation and deposit—in avoiding the necessity for blowing-off—in economising fuel, and saving tear and wear of boilers—it is not until very recently that engineers have practically adopted the principle of surface-condensation. As yet, very few engines have been fitted with surface-condensers; but it is probable, that before many years have elapsed, their adoption will become universal. Hereafter we shall have occasion to enter more fully into their construction. We only allude to them, at present, in connection with the subject of boilers, in order to show that the deposit and incrustation, which form the grounds of so many complaints and of so much danger, are by no means necessary evils.

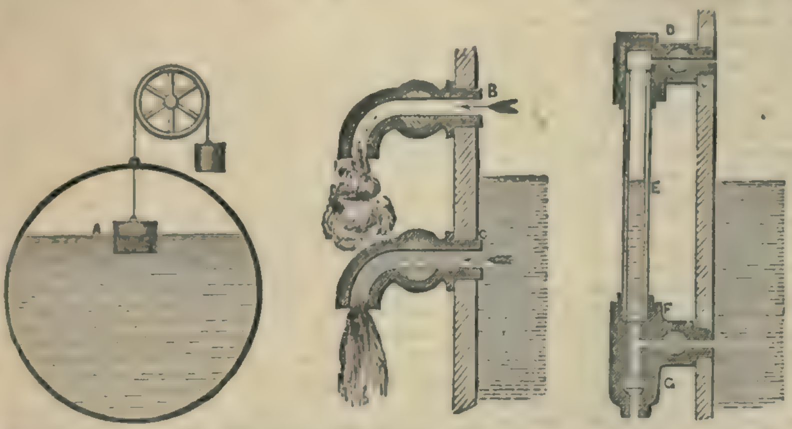
Mud-holes are small holes provided in the lower parts of boilers, and fitted with tight covers, which may be removed, when the boiler is not in use, for the admission of a rake to draw out the mud deposited from the water.

The man-hole, an opening sufficiently large to admit a man, is provided in the upper part of a boiler, and fitted with a tight cover, which may be removed, when the boiler is not in use, for the admission of a man for cleaning or repairs.

In boilers having considerable flue surface, but not great height of steam-room above the water-level, there is always a danger of priming—that is to say, the water in a rapid state of ebullition is often made to boil over, or blow in considerable quantities into the steam-pipes, and thence into the engine, where it is not only useless, but highly detrimental to the action of the machinery. It is usual, therefore, to provide a boiler with a steam-chest, or dome, in its highest part, to give greater space for steam, and greater height for the mouth of the steam-pipe above the surface of the boiling-water. In general, there should not be less than six inches of water above the flues, nor less than three feet of steam-room above the water. In marine boilers, where the water is surged about by the rolling of the vessel, there should be at least double the height named for water and steam. Should the level of the water be so low that the surface of a flue is no longer covered by it, the flue is liable to become over-heated, sometimes to redness; the iron is softened and weakened by the excessive heat, and rapidly deteriorated by oxidation and scaling off. But this is not the only danger of an over-heated flue; for on the water again covering it, the enormous volume of steam suddenly generated produces an excess of pressure, and causes an explosion of the boiler. We believe almost every case of explosion can be traced to some circumstance connected with a deficient supply of water; and, therefore, too much caution cannot be used in watching the condition of the water-level, and providing the proper supply or feed.

BOILER FITTINGS.—In order to ascertain the water-level within the boiler, several kinds of apparatus are employed. The *float* A (Fig. 158) consists of a stone suspended by a wire passing through the top of the boiler, and connected, by a chain led over a pulley, with a counter-balance, sufficient to balance so much of the weight of the float-stone that it shall always lie at the surface of the water. Should the water-level vary, the position of the float-stone, which rises or falls with it, is marked by an index on the pulley.

Fig. 158.



Gauge-cocks B C are two stop-cocks fitted into the face of a boiler, one above and the other below the proper water-level. When these cocks are opened, steam should blow through the upper, and water through the lower one. The objection to gauge-cocks consists in the circumstance that, in order by them to ascertain the level, the attendant must open them.

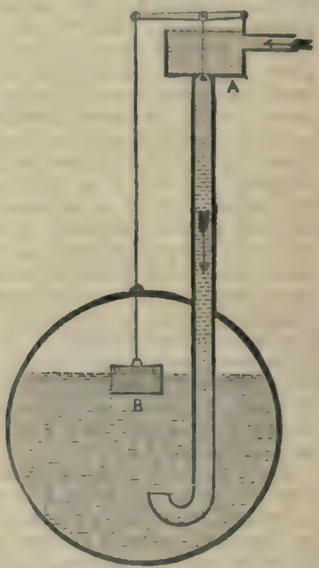
The *glass gauge* consists of two stop-cocks, D and F, fitted in the face of a boiler, one above and the other below the water-line. These cocks are connected by a glass tube E, in which the level of the water is distinctly seen. A stop-cock G is provided at the lower end of the glass tube; and this being occasionally opened, the sediment that may collect in the tube or passages of the cocks is blown out by the pressure within the boiler. Should the glass tube burst, the stop-cocks D and F can be closed until a new tube is fitted, or they can be employed as gauge-cocks.

Occasionally the float is connected with a whistle, in such a manner that when the level of the water becomes too low, it opens a small stop-cock, which permits steam to blow through the whistle, and thus give audible warning of the danger.

In many boilers a precaution is taken against the dangerous consequences of insufficiency of water, by the use of fusible metal plugs. A hole is made in the highest part of the fire-box or flue, which is filled up with a plug or rivet of metal fusible at a temperature not greatly exceeding that of boiling-water. So long as this plug is covered with water, its temperature cannot attain the melting point; but should it be left bare, the heat of the fire playing upon it causes it to melt out, and thus to leave a hole, through which the steam escapes into the flue. Not only does the rush of the escaping steam give warning of the circumstance, but it also relieves the steam-pressure within the boiler, and prevents the explosion which might otherwise result from the over-heating of the flue.

FEED.—In low-pressure boilers, the float occasionally is made to act as a self-feeding apparatus. A (Fig. 159) is a cistern constantly supplied with water, and fitted at bottom with a valve opening downwards into a pipe passing down to nearly the bottom of the boiler; the float B is connected by a wire and lever with the valve, so that when the water-level is too low, the descent of the float causes the valve to open, and thus permits the passage of water from the cistern into the boiler. This arrangement can only be adopted when the pressure of the steam within the boiler does not exceed that of the column of water in the feed-pipe. For high-pressure boilers, the height of cistern and feed-pipe would be inconveniently great to overcome the pressure; accordingly, for these and for marine boilers, the supply of water is effected by means of a force-pump, called the feed-pump, worked by the engine, and regulated by suitable cocks or valves. An apparatus for feeding boilers with water, called an Injector, has also been very successfully applied in many cases. The force of a certain quantity of steam issuing from the boiler by properly arranged channels and valves, is made to cause a proportional quantity of water to enter the boiler.

Fig. 159.



The quantity of water required for a boiler may be generally taken at 1 cubic foot per horse-power per hour; or, as 1 cubic foot contains about 6¼ gallons, and weighs about 63 lbs., we may take 1 lb. of water per minute, or 1 gallon every 10 minutes, as the necessary supply for each horse-power. The actual quantity of water required for generating steam to work an engine of 1 horse-power, depends much upon the construction of the engine, the extent to which the steam is used expansively, the amount of power derived from condensation, or the amount of resistance to the egress of waste steam, if not condensed; but the above estimate is tolerably correct for non-condensing engines worked without much expansion, and is in excess for condensing engines, and such engines as those in which the expansive force of the steam is taken advantage of, so as to produce great effect with small expenditure of steam.

It has been found by experiment, that 1 lb. of ordinary fuel—coal or coke—is capable of turning from 8 to 10 lbs. of water into steam, according to the capabilities of the boiler. Taking the lower estimate, we should reckon that as 63 lbs. of water are required per horse-power per hour, about 8 lbs. of fuel per horse-power per hour would be consumed. In condensing engines, working expansively and under the most advantageous circumstances, the consumption of fuel has been reduced so low as from 2½ lbs. to 3 lbs. per horse-power per hour. In non-condensing engines, when the steam is used to a considerable extent expansively, from 5 lbs. to 7 lbs. of fuel per horse-power per hour is nearly the average consumption.

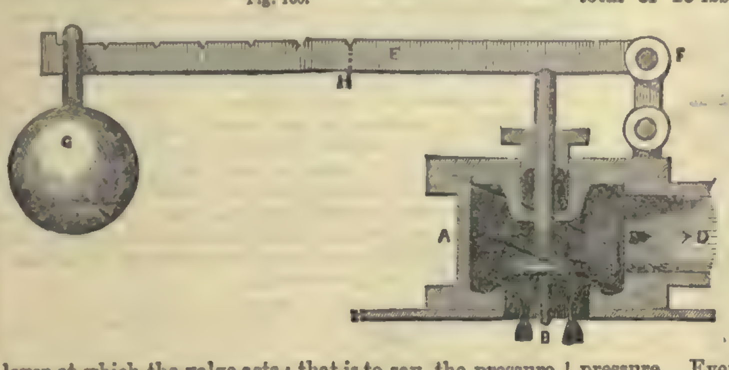
SAFETY-VALVE.—It may be readily conceived, that when water contained in a vessel or boiler of limited strength, is subjected to heat in such a manner as to turn part of it into steam, exerting unlimited pressure, there would be constant risk of explosion, unless some measures were taken for limiting the force of the steam, and preventing its pressure from exceeding that which the boiler could safely sustain. Every boiler is, therefore, fitted with one or more safety-valves, which constitute the most important of the boiler fittings.

The principle of the safety-valve is exceedingly simple. As fluids, and therefore steam, press equally in all directions, any part of the boiler-casing, such as 1 square inch, is subjected to the pressure of the steam. If, then, we make a hole in the upper part of a boiler 1 square inch in area, and cover it with a lid, laying on this lid a weight such as 50 lbs., we can apply heat and generate steam, which, as soon as its pressure exceeds 50 lbs. per square inch, will lift the loaded lid, and permit a portion to escape. Should the generating power of the boiler be moderate, the raising of the lid, and escape of a portion of steam, would prevent the pressure from ever exceeding that due to the weight on the lid; but should the steam be generated more rapidly than it can escape through the hole, the pressure must go on accumulating, until the strain to which it subjects the boiler exceeds the strength of the material of which it is made, and an explosive rupture is the consequence. It is, therefore, important to provide a safety-valve, with an opening of sufficient size to permit the escape of steam as rapidly as it can ever be generated, and to load it with a weight not greater than the pressure which the boiler can safely bear. Boilers are generally tested before use, under a pressure very much greater than that with which they are to be used. For condensing engines, the pressure seldom exceeds 20 lbs. per square inch; for ordinary non-condensing engines, 50 lbs. or 60 lbs. per square inch; and for locomotives, it is as high as 120 lbs. and 150 lbs. per square inch above that of the atmosphere. The area of the safety-valve should not be less than $\frac{1}{4}$th square inch per horse-power.

Fig. 160 represents a safety-valve of the ordinary construction. A is a box fixed over a hole B in the upper surface of the boiler, having a truly-faced seating on which the valve C can rest. The stem of the valve passes through the cover of the box, where there is a gland or stuffing-box to prevent the escape of steam round the stem. A pipe D conducts the steam that passes the valve, when it is lifted, to the chimney or elsewhere. The valve is kept down by a lever E, which works on a pin, or fulcrum, at F, and has a sliding weight suspended from it at any point such as G. The arm of the lever is graduated so that the weight can be placed to give such pressure on the valve as may be required. If we suppose, for example, that the area of the valve-opening is 1 square inch, that the length from the centre of the valve-stem to that of the pin F is 2 inches, and that a weight of 10 lbs. hangs at G, 16 inches from F; then the effect of the weight to press down the stem of the valve is as its weight multiplied by the length of lever at which it acts, divided by the length of lever at which the valve acts; that is to say, the pressure on the valve is $\frac{10 \text{ lbs.} \times 16 \text{ inches}}{2 \text{ inches}} = 80$ lbs. As the area of the valve is 1 square inch, this weight is capable of resisting a steam-pressure of 80 lbs. per square inch within the boiler. If the lever be graduated by divisions each 2 inches in length, each of these will correspond to a pressure of 10 lbs. on the valve; that is to say, the weight at—

16 inches gives a pressure of 80 lbs.
14 „ „ „ 70 „
12 „ „ „ 60 „ and so on.

Fig. 160.



We have not reckoned the effect of the weight of the valve and lever, which should generally be weighed, so that the pressure due to them, exclusive of the weight, may be estimated before graduating the lever. As a practical example, we will suppose that it is required to make a safety-valve of 4 inches diameter, and load it by a weight and lever graduated to steam-pressures varying from 20 lbs. to 50 lbs. per square inch above atmospheric pressure. We will suppose that the weight of the valve and stem is 5 lbs., that a convenient leverage for the valve is 3 inches, and that the whole lever from F to the end is 30 inches, the lever itself being of uniform depth and thickness, and weighing 9 lbs.

The area of the valve (a circle 4 inches in diameter) is $12\frac{1}{2}$ square inches; the effect of the weight of the lever is the same as if it were collected at its middle point H, 15 inches from F, and its

Pressure on the valve is therefore $\frac{9 \text{ lbs.} \times 15 \text{ in.}}{3 \text{ in.}} = 45$ lbs.

To which we add the weight of valve and stem . 5 „

Making a total constant weight on the valve = . 50 „

And as the area of the valve is $12\frac{1}{2}$ lbs., this gives a constant pressure of $\frac{50}{12\frac{1}{2}} = 4$ lbs. per square inch. For a pressure of 50 lbs. per square inch, or a load of $50 \times 12\frac{1}{2} = 625$ lbs. on the valve, the additional load must be 575 lbs. at a leverage of 30 inches against that of the valve at 3 inches; and therefore a weight of $57\frac{1}{2}$ lbs. at the end of the lever gives the required pressure; because $\frac{57\frac{1}{2} \text{ lbs.} \times 30}{3} = 575$ lbs. on the valve. The same weight, to give a pressure of 40 lbs. per square inch, should act on the valve with a force of $12\frac{1}{2} \times 40 - 50 = 450$ lbs; and its distance from F must be about 23·48 inches, because $\frac{57\frac{1}{2} \times 23{\cdot}48 \text{ inches}}{3} = 450$. Now the difference between 30 inches, the leverage for 50 lbs., and 23·48 inches, the leverage for 40 lbs., is 6·52 inches—a division that may be repeated along the lever for 30 and 20 respectively.

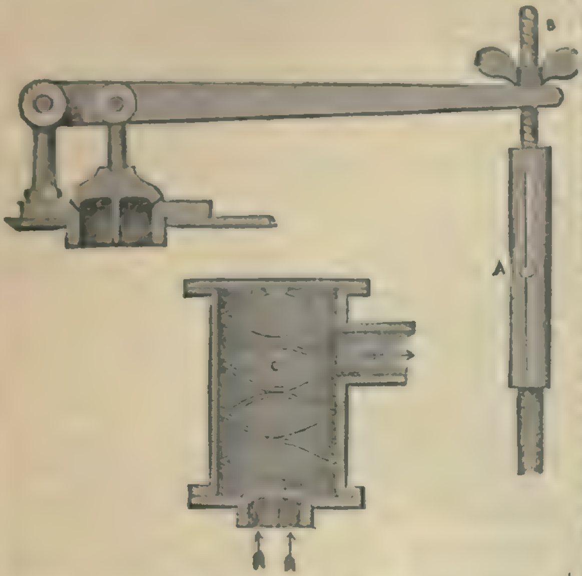
We might attain the same result by another process, thus:—Since the constant pressure due to the weight and valve is 4 lbs. per square inch, the additional pressure to be derived from the weight of $57\frac{1}{2}$ lbs., to make a total of 10 lbs. per square inch, would be 6 lbs. per square inch, or $6 \times 12\frac{1}{2} = 75$ lbs. in all. The leverage of the weight to produce this load would be found from the simple proportion:—

Weight.		Load on valve.		Leverage of valve.		Leverage of weight.
$57\frac{1}{2}$ lbs.	:	75	::	3 in.	:	3·913 in.

Repeating the same process for 50 lbs. pressure per square inch, we should find the leverage of the weight to be 30 inches. The difference of 30 inches and 3·913 inches, viz., 26·087 inches, being divided into 40 equal parts, each 0·652 inches—because 40 is the difference between 50 lbs. and 10 lbs.—would mark the lever for each lb. of pressure. Every 10 lbs. would thus be graduated by intervals of 10 × ·652 = 6·52 inches as before.

In locomotives and boilers where a weight sliding along a lever would be inconvenient, the lever is affixed to a spring-balance A (Fig. 161), graduated to the pressures per square inch due to the spring. By turning a nut B on the stem of the spring-balance, any required pressure can be thrown upon the valve, which is kept down by the spring acting on its lever. Should the pressure within the boiler exceed that to which the balance is adjusted, the valve is opened, and a portion of the steam escapes. The lock-up safety-valve consists of a valve pressed down by a set of strong springs C, the whole enclosed within a box under lock and key. While the engine-driver has command over the spring-balance valve, so as to increase or diminish the load at pleasure, the lock-up valve is inaccessible to him, and opens whenever he has loaded the open valve beyond the pressure to which the lock-up valve has been adjusted; thus serving as a check upon him in case of his working at a dangerous pressure.

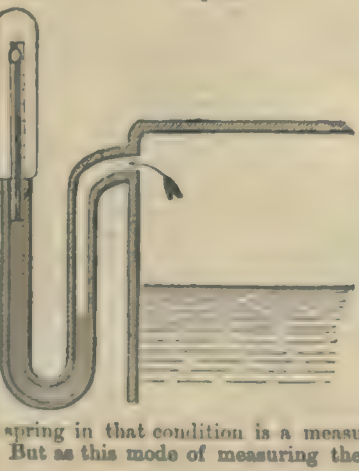
Fig. 161.



In a boiler for an engine working at very low pressures, there is frequently provided a vacuum valve, which is a safety-valve opening inwards, and admitting air into the boiler, in case the pressure within should fall so far below that of the atmosphere without, that there might be danger of collapse.

PRESSURE GAUGE.—The steam-gauge is an apparatus generally fitted to boilers for indicating the pressure of the steam. The safety-valve may be employed for this purpose; for if the weight be adjusted on the lever, or the spring of the balance released, until the valve begins to open and let steam escape, we know that the weight or spring in that condition is a measure of the pressure. But as this mode of measuring the pressure requires personal attendance, it is better to be provided with some self-acting instrument which shall show at a glance the condition of the steam in respect to pressure.

Fig. 162.



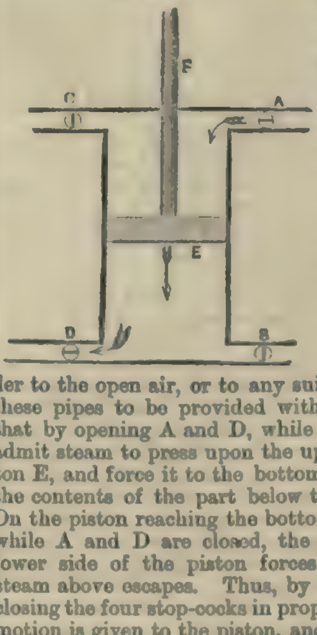
For low-pressure boilers, the mercurial steam-gauge is generally employed (Fig. 162). It consists of an iron pipe bent to a siphon form, connected with the boiler, and containing mercury, on which floats a rod of wood extending above the mouth of the tube, and pointing to divisions on a scale. As 2 cubic inches of mercury weigh very nearly 1 lb., the rise of the wooden index through 1 inch in height, indicates that the mercury in 1 limb of the siphon has risen 1 inch, and fallen 1 inch in the other, making a difference of level of 2 inches, equivalent to a pressure of 1 lb. per square inch in the boiler. Thus every inch on the scale corresponds to 1 lb. pressure per square inch.

For high-pressure boilers, the column of mercury necessary would be inconveniently high, and recourse is therefore had to gauges of other kinds. Among the most effective and ingenious of these may be mentioned that of Bourdon. It consists of a flattened elastic tube of copper or brass, bent into a spiral form. The pressure within the tube tends to bulge it, and uncoil it a little out of the spiral form; and the slight movement thus induced is communicated to an index, which points on a dial-plate to the pressure marked thereon from the result of experiments made for the purpose of determining the proper graduation of the dial.

CYLINDER.—The steam generated in the boiler at such pressure, and in such quantity as may be desired, is conveyed by the steam-pipe to the cylinder, which is a vessel closed at both ends, and fitted with a piston E (Fig. 163), capable of sliding tightly from end to end, and having a rod F passing tightly through one of the end covers, or the cylinder lid. If we suppose A and B two pipes communicating with the boiler, and opening into the cylinder at opposite ends, while two other pipes, C and D, lead from the ends of the cylinder to the open air, or to any suitable place; conceiving these pipes to be provided with stop-cocks, we can see that by opening A and D, while B and C are closed, we admit steam to press upon the upper surface of the piston E, and force it to the bottom of the cylinder, while the contents of the part below the piston escape by D. On the piston reaching the bottom, if we open B and C, while A and D are closed, the pressure acting on the lower side of the piston forces it upwards, while the steam above escapes. Thus, by alternately opening and closing the four stop-cocks in proper order, an alternating motion is given to the piston, and the force is communicated by the rod to any suitable machinery. The amount of force so communicated depends on the size of the piston, or number of square inches in its surface on which the steam-pressure acts, the intensity of that pressure, and the velocity at which the piston is caused to travel. If we suppose, for instance, that the diameter of the cylinder is 1 foot, the circular area of which is 113 square inches, that the pressure of the steam is 20 lbs. per square inch, and that the average speed of the piston is at the rate of 200 feet per minute, the power communicated through the rod is equivalent to

$$\frac{113 \text{ sq. ins.} \times 20 \text{ lbs.} \times 200 \text{ ft.}}{33000} = 13\cdot7 \text{ horse-power.}$$

Fig. 163.



Some of the first steam-engines had cocks arranged as we have described, which demanded the continual attendance of a workman to close or open them at the proper times. But it suggested itself that apparatus connected to the moving parts of the engine might be so adjusted as to perform this operation; and accordingly great ingenuity has been exhibited in contrivances for alternating the flow of steam to and from the opposite ends of the cylinder.

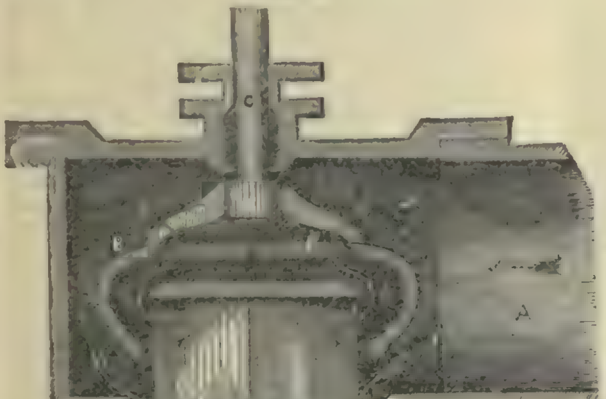
VALVES.—Instead of stop-cocks in the pipes, which are subject to considerable and unequal wear from constant working, and thereby become leaky, valves are often employed, similar to the safety-valve, and worked upwards and downwards by means of levers acting on their stems, which project beyond the steam-tight casing in which they are enclosed. Were these valves made in such a way that the steam pressed on their lower surfaces, and tended to raise them from their seats, it would be difficult to keep them tightly down without very considerable force. If, on the other hand, the pressure of the steam acted on their upper surfaces so as to keep them tightly down, considerable force would be required to lift them so as to permit the steam to pass at the proper times. Moreover, as, in large engines, these valves must be of considerable size to let sufficient steam pass through the openings they cover, and as, for effective working, they must be raised and lowered very rapidly, it becomes important to reduce, as low as possible, the pressure upon them, and thus diminish the force necessary for their movement.

The double-beat valve is a contrivance for covering a large area of steam-passage with a valve subjected to moderate pressure. The steam entering at A (Fig. 164), fills the valve-box, in which an annular or ring-shaped valve B is capable of being pressed upwards or downwards by a rod C passing through the cover of the valve-box. When the valve is down, its upper and lower conical surfaces rest on corresponding seats, to which they are nicely ground; while, the valve being raised, the steam flows between the surfaces into the pipe D. The force necessary to raise such a valve, is that of the steam-pressure on the excess of the area of the lower seating above that of the upper, and may be made much less than that on an ordinary valve of equivalent size. We shall suppose, for example, that a valve of 10 inches diameter is subjected to a pressure of 20 lbs. on the square inch. The force to lift it would be

(circ. area of 10 ins. diam. =) 78½ sq. ins. × 20 = 1570 lbs.;

and the height through which it must be lifted to give the full passage for steam round it, is 2½ inches; for 2½ × (circumf. of 10 ins. diam. = 31½) = 78½ sq. ins., the area of a circle 10 ins. in diameter. But were we to use a double-beat valve, having its lower seating 10½ ins. in diameter, and its upper 9 ins. in diameter, the area of the one being 86½ sq. ins. and that of the other 63½, the pressure on the difference, 23 sq. ins., is 23 × 20 = 460 lbs., little more than one-fourth of that on the single-seated valve. Again, the height through which the double-beat valve must be raised is only half that required for the single valve; because, when it is raised, passage is provided for the steam both above and below. Thus, to work the double-beat valve, only ⅐th or ⅛th part of the force requisite for the single-seated valve is required.

Fig. 164.



Wherever valves are used, especially in large engines, or under great pressures, for alternating the flow of steam, recourse is had to the double-beat valve, or some expedient of a similar character, by which considerable size of passage may be secured without having to lift a great weight, or move it through a great distance.

SLIDE.—But the frequent raising and lowering of any set of valves, however well balanced, would, in quickly-moving engines, be accompanied with noise, and would prove very inconvenient, on account of the complication of machinery required for the purpose, and the greater amount of wear and tear resulting from its use. To avoid these evils, the *slide* has been contrived; and it is almost universally employed for alternating the flow of steam to or from the ends of the cylinder, except in engines moving very slowly. There are various kinds of slides in use, but they are nearly all contrived on similar principles, with such differences in the details of construction as the peculiar views of makers, or the circumstances of their position, suggest. The most simple kind is called the D-*slide*, from the circumstance of its form resembling that of the letter D.

Fig. 165.

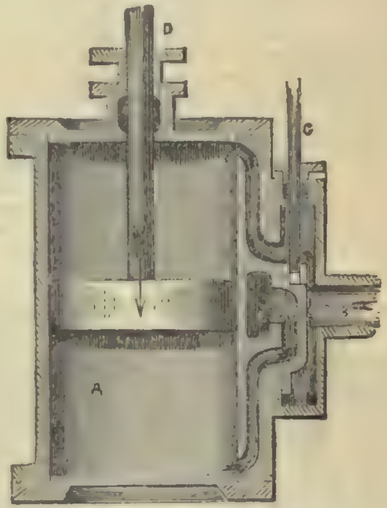
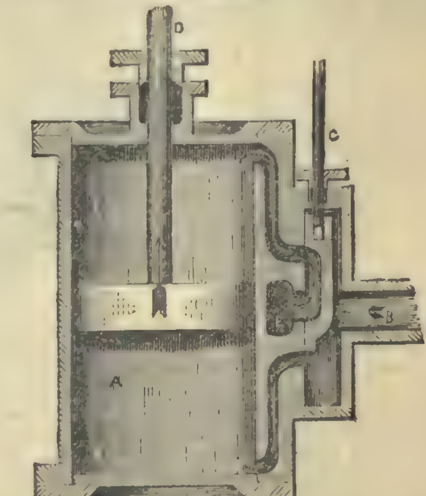


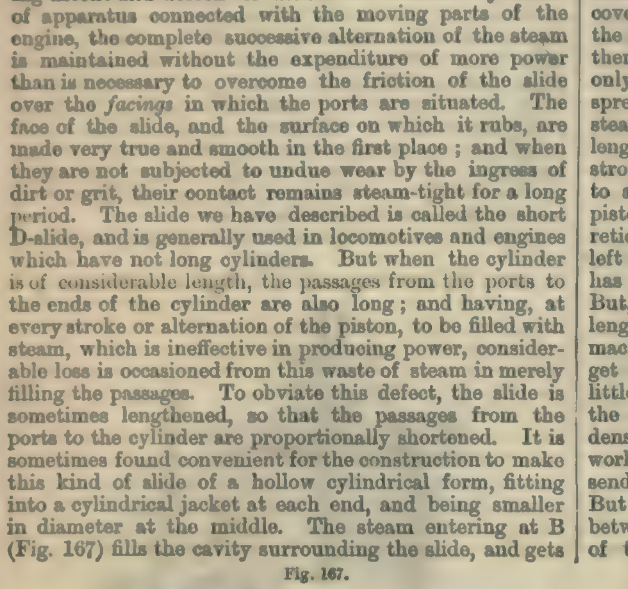
Fig. 166.



Figs. 165 and 166 represent two longitudinal sections of a cylinder fitted with a D-slide. The steam enters by a pipe B from the boiler into a cavity called the *slide-*

jacket, on the opposite side of which are three openings, the upper and lower, called the *ports*, communicating by tubes or passages with the upper and lower ends of the cylinder respectively; and the middle one, or exhaust channel, communicating by a cavity E with an opening at the side, by which steam can escape. These three openings are partially covered by a hollowed plate of metal, the *D-slide*, which, as its name implies, can be made to slide up or down by means of a rod C passing tightly through the jacket. The hollowed part of the D-slide is made to embrace the middle exhaust-passage and either of the ports, so as to let steam escape from the cylinder, while it leaves the other port open for the ingress of steam to the cylinder; and as for every ascent and descent of the piston in the cylinder, a corresponding ascent and descent of the slide is effected by means of apparatus connected with the moving parts of the engine, the complete successive alternation of the steam is maintained without the expenditure of more power than is necessary to overcome the friction of the slide over the *facings* in which the ports are situated. The face of the slide, and the surface on which it rubs, are made very true and smooth in the first place; and when they are not subjected to undue wear by the ingress of dirt or grit, their contact remains steam-tight for a long period. The slide we have described is called the short D-slide, and is generally used in locomotives and engines which have not long cylinders. But when the cylinder is of considerable length, the passages from the ports to the ends of the cylinder are also long; and having, at every stroke or alternation of the piston, to be filled with steam, which is ineffective in producing power, considerable loss is occasioned from this waste of steam in merely filling the passages. To obviate this defect, the slide is sometimes lengthened, so that the passages from the ports to the cylinder are proportionally shortened. It is sometimes found convenient for the construction to make this kind of slide of a hollow cylindrical form, fitting into a cylindrical jacket at each end, and being smaller in diameter at the middle. The steam entering at B (Fig. 167) fills the cavity surrounding the slide, and gets access to the cylinder by the upper or lower port as the slide is moved upwards or downwards, while the steam passes from either of these ports into the cavities at the upper or lower ends which communicate through the tubular body of the slide, and from one of which the waste-pipe C conveys the steam which has done its work.

Fig. 167.



We might mention many other varieties of slides; but all being constructed on similar principles to those we have described, we need not discuss them in detail. We will now proceed to describe the practical construction of a cylinder and piston suitable for a non-condensing engine.

CYLINDER AND PISTON.—Referring to Figs. 165 and 166, which give a general view of the cylinder and slide, we have to inquire into the proportions of the parts, and the details of their construction, in such a manner as to be economical, durable, and efficient.

The cylinder is made of cast-iron, bored in a suitable lathe, so that the interior is as nearly as possible perfectly cylindrical. The covers are also of cast-iron, having a projecting part turned to fit into the ends of the cylinder, to which they are secured by bolts and nuts. The mouths of the cylinder are generally bored somewhat larger than the rest, so that if, after some years' wear, it become necessary to bore out the cylinder afresh, thereby making its diameter a little larger, the same covers may still fit it. The flanges or projecting rims of the cylinder, and the faces of the covers which lie against them, are turned very true; and if well smoothed, require only a little thin flour-paste, or white and red lead paint, spread over them, to render the joints impervious to steam when the bolts are screwed tightly up. The length of the cylinder is determined by the length of stroke or movement of the piston: it should be such as to allow from ¼th to ½ an inch clearance between the piston and the cover at each end of the stroke. Theoretically, the less clearance the better, because any space left between the piston and cover at the end of the stroke, has to be filled at every stroke with steam uselessly. But, practically, it may be difficult to determine the length of stroke with perfect accuracy; the wear of the machinery may alter it a little, and water or dirt may get into the cylinder, so that it is necessary to allow a little space, such as we have named, for clearance. When the steam first enters a cold cylinder it becomes condensed into water; and not unfrequently, during the working of the engine, the boiler primes or boils over, sending water along with the steam into the cylinder. But as water is practically incompressible, its presence between the piston and the cylinder-covers at each end of the stroke, would be quite as detrimental as the presence of a mass of iron or any hard material, unless an exit were provided for it; for the sudden approach of the piston would be arrested by the water, and either the cover would be forced off, or the parts connecting the piston to the rest of the machinery would give way. Accordingly, cylinders are often fitted with relief-valves—small safety-valves loaded by weights or springs, and communicating with each end of the cylinder—so that whenever, by the presence of water, the pressure becomes increased to a dangerous extent, the valve is opened, and permits the water to escape. In the absence of relief-valves, *pet-cocks* or small stop-cocks are fitted for the same purpose. On first admitting steam to the cylinder, these are left open to permit the escape of the water arising from condensation; and during the working of the engine, they may generally be left a little open, especially when the presence of water is manifested by a sharp blow, heard when the piston strikes upon it. The thickness of the cylinder depends on its diameter, and the pressure to which it is subjected; and the strength of the flanges, covers, and bolts and nuts, must be determined on the same ground. For such details it is difficult to give precise rules, as experience and study of well-proportioned works can alone give the power of determining them.

The piston is constructed in various ways, one of which, being simple and effective, we will describe:—The body of the piston consists of a disc and boss A (Figs. 168 and 169), the outer edge of the disc fitting the cylinder, and the boss having a central conical hole, in which the piston-rod B is secured by means of a key, or thin bar of iron slightly tapered in width, driven through a slot or opening in the boss and rod, so as to tighten the conical end of the rod in the corresponding conical hole. To the body of the piston is secured by screws, a cover C fitting the cylinder, and leaving between

Fig. 168.

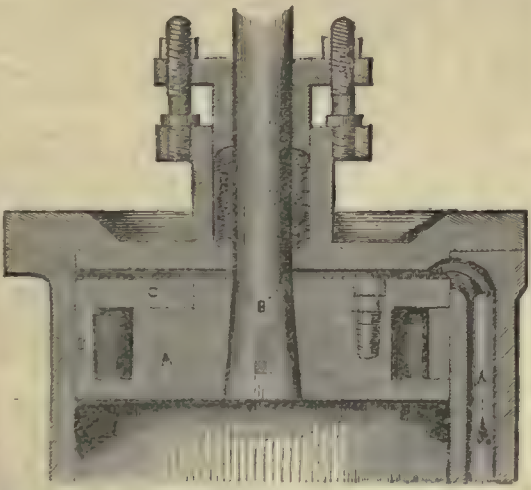
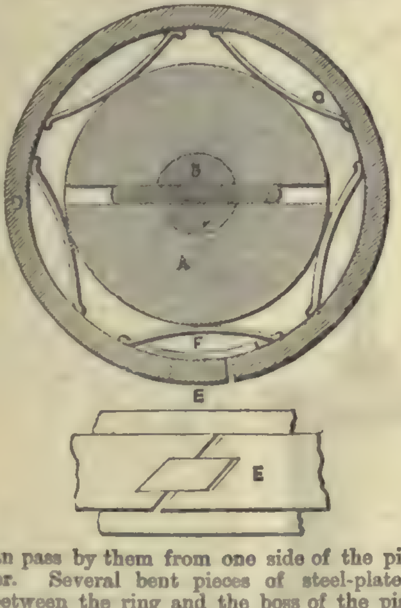


Fig. 169.

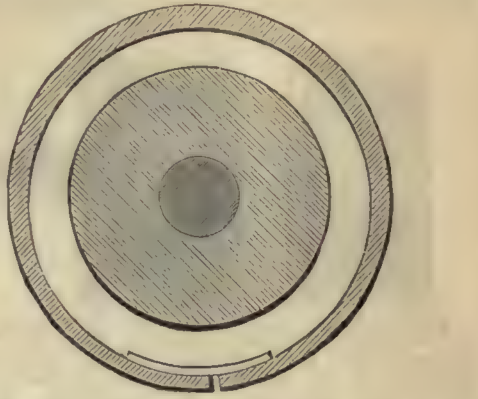


it and the edge of the disc below, a groove which contains the packing ring D. This ring is made to fit the cylinder, and is cut obliquely across at some point of its circumference E, a parallelogram-shaped hole being cut out of the middle, and filled with a piece of metal truly fitted to it. By thus cutting the ring across at E, it is permitted to expand in diameter ; and the slits made at E are covered by a plate F inside the ring so that no steam can pass by them from one side of the piston to the other. Several bent pieces of steel-plate G are placed between the ring and the boss of the piston, so as to push the circumference of the ring outwards.

As the inside of the cylinder and the edges of the piston and its cover become worn by constant rubbing, the packing-ring is made to expand, and still to work tightly in the cylinder, without permitting the flow of steam past the piston. Sometimes, for small pistons, the packing-ring is merely made thicker at the side opposite its slit ; and being at first slightly larger than the cylinder, so that it must be compressed when pushed into it, its own elasticity makes it expand to fit the cylinder even after considerable wear, without the necessity for steel springs within it (Fig. 170).

The piston-rod, in passing through the cylinder cover, is surrounded by a cavity called a stuffing-box, and filled with soft twisted hemp and tallow, called *packing*, which is compressed in the cavity by means of a *gland*, forced down upon it by tightening screws. By the use of this packing, while the rod travels upwards and downwards, steam cannot pass round it ; for even if the rod be worn somewhat irregularly, the elasticity of the packing serves to prevent the leakage of steam. Occasionally vulcanised india-rubber is employed for the purpose of packing.

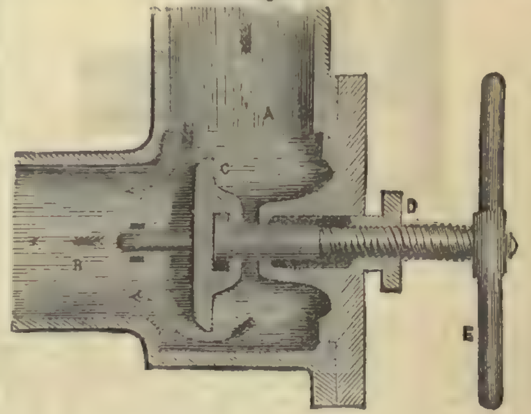
Fig. 170.



The slide-rod, and, indeed, all rods about an engine for moving valves or parts within cavities containing steam or water, have to pass through packing of this kind.

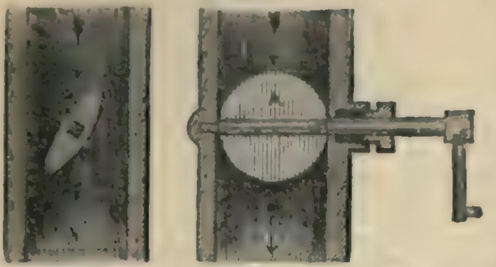
SHUT-OFF AND THROTTLE VALVES.—For opening or closing the communication between the boiler and the cylinder, so as to admit steam from the one to the other, to diminish the quantity or entirely arrest it, a stop-cock is generally employed for small engines, and for larger ones a *shut-off valve*. The stop-cock, being sufficiently well-known, we need not describe. The *shut-off* or *stop-valve* is generally made as indicated in Fig. 171. The steam-pipe A communicates with the valve-box, having a pipe B proceeding from it. The mouth of this pipe is fitted with a conical edging or seating, to which a conical-edged valve C is nicely fitted. Through a stuffing-box D in the cover of the valve-box, passes a screwed rod, connected by a free joint to the valve C ; when the rod is turned round by a handle E, so as to screw it inwards through the gland D, the valve is pressed firmly down on its seating, and thus all communication from A to B is cut off. By unscrewing the rod, the valve is raised from its seating as much as may be required for the passage of steam.

Fig. 171.



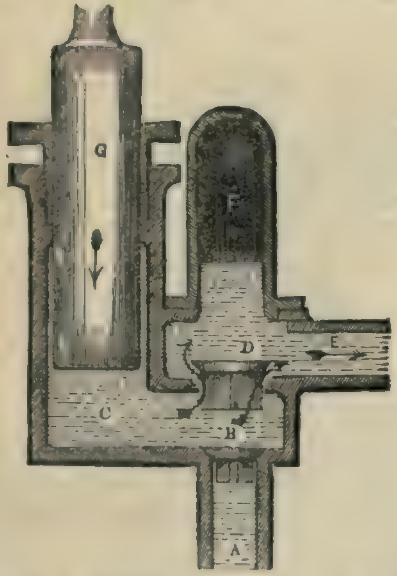
The *throttle-valve* is for the purpose of choking or throttling the passage of steam in a pipe, so as to regulate the quantity passing through it in a certain time, without perfectly arresting it. It consists of a disc A (Fig. 172) mounted on a rod passing across the pipe through a stuffing-box at one side. On turning the disc *edgeways* towards the current, the steam is allowed to pass ; but when it is turned *across* the pipe, none can pass except such a small quantity as can leak round the edges of the disc. This valve is almost universally applied to engines which are fitted with a governor, to which it is attached by means of proper connecting rods; and by its means the speed of the engine is regulated with great nicety, as more or less steam is permitted to pass the valve.

Fig. 172.



FEED-PUMP.—The feed-pump of an engine is for the purpose of supplying the boiler with water to take the place of that which is boiled off, and passes away in the form of steam, after working the engine. It consists of a barrel C (Fig. 173), in the upper part of which a plunger G is worked alternately upwards and downwards through a stuffing-box. At the lower end of the barrel is a valve-box, containing a suction-valve B, covering a pipe A by which the water is drawn from a convenient reservoir, and a discharge-valve D, past which the water has to flow in its progress to the boiler by the feed-pipe E. When the plunger is raised, the valve D being kept down in its seat by the pressure of water in the boiler (equivalent to that of the steam) communicated through the feed-pipe E, the vacuum left by the rise of the plunger is filled by water entering from A and raising the valve B for its passage. On the descent of the plunger, the water being forced out of the barrel, presses down the valve B, but raises the valve D, and flows onwards to the boiler. At some convenient part of the feed-pipe E there is always fitted a stop-cock, or shut-off valve, for completely cutting off communication between the pump and the boiler in case of the valves being deranged. But water being almost totally incompressible, it would be extremely hazardous to close this communication while the pump is in action; for in that case the barrel must be burst open, or some part of the machinery that works the pump must be broken. It is, therefore, usual to provide also a relief-valve, constructed exactly like a safety-valve on the feed-pipe, to permit the efflux of the water when its ordinary passage is closed. The cover F of the feed-valve box should be capable of being readily moved to give access to the valves; and it is often made of considerable size, hollowed out to contain air, which is compressed by the influx of water during the descent of the plunger, and reacts to force the water onwards to the boiler while the plunger ascends. In all cases, indeed, where water under considerable pressure is exposed to the recurring action of a propelling force, as in the feed-pump, an air-vessel should be provided to act as a spring, relieving the blow on the water, and regulating its motion to a gradual flow instead of a sudden movement.

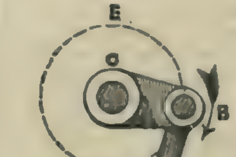
Fig. 173.



CRANK AND CONNECTING-ROD.—Having now described the *cylinder* in which, and the *piston* on which, the steam acts to put the machinery in motion—the *slide* by which the alternation of the course of the steam is effected—the *throttle-valve* by which its quantity is regulated—and the *feed-pump* by which the necessary supply of water is maintained in the boiler; we have to inquire how the reciprocating motion of the piston is converted into the rotary movement required for driving machinery, and how this rotary movement produces the reciprocating motion of the slide and feed-pump, and governs the action of the throttle-valve.

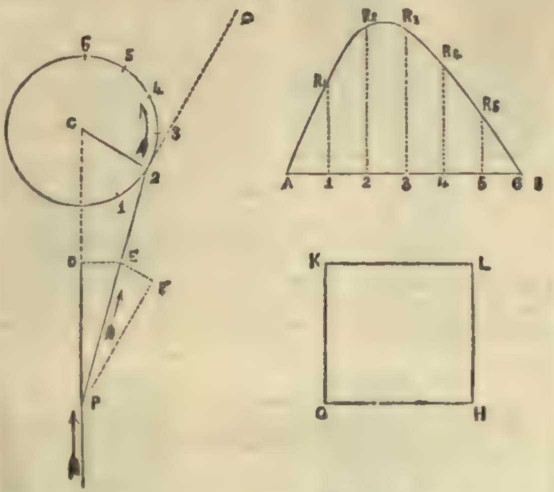
To the end A (Fig. 174) of the piston-rod there is jointed the connecting-rod A B, having an eye at B working on the *crank-pin*, or pin fixed to the *crank*—an arm B C projecting from the main shaft or spindle C. The crank-pin can move round in a circle, the diameter of which is exactly equal to the length of stroke, or distance through which the piston travels in the cylinder. As the piston descends from F to G, making the down-stroke, the crank is caused to revolve from E round to D, one half-revolution. Again, while the piston ascends from G to F, making the upstroke, the crank revolves from D round to E, another half-revolution. Each revolution, then, of the crank requires a double stroke of the piston; and to effect it, the upper and lower portions of the cylinder have each to be filled and emptied of steam; or, as their capacities are equal, the cylinder has for each revolution to be twice filled and emptied. It is obvious that, at every different point of its revolution, the crank is acted on by a different force, owing to the varying obliquity of the connecting-rod. At the two extreme points, D and E, where the crank is in a line with the connecting-rod, the effect of the piston to cause it to revolve is reduced to nothing; for it merely pushes or pulls it against the central shaft. These points are technically called the *dead centres*, because there the force of the piston is dead or ineffective. But to make up for the total want of action at those points, we find that at some other points the effect of the force passing through the connecting-rod to turn the crank, is greater than the pressure on the piston, in consequence of the obliquity of its action.

Fig. 174.



Again, the piston, during a revolution or double stroke, passes through a distance equivalent to twice the diameter of the crank-circle; while the crank-pin passes over the circumference of that circle, more than 3 times its diameter. The influence of the pressure on the piston to turn the crank may be best conceived by a graphical delineation of the force in the following manner:—If we divide the circle described by the crank-pin round the centre C into any number of equal parts (Fig. 175), and draw a straight line A B equal to the half-circumference divided into corresponding parts, A B represents the distance through which the crank-pin moves during half a revolution, developed into a straight line. If at any point, such as 2, we draw a line 2 C representing the crank, and a line 2 P representing the connecting-rod, and draw Q 2 touching the circle at 2, and therefore representing the direction in which the crank-pin is moving at the point 2, while the line P C represents the direction of the piston, we may take any length P D in that line representing the force of the piston (as, for instance, if the pressure on the piston were 5 tons, we might take P D = 5 inches), draw D E perpendicular to P D, P F parallel to Q 2, and E F perpendicular to P F, or parallel to C 2. Then, on the principle of resolved forces, the length of P E represents the force transmitted through the connecting-rod, and P F the force tending to turn the crank, while D E measures the side thrust on the piston-rod, and E F the longitudinal strain on the crank, pushing the shaft against its bearings. If, now, at the point 2 in the line A B we erect a perpendicular 2 R_2 equal in length to P F, and at the points 1, 3, 4, &c., erect others deduced by the same kind of construction, we can trace a curve through their summits, the height or ordinate of which at any point measures the force turning the crank-pin at the corresponding point of its circumference; and therefore the area or space enclosed between the curve and its base A B, which may be considered to be made up of an indefinite number of these ordinates, measures the total force expended on the crank during a half-revolution. If, now, we take a line G H equal to the diameter of the crank-circle or the stroke of the piston, and, dividing it into any number of equal parts, erect ordinates each equal to P D, and therefore representing the constant force of the piston, the area of the rectangular figure G H K L thus formed will, in like manner, measure the total force of the piston during one stroke. It will be found that the area of this figure is exactly equal to that of the curvilinear figure, as might be predicted from the knowledge of the *universal* mechanical principle, that by no combination of machinery can we create or annihilate force; and that consequently, whatever power the piston during its straight stroke impressed upon the crank, is found in the crank during its circular movement. By altering the length of the connecting-rod, as compared with that of the crank, we alter the figure of the *force-curve*, but we do not change its area; and whether the connecting-rod be long or short, the power conveyed through it during the half-revolution is the same. The other half-revolution being effected under similar circumstances, would give a *force-curve* precisely like that of the former.

Fig. 175.

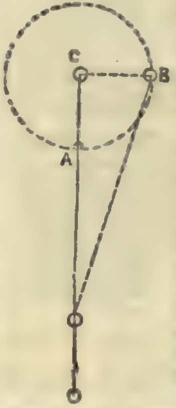


In this investigation we have observed that the oblique action of the connecting-rod causes a lateral thrust on the piston-rod, measured by the line D E. It will be found, that the longer the connecting-rod, the less will this lateral thrust be; and as it is a force not only useless to the machinery, but positively prejudicial, tending to bend the piston-rod or force it out of its straight path, it is advantageous to reduce it to as small a quantity as possible, and to provide means for counteracting its influence. For this reason, a long connecting-rod, three or four times the length of the crank radius at least, should be employed, and the end of the piston-rod should be made to move in guides so as to prevent it from being deflected.

As the piston at the dead centres has no influence in causing the crank to revolve, it is necessary to provide some means of making up for this deficiency. On the crank shaft there is fixed a large heavy wheel, which revolves with it, and acts as a reservoir of force to carry the crank round the dead centres, and otherwise to equalise the movement. A large mass of matter, such as the fly-wheel, weighing sometimes many tons, cannot be put in motion at a high velocity without the expenditure of great force; but when it is in motion it requires as great force to arrest it. In the motion of a crank connected with a fly-wheel, while the crank is receiving its most advantageous impulse from the piston at such points as 2, 3 or 4, it communicates some of its overplus to the fly-wheel, which is there stored up in the form of momentum or active power, to be given out upon the crank and any machinery connected with it, when, at the points 1 and 6, it is receiving no power from the piston.

In cases where a fly-wheel, on account of its weight and bulk, cannot be applied—as in marine and locomotive engines—the engine is made in duplicate, with two cylinders, pistons, connecting-rods, and cranks, both of the cranks being fixed on one shaft at right angles to each other. While the one crank is on its dead centre, as C A (Fig. 176), and receiving no rotatory impulse; the other crank C B is nearly at its best position for receiving the force of its piston.

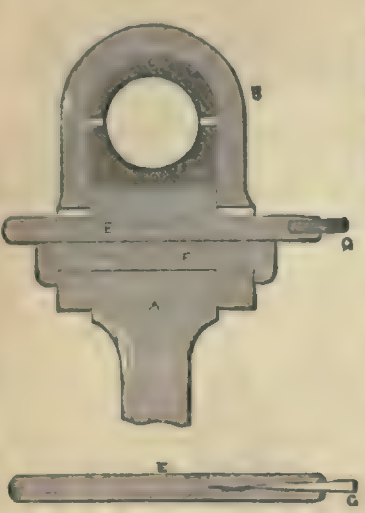
Fig. 176.



As the whole power of an engine has to pass through its connecting-rod, the joints which connect it with the piston-rod and the crank require to be made of great strength, and with precautions against friction and wear. The pins of the crank and piston-rod, on which these joints work, are made of wrought-iron or steel, for the sake of strength; and as the friction of like metals upon each other is found to exceed that of different metals, the eyes at the ends of the connecting-rod are lined or *bushed* with brass, gun-metal, or some soft metal, such as tin alloyed with copper. The special construction of those eyes depends upon circumstances, different engineers having preferences for different forms.

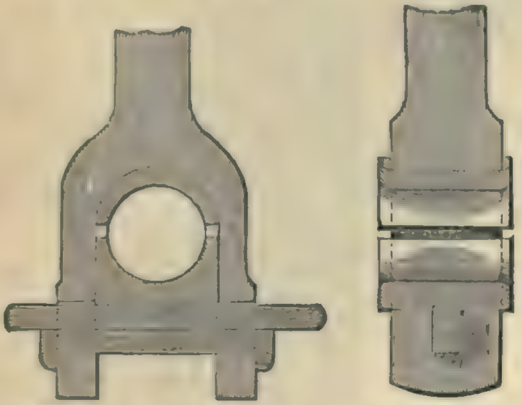
In Fig. 177 is represented what is called the *strap-eye.* The end A of the connecting-rod is squared, so that a wrought-iron strap B, bent to horse-shoe form, can slide on to it. Between the arch of the strap and the flat end of the rod are inserted the upper brass C and the lower brass D, generally made of gun-metal, sometimes lined with soft metal. The brasses have projecting lips or flanges at each side to prevent them from moving sideways within the eye. Through the sides of the strap and the head of the connecting-rod is cut a slot, into which are fitted two pieces of iron, the *key* E and the *gib* F, each slightly tapered, so that when the key is driven gently into its place by a hammer, it acts as a wedge, pulling the strap down, and thereby tightening the brasses on the pin to which they are fitted. A little space is left between the edges of the brasses to permit their closer approach as they or the pin become worn in the hole; and the slot for receiving the gib and the key is extended upwards in the strap and downwards in the connecting-rod, to permit the key to be driven. Lest the key should be shaken loose by the motion of the machinery, it is often split open at the small end, and a wedge G is driven in to spread it laterally.

Fig. 177.



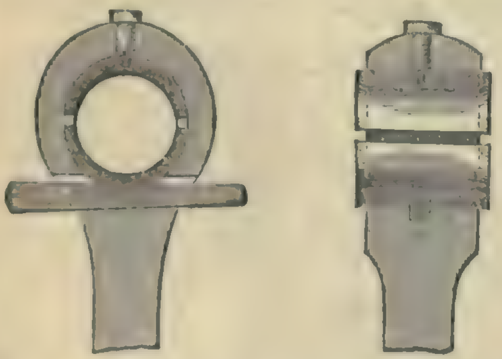
The fork-end (Fig. 178) is made by forming the end of the connecting-rod like a fork, fitting it with gun-metal bushes, and tightening them by means of a gib and key.

Fig. 178.



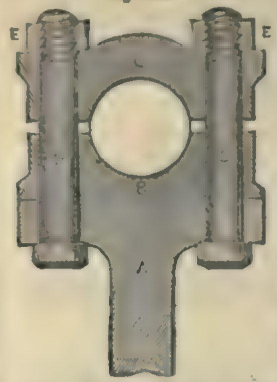
The bushed-eye (Fig. 179) is formed by shaping the end of the rod into an eye, in which gun-metal bushes are fitted, capable of being tightened by a key passing through a slot in the rod, and bearing against the lower bush, which is notched to receive it.

Fig. 179.



Sometimes the eyes are made as represented in Fig. 180. The end of the rod A is spread out to form a flat face, on which are fitted two brasses made with a hole to fit the pin, and secured by means of bolts and nuts D, E, passing through holes in the end of the rod, and in the brasses fitted accurately to receive them.

Fig. 180.



Such are the modes principally used in forming the eyes of connecting-rods, or of any joints through which considerable strain has to be communicated.

The crank A (Fig 181), is made, sometimes, of cast-iron for stationary engines, but, generally, for marine engines, of wrought iron, having a hole fitted to B the round end of the shaft, on which it is fixed, and prevented from turning by a key C, or tapered piece of iron, driven tightly into a slit, formed partly in the crank and partly in the shaft. Generally, the round hole in the crank is made slightly smaller in diameter than the round end of the shaft; the crank is heated so as to expand it, and permit it to be driven on the shaft; and as it cools it tends to contract, and thereby becomes very firmly bound on to the shaft, the end of which is riveted or hammered over the hole. The crank-pin D is made to fit truly into a hole in the crank, in which it is sometimes secured by a key or pin driven through it transversely. The distance E F, between the centre line or axis of the shaft and that of the pin, is called the *throw* of the crank, and it is exactly half the length of the *stroke* of the piston. For locomotive engines, and, indeed, for many of the largest marine engines, where double cranks are required, it is usual to forge both cranks and the shaft in one piece, as indicated in Fig. 182. By this means great strength and simplicity are secured, and all the risk of the loosening of parts, put together by keys or otherwise, is avoided.

Fig. 181.

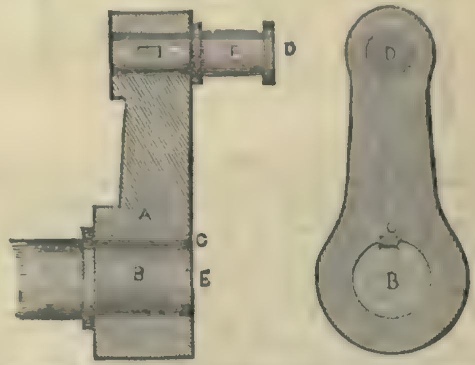
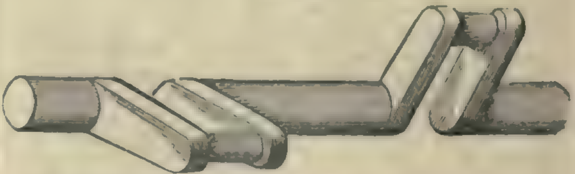
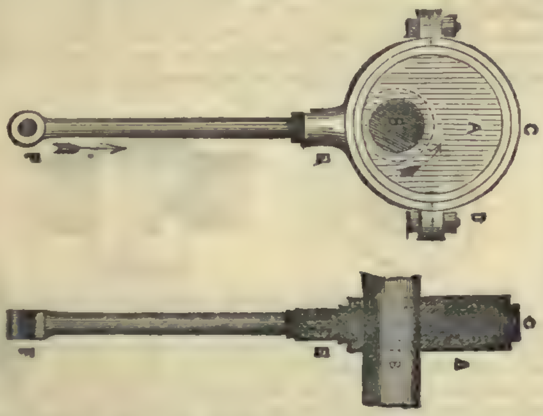


Fig. 182.



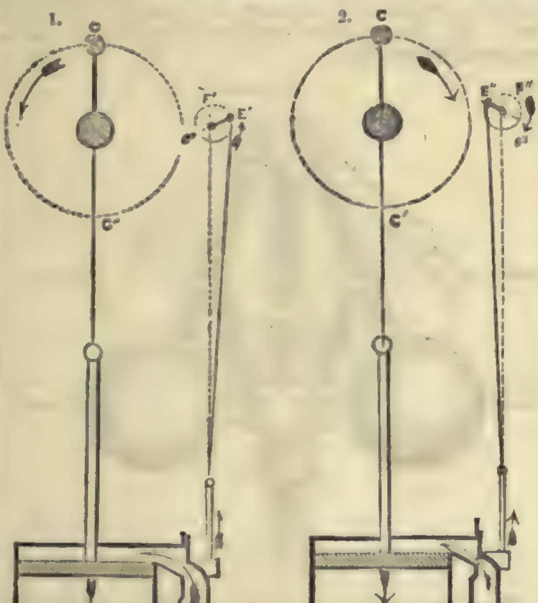
ECCENTRIC.—As, by means of a crank, the reciprocating motion of a piston is converted into a continuous rotary motion of the shaft, so the continuous revolution of the shaft may, by means of a crank, be converted into a reciprocating movement for the slide or feed-pump; but for this purpose, instead of a crank, an eccentric is generally employed. It consists of a circular disc A (Fig. 183), having a hole B, *not* in its centre, through which the shaft passes. Round the disc is fitted a ring C, generally made in halves, secured to each other by bolts and nuts at D; and to one side of the ring is attached the eccentric-rod E F. The eccentric disc, or *sheave*, being firmly fixed on the shaft, is caused to revolve with the latter, and its centre is thus made to describe a circle round the centre of the shaft. The whole sheave thus becomes a crank-pin of extended diameter; and as it can slip freely round within the ring, the end F of the eccentric-rod is caused to move upwards and downwards, during every revolution, through a distance equal to the diameter of the circle through which the centre of the sheave revolves. The radius of this circle is called the *throw* of the eccentric; and its diameter, or the distance through which F is caused to move during a half-revolution, is called the *stroke* of the eccentric-rod.

Fig. 183.



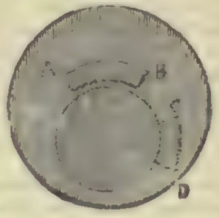
The setting of the eccentric upon the shaft, or the fixing of its position with respect to that of the crank, is a matter of nice adjustment for causing an engine to act well, as we shall endeavour to describe. Let A (Fig. 184) represent the outline of a cylinder and piston, with the ports at one side, and a long D-slide D fitted to them; C the circle in which the crank-pin revolves, and E that in which the centre of the eccentric revolves in connection with it. We suppose the piston at the top stroke, and the crank-pin at C.

Fig. 184.



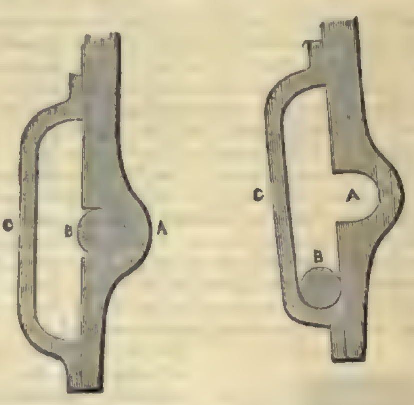
If it be intended that the crank shall revolve in the direction of the arrow in 1, the eccentric revolving in the same direction, the slide should be just opening to admit steam above the piston, and to permit its exit from below it. The centre of the eccentric must, therefore, be at some such position as E′—so that, as it continues to revolve, it may continue to open the slide to steam above and to eduction below for some time, and then be ready at e′ when the crank reaches C′ to reverse the movement. Again, if the rotation of the crank be in the opposite direction, as in 2, the eccentric centre must for the like reason be at E″, a point on the side of the vertical central line opposite to that occupied by E′. In an engine always moving in one direction, the eccentric can be fixed on the shaft in such a manner as to give the proper movement to the slide; but when it is desired that the engine should move in the opposite direction, some expedient must be devised for altering the position of the eccentric on the shaft through an angle measured by twice E′ F′, or E″ F″, so as to throw its centre as much to one side as it was to the other side of the crank-pin. The simplest mode of effecting this object is to make the eccentric sheave loose on the shaft, with a piece A B (Fig. 185) projecting from its face, and a piece, or stop, C D projecting from the shaft. When the shaft moves in one direction, the eccentric remains at rest until the end C of the stop comes against B, when it is caused to revolve with the shaft. But, if the shaft rotate in the opposite direction, it leaves the eccentric behind, until D comes round to A, when it is again caused to revolve, but having its centre changed with respect to any fixed point on the shaft, by a quantity determined by the extent of slip before the opposite edges of the stops come in contact.

Fig. 185.



When this arrangement is adopted, the end of the eccentric-rod is generally made of the form indicated in Fig. 186, having what is called a *gab* or round-bottomed notch A fitted to the pin of the slide-rod B.

Fig. 186.



When the eccentric-rod is pushed or drawn aside, so as to relieve the slide-rod pin from the gab, the slide can be moved upwards or downwards by hand, independently of any motion of the eccentric-rod, and the movement of the steam above or below the piston thus changed at pleasure. C is a guard or stop to prevent the withdrawal of the rod too far from the pin. The slide-rod is generally made capable of being worked by a lever conveniently situated for the hand of the attendant. When he wishes to reverse the motion of the engine, he quickly withdraws the gab from the slide-rod pin, and by means of the hand-lever moves the slide in the direction opposite to that in which it was formerly moved by the eccentric. He thus throws the steam-pressure on the opposite side of the piston—if it were formerly ascending, it begins to descend, and conversely—the movement of the crank and shaft is thus reversed; the stop that drives the eccentric is brought round to the opposite side; and when, after a few alternations of the slide by hand, the reversal of the engine is fairly established, the gab of the eccentric-rod is permitted again to drop over the slide-rod pin, and the contrary motion continues.

When there are two engines working together, such as marine or locomotive engines, the number of operations required for reversal makes it a rather difficult matter. There are two slides, two eccentrics, and two sets of hand-levers; both gabs have to be thrown out of gear, and both levers worked for a time by hand; while great care is demanded on the part of the attendant, lest he work them in such a way as to neutralise each other, or oppose the reversing effect which he desires to produce. To overcome these difficulties, an ingenious arrangement, called the *link motion*, has been introduced in marine and locomotive engines. For each of the engines there are two eccentrics fixed side by side on the shaft, and their rods are jointed to an arc A (Fig. 187) with a circular slot in it, in which the pin B of the slide-rod can freely slide. The two eccentrics are fixed on the shaft in such positions that one is adapted for the motion of the shaft in one direction, while the other suits its motion in the opposite direction. When the rod of the one is nearly in a direct line with the slide-rod, it gives it its reciprocating motion, while the other merely causes the arc to oscillate without affecting the motion of the slide; but when the arc and rods are pulled aside, so as to bring the slide-pin under the other eccentric rod, its motion is given to the slide, and the engine is thus reversed. With such an apparatus, then, one simple movement of a hand-lever, connected with the arcs of both engines, causes their immediate reversal. But this is not the only advantage of the link motion. It will be readily seen that the middle point A of the arc being brought round to the slide-rod pin, the latter will be left nearly at rest; for the opposite ends of the arc being moved nearly in opposite directions by the eccentrics, will merely oscillate round A, as a fulcrum or centre. By bringing the arc to this position the engines are stopped, because the slide being at rest, admits no alternation of steam above or below the piston. Again, by shifting the arc so as to bring the pin to any point on either side of A, more or less movement of the slide in either direction is produced at pleasure, and thus the quantity of steam passing through the ports into the cylinder may be varied, and consequently the speed of the engine, according as the slide is caused to expose a greater or less amount of opening at the ports for its admission.

Fig. 187.

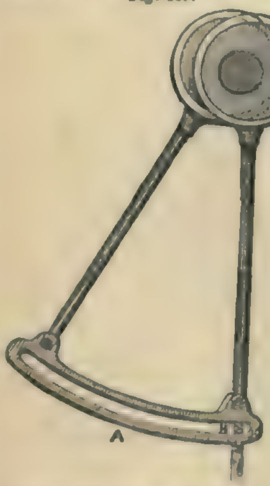
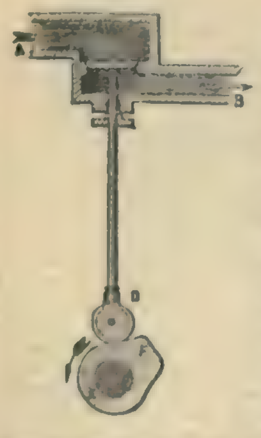
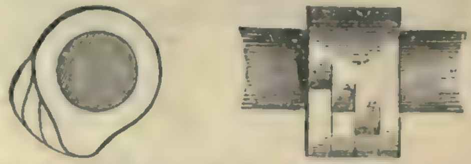


Fig. 188.



CAMS.—When valves are used instead of the slide, for alternating the flow of steam to the cylinder, it is not unusual to move them by apparatus called *cams.* A (Fig. 188) is a pipe leading from the boiler, B a port to the cylinder, and C a valve, either simple or of the double-beat kind, closing the passage from A to B. The valve-rod CD, passing through a stuffing-box in the cover of the valve-box, terminates in a roller D, which bears upon a cam F fixed on a shaft E, caused to rotate by the engine. This cam is a disc partly circular, and with part of it, F, projecting to a greater distance from the centre. As long as the roller D bears upon the circular portion, the valve C remains down upon its seat; but as the projecting part of the cam is brought by its revolution under the roller, the valve-rod is pushed up, and the valve lifted to permit the passage of the steam. By varying the extent of the projecting part, the valve can be kept open during a greater or less portion of a revolution of E. When it is desirable to vary this period, the cam is sometimes made with numerous steps of various extents, on any of which the roller may be made to bear at pleasure (Fig. 189).

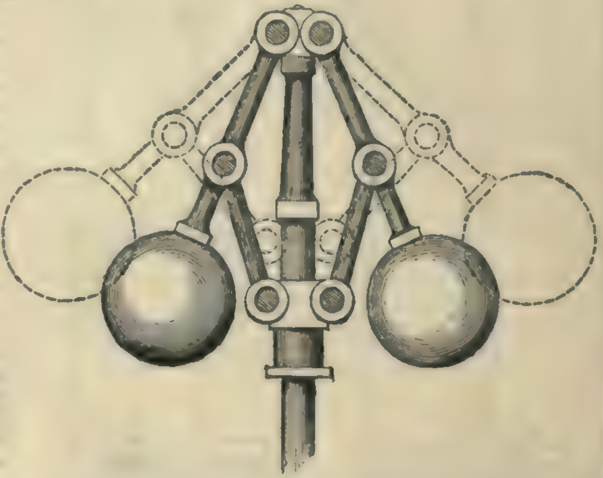
Fig. 189.



For engines revolving slowly, this mode of working valves is very valuable, as it gives the power of admitting steam to the cylinder during a less or greater part of a stroke, and thereby of taking advantage of its expansive power within the cylinder. But when the rate of revolution is rather rapid, it is difficult to make the cam and roller capable of working without noisy and injurious blows, resulting from their rapid alternations.

GOVERNOR.—In stationary engines, which are subjected to continual variations of work, and yet are required to move with great regularity, it is essential to provide some means of governing the speed. The most simple and efficacious apparatus for this purpose is the conical pendulum or governor, invented by Watt (Fig. 190). We have already described it in general terms as applied to windmills.

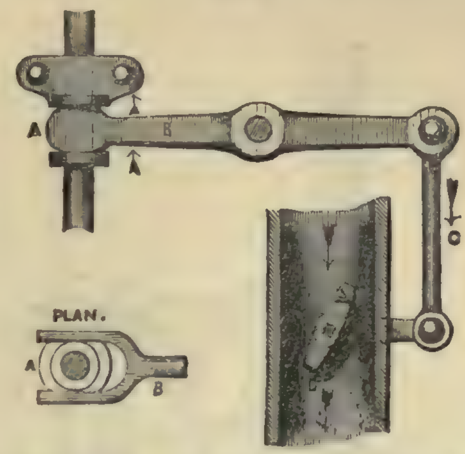
Fig. 190



For steam-engines it is used in a similar manner. The vertical spindle is put in motion by the engine, and revolves quickly or slowly, according to the velocity of the engine. When it revolves rapidly, the balls fly outwards, and raise the grooved brass which slides on the spindle A (Fig. 191), and thus moves a forked lever B, which, by proper rods and levers C, causes the throttle-valve D to turn round in the steam-pipe, and check the passage of the steam to the cylinder. When the engine revolves slowly, on the other hand, the balls fall in, the brass sinks, and the throttle-valve is presented edgeways to the steam, and permits a more free passage.

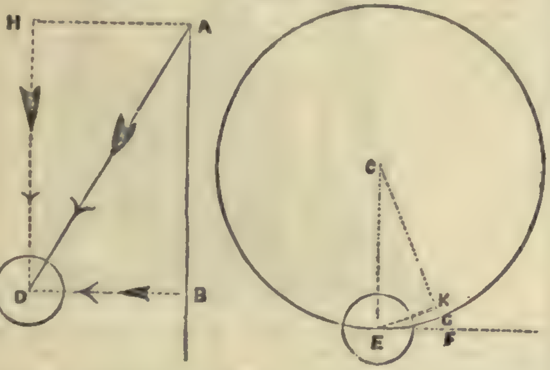
As, in the pendulum of a clock, the length from the point of suspension to the bob must be regulated to beat seconds, or half seconds, or any other intervals that may be required; so in the conical pendulum or governor,

Fig. 191.



the length of the arms that carry the balls must be regulated by the speed at which they revolve. In discussing this question, we need only consider one ball, as each is regulated by the same law, and they are generally made in duplicate for the sake of balancing the apparatus, and to give it symmetry. The force which tends to throw the ball outwards from the vertical spindle, is the centrifugal force of its revolution, or its tendency in obedience to the first law of motion to proceed in the straight line E F (Fig. 192), touching the

Fig. 192.



circle in which it revolves, rather than to be continually diverted from its straight to a circular path. The force which opposes the centrifugal force, and causes the ball to be continually deflected from the straight line, is the weight of the ball tending to push it close home to the vertical spindle. If we take any position of the ball, such as D, A B being the vertical spindle, and complete the parallelogram of forces A B D H, while the line D H or A B represents in quantity and direction the weight of the ball, drawing it down, and D B or A H the centrifugal force pushing it outwards, A D represents the resultant of those two forces as a tension on the rod by which the ball is suspended. If we know the velocity with which the ball moves in its path, and the radius of that path, we can estimate its centrifugal force in comparison with its weight, and can make the limb A D of such a length that these two forces shall be properly proportioned for a certain velocity.*

The length of the arm of the governor, measured from the point of suspension to the centre of the ball, may be found from the following rule:—

Divide 200 by the number of revolutions per minute, and multiply the quotient by itself for the length in inches.

Example.—Required the length of a governor arm suited to 50 revolutions per minute:

$$\frac{200}{50} = 4, \text{ and } 4 \times 4 = 16 \text{ inches.}$$

To find the speed suited to a governor of a given length of arm.

Divide 200 by the square root of the length (in inches), and the quotient will be the number of revolutions per minute.

Example.—Required the proper speed for a governor having an arm 16 inches long.

The square root of 16 is 4, and $\frac{200}{4} = 50$ revolutions per minute.

The rods and levers connecting the governor with the throttle-valve should be capable of adjustment, and it is useful to have an adjusting counterbalance to the centrifugal force of the balls, by changing which, the regulated speed of the engine can be varied at pleasure. Although the governor we have described is a most valuable accession to an engine, yet it is not a perfect regulator; for its very mode of action implies that the velocity of the engine must have undergone a change, before the governor can have begun to act on the throttle-valve. But within certain limits, the variation in speed of an engine thus regulated is inconsiderable, and there is no appa-

---

* E G being part of the circular path, which may be taken as small as we please, and E K perpendicular on C F, F K is nearly bisected in G, and E K is nearly equal to E F, and K F nearly parallel to C E. Also $CE : EF :: EK : KF = \frac{EK \cdot EF}{CE}$, or nearly $\frac{EF^2}{CE}$.

A body projected from E with such a velocity as would in a small period $\triangle t$ cause it to describe E F, if acted on during that period by a constant deflecting force parallel to C E, giving it during $\triangle t$ a velocity which would cause it to describe uniformly F K in the time $\triangle t$, would be deflected through F G, the half of F K in the time $\triangle t$.

If f = the deflecting force (measured by the velocity acquired in one second), $f \triangle t^2$ is the space traversed during $\triangle t$ at the velocity acquired during $\triangle t$, and if v be the velocity per second of the body in E F, and therefore $EF = v \triangle t$, and $r = CE$, the radius (measured in inches),

$$f \triangle t^2 = FK = \frac{EF^2}{CE} = \frac{v^2 \triangle t^2}{r} \therefore f = \frac{v^2}{r}$$

Taking n = the number of revolutions per minute, $\frac{n}{60}$ = number per second, and $2 \pi r$ = the circumference traversed in each revolution, $\therefore v = \frac{2 \pi r n}{60} = \frac{r n}{9{\cdot}545}$

Hence, $f = \frac{r^2 n^2}{(9{\cdot}545)^2 r}, = \frac{r n^2}{91{\cdot}1 \text{ \&c.}}$ very nearly.

The force of gravity is measured by a velocity of $32\frac{1}{2}$ feet or 385 inches acquired per second, and taking w = the weight of the body, $f : w :: \frac{r n^2}{91} : 385$

$$\therefore f = \frac{w r n^2}{91{\cdot}1 \text{ in.} \times 385} = \frac{w r n^2}{35344} \text{ very nearly.}$$

Now, as f is represented by B D, while w is measured by A B, and as $BD = r$

$$AB : BD :: w : f :: w \cdot \frac{w r n^2}{35344}$$

$$\text{or } AB = \frac{35344 \, BD}{n^2 \quad BD} = \frac{35344}{n^2} = \left(\frac{188}{n}\right)^2$$

It is convenient in practice to make the angle B A D about 30° when the governor is at its average speed, when $AD = 2 DB$; and as $AD^2 = AB^2 + DB^2$, we find

$$AD = \frac{2}{\sqrt{3}} \left(\frac{188}{n}\right)^2 = \left(\frac{200}{n}\right)^2 \text{ very nearly.}$$

ratus so simple and durable, that is capable of maintaining like regularity.

Fig. 193.

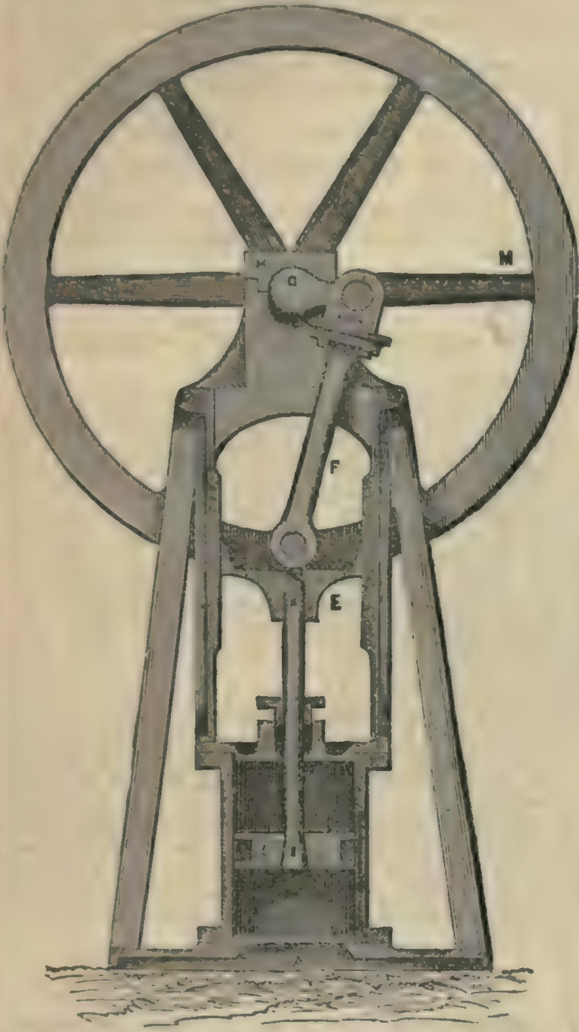
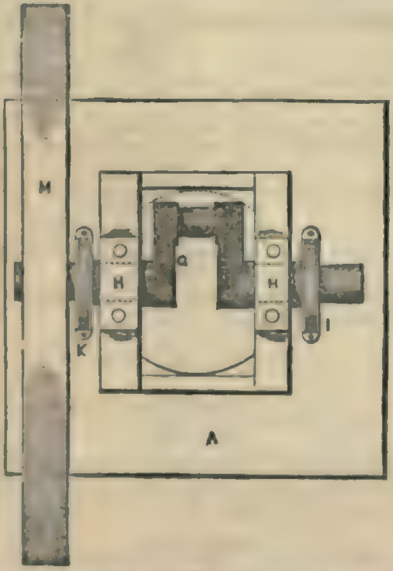
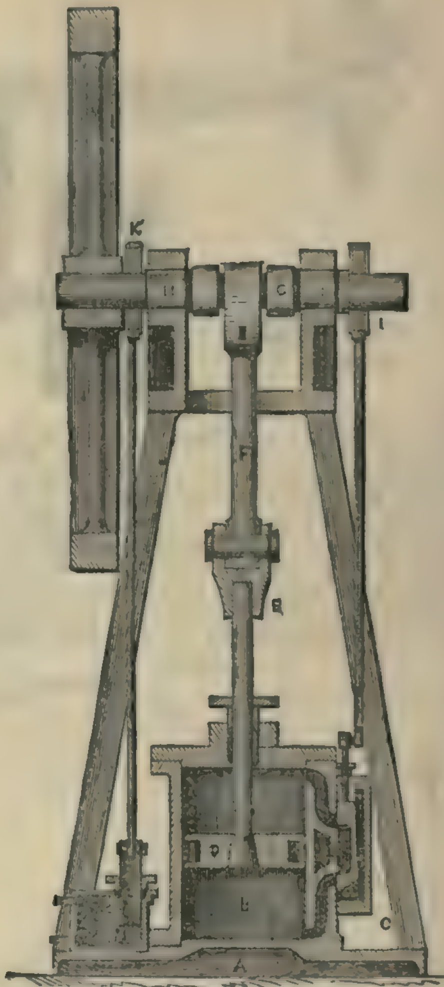


Fig. 195.



194.

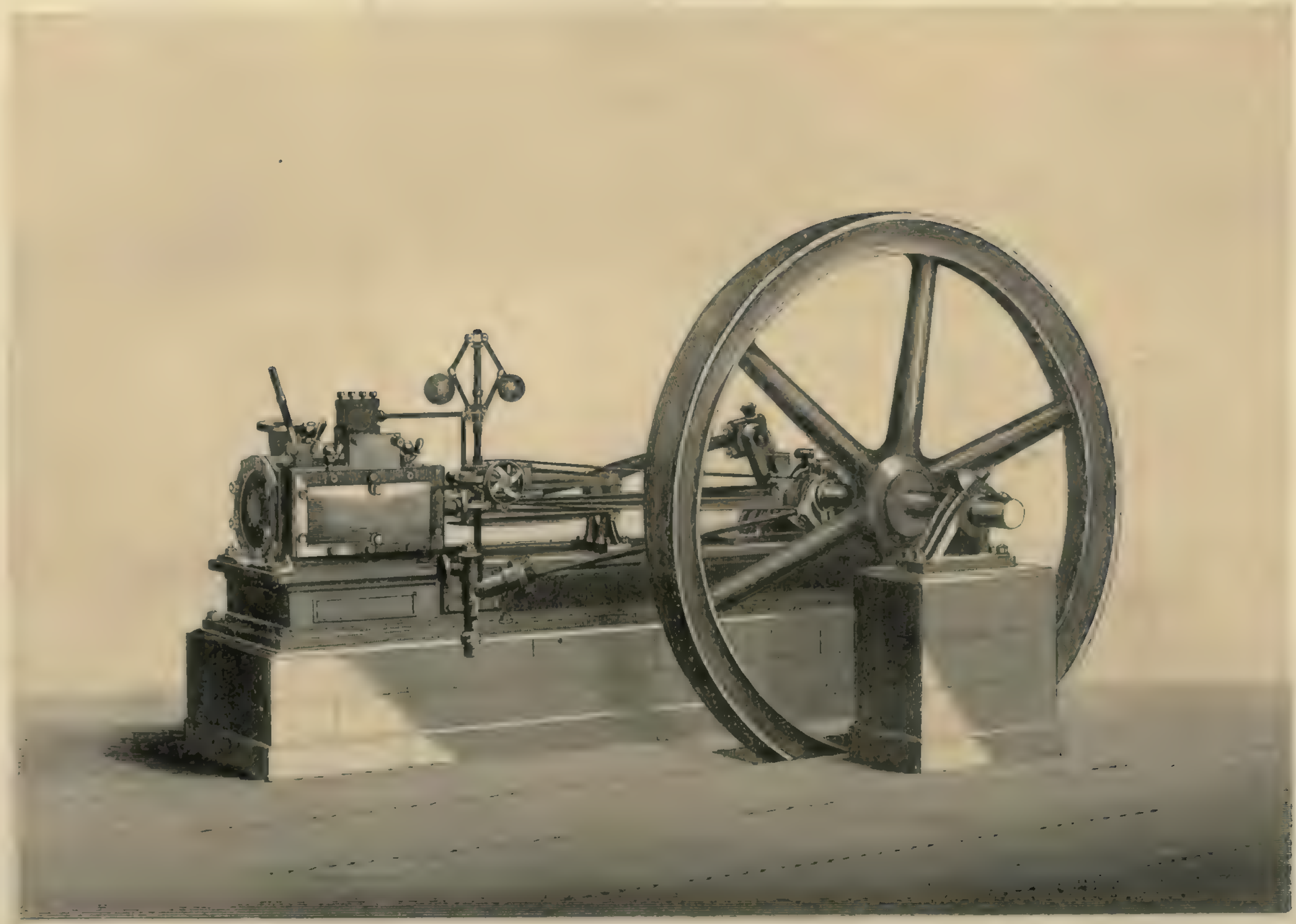


Another kind of governor has been applied, not without success. The lever of the throttle valve is connected by a rod with a small piston working in a cylinder; the engine puts in motion a small pump, forcing into this cylinder air or water, which escapes from it by an aperture, the size of which can be regulated at pleasure. If the speed of the engine increase, the small cylinder is filled more rapidly than it can empty itself, and the piston thus raised closes the throttle-valve. Should the engine move more slowly, the small cylinder becomes emptied more rapidly than it is filled by the pump; and the piston descending, opens the throttle-valve so as to admit more steam to the main cylinder.

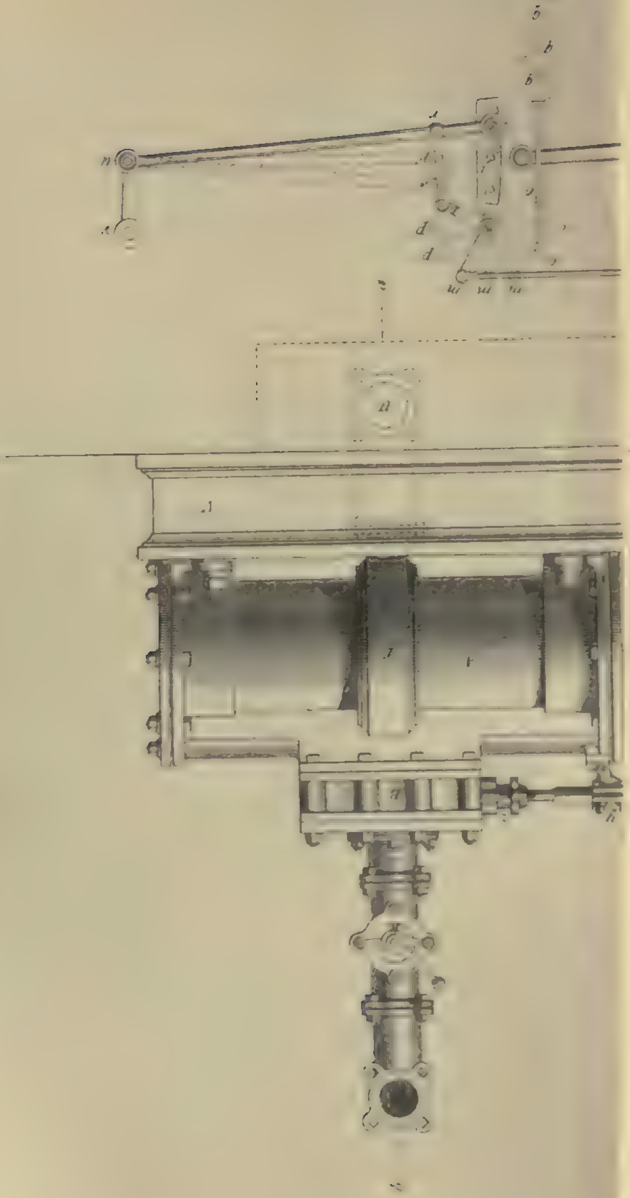
Other contrivances for governing the speed of engines have been applied with success. Among them we may mention the chronometric governor, of which our limited space does not permit us to give a detailed description.

Having now described separately the principal parts of which a non-condensing engine consists, we will discuss some of the modes in which these parts can be most conveniently arranged.

DIRECT-ACTING ENGINES.—Figs. 193 and 194 are sections, transverse to each other, of a vertical, direct-acting non-condensing engine; and Fig. 195 a plan of the same. A is the foundation-plate, forming the bottom of the cylinder B, which is secured to it by bolts and nuts. C, the slide and steam passages. D, the piston and rod, terminating in a cross-head E, which works between guides, and to which is jointed the connecting-rod F. G, the crank; and H H, the bearings in which the crank-shaft revolves. I, the eccentric and rod for working the slide; and K, an eccentric for working the feed-pump L. M, the fly-wheel. This engine is



[illegible] 15 HORSE POWER HORIZONTAL STATIONARY [illegible] STEAM ENGINE



lever, *m*, which
slotted lever, M,
the central pin,
ending a weight,
ended at *r*, by a
emity, *p*, of the
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carrying a fly-
-pump. The
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of very simple construction, and has no parts that are unnecessary to the efficient working.

The annexed plate represents a horizontal direct-acting non-condensing engine; and as this form of engine has of late years been very extensively used, we think its details may be studied with great advantage. The principal objects attained by adopting the horizontal arrangement are these:—

The framing (marked V V on the plate) is of a very simple character, and can generally be cast all in one piece.

The whole strain caused by the alternate pulling and pushing of the piston in giving motion to the crank, is thrown rectilineally on the framing, which can be readily made strong enough to bear it, and which, being subjected to no racking strain, is laid upon a bed of masonry, and secured in its place by a few bolts.

The whole machinery of the engine being spread out as it were under the eye, is visible and accessible in every part for cleaning, oiling, and repairs, and is in itself so simple that great economy of construction is secured, along with efficiency and durability.

The plate gives an external side elevation, in which—A is the cylinder surmounted by the slide jacket, B; C is the piston rod, which, attached at one end to the piston inside the cylinder, passes through a stuffing-box, O, in the cylinder cover, and terminates in a cross-head and pin, D, on which is jointed the connecting rod, E, by means of a strap-joint, as shown in the plate, or any other of the joints already described, that may be found most suitable. As the connecting rod, by its oblique action on the crank, throws a considerable lateral strain on the head of the piston-rod, tending to push it upwards or downwards, out of its direct line of action,* the cross-head, D, of the piston is made to slide between guides, N N, which are nicely smoothed and kept well lubricated.

F is the crank fixed on the main shaft, and to which the connecting-rod, E, is jointed by a strap-joint. The shaft revolves in bearings, one of which is fixed on the engine frame, and is generally made oblique (as indicated by the dotted lines on the plate), because the action of the piston being in the horizontal line, if the bearing were jointed horizontally, the strain would be thrown on the joints of the bushes instead of their solid bodies. The outer end of the shaft revolves in a bearing or plummer-block, which is fixed on a wall or other suitable masonry at the side of the engine. The fly-wheel, H, is fixed on the shaft, and revolves with it. A small crank, I, is mounted on a small shaft, J, concentric with the main shaft, in order to give motion to the governor-spindle, G, and to the eccentric, K, which regulate the supply of steam to the cylinder.

The governor and eccentric gear are connected by an ingenious arrangement, which we will endeavour to describe. From the eccentric, K, extends a rod, L, to the joint, M, of a lever, which vibrates on a pin, *n*. This lever contains a slit (which is part of a circular arc), in which slides a pin, *s*. A rod, *t u*, extends from this pin to the extremity of a lever, which vibrates on a spindle, *v*, and which has its other extremity, *w*, connected by a rod, *w x*, to the slide-rod. The slide-rod moves rectilineally through a guide, *y*, and through a stuffing-box, *z*, and puts the slide into alternating motion within the jacket, B, so as to admit steam to the opposite ends of the cylinder, and permit its egress from these ends successively.

The governor, which is mounted on a vertical spindle, G, is put in motion through bevil gearing, driven by the small spindle, J. The governor-balls, *e e*, are attached to equal rods, which work on pins mounted on the head of the governor-spindle, *d*. From these rods two bent rods extend downwards, and are jointed to a tubular boss, *f*, which slides upwards or downwards on the governor-spindle, according as the balls fly out from or fall in towards its axis. This boss, *f*, has a groove cut into it, which receives the ends of a forked lever, *g*, mounted on a fixed pin, *h*. A lever at right angles to this, terminating in a pin, *k*, communicates motion by the rod, *l*, to the extremity of another lever, *m*, which is mounted on the same pin, *n*, with the slotted lever, M, already described. From each side of the central pin, *n*, arms extend to *o* and *p*, the one suspending a weight, *q*, balancing the rod, *t u*, which is suspended at *r*, by a small vertical rod from the other extremity, *p*, of the double-ended lever. In order to understand the action of this apparatus in governing the engine, we shall first suppose that it is going at its proper speed, so that the governor-balls, *e e*, maintain their mean distance from the axis, G, neither flying out from it, as they would do if they revolved too fast, nor falling into it if they revolved too slowly. Under these circumstances, the eccentric-rod, M, works directly on the pin, *s*, and through it, and the rod, *t u*, and lever, *u v w*, on the slide-rod, *x*, giving the slide its mean amount of stroke backwards and forwards, so as alternately to uncover the steam ports to the extent necessary for the admission of such a quantity of steam into the cylinder as is required for working the engine at the speed to which it is timed. Let us now suppose that some additional work is thrown on the engine, more than the steam allowed to it is capable of performing. The first and immediate result of this is a reduction of speed.

The governor-balls, *e e*, no longer maintained in their position by their former centrifugal force, sink inwards towards the axis G; the tubular boss, *f*, is thus caused to slide down the axis, the bell crank lever, *g h k*, pushes the rod, *l*, towards the cylinder, and thereby throws the double-ended lever, *m o p*, into the position represented on the diagram (Fig. 2). The effect of this movement, by dropping the point *r*, is to let the pin, *s*, slide down the circular slot of the lever to a point farther from its centre, *n*. As the eccentric-rod, L, maintains the same amount of stroke as before, the pin, *s*, being removed farther from the fulcrum, *n*, of the lever, receives a more extensive vibration, which it gives to the slide-lever and rod, and the slide itself is thus caused to move over a larger extent of the facings within the jacket, B, and so to open the steam ports more widely, and admit into the cylinder the increased amount of steam required for the increased work thrown on the engine. If, on the other hand, work were thrown off the engine, the speed would increase, the governor-balls would fly outwards, their operation on the slotted lever would be reversed (as marked by the dotted lines in Fig. 2), and the stroke of the slide and supply of steam would be proportionally diminished, to suit the reduced work which the engine might have to perform.

P is the steam-pipe leading from the boiler to the cylinder; R the shut-off valve for stopping the passage of the steam, or regulating the quantity supplied; S a short piece of connecting-pipe, which may be removed by undoing the bolts, by which it is connected to the valve and to the slide-jacket, so that room may be afforded for lifting the slide-jacket itself off the cylinder without disturbing the steam-pipes, when it may be necessary to repair the slide or adjust the facings on which it works. The eduction passage is carried by a tubular jacket T round the cylinder to a convenient place, U, below the framing, whence the waste steam may be conducted away. The feed-pump X is placed in an excavation made to receive the lower portion of the fly-wheel. It is worked by an eccentric from the main shaft acting through the rod W, and draws by the pipe Z its supply of water, which, after passing through valves situated in the lower part of the pump X, is forced through the pipe Y to the boiler, and supplies the feed required for the generation of steam to work the engine.

Figs. 196 and 197 are views of what is called a table-engine. The cylinder is elevated on a table; the piston-rod terminates in a cross-head, having a roller at each end working in a guide, and a connecting-rod descending on each side of the cylinder to a shaft made with two cranks below, working in bearings, and carrying a fly-wheel and eccentrics for the slide and feed-pump. The governor is driven by bevil gearing from the crank-shaft.

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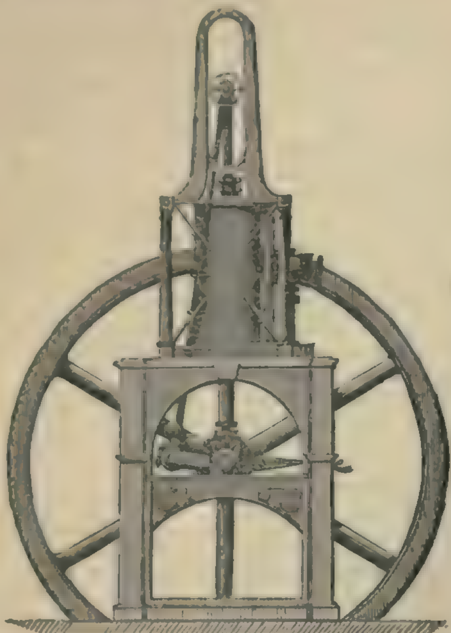
* See *ante*, p. 859.

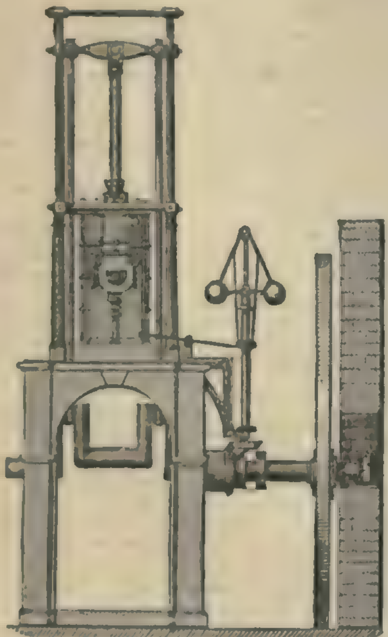
In the locomotive engine, the action is precisely similar to that of the direct-acting vertical engine, although the cylinder is laid horizontally, and the framing is combined in such a manner with the boiler, as to secure strength and lightness.

Fig. 196.



The three annexed plates represent respectively a side elevation, a plan, and a partial front elevation and section of Crampton's Locomotive. We omit a special description of these, because the drawings, if carefully examined, are sufficiently explanatory of the uses and connections of the various parts. Other locomotives differ somewhat in the details, both as to arrangement and proportions, but all embody the same principles of construction.

Fig. 197.



Of late years portable or locomoble engines have attained great importance, and likewise traction engines, or engines suited for travelling and drawing heavy loads on common roads. All these have so many features in common with the locomotive—such as tubular boilers, and cylinders placed horizontally, or at a slight angle to the horizon—that their construction may be readily understood without special drawings or description being necessary to illustrate it. Before quitting this portion of our subject, we may also refer to a particular form of engine, which has been largely adopted where economy of weight and of space is desirable :—we mean oscillating engines. At first these were made on a small scale suited for such boats as those which ply on the Thames ; but of late years they have been employed on a very large scale, and with success. The paddle-engines of the *Great Eastern* are of this kind, and many other large vessels are now fitted with engines of like construction. The cylinder of an oscillating engine, instead of being fixed firmly on its bed, is provided with trunnions at each side, about midway in its length, somewhat like a cannon, and these trunnions are fitted into bearings formed in the framing, in such a manner that the cylinder is free to oscillate upon them as on pivots. The piston-rod which passes through a long and strong stuffing-box on the cylinder cover, is made of larger diameter than usual, for the sake of giving it great strength transversely ; and it is jointed directly on to the crank without any intervening connecting-rod. As the crank-pin travels round in a circle, it carries with it the top end of the piston-rod, not only in a direction upwards and downwards (co-incident with the axis of the cylinder), but also from side to side (or transversely to the axis). The motion of the piston, from end to end of the cylinder, accords with the one direction of the crank's motion, and the oscillation of the cylinder on its trunnions allows for the other. As the steam has to be admitted to the oscillating cylinder, and has also to escape from it after having done its work, the trunnions are made hollow, and form passages for the steam to and from a jacket or steam space provided on the cylinder, whence it passes by suitable parts, commanded by a slide, to and from the opposite ends of the cylinder successively. For several years after the invention of this arrangement, it remained comparatively idle, because of the mechanical difficulties that were experienced in properly working the slide on an oscillating cylinder, itself oscillating with it. At length an ingenious arrangement for overcoming these difficulties suggested itself, and shortly afterwards the use of oscillating cylinders became very general on the small scale, and gradually came to be adopted for the very largest marine engines.

ESTIMATION OF POWER.—In estimating the power of a non-condensing engine, we have to ascertain the pressure on the surface of the piston causing it to move, and the distance passed over by it under this pressure in a given time, or the velocity with which the pressure acts. The product of the pressure by the velocity gives the power ; and if the former be taken in lbs., and the latter in feet per minute, we have to divide their product by 33,000 to reduce it to the standard of horse-power. For example, if the piston move over 200 feet every minute under an average pressure of 1,650 lbs. on its whole surface, the effect is the same as if 1650 × 200 = 330,000 lbs. moved over 1 foot per minute ; and as 1 horse-power is reckoned equivalent to 33,000 lbs. moved over 1 foot per minute, the power in the case assumed is $\frac{330000}{33000} = 10$ horse-power.

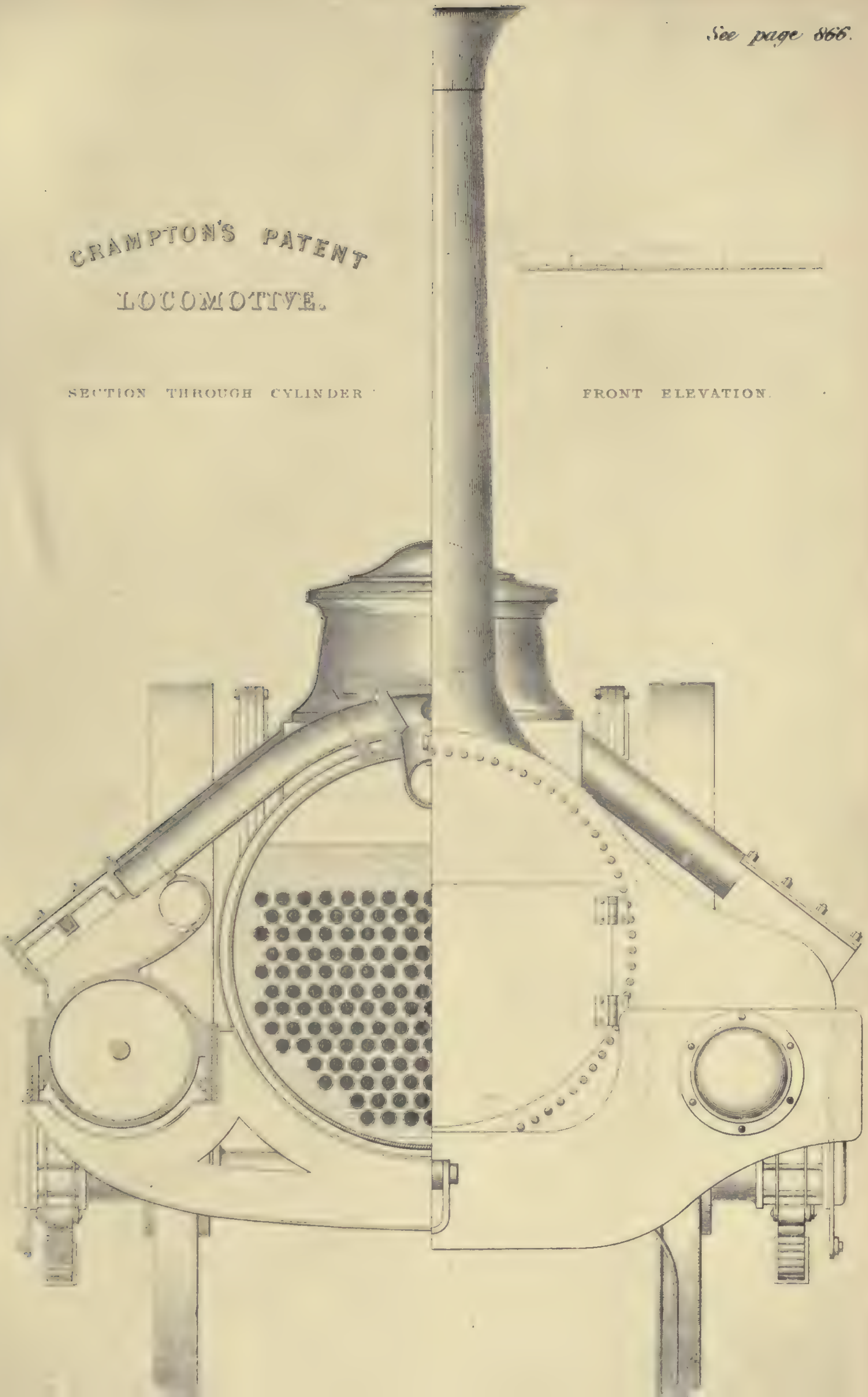
But in estimating the pressure on the piston, we are not entitled to assume it as that of the steam in the boiler, as measured by a steam-gauge. In the first place, there is always some length of pipe between the boiler and the cylinder, and the steam passing through this pipe loses some portion of its heat. Referring to the table of temperatures and corresponding pressures, we can see that the reduction, by a very few degrees of the former, produces a very considerable diminution of the latter. While it is usual to cover the steam-pipe with felt, or some such non-conducting casing, in order to prevent a loss of this kind as far as possible ; yet with

See page 866.

CRAMPTON'S PATENT LOCOMOTIVE.

SECTION THROUGH CYLINDER

FRONT ELEVATION.



every precaution, there is a diminution of pressure to the extent of several pounds per square inch, especially when the initial pressure is considerable.[*] Moreover, before the steam enters the cylinder, it has to pass the throttle-valve, which may be partially closed by the governor, and the steam is thus, as it were, *wire-drawn*, or made thinner, less dense, and consequently capable of pressing with less force on the piston. Were the piston at rest, however small might be the passage for the steam, yet, like water finding its level, its pressure would very quickly become uniform throughout every cavity to which it might have access. But as the piston is in motion, the steam has to flow along the pipe and passages with sufficient rapidity to follow up the piston in its progress; and if its course be arrested or impeded, that portion of it beyond the impediment must necessarily be of less density than that before it. Again, as the alternate flow of the steam above and below the piston is controlled by means of valves or a slide, the apertures covered by them cannot be opened or closed instantaneously, and there must, therefore, be at every alternation, moments of transition, during which the passages are throttled and the steam wire-drawn. When the slide is moved by an eccentric, the opening and closing of the ports is gradual, and the amount of passage open for the steam is continually changing. If the slide and eccentric be so adjusted that the upper port is just beginning to open when the piston is at the top, it continues to open for some time during the descent of the piston, until it attains its extreme width. It then begins to close; and some time before the piston reaches the bottom it must be closed by the slide, which has moved onwards a sufficient distance to be ready for opening the bottom port to admit the steam below the piston. The slide and eccentric are generally so adjusted that each port is closed when the piston has passed through $\frac{2}{3}$rds of its stroke. During the remaining $\frac{1}{3}$rd of the stroke, the pressure on the piston must therefore gradually diminish, for no fresh steam being admitted into the cylinder, that which is already in it becomes expanded in volume, and proportionally diminished in density and pressure. Moreover, when the double eccentric and link motion are employed to work the slide, or when a special valve is provided for cutting off or arresting the ingress of steam to the cylinder at an earlier period of the stroke than that which is determined from the motion of the slide alone, the engine is said to be worked expansively, and the gradual diminution of pressure during the stroke becomes still more marked.

Again, after the piston has completed its stroke, the cylinder being filled with steam of such final pressure as may result from the causes we have named, has to empty itself on the return of the piston, through the ports and waste-pipes. To move the steam through these passages demands some force, which acts as a back pressure or resistance on the piston, and thereby diminishes its effective working force.

Having in view, then, all these causes of change in the steam-pressure during each stroke of the piston, it becomes important to ascertain what is the mean or average pressure throughout, which may be reckoned as the working pressure, or the actual force applied to each part of the surface of the piston to move it against the resistance of the machinery on which it acts.

THE INDICATOR.—The *indicator*, to which we have already referred as having been invented by Watt (see p. 847), is a simple and beautiful instrument, by which this element of power can be ascertained with the greatest accuracy. A (Fig. 198), is a small cylinder open at top, fitted with a piston, and communicating by a pipe and stop-cock, with either the upper or lower part of the main cylinder. The piston is pressed down into the cylinder by a nicely adjusted spiral spring, and a pencil B is fixed to the piston-rod. C is a roller, round which a piece of paper is wound; on the axis of this roller is fitted a pulley D, connected by a string with some of the moving parts of the engine. The roller is also fitted with a spring, like the mainspring of a watch, in such a manner that, after being pulled round in one direction by the motion of the engine communicated through the string, it is made to recoil by the spring. If we suppose the stop-cock closed, the piston, being pressed on by the spring and the atmosphere, will remain at the bottom of the cylinder; and the pencil being stationary at its lowest point E, while the roller is made to rotate backwards and forwards, will describe a line on the paper which would appear straight on its being unfolded from the roller. But if, while the roller continues its motion, the stop-cock be opened, then the piston will be subjected to the pressure of steam in the main cylinder, and will be forced upwards in opposition to the pressure of the spring and of the atmosphere, and the pencil will trace a line on the paper, varying in height as the piston rises and falls. But farther, if the spiral spring be so adjusted that we know exactly how many pounds will compress it an inch, and if we know the area of the piston, we can, exactly, measure the amount of pressure on it by the height to which the pencil is raised above the neutral line E, where it remains when subjected to no upward pressure. And thus the position of the pencil on the paper, or the mark left by it at any point, furnishes the measure of the pressure on the main piston of the engine at the corresponding point of its stroke. On unfolding the paper from the roller, we should find a figure (Fig. 199) described on it by the pencil, which, when properly analysed, gives us the means of reckoning the varying pressure on the piston, and often points out defects in some of the adjustments, and suggests modes of remedying them.

Fig. 198.

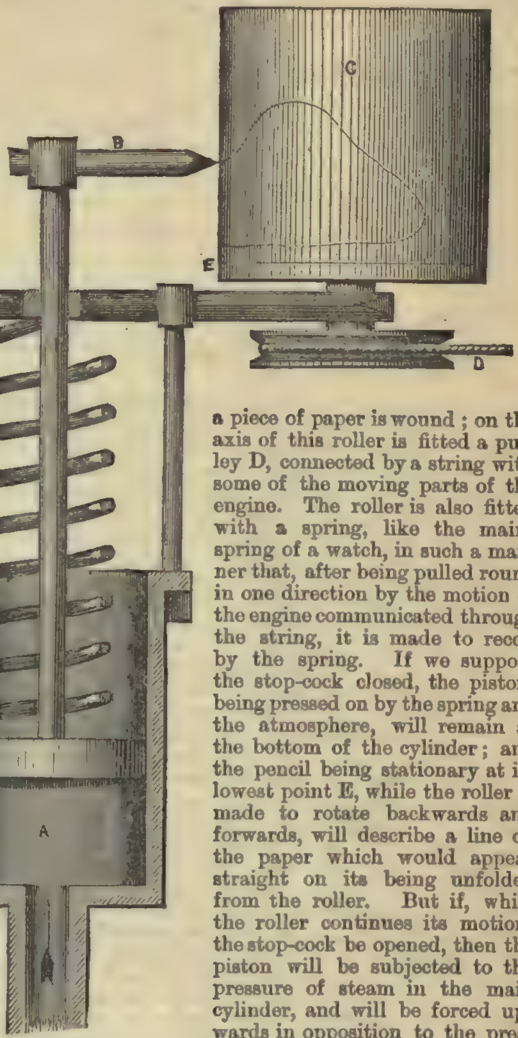
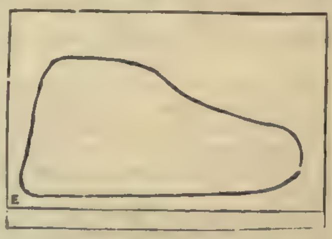


Fig. 199.



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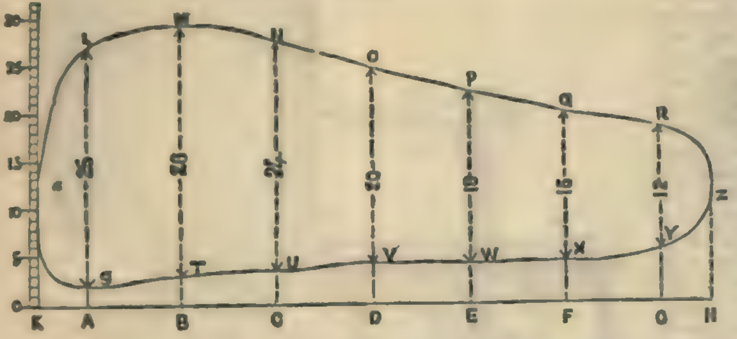
[*] We shall afterwards have occasion to describe the means of remedying this evil by super-heating the steam before it passes to the cylinder.

If we suppose that the area of the indicator-piston is 1 square inch, and that the spring is adjusted so that it requires a force of 10 lbs. to compress it 1 inch in length, or 1 lb. to compress it $\frac{1}{10}$th of an inch, we can form a scale of tenths of inches, and apply it to the indicator diagram at a number of points, A, B, C, &c. (Fig. 200), equally distant; and measuring off the lengths of ordinates A L, B M, &c., drawn through these points, we can thus estimate the pressures acting on the piston of the indicator-cylinder at equidistant points of the stroke through which the paper is made to travel. These pressures correspond exactly, per square inch, with those to which the main piston of the engine is subjected during its stroke, because the small cylinder of the indicator communicates freely with the cylinder of the engine. If we suppose the indicator to be fixed at the top of the cylinder, the upper part of the curve *a*, L, M, &c., R, Z, is that traced during the descent of the piston when the steam is pressing on it. The lower part of the curve Z, Y, &c., S *a*, is that traced during the ascent of the piston when the steam is escaping from the cylinder. Were the indicator fixed to the bottom of the cylinder, we should get corresponding curves for the steam-pressures there. Generally, when the eccentric-slide and its levers are properly adjusted, these figures are very nearly alike; and, if so, the upper part of the curve may be taken as that traced by the active pressure either above or below the piston, while the lower part of the curve may be taken as representing the corresponding resistance of steam during its egress from the cylinder. Now, as the total height C N of any ordinate, measures the total pressure on one side of the piston when it is at the point of its stroke corresponding to C; and as the part C U of the same ordinate represents the resisting pressure on the opposite side of the piston at the same point of its stroke; the difference U N, or the part of the ordinate intercepted between the upper and lower limbs of the curve, measures the effective pressure on the piston clear of all resistance. The same applies to all the ordinates; and, as we may suppose, the whole curved space made up of numerous equal, narrow, vertical strips, each measured in height by an ordinate, we may reckon the area of the figure contained within the curve as an expression of the power developed by the piston during its stroke. Or, having taken a considerable number of these pressure-ordinates, and found their average, we may consider this the mean effective pressure on the piston. For example, the average of those marked in the figure, found by adding them into one sum, and dividing it by their number, is $\frac{140}{7} = 20$ lbs., the mean effective pressure on every square inch of the piston. In taking the average in this way, the most correct method is to take the first and last ordinates, A L and G R, at distances, K A, H G, from the respective ends of the stroke *a* and Z, half of A B or B C the distance which separates the other ordinates.

Fig. 200.



If we suppose the engine from which this figure was taken had a cylinder 12 inches in diameter, a stroke of 15 inches, and that the crank made 80 revolutions per minute, we can readily calculate the effective power of the engine thus:—Each revolution of the crank requires an up-stroke and a down-stroke of the piston, or a travel through twice 15 inches, viz., 2½ feet; and as 80 revolutions are made per minute, the piston travels over 80 × 2½ = 200 feet per minute. Again, the area of the piston 12 inches diameter is 113 square inches, and as this is pressed on with an average load of 20 lbs. on every square inch, the total pressure on it is 113 × 20 = 2260 lbs. We have, therefore, 2260 lbs. moved over 200 feet per minute, equivalent to 2260 × 200 = 452000 lbs. moved over 1 foot per minute, which give $\frac{452000}{33000} = 13\frac{3}{4}$ horse-power nearly.

The complete rule for finding the power may therefore be thus stated:—Measure the ordinates (at least 6 or 8 in number) contained within the indicator figure, sum them up, and divide by their number for the mean pressure; multiply the area of the piston (in square inches) by the mean pressure (in lbs. per square inch) by twice the length of stroke (in feet) and by the number of revolutions per minute, and divide the product by 33,000 for the horse-power.

Example.—On the indicator being applied to an engine, having a cylinder 30 inches diameter, a stroke of 4 feet, and making 27 revolutions per minute, 8 ordinates of the figure were found to be respectively 34, 34, 34, 33, 24, 18, 14, and 9: required the power of the engine.

Sum of 8 ordinates = 200, which, divided by their number 8, gives 25 lbs. as the mean pressure.

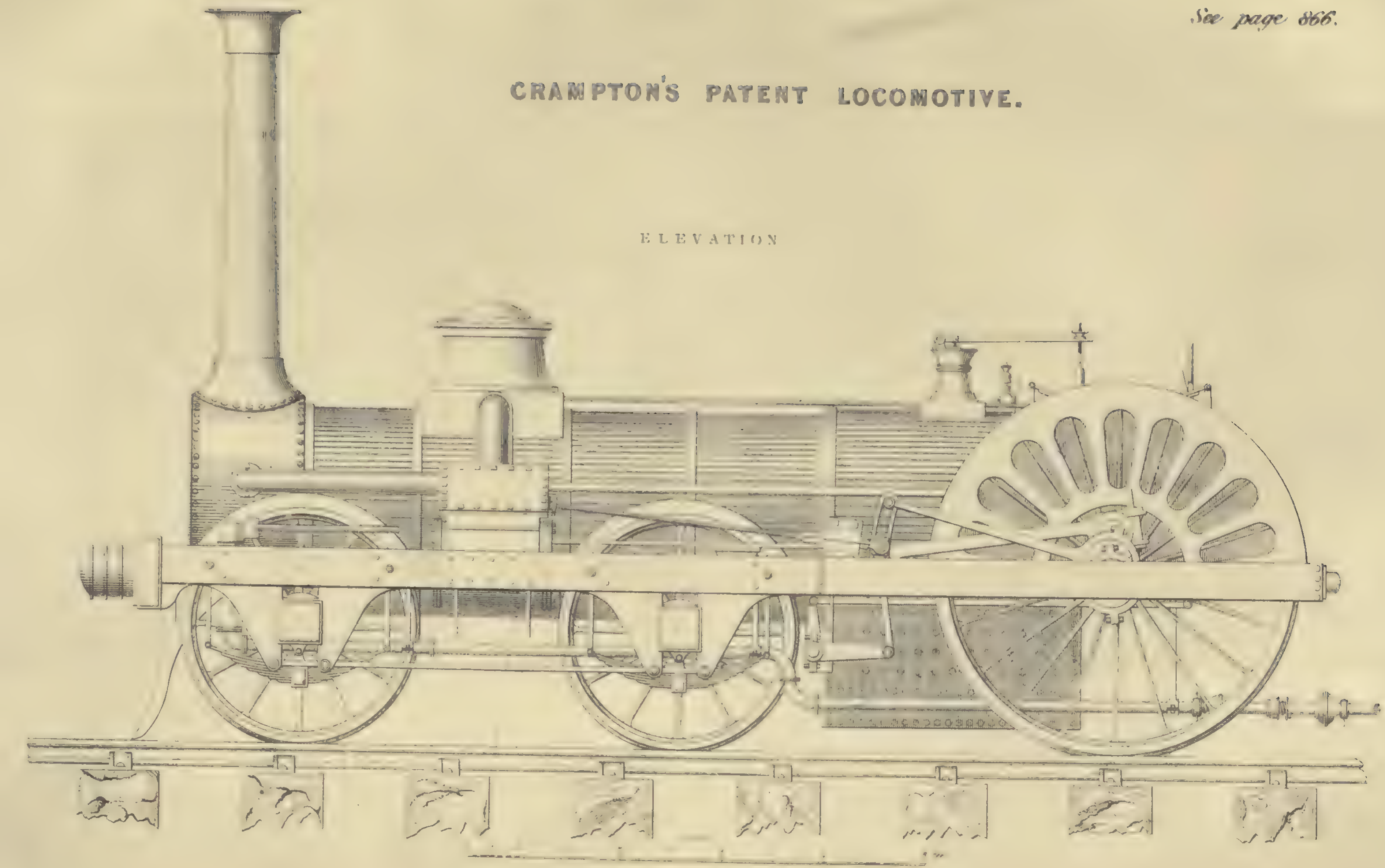
Area of cylinder, 30 ins. diameter =	707 square inches.
Multiply by mean pressure . .	25 lbs.
Total mean pressure on piston .	17675 lbs.
Double the stroke . . .	8 feet.
	141400
Number of revolutions . .	27 per minute.
Divide by	33000) 3817800
Horse-power	115¾ nearly.

But in making these calculations, it must not be forgotten that we only reckon the force with which the piston moves. In communicating this force to the crank-shaft, and thence to the machinery driven by the engine, there are losses from friction and other causes for which some allowance must be made. The piston rubs along the surface of the cylinder; its rod rubs through the stuffing-box; so with the slide. The end of the piston-rod rubs on the guides, which save it from yielding to the oblique action of the connecting-rod; the connecting-rod eyes rub on their pins, the crank-shaft rubs in its bearings, the eccentric and its rod also present rubbing surfaces, the fly-wheel encounters considerable resistance of the air to its rotation, and the feed-pump demands power for its working. All these resistances vary with the conditions of the rubbing surfaces, the accuracy or inaccuracy of their fitting, their state of lubrication, and other circumstances; so that it is difficult to state any constant deduction to be made from the calculated power on account of them. An engine in a very good state should thus waste not more than $\frac{1}{10}$th to $\frac{1}{8}$th of its power; while one in a bad state may lose as much as $\frac{1}{3}$rd. It may generally be fair to reckon the loss at $\frac{1}{5}$th or $\frac{1}{4}$th of the calculated power. Thus, in the example given, the calculated power being 115¾, we take the real power about 90, deducting rather more than $\frac{1}{5}$th for losses.

The indicator figure is not only a measure of power, it is also a picture of defects, and may often furnish useful hints as to the proper mode of improving the action of the engine. If A B (Fig. 201) represent the stroke, *See page 866.*

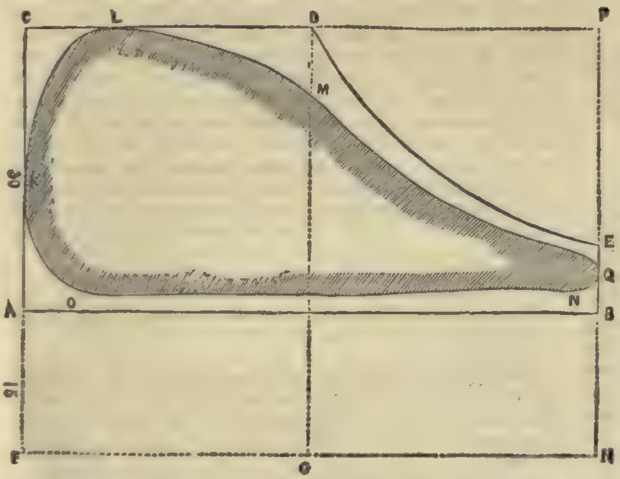
CRAMPTON'S PATENT LOCOMOTIVE.

ELEVATION



F H a line drawn at a distance A F below equal to 15 lbs., and if we suppose steam at 30 lbs. above atmospheric pressure is admitted to the cylinder during half the stroke, then suddenly cut off, we draw A C = 30 lbs., C D half the stroke; take E midway in P H, and fill in the hyperbolic curve D E. Then will the figure A C D E B represent the best possible effect that could be got under the conditions given; for the straight line A C represents the sudden rise of the pressure from that of the atmosphere to 30 lbs. above it at the beginning of the stroke; the straight line C D represents the continuance of that pressure during half the stroke; the curve D E indicates the gradual reduction of pressure, as the steam enclosed in the cylinder expands to fill its increasing capacity; the point E midway in P H marks the pressure at the end of the stroke, half that at the beginning or middle, because the capacity of the cylinder at the end has been doubled, while the quantity of steam within it has remained constant. The straight line E B marks the sudden fall of the final pressure to that of the atmosphere when the exit port is opened; and the straight line B A represents the constant resistance of the atmosphere to the issue of the steam during the return stroke of the piston. Such might be the theoretical figure. The practical figure inscribed within it must necessarily fall short, in some respects, of that which is theoretically perfect. For instance, at the beginning of the stroke, the port opening not suddenly, but gradually, produces a curved line from K to L, the piston having travelled some distance before the full pressure is attained; the gradual closing of the port or expansion valve, and some diminution of pressure from the cooling of the steam, or from leakage past the piston, are indicated by the inclined line L M. The cooling varies the expansion curve M Q from the true hyperbola, and the gradual opening of the exit port causes a curved turn from Q to N instead of a sudden drop E B. The line N O above the line of atmospheric pressure B A, indicates some additional resistance to the issue of the steam dependent on limited area, or bad form of opening, or leakage from the steam side of the piston, and the turn at O K marks the gradual closing of the exit port, and opening of the inlet port for the succeeding stroke. A careful analysis of a figure produced by an engine not working satisfactorily, will thus point out causes of loss, and suggest means of remedying them, by widening the ports, readjusting the eccentric and slide, clothing the cylinder to prevent cooling, and such other arrangements as may be found advantageous. In the hands of an experienced engineer, indicator diagrams become highly suggestive of merits and defects, and often furnish more information as to the economical working of an engine or the reverse, than continued observation of its structure and action could supply.

Fig. 201.



CALCULATION OF POWER.—In estimating the power of a non-condensing engine without reference to its indicator figure, we may generally make a very near approximation to the truth by the following mode of reckoning. We assume the engine to be in fair working order, and fitted with an ordinary slide, cutting off the steam at about $\frac{2}{3}$rds of the stroke; that the steam-pipe is not of great length, and well clothed with non-conducting material; that the ports are well-proportioned, and the piston and slide tight. We farther suppose the piston to travel at the velocity of 200 feet per minute, which is found to be practically a fair working rate; and that a horse-power, to be effective, after all allowances for friction, &c., should be estimated at 44,000 lbs. moved 1 foot per minute, or 220 lbs. moved 200 feet per minute. We observe the pressure in the boiler, and deduct from it $\frac{1}{4}$th for loss by cooling in the steam-pipe and expansion in the cylinder, and 2 lbs. for resistance to exit and other losses, the remainder being reckoned as the mean effective pressure. Multiplying this by the area of the piston, and dividing by 220, we get a fair estimate of the power.

Example.—An engine having a cylinder 30 inches in diameter, is worked at a pressure of 36 lbs. in the boiler: required its power.

From pressure in boiler . . .		36 lbs.
Deduct one-fourth for cooling, &c.	9 lbs.	
And for back pressure . .	2 lbs.	—11 ,,
Mean effective pressure		25
Multiply by area of 30 inches . .		707
Divide by		220)17675
Horse-power		80

In general it is the business of engineers to provide engines of certain powers without special reference to the pressure at which they should be worked. By employing very high pressures, the size, weight, and cost of an engine are certainly reduced; but, on the other hand, some danger is incurred, and the tear and wear are considerable. By using very low pressures, again, the cylinder must be large, the engine is generally cumbrous and heavy, and little advantage can be taken of the expansive power of the steam. We consider a boiler pressure of 40 to 50 lbs. to be a fair average on which to estimate the engine-power; and would suggest the following rules for calculating the power of a given engine, and the diameter of cylinder necessary to produce a given power.

1. To find the power of an engine when the diameter of the cylinder is given.

Rule.—Divide the diameter (in inches) by 3, and square it for the horse-power.

Example.—Required the power of an engine having a cylinder 15 inches in diameter.

$$\frac{15}{3} = 5, \text{ and } 5 \times 5 = 25 \text{ horse-power.}$$

2. To find the diameter of cylinder necessary for a given power.

Rule.—Multiply the square root of the power by 3; the product is the diameter of the cylinder in inches.

Example.—What should be the diameter of a cylinder for 100 horse-power?

Square root of 100 = 10, and 3 × 10 = 30 inches.

The length of stroke must depend on the number of revolutions made by the crank in a given time. It is convenient to assume that the piston, in engines going at a fair average speed, shall travel over 200 feet per minute. Sometimes it moves at the rate of 250, and even exceeding 300 feet per minute; but, upon the whole, 200 is a convenient and economical speed. This is the product of twice the stroke by the number of revolutions; and hence its half, 100, is the product of the stroke by the number of revolutions. If, then, either the length of stroke or the number of revolutions be given, the other may readily be thus found:—

1. Given the length of stroke to find the speed.

Rule.—Divide 100 by the stroke (in feet), the quotient is the number of revolutions per minute.

Example.—What is the speed of an engine having 2 ft. 6 ins. stroke?

$$\frac{100}{2\frac{1}{2}\text{ feet}} = 40 \text{ revolutions per minute.}$$

2. Given the speed to find the stroke.

Rule.—Divide 100 by the number of revolutions per minute, the quotient is the length of stroke in feet.

Example.—What must be the stroke of an engine making 35 revolutions per minute?

$$\frac{100}{35} = 2{\cdot}86 \text{ feet, or about 2 ft. 10 ins.}$$

STEAM PASSAGES.—The dimensions of the steam passages should be proportioned to the area of the cylinder; for while the piston travels at its quickest speed, increasing rapidly the space to be filled with steam, the passages should admit the steam with sufficient velocity to sustain the pressure on the retreating piston. It is found, practically, that the area of the steam-pipe may be advantageously $\frac{1}{36}$th of that of the cylinder, or the diameter of the one $\frac{1}{6}$th of that of the other. Thus, for an engine having a cylinder 30 ins. diameter, the steam-pipe should be 5 ins. diameter. When it is intended that the piston should travel more rapidly than the average rate of 200 feet per minute, the steam-pipe should be proportionally large. In such cases, its diameter may be advantageously reckoned at $\frac{1}{5}$th of the diameter of the cylinder. The ports which admit the steam into the cylinder should always present an area of passage considerably greater than that of the steam-pipe, for during a great part of every stroke they are partially closed by the slide. The exhaust-pipe, which conveys the steam from the cylinder, should be larger than the steam-pipe; for the waste steam should be permitted to become rapidly expanded in volume, in order that its back pressure on the piston may be diminished as much as possible.

FEED-PUMP.—The size of the feed-pump should always be greatly in excess of that which is absolutely required for supplying the amount of water boiled off in steam for the engine. Occasionally, the valves of the pump leak; there may be leaks in the boiler; some of the water may pass over in priming; and a good deal may be wasted in blowing off. The pump should, therefore, be capable of supplying at least 3 times as much water as is actually due to the steam supplied to the cylinder. The pump is generally arranged so as to make one stroke for each revolution of the engine, or each double stroke of the piston. If, then, we take a case where the diameter of the cylinder is 30 inches, the stroke 4 feet, and the average pressure 30 lbs. above that of the atmosphere—that is to say, the steam at 3 atmospheres—we find that, during each revolution, the cylinder, twice filled with steam, uses a volume of about 68,000 cubic inches. Steam at 1 atmosphere of pressure being about 1,600 times the volume of water, its volume at 3 atmospheres is $\frac{1}{3}$rd of that, or about 533 times that of water. The water necessary to generate 68,000 cubic inches of steam at 30 lbs. pressure, therefore, amounts to 128 cubic inches; and as the feed-pump should be capable of supplying thrice this quantity, it must deliver at each stroke 384 cubic inches. Should we make the stroke of the pump 2 feet, half that of the piston, its area must be $\frac{384}{24} = 16$ square inches, or its diameter $4\frac{1}{2}$ inches.

As a general rule, for the dimensions of the feed-pump in non-condensing engines, we may offer the following:—

Multiply the square of the diameter of the cylinder (in inches) by the length of stroke (in feet), divide by 90, and the quotient is the product of the square of the diameter (in inches) of the pump, by its length of stroke (in feet).

Example.—Required the size of the feed-pump for an engine having a cylinder 30 inches diameter, and a stroke of 4 feet.

$$\frac{30 \times 30 \times 4}{90} = 40,$$ the product of diameter squared by stroke. If we take the stroke of the pump 2 feet, then $\frac{40}{2} = 20$ is the square of the diameter, or the diameter is about $4\frac{1}{2}$ inches, because $4\frac{1}{2} \times 4\frac{1}{2} \times 2 = 40$ nearly.

Many treatises on the steam-engine give rules for the dimensions of all the principal parts of an engine. We think, however, that these rules are not, in many cases, practically available, because a difference in general design and arrangement, or in average pressure and speed, occasions very considerable variations in the proportions of the parts. Careful study of well-made engines, and actual experience in their construction and working, form the true sources of information as to their due proportions.

CHAPTER VI.

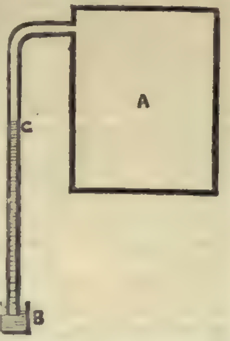
THE STEAM-ENGINE—(*Continued*).

Contents.—CONDENSING ENGINES—SAVERY'S—NEWCOMEN'S—WATT'S IMPROVEMENTS—CONDENSER AND AIR-PUMP—INJECTION—SURFACE CONDENSER—PARALLEL MOTION—BEAM ENGINE—MARINE ENGINE, PADDLE, SCREW—COMBINED ENGINES—ROTARY ENGINES—SUPER-HEATING STEAM—APPLICATIONS OF STEAM POWER—PUMPING—DRIVING MACHINERY—LOCOMOTION—PROPULSION OF VESSELS.

THE *Condensing Engine*, in all main points, resembles the non-condensing engine; but it requires some additional parts, in order that the vacuum produced by condensing the steam may be employed as a source of additional power. If we suppose that the steam, on leaving the bottom of the cylinder, instead of flowing out into the atmosphere, which resists its egress with a pressure of 15 lbs. per square inch, were conducted into a vessel totally void of air or steam, this resisting force would be entirely removed, and the effect of the steam pressing on the upper side of the piston, would be increased by that quantity. If the vacuum in the vessel were not perfect—that is to say, if there were contained in it some rare fluid, such as air or steam, or a mixture of both, greatly attenuated, and capable of pressing with a force of only 2 or 3 lbs. on the square inch—the pressure of the steam on the piston would be increased by a quantity 2 or 3 lbs. less than 15 lbs. per square inch. Generally, if we reckon the pressure of steam in the boiler as its absolute pressure, not its excess over the atmosphere, and deduct the pressure of fluid in the vacuum vessel, the difference will be the effective pressure on the piston. Thus, with steam in the cylinder exerting a pressure of 10 lbs. above that of the atmosphere, or having an absolute pressure of 25 lbs. per square inch, while the vacuum vessel contains a fluid pressing with a force of 2 lbs. per square inch, the effective pressure on the piston is 25 − 2 = 23 lbs. per square inch. The condition of the fluid in the vacuum vessel, as to pressure, is generally measured by a barometer.

The upper part of the vessel A (Fig. 202) is connected with a glass tube C, about 30 inches long, dipping into a cup of mercury B. Were the space in A an absolute vacuum, the atmospheric pressure on the surface of the mercury in the cup would force the mercury up the tube to the height of about 30 inches, because a column of mercury 30 inches in height, presses with a force of 15 lbs. on the square inch. But if A contained fluid pressing with 2 lbs. per square inch, the mercury would attain a height of only 26 inches; because the pressure of a column 26 inches high is 13 lbs., and the additional 2 lbs. of fluid pressure make up 15 lbs., the atmospheric pressure.

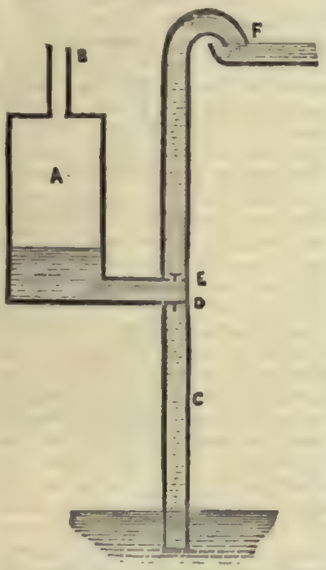
Fig. 202.



If, then, we know the pressure of steam in the cylinder (above that of the atmosphere), and the height of the mercurial column in the barometer, we can find the effective pressure on the piston by adding the steam pressure to half the height (in inches) of the column.

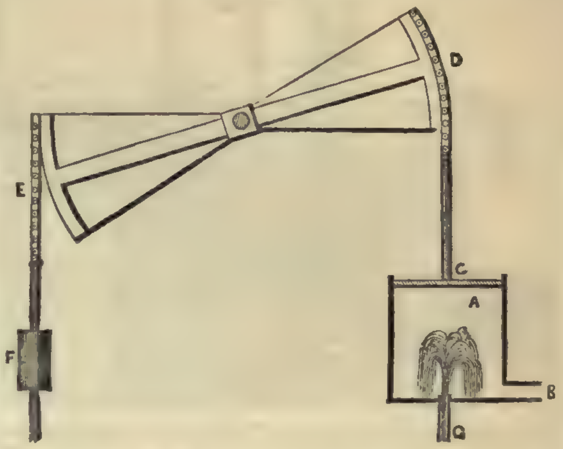
SAVERY'S AND NEWCOMEN'S ENGINES.—It is remarkable that some of the earliest efforts made for obtaining power from steam, were directed to the construction of apparatus in which its condensation, as well as its elasticity, should afford the force required. At the end of the 17th century, Captain Thomas Savery succeeded in constructing an engine for raising water by means of steam. A vessel, A (Fig. 203), connected by a steam-pipe and cock B, with a suitable boiler, communicated with a vertical water-pipe C, in which were fitted two valves, D and E, opening upwards. Steam being admitted into A, forced out the air contained in it by the upper part of the water-pipe, and occupied its place. The steam-cock B being closed, and a stream of cold water made to pour over A, the steam within it became condensed, and formed a partial vacuum; the pressure of the atmosphere acting on the water at the bottom of C, forced it up the pipe through the valve D, and into A, so as to fill the void space. The steam-cock being again opened, the pressure of steam on the water in A forced it through the valve E and up the pipe. The vessel A was thus successively filled and emptied by the alternate closing and opening of the steam-cock, and the water raised through the height of the pipe C.

Fig. 203.



A few years afterwards, Thomas Newcomen applied steam to give motion to a piston in a cylinder. The cylinder A (Fig. 204) communicated by a pipe and cock B with a boiler generating steam at low (or nearly atmospheric) pressure. A piston C, fitting the cylinder, was connected by a chain with one end of a beam or lever D, to the other end of which, E, was attached a pump-rod with a heavy weight F. The ends of the beam were made arcs of circles, so that while the chains were wound on them or unwound from them, the piston and pump-rod might move in straight lines. Steam being admitted into the cylinder below the piston, so as to balance the atmospheric pressure on its upper surface, the weight F caused it to ascend; and when it reached the top, the steam-cock was closed, and a water-cock G was opened to admit a jet of water into the midst of the steam. The coldness of the water rapidly condensed the steam, and formed a partial vacuum below the piston, and the atmospheric pressure forced it down to the bottom, raising the weight F. The water-cock being now closed, and the steam-cock opened, the ascent of the piston was repeated.

Fig. 204.



WATT'S IMPROVEMENTS.—Numerous inventors contributed to the improvement of Newcomen's engine; and about the beginning of the 18th century it had become a practically useful, but not economical apparatus for pumping. The principal objection to the use of this, as well as of Savery's engine, consisted in the circumstance that the alternated flow of cold water into the steam-cylinder cooled it down, and greatly diminished the force of the steam as it entered, a great portion of its heat being ineffectively expended on the cold metal of the cylinder. It was reserved for the illustrious James Watt, shortly after the middle of the last century, to introduce those improvements in the arrangement and construction of the steam-engine, which have rendered it the most useful and economical of all known sources of power. His principal improvement on Newcomen's engine was the use of a separate vessel for condensing the steam, with an air-pump for removing the water of condensation, and the air liberated from the water. But his ingenuity was likewise devoted to the improvement of every detail of arrangement and construction; and, having found the steam-engine in many respects rude, ineffective, and costly in its working, he left it an apparatus, as nearly perfect as any human work can be. Since his time, changes have been made in the arrangements of the parts to suit peculiar circumstances of operation, and new forms have been devised for particular purposes; but in all their leading features, the steam-engines of the present day are essentially the products of Watt's fertile genius.

The condensing engine, as improved by Watt, is of two kinds:—

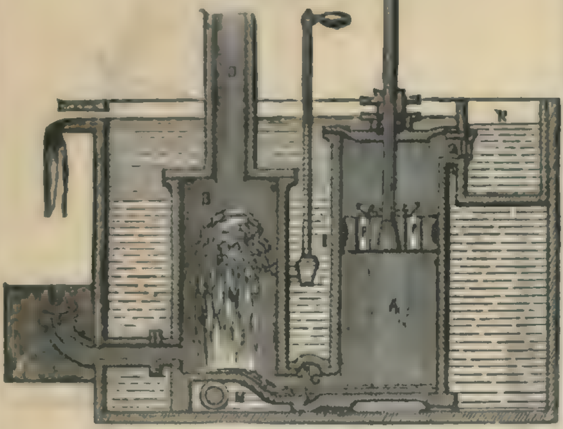
Single-acting, where the steam is permitted to press on one side of the piston only, so as to cause it to make a single stroke; the return stroke being effected by a counterbalance weight.

Double-acting, where the steam presses alternately on each side of the piston, and its reciprocating movement is converted, by a crank, into rotary motion.

The single-acting engine is well suited for such an operation as pumping, where the reciprocating movement of the pump-bucket corresponds with that of the piston. The double-acting engine, again, is adapted for driving machinery. In both these kinds of engine, where the steam is condensed, there are required a condenser and an air-pump, which we will now describe.

CONDENSER AND AIR-PUMP.—The condenser is a vessel B (Fig. 205), generally made of about $\frac{1}{8}$th of the capacity of the steam-cylinder, with which it communicates by the pipe D. The steam, after acting on the piston, instead of escaping into the atmosphere as in non-condensing engines, flows by this pipe into the condenser, which is placed in a cistern of cold water, and has a pipe and cock, I, for the admission of a jet of cold water to condense the steam. This jet is called the *injection*, and its quantity is regulated by means of the cock, worked by a rod passing upwards, with a handle in some place convenient for the attendant. The bottom of the condenser communicates by a passage, having a valve G, with the air-pump A. The bucket or piston P of the air-pump is fitted with valves opening upwards, and is moved upwards and downwards by a rod connected with some reciprocating part of the engine, and passing through a stuffing-box in the air-pump cover. Near the top of the air-pump there is a passage Q, fitted with a discharge valve opening into the *hot-well* K, from which the feed-water is pumped to the boiler, the overplus or waste being discharged by a waste-pipe. The cistern in which the condenser and air-pump are placed, is kept constantly filled with cold water by a pump called the *cold-water pump*, supplying it by a pipe N at the bottom, while the heated water overflows by a suitable waste-pipe. While the steam from the cylinder flows into the cold condenser, and meets the cold water diffused through it by the injection jet, it becomes condensed into water, and falls with the injection-water to the bottom, occupying very little volume compared with that which it occupied while in the state of steam, and leaving the space of the condenser as a partial vacuum. But water always contains a quantity of air mingled with it, which passes over with the steam from the boiler to the cylinder, and thence to the condenser, and the injection-water also parts with a portion of the air it contains, so that after a time the condenser would become filled with the air so liberated, and with the water of injection and condensation, unless means were taken to remove them.

Fig. 205.



As the air-pump bucket descends, the discharge-valve Q being closed, no air or water can enter from above to fill the space left void by the descent of the bucket, but the air and water from the condenser pass through the valves G and P, and enter this space from below. On the ascent of the bucket, the bucket-valves P being closed by the pressure of air and water above them, the contents of the pump are discharged through the valves Q into the hot well. Thus by the alternate descent and ascent of the air-pump bucket, when its capacity and the amount of injection are properly proportioned, almost a perfect vacuum is maintained in the condenser; and the effective pressure of the steam on every square inch of the piston is increased by nearly 15 lbs. above that which it would be, were the steam permitted to escape uncondensed into the atmosphere.

The condenser is generally fitted with a *blow-valve* H, which comes into play on starting the engine thus. The engine having been stopped, the condenser and air-pump may have become quite filled with water through the injection-cock; and on starting the engine again, no vacuum could be produced while they are thus *water-logged*. But by opening a small valve called the *blow-through valve*, a communication is made between the steam-pipe from the boiler, and the eduction-pipe leading from the cylinder to the condenser. The pressure of the steam in the boiler is thus brought to bear on the water in the condenser, and forces it out by the *blow-valve* H, the steam occupying its place. On shutting the blow-through valve and opening the injection-cock, the steam in the condenser is condensed, and a vacuum formed, so that the engine may be started. As long as there is even a partial vacuum in the condenser, the atmospheric pressure on the blow-valve keeps it closed.

The bottom-valve, bucket-valves, and discharge-valves of the air-pump, are frequently made of vulcanised caoutchouc sheet, cut into discs and laid over gratings. Water or air forced through the perforations in the gratings, raises the flexible discs of caoutchouc, and passes round their edges; but neither air nor water can return, for the atmospheric pressure forces the caoutchouc discs firmly down on the gratings, and thereby effectually closes their openings.

INJECTION.—It is a peculiar property of all vapours, that, besides their sensible heat, or the temperature to which they raise the thermometer, they contain a great amount of latent heat, not measured by the thermometer, but by its effect, when the condition of the vapour is changed. The latent heat of steam, when its temperature or sensible heat is 212°, is estimated to be about 1,000°. This does not mean that the latent heat could raise a thermometer 1,000°, but simply that a pound of steam at 212° being condensed by its mixture with 1,000 lbs. of water at any temperature, such as 60°, could raise the temperature of the whole mass of water 1°. In other words, if it were found that the combustion of a certain weight of fuel could raise the temperature of a given mass of water from 211° to 212°, it would require 1,000 times that quantity of fuel to convert the water into steam, having still the sensible temperature of 212°. This great latent heat is something essential to the condition of water in a state of vapour; for as soon as any portion of it is removed, by bringing the steam into contact with a cold substance, a part of the steam is immediately condensed into water; and the remainder, expanding to fill the space thus left void, loses density and pressure as it gains volume. In estimating the quantity of injection-water necessary for condensing the steam of an engine, we must therefore bear in mind that it is not alone the sensible temperature, but also the latent heat of the steam which we have to absorb by the cold water injected. Let us assume that 1 cubic foot of water, having been converted into steam in the boiler, and having acted on the piston in the cylinder, flows into the condenser at a temperature of 212°, and containing 1,000° of latent heat, and that it there mingles with a quantity of water at 62°, sufficient to condense it and produce an ultimate temperature of 112° throughout the mixture. The total heat of the steam being 1,212°, has to be reduced to 112°—that is, the steam has to lose 1,100° of temperature; the injection-water entering at 62°, and being raised to 112°, has to gain 50°. The quantity of injection-water must therefore be 22 cubic feet; for 22 cubic feet raised 50°, are equivalent to 1 cubic foot reduced 1,100°, because $22 \times 50 = 1100$.

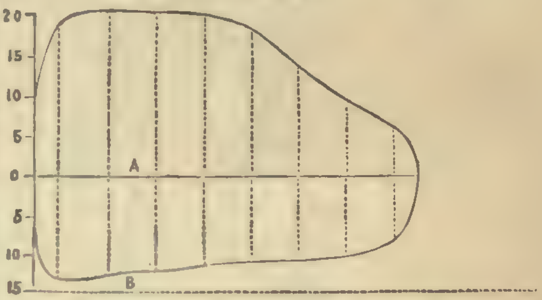
The temperatures we have assumed are such as would frequently occur in practice; and generally it will be found that the quantity of injection-water required for an engine is from 15 to 25 times the quantity required for feed.

The capacity of the air-pump is generally $\frac{1}{8}$th of that of the cylinder, the stroke being usually $\frac{1}{2}$, and the diameter $\frac{1}{2}$ that of the cylinder. The power necessary to work the air-pump of a condensing engine is about $\frac{1}{10}$th of the total power.

SURFACE-CONDENSER.—Condensation by injection, such as has been described above, is the system that has been almost universally adopted since Newcomen applied in his engine. We have already (see p. 851) alluded to another mode of condensation, by means of cold surfaces, which now begins to be adopted, and which, from the great advantages attending its use, will probably soon entirely supersede the *injection* method. On considering the mode in which a jet of cold water admitted into a vessel containing steam acts upon it, we readily see that the water, entering with considerable force through an aperture (usually fitted with a rose), presents to the steam a very great and minutely subdivided surface. Thus every minute particle of the water is at once surrounded by an atmosphere of steam, from which it abstracts so much heat, that the steam is almost immediately reduced to its liquid condition as soon as it comes in contact with the water: as a mode of condensing, then, rapidly and effectually, probably no better arrangement could be desired. But, after condensation, the injection-water, which, as we have seen, amounts to 15 or 20 times the quantity of water produced from the steam, forms the principal part of the water fed from the hot well to the boiler. If, therefore, the injection-water be impure, as it generally is; and still more, if it be salt, as it always is at sea, the boiler is fed with the impure water, and is thus subjected to all the evils of deposit and incrustation, to which we have before alluded. The quantity of water actually required for feeding the boiler, is precisely the quantity derived from the condensation of the steam, with a small allowance for unavoidable losses by leakages. If, then, the steam could be rapidly and effectually condensed without mixing it with impure water, it would itself supply almost enough water for feed, and that of the purest possible quality, because it would be distilled. There appears to be no mode of obtaining the rapidity of condensation required for the efficient working of an engine, except that of exposing the steam to a very extended cold surface, just as it is exposed to the minutely subdivided surface of the water of injection, but without mingling with it. The new condensers, called *surface-condensers*, are therefore constructed generally like tubular boilers. A box or casing, like the body of a boiler, has a great number of very small tubes passing from end to end, and tightly jointed, so that no communication can take place between the interiors of the tubes and the space through which they pass. In some condensers, cold water is made to flow through the tubes, while the steam from the cylinder is admitted to the box, and plays round the outside of the tubes in the spaces surrounding them. In other condensers the steam is made to pass through the tubes, which are surrounded by cold water supplied to the box in which they are contained. There are numerous modifications of form and arrangement of parts, and varieties in the modes of constructing the casing and of jointing the tubes to it, of causing the water to circulate, and in other details. But in all surface-condensers the general principle is the same—namely, that of exposing a very large cold surface to the steam, and of subdividing the steam so that every minute portion of its bulk is brought rapidly into contact with the cooling surface. In the best condensers of this kind now in use, a vacuum is obtained quite as good as was formerly obtained by the injection method; and, although an increased quantity of cold water may be demanded for keeping the surface cold, yet this is readily attainable at sea, and in most situations where condensing engines are employed. The very leakages are provided against by supplying for the loss of feed from this cause, by distilled water procured from a small supplementary boiler, so that scarcely a particle of impurity can enter the main boiler, and all the evils of deposit, incrustation and priming over of viscid water, are effectually remedied. As water, before attaining its boiling point, parts with the air which it may have held in solution, the condensed steam contains no air, and thus the vacuum in the surface-condenser is maintained by the use of a much smaller pump. With the ordinary condenser, worked by injection, the pump is properly called an *air-pump*, because a large portion of its capacity is at every stroke filled by the attenuated air given off from the steam and from the water of injection. But with the surface-condenser, the function of the pump is almost exclusively confined to the removal of condensed water, the quantity of air evolved from the distilled water in the condenser being so small as to be almost inappreciable. The size of the air-pump is thus considerably reduced when surface-condensation is adopted, and less of the engine power is consequently absorbed in working it.

The indicator applied to a condensing engine produces a figure similar to those we have already discussed. But as during one stroke of the piston the communication to the condenser is open, the pencil traces a line below that of atmospheric pressure. Thus if A (Fig. 206) be the line of atmospheric pressure, another line B, drawn at the pressure of 15 lbs. below A, would be the line of absolute vacuum, or of no pressure. The lower limb of the figure would more nearly approach this line, the more perfect the vacuum in the condenser. The area of the figure represents the power during a stroke; and the mean effective pressure is found by taking the average of the lengths of numerous ordinates drawn within the figure as before.*

Fig. 206.



PARALLEL MOTION.—Non-condensing engines are generally of the form called *beam-engines*. The piston-rod does not act directly on the crank in the case of the double-acting engine, nor on the pump in the case of the single-acting engine, but on one end of a lever or beam, working on central bearings, from the other end of which the crank or pump is worked by a connecting-rod. By jointed rods connected with pins on the beam, the air-pump is worked with half the stroke of the piston; the feed-pump and the cold-water pump are also worked with their proper strokes. As the piston-rod, and also the air-pump rod, pass through stuffing-boxes in the cylinder and air-pump covers respectively, and must therefore move in straight lines, while the parts of the beam to which they are connected vibrate in circular arcs, some arrangement is required for controlling the motion of the rods so that it shall not partake of the circular vibration. This could be effected by making them slide between rectilineal guides, but not without considerable friction and some difficulty in arranging the guides in a suitable manner. We are indebted to Watt for a simple and elegant manner of effecting this object by the apparatus called the *parallel motion*.

If A D_1 and B C_1 (Fig. 207) be two equal levers vibrating in circular arcs D_1 D D_2, C_1 C C_2 respectively, their extremities being connected by a rod D_1 C_1, it will be found that E, the middle point of this rod, moves from E_1 to E_2 in a line which is very nearly straight. Again, if F G, one of these levers, be prolonged to H so as to be doubled in length, and a parallelogram G H M L be formed of jointed rods so that F K and L are always in one straight line, then K being controlled so as to move in a straight line or nearly so, L will also move in nearly a straight line. Applying this principle to the beam of an engine, K is the point to which the air-pump rod is connected, and L that to which the piston-rod is connected; G M and H L are called *parallel motion links*, and M N, the subsidiary lever which controls the movement of K, is called the *radius rod*. In the case of single-acting engines, where the end of the beam opposite to that worked by the piston works a pump, a parallel motion is also fitted there to give rectilineal motion to the pump-rod.

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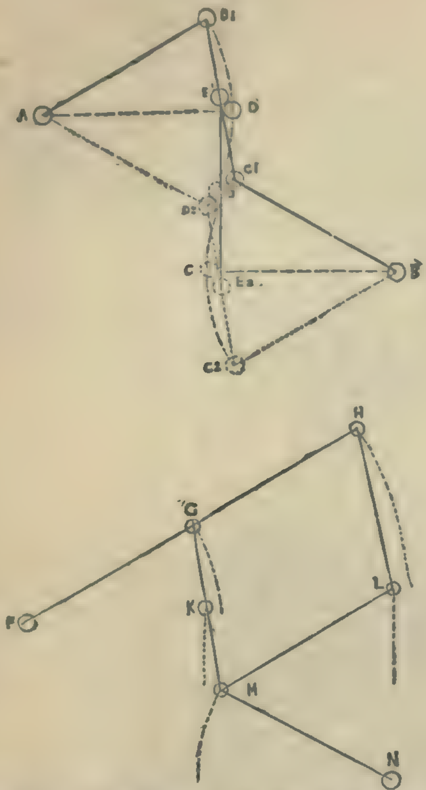
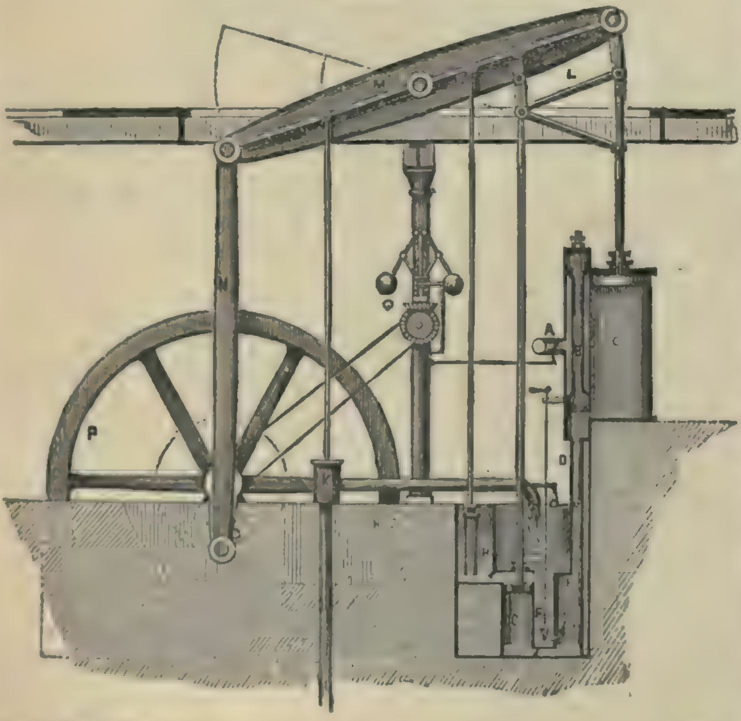
* See *ante*, p. 863.

Fig. 207.



BEAM ENGINES.—In marine engines, where it is desirable to keep the weight as low as possible, the arrangement of beam is different. Instead of one beam above the cylinder and crank, two beams are fitted below, one on each side of the cylinder, and the parallel motion employed in that case is somewhat different from that which we have described, although the principles embodied are very similar.

Fig. 208.



The alternate movements of the steam to and from the opposite ends of the cylinder, are effected in condensing engines in the same manner as non-condensing engines, by a slide worked by an eccentric, or by valves worked by cams. In single-acting engines valves are generally employed, and they are worked by levers struck by pins or tappets fixed to the air-pump rod, as it ascends and descends along with the piston.

The single-acting beam-engine is almost exclusively employed for pumping. The form of engine most commonly used for this purpose is what is called the Cornish engine, because at the mines in Cornwall these engines have been employed with great economy and efficiency for draining to great depths.

The steam is admitted above the piston at considerable pressure, and the supply is cut off when the piston has performed a small part of its down stroke, so that the rest of its stroke is effected by the expansion of the steam. When the piston has reached the bottom of the cylinder, the steam is permitted to pass from its upper to its lower side, so that it does not press it in either direction; but the weight of the pump-rods at the other end of the beam, loaded, if necessary, with additional weight, causes the piston again to ascend to the top of the cylinder. The steam is again admitted above the piston, while that which is below it flows into the condenser; and thus the alternation is continued, the valves which control the movements of the steam being worked by levers and tappets from the air-pump rod.

Fig. 208 represents a double-acting condensing beam-engine working a crank. Here the weight of the piston and air-pump bucket, and their rods at one end of the beam, is balanced by that of the connecting-rod at the other end, and the steam is alternately admitted by the slide to the upper and lower sides of the piston, and thence to the condenser, so that each end of the beam is alternately pushed upwards and pulled downwards by the pressure on the piston. A the steam-pipe from the boiler, supplying steam to the slide-jacket or space round the slide B. C the cylinder fitted with piston, and piston-rod connected to one end of the beam M by the parallel motion L. E the condenser, communicating with the slide-jacket by the eduction pipe D. F the injection-cock, admitting water into the condenser by a jet from the cold cistern in which it is placed. G the air-pump, worked by a rod from the parallel motion, and discharging into the hot well H, in which is placed the feed-pump, worked by a rod from the beam, so as to draw water from the hot well, and propel it by a feed-pipe to the boiler. K the cold-water pump, worked by a rod from the beam, drawing water from a well or stream, and supplying the cold cistern for condensation. N the connecting-rod, worked from one end of the beam, and giving rotary motion to the crank O. P the fly-wheel, fixed on the crank-shaft, and revolving with it, with the necessary momentum for bringing the crank over the dead centres, or highest and lowest points of its revolution, where the connecting-rod is ineffective to turn it. Q the governor, caused to revolve by a strap or band, and suitable gearing connecting it with the crank-shaft. The steam-pipe A is fitted with a throttle-valve connected with the governor by levers and rods, so that the velocity of the engine is controlled by the throttling of the steam, as already described.*

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* See *ante*, p. 862.

MAUDSLAY.

ELEVATION.

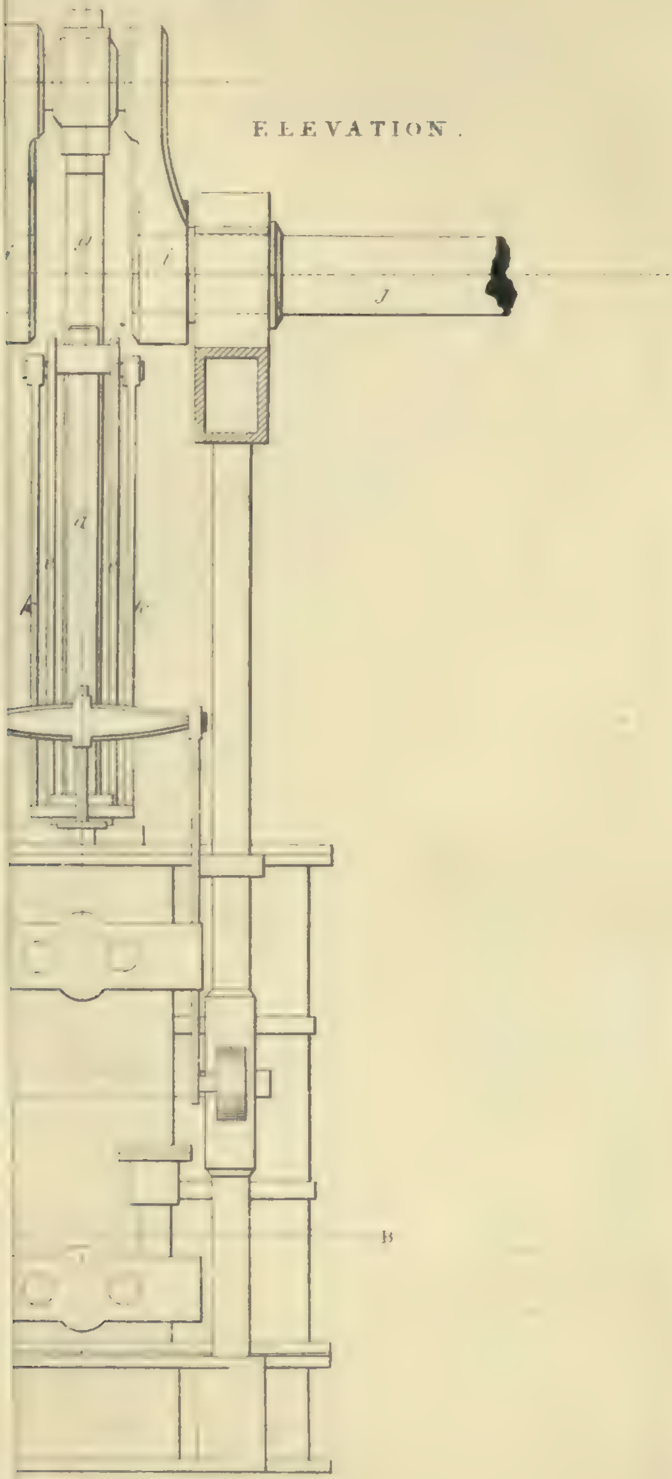
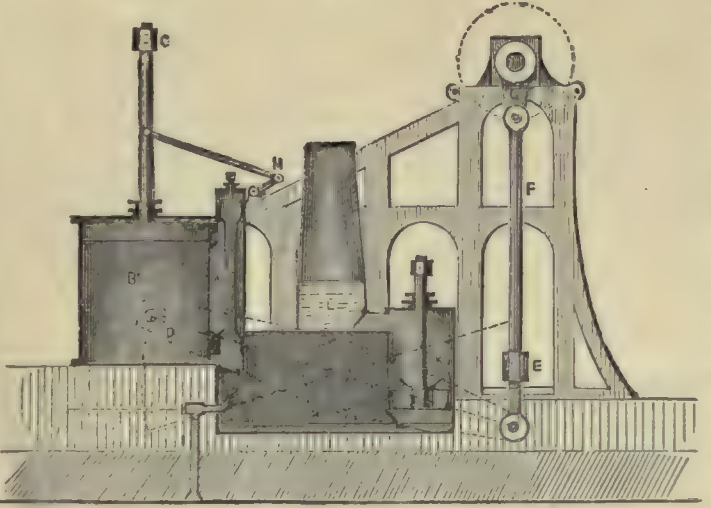


Fig. 209.



MARINE ENGINE.—Fig. 209 represents the arrangement of a condensing marine engine suited for driving paddle-wheels. A the slide-jacket supplied with steam from the boiler. B the cylinder fitted with piston and rod. C the piston cross-head, from the ends of which, side rods descend on each side of the cylinder to the extremities of two side beams D. H the condenser, supplied with injection-water, by the injection-pipe and cock I, passing through the ship's bottom. K the air-pump, worked by side rods from pins in the side beams D, and discharging into the cistern-head L, whence the supply of water for feed is drawn, and from which a discharge-pipe conveys the waste water through the ship's side into the sea. E the cross-head of connecting-rod, connected by side rods at each end to the beams D. F the connecting-rod, causing the revolution of the crank G, on the shaft of which are fixed the paddle-wheels. M is a parallel motion for constraining the cross-head C to move in a straight line. The feed-pump is generally placed on one side of the air-pump, and a bilge-pump (for emptying the bottom of the vessel of water leaking into it) on the other, both being worked by the cross-head which moves the air-pump bucket. The general arrangement of the marine engine is such as to keep the principal weight at as low a level as possible, lest the stability of the vessel should be endangered by too much weight above the floating line. These engines are generally in duplicate, placed side by side longitudinally in the vessel, the crank-shafts working the paddle-wheels being laid transversely.*

A great variety of arrangement has been adopted in marine paddle-engines, so as to secure compactness, simplicity, and lightness, consistently with the great strength required to sustain the shocks to which sea-going vessels are subjected. Among these arrangements we may instance the oscillating engine, to which we have already alluded (page 866), and a remarkably compact engine, called the Annular Cylinder Engine, represented in the annexed plate. In this engine, the cylinder *a* contains an inner cylinder *b*, open at top and bottom, and the piston *c* is of a ring or annular form, working tightly between the inner and outer cylinders. Two piston-rods *d d*, passing through stuffing-boxes in the cylinder cover, are firmly fixed to the opposite ends of a cross-head *e*, which is double, having two thicknesses with a space between them. The lower part of this cross-head extends downwards into the inner cylinder *b*, in which it is fitted to slide. To a pin *f* in this sliding part the connecting-rod *g* is jointed, and also to the crank-pin *h*; *i* is the crank fixed on the main paddle-shaft, which revolves in bearings in strong framing, secured to the wooden beams that extend across the vessel, and tied to the cylinder and foundation-plate by strong wrought-iron bolts or columns. It may be readily seen from the drawing that this arrangement permits the use of a long connecting-rod well guided at its lower end *f*, and a stroke of piston of considerable length, even when the height allowed for the engine is very limited, as is usually the case in vessels, especially such as are of light draught of water. The condenser is the box or hollow foundation on which the engine stands, and *m* is the air-pump, which is worked by a small double beam *l*, mounted on the framing, and receiving its motion from rods *j*, connected to the lower part of the cross-head at *f*. The feed and bilge-pumps are fixed at each side of the air-pump *m*, and are worked by rods connected to the same cross-head as that which works the air-pump rod, to which they serve as guides. The alternations of the steam to and from the opposite ends of the annular cylinder are effected by a long hollow D slide, which is worked by an eccentric from the main shaft. For stopping, starting, or reversing the engines, the eccentric-rod, having a gab end, as described (page 861), is thrown out of gear by a small lever, and the slide worked by a cross-handle, so as to make the required change of direction in the movement of the piston; and on this being effected the gab of the eccentric is again dropped into gear, and the motion of the engine continues.

The conditions of a marine engine suited for driving a screw-propeller, differ considerably from those of the engines we have hitherto described. The shaft of the screw-propeller is necessarily at a low level, and lies longitudinally in the vessel to which it is fitted, and it must be made to revolve at considerable velocity. When this system of propulsion was first tried, it was usual to employ ordinary marine engines placed transversely in the vessel, and to fit on their shaft a large cog-wheel, driving a pinion on the shaft of the screw-propeller below it. By this arrangement the low level of the latter was secured, as well as its high velocity; but the weight, bulk, shake, and noise of the toothed gearing were found objectionable. Screw-propeller engines are generally made now with very short stroke, so that the number of strokes made in a given time is much greater than in ordinary engines of equivalent power. The cylinders are arranged horizontally; and the piston-rods act on the cranks placed at a low level, without the intervention of side-beams.

The annexed plate represents the transverse section of a screw-steamer, with the engine placed in the lower part of its hold: *g* is the cylinder placed horizontally, and supplied with steam from the boiler by the steam-pipe *m*. The piston-rod *f* works a cross-head, which moves in horizontal guides, and acts by the connecting-rod *e* directly on the crank-shaft *d*, which is extended horizontally to the stern of the vessel, and projects through a stuffing-box into the water near the stern-post, carrying on it the screw-propeller, indicated by the dotted circle *s*. For working the air-pumps of screw engines, several different arrangements are employed. In some cases the air-pumps are placed horizontally, like the cylinders, on the opposite side of the vessel, and worked directly by the piston-rods, which are extended in length for that purpose. In the plate, another method of working the air-pumps is represented. A second or counter-shaft *i* is worked by gearing *c* from the main-shaft *d*; and this second shaft, by means of cranks and connecting-rods *j*, works the pistons of vertical air-pumps *l*. In order to secure, in a height so limited, a greater length of connecting-rod, and at the same time to furnish a guide to resist the transverse strain resulting from the obliquity of its action, the piston of the air-pump is made with a trunk or tubular rod *k*, which passes through a stuffing box in the pump-cover; and the lower end of the connecting-rod *j* is jointed within this trunk. The water discharged from the air-pump passes by a pipe *n* into a cistern, whence a portion is drawn by the feed-pump for the supply of the boiler, the rest escaping by the discharge-pipe *o*, which is carried through the side of the vessel above the level of the external water: *p* is the hand-lever, by which the engine-driver stops, starts, or reverses; it is connected by a rod *q* and bell-crank lever, with the link motion already described (page 862). When the lever *p* is held in its middle position by a catch dropping into a notch on an arc against which it slides, the engines remain at rest; when it is raised upwards, or depressed to either of the extreme notches, the engines work at full speed, either ahead or astern; and when it is set at either of the intermediate notches, they work at half speed, or are *eased* in either direction.

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* See *ante*, p. 859.

Many other forms of engine for screw-propulsion might be described; but as they all, more or less, display the same general features, we do not think it necessary, in our limited space, to enter on a full description of their details.

COMBINED ENGINES are those which embody the principles of both the condensing and non-condensing engines. In these, a small cylinder and large cylinder are placed side by side, and their piston-rods are connected so as to work together. High-pressure steam admitted to the small cylinder, works its piston as in a non-condensing engine; but, instead of being permitted to escape into the atmosphere, is conveyed to the larger cylinder, in which it expands to greater volume but lower pressure, and acts on a greater area of piston. Thence the expanded steam flows into a condenser, as in the ordinary condensing engine.

This arrangement gives the opportunity of securing a large amount of useful effect from the steam, and thereby economising fuel in the boiler. If we suppose, for example, that steam with an absolute pressure of 75 lbs. is admitted into the smaller cylinder during $\frac{1}{4}$th of the stroke, and allowed to expand during the remainder of the stroke, its final pressure, allowing for a little loss by cooling, will be about 18 lbs., and the mean pressure throughout the stroke will be about 42 lbs. The steam at 18 lbs. being now admitted into the larger cylinder during $\frac{1}{2}$ the stroke, will have a final pressure of about 8 lbs., and a mean pressure of about 13 lbs. throughout the stroke. The back pressure of rarefied air and vapour in the condenser may average 2 lbs. throughout the stroke, and the mean effective pressure on the larger piston would then be $13-2=11$ lbs., while that on the smaller is $42-13=29$ lbs. per square inch. Now, if the area of the small piston be to that of the large in the proportion of 11 to 29, the total mean pressure on each will be alike; and the power of the engine will be double that of either, with an expenditure of steam at 75 lbs. pressure, or 60 lbs. above that of the atmosphere, only sufficient to fill $\frac{1}{4}$th of the capacity of the smaller cylinder. By cutting off at an earlier period of the stroke, and using a still larger second cylinder, still greater economy of steam may be attained.

In determining the power of a condensing engine, it is necessary to know the pressure of the steam employed, the amount of vacuum produced in the condenser, the size and velocity of the piston, and the various losses of force occasioned by the friction of the machinery, and the resistance of the air-pump, feed-pump, and cold-water-pump. Assuming that, generally, the speed of the piston is about 200 feet per minute, and that the pressure in the boiler does not exceed 5 or 6 lbs. per square inch above that of the atmosphere, we may take the following rule as to power:—

Given the diameter of the cylinder to find the power.

Rule.—From the diameter (in inches) subtract 6, square the remainder, and divide by 20 for the power.

Example.—What is the power of a condensing engine having a cylinder 54 inches diameter?

$$54-6=48, \text{ and } \frac{48\times48}{20}=115 \text{ horse-power.}$$

Given the power to find the diameter of the cylinder.

Rule.—To the square root of 20 times the power add 6, for the diameter in inches.

Example.—Required the diameter of cylinder for 115 horse-power.

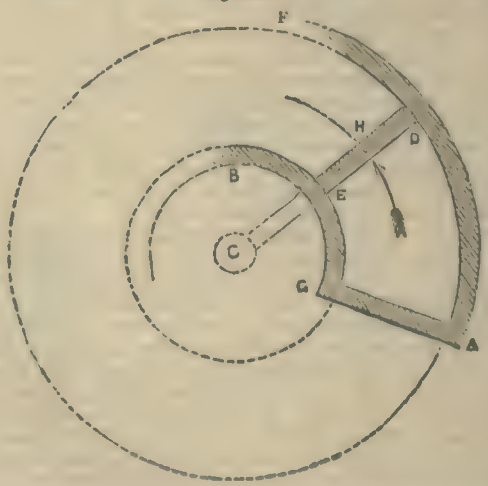
$20\times115=2300$, square-root $=48$ nearly, and $48+6=54$ inches.

These, however, merely furnish rough guesses at the power of any engine. The proper method of ascertaining the real power is to apply the indicator, when the engine is only moving itself, so as to ascertain the power necessary to overcome the friction and resistance of its working parts; and to deduct this quantity, with some allowance for additional strain, from the power indicated when the engine is in full work. No definite rule can be offered for estimating these allowances, as varieties in construction and workmanship introduce great differences among experimental results. In general, it is safe to reckon not more than $\frac{3}{4}$ths of the indicated power as really effective to move machinery, the remaining $\frac{1}{4}$th being absorbed in friction and moving the pumps.

ROTARY ENGINES.—In all steam-engines, except those that are single acting, a good deal of mechanism is necessary in order to convert the reciprocating rectilineal motion of the piston into the continuous circular motion of the crank. It has, therefore, been a great object with many mechanics to devise a rotary engine, or one in which the steam pressure shall at once give the required rotary motion, without the intervening machinery of connecting-rod, beam, parallel motion, and crank. Many of the arrangements devised for this purpose present great ingenuity, and it is not improbable that some may prove ultimately successful; but hitherto no rotary engine has proved so far satisfactory as to warrant its adoption in the place of those already in use.

A singular error regarding rotary steam-engines has crept into some of the best books on the subject; and as it is calculated to discourage inventors who may apply themselves to devising engines of this kind, we will endeavour to point it out. It is asserted that steam applied to give direct rotary motion to a piston is less effective, or loses part of the power which it would produce if applied to move a piston in a straight line. If we suppose E D (Fig. 210) to be a piston fitted to an annular cylindrical vessel B G A F, so that it may be moved round the centre C by the pressure of steam admitted behind it, we have to ascertain whether a certain quantity of steam applied to a piston thus arranged, will generate as much power as it would produce when applied to a piston in the ordinary way. Some authors (among them Tredgold, in his large work on the steam-engine) say that "the quantities of steam being equal, the power of rotary action will be less than that of rectilineal action;" and this fallacy is but too generally admitted among engineers. That it is a fallacy may be very easily shown by a practical example. Let us sup-

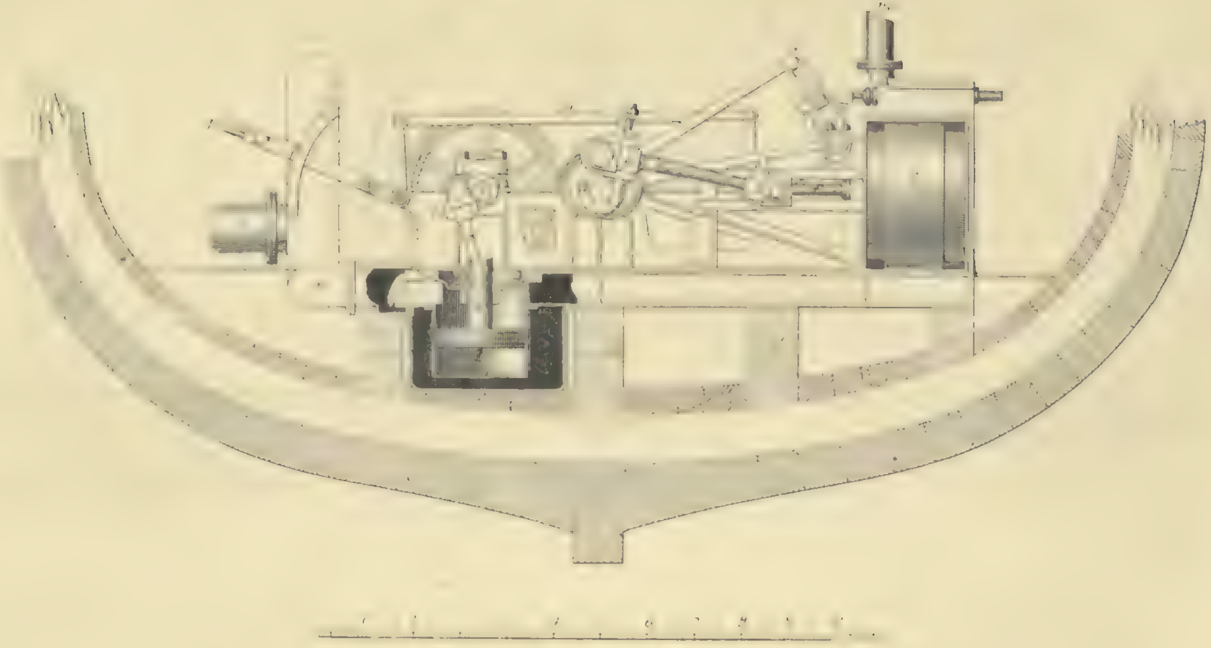
Fig. 210.



See page. 875

MARINE ENGINE.

SCREW PROPELLER



pose that the piston D E measures 10 inches each way, and therefore presents a surface of 100 square inches, and that it is acted on by steam having a pressure of 20 lbs. per square inch; so that the total pressure on its surface is 100 × 20 = 2000 lbs. Now, considering C D as the arm of a lever, of which a portion E D is loaded with pressures distributed uniformly over it, we know that their combined effect to turn the lever round its fulcrum C is the same as if the whole pressure were collected into one force at H, the centre of gravity of the part D E. We have, therefore, effectively a lever or arm of the length C H, pushed round C by a force of 2,000 lbs. applied at H. If we take C E = 9 inches, then as E H = 5 inches ($\frac{1}{2}$ of 10 inches), C H = 14 inches; and during one revolution, H passes over a distance equivalent to the circumference of a circle having a radius of 14 inches—that is to say, over 88 inches, or $7\frac{1}{3}$ feet. Hence, the work done during a revolution is 2000 lbs. × $7\frac{1}{3}$ feet = 14667 lbs. moved over 1 foot. Also the quantity of steam required to fill the annular space passed through by the piston is thus found, C E being 9 inches, and C D being 19 inches:—

Area of outer circle (38 inches dia.) =	1134·12 sq. ins.
Area of inner circle (18 inches dia.) =	254·47 ,,
Area of annular space (difference) .	879·65 ,,
Multiply by breadth	10 inches.
Volume of annular space . .	8796·5 cubic ins.

Were we to apply this quantity of steam in an ordinary cylinder, whatever be its dimensions, we should produce equivalent power. Let us, for example, take a cylinder 1 foot diameter, having an area of piston 113·1 square inches, subjected to a pressure of 113·1 square inches × 20 lbs. = 2262 lbs., moving through a stroke of 6·488 feet (a length which requires equivalent volume of steam), we find the work to be 2262 lbs. × 6·488 = 14667 lbs. moved over 1 foot as before. Instead of reckoning the whole revolution of the rotary piston, we might take any portion of its revolution, and we should find the power derived from the quantity of steam used, exactly equal to that which would be produced by the same quantity of steam, in any corresponding portion of a cylinder where the piston moves rectilineally. Without reference to any special numerical example, we observe these general laws. Whatever be the form of the rotating piston D E, steam pressing uniformly on its surface produces the same effect to turn it round its centre C, as if the whole pressure were collected into one force acting at the centre of gravity H of the surface of piston; and the work done by the steam during any part of a revolution, is equivalent to the total pressure in the piston, multiplied by the distance traversed by the centre of gravity H, or the portion of the circumference of a circle of which C H is the radius; and, further, the volume of steam required during the given portion of a revolution is (by the well-known law of mensuration of annular solids) measured by the area of the piston, multiplied by the distance passed over by its centre of gravity. In the case of a piston moving rectilineally, the work done by the steam, and the volume of steam used during any portion of a stroke, are measured in precisely the same way, and bear the same relation to each other. There is, therefore, no theoretical objection to the application of steam in such a way as to produce direct rotary motion. That there are considerable practical difficulties in the arrangement of the parts, and their construction, so as to present steam-tight rubbing surfaces, and to avoid undue friction and unequal wear, is doubtless true; but were these difficulties fairly surmounted, we should be in possession of an engine where simplicity, and economy of weight and bulk, might enable us to apply steam-power in many cases where it is not now applicable without inconvenience.

Before leaving the subject of rotary engines, we may mention that steam has been applied successfully to produce rotary motion on the same principle as that of Barker's mill, or the turbine applied to water-power. Steam of considerable pressure, passing through openings on the sides of several tubular arms mounted on an axis, causes them to revolve in the direction opposite to that in which the steam issues, on the same principle as the movement of a rocket, where the issue of the elastic gases, generated by the combustion of the charge at one end, leaves an unbalanced pressure to act on the other end, and thus to force it onward through the air; but we believe this mode of applying steam-pressure, though exceedingly simple, is by no means sufficiently economical to warrant its general adoption.

SUPER-HEATING STEAM.—For several years after the adoption of steam-power for marine propulsion, it was almost exclusively confined to vessels destined to convey passengers over short distances, along coasts or in rivers. The fuel required to maintain the steam-power was to be obtained easily and cheaply, and, as the distance was not great, the space required for the stowage of coals was not so large as to trench inconveniently on the accommodation of the vessel. Under these circumstances, engineers devoted their attention less to economy of fuel than to speed. But since steam-vessels have begun to make long voyages, lasting for several weeks—since the invention of the screw-propeller has rendered them even more servicable as sea-going vessels for carrying cargo than sailing vessels could be, and especially since the application of steam-power to ships of war, the economy of fuel has become a matter of the first importance. Many modifications in the construction of the boilers and of the engines have been introduced for the purpose of saving fuel, and still greater changes will probably be effected in the processes by which the combustion of fuel is applied to the production of motive power. Among these changes, which begin to receive general adoption, may be noted *surface-condensation*, to which we have already alluded (page 873), and *super-heating steam*. The latter process appears to promise great results in economy of fuel, especially when applied in connection to *combined engines* with *surface-condensers*, and we will therefore endeavour to describe how the process is effected, and what are the special advantages which it secures. On looking to the Table of the Temperature and corresponding pressure of steam (page 845), it will be seen that a very small reduction of temperature produces a very considerable diminution of pressure. Thus, steam at a pressure of 4 atmospheres, or exerting on every square inch a pressure of 45 lbs. above that of the atmosphere, has a sensible temperature of 290°. On diminishing the temperature by 15°, the pressure is reduced to three atmospheres, or a loss of pressure is incurred amounting to 15 lbs. on the square inch. At higher temperatures the loss is proportionally greater. Thus, steam at 344°, having a pressure of 8 atmospheres, when cooled down to 320°, has its pressure reduced to 6 atmospheres, losing 2 atmospheres, or 30 lbs. per square inch, by a reduction of its temperature to the extent of 24°.

It may be readily understood, then, that steam, in its passage from the boiler, where it is generated, to the cylinder, where its elasticity is required, exposed, as it is, to the cooling influence of an extended surface of pipe passages and cylinder, all made of rapidly conducting materials, must sustain a loss of effective pressure, which is absolute waste of power or of the fuel used to produce it. Nor is the loss of pressure the only evil resulting from the reduction of temperature. A cubic foot of steam at 290° of temperature and 4 atmospheres of pressure, on having its temperature reduced to 275°, loses, as already stated, 1 atmosphere of pressure; but, as it has still 1 cubic foot of volume, and as, by Mariotte's law, the density is always proportional to the pressure, the loss of pressure is accompanied by a corresponding diminution of density. That is to say, in the case we are examining, *one-fourth* part, or 25 *per cent.* of the steam, becomes actually condensed into water, leaving the remaining three-fourths, or 75 per cent., occupying the volume of 1 cubic foot at the reduced density and pressure. This water accompanies the steam into the cylinder, where it is of no assistance to the steam on the pressure side of the piston; and where, by evolving vapour on the vacuum side it produces a back or retarding pressure. It passes with the steam into the condenser, and there absorbs uselessly a large portion of the cooling power, whether of injection or of cold surface, and therefore, for a given power of engine, necessitates a proportionally greater cooling power, a proportionally larger air-pump, and proportionally increased waste of useful effect in working it. To avoid these evils it has long been usual to clothe the steam-pipes and cylinder with thick felt and wood casing, or non-conducting materials of a similar kind; and, in many engines, the cylinders have been made double, having a jacket or space all round them filled with steam from the boiler, which serves to keep the inner cylinder hot, and to prevent loss of pressure and condensation during the actual expansion of the steam within it. Notwithstanding these precautions, however, a great loss of effect is still incurred, amounting probably to 20 or 25 per cent. under the most favourable conditions, and, under average conditions, to a great deal more. Engineers accordingly begin to adopt the super-heating process, which is found to present an almost complete remedy against these losses. It may simply be described as a mode of surcharging the steam with heat, and raising its temperature considerably above what may be called its natural temperature, or the temperature due to its pressure. In order fully to understand this, it should be remembered that the temperatures given in the Table are those of the steam when it is in contact with water. For instance, when the pressure is equivalent to 4 atmospheres, the steam in contact with water, and the water, have the temperature of 290°; if the temperature were reduced, a portion of the steam would be condensed, and the remainder would present a proportionally diminished pressure. Were the temperature increased, a fresh portion of water would be converted into vapour, which, mingling with the rest, would proportionally increase the pressure. But if the steam were removed into a vessel separate from the water, while it would still be true that a reduction of temperature would cause a condensation and diminution of pressure, it would no longer be true that an accession of temperature would cause an increase of pressure from the generation of fresh steam, because there would be no water from which to generate it. In that case, the steam would probably follow the law of expansion, which is common to all gaseous bodies, increasing in bulk by a certain fraction of its volume, proportioned to its elevation of temperature; or, if its volume were confined, increasing in pressure in the same proportion. But steam, so elevated in temperature, would be in a condition to sustain a certain loss of heat without any part of it becoming condensed. It would, in fact, have a relay of heat stored within it, and might be exposed in pipes or cylinders to cooling surfaces without losing any appreciable per centage of its pressure, or depositing water of condensation. The practical modes by which this super-heating process is effected are various. Their general principle, however, is simply to expose the steam, in its passage from the boiler towards the cylinder, to the heat of the boiler flues; and, in order that this heat may be rendered effective as rapidly and completely as possible, the steam-pipe is subdivided into numerous small pipes or passages, exposing a very extended surface to the hot products of combustion which play upon it. Almost all the super-heating apparatus thus bear a close resemblance to the surface-condensers, though intended for an operation of an opposite character. As in the surface-condenser the steam is subdivided into numerous tubes or channels exposed to cold water for the purpose of cooling and condensing it, so in the super-heater, it is likewise subdivided into channels exposed to hot flame, smoke, and gases, for the purpose of surcharging it with heat.

It is said, that by the use of super-heating apparatus, an economy of fuel has been effected, amounting, in some cases, to 20 and 25 per cent. Probably a portion of this economy may be due to the circumstance that super-heating apparatuses are generally placed in the last part of the boiler flue leading to the chimney, where they utilise a quantity of heat which, under ordinary circumstances, passes up the chimney, and is totally wasted.

APPLICATIONS OF STEAM-POWER.—The chief purposes to which steam-power is applied are the following:—

I. Pumping water for the drainage of mines and docks, or at water-works for the supply of towns.

II. Driving machinery for raising ore from mines, for moving heavy weights, or for agricultural and manufacturing purposes.

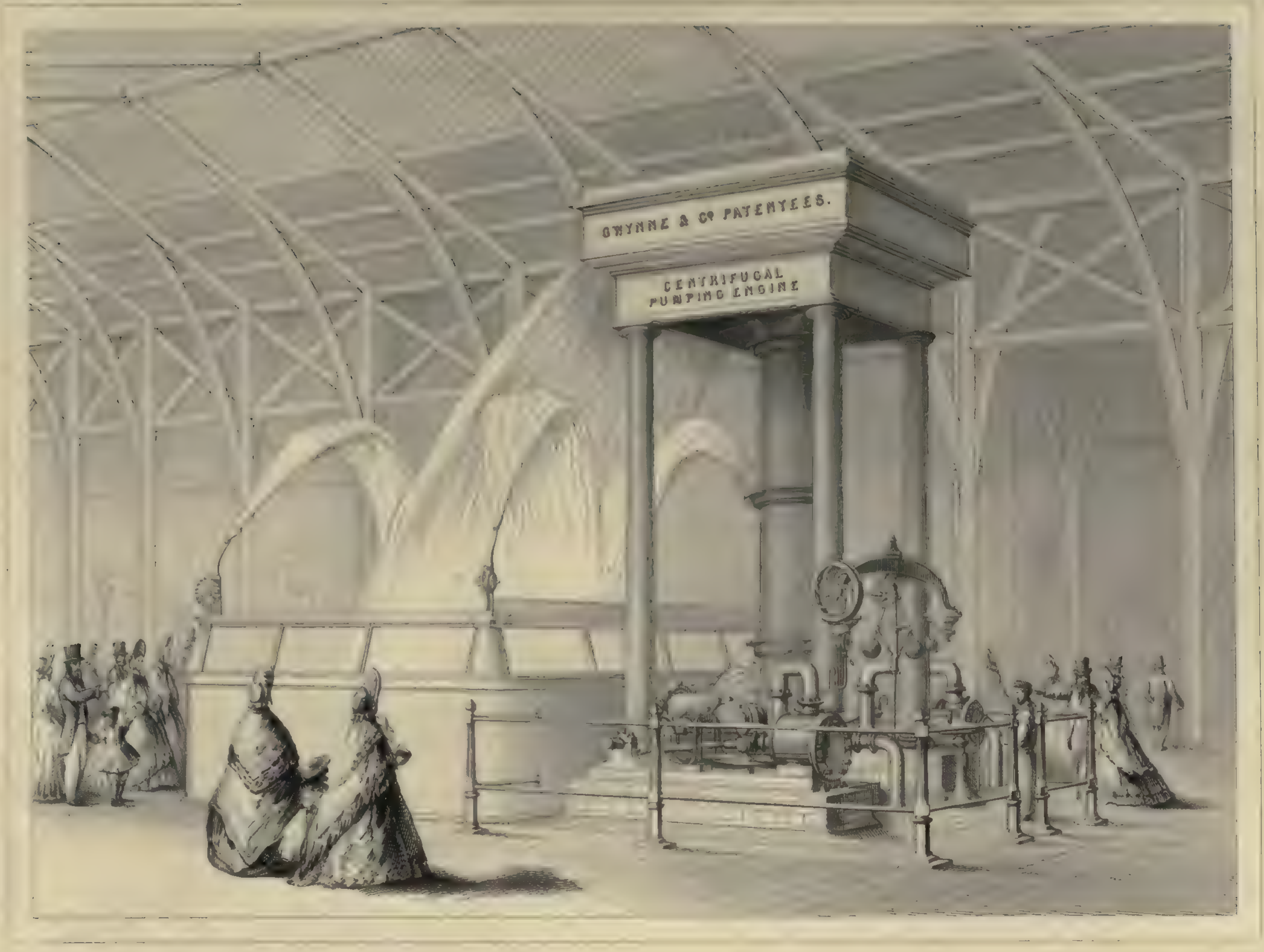
III. For locomotion on railways.

IV. For propelling steam-vessels.

We will briefly discuss the modes in which steam-power is generally made available for these different uses, and the forms of steam-engines most advantageously applied in each case.

I. Pumping.—In pumping water, until of late years, single-acting engines were almost universally applied. We may most readily account for this, not on the ground of any advantage derived from the use of single-acting engines, but from the circumstance that the earliest form of engine that was rendered practically available was that of Newcomen, which was single-acting, and suited only for working pumps; that Watt's improvements were first applied to engines of this kind, and that his engines were introduced at mines to supersede the labour of men and horses formerly applied to pumping; that these engines were of first-rate quality, effective and durable, and naturally impressed the miners with a preference for their form and arrangement; and that engineers in mining districts, applied themselves rather to the perfecting of forms already in use, than to the introduction of new arrangements. So strong, indeed, has been the preference for single-acting engines, especially those of the kind employed in Cornwall, when applied to pumping, that few double-acting engines have ever been employed for this purpose until very recently. We believe, however, that the results have shown decided advantages in favour of the double-acting engines, and that ere long they will supersede the more cumbrous and less advanced form of those that are single-acting.

The pumps employed in mines are of the lifting or forcing kind, having a stroke seldom exceeding 8 or 10 feet. When the mine is deep, the water is raised by stages, each 150 to 200 feet in height. The lowest pumps discharge into a reservoir about that height above the bottom of the mine; the second set of pumps draw from that reservoir, and discharge into the next higher; and so on, until the water is finally delivered at such a level that it may be permitted to flow off by natural drainage. As apparatus is generally required at mines for stamping and crushing the ore, it is not unusual to deliver the water at such a height above the general level of the ground, as may permit its use for driving a water-wheel, from which motion is given to the ore-crushing machinery. But as power is also required for raising the ore, it is generally more advantageous to employ a double-acting engine, besides the main pumping engine, for this purpose, as well as for the preparation of the ore. The power required for raising water depends upon the quantity raised in a given time, and the height to which it is raised. If we suppose that it is necessary to raise 100 cubic feet of water per minute 150 fathoms, or 900 feet, since each cubic foot of water weighs $62\frac{1}{2}$ lbs., the total to be raised is $100 \times 62\frac{1}{2} = 6250$ lbs. per minute. This load, raised through 900 feet, is equivalent to $6250 \times 900 = 5625000$ lbs. raised 1 foot, and would require $\frac{5625000}{33000} = 170$ horse-power, without allowance for friction and excess of power, to give the velocity of movement to the column of water. To estimate this quantity, we should add at least 10 per cent., or $\frac{1}{10}$th, making the power 187-horse, which would be the effective power, after making all allowances for the friction of the engine, and the various losses occasioned by the cooling or leakage of steam, and the working of the air-pump and feed-pump.



GWYNNE & Cº.s CENTRIFUGAL PUMP.

The performance of the Cornish engines has received special attention, from the circumstance that a number of mine proprietors placed their engines under the superintendence of engineers, and had monthly reports published as to the quantity of water lifted, the quantity of coals consumed, and other particulars. Six of the most effective engines reported on, presented the following average results :—

Diameters of cylinder from 60 to 90 inches.
Mean pressure of steam per square inch, 10 lbs.
Length of stroke from 8 to 10 feet.
Average number of strokes per minute, 7.
Diameter of pumps, 9 to 18 inches.
Average length of pump-stroke, 7 feet 6 inches.
Average work, reckoned in lbs. of water lifted 1 foot high by the consumption of 1 bushel of coals, 40,500,000 lbs., or the work of 1,227 horse-power.

In pumping docks, the quantity of water to be lifted in a given time is generally great, but the height inconsiderable, and the pumps may, therefore, be of large diameter. As a practical example, we may take the case of a rectangular dock, 200 feet long, 40 feet broad, and 20 feet deep, the water being lifted 2 feet above the highest level to give a fall for its drainage. The total quantity to be lifted is $200 \times 40 \times 20 = 160{,}000$ cubic feet, or $160{,}000 \times 62\frac{1}{2} = 10{,}000{,}000$ lbs. At the beginning of the work, the water has to be lifted only 2 feet; at the end it has to be lifted 22 feet; and the mean height of lift is, therefore, 11 feet, the work done being equivalent to $\frac{10000000 \text{ lbs.} \times 11}{33000} = 3333$ horse power; or, allowing $\frac{1}{10}$th for friction and velocity, about 3660 horse-power. If the work be done in 1 hour or 60 minutes, the power of the engine must be $\frac{3660}{60} = 61$ horse-power, clear of all losses. As in a case like this, the work at first is very light, and becomes greater as the level of the water is lowered, and the height to be lifted consequently increased, it is advisable to have several pumps, all of which may be kept in action at first, but which may be thrown out of action successively as the load on them becomes increased, the strain of the engine being thereby not greatly varied.

In water-works for supplying towns, not only has the water to be lifted to such a height that it may command the highest level to which it has to rise, but it has also to be conveyed through great lengths of pipe, often extending many miles. It is convenient to arrange, as near as possible to the pump, a high reservoir, of such altitude as may be sufficient to cause the necessary flow in the great length of pipe connected with it. This is effected by pumping the water through an elevated siphon, open at the top to permit the issue of air that may be mingled with the water. The pump, in making its stroke, has thus to put in motion only the column of water contained in the ascending limb of the siphon, instead of the whole mass contained in the pipes. But in order still farther to relieve the pump and engine from the strain required to put in sudden movement even this mass of water, the pump is provided with a capacious air-vessel—a dome-shaped vessel—the upper part of which contains air, and the lower part communicates with the discharge-pipe of the pump. When the pump discharges its contents, the air in the vessel is compressed by the influx of water below it; and while the pump is making its return stroke, the elasticity of the compressed air continues the flow of the water that had been forced into the vessel. The air thus acts as a spring, yielding to the force of the discharge from the pump, and sustaining the pressure on the flow-pipe at intervals when the pump is inactive. When the pump is double-acting, or discharges at both the up-stroke and the down-stroke, the intervals of inaction are only those occupied by the turn of the stroke, and the air-vessel has less to do than when the pump is single-acting. Two single-acting pumps, of which the one discharges while the other is drawing, produces a like effect. Upon the whole, we believe that for simplicity, economy of cost and of working, a good double-acting engine, with double-acting pump, is preferable to two single-acting engines and pumps, each of half the power, or to one single-acting of equivalent power. We know of no advantage possessed by the single-acting engine, which cannot be fully secured in the double-acting engine. It has so happened, that many single-acting engines have been made of large dimensions and long stroke, and that the expansive power of the steam has been employed by cutting it off at an early part of the stroke, to such an extent as to secure great economy of fuel. But in the double-acting engine, particularly when the duplicate cylinder, with high-pressure and low-pressure action, is applied, the same economy of fuel can be secured, and the bulk and weight of the whole are greatly reduced.

An operation, nearly allied to that of pumping water, is the production of the air-blast required for furnaces. At the iron-works, which constitute so important a part of our industry, a very great amount of power is employed for this purpose. The smelting furnaces, in which the metal is separated from the earthy ingredients of the ore, are supplied with the air required for intense combustion forced into them by pumps, which are worked generally by steam-engines. The volume of air supplied is very great, ranging from 1,500 to 2,000 cubic feet per minute; but its pressure is inconsiderable, seldom exceeding 3 lbs. per square inch above atmospheric pressure; and the pumps are therefore made of large capacity. In many places, the air, on its way to the furnace, is heated to a high temperature, so as to maintain a more intense combustion, and thereby extract a greater quantity of metal from the ore at a rapid rate. But it is found that the metal so produced, called *hot-blast* iron, though cheaper than *cold-blast* iron, is of inferior quality for all purposes where strength and tenacity are required. When the blast is not required at so high a pressure as that which is necessary for smelting furnaces—for ordinary foundry furnaces, and for blowing forge fires—the current of air is usually produced by a fan. The annexed plate represents a construction of fan which has obtained considerable, and we believe well-merited, repute for efficiency and durability. The fixed portion of this fan consists of a framing, which is generally bolted securely down to a block of stone or heavy timber, in order to avoid vibration, and a cylindrical casing, with circular openings on either side, and a mouth at one part of the circumference, which is connected to the pipe or channel for conveying the air-blast to the furnace. The framing is formed with long and carefully made bearings, in which a spindle revolves concentric with the cylindrical casing. On this spindle are fixed a number of recurved arms, fronted by plates bent to fit them, and the whole enclosed within two conical sides, which have lips projecting and fitting into the circular openings in the casing. The spindle and the arm-plates and conical sides being put in rotation at a high velocity —1,000, 1,500, or sometimes 2,000 revolutions per minute—air enters by the circular openings of the casing, passes between the two cones, and issues at their circumference into the casing, whence it proceeds with considerable velocity to the furnace. The force which produces the current of air is the centrifugal tendency of every particle to fly from the axis when it is put in rapid rotary motion, and the recurved blades act not only to carry the particles of air round in a circular direction, but also to drive them outwards from their inclined surfaces. The side casings of the blades are made conical, so that the area or channel through which the air passes should be nearly uniform throughout its course.

[On a similar principle, Messrs. Gwynne's centrifugal pump is constructed. The folio plate represents one of these engines, which was at work in the Exhibition at London, in 1862, and which raised a complete torrent of water by the great power it exerted. It is peculiarly suitable for draining purposes, and, generally, for lifting water in great quantities to moderate heights.—Ed.]

II. Driving Machinery.—Where the power required is small, or where fuel is cheap and water scarce, non-condensing engines are generally preferred. But, on the other hand, where there is a plentiful supply of water, and where fuel is expensive, condensing engines are employed; as by a certain expenditure of fuel, there is certainly a greater amount of power generated when the steam is condensed. For large manufactories, cotton, flax, and flour mills, breweries, and like establishments, the beam-engine is generally employed. Its advantages consist in the accessibility of all its parts in case of damage or repair, and the steadiness and regularity of its movement, resulting from its massiveness and solidity. The substantial and imposing look of a large beam-engine is certainly, however, with many persons, an argument for its use, where engines of less weight and bulk might be applied with quite as great advantage. The marine engine, having received great attention to its perfect construction, has often with advantage taken the place of ordinary beam-engines in manufacturing establishments; and we believe the only objection to their general use is, that they are rather more expensive in the first place. Where there is a deficiency of water for condensation, it is necessary to provide large reservoirs, in which the water discharged from the air-pump may have time to cool before it is again used in the condenser. Warm water, when used for injection, must either be admitted in quantities so great as to impede the engine by the additional work thrown on the air-pump—or, if limited in quantity, can only effect partial condensation of the steam, so that the piston is impeded by the imperfection of the vacuum.

The power of an engine required for manufacturing purposes, necessarily depends on the kind of work to be done, and its amount. A manufactory generally contains numerous machines, like and unlike; and unfortunately very few accurate experiments have been made as to the power required for working them. It is stated, that in the cotton manufacture, one horse-power is sufficient for 100 spindles of cotton yarn, and the machinery necessary for the preparation of the cotton; and, again, that one horse-power will work 12 power-looms. But, every day, improvements are being made in the construction of such machinery, and changes are introduced among the methods of preparing the material; and it therefore becomes difficult to assign any determinate method of estimating the power.

In the iron manufacture, steam-power is applied to the processes of rolling, shearing, flatting, punching, turning, planing, and the like. And so with operations on other metals, on wood, and generally all crude materials, the facility of deriving adequate power from the application of heat to water enables us to employ machinery instead of manual labour in a multitude of ways, so as not only to execute works that could not have been attempted without having such a force at command, but to do so with an economy and ease that could not otherwise be attained. In all these manufacturing operations it is essential to the safety as well as to the accurate working of the machinery, that the speed of the prime mover should be uniform.

Where there are numerous machines driven by the same power, it may often happen that several are in operation at once, while at another time none may be at work. To overcome the resistance caused by suddenly bringing a heavy machine into action, as in the case of a set of rollers for preparing iron, and at the same time to prevent the dangerous increase of speed that would result from suddenly throwing it out of work, it is essential that there should be a good governor and a very heavy fly-wheel. The fly-wheel acts as a reservoir of force in the one case, ready to give out a portion of its momentum when the additional resistance comes upon it; and as a reservoir of resistance or inertia, in the other case, ready to absorb a portion of the unnecessary power. The governor, again, permits the supply of additional steam to the cylinder as soon as the velocity of the engine begins to undergo diminution, and checks or throttles the supply whenever the speed rises above the average.

When the power required is small, or under circumstances where there is a difficulty in supplying water for condensation, or where there would be a great disadvantage in having any complicated machinery to be attended to, it is usual to employ non-condensing engines; these being more simple, are less costly in the first place, less liable to derangement and wear, and more easily understood and managed. It is true that for a given amount of power, the expenditure of fuel is somewhat greater than where the steam is condensed; but with a good boiler generating steam at a pressure of 40 to 60 lbs. if the cylinder be made of considerable size, and the steam cut off at an early part of the stroke so as to act expansively, the excess of fuel expenditure is not great. For agricultural purposes, in the colonies, or generally in districts removed from engineering establishments, simplicity is of more importance than even economy of fuel. Many of these non-condensing engines, from 5 to 15 horse-power, are made portable, so that the power may be readily brought to the work instead of the work being brought to the power. The portable engine consists of a cylindrical boiler, perforated with flues or tubes like that of a locomotive, mounted on wheels, with a direct-acting engine laid horizontally upon the boiler, or arranged in some convenient way for lightness and economy of space. For thrashing and winnowing corn, draining or irrigating land, brick and tile making, and field operations generally, these engines are very serviceable, especially in cases where the work of a farm is not enough to keep an engine constantly employed, and where the same engine may do duty throughout a considerable district. We believe, however, that as agriculture approaches more to the condition of a manufacturing art than it has hitherto done, the fixed engine will gradually supersede those that are portable; the farm buildings will become the head-quarters of the various operations; the materials will be brought thither to be operated on; and agricultural processes will be carried on with the regularity and precision of other branches of art. Of late years, many persons of high standing and energetic character have devoted themselves to the furtherance of this object; and their example is being generally followed throughout the country. His late Royal Highness the Prince Consort erected a very complete farming establishment near Windsor. A fixed engine there puts in motion a train of machines for thrashing, dressing, and grinding corn, cutting chaff, bruising beans and oats, steaming the food of cattle, and other processes, all systematically and economically arranged; and similar plans are adopted in many other places at home and abroad.

III. Locomotives.—In no application of steam-power has greater ingenuity been manifested than in locomotive engines. The great essentials of a locomotive are, lightness, compactness of strength of construction, rapidity of action, and facility of management. Some of the earliest attempts at these engines were necessarily rude and imperfect; but the rapid extension of the railway system, and the immense advantages derived from speed in the transmission of passengers and goods, have hurried on improvements so quickly, that in a very few years the locomotive has risen to a high degree of perfection. For some time it seemed difficult to contemplate the possibility of covering the land with a network of iron-ways, extending over many thousand miles in length; and accordingly attention was chiefly devoted to the construction of locomotives for running on ordinary roads. Some of the engines made for this purpose were tolerably successful in their operations, and embodied in their construction a great amount of ingenuity, which has not been without its use in leading to the more perfect locomotive of the present time. Even when railroads began to be formed, the employment of steam on them was a doubtful question, and their most sanguine promoters scarcely dared to hope for a speed exceeding 10 miles an hour, or to expect that passenger-traffic would equal in importance the conveyance of heavy goods. It was feared that the friction of an iron wheel on an iron road would not be sufficient to move onward the locomotive itself, much less a heavy train of carriages; and accordingly it was proposed to apply levers to act as legs and feet propelling the carriage, or to lay down a continuous rack, in which a toothed wheel driven by the engine might work, or an endless chain to be wound round a revolving barrel. It was not until the actual trial was made, that the simple expedient of causing a pair of wheels to revolve on the rails, was adopted without reserve.

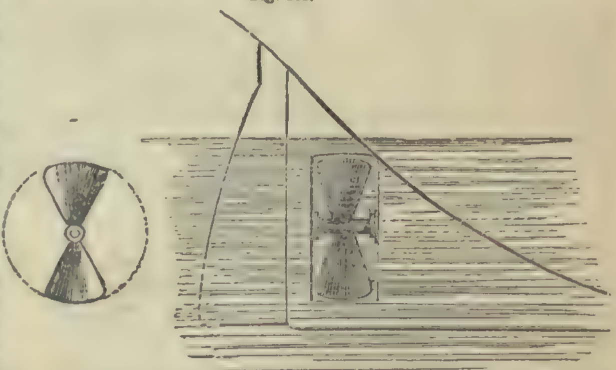
The construction of the boiler and engines of a locomotive has been already referred to (page 866). The whole power is applied to turn the crank-shaft, on which are fixed the driving-wheels. For each revolution of the engine, therefore, there is one revolution of the driving-wheels. Sometimes these are made 6 to 7 feet in diameter, and have therefore a circumference of 20 feet. If we suppose the engines to make 200 revolutions per minute, or 200 × 60 = 12,000 per hour, as the whole circumference of the driving-wheel is brought to bear on the rail during each revolution, if there be no slip, the locomotive must advance 20 feet for each revolution—that is to say, 12,000 × 20 = 240,000 feet, above 45½ miles per hour. When the rails are damp and slippery, the wheels occasionally slip round without taking sufficient hold of them to propel the train. In such cases, a little sand strewn on the rails generally causes sufficient friction to commence the movement of the train; and its momentum once established, the locomotive has only to keep it up against the resistance of the air, and the friction of the running wheels. When the locomotive is applied to the propulsion of heavy goods-trains, especially up inclinations, it is necessary to obtain more friction than could be derived from one pair of driving-wheels. For this purpose the locomotive is fitted with one, and sometimes two additional pairs of driving-wheels, of the same size as those worked by the engine. These wheels are coupled by connecting-rods working on pins projecting from their arms, so that they all revolve simultaneously, and thus present double or treble the amount of rubbing surface on the rails. The locomotive has attached to it a tender, which is an iron tank mounted on wheels, divided into two compartments, one containing fuel and the other water for the supply of the engine. The water-tank is connected by a flexible tube to the feed-pump of the locomotive, by which the pump draws its supply, and forces it into the boiler. In order to save fuel, the water in the tank is heated by steam blown through it from the boiler, while the train is stopping, when the steam is blowing off to waste by the safety-valve. Occasionally the water-tank is fixed on the top of the boiler, and the tender is dispensed with. The engine-driver and stoker stand on a stage at the fire-box end of the boiler, and have under their eye the water and steam-gauges. They have conveniently arranged levers for working the steam-valve, so as to permit more or less steam to enter the cylinder; for moving the link-motion of the slides, so as to reverse the motion of the engines; for adjusting the pressure on the safety-valve, the supply of feed-water, and the break on the wheels when it is necessary to stop or move more slowly. They have convenient means of oiling all the working parts, and of opening pet-cocks in the cylinders to permit the issue of water, and the cock for sounding the steam-whistle as a signal of their approach. Altogether, the locomotive is perhaps the most perfect apparatus that has been designed by engineers; and the materials and workmanship applied in its construction are necessarily of the best kind, to withstand the constantly reiterated shocks of the movement, and to convey power so great through parts so light and apparently so complex.

IV. Propulsion of Vessels.—Although numerous modes of applying steam to the propulsion of vessels have been proposed, only two have met with general adoption—viz., the paddle-wheel and the screw-propeller. The action of paddle-wheel engines is precisely similar to that of locomotives: the paddles in the one case occupying the place of the driving-wheels in the other, and having float-boards successively immersed in the water, and withdrawn from it as the wheels revolve. The paddle of a steam-vessel is, indeed, an undershot water-wheel reversed in its action; that is to say, instead of the wheel being put in motion by the current of water, the wheel put in motion by the steam-engine causes a current of water by its revolution; and the reaction of the water propelled by it backwards, forces the wheel forwards, and with it the vessel to which it is attached. As the float-boards enter and leave the water, they act on it obliquely, tending in some measure to press it down in front and raise it behind. In sea-going vessels, where the wheels often act on the undulating surface of the water, this obliquity of action becomes a very considerable resistance, and tends to retard the engines and give them severe shocks. To obviate this defect, paddles are frequently made in such a manner that while passing through the water the floats hang nearly vertical. Such a wheel is called a *feathering* paddle, because the float-boards *feather*, or enter and leave the water edgeways, like the oar in the hands of a practised rower. The power required for a steam-vessel depends upon its form and tonnage, and the speed at which it is moved.

Steam-vessels are generally made very long in proportion to their breadth, and finely wedge-shaped at each end, so as to cut through the water with as little resistance as possible. In vessels of similar form and proportions, the power required to produce a given speed is nearly as the tonnage. But a very slight increase of speed demands a very considerable augmentation of power, as may be thus estimated:—To double the speed of a vessel it is necessary to push aside double the quantity of water in a given time, and to impel this water with double the velocity, and therefore to encounter four times the resistance; and, as the speed of the engines must be at least doubled, the power expended in a given time must be at least eight times. Generally the power may be taken as the cube of the speed, or rather more. If, for example, a pair of engines working to 100 horse-power, propelled a vessel at the rate of 8 knots (nautical miles) per hour, we should have to work the engines up to 200 horse-power to attain a speed of 10 knots per hour; because while the cube of 8, or 8 × 8 × 8 = 512, the cube of 10 is 1,000, nearly double of 512, and therefore the power in the one case must be double of that in the other. The same law applies in the case of vessels propelled by a screw.

The principle upon which the screw acts as a propeller, may be best understood by considering the action of a windmill reversed. The screw generally consists of two inclined blades (Fig. 211) projecting from an axis mounted in bearings, in what is called the dead-wood of a vessel, or the part of the stern immediately before the rudder. The screw-shaft works through a water-tight tube fixed in the dead-wood, and is put in motion by the steam-engines in the body of the vessel. The whole screw is immersed under water, and the blades are of such size that, looking endways on the shaft, they appear to occupy each about one-sixth of the circle which circumscribes them.

Fig. 211.



The *pitch* of the screw depends on the obliquity of the blades to the plane of the circle, and this obliquity increases from the circumference towards the centre, because the parts nearer the centre travel with less circumferential velocity, and yet the water passes them with equal longitudinal speed. When we say that a screw has twenty-feet pitch, we mean that if the screw-blade were continued through a complete revolution round its axis, any straight line drawn parallel to the axis would cut two portions of the blade at points twenty feet apart. Did the screw, in revolving through the water, put the latter in motion at such a rate as precisely to glide along its blades in straight lines parallel to the axis, every revolution of the screw would move any portion of water exposed to it through a length of 20 feet. But, practically, the water is almost at rest, and the screw worms its way through it, having to put in motion the vessel to which it is connected, and a certain *slip* occurs, or the speed with which the screw and vessel travel through the water is less than that due to the obliquity of the screw by about 25 or 30 per cent. If we suppose that a screw of 20 feet pitch makes 60 revolutions per minute, any point in its surface without slip would move through 20 × 60 = 1200 feet longitudinally per minute, or 1200 × 60 = 72000 feet = 12 knots per hour nearly. But, allowing twenty-five per cent., or one-fourth, for slip, the actual speed would be nine knots per hour.

[THE STEAM HAMMER.—Of late years, the immense demand for heavy forgings has necessitated novel methods for reducing them to the proper shape, especially when required in connection with screw-propellers, one of the largest of which was that used in the *Great Eastern*. The steam hammer has been much employed for this purpose; and in the hands of Nasmyth, Condie, and others, has been brought to a high degree of perfection. The general principle of construction is that of a solid vertical piston, moved in a cylinder by steam; the motion being regulated in a similar manner to that of the ordinary steam-engine, by the action of a slide-valve moved by an attendant. The piston-rod is extended, so as to form the hammer-head; and below, on a suitable block, the iron to be forged is placed. The admission of steam beneath the piston raises it, and the hammer-head can then be driven, so as to impinge directly, at the extremity of the rod, on the article to be forged. By regulating the admission of steam, any amount of power may be exerted—a fact of great value to the practical engineer. The folio plate represents a machine of this kind, which was shown in action in the Exhibition at London, in 1862.—ED.]

CHAPTER VII.

THE COMMUNICATION OF POWER.

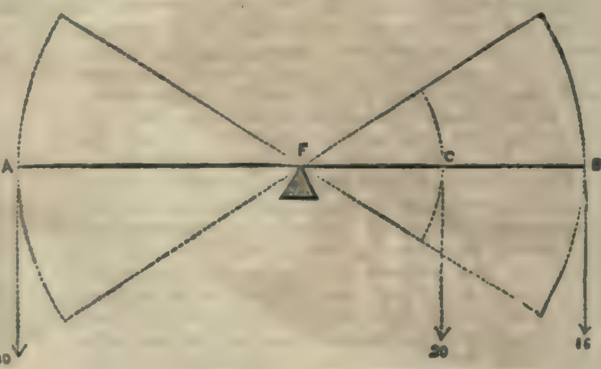
Contents.—WORK AND POWER—ROTARY MOTION—COUPLINGS—CLUTCHES—PLUMMER-BLOCKS—PULLEYS, OR DRUMS—TOOTHED WHEELS—FORMS OF TEETH—INVOLUTE OF A CIRCLE—BEVIL GEAR—RACKS—WORM AND WHEEL—EPICYCLOIDAL TEETH—RECIPROCATING MOTION—DISCONTINUOUS MOTION—RATCHET—CAM—MANGLE-MOTION—REVERSING GEAR—SCREW-CUTTING GEAR—ELLIPTICAL GEAR—SUN AND PLANET—FRICTION BREAKS—DYNAMOMETER—BREAKS—RESISTANCE OF FRICTION—FRICTION OF STRAPS—MOTION OF FLUIDS.

WORK AND POWER.—The communication of power is really the communication of motion, for power implies change, and change of place is motion. A column supporting a weight communicates the pressure of the weight to its foundation; and a string suspending a load from a hook, communicates its tension to the hook; but in neither of these cases is *power* communicated, for no change is effected: time forms no element in the question, the same strain being conveyed to the point of support in an instant, as in a century. But if the column or string be in motion, like the piston-rod of a steam-engine, pushing or pulling against a resisting force, not only is the pressure acting on the piston conveyed through the rod as a simple strain, but it is also conveyed through a certain distance in a certain time, or at a certain velocity, and can, by its motion, effect changes on materials presented to it, proportioned to the amount of power developed, and the time during which it acts. The *power* of any mechanical arrangement, therefore, means its capability of effecting change; while its *work* means the quantity of change effected. The most simple kind of change which we can contemplate is that of position; and we, therefore, take change of position or motion as the measure of one element of work done. The most simple notion as to quantity of change, is that of mass or weight of material moved; and we therefore take weight as the measure of another element of work. Lastly, we estimate the capability of effecting change—that is to say, power—by the time required for the work done. The greater the mass moved, the greater the distance over which it is moved; and the less the time occupied in the motion, the greater the power developed.

An engine of 10 horse-power, means an engine capable of moving 10 times the mass that can be moved by an engine of 1 horse-power over a given distance in a given time, or of moving the same mass over 10 times the distance in the same time, or of moving the same mass over the same distance in $\frac{1}{10}$th of the time. Power, indeed, is simply pressure multiplied by velocity; and if we know the pressures and velocities communicated through any trains of machinery, we compare their powers by comparing the products of these factors.

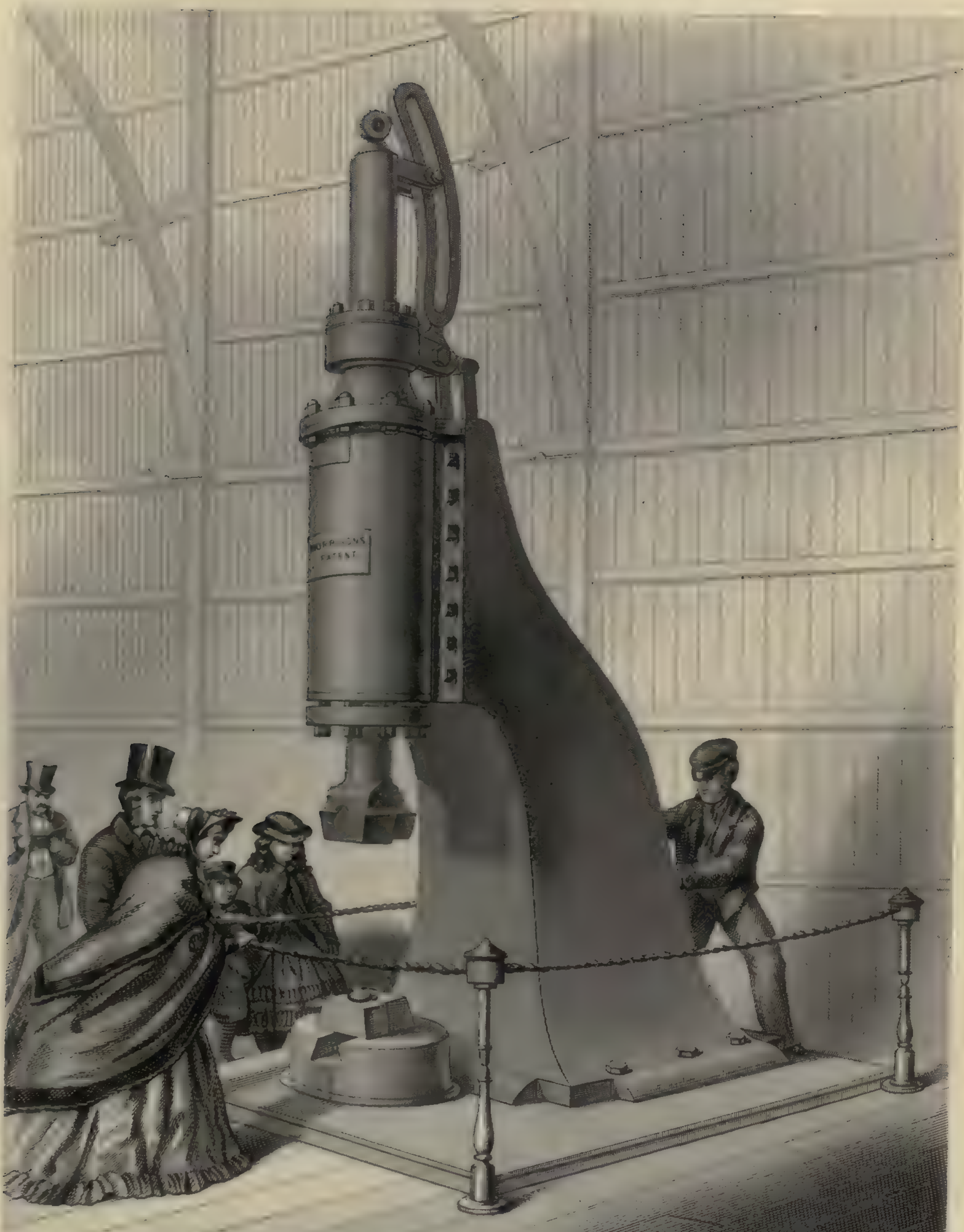
It is a simple law of mechanics, admitting of no exception, that, whatever be the nature or complication of any mechanical arrangement by which power is conveyed, whatever be the power applied to move it, the same power would be given out from every part of it, provided no waste occurred from friction. And the more perfect the construction of the machinery in respect of smoothness of rubbing surfaces, the more nearly is this law practically fulfilled.

Fig. 212.



If we take any of the simple mechanical powers, such as the lever A B (Fig. 212), capable of vibrating on the fulcrum F, we observe that when A F is equal in length to F B, a weight of 10 lbs. hung from A, balances the same weight hung from B; or, when A F is twice F C, a weight of 10 lbs. at A balances 20 lbs. at C.

Such is the law for mere pressure or weight without motion; but when we consider the question in respect of power, we have to imagine that the lever vibrates on its fulcrum, so that every point of it describes a portion of a circle round the centre F. We may suppose one such



MORRISON'S PATENT DOUBLE ACTING STEAM HAMMER.

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vibration effected in a second, and that the arc described by A measures 2 feet in length, then will the arc described by C be 1 foot long; and 10 lbs. at A, moving through 2 feet in 1 second, develops the same power as 20 lbs. at C moving through 1 foot in 1 second, because 10 × 2=20 × 1; the products of the pressures by the velocities are equal in both cases. We might trace the same law through all the simple mechanical powers, and through all possible combinations of them; that is to say, through all possible arrangements of machinery.

The practical application of this law in calculating the proper strengths and proportions of parts of machines, is exceedingly simple.

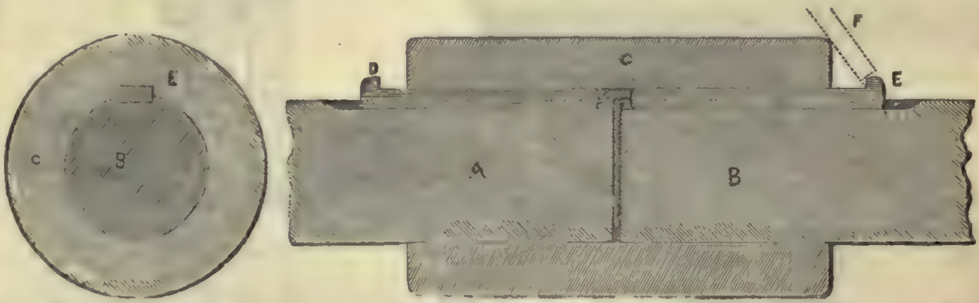
In a machine, for instance, moved by 1 horse-power, whenever we can ascertain the velocity of any part, we can find at once the pressure or strain passing through that part. One-horse power is reckoned as 33,000 lbs. moved 1 foot per minute; if, then, some part of the machine in question were found to move over 10 feet in 1 second, or 600 feet in 1 minute, we should at once know that the strain on that part is equivalent to a pressure of 55 lbs., for 55 lbs. × 600 feet=33000 lbs. × 1 foot. As a practical example, let us suppose that in some part of a machine worked by 5 horse-power, there is a wheel 7 feet in diameter, revolving 30 times in a minute, and that we desire to estimate the pressure on the rim of that wheel, or the resisting force which would have to be applied to balance its rotary force.

The circumference of a wheel 7 feet in diameter is 22 feet, and this moving 30 times round in a minute, has a velocity of 22 × 30=660 feet per minute; the power, 5-horse, is equivalent to 33000 × 5= 165000 lbs. moved through 1 foot per minute, or 250 lbs. moved through 660 feet per minute. The strain on the rim of the wheel is therefore equivalent to 250 lbs. Generally, the rule for estimating the strain (in lbs.) of any part, is to divide the power (expressed in lbs. moved over 1 foot per minute) by the velocity of the part (in feet per minute).

ROTARY MOTION.—The kind of motion most conveniently conveyed through machinery is rotary motion; and the chief subject of our enquiry will be as to how this can be conveyed and converted into motions of another kind, such as vibratory or reciprocating movements.

A shaft or spindle is a rigid bar of metal, or sometimes of wood, caused to rotate round its axis, and capable of conveying the rotary motion given to it along its whole length, and giving it off at any point to machinery connected with it. The lengths of shafts are limited in consequence of the difficulties attending their construction in great lengths. Large shafts, such as are used for the paddles of marine engines, are sometimes 20 or 30 feet long; but smaller shafts seldom exceed 10 or 12 feet in length. When greater lengths are required, the shafts have to be coupled or connected together.

Fig. 213.

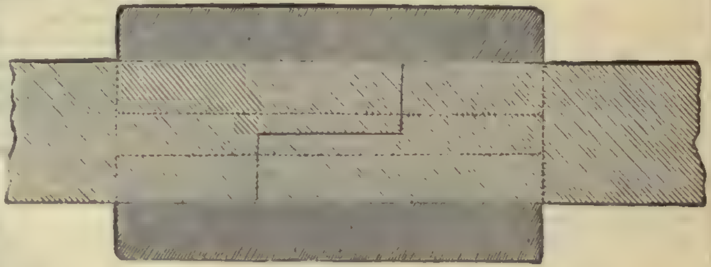


COUPLINGS.—Fig. 213 represents a simple coupling for round wrought-iron shafts. A and B, the ends of the two shafts, being nicely rounded, are inserted into a cylindrical box C bored to fit them, and having a groove or key-way cut along its interior surface, corresponding to grooves cut in the shafts. These grooves are filled by keys, D E, made slightly taper, and driven tightly in from each end. One of the shafts A being caused to rotate, must carry the coupling C round with it, and also the other shaft B, because the keys prevent either shaft from slipping round within the coupling, or the coupling from slipping round either. In making such a coupling of cast-iron, the following proportions will be found practically suitable :—External diameter of coupling double that of shaft; length of coupling three times the diameter shaft; width of key, one-fourth the diameter of shaft; mean depth of key, one-half its width.

The keys should be made with *gib-heads*, as shown in the figure, so that they may be driven out when required, by applying a piece of iron rod to the projecting part and striking it with a hammer, as marked by the dotted lines F, or by driving a wedge between the gib-head and the coupling.*

Some engineers, in coupling the shafts, halve them over each other, as in Fig. 214, and insert a tapered key in a groove formed in the coupling only, its inner surface fitting the shafts. Such a key, called technically a *hollow key*, from the hollowing or concavity of its inner surface, tightens the coupling very firmly on the shafts. The *face-coupling* (Fig. 215) consists of two discs, with projecting bosses keyed on the ends of the shafts facing each other, held together by four or more bolts and nuts passing through corresponding holes in the two discs.

Fig. 214.



Sometimes, when it is desired that by no accident shall the movement of any length of shafting be reversed, a ratchet-coupling is applied. *a* (Fig. 216) is a toothed boss, keyed firmly on the shaft *d*, and *b* a similar toothed boss capable of sliding longitudinally on the shaft *c* along a *feather* or parallel key, which prevents it from turning round independently of *c*. So long as the shaft *d* revolves in one direction, it carries the other shaft *c* round with it; but should the rotation of *d* be reversed, the pressure on the inclined faces of the teeth causes the boss *b* to slip out of gear with *a*, and thus to uncouple the two shafts.

CLUTCH.—When it is desired to couple or uncouple two lengths of shafting at pleasure, a *clutch*, like that represented in Fig. 217, is generally employed.

A, the driving shaft, on which is keyed firmly a half

* A "key," used for fixing a coupling or any other piece of machinery on a round shaft, is a piece of iron or steel of parallel breadth, but slightly tapered in thickness, so that on driving it into the "key-way" or recess provided for it, it acts as a wedge, pressing the opposite surface of the shaft against that of the hole. A "feather" is a key of equal breadth and thickness throughout, fitted into a recess in the shaft, and projecting from it into a slot formed in the hole of the coupling or other boss, which can thus slide along the shaft longitudinally, but must turn with it.

clutch C, having two projections from its face; B the driven shaft, the end of which rests in the hole of C, and on which is fitted, by a *feather* or parallel key, the other half clutch D, having also projections on its face similar to those on C. A groove is formed in D, and fitted with a ring E, put together in halves, and having pins projecting from its opposite sides, which may be taken hold of by hand or by a lever. The half clutch D being capable of sliding on the shaft B, can, by means of the ring, be pushed in or out of gear with C; and as the feather prevents it from turning on its shaft, when the projections on it are inserted between those in the other half clutch, the shaft B is caused to revolve by the rotation of A. Sometimes, when it is desired that the strain passing through any shaft shall be limited, recourse is had to *friction-clutches*. A very common mode of forming a friction-coupling is to key on one length of shaft, a boss with a conical hole bored in its face, and on the other to fit by a feather, a conical boss fitting nicely into the conical hole. When the one is pressed into the other, the friction of the conical surfaces causes them to revolve together; but should an excessive resistance be opposed to the rotation of the driven shaft, the friction of the surfaces proves insufficient to overcome it, and the cone on the driving shaft slips round that on the other without driving it.

Fig. 215.

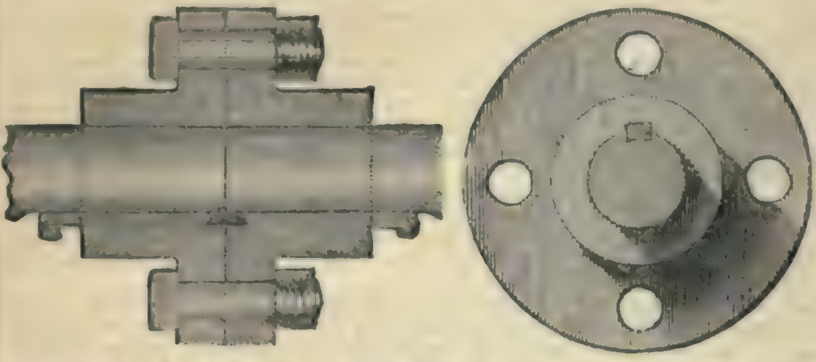


Fig. 216.

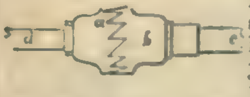
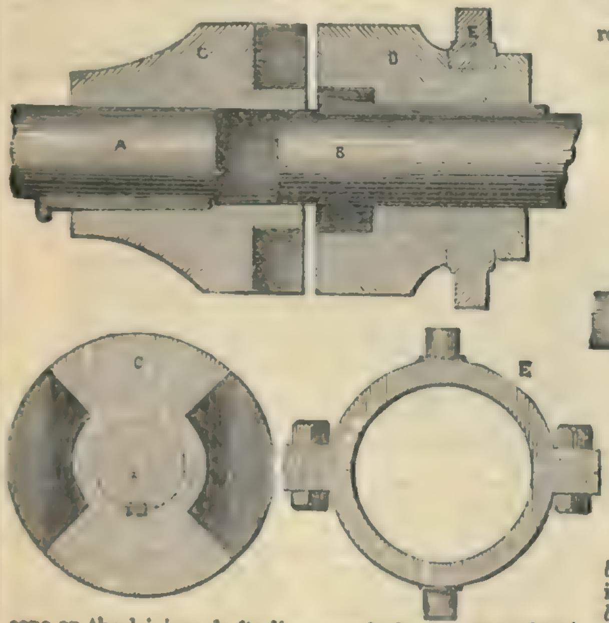


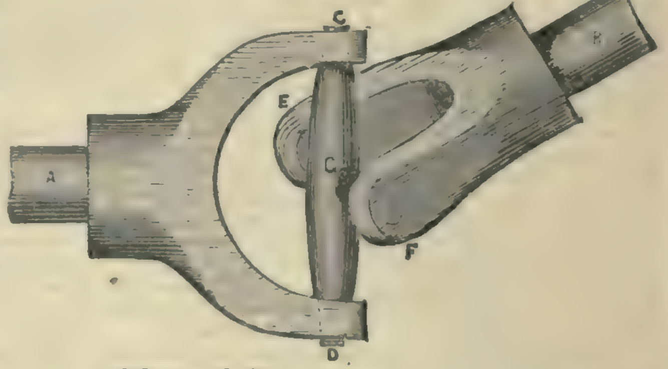
Fig. 217.



UNIVERSAL JOINT.—When two shafts, not lying in the same straight line, are to be coupled, recourse is had to a universal joint. On each of the two shafts A and B (Fig. 218) is keyed a fork, in the ends of which, at C, D, E and F, are pivoted the ends of a cross G, so as to permit its partial rotation on either of its two axes C D or F E. On A being caused to revolve, the cross is carried with it round its centre G, and its ends E F also rotate round the same centre, giving rotary motion to the other fork and shaft B, the pivoting of the cross permitting it to accommodate its position to the varying obliquity of the forks during their rotation.

When two shafts are not quite in the same straight line, but nearly so, they may be coupled by cranks. The shaft A (Fig. 219) has a crank C and pin keyed on to it, and the shaft B has also a crank D, with a slot cut in it to receive the crank-pin of C. The pin must be sufficiently long to allow for the departure of the two cranks from each other by obliquity of position (as indicated by the dotted lines representing them at the opposite point of their revolution); and the length of the slot must be such as to allow for the difference of level of the two shafts.

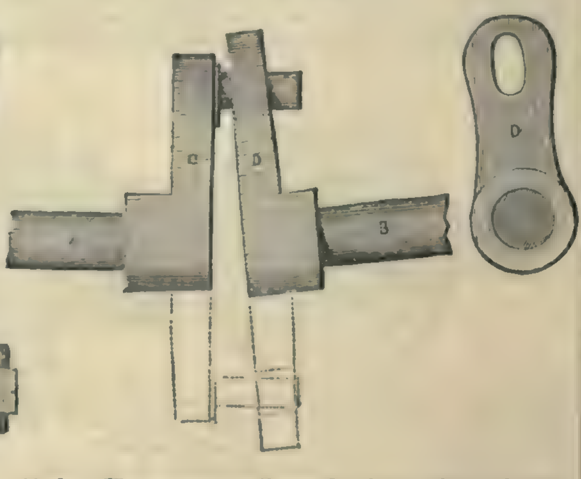
Fig. 218.



PLUMMER-BLOCKS.—The bearings in which shafts revolve, are technically called *pillow-blocks* or *plummer-blocks*. They are generally made of two pieces of cast-iron, the *base* A, and the *cap* B, lined with gun-metal *bushes* C, each half a cylinder (Fig. 220).

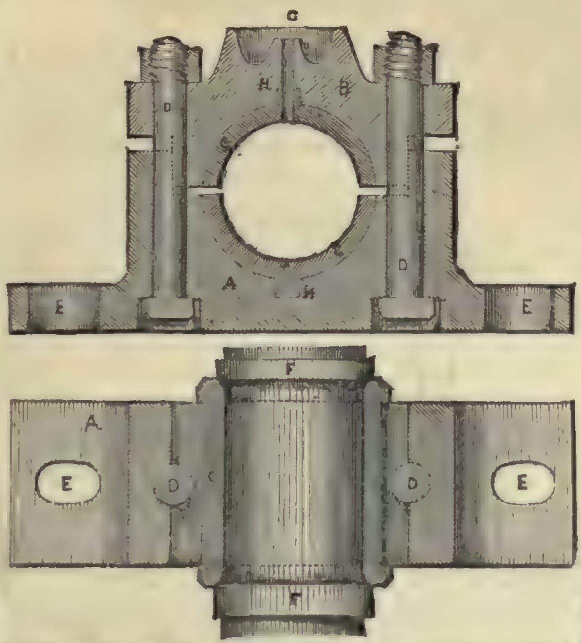
The base is recessed to receive projecting parts of the cap; and two bolts D, with nuts at their upper ends, pass through holes in both, so as to secure the cap firmly

Fig. 219.



on the base when the shaft has been laid in its place. The base has projecting flanges, with holes E, through which bolts pass for securing the *plummer-block* to the beam or frame on which it may be fixed. These holes are lengthened to permit a little transverse adjustment of the plummer-block. The cap has generally an oil-cup in its upper part, with a projecting nipple perforated, through which a cotton wick conveys a supply of oil to the surface of the shaft, acting like a siphon by capillary attraction. The *brasses* C are sometimes made square or octagonal in their outer surfaces, to prevent them from turning round within the plummer-block ; or, when they are quite cylindrical, small studs or steady-pins H, projecting from them into holes in the cap and base, answer the same purpose. The shaft has collars F, or parts of larger diameter outside the brasses, and the brasses have lips or flanges, to prevent the shaft from moving longitudinally. As the shaft or brasses wear from the constant friction, the cap and upper brass can be more tightly screwed down by the nuts of the bolts D, which are often made double, so that the one nut tightens the other in its place, and prevents it from becoming loosened by the shaking of the machinery.

Fig. 220.



Of late years, many bearings, instead of being fitted with gun-metal bushes, have been lined with a soft metal, in which tin is the principal ingredient.

Fig. 221 represents a transverse section of a bearing lined with soft metal ; A the cast-iron cap, and B the base, cast with a hole larger than the shaft. D D soft metal linings, filling the space between the surface of the shaft and that of the cast-iron. The shaft, or a piece of iron of equal size, being supported in the hole of the bearing, the soft metal, melted, is poured into the space ; and if the iron have been previously heated to nearly the melting temperature of the soft metal, the latter, on cooling, presents a smooth internal surface, in which the shaft revolves with little friction or wear. When it is inconvenient to form collars on the shaft to prevent longitudinal motion, rings E, bored to fit the shaft, are put on, and kept in their places by tightening-screws pressing against the shaft.

Fig. 221.

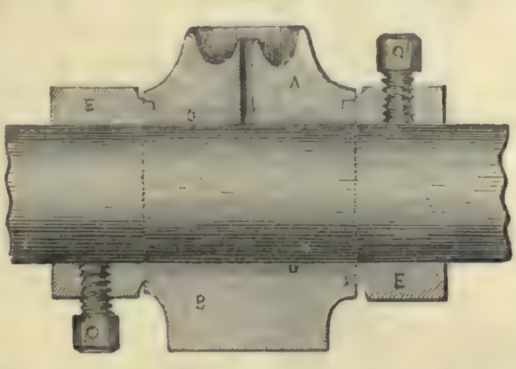
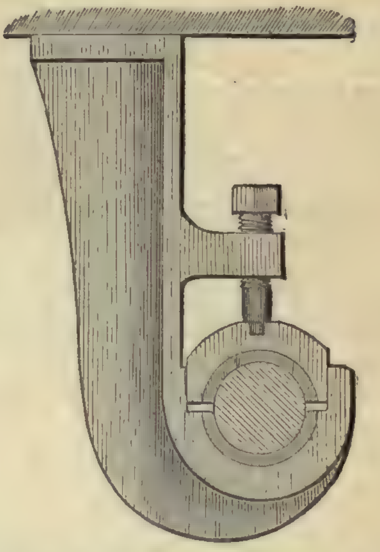


Fig. 222.



When it is not convenient to rest a shaft in plummer-blocks supported on walls or beams, they may be made to revolve in hanging bearings screwed up to a beam above. These bearings may be lined as plummer-blocks, with gun-metal or soft metal, and the cap tightened down by a setting-screw, as indicated in Fig. 222.

PULLEYS OR DRUMS.—When it is desired to convey motion from one shaft to another lying parallel to it, each shaft has a *pulley* or *drum* attached to it, and round their circumferences a flexible strap or band is tightly strained. Thus *e* and *f* (Fig. 223) being parallel shafts at some distance apart, a pulley *a* is keyed on the one, and a pulley *b* on the other, the flexible strap passing round both. On rotary motion being given to the shaft *a* and its pulley, the friction between the strap and its circumference puts the former in motion, and it conveys the rotation to *b* and its shaft in the same direction.

Fig. 223.

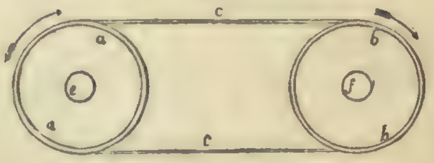
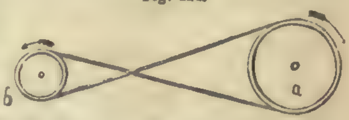


Fig. 224.



When it is desired to reverse the direction of rotation, the strap is crossed (Fig. 224). Also when a different angular velocity of rotation is desired, the pulleys are made of different sizes. If we suppose that the driving pulley *b* has a circumference of 6 feet, and the driven pulley *a* circumference of 3 feet, half the former ; since the strap travels at the same rate with the circumference of the driving pulley, and causes the circumference of the driven pulley to move round with equal velocity, it is clear that the latter must make two revolutions round its axis while the former makes one ; or that the angular velocity of the smaller pulley is just double that of the larger, because its circumference is half the length. If we take the circumferences in any other ratio, we should find the angular velocities in the inverse ratio. As the circumferences of circles are exactly proportional to their diameters, the angular velocities of pulleys connected by straps are inversely as their diameters, or the diameter of each pulley, multiplied by its speed, gives a like product. In calculating the speeds due to given dimensions of pulleys, or the dimensions suited for given speeds, we have therefore the following simple rules :—

Given the angular velocity of one of two pulleys (*i. e.*, the number of revolutions per minute), and the diameters of both, to find the speed of the other.

Rule.—Multiply the speed of the first by its diameter, and divide by the diameter of the other.

Example.—A pulley 36 inches in diameter, making 80 revolutions per minute, drives a pulley 24 inches in diameter : required the speed of the latter.

$$\frac{80 \times 36}{24} = 120 \text{ revolutions per minute.}$$

Given the angular velocities of two pulleys and the diameter of one, to find that of the other.

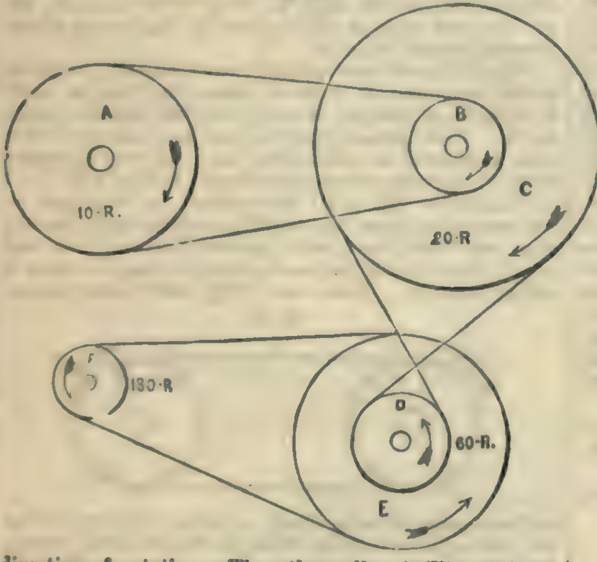
Rule.—Multiply the speed of the first by its diameter, and divide by the speed of the other.

Example.—A pulley 24 inches diameter, making 120 revolutions per minute, drives another at 80 revolutions per minute : required the diameter of the latter.

$$\frac{120 \times 24}{80} = 36 \text{ inches.}$$

Pulleys are often combined with direct or crossed straps, as may be required, in order to vary the velocity and direction of rotation. Thus the pulley A (Fig. 225), 24 inches diameter, revolving at the rate of 10 revolutions per minute, drives B, 12 inches diameter, at 20 revolutions, because $\frac{24 \times 10}{12} = 20$. On the shaft of B is fixed a pulley C, 36 inches diameter, revolving at the same speed with B, and driving D, 12 inches diameter, by a crossed strap at 60 revolutions, because $\frac{20 \times 36}{12} = 60$. Again, on the shaft of D is another pulley E, 27 inches diameter, revolving with a speed of 60, and driving F, 9 inches diameter, at a speed of 180, because $\frac{60 \times 27}{9} = 180$.

Fig. 225.



In estimating the final speed produced by such a train of pulleys, it is not necessary thus to calculate the velocity of each, as we find the same result by multiplying the initial speed of the driver A, by the product of the diameters of all the drivers A, C, and E, and dividing by the product of the diameters of those driven, B, D, and F ; or

$$\frac{10 \text{ revols.} \times 24 \times 36 \times 27}{12 \times 12 \times 9} = 180 \text{ revolutions, the speed of F.}$$

Sometimes, when it is desirable to have the power of varying the speed of machinery driven by a strap, the pulleys are elongated and made conical, and the strap is shifted longitudinally to such a position as shall give the speed required. Thus, if *a a* be the driving-shaft, revolving 24 times per minute, and it be desirable that *b b* shall be driven at any speed from 8 to 72 revolutions per minute, each pulley being made conical with the larger end 3 times the diameter of the smaller, the strap *c* may be shifted from one end to the other to give the desired variation of speed. Nor is the tightness of the strap materially altered ; for as it is made to pass round a larger circumference of the one, it passes round a smaller circumference of the other, and so gains nearly as much as it loses. Most of the machines that are driven by straps do not require so nice an adjustment of velocity, but act very well at various speeds within certain limits. In such cases the conical pulleys are made with regular steps, or consist each of a set of pulleys placed side by side, having diameters increasing as much in the one as they decrease in the other (Fig. 227). Thus, if the driving-shaft make 48 revolutions per minute, the driven shaft may be made to revolve at the following different speeds :—

Fig. 226.

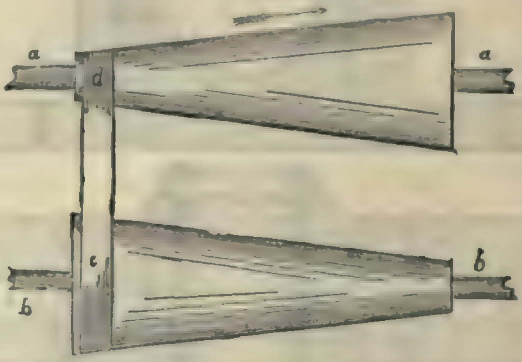


Fig. 227.

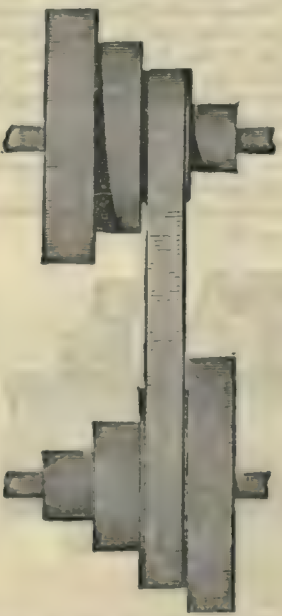
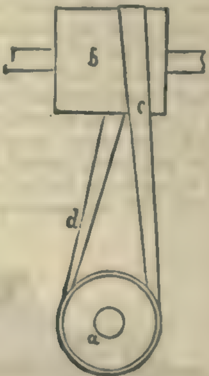


Fig. 228.



$$\frac{48 \times 16}{4} = 192,\ 48 \times \frac{12}{8} = 72,$$

$$48 \times \frac{8}{12} = 32, \text{ and } \frac{48 \times 4}{16} = 12$$

revolutions per minute; the diameters being 16, 12, 8, and 4 inches in each.

Occasionally, a strap may be employed when it is desired that the motion shall be changed transversely, as indicated in Fig. 228. In such a case it is necessary to have very wide pulleys, *a* and *b*, because the two parts of the strap *c* and *d* must pass obliquely over their surfaces. This mode of employing a strap is, however, very disadvantageous, for the strap must be continually slipping longitudinally along the face of each pulley, and thereby be subjected to friction and wear.

In a manufactory where numerous machines are employed, it is essential to have a ready means of throwing any of them into action or out of action at pleasure. When the machine is driven by a strap, this operation is conveniently effected by what are called the *fast* and *loose*, or the *live* and *dead* pulleys; that is to say, two pulleys of equal size placed side by side on the shaft, one being firmly fixed to the shaft by a key so as to revolve with it, while the other revolves freely on the shaft. When it is desired to put the machine in action, the band is put on the *fast* pulley, and thus gives motion to the shaft or receives motion from it. For throwing the machine out of action, the band is shifted to the circumference of the *loose* pulley, which revolves idly on the shaft. The shifting of the strap from the one pulley to the other is effected by means of the *striking gear*, a forked lever *f e d* (Fig. 230) moving on a pivot *e*, and holding the strap *c* within its forked part *d*. By pulling round the handle *f* of the lever, the band, being pressed edgeways by one side of the fork, is pushed gradually on to the fast pulley *a* or the loose pulley *b* at pleasure. It would be difficult thus to shift a tight strap when at rest; but while it is in motion, with the pressure of the fork directed on it, it is gradually edged off from its former position. For this reason, the *fast* and *loose* pulleys should always be placed on the *driven* shaft, because, were they placed on the *driving* shaft, the band when on the *loose* pulley, being at rest, could not easily be shifted.

Fig. 229.

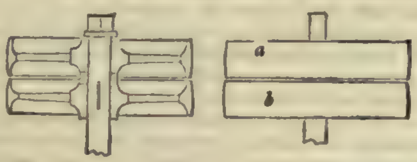
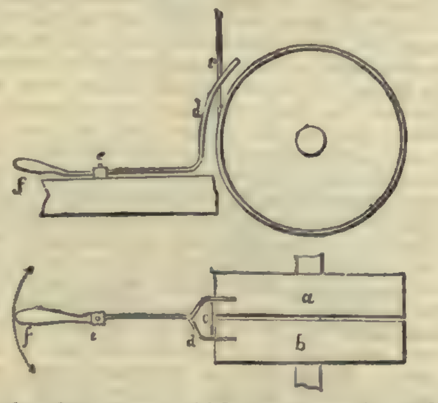


Fig. 230.

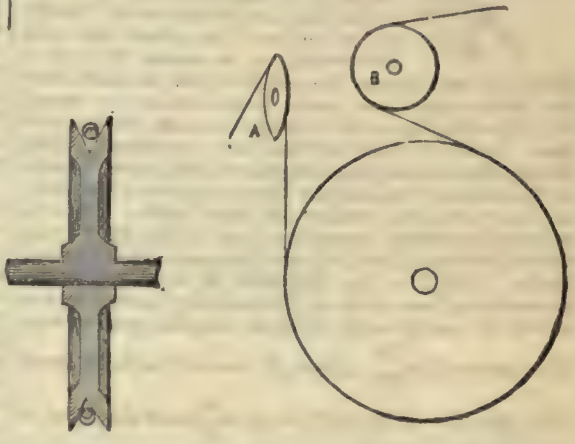


In order that straps, which are generally made of leather, india-rubber, or *gutta-percha*, may act well without slipping ineffectively along the surfaces of the pulleys, they should be tightly strained into their place. The pulleys should be turned true and smooth on their circumferences; for the smoother the pulley, the more driving friction appears to take place between its surface and that of the strap. When the strap is not sufficiently tight, or does not take sufficient hold of the pulley to drive the machinery with which it is connected, it must be made of greater breadth, and strained more tightly. Sometimes, when a strap slips, powdered resin is strewn upon its inner surface with good effect. The greater the arc of the circumference which a strap embraces, the firmer is its hold, or the less tendency has it to slip; and the greater the width of the strap, the firmer also is its hold. The diameter of the pulley does not affect the slip of the strap, except when the diameter is very small; while the strap is rigid, and not easily bent closely round the circumference.

Were straps perfectly flexible, it would be found that, with a given breadth and tightness, the driving power would be the same for all diameters of pulleys.

It is often convenient to use cylindrical bands or cords, made of catgut, or gutta-percha, instead of flat straps (Fig. 231). In such cases, the pulley is grooved in the circumference, and the band wedges itself into the groove, and thus exerts sufficient friction on the circumference for the communication of power from one pulley to another. The principal advantage of bands of this kind is, that by means of guide pulleys, A and B, grooved in the circumference, the band may be led in almost any direction on the plane of the pulley, or oblique to it.

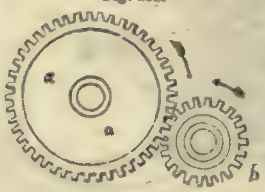
Fig. 231.



Straps are best suited for machinery driven at high speeds, with small pressures; they are cheap, convenient, and smooth and noiseless in their action; and when any undue strain comes upon the machinery, they slip, and thus act as a safeguard against damage. But for all machinery intended to communicate heavy strains through their parts, as in cranes, and whenever certainty of connection between one moving part and another is required, as in clockwork, recourse must be had to *gearing*, or toothed wheels.

TOOTHED WHEELS.—Were we to mount two wheels on parallel axes, so that their circumferences touch each other, by causing the one to revolve we should also cause the other to revolve, provided the friction between their surfaces at the point of contact were sufficient to overcome such resistance as might be presented to the rotation of the driven wheel. Practically, this friction is not sufficient in ordinary cases, and it becomes necessary to fit the circumferences with teeth or cogs, or projections and recesses at corresponding intervals, so that each tooth of the one fits successively in each recess of the other. By this arrangement, the direction of rotation is reversed, as indicated by the arrows in Fig. 232. When it is desired that the direction of rotation be retained the same, it is twice reversed by the intro-

Fig. 232.



duction of an intermediate wheel *b* (Fig. 233). Sometimes wheels are geared internally (Fig. 234), where *a* is a portion of the one, and *b b* part of the circumference of the other. In this case, the direction of rotation is not reversed.

Fig. 233.

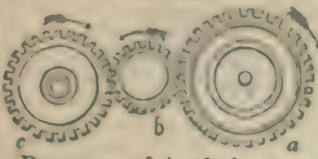
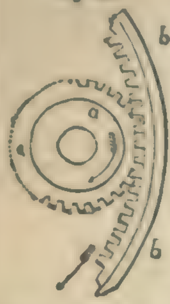


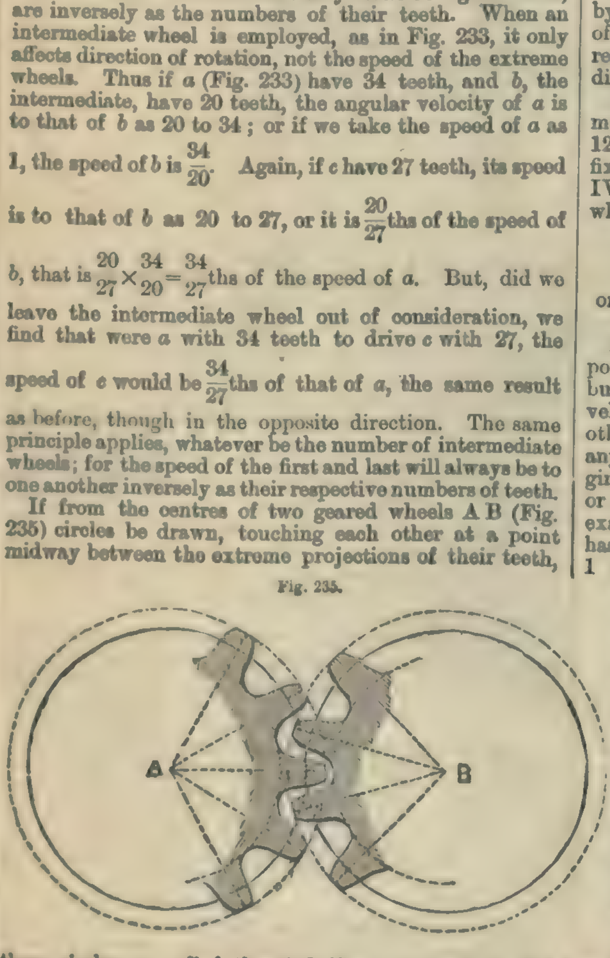
Fig. 234.



By means of toothed gearing, the angular speed of rotation may be altered at pleasure, as will be evident from the following considerations. The larger wheel (Fig. 232) has 45 teeth, and therefore, during each revolution, presents 45 successive recesses for the teeth of the smaller pinion, which number 22. For every revolution of the wheel, therefore, the pinion must make 2 revolutions and advance the space of 1 tooth, because $2 \times 22 + 1 = 45$; during 2 revolutions of the wheel, the pinion makes 4 revolutions and 2 teeth; and so on, until the wheel has made 22 revolutions, or caused $22 \times 45 = 990$ of its teeth to pass the point of gear, in which time the pinion must have made 45 revolutions, or caused $45 \times 22 = 990$, the same number, to pass the point of gear. And so it would be found with any other number of cogs, that the angular velocities of the geared wheels, or the number of revolutions they make in a given time, are inversely as the numbers of their teeth. When an intermediate wheel is employed, as in Fig. 233, it only affects direction of rotation, not the speed of the extreme wheels. Thus if *a* (Fig. 233) have 34 teeth, and *b*, the intermediate, have 20 teeth, the angular velocity of *a* is to that of *b* as 20 to 34; or if we take the speed of *a* as 1, the speed of *b* is $\frac{34}{20}$. Again, if *c* have 27 teeth, its speed is to that of *b* as 20 to 27, or it is $\frac{20}{27}$ths of the speed of *b*, that is $\frac{20}{27} \times \frac{34}{20} = \frac{34}{27}$ths of the speed of *a*. But, did we leave the intermediate wheel out of consideration, we find that were *a* with 34 teeth to drive *c* with 27, the speed of *c* would be $\frac{34}{27}$ths of that of *a*, the same result as before, though in the opposite direction. The same principle applies, whatever be the number of intermediate wheels; for the speed of the first and last will always be to one another inversely as their respective numbers of teeth.

If from the centres of two geared wheels A B (Fig. 235) circles be drawn, touching each other at a point midway between the extreme projections of their teeth,

Fig. 235.



these circles are called the *pitch* lines or circles of the teeth, and their circumferences being equally divided, give the intervals from tooth to tooth in each, or what is technically called the *pitch* of the teeth. When we speak of a wheel of 1 inch *pitch*, or 2 inches *pitch*, we mean that the distance measured from the centre of one tooth to that of the next (these centre points being taken on the pitch circle) is 1 inch or 2 inches, as the case may be. In Fig. 235, the distance between any two adjoining points, where the dotted radii from A and B cut the pitch circles, is the *pitch* of either wheel, and these *pitches* are necessarily equal in length, in order that the wheels may gear. But as this distance is the interval between the central points measured in a straight line, or along the chord of the small arc intercepted between them, it does not accurately correspond to the length of the arc. The larger the circle of which the arc is a portion, and the smaller the pitch or portions into which the circumference is divided, the more nearly do the circular arc and its chord approach to equality. Accordingly, in cases of nearly equal wheels, or whenever the pitch is small in proportion to the dimensions of either wheel, the more nearly do the numbers of teeth express the proportions of the circumferences. But the circumferences of circles being exactly proportional to their diameters, the numbers of teeth are very nearly in the same proportion; and the angular velocities of geared wheels are therefore inversely as the diameters of their pitch circles.

When there is a train of geared wheels, so arranged that the first shall drive the second—that the third, fixed on the shaft of the second, shall drive the fourth—the fifth, fixed on the shaft of the fourth, shall drive the sixth, and so on—we readily find the angular velocity of any wheel in the train thus.

Rule.—Multiply the angular velocity of the first driver by its number of teeth or diameter, and by the numbers of teeth or diameters of all the drivers, and divide the result by the product of the numbers of teeth or the diameters of all the driven wheels.

Example.—Wheel I., 36 inches diameter with 72 teeth, making 120 revolutions per minute, drives wheel II., 12 ins. diam. with 24 teeth; on the shaft of wheel II. is fixed wheel III., 10 ins. dia. with 30 teeth, driving wheel IV., 6 ins. diam. with 18 teeth: required the speed of wheel IV.

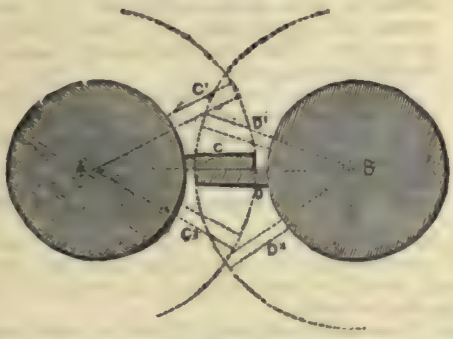
$$\frac{120 \text{ rev.} \times 36 \text{ ins.} \times 10 \text{ ins.}}{12 \text{ ins.} \times 6 \text{ ins.}}$$

$$\text{or } \frac{120 \text{ rev.} \times 72 \text{ teeth} \times 30 \text{ teeth}}{24 \text{ teeth} \times 18 \text{ teeth}} = 600 \text{ rev. per min.}$$

From the general principle of mechanics, that no power can be gained or lost in any train of mechanism, but that only the two elements of power, pressure and velocity, can be interchanged, irrespective of friction or other useless resistances, it follows that, at the end of any train of wheels, the power is the same as at the beginning; but the velocity being different, the pressure or strain on the teeth must be different also. In the example we have given, if we suppose that 1 horse-power has given motion to wheel I., we should expect to get 1 horse-power from wheel IV.; but as the speed of wheel IV. is five times that of wheel I., so the pressure exerted by it at any point is $\frac{1}{5}$th of that exerted by wheel I. at a point equally distant from its axis. To show that this is true, not only in general terms, but in the particular case, we shall suppose 1 horse-power passing through the teeth of wheel I., 18 ins. from its axis. The teeth of wheel II. being in contact with those of wheel I., receive its full power at 6 ins. from its axis, and convey it through its shaft to the teeth of wheel III., distant 5 ins. from its axis, and, therefore, sustaining a pressure of $\frac{6}{5}$ths of the original pressure. Again, this strain being given to the teeth of wheel IV. at 3 ins. from its axis, is equivalent to $\frac{3}{18}$ths of $\frac{6}{5} = \frac{1}{5}$th of the original pressure estimated at 18 ins. from the axis of the last wheel. In any train of wheelwork, then, we may safely diminish the sizes and strengths of the teeth as their velocity increases; and, conversely, we should increase their strengths as the velocity diminishes.

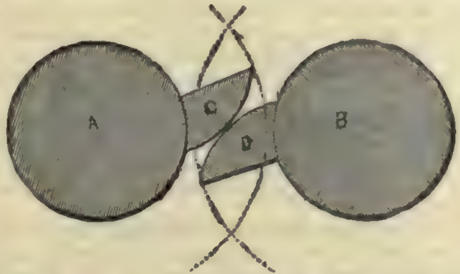
FORMS OF TEETH.—One of the most important matters connected with toothed wheels refers to the forms or outlines of the teeth. It is to be desired that engineers generally should determine on some standard form, so that all wheels of equal pitch should work or gear properly together. Unfortunately this is not the case: the wheel made by one machinist is unsuited to that made by another. It often becomes necessary to make costly patterns of wheels to suit some that may have been already made; these patterns, again, are not suited for other cases; and thus a great amount of time, labour, and material is misapplied in a matter where a little harmony among the views of machinists might do much to avoid these evils. That such a mutual understanding is not impossible, may be proved by the fact, that in a similar case, that of screw-threads, among which there once existed quite as great a variety, almost all engineers now adhere to certain forms and proportions; so that the screw which fits one nut, readily fits any other nut of equal diameter, wherever the screw or the nut may have been manufactured. Perhaps the principal cause of difference in the forms of teeth, has been the want of knowledge among practical men as to what the true forms should be. Much has been written on this subject, but a great deal is of too abstruse a character to be generally understood or appreciated; and the practical mechanic, desiring information, is discouraged by the difficulties with which the subject appears to be invested. We will endeavour to point out, as simply as possible, the laws which should govern the forms of teeth, and to lay down a few easy rules for delineating and executing them.

Fig. 236.



If we suppose A and B (Fig. 236) to be the bosses or central parts of two wheels having straight teeth or flat blades C and D projecting from them respectively, on tracing several positions of these, such as C C_2 and D_1 D_2, we observe that there must be considerable inequalities in their relative movements; or if C_1 C_2 make equal angles with the line of centres A B, D_1 and D_2 do not make equal angles with the same line. Again we observe that the points of each tooth must rub along the flat surface of the other in the course of the motion; and were this form of tooth practically carried out, the cutting and wear would be considerable. But if, as in Fig. 237, the wheel B have a tooth D of some curved outline, we may perhaps find some particular curve for the face of tooth C, such that an equable movement of the one wheel shall produce an equable movement of the other, and that the surfaces of the two teeth shall rather roll along each other than rub with a pushing or sliding movement. Now, although it may be generally possible to find a proper form for C, whatever be the form of D, yet it is desirable for many reasons that both these curves should be traced according to some fixed type, which shall be constant, whatever the dimensions of the wheels or their numbers of teeth. If we examine somewhat closely the relative motions of two toothed wheels, we may perhaps discover an appropriate form for their teeth.

Fig. 237.



INVOLUTE TEETH.—If A B (Fig. 238) represent a strap passing round the circumferences of two equal pulleys, and therefore touching both, and if through C, the middle point between their centres, two circular arcs be described, these may represent the pitch circles of two equal toothed wheels, whose relative rotation should be precisely the same from the gearing as it is from the motion of the strap, unwinding from the one pulley and winding on to the other. But farther, if we continue the lines A B and D E, and from some other point F in D E describe a circle touching A B in G, and another circle (dotted) through the point C;—since F C bears the same ratio to C E as F G to E B, it appears that the rotation of the large wheel, if geared with the other at C, should be precisely the same as if it were caused by the strap winding on to its pulley, of which F G is the radius. If we assume the angular velocity of the pulley D to be uniform, the rectilineal velocity of the strap A B G must also be uniform, and likewise the angular velocities of the wheels E and F. Whatever, then, be the relative sizes of the geared wheels, the rectilineal motion of the strap, with relation to the rotary motion of any one of them, is always constant; and if from this relation we can trace out a form of tooth, that form will apply in all cases, whatever be the diameters of the wheels that are geared together.

Fig. 238.

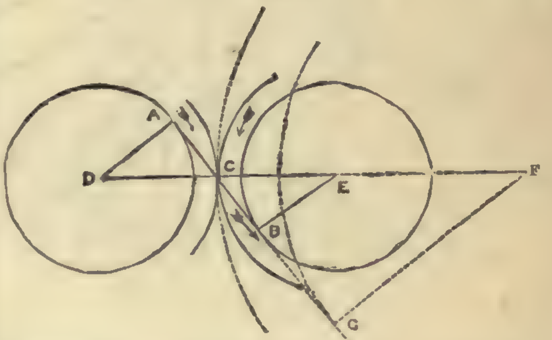
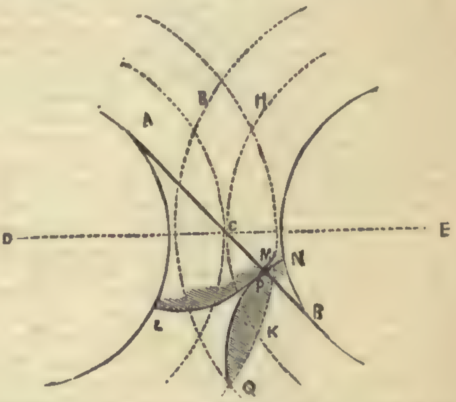


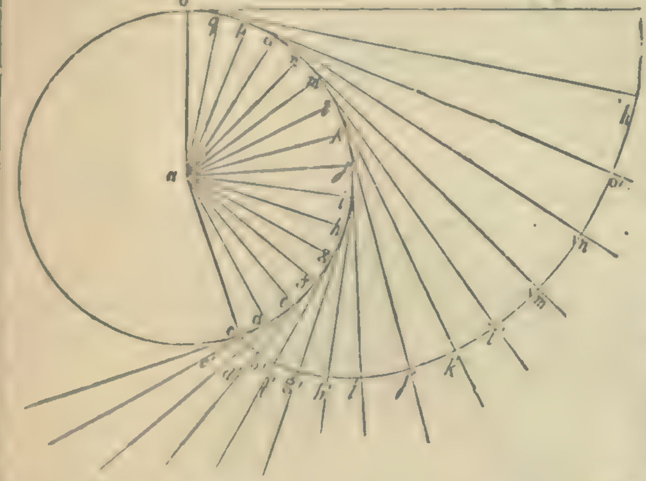
Fig. 239.



In order to avoid complexity, let us first take one wheel, supposing it a circular disc of paper, of which H M K (Fig. 239) is a portion, and let us suppose that the strap A B has a pencil projecting from it at some point P, so that as the disc rotates while the strap travels, the pencil shall trace on the disc a line L P M, compounded of these two motions. If, now, we take the other wheel (marked by the dotted lines), and suppose its disc extended to R Q overlapping the former, the same pencil would trace a corresponding line N P Q on it. Now, as the curves thus produced are traced by the same point in the band, and under precisely similar conditions of uniform rotation, we might cut the paper discs to their outlines, and making them rotate in contact, we should obtain that uniform relative motion which is required. The mathematical name of each of the curves so described is the *involute of the circle*, because it is produced by the point of a thread wound on to (or a circle, or wound off from a circle. The nature of the curve may be best understood by a reference to Fig. 240.

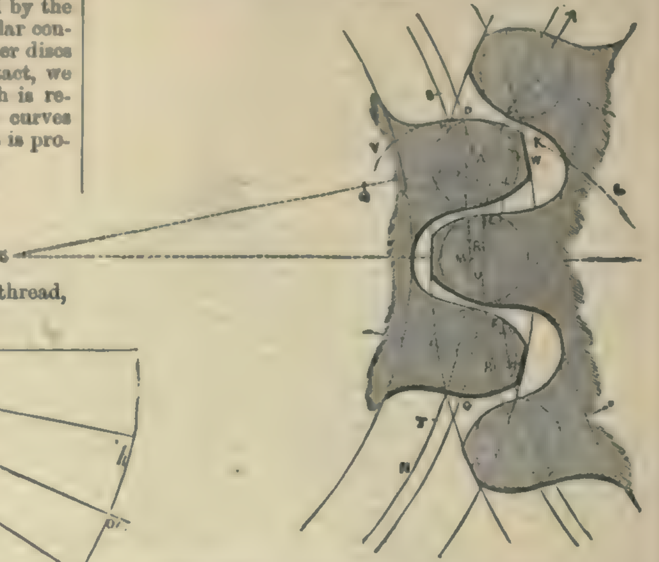
If *a* be the centre of a circle or plan of a roller, on which is wound a thread having its end at *c*, a pencil or tracing point being fixed at the end of the thread,

Fig. 240.



will trace the involute *c* to *q′*, as the thread is unwound; or *b q′* being the unwound thread, its point will trace the involute back to *c* as it is wound round the circle. If we divide the circumference into any number of equal parts at *d*, *e*, *f*, *g*, &c., and from these points draw tangents or lines touching the circle, at right angles to the radii *ad*, *ae*, *af*, &c., respectively, making the lengths of the tangents equal to the lengths of circumference measured round from *c*, the curve joining the extremities of the tangents is the involute—the proper outline for the teeth of wheels that we have just described. In applying this theory to the practical formation of teeth, we do not find it necessary to describe the actual involute form, because the portion of the curve that belongs to any tooth is so small, that we can draw a circular curve so near to the involute as to cause no material error in working. Thus, if C (Fig. 241) be the centre of the wheel, and A, B the centres of two adjoining teeth on the pitch circle, small circles being described round those centres to fix the breadth of each tooth D E and F G, and H M N being part of the generating circle of the involutes, the portion H K of any involute H K L very nearly corresponds with a circular arc of which M is the centre, and the radius D M is a tangent to the generating circle H M N at M. In practice, therefore, it is only necessary to determine the circle H M N, and the length of the radius D M, for describing the curved sides of the teeth. From what has preceded, it is clear that we may take any convenient generating circle H M N, but that whatever be the one we may select for any one wheel, that for any other wheel gearing with the former must be proportional to it. It is found practically convenient to make the radius D M, with which the side of the tooth is described, $\frac{1}{4}$th of the pitch radius C A, and the generating circle or locus of centres H M N should be described within the pitch circle, the interval between them A P being $\frac{1}{8}$th of D M or $\frac{1}{32}$nd of C A. If this rule be adhered to, all wheels of equal pitch will gear with one another, whatever be their diameters. In drawing or setting out teeth, then, the following process will be found convenient.

Fig. 241.



1st. From the centre C, with radius C A, describe the pitch circle, and divide it by the compasses into equal parts, A B, each equal to the given pitch. The pitch radius may be determined by the following rule:—

Multiply the number of teeth by 7 times their distance apart, or pitch, and divide by 44.

Example.—Required the pitch radius of a wheel having 28 teeth of $\frac{3}{4}$ inch pitch.

$$\frac{28 \times \frac{3}{4} \times 7}{44} = 3{\cdot}341 \text{ inches the pitch radius.}$$

NOTE.—When the number of teeth is small, and their pitch considerable, the pitch radius must be a little increased, as will be found necessary on trying the division of the circumference.

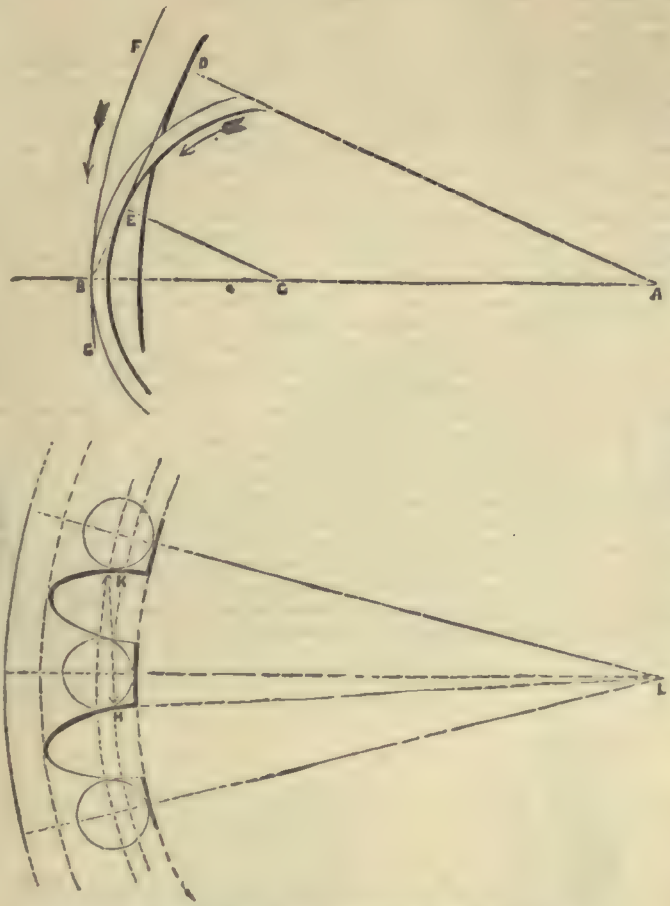
2nd. Round the centres A, B describe circles, each of diameter somewhat less than half the pitch: to determine the breadth of the teeth. The reason for making the breadth of teeth less than half the pitch, is to give a little room or clearance in the spaces between them, so that in the event of slight irregularities occurring in the workmanship, the teeth of two wheels may not become bound or locked into each other. When the teeth are cut by machinery to their exact form, this clearance is not necessary; but when they are merely cast and not shaped afterwards, there should be an allowance of about $\frac{1}{32}$nd of an inch for each inch of pitch—that is to say, if A B be 1 inch, D E should be $\frac{1}{2}$ an inch, wanting $\frac{1}{32}$nd of an inch, or $\frac{15}{32}$nds of an inch, while E F is $\frac{1}{2}$ of an inch and $\frac{1}{32}$nd of an inch, or $\frac{17}{32}$nds of an inch. Were A B 2 inches, then D E would be $\frac{15}{16}$ths of an inch, and E F $\frac{17}{16}$ths of an inch, and so on in like proportion.

3rd. Take A Q = $\frac{1}{4}$ of A C, and A P = $\frac{1}{8}$th of A Q, and from the centre C describe through P a circle of centres H M N.

4th. From the points D, E, F, G, with radius A Q in the compass, mark off the points R, S, T, U on the circle H M N, and from these points as centres, with the same radius A Q, describe the curved sides of the teeth as V H D K.

5th. It now only remains to determine the tops of the teeth and the bottoms of the spaces between them. The spaces should have somewhat greater depth below the pitch circle than the height of the teeth beyond it, to allow the teeth to clear; and it is convenient both for giving strength of form to the teeth and for providing this clearance of the teeth, especially in case of dirt getting between the gearing, to make the bottom of the spaces semicircular. It will be found convenient to make A W, the height of the point of the tooth above the pitch circle, half the space E F between two teeth, and A K the depth of the space below, greater than A W by $\frac{1}{8}$th of an inch for every inch of pitch; and circles described through W and X will determine the tops of teeth and bottoms of spaces.

Fig. 242.



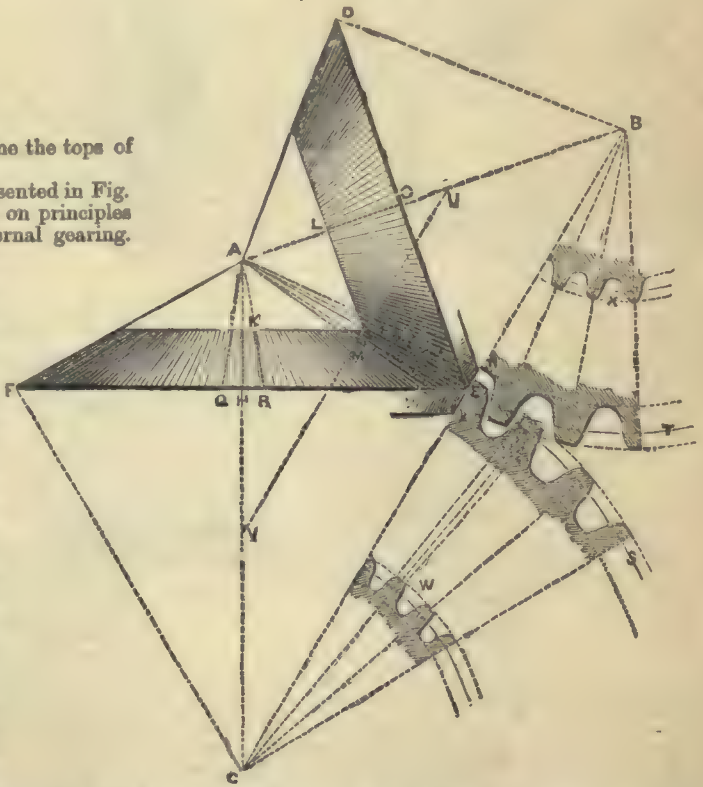
In the case of internal gearing (as represented in Fig. 242), the form of teeth may be developed on principles similar to those we have adopted for external gearing. If A B (Fig. 242) be the pitch radius of the wheel, F B G being a portion of the pitch circle, and C B the pitch radius of the pinion working within it, and if on the axes of the wheel and pinion we suppose pulleys to be fixed, having radii A D and C E, respectively proportional to the pitch radii, a band E D winding off the one and on to the other pulley would give them the same relative angular velocities as would be produced by the friction of their circumferences at B. And D E, the common tangent of the two pulleys, which, when prolonged, must always pass through B, may be supposed to have a pencil fixed to its prolonged part, tracing on the planes of the revolving wheel and pinion, the outlines of their respective teeth, which would manifestly be involutes of the generating circles D and E.

The inclination of D B being the same as that adapted for external gearing, the teeth of the pinion would evidently be the same, but the teeth of the wheel would be similar in form, not to the teeth of an externally geared wheel, but to the spaces between them. L being the centre of the wheel, L H = $\frac{3}{3}\frac{1}{2}$nds of the pitch radius, and H K = $\frac{1}{4}$th of the pitch radius, the side of a tooth at K is part of a circle described with radius K H. The tops of the teeth in this case project *within* the pitch circle, as far as in external gearing they project *beyond* it, and the bottoms of the spaces may be made semicircular, and of depth similar to that determined for external gearing.

The gearing we have hitherto described applies only in the case of wheels revolving in one plane or on parallel axes.

BEVIL GEAR.—When the axes are not parallel, it is necessary to employ *bevil gear.* Let A B and A C (Fig. 243) be the two axes meeting in A, and D E and E F the proper diameters of the wheels to give the required speed; if we suppose the cones D A E, F A E to roll upon one another touching along the line A E, then the velocities of their touching surfaces will be equal at any point, such as M, along the cone, because L M and K M, the radii of rotation at that point, bear the same proportion to each other as G E and H E the radii at any other point E, and therefore the circumferences at these points are in like proportion. Taking a portion of each cone (that shaded), if we conceive their meeting surfaces at M E to have sufficient friction, by causing one to rotate round its axis, we should also cause the other to rotate. But as practically the friction is not sufficient, it is necessary to cut the surfaces of the cones into teeth and spaces, as in plain gearing. It is evident that these teeth must taper towards A, both in width and in height. If a line C E B be perpendicular to A E, and N and P the tops of the teeth at E, the converging lines A N and A P will define the tops of the teeth along their whole extent. And again, if Q R be the breadth of a tooth at H, the converging lines A Q and A R will define the tapering breadth. The line P N is part of the boundary of coni-

Fig. 243.



cal surfaces, of which C is the apex for the one wheel, and B the apex for the other; and if from the centres C and B the circles E S and E T be described, they will represent the outlines of the developed surfaces of the cones F C E, D B E respectively, and become the pitch circles on which the outlines of the teeth at E may be described. Were we to cut these teeth in paper, and then wrap them round the cones F C E and D B E, we should have them interlacing and gearing into each other at E. Proceeding in the same manner at the point M, by drawing U V through M perpendicular to A E, and describing from centres C and B the pitch circles W and X, with radii respectively equal to V M and U M, we get the development of the teeth at M, which are precisely similar to those at E, but on a smaller scale, their outlines being defined by the radii converging from those in S and T towards C and B, the centres of development.

Bevil gearing applies when the motion is to be conveyed at any rate of speed from one axis to another, as in Fig 244, where *a a* is the large *bevil wheel*, and *b b* the smaller one, or the *bevil pinion;* or the motion may be communicated at any other angle, and in various directions by combinations of bevil wheels and pinions, as in Fig. 245, where the pinion *a* drives the wheel *b b* mounted obliquely to it; and the pinion *c*, fixed on the shaft of *b b*, drives the wheel *d d* at some other angle with it.

Fig. 244.

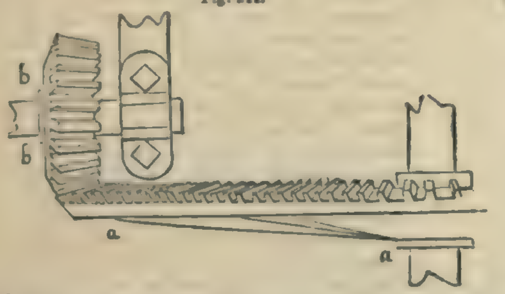


Fig. 245.

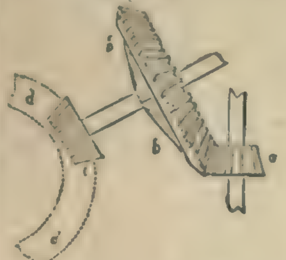
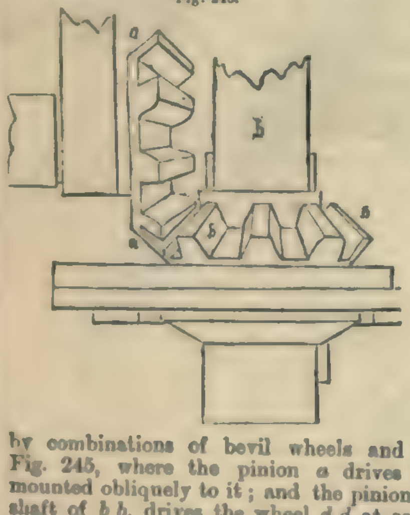


Fig. 246.



When the wheels are equal, and their axes at right angles to one another, as in Fig. 246, the wheels are technically called *mitre wheels.*

The general law as to velocities of rotation and pressure conveyed through bevil gearing, is precisely the same as in the case of plain gearing.

RACKS.—Gearing is sometimes used to convert a rotary into a rectilineal motion, by the use of a rack and pinion.

The pinion *a* (Fig. 247) has teeth which fit between the teeth of two racks *b* and *c*; and by giving *a* a reciprocating rotary motion round its centre, these two racks are put in reciprocating rectilineal motion.

The proper form for the teeth of a rack, so as to gear with those of pinions formed as we have described, may be ascertained thus (Fig. 248):—In describing the teeth of a wheel, the radius C F of the generating circle bears a certain proportion to C B the pitch radius; or, as we have taken it, C F is $\frac{3}{3}\frac{1}{2}$nds of C B. Again, F B, the radius of the side of the tooth, is $\frac{1}{4}$th of C B, or bears also a constant ratio to C B. Hence, for every wheel, B F makes a constant angle F B C with A C. Apply this to a rack N G, the radius of the tooth must be at the same angle to D E as that of F B to B C, or the angle N G E must be equal to the angle F B C; but as the pitch line of the rack is a straight line, or may be supposed a circle of infinitely great radius, so the radius N G of the tooth, being $\frac{1}{4}$th of the pitch radius, must also be infinitely great, and the portion of the circular side of the tooth described with this infinite radius must be a straight line H G perpendicular to N G. To find the amount of obliquity or inclination of H G to D E, we observe that it is the same with the inclination of B F to B M (which is perpendicular to A C). Now M F is very nearly bisected in L; and as F L is $\frac{1}{8}$th of F B, F M is very nearly $\frac{2}{8}$ths or $\frac{1}{4}$th of F B. So, in the rack, H K must be $\frac{1}{4}$th of K G; and hence we have a simple mode of setting out H G, the side of the rack tooth, by taking G K any length, such as 1 inch, along the line G D, and measuring up an offset K H = $\frac{1}{4}$th of K G. The line H G is the side of the tooth, of which the top and bottom may be determined as in wheel gearing.

Fig. 247.

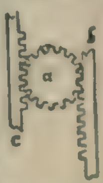
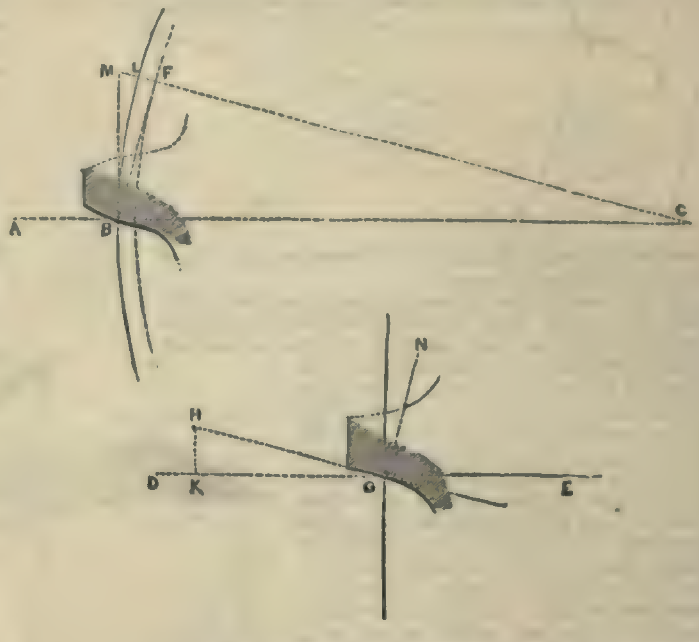
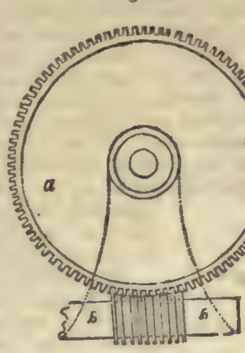


Fig. 248.



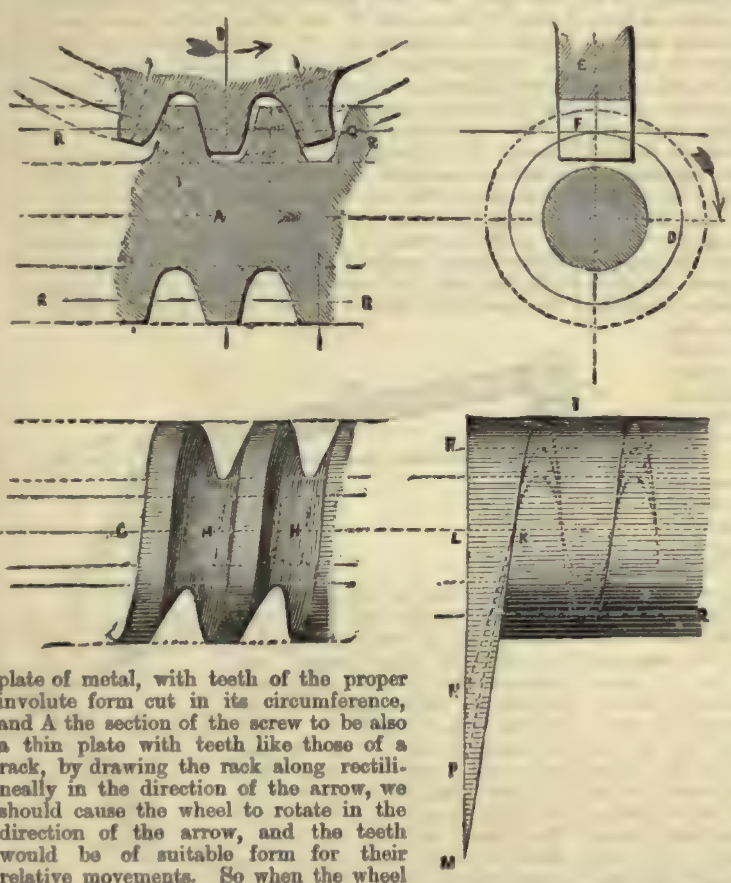
WORM AND WHEEL.—There still remains a kind of gearing most applicable in cases where it is desired to reduce very greatly the angular velocity, or to increase the moving pressure. We allude to the *perpetual screw*, or *worm and wheel*, as it is usually called (Fig. 249): *b b* the driving shaft has a screw or worm cut on its cylindrical surface, the coils of which fit between teeth on the driven wheel *a a*. These teeth have sides inclined to the plane of their wheel, in such a manner as to suit the obliquity of the screw-thread. When the worm is caused to revolve, one portion of the worm pressing against a tooth of the wheel, causes it to advance over a distance equal to the pitch of the thread or of the teeth (which are necessarily alike); and by the time one tooth is disengaged from the worm by the revolution of the wheel, another tooth has come into action, and is caused to advance in like manner. Every revolution of the screw round its axis thus causes the wheel to rotate through a portion of a revolution, equivalent to the distance of the centre of one tooth from that of the next; and if the wheel have, for example, 100 teeth, it thus requires 100 revolutions of the worm to cause one revolution of the wheel. It is evident that, whatever be the size of the worm, the same relation of angular velocities subsists, while the number of teeth in the wheel remains constant; but when we alter the *pitch* of the screw, that is to say, the distance from one of its coils to the next, we must alter, to a corresponding extent, the *pitch* of the toothed wheel; and, if its diameter be fixed, the number of its teeth must be altered accordingly.

Fig. 249.



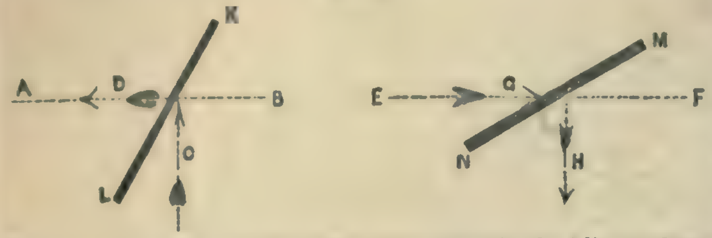
In deciding on the form of teeth for screw-gearing, we must be guided by considerations similar to those which we have dealt with in respect of ordinary gearing. If we suppose the screw-wheel B (Fig. 250) to be a thin plate of metal, with teeth of the proper involute form cut in its circumference, and A the section of the screw to be also a thin plate with teeth like those of a rack, by drawing the rack along rectilineally in the direction of the arrow, we should cause the wheel to rotate in the direction of the arrow, and the teeth would be of suitable form for their relative movements. So when the wheel is of some thickness, as shown in section at E, with teeth F projecting within the outer circumference of the screw, and clearing the solid central portion, we have only to make the teeth F inclined so as to suit the obliquity of the screw-thread, as indicated by the dotted lines H H on the plan of the screw G; and, instead of moving the screw longitudinally like a rack, we may cause it to rotate round its axis; and while we prevent its longitudinal motion, the teeth of the wheel will be caused to move onwards by the inclined action of the screw-thread. As no part of the screw-thread is a straight line, the teeth of the wheel, indicated by the dotted lines H H, ought theoretically to be curved; but when the diameter of the screw is large in proportion to its pitch, the portions of the thread, with which the teeth are in contact, approach very nearly to straight lines, and the oblique sides of the teeth may therefore, without much practical error, be made straight. Their proper obliquity may be found thus:—Let I be a cylinder of the same diameter with the screw, on which is wound a triangular piece of paper, so that its edge shall form the outline of the screw. If at any one of its convolutions K, instead of continuing to wind the paper on the cylinder, we stretch it out straight as K M L, if K L be half the pitch of the screw, or L half-way between K and the point where the next convolution after that at K would cross the axis, then L M must be equal to half the circumference of the cylinder (about 3$\frac{1}{7}$ times its radius), in order that the point M may be brought round to L when the paper is wound on to the cylinder. By taking K L half the pitch of the screw, and L M half its circumference, and joining K M, we get a line at the proper obliquity to the axis to suit the thread of the screw. By taking a cylinder equal to the inner part of the screw, or of diameter equal to that of the screw at the bottom of its thread, and proceeding in a similar way (as indicated by the dotted lines), we get the line K N of the proper obliquity to suit the bottom of the thread. Now, as the tops or points of the teeth of the wheel gear with the bottoms or hollows of the screw-threads, and the bottoms of the teeth gear with the tops of the screw-threads, we should make the tops of the teeth less inclined to the plane of the wheel than their bottoms, and we should gradually increase the obliquity from the tops of the teeth downwards. It would be very difficult to effect this in practice; and, indeed, even if it could be done with ease, it would be positively disadvantageous, because some of the teeth are always coming into such a position with respect to the thread as that marked Q, where the outer portion of the tooth is in contact with the upper part of the thread. Practically, then, it is best to make the obliquity of the teeth like K P a mean between those due to the upper and the lower parts of the screw-thread. In other words, tracing a pitch-line R R for the screw touching the pitch circle of the wheel, and then developing the obliquity K P due to the diameter of the screw measured to that pitch-line, we get an average inclination for the teeth, somewhat in error at points above and below the pitch-line, but correct where the principal contact and communication of power takes place. The greater the diameter of the screw, and the smaller its pitch, the greater is the inclination of the thread to its axis, and the less is the error of obliquity in the contact of the teeth and screw-thread above and below the pitch-line. Therefore, when the screw by rotating drives the wheel, the screw should be made as large in diameter as is consistent with

Fig. 250.



the necessary strength. But when it is intended that the rotation of the wheel shall cause the screw to rotate, the diameter of the screw should be made as small and its pitch as great as possible, so that the thread and the teeth may have small obliquity to the axis of the screw. In either case the screw-thread may be considered as a continuous inclined plane presented to the teeth of the wheel, and the effect of greater or less obliquity becomes very evident. In the first case, where the screw drives the wheel, A B (Fig. 251) being the axis of the screw, K L, part of its inclined surface, travelling in the direction of the arrow C, has to move the tooth of the wheel in the direction of the arrow D; the greater the inclination of K L to the axis, or the more nearly it approaches to a perpendicular to it, the better its effect to move the tooth, or the less is the lateral strain it produces on the tooth. In the second case, where a tooth moving in the direction of the arrow G, and pressing against the inclined side of the screw-thread M N, causes it to travel in the direction of the arrow H, the less the inclination of M N to the axis E F, or the more nearly it approaches to coincidence with it, the better is the action of the tooth to give it lateral motion, or the less is the longitudinal strain in the direction of the axis.

Fig. 251.

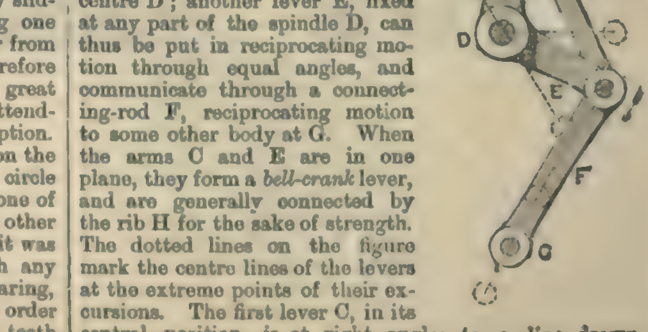


Before leaving the subject of toothed gearing, it is proper to remark, that the form of tooth which we have described—the involute of the circle—presents one disadvantage in use. The surfaces of the teeth do not roll over one another when in motion, but rub or slide along each other, and consequently absorb a portion of the power in mere useless friction, and in time wear each other out. There is a form for teeth which secures rolling contact of their surfaces, called the epicycloidal tooth, from the name of the mathematical curve, the epicycloid, adopted in its formation. If one circle be caused to roll round the circumference of another circle at rest, every point in the circumference of the rolling circle describes a curve, called the epicycloid, which is the resultant of its two motions, the rotation round its own centre and the revolution round the centre of the other circle. If now the circle which was fixed be caused to roll round the other, every point in its circumference would similarly describe the epicycloidal curve proper to its motion. And if both circles were caused to roll on each other at their due relative speeds, the *relative* motion of every point in their circumferences would be traced along their respective epicycloidal curves. If these curves were cut out in the circumferences of the two wheels, like teeth, projecting between and interlacing with each other, their surfaces would bear against each other without any sliding or rubbing motion, but simply rolling along one another like wheels upon rails. The loss of power from friction, and the amount of wear would be therefore very small. So far epicycloidal teeth are of great advantage. But there is one great difficulty attending them which practically forbids their general adoption. As the form of the epicycloidal depends not only on the size of the rolling circle, but also on that of the circle round which it rolls, a wheel made to gear with one of a given size cannot properly gear with one of any other size. With the involute tooth, on the contrary, it was shown that the same form applies to gearing with any size whatever. In a great many applications of gearing, it is necessary to change the wheels and pinions in order to change the rates of motion; and if epicycloidal teeth were employed, the number of wheels and pinions necessary to give the required changes would be enormous; whereas with involute teeth, a large number of changes can be effected by different combinations of a few wheels and pinions. Moreover, for large work, wheels and pinions are generally cast from patterns which are costly, and it therefore becomes important to make the teeth such as will suit all cases, instead of being limited to one particular case. When teeth are cut out of the solid metal, the form of the cutter can as readily be made to suit the epicycloid as the involute; and in such cases, especially for clock-work, and all work of an exact and light character, the epicycloid is certainly preferable to the involute.

RECIPROCATING MOTION.—Although continuous rotary movements are the most convenient for communicating power from one point to another, yet there are many operations to which machinery is adapted demanding motions of another character. These are chiefly of a reciprocating kind; and, whether the reciprocating movements take place in a straight line or about a centre or axis, it becomes important to inquire into the modes by which continuous rotary motion may be converted into them, or the converse. Sometimes it is desirable that these reciprocating movements should be continuous; at others that there should be intervals of rest between them; occasionally that they should be equal in velocity; and, again, that the time occupied by them should vary, according to the special character of the work to be done, and the intensity of the force transmitted. To describe all the known modes of effecting these objects would be to compile a list of almost all the mechanical inventions ever made, and, were it possible, would demand space far beyond the limits of a treatise like this. We will, therefore, merely draw attention to some of the modes of converting motion most generally applied in machinery. The arrangement best adapted to any particular case, or the special modification of action that may be most suitable, are matters that must be left to the ingenuity of the designer.

The most simple arrangement for converting continuous rotary motion into reciprocating motion, is the crank or eccentric, which we have already described in connection with the steam-engine. It is generally convenient to obtain the reciprocating movement in the arc of a circle instead of in a straight line. For instance, the revolving crank A (Fig. 252), connected by a rod B with the arm C of a lever mounted on an axis or spindle D, causes it to vibrate in the arc of a circle round the centre D; another lever E, fixed at any part of the spindle D, can thus be put in reciprocating motion through equal angles, and communicate through a connecting-rod F, reciprocating motion to some other body at G. When the arms C and E are in one plane, they form a *bell-crank* lever, and are generally connected by the rib H for the sake of strength. The dotted lines on the figure mark the centre lines of the levers at the extreme points of their excursions. The first lever C, in its central position, is at right angles to a line drawn

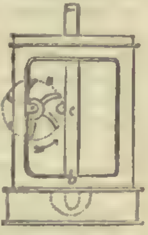
Fig. 252.



through the centre of the crank bisecting the vibration of the lever, or cutting the arc in which the pin of the lever vibrates in such a manner, that its deviations from the straight line at its middle and extreme points are equal. So also the line in which the point G is required to reciprocate, when prolonged, bisects the vibration of the arm E. By this arrangement the deviation of the pins moving in circular arcs from rectilineal motion is rendered as small as possible, and lateral strain from obliquity of connecting-rods is proportionally reduced. By varying the length of the arms C and E, the amount of vibration may be varied at pleasure; for while the angles in which they vibrate, remain equal, the lengths of the circular arcs in which their pins vibrate, are proportional to the lengths of their radii.

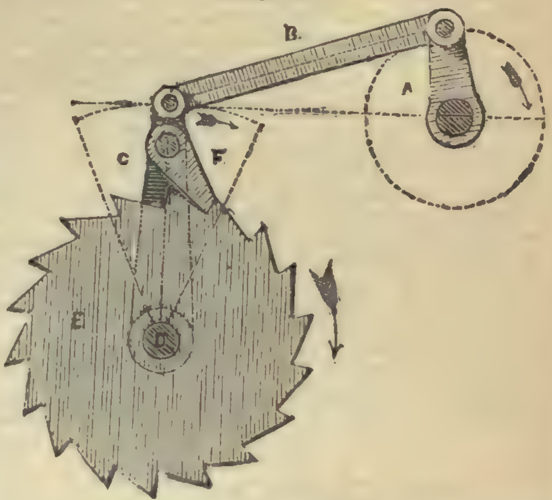
By proper arrangements of cranks, levers, and connecting-rods, in respect of lengths and relative positions, it is generally possible to convert a given rotary or reciprocating movement into one of greater or less extent, in a different plane or direction. When the reciprocating movement of a lever is applied to produce the rotary motion of a crank, it is necessary either to fit the spindle of the latter with a heavy fly-wheel to bring the crank over its *dead-centres*, or to adopt some other combination of cranks, as in the duplicate marine or locomotive engines, so that while one is on the *dead-centre*, the other is receiving motion from its connecting-rod. The crank or eccentric can give reciprocating movement to a lever, only in such a manner, that for each revolution of the crank the lever makes one complete double stroke, or an excursion from one extreme of its vibration to the other and back. Sometimes it is desirable that each revolution of the rotating shaft shall cause a number of reciprocating movements of a lever or rod connected with it. There are several methods of effecting this object. For moving a stamper *b* (Fig. 253) so as to crush or pulverise materials subjected to it, a wheel *a* is fitted with several curved arms or *wipers*, which, as the wheel revolves, come successively in contact with a pin or projection *c* on the stamper, lift it, and let it drop. If the wheel have six arms, the stamper makes six strokes during each revolution. Such an apparatus is frequently applied in cases where repeated strokes of falling heavy bodies are required for special operations, as for tilt-hammers used in iron manufacture, seed-crushers, fulling-mills, and washing-machines. For lighter work, such as the movement of pendulums or balance-wheels of timekeepers, there are numerous ingenious arrangements of escapement-wheels having their circumferences cut into teeth of suitable form, and capable of acting on the pallets presented to them, with the regularity and precision required.

Fig. 253.



INTERMITTENT MOTION.—Sometimes it is desirable to convert continuous rotary movement into one that shall proceed by fits and starts. A crank or eccentric A (Fig. 254) on the continuously rotating shaft connected by a rod B with a lever C, causes it to vibrate once during each revolution of the crank round a spindle D, on which is mounted a *ratchet*-wheel E, having teeth like those of a saw. The lever C is fitted with a *pall* F hanging freely from a pin, and formed so as to drop into the space between the ratchet teeth. One half-revolution of the crank A causes the *pall* to move over one or more teeth without putting them in motion, as it can slide freely along their inclined sides; but the other half-revolution bringing back the lever and *pall* which bears against the abrupt face of a tooth, causes the ratchet-wheel to rotate through the same arc with the point of the pall. In many cases where this arrangement of ratchet and pall is applied, it is necessary to vary the amount of movement of the ratchet-wheel at each stroke. This is effected either by varying the throw of the crank A, or the position of the connecting-rod pin on the lever C, so as to make the back stroke of the pall pass over two or more teeth, as may be required. In machines for boring, turning, planing, and shaping metals, in machinery for sawing timber and such like operations, this arrangement is applied for moving the material operated on, so as to present a new portion to the action of the tool or saw at each stroke.

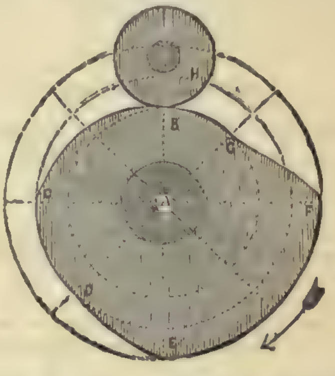
Fig. 254.



CAMS.—In describing the methods of regulating the flow of steam in engines, we alluded to the *cam* as an apparatus by which the continuous rotation of a shaft is made to lift a valve, sustain it for a certain period, and then let it drop. *Cams* are also applicable to many other mechanical movements of a similar character; and as their forms may be varied indefinitely, so almost any recurring movement can be effected by their use. As an example of what may be effected by a cam, let us suppose that, having a rotating shaft, we desire it, during $\frac{1}{4}$th of a revolution, to raise a weight 1 inch; during $\frac{1}{8}$th to retain it at that height; during the next $\frac{1}{8}$th to raise it 1 inch more; to retain it there during $\frac{1}{4}$th; during the next $\frac{1}{8}$th to drop it the two inches through which it had been raised; and during the remaining $\frac{1}{8}$th to retain it at its lowest position until it is again raised.

Let A (Fig. 255) be the shaft revolving in the direction of the arrow, and let three circles be described round its centre, the inner circle being of any convenient radius, and the others having radii respectively 1 inch and 2 inches greater. Let the circles be divided into eight equal segments, and let H be a roller connected with the weight to be lifted, bearing on the inner circle at B.

Fig. 255.

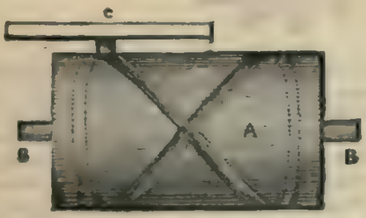


From B to C, $\frac{1}{4}$th of a revolution, let the circumference be part of a spiral curve touching the inner circle B and the middle circle at C; from C to D, $\frac{1}{8}$th of a revolution, let it follow the middle circle; from D to E, $\frac{1}{8}$th of a revolution, let it be another spiral curve touching the middle circle at D and the outer at E; from E to F, $\frac{1}{4}$th of a revolution, let it follow the outer circle; from F to G, $\frac{1}{8}$th of a revolution, let it again be a spiral touching the outer and inner circles; and from G to B, the remaining ¼th of a revolution, let it follow the inner circle. It is obvious that, as the shaft rotates and brings the different portions of the cam's circumference successively under the roller, the centre of which we suppose to be capable of vertical movement only, it successively lifts it one inch, retains it, lifts it again one inch, retains it, and permits it to drop, and remain down according to the conditions of the problem. The principal point to be attended to in the construction of a cam is, that the spiral portions leading from the one part of the circumference to the next be not too abrupt, and that they be carefully graduated from the one curvature to the other, so that the roller be not subjected to sudden jerks, but be made to rise from or fall towards the centre as gently and easily as possible. When the rotation of the cam is rapid, this is of the greatest importance: the amount of eccentric movement given to the roller should be as small as possible; or it should be spread over as large a part of the circumference as possible, so that the inclination of the revolving slope may be gentle.

In some kinds of printing machinery a peculiar form of screw-cam is applied, for the conversion of a continuous rotation into a reciprocating rectilineal motion.

A cylinder A (Fig. 256), rotating in bearings B B, has a double groove of a screw form cut in its surface, in which a pin C, projecting downwards from the table of the printing machine, is free to slide. The table being guided to slide backwards and forwards in a straight line, is caused to move by the revolution of the cylinder presenting always the inclined face of its screw-groove to the pin. The screw is double, of opposite direction, with a portion at each end not oblique to the axis of the cylinder. One half-revolution of the cylinder causes the pin to travel throughout the length of its screw; another half-revolution retains it at the end of its stroke; the next half-revolution carries it back to the opposite end; the next retains it; and so on successively to suit the alternate rest and motion of the printing-table to the successive impressions and intervals between them.

Fig. 256.



MANGLE MOTION.—Racks and partly-geared pinions are sometimes used for a similar purpose, thus:—A double rack *a a* (Fig. 257), guided at the ends *b* and *c* to move only longitudinally, is acted on by a partly-geared pinion *e* continuously rotating. When the teeth of the pinion gear with those of the upper limb of the rack, it is caused to move from *b* towards *c*, until the pinion, having left that limb, enters into gear with the other, giving it the contrary movement, and so on, successively.

Fig. 257.

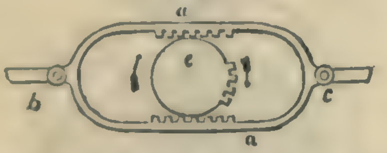
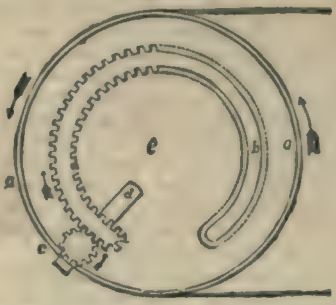


Fig. 258 represents an arrangement of a similar character, applied to produce alternating rotary motion. A pinion *c* continuously rotates on an axis fitted with a universal-joint, such as may permit the pinion to gear either with the exterior or interior cogs of the double circular rack *b*, fixed on the face of a drum or pulley *a*, and thus to give it rotary motion round its centre *e*, in directions alternately opposite. This apparatus having been applied to driving mangles, has received the name of the *Mangle Motion*.

Fig. 258.



REVERSING GEAR.—On referring to our remarks respecting the communication of motion by pulleys and straps, it will be seen, that while a pulley, driven from another by a direct strap, revolves in the same direction, one driven by a crossed strap revolves in the opposite direction. This principle is frequently applied in machinery where it is desired to reverse the direction of motion. Two pairs of fast and loose pulleys are arranged on a shaft, so that one pair may be connected by a direct, and the other pair by a crossed, strap, with pulleys on the prime mover; when the direct strap is on its fast pulley, while the crossed strap is on its loose pulley, the machine is driven in the direction of the prime mover; but when the direct strap is thrown on its loose pulley, while the crossed strap is brought on its fast pulley, the contrary motion is produced. In apparatus where an arrangement of this kind is applied, as in machines for planing iron, the movement of the table of the machine to each extreme of its stroke is made to shift the straps by very simple mechanism, which is capable of being adjusted so as to vary the amount of stroke at pleasure.

Fig. 259 represents another method of reversing rotary motion, frequently employed: *a* and *b* are two bevil pinions revolving loosely on a shaft *d e*, and gearing with a bevil wheel *c*. Between the pinions is fitted a clutch, sliding on, but revolving with, the shaft *d e*. The pinions being geared into opposite sides of the wheel, rotate in opposite directions; and as the clutch is thrown into gear with the one or the other, the shaft *d e* is caused to rotate in the one direction or the other accordingly. Instead of a toothed clutch, a conical friction clutch is occasionally employed with good effect, because, in the first place, a very slight movement of the lever pressing the cone into its seat on either side, is sufficient to couple the shaft with either wheel; and, in the next place, the shaft being driven solely by the friction of the conical surfaces, cannot be subjected to any strain exceeding the friction in amount, the cones slipping when subjected to extreme strain. In powerful rolling machinery, this plan prevents a sudden shock, always inseparable from the use of the clutch, and which may cause the breaking down of some part of the machinery.

Fig. 259.

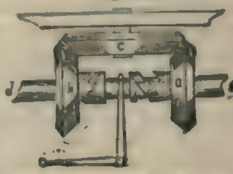
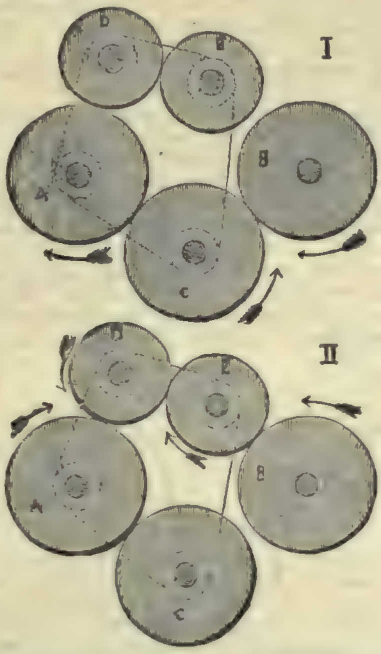


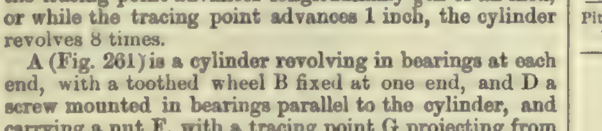
Fig. 260 indicates a mode of reversing the direction of rotation by means of toothed gearing. A being a toothed wheel on the driving-shaft, and B one on the shaft to which it is required to convey motion in either direction, C, D, and E are intermediate pinions mounted on a frame capable of vibrating on A as an axis. When C is geared with A and B, as in I., both revolve in the same direction; but when C is thrown out of gear with B, and E brought into gear with it, as in II., the direction of its rotation is reversed, as indicated by the arrows. By some such arrangements, or modifications of them, suited to the circumstances of any particular case, the direction of rotation may be readily varied.

Fig. 260.



SCREW-CUTTING GEAR.—It is often required to change a rectilineal motion, having a certain velocity, to one having another velocity, particularly in machinery for cutting screws. A screw, as we have already described, is a line traced on the surface of a cylinder by the motion of a point moving longitudinally along the cylinder, parallel to its axis, while the cylinder revolves. The pitch of the screw is the distance through which the tracing point moves longitudinally while the cylinder makes one revolution; and in practice, it is necessary to form screws of numerous different pitches, according to their dimensions, and the circumstances under which they are to be used. The pitch of a screw is generally named according to the number of turns or convolutions which the screw makes in a certain length of the cylinder. Thus, when a screw has 8 turns to the inch, we say that it has a pitch of $\frac{1}{8}$th of an inch—that is, during one revolution of the cylinder on which the screw is formed, the tracing point advances longitudinally $\frac{1}{8}$th of an inch, or while the tracing point advances 1 inch, the cylinder revolves 8 times.

Fig. 261.



A (Fig. 261) is a cylinder revolving in bearings at each end, with a toothed wheel B fixed at one end, and D a screw mounted in bearings parallel to the cylinder, and carrying a nut F, with a tracing point G projecting from it, to meet the surface of the cylinder, the screw having a toothed wheel E, gearing into an intermediate wheel C, which also gears into B. On causing the cylinder A to revolve, the screw D is also caused to revolve; the nut F and tracer G are made to move longitudinally parallel to the axis of A, and the screw-curve traced on A is that due to the velocity with which A revolves, as compared to the speed with which G advances. By altering the proportions or numbers of teeth in the wheels B and E, the relative velocities of the cylinder and screw may be changed at pleasure, and the pitch of the screw traced on A from a screw D of constant pitch may be proportionally varied. In lathes, or machines for cutting screws, the cylinder A is the piece of metal on which the screw is to be cut; its axis is what is technically called the *mandril* of the lathe; and the tracing point G is a steel tool cutting into the metal, so as to leave the screw-thread projecting. The screw D has generally a pitch which is some simple fraction of an inch, such as $\frac{1}{2}$ or $\frac{1}{3}$rd of an inch, and the lathe is furnished with numerous toothed wheels, which may be put in the place of B and E to vary the pitch of the cut-screw, as may be required. If we take the case of a lathe having a screw D of $\frac{1}{2}$ an inch pitch, or making 2 turns per inch, when the wheels B and E have equal numbers of teeth, whatever be the size of the intermediate wheel C, the screw D revolves with the same angular velocity, and in the same direction with the work A. The tool G, therefore, advances $\frac{1}{2}$ an inch along the surface of A during each revolution of A, and cuts a screw of precisely the same pitch, and lying in the same direction with that on the screw D. But if in place of two equal wheels B and E, we put wheels, B having 20 teeth, and E having 40, then the screw D will be caused to revolve at $\frac{20}{40}$ths, or $\frac{1}{2}$ the speed of A, and the tool will advance $\frac{1}{2}$ of $\frac{1}{2}$ an inch, or $\frac{1}{4}$th of an inch, during each revolution of A, and thus to cut a thread of $\frac{1}{4}$th of an inch pitch upon it. Screw-cutting lathes are generally furnished with a table of screw pitches, and the appropriate wheels for producing them, such, that the workman, by inspecting the table, can at once select the proper wheels for giving the desired pitch. The wheels belonging to the lathe have their numbers of teeth stamped upon them, to save the trouble of counting them. Sometimes, when the difference of the speed of the screw and of the work is required to be considerable, instead of the simple intermediate wheel C, it is necessary to introduce a wheel and pinion C and *c* fixed together. Thus, if it were required that the screw should revolve at $\frac{1}{25}$th of the velocity of the mandril, were the smallest possible wheel B to have 20 teeth, it would be necessary that E should have 500 teeth, because $\frac{20}{500} = \frac{1}{25}$th, if the simple intermediate wheel C were employed. This size of wheel for E might be extremely inconvenient, and it would be better to employ the compound intermediate wheel and pinion C and *c*. In this arrangement, B having 20 teeth, C 100, *c* 20, and E 100, the speed of E is $\frac{20 \times 20}{100 \times 100} = \frac{1}{25}$th of that of B. In the lathe table, the first column gives the pitch of the screw to be cut; the second gives the number of teeth on the mandril wheel B; the third and fourth give the numbers of teeth in the intermediate wheels C and *c* respectively; and the fifth gives the number on the screw-wheel E. When the compound intermediate wheels are not required, the columns of intermediates are left blank, as any simple intermediate may be employed without altering the relative velocities of the mandril and screw-wheels. The following is part of a table for a lathe having a screw of $\frac{1}{2}$ inch pitch:—

TABLE OF SCREW GEARING FOR LATHE.

Pitch: Turns per inch.	Teeth in Mandril Wheel.	Teeth in Intermediate and Pinion Wheel.		Teeth in Screw-wheel.
20	30	120	40	100
18	30	90	30	90
16	30	120	40	80
15	40	100	30	90
15	or 20	...	...	150
14	20	80	40	70
12	20	...	...	120
11	20	...	...	110
10	20	...	...	100
9	20	...	...	90
8	30	...	...	120
7	20	...	...	70
6	40	...	...	120
5	40	...	...	100
4	40	...	...	80

If we examine any part of the table, as, for instance, the numbers given for 14 turns to the inch, we find the following rule obtains :—The pitch in the first column is equal to twice the product of the numbers in the third and fifth, divided by the product of those in the second and fourth. Thus $14 = 2 \times \frac{80 \times 70}{20 \times 40}$. Again, where no compound intermediate is employed, the pitch is twice the number in the fifth column divided by that in the second. Thus $6 \text{ pitch} = 2 \times \frac{120 \text{ teeth.}}{40 \text{ teeth.}}$

MICROMETER.—For machinery by which straight lines or the circumferences of circles are divided into a number of equal parts, as for the marking out of scales for measuring or astronomical instruments, or for cutting teeth in racks and wheels, the screw and the worm and wheel are often employed in order to give the means of delicate subdivision. When a screw is used for such a purpose, it is generally called a micrometer (small measurer) screw, and the principle on which it acts may be thus described :—If we suppose an accurately cut screw, having $\frac{1}{10}$th of an inch pitch, to be fitted with a nut, the motion of a screw through one revolution would advance the nut through $\frac{1}{10}$th of an inch. If on one end of the screw there were fitted a wheel having its circumference divided into 100 equal parts, the screw might be turned round any number of revolutions or hundredths of a revolution, as marked by a fixed index pointing to the divisions on the wheel. But for every hundredth part of a revolution of the screw, the nut would be advanced $\frac{1}{100}$th of $\frac{1}{10}$th, that is, $\frac{1}{1000}$th part of an inch ; and by using a screw of still finer pitch, and having a wheel mounted upon it divided into still smaller and more numerous parts, the longitudinal advance of the nut through still smaller fractions of an inch could be effected and estimated. It is by such an arrangement that the fine divisions of mathematical instruments are traced, and the fine lines traced upon medallion drawings are engraved. For circular division, the screw, instead of moving a nut longitudinally, acts as a worm on the teeth of a wheel, and causes it to revolve through any required part of its circumference. Thus, if we had a worm-wheel with 360 teeth, and a worm with a wheel fitted on its axis, having 360 divisions, we could move the worm-wheel through $\frac{1}{360}$th of $\frac{1}{360}$th, that is, $\frac{1}{129600}$th of a revolution, or any number of such fractions of a revolution. And farther, if the wheel upon the screw were a toothed wheel, and we had numerous other toothed wheels, with various numbers of teeth like those we have described for a screw-cutting lathe, we might, by the proper selection of wheels gearing with that on the screw, effect the division of the circle into any number of required equal parts. Such apparatus are employed for dividing the circumferences of astronomical instruments, and also for cutting the teeth of wheels. When the number of teeth to be cut is a multiple of some simple numbers, such as the number 60, which is a multiple of some of the numbers 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, it is generally easy to select wheels which, in connection with the screw, shall give the required number of equal divisions ; but when the number of teeth is what is called a prime number, such as 59 or 61, which is not capable of subdivision, we must either provide wheels having such a number of teeth already cut in them, or resort to methods of approximation for their subdivision. Thus, if with the 360 teeth on the worm, and 360 divisions on the screw-wheel, we desired to divide a circle into 61 equal parts, we should for each part turn the screw through $\frac{129,600}{61} = 2124\frac{1}{2}$ divisions, or $\frac{2124\frac{1}{2}}{360} = 5$ revol. $324\frac{1}{2}$ div. nearly. The error at the end of the process would be found thus :

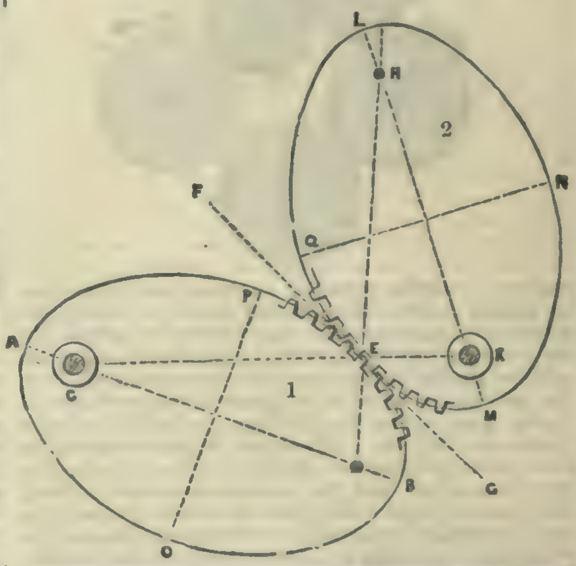
Since the total number of divisions due to a complete revolution are 129,600,

and since $2124\frac{1}{2} \times 61 = 129,594\frac{1}{2}$,

the revolution of the wheel would want $5\frac{1}{2}$ divisions of being complete, that is, $\frac{5\frac{1}{2}}{129,600}$, or about $\frac{1}{23564}$th part of its circumference : a quantity quite inappreciable.

ELLIPTICAL GEAR.—In several machines it is necessary to provide a rotary motion more rapid at one portion of a revolution than at another, as, for instance, in machines for slotting and planing iron. The iron to be planed, or the tool which planes it, makes a rectilinear stroke slowly in the one direction while the metal is being cut, but may be drawn back rapidly in the opposite direction when no work is done. For producing motion of this kind, elliptical geared wheels are sometimes employed (Fig. 262). The ellipse is a curve, of which the line A B is called the greater axis, and each of the two points C and D in that line is called a focus. If to any point E in the circumference of the ellipse, lines C E and D E be drawn from the foci, the sum of their lengths is equal to that of the axis A B. Farther, if the lines C E and D E be prolonged beyond the curve, and a line F G drawn so as to divide equally either of the angles formed by their intersection, the line F G is a tangent to the curve at E ; that is, it touches it, but does not cut it ; or every part of it, except merely the point E, lies entirely outside of the curve. If, in the prolonged portions of the lines C E and D E, lengths E K and E H be measured off equal to D E and C E respectively, H and K may be taken as the foci of another ellipse, precisely equal in every respect to the original ellipse, and having the axis L M, and the tangent F G touching it at E ; and as F G touches both ellipses, they touch each other at the same point E ; and the length of the line C K, which is made up of C E and E K, or its equal E D, is equal to A B or L M.

Fig. 262.

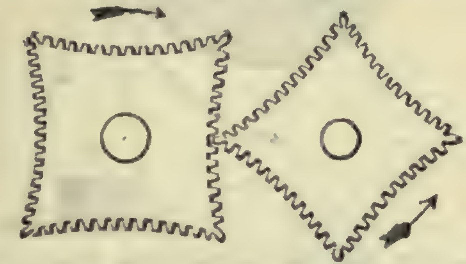


These peculiar properties of the ellipse enable us to employ elliptical wheels, each revolving round a focus as a centre, C for the one and K for the other, the two foci maintaining a constant distance apart, whatever be the relative position in which the wheels lie with respect to each other. The circumferences are cut into teeth and spaces like ordinary circular gearing, the elliptical curves being their pitch lines, on which the equal divisions are set out.

It will be observed that, during a revolution of these wheels, the point M gears with B, and the point L gears with A ; but as the radii of these points are C B and K M, and K L and C A respectively, the angular velocities of the wheels at these points vary inversely as the lengths of these radii. If, for instance, C A be $\frac{1}{4}$th of A B, and therefore C B = $\frac{3}{4}$ths of C A, K M being equal to C A, and K L to C B, when the points B and M are engaged, the wheel 2 is revolving with 3 times the angular velocity of 1, because the radius C B is three times the length of K M; but when the points A and L are engaged, the wheel 2 is revolving with $\frac{1}{3}$rd of the angular velocity of 1, because A C is $\frac{1}{3}$rd of K L. So at all points intermediate to these, except at N, O, P, and Q, where the radii are equal, the relative angular velocities of the wheels vary between the limits we have named.

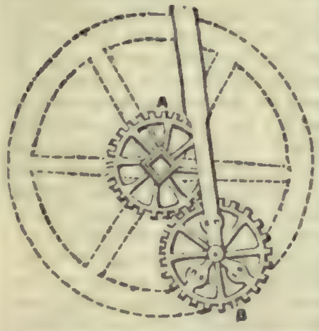
For some purposes in the manufacture of textile fabrics, gearing like that in Fig. 263 is employed. These vary the angular velocities 4 times in every revolution; and the principal condition of their construction is, that the distance between their centres must be a constant quantity.

Fig. 263.



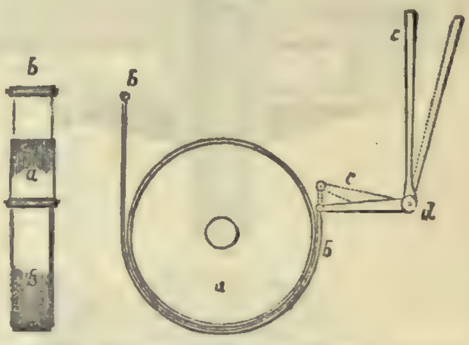
SUN AND PLANET.—The *sun and planet-wheel* is a contrivance for converting a reciprocating into a rotary motion. It was employed by Watt instead of the crank in a steam-engine, not because he preferred it to the latter, but because, through the bad faith of a workman who patented the crank as his own invention, he was precluded from employing it. The sun or central wheel A (Fig. 264) gears with the planet-wheel B, which is caused to revolve round A without rotating round its own centre, being fixed to the end of the connecting-rod, and retained in gear with A by means of a link which connects their centres, and revolves freely with B. When the sun and planet-wheels have equal numbers of teeth, every revolution of the latter causes 2 revolutions of the former, as may be understood by watching the relative positions of a tooth in each at different parts of a revolution. When the planet is vertically above the sun-wheel, the tooth B of the one is engaged with the space A of the other; and when the planet has made half a revolution, so as to be vertically below the sun, the sun has made a complete revolution, so as to bring the space A quite round to the point where it was at the commencement. One half-revolution of A is owing to the half-revolution of B propelling it round, while the other half-revolution of A is effected by the roll of the toothed half circumference of B presenting fresh teeth and spaces to A at the successive points of its revolution. When the numbers of teeth in the two wheels are different, the velocity of the sun-wheel varies accordingly, as may be best understood by an example. Let us suppose that the sun-wheel has 40 teeth, and that the planet has 50; then, during one revolution of the planet, it has presented the whole of its 50 teeth to the sun-wheel, and therefore turned it through 50 teeth, as well as its own complete revolution of 40 teeth. The sun-wheel, therefore, during 1 revolution of the planet-wheel, turns round a distance equivalent to 90 of its teeth, or $\frac{90}{40}$ths = $2\frac{1}{4}$ revolutions.

Fig. 264.



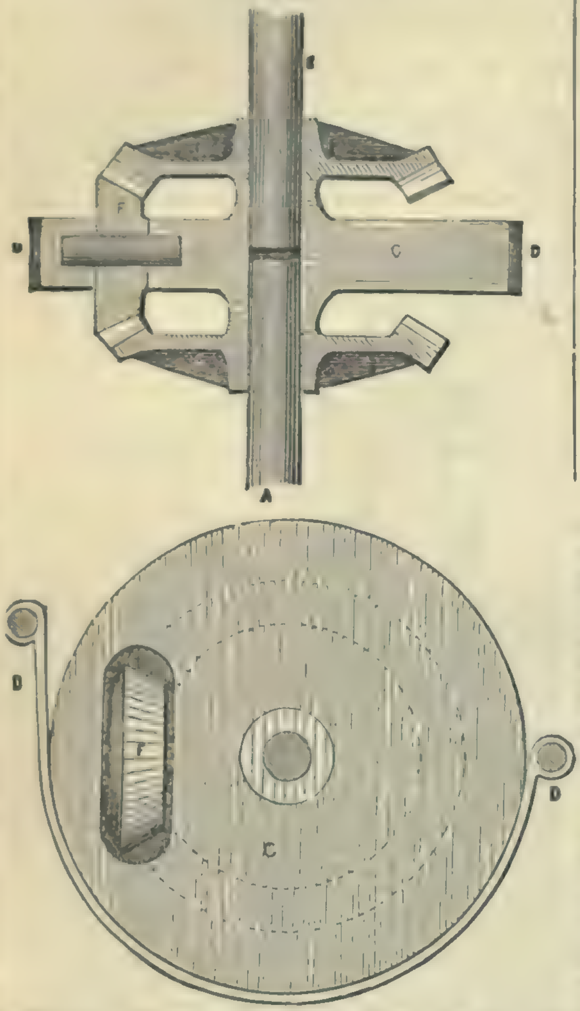
FRICTION-BREAKS.—It is often necessary to provide the means of stopping the motion of machinery when the mere cessation of power in the prime mover is not sufficient for the purpose. In a crane, when lowering a heavy weight, it may be desirable to lower it to a certain point and no farther, and therefore to stop the machinery of the crane when the weight has descended sufficiently far. Or, again, in any apparatus provided with a fly-wheel, or parts moving with considerable momentum, such as might continue the movement after the power has been withdrawn, it may be essential to provide the means of stopping the movement more suddenly. The most simple arrangement for this purpose is the *break* or friction-strap (Fig. 265). *a* is a wheel revolving with the rest of the machinery, and *b b* a flexible strap of iron passing round part of its circumference. One end of this strap, being fixed by a pin to some motionless part of the machine, and the other attached to a lever *c* pivoted on a fulcrum *d*, on applying force to the long arm of the lever, the strap is drawn tightly round the circumference of the wheel, and the friction caused by the close contact soon brings the wheel to rest. The great advantage of employing friction as a means of arresting motion, consists in the circumstance that it acts, not suddenly, but gradually. Were some solid obstacle presented to the motion of any part of a train of heavy or rapidly moving machinery, the momentum of all the moving parts would have to be suddenly destroyed; and as no time would be afforded for this operation, the strain would be incalculably great, and inevitable damage would ensue. But when the friction-break is employed, the wheel to which it is applied makes perhaps two or three revolutions before it comes finally to rest, and the time so occupied allows the momentum of all the parts connected with it to expend itself, in overcoming the great additional resistance caused by the friction.

Fig. 265.



In putting an extensive train of machinery in motion, the inertia of all the parts at rest has to be overcome in like manner; and were this done suddenly, the strain would be as great as in the opposite case of suddenly arresting their motion. This contingency is generally met by the use of pulleys and straps in communicating the power. A strap, communicating motion from one pulley to another, acts only by its friction on their circumferences; and when the strain which it has to overcome exceeds the force due to its friction, the strap slips at first to a considerable extent, but gradually less and less, until the proper velocity is attained, and the strap and circumference of the pulley move in unison. In cases where straps cannot be conveniently applied for driving a train, a friction coupling is employed. Fig. 266 is a diagram of one very generally used. A is the driving shaft, and B the driven shaft, the ends of which are free to revolve in the boss of a wheel C, true and smooth on its circumference, to which is applied a friction-strap D, worked by a suitable lever. Within the wheel C a bevel-pinion F is mounted in bearings, its axis being at right angles to that of the wheel and shafts; and bevel-wheels G and H, one on each shaft, are fitted to gear with the pinion F. If the friction-strap D be loose, so as to leave the wheel C free to revolve, the rotation of the shaft A, and its wheel G, gives motion to the pinion F, and causes its teeth to travel in those of the stationary wheel H, and thus to make the wheel C rotate at half the angular velocity of A, without putting B in motion. But if C be arrested by the friction of the strap D, the axis of F becomes fixed, and the rotation of A and G is communicated through F to B and H at the same speed, but in the opposite direction. The use of the friction-strap in this apparatus, presents the advantages in gradually generating momentum in the driven machinery, similar to those derived from its use in destroying the momentum of machinery in motion. Although, for the sake of simplicity, we have represented only one bevel-pinion F mounted in the friction-wheel, it is customary to provide at least two on opposite sides of the centre in order to balance it; and sometimes four are fitted for the sake of equilibrium and strength.

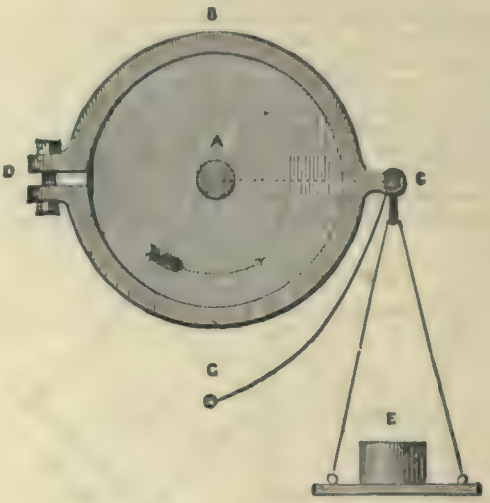
Fig. 266.



DYNAMOMETER.—It often becomes important to inquire what amount of power is communicated through a certain train of machinery. When a steam-engine is employed as the prime mover of any machine, the power communicated can be readily ascertained by the indicator. The engine is first worked alone, or with merely the train of wheel-work, in order that the power necessary to overcome friction may be estimated. It is then worked in connection with the machine, and the driving-power required for the machine is ascertained by subtracting the force necessary to overcome friction from the total power, including friction and the resistance of the machine. When machinery is driven by some other power, or when the indicator cannot be conveniently applied, the *dynamometer* (power-measurer) is employed.

The most simple kind of dynamometer consists of a pulley A (Fig. 267), fixed on the driving-shaft of the machinery whose power is required to be known. This pulley is surrounded by a flexible friction-strap C B D, the ends of which may be drawn close together by a screw at D, so as to tighten the strap as much as may be necessary on the circumference of the pulley. From a hook C, attached to the strap, is suspended a scale E, in which sufficient weights may be placed to prevent the strap from being carried round by the pulley in its revolution. A loose cord or chain C G, fixed at G, is also provided to keep the strap in its place, in case of the load E being insufficient to counterbalance the friction of the strap. Without this loose cord, a sudden increase of speed or friction might lift the scale and weights, and, whirling them round the pulley, do serious damage to the machinery. In using this machine, the speed of the shaft is carefully ascertained by counting the number of its revolutions per minute, and the screw D is gradually tightened until the scale and its load are just kept up by the friction, the imaginary line A C being horizontal. If the tightening of the screw cause the scale to be lifted, without reducing the speed of the pulley, more weight has to be added to the scale; but if the scale with its load cannot be lifted without retarding the pulley, the weight must be reduced. Having found the weight that is just supported when the velocity is correct, the power may be ascertained as follows:—The length of A C being the leverage at which the weight E acts to retard the rotation of the pulley, the power passing through the pulley must be such as would lift the given weight at the distance A C from its centre, or that would, during each revolution, move the weight E through a space equivalent to the circumference of a circle having A C for its radius. Since twice A C is the diameter of this imaginary circle, and the circumference is $3\frac{1}{7}$ times the diameter, $2 \times 3\frac{1}{7}$, or $6\frac{2}{7}$ times A C, is the space through which the weight is driven during each revolution; and this quantity multiplied by the number of revolutions per minute, is the total space through which the resistance is moved during each minute. The power is the weight multiplied by its velocity, or the distance through which it is moved per minute; and as 33,000 lbs. moved through 1 foot per minute is the standard horse-power, we have the following rule for estimating the horse-power as indicated by the dynamometer.

Fig. 267.



Rule.—Multiply the weight E (in lbs.) by the length of A C, the lever at which the weight acts (in feet), by $6\frac{2}{7}$ and by the number of revolutions of the pulley per minute, and divide by 33,000 for the horse-power.

Example.—The length of A C being 2 feet 4 inches, or $2\frac{1}{3}$ feet, the load E (including the weight of the scale) being 78 lbs., and the velocity of the pulley 120 revolutions per minute, required the power.

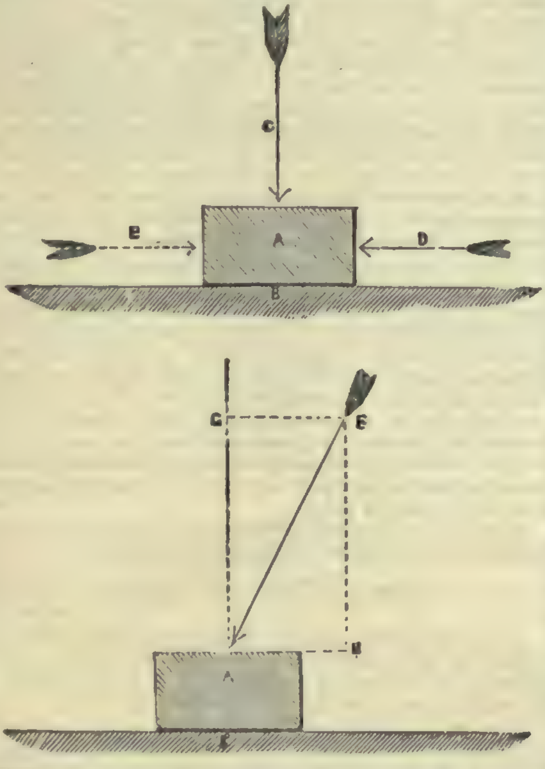
$$\frac{78 \text{ lbs.} \times 2\frac{1}{3} \times 6\frac{2}{7} \times 120}{33000} = 4{\cdot}16 \text{ horse-power.}$$

In dynamometers of this kind the friction-strap is plentifully supplied with oil, and it is found better to face the interior of the friction-strap with blocks of wood bearing on the surface of the pulley; because the friction of wood on iron is of a more constant character than that of iron on iron. When iron rubs on iron without the presence of oil or grease, great heat is produced, and the metal surfaces cut into each other and become roughened. But when wood bears on iron, if the surfaces are not oiled, the heat produced by the friction will only char the wood without damaging the iron, and will not effect any great variation in the amount of friction.

BREAKS.—Friction is also employed as a means of arresting motion in the case of carriages on declivities and railway trains. By the use of wheels to carriages, the friction is transferred from the surface of the road or rail to that of the axle. As the radius of the wheel is the length of lever at which any obstacle opposed to its progress acts, while the very much smaller radius of the axle is the lever at which its friction acts to oppose the rotation of the wheel, the larger the wheel and the smaller the axle, the less is the resistance from friction. If we were to suppose the axle extended to almost the whole dimensions of the wheel—that, in fact, the wheel were only the thin iron tyre revolving round a solid central part, the resistance to the motion of the carriage would be almost as great as if it were dragged along like a sledge. So if the wheel be prevented from revolving, it has to be dragged along the road, and the opposition which the friction thereby created presents, acts as a great retarding force. In ordinary carriages the wheel is generally prevented from revolving by placing under it a skid or plate of iron attached to the carriage by a chain, which has to be dragged along like a sledge. In railway carriages, the rotation of the wheels is arrested by means of blocks of wood pressed against their circumferences. These blocks are connected by levers and rods with screws conveniently situated, so that the engine-driver and guards can force them against the wheels, or remove the pressure at pleasure.

RESISTANCE OF FRICTION.—Except in these few instances, and that of pulleys and straps, in which friction is employed as a means of communicating power or arresting motion, it acts as a resistance in all mechanical arrangements. Although this resistance cannot be totally overcome, yet by carefully designing the arrangements of machinery with regard to the simplicity and proper formation of its parts, by good execution of the work, by the selection of suitable materials, and due provision of lubricating materials wherever surfaces move in contact with each other, it may be diminished to a very great extent.

Fig. 268.



In order that we may form a clear estimate of friction as a retarding force, we may suppose A (Fig. 268) to be a piece of material, such as iron, wood, brass, or the like, having a smooth lower surface in contact with a smooth table B of the same or any other material. If A be pressed down by a perpendicular force C, it will be found, that in order to move it laterally along the table, some force D will have to be impressed upon it, and this force will be greater the greater the perpendicular pressure C. There is, therefore, under these circumstances, a force acting in the direction of the arrow E, not tending to put A in motion, but resisting its motion in obedience to the force D. The resisting force E is the friction of the surfaces A and B. In order to measure the amount of this resistance, let us suppose a force F A acting obliquely at such an angle as just to cause A to slide along the table; then, on completing the parallelogram F G A H, while F H or G A measures the amount of force acting perpendicularly to the table, F G or H A measures the force acting parallel to the table, and causing A to slide along it.

Now it is found, practically, that whatever be the absolute force F A, and whatever be the extent of A's surface in contact with B, the obliquity of F A, or the angle F A G, which it makes with the perpendicular, when it just causes A to slide, is very nearly constant for any given material.

The ratio which A H bears to A G, or the fraction expressing the division of A H by A G, is called the *coefficient of friction*, and the angle F A G is called the *limiting angle of resistance*. Some very careful experiments have been made to determine the values of these for different materials; their results are embodied in the accompanying table. The coefficient of friction is the tangent of the limiting angle of resistance; and if we know the one, we can easily find the other from a trigonometrical table. We have, however, given an approximate value of both, to save the trouble of reference. As an example of the practical application of these numbers, we may take the case of brass and iron, for which the coefficient of friction is ·142, and the limiting angle 8°. If, then, a piece of smooth iron weighing 1 cwt. rested on a brass plate, it would require a lateral force of 16 lbs. to cause it to slide, for 112 lbs. (the vertical pressure) × ·143 = 16 lbs. Or, if the brass plate were inclined 8° to the horizon, the iron would slide along it from its own weight. Or, again, if we suppose a smooth round shaft of iron, weighing 1 ton, revolving in a brass bearing, the force necessary to be applied at the circumference of the shaft to overcome its friction, would be $2240 \times \cdot 143 = 320$ lbs. Assuming the shaft to be 6 inches diameter, or to have a radius of 3 inches at the bearing, the length of the radius, 3 inches, is the leverage at which the friction acts to resist its rotation, and the resistance, 320 lbs., at this leverage is equivalent to $320 \times \frac{12 \text{ in.}}{3 \text{ in.}} = 80$ lbs. at a radius of 1 foot. By a similar method of calculation, the resistance due to the sliding friction either of plane or of cylindrical surfaces of these or other materials may be readily estimated.

The numbers in the following table apply in the case of smooth surfaces, such as are employed in well-constructed machinery. When the surfaces are rough, the resistance may be increased indefinitely, and there can be no means of calculating its amount. When the surfaces are oiled, the friction is considerably reduced; but as, in all machinery, the rubbing surfaces are liable to become dry, or roughened by wear or the presence of grit, we think that in estimating the loss by friction, the numbers in the table should be employed without any allowance for lubrication.

TABLE OF FRICTION.

Materials of which the Rubbing Surfaces Consist.	Coefficient of Friction, Pressure being 1.	Limiting Angle of Resistance.
Steel and Ice	0·014	¾°
Ice and Ice	0·130	7½°
Hard Wood and Hard Wood .	0·135	7¾°
Brass and Steel or Iron . . .	0·142	8°
Soft Steel and Soft Steel . .	0·147	8¼°
Cast Iron and Steel	0·150	8½°
Wrought Iron and Wrought Iron	0·160	9°
Cast Iron and Cast Iron . .	0·163	9¼°
Hard Brass and Cast Iron . .	0·167	9½°
Cast Iron and Wrought Iron .	0·170	9¾°
Brass and Brass	0·175	10
Tin and Iron	0·180	10¼°
Soft Steel and Wrought Iron .	0·190	10¾°
Leather and Iron	0·250	14°
Tin and Tin	0·265	14¾°
Granite and Granite	0·300	16¾°
Yellow Deal and Yellow Deal	0·347	19¼°
Sandstone and Sandstone . .	0·364	20°
Woollen Cloth and Woollen Cloth	0·435	23½°

The best experiments made to fix these numbers are those by Rennie, who tested the materials under a pressure of 36 lbs. per square inch of rubbing surface. For greater pressures the friction is rather less in proportion, but the numbers given are sufficiently near for practical use. In general, it appears that when both surfaces are of the same material, the friction is greater than when they are of different materials. This is believed to be owing to the presence of a certain amount of that cohesive force which holds the molecules of any material together; and it is found to be greater the smoother the surfaces, and therefore the more intimate their contact. Independently of this circumstance, however, it is generally found inexpedient to make the rubbing surfaces in machinery of the same material, especially when there is any risk of their becoming heated by great pressure or rapidity of motion. In such cases, the particles of the one appear to blend with those of the other; the surfaces become cut into ridges and hollows, and sometimes cohesion takes place with such force that the materials themselves give way rather than separate from each other at their surfaces. This result is particularly observed in cases where iron and iron rub upon each other, especially when the iron is soft. When it is desired that the rubbing surfaces should be both of iron, it is better to case-harden the rubbing surfaces, as is usually done in the case of carriage axles. The process of case-hardening is effected by exposing the smooth surface to a red heat for several hours in a furnace, in contact with substances capable of furnishing it with carbon and nitrogen, such as prussiate of potash, leather shavings, and the like. The outer skin of the iron is penetrated by the carbon and nitrogen, and becomes a species of steel, which is rendered very hard by plunging it while still red-hot into cold water. After hardening, the surfaces have to be carefully ground smooth and true with oil and emery, or such-like polishing substances; and, before use, the emery must be carefully cleaned off, as its presence would otherwise cause the surfaces to cut, to become hot, and to cohere. In the case of shafts revolving in bearings, the shafts being generally of iron, are made to rest in bushes made of brass, gun-metal, tin, or some soft alloy. The rubbing surfaces are made to fit each other accurately, being turned, bored, filed or scraped where necessary, until the contact is made nearly perfect. When the contact is very imperfect, or only takes place at a few points of the surface, these become rapidly worn and cut, and the heat produced by their wear expands unequally other portions of the surfaces, which become abraded in their turn. In cases where great pressure is sustained on a bearing, the surfaces of contact are extended as much as possible, so that the intensity of pressure on any portion of the surface may be as small as possible. For shafts lying in cylindrical bearings, the extension of surface is best effected by lengthening the bearing, not by increasing its diameter. By increasing the length, the surface is proportionally increased; the pressure per square inch is proportionally diminished; the amount of friction is not altered, but the tendency to cutting or wear is reduced. But were the diameter increased, although the surface would be also increased, and the wear reduced, yet the friction would act at an increased leverage, and have greater effect as a resisting force. When the pressure is directed longitudinally along the shaft, the bearing surface is increased by forming on the shaft numerous collars or projecting rings, which are made to press against fixed ring-bearings fitted between them. The total pressure of the shaft is thus subdivided over the combined surface of all the collars, and the ring-bearings being made capable of adjustment by regulating screws, no one of them is unduly pressed upon. Such a form of bearing is particularly useful in steam-vessels fitted with screw-propellers. The propeller, in revolving, tends to throw the water backwards from the vessel, and the reaction of the water is the force which propels the vessel. But this force, sometimes amounting to several tons, is communicated through the shaft of the propeller to some fixed point in the vessel; and at this point is situated the bearing such as we have described, technically called the *pushing or thrust bearing*.

FRICTION OF STRAPS.—The friction of straps upon pulleys depends upon the extent to which they are tightened, the extent of circumference with which they are in contact, and their breadth. It is commonly believed, that the greater the diameter of pulley, the more surely does the strap cause it to revolve without slipping. Theoretically, however, and we believe practically, it will be found that, with equal degrees of tightness, equal breadths of strap, and equal circumstances as to perfection of contact, the friction of a strap on the circumference of a pulley is the same, whatever be its diameter. The only circumstance that can affect the constancy of the result is, that straps not being perfectly flexible, lie more closely to surfaces curved to a large radius than to those of smaller radius. When a certain amount of power has to be communicated through a strap, the speed at which the strap moves has to be taken into account, because power being pressure multiplied by velocity, the greater the velocity with which the power is transmitted, the less the pressure that has to be communicated at that speed. In this sense, then, it appears that the larger the pulley the less is the slip of the strap, because the greater the circumference of the pulley revolving at a given angular velocity, the greater is its absolute velocity through space, and therefore the less the pressure required to communicate a given power. It is found, practically, that a leather strap 8 inches wide, embracing half the circumference of a smoothly-turned iron pulley, and travelling at the rate of 100 feet per minute, can communicate 1 horse-power. For communicating any given power at any given velocity, the breadth of the strap may be found thus:—

Rule.—Multiply the power (horse) by 800, and divide by the speed (in feet per minute); the quotient is the breadth of strap in inches.

Example 1.—Required the breadth of strap, travelling 600 feet per minute, to communicate 12 horse-power.

$$\text{Width, } \frac{12\times800}{600}=16 \text{ inches.}$$

Note.—When the diameter of pulley (in feet) and the number of revolutions per minute are given, the speed of the strap is found by multiplying the given number of revolutions by the diameter (in feet), and by 3⅐.

Example 2.—Required the breadth of strap for communicating 10 horse-power to a pulley 3 ft. 6 ins. diameter, revolving 150 times per minute.

Speed of strap, $150 \times 3\frac{1}{2}$ ft. $\times 3\frac{1}{7} = 1650$ feet per minute.

$$\text{Breadth, } \frac{10 \times 800}{1650} = 5 \text{ inches nearly.}$$

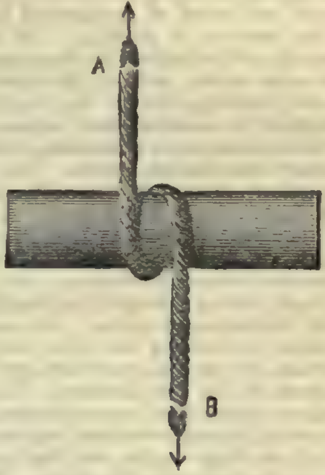
When less than half the circumference of the pulley is embraced, the strap must be proportionally wider; and when more than half the circumference is embraced, its width may be less.

The law according to which the friction of a strap increases with an increased arc of contact, is of a peculiar character, but may be readily understood by comparing the friction on arcs of different lengths. If a pulley (of any diameter whatever) were prevented from revolving, and a strap passing over part of its circumference were stretched by a certain weight at each end, additions might be made to the weight at one end until the strap began to slip over the pulley. The ratio which the weight so increased might bear to the weight at the other end, would measure the amount of friction. For example, in experiments made to test a theoretical investigation on this subject, a strap passing over a pulley in contact with 60°, or $\frac{1}{6}$th of its circumference, was stretched by a weight of 10 lbs. at each end. One of the weights was increased until it amounted to 16 lbs., when the strap began to slip. The ratio of 16 to 10, or $\frac{16}{10} = 1{\cdot}6$ was then the measure of the friction. When 20 lbs. at each end were used to stretch the strap, the one weight was increased to 32 lbs., giving the ratio of $\frac{32}{20} = \frac{16}{10} = 1{\cdot}6$, the same as before. And likewise when 5 lbs. were used for stretching, the weight at one end was increased to 8 lbs., giving still the same ratio $\frac{8}{5} = 1{\cdot}6$. So far, then, the friction was precisely proportional to the stretching weight, as might have been expected from the ordinarily received doctrine on the subject of friction. On extending the arc of contact to 120° = $\frac{2}{6}$ths or $\frac{1}{3}$rd of the circumference, the ratio was found to be 2·56, or 1·6 multiplied by 1·6, or the square of 1·6. And again, on embracing 180° = $\frac{3}{6}$ths or $\frac{1}{2}$ of the circumference, the ratio was found to be 4·1, very nearly the cube of 1·6. The theoretical investigation (given shortly in the note below*) brought out this result independently, and the following law may therefore be taken as established:—

If, for any given arc of contact, the one weight bears to the other, at the point of slipping, a certain ratio—for double the arc, the ratio will be squared; for triple the arc it will be cubed; for 4 times the arc it will be raised to the 4th power; and so on.

In all cases, however, much depends on the tightness of the strap, the limits to the force with which it is strained being, first, the tensive strength of the strap itself, and, secondly, the amount of pressure that it may be convenient to throw upon the shaft and its bearings. New straps become extended by use, and it is therefore frequently necessary to take them up or shorten them. Before use, they should be strained for some time by weights suspended from them, so as to leave less room for extension while in use. Wherever straps are employed, they should be of the greatest breadth, and travel at the greatest speed consistent with convenience, as it is most important to have the requisite strength in the form best suited for flexure, and the least possible strain on the shafts and bearings.

Fig. 269.



When ropes or chains are employed, as in cranes, capstans, windlasses, or the like, for raising heavy weights or resisting great strains, the requisite amount of friction is obtained by coiling them more than once round the barrel of the apparatus. It is found that one complete coil of a rope, as in Fig. 269, produces a friction equivalent to 9 times the tension on the rope, the barrel being fixed; that is to say, 1 lb. or 1 cwt. of tension on the end of a rope at A, can support 9 lbs. or 9 cwt. of tension at B. Were the end B of the rope coiled again round a barrel, it would support 9 times its tension; that is, 9 × 9, or 81 times the tension of A, and so on, coil after coil increasing the friction in a very high ratio, as may be calculated according to the law given above, thus:—

Multiply 9 by itself as many times as there are coils, and the product will be the number of times the tension at one end that will be supported at the other. For example, 1 cwt. at one end of a rope coiled three times round a barrel, would support $9 \times 9 \times 9 = 729$ cwt., or $36\frac{1}{2}$ tons at the other end of the rope. The diameter of the barrel does not affect the result. Having regard to these facts, we may readily understand the force with which a knot on a cord or rope resists the slip of the coils of which it consists, for the several parts of the cord act as small barrels, round which the other parts are coiled; and the yielding nature of the material of which the barrels are composed, permits the coils to become impressed into their substance on the application of force, and prevents them from slipping more effectually than if they were coiled on a hard and resisting barrel.

MOTION OF FLUIDS.—Before concluding this part of the subject, we may briefly allude to suggestions that have from time to time been made for communicating power by means of the movement of fluids. Some of these have been practically carried out with very good effect, and we believe that much may yet be done in this direction.

Manufactories, containing numerous machines, are generally arranged in such a manner that the power of a

---

* If w be a weight straining one end of a strap, and W a greater weight applied at the other end, just sufficient to cause the slipping of the strap when it is in contact over an arc of the circumference θ; then, while θ remains constant, the ratio $\frac{W}{w}$, which may be called f, is constant, whatever the absolute weights W and w may be, or the friction is proportional to the pressure; for $W - w$ is the measure of the friction; and while W and w maintain the same proportion, $W - w$ also maintains the same proportion to either of them. But if the strap supporting W be bent farther round the circumference, so as to embrace an arc increased by $\triangle\theta$, W will have to be increased by a quantity $\triangle W$ to produce slip, which must be proportional to W, the friction on any arc being proportional to the pressure. Hence ultimately

$\frac{dW}{d\theta} = aW$, when a is some constant depending on the nature of the rubbing surfaces. Integrating, we have $\log. W = a\theta + C$. When $\theta = o$, $W = w$, because the friction is $= o$. Hence $\log. \frac{W}{w} = a\theta$, or $\frac{W}{w} = f = \varepsilon^{a\theta}$. If when the arc is Θ, the friction be $F = \varepsilon^{a\Theta}$ and hence $f = \varepsilon^{a\theta} = \left(\varepsilon^{a\Theta}\right)^{\frac{\theta}{\Theta}} = F^{\frac{\theta}{\Theta}}$, or the ratio expressing the friction over any arc is, as the friction over any other arc raised, to the power expressed by the ratio of the one arc to the other great prime-mover, such as a steam-engine or water-wheel, is communicated by shafts, pulleys, straps, and wheel-work, to the different machines. However excellent may be the arrangements or workmanship of the connecting machinery, still there must be considerable waste of power in overcoming the mere friction attending its motion; and this waste is very much increased when the accuracy of arrangement is affected by wear, or by the unequal settlement of any part of the building supporting the bearings of shafting. Of late years, engineers have devoted attention to the communication of power, to some extent, without the intervention of wheel-work and shafting, but by the supply of steam from a central boiler to numerous small engines, arranged throughout the building, in the immediate neighbourhood of the machinery to be put in motion. The pipes that convey the steam are cheaper in first cost than shafting of equivalent length; their efficacy is not dependent on accuracy of levels, nor is it impaired by wear; and the friction attending the passage of steam through them is much less than the friction of shafting. It is true that the force of steam passing through great lengths of pipes is considerably diminished by cooling; but much of this loss may be saved by covering the pipes with non-conducting materials, such as felt or sawdust, and by superheating the steam in the first instance. The objection generally made to this mode of communicating power is, that the first cost of the numerous small engines generally exceeds that of one large engine, and that each of the small engines requires separate attendance. We believe, however, that, in an extensive work, the cost of steam-pipes and numerous small engines will not be found to exceed that of a large engine, with all the shafting and connecting machinery; and that, with proper arrangements, the attendance given by workmen to the separate machines will be sufficient for the engines that put them in motion. A number of machines are now made to act directly by steam-power, without the intervention of machinery. Among them we may cite, as examples, the steam-hammer and steam riveting machine. In both these cases, the direct pressure of steam upon a piston takes the place of the former complicated arrangements of wheel-work, eccentrics, cams, and levers, which were necessary for converting the rotary motion of shafting into the reciprocating movements required for hammering and riveting. To many other manufacturing operations we think the same principle of movement might be applied with good effect; and we believe that the simplicity, directness of action, and facility of control attending such arrangements, would soon cause them to take the place of the more complicated methods of deriving power from rotary motions.[*]

The atmospheric railway is an example of an arrangement for communicating power on a large scale by means of the fluid motion. A pipe, several miles in length, with a longitudinal slit or opening along its upper side, is laid down between the rails. The pipe is fitted with a piston, and the slit is covered by a flexible valve, which can be lifted so as to permit an upright rod attached to the piston to pass through the upper part of the pipe to a carriage mounted on the rails. When the carriage and piston are at one end of the pipe, the valve being closed tightly over the slit, and luted by grease melted over it, the air is extracted from the pipe by pumps worked by a steam-engine; and the pressure of air on one side of the piston being thus in a great measured removed, the atmospheric pressure, acting on the other side, forces it along the pipe with great velocity, and thus gives motion to the carriage which is connected with the piston and the train of carriages attached to it. The piston and connecting-rod are fitted with suitable rollers for lifting the valve in front of the upright rod, and for closing it and sealing it up behind it; so that after the passage of the train in one direction, the pipe may be ready for exhaustion to draw a train in the opposite direction. In the trials made with this system of propelling trains, it was found possible to secure great velocities, and to move very heavy trains over severe inclinations when the pipe and its valve were in good order. But the great practical difficulties attending the construction and tight-closing of the valve proved an almost insuperable objection to the use of the atmospheric railway, as even a small leakage of air into the vacuum-pipe necessitated the expenditure of great power in the pumping engine to sustain the required vacuum.

It was at one time proposed, that in the case of a city like London, with numerous small steam-engines and boilers scattered through it, power should be communicated to different establishments from some central source, by means of compressed air forced through pipes laid down like the gas and water-pipes in the streets. One large engine, continually sustaining a supply of air condensed to a pressure of several atmospheres, would thus take the place of the numerous separate boilers now supplying steam, and the air thus supplied would work the present engines as effectually as the steam. There is little doubt that, were such an arrangement carried out, the first cost of the air-pipes being met, there would be considerable advantages attending it, in respect of economy and the diminution of danger from boiler explosions, as well as the removal of numerous sources of offence from the chimneys of manufacturing establishments.

Arrangements of a similar character might at present be carried out in cities like London without any great outlay, where the supply of water is adequate. The pumping engines of the water companies might be made the means of supplying power to numerous establishments of machinery at a very cheap rate. It is found, practically, that in pumping water, 1 ton of fuel is capable of raising 25,000 gallons of water 100 feet high; and as water from a cistern 100 feet high would press with a force of 46 lbs. on every square inch of a piston exposed to its action, this power might be readily made available for raising weights, giving pressure, or even driving machinery by the intervention of properly constructed water-pressure engines.[†]

CHAPTER VIII.

RULES AND TABLES.

Contents.—SYMBOLS OF ARITHMETICAL OPERATIONS—WEIGHTS AND MEASURES—TEMPERATURE—ARCS AND ANGLES—CIRCUMFERENCES—SUPERFICIES—SOLIDS—SOLIDS OF REVOLUTION—DUODECIMALS—WEIGHTS OF MATERIALS.

THE practical mechanic has frequent occasion to calculate the lengths of lines, areas of figures, capacities of solids, and weights of masses of material. We think, therefore, that we cannot better conclude the general view we have endeavoured to give of mechanics practically applied, than by furnishing a few of those simple rules and methods of calculation, which most commonly occur in practice.

[Many of these rules have been dealt with already in a scientific form in the section on Mathematics, but this statement, in a simple and concise form, may render them more easily understood by persons not familiar with abstract science].

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[*] In Manchester and Glasgow it is usual to fix a separate steam engine to each calico-printing machine, partly for the reasons assigned above, and also that each machine may be independent of a main moving power, whose speed could not be readily varied to suit the exigencies of the work in hand.—ED.

[†] Since the completion of the Loch Katrine water-works, by means of which Glasgow is supplied with water at a high pressure, many small water engines have been erected for working cranes, &c., which are charged at a fixed annual rental.—ED.

We must presume that the mechanic is tolerably intimate with the ordinary operations of arithmetic—Addition, Subtraction, Multiplication, and Division; and that he will bear in mind the following symbols, as a kind of arithmetical short-hand, often useful in expressing rules and modes of operation.

+ placed before a quantity, means that it is to be added to the quantity preceding it.

— placed before a quantity, means that it is to be subtracted from the quantity preceding it.

× placed between two quantities, means that the first is to be multiplied by the other.

÷ placed between two quantities, means that the first is to be divided by the last.

When instead of two dots in this symbol of division, one quantity is written above the line and the other below it, the upper is to be divided by the lower. Thus $12 \div 4$ may be written $\frac{12}{4}$, which means that 12 is to be divided by 4.

= placed between two quantities, means that the one is equal to the other.

Example 1.—*Power = weight × velocity,* means that the power of any machine is equal to the weight raised by it, multiplied by the velocity with which it is raised. Thus, if an engine raise 330 lbs. at the rate or velocity of 100 feet per minute, we should say its power is equivalent to 330 lbs. × 100 feet = 33,000 lbs. at the rate of 1 foot per minute.

2. *Horse-power* $= \frac{\textit{weight (lbs)} \times \textit{velocity (ft. per min.)}}{33000}$

means that the power of a machine reduced to the standard of horse-power, is equivalent to the weight in lbs. multiplied by the velocity in feet per minute, and divided by 33000. Thus, if an engine raise 330 lbs. 100 feet high per minute, its horse-power is $\frac{330 \times 100}{33000} = 1$; that is to say, one-horse power.

WEIGHTS AND MEASURES.—The table of weights and measures most necessary in computations connected with Practical Mechanics, are those of Avoirdupois Weight, Lineal, Superficial, and Solid Measure, Time, Temperature, and the Division of the Circle. Unfortunately for ease of recollection and computation, the English Tables of measures have no regular system, and, therefore, require to be remembered separately, or to be constantly referred to. The French have adopted a regular and simple system, both in the names and in the relations of their different denominations. As their measures are frequently referred to in scientific works, we subjoin tables of them along with the English tables; and short rules for the reduction of quantities given in the one, to their corresponding values in the other.

TROY WEIGHT.

Used for the precious metals and for chemical analysis.

		Contracted.
24 Grains	= 1 Pennyweight	24 gr. = 1 dwt.
20 Pennyweights	= 1 Ounce	20 dwt. = 1 oz.
12 Ounces	= 1 Pound	12 oz. = 1 lb.

1 pound troy therefore contains 5760 grains.
1 pound avoirdupois contains 7000 troy grains.

AVOIRDUPOIS WEIGHT.

Used for weighing all materials except those to which troy weight is confined.

		Contracted.
16 Drachms	= 1 Ounce	16 dr. = 1 oz.
16 Ounces	= 1 Pound	16 oz. = 1 lb.
28 Pounds	= 1 Quarter	28 lbs. = 1 qr.
4 Quarters	= 1 Hundredweight	4 qrs. = 1 cwt.
20 Hundredweights	= 1 Ton	20 cwt. = 1 ton.

FRENCH DECIMAL SYSTEM OF WEIGHT.

10 Milligrammes = 1 Centigramme
10 Centigrammes = 1 Decigramme.
10 Decigrammes = 1 Gramme = 15·434 troy grains.
10 Grammes = 1 Decagramme.
10 Decagrammes = 1 Hectogramme.
10 Hectogrammes = 1 Kilogramme = 2·20486 lbs. avoirdupois.
10 Kilogrammes = 1 Myriagramme.
10 Myriagrammes = 1 Quintal = 1 cwt. 3 qrs. 25 lbs., nearly.
100 Quintals = 1 Millier or Bar = 9 tons. 16 cwt. 3 qrs. 12 lbs.

The general principle adopted in the French system, is that of the decimal scale. They settle on some unit of weight or measure—as the gramme for weight, and the mètre for measure. For the names of all fractions of that unit, proceeding by tenths, hundredths, and thousandths, downwards, they prefix the Latin numerals *deci* for tenth, *centi* for hundredth, *milli* for thousandth, to the name of the unit. Thus, a centigramme is the hundredth part of a gramme, and would be written in figures 0·01 gramme; a millimètre is a thousandth part of a mètre, and would be written 0·001 mètre. Again, for the names of all multiples of the unit, proceeding by tens, hundreds, thousands, and ten thousands upwards, they use the Greek numerals—*deca*, ten; *hecto*, hundred; *kilo*, thousand; *myria*, ten thousand, prefixed to the unit. Thus, for a thousand mètres, they say a kilomètre, written 1000 mètres; for ten thousand grammes, they say myriagramme, written 10,000 grammes. There are a few exceptions for the larger denominations in the scale of weights, which we have given in full. According to this system, each denomination finds its place in the ordinary decimal scale of notation, and arithmetical operations are reduced to the simple rules, without the necessities of complicated reduction.

For example, if we wished to ascertain the cost of 7 myriagrammes, 3 kilogrammes, 4 hectogrammes, 6 decagrammes, 5 grammes, 3 centigrammes, of a material at 2 francs 20 cents per kilogramme,

we should write the quantity . 73465·03 grammes
which is equivalent to . . . 73·46503 kilogrammes, pointing off 3 figures.
Multiply by the price per kilo. . 2·2

14693006
14693006

161·623066 francs.

Or, 161 francs, 62 cents.

In the English system, on the other hand, a similar question—as, for instance, the cost of 17 tons 4 cwt. 3 qrs. and 18 lbs., at 12*s.* 9*d.* per cwt.—would involve the necessity of reducing the several denominations, or of artifices for employing fractional parts as in the ordinary arithmetical rule of Practice.

To reduce kilogrammes to lbs. avoirdupois.

Rule.—Multiply by 2·20486.

Example 1.—To reduce 17 kilogrammes to lbs.

Multiply by 2·20486

37·48262 lbs.

For general practical computations, the decimal fraction, after its first figure, may be neglected, and the multiplier may be taken as 2·2 simply.

Example 2.—To reduce 23 kilogrammes to lbs.

$$23 \times 2{\cdot}2 = 50{\cdot}6 \text{ lbs.}$$

For closer approximation, after multiplying the number of kilogrammes by 2·2, add to the result the 200th part of the number, or the half with two decimal places pointed off.

Thus, $23 \times 2{\cdot}2 = 50{\cdot}6$
add $\frac{1}{200}$th of 23 ·115

50·715 lbs.
The accurate number 50·71178

To reduce lbs. avoirdupois to kilogrammes.

Rule.—Divide by 2·20486, or multiply by its reciprocal ·45355.

Example 1.—To reduce 37·48262 lbs. to kilogrammes.

Dividing by 2·20086) ———

17 kilogrammes.

The reduction may be effected with sufficient accuracy by the following artifice :—

Add together one-third of the given number of lbs , one-third of that third, and one-twelfth of the last third.[*]

Example 2.—To reduce 37·48262 lbs. to kilogrammes.

one-third	12·49421
one-third of one-third	4·16473
one-twelfth of the last	·34706
	17·00600

Example 3.—To reduce 5 cwts. 2 qrs. 12 lbs. to French weight.

4

22 qrs.
28

3)628 lbs.

3)209·33
12) 69·77
5·81

284·91 kilogrammes.

LINEAL MEASURES.

12 Inches	=1 Foot	6 Feet	=1 Fathom
3 Feet	=1 Yard		
$5\frac{1}{2}$ Yards	=1 Pole or Rod	7·92 Inches	=1 Link
40 Poles	=1 Furlong	100 Links	=1 Chain
8 Furlgs.	=1 Mile	80 Chains	=1 Mile

The measures employed for the smaller dimensions of mechanical work, are the foot, its twelfth part the inch, and the fractional divisions of the inch, dividing successively by 2 ; viz.—the half, the quarter, the eighth, the sixteenth, the thirty-second, &c. For larger works, as cuttings, embankments, shafts of mines, and the like, the yard of 3 feet, or the fathom of 6 feet, are the ordinary units. For surveying purposes, the chain of 11 fathoms, or 22 yards, or 66 feet, or 100 links is the unit. And for greater dimensions, the mile, or 1760 yards, or 5280 feet, and its fractional divisions by 2, the half-mile, the quarter mile, the eighth of a mile or furlong, are employed.

The nautical mile or knot measures 6080 feet.

The unit of French measure is the mètre, which is equal in length to 39·371 English inches, or about 3·281 English feet, or 3 feet $3\frac{3}{8}$ inches very nearly.

To reduce French measure to English.

Rule.—Multiply the number of French mètres by 3, and take the product as so many English feet, so many inches, and so many eighths of inches ; or add together the product, its twelfth part, and the eighth part of the twelfth, for the value in English feet.

Example 1.—To reduce 53 mètres to English measure.

	ft.	in.
53 × 3=	159	
159 inches=	13	3
159 eighths of inches= .	1	$7\frac{7}{8}$
	173	$10\frac{7}{8}$ nearly.

Example 2.—To reduce 42 kilomètres to English miles. The kilomètre is 1000 mètres, and 42 kilomètres are therefore equivalent to 42000 mètres.

	42000
Multiply by	3
	126000
One-twelfth . .	10500
One-eighth . .	1312·5

One mile contains 5280)137812·5 feet.

26·1 miles nearly.

1 kilomètre is exactly 1093·6389 yards.
1 mile is exactly 1760 yards.

Therefore, to reduce kilomètres to miles, multiply by $\frac{1093 \cdot 6389}{1760}$, or ·621386.

For rough calculations, take $\frac{5}{8}$ths decimally, ·625.

To reduce miles to kilomètres, multiply by $\frac{1760}{1093 \cdot 6389}$, or 1·60931 nearly.

For rough calculations take $1\frac{3}{5}$th decimally, 1·6.

The French pied or foot, is equal to 1·09 English foot.

The French pouce, or inch, is equal to 1·09 English inch.

SUPERFICIAL MEASURE.

144 square inches = 1 square foot.
9 square feet = 1 square yard.
$30\frac{1}{4}$ square yards = 1 square pole.
40 square poles = 1 rood.
4 roods = 1 acre.

The acre, therefore, contains 4840 square yards, or 10 square chains, each of 484 square yards.

The French superficial measure is, for small areas, reckoned by the squares of the mètre and its parts, the mètre carré, or square mètre, being 10·765 square feet, or 1·196 square yard. For larger areas, the French unit is the Are, which is a square decamètre, or 100 square mètres, equivalent to 119·605 square yards.

SOLID MEASURE.

1728 cubic inches = 1 cubic foot.
27 cubic feet = 1 cubic yard.
42 cubic feet = 1 ton of shipping.
40 ,, = 1 load of rough timber.
50 ,, = 1 load of squared timber.

The French solid measure is reckoned by the cube of the mètre and its parts.

1 cubic foot = ·0283 cubic mètre.
1 cubic yard = ·7645 ,,
1 cubic mètre = 35·32 cubic feet.

LIQUID MEASURE.

There are several varieties of English liquid measure, and provincial variations in their scales ; but the principal unit of liquid measure is the imperial gallon, which contains 277·274 cubic inches, and of which a cubic foot contains 6·2321, or nearly $6\frac{1}{4}$. The unit of French liquid measure is the litre, or cubic decimètre, the thousandth part of a cubic mètre, equivalent to 61·028 cubic inches, somewhat less than the imperial quart, which contains 69·3185 cubic inches.

TIME MEASURE.

60 seconds = 1 minute.
60 minutes = 1 hour.
24 hours = 1 day.
7 days = 1 week.

The French measurement of time is the same.

TEMPERATURE.

There are three scales of temperature in use :—

1. Fahrenheit's, used in England, in which the point of water freezing is 32°, and the point of water boiling is 212°.

2. The Centigrade, used in France, in which the freezing-point of water is the zero, or 0°, and its boiling-point is 100°.

3. Reaumur's, used among some Continental nations, in which the freezing-point of water is zero, or 0°, and its boiling-point 80°.

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[*] The reason of this rule is the following :—

One-third of 1, expressed decimally, is	·3333 &c.
One-third of ·333, &c., expressed decimally, is .	·1111 ,,
One-twelfth of ·111, &c., expressed decimally, is .	·0092 ,,
Their sum	·4536
is a near approximation to . . .	·45355

To reduce temperature by Fahrenheit to temperature by Centigrade.

Rule.—Subtract 32°, multiply by 5, and divide by 9.

Example 1.—To reduce 180° Fahrenheit to Centigrade.

180

Subtract . . 32

$$\frac{148 \times 5}{9} = 82^{\circ}{\cdot}222 \text{ Centigrade.}$$

Example 2.—To reduce 15° Fahrenheit to Centigrade.

15

Subtract . . 32

$$\frac{-17 \times 5}{9} = -9^{\circ}{\cdot}44, \text{ \&c.,}$$

or 9°·44, &c., below zero.

To reduce temperature by Centigrade to temperature by Fahrenheit.

Rule.—Multiply by 9, divide by 5, and add 32°.

Example 3.—To reduce 82°·2 Centigr. to Fah.

Multiply by . . 9

Divide by . . 5)740 nearly

148

Add 32

180° Fah.

Example 4.—To reduce — 9°·4 Centigr. to Fah.

9

Divide by . . . 5) − 85 nearly

−17

Add 32

15° Fah.

Note.—When the temperature has the mark — (*minus*) prefixed, it means *below* zero, and is to be subtracted in cases where it would be added if it had not the mark —.

Example 5.—To reduce − 100° Centigr. to Fah.

9

5) − 900

− 180

32

− 148° Fah., or 148° below zero.

To reduce temperature by Fahrenheit to temperature by Reaumur.

Rule.—Subtract 32, multiply by 4, and divide by 9.

Example 6.—To reduce 62° Fah. to Reau.

Subtract . . . 32

30

Multiply by . . 4

Divide by . . 9)120

13°·3 Reau.

Example 7.—To reduce 15° Fah. to Reau.

32

− 17

4

9) − 68

− 7°·5 Reau., or 7°·5 below zero.

To reduce temperature by Reaumur to temperature by Fahrenheit.

Rule.—Multiply by 9, divide by 4, and add 32.

Example 8.—To reduce 13°·3 Reau. to Fah.

9

4)120

30

32

62° Fah.

Example 9.—To reduce − 7°·5 Reau. to Fah.

9

4) − 68

− 17

32

15° Fah.

To reduce Centigrade to Reaumur.

Rule.—Subtract $\frac{1}{5}$th.

Example 10.—To reduce 85° Centigrade to Reaumur.

One-fifth 17

68° Reau.

To reduce Reaumur to Centigrade.

Rule.—Add $\frac{1}{4}$th.

Example 11.—To reduce 68° Reaumur to Centigrade.

One-fourth . . . 17

85° Centigr.

DIVISION OF THE CIRCLE.

60 Seconds = 1 Minute 60″ = 1′

60 Minutes = 1 Degree 60′ = 1°

90 Degrees = 1 Quadrant or right angle, often marked $\frac{\pi}{2}$

360 Degrees or 4 Quadrants = 1 Circumference, often marked 2 π.

THE FRENCH DIVISION IS

100 Seconds = 1 Minute.

100 Minutes = 1 Degree.

100 Degrees = 1 Quadrant.

4 Quadrants = 1 Circumference.

The quadrant, right angle, or quarter of the circumference, is the same for the French and English systems, and 100° French are therefore equivalent to 90° English.

To reduce French degrees to English.

Rule.—Subtract $\frac{1}{10}$th.

Example 1.—To express 73° French in English measure.

Subtract one-tenth . 7·3

65·7 65° 42′ English.

Multiply by . . 60

To find minutes . 42 0

Example 2.—To express 26° 7′ 35″ French in English measure.

The French number would be written 26°·0735

Subtract one-tenth . . . . . 2 ·60735

23 ·46615

60

27 ·969

60

58·14

Ans. 23° 27′ 58″·14 English.

To reduce English degrees to French.

Rule.—Add $\frac{1}{9}$th.

Example 3.—To express 65° 42′ English in French measure. $42' = \frac{42}{60}$ or 0°·7; therefore the English, expressed decimally, is 65°·7

Add one-ninth . 7 ·3

73°· French.

Example 4.—To express 23° 27′ 58″·14 English in French. The quantity, decimally expressed, is

$$\begin{array}{lr} & 23^{\circ}\cdot46615 \text{ English.} \\ \text{Add one-ninth} \quad . & 2\ \cdot60735 \\ \hline & 26^{\circ}\cdot0735 \text{ French.} \end{array}$$

The circumference of every circle is 3·1416 times its diameter, or 6·2832 times its radius or half diameter.

Hence to find the circumference of a circle whose diameter or whose radius is given,

Rule.—Multiply the diameter by 3·1416, or the radius by 6·2832.

Example 1.—To find the circumference of a pulley whose diameter is 4 feet 6 inches.

4 feet 6 ins., or 54 ins. × 3·1416 = 169·6464 ins., or 14 feet 1⅝ inches nearly.

Example 2.—To find the circumference of the pitch circle of a wheel, the pitch radius being 11 inches.

11 ins. × 6·2832 = 69·1152 ins., or nearly 69⅛ ins.

Since the decimal ·1416 is nearly equal to $\frac{1}{7}$th (which is accurately ·142587), we have the approximate rule for finding the circumference of a circle.

Rule.—Multiply the diameter by $3\frac{1}{7}$, or (since $3\frac{1}{7} = \frac{22}{7}$) multiply the diameter by 22 and divide by 7, or multiply the radius by 44, and divide by 7.

Example 3.—To find the circumference of a pulley 4 feet 6 inches diameter.

$$\begin{array}{lr} \text{4 feet 6 ins.} \times 3 & = \text{13 feet 6 ins.} \\ \text{4 feet 6 ins.} \times \frac{1}{7} (\text{or} \div 7) = & 7\tfrac{5}{8} \text{ ins. nearly.} \\ \hline \text{Circumference} & \text{14 feet } 1\tfrac{5}{8} \text{ ins. nearly.} \end{array}$$

Example 4.—To find the circumference of a wheel whose radius is 11 inches.

$$\frac{11 + 44}{7} = 69\tfrac{1}{8} \text{ ins. nearly.}$$

The circumference of an ellipse is that of a circle whose diameter is a mean between the two axes of the ellipse.

Example 5.—To find the circumference of an ellipse having a longer axis 18 ins., and a shorter axis 12 ins.

Mean $\frac{18+12}{2}$ = 15 ins. × 3·1416 = 47·124 ins. or 3 feet 11⅛ ins.

The converse operation of finding the diameter of a circle having a given circumference is,

Rule.—Divide the circumference by 3·1416, or multiply it by ·31831, the reciprocal of 3·1416.

Example 6.—To find the diameter of a pulley whose circumference is 14 feet 1⅝ in. or 169·625 ins.

$$\frac{169\cdot625}{3\cdot1416} = 54 \text{ ins., or 4 feet 6 ins. very nearly.}$$

or 169·625 × ·31831 = 54 ins. nearly.

The following approximation is generally sufficiently seen for practical purposes.

Rule.—Multiply the circumference by 7, and divide by 22, the quotient is the diameter.

Example 7.—To find the diameter of a wheel having 23 teeth, each of 3 ins. pitch.

23 × 3 ins. = 69 ins. the circumference of the pitch circle, and $\frac{69\times7}{22}$ = 22 ins. nearly.

The pitch radius is ½ of 22 ins. = 11 ins.

Mensuration of Superficies.

The object of Mensuration of Superficies is to discover the number of square units in a figure, the form and dimensions of whose boundary are known. The simplest kind of figure is that bounded by three straight lines, the triangle.

To find the area of a triangle.

Rule.—Multiply the length of any one of the sides by that of the perpendicular let fall upon it from the opposite angle (or, briefly, multiply the base by the height), and halve the result.

Example 1.—To find the area of a triangle having base 2 feet 3 ins., and height 1 foot 8 ins.

2 feet 3 ins. = 27 ins., and 1 foot 8 ins. = 20 ins.

$$\text{Area, } \frac{27\times20}{2} = 270 \text{ sq. ins.}$$

Example 2.—To find the area of a triangular field, having base 23 chains 8 links, and height 14 chains, 73 links.

$\frac{23\cdot08 \text{ ch.} \times 14\cdot73 \text{ ch.}}{2}$ = 169·9842 sq. chains; or 16·99842 acres, very nearly 17 acres.

The parallelogram is a figure bounded by four straight lines, the opposite sides being equal and parallel to one another. When the angles of the parallelogram are right angles, or, as it is said in ordinary language, when the sides are *square* to one another, the figure is called a rectangle or oblong.

To find the area of a parallelogram.

Rule.—Multiply the base by the perpendicular height. —When the figure is an oblong, we say, multiply the length by the breadth.

Example 3.—To find the area of a parallelogram, base 2 feet 3 ins., height 1 foot 8 ins.

27 ins. × 20 ins. = 540 sq. ins.

Example 4.—To find the surface of a cylinder, diameter 4 feet, length 14 feet.

The surface of a cylinder, if developed or unrolled, would form a rectangle whose length is the length of the cylinder, and breadth the circumference. Its area is therefore found by multiplying the length by the diameter, and by $3\frac{1}{7}$.

14 feet × 4 feet × $3\frac{1}{7}$ = 176 sq. feet.

The trapezoid is a figure bounded by four straight lines, two of which are parallel. To find the area of a trapezoid.

Rule.—Multiply half the sum of the two parallel sides by their perpendicular distance apart.

Example 5.—Required the area of a trapezoid having parallel sides, respectively 5 feet and 7 feet, 3 feet apart.

$$\frac{5+7}{2} \times 3 = 18 \text{ sq. ft.}$$

Any figure bounded by straight lines may be divided into triangles; and the sum of the areas of all the triangles into which it is divided, is the area of the whole figure.

Of surfaces bounded by curved lines, the most regular and frequent is the circle.

To find the area of a circle.

Rule.—Multiply the square of the diameter by ·7854. Or, multiply the circumference by the radius, and halve the product.

Example 6.—Required the area of a circle 4 ft. diam. 4 × 4 × ·7854 = 12·5664 sq. feet.

Otherwise, the radius is 2 feet, and the circumference is 4 × 3·1416 = 12·5664, and the area is

$$\frac{12\cdot5664 \text{ circumf.} \times \text{rad.}}{2} = 12\cdot5664 \text{ sq. ft.}$$

The following method furnishes a near approximation to the area.

Rule.—Multiply the diameter by itself.

Take half the product, to which add its half, and the seventh part of that half.*

Example 7.—Required the area of a circle 4 feet diameter.

$$\begin{array}{rl} 4 \times 4 = 16 \text{ and } & \frac{1}{2} \text{ of } 16 = 8 \\ & \frac{1}{2} \text{ of } \ 8 = 4 \\ & \frac{1}{7} \text{ of } \ 4 = 0\cdot57 \\ \hline \text{Area} & 12\cdot57 \text{ sq. ft. nearly.} \end{array}$$

Example 8.—Required the area of a circle 37⅝ ins. diameter.

---

* This approximate rule is thus derived:—

$$\begin{array}{lr} \text{Since one-half, decimally, is} & \cdot5000 \\ \text{One-half of } \cdot5000 & = \cdot2500 \\ \text{One-seventh of } \cdot2500 & = \cdot0357 \\ \hline \text{Their sum} & = \cdot7857 \end{array}$$

which is nearly equal to ·7854, the proper multiplier of the square of the diameter.

$37\frac{5}{8}$ or 37·625 × 37·625 = 1415·64 sq. ins.
$\frac{1}{2}$ of 1415·64 = 707·82 nearly.
$\frac{1}{2}$ of 707·82 = 353·91
$\frac{1}{7}$ of 353·91 = 50·56

Area 1112·29 nearly.
The correct area is 1111·84

Error in the approximate method 0·45 sq. ins.

The converse operation of finding the diameter of a circle when its area is given, is the following :—

Rule.—Divide the area by ·7854, or multiply the area by 1·273 (the reciprocal of ·7854), and extract the square root of the result.

Example 9.—Required the diameter of a circle having 12·5664 sq. ft. area.

$\frac{12\cdot5664}{\cdot7854}$ or 12·5664 × 1·273 = 16 ; and the square root of 16 is 4 feet.

Approximate method.

Rule.—To the given area add its fourth part, and $\frac{1}{10}$th of that fourth, or the fourth with one decimal place pointed off, and extract the square root.[*]

Example 10.—Required the diameter of a circle whose area is 1111·84 sq. ins.

Area = 1111·84
Add $\frac{1}{4}$ of 1111·84 = 277·96
$\frac{1}{10}$ of 277·96 = 27·796

1417·596

The square root of which is 37·65, or $37\frac{5}{8}$ inches nearly.

The area of an ellipse is ·7854 times the product of its two axes.

Example. 11.—Required the area of an ellipse whose axes are 2 feet 3 inches and 1 foot 8 inches respectively.

27 ins. × 20 ins. × ·7854 = 424·116 sq. in.

or, $\frac{424\cdot116}{144}$ = 2·9452 sq. feet.

The area of a sector of a circle is the same part of the area of the whole circle, as the arc of the sector is of the whole circumference.

Example 12.—Required the area of a sector ; radius 12 ins., and arc 67° 30′.

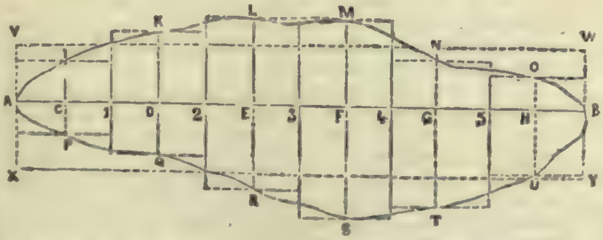
The area of the whole circle is 12 × 12 × 3·1416 = 452·39 sq. ins.

$$67^\circ\ 30' \text{ is } \frac{67\frac{1}{2}}{360}, \text{ or } \frac{67\frac{1}{2} \times 2}{360 \times 2} = \frac{135}{720},$$

or $\frac{135 \div 45}{720 \div 45} = \frac{3}{16}$ths of the whole circumference ; therefore

Area of sector is $\frac{3}{16}$ths of 452·39 sq. ins. = 84·823 sq. ins.

Fig. 270.



In measuring the area of figures bounded by lines of irregular curvature, as A L B S, the most convenient method is to draw through the figure a straight line A B, the longest possible, and to divide it into numerous equal parts at the points 1, 2, 3, 4, 5. If we bisect each of those parts at C, D, E, F, G, H, and draw perpendiculars to the line A B, meeting the curve in I, P, K, Q, &c., by drawing parallels through these points, meeting perpendiculars through the points A, C, D, &c., we form a number of rectangles, each of which is very nearly equal, in area, to the part of the curve contained between its two perpendicular sides. The area of the whole curved figure is, therefore, very nearly equal to the sum of the areas of all those rectangles. Now, as the area of each rectangle is equal to its base multiplied by its perpendicular height, and as the bases of all are equal, the area of all the rectangles is equal to the length of one of the bases, such as C D multiplied by the sum of all the heights I P, K Q, L R, &c. It is to be observed, that the distances A C and B H of the first and last perpendiculars from the extremities of the line, are each half the distance C D or D E between any two perpendiculars. Or, we may view the question in another way, thus :—Having divided the line A B as before, and set off the perpendiculars I P, K Q, &c. (technically called *offsets*), we may find a mean or average V X or W Y, by adding together the lengths of all the perpendiculars, and dividing their sum by their number, and then the whole rectangle V W Y Z has very nearly the same area with the curved figure. Hence the following—

Rule I.—Multiply the sum of all the offsets by the distance between two adjoining offsets ; or,

Rule II.—Add together the lengths of all the offsets, divide by their number, and multiply by the whole length of the base line.

Example 13.—The length of A B is 24, and the offsets are I P = 3, K Q = 5, L R = 7, M S = 8, N T = 6, O U = 4 : required the area.

By *Rule* I.—There are six offsets ; therefore the distance between any two adjoining is $\frac{24}{6}$ = 4 (note, A C and B H are each 2), and the area is

$$(3 + 5 + 7 + 8 + 6 + 4) \times 4 = 132.$$

By *Rule* II.—The mean of all the offsets is

$\frac{3 + 5 + 7 + 8 + 6 + 4}{6}$ = 5·5, and the area is 5·5 × 24 = 132.

The surface of a cone may be developed, or unrolled, into the form of a sector of a circle, and its area found by the rule. Or thus,

Rule.—Multiply the diameter of the base by the length of the side (from the apex to the edge of the base), and by 1·5708.

Example 14.—Required the surface of a cone, having a base 12 inches in diameter, and 18 inches length of side.

12 × 18 × 1·5708 = 339·2928.

The surface of a sphere is 4 times that of any of its circular sections passing through the centre. Hence, to compute the surface of a sphere—

Rule.—Multiply the square of the diameter by 3·1416.

Example 15.—Required the surface of a sphere 18 inches diameter.

18 × 18 × 3·1416 = 1017·8784 square inches.

MENSURATION OF SOLIDS.

By mensuration of solids we discover the number of cubic units in a body of given form and dimensions. A parallelopiped, or prism, has two opposite sides equal and parallel to one another. Either of these is called a base, and their perpendicular distance apart is called the height or altitude of the prism. A cylinder has a circular base.

For finding the solid contents of a prism or cylinder

Rule.—Multiply the area of the base by the height.

Example 1.—Required the capacity of a cistern, whose base is a rectangle 4 feet long by 3 feet wide, and whose height is 2 feet 6 inches.

---

[*] This rule, like the former, is thus derived,—

1 = 1·000
$\frac{1}{4}$ of 1 = 0·250
$\frac{1}{10}$ of 0·25 = 0·025

1·275, which is nearly 1·273.

Area of base $4 \times 3 \times 2\frac{1}{2}$ height = 30 cub. feet, or $30 \times 6{\cdot}2321 = 187$ gals. nearly.

Example 2.—Required the solid contents of a roller 22 ins. diameter and 42 ins. long.

$22 \times 22 \times {\cdot}7854 \times 42 = 15965{\cdot}6$ cubic inches.

When the prism or the cylinder is hollow.

Rule.—Subtract the area of the internal part of the base from that of the whole base, and multiply by the height.

Example 3.—Required the weight of a cast-iron hollow cylinder 5 feet high and 30 inches diameter internally, having metal 1 inch thick (reckoning 3·8 cubic inches of cast-iron equivalent to 1 lb.) The external diameter is the internal diameter increased by twice the thickness of metal; it is, therefore, 32 inches, and its area would be $32 \times 32 \times {\cdot}7854$. The area of the internal would be $30 \times 30 \times {\cdot}7854$; and, subtracting, we have $(32 \times 32 - 30 \times 30) \times {\cdot}7854 = 97{\cdot}436$ sq. inches for the area of the section of metal. The solid contents are

$$97{\cdot}436 \times 60 \text{ ins.} = 5846{\cdot}16 \text{ cub. ins., and } \frac{5846{\cdot}16}{3{\cdot}8} = 1538{\cdot}46$$

lbs., or 13 cwt. 2 qrs. $26\frac{1}{2}$ lbs.

When the diameter of a hollow cylinder is large in proportion to the thickness of its material, as in the case of a boiler, which may be several feet in diameter, while the thickness of its metal is less than half an inch, we may multiply the area of its surface by the thickness of material to ascertain the quantity of material. In this calculation the following points must be observed.

Boilers are generally made of wrought-iron plate; and it happens that a square foot of iron plate, $\frac{1}{8}$th of an inch thick, weighs 5 lbs. In order to ascertain the weight in pounds of a wrought-iron boiler $\frac{1}{8}$th of an inch thick, we should then have to multiply its surface by 5; and were the thickness $\frac{2}{8}$ths or $\frac{1}{4}$th, we should multiply by 2×5 or 10; for thickness $\frac{3}{8}$ths, multiply by 3×5 or 15, and so on. But in constructing boilers, there are many plates used, which overlap each other at their joints; and at the corners there are angle-irons, and numerous rivets are used to make the joints tight. Making allowance for all these additions to the weight, we may estimate the weight per superficial foot at about 6 lbs. for every $\frac{1}{8}$th of an inch in thickness, or 3 lbs. for every $\frac{1}{16}$th of an inch. Again, as $\frac{6}{112}$ is nearly $\frac{1}{20}$th, we may roughly approximate the weight in cwts. by the following rule :—

Rule.—Multiply the surface (in square feet) by the thickness (in $\frac{1}{8}$ths of an inch), and divide by 20 for the weight (in cwts.)

Example 4.—Required the weight of a Cornish boiler, external diameter 4 feet, diameter of flue-tube 2 feet, length 12 feet, thickness $\frac{3}{8}$ and $\frac{1}{16}$ of an inch (or $3\frac{1}{2}$ eighths of an inch).

$$\begin{array}{lll}
\text{External circumference . . } & 4 \times 3\frac{1}{7} & \\
\text{Circumference of flue . . . } & 2 \times 3\frac{1}{7} & \\
\hline
\text{Both . . . . } & 6 \times 3\frac{1}{7} & = \overset{\text{nearly}}{18{\cdot}9} \times \overset{\text{length}}{12} = \overset{\text{surface of casing and flue.}}{226{\cdot}8} \\
\text{Area of end of boiler . } & 4 \times 4 \times {\cdot}7854 & \\
\text{Area of flue . . . . } & 2 \times 2 \times {\cdot}7854 & \\
\hline
\text{Difference . . . . . } & 12 \times {\cdot}7854 & = 9{\cdot}4 \times 2 \text{ (ends)} = \overset{\text{surface of ends.}}{18{\cdot}8} \\
 & & \qquad\qquad\qquad\quad \overline{245{\cdot}6} \text{ total surface.}
\end{array}$$

$$\text{And } \frac{245{\cdot}6 \times 3\frac{1}{2}}{20} = 43 \text{ cwt. nearly, or 2 tons 3 cwt.}$$

The cubic contents of a sphere are $\frac{2}{3}$rds of that of a cylinder of the same diameter and altitude. But the altitude being equal to the diameter, and $\frac{2}{3}$rds of ·7854 being ·5236, the contents of the sphere may be found by the following method :—

Rule.—Multiply the cube of the diameter by ·5236.

Example 5.—Required the number of cubic inches in a ball 12 inches diameter.

$12 \times 12 \times 12 \times {\cdot}5236 = 904{\cdot}7808$ cubic inches.

When the sphere is hollow,

Rule.—Subtract the cube of the internal diameter from the cube of the external diameter, and multiply by ·5236.

Example 6.—Required the weight of a hollow ball of iron, 30 inches diameter externally, metal $1\frac{1}{2}$ inch thick, reckoning 3·8 cubic inches to the pound.

$$\begin{array}{ll}
\text{External diameter } 30 \times 30 \times 30 = & 27000 \text{ cubed} \\
\text{Internal diameter } 27 \times 27 \times 27 = & 19683 \\
\hline
\text{Difference . . . . } & 7317
\end{array}$$

$$\text{And } \frac{7317 \times {\cdot}5236}{3{\cdot}8} = 1008 \text{ lbs. or 9 cwt.}$$

The cubic contents of a cone are $\frac{1}{3}$rd of that of a cylinder of equal base and altitude. Hence, to find the contents of a cone :—

Rule.—Multiply the square of the base by the height and by ·2618.

Example 7.—Required the solid contents of a cone, having a base 18 inches diameter, and a height of 22 inches.

$18 \times 18 \times 22 \times {\cdot}2618 = 1866{\cdot}11$ cubic inches.

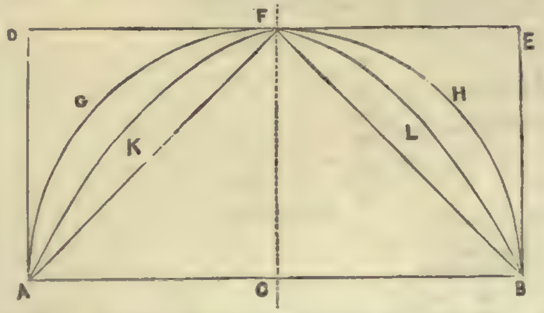
The solids with which the practical mechanic has to deal are chiefly those called solids of revolution, or such as may be conceived to be produced by the revolution of some figure round an axis. Any work that is formed in a turning-lathe is a solid of revolution. An imaginary line extending between the centres of the lathe is the axis; and if the solid were cut across by a plane passing through this line, the section or surface exposed on the removal of the half cut off, is symmetrical on each side of the axis or centre line. One-half of this symmetrical figure, or the part lying on either side of the axis, is called the generating figure. Thus, a cylinder is generated by the revolution of an oblong or rectangle round one side. A cone is generated by the revolution of a right-angled triangle round its perpendicular. A sphere is generated by the revolution of a semicircle round its diameter. A spindle, by the revolution of a segment of a circle, or other curve, round its chord. In the same way, other mathematical figures, such as the ellipse, or the parabola, are capable of producing solids of revolution by causing them to revolve round their axis. When the ellipse is supposed to revolve round its shorter axis, the solid produced is called an oblate or shortened spheroid; when it revolves round its longer axis, the solid is an oblong spheroid. The parabola, by revolution, produces the paraboloid.

There are singular relations among some of these solids of revolution, one of which it is useful to remember. If A B E D (Fig. 271) represent the section of a cylinder, whose base has a diameter A B double its height, or axis, C F—A F B the section of a cone on the same base, and of the same height with the cylinder—A G F H B the section of half a sphere,—and A K F L B the section of a paraboloid; then

The solid contents of the cone are $\frac{1}{3}$rd of that of the cylinder.
„ „ hemisphere are $\frac{2}{3}$rds „ „
„ „ paraboloid are $\frac{1}{2}$ „ „

As the paraboloid comes midway between the cone and the hemisphere in form of section, so its capacity comes midway between them in amount. In many cases of mensuration of solids of revolution, when the form of section is something between that of a cone and of a hemisphere, it may be taken as a paraboloid, and its capacity may be found thus :—

Fig. 271.



Rule.—Square the diameter of the base, multiply by the height and by ·3927 (half of ·7854). Or, multiply the area of the base by half the height.

Example 8.—Required the solid content of a paraboloid, base 2 feet 6 inches diameter, height 1 foot 8 inches.

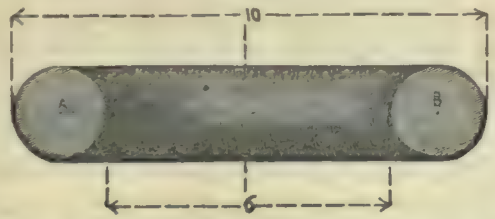
Area of a circle 2 feet 6 inches, or
30 inches diameter . . . . . 707 sq. ins., nearly.
Half of 1 foot 8 inches, or 20 inches 10

Solid content 7070 cubic inches.

In general, the capacity of any solid of revolution may be found by multiplying the area of its generating figure by the circumference described by the centre of gravity of that figure. In a ring, for instance, generated by the revolution of a circle round some centre without it, if we know the diameter of its generating circle or section, and the distance of its centre from that round which it revolves, we can compute its solid contents, as we may show by an example.

Example 9.—A ring of circular section measures 10 ins. external diameter, and 6 ins. internal: required its solid contents. Subtracting 6 from 10, there are left 4 ins., which must be double the diameter of the ring's section, giving 2 ins. as the actual diameter of the section, which has therefore an area 2 × 2 × ·7854 = 3·1416 sq. ins. Again, adding 6 to 10, and taking half, we get 8 ins. as the distance between A and B (Fig. 272), the centres of the circular sections, and the circumference of a circle 8 ins. diam. is 8 × 3·1416 = 25·1328 ins. The solid content of the ring is therefore 3·1416 sq. ins. × 25·1328 ins. = 78·9572 cub. ins. In this particular case, it happens that the centre of gravity of the section coincides with the centre of figure. When this is not the case, the position of the centre of gravity may readily be found by the following mechanical process :—

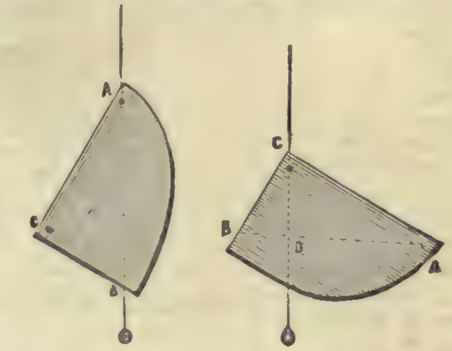
Fig. 272.



Draw upon a card or piece of thin wood, or plate of metal such as zinc, the figure, either of its full size or to any convenient scale ; and having cut it out to the shape, make two small holes in it, such as A and C (Fig. 273). Suspend it by a thread from one of the holes, and along the face of it let a plumb-line hang and mark B, the point where the plumb-line crosses its edge, tracing a line A B coinciding with the plumb-line. Suspend it now by the other hole C, with the plumb-line crossing the line formerly traced at D. The point where the lines cross is the centre of gravity, or that round which all parts are balanced. The position of the centre of gravity thus ascertained may be laid down upon the drawing of the work to be estimated, and the solid content may be found by the rule we have given.

For the measurement of a solid of irregular form, the most convenient method is to suppose it divided by numerous parallel planes into sections of equal thickness—to measure the area of each of those sections, and find the mean or average of them by summing them all and dividing by their number. This average area multiplied by the total thickness, measured perpendicularly across the planes of section, will give the solid content. In many cases of the measurement of cylindrical bodies, such as timber or round bars of metal, it is more convenient to use the girth or circumference as a known dimension for estimating the cubic contents. A near approximation to the capacity may be made thus :—

Fig. 273.



Rule.—Multiply the square of the girth by the length, and by 8, and point off two decimal places.

Example 10.—Required the cubic contents of a round log, having an average girth of 6 ft. 3 ins., and 12 ft. long.

6 ft. 3 ins., decimally expressed, is 6·25, and 6·25 × 6·25 × 12 × 8 = 3750. Pointing off two decimal places, the solid content is 37·5 cub. ft. The ordinary rule for timber gives a considerably smaller result, and is intended, not to furnish the accurate contents, but the contents estimated according to trade custom. It is,

Rule.—Multiply the square of ¼th the girth by the length.

According to this rule, the contents of the log. in *Example* 10 would be (¼th of 6·25 being 1·5625) 1·5625 × 1·5625 × 12 = 29·3 cubic feet.

DUODECIMALS.

In the mensuration of surfaces and solids, a system of computation is frequently adopted, called the duodecimal system, because the notation is reckoned by twelves (Latin, *duodecim*), instead of the ordinary decimal scale of tens (Latin, *decem*).

According to the duodecimal system, a square foot is supposed to be divided into 12 equal parts, each containing 12 square inches : and each of those parts again into twelfths, each 1 square inch ; and these again into twelfths. So, also, a cubic foot is divided into 12 equal parts, each 144 cubic inches ; each of those into 12 parts, each 12 cubic inches ; and each of those into 12 parts, each 1 cubic inch. To multiply a certain number of feet and inches by some other number of feet and inches, the one quantity is written below the other, as in ordinary multiplication.

1. The inches are multiplied by inches, the product divided by twelve, the remainder written one place back from the inches, and the quotient carried to the next operation.

2. The feet are multiplied by inches, and the number carried from the former operation added, the result divided by twelve, the remainder written in the place of inches, and the quotient written in the place of feet.

3. The inches are multiplied by feet, the result divided by twelve, the remainder written in the place of inches, and the quotient carried.

4. The feet are multiplied by feet and the carried number added, and the whole written in the place of feet.

5. The result of the two multiplications are added, carried by twelves.

For cubing, the same process is repeated, remembering that in the cubed result, or the result of multiplying three quantities of feet and inches, there are four places—viz., cubic feet; twelfths, each 144 cubic inches; twelfths of twelfths, each 12 cubic inches; and cubic inches.

Example.—Required the cubic contents of a cistern, the base of which is 4 ft. 8 ins. long by 3 ft. 10 ins. wide, and the height 5 ft. 7 ins.

We first find the area of the base.

4 ft. 8 in.
3 10
3*c*. 10*b*. 8*a*.
14*e*. 0*d*.
17 ft. 10 pts. 8 ins.

Operation. 8 ins. × 10 ins. = 80 sq. ins. and $\frac{80}{12}$ = 6 pts. 8 ins.
carry 6 and write 8, *a*.
4 ft. × 10 ins. = 40, add 6 = 46, and $\frac{46}{12}$ = 3 ft. 10 pts. *b*, *c*.
8 ins. × 3 ft. = 24, and $\frac{24}{12}$ = 2 ft. 0 pts.
carry 2, and write 0, *d*.
4 ft. × 3 ft. = 12 ft., add 2, 14 ft., *e*.
Sum the two lines.

We now multiply the area of base by the height.

	ft.	pts.	ins.	
Area	17	10	8	
		5	7	
	10	5	2	8
	89	5	4	
	99	10	6	8

Operation. 8 × 7 = 56, and $\frac{56}{12}$ = 4 ft. 8 ins., carry 4.
10 × 7 + 4 = 74, and $\frac{74}{12}$ = 6 ft. 2 ins., carry 6.
17 × 7 + 6 = 125, and $\frac{125}{12}$ = 10 ft. 5 ins.
8 × 5 = 40, and $\frac{40}{12}$ = 3 ft. 4 ins. carry 3.
10 × 5 + 3 = 53, and $\frac{53}{12}$ = 4 ft. 5 ins., carry 4.
17 × 5 + 4 = 89. Add up the lines.

The cubic contents are therefore

99 cub. ft. 10 twelfths, 6 hundred and forty-fourths, 8 cub. ins., or 99 ft. 1520 cub. ins., because 10 twelfths, each of 144 cubic inches . . . . = 1440 cubic inches.
6 hundred and forty-fourths, each 12 cub. ins. = 72 ,,
8 cubic inches . . . . 8 ,,
1520

Water is generally taken as the standard of specific gravity, and it fortunately happens that 1 cubic foot of water weighs very nearly 1000 ounces. By reference to any table of specific gravities, the weight of a cubic foot of each material is given in ounces; and from this may be readily derived—

1. The weight of 1 cubic foot in lbs., viz., $\frac{1}{16}$th of the specific gravity or weight in oz., because 1 oz. = $\frac{1}{16}$ of 1 lb.

2. The weight of 1 cubic inch in ounces—viz., the specific gravity divided by 1728, because there are 1728 cubic inches in 1 foot—and the weight of 1 cubic inch in lbs, viz., the specific gravity divided by 1728, and the quotient by 16, or the specific gravity divided by 27648.

3. The number of cubic inches in 1 lb., viz., 16 times 1728, or 27648 divided by the specific gravity.

4. The number of cubic feet in 1 ton, viz., 2240 lbs. × 16 oz., or 35840 oz. divided by the specific gravity.

Example 1.—Required the weight in lbs. of 7 cubic feet of English oak.

Spec. grav. $\frac{900}{16} \times 7 = 393\frac{3}{4}$ lbs., or 3 cwt. 2 qr. $1\frac{3}{4}$ lbs.

Example 2.—Required the weight of 178 cubic inches of cast-iron.

Spec. grav. $\frac{7200}{1728} \times 178 = 742$ oz.

Or $\frac{7200}{27468} \times 178 = 46\frac{1}{3}$ lbs.

Example 3.—Required the number of cubic inches in 2 cwt. (or 224 lbs.) of gun metal.

$\frac{27648}{\text{Spec. grav. } 8800} \times 224 = 703\frac{3}{4}$ cubic inches.

Example 4.—Required the number of cubic feet in 20 tons of granite.

$\frac{35840}{\text{Spec. grav. } 2700} \times 20 = 265\frac{1}{2}$ cubic feet.

In estimating the weight of wrought-iron cylindrical bars, the following method is a very near approximation:—

Rule.—Square the diameter reduced to eighths of an inch, multiply by the length in feet and by $4\frac{1}{7}$, and point off two decimal places for the weight in lbs.

Example 1.—Required the weight of a cylindrical bar of wrought-iron $5\frac{7}{8}$ ins. diameter, and 7 ft. 7 ins. long.

$5\frac{7}{8}$ ins. = 47 eighths, and 7 ft. 7 ins. = $7\frac{7}{12}$ ft.

$47 \times 47 \times 7\frac{7}{12} \times 4\frac{1}{7} = 69401$.

The weight is therefore 694 lbs., or 6 cwt. 0 qr. 22 lbs.

For wrought-iron bars of rectangular section:—The width of these advances by quarters, and the thickness by eighths of an inch.

Rule.—Multiply the breadth in quarters of an inch, by the thickness in eighths, by the length in feet, and by $10\frac{1}{2}$, and point off two decimal places for the weight in lbs.

Example 2.—Required the weight of a bar of wrought-iron $2\frac{1}{4}$ inches wide, $\frac{3}{8}$ in. thick, and 3 feet long.

Width 9 qrs. × thickness 3 eighths × length 3 ft. × $10\frac{1}{2}$ = 850; therefore the weight is 8·5 lbs.

For wrought-iron plates.

Rule.—Multiply the area in square feet by the thickness in eighths of an inch, and by 5 for the weight in lbs.

Example 3.—Required the weight of 123 square feet of boiler-plate $\frac{1}{4}$ and $\frac{1}{16}$ in. thick.

Thickness $\frac{1}{4}$ and $\frac{1}{16}$, or $2\frac{1}{2}$ eighths of an inch.

$123 \times 2\frac{1}{2} \times 5 = 1538$ lbs., or 13 cwt. 2 qr. 26 lbs.

For cast-iron balls.

Rule.—To the cube of half the diameter (in inches) add its eleventh part for the weight in lbs.

Example 4.—Required the weight of a cast-iron ball 6 ins. diameter. Half the diameter is 3 ins.

And 3 × 3 × 3 . . = 27
Add one-eleventh of 27 = 2·5
Weight . . . . 29·5 lbs.

SIR J. F. W. HERSCHEL, BART., F.R.S.

SECTION X.

ASTRONOMY.

CHAPTER I.

HISTORY OF THE SCIENCE, AND ITS DEFINITIONS.

Astronomy is that science which teaches the distribution and arrangement of the heavenly bodies ; their true and apparent motions in space ; their magnitude, distance, and physical condition ; and its history presents some of the most brilliant examples of the development of the human mind.

Astronomy essentially owes its existence as a science, as well as its present degree of perfection, to continuous observations of the heavenly bodies, from the first watchings of the Chaldean shepherds, to the refined instrumental measurements of the astronomers of the nineteenth century. But if the accumulated observations of centuries, which the genius of man would naturally endeavour to preserve, had not been made subservient for the prediction of recurrences of the same phenomena, Astronomy would exist only in name, and cease to hold its high position among the accurate sciences. Its course, however, has not always been progressive : it is only within the last few centuries that the errors of antiquity have been completely eradicated. To the invention of the telescope, and its application to graduated instruments, we are in a great measure indebted for this progress, which, in a physical point of view, it is impossible to over-estimate, for it has permitted great and permanent advances to be made in the accurate observations of the heavenly bodies. By the latest improvements in these instruments, astronomers have been enabled to note, with an incredible degree of precision, the apparent places of the fixed stars and planets. In this manner, as the accuracy of observation increased, the errors of the ancient theories exhibited themselves more clearly, and the mind of man has been enabled to explain, on sound and natural principles, the new phenomena which have been revealed. The discovery of the aberration of light and of the nutation of the earth's axis, two of the greatest discoveries of any age, have thus resulted, and afforded to Dr. Bradley the means of investigating, by accurate observation of the places of the stars, the preceding discovery by Roemer, of the successive transmission of light, which he had determined by observations of the eclipses of Jupiter's satellites.

The total overthrow of the Ptolemaic system is owing to the same cause—the observations of Tycho Brahé having furnished means of clearly exhibiting to Kepler, the errors of a circular hypothesis, in the same manner as the errors of the Alphonsine tables, based also on the Ptolemaic theory, pointed out to Copernicus the errors of the ancient philosophy, when it was discovered, about the epoch of 1500, that they differed more than two degrees of the truth. Calculation has also lent its aid in reducing to law, and in grappling with difficulties, problems, which the happy invention of logarithms has rendered accessible, and which, in other respects, without this assistance, would be almost insurmountable.

We find that, amongst the most ancient nations of antiquity, the appearance of the heavens was attentively watched. And, in the first place, the motions of the sun and moon commanded attention, in regard both to the duties of the husbandman, and appointments for the assemblage of large masses of the people. The Egyptians and Chaldeans discovered that the motions of these bodies, and the planets, were all performed within a certain compass of the heavens, which was termed the Zodiac ; the names of the constellations through which this belt passed, having especial reference to the motions of the sun. We also find that eclipses of the sun and moon were recorded with superstitious veneration ; whilst, in connection with the foolish study of judicial astrology, the positions of the planets, with reference to bright stars, were also carefully noted. The Egyptians and Chaldeans were acquainted with the time of the revolutions of the moon's nodes—an epoch which they termed the *Saros*—which would give them the means of predicting eclipses, recurring, as they naturally would, in the same eras. The Chaldeans were also acquainted with the lunar-solar period of 600 years, which Josephus attributed to the ancient patriarchs, although Pliny cites Hipparchus as its author. Their primitive observations appear to have consisted in the heliacal rising of bright stars, and probably in the use of the gnomon or sundial.

Thales, of Miletus, in Asia Minor, who had studied the astronomy of the Egyptians, is supposed to have predicted, by a knowledge of the period of 223 lunations, or the Saros, the famous eclipse which occurred about 585 B.C., and which is recorded by Herodotus.

Thales, on his return from Egypt, founded the Ionian school (B.C. 640). This school had for its followers Anaximander (B.C. 610), Anaximenes (B.C. 530), and Anaxagoras (B.C. 500). The first is known by a most useful invention, viz., that of geographical charts. The others added some fanciful notions, in addition to those of Thales, on the construction of the universe ; and they all appear to have believed in the plurality of worlds. The most enlightened disciple of this school, however, was Pythagoras (B.C. 580), who, having been advised by Thales to travel into Egypt to perfect his studies, became initiated in the secret mysteries of the priests in a greater degree than any of his predecessors. He founded a school in Italy on his return, where the doctrines of the Ionian school were more fully promulgated. He appears to have been acquainted with the earth's rotation on its axis, its annual revolution, and its spherical figure—doctrines which, in imitation of the Egyptian priests, were kept secret from the multitude.

But the first approach to a scientific system of astronomy was that of the Alexandrian school, where we find observations carried on regularly, and a theory which, though founded in error, rudely satisfied the observations of the period. The most distinguished members of this school were Aristarchus of Samos (B.C. 281), Eratosthenes of Cyrene (B.C. 276), and Hipparchus of Bythinia (B.C. 140). Aristarchus is famous for having made an approximation to the distance of the sun from the earth, by the angular measurement of the distance of the sun and moon, at the time at which the moon is half illuminated. Having found that the moon's angular distance at this period was 87°, he thence concluded that the sun was distant eighteen or twenty times that of the moon. This method, however ingenious, fails in consequence of the roughness of the moon's surface. By careful observations of the sun's path, he also made an approximation to the diameter of this body, which he considered to be $\frac{1}{720}$th part of the whole daily motion—an observation not very far from the truth. His tract on the magnitude and distances of the sun and moon has been preserved. In addition to the tenets of the Alexandrian school, he was of opinion that the stars were at distances immensely greater than the sun, on which account this astronomer must hold a high rank among those of antiquity.

Eratosthenes of Cyrene, who lived B.C. 276, is known for a determination of the obliquity of the ecliptic, by observations of the altitude of the sun at the summer and winter solstices—an observation of great value, as showing the progressive diminution of this element when compared with modern observations. He also made an approximation to the magnitude of the earth, on correct principles, by the measure of the celestial arc, included between Syene and Alexandria, combined with the known distance between the two places. Syene was one of the most southern cities of ancient Egypt, where it happened that at the summer solstice the sun was exactly vertical, which he determined by the fact that a deep well was wholly illuminated. At Alexandria, which was situated in the same meridian, and at the same period of time, the sun's zenith distance was 7° 12′, or one-fiftieth part of the whole circumference. The distance of the two places was known to be 5,000 stadia, whence he concluded that the circumference of the earth was 250,000 stadia. The great uncertainty of the length of the stadium does not permit us to make use of this determination, or to compare it with the method pursued with more accurate instrumental means by modern astronomers.

Apollonius of Perga deserves also to be mentioned as having invented the system of deferents and epicycles, for the purpose of accounting for the direct, stationary, and retrograde appearances of the planets.

But one of the greatest astronomers of antiquity, Hipparchus (B.C. 140), now appeared, famous alike as an accurate observer and geometer. To this distinguished man we are indebted for the discovery of the "precession of the equinoxes." The appearance of a new star led him to form a star catalogue, by a comparison of which, with a former catalogue of the same school, this important element became evident. He was also acquainted with the unequal motion of the sun in its orbit, and the inequality of the solar rays. The former he endeavoured to explain by a modification of the preceding system of deferents and epicycles, which will be explained in its proper place.

The revolution of the moon's nodes, and the inclination of its orbit to the ecliptic, were also known to Hipparchus, as well as the eccentricity of the sun's orbit and the motion of the apsides. By a comparison of his observations of the sun with the former astronomers, he determined the length of a tropical year. His solar tables were held in high estimation by Ptolemy three centuries after their construction.

Ptolemy (B.C. 130) collected the observations of the Alexandrian school in his *Almagest*, a work which existed as a text-book among astronomers for centuries. He is famous for having discovered the *evection* of the moon, imperfectly known to Hipparchus; and by a comparison of observations of eclipses, and at the other parts of the moon's orbit, he empirically determined its amount with a great degree of accuracy. He confirmed the value of the precession of the equinoxes, previously discovered by Hipparchus. He also proportioned the magnitude of the deferents and epicycles, for the purpose of explaining the planetary motions, which had been previously invented by Apollonius and Hipparchus, whence the name of the "Ptolemaic System." In his Geography, he collected the longitudes and latitudes of all known places. He also left behind him many other works, testifying to his skill and genius as a philosopher.

After the death of Ptolemy, the Alexandrian school existed only in name. During five centuries, the discoveries of Hipparchus and Ptolemy were not extended; and the labours of its followers consisted principally in commenting on their works, and in noting extraordinary phenomena.

After the dissolution of the Alexandrian school, the science of astronomy in Europe had almost disappeared; but among the Arabs, it had been cultivated with some success under the auspices of the more enlightened caliphs. The *Almagest* of Ptolemy was translated into their language, and their annals transmit to us many observations of the sun, moon, and planets, as well as the measure of a degree of latitude. Among the Arabian astronomers, Albategnius deserves honourable mention as an industrious and accurate observer. He confirmed and corrected the rate of the precession of the equinoxes, and also determined the obliquity of the ecliptic, which, from several collateral circumstances, is entitled to great confidence. He also paid considerable attention to the theory of the sun, and determined the eccentricity of its orbit with accuracy. He also found that the apogee of the sun was subject to a small annual displacement, according to the order of the signs, and determined its position at that period to be in the constellation Gemini, which, when compared with the modern elements, does not differ more than 40′ from the truth. The astronomy of the Arabians does not open to us any new theories, but is merely an extension of the system of Ptolemy. In the art of observation they had, however, improved; larger instruments and more refined calculations had been introduced by them, on which account their investigations are entitled to hold a high rank in the history of astronomy.

After the Persians had thrown off the yoke of the Arabians, Holeg Ilecoukhan, one of the Persian sovereigns, founded an academy of astronomers, where they formed new tables based on the Ptolemaic system, with trifling changes in their elements. But the greatest praise is due to Ulugh Beigh, one of their princes, who was a great patron of this science, as well as an observer, and who formed a catalogue of stars, and improved the solar and planetary tables. He also determined the obliquity of the ecliptic, and the precession of the equinoxes.

The next epoch of the history of astronomy commences with its revival in Europe, when, after the overthrow of the Arabian empire, it again became cultivated in Spain. Alphonso, the tenth king of Castile, collected a body of astronomers at Toledo; and having made use of the observations of the Arabians for the purpose of correcting the planetary motions, published the Alphonsine tables, which are more correct than any which preceded them in some respects; but their accuracy was not commensurate with the great trouble and expense they occasioned. The great confusion of the Ptolemaic system, caused Alphonso to say, "that if he had been consulted at the Creation, he could have devised a better arrangement." In Austria, the study of astronomy was cultivated under the auspices of Albert, the third duke, and Frederick the second emperor. In the 15th century, Purbach, Regiomontanus, and Waltho lived, known as assiduous cultivators of this science. Ephemerides of the planets and eclipses were published for the meridian of Vienna from 1475, first by Regiomontanus, who, being also an observer, was well acquainted with the errors of the Alphonsine tables, the place of Mars being above 2° in error. The art of measuring time received an important improvement by the substitution of clock-work motion for the clepsydræ of the ancients, which also increased the accuracy of observations; and about this period the true system of the universe forced itself on the attention of Copernicus.

This distinguished astronomer—who saw clearly the errors of the Ptolemaic system, and who, after thirty-six years of study, published, in a clear and connected form, the present system of the world—was born at Thorn in Prussia, in 1473, four years before the death of Regiomontanus. The ancient systems of astronomy, as related by Plutarch and Cicero, had, by their extreme simplicity, captivated his mind; and to this circumstance we owe his explanation of the celestial motions; an explanation which, however imperfect in several respects, shows his originality. In his system, he adopted the hypothesis of the earth's rotation on its axis, its annual revolution round the sun, in common with the other five known planets, whose distances from the great central body he proportioned. As regards the "precession of the equinoxes," he conceived it to arise from a small motion of the earth's axis. In consequence, however, of his ignorance of the elliptical motions of the planets, he was unable totally to abolish the system of epicycles. The work which contained his explanations was dedicated to Pope Paul III.; and it was owing only to the urgent representations of his scientific friends, that he was induced to publish it in the form of an hypothesis. He had only revised the last proof sheet when death put an end to his labours in his seventieth year.

One of the most important inventions ever introduced by the genius of man, was afterwards applied to the observations of the heavenly bodies by Galileo. This was the telescope, which has enabled the astronomer to overcome the feebleness of his natural vision, and to examine leisurely, and attentively, the physical constitution of the celestial bodies. Galileo, an ardent follower of Copernicus, was immediately convinced of the truth of the system, when, having observed the phases of Venus, he traced their connection to their elongation from the sun, and found that the orbit of this planet was included within the earth's orbit. Galileo also discovered the four satellites of Jupiter, and was acquainted with the existence of spots on the sun, from which he concluded the rotation of that body on its axis. He appears also to have first applied the pendulum to the clock.

The overthrow of the Ptolemaic system followed quickly on the footsteps of Copernicus and Galileo, but not till the new system had encountered much opposition. Tycho Brahé, who was born in 1546, is celebrated in having supplied observations of greater excellence than any of his predecessors. The cultivation of practical astronomy had made but little progress since the time of the Arabs, and the new theory required accurate observations to test its correctness. Tycho thus paved the way for Kepler to make his splendid discoveries. Personally, however, he resisted the Copernican theory, and invented an hypothesis of his own, retaining the earth as the centre of the universe, but making the sun the centre of the planetary motions. This system had few admirers; but the accuracy and number of his planetary observations, his star catalogue, his discovery of the "variation" of the moon, and his table of refractions, give him a position among the astronomers of his period. From the planetary observations of Mars, the immortal discovery of the elliptic motions of the planets has resulted. Kepler placed such confidence in the observations of Tycho, that he affirmed his belief that the true system of the universe might be founded on them. Taking at first the Copernican doctrine of a circular hypothesis, he found that a difference of 8′ existed between the theory and observation of Mars, which difference, from the accuracy of the observations, he had no doubt was attributive to an erroneous hypothesis. After labour infinitely great, extending over a period of twenty years, he discovered his three famous laws which serve as the foundation of modern astronomy. Kepler appears also to have had some notions on gravity; and he left also some tracts on optics. The laws which he had determined for the sun and planets, were found also to extend to the satellites of Jupiter, they having been just discovered by Galileo, Huygens, and Cassini.

Among the distinguished men who followed Kepler, mention must be made of Huygens, Hevelius, Cassini, and Roemer. The first, in addition to his optical and mechanical improvements, described the ring surrounding Saturn as well as one of its satellites, and is said to have applied the pendulum to clocks—a discovery also claimed for Galileo. Hevelius is known as a persevering observer. But all his observations were made with the naked eye; and although he was able to estimate angular spaces by these means to a wonderful degree of accuracy, which Dr. Halley confirmed by direct comparison, his observations cannot be made use of in any delicate inquiry. His researches on comets, and his tables of the sun, rank him in a high position among practical astronomers. Cassini formed tables of Jupiter's satellites from observation, and determined the rotation of Jupiter and Mars. He also occupied himself with refractions, and made a theory of the libration of the moon, which had been left in an imperfect state by Hevelius. Roemer is known as the inventor of the transit instrument and meridian circle, with which instruments he carried on a series of observations of the heavenly bodies for several years; but only those of three days have been preserved, the others having been destroyed in the great fire which occurred at Hafnia. Roemer also discovered, by comparison of Cassini's tables of Jupiter's first satellite, the successive transmission of light. The discovery of universal gravitation by Newton, at the latter end of the seventeenth century, produced a total revolution in astronomy, and enabled its illustrious author to account for the previous empirical deductions of Kepler.

Since the discovery of gravitation by Sir Isaac Newton, astronomy has been much advanced by the establishment of observatories. And, in this respect, England has great reason to be proud of her position, her astronomers having followed up this immortal discovery by a series of observations unexampled for their extent—an object which is essentially necessary, in order to continue the comparison of theory and observation, for the correction of the elements of the system. The accuracy of the science has been considerably increased by the institution of the Royal Observatory at Greenwich, both with regard to the eminent men who have directed this establishment, and to the continuous mass of observations which it has furnished. Founded by King Charles II., in 1675, it has had for its directors, successively, Flamsteed, Halley, Bradley, Bliss, Maskelyne, Pond, and Airy.

Flamsteed is known as the greatest observer of the age in which he lived. Patronised by Sir Jonas Moore, to whom the foundation of the Greenwich Observatory is due, he carried on for many years continuous observations of the planets and fixed stars, and contributed materially to furnish Sir Isaac Newton with the observations on which his system of the world is based. He was succeeded by Dr. Halley, an astronomer to whom we are indebted for many useful investigations, but whose province did not so much lie in practical astronomy as that of his predecessor. This distinguished man has the merit of adding considerably to our knowledge of cometary astronomy, and the lunar theory. But one of his greatest triumphs, which will always render his name illustrious, is his proposition of determining the sun's distance, by the transits of Venus over its disc.

Dr. Bradley, the next in succession, enriched this science by two important discoveries—viz., the aberration of light, and a more exact knowledge of the nutation of the earth's axis. Distinguished as the most accurate astronomer of his age, he reobserved, with better instruments, all the stars of Flamsteed, as well as the sun, moon, and planets, the former observations of which, have been incorporated in the *Fundamenta Astronomiæ* of Bessel. His observations of the moon and planets, are included in the Greenwich planetary and lunar reductions.

The same important class of observations was continued by Dr. Maskelyne, from 1765 to 1811, extending over an interval of forty-six years. In addition to other important works, this astronomer has the credit of establishing the *Nautical Almanac* for the use of mariners, and of introducing in that work, tabular elements for the improved method of finding the longitude at sea.

On the same regular system have been carried on, at the Royal Observatory, observations of the sun, moon, and planets, under the superintendence, successively, of Mr. Pond and Professor Airy. If it were necessary to discuss the importance of the immediate comparison of theory and observation, it would be merely requisite to mention, that the discovery of the planet Neptune has thus resulted—a discovery which, in the opinion of all competent judges, deserves to be ranked among the most brilliant triumphs of the human mind.

In the field of telescopic research, the labours of Sir W. Herschel deserve particular mention. His discovery of the planet Uranus and its satellites, as well as his investigation on Saturn's rings—his theory of the motion of the solar system in space and its direction, which have been verified recently by several independent researches, are sufficient monuments of his skill and industry.

In France, the cultivation of practical astronomy in the eighteenth century was well sustained by the celebrated astronomers Lalande and La Caille, the latter having observed the stars of the southern hemisphere about the epoch 1750; the publication and reduction of which, have since been undertaken by the British Association. He is also known as having determined the *constant* of lunar parallax, by corresponding meridional observations at the Cape of Good Hope and at European stations. Besides these important undertakings, he was an energetic calculator of ephemerides, and useful investigations on refraction, the figure of the earth, &c.

Lalande is known as an industrious and indefatigable observer. His immense catalogue of stars bears sufficient evidence of his unremitting industry for many years. The catalogue of stars formed from his observations, have been reduced and published by the British Association.

In Sicily, the labours of Piazzi have been principally occupied in an extensive catalogue of stars, the epoch of which is the beginning of the present century, and which is now one of the standard works of the day. His zeal was rewarded by the discovery of Ceres among the asteroids, in 1801. The discoveries of these bodies, during the last ten years, are too numerous to be mentioned here.

The progressive improvements of instruments during the last century, have been rewarded by increased accuracy of results. At Greenwich, the mural arc of Flamsteed is now replaced by the magnificent transit circle; and the aid of galvanism has been brought to bear on an essential and delicate element of time—viz., that of right ascensions. All these circumstances produce a corresponding improvement in theory. Successive comparisons of theory and observation engage the attention of our analysts; and to this we are in a great measure indebted for the important discoveries of the various inequalities, which have enriched the science of astronomy.

ASTRONOMICAL DEFINITIONS.

Having thus shown how the investigating disposition of man has been occupied in its endeavour to obtain a more correct and perfect knowledge of the universe, we proceed to explain the laws which have been established.

Astronomy is usually divided into three parts:—1. Spherical Astronomy, which teaches the knowledge of the various points and circles of the celestial sphere, the constellations, the position of the stars with respect to these points and circles, and the phenomena occurring in the sphere of the heavens.—2. Theoretical Astronomy, which enables us to determine, from observation, the path of the heavenly bodies.—3. Physical Astronomy, which gives the laws by which the heavenly bodies are regulated, teaches how their motions are to be calculated according to the rules of mechanics, and combines all that is known of their physical characters. Without the formality of so dividing our subject, which our space does not permit, we shall endeavour to combine all the useful and practical portions of the subject.

In order to facilitate the study of the heavens, artificial representations have been made, similar to those of the surface of the earth. These are called celestial globes, on which the stars are depicted in their natural positions, the observer being supposed to be in the centre, viewing them in the concave surface.

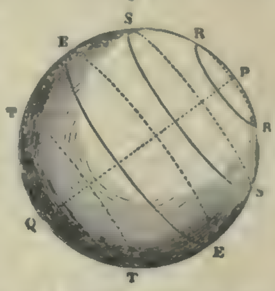
To represent the apparent diurnal motion of the heavenly bodies, the celestial globe must be turned from east to west.

Circles.—To designate with precision the situation of the sun, moon, and stars, imaginary circles have been considered as drawn in the heavens, most of which correspond to, and are in the same plane with similar circles supposed to be drawn for similar purposes on the surface of the earth. If a line be drawn on the sphere (Fig. 1) of the earth P Q, the plane cuts the surface of the sphere, and forms a great circle E E, which is the celestial equator, the sphere being divided in this circle into two hemispheres, in which one of the poles forms a central position; these form the northern and southern hemispheres.

A plane is that which has surface, but not thickness. The plane of a circle is that imaginary surface which the circle bounds.

The axis of the earth, P Q, is an imaginary line passing through its centre, north and south, about which its diurnal revolution is performed; the poles of the earth are the two extremities of the axis, where it is supposed to cut the surface. The axis of the heavens is the earth's axis, produced both ways to the concave of the sky; the poles of the heavens are two imaginary points exactly above the terrestrial poles.

Fig. 1.



Great circles are those which divide the globe into two equal parts, as the equator, the ecliptic, and the colures. R R, S S, and T T (Fig. 1) are small circles which divide the globe into two unequal parts, as the tropics, polar circles, and parallels of latitude.

Every circle is supposed to be divided into 360 equal parts or degrees. A degree is further subdivided into 60 equal parts or minutes; and a minute into 60 seconds. Degrees are marked°, minutes′, seconds″. The space included by a degree of a great circle in the heavens, is equal to nearly twice the apparent diameter of the sun or moon, when considerably above the horizon.

The equator of the earth, E E (Fig. 1), is an imaginary great circle passing round the globe, east and west, everywhere equidistant from the poles, dividing it into northern and southern hemispheres. The equator of the heavens, or the equinoctial, is the plane of the terrestrial equator extended to the concave surface of the heavens, and called the equinoctial, because, when the sun appears in it, the days and nights are equal all over the world.

The Ecliptic is the *via solis*, or sun's path, the great circle which he appears annually to describe among the fixed stars; though, more properly, it is the track which the earth actually describes among the stars, as viewed from the sun. The ecliptic is so called, because solar and lunar eclipses only can happen when the moon is in or very near this circle. It cuts the equinoctial obliquely at two opposite points, making an angle with it of 23½°, which is called the obliquity of the ecliptic. One half lies on the north side of the equinoctial; the other half on the south side. The points of crossing are the equinoctial points. A zone or girdle extending 8° on each side of the ecliptic, or 16° in breadth, is the zodiac, in which are the orbits of all the planets, with the exception of some of the asteroids. The ecliptic and zodiac are divided into twelve equal parts, called signs, each containing 30°. Their names, with the days on which the sun enters them, are as follows;—

Northern signs, being north of the equinoctial.

Spring Signs.	*Summer Signs.*
Aries, the Ram, March 21.	Cancer, the Crab, June 21.
Taurus, the Bull, April 19.	Leo, the Lion, July 23.
Gemini, the Twins, May 20.	Virgo, the Virgin, Aug. 22.

Southern signs, being south of the equinoctial.

Autumnal Signs.	*Winter Signs.*
Libra, the Balance, Sept 23.	Capricornus, the Goat, Dec. 21.
Scorpio, the Scorpio, Oct. 23.	Aquarius, the Water-bearer, Jan. 20.
Sagittarius, the Archer, Nov. 22.	Pisces, the Fishes, Feb. 19.

The Colures are two great circles passing through the poles of the heavens, dividing the ecliptic into four equal parts, and marking the seasons of the year.

One passes through the equinoctial points, Aries and Libra, and is therefore called the equinoctial colure. When the sun is in either of these points, the days and nights on every part of the globe are equal to each other. The other passes through the solstitial points, Cancer and Capricorn, which mark the sun's greatest declination, north and south of the equator, and is thence called the solstitial colure. When the sun is in or near these points, his meridian altitude undergoes scarcely any sensible variation for several days; and hence the term solstitial applied to them.

The Horizon is a great circle, whose plane passing through the centre of the earth, and extended to the sphere of the fixed stars, divides the heavens into two hemispheres, of which the upper is the visible, and the lower the invisible hemisphere. This is the rational or true horizon, which determines the rising and setting of the sun, planets, and stars. It is represented by the wooden horizon of the artificial globe. The sensible or apparent horizon is the circle which bounds our view, where the land, or water, and sky, seem to touch each other; more or less extensive according to the position of an observer.

The sensible horizon of a person changes as he moves, and, in an open country, enlarges or contracts as his station is high or low. Standing on a plain, the eye having an elevation of 5 feet above the surface, the radius of the sensible horizon will be less than 2¾ miles. At an elevation of 6 feet it will be just 3 miles.

Rule, to find the distance when the height is known—Increase the height in feet one half, and extract the square root for the distance in miles.

Thus, in the preceding case the eye is supposed to have an elevation of 6 feet above the surface of a plain, and 6 with its half is 9, the square root of which is 3; that gives the distance in miles which a person will be able to see in a right line upon that surface.

Again—a tower, 32 yards above the level of the ocean, may be seen along that level from a distance of 12 miles. For 32 yards = 96 feet, increased one half = 144, the square root of which is 12.

The poles of the horizon are the zenith and the nadir. The zenith is the point in the heavens which is directly over our heads; the nadir, that which is exactly under our feet. The zenith to us is the nadir to our antipodes, and the nadir to us is their zenith. Circles drawn through the zenith and nadir of any place, cutting the horizon at right angles, are called azimuth or vertical circles; and that which passes through the east and west points of the horizon, is the prime vertical.

Meridians are imaginary great circles passing through the terrestrial and celestial poles, cutting the equator and equinoctial at right angles.

A meridian is supposed to pass through every place on the earth, and every point in the heavens; but only 24 are drawn on the globes through every 15° of the equator and equinoctial, including altogether 360°. These meridians mark the space which, in consequence of the earth's diurnal rotation, the heavenly bodies appear to describe every hour through the 24 in the day. They are sometimes called, therefore, hour or horary circles. As 15° answer to an hour, 1° answers to four minutes of time, ½ to two minutes, and ¼ to one minute.

Longitude on the earth is distance east or west from a fixed meridian measured on the equator. The Fortunate Islands, supposed to be the Canaries, supplied the ancients with their first meridian. The western extremity of Africa, as then known, was taken by Abulfeda, the Arabian geographer. The meridian of Terceira was used by the Spanish and Portuguese in the sixteenth century; and that of Ferro by all nations in the seventeenth and eighteenth centuries. We now adopt the meridian of the Greenwich, and the French that of the Paris, observatories.

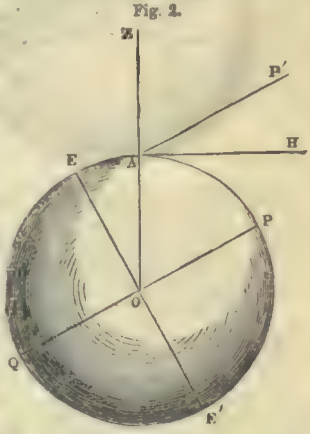
Longitude in the heavens, is distance east from the great meridian which passes through the first point of Aries, or the equinoctial colure, measured on the ecliptic.

Right ascension, is distance east from the same meridian measured on the equinoctial.

Terrestrial longitude being reckoned in two directions from a fixed point, east and west, can only extend to 180°. Celestial longitude, and right ascension, are only reckoned in one direction, east from the prime meridian, and may, therefore, extend to 360°.

Parallels of Latitude are small circles supposed to be drawn on the surface of the earth, north and south of the equator, and parallel to it, dividing the globe into two unequal parts. Let us suppose A (Fig. 2) to be placed under consideration, and P, E, Q, E′ its meridian, E E′ the line intersecting the equator, and P Q the line of the poles, it is here the arc A E, or, which is the same thing, the angle A, O, E, which represents the latitude sought. P, O, E being a right angle, the latitude is the complement of the angle A, O, P; but the angle A, O, P, is only another thing for the zenith distance Z, A, P, of the pole of the celestial sphere. For to an observer placed at A, P Q is a parallel to the earth's axis; the latitude of the point A then is the complement of the zenith distance from the pole at that point, to the height P′, A, H, of the pole above the horizon, A H, being equal to the distance Z, A, P′. We can, therefore, say that the latitude of a place is equal to the height of the pole above the horizon of that place. Parallels of declination are such circles produced in the heavens, north and south of the equinoctial, and parallel to it.

Fig. 2.



Latitude on the earth is the distance of a place from the equator, measured on a meridian, north or south.

Declination is the distance of the heavenly bodies from the equinoctial, measured on a meridian, north or south.

Latitude in the heavens is distance from the ecliptic, at a right angle, north or south.

Terrestrial latitude and declination may extend to 90°. The sun has no declination when in the equinoctial. His greatest declination is 23½° north or south. He has no latitude, being always in the ecliptic. The greatest declination of a planet is 30½°, and latitude 8° north or south, with the exception of the asteroids. It is more convenient to describe the position of the heavenly bodies by their declination and right ascension, than by their latitude and longitude, the former corresponding to terrestrial latitude and longitude.

The Tropic of Cancer is a small circle 23½° north of the equator, and parallel to it; and the tropic of Capricorn is a similar circle, at the same distance, on the south. The polar circles are also small circles, each 66½° from the equator, and at the same distance from the poles as the tropics are from the equator.

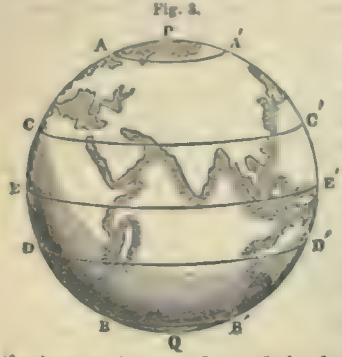
The tropics on the celestial sphere, mark the limits of the sun's farthest declination, north and south.

The tropics on the terrestrial sphere, divide the torrid from the two temperate zones, and the polar circles divide the temperate from the two frigid zones.

Zones.—Twice in the year the sun is vertical to those who dwell in the torrid zone. Consequently, at noon they deflect no shadow, and are hence styled *ascii*, meaning shadowless. At other times their shadows fall at noon, north or south, according as the sun is north or south of them. They are then called *amphiscii*, signifying that their shadows fall both ways.

The variation in the temperature has led to the division of the earth into zones. Between the latitude 66° 32′ north and south, the sun rises and sets every day. In all higher latitudes, however, there are certain periods of the year when the sun never rises, and others when it never sets. The two parallels of latitude A A′, B B′ (Fig. 3), which correspond to the latitude 66° 32′, divide the surface of the earth in three parts. The two zones at A P A′, B Q B′ are called the frozen zones. The circles A A′, B B′ are the polar circles; A A′ is the arctic; while the other is the antarctic circle: in the points between these two are the several zones, in which the sun sets and rises daily.

Fig. 3.



Those who dwell in the temperate zones have their shadows at noon always cast towards the north in the north temperate zone, and towards the south in the south temperate zone. They are styled *heteroscii*, meaning, that they have their shadows at noon, either north or south.

The inhabitants of the frigid zones are called *periscii*, signifying a shadow turning about, as, during a revolution of the earth on its axis, their shadows are projected towards every point of the compass.

The Disc of a planet is its apparent face, seemingly perfectly flat, though really spherical. The planets exhibit discs as seen with the telescope. The diameter of the disc of the sun and moon is considered to be divided into twelve parts called digits.

The Orbit of a planet is the path described by it in one revolution round the sun.

The Plane of a planet's orbit is an imaginary surface cutting through the centre of the sun and the planet, and reaching out to the stars.

Fig. 4.

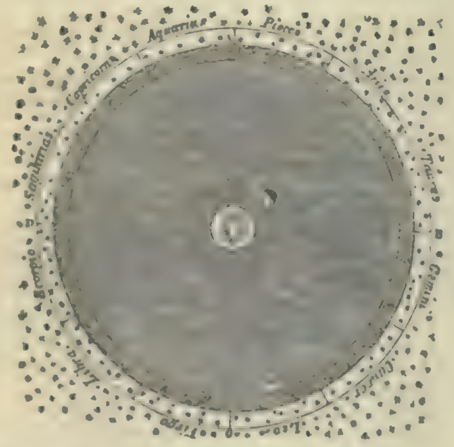


Figure 4 shows the plane of the earth's orbit. The stars to which it extends form the constellations of the zodiac. The circle A, B, C, D, is the ecliptic, the sun's place in the heavens, as seen from the earth, and the earth's place as seen from the sun.

The Inclination of the orbit of a planet is its plane referred to the plane of the earth's orbit, to which it is inclined.

If we suppose the shaded part of the diagram to be a surface of water, a ring hoop held inclined so as to be half immersed, will describe the relation of the planetary orbits to that of the earth. They are all inclined towards it, one half being below, and the other half above it. The angle of inclination varies, but exhibits, at the greatest, only a slight divergence.

Mercury . .	7°	0′	9″	Jupiter .	1°	18′	51″
Venus . .	3	23	28	Saturn . .	2	29	35
Mars . .	1	51	6	Uranus .	0	46	28

The Nodes of a planet are the two opposite points where its orbit appears to cut the orbit of the earth, or the ecliptic. Where the planet appears to rise above the orbit of the earth, it is called the ascending node; the opposite point, where it appears to go below it, is called the descending node.

Conjunction and *Opposition* are terms used to denote certain situations of the planets with respect to the sun and to each other.

A planet directly between the earth and the sun, is said to be in inferior conjunction with the sun. This can only take place in the case of Mercury and Venus, but along with them the earth may be in inferior conjunction with the sun to Mars, and with Mars to Jupiter, &c.

A planet with the sun directly between it and the earth, is said to be in superior conjunction with the sun. The terms inferior and superior refer to the smaller or greater distance of the planet from the earth.

A planet, with the earth directly between it and the sun, is said to be in opposition. This can never take place in the case of Mercury and Venus, whose orbits are included in that of the earth.

Planets are said to be in conjunction with each other when they are in the same sign and degree. This is a common occurrence in the case of two; but the very rare phenomenon of Mercury, Venus, Mars, Jupiter, and Saturn being in conjunction between the wheat ear of Virgo and Libra, took place, September 15, 1186. It will be ages before they cluster again in that part of the heavens. Venus, Jupiter, and the moon, were in conjunction in Leo in 1801.

The aspect of the planets to each other is said to be sextile, when they are two signs apart, or the sixth part of the zodiac; quartile, when they are three signs distant, the fourth part of the zodiac; trine, when they are four signs distant, the third of the zodiac; and in opposition, when they are six signs, or half the zodiac, from each other.

Apogee is that point of the moon or a planet's orbit which is farthest from the earth.

Perigee is that which is nearest.

Aphelion is that point of the orbit of the earth, or of any planet and comet, which is farthest from the sun.

Perihelion is that which is nearest. A straight line joining the points of aphelion and perihelion, is called the line of the apsides.

Culmination is the act of coming to the meridian in the case of any star or planet, when it attains, on any given day, its greatest altitude in the heavens.

Daily Acceleration is the interval of time that the stars rise, culminate, and set, sooner every succeeding day than on the one preceding. It amounts to about four minutes daily, or two hours a month.

Rotation.—Besides the apparent diurnal motion of the stars, caused by the earth's rotation upon its axis, they appear to have a motion westward, in consequence of the earth's orbital course eastward. They gain, therefore, on the sun, rising, culminating, and setting sooner, day after day. Thus, those stars and constellations that, on any given evening, rise at ten o'clock, will, at the same hour, a month afterwards, be 30° above the horizon; and three months afterwards, they will be advanced over our heads; and six months afterwards be setting in the west, having accomplished half of their apparent annual revolution.

Hence the same constellations are not always visible to us through the year. Some, not visible before, successively rise to view in the east; while others sink in the

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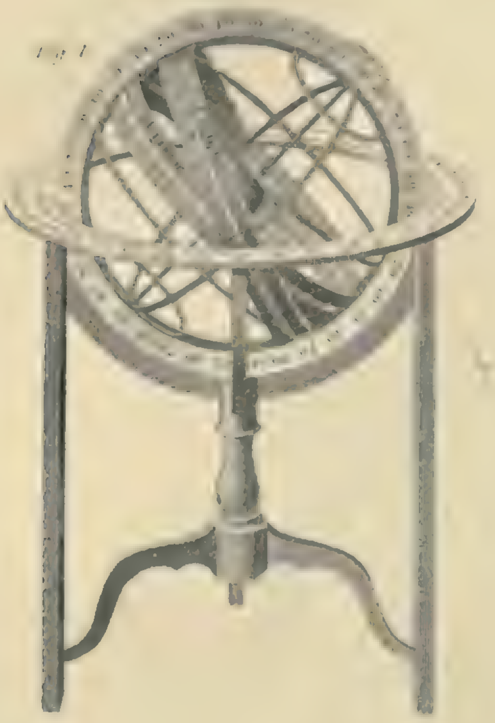
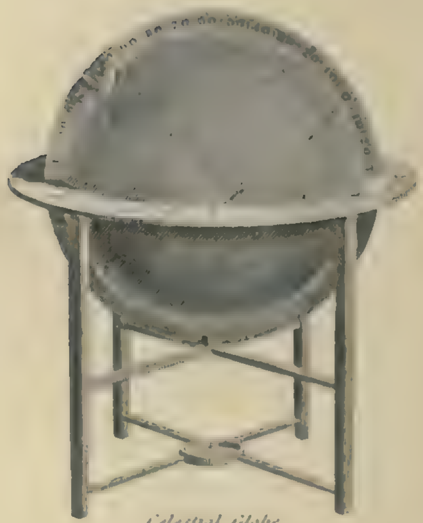
Armillary Sphere.

Fig. 1.





Celestial Globe.

Fig. 3.

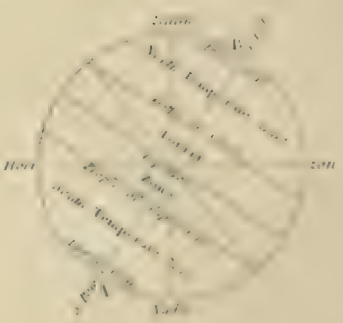
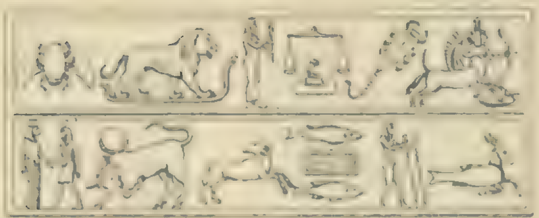
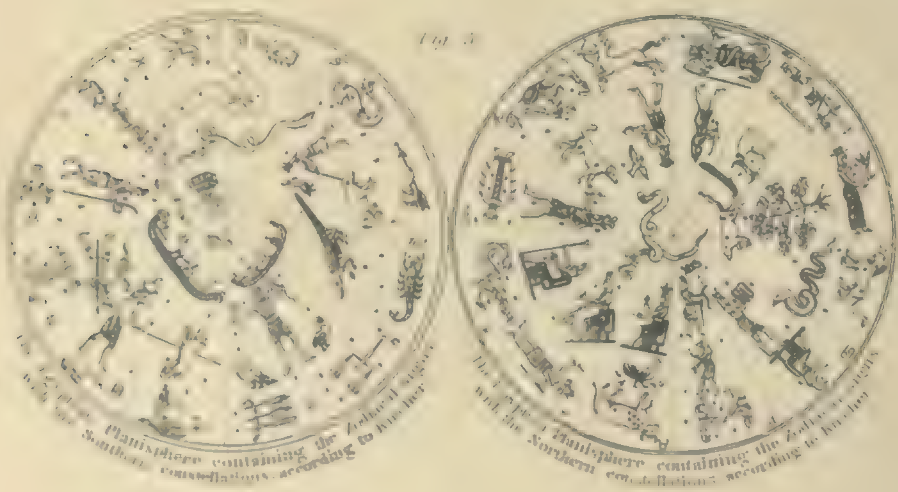


Fig. 4.



The Egyptian Zodiac from the Ceiling of the Grand Portico in the Temple of Isis at Dendera.

Fig. 5.



Planisphere containing the Zodiacal signs … Southern constellations according to Kircher

… Planisphere containing the Zodiacal signs and the Northern constellations according to Kircher west, and are not seen again, until, having passed through the lower hemisphere, they reappear in the east.

Rising and Setting.—Cosmical, achronical, and heliacal rising and setting of the stars and planets, are phrases of the old poets, who spoke of the phenomena in reference to the rising and setting of the sun. A star or planet rising and setting with the sun, was said to rise and set cosmically. A star or planet rising at sunset, or setting at sunrise, was said to rise and set achronically. A star or planet appearing a little before the sun, in the morning, after having been so near him as to be hid by his effulgence, was said to rise heliacally, and to set heliacally when it ceased to be visible after him in the evening, on account of its proximity to his orb.

Apparent Solar Day is the time included between the centre of the sun leaving the meridian of any place, to its return to the same meridian again.

It varies continually in length, owing to the unequal motion of the earth in its orbit, and the obliquity of the ecliptic, being sometimes more and sometimes less than twenty-four hours. The greatest variation occurs about November 1, when the solar day is 16′ 17″ less than twenty-four hours, as shown by a well-regulated clock.

Mean Solar Day is the time which would elapse between consecutive returns of the sun to the meridian of any place, if moving in the plane of the equator, with an equable motion. It is the mean of the true solar days throughout the year, and consists of twenty-four hours, as measured by a time-piece, which, on some days of the year, is as much faster than the sun-dial, as on other days the sun-dial is faster than the time-piece.

Sidereal Day is the time which elapses between consecutive returns of any fixed stars to the same meridian, or, in other words, the period which the earth takes to accomplish one rotation on its axis. This period is unvarying and immutable—23 hours, 56 minutes, 4 seconds—which would always be the length of the solar day, if the earth stood still in space, and only turned upon its axis.

In comparison with the immense distance of the stars, the diameter of the earth's orbit is but a point; and, consequently, in relation to them, the diurnal rotation is performed precisely the same as if our globe had no translation in space. This is not the case in relation to the sun, a nearer neighbour; and owing to the earth's change of place, somewhat more than one diurnal revolution, or twenty-four hours, is required to bring the sun round again to the same meridian.

Astronomical Day is reckoned from noon to noon; and, consisting of the same length of twenty-fours in all latitudes, is called a natural day.

Artificial Day is the time between sunrise and sunset, and varies with the latitude of places.

Solar or Tropical Year is the time which the earth takes in moving in its orbit, or, apparently, the sun in the ecliptic, from one equinox or tropic to the same again, consisting of 365 days, 5 hours, 48 minutes, 49 seconds.

Sidereal Year is the time occupied by the earth in moving in its orbit, or, apparently, the sun in the ecliptic, from a determinate point in relation to any fixed star, to the same point again, and consists of 365 days, 6 hours, 9 minutes, 12 seconds.

[The accompanying folio plate illustrates some subjects mentioned in the preceding pages, in reference to the history and technicalities of the science.—Fig. 1 represents an armillary sphere, showing the position of the equator, the ecliptic, and other circles found drawn on celestial globes. The general appearance of one of them is shown in Fig. 2.—Fig. 3 gives the names and positions of the equator, poles, zones, &c., of the sphere referred to in the foregoing definitions.—Figs. 4 and 5 afford an interesting illustration of the zodiac, and northern and southern constellations, as drawn and used by the ancient Egyptians.* At the present day, the representation of each sign used by astronomers, is almost identical with that figured in the plate.—Ed.]

CHAPTER II.

ON THE EARTH.

The earth, which to the eye of its inhabitants appears an immense plane, stretching out to an indefinite extent in all directions, is, by a variety of circumstances appealing to our reason and our senses, shown to be of a spherical form.

On the Figure of the Earth.—The notions of the ancients on the figure of the earth were very uncertain and vague. Xenophanes, who lived 500 or 600 years before the Christian era, supposed the earth to be a plane of indefinite extent, whose foundations were infinite in depth. There appears to have been a great repugnance to admit that a planet could remain suspended in space as the earth is; but as science progressed, and the art of observation became more certain, the ancient astronomers became well acquainted with its spherical figure. The shadow of the earth, projected by the moon in an eclipse of that body, clearly demonstrated its spherical form; and, on the other hand, the measures of its circumference, which the Grecian and Arabian astronomers have recorded, indicate clearly that, when reduced to a common modulus, their agreement is sufficiently striking to show that they possessed some knowledge of its dimensions. The natural pride of man, however, placed it in the centre of the universe; and according to the doctrines of the Ionian school, the solid crystalline orb, to which the stars were supposed to be attached, revolved around it in the space of 24 hours. The celebrated Aristotle, whose philosophy reigned for many centuries, was of this opinion, but he supposed that the motions of the sun, moon, and planets were performed with solid heavens, but at a nearer distance; and to explain their proper motions, he considered that a presiding genius was placed in each planet.

The usual arguments brought forward in favour of the spherical figure of the earth, are, that navigators sail round it, setting out in an easterly, and returning in a westerly, direction; but a happy illustration is given in the appearance of a vessel as it approaches the shore, or in leaving a harbour, or a succession of steam-vessels at sea. In the first case, we see at a distance the upper parts of the vessel, and gradually the lower parts, till finally the hull, which is the most conspicuous part of the vessel, in other circumstances, appears from beneath the waters. In the case of leaving a harbour, the contrary appearance takes place.

These appearances arise from the convexity of the water, between the eye and the object; for if the surface of the sea were a dead level, the largest objects would be visible the longest.

Other arguments are adduced:—Upon the bosom of the ocean, or in the midst of an extensive plain, the boundary of vision is a well-defined circle, and this circular horizon is a certain indication of the circular figure of the body to which it relates.

Navigators proceeding in the same general direction, east or west, have arrived at the same point from whence they have started. This enterprise, now so common, was first undertaken by Ferdinand Magellan, who sailed westerly from Seville, in Spain, August 10, 1519, passed the extremity of the South American continent, entered the Pacific, and reached the Philippine islands, where he was killed in a skirmish; but one of his ships arrived at St. Lucar, near Seville, September 7th, 1522.

Voyages of circumnavigation demonstrate the convexity of the earth, east or west, or that its form must be either globular or cylindrical. The convexity of the surface, north and south, is shown by the gradual declination and rise of the north and south circumpolar stars as the equator is approached and receded from, which proves the figure of the earth not to be that of a cylinder, but of a sphere.

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* See *ante*, p. 859.

Though spherical, like the other planets, whose round discs are defined by the telescope, yet the earth is not a perfect sphere, whose circumference is everywhere at an equal distance from the centre. It is more convex within the tropics than towards the poles, the equatorial diameter being longer than the polar; so that its general shape is that of an oblate spheroid, bulging out in the middle, and flattened at the two opposite sides.

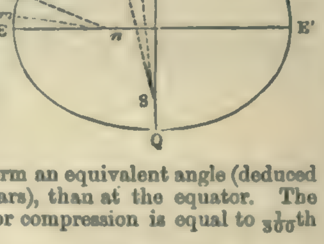
This proposition of Newton, the result alone of theory, has been amply confirmed by accurate measurements conducted by the most eminent mathematicians in various places, from the equator to the polar circle. He conceived that the velocity of the earth's daily rotation upon its axis being the greatest at the equator, the consequent greater action there, of the centrifugal force, would produce a bulging out of the surface in the equatorial regions, and a flattening at the poles. But as one of the first fundamental principles of astronomy consists in an accurate determination of the figure and dimensions of the earth, we shall briefly describe the methods adopted by astronomers for this purpose. The problem requires that all our operations must be carried on at the surface, as we cannot remove ourselves from the earth. In shifting our positions on the surface, we will take, as a point of reference, the successive appearances of the fixed stars. These bodies are so immensely distant, that at all positions on the earth, the rays emitted by them may be considered as parallel. If we shift our latitude 10° more southerly, other objects, invisible at the former station, will come under our notice near the southern horizon, whilst those near the north horizon will vanish from our view. If the difference of distance on the same parallel of longitude be measured, to produce a change of 10° in the apparent altitude of a star, we shall find, roughly speaking, that for this variation there will be a corresponding distance on the earth's surface of nearly 695 miles. And this is the principle on which the first recorded attempt of one of the ancient astronomers was founded.

Modern astronomers proceed on the same principle in a more accurate manner, and with every possible precaution to insure a correct result. In England, the first measurement of a degree was that by Norwood, in 1635, who found its length at the mean latitude of London and York, or 52° 45′, equal to 367,196 English feet, or 69 miles, 288 yards. This determination, however, is not entitled to much weight, as it appears that his latitudes were obtained by the solstitial zenith distances of the sun observed with a five-feet sextant; and it is feared, from other circumstances, that he was not sufficiently careful in his reductions to the meridian. Snell, Picard, and Cassini determined the length of a degree of latitude with a considerable accordance. It does not, however, appear that the deviation of the earth from a strictly spherical form was noticed till 1672, when Picard found that the pendulum of his transit clock, which beat seconds at Paris, required to be made shorter to beat seconds at the station at the island of Cayenne (latitude 4½° N.) This deviation of the earth from a spherical form was that which Newton and Huygens predicted, from the theoretical considerations of a revolving body, would be the case. But it is not from theoretical considerations alone that the form of the earth is adduced. In 1735, the French Academy fitted out an expedition for the purpose of determining its figure, with better instruments and methods than had been previously in use. Their proceedings became celebrated for the additions made to our astronomical knowledge. One of the stations fixed upon was near the equator at Peru, under the direction of Bouguer; the other at Lapland, under the superintendence of Maupertius, Clairault, and other celebrated men. All the measures taken by these astronomers denoted a decided ellipticity, but still the observations were not sufficiently numerous to infer its amount. At the Cape of Good Hope, in latitude 33° 18′ south, the celebrated La Caille measured an arc of the meridian; and in North America, in the plains of Pennsylvania, near the Alleghany Mountains, Mason and Dixon also measured a degree of the meridian in north latitude 39° 12′.

In Italy we have two measures of degrees—the first we owe to Boscovid and Le Maire, the other to Beccaria.

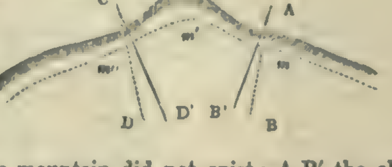
The surveys undertaken in places near the neighbourhood of mountains are very often affected by the attractions of those mountains. Thus, in the South American surveys, the attraction of the Chimborazo Mountains affected the deviation of the plumb-line by a quantity equal to 7½″. In latter times, in a survey undertaken by Plana and Castini, in Italy, the plumb-line was affected in a very perceptible manner, so much so that the resulting value of 1° in measure was 674 toises too large, from the known ellipticity of the earth deduced from a combination of observations at France and Peru. From all recorded measures, the length of a degree was greatest at the poles, and diminished gradually at the equator. If we lay down graphically the length of a degree from the different observations, we shall find that the intersections of these radii will form a curve, which will differ from the centre of the sphere in the following manner:—At the pole, the intersection or radius will touch the axis; it will afterwards recede from it, the convexity being averted to the polar axis; till, finally, at the equator, it will be perpendicular to that at the poles. The curve thus traced is termed the *evolute*. In the annexed (Fig. 5), P Q and E E are the minor and major axes of the ellipsoid of revolution; *m m′* and *n n* are measurements of a degree of latitude at the equator and the pole, which show that a longer arc, *n n′*, is required at the latter to form an equivalent angle (deduced from observations of stars), than at the equator. The effect of this flattening or compression is equal to $\frac{1}{300}$th part of the diameter.

Fig. 5.



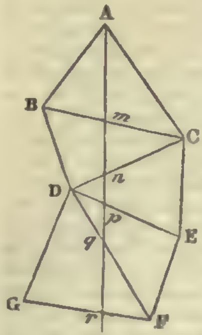
The practical method, then, of determining the length of a degree at the different positions of the earth's surface, is to find, first of all, the angle included between the verticals of the two stations, by the differences of zenith distances of certain selected stars. This is performed by means of a zenith sector, the stars being chosen near the zenith, in order to eliminate any uncertainty with regard to atmospherical refraction. But the adjustments of a zenith sector essentially depend on the accurate verticality of the plumb-line, which has been, as already stated, considerably affected by the attraction of neighbouring mountains. In Fig. 6, C M A is a surface of the earth; M a mountain; A B the direction of the plumb-line if the mountain did not exist; A B′ the observed direction of the plumb-line. In a similar manner, C D should be the real direction of the plumb-line, and C D′ the observed direction of the plumb-line. The effect of this attraction will be easily seen to affect observations by the zenith sector; and we may infer, from this circumstance, that the irreconcilable differences in the results of some surveys have been thus occasioned.

Fig. 6.



The measurement of the distance between the two stations, extending the whole length of a kingdom, is effected by a series of triangulations in the following manner :—In the figure (Fig. 7), the length of the meridian A *r* is required ; and for this purpose we select certain stations A, B, C, D, &c., as clock-towers or elevated objects. Conceive these to be joined as in the figure, and thus to form a series of triangles, as A B C, B C D, C D E, &c.

Fig. 7.



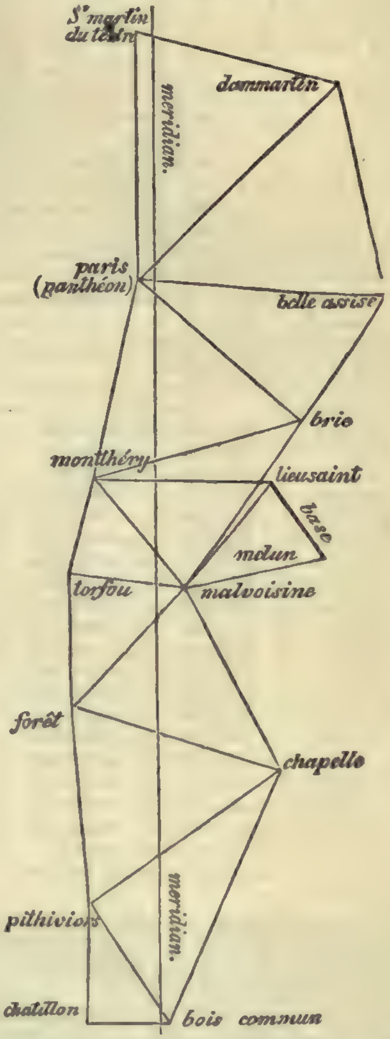
If we know all the sides and all the angles of these different triangles, as well as the angle formed with the meridian A *m n*, with the side A B, we can conclude easily the lengths of the different parts, A *m*, *m n*, *n p*, of this meridian. In fact, in the triangle A B *m*, we know the side A B, and the two adjacent angles A B *m*, B A *m*, whence we can readily find the side A *m*, which forms the first portion of the meridian, as well as B *m*, and the angle B *m* A. In the triangle *m* C *n* we know the side C *m*, which is the difference between B C and B *m*, and the two adjacent angles *m* C *n*, C *m n*, the second being equal to B *m* A, previously determined. We arrive at the same conclusion with the side *m n*, which forms the second portion of the meridian, and at the same time the side C *n*, and the angle C *n m*. In the same manner the triangle D *n p* will show us the third part of the meridian, and by this proceeding we have determined all the parts of the meridian of the point A.

But we do not require a measurement of all the sides —it will be necessary only to measure a certain part termed the *base*. Suppose that the side A B be the part measured, then the triangle A B C is entirely known, since we know one side and the three angles, and we can find the lengths of the two sides, A C, B C. In the same manner, the knowledge of the three angles of the triangle B C D, and of the side B C, that we find, permits the determination of the length of each of the two sides B D and C D ; and proceeding in this manner, we find all the sides and angles of all the triangles as well as if we had measured these lines directly.

In determining an arc of the meridian, we know well the point of departure, but we do not know where the second extremity is situated. We can find, it is true, after having determined conformably to what precedes, the length of the portion F *r* of the side F G, by measuring the distance F *r* ; but besides that this often presents great practical difficulties, it will frequently happen that the point *r* would not be favourably placed for erecting an instrument such as a repeating circle or theodolite. We have also sometimes occasion to know the latitude of the point *r*, as well as that of the point A, in order to deduce the angle included between the verticals drawn through the two points. To attain this, we observe the latitudes of the two extremities F G of the side on which the point *r* is situated ; we can easily compute the latitude of the point *r*, by the knowledge that we have ascertained of the distances comprised between this point *r* and the two points F G. Having the length of the line F G, relatively to the dimensions of the earth, we can admit that in going from F to G, along the line F G, the latitude varies proportionally to the distance passed over on this line.

MERIDIAN OF FRANCE.—One of the greatest measurements of an arc of the meridian was that performed at the end of the last century, by the celebrated astronomers Delambre and Méchain. The arc which they measured took its departure at Dunkirk and across France at its greatest length, terminating in Spain, near Barcelona, the Pantheon, at Paris, forming the summit level. Part of this survey is contained in the following figure ; the base line, on which the success of the whole measurement depends, was measured with every possible precaution by means of rods of platinum, and was found to be 6075·98 metres. The angles of the triangles were measured by means of the repeating circle, and the lengths of the different sides of the triangles were successively determined by the method before mentioned.

Fig. 8.



In order to have some independent check on the result, a second base was measured near Perpignan ; that is, near the southern extremity of the series of triangles. The length of the second base, reduced to the level of the sea, was found to be 6006·25 metres. In comparing the length thus obtained to that of this same base, deduced from the successive calculations, we have not found a greater difference between the two results than 10 inches 8 lines (cent. metres, 288). So small a difference on a length of more than 6,000 toises, is surprising, especially when we consider the immense distance which separates the base of Melun and Perpignan, a distance of more than 450,000 toises. This certainly shows that the operations had been executed with great care.

The results arrived at by these measurements showed at once, that the value of a degree was different at the equator and the poles, being least at the equator, and greatest near the poles.

From a judicious combination of the observations at all the stations, Bessel deduces the following elements of the earth's figure :—

Equatorial diameter, 41847199·9966 feet, or 7925·606 miles.

Polar diameter, 41707314·3324 feet, or 7899·113 miles.

Which shows an ellipticity of $\frac{1}{300}$th.

The following are the principal results on which the above elements are founded, viz. :—

Name of Place.	Mean Latitude.	Length of a Degree in English ft.
Peru	1° 31′ 1″ . .	362,808
India	12° 32′ 21″ . .	363,013
France and Spain .	46° 8′ 6″ . .	364,649
England	52° 2′ 20″ . .	364,914
Lapland	66° 20′ 10″ . .	365,782

In European latitudes, actual observation gives:—

Stations.	Mean Lat	English ft.
Formentera . . .	40° 0′ 50″ . .	364,206
Mountjouy . . .	42° 17′ 29″ . .	364,239
Carcassonne . . .	44° 41′ 49″ . .	364,347
Evaux	47° 30′ 46″ . .	364,935
Paris—Pantheon .	49° 56′ 29″ . .	365,052
Dunkirk	51° 15′ 25″ . .	365,116

From Bessel's Elements the next table is formed:—

Lat.	Length of a Degree in Latitude. English Feet.	Length of a Degree of Long. English Feet.	Radius Vector.	Angle of Vertical.	
°				′	″
0	362,749	365,186	1·000000	0	0·0
5	362,776	363,805	0·999975	1	59·5
10	362,858	359,674	0·999899	3	55·5
15	362,992	352,821	0·999778	5	44·3
20	363,174	343,296	0·999612	7	22·7
25	363,398	331,168	0·999407	8	47·9
30	363,658	316,524	0·999170	9	57·1
35	363,946	299,472	0·998907	10	48·3
40	364,254	280,135	0·998626	11	19·8
45	364,572	258,657	0·998336	11	30·5
50	364,590	435,198	0·998045	11	20·5
55	365,199	209,933	0·997763	10	49·7
60	365,489	183,052	0·997499	9	59·1
65	365,752	154,759	0·997259	8	50·2
70	365,979	125,271	0·997052	7	25·1
75	366,163	94,812	0·996884	5	46·3
80	366,300	63,620	0·996759	3	57·0
85	366,382	31,568	0·996683	2	0·3
90	366,411	0	0·996657	0	0·0

The elements of the earth's figure, deduced by Professor Airy,* are—

Equatorial diameter in miles .	7925·648
Polar diameter in miles . . .	7899·170.

The equatorial circumference being a little less than 25,000—accurately, 24,899.

To illustrate the very trifling proportion which subsists between the inequalities of the earth's surface and its entire volume, we may suppose an artificial ball eighteen inches in diameter, to represent our globe, when the proper proportionate elevation to be assigned to its highest mountains would be $\frac{1}{80}$th of an inch.

ROTATION OF THE EARTH.—Two principal motions belong to our planet; one of rotation upon its axis, called its diurnal motion, producing the succession of day and night; and another of progression in space, or revolution round the sun, called its annual motion, causing the vicissitude of the seasons. Both of these motions are to be understood of the whole earth, its interior substance, its superficial masses of land and water, the surrounding atmosphere, and the clouds in suspension over it; and both motions are in the same direction from west to east.

The exact time occupied by the diurnal rotation is 23 hours, 56 minutes, and 4·09 seconds. This forms a sidereal day; so called, because, in that time, the stars appear to complete one revolution round the earth. A star which is on the meridian of a place at a given period, will be on the meridian again after that interval. But as, while the earth rotates upon its axis, it is also moving in its orbit round the sun, it will require twenty-four hours, upon an average, throughout the year, for the sun to pass from the meridian of a place to the same meridian again. This forms a solar day, longer than a sidereal; and consequently, in the course of the earth's annual revolution round the sun, while we have 365 of the former, we have 366 of the latter, or that number of complete rotations of the earth upon its axis. Hence the well-known fact, that, in travelling round the globe, a person finds, on arriving at the point whence he set out, that he has gained or lost a day in his reckoning of time, according as he has travelled east or west, as compared with the reckoning of those who have remained at rest.

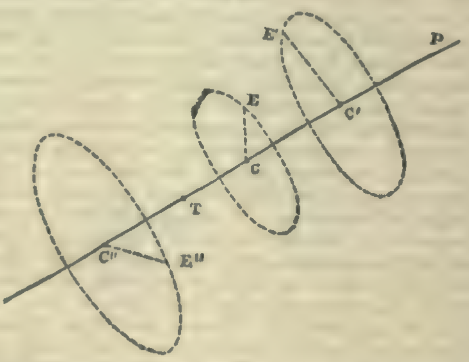
The earth's motion upon its axis is perfectly uniform and equable. Sidereal days, therefore, are always of the same length, every rotation being accomplished in the same time; but the motion of our planet in its orbit being unequal, sometimes faster, sometimes slower, solar days vary in length at different times of the year. Hence the hour shown by a well-regulated clock and a true sundial is scarcely ever the same; the difference between them, sometimes, amounting to 16¼ minutes, being called the equation of time. About the 21st of December, the solar day is half a minute longer, and about the 21st of September nearly as much shorter than 24 hours; which is an average of all the solar days throughout the year.

The rotation of the earth upon its axis is not susceptible of ocular evidence like that which the observation of spots upon the sun, and some of the planets, affords of the same fact in relation to those bodies. Nevertheless, the truth of the doctrine is established by various considerations.

Either the globe revolves upon an axis every twenty-four hours, or the whole universe, including the sun, moon, comets, and fixed stars, accomplishes a revolution round the earth in the same time. No third opinion upon the subject can possibly be held. In the latter case it is evident, from the distance of the celestial bodies, that their diurnal revolution around our planet must involve a rate of motion that is utterly inconceivable. The sun must travel at the rate of 400,000 miles a minute, the nearer stars with the velocity of upwards of 1,000,000,000 of miles a second, and the more distant with a rapidity which no numbers can express. It is absurd to suppose this, when the end to be gained requires only our little globe to revolve upon itself.

The following considerations present themselves. If the diurnal movement which we see, were attributable to a movement of the stars, E, E′, E″ (Fig. 9) would describe uniformly a circle situated in a plane, T P, perpendicular to the plane of the poles and the centres of the circle, C″, C, C′, for the reduced perpendiculars of the stars upon this line; that is to say, the most distant points from the earth T. But we know that when a body describes a circle with a uniform motion, it is attracted towards the centre of the circle by a constantly acting force, of which the magnitude depends at once upon its rate of motion and the radius of the circle which it describes. The stars E, E′ E″ cannot move in the circle of which we speak without being attracted towards the points C, C′, C″, situated upon the line of the poles.

Fig. 9.



But something like demonstrative proof of real circumstances may be adduced. If the earth rotate, the summit of a high tower, having a larger circle of rotation to describe in the same time than the base, must obviously move with greater rapidity; a stone, therefore, dropped from the summit, leaving it with a greater momentum, will move faster through the whole of its descent than the base, and reach the ground a little in advance, or easterly, of the foot of the perpendicular, the direction of the earth's rotation. Owing to the small height of buildings suitable for the purpose, this is a very difficult matter to test; but experiments have been conducted with this result, in 1804, in St. Michael's Tower, at Hamburg, and, in 1805, in a coal-pit at Schlebusch, in the county of Mark. Let the circle E (Fig. 10) be the equatorial circumference of the earth, the line T a tower perpendicular to C the centre, the circle M will then be the circumference described by the summit of the tower S, in the course of one rotation of the earth upon its axis. If we suppose the base of tower *b* to pass to *c*, the summit S will in the same time pass to *a*; and this being the larger arc, it follows that the summit must travel faster than the base. If then the earth rotate eastward, a ball dropped from the summit will leave it with its momentum, and move faster eastward through the whole of its descent than the base. The result will be that it will deviate a little from the plumb-line, and fall a little to the east of *c*.

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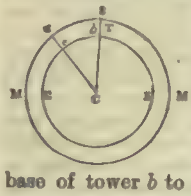
* *Encyclopædia Metrop., Art. Figure of the Earth.*

Fig. 10.



Again: a pendulum of a given length, which makes 86,535 vibrations in a day at London, will make only 86,400 in the same time if transported to the equator. This shows that the force of gravity which produces oscillations must be least where the movement is the slowest, or less at the equator than at London. Now assuming the earth's rotation, its equatorial regions, where the circle of the circumference is the greatest, must revolve with greater velocity than those which are situated towards the poles; consequently, the tendency to fly off from the centre is greater there, which proportionally neutralises the force of gravity, and accounts for the unequal action of the pendulum.

At the equator, the rate of the rotation is about 1,042 miles an hour, or 17 miles a minute; at 30° of north latitude it is 14 miles a minute; at 45°, or about the centre of France, it is 11.

ANNUAL MOTION OF THE EARTH.—The annual motion of the earth, or its orbital movement round the sun, occupies a period of 365 days, 5 hours, 48 minutes, 49·7 seconds. This forms the solar year, or the period which the sun appears to take, through the actual procession of our planet, in passing from a particular point of the ecliptic, say the first point of Aries, where the ecliptic and the equator intersect, to the same point again. It is also called the tropical year, because it is the interval occupied by the sun, in visiting the tropics and returning to the equator. The sidereal year, or the space of time which the sun takes in apparently passing from any fixed star till it returns to it again, is 365 days, 6 hours, 9 minutes, 9·7 seconds; rather more than 20 minutes longer than the tropical year, which is due to a slow annual displacement of the equinoctial points.

If the mean distance of the earth from the sun is 95,000,000 of miles, the diameter of the orbit is 190,000,000 of miles, and its linear extent near 600,000,000 of miles. This enormous distance is traversed at the rate of 68,000 miles an hour, or 19 miles in a second. This is the mean velocity for the year; but in January the earth travels at the rate of 69,600 miles an hour; which is more than 3,000 miles an hour its rate of motion in July, when it is only 66,400 miles an hour. This results from the second law of Kepler, to which we shall afterwards return.

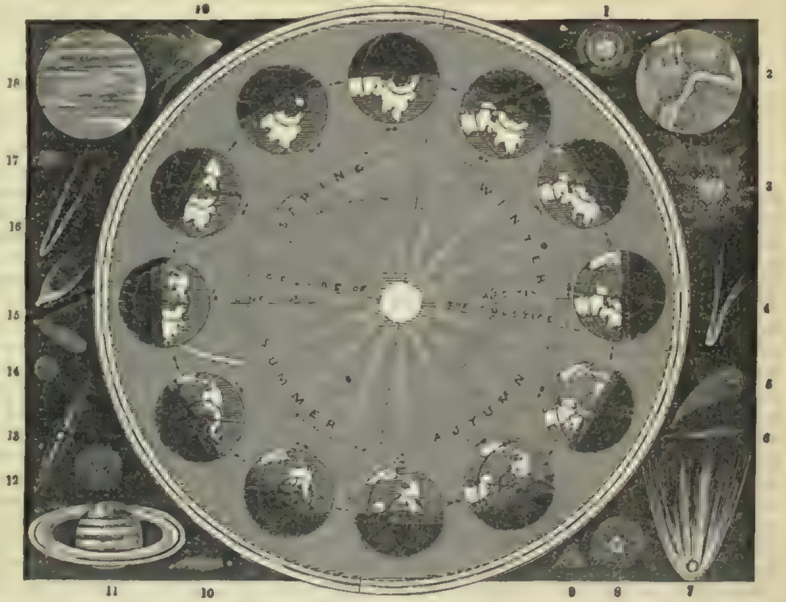
Hipparchus, two thousand years ago, was the first who closely approximated to the true length of the solar or tropical year. His determination of 365 days, 5 hours, 55 minutes, 12 seconds, exhibits a value but slightly in excess of the truth.

The earth's motion of translation in space, like that of its rotation, is imperceptible by us; resembling that we experience in calmly floating down a stream, when surrounding stationary objects appear to be in movement, and our senses are lulled into complete forgetfulness of our own progression. It appeals not to the eye, as in the case of the other planets, which are seen to be constantly changing their place; but, besides the evidence of strong probability in its favour, it has received sensible confirmation from the discovery of the aberration of the stars.

In its annual revolution round the sun, the axis of the earth, inclined 23½° from a line perpendicular to the plane of the orbit, maintains invariably the same position, causing the phenomena of the seasons. An attempt is made in the following engraving to delineate the annual revolutions of the earth about the sun, and the other phenomena attending it. (See Fig. 11).

By measuring the distances of the earth from the sun, at different times of the year, the shape of its orbit has been ascertained. These distances, as they were unequal, could not, of course, be semi-diameters of a circle, but they corresponded, taken together, to the radii vectores of an ellipse. The straight line connecting the perihelion and aphelion, passing through the centre of the sun, is the line of *apsides*. The inclination of the earth's orbit to its equator, or the so-called *obliquity of the ecliptic*, amounts to 23° 27'. The velocity of the earth is greatest at the perihelion, and least at the aphelion. It is further to be observed, that the mean distance of the earth from the sun is equal to half the major axis of the earth's orbit, and the line of apsides is itself the major axis. There are four noteworthy points in the earth's orbit in the engraving, viz., those which mark the beginning of the four seasons. Two of these points are called the *solstices*—they mark the beginning of winter and summer. The straight line uniting them, passing through the centre of the sun, is called the *solstitial colure*. The two other points are the *equinoxes*, vernal and autumnal, marking the commencement of spring and autumn. The straight line cutting them at right angles, and passing through the centre of the sun, is the *equinoctial colure*. This

Fig. 11.

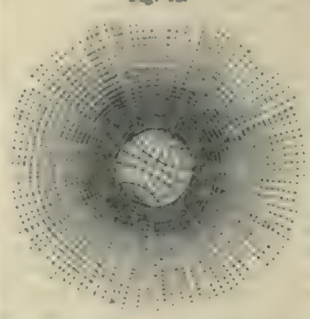


engraving also represents—1. Group of stars in Aquarius. —2. Mars, as seen Aug. 16, 1830, by Sir John Herschel at Slough.—3. Group of stars in the constellation of Hercules.—4. Great Comet, as seen Sept. 10, 1811.—5. Groups in Cancer.—6. A star in the middle of the elliptical nebula.—7. Comet of 1811.—8. Nebula in Ursa Major.—9. Bright elliptical nebula in Sagittarius.—10. Three stars in Auriga.—11. View of Saturn with his rings.—12. Nebula in Gemini.—13, 14. Nebula in Andromeda.—15. Nebula in Monoceros.—16. The Comet of 1810.—17. Nebula in Orion.—18. Streaks in Jupiter, observed Sept. 23, 1832.—19. Group of stars in Cancer.

The earth is represented on the first day of each of the twelve months of the year, the solar distances corresponding to these twelve positions, and the shape of the earth's orbit. The deeper circle surrounding the pole at a short distance, is intended to represent the parallel of latitude of Europe, or the hour circle of that place divided into twenty-four hours. Although at the end of December the earth is nearest the sun, yet, at that time in the northern hemisphere, the heat is less than at any other. The reason of this lies in the fact of the short days and long nights, as well as that the sun's rays fall very obliquely on the earth, traversing a longer path through the atmosphere, and consequently losing much of their heating power. At the beginning of July, on the contrary, although then the earth is at its greatest distance, the temperature of the northern hemisphere is greatest, on account of the long days and short nights, and the great altitude of the sun at noon. It must not be forgotten, however, that owing to the precession of the equinoxes, these signs no longer correspond to the constellations of the same name, so that now the sign Pisces corresponds to the constellation Aries, the sign Aries to the constellation Taurus, &c. It is further evident, that if the earth at the beginning of spring, summer, autumn, and winter, should be in the signs Aries, Cancer, Libra, and Capricornus, respectively, then the sun, as being always directly opposite in the ecliptic, will be in the signs Libra, Capricornus, Aries, and Cancer.

An atmosphere—the region of the winds, lightning, and meteoric corruscations—upon which respiring beings depend for vitality, surrounds our globe, and is one of its most important attributes; chiefly interesting to the astronomer, on account of its effect in displaying to us the heavenly bodies, and diffusing the rays of light in every direction around us. The extent of this wonderful and benign envelope is not precisely known; but its density diminishes as we ascend from the surface, and, at a very inconsiderable elevation, it becomes so rare as to interfere with the functions of existence. The diagram represents the engirdling atmosphere of our planet, various strata of air resting upon it, the upper pressing upon the lower, and causing the interior to be more dense than the exterior strata.*

Fig. 12.



The density of the earth is $4\frac{9}{10}$ that of water, so that our globe would counterpoise $4\frac{9}{10}$ globes of the same size, composed of materials of the same specific gravity as water. Yet, abandoned to the solar attraction, it would require 64 days, 13 hours, to fall upon the sun.

The varying intensity of the force of gravity at the surface of the earth, as shown by the unequal action of the pendulum, which vibrates slower at the equator than in other places, is as 1 at the equator to $1\frac{1}{194}$ at the poles. A body, therefore, weighing 194 pounds at the equator, would weigh 195 pounds at the north pole.

The place of the earth in the system is a favoured one, where nearly all the planets are visible to the naked eye. While they appear in our heavens, Jupiter, Venus, and occasionally Mars, shining with great splendour, our planet may be presumed to return the compliment, exhibiting to Venus, at the time of her inferior conjunction, when she is nearest to us, a full orb resplendent through her whole night. The apparent size of the earth will be greater than that of Venus to us.

CHAPTER III.

ON THE SUN.

THIS stupendous luminary, to which we are indebted for many of the blessings we enjoy—the source of light and heat, and which also contributes materially to the development of vegetation—will now claim our attention. It also serves as a standard for the regulation of our calendar. The orbit which it appears to describe about the earth fixes our year, whilst its displacements in the ecliptic regulate our seasons. A little attention will show us, that its motions in the heavens from day to day differ from that of a fixed star. We have only to mount firmly any line of sight, properly protected by coloured glasses from the glare of sun-light, and we can readily determine, by means of a watch or chronometer, that the interval between the successive returns of the sun will be different at different times of the year; whilst the invariable constancy of the returns of the fixed stars, on the other hand, impresses us at once with the notion that the earth rotates on its axis. The apparent path of the sun, however, in the heavens is not so easily traced as that of the moon or planets, whose position can be readily compared with the fixed stars. For any accurate investigations, we are obliged to make use of a transit instrument and mural circle, which will be described hereafter. By these instruments we find two elements necessary to define its path, namely, its right ascension and declination. Its diameter can also be readily known. We thus find, that, in addition to its diurnal motion, it partakes of a proper motion always in the same direction from west to east, and that it is six months above, and six months below the *equator*. Its path in the heavens is termed the ecliptic, and passes through the twelve constellations. Its orbit is inclined to the equator by an angle of 23° 27′, termed *the obliquity of the ecliptic*. The intersections of its orbit with the equator occur at two points, termed the vernal and autumnal equinoxes. The points at which the greatest and least declinations of the sun take place, are called the solstices.

MOTION OF THE SUN IN ITS ORBIT.—In order to obtain the accurate path of the sun in the ecliptic, it will be necessary to convert the right ascension and declination, obtained by means of the transit instrument and mural circle, into longitudes, which can be readily done, having given the obliquity of the ecliptic. It will then be easily seen that the arc described in the space of a day is not uniform, varying in amount at different parts of his orbit. The maximum change occurs on the 1st of January, when it amounts to 61′ 10″, and which gradually diminishes to the 1st of July, when the value is 57′ 12″, after which its motion is again quicker. The average velocity of 59′ 11″ takes place at the commencement of April and October. The observed diameter obtained by means of the passage of his eastern and western limb over the meridian, or the vertical diameter measured by the mural circle, will also exhibit fluctuations; the maximum occurring at the time of its greatest angular velocity. It would by this appear that its distance from the earth varied. The variable velocity of the sun in its orbit engaged the attention of Hipparchus, who lived about 140 years before our era, and who appears to have been a more accurate observer of the solar motions than his predecessors. This celebrated philosopher, who was of opinion that the motion in a circular orbit was the more natural, invented an *eccentric hypothesis*, which explained the greater angular velocity, and the smaller angular velocity, but which failed at the intermediate positions. He considered that the earth was removed some distance from the centre of the sun's motion, as in the annexed diagram. If the earth be supposed to be placed at T (Fig. 13), instead of at O, the centre of the circle M E N E, it will follow that the equal arcs described by the sun in equal times will not appear uniform, and thus the motion will be slower at N than at M, and will gradually increase from the former to the latter point. Hipparchus explained this change in the angular velocity of the sun, by means of the epicycle. Supposing that the earth is placed at T (Fig. 14), with a radius T C, describe the circle C C′ C″. If the sun be fixed at S, in the smaller circle C S (which is called the epicycle), and makes a complete revolution in the direction of the arrow, in the same time that the circle itself passes round the earth; it would follow that the sun would always remain at equal distances from the point T, as at a', a'', a'''. But if the sun be supposed to travel uniformly, and always preserve the same direction C S, C′ S′, C″ S″, &c., it is plain that its distance will vary from the point T. If we make T O equal to C S, and describe a circle from the centre O, this circle will pass through all the points S S′ S″ S‴. This agrees with the preceding explanation; for whilst the sun describes a circle round O, the earth T is placed eccentrically in this circle. The amount of this eccentricity may be determined by comparing the angular velocities of the sun when at apogee and perigee; and we thus find T M : T N :: 3431·5 : 3670·1; whence it would result that the greatest, the mean, and the least, distance, supposing the earth's orbit to be circular and the motion uniform, would be 1·0338 : 1·0000 : and 0·9662 respectively; the eccentricity of the earth's orbit would be 0·0338, or $\frac{1}{30}$th. We may also arrive at a knowledge of the eccentricity e, by comparing the diameters of the sun at different epochs.

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* See *ante, Mechanical Philosophy, Pneumatics*, p. 770.

Fig. 13.

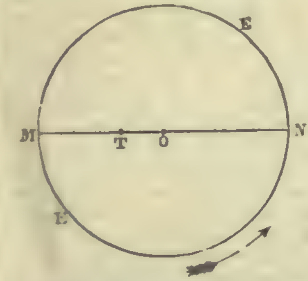
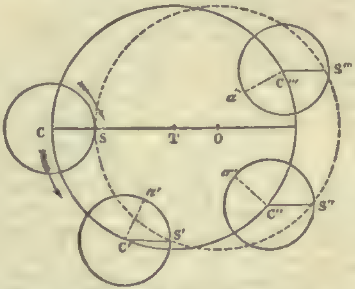


Fig. 14.



Thus, let d be the least diameter, D the greatest diameter, and δ the mean diameter, then

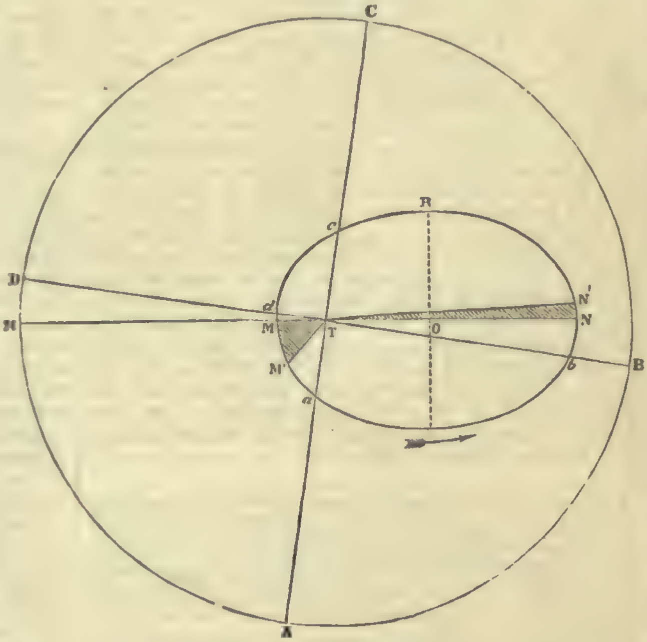
$$d = \frac{\delta}{1+e},\quad D = \frac{\delta}{1-e},\quad \frac{\delta}{D} = \frac{1-e}{1+e},\ \text{and}\ e = \frac{D-d}{D+d}$$

In this case D = 1955″·6, d = 1891″, and the resulting eccentricity is 0·0168; consequently the greatest and least distances would be 1·0168 and 0·9832. It would follow from this, that the motion of the sun in its orbit cannot be uniform, but that it must move really more rapid when at perigee than at apogee. The variation of its angular velocity is about twice as great as that of its distance. We may arrive at a knowledge of the true orbit of the earth round the sun by marking off the longitudes daily, and the proportional radii (estimated from the observed diameters). It will thus be found that the figure, roughly traced, will differ considerably from a circle, and will be an ellipse, having the earth in one of its foci. It was not, however, by these simple means that Kepler made the great discovery of the elliptic motion of the earth and planets, but by a long and laborious discussion of the observations of Mars, made by his master and patron, Tycho Brahé.

By further consideration on the motions of the planets, Kepler discovered that their angular velocity diminishes in the same proportion as the square of the distance increases, and that the areas described by the radius vector are proportionate to the times of description. If we suppose a planet to pass through M M′ (Fig. 15) and N N′ in the same space of time, it would follow from this that the areas M, T, M′, and N, T, N′, are equal, and that the angular velocities are greatest at perigee and least at apogee. The form of the ellipse which the earth describes about the sun, differs but little from that of a circle, as the distance O T is only one-sixtieth part of the semi-major axis O M; and if such an ellipse were described of a yard in diameter, the difference between the major and minor axis would be almost invisible.

The direction of the line of the apsides of the earth being known, its position in respect to the fixed stars has next to be determined. The line to which this is referred is that of D B of the equinoxes. At the present time, the inclination between the major axis M N of the earth's orbit, and the line D B of the solstices, is nearly 10°, and is gradually increasing—the major axis increasing in the same direction as the sun's motion, at the rate of 62″ annually. The duration of the seasons is consequently unequal, as may be seen by comparing the areas, a T b, b T c, c T d, and d T a, which being proportionate to the times, the sun will consequently remain longer in the quadrant b T c, than in d T a, &c. Whilst, therefore, the length of the year is constant, the duration of the seasons is subject to a slight change from year to year. At the present time, the lengths of the different seasons are as follows :—

Fig. 15.

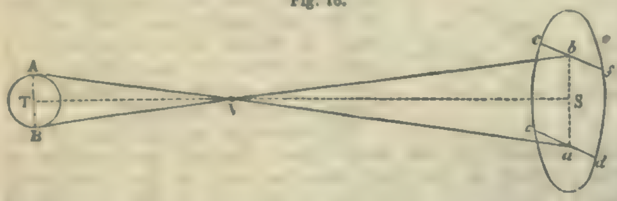


	d.	h.	m.
Spring	92	20	59
Summer	93	14	13
Autumn	89	18	35
Winter	89	0	2

The sun, therefore, remains nearly eight days longer in the northern than in the southern hemisphere. By taking into account the rate of the progressive motion of the apsides, M N, which is 61″·9 per annum, according both to theory and observation, the time when the line of solstices, D B, coincided with the line of apsides may be determined; and it has been concluded that this occurred in the year 1250. At that time the autumn and winter quarters were of equal length, and likewise the spring and summer; but the summer was longer than the winter.

Distance of the Sun.—Before we make any very accurate progress in the motion of the sun in its orbit, we must first discover the sun's actual distance, or the angle which the earth subtends at the sun. This angle is termed the parallax, and it is obtained by observing a transit of Venus, across the sun's disc, in the manner illustrated in the following diagram (Fig. 16).

Fig. 16.



The relative distances of the earth and Venus from the sun's centre are accurately known. Let us suppose that when Venus is situated between the sun and the earth, its distance from the former is 0·73, the earth's distance being 1·00. The earth being situated at T, and two observers stationed at the opposite diameters A and B, the observer at A will perceive Venus, V, crossing the sun's disc from *c* to *d*, whilst the observer at B will see it passing from *e* to *f*. The breadth of this zone, *a b*, will be considerably greater than the diameter of the earth, A B, in consequence of the greater proximity of the planet to the earth, V S being 2·7 times greater than the line V T, and consequently the line *a b* is 2·7 greater than the diameter of the earth, A B. If the angular measure *a b* could be obtained, this would therefore be 2·7 times greater than the earth's apparent diameter as seen from the sun, or 5·4 times the sun's horizontal parallax. This angular measure can be found by the observers A and B taking the distance of the limb or centre of Venus from the sun's limb. The line *a b*, as here represented, is drawn much greater than it really is, for the resulting diameter of the earth is only $8\frac{1}{2}$ seconds of arc; and consequently the breadth of this zone is only 46″·4, or about three-quarters of the diameter of Venus at the time of inferior conjunction, or $\frac{1}{42}$ of the mean diameter of the sun.

It is not only, however, by observing micrometrically the distance of Venus from the sun's limb that the relative distance *a b* may be deduced; it can likewise be determined by the *time* which the planet remains on the sun's disc at the several stations, or the interval elapsed between its entry at *c* and *e* to its disappearance at *d* and *f*. The motions of Venus are accurately known by means of the tables, and thus the lengths of the chords *c d* and *e f*, and therefore the distance *a b* also. It is usual, in these cases of the passage of the planets Venus and Mercury over the sun's disc, to note the time when the limbs of the planet and sun come in contact, and also, after the ingress has occurred, to note the time when they are last in contact. The same must be done at its egress. The times of interior contact can be perceived much more exactly than the exterior. If the transit be central, the duration of its passage may extend from 7h. 52m. to 7h. 54m.; and according as it is more distant from the centre, the chord will be shorter, and the time in the same proportion. In the passage of 1769, the duration at Wardhus was 6h. 29m.; at Hudson's Bay, 6h. 22m.; at California, 6h. 14m. 3s.; and by Captain Cook, at Tahiti, at 6h. $6\frac{1}{2}$m. From these measures the horizontal parallax of the sun was calculated by the astronomers of the time at 8″·5693; but the results differ considerably, the entrances being 8″·2 and 9″·2. The small altitude of the sun in the northern latitude, and its consequently bad definition, made it difficult to estimate the exact moment of entrance on the solar disc. Some observers saw it as much as twenty seconds sooner than others; but the effect of this would only entail an error of one-sixtieth, or probably of only one-half of that amount, on the total parallax; so that instead of 8″·5693, we should have 8″ 640 or 8″·498. These calculations have been performed anew with the utmost accuracy, by Professor Encke, of Berlin. From the first transit of June 5, 1761, that celebrated astronomer has obtained the value 8″·490525 comprised between the limits of 8″·429813 and 8″·551237. The discussion of the second passage of June 3, 1769, gives 8″·5776, not greatly different from the above: it is the value now generally adopted by astronomers. The resulting mean distance of the earth from the sun would, consequently, be

$$\frac{1}{\sin.\ 8''\cdot5776}, \text{ or } 24046\cdot9 \text{ radii of the terrestrial equator,}$$

which corresponds to 82,667,200 miles, of 60 to a degree. To pass through this distance light employs 8m. 13 15s., and in consequence we see the sun 20″·252 behind its real position. The radius of the sun at its mean distance being 960″·9, the real diameters of the two bodies will be in proportion to their apparent diameters, or as 8 578 to 960·9, or the diameter of the sun is 112·024 times greater than that of the earth. Their volumes being proportionally as the cubes of their radii, it would follow that the bulk of the sun is 1,405,845 times that of the earth.[*]

Passage of Venus across the Sun's Disc.—This phenomenon can only take place at intervals of eight years and 105 years, and the two next times will occur on Dec. 8, 1874, and Dec. 6, 1882. Since Venus comes nearly to the same point of the heavens every eight years, it may be expected that if it transits over the sun at one given epoch, it will pass over it on the eighth following year, and this generally takes place. As Venus, however, changes her latitudes during this period of eight years, it is impossible that three can follow each other in succession, as it must then pass beyond the disc of the sun, which is only 32′ in diameter; and as they can only take place when the plane is very near its nodes, it follows that for some centuries this will occur either in the months of June or December.

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[*] Although the calculations of the parallax of the sun, deduced from the transits of Venus in 1761 and 1769, were subsequently made with all possible care by Professor Encke, affording 8″·5776, still there was reason to doubt the accuracy of the data on which that value was founded. It was, therefore, extremely desirable to check the results thus obtained; and beyond the reasons just mentioned, the motion of the nodes of Venus, considered in relation with the other planets, the speed of light, as determined by M. Foucault, and other questions, required more accurate solution of the inquiry.

In 1857, Professor Airy called the attention of observers to the oppositions of Mars, occurring in 1860 and 1862. The former year was unfruitful, owing to the state of the weather precluding observations; but in 1862, sufficient data were gathered to show that the value 8″·5776 was underrated. In that year, observations of the parallaxes of Mars at Greenwich and Williamstown, gave 8″·94; whilst those of Pulkova and the Cape of Good Hope, resulted in 8″·97. Subsequently, M. Leverrier arrived at the conclusion, that 8″·95 would represent the solar parallax.

If this modified value be adopted, considerable differences will arise in the estimate of the distances, &c., of the planets. That between the sun and the earth must be reduced by about 4,030,000 miles, affording a mean distance of 91,330,000 miles. The mean distance from the sun, of the other planets, would consequently be, in round numbers—Mercury, 35,500,000; Venus, 65,500,000; Mars, 139,000,000; Jupiter, 475,000,000; Saturn, 871,000,000; Uranus, 1,752,000,000; and Neptune, 2,742,000,000. The diameter of the sun would be about 850,000 miles, and his volume 1,340,000 times that of the earth. The mass of our globe would be increased by one-tenth; the circumference of our orbit would be 599,194,000, and the mean hourly velocity of the earth, 65,460 miles. Each planet would be similarly affected in its mass, volume, &c., by changing the estimate of solar parallax from 8″·5776 to 8″·95.

No transit of Venus will occur, by which these results can be tested, until the years 1874 and 1882. Meanwhile, the care bestowed on the calculations, giving the new value, having been made by so eminent an astronomer as M. Leverrier, entitle them to the highest respect, if not the fullest confidence of observers. Indeed, incidentally, they receive considerable confirmation from the investigations of M. Foucault into the velocity of light. That philosopher, from careful experiment, proposes to reduce the estimated speed of light from 192,000 to 185,000 miles per second. For those not acquainted with the value of such data, in an astronomical point of view, we may observe, that in an eclipse, we never see the eclipsed body at the moment of last contact or emersion, obtained by calculation. It does not reappear for some time, owing to the fact that its light has to travel to the eye, and, in doing so, occupies frequently many minutes. By noting the time which elapses between the moment ending the eclipse, and that when the body reappears to the observer, the difference in seconds, multiplied by the speed of light, gives the distance of the body. On the other hand, if the known distance of the body be divided by the number of seconds which elapse between the actual and apparent conclusion of the eclipse, the speed of light, in miles per second, is obtained. The eclipses of the satellites of Jupiter afford an illustration of this fact, and will be referred to at page 961, *et seq.*, in connection with the laws of the aberration of light.—Ed.

The solid mass of the sun is found to be 354,936 times greater than that of the earth; and, comparing this with its volume, we find that the latter is only equal to one-fourth of that of the sun.* The gravity on the surface of the sun is 28·36 times greater than that on the surface of the earth; or a body which weighs one pound on the surface of our planet, weighs twenty-eight one-third pounds on the surface of the sun; and, consequently, a body will fall with twenty-eight one-third times the velocity during the first second of time.

Form of Sun's Disc.—In measuring the disc of the sun with the heliometer, or any graduated instrument, it is necessary to take precautions that it be favourably situated, and not too near the horizon, where the refraction has a very sensible effect in elevating the lower part of the disc in a greater degree than the upper. When the sun is 45° above the horizon, the difference between the vertical and horizontal diameters only amounts to 1″; but when it touches the horizon, the vertical diameter is one-sixth part of the whole diameter less than the horizontal. The figure which the sun takes under these circumstances will be seen by Figs. 17 and 18, which have been constructed with exact proportions. The refraction causes the sun, when really at S, to appear at S′, with its apparent lower limb touching the horizon. This is the usual effect of refraction; but at the horizon, many irregular local causes tend to produce changes, and it is generally found to be serrated, and with a continual whirling motion, due to the irregular motions of the atmosphere. It appears strange that the discs of the sun and moon, which, if anything, are smaller under those circumstances than when at a great altitude, are commonly remarked to be larger at the horizon than elsewhere. The ancient astronomers sought to explain this optical illusion, by the effect which the atmosphere had upon the luminous rays of the sun. It is, at the present time, more naturally attributed to the effect which a comparison with the various objects seen at the horizon produces; and, as we suppose the sun to be removed an immense distance beyond those bodies, its diameter appears greater than it really is. A similar effect takes place when the apparent areas of the constellations are compared at the zenith and the horizon; in the latter position, they appear considerably more extensive than they really are. It is, probably, due to a similar cause, that if we attempt to measure an altitude of 45° from the horizon, or the point half-way between the zenith and horizon, it frequently happens that when we come to measure it with an instrument, that which we consider to be the central point is situated some degrees nearer to the horizon than it appears to be.

Fig. 17.

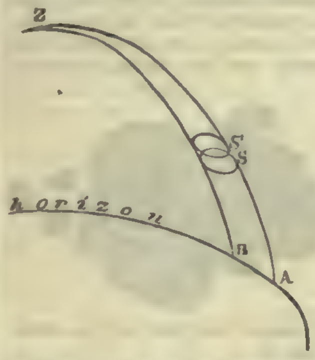
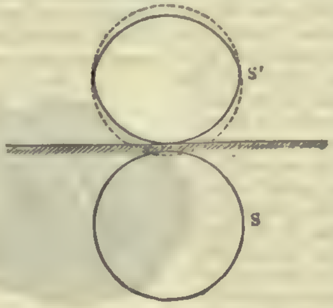


Fig. 18.



Telescopic Appearance of the Sun.—An observer, viewing the sun's surface with an instrument of moderate power, would probably not perceive any marked difference of light and shade over its general surface; but with higher powers, and a better telescope, a steady mottled appearance would become apparent, dark and light specks being softly intermingled, but not strikingly apparent when seen for the first time, or under favourable circumstances. Whilst the *ground* is not uniformly bright, there are very frequent portions which are decidedly brighter than the general surface, and others, which are much more remarkably darker, being as dark and black as the surrounding heavens, when viewed with the coloured glasses made use of. It is by means of these fleeting *maculæ* and *faculæ*, as they have been called, that nearly all our knowledge of the solar nature, its rotation, and the position of its axis, as well as the structure of its lurid and changeable atmosphere, is derived. These spots have constantly been perceived ever since the discovery of the telescope, and there is little doubt but that they were frequently seen before that time with the unaided eye, as they have occasionally been seen since. They sometimes cover large portions of the sun's surface, and are strikingly apparent from the contrast they afford with the bright surface surrounding them.

By whom these spots were first seen, after the invention of the telescope, is a matter which has found its way into the debateable land of scientific history. It would be most natural to suppose that Galileo, who had first scanned the heavens with the optic tube of his invention, and who was so capable of distinguishing every phenomenon which was apparent, would have been the first to notice these irregular appearances; but the first publication of them is due to Fabricius, who perceived them early in the year 1611; and his work, published in the same year, has a dedication bearing date June 13, 1611. In so far, however, as the mere observation of the spots is taken into account, they were seen by our countryman, Harriot, some time previous to any date fixed upon by the regular claimants to the discovery. Galileo appears to have seen them in April or May, 1611. Scheiner informs us that he perceived them during the same months, but he did not take them into consideration before the October of that year.

In the annals of China, it would appear that a large spot was visible on the sun in the year 321 of our era. In the year 807, a large spot was visible on the sun for the space of eight days, which was supposed by many to be a passage of Mercury over the sun's disc; but the length of time during which it remained visible is, of course, quite incompatible with such a supposition. Large spots which were seen by Averrhoes, Scaliger, and Kepler, were likewise supposed to be passages of this planet; but when it is remembered that a spot of the dimensions of Venus, with a diameter of five times that of Mercury, could not be detected on the sun's disc, we may be quite certain of their nature. Since that period, they have been frequently seen with the naked eye. It was, however, from telescopic observation of these spots, that the fact of the rotation of the sun on its axis was made apparent; and Fabricius was the first who surmised that they adhered to the sun, judging from their slow motion when they arrived at the edge of that luminary. This rotation, and its duration, were subsequently confirmed by Galileo. The inclination of the solar equator to the plane of the ecliptic was determined by Scheiner to be seven degrees.

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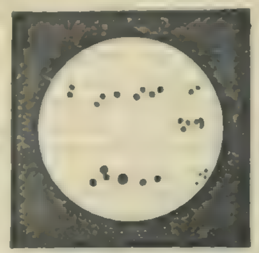
* This may be put in the following form:—To obtain a body having an attractive force equal to that of the sun, would require that 354,936 spheres each as large, and having the same specific gravity, as our earth, should be combined. The MASS of matter, however, forming the sun, occupies four times the BULK, or SPACE, of such imaginary spheres; and hence, the volume of the sun is equal to about 1,405,845 times that of the earth.—Ed.

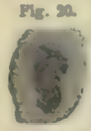
It has been remarked that the spots are confined to a certain equatorial zone, and that they rarely, if at all, extend beyond it. Their disposition in this respect may be seen from the accompanying diagram. By Galileo they were seen as far as 29 degrees of latitude north and south, but Scheiner extended this to a zone of 60 degrees in breadth, called by him the *royal zone*. They have frequently been seen, however, even beyond this limit, Messier having seen one of 31½ degrees, and Mechain one of 40½ degrees of north latitude; whilst another observer has detected one of 60 degrees of latitude. This, however, may have been one of the dark *pores*, as Herschel calls them, with which the whole surface of the sun is dotted, and tend to produce that mottled appearance which he compared to the skin of an orange. The rarity of the spots seen beyond 30 degrees of latitude quite confirm the limited nature of the equatorial zone, which the multitude of spots constantly seen on the sun renders more remarkable. It was surmised by Cassini that more spots were seen in the southern than in the northern hemisphere of the sun; but there does not appear to be any foundation for this, as they are generally equally disposed on each side of the equator. (See Fig. 19).

Fig. 19.



In the large dark spots, or *maculæ*, properly so called, it has constantly been noticed, that the very dark central portion, termed the nucleus, is fringed and surrounded on all sides with a shade of less intensity (Fig. 20), known by the name of the penumbra (pene-umbra), and which is nearly of the same degree of darkness throughout. Herschel found that even the largest of these spots commenced with one of these minute dark points, or pores, in the bright surface of the sun, which became larger and larger by degrees. When two spots were within a short distance of each other, they appeared to have a tendency to unite, and continued to expand until the moment of reunion. When the nucleus of a spot was going to disappear, it was noticed by Scheiner that the penumbra encroached gradually, but irregularly, upon it, so that the nucleus vanished before the penumbra. In modern times, and with better instruments, this breaking-up of a spot in the manner here specified, has been repeatedly noticed; the interior edge of the penumbra becomes ragged and irregular, the nucleus breaking up irregularly, and separating into many distinct nuclei, and presenting the appearance as if the luminous matter of the sun flowed in upon the dark spot. The exterior edges of the penumbra, although taking all shapes, are most frequently rounded off, and seldom appear with sharp projecting promontories. This, however, is not always the case, as was believed by Scheiner, since Herschel, on one or two occasions, perceived some prominent branches in the nucleus, which were equally apparent in the penumbra. Whilst observing it with his usual attention, he perceived six branches suddenly appear instead of the two at first observed, whilst a corresponding change took place in the penumbra. The same cause, therefore, may be regarded as having affected both equally. In regard to the proportional dimensions of nucleus, and penumbra, it is generally noticed that the latter is about three times the breadth of the former. It sometimes, though very rarely, happens, that even large nuclei are unaccompanied with any penumbra, and this was confirmed by Herschel on one or two occasions. A large path of penumbra was, on another occasion, perceived by him without any nucleus. It is but rarely that these phenomena take place in large spots; the smaller ones, on the contrary, seldom have any penumbra about them. Occasionally, the spot covers such a large space on the surface of the sun, that when, by virtue of the rotation, it comes near the edges, and is about to disappear, behind the disc, it forms a dark notch on its limb. This was seen in 1703, 1719, 1800, and 1846. It has been determined by Herschel, in respect to the relative brightness of the nucleus and penumbra, that the former may be estimated at 7, and the latter at 469; the intensity of the solar light being 1000. Many observers have noticed that the interior edge of the penumbra, or that immediately contiguous to the nucleus, is sometimes fainter than the exterior portion. At times, this is so apparent, that the nucleus appears quite detached from the penumbra, which has the appearance of a separate annulus. In many regularly round spots, the penumbra has been noticed to take a decidedly radiating appearance, like the iris of the eye, and the nucleus itself observed to be riddled with minute pores, presenting the appearance of wire gauze. Mr. Dawes has made still further observations on the telescopic appearance of the spots, and by means of eye-pieces, having a very small field of view, by which he gets rid of the surrounding glare of the sun's light, he has detected, within the part generally called the nucleus, a smaller spot, which may be considered as the nucleus proper. From his observations it would appear as if what has been termed the nucleus and penumbra, had a rotary motion around the nucleus power.

Fig. 20.



Figs. 21, 22, 23, are representations of spots really seen on the sun's disc, and it will be perceived under what irregular forms they appear. The long train of spots represent it as much broken up, with the nuclei and penumbra intermingled, and it is in this manner they are noticed when about to disappear. In Fig. 21, a bright streak of the surface of the sun is perceived in the central part. The spots are very capricious in their appearance, being sometimes very plentiful; and at other times the disc of the sun is entirely free from the slightest trace. Schroeter has recorded 81 separate spots perceived on the sun's disc at the same moment. For many years together no spot of any considerable size has been seen. Their dimensions, as already noticed, are frequently very considerable; and Schroeter saw one whose superficial extent was sixteen times greater than that of the earth.

Fig. 21.

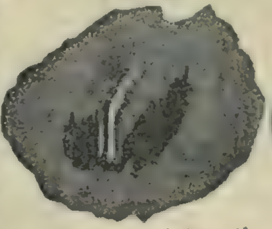


Fig. 22.

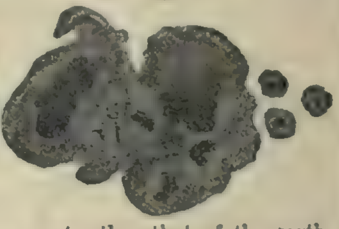
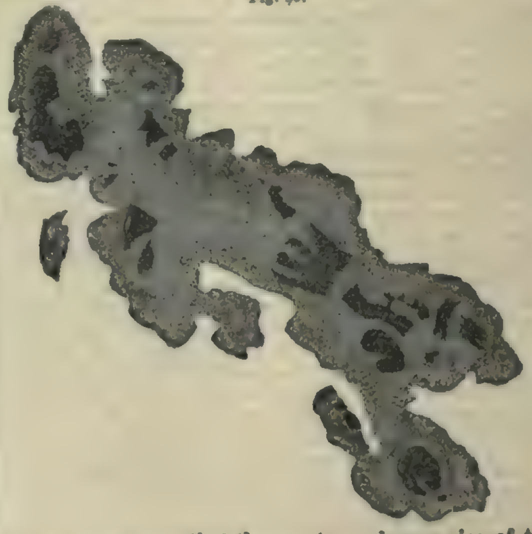


Fig. 23.

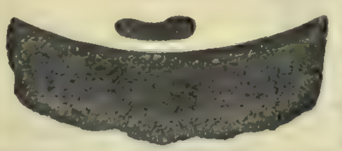


It does not appear that the great number or size of the spots have at any time been remarked to produce any degree of cold on the earth; for numerous and large as they seem to be, they bear but a small proportion to the total extent of the surface of the sun. The contrary opinion is much more generally regarded as true, and seems more accordant with the observed facts—viz., that the warmest seasons are those in which the sun is most plentifully covered with spots.

The bright spots which appear on the solar surface, either occur under the forms of long irregular veins, or minute specks, and are both perceptibly brighter than the general surface of the sun; the former being known as *faculæ*, and the latter as the *luculi*. The *luculi*, like the *pores*, are situated at all parts of the sun's disc, and assist in giving it that mottled appearance which it has. The faculæ are mostly confined to a zone of 60° in breadth, in which the spots make their appearance, and generally accompany the latter as invariably as the surrounding penumbra. The faculæ are only visible at the edges of the sun, as, when they are carried by the rotation of the sun to the centre, they are seldom, if ever visible. When the faculæ appear in great numbers and brightness on the limb of the sun, they are certain forerunners of large spots, which are almost sure to make their appearance in a few days afterwards. Cassini states that faculæ ordinarily show themselves in those places where spots have previously appeared, and that they have subsequently again become spots. Darham noticed a similar appearance, and, on one occasion, relates that he perceived changes in the spots with his eye at the telescope—a black spot appearing and disappearing successively in the centre of a brilliant faculæ. Herschel has likewise perceived extraordinary rapid changes with elongated faculæ; although Scheiner did not give credit to the existence of the *faculæ* or *luculi*, or anything on the sun brighter than its surface, yet they were detected with much less powerful instruments than he possessed, by earlier observers. Galileo first remarked the existence of the *faculæ*, and Scheiner that of the *luculi*. (See, also, page 934).

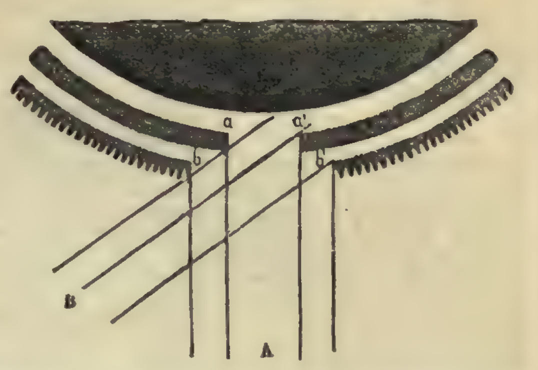
Physical Constitution of the Sun.—Until comparatively recent times, those spots were held to be the dark smoke or vapour which floated over the solar surface, although this did not explain the penumbra, or the shape which it takes when seen at different parts of the disc, when the penumbra surrounds a spot pretty equally at all sides near the centre of the sun, as at Fig. 24; for as it passes to the margin, and is received obliquely, the portion most distant would appear the narrowest. This, however, is directly contrary to observation, as it has been noticed that the nucleus and penumbra, when seen at the edge of the sun, appear as at Fig. 25, the part of the penumbra most distant from the sun's limb disappearing entirely, whilst the opposite side is only slightly diminished in breadth. Taking these facts into consideration, Dr. Wilson became convinced that the nucleus was in reality a deep hollow in the surface of the sun, and that the penumbra was the shelving sides surrounding it. If such a hollow as this be received obliquely, it is evident that it will appear as in Fig. 24. Dr. Herschel confirmed this theory in many respects; but, instead of holding the opinion that they were indentations on the surface, he considered, rather, that they were openings in the luminous atmosphere of the sun. The sun itself he considered to be a dark body surrounded by two envelopes, the interior one being formed of very luminous clouds and very bright, whilst the one lying between the photosphere and the body of the sun, is formed of clouds, very little, if at all luminous.

Fig. 24.



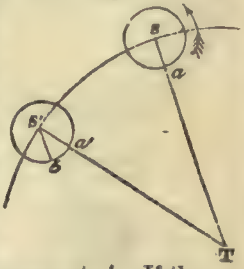
If we suppose an opening be formed in these envelopes, by a gas ascending from the body of the sun, and driving the atmosphere away, the opening at the centre of the sun (Fig. 25), to an observer at A, and the nucleus, will appear of the breadth *a a′*, surrounded equally on all sides by a penumbra, whose breadth is *b b′*. If, however, the observer is at B, he views the spot obliquely; the sides of the two openings, *a* and *b*, will coincide, and will lie in the same direction. On the opposite sides, however, at *a′* and *b′*, they will still be fully apparent. If the gas drive away the clouds of the two atmospheres, they will, of course, accumulate about the opening; and this may account for the faculæ which are to be perceived about the nucleus and penumbra. In regard to the luculi or points of light which cover the surface of the sun, they may be due to the roughness which would result from such a cloudy and irregular mass as the photosphere is imagined to be. An observation which tends to prove the unstable and cloudy nature of the outer envelope is, that they shift their positions from day to day; and it is very difficult to determine the exact period of rotation of the sun from these observations, some showing much longer periods than others.

Fig. 25.



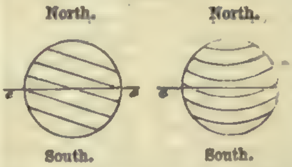
From observations on a great number of spots, M. Langier, of the Paris Observatory, has recently determined the exact period of rotation to be 25·34 days. Since, however, the earth is moving around the sun during this interval, we must wait nearly two days longer before we perceive the spot again at the centre of the sun. To explain this (Fig. 26): If T be the earth, S the sun, and *a* the spot as seen in its centre, whilst the spot appears to make a complete revolution, and to arrive again at the centre, the sun passes from S to S′; and when it arrives at S′, the spot appears at *a′*. If the sun had made exactly one revolution on its axis in the direction of the arrow, the radius S *a* would have taken the position S *b*, parallel to its first position. When it arrives in *a′*, it must, therefore, have made more than one revolution by the angle *b* S′ *a′*. To pass through 360° *plus* the angle *b* S′ *a′*, it requires 27·3 days; whilst to make a rotation on its own axis it only requires 25·34 days.

Fig. 26.



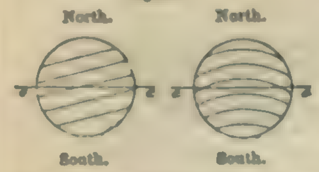
As the solar equator is inclined 7° 9′ 12″ to the ecliptic (according to M. Langier's investigations), the path which the spots appear to describe on the sun will vary at the different seasons of the year, the concavity being sometimes turned towards the north, and at other times to the south. In the beginning of December, the spots will appear to describe straight lines with reference to the ecliptic *e e*, the lines being inclined to it by an angle of 7° (Fig. 27). From the 1st of December to the 1st of June, they described curved lines, the convexity being turned to the north (Fig. 28). At the commencement of June they again describe straight lines, but in a con-

Fig. 27.



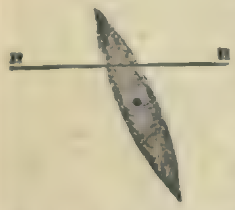
trary direction to the 1st of December. From June to December they describe curved lines, with the concavity turned to the north.

Fig. 28.



Zodiacal Light.—In the evenings of March and April, a cone of faint light is occasionally seen immediately after twilight in the western horizon, pointing in the direction, and sometimes reaching to the Pleiades, from the quarter in which the sun sets, and nearly along the ecliptic. The breadth at the base is from 20° to 30°, and its height occasionally fifty degrees. This is known as the *Zodiacal Light*, a name which was given to it by Cassini. It is very transparent, since the faintest stars can be perceived through it, equally as well as any other portion of the sky, although the glare of its bright light tends to extinguish very faint objects. Its nature is altogether unknown. It was formerly considered to be the atmosphere of the sun; but it has been proved by Laplace that this could not extend to such a distance as the zodiacal light has been observed to pass. An atmosphere to the sun could only extend to such a distance as the orbit of a planet could be situated, whose period of revolution would be equal to that of the sun on its axis, or 25 days, which would be placed considerably within the orbit of Mercury. Neither would the form of such an atmosphere agree with that observed with zodiacal light, as the equatorial and polar axes could not be beyond the ratio of three to two. Its appearance in the heavens may, however, be explained in this manner, although the cause is unknown; and it would seem as if a lenticular envelope surrounded the sun, a portion of which we see above the horizon. (See Fig. 29).

Fig. 29.

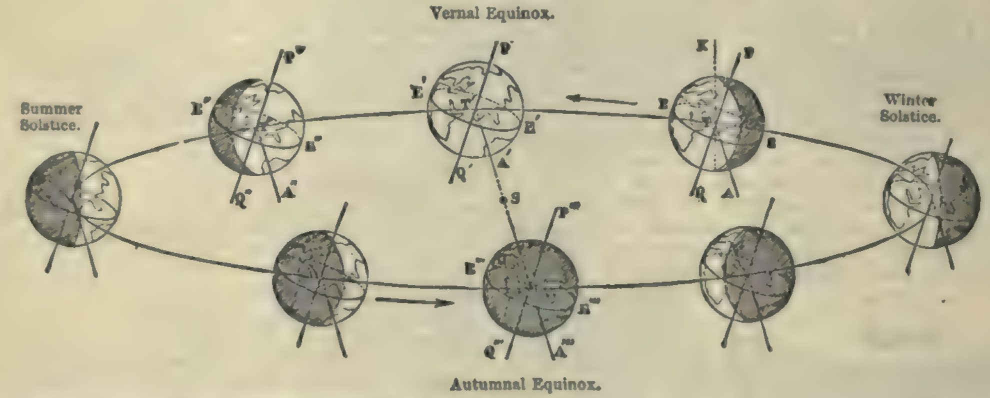


nomena of the seasons. Let S be the sun; T, T′, T″, &c., the different positions of the earth in its orbit; P Q its axis of rotation invariably parallel to itself; T A the line of the equinoxes; then at the vernal and autumnal equinoxes the sun is in the earth's equator, and the days and nights are of equal length, the sun illuminating at once one-half of the convex hemisphere. When the sun is at the winter solstice, the south pole of the earth is turned towards the sun, and the equator is above his path by an angle of 23° 27′. In our latitude, the cold northern hemisphere, the nights are longer than the days. At the summer solstice, the north pole is turned towards the sun, which is above the equator by its greatest inclination. In this latitude the days are longer than the nights. By an attentive examination of the figure, it will be seen that contrary appearances will take place in the southern hemisphere. Thus, if we consider the earth in its diurnal rotation at the summer solstice, the south pole will be constantly in darkness. At the vernal and autumnal equinoxes, the sun will shine on both poles of the earth.

Had the equator coincided with the ecliptic, the days and nights would be always equal; and had the inclination of the ecliptic to the equator been greater than at present, there would have been a corresponding difference in the seasons. Providence has wisely ordained that this inclination cannot exceed certain limits; the seasons will not, on this account, be sensibly different in occurrence and temperature.

The Earth's Equator.—In all the preceding investigations, we have supposed the pole of the earth's equator to remain fixed, but this is not the case; and, although it retains the same inclination to the ecliptic, it describes, in the course of a lapse of years, a complete revolution around the pole of the ecliptic. A very simple experiment will show, in the spinning top, that a motion of rotation around an axis may exist, without at the same time affecting its inclination, the axis of which will describe any figure. This phenomenon is termed the "precession of the equinoxes," and was discovered by Hipparchus, the astronomer, to whom we are indebted for one of our most ancient catalogues of stars. He found, by comparing the longitudes of his catalogue with those of some ancient catalogues, that whilst the latitudes of the stars were not changed, their longitudes were increased in the proportion, as he considered, of 1° in 72 years. The physical cause of the precession of the equinoxes was partly explained by Sir Isaac Newton, and afterwards more fully by the celebrated D'Alembert, and by La Place, in his *Mechanique Céleste*, on the theory of gravitation. It results from the effect of the attraction of the sun and moon on the excess of matter at the earth's equator, which produces a slow

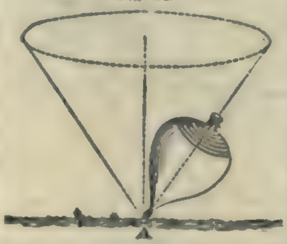
Fig. 30.



On the Seasons.—If we admit the annual revolution of the earth around the sun, its diurnal rotation on its axis, and the inclination of this axis to the ecliptic, we can readily account for the seasons, and the derivation of day and night. The annual revolution of the earth in its orbit is forced upon us by its extreme simplicity; we cannot imagine that the sun would revolve around a body 1,400,000 times smaller, as the earth is. We thus rank the earth as a planet, obeying all the laws of the other planets, describing an elliptic orbit around the sun, in the same manner as Mercury, Venus, Mars, &c. Another proof of the revolution of the earth around the sun is by the well-known existence of a phenomenon, termed the "Aberration of Light," arising from the velocity of light emanating from a star, combined with the orbital motions of the earth. The amount of this angular displacement has been determined with great accuracy by astronomers.

The above diagram (Fig. 30) will elucidate the phe-

Fig. 31.



angular motion at the plane of the equator, in a contrary direction to the earth's rotation.

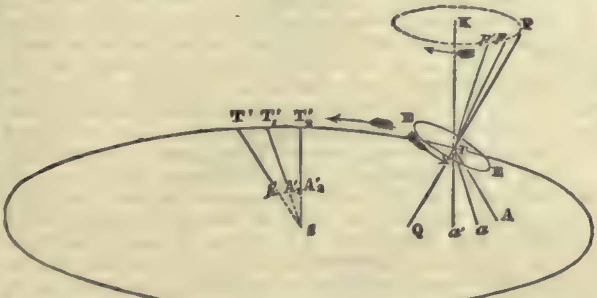
Precession of the Equinoxes.—This motion, though slow, being always in the same direction, and therefore continually accumulating, was, as we have seen, early remarked, and was one of the celestial appearances that suggested the idea of the *Annus Magnus*, or great astronomical period, by which so many days and years are circumscribed. As it affects the whole heavens, and as the changes produced are spread over a vast extent of years, it has proved a valuable guide amid the darkness of antiquity, and has enabled the astronomer to steer his course with tolerable certainty, and here and there to discover a truth in the midst of the traditions and fables of the heroic ages. The accurate analysis of the complicated effect thus produced, was a work that surpassed the power either of geometry or mechanics at the time when Newton wrote. His investigation accordingly was founded on the assumption that, though not destitute of probability, it could not be shown to be perfectly conformable to truth; and it even involved a mechanical principle, which was taken up without due consideration. The first who solved this difficult problem was D'Alembert. He employed the principle of the equilibrium among the forces destroyed, when any change of motion is produced; and by means of the equation this proportion furnished, this great mathematician was enabled to proceed with a solution that has never been surpassed for accuracy or depth of reasoning. Laplace proceeded on a more general principle, and with broader conclusions. He has shown that the phenomena of the precession and nutation must be the same; that, whatever may be the irregularity of the depth or currents of the sea, nothing can effect an alteration in the earth's rotation on its axis.

Wherever the sun, in his apparent annual course, crosses the equinoctial in spring, there is the vernal equinox; and wherever he crosses it in autumn, there is the autumnal equinox. The two points of intersection are not, however, the same year after year, but are subject to a slow annual displacement westward, so that the sun does not cross the equinoctial, spring and autumn, exactly in the same points, but every year a little behind those of the preceding year. This effect is termed the precession of the equinoxes, because it accelerates their time, though it is really their retrocession. It amounts to about 50¼″ in a year, or to 1° in 70½ years, to 30°, or a whole sign, in 2,140 years; so that in somewhat more than 25,000 years, the equinoctial points will complete a revolution westward along the ecliptic, and return to the same position.

One obvious effect of the falling back of the equinoctial points, is a progressive increase of longitude in all the heavenly bodies. Hence those stars which, in the time of Hipparchus, were in conjunction with the sun when he was in the equinox, are now 30°, or a whole sign, eastward of it; and the constellations and signs of the zodiac no longer correspond, as may be seen by reference to a celestial globe.

The annual precession of the equinoxes, apparently a change in the sun's passage across the equinoctial, is really a change in the point of the earth's orbit at which its two hemispheres are equally exposed to the sun. The cause of this remained unknown till the age of Newton, who showed that it resulted from the form of the earth, and the unequal attraction of the sun and moon on the unequal masses of matter at the equator and the poles, producing a slow reeling motion of the earth's axis from east to west, and the recession westward of the equinoctial points. Fig. 32 illustrates its effect.

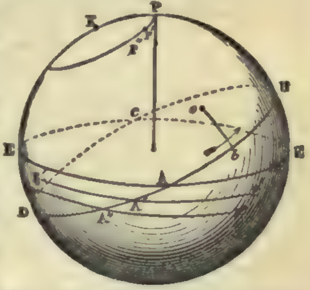
Fig. 32.



Let S be the sun, T K the pole of the ecliptic, T P the pole of the earth revolving around T K, in the direction of the arrow. In the space of one year, the revolution will change from T P, to T p; in the second year to T p'; and so on. Let E E be the earth's equator, then, when the pole of the earth moves from T P, to T p, the line of the equinoxes, or the intersection of the ecliptic and equator, will move from T A, to T a, and so on, turning slowly around the centre of the earth. It is evident that this progressive change of the direction of the poles of the earth will have a corresponding influence on the planes of the fixed stars, and on the commencements of the seasons. Thus, spring commences when the line of equinoxes is in T' A. The succeeding year would have the same commencement; but, in the meantime, the line of nodes has moved to T_1, A_1; in the following year it will have moved to T_2', A_2'—all passing through the sun S. But as the direction of the arrow shows direct motions, it is easily to be seen that the intersections *precede* at every year: for this reason this phenomenon is termed, as we have already seen, the precession of the equinoxes.

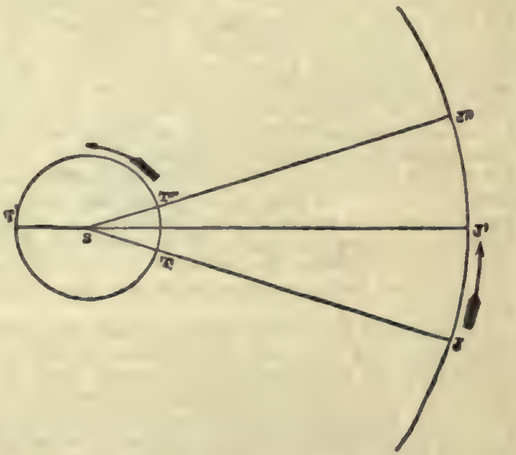
In order to exhibit the effect of this phenomenon on the planes of the stars, let E E (Fig. 33) be the earth's equator, A B C D the ecliptic, and let the intersections of the ecliptic and equator occur successively at A′, A″, &c., in an opposite direction to the sun's path. These successive changes of the equinoxes will cause corresponding changes in the pole P to P′ and P″, the right ascensions. Declinations of all bodies will be changed, as also their longitudes; but it will not have any effect on the position of the stars, the longitudes of which will be constantly the same.

Fig. 33.



On the Aberration of Light.—In ancient times it was supposed that the velocity of light was infinite and immeasurable. Roëmer, a Danish astronomer, pointed out, in the year 1675, that in comparing Cassini's tables of Jupiter's first satellite with observation, he discovered the following fact:—When Jupiter was near opposition,

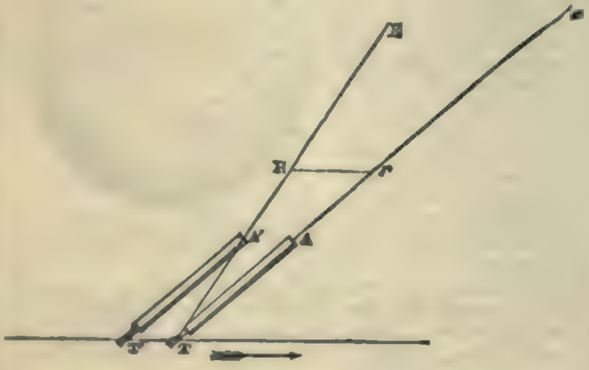
Fig. 34.



the eclipses always happened earlier than the predicted time; and when the planet was near conjunction, the eclipses happened later, the whole difference amounting to upwards of sixteen minutes. The tables of Cassini were founded on an extensive series of observations at all parts of the orbit of the planet, and would therefore give a result free from aberration at the mean distance of the planet from the earth. Thus, in Fig. 34, T T′ T″ represent the earth's orbit, S the sun, and J J′ J″ the orbit of Jupiter. Between T and T″ the planet is at opposition, and is at its least distance from the earth—the eclipses would, at this position, happen earlier—at T′ the planet is at conjunction, or at its greatest distance, when the eclipses would, of course, happen later about this period. Huygens and others confirmed this assertion of Roëmer, but the subject does not appear to have been further attended to till Dr. Bradley, in the year 1728, communicated to the Royal Society the theoretical cause of the displacement of a star, termed the "Aberration of Light." To the same astronomer we are likewise indebted for the discovery of the Nutation of the earth's axis. Without these two corrections of Aberration and Nutation, there would be a discordance in the prediction of the apparent place of a star to the amount of 1′ nearly of right ascension, and 30″ of North Polar distance. To the first of these discoveries, in chronological order, we now beg the reader's attention.

Picard, and other astronomers, in observations made for the purpose of determining the annual parallax of certain stars, found an unaccountable difference of 40″ (annually) after the application of the Prussian and all other known corrections. Dr. Bradley, who confirmed, by observations of several stars, this difference, explained the theoretical cause in the following manner:—Taking for granted that light took 8′ 3″ to pass from the sun to the earth at its mean distance, we have the motion of the earth in its orbit during this interval equal to 20″·5. In the preceding figure, when the earth is at T, the star being in the direction E T, it is evident that the telescope will not be in this position to see the star; it must have a certain inclination, T A or T′ A′, such that the cross of wires placed in T′ describe the distance T′ T by virtue of the motion of the earth, whilst that light describes the distance, A′ T. We see, in fact, that the light which passes the optical centre of the object-glass A′, when the telescope occupies the position T′ A′, arrives in T when the telescope has taken the position T A, and can, consequently, meet the cross wires, which are then found at the point T.

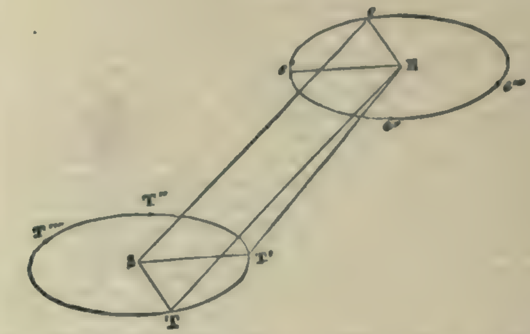
Fig. 35.



In order that the reader may see clearly the effect of the aberration of light on a star's position, we shall commence with the most simple appearances. And, in the first place, we will consider the effect of the varying position of the earth in its orbit on the place of a fixed star. For this purpose we shall take as a point of reference the place that a star would successively take, when seen from the centre of the sun, in the positions of the earth in its orbit. In the next place, we shall treat of the effect of the velocity of light combined with that of the earth in its orbit on a star's position.

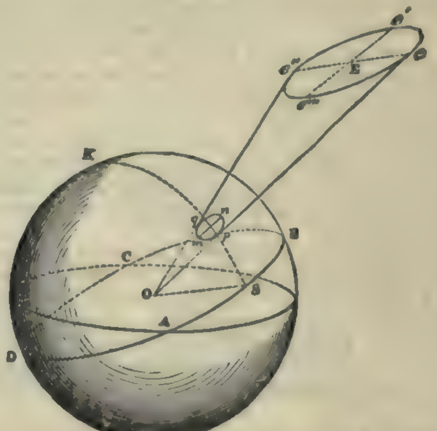
In Fig. 36 S represents the sun; T, T′, T″, &c., the earth in its orbit; E the place of a star. When the earth is at T, an observer at T will see it in the direction of T E; the corresponding direction that an observer at the sun would see it, would be found by making *e* E parallel and equal to T S, the radius of the earth's orbit; and drawing S *e* equal and parallel to T E, the star's place would then be, to an observer at S, in the position *e*: similarly when the earth has moved to T′, making *e*′ E equal and parallel to T′ S′, and completing the parallelogram, the star's place would be found in *e*′. In the same manner, tracing the position of the star throughout the year, it will be found to describe a curve equal as *e e*′ *e*″ *e*‴ is to the orbit of the earth, and parallel to its plane.

Fig. 36.



Now, to transfer these appearances to an example, let us take, in the next figure (Fig. 37), a sphere of which O is the centre, A B C D the ecliptic, K its pole, K S a circle of latitude passing through a star as γ Draconis; then, according to the preceding example, when the star is at S, the star will appear, with reference to the centre of the sphere, in a line E *e*, equal and parallel to O S, which cuts the cone in the direction *p* of the small ellipse *m p n q*. It may be remarked that each star will appear to describe an ellipse in the heavens, which will, for a star situated at the pole of the ecliptic, have its major and minor axes equal, or will become a circle, and will gradually become more and more elongated as it approaches the ecliptic, when it becomes merely a right line. The major axis, *m n*, of this ellipse is parallel to the ecliptic; and the minor axis *p q*, is perpendicular to it. We have thus found that when the sun is at S, or at the foot of the circle of latitude, the star should appear at *p*; and as the sun progresses in its orbit, it should successively be found in *n*, *q*, and *m*.

Fig. 37.

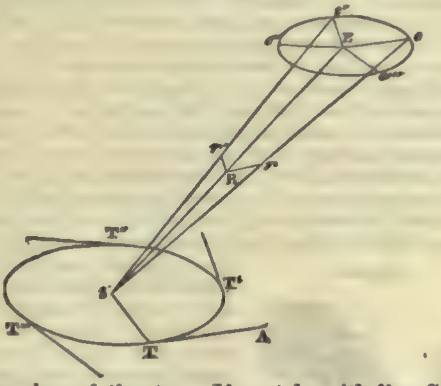


Such are the positions that Dr. Bradley calculated that the star should occupy in the annual revolution of the earth in its orbit. He was surprised to find that the most distant position of the star from the pole of the ecliptic did not occur when the sun was at S, but when it was distant 90° from this point. His observations of γ Draconis were commenced in December, 1725, when the sun would be about the position S of our figure, and the star in the position *p*; but he found that it gradually went towards the south, till it attained a position 20″ more southerly than in December. This occurred at the beginning of March, 1726. It afterwards went northerly, being in September more northerly by 39″ than it was in March. In the ensuing December the star was found in the position of the previous December, after making a proper allowance for the precession of the equinoxes.

Dr. Bradley did not, however, rest satisfied till he had repeated the observations with another instrument, made by one of the most celebrated artists of the day, Graham, which included a larger range of zenith distance. The same results as before mentioned were arrived at; the last instrument gave results identical with the three-foot quadrant of Picard, and the twenty-four foot sector of Molyneux. He endeavoured to trace the origin of the difference to the effect of refraction, or a *nutation* of the *earth's axis*, the latter of which corrections was then unknown, with the exception of the trifling effect of *solar nutation*, whose period was six months, and which at its maximum, as determined by Newton, only amounted to a fraction of a second. The cause was certainly proved by Bradley not to be owing to nutation, since a star, opposite in right ascension to γ Draconis, and at the same distance from the pole, observed at the same time of the year, exhibited differences only half the amount of γ Draconis. Had the cause been the effect of nutation of the earth's axis, the same result of fluctuation would have affected both stars by exactly the same quantity.

The effect of aberration may be thus explained. In the figure, T T′ T″, &c. (Fig. 38), represent as before; E is the place of the star. If we take with line S E, a part S R, and another line R *r*, parallel to the tangent to the earth's orbit at T, which would be the direction of the motion of the earth unless restrained by gravitation, the proportion of R *r* to S R is that of the motion of the earth in its orbit to the velocity of light. Thus when the earth is at T, the star, to an observer immovable at the centre of the sun, would, by the effect of the "aberration of light," be seen in the position S *e*, looking at the star as if he had been on the earth at T. Thus, by analogous reasoning, when the earth is at T′, by drawing R *r*′ parallel to the direction of the earth's motion, an observer at the sun will see the star in the direction S *e*′; and so on. The star would then appear to describe a curve in the heavens, *e*, *e*′, *e*″, *e*‴.

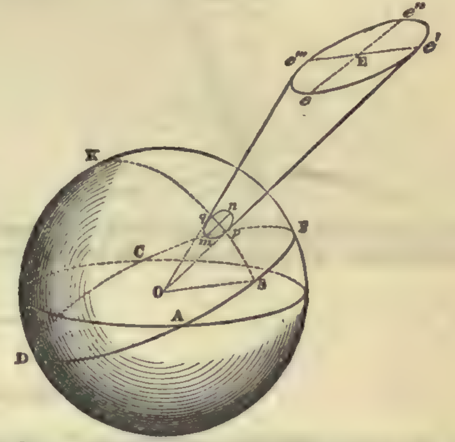
Fig. 38.



There is a great difference, therefore, between the two positions of the star, by taking into account the effect of aberration. In the former case, the direction of the visual ray was, for an observer, immovable at the sun, in a line parallel and equal to that joining the star with the earth in its orbit, and distant by the radius of the earth's orbit. In the latter case, the effect of aberration is to make the star appear in a line parallel to a tangent of the earth in its orbit, and distant by a quantity in proportion to the velocity of light, combined with that of the earth in its orbit.

Fig. 39 will show the real effect of aberration on the position of γ Draconis. By the preceding reasoning, when the sun is at S, the star should appear at *m*, in the small ellipse *m p n q*. The effect of aberration will therefore be to *retard* the position of the star by 90°, which will agree with Bradley's observation of December, 1725. At March, 1726, the star had increased its polar distance by 39″, or was found in the position *p*, and so on, the successive positions of the star agreeing precisely with observation.

Fig. 39.

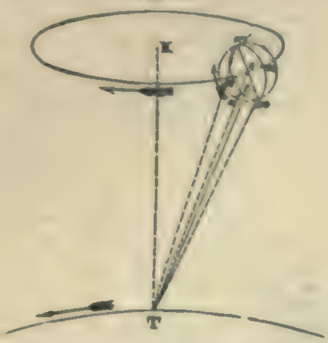


Nutation of the Earth's Axis.—After the discovery of the Aberration of Light, the indefatigable astronomer, Dr. Bradley, prosecuted his observations at Oxford and Greenwich, when his zeal and care were rewarded by the discovery of another important inequality—viz., Lunar Nutation. After applying Precession and Aberration, he found that, after a series of observations carried on from 1727 to 1745, an inequality existed, depending on the longitude of the nodes of the moon, whose period was 18 years, the existence of which had been previously mentioned by Roëmer, but of which no published account had been given. Dr. Bradley communicated this "Nutation of the Earth's Axis," in a Memoir, to the Royal Society, in 1748; in which he mentioned, that in order to reconcile it with his observations, it would be necessary to assume that the pole of the equator described a small ellipse about its mean plane, whose major and minor axes were respectively 18″ and 16″. It was afterwards explained on Sir Isaac Newton's Theory of Gravitation, as a necessary effect of the retrograde motion of the moon's nodes, by which the moon's inclination to the equator varies from 18° to 28°. By this it is evident that this variation of inclination will cause a considerable difference in the moon's attraction on the protuberant parts of the earth's equator. Nutation is, then, an irregularity of *Lunar Precession*, the mean value of which is 35″·9 annually—the remaining 14″·4 being principally due to the sun—or the whole amount of the precession of the equinoxes being equal to 50″·3.

The following explanation will clearly show the effect of a nutation of the earth's axis on the position of a star:—

It has been previously explained that the pole of the equator makes a complete revolution around the pole of the ecliptic in a term of about 25,000 years, and that it is termed the "Precession of the Equinoxes." In Fig. 40 (which is not in exact proportion to the phenomena), T K is the pole of the ecliptic, and T *o* the pole of the equator, the angle K T *o*, or the obliquity of the ecliptic, being 23° 28′. In consequence, however, of *nutation*, the pole of the equator, T *o*, does not preserve the same inclination to the pole of the ecliptic, but moves on the surface of a little cone at the elliptic base, T $m\ n\ m'\ n'$; still, however, the cone itself describing its revolution around the pole of the ecliptic, which is the "Precession of the Equinoxes," as before stated. The motion in the smaller cone, around its centre o, will evidently cause the pole of the equator to be alternately on one side and another of its mean plane. The major axis of this ellipse, $m\ m'$, is directed in the plane which passes through its axis, T o, and through the perpendicular, T K, to the plane of the ecliptic: its amplitude is 19·3″. The smaller axis of the ellipse is 14·4″. The pole, P, makes a revolution of this ellipse in the space of eighteen years, returning to m each time that the ascending node of the moon is found at the spring equinox. To know, at any time whatever, what is the position of the pole on the ellipse, we must imagine a circle described on the major axis, $m\ m'$, as a diameter, and to suppose that a point, z, describes uniformly this circle in the direction of the arrow, in a manner to come back again always to m at the times at which the pole ought to be found there. At any time whatever, the pole, P, is always situated at the point of meeting the ellipse, $m\ n\ m'\ n'$, with a perpendicular to its major axis, drawn through the position that the point z occupies at this instant. From this illustration the nature of nutation will be readily understood.

Fig. 40.



ADDENDA TO CHAPTER III.

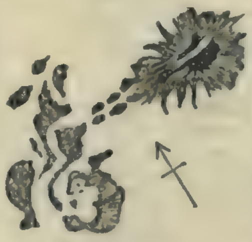
ONE of the most interesting speculations of Astronomical Science, is that which relates to the physical constitution of the sun; and we shall, therefore, venture to make some remarks in extension of those of Mr. Breen, given at page 929, on this subject.

The late advances made in Chemical and Optical Science, have not only permitted us to have a more accurate idea of the nature of solar light and heat, but, to a large extent, we have also become acquainted with some of the causes which produce them. The discovery of the polarisation of light, enables us to point out some of the characteristics of the solar luminous rays, as compared with other sources of light; but, since the year 1858, the observations of solar eclipses, and the discovery of the spectrum analysis, have proved the existence of an atmosphere encircling the sun, and shown that that atmosphere must be in a state of intense ignition. We will briefly point out how these facts have been ascertained.

At a subsequent page,* the reader will find an account of some singular appearances, which have been noticed during the eclipses of the sun that have occurred during the last few years. And as our means of observing and registering such appearances have improved,† the nature of the red flames has been directly traced to the existence of a solar atmosphere. At the time they have been noticed, they could not have occurred on the surface of the moon, because it is well known that our satellite possesses no atmosphere; and as there could be no other cause assigned for them, they were accounted for in the way we have named.

The discovery of the spectrum analysis, of which a full account will be found at the end of the section on Light, has satisfactorily set the question at rest. Our readers will understand us better if we give a brief outline of the manner of applying the spectrum method of investigation in this case.—If a ray of white light be decomposed by a glass prism, it is divided into seven colours, affording what is called the prismatic spectrum. Between each extremity of the spectrum a vast number of dark lines is distributed, which have definite positions for every ray of light. If these be viewed by means of the apparatus, illustrated and described at page 166, in the section on Light, it will be found that the vapour of any metal, passing through, or in combustion in, any luminous flame, will afford a distinctive coloured line for each metal—either of them being thus readily distinguished by its own colour and position, in the spectrum so afforded. If the reader will now peruse pages 167 and 168, in the section referred to, he will be prepared to see the application of these wonderful discoveries to the investigation of the nature of the solar atmosphere, in which, by these means, the existence of iron and other metals, in a vaporous state, has been observed. Reasoning on these facts, as we have there stated, the Astronomer Royal, Professor Airy, has pronounced a distinct opinion—that the surface of the sun is in a state of incandescence; and that immense bodies of vapour are continuously arising from it, doubtless being the cause of the intense light and heat emanating from that body. This, perhaps, is one of the most interesting applications of modern discovery with which we are acquainted.

Fig. 40*.



SOLAR SPOTS.—These curious appearances, which have been generally considered to exist *on* the disc of the sun, have generally attracted the attention of observers of late years. On July 25, 1862, a remarkable spot was noticed, having a length of 120,000 miles, and an area of 72,000,000 square miles. At its disappearance, on August 4, a distinct *notch* was afforded by the photographs of it, which seemed to prove that it was due to an indentation of the solar disc. Fig. 40* illustrates the shape of the spot as drawn on July 27.—ED.

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* See p. 984.

† See article "Celestial Photography"—Section *Light*.

CHAPTER IV.

ON THE MOON.

THIS object, the most valuable in relation to mankind—which serves as a guide to the mariner in the trackless path of the ocean, and whose influence, combined with the sun, produces the tides, will next claim our attention. Viewing its path in the heavens, we find its motion to be the most rapid of any other body; the earth's changes of position, which, with reference to the sun, amounted in the course of a day to 1°, will, for the moon, be more than twelve times that quantity. In consequence of this quick displacement with reference to the stars, its motions have become a most valuable means of determining, at any instant of time, the position of a vessel at sea.

Her appearance, and the breadth of her illuminated disc, vary every moment, showing that she is an opaque body which receives light from the sun. The disc, however, during each successive lunation, always presents the same physical appearances; and we have never been able to perceive the other hemisphere, which can only be seen when in conjunction with the sun, or at that time when the darkened hemisphere is directed to the earth. This arises from her rotation on an axis, in a period nearly coincident with the time of her sidereal revolution.

A most striking proof of the opacity of the body of the moon, is shown by a phenomenon frequently visible—viz., the occultation of a star by the moon. In consequence of the *proper* motion of the moon from west to east, she will pass over or occult in her march all the fixed stars in her path. Before full moon, we see the star disappear at her unenlightened hemisphere, and reappear at the illuminated limb. The time of the occultation will depend on the motion of the moon in her orbit, which can thus be calculated with great precision, and is found to agree exactly with observation.

PHASES OF THE MOON.—The phases are thus explained:—If we assume that the moon describes a circle around the earth, and that the sun is infinitely distant, we shall find that the rays of light from the sun will always be parallel, and will only illuminate one-half of the convex hemisphere limited by the line *m n* (Fig. 41). But to a spectator on the earth, at T, the illuminated portion will be limited by the line *p q*, directed perpendicularly to the radius of the moon's orbit. Thus at new moon, or A, the enlightened hemisphere is turned towards the sun, and the darkened part to the earth, when it is, consequently, invisible, except in peculiar circumstances, called "eclipses of the sun," when its diameter obscures for a time that luminary. At B a small portion of the enlightened hemisphere is seen from the earth. At C it is at its "first quarter." The enlightened portion increases gradually till it becomes a maximum, or is in opposition to the sun. At G it is again on the wane, or at its last quarter, decreasing gradually, but in a direction opposite to the first appearances. At A it is again found in conjunction. The above successive appearances are shown by the following diagrams. When the moon passes into the region of the heavens where the sun is, it is invisible. At the end of a few days, a careful observer will see the moon after sunset in the form of a crescent, with its convex side to the right, as it is represented in A (Fig. 42); this disappears, after a short period, under the horizon. For some days the same appearances present themselves, the crescent becoming broader at its centre, as in B, and remaining longer above the horizon. Six or seven days more and the crescent has nearly disappeared, and a half moon presents itself to us, as in C. At this period it is sufficiently distant from the sun, and crosses the meridian six hours after it. The moon continues to increase, first to three-fourths, and finally to a full circle, as in D and E. In another week, the waning moon, which is seen in the morning, assumes the appearance of F, G, H, and I, the concave side of the crescent being now to the left.

Fig. 41.

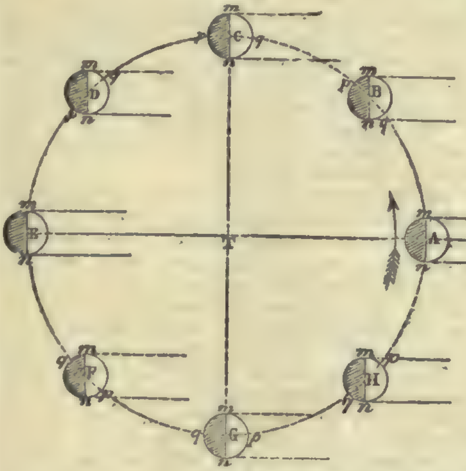
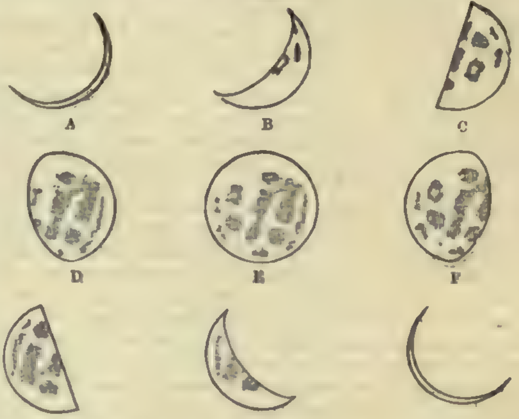
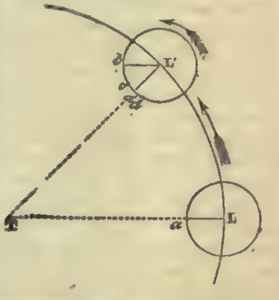


Fig. 42.


A B C
D E F
G H I

If the moon did not revolve on its axis during this progress, we should not have the same spots visible on its surface at each position in its orbit, in nearly the same direction. We are thus led to infer that the moon makes a corresponding turn on her axis, in the interval from conjunction to conjunction. Thus, if the surface were without motion, while she moved in her orbit from L to L′ (Fig. 43), the position L *a* would take a position L′ *b*; but during this time she has revolved on her axis through an angle *b* L′ *a*, equal to the angle L T L′ described in her orbit. Similarly, we find that when she has described a motion of 180° or 270°, she has rotated on her axis by an equivalent angle.*

Fig. 43.



LIBRATION OF THE MOON.—Seen through a telescope, there is another phenomenon called the *libration*, which enables us to perceive a small portion of the disc, otherwise invisible. The first, or the *libration* in longitude, arises from the two motions of rotation and translation not being exactly equal, and thus the appearance of the moon's borders will likewise partake of this inequality. The second arises from the circumstance that the axis of rotation is not exactly perpendicular to the plane of its orbit, but is inclined to it by an angle of 6° 37′. This is called the *libration in latitude*. Thus, when the moon is at L (Fig. 44), we see without difficulty the pole *q*, but cannot perceive the opposite pole *p*. But when the moon is at L′ the pole *p* becomes visible, while the pole *q* vanishes from our view. The position of a spot near the moon's equator at *a*, would, therefore, to a spectator on the earth, appear in different positions; this libration is directed perpendicularly to the plane of its orbit, or nearly to the plane of the ecliptic.

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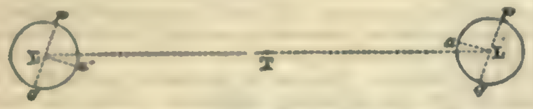
* A somewhat acrimonious discussion was commenced some years ago, by Mr. Jellinger Symons, who stoutly maintained that the moon did *not* rotate on her axis. The idea, however, was at last given up.—ED.

Fig. 44.

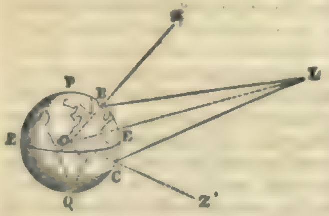


Finally, there is a third libration, called the diurnal libration, which depends on the positions of the moon to an observer on the earth's surface, in the interval from rising to setting. In consequence of the proximity of the moon, the spots will not retain the same positions at rising and passage across the meridian; but the effect at its maximum is only 32″, and is scarcely worth attention.

PARALLAX OF THE MOON.—Before attempting to deduce the orbit that the moon describes around the earth, we shall find it imperatively necessary to correct the results of observation for "Parallax." Parallax is the angle which the radius of the earth subtends at the moon. It is seen entirely in a vertical direction on the meridian, and can be determined in the following manner:—

If the zenith distance of the moon's centre be observed by two astronomers at two stations, differing considerably in latitude north and south, as Greenwich and the Cape of Good Hope, as (B and C) in Fig. 45, we shall have the angles Z B L and Z′ C L. The latitudes of the two stations, B O E and C O E, are well-known, and their difference of longitude is also accurately known. The motion of the moon in zenith distance is known to a great certainty, and thus the data are perfectly comparable. If the moon were as distant as the fixed stars, the sum of the zenith distances thus found would be precisely equal to the sum of the latitudes, north and south, of the two observatories, when proper allowance has been made for refraction. But the effect of parallax will be in both cases to increase the apparent zenith distances, and the observed sum will be greater than the sum of the latitudes by the whole amount of the two parallaxes. In this manner, by corresponding observations of the moon at her meridian passage, the constant of the horizontal parallax has been determined. (See Note at page 926).

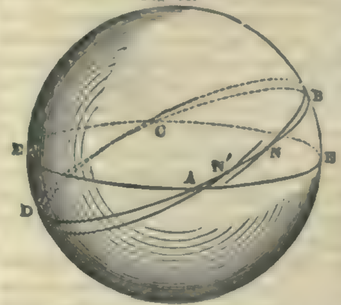
45.



In its practical application to the results of observation, it is customary to correct the zenith distances for the angle made by the direction of the plumb-line at each station, and a line drawn to the centre of the earth. This is termed the angle of the vertical; and its value, as well as the radius of the earth at the two stations, are known from the "figure of the earth." The point of reference to which horizontal parallaxes are referred, in the case of the moon, is the radius of the equator; and, in this manner, the individual results of observation are corrected for the effect of parallax in altitude, which varies as the line of the zenith distance.

THE MOON'S PATH IN THE HEAVENS.—By the transit instrument and mural circle, we are now able to trace the path of the moon in the heavens, after properly applying the corrections for parallax and semi-diameter before mentioned. We thus determine its angular displacements with respect to the equator, and find that it describes a revolution in a period of about 27 days, returning nearly to the point of departure; but if, during several lunations, its progress be watched, we shall find that its inclination to the equator differs by an angle varying from 18° to 20°. Instead, however, of referring to the equator, if we use the ecliptic, we shall find that it constantly preserves the same relation—viz., an inclination of about 5° 9′.

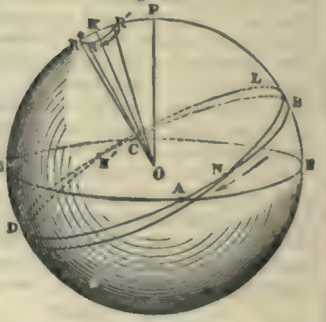
Fig 46.



The figure (Fig. 46) will show the orbit of the moon in its progress through two lunations. E E represents the earth's equator, A B C D the ecliptic, and near A B C D the orbit of the moon is seen inclined to the ecliptic by an angle of 5° 9′. The intersections of the orbit N N′ are termed the *nodes*. Supposing N to represent the node at one lunation, at the next it will be found to have retrograded to N′, and at each succeeding lunation in the same manner, always preserving the same inclination to the ecliptic, till, at the end of about 19 years, it returns to the position N. This will naturally cause a variation in its inclination with regard to the equator, which is shown by the figure (Fig. 47).

Let A B C D be the ecliptic, E E the equator, N L N′ the orbit of the moon, O P the axis of the earth, O K the axis of the ecliptic, and O R the axis of the moon's orbit. In the motion of the axis of the moon's orbit around the pole of the ecliptic, it will describe a small circle, R R′ R″, round the point K. The axis O R describes a cone of revolution, of which the axis of the figure is the line O K. In this motion the angle of O R with O K is always equal to the inclination of the moon's orbit, or 5° 9′. The inclination that the equator will make with the moon's orbit will be successively the angles P O R′, P O R, and P O R″; and as the angle P O K is the obliquity of the ecliptic, or 23° 28′, the angles P O R′ and P O R″ will be the minima and maxima inclinations, being 18° 19′ and 28° 37′ respectively. (See also Fig. 31).

Fig. 47.

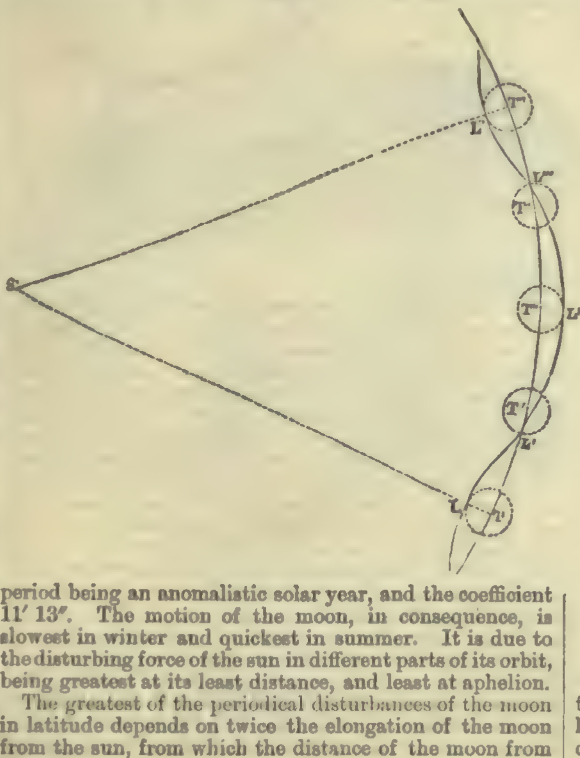


MOTION OF THE APSIDES.—The line of the apsides of the moon has been found to have a very rapid, though not uniform motion, amounting to 6′ 41″ daily, which is direct, or according to the order of the signs, and the major axis of the ellipse thus makes a complete revolution in 3,232½ days. In one hundred Julian years, or 36,525 days, the line of the apsides makes eleven complete revolutions + 109° 2′ 46″·6; and at the present time this motion decreases 50″·4203 in one hundred years. The longitude of the perigee may be calculated for any period—that in 1801, January 1 (Paris time), being 266° 10′ 7″·5. The true longitude of the moon, supposing it to move in an elliptic orbit, whose eccentricity is 0·0548442 in parts of the semi-major axis, or the greatest equation of the centre is 6° 17′ 12″·7, might thus be calculated for any period in the ordinary manner; but a great number of minute corrections must be applied to it before the tabular plane, thus obtained, would be found to agree with the observed plane of the moon.

EVECTION, VARIATION, AND ANNUAL EQUATION.—

These are the three greatest periodical fluctuations of the moon in longitude. The effect of the evection can be explained by a diminution of the equation of the centre, when the moon is at opposition and conjunction, and by an increase in its value when at quadratures; the amount of the correction in the latter case not, however, being so considerable as in the former. If we suppose the line of apsides to be in syzygy—that is, new or full moon—the observed longitude will be found to be 80′ greater than the calculation; and if the apsides are in quadratures, or first and last quarters, the observed will be found to be smaller by the same amount. The evection depends on the double of the elongation of the moon from the sun, the mean anomaly being subtracted from the product, and the coefficient of this quantity, amounting to 1° 20′ 30″. Hence, when the apsides are in syzygies, at which period the eccentricity is greatest, the greatest equation of the centre is 7° 39′; and when in quadrature, at which it is smallest, it is only 4° 58′. At the latter position of the apsides, the gravitation is greatest at apogee, and least at perigee; and in the former case, the gravitation is greatest at perigee, and least at apogee; and, consequently, the eccentricity of the orbit is increased. The discovery of the equation of the centre and evection is due to Hipparchus and Ptolemy. The more accurate observations of Tycho proved the existence of a second irregularity, which is termed the *variation*. The observed places disagreeing with the computed act of syzygies and quadratures, sometimes as much as 37′ when the line of apsides was in the octants, Tycho found that it depended on the elongation of the moon from the sun. The cause of this is the position and distance of the sun and the earth; for when the sun's disturbing force is at right angles with the radius vector, the moon's motion is accelerated from the quadratures to syzigies, and retarded in the contrary direction. The coefficient is 35′ 42″, and the increase is double the elongation of the moon and sun. The *annual equation* follows nearly the same law as the equation of the centre of the earth's orbit, only with opposite signs. This is greatest in the months of March and September, but almost vanishes in June and December. From this it derives its name—the period being an anomalistic solar year, and the coefficient 11′ 13″. The motion of the moon, in consequence, is slowest in winter and quickest in summer. It is due to the disturbing force of the sun in different parts of its orbit, being greatest at its least distance, and least at aphelion.

Fig. 48.



The greatest of the periodical disturbances of the moon in latitude depends on twice the elongation of the moon from the sun, from which the distance of the moon from its nodes being subtracted, the coefficient is 8′ 48″. In addition to this, there is another remarkable inequality, depending principally on the compression of the earth at the poles, and which may be termed the spheroidal inequality. The theory of universal gravitation has come to the aid of observation in detecting numerous inequalities, which observation alone would scarcely ever be able to discover, and has explained all those which were previously known to exist.

Motion of the Moon in Space.—The curve which the moon describes in space may be laid down in the following simple manner:—Let the earth, T (Fig. 48), pass round the sun, S′ in the orbit T, T′, T″, &c. An observer on the earth will see the moon in various positions in respect to the sun. When the earth is at T, the moon will be at L, or in conjunction; when at T′, the moon will be at L′, or at its first quarter; at L″ at full; at L‴ at last quarter; and as the earth describes its path around the sun, it follows that the curve described by the moon in space will pass through the points L, L′, L″, L‴, &c. This line is represented in the diagram as considerably more curved than it really is, the distance T L being only $\frac{1}{400}$th of the distance T S.

On the Harvest Moon.—The phenomenon of the harvest moon, when for some nights together, at that period of the year, the moon rises nearly at the same time, depends on the inclination of the ecliptic with the horizon. That part of the ecliptic in which this inclination makes the least possible angle lies in the constellation Aries (in north latitude); and when the sun is in Libra, as at the time of the autumnal equinox, the moon, when at full, will be near the first point of Aries, and but little distant from the ecliptic. It is clear that, when at this part of its orbit, as it travels from west to east, the times of successive rising must be within a short period of each other; and if the ecliptic were wholly parallel to the horizon, then it would rise exactly at the same time on each night. As the moon, however, is a little inclined to the ecliptic at some periods, this will make the difference even less. In the latitude of London, the least possible difference between the times of successive rising of the moon, has been calculated at seventeen minutes. When the constellation *Libra* rises, the ecliptic makes the greatest possible angle with the horizon, and the differences between the times of successive risings of the moon is then greatest. This takes

Fig. 49.

place with the full moon at the time of the vernal equinox, and the greatest possible difference in the latitude of London is 1h. 17m.

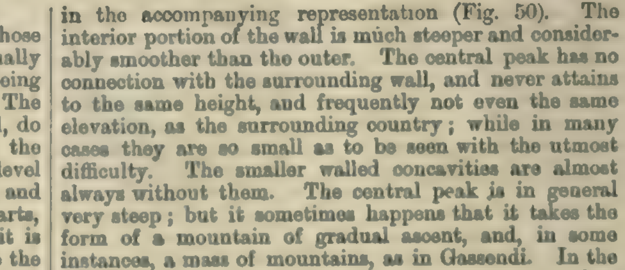
PHYSICAL CONSTITUTION AND TELESCOPIC APPEARANCE OF THE MOON.—No celestial object is better known to us, by means of the telescope, than the moon; and instruments have been constructed which shorten its distance by a thousand times, and thus brings it optically within less than two hundred and fifty miles of the earth.

In viewing it, the observer is immediately struck with the roughness of its surface, and the numerous circular formations, which appear as if surrounded by a steep and high wall: whilst those are by far the most numerous, there are also caves and hilly prominences. There exist also immense mountain chains, which for extent and height appear to surpass even those of the earth. Immense ridges, rising up from the open plains, are remarkable for their height and brightness, as well as for the numerous peaks and eminences into which the surface is separated. One of these is very visible in the interior of the crescent of the moon, when about half-full, and can even be seen as a *notch* in that portion, with the naked eye, from the shadow which it casts upon the open plain. This part is seen in the preceding representation of the moon when half-full (Fig. 49), and the mountain chain to which it belongs is a part of the *Lunar Apennines*, some of the peaks of which are calculated to be upwards of five thousand feet in height. Although, however, the general surface of the moon is rough and uneven, and the large circular mountains are extraordinarily plentiful, especially in the southern hemisphere, yet some portions are nearly level and of great extent. When the moon is full, these portions appear as dark grayish spots, and give it that appearance which has been likened to the human face; and as the moon cannot cast any shadows at this time, it follows that this must be their natural colour. These are known by the name of *seas*, and still retain that appellation; although when examined by means of the telescope, they have not any appearance of being sheets of water, but seem to be large, level, alluvial tracts of land, on which craters, peaks, cavities, and long banks are plentifully scattered. These seas take up a large portion of the lunar surface, although it is only the most extensive which are seen with the naked eye. The smaller ones are known by the names of bays, lakes, and marshes. They vary considerably in intensity of colour, and some of them have been even noticed of a grayish tinge. The greater number of these seas were named by Riccioli, who, according to the spirit of the age, called them, in the astrological manner, Mare Imbrium, Humorum, Serembates, &c. If we view the other portion of the moon when at the full, they might be mistaken for a crude, chaotic mass of crystallised or frozen fragments, without shape or termination. It is only when examined in its various crescent aspects, with good telescopes, that the various formations are seen, as it were, raised above its surface, which to all appearance they are when viewed in this bold relief.

The most remarkable of the lunar mountains are those of the circular shape, which are not, however, equally regularly defined all over its surface, some of them being quite perfect, and others misshapen and broken. The larger of the annular mountains, taken in general, do not possess the same regularity of structure as the smaller ones. For the greater part they have a level interior, which, in many instances, is very irregular; and the wall is but seldom equally high or broad at all parts, which will not, however, be perceived at first; and it is only when the sun has just commenced to illuminate the tops, that the shadows are noticed to be so rough and irregular. The exterior is frequently surrounded by high rocks, which are often more elevated than the wall. In some few instances, the interior surface is perfectly level, as in Plato, Archimedes, and the eastern portion of Stoflis; but, for the most part, they are covered with mountains, mountain chains, and craters of various depths and sizes. In several instances, the chain of mountains in the interior is quite straight and regular, and divides this surface into two parts more or less equal. These mountains are not in general so elevated as those which surround them. It is but rarely that those which are level in the interior, and may be denominated the *walled plains*, are quite circular in form. The wall which surrounds them is generally uneven, and in many parts is completely broken away: sometimes these appear as gaps, and in other cases the broken portion is completely levelled with the soil. In some cases the wall is wanting for a sixth part of the circumference; in others for one-fourth, or even one-third; but in these instances there is mostly some trace of a continuation of the real wall by a difference of brightness, or a succession of small hills. The walled plains may thus be considered to bear some affinity to the bays and some neighbouring portions of the great seas, as in the splendid *Sinus Iridum*, which, for a considerable portion of its boundary, bears a resemblance to some of the larger walled plains.

By far the greater number of the *walled plains* are seen in the southern hemisphere, where they are interlinked among each other in great plenty, and where it is a difficult matter to distinguish them properly, or to map them with the requisite distinctness. In the northern hemisphere they are less common, but appear to greater advantage from being more isolated; and it is here we see the more regular specimens of this class, as *Plato*, and *Archimedes*. It has been remarked that, in numerous instances, the walled plains appear ranged in a row in a north and south direction; and if the southern part be viewed at the first and last quarter, in that portion which separates the *Mare Nubium* from the hilly parts of the south-west quadrant, a row of these formations becomes plainly visible, some of which are of considerable magnitude and curiously entwined with one another. Similar to the circular-walled plains in shape, but mostly of smaller dimensions, are the circular-walled concavities, whose diameter varies considerably; and which, like the latter, are best seen in the northern part of the moon, though they are plentifully distributed in other parts, in the mountain chains, in the level seas, and even in the interior of the walled plains. The interior is mostly regularly concave; and in general they are furnished with a central peak, which occurs likewise, though not so frequently, in several of the walled plains, as may be seen in the accompanying representation (Fig. 50). The interior portion of the wall is much steeper and considerably smoother than the outer. The central peak has no connection with the surrounding wall, and never attains to the same height, and frequently not even the same elevation, as the surrounding country; while in many cases they are so small as to be seen with the utmost difficulty. The smaller walled concavities are almost always without them. The central peak is in general very steep; but it sometimes happens that it takes the form of a mountain of gradual ascent, and, in some instances, a mass of mountains, as in Gassendi. In the great annular mountain Tycho, which presents such a fine appearance at the time of full moon, the central peak is very steep and high. In some of the annular mountains, instead of the central part being occupied by a peak, it is frequently the locality of a deep crater, although this is not so exactly situated in the centre, as the peak generally is. It has been remarked that the very dark walled plains and concavities are mostly without any central mountain or peak, as is the case in

Fig. 50.



Julius Cæsar, Plato, Endymion, and many others of the same class. The central masses of mountains are, in general, very steep towards the summit, but are surrounded by low scattered mountains at the base. Several of the walled concavities are deficient in the same manner as the walled plains, a greater portion of the outer wall being broken down, and several gaps apparent in the circumference. The *pits* of the walled concavities are deeper than the surrounding country, the exterior height of the wall being generally one-half or one-third of the interior. But there is no proportion between the diameter and depth of the annular mountains, the smaller being sometimes deeper than the larger.

The walled mountains form but a small portion of the circular formations on the moon. Far more numerous than those, are the *craters*, by which name are designated the smaller concavities which present little or no appearance of a wall, and some of which are so minute that they can scarcely be perceived with the best telescopes. In some parts they are so numerous, that the lunar surface presents the appearance of a spongy mass covered with innumerable minute pores. Some of these craters, small as they appear, are furnished, like those of the former class, with a miniature peak in the centre. These are found at all portions of the moon, and are very plentiful in the interior of the walled concavities and plains, in the rocky chains and seas, and the level surfaces. When the moon is full, they appear as bright specks. They require a favourable time to be seen at all properly, for, unless the sun is at a favourable altitude, they can scarcely be perceived.

In viewing the moon when at full, several radiating bright streams will be noticed proceeding from many of the mountains, which pass to considerable distances from their apparent centres. None of the mountains, however, are surrounded by those bright radiating streaks to the same extent as Tycho, and many of them proceed to a distance of 500 miles. It is scarcely possible to imagine that these could be streams of lava, which flowed from the central eruption; and it is more plausibly conjectured that they must have flowed upwards through fissures, produced by the central explosion of the volcano. In either case, however, it may serve to give an idea of the tremendous force of action required in order to cause these appearances.

HEIGHT OF THE LUNAR MOUNTAINS.—The height of the lunar mountains is found either by observing the distance of the illumined summit of a mountain from the generally illuminated portion of its surface; or, otherwise, by the length of the shadow which it casts upon the plane. Both these methods will be easily understood from the following considerations. In determining their height by the former method, we find the distance of the illumined summit a (Fig. 51) from the moon's illumined edge $c\,o$, and we perceive that it is a tangent to the circle $p\,q\,r\,c'$. Knowing the two sides $o\,c'$ and $o\,a'$ of the right-angled triangles, we calculate $o\,a' = \sqrt{oc'^2 + a'c'^2}$, which is the distance of the summit of the mountain from the centre of the moon, and hence, subtracting the radius $o\,c'$, the remainder gives the height of the mountain above the surface. By this method we find its height in seconds of arcs, and this is nothing else than if we observe it resting on the limb of the moon. Its height may easily be reduced to feet. In general this method will give too small a value of the height, as the sun may have shone on the mountain for too great a length of time previously. It was by this means that Hevelius and Galileo ascertained the heights of many of the lunar mountains, some of which appeared to be as high as five English miles. More recent observations have shown them to be very high, though not of such an altitude as this.

Fig. 51.

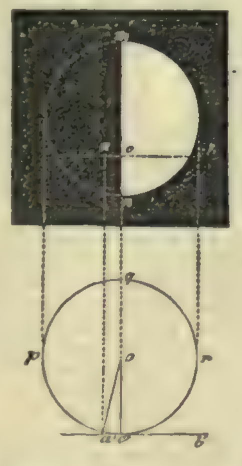
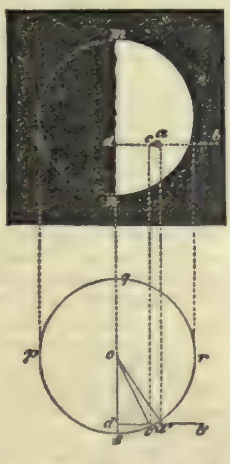


Fig. 52.



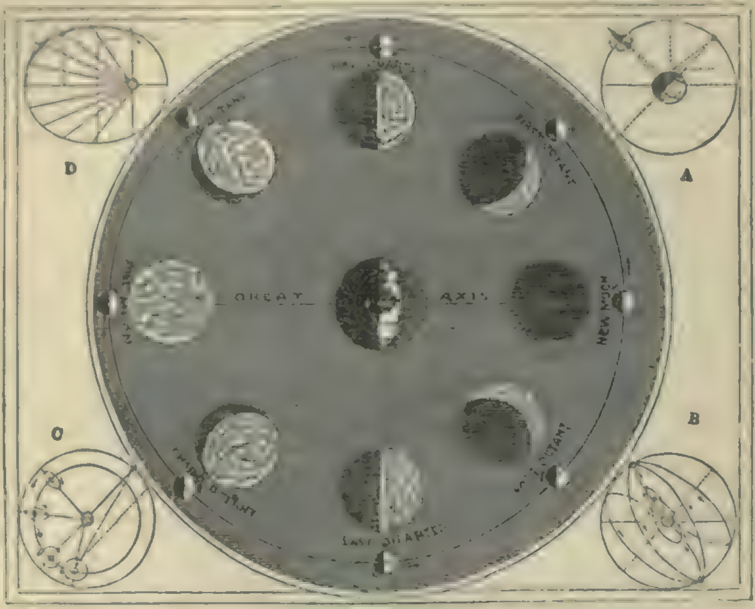
This method, however, is not applicable unless the moon is in quadrature, or half-full. The second method has been more commonly made use of on that account. As the mountain a (Fig. 52) casts the shadow $a\,c$, the length of this is measured by means of the micrometer. Describing the circle p, q, r, s, the solar ray directed towards $b\,c'$ will cut the circle in c'. The distance $c'd' = cd$ being measured, and the radius $o\,c'$ known, and the two sides of the right-angled triangle $o\,c'\,d$, the angle $o\,c'\,d$ is known, and consequently the angle $o\,c'\,a'$. In the triangle $o\,c'\,a'$ we thus know the sides $o\,c$, and $c'\,a$, and the included angle, and consequently the side $o\,a'$. Subtracting the radius $o\,c'$, the height of the mountain casting the shadow $a\,c'$ is determined.

Many observers have perceived bright spots on the *dark portion* of the lunar disc. Sir W. Herschel describes three which he once saw upon it, two of which appeared almost extinct, but the third was more luminous, and resembled a piece of burning charcoal covered with white ashes. A similar appearance was once noticed with the *naked eye* by two observers at Norwich and London. When the positions of those spots have been examined, it has been found that they are almost always situated at the brightest, and consequently the most reflective, portions of the lunar surface. When the disc of the moon is illumined by *earth-light*, as is the case when slightly crescent, or, as it is commonly called, the "old moon in the new moon's arms" (Fig. 53), those bright parts, which return most sun-light, also reflect the earth's light stronger than any others. The mountains Kepler, Tycho, and Copernicus are thus frequently visible in this manner, as also the bright mountainous districts. But, above all, Aristarchus is most commonly visible at those times. This is the brightest part of the moon, and was thought by Hevelius to be a burning volcano of sulphur and saltpetre.

Fig. 53.

In the following engraving, the cut B represents the earth when opposite the sun in the tropic of Capricorn, on the 21st of December, the longest night in the year, and moon full in the tropic of Cancer, the horizontal line being our horizon, showing that the diurnal arc of the sun is small, while the moon has a very large one to traverse. The contrary of this takes place in the summer solstice. This explains why the moon in summer seems to describe a very small, and in winter a very large arc. C represents the moon in various positions in reference to the earth and the sun. In the first and second figures she is in conjunction, or directly between the earth and the sun. In the next figures the earth is interposed between the sun and the moon. D shows how the elliptical orbit of planets is produced by the co-operation of attractive and tangential forces. The middle cut represents the phases of the moon.

Fig. 54.



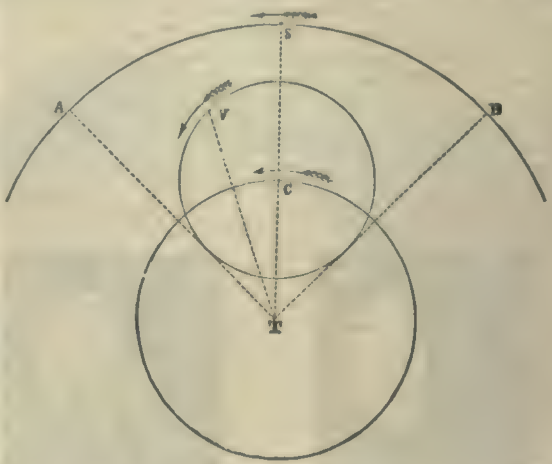
CHAPTER V.

ON THE PLANETS.

Among the more lustrous objects which arrest our observation in the heavens, are those which, since the earliest times, have been known by the name of *Planets*, from their erratic course among the fixed stars. The variable brightness of Venus, the morning and evening star, as well as those which appear at all times of the night, claim our attention equally with the sun and moon. Five of those objects were known to the ancients: Mercury, Venus, Mars, Jupiter, and Saturn; the first of which is but rarely seen with the naked eye in our latitudes, in consequence of its constant proximity to the sun, although a sparkling object, and much brighter than any of the fixed stars. These bodies, it will be perceived, do not, like the sun and moon, move in one constant direction from west to east, but sometimes direct, sometimes retrograde, and are at other times stationary. It was noticed, however, by the ancients, that whatever might be their movements in longitude, their latitudes did not depart much from the ecliptic; and a zone of 16° in breadth, or 8° north and south of the ecliptic, contained all the ancient planets. This was termed by them the zodiac, and was divided into twelve equal portions, the signs at which it cuts the equator, or the line of the equinoxes, being in the constellations Aries and Capricornus.

If Venus, the most brilliant of all the planets, be observed in its passage through the heavens, it will be seen that, in the course of a few months, it describes in the sky a very irregular circle, and that it oscillates to a certain distance on each side of the sun, sometimes rising before it in great brilliancy in the morning, and subsequently setting after it in the evening, and then for a length of time being altogether invisible to the naked eye (Fig. 55). It thus appears at an epoch nearly in the direction of the sun S; it then passes from S to A, and then returns from A to S; and having passed the sun, it goes from S to B, and finally returns to it in the contrary direction, or from B to S. When an evening star, it is seen between S and A, and between S and B in the morning. Its latitude varies in the same manner.

Fig. 55.



The greatest elongation from the sun never exceeds 47¾°, and is never less than 45°; and at those points its motion is considerably slower than when near the sun. In order to explain the oscillatory motion of Venus in respect to the sun, the ancients imagined an epicycle similar to that already mentioned in the case of the sun. If T (Fig. 55) be the position of the earth, it was supposed that V, the planet, described an orbit round the centre C, which had a motion itself around the earth, the three points, T C S, being always in a straight line. At the times of greatest elongation, the planet was in the direction of T A and T B; and in the intermediate times, its apparent motion was sometimes from west to east, and at others from east to west, or retrograde. If Venus be, however, examined with the telescope at its various distances from the sun, it will be seen that its diameter varies greatly, and that its disc undergoes a series of changes similar to those of the moon. By observing those in its different positions, it will be perfectly apparent that they agree with a motion of the planet round the sun, it being supposed that the planet is an opaque globular body, of which one hemisphere is illuminated by the sun. If the sun S (Fig. 56) be supposed to revolve round the earth T in the direction of the arrow, and the planet V around the sun, at the point V, where it is in a line with the earth and sun, and the hemisphere turned towards both illumined by the latter body, its disc will appear wholly illuminated by the latter. At the points V′ and V‴, where the angles S V′ T and S V‴ T are right angles, the planet will appear as half-full. At the point V″ the illuminated portion will be completely turned away from the east, and it will therefore be invisible, but would appear on the sun as a dark spot, and at its greatest diameter. In the positions between V and V′, and V and V‴, it will appear more or less gibbous; and between V′ and V″, and V″ and V‴, as crescentic. The appearances of the planet, and comparative sizes of its disc, under these circumstances, will be seen in the accompanying representation (Fig. 57).

Fig. 56.

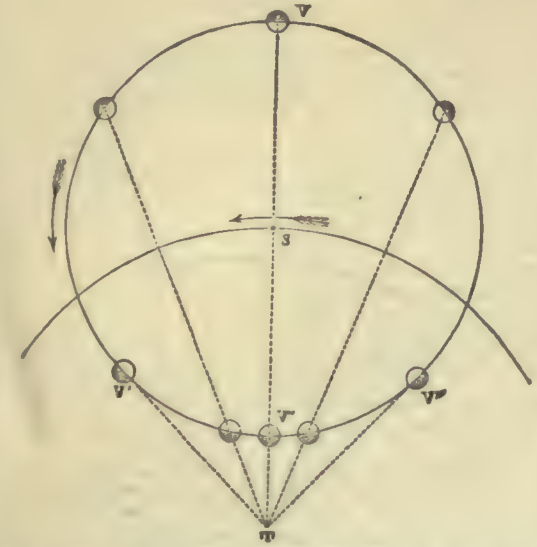
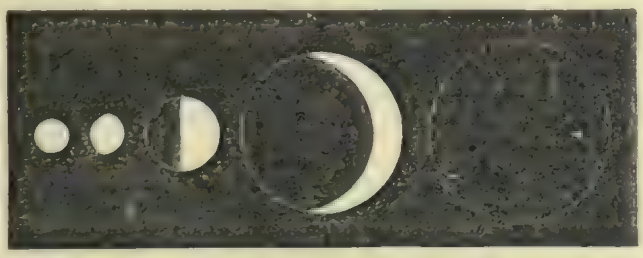


Fig. 57.



The magnitude of the illuminated part can readily be obtained for any epoch by the following simple construction:—The plane of projection being perpendicular to a line joining the earth and Venus, the boundary of the illuminated hemisphere will be projected into an ellipse more or less eccentric upon it, and the minor axis will be in the same proportion to the major, as the radius is to the cosine of inclination between the two planes. The inclination is the angle at the centre of Venus, formed by lines drawn to the sun and earth.

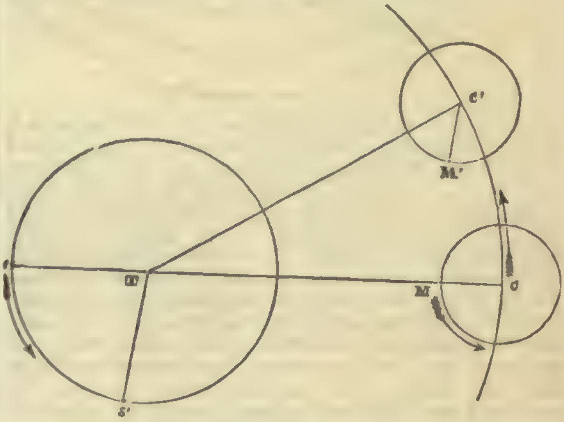
The greatest and least diameters of Venus are about 60″ and 10″, as seen from the earth; and it follows that if r be the distance between the sun and Venus, that the greatest and least distances will be $1 + r$, and $1 - r$ between Venus and the earth. The relative distances of Venus and the earth from the sun may therefore be found, as the extreme distances will be in proportion to the extreme diameters, or

$$1 + r : 1 - r :: 60 : 10, \text{ or } r = \frac{50}{70} = 0{\cdot}72.$$

The distance of Venus from the sun is therefore $\frac{72}{100}$ of that of the earth.

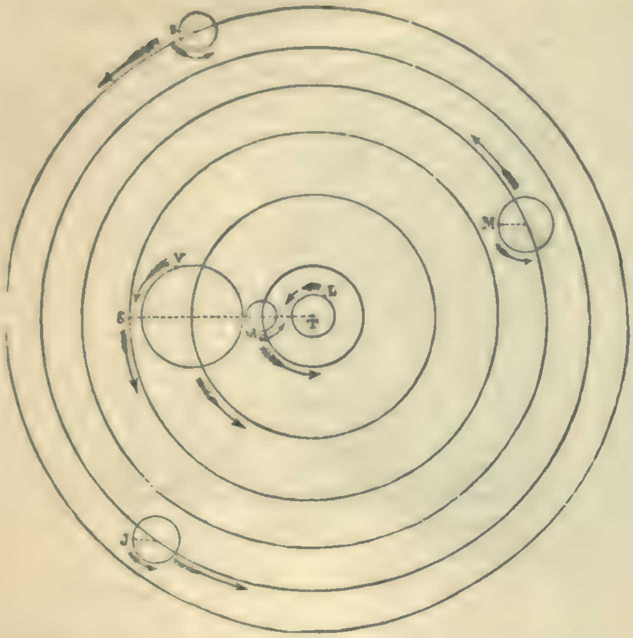
Whilst the planets Mercury and Venus can only be separated by a certain distance from the sun, and only perceived in the mornings and evenings, the other planets which are known at the present time can be seen at all parts of the sky, and pass the meridian at midnight when in opposition. The first are called the inferior, the latter the superior planets The apparent course of the latter in the sky is, however, equally irregular as the former; for the greater part of their revolution they are direct, and for a short time retrograde; their motion is sometimes slow and sometimes fast, and their latitude is equally variable. Mars returns to the same position in the heavens in 687 days, but preserves its direct motion 707 days; whilst its retrograde motion varies between 61 days and 81 days; but the direct motion, combined with the retrograde motion which follows, comprises about 780 days on the average. When in conjunction with the sun, its motion is always direct; but when in opposition, it becomes retrograde, and the arc of the latter may vary between 10° and 19½°. Jupiter, Saturn, Uranus, &c., are in a similar manner retrograde at opposition, and direct at conjunction. Jupiter's direct motion continues for 278 days, and the retrograde from 116½ to 122½ days, the arc of the latter being constantly at about 10°. In a similar manner, the period during which Saturn moves in the order of the signs is 239 days, and the duration of retrogradation may vary between 135½ and 138½ days, the arc described varying between 6° 40′ and 6° 55′. The manner in which these various motions were accounted for by the ancients was the same system of epicycle already described for Venus, but some slight modification was introduced. The planet M (Fig. 58) was supposed to describe an epicycle in such a manner that the radius C M, which joined it to the centre, was parallel to T S at all parts of its orbit, as C′ M′ to T S′, &c. If the different positions of Mars in reference to direct and retrograde motion be examined, it will be found to agree closely with this explanation.

Fig. 58.



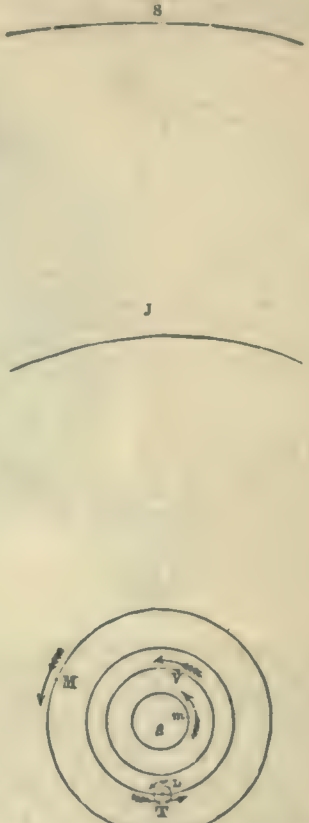
It was by this system of epicycle and deferent, that Ptolemy accounted for the apparent motions of the planets in their paths through the heavens; and he endeavoured to arrange the radii and motions of the epicycles in such a manner that they would agree with the observed appearances. As, however, new inequalities were brought to light, it was found to be impossible to explain them by those single epicycles; and, in most cases, a compound system of "wheel within wheel" had to be constructed, in order to obtain a more approximate similarity between the calculated and observed places. The more simple Ptolemaic system will be perceived from the following diagram (Fig. 59). The earth T is placed at the centre; next to which is the moon, L, which performs a revolution in 29½ days. Next to the moon is placed Mercury, *m*, whose period is 116 days; beyond which Venus, V, is situated, with a period of 584 days. The sun, S, next occurs in the order of distances; then Mars, Jupiter, and Saturn respectively, as their distances from the earth were judged to be greater or lesser, according to the length of time which elapsed between their successive returns to the same point of the heavens. It will be seen that the radii drawn from Mars, Jupiter, and Saturn, to the centres of their respective epicycles, always remain parallel to the line which joins the sun to the earth, and that the centres of the epicycles of Mercury and Venus are always in the same line with the earth and sun. Plato considered that the sun should be placed nearer to the earth than Mercury and Venus, as these two latter planets were never seen upon the disc of the sun; but it does not interfere with any of the foregoing explanations or appearances on which side it is supposed to be placed. At the present time, this could not remain arbitrary, as the phases and variable diameters of the planets would prove. It would have been much more natural for him to have made the planets Venus and Mercury revolve round the sun; but as it was not possible for him to do this in respect to the superior planets, it was likely that he would not adopt it on that account.

Fig 59.



Copernican System.—After the Ptolemaic system had been received among the learned for so many ages, who were educated in the belief of the heavens "with cycle and epicycle scribbled o'er," it was a difficult task which Copernicus had to contend with, when, with all those motions partly accounted for by the Ptolemaic system, he endeavoured to reconcile all, by far more simple machinery. How far the suppositions of former astronomers, however true they were found to be, essentially agreed with those subsequently enunciated by him, is of little consequence, as the idea of placing the sun at the centre of the system is but a small portion of the glory of his discoveries, which consisted in explaining all the apparent motions of the sun and planets, and the variety of seasons, on the most simple grounds. The Egyptians had conjectured that the planets Mercury and Venus might revolve about the sun; and others supposed that the earth was itself in motion around that body; but no further progress was made in this conjecture; and, although it might have arrested the attention of Copernicus, along with many other of the dreams and speculations, it could not afford him any insight to the system he developed.

Fig. 60.

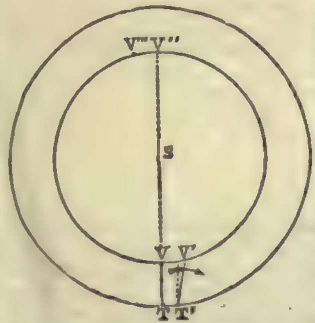


The system of Copernicus is represented in Fig. 60. The sun, S, is here the centre of the great orb, and the planets Mercury, Venus, the Earth, Mars, Jupiter, and Saturn, *m*, V, T, M, J, S, revolve about it in regular order, and all in the same direction. The moon was held, as in the Ptolemaic system, to have a direct motion around the earth, but was now made to accompany it in the same manner as a subsidiary planet; and, in order to explain the rising and setting of bodies, or the diurnal motion, the earth was supposed to have a motion on its axis in twenty-four hours. These, which have already been explained, all tend to confirm it as the true system of the heavens: the phases of Venus and Mercury, and many other proofs hereafter to be given, tended further to place it beyond doubt; while its simplicity and philosophical spirit led to its being generally acknowledged. The motion of the earth was, however, greatly opposed by many of the learned at that time; and, among others, Tycho Brahé endeavoured to form a system, in which all the apparent motions would be explained on the supposition of the immobility of the earth, which he considered was fully apparent from the absence of a sensible annual parallax of the fixed stars, as well as from what he judged to be the improbability, that an immense body, like the earth, would move at so great a rate in space.

The system framed by Tycho, which, at the present time, is preserved only as one of the curiosities of scientific history, is a combination of the Copernican and Ptolemaic systems. The earth is by it supposed to be stationary; the planets revolve about the sun; but the sun, accompanied by all these bodies, revolves around the earth, in exactly a similar manner to the moon. Whilst it explains the various motions of the sun, moon, and planets, it must, at the same time, be regarded as very unphilosophical, and as only fitted for the prejudices of the age.

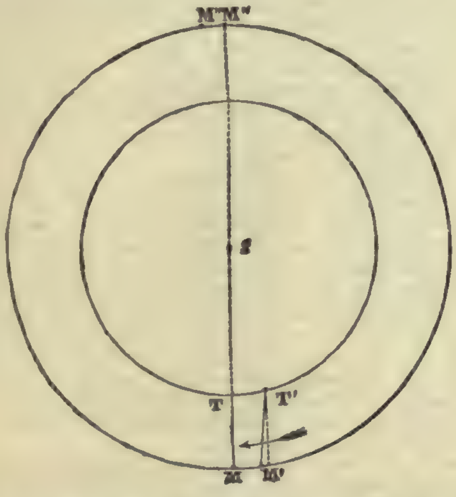
Stations and Retrogradations.—The various progressive and retrograde motions of the planets, explained by the cumbrous machinery of the Ptolemaic system, on the supposition of the earth being stationary, follow naturally from the circular motions of the planets themselves, combined with that of the earth, in the true system of the universe, and, indeed, led the illustrious Copernicus to his great discovery. The two inferior planets, Mercury and Venus, move round the sun in less time than the earth; and if S be the sun (Fig. 61), V, Venus at inferior conjunction, and T the earth, the planet will pass through an arc V V′ of its orbit, in the same time that the earth passes through the smaller arc T T′ of its orbit. The planet, therefore, has appeared to move quicker than the earth in the direction of the arrow, which is in a retrograde direction. If, however, Venus be found at superior conjunction with the sun, and it passes from V″ to V‴ in the same time that the earth moves from T to T′, it will appear to move in a contrary sense to the arrow, and, therefore, direct. Between these two points, the planet will, of course, be animated with different rates of motion, and, at certain times, will appear quite stationary in the heavens. If the earth were fixed at T, the planet would appear stationary at the point of greatest elongation. As it is, however, the planet and earth must be both moving at the same rate, and in the same direction, in order that the former may appear stationary. The duration of the retrograde movement of Venus is only three weeks before and after the time of inferior conjunction.

Fig. 61.



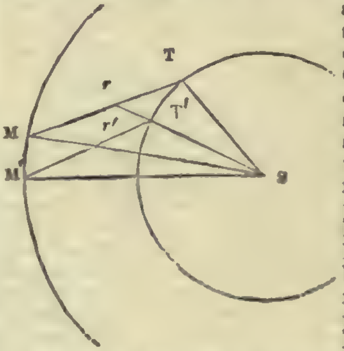
The superior planets bear the same relation to the earth as the latter does to Venus and Mercury; so that when the earth is stationary for any of those planets, the planets appear in the same manner to be stationary to the earth. The progressive and retrograde motions of the superior planets may, therefore, be explained in nearly the same manner. If M be Mars in opposition (Fig. 62), as the earth moves more rapidly in its orbit than the planet, it will pass from T to T′, whilst Mars only moves from M to M′; so that whilst it was first seen in the direction T M, it will afterwards appear in the direction T′ M; or, in other words, the planet will appear to have fallen back in the manner indicated by the arrow, or in retrograde direction. When the planet is in conjunction with the sun, or at M″, whilst it passes from M″ to M‴, the earth will have moved from T to T′ in the contrary direction, so that the planet will appear to move in a direct sense, or according to the order of the signs. Thus all the superior planets are retrograde at the time of opposition, because their motion is slower than that of the earth; and at the time of conjunction they are always direct, because moving in the contrary direction. Between those points the planet will appear as stationary at that part of its orbit, when the earth, passing from T to T′, will be so oblique in regard to M M′ that the lines T M and T′ M′ will be parallel; and as the distances of the stars are immense in comparison with any of the planets, the lines T M and T′ M′ will be directed to the same point of the heavens, and the planet will appear exactly in the same position in respect to any fixed point or star.

Fig. 62.



To Determine the Elongation of a Planet when Stationary is a difficult problem, when the inclinations and ellipticity of the orbit are taken into consideration; but as at the present time it is more a matter of curiosity than interest, the orbit may be considered as circular without much error, and situated in the same place as the ecliptic. Let S (Fig. 63) be the sun, T the earth, and M the position of Mars near its stationary point; then, when the earth passes from T to T′, the planet will move from M to M′, and the lines M′T, M′T′, may be considered as parallel, as before explained. Consequently M′ T′ S—M T S = M r S —M T S = T S T′; and in the same manner S M r — S M T′ = S r' T′— S M′ T′ = M S M′. Whence it follows that the angular variations at T and M are proportional to the angles T S T′ and M S M′; or as the angular velocities are inversely as the periodic times, then S $M^{\frac{3}{2}}$: S $T^{\frac{3}{2}}$; and putting a = S M, and S T = unity, the sines of the angles M and T will be in the ratio of their opposite sides, or as a ; 1, and the contemporary variations of the angles will be as their tangents; or,

Fig. 63.



$$\frac{\text{sin. T}}{\text{cos. S}} : \frac{\text{sin. M}}{\text{cos. M}} :: a^{\frac{3}{2}} : 1,$$

$$\text{whence } \frac{\text{sin. T}}{\sqrt{(1-\text{sin.}^2 \text{T})}} : \frac{\text{sin. M}}{\sqrt{(1-\text{sin.}^2 \text{M})}} :: a^{\frac{3}{2}} : 1$$

$$\text{And sin.}^2 \text{T} = \frac{a^3-a^2}{a^3-1} = \frac{a^2}{a^2+a+1},$$

$$\text{and sin. T} = \frac{a}{\sqrt{(a^2+a+1)}}$$

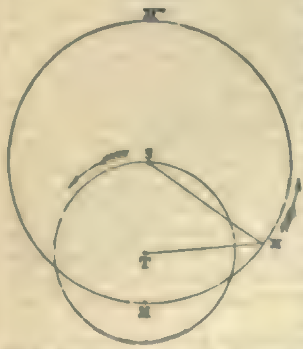
For an inferior planet the M and M′ may represent the place of the earth, and T and T′ the positions of Mercury and Venus. Thus if the mean distances of the earth and Venus be taken at 100,000 and 72,333, then sin. T = 0·48264, the sine of 28° 51′, or at this angle of elongation from the sun, the planet is stationary.

To determine the time when a planet is stationary, the time of conjunction or opposition must be known. If m and n are the daily motions of the earth and planet, then $m-n$, or $n-m$ is the daily variation at the angle T S M, as it is a superior or inferior planet; then,

$$\left.\begin{matrix} m-n \\ n-m \end{matrix}\right\} : \angle \text{TSM} :: 1 \text{ day} ; \frac{\angle \text{TSM}}{m-n}$$

Phases of the Superior Planets.—None of the superior planets show the same succession of phases as Mercury and Venus; and only one of them, Mars, is near enough to the earth to show any appearance of departure from a full disc, the illuminated hemisphere being turned nearly always towards the earth. The point of its orbit, at which Mars will appear most gibbous, will be at M (Fig. 64); at the points M′ and M″, when at opposition and conjunction, its illuminated hemisphere will be wholly turned towards the earth. The diminution of its disc from a complete circle is, however, very sensible; and a small table of its defective illumination is given in the *Nautical Almanac* for every month in the year. It is wholly insensible in Jupiter and Saturn; and to reduce the observation of the limb to the centre, it is only requisite to apply the semi-diameter.

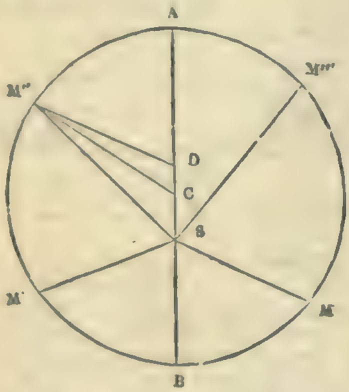
Fig. 64.



Kepler's Laws.—Copernicus, in explaining the apparent motions of the planets in the heavens, confined himself, it will be seen, to circular motion, and all the planets were supposed to revolve round the sun, as a centre in circular orbits, which he considered the most natural of any. It has already, however, been seen that the sun's motion in its orbit is not equable; nor will it be explained by supposing the earth to revolve in a circle, of which the sun is placed eccentrically to the orbit, and that it can only be accounted for by supposing that our planet moves in an ellipse, of which the sun is placed in one of the foci, whilst rejecting all the other cumbrous hypotheses of the Ptolemaic system. In order to explain the various inequalities caused by this elliptic motion, Copernicus was forced to retain the supposition of epicycle and eccentric. The observations of Tycho Brahé, in the hands of the immortal Kepler, however, threw off this last obstruction, and discovered not only the true figure of the orbits of the planets, but likewise other wonderful laws in relation to them.

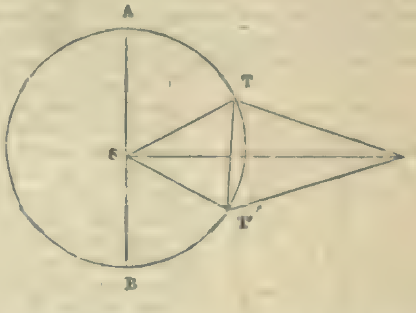
In examining the orbit of Mars, he endeavoured to determine the eccentricity in respect to the sun, supposing it to be a circle, in the following manner:—Supposing that D (Fig. 65) was the point round which the motion of the planet was uniform, C the centre of the circle, and S the position of the sun, M, M′, M″, M‴, being four places of Mars at opposition, he endeavoured to determine the angles A D M″, A S M″, in such a manner that the four points, M, M′, M″, M‴, were situated on the circumference of the circle with the centre C. By assuming the distance S D, and the two angles, A D M″, and A S M″, he calculated, trigonometrically, all the other parts of the figure, in order to determine if the four angles at S made up 360°, and the points, S, C, and D, on the same straight line. After seventy most laborious trials, he at length arrived at one which agreed so well with observation, that out of twelve oppositions, none of them differed more than 1′ 47″ in longitude; and he thought that Tycho's observations might be in error by that, or even a quantity of 2′. When compared with observations of the planet out of opposition, it, however, became apparent that this orbit would not answer, the errors of longitude sometimes amounting to 8′, which he was persuaded could not be due to the observation of Tycho. He was consequently led to doubt if the observations could be satisfied by any circular hypothesis, and further computation served to confirm his doubts.

Fig. 65.



Supposing S to be the sun (Fig. 66), M, Mars, and T and T′, two positions of the earth, when Mars returned to the same part of its orbit, whilst the earth was at T, he determined by observation the angle M T S, and when at T′, the angle M T′ S. The distances T S, T′ S being known, and the angle T S T′, the side T T′, and the angles S T T′ and S T′ T will be known, and consequently the angles M T T′ and M T′ T. The side M T may be found in the triangle M T T′, and finally the side M S in the triangle M T S, or the distance of Mars from the sun. By this method Kepler determined the distance M S, at perihelion and aphelion, the former of which he found to be 138500, and the latter 166780; the mean distance being 152640; that of the earth being supposed to be 100000. In the same manner Kepler determined three other distances of Mars, from observation at different parts of his orbit, which he found to be 166255; 163100; 147750. But by calculating these distances on the supposition that the mean distance was 152640, and the eccentricity 14140, the distances were found to be respectively 166605; 163883; 148539. Whence the errors were found to be 350, 783, and 789. The true distances of Mars from the sun were, therefore, shorter than those calculated; and as the line of apsides and the perihelion and aphelion distance were truly known, it followed that the orbit was of an oval form; and the ellipse being the simplest of all ovals, he found that this was the true curve. By examining Mars at many other parts of its orbit, he found that it agreed closely with this supposition. Thus the first law of Kepler was discovered by infinite labour and sagacity, viz., *that the planets revolve about the sun in elliptic orbits, the sun being situated in one of the foci.*

Fig. 66.



The second great law of Kepler was likewise discovered by him from observation, by comparing the velocity of the planets in their orbit with their distances from the sun, or rather the areas of the sectors, and the arcs included between the radii vectores bounding them. He found that when at their apsides, the velocity of their motion was inversely as their distances from the sun, or that the *planets describe equal areas in equal times* at those points; and he was of opinion that this was true at all parts of the orbit, although he could not prove such to be the case. This, however, has since been proved to be a necessary law, and to follow consequently from the doctrine of universal gravitation.

The third great law of Kepler connects the distances of the planets from the sun, with the time in which they make a complete revolution about that luminary. The further a planet is removed from the sun, the slower he found its motion on comparing it with the absolute motion of the earth. The distance of Saturn from the sun being nine and a-half times that of the earth; consequently if their rates of motion were alike, the period in which Saturn would perform a complete revolution would be nine and a-half years. It is apparent that the periodic times do not, however, increase according as the squares of the distances, as in that case the periodic time of Saturn would be upwards of ninety years, whilst in reality it is about thirty. After numerous and laborious trials, Kepler at length found out the remarkable analogy that *the squares of the numbers representing the periodic times were exactly proportional to the cubes of their mean distances from the sun.* Comparing the earth and Mars, for example, we find that $(365 \cdot 2564)^2 : (686 \cdot 9796)^2 :: 100000^3 : 152369^3$. All of those remarkable laws were afterwards found by Sir Isaac Newton to be necessary consequences of the laws of gravitation; and thus what was deduced by Kepler from observations on one particular planet, and at one particular point of its orbit, was verified, and found to be universally applicable to the whole solar system, and to form a connecting link between all its members.

From the third law of Kepler the mean distance of a planet from the sun can be readily calculated, and with much less difficulty and more accuracy than by direct observations. The periodic time may be found by observing the time of the planet's passage through the nodes, and by observing the intervals between the ascending and descending nodes, and the descending and ascending (which, if the orbit be an ellipse, will not be exactly alike); a value of the eccentricity may be obtained from the greatest difference, the true and mean anomalies, or the equation of the centre. To determine the mean distance—supposing t to be the periodic time of the earth, and r that of the planet, D being the mean distance of the earth, and $\triangle$ that of the planet, we have

$$t^2 : r^2 :: D^3 : \triangle^3, \text{ whence } \triangle = D \times \left(\frac{r}{t}\right)^{\frac{2}{3}}$$

The longitude of the planet in the nodes, or the longitude of the node, may be determined from two heliocentric latitudes and longitudes.

Telescopic Appearance of the Planets.—The planet Mercury appears under all the phases of the moon and Venus, but is seldom visible in our latitudes to the naked eye, as, when most favourable for that purpose, there is always a strong twilight, and it is situated too near the mists of the horizon. At its nearest approach to the earth, or at the time of inferior conjunction, its apparent diameter amounts to 12″, and at its superior conjunction this decreases to 4″; at its mean distance from the earth its apparent diameter is 6″·7. Its diameter is upwards of 3,000 English miles; but observers differ considerably as to its apparent diameter as measured by the micrometer. In such a small body as this, it is almost impossible to perceive anything beyond the mere form of the disc by means of the most perfect and powerful telescopes; and even these are sometimes rendered useless by the scintillation and bad definition of the planet. The only observer who has followed this planet with the requisite attention, is Schroeter, who, by means of powerful reflecting telescopes, was able to perceive that the crescent was not always regular, but that sometimes one horn was blunter than another. This, he naturally considered, was due to their irregularities on the surface of the planet—a mountain, or chain of mountains, situated in the southern hemisphere, intercepting the rays of the sun from proceeding onwards. Schroeter endeavoured to make use of this appearance in determining the time of rotation of the planet. He attentively observed the form of the disc, and the degree of bluntness of the southern horn at one particular time; and the changes which it underwent in a few hours were found to be plainly perceptible. When the planet reappeared on the following day, or any number of days afterwards, the time at which its disc presented the same form was again recorded, and so on for any length of time. Schroeter concluded, from these observations, that the time of rotation of the planet on its axis must be 24h. 5m.; and by taking the extreme times of observation, and dividing the interval by the number of revolutions, he concluded it must have one rotation on its axis in the space of 24h. His assistant, M. Harding, on one occasion, perceived a faint spot on the disc, and, by following it attentively, he arrived at the same conclusion as Schroeter. The latter attempted to arrive at a knowledge of the height of the mountains by measuring the deficient part of the horn; and his observations go to prove that some of those were upwards of twelve miles in altitude, which is three times that of any on the earth, and, compared with the actual size of the two planets, is out of all proportion. During the passage of Mercury over the sun's disc on November 9, 1802, Sir W. Herschel could not perceive any indications of an atmosphere, nor the least departure from the circular shape in its form. Even if it were as much compressed at the poles as the earth, the ellipticity of its outline would not, however, be visible in such a small body. Many observers have perceived a dusky ring of considerable extent surrounding the planet at those times, and others have noticed a bright spot on its disc on the same occasions; but this has been explained on optical grounds. During the passage, Mercury appeared as a very dark spot, and considerably more so than any of the spots which appeared on the sun at the time. The transits of Mercury over the sun's disc, which will occur during the remainder of the present century, are as follows:—

1868, November 4. 1891, May 9.
1870, May 6. 1894, November 10.
1881, November 7.

Those in 1881, November 7, and 1891, May 9, are invisible in the northern parts of Europe.

Venus.—The phases which Mercury undergoes are seen in a much more perceptible manner in those of Venus, from the great size and brilliancy of the planet at those times. The same appearance of the different sharpness of the horns has likewise been noticed; and from these circumstances Schroeter endeavoured to determine the period of rotation in the same manner as pursued by him in Mercury, and apparently with more chance of success. Not only was the southern horn noticed to be very blunt, but a detached point of light was perceived by him, which he concluded to be the summit of a mountain illumined by the setting rays of the sun. By numerous consecutive observations on this planet (continued for many years), Schroeter came to the conclusion that it performed a rotation on its axis in the space of 23h. 20m. 59·04s., or, in round numbers, 23h. 21m. Previous to this time, Cassini had perceived, in the clear sky of Italy, a small bright spot on the surface of the planet; by the observation of which, on several consecutive mornings, in the summer of the year 1667, he arrived at the conclusion that its period of rotation was 23h. 21m. or 22m. A far more extensive series of observations on several *dusky* spots was made during 1726-'27 by Bianchini, at Rome, with excellent telescopes. The period of rotation, as determined by this observer, from those numerous spots, was very different from those of Cassini and Schroeter, amounting to 24h. 8m. It had been suspected, by recent observers, that a mistake arose from the spots so closely resembling each other; and as they were only observed during the evenings, it was impossible to recognise them by their appearance alone. The late De Vico has reobserved all the effects perceived by Bianchini (from which it is certain that they were not of the fleeting, cloudy nature, suspected by Herschel and Schroeter); and from some thousands of micrometrical observations, made both during the day and evening, he concluded that it performs its rotation on its axis in 23h. 21m. 21 9s. The axis of rotation was supposed by Cassini, Schroeter, and Bianchini to be inclined as much as 70° to the pole of the ecliptic. The latter determination of De Vico, although it does not show an inclination so considerable as this would prove to be, shows one nearly twice as great as that of the earth—*viz.*, 53° 11′. Sir W. Herschel, on one or two occasions, perceived spots, but they were too faint and uncertain to give any idea of the time of rotation. He was never, however, able to perceive the bluntness of the southern horn observed by Schroeter; which is the more remarkable, as it has plainly been seen by most observers who have examined it; and Maedler has perceived the quick changes which they undergo. Herschel, however, confirmed other appearances detected by Schroeter—viz., the brightness of the outer or circular edge of the crescent, and the dimness of the elliptical boundary. Herschel considered that the atmosphere of Venus was like our own, refracting and reflecting light; and as we view the circular part more obliquely than the elliptical boundary, and consequently a greater thickness of the atmosphere comes into view, this explanation would be more in accordance with the appearances observed by Schroeter, where the outer bright portion gradually melted into the interior dusky region that was observed by Herschel himself—the outer bright part being distinctly separated from the inner, and appearing as a bright colour.

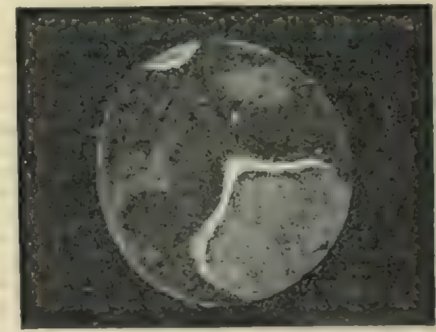
In observing Venus when near its inferior conjunction, Schroeter perceived, in the very slender crescent seen on such occasions, a very faint light stretching beyond the pointed horns, which, as seen in an ordinary telescope, would be the extreme points of the illuminated semicircle; and Herschel confirmed this curious fact. This twilight affords further proof of the atmosphere of Venus, which Schroeter supposed to be of about the same density as that of the earth, having concluded that the horizontal refraction was 30½′. In the inferior conjunction of 1849, Professor Maedler plainly perceived this faint extension of the horns; and he found, for the horizontal refractions, quantities varying from 39 to 48 minutes, but whose most probable value was 43′·7. It is consequently about one-sixth greater than that of the earth's atmosphere. From appearances noted during the transits of Venus across the sun's disc, it would appear probable that the atmosphere is very considerable. The two next transits of Venus will occur on December 8, 1874, and December 6, 1882.

The apparent diameter of Venus, at its mean distance, is 16″·9; but, at the time of inferior conjunction, this increases to 62″, and at the time of superior conjunction, decreases to 9½″. It resembles the earth, in volume and density, more than any other of the planets. Its diameter is 0·985 of that of the earth; its volume 0·957, and its density 0·923. A body which would weigh one pound on the earth, would weigh 0·91 on the surface of Venus. Light and heat would be nearly twice as great at Venus as on the earth. No compression at the poles has been perceived in Venus.

MARS.—This planet shines with a ruddy and dusky light, quite different from any of the other heavenly objects. When at favourable opposition, or when near its perihelion and opposition at the same time, it is a very bright object, and so different from its ordinary appearance, that it has been frequently mistaken for a new star. Its apparent diameter, at its mean distance, is 5″·8; but at the time of opposition this can increase to 23″, and at conjunction decrease to 3″·3. The true diameter of this planet is 0·519 of that of the earth, and consequently its volume is only 0·140 of that of this earth. Its density, compared with that of the earth, is 0.948. A body which would weigh one pound on the earth, would only weigh half a pound in Mars. The light and heat which it receives at its mean distance from the sun is 0·43 of the earth.

When examined with powerful telescopes, many dark spots are perceived on the surface of this planet, which then loses much of that red colour so apparent to the naked eye. These dusky portions, it has been found, are quite fixed and constant in their position, and they have thus been supposed to be the seas and continents of this planet (see Fig. 67). But not only have the land and water become visible on its surface, but likewise its climate; for we perceive, at its north and south poles, bright white patches of light, which are naturally held to be the snows of Mars collected in an immense mass. This conjecture is considerably strengthened by the fact, that when, by the position of the axis of Mars in respect to the sun, those luminous spots are turned towards that luminary, they diminish in size rapidly. In 1781, Sir W. Herschel noticed that the southern white spots were extremely brilliant and extended; this was after a winter of a year's duration on this part of the planet. In 1783 this same spot was very small, but the sun had continued for nearly eight months above the horizon of this part, and melted it away. The existence of an atmosphere to this planet is apparent from other considerations, as, at various times, dark, extraneous, cloudy patches have been noticed, which have obscured the planet and hid the spots on its surface. When the snow melts away, a dusky and densely-clouded atmosphere appears to hang over the planet at those parts.

Fig. 67.



The rotation of the planet has been determined by several observers; and from the fixed nature of the spots observed, and the good definition with which they appear under favourable circumstances, it has been found with considerable accuracy. Cassini, in 1666, determined it to be performed in 24h. 40m.; and, in 1704, Maraldi repeated the observation, and found the period of rotation as 23h. 39m. Herschel made many observations relative to the telescopic appearances of this planet. He determined the sidereal rotation as 24h. 39m. 21·7s., and the synodical at 24h. 38m. 20·3s. The equator of Mars he found was inclined at an angle of 28° 42′ to his orbit, and the node was directed to the constellation Sagittarius. The seasons on this planet would not, therefore, be much different from those on the earth; but, on account of his great eccentricity, their duration is very different.

The compression of Mars at the poles appears to be considerably greater than that of the earth, which, considering his small diameter, and that the rotation and density are nearly similar to those of the earth, is rather remarkable. It does not appear, however, to be so considerable, as determined by Sir W. Herschel, according to whom the ratio of the polar and equatorial axes was as 98 to 103, or as 15 to 16. Schroeter estimates them at 80 to 81. According to M. Arago, it is more than $\frac{1}{30}$. Harding has noticed that the equatorial sides of Mars sometimes appear very bright; and he thinks that the irradiation caused by this circumstance produces discrepancies in the measurement of the equatorial and polar diameters.

THE ASTEROIDS.—The present century opened with the discovery of four small planets—Vesta, Juno, Ceres, and Pallas, situate between Mars and Jupiter; called asteroids (the appearance of stars), because of their stellar aspect under telescopic examination. The order of their discovery, with the names of their discoverers, is as follows:—

Ceres . .	January 1, 1801, by M. Piazzi, of Palermo, in Sicily.
Pallas . .	March 28, 1802, by M. Olbers, of Bremen, in Saxony.
Juno . .	September 2, 1804, by M. Harding, of Lilienthal, in Hanover.
Vesta . .	March 29, 1807, by M. Olbers, of Bremen.

These bodies are sometimes styled planetoids, as more expressive of their character, and extra-zodiacal planets, because their orbits are not confined within the zodiac like those of other planets. They are exclusively telescopic objects, and require the very best instruments to be caught, with the exception of Vesta, which, under favourable circumstances, has been seen by the naked eye.

Vesta, before which, however, there are others in the order of succession in the system, is at the mean distance of 225,000,000 of miles from the sun, and performs a revolution in 1,325 days, somewhat more than three years and a-half.

This planet is much brighter than its compeers, and appears like a star of the fourth or fifth magnitude.

Juno is at the mean distance of 254,000,000 of miles, and accomplishes an orbital revolution in four years and a hundred and twenty-eight days.

Juno has a very eccentric orbit, varying in her distance from the sun to the extent of 130,000,000 of miles. This eccentricity has a remarkable effect upon her motion, for she goes through that half of her orbit which is nearest the sun in nearly half the time that she travels through the remainder. The planet has a reddish colour, and appears like a star of the eighth magnitude.

Ceres, following Leda, revolves round the sun in about four years and two-thirds, at the mean distance of 263,000,000 of miles.

The telescope reveals Ceres as a ruddy star of the eighth magnitude, under circumstances which leave the impression of an extensive atmosphere.

Pallas is at nearly the same mean distance from the sun as Ceres, and has nearly the same period of revolution.

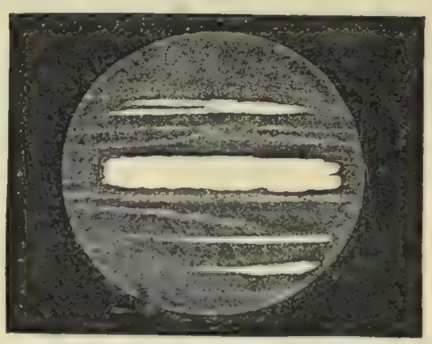
The discovery of the asteroids was not an accidental circumstance, but the result of a search conducted upon the presumption that the harmony of the solar system required the presence of a planetary body between Mars and Jupiter. It was observed, that the distance between the orbits of Mars and the earth is about double that between those of the earth and Venus, or 50,000,000 of miles; whereas, between the orbits of Mars and Jupiter there is the tremendous interval of 340,000,000 of miles, nearly two-and-a-half times the whole distance of the former from the sun. Astronomers, therefore, became thoroughly imbued with the notion that an undiscovered planet existed in the gap; and commencing an active search of the heavens, the asteroids were met with in the vacancy.

On the ground of these peculiarities, it has been surmised that these inferior bodies have diverged from a common node, and therefore originally formed a single large planet, which some mighty convulsion shattered. This bold hypothesis, first started by Olbers, has received a very general sanction. It obtains evidence from the powerful explosive forces in action in the interior of our own globe, to which its volcanic vents act as safety-valves; from the phenomena of meteoric showers, respecting which we have no supposition better than that they are *débris* which the earth encounters in its orbit, fusing upon contact with its atmosphere.

Of the telescopic appearance of this group of planets, now increased to above eighty in number, it would be difficult to say anything. A keen eye can detect, in a good telescope, the difference between one of these asteroids (when not less than the eighth or ninth magnitude) and a star of the same degree of brightness; but this perhaps is the extent of the power of the telescope. Their diameter is very doubtful. Some observers consider them as a few hundred miles in circumference, others as many thousands. The hazy appearance surrounding Ceres, which was noticed by Herschel and Schroeter, whence they surmised that the asteroids were partly of a cometary nature, has not been noticed by modern observers, and may be due to the imperfection of their reflecting telescopes.

JUPITER.—This is one of the finest planetary objects to which the telescope can be directed; and even with a small instrument, its disc, band, and satellites can be plainly seen; and at each opposition there is but little difference in the brightness. Whenever viewed with a telescope, the singular appearance which its disc presents, being always surrounded by two or more dark belts or bands, cannot fail to strike the beholder, and the changes to which they are subject are still more curious. The equatorial portions are generally the brightest, and it is at this part the bands are most distinctly visible. They are generally directed parallel to the equator; but this is not always the case. The polar portions are for the most part darker than any other, but it is only on rare occasions they are equally so; and indeed the whole cloudy covering of the planet is of such a shifting and unsteady character, that it may almost be said to vary from night to night. In addition to the dark bands, there are occasionally to be seen, on the disc of the planets, dark spots of irregular form, which are subject to the same changes as the bands, though they are frequently of long duration, and remain for months together at nearly the same place. A telescopic view of Jupiter, and the appearance of its surface under favourable circumstances, will be seen at Fig. 68.

Fig. 68.



By attending to the motions of those spots, it has been found that Jupiter, like the planets hitherto described, rotates upon its axis; but the length of its day is much shorter than that of the planets in the vicinity of the earth. It is rather difficult to tell the exact duration, for the spots themselves have got a motion on the disc; and Herschel found that the time of rotation appeared to be sometimes 9h. 55m. 40s., at other times 9h. 54m. 53s., from observations of the same spot; whilst, by a different spot, it was sometimes 9h. 51m. 45s., and at other times 9h. 50m. 48s. As the equator of the planet is but very slightly inclined to the plane of its orbit round the sun, it follows that there is but very little variation of the seasons; so that during its long year, the length of the day and night varies very slightly, either being at any latitude about five hours in length.

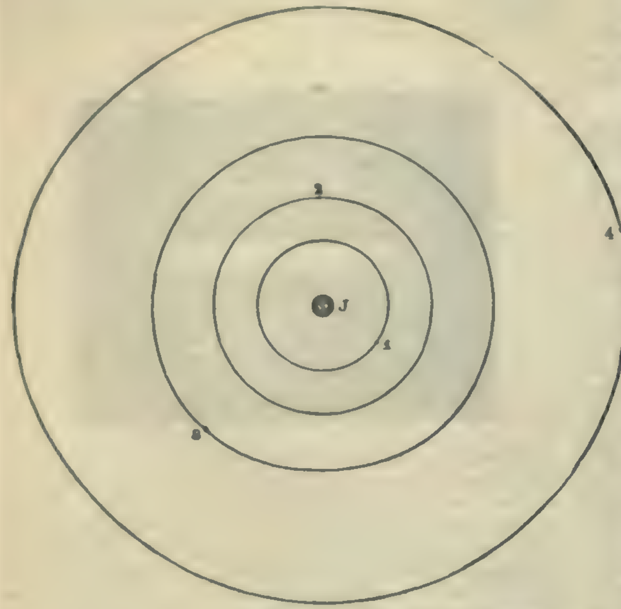
From the rapid motion which those spots undergo near the equator, Herschel thought it was not improbable that at these portions of the planet's disc there existed currents of air similar to the trade winds, the effect of which would likewise be to form the loose vapours on its surface into the parallel belts. From Herschel's observations on these accidental clouds—as he was of opinion the spots were—he considered that they were occasionally driven along at the rate of ninety-six leagues per hour, a velocity far exceeding that of our most violent hurricanes.

It might be thought, at first sight, that the dark bands represented in Fig. 68, were the clouds, the overcast portion of the planet's disc, and the brighter belts were parts of its surface. Herschel, however, considered that the contrary was the case, and in this he is followed by all modern observers. He supposed that the more brilliant portions were the zones, in which the atmosphere was charged with clouds, the darker parts those on which it was quite clear and serene. The latter allow the solar rays to pass through to the surface of the planet, where the reflection being less powerful than from the clouds, less light was consequently returned.

The disc of Jupiter is considerably flattened at the poles, and bulged out at the equator. This is immediately seen when the eye is directed through a telescope to the planet; and the extent to which it occurs will be seen by the diagram (Fig. 68). The belts appear always parallel to the greater axis.

The four satellites of Jupiter are constantly seen in the same direction, and almost in the same straight line; and the motions of the two inner ones round their primary are very rapid. They almost describe circles around the planet; the orbits of the third and fourth satellites are, however, slightly eccentric. The following table shows their mean distances from the centre of the planet, expressed in the equatorial diameter of Jupiter. The diagram (Fig. 69) represents the relative orbits of the four satellites in exact proportion.

Fig. 69.

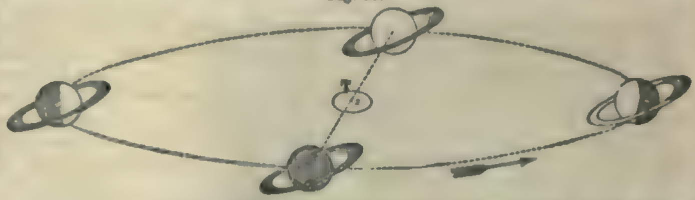


	Mean distance.	Period of revolution.
1st satellite . .	3·03 . .	1·77 days.
2nd „ . .	4·81 . .	3·55 „
3rd „ . .	7·68 . .	7·15 „
4th „ . .	13·50 . .	16·69 „

The laws of Kepler are as strictly fulfilled in the passage of the satellites round their primary, as in the case of the motions of the planets round the sun. They are subject to eclipses and occultations, and are sometimes seen projected on the limb in the form of small white spots. As the cone of the shadow is of considerable length, Jupiter being at a great distance from the sun, and the satellites comparatively near it, it follows that, at every revolution, the first three satellites are immersed in the shadow, and it is only the fourth which occasionally escapes. The use made of the eclipses of Jupiter's satellites, is represented by A, of the engraving at the end of Chapter IV., page 940. The centre globe being our earth, two lines are drawn from places on its surface, forming an angle, and meeting at Jupiter; an observer at each place marks the time at which the satellite enters the cone of the shadow. Assuming the time at the nearest line to be ten, and the farthest eight o'clock, the two hours = 30° show the one place to be distant 30° from the other. These observations of the eclipses give the readiest means of determining the longitude of any place at which they are observed. This accurate observation of the moment of disappearance depends partly on the excellence of the telescope used, and partly on the keenness of sight and experience of the observer. When the satellites pass between the sun and Jupiter, they produce solar eclipses; and the shadow of the satellite, in addition to the satellite itself, may be seen slowly traversing the disc of the planet, in a powerful instrument.

Saturn.—The most wonderfully constituted body of the solar system is Saturn. Surrounded by numerous rings, of different form and brilliancy, and eight satellites, with the varied bands which, as in the case of Jupiter, cross the ball of the planet, it forms the most curious of telescopic objects. The ring was first detected by Huygens, by means of his powerful telescopes, and he was likewise the first to perceive the brightest of the satellites. He explained the various appearances at different times by taking into account the obliquity of the ring, and the parallelism which it retains at all parts of its orbit; which is, indeed, similar in all respects to the change of seasons on the earth. This will easily be seen by the diagram (Fig. 70), where S is the sun, and T the earth, and the outer circle the extent of Saturn's orbit. It is plain, that to an observer at T, the upper side of the ring, and the northern pole of Saturn, will be visible, as at Fig. 71. At the opposite part of the orbit of Saturn, the *southern* pole of the planet and the *under* side of the ring is that which will be illuminated by the sun, and seen by an observer on the earth. As Saturn passes from the one to the other of these points, the ring will become gradually less open; and at the points between the two it will only be visible by the illuminated edge, as in Figs. 72 and 73; whilst at other times, the unillumined plane of the ring will be turned to an observer at T, and the outer edges of the ring will not be visible, as at Fig. 70. This disappearance of the ring will take place twice during every revolution of Saturn, and it may be compared to the position of the earth's axis at the times of the equinoxes in respect to the sun, both poles coming into sight, for the ring of Saturn surrounds the equator of the plane.

Fig. 70.



The ring of Saturn is seen, with an instrument of moderate power, to be divided into two parts; but when a stronger telescope is made use of, under favourable circumstances, the outer portion is found to be divided into two likewise; thus making three in all, as seen in the diagram, Fig. 71. Some observers have been able to detect other divisions in the bright ring when it is most open. The most remarkable appearance detected of late years is, however, the dusky ring, situated between the ball and the bright ring, and adjoining the latter. The existence of this curious appendage is now placed beyond doubt, by the testimony of different observers; and it appears to be of a misty, semi-transparent nature, as the ball of the planet can be seen through it. From the observations of the elder Herschel, it would follow that the globe of Saturn revolves on its axis once in 10h. 16m., and that the ring makes a rotation in its plane once in 10h. 32m. 15s. Both move in the direction of the other planetary bodies—viz., from west to east.

Fig. 71.

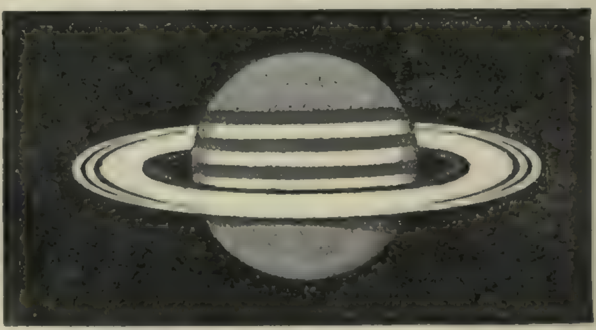
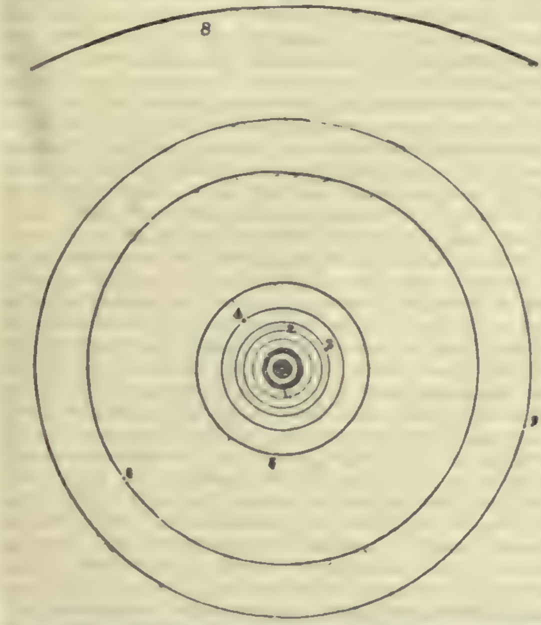


Fig. 72.

Fig. 73.

The system of satellites which surround Saturn and his ring will be seen from the accompanying diagram (Fig. 74). Of these the brightest is that discovered by Huygens, and resembles a star of the ninth magnitude.

Fig. 74.



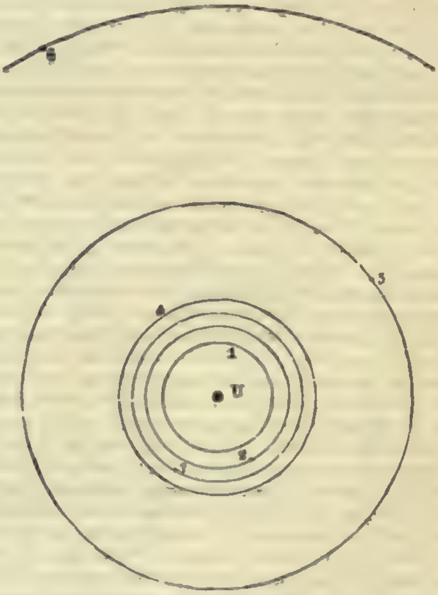
It is the sixth in the order of distance, counting from the planet, and its orbit is the best determined of any of them. The third, fourth, fifth, and eighth, discovered by Dominique Cassini, are likewise comparatively bright; but the first and second, discovered by Herschel, and the seventh, discovered by M. Sassel in 1848, are the faintest of telescopic objects, and require both great light and high power to be seen at all. The orbit of the eighth satellite is considerably inclined to the equator of the planet; those of the other seven lie nearly in the same plane as the ring. The mean distances, expressed in equatorial radii of the planet, and the periods of their revolutions, are seen from the following table :—

	Mean distance.	Time of Revolutions.
1st Satellite	3·35	0·94
2nd „	4·30	1·37
3rd „	5·28	1·89
4th „	6·82	2·74
5th „	9·52	4·52
6th „	22·08	15·94
7th „	27·78	22·50
8th „	64·36	79·33

Uranus and Neptune.—Of the telescopic appearance of the two exterior planets, Uranus and Neptune, very little can be said. The former appears in the telescope as a star of the sixth magnitude, of a planetary aspect, and with the appearance of a well-defined disc. The latter is of the same faint blue colour, but the disc is more difficult to be perceived; but, if compared with a neighbouring star of the same magnitude, the difference in their definition is more apparent. Herschel considered that he was able to detect a slight ellipticity in the figure of Uranus, from which it appeared that its axis is very little inclined to the plane of the elliptic, and that consequently the sun is hid for many years from its poles.

The satellite system of Uranus (according to Herschel) is represented in Fig. 75; but only four of those satellites, two of which do not agree in their periods with that set down by Herschel, have been perceived by M. Sassel; and he seems inclined to believe that only that number exists. The second and fourth have frequently been seen, and their orbits are pretty well determined by the observations of Sir J. Herschel and M. Lamont, of Munich. The orbits of those satellites are very greatly inclined to the plane of the elliptic, the angle between the two being nearly 80°. The periods of revolutions and mean distances of the several satellites are seen from the following table :—

Fig. 75.



	Mean Distances.	Time of Revolutions.
1st Satellite	13·12	5·89
2nd „	17·02	8·71
3rd „	19·85	10·96
4th „	22·75	13·46
5th „	45·51	38·07
6th „	91·01	107·69

The only satellite to Neptune which has yet been perceived is that discovered by M. Sassel, and it is represented in Fig. 76, on the same scale as that of the satellite systems of Jupiter, Saturn, and Uranus. It will be seen that it differs but slightly from that of the moon to the earth (Fig. 77) in the dimensions of its orbit. The period of revolution is 5·87 days, and its orbit is inclined at an angle of 35° on the elliptic. It is excessively minute, and has hitherto been seen only by Messrs. Sassel and Bond.

Fig. 76.

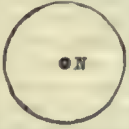
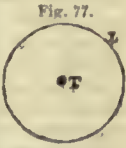


Fig. 77.



Having thus described the leading phenomena presented in the planetary system, we shall next proceed to consider the constellations, and the fixed stars generally.

CHAPTER VI.

STELLAR ASTRONOMY.

In no other subject to which the mind of man can be directed, is the grandeur so imposing or inexhaustible as in that relating to the distant regions of space known as the sidereal heavens. This is not merely made manifest by the help of gigantic telescopes, and other artificial appliances which the intellect of man has devised; even with the naked eye, we can perceive how multitudinous are the brilliant specks of light so lavishly scattered in every direction, and congregated in such numbers. In the constellations of Orion or Taurus, however, in the Hyades, and, above all, in the great southern Magellanic Clouds, or the wonderful zone of the Milky Way, the stars are so numerous that they cannot be separately seen by unassisted vision. But although the prospect is so magnificent, and apparently so boundless, even in the most simple and universal view, it becomes infinitely more so when the depths of the heavens are sounded by means of the telescope. When we consider, furthermore, that our sun is but one out of those myriads of objects, shining by their own light, and that each is performing in its sphere the same important functions to worlds receiving its radiated heat and light, which the sun performs for us, we become more and more deeply impressed with the boundless extent and variety of celestial objects.

Viewing the heavens with the unassisted eye, we perceive that the stars are of very different degrees of brightness; and, for the purposes of classification, astronomers have chosen to divide those visible to the naked eye into six degrees of lustre; the faintest, or those just seen by persons of ordinary eyesight, being termed stars of the sixth magnitude. This classification, however, gives but a rude approximation to the truth, as it would be difficult to find a number of stars of exactly the same degree of brilliancy, while it would be easy to count twenty or thirty stars of perceptibly different degrees of brightness. With the help of the telescope, ten additional degrees of magnitude—each ranging from the seventh to the sixteenth magnitude—have been described by astronomers. Stars of the seventh magnitude can be perceived by persons of keen eyesight; but those of the fifteenth and sixteenth magnitudes are only visible in the largest telescopes: even in the gigantic reflectors of Herschel, of twenty feet focal length and two feet aperture, they are very faint objects. The satellites of Uranus, and the faintest of Saturn's moons, are estimated to be of this degree of brightness. When the magnitudes of the fixed stars are expressed in this manner, it should be premised that their *apparent* magnitudes are understood by the expression, and not their intrinsic and absolute value. The relative intensity of the light of the stars is still a matter of considerable doubt; they have not yet been determined phonometrically with any exactitude. Sir W. Herschel endeavoured to compare the light of a star of the sixth magnitude with that emitted by Sirius, by covering the speculum of the telescope, when pointed at the latter object, with a disc having a circular opening; and the aperture was so diminished that Sirius appeared as a star of the sixth magnitude as seen with the full opening. When the magnitudes were thus rendered artificially equal, he found that Sirius, viewed with a circular aperture of one inch in diameter, was reduced to the same intensity of light as a star of the sixth magnitude when viewed with the full aperture of eighteen inches; and he concluded from this, that if the light of the latter be supposed equal to unity, that of Sirius would be eighteen times eighteen, or three hundred and twenty-four. As the light of Sirius is fully three times that of a star of the ordinary first-class magnitude, he considered that, in general, the light of a star of the first, in proportion to that of one of the sixth magnitude, was as a hundred to one. If, in the intermediate classes, the brightness is supposed to be inversely as the squares of the distances, we have—

1st magnitude . . the brightness = 100
2nd ,, ,, $\frac{100}{4}$ = 25
3rd ,, ,, $\frac{100}{8}$ = 12
4th ,, ,, $\frac{100}{16}$ = 6

And in the two remaining classes, he concluded, without reference to this law,

5th magnitude . . the brightness = 2
6th ,, ,, = 1

This determination of the relative intensities of the brightness of stars, rests, however, on too narrow a basis to be regarded as more than an approximation; and it is, perhaps, impossible to determine the brightness of one class of magnitudes in fractional parts of that of another. All that astronomical observers have hitherto accomplished on this subject, has been to arrange and catalogue them in the order of brightness, which, for the first six magnitudes, is best done with the naked eye. When they are estimated by means of the telescope, great discrepancies have occurred between different observers; and the fault of our celestial globes and charts has been, that a number of faint telescopic stars are inserted as among those visible to the naked eye; whilst others, equally bright, are completely omitted, their magnitudes being set down at the time their positions were first determined, and, perhaps, wrongly estimated from the state of the atmosphere, position of the moon, or other causes. Those visible to the naked eye in the northern hemisphere have been thoroughly revised by Argelander, who has subdivided the six classes formerly reckoned into sixteen, inserting two new divisions between each of the ancient classes; thus, for instance, between the fourth and fifth magnitudes he finds many stars a little fainter than the fourth, which he terms the 4·5 magnitude; and others a little brighter than the fifth (yet not so bright as those of the 4·5 class), which he denominates the 4·8 magnitude. Sir John Herschel, during his sojourn at the Cape, has instituted a further approximation to correctness in the relative brightness of stars, which will be of the utmost importance to future ages in arriving at a knowledge of the constancy of their brightness. This method of *sequences*, as he terms it, consists in choosing one of the brightest stars in any region of the heavens; secondly, one just inferior to it in lustre; thirdly, one immediately inferior to that of the supposititious *second* magnitude; and so on for twenty or thirty times, until a catalogue is formed for this particular region, with the stars ranged merely in regard to their brightness. On any subsequent opportunity this is repeated with a new set of stars, introducing, however, as many as possible of the former series. By this means, the relative intensity of all such stars as are visible to the naked eye, may be determined with the most rigorous accuracy. A valuable catalogue of such objects, and various proofs of the almost absolute certainty of the process, may be seen in Sir J. Herschel's *Result of Observations*, made at the Cape. The want of such a class of observations has led many astronomers to conjecture that great changes have taken place in the brightness of stars since the time of Bayer, at the commencement of the seventeenth century. Among others, Sir W. Herschel was of opinion, that at least one out of every thirty stars observed and mapped by Bayer, had diminished or increased in brightness in the two centuries which intervened between the end of the sixteenth and commencement of the nineteenth. It was supposed that Bayer's practice was to call the brightest star of *any* constellation by the first letter of the Greek alphabet, α; the next brightest, β; the third, γ; and so on. But Sir W. Herschel found that, instead of the stars in the constellation of Cygnus preserving this order, α, β, γ, δ, ϵ, &c., they now appeared according to the order α, γ, ϵ, β, and δ; in Aquila they had changed to the order α, γ, δ, β, ϵ; in Draco, to γ, β, δ, α; in Leo, to α, γ, β, δ, ϵ; and this change was apparent in numerous other instances. But it would now appear as if the magnitudes of the stars had been carelessly inserted by Bayer, no such change being due to the objects themselves.

Number of Stars.—In our estimation of the number of the fixed stars, it would appear that we are liable to an illusion; and that the generality of individuals suppose a much greater number to be visible than is found to be the case. The attention is probably directed to the richest portions of the heavens; and it may not unfrequently happen that the judgment is biased by what we have heard or read, rather than by what we see. In this case, it may naturally be supposed that other parts of the surface of the heavens are equally crowded with stars, though they are not so bright or apparent as the part which we really have in view; and it may thus be erroneously concluded that thousands of stars are visible at the same moment, which only exist in imagination. When we come to test this conclusion by absolute proof, it is found to be vastly out of proportion to these impressions. The *whole number* of stars visible to the naked eye in the central parts of Europe—viz., those included in the whole surface of the heavens north of the equator, and including a zone of 30° of south declination, comprising nearly eight-tenths of the whole sky—only amounts to 3,256; so that scarcely more than 2,000 stars can be visible to the naked eye, and above the horizon, at the same moment.

An attempt has been made to ascertain the number of stars of different magnitudes by supposing them to be situated at equal distances from one another, and that they are all of the same absolute magnitude, but appearing differently in consequence of their various distances. There being fourteen stars of the first magnitude, we are to suppose them arranged at equal distances upon a sphere: supposing the stars of the second magnitude to be twice the distance of those of the first, the surface over which they would be scattered would be four times that of the former; and if placed at the same distances from each other, it would take fifty-six stars to cover this area. The sphere of stars of the third magnitude would be nine times the area of that of the first, and it would consequently take one hundred and twenty-six stars to fill that surface; and in a similar manner there would be two hundred and twenty-four on the fourth, three hundred and fifty on the fifth, and so on. This law, however, does not correspond with the observed number of stars of different magnitudes, there being seventy stars of the second, and three hundred of the third—a much greater number than would exist on this supposition. Various other hypotheses have been formed to show the probable number of stars, of different magnitudes, visible on a given portion of the sky; but the data are too inconclusive and vague to secure any degree of accuracy in the result.

It would be difficult to determine the number of stars fainter than those of the sixth magnitude. The catalogue of Lalande (called the *Histoiré Céleste*, and published at the latter part of the last century) contains the places of about 50,000 stars visible from the north pole to 25° of south declination, and including those from the first to the ninth magnitude. The zone included between 15° of north, and 15° of south declination, contains, according to the more modern observations of the illustrious Bessel, 31,085 stars, viz.:—

664	bright stars from the 1st to the 6th magnitude.
2,500	of the 7th magnitude.
8,183	,, 8th ,,
19,738	,, 9th ,,

Stars fainter than those of the ninth magnitude increase in number in a wonderful degree; and Struve concludes that the number of stars visible in the twenty-feet telescope of Herschel, in the same zone of 30° in breadth observed by Bessel, amounts to the enormous number of 5,819,000, by far the greater number of which are situated at those parts where the Milky Way intersects the equator at six and eighteen hours of right ascension. Nor will this appear overrated, when we recollect that on one occasion Herschel perceived that nearly 120,000 stars passed through the field of view of his telescope (15′ in diameter) in a quarter of an hour. The stars observed by Herschel appear to have been situated in the following order in respect to their right ascension; and their density as they approach the Milky Way becomes immediately apparent.

From 1h. to 5h.	of right	ascension	391,700	stars.
,, 5h. ,, 9h.	,,	,,	1,984,200	,,
,, 9h. ,, 13h.	,,	,,	235,400	,,
,, 13h. ,, 17h.	,,	,,	387,000	,,
,, 17h. ,, 21h.	,,	,,	2,365,100	,,
,, 21h. ,, 1h.	,,	,,	455,600	,,
			5,819,000	

Thus, in respect to the distribution of stars of different magnitudes in the heavens, we perceive that the least crowded regions lie between nine and seventeen hours, and twenty-one, and five hours of right ascension. If we take the most brilliant stars, or those of the three first magnitudes only into account, we find them to be pretty evenly distributed over the surface of the heavens; but those of the fourth, fifth, and sixth magnitudes are congregated more densely as they approach the Milky Way. At every succeeding class this becomes more and more apparent, and the faintest stars are most thickly crowded in and near this zone. This would naturally lead us to imagine that there is some connection between the great galactic circle and the other portions of the heavens, and that they might form one great system.

The Milky Way extends completely round the heavens, and makes almost a great circle upon its surface. The breadth is very unequal: in some regions it is not more than 5°; in other parts it is two or three times that breadth. For one-third of its extent, viz., between Serpentarius and Antinous, it is divided into two branches; but for this distance of about 120°, the dark opening is of no great breadth. The resolvability of the Milky Way into distinct individual stars, which was proved immediately on the invention of the telescope, had been long previously conjectured by some of the ancient philosophers; whilst the absurd suppositions of others on its structure—if they were ever supposed worthy of examination—were dissipated by the same discovery. No other theory was started, however, to supply their place, or to explain this phenomenon, until about the middle of the eighteenth century, when Thomas Wright, of Durham, author of the *Clavis Cœlestis*, endeavoured, in his *Theory of the Universe*, to account for this appearance, by supposing that the stars were ranged in regular *strata*, and not dispersed fortuitously throughout space, as was previously supposed. By this arrangement, although the individual stars composing the stratum were at vast distances apart, yet, supposing our sun and its attendant satellites to be situated near the centre of this plane, we should witness such an appearance as the Milky Way presents. In the direction of the plane, the stars would be seen in such vast numbers, although the more distant ones would be so extremely small, that it would appear as a white and confused zone of light projected on the dark space surrounding us on every side. This idea received further development from the celebrated Immanuel Kant, who considered that it was rendered probable, from the arrangement of the three or four thousand stars visible to the naked eye, the greater number of which were contained in a zone within a short distance of the galactic circle. Lambert was likewise of opinion, that all the stars visible through the best telescopes lay in one vast stratum; but he considered that many of the clusters in the Milky Way were separate and individual systems, but nevertheless subsidiary members of one great system. Those immense clusters, each containing millions of stars, were connected and held together by the same power which predominates in inferior and more simple systems; and Lambert went so far as to imagine that the separate clusters performed revolutions round a great central body, in the same manner as our planet and satellite move round their primary.

The attention of Sir W. Herschel was particularly directed to the subject of the construction of the heavens; and his labours in this field must be regarded as among the most important of his works. In order to bring the theories started by his predecessors to a test, and to obtain an idea of the form and dimensions of the stellar universe, Herschel had recourse to a long and laborious method, which he properly termed "gauging the heavens." This was done by directing a powerful telescope (the twenty-foot reflector) to different parts of the sky, and counting the number of stars in each field of view. In order to insure greater accuracy, he counted the number of stars in ten contiguous fields, and took the average to express the comparative riches or poverty of the district. In some portions of the sky, only three or four stars of all magnitudes were seen *per* field for a considerable distance round; whilst at other times the field was crowded with many hundreds; and those latter portions were always found in or near the Milky Way. By combining the numerous "gauges" which he made, he endeavoured to determine the various depths (from the different degrees of obliquity in which the stars were viewed) of the Milky Way. Supposing the individuals composing it to be placed at pretty equal distances, he concluded the whole visible heavens to be of a lenticular form, and not a stratum of stars inclosed by plane surfaces. The proportion the thickness bears to the diameter of this *lens*, he considered was as one to five and a half; and he further concluded that the sun was removed but little from the centre of the group. Subsequently, when the motion of various double stars, consequent on their physical connection, was discovered by Herschel, his ideas were considerably modified; he now imagined that he perceived evidences of this physical connection in the great groups of the Milky Way, and that this "clustering power," as he termed it, tended to break it up into fragments.

Nor was this the only evidence of the motion of the stars, for many of them are endowed with an indubitable motion, as was apparent from their positions, compared with neighbouring stars; and, whether due to their own proper motion, or to that of the sun, it must be considered as an absolute proof of the instability of these bodies in space. Among the stars which are thus known to have considerable proper motion, we may mention μ Cassiopeiæ, which has a proper motion of 5″·82 annually, in right ascension, and 1″·55 in north polar distance. The star 61 Cygnus, whose distance has been determined with some exactness, is one of those which has a large proper motion; and it was in consequence of this circumstance that it was chosen by Bessel for the determination of the annual parallax. The proper motion, in right ascension, amounts to 5″·39, and in north polar distance, to 3″·30. In 40 Eridani, the proper motion, in right ascension, amounts to 2″·16, and in north polar distance, to 3″·45. The star 1830 of Groombridge's, which was considered to show the greatest amount of parallax, is one of those whose proper motion is most considerable, amounting to 5″·16 in right ascension, and 5″·70 in north polar distance. Other stars, in almost every constellation, have large proper motions; but the quantities are, as yet, somewhat doubtful, as it is only by the comparison of accurate modern observations with ancient authorities nearly as accurate, that these small quantities can be deduced; and it is but since the times of Bradley that the places of the objects can be depended upon.

Enough, however, has been effected to show the reality of their movement, and its direction. If this were due to the motion of the stars themselves, it might be supposed that they would move in all directions, north or south, east or west; and though this is found to be the case with some, yet, with far the greater number, the direction taken is much more regular. It has hence been conjectured that it is the sun itself which is in motion—the consequence of which would be, that those stars which are situated in that part of the sky which we approach, would appear to be separated more and more, as the angular distance would increase the nearer we approach to the objects. In that part of the celestial regions which we are leaving behind us, the stars would appear to be falling closer together, for the contrary reason. Although this supposition, explaining the proper motions of stars as being due to the simple effect of parallax, was made by both Mayer and Lambert, it was Sir W. Herschel who first attempted to show the direction of the solar motion, which he concluded was towards a point in the constellation of Hercules, whose position, in 1783, was at 257° of right ascension, and 65° of north polar distance. An attempt has since been made, by Argelander, to solve this problem; and that celebrated astronomer arrives nearly at the same result as Herschel, finding, from an examination of the proper motions of three hundred and ninety stars, that the part of the heavens towards which the sun is progressing, is probably situated at 260° 51′ of right ascension, and 58° 43′ of north polar distance.

Nature and Different Species of Stars.—It would, of course, be impossible to form any idea of the stars; but analogy would lead us to imagine that the heat and light which they emit, is in every respect similar to that of our own sun; and some photometrical experiments, which have been made on the latter object, seem to show that, if it were transported to the same distances as some of the stars whose distances are pretty well known, its apparent magnitude would not be so great as that of many which we see around us. From the distance and orbit of 61 Cygni, Bessel was able to arrive at a rough idea of the masses of those stars, which he found were not greatly under that of the sun; and if this were the case with a star of that magnitude, we may conclude that the brighter objects are considerably larger than our luminary. It is certain that reflected light could not reach us from such immense distances; and we must conclude that, like our own sun, they are self-luminous, although we know nothing of the agency at work to produce those extraordinary effects. It would be rash to imagine that, like it, they are all accompanied by a *cortége* of planets; for some of the latter bodies in the solar system have numerous satellites; our own earth but one; and others none whatever. But it would be equally rash to conclude that their heat and light expend themselves in the unprofitable and dark voids of the celestial spaces; and, in addition to the slight evidence which analogy affords, we have the further proofs of their being the centres of great systems from the remarkable phenomena of *double* and *changeable* stars, the latter of which, in a different point of view, may be regarded as evidence of their rotation on their axes, similar to that of our own sun.

Double Stars.—A few of those curious objects have been known ever since the invention of the telescope; and in a cursory examination of some of the brighter stars, as Castor, ζ Ursæ Minoris, α Herculis, γ Virginis, an observer could not help detecting the strange appearance of two stars close together, and almost blending their light, forming, apparently, but one star to the naked eye. In the middle of the seventeeth century further importance was attached to those curious objects, and several were closely examined, although without any result, for the purpose of determining their annual parallax, for which they offered peculiar advantages. It was then supposed that all these objects were fortuitously or optically double; one of the components—most probably the fainter—being situated at a much greater distance from us than the brighter, but both appearing in the same direction. It was for this purpose, likewise, that Sir W. Herschel commenced, in 1779, to apply his powerful telescopes and delicate micrometers to the task of recording their distances and positions in respect to one another. But in looking for one thing, as sometimes happens, another was found; and it became apparent that the components were not only at the same distance from the sun, but that the smaller body, in many instances, described an orbit round the larger star. It should not be forgotten, however, and it tends much to the credit of philosophical conjecture, that this remarkable law was previously surmised by the celebrated Lambert, who considered it possible that there were some groups in which the stars might make complete revolutions round a common centre of gravity in a comparatively short period of time. Mitchell conjectured the same law to apply to the more simple case of a double star; and, in 1784, he supposes that in a few years this question would be resolved by the stars whose positions and distances were ascertained by Herschel. The great discovery was first published by Herschel in 1803, who had then perceived a decided change in the positions and distances of several stars, as ξ Ursæ Majoris, Castor, ξ Bootis, 70 Ophiuchi, ξ Cancri, ξ Herculis, &c., &c. The labours of the elder Herschel were resumed in more modern times by his celebrated son, who, in conjunction with Sir J. South, reobserved all the stars in the northern hemisphere, and who has also observed independently, and measured all those discovered by him in the southern hemisphere. Struve, at Dorpat and St. Petersburg, also made the subject his constant care since the year 1814, and, with the powerful means at his command, has observed and reobserved their positions with the greatest possible accuracy; indeed, the publication of his great work, *Mensuræ Micrometricæ Stellarum Duplicium*, forms an era in this subject. This work contains observations of three thousand one hundred and twelve double stars, nearly three-fourths of which were discovered by himself.

The combinations of the components of double stars take every variety, both in regard to magnitude, distance, and, we might add, colour. The components of Castor are each of the third magnitude, and of the same ashy white colour, and close together, as are likewise those of γ Arietis in the same respects; whilst the stars Polaris, α in Herculis, γ in Delphini, are very different either in magnitude, distance, or colour. Out of the 3,000 stars detected by Struve, only a small portion have as yet been determined to be physically connected, or to form *binary systems*, as it has been termed; and the numbers whose orbits and periods have been even approximately determined, are fewer still. The time which has elapsed since the discovery, has been too inconsiderable to determine their true periods; some of which, as that of ε Lyræ, cannot be less than 2,000 years; and in others, as in 61 Cygni, or γ Leonis, the period is several hundred years. Among those whose orbits have been determined with more or less exactness, we may mention the following:—

Name of the Star.	Mean Distance of the Components	Eccentricity.	Period.	Direction of Motion.
			Years.	
ξ Herculis . .	1·25	0·448	36·4	Retrograde.
η Ursæ Minoris	2·44	0·431	61·6	Retrograde.
η Cor. Bor. .	1·20	0·404	67·3	Direct.
70 Ophiuchi .	4·97	0·444	92·3	Retrograde.
σ Coronæ . .	2·93	0·577	199·9	Direct.
γ Virginis . .	5·35	0·868	157·6	Retrograde.
Castor . . .	7·01	0·797	230·3	Retrograde.

M. Savary was the first who determined that the revolution of a star round its primary was performed in the same manner as that of a planet round the sun, and conformably to the laws of gravitation; and the same principles applied to other stars, have shown that the first two laws of Kepler, founded upon the motions of Mars, extend to the revolution of sun around sun. In determining the orbit, supposing it to be an ellipse, the same elements have to be deduced as in a planetary orbit, with the exception that the period of revolution and the semi-major axis are here two distinct elements, and that the latter is not expressed in linear measure, but in an arc of a great circle. From four observations of the angles of position and the distances, knowing the intervals and taking the Keplerian laws as the basis of calculation, the seven elements of period, semi-major axis, eccentricity, nodes, inclination, position of peri-astre, and epoch of the peri-astre, can be determined. In the first place, however, the *apparent* ellipse must be determined before the *real* one can be arrived at. The first is that which the star describes around its primary on the plane of the sky; but the *orbit* may be inclined at any angle to this plane; and it is only when the plane of the orbit is perpendicular to the line of sight, that the true and apparent ellipses are identical. The projection of the circular or elliptic orbit on the plane of the heavens, remains always an ellipse, but the projection of the focus and the major and minor axes will take different positions; and the proportions of the latter to one another, and consequently the eccentricity of the orbit, will be different. In consequence of this difference between the apparent and true orbits, we sometimes see the one star projected on, or occulted by, its companion. Such was the case with γ Virginis, which, from 1834 to 1836, appeared as a well-defined single star. The plane of the orbit of the companion of 44 Bootis, is almost perpendicular to the plane of the heavens; and between 1802 and 1819 an *almost central occultation* must have taken place. The companion of ξ Herculis has twice undergone an eclipse, in consequence of the great inclination of the orbit to the plane of vision—once in 1802, and again in 1831.

In addition to the double stars, triple and quadruple stars are sometimes met with; and if the fortuitous combination of the former is so little to be expected, that of the latter is much less probable. Among the more conspicuous of the triple stars, that of ξ Cancri holds the most prominent position; the three stars, all of which are nearly of the same magnitude, being physically connected. The close double star has made upwards of a revolution in a retrograde direction since 1782, and the more distant one has moved fifty degrees in the same time. In the list of quadruple stars, the most remarkable is that of ε and 5 Lyræ, which can be detected as *double*, by a keen eye, without the aid of any instrument; but each of which, when examined with a power of two hundred, appear as a double star, the four components being nearly of the same magnitude and colour. This system likewise seems to be physically connected. Among the multiple stars, that of θ Orionis, situated in the centre of the great nebula, is the most apparent, six stars being situated in a circle of twenty-three seconds in diameter.

The colours seen in the components of many double stars have been described by some as an optical illusion, on the ground that it is always the complementary colours which are thus perceived, as, in a similar manner, a white spot seen on a red ground will appear green. It is, however, impossible to view the components of α Herculis or γ Andromeda, without coming to the conclusion that they have a proper colour of their own, and that the blue and orange so vividly distinct is something more than the effect of contrast. In those stars which show high and brilliant colours, the larger star is always of a golden orange, the smaller greenish or bluish. In many, however, the components are of the same colour; but it seldom happens that the two colours are of a bluish or greenish tint; most frequently they are white and yellow.

TEMPORARY AND VARIABLE STARS.—Those stars which vary in brilliancy from time to time, and others which suddenly seem to start into existence and disappear as precipitately as they have become visible, must be regarded as the most curious objects in the heavens, for they prove manifestations of life and motion in the immeasurably distant regions of space; they form one of the subjects, too, in which even a lover of science who does not possess a telescope, can, by simple observation of the relative brightness of the various stars, recorded on favourable opportunities, confer considerable benefit on this branch of sidereal astronomy. A telescope, in this case, is of little value; the field of view is much too small to include the stars proper for comparison with the object whose light is suspected to be variable, and the photometer has hitherto been of very little service. The most accurate results which have been obtained as yet, are due to observations with the naked eye, compared with stars in its vicinity; by this means the interval between the faintest and brightest phases have been found with far more accuracy than would be imagined.

There are numerous instances of the appearances of temporary stars recorded in the Chinese annals; but the two most remarkable of those phenomena have occurred in comparatively modern times, and have been minutely described by the two great contemporaries, Tycho Brahé and Kepler. The first appeared in the year 1572, in the constellation of Cassiopeiæ. On the 11th of November of that year, when seen for the first time by Tycho, it surpassed in brilliancy both Sirius and Jupiter, the most lustrous objects in the heavens. In the following month it had diminished slightly in brightness, but was still equal to Jupiter. At the beginning of 1573, it was inferior to that planet in brightness; and by the end of March it was not brighter than the principal star in Taurus, although still a good first magnitude. It continued gradually to decrease until the end of the year, but remained visible to the naked eye until March, 1574. If its period of increase were equal to its period of decrease, it must have been visible to the naked eye for nearly three years, as it was not noticed until it had attained its maximum brightness. For a description of the second instance we are indebted to Kepler. The star was perceived suddenly in the constellation Serpentarius, on October 10th, 1604, and appears to have been nearly as brilliant as it was formerly, though not so favourably situated for observation. It remained visible to the naked eye (the telescope had not yet been invented) for upwards of a year. A star of the third magnitude also appeared suddenly in the constellation of Cygnus, in the year 1670, which soon afterwards disappeared; it again made its appearance, and again disappeared, and has not been since seen. It underwent several changes during the two years in which it was observed.

VARIABLE PERIODIC STARS.—The appearance of temporary stars is as rare as the phenomena observed are extraordinary. Of a similar nature, however, are the class of periodic variable stars, whose changes of lustre are equally as decided and curious as those of the objects just mentioned; being more known, however, they excite less attention. The most singular of those objects is that of *ο* Ceti, whose variability was first discovered by Holwarda in 1639, and which has a period of about eleven months. Although at its maximum brightness it reaches to the second magnitude, it does not appear to have been at all noticed by any observer previous to 1596; but this may be partly in consequence of the length of its period; for if the maximum intensity of light fall in the summer months, it may sometimes remain invisible for three or four years together. During the winter months this is not the case; but, as happened in February and December, 1847, two maxima may take place during the same year. There is another circumstance which has been noticed, and which may tend to explain the silence of ancient authors and observers. This is, that the star does not always reach the same brilliancy, sometimes approaching in brightness to stars of the first or second magnitudes at this period, whilst at other times it is not brighter than the fourth maximum. It has generally been supposed that it disappears entirely at the period of its minimum, even in the best telescopes; but this is not always the case, for at these times it has occasionally been observed to be not fainter than stars of the eleventh magnitude. What its variations of light may be when visible only in the telescope is not very well known, as it has not been closely observed at those times; but it is certain that, whilst visible to the naked eye, its fluctuations of brightness are very remarkable. It is visible to the naked eye, on an average, for about two months previous to the period of its maximum brightness; but the period of its diminution of brightness is generally longer than that of its increase; and it has been visible for three months after its maximum, the average duration being, however, only seventy days. It sometimes, but rarely, happens that the interval which elapses between its coming into sight and its maximum brightness, is greater than the time between its maximum and disappearance. Some attempt has been made to reduce the fluctuations which its light undergoes, and to establish a law; but the changes are too abrupt and irregular to be dealt with in this manner, or to be foretold with any accuracy.

Among the other stars of long period, which are visible to the naked eye for a length of time, and then entirely disappear from sight, is χ in the neck of Cygnus, which is almost as remarkable as the preceding. Its light varies between the fifth and eleventh magnitude. Its maximum brightness, like that of *ο* Ceti, is, however, very variable; sometimes it reaches to the fourth magnitude, and at other times its maximum is not more than the 6·7 magnitude, when it is quite invisible to the naked eye. The variability of the light of this star was discovered by Gottfried Kirch; but the period was first found by Maraldi. The interval between its successive *maxima* and *minima*, as well as its intensity of light, at those times, has since been discovered to be very irregular. On some occasions it has been visible to unassisted vision for a period of nearly three months; but the average duration, according to Argelander, is only fifty-two days, being twenty days of an increase, and thirty-two of a decrease. The longest period which has yet been recorded in the class of variable stars, occurs with the star 30 Hydræ, whose period has been determined, by Maraldi, at four hundred and ninety-four days; this period, however, is very irregular. At its maximum brightness it sometimes arrives at the fourth magnitude, and at other times only at the fifth, and thence it decreases in brilliancy until its entire disappearance. The star 19 Leonis is another with a very long period, the interval between its successive maxima being three hundred and eleven days. This period, however, is also somewhat irregular, and the changes in its brilliancy occasionally abrupt. At its maximum brilliancy, it is equal to stars of the fifth magnitude, and is invisible at its minimum. These are the only four stars hitherto discovered which are invisible to the naked eye at their *maxima*, and vanish out of sight at their *minima*, except to the most powerful telescopes. There are a few other stars discovered by Harding, Schwerd, and Hind, with periods which appear to be upwards of a year; but, even at their maxima, these stars are only visible in a telescope, and not the slightest trace of them can be perceived in the best instruments at the epoch of their minima.

In others of the variable stars, the period is much shorter; but the change of light is not so considerable. The most remarkable of those is the star β Persei, in which the interval between two successive maxima is only 2d. 20h. 48m. 58s., and is otherwise a very curious object, from the abrupt and variable changes of brilliancy which it exhibits. For the greater portion of its period it remains at nearly its maximum brightness, or of the two-and-a-half magnitude. In about four hours it suddenly decreases to the fourth magnitude, which is its minimum brightness; and in a like period of four hours it regains its former brightness. There are other but less remarkable changes in its light; it is one of the best determined of the variable class. The majority of the periodic stars shine with a reddish light, but this shows no sign of colour, being of a pure white. The star δ Cephei has a period of 5d. 8h. 37m., with a variation of brilliancy from the third to the fifth magnitude. This star exhibits a great regularity in its successive changes of brilliancy, which have been determined with considerable exactness; but the periods of increase and decrease are very dissimilar, as it takes 3d. 18h. to pass from its maximum to its minimum, and only 1d. 15h. to return from its minimum to its maximum, according to the careful observations of Argelander. During eight hours of its decrease, it scarcely, if at all, changes; and for a whole day its diminution of brightness is scarcely perceptible. Two other variable stars of short period—η Aquilæ and β Lyræ—are known at present. Both of those stars change from the three-and-a-half to the four-and-a-half magnitude; the former in 7·18 days, the latter in 12·8 days. The period of the latter star was first stated to be 6d. 9h. by Goodricke, who detected its variation, which he at first supposed to be from the third to the fifth magnitude. In this determination of the length of the period he was partly correct; for it is one of those stars in which there is a smaller variation of brilliancy within the larger period. After its minimum brilliancy, it passes in 3d. 5h. to its first maximum, and then in 3d. 3h. to its second minimum. In 3d. 2h. it again rises to its second maximum, and after this, in 3d. 12h., to its former minimum. The whole period, according to Argelander, is 12d. 21h. 46m. 40s., in which period it performs all the preceding changes. This celebrated astronomer is of opinion that the period is diminishing, and that it is now some hours shorter than when first discovered. Even from recent observations it is apparent that there is a decrease in the period.

Previous to the time of Sir W. Herschel, it was only stars of very long and very short periods which were known to be variable. The period of α Herculis, discovered by him, proved, however, that there was an intermediate class. This star is remarkable for its deep orange-red colour, which is very strikingly contrasted with its small dark blue companion. The period was considered by Herschel to be 63 days; but the change of lustre is very slight, being only from the third to the third and a-half magnitude. There is still some doubt respecting its period, some observers considering that it is as much as ninety-five days; but from seven years' observations, it would appear that a period of sixty-six days would agree better with the changes which it undergoes. The observer Heis, who has particularly attended to this species of phenomena, is of opinion that the variations of light are best satisfied by a period of 184·9 days with two maxima and two minima. Since the time of Herschel, various other stars of *medium* period have been added to that of α Herculis, nearly all of which are, like it, stars of considerable magnitude. Thus α Cassiopeiæ changes from the second to the third magnitude in 79d. 3h., according to Biot; but it is very difficult to determine the exact period, the change of brilliancy being very slight, and apparently irregular. This is equally the case with α Orionis, which, at its minimum, becomes slightly inferior to stars of the first magnitude; and, according to Argelander, the period of increase of light takes place in ninety-one days and a-half, whilst it continues to decrease in brilliancy during a hundred and four days and a-half. The period of α Hydræ is supposed by some to be performed in fifty-five days; but the change of brightness being minute, as in other instances, it is somewhat doubtful. Sir J. Herschel considers it to be performed in twenty-nine or thirty days. The star commonly designated as *l* in the constellation of *Sobieski's Shield*, is remarkable for the great variation of its magnitude, particularly at its minimum, when it varies between the sixth and ninth magnitudes. At its maximum it only varies between the fifth and sixth. Among those of shorter period may be counted that of β Pegasi, which varies between the second and second and a-half magnitude, but whose period is not more than forty days. At the present time, we thus know stars of almost every period from 2d. 20h. to 406 days. There are ten whose period varies between 1 and 100 days; two between 100 and 200; eight from 300 to 400; and two above 400. It is curious that there are none at present known whose period is between 200 and 300 days.

Herschel considered that, by introducing a medium period between those which were very long and very short, he proved the rotation of those objects on an axis, by which means such portions of their surfaces as were dark and obscure came into sight at stated times, and hence their apparent faintness. To the astronomers of the 16th and 17th centuries—by many of whom the rotation of the earth was doubted, and that of the other planets unknown—this method of accounting for it would have appeared strange. It was attempted, however, by Bouilland to explain the changes seen in ο Ceti on this principle: the larger portion of the surface of this star was, he considered, non-luminous, which rendered it invisible for the greater part of its period. Another supposition in respect to those changes was, that they were due to the interposition of opaque bodies, by which their light was more or less eclipsed at regular intervals. Maupertius considered that, among the many stars which were scattered through space, there were some whose figure was not regularly spherical, but greatly flattened at the polar regions, and that the variations of light were due to the thin edges of those being sometimes presented to view, and at other times the large surface of the flattened disc. There is yet no *experimentum crucis* known, by which it can be said whether either or any of these hypotheses are correct.

CLUSTERS AND NEBULÆ.—The most striking and magnificent objects which the sidereal heavens present to view, are the clusters and nebulæ, where, in a telescopic field of view of a few minutes in diameter, we may perceive a crowd of stars collected together, and forming a small patch of light, the individuals composing which must be some thousands in number. To those wherein the stars are plainly seen, have been given the name of *clusters;* but in others, which are unresolved, the terms *nebulæ* and *unresolved nebulosities* have been applied. The nebulæ and clusters which are at present known are of every degree of resolvability; and though some of those which are generally called nebulæ have been resolved by the most powerful instruments, yet there are others which have baffled all the attempts of opticians to separate them into individuals. They are of every conceivable shape and size, from the regular and minute form of the planetary nebula of a few seconds in diameter, to the amorphous and large surface of the great nebula in Orion. Numerous clusters of stars are visible to the naked eye, as the Pleiades, Hyades, the Presepe of Cancer, and that in the sword-handle of Perseus; and in the southern hemisphere we have the Magellanic Clouds, which cover a space of some degrees square.

The most magnificent of the clusters which have been resolved by the telescope, is that in the constellation of Hercules (Fig. 78), which can be detected on a clear night by the naked eye, and which, in the telescope, presents one of the finest instances of dense isolated clustering in the whole of the heavens. The stars are from the tenth to the fifteenth magnitudes, and are so densely congregated round the centre, that it presents the appearance of a blaze of light in that part. The stars are pretty evenly distributed in the interior, but are rather irregular at the edges, where they are formed into curvilinear branches. Sir J. Herschel is of opinion that there are many thousands of them, some of which are the faintest the telescope can show. This object is only seven or eight minutes in diameter, and, with the telescope which Sir J. Herschel employed, no appearance of a nucleus was perceptible; although, with a smaller one, such might be suspected.

Fig. 78.

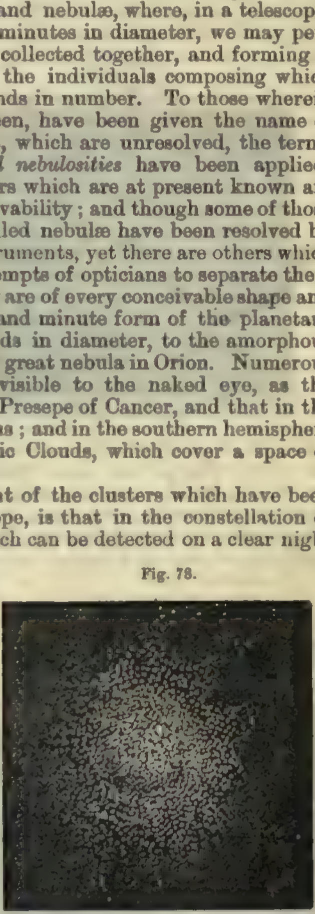
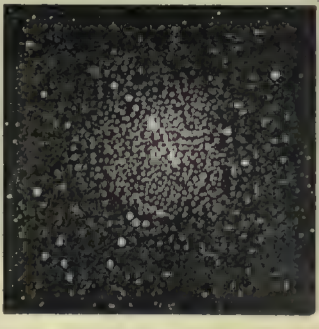


Fig. 79.



The fifth object in Messier's catalogue, is another very rich but compressed specimen of a globular cluster (right ascension 15h. 10m., north polar distance 87° 16′). In this case (Fig. 79) the density towards the centre is not gradual; the condensation commences suddenly, and the blaze of light into which it is formed, appears projected on a loose irregular ground of stars. The neighbourhood of this cluster is very poor in stars—a circumstance which Sir W. Herschel remarked in other cases, and which he conceived might be due to the stars being attracted to one point, and formed into one cluster. The cluster situated at 21h. 25m. right ascension, and 91° 34′ of north polar distance, appears like an unresolved nebula with instruments of even considerable power; and it is only when viewed with the great light of high magnifying powers, that faint traces of stars become apparent on its surface, and it takes a very fine sky and splendid instrument to show the grains of star-dust faintly visible (Fig. 80). *Finer* specimens of clusters, or rather nebulæ, *merely* resolved, are seen at Figs. 81 and 82. We may consider the latter of those as the extreme boundary of the clustering species; after which the nebulæ, properly so called, begin. As the reflecting telescope advances more and more to perfection, we may, however, expect that many objects will be won from the mysterious domain of nebulæ, and placed to the account of the more explicable clusters.

Fig. 80.

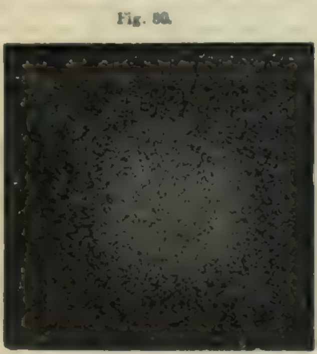


Fig. 81.

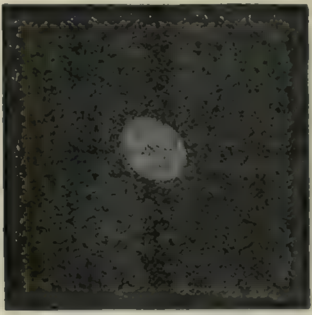
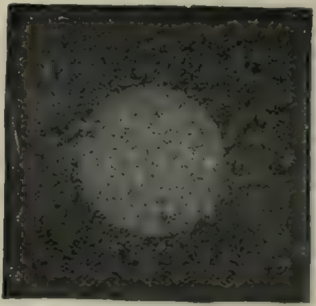
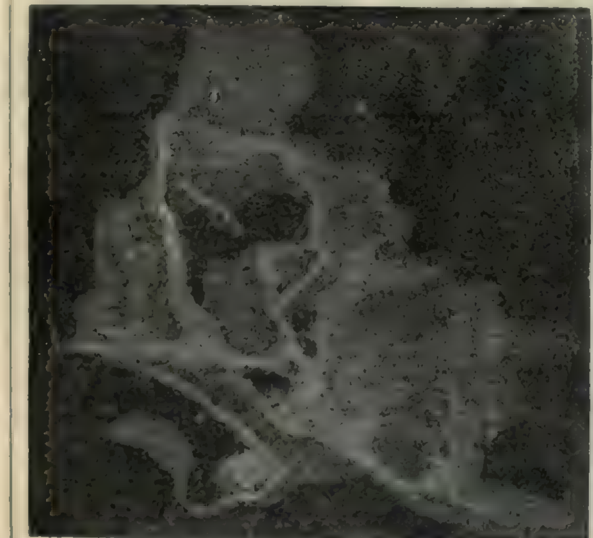


Fig 82.



The large clusters have some approach to regularity, but this can scarcely be said of the more considerable nebulæ. The finest of those is, without doubt, that of Orion (Fig. 83), which, for its various inequalities of light and shade, its great extent, and the capricious irregularities in the form of the great promontories which jut out from the main body, constitute it one of the most noble and mysterious of those objects. When examined with the two-feet mirror of Sir John Herschel, and some powerful refracting telescopes, it shows some signs of resolvability, being likened to a curdling mass, or the breaking up of a mackerel sky. But with the great mirror of Lord Rosse, the resolution is complete, and it is broken up into distinct stars, which are scattered in irregular masses through its body. The great nebula of Andromeda (Fig. 84), is another very conspicuous object of the class, and may even be perceived with the naked eye on a fine dark night. It is a long, elliptical ray of light, rather broader towards the centre, and was compared, by its discoverer, Simon Marius, to the flame of a lamp shining through a piece of horn. With the improved optical means of more modern times, equally as with the imperfect telescope of Marius, no signs of resolvability have as yet appeared, though the surface has been noticed to be dotted over with innumerable stars, which are not, however, supposed to be directly connected with it, being merely casual groups interposed between it and the sun.

Fig. 83.



The majority of the nebulæ hitherto discovered—now upwards of 3,000 in number—are mostly of a regular form, being either circular or more or less elliptic, like that of Andromeda. But there are many others; and

Fig. 84.

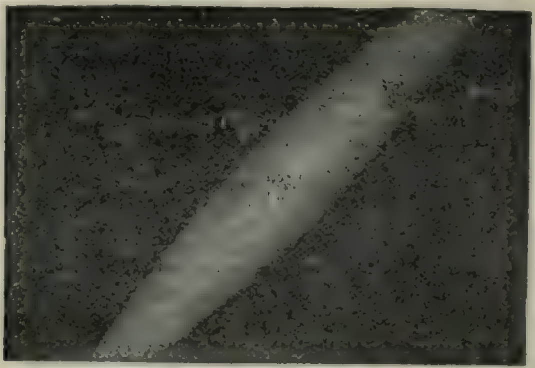
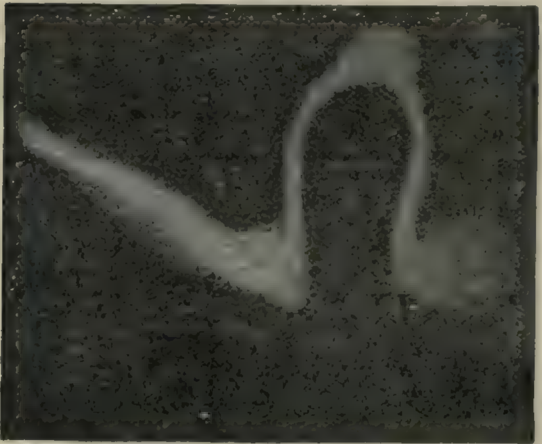
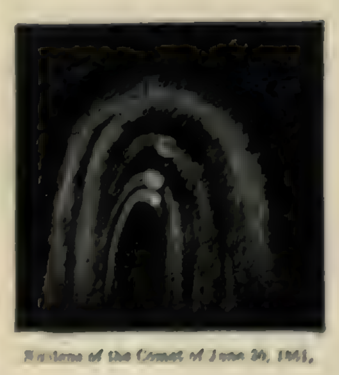


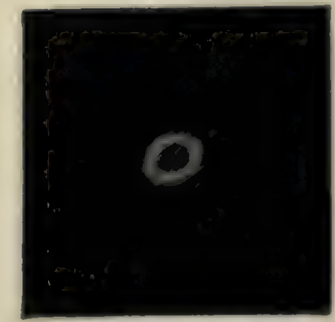
Fig. 85.





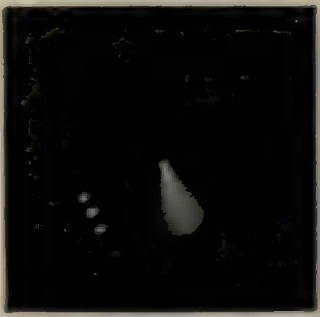
Sections of the Comet of June 30, 1861.

Fig. 87.



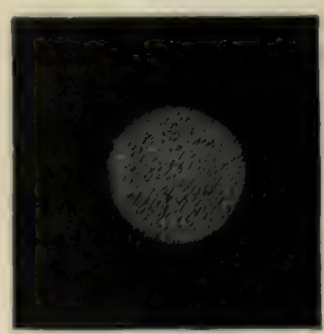
R.A. 281° 40′; N.P.D. 57° 11′.

Fig. 88.



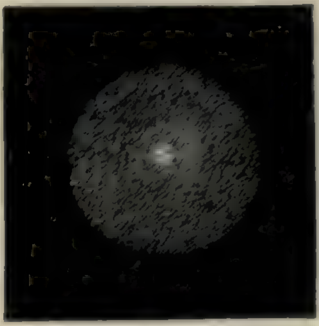
Cometary Nebula.

Fig. 89.



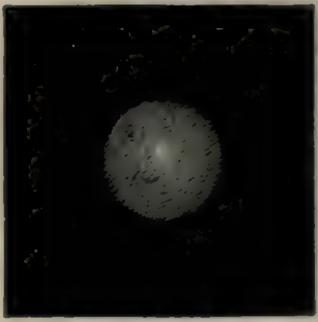
R.A. 19° 47′; N.P.D. 77° 10′.

Fig. 90.



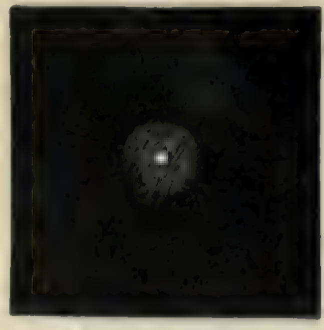
R.A. 202° 13′; N.P.D. 107° 1′.

Fig. 91.



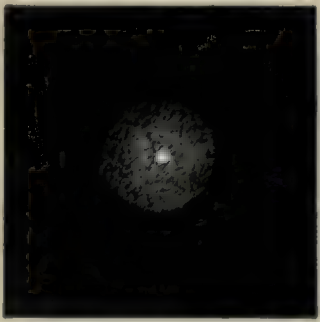
R.A. 50° 43′; N.P.D. 47° 37′

Fig. 92.



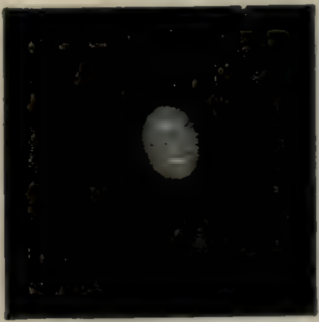
R A. 50° 30′; N.P.D. 59° 40′.

Fig. 93.



R.A 205° 5; N.P.D. 39° 54.

Fig. 94.

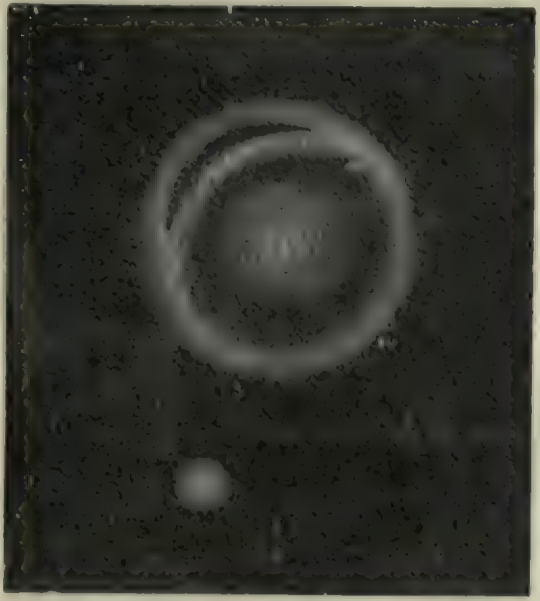


R.A. 271° 45′; N.P.D. 109° 56′

NEBULÆ.—(See page 957). COMET OF 1861.—(See page 972).

among those that are the largest and brightest, in which the nebulous matter is dispersed very capriciously, no regularity is perceptible. Among the latter, the one situated at 272° 42′ of right ascension, and 106° 15′ of north polar distance (Fig. 85), may be instanced, being of the shape of the large Greek omega, and having no symptoms of resolvability apparent. Among the more regular forms, are those termed *annular* nebulæ and *cometary* nebulæ. The fine object situated at 200° 40′ of right ascension, and 41° 56′ of north polar distance (Fig. 86), and which is large enough to be perceived as a faint patch of light in a small telescope, is among the finest instances of the former kind. In this, the surrounding ring appeared, in the two-foot mirror of Sir J. Herschel, to be divided into two branches; but in the great telescope of Lord Rosse, the whole takes the form of a spiral, and many curious details become visible. The smaller nebula below (here represented as quite detached), appears to be connected with the larger body by a projecting branch of the spiral. A fine specimen of the more simple and rare form of annular nebulæ, is that situated in the constellation of Lyra (Fig. 87), at 281° 49′ of right ascension, and 57° 11′ of north polar distance. It is not quite dark in the centre. The form of cometary nebulæ will be seen by Fig. 88, the pointed end being attached apparently to a star, which appears like a stellar nucleus. (See folio plates).

Fig. 86.



In the circular, or globular form of nebulæ, the degrees of condensation of the nebulous matter towards their centre is very different. In Fig. 89, right ascension 18° 45′, north polar distance 77° 59′, there is little or no condensation towards the middle. In Fig. 90, right ascension 202° 13′, north polar distance 107° 1′, the condensation is considerably more marked. In Fig. 91, right ascension 59° 43′, north polar distance 47° 57′, the central bright portion is of smaller extent than in the last instance, but better defined and much more lustrous. In Fig 92, right ascension 59° 39′, north polar distance 59° 40′, the central portion becomes almost stellar, appearing like a well-defined star surrounded by a halo. The different degrees of condensation recognisable in these objects, led Herschel to imagine that the nebulous matter, in the course of ages, underwent considerable changes, and that, having condensed gradually, the nebulæ finally ended by becoming stars. These changes were so slow that they had not yet been perceived; but it was visible, by comparing them together, that such probably was the case. In some cases, the central star or stars was distinctly visible. Such an object is perceived at right ascension 295° 5′, north polar distance 39° 54′ (Fig. 93), in which the nebula is evenly distributed round the star. At right ascension 271° 45′, north polar distance 109° 56′ (Fig. 94), an elliptic nebula is visible, with a small star situated in each of the foci of the ellipse; and at Fig. 95, right ascension 80° 3′, north polar distance 55° 54′, a triangle of stars is congregated at the centre of a small nebula. In those which were equally bright throughout the whole disc, Herschel conjectured that the star was not visible on account of its faintness, or from being surrounded by a dense nebulosity. Such an one is situated at right ascension 166° 12′, north polar distance 34° 4′ (Fig. 96).

Among the most curious forms of the nebulæ, those which are double take a prominent place. In those, both the components are sometimes of equal magnitude (Fig. 97), right ascension 174° 20′, north polar distance 55° 31′; and similar in every respect, or like those at Fig. 98, right ascension 140° 38′, north polar distance 67° 45′; and Fig. 99, right ascension 183° 18′, north polar distance 84° 35′, where they are of irregular size and condensation. In all these examples the components are joined together by the surrounding nebulosity; but in some cases the two nebulæ are quite distinct, as in Fig. 100, right ascension 342° 48′, north polar distance 103° 43′. In such cases, Herschel imagined he perceived symptoms of the gradual formation of a *double* star, the nebulous matter of which they are formed, condensing towards two centres of attraction. It will appear from this that the *nebulæ*, properly so called, according to Herschel's ideas, were far from being such important objects as the clusters, the latter being great agglomerations of stars, the nebulæ only the material of a single, or, at most, a double star. But it would be hazardous to say that any particular nebula was irresolvable. The gigantic telescope of Lord Rosse has shown that many such, hitherto deemed of this nature, consist of innumerable stars, very different from the single individual star which Herschel thought it was probable it would finally become.

The suppositions relative to this nebulous substance, which Herschel thought to be so plentifully scattered through space, and the changes which he surmised that it underwent in the course of time, led the celebrated astronomer Laplace to form a similar hypothesis on the formation of the solar system, and the progressive development of the bodies of which it is composed, from one single and primitive mass of nebulous matter. He supposed that the sun was originally included among the nebulæ, which extended as far as the most distant planet, and was endowed with a rotary motion round the centre; and that, in the course of ages, this gaseous substance became more condensed towards the centre, leaving the exterior portions in the form of immense rings, still preserving their revolving motion, precisely similar to the ring of Saturn as at present existing. From many different causes it was impossible for those rings to exist in that form, and they finally broke up into globular masses forming the different planets. By this means he explains the revolution and rotation of the planetary bodies in one direction, as well as of their satellites, formation of comets, &c. Even the zodiacal light was fully explained on this hypothesis. The nebulous matter, as he conjectured, was not quite condensed, but still surrounded the sun in a very diffused state, and by the rotation of that body on its axis, took a lenticular form, as was observed to be the case with many of the nebulæ already discovered. Thus the great nebula of Andromeda is of this elliptic form; and different degrees of ellipticity will be seen from the figures in the folio plate (Figs. 101, 102, and 103), which are some of those actually observed by Sir J. Herschel.

CHAPTER VII.

ON THE CONSTELLATIONS AND FIXED STARS

OUR present arrangement of the constellations into groups or clusters, representing figures of men and animals, dates from the most remote antiquity. Aratus, a Cilician of Eudoxus, B.C. 370, a contemporary of Plato, enumerates forty-five as being then in use, all of which still remain on our celestial globes. In the majority of instances, however, there is no similarity between these configurations, and the figures to which they are supposed to bear some resemblance. Of the fixed stars, in both hemispheres, nearly 5,000 are visible to the naked eye, the greater number of which have been catalogued by Argelander, in his standard work, the *Manometria Nova*. They are divided into classes of magnitude, the *first* consisting of stars pre-eminently bright, and which amount in our latitude to about 14; the second magnitude consisting of about 50; the third of 140; the fourth of about 320; the fifth of 809; and the sixth of 2,000 (roughly speaking). Objects of a magnitude greater in class, but apparently less in size than the sixth, are only seen by the help of telescopes, and are reckoned on to the thirteenth or fourteenth magnitude.

Bayer appears to have been the first astronomer who systematically arranged the stars of each constellation in the order of magnitude. In his catalogue, of which the epoch is 1603, he classed the brightest stars in each constellation according to the first letter of the Greek alphabet; the second by the second letter, and so on; afterwards using for smaller objects the *Roman letters*. Of the first magnitude in these latitudes, we have nine north of the equator, and five south, viz. :—

North.

Vega, or α Lyræ
Capella, or α Aurigæ
Arcturus, or α Bootis.
Aldebaran, or α Tauri.
Betelgeux, or α Orionis.
Regulus, or α Leonis.
Altair, or α Aquilæ.
Deneb, or α Cygni.
Procyon, or α Canis Minoris

South.

Sirius, or α Canis Majoris.
Rigêl, or β Orionis.
Spica, or α Virginis.
Antares, or α Scorpio.
Fomalhaut, or α Piscis Australis.

Bayer's arrangement has since been used by astronomers.

The most ancient catalogues we possess of the fixed stars are those of Ptolemy, Ulugh Beigh, Tycho, and Hevelius, all of which have been formed from observations by the naked eye. The catalogue of Ptolemy is based on the observations of Hipparchus, and amounts to upwards of 1,000, which are arranged in the order of longitude and latitude. That of Ulugh Beigh contains 1,019 stars for the epoch of 1437, observed at Samarcand in Persia, north latitude 39° 52′. The epoch of Tycho's catalogue is for the end of the sixteenth century, and contains 777 stars, reduced and edited by his pupil, Kepler. The catalogue of Hevelius contains 1,564 positions of stars for the epoch 1660. All the preceding works have been re-edited by Mr. Baily, to whom this branch of astronomy is much indebted, and are given in the thirteenth volume of the *Memoirs of the Royal Astronomical Society*.

Since the application of the telescope to astronomical observations, we have the catalogues of Halley and La Caille, observed in the southern hemisphere; and those of Flamsteed, Bradley, and Mayer, observed in the European observatories. The work of Flamsteed is the basis of that of Bradley, and they both, in common, contain nearly the same stars, but, in the latter case, observed with more improved instruments. The numbers in the catalogues of Flamsteed, Bradley, and Mayer, are respectively 3,400, 3,222, and 998 stars.

At the commencement of the present century we possessed the catalogues of Groombridge, Piazzi, and Lalande, of which the first consists of circumpolar stars numbering upwards of 4,000; the second of 7,646 stars; and the last of 50,000 stars.

In modern times we have the catalogues of Busham and Taylor, observed at New South Wales and Madras respectively; the Greenwich catalogues of Pond and Airey; and the continental catalogues of Remiker, Bessel, and Argelander. The British Association have also published a standard catalogue of 8,377 stars.

THE ZODIACAL CONSTELLATIONS are twelve in number, all of which occur in the list prepared by Aratus.

I. *Aries* has two very conspicuous stars in the head of the Ram, about 4° apart. They are the nautical star, α Arietis, called *Hamal* by the Arabs, of the second magnitude, and *Sheratan*, of the third.

II. *Taurus*, one of the finest of zodiacal Asterisms, is just rising in the east when Aries is 27° above the horizon. It includes Aldiebaran, a star of the first magnitude, forming with Hyades the letter Y in the face of the Bull; on the left shoulder is the well-known cluster of the Pleaides.

III. *Gemini* has two principal stars, Castor of the first, and Pollux of the second magnitudes, about 4½° apart, in the neighbourhood of Propus, a small star of the fifth magnitude. In this constellation Sir John Herschel found Uranus, and it served for many years to guide astronomers to that planet.

IV. *Cancer* has no very conspicuous star. Two of the fourth magnitude, the Aselli (the Asses) of the Romans, and Presepe (the Manger). A nebulous cluster, at the distance of 2°, distinguishes this constellation.

V. *Leo*, a brilliant constellation, has Regulus, of the first magnitude, in the breast of the Lion, and Denebola, of the second magnitude, in the tail, about 25° apart.

VI. *Virgo* has Spica Virginis, a star of the fourth magnitude, in the Wheat-ear, remarkable for its solitary splendour, having only one other star, of the fourth magnitude, near it. This star is called *Al-simak-al-a-zal* (the Defenceless) by the Arabs.

VII. *Libra* has four subordinate but bright stars, which form a quadrilateral figure. Two in the northern and two in the southern scale, 7° and 6° apart.

VIII. *Scorpio* is a beautiful collection of stars, among which Antares (in the heart) is of the first magnitude, and is distinguished by a remarkably red appearance.

IX. *Sagittarius* has five stars of the third and fourth magnitudes, which form a figure resembling a straight-handled dipper, familiarly called the milk-dipper, because situated in the Milky Way.

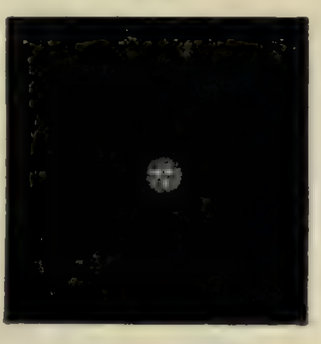
X. *Capricornus* has only stars of the third and fourth magnitudes. The sun was formerly in this constellation, when at mid-winter he attained his greatest southern declination; hence it was called the southern gate of the sun, as Cancer was the northern. Now, owing to the precession of the equinoxes, the sun does not reach the constellation till the middle of January.

XI. *Aquarius* is recognisable by four stars of the fourth magnitude, so placed as distinctly to form the letter Y, which is visible about the urn of the Water-bearer.

XII. *Pisces* is a loose assemblage of small stars, not readily traced, occupying a large triangular space in the heavens. This is the first constellation in the order of the zodiac, opening the astronomical year, and preceding our vernal equinox.

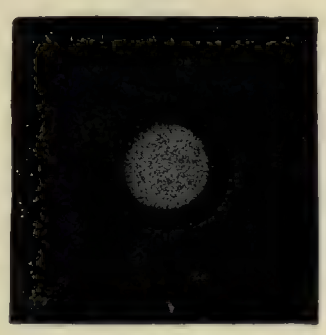
In naming and figuring these zodiacal groups, the ancients are supposed to have been guided by the rural occupations coincident with the sun's appearance in particular parts of the heavens, or by other analogous phenomena presented to them. Thus, the RAM, the BULL, and the TWINS (originally two goats), relate to

Fig. 95.



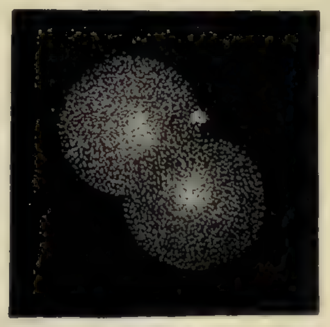
R.A. 80° 3′ ; N.P.D. 55° 54

Fig. 96.



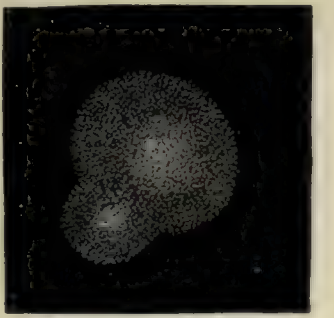
R.A. 166° 12′ ; N.P.D. 34° 4′.

Fig. 97.



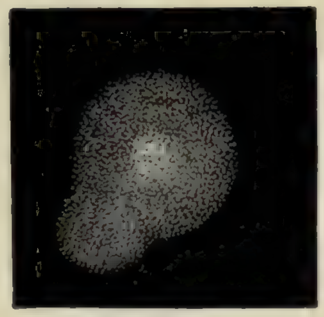
R.A. 174° 20′ ; N.P.D. 55° 31′

Fig. 98.



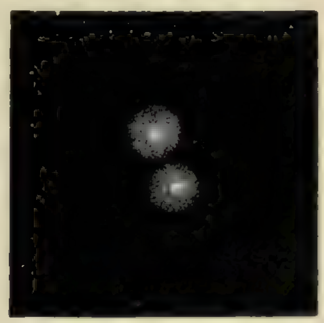
R.A. 140° 38′ ; N.P.D. 67° 45.

Fig. 99.



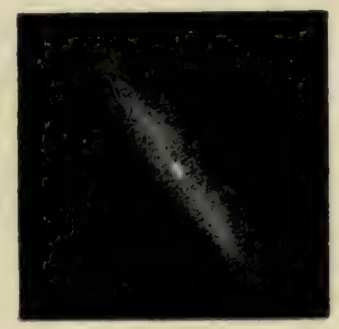
R.A. 183° 18 ; N.P.D. 84° 35′.

Fig. 100.



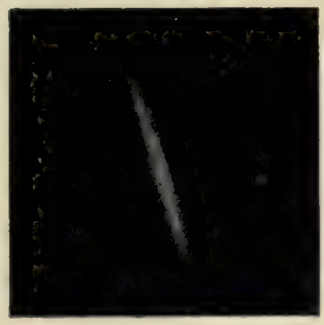
R.A. 342° 48′ ; N.P.D. 103° 43′.

Fig. 101.



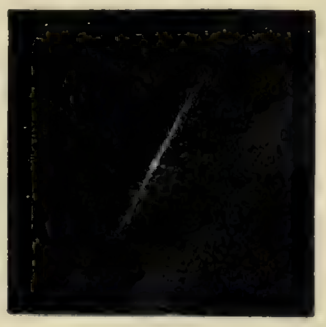
Andromeda.

Fig. 102.



Andromeda.

Fig. 103.



Andromeda.

NEBULÆ.—(See page 957).

animals most useful to them during the spring months. The Crab, walking backwards, is typical of the retreat of the sun from the northern tropics. The fierce Lion represents the intensity of summer heat. The Virgin, holding an ear of corn, refers to a girl gleaning. The Balance indicates the equality of day and night at the equinox; while the Scorpion indicates disease as the incident of the season.

Northern Constellations.—The constellations of the northern hemisphere are thirty-five in number, of which twenty were enumerated by Aratus. Of these Ursa Major is the most conspicuous, consisting of three principal stars, forming a triangle in the tail, and four forming a quadrangle on the body of the Bear. Commencing at the tip of the tail we have Benetnasch, a star of the second magnitude; Mizar, 7° distant west; and Alioth, about 4½° further off; 5½° from Alioth, at the root of the tail, is Megrez; south of it Phad, forming the shorter side of a quadrangle. On the opposite side, 8° west of Phad, is Merak; and 5° north, towards the pole, is Dubhe, the brightest star of the constellation.

Dubhe and Merak are called the pointers, because a line drawn through them, and carried about 29° in the same direction, passes almost over Polaris (the pole star), which is close to the north polar point in the heavens.

Ursa Minor, while it is inferior to the preceding in point of size, is more important from its position indicating the north polar point, and its utility as a guide in finding the latitude of places. Like Ursa Major, it consists of seven stars; three of the third magnitude, and four of the fourth. Polaris is the important star of this group. It is between the second and third magnitudes. It is at the tip of the tail of Ursa Minor, and appears stationary, the rest of the constellations appearing to swing round it in the diurnal revolution of the sphere. All the stars appear to revolve round the pole of the ecliptic, owing to the real revolution of the pole of the earth round it; a revolution, however, which requires the long cycle of 26,000 years, or thereabouts, for its performance.

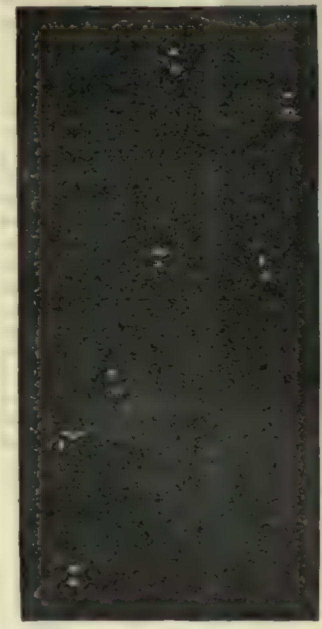
Bootes appears among the stellar groups to be driving on Ursa Major; hence it has sometimes been called the Bear Driver. Bootes has Arcturus, a star of the first magnitude, long supposed by the ancients to be the nearest star to the earth.

Southern Constellations.—The constellations of the southern hemisphere are forty-six in number; the most important being Orion, which constitutes the richest part of the visible heavens; and when on the meridian (which occurs about 10 p.m., January 1), presents the most magnificent view the starry heavens offer. Orion is visible, in its turn, to all the habitable world, the equinoctial passing through the centre of it. Four principal stars, in the form of a long square or parallelogram, form its outline. Betelgeux, of the first magnitude, is 7½° of Bellatrix of the second; Saiph of the third magnitude, and Rigel of the first, 8½° west of Saiph, and 15° of Bellatrix. Canis Major, on the south-east of Orion, contains one star of the first magnitude, four of the second, and two of the third; the former, Sirius, glowing in our winter hemisphere with a lustre unequalled by any other star in the firmament. Canis Minor, east of Orion, and north of Canis Major, has two brilliant stars—Procyon, of the first magnitude, and Gomelza, of the second, about 4° to the south-east. A knowledge of these constellations will enable us to find our path in the heavens.

When a few particular stars have been recognised, they will serve as starting-points; and by alignments, or imaginary lines, drawn from them, other stars and groups of stars will be found, and thus a general knowledge of celestial objects acquired. For this purpose, one of the most conspicuous objects is the constellation of Ursa Major, which never sets in our latitudes.

The seven stars (shown in Fig. 104) are nearly all of the same magnitude; a line drawn through β and α, and produced to a distance equal to that from α to η, will point out Polaris, an object of the 2·3 magnitude. This star will be easily distinguished, since it is not surrounded by any others of the same magnitude. The alignment to find the Pole Star is as follows:—A line drawn from δ Ursæ Majoris, or from ε Ursæ Majoris through Polaris, and produced the same distance as that from δ Ursæ Majoris to Polaris, will reach the centre of Cassiopeia. This remarkable group of stars, which, in these latitudes, is circumpolar, is always on the opposite side of Polaris to Ursa Major. When Ursa Major is near the zenith, Cassiopeia is near the horizon, and when Ursa Major is on the eastern side, Cassiopeia is on the western side of Polaris.

Fig. 104.



The right lines, which direct α and β Ursæ Majoris to Polaris, when produced in the same direction, will point out the square of Pegasus, formed of four stars of the second magnitude, the two upper of this group being respectively α Pegasi and α Andromedæ. A line drawn from the two latter will pass through β and γ Andromedæ, and finally to α Persei, a star of the second magnitude nearer the pole. It will be remarked that α Persei may also be found by drawing a line from δ, α, Ursæ Majoris through Polaris.

On the opposite sides of α Persei are situated γ and δ Persei, stars of the fourth and third magnitudes respectively; a line drawn from α, δ Ursæ Majoris will meet (after passing through α Persei) Algol, or β Persei, a star remarkable for its variability. Producing the arcs γ and δ Persei, we find α Arietis. Below these, the conspicuous cluster of stars, the Pleiades, are situated. If we join Polaris and α Arietes, and produce it beyond the latter, we shall meet with the constellation Orion, which is well known by its remarkable brilliancy and form. Of this group of stars, δ, ε, and ζ are called the belt; the remaining four stars of this group form an irregular quadrilateral, α and γ being the upper, and β and δ being the lower sides. α and β Orionis are of the first magnitude, and all the others are of the second magnitude.

The most remarkable object in the heavens, Sirius, or α Canis Majoris, is pointed out by producing the belt of Orion on the eastern side. By producing it on the western side, we meet with Aldebaran, or α Tauri, a star of the first magnitude. But this star may also be found by producing the line which connects α Ursæ Majoris and α Arietis (Fig. 105).

The diagonal of the square [α, β, γ, δ, Ursæ Majoris], or δ, β, being produced sufficiently, will pass through the bright stars of the constellation Gemini, α and β, or Castor and Pollux. A short distance in the same line, between Castor and Sirius, we find Procyon, or α Canis Minoris. This may also be found by prolonging the line passing through Polaris and

Fig. 105.

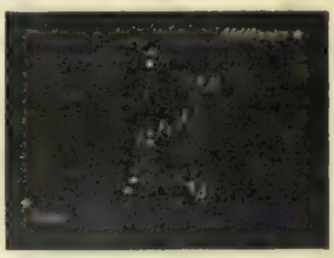


Fig. 106.

Pollux. The diagonal α and γ of the same square, produced on the side of γ above the pole, will point out α, or Spica Virginis, which forms an equilateral triangle with Arcturus and β Leonis.

The line connecting α and β Ursæ Majoris, which points so accurately to the Pole Star, when produced, will pass through the constellation Leo. This constellation consists of four principal stars, in the form of a trapezium. The most brilliant is of the first magnitude; all the others are of the second magnitude. ζ and η Ursæ Majoris, being connected in a line, will meet with a remarkable star of the first magnitude, Arcturus, or α Bootis.

At the side of Arcturus, and in the direction of the stars β, δ, ϵ, ζ of Ursæ Majoris, we find the constellation of Corona Borealis, composed of many stars, arranged in a semicircle, and of which the most brilliant is of the second magnitude.

Vega, or α Lyræ, is a conspicuous object, passing the meridian of Greenwich 13° south of the zenith. It forms a great triangle with Arcturus and Polaris, of which it occupies the summit of the right angle. At the side of α Lyræ are two stars of the third magnitude, β and γ, and three of the fourth magnitude, δ, ϵ, ζ. The four stars, β, γ, δ, ζ, form a parellogram, easily distinguished.

Between Lyræ and Pegasus the constellation Cygnus is found, composed of five principal stars, in the form of a cross. The line which joins Cygnus to Gemini is cut in two equal parts by Polaris. The same line, produced beyond Cygnus, passes through Altair, or α Aquilæ, a star of the first magnitude. α Aquilæ, is situated between γ and β Aquilæ, of about the third and fourth magnitudes respectively.

The folio plates afford a general view of the northern and southern constellations, with their most remarkable stars.

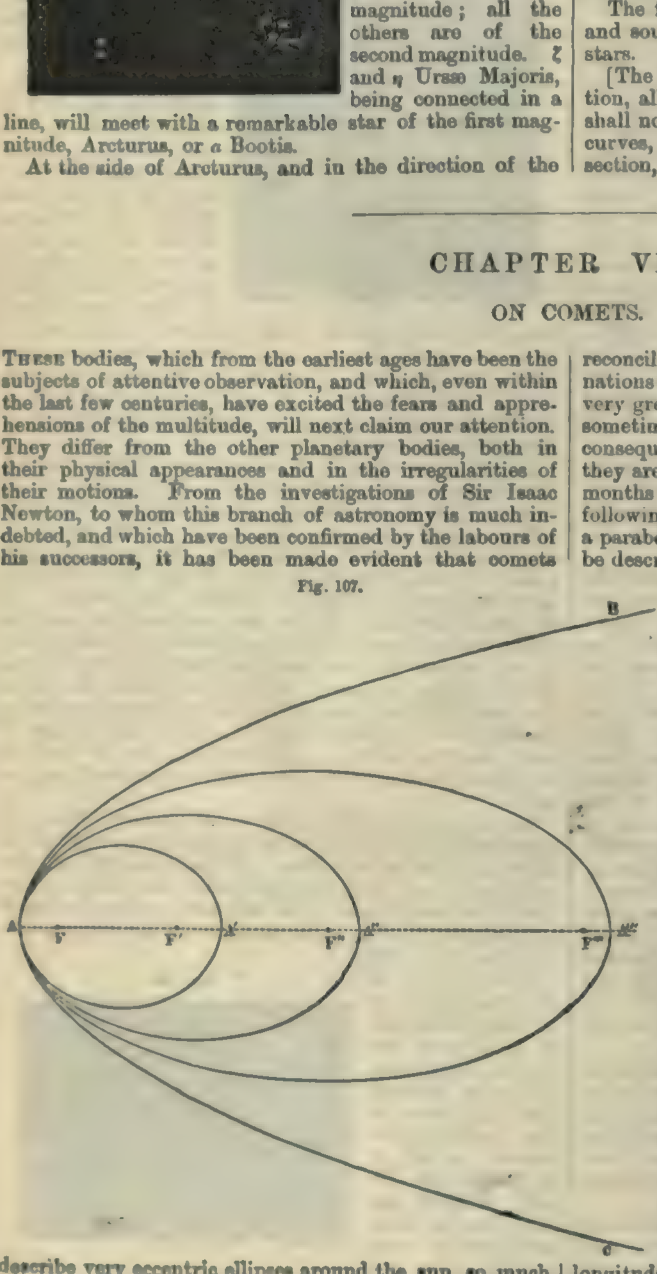
[The heavenly bodies, to which we have directed attention, all revolve in orbits of an elliptic character. We shall now pass to others moving in essentially different curves, which have been described in the Mathematical section, p. 608, *et seq.*]

CHAPTER VIII.

ON COMETS.

These bodies, which from the earliest ages have been the subjects of attentive observation, and which, even within the last few centuries, have excited the fears and apprehensions of the multitude, will next claim our attention. They differ from the other planetary bodies, both in their physical appearances and in the irregularities of their motions. From the investigations of Sir Isaac Newton, to whom this branch of astronomy is much indebted, and which have been confirmed by the labours of his successors, it has been made evident that comets describe very eccentric ellipses around the sun, so much so that, in the majority of cases, a parabola will perfectly reconcile their observed path in the heavens. The inclinations of their orbits to the ecliptic are also frequently very great, and the directions of their orbital motions are sometimes direct, and in other instances retrograde. In consequence of the great eccentricities of their orbits, they are seldom visible at the utmost more than a few months about the time of the perihelion passage. The following diagram (Fig. 107) will show, in a clear manner, a parabolic orbit. If with the two foci, F F′, an ellipse be described in the usual manner, we shall find that the major axis of this ellipse will be A A′, and that we shall obtain a curve differing little from an arch, and presented in a sense perpendicular to the axis A A′, or with a very slight degree of ellipticity. A second ellipse, with the foci F F″, will exhibit a greater degree of ellipticity, and will enclose the preceding one. A third ellipse, with the foci F F‴, will (still leaving A for the summit of the ellipse) show a still greater degree of ellipticity, and so on. By describing ellipses successively, we shall find the circumference of the curve more and more removed from the direction of the major axis A A″, till finally, we shall arrive at a point beyond which there is no variation. The curve B A C, thus described, is called a parabola. The point A is called the summit of this parabola, and the point F its focus. A parabola is then formed of two parts, A B and A C, exactly alike to one another, and extending infinitely on each side of the major axis, A F F′ F″. We can easily perceive that, as the comet recedes from the sun into space, it will be necessarily lost to our view. For the determination of the elements of the orbit of a comet, three complete observations are necessary as a first approximation, which can be afterwards brought to greater accuracy by means of the other additional observations during the time of its visibility. In a parabola the following elements are required:—1st, the longitude of the perihelion; 2nd, the longitude of the node; 3rd, the inclination of the orbit to the plane of the ecliptic; 4th, the time of perihelion

Fig. 107.



passage; and 5th, the perihelion distance. The observations will also give the direction of motion. Dr. Halley alone, acting according to the method of Sir Isaac Newton, calculated twenty-four orbits, by a comparison of which he was led to infer the periodic nature of the body which bears his name; and its certainty has been confirmed by the well-observed and authentic observations of successive returns.

Fig. 108.

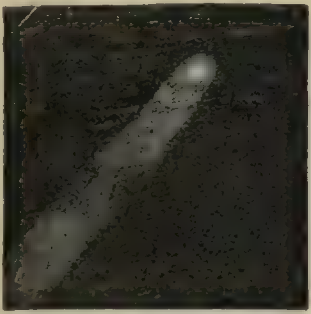


Fig. 109.

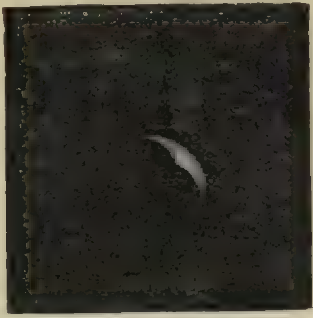
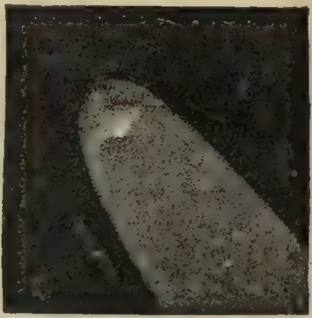


Fig. 110.



Modern astronomers have, with great zeal, applied the methods of Olbers and Gauss for this purpose, not only to those comets which have appeared in their own times, but also to others, whose paths in the heavens have been recorded by the ancient Chinese and European annalists. M. Langier has thus calculated, with a strong degree of probability, that the comets of 451 and 760, mentioned by the Chinese annalists, were returns of Halley's comet. Mr. Hind has also been led, by calculation, to consider the comets of 218 and 295 as appearances of the same body. But no certainty can be attached to any elements of this comet till the year 1378, the observations of which have been discussed by M. Langier, and of which the resulting elements are given with the others in the following table :—

		Long. of Perih.	Long. of Node.	Inclination.	Long. Perih. Dist.
1378, Nov.	8·764...	299° 31'...	47° 17'...	17° 56' ...	9·7560 R.
1456, June,	8·917...	301° 0'...	48° 30'...	17° 56' ...	9·7675 R.
1531, Aug.	24·888...	301° 39'...	49° 25'...	17° 56' ...	9·7538 R.
1607, Oct.	26·716...	301° 38'...	48° 40'...	17° 12' ...	9 7694 R.
1682, Sept.	14·795...	301° 56'...	51° 11'...	17° 44' ...	9·7656 R.
1759, March,	12·552...	303° 10'...	53° 50'...	17° 37' ...	9·7668 R.
1835, Nov.	15·939...	304° 32'...	55° 10'...	17° 45' ...	9·7683 R.

Fig. 111.

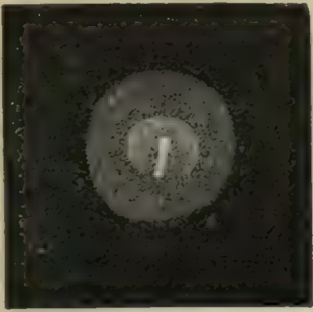


Fig. 112.

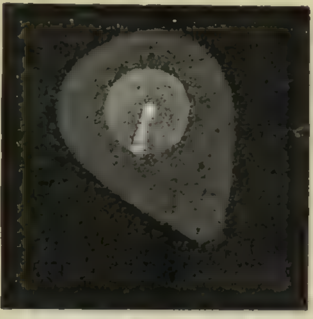
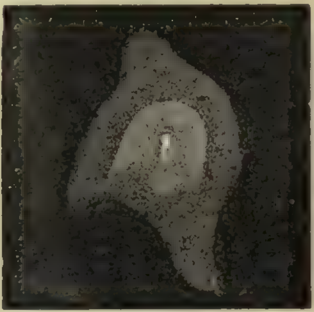


Fig. 113.



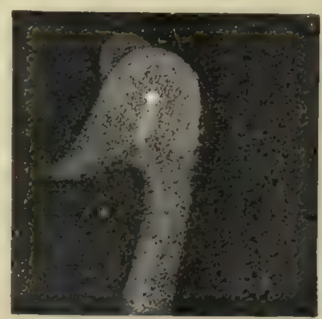
The striking similarity of the elements of the comets of 1531, 1607, and 1682, immediately presented itself to the mind of Dr. Halley, who was also led to infer that, by the influence of the planet Jupiter, the period of its next return would be considerably retarded. This prediction was found to be accurate by the laborious calculations of Clairaut, who, after minutely computing the disturbing effect of Jupiter and Saturn, found that its next return would be delayed by 618 days, as he concluded that the comet would arrive at its perihelion on April 13th, 1759.

The actual return, from the best elements, took place on March 12th, 1759, and the comet was seen for the first time at Dresden, by Pazlitch, on Christmas Day, 1758. This body being thus ranked in the planetary system, the investigation of the period of its next arrival at the perihelion was made the subject of a prize in several European academies. The investigations of Damoiseau and Pentécoulant, on this question, gave only the difference of one week in the predicted time of the arrival at the perihelion—an agreement very accordant, when the great intricacy of the problem is considered. The predicted return was stated to be on November 13th, 1835; and although the comet was attentively looked for during several months previously, it was not seen till August, when it was discovered at the Collegio Romano, of Rome. From the best elements, the return to perihelion took place on November 16th, the predicted time being only three days in error.

With regard to the physical appearances of these bodies, it is generally admitted, at present, that comets possess a very small degree of density.

The above diagram (Fig. 108) will show the appearance of Halley's comet, as observed by Sir J. Herschel, at the Cape of of Good Hope, on October 28, 1835. The bright spot situated in the head of the comet, is called the *nucleus*, and the luminous appendage which accompanies and surrounds this nucleus, is called the *tail*. On October 29, Sir J. Herschel observed the comet with a twenty-foot telescope, and noted its appearance, as in Fig. 109. A little later, on the same evening, the appearance was in Fig. 110. On January 25, 1836, it had an appearance as in Fig. 111; in the intermediate time the comet was not visible, in consequence of its proximity to the solar rays. On January 26, 27, and 28, 1836, it presented the appearances indicated in the Figures 112, 113, 114.

Fig. 114.



As to the light which these bodies possess, the late M. Arago inferred, from his experiments, that they shine by reflected light. But, although his experiments on the comet of 1819, and that of Halley in 1835, showed that they shone by reflected light, which he satisfactorily confirmed by experiments with a polariscope on the light of stars of the same magnitude in their neighbourhood, he was rather doubtful whether they had not a proper light of their own, since, without losing their property of reflection, some independent light might at the same time reach the earth.

So long ago as the year 837 of our era, the circumstance was remarked by the Chinese annalists; and again noticed by Appian, in the 16th century, that the tails of comets were generally directed from the sun. The appearance of these accompaniments are variable, both in regard to magnitude and brilliancy. Of the comet of 1689, the contemporary historians state that its tail occupied an extent of 90°, and presented an appearance similar to a Turkish sabre. The tail of the comet of 1769, was observed to be 97° in length. The comet of 1744 had as many as six tails, each 4° in breadth, and 30° to 44° in length. In our own days, the appearance of the comet of 1861 cannot be forgotten. At London, in July, its length, by observation, was found to be from 60° to 80°. The comet of 1618 had also a tail 104° in length.

In addition to Halley's comet, others of nearly similar period present themselves to our notice, and this remarkable group consists of the following:—

1st. The comet of 1812, which was discovered by Pons in that year, and only visible with the aid of a telescope. Encke, from all the observations, calculated its period to be 70·7 years.

2nd. The comet of 1815, discovered by Olbers, the period of which has been calculated by Bessel to be 74·05 years. According to the investigations of Bessel, the period of the next return will be retarded by two years, in consequence of planetary perturbations, or it would appear in 1891.

3rd. The comet discovered by De Vico in Rome, in 1846, Feb. 20, and observed till April. The calculations of De Vico and Peira, show that its orbit is decidedly elliptical, and that its period is upwards of 70 years. The only comet in the list of calculated orbits, similar to this body, is that of 1707.

4th. The next comet, nearly identical in period, is that discovered by Brorsen on July 20, in Triangularus, and observed at several places for the space of one month. Elliptic elements, compiled by D'Arrest, give its period as 75 years.

Another comet of long period, but the elements of which are not so well worthy of confidence, is that discovered by Flamsteed in 1663. Elliptical elements of this body, which are, however, too uncertain to be much relied on, assign to it a period of 187·5 years.

Several other elliptical orbits have been calculated, which will appear in a table at a subsequent page.

The most remarkable periodical comet (which there is a great reason to believe to be such, from the strong identity of the elements) is that discovered by Fabricius, in 1556, and which is considered to be a second appearance of the fine comet of 1264. Necessarily rude as the observations of this period were, Pingré and Dunthorne have calculated its elements from its recorded path, which agree in every particular with the comet of 1556. This subject has recently occupied the attention of M. Baume, who infers, from his investigations, that the comet will be retarded in its return to the perihelion by a period of some years. The last appearance, in 1556, was magnificent in the extreme. The tail was more than 100° in length; and, according to the Chinese accounts, it presented a curvature similar to a sabre. It remained visible till the 2nd of October of that year. There was a strong probability in the inference that this body would have returned to its perihelion, in the period from 1858 to 1860; but, as the elements are based on such uncertain data, we must not be astonished at the error which its absence has proved to exist.

After thus mentioning two comets, whose periods are the longest on record, and the first of which has been made evident by many successive returns, we shall now proceed to others, whose revolutions around the sun are of shorter duration. The first, and most celebrated, is that of Encke. This body was discovered by Pons, at Marseilles, on November 26, 1818; but the credit of being the first calculator of its elliptic elements is due to Encke, from which circumstance it bears his name. The similarity of the approximate parabolic elements to those comets observed in 1786, 1795, and 1805, immediately presented itself to the mind of Encke, who likewise found that a parabolic orbit would not satisfy the observations of 1818, 1819. Its period is approximately three years and a quarter. Since 1819, no return to the perihelion has passed without observation; and for this, in a great measure, astronomy is indebted to the laborious calculations of Professor Encke, who furnished the ephemerides of its predicted places. On June 2, 1822, the comet was seen by Rümker, for the first time, at the Observatory of Paramatta, in New South Wales, which had then been recently founded by General Busham. Although it was only observed there for a short space of time, the observations were found of considerable value in determining its next return in 1825. True to prediction, the comet was discovered by Valz, on July 13, within a short range of the calculated place. At this period the comet was well observed at Naples, and other European observatories. On its next arrival at its perihelion, 1828, it was first seen at Dorpat, by Struve, on October 13, and was observed in European observatories till December 25.

Although very unfavourably situated for observation in the northern hemisphere, in 1832, the comet was observed by Harding, at Göttingen, on August 21. Professor Henderson also observed it at the Cape of Good Hope; but the observations of this return were so few and scattered, that Professor Encke, in his investigations for its return in 1835, made no use of any of them. In 1835, the comet was observed, on July 26, by Boguslanzki, and previously by Kiel, at Milan, on July 22.

In 1838, it was again seen by Boguslanzki, on August 14, as a very faint, shapeless object, which afterwards increased in brightness till November. Professor Encke, in combining all the preceding returns at this period, found it necessary to alter the received mass of the planet Mercury, and also to allow for the effect of a resisting ethereal medium, pervading all space, and which had an appreciable resistance to the cometary motions. With these two important discoveries, Professor Encke published an ephemeris for its reappearance in 1842, on the 8th of February of which year it was observed by Dr. Galle.

Although the return of 1845 was very unfavourable, the comet was seen at Rome, and in America; but the observations were so few, that no use has been made of them in any investigation.

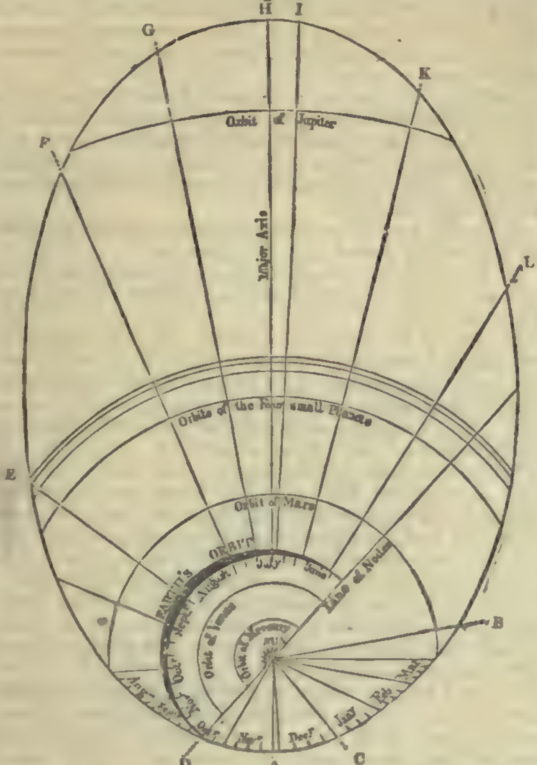
In 1848, it was observed for a period of three months, extending from August to November. The comet was very faint at first, but towards November its brightness increased considerably.

In 1852, the perihelion passage took place on March 14, and the comet was visible on the 2nd of January.

In 1855, the return of the comet was not favourable for observation in the northern hemisphere, but it was seen at the Cape of Good Hope, by Maclear. It was seen in November, 1861, by Valz, at Marseilles.

GAMBART'S COMET.—This body was discovered by Biela, on the 27th of February, 1826; but, as in the preceding case, it bears the name of Gambart, who first computed its orbit, and noticed its similarity with the comets of 1772 and 1805. Elliptical elements calculated by him and other astronomers, assigned to it a period of 2,460 days. In the interval of time from 1772 to 1826 it had performed six revolutions. The return to the perihelion, in 1832, was observed generally at all the European observatories. It was first seen at Rome, on August 25th, as a faint object, the light of which, however, increased afterwards. Professor Henderson likewise observed it at the Cape of Good Hope. On a calculation of the elements of its orbit—which are represented, with the relative position of the earth, in the following engraving—in order to ascertain the precise period of its return, it was, of course, necessary to study the perturbing influence of the planets; and when their places were ascertained and compared with its course, it was supposed that it would cross the earth's orbit about a month before our planet was at the point of intersection. This announcement excited much curiosity and some alarm. It will be evident, however, from a consideration of its orbit, that it never came within sixty millions of miles. It was a small, insignificant comet, looking like a collection of nebulous matter, and having neither tail, nor any appearance of a solid nucleus. In the engraving, A is its perihelion and H its aphelion, B its position, January 1st, 1840; C, on January 1st, 1833; D, its December node; E, its position January 1st, 1839; F, January 1st, 1838; G, January 1st, 1837; I, January 1st 1836; K, January 1st, 1835; L, January 1st, 1834. It will be seen from this diagram the point where the earth's orbit and that of the comet intersect is very near the descending node. The time at which the comet reached its perihelion in 1832 was the middle of November; it crossed the earth's orbit on the 29th of October, but the earth did not reach the same point until the 30th of November. At its return, in 1839, the comet was not seen from any part of the globe. In 1845, however, by means of an ephemeris calculated by Professor Santini, the comet was discovered by De Vico, at Rome, on November 26th. In a very short time afterwards a most singular circumstance took place; the comet appeared to have been divided into two portions. This appearance was first seen by Professor Wichmann, about the middle of December, and generally confirmed by other observers. The two nuclei were also observed till the 22nd of March, 1846.

Fig. 115.



On its return to the perihelion, in 1852, the comet was again seen by Professor Secchi at Rome, and at the Cambridge Observatory in England. At this time, although the predicted ephemeris was in error, both nuclei were again observed, but they had separated considerably. The comet at this return was exceedingly faint, and was only visible on nine different occasions. This and the comet of Encke are comprised within the limits of the solar system, and do not extend beyond the orbit of Saturn.

Faye's Comet.—The next known periodical comet, whose orbit is comprised within that of the planet Neptune, is that discovered by M. Faye, at the Paris Observatory, on the 22nd of November, 1843. At its first appearance it had a brilliant nucleus and tail. Faye and others (Gauss included) discovered that its orbit was elliptical, and that its period was 7·2 years. At first it was imagined by Argelander and Professor Henderson to be the lost comet of 1770, or that of Levell; but Leverrier has since proved that this could not be the case. Its orbit at aphelion and perihelion approaches near to Mars and Jupiter, and must have experienced considerable perturbations from the latter.

M. Leverrier calculated an ephemeris for its return in 1850, and, by means of this, the comet was observed on November 28th, of extreme faintness, at the Cambridge Observatory in England. With the exception of two other observations obtained in the Cambridge Observatory (U.S), by Mr. Bond, it was not seen at any other observatory. Mr. Bond described it as a very faint object in the twenty-three feet refractor.

The last four periodic comets are known as such by the frequent observations of their returns to the perihelion. But others exist, whose ellipticity is equally decided, although their returns have not been observed. Of these, the first is the comet of De Vico, comprised, as in the other case, within the planetary system, and discovered by him at Rome, in 1844. Its period was found to be 1980 days; but although its orbit has been ably investigated by Dr. Brünnow, it has not been seen either in 1850 or in 1855.

Brorsen's Comet.—This comet was discovered at Kiel, in Denmark, by Brorsen, on the 26th of February, 1846, and found to be elliptic, its period being upwards of five years and six months; Drs. Brünnow and Petersen consider that it was identical with the comets of 1532 and 1661. Dr. Halley, likewise, inferred the similarity of these comets; but, although expected, it was not visible in 1790. It appears probable that the influence of the planet Jupiter has a great effect on the elements of this body. Mr. Hind is of opinion that it experienced considerable perturbations from this planet in 1842.

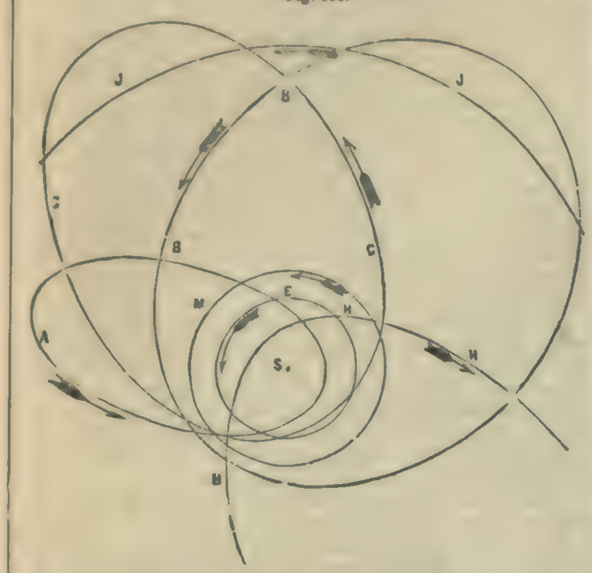
D'Arrest's Comet.—This comet, whose period is six years and five months, was discovered by D'Arrest in 1851, June 27th. It suffers great perturbation from the planet Jupiter in its path, and the time of its next return is not yet fixed.

Another remarkable comet of this class is that discovered by Messier in 1770, the orbit of which had been calculated by Levell, indicating a period of about five or six years. The planetary perturbations have, however, altered the path of this comet so considerably that it has never been since observed.

Fig. 116 shows the relative orbits of four periodic comets, and the comet of Halley. It must be remarked, however, that the orbits are not in the same plane, and that the cometary inclinations are sometimes very considerable. In Fig. 116, A is an orbit of comets of short period; B B, the comet of seven and a half years; C C, Gambart's comet; E, the orbit of the earth; H H, Halley's comet; J J, the orbit of Jupiter; M, the orbit of Mars; S, the position of the sun.

Nature of Comets.—Some attempts have been made to define with precision the difference between a star and a comet, and determine by what characteristics a newly discovered star is to be distinguished from one of these wanderers of the firmament. The planets always move in the same direction; the plane of their orbits being slightly inclined one to the other, and the eccentricity of these orbits are trifling. Comets on the contrary, for the most part, describe orbits so much elongated that they seem to move in a sort of parabola; and, in the case of the few with whose movements we are best acquainted, the eccentricity of their orbits distinguishes them in a remarkable degree from the planets. Each newly discovered object that is observed in an ellipse, only slightly eccentric, has been classed among the planets; while those which do not satisfy these conditions are regarded as comets. The comets we have described, while differing in many respects, seem to agree in the great degree of eccentricity of their orbits, and would seem to satisfy the inquirer that they cannot have the same origin.

Fig. 116.



Comets generally present the appearance of a brilliant nucleus, surrounded with a nebulosity which extends on one side more or less distant from the centre. This nebulous matter is so transparent, that stars, even of small magnitude, can be observed through the tail, and even through its thicker parts. The nebulosity of a comet, therefore, may be regarded simply as an extremely light vapour, which accompanies the central nucleus. The rapid changes which have been occasionally observed in the form of comets would seem to confirm this impression. For example, Halley's comet was observed with much care by Herschel at the Cape of Good Hope at the end of 1835, and we have already noted the changes he registered; but the most striking instance of change on record occurred in January, 1846, when Gambart's comet was observed to be separated into two parts, each having formed an independent nucleus, with the usual nebulous accompaniment. When the comet again appeared in 1852, the two objects also appeared, but at an increased distance.

Notes on Comets.—371 b.c. According to Diodorus of Sicily, about the end of the year 371, a comet appeared of prodigious magnitude, which was accompanied at the same time by inundations and an earthquake. Its apparent motion was from west to east, but its real motion was probably retrograde. From all recorded circumstances, it appears when observed to have been near its perihelion, and the inclination of its orbit to have been very great.

Seneca relates that the comet was divided into two portions near the end of its appearance. Stuyck is of opinion that the comet was identical with that of 1664, but there are no grounds for this supposition.

344 b.c. At this period, Diodorus relates that Fabius and Sulpicius, being consuls, on the occasion of Timoleus of Corinth undertaking an expedition to Sicily, a burning torch appeared in the heavens the whole night, and preceded the train of Timoleus, even till its arrival in Sicily. We may then infer that it had a considerable north declination, and that it appeared in the western heavens.

203 b.c. M. Cornelius Cethegus and P. Sempronius Iuditanus being consuls a comet was seen at Sethia, extending from east to west. The Chinese annals confirm this by its appearance, about the month of August, near Arcturus.

172 b.c. P. De Maille relates that a great comet appeared in China, at the end of the summer of this year, which had a tail.

156 b.c. In China, near the end of October, and visible for a period of twenty-one days, a comet in the west, which traversed the constellations of Aquarius, Equuleus, and Pegasus.

156 b.c. In this year, Seneca relates that, after the death of Demetrius, king of Syria, a comet appeared as large as the sun. At first its appearance was red and fiery, emitting sufficient light to dissipate the darkness of night. Gradually, however, its magnitude diminished, till it finally vanished.

136 b.c. In this year three comets were seen, which all may belong probably to the same body.

I. S. M. Emilius and C. Hostelius Mancinus being consuls, a burning torch was seen in the heavens at Prenéste.

II. Under the reign of Attalus, king of Pergamus, who reigned from 138 or 137 to 132 b.c., a comet was seen, which is supposed to be that of 136 b.c., and which, small at first, gradually increased till it reached the equator. Its tail equalled in length the parts of the heavens whose extent is the "Milky Way."

III. At the birth of Mithridates, who lived from this year to about 65 b.c., a comet, whose brightness was greater than that of the sun, lasted for about seventy days; the heavens appeared on fire, and the comet appeared to occupy one-fourth part of the sky; but it is doubtful whether this occurred in this year or 134 b.c.

133 b.c. In this year at Anisterne, the sun was seen during the night, and this lasted for some time; but it is likely that these appearances were due to the meteors which we so frequently observe.

48 b.c. Lucian mentions that in this year a terrible comet appeared; the darkness of the night was illuminated by it. In 49 b.c., or probably 48, a comet was seen in China whose paths were extended from β Cassiopeiæ through ι Cassiopeiæ, till it was lost finally among those stars which never set.

43 b.c. In the latter part of September of this year, during the feasts in honour of Venus, a comet was seen which was of great brightness, and was visible in all parts of the earth. It was visible about 5 p.m., when the brightness of the sun permitted it to be observed. It is very probable that it was in the sign Leo, with a north latitude of 35° to 40°, and that it was identical with the comet of 1680. In China, in this year also, a comet was seen in the months of May or June, which was in the constellation Orion; but, if this be the same comet as the preceding, it will not agree with the elements of the comet of 1680.

Since the Christian Era.—14. According to Dio Cassius, S. Pompeius Magnus, and S. Apuleius being consuls, many comets of the colour of blood were seen to shine.

39. In the Chinese annals mention is made of a comet in this year, which was visible from the 13th of March to the 30th of April, or for forty-nine days. Its path was from the Pleiades, through Pegasus, and finally to the head of Andromeda.

60. In China a comet was seen in the constellation of Capricornus, whose tail was eight degrees in length; its appearance lasted for fifty days, and it was last observed in the south of the head of the constellation Scorpio.

76. Pliny relates of a comet being observed this year, which was described by Titus Cæsar as having occurred in his fifth consulate. In China it was observed from the 7th of September, and was visible for forty days. Its tail was 3° in length, and the path of the comet was from the head of the constellation Hercules to 3° of the east of β Capricorn.

117. In the 9th of January of this year a comet was seen towards the west. On January 14th it was seen near β Libræ and α Equulei; it afterwards passed to the constellation Musca.

141. According to the Chinese annals, on March 27th

of this year, a comet, whose tail was 6° or 7° in length, was seen near α and β Pegasi. On the following 16th of April it was near ξ Andromeda. On the evening of the 22nd of April it had passed over the Pleiades, or about 11° of right ascension. On the 23rd of April, it was seen in the feet and legs of Gemini, and finally disappeared in Leo.

240. Seen in China, on the 10th of November, a comet with a tail 30° in length in Scorpio; it passed through Capricornus, and on the 5th of December entered into Libra. Its latitude was 2° south on the 10th of November, but at the beginning of December it was not an entire degree. On December 19th the comet was south of the ecliptic, and was in the constellations of Aquarius and Cetus.

252. In this year two comets were seen in China. The first was seen in the wing of Pegasus on January 10th. It was to the west, and appeared seventy days.

On the 25th of March, a comet was visible in the constellation Musca; it was seen for twenty days, and the length of its tail was 50° or 60°.

336. In China, on the 16th of February, in the evening, a comet was seen in the southern arm and the girdle of Andromeda, and in the northern part of the constellation Pisces. It is probable that this is the comet which is mentioned as having been seen at the death of the Emperor Constantine, who, however, died on May 22nd, in the following year.

363. At the end of August of this year, according to the Chinese annals, a comet was seen near the stars, α, ξ, κ, λ, ι, υ Virginis. Its brightness was so great that it was visible in full day.

389. In this year a brilliant comet was visible, in the month of August, in the morning, which Mascellin, Philosotorgus, and Nicephorus mention. It equalled Venus in brightness, and was erroneously placed near this planet. Payne has calculated that the planet Venus was at this time in inferior conjunction, and that it did not become visible in the morning till November; but he thinks that it was very likely that it was near Jupiter. Jupiter was, at this time, in August, to the west of the sun, and would rise a short time after midnight. The comet lasted for forty days, and finally disappeared in the constellation of Ursa Major.

390. In this year a comet appeared similar to a hanging column, which lasted for thirty days. There appears to be no doubt that this was a different body from that of the preceding year.

400. The historians of this year, Socrates and Sozerames, make mention of a terrible comet which occurred, and was seen in this year. Its form was that of a sword. It was also observed in China on the 19th of March, near the northern part of Pisces, and the girdle and southern arm of Andromeda; its tail was 30° in length. It successively passed through the constellations of Cassiopeiæ, and to the square of the Great Bear, where it was near χ Ursa Majoris, then to α Hercules, and finally between γ and β Virginis. But this does not appear probable, for it would have been more so, if, instead of the head of Hercules, it had been written the tail of the Lion.

418. During an eclipse of the sun of this year, on the 19th of July, a comet was seen which had a conical form, and was visible for a period of four months. This is confirmed by an actual calculation of this eclipse, which occurred at this time. It was also seen in China, and its path was as follows:—At its first appearance, it was near the star δ Cygni, from which it went to the square of the Great Bear; it afterwards proceeded to that part of the heavens which always remains above the horizon, to the constellations Bootes, Virgo, Leo, and Corona. Stuyck was of opinion, at first, that this comet was identical with that of 1596, but afterwards renounced this idea.

467. Perseus and John being consuls, a great prodigy was seen for some days in the heavens. It was observed for forty days, but in some places it was only seen for ten days.

504. In this year a star or comet of great brilliancy was seen, but the Saxon annalist describes it in exaggerated terms.

530 or 531. The Chinese annals make mention of the first, which appeared in the month of October, 530. Its course was from Arcturus to λ Ursæ Majoris. This would agree with the motion of the comet of 1680, which period Halley had assigned to it; and we may, therefore, infer its probability. The second, which appears to be a different body, was seen at Constantinople in 531. All the Byzantine authors speak of the last, which is described as having been a large and terrific body, and was visible with them for twenty days. Its rays extended to the zenith, and it was compared to a lamp.

539. This comet, which was visible for forty days, occurred in the 13th year of Justinian. Its head was in the east, and its tail extended to the west. Its appearance is described as being equal in magnitude to a great man, which afterwards increased. It was subsequently observed in China, in the constellation Sagittarius, near μ, τ, φ, σ, τ, and ζ. Its tail was 10° long. On the 1st of January, 540, the comet was within 3° of Venus; but this appears improbable.

565. On the 4th of August of this year, a comet was visible in China, and remained so for 100 days. It was seen in ε, f, θ, φ, o, and h of the Great Bear. Its tail was not a whole degree in length. It then approached γ Aquarii, and ε and θ Pegasi, at which time its tail was 10° long. Its last appearance was in Equuleus.

566. In this year a comet was seen which, according to one chronicler, lasted for a whole year; but, according to another, which is more probable, only remained for sixty-six days. It is described as throwing out a long flame, and appeared in the Arctic circle.

568 (first comet). On the 10th of July of this year, a comet was seen in China in the feet and thighs of Gemini.

568 (second comet). Another comet was observed by the Chinese this year, on the 3rd of September, in the face and heart of Scorpio, and proceeded eventually to the east. On the 8th of September, it had a tail of 40°, near the northern star of Delphinus. It then passed through the constellations near β Aquarii and α Equulei, and α Aquarii, and ε and θ Pegasi, and entered into the constellation of the Eye of Pegasus. On the 8th of October it was in the head of Aries. It was observed altogether for sixty-nine days.

582. At Easter day, which occurred on the 29th of March, a great comet is mentioned by many historians as having been seen at Soissons.

607. In China, two comets are mentioned as having been seen, which some astronomers consider identical. The first appeared on the 4th of April, in the west. It passed through the constellations, the Northern Fish, the girdle and the southern arm of Andromeda, the head of Aries, α and ζ Virginis, and the feet of the Virgin, where it disappeared.

The second was seen on the 21st of October, in the same constellations. It passed over the northern part of Leo, of Virgo, and of others stars more to the north. The comet did not attain the constellation α, β, γ, δ, ε, ζ, η, θ, κ, Orionis.

615. In July a comet was seen in China, in the Great Bear. Its tail was 50° or 60° in length. During the night its head or nucleus had, as it were, a motion of libration.

676. In the beginning of this year a comet was discovered in China, whose tail was 5° in length, to the south of the constellations α and ζ Virginis, and the feet of the Virgo.

729. Bede and other chroniclers mention a comet as having appeared in the month of January of this year, which was near the sun for 14 days. But some mention two having been seen, which is improbable. It is likely that it had a right ascension little different from the sun, with a northern declination, which would account for its setting after the sun, and rising before it.

837. Father de Maille, according to Pingré, relates that on the 22nd of March, in China, a comet was seen near α Aquarii, ε and θ Pegasi. The tail of the comet was 7° long.

On March 29th, it was seen in β Aquarii and α Equulei. On April 6th, it had a tail 10° long, and its motion was towards the west. On April 10th, the tail was 50° long, and separated into two portions. On April 6th, its tail was undivided, and its length was 60°. On April 14th, the length of the tail was 80°, at which time the comet was in the constellation Hydra. After this time the length of the tail decreased considerably, and on April 28th, it was seen for the last time, with a tail only 3° long. Pingré, after having carefully investigated all the preceding observations, deduces an orbit which appears in a table at the end of this section: there are reasons for inferring that it was probably an apparition of Halley's comet, which, from some errors in the published accounts, will not agree with the observations. There also appears to be a doubt as to the year of this appearance—the Latin accounts agreeing consistently in placing it in 838, whilst the Chinese give 837. Were two comets seen, or is there an error in the Chinese account? The account of an anonymous Latin historian differs, both in date and position. It is nearly as follows:—It was first seen in Virgo, and, pursuing a retrograde course, it traversed successively Leo, Cancer, and Gemini, all in the space of twenty-five days, and finally disappeared in Taurus, under the feet of Auriga. Pingré's orbit, which is based on the Chinese observations, will not agree with the latter path; and we may infer, with some reason, that the year was 838, and the comet of which the Latin historian speaks was that of Halley.

855. In this year, an ancient chronicler mentions two stars being seen in the month of August, on ten successive occasions. They were possibly comets, the larger being always visible, but the smaller not so freqently. A comet was also seen in France during 20 days.

875. On June 6th, an extraordinary blazing comet was seen, with a fine tail. It appeared first in Aries, and was visible during the whole month of June. This comet was supposed, at the period, to be the announcement of the death of the Emperor Louis II.

891. In China, on May 12th, a fine comet was seen in Ursa Major, with a tail 100° in length. This comet is also mentioned in the Saxon annals. Pingré was of opinion that this comet was identical with that of 1532 and 1661.

895. A fine comet, whose tail was 100° long, was visible in China, on June 25th. It was first seen near ι and κ of the Great Bear, and in the course of its appearance it passed over Corona, part of Hercules, and Serpentarius. The length of the tail increased to 200°, which, however, Pingré asserts is difficult to believe.

905. In this year, the European and Chinese annals mention the appearance of a great comet in the months of May and June.

912. In this year the Latin authors relate that a comet appeared in the month of March, for fourteen days, in the north-west. A Greek author mentions its duration as being forty days.

931. A comet was visible in Cancer, in the months of May, June, and July of this year.

939. On the 19th of July of this year a long eclipse of the sun was seen; and in Italy, during eight successive nights, a comet of great splendour was beheld.

942. A comet was seen in October of this year, for upwards of twenty days, in the western heavens. Its motion was towards the east. Its head was faint, but the tail projected and resembled smoke.

975. In the autumn of this year a comet is mentioned by the Latin and Chinese historians. In China it was seen on August 3rd in Hydra; its tail was 40° in length. The comet during the time of its appearance, which was for eighty-three days, passed through the constellation Cancer to the space between γ Pegasi and α Andromedæ. Pingré remarks that there is a considerable similarity in the above path to that of 1556, which is supposed, with great probability, to be identical with that of 1264. Taking its perihelion passage a few days before the end of 975, July, Pingré finds that it might have been seen about the 11th of July, which Father Ganbil states to be the case in China. On August 3rd it would have been in conjunction with the sun, but its northerly latitude being considerable, it would rise some time before it, and might leave, as mentioned, a tail 40° long. Its course would then be retrograde, apparently through the constellations of Cancer, Gemini, Taurus, and Aries, in which latter sign it would probably be visible about the month of October. There is sufficient accordance in the above, to give this opinion of Pingré's considerable weight.

989. An appearance of Halley's comet is supposed to have occurred in the autumn of this year. On August 5th it was seen in the constellation of Gemini, and, pursuing a retrograde direction, it passed through the constellations of Leo and Cancer. Buckhardt has computed an orbit from the rough account of the Chinese annalist, which appears in our table.

1000. A comet of extraordinary brilliancy was seen this year for nine days; but there appears to be some doubt as to the exact period.

1066. In the April of this year the Chinese and European annalists mention that a comet of great brilliancy was seen. It was supposed to be the forerunner of the conquest of England by William duke of Normandy. The orbit has been computed by Pingré, from the Chinese annalists, and there appears a strong probability of its identity with that of 1677.

1097. Although the comet of this year was only seen for a very short time (in Europe only fifteen days), it is remarkable as projecting two rays or tails, which were directed to the east and south-east respectively. In China, on October 6th and 9th, its tail was noted as being respectively 30° and 50° in length. On October 6th it was seen in China near α and μ Libræ. On October 16th it was seen near the head of Hercules. It ceased to be visible in China on October 25th.

1106. A great and fine comet was seen in Palestine on February 7th, and in China on February 10th. It was seen on February 7th in Pisces. Its tail was similar in colour to the whiteness of snow. Its appearance lasted for fifty days. In China, on February 10th, its tail was 60° in length, and extended from Gemini to Orion. During its apparition, it traversed the path from the end of Pisces to the end of Taurus. There is some similarity supposed to exist between this comet and that of 1686.

1222. In the months of August and September, a comet of extraordinary magnitude, very red, and accompanied with a great tail, was seen. In China it was observed on September 10th, beneath the feet of the Virgin, Arcturus, and Berenice. It disappeared on October 8th.

1231. A comet was seen in China, on February 6th, in Cygnus; its magnitude was equal to Saturn. From the recorded path, Pingré has calculated an orbit, which appears in our table.

1264. A great and celebrated comet, of which all the historians make mention. It is supposed to be identical with the comets of 975 and 1556. It was visible for some months—at least three, probably four. Its disappearance occurred the same day as the death of Pope Urban IV., or on October 3. Pingré and Dunthorne have calculated its orbit.

1265. An historian mentions that, at the commencement of autumn of this year, a comet was seen with a long tail, which commenced to shine after midnight.

1266. In the month of August of this year a comet was visible in France. It was also seen at Constantinople, near the sign of Taurus; there appears some doubt, however, with respect to this last account. It is questionable whether the comets of 1265 and 1266 be not identical.

1299. On January 24th of this year a comet was seen near Columba. It was visible for sixty-three days. Pingré has deduced its orbit from all the observations.

1301. A great comet, which is mentioned by the Latin historians as having occurred at the time of the autumnal equinox. It was also seen in China for forty-six days, commencing with September 16th. The elements deduced by Pingré are very uncertain, but Mr.

Hind is of opinion that this is identical with the comet of Halley.

1337. At the summer solstice a comet is noted as having occurred about the time of the death of Frederic, king of Sicily. It was also seen in China. The European historians give its duration for three or four months.

1337. A second comet was seen in this year, but it was not observed in China. The European historians mention it as having lasted two months.

1351. In the month of December a comet was seen in Cancer in the east. The account, however, is obscure, and gives its position with uncertainty. From the Chinese positions of November 24th, 26th, 29th, and 30th, Burkhardt has determined the time of the perihelion passage in November, as 26 days 12 hours.

1362. On March 5th a comet was seen in China, near the constellation of α Aquarii and ε and θ Pegasi. It disappeared on the 7th of April. It was visible in Europe during Lent as a very great and brilliant star.

1362. On June 29th this comet was seen in China for forty days. Its first appearance was in Capricornus, and it had an extensive tail.

1366. On August 26th a comet was seen in China, in Ursa Major. On August 27th it was visible in the tail of Scorpio. On August 29th it was near ε and μ Aquarii. On August 30th it was near β Aquarii and α Equulei.

1378. On the 22nd of September of this year, a comet was seen in China to pass over the stars in the western foot of Antinous. It was also seen in Europe. On September 29th it was near the constellation Ursa Major, between Aries and Taurus. (This is certainly an error—the Great Bear is not near these constellations). It was visible for five days, and its motion was in an opposite direction to the apparent diurnal motion. This is an appearance of Halley's comet.

1385. On October 23rd a comet was seen in the constellations of Leo and Virgo, to the south of the constellation Ursa Major. On the morning of October 30th, it was seen near χ Hydræ. It afterwards passed to the south of the constellations φ, μ, λ, υ, and κ Hydræ. The tail was 10° long.

1402. A very large and very brilliant comet, which commenced to appear on the first day of Lent, on the 8th day of February, and which remained to the beginning of March.

1433. At Bologna a remarkable and brilliant comet was seen, which appeared from evening to morning. It lasted more than a month, or rather less than three months. Some dated its first appearance on October 12th.

1456. A return of the celebrated comet of Halley. It is represented by all the historians as being grand, terrible, and of extraordinary magnitude, with a tail equivalent in extent to 66°. But the extent of this appendage was fleeting and uncertain, varying to only 7°. The period of its durability was about one month.

1457. In the month of June of this year a comet was seen in Pisces. Its body was small. Its tail, at first very long, was equal to 15° of a great circle. The colour of the comet was of the appearance of lead.

1468. The second comet of this year, which appeared in September, October, and November. It was first seen in Leo, near the tail of Leo. Its colour was blue, with some mixture of paleness. It was a very small body.

1472. In the month of December, 1471, a fine comet was discovered in China. At Japan it was observed on the 9th of January, 1472, where it is described as being very great, its tail being equal in length to a street. In Europe it appears to have been first seen about Christmas—at first small, but afterwards very large. The greater number of historians represent it as very fine, but as altogether fearful.

1490. At the commencement of this year a great comet, with a very white and very long tail, was seen. It was visible at Bologna about the middle of February. Its head was small, and its tail long, but of little brightness. Pingré makes the year of this appearance to be 1491; but another was seen in China in January, 1491, in Cygnus, which cannot be reconciled with the European observations.

1506. In the month of August of this year a comet was visible with a long and bright tail, which extended between the stars of Ursa Major. It was observed in China and Japan, and appeared for a period of eighteen or twenty days.

1531. On July 31st, the comet of Halley again made its appearance in Europe. In China and Japan it was seen on the 13th of July. Apian, astronomer at Ingoldstadt, observed it from August 13th to August 23rd; from which observations, however rude and imperfect, Halley has computed elements. Apian also observed that the tail was always directed from the sun. It appears, from the Chinese observations, that the tail was 7° in extent, and that the comet was visible for thirty-four days.

1532. A comet was visible this year, which is supposed to be identical with that of 1661. The orbit has been deduced from Apian's observations, which, as in the preceding case, are not worthy of much confidence. Its head appeared constantly in the morning before the sun, and was three times larger than Jupiter. Its duration was about seventy days. According to the Chinese accounts, it was visible altogether for 115 days, and its tail varied from 1° to 10° in extent.

1533. A comet appeared this year about the middle of June. It was seen in the summer solstice in Taurus, with a very long tail. Its appearance is generally dated at the end of June, and it was seen to the first days of September.

1556. This is the celebrated comet whose return is so anxiously expected. It began to appear about the end of February, at which time it equalled in magnitude one-half of the full moon. Its tail, however, was short and variable. It was not more, at the greatest, than 4° in length, and its flickering nature resembled much a flame agitated by the wind. The comet disappeared altogether on the 23rd of April, near the chain of Cassiopeia, after having been seen for many days; its brightness being effaced by the rays of the sun, which it was very near. It created great terror in the mind of Charles V.

1558. This comet appeared on July 14th, in Leo. It disappeared on August 24th and 25th, apparently behind the clouds. It was at first not brilliant, but its brightness increased in the latter part of its appearance.

1569. In the November of this year a comet was seen in Serpentarius, and in the signs of Sagittarius and Capricornus. Its motion, in longitude, equalled the extent of these two signs. The comet was discovered in November, and its tail was directed towards the east.

1577. A great comet, which was discovered at Peru on November 1st. It was seen in Europe by Tycho on November 13th, at Uraniburg, in the island of Huen;* on this day he estimated the diameter of the head to be 7′, and the length of its tail 22°; the head was white, but less bright than the fixed stars. It was observed to January 26th of the following year.

1580. This comet was discovered at Tubingen, by Mestlim, on October 2nd. Tycho discovered it on the 10th of the same month, and his observations have been used by Pingré in calculating the orbit. Tycho notes that the diameter of the head was 8′, its light faint, its colour livid, and its tail difficult to be distinguished. Dr. Halley's orbit was based on the uncertain observations of Mestlim. Tycho observed the comet to December 12th.

1582. The second comet of this year, which was seen for only fifteen days at the longest. Tycho discovered the comet, and observed it at Uraniburg, from May 12th to May 18th. On May 17th, the magnitude of the head was scarcely equal to stars of the fourth magnitude. Its tail was more than 3° long, and very faint. Pingré computed two orbits from Tycho's observations, which are both necessarily uncertain.

1585. This comet was discovered by the Landgrave of

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* Uraniburg was a castle built by Tycho Brahé. Its name signifies "The Castle of the Heavens." It is now in ruins.—Ed.

Hesse, and Rothmaun, his astronomer, on October 18th, and afterwards observed by Tycho to November 22nd. At its first appearance it equalled Jupiter in magnitude, but of less brightness. This comet had neither coma nor tail.

1590. This comet was discovered on March 5th by Tycho, and observed to March 16. It is noted as being of a medium magnitude; but it had a great tail, which extended to the zenith. On the day of its discovery, the comet appeared as a star of the second magnitude, and shortly afterwards, on the same night, of the first magnitude. Its brightness, however, was not so great as stars of this description.

1593. This comet, which escaped entirely Tycho's scrutiny and observations, was discovered by his pupil, De Ripen, at Zerbst, in Anhault. On August 4th, it was noted as being of a livid and reddish colour; its head equalled in magnitude stars of the third class, with a tail 4½° in length. On August 9th, it was equal in brightness to stars of the fourth magnitude, and a very slight vestige of a tail could be perceived.

1596. This comet appears to have been discovered on July 11th, and observed to the 12th of August. Its colour was feeble and pale, with a tail small, fleeting, and undefined. Valz considered that this comet was identical with that of 1845.

1607. The third apparition of Halley's comet, as ascertained by the elements, which, however, are not considered so accurate as could be wished. It was observed by Kepler in Prague, by Longemontanus in Copenhagen, and Malmoe in Scania (a province of Sweden). Bessel's orbit is deduced from the observations of the English astronomer Harriott.

1618. The first comet of this year was discovered at Caschau, in Hungary, on August 25th, and observed by Kepler from September 1st to September 25th.

1618. The second comet of this year was discovered at Silesia, on November 10th, and at Rome on the same day. At Ispahan, in Persia, the Spanish ambassador saw this comet for the space of fifteen or sixteen days, commencing with November 10th, two hours before the rising of the sun. The length of its tail equalled in extent one-sixth part of the zodiac. On November 18th, the Jesuits at the Roman College noted the extent of the tail to be 40°.

1647. Seen in Prussia on September 29th, soon after the setting of the sun. This comet was small, and remained visible for a very short time.

1652. This comet, which was of a pale and livid colour, almost equalled the moon in magnitude, according to Hevelius and De Caniers. It was observed by many astronomers, but, with the exception of those by Hevelius at Dantzic, the observations are very rough. Dr. Halley has computed his orbit from the observations of Hevelius.

1661. This comet, which was considered at first to be a return of the comet of 1532, was discovered by Hevelius from February 3rd to March 10th. On these observations Dr. Halley has calculated its orbit; but no body, which bears any similarity to these elements, was observed in 1789 or 1790.

1664. This comet appears to have been discovered on November 17th, in Spain. It was observed by Huygens on December 2nd, and by Hevelius on December 14th. The observations were continued by Hevelius to February 18th of 1666. At its first appearance it was noted as large as a star of the first magnitude, but not so bright. The length of the tail was from 5° or 6° to 10°.

1665. This comet was seen at Aix on March 27th. From the observations of Hevelius, from April 6th to April 20th, Dr. Halley has calculated its orbit.

1668. This comet, of which the tail appears to have been only seen at Bologna, was observed principally in the southern hemisphere. There appears to be some similarity of elements with the comet of 1843.

1672. This comet was observed by Hevelius at Dantzic on March 2nd, and seen by him till April 21st. On these data Dr. Halley has computed his orbit. The comet was small, having a tail only 1° or 1½° in length.

1676. Father Fontenoy, of the Society of the Jesuits, observed a comet at Nantes on the 14th of February. It was visible to March 9th, and was equal to stars of the third magnitude. This comet had no tail.

1677. This comet was discovered at Dantzic, on April 27th, by Hevelius. It was observed to the 8th of May, and it is on these observations Dr. Halley has calculated his orbit. The magnitude of this body was equal to Jupiter, and it had a tail about 2° in length.

1678. La Hire discovered this comet on September 11th, and observed it to October 7th. The orbit, however, deduced by Dawes is very rough, principally in consequence of the uncertainty of the observations.

1680. This great and celebrated comet was discovered by a person whose name is unknown, at Coburg in Saxony, on November 4th. It was, independently, discovered by Godfrey Kirch, on the 14th of November, while about to observe the Moon and Mars. Flamsteed, at Greenwich, saw the comet on the 20th of December, and observed it to the 15th of February, 1681. The length of the tail on December 16th was 70°. Several elliptic orbits have been calculated for this comet.

1682. Halley's comet. It was first observed at Paris by Picard and La Hire, on August 26th, and last observed by Flamsteed on September 19th. Its head was 2′ in diameter, and the length of the tail varied from 12° to 15°.

1683. Hevelius observed this comet at Dantzic, from the 30th of July to the 4th of September. Clausen's elliptic elements, from Flamsteed's observations, give its period as 190 years. Its nucleus was equal to a star of the fourth magnitude, with a tail varying from 2° to 4°.

1684. This comet was discovered by Bianchini, at Rome. It was observed from July 1st to July 17th; on which observations Halley has calculated its orbit.

1686. In the August of this year a comet was visible at Para, in Brazil, during the whole month. Its head equalled stars of the first magnitude, and its tail was 18° in length.

1689. This comet was discovered at Pekin on the 11th of December. It was not seen at all in Europe. On December 11th, the part visible of the tail was from 10° to 12°. In the southern hemisphere it was regularly observed, and its tail, at the greatest, was 60° long.

1695. This comet was almost obscured by the atmosphere or coma by which it was surrounded, so that the nucleus was scarcely distinguishable. It was seen in the southern hemisphere, in the Brazils, by Father Jacob, a French Jesuit, on October 28th. The length of the tail was then 18°.

1698. The orbit of this comet is very uncertain, in consequence of the roughness of the observations. Cassini discovered it at Paris, in the beginning of September, in the constellation Cassiopeia. La Hire observed it to Septembr 28th. This comet was only seen at Paris, and was not larger than a star of the third magnitude.

1699. Discovered at Pekin, by Father de Fontenoy, on February 17th, and observed to February 26th. It was seen at Paris from February 20th to March 2nd.

1701. This comet was discovered at Pau, by Father Palla, from October 28th to November 1st. It was a small body without tail, and diminished sensibly in magnitude.

1702. This comet, which was the second discovered this year, was observed from April 20th to May 5th, at Paris, Rome, and Berlin. The orbit is uncertain, in consequence of the roughness of the observations. This comet was compared to a nebulous star on April 20th.

1706. Observed at Paris by Cassini and Maraldi, from March 18th to April 16th. This comet was also similar in appearance to a nebulous star.

1707. This comet appears to have been discovered at Bologna on November 25th, and observed to January 23rd of the following year. Of all known comets, this body has the greatest inclination. Viewed with a telescope, it appeared to be nebulous, and of the second magnitude.

1718. This comet was observed at Berlin by Kirch, from January 18th to February 5th. It appeared to be equal to a star of the fourth or fifth magnitude, with a nebulous diameter of 5′ or 7′.

1723. Discovered at Bombay on October 12th, and observed at Lisbon and other European stations to the middle of December. It appeared of about the third magnitude, with a very faint tail, not more than 1° in length.

1729. This comet, which is remarkable in the length of its visibility, and the greatness of its distance from the sun and earth, was discovered at Nîmes, by P. Sarabat, on July 31st. It was a small nebulous body, scarcely visible to the naked eye.

1737. The observations of this comet were made at Paris, Rome, Bologna, Oxford, Lisbon, and also in Jamaica and Madras.

1739. This comet was observed at Bologna, from May 28th to August 18th.

1742. First comet. Seen at the Cape of Good Hope on February 5th. It was also seen in Europe in March, and was visible to the naked eye. The length of the tail was 5° to 8°.

1743. First comet. This comet was observed at Bologna, Paris, Vienna, and Berlin. In Berlin, it appears to have been discovered, on February 10th, by M. Grischon. It was a small body.

1743. Second comet. Observed at Haarlem from the 18th of August to the 13th of September. This comet was small, but was seen, notwithstanding, with the naked eye.

1744. This comet appears to have been discovered at Haarlem on December 9th, 1743, and was seen to March, 1744, at several observatories. It was one of the finest bodies which had occurred since the comet of 1680. In February, M. Cassini noticed that the head was divided into two portions. The tail was also divided shortly after into two branches.

1747. Discovered at Lausanne, by Cheseaux, on the 13th of August, 1746, and observed there to September 22nd. His observations were then interrupted by illness; but it was last seen on the 23rd of November, near a star in Capricornus.

1748. The first comet of this year was discovered at Paris about the latter end of April. It was also observed in South America, and in Pekin in China, as well as the Royal Observatory, Greenwich. It appears to have been a fine object, easily seen with the naked eye, and having a tail 20° long. This comet was observed at Paris, by Maraldi, to the 30th of June.

1748. The second comet of this year was seen at the same time as the first, but in a different part of the heavens. Its nucleus was brighter than the preceding, but there was no appearance of tail. The orbit depends on three approximate observations made at Haarlem, and is consequently very uncertain.

1757. Observed at Greenwich from September 13th to October 18th. The orbit of this comet is very approximate.

1758. Discovered in the island of Bourbon on the 26th of May, and seen in London on June 18th. Messier first observed the comet on August 15th, and continued his observations to the 2nd of November. It appears to have been a diffused body, equal in diameter to Jupiter.

1759. Halley's comet, first seen on Christmas day, 1758, by Pazlitch, near Dresden, and on the 28th by Dr. Hoffmann. This return is very celebrated in the history of astronomy, as being the first predicted appearance of a comet. Dr. Halley roughly estimated that the time of arriving at its perihelion would be at least one year longer than the interval between the two preceding returns. Clairaut, by a laborious calculation of the planetary perturbations, fixed its return on the 13th of April, the true time having been determined by Rosenberger, from a minute discussion of all the observations, to have taken place on March 12th. From the beginning of 1759 to the middle of February, the comet was regularly observed, till it became plunged in the solar rays about the latter period. On its reappearance, at the end of March, it was again observed to the 17th of April, till its declination became too great to permit its observation at European stations. It was observed, however, from the 20th of April to the middle of May in southern stations. La Nux, at the Isle of Bourbon, found the length of its tail on March 21st to be 8°; on the 28th of March, 25°; on May 1st, 33° to 34°; and on the 5th of May, 47°. It afterwards diminished, till, on the 14th of May, it was not more than 19°. It afterwards reappeared above the European horizon, and was observed in France and Portugal to the 3rd of June.

1759. The second comet of this year is reckoned in the order of its time of perihelion passage. It was discovered on the 25th of January, 1760, in the constellation Leo, by the celebrated astronomer Messier, and regularly observed to the 16th of March.

1759. The third comet of this year (reckoning, as before, in the order of perihelion passage) was discovered on the 1st January, 1760, by all the astronomers of the French Academy, and observed till the 8th of February. Its motion was exceedingly rapid, being, on the day of its discovery, 2° 25′ of a great circle in two hours, or at the rate of 29° daily. The diameter of the nebulosity was found to vary from 20″ to 30″. Its tail was about 4° in length.

1762. This comet was discovered by Klinkenberg on the 17th of May, and observed by Messier and Maraldi to the 2nd of July. Its appearance at the time of its discovery was similar to a star of the fourth or fifth magnitude, with a slight tail. The nucleus was readily visible to the naked eye, but in a telescope was bright and ill-defined.

1763. Discovered by Messier on September 28th, and observed to the 25th of November. Its appearance was that of a nebulous star. On the 4th of October, its diameter was 7′ or 8′. The orbit of this comet gave Pingré considerable trouble. That of Burckhardt is founded on unpublished observations.

1764. This comet was also discovered by Messier, and observed by him from January 3rd to February 11th. It was a bright nebulous object, visible to the naked eye, with a tail 2½° in length.

1766. Discovered by Messier on the 8th of March, when looking for the supposed satellite of Venus, which had created great sensation about this period. It appeared as a small nebulosity, with a luminous centre.

1766. Second comet. This comet appears to have been discovered by Helfenzriella, of Dillengen, in Suabia, on the 1st of April. On April 9th, it presented a tail from 3° to 4°, its nucleus being similar to a star of the fourth magnitude. Pingré supposed that it would be visible after the perihelion passage, which proved to be the case; but not, however, in European latitudes. La Nux observed it in the Isle of Bourbon, from April 29th to May 13th.

1769. This remarkable comet was discovered by Messier, on the 8th of August, and observed by nearly all the astronomers. La Nux observed it at the Isle of Bourbon, from August 26th to September 26th. On September 11th, he found its tail by measurement 97° long. Pingré, the same day, measured its length to be 90°. On August 28th, Dr. Maskelyne noticed its tail as 7°; Messier as 15°; La Nux and Pingré as 19° to 20°. On September 9th, Dr. Maskelyne estimated its length 43°; Messier as 55°; La Nux upwards of 60°; and Pingré as 75°. This comet was also observed after its perihelion passage, from October 24th to the 1st of December.

1770. First comet. Messier discovered this comet on the 14th of June, and observed it regularly to the 2nd of October. This comet is celebrated for the trouble it has successively given astronomers in the attempts to investigate its orbit. Leverrier has recently determined its period to 5·6 years nearly, but it has never been observed since the time of its discovery.

1770. Second comet. This comet appeared in 1771, on the 10th of January of which year it was seen at Paris by Messier, and at Milan by Boscovich. It appears, however, to have been discovered by La Nux, at the Isle of Bourbon, on January 9th.

1771. Messier discovered this comet on the 1st of April, and observed it to June 19th. It was also observed by Dr. Maskelyne, at Greenwich, from April 14th to May 30th. Its appearance was similar in brightness to a star of the third magnitude, with a train varying in length from 1° to 3°. The orbit is supposed, with some probability, to be hyperbolic.

1772. BIELA'S COMET.—This body, of which there is no doubt as to its identity with the comet of Gambart, or Biéla, was discovered by Montague, at Limoges, on the 8th of March. Messier observed it on three occasions, on March 27th, March 30th, and April 3rd.

1773. Discovered by Messier on the 12th of October, and observed to the 14th of April, 1774. It was with difficulty seen by the naked eye.

1774. Montague discovered this comet on the 11th of August, at Limoges. This was a small body, not visible to the naked eye.

1779. The small comet of this year was discovered by Bode, on January 6th, at Berlin; and independently by Messier, on January 18th, who observed it on the 17th of May.

7780. First comet. This comet was discovered by Messier, on October 26th.

1780. Second comet. This was discovered by Montague, at Limoges, on October 18th. It was also seen by Olbers, on the same day.

1781. First comet. This small comet, which was not visible to the naked eye, was discovered by Méchain on the 28th of June. It had no tail. The diameter of the nebulosity did not exceed 3′ or 4′.

1781. Second comet, discovered by Méchain on the 9th of October, and observed to December 25th. On November 9th it was visible to the naked eye, the nebulosity being 4′ or 5′ in diameter, and its tail 3° or 4° in length.

1783. Discovered by Pigott at York, on November 19th, as a small faint body, with difficulty bearing any illumination of the telescope. There is little doubt as to the ellipticity of its orbit, the period varying from five years and ten years, in Burckhardt's two orbits.

1784. La Nux first observed this comet on the 15th of December, 1783, at the Isle of Bourbon. At Paris, in the month of January, 1784, it was visible to the naked eye, with a tail from 2° to 3° in length.

1785. First comet discovered by Messier and Méchain on the 7th of January. It was invisible to the naked eye.

1785. Second comet discovered by Méchain on the 11th of March, and observed to the 7th of April. On April 4th, the nucleus was visible to the naked eye, with a tail 5° in length.

1786. First comet. This is a well-ascertained appearance of Encke's comet, which was discovered by Méchain on January 17th, and only observed on two occasions.

1786. Second comet. This body was discovered at Slough, on the 1st of August.

1787. This comet was discovered by Méchain on the 10th of April. It was a small luminous body, only visible in a telescope.

1788. First comet. This comet, which at its first appearance was not visible to the unaided vision, was discovered by Messier on November 25th. Its brightness increased so much that it was visible to the naked eye on November 30th. It appeared with a train 2° or 3° in length. The last observation was on December 29th, when the comet had diminished considerably in brightness.

1788. Second Comet. This was discovered by Miss Caroline Herschel on December 21st. It was a small telescopic body, presenting, at its first appearance, a nebulosity of five or six minutes in diameter.

1790. First comet. Discovered by Miss Herschel on January 7th. This was also a nebulous body, only visible by the help of a telescope, and of a diameter equal to about five or six minutes of arc.

1790. Second comet. Discovered by Méchain on January 9th. It appears to have been nearly similar to the preceding, and visible at the same time.

1790. Third comet. This was discovered by Miss Herschel on the 18th of April. It was observed by Messier from the 1st of May to the 29th of June. Its brightness on the 17th of May was so great that it was visible to the naked eye; the length of the tail varied from 2° to 4°.

1792. First comet. Discovered by Miss Herschel on the 15th of December, 1791. It was a faint nebulous body.

1792. Second comet. Discovered by the Rev. E. Gregory on the evening of January 8th, 1793. It was a dull nebulous body, with a faint appearance of tail.

1795. First comet. Discovered by Messier or Méchain on the 27th of September.

1795. Second comet. Discovered by Peray on September 24th. Miss Herschel detected it on the 7th of October. Burckhardt estimated its period to be twelve years, which, from a revision of the elements by D'Arrest, is improbable. The latter astronomer makes its period to be 422 years.

1795. Encke's comet. Discovered by Miss Herschel on November 7th, and observed by Dr. Maskelyne, and other astronomers, to the end of the month. It was a round, ill-defined body, without a nucleus.

1796. This comet was discovered by Olbers on March 31st. It was a very faint body, and would bear no illumination of the field.

1797. This comet was discovered by Bouvard on August 14th. It was a nebulous body from 3′ to 5′ in diameter.

1798. First comet. Discovered by Messier on April 12th.

1798. Second comet. Discovered by Bouvard on December 6th. It was a faint body, with a slight increase of light in the centre.

1799. First comet. This body was discovered by Méchain on August 7th, and visible to October. It was very faint when first seen; but at the end of August it was visible to the naked eye, with a tail about 1° in length.

1799. Second comet. Discovered by Méchain on December 26th; it was visible to the naked eye as a star of the fourth or fifth magnitude. It had a tail from 1° to 3° in length.

1801. Discovered by Reissig, jun., of Cassel, on June 30th, and observed by Messier to July 23rd. It was a small body.

1802. Discovered by Pons, at Marseilles, on August 26th; it was not visible to the naked eye.

1804. This comet, which had neither nucleus nor tail, was discovered by Pons on March 7th. It was seen the last time on the 1st of April.

1805. (1) Encke's comet. It was visible to the naked eye, and much resembled the nebula in Andromeda. It was discovered by Thulis, of Marseilles, on October 19th. One observer mentions its tail as being 3° in length.

1805. (2) Biéla's comet. Discovered by Pons, on November 10th, as a very faint body, the nebulosity scarcely bearing the slightest illumination of the field.

1806. Discovered by Pons on the 10th of November. The coma was from 5′ to 7′ in diameter.

1807. Discovered by Pons on September 20th; but it appears to have been first seen eight days previously by an Augustine monk in Italy. This was the finest comet that had appeared since 1769. On the 30th of September, the nucleus was equal to a star of the first magnitude. Olbers, on November 7th, noted the division of the tail into two branches.

1808. (2) Discovered by Pons, in Camelopardus, on March 25th, and observed to the 2nd of April.

1810. Discovered by Pons, on the 22nd of August, and observed to October 8th. It was a faint and small round nebulous body.

1811. The great comet of this year, which was first discovered by Flangergues on March 25th, and visible to the end of October.

The great comet of this year, so remarkable in several particulars, was discovered by Flangergues at Vivières, on the 26th of March, and observed to August 17th, 1812. From the laborious investigations of Argelander. its orbit was found to be elliptical, the period being 3065 years. This comet, in addition to its magnitude and the remarkable duration of its visibility, presented, on September 7th, a tail bent off in two branches; but these branches did not proceed from the comet itself, but were hung together at a slight distance from it, and separated from it by a dark interval, so that they enclosed the comet as a parabola does its focus. At this period its tail was 5° in length. On September 20th, Bode found the length of the tail to be 10°. On October 11th, the tail was about 13° long; the diameter of the nebulosity being 1′ 20″. Its maximum length appears to have occurred in the first week of October, when the tail was found to be 25° long, and about 6° broad. Sir W. Herschel paid considerable attention to the physical appearance of this comet, and the reader is referred to the *Philosophical Transactions*, 1812, for his remarks, which are too extensive to be given here.

1811. The second comet of this year was discovered by Pons on November 16th, and observed to the end of January, 1812. It had a well-defined nucleus, with a faint surrounding coma.

1812. This comet was discovered by Pons on July 20th, and observed to the end of September. At first it was only visible in a telescope; but in September it increased in brightness, and had a tail 2° in length. Professor Encke found its orbit elliptical, the period being 70·7 years.

1813. First comet. This comet was discovered on the 4th of February, by the preceding observer; and its appearance was that of a small confused nebula.

1813. Second comet. This comet, the eighteenth discovered by Pons, was first seen on March 28th. It appears to have been visible to the naked eye on April 24th and 25th, as a small round nebulosity without tail.

1815. This comet was telescopic, and was discovered by Olbers on March 6th. It is remarkable as being one whose elliptical orbit is decided; the periods of revolution, by several computers, varying from 72 years to 77 years.

1818. First comet. Discovered by Pons, in Cetus, on the 23rd of February. It was very faint, and has recently been supposed to have been an appearance of the comet of Bièla.

1818. Second comet. Discovered by Pons, on December 26th, 1817. It was a faint telescopic body.

1818. Third comet. Discovered by Pons, on November 28th, as a small, round, and well-defined body. Bessel, independently, discovered a comet on December 22nd, which proved to be the same body.

1819. The first comet of this year was remarkable in affording to Encke the discovery of its periodic nature, and the identity of it with the comets of 1786, 1795, and 1805. It was discovered by Pons, on November 26th, 1818, as a small, ill-defined nebulosity. The history of this body has been given in the preceding part of this chapter. (See p. 962).

1819. Second comet. This was discovered by Professor Tralles, at Berlin, on July 1st. It appeared with a well-defined planetary nucleus, and a tail from 7° to 8° in length.

1819. Third comet. Discovered by Pons, on July 12th. Encke computed its orbit, which he found to be elliptical, with a period of 2052 days.

1819. Fourth comet. Discovered by Pons, on December 4th; and previously at Marseilles, by Blanpain, on November 28th. Encke found its orbit elliptical, and its period 4·8 years. Clausen was of opinion that it was identical with the comet of 1743.

From the year 1821 we can do little more than record the various comets which have appeared in our hemisphere, with their dates, and names of their discoverers:—1821. Discovered by Pons on January 28th. Santini saw it with the naked eye on February 19th, and estimated its tail to be 2½° long. It was observed to May 3rd.—1822 (1). Gambart discovered this comet on May 12th.—1822 (2). Encke's comet. Discovered at Paramatta by Rümker on June 2nd.—1822 (3). Discovered by Pons on May 31st, but not observed much in Europe in consequence of its southerly declination.—1822 (4). Discovered by Pons on July 13th. On August 21st it was visible to the naked eye, and had a tail 1½° long.—1823. Discovered by Pons on December 29th. Santini found it on January 3rd. De Zach, on January 23rd, noticed that in addition to the usual tail, directed from the sun, it had another in a contrary direction, varying from 4° to 7° in length.—1824 (1). Discovered by Rümker on July 15th.—1824 (2). Discovered by Scheithaner, at Chemitz, on July 23rd.—1825 (1). Discovered by Gambart on May 19th, in Cassiopeia. This comet has some resemblance in its elements with the third comet of 1790.—1825 (2). This small comet was discovered by Pons on the 9th of August.—1825 (3). Encke's comet. Discovered by Valz on July 13th.—1825 (4). Discovered by Pons on July 15th. It was visible to the naked eye, and in October presented a remarkable appearance in the heavens.—1826 (1). Gambart's or Bièla's comet. Discovered by Bièla on February 27th.—1826 (2). Discovered by Pons on November 6th, 1825.—1826 (3). Discovered by Flangergues on March 29th.—1826 (4). Discovered by Pons on August 7th, and observed to October.—1826 (5). Discovered by Pons on October 22nd. It was visible to the naked eye in December, with a beautiful train.—1827 (1). Discovered by Pons on December 26th, 1826.—1827 (2). Discovered by Pons on June 20th.—1827 (3). Discovered by Pons on August 2nd.—1829. Encke's comet. Discovered by Struve in 1828, October 13th.—1830 (1). Discovered by Professor D'Abbadie in the Mauritius on March 16th. It was also seen on March 17th, during a voyage from Calcutta to Boston, as a bright object, with a tail 8° in extent.—1830 (2). Discovered by Herapath on January 7th, 1831. It exhibited a tail 2½° long; the diameter of the nucleus was 3′ or 4′.

1832 (1). Encke's comet. Discovered by Mossotti, on June 1, at Buenos Ayres, and at the Royal Observatory, Cape of Good Hope, on the next day. It was only seen once in Europe, by Professor Harding, on August 21st.—1832 (2). Discovered by Gambart, on July 19th.—1832 (3). Gambart's or Bièla's comet. Discovered at Rome, on August 25th.—1833. Discovered at Paramatta, by Dunlop, in September.—1834. Discovered by Gambart, on March 8th, in Sagittarius.—1835 (1). Discovered by Boguslanski, on April 20th.—1835 (2). Encke's comet. Discovered by Boguslanski, on July 30th.—1835 (3). Halley's comet. Discovered by Dumonchel, at Rome, on August 6th. (See the history of this comet, at the commencement of the chapter.)—1838. Encke's comet. Discovered by Boguslanski, on August 14th.—1840 (1). Discovered by Galle on December 3rd, 1839.—1840 (2). Discovered by Galle on January 25th. It was very faint, without any appearance of tail.—1840 (3). Discovered by Galle, on March 6th. It had a tail several degrees in length.—1840 (4). Discovered by Bremieker, on October 26th.

1842. Encke's comet. Discovered on March 9th, by Valz.—1842 (2). Discovered by Langier on October 28th. It was very faint, and without tail.—1843. The great comet of this year is within the recollection of many readers of this work. It was seen in South America, on Febuary 27th, and observed afterwards in Europe to the middle of April. This was the finest body since the great comet of 1811.—1843 (2). Discovered by Mauvais, on May 2nd.—1843 (3). Discovered by Faye, on November 22nd. Its orbit is elliptical, and it was again observed at the Cambridge Observatory in 1850.—1844 (1). Discovered by De Vico, on August 22nd.—1844 (2). Discovered by Mauvais, on July 7th. It had a small brilliant nucleus of 3′ in diameter.—1844 (3). Detected by Captain Wilmot, at the Cape of Good Hope, on December 19th.—1845 (1). Discovered by D'Arrest, on December 28th, 1844.—1845 (2). Discovered by De Vico, on February 25th.—1845 (3). Discovered by Coller, on June 2nd. It was a beautiful object, visible to the naked eye, with a tail 2½° in length.—1845 (4). Encke's comet. Discovered by

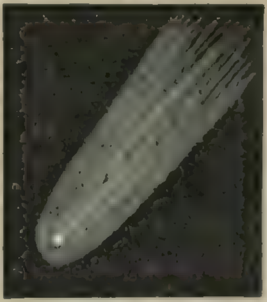
De Vico, at Rome, on July 10th.—1846 (1). Discovered by De Vico, on January 24th.—1846 (2). Gambart's or Biela's comet. Discovered by De Vico, on November 26th, 1845. At this appearance, the singular phenomenon of a double comet was seen.—1846 (3). Discovered by Brorsen, on February 26th. Its elements have some resemblance to the comets of 1532 and 1661.—1846 (4). Discovered by De Vico, on February 20th. There is no doubt as to the elliptical orbit of this comet, the period being 73 years.—1846 (5). Discovered by Hind and De Vico, on July 29th.—1846 (6). Discovered by Peters, on June 26th.—1846 (7). Discovered by Brorsen, on April 30th.—1846 (8). Discovered by De Vico, on December 23rd.—1847 (1). Discovered by Hind, on February 6th. —1847 (2). Discovered by Coller, on May 7th.—1847 (3). Discovered by Schweizer, on August 31st.—1847 (4). Discovered by Mauvais, on July 4th.—1847 (5). Discovered by Brorsen, on July 20th.—1847 (6). Discovered by Miss Mitchell, on October 1st.—1848 (1). Discovered by Petersen, on August 7th.—1848 (2). Encke's comet. Discovered by Hind, on September 13th.—1849 (1). Discovered by Petersen, on October 26th, 1848.—1849 (2). Discovered by Gonjon, on April 15th.—1849 (3). Discovered by Schweizer, on April 11th.—1849 (4). Discovered by Mr. Jenkins, on November 28th, at sea.—1850 (1). Discovered by Petersen, on May 1st.—1850 (2). Discovered by Bond, on August 29th.

1851 (1). Faye's comet. Discovered at the Cambridge Observatory, on November 28th 1850.—1851 (2). Discovered by D'Arrest, on June 28th.—1851 (3). Discovered by Brorsen, on August 1st.—1851 (4). Discovered by Brorsen, on October 22nd.

[Omitting various minor cometary appearances, we may give a short description of two remarkable comets, which appeared in 1858 and 1861.

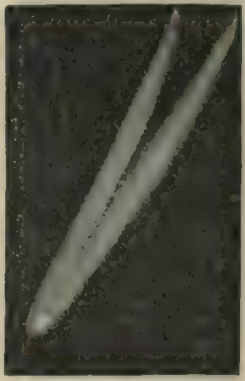
On June 2nd, 1858, M. Donati discovered the comet which has since borne his name. Until August 23rd, it did not present any marked appearance to observers in this kingdom, but was a round, bright object, like a nebulous patch of light, with a condensed centre, and without a tail. It was not visible to the naked eye till September, when it suddenly presented a most magnificent appearance. On October 5th, it passed near Arcturus, its tail being very brilliant, and of a length of full 40°, through which bright stars could be easily distinguished. The appearance of the tail continually varied. We observed it about the beginning of October, at Campbelton, in Argylshire, where the clearness of the air was extremely favourable, and, on one evening, a curious contrast was presented to it by the display of a brilliant aurora borealis, which it however exceeded in brightness. Its periodic time has been calculated to be 2,100 years. It was, at first, supposed to be the return of the comet of 1556, but its elements showed such an opinion to be erroneous. The annexed cut will give an idea of its general appearance at the commencement of October, as seen and sketched at the time.

Fig. 117.



In the latter portion of June, 1861, every one was startled by the sudden appearance of a comet, at dusk, on Sunday evening, June 30th. The writer observed it, before sunset, as a kind of nebulous patch, mistaking it for a star hid by haze. But, about eleven o'clock, all clouds having cleared away from the south-eastern sky, a magnificent comet, with a bright nucleus, and a tail of nearly 60° in length, presented itself. Its image was distinctly reflected from the sea (Ramsgate pier being the point of our observation). The tail had a quivering motion, very much resembling that of the streamers of the aurora borealis, and was divided near the head into two parts.

Fig. 118.



The annexed cut gives an idea of its appearance. (See, also, folio plate, page 957).

It was calculated, on June 25th, by Professor Challis, of Cambridge, that the comet was distant 82 millions of miles from the sun. And that its tail had a length of 10 millions of miles. Its brightness rapidly diminished as it receded from the sun; and, on August 24th, its distance from that body was about 140 millions of miles, increasing, in a week, to 152 millions, by which time it had arrived in the northern part of the constellation Bootes. Its tail had almost disappeared by this date, and in a few days powerful telescopes failed to discover it.

The most remarkable fact respecting this comet was its sudden appearance. It had not been seen by any of the observatories in Europe, till it so suddenly burst on the eye, of a size which was its maximum. For three days, however, previously to its discovery, the weather had been very cloudy, and the sky almost entirely overcast.] —EDITOR.

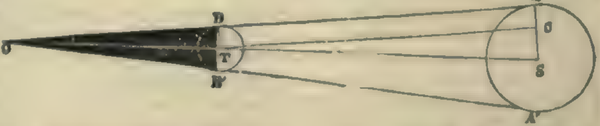
CHAPTER IX.

ECLIPSES AND OCCULTATIONS.

THE striking phenomena attending eclipses of the sun and moon, in which, apparently, the course of nature is interrupted, and for a time the discs of the sun and full moon lose their wonted lustre, and their light is almost extinguished in the mid-heavens, have ever been a source of curiosity, and occasionally of terror, to mankind. Whilst they have furnished many examples of credulity which have been taken advantage of by superior minds, they have sometimes served a nobler purpose; and the biography of astronomers is filled with instances in which the accidental occurrence of a solar or lunar eclipse, has been the cause of their first directing attention to the science that has been subsequently enriched by their investigations. *Eclipses* may be defined as a short interruption in the passage of light to the earth, which is either *real*, as in the case of the interposition of an opaque body between the earth and sun; or *apparent*, as takes place when the earth itself passes between the sun and the object previously illumined. The first are termed solar eclipses, and take place at the time of the new moon; the latter are lunar eclipses, and occur at the time of full moon, or when the earth is interposed between the sun and her satellite; the earth being a body of such dimensions that its shadow is sufficient to reach to the moon.

LUNAR ECLIPSES.—Taking the latter case, or that of a lunar eclipse, we readily see by Fig. 119 the form of the shadow cast by the earth. The sun, S, scatters its rays in all directions, and such as are directed towards the earth, T, will be interrupted in their onward passage by

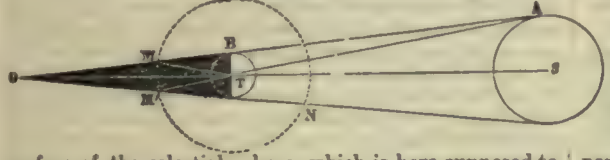
Fig. 119.



the interposition of this body; and we perceive that if the rays of light, A B, A′ B′, preserve their straight path they cannot penetrate within the portion B O B′, or between the summit of the cone A O A′ and the earth, and that this is the form and dimensions of the shadow cast by the globe T. In order that the moon may be eclipsed, it must be placed between the earth T and the summit of the cone O; and the distance of those two points is easily determined in the following manner:—Draw the line T C parallel to O A, and in the two similar triangles, O T B and T S C, we have the proportion S C : T S : : T B : O T. The radius of the earth, T B, being taken for unity, the line S C will be equal to the radius of the solar sphere, diminished by the former quantity, or to 111 radii of the earth. As the mean distance, T S, of the earth and sun are equal to 24,000 radii of the earth, we conclude that the distance O T is equal to about 216 terrestrial radii; or, more exactly, that the length of the axis of the shadow is 216·531 radii of the earth. At the time when the sun is in perigee this will decrease to 212·896 radii of the earth, and at the time of apogee will be 220·238 radii. As the distance of the moon from the earth never exceeds 64 terrestrial radii, and the least length of the shadow is 212 terrestrial radii, it is obvious that the moon must be obscured when the earth is situated between it and the sun. The breadth of the shadow of the earth at the distance of 108 terrestrial radii will be equal to half the diameter of the earth, and considerably greater at the distance of only 60 radii; but the exact dimensions of the latter point may be obtained as follows:—

Dimensions of the Shadow.—Let M N (Fig. 120) be the surface of the celestial sphere, which is here supposed to pass through the centre of the moon; this surface will cut the cone of the earth's shadow at M M′, and the angle M T M′ is the apparent angle of the shadow which it is required to determine. The half of this angle, or M T O, is equal to the angle B M T, diminished by the angle M O T; but the former is the parallax of the moon (since M T is the distance of the moon from the earth), and the latter is equal to the angle A T S (the semi-diameter of the sun) diminished by the angle B A T (the parallax of the sun). In order, therefore, to obtain the apparent semi-diameter of the shadow at the distance of the centre of the moon, we must add the parallax of the sun to that of the moon, and subtract the apparent semi-diameter of the sun.

Fig. 120.



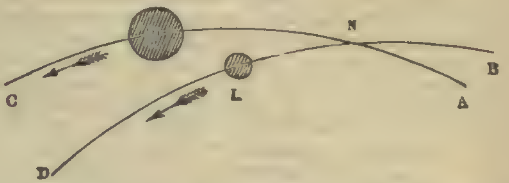
As the moon moves over a space about equal to its diameter in an hour, it follows that it may be entirely within the shadow for two hours. The greatest value of the dimensions of the earth's shadow at the moon, may be readily obtained by taking the greatest parallax of the moon, and the least of the sun; and as the former is 61′ 29″ at its maximum, and the sun's least semi-diameter and corresponding parallax, respectively, are 15′ 46″ and 8″·6, it would follow that the greatest semi-diameter of the earth's shadow at the distance of the moon is 45′ 52″.

The apparent semi-diameters of the shadow for other distances of the sun and moon may be readily calculated, and are here annexed:—

		′	″
Sun in perigee . .	Moon in apogee . .	37	42
	,, at mean distance	41	31
	,, in perigee . .	45	20
Sun at mean distance	Moon in apogee . .	37	58
	,, at mean distance	41	48
	,, in perigee . .	45	37
Sun in apogee . .	Moon in apogee . .	38	14
	,, at mean distance	42	3
	,, in perigee . .	45	52

Respective Positions of Moon and Shadow.—Let A C (Fig. 121) be the great circle of the ecliptic, and B D the orbit of the moon, N will be the nodes of this orbit. The shadow O moves along the first circle with a velocity equal to that of the sun, and the moon passes along the second circle with a velocity about thirteen times greater. In order that the moon may meet the shadow, it must happen that the centre of the shadow is sufficiently near the nodes N at the moment of opposition. By taking into consideration the above facts—that the apparent diameters of the moon and the earth's shadow vary from one epoch to another—and remarking that the distance of the centre of the shadow from the node N is precisely equal to the distance of the centre of the sun from the opposite node of the moon, we find that if the distance of the centre of the sun from the node at the time of full moon be greater than 12° 3′, there *cannot* be an eclipse; and if the distance be less than 9° 31′, there *must* certainly be an eclipse. Between these two extremes the case is doubtful; but a more exact calculation will show whether an eclipse really does occur.

Fig. 121.



PARTIAL ECLIPSES OF THE MOON.—When the moon is wholly obscured, as we see it can be, the eclipse is termed total; but when only a portion of the lunar disc enters into the shadow, the eclipse is partial. In the latter case, the outline of the earth's shadow projected upon the disc of the full moon, clearly shows its round and globular figure, although the diameter of the conical shadow is large in proportion to that of the moon. The accompanying diagram (Fig. 122) gives an idea of the relative proportions of the shadow of the earth and the disc of the moon at those times, and of the curvature of the circumference of the shadow *a b c*. But the definition of the earth's shadow is far from being so sharp as is here represented, and, like that of the shadow of any other opaque body, is edged with a cloudy and imperfectly-defined penumbra, the dimensions of which may be estimated by the following considerations:—Let the lines A O′ B′ C′ and A′ O′ B C be drawn (Fig. 123), another cone, A O′ A′, having its summit O′ between the sun and the earth, is formed, and enveloping the sun and the earth in its opposite parts A O′ A′, B O′ B′. It is clearly seen that all the part situated within the space C B B′ C′, and without the real shadow B O B′, would only receive a portion of the rays of the sun coming from the part of the hemisphere turned towards it, the other part being hid by the earth; and that the portion of the sun which is visible is greater according as this is nearer to the exterior surface of the space C B B′ C′, and, on the contrary, smaller as it is nearer to the real shadow of the earth, or B O B′.

Fig. 122.

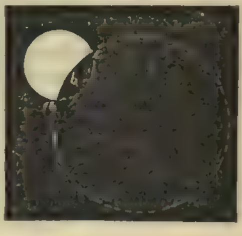
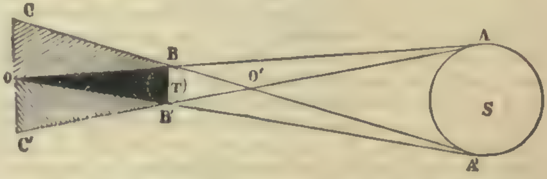


Fig. 123.



The consequence would be, that as the moon passed into the part C B B′ C′ before meeting with the real shadow, it would insensibly lose a portion of its light, and its lustre would become gradually less intense as it approached the real shadow. As, however, the breadth of the penumbra is equal to the angle O B C = A B A′, or the diameter of the sun as seen from the earth; and as the apparent diameters of the moon and sun are nearly equal, it follows that the whole disc of the moon may be within the penumbra C B O. When *any part* of the disc of the moon is obscured by the real shadow, it follows that the *whole* of its surface is more or less hidden by the penumbra. The insensible melting away of the real shadow into the penumbra is noticed at every lunar eclipse, but it is impossible to tell precisely where the penumbra ends, and the shadow begins.

EFFECT OF THE ATMOSPHERE OF THE EARTH.—Hitherto we have supposed the rays proceeding from the sun to pass in straight lines until they meet the lunar disc, but this is not the case; for those which pass through the earth's atmosphere are subject to refraction, and change their direction in the manner already explained in the case of a star. This will be more apparent when we consider the direction of the ray S A (Fig. 124), which traverses the atmosphere of the earth, and passes beyond it. The direction which it takes previous to entering the atmosphere of the earth makes an angle of 33′ with its direction when it arrives at that point; and after passing through it, it is still further deflected from its original course, and by the same amount as before. The consequence of this will be, that it is altogether deflected more than a degree, and that the solar rays will meet before arriving at the point O (Fig. 125); and, instead of the rays A B and A′ B′ taking a straight direction, they will be deflected to the point D (Fig. 125), which is considerably nearer the earth than the point O. Thus it will be only the inner cone B D B′, which is the real cone of the shadow; the remaining parts of the *mathematical* cone B O B′ will be traversed by the solar rays, which are bent from their primitive direction by the action of the terrestrial atmosphere. The distance of the point D from the centre of the earth can be calculated as in the former case, and it has been found that its average is forty-two terrestrial radii; and we conclude from this, that as the moon's mean distance is equal to sixty-four radii of the earth, it can never be inclosed within the real shadow of the earth B D B′; but at the time of total eclipse it falls within that part of the shadow where the rays, refracted by the atmosphere, penetrate. This is the cause that when the moon is wholly eclipsed, it still shines with a reddish light, being illumined by the faint rays refracted by the earth's atmosphere. This, however, is not the sole effect of the atmosphere, for the apparent diameter of the earth's shadow has been found by observation to be *greater* than might be expected from the preceding investigation; and it has been accounted for, by supposing that the solar rays do not really touch the surface of the earth, but that the lower strata of the air absorbs those which approach its margin; and if such be the case, the diameter of the globe of the earth and its shadow would accordingly be greater.

Fig. 124.

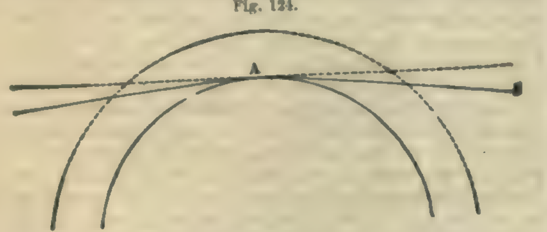
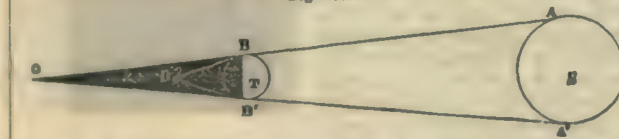


Fig. 125.

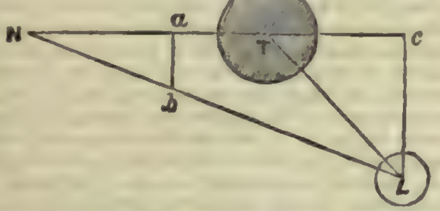


The *condition* of the atmosphere through which the rays of the sun pass, produces considerable changes in the appearance of the lunar disc at the times of total eclipse. In ordinary cases, as before mentioned, the colour of the moon is of a red or coppery tint, similar to that frequently assumed by the setting sun, and produced by the same causes—viz., an absorption of the blue rays of light when passing through great depths of the atmosphere. If that portion of the atmosphere be charged in addition with cloud and dense vapours, the whole of the elementary rays will be stopped in their passage to the lunar surface; and several instances are on record in which the disc of the moon was completely invisible at the time of total eclipse. In the eclipses of 1601 and of June 1620, this was the case, although the air was sufficiently clear to allow the light of stars of the fifth magnitude to be distinguishable; and, in a similar manner, the moon could not be perceived (even with the aid of a telescope) during the eclipse of April, 1642, although the sky was equally as clear as in the former case. When, however, the sky is clear at those portions of the earth's surface through which the solar rays pass, the red rays are transmitted in great number, and the lustre of the lunar surface is but slightly dimmed in consequence. Such was the case with the eclipse of the moon which happened in March, 1848, when the lunar disc was almost as apparent as on ordinary dull nights, and when many persons who were observing it could scarcely be persuaded that it was really eclipsed. The dark spots and bright places were as well seen on its surface as if viewed through thin cloud or vapour; and one observer, who had witnessed lunar eclipses for more than sixty years, never remembered one in which the illumination was so strong—appearing like the glowing heat of fire from the furnace, and tinged with a deep red. Sometimes the moon becomes invisible during the progress of its immersion in the shadow, as was the case with that observed by Wargentin, in 1761, when the moon was very bright for ten minutes after its total immersion; but for an hour afterwards became so completely invisible that not the slightest trace of it could be detected, either with the naked eye or telescope, although the sky was clear, and stars, in the immediate vicinity of the moon, appeared bright and distinct. In other cases, the region of the atmosphere, through which the rays have passed, has been clear at some parts, and obscured at others: and the consequence has been, that whilst the lunar disc appeared clear and bright in some portions, it was very dark at others. Kepler states that, during the total eclipse of August, 1598, one half of the moon appeared so bright that it seemed doubtful if it was eclipsed at all, whilst the other was seen with the utmost difficulty. A similar variegated appearance was presented by the lunar disc in October, 1837. Sir W. Herschel perceived many bright spots on the occasion of a total eclipse, and was induced to believe that they were volcanoes in action. Such an appearance, however, it will plainly be seen, may be supposed due to the various reflective qualities of the surface of the moon, and that the bright spots noticed by Herschel were only the bright mountainous districts illuminated by the red light of the sun. (See Note, p. 984).

PREDICTION, DURATION, AND MAGNITUDE OF A LUNAR ECLIPSE.—If the moon did not depart from the plane of the ecliptic, there would be a lunar eclipse at each full moon, and a solar one at new moon; but as it is sometimes above and sometimes below this plane, an eclipse can only take place when the moon approaches the nodes of its orbit. It was known to the ancients that at certain intervals the new and full moon returned again on the same day of the month; and they had observed that at the end of eighteen years, eleven days, or a period of 223 lunations, there was a return of the same eclipses, and were thus enabled to foretel them with considerable, but not with perfect accuracy; for the exact recurrence, if it took place, would depend upon the return of the sun's place, the moon's place, the position of the moon's apogee, and that of the ascending node of the moon to exactly the same situation. This exact return cannot happen, but is sufficiently approximate to permit us to foretel eclipses with a remarkable degree of accuracy; and Dr. Halley found that if the period of 18 years, 10 days, 7 hours, 43¾ minutes, were added to the middle of the time of any eclipse, the return of the corresponding one might be predicted within 1h. 30m. If only four leap-years occur in the interval, the period would be enlarged to 18y. 11d. 7h. 43¾m. It may, however, happen that when the eclipse is very slight, or an *appulse* occurs, that at the succeeding period of 18y. 11d. it will not be observed. Since the position of the heavenly bodies at any epoch, as well as the laws which govern their motions, are at present equally well known, the eclipses of the sun and moon are now rigorously calculated, although, to save time and trouble, the above period of 18y. 11d. (the period of 521y. 3h. 3m. would be still more exact) is still made use of. The *Nautical Almanac* contains the exact positions of the sun and moon in relation to the earth at any given time; and it is from these data that we shall determine the beginning and end of an eclipse at any given time.

In order to *calculate* an eclipse of the moon, the following data are necessary, and the manner in which they are used will be seen by the diagram (Fig. 126) and explanation.

Fig. 126.



Suppose the moon's place at opposition to be at b, the centre of the earth's shadow being at a at the same time, the latter passing through the space T a, the path of the ecliptic, whilst the moon's centre describes the line L b, the path of the moon's motion (which may be regarded as straight lines, as well as those of L c, a, b):

Let m = moon's horary motion in longitude.
n = moon's horary motion in latitude.
s = sun's horary motion in longitude.
λ = moon's latitude when in opposition at b.
t = time taken by moon to pass from L to b.
d = distance from T to L.

The moon's motion in longitude will be $a\,c$ in the interval it takes to pass from L to $b = m\,t$. Its motion in latitude, during the same time, or $\mathrm{L}\,c - a\,b = n\,t$. The sun's motion in longitude, or the distance $\mathrm{T}\,a = s\,t$ in the same manner. Then

$$\mathrm{L}\,c = a\,b + n\,t = \lambda + n\,t \text{ and } \mathrm{T}\,c = a\,c - \mathrm{T}\,a = m\,t - s\,t$$
$$d^2 = \mathrm{L\,T}^2 = \mathrm{L}\,c^2 + \mathrm{T}\,c^2 = (\lambda + n\,t)^2 + (m\,t - s\,t)^2.$$

Expanding this expression, we obtain a quadratic equation of which t is the unknown quantity (the others being derived from the above data), and will depend on the value given to d. Such values may be given to d as correspond to the phases of the eclipse, and the interval between the time of opposition, and the occurrence of the phases will thus readily be obtained, the time of *opposition* being known, from the position of the sun and moon as given by the tables. Arranging them, we obtain

$$d^2 - \lambda^2 = t^2\left[(m-s)^2 + n^2\right] + 2\,t\,\lambda\,n.$$

Substituting $\tan.^2\,\theta = \dfrac{\sin.^2\theta}{1-\sin.^2\theta}$ for $\dfrac{n^2}{(m-s)^2}$, we obtain, instead of the above,

$n^2\,t^2 + 2\,\lambda\,n\,\sin.^2\theta\;t = (d^2-\lambda^2)\sin.^2\theta$; or, completing the quadratic,

$$n^2t^2 + 2\,\lambda\,n\,\sin.^2\theta\;t + \lambda^2\sin.^4\theta = (d^2-\lambda^2)\sin.^2\theta + \lambda^2\sin.^4\theta$$
$$= \sin.^2\theta\,(d^2-\lambda^2)\,(1-\sin.^2\theta)$$
$$= \sin.^2\theta\,(d^2-\lambda^2\cos.^2\theta).$$

$$\therefore t = \frac{1}{n}\left(-\lambda\sin.^2\theta \pm \sin.\theta\,\sqrt{d^2-\lambda^2\cos.^2\theta}\right).$$

And from this expression the values of the times for any values of d may be obtained. For the determination of the time, t, at which the moon immerges into the earth's penumbra, d = moon's horizontal parallax + sun's horizontal parallax + the sun's semi-diameter + moon's semi-diameter $= \mathrm{P} + p + \frac{\mathrm{D}}{2} + \frac{\delta}{2}$; the first value of t giving the instant of immersion, the second that of emersion. At the time of immersion in the umbra $d = \mathrm{P} + p + \frac{\delta}{2} - \frac{\mathrm{D}}{2}$. At the period when the whole disc has entered the shadow d = sum of the parallaxes — sum of the semi-diameters, or $\mathrm{P} + p - \frac{\delta}{2} - \frac{\mathrm{D}}{2}$. The time at which the middle of the eclipse occurs (or the *greatest phase*) is when the two halves of t are equal, or when $d^2 - \lambda^2\cos.^2\theta = 0$, and $t = \frac{\lambda\sin.^2\theta}{n}$. In this case the distance $d = \lambda\cos.\theta$. When the latter term is known, the *magnitude* of the eclipse may be determined. Some part must be eclipsed when $\lambda\cos.\theta$ is less than the distance $\mathrm{P} + p + \frac{\delta}{2} - \frac{\mathrm{D}}{2}$, as the latter quantity shows when the moon's limb just touches the shadow, and the portion of diameter eclipsed is $\left(\mathrm{P} + p + \frac{\delta}{2} - \frac{\mathrm{D}}{2}\right) - \lambda\cos.\theta$. The portion of the diameter of the moon which is not eclipsed will, therefore, be $\lambda\cos.\theta + \frac{\delta}{2} + \frac{\mathrm{D}}{2} - \mathrm{P} - p$. The eclipse will be *exactly* total when this is nothing, and will be *more* than total when it is negative. The part eclipsed may be expressed either in digits or twelfths of the lunar diameter (δ), or in decimal parts, the moon being taken for unity.

Example.—Partial eclipse of the moon, April 20th, 1856.

By the tables of the sun and moon, the time of opposition is at 9h. 13m. ·6 morning, Greeenwich mean time.

Moon's lat. at time of opposition $\lambda = 33'\;18''$
,, horary motion in lat. . $n = \;\;2\;46$ lat. increasing.
,, horary motion in long. . $m = 30\;\;8$
Sun's horary motion in long. . $s = \;\;2\;27$
Moon's apparent diameter . . $\delta = 29\;44$
,, horizontal parallax . . $\mathrm{P} = 54\;33$
Sun's apparent diameter . . $\mathrm{D} = 31\;52$
,, horizontal parallax . . $p = \;\;\;\;9$

$$\mathrm{Tan.}\,\theta = \frac{n}{m-s} = +\frac{166''}{1661} \;\therefore\; \theta = +\,5^\circ\,43'\,10''.$$

The middle of the eclipse, or

$$-\frac{\lambda\sin.^2\theta}{n} = +\frac{1998}{166}\sin.^2(5^\circ\,43'\,10'') = -7\text{m.}\cdot16.$$

Therefore the midddle of the eclipse will occur at 9h. 6m. ·4 morning, reckoning from opposition. The times at which it entered the shadow and emerged from it are calculated by finding the value of d at those times:—

$d = \frac{\delta}{2} - \frac{\mathrm{D}}{2} + p + \mathrm{P} = 53'\;38''$, and adding the one-sixtieth part of $\left(\mathrm{P} + p - \frac{\mathrm{D}}{2}\right)$ for the effect of the earth's atmosphere, $d = 54'\;17''$. Introducing this value into the expression $-\frac{\lambda\sin.^2\theta}{n} \pm \frac{\sin.\theta\,\sqrt{(d^2-\lambda^2\cos.^2\theta)}}{n}$ we obtain for the two values of t:

	m.		h. m.		h. m.
End of eclipse	= 7·16	+	1 32·68	=	1 39·84
Beginning	= 7·16	—	1 32·68	=	1 25·52

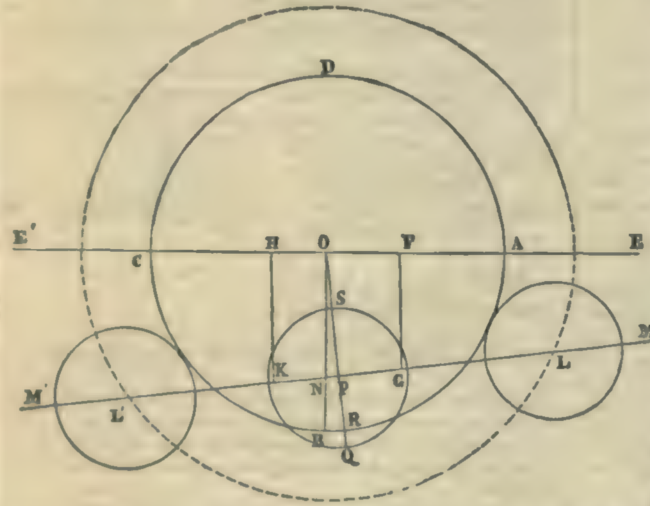
The commencement of the eclipse accordingly took place at 7h. 33m. ·8 morning, and the end at 10h. 39m. morning. The distance of the centres corresponding to the middle of the eclipse, or $\lambda \cos. \theta = 33' 8''$; and the eclipsed part, or $\frac{\delta}{2} - \frac{D}{2} + p + P - \lambda \cos. \theta = 21' 9''$, (allowing for the effect of atmosphere), or the magnitude of the eclipse, reckoning the moon's diameter at 1 = 0·71.

In order to determine the time at which the moon entered the *penumbra*, we assume, $d = P + p + \frac{D}{2} + \frac{\delta}{2} =$ 86′ 9″·8, whence the two values of t are 2h. 44m. ·32 and 2h. 58m. ·64, which, applied to the time of opposition, 9h. 13m. ·6, gives the time of commencement at 6h. 15m. ·0 a.m., and the time of ending at 11h. 57m. ·9 a.m.

GRAPHICAL CONSTRUCTION OF A LUNAR ECLIPSE.—Instead of calculating the various phases of a lunar eclipse, they may be obtained by the following graphical process, applied to the eclipse of November 13th and 14th, 1845.

At Paris, mean noon of November 13th, the longitude of the sun exceeded that of the moon by 186° 20′ 7″·4. On the 14th, at the same hour, it exceeded it by 174° 45′ 8″·6. Finding, by interpolation, the instant at which the difference of longitudes was exactly 180°, or the moment at which the moon was in opposition, we obtain, November 14th, at 1h. 4m. 20s. ·9 morning. At this time the parallax of the moon was 55′ 39″·6; that of the sun, 8″·7; the semi-diameter of the moon, 15′ 10″·1; and that of the sun 16′ 12″·3; whence, as before, we conclude that the semi-diameter of the shadow of the earth was 39′ 36″, or increasing it by $\frac{1}{60}$ = 2415″·6. In addition to this, we find that at 0h. 30m. on the morning of the 14th, the longitude of the sun exceeded that of the moon by 180° 16′ 33″·7; and that the latitude of the moon was 0° 25′ 57″·6 S. At 1h. 30m. of the same morning, the excess of the longitude of the sun was 179° 47′ 37″·7, and the latitude of the moon 0° 28′ 51″·5 S. Let the circle A B C D (Fig. 127) represent the dimensions of the shadow of the earth at the distance of the moon (semid. = O A = 2415″·6), let E E′ be the ecliptic.

Fig. 127.



At 0h. 30m. of the morning of the 14th, the longitude of the sun surpassed that of the moon by 180° 16′ 33″·7, the longitude of the centre, O, of the shadow, therefore, only exceeded that of the moon by 993″·7. According to the scale adopted, set down O F = 993″·7. Make F G perpendicular to E E′, and mark off F G = 25′ 57″·6, which is the latitude of the moon at the time, and G is the position of the centre of the moon at 0h. 30m. morning. In the same manner, take O H = 12′ 22″·3, or 742″·3, and H K = 28′ 51″·5, or 1731″·5; and K will be the position of the centre of the moon at 1h. 30m. morning. Draw a right line, M M′, through the points G and K; this line will represent the path of the moon during the eclipse, in reference to the shadow of the earth. The point N is the position of the moon at the moment of opposition, or at 1h. 4m. 20s.·9 morn.

With a radius equal to the sum of the radii of the shadow and of the moon, or 3325″·7 describe a circle with the centre O. The circumference will cut the line M M′ at two points, L L′; and it is plain that, if with those two points as centres, we describe two lesser circles with the radius of the moon, or 910″·1, these two circles will touch the circumference of the shadow A B C D, and represent the position of the moon at the beginning and end of the eclipse. If, moreover, the perpendicular, P O, be raised on M M′, the point P represents the position of the centre of the moon, at the middle of the eclipse.

As the moon employs an hour in passing from G to K, we may, by proportion, determine the time it takes in passing from N to P (if we make the figure large enough); and we thus find that it takes 5m. 41s. in going through this distance, or 5m. 41s. before the opposition, or it is at 0h. 58m. 40s. that the middle of the eclipse happens. In the same manner, we find that the moon should be 1h. 39m. 19s. in passing from P to L′; and it is therefore at 11h. 29m. 21s. of the evening of November 13th that the eclipse commences, and at 2h. 38m. of the morning of the 14th that it ends. The *magnitude* of the eclipse is found, likewise, by this graphical construction; and in this case we perceive that it is partial, since at the moment when the centre of the moon is nearest the centre of the shadow, a portion of its disc lies *without* the latter circle. If we draw the diameter Q S, directed towards the point O, and we take the proportion of this diameter S R, which is in the shadow, and the diameter itself, we find the magnitude. In the present case this amounts to 0·92, the diameter being 1·00. Should the diameter Q S be within the circle of the shadow, or the eclipse be total, the various phases, it will be seen, can be measured off with equal facility, as in the instance here given.

Instead of making use of latitudes and longitudes, the right ascensions and declinations of the sun and moon may be employed, as in the following example of the total eclipse of October 24, 1855 (Loomis Ast.) The Greenwich time of opposition in right ascension is 19h. 17m. 55s. ·6, or the Washington time = 14h. 10m. 29s. ·6, for which time the *Nautical Almanac* furnishes the following data :—

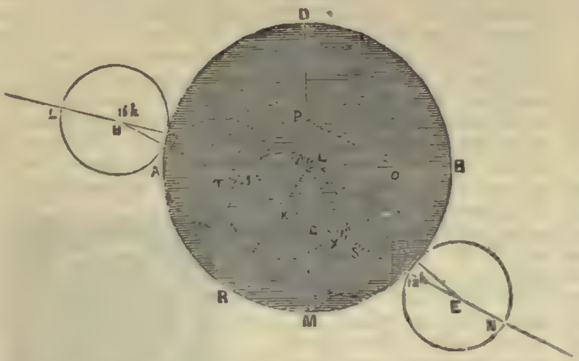
	°	′	″	
Declination of moon . . =	11	42	24·3	north.
Declination of sun . . . =	11	56	48·0	south.
Moon's horizontal parallax (lat. 45°) . . . . . =		59	39·0	
Sun's horizontal parallax =			8·6	
Moon's semi-diameter . . =		16	18·8	
Sun's ,, ,, . . . =		16	7·9	
Moon's hourly motion in R. A. . . . . . =		33	22·1	
Sun's ,, ,, ,, =		2	23·5	
Moon's hourly motion in declination . . . . =		15	40·1	north.
Sun's ,, ,, ,, =		0	52·0	south.

The semi-diameter of the earth's shadow, or CB (Fig. 128) = 59′39″·0 + 8″·6 — 16′7″·9 = 43′39″·7, to which adding one-sixtieth for refraction, C B = 44′23″·4, and the distance of the moon's centre, or C E = 60′ 42″·2. The hourly motion of the moon from the sun in right ascension is 30′ 58″·6, and this reduced to the arc of a great circle by multiplying it by the cosine of the moon's declination = 1819″·9.

From the centre C (Fig. 128), with the radius C B describe the circle A R B, or the shadow of the earth. Draw the line A C B to represent a parallel to the equator, and make C G perpendicular to it, equal to 14′ 23″·7, or the sun's declination minus the moon's declination, the point G being taken below C as the centre of the moon is south of the centre of the earth's shadow. Make C O

= 1819″·9 and C P, to the hourly motion of the moon in declination minus hourly motion of the sun in declination, or 14′ 48″·1; the point P being placed above C, as the moon was moving northward. Join O P and draw parallel to it through G the line N G H, which represents the path of the moon, and on N H let fall the perpendicular C K. At 14h. 10m. 29s. ·6 the moon was at G. To find its position at 14h. we make the proportion :—As 60m. : 10m. 29s. ·6 : : O P : G X. The position of the moon's centre at 14h. is set at X, and its position at 16h., 15h., &c., may be laid down in the same manner by adding the distance O P. Subdividing those into sixty parts, the beginning, end, and middle of the eclipse will be ascertained, answering to the times when the moon's centre is at E, H, and K.

Fig. 128.



The calculation of the eclipse may be performed accurately as follows :—In the right-angled triangle O C P we have C O = 1819″·9 and C P = 888″·1, then C P : R : : C O : tan C P O.

C O = 1819″·9 = 3·2600475
888″·1 = 2·9484619

C P O = 63° 59′ 16″ = 0·3115856 log. tan.

Sin. C P O : R : : C O : O P.

C O = 3·2600475
Sin. C P O = 9·9536150

O P = 2025″·0 = 3·3064325

The angle C P O = angle C G K, G K and O P being parallel. The angle C G E = 116° 0′ 44″; the side C G =863″·7, and the line C E=3642″·2. The angles C E G and the side E G of the triangle C E G are found thus :—

C E : sin. C G E : : C G : sin. C E G.

Comp. C E = 6·4386362
Sin. C G E = 9·9536150
C G = 2·9363629

C E G = 12° 18′ 18″ Sin. 9·3286141

Therefore the angle E C G = 51° 40′ 58″.

Sin. C G E : C E : : sin. E C G : E G.

Comp. sin. C G E = 0·0463850
C E = 3·5613638
Sin. E C G = 9·8946428

E G = 3179″·7 = 3·5023916

To Determine the Time of Describing E G.

As O P : 1h. : : E G (= 3179s. ·7) : 1h. 34m. 12s. ·8, which, subtracted from the time of opposition in right ascension, or 19h. 17m. 55s. ·6 (Greenwich time), gives the time of the beginning of the eclipse, or first contact with shadow, at 17h. 43m. 42s. ·8. The middle of the eclipse is found by the triangle C G K similar to C P O, in which the angles and hypothenuse are given to find C K and K G.

R : C G : : sin. C G K : C K : : cos. C G K : G K.

Sin. C G K=9·9536150 Cos. C G K=9·6420319
C G=2·9363629 C G=2·9363629

C K = 776″·2=2·8899779 G K=378″·8=2·5783948

To Determine the Time of Describing G K.

2025″ : 1h. : : 378″·8 : 673s. ·4 = 11m. 13s. ·4, which, added to the Greenwich mean time of opposition, gives 19h. 29m. 9s. for the middle of the eclipse. The duration of the eclipse will consequently be 3h. 30m. 52s. ·4, and the end 21h. 14m. 35s. ·2 Greenwich mean time. The magnitude of the eclipse is found by subtracting C K = 12′ 56″·2 from C R = 44′ 23″·4, whence K R = 31′ 27″·2; to which adding the moon's semi-diameter, we obtain R L = 47′ 46″·0. Dividing this by the moon's diameter, or 32′ 37″·6, the magnitude of the eclipse=1·464 on the northern limb.

The beginning and end of total darkness is found as follows :—With the radius of the earth's shadow, or C B diminished by the moon's semi-diameter, or 44′ 23″·4—16′ 18″·8=28′ 4″·6, or 1684″·6, describe a circle about the centre C cutting H N in the points S and T, which will represent the points of beginning and end of total darkness.

In the triangle C G S, C G=863·7, C S = 1684·6, and the angle C G S=116° 0′ 44″. Hence C S : sin. C G S : : C G : sin. C S G.

Comp. C S = 6 7735032
Sin. C G S = 9 9536150
C G = 2·9363629

C S G=27° 26′ 12″. Sin. = 9 6634811

And the angle S C G = 36° 33′ 4″.

Sin. C G S : C S : : sin. S C G : S G.

Comp. sin. C G S = 0·0463850
C S = 3 2264968
Sin. S C G = 9 7749107

G S=1116·3 = 3·0477925

To Determine the Time of Describing G S.

2025·0 : 1h. : : 1116·3 : 1984·6 = 33m. 4s. ·6, which being subtracted from 19h. 17m. 55s. ·6, gives 18h. 44m. 51s. for the time of disappearance or beginning of total darkness. The duration of total darkness is therefore 1h. 28m. 36s., and the time of the end of total darkness or reappearance = 20h. 13m. 27s., the *middle* of the eclipse occurring at 19h. 29m. 9s.

The semi-diameter of the penumbra is equal to the semi-diameter of the shadow + the sun's diameter, or 44′ 23″·4 + 32′ 15·8=76′ 39″·2. Let the circle A R B represent, in this case, the limits of the penumbra, then C E=76′ 39″·2 + 16′ 18″·8=92′ 58″·0. In the triangle C G E, the angle C G E=116° 0′ 44″. C G=863·7 and C E=5578″·0.

C E : Sin. C G E : : C G : sin. C E G.

Comp. log. C E = 6·2535215
Sin. C G E = 9·9536150
C G = 2·9363629

C E G=7° 59′ 56″ Sin. = 9·1434994

And angle E G G=55° 59′ 20″.

Sin. C G E : C E : : sin. E C G : E G.

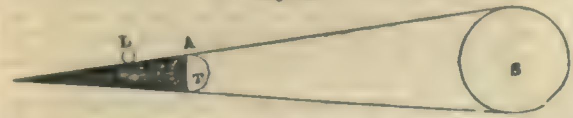
Comp. sin. C E G = 0·0463850
C E = 3 7464785
Sin. E C G = 9·9185174

E G=5144″·9 = 3·7113809

As 2025 : 1 h. : : 5144·9 : 2h. 32m. 26s. ·5, which gives the time of describing E G. This subtracted from 19h. 17m. 55s. ·6, gives 16h. 45m. 29s. ·1 for the time of first contact with the penumbra; and the whole duration being 5h. 27m. 19s. ·8, we get the time of ending, 22h. 12m. 48s. ·9 for the last contact with the penumbra.

An eclipse of the moon can only be seen at those parts of the earth where the moon is above the horizon, as well as the shadow of the earth, or at least a portion of this shadow. As this can only take place when the sun is below the horizon, it is only during the night that eclipses of the moon are visible. It may happen, however, that the eclipses may be seen for a few moments before the setting of the sun, or after its rising. This is due to the refraction of the atmosphere, which, when the sun is altogether below the horizon, as well as that part of the moon which is eclipsed, elevates both objects above the horizon, and an observer at A (Fig. 129) will be able to see the sun at one side, and the eclipsed moon on the other.

Fig. 129.



Eclipses of the Sun.—Whilst eclipses of the moon follow from the interposition of the earth, and prevent the light of the sun from reaching to, and illuminating the surface of our satellite in the usual manner; eclipses of the sun are due to the contrary cause, being the shadow of the moon falling on certain parts of the earth (passing over it like the shadow of a cloud), and hiding the disc of the sun at those districts. The dimensions of the shadow at the surface of the earth may be found in the same manner as in the former case; and an idea of the different kinds of solar eclipses may be gathered from the following diagrams. Let L be the position of the moon (Fig. 130) situated exactly in a line with the sun S, and the earth T, and at a distance L T, which corresponds to the distance of the sun and moon at the time. The smallest distance of the centres of the earth and moon being equal to 55·947 semi-diameters of the former body, and the greatest value of the shadow O L (calculated in the manner already mentioned) being 59·73 semi-diameters of the earth, it would follow that, under those circumstances, the shadow of the moon would extend beyond the centre of the earth, and consequently, that at that region of the earth where the shadow would fall (as at *a b* Fig. 131), the eclipse would be total—the moon completely obscuring the sun's disc.

Fig. 130.

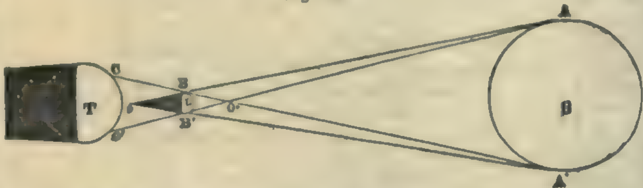
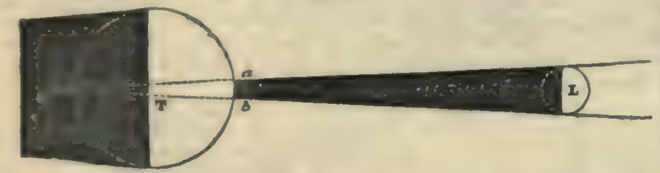
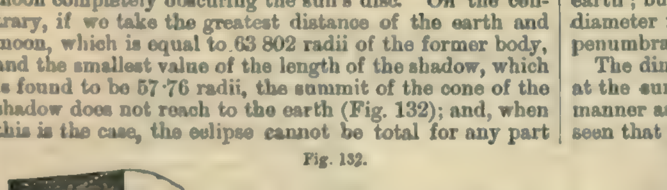


Fig. 131.



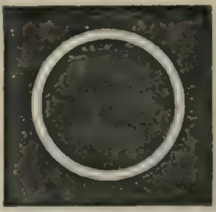
On the contrary, if we take the greatest distance of the earth and moon, which is equal to 63·802 radii of the former body, and the smallest value of the length of the shadow, which is found to be 57·76 radii, the summit of the cone of the shadow does not reach to the earth (Fig. 132); and, when this is the case, the eclipse cannot be total for any part of the earth; on all that hemisphere turned towards the sun, we perceive a portion, though not the whole of its disc.

Fig. 132.



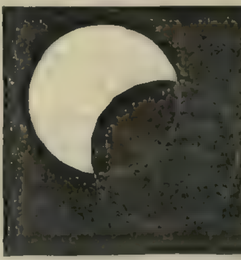
There is, however, a peculiarity to be noted; which is, that if we prolong the cone of the shadow of the moon beyond the summit O, the base of the second cone formed will be situated on the portion of the surface of the earth *c d*, and for all this part the eclipse will appear *annular*—*i. e.*, the moon will appear projected upon the disc of the sun at all those parts of the earth, and the exterior portion of the disc of the sun will consequently appear under the form of a luminous ring. When the moon is placed between the sun and the earth, the eclipse will be either total or annular for certain parts of the earth, according as the distances of the sun and moon are more or less great. This is also apparent from considering the respective dimensions of the discs of the sun and moon at different periods. The apparent diameter of the moon, as seen from the surface of the earth, may increase to 34′ 6″, whilst the diameter of the sun may decrease to 31′ 31″; and if an eclipse take place under those circumstances it must be total. As the diameter of the moon, on other occasions, may decrease to 29′ 22″, whilst that of the sun can attain its maximum value of 32′ 35″·6, it will follow that when an eclipse happens under such circumstances, it must be annular, and that for a few moments, at such parts of the earth as are exactly in a line with the centres of the sun and moon, the moon will be seen projected upon the sun; the exterior parts of the sun will continue visible, and shine with their usual brilliancy (Fig. 133).

Fig. 133.



At the parts of the earth adjoining those which are exactly in a line with the centres of the sun and moon, the eclipse cannot be total or annular, as only a portion of the moon can be projected on the disc of the sun. This will be apparent from Fig. 130, in which it is easy to perceive that at all those points situated to the interior of the cone C O′ C′, and not comprised within the real shadow B O B′, a similar appearance will be produced as in the case of the *penumbra* of the earth's shadow, on the lunar disc, during an eclipse of the moon. From any such point we should consequently perceive the circular disc of the moon intercepting a portion of the surface of the sun (as in Fig. 134); and the nearer the observer is situated to the real cone of shadow B O B′, or the further he is distant from the surface of the cone C O′ C′, the greater will be the portion of the solar surface covered by the moon.

Fig. 134.

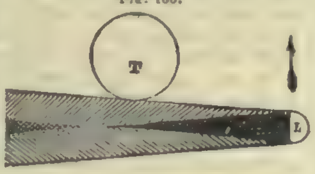


We have seen, in the case of lunar eclipses, that the whole disc of the moon can be hid by the shadow of the earth; but the moon's shadow only covers an area whose diameter is 180 English miles at the greatest; while the penumbra may reach to nearly 5,000 miles in diameter.

The dimensions of the lunar shadow, here mentioned at the surface of the earth, are calculated in the same manner as that of the earth's shadow. We have already seen that the semi-diameter of the latter is equal to the parallax of the sun added to that of the moon, and from which the apparent semi-diameter of the sun is to be subtracted; in the same way the semi-diameter of the moon's shadow at the earth is equal to the parallax of the earth added to the parallax of the sun at the distance of the moon, from which sum we subtract the apparent semi-diameter of the sun, as seen from the moon. The latter quantity is found by increasing the semi-diameter as seen from the earth, in the proportion of the distances of the moon and earth from the sun at those times, or the apparent semi-diameter of the sun seen from the moon, = apparent semi-diameter of the sun $\left(\frac{D}{2}\right)$ as seen from the earth $\times \frac{K}{k}$, K being the distance of the earth from the sun, and k the distance of the moon from the sun. The sun's horizontal parallax at the moon is equal to the sun's horizontal parallax at the earth, increased by the ratio of the distances, and diminished by the ratio of the diameters of the sun and earth; or the sun's horizontal parallax at the moon = sun's horizontal parallax on the earth (p) $\times \frac{r}{R} \times \frac{K}{k}$, r being the moon's *true* semi-diameter, and R the earth's *true* semi-diameter. Substituting those values in the preceding expression, the semi-diameter of the moon's shadow will be $\frac{\delta}{2} - \frac{D}{2} \times \frac{K}{k} + p \cdot \frac{r}{R} \cdot \frac{K}{k}$. The parallaxes of the sun and moon may be used instead of the ratio of their distances in this expression, and as $p = \frac{R}{K}$ and $P = \frac{R}{K-k}$, consequently $\frac{K}{k} = \frac{P}{P-p}$. As $\frac{r}{R} = \frac{\delta}{2 P}$, we also find $\frac{\delta}{2} + p \frac{r}{R} \cdot \frac{K}{k} = \frac{\delta P}{2 (P-p)}$; and the apparent semi-diameter of the moon's shadow $= \frac{\delta - D}{2} \times \frac{P}{P-p}$. We perceive from this that when the moon's diameter is equal to that of the sun, the apparent diameter of the moon's shadow is nothing, or the apex of the cone just reaches to the earth. When the moon's diameter is less than that of the sun, the expression is negative, and the shadow cannot reach the earth's surface. The formulæ for the dimensions of the penumbra may be adapted to the moon's shadow in a similar manner. If the apparent semi-diameter of the earth, or the moon's horizontal parallax, be added to the apparent semi-diameter of the shadow, the distance of the centre of the moon's shadow and the centre of the earth is found; and the limits of the distance of the moon from her nodes, when an eclipse can happen, will readily be obtained, and is found to be 17° 27′.

It is thus seen that the same formulæ apply to the calculation of both lunar and solar eclipses, in so far as the times at which the earth in general would become enveloped in the shadow of the moon, are concerned; but this would only have reference to an observer situated on the moon, who would thus be enabled to foretel and note the duration, times, and magnitude of an eclipse, the instant at which the penumbra entered upon the enlightened disc of the earth, and the period of total eclipse. In Figure 135 we perceive the phenomena which would follow from the interposition of the moon between the sun and earth; the fictitious observer situated at L, as the moon moved in the direction of the arrow, carrying with it the real shadow and the penumbra, would first see the penumbra touch the disc of the earth T, then (if the eclipse were total) the contact of the real shadow; and the above formulæ would give the instant at which those appearances would happen. The two cones, that of the real shadow and the penumbra, would continue to traverse the portion of the hemisphere turned towards the moon, covering, successively, the various parts of its disc, advancing on new regions, and leaving those which it had previously shadowed. Finally, the cone of the shadow, then that of the penumbra, would again become tangents to its surface; and the instant at which this would take place, would mark the end of the eclipse. But in the case of a solar eclipse on the earth's surface, the time at which it became generally visible on the earth, or the period of first contact, would not be of such interest to any observer, as the time at which it would become visible, or total, at his own particular locality; and the position of the observer at different parts of the moon was not taken into consideration in the foregoing calculations, the latitude and longitude of the several places on the moon being a matter of indifference to an observer on the earth. But to an observer on the earth it is a matter of great consequence to know the exact time at which the eclipse would commence at his own station (for it will not begin for him until such period), and also the extent and duration of the shadow of the moon, when it passes over a known station of any assigned latitude and longitude. The calculation of an eclipse of the sun is, therefore, more difficult than that of the moon, because more is required to be determined. The foregoing formulæ may be applied to the case of an eclipse of the sun, viewed from the centre of the earth, to which they will exactly apply. But to the position of an observer on the surface of the earth, various corrections of parallax have to be applied on account of the angle which he makes with the centre of the earth, whether in latitude or longitude, as the various phases of the eclipse at the different parts of the earth, by which it is visible at some parts and not at others, are due altogether to parallax. In calculating a solar eclipse for any assigned position, it is necessary to correct the angular distances, or the longitudes and latitudes, for the effects of parallax. This requires a long and intricate calculation; but the principal appearances and phenomena, as likewise the times of contact, greatest eclipse, duration, and magnitude, may be determined in the following simple manner :—

Fig. 135.



GEOMETRICAL OR GRAPHICAL CONSTRUCTION OF A SOLAR ECLIPSE.—If an observer be supposed to be situated at the centre of the sun, and the moon be interposed between the sun and earth, he will observe the moon passing across the disc of the earth in the same manner as an observer on the earth sometimes perceives the satellites of Jupiter traversing the bright disc of the latter planet. In the same manner, likewise, as an observer on the earth perceives the equator and axis of the sun inclined at different angles to the north and south points and to the ecliptic, at the various seasons of the year (pages 929, 930, Figs. 27 and 28), the observer at the sun would perceive the poles and equator of the earth change their situations in respect to the ecliptic, but in a greater degree, the solar equator being only inclined at an angle of 7° to the ecliptic, whilst that of the earth is inclined at 23½°. At the time of the vernal equinox, the plane of the equator passes through the sun, and the north and south poles of the earth will consequently appear to an observer at the sun situated at the margin of the disc, the parallels of latitude describing straight lines, as in Fig. 136. At the summer solstice, the north pole of the earth will be turned towards the sun, the south pole will be invisible, and the parallels of latitude will, to an observer on the sun, be projected into ellipses, as at Fig. 137. At the autumnal equinox, the parallels of latitude will again appear as straight lines, but the poles of the earth will be on the opposite side of the poles of the ecliptic, as at Fig. 138. Finally, at the winter solstice, the south pole will be turned towards an observer on the sun, and the parallels of latitude projected into ellipses, as in the former case, but in the contrary direction (Fig. 139).

Fig. 136.

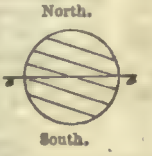


Fig. 137.

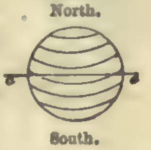


Fig. 138.

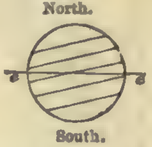
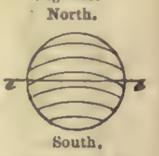
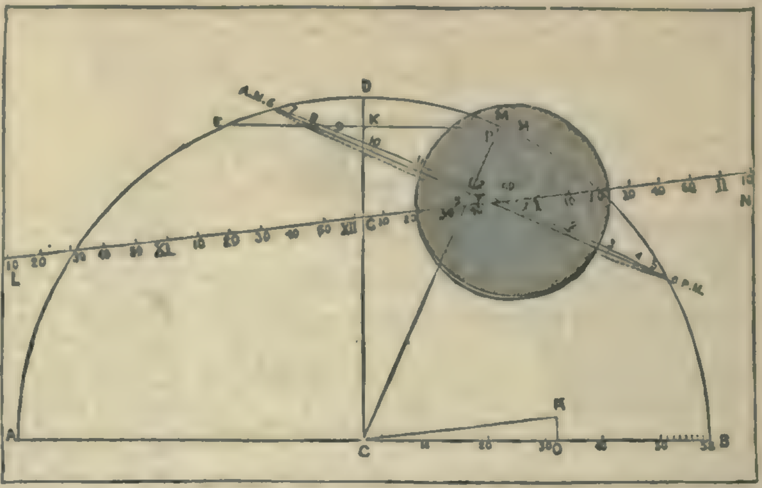


Fig. 139.



By projecting in this manner the surface of the earth at the various seasons of the year, as it appears to a spectator on the sun, we can represent any parallels of latitude. By doing so for the day on which the eclipse is expected to occur, we can mark along the projection, the position of the place at the different hours of the day; and the moon's apparent path across the earth's disc may be laid down in a similar way, and its position from hour to hour noted. In order to determine the time at which the eclipse appears greatest at this locality, we must find that point in the moon's path and the path of the observer marked with the same times, and which are at the *least* distance from each other. The beginning or end of the eclipse will be determined by finding those points in the moon's path and in the path of the spectator which are marked with the same times, and whose distance is equal to the sum of the semi-diameters of the sun and moon. As an illustration of this method we will choose the solar eclipse which was visible in the British Islands, March 15th, 1858 (Figs. 140 and 141), and generally seen throughout the north of Europe. This is the greatest which has occurred or will happen in this country for many years. The times and phases of the eclipse are for the latitude of Greenwich, or 51° 28′ 38″.

Fig. 140.



By the *Nautical Almanac* the time of new moon is March 15d. 0h. 12m. 0s., mean time, corresponding to March 15d. 0h. 2·9m. apparent time, the equation of time being — 9·1m. The following elements are calculated for that epoch ;—

	°	′	″
Sun's longitude	354	38	38
Sun's declination	2	7	15 S
Moon's latitude	0	37	43 N
Moon's hourly motion in longitude .	0	34	18
Sun's hourly motion in longitude .	0	2	29
Moon's hourly motion in latitude .	0	3	9
Moon's equatorial horizontal parallax	0	58	15·2
Sun's equatorial horizontal parallax .	0	0	8·6
Moon's true semi-diameter . .	0	15	54·6
Sun's true semi-diameter . .	0	16	6 5

The *geocentric* latitude, which is found by applying the angle of the vertical (p. 922) to the *geographical* latitude (and which in this case is 11′ 13″), = 51° 17′·4. The moon's *equatorial* horizontal parallax being that which is given by the tables, the horizontal parallax for any other latitude will be found by applying the following correction :—

Latitude.	Moon's Horizontal Equatorial Parallax.		
	53′	57′	61″
°	″	″	″
15	0·7	0·8	0·8
20	1·2	1·3	1·4
25	1·9	2·0	2·2
30	2·6	2·8	3·0
35	3·5	3·7	4·0
40	4·4	4·7	5·0
45	5·3	5·7	6·1
50	6·2	6·7	7·2
55	7·1	7·7	8·2
60	8·0	8·6	9·2
65	8·7	9·4	10·0
70	9·4	10·0	10·8
75	9·9	10·7	11·4
80	10·3	11·0	11·9
85	10·6	11·4	12·1
90	10·6	11·4	12·2

The moon's horizontal parallax for Greenwich will therefore be 58′ 8″·0 ; and as the relative positions of the sun and moon will remain the same if the sun be supposed to retain a fixed position, and the moon be supposed to be affected with the difference of both their parallaxes, the relative parallax, or that which affects the relative positions of the two bodies, will be 57′ 59″·4, which is the semi-diameter of the earth as seen from the moon, that of the moon seen from the earth being 15′ 54″·6. The relative sizes of their discs will be in the same proportion if viewed from any distance. With a radius A C (Fig. 140) equal to 57′ 59″·4, or if practicable with one of 3,479 parts from any scale of equal parts, describe the semicircle A D B with the centre C, which represents the northern half of the enlightened part of the globe of the earth viewed from the sun. If the place were in south latitude, the lower half of the earth's disc should be that represented. The line A C B represents the ecliptic, and the perpendicular C D the axis of the ecliptic. Take from a sector, or any other convenient scale, the chord of 23° 28′ (which is the obliquity of the ecliptic) to the radius C B, and set it off from D to H and E in the periphery of the circle, or the angle D H = 23° 28′ may be at once set off. Draw the straight line E H, cutting C D in K. By considering Figs. 136, 137, 138, 139, it will be seen that the north pole of the earth, as viewed from the sun, will constantly appear on this line E K H, E being its position at the autumnal, H at the equinox, and K at the solstices. Its distance from H at any time will be equal to the versed sine of the sun's longitude, or its distance from K will be the sine of the difference between the sun's longitude and 90° or 270°. To the radius E K take the sine of 354° 38′·6 – 270° = 84° 38′·6, and set it off from K to P, the north pole of the earth ; for when the sun's longitude is between 270° and 90°, the north pole lies to the right of the pole of the ecliptic, and to the left when it is in the other six signs. Draw C P, which is the northern half of the earth's axis, and produce it until it cuts the circle A D B in M.

In order to draw the parallel of latitude of Greenwich on the earth's disc as seen from the sun, from sunrise to sunset, the following method may be taken :—If the latitude of the place were exactly equal to the sun's declination, it would be vertical at noon, and the place of observation would be at the centre C. But as the latitude of the place exceeds the sun's declination by 53° 24′, it must be seen that distance north of the place where the sun is vertical, or when projected on the disc ; this will be the sine of that angle which is set off from C to 12, which is the apparent position of Greenwich at noon of March 15. The position of Greenwich at midnight is determined in the same manner by taking C S equal to the sine of 49° 10′ to the radius A C, and the point S will be its position at midnight. It is easily seen that the line 12 S will be the shortest diameter of the ellipse into which the circle of latitude is projected. To determine the length of the longest diameter, as it is not shortened by being viewed obliquely, it will be equal to the diameter of the parallel of latitude, or to the cosine of the co-latitude = 38° 43′ ; and setting off this distance from the point T (which is situated exactly between S and 12) on the line perpendicular to C M, we obtain the extremities of the longest axis of the ellipse, or the points at which Greenwich will appear at 6 A.M. and 6 P.M. At the times of the equinoxes these hours will be projected at the edges of the disc (as they are almost so in the present instance), but at all the other times of the year they will be more or less distant.

In order to obtain the position of Greenwich at any other hour of the day, we take, with a radius equal to T 6, the sine of 15°, or one hour of time, which is set off on each side of the point T. The sines of 30,° 45°, 60°, 75° may be obtained and set off in the same way. Through these points draw lines parallel to C M. With a radius equal to S T take the sines of 75, 60, 45, 30, 15, which are marked off on each side of the line T 6 respectively at the points 15°, 30°, 45°, &c., as before. By drawing an elliptic curve through those points, the several positions of Greenwich at each hour of the day from sunrise to sunset are readily obtained; and by continuing it on the opposite side, its position (here represented by the dotted line) is represented during the night. The points at which the ellipse touches the circle A D B show the time of sunrising and sunsetting. It will be seen that the ellipse will be more open near the time of the solstices, and more eccentric near the equinoxes (as is the case in the present instance). The points 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6, represent the position of Greenwich at those hours, which are marked from noon towards the right. If the sun's declination had been north, the diurnal path of Greenwich would be on the lower or dotted portion of the ellipse. If possible, the figure should be constructed on such a scale that the hours might be divided into minutes with sufficient certainty.

Having represented the position of Greenwich at the various hours of the day, as they appear to a spectator on the sun, the next thing to be done is to draw the moon's apparent path across the earth's disc. From the same scale of equal parts used in the radius C B, measure off an interval equal to the moon's latitude, or 37′ 43″; and, as the moon's latitude is north, make it equal to C G, above the ecliptic A B. Take in the same scale, C O, equal to 31′ 49″, and apply it on the line C B, from C to O. This quantity represents the hourly motion of the moon from the sun in longitude. Draw O R perpendicular to C B, and make it equal to 3′ 9″, the hourly motion of the moon in latitude; and as the moon is moving northwards, set it above the line C B. The line C R, joining the points C and R, represents the relative hourly motion of the sun and moon; and parallel to this line draw the orbit of the moon, or L G N, on which, by means of the hourly motion C R, we can mark the position of the moon from hour to hour; and as the moment of new moon at Greenwich is at three minutes past noon of apparent time, the position of the moon at apparent noon is found by taking the proportion of 60m. : 3m. : : the distance C R, to the distance G XII., which is measured to the left of the point G, and shows the position of the moon at twelve o'clock, or noon. By taking the line C R in the compasses, the other hours may be marked off which represent the places of the moon at those times. If the scale be made sufficiently large, those spaces may be subdivided into sixty parts or minutes.

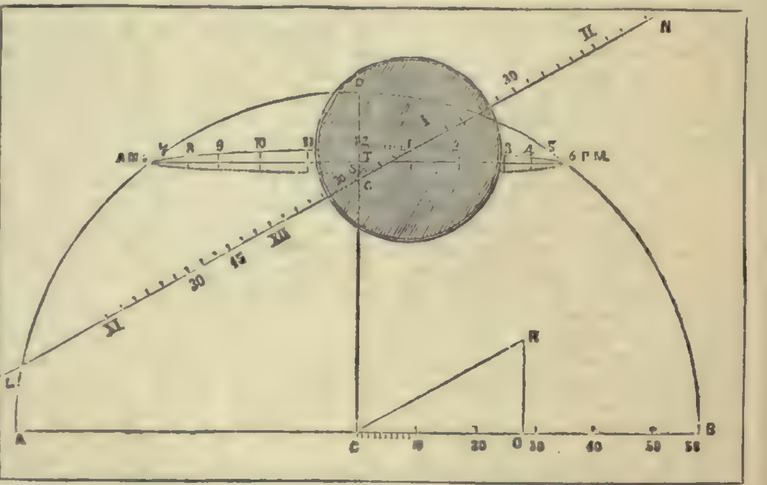
In order to find the times at which the eclipse is greatest, those times must be found on the path of the spectator, and the path of the moon, which are marked with the same times, and which are at the *least* distance from each other. The times which best correspond to those conditions in the present case are those of 12h. 52m., or 52 minutes past noon of apparent time. The appearance of the moon at any hour may be shown by taking its semi-diameter to the adopted scale of equal parts—and which in this instance = 15′ 54″·6—and with this radius to describe a circle whose centre is the point where the moon's centre is at the given time. The position of the disc of the sun at the same moment may be represented in the same manner, by taking a radius equal to the semi-diameter of the sun = 16′ 6″·5, and describing a circle taking the *same* time on the path of the spectator for a centre. The relative positions given in the diagram are those at which the eclipse are greatest, when the magnitude of the eclipse = 0·976, on the northern limb, the sun's diameter being 1·000; and as the moon's semi-diameter is less than that of the sun, the eclipse will be annular and partial. The sun's centre must be invisible over all that portion of the earth, represented as being covered by the moon's disc, and throughout a much larger area some portion of the sun's disc will be obscured. The extent of this area will be found by describing a circle with the same centre, and whose radius is equal to the sum of the semi-diameters of the sun and moon, measured on the adopted scale.

In order to determine the times of the beginning and end of the eclipse, we take from the scale of equal parts a distance equal to the sum of the semi-diameters of the sun and moon = 32′ 1″·1, and beginning near L, place one foot of the compasses on the moon's path, and the other on the path of the spectator, and shift them backwards and forwards until the *same times* are found on the two paths, which are this distance apart. This will give the time of the beginning of the eclipse, which in the present case is 11h. 32m. A.M., *apparent* time. In the same manner find the corresponding times on the other side of the moon's path, and the end of the eclipse will be determined, which in the present case is 2h. 9m. P.M., of apparent time. We thus determine :—

	Apparent Time.	Mean Time.
Beginning of Eclipse .	11h. 32m. A.M.	11h. 41m. A.M.
Greatest Obscuration .	0h. 52m. P.M.	1h. 1m. P.M.
End of Eclipse . . .	2h. 9m. P.M.	2h. 18m. P.M.

When this projection is made on a sufficient scale, and carefully finished, it will give the above times accurately to one or two minutes, and furnish a useful chart of the various phases of the eclipse. By drawing different parallels of latitude, we can determine the phases of the eclipse for any place. The results are obtained in apparent time, because both the moon's path and the path of the spectator correspond to apparent time.

Fig. 141.



Projection by Right Ascensions and Declinations.—Instead of employing longitudes and latitudes in the projection of a solar eclipse, we may make use of right ascensions and declinations; and as those are the elements given in the *Nautical Almanac* for these phenomena, it may be convenient to append an example for this eclipse, but for a different latitude and longitude. The radius of projection, C B (Fig. 141), is the same in this case as in the last, being the difference of the horizontal parallaxes of the sun and moon ; C D is the meridian or circle of declination ; the ellipse represents the projection of the parallel of latitude ; L G N the moon's apparent path ; C G the difference of *declination* of the sun and moon at the conjunction in right ascension, and C 12 the sine of the sun's zenith distance at noon. The elements for the above eclipse, given in the *Nautical Almanac*, are as follows, and the place may be considered as situated 0° 6′ west of Greenwich, and at 52° 41′ of north latitude :—

Greenwich mean time of conjunction in right ascension = March 15, 0h. 44m. 7s. ·6, or March 15, 0h. 43m. 44s.·2 of the mean time at the given place.

The apparent time will be March 15, 0h. 34m. 37s. ·8, the equation of time being—9m. 6s. ·4. The moon's hourly motion in right ascension = 30′ 16″·4 ; and the sun's, 2′ 17″·1 ; and the relative hourly motion of the moon from the sun on right ascension 27′ 59″ 3, = which must be reduced to the arc of a great circle by being multiplied by the cosine of the declination, or 1° 24′ 21″·8 = 27′·98.

The radius A C = 58′·0 as in the last case.

Hourly motion of moon from sun . .	= 27′·98
Moon's hourly motion from sun in declination	= 15′ 31″
Moon north of sun	= 42′ 53″
Sum of semi-diameter of sun and moon	= 32′ 1″
Difference	= 11″·9

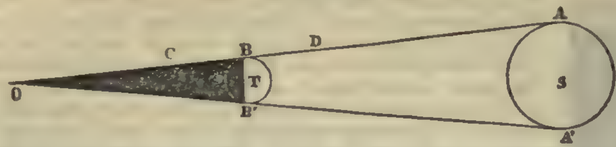
The semicircle A D B represents the northern half of the earth's disc drawn with a radius, A C, equal to 58′ 0, as in the last case C D is the axis of the earth. The position of a point of 52° 41′ of north latitude, or 52° 30′ of geocentric latitude, will be obtained in a similar manner to the last, and its place on the axis of the earth at noon and midnight will be 54° 37′ and 50° 23′ to the north of the point where the sun is vertical. Obtain, by means of a sector or graphically, the sines of those angles to the radius C B, and set them off on the line C D from C to S. The position of the locality whose latitude is 52° 30′ at 6h. in the evening, will be obtained by taking the cosine of the latitude, and marking it off on the line T 6 — T 6, perpendicular to C D on each side of T, that point being the bisection of the two points S 12. These distances may likewise be obtained, and with more exactness, by multiplying the radius C B = 58′ by the sines of 54° 37′, and 50° 23′, the former of which gives the distance C 12, and the latter C S, the distance T 6 being obtained by multiplying 59′ by the cosine of 52° 30′ ; and those quantities may be marked off by means of the same scale of equal parts, as adopted in the radius C B. The other times in path of the observer, are determined as in the former instance. The moon's path, according to right ascension and declination, is laid down as before. Take the difference of the declinations of the sun and moon = 42′ 53″, and set it off, from C to G above the line A C B, because the moon is north of the sun. Take C O equal to the hourly motion of the moon from the sun in right ascension, reduced to a great circle, and set it off from C to O. Make O R, which is perpendicular to C B, equal to 15′ 31″, the moon's hourly motion in declination from the sun. The line C R represents the hourly motion of the moon from the sun in regard to right ascension and declination, and the line L G N, drawn parallel to this, represents the moon's path across the earth, the point G being the position of the moon's centre at the moment of conjunction in right ascension. The even hours may therefore be marked off on this point, by taking the point G to be 34m. 38s. past noon on March 15th ; and the line C R to be the hourly motion of the moon, and the position of the moon at noon will be found by the proportion as 60m. : 34m. 38s. : : C R : G XII. : : 31·6 : 18·2. The hours on the moon's path should, if possible, be subdivided to minutes, and the same on the path of the spectator. The sum of the semi-diameters of the sun and moon being taken on the adopted scale = 32′ 1″, that distance is taken in the compasses, and those are shifted backwards and forwards, as in the former case, until the *same* times are found on the moon's path and the path of the spectator, which are exactly at this distance apart. When the left leg of the compass is on the moon's path, and the right leg on that of the spectator, this will give the commencement of the eclipse ; and when the *right* leg of the instrument is on the moon's path, the left on that of the spectator, and the corresponding times are again found at the same distance apart, we obtain the end of the eclipse in the same manner. By applying the side of a small square to the moon's path, and moving it along until the other side cuts the same hour and minute on both lines, we obtain the nearest approach to the centre of the sun and moon. If the scale of the projection be made of sufficient dimensions, the distance of those two points will show whether the eclipse be total or annular ; if less than the difference of the semi-diameters of the sun and moon, it will be annular at that locality. By describing a circle whose radius is equal to that of the moon, and whose centre is situated on the moon's path at the time of the nearest approach of the centres, we have a circle representing the moon's disc. Taking the radius of the sun in the same manner, and the corresponding time on the path of the spectator for a centre, we describe the position of the solar disc, and their intersection will represent the phase of the eclipse at the moment of greatest darkness. With the points of the compasses at a distance apart, equal to the differences of the semi-diameters of the sun and moon, the times of the formation and rupture of the annulus may be determined in a similar manner, as the beginning and end of the eclipse. In the present case we obtain for the beginning and end of the eclipse at the given position 11h. 32m. A.M., and 2h. 8m. P.M. of apparent time, which is equal to 11h. 41m. and 2h. 16m. of mean time. The time of greatest obscuration is 12h. 52m., found by taking the shortest distance between the corresponding times on the paths of the moon and spectator, answering to 1h. 1m. P.M. of mean time as before.

From the relative dimensions and positions of the sun and moon, it will be seen that the eclipse is central and annular at this locality—viz., at 0° 6′ (of arc) of west longitude and at 52° 41′ of north latitude, or at a point which is a little to the east of Peterborough. The line of central eclipse passed across England from Bridport in Dorsetshire (at 2° 51′ of west longitude, and 50° 43′ of north latitude) to the Wash, and a little to the north of the towns of Sherborne, Marlborough, Oxford, Buckingham, and Wisbeach. The diameter of the moon so closely approaches to that of the sun, that this eclipse might be *total* in the vicinity of the island of Madeira by the augmentation of the former value. At the station for which the above calculation is made, the duration of the annulus was only 12s. ·3.

Annular eclipses are more common than total eclipses, and the prediction of their times and duration shows more palpably than anything else, the great accuracy to which the tables of the sun and moon have been carried ; but astronomers still rely upon the careful observation of those phenomena, in order to correct the places of those bodies as given by theory ; and the times of the eclipse of March 15th, 1858, are liable to be slightly affected by small errors in the tabular places of the sun and moon ; whilst errors which are known to exist in the received diameters of those bodies affect the duration of the annulus. These errors, however, only amount to a fractional part of a second at most. How different is this from the prediction of the eclipse which was to have happened at Rome in 1684, which the tables at that time in use predicted would be *total*, whilst in reality only three-fourths of the sun disappeared. In the prediction of the eclipse of 1706, the tables of La Hire were four or five minutes in error.

Annual Number of Eclipses of the Sun.—Eclipses of the sun on the earth generally are more frequent than eclipses of the moon, in the proportion of three to two, as it will be seen should follow from the following considerations. In order that there be an eclipse of the moon, it must happen that the moon penetrates the cone of shadow B O B′ at C (Fig. 142); and it is easily seen, that in order that there be an eclipse of the sun at *some portion* of the earth, it must penetrate into the cone A O A′ at the point D. As the diameter of the cone is larger at D than at C, it consequently happens that the moon passes more frequently through the former than the latter, and that eclipses of the sun will be more frequent than those of the moon. Theory and observation alike prove, that in the period of 18 years 11 days, in which, as before mentioned, the moon passes to the same position in respect to the sun and its nodes, there are in general seventy eclipses, of which forty-one are of the sun, and twenty-nine of the moon. There cannot be more than seven eclipses in one year, nor less than two; and if the latter, they must both be eclipses of the sun. In any *given place*, however, the eclipses of the moon will appear to be more frequent than those of the sun, as will be apparent when we come to consider that the former are due to the total or partial extinction of light on the surface of our satellite, and are seen on every occasion on that hemisphere of the earth, where the moon is above the horizon. Eclipses of the sun, on the contrary, can only be seen at one zone of the earth, and that a very narrow one; for, in addition, it will be seen that the dimensions of the shadow of the moon are small as compared with that of the earth. Out of the forty-one solar eclipses, which may occur in eighteen years, there are in general twenty-eight of those which may become central according to circumstances—that is, either total or annular. It results from Du Séjour's calculations, that the greatest possible duration of an eclipse of the sun cannot be more than 4h. 29m. 44s. for a place situated at the equator, or 3h. 26m. 32s. for the latitude of Paris. In total eclipses, the greatest possible duration of total obscuration may only be 7m. 58s. at the equator, and 6m. 10s. at the latitude of Paris. In the case of annular eclipses, the greatest possible duration of the phase is 12m. 24s. at the equator, and 9m. 56s. at the latitude of Paris. Such combinations are of course very rare. The total eclipse of 1706 lasted for 4m. 10s.; that of 1715, at London, 3m. 57s.; the total eclipse of 1806, at Kinderhook, in America, continued for 4m. 37s.; and that of 1724, at Paris, for 2m. 16s. In the eclipse of 1778, the darkness continued for four minutes.

Fig. 142.



Phenomena observed during a Total Eclipse.—The great and sudden darkness which takes place at the moment of a central eclipse is, of course, the great phenomenon to be witnessed in solar eclipses, but many other curious circumstances have been unexpectedly noticed by the telescopic observers during the eclipses of 1842 and 1851, which serve to throw some light on the constitution of the sun; so true is it, as Arago observes, that, in the intellectual world as in the terrestrial, we cannot advance a step without discovering a new horizon. It is not, however, to be strictly understood that those appearances were first perceived on these occasions, but rather that they were observed with greater care; and from the number of observers engaged, and the different instruments they made use of, the results are placed beyond doubt, and confirm the desultory notes of former astronomers.

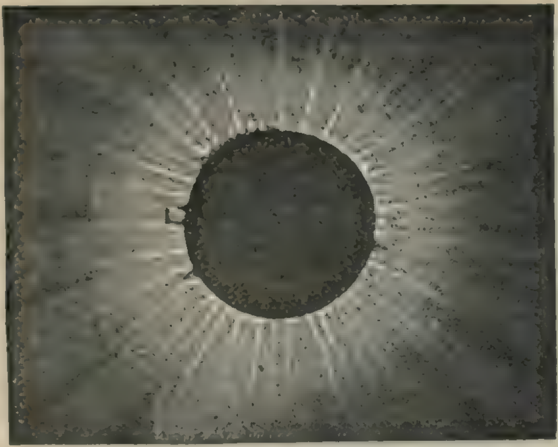
The darkness which follows the total obscuration of the sun's disc, though momentary, is generally very great, though not complete. The total eclipse which occurred in England, on June 17th, 1433, was long remembered under the name of the "Black Hour." That of 1598 was equally imprinted on the memories of the peasantry, and called to mind by the name of the "Black Saturday;" whilst the total eclipse of 1652, is recorded in Scotland by the name of "Mirk Monday." Nor, if we are to allow the testimony of Dr. Halley, are those titles misapplied in such cases. "I forbear," he says, in his communication to the Royal Society, on the eclipses of 1715, "to mention the chill and damp which attended the darkness of this eclipse, of which most spectators were sensible and equally judges. Nor shall I trouble you with the concern that appeared in all sorts of animals, birds, beasts, and fishes, upon the extinction of the sun, since ourselves could not behold it without some sense of horror." In the eclipse of 1706, during the period of total darkness at Geneva, bats flew about as at dusk, swallows were seen flying about, and cage-birds put their heads under their wings. The stars appeared as thickly strewed as at the time of full moon. In the eclipse of 1842, several stars of the first magnitude, as well as the planet Mars, were distinctly visible. The colour of the sky at those times does not appear, however, either to belong to the darkness of night nor the hue of twilight; in some places it was even noticed to be of a violet tint; and *red* stars, such as those of *a* Orionis, Aldebaran, and others, were noticed to be *white*. At Montpellier, in the eclipse of 1842, the light had acquired a livid tint, imparting to the human countenance an aspect which it was painful to contemplate; and the spectacle was generally allowed to be of an extraordinary and appalling character. An owl was seen to leave the tower of St. Peter, the bats left their retreats, the swallows disappeared, the fowls went to roost, and the cattle stood still in the field. A heavy dew fell at Perpignan, Turin, and Vienna, during the obscuration. Similar appearances were noticed during the total eclipse of 1851, by the numerous observers stationed in the path of central eclipse in Sweden.

Even during the greatest obscuration, and when the disc of the sun is completely hidden by that of the moon, its place is still made apparent by the brightness in the part of the heavens in which it is situated. This light appears in the remarkable form of a corona or lustrous ring, and it has sometimes been so bright as to be observed and mistaken for an *annular* eclipse. This luminous ring was seen during the eclipse of 1567, and was mistaken for the margin of the solar disc; it was equally visible in that of 1598. Its *radiating* appearance seems to have been first noticed in the eclipse of 1652, and appeared to be endowed with a sort of rotatory motion. In every subsequent eclipse, observed under any favourable circumstances, the corona has been observed with equal plainness, generally of about one-tenth or one-twelfth of the diameter of the moon, and of a pearl-white colour, or, in the words of an observer, "of that bluish tint which distinguishes the colour of quicksilver from that of a dead white." In the eclipses of 1842 and 1851, it was the principal circumstance to which the attention of the observer was directed; and M. Arago was able to perceive what might be termed *two* rings—the inner one, or that which bordered on the moon's limb, being of a uniform brightness, whence it faded imperceptibly outwards, and terminated irregularly. The inner ring was 3′ or 4′ in breadth; but the whole breadth of the corona was differently estimated by various observers, as 8′, 16′, and 25′ in breadth. Several luminous jets of light were noticed in the corona by M. Struve, at Lipesk, as in the following diagram (Fig. 143); and it had a decidedly radiating appearance; but the observers in France only perceived one or two radiations, and in some cases only the brighter or inner ring of light was observed. In the eclipse of 1851, the same discordance in the estimation of the breadth, and in the radiating aspect of the corona, existed between the several observers. The direction of the diverging rays did not seem to be perpendicular to the circumference of the moon, but were greatly inclined; and M. Arago noticed, that at one part of the corona, the rays were entwined within each other, like a skein of thread which was entangled. (See, also, page 165, *ante*).

The appearance of this ring of light is now generally attributed to the effect of the atmosphere of the sun, rather than to anything of the same nature in the case of the moon. The latter was the supposition of Kepler;

and before the want of an atmosphere to our satellite was established by numerous proofs, it was the most natural one which could be imagined; although the great breadth of the ring would go to prove that its height must be three or four hundred miles, or ten times the extent of the earth's atmosphere. No such difficulties exist in the case of its being supposed to be the solar atmosphere, while its concentric form, in regard to the sun at the different periods of observation, tends still further to confirm this conjecture. By what has already been stated of the photosphere of the sun, and the different strata which surround its surface, as well as the appearance of the zodiacal light, this may be deemed to be, in every respect, the most probable conjecture.* The corona, however, cannot be identified as forming the zodiacal light, for it has mostly been observed of a circular form, whilst the latter is elliptical; even where the corona has been noticed to be protuberant on opposite sides, the greater axis does not take the direction of the ecliptic.

Fig. 143.



Red Flames.—In the eclipses of 1715 and 1733, when the sun was wholly eclipsed, and only the corona was visible, the margins of the moon were noticed to be marked with some red spots, which remained visible for some seconds. Whilst observing the corona in the eclipse of 1842, these appearances again became unexpectedly conspicuous to the different observers. They were first observed a few seconds after the time of total obscuration, and were of a fine crimson tint—being compared by one observer to the peaks of the Alps illumined by the setting sun; and, by another, to beautiful sheaves of flames, which remained visible even after the sun had emerged. The length of the most considerable was, by actual measurement, found to be equal to 1¾ minutes of arc. Only two or three isolated prominences were perceived by the observers stationed in the south of France; but to MM. Struve and Schidlowsky, these rose-coloured flames burst out at several parts of the lunar disc—a *very large portion* of the periphery being garnished with this reddish bordering. M. Littrow noticed that they changed their colour as the eclipse advanced, being at first white, then rose colour, and finally violet, passing afterwards in a reverse order through the same tints. In the eclipse of 1851, these crimson projections were beautifully seen, and by some observers in great number. Some seen were, in this instance, crooked, and resembled a flame bent aside by the wind, but their colour was of the same remarkable tint as in the former case. In the eclipse of 1706, observed by Captain Stannyann, at Berne, "a blood-red streak of light" was noticed; and Halley, in 1715, saw a similar "long, and very narrow streak, of a dusky but strong red light" Equally with the corona, these appearances have been conjectured to be in the atmosphere of the sun, and to be in no way connected with the moon. Their reddish light would seem to show that they were of a cloudy nature, resembling our terrestrial clouds, and absorbing all the rays of the spectrum, with the exception of the red. (See, also, page 165, *ante*).

Baily's Beads.—In some cases when the margin of the moon comes in contact with that of the sun, instead of the faint and regular thread of light which would be supposed to ensue under those circumstances, the appearance presented is a broken glimmer of light, which was first noticed by the late Mr. Baily, who compared it to *beads* of light. These are noticed in total as well as annular eclipses, and were seen by different observers in the eclipse of 1851. They are generally regarded as being due to the rough and mountainous edges of the moon coming in contact with the margin of the sun, and the light proceeding from the latter shining through the *chinks* or valleys of the moon, the effect being greatly increased by irradiation.

Method of Observation.—We can follow the different phases of a solar eclipse without the help of a telescope, and by merely making use of a piece of coloured glass; that colour which is technically termed by opticians *London Smoke* being the most convenient for this purpose. If a pin-hole be made in a piece of card, and the image received on a screen, the disc of the sun, and the various phases it assumes from the interposition of the moon, likewise become very apparent, and we have thus a still more simple method of following the phases of an eclipse. The *form* of the aperture, or the pin-hole, is of little consequence in this instance, provided it be small.

Application of Photography.—For particulars on this subject, the reader may refer to the article on Celestial Photography, in the section on Light (page 164).

Occultations of Stars by the Moon.—The occultations of stars by the moon are analogous to eclipses of the sun, and their graphical projection may be made with some slight modifications as in that case, and the same formulæ may be applied to their prediction. All the planets may be occulted by the moon, but only such of the fixed stars as are situated near the ecliptic, and within the limits of the moon's latitude. Among the brighter stars which can suffer eclipse—those of Regulus, Spica, Aldebaran, Antares, and the group of the Pleiades, may be mentioned. An occultation can be seen at many different parts of the earth's surface, and their accurate observation forms one of the best means of determining the longitude, being much more exact than the occultations of Jupiter's satellites, as the moments of immersion and emersion are instantaneous. The portion of the earth's surface at which the star appears to be occulted at the same instant, will have a diameter equal to that of the moon, and will pass, like the shadow of the moon, over a certain zone of the earth. As the star has no perceptible diameter, there can be no penumbra in such a case, even if the light of the star were sufficient to cast a shadow. This circumstance renders the calculation more simple, which likewise follows from the star having neither motion nor parallax.

[The lunar eclipse of June 1, 1863, afforded some beautiful phenomena of the kind described at page 974; and the evening being remarkably fine, was highly favourable for observation. At 10.45 P.M., being the commencement of the total eclipse, the limb last occulted showed a blood-red colour. On complete immersion, the disc was still visible, and continued so to the end of the eclipse, appearing occasionally red, with a strange flickering appearance, like that produced by a "London fog" on the sun, in November. A beautiful phosphorescence was noticed, at 11 P.M., over the whole disc, and this alternated with a red, or rather rose-coloured tint, until the reappearance of the illuminated edge. The course of the phosphorescence was the same as that of the earth's shadow, for it immediately preceded both the immersion and emersion, and was greatest on the limb of the moon on which each took place. In observing the above, we preferred the naked eye to the telescope, so distinctly were the phenomena visible.—ED.]

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* See *ante*, p. 929.

CHAPTER X.

ON UNIVERSAL GRAVITATION.

All experience shows, that every heavy body requires force, effort, or support, in order to prevent its falling to the earth. What are called light bodies, also require a similar counteracting force when their tendency to fall is not resisted by the presence of the air, as numerous experiments with the air-pump have sufficiently proved.

This influence of the earth upon all bodies suspended over it, and which renders force necessary to *keep* them suspended, is itself called force—the *force of gravitation.* Such is the name which philosophers give to the hidden cause, whatever it be, which forces bodies to fall to the earth when left to themselves, instead of remaining suspended wherever they may be placed, or moving in any other direction.

That this force of gravitation or attraction is not exclusively expended on terrestrial bodies, was a doctrine entertained by several astronomers before Newton's time. Kepler had a steady conviction that the moon gravitated towards the earth; but he conceived that some additional force, animal or spiritual, was in operation to check the gravitating influence, and prevent the two bodies from rushing into collision.

It was reserved for Newton to prove that the single principle of gravitation, combined with one single impulse given to each planet, was sufficient to account for all the celestial movements, and to render clear and intelligible the entire mechanism of the heavens.

In addition to the first distinct enunciation of the *principle*, Newton also discovered the *law* of gravitation. The principle is, that all particles of matter mutually attract each another; the law is, that the intensity of the attraction varies inversely as the square of the distance at which it acts.

To verify these important propositions, observations of a peculiar kind were indispensable. These, fortunately, had already been supplied by Kepler; they are embodied in the three following statements, which have, however, a greater degree of generality than Kepler's observations strictly warrant. His *laws*, as they are called, thus generalised, have been mentioned at page 944: it is convenient to repeat them here.

1. If a line be supposed to connect any planet with the sun, this line will describe equal areas in equal times.

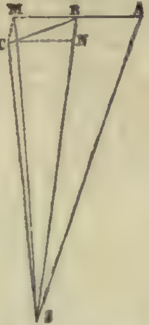
2. The planets revolve about the sun in elliptic orbits, the sun occupying one focus of the ellipse.

3. The squares of the times in which any two planets perform their revolutions about the sun, are as the cubes of their mean distances from it.

The first of these three laws is not peculiarly connected with astronomy, and is of no special importance in establishing the physical theory of the planetary motions; for, as shown by Newton, it is a universal truth, that a body, moving round a point of attraction in any orbit and attracted by any force, *must* describe equal sectorial areas in equal times. This truth, therefore, is quite independent of every specified law of attraction, and is irrespective of all considerations as to the distance of the revolving body, or the form of its orbit. If only the direction of the force be towards a fixed point, the equable description of areas necessarily has place; and, conversely, if this latter have place, the direction of the attractive force is always towards the same fixed point. This is proved as follows, after the manner of Newton:—

Suppose that a planet, moving at any distance from the sun, be subjected to the influence of an attractive force tending to draw it directly to that body; and imagine this force, instead of acting continuously, to act by successive impulses, after certain equal intervals of time. Let A B (Fig. 144) be the path described by the planet during one of these intervals, which path will, of course, be a straight line, since in describing it the planet is not acted upon by any force.

Fig. 144.



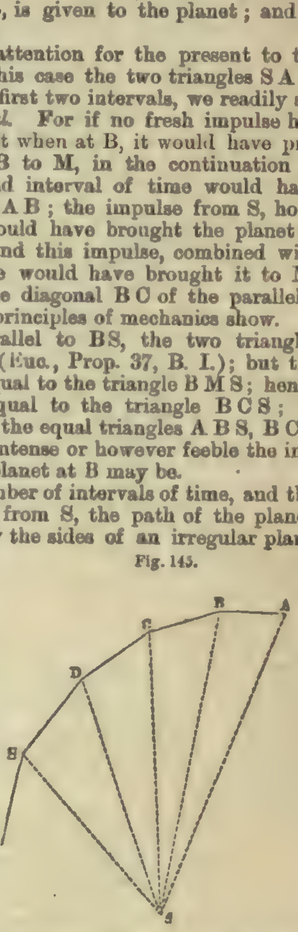
Arrived at the point B, the attractive force at S, acting merely as an instantaneous impulse, changes both the direction and velocity of the planet, which takes another rectilinear path, B C, described in an interval of time equal to the former interval; at the end of which, another impulse, in the direction C S, is given to the planet; and so on.

Now, confining our attention for the present to the two sectorial areas (in this case the two triangles S A B, S B C) described in the first two intervals, we readily see that they must be *equal.* For if no fresh impulse had been given to the planet when at B, it would have proceeded onwards from B to M, in the continuation of A B, and in the second interval of time would have described B M equal to A B; the impulse from S, however, if acting alone, would have brought the planet to some point N in B S, and this impulse, combined with that which acting alone would have brought it to M, causes it to describe the diagonal B C of the parallelogram M N, as the first principles of mechanics show.

Now M C being parallel to B S, the two triangles B M S, B C S are equal (Euc., Prop. 37, B. I.); but the triangle A B S is also equal to the triangle B M S; hence the triangle A B S is equal to the triangle B C S; so that in the equal times, the equal triangles A B S, B C S are described, however intense or however feeble the impulse from S upon the planet at B may be.

By increasing the number of intervals of time, and the corresponding impulses from S, the path of the planet would be represented by the sides of an irregular plane polygon, as in the diagram (Fig. 145); and however these sides may differ in length and direction, it is plain, from what is shown above, that they are the bases of *equal triangles* of which the point S is the common vertex. As each triangle is described in the same time, by the line joining the planet and the sun, it follows that equal areas are always described by this line in equal times.

Fig. 145.



It is obvious that this conclusion has nothing to do with the length of time which has been supposed to intervene between the impulses upon the planet, but only upon the equality of the intervals. We may therefore imagine these equal intervals to be as small as we please, and therefore the impulses to succeed one another with the utmost rapidity, and, in fact, to unite in one continuous force; in which case the sides of the polygon, becoming shorter and shorter, must unite in one continuous plane curve, the centre of attraction being itself in that plane. We infer, therefore, that a planet moving in virtue of a primitive impulse, and diverted from its rectilinear path by an attractive force residing in the sun, describes a curvilinear orbit round the source of attraction, such that equal sectorial areas are generated by the *radius vector* of the planet in equal times.

However accordant this description of equal areas in equal times may be with actual observation, we could not fairly infer that the centre of force is necessarily in the sun, unless it were further proved that such equal areas could not be described if the centre of attraction were situated elsewhere than at the common vertex of the equal sectors. And this may be done thus :—If the force acting on the planet when at B (see Fig. 144) had a direction out of the line B S, B N would make some angle with B S, and C M, which is parallel to B N, would not then be parallel to B S; and the two triangles B C S, B M S, on the same base B S, would have their vertices at unequal distances from that base; so that the surfaces of those triangles would be unequal. The triangle A B S, always equal to B M S, would no longer be equal to the triangle B C S; nor, consequently, would the triangles A B S, B C S, C D S, &c. (Fig. 145), described by the radius vector in equal times, be equal. We conclude, therefore, that, 1st, if the force acting on a planet be constantly directed towards the sun, the areas described by the straight line joining the planet and sun are proportional to the times of describing them; and, 2nd, if the force which acts on the planet be not directed towards the sun, the proportionality of the sectorial areas to the times does not exist. The first law of Kepler, therefore, warrants the conclusion that the attractive force acting on a planet, is constantly directed towards the sun; and this is the only conclusion derivable from that law. As respects the intensity of the attractive force—how it varies with the distance of the planet, or whether it varies at all, are particulars of which nothing can be learnt from the observed fact of the equal description of areas; for, as the preceding reasoning shows, equal areas are always described about a point, provided only the force acting on the moving body be in the direction of the line from the body to the point, however the intensity of the force may vary. But if the body move in a circle round the centre of attraction, then we may infer, from the law of equal areas, in equal times, that its motion must be uniform; for only equal arcs of circles can belong to equal sectors; and from the constancy of the distance, or radius vector, we cannot but infer the constancy of the attractive force.

Kepler's other two observed laws of planetary motion evidently imply a certain law of attractive force. If the attraction be such as to cause the planet to describe an elliptic orbit, any alteration in the force acting upon the planet at any particular point in that orbit, would necessarily alter its path. And if two planets revolve round a centre of force, and it be observed that their mean distances from that centre and their times of revolution bear the same relation to one another, a uniform law of force is again implied. Let us endeavour to discover what this law is from Kepler's third proposition; namely, that the squares of the times are as the cubes of the mean distances.

It will be observed that the enunciation of this truth involves no condition in reference to the *eccentricities* of the planets; the proposition should hold, therefore, for all eccentricities, and even for *circles*, in which the eccentricities are zero. For simplicity, then, we shall consider the case of planets revolving in circular orbits, and consequently, as shown above, moving uniformly; and shall thence endeavour to discover the law of force necessary to justify the proportion, that the squares of the times of revolution are as the cubes of the radii of the circular orbits.

The intensity with which a force acts upon a body, is measured by the addition it imparts to the body's velocity in the direction of that force in a second of time. If no force act, no addition can be made to velocity; if a body be at rest, or move uniformly in a straight line, we infer at once that it is not acted upon by any force. If a force act continuously, and in the same direction, it must generate equal increments of velocity in equal times; and the motion of the body is thus said to be uniformly accelerated. It must be borne in mind, however, that the force must act with the same intensity upon the body throughout. The attraction of the earth, or terrestrial gravity, may be regarded as such a constant force in all our experiments and observations respecting bodies falling to its surface, because these observations and experiments can never be made in a region so remote from the surface as to render the variation in the force of the earth's attraction sensible.

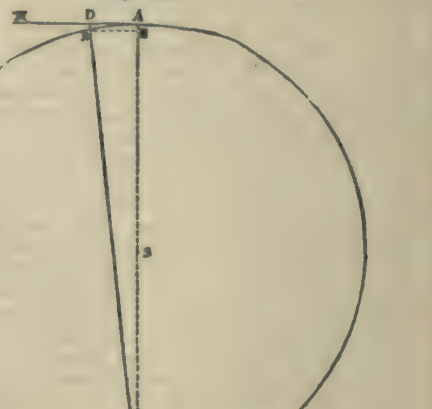
If a body fall from rest to the surface of the earth, it will acquire a velocity at the end of the first second, which, as stated above, is the measure of the earth's attraction; throughout this second, the velocity increases uniformly from nothing up to the final velocity which measures the force. The velocity midway is, therefore, half this final velocity; and if the body were to move *uniformly* with this midway velocity, it is pretty obvious that in one second it would describe the same length of path, as in falling from rest as above supposed.*

The length of path in uniform motion is evidently found by multiplying the uniform velocity (the length passed through in a second) by the number of seconds; that is, calling the length of space s, the velocity v, and the number of seconds t, we have $s = v\ t$, or if $t = 1$, $s = v$. Now, if v be the final velocity at the end of the second of accelerated motion, adverted to above, and S the space actually passed through in that second; then, putting $\frac{1}{2}$ V for v, and S for s, we have $S = \frac{1}{2}\ V$; therefore $V = 2\ S$; that is, the final velocity, or the measure of the force of gravity, is twice the space described by a body falling from rest in the first second of time.

The same conclusion applies, however remote from the surface of the earth the falling body be conceived to be placed; the intensity of gravity *there* would still be measured by double the space through which the body would fall from rest in one second; for during so short a time, the force may, as before, be considered as constant. And a like conclusion applies whatever centre of force be considered; the measure of its intensity on a body anywhere subjected to its action, will always be accurately expressed by twice the space through which the body falls towards that centre in one second.

Let the circle in Fig. 146 represent the path of a planet revolving about the sun at S. When at A, the planet is actuated by a velocity in the direction of the tangent A M, and in this direction it would, of course, continue to move uniformly if no force diverted it from its path. But it is acted upon by a force which is constantly directed towards the sun, S, and which tends to draw it towards that body, or to deflect it more and more from the tangent. It may thus be said to fall towards the sun, just as a body near the earth, projected forwards in a straight line, A M (Fig. 147), falls towards

Fig. 146.



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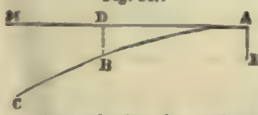
* See *ante*, *Mechanics*, p. 734, *et seq.*

the earth's surface, the actual motion in either case being in a curved line, A B. Let A B, on the orbit of the planet, be the portion of that orbit described in one second of time; the distance D B will be that which the planet has fallen, from its wonted path, towards the sun, in this second of time, and the double of D B will measure the attractive force of the sun at that distance.

Now, to find the length of D B we proceed as follows:—From the point B draw B E perpendicular to the radius S A; draw also from B a straight line to the point A′ of the orbit diametrically opposite to the point A, and make A E equal to D B. It is plain that the arc A B differs insensibly from its chord, and may be replaced by it; so that the angle A B A′, being in a semicircle, is a right angle. In the right-angled triangle A B A′, we have the proportion A E : A B :: A B : AA′ (Euc. 8 of VI.), for B E cannot differ sensibly from parallelism with D A; therefore, $\frac{AE}{AB} = \frac{AB}{AA'}$, $\therefore AE = \frac{AB^2}{AA'} = \frac{AB^2}{2AS}$ As the arc A B is the portion of the orbit described by the planet in a second of time, it measures the uniform velocity with which it moves; and we see that the distance A E, or B D, which the planet falls in that time towards the sun, is found by dividing the square of the velocity by twice the radius of the orbit. We have seen that the attractive force of the sun, acting at that distance, is measured by double the distance fallen in a second. The intensity of the force, therefore, at the planet, is expressed by the quotient of the square of its velocity by the radius of the planet's orbit.

Fig. 147.



We may now compare together the intensities of the forces which act at different distances, or on different planets, by means of the third law of Kepler. In order to this, let us suppose that planets, moving uniformly in circles round the sun, be situated at distances from that central body proportional to the numbers

1, 2, 3, 4, 5, &c. . . . . (A).

To obtain the velocity of one of these planets in its circular orbit, we must divide the length of the circumference by the number of seconds occupied in describing it; and to get the square of the velocity, we must divide the square of the circumference by the square of the number of seconds. But the squares of circumferences are as the squares of their radii; that is, in the present case, as the numbers

1, 4, 9, 16, 25, &c.

Also, from the third law of Kepler, the squares of the times of revolution are as the cubes of the radii of the orbits; that is, in the present case, as the numbers

1, 8, 27, 64, 125, &c.

The squares of the velocities of the several planets will, therefore, be related to one another as the numbers

$$1,\ \frac{4}{8},\ \frac{9}{27},\ \frac{16}{64},\ \frac{25}{125},\ \text{\&c.}$$

Or as the numbers

$$1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \text{\&c.} \quad . \quad . \quad . \quad . \quad \text{(B).}$$

And as the force acting on each planet is measured by the square of its velocity divided by its distance, we shall evidently discover the law by which the intensity varies, from one planet to another, by dividing the several numbers (B) by the corresponding numbers (A); that is to say, the forces at the several distances (A) will be as the numbers

$$1,\ \frac{1}{4},\ \frac{1}{9},\ \frac{1}{16},\ \frac{1}{25};$$

or inversely as the *squares* of the distances (A). And in this way is the law of solar attraction established; and not only does this follow from the third law of Kepler, but this law itself may be shown to be a necessary consequence of the law of force just established. For, since the length of orbit, divided by the uniform velocity, gives the periodic time in seconds, and that circumferences vary as their radii, the periodic time varies as the radius divided by the velocity. Hence, calling the periodic time P, the radius R, the velocity v, and the force F, we have in symbols,

$$P \propto \frac{R}{v} \therefore P^2 \propto \frac{R^2}{v^2}. \quad \text{Now } F = \frac{v^2}{R}.$$

$$\text{If, then, } F \propto \frac{1}{R^2} \therefore \frac{v^2\ 1}{R^2\ R^2} \therefore \frac{R^2}{v^2} \propto R^3 \therefore P^2 \propto R^3; \text{ so}$$

that if the force vary inversely as the square of the distance, and bodies move round the common centre of attraction in circles of different radii, the squares of the periods of revolution will be to one another as the cubes of the distances.

Having proved that the relation observed by Kepler between the periodic times and the distances is a necessary consequence of the above-named law of gravitation, Newton sought to determine whether the forms of the planetary orbits were not also a necessary consequence of the same law; and he accordingly found that, under the influence of that law of attraction, it was impossible for a body to move in any other curve than a *conic section;* that is to say, the orbit must be either an ellipse (of which a circle is a particular case), a parabola, or an hyperbola.* This may be proved as follows:—

Let v be the velocity in a circle of radius R; v' the velocity in a parabola, ellipse, or hyperbola, whose radius of curvature at perihelion is R′; then

$$F = \frac{v^2}{R} = \frac{v'^2}{R'}; \therefore R' = R\frac{v'^2}{v^2}$$

Now R′ is what, in the doctrine of the conic sections, is called the *semi-parameter* of the curve; and R being the distance of the focus from the vertex of the curve, it is also proved in that doctrine, that the curve will be a parabola, an ellipse, or an hyperbola, according as R′ is $= 2$ R, $\angle\ 2$ R, or $>\ 2$ R. Hence, if $\frac{v'^2}{v^2} = 2$, the orbit is a parabola; if $\frac{v'^2}{v^2} \angle\ 2$, the orbit is an ellipse; and if $\frac{v'^2}{v^2} > 2$, the orbit is an hyperbola. And as one or other of these conditions must necessarily have place, whatever be the velocity v', it follows that the planets and comets must all move in one or other of these three curves.

The above established law of force is thus competent to account for the revolutions of all the planets and comets round the sun, as also for the motions of those comets in the system, if any there be, which describe parabolic or hyperbolic orbits, and which consequently proceed onward in space continually, and never return.

The same law of force accounts, in like manner, for the revolutions of the satellites round the planets to which they belong, for these all describe elliptic orbits.

This general statement, however, is, in strictness, only an approximation to the truth, although an approximation so close, as to be but very little at fault in explaining the great phenomena of the planetary motions, and therefore regarded by Kepler as strictly accurate. In the foregoing reasonings the attracting body has been supposed to be absolutely fixed in space. This is not entirely consistent with Newton's laws of *universal gravitation*, according to which all bodies in the universe mutually attract one another with a force directly proportional to their masses, as well as inversely proportional to the squares of their distances. On account of the masses of the planets being very small in comparison with the mass of the sun, the rejection of the planet's attraction on the latter body involves an error comparatively insignificant.

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* See *ante*, *Mathematics*, p. 608, *et seq.*

"It is true, that when observations are carried to a high degree of precision, and when each planet is traced through many successive revolutions, and its history carried back, by the aid of calculations founded on these data, we learn to regard the laws of Kepler as only *first approximations* to the much more complicated ones which actually prevail; and that to bring remote observations into rigorous mathematical accordance with each other, and at the same time to retain the extremely convenient nomenclature and relations of the ELLIPTIC SYSTEM, it becomes necessary to modify, to a certain extent, our verbal expression of the laws, and to regard the numerical data or *elliptic elements* of the planetary orbits as not absolutely permanent, but subject to a series of extremely slow and almost imperceptible changes. These changes may be neglected when we consider only a few revolutions; but going on from century to century, and continually accumulating, they at length produce considerable departures in the orbits from their original state."—*Herschel.*

From the principle of universal gravitation, the third law of Kepler, from which the law of attractive force has been deduced above, requires to be modified as follows:—The cubes of the mean distances of the planets from the sun are as the products of the squares of the periodic times, by the sum of the masses of the attracting and attracted bodies. In the original enunciation of the law, the mass of the attracted body was neglected; the trifling amount of error thus introduced—seeing that even Jupiter, the largest of the planets, has less than a thousandth part of the matter contained in the sun—is evidently such as to render the departure from strict accuracy too minute to be detected, except by delicate observations extended over a long period of time.

PERTURBATIONS OF THE PLANETARY MOTIONS.—In consequence of the law of universal gravitation, each planet exercises an influence over every other planet; these mutual disturbances necessarily modify, in a slight degree, as just noticed, the orbits of all, and occasion what are called *perturbations* and *inequalities*. But all these are fully accounted for, and satisfactorily explained, by referring them to Newton's great principle. Calculations founded upon this principle, enable us to predict the position of a planet at any future time with a degree of accuracy that appears little short of marvellous, when we consider the complication involved in the mutal actions of the sun and all the planets on one another. "The motion of Jupiter, for instance, is so perfectly calculated, that astronomers have computed, ten years before hand, the time at which it will pass the meridian of any specified place; and we find the prediction correct within half a second."—*Airy.*

No physical law has ever been enunciated which long and careful observation more completely verifies than Newton's law of universal gravitation. The distances and masses of the planets being known, the effect of the attractions of all, in modifying the path of each, can be correctly ascertained, though only by aid of investigations of the most difficult and recondite character; but one of the most remarkable attestations to the truth of this universal principle, is that which has been furnished in our own day by Mr. Adams, of Cambridge. Knowing that there were certain small perturbations of the orbit of Uranus that the combined attractions of all the other planets of the system were insufficient to account for, and fully confiding in the competency of Newton's law to explain every movement of the heavenly bodies, he had the boldness to pronounce that a yet unseen planet existed beyond the bounds of what had till then been regarded as the remotest planet in our system; and taking the unaccounted-for perturbations of Uranus as data, he had science enough to assign the place where, at a specified time, the new planet would be found. An eminent French mathematician, Leverrier, had independently, and but a short time afterwards, arrived at a similar conclusion; and upon transmitting the necessary instructions to a German astronomer, Galle, the predicted planet, Neptune, was revealed to his telescope on the evening of the day that the communication reached him.

One of the most important results to which the various researches into the planetary perturbations have conducted is, that the major axes of the variable elliptic orbits always preserve the same values. The disturbances to which each planet is subjected, by the action of the others, affect all the elements of its elliptic path, except its major axis, which throughout every other change continues itself invariable; this invariability insures at the same time the invariability of the period of revolution, agreeably to the third law of Kepler. And thus the stability of the system is secured. We are indebted for this great truth to the genius and researches of Lagrange.

MASSES OF THE PLANETS AS COMPARED WITH THE SUN.—The mass of a planet is ascertained by observing the disturbances which its attraction produces in the orbits of other planets, or, if it have a satellite, by determining the force exerted upon that satellite. The mass of the sun being represented by unit, the masses of the planets have been estimated as in the following table.*

Names of the Planets.	Masses.	Names of the Planets.	Masses.
Mercury	$\frac{1}{2025810}$	Jupiter	$\frac{1}{1050}$
Venus...	$\frac{1}{401841}$	Saturn	$\frac{1}{3500}$
Earth	$\frac{1}{354936}$	Uranus	$\frac{1}{21000}$
Mars	$\frac{1}{2680337}$	Neptune	$\frac{1}{14446}$

It must be understood that the *mass* of a body is not the same thing as its bulk or volume. The mass is estimated by volume and density; if one body have only half the volume of another, but be twice as dense, their masses are equal. If a planetary body be so minute in volume, or so feeble in density, as to render its attractive energy too small to perturb sensibly the motions of the other planets, the mass of that body cannot be determined. On these accounts there is still some uncertainty as to the mass of Mercury; and respecting the masses of the small planets between Mars and Jupiter—asteroids, as they have been called—we know nothing. In like manner, as respects the comets, all we know is, that their masses must be exceedingly small; that is, that they contain but a very small quantity of matter, as they produce no appreciable disturbance of the planetary movements. It has happened that a comet has crossed among the satellites of Jupiter without occasioning any observable disturbance even in the motions of these comparatively small bodies.

Before concluding this section on the principle of universal gravitation, we must devote a short article to one of the most noticeable effects of that principle on our own planet—the phenomena of the tides.

THE TIDES.—The principal physical cause of those periodical oscillations of the surface of the ocean called *tides*, is the attraction of the moon; the sun contributes to the general results, but in a far less degree.

Suppose the earth were entirely covered with water: from the mutual attraction of all its parts, its surface would assume an exactly spherical form. Its rotation on its axis, however, would cause the equatorial parts to recede from the centre, and to bulge out.

This receding from the centre, of the equatorial regions of the earth, is caused by a new force, opposed to that of gravitation, being brought into operation by the planet's rotation. It is called the *centrifugal force*, from its imparting a tendency in the outer particles of the rotating mass to fly off or recede from the centre. Just such a tendency would be excited if the earth were at rest, and an attractive external influence were uniformly diffused round its equatorial regions, and penetrating only to a small depth below the surface. If, however, the external attractive influence, instead of being thus diffused as a ring round the earth, were confined to a limited place, the bulging out of the waters would of course be confined to the place immediately under the attracting body; and if the force of attraction extended to the centre of the earth, the elevation of the waters would evidently be due, not to the force acting on their surface, but only to the difference of the forces acting on the surface and on the centre. The waters would, as it were, be drawn away from the earth by the influence of this difference.

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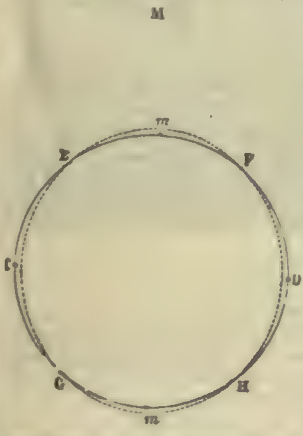
* See Note, *ante*, p. 926.

If the attractive force of the external body extended to the opposite part of the earth's surface, then the force on the centre being greater than that on the remote surface, the earth would, as it were, be drawn away from the waters at the further surface, and there would in consequence be also a bulging out there; and as the difference of the forces on the centre and near surface must be very nearly the same as the difference of the forces on the centre and remote surface, the waters will be heaped up to nearly the same extent at both places.

The application of these general considerations to the moon and the tides, will sufficiently show that the bulging out of the waters by the attraction of the moon M (Fig. 148), is nearly equal at the two places under the moon marked *m* on the earth's surface, diametrically opposite to each other, causing the waters of the entire globe to assume a spheroidal form, the elevations at *m* necessitating a subsidence or depression at C and D. As different portions of the earth's aqueous surface are brought under the direct influence of the moon, those portions in like manner bulge out, producing a continuous succession of tides.

Fig 148.



If the moon alone acted, and always moved directly over the equator, the interval between two consecutive high tides would be just half a lunar day. In like manner, did the sun alone act on the waters, its motion being supposed to be directly over the equator, the interval would be just half a solar day: but the combined action of both these bodies causes, of course, the intervals to vary.

In the actual state of the earth, whose surface presents both land and water, the phenomena of the tides cannot be expected to be in strict agreement with what they would be if no solid matter existed on it. Headlands, coasts, the shallowness and contraction of channels, &c., continually obstruct the free motion of the tide-wave; these obstructions often greatly delay the time of high water, and cause a much greater rise than would take place if the advance were unimpeded. Such local influences, indeed, greatly modify the results of pure theory, both as to the time and height of high water at different ports. Special observations, therefore, at each place are necessary to supply the proper data for predicting the time of high water, and to determine what is called the "establishment of the port." The height of the tide, too, often depends upon the set and force of the wind; but, "of all the causes of difference in the height of the tides, local situation is the most influential. In some places, the tide-wave, rushing up a narrow channel, is suddenly raised to an extraordinary height. At Annapolis, for instance, in the Bay of Fundy, in Nova Scotia, it is said to rise 120 feet. Even at Bristol, the difference of high and low water occasionally amounts to fifty feet."—*Herschel.*

The same influences which produce the tides of the ocean, operate also on the surrounding atmosphere of the earth; and produce tides in *it.* It is popularly supposed, too, that the weather is subjected to the moon's influence; but careful and long-continued observation, by competent persons, has shown this supposition to be fallacious. A change in the moon is imagined to be attended with an immediate change in the weather; it being overlooked that the moon changes her position, and passes through her several phases, by imperceptible gradations; which is incompatible with a sudden change in the weather. The predictions frequently given in almanacks, in reference to this point, will generally be found to prove fallacious.

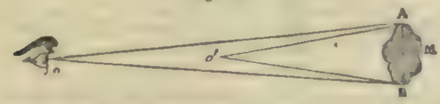
CHAPTER XI.

PRACTICAL ASTRONOMY.

We have now reached—perhaps we are somewhat beyond—the point where the astronomer's labours must have ceased but for the intervention of instruments, by which he has been enabled to sweep the heavens, and reveal some of their hidden mysteries. We propose, at this stage of our labours, to give some description of these instruments, of the principles on which they are constructed, of the manner in which they tend to the augmentation of our power of sight, and the most convenient methods of mounting them, with directions for their practical application.

The apparent size of an object, as perceived by its image on the retina, depends on the distance between the eye and the object. Thus, for instance, if the object be M (Fig. 149), and the eye be placed at *o*, the right line which joins the extreme points, A and B, will be seen under the angle A *o* B. If the eye be placed at the point *o'*, or at half the distance at which it was previously situated, the line A B will, in the same manner, be viewed under the angle A *o'* B, which will be the double of the preceding, supposing that the line A B is small in comparison with the distance of the object M from the eye of the observer. In the same manner, the angle A B will be three or four times greater, as the distance of the object is one-third or one-fourth of the primitive distance. The angle A *o* B is, therefore, the measure of its *apparent* size (which is different from its *real*, the latter being invariable), and is called the *visual* angle, or the apparent *diameter* of the object. The surface of the body becomes four, nine, and sixteen times greater, as the distance is decreased to one-half, one-third, and one-fourth of the primitive distance. In other words, the diameter of an object varies inversely as the distance of the eye from it, and its apparent area varies inversely as the square of this distance. The power of the eyesight is limited; and it is generally held, that when the diameter of an object is less than a minute, the object ceases to be visible. But, the *brightness* of the image performs as important a part in this respect as its *size*, as is easily proved by the visibility of the fixed stars and of the planets, none of the former of which can subtend an angle as great as a second. Where the colour or brightness of an object, however, differs less from the ground on which it is placed, the above rule holds true.

Fig. 149.



The *distinctness of vision*, as every one is aware, depends likewise on the distance of the eye from the object; and this distance is variable for different persons. Even the one eye of the same individual may have a longer range of vision than the other. Every one knows, also, that if an object be held at a greater or less distance from the eye than that which is customary or natural, in either case it becomes indistinct.

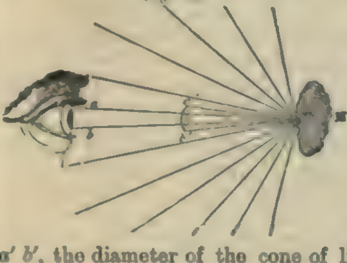


Fig. 150.

In regard to the *brightness* of any object, according as it is more or less removed from the eye, the following considerations are to be taken into account. From any bright object as M, rays of light are sent in all directions; and from the point *m*, as from any other point on its surface, these rays are sent before it, as represented in Fig. 150. Of all the rays of light, the eye only receives those which penetrate through the pupil *a b*. If the eye now approach the object M, so that its distance is one-half of what it was in the previous case, and its position *a′ b′*, the diameter of the cone of light will be only one-half of what it was in the previous case, and consequently the area of the section in that part, is only one-quarter of that at the base *a b*, where it was equal in area to the surface of the pupil of the eye. Hence it follows that the opening of the pupil at *a′ b′* will take in a greater number, by four times, of rays, than when it was situated at *a b*, so that the light received from the point *m* will be four times as much in the latter case, as in the former. At the same time, however, as the eye approaches the object M, a greater portion of its surface is taken in, and the position *m* becomes four times as great. The bright light is thus spread over a greater surface, and in a like proportion; and it follows that the brightness of the surface is unchangeable. If it happen that this is not the case in nature, it must be remembered that the interposition of the air causes a loss of light, and even a change in the colour of objects; for distant mountains appear blue, as likewise does the sky—being the effect when viewed through great depths of the atmosphere. Without this, a white object would appear of equal brightness whether it were seen close at hand or at a distance.

LENSES.*—Many solid bodies, and a greater portion of liquid ones, are transparent; but, in order that the former become perfectly so, it is necessary that their surfaces should be highly polished, which condition is fulfilled, in liquids, by the natural effect of gravity. In the grinding and polishing of glass and crystal, or even reflecting metals, into various shapes, it was early discovered that various effects were produced; but none more extraordinary than those which took place in the apparent magnitude of objects when viewed through these artificial media. The fundamental law on which all those changes are based is, that when a luminous ray passes obliquely from one transparent medium into another, it departs from its primitive direction, and undergoes a *refraction*. On this law the theory of lenses is based.

Fig. 151.

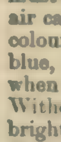


Fig. 152.

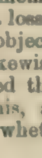
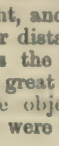


Fig. 153.



There are two different species of lenses, the surfaces being, in both cases, spherical, or one of the faces may be plane. The first are those in which the surface is convex or converging, and the other in which it is concave and diverging. The converging glasses are those which are doubly convex (Fig. 151), the form being *lenticular* (whence they derive their general name of lenses)—2° those which are plano-convex (Fig. 152), and 3° concave-convex (Fig. 153), the convexity being, in this case, more considerable than the concavity—and which are likewise known by the name of *menisci*, in consequence of the form of their section. The converging glasses are those which are doubly concave, as in Fig. 154; plano-concave, or convex-concave, the concavity being more considerable than the convexity, and likewise known as menisci. The *radii* of lenses are the radii of the spheres of which their surfaces are the segments.

Fig. 154.

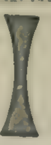


Fig. 155.

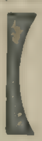
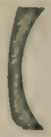


Fig. 156.



If we expose a convex glass to the rays proceeding from the sun, they will be converged to a point, A (Fig. 157); and if we remove the lens more or less away from the surface on which the converging rays are received, this bright spot (A) will be of greater or smaller extent. To that point where it is of the smallest dimensions, the name of *focus* has been given, and its distance from the nearest surface of the lens is called the *focal distance*. If we turn the lens round, the same effect is produced; and if both the surfaces have the same radius, the distance will be alike in both cases, though it will vary but very slightly should they be different—even in the case of a *meniscus* lens.

Fig. 157.

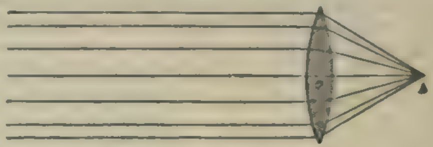
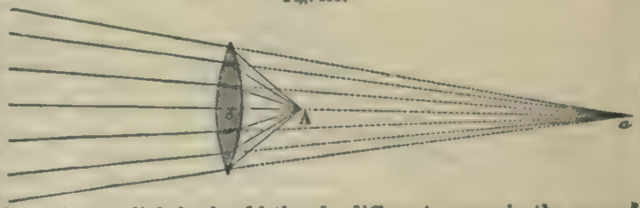


Fig. 158.



The line *a* A (Fig. 158) is termed the *axis* of the lens, and the point *o* its optical centre. In the above case the rays are supposed to pass parallel to the optical axis; but if they proceed from a luminous point, A, a short distance from the axis (Fig. 159), or rather, if the parallel rays make a small angle with the optical axis, the manner in which they converge to a focus, *a*, is the same. In both cases they pass through the optical centre *o* of the lens (Fig. 160). In concave lenses, the parallel rays of light which pass from an object are rendered divergent; or if already divergent, are rendered more so by their interposition, as in Fig. 166, p. 992.

Fig. 159.

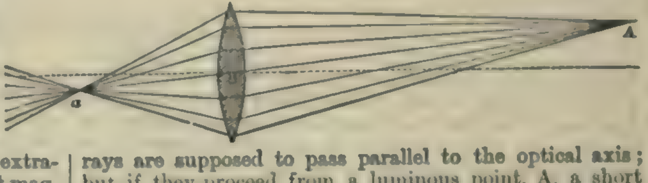
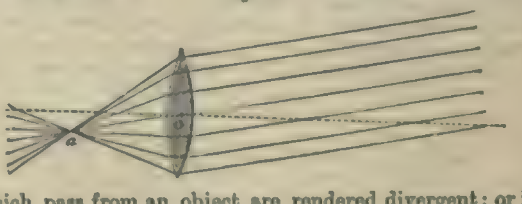


Fig. 160.



Supposing that, instead of proceeding from a point, A, the rays of light from a luminous object, A B (Fig. 161), pass through a convex lens. In this case the rays of light coming from the point A, will, as in Fig. 161, converge to the point *a*; and those from the point B to the

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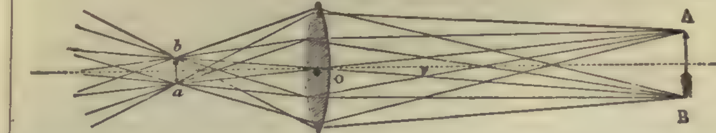
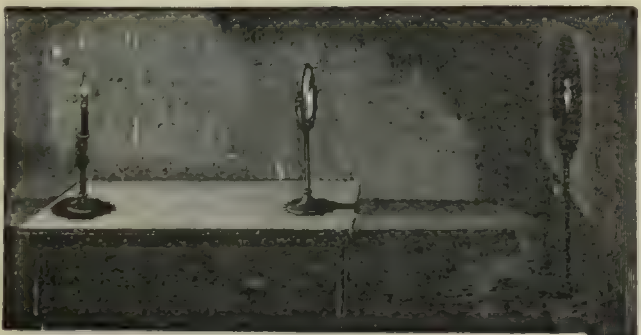
* The construction of many of the following arrangements has already been described, under the head of Optical Instruments, in the section on Light, but not specially in reference to astronomy, to which the succeeding remarks are mainly directed.—ED.

Fig. 161.



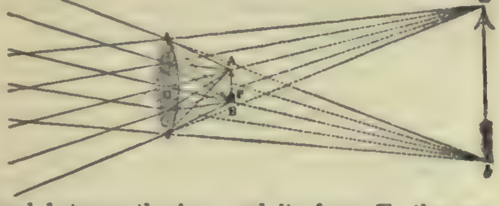
focus *b*; and in the same manner, all the other points of the object A B will be represented by opposite points in the image formed by the lens. It thus follows that the image is reversed; and if the eye be placed at the focus *a b*, the object A B will appear turned *upside down;* but in all other respects, a perfect picture of the object A B will be perceived. This effect may likewise be seen by placing a candle, A (Fig. 162), in a dark chamber, at a certain distance from the mounted lens B, the light from the candle passing through the optical axis of the lens, to the opposite wall, C, which is at the proper distance from the lens to receive a well-defined image of the flame of the candle. A reversed image of the candle may here be perceived, as well as of that portion of the candle which is illumined by the flame.

Fig. 162.



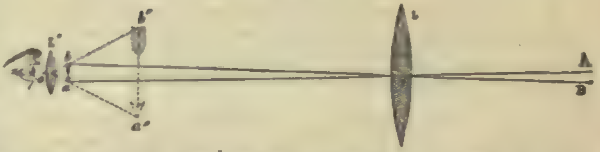
The well-known effect of convex glasses in magnifying small objects, will be seen from the following diagram (Fig. 163). The object A B being placed between the lens and its focus E, the rays of light proceeding from the point A do not lose all their divergence, but appear to come from a more distant point *a*, formed by the prolongation of the line O A; and in the same manner the point B seems to be situated at *b*, formed by the prolongation of the line B O. To the eye situated at the other side of the lens, therefore, it will appear as if the object A B were replaced by the image *a b*, and the latter object will appear more or less distant from the eye, according as the object A B is nearer or more removed from the focus of the lens. The lens can be so shifted in respect to the object, that the image *a b* will appear distinct and well defined, as well as magnified. We can readily perceive that the image *a b* is greater and more distant when the focal distance is smaller, and, in consequence, that a lens magnifies so much the more as its focal distance is less. The magnifying power of lenses is somewhat illusory, and nearly every one judges differently in estimating the magnified size of the object under examination. This may arise from two causes—in the first place, the distance of distinct vision is different for almost every person; and, in the second place, when we look through magnifying glasses, almost every guide which serves to regulate the judgment, on the distance and size of the object looked at, is removed.

Fig. 163.



Telescopes.—Every convex glass, as we have seen, produces an image of the object from which it receives rays of light; and by a combination of such glasses, viz., by examining the image formed by one lens (as if it were a real object) by means of another, and thereby magnifying the image as explained in the last paragraph, distant objects can be seen with a distinctness unapproachable with the naked eye. The most simple and most common description of the telescope, used by the astronomer, is that formed by two convex lenses, L and L′ (Fig. 164); the first of which, from being turned in the direction of the object, is termed the *object-glass;* the second the *eye-piece.* In this, the rays of the object A B, passing through the convex lens L, form an image *a b* in the focus of the lens. The second lens serves to magnify the reversed image *b a* in the same manner as if it were a tangible object. The image *b a* is not always at the same distance from the object-glass, but varies more or less according to the distance of the object; and when the latter is so far removed that the rays which fall on the object-glass may be considered as parallel—which is the case with all celestial objects—the image *b a* falls in the focus of the lens. As the object should be seen with the requisite distinctness at *b′ a′*, which is the distance of distinct vision, and as the latter varies almost with every person, it is necessary that the eye-piece should be drawn in or out from the image *b a*, in order to give perfect definition. It will be seen from the diagram, that the telescope does not, like the microscope or magnifying-glass, increase the *natural* size of the objects viewed; for the image *b′ a′* is much smaller than the object A B, which is at a great distance from the object-glass: it only tends to increase the *apparent* size of an object when viewed from a distance.

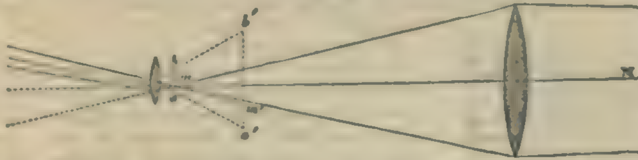
Fig. 164.



To compare the size of a distant object when viewed with natural and telescopic sight, we have only to compare the angles A *o* B (Fig. 164), or *a o b*, which is the angle subtended by the object to the unassisted vision, and the angle *a′ o′ b′*, which is the angle subtended by the image to the eye of the observer. The proportion between the relative sizes of the object A B, and the image *a′ b′*, is consequently the same as that between the angles *a o b* and *a o′ b;* and this is what is termed the magnifying power of the telescope. The angle *a′o′b′* and *a o b* being always small, they may be regarded as in the inverse proportion of the two points *o* and *o′* from the image *a b;* or, in other words, that as these distances may be regarded as the focal lengths of the object and eye-glasses, that the magnifying power of the telescope is in the same proportion. This instrument allows a large *field of view* (or the circular space which the eye would take in without the assistance of any telescope), for this depends on the dimensions of the instrument at *a b*, or the common focus of both lenses. Equally important with the perfect definition of the object, is, as already stated, the *brightness* with which it appears when under examination. If a ray of light pass from any point, M (Fig. 165), of an object, it will fall on the whole surface of the object-glass, and thence through the same point,

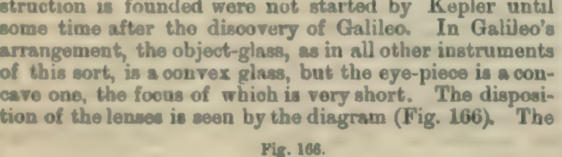
m, of the focus, $a\,b$, after which it will form a pyramid of rays, the divergence of which will be lessened by passing through the eye-piece in such a manner that they will appear to come from the point m' in the image $a'\,b'$. If the eye viewed the image $a\,b$, without the interposition of the eye-piece, it would only receive a portion of the light and rays emitted from the point M, many of the outer diverging rays having been lost, the opening of the eye being too small to take the whole in. But by employing a second convex glass, the divergence is diminished, and the eye receives the whole of the light emitted from the point M, which falls on the object-glass. Comparing, therefore, the brightness of an object, as seen with the naked eye and seen through a telescope, and supposing, in the latter case, that all the light passes through the lenses, it would follow that the light received by the eye, in using a telescope, would be as many times greater than that received by the unassisted eye, as the surface of the object-glass exceeds that of the pupil of the eye. This, however, is only theoretically true, for there is a considerable absorption of light effected by its passing through the lenses, as well as by reflection from their surfaces. If the size of the objects were increased by the telescope in the same proportion as the brightness of their surfaces, it would follow that the brightness would always remain the same. The magnifying power, however, is quite distinct from the illuminative power of the telescope, the first depending on the proportion between the focal distance of the two lenses, the latter upon the area of the object-glass. It thus happens that the brightness of the image will vary greatly with the power applied; and, with high powers applied to the same telescope, the objects become dim and indefinite, and the two excellences of brightness and good definition are very difficult to be obtained.

Fig. 165.



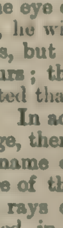
This description of telescope was not, however, the first invented. The optical principles on which its construction is founded were not started by Kepler until some time after the discovery of Galileo. In Galileo's arrangement, the object-glass, as in all other instruments of this sort, is a convex glass, but the eye-piece is a concave one, the focus of which is very short. The disposition of the lenses is seen by the diagram (Fig. 166). The reversed image of distant objects produced by the object-glass L is formed in $a\,b$, but the concave eye-piece is placed between the two, in such a manner that the eye placed behind the lens L′, will see $a'\,b'$, instead of the image $b\,a$; and the eye-piece can be so adjusted that the image $a'\,b'$ will appear well defined. This telescope magnifies the apparent diameter of objects as many times as the focal distance of the eye-piece is contained in the focal distance of the object-glass. It will not admit of any considerable magnifying powers, the field of view being very small, on which account it is never used at the present time as an astronomical telescope. On the other hand, it is very convenient as a pocket telescope, as it does not reverse objects; and as the eye-piece is placed between the object glass and its focus, it is shorter than the common telescope. In the common opera-glasses, which are of this construction, the magnifying power does not exceed three; but in some of those made by Galileo the magnifying power reached to thirty-two. Another form of the erecting or terrestrial telescope is that imagined by Rheita, in which three convex glasses, of equal but short focus, are fixed in a tube, and form the eye-piece. The first of those lenses, or that most distant from the eye, would give a reversed image, which, passing through the second one, produces an erect image, and finally, as it is before the focus of the third lens, it is viewed and magnified by this one. The magnifying power of this is measured as in the preceding ones, being in the proportion of the focal distances of the object-glasses and one of the lenses of the eye-piece. The magnifying power of a telescope may be roughly determined by comparing the sizes of an object, such as the moon, as seen with it and with the naked eye, and the extent of the field of view may be found by taking the diameter of the moon at about half a degree.* The former, however, is determined more exactly by an instrument called the *dyanometer*, which serves, when a telescope is directed towards the sky, and a sheet of paper is held behind the eye-piece, to measure the diameter of the luminous circle which falls upon the white surface, and the best definition of which is found by trial. By dividing the diameter of the object-glass by the diameter of this circle, the magnifying power of the telescope is obtained. The luminous circle is itself an image of the object-glass, and is contained as many times in the latter as the telescope magnifies distant objects.

Fig. 166.



Hitherto the object-glass has been supposed to be formed of one piece and of one sort of glass, but in all modern instruments this is not the case. The image formed by a convex glass is situated at a greater or a less distance from it, according to the refracting power of the glass, and is so much shorter as the refracting power is more considerable. In addition to this, the different colours of which white light is composed, are not equally refracted, and cannot have the same focus. If the eye of an observer be situated at the focus of the lens, he will not see the image perfectly defined and colourless, but it will appear surrounded with the prismatic colours; the violet rays of the object are more strongly refracted than the red ones, and will be thrown nearer the lens. In addition to this source of indistinctness of the image, there is another less important one known by the name of *spherical aberration*, which depends on the figure of the lens, there being no curvature in which all the rays of light coming from any object are *exactly* united in a common focus. In order to get rid of the first source of indistinctness, in using a single lens for the object-glass, it was found necessary, with high magnifying powers, to have the lens of very long focus (some of those used by the earliest observers were 300 feet in length); for by this means the causes which led to the formation of the colour, viz., the considerable curvature of the lens, were diminished, and the larger images were formed. By an examination of different species of glass, Dollond found that some sorts refracted light, and dispersed the colours, much more than ordinary glass; and by passing a ray of light through two prisms made of different glass, he had the satisfaction of finding that a white light was transmitted. After this, he employed the two sorts of glass in forming an object-glass, which would transmit light without decomposing it. This he effected by uniting a convex lens of crown-glass, of a greenish colour, with a concave lens of flint-glass, of a white tint (Fig. 167). When a ray of light falls upon this object-glass, it is acted upon by both—that of the crown-glass renders it convergent, and decomposes it; the concave flint lens, on the contrary, destroys the effect of the first, and an uncoloured image is formed at the focus. By this beautiful discovery, the refracting telescope has been made a convenient and easily managed instrument, to which higher powers can now

Fig. 167.



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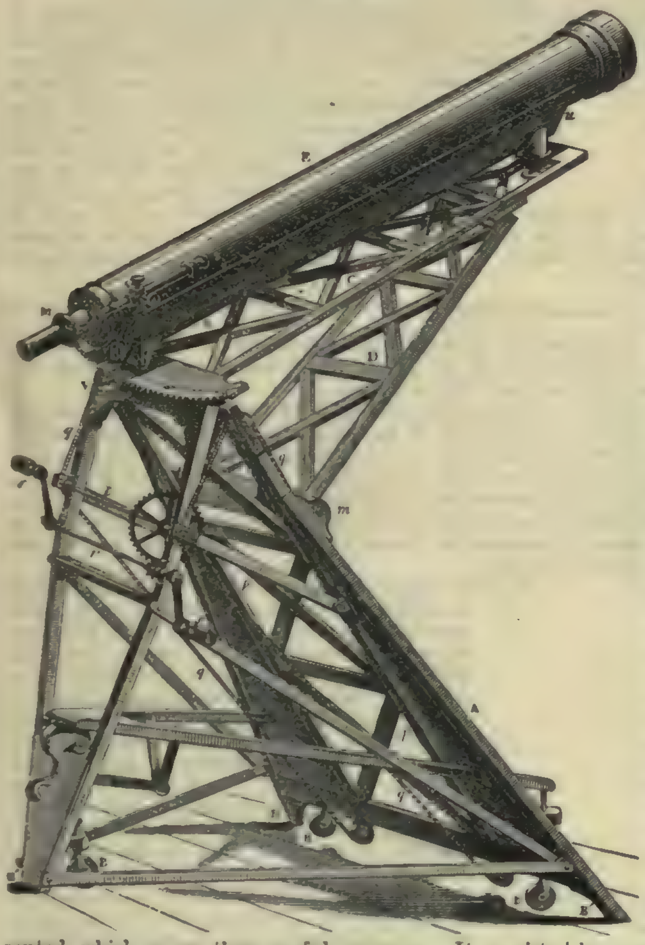
* The observer should not try this experiment by directing the telescope to the sun, as, if shining fully, its light and heat would destroy the pupil of the eye.—EDITOR.

be applied, and with better effect in one of 10 feet in length, than to those formerly made, which were some hundreds of feet in focal distance.

It is a matter of considerable moment to have the telescope mounted as steadily as possible, and to be able to direct it easily to any particular part of the heavens, so as to follow any object with the requisite facility. In the following figure (Fig. 168) a telescope-stand is represented, which serves these useful purposes. It consists of a firm support, A, resting on the ground by means of the feet B′ B′ B′ and the rollers B B B, and two movable supports, C and D, the former of which directly supports the telescope E, and is connected with the stand A by means of hinges at M M, by which it can be inclined more or less to the horizon. The second support, D, is connected with the first in a similar manner by the hinge *n*, by which means the angle between the two supports, C and D, may be varied at will. The inferior beam, *o o*, of the branch D, should slide along the inclined side of the stand A, and this motion produces the requisite elevation or depression of the extremity *n*, which turns on the hinge *m*. Two endless chains, *q q*, pass round the stand A, and are attached to the ends *o o* of the branch D. An axis, *r*, terminated by two handles, *s s*, carries a pinion which works in a wheel mounted on a second axis, *t*; this second axis is furnished with two pinions at both ends, of which the teeth catch in the links of the endless chain *q q*. In turning round the handle *s s*, the chain is thus worked up and down; the portion *o o* of the support D slides on the inclined plane, *p p*, and the projecting branch is raised more or less. The telescope, E, rests in *u u*, the latter part being connected with the support C by a pin at the object end, round which it can easily be turned as on a pivot. At the eye end of the telescope, the support *u u* rests upon two wheels, and a milled head, V, attached to a pinion, works in the toothed edge of the support C in such a manner, that in turning the milled head it turns the telescope to the right or left. The observer, by turning one of the handles, *s s*, with the one hand, and the milled head, V, with the other, is thus able to change the direction of the instrument according to his wish. A smaller telescope, Z, is commonly attached to the tube of the larger one, and is known by the name of the *finder*. Having a larger field of view than the principal telescope, the observer can discover the object he is in search of with greater facility; and by placing it in the centre of the finder, he guides the larger telescope directly upon it.

Fig 168.



REFLECTING TELESCOPES.—If an object A B be placed before a concave reflecting mirror M (Fig. 169), a similar effect is produced as when the rays of light proceeding from such an object pass through a convex lens. The ray of light passing from the point A will be reflected to the point *a*, while those passing from B will take the direction *b*, and the inverted image *a b* will be produced. The image *a b* can be magnified in the same manner as the image formed by a convex lens, though evidently not with the same facility; for here the image is formed between the object and the mirror, and there is a difficulty in knowing in what manner the eye-piece is to be placed. To obviate this difficulty, the inventor of this telescope, the celebrated Gregory, thought of placing a smaller concave lens N (Fig. 170)—which would only intercept a portion of the rays proceeding from any object—after their passage through the image *a b*, and to send them through the opening at the centre of the mirror M, in such a manner as to produce a second image, *a′ b′*, which could be examined with the requisite distinctness by the eye-piece *o*. By this combination an erect image would be produced, and the telescope would be used in precisely the same manner as the Galilean telescope. The telescope independently invented a few years later by Sir Isaac Newton, is more simple than the former, receiving the rays which pass from the great mirror on a plane reflector N (Fig. 171), inclined at an angle of forty-five degrees, before they arrive at the focus *a b*, and produce the image *a′ b′*, which can be examined by means of the eye-piece O. In this form of the reflecting telescope, the observer looks in a direction perpendicular to that of the object observed. In order to obtain as much light as possible, a considerable portion of which was lost by being reflected from successive surfaces, Herschel conceived the plan of inclining the great

Fig. 169.

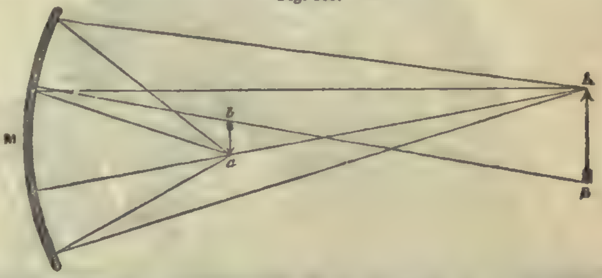
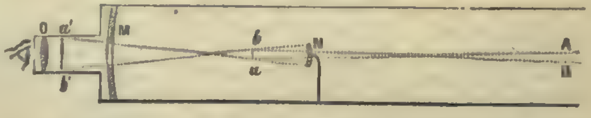
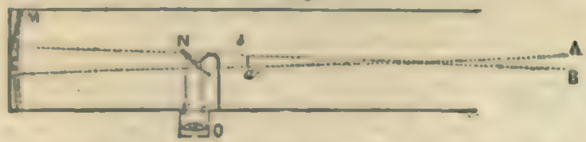


Fig. 170.



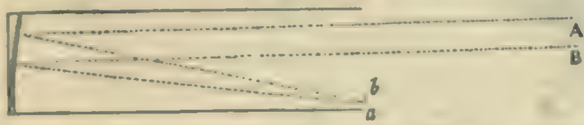
concave mirror at a small angle to the object observed, by which means the rays of light proceeding from the

Fig. 171.



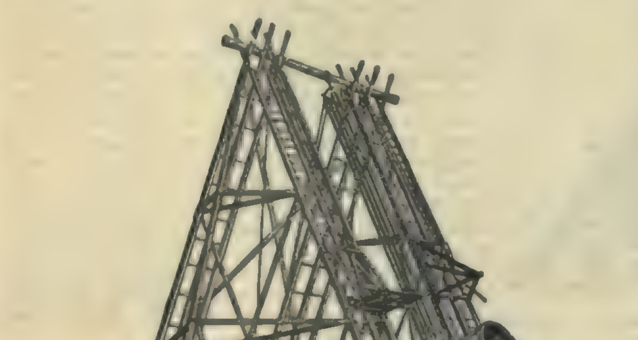
object A B (Fig. 172) were reflected to *b a*, and the image thus formed could be examined in the ordinary manner by the eye-piece, the back of the observer being turned to the object. This disposition (termed by Herschel *front view*) is, however, only suited for instruments with large apertures, in which the loss of light is less than when reflected from a second mirror.

Fig. 172.



The great forty-feet telescope of Herschel, with a mirror of four feet in diameter, was mostly used in this manner, the observer being placed in a gallery at the end. The engraving below gives a view of this magnificent instrument, which was moved upwards and downwards by means of numerous ropes and pulleys, whilst the motion to the right and left was facilitated by rollers. With this mighty tube Herschel was able to apply a magnifying power of six thousand! Unfortunately it was too much exposed to the weather; the polished mirror was dimmed in one night by the damp atmosphere; and the instrument remained in use for only a very few years, when it was found unfit for service.

Fig. 173.



INSTRUMENTS FOR MEASURING THE ANGLE.—Hitherto our attention has been directed to instruments of great magnifying power, intended to bring distant objects apparently nearer to the observer. We have now to speak of graduated instruments of more delicate construction, by which the astronomer is enabled to measure, with wonderful accuracy, the size and distance of celestial objects. The true solar time is also obtained, by comparing their angular distances from other well-known objects; in other words, by measuring their angle.

The length of the arc of a circle is described between the two sides of an angle, valued by means of a particular arc taken as unity. This arc, by common consent, is taken as the three hundred and sixtieth part of the entire circumference, which is called a degree, marked thus, 1°; and the angle which corresponds with it is the angle of a degree (Fig. 174).

It rarely happens, however, that the angle we seek to determine comes out exactly in degrees. It becomes necessary, therefore, to divide the degrees into fractional parts; and for this purpose the degree is divided into sixty parts, called minutes, written thus, 1′. For further accuracy the minute is again subdivided into sixty parts, called seconds, written thus, 1″. When the instruments which we are about to describe are properly arranged, and the two telescopes are presented to their proper side of the object whose position or magnitude is to be measured, the angle is read off the graduated scale, which surrounds the circumference of the circle, by means of an index, which extends its limb over the whole length of the arc.

Fig. 174.

In measuring the angle by the visual rays which abut on each side of an object, two operations are required. The two rays of the graduated circle must coincide with the two sides of the angle. This is effected by viewing it successively in the direction of each of its sides; the object being to value the degrees, minutes, and seconds contained in the arc of a circle comprised between the two rays.

THE REPEATING CIRCLE consists of a graduated circle A A (Fig. 175) divided into 360°, with their respective fractional parts, and fitted with two telescopes furnished with micrometers, the telescopes varying in power according to the size and value of the instrument, and the whole mounted on a pedestal, as represented in the figure. It is also so arranged as to permit of its being turned in any required direction. The circle, A A, turns on its own plane round an axis implanted in it perpendicularly, and in its centre. This axis traverses a socket, B, which is fixed to the horizontal axis, C, and is terminated by the weighted drum of the pedestal D. This weighted drum is intended as a counterpoise to the circle and its telescopes, and to prevent it from swinging while turning on the axis C; the extremities of the axis C are supported by the mounted frame E, which is arranged so as to turn freely in the openings left by the mounting. In short, the pedestal, F, can itself turn, with all it carries, round an axis which penetrates its interior, and which is fixed in the foot of the instrument. By this disposition of its parts in turning the circle round the axis C, and at the same time turning all the instrument round the axis of the pedestal E, the plane of the circle can be trained in any desired direction.

A telescope, S S, is fixed on the upper face of the circle in the direction of one of its diameters, and turns freely round its centre. A second glass, I I, is adapted, in the same manner, to the under-face of the circle at an angle with the other, but so adapted as to turn freely and independently of the circle. The eccentricity of the under telescope, and the great distance of the objects usually under observation, render the errors scarcely appreciable.

In Fig. 176 an enlarged portion of the circle is represented, showing the graduated scale and the extremity of the piece K (Fig. 175). The piece *a* (Fig. 176) is pierced with a rectangular opening, traversed in its middle by the screw *b*, which is formed to turn upon itself in the collars fixed at the two extremities of the opening. Screw *c* acts upon *b*, and is besides attached to the clamp *d*, which clasps the edge of the limb above and below. The screw *c* is to regulate the place of the clamp on the opening, pressing the edge of the limb between its jaws, and thus fixing the screw, *c*, to the limb. When it is required to displace the telescope rapidly, it is necessary to press the screw *c*, which throws it loose, so as to move round the circle under the impulse of the hand. When the glass has nearly attained its proper position, it is trained more exactly by turning the screw *a*. The vice *d* and the screw *c* are acted on and fixed again to the limb, and by turning the screw *b*, a slow movement is communicated to the screw *a*, which trains the telescope to the exact spot required.

Fig. 175.

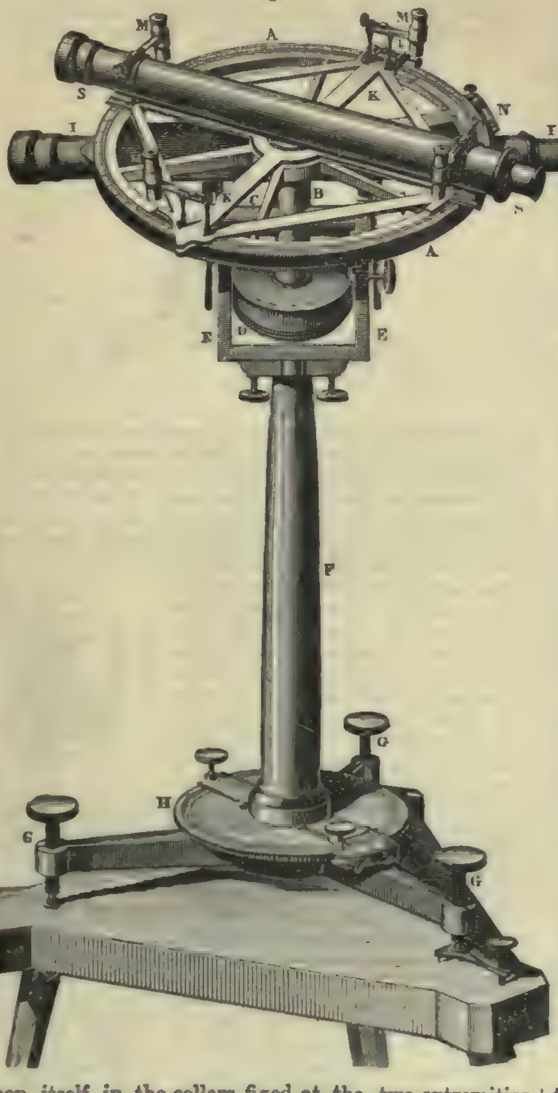
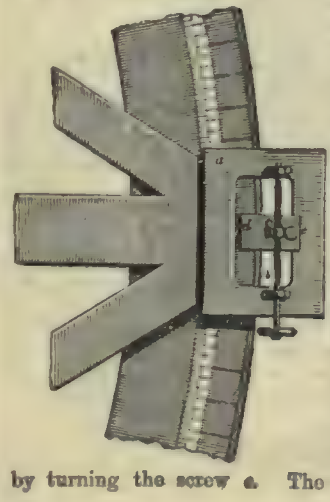


Fig. 176.



Another disposition has been adopted for producing, in an analogous manner, the movement of the circle round its centre. The axis, after having traversed the socket B B (Fig. 175), and the cylindrical counterpoise D slightly inclined from below, carries a toothed wheel of the same diameter as the counter-weights. An endless screw is connected with this wheel, and is carried by a collar fixed to the counterpoise D, as is shown in Fig. 175. If the endless screw *a b* be moved by turning one of the two milled heads, the toothed wheel with which it is connected will receive a rotatory motion, in which the circle to which it is fixed by the same axis will necessarily participate. But the screw *a b* may be withdrawn from the toothed wheel in such a manner as to stop all communication between them. To do this, it is only necessary to turn the finger *a* of the screw when the two are disengaged, as in Fig. 178. Having thus explained the mechanical details of the repeating circle, its application to measuring the angle is as follows:—

Fig. 177.

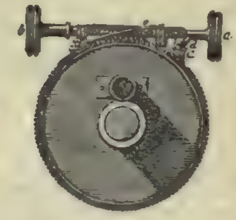
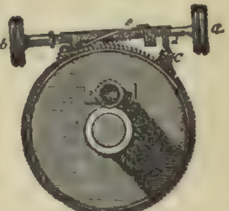


Fig. 178.



Let A and B (Fig. 179) be the two distant points, which are the sides of the angle to be measured. Having placed the upper glass at zero of the graduated scale, and fixed it to the circle in that position, the circle is now placed in the plane of the angle and made to turn in this plane, so that the telescope of the face is directed towards the point A, the other telescope being directed to the point B, the circle remaining in the position Fig. 179. The circle is now to be turned until the lower instrument is directed towards the point A, as in Fig. 180. The circle being again fixed, the upper telescope is detached and trained so as to be directed to the point B, as in Fig. 181. The upper instrument is thus made to describe an angle exactly double that sought for, and the index travels over the limb the exact measure of this double angle, which it represents in degrees, minutes, and seconds; dividing this number by two, the exact value of the angle is obtained. Supposing the operation to require confirmation, the same process may be repeated, turning the whole instrument round the axis of the circle, so that the upper telescope is again directed towards the point

Fig. 183.

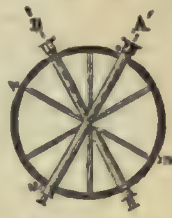


Fig. 182.

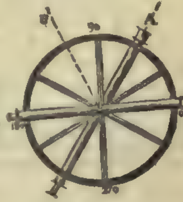


Fig. 181.

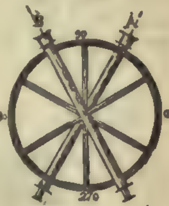


Fig. 180.

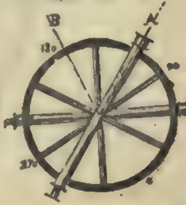
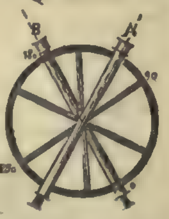


Fig. 179.



A, as in Fig. 182, the other detached and turned towards point B, as in Fig. 183, when the instrument stands exactly in its first position, as at Fig. 179; the only difference being, that the index, which at the beginning of the first operation was at zero of the graduated scale, has, at the end of the first operation, travelled over it double the distance of the angle sought; and, at the end of the second operation, this distance from zero is quadrupled; therefore, to get the exact value of the angle, the portion of the circle thus travelled over has to be divided by four. In order to diminish the chances of error in reading off the scale at the end of these operations, four different verniers, regularly subdivided, are fixed upon the circle; one of the indexes only which accompany these verniers, is employed to determine the entire number of divisions of the limb which the telescope has turned; but the four verniers give each a value of the fraction of division which is to be added to the entire number, and it is the mean of their indications which is given as the value of this fraction of division. The microscopes M M (Fig. 175) are so disposed that the divisions of the vernier can be easily observed, as well as the coincidence of one of them with the scale of the limb itself. It is hardly necessary to add that the repetition of the operation secures immunity from error, and that it also gives its name to the instrument.

To Measure the Zenith Distance.—In measuring the angle, we have real objects to deal with; it is otherwise in the present operation. We name that point of the heavens the zenith to which the vertical line is directed, and this direction is indicated by the plumb-line (Fig. 184). The zenithal distance of a point is the angular distance of this point and of the zenith—that is to say, the angle that the ray directs towards the point made in the vertical, at the place of observation. The zenith is not, as we have said, a point that we can observe with an instrument; but the zenith distance is, nevertheless, to be found by the following operation:—In order to render the repeating circle available for measuring the zenith distance, the axis of the column F (Fig. 186) is rendered perfectly vertical. For this purpose three powerful screws, G, are required, on which the instrument is to rest; these screws are turned until it is perfectly level, which is ascertained by means of a spirit-level (Fig. 185) fixed to the tube of the lower telescope, in a horizontal position, while the circle itself is nearly vertical, as in Fig. 187; the screws, M *n q* (Fig. 187), being used to regulate the position, and render the whole perfectly square.

Fig. 184.

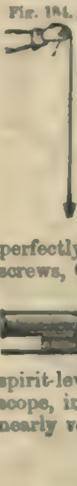
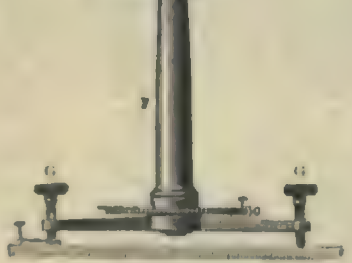


Fig. 185.

Fig. 186.



This position being attained, turn the whole instrument round the axis, F (Fig. 186), until the spirit-level is parallel to the line *p q*, and perpendicular to the line *m n* (Fig. 187).

The axis of the column of the circle and the plane of the circle being thus rendered perfectly vertical, the upper telescope is to be turned upon the circle so that its index stands at zero of the graduated scale, and the circle is fixed in this position. The circle is then turned with the telescope, first around the axis of the column so as to lead the vertical plane of the circle by the point A Fig. 188), and afterwards round the axis of the circle so as to train the optical axis of the glass exactly towards the same point. The circle being fixed in this position by means of the tangent screw acting on the extremity of its axis, a half turn is given to all the instrument, which carries it round the axis of the column, placing it in the position indicated (Fig. 189), detaching the instrument, and training it along round the axis of the circle in such a manner as to point towards A (Fig. 190). It is clear that in this movement the telescope has made an angle double the zenithal distance A O Z (Fig. 193), which is to be determined; and that by reading the number of degrees, minutes, and seconds of the graduated scale to which the index, which accompanies the telescope, corresponds, we have only to take the half of this number to have the value of zenithal distance.

Fig. 187.

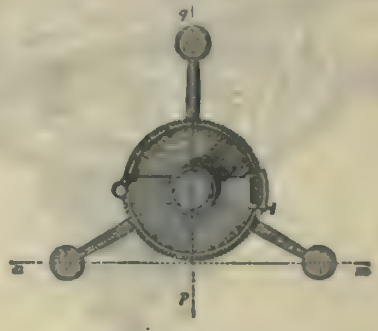


Fig. 192.

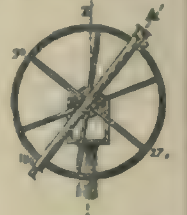


Fig. 191.

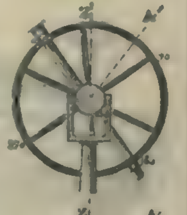


Fig. 190.

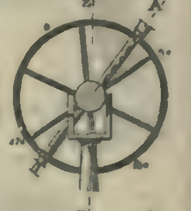


Fig. 189.

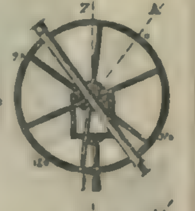
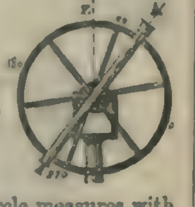


Fig. 188.

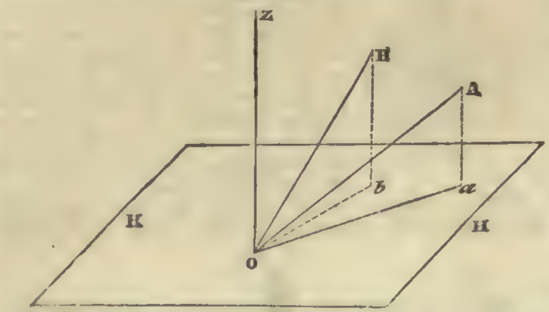


The operation may here terminate, if the operator be satisfied with the measure obtained; but should he wish to verify the calculations by increasing the multiple of the angle, he may continue the operations by making a half turn round the axis of the column, crossing the circle in its plane, so that the telescope, which remains fixed, is again directed towards the point A (Fig. 190).

The instrument is now in exactly the same position as in Fig. 188, in the first operation; but the index, in place of being at zero, is now at an angular distance from zero just double the data we seek to value. A new operation, exactly like the first, consequently gives, at its termination, a value of four times the value of the zenithal distance, and the sum found, divided by four, will give the precise value.

The Theodolite.—The repeating circle measures with great exactness the angle A O B (Fig. 193), formed by the right lines joining the two points A and B at the point O. But it frequently happens that the angle comprised between the vertical plane Z O A, Z O B, which passes between these two points, is required—that is, the angle *a* O *b*, formed by the intersections, O *a* O *b*, of the two planes, vertical with the horizontal plane, H H. Our knowledge of the angle A O B, measured by the repeating circle, added to that of the angles, Z O A, Z O B, which are the zenith distances of the points A and B, enables us, by deducting the angle *a* O *b*, either by geometrical construction, or by trigonometrical calculation, to determine the angle. But it is sometimes more convenient to ascertain its value by direct measurement; and the theodolite we are about to describe enables us to do this with great exactness.

Fig. 193.



This instrument is represented in Fig. 194. It is composed of two graduated circles, of which the one is vertical, the other horizontal. The first of these two circles, A, is adapted to the horizontal axis B, round which it turns. The axis B is carried by the upper extremity of the vertical axis C, round which the circle A and the axis B can be turned by one common movement. The weight D acts as a counterpoise to balance the circle A, preserving it steadily on the vertical axis C. The second circle, E, has its centre upon the same vertical axis C, and turns on its plane round the axis. The foot of the instrument is furnished with three double screws like the repeater, by means of which the instrument can be rendered perfectly vertical, which is ascertained by means of the spirit-level, F, on the axis B. This spirit-level is not carried round with the circle, as in the repeater, but a slight motion can be given to it by means of the screw *a*, thus slightly raising or depressing one of its extremities, and making it turn round a small axis at its other extremity. In this manner it is so managed that the bead of the level will be exactly in the centre of its tube when the axis C is in a vertical position; and, consequently, the plane of the circle A will be vertical also. As a test of the circle and the axis C being perfectly vertical, and the axis B horizontal, the movable spirit-level (Fig. 195) is used. This instrument is made so as to rest on the axis B; the feet of the spirit-level being adapted to it for that purpose, while it is supported in its place by the fork *h* (Fig. 194).

Fig. 194.

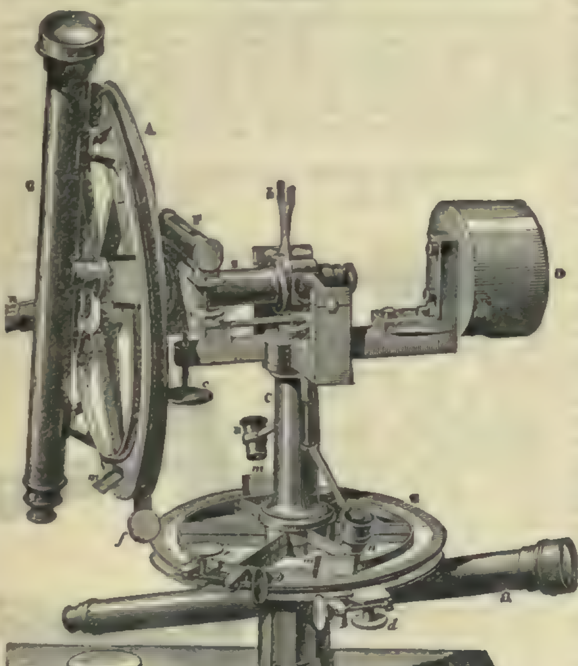
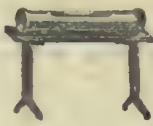


Fig. 195.



The telescope G is adapted to an inner circle, which moves round the interior of the circle A. In like manner the parts of the instrument attached to the horizontal circle, E, are fixed to an inner circle, which moves round its interior, the outer extremity of each circle carrying a graduated scale, to which the inner circle acts as an index guide. A clamp, *d*, having an adjusting and guiding screw, serves to fix the circle E to the pedestal; while another clamp, *e*, in a similar manner, serves to fix the upper parts to it; and a third clamp, *f*, serves to fix the limb A in such a manner as to be in opposition to it when turned round its centre. A fourth clamp, not seen in the figure, attaches the telescope G to the circle A.

A second telescope, H, is adapted to the pedestal of the instrument, having movements which, from its position, are very limited; it serves no other purpose, however, than to show that the foot of the instrument is steady while the observation is in progress. With this object, profiting by the limited movements it can take, its optical axis is trained in the direction of a point which can be easily recognised; and from time to time, while shifting the instrument, it serves to assure the observer that the telescope is directed to the same point as at the commencement of the operation. A guiding-screw, *g*, in the foot of the pedestal, serves as a slow movement, by which the optical axis of this glass may be trained to the required point.

To measure the angle comprised between the two vertical planes passing between two objects, turn the whole upper part of the instrument, independently of the graduated limb, E, so that the index traced upon the inner circle coincides with zero of the graduation, and fix this circle to the limb E in this position by means of the clamp *e*. By turning the limb E and all it carries, and at the same time moving the telescope G round the centre of the circle A, so that the optical axis is directed to the first of the two objects to be observed, the first operation is completed. Fix the limb E in this position, by means of the clamp *d*, for the second operation, throwing loose at the same time the clamp *e*, and turning the upper instrument round its axis so as to lead the telescope G direct to the second object. The index of the circle, moving in the interior of the limb E, has described upon this limb an arc representing the value of the angle sought, and of which we can read the amount on the graduated scale.

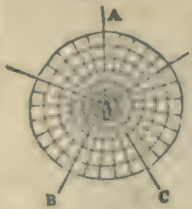
If it be intended to apply the principle of repeating the angle, fix the higher instrument to the limb E in the position it has now attained; loosen the clamp *d*, and turn the limb E, with all above it, so that the telescope G is again directed to the first object. By fixing the circle E in this position by means of the clamp *d*, and turning the upper part of the instrument until the telescope G is directed to the second object, it is clear that the index of the inner circle has described another arc equal to that already described in the first operation; and so on, by continuing the same series of operations until the multiple is sufficiently enlarged, a result may be secured free from error when it is reduced to its real value, which is done by dividing the gross result by the number of multiples. The readings on this instrument are accomplished by secured verniers, whose divisions are enlightened by small plates of polished glass, *m m*, and by the microscopes, *n n*, trained on the scale; by these means indications are easily read off.

THE ALTITUDE AND AZIMUTH INSTRUMENT.—This is the most useful of all the portable instruments, and to the scientific traveller an invaluable one, measuring with great accuracy both vertical and horizontal angles. (See folio plate).

In applying the instrument to astronomical purposes, it was formerly the custom to clamp it in the direction of the meridian, and, after taking an observation, or series of observations, with the face of the instrument one way, to wait till the next night, or till opportunity permitted, and then take a corresponding series of observations of the same object with the face reversed, by way of verification. This is now seldom practised, being obviously imperfect. The instrument consists of a central tripod, to which is fixed the azimuth circle, having a horizontal motion of about three degrees, so that its zero can be brought exactly in the meridian by means of a slow moving screw beneath the circle. The tripod rests upon foot-screws, which are described by Mr. Troughton, in the *Memoirs of the Astronomical Society*, as "being double; that is, a screw within a screw, the exterior one having its female in the end of the tripod, and the female of the interior screw being within that of the exterior," by which ingenious contrivance, three distinct motions are gained for regulating the azimuth. Brass cups are placed under the spherical ends of the foot-screws; this screw, invented by Mr. Troughton, is intended to give a very slow motion to one of the feet, and the foot of the tripod is designed to be placed either north or south. Above the azimuth circle, and concentric with it, is a strong circular plate, which carries the whole of the upper works. This plate rests on the axis of the azimuth, and moves concentrically with it. Rising from this plate are two strong conical pillars, on which the transit instrument is supported. Upon the axis, as a centre, is fixed a double vertical circle with the telescope between them, the circles being fastened together by small brass pillars, while the graduated scale is made on a narrow silver ring, inlaid on the outer face of one of the circles. Two reading microscopes are placed at each extremity of the circle, supported by two attached near the top of the pillars. The adjustments required are as follows:—The horizontal circle is first to be leveled, which is to be effected in the same manner as with a theodolite. The axis of the telescope must also be leveled, as in the transit instrument, and the spider-lines adjusted for collimation and verticallity. The meridional point on the azimuth is its reading when the telescope is pointed north or south. This may be determined by observing a star at equal altitudes, east and west of the meridian, and finding the point midway between the two observed azimuths; or the instrument may be adjusted to the meridian, in the same manner as a transit instrument. The horizontal point of the altitude circle is its reading, when the axis of the telescope is horizontal, and may be found, as with the mural circle, by alternate observations of a star directly, and reflected from the surface of mercury. The telescope usually carries, in its principal focus, a spider-like micrometer, as in the transit instrument, to be described presently.

THE MICROMETER.—We have had occasion to mention this instrument, and a short account of it may not be out of place here. It consists of a system of very fine wires (Fig. 196), by which the apparent magnitude of heavenly bodies may be measured, and the exact moment of their transit across the wires observed with great exactness. In the case of refracting telescopes, the micrometer is fitted in the instrument itself. In circular instruments, and the larger instruments of observatories, it forms a part of the microscope attached to the outer edge limb for reading off the graduated scale.

Fig. 196.



When a telescope is fitted with the micrometer, the right line of the optical axis *o* (Fig. 197), exactly coincides with the centre of the crossed lines B. When it receives the ray A, it is very obvious that, by a proper arrangement of wires, the utmost exactness of observation may be attained. The micrometer is usually formed of the finest platina wire, having three horizontal and four transverse wires. When the telescope is furnished with this instrument, the reticulated frame A (Fig 198) is placed at the end of the tube B C, near the eye-piece, which again shuts into the principal tube D of the telescope. On the other hand, the object-glass E, F, formed of two lenses, also shuts into the same tube, each requiring to have their focal distance from the object-glass adjusted either by the hand or by means of a guiding screw.

Fig. 197.

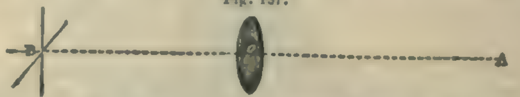
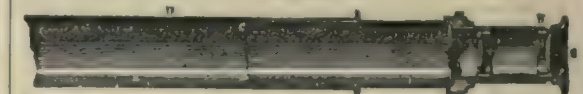


Fig. 198.



In the case of circular instruments, the micrometer is fitted in a microscope, as represented in Fig. 199. It is firmly fixed in its position, so as to enable the observer to read off the angle from the graduated scale on the outer side of the limb C D, Fig. 200. In small instruments, the screws which serve to fix the microscope in its position are usually arranged so as to regulate its distance from the limb of the circle. In the body of the microscope at A are placed two wires, crossing each other diagonally, which may be made to traverse the field of view, either horizontally or perpendicularly, by turning the micrometer screw *a*, working in the box underneath it. Figure 201 represents the field of view, with the magnified bars of the graduated scale as seen through the microscope. A small mirror, *l* (Fig. 199), is sometimes fixed to the micrometer, arranged in such a manner as to throw the light of a lamp, or a jet of gas upon the part of the limb which is opposite to the microscope; it is sometimes pierced with an opening in the centre, which receives the rays, and serves to illumine the microscope itself. In the larger instruments of observatories, they are fixed in the wall which supports the circle, the graduated scale being illuminated by a lamp

Fig. 199.

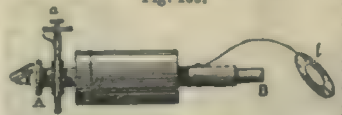


Fig. 200.

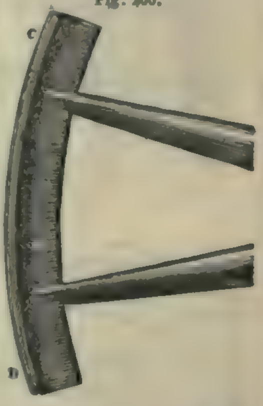
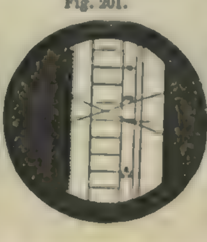


Fig. 201.



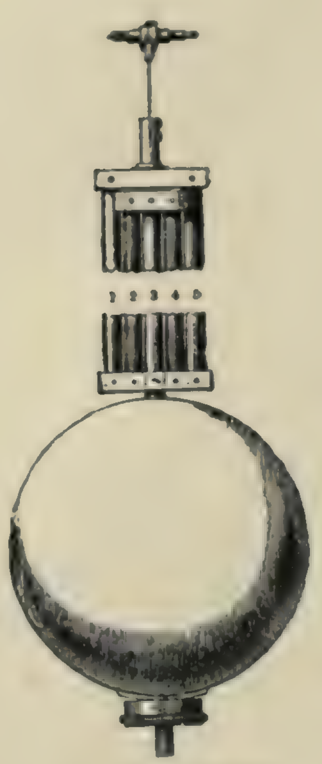


Fig. 1.—Gridiron Pendulum.

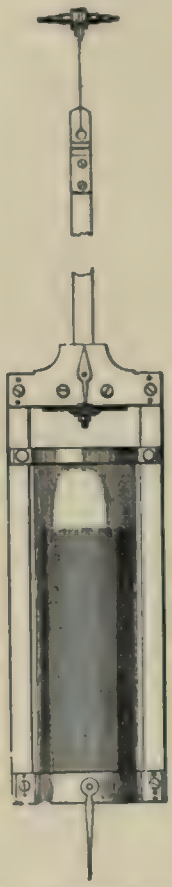


Fig. 2.—Mercurial Pendulum.

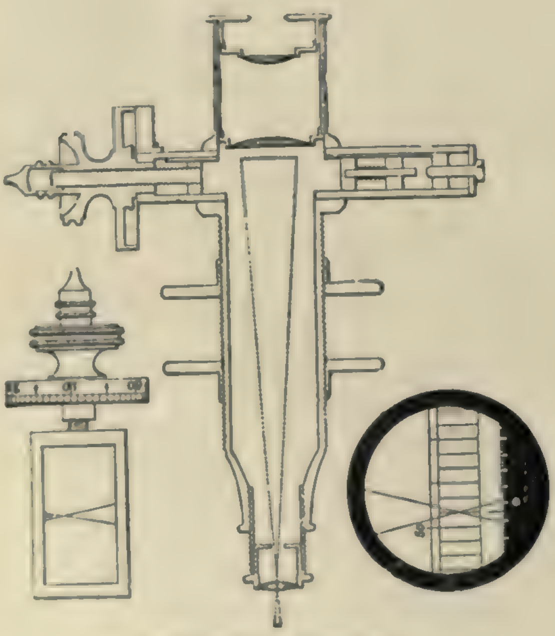
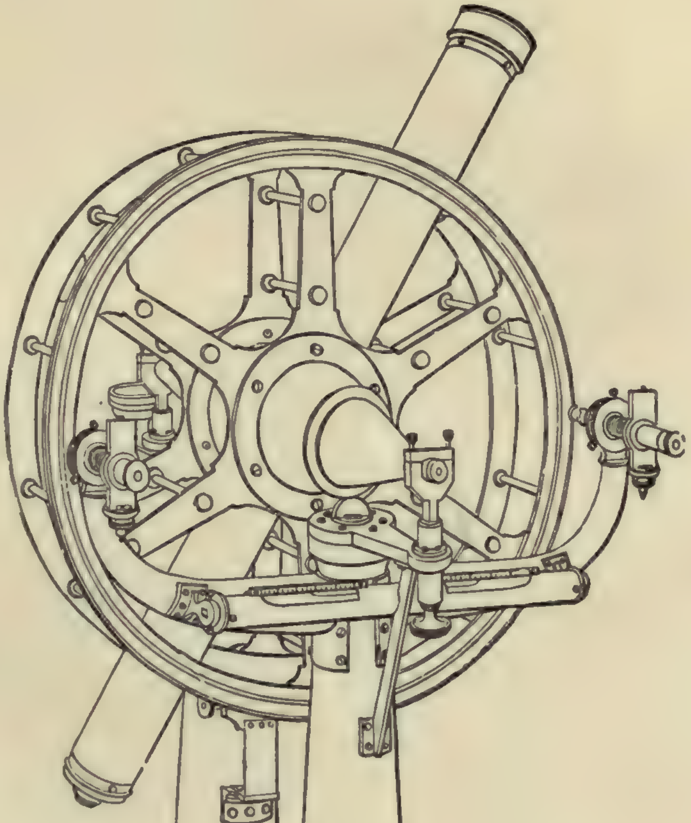


Fig. 3.—Micrometer-Microscope.

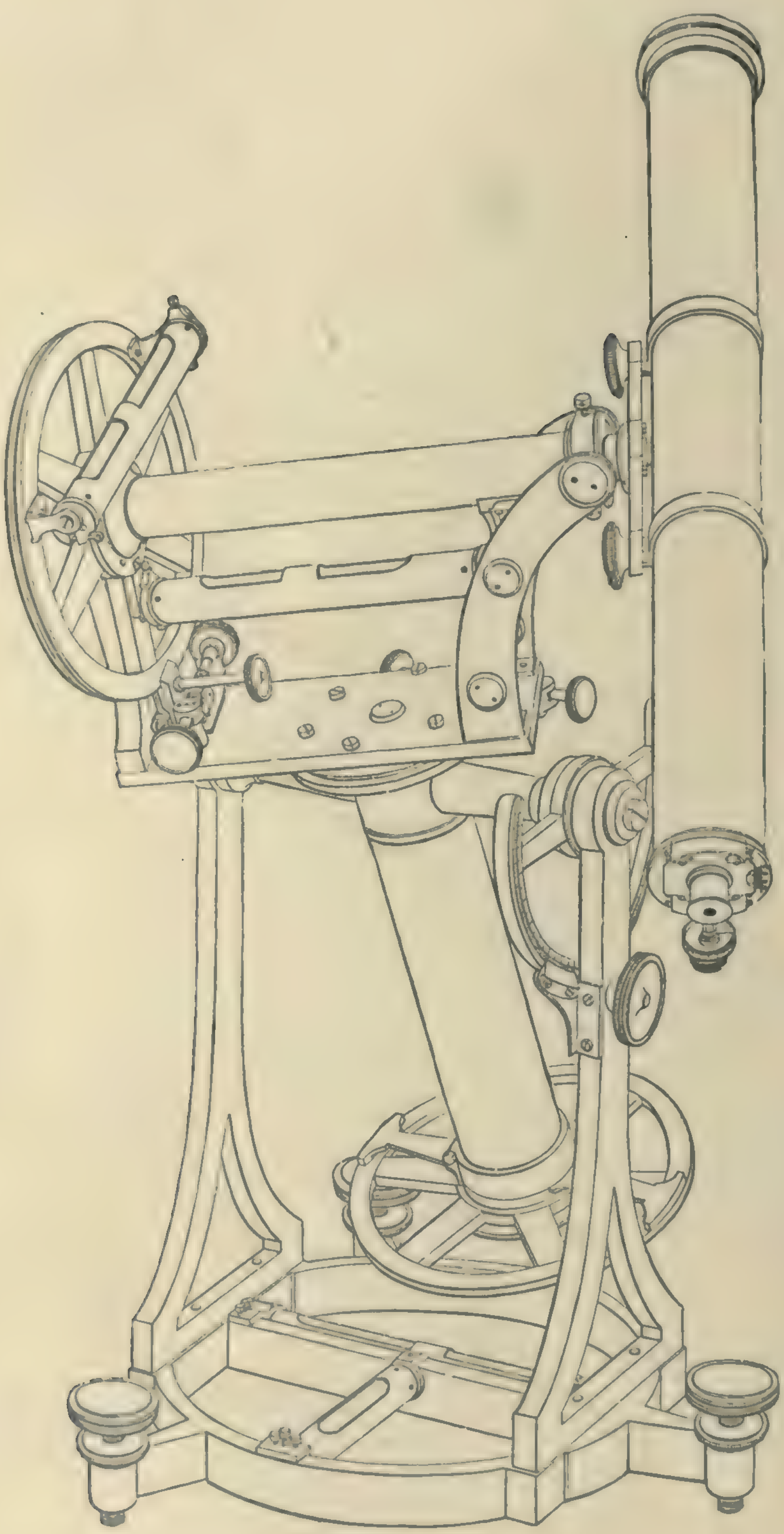
Figures 1 and 2 illustrate contrivances for overcoming the changes, produced by heat, in the pendula of clocks referred to at page 1001. In the Gridiron Pendulum, this is effected by employing rods of different metals, or the same metals, so fixed, as that whilst one series expands or contracts in one direction, the other shall do so in an opposite one. In the Mercurial Pendulum, a cistern containing mercury is employed, which, by its changes of length, compensates for those of the rod of the pendulum; that being the object of each arrangement.

Fig. 3 gives a sectional view of the Micrometer-Microscope, which has been already described at page 998.



The Altitude and Azimuth Instrument.

ASTRONOMICAL INSTRUMENTS.—(See page 998).



The Equatorial Instrument.

ASTRONOMICAL INSTRUMENTS.—(See page 999).

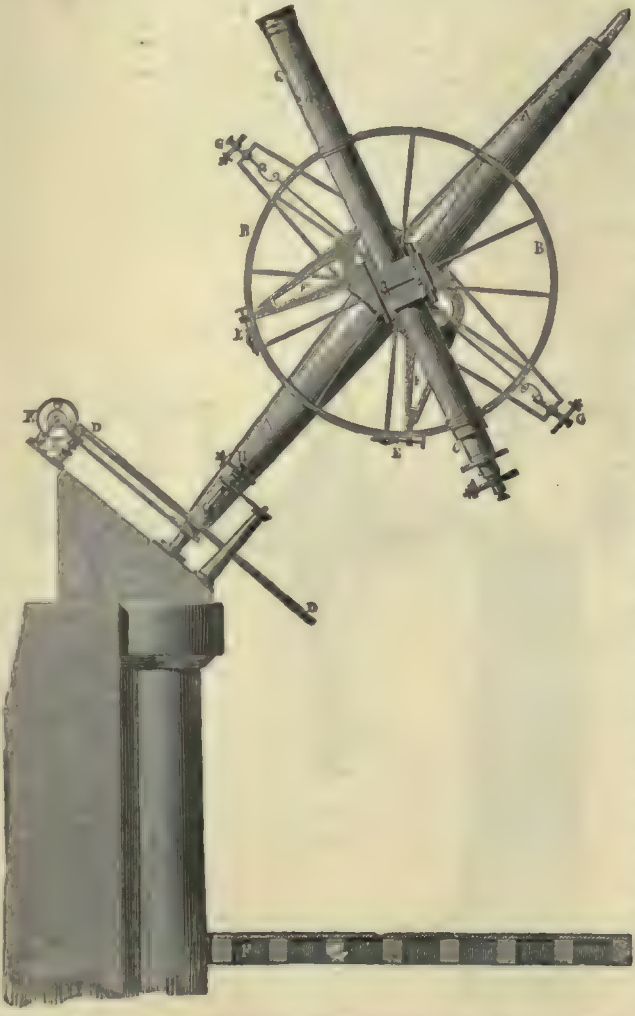
or jet of gas. The shaded part (Fig. 201) represents a diaphragm with the cross wires, *m*, the angle of which may, by turning the micrometer screw, *a*, be bisected by any line on the circle in the field of view. On the left hand of the diaphragm is a comb, or scale, each tooth of which represents one minute; and one revolution of the screw moving the wires over one tooth of the comb is equal to one minute of space.

The adjustment of the microscope consists in making the cross wires in its focus, and the divisions on the circle, both appear at the same instant of time, and free from parallax: the adjustment is such that five revolutions of the micrometer screw shall measure a five-minute space on the graduated circle. For the former of these adjustments in the telescope, the eye-piece is drawn out until the distinct vision of the wires is obtained, and the divisions or bars of the instrument are well defined.

The motion of the comb, or scale of minutes, is regulated by a screw; and the micrometer head, by friction, can be made to read either zero, or any required second when the cross wires bisect any particular bar, by holding the milled head of the micrometer screw.

THE EQUATORIAL.—The instruments we now come to describe belong exclusively to observatories, and in these establishments the equatorial is specially adapted to mark the diurnal movements of the heavens. The axis A A (Fig. 202), round which the whole of the instrument can turn, is so arranged as to follow the axis of our planet the earth, and it carries, laterally, the graduated scale B B, which can turn both in its plane and round its centre. The telescope C C, fixed to this circle, follows its movements, and its optical axis thus makes an angle variable with the earth's axis. A second graduated circle D D, whose plane is parallel to the celestial equator, and having the axis A A as its centre, is fixed in such a manner, that it follows all the movements of the instrument in its rotations round this axis; the position of this second circle determines the altitudes, and gives name to the instrument. The clamp E E, with guiding and reversing screws, intended to fix the circle and the telescope to the axis A A, when the circle turns round its centre, is carried by the pieces F F attached to the axis. Two micrometers, G G, are adapted to the extremities of the cross beam, firmly attached to the axis in such a manner as to permit of the divisions of the graduated scale, carried on the side of the limb of the circle, being read off. Other micrometers are attached to the masonwork which carries the extremities of the axis, and are intended to read the graduated scale of the second circle D D, which, in this instance, is on the upper face, and not on the outside of the limb. Such an instrument affords great advantages in measuring the relative position of two contiguous bodies, in measuring the diameters of the planets, &c. The circle which is connected with the polar axis is graduated into hours, minutes, and seconds of time, to indicate the right of ascension of the object under examination; while the circle connected with the declination axis is graduated into degrees, minutes, and seconds of arc, to indicate declination, or polar distance. (See, also, the folio plate.)

Fig. 202.



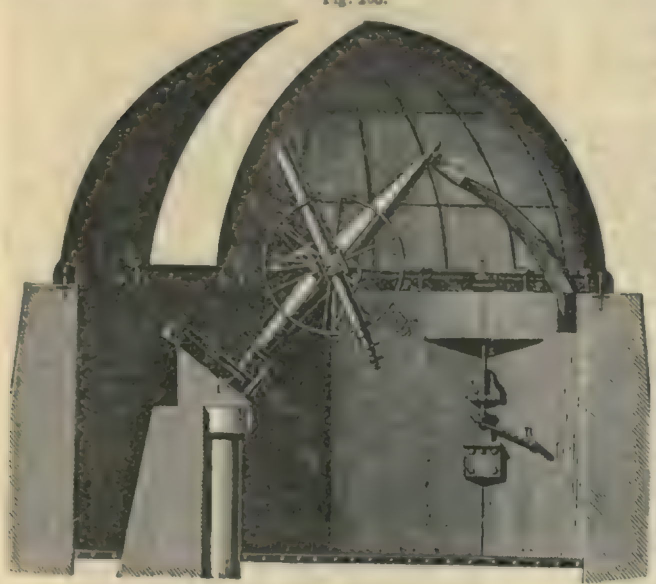
From this disposition of the instrument it will be seen that the optical axis or line of collimation of the telescope turns towards all points of the heavens, and, in making it turn with the circle B B round its centre, it can be made to form any angle with the axis of the earth. If the circle B B be fixed in one particular position, by means of the clamp E E, and an entire turn be made all round the axis A A, it is evident that the optical axis of the telescope meets the celestial sphere at various points of the same parallel.

A particular piece of mechanism, K, permits the equatorial circle, D D, to be put in communication with a clock movement, so disposed that the circle, D D, performs an entire turn on its axis in a sidereal day. If the optical axis of the telescope be directed to a star after the circle B B is fixed to the axis A A, and the circle D D put in communication with the clock-work, the whole instrument will be carried along with it, and the telescope will continue to follow the movements of the star; thus affording a means of verifying the uniform rotatory movement of the celestial sphere, after having made due allowance for the effects of atmospheric refraction.

The telescope of the equatorial requires to be in such a position that it can be directed to every part of the heavens above the horizon. This renders it necessary that the instrument should be so placed as to be clear of interruption from neighbouring objects. It is usual, therefore, to place it in the upper part of the observatory. The following engraving represents a section of the equatorial room of the Paris observatory, Fig. 203 representing a part of the instrument, of which we shall here give a brief description.

The axis is supported at its lower extremity by a massive piece of masonry, L, its upper extremity being supported by the iron beam M, made as light as the safety of the instrument, and its required steadiness will permit. The instrument is protected from the weather by a roof in the form of a hemisphere, having a long opening following its vertical plane, and formed with doors sliding laterally so as to leave a free opening in any required direction. By this arrangement the glass can sweep the heavens on its vertical plane from the zenith to the horizon; and by sliding the other parts of the roof, the whole of which is placed on friction rollers, and fur-

Fig. 203.



nished with a crank, R, for the purpose of moving it, every portion of the heavens can be swept with the instrument. The equatorial, consisting, as we have seen, of one circle, parallel to the plane of the equator, and of another circle which follows the axis of the earth, is arranged so as to coincide with any circle of declination, and is well adapted for measuring the differences of right ascension and declinations of two neighbouring stars.

The Transit Instrument.—The place of an object in the heavens is mostly defined by two elements; viz., by its right ascension and north polar distance. In the first approaches to accurate results of observation, these elements were generally determined by a mural arch or quadrant, firmly fixed in the direction of the meridian, the position of which was frequently checked by equal altitudes of stars. It was by means nearly similar, that Flamsteed observed his catalogue of three thousand stars, which has been so ably reduced and edited by the late Mr. Baily, and is generally known as the *British Catalogue*.

La Caille, at the Cape of Good Hope, observed his zones in like manner, using certain well-determined stars as "zero points," whose positions had been obtained by independent methods. The first astronomer was content with noting his observations to seconds of time—an accuracy, which, from the imperfection of his instruments, he considered to be quite sufficient. The invention of the transit instrument by Roëmer, in the year 1690, enabled astronomers to make great advances in the accurate determinations of right ascensions. This element, the most delicate and important, essentially depends upon time, and is, therefore, more difficult of direct estimation than the other, which consists of angular measurement, *read off* on graduated instruments, which modern artists have been enabled to bring to an astonishing degree of perfection. The transit instrument has been successively improved since the first idea of Roëmer. Dr. Halley, when Astronomer Royal, introduced its use at Greenwich, the construction being similar to that given by *Lerrebow* in his *Praxis Astronomiæ*. In this arrangement the telescope was placed twenty-six inches nearer one end of the axis, instead of being central—a construction which evidently rendered the determination of error of collimation very troublesome.

The form of the transit instrument used by Dr. Bradley (the next Astronomer Royal) differs little from that adopted in modern times—viz., a telescope fixed at right angles to a cross axis, and capable of taking all positions in the meridian of the place. This axis terminates in two cylindrical pivots, resting in Y's so that they only touch in two points for the purpose of diminishing the friction. One of these Y's is movable in such a manner as to correct either an error of horizontality of the axis, or an error of deviation from the meridian. The instrument is furnished with a graduated setting circle, sometimes placed at one extremity of the axis concentric to this axis (as was the case formerly with Troughton's Greenwich transit instrument), or two small divided circles placed near the eye end of the telescope. In the common focus of the object-glass and eye-glass, a system of wires is placed for the purpose of defining the position of an object as it passes through the telescope. These wires are of extreme tenuity, and generally consist of five or seven vertical wires and two horizontal (Fig. 204), the central vertical wire closely approaching to the meridian. The object of the lateral wires is to diminish the error of estimation of a star's transit, by taking the mean of all, since the time of transit over a single wire would be liable to some uncertainty. To prevent any error arising from parallax in the observation of the wires, the eye-piece is movable by a screw, so that the transit over each wire may be observed in the centre of the field. The adjustments of a transit instrument are, first, that its optical axis, or the line joining the centre of the object-glass and the middle wire, be perpendicular to the axis of rotation; secondly, that the axis be horizontal; and, lastly, that when corrected for the two preceding

Fig. 204.

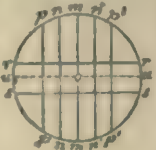
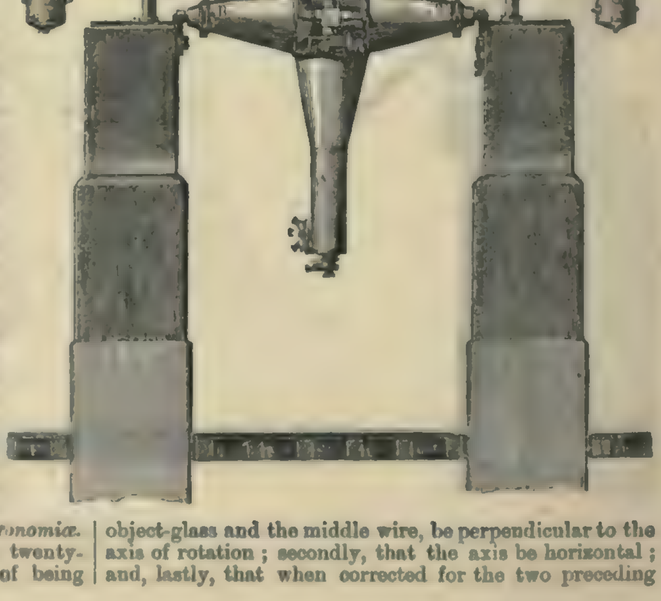
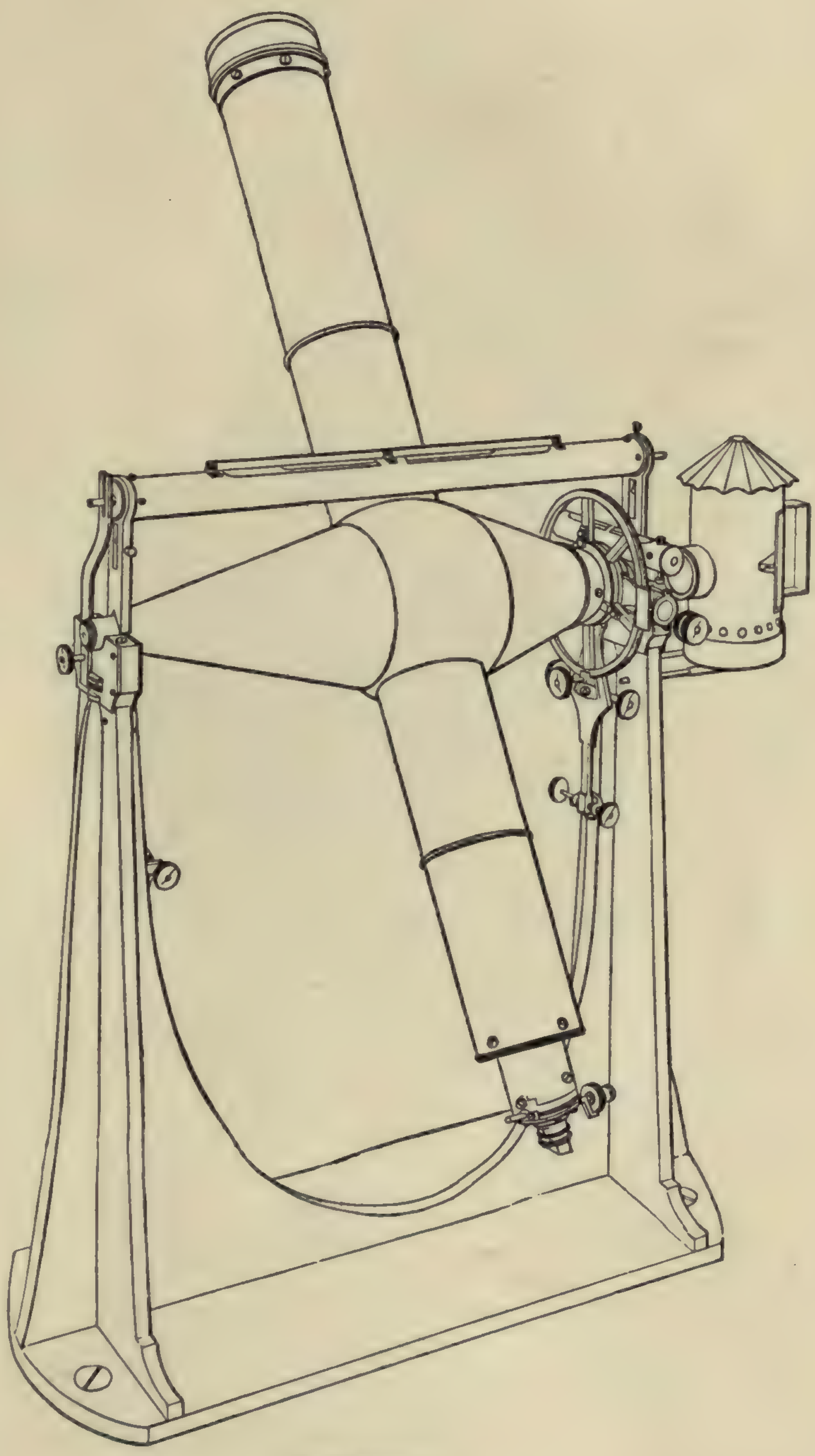


Fig. 205.





Transit Instrument.

ASTRONOMICAL INSTRUMENTS.—(See page 1000).

adjustments, the line of collimation describes the meridian. (See, also, the folio plate).

The other instrument, equally important in connection with a transit instrument, is a sidereal clock, beating seconds, and so regulated that the hour-hand will describe a complete revolution, or 86,400 seconds, from the time of a star's transit over the meridian fibre, to its return to the same point. The difference of the clock's gain or loss from this number is termed *its rate;* the excellency of a clock, therefore, depends on the steadiness of its rate, and every means are taken by mechanical refinements to preserve its uniformity. Astronomical clocks are generally furnished with mercurial pendulums for the purpose of compensating for extremes of temperature.

The transit clock requires to be mounted on a pier of solid masonry for the purpose of insuring steadiness.

The method of observing transits is as follows :—The observer, being comfortably seated (the telescope having been previously directed to the collimating star), notes carefully the minute, second, and fractional part of a second at which a star transits each wire. The hour is afterwards noted, and the counting of the clock verified when the transit is completed. This is done for several objects, the mean of the wires is taken, and thus, the differences of their right ascensions, subject to the rate of the clock, can be found. The fractional part is estimated by carefully noting the position of an object before its passage over the wire, and after its passage over the same wire. By a little practice, the instant when it was exactly behind the wire can be very accurately proportioned. In good observations the mean of wires is seldom liable to an error greater than one-tenth part of a second.

Recently, a new method of recording transit observations, by the agency of galvanism, has been brought into use at the Royal Observatory, Greenwich, which promises to supersede the old systems of observing. Its first introduction appears to have been in the coast survey of the United States; and in the hands of Mr. Walker, Dr. Locke, and Professor Mitchell, it has been considerably improved. The form of the invention, as described by the Astronomer Royal, is as follows :—From the principal instruments of the observatory, galvanic wires are connected with a recording surface placed in another part of the building. The transit clock, at each beat, completes a galvanic circuit, which is registered by proper apparatus on the recording surface, consisting of a cylindrical barrel, covered with prepared paper. Above the barrel (which is driven by clockwork regulated by a conical pendulum), a system of prickers is placed, carried by a travelling frame, moving slowly in the direction of the barrel axis. Thus, the pricker, which is connected with the transit clock, will register seconds on this barrel, and will form a series of spiral lines as it revolves. The method of arrangement is as follows :—A wheel of 60 teeth is fixed on the escape-wheel axis, and the teeth of this wheel, in succession, make momentary contacts of the galvanic springs. The position of the springs is so adjusted, that the effect of the wheel-tooth upon them occurs only when an escape-tooth has passed the sloping surface of the pallet, and the other escape-tooth is dropping upon its bearing; and thus the resistance of the springs does in no way affect the legitimate action of the train upon the pendulum.

Another pricker, carried by the same travelling frame, is connected, by arbitrary touch, with an index at the eye end of the transit circle. At the instant of a star's passage behind the wire, the observer touches this index, which will, therefore, register on the barrel a series of punctures equivalent to a transit observation. An advantage is gained by this method, insomuch that the equatorial intervals of the wires may be reduced to three seconds of time—the duty of the observer merely consisting in writing down, in addition to the preceding signals, the name of the object observed. For the purpose of registering on the barrel the beginnings of some minutes, and hours and minutes, from which the rest may be inferred, a provision is made by certain prearranged signals at known instants of the transit clock. In this manner the paper, when taken off the barrel, may be easily translated. This system has so far succeeded admirably at Greenwich, requiring only the use of the nerves of sight and sensations of touch, of which there is a more intimate connection than those of the eye and the ear. It will, therefore, most probably reduce the amount of "personal equation."

It frequently happens that two observers will not give the same time of the occurrence of the same phenomenon; as, for instance, the passage of a star over the wires of a transit instrument. The amount of this difference, which is termed "personal equation," varies from two-tenths to half a second, though instances have been known of this quantity exceeding one second of time. Dr. Maskelyne, and Kinnebrook his assistant, differed in this manner seven-tenths of a second. In modern times, Bessel and Argelander have differed upwards of a second. The method of determining it is to compare the "clock error" of one observer with that of another independently obtained (at a short interval of time). Or it can be deduced by taking transits of the same star over some wires of a transit instrument, by one observer, compared with the transits over other wires by another observer. Reducing each separately to the middle wire, the difference of their methods of observing may be at once deduced. We refer to the annual volumes of the Greenwich observations for examples of the first of these methods.

It can also be obtained by a binocular eye-piece, an instrument which permits two observers to see the star in its passage over the wires at the same time, and thus to observe the whole transit. This instrument is the invention of the late Mr. J. Jones.

In order to determine the relative accuracy of transit observations at the separate wires, the following method, first introduced by Bessel, and recently used by Struve and Dr. Oudemans, may be applied. Let x be the mean error of one observed wire, arising from a defect of hearing, y the mean error produced by an imperfection in the sight, which may be caused by a wrong estimation of the time at which the star passes behind the wire (varying with the thickness of the wires); then the mean error of an observed transit at one wire will be

$$M = \sqrt{(x^2 + y^2 \sec.^2 \delta)}.$$

The error of hearing, or x, will arise from the estimation of the precise instant of the origin of second, and will, therefore, be the same for all stars. The other will vary as the secant of the declination.

Then, if the transits of several stars are reduced to the middle wire by the known intervals of the wires, and the difference of each result from the mean is taken, the squares of these differences will be $= (n - m) M^2$, where n = the number of wires, m = the number of observed culminations. The probable error of M, determined by this equation, will be $M \frac{0{\cdot}4769}{\sqrt{n}}$, the probable error of the value M^2 will be $M^2 \frac{0{\cdot}9538}{\sqrt{n}}$.

Forming for each culmination $\Sigma(\epsilon^2)$, and their sums for all the culminations of the same star, the equation will become

$$(n - m)\, x^2 + (n - m)\, y^2 \sec.^2 \delta = \Sigma(\epsilon^2).$$

Proceeding in this manner, Dr. Oudemans has found, from 228 culminations, observed from October, 1847, to October, 1848, the mean error of a single wire =

$$\text{or } M = \sqrt{(\overset{s.}{0}{\cdot}1242^2 + \overset{s.}{0}{\cdot}0603^2 \sec.^2 \delta)}.$$

But the probable error

$$w = \sqrt{(\overset{s.}{0}{\cdot}838^2 + \overset{s.}{0}{\cdot}0407^2 \sec.^2 \delta)}.$$

A similar investigation from October, 1848, to October, 1849, gave a mean error, or,

$$M = \sqrt{(\overset{s.}{0}{\cdot}0062^2 + \overset{s.}{0}{\cdot}0602^2 \sec.^2 \delta)};$$

and the probable error

$$w = \sqrt{(\overset{s.}{0}{\cdot}649^2 + \overset{s.}{0}406 \sec.^2 \delta)}.$$

It would, therefore, appear that his hearing had improved, whilst his sight had remained the same as before. This determination rests on 360 culminations.

Dr. Robinson has recently investigated this subject, and has introduced another error arising from atmospheric tremor, which he considers to vary as the secant of the zenith distance. From his observations in 1830, he deduces the following probable errors for a star whose declination is δ :—

$x = \pm \overset{s.}{0}{\cdot}0445$ $y = \pm \overset{s.}{0}{\cdot}0619$, Z, or the atmospheric tremor $= \pm \overset{s.}{0}{\cdot}0331$.

Several years afterwards, he found the following values :—

$$x = \pm \overset{s.}{0}{\cdot}0732 \qquad y = \pm \overset{s.}{0}{\cdot}0554 \qquad Z = \pm \overset{s.}{0}{\cdot}049.$$

But between the periods of the two observations, new and more numerous wires had been inserted in the transit instrument.

Astronomers are very careful that the three adjustments of the transit instrument, before mentioned, are satisfied. And, firstly, with regard to the line of collimation. This adjustment is generally made by the observation of a well-defined terrestrial mark in one position of the axis, measuring its distance from the central vertical wire (in terms of the micrometer) with which the telescope is furnished. If, on reversal of the axis, the mark be again seen at the same distance from the central wire, the collimation of the instrument is correct; but, if not, the wire frame must be moved to one-half the difference of the two bisections. In the practice of modern observatories, the complete correction of this error is seldom attempted; this adjustment, though probably in large instruments the most permanent, being nearly corrected, and then left for occasional verification.

The following extract from the *Greenwich Observations*, 1850 (Introduction, page xvi.), will show the method of determining the error of collimation :—

"The value of this error, which is given in seconds of space, is supposed positive when it implies an additive correction to the transits of stars above the pole. For its measurement the following method has been used :—

"A small transit instrument is fixed on temporary wooden piers in the north opening of the transit room, and its object-glass is turned towards the principal transit instrument. In this position, when the principal transit instrument is turned towards the small one, the wires of the latter are seen as well-defined marks at an infinite distance, and the error of collimation of the principal transit may be determined by repeatedly reversing it, and viewing these wires.

"In November, 1846, a new determination of the value of the micrometer-screw (of the transit telescope) was made by means of six transits of Polaris over the two wires moved by the micrometer. The mean of the intervals of the times of transit was found to be 3m. 1s. ·17; and the north polar distance of Polaris being 1° 30′ 9″·3, this corresponds to an interval of space = 71″·188. But, by bringing each micrometer wire several times into contact with the fixed central wire, this interval was found to correspond also to 4r. ·368 of the micrometer. Hence, one revolution = 16″·297.

"April, 29d. 22h. The St. Helena transit instrument was used in the manner stated above. The wire appeared as a broad white line, there being no reflector; and the edges of this line were observed alternately by placing upon them the micrometer wire of the transit instrument.

"OBSERVER M.

Illuminated End East.

	s.
Micrometer reading on coincidence with collimator (12 measures)	10·494

Illuminated End West.

Micrometer reading on coincidence with collimator (12 measures)	8·895

Illuminated End East.

Micrometer reading on coincidence with collimator (12 measures)	10·500

Illuminated End West.

Micrometer reading on coincidence with collimator (12 measures)	8·873
Hence reading for true line of collimation by 1st and 2nd sets	9·695
Hence reading for true line of collimation by 3rd and 4th sets	9·687
Reading for true line of collimation . . .	9·691
Micrometer reading on coincidence with D or central vertical wire	9·674
Hence apparent error of collimation for D .	0·017

"The illuminated end of the axis was left east, which was also the position of the micrometer head; and, as the readings of the micrometer increase as the wire moves from the head, D (or central vertical wire) was therefore east of the line of collimation, and stars pass it too late, or the error of collimation of D is —0r. ·017, or — 0″·277. Also, with the illuminated end of axis east, Polaris passes the mean of wires earlier than it passes D by 0s. ·07, which in arc is equivalent to 0″·03; and the correction for diurnal observation is — 0″·19. Hence the corrected error of collimation is — 0″·44."

The second method of determining the error of collimation is by observations of the transit of Polaris, or any other close circumpolar star, in one position of the axis, compared with its transit in a reversed position of the axis. By proper apparatus, this reversal of the instrument can be effected during the time of transit. If, therefore, three wires of Polaris, or λ Ursæ Minoris, be observed in one position of the axis, and the remainder of the transit in a reversed position, and if each set of wires be separately reduced to the central wire, half the difference of the two results will be the error of collimation for the object observed, which can be reduced to the error of collimation (in arc) for an equatorial star (dividing by the proper factor). It is necessary, however, in the use of this method, that the reversal be very carefully made, and that the mean of several results of separate stars be taken.

The following example of this method is extracted from the *Georgetown Observations* :—

Transit of λ Ursæ Minoris on August 7th, 1845.
End C of the axis to the east.

	h.	m.	s.	
At 3rd wire .	20	4	24	
Movable wire— x	20	11	3	
4th wire .	20	17	48	Here the instrument was reversed.
x	20	24	31	
3rd wire .	20	31	10	

	m.	s.
Interval from 3rd to 4th wire . . .	13	24 = t
" 4th to 3rd wire . . .	13	22 = $t\,o$

$$\text{Difference} = \qquad 2 = t - t\,o$$

$$1 = \frac{t - t\,o}{2}$$

m. s.
Interval from x to 4th wire . . . . 6 45 = t'
" 4th wire to x . . . . 6 43 = $t'o$

$$t' - t'o = 2$$

$$\frac{t' - t'o}{2} = 1$$

$$\text{Error of collimation or } C = \frac{(t - t\,o)\,15 \sin. p}{2} = 0''\cdot303$$

The third method was originally introduced by Professor Struve, at the Pulkowa Observatory. It consists in mounting firmly two telescopes in the direction of the meridian, the wires of which can be adjusted on each other. It is evident that by comparing the principal instrument (transit or meridian circle) with each successively, the true line of collimation may be obtained without reversal.

For the purpose of determining the line of collimation of the Greenwich transit circle (as the instrument is not capable of reversion), two collimating telescopes, the axes of which are as nearly as possible in the same line with the principal instrument, are firmly mounted on brick piers, a few feet north and south of it. These collimating telescopes are furnished with proper wires, which can be accurately adjusted to coincidence, and will, therefore, represent (when viewed with the telescope of the transit circle) two objects at an infinite distance, and exactly opposed to each other. When a series of bisections is taken of each collimator in terms of the micrometer of the transit circle telescope, the mean of these will give the geometrical line of collimation. The annexed form will show the work :—

ROYAL OBSERVATORY, GREENWICH.

Readings of the Transit Micrometer at coincidence of central wire with the wires of the Collimators for determining the position of the Line of Collimation of the Transit Circle, in the year 1851.

Approx. Solar Time.	Oct. 11, 7h		12, 23h		13, 8h		13, 22h	
Observer.	H.		R.		R.		H. B.	
Collimator observed.	South.	North.	South.	North.	South.	North.	South.	North.
Readings of Transit Micrometer	r 28·358 349 347 346 345 340	r 34·572 586 578 580 581 582	r 28·296 294 293 290 300 294	r 34·602 600 595 592 590 598	r 28·312 310 307 314 315 310	r 34·580 596 590 592 585 600	r 28·302 318 306 301 303 307	r 34·563 587 594 556 575 584
Sum of Readings	85	479	67	577	68	543	37	459
Mean	28·348	34·580	28·295	34·596	28·311	34·591	28·306	34·577
Mean of South Readings...		28·348		28·295		28·311		28·306
Sum.............................		62·928		62·891		62·902		62·883
Concluded Reading for line of Collimation ...	r 31·464		r 31·446		r 31·451		r 31·442	
Approx. Solar Time.	Oct. 14, 8h		14, 22h		15, 6h		15, 22h	
Observer.	H. B.		J. H.				E.	
Collimator observed.	South.	North.	South.	North.	South.	North.	South.	North.
Readings of Transit Micrometer	r 28·327 291 330 312 337 331	r 34·582 614 600 593 612 604	r 28·355 345 330 337 348 342	r 34·592 600 578 598 585 602	r 28·332 323 329 340 350 342	r 34·626 626 618 592 590 605	r 28·284 278 290 284 285 280	r 34·692 683 695 695 685 686
Sum of Readings	28	05	57	555	16	57	501	536
Mean	28·321	34·601	28·343	34·593	28·336	34·610	28·284	34·689
Mean of South Readings...		28·321		28·343		28·336		28·284
Sum.............................		62·922		62·936		62·946		62·973
Concluded Reading for line of Collimation ...	r 31·461		r 31·468		r 31·473		r 31·487	

The error of horizontality of the axis is generally determined by the application of a spirit level to the pivots. But it must be previously known whether the pivots are perfectly circular, and of equal diameter. For this purpose, careful series of levellings are taken in reversed positions of the axis of the transit instrument. If we assume the Y's to be perfectly horizontal, but that on application of the level, the same pivot shows a quantity too high in reversed positions of the axis, it is evident that this pivot is the larger of the two ; and it is equally evident that if the Y's are not perfectly horizontal, we can determine the difference of the pivots from level readings in reversed positions of the axis. The following example, from the *Oxford Observations*, 1840, page v., will illustrate the whole process. The sign + is used when the west end of the axis is highest ; — when the east end is highest :—

Day, 1840.	Position of Lamp.	Level Reading.	E.—W.
		″	″
Jan. 16	East	+ 2·31	+ 9·14
	West	— 6·83	
17	East	+ 1·90	+ 5·60
	West	— 3·70	
20	West	— 3·86	+ 4 91
	East	+ 1·05	
March 14	West	— 2·60	+ 5·61
	East	+ 3·01	
April 14	East	+ 2·37	+ 4·94
	West	— 2·57	
15	East	+ 4 50	+ 5·10
	West	— 0·60	
July 7	East	+ 2·50	+ 3·60
	West	— 1·10	
10	West	— 0 43	+ 3·54
	East	+ 3 11	
11	West	— 1·16	+ 5·20
	West	— 0·22	
	East	+ 4·35	+ 4·86
	East	+ 4·12	
	West	— 0 62	
	West	— 1·02	
Aug. 10	West	— 1·09	+ 4·89
	East	+ 3·80	
Nov. 26	West	— 5·43	+ 5·76
27	East	+ 0·33	
28	West	— 5·83	+ 6·31
29	East	+ 0·48	

"From these it results, that when the lamp is east the west end of the axis is highest, and when the lamp is west the east end is highest ; thus they show that the solid pivot is larger than the perforated one.

"The angle of the level's Y's is the same as that of the pivot's Y's, both being rectangular, or very nearly so : therefore the difference of the radii of the pivots

$$= \frac{\text{Difference of the Level Readings in Opposite Positions of the Axis.}}{4\sqrt{2}}$$

"The column E — W shows the difference of the level readings, the arithmetical mean of which = 5″·445 ; therefore $\frac{5''{\cdot}445}{5{\cdot}660} = 0''{\cdot}902$ is the mass of the radius of the solid above that of the perforated pivot, and 0″·962 $\times \sqrt{2} = 1''{\cdot}361$ is the correction to be applied to the level readings to obtain the inclination of the pivot centres. This correction is to be added *algebraically* to the level readings when the lamp is west, and subtracted when the lamp is *east.*"

A careful series of levellings at different elevations of the object-glass, showed that there was no appreciable difference in the circularity of the pivots, as the reader will observe by reference to the descriptions of the Altazimuth and Transit Circle in the *Greenwich Observations*, 1847 and 1852, for the method adopted by the Astronomer Royal to insure perfect circularity of the pivots.

Dr. Oudemans, in his essay on the Transit Instrument, gives the following formulæ for the correction of the level indication caused by unequal diameters of the pivots. The error of level being determined after frequent reversals of the transit instrument, this correction may be easily deduced.

Let 2 g be the angles between the planes of the Y's, on which the pivots are placed ; 2 f, the angles between the feet of the level, by which it is placed on the pivots ; u, the difference of inclinations observed in both positions of the axis ; r and r', the two radii of the pivots ; L, the distance between the points, where the two pivots rest on the planes of the Y's. Now g, f, u, and L being known, the difference of the radii r and r' is made known by the following formulæ :—

$$r - r' = \tfrac{1}{2}\, u \, L \sin. 1'' \cdot \frac{\sin. g \sin.}{\sin. g + \sin. f}$$

He then gives a determination of the values before and after reversal, and finds an arithmetical mean of the differences, or of $u = + 4''{\cdot}15$. Since this value of u found will depend on one unequal breadth of the extremities of the axis, the correction of any inclination can be obtained by the level.

$$s = \pm \tfrac{1}{2}\, u \cdot \frac{\sin. g}{\sin. g + \sin. f}$$

In his transit circle, $2f = 98° 40'$, $2g = 91° 25'$; therefore

$$s = \pm \tfrac{1}{2} \times 0{\cdot}4854 \times 4''{\cdot}15 = \pm 1''{\cdot}01 = \pm \overset{s.}{0{\cdot}067}.$$

In order to eliminate any errors in the level itself, it is the practice to reverse it several times, or to take a series of readings in one position, and then to turn it, end for end, and to take a set of readings in an opposite position. The following is an example :—

ROYAL OBSERVATORY, GREENWICH.

1848.	*d. h.* April 14, 21.	
Position of Illum. End.	East.	
	East Scale.	West Scale.
Cross Level.	div.	div.
E	115·2	110·3
W	107·0	119·0
E	115·3	110·4
W	107·0	119·0
E	115·4	110·5
W	106 6	119·0
Sums	666·5	688·2
Subtract Sum of East Scale .		666·5
		12)21·7
		1·81
Add for inequality of Pivots .		
West End high in Scale Divisions		1·81
Multiply by 1″·2		1·2
		362 181
West End high in seconds .	......	2·172

The value of one division of the level scale may be determined by the application of the level to a vertical circle, moving it so as to produce changes in the circle reading, and in the position of the bubble. Thus, in the *Greenwich Observations*, 1836, Introduction, pages xvi. and xvii., "on January 27, the level of the transit instrument was lashed to Troughton's circle, and the circle being moved till the bubble was alternately near the end of one and the other scale, the microscopes A and B were read. This operation was repeated sixteen times. The mean of the results gave 2′ 0″·03 = 112·18 parts at one end, = 108·93 parts at the other end of the scale ; the mean value of one part being, therefore, 1″·086."

It can also be determined by the comparison of the results of the transit of a star observed by reflection and direct vision. A slowly moving star is generally selected for this purpose, four or more wires being observed in each position. The difference of the two results (separately reduced) is equal to double the level error of the object observed, which can be converted into "error of inclination" for an equatorial star by the proper factor. By comparing this result with the level indication, the value of one division of the scale may be found.

Lastly, it can be obtained by elevating or depressing the Y's by the proper screw, and finding the effect of a change of level indicative of the transits of a close circumpolar star. The level error may also be obtained by a Bohneberger's eye-piece, an instrument consisting of an ordinary eye-piece, perforated at the side, and so arranged that, by means of a glass reflector, placed near the eye lens, inside the tube, the light of a lamp may be thrown into the instrument. In this manner the system of vertical and horizontal wires may be seen, by directing the telescope nearly to its nadir position, and placing underneath the object-glass a trough of mercury. By causing the two images to overlap, by means of the right ascension micrometer (noting its readings), and comparing these with the corresponding readings for the line of collimation of the instrument previously obtained, the level error may be easily deduced. This method is now practised in the use of the Greenwich transit circle. The following form shows the readings of the transit micrometer for coincidence of the central wire, for determination of error of level of the axis of the transit circle of 1851:—

Approx. Solar Time.	Oct. 11, 7^{h}	12, 22^{h}	13, 8^{h}	13, 22^{h}	14, 8^{h}	14, 22^{h}
Observers.	H.	R.	R.	H. B.		J. H.
Readings of Transit Micrometer.	^{r} 31·634 ·635 ·640 ·643 ·635 ·650	^{r} 31·650 ·642 ·650 ·640 ·639 ·650	^{r} 31·650 ·658 ·660 ·650 ·643 ·648	^{r} 31·625 ·660 ·625 ·669 ·655 ·667	^{r} 31·620 ·675 ·642 ·662 ·621 ·661	^{r} 31·589 ·580 ·583 ·578 ·582 ·580
Sum of Readings	·237	·271	·309	·301	·281	·492
Mean	31·639	31·645	31·652	31·650	31·647	31·582
Subtract from Reading for line of Collimation.	31·449	31 455	31·455	31·455	31·455	31 455
Remainder.........	0·190	0·190	0·197	0·195	0·192	0·127
Reduction for Value in Arc.	....	....	....	....	....	....
Error of Level...	″ 2·81	″ 2·81	″ 2·91	″ 2 88	″ 2·84	″ 1 88

The azimuthal error may be determined by the transits, above and below the pole, of any close circumpolar star. In observatories, where the adjustments of an instrument are sufficiently steady to be relied on, the successive passages of Polaris, or δ Ursæ Minoris, for several days, are used for this purpose. But, in small instruments, whose foundations are not so firm, it is usual to obtain an azimuth error by corresponding observations of high and low stars, or an observation of Polaris, or δ Ursæ Minoris, compared with the transit of a star south of the zenith. Thus, Polaris and θ Ceti can be used advantageously for this purpose, the difference of their right ascension being only thirteen minutes, and their tabular right ascensions being known with sufficient precision.

Mr. Johnson, in his *Oxford Observations*, uses certain pairs of stars for azimuthal errors, whose positions have been determined with considerable accuracy. These stars are distributed in such a manner, that the observer can have little trouble in finding one transiting above the pole, and another passing below the pole, within a short interval of time of each other. Thus, the result is entirely free from any doubt as to the rate of the clock, or other causes, and the principal error can only arise from the unavoidable uncertainty of observation.

It is frequently necessary, in the use of a small transit instrument, to have a fixed mark in the direction of the meridian, which can be easily referred to for the purpose of determining the error of collimation, and also its deviation from the meridian, in cases when, from cloudy weather, observations from circumpolar stars cannot be obtained.

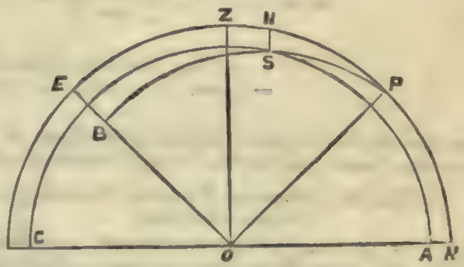
At the distance of 286 feet, a circle of two-tenths of an inch in diameter will subtend an angle of 12″ of azimuth. By noting the positions of the central wire of the transit in reversed positions of the axis, an approximation may be made at once to the error of collimation, from its relative positions in this circle. This meridian mark may be frequently verified by observations of circumpolar stars.

If the instrument be furnished with a micrometer, this method of determining the collimation will not be necessary.

We will now proceed with the investigation of the formulæ for the three preceding errors, and will show the method of application to the results of observations.

Error of Collimation.—The error of collimation is the distance of the middle line from the true line of collimation, east or west. When the middle wire is east of the true line of collimation, the transit is observed too soon, and consequently the correction for collimation is additive for stars above the pole, and subtractive for those below the pole. The contrary takes place when the middle wire is west of the line of collimation. According to this, the transit telescope may be supposed to describe a circle parallel to the meridian. Thus, let C S A (Fig. 206) be the circle described by the telescope, or the circle in whose plane the middle wire is. Then A N, or S N, is the error of collimation, which is generally expressed in seconds of arc. Let S be the place of a star, P S B a great circle passing through it. We have now to find B E, the corresponding part of the equator; but

Fig. 206.



$$\frac{S N}{\sin. P S} = B E \text{ in arc, or } \frac{S N}{15 \sin. P S} = B E \text{ in time,}$$

which is the correction to be applied to the observed transit. In that investigation, stars, however, near the pole, are supposed to transit the middle wire; that is, their distance from the pole is supposed to be greater than the error of collimation. If this were not the case, such stars could not pass the middle wire.

The correction for the error of collimation is written thus:—

$$\text{Correction} = \text{error} \times \frac{1}{15 \times \sin. \text{ star's } N P D}.$$

Correction for Level.—The error of level is considered positive when the western pivot is too high. In that position the great circle, passing through the north and south points of the horizon, and which is the circle on which observations are made, lies to the east of the meridian, and the observed time of passage is less than the true time for stars above the pole.

Thus (Fig. 207), O Z = the error of level.

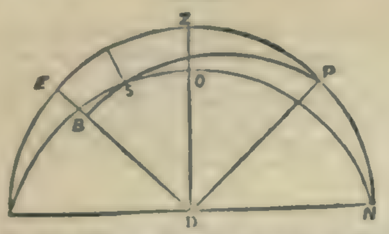
$$Z O \times \cos. S O = S x, \text{ and}$$

$$B E = \frac{S x}{15 \sin. N P D} = \frac{S x}{15 \sin. P S}$$

$$\therefore B E = \frac{Z O \times \cos. \text{ zenith distance south}}{15 \sin. N P D}.$$

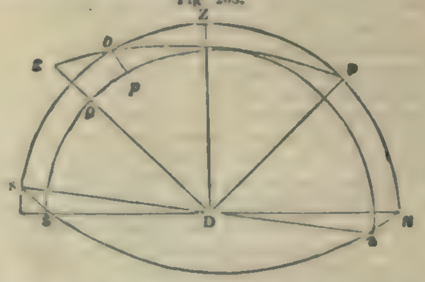
Azimuthal Error.—This error is considered positive when the eastern pivot is too far north. In this position, stars to the south of the zenith are observed before they come to the true meridian, and those north of the zenith after they pass the true meridian.

Fig. 207.



Let N *n* or S *s* (Fig. 208) be the azimuthal error, O the observed place of the star, then S *s* × sin. Z *o* = *o p*, draw the great circle P O E from the pole to the equator, passing through the star O, then E Q is the correction in R. Asc. in arc, which, being divided by 15, will give the correction in time; but E Q = O *p* × $\frac{1}{\text{sin. P O}}$; therefore, E Q *s* = S *s* × $\frac{\text{sin. Z } o}{15 \text{ sin. P } o}$ = azimuthal error × $\frac{\text{sin. Z D south}}{15 \text{ sin. N P D}}$. Since the sign of the correction to the north of the zenith is contrary to that of the south, if zenith distances *south* have the sign *plus*, and those to the north the sign *minus*, the above expression will give the correction in either case.

Fig. 208.



The preceding corrections for errors of level, azimuth, and collimation being all supposed positive, may be represented by

$$\frac{l \text{ cos. Z D}}{\text{sin. N P D}} + \frac{a \text{ sin. Z D}}{\text{sin. N P D}} + \frac{c}{\text{sin. N P D}}$$

where *l* represents the error of level, *a* that of azimuth, and *c* the error of collimation. By introducing the latitude L, the expression for the sum of these corrections may be written thus (δ being the declination),

$$\text{or} = \frac{l \text{ cos. } (\text{L} - \delta) + a \text{ sin. } (\text{L} \overset{*}{\mp} \delta) + c}{\text{cos. } \delta} \left\} \begin{array}{l} \dfrac{l\,(\text{cos. L cos. } \delta \overset{*}{\mp} \text{sin. L sin. } \delta)}{\text{cos. } \delta} \\ + \dfrac{a\,(\text{sin. L cos. } \delta \pm \text{cos. L sin. } \delta)}{\text{cos. } \delta} \end{array} \right.$$

$$= l \text{ cos. L} + l \text{ sin. L tan. } \delta + a \text{ sin. L} - a \text{ cos. L tan. } \delta + c \text{ sec. } \delta.$$

$$= (l \text{ cos. L} + a \text{ sin. L}) + (- a \text{ cos. L} + a \text{ cos. L}) \text{ tan. } \delta + c \text{ sec. } \delta.$$

= *m* + *n* sin. δ + *c* sec. δ, which is the form of these corrections Bessel gives in the *Tabulæ Regiomontanæ.*

To Find the Azimuthal Errors by Transits of a Circumpolar Star.

Let T = the time of superior transit, supposed too late; *t* = the time of an inferior transit, which will be too early; then, since the difference between these transits corrected for motion in R. A. should be 12h., all the other corrections being made, we shall have—

$$\text{T} - \frac{a \text{ sin. Z D}}{15 \text{ sin. N P D}} = 12\text{h.} + t + \frac{a \text{ sin. Z D}'}{15 \text{ sin. N P D}}$$

Let T − *t* = 12h. + *b*, then—

$$b = \frac{a\,(\text{sin. Z D} + \text{sin. Z D}')}{15 \text{ sin. N P D}}$$

$$= \frac{a\,(\text{sin. (colat. − N P D)} + \text{sin. (colat. + N P D)})}{15 \text{ sin. N P D}}$$

$$\text{or } b = \frac{2\,a \text{ sin. colat. cos. N P D}}{15 \text{ sin. N P D}} \therefore a = \frac{15\,b \times \text{tan. N P D}}{2 \text{ sin. colatitude}}$$

In a future page we shall afford numerical examples of the determination of azimuthal errors.

The preceding factors for collimation, level, and azimuth, viz.—

$$\text{Error of collimation} \times \frac{1}{15 \text{ sin. star's N P D}}$$

$$\text{Error of level} \times \frac{\text{cos. Z D south}}{15 \text{ sin. N P D}}$$

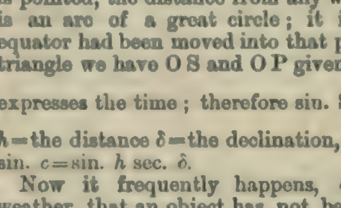
$$\text{Error of azimuth} \times \frac{\text{sin. Z D south}}{15 \text{ sin. N P D}}$$

are tabulated for different north polar distances at the principal observatories, the multiplication being easily made by means of a sliding ruler. Several other contrivances have been invented for forming these corrections by mechanical methods, by the Astronomer Royal, Professor Challis, and Mr. Carrington, for which we beg to refer to the Memoirs of the Royal Astronomical Society.

Having thus explained the formulæ and methods of determining the three preceding errors, we shall now proceed with the investigation for a determination of the equatorial intervals of the wires, which are immediately necessary for the reduction of imperfect transits.

When a heavenly body is observed with a transit instrument at a distance from the meridian, if this distance be known, and also the distance of the object from the equator, the time to be added to or subtracted from the sidereal time of observation can be computed thus:—Let P be the pole, Q E a portion of the equator, O the object at the distance O S measured on a great circle. (It may be well to remark that to whatever part of the heavens a transit instrument is pointed, the distance from any wire to the middle wire is an arc of a great circle; it is the same as if the equator had been moved into that position). Now in the triangle we have O S and O P given to find S P O, which expresses the time; therefore sin. S P O = $\frac{\text{sin. S O}}{\text{sin. O P}}$, or if h = the distance δ = the declination, c = angle S P O; then sin. c = sin. h sec. δ.

Fig. 209.



Now it frequently happens, especially in cloudy weather, that an object has not been, or cannot be, observed at all the wires of the transit instrument; but if the intervals of the wires be known—that is, the times employed by an equatorial star in passing from the first to the second, from the second to the third, &c.—the true time at which an object, which has been observed on any wire or wires, would transit the meridian, can be found. The manner in which the equatorial intervals are given, is the distance of the first wire from the mean of wires, which is called the first interval; the distances of the second and third from the same point are the second and third intervals—that is, the distance of the mean of all the wires from each wire is the interval: the intervals before the meridian point having the sign *plus*, and those after it the sign *minus*, for stars above the pole—the signs to be changed for stars below the pole. If the instrument be reversed, the signs are to be changed. For the determination of these intervals, Polaris, or some other star near the pole, is observed; then for each transit, the difference between each wire and the mean of all the wires is taken; and the mean of all these differences for the same wire for all the observations; then, if we call this mean for any wire$=c$, we have

$$\text{Sin. } h = \text{sin. } c \text{ cos. } \delta = \text{sin. } c \text{ sin. star's N P D}$$

$$\text{therefore } h = \frac{\text{sin. } c \times \text{sin. star's N P D}}{15 \text{ sin. } 1''}$$

This gives the equatorial distance in seconds of time from the wire under consideration to the mean of all the wires.

Now let $+a, +b, +c, +d, -e, -f, -g$, be the equatorial intervals for a telescope having seven wires, and suppose the times of transit of a star to have been observed on two of these wires; if the mean of these two observed times of transit be taken, it will be the time at which a star passed a point equally distant from both wires. We shall suppose the two wires to be the first and second; to find the distance of this point from the mean of wires, or from what we consider to be the meridian point. The distance between the first and second wires is $a-b$; hence the distance of the first wire from the required point is $\frac{a-b}{2}$, the distance of the first wire from the meridian point is a, therefore $a-\frac{a-b}{2}$ $=\frac{a+b}{2}$ is the distance of the point, for which we have the time of transit to the meridian point. This shows that we have only to take the sum of the equatorial intervals for the wires observed; this sum, divided by the number of wires, will be the distance of the point for which we have the time of transit from the meridional point. This distance being called h, and c representing the correction required, we have—

$$(1)\ \text{sin. } c = \text{sin. } h \text{ sec. } \delta, \text{ or}$$

$$(2)\ c \text{ sec.} = h' \text{ sec. } \delta.$$

For the pole star and others near it (1) must be used.

Determination of Intervals of the Greenwich Transit Instrument by the following Transits of Polaris:—

Distance of each Wire from the Mean of the 7th Wires (Polaris), 1848, May 2 S.P. to May 12 S.P.

1848.	A	B	C	D	E	F	G	Decl. of Polaris.
								88° 29′
	m. s.	m. s.	m. s.	s.	m. s.	m. s.	m. s.	
May 2 S. P.	—24 29·86	—16 20·86	—8 9·86	—0·86	+8 10·14	+16 17·14	+24 34·14	50″·3
3	32·00	21·00	9·00	+0·50	9 50	17·50	34·50	50·0
4 S. P.	32·14	22·14	10·14	+1·36	10·36	17·86	34·81	49·8
4	32·57	20·57	8·57	+0 43	10·43	18·43	32·43	49·7
5 S. P.	33·29	21·29	11·29	—0 29	9·71	20 71	35·71	49·5
5	30·14	19·14	10·14	—1·14	9·36	17·36	33·86	49·4
6 S. P.	33·14	21·64	8·14	+0·36	8·86	18 36	35·36	49·2
7	28·57	22 57	8·57	—0·57	6·43	18·43	35·43	48·9
8	30·86	20·86	11·86	+1·14	12·14	19·14	31·14	48·7
9 S. P.	31·57	21·57	9·57	+0 43	8·43	19·43	34·43	48·6
9	29·57	19·07	8·57	+0·43	8·93	15·43	32·43	48·5
May 12 S. P.	—24 29·00	22·00	10·00	—1 00	8 00	16 19·00	35·00	48·0
Means.	—24 31·06	—16 21·06	— 8 9·64	+0·07	+ 8 9·36	+16 18·23	m. s. +24 34·11	49″·22
Or in Arc	6° 7′ 45″·9	4° 5′ 15″·90	2° 2′ 24″·60	+1·05	+2° 2′ 20″·40	+4° 4′ 33″·45	+6° 8′ 31″·65	
Log. Sines Constant	9·0284678 2·5571186	8·8529932 2·5571186	8·5514513 2·5571186	4·6855749 2·5571186	8·5512029 2·5571186	8·8517407 2·5571186	9·0293639 2·5571186	
Log. Equat. Interval	1·5855864	1·4101118	1·1085699	7·2426935	1·1083215	1·4088593	1·5864825	
	s.	s.	s.	s.	s.	s.	s.	
Equatorial Interval	+ 38·511	25·711	12·840	0·002	12·633	25·636	38·591	
In Arc	9′ 37″·67	6′ 25″·67	3′ 12″·60	+ 0″·03	3′ 12″·50	6′ 24″·54	9′ 38″·87	

Log. cos. 88° 29′ 49″·22 . . . . . 8·4187848
Ar. co. log. sin. 1″ . . . . . . 5·3144251
Ar. co. log. 15 . . . . 8·8239087

Sum = constant . . . . 2·5571186

Computation of Intervals from Middle Wire for Polaris for Declinations 88° 29′, 88° 30′, and 88° 31′.

	A	B	C	D	E	F	G
Log. sin. equatorial interval Ar. co. log. cos. 88° 29′	7·4472539 1·5772832	7·2717901 1·5772832	6·9702297 1·5772832		6·9700041 1·5772832	7·2705157 1·5772832	7·4481553 1·5772832
Log. sin. interval	9·0245371	8·8490733	8·5475129		8·5472873	8·8477989	9·0254385
Interval	6° 4′ 26·33″	4° 3′ 3·45″	2° 1′ 18·27″		2° 1′ 14·50″	4° 2′ 20·65″	6° 5′ 11·94″
In. time	24′ 17·76″	16′ 12·23″	8′ 5·22″	0·07″	8′ 4·97″	16′ 9·37″	24′ 20·80″
Log. sin. equatorial interval Ar. co. log. cos. 88° 30′	7·4472539 1·5820810	7·2717901 1·5820810	6·9702297 1·5820810		6·9700041 1·5820810	7·2705157 1·5820810	7·4481553 1·5820810
Log. sin. interval	9·0293349	8·8538711	8·5523107		8·5520851	8·8525967	9·0302363
Interval	6° 8′ 30·17″	4° 5′ 45·73″	2° 2′ 39·16″		2° 2′ 35·33″	4° 5′ 2·45″	6° 9′ 16·29″
In. time	24′ 34·01″	16′ 23·05″	8′ 10·61″	0·08″	8′ 10·36″	16′ 20·16″	24′ 37·09″
Log. sin. equatorial interval Ar. co. log. cos. 88° 31′	7·4472539 1·5869324	7·2717901 1·5869324	6·9702297 1·5869324		6·9700041 1·5869324	7·2705157 1·5869324	7·4481553 1·5869324
Log. sin. interval	9·0341863	8·8587225	8·5571621		8·5569365	8·8574481	9·0350877
Interval	6° 12′ 39·52″	4° 8′ 31·66″	2° 4′ 1·86″		2° 3′ 57·99″	4° 7′ 47·89″	6° 13′ 26·15″
In. time	24′ 50·63″	16′ 34·11″	8′ 16·12″		8′ 15·87″	16′ 31·19″	24′ 53·74″

Comparing the intervals for 88° 29′ and 88° 30′ we have—

+ 16·25*s.* | + 10·82*s.* | + 5·39*s.* | | − 5·39*s.* | − 10·79*s.* | − 16·29*s.*

And dividing these by 60, we have the change for one second of arc—

+ 0·271 | + 0·180 | + 0·090 | | − 0·090 | − 0·180 | − 0·271

Therefore for Polaris, declination = 88° 29′ + n''.

	m.	*s.*		*m.*	*s.*
A	+ 24	17·76 + $\overset{''}{n}$ × 0·271	E	− 8	4·97 − $\overset{''}{n}$ × 0·090
B	+ 16	12·23 + n × 0·180	F	− 16	9·37 − n × 0·180
C	+ 8	5·22 + n × 0·090	G	− 24	20·80 − n × 0·271
D	—	0·07			

Zenith Point of Transit Circle for 1849. (See page 1026).

Day and Hour.	Object.	Circle Reading. Direct.	Circle Reading. Reflection.	Sum of Seconds & Mean.	Half of Mean Sum.	Adoption of Zenith Point.
		North Paral.				
		° ′ ″	° ′ ″			
Oct. 14 8	α Cephei	349 22 15·10	190 20 11·35	26·45		
16 9	B. A. C 7851	325 58 47·99	213 43 40·48	28·47		
17 8	α Cephei	349 22 12·87	190 20 11·86	24·73		
17 8	β Cephei	341 25 18·15	198 17 8·48	26·63		
				4) 106 28		
		Nadir.		26·57	13·29	13·16
Oct. 12 23	Wire R.		179 51 14·08			
13 8	"		14·51			
13 22	"		14·09			
14 8	"		14·12			
14 22	"		13·18			
15 6	"		13·72			
15 22	"		14·70			
16 6	"		14·56			
16 22	"		14·32			
17 6	"		13·43			
17 22	"		13·76			
18 7	"		14·45			
			12) 168·92			
			179 51 14·08			16·32
		South Paral.				
Oct. 16 8	ι Pegasi	26 41 52·07	153 0 38·51	30·58		
17 12	α Trianguli	22 28 17·15	157 14 15·10	32·75		
				2) 63·33		
				31·67	15·84	11·68
						10) 41·16
						179 51 14·12

Or Z. P. correction . . . . . 8 45·88

The following examples will illustrate the whole method of completing imperfect transits:—

SPECIMEN PAGE OF THE COMPLETION OF IMPERFECT TRANSITS.

Day.	1849, July 20.	July 20.	July 20.	July 20.	July 20.
Object.	B. A. C. 5509.	*a* Herculis.	*β* Draconis.	*a* Ophiuchi.	B. A. C. 6125
N. P. D.	27° 57′	75° 26′	37° 35′	77° 20′	111° 27′
1st wire.		51·5	50·0		
2 ,,		9·6	10·8		
3 ,,		22·8	32·0	32·7	
4 ,,	21 39·5	36·4	52·7	46·0	
5 ,,	22 6·7			59·5	11·6
6 ,,	22 34·0			12·5	25·7
7 ,,	16 23 1·6	17 7	17 26	17 28 25·6	17 58 39·6
	4) 89 21·8	4) 65·3	4) 85·5	5) 29 6·3	3) 76·9
Mean of wires.	16 22 20·45	17 7 16·33	17 26 21·38	17 27 59·26	17 58 25·63
Correction to imperfect Transit.*	— 41·11	+ 19·91	+ 31·59	— 13·16	— 27·6
Concluded transit over mean of the seven wires.	16 21 39·34	17 7 36·24	17 26 52·97	17 27 46·10	17 57 58·03

* See the Form at page 1011.

COMPLETION OF IMPERFECT TRANSITS OF POLARIS.

Day.	1848, July 21.	July 22.	Sept. 5.	Sept. 21.	1848, Oct. 2.
Object.	Polaris.	Polaris, S. P.	Polaris, S. P.	Polaris, S. P.	Polaris, S. P.
1st wire.	40 26·0				40 29·0
2 ,,	48 34·0	48 40·0			48 47·0
3 ,,	56 46·5	56 47·0	56 24·0		57 0·0
4 ,,	4 54·0	57·0	4 35·0	5 19·0	5 10·0
5 ,,	4·0		12 40·0	13 28·0	13 21·0
6 ,,			52·0	21 40·0	
7 ,,	1 13	13 4	13 20	13 29 50·5	13
	5)28 3 44·5	3) 144·0	4) 151·0	4) 137·5	28 4 47·0
Mean of observed wires.	56 44·90	12 56 48·00	13 8 37·75	13 17 34·38	12 56 57·40
Reduction.*	+ 8 10·09	+ 8 8·85	— 4 5·64	— 12 17·14	+ 8 11·94
Concluded transit over mean of the seven wires.	1 4 54·99	13 4 56·85	13 4 32·11	13 5 17·24	13 5 9·34

* See the following Form.

FOR POLARIS AND δ URSÆ MINORIS.*

Day.	July 21.		July 22.		September 5.		September 21.		October 2.	
Object.	Polaris.		Polaris S. P.		Polaris S. P.		Polaris S. P.		Polaris S. P.	
For observation on A	+ 24 17·76	+ n × 271					+ 24 17·76	+ n × 271		
,, ,, B	+ 16 12·23	+ n × 181			+ 16 12·23	+ n × 181	+ 16 12·23	+ n × 181		
,, ,, C	+ 8 5·22	+ n × 090			+ 8 5·22	+ n × 090	+ 8 5·22	+ n × 090	+ 8 5·22	n × 090
,, ,, D	— 0·07		— 0·07		— 0·07		— 0·07		0·07	
	48 35·14	n × 542	—	— n ×	24 17·38	n × 271	48 35·14	n × 542	8 5·15	n × 090
,, ,, E	— 8 4·97	n × 090	— 8 4·97	— n × 0·090	— 8 4·97	+ n × 090	—	— n ×	— 8 4·97	— n × 090
	40 30·17	n × 452	— 16 9·37	— n × 180	16 12·41	n × 181	—	— n ×	0·18	— n × 000
,, ,,	—	— n ×	—	— n ×	—	— n ×	—	— n ×	— 16 9·37	— n × 180
	—	— n ×	—	— n ×	—	— n ×	—	— n ×	— 16 9·19	— n ×
,, ,, G	—	— n ×	—	— n ×	—	— n ×	—	— n ×	— 24 20·80	— n × 272
Sum	+ 40 30·17	n × 452	— 24 14·41	— n × 270	16 12·41	n × 181	48 35·14	n × 542	— 40 29·99	— n × 452
Value of n		9·44		59·44		51·65		7·16		7·56
Decl. = 88° 29' + n'		1808 181 41		1080 108 24 1		905 109 2 1		3252 54 38		2712 226 32
Product		20·30		— 12·13		10·17		+ 33·44		— 29·70
Add Sum independent of n		40 30·17		— 24 14·41		16 12·41		48 35·14		— 40 29·99
Sum		5) 40 50·47		3) — 24 26·54		4) + 16 22·58		4) 49 8·58		5) — 40 59·69
Mean		8 10·09		— 8 8·85		4 5·64		12 17·14		— 8 11·94

* See the preceding page.

ROYAL OBSERVATORY, GREENWICH.

Calculations for Completing Imperfect Transits in the Year 1848.

For Stars and Planets in general. (See, also, page 1009).

Day.	July 21.									
Object.	B.A.C. 5509.		α Herculis.		β Draconis.		α Ophiuchi.		B.A.C. 6125	
For observation on . . A			+ 38·511							
" B			+ 25·711							
" C			+ 12·840				+ 12·840			
" D	− 0·002		− 0·002				− 0·002			
							12·838			
" E	− 12·833						− 12·833		− 12·833	
							0·005			
" F	− 25·636						− 25·636		− 25·636	
							− 25·631			
" G	− 38·591						− 38·591		− 38·591	
Sun and mean .	− 77·062	− 19 266	77·060	19·265		19·265	− 64·222	− 12·844	− 77·060	
										− 25·687
Declination and nat. sec. .	62° 3′	553·312	14° 34′	23·301	52° 25′	69·361	12° 40′	74·201	21° 27′	44·701
		38 532		19 265		19 265		12 844		25 687
		1 927		578		11 559		257		1·798
		578		58		578		51		103
		58		4		173		9		10
		10				12				
		1								
Product . .		41·106		19·905		31·587		13·161		− 27·598

Another method of reducing imperfect transits is to form a table of intervals from the middle wire for different north polar distances, by multiplying the equatorial intervals by the secant of the declination (see preceding formulæ). When the wires observed of any object have been separately reduced to the middle wire by this table, the mean of all the results will be the correct number. For a planet, the change of right ascension in the interval, may be applied in the formation of the results for each wire.

The following is an example of Polaris by this method:
1848, October 2. Wires observed—A, B, C, D, E.
Declination of Polaris 88° 29′ 65″·7.

	h.	m.	s.		Reduction to Mean of Wires.	m. s.
A ...		40	29·0 ...	+ 24 17·76	− 24 35·56	= 5 4·56
				+ 17·80		
B ...		48	47·0 ...	+ 16 12·23	−	5 11·06
				+ 11·83		
C ...		57	0·0 ...	+ 8 5·22	−	5 11·13
				+ 5·91		
D ...		5	10·0 ...	− 0·07	−	5 9·93
E ...	13	13	21·0 ...	− 8 4.97	−	5 10·12
				5·91		
					5)	46·80
						13 5 9·36

The intervals of the transit wires may also be determined by directing the telescope of a theodolite to the object-glass of a transit instrument. In this manner the wires may be distinctly seen; but it is also necessary that the axes of the two instruments should be exactly in the same parallel. Gauss first introduced this method at Gottingen. The following is an example:—It is to be premised that the axis of the theodolite was higher than that of the centre of the circle, and therefore that the measured readings must be diminished by multiplying by the cosine of the inclination, or 23° 58′ 53″.

1823, October 23.

	Horizontal Distance. ′ ″	′ ″	True Distance. ′ ″	′ ″
Wire I.	11 25·050	11 23·000	10 25·93	10 24·06
" II.	7 45·900	7 46·275	7 5·69	7 6·03
" III.	3 58·225	3 57·750	3 37·66	3 37·23
" V.	3 58·850	3 59·375	3 38·24	3 83·71
" VI.	7 53·625	7 53·875	7 12·74	7 12·98
" VII.	11 17·600	11 17·775	10 19·11	10 19·28

At the Royal Observatory, Greenwich, the Astronomer Royal has made use of the south collimator for the determination of the wires of the transit circle. "The eye-piece of the transit circle was turned round 90°, so as to bring the wires into a sensibly horizontal position, and with the head of the micrometer upwards when the telescope was directed to the south. Each of the wires, in succession, was then brought upon a determinate point of the image of one of the nearly horizontal wires of the south collimator, partly by turning the instrument, and partly by the right ascension micrometer, and the instrument being clamped, the six microscopes of the zenith distance circle were read as well as the right ascension micrometer."

In this manner, the following concluded circle readings for each wire were obtained:—

	′ ″
Wire I.	59 33·696
" II.	56 6·746
" III.	52 39·290
" IV.	49 12·018
" V.	45 44·880
" VI.	42 17·242
" VII.	38 50·482

The mean of these numbers is 49′ 12″·051; and, comparing this with each number separately, we finally get for the distance in arc of each wire from the mean of all :—

	′ ″	s.
For Wire I.	+ 10 21·645	or + 41·443
,, II.	+ 6 54·659	+ 27·646
,, III.	+ 3 27·239	+ 13·816
,, IV.	− 0·033	− 0·002
,, V.	− 3 27·171	− 13 811
,, VI.	− 6 54·809	− 27·654
,, VII.	− 10 21·569	− 41·438

Dr. Drew has usefully applied a wire micrometer, attached to a five feet telescope, for the determination of the intervals of his transit circle—an instrument of forty-two inches focal distance. The value of one revolution of the micrometer being known, he was able to measure easily the distances of the wires, by directing the two object glasses towards each other. The result agreed closely with that deduced from a transit of δ Ursæ Minoris.

To find the correction to an imperfect transit of the moon or a planet :—

Let h'' be the apparent distance from the middle wire.
,, μ = the right ascension of the zenith.
,, α' = the apparent right ascension of the interval.
,, δ' = the apparent declination.

Then we have—

$$\text{Sin. } h = \cos.\ \delta' \sin.\ \overline{\mu-\alpha'} = \frac{d}{d'}\cos.\ \delta \sin.\ (\mu-\alpha),$$

Put $\sin.\ h = h \sin.\ 1''$, and $\sin.\ \overline{\mu-\alpha} = \overline{\mu-\alpha} \sin.\ 1''$. Then we shall have—

$$h = \frac{d \cos.\ \delta}{d'}(\mu-\alpha)\ .\ .\ .\ \mu-\alpha = \frac{h \times d'}{d \cos.\ \delta}\ .\ .\ .\ (36).$$

This is the expression for all those bodies which do not change their right ascension. The fixed stars are the only ones which do not, and their distances are so immense, that a' and d may be regarded as equal, which is also the case with the planets, on account of their distance. Let λ^{s} be the moon's or planet's increase of R. A. in one second of time, then $(1-\lambda)^{s}$ is the equatorial space described by the moon or planet in one second of sidereal time ; consequently, the sidereal time of describing $\mu-\alpha$, will be represented by—

$$\frac{1}{1-\lambda}\left(\frac{d'\,h}{d \cos.\ \delta}\right) = (1+\lambda)\frac{d'\,h}{d \cos.\ \delta}$$

$$= (1+\lambda) \times \frac{\sin.\ \text{Geo. Z. D.}}{\sin.\ \text{app. Z. D.}} \times \frac{h}{\cos.\ \delta'}, \text{ nearly } (37).$$

This is the formula given in the *Greenwich Observations* for the same purpose. In the case of the planets d' is very nearly equal to d, and the formula is—

$(1+\lambda) \times \frac{h}{\cos.\ \delta'}$, where λ is the increase of the planet's right ascension in one second of sidereal time.

COMPLETION OF AN IMPERFECT TRANSIT OF THE MOON.

Day.	1848, August 9.	
Object.	☽ 1 L.	
1st wire.		47·0
2 ,,		0·9
3 ,,		14·7
4 ,,		
5 ,,		
6 ,,		
7 ,,	17·4	
	3)	2·6
Observed mean of wires	17·4	0·87
Reduction		27·79
Concluded transit over mean of seven wires	17·4	28·66

1848, August 9.

	°	′
☽'s geo. N.P.D.	107	41
Co. lat. of Greenwich	38	31
☽'s geo. Z. D.	69	10
Parallax in altitude		49
☽'s app. Z.D.	69	59

I = 131″·62 = the increase (in seconds of time) of the Moon's right ascension for a transit over a meridian upon the earth, distant by 1h. of terrestrial longitude (*Nautical Almanac*, 1848, Section—Moon-Culminating Stars).

Log. $\frac{3600+\text{I}}{3600}$	0·01559
Log. sin. ☽'s geo. Z.D.	9·97064
Ar. co. log. sin. ☽'s app. Z.D.	0·02706
Log. sec. ☽'s geo. decl.	0·02102
	0·03431
Log. 25·687 s. (see below*)	1·40972
	1·44403
Completion of imperfect transit, to reduce to mean of seven wires	s. 27·79

1848, August 9th.

Wires observed.

		s.
For an Equatorial Star.	A	+ 38·511
	B	25·711
	C	12·840
		3) 77·062
		+ 25·687*

COMPLETION OF AN IMPERFECT TRANSIT OF A PLANET.

Day,	1848, August 9th.	
Object,	Mercury 2 L.	
1st wire.		27·2
2 "		40·2
3 "		54·2
4 "		8·1
5 "		
6 "		
7 "	8·6	
		4) 189·7
Mean of observed wires	8·3	47·42
Reduction . . .		+ 20·53
Concluded transit .	8·4	7·95
Object	Mercury 2 L.	

Planets, N.P.D. = 70° 22′

I = variation in R.A. in 1h. = + 12 92 s.

Wires observed :—

A	+ 38·511
B	+ 25·711
C	+ 12·840
D	— ·002
	4) + 77·060
	+ 19·265
Log. 19·265	+ 1·28477
Log. co. sec. planets, N.P.D.	0·02601
$\text{Log.}\ \frac{3600 + I}{3600} =$	0·01559
L. No. 20·53* s. = ...	1·31233

Formula for Determining Collimation.—In the operations of the Brussels Observatory, the following formula for the determination of the collimation, independently of the other errors of the instrument, is used. The investigation is to be found in M. Lingré's memoir, *Sur les Corrections de la Lunette Méridienne*—one of the prize essays of the Brussels Academy, vol. xviii., 1844—'45. The stars best suited for this method are, one near the pole, another near the zenith, and a third of south declination. The example and formula are as follows :—

Example.—Greenwich Observations, 1842, May 10th.

		h.	m.	s.		h.	m.	s.		°	′	″	
β Corvi . . .	H =	12	25	39·53	R A =	12	26	8·86	p =	112	31	39	Observer Henry.
γ Ursæ Majoris .	H° =	11	45	3·88	R A° =	11	45	33·11	p° =	55	22	39	
Polaris S.P. . .	H′ =	13	1	30·42	R A′ =	13	2	6·52	p′ =	1	31	59	

Calculation of the Collimation.

$$C = D^\circ \frac{\sin. p^\circ \cos. \frac{1}{2}(p - p')}{2 \sin. \frac{1}{2}(p - p^\circ) \sin. \frac{1}{2}(p^\circ - p')} + D' \frac{\sin. p' \cos. \frac{1}{2}(p - p^\circ)}{2 \sin. \frac{1}{2}(p - p') \sin. \frac{1}{2}(p' - p^\circ)}$$

$$D^\circ = 15\left[(H + R A^\circ) - (H^\circ + R A)\right] = -1''\cdot50.$$

$$D' = 15\left[(H + R A') - (H' + R A)\right] = +101''\cdot40.$$

$$\tfrac{1}{2}(p - p') = 57^\circ\ 1'\ 49'' : \tfrac{1}{2}(p - p^\circ) = 38^\circ\ 33'\ 0'' : \tfrac{1}{2}(p^\circ - p') = 18^\circ\ 28'\ 40''$$

Log. D°	−0·17609
Log. sin. p°	9·76317
Log. cos. ½ (p − p′)	9·73516
Ar. co. log. 2	9·69897
Ar. co. log. sin. ½ (p − p°)	0·20537
Ar. co. log. sin. ½ (p° − p′)	0·49897
	−0·07773
1st Number =	− 1″·20

Log. D′	2·00604
Log. sin. p′	− 8·42738
Log. cos. ½ (p − p°)	9·89324
Ar. co. log. 2	9·69897
Ar. co. log. sin. ½ (p − p′)	0·07626
Ar. co. log. sin. ½ (p′ − p°)	− 0·49897
	+ 0·60086
2nd Number .	+ 3·99
	− 1·20
C =	+ 2·79

A similar calculation, made on May 13th, 1842, using β Corvi, γ Ursæ Majoris, and Polaris, gave C = + 2″·67. This agrees very satisfactorily with the preceding result.

The error determined (by reversal of the instrument in the usual way practised at Greenwich) was found to be, on April 6th, = + 1″·51, which is a very good agreement.

The Astronomer Royal, in the *Cambridge Observations*, 1828, proposed a method, nearly similar, for determining the clock errors, and the errors of collimation and azimuth, but was obliged to discontinue it on account of the imperfect places of the star catalogues, and the difficulty of making observations sufficiently accurate. The following is the investigation :—" Let x, y, z, be the clock error, and the errors of collimation and deviation (the clock being supposed slow). O, O′, O″, the observed times of transit corrected for the error of level and the clock rate; T, T′, T″, the true sidereal times at which they pass the meridian; Y, Y′, Y″, the respective multipliers of the error of collimation, which give results additive to the observed transits; Z, Z′, Z″, the same for the error of meridian. Then the following equations subsist :—

$$O + x + Yy + Zz = T$$
$$O' + x + Y'y + Z'z = T'$$
$$O'' + x + Y''y + Z''z = T''$$

And subtracting the 2nd from the 1st, and the 3rd from the 2nd—

$$(Y - Y')\,y + (Z - Z')\,z = (T - O) - (T' - O')$$
$$(Y' - Y'')\,y + (Z' - Z'')\,z = (T' - O') - (T'' - O'')$$

" The catalogued R. A. and the observation of the transit must be extremely accurate, in order that any result worthy of confidence may be obtained from these equations. The most favourable stars are Polaris, a star below the pole, and a star above the pole, whose N. P. D. is considerable."

Method of Finding the Time.—The method of finding the time given in the following pages is capable of so many useful applications to travelling astronomers, that, in a treatise on this subject, it would be unwise to neglect it.

Professor Hansteen, in *Schumacher's Astronomische Nachrichten*, vol. vi., p. 189, gives a simple and expeditious means of determining the time by a small transit instrument, which requires only the observed transit of Polaris in a certain parallel, and the transit of a known fundamental star in the same vertical. The formulæ of reduction are as follow :—

$$A = \frac{\tan.\ \phi - \tan.\ \delta}{\tan.\ \delta - \tan.\ \delta' \cos.\ P}$$

$$B = A^2 \frac{\tan.\ \phi + 2 \tan.\ \delta'}{\tan.\ \delta - \tan.\ \delta' \cos.\ P}$$

$$P = 15\ (\alpha - \alpha' + T)$$

$$O = \frac{A \sin.\ P}{\sin.\ 15''} + \frac{B \sin.\ P^2}{\sin.\ 90}$$

In which δ and α are respectively the declination and right ascension of the pole star; δ' and α' are the same co-ordinates of another fundamental star; ϕ is the latitude of the place, and T is the sidereal interval of the observations.

Example.—At Christiana, on the 8th of November, 1823, the following transits of stars were observed with a universal instrument of Reichenbach :—

Star.		Eye-piece towards the East. h.	m.	s.
Polaris	τ . .	13	36	31·5
η Ursæ Majoris . . .	τ . .	13	42	49·6

Star.		Eye-piece towards the West. h.	m.	s.
Polaris	τ' . .	14	0	28·50
Polaris	τ' . .	14	10	47·46

The apparent places of the stars are—

		h.	m.	s.		°	′	″
Polaris . .	α =	24	58	32·90	δ =	88	22	26·68
η Ursæ Majoris .	α' =	13	40	34·47	δ' =	50	11	34·79
α Bootis . .	α'' =	14	7	37·38	δ'' =	20	6	8·66

Therefore we have—

	h.	m.	s.
$\alpha - \alpha'$ =	11	17	58·43
τ =	+	5	33·46
$\alpha - \alpha' + \tau$ =	11	23	31·89

$P = 170°\ 52'\ 58''$

	h.	m.	s.
$\alpha - \alpha''$ =	10	50	55·52
τ' =	+	10	18·96
$\alpha - \alpha'' + \tau'$ =	11	1	14·48

$P' = 165°\ 18'\ 35''$

The latitude ϕ is assumed to be 59° 54′ 8″·4.

Log. tan. δ' . .	0·0791592
Log. cos. P . .	9·9944783
	— 0·0736375
Log. (tan. ϕ — tan. δ') .	9·7204185
Log. N . . —	1·5612716
Log. A . . .	8·1591469
Log. sin. 15″ . .	4·1383339
Log. sin. P . . .	9·1999056
	1·4973864
Log. A^2 . . .	6·31829
Log. (tan. ϕ + 2 tan. δ')	0·61543
Ar. co log. N . .	8·43873
Ar. co. log. sin. 90″ .	3·36018
Log. sin. P^2 . . .	7·59971
	6·33234

tan. δ' . . =	35·22949
	1·18478
N . .	36·41427
Log. tan. ϕ . . =	1·7752525
Log. tan. δ' . — =	1·1999391
(Tan. ϕ — tan. δ') .	0·5753134
Tan. ϕ + 2 tan. δ' .	4·1251307
. . . .	s. + 31·433
. . . =	0·000
	31·433

In the same manner, if a calculation be made for α Bootis, we shall have—

$$\theta' = \overset{s.}{133\cdot205} + \overset{s.}{0\cdot004} = +\overset{m.}{2}\ \overset{s.}{13\cdot21}$$

From these values of the quantities θ and θ' are found the binary angles t and t' for these two stars; and the zenith distance of the pole star by this formula—

Cos. ξ = cos. ϕ cos. δ cos. $(P - t)$ + sin. ϕ sin. δ.

Therefore, by calculation—

	°	′		°	′
$P - t$ =	170	45	$P' - t'$ =	164	45
ξ =	31	43	ξ' =	31	40
ϵ =	0	23	ϵ' =	0	25
$\delta - \phi + Z$ =	—9	42	$\delta' - \phi' + Z'$ =	— 9	48
C =	0·13754		C′ =	0·15755	

The daily gain of the clock was 2s.·5; therefore for the time $\tau' - \tau$ or 28m., a was = 0s.·047. Therefore, if ϵ and $\epsilon' = 0$, C + C′ will be equal to 0·29509.

	m.	s.
$\tau' - \tau$ =	28	42·50
$- a$ =	—	0·05
$-(\alpha'' - \alpha)$ =	— 27	2·91
$-\ \theta' - \theta)$ =	— 1	41·78
S =		— 2·24

$$\frac{S}{C + C'} = \gamma = \frac{-\ 2\cdot24}{0\cdot29509} = \overset{s.}{7\cdot59}$$

By this value of γ itself we find,

for η Ursæ Majoris $C\gamma$ = − 1·044 s.

α Bootis $C'\gamma$ = − 1·196

Wherefore the state of the clock is as follows :—

η Ursæ Majoris.

	h.	m.	s.
α =	13	40	34·47
θ =		+	31·43
$- C\gamma$ =		+	1·04
Sidereal time =	13	41	6·94
Clock time =	13	42	4·96
Error of Clock		+	58·02

Arcturus.	$\alpha' =$	14h.	7′	37·38″
	$\theta' =$		2	13·21
	$+ C' \gamma =$			— 1·20
	Sidereal time =	14	9	49·39
	Clock time =	14	10	47·46
	Error of Clock		+	58·07

This method is likely to be of considerable value to the astronomer travelling from place to place; and for this purpose we give it here.

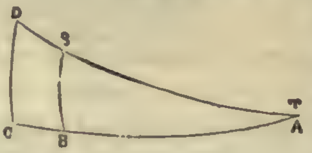
On the Determination of Right Ascension by the Transit Instrument.—The origin of right ascensions is the intersection of the ecliptic and equator at the vernal equinox. For the purpose of determining this origin, Dr. Maskelyne selected α Aquilæ, the place of which he arbitrarily assumed, and with which he compared thirty-five other principal stars, applying the clock error and rate; and, by these means, forming apparent right ascensions. These were afterwards converted into mean right ascensions, for the beginning of each year, by the application of precession, nutation, and aberration, from certain assumed constants. The places of all the others were, therefore, affected with the error of the assumed right ascension of α Aquilæ. To determine the amount of this error, he computed the sun's right ascension from the transit observations depending on the assumed place of α Aquilæ; and he also computed a right ascension of the sun, from observations of declination by the quadrant, by the formula,

Rad. × sin. R. A. = cotan. obliq. × tan. declination.

And comparing the two results for certain periods of time, on each side of the equinoxes, so as to eliminate any errors arising from refraction, parallax, obliquity of the ecliptic, and latitude of the place, he obtained a correction to his assumed place of α Aquilæ, which was common to the remaining stars of the catalogue.

When a number of stars have had their right ascensions determined by referring them to some fundamental star, they will all be charged with the error which may happen to belong to this star; and it is an object of the utmost importance to ascertain the existence and quantity of such error. The difficulty lies in the determining accurately the position of the first point of Aries, from which the right ascensions of all the stars are counted. The course pursued, therefore, by astronomers, is first to find the sun's right ascension, by comparing the transit of his centre with the transit of the fundamental star, or with the transits of several principal stars, related to it by known differences; and, secondly, to compute from his observed declination the right ascension belonging to the moment of the meridian passage. These operations should be performed on several days, near both the vernal and autumnal equinox. The right ascensions derived from a comparison with the stars, should agree with those derived from the observed declinations of the sun. If there be a constant difference, this will be the correction to be applied to the assumed right ascension of the fundamental star. The sun's right ascension is deduced from his declination in the following manner:—Let A C (Fig. 210) represent a part of the equator, A D a part of the ecliptic, and A be the first point of Aries. Suppose the sun to be at S, and draw S B perpendicular to A C; then A B will be the right ascension of the sun, and S B his declination. But, by Napier's rule, rad. × sin. A B = cotan. S A B × tan. S B; that is, sin. R. A. = cotan. obliquity × tan. dec.

Fig. 210.



Or, representing the sun's declination by δ, and the obliquity of the ecliptic by ω, we have

$$\text{sin. R. A.} = \frac{\text{tan. } \delta}{\text{tan. } \omega}$$

Example.—The following observations of the sun's centre were made at Greenwich in 1851 :—

Date.	Sun's R. A. Observed.	Sun's Dec. Observed.
	h. m. s.	° ′ ″
Sept. 15	11 34 16·15	2 47 0·18 N.
16	37 51·22	2 23 50·70 N.
21	55 48·55	0 27 15 35 N.
22	59 24·19	0 3 51·53 N.
23	12 3 0·32	0 19 33·76 S.

The following example, from the Appendix to the *Greenwich Observations*, 1851, will explain this process :—

Errors of the Assumed Position of the First Point of Aries, by Observations made near the Vernal and Autumnal Equinoxes of 1802. Assumed lat. of Observatory, 51° 28′ 38″·9. Assumed mean obliquity, 1802, Jan. 1, 23° 27′ 55″·58. Assumed mean R. A. of α Aquilæ, 1802, January 1, 19h. 41m. 7s. ·00.

Month and Day. 1802.	Sun's R. A. from observation of Transits.	Sun's R. A. computed from observed Declination.	Excess of computed R. A.	Month and Day. 1802.	Sun's R. A. from observation of Transits.	Sun's R. A. computed from observed Declination.	Excess of computed R. A.
Feb. 13	326° 27′ 48″·6	326° 28′ 17″·6	+ 29″·0	Oct. 30	214° 2′ 14″·9	214° 1′ 40″·7	— 34″·2
14	327 26 28·5	327 26 52·5	+ 24·0	28	212 5 53·7	212 5 29·5	— 24·2
26	338 56 40·4	338 57 7·2	+ 26·8	16	200 42 44·8	200 42 19·9	— 24·9
27	339 53 17·3	339 53 38·5	+ 21·2	15	199 46 52·5	199 46 30·2	— 22·3
Mar. 1	341 45 54·8	341 46 21·1	+ 26·3	14	198 51 5·5	198 50 45·0	— 20·5
7	347 20 51·8	347 21 18·5	+ 26·7	7	192 24 31·6	192 24 12·0	— 19·6
15	354 41 56·4	354 42 12·9	+ 16·5	Sept. 29	185 8 23·6	185 8 10·4	— 13·2
18	357 26 4·0	357 26 10·4	+ 6·4	26	182 25 52·5	182 55 40·3	— 12·2
20	359 15 18·0	359 15 33·5	+ 15·5	24	180 37 54·6	180 37 35·1	— 19·5
23	1 58 53·7	1 59 7·8	+ 14·1	31	177 55 57·8	177 55 43·9	— 13·9
25	3 47 46·4	3 47 57·5	+ 11·1	19	176 8 15·6	176 7 55·2	— 20·4
26	4 42 14·7	4 42 26·8	+ 12·1	18	175 14 20·3	175 14 11·8	— 8·5
27	5 36 41·3	5 36 50·7	+ 9·4	17	174 20 33·3	174 20 10·4	— 22·9
28	6 31 11·0	6 31 24·1	+ 13·1	16	173 26 38·7	173 26 22·6	— 16·1
29	7 25 40·2	7 25 57·5	+ 17·3	15	172 32 44·3	172 32 33·3	— 11·0
31	9 14 39·2	9 14 53·9	+ 14·7	13	170 45 1·6	170 44 41·6	— 20·0
Apr. 1	10 9 9·7	10 9 22·0	+ 12·3	12	169 51 7·5	169 50 51·9	— 15·6
4	12 52 57·6	12 53 11·5	+ 13·9	9	167 9 17·0	167 91 5·3	— 1·7
5	13 47 37·1	13 47 53·9	+ 16·8	8	166 15 12·8	166 14 56·4	— 16·4
10	18 21 36·9	18 21 46·3	+ 9·4	3	161 44 9·0	161 44 16·5	+ 7·5
12	20 11 37·5	20 11 58·3	+ 20·8	1	159 55 15·9	159 55 9·0	— 6·9
14	22 1 58·9	22 2 9·4	+ 10·5	Aug. 30	158 6 5·0	158 5 58·7	— 6·3
		Mean	+ 16·72			Mean	— 15·58

[The correction applicable to the right ascensions of all stars observed in 1802, is (+ 16·72 — 15·58) ÷ 2 = + 0·57 = + 0·04s.]

Errors of collimation may be determined by a micrometer attached to the eye-piece of the telescope, by which the angular distance of the object from the central wire is first measured. The instrument is then reversed, and the distance of the same object from the central wire again measured. The azimuthal direction may be found by comparing any two stars, differing in declination, whose places are known, Polaris being generally one of these stars. At the present time, the places of certain fixed stars are used for these purposes, and a correction of the assumed equinox is obtained from all the right ascensions and north polar distances of the sun. (See the annual volumes of the *Greenwich* and *Cambridge Observations*).

When the preceding errors of collimation, level, and azimuth, have been applied to the results of observations, the transit instrument can be usefully employed in the determination of accurate time and the investigation of apparent right ascensions. The *Nautical Almanac* contains the apparent places, for every tenth day in the year, of certain fixed stars called clock stars, which can be immediately compared with the corrected instrumental results. A series of clock errors, for certain epochs of time, will thus be obtained (by taking the differences of the foregoing numbers); which, being grouped on successive days, will give at once a *rate* of the clock. In adopting the *rate* for use, due weight must be allowed for the interval of time on which each *deduced rate* depends, and also on its relative accuracy with regard to the number and distribution of the clock stars.

Apparent Errors of the Transit Clock, and Adopted Clock Rates and Clock Errors at 0h. Sidereal, in the Year 1848.

Month, Day, and Hour.	Observer.	Apparent R. A.	True Transit and Mean of each Group.	Apparent Error, and Mean, and Reduction to 0h. by adopted Rate.	Mean Error reduced to Standard Observer.	Interval of consecutive Means of True Transits.	Difference of consecutive Means of Apparent Errors.	Clock's loss in 24h.	Adopted Losing Rate.
		h. m. s.	h. m. s.	s.	s.	s.	s.	s.	s.
March 14, 7	H.	6 38 28·04	6 38 23·65	4·39					0·34
7	”	7 11 3·66	7 10 59·10	4·56					
8	”	7 24 54·90	7 24 50·40	4·50					
8	”	7 31 21·74	7 31 17·36	4·38					
9	”	8 38 44·87	8 38 40·32	4·55					
5)39			37 21	22·38					
14·7			7 28	4·48 ·11	4.49				
		d. h. h. March 13 23 at 0 sidereal.		4·37		74.32	1·17	0·37	
March 17, 10	H.	10 0 17·94	10 0 12·29	5·66					
	”					22·21	0·59	0·64	
March 18, 7	R.	6 52 40·08	6 52 33·39	6·69					0·54
7	”	7 11 3·58	7 10 56·77	6·81					
8	”	7 24 54·83	7 24 48·20	6·63					
8	”	7 31 21·68	7 31 15·01	6·67					
8	”	7 36 1·68	7 35 55·03	6·65					
8	”	8 1 5·66	8 0 58·96	6·70					
11	”	11 6 2·77	11 5 56·16	6·61					
11	”	11 11 46·68	11 11 40·22	6·46					
8)68			66 48	53·22					
18·8			8 21	6·65 —·18	6.25				
		March 17d. 23h. at 0h. sidereal.		6·47		26·47	0·48	0·44	
March 19, 11	H.	11 6 2·77	11 5 55·87	6·90					0·47
11	”	11 11 46·68	11 11 40·15	6·53					
19·11			11 8	6·72 ·21	6.73				
		March 19d. 0h. at 0h. sidereal.		6·51					

The following forms will show the method of reducing transit observations at the Royal Observatory, Greenwich, as well as the formation of clock errors at 0h. sidereal and adopted rates:—

Example I.—Observations of Transits in the Year 1851 with the Transit Circle, and Computations of Right Ascension.

Approx. Sol. Time.	d. h. m. August 15 8 27	d. h. m. 15 10 26	d. h. m. 15 10 32	d. h. m. 15 10 49
Name of Object.	μ Sagittarii.	θ Aquilæ.	α² Capricorni.	ε Delphini.
Approx. N.P.D. of Object.		° ′ 91 16		° ′ 79 12
Observer.	H. B.			
Reading of Transit Micrometer.	r 31·500			
	h. m. s.	h. m. s.	h. m. s.	h. m. s.
Trans. over wire I.	26·2	15·2	24·4	43·1
" " II.	41·2	29·1	38·3	57·3
" " III.	55·9	42·8	52·3	11·4
" " IV.	10·7	56·7	6·4	25·3
" " V.	25·6	10·4	20·9	39·6
" " VI.	40 7	24·4	35·2	53·6
" " VII.	18 4 54·7	20 3 38·0	20 9 49·4	20 25 7 8
	7) 25·0	7) 36·6	7) 36·9	7) 38·1
Add	3·57 7·14	5·23 1·43	5·27 1·43	5·44
Observed Transit.........	18 4 10·71	20 3 56·66	20 9 6·70	20 25 25·44
Col. error — 0″·76 ×	+ ·071 = — ·05	+ ·067 = — ·05	+ ·068 = — ·05	+ ·068 = — ·05
Level error — 4·34 ×	+ ·021 = — ·09	+ ·040 = — ·17	+ ·029 = — ·13	+ ·052 = — ·23
Azim. error — 0·36 ×	+ ·068 = — ·03	+ ·053 = — ·02	+ ·062 = — ·02	+ ·044 = — ·02
True Transit over Meridian......................	18 4 10·54	20 2 56·42	20 9 6·50	20 25 25·14
Clock slow at 0h. Siderl. preceding	41·83	41·83	41·83	41·83
Adop. los. rate 1·14......	·86	·95	·96	·97
Ob. R. A. of Object	18 4 53·23	20 3 39·20	20 9 49·29	20 26 7·94
Star's Correc. with sign changed	— 2·09	— 2·24	— 2·33	— 2·25
Obser. Mean R. A. Jan. 1, for Stars	18 4 51·14	20 3 36·96	20 9 46·96	20 26 5·69

Example II.—Computation of Star Corrections applicable to Mean R. A. by the New Constants, in the year 1851.

Day.	August 15.							
Star's Name.	μ Sagittarii.		θ Aquilæ.		α² Capricorni.		ε Delphini.	
Star's R. A.								
Star's N. P. D.								
Reference for Star Constants.	1578 Gr. 12 yr. Cat.		1804 Gr. 12 yr. Cat.		1816 Gr. 12 yr. Cat.		1836 Gr. 12 yr. Cat.	
Log. *E*	1·60053		1·60053		1·60053		1·60053	
Log. *e*	0·07971		0·09141		0·09228		0·09355	
Sum and Number	1·68024	s. 47·890	1·69194	s. 49·197	1·69281	s. 49·296	1·69408	s. 49·440
Log. *F*	1·09921		1·09921		1·09921		1·09921	
Log. *f*	0·05253		0·05794		0·05774		0·05897	
Sum and Number.........	1·15174	14·182	1·15715	14·360	1·15695	14·353	1·15818	14·394
Log. *G*	0·18447		0·18447		0·18447		0·18447	
Log. *g*	1·45615		1·44865		1·45226		1·44507	
Sum and Number.........	1·64062	43·714	1·63312	42·966	1·63673	43·324	1·62954	42·613
Log. *H*	1·45769		1·45769		1·45769		1·45769	
Log. *h*	0·07899		0·07891		0·07618		0·08191	
Sum and Number.........	1·53668	34·410	1·53660	34·403	1·53387	34·188	1·53960	34·642
L		74·433		74·433		74·433		74·433
l		87·459		86·878		86·736		86·727
Sum of Numbers		302·088		302·237		302·330		302·249
Subtract Constant		300·000		300·000		300·000		300·000
Star Correc.=Remainder		2·088		2·237		2·330		2·249

Having obtained the error of the clock at any certain period, and also the rate, by a simple interpolation we can find its error at any other time. When this is applied to the observed transit (properly corrected), the apparent right ascension is at once obtained. This can be converted into the mean right ascension for the beginning of each year by the day numbers, A, B, C, D, given in the *Nautical Almanac*, combined with the star constants, *a*, *b*, *c*, *d*, of the *British Association Catalogue*. This method of determining the corrections for precession, nutation, and aberration, is so lucidly explained in the latter work, that we shall content ourselves by simply referring to it; but we may mention that, for the purpose of avoiding *negative* signs in their computations, the Astronomer Royal has ingeniously transformed these formulæ. (See the Introduction to the *Greenwich Twelve Year Catalogue*). The two previous and following examples worked out, will illustrate all the preceding formulæ.

SPECIMEN OF THE GREENWICH METHOD OF RECORDING TRANSIT OBSERVATIONS.

Transits observed with the Transit Circle, and Computations of Apparent Right Ascension, at the Royal Observatory, Greenwich, in the year 1851.

Month and Day.	No. for Reference.	Name of Object.	Observer.	Seconds of Transit over the Wire.							Concluded Transit over the Mean of the Wires.	Error of Collimation (Level) (Azimuth.)	Seconds of Transit Corrected.	Tabular R. A. of known Stars.	Clock apparently Slow.	Adopted Clock Slow at 0h. Sidereal and (Rate).	Correction for Semi-diameter.	Apparent R.A. of Centre from the Observation.	Correction to Mean R.A. 1851. Jan. 1.
				I.	II.	III.	IV.	V.	VI.	VII.									
				s.	s.	s.	s.	s.	s.	s.	h. m. s.	"	s.	s.	s.	s.	s.	h. m. s.	s.
June 17.	1	⊙ 1 L.	H.B.	49·2	4·2	19·5	34·7	49·7			5 39 34·56	−0·73	34·10			16·05	+ 68·95	5 40 59·17	
	2	Polaris S.P.		12·0	6·5	59·5	52·5				13 4 51·12	(−4·97)	58·90	14·84	15·94	(0·32)		13 5 15·12	+ 3·41
	3	Spica.		23·7	38·0	51·9	6·2	20·4	34·4	48·3	13 17 6·13	(−2·50)	5·77	22·01	16·24			13 17 22·00	— 1·06
	4	Arcturus.		53·2	7·9	22·7	37·5	52·2	6·6	21·4	14 8 37·36		36·92	53·12	16·20			14 8 53·16	— 1·13
	5	μ Bootis.		21·7	37·8	53·7	10·2	26·0	42·2	58·5	14 25 10·01		9·51					14 25 25·75	— 1·25
	6	ι Bootis.		27·5	43·0	58·7	14·4	29·7	45·6	1·2	14 38 14·30		13·81					14 38 30·06	— 1·10
	7	ξ 2 Bootis.			47·2	1·8	16·8	31·7	45·9	0·7	14 44 16·68		16·24					14 44 32·49	— 1·34
	8	β Ursæ Minoris.				5·7	57·9	51·2	44·5		14 50 58·52		57·40					14 51 13·65	— 2·84
	9	ψ Bootis.					49·2	5·2	20·9	36·2	14 57 49·51		49·03					14 58 5 28	— 1·40
	10	β Libræ.		3·5	17·4	31·5	45·4	59·5	13·3	27·3	15 8 45·41		45·04	1·29	16·25			15 9 1·29	— 1·58
	11	ζ Libræ.			8·8	23·2	37·7	51·7	6·5	20·8	15 19 37·59		37·25					15 19 53·50	— 1·70
	12	α Coronæ.		21·8	37·7	53·1	8·6	24·2	39·7	55·4	15 28 8·65		8·16	24·35	16·19			15 28 24·42	— 1·53
	13	B.A.C. 5188.				35·8	50·2	4·2	18·7	32·8	15 34 50·07		49·72					15 35 5·98	— 1·73
	14	Irene.			15·7	30·2	44·5	58·8	13·2	27·5	15 38 44·52		44·17					15 39 0·43	
	15	γ Serpentis.				5·8	20·3	34·5	49·2	3·6	15 49 20·29		19·85					15 49 36·11	— 1·60
	16	β′ Scorpii.			3·4	18·2	32·7	47·4	2·2	16·8	15 56 32·80		32·46	48·68	16·22			15 56 48·72	— 1·85
	17	Lalande 29, 490.		4·0	18·2	32·4	46·4	0·7	15·2	29·3	16 3 46·60		46·25					16 4 2·51	— 1·80
	18	ι Ophiuchi.			44·7	58·6	12·4	26·5	40·2	53·9	16 10 12·46		12·08					16 10 28·35	— 1·73
	19	γ Herculis.			37·	52·2	6·7	21·6	36·4	50·7	16 15 6·87		6·43					16 15 22·70	— 1·67
	20	Antares.			32·0	47·2	2·4	18·2	34·1	49·0	16 20 2·78		2·47	18·77	16·30			16 20 18·74	— 2·01
	21	ζ Ophiuchi.		1·2	15·5	29·4	43·5	57·6	11·7	25·8	16 28 43·53		43·17					16 28 59·44	— 1·80
	22	ζ Herculis.		37·5	53·6	9·8	26·2	42·5	58·8	15·2	16 35 26·23		25·73					16 35 42·00	— 1·80
	23	* N.P.D. 74° 29.		58·1	12·2	26·5	41·0		9·3	24·2	16 44 40·94		40·51					16 44 56·78	— 1·74
	24	ι Herculis.			49·4	5·2	21·2	37·7	53·7	10·0	16 54 21·46		20·96					16 54 37·24	— 1·84
	25	Vesta.		30·2	44·9	59·4	13·8	28·5	43·0	57·4	17 1 13·88		13·54					17 1 29·82	
	26	θ Ophiuchi.		51·7	7·6	22·7	37·7	53·2	8·4	23·6	17 12 37·85		37·53					17 12 53·81	— 2·06
	27	Juno.		56·5	10·0	24·4	38·2			19·7	17 18 38·07		37·69					17 18 53·97	

Illuminated End of Axis West. Order of Wires for Stars above the Pole, A, B, C, D, E, F, G.

Correction to Equatorial Transits :—For I. + 41·443s.; II. + 27·646s.; III. + 13·816s; IV. + 0·000s.; V. — 13·811s.; VI. — 27·654s.; VII. — 41·438s.
20. The image very confused and coloured.

The following examples will show the methods of determining azimuthal errors of the transit instrument at the Royal Observatory, Greenwich.

Approximate Solar Time. 1848.	Transits Corrected for Error of Collimation and Level Error, with the Corrections for Clock Rate and Change of R. A., and inferred Azimuthal Errors.	
d. h.		h. m. s.
April 28, 9	Transit of β Leonis corrected for two errors . . .	—. 11 40 45·45 + z × 0·041
	Tabular Right Ascension . .	11 41 19·74
	Clock Slow	34·29
	Rate [1·25 s.] for 1 h. 20 m. . . .	+·07
		34·36 — z × 0·041
		h. m. s.
April 28, 10½	Transit of Polaris S. P. . .	— 13 3 25·85 + z × 1·638
	Tabular Right Ascension . .	1 4 8·76
		42·91 — z × 1·638
	$\therefore Z = \frac{+8{\cdot}55}{1{\cdot}597} = +5'' 54$	
		h. m. s.
June 13, 8	Transit of Polaris S. P. —. .	13 4 5·46 + z × 1.638
20	Transit of Polaris	1 4 15·84 — z × 1·533
	Clock Rate [1·06 s.] for 12 h. . .	+ 0·53
	Subtract Increase of R. A. in 12h. . . .	— 0·40
		1 4 15·97 + z + 1·533
	$\therefore = + \frac{10{\cdot}51}{3{\cdot}171} = +3''{\cdot}314$	

The practical directions for using them, as given in the *Greenwich Observations*, are as follows (1848, p. xx.) :—

"If two consecutive transits of Polaris have been observed, the first transit (as corrected for error of collimation and level error) is altered by the change of the star's right ascension, and by the estimated clock error for 12h.; and the difference, in seconds of time, between the altered first transit and the second transit (rejecting 12h.) is divided by 3·171 to obtain the azimuthal error in seconds of space.

"If three transits have been observed, the difference between the first and second is taken (rejecting 12h.), and the difference between the second and third in like manner; and the mean between these is supposed to be independent of the change of right ascension and the clock's rate, and is then divided by 3·171.

"If several transits have been observed, the same process is used for every successive set of three, and the results are used separately, or the mean of the results is taken, according as there appears reason to think that the position of the instrument has or has not undergone change."

Example of the Determination of Azimuthal Error by Three Consecutive Transits of Polaris :—

Transit corrected for two errors.

			h. m. s.
1846.	Dec. 15.	Polaris . . .	1 4 49·39
			+ 14·09
	Dec. 15.	Polaris, S.P. .	13 4 35·30
			+ 11·78
	Dec. 16.	Polaris . . .	1 4 47·08
			25·87
			+ 12·94

$$\therefore Z = \frac{+12{\cdot}94}{3{\cdot}148} = +4''{\cdot}12.$$

On the Determination of Terrestrial Longitudes.—In this section we shall explain the principal methods used in the determination of terrestrial longitudes :—

Longitudes by transits of the moon and moon culminating stars.

The first introduction of this method appears to be entirely due to the German astronomers. About the year 1820, it was agreed on by Gauss, Bessel, Struve, Nicolai, and others, to observe transits of the moon, and certain prearranged stars lying in its parallel, for the purpose of determining longitudes. The celebrated Schumacher gave his zealous co-operation by the immediate publication of the results in the *Astronomisch Nachrichten*. In these kingdoms, the method was cultivated by Dr. Brinkley and the late Mr. Baily; the latter astronomer having contributed to its advancement by his memoir on the subject, in the *Transactions of the Royal Astronomical Society*, which led to the insertion of the requisite data in the pages of the *Nautical Almanac*.

The stars with which the moon is to be compared, are selected in the same parallel of declination, for the purpose of nullifying any small errors which may be left on the transit instrument; but, notwithstanding all the care which was taken by these refinements, there are still uncertainties produced by irradiation and a peculiar personal equation of the limbs of the moon, which tend to throw doubt on the results. The former quantity, or "irradiation," may be eliminated by the results of the moon's 1 L. and 2 L.; but the latter is a variable and unknown quantity, in some instances affecting the transits of the first and second limb in a different proportion. In the following pages we will show the method pursued by the Astronomer Royal in the longitudes of the North American boundary, and also those of MM. Rumker and Struve. The method of the Astronomer Royal is to be used in cases where there are no corresponding observations, and the results will consequently be affected with the tabular errors of the moon's right ascension in the *Nautical Almanac*, an approximation to which, however, may be made by observations on other days at any standard observatory. The methods of M. Rümker and Professor Struve are intended for corresponding observations; the former having been applied in the determination of the longitude of Port Stephens, New South Wales, and the latter in the longitudes of several places in Turkey, during the years 1828 to 1832 inclusive.

The first method of determining longitudes is by the absolute right ascensions of the moon's limb, and is only to be used when there are no corresponding observations of moon-culminating stars. It consists in making two assumptions of longitude, differing by any known quantity, and interpolating, for these times, the right ascensions of the moon's limb from the numbers given in the section of "Moon-Culminating Stars," in the *Nautical Almanac*. These tabular right ascensions are given for the time of transit of the upper and lower meridian at Greenwich. By comparing the interpolated data with the result of the meridian observation at the place, the exact difference of longitude may be found by a simple proportion. The true difference of longitude being supposed to lie between two assumptions 1′ different, the following will be the proportion:—

As the difference of the two interpolated R. A.'s : 60 sec. : : the difference between the R. A. of observation, and the computed R. A. : the correction of the first assumption.

The data from the *Nautical Almanac* are as follows:—

Moon's R. A. on preceding day.
R. A. at lower passage, preceding day.
R. A. of corresponding day.
Moon's R. A. at next lower passage.
Moon's R. A. at following day.

The following method of interpolation may be used, the fourth differences being considered constant:—

Suppose $a_h = a_0 + bh + ch^2 + dh^3 + eh^4$. In this, for h write $-2, -1, 0, +1, +2$, and take the differences. This gives—

	1st differences.	2nd differences.	3rd differences.	4th differences.
$a_{-2} = a_0 - 2b + 4c - 8d + 16e$	$+b - 3c + 7d - 15e$			
$a_{-1} = a_0 - b + c - d + e$		$2c - 6d + 14e$		
$a_0 = a_0$	$-b - c + d - e$	$2c + 2e$	$6d - 12e$	
$a_1 = a_0 + b + c - d + e$	$+b + c + d + e$			$24e$
$a_2 = a_0 + 2b + 4c + 8d + 16e$	$b + 3c + 7d + 15e$	$2c + 6d + 14e$	$6d + 12e$	$= \Delta''''$

Now take the differences of the terms a_{-2}, a_{-1}, a_0, a_1, a^2, thus—

$$\left.\begin{array}{lllll}
a_{-2} & & & & \\
 & +\Delta'_1 & & & \\
a_{-1} & & +\Delta''_{-1} & & \\
 & +\Delta'_0 & & +\Delta'''_{-1} & \\
a_{-0} & & +\Delta''_0 & & +\Delta'''' \\
 & +\Delta'_1 & & +\Delta'''_0 & \\
a_{-1} & & +\Delta''_1 & & \\
 & +\Delta'_2 & & & \\
a_{-2} & & & &
\end{array}\right\}$$

Comparing this with the foregoing difference, we shall have

$$24e = \Delta'''' \therefore e = \frac{\Delta''''}{24}$$

$$6d + 12e + 6d - 12e = \Delta'''_{-1} + \Delta'''_0 \therefore d = \frac{\Delta'''_{-1} + \Delta''_0}{12}$$

$$2c + 2e = \Delta''_0 \therefore c = \frac{\Delta''_0}{2}$$

$$b + c + d + e + b - c + d - e = \Delta_0' + \Delta'_1 \therefore b = \frac{\Delta'_0 + \Delta'_1}{2} - d.$$

$$\therefore a_h = a_0 + \left(\frac{\Delta'_0 + \Delta'_1}{2} - d\right)h^2 + \left(\frac{\Delta''_0}{2} - e\right)h^2 + \left(\frac{\Delta'''_{-1} + \Delta'''_0}{12}\right)h^3 + \frac{\Delta''''}{24}h^4$$

The interval between any two consecutive terms of the given quantities, as a_{-2}, a_{-1}, is supposed to be unity, and therefore h must be a fractional part of this unit. Thus, if this interval be 12h., and the time for interpolation 3h. 20m., then $h = \frac{3h.\ 20m.}{12h.}$. If the interval between the consecutive quantities be 24h., then we should have $h = \frac{3h.\ 20m.}{24h.}$; and so on for other intervals.

The following example will illustrate the whole process:—

Longitude of Port Essington, north-east coast of Australia, deduced from lunar transits observed there in 1839 by Captain Owen Stanley, R.N.

The following meridian right ascensions of the moon's 1 L. at Port Essington are found by the usual means, applying carefully the error and rate of the chronometer:—

	R.A. h.	R.A. m.	R.A. s.
1839, June 20, ☽ 1 L. . .	12	50	14·38
,, June 22, ☽ 1 L. . .	14	20	57·01
,, June 24, ☽ 1 L. . .	16	2	59·99
,, June 25, ☽ 1 L. . .	16	59	11·11
&c.		&c.	

By assuming two longitudes, 8h. 49m. east, and 8h. 48m. east, and interpolating, with fourth differences, the right ascension of the moon's limb, from the data for upper and lower transit over the Greenwich meridian, we find for longitude 8h. 49m. east, right ascension of the moon's 1 L., 12h. 50m. 14s.·64; and for longitude 8h. 48m. east, right ascension of the moon's 1 L., 12h. 50m. 16s.·46. The correction to the tabular right ascension of the moon's 1 L. is — 0s.·27, from an observation made at the Royal Observatory, Edinburgh.

The observed right ascension of the moon's 1 L. is

12h. 50m. 14s.·38; therefore, by a proportion, the longitude from this observation is 8h. 48m. 59s.·67 east.

Proceeding in a similar manner, we find for the observation in 1839, June 25, for assumptions of longitude 8h. 49m. east and 8h. 48m. east, the right ascensions of the moon's 1 L. 16h. 59m. 9s.·95 and 16h. 59m. 12s.·36 respectively (a correction of + 0s.·09 being applied from the Hamburg meridian observation).

The result of the meridian observation at Port Essington is 16h. 59m. 11s.·11; therefore the resulting longitude is 8h. 48m. 31s.·12 east.

The second method will include the use of moon-culminating stars observed at other stations. The simplest way is to form differences at each observatory between the transits of the moon's limb and each moon-culminating star (properly corrected for the rate of the clock in the interval). Comparing these differences with the respective differences at other stations, the increase of the right ascension of the moon's limb will be obtained, which will, however, differ from the increase in the right ascension of the moon's centre, in consequence of the change of the moon's geocentric semi-diameter in the interval of the two passages. This correction may be computed as follows:—If the geocentric radius of the moon culminating in A be called r, and the true declination d; and for a culmination at another station, B, these quantities are called ρ and δ respectively, the correction will be

$$\pm \frac{1}{15}\left(\frac{r}{\cos.\ d} - \frac{\rho}{\cos.\ \delta}\right)$$

the true difference of the right ascension of the centre for both times of observation. The sign — applies to the western limb, and the sign + to the eastern limb.

But, if the apparent right ascensions of all the objects observed are given at the stations, the more direct way will be as follows (Struve's method):—

Example.—1846, March 10. The following corresponding observations of the moon 1 L. and moon culminators were observed at Greenwich, Oxford, Hamburg, and Georgetown (U.S.) Observatories. The differences of longitude of Greenwich, Oxford, and Hamburg are supposed to be accurately known; and it is proposed to determine the longitude of the Georgetown Observatory.

The longitude of Oxford is . . 5m. 2s.·6 W.
" " Hamburg . . 39m. 54s.·1 E.
" " Georgetown. 5h. 8m. 20s. ± West.

1846, March 10. Apparent Right Ascensions Observed:—

	Greenwich.	Oxford.	Hamburg.	Georgetown.
	h. m. s.	h. m. s.	h. m. s.	h. m. s.
ξ Leonis	9 23 40·73	9 23 40·94	9 23 40·773	
ο Leonis	9 32 57·99	9 32 58·08	9 32 57·996	9 32 57·995
☽ 1 L.	9 44 4·21	9 44 14·22	9 42 44·850	9 54 13·876
Regulus	10 0 12·39	10 0 12·42	10 0 12·327	10 0 12·380
ρ Leonis	10 24 44·47	10 24 44·46	10 24 44·358	

Then, comparing each star with the Greenwich Observation, we have the following corrections:—

Name of Star.	Oxford.	Hamburg.	Georgetown.
	s.	s.	s.
ξ Leonis	— 0·21	— 0·043	
ο Leonis	— 0·09	— 0·006	— 0·005
Regulus	— 0·03	+ 0·063	+ 0·010
ρ Leonis	+ 0·01	+ 0·112	
Mean	— 0·08	+ 0·032	+ 0·003

When the above corrections are applied to the right ascension of the moon's limb, we shall be enabled to give its place independent of any errors, either in the assumed equinox, or of any small residual instrumental correction. Thus, the following sidereal times of observation are formed:—

	Greenwich.	Oxford.	Hamburg.	Georgetown.
	h. m. s.	h. m. s.	h. m. s.	h. m. s.
Sidereal time of observation	9 44 4·210	9 44 14·140	9 42 44·882	9 54 13·879
Longitude . . .	0 0·000	+ 5 2·600	— 39 54·100	+ 5 8 20·000
Greenwich sidereal time .	9 44 4·210	9 49 16·740	9 2 50·782	15 2 33·879
Greenwich mean solar time	10 31 13·45	10 37 26·94	9 50 6·77	15 48 50·95

Interpolating for these times, from the *Nautical Almanac*, we find the following right ascensions, declinations, and semi-diameters of the moon:—

	☽'s R. A. of Centre.	☽'s Semid.	☽'s Geo. Decl.
	h. m. s.	″ ′	° ′ ″
For the Greenwich Meridian Passage	9 45 3·86	14 48 06	+ 8 22 53
" Oxford "	9 45 13·84	14 48·06	+ 8 21 48
" Hamburg "	9 43 44·87	14 47·95	+ 8 29 10
" Georgetown "	9 55 13·54	14 48·93	+ 7 33 40

Computing the moon's geocentric radius, by the formula $\frac{\text{☽'s semid.}}{\text{15 cos. decl.}}$, we have for the four observations :—

	Observed R. A. of ☽'s 1 L.	$\frac{\text{☽'s Semid.}}{\text{15 cos. Decl.}}$	Observed R. A. of Centre.	Seconds of N. A.	Corrections of N. A.	Means.
	h. m. s.	s.	h. m. s.	s.	s.	
Greenwich .	9 44 4·210	59·843	9 45 4·053	3·86	+0·193	D. A.
Oxford . .	9 44 14·140	59·843	9 45 13·983	13·84	+0.143	s. +0·067
Hamburg .	9 42 44·882	59·852	9 43 44·734	44·87	−0·136	
Georgetown .	9 54 13·879	59·782	9 55 13·661	13·54	+0·121	D. A.

The longitudes of the first three observatories are considered well known, and, by taking their means, we obtain + 0s.·067. The correction from the assumed longitude of Georgetown is + 0s.·121; therefore the difference, + 0s.·54, is to be converted into an error of longitude.

The motion of the moon in R. A. at the time of Georgetown transit, by the formula $d\,L = \frac{d\,A' - d\,A''}{\mu}$, where d is + 115s.·08 in 1m., or + 0s.·03197 in 1 second of sidereal time — hence the correction of the assumed longitude is $\frac{+\ 0s.\cdot054}{0s.\cdot03197}$ = 1s.·69, or the true longitude from this observation is 5h. 8m. 21s.·69 west.

The following additional example of the determination of longitude, by observation of the moon and moon-culminating stars, is from Struve's *Geographical Investigations in Turkey*.

1831, May 22, the following transits of the moon and moon-culminating stars were observed :—

APPARENT RIGHT ASCENSIONS.

Object.	Schurscha.	Dorpat.	Cracow.	Greenwich.
	h. m. s.	h. m. s.	h. m. s.	h. m. s.
θ Virginis .	13 1 14·16	13 1 13·83	13 1 13·90	
☽ L. . . .	13 46 60	13 40 93	14 34·05	13 17 10·37
ι^3 Virginis .	26 45 64	26 45·80	26 45·75	26 45·77
174 Virginis	35 8·90	35 8 61	35 8·87	35 8·81

Taking the mean of the apparent right ascension at Dorpat and Cracow, we obtain—

h. m. s.
θ Virginis . . 13 1 13·86
ι^3 . . . . . 26 45·77
174 . . . . . 35 8·74

The following table of reductions is formed :—

Object.	Schurscha.	Dorpat.	Cracow	Greenwich.
	s.	s.	s.	s.
θ Virginis .	− 0 30	+ 0·03	− 0 04	
ι^3 Virginis .	+ 0 13	− 0·03	+ 0 02	− 0 00
174 Virginis	− 0 16	+ 0 13	− 0 13	− 0·07
Means . .	− 0·41	+ 0·04	− 0·05	− 0·03

By applying which to the observed right ascension of the moon's 1 L., the right ascensions of the moon's 1 L. are obtained, viz.—

	h. m. s.
At Schurscha . . .	13 13 46·49
„ Dorpat . . .	13 13 40·97
„ Cracow . . .	13 14 34·00
„ Greenwich . . .	13 17 10·34

Proceeding, then, with an assumption of longitude of Schurscha = 4, we obtain a correction of the ephemeris $d\,A'$, which includes both the error of the ephemeris, and also the motion of the moon in R. A. produced by an error in the assumed longitude. The error of the ephemeris having been determined by the comparison of the observations at Dorpat, Cracow, and Greenwich, the residual error at Schurscha will be entirely due to the erroneous assumption of the longitude of the latter place. Thus, if $d\,A$ be the correction of the ephemeris determined at the three standard observatories; and, if the observed right ascension of the moon at Schurscha be a', and that from the ephemeris A', so that $a' - A' = d\,A'$, then the correction of the assumed west longitude—

$$d\,L = \frac{d\,A' - d\,A}{\mu}.$$

The longitude of the place deduced from the lunar culmination will be then $L = L + d\,L$.

μ is the increase in R. A. (in arc) for the time of observation at the required place.

Assuming L′ = − 1h. 34m. 0s.·0 from Paris, and computing the value of the geocentric radius of the moon, by the preceding method the following table is formed :

Place.	Mean Time at Berlin.	Observed R. A. of ☽'s 1 L.	r sec. δ.	R. A. of Moon's Centre. Observed.	R. A. of Moon's Centre. From the Berlin Ephemeris.	Corrections.
	h. m. s	° ′ ″	′ ″	° ′ ″	° ′ ″	″
Schurscha .	8 25 17·39	198 26 37·35	+ 14 54·49	198 41 31·84	198 41 53·57	d A = − 21·73
Dorpat .	8 21 38 48	198 25 14 55	14 54·50	198 40 9·05	198 40 9·46	d A = − 9·41
Cracow .	8 49 29·93	198 38 30·00	14 54·46	198 53 24·46	198 53 24·47	d A = − 0·01
Greenwich	10 11 45·25	199 17 35·10	+ 14 54·34	199 32 29·44	198 32 31·68	d A = − 2·24

The three observatories give an error of the ephemeris very accordant; the mean is $d\,A = -0''\cdot88$, with $\mu = 0''\cdot4744$.

$$d\,L = \frac{-21\,73 + 0\,88}{0\,4744} = \frac{-20''\cdot85}{0\,4744} = -44\cdot0 \text{ sec.}$$

or the longitude of Schurscha = − 1h. 34m. 44s.·0 from Paris.

The comparison with the individual observatories gives—

With Dorpat $d\,L = \frac{-21''\cdot32}{0\cdot4744} = -44^{s}\cdot9$ lon. = 1h. 34m. 44s.·9.

Cracow $= \frac{-21\cdot72}{0\cdot4744} = -45\cdot9$ . . = 45·9.

Greenwich $= \frac{-19\cdot49}{0\cdot4744} = -41\cdot1$ . . = 41·1.

The Mean is . . = − 1 34 44·0.

M. Struve then proceeds to investigate the differences of determinations of longitude by the moon's 1 L., and moon's 2 L., and finds it necessary, in order to reconcile the observations made at twelve stations, to increase determinations of longitude by the moon's 1 L. 7s.·2, and to diminish those of the moon's 2 L. by the like quantity.

The following additional examples of the determination of longitudes by moon culminators, are extracted from Rümker's *Langen Bestimmung durch den Mond*, Hamburg, 1849.

The longitude of Hamburg, in three examples, being supposed to be well known, let p be the sidereal time of the earlier, and π that of the later culmination; t the mean time of the earlier culmination, and τ that of the later, deduced from the known longitude. Then for these times, the moon's right ascensions, declinations, and semi-diameters (viz., a and α, d and δ, r and ρ respectively), are to be interpolated from the ephemeris. There will thus be obtained from the ephemeris, the difference of the moon's right ascension $\alpha - a$, and there will be deduced from the observations the difference $\pi - p \pm (\rho \sec. \delta - r \sec. d) = A$, the *upper* sign applying to the *first*, and the *lower* to the second limb of the moon. Hence there will be this proportion—

$$\alpha - a : A = \tau - t : \lambda - L ;$$

consequently $(A - (\alpha - a)) \dfrac{(\tau - t)}{\alpha - a} = c$, the correction of the assumed longitude. The westerly place in these examples being the first, the sign of c is to be changed.

The observations at Port Stephens were made by Captain King, R.N., and have been published in the monthly notices of the Royal Astronomical Society; those at Hamburg, by M. Rümker, are given in the twenty-second volume of the *Astronomische Nachrichten.*

1843, April 12, Hamburg, East.

	h.	m.	s.
Moon's 1 L. . . .	11	43	39·797
η Virginis . . .	12	11	55·612

April 13, Port Stephens, West.

	h.	m.	s.
η Virginis . . .	12	12	21·38
Moon's 1 L. . . .	12	19	12·16

Then, reducing the sidereal clock-times of the star's passage to one standard, as Hamburg, we have the right ascension of the moon's 1 L. at Port Stephens, 12h. 18m. 46s.·392.

	h.	m.	s.
Moon's 1 L. p . . .	11	43	39·797
Sid. time at mean noon .	− 1	20	2·455
	10	23	37·342
Acceleration . . .		− 1	42·165
Mean Hamburg time .	10	21	55·177
Long L.	−	39	54·000
Mean Greenwich time t. . April 12.	9	42	1·777

	h.	m.	s.
Moon's 1 L. π . . .	12	18	46·392
Sid. time at mean noon .	− 1	22	25·662
	10	56	20·730
Acceleration . . .		− 1	47·426
Mean time at Port Stephen	10	54	33·204
λ	−10	8	7·000
Mean Greenwich time τ . April 13.	0	46	26·204

The right ascensions of the moon's limbs, accurately interpolated from the hourly ephemeris in the *Nautical Almanac*, are—

	h.	m.	s.	
	−11	44	47·1369	$= \; - a$
And	12	19	54·1846	$= \alpha$
		35	7·0477	$= \alpha - a = 2107''·0477.$

The moon's semi-diameters accurately interpolated are, respectively, $r = 16' \; 39''·414$, $\rho = 16' \; 41''·8745$. The declinations are respectively $d = 3° \; 47' \; 5''$, $\delta = 7° \; 37' \; 34''·5$.

Log. $r = 999·414 = 2·9997454$

Log. sec. $d = 3° \; 47' \; 5'' = 0·0009482$

3·0006936

r sec. $d = 1001''·5985$

$\rho = 1001·8742$ log. 3·0008133

$\delta = 7° \; 37' \; 34''·5$ log. sec 0·0038585

3·0046718

ρ sec. $\delta = 1010''·815$

r sec. $d = 1001·5985''$

ρ sec. $\delta = 1010 \, 8150''$

$$\frac{\rho \sec. \delta - r \sec. d}{15} = \frac{9·2165}{15} = +\overset{s.}{0} \, 6144$$

	h.	m.	s.	
$\pi =$	12	18	46·392	
$p =$	11	43	39 797	
$\pi - p =$		35	6 595	= 2106·5950

$A = 2107·2094$

$\alpha - a = 2107·0477$ Ar. co. log. = 6·676326

$A - (\alpha - a) = + 0·1617$ log. = 9·208791

$\tau - t = \overset{h.}{15} \; \overset{m.}{4} \; \overset{s.}{25}·027 = 54265·027$ log. = 4·734520

log. = 0·619637

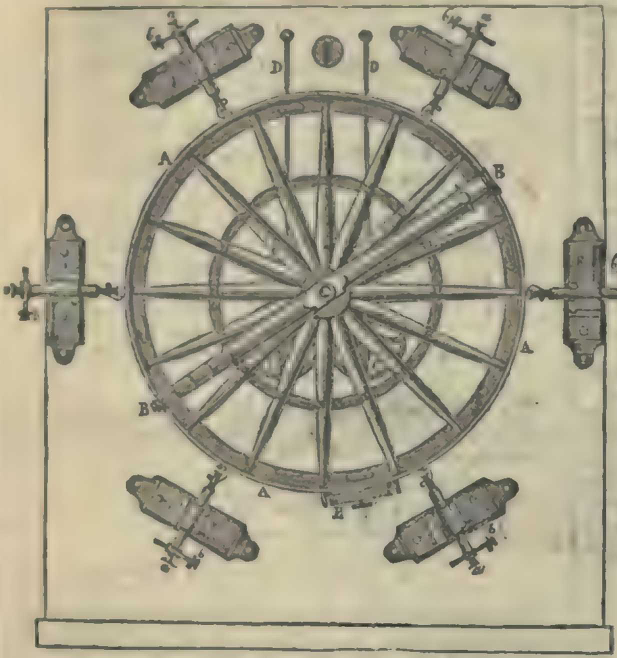
Correction = $- \overset{s.}{4}·165$

$\lambda = 10 \; 8 \; 7·000$

Sine longitude = 10 8 2·835

Meridian or Mural Circle.—The north polar distance of an object is determined by means of the meridian or mural circle. Previous to the introduction of the circle, the astronomical quadrant was used for this purpose; but it was discovered, about the beginning of the present century, that its results had been in a great measure vitiated by the expansion of the instrument, and consequent change of figure. The late Astronomer Royal (Mr. Pond), by aid of a circle only two feet in diameter, was enabled to detect and point out the errors of the Greenwich quadrant by a direct comparison of the results of the two instruments. Since that time, circular instruments have been very generally employed. The improved method of graduation, first introduced by Troughton, in 1806, and the previous invention of the micrometer-microscope by Ramsden, have allowed artists to give to this instrument the greatest degree of perfection. The first large instrument of this class appears to have been the Greenwich mural circle, which was completed by Troughton in 1811, and which, in several details, may almost be considered as a new instrument.

Fig. 211.



Some years previously, the celebrated Ramsden made a circle of five feet in diameter, with which Piazzi framed his great catalogue. The idea, however, is an old one, as Roëmer, although better known as the inventor of the transit instrument, used a meridian circle for several years, similar in construction to the modern form. In his letter to Leibnitz, December 15th, 1700, he mentions that he preferred an entire circle of four feet in diameter to a quadrant of eight feet radius. Of his observations with the meridian circle, extending from December, 1704, to December, 1710, only three days remain, the others having been destroyed by the fire which occurred at Hafnia, October 2nd, 1728. These three days' observations, from October 20th to October 23rd, 1706, have been carefully reduced by Dr. Galle, and contained in his *Inaugural Dissertation*, printed at Berlin, in 1845. The diameter of this meridian circle and the length of the axis was five feet and a quarter; the length of the telescope fixed to the axis was five feet. The circle was "read off" by two microscopes placed at an interval of 10°.

The mural circle consists of the graduated circle (Fig. 211) fixed in the meridian, and supported by a long axis, on which it revolves. This axis passes through a block of stone, or solid masonry, the weight of the instrument being taken off the pivots by means of counterpoises placed at the back of the pier. The circle is generally graduated on its rim from 0° to 360°, and also into smaller subdivisions of 5′ of arc.

The instrument is furnished with a telescope, B B, firmly fixed to the axis, and perpendicular to its plane, with which it revolves. The divisions are read by micrometer-microscopes, F, generally six in number, placed at equal distances around its circumference, their object being to eliminate, by readings of opposite diameters, errors arising from eccentricity, flexure of the axis, and expansion. The size of the instrument, at the principal observatories, varies from four to eight feet in diameter. The Greenwich circles were six feet; that at Cambridge is eight feet in diameter. The number of microscopes made use of at Oxford and Edinburgh is four; at Greenwich and Cambridge six are constantly *read off*. It was formerly thought necessary to have two instruments at Greenwich for determining a star's place—namely, a transit instrument and Troughton's mural circle; but a larger transit circle has been constructed for the Observatory, under the direction of the present Astronomer Royal, which has been in use since 1851. The telescope of this instrument has an aperture of eight inches, and a focal length of eleven feet and a-half; a length of axis between the extremities of the pivots of six feet, the diameter of each pivot being six inches. The circle is six feet in diameter, and of cast iron.

The axis of the circle is made horizontal by the aid of a plumb-line suspended in front of the circle, and viewed by two microscopes—one near the top, the other near the bottom of the circle; or, as this instrument is supposed to be used in conjunction with a transit instrument, the axis may be made horizontal by moving the adjusting screw, so as to make a zenith star pass the middle wire at the instant the star is passing the middle wire of the transit instrument.

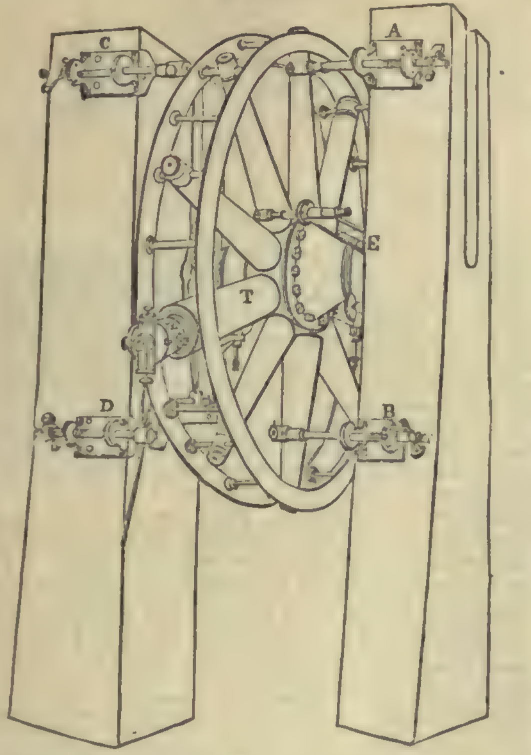
The adjustments of the meridional position of the mural circle are made by the observations of certain stars in conjunction with the transit instrument. The reading microscopes should accurately describe 300″, or five revolutions between each graduated space on the limb of the circle. This can be approximated to by proper adjusting-screws, which regulate the distance of the microscope from the limb; but, notwithstanding the accuracy with which the adjustment may be made, it is found that unequal temperature will alter these numbers considerably. The error arising from this circumstance is called the "correction for runs." It is the practice in observatories to determine the amount of this quantity frequently—which applies to a microscope reading of 5′—and to form corrections proportional for other circle readings. The following is an example (*Cambridge Observations*, May 3rd, 1834):—

	A	B	C	D	E	F	Sums of Excesses.
Negative side .	23·5	18·7	19·2	24·7	19·0	27·3	
Positive . .	25·1	19·4	20·3	26·5	20·3	26·1	
	+1·6	+0·7	+1·1	+1·8	+1·3	−1·2	+5″·3

The telescope is furnished with a system of vertical wires similar to the transit instrument, and a fixed horizontal wire placed at right angles to the others. This wire is generally adjusted so that an equatorial star will continue bisected during its transit; but it is sometimes left with a small inclination, which can be easily observed by comparison with the movable horizontal wire. Several stars being bisected at the first and seventh vertical wires, the effect of inclination can be readily determined for the whole, by comparison with the micrometer of the movable horizontal wire. To determine the value of one revolution of this micrometer, any distinct terrestrial object may be bisected at one position of the micrometer, and the circle carefully "read off." The same mark is again to be bisected at another position of the micrometer, and the circle is again to be "read off." When the two circle readings have been properly reduced, and the runs of the microscope having been applied, their difference will be a value of circle reading, in minutes and seconds, equal to a certain number of revolutions of the micrometer. This operation being repeated at different micrometer readings, a current value of one revolution of the telescope micrometer can be obtained. The more distant and distinct the terrestrial object selected is, the better.

A very convenient transit circle has been erected for the Observatory at Cambridge, Massachusetts, of which the accompanying engraving is a representation. With this instrument one observer can, at the same time, determine the right ascension and declination of a star with great precision. The telescope T (Fig. 212) has an object-glass of four and one-eighth inches of an aperture, and of four feet focal length. The length of the axes between the shoulders of the pivot is ninety-six inches; the pivots are of steel, and two-and-a-half inches in diameter, and the same in length. The eye-piece is provided with two micrometers, one having a vertical, and the other a horizontal movement. Besides the usual mode of illuminating the field through the axis, there are also facilities for illuminating the circus in a dark field. The circles are four feet in diameter, and cast in one piece, and both circles are graduated on silver from 0° to 360°, divided into five minute spaces. There are eight micrometer-reading microscopes attached to the granite piers, being four for each circle. Four of these are seen at A B C D, the other four being on the opposite side of the pier. These micrometers bisect diametrically both piers. The arm E, attached to the pier, supports an additional microscope, which serves as a pointer to indicate approximately the degrees and minutes. For levelling the axis a striding level is employed; and this, combined with a method of reflecting from quicksilver at the nadir point, affords means of ascertaining the amount of collimation of the middle wire without reversal of the pivots. There is, however, an apparatus for reversing the instrument when required. The object-glass of this circle is by Merz of Munich; the fitting by Simms of London.

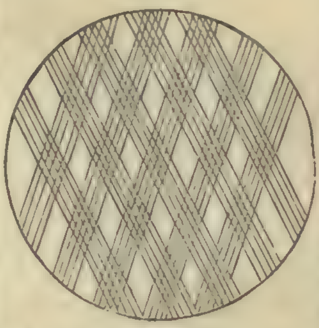
Fig. 212.



The mode of observation with the mural circle is as follows:—The telescope having been set approximately to the culminating star, the observer bisects the object by the movable horizontal wire as it passes the meridional vertical fibre. He then "reads off" the points and the six microscopes, as well as the telescope micrometer, by a combination of which a concluded circle reading may be obtained. It may be necessary to mention that the instrument is furnished with a clamp and a slow motion screw, by which the horizontal wire may be brought on the star, after the telescope has been approximately directed to it. If, by inadvertence, or other causes, the object is not observed at the meridian wire, it will involve a "correction of curvature," which is thus investigated.

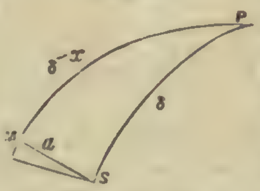
For the purpose of recording the position of every star within range by means of electro-magnetism, the telescope is firmly clamped to remain in its position, while the observer, sitting with his eye at the telescope, has but to press his finger upon a key at the instant a star is seen to pass the wires, which, for this purpose, are divided into two systems (Fig. 213). Those for right ascension are thirty-five in number, divided into groups of five each, the intervals between the wires being from two to three seconds. The wires for difference of declination are also thirty-five, and arranged in similar groups. In order to prevent confusion between observations for right ascension and declination, the rule is, to observe for right ascension on one set of wires first, and denote the magnitude of the star by telegraphic symbol; and afterwards to observe for declination on the inclined wires.

Fig. 213.



When an object is observed by the mural circle off the meridian, the north polar distance found from the circle reading is the N. P. distance of a point of the meridian, which is intersected by a great circle passing through the place of the object at the time of observation. If the object moved in a great circle, this would be its north polar distance when on the meridian; but as it moves in a parallel to the equator, a correction is required, which, as it is evident, will be positive for stars north of the equator, and negative for those south of it. It is thus computed:—

Fig. 214.



Let P be the pole (Fig. 214); S the place of the object; δ = its north polar distance; a = its distance from the meridian; $\delta - x$ = the observed N. P. D.; then we have

$$\cos.\ \delta = \cos.\ a \cos.\ (\delta - x) = (1 - 2 \sin.^2 \tfrac{1}{2} a)\ (\cos.\ \delta \cos.\ x + \sin.\ \delta \sin.\ x)$$

$$\cos.\ \delta = (1 - 2 \sin^2 \tfrac{1}{2} a)\ (\cos.\ \delta \times \sin.\ \delta \times x \sin\ 1'')$$
$$= \cos.\ \delta + \sin.\ \delta \times x \sin.\ 1'' - 2 \sin.^2 \tfrac{1}{2} a \cos.\ \delta$$
$$- 2 \sin.\ \delta \sin.^2 \tfrac{1}{2} a \times x \sin.\ 1''$$

If a be small, the last term will vanish, and by making $\sin.\ \tfrac{1}{2} a = \tfrac{1}{2} a \sin.\ 1''$, we have $2 \sin.^2 \tfrac{1}{2} a = \tfrac{1}{2} a^2 \sin.^2 1''$

$$\therefore \sin.\ \delta \times x \sin.\ 1'' = \cos.\ \delta \times \tfrac{1}{2} a^2 \sin.^2 1''.$$

$$x = \cos.\ \delta,\ \frac{\sin.\ 1''}{2}\ a^2.$$

The distance between the wires is supposed to be 20s. = one interval; and if n be the number of intervals, $a = n \times 20$;

$$\therefore a^2 = n^2 \times 400, \text{ consequently } x = \cot.\ \delta \frac{\sin.\ 1'' \times 400}{2} n.$$

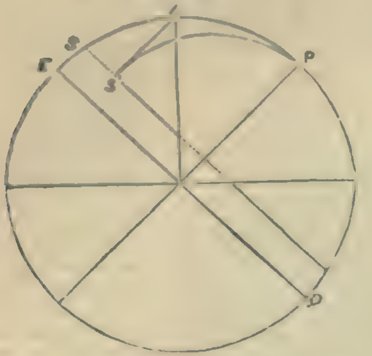
$$x = \tan.\ \text{declination} \times (\sin.\ 1'' \times 200)\ n^2.$$

In the case of a star near the pole, the following is the investigation for curvature of path (Fig. 215):—

Let Z S′ = Z = the observed zenith distance ; Z S — x = zenith distance on meridian; Z P S = p' the difference between the star's R. A. and the sidereal time of observation; d = star's declination; L = the latitude of the place of observation. Then in the triangle Z S′ P, we have

$$\cos. Z = \sin. L \sin. d + \cos. L \cos. d \cos. p$$
$$= \sin. L \sin. d + \cos. L \cos. d \left(1 - \frac{p^2}{2}\right)$$

Fig. 215.



But $\cos. Z - x = \sin. L \sin. d + \cos. L \cos. d$;

or since x is small,

$$\cos. Z + \sin. Z \times x = \sin. L \sin. d + \cos. L \cos. d;$$

therefore $x \sin. Z = \cos. L \cos. d \times \frac{p^2}{2}$

$$\sin. Z \times x \sin. 1'' = \cos. L \cos. d \times \frac{(15\,p)^2}{2} \sin.^2 1''$$

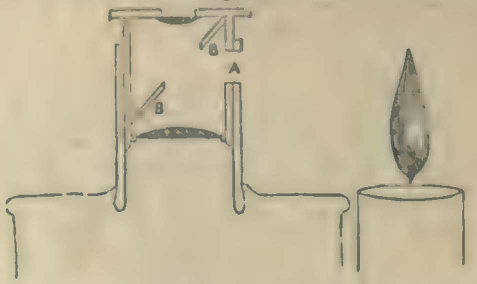
$$\therefore x'' = \frac{\cos. L \cos. d}{\sin. Z} p^2 \times \text{number whose log.} = 6{\cdot}73673$$

$$\text{or } \log. x'' = \log. \cos. L + \log. \cos. d - \log. \sin. Z + 6{\cdot}73673 + 2 \log. p$$
$$= \log. \cos. L + \log. \cos. d + \text{ar. co. } \log. \sin. Z + 6{\cdot}73673 + 2 \log. p.$$

For the purpose of obtaining a zero point of the mural circle, we make use of two different methods, of which the first consists in the observations of stars by reflection from a surface of mercury. The mode of operating is as follows:—Point the telescope to any known star when it crosses the meridian, and record the reading of the circle; on the next night observe the same star as it crosses the meridian, by pointing the telescope upon the image of the star reflected from the surface of the mercury. As the surface of a fluid at rest must be horizontal, and as the angle of reflection is equal to the angle of incidence, this image will be just as much depressed below the horizon as the star itself is above it. The arc intercepted on the limb of the circle, between the star and its reflected image, is the double altitude of the star, and its middle point is the horizontal point of the circle, allowing for the difference of refraction at the moment of observation. It is evident that the image of a star seen by reflection will be as much depressed below the horizon, as the object is really above it; and by combining the two readings of the circle for the same star, observed by reflection and direct vision, the horizontal point of the instrument may be readily determined. This method of obtaining a reflection and direct observation of a star, at the same transit, is daily practised at our principal observatories.

The second method is by the use of a Bohnenberger's eye-piece, previously explained. By causing the two images of the wire of the declination micrometer to coincide, and noting its readings, the *nadir*, and consequently the *zenith*, points of the circle may be determined by directing the telescope vertically downward upon a basin of mercury, the reflected image of the horizontal wire being made to coincide with its direct image. The telescope is directed towards the *nadir*, which is distant 90° from the horizontal point, or 180° from the zenith point. As this observation can be made at any time, independently of the weather, it is a most valuable method, and in many observatories is the one exclusively employed. The horizontal point determined by direct and reflected observations, should differ exactly 90° from the zenith point, as determined by the collimating eye-piece. This eye-piece, first suggested by Bohnenberger, consists of an ordinary eye-piece with an aperture A (Fig. 216), cut in its side, and a plane perforated speculum B B inserted between the two lenses, at an angle of 45°, with the optical axis, having a lamp held so as to throw a strong light on the speculum. The reflected images of the wires may be seen with great distinctness. Instead of a perforated opaque speculum, a piece of plane glass with parallel faces, and without any perforation, is sometimes used. The observer looks through the plane glass without difficulty, while sufficient light is reflected from the lower surface to render the lines visible. If the axis of the telescope be not horizontal, half the distance between the middle wire and its image, corrected for error of level, will give the error of collimation of the middle wire.

Fig. 216.



The tables of refraction, made use of at the Royal Observatory, Greenwich, are those of Bessel, modified and expanded from the *Tabulæ Regiomontanæ*. A numerical example of their application is given further on.

Example of a zenith point determination from the Greenwich Observatory, 1849. (See, also, page 1008).

Day. 1849.	Name of Star.	Concluded Circle Reading. (Reflection.)	Concluded Circle Reading. (Direct.)	R ÷ D 367° 5′	° ′ 130 32
		North Paral.			
		° ′ ″	° ′ ″		
April 5	B.A.C. ·3528	305 23 2·49	61 42 38·68	41 17	
	B.A.C. ·3593	295 33 2·60	68 32 33·53	39 13	
	δ Ursæ Majoris	279 56 24 11	87 9 17·96	42·07	$\frac{R+D}{2}$
				3) 122·37	
				40·79	50″·40
		South Paral.			
April 3	Pollux	250 27 39·07	116 37 59·10	38·17	
3	26 Lyncis	270 1 15 66	97 4 24·29	39·95	
5	Regulus	234 46 57·72	132 18 41·52	39·24	
5	9 Cancri in Va.	263 46 41·08	103 19 0·48	41·56	
				4) 158 92	
				39·73	49 87

183° 32″
Horizontal Point = Mean 50″·14
or Zenith Point = 93° 32′ 50″·14

The adopted Zenith Point is 93° 32′ 49″·89.

The printed form given as follows, will show fully the method of reducing observations of zenith distance. It is arranged for the reduction of the zenith distance observations with the transit circle, which differs from the above specimens, principally in the valuation of the micrometer-microscopes. The manner in which the revolutions and parts of revolutions of the microscope-micrometer are converted into arc, are explained as follows in the *Greenwich Observations*, 1852, page iv.:—

"I must premise that, as one revolution of each microscope-micrometer does not differ extravagantly from 1′ on the limb of the circle, we may consider each revolution as a nominal minute. Next, supposing the number of integral revolutions, as shown by each microscope, to be the same for all, we ought, in order to obtain the mean of the fractions of a revolution, to add together the subdivisions as shown by the different micrometer-heads, and divide the sum by 6. Thirdly, as the subdivisions are in the decimal scale, we should then multiply this mean by 60 to reduce it to seconds. It is evident thus that the number of nominal seconds to be attached to the nominal minute will be found by simply adding together the subdivisions on the micrometer-heads, and shifting the decimal point.

"The following were found by trial to be the *runs* of the six microscopes :—

"NUMBER OF REVOLUTIONS CORRESPONDING TO 5′ FOR MICROSCOPES.

	A	B	C	D	E	F
Means . . .	r 4·912	r 4·882	r 4·882	r 4·879	r 4·886	r 4·878

"The results given above show that the screws of the micrometers are so sensibly equal that, in the reduction of the observations, it is sufficient to take the mean of the six readings, and to apply to it the mean correction for runs. To do this, the following process is employed :— The sum of the runs for the six microscopes above given is 29r·320. Now, if the correction by which each revolution of a micrometer may be converted into a minute of arc were exactly $\frac{1}{50}$ part of the reading, then the sum of the runs would be $\frac{50}{51} + 30^{r}{\cdot}000$, or $29^{r}{\cdot}4118$. It is not

ROYAL OBSERVATORY, GREENWICH.

Observations of Zenith Distance with the Transit Circle in the Year 1851, and Computations of Geocentric North Polar Distance. (See, also, page 1008).

Approximate Solar Time.	Oct. 24.	24.	24.	24.	24.
Name of Object.	Wire (Nadir obs.)	ζ Eridani.	o Tauri.	ι Eridani.	Pallas.
Mode of Observation.	R.				
No. of Vertical Wire at which observation is made	4.	4.	4.	4.	4.
Observer.	H. B.				
Pointer.	179° 40′	60° 25′	42° 35′	61° 0′	74° 30′
Micrometer reading of Microscope A	r. 0·652	r. 5·102	r. 2·410	r. 4·196	r. 1·462
B	·637	5·046	2·385	4·137	·445
C	37 ·712	302 5·154	141 2·443	247 4·222	85 ·539
D	·414	4·822	2·146	3·940	·234
E	·497	4·872	2.221	3·973	·270
F	·775	5·168	2·498	4·252	·575
Uncorrected Mean of Microscopes . .	179 40 36·87	60 30 1·65	42 37 21·03	61 4 7·20	74 31 25·25
First part of Correction for Runs	0·74	6·04	2·82	4·94	1·70
Second Part, +0″·381 for 100″	·14	1·15	·53	·94	·32
Micrometer reading, and equivalents . . .	21·428 | 10 22·15 12·44 ·24	20·923 | 9 52·52 27·26 ·09	22·845 | 10 51·77 27·88 ·15	23·425 | 11 21·40 12·44 ·15	24·730 | 11 51·02 21·63
Correction for flexure and errors of division . .	0·86	1·49	1·94	1·52	2·03
Circle reading at observation	179 51 13·44	60 40 30·20	42 48 43·12	61 15 48·59	74 43 41·95
Add zenith point correction	8 46·25	8 46·25	8 46·25	8 46·25	8 46·25
Apparent zenith distance south		60 49 16·45	42 57 29·37	61 24 34·84	74 57 28·20
Add refraction (from below)		1 45·37	54·94	1 47·96	3 36·54
True zenith distance south		60 51 1·82	42 58 24·31	61 26 22·80	75 1 4·74
Subtract parallax (see page 1028)					4·97
Geocentric Distance from astronomical zenith .					75 0 59·77
Add colatitude . . .		38 31 21·80	38 31 21·80	38 31 21·80	38 31 21·80
Observed north polar distance		99 22 23·62	81 29 46·11	99 57 44·60	113 32 21·57
Star's correction with sign changed . . .		11·79	9·95	11·38	
Mean N.P.D. Jan. 1, for stars		99 22 35·41	81 29 56·06	99 57 55·98	
From Appendix No. 1 to Greenwich Observations, 1836. Barometer and B (Table I.)		in. 30 30 | ·03720	in. ·03720	in. 30 29 | ·03706	·03706
Thermometer and T. (Table II.)		48°·5 | ·04257	·04257	48°·3 | ·04275	47°·8 | ·04319
Z. (Table III.)		1·94005	1·65823	1·95186	2·25086
Proportional parts . (Table IV.)		·00264 ·00007 ·00020	·00178 ·00012	·00119 ·00018 ·00021	·00341 ·00023 ·00080
Sum or log. Refraction in seconds . . .		2·02273	1·73990	2·03325	2·33555
Refraction . . .		105″·37	54″·94	107″·96	216″·54
Interior Thermometer.		53·0			52°·0

likely that the sum of the runs will, in ordinary cases, amount to this quantity; and the variable correction, after adding $\frac{1}{50}$ part to the mean of the readings of the microscopes, will therefore always be additive. To determine this quantity, let the sum of the runs be $29'\cdot412 - x$. Then the true reading, in seconds of space, corresponding to a nominal reading of r'' (including the value in seconds of the nominal minutes), will be

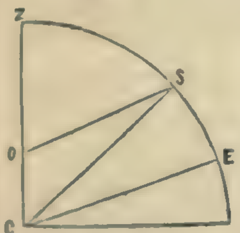
$$r'' \times \frac{30\cdot000}{29\cdot412 - x}$$

Or $$r'' \left\{ 1 + \frac{1}{50} + 0\cdot03468 \times x \right\}$$

Hence the correction, after adding $\frac{1}{50}$ part of the mean of readings, will be $+ r x \times 0''\cdot03468$. For the purpose of calculating this quantity easily, by the ordinary proportioned scale, it is convenient, in the first place, to compute its value, where $r = 100''$. This value, which is evidently $+ 3''\cdot468 \times x$, is tabulated for different values of x. Let X be the number taken from this table; then, for any actual reading r, the correction will be $\frac{r\text{X}}{100}$; a quantity which can easily be taken from the ordinary sliding rule.

PARALLAX.—The change which takes place in the position of a heavenly body on account of its having been observed from a point which is not the centre of motion, is called its parallax. All the heavenly bodies appear to move in the concave surface of a sphere concentric with the earth; hence the centre of the earth is considered to be the centre of motion. We shall now inquire into the change which would take place in the position of a body which has been observed at the surface of the earth, and its position as seen from the centre. Or, in other words, to find the correction which should be applied to an observation made at the earth's surface to reduce it to what it would be if made at the centre. Let C (Fig. 217) be the centre of the earth, O the place of observation on its surface, Z the geocentric zenith, S the object observed; then Z O S is the observed zenith distance, Z C S the zenith distance as seen from the centre, which is called the true zenith distance, because astronomers reduce all their observations to this point. The difference of these angles is the parallax; it is, therefore, the angle subtended by the earth's semi-diameter at the object observed.

Fig. 217.



Put Z C S the true zenith distance = Z.
Z O S the apparent zenith distance = Z′.
C O the earth's semi-diameter = r.
C S the distance of the object from the earth's centre = D.
The parallax O S C = p.

Then $$Z + p = Z', \text{ or } Z = Z' - p.$$

This shows that the observed zenith distance is greater than the true zenith distance by the quantity p. Hence the true zenith distance is found by subtracting the parallax from the observed zenith distance.

It is evident that parallax exerts its influence in a vertical plane passing through the object; consequently it has no effect on the right ascension of an object when observed on the meridian; but if the observation be made on either side of the meridian, the right ascension will be affected by it, as it raises the object to an hour circle nearer to the meridian. Hence, if the object lay east of the meridian, the apparent right ascension is diminished; but, if west, it is increased by parallax. Refraction has a contrary effect.

Now we have $\frac{\sin. Z'}{\sin. p} = \frac{D}{r}$ $\therefore \sin. p = \frac{r}{D} \sin. Z' = \frac{r}{D} \sin. (Z + p) \quad . \quad . \quad . \quad (1)$

The equation $\sin. p = \frac{r}{D} \sin. Z'$, gives $p = 0$ when $Z' = 0$, and $\sin. p = \frac{r}{D}$ when $Z' = 90°$. As this is the greatest value of Z', $\frac{r}{D}$ is the greatest value of $\sin. p$.

Therefore, in the geocentric zenith the parallax is nothing, whereas in the horizon it is the greatest possible.

Put $\frac{r}{D} = \sin. \pi$, then $\sin. p = \sin. \pi \sin. Z'$. This equation may be put under the form

$$p \left(\frac{\sin. p}{p} \right) = \pi \left(\frac{\sin. \pi}{\pi} \right) \sin. Z$$

$$\therefore p = \pi \left(\frac{\sin. \pi}{\pi} \right) \left(\frac{p}{\sin. p} \right) \sin. Z' \quad . \quad . \quad . \quad . \quad (2)$$

The horizontal parallax π, or rather the horizontal equatorial parallax (for the moon), given in the *Nautical Almanac*, is the angle subtended by the earth's equatorial semi-diameter.

Let r' be the semi-diameter of the place of observation, then $r' \sin. \pi$ will be the sine of the horizontal parallax at that place, and we shall have

$$p = r' \pi \left(\frac{\sin. \pi}{\pi} \right) \left(\frac{p}{\sin. p} \right) \sin. Z' \quad . \quad . \quad . \quad . \quad (3)$$

This is the formula given in the Introduction to the *Greenwich Observations* for computing the moon's parallax when observed on the meridian.

The equation $\sin. p'' = r' \sin. \pi \sin. Z$ may be written

$$p'' \sin. 1'' = r' \pi'' \sin. 1'' \sin. Z'$$

Or, $$p'' = r' \pi'' \sin. Z' \quad . \quad . \quad . \quad (4)$$

This will be sufficiently exact for the planets. In the

Example of the Reduction of an Observation with the Mural Circle. (See page 1027).

	1848, Dec. 21.			
	Uranus.			
	°	′	″	
Mic.	118	0		
A		3	37·7	
B			44·8	
C			65·8	
D			39·9	
E			42·3	
F			27·9	
Correction for Runs − 9″3			258·4	
for an arc of 5′			—0·2	
			258·2	
	118	3	43·03	Concluded Circle Reading.
Adopted Zenith Point ...	73	12	52·03	
Apparent Zenith Distance	44	50	51·00	
Add Refraction		1	0·73	
	44	51	51·73	
Subtract Parallax ...			0·31	
	44	51	51·42	
Add Colatitude	38	31	21·80	
Geo. N.P.D. of Centre...	83	23	13·22	

case of the moon, a correction will be necessary, which is found thus—

$$\sin. p'' = p'' \sin. 1'' - \frac{p''^3 \sin.^3 1''}{6} \text{ nearly ; hence}$$

$$p'' \sin. 1'' - \frac{p''^3 \sin.^3 1''}{6} = \left(\pi'' \sin. 1'' - \frac{\pi''^3 \sin.^3 1''}{6}\right) \sin. Z'$$

but $p''^3 \sin.^{1''} = \pi''^3 \sin.^3 1'' \sin.^3 Z'$, by neglecting powers higher than the third ; hence, by substitution,

$$p'' \sin. 1'' = \pi'' \sin. 1'' \sin. Z' - \frac{\pi''^3 \sin.^3 1''}{6} (\sin. Z' - \sin.^3 Z')$$

$$= \pi'' \sin. 1'' \sin. Z' - \frac{\pi''^3 \sin.^3 1''}{6} \sin. Z' \cos.^2 Z'$$

Consequently the correction is

$$-\frac{\pi''^3 \sin.^3 1''}{6} \sin. Z' \cos.^2 Z' \quad . \quad . \quad . \; (5)$$

This correction is applied in the Greenwich Lunar Reductions from 1750 to 1830.

Again, since $\sin. \pi = \frac{r}{D}$, for any other distance D′, we shall have $\sin. \pi' = \frac{r}{D}$; therefore,

$$\sin. \pi : \sin. \pi' \; \frac{1}{D} : \frac{1}{D}, \text{ or } \frac{\pi'}{D} = \frac{\pi}{D}.$$

If D′ be expressed in parts of D taken as unit, we have $\pi = \frac{\pi}{D}$; therefore, if π be the sun's horizontal parallax at the mean distance from the earth, and D′ the distance of any planet from the centre of the earth, expressed in terms of the sun's mean distance, the planet's horizontal parallax will be represented by $\pi \times \frac{1}{D'}$, and for a planet we shall have

$$p = \pi \times \frac{1}{D} \sin. Z' \quad . \quad . \quad . \quad . \; (6)$$

It is by this method the parallax of the planets has been computed in the Greenwich Planetary Reductions from 1750 to 1830.

Equation (1) gives the means of finding the parallax when the true zenith distance is given ; thus

$$\sin. p = \sin. \pi \sin. (Z + p) = \sin. \pi (\sin. Z \cos. p + \cos. Z \sin. p)$$

$$\tan. p = \sin. \pi \sin. Z + \tan. p \cos. Z \sin. \pi$$

$$\tan. p = \frac{\sin. \pi \sin. Z}{1 - \sin. \pi \cos. Z} \quad . \quad . \quad . \quad . \; (7)$$

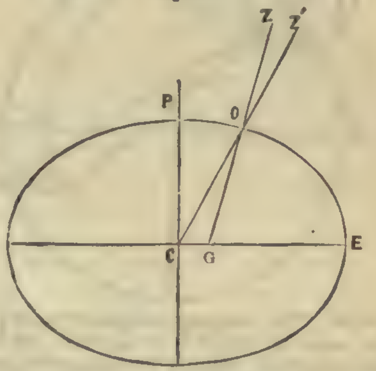
And by the usual method of development,

$$p = \frac{\sin. \pi \sin. Z}{\sin. 1''} + \frac{\sin.^2 \pi \sin. 2 Z}{2 \sin. 1''} + \frac{\sin.^3 \pi \sin. 3 Z}{3 \sin. 1''}, \text{ \&c. } (8)$$

We have seen that $\sin. \pi = \frac{r}{D}$; hence, if D be very great, π will be very small ; and, therefore, bodies at an extremely great distance, compared with the radius of the earth, have no sensible parallax.

Before proceeding any further, it may be well to remark that the value of sin. p depends on lines drawn from the centre of the earth, or from what is called the geocentric zenith, which differs from the astronomical zenith for this reason :—All astronomical instruments used for determining the north polar distance of an object depend on a zenith point, or something equivalent to one ; and this is nothing more than the instrumental reading corresponding to a vertical position of the telescope, and is determined by observing the same star by direct vision and reflection. The zenith point is therefore the reading for a point perpendicular to the horizon ; or, which is the same, a line drawn from the point thus determined in the heavens to the place of observation, is a normal to the curve by which the earth is generated, supposing it to be a surface of revolution. The earth is supposed to be a surface of revolution, generated by an ellipse revolving round its shorter axis ; the ratio of the axis being 300 : 299. Now, it is well known that normals to an ellipse do not pass through its centre ; hence the lines drawn from the centre of the earth to the place of observation, which determines the geocentric zenith, will make a small angle with the normal to the earth's surface at the same place, which determines the astronomical zenith ; and this angle is called the angle of the vertical. The geocentric zenith is nearer to the equator than the astronomical zenith by the value of this angle ; and as all the bodies (some comets excepted) for which we have to compute parallax, lie to the south of the zenith, the angle of the vertical is subtracted from the astronomical zenith distance, in order to have the geocentric zenith distance. This angle is easily determined, for it is the difference between the astronomical and geocentric latitude of the place of observation, and is thus formed : —Let P (Fig. 218) be the pole of the earth ; E a point of the equator ; O the place of observation ; Z′ the geocentric zenith ; Z the astronomical zenith ; then Z′ C E is the geocentric latitude = L g, and Z G E is the astronomical latitude = L a, and Z′ O Z = C O G is the angle of the vertical. Let a and b be the axis of the ellipse, the equation of the normal Z G is

Fig. 218.



$$y - y' = \frac{a^2}{b^2} \cdot \frac{y'}{x'} (x - x') = \frac{a^2}{b^2} \tan. L g (x - x') ;$$

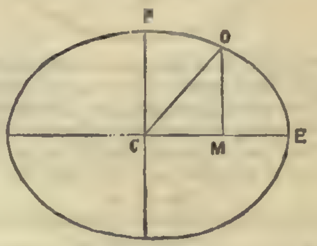
therefore,

$$\tan. L a = \frac{a^2}{b^2} \tan. L g, \text{ or } \tan. L g = \frac{b^2}{a^2} \tan. L a \; . \; . \; . \; . \; (9) ;$$

hence, L g becomes known. For an ellipticity of $\frac{\ }{300}$, and the astronomical latitude of Greenwich, viz., 51° 28′ 39″, we find the geocentric latitude, 51° 17′ 29″, and consequently the angle of the vertical = 11′ 12″.

The value of r', by which the moon's horizontal parallax must be multiplied, to have the horizontal parallax at the place of observation, is thus computed (Fig. 219) :

Fig. 219.



C O = r', O C E = L g ; then by the property of the ellipse $b^2 x^2 + a^2 y^2 = a^2 b^2$, we easily find

$$r^2 = \frac{a^2 b^2}{a^2 \sin.^2 L g + b^2 \cos.^2 L g}$$

$$= \frac{a^2 b^2}{a^2 - (a^2 - b^2) \cos.^2 L g}$$

$$r^2 = \frac{b^2}{1 - \frac{a^2 - b^2}{a^2} \cos^2. L g} \; \therefore r = b \sec. \theta$$

$$\text{where } \sin.^2 \theta = \frac{a^2 - b^2}{a^2} \cos.^2 L g.$$

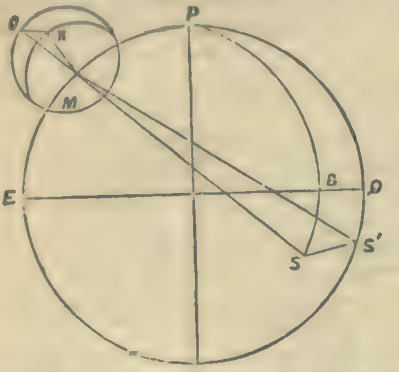
The reductions from the apparent to the mean places are performed in a manner similar to that previously explained for right ascension.

The other corrections for circle observations, viz., defective illumination of Moon and Venus, &c., are so fully explained in the Introduction to the *Greenwich Observations*, that a short investigation will only be necessary.*

Correction for Defective Illumination.—The following is the investigation of the correction for the defective illumination of the moon in the right ascension :—

Let S (Fig. 220) be the place of the sun, M that of the moon ; if the sun were at S′ both limbs of the moon would be full 12h. ∽ apparent time of moon's transit = B P Q = arc B Q, but B Q × cos. S B = S S = S M S′ = O M *n* = arc O *n*. This is the arc on the moon's disc, which is not illuminated, and its versed sin. is the part of the surface not enlightened.

Fig. 220.

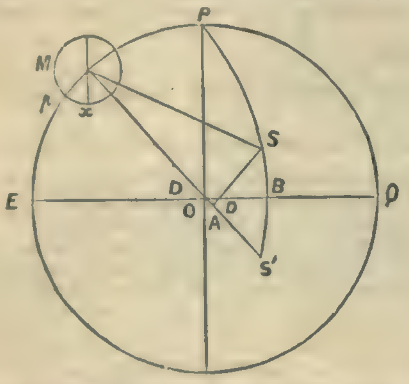


Moon's semi-diameter × versed sine S S′ = correction.

Defective Illumination in N. P. D.—Moon gibbous.

Let E Q (Fig. 221) be the equator, P the pole, E M = E O M = S′O B = D = the moon's declination. If the sun were at S′ both limbs of the moon would be enlightened. E P S = sun's hour angle = the difference between the times of passage of the sun and moon without regard to sign. Then in O B = cot. B O S′ × tan. B S′.

Fig. 221.



∴ tan. B S = sin. O B tan. D = sin. (E P S − 90°) tan. D =

— cos. sun's hour angle × tan. moon's apparent declination.

B S′ is called the new declination.

New declination — sun's declination = angle required. When this angle is positive, the north limb is full.

From S′ let fall the perpendicular arc S A, then cos. B O S′ = cos. B S′ sin. B S′ O. ∴ sin. B S′ O = $\frac{\text{cos. B O S}'}{\text{cos. B S}'}$ sin. S A = sin. S S′ sin. AS′S = sin. SS′; sin. AS′ S = sin. S S′ × $\frac{\text{cos. B O S}'}{\text{cos. B S}'}$ = sin. S S′ cos. B O S′ sec. B S′. S A s called the new angle, therefore sine new angle = cos. app. decl. × sec. new decl. × sine angle required.

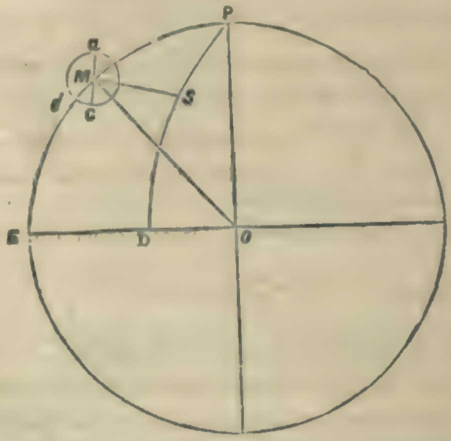
The new angle, or S A, is taken for the measure of S M A or *p* M *x*, the angle of defective illumination.

Defective Illumination in N. P. D.—Moon horned.

Mean time of passage 3h. to 6h.

Let S (Fig. 222) be the sun, M the moon on the meridian, *a c* perpendicular to S M. Join M O, D = E M = the moon's apparent declination, *d* = the sun's apparent declination. The angle M P S = T = the difference between the mean times of passage of the sun and moon ; the angle S M O is the angle of defective illumination = 90° — P M S.

Fig. 222.

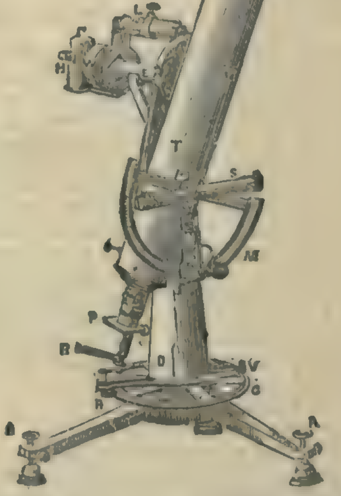


$$\text{tan. P M S} = \frac{\text{sin. T}}{\text{cos. D tan. } d - \text{sin. D cos. T}}$$

$$\text{tan. S M O} = \text{cot. P M S} = \frac{\text{cos. D tan. } d}{\text{sin. T}} - \frac{\text{sin. D cos. T}}{\text{sin. T}}$$

The Zenith Telescope.—The instrument employed in measuring the difference of the zenith distance and hour, called the zenith telescope, may be described here, although it is scarcely the proper place for it. A A (Fig.

Fig. 223.



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* The subject of parallax has already been examined, in reference to the sun, moon, &c., at page 926: to which the reader is referred for the practical application of the preceding remarks and formulæ.

223) are the screws which support the entire instrument, and by which the column carrying the telescope-screw is rendered truly vertical; C is the horizontal circle twelve inches in diameter, graduated to 10′, and reading to 10″, by means of its Vernier and microscope, V. B is the tangent-screw for slow motion.

This circle serves to mark the position of the meridian when it has once been determined. It likewise enables the observer to turn the telescope promptly through 180° of azimuth. D is the vertical column which supports the telescope, and about which the telescope turns in azimuth. E is a horizontal axis, to one end of which is attached the telescope T T, which is counterpoised by the weight, W, at the other end; this axis is hollow, and through it passes the light of the lamp, H, to illumine the wires of the telescope. The telescope has a focal length of about forty inches, and an aperture of three inches. L is the level by means of which the column D is rendered truly vertical, riding on the horizontal axis. M is a graduated semicircle attached to the telescope, and having a Vernier, N, with a microscope. This semicircle serves as a finder for setting the telescope to the altitude of the stars to be observed. S S is a very delicate level attached to the semicircle. P is the parallel wire micrometer for measuring small differences of altitude; and R is the diagonal eye-piece, so constructed that the micrometer may not interfere with the field of observation.

The stars selected for observation by this instrument should be within a convenient range of the micrometer—say 10′—one culminating to the north, and the other south of the zenith. Having levelled the instrument, set the telescope to an altitude midway between the two stars, and bring the bubble of the level S to the middle of its scale.

Bring the telescope into the plane of the meridian by setting the Vernier of the horizontal circle to the point previously determined.

As the first star enters the field of view, follow its image with one of the horizontal wires, and bisect it at the instant it crosses the middle wire. Record the position of the level S, noting the divisions corresponding to each extremity of the bubble. Turn the telescope 180° in azimuth, taking care to preserve the same inclination to the horizon, and make a similar observation on the second star, bisecting it with the horizontal wire. A comparison of the readings of the two micrometers will give the zenith distance of the two stars after correction by readings of the level, and also for the difference of refraction of the two stars.

We here subjoin a table of calculated orbits of comets, from the best authorities.

TABLE OF CALCULATED ORBITS OF COMETS.

No.	Greenwich Mean Time of Perihelion Passage.	Longitude of Perihelion.	Longitude of Node.	Inclination.	Logarithm of per Dist.	Eccentricity.	Direction of Motion	Calculator.
	OLD STYLE.	° ′ ″	° ′ ″	° ′ ″				
1	371 Winter. B.C.	150 to 210	270 to 330	Ab. 30°	very small	1	R	Pingré
2	137 Apr. 29	230 0 0	220 0 0	20 0 0	0·0043	1	R	Peirce
3	69 July	315 0 0	165 0 0	70 0 0	9·90	1	D	Peirce
4	12 Sep. 15. A.D. ...	310 0 0	35 0 0	67 0 0	9·949	1	R	Peirce
5	66 Jan. 14·2...	325 0 0	32 42 0	40 30 0	9·6480	1	R	Hind
6	240 Nov. 10	271 0 0	189 0 0	44 0 0	9·570	1	D	Burckhardt
7	451 July 3·5	Halley's	Comet.					Langier
8	539 Oct. 20·62	313 30 0	58° or 238°	10 0 0	9·53307	1	D	Burckhardt
9	565 July 14·5	80 0 0	159 30 0	59 0 0	9·92000	1	R	Burckhardt
10	568 Aug. 29·32	318 35 0	294 15 0	4 8 0	9·95771	1	D	Langier
11	574 Apr. 7·28	143 39 0	128 17 0	46 31 0	9·9836	1	D	Hind
(7)	760 June 11	Halley's	Comet.					
12	770 June 6·588	257 7 0	90 59 0	61 49 0	9·80766	1	R	Langier
13	837 March 1	289 3 0	206 33 0	10° to 12°	9·763428	1	R	Pingré
14	961 Dec. 30·16	268 3 0	350 35 0	79 33 0	9·7418	1	R	Hind
15	989 Sep. 12	264 0 0	84 0 0	17 0 0	9·7546	1	R	Burckhardt
16	1066 May 30 or 31	120 0 0	230 0 0	70° to 80°	9·53	1	R	Pingré
17	1092 Feb. 15	156 20 0	125 40 0	28 55 0	9·9676	1	D	Hind
18	1097 Sep. 21·9	332 30 0	207 30 0	73 30 0	9·86832	1	D	Burckhardt
19	1231 Jan. 30·3	134 48 0	13 30 0	6 5 0	9·976698	1	D	Pingré
20	1264 July 17·25	275 45 0	178 45 0	30 25 0	9·61364	1	D	Pingré
21	1299 March 31·312	3 20 0	107 8 0	68 57 0	9·50233	1	R	Pingré
22	1301 Oct. 24	312 0 0	138 0 0	13 0 0	9·806	1	R	Pingré
23	1337 June 15·074	2 20 0	93 1 0	40 28 0	9·91815	1	R	Langier
24	1351 Nov. 26·5	69 0 0	Indeterminate.				D	Burckhardt
25	1362 March 2·33	227 0 0	237 0 0	32 0 0	9·67214	1	R	Burckhardt
26	1366 Oct. 13	66 0 0	212 0 0	6 0 0	9·9814	1		Peirce
(7)	1378 Nov. 8·764 Halley's Comet	299 31 0	47 17 0	17 56 0	9·76604	1	R	Langier
27	1385 Oct. 16·26	101 47 0	268 31 0	52 15 0	9·8886	1	R	Hind
28	1433 Nov. 4·42	281 2 0	133 49 0	79 1 0	9·53079	1	R	Langier
(7)	1456 June 8·917 Halley's Comet	301 0 0	48 30 0	17 56 0	9·76754	1	R	Pingré
29	1457 Sep. 3·7	92 50 0	256 5 0	20 20 0	0·3229	1	D	Hind
30	1468 Oct. 7·4265	1 22 0	71 5 0	38 1 0	9·91893	1	R	Valz
31	1472 Feb. 28·218	48 3 0	207 32 0	1 55 0	9·751718	1	R	Langier
32	1490 Dec. 35·9...	113 0 0	268 0 0	75 0 0	9·878	1	R	Peirce
33	1506 Sep. 3·662	250 37 0	132 50 0	45 1 0	9·58656	1	R	Langier
(7)	1531 Aug. 25·722 Halley's Comet	301 12 0	45 30 0	17 0 0	9·76338	0·967391	R	Halley
34	1532 Oct. 18·3324	111 48 0	87 23 0	32 30 0	9·715351	1	D	Olbers
35	1533 June 14·883	217 40 0	299 19 0	28 14 0	9·514362	1	D	Olbers
(20)	1556 Apr. 22·0233	274 14 54	175 25 48	30 12 42	9·70323	1	D	Hind
36	1558 Aug. 10·52	329 49 0	332 36 0	73 29 0	9·76140	1	R	Olbers
37	1577 Oct. 26·9476	129 42 0	25 20 24	75 9 42	9·24920	1	R	Wolstedt

No.	Greenwich Mean Time of Perihelion Passage.	Longitude of Perihelion.	Longitude of Node.	Inclination.	Logarithm of per Dist.	Eccentricity.	Direction of Motion	Calculator.
		° ′ ″	° ′ ″	° ′ ″				
38	1580 Nov. 28·5727	109 11 55	19 7 37	64 51 50	9·774903	1	D	Pingré
39	1582 May 7·348	281 26 45	214 42 35	59 29 5	8·602754	1	R	Pingré
	NEW STYLE.							
40	1585 Oct. 8·0262	9 8 21	37 44 10	6 5 52	0·0393530	1	D	Peters and Sawitsch
41	1590 Feb. 8·0271	217 57 21	165 37 5	29 29 44	9·7541386	1	R	Hind
42	1593 July 18·5685	176 19 0	164 15 0	87 58 0	8·94940	1	D	Lacaille
43	1596 July 23·612	274 24 0	335 39 0	52 48 0	9·75258	1	R	Vaiz
(7)	1607 Oct. 26·71594 Halley's Comet	301 38 10	48 40 28	17 12 17	9·769358	0·9670887	R	Bessel
44	1618 (1) Aug. 17·1268 ...	318 20 0	293 25 0	21 28 0	9·71010	1	D	Pingré
45	1618 (2) Nov. 83·507 ...	3 5 21	75 44 10	37 11 31	9·590556	1	D	Bessel
46	1652 Nov. 12·653	28 18 40	88 10 0	79 28 0	9·928140	1	D	Halley
47	1661 Jan. 26·881 (See 34; 1532, *ante*) ...	115 16 8	81 54 0	33 0 55	9·646131	1	D	Méchain
48	1664 Dec. 4·494	130 41 25	81 14 0	21 18 30	0·011044	1	R	Halley
49	1665 Apr. 24·219	71 54 30	228 2 0	76 5 0	9·027309	1	R	Halley
50	1668 Feb. 28·8...	277 2 0	357 17 0	35 58 0	7·680000	1	R	Henderson
51	1672 March 1·359	46 59 30	297 30 30	83 22 10	9·843476	1	D	Halley
52	1677 May 6·026	137 37 5	236 49 10	78 3 15	9·448072	1	R	Halley
53	1678 Aug. 26·586	327 46 0	161 40 0	3 4 20	0·092727	1	D	Derowes
54	1680 Dec. 17·99338... ...	262 49 5	272 9 29	60 40 16	7·793551	0·999985417	D	Encke
(7)	1682 Sep. 14·79505 Halley's Comet	301 55 37	51 11 18	17 44 45	9·7655898	0·96792019	R	Rosenberger
55	1683 July 12·72586... ...	86 31 42	173 18 15	83 47 46	9·7430148	0·9832470	R	Clausen
56	1684 June 8·428	238 52 0	268 15 0	65 48 40	9·982339	1	D	Halley
57	1686 Sep. 16·606	77 0 30	350 34 40	31 21 40	9·511883	1	D	Halley
58	1689 Dec. 2·1403	270 16 0	344 18 0	30 25 0	8·01284	1	R	Peirce
59	1695 Nov. 9·7018	60 0 0	216 0 0	22 0 0	9·9261	1	D	Burckhardt
60	1698 Oct. 18·706	270 51 15	267 44 15	11 46 0	9·839660	1	R	Halley
61	1699 Jan. 13·349	212 31 6	321 45 35	69 20 0	9·871570	1	R	Lacaille
62	1701 Oct. 17·410	133 41 0	298 41 0	41 39 0	9·772784	1	R	Burckhardt
63	1702 March 13·6065 ...	138 46 34	188 59 10	4 24 44	9·810790	1	D	Burckhardt
64	1706 Jan. 30·2060	72 36 25	13 11 23	55 14 5	9·630290	1	D	Struyck
65	1707 Dec. 11·9885	79 59 8	52 50 29	88 37 40	9·934013	1	D	Struyck
66	1718 Jan. 14·90573... ...	121 39 57	127 55 31	31 8 6	0·010008	1	R	Argelander
67	1723 Sep. 27·62788... ...	42 52 35	14 14 17	50 0 18	9·9994743	1	R	Spörer
68	1729 June 13·26350 ...	320 31 22	310 38 0	77 5 18	0·6067570	1·0050334	D	Burckhardt
69	1737 (1) Jan. 30·34767 ...	325 55 0	226 22 0	18 20 45	9·347960	1	D	Bradley
70	1737 (2) June 8·3185 ...	262 36 39	123 53 43	39 14 5	9·93802	1	D	Daussy
71	1739 June 17·4164	102 38 40	207 25 14	55 42 44	9·828389	1	R	Lacaille
72	1742 Feb. 8·6192	216 39 20	185 9 30	67 31 40	9·886523	1	R	Barker
73	1743 (1) Jan. 8·19403 ...	93 19 37	86 54 30	1 53 43	9·9352858	0·7213086	D	Clausen
74	1743 (2) Sep. 20·8866 ...	246 33 52	5 16 25	45 48 21	9·717310	1	R	Klinkenberg
75	1744 March 1·3282	197 11 58	45 46 6	47 10 53	9·346783	0·999100	D	Euler
76	1747 March 3·2991	277 2 0	147 18 50	79 6 20	0·342146	1	R	Lacaille
77	1748 (1) Apr. 28·78065 ...	215 23 29	232 51 50	85 28 23	9·94486	1	R	Lémonier
78	1748 (2) June 18·88751 ...	278 47 10	33 8 29	67 3 28	9·796128	1	D	Bessel
79	1757 Oct. 21·38450	122 36 29	214 7 11	12 41 17	0·530610	1	D	De Ratte
80	1758 June 11·1373	267 38 0	230 50 0	68 19 0	9·333148	1	D	Pingré
(7)	1759 (1) Mar. 12·5517 Halley's Comet	303 10 28	53 50 27	17 36 52	9·7667990	0·9676844	R	Rosenberger
81	1759 (2) Nov. 27·02358 ...	53 38 4	139 40 15	79 3 19	9·904218	1	D	Chappe
82	1759 (3) Dec. 16·53392 ...	139 3 52	79 20 24	4 42 10	9·983064	1	R	Chappe
83	1762 May 28·33451... ...	104 2 0	348 33 5	85 38 13	0·033912	1	D	Burckhardt
84	1763 Nov. 1·86141	84 58 58	356 24 4	72 31 52	9·6974784	0·998680	D	Burckhardt
85	1764 Feb. 12·57100... ...	15 14 52	120 4 33	52 53 31	9·744462	1	R	Pingré
86	1766 (1) Feb. 17·3616 ...	143 15 25	244 10 50	40 50 20	9·703570	1	R	Pingré
87	1766 (2) Apr. 26·98882 ...	251 13 0	74 11 0	8 1 45	9·6009521	0·864000	D	Burckhardt
88	1769 Oct. 7·62039	144 11 29	175 3 59	40 45 55	9·0890392	0·9992490	D	Bessel
89	1770 (1) Aug. 13·54085 ...	356 16 51	131 58 56	1 34 28	9·8289491	0·786119	D	Leverrier
90	1770 (2) Nov. 22·2352 ...	298 22 44	108 42 10	31 25 55	9·722833	1	R	Pingré
91	1771 Apr. 19·21271... ...	104 3 10	27 51 49	11 15 19	9·9559104	1·0093698	D	Encke
92	1772 Feb. 19·09933 Biela's Comet (?)	110 14 54	254 0 1	13 17 38	0·0058652	0·9931481 (?)	D	Bessel
93	1773 Sep. 5·37685	74 57 41	121 4 49	61 13 19	0·0512720	0·9935023	D	Pingré
94	1774 Aug. 15·83010 ...	317 27 40	180 44 34	83 20 6	0·1562066	1·0252955	D	Burckhardt
95	1779 Jan. 4·10445	86 53 0	23 40 0	32 43 0	9·853090	1	D	Olbers
96	1780 (1) Sep. 30·92630 ...	246 35 56	123 41 15	54 23 12	8·9836118	9·9099640	R	Clüver
97	1780 (2) Nov. 28·84790 ...	246 52 0	141 1 0	72 3 30	9·7120410	1	R	Olbers
98	1781 (1) July 7·18887 ...	239 11 25	83 0 38	81 43 26	9·8897840	1	D	Méchain
99	1781 (2) Nov. 29·52320 ...	16 3 7	77 22 55	27 12 4	9·9827230	1	R	Legendre
100	1783 Nov. 19·56218 ...	49 31 55	55 12 0	47 42 0	0·1747341	0·6784	D	Burckhardt
101	1784 Jan. 21·19960... ...	80 44 24	56 49 21	51 9 12	9·8499460	1	R	Méchain

No.	Greenwich Mean Time of Perihelion Passage.	Longitude of Perihelion.			Longitude of Node.			Inclination.			Logarithm of per Dist.	Eccentricity.	Direction of Motion	Calculator.
		°	′	″	°	′	″	°	′	″				
102	1785 (1) Jan. 27·32549 …	109	51	56	264	12	15	70	14	12	0·0581980	1	D	Méchain
103	1785 (2) Apr. 8·47200 …	297	34	30	64	44	40	87	7	0	9·6310240	1	R	Laron
104	1786 (1) Jan 30·87350 Encke's Comet … …	156	38	0	334	8	0	13	36	0	9·5248100	0·84836	D	Encke
105	1786 (2) July 8·56747 …	158	38	30	195	23	32	50	58	33	9·5957630	1	D	Reggio
106	1787 May 10·82545 … …	7	44	9	106	51	35	48	15	51	9·5427140	1	R	Laron
107	1788 (1) Nov. 10·36932 …	99	8	7	156	56	43	12	27	40	0·0265380	1	R	Méchain
108	1788 (2) Nov. 20·30254 …	22	49	54	352	24	26	64	30	24	9·8792760	1	D	Méchain
109	1790 (1) Jan. 16·79038 …	58	24	45	172	50	2	29	44	7	9·8735160	1	R	Laron
110	1790 (2) Jan. 28·31677 …	111	44	37	267	8	37	56	58	13	0·0266503	1	D	Méchain
111	1790 (3) May 21·24090 …	273	43	27	33	11	2	63	52	27	9·9019814	1	R	Méchain
112	1792 Jan. 13·53489… …	36	20	32	190	42	9	39	45	47	0·1114563	1	R	Zach
113	1792 (2) Dec. 27·25360 …	135	59	24	283	15	17	49	1	45	9·9851060	1	R	Prosperin
114	1793 (1) Nov. 4·84140 …	228	42	0	108	29	0	60	21	0	9·6057360	1	R	Laron
115	1793 (2) Nov. 28·59981 …	75	58	58	359	4	48	47	35	5	0·1461360	0·7347635	D	Burckhardt
(104)	1795 Dec. 21·44098 Encke's Comet . . …	156	41	24	334	39	26	13	42	30	9·5243046	0·8488828	D	Encke
116	1796 April 2·82478 … …	192	44	13	17	2	16	64	54	33	0·1981510	1	R	Olbers
117	1797 July 9·11424 … …	49	34	42	329	16	30	50	35	50	9·7205310	1	R	Bouvard
118	1798 (1) April 4·49879 …	105	6	57	122	12	21	43	44	42	9·6853710	1	D	Olbers
119	1798 (2) Dec. 31·55350 …	34	27	27	249	30	30	42	26	4	9·8918290	1	R	Burckhardt
120	1799 (1) Sep. 7·22626 …	3	38	9	99	21	11	51	1	29	9·9244710	1	R	Olbers
121	1799 (2) Dec. 25·89640 …	190	20	12	326	49	11	77	1	38	9·7964370	1	R	Méchain
122	1801 Aug. 8·5574 … …	183	49	0	44	28	0	21	20	0	9·417804	1	R	Burckhardt
123	1802 Sep. 9·89102 … …	332	9	4	310	15	39	57	0	47	0·039061	1	D	Olbers
124	1804 Feb. 13·58813… …	148	44	51	176	47	58	56	28	40	0·029858	1	D	Gauss
(104)	1805 (1) Nov. 21·49987 Encke's Comet … …	156	47	19	334	20	5	13	33	30	9·5320168	0·84617529	D	Encke
(92)	1805 (2) Dec. 32·97420 Biéla's Comet … …	109	32	23	251	15	15	13	38	45	9·9575120	0·745784	D	Gambart
125	1806 Dec. 28·93150… …	97	2	3	322	19	15	35	2	50	0·0340550	1	R	Burckhardt
126	1807 Sep. 18·738870 …	270	54	42	266	47	11	63	10	28	9·81031575	0·9954878	D	Bessel
127	1808 (1) May 12·953 …	69	12	57	322	58	36	45	43	7	9·59091	1	R	Encke
128	1808 (2) July 12·16768 …	252	38	50	24	11	15	39	18	59	9·783870	1	R	Bessel
129	1810 Sep. 29·0997 … …	52	44	42	310	21	2	61	11	15	9·9893549	1	D	Iriesnecker
130	1811 (1) Sep. 12·25731 …	75	0	34	140	24	44	73	2	21	0·0151178	0·9950933	R	Argelander
131	1811 (2) Nov. 10·99048 …	47	27	20	93	1	45	31	17	11	0·1992359	0·9827109	D	Nicolai
132	1812 Sep. 15·31356… …	92	18	46	253	1	3	73	57	3	9·8904995	0·9945412	D	Encke
133	1813 (1) Mar. 4·52650 …	69	56	8	60	48	24	21	13	33	9·8445579	1	R	Nicollet
134	1813 (2) May 19·62378 …	197	28	37	42	39	36	80	55	5	0·081364	1	R	Olbers
135	1815 Apr. 25·99217… …	149	2	12	83	28	50	44	29	55	0·0838109	0·93921968	D	Bessel
(92)?	1818 (1) Feb. 7·397 …	95	7	0	254	0	0	20	2	24	0·86526	1	D	Pogson
136	1818 (2) Feb. 25·95890 …	182	45	22	70	26	11	89	43	48	0·0783711	1	D	Encke
(104)	1819 (1) Jan. 27·25616 Encke's Comet … …	156	59	12	334	33	19	13	36	54	9·5253771	0·8485841	D	Encke
138	1819 (2) June 27·73998 …	287	13	4	273	42	28	80	43	56	9·5340268	1	D	Nicolai
139	1819 (3) July 18·90020 …	274	40	51	113	10	48	10	42	48	9·8885382	0·75519035	D	Encke
140	1819 (4) Nov. 20·24553 …	67	18	42	77	13	51	9	1	16	9·9506368	0·6867458	D	Encke
141	1821 Mar. 21·53656 …	239	29	25	48	40	56	73	33	7	8·9629523	1	R	Rosenberger
142	1822 (1) May 5·57938 …	192	45	48	177	27	22	53	34	48	9·70280	1	R	Encke
(104)	1822 (2) May 23·96296 Encke's Comet … …	157	11	44	334	25	9	13	20	17	9·5390382	0·8444643	D	Encke
143	1822 (3) July 16·02433 …	219	53	48	97	51	23	37	43	4	9·92743	1	R	Heilligenstein
144	1822 (4) Oct. 23·76978 …	271	40	17	92	44	42	52	39	10	0·0588305	0·9963021	R	Encke
145	1823 Dec. 9·44408 … …	274	34	30	303	3	1	76	11	57	9·3550726	1	R	Encke
146	1824 (1) July 11·51296 …	260	16	32	234	19	9	54	34	19	9·7717807	1	R	Rümker
147	1824 (2) Sep. 19·2440 …	4	44	24	279	5	49	54	22	3	0·0200454	1	D	Bouvard
148	1825 (1) May 30·55880 …	273	55	48	20	7	53	56	41	30	9·948964	1	R	Hansen
149	1825 (2) Aug. 18·71105 …	10	14	57	192	56	41	89	41	47	9·9461924	1	D	Clausen
(104)	1825 (3) Sep. 16·27312 Encke's Comet … …	157	14	31	334	27	30	13	21	24	9·5376348	0·8448885	D	Encke
150	1825 (4) Dec. 10·68186 …	318	46	41	215	43	14	33	32	39	0·0937189	0·9953690	R	Hansen
(92)	1826 (1) Mar. 18·41218 Biéla's Comet … …	109	46	0	251	28	23	13	33	51	9·9554571	0·7465727	D	Santini
151	1826 (2) Apr. 21·97761 …	117	11	30	197	30	34	39	57	24	0·3016581	1·0089597	D	Nicolai
152	1826 (3) Apr. 29·03904 …	35	48	1	40	29	1	5	17	2	9·2744275	1	R	Clüver
153	1826 (4) Oct. 8·95224 …	57	48	24	44	6	28	25	57	18	9·9398520	1	D	Argelander
154	1826 (5) Nov. 18·41206 …	315	31	38	235	7	48	89	22	10	8·4296128	1	R	Clüver
155	1827 (1) Feb. 4·92156 …	33	30	16	184	27	49	77	35	35	6·746000	1	R	Heilligenstein
156	1827 (2) June 8·41120 …	297	32	42	318	10	28	43	38	45	9·9074940	1	R	Heilligenstein
157	1827 (3) Sep. 11·69286 …	250	57	16	149	39	14	54	4	42	9·1393857	0·9992731	R	Clüver
(104)	1829 Jan. 9·74591 Encke's Comet … …	157	17	53	334	29	32	13	20	34	9·5385038	0·8446245	D	Encke
158	1830 (1) Apr. 9·60506 …	212	23	19	206	22	43	21	11	9	9·9650486	1	D	Santini

No	Greenwich Mean Time of Perihelion Passage	Longitude of Perihelion	Longitude of Node.	Inclination.	Logarithm of per. Dist.	Eccentricity.	Direction of Motion	Calculator.
		° ′ ″	° ′ ″	° ′ ″				
159	1830 (2) Dec. 27·66040 ...	310 59 19	337 53 7	44 45 30	9·0999822	1	R	Wolfers
(104)	1832 (1) May 3·97551 Encke's Comet	157 21 1	334 32 9	13 22 9	9·5358905	0·8454141	D	Encke
160	1832 (2) Sep. 25·52156 ...	227 55 36	72 26 42	63 18 3	0·0732061	1	R	E. Bouvard
(92)	1832 (3) Biela's Comet	See p. 963.						
161	1833 Sep. 10·186	222 51 14	323 0 48	7 21 2	9·6612600	1	D	Peters
162	1834 April 2·821	276 27 3	226 1 31	5 59 48	9·7096600	1	D	Peters
163	1835 (1) Mar. 27·57651 ...	207 42 55	58 19 46	9 7 39	0·3099084	1	R	W. Bessel
(104)	1835 (2) Aug. 26·35078 Encke's Comet	157 23 29	334 34 59	13 21 15	9·5371089	0·8450356	D	Encke
(7)	1835 (3) Nov. 15·93889 Halley's Comet	304 31 32	55 9 59	17 45 5	9·7683194	0·9673909	R	H. Westphalen
(104)	1838 Dec. 19·01224 Encke's Comet	157 27 4	334 36 41	13 21 28	9·5366085	0·8451775	D	Encke
164	1840 (1) Jan. 4·47111 ...	192 12 17	119 57 54	53 5 41	9·7913112	1	D	Lundhal
165	1840 (2) Mar. 12·99064 ...	80 18 20	236 49 16	59 13 20	0·0868563	0·9978836	R	Plantamour
166	1840 (3) Apr. 2·49544 ...	324 12 40	186 2 57	79 51 52	9·8740948	1	D	Rümker
167	1840 (4) Nov. 13·64438 ...	22 31 34	248 56 15	57 57 23	0·1705070	0·9698527	D	Götze
(104)	1842 (1) Apr. 12·01815 Encke's Comet	157 29 27	334 39 10	13 20 26	9·5378361	0·8447904	D	Encke
168	1842 (2) Dec. 15·95670 ...	327 16 13	207 49 1	73 33 37	9·7020605	1	R	Langier
(50)	1843 (1) Feb. 27·414015 ...	278 43 12	1 26 59	35 38 34	7·7392979	0·9999365	R	Hubbard
169	1843 (2) May 6·05593 ...	281 29 35	157 14 46	52 44 46	0·2085315	1·0001798	D	Götze
171	1843 (3) Oct. 17·14786 ...	49 34 9	209 29 9	11 22 31	0·2285488	0·5559623	D	Leverrier
(53)	1844 (1) Sep. 2·47402 ...	342 30 50	63 49 0	2 54 50	0·0742309	0·6176539	D	Brünnow
172	1844 (2) Oct. 17·33613 ...	180 23 59	31 38 55	48 36 22	9·9321180	1	R	Hind
173	1844 (3) Dec. 13·68294 ...	296 0 30	118 22 23	45 36 34	9·4001230	1	D	Hind
174	1845 (1) Jan. 8·15698 ...	91 19 42	336 44 31	46 50 36	9·9567272	1	D	Gengon
175	1845 (2) Apr. 21·03098 ...	192 33 34	347 7 0	56 23 36	0·0985330	1	D	Faye
176	1845 (3) June 5·67342 ...	262 3 17	337 49 18	48 41 59	9·6038230	0·9898743	R	D'Arrest
(104)	1845 (4) Aug. 9·62627 Encke's Comet	157 44 21	334 19 33	13 7 34	9·5291008	0·8474362	D	Encke
177	1846 (1) Jan. 22·09387 ...	89 6 25	111 8 29	47 26 6	0·1704680	0·9924026	D	Jelinck
(92)	1846 (2) Feb. 10·98766 ...	109 2 26	245 54 45	12 34 53	9·9327011	0·7570030	D	Plantamour
(92)	1846 (2) Feb. 11·05401 Biela's Comet	109 2 45	245 56 7	12 34 13	9·9326965	0·7558991	D	Plantamour
178	1846 (3) Feb. 25·37406 ...	116 28 22	102 41 6	30 55 53	9·8129827	0·7933879	D	Brünnow
179	1846 (4) Mar. 5·35599 ...	89 16 50	76 49 35	85 34 58	9·8245330	1	D	Van Denise
180	1846 (5) May 25·94071 ...	84 48 14	161 31 51	57 43 40	0·1314317	1	R	Brorsen
181	1846 (6) June 1·21242 ...	240 7 35	260 28 59	30 24 24	0·1842997	0·7213385	D	C. H. A. Peters
182	1846 (7) June 5·47921 ...	162 6 2	261 53 12	29 18 47	9·8018857	0·9899389	R	Oudemans
183	1846 (8) Oct. 29·91663 ...	98 47 19	4 38 22	49 39 3	9·9187601	0·9953127	D	Quirling
184	1847 (1) Mar. 30·32230 ...	275 46 42	21 37 44	48 32 23	8·6457238	0·9993425	D	Villarceau
185	1847 (2) June 4·69052 ...	141 37 13	173 57 43	79 33 43	0·3253373	1	R	Gauttier
186	1847 (3) Aug. 9·25914 ...	21 20 33	76 42 2	32 38 24	0·1715239	0·9974348	R	Schweizer
187	1847 (4) Aug. 9·44226 ...	246 42 4	338 18 1	83 27 1	0·2472789	1	R	V. Littrow
188	1847 (5) Sep. 9·54272 ...	79 12 41	309 49 24	19 8 25	9·6882986	0·9725603	D	D'Arrest
189	1847 (6) Nov. 14·41856 ...	274 23 35	190 51 37	71 57 46	9·5187597	1	R	Rümker
190	1848 (1) Sep. 8·04532 ...	310 34 41	211 32 31	84 24 50	9·5050568	1	R	Sonnetag & Quirling
(104)	1848 (2) Nov. 26·11567 Encke's Comet ...	157 47 8	334 22 12	13 8 36	9·5276717	0·8478281	D	Encke
191	1849 (1) Jan. 19·34781 ...	63 14 18	215 12 56	85 2 54	9·9821497	1	D	Petersen & Sonnetag
192	1849 (2) May 26·49803 ...	235 43 48	202 33 15	67 9 19	0·0642078	1	D	Gongon
193	1849 (3) June 8·20364 ...	267 6 30	30 32 22	66 55 19	9·9515250	0·9978312	D	D'Arrest
194	1850 (1) July 23·51966 ...	273 24 27	92 53 25	68 12 7	0·0340307	0·9999868	D	Sonnetag
195	1850 (2) Oct. 19·34851 ...	89 20 31	206 1 0	40 6 53	9·7518260	1	D	Roslhuher
(171)	1851 (1) Apr. 1·80560 ...	49 41 38	209 31 19	11 21 39	0·2303547	0·5549226	D	Leverrier
196	1851 (2) July 8·69157 ...	322 59 49	148 27 41	13 56 4	0·0696476	0·6007426	D	Villarceau
197	1851 (3) Aug. 26·23463 ...	310 59 22	223 41 6	38 9 2	9·9931273	0·9968516	D	Brorsen
198	1851 (4) Sep. 30·79993 ...	338 45 56	44 26 14	73 59 44	9·1503928	1	D	Götze and Sonnetag

SECTION XI.

NAVIGATION AND NAUTICAL ASTRONOMY.

CHAPTER I.

INTRODUCTION.

The principles of Navigation rest almost entirely upon that part of Plane Trigonometry which is limited to the doctrine of plane triangles. A person unacquainted with the mathematical theory upon which the practical rules followed by the navigator are based, would naturally imagine, as the track of a ship is a path marked out on the surface of a *sphere*, that to calculate, from the necessary data, the length of this track, the aid of *Spherical* Trigonometry would be required. But, in general, such is not the case; and for this reason:—spherical trigonometry is wholly concerned with the arcs of great circles of the sphere, and with the angles formed by such arcs; whereas, the course of a ship at sea, unless it sail on the equator, or on a meridian, is never a great circle;—it is either a small circle, a parallel to the equator, or else a line cutting the successive meridians over which it sails, obliquely, and at the same invariable angle, so long as its *course* remains unchanged. A ship, continuing on this unchanged course, would trace out, on the surface of the sphere, a winding or spiral curve, called in navigation a *rhumb line*, and which is widely different in figure from a circle. If a vessel were to start from any point between the equator and either pole, and on a course inclined ever so little towards that pole, it would wind round the globe in this spiral path, approaching nearer and nearer to the pole, but actually arriving at it only after it had circulated round it an infinite number of times.

It is the length of a portion of such a spiral line that is one of the objects of navigation to calculate; and it is pretty obvious that rules and formulæ, supplied by *spherical* trigonometry, can give no aid in such a calculation; since the latter science has nothing to do with the spiral curves, which vessels at sea trace out.

It would seem, however, that these spirals on a *sphere*, are equally external to the proper objects of *plane* trigonometry, which recognises only *straight* lines, drawn on a flat surface. But it must be observed that navigators are not interested in any investigations respecting the form or shape of the spiral path of a ship; but only in its *length*, and in the angle it makes with the meridians crossed by it. *Lengths*, and the angles formed by them, are, of course, the proper objects of consideration in plane trigonometry; and some notion may, therefore, be formed as to how it happens that a ship's course and distance sailed, are matters for investigation by plane, and not by spherical trigonometry. In the latter subject, the *form* of the lines concerned cannot possibly be disregarded, any more than their lengths; it would not be spherical trigonometry unless the lines of which it treats were all of them portions of great circles of the same sphere. Plane trigonometry regards lengths and angles only; and these alone are all that navigators require to be calculated.

It is obvious, therefore, that, in order to understand the *science* of navigation, the theory and practice of plane trigonometry—as far, at least, as the triangle is concerned—must be previously acquired; but modern books on Trigonometry, with but few exceptions, are very deficient in this needful preparatory instruction; so that it becomes almost imperative upon a writer on navigation, who wishes to be clear and intelligible, to conduct his reader through some amount of preliminary matter before formally entering upon the special object of his work. We shall, therefore, introduce the subject by an article on the calculation of plane triangles; referring to the chapters on Trigonometry, in the Section of MATHEMATICS, for the necessary theoretical principles; except in a few cases where it may be thought expedient to use rules of operation not expressly provided for in that part of this work: in such cases the theoretical investigation will precede the practical rule.

Most persons who read this Section, will very likely do so for the sole purpose of becoming acquainted with the general principles of a most valuable part of practical mathematics; and more especially for learning in what way a few elementary theories, in pure mathematics, are made available for objects of such commercial and national importance, as the conducting a ship across the ocean from one port to another; the daily registry of its position on the globe; the determination of its direct distance from any preceding position, however irregular its actual course between the two positions may have been; of its distance from the equator at any time, and from the meridian of Greenwich, &c., &c. These things are well worth inquiring into even by the non-professional student of science; as they give a practical value, of the highest kind, to what might otherwise seem to be but barren speculations.

Nothing can more forcibly illustrate the importance of the theoretical principles developed in the Mathematical Section above referred to, than the application of those principles to Navigation and Nautical Astronomy; for, aided by the Compass and the Log, and by a simple optical instrument—the construction of which those principles have suggested—they instruct the mariner to read aright the intelligence of the stars—those faithful finger-posts of heaven, which direct him unerringly on his course across a vast and dreary expanse, where no human skill can erect a signal to guide, or a beacon to warn; and where even the busy traffic of ages has left no beaten pathway behind it.

Our object is to describe, in simple language, but in a strictly scientific manner, how all this is brought about; and while we shall explain everything that can be reasonably desired by the mathematical student, we shall also furnish, to the practical seaman, the reasons on which his rules of calculation are founded, and the principles on which his tables are constructed; and will thus, we hope, afford a useful accompaniment to whatever book of rules and tables he may choose to consult in the ordinary exercise of his profession. It is in the highest degree desirable that persons engaged in a calling so responsible, should be enabled to judge for themselves of the soundness and accuracy of the directions which guide them, and not deliver themselves blindly up to authoritative rules. The amount of mathematical knowledge necessary to convert their art into a science is really very small; and the satisfaction arising from knowing a thing as an ascertained truth, instead of receiving it upon trust, is alone sufficient to compensate for the time expended on the study of it. Besides, mere printed directions can never suffice for unforeseen emergencies; and in no occupation are these more likely to arise than amidst the hazardous duties of the sailor's life. In such circumstances a little science may often be of more avail than volumes of prescribed directions; and even in the ordinary calculations, which form part of the every-day routine on shipboard, time and figures are often both thrown away by the unscientific mariner, under the impression that he is increasing the accuracy of his work, when he is, in reality, only encumbering it with errors. A very meritorious writer, Lieut. Raper, justly remarks, that "very indistinct and erroneous notions prevail among practical persons on the subject of accuracy of computation, and much time is, in consequence, often lost in computing to a degree of precision wholly inconsistent with that of the elements themselves. The mere habit of working invariably to a useless precision, while it can never advance the computer's knowledge of the subject, has the unfavourable tendency of deceiving those who are not aware of the true nature of such questions, into the persuasion that a result is always as correct as the computer chooses to make it; and thus leads them to place the same confidence in all observations, provided only they are *worked* to the same degree of accuracy."* These are very judicious observations; and Lieut. Raper is, as far as we know, the only person who has drawn attention, in print, to this customary waste of calculation.

The tables which accompany books on navigation are, in general, computed to an extent of decimals—usually six or seven places—much beyond what the ordinary calculations of navigation require; and the unscientific seaman uses them *all*, when half the number would, in most cases, be amply sufficient: his fault is analogous to that of the ill-taught school-boy, who, having to multiply together two numbers of five decimals—the final decimal of each number being confessedly inaccurate—is at the trouble of computing ten decimals in his product, and fancies, if there be no error in his operation, that they are all correct, whereas five of them at least are wrong, and, therefore, worse than useless.

We shall now proceed to those preliminary matters which furnish the suitable introduction to the different sailings—plane sailing, parallel sailing, middle latitude sailing, and Mercator's sailing.

ON LOGARITHMS.—Logarithms are a peculiar kind of numbers, invented by Lord Napier, for the purpose of shortening the calculations by common figures, whenever these calculations require the operations of multiplication or division, the raising of powers, or the extracting of roots.

Logarithms are of great importance in the computations of Navigation and Nautical Astronomy; and it was mainly for the purpose of reducing the labour of such computations, that Napier was led to construct a *table of logarithms.*

The peculiar character, and the practical use of these important numbers may be briefly explained as follows, the principles of Algebra, as given in our mathematical section, being previously understood.

It is therein shown (see Algebra, page 465) that, a being any *base*, and x and y any exponents,

$$a^x \times a^y = a^{x+y}, \text{ and } a^x \div a^y = a^{x-y};$$

and, moreover, that whatever exponents m and n may be, $(a^m)^n = a^{mn}$.

Now let us suppose that any number N could be written in the form a^x, a being some chosen number fixed upon as a *base*, and x the suitable exponent to justify the equation $a^x = \text{N}$; thus, the base a might be the number 2, or 3, or 4, or any other positive number different from unity; and it is plain that, whether we can find it or not, *some* value exists for x which would satisfy the equation just written, whatever be the positive number N. There is no doubt, for instance, that a numerical value for x exists such that $2^x = 12$: the value of x must evidently lie between 3 and 4. Again, the value of x, which satisfies the condition $2^x = 6$, must evidently lie between 2 and 3. Now imagine two numbers thus represented, one of them N by a^x, and the other N′ by a^y; then, from the principles stated above, we should have, by multiplication and division,

$$a^x \times a^y, \text{ that is, } a^{x+y} = \text{N N}'; \text{ and } a^x \div a^y, \text{ that is, } a^{x-y} = \text{N} \div \text{N}'.$$

where we see that the *multiplication* of the two numbers N, N′ is replaced by the simple *addition* of the two exponents x and y, in the proposed notation for those numbers, and that the *division* of N by N′ is replaced by the simple *subtraction* of the exponent y from the exponent x. The proper exponent s, to be placed over the base a, so that a^s may replace the number N N′, is therefore $s = x + y$; and the proper exponent, for the number $\frac{\text{N}}{\text{N}'}$, is $s = x - y$. It appears from this, that if a table were constructed in which the numbers 1, 2, 3, &c., up to the highest number likely to occur in calculation, were inserted, and against each number n were placed the proper exponent x—that is, such a value of x that the equation $a^x = n$ should, in each case, be satisfied—the operations of multiplication and division might, by aid of the table, be converted into the simpler operations of addition and subtraction.

Such a table was actually constructed by Napier; it was afterwards improved by Briggs, who found that 10 was the most convenient number to choose for the base a; and the tables constructed to this base, and in which are inserted against each number n the value of x proper to satisfy the equation $10^x = n$, are the modern *tables of logarithms*:—the exponent x, adapted to any proposed number n, being called the *logarithm* of n; so that a table of logarithms is nothing more than a table of exponents—the base-number, to which each exponent is conceived to be attached, being 10.

For example, suppose we want the logarithm of 67:—in other words, that we require the value of the exponent x, fitted to satisfy the condition $10^x = 67$, we turn to the tables, and find the value to be $x = 1{\cdot}826075$.

In like manner, if we want the logarithm of 58, or to solve the equation $10^x = 58$, we find, from the table, that $x = 1{\cdot}763428$. We therefore say that log. 67 = 1·826075, and log. 58 = 1·763428; which is only another way of expressing that

$$67 = 10^{1\cdot826075}, \text{ and } 58 = 10^{1\cdot763428}.$$

The product of these numbers is $67 \times 58 = 10^{1\cdot826075+1\cdot763428}$; that is $10^{3\cdot589503}$. The quotient of the first by the second is $67 \div 58 = 10^{1\cdot826075-1\cdot763428}$; that is $10^{\cdot062647}$. In other words, log. $(67 \times 58) = 3{\cdot}589503$, and log. $(67 \div 58) = {\cdot}062647$. Turning now to the table, we find, against the logarithm 3·589503, the number 3886, and against the logarithm ·362647, the number 1·155 . . . : we conclude, therefore, without actually performing the operation of multiplication or division, that

$$67 \times 58 = 3886, \text{ and } 67 \div 58 = 1{\cdot}155 \ldots.$$

And in a similar way may the product or quotient of any two numbers, within the limits of the table, be found: for the *product*, we *add* the logs. of the factors; for the *quotient*, we *subtract* log. of divisor from log. of dividend. The result, in each case, is a log., against which, in the table, is the product or quotient sought; and in this way addition and subtraction is made to replace the more lengthy operations of multiplication and division.

It must be noticed, however, that the *integral* part of the logarithm or exponent is not inserted in the table—only the *decimal* part: the insertion of the integral part, or *index*, as it is called, is unnecessary, because this index is at once known from inspection of the number to which the logarithm belongs: thus, since

$$10^0 = 1,\ 10^1 = 10,\ 10^2 = 100,\ 10^3 = 1000,\ 10^4 = 10000, \text{ \&c.},$$

we know that the log. of a number between 1 and 10 must lie between 0 and 1; we know, therefore, that the index of a positive number, anywhere between these limits, is 0. We see, also, that the log. of a number between 10 and 100 must lie between 1 and 2; the index of such a number must therefore be 1. In like manner, since the log. of a number between 100 and 1000 must lie between 2 and 3, we know that the index of such a number must be 2; and so on.

Hence the index of the log of a number consisting of but one integer—however many decimals may follow that integer—is 0; the index of a log. of a number consisting of two integers, however many decimals may follow, is 1; of a number consisting of three integers, the index of the log. is 2; of a number consisting of four integers, the index is 3; and, generally, of a number consisting of n integers, the index of the log. is $n-1$.

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* *The Practice of Navigation and Nautical Astronomy.* By H. Raper, Esq., Secretary to the Royal Astronomical Society.

We have, therefore, only to count the figures in the integral part of a number, disregarding the decimals, and then to write down, for the index of the log. of that number, the figure denoting what was counted, diminished by 1: to this index the proper decimal part of the log., called the *mantissa*, which is furnished by the table, is to be united, when the complete log. will be exhibited. Thus, for the numbers 43, 58, 32·47, 67·813, &c., the index, in each case, is 1; for each of the numbers 246, 835, 647·29, 158·72, &c., the index is 2; and so on.

If, however, the number proposed have no integral figures—that is, if it consist wholly of decimals—then the index of the log. will be *negative*. For instance, suppose the number is ·3247, we may write it thus $\frac{3{\cdot}247}{10}$; and, from the principles already explained, the log. of the quotient thus indicated is log. 3·247 — log. 10; that is, log. 3·247 — 1. Consequently, the index of the log. of ·3247 is the index of log. 3·247 diminished by 1: but as there is only one integer in the number 3·247, the index of its log. is 0; therefore the index of log. ·3247 is —1. Suppose the number is ·03247; this may be written $\frac{{\cdot}3247}{10}$: hence log. ·03247 = log. ·3247 — log. 10; that is, = log. ·3247 — 1: but the index of log. ·3247, as just shown, is —1; hence the index of log. ·03247 is —2. Similarly, the index of log. ·003247 is −3, that of log. ·0003247 is −4, and so on. Consequently, whatever be the number whose log. is required, we find the index of that log. thus:—

Count how many places the *leading figure* of the number is from the unit's place, and put down what is counted, for index: the figure thus put down will be *plus* if the counting is towards the right, and *minus* if towards the left; or the rule is expressed otherwise, thus:—"Place your pen between the first and second FIGURE (NOT CIPHER), and count *one* for each figure or cipher, until you come to the decimal point; the number this gives will be the index. If you count to the right the index is positive, if to the left it is negative." (See page 522).

The decimal part of the log. of a number is always found against that number in the table, which is to be referred to so soon as the index is written down. Because of this ready way of ascertaining what the index of the log. of a number is, it is not necessary to insert more than the mantissa, or decimal part of the log., in the table: but it is to be observed that this decimal part is always *plus*. For instance, having first written down the proper index, we find, on referring to the table, that

$$\text{log. } {\cdot}3247 = -1 + {\cdot}511482,\ \text{log. } {\cdot}03247 = -2 + {\cdot}511482,$$
$$\text{log. } {\cdot}003247 = -3 + {\cdot}511482,\ \&c.$$

But a more compact and convenient way of writing these logs. is this, namely:

$$\text{log. } {\cdot}3247 = \bar{1}{\cdot}511482,\ \text{log. } {\cdot}03247 = \bar{2}{\cdot}511482,\ \text{log.}$$
$${\cdot}003247 = 3{\cdot}511482,\ \&c.$$

And in this form the logarithms of numbers less than unity—that is, of numbers of which all the figures are decimals—are always used in practice.

The logarithmic operation for finding a product from its factors suggests that for finding any power of a known root, or any root of a known power: thus, since n^p merely denotes the product of p factors, each equal to the number n, we have log. $n^p = p$ log. n; and since if $n^{\frac{1}{p}} = r$, we must have $n = r^p$, it follows that log. $n = p$ log. r, and consequently that log. $r = \frac{\text{log. } n}{p}$. We thus derive the following practical rules for performing the more troublesome operations of arithmetic in a short and easy manner by help of a table of logarithms.

1. *Multiplication.*—From the table take the log. of each factor; add these logs. together: the *sum* will be the log. of the *product* of the factors; and against this log. in the tables, will be found the product sought.

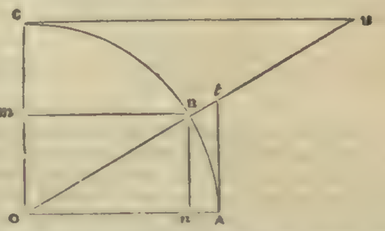
2. *Division.*—Subtract the log. of the divisor from the log. of the dividend: the remainder is the log. of the *quotient.*

3. *Powers, and Roots.*—Multiply the log. of the number to which the exponent is attached, by that exponent, whether it be integral or fractional: the result will be the log. of the power or root.

As to the means which algebraists have devised for constructing such a table as that here referred to, the student may consult the Chapter on Logarithms and Series in the Section on Mathematics: it has been thought sufficient, in this introductory article, to show the general principles on which tables of logs. are based, without entering into details as to their actual formation. These tables are given at page 525, *et seq.*

ON THE SINES, COSINES, TANGENTS, &c., OF ANGLES.—In the Section on Mathematics (p. 613) the trigonometrical terms, sine, cosine, tangent, &c., are defined in two different ways. For the purposes of navigation, the first of these ways will be found to be the more convenient, as well as the more intelligible to the learner. Taking the diagram at page 613, we may explain the lines there figured as follows (Fig. 1):—

Fig. 1.



Sine.—The sine of an angle A O B, or of the arc A B, which measures that angle, is the straight line B*n* drawn from the extremity B of the arc, at right angles to, and terminating in, the radius O A, drawn to the commencement of the arc.

Cosine.—And the cosine of the same angle is the portion O*n* of the radius between the centre O and the foot *n* of the sine.

Tangent.—The tangent of the same angle is the line A*t*, touching the measuring arc A B at its commencement A, and terminating in the prolonged radius drawn through the extremity B.

Secant.—And this prolonged radius, that is, the line O*t*, is the secant of the angle A O B.

As A is regarded as the origin or commencement of the arc which measures the angle A O B, so C is regarded as the commencement of the *complement* of that arc; that is, of the arc which, united to the former, makes up a quadrant or 90°. The sine, cosine, &c., of *this* arc (or of the angle which it measures) are the cosine, sine, &c., of the former, as is obvious; and the tangent and secant of the latter are called the cotangent and cosecant of the former; thus:—

Cotangent.—The cotangent of the angle A O B, or of the arc A B which measures it, is the line C*u*, drawn to touch the complement of that arc at C, and terminating in the secant O*t*, or in that secant prolonged.

Cosecant.—And O*u*, drawn from the centre through B, up to the cotangent, is the cosecant of the angle A O B, or of the arc A B.

The lines here defined refer to circular arcs and the angles which they measure. The sides O A, O B of the angle at O, are each equal to the radius of the circle here considered: if these sides or radii had been of any other length, no change would have taken place in the angle at O; neither would any change have taken place in the *number of degrees* contained in the measuring arc; but the actual length of that arc would have varied, being longer than A B in a circle of longer radius, and shorter than A B in a circle of shorter radius. On this account the sine, cosine, &c., of an *arc* are not lines of fixed lengths for a fixed number of degrees: a degree of one circle is double the length of a degree of another of only half the radius of the former. Now, in trigonometry, angles alone, independently of all connection with their measuring arcs, are almost exclusively the subjects of consideration; and it is therefore necessary that an invariable angle should have an invariable sine and cosine, &c. It is agreed, therefore, that the trigonometrical sine, cosine, &c., should be estimated according to the scale O A = 1: in other words, that the abstract number 1, being the numerical representative of the radius O A, the abstract numbers, which, conformably to this scale, are the numerical expressions for B*n*, O*n*, &c., should be called the *trigonometrical* sine, cosine, &c., of the angle A O B. The lines in the preceding diagram, considered *as* lines, are the *geometrical* sine, cosine, tangent, &c., of the arc A B, or of the angle A O B *to radius* O A; but viewed as merely linear representations of abstract numbers, O A denoting the abstract unit, they are the trigonometrical sine, cosine, tangent, &c., of the angle A O B.

It is thus that the terms sine, cosine, tangent, &c., as employed in trigonometry, are regarded as *ratios;* for from what has just been said,

$$\text{O A} : \text{B}n :: 1 : \frac{\text{B}n}{\text{OA}}, \text{ the trigonometrical sine of O.}$$

$$\text{O A} : \text{O}n :: 1 : \frac{\text{O}n}{\text{OA}}, \ldots\ldots\ldots \text{ cosine } \ldots$$

$$\text{O A} : \text{A}t :: 1 : \frac{\text{A}t}{\text{OA}}, \ldots\ldots\ldots \text{ tangent } \ldots$$

&c. &c.

And generally any geometrical sine, cosine, &c., divided by the radius of the arc with which such sine, cosine, &c., is connected, is the trigonometrical sine, cosine, &c., of the angle measured by that arc: but in trigonometry, the terms sine of an angle, cosine of an angle, &c.—without any prefix—are sufficiently explicit, because when geometrical lines are meant, the fact is always stated.

The learner will find it an assistance to keep the diagram at page 1037 before his mind in investigating the relations among the trigonometrical quantities now considered, regarding the lines defined above as merely the linear representatives of the abstract numbers, sine, cosine, &c., the radius being the representative of the abstract number 1; for he will have the aid of geometry to assist him in establishing the fundamental principles of trigonometry. Thus, from Euclid, Prop. 47, Book I., we know that

$$\text{B}n^2 + \text{O}n^2 = \text{O B}^2,\ \text{O A}^2 + \text{A}t^2 = \text{O}t^2,\ \text{O C}^2 + \text{C}u^2 = \text{O}u^2,$$

so that we have from the first of these,

$$\text{sin.}^2\text{O} + \text{cos.}^2\text{O} = 1 \quad \ldots \ (1)$$

from the second,
$$1 + \text{tan.}^2\text{O} = \text{sec.}^2\text{O} \quad \ldots \ (2)$$

and from the third,
$$1 + \text{cot.}^2\text{O} = \text{cosec.}^2\text{O} \quad \ldots \ (3)$$

Also from the property that the sides about the equal angles of equiangular triangles are proportional (Euc., Prop. 4, B. VI.), we find, upon comparing together the equiangular right-angled triangles B O*n*, *t*O A, as also the equiangular right-angled triangles B O*m*, *u*O C, that

$$\frac{\text{sin. O}}{\text{cos. O}} = \frac{\text{tan. O}}{1}, \text{ that is, tan. O} = \frac{\text{sin. O}}{\text{cos. O}} \quad \ldots \ (4)$$

$$\frac{\text{cos. O}}{\text{sin. O}} = \frac{\text{cot O}}{1}, \ \ldots\ldots \text{ cot. O} = \frac{\text{cos. O}}{\text{sin. O}} \quad \ldots \ (5)$$

$$\frac{\text{sin. O}}{1} = \frac{1}{\text{cosec. O}}, \ \ldots \text{ sin. O} = \frac{1}{\text{cosec. O}}; \quad \ldots \ (6)$$

$$\frac{\text{cos O}}{1} = \frac{1}{\text{sec. O}}, \ \ldots\ldots \text{ cos. O} = \frac{1}{\text{sec. O}} \quad \ldots \ (7)$$

These fundamental relations will suffice for our present purpose. We learn—

From (1), that sin. O $= \sqrt{(1 - \text{cos.}^2\text{O})}$; and cos. O $= \sqrt{(1 - \text{sin.}^2\text{O})}$.

From (2), that sec. O $= \sqrt{(1 + \text{tan.}^2\text{O})}$; and tan. O $= \sqrt{(\text{sec.}^2\text{O} - 1)}$.

From (3), that cosec. O $= \sqrt{(1 + \text{cot.}^2\text{O})}$; and cot. O $= \sqrt{(\text{cosec.}^2\text{O} - 1)}$.

From (4) (5), that tan. O $= \frac{1}{\text{cot. O}}$; and cot. O $= \frac{1}{\text{tan. O}}$.

And from (6) (7), that cosec. O $= \frac{1}{\text{sin. O}}$; and sec. O $= \frac{1}{\text{cos. O}}$.

For the more general theory of the trigonometrical ratios, the student is referred to the Chapters on Trigonometry.

ON THE TABLES OF SINES, COSINES, TANGENTS, &c.—The numerical values of the trigonometrical lines considered above are computed according to the scale radius = 1. and are arranged in tables called tables of *natural sines, cosines,* &c. As tangents, cotangents, secants, and cosecants, may be deduced with but little trouble from sines and cosines, as shown above, and as, moreover, the *natural* sines, cosines, &c., are seldom employed in navigation, the tables of these, which accompany books on that subject, usually give only the sines and cosines. But the tables which furnish the logarithms of these, being indispensable in most of the calculations performed by the mariner, are always given in a complete form, and contain the logarithmic sines, cosines, tangents. cotangents, and cosecants of all angles from 0° up to 90°. In the former tables the numerical values of the sine and cosine of every angle between 0° and 90° is less than 1, because the radius itself is only 1: the logarithms of all these sines and cosines would, therefore, have negative indices (p. 1037), which is an inconvenience in practice. To remedy this, the logarithmic tables are computed, not to the scale rad. = 1, but to the scale rad. = 10^{10}, which is so large as to preclude the possibility of any logarithm in the tables requiring a negative index: in these tables, the index, or integral part of the log., is always inserted before the decimal part; while in the logs. of numbers, as already stated, the index is omitted.*

Now, in investigating the rules and formulæ of trigonometry, the argument always proceeds on the supposition that the numerical value of the radius is 1; and consequently, in the practical application of those rules and formulæ, it must be borne in remembrance, when logs. are used, that the radius is taken 10^{10} times as great—that is, instead of the log. of a natural sine, cosine, &c., we employ in reality the log. of 10^{10} times that natural sine, cosine, &c. For instance, if we call any natural sine, cosine, &c., p, the logarithmic tables will give us, not log. p, but log. $10^{10}p$—that is, log. 10^{10} + log. p; or since log. 10^{10} is 10 log. $10 = 10 \times 1$, the tables will give us 10 + log. p, so that the log. of every natural sine, cosine, &c., is increased by 10. In order, therefore, that our practical results may agree with our theoretical formulæ, these superfluous 10's—introduced merely to avoid negative logs.—must be suppressed at the close of our operation. If, in our work, as many of these logarithmic trigonometrical quantities are used subtractively as are used additively, then, of course, as the superfluous 10's destroy one another, no correction of the final result becomes necessary; but if more of these logs. are additive than subtractive, as many 10's as mark the excess of additive over subtractive logs. must be suppressed in the result; and if the excess be in favour of the subtractive logs., so many 10's must be introduced. The suppression or the introduction of a 10, in any amount, is so easy a matter, that the trouble attending it is not worth consideration, in comparison with the advantages gained by the avoidance of logs. with negative indices.

What has now been said being understood, let us suppose that, by aid of the algebraic series and formulæ in our Section on Mathematical Science, the two sets of trigonometrical tables have been constructed—namely, tables of natural sines, cosines, &c., and tables of logarithmic sines, cosines, &c.; and let us endeavour to see a little into the use and advantages of what is thus supplied, in the calculations of plane triangles.

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* For tables of logarithmic sines, cosines, &c., see p. 666, *et seq.*

And first as to the table of natural sines, cosines, &c. By referring to the diagram at p. 1037, we see that this table supplies, already computed for us, the numerical lengths of the sides of two similar right-angled triangles O B*n*, O*t*A, whatever be the angle O, from O = 0°, up to O = 90°. We may be quite sure, therefore, whatever be the magnitude or shape of any right-angled triangle met with in practice, that *two* triangles, *equiangular to it*, will always be found in the table; the sides of the one, O B*n*, being computed to the scale O B, the hypotenuse, = 1, and the sides of the other to the same scale, the base O A, = 1. The table thus furnishes us with the perpendicular and base of every possible right-angled triangle whose hypotenuse is 1, and also with the perpendicular and hypotenuse of every possible right-angled triangle whose base is 1.

Now the sides about the equal angles of equiangular triangles are proportional (Euc., Prop. 4, Book VI.), so that if *one* side only of a triangle be known, and *all* the sides of another, equiangular to it, be given, we can compute the unknown sides of the former by simple proportion. It follows, therefore, that if one side only, of any right-angled triangle, with which we may be engaged in practice, be measured, we shall be able to calculate each of the other sides, in this manner, provided only we know where to look for the triangle, equiangular to the proposed triangle, in the table.

As already observed, there are always *two* such equiangular triangles in every complete table of natural sines, cosines, &c., so that we have a choice as to which shall be compared with the triangle to be calculated; but we must, of course—take which ever we may—be careful to compare base with base, perpendicular with perpendicular, and hypotenuse with hypotenuse. The sides of the two triangles O B*n*, O*t*A, computed in the table, do not, however, go by these names: the base of the triangle O B*n* is called *cosine* of the angle O, the perpendicular is called *sine* of the angle O, and the hypotenuse is called *radius*, or 1. Also the base of the similar triangle O*t*A is *radius* or 1, the perpendicular is *tangent* of the angle O, and the hypotenuse is *secant* of the angle O: the angle O, expressed in degrees and minutes, is thus a sufficient guide to direct us to that part of the table where the equiangular triangle we are in search of is to be found.

It is always best to fix upon that one of the two tabular triangles in which the unit-length, or radius, corresponds to the given side of the triangle proposed for calculation. Thus, if the given side be the hypotenuse, we should compare our triangle with the tabular triangle O B*n*, because in this the hypotenuse is 1; we should then have to compare the perpendicular of the proposed triangle with B*n*, that is, with sin. O, and the base with O*n*, that is, with cos. O. For example, if the angle equivalent to O, and the hypotenuse were given to find the perpendicular and the base, then, referring to the necessary particulars, on the page of the table headed by the degrees and minutes in the given angle, we should be supplied with the first three terms of the proportion.

1 : sin. O : : given hyp. : required perp.,

Consequently the required perp. $= \dfrac{\text{hyp.} \times \text{sin. O}}{1}$ = hyp. × sin. O.

Also, 1 : cos. O : : given hyp. : required base,

Consequently the required base $= \dfrac{\text{hyp.} \times \text{cos. O}}{1}$ = hyp. × cos. O.

As the first term of each proportion is 1, all division is thus avoided; and the operation reduced merely to the multiplication together of two factors.

On the other hand, if the base were given to find the perpendicular and hypotenuse, the other tabular triangle, O*t*A, would be the better to compare with the proposed one, because the radius or unit-line here is O A, the *base*, corresponding to the side given, and the proportions would be 1 : tan. O : : given base : required perp.,

and 1 : sec. O : : given base : required hyp.,

∴ required perp. = base × tan. O, and required hyp. = base × sec. O.

As to the tables of logarithmic sines, cosines, &c., little need be said in addition to what has already been stated; they furnish the logarithms of the tabular numbers in the table of natural sines, cosines, &c., with the addition of 10 to each logarithm, as before explained. By using the logarithmic tables, we convert the multiplication of the two factors adverted to above, into the addition of the corresponding logarithms. Sufficient practical details of these operations will be given in the next article.

It may be noticed here, however, that in what is said above, as to the assistance afforded by trigonometrical tables, our observations have had exclusive reference to *right-angled* triangles; we shall shortly see that they are equally available for oblique-angled triangles; but, it will strike the learner as a remarkable fact, that, notwithstanding the complicated character of the curve, traced on the surface of the globe by the course of a ship, all the calculations in reference to the different sailings, involve only *right-angled triangles*—the simplest kind of triangles with which trigonometry has to deal; we shall, therefore, give a distinct article on the calculation of the sides and angles of right-angled triangles.

TO CALCULATE THE SIDES AND ANGLES OF RIGHT-ANGLED TRIANGLES.—The sides and angles of any triangle make up what are called the six *parts* of the triangle; and if any three of these six parts, provided they are not the three angles, be given, the remaining three can be found by calculation. In a *right-angled* triangle, one of the six parts is always known—namely, the right angle; so that a side and one of the acute angles, or two sides, being measured, we can always calculate the remaining parts.

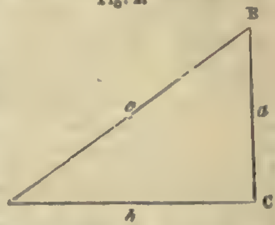
The reason that the three angles alone will not suffice for the determination of the other three parts—the three sides—is obvious from the simplest principles of geometry; for all *equiangular* triangles are alike, as respects the equality of the angles; so that, from the angles alone, we could not conclude to which one, out of an infinite variety of similar triangles, the proposed angles are considered to belong, for they belong, in reality, equally to all.

In a right-angled triangle the given parts must therefore be either :—

1. A side and one of the acute angles, or,
2. Two of the sides, to find the remaining parts.

I. *When the Hypotenuse and one of the Acute Angles are given.*

Fig. 2.



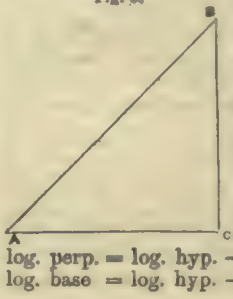
Let the hypotenuse A B (Fig. 2), and the angle A be given; then turning to the diagram at page 1037, we should compare the given side A B, with the radius O B, as directed above; so that O B *n* would be the tabular triangle equiangular to the triangle A B C. And, therefore, if *c* denote the numerical length of A B, since the numerical length of O B is 1, A B = O B × *c*; consequently, *a* and *b* being the numerical lengths of B C, and A C, we have, since the like sides are proportional,

B C = B *n* × *c*, and A C = O *n* × *c*, that is,

a = *c* sin. A, and *b* = *c* cos. A.

Hence the following rule :—

Fig. 3.



RULE.—Multiply the given hypotenuse by the sine of the angle at the base; the product is the perpendicular. Multiply the given hypotenuse by the cosine of the angle at the base: the product is the base.

Or work by the following formula,

perp. = hyp. × sin., ang. at base; base = hyp. × cos., ang. at base,

which by logarithms will be,

log. perp. = log. hyp. + log. sin., ang. at base − 10;
log. base = log. hyp. + log. cos., ang. at base − 10.

Example 1.—In the right-angled triangle A B C (Fig. 3), are given A B = 480 feet, and the angle A = 53° 5′; required the perpendicular B C, and the base A C.

1. *To find the perpendicular* B C.

Without logarithms.

B C = A B sin. A.

Sin. A, 53° 5′, = ·7995

A B = 480

639600

31980

∴ B C = 383·7600

With logarithms.

Log. B C = log. A B + log. sin. A − 10.

−10

Log. A B = log. 480 = 2·6812

Log. sin. A, 53° 5′, = 9·9028

Log. B C log. = 383·8 = 2·5840

Hence B C is 383·8 feet.

2. *To find the base* A C.

Without logarithms.

A C = A B cos. A.

Cos. A, 53° 5′, = ·6006

A B = 480

480480

24024

∴ A C = 288·2880

With logarithms.

Log. A C = log. A B + log. cos. A − 10.

−10

Log. A B = log. 480 = 2·6812

Log. cos. A, 53° 5′, = 9·7786

Log. A C = log. 288 3 = 2·4598

Hence A C is 288·3 feet.

In solving the preceding example we have taken only *four* places of decimals from the tables; this is a number quite sufficient for all the ordinary purposes of navigation, and it will be remembered that the present introductory article is only preparatory to the discussion of that subject.

If, instead of the angle A at the base, the angle B at the vertex had been given, the operation would have been much the same as that above; for, since A = 90° — B, sin. A = cos. B; and cos. A = sin. B, because the sine and cosine of an angle are respectively the same as the cosine and sine of the *complement* of that angle; so that where we have used above sin. A, we should have used its equal cos. B; and where we have used cos. A we should have substituted its equal sin. B. This must be observed in solving the last three of the following examples.

2. In the triangle A B C (last figure) are given A B = 291, angle A = 47° 55′, to find A C, B C.
Ans. A C = 195, B C = 216.

3. Given A B = 480, angle A = 53° 8′, to find A C, B C.
Ans. A C = 288, B C = 384.

4. Given A B = 521, angle B = 36° 6′, to find A C, B C.
Ans. A C = 307, B C = 421.

5. Given A B = 645, angle B = 50° 50′ to find A C, B C.
Ans. A C = 500, B C = 407·4.

6. Given A B = 98, angle B = 33° 12′, to find A C, B C.
Ans. A C = 53·66, B C = 82.01.

II.—*When the base or perpendicular, and one of the acute angles are given.*

Let the base A C, and the angle A be given; then, referring to the diagram at p. 1037, Fig. 1, we are to compare the given side A C with the radius, or unit-line, O A; so that O*t*A is the tabular triangle now to be referred to. As A C is *b* times O A, therefore, because the proposed and tabular triangles are equiangular, B C must be *b* times A*t*, and A B *b* times O*t*; that is,

$$a = b \text{ tan. A, and } c = b \text{ sec. A.}$$

Hence the following rule:—

RULE.—Multiply the given base by the tangent of the angle at the base: the product will be the perpendicular. Multiply the given base by the secant of the angle at the base: the product will be the hypotenuse.

In a right-angled triangle either of the two perpendicular sides may be regarded as the base, and the other as the perpendicular; so that two distinct rules would be unnecessary. And as in the former case, when the vertical angle B is given instead of the base angle A, the former may take the place of the latter in the operation, provided only that we write cotan. for tan., and cosec. for sec.

But, as already observed at page 1038, the tables of *natural* sines and cosines, which accompany books on navigation, do not, in general, furnish the natural tangents and secants; so that, if this table be employed in the work, we must make use of the relations established at page 1038, namely,

$$\text{tan. A} = \frac{\text{sin. A}}{\text{cos. A}},\ \text{sec. A} = \frac{1}{\text{cos. A}};\ \text{cotan. B} = \frac{\text{cos. B}}{\text{sin. B}},$$

$$\text{cosec. B} = \frac{1}{\text{sin. B}}.$$

The working formulæ for the present case are as follows:—

perp. = base × tan. ang. at base; hyp. = base × sec. ang. at base.

Or, by logarithms,

log. perp. = log. base + log. tan. ang. at base — 10;

log. hyp. = log. base + log. sec. ang. at base — 10.

The first of these formulæ, if the table referred to be limited to natural sines and cosines only, must be replaced by

$$\text{perp.} = \frac{\text{base} \times \text{sin. ang. at base}}{\text{cos. ang. at base}};\ \text{hyp.} = \frac{\text{base}}{\text{cos. ang. at base}},$$

agreeably to the relations given above.

EXAMPLES.

1. In the right-angled triangle A B C, Fig. 3, are given the base A C = 327, and the angle A = 54° 17′: required the perpendicular B C, and the hypotenuse A B.

1. *To find the perpendicular* B C.

Without logarithms.

$$\text{B C} = \frac{\text{A C sin. A}}{\text{cos. A}}$$

sin. A 54° 17′, = ·8119

A C = 327

56833 (See the remarks at page 1042.)

16238

24357

cos. 54° 17′ = ·5838)265·4913(454·8 = B C.

With logarithms.

log. B C = log. A C + log. tan. A — 10.

— 10

A C = 327 . . . . . . 2·5145

tan. A, 54° 17′ . . . . . 10·1433

log. B C, 454·8 . . . . . 2·6578

Hence B C = 454·8

2. *To find the hypotenuse* A B.

$$\text{A B} = \frac{\text{A C}}{\text{cos. A}}$$

·58,3,8) 327 (560 = A B

2919

351

350

1

$$\text{log. A B} = \text{log. A C} + \text{log. sec. A} - 10.$$

$$\begin{array}{lr}
 & -10 \\
\text{A C} = 327 \;\ldots\ldots & 2.5145 \\
\text{sec. A, } 54^\circ\, 17' \;\ldots\ldots & 10{\cdot}2338 \\
\hline
\text{A B, } 560 \;\ldots\ldots & 2{\cdot}7483 \\
\hline
\end{array}$$

The logarithmic operation might have been performed as readily by using the formula for A B, as before, which gives

$$\text{log. A B} = \text{log. A C} - \text{log. cos. A} + 10.$$

The work by this formula will be as follows :—

$$\begin{array}{lr}
 & 10 \\
\text{A C} = 327 \;\ldots\ldots & 2\,5145 \\
\text{cos. A, } 54^\circ\, 17' \;\ldots\ldots & -9{\cdot}7662 \\
\hline
\text{A B} = 560 \;\ldots\ldots & 2{\cdot}7483 \\
\hline
\end{array}$$

2. In the triangle A B C (Fig. 3), are given A C = 195, and the angle A = 47° 55′, to find B C and A B.
Ans. B C = 216, A B = 291.

3. Given A C = 288, and the angle A = 53° 8′, to find B C and A B. Ans. B C = 384, A B = 480.

4. Given A C = 421, and the angle A = 36° 6′, to find B C and A B. Ans. B C = 307, A B = 521.

5. Given A C = 625, and the angle B = 41° 15′, to find B C and A B. Ans. B C = 713, A B = 948.

6. Given the base of a right-angled triangle 346½, and the opposite angle 54° 36′ : required the perpendicular and hypotenuse. Ans. Perp. 246·2, Hyp. 425·1.

III. *When the hypotenuse and one of the other sides are given.*

Let the hypotenuse A B (Fig. 3), and the perpendicular B C be given ; then comparing A B with the radius or unit line O B in the diagram at page 1037 (Fig. 1), the tabular triangle O B*n* will be that to which the proposed triangle is to be compared : and since A B is *c* times O B, therefore B C is *c* times B*n* ; that is to say,

$$a = \text{B C} = c \text{ sin. A, therefore sin. A} = \frac{a}{c}.$$

If the base were given instead of the perpendicular, then we should have

$$b = \text{A C} = c \text{ cos. A, therefore cos. A} = \frac{b}{c}.$$

The rule is therefore as follows :—

RULE.—Divide the perpendicular by the hypotenuse : the quotient will be the sine of the angle at the base. Divide the base by the hypotenuse : the quotient will be the cosine of the angle at the base.

An angle being thus determined, the remaining side of the triangle may be found by either of the preceding rules. Or, without first finding an angle, the third side of any right-angled triangle may be determined from the other two, by the 47th of Book I. of *Euclid ;* for since $c^2 = a^2 + b^2$; $\therefore a^2 = c^2 - b^2$, and $b^2 = c^2 - a^2$; the working formulæ for finding the angles, when the hypotenuse and a side are given, are

$$\text{sin. ang. at base} = \frac{\text{perp.}}{\text{hyp.}} : \text{cos. ang. at base} = \frac{\text{base}}{\text{hyp.}}.$$

Or, by logarithms,

log. sin. ang. at base = log. perp. − log. hyp. + 10 ;
log. cos. ang. at base = log. base − log. hyp. + 10.

EXAMPLES.

1. In the right-angled triangle A B C (Fig. 3), are given the hypotenuse A B = 480, and the perpendicular B C = 384, to find the angles A, B, and the base A C.

1. *To find the angles* A, B.

Without logarithms.

$$\text{sin. A} = \frac{\text{BC}}{\text{AB}}.$$

$$\begin{array}{l}
480)384({\cdot}8 = \text{sin. } 53^\circ\, 8' \\
384 \\
\hline
\end{array}$$

With logarithms.

$$\text{log. sin. A} = \text{log. B C} - \text{log. A B} + 10$$

$$\begin{array}{lr}
 & 10 \\
\text{B C, } 384 \;\ldots\ldots & 2{\cdot}5843 \\
\text{A B, } 480 \;\ldots\ldots & -2{\cdot}6812 \\
\hline
\text{sin. A, } 53^\circ\, 8' \;\ldots & 9{\cdot}9031 \\
\hline
\end{array}$$

Consequently A = 53° 8′, ∴ B = 90° − 53° 8′ = 36° 52′.

2. *To find the base* A C.

A C = A B cos. A
cos. A, 53° 8′, = ·6
∴ 480 × ·6 = 288 = A C

A C is thus determined with very little troublo.

$$\text{log. A C} = \text{log. A B} + \text{log. cos. A} - 10$$

$$\begin{array}{lr}
 & -10 \\
\text{A B, } 480 \;\ldots\ldots & 2{\cdot}6812 \\
\text{cos. A, } 53^\circ\, 8' \;\ldots\ldots & 9{\cdot}7781 \\
\hline
\text{A C, } 288 \;\ldots\ldots & 2{\cdot}4593 \\
\hline
\end{array}$$

To find the base without the aid of the angle, we have $\text{A C}^2 = \text{A B}^2 - \text{B C}^2 = (\text{A B} + \text{B C})(\text{A B} - \text{B C}) = 864 \times 96 = 82944$

$$\begin{array}{lr}
\dot{8}2\dot{9}4\dot{4}\,(288 \text{ ; or by logs., log. } 864 = & 2{\cdot}9365 \\
4 \quad \ldots\ldots \quad \text{log. } 96 = & 1{\cdot}9823 \\
\hline
48)429 & 2)4{\cdot}9188 \\
384 & \\
\hline
568)4544 & \text{log. } 288 = 2{\cdot}4594 \\
4544 & \\
\hline
\end{array}$$

Consequently A C = 288. The sum of the two logs. on the right is divided by 2, because the half of the log. of a number is the log. of the square root of that number.

2. Given the hypotenuse A B = 237, and the perpendicular B C = 197, to find the angles and the base.
Ans. A = 56° 13′, B = 33° 47′, Base = 132.

3. Given the hypotenuse A B = 645, and the perpendicular B C = 407·4 to find the angles and the base.
Ans. A = 39° 10′, B = 50° 50′, Base = 500.

3. Given the hypotenuse A B = 400, and the base A C = 236, to find the angles and the perpendicular.
Ans. A = 53° 51′, B = 36° 9′, Perp. = 323.

4. Given the hypotenuse A B = 54·68, and the base A C = 35·5, to find the angles and the perpendicular.
Ans. = 49° 31′, B = 40° 29′, Perp. = 41·6.

5. Given the hypotenuse of a right-angled triangle = 779·8, and the perpendicular = 725, to find the vertical angle and the base. Ans. 21° 36′, and 287.1.

6. Given the hypotenuse = 780·6, and the base = 607·6, to find the angle at the base, and the perpendicular. Ans. 38° 53′, and 490.

IV. *When the base and perpendicular are given.*

The base A C = *b*, and the perpendicular B C = *a* being given, we are to compare the triangle with the tabular triangle O A*t*, of which the base O A = 1 : we thus have B C = *b* times A*t*, that is

$$a = b \text{ tan. A } \therefore \text{tan. A} = \frac{a}{b}.$$

Consequently the rule for finding the angle A at the base is as follows :—

RULE.—Divide the perpendicular by the base : the quotient will be the tangent of the angle at the base. Or, divide the base by the perpendicular : the quotient will be the tangent of the angle at the vertex. The hypotenuse may be found without first finding the angles by *Euclid*, 47th of Book I. : thus $c^2 = a^2 + b^2$. The formulæ for finding either angle is therefore

$$\text{tan. ang. at base} = \frac{\text{perp.}}{\text{base}} \text{ ; tan. ang. at vertex} = \frac{\text{base}}{\text{perp.}}$$

Or, by logarithms,

log. tan. ang. at base = log. perp. — log. base + 10;
log. tan. ang. at vertex = log. base — log. perp. + 10.

EXAMPLES.

1. In the right-angled triangle A B C (Fig. 3), page 1039, are given the base A C = 288, and the perpendicular B C = 384, to find the angles and the hypotenuse.

1. *To find the angles* A, B.

Without logarithms.

$$\text{tan. A} = \frac{\text{BC}}{\text{AC}}$$

$$\begin{array}{l} 288)384(1{\cdot}3333\ \ldots = \text{tan. A, } 53^\circ\,8'. \\ 288 \\ \hline 96 \\ 864 \\ \hline 96 \end{array}$$

With logarithms.

$$\text{log. tan. A} = \text{log. B C} - \text{log. A C} + 10.$$

$$\begin{array}{lr} & 10 \\ \text{B C, } 384\ \ldots\ldots\ldots & 2{\cdot}5843 \\ \text{A C, } 288\ \ldots\ldots\ldots & -\ 2{\cdot}4594 \\ \hline \text{tan. A, } 53^\circ\,8'\ \ldots\ldots & 10{\cdot}1249 \end{array}$$

$\therefore$ A = 53° 8′, and B = 90° − A = 36° 52′.

2. *To find the hypotenuse* A B.

$$\text{A B} = \sqrt{(\text{A C}^2 + \text{B C}^2)}$$

$$\begin{array}{rl} 288^2 = & 82944 \\ 384^2 = & 147456 \\ \hline & 230400(480 = \text{A B} \\ & 16 \\ \hline 88) & 704 \\ & 704 \\ \hline & 00 \end{array}$$

$$\text{A B} = \text{A C sec. A}.$$

$$\therefore \text{log. A B} = \text{log. A C} + \text{log. sec. A} - 10.$$

$$\begin{array}{lr} & -10 \\ \text{A C, } 288\ \ldots\ldots\ldots & 2{\cdot}4594 \\ \text{sec. A, } 53^\circ\,8'\ \ldots\ldots & 10{\cdot}2219 \\ \hline \text{A B, } 480\ \ldots\ldots\ldots & 2{\cdot}6813 \end{array}$$

In the foregoing example, a table of natural tangents and secants is necessary to enable us to find the required parts without logarithms, and in the shortest manner; tan. A being determined as above, and then A B from the formula A C sec. A.

In all works on Navigation, as also in most books on Trigonometry, the calculation of the several cases of right-angled triangles is performed exclusively by logarithms. The learner will perceive, from the illustrations furnished above, that, in general, the work is more expeditious without logarithms than with them; and that it would be of advantage to the practical navigator if tables more comprehensive than those generally given—that is, tables including the natural tangents and secants—were bound up with books on navigation. As the preceding examples show, a single reference to such a table suffices for the discovery of the unknown part; whereas *three* references to the logarithmic tables are necessary. In the latter mode of working, fewer figures may appear on the paper, but the time occupied in two searchings in the table, out of three, is saved; and, in operations of this kind, referring to tables, and transcribing the figures, constitute the principal part of the trouble.

The greatest amount of arithmetical work involved in any of the preceding solutions occurs in the example at p. 1040; but the multiplication and division operations there indicated, according to the common beaten track, may be pruned of many superfluous figures by using the *contracted* methods: thus to multiply ·8119 by 327, in the shortest way, without sacrificing accuracy in the result, write the figures of the multiplier in reverse order thus, 723; and multiply as in the margin, rejecting from the multiplicand a figure at every step, after the first, but attending to the *carryings* from the rejected figures. And similarly for contracted division, as exemplified at the page referred to, and again here in the margin. It will be observed, however, that the whole of the division part of this work would have been saved if tan. 54° 17′ had been supplied by the tables: the entire work would then have stood as here annexed.

$$\begin{array}{rr} \text{sin. } 54^\circ\,17' = & {\cdot}8119 \\ & {\cdot}723 \\ \hline & 24357 \\ & 1624 \\ & 568 \\ \hline \text{cos. } 54^\circ\,17' = {\cdot}5838) & 265{\cdot}49(454{\cdot}8 \\ & 23352 \\ \hline & 3197 \\ & 2919 \\ \hline & 278 \\ & 233 \\ \hline & 45 \\ & 46 \end{array}$$

A person but moderately expert in the simple operations of arithmetic could execute the work in the margin in less time than it would take to refer to the logarithmic tables, and to transcribe therefrom a single number. And then the numerical process, being placed permanently before the eye, is very easily revised if error be suspected. These obvious advantages are of sufficient importance to justify the entire abandonment of logarithms in those calculations of Navigation—and they include nearly all—in which right-angled triangles only are concerned. When oblique-angled triangles are the subjects of computation, the case is different; as the logarithmic operation has then, with few exceptions, the advantage over that with the natural numbers, as will be sufficiently seen in the next article.

$$\begin{array}{rr} \text{tan. } 54^\circ\,17' = & 1{\cdot}3908 \\ & 723 \\ \hline & 41724 \\ & 2782 \\ & 974 \\ \hline \text{B C} = & 451{\cdot}80 \\ \hline \end{array}$$

2. In the right-angled triangle A B C (Fig. 3), are given the base A C = 101·9, and the perpendicular B C = 195·4, to find the angles and the hypotenuse.
   Ans. A = 62° 27′, B = 27° 33′, A B = 220·3.

3. Given the base A C = 659·8, and the perpendicular B C = 520·5, to find the other parts.
   Ans. A = 38° 16′, B = 51° 44′, A B = 840·4.

4. Given the base A C = 35·5, and the perpendicular B C = 41·6, to find the other parts.
   Ans. A = 49° 31′, B = 40° 29′, A B = 54·68.

5. Given the base A C = 32·76, and the perpendicular B C = 46·58, to find the vertical angle B.
   Ans. B = 35° 7′.

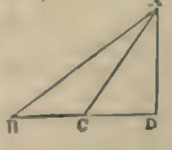
6. Given the base A C = 53·66, and the perpendicular B C = 82·01, to find the angles and the hypotenuse.
   Ans. A = 56° 48′, B = 33° 12′, A C = 98.

Miscellaneous Examples in the Calculation of Right-angled Triangles.

1. The angle of elevation A C D (Fig. 4), of the top A, of a tower, was found to be 55° 54′; and from the station B, 100 feet from C in the same straight line D B, the angle of elevation A B D was found to be 33° 20′: what was the height of the tower?

Solution without Logarithms.

Fig. 4.



By right-angled triangles,

D C = A D tan. C A D, D B = A D tan. B A D,

$\therefore$ D B — D C = B C = A D (tan. B A D — tan. C A D).

But B A D = 90° — 33° 20′ = 56° 40′, and C A D = 90° − 55° 54′ = 34° 6′,

$\therefore$ B C = 100 = A D (tan. 56° 40′ — tan. 34° 6′).

By referring to a table of natural tangents, we find

Tan. 56° 40′ = 1·5204
Tan. 34° 6′ = ·6771

The difference = 8,4,3,3) 100 (118·6 = A D.
8433
1567
843
724
675
49
51

The division here is by the contracted method, recommended before.

Hence the height of the tower is 118·6 feet, very nearly; it is accurately between 118·5 feet and 118·6 feet: but nearer the latter than the former.

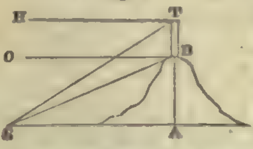
To solve such examples as this by logarithms, as is done in the books, the computation of an oblique-angled triangle—namely, of the triangle A B C—is necessary; and there will be five references to the logarithmic tables. In the above operation two references to a table suffice; and the subsequent arithmetical work, whatever number the dividend in the division may be, will always be trifling, if the contracted method of dividing be employed.

2. From the top of a castle (Fig. 5), 60 feet high, standing upon a hill near the sea-shore, the angle of depression H T S of a ship at anchor was observed to be 4° 52′; and the angle of depression O B S, taken from the bottom of the castle, was found to be 4° 2′. What was the distance A S of the ship?

Solution without Logarithms.

Since T H (Fig. 5) is parallel to A S, the angle A S T is equal to the angle H T S (Euc. 29, I.) For a like reason the angle A S B is equal to the angle O B S. Consequently, the angles of elevation A S T, A S B are known—namely, A S T = 4° 52′, and A S B = 4° 2′.

Fig. 5.



By right-angled triangles,

A T = S A tan. A S T, A B = S A tan. A S B,
∴ A T − A B = 60 = S A (tan. A S T − tan. A S B).

By the table of natural tangents, we find

Tan. A S T = tan. 4° 52′ = ·0851
Tan. A S B = tan. 4° 2′ = ·0705

The difference = ·01,4,6)60 (4100 = S A,
584
16
15
1

Consequently, the horizontal distance of the ship is 4100 feet, or 1367 yards, very nearly. By multiplying S A by tan. A S B, we shall get the height of the hill—namely,

·0705 × 4100 = 289 = A B.

3. The angle of elevation of the top of a tower was found to be 46° 30′, the place of observation from the bottom being 220 feet distant in a horizontal line: required the height of the tower. Ans. 232 feet.

4. At a horizontal distance of 45 yards from the bottom of a steeple, the angle of elevation of the top was found to be 48° 12′; the height of the observer's eye was 5 feet: required the height of the steeple. Ans. 52 yards.

5. In order to find the height of a castle surrounded by a moat, the angle of elevation of its top was taken from a convenient station, and found to be 46° 10′; then at a station 110 yards further off, but in the same horizontal line as the former station and the bottom of the castle, the angle of elevation was again taken, and found to be 29° 56′: required the height of the castle. Ans. 141·6 yards.

6. The height of an inaccessible object being required, the angle of elevation was taken at some distance from it, and found to be 51° 30′; and then a further distance of 75 feet being measured in the same horizontal line, the angle of elevation was again taken, and found to be 26° 30′. What was the height of the object, and at what horizontal distance was it from the first place of observation? Ans. Height, 61·97 feet; distance, 49·29 feet.

7. From the top of a ship's mast, 80 feet above the water, the angle of depression of another ship's hull was taken, and found to be 20°. What was the distance between the ships? Ans. 220 feet.

8. From the top of a lighthouse, 85 feet high, reckoning from the summit of the rock on which it stands, the angle of depression of a ship at anchor was found to be 3° 38′, and the angle at the bottom of the lighthouse, or top of the rock, was found to be 2° 43′: required the horizontal distance of the ship and the height of the rock above the level of the sea.

Ans. Distance, 5296 feet; height of rock, 251 feet.

9. From the edge of a ditch, 36 feet wide, surrounding a fort, the angle of elevation of the top of the wall was found to be 62° 40′: required the height of the wall, and the length of a scaling-ladder to reach from the edge of the ditch to the top.

Ans. Wall, 69·6 feet; ladder, 78·4 feet.

10. In the year 1784, two observers on Blackheath, at the distance of exactly a mile one behind the other, observed the angle of elevation of Lunardi's balloon, at the same instant of time; these angles were 36° 52′ and 30° 58′ respectively: required the perpendicular height of the balloon. Ans. 3 miles.

To Calculate the Sides and Angles of Oblique-angled Triangles.—In an oblique triangle the given parts must be either

1. Two angles and a side;
2. Two sides and an angle; or
3. The three sides, to find the remaining three parts.

I. *When the given parts are either two angles and an opposite side, or two sides and an opposite angle.*

Investigation of the Rule.

Let the triangle be A B C, and let it be either acute-angled or obtuse-angled, as in Figs. 6 and 7; and let the perpendicular A D be drawn, meeting the base or the base produced in D.

Fig. 6.

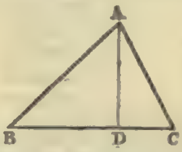
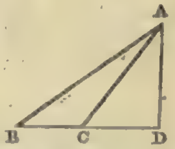


Fig. 7.



As in the former investigations, let the sides of the triangle, or rather their numerical measures, be represented by a, b, c, these letters applying to the sides opposite to the angles A, B, C, respectively. Each triangle presents now two right-angled triangles—namely, the triangles A B D, A C D; and from each we get a distinct expression for the perpendicular A D common to both; namely,

$$AD = AB \sin. B, \text{ and } AD = AC \sin. C;$$

so that A B sin. B = A C sin. C, that is,

$$c \sin. B = b \sin. C, \therefore \frac{c}{b} = \frac{\sin. C}{\sin. B}$$

which shows that *any one side of a triangle is to any other, as the sine of the angle opposite to the former, is to the sine of the angle opposite to the latter.* The angle C, in the preceding equations, belongs to the right-angled triangle A C D, which, in the second diagram, is not the angle C of the proposed triangle A B C, but the supplement of that angle; yet, as the sine of an angle is the same as the sine of its supplement, we may regard the C, in sin. C above, as the angle of the proposed triangle in each diagram.

The property enounced in italics above, supplies the following rule, when of the three given parts two are opposite to one another :—

RULE.—*To find an angle.* As one of the given sides is to the other, so is the sine of the angle opposite to the former, to the sine of the angle opposite to the latter.

To find a side. The sines of the given angles are to each other as the sides opposite to those angles.

If a, b, be any two sides, and A, B, their opposite angles,

$$\therefore a : b :: \text{sin. A} : \text{sin. B} = \frac{b \text{ sin. A}}{a},$$

$$\text{Or, sin. A} : \text{sin. B} :: a : b = \frac{a \text{ sin. B}}{\text{sin. A}},$$

$$\therefore \text{log. sin. B} = \text{log. } b + \text{log. sin. A} - \text{log. } a\text{; log. } b = \text{log. } a + \text{log. sin. B} - \text{log. sin. A.}$$

When an *angle* is to be determined by this rule, there is sometimes a choice of *two* angles ; and the case is then called the *ambiguous case.* The ambiguity arises from the circumstance that the sought angle is to be inferred from its *sine ;* and in the absence of all overruling restrictions, *two* angles have each equal claim to the *same sine :* an angle and its supplement. But if it happen that one of the given sides which is opposite to the given angle, is *greater* than the other given side, the angle opposite to the latter must be *acute*, otherwise the triangle would have two obtuse angles, which is impossible. Also, if either of two angles are known to be obtuse, the others must, of course, each be acute. Except under these conditions, the angle sought, may be either acute or obtuse ; so that there are two distinct triangles determinable from the proposed given parts, these parts belonging equally to both triangles.

EXAMPLES.

1. In a triangle two of whose sides are a = 95·12, and b = 98, and of which the angle A opposite to the former is 32° 15′, it is required to find the angle B opposite to the latter, as also the third side c.

To find the angle B.

As a = 95·12	−1·9783
: b = 98	1·9912
: : sin. A, 32° 15′	9·7272
: sin. B, 33° 21′	9·7401

To find the side c.

As sin. A, 32° 15′	−9·7272
: sin. C, 114° 24′	9·9594
: : a = 95·12	1·9783
: c = 162·3	2·2105

This example comes under the ambiguous case noticed above, for the side b, opposite to the sought angle, being greater than the side a, opposite the given angle, the angle B may be either 33° 21′ or its supplement 146° 39′, as the sine determined above belongs equally to both ; and each of these angles is greater than 32° 15′. The side c is calculated here for the acute angle B = 33° 21′, for which C = 180° − (32° 15′ + 33° 21′) = 114° 24′. If it be calculated for the obtuse angle B = 146° 39′, then C will be C = 180° − (32° 15′ + 146° 39′) = 1° 6′.

It will be seen that there is an inconvenience in having a *subtractive* logarithm in each of the columns ; it may be replaced by an additive quantity as follows :—Instead of writing down 1·9783 from the table, write down what this number wants of 10, which is 8·0217 ; but instead of subtracting from 10, to get this remainder, in the ordinary way, commence with the leading figure 1 on the left, and proceed from figure to figure towards the right, subtracting each from 9, till the last, 3, is reached, which subtract from 10 : this is of course the same, in effect, as subtracting the 3 from 10, in the common way, and carrying 1 from every figure in proceeding from right to left ; but it is easier to write down the remainder by the former way than by the latter : thus, pointing to the 1 in the number 1·9783, we write down 8 ; to the 9 we write down 0 ; to the 7 we write down 2 ; to the 8 we write down 1 ; and, lastly, to the 3 we write down 7. The remainder thus written in place of the tabular logarithm, is called the *arithmetical complement* of that logarithm. The arithmetical complement should always be written instead of a subtractive log., and *added ;* and the error of 10, thus committed, is to be corrected by suppressing 10 in the result of the addition.

The foregoing work should, therefore, be modified thus :—

To find the angle B.

As a = 95·12, Arith. Comp.	8·0217
: b = 98	1·9912
: : sin. A, 32° 15′	9·7272
: sin B, 33° 21′	9·7401

To find the side c.

As sin. A, 32° 15′, Arith. Comp.	·2728
: sin. C, 114° 24′	9·9594
: : a = 95·12	1·9783
: c = 162·3	2·2105

2. Given A = 48° 3′, B = 40° 14′, and c = 376, to find a and b.

C = 180° − (48° 3′ + 40° 14′) = 91° 43′.

To find a.

As sin. C, 91° 43′, Arith. Comp.	·0002
: sin. A, 48° 3′	9·8714
: : c = 376	2·5752
: a = 279·8	2·4468

To find b.

As sin. C, 91° 43′, Arith. Comp.	·0002
: sin. B, 40° 14′	9·8102
: : c = 376	2·5752
: b = 243	2·3856

3. Given a = 355, c = 336, and A = 49° 26′, to find b and C.

Ans. b = 465·3, C = 45° 58′.

4. Given a = 310, B = 62° 9′, and C = 41° 13′.

b = 281·7, c = 210, A = 76° 38′.

5. Given a = 70, b = 104, and B = 44° 12′.

A = 27° 59′, C = 107° 49′, c = 142.

6. Given b = 104, c = 142, and B = 44° 12′.

There are *two* triangles having these parts in each; the remaining parts being as here annexed.

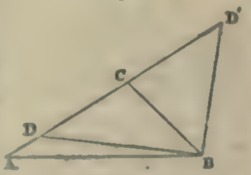
C = 72° 11′ or 107° 49′.
A = 63° 37′ or 27° 59′.
a = 134·8 or 70°.

II. *When the given parts are two sides and the included angle.*

Investigation of the Rule.

Let the two sides A C, B C (Fig. 8), and their included angle A C B, be given, to find the remaining angles A, B, of the triangle A B C.

Fig. 8.



From the greater C A, of the two given sides, cut off a part C D, equal to the less C B ; and also prolong A C, till C D′ = C B : then C B = C D = C D′, and consequently, with centre C, and radius C B, a circle may be circumscribed about D B D′, so that the angle D B D′ is a right angle (Euc. 31 of III.), and therefore C D B is the complement of D′.

Now C D B = A + A B D : add C B D = C D B to each ;

$$\left.\begin{array}{c} \text{then } 2\text{ C D B} = \text{B} + \text{A}, \therefore \text{C D B} = \tfrac{1}{2}(\text{B} + \text{A}), \\ \therefore \text{sin. A D B} = \text{sin. } \tfrac{1}{2}(\text{B} + \text{A}) \\ \text{And consequently, since D}' \text{ is the comp. of D,} \\ \text{sin. A D}'\text{B} = \text{cos. } \tfrac{1}{2}(\text{B} + \text{A}) \end{array}\right\} \cdots (1)$$

$$\left.\begin{array}{l}\text{But C B D, the half sum of A and B, added to}\\ \quad\text{A B D, gives B, the greater,}\\ \therefore \text{A B D} = \tfrac{1}{2}(\text{B}-\text{A}),\ \therefore \sin.\ \text{A B D}\\ \qquad = \sin.\ \tfrac{1}{2}(\text{B}-\text{A})\\ \text{But sin. A B D}', \text{ greater than } 90^\circ, \text{ is the same as}\\ \quad\text{the sine of an angle as much less than } 90^\circ\\ \therefore \sin.\ \text{A B D}' = \cos.\ \tfrac{1}{2}(\text{B}-\text{A})\end{array}\right\}\ldots(2)$$

These relations being established, let us refer to Case I., which gives

$$\frac{\text{AD}'}{\text{AB}} = \frac{\text{AC}+\text{CB}}{\text{AB}} = \frac{\sin.\ \text{ABD}'}{\sin.\ \text{AD}'\text{B}}\ldots(3)$$

$$\text{and}\quad \frac{\text{AD}}{\text{AB}} = \frac{\text{AC}-\text{CB}}{\text{AB}} = \frac{\sin.\ \text{ABD}}{\sin.\ \text{ADB}}\ldots(4)$$

Hence, dividing (3) by (4), we have

$$\frac{\text{AC}+\text{CB}}{\text{AC}-\text{CB}} = \frac{\sin.\ \text{ABD}'\ \sin.\ \text{ADB}}{\sin.\ \text{ABD}\ \sin.\ \text{AD}'\text{B}}\ldots(5)$$

that is, by substituting for these sines their values in (1) and (2),

$$\frac{\text{AC}+\text{CB}}{\text{AC}-\text{CB}} = \frac{\cos.\ \frac{1}{2}(\text{B}-\text{A})\ \sin.\ \frac{1}{2}(\text{B}+\text{A})}{\sin.\ \frac{1}{2}(\text{B}-\text{A})\ \cos.\ \frac{1}{2}(\text{B}+\text{A})}$$

$$= \frac{\tan.\ \frac{1}{2}(\text{B}+\text{A})}{\tan.\ \frac{1}{2}(\text{B}-\text{A})}\quad\text{(page 1038);}$$

so that in any plane triangle, *the sum of two sides is to their difference, as the tangent of half the sum of the opposite angles is to the tangent of half their difference.* The following, therefore, is the rule :—

RULE.—As the sum of the two given sides
Is to their difference,
So is the tangent of half the sum of the opposite angles
To the tangent of half their difference.

$$\text{or,}\quad a+b : a-b :: \tan.\ \frac{\text{A}+\text{B}}{2} : \tan.\ \frac{\text{A}-\text{B}}{2}.$$

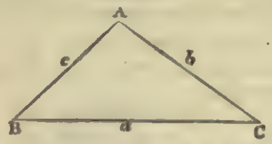
The half difference of the unknown angles thus becomes known; and as their half sum is also known—for the whole sum is 180° minus the given angle (Euc. 32 I.)—the two angles themselves are readily discovered; the greater of the two is the half sum increased by the half difference, and the less, the half sum diminished by the half difference.

NOTE.—Instead of using the tangent of half the sum of the opposite angles, we may use the cotangent of half the included angle, as is obvious.

EXAMPLES.

1. In the triangle A B C (Fig. 9), are given $b = 47$, $c = 85$, and the angle A = 52° 40′; required the remaining parts.

Fig. 9.



$\frac{1}{2}(\text{C}+\text{B}) = 90^\circ - 26^\circ\ 20' = 63^\circ\ 40'$, and the formula for finding $\frac{1}{2}(\text{C}-\text{B})$ is

$$c+b : c-b :: \tan.\ \tfrac{1}{2}(\text{C}+\text{B}) : \tan.\ \tfrac{1}{2}(\text{C}-\text{B})$$

As $c+b$ = 132 Arith. Comp.	7·8794
: $c-b$ = 38	1·5798
: : tan. $\frac{1}{2}$(C + B), 63° 40′	10·3054
: tan. $\frac{1}{2}$(C − B), 30° 11′	9·7646

∴ C = 93° 51′, the sum of $\frac{1}{2}$(C + B) and $\frac{1}{2}$(C − B)
B = 33° 29′, the difference.

As all the angles of the triangle are now known, the remaining side a may be found by Case I., thus :—

To find the side a,

As sin. B, 33° 29′ Arith. Comp.	·2583
: sin. A, 52° 40′	9·9004
: : b = 47	1·6721
: a = 67·74	1·8308

It may be observed here that the third side of the triangle may always be determined independently of Case I., and in a manner somewhat more easily, by employing a relation furnished by the foregoing investigation, thus : the equations (3) and (4), after putting for the sines the values previously deduced, are

$$\frac{\text{AC}+\text{CB}}{\text{AB}} = \frac{\cos.\ \frac{1}{2}(\text{B}-\text{A})}{\cos.\ \frac{1}{2}(\text{B}+\text{A})};\quad \frac{\text{AC}-\text{CB}}{\text{AB}} = \frac{\sin.\ \frac{1}{2}(\text{B}-\text{A})}{\sin.\ \frac{1}{2}(\text{B}+\text{A})}$$

which furnish the proportions

$$\cos.\ \tfrac{1}{2}(\text{B}-\text{A}) : \cos.\ \tfrac{1}{2}(\text{B}+\text{A}) :: b+a : c\ \ldots\ (6)$$
$$\sin.\ \tfrac{1}{2}(\text{B}-\text{A}) : \sin.\ \tfrac{1}{2}(\text{B}+\text{A}) :: b-a : c\ \ldots\ (7)$$

The advantage of using either of these proportions instead of the rule in Case I. is, that all the three logarithms employed occur at the same openings of the table as the logarithms in the process for finding the angles, and they may, therefore, be readily taken out at the same time : *one* of the three—that for the sum or difference of the sides—has only to be repeated.

For the particular triangle solved above, the second of these proportions is

$$\sin.\ \tfrac{1}{2}(\text{C}-\text{B}) : \sin.\ \tfrac{1}{2}(\text{C}+\text{B}) :: c-b : a;$$

and in order to show the facilities gained by employing it, we shall re-work the example just given by its aid.

As $c+b$ = 132 Arith. Comp.	7·8794	To find the side a.	
: $c-b$ = 38	1·5798		1·5798
: : tan. $\frac{1}{2}$(C + B), 63° 40′	10·3054	sin.	9·9524
: tan. $\frac{1}{2}$(C − B), 30° 11′	9·7646	Arith. Comp. sin.	·2986
∴ C = 93° 51′ B = 33° 29′		a = 67·74	1·8308

It may assist the memory in computing the side by this method, to observe that the three terms employed in the proportion for the side are merely those in the proportion for the angles *reversed*, with sine in the place of tangent.

Another use, too, may be made of the proportions (6) and (7)—namely, in the case where two angles and the interjacent side are given to find the remaining sides, as in the following example.

2. Given A = 41° 13′, B = 62° 9′, and $c = 310$, to find a and b.

Inverting the proportions (6) and (7), we have

$$\sin.\ \tfrac{1}{2}(\text{B}+\text{A}) : \sin.\ \tfrac{1}{2}(\text{B}-\text{A}) :: c : b-a$$
$$\cos.\ \tfrac{1}{2}(\text{B}+\text{A}) : \cos.\ \tfrac{1}{2}(\text{B}-\text{A}) :: c : b+a.$$

Hence the work will stand as follows :—

As sin. $\frac{1}{2}$(B + A), 51° 41′ Arith. Comp.	·1054	Arith. Comp. cos.	·2076
: sin. $\frac{1}{2}$(B − A), 10° 28′	9·2593	cos.	9·9927
: : c = 310	2·4914		2·4914
: $b-a$ = 71·8 or nearly 71·7	1·8561	$b+a$ = 491·7	2·6917
		$b-a$ = 71·7	
(See Example 4, page 1044.)		2 | 420 , 210 = a 563·4, 281·7 = b	

3. Given $a = 512$, $c = 907$, and B = 49° 10′, to find the remaining parts of the triangle.
Ans. A = 34° 6′, C = 96° 44′, b = 691.

4. Given $a = 95{\cdot}12$, $c = 98$, and B = 114° 24′, to find the remaining parts.
Ans. A = 32° 15′, C = 33° 21′, b = 162·34.

5. Given $a = 112$, $b = 120$, and C = 57° 57′, to find the remaining parts.
Ans. A = 57° 28′ B, = 64° 35′, c = 112 6.

6. Given $b = 154{\cdot}3$, $c = 365$, and A = 57° 12′, to find the remaining parts.
Ans. B = 24° 45′, C = 98° 3′, a = 310.

III.—*When the given parts are the three sides.*

Investigation of the Rules.

Returning to the diagrams of Case I., page 1043, we have by right-angled triangles,

$$\text{B D} = c\cos.\ \text{B, and C D} = b\cos.\ \text{C}.$$

If the angle C of the triangle be acute, the cosine of it will be positive; if it be obtuse, as in the second of the diagrams referred to, it will be negative. In the equation above, the angle C is A C D, which is acute in the second diagram; but if we replace it by the angle C of the triangle, thus making cos. C subtractive, we shall have, equally for both triangles, the condition

$$a = b \cos. C + c \cos. B$$

And similarly, $b = c \cos. A + a \cos. C$

$$c = a \cos B + b \cos A.$$

Let these be regarded as three algebraical equations, in which the unknown quantities (usually x, y, and z) are cos. A, cos. B, and cos. C; then, multiplying them by a, b, c respectively, we have—

$$a^2 = ab \cos. C + ac \cos B$$
$$b^2 = bc \cos. A + ab \cos. C$$
$$c^2 = ac \cos. B + bc \cos. A;$$

and subtracting each of these from the sum of the other two, there results—

$$\left.\begin{array}{l} b^2 + c^2 - a^2 = 2bc \cos. A \\ a^2 + c^2 - b^2 = 2ac \cos. B \\ a^2 + b^2 - c^2 = 2ab \cos. C \end{array}\right\} \therefore \left\{\begin{array}{l} \cos. A = \dfrac{b^2 + c^2 - a^2}{2bc} \\ \cos. B = \dfrac{a^2 + c^2 - b^2}{2ac} \\ \cos. C = \dfrac{a^2 + b^2 - c^2}{2ab} \end{array}\right.$$

The expressions on the right furnish a complete solution to the present problem; but when the given sides a, b, c, consist each of several places of figures, the calculation of the fractions involves a good deal of arithmetical work. On this account, means have been contrived to change the preceding forms into others, that shall consist exclusively of *factors* and *divisors*, without any addition or subtraction operations; so that the expressions may be fitted for computation by logarithms: how this is brought about we shall explain presently. But, as logarithms may be dispensed with whenever a, b, c are conveniently small numbers, we shall first show the best form of using the above expression for cos. A, in such a case.

Subtract 1 from the fraction for cos. A; it then becomes,

$$\frac{b^2 + c^2 - a^2}{2bc} - 1 = \frac{(b - c)^2 - a^2}{2bc}$$
$$= \frac{(b - c + a)(b - c - a)}{2bc}$$

and now adding the 1 subtracted, we have—

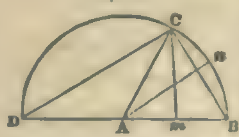
$$\cos. A = \frac{(b - c + a)(b - c - a)}{2bc} + 1$$
$$= 1 - \frac{(a + b - c)(a - b + c)}{2bc} \ldots (1)$$

which is tolerably convenient for calculation without logarithms: but a form somewhat preferable is given subsequently.

If the 1 were transposed and signs changed, the right-hand member of the last equation would be equally convenient for calculation *with* logarithms; but the left-hand member, which would then be $\overline{1 - \cos. A}$, has no corresponding logarithm in the ordinary tables;* the object is then to change $\overline{1 - \cos. A}$ into some equivalent trigonometrical quantity, the log. of which is to be found in the tables. A diagram will assist in showing how this is to be effected

Suppose A B, Fig. 10, to be the trigonometrical radius—namely, A B = 1, and let B C D be the arc of a semicircle to that radius, and B A C the angle A; then An bisecting the chord B C, also bisects the arc B C and the angle A, ∴ B C = 2 sin. ½ A.

Fig. 10.



Now D C B, in a semicircle, being a right angle, we have (Euc. 8 of VI.) B D. Bm = B C²; that is, since B D = 2, and Am = cos. A,

$$2(1 - \cos. A) = (2 \sin. \tfrac{1}{2} A)^2 \therefore \overline{1 - \cos. A} = 2 \sin.^2 \tfrac{1}{2} A,$$

$$\therefore (1),\ \sin.^2 \tfrac{1}{2} A = \frac{(a - b + c)(a + b - c)}{4bc}$$

$$\therefore \sin. \tfrac{1}{2} A = \sqrt{\frac{\frac{1}{2}(a - b + c)\frac{1}{2}(a + b - c)}{bc}}$$

a formula well adapted to computation by logarithms. If s be put to represent $\frac{1}{2}(a + b + c)$, then the two factors in the numerator will be respectively $s - b$ and $s - c$, and we shall have the expression for sin. ½ A in the more compact form,

$$\sin. \tfrac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{bc}} \quad . \quad . \quad . \quad . \quad (I)$$

Again: because as proved above $\frac{1}{2}(1 - \cos. A) = \sin.^2 \frac{1}{2} A$, if each be subtracted from 1, we shall have,

$$\tfrac{1}{2}(1 + \cos. A) = 1 - \sin.^2 \tfrac{1}{2} A = \cos.^2 \tfrac{1}{2} A,$$

$$\therefore 2 \cos.^2 \tfrac{1}{2} A = 1 + \cos. A = 1 + \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(b + c)^2 - a^2}{2bc} = \frac{(a + b + c)(b + c - a)}{2bc}$$

$$\therefore \cos. \tfrac{1}{2} A = \sqrt{\frac{s(s - a)}{bc}} \quad . \quad . \quad . \quad . \quad (II)$$

and dividing (I) by (II), we have finally

$$\tan. \tfrac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{s(s - a)}} \quad . \quad . \quad . \quad . \quad (III)†$$

Either by the formulæ marked (I), (II), (III), will supply a rule for finding an angle from the sides being given; but it is better to work by the formula itself than to be guided by written directions. That marked (II) is a little the shortest; but when ½ A, that is, half the sought angle, is foreseen to be very small, it will be better to use the formula (I): the reason is that the sines of angles very near 90°, or, which is the same thing, the cosines of angles very near 0°, vary from each other by such slight differences, that several of such angles have equal claim to the same sine or cosine. For instance, suppose we were led to 9·9999998 for the logarithmic sine or cosine of an angle to be found. By tables calculated to seven places of decimals, this number is the log. sine of every angle, indifferently, between 89° 56′ 19″ and 80° 57′ 8″; and, consequently, the log. cosine of every angle between 2′ 52″ and 3′ 41″. In such extreme cases the formula (III) is to be preferred, although for a small angle (I) may be safely employed, and (II) for a large one. In practice, however, these ill-conditioned triangles, as they are called, are always avoided, if possible; and in Navigation they are but little likely to be met with. An error of a few seconds in *this* subject is, however, a matter of no moment; and we advert to the peculiarity here, merely to explain to the learner how it happens that, although the three formulæ above are all equally correct, yet in inquiries where the minutest accuracy is necessary, one of them may, in certain rare cases, be preferable to another. The defect is not in the formulæ, but in the tables, which being calculated to only six or seven places of decimals, cannot, of course, mark distinctions of value that affect only the decimals more remote.

EXAMPLES.

1. Given the sides a = 95·12, b = 162·34, and c = 98, to find the angle A.

By formula (I).

$$\sin. \tfrac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{bc}}$$

a = 95·12		
b = 162 34	Arith. Comp.	7·7896
c = 98	Arith. Comp.	8·0088
2)355·46		
s = 177·73		
$s - b$ = 15·39	. . . .	1·1872
$s - c$ = 79·73	. . . .	1·9016
		2)18 8872
sin. ½ A, 16° 7½	. . . .	9·4436
2		
∴ A = 32° 15′		

---

* The expression 1 – cos. A is called the *versed-sine* of the angle A. A table of *natural* versed-sines is given in Vol. II. of Dr. Mackay's valuable work on *The Theory and Practice of finding the Longitude.*

† The student may compare the above formulæ with similar expressions at page 624, in connection with the calculation of sines, &c., from derived formulæ.—ED.

By formula (II).

$$\cos.\ \tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$$

$a = 95{\cdot}12$
$b = 162{\cdot}34$ Arith. Comp. 7·7896
$c = 98$ Arith. Comp. 8·0088

2)355·46

$s = 177{\cdot}73$ . . . . 2·2498
$s - a = 82{\cdot}61$ . . . . 1·9170

2)19·9652

cos. ½ A, 16° 7½′ . . . . 9·9826
2

∴ A = 32° 15′

In the first of these logarithmic operations there enters a log. sine, and in the second a log. cosine: each, therefore, requires a correction, as noticed at page 1038; 10 should be added to the right-hand member of each of the formulæ (I), (II), when expressed logarithmically. Now *two* tens have been added to the sum of the four logs. above, on account of the two complements; so that, after dividing this sum by 2, for the square root, *one* ten, in excess, affects the result, which therefore supplies the 10 that ought to have been added, and thus the proper correction is provided for.

We shall now exhibit the work of the preceding example without logarithms. The most convenient formula for this purpose is, perhaps, that immediately derived from the expression for cos. A at page 1046, by first adding and then subtracting 1; we thus get

$$\cos.\ A = \frac{(b^2 + 2bc + c^2) - a^2}{2bc} - 1 = \frac{(b + c)^2 - a^2}{2bc} - 1$$
$$= \frac{(a + b + c)(b + c - a)}{2bc} - 1$$

$a = 95{\cdot}12$
$b = 162\ 34$
$c = 98$

$a + b + c = 355{\cdot}46$ . . . 355·46
$b + c - a = 165{\cdot}22$ reversed = 22561

$b = 162{\cdot}34$ 35546
$c = 98$ 21328
1777
129872 71
146106 7

15909·32 × 2 = 31818,64)58729(1·8457
31819

·8457 = cos. A, 32° 15′ 26910
25455

The contracted method, so often recommended in these pages, is used throughout this operation. 1455
1273

182
159

23
22

It may be remarked here, that in computing, as in this specimen, the first figure in the quotient will always be 1, followed by decimals; so that there will be no occasion to take any notice of decimal points, either in the dividend or divisor: it is sufficient that we know that the first figure of the quotient is always an integer (unit), and the following figures decimals. But there is no question that, in general, the operation by logarithms would be preferred. It is worthy of notice, however, that whenever *a*, *b*, *c* are all divisible by the same number, we may divide accordingly, and use the quotients: thus, instead of the above values, we might have taken $a = 47{\cdot}56$, $b = 81{\cdot}17$, and $c = 49$; these being the halves of those values.

2. Given $a = 174{\cdot}1$, $b = 232$, and $c = 345$, to find the angle A. Ans. A = 27° 4′.

3. Given $a = 95{\cdot}12$, $b = 162{\cdot}34$, and $c = 98$, to find the angle B. Ans. B = 114° 24′.

4. Given $a = 3388$, $b = 2065$, and $c = 1637$, to find the angle A. Ans. 132° 7′.

5. Given $a = 112$, $b = 112{\cdot}6$, and $c = 120$, to find all the angles. Ans. A = 57° 28′, B = 57° 57′, C = 64° 35′.

6. Given $a = 698$, $b = 352$, and $c = 467$, to find all the angles. Ans. A = 116° 12′, B = 26° 54′, C = 36° 54′.

MISCELLANEOUS QUESTIONS REQUIRING THE SOLUTION OF PLANE TRIANGLES.—1. Being on one side of a river, and wishing to know the distance of a tree (C—Fig. 10, *ante*) on the other side, I measured a length of 500 yards along the side of the river, and set up a rod at each end (A and B): the angle at A, subtended by B C, was then measured, and found to be 74° 14′; and the angle at B, subtended by A C, was found to be 49° 23′: required the distance of the tree from each station.

Here the two angles at A and B are given, as also the interjacent side A B; the third angle at C is therefore known; namely, C = 180° — (74° 14′ + 49° 23′) = 56° 23′.

As sin. C, 56° 23′ Arith. Comp. . ·0795
: sin. B, 49° 23′ . . . 9·8803
: : A B = 500 . . . . 2·6990

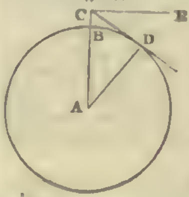
: A C = 455·8 . . . . 2·6588

As sin. C, 56° 23′ Arith. Comp. . ·0795
: sin. A, 74° 14′ . . . 9·9833
: : A B = 500 . . . . 2·6990

: B C = 577·8 . . . . 2·7618

2. From the top of a mountain 3 miles high, the angle of depression of the visible horizon—that is, of the circle where sky and sea appeared to meet—was found to be 2° 13′½: it is required from this to determine the diameter of the earth.

Fig. 11.



Let A (Fig. 11) be the centre of the earth, C the summit of the mountain B C, and E C D the angle of depression of the remotest visible point of the surface of the sea, below the horizontal line C E.

The angles A C D, D C E, make up a right angle; so do the angles A C D and A, because D is a right angle (Euc. 18 of III.); therefore the angle A = the angle E C D. Now by right-angled triangles A C = A D sec. A.

∴ A C − A B = B C = A D (sec. A − 1); that is,

$$AD\,(\sec.\ 2^\circ\ 13'\tfrac{1}{2} - 1) = 3,\ \therefore AD = \frac{3}{\sec.\ 2^\circ\ 13'\tfrac{1}{2} - 1}$$

By help of a table of natural secants this expression for the semi-diameter AD is very readily calculated. To convert it into a form adapted to logarithms, we have, by putting $\frac{1}{\cos.\ A}$ for sec. A,

$$AD = \frac{BC}{\sec.\ A - 1} = \frac{BC\cos.\ A}{1 - \cos.\ A}$$

But it was shown at page 1046 that $1 - \cos.\ A = 2\sin.^2 \tfrac{1}{2}A$,

$$\therefore AD = \frac{BC\cos.\ A}{2\sin.^2\tfrac{1}{2}A},\ \therefore 2\,AD = \frac{BC\cos.\ A}{\sin.^2\tfrac{1}{2}A}$$

∴ log. diameter = log. 3 + log. cos. 2° 13′½ − 2 log. sin. 1° 6′¾ + 10,

the 10 being added because there are two subtractive log. sines—namely, log. sin. 1° 6′¾ + log. sin. 1° 6′¾, and only one additive trigonometrical quantity—namely, log. cos. 2° 13′½.

log. 3 . . . . ·477121
log. cos. 2° 13′½ . . . . 9·999672
log. sin. 1° 6′¾ Arith. Comp. 1·711859
Repeated . . . . 1·711859

log. diam., 7952½ . . . . 3·900511

Hence the diameter of the earth, as deduced from these observations, is 7952½ miles.

Otherwise by a table of natural cosines.

Since, as shown above, $A\,D = \frac{BC \text{ cos. } A}{1 - \text{cos. } A}$, we may compute by the table of natural sines and cosines as follows:—

$$\begin{array}{lrl}
\text{nat. cos. } 2^\circ\,13'\,\tfrac{1}{2} = & \cdot 999246 & \\
 & 3 & \\
\hline
1 - \cdot 999246 = \cdot 000754\;) & 2\cdot 997738\,(& 3976 \\
 & 2262 & \quad\;\; 2 \\
\hline
 & 7357 & 7952 \text{ miles the diameter.} \\
 & 6786 & \\
\hline
 & 5713 & \\
 & 5278 & \\
\hline
 & 4358 & \\
 & 4524 &
\end{array}$$

3. Wanting to know the breadth of a river, I measured a base of 500 yards in a straight line close to its edge, and at each end of it I found the angles subtended by the other end, and a tree close to the opposite margin of the river to be 53° and 79° 12′: what was the breadth of the river? Ans. 529·5 yards.

4. Two ships of war intending to cannonade a fort separated from each other 500 yards, as near to the shore as possible: the angles subtended by each ship and the fort were observed from the other ship to be 38° 16′ and 37° 9′: required the distance of the fort from each place of observation. Ans. 312 yards and 320 yards.

5. The peak of Teneriffe is said to be 2½ miles above the surface of the sea: the angle formed at the top between a plumb-line, which, of course, hangs perpendicular to the horizon, and the line from the eye to touch the sea at the remotest visible point, is found to be 87° 58′: required the diameter of the earth. Ans. 7936 miles.

6. Three objects, A, B, C, whose distances apart were A C = 8 miles, B C = 7½ miles, and A B = 12 miles, were visible from a station D, in the line joining A and B; and the angle at that station, subtended by AC, was observed to be 107° 56′ 13″; required the distance of the three objects from D.

Ans. A D = 5 miles, D B = 7 miles, D C = 4·89 miles.

We here conclude the introductory chapter on the solution of plane triangles: it is intended solely to familiarise the learner with the business of practical calculation, and to illustrate the best and shortest methods of arranging his arithmetical operations. Many instances have been given, more especially in what concerns right-angled triangles, of the advantage of making more use of the table of natural sines and cosines than is at present customary. Valuable as logarithms unquestionably are, they do not always effect a saving of time or trouble, even in cases well fitted for their application; and we would, therefore, recommend that the example we have here set him be followed by the learner—that he would exercise his own deliberate judgment as to which kind of table will aid him in reaching his result in the readiest way, and not in all cases resort to logarithms, as is almost invariably done in works of this kind.

But whatever table he uses he should use with deliberation and care: tabular numbers should never be transcribed in a hurry, as everything depends on the accuracy with which they are written down. It should also be the habitual practice of the calculator to make all the use he can of his table when it is in his hand; and when it is not in use, he should advance his work as much as practicable before he refers to it. In order to this, as many as possible of the *arguments*, as they are called—that is, of the quantities with which the tabular numbers are to be connected—should be written down before the tables are touched: thus, in Example 1, for instance, page 1046, the entire skeleton of the operation should be formed before the table of logarithms is referred to, to fill it up. Again, in Example 2, page 1045, every item, in both columns of the work, should be written down while the tables are in hand, and they should be again referred to only after the amount of each column is found.

In the chapters on TRIGONOMETRY in the mathematical section, the main object has been to develop the analytical theory of angular magnitude, and not to enter at any length into arithmetical details; the present chapter, independently of the special purpose it is intended to serve in this section, may, therefore, be acceptable to the student as a praxis on certain theoretical principles there delivered. This collateral object has not been lost sight of in the preparation of the foregoing pages; and we have, accordingly, introduced examples in sufficient number and variety to supply the learner with all needful materials for exercise, in the ordinary calculations of Practical Trigonometry.

Before closing the introductory portion of our subject, it may be well to collect into one place the several formulæ for the solution of oblique triangles. By thus bringing them together, future reference to them will be facilitated; and the learner, by having the whole more frequently under his eye, will, at length, get them so impressed on his mind as eventually to dispense altogether with a formal reference to them. This amount of familiarity with his tools is a necessary qualification in a good workman.

Formulæ for the Solution of Plane Triangles.

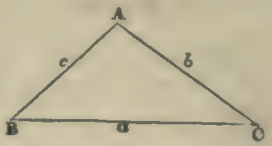
I. $a : b :: \text{sin. } A : \text{sin. } B$

II. $a + b : a \backsim b :: \text{tan. } \tfrac{1}{2}(A + B) : \text{tan. } \tfrac{1}{2}(A \backsim B)$

$\text{sin. } \tfrac{1}{2}(A \backsim B) : \text{sin. } \tfrac{1}{2}(A + B) :: a \backsim b : c$

$\text{cos. } \tfrac{1}{2}(A \backsim B) : \text{cos. } \tfrac{1}{2}(A + B) :: a + b : c$

Fig. 12.



$$\text{III. sin. } \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}, \quad \text{cos. } \tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$$

$$\text{tan. } \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\text{cos. } A = \frac{(a+b+c)(b+c-a)}{2bc} - 1.$$

The last of these expressions is to be computed by common arithmetic, and the cosine of A to be found in the table of natural sines and cosines; all the other formulæ are adapted to logarithmic computation. Whenever the sides *a*, *b*, *c*, have a factor common to all, it may be cancelled, and the results used instead, in any of the formulæ III.

CHAPTER II.

NAVIGATION.

GENERAL NOTIONS OF THE FIGURE AND ROTATION OF THE EARTH.—The surface of the sea is very nearly that of a perfect sphere or globe. For all the purposes of Navigation, it may be regarded as accurately so; for if the very small deviation from this figure were to be removed, no sensible difference would be made in any of the rules and operations by which the sailings of a ship are regulated, and its position determined.

That the surface of the ocean is globular may be inferred from the evidence of the senses. On whatever part of it an observer be placed, or however high above it he be raised, he sees around him an expanse in which no defect from sphericity can be discovered anywhere within the compass of his vision. He sees a receding vessel, in the distance, gradually disappearing behind the rotundity; he first loses sight of the hull, then more and more of the upper works, till at length even the most elevated parts sink below the circle that limits his field of view. These effects being observed, at whatever spot on the surface the spectator may be, lead to the natural impression that it is a portion of a globe that is everywhere spread around him. This impression is confirmed by the unquestionable fact that, on the assumption of the surface being uniformly spherical, and under the sole guidance of rules and directions based exclusively on that assumption, ships have actually circumnavigated the earth, and that by routes so various, as to leave on the mind not the slightest doubt that the inference, as to the general figure of the ocean's surface, drawn from observation of its appearance to the eye, is correct.

Again: an eclipse of the moon is caused by the passage of that body through the shadow cast by the earth, the earth being then between the sun and moon. As soon as the moon enters the shadow, the small part of her face thus obscured always presents a circular boundary; as she advances, and the obscuration increases, the boundary of the shadow enlarges, but it continues circular; and whenever the centres of the sun, earth, and moon are so nearly in the same straight line as to render the eclipse *annular*, as it is called, then the shadow projected on the moon's surface is observed to be a complete circle. These appearances being uniform, whatever part of the earth's surface is exposed to the sun's rays, we cannot resist the conclusion that the planet we inhabit does not differ in external figure from a perfect sphere, except in a very slight degree.

The earth rotates—turning once round every day; it revolves invariably about the same diameter, and completes each revolution invariably in the same time. That the rising and setting of the sun and stars are appearances really due to the diurnal rotation of the earth, and not to the motions of the celestial bodies themselves, is a truth not so obvious to the senses as the spherical figure of the earth—at least, till very lately, no contrivance had been thought of to render this rotation visible to our eyes. But there are means now of *showing* that the earth turns round; so that we may have the same visible proof of its diurnal revolution that we have of its general figure. This will be explained when we come to treat of NAUTICAL ASTRONOMY.

That the rotation is daily performed in exactly the same invariable period of time—without the difference of a single second—is proved by innumerable observations. Age after age, the same spot on the earth, after the lapse of the same interval of time, invariably returns to the same fixed star; which could not be the case if there were the slightest irregularity in the diurnal rotation of the earth. Nor could such be the case, if the diameter round which the rotation is performed, were shifted.

Admitting, then, from facts such as these, that the earth is a sphere—or that, at least, it differs from a sphere so little as for the purposes of navigation to be of no moment—that it revolves uniformly once in twenty-four hours—and that the diameter about which it turns is invariable, we may proceed to the following definitions:—

DEFINITIONS.

1. *Axis.*—The diameter about which the earth performs its daily revolution is called the axis of the earth: the revolution about this axis is from west to east.

2. *Poles.*—The extremities of this diameter are called the *poles* of the earth. The extremities of *any* diameter of a sphere are also called poles—the poles, namely, of that great circle of the sphere the circumference of which is at every point of it, at the same distance from each, this distance being a quadrant, or 90°: these same points are also spoken of as the poles of every *small* circle parallel to the great circle just mentioned; but from the circumference of any small circle, the poles are, of course, at unequal distances.

3. *Equator.*—The *equator* is that great circle of the earth, the axis of which is perpendicular to the plane of that circle: the poles of the equator are the poles of the earth. These poles are called—one of them the NORTH POLE, and the other the SOUTH POLE. Whenever, in navigation, we speak of the poles, the poles of the *earth*—that is, of the Equator—are always to be understood.

4. *Meridians.*—Every semicircle which terminates at the two poles, and which is therefore perpendicular to the equator, is called a *meridian:* it is said to be the meridian of each place on the earth through which it passes. For the convenience of Navigation and Geography, every civilised kingdom selects one of these innumerable meridians as a meridian of reference; it is usually that which passes through the national observatory, or the principal city, and is called the *first meridian*. In this country the first meridian is that of Greenwich; in France, it is that of Paris.

The plane of every meridian (as also the plane of the equator) is conceived to be extended to the heavens, and to mark out, on the celestial sphere, the celestial meridian of the place. The apparent daily motion of the sun and stars is across the celestial meridians; the path of a ship, easterly or westerly, is across the terrestrial meridian; but, in general, the distinctions *terrestrial* and *celestial* are dropped: a ship cannot be on a celestial meridian, nor a star on a terrestrial meridian.

5. *Latitude.*—The latitude of a place on the surface of the earth, is the distance of that place from the equator, measured on the meridian of the place. Latitude is, therefore, either north or south: a place cannot exceed 90° in latitude, this being the latitude of each pole.

6. *Parallels of Latitude.*—A small circle on the globe, parallel to the equator, is called a *parallel of latitude:* every point on the circumference of such a circle has the same latitude, as the parallel is everywhere equi-distant from the equator. The arc of a meridian, intercepted between two such parallels drawn through any two places on the globe, measures the *difference of latitude* of those places. When the places are both on the same side of the equator—that is, both north or both south—their difference of latitude is found by subtraction; when they are on opposite sides of the equator, their difference of latitude is found by addition.

7. *Longitude.*—The longitude of any place on the earth's surface is the arc of the *equator* intercepted between the meridian of that place and the *first meridian*. It is estimated, like latitude, in degrees and parts of a degree, and is, of course, the measure of the angle at the pole, included between the two meridians spoken of. When the place is to the east of the first meridian, it has east longitude; when to the west, west longitude.

As every place on the surface of the earth performs a complete revolution in twenty-four hours, 15° of longitude become the measure of one hour of time; the whole 360° of the equator, and of every parallel to it, having the measure of twenty-four hours. Longitude is thus sometimes expressed in *time;* a place 45° east of the meridian of Greenwich is, for instance, said to be three hours east of Greenwich. It is necessary to precision, that the *latitude* of a place should be designated either north or south, according as it is situated in the northern or southern hemisphere; but the distinction of longitude into east or west is unnecessary and inconvenient. "It would add greatly to systematic regularity, and tend much to avoid confusion and ambiguity in computations, were this mode of expression abandoned, and longitudes reckoned invariably *westward* from their origin round the whole circle from 0° to 360°."* In the present mode of reckoning, however, longitude, like latitude, is of two denominations; so that the difference of longitude of two places is found sometimes by subtraction and sometimes by addition: by subtraction, if both places are on the same side of the first meridian; and by addition, if they lie on opposite sides: the limit of longitude, whether east or west, is of course 180°. A place of which the longitude is 180° is on the meridian opposite to that of Greenwich.

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* Sir John Herschel.

The learner must bear in mind that, in Geography and Navigation, the meridian of a place is limited by the poles of the earth: it is the semicircle passing through the place, and terminating in the north and south poles. As already remarked, the terrestrial meridian is extended to the concavity above us, and marks out the corresponding celestial meridian. Certain stars which are observed to come to this meridian are seen to pass the opposite meridian without setting; but the distinction, noticed above, between a meridian and the opposite meridian is not preserved here: the star is generally said to come to the meridian twice—once above and once below the pole. In the case of a ship, however, sailing round the pole from one meridian to its continuation, it is always said to have arrived at the *opposite* meridian, and to have advanced 180° in longitude.

8. *Horizon.*—The horizon of any place is a plane conceived to touch the surface of the earth at that place, and to be extended to the heavens—that is, to the region of the remotest of the stars. This plane is called the *sensible horizon.* A plane parallel to this, but passing through the centre of the earth, is called the *rational horizon.* These two imaginary planes, though separated from each other by an interval equal to the semi-diameter of the earth, cannot but be regarded as coincident at the distance of the stars. An eye, whether at the centre of the earth, or at the point on its surface immediately over it, would see a star in precisely one and the same direction: the altitude of it, referred to the rational horizon, would be exactly the same as the altitude referred to the sensible horizon. The observer, at the centre, sees the star higher above *his* horizon (that is, the parallel to the sensible horizon) than the observer at the surface sees it above his, by the apparent interval between the two horizons, at the distance of the star; which interval, however, although we here call it *apparent*, is in reality too minute to appear as any interval at all.

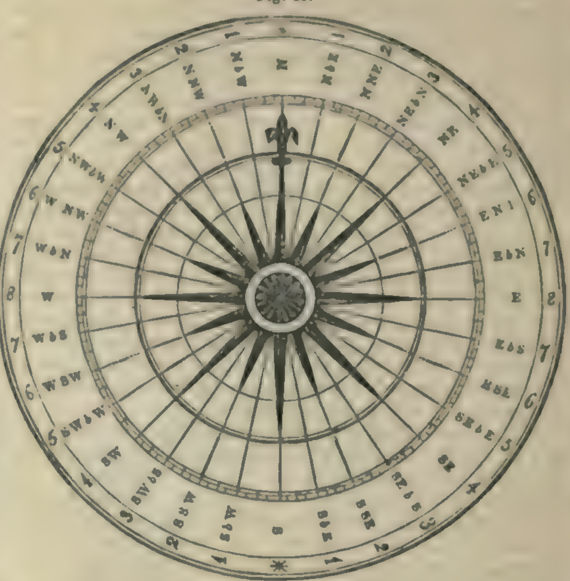
An eye situated *above* the surface of the earth, as is always practically the case, has an horizon different from the sensible horizon: the latter is an extended *plane;* the former is a *conical surface*, everywhere *dipping* below the sensible horizon, and of wider visible boundary. A very moderate elevation above the surface will give a sensible increase to the observed altitude of a heavenly body; namely, the whole of the angular distance between the two horizons—the whole of what may be called the angle of the dip.

The apparent circular boundary of the sea, as seen by an eye thus elevated above its surface, is called the visible, or *sea-horizon;* or, more frequently by sailors, the *offing.*

9. *The Compass.*—The straight line through any place, in which the plane of the meridian of that place cuts the sensible horizon, is called the horizontal meridian, or simply the meridian line, or the north and south line; and the horizontal straight line perpendicular to this is the east and west line of the horizon. The horizon is represented in miniature by a circular card, connected with a magnetised needle, which points out the direction of the horizontal meridian, and consequently also that of the east and west line: the *points* thus marked out on the rim of the card, namely, the North, South, East, and West points, are called the four Cardinal Points of the compass; the quadrantal arcs, intermediate between these, are subdivided each into *eight* equal parts, called also *points*, and these again each into four equal parts, called quarter-points. The above engraving is a representation of the *Mariners' Compass*, divided into its thirty-two points, with the intermediate quarter-points also marked.

This important instrument is so suspended on shipboard as always to assume a horizontal position under every change of motion in the vessel, so that the direction in which the ship is sailing, at any time, is always known by observing what *point* is in that direction; that is, in general, what point coincides with the line, through the centre of the compass-card, from stern to stem of the ship.

Fig. 13.



It is of importance to mention, however, that the magnetic needle does not point accurately north and south; the points to which it *is* directed are called the *magnetic* north and south: and the angular departure from the true north and south, at any place, is called the *variation* of the compass at that place. Its amount may be discovered, and the necessary corrections for it made, by Nautical Astronomy.

10. *Courses.*—A line on the globe, which cuts the successive meridians at the same angle, is called a *rhumb-line;* it marks the track of a ship, the constant angle referred to being called the ship's *course.* It is indicated by the compass; but if no corrections be made for variation, the course thus indicated is called the *compass-course;* after correction and allowance for what is called *deviation*, it is the *true-course.* These modifications of the compass-courses will be more fully noticed as we proceed.

11. *Leeway.*—Another correction of the course is also frequently requisite. The ship's progress is not always in the direction of her length; the wind often impels her sideways, or, as it is called, to *leeward* of the line from fore to aft. The necessary allowance for this divergence from the path indicated by the compass is the correction for *leeway;* its amount can be estimated only by practical experience.

12. *Rate of Sailing.*—The rate at which a ship sails on any course is measured by an instrument called the *log*, and a line attached to it called the *log-line*, about 120 fathoms in length. The log is a piece of wood in shape of a sector of a circle, and with its arc or rim so loaded with lead that, when thrown into the sea, it stands vertically in the water, with only its centre just above the surface.* The log-line is so attached as to keep the face of the log towards the ship, in order that it may offer the greater resistance to being dragged after it; the length of line unwound from a reel, by the advancing motion of the vessel, *half a minute*, gives the distance run in that time, and thus is inferred the rate of sailing. The log-line is divided into equal parts by means of a bit of string passed through the strands and *knotted*, the number of knots showing the number of parts—each part, which is the 120th of a nautical mile, is hence called a *Knot;* so that as many knots as are run out in half a minute, or the 120th of an hour, so many nautical miles per hour is the ship's rate of sailing. Sailors thus say that the rate is so many knots an hour, meaning so many nautical miles an hour. A nautical mile is the 60th

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* This is the common log, still too much used; a more accurate instrument is Massey's log.

part of a degree of the equator or of the meridian; that is, about 6,080 feet. When the log is *hove*, about ten or twelve fathoms of line are suffered to run out before the counting commences; this is called the *stray-line*, which is an allowance for letting the log go clear of the ship, and settle in the water. As soon as the end of the stray-line, which is marked by a bit of red cloth, passes from the reel, the half-minute commences. The time is measured by a sand-glass, which runs out in thirty seconds, and which is turned when the end of the stray-line passes. The line is stopped as soon as the sand has run out.

13. *A Day's Work.*—By a day's work is meant the ordinary daily operations at sea to determine the position of a ship, and the advances made from noon till noon; it consists in keeping a record of the compass-courses sailed in the interim, of the different rates of sailing, of the variation of the compass, the velocity and direction of currents, &c., &c., and, finally, of the latitude and longitude. These particulars, as they become known, are recorded on the *log-board;* and the latitude and longitude, finally deduced, is called the *dead-reckoning*, or the latitude and longitude *by account.* The particulars of the log-board are transferred, from day to day, to the *log-book;* and, with the addition of whatever else may give the necessary completeness to the record—the astronomical observations made to correct the dead-reckoning, the state of the wind and weather, the bearings of points of land, &c., &c.—form the *journal* of the voyage.

POINTS OF THE COMPASS.—The following table gives the several angles which the different points of the compass make with the meridian, as also the angles for the quarter-points:—

North.		Points.	Angles.	South.	
		¼	2° 49′		
		½	5° 37½′		
		¾	8° 26′		
N. *b.* E.	N. *b.* W.	1	11° 15′	S. *b.* E.	S. *b.* W.
		1¼	14° 4′		
		1½	16° 52½′		
		1¾	19° 41′		
N. N.E.	N. N.W.	2	22° 30′	S. S.E.	S. S. W.
		2¼	25° 19′		
		2½	28° 7½′		
		2¾	30° 56′		
N.E. *b.* N.	N.W. *b.* N.	3	33° 45′	S.E. *b.* S.	S.W. *b.* S.
		3¼	36° 34′		
		3½	39° 22½′		
		3¾	42° 11′		
N.E.	N.W.	4	45° 0′	S.E.	S.W.
		4¼	47° 49′		
		4½	50° 37½′		
		4¾	53° 26′		
N.E. *b.* E.	N.W. *b.* W.	5	56° 15′	S.E. *b.* E.	S.W. *b.* W.
		5¼	59° 4′		
		5½	61° 52½′		
		5¾	64° 41′		
E.N.E.	W. N.W.	6	67° 30′	E. S.E.	W. S.W.
		6¼	70° 19′		
		6½	73° 7½′		
		6¾	75° 56′		
E. *b.* N.	W. *b.* N.	7	78° 45′	E. *b.* S.	W. *b.* S.
		7¼	81° 34′		
		7½	84° 22½′		
		7¾	87° 11′		
E.	W.	8	90° 0′	E.	W.

The angles in the above table nowhere differ from the truth by more than a quarter of a minute; a degree of accuracy amply sufficient for all the purposes for which it is used. It is necessary that the learner should commit this table to memory—at least, so far as to be able to state, without reference to it, how many points any of these courses, or rhumbs, are distant from the meridian, or north and south line; as each point is 11° 15′, the angle corresponding to any number of points is easily deduced. Repeating the points in order, completely round the compass, as figured at page 1050, or according to this table, is called by sailors *boxing the compass.*

We shall now proceed to consider the various sailings, first, however, making a few brief remarks in reference to the data, or observed conditions upon which the computations in the following articles are, in general, to be founded.

From what has been explained in the preceding pages, the learner will perceive that the principal measurements made at sea to determine the place of a ship, independently of astronomical observations, are the measurements of the *course* sailed on, and the *rate* of sailing. From the account given of the means and instruments employed for these purposes, he must see that the results furnished by them can scarcely ever be regarded as rigorously correct. The compass-card, valuable and indispensable as it is, has no divisions upon it to distinguish angles which differ from one another by less than a degree;* such differences, therefore, in steering a ship, have to be roughly estimated by guess—the course is thus liable to error to some extent. Again, the rate of sailing, as measured by the log, is equally exposed to error from the very nature of the operation, and thus the distance run, on any course, cannot be determined with strict accuracy. Even if these sources of error could be obviated, yet winds, currents, swells of the sea, and the various other accidents to which a ship is exposed, and which all act as disturbing causes, would often seriously affect the correctness of the position of the vessel, as deduced from the dead-reckoning.

The aim of the careful and experienced mariner is to be on the look-out for these external influences, and to allow, as best he can, for their effects; matters in reference to which there is room for the exercise of much practical tact and judgment.

These allowances being made, the courses on the log-board are modified accordingly, and, being corrected for the variation and local deviation of the compass, the actual distance made, as also the difference of latitude and longitude since the preceding noon, is computed, and the latitude and longitude, by *account*, are thus ascertained.

These last important particulars—the latitude and longitude—being derived from data so exposed to error, can be regarded, at best, as only approximately true; and therefore the properly qualified navigator loses no opportunity to correct his dead-reckoning by employing the more sure and certain methods which nautical astronomy supplies. In the following articles, however, none of these can be introduced; the object of this part of our subject is to treat exclusively on what concerns the dead-reckoning; and we shall, throughout, suppose that proper allowance for the variation of the compass, for the leeway, &c., have been made, and that the courses concerned are the *true* courses. The methods for ascertaining the variation of the compass must be deferred till we come to treat of Nautical Astronomy.

PLANE SAILING.—*Investigation of the Theoretical Principles.*—Let the following diagram represent the globe of the earth, P being one of its poles, and E Q the equator. Let A B be a rhumb-line or track of a ship on a single course. Imagine this oblique path to be divided, by equidistant meridians, into portions Ab, bc, cd, &c., so small that each portion may differ insensibly from a straight line; and, as in the figure, let the parallels of latitude $b'b$, $c'c$, $d'd$, &c., be drawn. A series of triangles Abb', bcc', cdd', &c., will thus be formed on the surface of the sphere, so small that each may be practically regarded as a *plane* triangle. It is obvious that this may be conceived without any sensible violation of strict accuracy. The triangles thus described, and thus assumed to be plane triangles, are all similar; for the angles at b', c', d', &c., are all right angles, and the ship's track cuts every meridian which it crosses at the same angle. Consequently, by Euclid, Prop. 4, Book VI., we have the proportions,

$$Ab : Ab' :: bc : bc' :: cd : cd' \text{ \&c.};$$

---

* The outer edge of the card is divided into 360 degrees; but as it is difficult to steer a ship to the nicety which these divisions imply, the marks on the extreme rim of the compass-card are seldom much attended to by mariners.

and therefore (Euclid, Prop. 5, Book V., or Prop. 10 of the Treatise on Proportion),[*]

$$Ab : Ab' :: Ab + bc + cd + \&c., : Ab' + bc' + cd' + \&c.$$

But $Ab + bc + cd + \&c.$, is the whole distance sailed on the course, and $Ab' + bc' + cd' + \&c.$, is the difference of latitude A C between A, the place left, and B, the place arrived at. Consequently, if a right-angled triangle, similar to the little right-angled triangle Abb', be constructed—that is, a right-angled triangle, in which A, Fig. 15, is the angle of the course, and if the hypotenuse A B be made to represent the distance sailed—that is, the track A B on the globe—then, obviously, the perpendicular A C will represent the difference of latitude, while the base C B—the side opposite to the course—will represent the sum of all the minute *departures* which the ship makes from the successive meridians which it crosses; for since

Fig. 14.

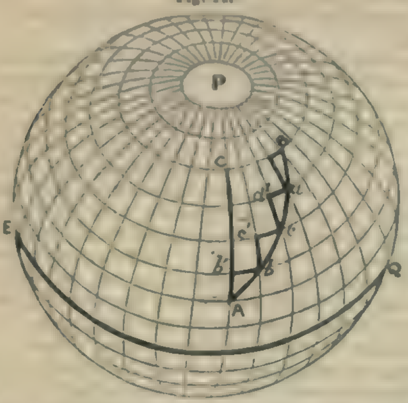
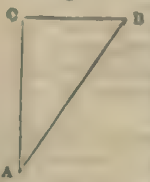


Fig. 15.



$$Ab : bb' :: A B : B C$$
$$Ab : bb' :: bc : cc' :: cd : dd', \&c.$$
$$\therefore Ab : bb' :: Ab + bc + cd + \&c. : bb' + cc' + dd' + \&c.$$

and since by construction $A B = Ab + bc + cd + \&c.$, therefore $B C = bb' + cc' + dd' + \&c.$ The length B C is called the *departure* made by the ship in sailing from A to B; and it therefore follows that the distance sailed, the difference of latitude made, and the departure, may be represented by the sides of a right-angled *plane* triangle, the angle opposite to the departure being the angle of the course.

Hence, when any two of the four things—Distance, Difference of Latitude, Departure, and Course—are known, the remaining two may be determined by the solution of a right-angled plane triangle; so that, as far as these particulars are concerned, the results are the same as if the ship were sailing on a plane surface, the meridians being replaced by parallel straight lines, and the perpendiculars to these, taken for the parallels of latitude. It is thus that that part of Navigation, which is concerned only with the four things just mentioned, is called PLANE SAILING.

The line B C, or the side of the right-angled triangle opposite to the course, is not the representation of any corresponding line on the globe: it is the entire sum of all the minute departures made by the ship in passing from meridian to meridian, from A up to B.

The foregoing investigation comprehends the whole mathematical theory of Plane Sailing, into which, it will be observed, the consideration of *longitude* does not enter.

The attentive reader will perceive, that in replacing the spiral track of a ship's run, and the great circle arc which measures the difference of latitude made good in that run, by the hypotenuse and perpendicular of a right-angled triangle drawn upon a *plane surface*, we sacrifice not the slightest amount of accuracy. It is shown above, that if this spiral track were unbent into a straight line A B, and at one extremity, A, of this straight line a plane angle equal to that of the course were made by A C, and the perpendicular B C drawn—it is shown that A C must accurately represent the difference of latitude, and B C the departure.

It cannot be any objection to this conclusion that we have taken small triangles on the sphere to be *plane* triangles; for the reasoning fixes no limit to the degree of smallness of the sides; nor must it be understood that we have assumed the sphere, on which these triangles are assumed to be figured, to be *itself* a plane. The assumption extends only to the length of supposing the sphere to present a succession of triangular plane faces; and as each face is contracted to any degree of minuteness, the error of this supposition ultimately disappears.

As a corollary to what is proved above, we may add that—In sailing upon a single rhumb, the differences of latitude made, are proportional to the distances run. And, from the theory of the right-angled triangle established in the INTRODUCTION,[†] we have all the proportions usually given on this part of the subject in books on Navigation; they are expressed as follows;[‡]—

I.	Radius	:	Sin. Course	::	Distance sailed	:	Departure.
II.	Radius	:	Cos. Course	::	Distance sailed	:	Diff. Latitude.
III.	Cos. Course	:	Radius	::	Diff. Latitude	:	Distance sailed.
IV.	Radius	:	Tan. Course	::	Diff. Latitude	:	Departure.
V.	Distance sailed	:	Diff. Latitude	::	Radius	:	Cos. Course.
VI.	Distance sailed	:	Departure	::	Radius	:	Sin. Course.
VII.	Sin. course	:	Radius	::	Departure	:	Distance sailed.
VIII.	Tan. course	:	Radius	::	Departure	:	Diff. Latitude.
IX.	Diff. Latitude	:	Departure	::	Radius	:	Tan. course.

If the table of natural sines, cosines,[§] &c., be used, then Radius = 1; if the table of logarithmic sines, cosines, &c., be used—and they are employed by seamen too indiscriminately—then log. Radius = 10.[‖]

In all books on Navigation, the latter tables, exclusively, are referred to; but, as already stated in the INTRODUCTION, we would recommend a departure from this practice; we shall, therefore, in general, exhibit the calculations by *both* tables.

It is not considered necessary, in the examples that follow, to introduce *diagrams* of the several triangles; but the learner should always roughly sketch the suitable triangles himself, observing that, as is usual in maps, the top of the page is to be regarded as North, and the bottom as South; the right-hand side East, and the left West. In sketching his right-angled triangle, therefore, he should first draw the N. and S. line, or the horizontal meridian, and then take a portion of it for the difference of latitude, drawing, from the latitude reached, the base of the triangle, to represent the departure—to the *right*, if the departure be east, to the *left* if it be west. The hypotenuse will then represent the distance sailed, and the angle between it and the difference of latitude, the course. It will be as well to regard the vertex of this angle as at the centre of the compass-card, since it is the centre of the sensible horizon at starting on the course, and thus no mistake can be made as to which side of the meridian line the angle of the course is to lie on, or whether its opening be upward or downward.

Examples.

1. A ship from latitude 47° 30′ N., has sailed S.W. by S., a distance of 98 miles: what latitude is she in, and what departure has she made?

---

[*] See *ante*, p. 581.

[†] See *ante*, p. 1037.

[‡] We have thought it as well to express the rules and formulæ given in the INTRODUCTION, in the form of proportions here, as seamen are more accustomed to use them in this shape; but the reader of the Introduction will see that this rule-of-three arrangement of the terms employed is not necessary, and he may therefore adopt it or not, as he pleases.

[§] See *ante*, p. 665.

[‖] See *ante*, p. 666, *et seq.*

The course being 3 points is 33° 45′; hence—

1. *To find the diff. lat.*

By logarithms.

As rad. . . . . . . −10
: cos. course, 33° 45′ . . 9·9199
: : distance = 98 . . . 1·9912

: diff. lat. = 81·49 . . . 1·9111

Without logarithms.

diff. lat. = cos. course × distance.

cos. course, 33° 45′ . . . ·8315
distance 98

66520
74835

Diff. lat. . . . . . = 81·487

Hence the difference of latitude is 81·49 miles, S.

2. *To find the departure.*

By logarithms.

As rad. . . . . . −10
: sin. course, 33° 45′ . . 9·7447
: : distance = 98 . . . 1·9912

: departure = 54·44 . . 1·7359

Without logarithms.

dep. = sin. course × distance.

sin. course, 33° 45′ . . . ·5556
distance 98

44448
50004

54·4488

Hence the deparure is 54·45 miles, W.

The latitude from . . . 47° .30′ N.
The difference of lat. . . 1° 21′ S.

Latitude in . . . . . 46° 9′ N.

☞ $\frac{81}{60}$ degrees = 1° 21′.

Departure 54½ miles W.

The example is here worked by *computation*; there is another way of obtaining the results—namely, by *inspection*. This latter method requires reference to a table called the Traverse Table, and which is to be found in every collection of navigation tables. The table is arranged much like a table of sines and cosines; the angles of the courses are inserted at the top of the page, when they do not exceed 45°, and at the bottom when they do, and the distances are placed down the margin. By entering the table with any given course and distance, the corresponding difference of latitude and departure can be taken out from the body of the table.*

This table is a very useful one for seamen, as it computes for him the two formulæ worked by in the right-hand column of operations above; that is, entering his table with the proper course and distance, he finds, under the respective heads "Lat." and "Dep.," the values of

cos. course × distance, and sin. course × distance

already worked out for him, usually as far as two places of decimals, which is an extent amply sufficient. A table of natural sines and cosines is thus all that is wanted to construct a traverse table. In such a table each sine and cosine is multiplied by all distances from 1 up to 120, which is the ordinary limit to which the columns of distances are carried. Distances which exceed 120 miles may be cut up into smaller distances, and the portions brought within the compass of the table.

2. A ship from latitude 47° 30′ N., sailing N.W. *b* W., finds that she has made 82 miles of departure: What is her distance run, and her latitude in?

The course being 5 points is 56° 15′; hence—

1. *To find the distance.*

By logarithms.

As sin. course, 56° 15′ . . − 9·9198
: radius 10
: : departure = 82 . . 1·9138

: distance = 98·62 . . 1·9940

Without logarithms.

dist. = dep. ÷ sin. course.

$$\text{sin. course, } 56^\circ\ 15' \quad \cdot8{,}3{,}1{,}5)\overset{\text{dep.}}{82}\ (98\cdot6$$
7484

716
665

51
∴ distance = 98·6 miles . . 50

2. *To find the diff. lat.*

By logarithms.

As tan. course, 56° 15′ . −10·1751
: radius 10
: : departure = 82 . . 1·9138

: diff. lat. = 54·79 . . 1·7387

Without logarithms.

diff. lat. = dep. ÷ tan. course.

tan. course, 56° 15′ . 1·4,9,6,6)82 (54·8
7483

717
599

118
119

∴ Diff. lat. = 54·8 miles = 55′ N.
Latitude from . . . 47° 30′ N.

Latitude in . . . 48° 25′ N.

3. A ship has sailed S.E. *b* S., from lat. 47° 30′ N. to lat. 46° 8′ N. Required the distance run and the departure made.

The course being 3 points is 33° 45′;

also { lat. from 47° 30′ N.
lat. in 46° 8′ N.

diff. lat. 1° 22′ N. = 82 miles.

1. *To find the distance.*

By logarithms.

As cos. course, 33° 45′ . . − 9·9198
: radius 10
: : diff. lat. = 82 . . 1·9138

: distance = 98·62 . . 1·9940

Without logarithms.

dist. = diff. lat. ÷ cos. course.

cos. course, 33° 45′ . ·8,3,1,5)82 (98·62
74835

7165
6652

∴ dist. = 98·62 miles 513
499

14
17

2. *To find the departure.*

By logarithms.

As radius . . . . −10
: tan. course, 33° 45′ . . 9·8249
: : diff. lat. = 82 . . 1·9138

: departure = 54·79 . . 1·7387

---

* See Tables at pages 1063 and 1069.

Without logarithms.

dep. = diff. lat. × tan. course.

tan. course, 33° 45′ . . ·06682
diff. lat. = 82, reversed . 28

53456
1336

departure = 54·792 miles.

4. A ship from lat. 50° 13′ N., while sailing on a course between south and east, a distance of 98 miles, makes 82 miles of departure: What course did she keep, and what latitude did she arrive at?

1. *To find the course.*

By logarithms.

As dist. = 98 . . . −1·9912
: dep. = 82. . . . 1·9138
: : rad. 10

: sin. course, 56° 48′ . . 9·9226

Without logarithms.

$$\text{sin. course} = \text{dep.} \div \text{dist.} = \frac{82}{98} = \frac{41}{49}$$

7)41

7) 5·8571

sin. 56° 48′ . . ·8367

Hence the course is S. 56° 48′ E.

2. *To find the diff. lat.*

By logarithms.

As rad. . . . . . − 10
: cos. course, 56° 48′ . . 9·7384
: : dist. = 98 . . . . 1·9912

: diff. lat. = 53·66 . . 1·7296

Without logarithms.

diff. lat. = dist. × cos. course.

cos. course, 56° 48′ . ·5476
dist. = 98, reversed . 89

49284
4381

diff. lat. = 53·665 miles = 54′

And 50° 13′ N.—54′ N. = 49° 19′ N., the lat. in.

5. Yesterday at noon we were in lat. 38° 32′ N.; and this day at noon we were in lat. 36° 56′ N. We have run on a single course between S. and E., at 5½ knots an hour: required our course and departure.

Lat. from 38° 32′ N.	24, number of hours.
Lat. in 36° 56′ N.	5½
Diff. lat. 1° 36′ N. = 96 miles	120
	12
	132 miles, the distance.

1. *To find the course.*

By logarithms.

As dist. = 132 . . . −2·1206
: diff. lat. = 96 . . . 1·9823
: : rad. 10

cos. course, 43° 20′ . . . 9·8617

Without logarithms.

$$\text{cos. course} = \text{diff. lat.} \div \text{dist.} = \frac{96}{132} = \frac{8}{11}$$

11)8

cos. course, 43° 20′ . . ·7273

∴ the course is S. 43° 20′ E. = S.E. *b.* S. ¾ E. nearly.

2. *To find the departure.*

By logarithms.

As rad. . . . . . — 10
: sin. course, 43° 20′ . . 9·8365
: : dist. = 132 . . . . 2·1206

: dep. = 90·58 . . . 1·9571

∴ departure = 90 58 miles E.

Without logarithms.

dep. = sin. course × dist.

sin. course, 43° 20′ . . . ·6862
dist. = 132, reversed . . . 231

6862
2059
137

departure = 90·58 miles E.

The foregoing examples have all been solved by computation. As remarked at page 1053, the same results, though with not precisely the same amount of accuracy, may be obtained by inspection of the Traverse Table. There is also a third method of proceeding, much practised by seamen, though less accurate still, by which the required conclusions may be reached: it is the method of *construction.*[*] A circle is described, and the north and south line, or the horizontal meridian, is drawn through its centre; then, from a scale of chords, constructed agreeably to the radius used, which radius is of course the chord of 60° on the scale, the chord of the course is pricked off in its proper direction from the N. or S. extremity of the meridional diameter; from the same extremity, a line is then drawn through the point of the circumference before marked: this is the line of distance, or hypotenusal line, and the line already drawn through the centre is the line of difference of latitude: whichever of these is given is now to be measured off from any scale of equal parts, and the right-angled triangle is then to be completed by introducing the third side, which, measured from the same scale, will give the length sought.

In this illustration, the course and one of the sides including it, are supposed to be given; but if the course be unknown, and any two of the sides of the triangle given, the mode of proceeding is readily suggested from that above: in all cases we have two parts of a right-angled triangle given to construct the triangle—a simple geometrical problem. The unmeasured parts are then to be measured, the angle of the course from the scale of chords, and the sides from the scale of equal parts already employed. We shall conclude this article with a few examples for the exercise of the learner: he will not forget that the given courses are always understood to be the true courses, that is, the compass courses corrected for variation, local deviation, and leeway. The means by which the variation of the compass is ascertained cannot be considered here, as the subject belongs to Nautical Astronomy. (See next Chapter.)

Examples for Exercises in Single Courses.

1. A ship from latitude 48° 40′ N. sails N.E. by N. 296 miles. Required the departure made and the latitude in.

Ans. Departure, 164·4 miles E. Latitude in, 52° 46′ N.

2. A ship from latitude 49° 30′ N. sails N.W. by N. 103 miles. Required her departure and the latitude in.

Ans. Departure, 57·2 miles W. Latitude in, 50° 56′ N.

3. A ship from latitude 47° 20′ N., sails on a course between N. and E. a distance of 98 miles, and arrives at lat. 48° 42′ N. Required the course steered and the departure made.

Ans. Course, N. 33° 12′ E. Departure, 53·7 miles E.

4. A ship sails S.E. ½ E. from latitude 15° 55′ S. till

---

[*] There is also a fourth method—by GUNTER'S SCALE—which it is not worth while to dwell upon. Any mechanical operation with scale and compasses is necessarily affected with inaccuracy; and as reference to the Traverse Table requires less time, and furnishes truer results, it is always to be preferred next to computation.

she is found by observation to be in latitude 18° 49′ S.: required her distance run, and departure.

Ans. Distance, 274 miles. Departure, 212 miles E.

5. A ship from latitude 34° 23′ S. sails between the south and west till she reaches latitude 36° 34′ S. and finds that she has made75 miles of departure. Required her course and distance run.

Ans. Course S. 29° 47′ W. Distance, 151 miles.

6. A ship from latitude 3° 16′ N. sails S.W. by W.¼W., till she has made 356 miles of departure. Required the distance sailed and latitude in.

Ans. Distance, 415 miles. Lat. in, 17′ S.

7. A ship in latitude 3° 52′ S., is bound to a port bearing N.W. by W. ½ W., in latitude 4° 30′ N. How far does that port lie to the westward, and what is the ship's distance from it? in other words, what departure and distance must the ship make to reach it?

Ans. Departure, 939 miles W. Distance, 1065 miles.

8. A ship, from latitude 42° 18′ N., sails S. 25° W. a distance of 150 miles. Required her departure and latitude in.

Ans. Departure 63⅓ miles W. Lat. in 40° 2′ N.

9. If a ship sail from latitude 48° 27′ S. on a S.W. by W. course at the rate of 7 knots an hour, in how many hours will she arrive at latitude 50° S.?

Ans. In 23$\frac{9}{10}$ hours.

10. A ship, from latitude 55° 30′ N., sails S.W. by S. for 20 hours, and then finds by observation that she is in latitude 53° 17′ N. Required her hourly rate of sailing, and the departure she has made.

Ans. Rate, 8 miles an hour. Departure, 88·87 miles W.

11. A ship sails for 18 hours on a single course between the S. and W. from latitude 38° 32′ N. to latitude 36° 56′ N., at the rate of 7⅓ miles an hour. Required the course, distance, and departure.

Ans. Course, S. 43° 20′ W. Distance, 132 miles. Departure, 90·58 W.

12. A ship sails for 53¾ hours on a S.E. ¾ E. course from latitude 52° 30′ N. to latitude 47° 10′ N. Required the average rate of sailing per hour, and the departure made.

Ans. Rate, 10 miles an hour. Departure, 432 miles E.

COMPOUND COURSES, OR TRAVERSE SAILING.—When, from contrary winds or other causes, a ship's track from one place to another is made up of several single courses, the zig-zag path it takes is called a Traverse, or a Compound Course; and the determination of the single course and distance, from the place left to the place arrived at, is called *working* or *resolving* the traverse.

To work a traverse, it is only necessary to find the difference of latitude and departure for each distinct course, as in the foregoing article; to take the aggregate of these for the whole difference of latitude and departure, and thence to find the corresponding single course and distance.

The most orderly way of proceeding is to form a little traverse table, consisting of six columns, to receive the proper entries for course, distance, diff. lat. N. and S., and dep. E. and W., as in the specimen in Example 1 following. When the entries are completed, the two diff. lat. columns are added up separately, and the difference of the results taken: this difference is the whole diff. lat., which is N. or S. according as the N. or S. column gives the greater result. In like manner, the results of the two departure columns being found, their difference is the resultant departure, to be used with the whole difference of latitude, to determine the direct course and distance.

Examples.

1. A ship from latitude 51° 25′ N. has sailed the following courses, namely—

1st, S.S.E. ¼ E., 16 miles.
2nd, E.S.E., 23 miles.
3rd, S.W. *b* W. ½ W., 36 miles.
4th, W. ¾ N., 12 miles.
5th, S.E. *b* E ¼ E., 41 miles.

Required the latitude in, and the direct course and distance to reach it.

TRAVERSE TABLE.

Courses.	Dist.	Diff. of latitude.		Departure.	
		N.	S.	E.	W.
S.S.E. ¼ E.	16		14·5	6·8	
E.S.E.	23		8·8	21·3	
S.W. *b* W ½ W.	36		17		31·8
W. ¾ N.	12	1·8			11·9
S.E. *b* E. ¼ E.	41		21·1	35·2	
Equivalent course, S. 18° 12′ E.		1·8	61.4	63·3	43·7
			1·8	43·7	
Direct distance 63 miles.			59·6	19·6	

The first two columns of this table are occupied with the given courses and distances; in the other four are inserted the diff. lat. and dep. corresponding to each course and distance, taken by inspection from the Traverse Table. The results of these latter columns show that the difference of latitude made is 59·6 miles S., and the departure 19·6 miles E. And from these the lat. in, and the direct course and distance from the lat. left to the place reached, is found by computation as follows:—

```
Latitude left.   .   .   .   51° 25′ N.
Diff. lat. 59·6 miles  .  .   1°  0′ S.
                             ----------
Latitude in  .   .   .   .   50° 25′ N.
```

1. *To find the direct course.*

By logarithms.

```
As diff. lat. = 59·6  .   .   .  −1·7752
  : departure = 19 6  .   .   .   1·2923
: : radius  .   .   .   .   .   . 10
                                 -------
   tan. course, 18° 12′  .   .    9·5171
```

Without logarithms.

tan. course = dep. ÷ diff. lat.

```
5,9·6)19·6(·3288 = tan. 18° 12′
      1788
      ----
       172
      1192
      ----
       528
       477
      ----
        51
        48
```

Hence the direct course is S. 18° 12′ E.

It thus appears that if the ship had left her port on the course S. 18° 12′ E., and had kept this course unaltered for a run of 63 miles (see next page), she would have arrived at the place reached by the above traverses. This conclusion, however, is not rigorously true, though near enough for practice. (See the remarks subjoined to Example 8).

2. *To find the distance.*

By logarithms.

```
As sin. course, 18° 12′   .   .  −9·4946
  : radius  .   .   .   .   .   . 10
: : departure = 19·6  .   .   .   1·2923
                                 -------
  : distance = 62·75  .   .   .   1·7977
```

Without logarithms.

dist. = dep. ÷ sin. course

```
sin. 18° 12′ = ·3,1,2,3)19·6   (62·75
                        18738
                        -----
                          862
                          625
                          ---
                          237
                          219
                          ---
                           13
                           16
```

Hence the nautical distance is 62·75 miles.

NOTE.—It will save time, and be a guard against mistake in the filling up the several columns from the Traverse Table, if, before that table is opened, a mark be put opposite to each course, and in each of the columns where the entries connected with that course are to be inserted. Thus, if N. occur in the course, mark a little cross against it in the N. column, near enough to the right-hand margin of that column to allow of room for the extract from the Traverse Table; if S. occur in the course, put a like mark in the S. column. If E. occur, mark the E. column; and if W. occur, mark the W. column. Then, when the Traverse Table is consulted, we shall have precluded the risk of writing the particulars from it in the wrong column.

2. A ship sails S.W. *b* S. 24 miles; N.N.W. 57 miles; S.E. *b* E. ½ E. 84 miles; and S. 35 miles. Required the direct course and distance.

Ans. Course, S. 43° E. Distance, 57 miles.

3. A ship from latitude 50° 13′ N. has sailed the following courses, namely—

1st. W.S.W., 51 miles. 2nd. W. *b* N., 35 miles. 3rd. S. *b* E., 45 miles. 4th. S.W. *b* W., 55 miles. 5th. S.S.E. 41 miles. Required the latitude in, and the direct course and distance sailed.

Ans. Lat. in, 48° 8′ N. Course, S. 39° 19′ W., or S.W. *b* S. ½ W. Distance, 162 miles.

4. A ship from latitude 28° 32′ N. has run the following courses, namely—

1st. N.W. *b* N., 20 miles. 2nd. S.W., 40 miles. 3rd. N.E *b* E., 60 miles. 4th. S.E., 55 miles. 5th. W. *b* S., 41 miles. 6th. E.N.E., 66 miles. Required the latitude in, and the direct course and distance.

Ans. The same latitude. Course, due E. Distance, 70·2 miles.

5. From noon to noon the following courses were run, namely—

1st. S.W. *b* S., 20 miles. 2nd. W., 16 miles. 3rd. N.W. *b* W., 28 miles. 4th. S.S.E., 32 miles. 5th. E.N.E, 14 miles. 6th. S.W., 36 miles. What difference of latitude has the ship made, and what is her direct course and distance?

Ans. Diff. lat., 50·7 miles S. Course, S.W. Distance, 71·7 miles.

6. A ship sails from latitude 10° 6′ S., the following courses, namely—

1st. N.N.E., 86 miles. 2nd. N., 74 miles. 3rd. E. *b* N., 53 miles. 4th. N.N.W. ¼ N., 40 miles. 5th. E.N.E. ½ N., 21 miles. Required the latitude in, and direct course and distance.

Ans. Lat. in, 6° 34′ S. Course, N. 23° 25′ E. Distance, 231 miles.

7. A ship from latitude 51° 30′ N., running at the rate of 8 knots an hour, sails W.S.W., 3 hours; N.W., 2½ hours; W., 4 hours; S.W. *b* S., 2¼ hours; and N.W. ½ W., 2 hours. Required her latitude in, and her direct course and distance.

Ans. Lat. in, 51° 30′ N. Course, W. Distance, 90·7 miles.

8. A ship from latitude 24° 32′ N., sails the following courses, namely—

1st. S.W. *b* W., 45 miles. 2nd. E.S.E., 50 miles. 3rd. S.W., 30 miles. 4th. S.E. *b* E., 60 miles. 5th S.W. *b* S. ¼ W., 63 miles. Required her latitude in, her departure, and the direct course and distance.

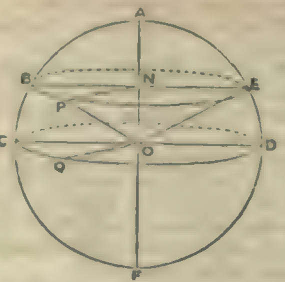
Ans. Lat. in, 22° 3′ N. Departure, 0. Course, S. Distance, 149·2 miles.

In this last example, the ship is said to have returned to the meridian from which she sailed, so that her course from the place arrived at to that left, is concluded to be due south. In the present case, the latitude being low, the error of this conclusion is practically of no consequence; but the balance, or aggregate of the several departures made on a traverse, is not, in strictness, the departure due to the direct course—a fact that sailors, in general, are not sufficiently sensible of. We shall advert more at length to this matter in our introductory observations to MERCATOR'S SAILING.

PARALLEL SAILING.—When a ship sails upon a parallel of latitude, her distance run is then the same as her departure; her difference of latitude is nothing, and her difference of longitude may be easily determined. The case is one of *parallel sailing*, the theory of which may be established in the following manner:

Taking the diagram at p. 596, Chapter VIII., *Spherical Geometry*, let C Q represent a portion of the equator corresponding to the portion B P of a parallel of latitude sailed over by a ship, the points P Q being on the same meridian. Then O C is the radius of the equator, and N B the radius of the parallel, and the difference of longitude of B and P will be measured by the arc C Q.

Fig. 16.



Now, since similar arcs are to one another as the radii of the circles to which they belong, we have

$$N B : O C :: \text{dist. } B P : \text{diff. long. } C Q.$$

But N B is the geometrical cosine of the latitude C B, to the radius O C; consequently N B is equal to O C multiplied by the trigonometrical cosine of the angle Q O P of the latitude; that is, expressing the latitude in degrees and minutes, and not in linear measure, we have

$$O C \cos. \text{lat.} : O C :: \text{dist. sailed} : \text{diff. long.} \quad (1)$$

$$\therefore \cos. \text{lat.} : 1 :: \text{dist. sailed} : \text{diff. long.} \quad (2)$$

And it follows from this, that if the distance between any two meridians on a parallel in latitude L, be D, and the distance of the same meridians on a parallel in latitude L′ be D′, then alternating the proportion (1)

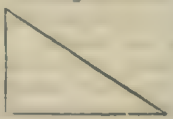
$$\cos. L : \cos. L' :: D : D' \quad . \ . \quad . \quad (A)$$

The proportion (2) evidently solves the problem—Given the latitude of the parallel and the distance sailed on it, to find the difference of longitude; the solution being

$$\text{difference of longitude} = \frac{\text{distance sailed}}{\cos. \text{latitude}} \quad . \ . \quad (3)$$

So that, as in the former cases, we may connect the three things concerned in a right-angled plane triangle (Fig. 17), the base representing the distance sailed, the hypotenuse, the difference of longitude, and the angle between the two the latitude of the parallel, because we know from the theory of the right-angled triangle that these three parts are related as in condition (3). Any problem in parallel sailing may, therefore, always be reduced to a case of right-angled triangles; and consequently may be solved, like a problem in plane sailing, by inspection of the Traverse Table. We shall only have to consider the latitude of the parallel as *course*, and the distance as *diff. lat.*; the corresponding distance in the Traverse Table will be the *diff. long.*

Fig. 17.



It must be observed, however, that the perpendicular of our right-angled triangle merely serves to connect together the three things here mentioned; it has no signification in navigation. The distance sailed on a parallel, the latitude of that parallel, and the difference of longitude between the place left and that arrived at, are related to one another as the three parts, noticed above, of a right-angled triangle, the perpendicular of which merely serves to complete the diagram in which these relations are geometrically embodied.

The proportion (2) above, is usually expressed thus:—

$$\cos. \text{lat.} : \text{radius} :: \text{dist. sailed} : \text{diff. long.}$$

where rad. = 1 when the table of natural sines and cosines is used; and log. rad. = 10 when the logarithmic table is used. As usual, we shall exhibit the working by both tables; but it will be perceived, as in most of the computations already given, that the former table is in general to be preferred.

Examples.

1. A ship from latitude 53° 36′ N. longitude 10° 18′ E., sails due west 236 miles: required the longitude in.

To find the diff. long.

By logarithms.

As cos. lat., 53° 36′ . . . –9·7734
: radius . . . 10
: : distance=236 . . . 2·3729
: diff. long.=397·7 . 2·5995

Without logarithms.

diff. long.=dist. ÷ cos. lat.
cos. 53° 36′=·5,9,3,4)236 (397·7
17802
5798
5341
457
415
42
42

Hence the diff. long. is 397·7 miles=398 miles nearly.

Reducing this to degrees,
60)398

diff. long.= 6° 38′ E. } The difference to be taken,
long. left . . 10° 18′ W. } as the longitudes are E. and W.
long. in . . 3° 40′ E.

2. A ship from latitude 32° N., sails due east till her difference of longitude is found to be 384 miles: what distance has she run?

To find the distance.

By logarithms.

As radius . . . –10
: cos. lat., 32° . . . 9·9284
: : diff. long=384 . . . 2·5843
: distance=325·6 . . 2·5127

Without logarithms.

Dist.=diff. long. × cos. lat.
cos. 32°=·8480
384 reversed, 483
2544
678
34
325·6

Hence the distance run is 325·6 miles.

3. From two ports in latitude 32° 20′ N., distant 256 miles, measured on the parallel, two ships sail directly north, till they come to the latitude 44° 30′ N.: how many miles, measured on the parallel arrived at, are they apart?

This question is to be worked by using the proportion (A).

By logarithms.

As cos. lat. from, 32° 20′ Arith. Comp. ·0732
: cos. lat. in, 44° 30′ . . . 9·8532
: : first dist.=256 . . . 2·4082
: secd. dist.=216·1 . . . 2·3346

Without logarithms.

D′=D cos. L′÷cos. L
cos. L′=cos. 44° 30′=·7133
D=256, reversed, 652
14266
3567
428
cos. L=cos. 32° 20′,=·8,4,5,0)182·61(216·1
1690
1361
845
516
507
9

Hence, measured on the parallel of 44° 30′ N., the ships are 216 miles apart. Their *least* distance apart is the arc of the great circle from one to the other; because an arc of a great circle of the sphere is the shortest distance between its extremities.

4. A ship from latitude 42° 54′ S., longitude 9° 16′ W., sails due west 196 miles: required her longitude in.
Ans. 13° 44′ W.

5. A ship has sailed due east for 3 days on the parallel of 43° 28′; her rate of sailing has been, on the average, 5 knots an hour. What difference of longitude has she made?
Ans. 8° 16′ E.

6. A ship from longitude 81° 36′ W. sails due west 310 miles, and is then found by observation to be in longitude 91° 50′ W.: on what parallel has she sailed?
Ans. The parallel of 59° 41′.

7. In what parallel of latitude is the length of a degree only one-third the length of a degree at the equator?
Ans. Lat. 70° 32′.

8. Two ships in latitude 47° 54′ N., but separated by 9° 35′ of longitude, both sail directly south 836 miles, and at the same rate: how many miles were they apart at starting, and how many after running the 836 miles?
Ans. First distance, 385½ miles; second distance, 477 miles.

9. If two ships in latitude 44° 30′ N., and distant from each other 216 miles, were both to sail, at the same rate, directly south until their distance on the parallel arrived at, became 256 miles, what latitude would they be in?
Ans. 32° 17′ N.

10. If a ship sail due east 126 miles, from the North Cape in latitude 71° 10′ N., and then due north, till she reaches latitude 73° 26′ N., how far must she sail west to reach the meridian from which she started?
Ans. 111·3 miles.

MIDDLE LATITUDE SAILING.—It has already been sufficiently seen that the principal object of plane sailing is to determine the difference of latitude made by a ship sailing upon an oblique rhumb. This sailing gives us no information respecting the change made in the ship's longitude; but if the rhumb sailed upon, instead of being oblique to the meridians crossed by it, cuts them all at right angles, as in parallel sailing, then, as just shown, the difference of longitude made may be accurately ascertained. Except in this particular case, the determination of the change of longitude made by a ship in sailing from one place to another, is a problem the strict solution of which is by no means easy. Independently of astronomical observations, there are two modes of proceeding:—one is called *Middle Latitude Sailing*, and can be regarded only as a close approximation to the truth, unless a certain correction, hereafter given, be applied to it.[*] The other is called *Mercator's Sailing*, and by this the problem is solved upon strict mathematical principles. Middle latitude sailing is a combination of plane sailing and parallel sailing, which are united in the following way :—

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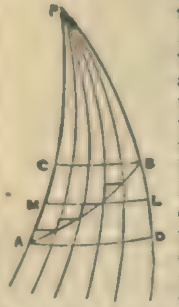
[*] In the year 1805, Mr. Workman published, under the sanction of the then Astronomer Royal, Dr. Maskelyne, a small and very useful table for correcting the errors of middle latitude sailing. The table is even now scarcely so well known as it ought to be; and as it removes the only objection to this mode of finding the difference of longitude, we have inserted it, a little abridged, at the end of these remarks on mid. lat. sailing, and would strongly recommend it to the notice of the practical navigator. (See page 1060).

It has been seen, in the theory of plane sailing, that the line called the departure, is a line equal to the sum of all the elementary departures made by a ship in sailing on an oblique rhumb. Thus, if A B, in Fig. 18, be the distance sailed, the departure is made up of all the elementary portions of the parallels of latitude lying between A D and C B; it is plain, therefore, that the departure is less than A D, and greater than C B, since the meridians approach closer together, the nearer the parallel is to the pole : there is, therefore, some parallel, M L, between A and C, such that the portion M L would be exactly equal to the departure ; and that, in latitudes near the equator, this parallel M L cannot differ materially from the *middle* parallel. It is on the supposition that the departure is equal to the distance between the meridians left and arrived at, measured on the middle parallel, that middle latitude sailing is founded.

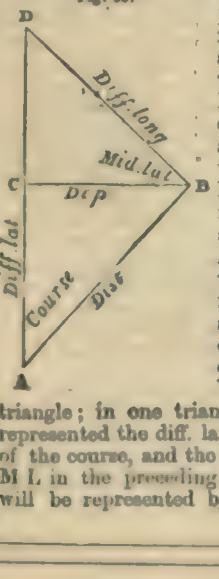
Fig. 18.



From a mere inspection of the diagram—or, better still, of a common globe—it is obvious that this supposition can differ but very little from the truth for low latitudes, and for such short distances, A B, as a day or two's run ; and more especially if the angle of the course be large, so that but little difference of latitude is made, and therefore the parallels A D, C B, are pretty close together. In such favourable cases, the method of middle latitude sailing—though, if uncorrected, only an approximation to the truth—is preferable to the method of Mercator's sailing, though this is theoretically accurate, for reasons that will be hereafter shown. In high latitudes, however, this method is not to be depended on, at least for more than a single day's run, if the angle of the course be small, and the middle latitude be uncorrected ; because the interval between the latitude left and that reached, may be too wide to warrant the supposition that the departure is equal to the middle parallel, between the meridians left and arrived at. But if the middle latitude be corrected by Workman's table, given at page 1060, all objection will be obviated.

Investigation of the Rules for Middle Latitude Sailing.—Let A B in the preceding diagram be the track of the ship, then the difference of longitude made will be the same as if the ship had sailed from M to L, along the middle parallel M L. By the present hypothesis, the distance on this middle parallel is the departure made in running from A to B; hence, the departure being known by plane sailing, we know the length of the parallel M L : and we know, also, the latitude of that parallel. Consequently, the difference of longitude may be found as in parallel sailing. Thus, if in the right-angled triangle B C D, Fig. 19, the base represent the departure, that is, M L, and the angle at the base be made equal to the latitude of M L, then the hypotenuse will represent the difference of longitude between M and L; that is, between A and B. As the base of the right-angled triangle represents the departure made, we may connect with it, as in the annexed diagram, the difference of latitude A C, and the distance A B, as in plane sailing. We shall thus have a sort of double triangle; in one triangle (the lower one here) will be represented the diff. lat. A C, the dist. A B, the angle A of the course, and the departure C B, equal in length to M L in the preceding diagram. In the other triangle will be represented by C B, the distance M L in the preceding diagram, the diff. long. B D, of M and L, and the angle C B D of the mid. lat. ; the lower triangles being constructed conformably to the principles of plane sailing, and the upper conformably to the principles of parallel sailing. In the latter the perpendicular C D is of no significance.

Fig. 19.



1. In the triangle D C B we have

cos. D B C : radius :: B C : D B,

that is, cos. mid. lat. : radius : : depart. : diff. long. . . . (1).

2. In the triangle D B A we have,

sin. D : sin. A : : A B : B D (INTRODUCTION, p. 1044).

that is, cos. mid. lat. : sin. course : : dist. : diff. long. . . (2).

3. Also, in the two triangles A B C, D B C, we have

A C tan. A = B C, B D cos. D B C = B C,

∴ A C tan. A = B D cos. D B C, consequently

A C : B D : : cos. D B C : tan. A ; that is,

diff. lat. : diff. long. : : cos. mid. lat. : tan. course. . . (3).

The proportions marked (1) (2) (3) comprehend the entire theory of middle latitude sailing. It is scarcely necessary to mention that the middle latitude is half the difference of the extreme latitudes when both are north or both south, and half their sum when one is north and the other south.

The three proportions given above may be united in one set of equations, by which mode of expressing them, they will, perhaps, appear in a form more convenient for memory. They are as follows :—

$$\text{diff. long.} = \frac{\text{departure}}{\text{cos. mid. lat.}} = \text{departure} \times \text{sec. mid. lat.}$$

$$= \frac{\text{dist.} \times \text{sin. course}}{\text{cos. mid. lat.}} =$$

$$\text{dist. sin. course sec. mid. lat.}$$

$$= \frac{\text{diff. lat.} \times \text{tan. course}}{\text{cos. mid. lat.}} =$$

$$\text{diff. lat.} \times \text{tan. course} \times \text{sec. mid. lat.}$$

But, by imprinting upon the mind the two connected triangles in the preceding diagram, any recurrence to formulæ will not be necessary on the part of any one familiar with what is taught in the INTRODUCTION to this Section ; and this is one advantage of making the plane triangle subservient to the purposes of Navigation. However, without reference to the triangle, the single equation, $\text{diff. long.} = \frac{\text{departure}}{\text{cos. mid. lat.}}$ embodies all that is peculiar in middle latitude sailing ; the other equations are, in fact, implied in this ; instead of *departure*, the equivalent to it is substituted.

1. A ship from latitude 51° 18′ N., longitude 9° 50′ W., steers S. 33° 8′ W., till she has run 1024 miles : required the latitude and longitude in.

1. *To find the difference of latitude.*

By logarithms.

As radius	— 10
: cos. course, 33° 8′ . .	9·9229
: : dist. = 1024 . . .	3·0103
: diff. lat. = 857·5 . .	2·9332

Without logarithms.

diff. lat. = dist. × cos. course

cos. 33° 8′ . . .	·8374
1024 reversed	4201
	·8374
	167
	34
	857·5

∴ the difference of latitude is 857½ miles.

2. *To find the middle latitude.*

6,0) 85,7
14° 17′ S. = diff. lat
51° 18′ N. = lat. left
37° 1′ N. = latitude in

$\left\{\begin{matrix}51° 18′\\37° \ 1′\end{matrix}\right\}$ = 88° 19′ = Sum of the latitudes

$\frac{88° \ 19′}{2}$ = 44° 9½′ = middle latitude.

3. *To find the difference of longitude* (Proportion 3).

As cos. mid. lat.,	44° 9½′	Arith. Comp.	·1442
: tan. course	33° 8′	. .	9·8147
: : diff. lat.	= 857·5	. .	2·9332
: diff. long.	= 780·1	. .	2·8921

6,0)78,0
13° W. diff. long
9° 50′ W. long. left
22° 50′ W. long. in

Hence the place of the ship is lat. 37° 1′ N., long. 22° 50′ W. We shall now work the last proportion by using the middle latitude as corrected by Workman's table on a following page.

Under the diff. lat. 14°, at the top of the table, and in a line with 44°, the middle latitude, we find the correction 27′; which, added to 47° 9½, gives 44° 36½′ for the corrected middle latitude.

As cos. corrected mid. lat.,	44° 36½′	Arith. Comp.	·1476
: tan. course	33° 8′	. .	9 8147
: : diff. lat.	= 857·5	. .	2·9332
: diff. long.	= 786·2	. .	2·8955

∴ diff. long. = 13° 6′ ∴ 9° 50′ + 13° 6′ = 22° 56′ W. long. in.

Hence the error in longitude, by taking the uncorrected middle latitude, is 6 miles.

The construction of the table here made use of, cannot be explained till we come to Mercator's Sailing. But the necessity for some such table may be readily made apparent; for since, as shown before,

diff. long. = departure × sec. mid. lat.,

it follows, that if there be any error in our estimation of the departure—that is, in our considering it to be exactly equivalent to the middle latitude distance between the meridians—there will be a still greater error in the resulting difference of longitude, because a *secant* is always greater than unity, when the angle to which it relates is of any value at all; and the greater the angle the greater the error. In high latitudes, therefore, where the middle latitude is considerable, the error in longitude, if left uncorrected, may be seriously wide of the truth. For example, when the difference of latitude is 20°, and the middle latitude 72°, the error in longitude would amount to nearly half a degree; that is, to nearly 30 miles. Mr. Workman's table shifts the middle latitude parallel a little higher up, as the mid. latitude parallel a little exceeds the departure in length; as will be shown presently.

2. A ship from latitude 49° 57′ N., and longitude 5° 11′ W., sails between the south and west till she arrives in latitude 38° 27′ N., and finds that she has made 440 miles of departure: required the course steered, the distance run, and the longitude in.

1. *To find the diff. lat. and mid. lat.*

Latitude left, 49° 57′ N.
Latitude in, 38° 27′ N.

Sum 88° 24′ ÷ 2 = 44° 12′ = mid. lat.
Diff. 11° 30′ = 690 miles = diff. lat.

2. *To find the course.*

By logarithms.

As diff. lat. = 690	. . .	— 2·8388
: departure = 440	. . .	2·6435
: : radius	. . . . .	10
: tan. course 32° 32′	. . .	9·8047

Hence the course is 32° 32′.

Without logarithms.

tan. course — dep. ÷ diff. lat. = $\frac{44}{69}$

69)44 (·6377 = tan. 32° 32′
414
26
207
53
483
47

3. *To find the distance.*

By logarithms.

As sin. course, 32° 32′	. .	— 9·7306
: radius	. . . . .	10
: : departure = 440	. . .	2·6435
: distance = 818·3	. . .	2·9129

Hence the distance is 818 miles.

Without logarithms.

dist. = dep. ÷ sin. course
sin. 32° 32′ = 5,3,7,7,9) 440 (818·2
43023
977
538
439
430
9

4. *To find the difference of Longitude* (Proportion 1).

As cos. correct mid. lat. 44° 28′		— 9·8535
: radius	. . . . .	10
: : departure = 440	. . .	2·6435
: diff. long. = 616·6	. .	2·7900

Longitude left	. . .	5° 11′ W.
Diff. long. 617 miles, =	. .	10° 17′ W.
Longitude in	. . . .	15° 28′ W.

If the mid. lat. had not been corrected, the long. would have been about 3′ in error.

3. A ship from latitude 51° 18′ N., longitude 9° 50′ W., sails S. 33° 19′ W. until her departure is 564 miles: required the latitude and longitude in, and the distance sailed.

1. *To find the difference of latitude.*

By logarithms.

As tan. course 33° 19′	. .	— 9·8178
: radius	. . . . .	10
: : departure = 564	. .	2·7513
: diff. lat. = 858	. .	2·9335

Hence the diff. lat. is 858 miles.

Without logarithms.

diff. lat. = dep. ÷ tan. course.
tan. 33° 19′ = ·06,5,7)564 (858
5256
384
329
55
52

2. *To find the middle latitude.*

6,0)858

14° 18′ S. = diff. latitude
51° 18′ N. = latitude left

37° 0′ = latitude in

$\left.\begin{matrix}51^\circ\ 18'\\37^\circ\ 0'\end{matrix}\right\} = 88^\circ\ 18'$ = sum of the latitudes

$\frac{88^\circ\ 18'}{2} = 44^\circ\ 9'$ = middle latitude.

3. *To find the distance sailed.*

By logarithms.

As sin. course, 33° 19′ . . — 9·7398
: radius 10
: : departure = 564 . . . 2·7513

: distance = 1027 . . 3·0115

Without logarithms.

dist. = dep. ÷ sin. course
sin. 33° 19′ = ·5,4,9,3)564 (1027
5493

147
110

37
38

Hence the distance sailed is 1027 miles.

4. *To find the difference of longitude* (Proportion 1).

As cos. correct mid. lat. 44° 36′ — 9·8525
: radius 10
: : departure = 564 . . . 2·7513

: diff. long. = 792 1 . . 2·8988
6,0)79,2

13° 12′ W. diff. long.
9° 50′ W. long. left

23° 2′ W. long. in

The error in longitude here, by using the uncorrected middle latitude, would be six miles.

4. A ship in latitude 51° 18′ N., longitude 22° 6′ W., is bound to a place in the S.E. quarter, of which the nautical distance is 1024 miles, and the latitude of it 37° N. Required the direct course, as also the difference of longitude between the two places.

Ans. Course, S. 33° 5′ E. Diff. long., 786½ miles.

5. A ship from St. Michael's in the Azores, lat. 37° 48′ N., long. 25° 10′ W., is bound for the Start Point, lat. 50° 13′ N., long. 3° 38′ W. Required her course and nautical distance.

Ans. Course, N. 51° 7′ E. Distance, 1187 miles.

6. A ship sails in the N.W. quarter 248 miles, till her departure is 135 miles, and her difference of longitude 310 miles. Required her course, the latitude left, and the latitude in.

Ans. Course, N. 32° 59′ W. Lat. left, 62° 27′ N. Lat. in, 65° 55′ N.

7. A ship from latitude 38° 42′ ½ N., long. 9° 8′ ½ W., sails W.S.W.½W. 168 miles. Required her latitude and longitude. Ans. Lat. 37° 51′ N., long. 12° 33′ W.

8. A ship sails from the Lizard, lat. 49° 57′ 44″ N., long. 5° 11′ 5″ W., for a distance of 700 miles, on a W.S.W. course. Required the latitude and longitude in.

Ans. Lat. in, 45° 29′ 50″ N. Long. in, 21° 13′ 33″ W.

Mercator's Sailing.—The two great problems of navigation are the determining the latitude and longitude of a ship at sea; the other demands of the science are easily provided for. Of the two leading problems mentioned, the former is, in general, by far the more simple; as, by the aid of the imaginary line, called the departure, we may readily discover it, either by construction or computation, by means of the common theory of the right-angled triangle, whenever the course and distance sailed are taken account of.

WORKMAN'S TABLE OF CORRECTIONS TO BE ADDED TO THE MIDDLE LATITUDE—DIFFERENCE OF LATITUDE.*

Mid Lat.	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°
°	′	′	′	′	′	′	′	′	′	′	′	′	° ′	° ′
15	2	3	4	6	9	12	15	19	23	27	31	35	0 40	0 45
16	2	3	4	6	9	12	15	18	22	26	30	34	0 38	0 43
17	2	3	4	6	8	11	14	17	21	25	28	32	0 37	0 42
18	2	3	4	6	8	11	14	17	20	24	27	31	0 36	0 41
19	2	3	4	6	7	10	13	16	19	23	26	30	0 34	0 40
20	2	3	4	6	7	9	12	15	18	22	25	29	0 33	0 38
21	2	3	4	6	7	9	12	15	18	21	25	29	0 33	0 37
22	2	3	4	6	7	9	12	15	17	20	24	28	0 32	0 36
23	2	3	4	6	7	9	12	15	17	20	24	28	0 32	0 36
24	2	3	4	6	7	9	11	14	16	19	23	27	0 31	0 35
25	2	3	4	5	7	9	11	14	16	19	23	27	0 31	0 35
26	2	3	4	5	7	9	11	14	16	19	22	26	0 30	0 34
27	2	3	4	5	7	8	11	14	16	19	22	26	0 30	0 33
28	2	3	4	5	6	8	10	13	15	18	21	25	0 29	0 33
29	2	3	4	5	6	8	10	13	15	18	21	25	0 29	0 32
30	2	3	4	5	6	8	10	13	15	18	21	25	0 28	0 32
31	2	3	4	5	6	8	10	13	15	18	21	25	0 28	0 32
32	2	3	4	5	6	8	10	13	15	18	21	25	0 28	0 31
33	2	3	4	5	6	8	10	13	15	18	21	24	0 27	0 31
34	2	3	4	5	6	8	10	13	15	18	21	24	0 27	0 31
35	2	3	4	5	6	8	10	13	15	18	21	24	0 27	0 31
36	2	3	4	5	6	8	10	13	15	18	21	24	0 27	0 31
37	2	3	4	5	6	8	10	13	15	18	21	24	0 27	0 31
38	2	3	4	5	6	8	10	13	15	18	21	24	0 27	0 31
39	2	3	4	5	6	8	10	13	15	18	21	25	0 28	0 32
40	2	3	4	5	6	8	10	13	15	18	22	25	0 28	0 32
41	2	3	4	5	6	8	10	13	15	18	22	25	0 28	0 32
42	2	3	4	5	6	8	10	13	15	18	22	26	0 29	0 33
43	2	3	4	5	7	9	11	14	16	19	23	26	0 30	0 34
44	2	3	4	5	7	9	11	14	16	19	23	27	0 30	0 34
45	2	3	4	5	7	9	11	14	16	19	23	27	0 31	0 35
46	2	3	4	5	7	9	11	14	16	19	23	27	0 31	0 35
47	2	3	4	5	7	9	11	14	16	20	23	27	0 31	0 35
48	2	3	4	5	7	9	11	14	16	20	23	27	0 31	0 35
49	2	3	4	5	7	9	11	14	17	21	24	28	0 32	0 36
50	2	3	4	5	7	9	11	14	17	21	24	28	0 32	0 36
51	2	3	4	5	7	9	11	14	17	21	24	28	0 32	0 37
52	2	3	4	5	7	9	12	15	18	22	25	29	0 33	0 37
53	2	3	4	6	7	9	12	15	18	22	25	29	0 33	0 38
54	2	3	4	6	8	10	13	16	19	23	26	20	0 34	0 39
55	2	3	4	6	8	10	13	16	19	23	26	30	0 35	0 40
56	2	3	4	6	8	10	13	16	20	24	27	31	0 36	0 41
57	2	3	4	6	8	11	14	17	20	24	28	32	0 37	0 42
58	2	3	4	6	9	11	14	17	21	25	29	33	0 38	0 43
59	2	3	4	6	9	12	15	18	22	26	30	34	0 39	0 44
60	2	3	4	6	9	12	15	19	23	27	31	35	0 40	0 46
61	2	3	5	7	9	12	15	19	23	27	31	36	0 41	0 47
62	2	3	5	7	9	12	16	20	24	28	32	37	0 42	0 49
63	2	4	5	7	9	13	16	20	24	29	33	39	0 44	0 51
64	2	4	6	8	9	13	17	21	25	29	34	40	0 46	0 53
65	2	4	6	8	10	13	17	21	25	30	35	41	0 48	0 55
66	2	4	6	8	10	14	18	22	26	31	37	43	0 50	0 58
67	2	4	6	8	11	15	18	23	27	33	38	45	0 53	1 0
68	2	4	6	8	11	15	19	24	28	34	40	48	0 55	1 2
69	2	5	6	9	12	16	20	25	30	36	44	50	0 58	1 5
70	3	5	6	9	13	17	21	26	31	38	44	52	1 0	1 8
71	4	6	7	9	13	18	22	27	33	40	46	55	1 3	1 12
72	4	6	8	10	14	19	23	29	35	43	49	58	1 6	1 16

The departure, as used in plane sailing, is the occasion of some error in certain of the results of navigation: for instance, in the determination of the direct course and distance, as the result of a traverse. If a ship sail on several courses, and her departures east just balance her departures west, we conclude that she has returned to her meridian; but it is plain, from the principles of middle latitude sailing, that the amount of departure that would bring a ship in north latitude to any given meridian on a course inclined to the *north*, would be less than the departure necessary to bring her to the same meridian on a course inclined to the *south:* indeed, the first principles of plane sailing show that the elementary departures, in the latter case, exceed those in the former. And hence, in a traverse, the resultant of the departures will not be the correct departure due to the resultant difference of latitude; and consequently the resultant single course will not, in strictness, be the true one.

---

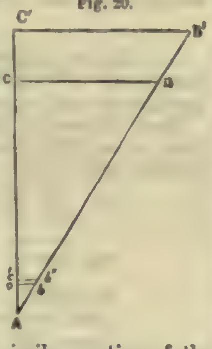
* When the difference of latitude is 2° and under 3°, add 1′.

In a day's run, the error will no doubt be in general of but little moment, and not worth taking note of practically, when we consider that the courses steered are not rigidly determinable; but it is well to apprise the learner of the mathematical shortcomings of our proceedings; and more especially to forewarn him, that if *longitude* be deduced from the balance of departures, even for a single day's run, in high latitudes—latitudes, for instance, above 52° or 53°—the result may be sensibly erroneous; and in latitudes of from 60° to 70°, the error may be such as to endanger the safety of the ship. Unfortunately, sailors are, in general, so much guided by prejudice, and so unwilling to adopt innovations, as they think certain improvements to be, that the common practice still is to confide as implicitly in the resultant departure of a set of departures, as in the departure made in a single course. And it is probably in deference to this erroneous impression, that even the most popular books on navigation are silent on the subject. In such books, the middle latitude distance of the meridians is still taken for the correct departure; though, as already noticed, the resulting error in longitude may amount to so much as 30 miles.

The place of a ship in reference to any given meridian, can be determined correctly only by correctly finding her longitude; and this is done by Mercator's sailing, in a very ingenious manner, as we shall now show.

Investigation of the Theory of Mercator's Sailing.—When a ship sails upon an oblique rhumb, it has already been shown that the difference of latitude, the departure, and the distance run, are all truly represented by the sides A C, C B, A B of a plane triangle, the angle A being that of the course (Fig. 20). The departure, C B, is not the representative of any line on the sphere: it is the equivalent of all the minute departures in the diagram at page 1052, united in one continuous line. Let A *b c*, in the annexed diagram, be one of the elementary triangles figured in the representation just referred to, *c b* being one of the elementary departures, and A *c* the corresponding difference of latitude. Now *c b* being a small portion of a parallel of latitude, it will be to a similar portion of the equator, or of the meridian, as the cosine of its latitude to radius, as was proved at page 1052; and this similar portion of the equator, or of the meridian, measures the difference of longitude between *c* and *b*. Suppose the distance A *b* prolonged to *b′*, till the departure *c′ b′* is equal to this difference of longitude: then we shall have

Fig. 20.



$$cb : c'b' :: \text{cos. lat. of } cb : 1 \text{ (the trig. radius).}$$

$$\text{But } cb : c'b' :: \text{A}c : \text{A}c' \text{ (Euclid, Prop. 4, VI.)}$$

$$\therefore \text{A}c : \text{A}c' :: \text{cos. lat. of } cb : 1 \;\therefore$$

$$\therefore \text{A}c' = \frac{\text{A}c}{\text{cos. lat. of } cb} = \text{A}c \times \text{sec. lat. of } cb \;.\;.\;.\;. \text{ (I)}$$

That is, the proper difference of latitude, A *c*, must be increased to A *c′* = A *c* × sec. lat., in order that the proper departure, *c b*, may be increased to an amount *c′ b′*, equal to the difference of longitude of *c* and *b*; in other words, the ship having made the small difference of latitude A *c*, and the corresponding departure *c b*, must continue her course till the difference of latitude has increased to A *c* × sec. lat. of *c*, in order that her increased departure, *c′ b′*, may give her difference of longitude made in sailing from A to *b*.

Suppose, now, that all the elementary distances are prolonged in this manner; it is then obvious that the sum of all the corresponding increased departures will necessarily be the whole difference of longitude made by the ship during its course from A to B. Hence, to represent the difference of longitude between A and B, we must prolong the difference of latitude A C, till the length A C′ becomes equal to the sum of all the increased elementary differences of latitude; this done, it follows that the departure C′ B′, due to this increased difference of latitude, will represent the difference of longitude made in sailing from A to B.

It is self-evident that, as the departure C B, actually made by the ship, is always *less* than the difference of longitude made, there must be some more advanced departure C′ B′, that would be exactly *equal* to this difference of longitude. The object of the foregoing reasoning, is to discover how the increased difference of latitude, due to this departure, is to be ascertained.

Now the finding the length of A C′ implies the finding of all its elementary parts: suppose we take each of the elementary parts of A C equal to 1′, that is, that we regard A *c* to be 1 nautical mile; then A *c′* will be equal to 1′ × sec. lat of *c*. And, generally, if *l* be the latitude of any point in A C, the length of a minute of latitude, terminating in that point, will be increased to 1′ × sec. *l*.

These enlarged portions of latitude are called **meridional parts**; so that we have

Meridional parts of
$1' = \text{sec. } 1'$
$2' = \text{sec. } 1' + \text{sec. } 2'$
$3' = \text{sec. } 1' + \text{sec. } 2' + \text{sec. } 3'$
$4' = \text{sec. } 1' + \text{sec. } 2' + \text{sec } 3' + \text{sec. } 4'$
$5' = \text{sec. } 1' + \text{sec. } 2' + \text{sec. } 3' + \text{sec. } 4' + \text{sec. } 5'$
&c. &c.

Consequently the meridional parts, on the proper enlargement of every portion of the meridian, measured from the equator up to any latitude, may be calculated by help of a table of natural secants: thus—

Meridional parts of
Nat. Sec. Mer. Parts.
$1' = 1{\cdot}0000000 = 1{\cdot}0000000$
$2' = 1{\cdot}0000000 + 1{\cdot}0000002 = 2{\cdot}0000002$
$3' = 2{\cdot}0000002 + 1{\cdot}0000004 = 3{\cdot}0000006$
$4' = 3{\cdot}0000006 + 1{\cdot}0000007 = 4{\cdot}0000013$
$5' = 4{\cdot}0000013 + 1{\cdot}0000011 = 5{\cdot}0000024$
&c. &c. &c.

If, therefore, a ship leave the equator, and sail upon any course till her latitude becomes 1′, and we desire to know how much she must increase her latitude in order that her corresponding increased departure may be equal to her advance in longitude, we find, from the above, that no further advance in latitude is to be made; for the difference between the departure corresponding to the 1′ of latitude already made, and the advance in longitude is, as might be expected, insensible; or, to speak more rigorously, the difference is too minute to have any numerical value within the limits of seven places of decimals. If her latitude become 2′, her corresponding departure falls short of her advance in longitude by a quantity so small that she has only to increase her latitude by ·0000002 miles to render her departure exactly equal to her difference of longitude.

In like manner, when 3′ of latitude are made, a further advance in latitude to the extent of ·0000006 miles is all that must be made to render her departure the same as her difference of longitude due to the 3′ of latitude. And in this way may a table of meridional parts be calculated, minute by minute. If we enter such a table with the latitude sailed from, and the latitude arrived at, and subtract the meridional parts for the lower latitude from the meridional parts for the higher, the remainder will be the *meridional difference of latitude*, or the line A C′, in the preceding diagram. If the latitude in be on the contrary side of the equator to the latitude left, then, of course, the *sum* of the meridional parts for the two latitudes will be the meridional difference of latitude.

Having thus obtained from the table the meridional difference of latitude (that is, the line A C′), the difference of longitude (that is, the line C′ B′) is then deduced by this proportion; namely—

As radius (1) is to the tangent of the course, so is the meridional difference of latitude to the difference of longitude; or, if instead of the course, the departure (C B) be given, then the proportion will be—

As the proper difference of latitude is to the departure, so is the meridional difference of latitude to the difference of longitude.

Such are the correct principles upon which a true sea-chart is constructed; but whether Gerrard Mercator, the Flemish Chart-maker, was really in possession of these principles, or whether he arrived at their practical results by a series of happy mechanical experiments, is not positively known; he never divulged the methods by which he proceeded, and his secret descended with him to the grave. The first chart he published was in 1556; and it was not till the year 1590 that the true theory of its construction was expounded. This was done by Edward Wright, Fellow of Caius College, Cambridge, who did not, however, communicate his discovery to the public till the year 1590, when his *Certain Errors in Navigation Detected and Corrected*, appeared; yet the key to the entire secret is furnished by the single equation marked (1) at page 1061; a property so obvious, that we cannot but suppose it must have been noticed by numbers of persons engaged in these investigations, before that time; but, like the common occurrence of the falling of an apple from a tree, it was not perceived to be the germ of a theory destined to change the aspect of an important department of science, till it arrested the attention of a superior mind.

Edward Wright constructed his table of meridional parts after the process already described; that is, by actually adding secant after secant through every minute of the quadrant. But it was subsequently shown, by Dr. Halley, that the meridional parts might be actually obtained in another way—we shall explain how at the end of the present article.

It is proper to remark that a table, constructed agreeably to Mr. Wright's plan, will be more mathematically correct the smaller the elementary portions of the meridian are made; as, for instance, if the portions, instead of a minute, be only half a minute in length. Such a table was accordingly constructed by Oughtred, and further extended by Sir Jonas Moore; but the modern table of meridional parts is based upon the strictly accurate theory of Dr. Halley. Mr. Wright, however, was fully sensible that, from the intervals in his table being so great as 1′, his meridional parts, for high latitudes, erred a little in excess, and he pointed out the method of diminishing the error to any extent.

It behoves every writer on Navigation to bring the name of Edward Wright prominently forward when treating of what is called Mercator's Sailing. Mercator has, no doubt, the merit of originating the scheme of enlarging the degrees of the meridian more and more as they approach the pole, or of widening the intervals between the successive parallels of latitude; he was thus instrumental in awakening attention to a more correct way of exhibiting longitudes on a chart than was known before his time; but that he was acquainted with the true principles discovered by Wright, there is strong reason to doubt: for the degrees in his chart were not lengthened in the due proportion which those principles pointed out.

It will occur to the reader that, when the course A is a large angle, a small inaccuracy in the measure of that angle may considerably affect the length of C′ B′; and, therefore, to diminish the influence of this error in the course, it is better and safer, when it differs but little from E. or W., to use the middle latitude method: there is no defect in the table of meridional parts, when the proper decimals are inserted, but in the *data* employed in connection with it.

In most collections of navigation tables, however, the decimals of the meridional parts are omitted, and the meridional parts given only to the nearest unit. It is possible, therefore, that in taking the *difference* between two numbers in the table, there may be an error of nearly a mile in the result. The equation

diff. long. = merid. diff. lat. × tan. course,

shows that the corresponding error in longitude will be proportional to the error in tan. course. Suppose, for instance, the course be 7 points, the tangent of which is about 5; then the error in longitude may *possibly* be 5 miles, which is of some consideration; but, as observed before, in such a large course the middle latitude method, for other reasons, should be employed. It would, however, be an improvement if the leading decimal belonging to each meridional part were actually inserted in the tables. Robertson, one of the most able and instructive writers on Navigation, *does* introduce the leading decimal;* and so, likewise, does Dr. Inman in his *Nautical Tables*. An extensive table of meridional parts will also be found in the quarto collection of *Tables* of J. De Mendoza Rios. The following short table will show the *true* meridional parts calculated agreeably to the mathematically-correct formulæ given at the end of the present article.

deg.	Wright's Mer. Parts.	True Mer. Parts.	deg.	Wright's Mer. Parts.	True Mer. Parts.
5	300·369	300·382	50	3474·605	3474·472
10	603·048	603·070	55	3968·188	3967·966
15	910·433	910·461	60	4527·711	4527·368
20	1225·129	1225·139	65	5179·308	5178·808
25	1549·988	1549·995	70	5966·681	5965·918
30	1888·377	1888·375	75	6971·549	6970·340
35	2244·305	2244·287	80	8377·342	8375·197
40	2622·756	2622·690	85	10769·620	10764·621
45	3030·127	3020·939	89	16317·532	16299·556

We shall now re-state the proportions given in words above, and proceed to examples.

As radius : tan. course : : Mer. diff. lat. : diff. long
As proper diff. lat. : departure : : Mer. diff. lat. : diff. long

Examples.

1. A ship from latitude 51° 18′ N., longitude 9° 50′ W., steers S. 33° 8′ W., till she has run 1024 miles. Required the latitude and longitude in.

The difference of latitude is found in Example 1, page 1058, to be 857½ miles, or 14° 17′ S.; and therefore the latitude in 37° 1′ N.

51° 18′ mer. parts . . 3597·5
37° 1′ . . . . 2393·9
Diff. 14° 17′ 1203·6

As radius . . . . – 10
: tan. course, 33° 8′ . . 9·8147
: : mer. diff. lat. 1204 . . 3·0806
: diff. long. 785·8 . . . 2·8953

Hence the diff. long. = 13° 6′ W.
long. left = 9° 50′ W.
∴ long. in = 22° 56′

2. A ship from latitude 49° 57′ N., and longitude 5° 11′ W., sails between the south and west till she arrives in latitude 38° 27′ N., and finds that she has made 440 miles of departure. Required the course steered, the distance run, and the longitude in.

The course and distance are found in Example 2, page 1059; and the difference of longitude is found by the table of meridional parts, as follows:—

lat. left 49° 57′, mer. parts . 3470
lat. in 38° 27′ . . . 2503
diff. lat. 11° 30′ 967

---

* We do not understand what Robertson means when he says—"But a table of meridional parts, constructed by the most accurate method, has only showed that Mr. Wright's table does nowhere exceed the true meridian parts by half a minute, and this only near the pole; for in latitudes as far as navigation is practicable, the difference is scarcely sensible."—*Elements of Navigation*, by John Robertson, formerly Head Master of the Royal Academy at Portsmouth; vol. 2. p. 136.

Robertson's work is a very valuable performance, and is well deserving a place in the seaman's library. It may be frequently picked up, at the book-stalls, at a very low price. For a correct table of meridional parts (the decimals omitted), the tables of Norie, Riddle, or Raper may be onsulted.

As diff. lat. 690 Arith. Comp.	7·1612
: departure 440 . . .	2·6435
: : mer. diff. lat. 967 . .	2·9854
: diff. long. 616·7 . .	2·7901

Hence diff. long. =	10° 17′ W.
Long. left	5° 11′ W.
Long. in	15° 28′.

3. A ship from latitude 51° 18′ N., longitude 9° 50′ W., sails S. 33° 19′ W., till her departure is 564 miles. Required her longitude in.

We must first find the difference of latitude made; this, by Example 3, page 1059, is 858 miles, or 14° 18′ S.: the latitude left and latitude in will then become known; and thence the meridional difference of latitude may be found by the table of meridional parts, by aid of which the difference of longitude is determined as below.

lat. left	51° 18′ mer. parts	3598
diff. lat.	14° 18	
lat. in	37° 0′	2393
	mer. parts for diff. lat.	1205

As diff. lat. 854 Arith. Comp. ...	7·0665
: departure 564	2·7513
: : mer. diff. lat. 1205	3·0810
: diff. long. 792·1	2·8988

Difference of longitude ...	13° 12′ W.
Longitude left	9° 50′ W.
Longitude in ...	23° 2′ W.

Hence the longitude reached is 23° 2′ W.

4. Required the course and distance from the east point of St. Michael's to the Start Point.

To find the course we must know the meridional difference of latitude and the difference of longitude; the course being determined, the distance will be found by combining it with the proper difference of latitude, as below.

Start Point lat.	50° 13′ N.	Mer. pts. 3495	Lon. 3° 38′ W.
St. Michael's lat.	37° 48′ N.	Mer. pts. 2453	Lon. 25° 13′ W.
Diff. lat.	12° 25′	1042	21° 35′ W.
	60		60
Proper diff. lat.	745 miles.		1295 m.

To find the course.

As mer. diff. lat.	1042	—3·0178
: diff. long.	1295	3·1123
: radius	... ...	10
: tan. course, 51° 11′	... ...	10·0945

To find the distance.

dist. = diff. lat. ÷ cos. course

$$\begin{array}{rr}
\text{cos. } 51° 11' = \cdot 6{,}2{,}6{,}8) & 745\ (1189 \\
 & 6268 \\ \hline
 & 1182 \\
 & 627 \\ \hline
 & 555 \\
 & 501 \\ \hline
 & 54
\end{array}$$

Hence the course is N. 51° 11′ E., and the distance 1189 miles.

It may be as well to exhibit here the work of finding the difference of longitude in Example 2, and the course in this last example, without logarithms.

For the diff. long. Ex. 2.

$$\text{Diff. long.} = \frac{\text{mer. diff. lat.} \times \text{dep.}}{\text{diff. lat.}}$$

$$\begin{array}{rr}
\text{mer. diff. lat.} = & 967 \\
 & 44 \\ \hline
 & 3868 \\
 & 3868 \\ \hline
6{,}9) & 42548(616\cdot 6 \text{ miles} \\
 & 414 \\ \hline
 & 114 \\
 & 69 \\ \hline
 & 458 \\
 & 414 \\ \hline
 & 44 \\
 & 41 \\ \hline
\end{array}$$

∴ diff. long. = 10° 17′ W.

For the course Ex. 4.

$$\text{tan. course} = \frac{\text{diff. long.}}{\text{mer. diff. lat.}}$$

$$\begin{array}{rl}
1{,}0{,}4{,}2) & 1295(1\cdot 243 = \text{tan. } 51° 11' \\
 & 1042 \\ \hline
 & 253 \\
 & 208 \\ \hline
 & 45 \\
 & 42 \\ \hline
 & 3
\end{array}$$

Hence the course is N. 51° 11′ E.

The examples already given, under the head of Middle Latitude Sailing, may serve for exercises in Mercator's Sailing. A comparison of the results of the two methods will also show the value of Workman's table for correcting the middle latitude. It is necessary, however, that the learner should notice, that the two reasons given at page 1061, both conspire to render it unadvisable to use the ordinary table of meridional parts when the course much exceeds 45°; from 50° and upwards the middle latitude method is to be preferred; and, for the first of the reasons mentioned, a small error in the course had better be in defect than in excess. We shall now explain the principles on which that table is constructed.

CONSTRUCTION OF THE TABLE FOR CORRECTING THE MIDDLE LATITUDE.—Let l represent the proper difference of latitude, l' the meridional difference of latitude, L the difference of longitude, and m the latitude in which the distance measured on the parallel between the two meridians is exactly equal to the departure. Then, first by middle latitude and then by Mercator's sailing, we have for the tangent of the course

$$\text{tan. course} = \frac{L \times \cos. m}{l} = \frac{L}{l'}, \therefore \cos. m = \frac{l}{l'}$$

Consequently, by dividing the difference of latitude l taken in miles, by the meridional parts corresponding to that latitude, we shall get cos. m, and thence m, the degrees of latitude in which the parallel between the two meridians is exactly equal to the departure. It is the difference between m and the latitude of the middle parallel that is given in the table at page 1060.

By means of the corrected middle latitude, the difference of longitude, and the course, the nautical distance is found thus:—

The expressions for the departure by plane sailing and by middle latitude sailing, are—

departure = dist. × sin. course = diff. long. × cos. m
∴ sin. course : cos. corrected mid. lat. : : diff. long. : distance.

Construction of a Table of Meridional Parts by Dr Halley's Method.—In Mr. Wright's method of constructing a table of meridional parts, every tabular number, after the first, depends upon the number previously calculated, as already shown at page 1061. Dr. Halley proposed another plan, by which the meridional parts corresponding to any given latitude may be computed independently of previous calculations. As the matter is of so much practical interest, we shall here show how this may be effected.

To transcribe Dr. Halley's investigation (*Philosophical Transactions*, No. 219) would be to occupy more room than we can spare: we must therefore employ a different process; and, if the reader be unacquainted with the first principles of the Differential and Integral Calculus, he may pass it over, and omit this article altogether.

It is shown at page 1061, that if a ship in latitude x vary her latitude by a small portion of the meridian $\triangle x$, and that she continue her course till her departure becomes equal to the difference of longitude due to the difference of latitude $\triangle x$, then the increased difference of latitude $\triangle y$, necessary to produce this effect, will be

$$\triangle y = \sec. x \triangle x, \therefore \frac{\triangle y}{\triangle x} = \sec. x.$$

This equation is rigorously true only when $\triangle x$, and consequently $\triangle y$ is diminished to the last degree of smallness; so that, employing the notation of the differential calculus, we strictly have

$$\frac{dy}{dx} = \sec. x \therefore dy = \sec. x\, dx$$

and the integral of this is the equation

$$y = \log. \tan. (45^\circ + \tfrac{1}{2} x) \quad . \quad . \quad . \quad (1)$$

The logarithm here implied, is the *Napierian* logarithm. To change it into Brigg's, or a common logarithm, we must multiply it by the modulus $\frac{1}{2 \cdot 302585 \ldots}$ (see the article on Logarithms, page 519); we shall then get the logarithm of the *natural* tangent of $45^\circ + \frac{1}{2} x$, according to Brigg's system. But in the tables it is not the log. of the *natural* tangent of $45^\circ + \frac{1}{2} x$, but this log. increased by 10, that is inserted: hence, for the *common* logarithms, the equation (1) is

$$\frac{y}{2 \cdot 302585 \ldots} = \log. \tan. (45^\circ + \tfrac{1}{2} x) - 10$$

$$\therefore y = 2 \cdot 302585 \left\{ \log. \tan. (45^\circ + \tfrac{1}{2} x) - 10 \right\}$$

This is the correct expression for the increased meridian length, measured in miles, in reference to a globe whose radius is 1 mile; to adapt it to the globe of the earth, we must multiply the expression by the radius of the earth, or by 3437·74679 nautical miles; for in every circle the radius is equal to 3437·74679 minutes of that circle, thus:—

Rad. earth × 3·14159 . . . = length of 180° = 10,800 nautical miles.

$$\therefore \text{Rad. earth} = \frac{10800}{3 \cdot 14159 \ldots} = 3437 \cdot 74679 \text{ nautical miles.}$$

Hence, for the number of miles in the lengthened meridian y, from the equator to the latitude x, we have the formula

$$y = 7915 \cdot 7044679 \left\{ \log. \tan. (45^\circ + \tfrac{1}{2} x) - 10 \right\}$$

or, since $\tan. (45^\circ + \frac{1}{2} x) = \cot. (45^\circ - \frac{1}{2} x)$ and that

$$\cot. = \frac{R}{\tan.}, \text{ and } \therefore \log. \cot. = 20 - \log. \tan.$$

the preceding formula may be otherwise written, as follows:—

Meridional lat. = 7915·7044679 {10 − log. tan. (45° − ½ proper lat.)} . . . (2)

It thus appears, that if the log. tan. of half the complement of any latitude be subtracted from 10, and the remainder be multiplied by 7915·7044679, the product will be the meridional parts, in nautical miles, corresponding to that latitude; and, therefore, as observed already, the meridional parts for any latitude may be computed independently of previous results.

The logarithm of the constant multiplier 7915·704 . . . is 3·8984895; so that from (2) we have

$$\log. \text{ merid. lat.} = 3 \cdot 8984895 + \log. \left\{ 10 - \log. \tan. \tfrac{1}{2} \text{ comp. lat.} \right\} \ldots (3)$$

from which formula, the *true* meridional parts for all latitudes may be calculated.

We shall conclude this part of our subject with a few remarks upon the peculiar character of the *Rhumb line*, or, as it is sometimes called, the *Loxodromic curve*.

On the Continued Rhumb Line.—From the principles of Mercator's sailing, or from the diagram at page 1061, which connects the enlarged meridian with the difference of longitude, it is clear that if a ship set out from any point on the globe, and sail on the same oblique rhumb towards the pole, it can reach it only after circulating an infinite number of times round it; for, from any point to the pole, the enlarged meridian is infinite in length, and so, therefore, is the difference of longitude due to this advance in latitude, provided longitude be measured round the globe in one uniform direction: the longitude, thus measured, being infinite, the ship must wind round the pole an infinite number of times.

But—paradoxical as the statement may appear—it is nevertheless true, that the infinite number of revolutions about the pole are performed in a finite time, and that the entire length of the spiral track of the ship is a finite line. However strange this may seem, it follows, as a necessary consequence, from the principles of *plane sailing*; for these principles correctly give,

$$\text{length of track} = \frac{\text{diff. lat.}}{\text{cos. course}}$$

which is finite; and, therefore, the rate being uniform, is described in a finite time.

The matter may be explained as follows:—Whatever be the progressive rate of the ship along its undeviating course, the *times* of performing the successive revolutions about the pole continually diminish as the latitude increases; both the extent of circuit, and the time of performing it, evidently tending to zero—the limit is actually attained, only at the pole itself. Consequently, an infinite number of circuits must ultimately be performed to occupy a finite portion of time.

The case is somewhat analogous to those infinite descending series so frequently met with in arithmetic and algebra; they are infinite—not in their aggregate amount, but only in the number of the continually-diminishing parts into which that amount is divided, these parts becoming less and less, and ultimately vanishing altogether.

In like manner, every additional circuit the ship makes round the pole, increases the length of the previously-described track by a quantity less and less, the successive increments continually diminishing, and ultimately vanishing altogether; so that, just as in a common decreasing geometrical series, the sum of the increments —thus continually tending to, and ultimately terminating in zero—is finite; and hence the time of describing the track must be finite too.

Traverses by Mid-Latitude and Mercator's Sailing.—In order to work a traverse with a view to finding the ship's place, and the direct course and distance to it, we may proceed in either of the two ways following:—

1. Form a traverse table, in the first two columns of which insert the several courses and distances, and in the remaining columns put the corresponding differences of latitude and departures, found either by computation or by reference to the already computed traverse table,

and thence determine the whole difference of latitude and departure, as also the corresponding direct course and distance, exactly as in plane sailing. (See page 1052).

The traverse now being reduced to a single course, find the corresponding difference of longitude, either by mid-latitude or Mercator's sailing.

This method is less accurate than that which follows; because, as already observed (page 1056), the aggregate of the several departures is not, in general, the same as the single departure due to the correct course and distance. It is better, therefore, to take the several differences of latitude and departures separately, and thus to determine the difference of longitude due to each distinct course, and thence the whole difference of longitude made. The second method is, therefore, this:—

2. Fill up the columns of the traverse table, as directed above; but find the aggregate of the diff. latitude columns only. By means of the successive latitudes reached at the end of each course, find the corresponding mid-latitude; and with this, and the departure, deduce the difference of longitude made at the end of each course by mid-latitude sailing, and thence the whole diff. long. made good.

But if Mercator's, instead of the mid-latitude method be employed, then the several departures need not be inserted in the traverse table at all. An example will sufficiently show the mode of proceeding by each method.

Examples.

1. A ship from latitude 38° 14′ N., longitude 25° 56′ W., has sailed the following courses and distances, namely—1st. N.E. *b* N. ½ E., 56 miles. 2nd. N.N.W., 38 miles. 3rd. N.W. *b* W., 46 miles. 4th. S.S.E., 30 miles. 5th. S. *b* W., 20 miles. 6th. N.E. *b* N., 60 miles. Required the latitude and longitude in, and the direct course and distance to it.

BY THE FIRST METHOD.

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
N.E. *b* N. ½ E.	56	45		33·4	
N.N.W.	38	35·1			14·5
N.W. *b* W.	46	25·6			38·2
S.S.E.	30		27·7	11·5	
S. *b* W.	20		19·6		3·9
N.E. *b* N.	60	50		33·3	
		155·7	47·3	78·2	56·6
		47·3		56·6	
		108·4		21·6	

∴ Direct course, 11° 15′, or N. *b* E.
Distance, 111 miles.

The direct course and distance, due to diff. lat. 108·4, and dep. 21·6, are found by reference to the traverse table, or by computation, as at page 1055, to be as above.

To find the diff long. by Mid-Latitude Sailing.

Latitude left . . . 38° 14′ N.
Diff. lat. 108 m. . . . 1° 48′ N.
Latitude in . . . 40° 2′ N.
Sum of lats. . . . 78° 16′
Half sum = mid. lat. . . 39° 8′
Diff. long. = dep. ÷ cos. mid. lat.
= 21 6 ÷ ·7757 . . = 28′ E.
Longitude left . . . . 25° 56′ W.
Longitude in . . . . 25° 28′ W.

BY THE SECOND METHOD (MID-LATITUDE).

Courses.	Dist.	Diff. Lat.		Departure.		Lats. left.	Lats. in.	Sum. Lats.	Mid. Lats.	Diff. Long.	
		N.	S.	E.	W.					E.	W.
1st.	56	45		33·4		38° 14′	38° 59′	77° 13′	38° 36′	43′	
2nd.	38	35·1			14·5	38° 59′	39° 34′	78° 33′	39° 16′		19′
3rd.	46	25·6			38·2	39° 34′	40°	79° 34′	39° 47′		50′
4th.	30		27·7	11·5		40°	39° 32′	79° 32′	39° 46′	15′	
5th.	20		19·6		3·9	39° 32′	39° 12′	78° 44′	39° 22′		5′
6th.	60	50		33·3		39° 12′	40° 2′	79° 14′	39° 37′	43′	
		155·7	47·3							101′	74
		47·3								74′	

Diff. latitude 108·4 N. Difference of longitude 27′ E.

To find the same by Mercator's Sailing.

Latitude left, 38° 14′ mer. pts. . 2486
Latitude in, 40° 2′ . . 2625
Meridional diff. lat. . . . 139

Diff. long. = mer diff. lat. × tan. course.
= 139 × ·199 = . . 28′ E.
Longitude left . . . . 25° 56′ W.
Longitude in . . . . . 25° 28′ W.

The diff. long. columns may be filled up from the traverse table, by taking the complement of each mid. lat. as a course, and, with the corresponding departure, finding the proper number of miles or minutes in the *distance* column of the table; or, which is somewhat simpler, take the middle latitude itself as a course, and seek the corresponding departure in the *diff. lat.* column of the table, against which, in the *dist.* column will be found the number of miles of diff. long. In this way the two diff. long. columns, as exhibited above, have been formed.

BY THE SECOND METHOD (MERCATOR).

Courses.	Dist.	Diff. Lat.		Lats.	M. parts.	M diff. l.	Diff. Long.	
		N.	S.	38° 14′	2486		E.	W.
N.E. *b* N. ½ E.	56	45		38° 59′	2544	58	43	
N.N.W.	38	35·1		39° 34′	2589	45		18·7
N.W. *b* W.	46	25·6		40°	2623	34		50·5*
S.S.E.	30		27·7	39° 32′	2586	37	15	
S. *b* W.	20		19·6	39° 12′	2560	26		5·1
N.E. *b* N.	60	50		40° 2′	2625	65	43·4	
			47·3					
		155·7					101·4	74·3
		47·3					74·3	

Difference of latitude 108·4 N. Difference of longitude 27·1 E.

The diff. long. columns are here also supplied from the traverse table by entering it with the given course and mer. diff. lat. taken as the proper diff. lat.: the corresponding departure, furnished by the table, is the diff. long.

The longitude in, by the second method, whether the mid. latitude, or Mercator's sailing be employed, is 25° 20′ W., and the latitude and longitude left being given, the direct course and distance, and the corresponding departure, may be readily found by Mercator's sailing.

---

* The 40 corresponding to this in the "Lat." column is more nearly 39¾, so that 50·5 is written instead of 50·7, given in the table

2. A ship, from latitude 52° 20′ N., and longitude 14° 38′ W., has sailed the following courses, namely—

1st, E.S.E., 43 miles ; 2nd, S.W., 32 miles ; 3rd, S.E. *b* S., 58 miles ; 4th, S.S.W., 60 miles. Required the latitude and longitude in.

Ans. Latitude, 49° 57′ N. Longitude, 13° 57′ W.

3. A ship, from latitude 52° 36′ N., and longitude 21° 45′ W., has sailed the following courses, namely—

1st, N.E., 36 miles ; 2nd, N. *b* W., 14 miles ; 3rd, N.E. *b* E. ½ E., 58 miles ; 4th, N. *b* E., 42 miles ; 5th, E.N.E., 29 miles. Required the latitude and longitude in.

Ans. Latitude, 54° 35′ N. Longitude, 18° 4′ W.

4. A ship, from latitude 66° 14′ N., and longitude 3° 12′ E., has sailed as follows :—

1st, N.N.E. ½ E., 46 miles ; 2nd, N.E. ½ E., 28 miles ; 3rd, N. ¾ W., 52 miles ; 4th, N.E. *b* E. ¼ E., 57 miles ; 5th, E.S.E., 24 miles. Required the latitude and longitude in.

Ans. Latitude, 68° 24′ N. Longitude, 7° 54′ E.

5. A ship, from latitude 67° 30′ N., longitude 8° 46′ W., has sailed the following courses, namely—

1st, N.E., 64 miles ; 2nd, N.N.E., 50 miles ; 3rd, N.W. *b* N., 58 miles ; 4th, W.N.W., 72 miles ; 5th, W., 48 miles ; 6th, S.S.W., 38 miles ; 7th, S. *b* E., 45 miles ; 8th, E.S.E., 40 miles. Required her present latitude and longitude by Mercator's sailing.

Ans. Latitude, 68° 43′ N. Longitude, 11° 43′ W.

SAILING IN CURRENTS.—In what has preceded, the course of the ship is supposed to have been indicated by the compass, and that she has actually moved through the water in the direction of her length. It is plain, however, that if a current act upon the ship, she will be diverted from the course shown by the compass, unless the set of the current be itself in the direction of the ship's length, when the rate of sailing only will be interfered with.

The *set* of a current is the point of the compass *towards* which the stream runs ; its rate, or velocity, is called the *drift* of the current.

The common way of ascertaining the set and drift of a current unexpectedly met with at sea, is to take a boat, if the weather permit, a small distance from the ship ; and, in order to keep it from driving with the stream, to let down, to the depth of about 100 fathoms, a heavy weight attached to a rope fastened to the stem of the boat ; steadiness being in this way secured, the log is hove into the current ; the direction in which it is carried is observed by means of a boat-compass, and the number of knots run out in half a minute gives the hourly drift. In this way the set and velocity, or drift, of the current is estimated.

The set of the current being ascertained, we can apply it to correct the compass-course of the ship ; and the drift, or velocity, of the current being known, we can apply it to correct the ship's rate of sailing as indicated by the log. The rate indicated by the log, it will be observed, is only the rate at which the ship moves faster than the log moves in the ship's direction, and, in still water, is the velocity of the ship itself, since the log remains stationary ; but, as in a current, the log also moves in the direction and with the rate of the current, the absolute velocity of the ship cannot be directly determined by the log in the customary manner : yet the distance sailed, as indicated in this way by the log, and the course steered as indicated by the compass, though both modified by the action of the current, and therefore both inaccurate, are necessary helps to the determination of the true course and distance.

For the ship has, in fact, been making a sort of double course : the wind has carried her on a certain course (shown by the compass) a certain distance ; and the current has carried her a certain other course and distance in the same time ; the final effect being the same as if the two courses and distances had been sailed in succession ; so that the case becomes a very simple one of traverse sailing, as in the following example.

Examples.

1. A ship runs N.E. *b* N. 18 miles in three hours, in a current setting W. *b* S. two miles an hour. Required the course and distance made good.

This is evidently the same as saying—A ship sails the following courses, namely—

1st, N.E. *b* N., 18 miles ; 2nd, W. *b* S., 6 miles. Required the course and distance.

TRAVERSE TABLE.

Courses.	Dist.	Diff. Latitude.		Departure.	
		N.	S.	E.	W.
N.E. *b* N.	18	15		10	
W. *b* S.	6		1·2		5·9
	Diff. lat. 13·8			Dep. 4·1	

And, with the difference of latitude and departure thus found, we have, by plane sailing, or by the traverse table, the distance made good, 14 miles, and the course 1½ points, or N. *b* E. ½ E.

2. A ship sails N.W. 60 miles, in a current that sets S.S.W. 25 miles in the same time. Required the course and distance made good.

Ans. Course, N. 69° 38′ W. Distance, 55½ miles.

3. A ship has sailed the following courses and distances in twenty-four hours :—

1st, S.W., 40 miles ; 2nd, W.S.W., 27 miles ; 3rd, S. *b* E., 47 miles ; but she has been the whole time in a current setting S.E. *b* S. at the rate of 1½ mile an hour. Required the ship's direct course and her distance made good. Ans. Course, S. *b* W. Distance, 117 miles.

4. A ship has sailed by reckoning N. ½ W. 20 miles; but it is found by observation, that, owing to a current, she has actually sailed N.N.E. 28 miles. Required the set of the current and the amount of drift upon the ship.

Ans. Set N., 64° 48′ E. Drift, 14 miles.

The method described above, of estimating the set and drift of a surface-current, by sinking a weight, and observing the direction and velocity of the log, is often likely to lead to conclusions not strictly correct. Lieut. Walsh and Lieut. Lee, of the United States' Navy, while carrying on a system of observations in connection with the wind and current charts, had their attention directed to the subject of *submarine currents*, upon which they made some interesting experiments. A block of wood was loaded to sinking, and, by means of a fishing-line, was let down to the depth of one hundred and five hundred fathoms. A small float, just sufficient to keep the block from sinking farther, was then tied to the line, and the whole let go from the boat.

To use their own expressions—"It was wonderful, indeed, to see this *barrega* move off, against wind, and sea, and surface-current, at the rate of over one knot an hour, as was generally the case, and on one occasion as much as 1¾ knot. The men in the boat could not repress exclamations of surprise, for it really appeared as if some monster of the deep had hold of the weight below, and was walking off with it."

The effects of such under-currents must sometimes interfere with the results, deduced in the ordinary way, for the set and drift of a surface-current, since to hidden influences, of the existence of which the observer has no suspicion, part of what he attributes to the surface-current may be really due ; the boat, though apparently steady, may have an imperceptible drift. Such drifts are not caused so much by the action of the under-current upon the sunken weight, as by the bellying of the line in the direction of the set. The surface-current, too, if of any considerable depth, may operate in a similar way.

Admiral Sir Francis Beaufort, when in the Mediterranean, made several experiments on this subject. He says—

"The counter-currents, or those which return beneath the surface of the water, are very remarkable. In some parts of the Archipelago they are at times so strong as to prevent the steering of the ship; and, in one instance, on sinking the lead, when the sea was calm and clear, with shreds of bunting of various colours attached to every yard of the line, they pointed in different directions all round the compass."

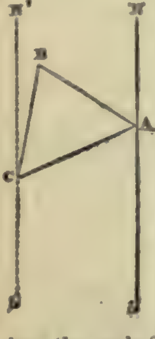
For the foregoing quotations, on the under-currents of the ocean, we are indebted to a recent publication of great value and ability—*The Physical Geography of the Sea*, by Lieut. Maury, of the United States' Navy, the author of *The Wind and Current Charts*, in which the results of extensive observation and experience, in different seasons, and during a course of years, in all parts of the globe, are recorded. Speaking of these charts, the distinguished writer says—"The young mariner, instead of groping his way along till the lights of experience should come to him by the slow teachings of the dearest of all schools, would here find, at once, that he had already the experience of a thousand navigators to guide him on his voyage. He might, therefore, set out upon his first voyage, with as much confidence in his knowledge as to the winds and currents he might expect to meet with, as though he himself had already been that way a thousand times before."*

Oblique Sailing—Taking Departures.—From what has hitherto been explained, the learner has perceived that oblique-angled triangles may be dispensed with in all the ordinary computations which enter into the dead reckoning. But, at the commencement of her voyage, before the ship is out of sight of some known point of land, or other object on shore, the distance and bearing of that object is found, and thus a first course and distance obtained. This is called taking a *departure*. It is a common practice to observe the bearing by compass, and to estimate the distance by guess; and experienced navigators can, in this way, come pretty near the truth: but as our object is to describe the most accurate methods of performing the various calculations of Navigation, we shall devote a short article to the more correct way of taking a departure, which is by observing two bearings of the object, and measuring, by the log, the distance sailed by the ship in the interim between the observations. The solution of an oblique-angled triangle will thus become necessary, as in the following example.

Examples.

1. In sailing down the Channel the Eddystone Lighthouse bore N.W., and after running W. *b* S. 8 miles, it bore N.N.E. Required the ship's course and distance from the Eddystone at the last place of observation.

Fig. 21.



In the annexed diagram (Fig. 21), A represents the first position of the ship, and B the lighthouse, the bearing of which from A, that is, the angle of N A B, is N.W., or 4 points. C is the second position of the ship; and the bearing of B from this position, that is, the angle N'C B, is N.N.E., or 2 points. Also the course of the ship in the interim, that is, the angle S A C, is W. *b* S., or 7 points. Hence the angle B A C is 16 points—11 points + 5 points; also, since the angle S A C = N'C A, we have B C A = 7 points − 2 points = 5 points. Consequently, the angle B is 16 points — 10 points = 6 points; so that in the triangle A B C are given A C = 8, A = 56° 15′, and B = 67° 30′, to find B C as follows:—

As sin. B, 67° 30′ Arith. Comp. .	·0344
: sin. A, 56° 15′	9·9198
: : A C = 8°	·9031
: C B = 7·2	·8573

Consequently, the distance of the Eddystone from the last place of observation is 7·2 miles; and, as the course *to* the Eddystone is N.N.E., the course *from* the Eddystone to the place of the ship must be S.S.W., the opposite point.

As the triangle B C A, in this example, happens to be isosceles, the angle at A and C being equal, a perpendicular upon A C from B will bisect A C, so that we shall have

$$C\,B = 4 \text{ sec. } 56° 15' = 4 \div \text{cos. } 56° 15';$$

that is, C B = 4 ÷ ·556 = 7·2, as in the margin. And this is an easier way of finding the distance than that exhibited above; but when, as is usual, the triangle is scalene, the operation by logarithms, as in the specimen above, is to be preferred.

```
cos. 56° 15′ = ·5,5,6)4   (7·2
                     389
                     ———
                      11
                      11
```

2. Sailing down the Channel the Eddystone bore N.W. *b* N., and after running W.S.W. 18 miles, it bore N. *b* E.: required the course and distance from the Eddystone to both stations.

Ans. Course, S.E. *b* S. Distance, 21·2 miles from the first station. Course, S. *b* W. Distance, 25 miles from second.

3. Coasting along shore, a headland bore N.E. *b* N.; then having run 15 miles E. *b* N., the headland bore W.N.W. Required the distance from the headland at each time of observation.

Ans. First distance, 8½ miles; second, 10·8 miles.

4. Two ships sail at the same time from one port; one sails E.S.E., and the other S.S.E. Required their bearing and distance when each has run 37½ miles.

Bearing from first ship, N. 45° E. Distance, 28·7 miles.

Great Circle Sailing—Shortening of Passages.—The great object of the navigator is to reach the port he intends to make by the shortest route. The path described by a ship, sailing on what is called a direct course from one place to another, is not the *shortest* path, unless the ship sail either on the equator or on a meridian. The oblique spiral track, or rhumb-line, of a ship, sailing in any other direction, exceeds the arc of the great circle, joining the two places, by an amount that becomes more and more considerable as the distance increases. The shortest distance between any two points on the globe is the arc of the great circle between them; for the curvature of this arc is less than that of any other line drawn on the surface, from point to point, and, therefore, approaches more nearly to a straight line.

If, therefore, a ship could sail accurately on the arc of a great circle, voyages might, in general, be considerably shortened. But a great circle cuts all the meridians it crosses in different angles; so that to keep on such a circle the course must be continually varying. The practical impossibility of changing the course every instant, renders Great Circle Sailing—strictly speaking—a matter of mere theoretical speculation. Much advantage, however, has been found to arise from dividing the great circle path, from one place to another, into short portions, or stages, and to reach these in succession by the ordinary methods of sailing. A ship making these several stages, though never actually moving on the great circle, always keeps so closely in its neighbourhood as to be a considerable gainer, in point of time, in a long voyage.

Mr. Towson, of Devonport, has been at the pains of constructing tables, by which the proper successive courses, necessary to cause the entire track of the ship to approximate as closely as conveniently practicable to the great circle path, may be found; they are published by the Admiralty, under the title of *Tables to Facilitate the Practice of Great Circle Sailing*. We can do no more than allude to them here, and recommend them to the notice of the practical seaman.

But, without the aid of tables, a twelve or eighteen-inch globe would be of service in suggesting how to shape the different courses: a line stretched from one point to another, on such a globe, would show the great circle track between them. If a brass circular rim, two or three inches in diameter, made to lie close to the surface of the globe, were divided, like a compass-card, and if a tubular box inclosing a reel, with a yard of thread, were to rise from its centre, this centre might be applied to any proposed spot; and, by means of the thread, pulled out under the disc, each successive course could be ascertained; the only care being, that the north and south line of the disc—which should be marked on the only metallic diameter it need to have—be made to coincide with the meridian of the place under the centre of the instrument. We offer this suggestion as one that might, perhaps, be turned to practical account in long voyages. But, to give perfection to the problem of shortening passages, the *Sailing Directions*, and *The Wind and Current Charts*, by Lieutenant Maury, are of the greatest use. The winds blow and the currents run in all parts of the ocean. "These control the mariner in his course; and to know how to steer his ship on this or that voyage, so as always to make the most of them, is the perfection of navigation. When one looks seaward from the shore, and sees a ship disappear in the horizon as she gains the offing, on a voyage to India, or the antipodes, perhaps, the common idea is that she is bound over a trackless waste; and the chances of another ship, sailing with the same destination the next day, or the next week, coming up and speaking with her on the 'pathless ocean,' would, to most minds, seem slender indeed. Yet the truth is, the winds and the currents are now becoming to be so well understood, that the navigator, like the backwoodsman in the wilderness, is enabled literally 'to blaze his way' across the ocean—not, indeed, upon trees, but upon the wings of the wind. The results of scientific inquiry have so taught him how to use these invisible couriers, that they, with the calm belts of the air, serve as sign-boards to indicate to him the turnings, forks, and crossings by the way."*

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* *The Physical Geography of the Sea.* By M. F. Maury, LL.D. Lieut. U.S. Navy. 1855. Introduction, p. vi.

We here terminate our exposition of the Principles of NAVIGATION, in so far as these are independent of astronomical observations. What has hitherto been taught, the learner will remember, concerns only what is called the dead-reckoning, which is generally affected with errors, more or less, on account of the practical difficulty of measuring course and distance with strict precision, as well as on account of disturbing causes, either operating unseen, or, where observed, but imperfectly estimated. It is the business of Nautical Astronomy to rectify these unavoidable defects, to adjust the place of the ship, from time to time, to its true position, and thus to enable the mariner to take, as it were, a new departure, and start afresh upon each successive stage of his journey.

As an introduction to this important branch of our subject, we shall treat, in the next chapter, on the rotation of the earth, agreeable to our promise at page 1049.

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* Maury's *Physical Geography of the Sea*, p. 263. (1855.)

[The following Tables of Differences of Latitude and Departure have been frequently referred to in the previous pages. They are given for each degree, and quarter-point of the Compass.—ED.]

A TABLE of Difference of Latitude and Departure to every Degree and Quarter-point of the Compass.

Cour.		Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		Dist. 6.		Cour.	
Pts.	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts.
	1	0,9998	0,0175	1,9997	0,0349	2,9995	0,0524	3,9994	0,0698	4,9992	0,0873	5,9991	0,1047	89	
	2	0,9994	0,0349	1,9988	0,0698	2,9982	0,1047	3,9976	0,1396	4,9970	0,1745	5,9963	0,2094	88	
0.¼		0,9988	0,0491	1,9976	0,0981	2,9964	0,1472	3,9952	0,1963	4,9940	0,2453	5,9928	0,2944		7.¾
	3	0,9986	0,0523	1,9973	0,1047	2,9959	0,1570	3,9945	0,2093	4,9931	0,2617	5,9918	0,3140	87	
	4	0,9976	0,0698	1,9951	0,1395	2,9927	0,2093	3,9903	0,2790	4,9878	0,3488	5,9854	0,4185	86	
	5	0,9962	0,0872	1,9924	0,1743	2,9886	0,2615	3,9848	0,3486	4,9810	0,4358	5,9772	0,5229	85	
0.½		0,9952	0,0980	1,9904	0,1960	2,9856	0,2940	3,9807	0,3921	4,9759	0,4901	5,9711	0,5881		7.½
	6	0,9945	0,1045	1,9890	0,2091	2,9836	0,3136	3,9781	0,4181	4,9726	0,5226	5,9671	0,6272	84	
	7	0,9925	0,1219	1,9851	0,2437	2,9776	0,3656	3,9702	0,4875	4,9627	0,6093	5,9553	0,7312	83	
	8	0,9903	0,1392	1,9805	0,2783	2,9708	0,4175	3,9611	0,5567	4,9513	0,6959	5,9416	0,8350	82	
0.¾		0,9892	0,1467	1,9784	0,2935	2,9675	0,4402	3,9567	0,5869	4,9459	0,7337	5,9351	0,8804		7.¼
	9	0,9877	0,1564	1,9754	0,3129	2,9631	0,4693	3,9508	0,6257	4,9384	0,7822	5,9261	0,9386	81	
	10	0,9848	0,1736	1,9696	0,3473	2,9544	0,5209	3,9392	0,6946	4,9240	0,8682	5,9088	1,0419	80	
	11	0,9816	0,1908	1,9633	0,3816	2,9449	0,5724	3,9265	0,7632	4,9081	0,9540	5,8898	1,1449	79	
1.		0,9808	0,1951	1,9616	0,3902	2,9424	0,5853	3,9231	0,7804	4,9039	0,9754	1,8847	1,1705		7.
	12	0,9781	0,2079	1,9563	0,4158	2,9344	0,6237	3,9126	0,8316	4,8907	1,0396	5,8689	1,2475	78	
	13	0,9744	0,2250	1,9487	0,4499	2,9231	0,6749	3,8975	0,8998	4,8718	1,1248	5,8462	1,3497	77	
	14	0,9703	0,2419	1,9405	0,4838	2,9108	0,7258	3,8812	0,9677	4,8515	1,2096	5,8218	1,4515	76	
1.¼		0,9700	0,2430	1,9401	0,4860	2,9101	0,7289	3,8801	0,9719	4,8502	1,2149	5,8202	1,4579		6.¾
	15	0,9659	0,2588	1,9319	0,5176	2,8978	0,7765	3,8637	1,0353	4,8296	1,2940	5,7956	1,5529	75	
	16	0,9613	0,2756	1,9225	0,5513	2,8838	0,8269	3,8450	1,1025	4,8063	1,3782	9,7676	1,6538	74	
1.½		0,9569	0,2903	1,9139	0,5806	2,8708	0,8709	3,8278	1,1611	4,7847	1,4514	5,7416	1,7417		6.½
	17	0,9563	0,2924	1,9126	0,5847	2,8689	0,8771	3,8252	1,1695	4,7815	1,4619	5,7378	1,7542	73	
	18	0,9511	0,3090	1,9021	0,6180	2,8532	0,9271	3,8042	1,2361	4,7553	1,5451	5,7063	1,8541	72	
	19	0,9455	0,3256	1,8910	0,6511	2,8364	0,9767	3,7821	1,3023	4,7276	1,6278	5,6731	1,9534	71	
1.¾		0,9415	0,3369	1,8831	0,6738	2,8246	1,0107	3,7662	1,3476	4,7077	1,6844	5,6493	2,0213		6.¼
	20	0,9397	0,3420	1,8794	0,6840	2,8191	1,0261	3,7588	1,3681	4,6985	1,7101	5,6382	2,0521	70	
	21	0,9336	0,3584	1,8672	0,7167	2,8007	1,0751	3,7343	1,4335	4,6679	1,7918	5,6015	2,1502	69	
	22	0,9272	0,3746	1,8544	0,7492	2,7816	1,1238	3,7087	1,4984	4,6359	1,8730	5,5631	2,2476	68	
2.		0,9239	0,3827	1,8478	0,7654	2,7716	1,1480	3,6955	1,5307	4,6194	1,9134	5,5433	2,2961		6.
Pts.	D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.	Pts.
		Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		Dist. 6.			

Cour.		Dist. 7.		Dist. 8.		Dist. 9.		Cour.	
Pts.	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts
	1	6,9989	0,1222	7,9988	0,1396	8,9986	0,1571	89	
	2	6,9957	0,2443	7,9951	0,2792	8,9945	0,3141	88	
0.¼		6,9916	0,3435	7,9904	0,3925	8,9892	0,4416		7.¾
	3	6,9904	0,3664	7,9890	0,4187	8,9877	0,4710	87	
	4	6,9829	0,4883	7,9805	0,5580	8,9781	0,6278	86	
	5	6,9734	0,6101	7,9696	0,6972	8,9658	0,7844	85	
0.½		6,9663	0,6861	7,9615	0,7841	8,9567	0,8822		7.½
	6	6,9617	0,7317	7,9562	0,8362	8,9507	0,9408	84	
	7	6,9478	0,8531	7,9404	0,9750	8,9329	1,0968	83	
	8	6,9319	0,9742	7,9221	1,1134	8,9124	1,2526	82	
0.¾		6,9242	1,0271	7,9134	1,1738	8,9026	1,3206		7.¼
	9	6,9138	1,0950	7,9015	1,2515	8,8892	1,4079	81	
	10	6,8937	1,2155	7,8785	1,3892	8,8633	1,5628	80	
	11	6,8714	1,3357	7,8530	1,5265	8,8346	1,7173	79	
1.		6,8655	1,3656	7,8463	1,5607	8,8271	1,7558		7.
	12	6,8470	1,4554	7,8252	1,6633	8,8033	1,8712	78	
	13	6,8206	1,5746	7,7950	1,7996	8,7693	2,0246	77	
	14	6,7921	1,6935	7,7624	1,9354	8,7327	2,1773	76	
1.¼		6,7902	1,7009	7,7602	1,9438	8,7303	2,1868		6.¾
	15	6,7615	1,8117	7,7274	2,0706	8,6933	2,3294	75	
	16	6,7288	1,9295	7,6901	2,2051	8,6513	2,4807	74	
1.½		6,6986	2,0320	7,6555	2,3223	8,6125	2,6126		6.½
	17	6,6941	2,0466	7,6504	2,3390	8,6067	2,6313	73	
	18	6,6574	2,1631	7,6084	2,4721	8,5595	2,7812	72	
	19	6,6186	2,2790	7,5642	2,6045	8,5097	2,9301	71	
1.¾		6,5908	2,3582	7,5324	2,6951	8,4739	3,0320		6.¼
	20	6,5779	2,3941	7,5175	2,7362	8,4572	3,0782	70	
	21	6,5351	2,5086	7,4686	2,8669	8,4022	3,2253	69	
	22	6,4903	2,6222	7,4175	2,9969	8,3447	3,3715	68	
2.		6,4672	2,6788	7,3910	3,0615	8,3149	3,4441		6.
Pts	D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.	Pts.
		Dist. 7.		Dist. 8.		Dist. 9.			

Cour.		Dist. 1.		Dist. 2.		Dist. 3.		Cour.	
Pts.	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts.
	23	0,9205	0,3907	1,8410	0,7815	2,7615	1,1722	67	
	24	0,9135	0,4067	1,8270	0,8135	2,7406	1,2202	66	
	25	0,9063	0,4226	1,8126	0,8452	2,7189	1,2679	65	
2.¼		0,9040	0,4276	1,8080	0,8551	2,7120	1,2827		5.¾
	26	0,8988	0,4384	1,7976	0,8767	2,6964	1,3151	64	
	27	0,8910	0,4540	1.7820	0,9080	2,6730	1,3620	63	
	28	0,8829	0,4696	1,7659	0,9389	2,6488	1,4084	62	
2.½		0,8819	0,4714	1,7638	0,9428	2,6458	1,4142		5.½
	29	0,8746	0,4848	1,7492	0,9696	2,6239	1,4544	61	
	30	0,8660	0,5000	1,7320	1,0000	2,5981	1,5000	60	
2.¾		0,8577	0,5141	1,7155	1,0282	2,5732	1,5423		5.¼
	31	0,8572	0,5150	1,7143	1,0301	2,5715	1,5451	59	
	32	0,8480	0,5299	1,6961	1,0598	2,5441	1,5896	58	
	33	0,8387	0,5446	1,6773	1,0893	2,5160	1,6339	57	
3.		0,8315	0,5556	1,6629	1,1111	2,4944	1,6667		5.
	34	0,8290	0,5592	1,6581	1,1184	2,4871	1,6776	56	
	35	0,8192	0,5736	1,6383	1,1472	2,4575	1,7207	55	
	36	0,8090	0,5878	1,6180	1,1756	2,4271	1,7634	54	
3.¼		0,8032	0,5957	1,6064	1,1914	2,4096	1,7871		5.¼
	37	0,7986	0,6018	1,5973	1,2036	2,3959	1,8054	53	
	38	0,7880	0,6157	1,5760	1,2313	2,3640	1,8470	52	
	39	0,7771	0,6293	1,5543	1,2586	2,3314	1,8880	51	
3.½		0,7730	0,6344	1,5460	1,2688	2,3190	1,9032		4.½
	40	0,7660	0,6428	1,5321	1,2856	2,2981	1,9284	50	
	41	0,7547	0,6561	1,5094	1,3121	2,2641	1,9682	49	
	42	0,7431	0,6691	1,4863	1,3383	2,2294	2,0074	48	
3.¾		0,7410	0,6716	1,4819	1,3431	2,2229	2,0147		4.¼
	43	0,7314	0,6820	1,4628	1,3640	2,1941	2,0460	47	
	44	0,7193	0,6947	1,4387	1,3894	2,1580	2,0840	46	
4.	45	0,7071	0,7071	1,4142	1,4142	2,1213	2,1213	45	4.
Pts.	D	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D.	Pts.
		Dist. 1.		Dist. 2.		Dist. 3.			

Cour.		Dist. 4.		Dist. 5.		Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Cour.	
Pts.	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts.
	23	3,6820	1,5629	4,6025	1,9537	5,5230	2,3444	6,4435	2,7351	7,3640	3,1258	8,2845	3,5166	67	
	24	3,6542	1,6269	4,5677	2,0337	5,4813	2,4404	6,3948	2,8472	7,3084	3,1539	8,2219	3,6606	66	
	25	3,6252	1,6905	4,5315	2,1131	5,4378	2,5357	6,3442	2,9583	7,2505	3,3809	8,1568	3,8036	65	
2.¼		3,6160	1,7102	4,5199	2,1378	5,4239	2,5653	6,3279	2,9929	7,1319	3,4204	8,1359	3,8480		5.¾
	26	3,5952	1,7535	4,4940	2,1919	5,3928	2,6302	6,2916	3,0686	7,1904	3,5070	8,0891	3,9453	64	
	27	3,5640	1,8160	4,4550	2,2699	5,3460	2,7239	6,2370	3,1779	7,1280	3,6319	8,0191	4,0859	63	
	28	3,5318	1,8779	4,4147	2,3474	5,2977	2,8168	6,1806	3,2863	7,0636	3,7558	7,9465	4,2252	62	
2.½		3,5277	1,8856	4,4096	2,3570	5,2915	2,8284	6,1734	3,2998	7,0554	3,7712	7,9373	4,2426		5.½
	29	3,4985	1,9392	4,3731	2,4240	5,2477	2,9089	6,1223	3,3937	6,9970	3,8785	7,8716	4,3633	61	
	30	3,4641	2,0000	4,3301	2,5000	5,1961	3,0000	6,0622	3,5000	6,9282	4,0000	7,7942	4,5000	60	
2.¾		3,4309	2,0564	4,2886	2,5705	5,1464	3,0846	6,0041	3,5987	6,8618	4,1128	7,7196	4,6269		5.¼
	31	3,4287	2,0602	4,2858	2,5752	5,1430	3,0902	6,0002	3,6052	6,8573	4,1203	7,7145	4,6353	59	
	32	3,3922	2,1197	4,2402	2,6496	5,0883	3,1795	5,9363	3,7094	6,7843	4,2394	7,6324	4,7693	58	
	33	3,3547	2,1786	4,1934	2,7232	5,0320	3,2678	5,8707	3,8125	6,7094	4,3571	7,5480	4,9018	57	
3.		3,3259	2,2223	4,1573	2,7778	4,9888	3,3334	5,8203	3,8890	6,6518	4,4446	7,4832	5,0001		5.
	34	3,3162	2,2368	4,1452	2,7960	4,9742	3,3552	5,8033	3,9144	6,6323	4,4735	7,4613	5,0327	56	
	35	3,2766	2,2943	4,0958	2,8679	4,9149	3,4415	5,7341	4,0150	6,5532	4,5886	7,3724	5,1622	55	
	36	3,2361	2,3511	4,0451	2,9389	4,8541	3,5267	5,6631	4,1145	6,4721	4,7023	7,2812	5,2901	54	
3.¼		3,2128	2,3828	4,0160	2,9785	4,8192	3,5742	5,6224	4,1699	6,4257	4,7656	7,2289	5,3613		4.¾
	37	3,1945	2,4073	3,9932	3,0091	4,7918	3,6109	5,5904	4,2127	6,3891	4,8145	7,1877	5,4163	53	
	38	3,1520	2,4626	3,9401	3,0783	4,7281	3,6940	5,5161	4,3096	6,3041	4,9253	7,0921	5,5409	52	
	39	3,1086	2,5173	3,8857	3,1466	4,6629	3,7759	5,4400	4,4052	6,1272	5,0346	6,9943	5,6639	51	
3,½		3,0920	2,5376	3,8650	3,1720	4,6381	3,8064	5,4111	4,4408	6,1841	5,0751	6,9571	5,7095		4.½
	40	3,0642	2,5712	3,8302	3,2139	4,5963	3,8567	5,3623	4,4995	6,1284	5,1423	6,8944	5,7851	50	
	41	3,0188	2,6242	3,7736	3,2803	4,5283	3,9363	5,2830	4,5924	6,0377	5,2485	6,7924	5,9045	49	
	42	2,9726	2,6765	3,7157	3,3457	4,4589	4,0148	5,2020	4,6839	5,9452	5,3530	6,6883	6,0222	48	
3.¾		2,9638	2,6862	3,7048	3,3578	4,4457	4,0294	5,1867	4,7009	5,9276	5,3725	6,6686	6,0440		4.¼
	43	2,9254	2,7280	3,6568	3,4100	4,3881	4,0920	5,1195	4,7740	5,8508	5,4560	6,5822	6,1380	47	
	44	2,8774	2,7786	3,5967	3,4733	4,3160	4,1679	5,0354	4,8626	5,7547	5,5573	6,4741	6,2519	46	
4.	45	2,8284	2,8284	3,5355	3,5355	4,2426	4,2426	4,9497	4,9497	5,6569	5,6569	6,3640	6,3640	45	4.
Pts	D.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	D	Pts.
		Dist. 4.		Dist. 5.		Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.			

CHAPTER III.

NAUTICAL ASTRONOMY.

On the Rotation of the Earth.—The common arguments in support of the doctrine that the earth has a diurnal rotation about one of its diameters, give to that doctrine a degree of probability, so nearly approaching to absolute certainty, that the mind readily acquiesces in the reality of the phenomenon. Since the time of Copernicus, the evidence for the earth's rotation has been continually increasing; but still this evidence is not of that direct and positive kind that is necessary to give to it the character of demonstration. All the other hitherto-discovered planets of our system revolve on their axes, and, as might be expected as a consequence of this revolution, those of them upon which the examination can be made are seen to be flattened at their poles. It is probable, therefore, that the planet we inhabit *also* revolves on an axis; if so, it too may be expected to be flattened at its poles. Whether or not *this* is the case can be actually ascertained by experiments: these have been undertaken and repeated, again and again, with the greatest care, and by independent and widely different means: the results all show that the earth *is* flattened at the poles. There is thus a very high degree of probability that the earth rotates; and this is further increased by the fact that all the phenomena of the heavens are completely consistent with the hypothesis of such a rotation; that it is, moreover, the simplest hypothesis upon which the celestial appearances can be explained; and that to attempt to account for them on any other hypothesis, involves the system of the universe in such intricacy and extreme complication, that, judging from all the other operations of nature, we cannot bring ourselves to suppose that such complex machinery should really be the "handiwork" of an all-wise Creator, when means so immeasurably simpler presented themselves, of bringing about the same ends. It can scarcely be charged to the King Alphonso, as a sentiment of impiety, when he exclaimed, in reference to the confused astronomical systems of the ancients—"If I had been the Almighty's counsellor when he framed the universe, I would have advised him better."

But, notwithstanding all this, a sensible or experimental *proof* that the earth rotates was still wanting: its general figure, as already declared (page 1049), can be experimentally discovered: its superficial rotundity can be *seen*. It is very desirable that we should have the same ocular evidence of its rotation. A profound writer on Physical Astronomy has observed—"We must, however, be content, at present, to take for granted the truth of the hypothesis of the earth's rotation. If it continue to explain, simply and satisfactorily, other astronomical phenomena than those already noted, the probability of its being a true hypothesis will go on increasing.

"We shall never, indeed, arrive at a term when we shall be able to pronounce it absolutely *proved* to be true. The nature of the subject excludes such a possibility."

This prediction of Professor Woodhouse has been falsified: we can *now* obtain sensible evidence of the rotation of the earth.

The idea of proving this interesting phenomenon to the senses occurred a few years ago to M. Foucault, of Paris: it was suggested to him by accidentally observing the motion of a weight suspended by a string to a high ceiling, and which, by chance, had been set vibrating. Most of our readers must remember the sensation produced in the scientific world, in the spring of 1851, by the remarkable **Pendulum Experiment** of Foucault. We here propose to submit to them, without going into any minute mathematical details, what appears to us to be a conclusive and satisfactory explanation of the manner in which this experiment renders the rotation of the earth a matter of personal observation; but as we write chiefly for the young, and for those who may be supposed to be but little habituated to scientific research, we shall previously offer to their attention a few general remarks in reference to what may be called the two great postulates of astronomy: these are the rotation of the earth, and the hypothesis of gravitation.

We speak of gravitation as a physical *hypothesis*: it is not, like a proposition in Geometry, a necessary truth; nor is it an observed fact recognisable by our senses. Certain phenomena in nature are observed: they exhibit a regularity of succession, and a mutual dependence, that suggest the idea of a connecting principle and a governing law. The phenomena are *seen*, their proximate cause is *inferred*. The philosopher looks abroad upon nature, and carefully studies the facts presented to him, the order of their recurrence, and the measure of their intensities; he retires to his closet, and endeavours to frame a law, of which, the appearances he has been studying, shall be the outward—the practical manifestation. This is an *hypothesis*. An hypothesis, therefore, need not be more than co-extensive with the phenomena actually observed; but it is a strong confirmation of the soundness of an hypothesis when it is found that new and unexpected phenomena are equally comprehended in it, and a stronger confirmation still, when such phenomena can actually be predicted. The two fundamental hypotheses of Astronomy—the rotation of the earth, and the law of gravitation—have this character in the highest degree: every new discovery in the science has only the more firmly established the truth of both.

The learner will perceive that we could not, with any propriety, speak in this way of the truths of geometry: *they* are quite independent of the confirmations of experience, and hence the marked difference between physical and geometrical science. When Dr. Halley had predicted the return of the comet of 1682 in 1759, and Clairault had computed, from the hypothesis of gravitation, the time when it ought to appear, its return was watched for by astronomers with the greatest interest—not from any anxiety to see the comet, but to learn how the hypothesis of gravitation would stand so severe a test; and the reappearance of the same comet, in 1835, was anticipated with like interest, solely in reference to the planetary attractions—that is, to the general theory of gravitation.

"The rude supposition of the uniform revolution of the moon in a circle about the earth as a centre, led Newton at once to the true law of gravity, as extending from the earth to its companion. The uniform circular motions of the planets about the sun, in times following the progression assigned by observation in Kepler's rule, confirmed the law, and extended its influence to the boundaries of our system. Everything more refined than this—the elliptic motions of the planets and satellites—their mutual perturbations—the slow changes of their orbits and motions, denominated secular variations—the deviation of their figures from the spherical form—the oscillatory motions of their axes, which produce nutation and precession of the equinoxes—the theory of the tides, both of the ocean and the atmosphere—have all in succession been so many trials for life and death, in which this law has been, as it were, pitted against nature: trials, whose event no human foresight could predict, and where it was impossible even to conjecture what modifications it might be found to need. Even at this moment, if, among the innumerable inequalities of the lunar or planetary motions, any one, however small, should be discovered decidedly not explicable on the hypothesis of a force varying as the inverse square of the distance, that hypothesis must be modified till it accounts for it."*

From these statements the student will perceive that one of the two fundamental hypotheses of astronomy—the hypothesis of gravitation—is not irrefragably established like a proposition in Euclid; nor is it a truth set at rest, once and for ever, by observation and experiment. Indeed, no physical truth can be regarded as thus unalterably fixed, like a *necessary* truth of geometry. The laws of nature *may* change; the supposition of such a change would involve no such absurdity as that which would be implied in the supposition that the three angles of a triangle could ever exceed or fall short of two right angles. This truth would remain undisturbed, however the properties of matter might be modified, and even though matter were to be altogether annihilated. It is obvious that we reckon upon the continuance of the properties of matter, and the return of natural phenomena, only to the extent to which we reckon upon the permanence of the existing natural laws. And Laplace, in extension of this idea, has even calculated the probability that the sun will not rise to-morrow.

But, assuming the unchangeableness of nature's laws, we are authorised in regarding certain of its phenomena as unalterable truths. For instance, if the planet we inhabit is clearly ascertained *now* to be a round body, we conclude that it will remain round as long as it lasts. If it be as clearly seen to rotate, we conclude, in like manner, that it will always rotate; its rotation ceases then to be an hypothesis—it becomes an observed fact, the evidence for the truth of which is not increased by the confirmations of future experience, nor by its satisfactorily accounting for whatever phenomena may be referred to such rotation. It is a matter of some importance, therefore, that the rotation of the earth is taken out of the category of hypothesis, and classed among observed physical truths, as we now proceed to show.

Foucault's Pendulum Experiment.—Let the reader conceive before him a circular table, upon which, passing through its centre, the meridian line is drawn. If the earth have no rotation about an axis, this line can never change its direction; if it do rotate, the direction must continually vary, except the place of observation be at the equator: this will readily appear from the following considerations.

Let our horizontal meridian line be indefinitely extended; we shall thus have an indefinite straight line, in the plane of the terrestrial meridian, and touching the surface of the earth, the point of contact being the centre of the table;—we may, of course, regard the table-top as lying horizontally on the ground.

For any place of observation between the equator and the pole, it is obvious that if the earth turned round its axis, this tangent line will, in one complete rotation, describe a conical surface enveloping the globe; and, as the vertex of the cone is necessarily at a finite distance, the line which generates its surface—thus always pointing to a fixed determinate point (the vertex)—must continually change its direction, which, however, it cannot do if the earth be at rest.

But if the place of observation be at the equator, what, in the case just considered, is a *conical* surface, would evidently be a *cylindrical* surface; the straight line generating it would thus always be parallel to itself, and, therefore, though the earth should really rotate, there would be no more change of direction in the meridian line than if it were at rest.

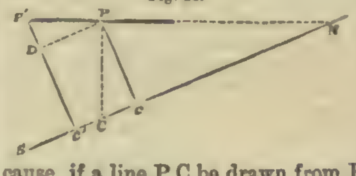
If, however, the centre of the table were directly over the pole, then, taking any diameter of the table for a meridian line, the changes in its direction—if the earth rotate—would clearly be more rapid and more considerable; it would pass through a revolution of 360° for every complete rotation, and the surface described by the line would be a plane surface.

It is thus easy to see what must necessarily happen, as to the direction of the horizontal meridian, if the earth have any rotation about its axis. At the equator, the direction would remain undisturbed, and the line would generate a cylindrical surface; at a small distance from the equator, the cylinder would become a cone, and the direction of the line would regularly, but only slightly change. At a greater distance from the equator the cone would differ more palpably from the cylinder, and the angular deviation of the line—always directed to the vertex of the cone—would be more considerable. As the place of observation approaches the pole, the cone would, as it were, widen out more and more, the deviation of the line would become greater and greater, in a given time, till at length, when the pole is reached, the cone would spread into a plane, and the change of direction would be the greatest possible.

The reader will find it of much assistance, towards clearly apprehending what is to follow, if he keep constantly before his mind the idea of the enveloping cones. Every parallel of altitude is to be regarded as the circle of contact of a cone, the apex of which becomes less and less remote as the parallel approaches the pole; we shall call the cone adapted to any particular parallel, the *cone of latitude* of that parallel—a designation which is sufficiently significant. The extreme cases of the cone of latitude are the cylinder and the plane—the former belongs to the equator, the latter to the pole; the angle of the former is 0, that of the latter 180°. The straight line from a point of contact to the vertex of the cone, is the horizontal meridian of that point of contact. The imaginary cone, of course, rotates with the earth, if the latter rotate, but not else; and the axis of the earth is, when prolonged, also the axis of the cone. All this is obvious.

Let now P (Fig. 22), be the centre of the circular top of the table, P P′ half its meridional diameter, N S the axis of the earth, prolonged to meet the vertex of the cone of latitude in N. The angle N will be equal to the latitude of the point of contact P; because, if a line P C be drawn from P to the centre of the earth, the angle P C N will evidently be the complement of the latitude; consequently, since N P C is a right angle, the angle N is the latitude.

Fig. 22.



Over the point P, let now the bob of a pendulum freely hang—the longer the wire or cord by which it is suspended the better; but it must be so attached to the support above, as to be equally free to vibrate in any direction. Let the pendulum be made to vibrate in the direction of the meridian line N P′; and, *if* the earth rotate, let us try to anticipate the phenomena that will be presented to us.

The point P immediately under the bob, at rest, must rotate *slower* than the points in the meridian line more remote from the apex of the cone; so that, as the bob sweeps over these points, they must keep proceeding in *advance* of it, that is, towards the east, if P be in the north latitude, as here supposed. The rate of rotation of the bob round the cone must be the same whether it oscillate or not; because the fact of its oscillating cannot interfere with any motion of revolution round the cone it may have had, when hanging freely. We see, therefore, that the path of the pendulum, that is, its line of vibration, must *appear* to be gradually receding from the meridian line P P′ towards the *west*, making with P P′ an angle of deviation, which increases at every oscillation.

Actual experiment fully justifies these anticipations: the deviation of the track of the bob is *seen* to be in complete accordance with them; and although, in the following mathematical investigation of the exact amount of deviation in a given latitude and in a given time, allowance must be made for mechanical imperfections, and for the accidental impulse to the right or left communicated to the bob by the hand in giving the initial impulse, and which will give rise to what is called an apsidal motion, yet, with only very ordinary care, the natural tendency of the pendulum sufficiently overrules these hindrances to render the deviation of its path towards the west always palpable, and always accumulative.[*] The rotation of the earth from west to east is the only way of accounting for this apparent deviation of the path of the bob; what we really *see* is the motion of the line P P′, receding from the track of the bob, with an angular movement about the centre P, towards the east. If the pendulum can be kept swinging sufficiently long, it is plain that, as the angular deviation is always accumulative, a time will come when it shall have described a complete revolution, or 360°; or, to speak more accurately, when the radius P P′ of the table shall have revolved completely round. Let us seek to determine this time.

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* "Physical Astronomy," in the *Encyclopædia Metropolitana*, by Sir J. F. W. Herschel; p. 648.

Time in which the Meridian Line completes a Revolution.—As the absolute direction of the oscillations remains invariable, it follows that, when the cone of latitude has turned through any angle, the original horizontal line must have turned through an equal angle; so that, when the cone has made a complete revolution, the deviation of the path of the bob, from the horizontal line, amounts to an angle equal to the plane angle given by developing the conical angle.[†] This, it is obvious, can never amount to so much as 360°, or a complete revolution of the table, except when the bob is suspended over one of the poles of the earth, the conic surface then becoming a *plane*, tangential to the sphere of the pole. Proceeding from this extreme limit towards the equator, the angle of the cone becomes less and less, till, on reaching the equator, it vanishes altogether, as before remarked; the cone then becomes a cylinder, and no deviation can take place.

It thus appears that the angle of deviation in any time, at any place, is a plane angle exactly equal to the development of the corresponding conical angle, turned through in that time by the rotation of the cone of latitude. It remains to be ascertained what part this angle of deviation is of a complete rotation, or 360°. In other words, we have to ascertain what part of 360° the developed angle at the apex of the cone of latitude amounts to.

Conceive an upright cone, unconnected with the globe, and from its apex let any two straight lines be drawn along the slant side, intercepting an arc of the base; this arc will measure a certain angle at the centre of the base. Measure from the apex along either of the two lines drawn from it, a length equal to the radius of the base, and describe with this length, as radius, a circular arc, limited by the two straight lines on the cone: this arc will evidently measure the conic angle; it will therefore be to the corresponding arc of the base, as the angle at the apex to the corresponding angle at the centre of the base; but the developed arcs are also to one another as their distances from the apex: for these distances are their radii. Hence, the conic angle is to the corresponding plane angle—or angle at the base on the horizontal plane—as the radius of the base of the cone to the length of the slant side; that is, as the sine of half the plane angle of the cone to unity. Consequently, the whole conic angle is to 360° as the sine of half the plane angle of the cone—that is, as the sine of the latitude is to unity. Hence, from what is shown above, the angle of deviation, or of the path of the bob, in one entire rotation of the earth, is to 360° as the sine of the latitude is to unity. The *time* of this rotation is 24 hours; consequently, 24 hours divided by the sine of the latitude, is the time in which the path of the pendulum makes a complete revolution in that latitude. Thus, assuming the deviation to be 360°, we have—the angle of rotation of the earth in any time being x—

$$x = \frac{\text{angle of deviation}}{\text{angle of rotation}}$$

$$360 = x \text{ sin. lat.}, \therefore x = \frac{360^\circ}{\text{sin. lat.}}, \text{ in } \textit{degrees.}$$

$$\therefore x = \frac{24 \text{ hours}}{\text{sin. lat.}}, \text{ in } \textit{time.}$$

And all the more carefully conducted experiments justify this result, within those limits of difference that they may reasonably be attributed to the disturbing causes adverted to above.

We have further obtained, from a few simple considerations, the following interesting proposition, namely:—

The length of the arc of the rim of the table, subtending the angle of deviation at its centre, which a pendulum oscillating over it makes during one rotation of the earth, is exactly equal to the difference between the parallel of latitude described by that centre, and the parallel described by the extremity P′ of the meridional diameter of the table.

Draw P*c* (Fig. 22), P′*c*′ perpendiculars upon the axis of the earth, and P D parallel to the axis N S. It has been proved above that the angle of deviation in one revolution of the earth is

$$360^\circ \text{ sin. N} = 360^\circ \frac{P'c'}{P'N} = 360^\circ \frac{P'D}{P'P}$$

This angle, multiplied by the radius P P′ of the table, and by 3·1416, and the product divided by 180°, is the arc of the rim of the table subtending it; that is, the measure of this arc is 2 P′ D × 3·1416. But twice P′ D is the difference between the diameters of the two parallels described by P and P′. Hence the arc that measures the angle of deviation in one rotation of the earth, is equal to the difference between the two circumferences described by P and P′. And the arc of deviation, due to any portion of a complete rotation of the earth, is equal to the difference between the two portions of parallels described by P and P′.

The same conclusion may be obtained by aid of considerations still more simple. It is plain that the difference between the circumferences of any two equidistant circles on the surface of a cone is always the same; hence, if a circle be described about the apex, with a radius equal to P P′, the circumference of it will be equal to the difference between the two circumferences described by P and P′. But this same circumference, when the cone is developed, is the arc of deviation, on the table, due to a complete rotation of the earth: hence, this arc must be equal to the difference between the two parallels described by P and P′ in a complete rotation.

It thus appears, not only that the pendulum experiment affords ocular demonstration of the rotation of the earth, but that it moreover exhibits to us the actual velocity, in linear measure, with which the point P′ proceeds in advance of P. It is the velocity with which the arc of deviation increases.

If the length of this arc, described in any interval of time, be measured, we may readily deduce the arc that would be described in a complete rotation of the earth. If the length of this arc be taken for the circumference of an entire circle, the diameter of that circle may be inferred; this diameter, applied as a chord to the circle of deviation, will subtend an arc of it, the degrees and minutes of which will be double the latitude of the place. And thus we may conceive it possible that a person, conveyed to a dungeon in some unknown part of the world, with a piece of string and a weight at hand, might form an estimate of the latitude of his position. The following diagram shows the method of trying the experiment. The path of the bob is between the points *a a* (Fig. 23):—

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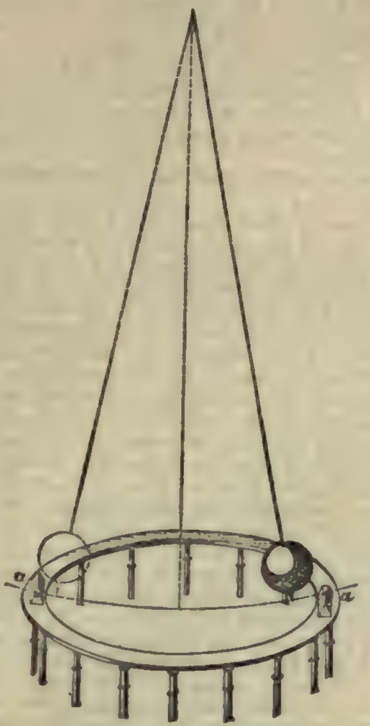
[*] In the experiment as performed at the Royal Institution of Great Britain, the impulse was not given by the hand; the bob was drawn out of the vertical by a silken thread, the end of which was fastened to a peg in the floor; a flame was then applied to the thread, and the ball set free. The author of this treatise first witnessed the experiment at the house of a scientific friend, W. H. Spiller, Esq., of Highgate: in the different repetitions of it the suspending wire, nearly twenty feet in length, was of different metal, and the hand was employed to let the bob go; the deviation was quite palpable in eight minutes.

[†] The proper distinction must, of course, be observed by the reader between the angle of the cone and the conical angle. The angle of the cone is the plane angle of the isosceles triangle, which the section of the cone through its axis presents; the conical angle is the angle at the vertex, formed by the surface of the cone: if this surface be cut along the straight line which may be conceived to generate it, and then the surface developed or unfolded into a plane, the conical angle will become an equivalent plane angle.

Fig. 23.



Application to an Unexplained Phenomenon in Falling Bodies.—The idea of the cone of latitude will subserve the purpose of accounting for circumstances in the late M. Oersted's experiments on falling bodies, hitherto, we believe, involved in some obscurity. The following quotation is from the *Literary Gazette* of March 22, 1851:—"One of the most important observations first made by Oersted, and since then confirmed by others, was, that a body falling from a height, not only fell a little to the east of the true perpendicular—which is, no doubt, due to the earth's motion—but that it fell to the *south* of that line: the cause of this is, at present, unexplained. It is no doubt connected with some great phenomenon of gravitation which yet remains to be discovered."

The explanation of this phenomenon is very easy. Suppose a heavy body to be let fall from a point at a considerable height vertically over P (Fig. 22): when it is let go, the body will have a progressive velocity towards the east, *greater* than the velocity of P at the foot of the vertical; and this velocity it will preserve throughout its descent, which, from the nature of the gravity, must be in a vertical plane through P C, C being the centre of the earth. Now the point P, at the foot of the vertical line, *recedes* from this plane, towards the north, during the descent of the body: it always keeps in the plane through P*c*, and perpendicular to the axis of the earth, and describes a circle whose radius is *c*P on the cone of latitude. The body, therefore, must necessarily fall towards the *south* of P, as well as towards the east. If the experiment be made in south latitude, the deviation will, of course, be north instead of south.

It is plain that, in Oersted's experiment, the falling body, by the rotation of the earth, and, therefore, by its own more rapid easterly motion, had advanced more towards the east, when it reached the ground, than the point P at the foot of the vertical, but it had not advanced at all towards the south; it was the foot of the vertical—the point P—that had receded towards the *north*.

These experiments prove, in the most satisfactory manner, that the earth really rotates on its axis: we are made sensible of this rotation in other ways also. "It is well known to engineers, that when railway carriages are going north, their tendency is to run off the rails on the east side; but when the train is going south, their tendency is to run off on the west side of the track; that is, always on the right hand."* In the former case, the train, at starting, is moving eastward with a velocity greater than that with which any more northerly point of the track moves; and in the latter case, it is moving more slowly towards the east than any more southerly point of the track, and hence the uniform tendency to escape the confinement of the rails towards the right.†

Explanation of Terms in Nautical Astronomy.—What are called the heavenly bodies, appear, to an observer on the earth, to occupy a surrounding spherical concavity, at the centre of which our planet is placed: the phenomena of their rising and setting are appearances which necessarily present themselves in consequence of the rotation of the earth about its axis. This apparent concavity is called the celestial sphere, and the imagination traces upon it a variety of circles, analogous to those conceived to be traced on the terrestrial globe.

To assume, however, that what we call the starry heavens is really a concave sphere, whose centre coincides with that of the earth; and, therefore, that all the celestial bodies situated in it are at equal distances from that centre, would be to oppose what is well known to be truth; but the part of Astronomy with which we are at present concerned is occupied mainly with appearances, not with realities; or, we should rather say, it is chiefly occupied with the consideration of those astronomical phenomena which are independent of actual distances, and which would equally present themselves were these distances other than what they are, or, as they all appear to be, the same.

The learner will readily perceive how this assumption of a surrounding celestial sphere is perfectly consistent with correct deductions in certain departments of astronomy—all those departments, for instance, which regard only the angular distances of the stars from one another, or from the imaginary circles before alluded to. The angular distance of two objects—whether on the earth or in the heavens—is the angle formed at the eye of the observer by lines drawn to it from the objects observed. If one or both of these objects move nearer to the eye, along the line of vision, or recede further from it, it is plain that the *angular* distance of the two must remain the same;—the objects cannot, in this way, increase or diminish their angular separation. The observer therefore may, if he please, consider the linear distances of the two objects from his eye to be the same.

Nautical Astronomy has a good deal to do with observations of this kind—that is, with the measurement of angular distances, and but very little with linear distances. It would have *nothing* to do with linear distances if the earth were really a point, or if observations were all carried on at the centre instead of on the surface; but, as it is, the semi-diameter of the earth is a linear measure of which cognizance must be taken, simply because *appearances*—in reference to the sun and moon, but not in reference to the stars—are different at the surface, from what they would be at the centre. Hence, the angular distances are not exactly the same from the two points of observation.

What is here said of distances applies equally to magnitudes. The linear diameters of the sun and moon are not matters of concernment in Nautical Astronomy, only their apparent diameters, the diameters (taken in angular measure) they would appear to have, to an observer at the centre of the earth.

All observations made upon these two bodies, for the purpose of determining the latitude and longitude at sea, are reduced to what they would be if the place of observation were the centre of the earth. As to the stars, it is found that observations on them, though made at the surface, would require no modification if made at the centre—the radius of the earth being a mere point in comparison to their immense distance. We shall now define the principal circles of the celestial sphere.

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* Maury's *Physical Geography of the Sea*, p. 39.

† We may observe, that whilst personally agreeing with most of the arguments which Professor Young has advanced, and having made the experiment repeatedly, the whole subject has excited great difference of opinion; and we are inclined to suggest, that the theory of the pendulum experiment is more satisfactorily established than its facts.—Ed.

Axis.—The axis of the heavens is the diameter of the celestial sphere, about which the apparent diurnal revolution of the celestial sphere takes place, and which, as we have seen, is due to the real rotation of the earth. The axis of the heavens is, therefore, only the axis of the earth prolonged; and the extremities of this axis—of course the *imaginary* extremities—are the poles of the heavens.

Equinoctial.—The celestial great circle, to the plane of which the axis of the heavens is perpendicular, is called the *equinoctial*, or the celestial equator. It is traced out merely by extending the plane of the terrestrial equator to the heavens.

Meridians.—The celestial meridians are, in like manner, marked out by extending the planes of the terrestrial meridians; or they are semicircles terminating in the poles of the heavens, and perpendicular to the equinoctial.

Zenith, Nadir.—The zenith is that point in the heavens which is directly over the head of the spectator; or, if a straight line be drawn from the centre of the earth to any spot on its surface, and then prolonged to the heavens, the point on the celestial sphere which it would mark out is the *zenith* of that spot. The same line continued in the contrary direction would mark a point in the celestial sphere called the *nadir*. These two points, therefore, in reference to any place on the earth, are at the extremities of that diameter of the celestial sphere which is perpendicular to the plane of the horizon of that place:—that is, they are the *poles* of the horizon.

Vertical Circles.—Vertical circles of any place are those which pass through the zenith and nadir of that place; they are all perpendicular to the horizon of that place. They are hence, also, called circles of altitude.

The *altitude* of a celestial body is its distance above the horizon, measured on the vertical circle passing through the body. The complement of the altitude is the *zenith distance*. In the case of the sun and moon, the *true* altitude is measured from the rational horizon, and is a little greater than the altitude measured from the sensible horizon. In the case of the stars, as observed above, the difference in altitude is insensible, whichever horizon be referred to.

The most important of all the vertical circles of any place is the *meridian*. When a celestial object is on the meridian, its altitude is the greatest which that object can possibly have; it is called the meridian-altitude of the object.

The vertical circle which cuts the meridian at right angles, and which, therefore, passes through the east and west points of the horizon, is distinguished next to the meridian. It is called the *prime vertical*. When a celestial object arrives at the prime vertical, it is either due east or due west.

Azimuth.—The azimuth of a celestial object is the arc of the horizon, comprehended between the meridian of the observer and the vertical circle passing through the object. The arc of the horizon here spoken of is, of course, the measure of the angle at the zenith, between the meridian and the vertical, through the object. Vertical circles are, sometimes, called azimuth circles.

Amplitude.—Amplitude is also an arc of the horizon. It is the arc comprised between the east point of the horizon and the point where the body rises, or between the west point and where it sets; the former arc is called the rising amplitude of the body, and the latter its setting amplitude. Azimuth is measured either from the north or south points of the horizon; amplitude either from the east or west. When we speak of the azimuth of a body, we refer merely to the azimuth of the vertical on which the body is, whatever its altitude on that vertical may be; when we speak of its amplitude, we refer exclusively to its position with respect to the east or west point of the horizon at rising or setting.

Declination.—The declination of a celestial object is its distance from the equinoctial, measured on the celestial meridian passing through it, and is either north or south. What is latitude as respects a point on the earth, is declination in reference to a point in the heavens. Celestial meridians are thus, sometimes, called circles of declination; and what are parallels of latitude on the earth, become parallels of declination on the celestial sphere.

The distance of an object from the elevated pole is the polar distance of it. It is the complement of the declination when the elevated pole and the object are both on the same side of the equinoctial; but when they are on contrary sides the polar distance is the declination plus 90°. The elevation of the pole above the rational horizon of any place is always equal to the latitude of that place, for the latitude is equal to the distance of the zenith of the place from the equinoctial; the distance between the zenith and the elevated pole is, therefore, the complement of the latitude, and it is equally the complement of the elevation of the pole above the rational horizon: this elevation, therefore, is equal to the latitude of the place. Consequently, the depression of the equator below the horizon, or its elevation above the horizon, in the opposite quarter, is the complement of the latitude, or, which is the same thing, the latitude is the measure of the angle which the horizon makes with the equator.

The celestial circles now defined, have especial reference to the earth. The meridian and the equinoctial are merely extensions to the heavens of corresponding circles on the earth; and the vertical circles, or perpendiculars to the horizon, are imagined for the purpose of recording altitudes above the horizon, measured on the earth. But there are some circles peculiar to the celestial sphere; the principal of these are the *ecliptic*, or the circle of celestial longitude and the perpendiculars to it—the circles of celestial latitude.

The Ecliptic.—The ecliptic is the great circle described on the celestial sphere by the sun in its apparent annual motion about the earth: in reality, it is the path of the earth about the sun in the contrary direction; but, as already remarked, we are in this subject only concerned with the appearances. The ecliptic crosses the equinoctial at an angle subject to continuous but very small variation, determinable by observation. It is always given, with the utmost attainable accuracy, in the *Nautical Almanac*. The obliquity at present is about 23° 27½′.

The two points where the ecliptic crosses the equinoctial are called the equinoctial points. The sun, in its apparent annual course, passes through these points about the 21st of March and the 23rd of September; the former being the time of the vernal equinox, and the latter of the autumnal equinox: these names being given because the night is then equal to the day at all places where the sun rises and sets. This is obvious, because any point in the equinoctial, by the diurnal rotation of the earth—or the apparent rotation of the heavens—is just as long below the horizon of any place as it is above it.

Celestial Longitude.—The circle on which the longitude of any heavenly body is measured is the ecliptic, not the equinoctial; and as terrestrial longitude is measured from a fixed point of the equator, the point (with *us*) where the meridian of Greenwich crosses it, so celestial longitude is measured from a fixed point in the ecliptic—namely, the vernal equinoctial point, which is called the first point of the constellation Aries.

As respects terrestrial longitude, the fixed point from which the reckoning commences is only fixed for particular nations, each kingdom choosing its own: this is some inconvenience. But, as respects celestial longitude, there is perfect uniformity of reckoning among astronomers; and this reckoning—unlike that for terrestrial longitude—is carried on in one direction round the celestial sphere; so that a body may have any longitude short of 360°.

The ecliptic is conceived to be divided into twelve equal parts, called *signs;* a sign is therefore an arc of 30°. The twelve signs have the names and symbols following:—

1. ♈ Aries (The Ram).
2. ♉ Taurus (The Bull).
3. ♊ Gemini (The Twins).
4. ♋ Cancer (The Crab).
5. ♌ Leo (The Lion).
6. ♍ Virgo (The Virgin).
7. ♎ Libra (The Balance).
8. ♏ Scorpio (The Scorpion).
9. ♐ Sagittarius (The Archer).
10. ♑ Capricornus (The Goat).
11. ♒ Aquarius (The Water-bearer).
12. ♓ Pisces (The Fishes).

The first six of these lines are to the north of the equinoctial, and the others to the south: they are also called Signs of the *Zodiac*—the name given to a belt of the heavens 8° on each side of the ecliptic.

Celestial Latitude.—As the ecliptic is the circle of longitude, the perpendiculars to it, that is, the great circles through the poles of the ecliptic, are the circles of latitude. The distance of a celestial object from the ecliptic, measured on one of these perpendiculars, is its latitude; it is north or south, according as the object is on the north or south side of the ecliptic.

Right Ascension.—The right ascension of a celestial object is the arc of the equinoctial intercepted between the first point of Aries and the declination circle, or meridian passing the object.

The learner will perceive that the first point of Aries is the starting-point from which both longitude and right ascension are measured; and that, what on the terrestrial globe would be longitude and latitude, on the celestial globe are right ascension and declination—the first point of Aries being substituted for the meridian of Greenwich.

Great circles, all of which pass through the poles of any of the more important great circles of the sphere, are frequently called *secondaries* to the latter. This is a very convenient term: thus, vertical circles are secondaries to the horizon; meridians, or declination circles, are secondaries to the equinoctial; and circles of celestial latitude are secondaries to the ecliptic.

ON TIME.—The most important portion of time, in matters connected with nautical astronomy, is the *day* and its subdivisions. There are several kinds of day referred to in astronomy; but the period occupied by a single rotation of the earth, comprises, in each case, nearly the whole of the time so designated. If the heavenly bodies were all fixed, and the earth had no progressive motion, but only its present diurnal rotation on its axis, all days would be alike as to length, since the diurnal rotation is always performed in the same time; the interval between the departure from, and the return to, the meridian of any heavenly body would then be invariably the same. But, as the earth is continually shifting its place in its orbit, and that by an amount which is not uniform, the interval between two successive passages of the sun over the meridian of any place is variable. This interval is called an *apparent solar day.*

Apparent Time.—When the sun is on the meridian of any place, it is *apparent noon* at that place; when it is in any other position, the angle between the meridian of the place and that on which the sun is, is called the *hour angle* from noon at that place and instant; this angle, converted into time, at the rate of 15° to an hour, is the apparent time at the place.

Mean Time.—As, on account of the inequality of the earth's motion in its orbit, the solar day is continually varying in length, a day that is the average, or mean, of these variable days is fixed upon for civil reckoning; and it is the length of such a mean day that is marked out by the twenty-four hours of a common clock or watch. This length of time is called a *mean solar day;* and any time shown by a correct clock or watch is mean solar time, or simply *mean time.* At certain periods of the year the sun will thus arrive at the meridian before the clock points to XII.; and at other periods the clock will be in advance of the sun: the interval between the arrival of the index of the clock to XII., and of the sun to the meridian, is called the *equation of time.* It is given for every day in the year, at page 1 of the *Nautical Almanac,* for the meridian of Greenwich; that is to say, when it is apparent noon at Greenwich, on any day of the year, the almanac shows the time to be added or subtracted to obtain the corresponding mean time at that meridian.

Sidereal Time.—A sidereal day is the time occupied by one complete rotation of the earth on its axis. This interval is ascertained by observing the time elapsed between two successive passages of the same fixed star over the meridian. Such is the immense distance of the stars, that the earth's change of place, from day to day, produces not the slightest effect upon their apparent positions, which are preserved the same as if the earth were at rest. Whatever star be observed, and whatever be the place of the earth in its orbit, it is uniformly found that the interval of two successive passages of the star over the meridian is invariable—namely, 23h. 56′ 4·09″ of mean time.[*]

Besides the *three* kinds of day here described, there is also the *lunar day,* which is the interval between two successive passages or transits of the moon over the meridian; the *average* length of it is 24h. 54m. But navigators have nothing to do with lunar time; what they are most concerned with are apparent time and mean time—the time that would be shown by a properly constructed sun-dial, and the time shown by a well-regulated chronometer. The time determined by observations at sea, which is in general deduced from the sun's hour-angle with the meridian of the place, is, of course, apparent time. It is turned into mean time by help of the table of the equation of time at page 1 of the *Nautical Almanac*—the phenomena, predicted in that important publication, for the use of seamen in finding the latitude and longitude at sea, being recorded in mean time, just like the transactions of common life.

But there is this difference between the civil and the astronomical mode of reckoning: the civil day reckons from twelve o'clock, at *midnight,* and the whole twenty-four hours is divided into two sets of twelve, the counting recommencing at twelve o'clock, noon; but astronomers commence their day at noon, and count on through the twenty-four hours, from 0 hours up to 24 hours, when another day begins. Consequently, the common or civil reckoning is always twelve hours in advance of the astronomical reckoning, both reckonings being in reference to mean time: so that, to deduce the civil from the astronomical time at any instant, we have only to add twelve hours to the latter. For instance, Jan. 1, 15h. 35m., astronomical time, is Jan. 2, 3h. 35m., in the morning, civil time.

It is indispensably necessary that the learner have clear conceptions of apparent and mean time: the former is at once ascertained by observation of the true sun's hour-angle from the meridian; the latter is not pointed out by nature, but is arbitrarily chosen for practical convenience: its measure is not ascertained immediately from observation, but computed from the actual phenomena. Astronomers conceive an imaginary sun, called the *mean* sun, to move uniformly in the equinoctial, and with a motion in right ascension exactly equal to the real sun's mean or average motion in right ascension, so that the interval between two consecutive transits of the mean sun is a mean solar day—the mean, that is, of all the variable solar days of the year of the true sun. It is the motion of this imaginary sun that is measured by a chronometer; it completes every revolution in exactly twenty-four common hours; the twenty-four hours completed by the real sun—which twenty-four hours is a variable interval—is the apparent solar day. The twenty-four hours completed in one revolution of a star is a fixed and invariable period; and, as already remarked, is about 3m. 56s., mean time, less than a mean solar day. The sidereal day commences when the first point of Aries is on the meridian, and continues till its return.

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[*] The whole starry heavens have, however, a slow apparent movement, arising from a real motion of the earth distinct from its rotation on its axis. This motion causes the axis to describe a minute circle round the poles of the ecliptic in about 26,000 years; the effect is to cause the apparent approach of some stars towards the pole, and the recession of others. Thus, the pole-star, as it is called, has, for many centuries, been getting nearer to the pole; it is now about 1° 34′ from it: the star will continue its approach till within about 30′, and will then recede. The physical cause of the phenomena has been noticed in treating of the Precession and Nutation, in the previous section on Astronomy.

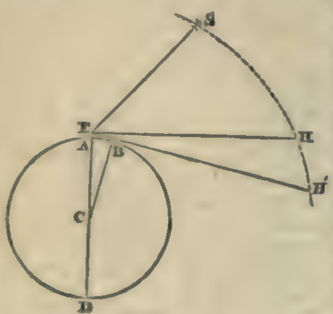
On the Corrections to be Applied to the Observed Altitudes of Celestial Objects.—The true altitude of a celestial object is the angular distance of it above the horizon of the place of observation. It is of course measured (by a quadrant or sextant) in degrees and minutes, the altitude of the zenith being 90°.

The altitude is estimated, not from the visible, but from the sensible horizon of the observer; and in the case of the sun or moon it is measured up to the centre of the body. The *observed* altitude is the angular distance of the visible horizon from the lower or upper limb of the body, so that a correction has to be made for the dip or depression of the horizon, and for the semi-diameter of the body. If the object be a star, then there is no correction for semi-diameter. These corrections being made, the result is called the *apparent altitude* of the body.

Some other corrections are necessary to obtain the *true* altitude from the apparent; these we shall speak of presently.

Dip of the Horizon.—As the eye is always elevated above the surface of the sea, the visible horizon dips below the sensible horizon, and forms an angle with it. It is the amount of this angle which must be subtracted from the observed altitude. Let E (Fig. 24) be the place of the observer's eye, and S the position of the celestial object whose altitude is to be found. The visible horizontal line is E H′, the true horizontal line E H; the altitude of S, as shown by the instrument, is the angle S E H′, instead of the angle S E H; the angle H E H′, by which the latter angle is increased, is the *dip* which must be subtracted from the observed altitude to give the apparent altitude S E H.

Fig. 24.



The angle H E H′ is equal to the angle C, since the angle C E B is the complement of each. The height, A E, of the eye being known, as also the radius C A of the earth, E B becomes known for E B² = E D × E A (Euclid, Prop. 36, Book III.), so that the amount of the angle of depression can always be found when the height of the eye above the surface is given.

Thus, let r be put for the radius of the earth, and h for the height of the eye above its surface; then, as just shown,

$$EB^2 = (2r + h)\,h = 2rh, \text{ very nearly,}$$

the quantity h^2 being omitted as insignificant in relation to $2rh$. Hence, because by right-angled triangles, $\sin. C = \frac{EB}{EC}$, and since C being always very small—only a few minutes—the arc may be taken for its sine, we have

$$\text{Dip.} = \frac{\sqrt{2rh}}{r+h} = \frac{\sqrt{2rh}}{r} \text{ (very nearly)} = \sqrt{\frac{2h}{r}}$$

which is the length of the arc (to radius 1) that measures the angle of the dip due to the height h. This length, for different values of h, is converted into minutes, and in this way the correction for dip is calculated for different altitudes of the eye, and the results arranged in a table.

Semi-diameter.—The foregoing correction for the dip of the horizon having been applied to the altitude of the point observed, if this point be the uppermost or lowermost point of the disc of the sun or moon, a correction for the semi-diameter of the body must be applied in order to obtain the apparent altitude of the centre. As the measurements in the present subject are all *angular* measurements, the correction here adverted to is the angle subtended at the eye, by the semi-diameter of the observed body. This angle, both for the sun and moon, is given in the *Nautical Almanac*. In the case of the moon, the diameter is seen under a greater angle as she approaches towards the zenith; for at the zenith she is nearer to the observer than when she is in the horizon, by a semi-diameter of the earth; and such is the comparative nearness of the moon that this difference in her distance makes a sensible difference in her apparent magnitude. The semi-diameter given in the *Nautical Almanac* is the horizontal semi-diameter, or that under which she would be seen when in the horizon; or, which is the same thing, it is the angle subtended by the semi-diameter at the centre of the earth. As this semi-diameter increases with her altitude, the increase being so much as one-sixtieth part of the whole when the moon is in the zenith—for she is about sixty semi-diameters of the earth off—the amount of increase for any altitude is found by multiplying one-sixtieth of the moon's linear semi-diameter by the sine of the altitude; and in this way the table entitled *Augmentation of the Moon's Semi-diameter*, and given in some collections of Nautical Tables, is constructed: it supplies the proper correction, to be applied additively to the horizonal semi-diameter, to obtain the semi-diameter at the given altitude.[*]

On account of the great distance of the sun, the variation of his semi-diameter, as he increases in altitude, is too minute to give any correction: it is practically insensible.

The corrections for dip and semi-diameter being thus applied, the result is the apparent altitude of the centre. As to the stars, the only correction of the observed altitude of a star, to reduce it to the apparent altitude, is the correction for dip. It remains to be shown how the true altitude is obtained from the apparent altitude: this requires two additional corrections—one for refraction, and the other for parallax.

Refraction.—The atmosphere which surrounds the earth is of variable density, the lower parts being compressed by the weight of the upper. A ray of light, therefore, from a celestial object passes through a medium, which opposes some obstruction to its free passage, the density of the medium increasing as the ray advances from the upper to the lower regions of the atmosphere, where it meets the eye of the observer. This disturbance causes the ray to be deflected or bent, the deflection being greater as the density of the medium through which it passes increases. Instead, therefore, of reaching the eye in a direct line from the object, it begins to curve as soon as it enters the atmosphere, and the curvature increases as it reaches the earth: the direction of the object from which it proceeds—which direction is judged of by the last direction the ray takes, and in which it enters the eye—thus appears to be different from its true direction: the object always, except when in the zenith, seems *higher* than it really is. Consequently, the correction for this *refraction*, as it is called, of the rays of light, like that for dip, is always *subtractive*. At the horizon it is greatest, for the rays become more bent the more obliquely they enter a refracting medium; when they strike upon it perpendicularly they are not bent at all. These facts are proved by many optical experiments.[†]

The refraction takes place entirely in the vertical plane; for contiguous to this plane, to the right and left, the medium being the same, there is nothing to divert it from its path either on one side or the other. Refraction, therefore, like dip, affects *altitudes* only. Tables for the corresponding correction of the altitude, from the horizon to the zenith, are given in all nautical collections: these, however, are computed for the *mean* state of the atmosphere; and it must occur to the reader that, as this state is continually varying in certain latitudes, it becomes necessary to modify the numbers in the table, when the true altitude of a celestial object is required with the utmost accuracy, by taking note of the actual state of the atmosphere, as indicated by the thermometer and barometer. To the table of mean refractions a table of these corrections is generally annexed. When the *latitude* only of the ship is required, the correction of the mean refraction is of comparatively little consequence; but, in determining the longitude by lunar observations, it is deserving of attention.

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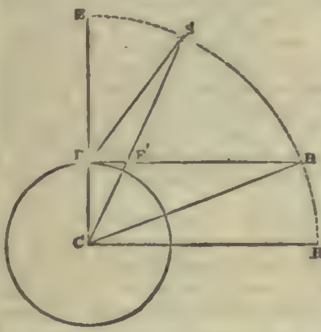
[*] This correction, amounting only to a few seconds, is frequently omitted in nautical tables, and, therefore, often neglected at sea.

[†] The student may refer to the article Refraction, in the section on Light, for fuller details on this subject.—Ed.

When the corrections, now explained, are applied to the observed altitude of a celestial object, the result is the true altitude of it above the sensible horizon of the observer; and it now only remains to reduce this to the altitude above the *rational* horizon of the place, as if the object were observed from the centre of the earth, instead of from the point on its surface immediately above the centre. In the case of a star, the altitude would be the same, whether the observation be made on the surface or at the centre—the change of position being insensible in reference to objects so remote as the stars; but, for the sun and moon, especially for the latter, the angle subtended at the body by the radius of the earth, called its *parallax in altitude*, is of sensible amount.

PARALLAX.—Let S (Fig. 25) represent the place of the celestial body observed from the surface of the earth at E; the observed angle S E H, when corrected for dip, semi-diameter, and refraction, will be the true altitude of the centre of S above the sensible horizon E H; and the angle S C R, will be the true altitude of the centre above the rational horizon C R. It is the difference of these angles that is called the parallax in altitude. If the body be in the sensible horizon, as at H, then the difference spoken of will be the whole angle H C R: this is called the horizontal parallax. For any other position of the body the parallax is less—diminishing as the object approaches the zenith, and vanishing at that elevation. Since parallax in altitude = S E′ H — S E H = E S C, the parallax is always the angle subtended by the semi-diameter of the earth at the object; and since the true altitude above the rational horizon is

Fig. 25.



$$S C R = S E' H = S E H + E S C,$$

the correction for parallax in altitude must be applied *additively* to the true altitude above the sensible horizon, to obtain the true altitude above the rational horizon. The sun's horizontal parallax is always about 9″; the moon's horizontal parallax varies considerably, and is given, together with her semi-diameter, for every noon and midnight, at page 3 of the *Nautical Almanac.* And from the horizontal parallax being known, the parallax in altitude is easily found thus. Referring to the triangle S E C, we have the proportion

$$S C : E C :: \text{sin. } S E C : E S C = \frac{E C}{S C} \text{ sin. } S E C;$$

but sin. S E C = sin. S E Z = cos. S E H; and as E C, S C are constant, it follows that the sine of the parallax in altitude varies as the cosine of the altitude; that is,

1 : cos. alt. : : sin. hor. par. : sin. par. in alt.

The parallax being always a very small angle, it is usual to substitute the seconds in the arc for the sine, so that we have, as above,

par. in alt. in seconds = hor. par. in seconds × cos. alt. And in this way the table for parallax in altitude is constructed.

We have now explained the necessary corrections for reducing the observed altitude of a celestial object to its true altitude, as seen from the centre of the earth. When the object is the sun or moon, these corrections are four in number—namely, for dip, semi-diameter, refraction, and parallax in altitude; when it is a star, there are only two corrections—namely, those for dip and refraction. The *Nautical Almanac* furnishes the necessary particulars for the other two corrections when either the sun or moon is observed: the semi-diameter of the moon, as seen from the centre of the earth, is given for intervals of twelve hours throughout the year; its value for any intermediate time is to be found in proportion, and it is the same for the horizontal parallax. In the "Explanation" which accompanies the *Nautical Almanac*, every needful information is given as to how values which vary continuously, may be determined for any proposed time, from the recorded values at stated intervals: thus—

To find the moon's semi-diameter and horizontal parallax at 6h. A.M. (that is, *before* noon) on February 23rd, 1846, at a place 15°, or 1h., to the east of Greenwich.

The civil time at the place, expressed in mean astronomical time, is February 22nd, 18h., from which, subtracting 1h., because the place is to the east of Greenwich, we have, February 22nd, 17h. for the corresponding time at Greenwich, or 5h. after midnight. Proceeding from the semi-diameter given for midnight of the 22nd, we must compute the proportional part of the variation in 12 hours, due to the time elapsed, viz., 5h.; thus, the semi-diameter for midnight, or 12h. of the 22nd, is 16′ 31″·6, and for the 23rd, at noon, or 24h., it is 16′ 34″·7; the difference, 3″·1, is the variation in 12 hours. Therefore,

12h. : 5h. : : 3″·1 : 1″·3,

which *added* (because the quantities are increasing) to 16′ 31″·6, gives 16′ 32″·9 for the moon's semi-diameter at the time proposed. Similarly, the horizontal parallax at midnight of the 22nd, is 60′ 39″, and at noon of the 23rd, it is 60′ 50″·4; the difference 11″·4 is the variation in the 12 hours, which include the given time: therefore,

12h. : 5h. : : 11″·4 : 4″·75 or 4″·8,

which *added* (because the quantities are increasing) to 60′ 39″, gives 60′ 43″·8 for the horizontal parallax required. And if with this horizontal parallax, and the apparent altitude of the moon, we enter the table entitled "Moon's Parallax in Altitude," we shall obtain the parallax in altitude. But in most nautical tables, the two corrections for refraction in altitude and parallax in altitude are combined, and the results tabulated under the head of "Correction of the Moon's Apparent Altitude," and this is the preferable arrangement when the true altitude is to be deduced.

Besides the foregoing corrections for obtaining the true altitude of a celestial object from the observed altitude, the observed altitude itself generally requires a little correction for the known error of the instrument (quadrant or sextant) employed in taking the altitude. "Human hands, or machines, never formed a circle, drew a straight line, or erected a perpendicular;"* there are, in consequence, unavoidable departures from strict mathematical accuracy in all mechanical constructions. The shortcomings may be discovered and allowed for, though not remedied—just as the gain or loss of a chronometer may be discovered, though to construct one, without gain or loss, be a practical impossibility. The *index error*—as it is usually called—of the instrument, is to be allowed for before any of the astronomical corrections are introduced; it is not constant, but varies with temperature. The following examples will sufficiently show how the several corrections are supplied.

EXAMPLE OF CORRECTING ALTITUDES AT SEA.—1. Suppose the observed altitude of a star to be 47° 10′, the height of the eye 18 feet, and the index error of the instrument to be 3′ 12″ *subtractive*. Required the true altitude.

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* Sir John Herschel's *Treatise on Astronomy.*

Observed altitude . . .	47° 10′ 0″
Index error	−3′ 12″
	47° 6′ 48″
Dip of the horizon . .	−4′ 11″
Apparent altitude . .	47° 2′ 37″
Refraction	− 53″
True altitude . . .	47° 1′ 44″

The refraction here taken from the table is that for the mean state of the atmosphere. If the height of the barometer and thermometer be observed, the mean refraction may be corrected accordingly, by aid of a table usually placed beside that for the mean state of the atmosphere.

2. The observed altitude of the sun's lower limb on a certain day was 16° 33′, the height of the eye was 17 feet, the index error was 3′ *additive*; the barometer stood at 29 inches, and the thermometer at 58°. Required the true altitude of the sun's centre, his semi-diameter, as given in the *Nautical Almanac*, for the day, being 16′ 17″.

Observed alt. sun's L. L. . .		16° 33′ 0″
Index error		+3′ 0″
		16° 36′ 0″
Dip		−4′ 4″
Apparent alt L.L. . . .		16° 31′ 56″
Refraction .	−3′ 10″	
Correction for barom. −	7″	
„ „ therm. −	3″	
	=	−3′ 20″
True altitude of L. L. above sensible horizon . . .		16° 28′ 36″
Semi-diameter (*Naut. Alm*).		+ 16′ 17″
Parallax in altitude . . .		+ 8″
True alt. of sun's centre . .		16° 45′ 1″

The corrections for the barometer and thermometer being, as in this example, always very small, they are not attended to at sea when the latitude is the only thing to be determined. The error in the latitude arising from omitting these small quantities is too trifling to be of any consequence.

3. The observed altitude of the moon's lower limb (index error allowed for) is 31° 18; the horizontal parallax, from the *Nautical Almanac*, 58′ 37″; semi-diameter, 15′ 58″; and the height of the eye 16 feet. Required the true altitude of the moon's centre.

Observed alt. moon's L. L. . . .			31° 18′ 0″
Dip		. − 3′ 50″	
Semi-diam.	15′ 58″	. +16′ 6″	
Augmentation	8″		
		=	+ 12′ 16″
App. alt. moon's centre			31° 30′ 16″
Cor. for par. and ref. (hor. par. 58′ 37″, alt. 31½°)			+ 48′ 26″
True alt. moon's centre . . .			32° 18′ 42″

4. The observed altitude of the moon's upper limb, corrected for index error, was 41° 25′; the horizontal parallax, 55′ 40″; semi-diameter, 15′ 10″; and the height of the eye, 15 feet. Required the true altitude of the moon's centre.

Observed alt. moon's U. L. . . .			41° 25′ 0″
Dip		. − 3′ 42″	
Semi-diam.	15′ 10″	. −15′ 20″	
Augmentation	10″		
		=	−19′ 2″
App. alt. moon's centre			41° 5′ 58″
Correction for parallax and refraction .			+ 40 51″
True alt. moon's centre . . .			41° 46′ 49″

To Determine the Latitude at Sea from the Meridian Altitude of a Celestial Object whose Declination is Known.—The determination of the latitude of the ship by means of the altitude, when on the meridian, of a celestial object of known declination, is the easiest, and in general the safest, method for the purpose. The observations and the subsequent calculations being but few, they may be readily accomplished, and with but little liability to error in the result. This method, therefore, is always used at sea, whenever foggy or cloudy weather does not render it impracticable.

The celestial object observed must be one of which the declination is given in the *Nautical Almanac*, for the meridian of Greenwich. This declination may be reduced to the meridian of the ship—or rather to the time at Greenwich corresponding to that at the ship—by turning the longitude by account into time, and then applying the variation for declination due to that time, as explained at page 1077; the hourly variation of the sun's declination is given in the *Nautical Almanac*, at page 1, of each month. The longitude by account is always sufficiently near the truth for the determination of this element, though greater precision is required for the moon than for the sun, as the declination of the former changes more rapidly. The declination of the object observed being thus known at the time of observation, and, from its altitude, the zenith distance of it being also known, we have the distance of the object both from the equinoctial and from the zenith of the ship: consequently, the distance of the zenith from the equinoctial—that is, the distance of the ship from the equator—becomes known either by simple addition or subtraction; and thus the latitude is found.

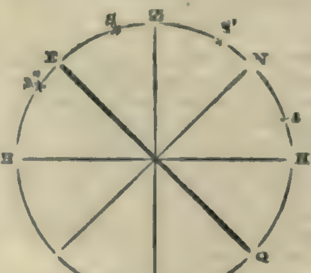
The object observed may be either above the pole or below it; that is, it may be on either the mid-day or the mid-night portion of the celestial meridian of the place; and, in the former case, it may be in either of the three following positions in reference to the equinoctial and the zenith, namely—

1. The zenith, and the object observed, may both be on the same side of the equinoctial, and the object nearer to the equinoctial than the zenith is, as at S, in the diagram below.

2. The zenith and the object being on the same side of the equinoctial, the zenith may be nearer to the equinoctial than the object is, as at S′.

3. The zenith and the object may be on different sides of the equinoctial, as at S″.

Fig. 26.



Let N Z H be the meridian, Z the zenith of the ship, N the elevated pole—the north pole suppose—and E Q the equinoctial.

First.—Let the object observed be at S, between the zenith and the equinoctial, and both *north* of the equinoctial: then for the latitude E Z, we have E Z = E S + S Z: that is,

(1). The latitude is equal to the declination, *plus* the co-altitude.

Second.—Let the object be at S′, on the contrary side of the zenith, then the latitude E Z is E Z = E S′ − S′ Z; that is, (2). The latitude is equal to the declination, *minus* the co-altitude.

Third.—Let the object be at S″, on the contrary side of the equinoctial; that is, let its declination be *south*, then the latitude is E Z = S″ Z − E S″: that is,

(3). The latitude is equal to the co-altitude, *minus* the declination.

Hence, if we call the co-altitude, or zenith distance of the object, *north*, when the zenith is north of it, and *south*, when the zenith is south of it, we shall have the following rule for all the three cases—namely,

RULE.—When the object observed is above the pole, if the zenith distance and the declination have the *same* name, that is, be both north or both south, their *sum* will be the latitude. If the zenith distance and the declination have *different* names, their *difference* will be the latitude, of the same name as the greater.

It is assumed above that the north is the elevated pole; but if it be the south, then, by merely writing south for north and north for south, the reasoning will remain the same: the rule will require no modification.

It is obvious that the elevation of the pole above the horizon of any place is always equal to the latitude of that place, for Z N is equally the complement of the latitude E Z, and of H N the elevation of the pole. The elevation of the equator, or the angle it makes with the horizon, is also clearly equal to the co-latitude.

It should be remarked, that when celestial objects are near the horizon, they are in a position less favourable for observation than when they are more elevated, because the refraction near the horizon is very variable in its effects.

LATITUDE FROM THE SUN WHEN ABOVE THE POLE.—The sun is the celestial object most commonly appealed to by mariners for the determination of the latitude of the ship: it is more frequently visible in the daytime, when, except in bad weather, the sea horizon is more clearly defined, and the corrections to be applied to the observed, in order to get the true altitude, are few and simple. The corrections for a star are still fewer; but, as the horizon is usually getting obscure when the stars begin to appear, a star is, in general, less favourable for the purpose than the sun: the moon, however, is often on the meridian under favourable circumstances, but the corrections necessary are more in number, and require to be made with greater care. We shall give suitable directions for each of these objects separately, detailing the proper corrections to be made preparatively to inferring the latitude, as briefly indicated by the general rule given above.

RULE 1.—From the longitude by account, find the *apparent* time at Greenwich; this is called the Greenwich *date.*

2. From page 1 of the *Nautical Almanac* get the noon declination at Greenwich; and, by means of the hourly difference in declination there given, and the previously found Greenwich date, find the correction due to that date; the declination at the time of observation will thus be discovered.

3. Apply to the observed altitude the proper corrections for dip and semi-diameter; the apparent altitude of the centre will then be obtained; and the corrections for refraction and parallax will reduce the apparent to the true altitude.

4. Mark the zenith distance N. or S. according as it is N. or S. of the sun: then, if the declination and zenith distance have the same marks, their sum will be the latitude; if they have different marks, their *difference* will be the latitude.

NOTE.—The first step in the foregoing rule requires the conversion of degrees, minutes, &c., of longitude into time, which is readily done from the known relations—

15° = 1h., 15′ = 1m., 15″ = 1s.;

for from these it is evident that if we multiply the degrees of longitude by 2, and then divide by 30, the quotient will be hours, and so many *thirtieths* of an hour; so that twice these thirtieths will be the additional minutes of time.

In like manner, if we multiply the minutes of longitude by 2, and divide by 30, the quotient will be the minutes of time, and so many thirtieths; that is, twice that number of seconds of time.

Suppose, for instance, the longitude is 93° 37′ 41″, we may easily convert it into time, as in the margin, thus: multiplying the degrees, minutes, and seconds by 2, we have 186° 74′ 82″.

Dividing each, separately, by 3, cutting off the unit figure of each for the 0 suppressed in the 30, we have, for the first quotient, 6 hours and 6 thirtieths—that is, 6h. 12m.; for the second quotient, 2 minutes and 14 thirtieths—that is, 2m. 28s.; and for the third quotient, 2·73s.—decimals of a second being always used instead of *thirds:* hence the time corresponding to 93° 37′ 41″ of longitude is 6h. 14m. 30·73s.

3)18,6° 7,4′ 8,2″

6 12

2 28

2·73

6h. 14′ 30·73″

Examples.

1. On April 27, 1853, in north latitude, and in longitude 87° 42′ W., the observed meridian altitude of the sun's lower limb was 48° 42′ 30″ (zenith N.), the index correction was + 1′ 42″, and the height of the eye 18 feet. Required the latitude.

1. *For the apparent time at Greenwich.*

Long. by account . . . 87° 42′ W.

2

3)17,4 8,4

5 48

2 48

5h. 50′ 48″.

The variation for this time to be added as the declination is increasing.

2. *For the sun's declination.*

Dec. at app. noon (*Naut. Alm.*)

13° 43′ 53″ N. var. in 1h. 47″·7 increasing

+ 4′ 38″ 6

13° 48′ 31″ N. in 6h. . 286·2

in 10m. . 7·9

in 5h. 50m. 278·3 second

or 4′ 38″

Observed altitude of sun's L. L. .	48° 42′ 30″
Index cor. + 1′ 42″ Dip . . . − 4′ 11″ Semi-diam. + 15′ 54″	+ 13′ 25″
App. alt. of sun's centre . . .	48° 55′ 55″
Refraction and parallax . . .	− 44″
True alt. of centre	48° 55′ 11″
	90°
True zenith distance . . .	41° 4′ 49″ N.
Sun's reduced declination . . .	13° 48′ 31″ N.
Latitude	54° 53′ 20″ N.

As noticed in the Introduction, page 1048, it is always advisable to make all the use we can of Tables when they are once in hand. The first Table referred to in the foregoing operation, is that given at page 1 of the *Nautical Almanac* for the sun's declination at apparent noon at Greenwich, with the hourly variation; the semi-diameter should be taken out at the same time, and inserted in its proper place; but seamen, in general, use invariably 16′ for the sun's semi-diameter. A blank form of the several particulars in such operations is of considerable assistance, as the work is thereby facilitated, and the risk of mistake diminished. The "correction in altitude"—that is, the allowance for refraction and parallax—should be taken from the Nautical Tables at the same time as that for dip in working examples;

though at sea the dip generally differs but little throughout the voyage.*

It may be remarked, that in computing the declination of the sun for the time of observation, seconds of time may be disregarded, and even a few minutes is of no practical consequence: indeed, the longitude by account is only an approximation to the correct longitude.

But, as a little consideration will be sufficient to show, the error in the estimated longitude must be very great indeed to occasion any error of practical importance in the declination of the sun at the instant of observation. If only the altitude be observed with proper care, the computer may trust to the accuracy of the resulting latitude without any nice determination of his longitude: 1° of longitude is only 4′ of time.

2. On August 14th, 1846, in north latitude, and in longitude 51° W., the meridian altitude of the sun's upper limb was 47° 26′ (zenith N.); the index correction — 1′ 47″, and the height of the eye 20 feet. Required the latitude.

1. *For the apparent time at Greenwich.*

Long. by account	51° W.
	2
	3)10,2
App. time at Greenwich .	3h. 24m.

The variation for this time must be subtracted, as the declination is decreasing.

2. *For the sun's declination.*

Dec. at app. noon (*Naut. Alm.*)

14° 25′ 28″ N.	var. in 1h. . .	46″·51 dec.
— 2′ 38″	. .	3
14° 22′ 50″ N.	in 3h. . .	139·53
	in 12m. . .	9·30
	in 12m. . .	9·30
	2′ 38″ =	158·13 sec.

Observed altitude of sun's U. L. .		47° 26′ 0″
Index cor.	— 1′ 47″	
Dip . .	— 4′ 24″	— 22′ 0″
Semi-diam.	— 15′ 49″	
App. Alt. of centre		47° 4′ 0″
Refraction and parallax . .		— 47″
True alt. of centre . . .		47° 3′ 13″
		90°
True zenith distance . . .		42° 56′ 47″ N.
Sun's reduced declination . .		14° 22′ 50″ N.
Latitude . . .		57° 19′ 37″ N.

3. On November 8th, 1846, in longitude 62° E., the meridian altitude of the sun's lower limb was 57° 12′ 30″ (zenith S.); the index correction was + 1′ 36″, and the height of the eye 30 feet. Required the latitude.

1. *For the apparent time at Greenwich.*

Long. by account . . .	62° E.
	2
	3)12,4

App. time at Greenwich 4h. 8m. before noon.

The variation for this time must be subtracted, as the declination is greater at noon than before noon.

2. *For the sun's declination.*

Dec. at app. noon (*Naut. Alm.*)

16° 33′ 34″ S.	var. in 1h. . .	43″·36 inc.
— 3′ 0″	. .	4
16° 30′ 34″ S.	in 4h. . .	173·44
	in 8m. . .	6·19
	3′ =	179·63 sec.

Observed altitude of sun's L. L. .		57° 12′ 30″
Index cor.	+ 1′ 36″	
Dip . .	— 5′ 24″	+ 12′ 22″
Semi-diam.	+ 16′ 10″	
App. alt. of centre		57° 24′ 52″
Refraction and parallax . . .		— 32″
True alt. of centre		57° 24′ 20″
		90°
True zenith distance . . .		32° 35′ 40″ S.
Sun's reduced declination . .		16° 30′ 34″ S.
Latitude . . .		49° 6′ 14″ S.

It may be remarked here, that in reducing the declination to the time at Greenwich when the observation is made, the hourly variation should, in strictness, be taken equal to 44·06, which is the average variation during the *preceding* 24 hours, or from the noon of Nov. 7th to the noon of Nov. 8th, because the 4h. 8m., for which the correction is made, is a portion of *this* 24 hours; but the difference in the correction would amount only to about 2½ seconds. Such small quantities are not worth attending to in the present problem, as the altitude of a celestial object, taken at sea, cannot be measured to within a few seconds of the truth; and if the latitude can be deduced to the nearest minute, it is all that can be reasonably expected.

To save the trouble of the preliminary reductions here adverted to, a table is given in most nautical collections, by entering which, with the sun's noon-declination and the longitude by account, the correction for declination is found with accuracy sufficient for the purposes of navigation. The table referred to is, in Norie's *Tables*, the twenty-first; we shall use it in the following example.

4. On November 21st, 1841, in longitude 165° E., the meridian altitude of the sun's lower limb was observed to be 47° 38′ (zenith N.); the index correction was — 1′ 15″, and the height of the eye 17 feet. Required the latitude.

For the declination at the Greenwich time of observation.

Sun's declination at noon, Nov. 21 (*Naut. Alm.*)		19° 57′ 55″ S.
Correction for longitude 165° E. .		— 5′ 52″
Sun's reduced declination .		19° 52′ 3″ S.
Observed altitude of sun's L. L. .		47° 38′ 0″
Index cor.	— 1′ 15″	
Dip . .	— 4′ 4″	+ 10′ 54″
Semi-diam.	+ 16′ 13″	
App. alt. of centre		47° 48′ 54″
Refraction and parallax . . .		— 46″
True alt. of centre		47° 48′ 8″
		90°
True zenith distance . . .		42° 11′ 52″ N.
Sun's reduced declination . .		19° 52′ 3″ S.
Latitude		22° 19′ 49″ N.

LATITUDE FROM A STAR ABOVE THE POLE.—A fixed star changes its declination so slowly, that its variation, even in a month, is scarcely sensible; no correction, therefore, for longitude, will be necessary; the declination, as given in the *Nautical Almanac*, on the day of observation, may be taken as that at the time of observation. As a fixed star has no parallax in altitude, the only corrections will be those for dip and refraction; the rule for deducing the latitude is therefore as follows:—

---

* In some cases, however, when the voyage is long, and the complement of men considerable, the dip may sensibly increase. Captain Parry, after wintering in "Winter Harbour," in 1820, estimated the consumption of stores and provisions to have amounted, in the *Hecla*, to about seventy tons: the dip must, therefore, have increased.

Rule.—1. Apply to the observed altitude the corrections for dip and refraction; the result will be the true altitude; and this, subtracted from 90°, will give the true zenith distance.

2. Mark the zenith distance N. or S. according as it is N. or S. of the star; then, if the declination and zenith distance have the same marks, their sum will be the latitude; if they have different marks their difference will be the latitude.

Examples.

1. January 22, 1846, the meridian altitude of Arcturus was observed at sea to be 43° 27′ (zenith north); the index correction was + 2′ 10″, and the height of the eye 20 feet. Required the latitude.

Observed altitude of star . .	43° 27′ 0″
Index cor. + 2′ 10″ } Dip . . — 4′ 24″ } . . .	— 2′ 14″
App. alt. of star	43° 24′ 46″
Refraction	— 1′ 0″
True alt. of star	43° 23′ 46″ 90°
True zenith distance . . .	46° 36′ 14″N.
Star's declination, Jan. 22, 1846 .	19° 58′ 59″N.
Latitude	66° 35′ 13″N.

2. On February 12, 1853, the meridian altitude of α Hydræ was observed to be 47° 24′ 30″ (zenith north); the index correction was − 2′ 10″, and the height of the eye 17 feet. Required the latitude.

Observed altitude of star . .	47° 24′ 30″
Index cor. − 2′ 10″ } Dip . . − 4′ 4″ } . . .	− 6′ 14″
App. alt. of star	47° 18′ 16″
Refraction	− 52″
True altitude	47° 17′ 24″ 90°
True zenith distance . . .	42° 42′ 36″N.
Star's declination, Feb. 12, 1853 .	8° 1′ 29″ S.
Latitude	34° 41′ 7″N.

3. On July 16, 1845, the meridian altitude of Fomalhaut was found to be 73° 36½′ (zenith north); the index correction — 30″, and the height of the eye 24 feet. Required the latitude.

Observed altitude of the star . .	73° 36′ 30″
Index cor. − 30″ } Dip . − 4′ 49″ } . . .	− 5′ 19″
App. alt. of star	73° 31′ 11″
Refraction	− 16″
True altitude	73° 30′ 55″ 90°
True zenith distance . . .	16° 29′ 5″N.
Star's declination, July 16, 1845 .	30° 26′ 20″S.
Latitude	13° 57′ 15″S.

Note.—In some nautical tables the corrections for dip and refraction are united under the head of *Correction of star's observed altitude.*

Latitude from the Moon above the Pole.—The moon's declination is given in the *Nautical Almanac* for every hour of the day, and the time of her meridian passage from day to day. These elements are computed for *mean time*, the reckoning being from mean noon at Greenwich. Hence, to find the moon's declination corresponding to the time of taking her meridian altitude at sea, we must know the time at Greenwich at the instant of observation. This is ascertained as follows:—

As the motion of the moon in her orbit is eastward, her transit over the meridian of any place is delayed from day to day. In consequence of this retardation, she will pass the meridian of a place to the west of Greenwich later in the day at that place than she passes the meridian of Greenwich, and her transit over a meridian to the east of Greenwich will take place earlier in the day. How much later, or how much earlier, will be ascertained by converting the longitude of the meridian into time, and applying the corresponding proportional part of the daily variation as furnished by the *Nautical Almanac.* By help of the following short table, and the daily change in the time of transit, the correction to be added to the Greenwich time of transit, to obtain the time of transit over a meridian west of Greenwich, or to be subtracted to obtain the time of transit over a meridian east of Greenwich, may be at once found.

The daily variation, as given by the *Nautical Almanac*, is to be sought for in the top row of figures, and the longitude of the place in the marginal column on the left; the proper correction of the Greenwich mean time, to reduce it to the mean time of transit at the place, is then to be taken out from the body of the table. It is sufficient that the daily variation be taken to the nearest minute.

It must be observed, however, that in the case of the moon, there must not be the same indifference as to the accuracy of the longitude by account as is allowable in the case of the sun; the following examples will show, that on account of the moon's more rapid change in declination, a comparatively short interval of time makes a sensible difference in this element.

Table for finding the Time of the Moon's Transit over a given Meridian when the Time of the Transit at Greenwich is known, and the Daily Variation of the Time.

Longitude.	Daily Variation in Minutes of Time of Moon's Transit.													
	40m.	42m.	44m	46m.	48m.	50m.	52m.	54m.	56m	58m.	60m.	62m	64m.	66m.
10	1	1	1	1	1	1	1	1	1	2	2	2	2	2
20	2	2	2	2	3	3	3	3	3	3	3	3	3	4
30	3	3	4	4	4	4	4	4	4	5	5	5	5	5
40	4	4	5	5	5	5	6	6	6	6	6	7	7	7
50	5	6	6	6	6	7	7	7	7	8	8	8	9	9
60	6	7	7	7	8	8	8	9	9	9	10	10	10	11
70	7	8	8	9	9	9	10	10	10	11	11	12	12	12
80	9	9	9	10	10	11	11	12	12	12	13	13	14	14
90	10	10	11	11	12	12	13	13	13	14	14	15	15	16
100	11	11	12	12	13	13	14	14	15	15	16	17	17	18
110	12	12	13	14	14	15	15	16	16	17	18	18	19	19
120	13	14	14	15	15	16	17	17	18	19	19	20	20	21
130	14	15	15	16	17	17	18	19	19	20	21	21	22	23
140	15	16	17	17	18	19	20	20	21	22	22	23	24	25
150	16	17	18	19	19	20	21	22	22	23	24	25	26	26
160	17	18	19	20	21	21	22	23	24	25	26	26	27	28
170	18	19	20	21	22	23	24	25	25	26	27	28	29	30
180	19	20	21	22	23	24	25	26	27	28	29	30	31	32

By the aid of this table, the latitude from a meridian altitude of the moon, when above the pole, may be found as follows:—

Rule.—1. From the *Nautical Almanac* take out the time of the moon's passage over the meridian of Greenwich on the given day, as also the daily variation.

2. From the longitude by account, and the foregoing table, reduce this to the time, at the place, of the moon's passage over the meridian of the ship; the time of observation, at the place where that observation is made, will thus be found.

3. From the ship's time and longitude, find the corresponding time at Greenwich.

4. Find now the moon's declination at that time from the *Nautical Almanac*, computing the variation for the odd minutes by means of the difference in declination for 10m.

5. From page 3 of the month, take out the moon's

semi-diameter, and increase it by the "Augmentation" given in the Nautical Tables. The correction for index error, dip, and semi-diameter, will reduce the observed altitude of the limb to the apparent altitude of the centre.

6. To the apparent altitude of the centre add the correction in altitude, that is, the parallax in altitude *minus* the refraction, and the true altitude of the centre will be obtained. Subtract this from 90°, and then, by adding or subtracting the moon's declination at the time, as in the case of the sun, the latitude of the place will be ascertained.

Examples.

1. On May 27th, 1846, in longitude 49° W., the meridian altitude of the moon's lower limb was found to be 47° 18′ 30″ (zenith S.); the index correction was + 1′ 40″, and the height of the eye 20 feet. Required the latitude of the ship.

1. *For the mean time at Greenwich when the observation was made.*

Moon's transit at Greenwich, May 27th	1h. 56m.	Daily variation, 48·7m.
Cor. for long. 49° W. and 48m. variation	+ 6m.	
Time at ship when alt. was taken	2h. 2m.	
Long. 49 W. in time	3h. 16m.	
Time at Greenwich when alt. was taken	5h. 18m.	

2. *For the moon's declination at that time.*

Declination May 27th, at 5h.	18° 56′ 16″ N.	Var. in 10m., 26″·55 (dec.)
Decrease in 18m. = 2″·655 × 18 =	− 48″	
Declination at 5h. 18′.	18° 55′ 28″ N.	

3. *For the true altitude of the Moon's centre, and thence the latitude.*—At page 3, of MAY, in the *Nautical Almanac*, the moon's semi-diameter at noon on the 27th is 15′ 1″; and her horizontal parallax at the same time is 55′ 16″. Hence we get the latitude thus:—

Observed alt. of moon's L. L.		47° 18′ 30″
Index cor.	+ 1′ 40″	+ 12′ 32″
Dip	− 4′ 24″	
Semi-diameter + 11″ for augmen.	+ 15′ 16″	
App. alt. of moon's centre		47° 31′ 2″
Parallax and refraction		+ 36′ 10″
True alt. of moon's centre		48° 7′ 12″
		90°
True zenith distance		41° 52′ 48″ S.
Moon's declination		18° 55′ 28″ N.
Latitude		22° 57′ 20″ S.

It is plain that whatever be the celestial object observed, the error in the latitude will be the sum of the errors in the zenith distance and declination when both are of the same name, and the difference when they are of contrary names.

The latitude at sea is seldom computed to seconds, as the exact longitude and time of the ship, as inferred from the dead-reckoning, generally deviates from the truth. It is customary, therefore, to aim at deducing the latitude only to the nearest minute: thus, in the present example, the latitude would be concluded to be 22° 57′ S.

2. On Dec. 7th, 1840, in longitude 16° W., at 10h. 43m. apparent time, the meridian altitude of the moon's lower limb was 83° 7′ (zenith S.); the index correction was − 1′ 55″, and the height of the eye 16 feet. Required the latitude to the nearest minute. Here the time at the ship being given, we shall not require to take out of the *Nautical Almanac* the time of the moon's meridian passage at Greenwich: the Greenwich date, or mean time at Greenwich when the altitude was taken, is found thus:—

1. *For the mean time at Greenwich when the observation was made.*

Apparent time at ship, Dec. 7	10h. 43m.
Long. 16° W. in time	1h. 4m.
App. time at Greenwich	11h. 47m.
Equation of time (*Naut. Alm.*, p. 1)	−8m.
Mean time at Greenwich	11h. 39m.

2. *For the Moon's declination at that time.*

Declination, Dec. 7, at 11h. (inc.) Var, in 10′, 8″.	24° 45′ N.
Increase in 39m.	+ 6′
Declination at 11h. 39m.	24° 51′ N.

3. *For the true altitude of the Moon's centre, and thence the latitude.*

Observed alt. of moon's L. L.		83° 7′
Index cor.	− 2′	+ 11′
Dip	− 4′	
Semi-diam.	+ 17′	
App. alt. moon's centre		83° 18′
Parallax and refraction		+ 7′
True alt. of moon's centre		83° 25′
		90°
True zenith distance		6° 35′ S.
Moon's declination		24° 51′ N.
Latitude		18° 15′ N.

Examples for Exercise.

NOTE.—In the following examples the latitude is to be determined to the nearest minute.

1. On the 2nd of May, 1833, the meridian altitude of the sun's lower limb was 47° 20′ (zenith N.); the index correction − 2′; the height of the eye 20 feet, and the longitude by account 32° E.

Also sun's dec. May 2nd, 15° 23′ 21″ N. (inc.) Hourly difference 45″.

Semi-diameter 15′ 53″ (*Nautical Almanac*).

Required the latitude to the nearest minute.

Ans. Latitude, 58° 53′ N.

2. On Jan. 9th, 1840, in longitude 116° W., the meridian altitude of the sun's upper limb was found to be 69° 14′ (zenith N.); index error 0, and height of the eye 27 feet. Required the latitude to the nearest minute.

Sun's dec. Jan. 9, 22° 12′ S. (dec.) Correction for long. 116 W., − 3′. Semi-diam. 16′.

Ans. Latitude, 1° 2′ S.

3. On May 15th, 1828, the meridian altitude of the star, Spica, was observed to be 30° 17′ (zenith N.); index correction + 1′ 10″; the star's declination was 10° 16′ S. Required the latitude to the nearest minute, the dip being − 5.

Ans. Latitude, 46° 32′ N.

4. The meridian altitude of the star Rigel was observed to be 85° 4′ (zenith N.); the index correction was + 2′, and the star's declination 8° 22′ 45″ S.; the height of the eye was 20 feet. Required the latitude to the latest minute.

Ans. Latitude, 3° 24′ S.

5. On February 19th, 1823, in longitude 40° W., the meridian altitude of the moon's lower limb was observed to be 55° 8′ (zenith N.); the index correction was − 2′, and the height of the eye 16 feet.

The moon's passage over the merid. of Greenwich, Feb. 19th . .	was 6h. 56m.
The moon's passage over the merid. of Greenwich, Feb. 20th . .	was 7h. 59m.
Declination, Feb. 19th, at noon .	26° 38′ 17″ N.
,, ,, midnight	26° 54′ 39″ N.
Semi-diameter	16′ 12″
Horizontal parallax . . .	59′ 32″

Required the latitude to the nearest minute.

Ans. Latitude, 61° 1′ N.

6. On November 12th, 1853, at 2h. 20m. mean time, in longitude 60° 42′ W., the meridian altitude of the moon's lower limb was observed to be 30° 30′ 40″ (zenith N.); the index correction was + 10′ 42″; and the height of the eye 16 feet.

Moon's declination, Nov. 12, at 6h.	2° 44′ 20″ N.
,, ,, ,, 7h.	2° 57′ 38″ N.
Semi-diameter	15′ 12″
Horizontal parallax	55′ 13″

Required the latitude to the nearest minute.

Ans. Latitude, 61° 11′ N.

To determine the latitude from the meridian altitude of a celestial object when below the pole.—In the diagram (Fig. 26), at page 1078, let *s* be the position of a celestial object when on the meridian of the place whose zenith is Z, and below the elevated pole N; the altitude of it will be H *s*, and *s* N will be the complement of its declination Q *s*. Consequently, the altitude added to the co-declination will be the latitude H N of the place whose zenith is Z (page 1078). The sun is on the meridian of any place, below the pole, 12 hours after the apparent noon at that place; consequently, 12 hours, increased or diminished by the longitude in time, according as the place is W. or E. of Greenwich, will be the apparent time at Greenwich when the observation was made; and the declination corresponding to this time may be found as in the foregoing examples.

For a star, the declination will be the same as that given for the day, in the *Nautical Almanac*; since, as before remarked, the change in the declination of a fixed star is insensible till after the lapse of several days.

For the moon, the time of transit over the mid-day portion of the meridian of the place may be found as at page 1081; and this time increased by 12 hours, and by half the daily difference of time, will be the time of her returning to the meridian below the pole; and the proper reduction being made for longitude, as in the case of the sun, the time at Greenwich, and thence the corresponding declination, may be found. The rule for computing the latitude is therefore as follows:

RULE.—1. Find the declination of the object at the time of observation.

2. To the observed altitude apply the proper corrections for deducing the true altitude.

3. To the true altitude add the co-declination; the sum will be the latitude, of the same name as the declination.

LATITUDE FROM THE SUN WHEN BELOW THE POLE.—As the rule just given applies equally to the sun, moon, or star, special directions for each case will be unnecessary; we shall, therefore, give a practical illustration of the mode of working for each object separately, and then add an example or two for exercise. The following is an example when the object is the sun:—

Example.—On June 18th, 1853, in north latitude, and in longitude 96° W., the meridian altitude of the sun's lower limb, at apparent midnight, was observed to be 8° 36′; the index correction was − 2′; and the height of the eye 20 feet. Required the latitude.

For the declination at the Greenwich time of observation.

Apparent time at ship, June 18 . .	12h. 0m.
Longitude 96° W. in time	6h. 24m.
Apparent time at Greenwich . . .	18h. 24m.
Sun's declination at noon, June 18 .	23° 25′ 36″ N. (inc.)
Correction for 18h. 24m.	+ 40″
Declination at time of observation .	23° 26′ 25″ N.
	90°
Co-declination	66° 33′ 35″

Observed alt. of sun's L.L. . . .		8° 36′ 0″
Index cor. − 2′	. . . .	+ 9′ 22″
Dip . . − 4′ 24″		
Semi-diam. + 15′ 46″		
App. alt. of centre		8° 45′ 22″
Refraction and parallax		− 5′ 49″
True alt. of centre		8° 39′ 33″
Sun's co-declination		66° 33′ 35″
Latitude		75° 13′ 8″ N.

LATITUDE FROM A STAR WHEN BELOW THE POLE.—In northern latitudes, the star called the pole star is very convenient for the purpose of finding the latitude of the ship, and is, therefore, frequently observed by mariners for this object. The following is an example:—

Example.—On March 1st, 1823, the observed altitude of the pole star when on the meridian below the pole, was 30° 7′; the index correction was + 2′; and the height of the eye 18 feet. Required the latitude.

Observed altitude of pole star . .		30° 7′ 0″
Index cor. + 2′	. . . . .	− 2′ 11″
Dip . . − 4′ 11″		
		30° 4′ 49″
Refraction		− 1′ 38″
True altitude of star		30° 3′ 11″
Co-declination Mar. 1, 1823 . . .		1° 38′ 2″
Latitude		31° 41′ 13″ N.

LATITUDE FROM THE MOON WHEN BELOW THE POLE.—The preliminary corrections for the moon, as already seen, are more numerous than those for the sun or for a star; a specimen of them is given in Example 1, p. 1082. These corrections are of course the same, whether the moon be observed on the meridian above the pole or below it. In the following example the moon's declination is found as at the page just referred to, so that the reductions need not be repeated here.

Example.—On the 27th of May, 1846, in longitude 49° W., the meridian altitude of the moon's lower limb, when below the pole, was found to be 7° 12′; the index correction was − 1′ 40″; and the height of the eye 20 feet. Required the latitude.

The moon's declination at the time when the observation was made, was found, at page 1082, to be 18° 55′ 28″ N.: hence the operation for finding the true altitude of the moon's centre, and thence the latitude of the place, is as follows:—

Observed alt. of moon's L.L. . .		7° 12′ 0″
Index cor. . — 1′ 40″	. . .	+ 9′ 12″
Dip . . . . — 4′ 24″		
Semi-diam. augmented + 15′ 16″		
App. alt. of centre		7° 21′ 12″
Parallax and refraction		+ 47′ 31″
True alt. of moon's centre . . .		8° 8′ 43″
Co-declination		71° 4′ 32″
Latitude		79° 13′ 15″ N.

Examples for Exercise.

1. On June 28th, 1841, in longitude 126° W., the altitude of the sun's lower limb at midnight was 6° 28′; the index correction was + 2′ 15″; and the height of the eye 20 feet. Required the latitude to the nearest minute, the sun's declination at noon, Greenwich time, on the 28th, being 23° 17′ 59″ N. decreasing, and his semi-diameter 15′ 45″. Ans. Latitude, 73° 19′ N.

2. The observed altitude of the sun's lower limb, when on the meridian below the pole, was 7° 5′; its declination at the time of observation was 23° 8′ 17″ N.; its semi-diameter 15′ 45″, and the height of the eye 20 feet. Required the latitude, the index error of the instrument being 0. Ans. Latitude, 74° 1′ N.

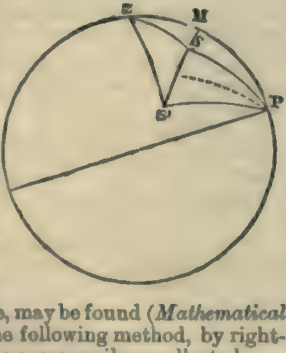
3. June 1st, 1823, the observed altitude of the star Capella, when on the meridian below the pole, was 10° 1′ 30″; its declination was 45° 48′ 27″ N.; and the height of the eye 17 feet. Required the latitude.

Ans. Latitude, 54° 4′ N.

The foregoing examples exhibit the different means of determining the latitude at sea, by taking the altitude of a celestial object when upon the meridian of the ship. Whenever practicable, the sun is always the object observed, and its mid-day altitude is that which is to be preferred, because at mid-day it attains its greatest elevation, and the refraction is less liable to variation from the mean state of the atmosphere. The altitude below the elevated pole can be taken only in high latitudes, where the sun is above the horizon during the whole twenty-four hours for a part of the summer, and, therefore, the horizon clearly visible: the obscurity of the horizon often precludes the possibility of accurately measuring the altitude of a star; and on account of its rapid change in declination, the moon is less suited for the purpose of deducing the latitude, when there is much uncertainty as to the longitude of the ship, or the time at Greenwich when the observation is made. In preparing for a meridian altitude, the observer holds himself in readiness, before the object attains its greatest elevation, and continues to observe it till it ceases to rise and appears for a moment stationary; at this instant its altitude is noted, and it is regarded as upon the meridian. Strictly speaking, however—at least as respects the sun and moon—the centre of the object is not necessarily exactly on the meridian when its altitude is the greatest or least; for the change in declination may more than counterbalance its change in altitude during the few minutes which precede its meridian transit, especially in the case of the moon; but these differences are too trifling to lead to any error of practical importance. It may, however, happen that the object becomes hidden by a cloud at the time of transit, so that the meridian observation cannot be made; it is of importance, therefore, to take note of the altitudes before the transit, as an altitude *near* the meridian may be made available for the determination of the latitude, as we are now about to show: we shall first, however, explain how to find the latitude generally when the object observed is off the meridian, provided the hour angle, which its declination-circle makes with the meridian of the ship, is known.

To find the Latitude from the Declination, Altitude, and Hour Angle.—Let Z be the zenith, and P M Z the celestial meridian of the ship, P the elevated pole, and S the place of the heavenly body off the meridian; then P S is the co-declination, Z S the co-altitude, and P Z the co-latitude. In the spherical triangle P Z S, there is supposed to be known the three parts, P S, Z S, and the hour angle P, which, for the sun, is the time from noon. Hence, by Spherical Trigonometry, P Z, the co-latitude, may be found (*Mathematical Sciences*, page 661). But the following method, by right-angled triangles only, will be more easily recollected.

Fig. 27.



Draw S M, perpendicular to the meridian: we shall then have two right-angled triangles P M S and Z M S; and applying Napier's rules to these (page 660), we have—

From the triangle P M S, taking P for middle part, and P S, P M, for adjacent parts,

$$\text{cos. } P = \text{cot. } PS \text{ tan. } PM \therefore \text{tan. } PM = \text{cos. } P \text{ tan. } PS \quad . \quad . \quad (1)$$

Also, from the same triangle, taking the hypotenuse P S for middle part, and P M, S M, for opposite parts,

$$\text{cos. } PS = \text{cos. } PM \text{ cos. } SM \quad . \quad . \quad (2)$$

And from the triangle Z M S, taking in like manner the hypotenuse Z S for middle part, and Z M, S M for opposite parts,

$$\text{cos. } ZS = \text{cos. } ZM \text{ cos. } SM \quad . \quad . \quad (3)$$

Therefore, dividing (2) by (3), in order to get rid of S M we have—

$$\frac{\text{cos. } PS}{\text{cos. } ZS} = \frac{\text{cos. } PM}{\text{cos. } ZM} \therefore \text{cos. } ZM = \text{cos. } PM \text{ cos. } ZS \text{ sec. } PS \quad . \quad . \quad (4)$$

As Z M is thus expressed in terms of the given quantities, and as P M is also in like manner known from (1), the difference (or the sum, if M fall between P and Z) P Z, that is, the co-latitude becomes known.

The formulæ (1) and (4) to be combined are

$$\left.\begin{array}{l}\text{tan. } PM = \text{cos. hour angle} \times \text{cotan. dec.} \\ \text{cos. } ZM = \text{cos. } PM \text{ sin. alt.} \times \text{cosec. dec.}\end{array}\right\} (A)$$

To know the hour-angle P, it is necessary to know the time at the ship. The chronometer shows the mean time at Greenwich; and hence, by help of the longitude by account, we may find the mean time of observation nearly; this, reduced to apparent time, by applying the correction for the equation of time taken from the *Nautical Almanac*, will make known the time from apparent noon at the ship. The longitude by account is, however, most likely affected with error, and it is therefore desirable that the altitude off the meridian be taken at such a time as that a small error in the hour angle may have the least influence on the determination of tan. P M. Now, whenever we have to employ the cosine of a small angle, and have reason to suspect that the angle itself has not been accurately determined, the error in the cosine will be smaller as the angle itself is smaller; for the cosines of arcs near the beginning of the quadrant differ very little from one another within the limits of several seconds, and the difference becomes less as the arc approaches to zero (see *Introduction*, page 1046).

It follows, therefore, that when the time at the ship is only approximately known, the altitude should be taken when the object is as near to the meridian as it is likely to be, before being obscured by clouds.

The formulæ just established is, we see, generally applicable, however distant from the meridian the observed object may be, provided we know the apparent time at the ship; and we see, also, that they may be employed when the time *nearly* is known, provided the object be pretty close to the meridian. But for this latter case there is a special method somewhat more convenient, which may be investigated as follows:—

Referring to the preceding diagram, we have, by the fundamental theorem of Spherical Trigonometry, which expresses the relations among the three sides and one of the angles of a spherical triangle,

$$\text{cos. } ZS = \text{cos. } PZ \text{ cos. } PS - \text{sin. } PZ \text{ sin. } PS \text{ cos. } P.$$

$$\therefore \text{cos. } P = \frac{\text{cos. } ZS - \text{cos. } PZ \text{ cos. } PS}{\text{sin. } PZ \text{ sin. } PS}$$

Let z be the zenith distance that S would have when upon the meridian, and z' the difference between this meridian zenith distance and that Z S actually observed; that is, let $ZS = z + z'$; then the equation above is

$$\text{cos. } P = \frac{\text{cos. } (z + z') - \text{cos. } PZ \text{ cos. } PS}{\text{sin. } PZ \text{ sin. } PS}$$

Subtracting each side from 1, we have

$$1 - \text{cos. } P = 2 \text{ sin.}^2 \tfrac{1}{2} P$$

$$= \frac{\text{sin. } PZ \text{ sin. } PS + \text{cos. } PZ \text{ cos. } PS - \text{cos. } (z + z')}{\text{sin. } PZ \text{ sin. } PS}$$

$$= \frac{\text{cos. } (PZ \backsim PS) - \text{cos. } (z + z')}{\text{sin. } PZ \text{ sin. } PS}.$$

Now $PZ \backsim PS$, that is, the difference between the co-latitude and the co-declination is equal to Z, the meridian zenith distance, because the co-latitude P Z is always equal either to $PS + z$, or $PS - z$, or $z - PS$, the latter being the case when S is below the pole (see Fig. 26, at page 1078). Consequently

$$2\ \text{sin.}^2\tfrac{1}{2}\,\text{P}=\frac{\text{cos. z}-\text{cos. }(\text{z}+\text{z}')}{\text{sin. PZ sin. PS}}$$

$$=\frac{\text{cos. z}-\text{cos. z cos. z}'+\text{sin. z sin. z}'}{\text{sin. PZ sin. PS}}.$$

If the difference z′, between the meridian zenith distance and that actually observed, be so small that cos. z′ may be regarded as equal to 1, then we shall have

$$2\ \text{sin.}^2\tfrac{1}{2}\,\text{P}=\frac{\text{sin. z sin. z}'}{\text{sin. PZ sin. PS}}$$

$$\therefore\ \text{sin. z}'=2\ \text{sin. PZ sin. PS cosec. z sin.}^2\tfrac{1}{2}\,\text{P}.$$

The number of seconds in the arc z′ is very nearly equal to the number of times sin. z′ contains sin. 1″: consequently, No. of seconds in

$$\text{z}'=\frac{2}{\text{sin. }1''}\ \text{sin. PZ sin. PS cosec. z sin.}^2\tfrac{1}{2}\,\text{P}$$

$$=\frac{2}{\text{sin. }1''}\ \text{cos. lat. cos. dec. cosec. z sin.}^2\tfrac{1}{2}\ \text{hour-angle.}$$

To apply this formula, we must, of course, know z approximately: this is deduced from the latitude by account. By the aid of this approximate latitude and the small hour-angle P, we may, therefore, discover what correction z′ must be applied to the zenith distance actually observed, to reduce it to the meridian zenith distance, from which the corrected latitude is easily obtained, as in the examples already given.

A result still more perfect may in general be arrived at, by proceeding anew with this corrected latitude, writing it in the formula in place of the latitude by account, and thus getting a more accurate correction for the reduction of the observed, to the true meridian zenith distance. The additional work for this purpose will be but trifling.

The formula just deduced, furnishes the following rule for deriving the latitude by aid of the latitude by account, and from the observed altitude when near the meridian of a celestial object whose declination is known. The number 5·615455 is the logarithm of $\frac{2}{\text{sin. }1''}$.

LATITUDE FROM AN ALTITUDE NEAR THE MERIDIAN, THE DECLINATION, THE HOUR-ANGLE, AND THE LATITUDE BY ACCOUNT.—Rule 1. Take the declination of the object for the Greenwich time by account, and add it to the latitude by account when they are of different names; otherwise, take the difference of the two: the result is the meridian zenith distance by account.

2. If the object be the sun, the apparent time from noon is the hour-angle: for any other object, add the sun's right ascension to the apparent time, since preceding noon. The difference between this sum and the object's right ascension is the hour-angle.

3. Add together the following logarithms:—

The constant logarithm 5·615455,
log. cosine of the latitude by account,
log. cosine of the declination,
log. cosine of the mer. zenith dist., as deduced from the two latter,
twice log. sine of half the hour-angle.

The sum, rejecting the tens from the index, is the logarithm of a number of seconds, which, subtracted from the true zenith distance, deduced from the observation, gives the meridian zenith distance. If this and the declination are of the same name, their sum, otherwise their difference, is the latitude, of the same name as the greater.

Examples.

1. In latitude 48° 12′ N. by account, when the sun's declination was 16° 10′ S., at 0h. 16m. P.M., apparent time, the sun's true zenith distance was 64° 40′ N. Required the latitude.

Constant log.			5·615455
Latitude by acct.	48° 12′ N.	cos.	9·823821
Declination	16° 10′ S.	cos.	9·982477
∴ Mer. zen. dist. acct.	64° 22′	cosec.	10·044995
Half hour-angle in deg.	2° 0′	2 sin.	17·085638
			log. 2·552386
			60)357″

Reduction	— 5′ 57″
Zen. dist. from observation	64° 40′ 0″ N.
True mer. zen. dist.	64° 34′ 3″ N.
Declination	16° 10′ 0″ S.
Latitude	48° 24′ 3″ N.

This example is from Mr. Riddle's *Treatise on Navigation and Nautical Astronomy.* We shall now solve it anew by putting the latitude here deduced, in place of the latitude by account.

Constant log.			5·615455
Latitude	48° 24′ 3″ N.	cos.	9·822117
Declination	16° 10′ 0″ S.	cos.	9·982477
Mer. zen. dist.	64° 34′ 3″	cosec.	10·044268
Half hour-angle	2° 0′ 0″	2 sin.	17·085638
Reduction	355″	log.	2·549955

The difference between this and the former reduction is only 2″, so that the corrected latitude is 48° 24′ 5″ N.

It may here be remarked, that as fractions of a second are disregarded in the *Reduction,* the logarithms used in finding it, need be taken out of the table only to the nearest minute of each of the angular quantities.

If the formulæ marked (A), at page 1084, be applied to the preceding example, the work will be as follows:—

Referring to Fig. 27, at page 1084, in connection with the formulæ (A), we have

1. tan. P M = cos. hour ang. cotan. dec.

cos.	4° 0′	9·998941
cotan.	16° 10′	10·537758
tan. P M	73° 47′ 45″	10·536699
Z M	64° 36′ 20″	

138° 24′ 5″ = 90° + lat., since lat. and dec. have different names.
90°
48° 24′ 5″ = lat. N., the same as before.

2, cos. Z M = cos. P M cosec. dec. sin. alt.

cos.	73° 47′ 45″	9·445699
cosec.	16° 10′	10·555280
sin.	25° 20′	9·631326
cos. Z M	64° 36′ 20″	9·632305

In this example the latitude is north, and the declination south, so that P M, in the general investigation of the formulæ, is in this particular case P′ M, and therefore P′ M + Z M — 90° is the distance of Z above the equinoctial; that is, it is the latitude of the ship.

In finding the latitude by the above formulæ (A), it is of course necessary to ascertain on which side of Z the foot M of the perpendicular from S falls; that is, whether Z M is to be added to or subtracted from P M; but whether the correct co-latitude is P M — Z M or P M + Z M, can be matter of doubt only when Z M is so small as to make it of little consequence which be taken. But as the method by the rule is free from all ambiguity, it is to be preferred when the object is near the meridian.

2. In latitude 50° 50′ N. by account, when the sun's declination was 11° 41′ 58″ N. at 12m. 3s. from apparent noon, the sun's true altitude was 50° 52′ 29″. Required the latitude. Ans. Latitude, 50° 47′ 49″ N.

3. At 3h. 5m. 36s. apparent time, the sun's true alti-

tude was 35° 4′ 7″, and his declination 10° 54′ 26″ N. Required the latitude from the formulæ (A).

Ans. Latitude, 50° 48′ 23″ N.

4. At 18m. 45s. from apparent noon, in latitude 8° S. by account, the sun's true altitude was 74° 16′, and his declination 23° 27′ S. Required the latitude to the nearest minute. Ans. Latitude, 8° 23′ S.

5. In longitude 0° 45″ W., at 11h. 2m. 32s. apparent time, the observed altitude of the pole star was 51° 22′, the index correction being — 2′. For apparent noon at Greenwich, on the day of observation, the *Nautical Almanac* gave the following particulars :—

Sun's right ascension, 6h. 51m. 11⅓s.
Star's right ascension, 1h. 1m. 41s.
Star's declination, 88° 26′ 56″.

Required the latitude to the nearest minute by the formulæ (A). Ans. Latitude, 51° 47′ N.

ARTIFICIAL HORIZON.—The problem just discussed will be found very useful at sea, whenever the mariner, on account of cloudy weather coming on near noon, is disappointed of a meridian altitude. As the object may be obscured though the horizon may be clear and well defined, so, on the other hand, the celestial body may be visible and in a position well suited for observation, and yet a haze may obscure the horizon. In such a case an artificial horizon is employed: this is a shallow trough of quicksilver, protected from agitation from the air by a glass covering or roof.

The observer, placing himself at a convenient distance from this horizon, so that both the celestial object and its reflected image may be distinctly seen, measures with his sextant the angular distance between the two ; and as the real object is as much above the horizontal plane as the image is below it, he thus gets *double* the altitude, and has no correction to make for *dip:* the angle read off from the instrument, being corrected for the index error and divided by 2, gives the apparent altitude of the point observed.

To get a *meridian* altitude of the sun is one of the principal items in a "day's work" at sea, for correcting the latitude by the dead-reckoning ; if the weather interfere with this operation, then an observation off the meridian is sought for, and the latitude inferred by help of the apparent time at the ship, as explained in the preceding article. The reader will bear in remembrance that a ship carries the mean time at Greenwich with her : the ship's chronometer, when the known daily gain or loss is applied to it, supplies this important information. The *apparent* time is deduced therefrom, by means of the equation of time given in the *Nautical Almanac;* and the apparent time at the ship is ascertained by turning the longitude into time ; and thus the hour-angle of the object, observed from the meridian, is found. The longitude by account may be somewhat in error ; but the trifling inaccuracy in the resulting time at the ship, will have no important influence on the latitude deduced from it.

But valuable as the chronometer is, yet, like all human contrivances, it is subject to accidents and exposed to derangements from circumstances beyond our control. It is of great importance, therefore, to be able to find the position of a ship at sea, independently of its aid : this we have seen, as far as *latitude* is concerned, may be done by means of meridian altitudes. It may also be done by aid of *two* altitudes of the same celestial objects taken off the meridian. This is technically called the problem of *double altitudes:* we proceed now to investigate its principles.

LATITUDE FROM TWO ALTITUDES OF THE SUN, AND THE TIME BETWEEN THE OBSERVATIONS.—Scarcely any problem in nautical astronomy has received more attention from scientific men than the problem of *double altitudes* ; and, as the calculations involved in it are much longer than those for finding the latitude from a single altitude, various tables, to facilitate the operations, have been constructed. The determination of the latitude, by help of such tables, is what is called the *indirect* method of solution ; and, like all such methods, it is not so strictly correct as the direct method by trigonometry. Delambre carefully examined all the rules he was acquainted with for the solution of the present problem, and he came to the conclusion that the rigorous process by spherical trigonometry was to be preferred, as well for brevity as for accuracy of result. The investigation of the direct method is as follows :—

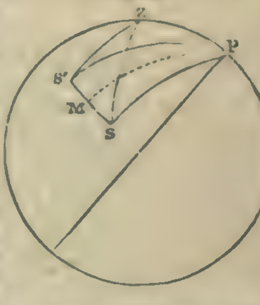
Let P (Fig. 28) be the elevated pole, Z the zenith of the ship, and therefore Z P its co-latitude. Let S, S′ be the two places of the sun when the altitudes are taken ; then, drawing the great circle arcs, as in the diagram, or in that of Fig. 27, we shall have the following quantities given, namely—

$$\left.\begin{array}{ll}\text{The co-declinations} & PS,\ PS' \\ \text{The co-altitudes} & ZS,\ ZS' \\ \text{The hour-angle} & SPS'\end{array}\right\}\ \text{To find } ZP.$$

There are three spherical triangles to consider, viz.—

1. The triangle P S S′, in which are given the two sides P S, P S′, and their included angle, to find the third side S S′, and one of the remaining angles, as, for instance, the angle P S S′.

Fig. 28.



2. The triangle Z S S′, in which are given the three sides, to find the angle S′ S Z ; from which, and the previously-found angle P S S′, the angle Z S P becomes known, so that we have,

3. The triangle Z S P, in which are given two sides and their included angle, to find the third side Z P.

Before proceeding to the solution of these triangles, the observed altitudes must, of course, be reduced to the true altitudes, as in the former examples ; and since the ship most probably sails on during the interval between the two observations, an additional correction becomes necessary, in order to reduce the first altitude to what it would have been, if taken at the place of the second observation. This correction for the ship's run will become known, provided we know the number of minutes the ship has sailed directly towards or directly from the sun, in the time between the two observations ; and they may be ascertained as follows ;—

Take the angle included between the ship's course and the sun's bearing at the first observation ; and considering this angle as a course, and the distance sailed as the corresponding distance, find by the traverse table, or by calculation, as in plane sailing, the difference of latitude ; this difference of latitude, expressed in minutes, will be the number of minutes by which the ship has approached to or receded from the sun, so that we shall know how many minutes must be added to or subtracted from the first altitude, to reduce it to what it would have been if taken by another observer, at the place of the second observation.

If the angle between the ship's track and the bearing of the sun be less than 90°, the ship will obviously be approaching towards the sun, in which case the correction of the altitude, determined as above, must be added ; but if the angle exceed 90°, it must be subtracted : if it be exactly 90°, no correction of the altitude will be necessary for the ship's change of place.

But a correction of the elapsed time may be requisite for the change of longitude ; this change, converted into time, must be added to the time elapsed between the two observations, if the ship have sailed eastward, and subtracted if she have sailed westward.

These are the corrections necessary to fit the three triangles above for trigonometrical calculation. And to simplify the work of finding S S′, without any material sacrifice of accuracy, we may regard the declinations of the sun at the times of observation, as both equal to the declination it has at the middle time between them ; the shorter this time is, the less will the supposition affect the precision of the result.

Regarding the above mentioned corrections to have been made, we shall now give an example of the trigonometrical operation.

Examples.

The two corrected zenith distances are,

Z S = 73° 54′ 13″, and Z S′ = 47° 42′ 51″;

the corresponding co-declinations of the sun are

P S = 81° 42′ N., and P S′ = 81° 45′ N.;

and the interval of time between the observations is three hours. Required the latitude.

For the more easy determination of S S′, let it be regarded as the base of an isosceles spherical triangle, of which each of the equal sides is ½ (P S + P S′) = 81° 43′ 30″, the vertical angle at P being 3h. or 45°; then if the perpendicular P M be drawn, the triangle P M S will be right-angled, and we shall have given

$$\text{P S} = 81°\,43'\,30'' \text{ and P} = \frac{4°}{2} = 22°\,30'$$

to find S M = ½ S S′ as follows :—

1. *In the triangle* P M S *to find* S M.

sin. P S,	81° 43′ 30″	. . .	9·9954547
sin. P,	22° 30′ 0″	. . :	9·5828397
sin. S M,	22° 15′ 11″·3	. . .	9·5782944
	2		
∴ S S′ =	44° 30′ 22″·6		

2. *In the triangle* P S S′ *to find the angle* P S S′.

sin. S S′,	44° 30′ 22″·6	Arith. Comp.	·1542898
sin. P S′,	81° 45′ 0″	„ „	9·9954822
sin. S P S′	45° 0′ 0″	„ „	9·8494850
sin. P S S′	86° 38′ 58″	„ „	9·9992570

3. *In the triangle* Z S S′ *to find the angle* Z S S′.

Z S′,	47° 45′ 51		
sin. Z S′,	73° 54′ 13″	Arith. Comp.	·0173686
sin. S S′,	44° 30′ 22″·6	„ „	·1542898
	2)166° 10′ 26″·6		
½ sum =	83° 5′ 13″·3		
sin. (½ sum − Z S),	9° 11′ 0″·3	. . .	9·2030206
sin. (½ sum − S S′),	38° 34′ 50″·7	. .	9·7949179
			2)19·1695969
sin. (½ Z S S′,	22° 36′ 26″·4	. . .	9·5847985

∴ Subtract

Z S S′ = 45° 12′ 52″·8

from P S S′ = 86° 38′ 58″

∴ P S Z′ = 41° 26′ 5″·2

4. *In the triangle* Z S P *to find* Z P. (See the following Investigation.)

tan. P S,	81° 42′ 0″	. . .	10·8359917
cos. P S Z,	41° 26′ 5″·2	. . .	9·8748930
cot. ω =	11° 0′ 41″·2	. . .	10·7108847
Z S =	73° 54′ 13″		
ω + Z S =	84° 54′ 54″·2		
cos. P S		. . . . .	9·1594354
sin. ω Ar. Comp.		. . .	·7189551
sin. (ω + Z S).		. . . .	9·9982874
sin. Z P,	48° 49′ 59″·7	. . .	9·8766779

Hence the latitude is 48° 50.′

As Z P is here found by a method not always given in works on Spherical Trigonometry, we shall subjoin the investigation of it. Representing, as usual, the three sides of a spherical triangle by *a*, *b*, *c*, and the angle included by the first two by C, we have, by the fundamental formula of Spherical Trigonometry (page 658),

$$\cos. c = \cos. a \cos. b + \sin. a \sin. b \cos. C.$$

$$\text{Or, since } \sin. a = \cos. a \frac{\sin. a}{\cos. a} = \cos. a \tan. a,$$

$$\cos. c = \cos. a\,(\cos. b + \tan. a \sin. b \cos. C).$$

Put cot. ω for tan. *a* cos. C; that is, let

$$\tan. a \cos. C = \cot. \omega = \frac{\cos. \omega}{\sin. \omega} \quad . \; . \; . \; . \; . \quad (1)$$

Then we shall have—

$$\cos. c = \cos. a \frac{\sin. \omega \cos. b + \sin. b \cos. \omega}{\sin. \omega}$$

$$\text{that is, } \cos. c = \frac{\cos\ a \sin. (\omega + b)}{\sin. \omega} \quad . \; . \; . \; . \; . \quad (2)$$

The expressions (1) and (2) are those already calculated. In the preceding solution more attention is paid to minute quantities than is at all necessary in actual practice at sea, where fractions of a second are of course disregarded. Yet in lengthy operations seconds themselves ought not to be entirely neglected, except in those confessedly approximative methods which the indirect processes, by peculiar *tables*, generally are. The only departure from strict mathematical rigour in the foregoing work, is in the first part of it, where the arc S S′ is supposed to be equal to an arc subtending the same angle at P, the sides of this angle being the mean between the two slightly differing co-declinations P S, P S′. It is plain, from the small amount of this difference, that the fictitious arc cannot vary in length from the real arc S S′ by a quantity deserving of any consideration, when the time between the observations is not unreasonably great.

By any one familiar with the practical solution of spherical triangles, the method here illustrated will be preferred to the indirect methods adverted to above; for the operation, though rather long, lays no burden upon the memory, and is, moreover, free from the inaccuracies —small inaccuracies, no doubt—which indirect methods are always affected with. A distinguished practical navigator, Captain Kater, after the example of Delambre, gives the preference to the direct process : the preceding Exercise is taken from the former writer; but the latter part of the solution is conducted differently.

2. On the 7th of February, 1846, in latitude by account 35° N., and longitude 47° W., at 8h. 9m. 4s. A.M. mean time, the altitude of the sun's lower limb was 36° 10′, and his bearing S. ¼ E.; after running N.E. 27 miles, the altitude of the lower limb, at 11h. 30m. 18s., was observed to be 41° 20′; the height of the eye was 20 feet. Required the latitude of the ship when the second observation was made.

From the *Naut. Alm.*	declination.		
	Feb. 7, noon	15° 19′ 32″ S.	Semi-diam. 16°14′.
	„ 8, .	15° 0′ 40″ S.	

1. *For the true altitude of sun's centre.*

First alt. sun's L. L.	. .	36° 10′ 0″
Dip . . . . −4′ 24″ Semi-di. . 16′ 14″	. .	+ 11′ 50″
App. alt.	. . . .	36° 21′ 50″
Refraction and par.	. . .	− 1′ 10″
True altitude	. . . .	36° 20′ 40″
Second alt. sun's L. L.	. .	41° 20′ 0″
Dip. and semi-diam.	. .	+ 11′ 50″
Apparent alt.	. . .	41° 31′ 50″
Refraction and par.	. .	— 58″
True altitude	. . .	41° 30′ 52″

The angle between the sun's bearing S. ¼ E. and the course N.E. is 11¾ points, so that the ship has sailed within 4¼ points of the direction opposite to the sun, a distance of 27 miles. With this distance and 4¼ points as a course, the Traverse Table gives 18′ for the corresponding difference of latitude, which is the number of minutes the ship has receded from the sun during the interval of the observations; hence these 18′ must be *subtracted* from the first true altitude, to reduce it to what it would have been if taken by another observer at the place of the second observation; consequently, the true altitudes at this second place, taken at the stated times, are 36° 2′ 40″ and 41° 30′ 52″

∴ Z S = 53° 57′ 20″ and Z S′ = 48° 29′ 8″.

2. *For the co-declinations* P S, P S′.

Ship's time, Feb. 6 . .		20h. 9m. 4s.
Long. in time . . .		3h. 8m. 0s. W.
Time at Greenwich . .		23h. 17m. 4s.
Dec., Feb. 6.	15° 38′ 0″	var. — 46″·5
	17′ 49″	23
Declination .	15° 20′ 20″ S.	1395
	90°	930
	P S = 105° 20′ 20″	6)106·95
		17·82

Ship's time, Feb. 6 . .		23h. 30m. 18s.
Long. in time . . .		3h. 8m. 0s.
Gr. time, Feb. 7 . .		2h. 38m. 18s.
Dec. Feb. 7 .	15° 19′ 32″	var. — 47″·2
	1′ 58″	2½
Declination .	15° 17′ 34″ S.	944
	90°	236
	P S′ = 105° 17′ 34″	6)11·80
		1·97

The decimal part of the minutes, in the correction for declination, is reduced to seconds by multiplying by 60.

3. *For the angle* S P S.′

Time of first observation . . .	8h. 9m. 4s.	3h.	= 45°
Time of second observation . .	11h. 30m. 18s.	21m.*	= 5° 15′
Interval of time	3h. 21m. 14s.	14s.	= 3′ 30″

∴ the angle S P S′, in degrees, = 50° 18′ 30″

Hence in the isosceles triangle P M S, we have for each of the equal sides ½ (P S + P S′) = 105° 18′ 57″, and for half the vertical angle, 25° 9′ 15″.

1. *In the triangle* P M S *to find* S M.

sin. P S,	105° 20′ 20″ . .	9·9842479
sin. P,	25° 9′ 15″ . .	9·6284454
sin. S M,	24° 11′ 58″ . .	9·6126934
	2	
S S′ =	48° 23′ 56″	

2. *In the triangle* P S S′ *to find the angle* P S S′.

sin. S S′,	48° 23′ 56″	Arith. Comp.	·1262231
sin. P S′,	105° 17′ 34″	” ”	9·9843431
sin. S P S′,	50° 18′ 30″	” ”	9·8862044
sin. P S S′,	83° 1′ 17″	” ”	9·9967706

3. *In the triangle* Z S S′, *find the angle* Z S S′.

Z S,	48° 29′ 8″		
sin. Z S,	53° 57′ 20″	Arith. Comp.	·0922566
sin. S S′,	48° 23′ 56″	” ”	·1262230
	2)150° 50′ 24″		
½ sum =	75° 25′ 12″		
sin. (½ sum − Z S),	21° 27′ 52″	” ”	9·5633906
sin. (½ sum − S S′),	27° 1′ 16″	” ”	9·6573607
			2)19·4392309
sin. ½ Z S S′	31° 37′ 27″	” ”	9·7196154

∴ Z S S′ = 63° 14′ 54″

P S S′ = 83° 1′ 17″

∴ P S Z = 19° 46′ 23″

4. *In the triangle* Z S P, *to find* Z P.

tan. P S	105° 20′ 20″ .	10·5617760
cos. P S Z	19° 46′ 23″ .	9·9736081
cot. ω	16° 15′ 2″ .	10·5353841
Z S =	53° 57′ 20″	
− ω + Z S =	37° 42′ 18″	

cos. P S . . .	9·4226245
sin. ω, Ar. Comp . .	·5530929
sin. (ω, + Z S) . .	9·7864647
sin. Z P, 35° 20′ . .	9·7621821

Hence the latitude is 35° 20′ N.

Note.—It *may* happen, in low latitudes, that the arc S S′, if prolonged, would cut the meridian P Z between P and Z, in which case the angle P S Z will not be the difference between the angles P S S′, Z S S′; and the sum of these angles must be taken instead. In cases of doubt, therefore, the last portion of the work should be modified on this second supposition, and that one of the two resulting latitudes taken which best agrees with the latitude by account. (See Fig. 27, page 1084).

It may be also noticed that the error of a few minutes in the estimated mean time at the ship will be of no consequence in deducing the declination; but as an error in the elapsed time, and therefore in the corresponding polar angle, is to be avoided, the elapsed time should be taken from the chronometer, or a good watch.

3. The two corrected altitudes of the sun are 42° 14′ and 16° 5′ 47″; the corresponding declinations are 8° 16′ 30″ N., and 8° 15′ N., and the time between the observations is three hours. Required the latitude of the place. Ans. Latitude, 48° 54′ 27″ N.

4. The two corrected zenith distances are 54° 39′ and 19° 59′; the corresponding declinations are 5° 31′ 6″ S. and 5° 28′ 54″ S., and the interval of time 2h. 20m. Required the N. latitude. Ans. Latitude, 1° 29′ 28″ N.

5. In latitude 29° 10′ S. by account, and longitude 124° W., the sun being obscured at noon, its altitude was taken at about 20m. past noon, the chronometer at the time showing 9h. 49m. 20s.: at 10h. 44m. 45s., by the same chronometer, the altitude was again taken. In the first observation the altitude of the upper limb was found to be 45° 33′; in the second, the altitude of the lower limb was 42° 8′ 30″, at which second observation the sun bore N. ½ E. The ship's course between the observations was N.W. ¾ W., and her run 6 miles; the allowance for dip was −4′ 30″, and the *Nautical Almanac* gave the following particulars for noon of the day at Greenwich:—

Sun's declination from *Naut. Alm.*, 16° 34′ 4″ N.; hourly var. 42″·8.

Semi-diameter from ditto, 15′ 52″.

Required the latitude of the ship, when the greater altitude was taken. Ans. Latitude, 28° 0′ S.

Note.—When, as in this last example, the latitude at the first observation is to be found, and the sun's bearing is taken at the second observation, the point *opposite* to that of the ship's course from the first position, is to be used in reducing the second altitude to what it would have been, if taken at the first position of the ship.

If the sun's true bearing, or *azimuth*, could be taken with precision at either place of observation, there would be no necessity for a second altitude, because in the spherical triangle Z P S, formed by the co-latitude Z P, the co-declination P S, and the co-altitude Z S, the angle

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* Divide by 4, and reckon every unit of remainder as 15 *minutes*, if minutes be the dividend; or *seconds*, if seconds be the dividend.

Z would then be given; and this, together with Z S and P S, is sufficient for the determination of Z P.

It is more convenient to observe the bearing when the sun is low than when it is high, and the result can be the better depended upon; therefore, in the problem of double altitudes, the bearing of the sun is always taken when the less of the two altitudes is taken.

The method of finding the latitude here explained, may of course be applied to a star as well as to the sun, the interval between the observations being expressed in sidereal instead of in solar time; but as, in general, the horizon increases in obscurity as the star becomes more clearly visible, *two* observations of a star, with a sufficient interval of time between them, can seldom be accurately made. The following, therefore, is a more suitable problem for the purpose in view.

Latitude from the Altitudes of two Fixed Stars observed at the same time.—There are two advantages connected with this method of deducing the latitude: the first is, that as no allowance is to be made for change of place in the ship, all error arising from inaccuracies in the sun's bearing, and the course and distance steered, is avoided; the second is, that the risk of losing another observation, from unfavourable weather, is not incurred when both observations are made at the same time.

When the altitudes of the two stars are to be taken by one person, the mode of proceeding is this:—The altitude of one star is taken, and the time by the watch noted; the altitude of the other star is then taken, and the time noted; after a short interval, the altitude of the second star is again taken, and the time noted. Thus the change of altitude of the second star, in a known interval of time, will be found; and therefore the correction for the time, when the first star was observed, may be found by proportion.

As to the principles of solution, they are the same as in the former problem: the co-declinations or polar distances P S, P S′ of the two stars are given; the angle P between these is also given, this angle in time being the difference of the right ascensions of the two stars. We may hence compute the third side S S′ of the spherical triangle. As the co-declinations may differ considerably, this third side must not be calculated on the supposition that the mean of the co-declinations may be taken for each of the other two sides. It must be found from the distinct co-declinations themselves. The remaining part of the operation is the same as in the former problem.

Examples.

1. In north latitude 52° 30′ by account, the corrected zenith distances of Capella and Sirius were as follows:—

Capella, Z S = 29° 14′ 24″.
Pol. Dist., P S = 44° 11′ 39″.
Sirius, Z S′ = 72° 5′ 48″.
Pol. dist., P S′ = 106° 28′ 40″.

Difference of the two right ascensions, 1h. 33m. 45s. = 23° 26′ 11″ = P. Required the latitude of the ship.

1. *In the triangle* S P S′ *to find* S S′.

tan. P S,	44° 11′ 39″ . . .	9·9877822
cos. P,	23° 26′ 11″ . . .	9·9626072
cot. ω,	48° 15′ 56″ . . .	9·9503894
	106° 28′ 40″ . . .	
ω + P S′ =	154° 44′ 36″	

cos. P S	9·8555080
sin. ω Ar. Comp.	·1271225
sin. (ω + P S′)	9·6300961
cos. S S′, 65° 47′ 55″	9·6127266

2. *In the triangle* P S S′ *to find the angle* P S S′.

sin. S S′,	65° 47′ 55″	Arith Comp.	·0399527
sin. P S′,	106° 28′ 40″	„ „	9·9817868
sin. S P S′,	23° 26′ 11″	„ „	9·5995891
sin. P S S′,	24° 43′ 3″	„ „	9·6213286

3. *In the triangle* Z S S′ *to find the angle* Z S S′.

Z S′ = 72° 5′ 48″		
sin. Z S′ = 29° 14′ 24″	Arith. Comp.	·3111631
sin. S S′ = 65° 47′ 54″*	„ „	·0399536
2)167° 8′ 6″		
½ sum = 83° 34′ 3″		
sin. (½ sum − Z S) = 54° 19′ 39″	„ „	9·9097504
sin. (½ sum − S S′) = 17° 46′ 9″	„ „	9·4845601
		2)19·7454272
sin. ½ Z S S′ = 48° 14′ 29″	„ „	9·8727136

∴ Z S S′ = 96° 28′ 58″ } to be added
P S S′ = 24° 43′ 3″ }
121° 12′ 1″
supplement = 58° 47′ 59″ = P S Z.

3. *In the triangle* Z S P *to find* Z P.

tan. P S,	44° 11′ 39″ . . .	9·9877822
cos. P S Z,	58° 47′ 59″ . . .	9·7143559
cot. ω,	63° 16′ 3″ . . .	9·7021381
Z S =	29° 14′ 24″ . . .	
(ω + Z S) =	92° 30′ 27″	

cos. P S	9·8555030
sin. ω Ar. Comp.	·0490919
sin. (ω + Z S)	9·9995830
sin. Z P, 53° 19′ 23″	9·9041829

Hence the latitude is 53° 19′ 23″ N.

In connection with the foregoing method of solving the problem of double altitudes, there are some points of theoretical interest to which the learner should attend. In the second step of the work the object is to find the angle P S S′; the operation conducts us to the *sine* of this angle; and as the angle answering to a given sine may be either acute or obtuse, we have no right, independently of controlling conditions, to assume it to be the one any more than the other.

In the example just solved, we have arbitrarily taken it to be acute, simply for convenience, without examining into any overruling circumstances. Now, in the present problem we may always take a similar liberty; for although the proper angle may really be the supplement of that we put down, the subsequent step will not be affected by this circumstance: the angle P S Z, which it is the object of that step to find, will still be the sum or difference of Z S S′, P S S′ or the supplement of the sum.

But it is well to show how all doubt respecting the species of the angle P S S′ may be avoided.

The fundamental formula of spherical trigonometry, applied to the triangle P S S′, gives

$$\cos.\ PSS' = \frac{\cos.\ PS' - \cos.\ PS \cos.\ SS'}{\sin.\ PS \sin.\ SS'};$$

and as it is matter of indifference which of the two stars, or which of the two places of the sun we mark S or S′, we may always consider, in this formula, that cos. P S′ is *numerically greater* than cos. P S, in which case the numerator of the above fraction, and consequently the fraction itself (since its denominator is positive), will have the same sign as cos. P S′; hence *taking* P S′ *for that of the two co-declinations whose sine is the less, the opposite angle* P S S, *will always be of the same species as* P S′—that is, they will either be both acute or both obtuse.

In the example just solved, cos. P S′ is negative, and cos. P S, cos. S S′ are both positive; consequently the fraction is negative, and therefore the angle P S S′ is obtuse—namely, 155° 16′ 57″, instead of 24° 43′ 3″, as we have taken it above; and we see that by *subtracting* 96° 28′ 58″ we get the same result—namely, 58° 47′ 59″.

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* A second is deducted from this arc, in order that there may be an even number of seconds in the sum of the three, so as that, in taking the half, fractions of a second may be avoided; the omission of a second can have no sensible influence on the result.

Whatever method of solving the problem of double altitudes be employed, there is always, in particular instances, a degree of uncertainty as to whether the sum or difference of a particular pair of arcs or angles is to be taken, and which uncertainty can be removed only by reference to the latitude by account.

The method proposed by Mr. Ivory, and which has been put in a very commodious form by Mr. Riddle, in his *Navigation and Nautical Astronomy*, is perhaps the shortest of the correct processes, when the object observed is the sun; but the investigation of it is very long and complicated, and the several steps of the work are far less easy of recollection than those above: it has the advantage, however, of deciding the ambiguity here mentioned in the case of the sun, with but very little trouble.

But in the case of a pair of stars, the uncertainty is not so easily removed: an amount of calculation, equivalent to a repetition of the work involved in the last step above, has to be gone through.

Now it occurs to us, that the best and most satisfactory way of coming at the proper value of the angle P S Z, is to directly compute this angle (in imitation of step 2 before) from the three sides of the triangle P S Z—namely the co-declination P S, the co-altitude Z S, and the co-latitude by account P Z. This extra work may be regarded as at least an approximate verification of the entire process up to the end of step 3; and, in executing it, seconds in the several arcs need not be attended to: it will be sufficient to take each side to the nearest minute.

The resulting value of the angle P S Z would be a safe guide to the value of it, which step 3 ought to give; and the concluding step might then be worked without any misgiving.

It may be further remarked here, in reference to a double altitude of the *sun*, that when the true co-latitude P Z is obtained, we may combine it with co-declination P S, and the co-altitude Z S, to determine the hour-angle Z P S, the apparent time from noon when the altitude furthest from the meridian was taken: the correction for the equation of time being applied, the result will be the mean time at the ship when S was observed. The chronometer, proper allowance being made for its daily loss or gain, will show the mean time at Greenwich; and the difference between the two will be the longitude in time.

But this important subject will be more fully discussed in the next chapter; we shall only add here, that an altitude near the meridian is not eligible for determining *time*, because, in that position, altitudes vary so slowly, that a small error in altitude may occasion a comparatively large error in time.

Example 2.—In latitude 58° N. by account, the true altitude of Capella was 69° 23′; and at the same time the true altitude of Sirius was 16° 19′; the following particulars were also given by the *Nautical Almanac*:—

Right Ascension of Capella 5h. 2m. 18s. Dec., 45° 47′ N.
" Sirius 6h. 36m. 33s. " 16° 27′ S.
Required the latitude. Ans. Latitude, 57° 8′ N.

THE DEVIATION OF THE COMPASS.—The mariner's compass is subject to two kinds of disturbance, which operate against its true north and south direction. One is a local disturbance attributable to the attractive influence of the iron in the ship, which, when unequally distributed in reference to the magnetic meridian, or the line along which the compass would otherwise settle, causes a deviation from that line. The effect of this local attraction is called the *deviation* of the compass. The amount of the angular disturbance or aberration is ascertained by experiment, before the ship proceeds on her voyage; and a table is then formed, which gives the proper correction for different positions of the ship's head. But Mr. Barlow, of the Royal Military Academy, Woolwich, has devised an apparatus for counteracting the effect of local attraction: a small iron plate is so placed near the compass, vertically and behind it, as that its influence upon it may neutralise the action of the iron in the ship; the proper situation for the plate being ascertained by experiment when the ship is in harbour. For an account of this contrivance, and the mode of applying it, the reader is referred to Barlow's *Essay on Magnetic Attractions*. It may be observed, however, that much care is requisite in fixing the plate; and that if the distribution of the iron in the ship be afterwards changed, the plate must be readjusted; but this inconvenience must attach to every method of estimating the deviation from the results of observations previously made, under particular conditions, as to the amount and position of the disturbing materials. Local deviation has sometimes been found to be so great, that, in a ship loaded with iron, the course by compass has been known to differ from the proper course by so much as 30°.*

THE VARIATION OF THE COMPASS.—The vertical plane passing through the direction of the horizontal needle at any place, marks out the *magnetic meridian* of that place, just as the vertical plane through the north and south points of the horizon marks out the geographical meridian of the place. The angle traced on the horizontal plane by the vertical planes just mentioned, measures the *variation of the compass* at the place. There are but very few places on the surface of the earth where the needle points directly to the north and south points of the horizon, or where the variation is zero. The line in which such places lie is called the line of no variation, and is a curve of very complicated form. It was not till probably some centuries after the discovery of the compass that the needle was suspected ever to lie out of the plane of the geographical meridian: it seems first to have been observed by Columbus, in 1492.

The variation of the compass is not only different at different places; but, what is more remarkable, it is not constant even at the same place. At London the variation was *eastward* till about the middle of the seventeenth century; in 1659 the needle pointed due north and south, the variation being zero, so that at that time the magnetic coincided with the geographical meridian. After this the variation became *westerly*, and it continued to increase till the year 1818, when it appears to have attained it greatest limit of westerly variation—namely, 24° 30′. Since then the variation has been slowly diminishing, and it is now about 23° *west*.

In order to ascertain the variation of the compass at any place, it is necessary that we find by computation, or, by some means independent of the compass, the *true bearing* of a celestial object; then observe the magnetic or compass bearing; the difference of the two will be the variation of the compass; including, however, the local deviation, unless this have been neutralised, as explained above.

There are two forms of the problem: the object whose bearing is observed may be either in the horizon or above it. In the former case it is the magnetic *amplitude* that is observed; in the latter case it is the *azimuth*: in both cases the latitude of the place is supposed to be known.

TO FIND THE VARIATION FROM AN OBSERVED AMPLITUDE.—It has already been seen that the angle which the equinoctial makes with the horizon of any place is the co-latitude of that place (page 1074). This angle is therefore given; and as the object observed is in the horizon, and its declination or distance from the equinoctial known, we have given the perpendicular (the declination) and the opposite angle (the co-altitude) of a right-angled spherical triangle to find the hypotenuse (the true amplitude); and the difference between this and the observed amplitude is the variation, including the local deviation when not previously counteracted. Refraction causes objects to appear in the horizon when, on an average, they are 33′ below it; consequently, the compass-amplitude should be taken when the sun's centre, or the star selected, is about 33′ *plus* the dip, above the sea-horizon; or, allowing 16′ for the sun's semi-diameter, the altitude of the sun's lower limb should be about 17′ + dip.

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* This subject, and that of the Variation, is fully entered into in the section on Magnetism in this volume.—ED.

Examples.

1. In January, 1830, in latitude 27° 36′ N., the rising amplitude of Aldebaran was observed, by compass, to be E. 23° 30′ N. Required the variation of the compass.

By the *Nautical Almanac* the declination of Aldebaran was 16° 9′ 37″ N.; and since by right-angled triangles, we have

sin. declination = sin. amplitude × cos. latitude;

$$\text{therefore, sin. amplitude} = \frac{\text{sin. declination}}{\text{cos. latitude}}$$

so that the logarithmic computation is as follows:

		10
sin. declination	16° 9′ 37″ . . .	9·44455
cos. latitude	27° 36′ 0″ . . .	−9·94753
sin. amplitude	18° 18′ 16″ . . .	9·49702
Magnetic amplitude	23° 30′ 0″	
	5° 11′ 44″	

As the star is thus farther by 5° 11′ 44″ from the magnetic east towards the north than from the true east, the magnetic east has therefore receded thus much towards the south, and consequently the magnetic north towards the east, hence the variation of the compass is 5° 11′ 44″ E. But the following directions will serve for all cases.

When the object is rising, the true amplitude is always estimated from the east, and when it is setting, from the west, and towards the north or south, according as the declination is north or south.

If the true amplitude, and that by compass, be both north or both south, their difference will be the variation; but if one be north and the other south, their sum will be the variation, easterly when the true amplitude is to the right, and westerly when it is to the left of the observed amplitude.

2. On Feb. 15th, 1841, in latitude 43° 36′ N., and longitude 20° W., the setting amplitude of the sun's centre was observed to be W. 6° 45′ N., at 6h. 52m. P.M., apparent time. Required the variation of the compass.

Time at ship . . .	6h. 52m.
Long. in time . . .	1h. 20m. W.
Time at Greenwich .	8h. 12m.

Sun's dec. at noon, *Naut. Alm.*	12° 37′ 18″ S.
Correction for 8h. 12m. .	−6′ 49″
Sun's dec. at time of obs.	12° 30′ 29″ S.

		10
sin. declination	12° 30′ S. . .	9·33534
cos. latitude	43° 36′ . .	−9·85984
sin. amplitude W.	17° 23′ S. . .	9·47550
Magnetic amplitude W.	6° 45′ N.	

Variation . 24° 8′ West, the true amplitude being to the left of the magnetic.

The true amplitude is always of the same name—north or south, as the declination, as is obvious.

3. In latitude 21° 14′ N., when the sun's declination reduced to the time at ship, was 19° 18′ 6″ S., its magnetic amplitude at rising was E. 35° 20′ S. Required the variation of the compass.

Ans. Variation, 14° 34′ West.

4. In latitude 26° 32′ N., and longitude 79° W., the sun's centre was observed to set W. 4° 17′ S.; the time at the ship was about 6h. P.M.: the sun's declination at noon Greenwich time, was 30′, and the hourly increase of declination 1′. Required the variation of the compass.

Ans. Variation, 3° 31′ E.

5. In latitude 48° 20′ N. the star Rigel was observed to set 9° 50′ to the northward of the west point of the compass: the star's declination was 8° 25′ S. Required the variation of the compass.

Ans. Variation, 22° 33′ W.

VARIATION OF THE COMPASS FROM AN AZIMUTH.—The computation of the azimuth of a celestial object requires the solution of an oblique-angled spherical triangle, the three sides being given to find an angle. The three sides are the co-latitude, the co-declination, and the zenith distance, or co-altitude of the object: the azimuth is measured by the angle at the zenith, or that of which the sides are the co-latitude and the co-altitude; in other words, it is the angle at the zenith between the meridian and the vertical circle through the object, agreeably to the definitions (page 1074). In north latitude, the true azimuth is reckoned from the south point of the horizon; and in the south latitude, from the north point, towards the east, when the altitude is increasing, and towards the west when the altitude is decreasing.

The observed or magnetic azimuth being reckoned from the same point as the true azimuth, if both are east or both west, their difference will be the variation; but if one is east and the other west, their sum will be the variation. The variation is east or west, according as the true azimuth is to the right or to the left of the observed azimuth.

Examples.

1. On June 9, 1853, at about 5h. 50m. A.M., in latitude 50° 47′ N. and longitude 99° 45′ W., the bearing of the sun by compass was S. 92° 36′ E.; when the altitude of his lower limb was 18° 35′ 20″, the index correction was + 3′ 10″, and the height of eye 19 feet. Required the variation of the compass.

1. *For the Co-declination and the Co-altitude.*

Time at ship, June 8 . .	17h. 50m.
Long. 99° 45′ W. in time . .	6h. 39m.
Time at Greenwich, June 9 .	0h. 29m.
Sun's dec. June 9, at noon, Gr. time	22° 57′ 36″
Hourly var. + 11″·8 ∴ cor. for 29m.	+ 6″
Dec. at time of observation .	22° 57′ 42″
	90°
Co-declination	67° 2′ 18″
Obs. alt.	18° 35′ 20″
Index cor.	+ 3′ 10″
	18° 38′ 30″
Dip — 4′ 17″ Semi. 15′ 46″ . . .	+ 11′ 29″
App. alt.	18° 49′ 59″
Ref. and par.	— 2′ 41″
True alt.	18° 47′ 18″
	90°
Co-alt.	71° 12′ 42″

2. *For the true Azimuth.*

Co-declination	67° 2′ 18″		
Sin. co-alt. .	71° 12′ 42″	Arith. Comp.	·0238109
Sin. co-lat. .	39° 13′ 0″	Arith. Comp.	·1991079
	2)177° 28′ 0″		
Sin.	88° 44′ 0″	,, ,,	9·9998939
Sin. (88° 44′ — co-dec.)	21° 41′ 42″	,, ,,	9·5678091
Cos.	51° 52′ 4″	,, ,,	9·7906218
	2		

True azimuth S.	103° 44′ 8″ E.
Observed azi. S.	92° 36′ 0″ E.

Variation . . 11° 8′ 8″ W., the true azimuth being to the left of the observed.

2. In latitude 48° 50′ N. the true altitude of the sun's centre was 22° 2′; the declination at the time of the observation was 10° 12′ S.; and the magnetic bearing S. 161° 32′ E. Required the variation of the compass.

Ans. Variation, 22° 40′ 40″ W.

3. November 19, 1835, in latitude 50° 22′ N., longitude 24° 30′ W., at about 9 o'clock in the morning, the altitude of the sun's lower limb was 8° 10′, and the bearing by compass S. 21° 18′ E.; also the height of the eye was 20 feet. Required the variation to the nearest minute.

Sun's dec. at noon, Nov. 19, G. T., 19° 23′ S. Semidiameter, 16′.

Variation of dec. in 1h. + 40″, so that 2′ must be subtracted for three hours before noon.

Ans. Variation, 24° 12′ West.

NOTE.—In the foregoing examples, if the local deviation of the compass remain uncorrected, what is named *variation* will be compounded of variation proper and deviation. Each result must then be regarded as the difference between the uncorrected compass-bearing and the true bearing.

It is proper to remark also, in concluding this division of our subject, that a single altitude or a single azimuth taken at sea is not considered so trustworthy as the mean of several; consequently, when all attainable accuracy is desired, the observations are repeated at short intervals of time (a minute or so), and the mean result of the set is taken for *the* observation employed in the calculation, the mean of the times being the corresponding time of that observation.

Attention to the local deviation of the compass is a matter of great practical importance; and Barlow's correcting plate is a valuable check to its influence. About a quarter of a century ago the *Thetis*, with a million of dollars and other treasure on board, sailed under the most favourable prospects from Rio Janeiro; the next day, in consequence of unfavourable wind, they tacked ship, in full confidence of being clear of land; the fatal mistake was first discovered by the jib-boom striking a high perpendicular cliff; all the three masts were at once sent over the side, and vessel and cargo were lost. In a paper in the *Phil. Trans.* for 1831, Mr. Barlow shows that the local deviation of the compass was exactly such as to be likely to occasion this great mistake in the ship's reckoning. The distance the vessel had run was about 80 miles; and assuming the local attraction to have been equal to that of the *Gloucester*—a similar ship—she would have passed five miles nearer to the scene of her destruction—Cape Frio—than she would have done had the compass been undisturbed by the attraction of the vessel, or had this disturbance been counteracted by any neutralising apparatus.

CHAPTER IV.

ON FINDING THE LONGITUDE AT SEA.

INTRODUCTION.—To determine the longitude of a ship at sea, is justly regarded as the greatest achievement of Nautical Astronomy; it is often considered, too, as the most important achievement; but since both the latitude and the longitude are equally necessary to enable the mariner to ascertain the position of his ship on the ocean, one of these determinations has, in fact, no claim to superior importance over the other, as respects its practical value to the navigator.

A higher degree of consequence has been attached to the problem of finding the longitude, solely because of the greater difficulties with which the solution of it is beset, and of the larger demand made upon the resources of science—both mechanical and astronomical—for the means of overcoming them.

We have seen, in the preceding Chapter, that there are several ways of determining latitude: of these the simplest is that wherein the data are the altitude and declination of a celestial object when on the meridian. So there are several methods of determining longitude; but in each of these we have two distinct problems to solve instead of one: the problems are, 1st, to find the time at the ship; and, 2nd, to find the time at Greenwich. The difference of the times is the longitude of the ship in time. The first of these problems is comparatively easy; the second has exercised the ingenuity and tasked the exertions of the greatest minds; and two very different forms of solution have resulted from these efforts.

As the great object to be accomplished is to discover, at any instant at the ship, what time it is at Greenwich, or at the meridian from which longitude is reckoned, it is natural that the problem of finding the longitude should have more especially engaged the attention of chronometer-makers; and, accordingly, when, in 1714, the first parliamentary reward was offered for the solution of this problem, within certain limits, a most laudable emulation was called forth among the more scientific of this class of artists. The pecuniary stimulus offered by government was this—namely, £10,000 for a method which should determine the longitude to within 60 miles of the truth; £15,000 if the method should give the longitude within 40 miles; and £20,000 if within 30 miles. The most successful competitor for these rewards was John Harrison, who, with indefatigable industry, applied himself to the construction of a chronometer that should keep time with sufficient accuracy to accomplish the last of these objects. In 1736 his first chronometer was subjected to trial in a voyage to Lisbon; and in 1739 a second, still more perfect, was produced; but in 1758 he completed a third, which he affirmed to be so accurate as to entitle him to the highest reward offered by the Commissioners of Longitude. In compliance with Mr. Harrison's request, the Admiralty directed the watch to be tried on a voyage to Jamaica, in the ship *Deptford*, which sailed from Portsmouth, November 18, 1761, and arrived at Port Royal, Jamaica, January 19, 1762. The time at Portsmouth—for the watch had been set to Portsmouth time—was found, on the 26th of the same month, to be 5h. 2m. 47s., as shown by the chronometer, at the instant of noon at Port Royal; and the exact difference of longitude, in time, between the same two places, as found by careful astronomical observations, was 5h. 2m. 51s.; differing from the determination of the chronometer by only 4s., which, in the parallel of Jamaica, is less than one nautical mile.

On January 28, 1762, the chronometer was sent back in the *Merlin*, which arrived at Portsmouth on March 26. On the passage, the ship encountered a violent storm, and the chronometer had to be removed to a place where it was likely to suffer from exposure. The time of mean noon, as shown by the chronometer, April 2, was 11h. 58m. 6s·5. Hence, from Nov. 6, 1761, to April 2, 1762, during which period the chronometer had passed through a variety of climates, and been subjected to violent agitations of the sea, its error was no more than 1m. 53s·5, or 28½ minutes of longitude in time, which, in the parallel of Portsmouth, does not amount to 18 nautical miles.

On this occasion Mr. Harrison received £5000; and another trial voyage was proposed—namely, from Portsmouth to Barbadoes—which was made in 1764, and which proved so satisfactory that an additional £5000 was ordered to be paid to him, and the remainder of the highest reward promised as soon as he had sufficiently explained the principles of his time-keeper to enable another artist to construct one as good. Mr. Harrison accordingly explained fully the whole of his mechanism to a properly qualified committee; and Mr. Kendal, a member of that committee, was directed to construct a chronometer on Harrison's principles. The instrument he produced was committed to the care of Mr. Wales, on his voyage round the world with Captain Cook, in the years 1772, 1773, and 1774; and it so fully justified Harrison's expectations in this severe trial, that the House of Commons ordered the remaining £10,000 to be paid to him.

Subsequently to this, chronometers were constructed by various persons, upon independent principles and of equal merit; those by Mudge, Arnold, and Earnshaw were considered to deserve special distinction, and accordingly £3000 were paid by government to each of these makers; but no parliamentary reward has since been given for chronometers.

At the present day, chronometers fully equal to any of those here adverted to are readily to be procured, and from the best makers; even superior time-keepers may at all times be obtained. The difference of longitude between Greenwich and New York, as recently determined by the chronometers of Mr. Dent, agreed with the results of astronomical observations, to within three-quarters of a mile!

Such, then, is the degree of perfection to which this department of mechanical art has been brought; so that the determination of the time at Greenwich, at any instant, in any part of the globe, would be an easy matter, provided only that a chronometer could be insured against all external interference with its action. But at sea, a piece of machinery so delicate is peculiarly exposed to deranging influences—changes of climate—the jerks and vibrations of the ship—local attraction, &c., &c., all have their influence; and however these ordinary hindrances to the correct performance of the watch may be provided against, yet an unavoidable accident may at any time render the machine useless. The safety of navigation requires, therefore, that—valuable as the chronometer is—other means of finding the longitude, independent of its aid, should be devised; and these are furnished by what are called the LUNAR OBSERVATIONS.

By aid of tables of the moon's motion—first supplied to the British Admiralty by Professor Mayer of Göttingen (1755—'62), and subsequently improved by Mason, Bradley, and others—the angular distances of the moon from the sun, and from the principal stars that lie near her path, are predicted for every three hours of time, and for several years in advance. These "Lunar Distances" are given in the *Nautical Almanac*. The motion of the moon in her orbit is so rapid—about 13° in 24 hours—that her advance to, or recession from a star in her path, is a measurable quantity even in the lapse of a few seconds of time: by means of the three-hourly distances computed in the *Nautical Almanac* for Greenwich time, her distance from any one of the selected stars, at any intermediate instant of Greenwich time, can be easily found by simple proportion; and, conversely, any intermediate distance being known, the corresponding Greenwich time can, in a similar manner, be found.

Like as in all the other angular measurements of Nautical Astronomy, the observer is considered to be situated at the centre of the earth: a lunar distance, therefore, being taken at sea, and reduced to this point, the mariner has only to turn to his *Almanac*, and from the distances there given, to compute, by proportion, the time at Greenwich, when the distance of the same objects was what he has found it to be. The Greenwich time at the instant of observation is thus discovered, and thence—his own time being known—the longitude of the ship is ascertained.

The heavens thus supply the navigator with an unerring chronometer—a time-piece that needs no winding up—that never gets out of repair, nor ever requires re-adjustment—that can neither be deranged by accident, nor be deteriorated by neglect. Placed beyond the reach of sublunary vicissitudes, heat and cold, storm and tempest affect it not: these, indeed, may temporarily cloud and obscure its face, but they can neither disturb its mechanism nor alter its rate; and we know that it will run down only when our concerns with TIME are at an end.

From these introductory remarks, the student will readily anticipate the business that is now before us. Whether we consult a chronometer, or take a lunar distance, for the purpose of finding the time at Greenwich, the time at the ship is an indispensable element in the determination of the longitude; the problem first to be solved, therefore, is *to find the local time.*

ON FINDING THE TIME AT SEA.—At first sight it would appear that the time at the ship, most easily determined, is the time of *noon*, or when the sun comes to the meridian; but, for a minute or two before and after the sun's transit, the change in altitude is so small, except in very low latitudes, that it is not possible at sea to detect the instant of the meridian passage; and the same may of course be said in reference to any other celestial object.*

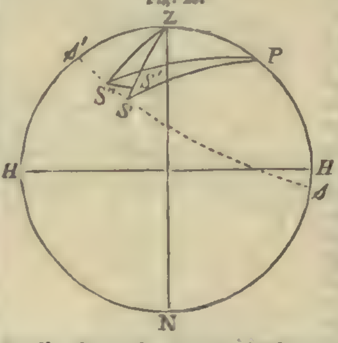
In determining the *latitude* by a meridian observation, the error of a minute or so in the time of apparent noon is of no consequence; for, as just remarked, the altitude of the object—which is all that the latitude is concerned with—is pretty nearly constant for a few minutes in the neighbourhood of the meridian; but, as respects the *longitude*, the error of a single minute in the time would occasion an error of fifteen minutes of longitude. It is obvious, therefore, that when time is to be deduced from altitude, with a view to the determination of the longitude, the position of the celestial object to be observed should be so chosen—when circumstances are such as to permit of a change of position—as that a small error in the altitude may have the least effect possible on the hour-angle, or time from noon. It is of importance, therefore, to consider the following preliminary problem:—

To determine upon what vertical a Celestial Object must be, in order that a small error in the Altitude may have the least effect on the Time.

Let S (Fig. 29) be the place of the celestial object observed; but by a small error in taking the altitude, let it be referred to S'. Draw $S' S''$ parallel to the horizon, and meeting the parallel of declination $s s'$, in the point S''.

Then, when the object is at S'' it will really have the co-altitude $Z S''$ equal to its supposed co-altitude $Z S'$, when it was actually not at S', but at S; so that in the determination of the hour-angle P from the co-latitude $P Z$, the co-declination $P S''$, and the erroneous co-altitude $Z S''$, the small angle $S P S''$ will measure the amount of error in the time.

Fig. 29.



As the triangle $S S' S''$ is of course exceedingly small, since the error of observation is not purposely made, it may be regarded as a rectilinear triangle, right-angled at S'; therefore $S S' = S S'' \sin. S''$, and $S S'' = \sin. PS \angle S P S''$, for $S S''$, the distance passed over by the object, may be compared to the distance sailed on a parallel of latitude by a ship, making the difference of longitude $S P S''$; and we know that in this case (page 1056),

$$\text{cos. lat.} : 1 :: \text{dist.} : \text{diff. long.} \therefore \text{dist.} = \text{cos. lat.} \times \text{diff. long.}, \therefore S S' = \sin. PS \angle S P S''.$$

$$\text{Hence } S S' = \sin. S'' \sin. PS \angle S P S'' \therefore S P S'' = \frac{S S'}{\sin. S'' \sin. PS} \quad . \quad . \quad . \quad (1)$$

Now the angle S'', that is, the angle $S' S'' S$, is equal to the angle Z S P, because S′ being a right angle, S′ S S′ is the complement of each; and therefore, from the relation between the sides and angles of a spherical triangle, we have

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* It may be remarked, moreover, that the altitude of the sun, the moon, or a planet, is not necessarily always the greatest when on the mid-day meridian of the place of observation; nor necessarily the least, when on the midnight meridian: the body's change of declination may be such as to prevent this.

$$\text{sin. } S' : \text{sin. } SZP :: \text{sin. } PZ : \text{sin. } PS$$

$$\therefore \text{sin. } S' \text{ sin. } PS = \text{sin. } PZ \text{ sin. } SPZ \quad . \; . \; . \; (2)$$

Substituting the second member of (2) in (1), we therefore have

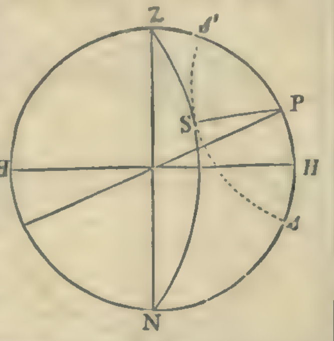
$$SPS' = \frac{SS}{\text{sin. } PZ \text{ sin. } SZP} = \frac{\text{error in altitude}}{\text{cos. lat. sin. azimuth}} \; . \; . \; . \; (3)$$

This expression for the error in time will obviously be the least possible, when the sine of the azimuth is the greatest possible; hence, if the co-declination be sufficiently great, or the co-latitude sufficiently small for the object to cross the prime vertical Z N, above the horizon, in its progress towards the meridian or towards the horizon, the most favourable time for an observation of the altitude will be when the prime vertical is reached—that is, when the object is due east or due west—the azimuth being then 90°. If other circumstances—weather, proximity to the horizon, &c.—be unfavourable, then the nearer the position of the object to the prime vertical the better.

But if the co-latitude P Z exceed the co-declination P s′, then the object cannot arrive at the prime vertical; the azimuth P Z N will then be the greatest, and have the greatest sine, when the object is at S, the point in which the azimuth circle Z S N *touches* the parallel of declination s s′. It is plain that in this position the apparent motion of the object is nearly perpendicular to the horizon. (See Fig. 30).

The triangle P S Z (Fig. 29) is right-angled at S, for P S is the shortest arc from P to Z N, and is, therefore, at right angles to Z N; in this triangle the two sides P S, P Z, the co-declination and the co-latitude, are supposed to be given: we may, therefore, find the angle Z, or the bearing the object ought to have, so that the error in altitude may affect the time as little as possible; or we may find the angle P, the time from apparent noon, when the observation should be made. If it be not practicable to take the altitude when the object is in the most favourable position, then a position as near to it as possible should be chosen; and it must be remembered that, throughout the interval between the best position and the meridian, the nearer the object is to the meridian, the more unfavourably is it situated for the purpose of computing the time from an altitude; the formula (3) sufficiently shows this.

Fig. 30.



This formula (3) will of course serve for determining the error in the time consequent upon any assumed error in the altitude, at any observed azimuth; but as the angle S P S′ is expressed in minutes of the equinoctial, as appears from (1) above, it is necessary to divide by 15, to bring the measure into minutes of time: so that

$$\text{Error of app. time in minutes} = \frac{1}{15^{\circ}} \frac{\text{error of altitude}}{\text{cos. lat. sin. azimuth}}$$

For example. Suppose the latitude to be 51’ 31′ N.’ the azimuth of the object S. 48° 10′ E., and that the error of altitude is estimated at 10′. Required the error in the apparent time.

The above formula, put into logarithms, is,

$$\text{log. error of time} = \text{log. error of alt.} - \text{log. cos. lat.} - \text{log. sin. azimuth} - \text{log. } 15 + 20.$$

10	1·0000
cos. 51° 31′ Arith. Comp.	·2060
sin. 48° 10′ Arith. Comp.	·1278
15 Arith. Comp.	8·8239
1′·438	·1577

Hence the error in time is 1′·438 = 1′ 26″

It will, of course, be observed that if the latitude and azimuth remain the same, as also the error in altitude, the error in time will remain the same, however the altitude may vary.

TIME AT SHIP DEDUCED FROM A SINGLE ALTITUDE.—If the object observed be the sun, the hour-angle which its circle of declination, at the instant of observation, makes with the meridian, is the apparent time from that meridian. By applying the equation of time, given in the *Nautical Almanac*, and reduced by means of the longitude by account, to the estimated instant of observation, the apparent may be converted into mean time.

If the object be a star, the hour-angle its circle of declination makes with the meridian is a portion of sidereal time; it is the difference between the right ascension of the star, and the right ascension of the meridian. When the star is to the west of the meridian, its R. A. (right ascension) must be increased by the hour-angle; when it is to the east, its R. A. must be diminished by the hour-angle: the result is the R. A. of the meridian; and the difference between this and the sun's R. A. at the time of observation, is the time from the same meridian, and which is apparent, or mean time, according as the sun's R. A. is taken from p. 1 or p. 2 of the *Naut. Alm.*

If the object be the moon or a planet, the apparent and mean time are obtained just as in the case of a star.

To determine the hour-angle P from which the mean time at the ship is thus deduced, there are given in the spherical triangle P Z S, the co-latitude P Z, the co-declination P S, and the co-altitude Z S; hence, putting *s* for half the sum of these three sides, we have for cos. ½ P.

$$\text{cos. } \tfrac{1}{2} P = \sqrt{\frac{\text{sin. } s \text{ sin. } (s - ZS)}{\text{sin. } PZ \text{ sin. } PS}} \quad \text{(TRIGONOMETRY, p. 658).}$$

The co-altitude Z S, used in working this formula, is usually deduced from a set of altitudes taken within a minute or two of each other, as in the following examples:—

Examples.

1. *Time deduced from the Sun.*—On September 23, 1845, in latitude 50° 30′ N., and longitude by account 110° W., a set of altitudes of the sun were taken as below; the index correction was — 3′ 20″, and the height of the eye 20 feet. Required the mean time at the ship.

NOTE.—Besides a chronometer or two a ship always carries a good common watch, marking seconds, by which, what may be called the time at ship *by account*, or the estimated time, is kept. This may be regulated by the meridian altitudes, or by the double altitudes, or, as already noticed, by means of the chronometer and estimated longitude. For the purposes to which the watch is applied, an error of a few minutes is of no consequence; in the present problem the estimated time is used to get the declination and equation of time. The times recorded below are the mean times at the ship, as shown by the *watch*.

Altitudes of the Sun.	*Times per Watch.*
11° 4′ 40″	4h. 43m. 42s.
11° 2′ 25″	4h. 44m. 35s.
10° 59′ 45″	4h. 45m. 24s.
10° 56′ 30″	4h. 46m. 19s.
4)44° 3′ 20″	4)19h. 0m. 0s.
11° 0′ 50″	4h. 45m. 0s.

Mean obs. alt.		11° 0′ 50″
Index	−3′ 20″	
Dip	−4′ 24″	+8′ 14″
Semi-diam.	15′ 58″	
App. alt.		11° 9′ 4″
Ref. and Par.		−4′ 38″
True alt.		11° 4′ 26″
		90°
	∴ Z S =	78° 55′ 34″

Time nearly	4h. 45m. 0s. P.M.
Long. 110 W.	7h. 20m.
Mean time at Greenwich	12h. 5m.

Sun's dec. at noon Greenwich time	0° 6′ 56″ S. v. + $58\frac{1}{2}''$
Cor. for 12h. 5m. ($58\frac{1}{2}'' \times 12\frac{1}{12}$)	11′ 47″
	0° 18′ 43″ S.
	90°
	P S = 90° 18′ 43″

Also since P Z = 39° 30′, the computation of the hour-angle P, will be as follows, namely :—

Z S,	78° 55′ 34″		
Sin. P Z,	39° 30′ 0″	Arith. Comp.	·1964895
Sin. P S,	90° 18′ 43″	Arith. Comp.	·0000064
	2)208° 44′ 17″		
Sin. s,	104° 22′ 9″		9·9861968
Sin. (s − Z S),	25° 26′ 35″		9·6330782
			2)19·8157709
			9·9078854
cos. $\frac{1}{2}$ P,	36° 0′ 47″		
	2		
∴ P =	72° 1′ 34″		

Equation of Time.
Sep. 23 7m. 42s. var. 0″·858,*
Cor. for 12h., + 10s.
7m. 52s.

In time =	4h. 48m. 6s.	Apparent time
Equation of time	− 7m. 52s.	at ship.
	4h. 40m. 14s.	Mean time at ship.

In this result a quarter of a second has been disregarded, and we conclude that the watch is about 5m. *fast.*

It is of importance in discussing operations of this kind, the results of which are required to be brought out with the utmost attainable accuracy, that the student's confidence in such results be not shaken by the fact that certain of the data with which he works are confessedly erroneous, and are never more than approximations to the truth ; the estimated longitude, and the estimated time at the ship are, of course, more or less affected with error. He must take notice that these two elements do not enter directly into the trigonometrical process : they merely affect the preparatory reductions for declination and equation of time ; quantities that vary so little, even in a large interval of time, that a considerable error in the estimation of this interval, occasions but an inconsiderable error in the proper corrections. Hence, with but ordinary care in the dead-reckoning, and the proper accuracy in taking the altitude, the time at sea, determined as above, may be depended upon as correct, not only to the nearest *minute,* but even to the nearest *second* notwithstanding the acknowledged fact that certain of the data are only approximately true.

To illustrate this, let us modify the foregoing operation by applying the above correction to the watch, or by putting it back 5m. ; then the mean time at Greenwich, when the observed altitude of the sun at the ship was 11° 0′ 50″, will be 12h. The correction for declination will therefore be $58'' \frac{1}{2} \times 12 = 11' 42''$; hence, subtracting 5″ from the above value of P S, we shall have, for the more correct value, P S = 90° 18′ 38″ ; and the work, modified in accordance with this change in the co-declination of the sun, will stand as follows :—

Z S,	78° 55′ 34″		
sin. P Z,	39° 30′ 0″	Arith. Comp.	·1964895
sin. P S,	90° 18′ 38″	Arith. Comp.	·0000064
	2)208° 44′ 12″		
sin. s,	104° 22′ 6″		9·9861985
sin. (− Z S)	25° 26′ 32″		9·6330650
			2)19·8157594
cos. $\frac{1}{2}$ P,	36° 0′ 51″		9·9078797
	2		
∴ P =	72° 1′ 42″		

In time = 4h. 48m. 6s. $\frac{3}{4}$ } ∴ 4h. 40m. 14s. $\frac{3}{4}$, is
Equation of time — 7m. 52s. } the correct mean time at the ship.

The time, according to the former result, was 4h. 40m. 14s. $\frac{1}{4}$; for, as stated above, the fraction of a second was suppressed : we see, therefore, that the error of 5m. in the *estimated* time occasions an error of only half a second in the *correct* time. Suppose now that not only the estimated time, but that the estimated longitude is also affected with an error equivalent to 5m. of time—that is, an error of $1\frac{1}{4}$° in longitude, which is a large error ; and suppose that both errors conspire, making the mean time at Greenwich 12h. 10m. The correction for declination will then be $58\frac{1}{2}'' \times 12\frac{1}{6}$ = less than 11′ 52″ ; then P S will be 90° 18′ 48″, and the modified work will be as follows :—

Z S,	78° 55′ 34″		
sin. P Z,	39° 30′ 0″	Arith. Comp.	·1964895
sin. P S,	90° 18′ 48″	Arith. Comp.	·0000065
	2)208° 44′ 22″		
sin. s,	104° 22′ 11″		9·9861958
sin. (s − Z S),	25° 26′ 37″		9·6330871
			2)19·8157789
cos. $\frac{1}{2}$ P,	36° 0′ 45″		9·9078894
	2		
∴ P =	72° 1′ 30″		

In time = 4h. 48m. 6s. } ∴ 4h. 40m. 14s., is the
Equation of time − 7m. 52s. } mean time at the ship.

Hence, notwithstanding the above errors in the estimated time and longitude, the time at the ship is correctly deduced to within three-quarters of a second of the truth. Whenever there is a very considerable difference between the time per watch, and the calculated time at the ship, it will be prudent to repeat the work with the corrected time, as in the second of the foregoing operations.

2. *Time deduced from a Star.*—On June 3, 1842, at 12h. 9m. P.M., nearly (mean time) in latitude 50° 48′ N., and longitude by account 1° 6′ 3″ W., the observed altitude (or the mean of a set of altitudes) of *a* Bootis, west of the meridian, was 89° 53′ 30″, with the artificial horizon : the index correction was — 10″. Required the mean time at the ship.

As the artificial horizon was used, there will be no correction for dip (page 1076) : the angle shown by the instrument, when corrected for index error, is double the altitude from the rational horizon.

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* This is the variation of the equation of time in one hour, as given in the *Nautical Almanac.*

Angle by Instrument . . .	89° 53′ 30″
Index correction	— 10
	2)89 53 20
App. alt. of star	44 56 40
Refraction	— 58
True alt.	44 55 42
	90
∴ Z S =	45 4 18

Estimated time . . .	12h. 9m.
Long. 1° 6′ W. . . .	4m.
Mean time at Greenwich .	12h. 13m.
Sun's R. A. at noon, or the sidereal time . . .	4h. 46m. 7s.·1
Correction for 12h. 13m. .	+ 2m. 0s.·4
Sun's R. A. at time of obs. .	4h. 48m. 7s.·5

Stars R. A. 14h. 8m. 30s.·5, Dec. 20° 0′ 15″ N., ∴ P S = 69° 59′ 45″. Also P Z = 39° 12′.

Z S, 45° 4′ 18″		
sin. P Z, 39 12 0	Arith. Comp.	·1992628
sin. P S, 69 59 45	Arith. Comp.	·0270256
2)154 16 3		
sin. s, 77 8 2	. .	9·9889570
sin. (s — Z S), 32 3 44	. .	9·7249635
		2)19·9402089
cos. ½P, 21 1 1½	. .	9·9701044

∴ P = 42 2 3 or 2h. 48m. 8s·2 of sid. time.

Star's R. A. . .	14h. 8m. 30s.·5
R. A. of meridian .	16h. 56m. 38s.·7
R. A. of sun . .	4h. 48m. 7s.·5
Mean time at ship .	12h. 8m. 31s.·2

It will be observed here that the hour-angle P is expressed in sidereal time; and in like manner the right ascensions of the star, the meridian, and the mean sun, are all expressed in sidereal time: hence, 12h. 8m. 31s.·2 is the distance of the mean sun from the meridian in sidereal time; and this time in reference to the *mean* sun is mean time.

Examples for Exercise.

1. On January 12th, 1840, in latitude 30° 55′ N. and longitude 14° W., by account, the mean of a set of altitudes of the sun's lower limb was 22° 55′ 33″; the corresponding time by watch was 9h. 31′ 33″ A.M.; the index correction was + 4′ 30″, and the height of the eye 30 feet. Also the *Nautical Almanac* gave the following particulars, namely—

Sun's dec., Jan. 11th . .	21° 54′ 30″ S.
Hourly variation . . .	— 23″·42
Equa. of time, Jan. 11th .	8′ 3″
Hourly variation . . .	+ 0′ 1″
Sun's semi-diameter . .	16′ 16″

Required the mean time at the place of observation.

Ans. 9h. 41′ 24″ A.M.

2. On April 18th, 1844, in latitude 50° 48′ N., and longitude by account 1° 0′ W., the mean of a set of altitudes of the sun's lower limb (with artificial horizon) was 76° 16′ 46″; the corresponding time by watch was 9h. 18m. A.M.; the index correction was — 3′ 46″; and the *Nautical Almanac* gave the following particulars, namely—

Sun's dec., April 17th . .	10° 36′ 49″ N.
Hourly variation . . .	52″ 38
Equa. of time, April 17th .	0′ 31″·3
Hourly variation . . .	0′ 0″·573
Sun's semi-diameter . .	15′ 56″.

Required the mean time at the place of observation.

Ans. 9h. 18m. 16s. A.M.

3. On April 26th, 1840, in latitude 29° 47′ 45″ S., and longitude by account 31° 7′ E., at 2h. 19m. 41s. A.M. by watch, the true altitude of the star Altair was 25° 14′ 20″ to the east and north. Required the mean time at the place of observation.

R. A. of Altair 19h. 43m. Decli. 8° 26′ 47″ N. Sun's R. A., or sidereal time at mean noon, 2h. 18m. 21s.·1.

Ans. 1h. 45m. 11s. A.M.

NOTE.—As remarked at page 1093, the time may be found in a similar way when the object is the moon instead of the sun or a star; but, on account of the rapidity of the moon's motion in right ascension and declination, this body is the least eligible for the purpose of discovering the time at sea. The approximate time and the approximate longitude which, as we have seen in the case of the sun (page 1095), may be employed with safety, may lead to considerable uncertainty when the object is the moon, of which the declination changes sometimes more than 2′ in 10m. of time.

TIME DEDUCED FROM EQUAL ALTITUDES.—There is another way of determining the time, which ought to be briefly noticed. It is called the method of *equal altitudes:* the following exposition of it is principally from Lieut. Raper's *Practice of Navigation and Nautical Astronomy*—a work which well deserves the attention of the practical seaman.

Since the altitude of a body which does not change its declination, varies exactly at the same rate while rising on the E. side of the meridian, as while falling on the W. side, the same altitude occurs at the same hour-angle on each side of the meridian, and the middle point of time, between the instants of two equal altitudes, is the instant at which the body passes the meridian.

In the case of the sun, the middle point of time, or the mean of the observed times of equal altitudes, A.M. and P.M., is apparent noon. In the case of a star, or other body, the mean of these times corresponds to the R. A. of the star when upon the meridian, or the sidereal time, which may easily be converted into apparent time, or mean time.

Since the sun changes his declination sensibly in large intervals of time, two equal altitudes, A.M. and P.M., do not correspond to equal hour-angles; and it therefore becomes necessary to apply to the mean of the observed times a correction which is called the *Equation of Equal Altitudes.* A table of the equation of equal altitudes is to be found in all nautical tables. Also at sea, the change of place of the ship, in the intervals between the altitudes, will generally render another correction necessary. When the course made good is true E. or W., the ship changes in her longitude only, by the portion of time which she gains or loses on the sun in intervals. This change, provided it is made good equally on both sides of the meridian, introduces no correction; and the only question is, the time by watch when the interval is half expired.

But when the ship changes her latitude, the same altitude no longer corresponds to the same time from noon, and the correction adverted to becomes necessary.

This method has some advantages:—it is independent of the terrestrial refraction, provided this remains unchanged in the interval employed; and the correction, when necessary, requires the latitude and altitude to be but roughly known. Within the tropics, the interval may in general be very small, on account of the rapid change of the altitude; and the correction for change of latitude may, in such cases, be omitted. In high latitudes, however, the ship's change of latitude considerably alters the time from noon, at which the second altitude (equal to the first) is taken; hence, in such high latitudes the method is less useful.

The general mode of proceeding is this:—Observe the sun's altitude shortly before noon, and note the time. Note the instant of the same altitude in the afternoon. For great accuracy, several pairs of equal altitudes should be obtained. Take the mean of the A.M. and P.M. times by watch: this, when the ship does not change her latitude, is the time by watch of apparent noon. To render any correction for change of latitude unnecessary, it would be desirable to alter the course of the ship, so as to preclude this change, when the interval between the observations is but short, as it should be to warrant the dispensing with correction for change of declination.

On account of the constancy of its declination, a star is the best suited for the purpose of the present problem; but a change in the state of the atmosphere, during the interval of the observations, may interfere with the precision of the result, especially in the case of low altitudes; attention to the indications of the barometer and thermometer is therefore necessary to the attainment of accuracy. But the observations, to be free from all source of error, should be made on shore; for if the interval be large, there is, at sea, some uncertainty as to the correction for change of latitude; and if the interval be small, there is some uncertainty as to the altitudes, especially in high latitudes, on account of the slowness with which altitude changes near the meridian.

ON FINDING THE LONGITUDE BY A CHRONOMETER.—From what is shown in the preceding articles, it appears that the time at the ship can be accurately obtained by means of altitudes of the sun or fixed stars, assisted by the longitude, by dead-reckoning, and the time by estimation; and therefore all that is necessary for the determination of the ship's longitude is, that we know also the time at Greenwich at the same instant. If we can only discover what o'clock it is at the same instant at two different places on the globe, however distant, we may at once infer the difference of longitude of those places. We have only to convert the difference of time into degrees, at the rate of 15° to one hour, to effect this object. To find the time at Greenwich at any instant is therefore to solve the problem of the longitude; and as noticed at page 1092, it is to supply this important information that so much skill and perseverance have been bestowed on the *chronometer.*

It is not necessary to the perfection of this instrument that it should show the exact time at Greenwich, which, in fact, a chronometer never does; what is required is, that its action be uniform and regular. The difference between Greenwich mean time at any instant and the time shown by the chronometer, is the *error* of the chronometer at that instant; and its average gain or loss in 24 hours is its *mean daily rate.* There is a depôt established at the Royal Observatory, Greenwich, for the reception of such chronometers as the makers choose to send, and where their errors and rates are determined by the Astronomer Royal; and few persons would now purchase a chronometer for use at sea, without the proper official certificate of these particulars. If the original error on Greenwich mean time be allowed for, and the proper correction be made for the accumulated daily rate, the Greenwich time at any instant will of course be correctly obtained, provided that nothing has disturbed the action of the chronometer after it left the observatory.

It will have been observed in the foregoing articles, that the estimated longitude and time at the ship have been employed solely for the purpose of getting the Greenwich time when the altitude was taken, with sufficient accuracy to enable us to get the co-declination or polar distance of the object observed; and we have seen that even a considerable error in the estimated quantities will not sensibly affect the precision of the result. But if the chronometer can at all be depended upon, the Greenwich time of the observation will become known at once, and, unless great derangement of the chronometer have taken place, with more accuracy than it can be found by the estimated longitude and time at the ship. The problem just disposed of may therefore be correctly solved by aid of even an indifferent chronometer; but if the time at Greenwich, after the corrections for error and rate, be not correctly furnished by the instrument, there will of course be a proportionate error in the longitude inferred from it.

We are anxious that the student should clearly understand how it happens that, from erroneous data, we obtain accurate results, even in a matter of so much delicacy as the determination of the time at sea. The errors referred to can affect only two elements of the calculation—the declination and the equation of time; and from the slowness with which these vary, a few minutes, more or less, can have no sensible influence.

ON FINDING THE RATE OF A CHRONOMETER.—It is evident that if the rate of a chronometer remained invariable, the time at Greenwich could always be accurately deduced from it; but a sudden jerk or concussion, change of climate, local attraction, and even the motion of transporting it from the observatory to the ship, may cause the rate to vary. The scientific navigator, therefore, avails himself of every favourable opportunity of ascertaining the rate of his chronometer. This is best done when the ship is in harbour, and in the neighbourhood of a fixed observatory, at which the rate may be ascertained like as it was at Greenwich. For this purpose the chronometer need not be carried on shore. The time may be taken from it by a good seconds-watch, and the watch then carried to the observatory and compared with the mean-time clock—the error of the chronometer, on the mean time at that place, will thus be seen; and this comparison being repeated day after day, the daily deviation from the exact amount of the error will be discovered, and hence the mean daily rate. The ship then sails afresh with the time at the new meridian, and therefore—the longitude of that meridian being known—with the Greenwich time.

Or, without resorting to a fixed observatory, the rate may be ascertained, when the ship is in harbour, by finding the mean time at the place, as explained in last problem. A comparison of this with the mean time shown by the chronometer will, in like manner, show the error on mean time at the place; and the observations being repeated day after day, the rate will become known. The altitudes may in general be obtained with greater precision by taking them on shore, by means of an artificial horizon, the chronometer time being carried there by the watch.

Every attention should be paid on ship-board to circumstances likely to interfere with the rate of the chronometer: it should be kept as much as possible out of the influence of changes of temperature, and its horizontal position should remain undisturbed: to secure this permanence of position, it is frequently suspended on gimbals, like the compass. It should be wound up by the action of the key alone, the chronometer itself being kept steady; if the instrument be turned as well as the key, the motion of the balance-wheel will be affected, and the rate disturbed. We have hitherto spoken of *the* chronometer, as if ships in general carried only one; but in long voyages, or in voyages undertaken for scientific purposes, ships usually take several chronometers as checks to one another, and to provide against accident. Captain Fitzroy, in his surveys of the coast of South America, took so many as twenty-two chronometers. The apartment in which the chronometers are kept is called the chronometer-room, and the temperature of it is sometimes regulated by lamps, with the aid of the thermometer. This should be the permanent depository of the time-keepers, and they should be meddled with only for the purpose of winding them up, which should be at the same regular interval; and, of course, they should not be allowed to run down.

The error and rate of a chronometer being found, as explained above, the longitude is determined, as in the following example, from Captain Kater's article in the *Encyclopædia Metropolitana.*

Examples.

1. On the 2nd of June, 1823, the true altitude of the sun's centre was 30° 2′, when the chronometer showed 5h. 1m. 0s.; the latitude was 40° 5 N.; and the sun's

declination at the time of observation, 22° 9′ 17″ N. The chronometer on the 20th of May preceding was 45s. slow for Greenwich time: its daily rate was 2s.·1 losing. Required the longitude of the ship.

1. *Apparent Time at Greenwich from Chronometer.*

	h. m. s.
Time by chronometer . :	5 1 0
Slow, May 20 . . .	+ 45
Loss from May 20 to June 2 .	+ 25·2
Mean time at Greenwich . .	5 2 10·2
Corresponding equation of time .	+ 2 31·4
App. time at Greenwich . .	5 4 41·6

2. *Apparent Time at Ship from Triangle* P Z S, Fig. 29 Here Z S = 59° 58′, P S = 67° 50 43′, PZ = 49° 55′, to find P.

Z S, 59° 58′ 0″	Arith. Comp.	·0333098
sin. P S, 67 50 43	Arith. Comp.	·1162768
sin. P Z, 49 55 0		
2)177 43 43		
sin. s, 88 51 51		9·9999146
sin. (s — Z S), 28 53 51		9·6841666
		2)19·8336678
cos. ½ P, 34° 20′ 17·6″		9·9168339
2		
∴ P = 68 40 35·2		
2		
3)13,6 8,0 7,0·4		
4 32		
2 40	(See page 1079).	
2·3		

App. time from noon at ship . .	4h.	34′	42·3″
App. time at Greenwich (by chron.)	5	4	41·6
Longitude W. in time	0	29	59·3

29m. =	7°	15′		
59s. ·3 =		14	49·5	(See page 1088).
LONGITUDE W.	7	29	49·5	

2. On the 28th of May, 1823, in latitude 0° 50′ N., the mean of several observed altitudes of the star Antares, when eastward of the meridian, was 30° 42′, and the corresponding time by the chronometer was 9h 35m. 43s. The chronometer was too fast 1m. 5s. on April 20, at noon, at Greenwich, and its daily gain was 5s. ·4. The height of the eye was 16 feet. Required the longitude of the ship.

Apparent Time at Greenwich from Chronometer.

		h. m. s.
Time by chronometer . .		9 35 43
Fast, April 20 . .	— 1′ 5″	
Gain, from Apr. 20 to May 28	— 3 27·5	
	=	— 4 32·5
Meantime at Greenwich . .		9 31 10·5
Corresponding equation of time .		+ 8 9·2
App. time at Greenwich . .		9 34 19·7

2. *Apparent Time at Ship from Triangle* P Z S.

Mean of observed altitudes . .	30° 42′ 0″
Dip .	— 3 50
Star's apparent altitudes . .	30 38 10
Refraction . .	— 1 39
Star's true altitude	30 36 31

Z S, 59° 23′ 29″		
sin. P S, 116 1 37	Arith. Comp.	·0464395
sin. P Z, 89 10 0	Arith. Comp.	·0000459
2)264 35 6		
sin. s, 132 17 33		9·8690669
sin. (s — Z S), 72 54 4		9·9803665
		2)19·8959188
cos. ½ P, 27 29 32		9·9479594
2		
∴ P = 54 59 4		
2		
3)10,8 11,8 ,8		
3 36		
3 56·3	(See page 1079).	

Polar angle in sid. time .	3h.	39′	56·3″
Star's R.A. . .	16	18	31·2
R.A. of merid. .	12	38	34·9
Sun's R.A May 28, at 9h 34m. 20s. app. time	4	19	17
App. time at ship	8	18	17·9
App. time at Greenwich	9	34	19·7
Long. W. in time	1	16	1·8

Long. W. = 19° 0′ 28″.

Examples for Exercise.

1. On September 23rd, 1845, in the afternoon, and in latitude 50° 30′ N., the observed altitude of the sun's lower limb was 11° 0′ 50″, the index correction being— 3′ 20″, and the height of the eye 20 feet. The chronometer showed 11h. 59m. 30s.; it was fast on Greenwich mean time 45s. ·5 on August 21st, and its daily rate was 5s. ·7 losing; the sun's declination at the time of observation being 0° 18′ 41″ S., and the equation of time 7m. 52s. ·3; also the sun's semi-diameter was 15′ 58″. Required the longitude. Ans. Long. 110° 18′ 10″ W.

2. In the afternoon of October 18th, 1841, in latitude 15° 46′ N., the observed altitude of the sun's lower limb was 12° 40′: the chronometer showed 11h. 12m. 42s. Greenwich time of October 17th: its error August 12th, at noon, was 5m. 26s. slow, and its daily rate was 12s. ·5 gaining. Required the longitude, the index correction being + 2′ 30″, the height of the eye 16 feet, and the following particulars being furnished by the *Nautical Almanac*, namely—

Sun's dec. October 17th, 9° 18′ 8″ S. Hourly diff. + 54″·81. Eq. of time 14m. 34s. ·3. Hourly diff. + 0″·483. Sun's diameter 16′ 5″. Ans. Long. 83° 51′ 45″ E.

3. On September 10th, 1844, in latitude 48° 20′ N., an afternoon altitude of Arcturus was observed to be 31° 5′ 40″, the star west of the meridian: the chronometer showed 5h. 1m. 28s.: its rate August 25th, was 4s. ·3 gaining, and it was 2m. 40s. slow on Greenwich mean time. The index correction was — 4′ 10″, and the height of the eye 20 feet: also the sun's R. A. at the time of observation was 11h. 19m. 18s.; the star's R. A. 14h. 8m. 34s. ·65, and its declination 14° 59′ 44″ N. Required the longitude. Ans. Long. 32° 8′ 20″ E.

ON FINDING THE LONGITUDE AT SEA BY LUNAR OBSERVATIONS.—The reader has sufficiently seen that the difference of longitude between two places on the surface of the earth, is virtually the same as the difference of time between those two places at the same absolute instant. Suppose, for example, that a rocket could be projected so high, and give out light of such intensity, as to be seen at the instant of explosion by two observers thirty or forty miles apart: the explosion takes place, of course, at a certain instant of absolute time: the mean-time clock at Greenwich might mark this instant—in the local reckoning of that place—as 3h. 20m.: an observer to the east of Greenwich might find the time at *his* place to be 3h. 22m.: the difference in the local reckoning would thus give a difference of 2m. in the time, although both observers saw the flash at the same absolute instant; and we should conclude that the easterly place of observation had half a degree, or 30 nautical miles of east longitude. It is the same in reference to any other phenomenon: let only the time at Greenwich be noted which marks the instant of its occurrence, and also the time at any other locality which marks the same instant; the difference of the times thus noted will be the longitude, in time, of that other locality from Greenwich.*

The heavenly bodies furnish signals in abundance analogous to the rocket-signal here imagined, and astronomers have only to make their selections suitably to the circumstances of the observers: what are called *the lunar observations* are the best adapted to determine the longitude at *sea*. Certain stars, lying in or very near to the moon's path, are chosen, and the distance of the moon from each of these, as also from the sun at noon (Greenwich time), and at every interval of three hours, is predicted and recorded in the *Nautical Almanac* for every day in the year. An observer at sea measures one of these distances, and refers to the *Nautical Almanac* for the Greenwich time when the same phenomenon happened; he finds, most likely, that *his* distance is intermediate between a certain *pair* of the three-hourly distances; he knows, therefore, that the Greenwich time is intermediate between the recorded hours; and, just as in the case of any other varying quantity—given in the *Almanac* for regular intervals of time—he calculates the intermediate time corresponding to the intermediate distance, by proportion. The motion of the moon is found to be sufficiently uniform, for a period of three hours, to justify an intermediate position being inferred in this way, and it is sufficiently rapid to render her change of place sensible even in two or three seconds of time; a trifling correction for variable motion will, however, be noticed hereafter.

The lunar observations thus enable the mariner to discover the time at Greenwich independently of the chronometer, and thence to determine the error of his time-keeper; and, therefore, by taking the difference between this error and that last recorded, to apply the proper correction to the mean daily rate. The lunar observations *alone* will not enable the navigator to find his longitude; but they enable him to find what o'clock it is at Greenwich at the instant the lunar distance was taken, and thus to correct his chronometer.

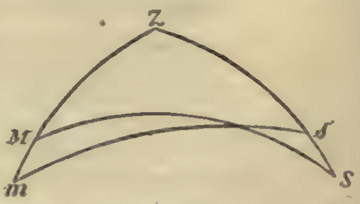
Knowing, in this way, the time at Greenwich, we can very accurately compute the declination of the sun at the time of observation, or its right ascension, if the distance be that between the moon and a star, and thence, by means of the latitude of the place, supposed to be already known, we can find, as explained at page 1094, *et seq.*, the time at that place, and thence the longitude.

It must be remarked that the *observed* distance between two celestial objects is not the *true* distance, any more than their observed altitudes are the true altitudes; these latter—the true altitudes—it is necessary to find, in order, from the observed, to compute the true distance; for it is the true distance, as measured from the centre of the earth, that is predicted in the *Nautical Almanac*: in other words, it is the observed distance cleared from the effects of parallax and refraction. Such being the case, the reader will perceive that the deduction of the Greenwich time, from an observed distance, is not wholly independent of all local information to be afforded by the ship's account: the estimated time at the ship, and the estimated longitude, are evidently useful for the purpose of enabling us to get the semi-diameter and horizontal parallax of the *moon* with greater precision; for these quantities, especially the latter, sensibly vary from noon to midnight, and they are elements which necessarily enter the calculations for reducing the moon's observed altitude to the true altitude.

We shall now investigate formulæ for computing the true distance of the moon's centre from that of the sun, or from a fixed star, by means of the apparent and true altitudes of the two objects, and their apparent distance—that is, the observed distance corrected for semi-diameter.

INVESTIGATION OF FORMULA FOR CLEARING THE LUNAR DISTANCE.—In the annexed diagram (Fig. 31), let Z represent the zenith of the place of observation, and Z M, Z S the two verticals on which the objects are observed. Let *m*, *s* be the observed places of the moon and sun, or of the moon and a fixed star, and let M, S be their true places. As the moon is depressed by parallax more than it is elevated by refraction, M will be above *m*; but the sun, on the contrary, being more elevated by refraction than depressed by parallax, S will be below *s*.

Fig. 31.



The corrections for dip, index error, and semi-diameter being applied, observation gives the apparent zenith distances, Z *m*, Z *s*, and the apparent distance *m s* of the objects themselves; and the proper corrections for parallax and refraction being applied to the apparent altitudes, we get the true zenith distances Z M, Z S, and the object of the present investigation is to determine the true distance M S by computation.

Let d stand for the apparent distance
D . . . true distance
a, a' . . . apparent altitudes
A, A' . . . true altitudes

By Spherical Trigonometry (page 658), the triangle M Z S gives for cos. Z, the following expression, namely:—

$$\text{ocs. Z} = \frac{\text{cos. D} - \text{sin. A sin. A}'}{\text{cos. A cos. A}'}$$

and the triangle *m* Z *s* gives in like manner

$$\text{cos. Z} = \frac{\text{cos. } d - \text{sin. } a \text{ sin. } a'}{\text{cos. } a \text{ cos. } a'}$$

Hence, for the determination of D we have the equation

$$\frac{\text{cos. D} - \text{sin. A sin. A}}{\text{cos. A cos. A}'} = \frac{\text{cos. } d - \text{sin. } a \text{ sin. } a'}{\text{cos. } a \text{ cos. } a'}$$

from which we have for cos. D the expression

$$\text{cos. D} = \frac{\text{cos. } d - \text{sin. } a \text{ sin. } a'}{\text{cos. } a \text{ cos. } a'} \text{cos. A cos. A}' + \text{sin. A sin. A}.$$

But since cos. $(a + a') = \text{cos. } a \text{ cos. } a' - \text{sin. } a \text{ sin. } a'$, this is the same as

$$\text{cos. D} = \frac{\text{cos. } d + \text{cos. } (a + a') - \text{cos. } a \text{ cos. } a'}{\text{cos. } a \text{ cos. } a'}$$

$$\text{cos. A cos. A}' + \text{sin. A sin. A}'$$

$$= \left\{ \frac{\text{cos. } d + \text{cos. } (a + a)}{\text{cos. } a \text{ cos. } a'} - 1 \right\} \text{cos. A cos. A}' + \text{sin. A sin. A}'.$$

If the -1 within the braces be suppressed, we may restore it by adding cos. A cos. A' at the end, as is obvious. Hence

$$\text{cos. D} = \frac{\text{cos. } d + \text{cos. } (a + a')}{\text{cos. } a \text{ cos. } a'} \text{cos. A cos. A}' - \text{cos. } (\text{A} + \text{A}')$$

The numerator of the fraction in this expression being

---

[*] The electric telegraph has been largely used on similar principles throughout Europe during the last few years, for accurately determining the longitudes of various stations.—ED.

the sum of two cosines, we may replace it by a product from the formula (page 628), namely :—

$$\cos.\ \theta + \cos.\ \phi = 2 \cos.\ \tfrac{1}{2}(\theta + \phi) \cos.\ \tfrac{1}{2}(\theta \backsim \phi)$$

so that we have

$$\cos.\ D = \frac{2 \cos.\ \frac{1}{2}\left\{(a + a') + d\right\} \cos.\ \frac{1}{2}\left\{(a + a') \backsim d\right\}}{\cos.\ a \cos.\ a'}$$

$$\cos.\ A \cos.\ A' - \cos.\ (A + A')$$

Subtract each side of this equation from 1, then since by Trigonometry (page 628),

$$1 - \cos.\ D = 2 \sin.^2 \tfrac{1}{2} D, \text{ and } 1 + \cos.\ (A + A') = 2 \cos.^2 \tfrac{1}{2}(A + A')$$

the equation after division by 2 becomes

$$\sin.^2 \tfrac{1}{2} D = \cos.^2 \tfrac{1}{2}(A + A')$$

$$- \frac{\cos.\ \frac{1}{2}\left\{(a + a') + d\right\} \cos.\ \frac{1}{2}\left\{(a + a') \backsim d\right\} \cos.\ A \cos\ A'}{\cos.\ a \cos.\ a'}$$

$$= \cos.^2 \tfrac{1}{2}(A + A')$$

$$\left\{1 - \frac{\cos.\ \frac{1}{2}\left\{(a + a') + d\right\} \cos.\ \frac{1}{2}\left\{(a + a') \backsim d\right\} \cos.\ A \cos.\ A'}{\cos.\ a \cos.\ a' \cos.^2 \frac{1}{2}(A + A')}\right\}$$

Or, calling the second of these factors $1 - \sin.^2 \theta$, that is, $\cos.^2 \theta$, we have

$$\sin.^2 \tfrac{1}{2} D = \cos.^2 \tfrac{1}{2}(A + A') \cos.^2 \theta$$

$$\therefore \sin.\ \tfrac{1}{2} D = \cos.\ \tfrac{1}{2}(A + A') \cos.\ \theta$$

which is known by the name of the formula of Borda. This formula consists, we see, of two parts: the first part determines the arc θ, and the second by means of θ computes D. The working model is therefore this, namely :—

$$\left.\begin{array}{l} \sin.\ \theta = \sqrt{\dfrac{\cos.\ \frac{1}{2}\left\{(a + a') + d\right\} \cos.\ \frac{1}{2}\left\{(a + a') \backsim d\right\} \cos.\ A \cos.\ A'}{\cos.\ a \cos.\ a' \cos.^2 \frac{1}{2}(A + A')}} \\ \sin.\ \frac{1}{2} D = \cos.\ \frac{1}{2}(A + A') \cos.\ \theta \end{array}\right\} (1)$$

from which we see that the entire work for determining the true distance D may be conducted by logarithms.

It may be worth while to remark, that we were fully justified in assuming that the fraction in the foregoing expression for $\sin.^2 \frac{1}{2} D$ could be represented by $\sin.^2 \theta$; for this was only assuming that the fraction must be less than unity, or that the expression within the braces is positive, which it necessarily is, because $\sin.^2 \frac{1}{2} D$ is necessarily positive. A desirable feature in this method of clearing the lunar distance is, that sines and cosines are the only trigonometrical quantities employed in it; a circumstance which not only affords relief to the memory, but, at the same time, facilitates reference to the tables.

Rule for Clearing the Lunar Distance.—The steps for calculating D from the formula just established may be described in words as follows :—

Rule.—1. Add together the apparent distance and the apparent altitudes; take the difference between half their sum and the apparent distance, and underneath the result write the true altitudes; and, leaving a gap for two lines, then write half the sum of the true altitudes: the *whole* sum may be interposed in the space thus left.

2. These directions having been complied with, it will be found that *nine* arcs have been written down, the apparent distance being the first. Then, disregarding the first arc, as also the fourth arc, which is the sum of the preceding three, write opposite to each of the other *seven*, towards the right, the word *cosine*, prefixing, however, "*complement*" to the first two. Refer to the table of sines and cosines for the quantities thus indicated, putting a *plus* mark after the last cosine, and a *minus* mark in front of it.

3. Add up the first six of the numbers thus placed in column, divide the sum by 2, and subtract the cosine previously marked with the *minus*; the remainder will be sin. θ

[These three precepts conduct us, therefore, to the value of the first of the two expressions in the formula (1) above: the fourth precept, following, enables us to compute the second expression].

4. Take out cos. θ, putting a *plus* mark after it, and add it to the cosine with the + after it; the sum will be sin. $\frac{1}{2}$ D; taking, therefore, the corresponding arc $\frac{1}{2}$ D, out of the tables, and multiplying it by 2, we have the true distance sought.

Examples.

1. Suppose the apparent distance between the centres of the sun and moon to be 83° 57′ 33″; the apparent altitude of the moon's centre 27° 34′ 5″; the apparent altitude of the sun's centre 48° 27′ 32″; the true altitude of the moon's centre 28° 20′ 48″; and the true altitude of the sun's centre 48° 26′ 49″. Required the true distance.

Here $d = 83° 57' 33''$, $a = 27° 34' 5''$, $a' = 48° 27' 32''$
$A = 28° 20' 48''$, $A' = 48° 26' 49''$;

and proceeding by the rule, the work will stand thus :—

d	83°	57′	33″			
a	27	34	5	comp. cos.	·0523390	
a'	48	27	32	comp. cos.	·1783835	
2)	159	59	10			
½ sum	79	59	35	cos.	9·2399686	
½ sum ∽ d	3	57	58	cos.	9·9989587	
A	28	20	48	cos.	9.9445275	
A′	48	26	49	cos.	9·8217187	
A + A′	76	47	37		2)39·2358960	
					19·6179480	
½ (A + A′)	38	23	48½	−cos.	9·8941654 +	These two being added together
θ	31	57	53½	sin.	9 7237826	
θ	.	.	,	cos.	9·9285870 +	
½ D	41	40	27	sin.	9·8227524	= their sum.
∴ D =	83	20	54			

In the foregoing operation more attention is paid to minute quantities than is necessary in actual practice. Fractions of a second are of course always disregarded at sea: seconds themselves are, indeed, very frequently neglected; but this negligence in problems like the present is by no means to be commended. A table of sines and cosines computed to every ten seconds—such as the *Tables Portatives* of Callet—will enable the computer to take account of his seconds, without entailing upon him any extra work worth mentioning. Tables of logarithmic sines and cosines, computed to every ten seconds, will also be found in the *Nautical Tables* of Inman and Mackay.

The reader will have observed that, agreeably to what is recommended at page 1048 of the Introduction, the preceding rule directs that the table employed should not be applied to, till the work can be advanced no further without its aid; and that when it is once in hand, all the extracts from it, previously to the determination of θ, should be made before it is laid down. As a and A are always pretty close together, cos. A may be taken out of the table immediately after comp. cos. a; and as a' is, in like manner, in close neighbourhood to A′, cos. A′ may be taken out immediately after comp. cos. a'.

The above is, of course, a rigorous method of clearing the lunar distance from the effects of parallax and refraction, and provided only that the data furnished by the observations be correct, is perfectly accurate. By help of an auxiliary table, like that which forms Table XLII., in the second volume of Dr. Mackay's *Treatise on the Longitude*, the operation may be shortened. The table alluded to supplies the value of the expression log. $\frac{\cos.\ A \cos.\ A'}{\cos.\ a \cos.\ a'}$, which is called the *Logarithmic Difference*, and unlike some of the subsidiary tables, for abridging the calculations required in the direct methods of working the present problem, it may always be used with confidence; this part of the trigonometrical operation may therefore be shortened without any sacrifice of accuracy. In general, however, the indirect methods of clearing the lunar distance, though often shorter than the direct methods, are proportionately deficient in precision. The author conceives that the strict accuracy of the results, and the unhesitating confidence that may consequently be always placed in the conclusions of the direct methods, by Spherical Trigonometry, produce a degree of satisfaction which more than compensates for the trouble of a few additional references to those common tables, with which every calculator may reasonably be expected to be abundantly familiar.

It will, no doubt, occur to the reader, that instead of comp. cos., secant may be used; observing to reject the index 10 from each secant.

2. The apparent distance of the moon's centre from the star Regulus was 63° 35′ 14″; the apparent altitude of the moon's centre 24° 29′ 44″; the apparent altitude of the star 45° 9′ 12″; the true altitude of the moon's centre 25° 17′ 45″; and the true altitude of the star 45° 8′ 15″. Required the true distance.

d 63° 35′ 14″			
a 24 29 44	comp. cos.	·0409617	
a' 45 9 12	comp. cos.	·1516803	
2)133 14 10			
½ sum 66 37 5	cos.	9·5986359	
½ sum ∽ d 3 1 51	cos.	9·9993921	
A 25 17 45	cos.	9·9562230	
A′ 45 8 15	cos.	9·8484402	
A + A′ 70 26 0		2)39·5953332	
		19·7976666	
½(A + A′) 35 13 0	− cos.	9·9122099+	These two being added together
θ 50 11 23	sin.	9·8854567	
θ . . .	cos.	9·8063479+	
½ D 31 32 17½	sin.	9·7185578 = their sum.	
∴ D = 63 4 35			

On Making the Observations.—In taking a lunar distance at sea, it is desirable that there should be *three* observers; one of these measures the distance between the limbs of the sun and moon, or between the limb of the moon and the selected star, and the other two take the altitudes of the objects, at the instant the distance is obtained. A single distance, and a single corresponding pair of altitudes, are not in general considered as sufficient; but a set of distances, and a set of corresponding altitudes, are usually taken, allowing as little time as possible to intervene between each observation: the mean of the distance, and the corresponding means of the altitudes, are then employed in the calculation.

If, however, there be but one observer, it will be necessary that he have the aid of an assistant, to note by the watch intervals of time, the observer proceeding as follows. 1, Let the altitude of the sun or star be taken; 2, then the altitude of the moon; 3, a set of distances; 4, another altitude of the moon; 5, another altitude of the sun or star; the times of each observation being noted. Then let the means of the distances and times of observing them be taken; and reduce the altitudes to the mean time thus found by proportion. These are the directions given by Norie; but Lieut. Raper, an experienced practical navigator, recommends as follows:—When the observer is alone, he will first observe the altitude of the object farthest from the meridian, then that of the other object, and then the distance, concluding with the altitudes in the reverse order; the reason of this order is, that the outer object preserves uniformity in its change of altitude for a longer time than the other, and consequently its altitude may be reduced, by simple proportion, to an intermediate time, with less error than the altitude of the other object; we may add, however, that it is desirable that the outer object should not be the moon.

When the ship has much motion, the observer fixes himself in a corner, or lies on his back on the deck, to take the distance, in order to remove, as much as possible, the sense of bodily effort and inconvenience which disturbs the eye and the attention. Three or five distances, at least, should be taken; less precision is necessary in taking the altitudes than in observing the distance, so that one altitude of each object, taken with ordinary care, will in general suffice, when the time at the ship is not to be deduced as well as the true distance.

Other Methods of Clearing the Distance.—As the problem of deducing the true lunar distance from the observed distance is one of such note and importance in Nautical Astronomy, we shall present the reader with some other methods, all, like that of Borda just given, conducted by the rigorous principles of Spherical Trigonometry, and requiring only the common logarithmic and trigonometrical tables; that is, only the logarithms of numbers, and the natural and logarithmic sines and cosines.

The first of these additional methods which we shall offer is that of Delambre, the formula for which occurs in the investigation of the method given at page 1100; namely—cos. D

$$= \frac{2 \cos. \tfrac{1}{2}\{(a + a') + d\} \cos. \tfrac{1}{2}\{(a + a') \backsim d\} \cos. A \cos. A}{\cos. a \cos. a'}$$

$$- \cos. (A + A').$$

The logarithm of the first expression in this formula is calculated as in the method of Borda; then the natural number answering to that logarithm is taken out of the table, and the natural cosine of (A + A′) subtracted from it; the remainder is the natural cosine of the true distance. As this is a sort of mixed method, requiring reference to both logarithmic and natural cosines, attention must be paid to the change of radius in passing from one to the other: the expression which it is here proposed to find the common logarithm of, will supply four additive logarithms and two subtractive ones; hence two tens, or 20, must be suppressed in the result (page 1038); but if the arithmetical complements of log. cos. a, log. cos. a', be *added*, then four tens, or 40, must be suppressed; and the result will then be, the ordinary logarithm of the expression; and from the number answering thereto, the nat. cos. (A + A′) is to be subtracted, as the formula indicates.

Taking the example at page 1100, in which

$$d = 83° 57' 33'', \ a = 27° 34' 5'', \ a' = 48° 27' 32''$$
$$A = 28° 20' 48'', \ A' = 48° 26' 49''$$

The operation by the above method of Delambre will stand arranged as follows:—

d 83° 57′ 33″ . .	comp. cos.	·0523390
a 27 34 5 . .	comp. cos.	·1783835
a' 48 27 32 . .	log. 2	·3010300
2)159 59 10		
½ sum 79 59 35 . .	cos.	9·2399686
½ sum ∽ d 3 57 58 . .	cos.	9·9989587
A 28 20 48 . .	cos.	9·9445275
A′ 48 26 49 . .	cos.	9·8217187
	log.	·3442921 = $\bar{1}$·5369260 (40 suppressed).
A + A′ 76 47 37	nat. cos.	·2284595
D 83 20 54	nat. cos.	·1158326

Other formulæ may be investigated as follows:—

Referring to the two expressions for cos. Z at page 1099, we have

$$1 + \cos. Z = \frac{\cos. D + \cos. A \cos. A' - \sin. A \sin. A'}{\cos. A \cos. A'}$$

$$= \frac{\cos. D + \cos. (A + A}{\cos. A \cos. A'}$$

$$1 + \cos. Z = \frac{\cos. d + \cos. a \cos. a' - \sin. a \sin. a}{\cos. a \cos. a'}$$

$$= \frac{\cos. d + \cos. (a + a')}{\cos. a \cos. a'}$$

$$\therefore \frac{\cos. D. + \cos. (A + A')}{\cos. A \cos. A'} = \frac{\cos. d + (\cos. a + a')}{\cos. a \cos. a'}$$

$$\therefore \cos. D = \left\{ \cos. d + \cos. (a + a') \right\} \frac{\cos. A \cos. A'}{\cos. a \cos. a'}$$

$$- \cos. (A + A') \quad . \quad . \quad . \quad (1)$$

From the same two expressions for cos. Z, we also have

$$1 - \cos. Z = \frac{\cos. A \cos. A' + \sin. A \sin. A' - \cos. D}{\cos. A \cos. A'}$$

$$= \frac{\cos. (A \backsim A') - \cos. D}{\cos. A \cos. A'}$$

$$1 - \cos. Z = \frac{\cos. a \cos. a' + \sin. a \sin. a' - \cos. d}{\cos. a \cos. a'}$$

$$= \frac{\cos. (a \backsim a') - \cos. d}{\cos. a \cos. a'}$$

$$\therefore \frac{\cos. (A \backsim A') - \cos. D}{\cos. A \cos. A'} = \frac{\cos. (a \backsim a') - \cos. d}{\cos. a \cos. a'}$$

$$\therefore \cos. D = \left\{ \cos. d - \cos. (a \backsim a') \right\} \frac{\cos. A \cos. A'}{\cos. a \cos. a'}$$

$$+ \cos. (A \backsim A') \quad . \quad . \quad . \quad . \quad (2)$$

The formula marked (1) and (2) are tolerably commodious by help of the common logarithmic tables; as an exemplification of their advantages, we shall solve the example at page 1101 by each. In this example the following quantities are given, namely—

$$d = 63° 35' 14'', \; a = 24° 29' 44'' \; a' = 45° 9' 12''$$
$$A = 25° 17' 45'', \; A' = 45° 8' 15''.$$

Determination of D *by formula* (1).

			Logarithms.
d 63° 35′ 14″ nat. cos.	·4448349	+	
a 24 29 44	.	.	comp. cos. ·0409617
a′ 45 9 12	.	.	comp. cos. ·1516803
a + *a*′ 69 38 56 nat. cos.	·3477722	+	
natural number	·7926071	.	$\bar{1}$·8990579
A 25 17 45	.	.	cos. 9·9562230
A′ 45 8 15	.	.	cos. 9·8484402
natural number	·7877042	.	$\bar{1}$·8963631
A+A′ 70 26 0 nat. cos.	·3349034	—	
D 63° 4′ 35″ nat. cos.	·4528008		

With the exception of the middle logarithm in the column on the right, the sum of the numbers in that column may be found at once by a table of the Logarithmic Difference, as already noticed at page 1100. Of all the auxiliary tables employed in Nautical Astronomy, or at least in that part of it with which we are here occupied, the table alluded to is perhaps the most useful; the element it furnishes enters into nearly all the methods of clearing the lunar distance.

As far as the author knows, the method just given is new; but rules for clearing the distance are so numerous, that it is more than probable it has appeared elsewhere. In working by this method, it is best to take the formula itself as a guide, and to attend to the algebraic sign of the factor $\left\{ \cos. d + \cos. (a + a') \right\}$: if this should be negative, the first natural number above, is still to be treated as positive; the second natural number, which is the product of the two factors in the formula, will then, however, be negative, and from this negative number the nat. cos. (A + A′) is to be subtracted as above.

Determination of D *by formula* (2).

d 63° 35′ 14″ nat. cos.	·4448349	+	
a 24 29 44	.	.	comp. cos. ·0409617
a′ 45 9 12	.	.	comp. cos. ·1516803
a ∽ *a*′ 20 39 28 nat. cos.	·9357042	—	
natural number	·4908693	.	$\bar{1}$·6909659
A 25° 17′ 45″	.	.	cos. 9·9562230
A′ 45 8 15	.	.	cos. 9·8484402
natural number	·4878329	.	$\bar{1}$·6882711
A ∽ A′ 19 50 30 nat. cos.	·9406341	+	
D 63 4 35 nat. cos.	·4528012		

The first of the preceding natural numbers is, by the formula, negative, though treated as positive. In consequence of this, the second natural number—which is the product of the two factors in the formula—is also negative, so that cos. D is the *difference* between the last nat. number and cos. (A ∽ A′).

This last method of clearing the distance is the same as that proposed by Mr. Keith, in his *Treatise on Trigonometry*, and, like the one previously given, is capable of abridgment by means of the table of "Logarithmic Differences."

The reader will observe that the natural cosine of D, arrived at above, differs by four units in the seventh place of decimals, from the natural cosine of D in the former method, although the resulting arcs differ only by a fraction of a second. Occasional insignificant discrepancies of this kind must be expected in working the same example by different methods with tables. In looking at the "differences" in tables, it will be seen that, in some parts, the difference between two consecutive numbers is large, and in other parts small. In proportioning for an intermediate number, in the former case a comparatively large difference will have but small influence, while in the latter case a comparatively small difference may have a very sensible effect. But a discrepancy in the seventh place of decimals is of no moment even in working to seconds.

All the preceding methods of clearing the lunar distance are independent of subsidiary tables; a large collection of compendious rules for working the problem by special tables will be found in Dr. Mackay's valuable *Treatise on the Longitude;* and a very short form of operation is also given by Mr. Woodhouse in the *Appendix* to White's *Ephemeris*, 1855.

Before leaving the present problem, it may be useful to observe that the altitudes of the objects are not required to that precision with which the distance should be taken: this is a desirable circumstance, because, from the frequent obscurity of the sea horizon, it is more difficult to get the altitudes accurately than the distance. If reference be made to any of the formulæ given in the preceding pages, it will be seen that *d* remaining the same, a small alteration in the values of *a*, *a*′, and the same alteration in those of A, A′, cannot produce any sensible effect upon the value of D: the factor $\frac{\cos. A \cos. A'}{\cos. a \cos a'}$, the logarithm of which is, in nautical tables, called the Logarithmic Difference, is always very nearly equal to unit, as is obvious; and this is the principal reason why a small error in the altitudes does not sensibly affect the distance. It is of importance, however, that the proper corrections be carefully applied to the observed altitudes to obtain the true altitudes, even though the former should not have been taken with precision—the *relative* values of the observed and true altitudes must still be preserved. With a view to the determination of the longitude from the lunar distance, accuracy in these corrections is of much importance; and the neglect even of those depending upon the state of the atmosphere, as indicated by the barometer and thermometer, will sometimes occasion an error of more than thirty minutes of longitude.

From what is here said, the learner will perceive that a few seconds may always be safely added to or subtracted from the observed altitudes, if such a modification be found to facilitate the calculation; the seconds may, for instance, be always made a multiple of 10. The observed altitudes may, indeed, without occasioning any appreciable error in the result, be taken to the nearest minute; and even an imperfect altitude, that may err from the truth by so much as two or three minutes, may still be employed with safety, provided the time is not to be computed as well as the distance. In most cases an error of altitude to the extent of 10 minutes will affect the resulting distance by less than that number of seconds. It is also worthy of notice that the apparent distance itself may be so modified as to be rendered free from seconds, provided that when D is deduced, the seconds of error, whether in defect or excess, be applied the reverse way to D.

But except in the merely approximative methods of solution, seconds cannot be wholly dispensed with throughout the work: the corrections for deducing the observed altitudes to the true will always introduce them. By means, however, of the tables already referred to, in which the trigonometrical quantities are computed to every ten seconds of the arcs or angle, the proper allowance for the odd seconds may always be made with but comparatively little trouble.

Examples for Exercise.

1. The apparent distance between the moon's centre and a star is 64° 36′ 40″; the apparent altitude of the moon's centre 44° 33′; the apparent altitude of the star 11° 51′; the true altitude of the moon's centre 45° 15′ 38″; and the true altitude of the star 11° 46′ 33″. Required the true distance. Ans. D = 64° 46′ 14″.

2. The apparent distance between the centres of the sun and moon is 108° 14′ 34″; the apparent altitude of the moon's centre 24° 50′; the apparent altitude of the sun's centre 36° 25′; the true altitude of the moon's centre 25° 41′ 39″; and the true altitude of the sun's centre 36′ 23′ 50″. Required the true distance. Ans. D = 107° 32′ 1″.

3. The apparent distance between the centre of the moon and a star is 51° 28′ 30″; the apparent altitude of the moon's centre 12° 30′ 4″; the apparent altitude of the star 24° 48′ 17″; the true altitude of the moon's centre 13° 20′ 40″; and the true altitude of the star 24° 46′ 14″. Required the true distance. Ans. D = 51° 9′ 48″.

4. The apparent distance between the moon's centre and a star is 31° 13′ 26″; the apparent altitude of the moon's centre 8° 26′ 13″; the apparent altitude of the star 35° 40′; the true altitude of the moon's centre 9° 20′ 45″; and the true altitude of the star 35° 38′ 49″. Required the true distance. Ans. D = 30° 23′ 56″.

5. The apparent distance of the centres of the sun and moon is 90° 21′ 17″; the apparent altitude of the moon's centre 5° 17′ 9″; the apparent altitude of the sun's centre 84° 7′ 20″; the true altitude of the moon's centre 6° 9′ 14″; and the true altitude of the sun's centre 84° 7′ 15″. Required the true distance. Ans. D = 89° 29′ 18″.

NOTE.—The method of finding the true distance between the centre of the moon and a star, is the same as that for the distance between the moon's centre and a planet: in the case of a star, the true altitude is deduced from the apparent altitude, by applying the correction for refraction merely; but in the case of a planet, a correction may be necessary for the parallax in altitude. But in general this correction may be neglected, as it is usually too small to be of much importance.

ON COMPUTING THE ALTITUDES.—It sometimes happens that, though circumstances may be favourable for taking a lunar distance, yet the obscurity of the horizon may present an obstacle to the observations for altitude; in such a case the true altitudes will have to be computed, and from these the apparent altitudes may be deduced, by applying the corrections the contrary way. To compute an altitude it is necessary to know the hour-angle, or angular distance, of the object from the meridian. If the object be the sun, this angle is the apparent time for the meridian; and as the altitude, as already remarked, is not required with precision, the estimated time at the ship will answer for the purpose. If the object be the moon or a star, the sun's right ascension, at the instant, must be increased or diminished by the apparent time, according as this time is P.M. or A.M., to get the right ascension of the meridian; the difference between this and the right ascension of the object, is the hour-angle or the meridian distance of the object. The hour-angle being thus found, a formula for the altitude may be investigated as follows:—

Let P be the hour-angle, l the co-latitude, z the co-altitude, and p the polar distance or co-declination. Then the fundamental formula of spherical trigonometry gives,

$$\cos. \mathrm{P} = \frac{\cos. z - \cos. l \cos. p}{\sin. l \sin. p}$$

$$\therefore \cos. z = \cos. l \cos. p + \sin l \sin. p \cos. \mathrm{P};$$

$$\text{but } \cos. \mathrm{P} = 1 - 2 \sin.^2 \tfrac{1}{2} \mathrm{P}$$

$$\therefore \cos. z = \cos. l \cos. p + \sin. l \sin. p - 2 \sin. l \sin. p \sin.^2 \tfrac{1}{2} \mathrm{P}$$

$$\therefore \cos. z = \cos. (l \backsim p) - 2 \sin. l \sin. p \sin.^2 \tfrac{1}{2} \mathrm{P} \quad \ldots (1)$$

By means of these tables of natural and logarithmic sines and cosines, the co-altitude z may be obtained from this formula; but to adapt it wholly to logarithms, let 1 be added to each side of the equation; then remembering that $1 + \cos. \mathrm{A} = 2 \cos.^2 \tfrac{1}{2} \mathrm{A}$, we have, upon dividing by 2,

$$\begin{aligned} \cos.^2 \tfrac{1}{2} z &= \cos.^2 \tfrac{1}{2} (l \backsim p) - \sin. l \sin. p \sin.^2 \tfrac{1}{2} \mathrm{P} \\ &= \cos.^2 \tfrac{1}{2} (l \backsim p) \left\{ 1 - \sin. l \sin. p \sin.^2 \tfrac{1}{2} \mathrm{P} \times \sec.^2 \tfrac{1}{2} (l \backsim p) \right\} \end{aligned}$$

Put $\cos.^2 \mathrm{M}$ for the expression within the braces; then for computing the altitude a, that is, the complement of z, we have the following formula—namely,

$$\left. \begin{aligned} \sin. \mathrm{M} &= \sin. \tfrac{1}{2} \mathrm{P} \sec. \tfrac{1}{2} (l \backsim p) \sqrt{\sin. l \sin. p} \\ \sin. \tfrac{1}{2} (a + 90^\circ) &= \cos. \tfrac{1}{2} (l \backsim p) \cos. \mathrm{M} \end{aligned} \right\}$$

or, which is perhaps a little more convenient,

$$\left. \begin{aligned} \sin. \mathrm{M} &= \frac{\sin. \tfrac{1}{2} \mathrm{P}}{\cos. \tfrac{1}{2}(l \backsim p)} \sqrt{\sin. l \sin. p} \\ \sin. \tfrac{1}{2}(a + 90^\circ) &= \cos. \tfrac{1}{2} (l \backsim p) \cos. \mathrm{M} \end{aligned} \right\} \quad \ldots\ldots (2)$$

Suppose, for example, that by means of the estimated time at the ship, and the longitude by account, the moon's angular distance from the meridian is found to be 33° 30′, at the time of taking a lunar distance in latitude 38° 14′, the moon's co-declination reduced to the time, being 64° 13′ 13″. Required the apparent altitude by computation, the obscurity of the horizon preventing an observation. Working by the formula (2) the operation is as follows, where P = 33° 30′ p = 64° 13′ l = 51° 46′:

l	51° 46′	½ sin.	4·947572		
p	64° 13′	½ sin.	4·977228		
½ P	16° 45′	sin.	9·459688		
½($l \backsim p$)	6° 13½′	comp. cos.	·002568	cos.	9·997432
		sin.	9·387056	cos.	9·986683
½(a + 90°) = 74° 36′				sin.	9·984115
	2				

∴ a = 59° 12′ the true altitude

cor. for par. and refraction — 28′

58° 44′ the apparent alt.

It is obvious, that in strictness, the correction for parallax and refraction should be taken out of the tables in accordance with the *apparent* altitude, not the true altitude: the correction above, therefore, belongs to an altitude somewhat too great, so that only an approximate apparent altitude is in reality deduced from the true altitude. The proper correction may, however, be found by again entering the table with this approximate apparent altitude; we shall thus get 27′ 54″ for the correction, instead of 28″, so that the correct apparent altitude is 58° 44′ 6″.

Unless the time at the ship is to be deduced, precision is not necessary in computing the true altitude:—seconds may always be disregarded, and the result found to the nearest minute as above. But seconds should not be neglected in the corrections.

It is of importance that the practical navigator have a due appreciation of the value of small quantities in the computation of the true lunar distance: it is only in the apparent altitudes that precision can be dispensed with. In finding *latitude*, if the result be brought out to the nearest minute, the demands of practice will be fully satisfied, as the error cannot exceed half a mile; but an error of only twenty-two seconds in the lunar distance will, on the average, occasion an error of ten minutes of longitude, which, except in high latitudes, is equivalent to seven or eight miles; and within 36° of latitude the error would range from eight to ten miles. Too much pains and caution cannot therefore be exercised in observing a lunar distance: the instrument should be of the very best description, and the observer should have a well-disciplined eye; but the corresponding altitudes may be safely taken by a less skilful person.

An interesting incident is related by the late Captain Basil Hall, an officer who was highly accomplished in the science of his profession. He once sailed from San Blas, on the west coast of Mexico; and after a voyage of eight thousand miles, occupying eighty-nine days, he arrived off Rio Janeiro, having in this interval passed through the Pacific Ocean, rounded Cape Horn, and crossed the South Atlantic, without making any land, or even seeing a single sail, with the exception of an American whaler off Cape Horn. When within about a week's sail of Rio, he set seriously about determining, by lunar observations, the longitude of his ship, and then steered his course accordingly by those common principles of navigation which may be safely employed for short distances between one known position and another.

Having arrived to within what he considered, from his computations, to be about fifteen or twenty miles of the coast, he hove to, at four o'clock in the morning, to await the break of day, and then bore up, proceeding cautiously onward on account of a thick fog which enveloped the ship. As this cleared away, the people on board had the satisfaction of seeing the great Sugar-loaf Rock, which stands on one side of the harbour's mouth, so nearly right a-head that they had not to alter their course above a point, in order to hit the entrance of the harbour! This was the first land they had seen for three months, after crossing so many seas, and being set backwards and forwards by innumerable currents and foul winds. The effect on all on board was electric, and the admiration of the sailors was unbounded.

Something in this remarkable case may have been due to a compensation of small errors; but it is only fair to conclude, that the accuracy with which the ship's position was ascertained, was almost entirely attributable to the precision with which the lunar distances were taken, and the care with which the computations were executed.

To Determine the Longitude from the Lunar Observations.—In the preceding articles, we have shown at considerable length how the true distance between the moon and the sun, or a fixed star, as seen from the centre of the earth, may be determined from the observed distance taken from the surface. It was also shown, by means of the altitude of a celestial object whose declination is known, and the latitude of the place where the altitude is taken, how the time at that place may be found.

The determination of a lunar distance necessitates the determination of the altitudes of the objects whose distance is observed, so that the data for finding the true lunar distance, involves likewise the data for finding the time when that distance had place. But, as already remarked, if these two objects be sought to be accomplished, the altitude employed for the purpose of ascertaining the time at the ship, must be taken with a degree of precision which is not indispensable in "working the lunar:" much more care and accuracy is requisite in observations of altitude for *time* than for either latitude or lunar distance; and on this account the time at ship is, in general, determined independently by one or other of the methods explained at page 1094, *et seq.*, either shortly before or shortly after the distance is taken. By means of the chronometer or watch, the interval between the time thus found, and the instant of taking the distance, is known, and the proper correction for change of longitude in that interval being made, we get the time corresponding to the distance.

We have just said that the altitude—or rather the set of altitudes—for determining the time, are taken *shortly* before or after the observations for the distance, whenever these observations are not themselves sufficiently accurate for the purpose. It is desirable that the interval should not be large, because the difference of longitude in that interval may be large too, and our estimation of its amount is, therefore, liable to a larger error. But as it is important to get the altitude as precise as possible, the situation of the object should be as near the prime vertical; that is, as nearly due east or due west as possible (page 1093). There is thus room for the exercise of some judgment: the object selected for the determination of the time at ship should be near the prime vertical, and it should reach this position shortly before or shortly after the observation for the lunar distance. Whenever the weather is so favourable as to render all risk of losing the anticipated observation but very small, it is prudent to wait till these conditions are fulfilled, provided other circumstances are such as to allow of the delay. In general, the object best suited to the determination of the time at sea is the sun, as its declination, though varying with the time, changes so slowly, as to be deducible accurately enough for the purpose from the estimated time, or time by account, as sufficiently shown at page 1095. A fixed star, however, is very suitable for the purpose, whenever it is nearly due east or west, and the horizon clearly enough defined to admit of an accurate observation of its altitude.

The object of the lunar distance is to find the time at Greenwich at the instant that distance has place; and the object of the other, or extra observation, is to find the time at the same instant at the ship. The difference of the two times gives the longitude in time. The Greenwich time is obtained by comparing the true distance, deduced from the observation, with the nearest predicted distance, supplied by the *Nautical Almanac*, in the way that will be explained presently. It may be well, however, first to show what will be the average effect on the inferred longitude of a given error, in the determination of the lunar distance.

Effect on the Longitude of an Error in the Lunar Distance.—The mean diurnal motion of the moon in her orbit is 13°·1764: at certain times it is about 2° slower, and at other times 2° quicker; but this is her average rate of motion; so that, on the average, 360° of longitude (or 24h. of time) correspond to 13°·1764 of the moon's progress. Hence, to find the error in longitude produced by an error of a seconds in the distance between the moon and a fixed star in her path, we have the proportion

$$13^\circ{\cdot}1764 : 360^\circ :: a : \frac{360a}{13{\cdot}1764} = 27{\cdot}322a\,;$$

which show that the error in the longitude is 27·322 times the error in the distance. Thus an error of 10″ in the lunar distance, causes, on the average, an error of 273″·22 in the resulting longitude, or 4′ 33″·22; and an error of 1′ in the distance, causes, on the average, an error of 27′ 19″·3 in the longitude.

The more rapidly the moon moves, the less is the effect upon the longitude of a given error in the lunar distance: the most rapid change in three hours is very nearly 1° 48′, or 1°·8; so that for any error a in the distance, the corresponding error in longitude, in the most favourable case is $\frac{45a}{1{\cdot}8} = 25a$. Hence, in the most favourable case, the error in longitude corresponding to an error of 10″ in the lunar distance, is 250″ = 4′ 10″; and the error in longitude caused by an error of 1′ in the lunar distance, is 25′. As every inaccuracy in the distance becomes thus increased twenty-five fold in the longitude deduced from it, even in the most favourable case, the reader will at once perceive the importance of securing precision in this element. Several distances ought to be carefully observed, and the mean of them all employed; and scrupulous attention should be paid to the *corrections* of the altitudes; though, as already remarked, the altitudes themselves need not be taken with the utmost nicety.

To obtain the time at Greenwich, corresponding to the true distance determined at sea, requires an operation in simple proportion. To facilitate this operation, Dr. Maskelyne—to whom, indeed, navigators are indebted for originating the *Nautical Almanac*—contrived the table of *Proportional Logarithms*, to be found in every collection of nautical tables. As Proportional Logarithms have not as yet been alluded to in the present section, it will be necessary to say a word or two about them here.

Proportional Logarithms.—These logarithms are derived from the logarithms in common use, thus:— From the logarithm of 10800, the number of seconds in 3 hours, subtract the logarithm of a, any portion of time, in seconds, less than 3 hours: the remainder is called the proportional logarithm of a; in other words,

$$\text{Prop. log. } a = \text{common log. } \frac{10800}{a}$$

If $a = 10800$, then Prop. log. a = com. log. $1 = 0$; so that proportional logarithms are, in fact, complements of the ordinary logarithms to the number log. 10800; just as the arithmetical complements of log. sines and log. cosines, are complements to 10.

As already remarked, the lunar distances, given in the *Nautical Almanac*, are calculated for every three hours of interval. Suppose a distance is determined at sea, at some Greenwich time within one of these intervals; we seek in the *Almanac* for the nearest distance, preceding, in order of time, the given distance, and take the difference between it and the given distance: call this d, and the difference between the two three-hourly distances, intermediate to which the given distance occurs, call D: then, by proportion, we should have for the time x^h, corresponding to the given distance,

$$D : d : : 3^h : x^h ;$$

and therefore, in common logarithms, we should have

$$\text{log. } x^h = \text{log. } 3^h + \text{log. } d - \text{log. D.}$$

But since $3^h = 10800''$, the proportional logarithm of which is 1, we should have, in proportional logarithms,

$$\text{Prop. log. } x^h = \text{Prop. log. } d - \text{Prop. log. D.}$$

In the *Nautical Almanac*, Prop. log. D is inserted between the distances there given, at the beginning and end of every three hours; so that by subtracting this given proportional logarithm from the proportional logarithm of d taken out of the table, we get a proportional logarithm, answering to which, in the table, is the portion of time to be added to the hour of the earliest distance: the result is the Greenwich mean time corresponding to the given distance.

For example—Suppose it were required to find the Greenwich mean time, at which the true distance between the Moon and Pollux was 32° 30′ 25″, on January 14, 1846.

By inspecting the distances in the *Nautical Almanac* for that year, we find against January 14, and opposite the name of the proposed star, Pollux, the following row of lunar distances:—

Midnight.	Pr. log.	xv$^{h.}$	Pr. log.	xviii$^{h.}$	Pr. log.	xxi$^{h.}$	Pr. log.
31° 49′ 25″	3368	33° 12′ 18″	3342	34° 35′ 41″	3319	35° 59′ 31″	3298

From which it appears that the time at Greenwich corresponding to the given lunar distance was between midnight and XV hours; the nearest distance preceding, in order of time, the given distance, is therefore the distance at midnight: we therefore proceed as follows:—

Distance at midnight 31° 49′ 25″ Prop. log. of diff. 3368 –
Given distance 32 30 25
Difference 0 41 0 . . Prop. log. 6425
Portion of time after midnight } 1h. 29m. 2s. . . Prop. log. 3057

Hence the Greenwich mean time, when the distance was as stated above, was 13h. 29m. 2s.

If the distance increased with perfect uniformity during the interval within which the given distance is found, the Greenwich time corresponding to that given distance, determined as above, would be strictly correct, but, as such is not the case, a correction should be applied to the time so found, for the variation of the differences of the distances. A table for obtaining such corrections of the approximate interval of time as found above, is given in the *Nautical Almanac*. In the example above, the correction comes out 8s. additive, so that the correct Greenwich mean time is 13h. 29m. 10s.; the neglect of this correction would, however, occasion an error of only 2′ in the longitude; but in extreme, and therefore, of course, unusual cases, the error in longitude, from a neglect of this correction, might amount to so much as 12′ in the longitude.

Besides the use of proportional logarithms in connection with the lunar problem, they also serve to point out the star which is most favourably circumstanced for accurate observation; that star being to be preferred which has the *least* proportional logarithm opposite to it: for, as already shown (page 1104), the greater the velocity of the moon from or towards a star, the greater is the reliance to be placed on an observation of the distance; in other words, the less is the effect of a small error in the distance upon the longitude. It is a property of proportional logarithms, to decrease as the natural numbers answering to them increase; a smaller proportional logarithm, therefore, indicates a greater velocity of the moon, or greater variation of distance in the interval, upon which the value of the observation depends.

We shall add another example or two of finding the Greenwich mean time, corresponding to a given lunar distance on a given day.

2. On August 2nd, 1836, the distance of the moon from the planet Mars was found, from an observation at sea, to be 56° 30′ 8″: the *Nautical Almanac* gave,

Aug. 2, at 3h. distance 57° 43′ 59″ Pr. log of diff. 2948 –
Given distance . . 56 30 8
Difference . . . . 1 13 51 Pr. log. . . 3869
Portion of time aft. 3h., 2h. 25m. 36s. Pr. log. . 921

Hence the mean time at Greenwich was 5h. 25m. 36s.

3. On April 7, 1831, the true distance between the sun and moon was found to be 65° 54′ 48″: the *Nautical Almanac* gave,

April 7 at noon, Dist. 67° 0′ 8″ P. L. of diff. 3051 –
Given distance . . . 65 54 48
Difference . . . . . 1 5 10 P. L. . . 4412
Portion of time aft. n., 2h. 11m. 35s. P. L. . . 1361

Hence the Greenwich mean time was 2h. 11m. 35s.

4. On November 22, 1853, the true distance of Saturn from the moon was found to be 77° 52′ 45″: the *Nautical Almanac* gave,

Nov. 22, at 3h. Distance, 77° 14′ 40″, and P. L. = 6745.

Required the Greenwich mean time.

Ans. 4h. 13m. 55s.

The proportional logarithms annexed to the Lunar Distances in the *Nautical Almanac*, and most of the tables of them inserted in books on Navigation, are limited to four places of figures; in certain parts of the table, this number of places is too few to show any difference for two, or even three consecutive *arguments;* thus the proportional logarithms of 2h. 41m., 2h. 41m. 1s., 2h. 41m. 2s., are all the same—namely, 484: this is a defect. In Dr. Inman's *Nautical Tables*, the proportional logarithms are given to five places of figures, so that the logarithms of consecutive quantities are not confounded. If five-figure proportional logarithms are used in finding the Greenwich mean time, corresponding to a lunar distance, then the proportional logarithm of the difference between the consecutive lunar distances in the *Almanac* must be sought for in the table, as the *Almanac* gives it to only four places of figures. Thus, taking the example just given,

Given distance .	77° 52′ 45″	
Nov. 22, at 3h Dist.	77 14 40	from *Naut. Alm.*
,, 6h. ,,	78 47 24	
Diff. of dist. in *Alm.*	1 32 44 .	Prop. log. 28804—
Diff. of given dist. and that at 3h.	0 38 5 .	Prop. log. 67454
Time after 3 hours,	1h. 13m. 55s. .	Prop. log. 38650

Hence the Greenwich mean time was 1h. 13m. 55s.

Examples of Determining the Longitude from Lunar Observations.

The foregoing articles contain all that is necessary for the determination of the time at the ship, and the time at Greenwich, at the same absolute instant; and it has been sufficiently explained how these two determinations lead immediately to the discovery of the longitude, as in the following examples :—

Example 1.—On the 1st of September, 1846, the true distance of the star Antares from the moon's centre was 39° 48′ 20″, the mean time at the ship being 8h. 14m. 12s. Required the longitude.

The time at the ship at the instant the lunar distance was taken being known, we have merely to find the Greenwich time corresponding to that distance. We seek, therefore, among the predicted lunar distances in the *Nautical Almanac*, for that one preceding it in order of time, which is the nearest to it, and take the difference between the two. The proportional logarithm, annexed to the distance extracted from the *Nautical Almanac*, subtracted from the proportional logarithm of this difference, will give the proportional logarithm corresponding to a portion of time, to be added to the time at which the distance taken from the *Almanac* had place. The result will be the time at Greenwich corresponding to the given time at the ship. Thus :—

Sept. 1, at 6h., dist. (*Naut. Alm.*)	38° 50′ 28″	P.L. 2287 –
Given distance . . .	39 48 20	
Difference	0 57 52	P.L. 4928
Time after 6h. .	1h. 37m. 59s.	P.L. 2641
∴ Time at Greenwich	7h. 37m. 59s.	
Time at ship . .	8h. 14m. 12s.	
Long. E. in time	0h. 36m. 13s.	
Then, page 1104, 36′ in time =	9° 0′ 0′	In Longitude.
,, 13″ ,, =	3 15	
∴ Longitude	9 3 15	East.

In this example the mean time at the ship is supposed to have been determined by one or other of the methods already explained, and the true lunar distance to have been deduced from the observed, as shown in the preceding pages. But it will be instructive to exhibit, in a connected form, the operations necessary for ascertaining the longitude from the ship's account, in combination with the astronomical observations: we shall, therefore, proceed to an illustration or two of this kind, conducting the process by the following steps :—

1. The first object will be to get, from the ship's account, the approximate time at Greenwich when the observations are made.

By help of this approximate Greenwich date, we shall find semi diameter, horizontal parallax, declination, &c., at the instant of observation, sufficiently near the truth for our ultimate purpose; because, as already seen, these elements vary so slowly, that even a large error in the time can never affect their values, except in a very trifling degree.

2. The next thing will be to apply the necessary corrections to the observed, in order to obtain the apparent and true altitudes, and the apparent distance. Sufficient data will thus be obtained for deducing both the time at ship and the time at Greenwich; and it is matter of indifference which of these requisites is determined first, when the time at the ship is deduced from either the sun or a star.

An error in the ship's account will but very slightly affect the time at ship when the observation is made, unless this time be determined from the moon (page 1105); but the altitude employed must be taken with care.

An error in the ship's account will, in like manner, but very slightly affect the lunar distance, and thence the time at Greenwich; for the moon's semi-diameter, though dependent on the time, varies too slowly to cause its approximate value, deduced from the estimated Greenwich time, to differ from its true value, at the instant of taking the distance, by any appreciable amount; and as far as the *altitude* is concerned, it has been seen that this difference is of no moment.

Example 2.—On February 12, 1848, at 2h. 36m. P.M., mean time by estimation, in latitude 53° 30′ S., and longitude by account 15° 30′ E., the following lunar observation was taken :—

Obs. alt. sun's L. L.	29° 17′ 26″	Obs. alt. moon's L. L.	25° 40′ 20″
Index cor.	—2 10	Ind. cor.	—1 10
	29 15 16		25 39 10

Obs. dist. nearest limbs.	99° 27′ 30″
Ind. cor.	— 50
	99 26 40

The height of the eye above the sea was 20 feet. Required the longitude.

Ship time, Feb. 12 . . .	2h. 36m.
Longitude E. in time . . .	1h. 3m.
Approximate Greenwich time .	1h. 33m.

Referring now to the *Nautical Almanac*, we take out the two semi-diameters for noon of Feb. 12: the approximate Greenwich time differs too little from noon to render any correction necessary; the sun's declination and the moon's horizontal parallax are taken out at the same time: we thus have—

From the Nautical Almanac.

Sun's semi-diam.		16′ 13″
Moon's at noon		15 58
Hor. par. noon 58′ 36″ var. in 12h.,		— 13
Var. in 1½h.	1″·6	
Hor. par. at est. time . . .	58 34	
Sun's dec. noon . . .		13° 52′ 18″ S.
Cor. for 1h. 33m. . . .		—1 17
Dec. at estimated time . .		13 51 1 S.
		90
Polar dist. P S =		76 8 59

Hourly diff. of sun's dec. . .		– 49″·87
30m.	$\frac{1}{2}$	– 24 ·94
3	$\frac{1}{10}$	– 2 ·49
		– 77 ·30

For the Apparent and True Altitudes.

Sun's altitude.

Obs. alt. L. L.	29° 15′ 16″
Dip − 4′ 24″ Semi. + 16 13	+ 11 49
App. alt.	29 27 5 (*a*)
Ref. and par.	− 1 35
True alt.	29 25 30 (A)
	90
Co-alt. Z S =	60 34 30

Moon's altitude.

Obs. alt. L. L.	25° 39′ 10″
Dip − 4′ 24″ Semi. + 7″ aug. +16 5	+ 11 41
App. alt.	25 50 51 (*a*′)
Par. and ref.	+ 50 44
True alt.	26 41 35 (A′)

As it is proposed to deduce the time from the sun, the co-altitude of the moon is not wanted.

*For the Time at the Ship.**

To compute the time, we have the co-altitude Z S = 60° 34′ 30″, the co-latitude P Z = 36° 30′, and the co-declination, or polar distance, P S = 76° 8′ 59″; therefore, page 1095, the work is as follows:—

Z S,	60° 34′ 30″		
sin. P Z,	36 30 0	Arith. Comp.	·2256124
sin. P S,	76 9 0	Arith. Comp.	·0128140
	2)173 13 30		
sin. *s*,	86 36 45		9·9992405
sin. (*s*.−Z S),	26 2 15		9·6424232
			2)19·8800901
cos. ½ P,	29 25 7		9·9400450
	2		
P =	58 50 14	Equa. of time Feb. 12, 14m. 33s. +	
	2	Equa. of time Feb. 13, 14m. 32s.	
	3)11,6 10,0′ 2,8″	∴ correction . . 0	
	3 52		
	3 20·9		
App. time at ship	3h. 55′ 21″	∴ Mean time 4h. 9′ 54″.	
Equation of time	14 33		
Mean time at ship	4h. 9 54		

From this result it appears that the ship's account must be very considerably in error. It may be advisable therefore, as recommended at page 1095, to repeat the foregoing operation with the corrected time here determined. Whether or not this repetition be absolutely necessary, we shall be able to ascertain after finding the mean time at Greenwich, as follows:—

For the Time at Greenwich.

Obs. dist.	99° 26′ 40″
Sun's semi.	16 13
Moon's semi.	16 5
d =	99 58 58

d,	99° 58′ 58″		
a,	29 27 5	comp. cos.	·0600950
a′,	25 50 51	comp. cos.	·0457779
	2)155 16 54		
½ sum,	77 38 27	cos.	9·3304933
½ sum ∽ *d*,	22 20 31	cos.	9·9661096
A,	29 25 30	cos.	9·9400179
A′,	26 41 35	cos.	9·9510585
A + A′,	56 7 5		2)39·2935522
			19·6467761
½ (A + A′),	28 3 32½	− cos.	9·9456967 +
θ	30 9 40	sin.	9·7010794
θ		cos.	9·9368233 + These two to be added together.
½ D =	49 43 43	sin.	9·8825200 = their sum.
∴ D =	99 27 26	the true distance	
Distance at noon,	98 38 0	Prop. log. diff.	·2725 −
	49 26	Prop. log. ,,	·5612

Mean time after noon at Greenwich	1h. 32m. 35s.	Prop. log. ,,	·2887
Mean time at ship	4h. 9m. 54s.		
Longitude E. in time	2h. 37m. 19s.		

2h. in time = 30° in longitude
37m. ,, = 9 15′ ,,
19s. ,, = 4 45″ ,,

Longitude E. 39 19 45

It appears from this result, that although the ship's longitude by the dead-reckoning is nearly 24° in error, yet the error in the estimated time happens to be such, that the supposed time at Greenwich, namely, 1h. 33m., is so nearly equal to the correct time there, that the operation for the time at the ship need not be repeated with the corrected data. We may safely conclude from what has just been done, that the mean time at the ship at the instant of observation, is 4h. 9m. 54s. P.M., and that the longitude is 39° 19′ 45″ east. Such a large error as the above, as well in the estimated time as in the longitude, could arise only from a long continuance of foul weather, which, preventing observations of the heavenly bodies, left the ship entirely dependent upon the dead-reckoning. And without taking into consideration the effect of unknown currents, leeway, &c., even the deviation of the compass alone, from the magnetic action of the vessel, might in a few weeks lead a ship astray to the above extent.*

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* See *ante*, Fig. 29, p. 1093.

Under the circumstances here imagined, it would be better to take the Greenwich time at once from the chronometer, if this can at all be depended on, as already recommended at page 1097; for the longitude and time by account are employed, only as subsidiary, to finding an approximation to the time at Greenwich when the observations are made. If the Greenwich time, as determined from the lunar distance, differ considerably from the chronometer time, the operation for the time at the ship should be repeated with the more correct Greenwich time thus obtained, and thence the longitude accurately deduced.

2. On June 2, 1849, at 10h. 17m. P.M., mean time by estimation, in latitude 50° 51′ N., and longitude by account 41° W., the following observations were taken:—

Regulus W. of meridian.

	Observed altitude.		Observed alt. moon's L. L.
	20° 21′ 40″		31° 11′ 0″
Index	— 3′ 50″	Index	— 4′ 10″
	20° 17′ 50″		31° 6′ 50″

Obs. dist. nearest limb.

	72° 36′ 30″
Index	— 9′ 10″
	72° 27′ 20″

The height of the eye was 20 feet. Required the longitude.

Ship time, June 2 . . .	10h. 17m.
Longitude W. in time . .	2h. 44m.
Approximate Greenwich time	13h. 1m.

Referring now to the *Nautical Almanac*, we take out the moon's semi-diameter and horizontal parallax for midnight of June 2, the right ascension of the mean sun at noon, and the right ascension and declination of the star.

From the Nautical Almanac.

Moon's semi-diam. midnight	14′ 49″
Horizontal parallax . . .	54′ 23″
Star's R.A. 10h. 0m. 20s.	
Decl. . . .	12° 42′ 4″ N.
Sun's R.A. noon . .	4h. 43m. 21s.
Cor. for 13h. . .	+ 2m. 8s.
Sun's R.A. at 13h. .	4h. 45m. 29s.

For the Apparent and True Altitudes.

Star's Altitude.

Obs. alt. . . .	20° 17′ 50″
Dip . . .	— 4′ 24″
App. alt. . . .	20° 13′ 26″ (*a*)
Ref.	— 2′ 37″
True alt. . . .	20° 10′ 49″ (A)
	90°
Co. alt. Z S = . .	69° 49′ 11″

Moon's Altitude.

Obs. alt. . . .		31° 6′ 50″
Dip	— 4′ 24″	+ 10′ 32″
Semi-dia. + 7 Aug.	14′ 56″	
		31° 17′ 22″ (*a*′) App. alt.
Par. and Ref. . .		+ 44′ 54″
		32° 2′ 16″ (A′) True alt.

For the Time at the Ship.

To compute the time we have the star's co-altitude Z S = 69° 49′ 11″, its polar distance, or co-declination P S = 77° 17′ 56″, and the co-latitude P Z = 39° 10′, to find the star's hour angle P, which, since the star is west of the meridian, if added to the star's right ascension, will be the right ascension of the meridian; and since the time is P.M., this right ascension, diminished by that of the mean sun, will be the mean time at the ship. (See page 1096).

Z S,	69° 49′ 11″	
sin. P S,	77° 17′ 56″ Arith. Comp.	·0107592
sin. P Z,	39° 10′ 0″ Arith. Comp.	·1995728
	2)186° 17′ 7″	
sin. *s*,	93° 8′ 33″ . .	9·9993464
sin. (*s* — Z S),	23° 19′ 22″ . .	9·5975972
		2)19·8072756
cos. ½ P,	36° 46′ 24″ . .	9·9036378
	2	
P =	73° 32′ 48″	
	2	
	3)14,6 6,4 9,6	
	4 52	
	2 8	
	3	

Star's hour-angle in time . .	4h.	54m.	11s.
Star's right ascension . .	10h.	0m.	20s.
R.A. of meridian	14	54	31
R.A. of mean sun	4	45	29
Mean time at ship	10	9	2

For the Time at Greenwich.

In computing the true lunar distance in this example, we shall adopt the recommendation at the bottom of page 1103, and so modify the observed, or rather the apparent altitudes, that the seconds in each altitude may be a multiple of 10: whatever be added or subtracted in this way, must of course be in like manner applied to the true altitudes. We shall also introduce a similar modification into the apparent distance, and apply the necessary correction to the true distance: the object of these changes is to reduce the work of computing for odd seconds.

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* In iron ships, propelled by steam, the local attraction is subject to great and frequent changes, and the compass is so powerfully acted upon by this fluctuating influence—the effect of which is often unsuspected—that the most fearful consequences sometimes result—the true course of the vessel being, from this cause, widely different from that indicated. At the conclusion of the next Chapter, we shall give a brief abstract of Dr. Scoresby's researches and suggestions in reference to this very important subject. If science fail to apply a remedy to counteract the ever-changing influence of iron steamers on the compass, this useful instrument had almost as well be abandoned in such ships. The mere heeling of the vessel will greatly affect the local deviation, and the continual strain and vibration to which a vessel under steam is subjected, so influences its magnetic condition, as to render the compass a dangerous guide. The ship *Tayleur*, which was wrecked in the Irish Channel in January, 1854, was led out of her proper course by these unsuspected influences. The captain relied upon his helm compass, and 290 persons perished. The original deviation in this iron steamer was so great as 60°, and this was subjected to changes that might amount to as much as between 40 and 50 degrees. It is easy to conceive that, in the open ocean, a ship misguided in this way, and precluded from adjusting her position by astronomical observations, through a long continuance of bad weather, might err even to the extent supposed in the above example.

Obs. dist.	72° 27′ 20″		
Moon's semi.	14 56		
d =	72 42 16		

d,	72° 42′ 20″		
a,	20 13 20	comp. cos.	·0276310
a′,	31 17 20	comp. cos.	·0682577
	2)124 13 0		
½ sum,	62 6 30	cos.	9·6700614
½ sum ∽ d,	10 35 50	cos.	9·9925289
A,	20 10 43	cos.	9·9724907
A′,	32 2 14	cos.	9·9282440
A + A′,	52 12 57		2)39·6592137
			19·8296068
½ (A + A′),	26 6 28	− cos.	9·9532608 +
θ,	48 47 0	sin.	9·8763460
θ,		cos.	9·8188250 +
½ D =	36 16 34	sin.	9.7720858 = their sum.

(9·9532608 + and 9·8188250 +: These two to be added together.)

∴ subtracting the 4″ added to *d*, D = 72 33 4 the true distance.

Distance at 12h.	72 19 35	Prop. log. diff.	·3017 −
Difference	13 29	Prop. log.	1·1255
Mean time after 12h. at Greenwich	0h. 27m. 0s.	Prop. log.	·8238
∴ Greenwich mean time	12h. 27m. 0s.		
Ship mean time	10h. 9m. 2s.		
Longitude W. in time	2h. 17m. 58s.		

2h. in time = 30° in longitude.
17m. " = 4° 15′ "
58s. " = 14′ 30″ "

Longitude W. 34° 29′ 30″

In each of the foregoing examples the mean time at the ship is obtained from an altitude of the sun or a star, and, as remarked at page 1106, it is matter of indifference, when such is the case, whether the time at the ship or the time at Greenwich is determined first; but as the moon's right ascension and declination change very rapidly, comparatively to the sun's in a given interval, if the estimated time at ship be much in error, it is plain that the elements just referred to, computed to this estimated time, may differ so much from the truth as to very sensibly affect the accuracy of the ship's time, and thence of the longitude. We would, therefore, recommend when, in consequence of the other object, the sun or a star, being too near the meridian for the purpose of finding from it the time, the moon's altitude must be employed, that the time at Greenwich be found before that at the ship is computed, as in the following example:—

3. On May 22nd, 1844, at 11h. 15m. A.M. mean time nearly, in latitude 50° 48′ N., and longitude by account 1° W., the following lunar observation was taken when the moon was E. of the meridian:—

Obs. alt. sun's L. L.	57° 53′ 0″
Index Cor.	+ 35″
	57° 53′ 35″
Obs. alt. moon's L. L.	22° 53′ 2″
Index Cor.	− 20″
	22° 52′ 42″
Obs. distance N. L.	56° 26′ 6″
Index cor.	− 35″
	56° 25′ 31″

The height of the eye was 24 feet; required the longitude:—

Ship time, May 21st	23h. 15m.
Longitude W. in time	4m.
Approximate Greenwich time	23h. 19m.

From the Nautical Almanac.

Moon's semi-diam., 21st, midnight	15′ 1·3″	Hor. par. 55′ 7·6″
Moon's semi-diam., 22nd, noon	15′ 5·7″	55′ 23·7″
	4·4″	16·1″
Correction for 41m. before noon, 22nd	·25	·9
Semi-diam. at 23h. 19m. May 21	15′ 5·4″	Hor. par. 55′ 22·8″

Sun's semi-diameter at noon, May 22nd, 15′ 49″.

For the Apparent and True Altitudes.

Sun's altitude.

Obs. alt. L. L.	57° 53′ 35″
Dip − 4′ 49″ Semi. + 15′ 49″	+ 11′ 0″
App. alt.	58° 4′ 35″ (a)
Ref. and par.	− 36″
True alt.	58° 3′ 59″ (A)

As it is proposed to deduce the time from the moon, the co-altitude of the sun is not wanted.

Moon's altitude.

Obs. alt. L. L. . . . 22° 52′ 42″
Dip . — 4′ 49″ }
Semi. . + 15′ 11″ } . . 10′ 22″
Aug. + 5″·5 }

App. alt. . . . . 23° 3′ 4″ (a′)
Par. and ref. . . . + 48′ 38″

True alt. . . . . 23° 51′ 42″ (A′)
90°

Co-altitude . . . . 66° 8′ 18″ = Z M

For the Time at Greenwich.

Obs. dist. . . . 56° 25′ 31″
Moon's semi. . . . 15 11
Sun's semi. . . . 15 49

d = 56 56 31

d, 56° 56′ 31″
a, 58 4 35 comp. cos. ·2767183
a′, 23 3 4 comp. cos. ·0361386

2)138 4 10

½ sum, 69 2 5 cos. 9·5336429
½ sum ∽ *d*, 12 5 34 cos. 9·9902543

A, 58 3 59 cos. 9·7234034
A′, 23 51 42 cos. 9·9611955

A + A′, 81 55 41 2)39·5413530

19·7706765
½ (A + A′), 40 57 50½ − cos. 9·8780168 + }

θ, 51 21 13 sin. 9·8926597 } These two to be added together.

θ, . . cos. 9·7955409 + }

½ D = 28 8 13½ sin. 9·6735577 = their sum.

∴ D = 56 16 27 the true distance.
Dis. at 21h. 55 15 36 Prop. log. diff. ·3221 −

Difference 1 0 51 . . . ·4710

Mean time after 21h. at Greenwich . } 2h. 7m. 45s. ·1489
∴ Greenwich mean time . . . } 23h. 7m. 45s.

Having thus determined the time at Greenwich when the lunar distance was taken, we can compute the right ascension and declination of the moon to greater precision than the time by account could be expected to give. It so happens that, in this particular example, the time, by account, is very nearly the same as that deduced from the observations; but yet there would be a sensible difference in the resulting longitude, if the computation of the ship time had preceded that for the Greenwich time.

It is only in reference to the moon that much accuracy in the time and longitude, by account, is of any consequence.

For the Mean Time at the Ship.

Sun's R.A.		Moon's R.A.	
21st . .	3h. 56m. 53·8s.	23h. . .	7h. 55m. 24s.
23h. . .	+ 3m. 46s.	7m. 45s. .	+ 16s.
7m. 45s. .	1 3s.		R. A. = 7h. 55m. 40s.
	R. A. = 4h. 0m. 41s.		

Moon's declination.

23h. . . . 17° 5′ 12″ N.
7m. 45s. . . — 1 2

Dec. = 17 4 10 N.

The data for computing the moon's hour-angle with the meridian are therefore as follows: namely—

Z M = 66° 8′ 18″, P Z = 39° 12′, P M = 72° 55′ 50″
Z M, 66° 8′ 18″
P Z, 39 12 0 comp. sin. ·1992628
P M, 72 55 50 comp. sin. ·0195650

2)178 16 8

s, 89 8 4 . . sin. 9·9999503
s − Z M, 22 59 46 . . sin. 9·5918086

2)19·8105867

½ P, 36 28 46 . . cos. 9·9052933

∴ P, = 72 57 32 the moon's hour-angle
2

3)14,5° 5,5′ ,4″

4h. 50m.
1m. 50s.

Moon's hour-angle in time . . . . } 4h. 51m. 50s. east of the merid.
Moon's R.A. 7h. 55m. 40s.

R.A. of meridian 3h. 3m. 50s.
Sun's R.A. 4h. 0m. 41s.

Sun's hour-angle with meridian . . . } 0h. 56m. 51s. { before noon of the 22nd.
24h.

Mean time at ship 23h. 3m. 9s. May 21st.
Mean time at Greenwich 23h. 7m. 45s.

Longitude W. in time 0h. 4m. 36s. ∴ Long. W. 1° 9′.

Although the Greenwich time, by account, differs by only a few minutes from the Greenwich time as determined by the observation, yet the longitude would have differed by more than 4′ from the result here arrived at, if the ship time had been determined before the Greenwich time. It thus appears, that it is in all cases more prudent to compute the time at Greenwich before that at the ship, when the latter is to be deduced from the *moon*.

Examples for Exercise.

1. On March 25, 1847, at 3h. 30m. P.M., mean time nearly, in latitude 52° N., and longitude by account 33° W., the following lunar observation was taken:—

Obs. alt. sun's L.L.	Obs. alt. moon's L.L.
23° 10′ 20″	23° 50′ 10″
Index cor. − 6 10	Index cor. + 5 0

Obs. dist. nearest limbs.
112° 56′ 30″
Index cor. − 4 20

The height of the eye was 20 feet, and the following particulars were furnished by the *Nautical Almanac:* namely,

	Sun's dec.	Equa. of time.
Mar. 25,	1° 40′ 56″ N.	6m. 13·8s. *additive.*
26,	2° 4′ 29″ N.	5m. 55·2s. ,,

Moon's semi.	Hor. par.	Sun's semi.
noon 25th, 14′ 59″	noon 54′ 59″	16′ 3″
midnight 14′ 55″	mid. 54′ 44″	

Also distance at 3h., 111° 33′ 34″; at 6h., 112° 57′ 16″. Required the longitude of the ship. Ans. 32° 59½′ W.

2. On January 9, 1851, at 7h. 50m. P.M., mean time nearly, in latitude 49° 40′ N., and longitude by account 10° E., the following lunar observation was taken:—

Obs. alt. of Pollux E. of merid.	Obs. alt. moon's L.L.
37° 10′ 10″	31° 50′ 10″
Index cor. − 1′ 10″	Index cor. + 1′ 20″

Obs. dist. farthest limb.
103° 20′ 0″
Index cor. + 1′ 30″

The height of the eye was 18 feet, and the following particulars were supplied by the *Nautical Almanac:* namely,

Right ascen. mean sun.	Moon's semi.	Hor. par.
Jan. 9, 19h. 13m. 42·27s.	noon 14′ 55·8″	54′ 47·2″
	mid. 15′ 0·2″	55′ 3·6″

The star's R.A. was 7h. 36m. 12s., and its dec. 28° 22′ 46″ S. Also, the distance at 6h. was 103° 8′ 4″, and at 9h., 101° 37′ 51″. Required the longitude of the ship.

Ans. 10° 19′ 15″ E.*

The examples now given will, we think, sufficiently illustrate the practical operations for determining the longitude at sea by a lunar observation. In the *Nautical Almanac*, the lunar distances are given for the planets Mars, Venus, Jupiter, and Saturn, as well as for the fixed stars near the moon's path. The calculations for the true distance are, of course, the same for a planet as for a fixed star; and, in deducing the time at ship from the planet, we must proceed exactly as in deducing the time from the moon; that is to say, we must find the planet's hour-angle, and thence, by means of its right ascension, taken from the *Nautical Almanac*, we must find the right ascension of the meridian: the difference between this and the right ascension of the mean sun will—as in last example—be the sun's hour-angle with the meridian of the place; that is to say, the time at the ship.

If either of the celestial objects observed be near the horizon, where the refraction is subject to considerable irregularities, the *mean* refraction, which is that generally employed, should be modified according to the state of the atmosphere, as shown by the barometer and thermometer. A table for properly correcting the mean refraction is to be found in every collection of nautical tables. A neglect of this correction, when one of the objects observed is not more than 8 or 9 degrees above the horizon, may occasion an error of 1′ in the lunar distance; and this error may be sufficient to introduce one of upwards of 30′ in in the longitude.

Longitude from Occultations and Eclipses of Jupiter's Satellites.—What has now been mentioned, comprehends all the essentials for finding the latitude and longitude at sea. Other celestial phenomena besides those here dwelt upon, may be occasionally made available for determining the longitude; as, for instance, an eclipse, or an occultation of a fixed star or planet by the moon. Eclipses are of too infrequent occurrence to be of much service to the navigator; but the passage of the moon over the stars and planets, in her path, is continually occurring; and this occultation of the object by the moon would furnish a very convenient means of finding the Greenwich time, and thence the longitude, if the motion of the ship did not, in general, preclude the possibility of keeping the telescope steadily directed to the moon's edge.

It is plain that at the instant of the occultation, that is, at the instant of the disappearance of the star or planet by the interposition of the moon—called the *immersion*—the apparent right ascension of the moon's occulting limb must be the same as the right ascension of the occulted star. By removing the effect of parallax, the moon's true right ascension at the instant of the star's immersion may therefore be found, and the Greenwich time corresponding to this right ascension may thence be deduced.

The eclipses of Jupiter's satellites answer a similar purpose, since the entrance of a satellite into the shadow of the planet, is a phenomenon which takes place at the same absolute instant, wherever on the surface of the earth the immersion be observed; and so likewise does the reappearance of the satellite, or its emersion.

The Greenwich time, when these immersions and emersions are predicted to happen, are given in the *Nautical Almanac;* so that if the ship time, when any such phenomenon occurs, be known, the longitude may be at once obtained.

But here again, as in the case of occultations of the stars by the moon, the frequent impracticability of keeping a telescope sufficiently steady for the accurate observation of the phenomena at sea, renders this short and otherwise convenient method of finding the longitude of but very limited application. In a calm sea, or in harbour, a telescope of sufficient magnifying power may, of course, be used without inconvenience. Of the four satellites, the *first*, or that which is at the least distance from the planet, is the best adapted for the purpose of determining the longitude, on account of its more rapid motion; it revolves round Jupiter, and is eclipsed by the shadow of the planet once in every forty-two hours: and the instants of immersion and emersion are capable, in general, of being much more accurately noted, than the instants of contact of the earth's shadow with the moon's limb.

CHAPTER V.

NAUTICAL INSTRUMENTS.

The Quadrant and Sextant.—These two instruments are the same in principle—both are equally employed to measure angular distances; but as the distance between a celestial object and the horizon, for the purpose of determining the latitude at sea, is a measurement more frequently made than any other, the former of the two above-mentioned instruments—the quadrant—is constructed with exclusive reference to this purpose, and, being less elaborate in its fittings and workmanship, is by far the cheaper instrument of the two.

The arc of the sea quadrant is the eighth part of an entire circumference, or 45°. This arc is therefore, strictly speaking, not a quadrant, but an *octant;* but as it is capable of measuring all altitudes from the horizon to the zenith—as will presently be explained—a greater extent of arc is unnecessary.

The sextant is a more delicate instrument. Its arc is the sixth part of an entire circumference, or 60°, and it is capable of measuring angular distances up to 120°. The arc of the more common quadrants is divided—and that by means of an auxiliary scale attached to the index-limb—into minutes only: those of a superior kind are thus divided into half minutes; but the arc of a sextant is frequently subdivided—by aid of the Vernier scale just alluded to, and hereafter explained—to every 10″.†

Artists generally extend the arc of a quadrant to a few degrees beyond 45°, and the arc of a sextant to a few

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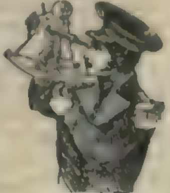
* For more examples, see Mr. Jean's *Navigation and Nautical Astronomy*, whence the above have been taken.

† Subdivisions of less extent than this must be estimated by the eye. A skilful observer can usually make this estimate within two or three seconds of the truth. Such an error, in the measurement of a lunar distance, would not occasion an error of more than about a mile in the resulting longitude. (See *Astronomical Instruments*, page 980, *et seq.*)

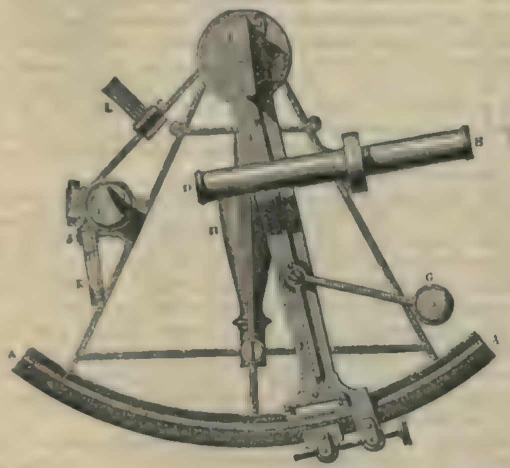
Fig. 32.



degrees beyond 60°. With either of these instruments an altitude may be taken; but usually with more precision with the sextant than with the quadrant, on account of the more minute subdivisions of the arc. The manner of holding the instrument in taking an altitude is figured in the margin. (See Fig. 32).

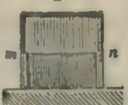
The sextant, however, is almost exclusively used for taking a lunar distance. All the essential parts of this valuable instrument are represented in the accompanying figure. H (Fig. 33) is the handle by which the sextant is held while taking the angular distance; D E is a small telescope fixed to the frame of the instrument, and directed towards the plane reflector or mirror, C. This reflector is immovable; the other plane reflector or speculum, B, is fixed to the movable radius, or movable limb, F, at the extremity of which is the index for pointing out the angle measured on the graduated limb, A A.

Fig. 33.



The two reflectors, B C, are perpendicular to the plane of the instrument—that is, to the plane in which the arc, or graduated limb, lies. The immovable reflector, C, is called *the horizon glass*; the upper half of it is transparent and unsilvered; and it is through this part that the horizon is viewed in taking an altitude; the lower half only, strictly speaking, is a reflector, being the only part coated with quicksilver (Fig. 34). The movable reflector, B, is called *the index glass*; it turns with the radius, or limb carrying the index, round the centre of the graduated limb.

Fig. 34.



Suppose this radius to be turned into such a position that the planes of the two reflectors, B, C, may be exactly parallel to one another: in this position of the index limb, the point on the graduated arc shown by the index is to be marked 0. It is the perfection of the instrument that, when the index points to 0, the two reflectors should be accurately parallel to one another. And here we pause for a moment to explain the cause of what the learner may have hitherto considered as the result of indifferent workmanship in the instruments employed in the preceding problems of Nautical Astronomy. We have repeatedly spoken of the *index error*, or the *error of the instrument*.

These terms do not imply faulty workmanship; all instruments whatever—the most elaborately-finished specimens in the most richly-furnished observatories—have, without exception, their *instrumental errors*, which are very different things, however, from errors of workmanship.

In the section on Mathematics (Commentary on Euclid), the reader has been frequently reminded, that it is beyond the reach of art to draw a line of a prescribed length *accurately*; to raise a perpendicular; or to draw a pair of lines that shall be accurately parallel. The perfect perpendicularity and parallelism, therefore, of the reflectors, B, C, are things that cannot be practically brought about; some minute departure from strict geometrical precision always exists, and hence, what is called "the index error." How this index error may be discovered and allowed for, will be seen presently; it is always furnished to the purchaser by the maker of the instrument.

In reference to this subject, it may not be unprofitable to the learner, to add to the present digression the following observations of Sir John Herschel:—

"Astronomical instrument-making may be justly regarded as the most refined of the mechanical arts, and that in which the nearest approach to geometrical precision is required, and has been attained.

"It may be thought an easy thing, by one unacquainted with the niceties required, to turn a circle in metal, to divide its circumference into 360 equal parts, and these again into smaller subdivisions—to place it accurately on its centre, and to adjust it in a given position; but, practically, it is found to be one of the most difficult. Nor will this appear extraordinary when it is considered that, owing to the application of telescopes to the purposes of angular measurement, every imperfection of structure or division becomes magnified by the whole optical power of that instrument, and that thus, not only direct errors of workmanship, arising from unsteadiness of hand or imperfection of tools, but those inaccuracies which originate in far more uncontrollable causes, such as the unequal expansion and contraction of metallic masses, by a change of temperature, and their unavoidable flexure or bending by their own weight, become perceptible and measurable.

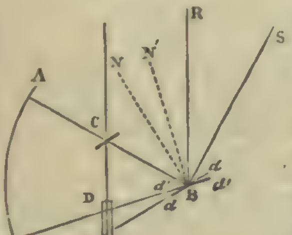
"An angle of one minute occupies, on the circumference of a circle of ten inches in radius, only about $\frac{1}{350}$th part of an inch—a quantity too small to be *certainly* dealt with without the use of magnifying glasses; yet one minute is a gross quantity in the astronomical measurement of an angle. With the instruments now employed in observatories, a single second, or the 60th part of a minute, is rendered a distinctly visible and appreciable quantity. Now the arc of a circle, subtended by one second, is less than the 200,000th part of the radius; so that on a circle of six feet in diameter, it would occupy no greater linear extent than $\frac{1}{5700}$th part of an inch—a quantity requiring a powerful microscope to be *discerned* at all.

"Let any one figure to himself, therefore, the difficulty of placing on the circumference of a metallic circle of such dimensions (supposing the difficulty of its construction surmounted), 360 marks, dots, or cognisable divisions, which shall be true to their places within such minute limits, to say nothing of the subdivision of the degrees so marked off into minutes, and of these again into seconds. Such a work has probably baffled, and will probably for ever continue to baffle, the utmost stretch of human skill and industry; nor, if executed, could it endure. The ever-varying fluctuations of heat and cold have a tendency to produce not merely temporary and transient, but permanent, uncompensated changes of form in all considerable masses of those metals which alone are applicable to such uses; and their own weight, however symmetrically formed, must always be unequally sustained, since it is impossible to apply the sustaining power to *every part* separately: even could this be done, at all events force must be used to move and to fix them, which can never be done without producing temporary, and risking permanent, change of form. It is true, by dividing them on their centres, and in the identical places they are destined to occupy, and by a thousand ingenious and delicate contrivances, wonders have been accomplished in this department of art, and a degree of perfection has been given, not merely to *chefs-d'œuvre*, but to instruments of moderate prices and dimensions, and in ordinary use, which, on due consideration, must appear very surprising. But though we are entitled to look for *wonders* at the hands of scientific artists, we are not to expect *miracles*. The demands of the astronomer will always surpass the power of the artist; and it must, therefore, be constantly the aim of the former to make himself, as far as possible, independent of the imperfections incident to every work the latter can place in his hands. He must, therefore, endeavour so to combine his observations, so to choose his opportunities, and so to familiarise himself with all the causes which may produce instrumental derangement, and with all the peculiarities of structure and material of each instrument he possesses, as not to allow himself to be misled by their errors, but to extract from their indications, as far as possible, all that is *true*, and reject all that is erroneous. It is in this that the art of the practical astronomer consists—an art of itself of a curious and intricate nature, and of which we can here only notice some of the leading and general features." (See Herschel's *Astronomy*, Lardner's *Cyclopædia.*)*

Returning now to the sextant:—To understand the way in which an angular distance is measured by this instrument, we must first assent to the following simple optical principle—that is, when a ray of light from a luminous body falls upon a reflecting surface, and is thence received by the eye, the *incident* ray—or that direct from the object to the reflector—makes, with the perpendicular to the surface of the reflector, drawn from the point where it impinges on it, an angle equal to the angle made with the same perpendicular and the *reflected* ray, or that received by the eye. This property is briefly expressed thus:—the angle of incidence is equal to the angle of reflection. It is the same with an elastic sphere striking a smooth hard surface, as, for instance, a common marble, shot against a smooth wall; if the marble be shot perpendicularly, it will rebound along the same path, and return to the hand, the angle of incidence and the angle of reflection being *nothing;* but if the marble be projected obliquely, it will rebound on the other side of the perpendicular; and the oblique incident path, and the oblique reflected path, will make equal angles with that perpendicular.

This principle being admitted, conceive the limb F to be moved so that the attached index points to zero on the graduated arc (the point A on the right in the figure 33). In this position of the index and of the reflector B, if the eye at E look through the upper or unsilvered part of the horizon glass at C, and perceive a celestial object, such as a star, it will at the same time also perceive the *image* of that star reflected from the silvered part of C. For as the reflectors are, by hypothesis, parallel, and the star so distant that two rays from it falling, the one on the glass B and the other on C, must differ only insensibly from parallelism, it follows that the ray from the star, reflected at B and thence proceeding to C, from which it is again reflected at E, must proceed to the eye E in the same direction as the direct ray from the star through the unsilvered part of C. But this will be better illustrated by a distinct diagram.

Fig. 35.



Let *d d* (Fig. 35) be the position of the index glass when the index points to zero, which in the diagram is marked by the letter *e*. By hypothesis, the surface of the half-silvered glass C is parallel to that of *d d*. An object, R, so remote as one of the heavenly bodies, would be equally seen to an eye at E in the direction of E D, parallel to B R, as to an eye at B, in the direction B R. If the dotted line B N be perpendicular to *d d*, R B N will be the angle of incidence, and C B N the angle of reflection: the reflected ray B C is now incident on C—a surface *parallel* to *d d*; therefore the reflected ray C D must be parallel to R B, so that an eye at E will see the object directly through the transparent part of C, and the image of it, after two reflections, in the same direction; or rather the object and its image would become confounded and superposed. Suppose now, while the eye is still looking at the object R through the telescope E D, the index-limb be moved from *e* to *e′*; the reflector *d d* turning round with this limb will take the new position *d′ d′*, and the *image* of R will disappear, and that of some other object S, in reference to a ray from which, B C will still be the reflection, will take its place.

In this way, two luminous points, R, S, as, for instance, two stars, or a star and an edge of the moon, or the edges of the sun and moon, may be brought together; one of the two, R, being seen by direct vision; the other, S, after two reflections at the mirrors.

The movement of the index from *e* to *e′*, necessary to bring the two objects R, S into contact, moves the reflector *d d* into the position *d′ d′*, and the perpendicular B N into the position B N′. Now B N, by the above-mentioned optical principle, bisects the angle R B C, and B N′, in like manner, bisects the angle S B C. Hence

$$\mathrm{NBN'} = \tfrac{1}{2}\,(\mathrm{SBC} - \mathrm{RBC}) = \tfrac{1}{2}\,\mathrm{SBR}.$$

And since the angles N B N′, *e* B *e′*, measured by the arc *e e′*, are obviously equal, it follows that the arc *e e′* measures half the angle S B R, formed by the two incident rays S B, R B; that is to say, it measures half the angular distance of the two objects S, R.

If, therefore, the arc A A, supposed to be 60°, be divided into twice that number—that is, 120 equal parts—then, by considering each part as a whole degree, the index at *e′* will show the number of degrees in the angular distance of S and R. In like manner, the degrees and minutes will be shown if each division be subdivided into 60 equal parts. This is the important principle in the construction of the sextant.

Besides the two reflectors, B and C, in figure 33, several other glasses—called *screens* or *shades*—are attached to the framework of the instrument, as shown at K and L. These are merely stained or coloured glasses to be interposed in the path of the rays from luminous objects, as the sun, and sometimes the moon, to reduce the intensity of the glare, which might be too strong for the sight, and, in the case of the sun, could not be endured with a clear sky. The principal use of these glasses, therefore, is to screen the eye; but they also serve to distinguish the object from its image by difference of colour. The movable glass, G, is a microscope, supported by a slip of metal turning about its extremity *a*, so as to allow of its being brought over the divisions of the *Vernier*—a small and important scale attached to the index, for the purpose of marking subdivisions too minute to be engraved on the circular limb of the instrument. We shall explain the Vernier presently.

To Use the Instrument.—The plane of the quadrant or sextant must always be held in the plane of the two objects, of which the angular distance is to be taken; it is grasped by the handle H, usually with the right hand (see Fig. 33, page 1112), the other hand being employed in moving and adjusting the index: when, as in common quadrants, there is no handle, the instrument is held by the framework itself. If the sun's altitude is to be taken, the instrument is to be held in a vertical position, the index set to 0 on the limb, and the eye applied to the telescope, or—removing this—to the sight-vane, which supplies its place, and directed through the horizon glass to that part of the horizon which is vertically under the sun. The index is now to be moved forward till the limb of the image of the sun, which we shall see to be gradually descending, just touches the horizon: the observed altitude of that limb will thus be obtained.

The sight-vane may now be turned down and the telescope introduced, which, by magnifying the image, will render the contact more distinct. It is, in general, more easy to get a contact, though with less precision, without the telescope than with it, as the telescope greatly limits the field of view; but after the index is adjusted to the approximate contact, the telescope, previously set to distinct vision, will at once show the object more clearly defined, and give the contact more accurately; of course, whatever shades may be necessary to protect the eye, and to distinguish the object from the image, are to be put in front.

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* The reader may refer, on this subject, to the remarks on individual errors of observation, &c., in the section on Astronomy, as another source of error, independent of the instrument.—Ed.

If the altitude of a star is to be taken, the operation is just the same, care being taken to keep the star's image in view during the whole of its descent to the horizon, to avoid the mistake of bringing down the wrong star. The moon's altitude is taken in the same way as the sun's, using such shades as may be found necessary.

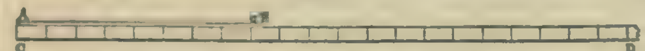
When a lunar distance is to be observed, the plane of the sextant must be held so that both objects may lie in that plane, and the sight is to be directed, through the horizon glass, to the fainter of the two, so that when the brighter object is to the left, the instrument must be held face downwards.

The practical management of the instrument, in making observations at sea, can be efficiently acquired only on ship-board; the movements of the body must be accommodated to the motion of the vessel, and peculiar attitudes and positions will be necessary in peculiar circumstances: the observer sometimes stands erect, sometimes reclines against a support, and sometimes lies on his back on the deck, when taking a lunar distance. It is plain that nothing but experience can dictate to him the best way of handling his instrument on the various occasions that may require its use. Supposing the observation to have been made, it remains to *read off* the angular measure; this is done by aid of the *Vernier*, a contrivance so called from the name of its inventor.

THE VERNIER.—This is a small scale attached to the index-limb, F, Fig. 33, of the instrument; it is slightly inclined to the face of the divided limb A A, and moves, with the index-limb, in close contact with the divided arc A A. It is attached to many other scales—as, for instance, to those of the barometer and thermometer—as well as to the scales of the quadrant and sextant, and is, in fact, an appendage to many astronomical instruments used for angular measurement; its object and utility may be explained as follows:—

Let C D (Fig. 36) represent any graduated scale, and A B a line which we wish to measure by it; the scale and line must of course be of the same character—both straight or both circular. If, upon applying the scale to the line, as in the following figure, we find the extremity B to fall accurately upon one of the divisions of the scale, we, of course, obtain the measure without any fractional parts of a division: we may, for illustration, call the divisions degrees, and we shall conclude the measure to be so many degrees exactly.

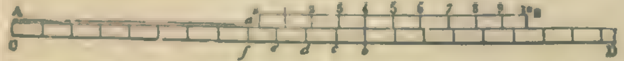
Fig. 36.



But if, as would be most likely, the extremity, B, of the line project beyond the boundary of a division without reaching the next, the length would be so many degrees and some fractional parts of a degree, which the scale affords us no efficient means of measuring. In the figure, the measure of A B is eight degrees, with a fractional part, *a* B, of the ninth degree, the exact amount of which can only be guessed at. The object of the Vernier is to make known the value of this fractional part.

Imagine a second scale, B E (Fig. 37), with its commencement placed in contact with the extremity, B, of the proposed line, to be applied to the scale C D: suppose the whole length of this second scale to be 8 degrees, but that it is divided, as in the figure, into 9 equal parts; and let us assume, moreover, that *a* B may be expressed in tenths of degrees.

Fig 37.

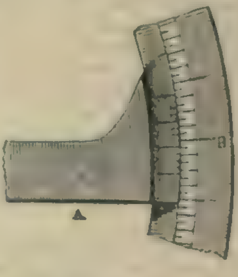


Among the divisional marks of the second scale B E, thus placed, there will necessarily be found one which exactly corresponds to a divisional mark on the first scale, C D: in the figure below it is the *fourth* mark; and we accordingly conclude that *a* B measures four-tenths of a degree, so that the whole measure of A B is $8\frac{4}{10}$. That such is the case will be readily seen from considering that one of the divisions of B E is only $\frac{9}{10}$ths of one of the divisions of C D, so that one of the latter divisions exceeds one of the former by $\frac{1}{10}$th of a degree, two of the latter exceed two of the former by $\frac{2}{10}$ths, and so on.

Now from 4 to B, on B E, there are four divisions; and from *b* to *f*, on C D, there are also four divisions; the latter four, in their whole length, exceed the former four by $\frac{4}{10}$ths of a degree; but this excess is the length *a* B, consequently *a* B = $\frac{4}{10}$ths. It follows, therefore, that if *a* B be only an exact number of tenths of a degree; we shall be able to measure those tenths by this contrivance; and the error of measurement, if *a* B be not an exact number of tenths, and therefore the mark 4 not strictly the continuation of the mark *b*, must be less than $\frac{1}{10}$th. In like manner, if the scale B E had the length of 19 degrees of C D, and were divided into 20 equal parts, *a* B could have been measured accurately to within $\frac{1}{20}$th of a degree, and so on. The scale B E is the *Vernier*.

The annexed figure (Fig. 38) will give an idea how the Vernier, attached to the index-limb of the quadrant or sextant, adapts itself to the circular graduated limb of the instrument. The point marked *a* on the Vernier is the *index* of the graduated arc of the limb, and is that which marks out the integral part of the measure of the angle, the fractional part being indicated by the Vernier divisions, as already explained. It is the mark *a* which ought to correspond with the mark 0 on the graduated limb, when the index and horizon glasses of the instrument are parallel: it is common, however, to speak of the whole movable limb A as the *index*.

Fig. [illegible]



Suppose each of the divisions on the graduated limb to denote n minutes, and let m be the number of those divisions which make up the whole extent of the Vernier scale, then the Vernier will contain $m\,n$ of the minutes of the graduated limb. If this extent be divided into $m + 1$ equal parts, then the difference between one division on the graduated limb and one division on the Vernier, will be

$$n - \frac{m\,n}{m+1} = \frac{n}{m+1}$$

If $n = 20'$, and $m = 19'$, $\therefore \frac{n}{m+1} = 1'$. If $n = 10'$, and $m = 59'$, then $\frac{n}{m+1} = 10''$, &c.

THE INDEX ERROR.—If when the index and horizon glasses are parallel, the beginning *a* of the divisions on the Vernier does not coincide with the mark 0 on the graduated limb, the distance between them is the *index error*—subtractive when the Vernier mark is to the left of the 0, and additive when it is to the right. To discover the amount of index error, move the index till a point of the horizon, or some more distant object, coincides with its image: the distance of the index mark *a*, from 0 on the graduated limb, is the amount of index error.

The sextant just described is only a modification of the quadrant; both instruments are in principle the same. The first published description of the quadrant appeared in the *Philosophical Transactions*, No. 420, for the year 1738, the paper communicating it having been laid before the Society in May, 1731. It was the production of John Hadley, and the instrument he described was consequently called Hadley's Quadrant; there is little reason to suspect that the invention was not his own. But a similar instrument had long previously been invented by Newton, and communicated to Halley, who kept the contrivance to himself. "The description of the instrument was found, after the death of Halley, among his papers, in Newton's own handwriting, by his executor, who communicated his papers to the Royal Society, twenty-five years after Newton's death, and eleven after the publication of Hadley's invention, which might be, and probably was, independent of any knowledge of Newton's, though Hutton insinuates the contrary." "But," adds Sir John Herschel, "the priority of invention belongs undoubtedly to Newton, whose claims to the gratitude of the navigator are thus doubled, by his having furnished at once the only theory by which his vessel can be securely guided, and the only instrument which has ever been found to avail in applying that theory to its nautical uses."

CHARTS.—A chart is a map upon a plane surface of a portion of the sea, including whatever within its limits it may be useful to the mariner to have exhibited, such as rocks, shoals, &c.; and in some, the directions of currents, and the variations of the compass. Charts are of two kinds—the Plane Chart and Mercator's Chart. As in the former the meridians are all parallel lines, and the degrees of latitude all of equal length, the proper relations of latitude and longitude are grossly violated, and such charts are of no use except for mere coasting purposes. At sea, the only charts of any use are those constructed on Mercator's principles, described at page 1062, the degrees of longitude remaining invariable; while those of latitude are enlarged more and more from the equator to the pole, agreeably to the law investigated and explained at the page referred to.

The accompanying map is a representation of the globe projected upon Mercator's plan, but on a scale far too minute, of course, to serve any other purpose than mere illustration, for which alone it is here introduced. On the actual sea chart, representations of the compass are placed at intervals, for the purpose of setting off courses.

The meridians which limit the chart employed at sea are graduated, as also the parallel of latitude which bounds the extent of the chart north or south. A point on the chart being marked, we find its latitude by simply observing where the edge of a straight ruler applied to the point, and held parallel to any parallel of latitude, cuts the graduated meridian; and the longitude is found by placing the edge, while on the point parallel to any of the meridians, and observing where the edge cuts the graduated parallel.

To find the course between two places on the chart, apply the edge of a parallel ruler to the two places, and, holding it there, move the other part of the ruler till an edge passes through the centre of one of the compasses; the course will thus be indicated.

To lay down on the chart the position of the ship by dead-reckoning—that is, from her course and distance from a given point of departure, as the preceding noon—place the ruler on the point of departure, and parallel to the given course: from the graduated meridian at the side of the chart, and in the latitude of the ship, take the distance, in degrees, &c., by the compasses: this distance, applied from the point of departure along the edge of the ruler in the direction of the course, will mark out the position of the ship by dead-reckoning. To lay down the position as given by the latitude and longitude is sufficiently obvious: the intersection of two pencil lines, through the given points of latitude and longitude, and parallel to the boundaries of the chart, will be the position sought.

It will be perceived by the reader who has gone over what has been stated in the Navigation respecting Mercator's sailing, that although positions are correctly exhibited on the chart, as regards latitude, longitude, and course, yet *distances* are exaggerated. Distances which are the same on the globe, become more and more elongated on the chart as we approach the pole.

THE SHIP'S JOURNAL.—A Sea Journal is a record of the daily transactions and occurrences in connection with the navigation of the ship, including whatever observations and remarks that may be necessary to give a brief but connected professional history of the voyage.

The entries in the Journal are made hourly after the departure is taken: the ship shapes her course towards a definite point; and to do this, either a chart is consulted, or the angle determined by computation; allowance then being made for the variation of the compass, and the local deviation, the compass or steering course for the first stage is obtained. The ship, however, is usually considered to depart from the point of land or other conspicuous object last seen; and the bearing opposite to that, of this point of departure, is regarded as the first course, and the distance of it as the first distance. The ship is not, however, considered as having fairly commenced her voyage till her final departure has been taken.

Time is generally recorded as in the affairs of civil life, and not according to astronomical reckoning: noon and midnight equally divide the twenty-four hours, as on land. From the hourly registry of the course and distance, the leeway and the variation of the compass being properly allowed for, the ship's position is determined every day at *noon*. If no astronomical observations have been made, the position thus determined is the place of the ship by *dead-reckoning;* but if the latitude or longitude, one or both, have been computed from observations, a distinct entry to that effect is made, although the day's account by dead-reckoning is still preserved.

This clearing up the ship's account every day at noon, so as to enable her to take a fresh departure daily at that hour from a known position, just as she took her departure at first, is called a *day's work*. As noon is the time invariably fixed upon for ascertaining the resultant of all the preceding twenty-four hours' sailings, whatever latitude or longitude may have been determined by astronomical observations in the interval, is brought up to that hour by help of the dead-reckoning. Hence the entries, "Latitude by Observation," and "Longitude by Observation," frequently inserted in the noon results, are in general made up in some small part of the latitude and longitude by account.

In keeping the ship's reckoning, the position departed from at each noon, is considered to be that which nautical astronomy assigns; so that when observations have been made in the interim, the dead-reckoning commences afresh, and is not a continuation of the yesterday's account. The record is then carried uninterruptedly on till a noon arrives, immediately before which, the true position of the ship has again been settled by observations, and so on. The meridian observation for latitude is of course recorded for the noon on which it is made.

The working of a day's work may always be effected by the Traverse Table, after the manner shown at page 1055; and as the twenty-four hours' sailings should be regarded only as furnishing data for finding the position of the ship at the end of that time approximately, it is not considered as in general necessary to attend to minutes in the courses. "It is mere waste of time," observes Lieut. Raper, "to work the course nearer than to the whole degree; for if even the compass could be depended upon, as it cannot be, to 1°, the ship cannot be steered to twice that quantity." We shall now give a very short specimen of a Ship's Journal: there is no settled and uniform plan of arranging all the entries; but there are certain prominent features in which all sea journals are alike. The specimen here offered is, with some slight modification, extracted from that given by Mr. Riddle in the work before referred to.

The reader is already aware that the principal entries in the Journal are, in the first instance, written in chalk on the black board called the *log-board*, from which they are afterwards transferred to the *log-book*. The correction of the several courses for leeway and variation being made, and the position of the ship at noon, as deduced from the sailings, and as determined by observation when observations are taken, completes the day's work, and renders the log-book a *journal*.

EXTRACT FROM A JOURNAL OF A VOYAGE FROM ST. MICHAEL TOWARDS ENGLAND.

H	K	F	Courses.	Winds.	Lee-way.	Remarks, Sunday, Sept. 11, 18—.
1				W.		Moderate and clear weather. P.M.
2						At 3 P.M. the eastern end of St. Michael's, lat. 37° 48′ N., long. 25° 13′ W., bore W.S W., dist. 6 leagues, from which I take my departure.
3				N.N.W.		
4	5	8	E.N.E. ½ E.			Ditto weather.
5	5	7		N.		Fresh breezes and cloudy weather. In top-gallant sails.
6	5	4				Ditto weather.
7	4	2	E. *b.* N.	N. *b.* E.	½	In second reef topsails, hauled down jib, and set fore topmast staysail.
8	4	1				Strong breezes and hazy weather; brailed up spanker, and set mizen staysail.
9	3	0	E. *b.* S.	N.E. *b.* N.	2	
10	3	0				In third reef topsail, and set trysail.
11	2	8				
mid-night	2	6				Ditto weather.
						Monday, Sept. 12. A.M.
1	3	4	E.S.E.	N.E.	3½	Fresh gales and hazy weather,
2	3	6			3	More moderate.
3	3	9				
4	4	2				Out fourth reef of topsails.
5	4	3	N.N.E.	E. *b.* S.	2¼	Tacked; strong breezes and cloudy weather.
6	4	0			2	Out third reef topsail; set jib and spanker.
7	4	5				Set top-gal. sails. Fresh breezes and cloudy weather.
8	5	0			1½	
9	4	7				Out second reef topsails.
10	4	2				Lat. by doub. alts. at 11 A.M. . . 38° 15′ N. Diff. lat. up to noon . . . . 5′ N. Latitude at noon 38° 20′ N.
11	4	8				Moderate and clear weather.
noon	4	8				Variation by amplitude 1¼ points W.

Course	Dist.	Lat. acct.	Lat. obs.	Long. acct.	Long. obs.	Bearing and dist. of Lizard at noon,
N 58° E.	Dist. 57m.	38° 19 N.	38° 20′ N.	24° 11′ W.		N. 49¾° E. Dist. 1074m.

H	K	F	Courses.	Winds.	Lee-way.	Remarks, Monday, Sept. 12, 18—.
1	4	6	N.N.E.	E.	1¼	Moderate and clear weather. P.M.
2	5	0				Out first reef topsails, set royals, and flying jib.
3	5	3	N. *b.* E. ¾ E.	E. ¼ N.	½	Light breezes and clear weather.
4	5	6				Ditto weather. Swell from E. from 4 P.M. till 8, for which allow a drift of 24 miles.
5	6	0				
6	6	1				In royals and flying gib,
7	5	8	E.S.E.	N.E.	1	Tacked.
8	5	7				
9	5	0			½	Ditto weather.
10	5	3				
11	5	8			0	
mid-night	6	2				Ditto weather.
						Tuesday, Sept. 13th. A.M.
1	5	9	E.S.E.	N.E.	1¼	Moderate and clear weather.
2	5	7				
3	5	3			1	Fresh breezes. In top-gallant sails.
4	5	4				
5	5	0	E.N.E.	N.	1½	In first reef topsails.
6	5	0				
7	4	8				Strong breezes and cloudy.
8	4	3				In second reef topsails.
0	3	9			2	Long. by chron. at 8 A.M. . . . 23° 2′ W. Diff. long. up to noon . . . . 17′ E. Long. at noon . . . 22° 45′ W.
10	3	4				Flying clouds, with light showers.
11	3	3			2½	Fresh gales and squally: down jib and in spanker.
noon	3	5				Lat. at noon by mer. alt. 38° 46′ N. Var. 20° W. by azimuth.

Course.	Dist.	Lat. acc.	Lat. obs.	Long. acct.	Long. obs.	Bearing and dist. at noon, Lizard.
N. 69 E.	Dist. 63m.	38° 43′ N.	38° 46′ N.	22° 56′ W.	22° 45′ W.	N. 48¾° E. Dist. 1013m.

From what has already been said as to the unavoidable imperfections in even the most careful measurements of a ship's course and distance, the difficulty of making the proper allowances for the leeway, &c., and of estimating the effects of squalls, currents, &c., it will readily be inferred that a ship's journal would soon become so erroneous, as a registry of actual facts, as to be quite valueless, unless repeatedly rectified by astronomical observations. In the absence of these, such a journal would describe little other than the imaginary route of an imaginary ship, which, after a voyage of any length, might terminate at almost any point of the globe as likely as at the real point reached by the real ship. No opportunity, therefore, should be lost to check this increasing tendency to error in the dead-reckoning, by carefully determining the position of the ship from the safe principles of nautical astronomy; and the ship's account should be regarded as of value, only in so far as it can be made auxiliary to the application of those principles.

The letters H, K, F, at the heads of the first three columns in the preceding specimen of a journal, stand for hours, knots, and fathoms, or tenths of a nautical mile. The entries between noon and midnight are marked P.M. (*post meridiem*), and those between midnight and noon are marked A.M. (*ante meridiem*).

The result of each day's work is inserted at the bottom of the page; the courses being corrected for leeway and variation, the result exhibits the true course and distance from the point of departure; together with the diff. lat. and diff. long. made in the twenty-four hours, or rather the latitude and longitude reached.

In reference to the two preceding days' work, it will be observed, that as the variation of the compass is westerly, it must be allowed to the left of the compass courses; and, therefore, when the ship makes leeway on the larboard tack, the difference between the leeway and variation is the correction to be applied to the course—to the *left*, if the variation be greater; but to the *right* if the leeway be the greater.

When the ship makes leeway on the starboard tack, the allowance for it, as well as that for variation, being to the left, their sum will be the correction to be applied to the compass course; and when no leeway is made, the only correction is for the variation. Now E. N. E., the opposite point to the bearing of the land from which the departure is taken, is the first course; and this, and the drift, being corrected for variation, and the other courses for both variation and leeway (when there is any), and the distances on each of these courses added up, we have the following Traverse Table for the first day's work:—

TRAVERSE TABLE.

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
N.E. ¼ E.	18	12·1		13·3	
N.E. ¾ E.	16·9	10·1		13·6	
N.E. *b.* E. ¾ E.	8·3	3·5		7·5	
E.S.E. ¾ E.	11·4		2·8	11·1	
S.E. ¼ E.	7·0		4·7	5·2	
S.E. ¾ E.	8·1		4·8	6·5	
N.N.W.	4·3	4·0			1·6
N. *b.* W. ¾ W.	8·5	8·0			2·9
N. *b.* W. ¼ W.	9·7	9·4			2·4
N. ¾ W.	4·2	4·2			·6
N. ¼ W.	9·6	9·6			·5
S. ¼ W.	18·0		18·0		·9
		60·9	30·3	57·2	8·9
		30·3		8·9	
Course, N. 58° E. Distance, 57 miles.		30·6		48·3	

Lat. left . . 37° 48′ Mer. parts . . 2453
Diff. lat. . . 31′
Lat. in. . . 38° 19′ 2492
M. diff. . . 39

Hence, by Mercator's sailing, page 1061, the difference of longitude is 1° 2′ E.:—

Long. left . . .	25° 13′ W.
Diff. long. . . .	1 2 E.
	24 11 W.

Therefore the ship's place, by account, is

Lat. 38° 19′ N. Long. 24° 11′ W.

From 11 A.M. till noon, the ship's true course was N. ¼ W., 5 miles nearly: hence the difference of latitude also is 5 miles nearly; and this added to the latitude, as determined by double altitudes, at 11 A.M., gives 38° 20′ N. for the latitude by observation at noon.

With this latitude and the longitude by account, the bearing and distance of the Lizard are found to be N. 49° ½ E., 1074 miles.

The courses for the second day's work, being first corrected for leeway only, as the variation is given degrees, we have the following traverse table.

TRAVERSE TABLE.

Courses,	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
N. ¾ E.	9·6	9·5		1·4	
N. *b.* E. ¼ E.	23·0	22·3		5·6	
S.E. *b.* E.	11·5		6·4	9·6	
S.E. *b.* E. ½ E.	10·3		4·8	9·1	
E.S.E.	12·0		4·6	11·1	
S.E. ¾ E.	11 6		6·9	9 3	
S.E. *b.* E.	15·7		8·7	13·0	
E. ½ N.	14·1	1·4		14·0	
E.	7·3			7·3	
E. ½ S.	6·8		·7	6·8	
W. (swell).	24·0				24
		33·2	32·1	87·2	24
		32·1		24·0	
Compass Co., N. 89° E. Distance, 63 miles.		1·1		63·2	

Compass course . . .	N. 89° E.
Variation	20° W.
True course . . .	N. 69° E.
Distance	63 miles.

With this course and distance, 63 miles, found from the Traverse Table, the diff. lat. and departure are found to be 22·8 N. and 58·8 E.

Lat. left	38° 20′ N.
Diff. lat.	23 N.
Lat. by account . .	38 43 N.
Long. left	24 11 W.
Diff. long.	1 15 E.
Long. by account . .	22 56 W.

The departure made, from 9 A.M. till noon, is nearly 14 miles, with which, and the mid-latitude, about 38½°, the difference of longitude is found to be 17 miles; which, taken from 23° 2′, the longitude by the chronometer at 9 A.M., gives 22° 45′ for the longitude by the chronometer at noon.

CONCLUSION.—In here terminating this section on the Principles of Navigation and Nautical Astronomy, we would remind the reader of the objects proposed in undertaking it, as sufficiently declared in the INTRODUCTION. This object was twofold: first, to furnish to the mathematical student a convincing proof of the great practical value of the abstract sciences which he cultivates; and, second, to supply the professional navigator with the theoretical principles on which his rules of operation are founded. It did not enter into our plan to go into all the practical details of navigating a ship, nor to dwell upon those facilitating expedients which could be rendered intelligible only by the aid of extensive nautical tables. The present treatise is offered to the notice of the mariner, more in the character of a companion or supplement to the books of rules and tables in every-day use at sea, and as serving to show him the scientific theory on which his practice is based. But as far as could be done, without special tables, we have fully gone into the calculations necessary for determining the position of a ship on the ocean; and have shown that all the practical demands of navigation may be amply satisfied by help of only the common logarithmic tables, and that knowledge of turning them to account, which the elementary principles of plane and spherical trigonometry supplies.

We shall now add a few general remarks by way of comment, subjoining some interesting and valuable information respecting the action of iron ships on the compass.

It cannot fail to strike an attentive reader, that the subject of Navigation presents a forcible example of the value of abstract science, even in circumstances where the practical application of its principles would seem to be almost precluded, on account of the unavoidable imperfections of our observations and experiments; of the instruments with which we work, and of the materials upon which we operate.

The mechanical tools or implements of the navigator, are the log, the compass, the chronometer, and the sextant; these are to furnish him with the materials upon which his science is to work, and from which he is to extract all his information in situations where no external aid can reach him, and where to err may involve life and property in sudden destruction. Yet the mechanical means upon which he thus depends for guidance and safety, are all confessedly imperfect; he can measure with accuracy neither the rate at which he sails, nor the course upon which he steers; and even if the log and compass were perfect, hidden and unsuspected agencies may vitiate, and falsify the indications of both. The sextant, fortunately, is beyond the operation of these disturbing causes; it is, moreover, the least imperfect of all his nautical appliances, and accomplishes the important end of rectifying and adjusting, to a very close approach to accuracy, what the other instruments may have done amiss. It is among the most valuable gifts that science has ever presented to man to aid him in his necessities. Nautical Astronomy could not exist without it: and to say that it is not perfect, is only to repeat what has been applied to every work of man's hands. The best sextants, however, are subdivided to no smaller arc than 10″, so that fewer seconds than 10 must be estimated, by help of the microscope, entirely by the eye.*

As just noticed, the sextant—including, of course, in this term the quadrant—is of the utmost use in correcting the results of the dead-reckoning. But the ship's account continuously accumulates, and its errors must run on till the weather and the sky furnish opportunities for celestial observations. In the interim, the vessel is trusted almost entirely to the guidance of the compass; and it most unfortunately happens that, from the local attraction, the ship may often be said rather to direct the compass than the compass to direct the ship. It is most important, therefore, that the intervals between observations at sea be shortened as much as possible by seizing every occasion that offers for making them. From what is taught in the preceding pages, the reader will easily perceive how it happens, that even very gross errors in the dead-reckoning become comparatively inoperative in the results deduced from astronomical observation, although the calculations founded upon these observations virtually involve the data furnished by the ship's account. These data, however, do not directly enter into the work; it is the *time* which corresponds to them that is employed; and, fortunately, the astronomical elements, taken from the *Nautical Almanac*, in reference to this time—semi-diameter—declination—right ascension—horizontal parallax, &c., vary so little, even in a large interval, that an error in the ship's place to the extent of a quarter of the globe, would not, in general, entail an error of a quarter of a degree in the adjustment of that place by the lunar observations, provided, at least, that the time be not deduced from the moon.

As already remarked at page 553, in the MATHEMATICAL SECTION, the reasonings of pure geometry tolerate no errors in the premises; but practice can never satisfy these rigorous conditions; and in proportion as they are departed from, will be the geometrical shortcoming of our conclusions. At first sight, therefore, it would seem chimerical to hope for any close approach to accuracy from data so widely erroneous; but when it is considered that these data connect themselves with other dependent data, which are incapable of error beyond a very limited range, we at once perceive that these latter may be very near the truth, though the former may greatly depart from it; and that if the inquiry involve the dependent data only, and not in a direct manner the original, the inaccuracies of *these* need give us but comparatively little concern.

It is thus that a very close approximation may be made to the true position of a ship, though the dead-reckoning may displace her many degrees, and though, at the same time, we employ this reckoning in the operation, as if it involved no error at all. But in the intervals between these adjusting observations, the safety of the ship is often wholly dependent on the trustworthiness of the compasses; and, of late, much mischief has arisen from placing too implicit a confidence in them in certain circumstances.

Since the prevalence of iron vessels, the disturbances of the compass have been seriously forced upon the attention of scientific men; and the subject of local deviation, still involved in considerable obscurity, is becoming more and more a matter of anxious scrutiny and investigation. The importance of the inquiry was strongly urged by Dr. Scoresby, at the meeting of the British Association, at Liverpool, in 1854. "There were certain principles," observes the Rev. Dr., "connected with the navigation of iron ships, which were universally admitted. Those principles were, that iron, being more especially disposed to the magnetic condition, was a material, of course, calculated above all others to disturb the action of the compass on board the ship. Again, it was admitted that there were difficulties in the navigation of iron ships, arising, not merely out of the original or primarily magnetic condition and disturbing influence of the iron, but also in respect of certain changes which had been held as mysterious—changes which took place not unfrequently in regard to ships whose magnetic condition had been supposed to be very well ascertained." Dr. Scoresby then adverts to the circumstances connected with the melancholy wreck of the *Tayleur*, the fate of which must be in the recollection of all our readers. The ship *Tayleur*, a new vessel, bound to Australia, sailed from Liverpool on Thursday, January 19th, 1844. She was 1,979 tons burthen, new measurement, and she had on board about 458 passengers—the crew and passengers, altogether, making a total of 528 persons. She left the Mersey about noon on the above-named day. The pilot left her between seven and eight o'clock in the evening, in a position between Point Lynas and the Skerries. On Friday she encountered very heavy weather; and about eight o'clock on the following morning (Saturday), it was for the first time ascertained that there was any material difference between the compasses.

There were three compasses on board. Dr. Scoresby makes special reference to two of these. One of the two was near the helmsman, and was the one by which he steered; and the other was near the mizen-mast. Both of these compasses had been what is called adjusted, by permanent magnets; so that if the principle of adjustment had been correct, they should not either of them have changed or differed from the other.

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* The error of this estimation can never reach *five* seconds, so that the corresponding error in longitude cannot be more than about two miles; an error that is quite compatible with perfect safety, except in very extraordinary circumstances. (See page 1103).

Trusting to the compass near the helmsman, the captain had the idea firmly impressed upon his mind that he was sailing fairly down almost mid-channel; at all events, in a good position for navigating the Irish Channel. The other compass indicated a difference of about two points. The captain, however, judging from certain indications which he had noticed previously, assumed that the wheel compass was the correct one.

In the course of a few hours, about half-past eleven o'clock on the same morning, the wind having increased, and a heavy sea setting up the channel, the ship made rather a rapid progress, when they suddenly came in sight of land on the lee beam, in such a position that there was necessarily a great difficulty—in this case (according to the measures pursued) an insurmountable difficulty—in avoiding the land. An attempt was made to wear the ship round. This failed: and then an attempt was made to use the anchors to bring her up. Both the cables snapped on the occasion, and the ship was then left helpless, driving broadside upon the rocks of Lambay Island. The result was the fearful catastrophe—namely, the loss of about 290 lives: out of 100 females who were on the ship, only three escaped upon that melancholy occasion.

Investigations into the cause of the calamity were undertaken; and the Marine Board of Liverpool, after stating that Captain Noble had given very great attention to the ascertaining of the correctness of his compasses, and verifying their action on different occasions—report that, "notwithstanding these precautions, it appears to this Board that the *Tayleur* was brought into the dangerous position in which the wreck took place through the deviation of the compasses, the cause of which they (the Marine Board) had been unable to determine."

To these important matters Dr. Scoresby has given much thought and attention; and he finds, from numerous experiments, some of which are very simple, that mechanical violence has a very considerable influence upon the magnetic condition of iron. When an iron bar, entirely neutral as to its molecular magnetism, if held in an upright position, or inclined in the axial direction of the earth's magnetism, was subjected to percussion or other mechanical violence, not only did its magnetism become much more powerful than that of simple induction, but it strongly exhibited augmented polarity, when placed in the east and west equatorial position; and however it might be moved about and swung round, its polarity remained the same. Dr. Scoresby applies these facts to iron ships, and points out that, in consequence of the percussive action to which the material is exposed while the ships are in course of construction, it became as intensely magnetic as it is possible for malleable iron to be. This augmented magnetism, however, is not permanent or fixed, but, under different circumstances, as to the relative directions of the ship's magnetism and that of the earth, is easily changeable, and liable necessarily to be changed. The magnetism developed by mechanical violence can be readily neutralised or changed, under a proper change of conditions, by other processes of mechanical violence.

The general result of his experiments went to the establishing of the fact, that besides the two denominations of magnetism ordinarily received—that of simple terrestrial induction, and that of permanent independent magnetism—there is another denomination corresponding with neither; not being absolutely controllable, like the former, by terrestrial influences, nor capable, like the latter, of resisting all kinds of mechanical violence. To this third denomination he gives the name of Retentive Magnetism, and which he proves to be a fluctuating quality, though hitherto considered as permanent. On the contrary, the long-continued vibration of a ship under steam, and, much more so, the straining of the vessel in a heavy sea, under the circumstances when the terrestrial induction might be acting in a very different direction from the original axial polarities of the ship, would be sufficient to change the direction of the magnetism originally developed in the course of her construction. Hence, he observes, much would depend, in respect of the mechanical action of the sea, on the position in which the ship had been built. In the case of the *Tayleur*, when he first heard of the catastrophe and had read the evidence, he stated to some friends at Torquay, that he would venture to affirm that she had been built with her head *northward*; he found, on inquiry, that she had been built with her head nearly north-east. Here then, he adds, were the precise circumstances for expecting a change in the ship's magnetic distribution. Having been built with her head to the north-east, she had a certain magnetic distribution accordingly; and when she began to strain, with her head to the south-west, that distribution was necessarily changed, and the first effect of it had been to produce a great difference in the two compasses adjusted by fixed magnets. If the captain had been aware of the changes which might, and most probably would, take place when the ship began to strain in a different position from that in which she had been built; if he had known that the compasses, having so large an original deviation as 60°, might vary as much as two, three, or even four points, he would have known, of course, that he must place no reliance upon them.

It is most important, therefore, continues Dr. Scoresby, for safety in navigating these vessels, that captains should be made aware of the liability of the compasses to change, and so to mislead them; that they should know the circumstances under which, in accordance with natural laws regulating and applying the earth's inductive action, changes were most likely to occur; that they should be always watchful of opportunities for determining the true magnetic direction, with reference to their compasses, by observations of the sun and stars; and that by providing a place for a standard compass aloft, as far from the deviating influence of the body of the ship as possible, they might have guidance sufficient, with some small allowances, for steering a correct magnetic course. With such precautions, Dr. Scoresby did not doubt that the difficulties in respect of compass guidance, in the navigation of iron ships, might be mainly and practically overcome.

These remarks and suggestions, from an experienced navigator so well acquainted with his subject, deserve the serious attention of mariners; and there is no doubt that, even in wooden ships, such a locality for a standard compass, as he here recommends, would prove of service. In iron ships, as sufficiently shown above, any compass-adjusting apparatus, applied at the outset of a voyage, becomes of little or no avail when the vibration and strain of the vessel are thus known to change its magnetic condition; and any confidence placed in such adjustments is likely to beget a feeling of security, and to allay apprehension, even in situations of the most imminent peril.

For a full account of Dr. Scoresby's views, and of those of Mr. Towson, another very competent authority, the reader is referred to *The Proceedings of the Twenty-fourth Meeting of the British Association for the Advancement of Science*. (See, also, on this subject, p. 246, *ante*).

SECTION XII.

METEOROLOGY.

[THIS Section may be properly divided into three parts. That by Dr. Scoffern, affords a popular and comprehensive description of the science. Such portions of the original paper as referred to Light, Electricity, and Magnetism, have been omitted, for those subjects have already been discussed in detail, under their respective sections. The remarks on Heat and its laws have been left untouched, because, although they have been anticipated under that section, it was thought desirable to bring them again into prominence, in connection with Meteorological phenomena. The second part (Chapter IV.), by Mr. Lowe, is devoted entirely to Practical Meteorology, and affords valuable aid and directions to those who may be disposed to make accurate observations. The third part (Chapter V.), by the Editor, gives a *résumé* of the proceedings of Admiral Fitzroy and Mr. Glaisher, especially in reference to the balloon ascents of that eminent meteorologist; with other recent and important particulars.]

CHAPTER I.

INTRODUCTION.

DEFINITION AND LIMITATIONS.—The term Meteorology, derived from *μετεωρος*, elevated, was applied by Aristotle to signify phenomena occurring in elevated regions. It may be considered synonymous with the study of atmospheric phenomena; though all which concerns meteors proper is very nearly allied with astronomy.

In many respects, Aristotle's opinions on meteorologic subjects display the usual acumen of that deep thinker; his remarks on the subject of dew are particularly interesting. Deprived, however, of the barometer and thermometer—deprived of optical instruments—the nature of electricity yet undeveloped, and the composition and functions of the atmosphere unknown, ancient speculations on meteorologic subjects were necessarily unsatisfactory and vague.

The meteorological writings of Theophrastus, Aristotle's pupil, were more diffuse than those of that great philosopher, and were long recognised as constituting a text-book. They constituted the groundwork of the *Διοσημεια*, or prognostics of Aratus, and were embodied in a versified rendering by Cicero in his youthful days. Portions of these attempts at versification are still in existence, and do but little credit to the great Roman orator in a poetical capacity.

Meteorology, in its most common and restricted sense, may be considered synonymous with *knowledge of the weather*. It therefore involves a full acquaintance with the nature and composition of the atmosphere; with the laws of gaseous and vaporous elasticity; with the conditions determining the production of fogs, dew, snow, and hail; also with the laws of atmospheric, optical, and electrical phenomena. It is the province of meteorology, also, to study the phenomena of aërolites, and the relations which subsist between atmospheric conditions, and the development of organic species.

The above is a general outline of the scope and limits of meteorology. Its successful study will be seen to involve a pre-acquaintance with many sciences, more especially those of chemistry and electricity. There does not, indeed, exist any science having limits so undefined as meteorology. From a consideration of the theory of shooting stars, to a contemplation of the mutual alliance subsisting between certain forms of disease and atmospheric conditions, or the relation between certain animal and vegetable tribes and given atmospheric conditions, the divergency is wide. Nevertheless, all these branches of study are intimately allied with meteorology; and, perhaps, the most delightful part of botanical science is that which seeks to establish connections between the localisation of certain vegetable families, in districts characterised by some peculiarity of meteorologic condition. Horticulturists have been too ready to overlook the influence of remote atmospheric conditions on certain vegetable families. Too frequently it has been considered that a vegetable surrounded by an atmosphere having a temperature similar to that of its native region, and planted in a soil of similar chemical composition, must necessarily thrive. There are, nevertheless, meteoric conditions beyond these. Why is it that many species of the palm tribe refuse to grow very far away from their native regions, although transplanted to localities seemingly identical in all respects? Why is it that the cocoa-palm refuses to grow in regions very far distant from the sea? These questions involve meteorologic considerations of great interest: and not less interesting to a meteorologist, is the partiality evinced to a restricted region by the cinchona tribe. An atmosphere very much rarefied, and perpetually moist, is so essential to their existence, that they cannot live without it.

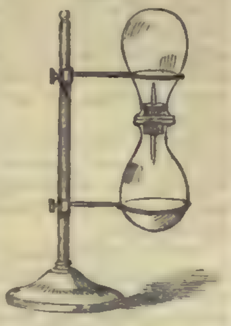
The natural approach to meteorology is the study of the atmosphere, which admits of being contemplated under many aspects. It may be contemplated either as the atmosphere proper or theoretical, composed of two gases—oxygen and nitrogen; or as the practical atmosphere or mixture of the gaseous theoretical atmosphere, with numerous vapours, extraneous gases, and fleeting undetermined miasmata. The atmosphere, too, admits of being regarded statically, *i.e.*, at rest; and dynamically, *i.e.*, in motion; the latter involving a study of the causes of winds. The atmosphere, lastly, may be considered in relation to the imponderable agents, to heat, light, electricity, and magnetism. We shall begin by investigating the nature of our atmosphere regarded chemically.

CHEMICAL CONSTITUTION OF THE ATMOSPHERE.—By the ancients, air was considered to be an elementary substance. Chemistry at length demonstrated it to consist of two gases, oxygen and nitrogen combined, or rather mixed in the proportions of about eighty parts, by measure, of nitrogen, and twenty of oxygen; or, in other words, one volume of oxygen to four of nitrogen.

Considerable difference of opinion once existed on the question, whether the atmosphere be a chemical or a mechanical compound. To adduce evidence bearing on this discussion, would be foreign to the subject of meteorology. The generally received opinion is in favour of the mechanical constitution of the atmosphere. The law of the diffusion of gases perfectly accounts for the intimate mixture with oxygen and nitrogen in the atmosphere, without having recourse to the assumption of chemical union. An outline of the law in question it will be proper to give.

Gaseous Diffusion.—If two glass vessels be taken, as represented in the accompanying diagram (Fig. 1), the upper one being filled with hydrogen gas, the lower one with oxygen gas, placed in communication with each other by a capillary tube passing through the cork stopper of both, and allowed to remain at rest for about half an hour, perfect mixture of the oxygen and hydrogen gases will have ensued. Inasmuch as no chemical union takes place between the two gases thus circumstanced, and as hydrogen gas, filling the upper vessel, weighs but $\frac{1}{16}$th of its bulk of the oxygen gas filling the lower vessel, some new cause of admixture has to be sought; it depends upon the mutual tendency of the two gases to become diffused through each other.

Fig. 1.



The above is an individual case exemplifying a general principle; any two gases might have been selected, and mutual diffusion would have ensued. Oxygen and hydrogen gases have been here chosen, because of the facility wherewith the circumstance of these having become diffused, may be determined. It is well known that neither oxygen nor hydrogen gas, taken separately, will explode on the application of flame, whereas a mixture of the two readily explodes; hence the propriety of selecting these two gases for illustration of the principle in question, will be obvious.

Faraday appears to have been the first to direct attention to the mutual diffusibility of gases. He noticed that bottles filled with gases, and corked or stoppered, or vessels filled with gases and inverted over mercury, in either case enclosed to all appearance accurately, nevertheless almost always permitted mutual admixture of the air without and the gas within. Döbereiner, Mitchell, and Graham, more especially the latter, have since investigated this class of phenomena more narrowly; and Graham has succeeded in determining the law which regulates this diffusion. He finds that the relative diffusiveness of any two gases is expressed by the reciprocals of the square root of their densities. Thus, the density of air being one, its diffusiveness is one also. The density of hydrogen being 0·693, its diffusiveness is $\frac{1}{\sqrt{0 \cdot 693}} = \frac{1}{0 \cdot 2633} = 3 \cdot 8$; the density of ammonia being 0·5898, its diffusiveness is $\frac{1}{\sqrt{0 \cdot 5898}} = \frac{1}{0 \cdot 7681} = 1 \cdot 30$; and generally representing the density of a gas by d, its diffusiveness is $= \frac{1}{\sqrt{d}}$ Applying this rule to practice, it appears that, supposing hydrogen and ammonia placed under circumstances promoting their mutual diffusion, 3·8 volumes of hydrogen will become mingled with 1·30 of ammonia.

It will be remarked that the atmosphere has hitherto been treated of, in a theoretical sense, as a mere mixture of nitrogen and oxygen gases. Practically, however, the atmosphere is far more complex. It invariably contains portions of carbonic acid (about one part in a thousand); also extraneous gases, besides aqueous and other vapours. A mixture of all these things may be termed, distinctively, the actual or *practical* atmosphere. They will hereafter come under our notice *seriatim*; but even a theoretical mixture of oxygen and nitrogen gases is subject to remarkable variations of property, its chemical composition remaining unchanged; portions of its oxygen are subject to be converted into *ozone*.

The change of common oxygen into ozone, furnishes an illustration of one of the most remarkable discoveries of modern science. It displays what is, perhaps, the most extraordinary example of the condition of *allotropism*, or the existence of one body under two different aspects; it promises to render evident some of these occult atmospheric causes which determine the progress of epidemics, and promote the existence of endemic diseases.

A summary of our knowledge relative to ozone may be briefly stated as follows:—Oxygen gas is susceptible of undergoing a change, the nature of which is altogether veiled in mystery. It is susceptible of becoming odorous, corrosive, and irritating when breathed; its chemical action may be exalted and modified, so that, whilst ordinary oxygen gas neither bleaches nor corrodes silver, nor decomposes iodide of potassium, the allotropic, or second form of oxygen gas, will accomplish all these results, and many more too numerous for mention here. The general conclusion to which it is desired to bring the reader is this: if causes can be proved to exist capable of changing atmospheric oxygen gas, in its ordinary state, into oxygen gas in its extraordinary state, how vast, how complicated must be the meteoric results determined thereby! That such natural causes do exist, will be readily inferred from a consideration of the artificial methods to which the chemist has recourse, for changing ordinary oxygen into ozone.

METHOD OF OZONISING OXYGEN GAS.—The most ready method of ozonising oxygen gas is as follows:—Take a few sticks of phosphorus; scrape them free from all superficial oxide, &c.; place them in a wide-mouthed bottle containing a little water, but not enough to cover the phosphorus. Let the whole remain at rest for about ten or fifteen minutes, and a considerable portion of the atmospheric oxygen will have been converted into ozone. The ozonised air thus generated will be at present mixed with vapours of phosphorus acid; washing will free the air from these, however, without removing the ozone. That the air thus treated has become considerably modified in some way, will, in the first place, be rendered evident by the smell. Atmospheric air, when pure and in its ordinary state, is devoid of smell; but the atmospheric air, the product of the experiment just detailed, will be found to have a very peculiar odour. It will be found, moreover, to be capable of removing the colour of sulphate of indigo, and other vegetable and animal colouring bodies. All this is due to the modification which ordinary oxygen gas assumed—due to its assumption of the allotropic state—to its conversion into ozone.

Another ready method of generating ozone is this:—Moisten the interior of a bell-glass receiver, or a large-mouthed bottle, with ether; then take a glass rod, heat it in the flame of a spirit-lamp, and plunge it into the bottle or bell-glass: under these circumstances ozone will be formed, provided the glass rod has not been heated to a very high temperature; for the circumstance has to be mentioned, that ozone is reconverted into ordinary oxygen gas by contact with any body heated above a certain, but not very well-determined point.

We have seen that the ordinary method of generating ozone, consists in bringing phosphorus into contact with atmospheric air (or oxygen) under certain conditions. Many other substances besides phosphorus are capable of generating ozone by contact. Oil of turpentine, and many other essential oils, will accomplish this; and the fact in question cannot be too forcibly remembered by the painter, who may discover, in the philosophy of ozone, the reason why certain pigments fade, or are bleached, thus destroying the general effect which he desired to produce. Meteorologically considered, however, the most important source of ozone remains to be described: we refer to the production of ozone by means of electricity. Every person who has been much accustomed to work with the electrical machine, must have noticed the generation of something powerfully odorous during the friction of the cylinder, or plate, against the rubber. This odour has, in point of fact, been called the *electric smell.* Now, if this electric smell be compared with the smell of the atmospheric air which has been treated with phosphorus, as just described, and the air washed, the two odours will be found to be identical, which is a presumptive evidence that electricity has in some way been concerned in the formation of ozone—an idea which extended experiment fully confirms.

It has already been stated that the most prominent quality of ozone is its highly developed oxidising power. By taking advantage of this property, we are supplied with an easy method of recognising it, by means of a test-paper, imbued with a mixture of iodide of potassium and starch. The chemical reader need not be informed that iodine colours starch blue, whereas iodide of potassium does not; therefore test-paper, containing iodide of potassium and starch, may occasionally be resorted to for indicating the presence of certain bodies, which have the faculty of decomposing iodide of potassium, and liberating free iodine. Ozone is one of these; which fact remembered, will render the following experiment intelligible:—If a piece of this paper be held between the prime conductor of an electrical machine and the knuckle, or a metallic ball, and electrical sparks transmitted through it, spots of blue discoloration will be seen on it, corresponding to each electrical spark.

The method of detecting the presence of atmospheric ozone is now readily indicated. If a strip of prepared paper turn blue, the existence of ozone is demonstrated. The experiment is very striking when performed near the sea. During the prevalence of a land wind, the test-paper will generally afford slight indications, or none at all; during the prevalence of a sea wind, however, ozone can generally be detected.

The reader will now be prepared to form some idea of the natural causes which may generate ozone. We need assume no other agency than that of electricity, to be assured that the production of ozone must be universal, and, looking on the world as an aggregate, continuous; and when we consider the potent nature of ozone, the irritation it produces when breathed, the facility with which it bleaches, corrodes, and destroys, we shall not be at a loss to understand that the consequences to living beings, of its excess or diminution, must be all-important. A most important function of ozone has yet to be indicated: it removes, almost more rapidly than chlorine itself, the bad odours resulting from the decomposition of animal or vegetable bodies. If a piece of putrid flesh be immersed for a few minutes in a bottle of ozonised air, the odour of decomposition is totally destroyed. With these facts before us, we may form some idea to ourselves of the important functions which ozone is designed to accomplish. Lessen the amount of atmospheric ozone, lower it below given limits, and increase the atmospheric temperature to the degree most congenial to organic decomposition, and the air will soon be charged with disease-bearing and putrid odours. It is in accordance with all that philosophy has been able to teach us, in relation to the laws of epidemic and endemic maladies, that the presence of such gaseous odours of organic decomposition as are here assumed, must be the fruitful source of disease; and it is not possible, after having studied the qualities of ozone, to refuse assent to the proposition, that the existence of this agent, in competent amount, must be followed by the destruction of the pestiferous odours of organic decomposition. If, however, ozone be naturally formed at any time in excessive amount, it is not difficult to foresee that other serious consequences must result to animal life. The inhalation of an irritating gas cannot but produce injurious effects on organs so delicate as the lungs; and, perhaps, many of the now anomalous and inexplicable effects of change of air, to patients suffering from chest diseases, may hereafter receive their solution in a more intimate acquaintance with the laws of ozone.

PHYSICAL PROPERTIES OF GASEOUS BODIES.—The word *gas* is of German origin, and was first employed by Van Helmont, to signify the vapour which escaped from liquids undergoing vinous fermentation. At later periods, the term was applied to designate every invisible substance disengaged from bodies by the application of fire. Macquer, a celebrated French chemist of the eighteenth century, extended the meaning of the term gas to signify every kind of air besides atmospheric air; and modern chemists have extended the meaning of the term still further, to indicate elastic fluids, whatever their colour may be, which are not readily condensible. At one time it was erroneously imagined that gases did not admit of condensation: in accordance with this belief, a gas was defined as being *a permanently elastic* fluid, thus distinguishing this class of bodies from mere vapours, which, so far from being permanently elastic, are very readily condensible. Modern discovery has proved this distinction to be untenable. All the known gases, except oxygen, nitrogen, hydrogen, nitric oxide, carbonic oxide, and coal gas, have been liquefied, and a great number of them solidified, by subjecting them to extreme cold and pressure.

Law of Marriotte—The Volumes of Gases are inversely to the Pressure applied.—This is a very celebrated law, and one that intimately concerns the meteorologist; it may be otherwise termed the law of compressibility of elastic fluids.

It will be recognised that the law in question, according to the exposition of it just given, is a general law, applying to a vast number of gaseous and vaporous bodies. For a long time its absolute truth remained unquestioned; but, more recently, M. Regnault and others have demonstrated it to be not of such universality.

Even atmospheric air and nitrogen do not rigorously conform to the law; and carbonic acid, and liquefiable gases generally, are so little amenable to the law, that, as applied to them, it cannot be regarded as approximately correct. Even the rate of compressibility of hydrogen is not strictly accordant with the law, although the deviation in this case is in the opposite direction to the deviation when atmospheric air is concerned; for it suffers less compression than, according to the law, should take place. Carbonic acid and nitrogen, compressed by a force of forty-five atmospheres, only fill seven-tenths of the space they ought to occupy according to the law.

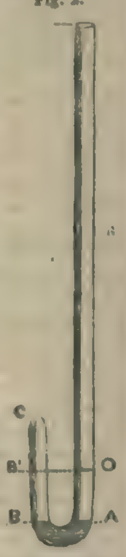
Now, inasmuch as the philosophy of estimating the height of mountains barometrically, is intimately associated with the law of Marriotte, it is well to indicate that this law is not quite correct; nevertheless, as regards atmospheric air, it is so nearly correct that we may accept it without demur.

The experiment by which the law of Marriotte was deduced is as follows:—Into a strong glass tube, of equal diameter throughout, bent on itself (as represented in Fig. 2), open at the long, and closed at the short extremity, a little mercury is poured, in such a manner that it shall be perfectly level in the two legs, as represented by the line A B. Under these circumstances, it will be evident that the air inclosed between B C and A D will be equally compressed. Let us assume the amount of compression to be represented by the weight x. Let us furthermore assume that the weight of a column of mercury between O and D to be $= x$, and let us call it m. If, then, on filling the long arm of the syphon with mercury, the column of air originally extending from B to C be diminished to the column extending from B′ to C, that is to say, to one-half, then we have $x + m = \frac{1}{2}$ B C, or

$$2x = \tfrac{1}{2}\,\mathrm{B\,C};$$

proving that the compression of air within the limits of the experiment is inversely to the pressure applied. In like manner, when the pressure is triple, the volume of gas is reduced to one-third; when quadruple, to one-fourth, &c. M. Arago has experimentally determined that the law rigorously applies to atmospheric air up to a pressure of twenty-seven atmospheres.*

Fig. 2.



The Atmosphere as a Ponderable Agent.—Owing to the equality of pressure which the atmosphere exercises on every side, we are not ordinarily conscious of its possessing weight. Nevertheless, its weight is no less definite, under proper limitations of demonstration, than the weight of any solid or liquid. The weight of the atmosphere may be contemplated under two conditions:

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* The student will find a more extended mathematical investigation of this and the following subjects in the Section on *Pneumatics*, p. 709, *et seq.* —ED.

firstly, as the equivalent weight of an atmospheric column of known sectional area, but unknown height; secondly, as the equivalent weight of a known atmospheric volume under proper limitations, presently to be indicated. Both these investigations will now have to be considered.

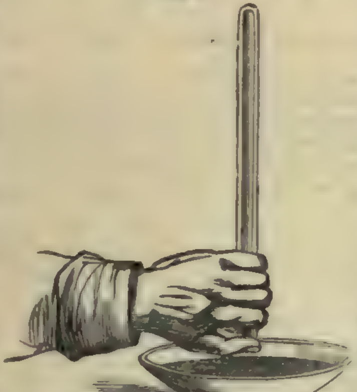


Fig. 3.

Determination of the Weight of an Atmospheric Column of known Sectional Area, but unknown Elevation. The Barometer.—*Experiment* 1.—If a piece of glass tube be taken equal in bore throughout, having a length of some thirty-three or thirty-four inches, closed at one extremity, and open at the other; if it be filled with mercury, then closed temporarily by the thumb, and inverted in a basin of mercury, as represented in the accompanying sketch (Fig. 3), we shall obtain a barometer of perhaps the most simple form this useful instrument can assume. We will proceed to study its philosophy. Firstly, let it be observed that though the tube *was* quite full of mercury, it does not remain quite full. No sooner is the restraining thumb removed, than a portion of the mercury sinks into the basin. If, instead of holding the inverted tube in the hand, as represented, some permanent support be devised for it; if the point corresponding with the present elevation or level of the mercury be marked on the tube; and if the tube be examined from day to day, the observer will soon find the level to fluctuate; sometimes it will rise, at other times fall. He will find, moreover, that the mercury will become depressed previous to stormy weather.

The instrument thus roughly extemporised is, in point of fact, a measurer of the weight of an atmospheric column of known sectional area (*i.e.*, the area of the interior diameter of the tube), but of unknown elevation. It is a barometer; and colaterally—inasmuch as depression of the mercurial column usually precedes stormy weather—the roughly-extemporised instrument is a weather-glass.

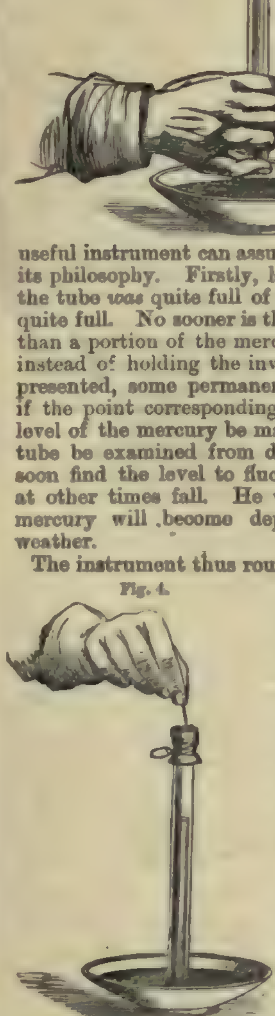


Fig. 4.

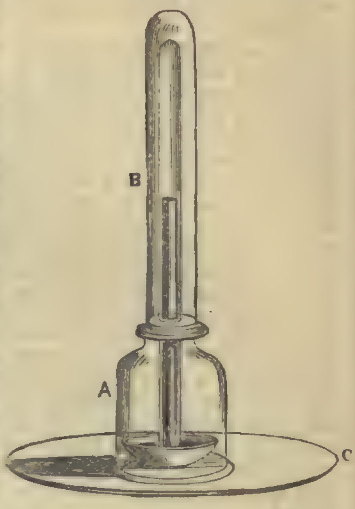
Demonstration.—It is desirable occasionally, when treating of natural phenomena, to violate the mathematical rule of accepting no fact until it has been demonstrated. Thus, in the present instance, we have taken the fact for granted, that the cause operating to maintain a mercurial column in the closed tube, is atmospheric pressure; and the cause to which variations of the height of that column is referable, is fluctuation of atmospheric pressure. The first proposition shall now be demonstrated, when the second will be accepted by inferential reasoning.

Experiment 2.—If, instead of a tube closed at one extremity, an open tube be taken, and one end be closed, by tying securely over it two or three strong pieces of moistened bladder—if the tube thus prepared be filled with mercury, as before, and inverted in a basin of mercury, we shall be in a position to demonstrate the proposition, that it is owing to atmospheric pressure—and that alone—that the mercurial column is supported. The operator has simply to prick the bladder, and let in air, when the whole column will suddenly descend to the level of the mercury in the basin (Fig. 4).

There is another form of demonstration, as follows; but it is not so simple as the last, inasmuch as it requires the aid of the air-pump:—

A is a glass air-pump receiver (Fig. 5), through the neck of which the tube B passes, inclosed in an exterior tube, which may be regarded as a prolongation of the receiver. C is the plate of an air-pump, on which the receiving jar is laid.

Fig. 5.



Let us assume that the barometer tube has been filled with mercury as before—plunged in the basin containing mercury underneath, the receiver A slipped over it, and finally extension made by the long glass sheath. Let us assume now that the air-pump is worked, and exhaustion gradually effected. Under these circumstances, the mercurial column will be seen to fall, and it will fall by jerks, each jerk corresponding with a stroke of the pump-handle. Hence, by the two preceding experiments, it is demonstrated, that to atmospheric pressure, and atmospheric pressure alone, is the variation of the mercurial column due. It is also proved, inferentially, that the fluctuations of height witnessed from day to day in a barometric column, are due to variations of atmospheric pressure. It appears, then, that by the barometer we actually weigh a column of atmospheric air, equal in height to the whole elevation of the atmosphere, whatever that may be, and equal in area to the internal sectional area of the barometer tube. The barometer, however, gives us no information in terms of cubic dimensions of the barometric mercury. Thus, supposing the barometric tube employed, to have a sectional area of one square inch, and supposing the mercurial column to be thirty inches high, then we should be correct in averring the pressure of an atmospheric column a square inch in sectional area, and extending the whole height of the atmosphere, to be equal to the weight of thirty cubic inches of mercury. Now the weight of thirty cubic inches of mercury will be about fifteen pounds; hence the atmosphere is said to exert a mean pressure of fifteen pounds on every square inch at the level of the sea.

INFLUENCE OF ELEVATION ON THE BAROMETRIC COLUMN.—It will be evident, on reflection, that the atmospheric pressure must decrease with every increment of elevation above the level of the sea. Founded on this principle, the barometer is frequently employed to determine the height of mountains; and, reversing the application, it might also be employed to determine the depth of mines and wells. At the elevation of about thirty-six miles, the pressure of the atmosphere cannot amount to more than 0·001 of an inch of the barometric column; and conversely, at a depth of about sixty-six miles, the density of the atmosphere would be about 100,000 times greater than at the surface of the earth, being six times more than the density of gold and platinum; so that, supposing either of these metals to be plunged into such an atmosphere, they would actually float.

If the atmospheric density were uniform, a barometric fall of one inch would correspond to 11,065 inches, or 922 feet of air. But it is not uniform, as we have already seen; therefore such deviation from the standard of uniformity, and indeed many other circumstances, must be taken cognizance of before we are enabled to employ the barometer as an indicator of atmospheric elevations.

The law of Marriotte teaches that the dilation of a volume of air is proportional to its density, so long as the temperature to which it is exposed is constant; whence it follows, that the density of the atmosphere diminishes from below upwards, in geometrical progression. Reasoning on this basis, it appears that, assuming any particular elevation above the point of observation, its numeral exponent may be regarded as the logarithm of the density of the lowest atmospheric layer, or, in other words, of the barometric column. A consideration of the mutual relation subsisting between numbers and their logarithms will render this evident, for logarithms are nothing more than numbers increasing by arithmetical progression, corresponding to other numbers, the increase of which is also in geometrical progression. Being possessed of a table of logarithms, expressing the densities of atmospheric layers, one might calculate the height of a mountain by two observations made at two stations; but the same result may be arrived at by using a common table of logarithms, and multiplying them by a constant factor. According to Deluc, the factor is 10,000.

Many other considerations have to be allowed for before the barometer can be accurately applied as a measure of mountain elevations: they are temperature, latitude, relation subsisting between the specific gravities of air, and mercury, and dilation of mercury for each degree of the thermometer = 0·0001 for each degree of Fahrenheit's scale. All these general considerations have been embraced in tables.

Although the circumstances necessary to be taken cognizance of when employing the barometer as an indicator of mountain elevation, are numerous and complicated, nevertheless, the results obtained are susceptible of a considerable degree of accuracy. Subjoined is a table of comparative results between trigonometric and barometric observations. The difference, it will be seen, is only trifling.

COMPARISON OF TRIGONOMETRIC AND BAROMETRIC MEASUREMENTS OF MOUNTAIN HEIGHTS.

Observers.	Place of Observation.	Latitude.	Longitude.	Trigonometric height in feet.	Barometric height in feet.
Webb	Gunna Nath, Stockdale	29° 45′ 56″	79° 30′ 29″	6.828	6.831
Borda	Barha Ling, Temple ...	29 47 30	80 2 27	7.646	7,635
Von Buch	Teneriffe	28 30 0	16 13 0	12,188	....
	Ditto		....	...	12,131
Buckle ...	Sugar-loaf, Sierra Leone	8 29 40	13 15 0	2,493	...
Sabine ...	Ditto Ditto	....	....	....	2.521
Sabine ...	Spitzbergen	73 0 0	10 0 0	1,644	1,640

When an approximate result is alone required, and the height is inconsiderable, a fall of $\frac{1}{10}$th of an inch may be allowed for every ninety feet of elevation, or $\frac{1}{1000}$th of an inch for every foot. This rule suffices for the small differences of elevation at which barometers are hung, and enables the observer to institute a comparison between them. Correction for temperature must, however, not be omitted. The ratio of expansion for mercury, glass, and brass—the materials employed in the manufacture of barometers—will be pointed out hereafter; meantime we may as well indicate, that perhaps, after all, it is well in practice to ignore these complex elements, and to consider $\frac{3}{1000}$ths of an inch as the allowance for mercurial expansion for every degree above 32, and *vice versâ*. Applying the above corrections for temperature and pressure to practice, let it be required to know what height the mercury would stand at the level of the sea and at 32° Fah., if at an altitude of 150 feet, and temperature 55°, it indicated 29·565 inches?

Inches.

Actual height of mercurial column . . . 29·565

Deduct for 23° of temperature above 32

$\frac{3}{1000} \times \frac{23}{1} = \frac{69}{1000}$ . . . . . = ·069

Altitude of mercury . . . . . = 29·496

Add for elevation ·001 × 150 . . . = ·150

Altitude of level at sea, at temperature 32 Fah. = 29·646

The reader will remember that the previous remarks have reference to the barometer as affected by air at rest, this being the simplest atmospheric condition which theory can assume. Hereafter we shall discover that the barometer, when influenced by the atmosphere in motion —by winds, in other words—is subject to great variations.

Further Improvements of the Barometer.—The barometer in its simplest form, as already described, is a more perfect instrument than many in which that simplicity of form is departed from, in deference to portability; nevertheless, it is not quite correct. To be absolutely correct, it is indispensable that the mercurial level in the basin should bear a constant ratio to the mercury remaining in the tube, a condition which evidently cannot be obtained in the instrument just described. In proportion as mercury descends out of the inverted tube, the level of the mercury in the basin will be elevated, and to the extent of such elevation the indications of the instrument will be prejudiced. Various means are had recourse to for lessening, or absolutely removing, this evil. It may be lessened by increasing the width of the basin to such extent, that the ratio of elevation of the mercurial surface may be so greatly diminished that it will practically cease to impart errors. It may be absolutely removed by one of two devices. One consists in mounting the receiving basin on a screw, which, by elevation and depression, regulates the quicksilver to any desired level. Such is the contrivance of M. Fortin, whose construction of a barometer is here annexed (Fig. 6); but it is more usual to attach to the barometer, a long scale having a slide motion, so that the lower end, or commencement of the scale, may be made to coincide with the level of the mercury in the basin. Practically, however, the basin is usually dispensed with, the reservoir for mercury being a mere extension of the barometric tube, in some cases bulbed, and in others quite plain. Both these forms of construction are shown in figures 7 and 8.

Fig. 6.

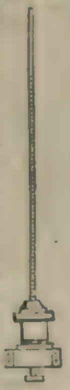


Fig. 7.

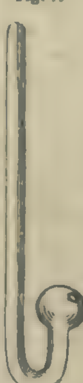
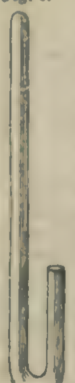
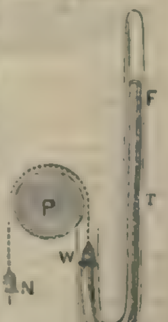


Fig. 8.



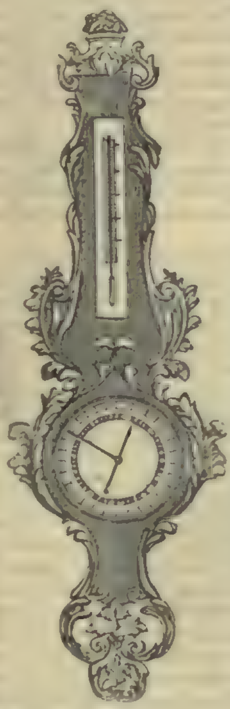
The Weather-glass.—The primary and only *direct* function of the barometer is, that its mercurial column indicates, by its rising and falling, the varying weight of the superincumbent atmosphere. Very frequently this direct function is taken no account of; variation in the state of the weather being all which the observer desires to make himself acquainted with. Subserviently to this intention, all direct rise and fall of the barometric column is lost sight of, and the indications of a dial-plate with movable hand are substituted. Such an instrument is termed the dial weather-glass, the construction of which is as follows:—T (Fig. 9) is a barometer tube; W is a small float attached to one end of a cord, the other extremity

Fig. 9.



of which is attached to a small weight, N. From this arrangement it will be seen that every rise and fall of the real barometric column, in the long arm of the tube, will correspond with a parallel fall and rise of the mercurial column in the short arm. It will be seen, moreover, how the small float, W, is raised and lowered; how the pulley, P, will be caused to revolve, and the index-hand to traverse the dial-plate of the instrument. The exterior of the wheel barometer is represented in Fig. 10.

Fig. 10.



We need scarcely state that the wheel barometer is considered, pneumatically, a very imperfect instrument. Not only is the varying ratio between the mercurial column and the level of the mercury in the reservoir here a necessity—the very index motion depending upon it—but the presence of the float, W, tends also to embarrass the free ascent and descent of the barometric column of mercury.

Manufacture of a Correct Barometer.—Not to render the principles concerned in the barometer complex, we have hitherto assumed that the act of charging a tube with mercury is simple and free from difficulties. Practically this is not so; many precautions have to be taken, otherwise the resulting barometer will be anything but correct. The tube selected must not be too small; many instruments are rendered incorrect owing to neglect of this precaution. The internal diameter of the barometer tube should never be less than a quarter of an inch; it may be even more with advantage. A small barometer tube prejudices the correctness of the instrument in two respects, for the variations of expansion and contraction of the mercury, due to variations of temperature, are more considerable; and the motion of the quicksilver, up and down, is impeded by friction against the glass.

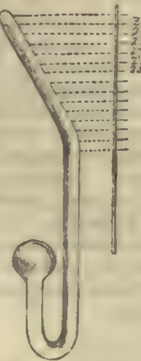
The Mercury must be Pure, and deprived of Atmospheric Air.—Mercury, as commonly existing, is generally impure. It contains uncertain quantities of tin, lead, and sometimes zinc. Of course these admixtures damage the mercury for barometric purposes. The observer desires to read off his atmospheric pressure in terms of inches of mercury, not in terms of inches of a mercurial compound. It is indispensable, therefore, that the impurities be discharged or extracted. Various processes are used to this end; but that usually followed by makers of barometers, consists in agitating the mercury to be purified, with dilute nitric acid, which gradually dissolves out the extraneous metal, and leaves the mercury pure.

Far greater difficulties are encountered in discharging atmospheric air from the mercury employed. This is accomplished by boiling the mercury after it has been poured into the tube. The operation requires great delicacy, and the instrument is frequently broken in the operation.

Method of Reading off Barometric Indications correctly.—It is not possible to read off, by referring to a common scale of inches and parts of inches, the various small elevations and depressions of a barometric column. Two methods are had recourse to for obviating this difficulty: one is the diagonal tube, a contrivance altogether peculiar to the barometer; the other is the Vernier or Nonius scale, employed for the general purposes of facilitating the reading of minute scale divisions.

The diagonal-tubed barometer is represented by the accompanying diagram (Fig. 11).

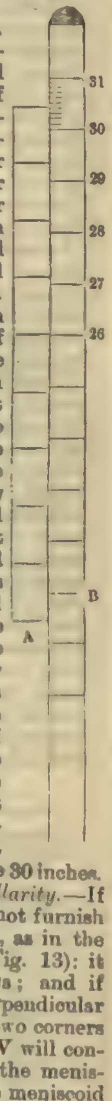
Fig. 11.



The Nonius or Vernier Scale.—This is an ingenious contrivance for measuring small linear divisions by means of larger divisions, and, consequently, more easily recognisable by the eye than larger divisions would be. Thus, for example, by means of a Vernier graduated in divisions of an inch and one-ninth, we can read off tenths of an inch, as will be seen by reference to Fig. 12.

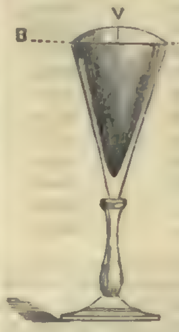
Let A be a scale sliding in proximity to B. Let each of the divisions on B be = one inch, and each of the divisions on A = one inch and one-ninth. From these considerations it follows that nine divisions on A are equal to ten divisions on B. Directing the eye to the upper limit of A, it will be seen that its edge corresponds to thirty inches, and *something more* on B. In this case the observer would have no difficulty in recognising the amount over and above thirty inches on B to be equal to four tenths of an inch. Our scale divisions are so large that a Vernier scale is not required for conveying that information. But, assume the tenth division between 30 and 31 to be obliterated, still we should be able to discover the overplus beyond 30 inches to be four-tenths by means of the Vernier scale A, inasmuch as the number of tenths will be equal to the number of whole parts on the Vernier scale A, above the first line of coincidence between it and the scale B. Now the line of coincidence in question is at 26, counting upwards; from which, to the extremity, we have four divisions, which indicate a fraction of four-tenths of an inch over and above 30 inches.

Fig. 12.



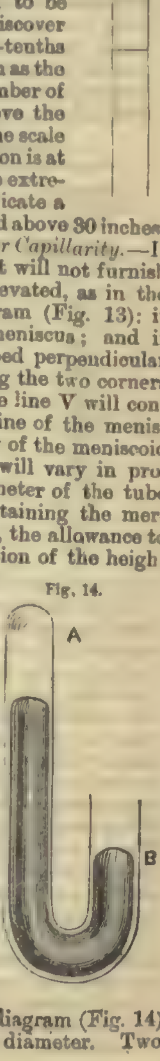
Correction of the Barometric Column for Capillarity.—If mercury be poured into a glass vessel, it will not furnish a perfectly level surface, but will be elevated, as in the accompanying diagram (Fig. 13): it will constitute a meniscus; and if the line V be dropped perpendicular to the line B, joining the two corners of the meniscus, the line V will constitute the versed sine of the meniscus. The convexity of the meniscoid surface of mercury will vary in proportion as the diameter of the tube or other vessel containing the mercury varies. Hence, the allowance to be made for diminution of the height of a mercurial column, owing to capillarity, is determined by two considerations—the diameter of the tube, and the length of the versed sine of the corresponding meniscus.

Fig. 13.



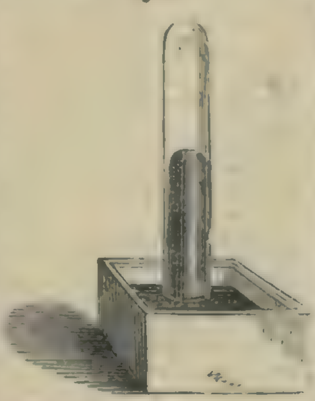
The necessity for such allowance does not apply, as will hereafter be seen, to barometers of every form; neither is it imperative, so long as the indications of one barometer are to be compared amongst themselves; but it is indispensable if we would compare the indications of barometers with each other.

Fig. 14.



First, let us consider the cases to which the correction for capillarity does not apply. It does not apply to any barometer, the reservoir of which constitutes part of the tube itself. The diagram (Fig. 14) represents a barometer of great tubular diameter. Two meniscoid surfaces of mercury are there apparent—one at A, another at B. Now it is evident that the columnar interference at A will be exactly equal to the columnar interference at B, provided the tubular diameter be equal throughout. Hence, whether the degree of elevation be counted from the summit of the lower to the summit of the upper meniscus, or from the base line joining the two corners of the lower meniscus to the base line joining the two corners of the upper one, the two resulting columnar estimations will be strictly equal and comparable. But it is different when the observer has to do with barometers constructed on the original, or Toricellian, plan of a separate reservoir; in this case the meniscoid elevation of the mercury in the reservoir is practically ignored, being so inconsiderable that it amounts practically to nothing, as will be seen by reference to the diagram (Fig. 15).

Fig. 15.



The following is a small table of depressions due to capillarity. Larger and more elaborate tables have been calculated; but the one given will suffice for, perhaps, every occasion. The meteorological student cannot have the fact too strongly impressed upon him, however, that the barometer is confessedly a very imperfect instrument; therefore correct results from barometric observations are to be looked for only as the mean resultant of a number of accumulated observations.

DEPRESSION DUE TO CAPILLARY ACTION.

Diameter. Hundredths of inches.	Depression in Decimals of Inch.		
	Ivory.	Young.	Laplace.
5	·2949	·2964	......
10	·1404	·1424	·1394
15	·0865	·0830	·0854
20	·0583	·0589	·0580
25	·0409	·0404	·0412
30	·0293	·0280	·0296
35	·0212	·0196	·0216
40	·0154	·0139	·0159
45	·0112	·0100	·0117
50	·0082	·0074	·0087
60	·0043	·0045	·0046
70	·0023	......	·0024
80	·0012	......	·0013

The Aneroid Barometer.—Of comparatively recent date is the invention of the *Aneroid* barometer. It consists of two discs of metal, accurately joined together, so as to enclose a space, from which the air is removed, and a vacuum produced. An increase or decrease of atmospheric pressure will cause the contraction or expansion of the sides of this vessel; and by means of tooth-wheels and rack-work, an index is thus moved on the face of the instrument, so as to indicate the pressure on a circular plate, divided into inches, &c., corresponding with the gauge of the mercurial barometer. The "aneroid" has been so greatly improved, that, according to Mr. Glaisher, a good one may be considered correct at a pressure so low as that equal to five inches of mercury. (See Chapter V.)

Thermal Expansion.—We will now examine the function of thermal expansion, which not only intimately concerns the barometer, but is the fundamental basis of the thermometer; and besides, it enters as an element into so many meteorologic calculations, that a thorough investigation of its laws cannot be omitted. We shall, therefore, embody the laws of thermal expansion in a few propositions for successive demonstration.

Heat may be regarded in the two senses: one signifying temperature, or that sort of heat which is recognisable to the sense of touch, and which affects the thermometer; and heat, which is devoid of these manifestations, which neither creates the sensation of warmth, nor is amenable to thermometric demonstration. The former we may express by the term *sensible heat*; and the second by the term *latent*, or *insensible* heat.

On the supposition that all the functions of heat, sensible as well as insensible, are referable to a real physical agent, the term *caloric* has commonly been applied as the representative of such agent; but though the term be in general use, it is perhaps objectionable—modern science leading us to infer that the functions of heat are due to a condition of matter, rather than to a separate agency. It is to evident heat, recognisable to the touch, that we shall now direct the reader's attention.

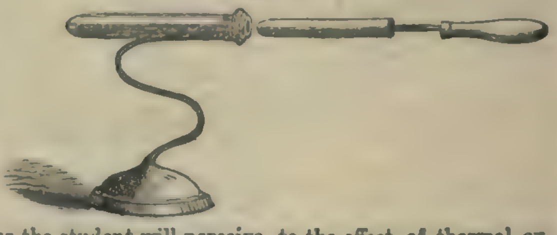
I. *Heat affects the Volume of all Bodies.*—The general effect of heat on bodies is to cause their expansion. So general is this rule, that we shall do well to consider it as universal; treating all deviations from it hereafter as so many exceptions.

II. *The Volume of all Solids is Increased by Increase of Heat.*—This proposition is demonstrated by so many instances commonly occurring, that specific experiments are hardly required. The wheelwright takes advantage of this property to bind tightly together the wood-work of his carriage-wheels. He heats the annular tire, by which he expands it; he then slips the tire over and around the wood-work, and, allowing the tire to cool, the wood-work is tightly braced together by an immense force.

Some years since, the walls of the *Conservatoire des Arts et Métiers*, at Paris, were found to be diverging from the perpendicular, and were restored to their original lines by the following beautiful expedient:—They were perforated transversely; copper bars were thrust through the perforations, each bar at either extremity being supplied with a nut and screw. Every alternate bar was heated by means of a spirit-lamp flame; being heated, the bars expanded, and the screw-nuts being now turned close up to the wall on either side, the bars were allowed to cool. By cooling they contracted—pulled the walls to some extent together, leaving the ends of the unheated bars protruding; their screw-nuts were now turned close up to the wall on either side, and the heating process repeated. Thus, little by little, the walls were restored to their original position.

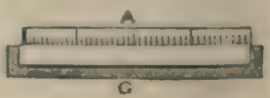
An exemplification of the expansion of iron by heat sometimes occurs to the laundress. Occasionally she is surprised to find that the heater of her Italian iron will not enter its corresponding sheath when red-hot, though it enters readily enough when cold. This is attributable, as the student will perceive, to the effect of thermal expansion on the iron (Fig. 16).

Fig. 16.



Thousands of familiar instances might be cited, all illustrative of the same property. We shall leave their consideration to the reader, concluding the remarks on this part of the subject by bringing before his notice a common lecture experiment, illustrative of solid thermal expansion. Let G (Fig. 17) be a gauge, into which the metallic bar, A, accurately fits, whilst cold; it will be found that when the bar A is heated—moderate heating will suffice, such as may be accomplished by means of

Fig. 17.



the flame of a spirit-lamp, or a basin of hot water—the heated bar will no longer fit into the gauge. In this way the student may demonstrate the fact that each different solid possesses its own definite rate of expansion for equal degrees of temperature.

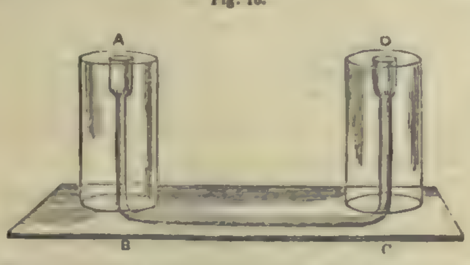
Investigations of the law regulating the thermal expansion of bodies, under every condition, are attended with extreme difficulty. They have been conducted by Regnault, Rudberg, and others, with great industry and much success; but, as in most cases where the investigation of natural phenomena through long ranges are concerned, the results are merely approximate. To give the reader a general notion of one of these difficulties, let it be assumed that A B C D stand for successive equal intervals of thermometric graduation. Let it be assumed that the rate of expansion of a substance from A to D be known as equal to a quantity expressed by Y. It by no means, however, follows—nor do philosophers believe—that, because the rate of expansion of the body between A and D is equal to Y, therefore the rate of expansion of the same body between A and B is equal to $\frac{Y}{3}$.

Further consideration of this matter may, however, be omitted in a Treatise on Meteorology. We merely want to be acquainted with approximate results of the law, in order to allow for practical discrepancies between the apparent and the actual indications of the instruments employed in the course of our researches.

To convey a general notion of this kind of knowledge to the meteorological observer, the student's attention may be directed to the fact, that, supposing a barometer-scale to be made of brass, and the tube to be filled, as usual, with mercury, then the amount of expansion of mercury by heat, and for which allowance has to be made, will be determined by the ratio $\frac{M}{B}$, if M stand for the co-efficient of expansion of mercury, and B for the co-efficient of expansion of brass. By the term, "co-efficient of expansion," is meant the number indicating the amount of expansion peculiar to any body for given ranges of temperature.

III. *The Volume of all Liquids is Increased by Addition of Heat.*—Investigations prosecuted for demonstrating this law, are attended with a difficulty which does not apply to the previous case. Liquids require vessels to hold them, and these vessels are themselves amenable to expansion. This difficulty has been very ingeniously avoided by MM. Petit and Dulong, who determined the expansion of liquids by a method founded upon the well-known hydrostatic principle, that the vertical heights of two fluids, communicating by a horizontal tube, are in inverse ratio to their densities. The apparatus shown in Fig. 18 was employed in their experiments. A, B, C, D is a tube bent twice at right angles, and enlarged at either extremity; the two vertical tubes are connected inferiorly, by a horizontal tube of exceedingly fine bore. By virtue of the hydrostatic law just mentioned, it follows, that if any liquid of homogeneous density be poured into the vertical leg of one side, it will rise to a corresponding elevation in the vertical leg of the other; and if the fluid in one vertical leg be now heated, and consequently expanded, its height will be in excess of the columnar height of the other, by a definite quantity. By an easy train of mathematical reasoning, the expansion due to heat can be deduced from a consideration of the different levels and the different temperatures of the two vertical tubes. Our diagram represents each vertical tube surrounded with a cylindrical vessel. These vessels are for the purpose of commanding variations of temperature; one tube filled with ice, whilst the other is filled with hot water.

Fig. 18.



By an experiment of this kind, the co-efficient of thermal expansion of mercury from 0 to 100 of the centigrade scale, is $\frac{100}{5550}$; whence, assuming its rate of expansion equal throughout, the expansion for every centigrade degree will be $\frac{1}{5550}$, or, for every degree of Fahrenheit, $\frac{1}{9990}$; in decimals = 0·000101; or, expressed in round numbers, one ten-thousandth part of its bulk.

IV. *The Volume of all Gases is Increased by Addition of Heat.*—Though the consideration of this law is not related, like the two preceding, to the *construction* of the barometer, it is intimately connected with the functions of that instrument, more especially as regards its application to the measuring of elevations; we shall do well, therefore, to consider it at once.

It has already been remarked, that the rate of expansion of all solid bodies for giving increments of temperature is various; and a similar remark applies to liquids. The rate of expansion of gases was first closely investigated by the immortal Dalton, who arrived at the conclusion, that all gases were equally affected by equal increments of heat, expanding to $\frac{1}{480}$th parts of their volumes at 32° Fah. for each of the 180°, between 32° Fah. and 212° Fah. As regards temperature, above 212° Fah. and below 32° Fah., it was imagined by Dalton that the same law of expansion held good. Various theoretical reasons exist of a nature to create a doubt as to the large generalisation of Dalton; and these doubts have been fully substantiated by M. Regnault. He finds that all gases do not dilate to the same extent between equal limits of temperature, neither is the dilatation of the same gas, between the same limits, independent of its primitive density. These are interesting facts: they prove that to assume one co-efficient of dilatation for all gases is obviously incorrect; nevertheless, his experiments go to prove that such a universal gaseous co-efficient of dilatation may be adopted for convenience, without appreciable error, within the theoretical limits of ordinary experiment. We must not, however, continue to adopt $\frac{1}{480}$th as our working gaseous co-efficient of dilatation for every degree of Fah. between 32° and 212°; nor $\frac{1}{483}$, as subsequently adopted by MM. Petit and Dulong, but $\frac{1}{491}$.

The Atmosphere Actually or Practically Considered.—We have hitherto regarded the atmosphere as a compound or mixture of nitrogen and oxygen gases only; but the reader need not be informed that such an atmosphere is altogether theoretical. Besides nitrogen and oxygen gases, there always exists a portion of carbonic acid, and aqueous vapour, either visible or invisible. These are invariable components of the atmosphere, indispensable to its functions, and adapting it to the purpose of this world's economy. The atmosphere contains also other materials, the results of local operations in nature, no less than the operations of man. As the subject of our present investigations, let us consider the atmosphere as a mixture of the theoretical atmosphere, *plus* aqueous moisture and carbonic acid.

Limits of the Atmosphere.—It follows, from a consideration of the laws of elasticity, that the atmosphere must vary in density for every difference of elevation. The atmospheric layer nearest to the earth must be pressed upon by the superincumbent atmosphere above it; whence the deduction follows, that when we speak of 100 cubic inches, or *any* definite measure of the atmosphere, weighing a certain number of grains, certain conditions and limitations are implied. Some of these have reference to the composition of the atmosphere chemically considered; others have reference to the atmosphere merely regarded as an elastic medium. To the latter consideration alone the reader's attention will be now directed.

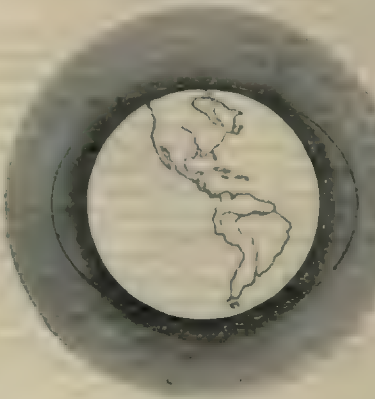
Seeing that the atmosphere, in accordance with the laws of elasticity, must necessarily expand, the higher we ascend above the normal level of the surface of our globe—or, in other words, the level of the sea—the first question which arises is this—To what extent do the atmospheric limits reach? does the expansion go on, *ad infinitum*, to the farthest realms of space, or are these limits definite? and if definite, what is the cause, or what are the causes, of limitation? Two theories have been adopted in reference to this question. According to one theory, the atmosphere is illimitable; according to the other, it is limited. Some of the arguments for and against we shall now proceed to give.*

If the atmosphere be really illimitable, let us see what should follow, to be in accordance with recognised laws, to which all ponderable matter, or matter subject to gravitating influences, is amenable. Gravitation being directly as the mass of gravitating bodies, it should follow that, were the atmosphere illimitable, each of the heavenly bodies should be surrounded with an atmosphere proportionate to its mass—an assumption which astronomy disproves. Thus, astronomy furnishes strong proof in favour of the finite extension of the atmosphere. A consideration of the laws of the atomic constitution of matter, lends further, and perhaps stronger, proofs. Chemistry is full of evidence in favour of the atomic constitution of matter; or, in other words, is full of proofs that all material substances are composed of molecules or particles; to which extent they can alone be divided, and not beyond. The mathematical reasoning, which has been employed against this atomic theory, as chemists term it, is specious at a first glance, but really untenable. To argue that the theoretical space, occupied by any material particle, may be supposed capable of division and subdivision, *ad infinitum*, is really not to the point. Space is one thing; the matter occupying such space is another. The mathematical objection touches the space alone, not the matter; therefore the chemical evidence in favour of the atomic constitution of matter is unanswered, and is apparently unanswerable. Let us now regard the consequences of this assumption as it relates to atmospheric air. The late Dr. Wollaston was the first person who directed attention to the limitation which should, theoretically, be imposed on atmospheric expansion, supposing the assumption of its atomic constitution to be correct. The atmosphere, like other ponderable material bodies, is subject to gravitating influences, thus imparting a tendency of descent towards the earth. On the other hand, the atmosphere being elastic, its particles are mutually separated from each other by the operation of this force. Now, assuming the atomic constitution of the atmosphere to hold good, there must be some finite distance from the earth's surface, at which the force of elasticity would be counterbalanced by the force of gravitation, which distance would correspond with the farthest limits of the atmosphere. The mean distance of this limit is assumed to be about forty-five miles, though it must differ from every point north and south; being greatest over the equator, and least at either pole, as indicated by the accompanying diagram (Fig. 19).

The reason of its being greatest at the equator is immediately referable to the diurnal rotation of our globe on its axis, thus generating a centrifugal force, which has determined the oblate spheroidal form of the earth. Material bodies will be affected by this centrifugal force, *cæteris paribus*, directly as their attenuation; whence it follows that the atmosphere, being a gas, must be affected to an extreme degree. It will be sufficiently evident, however, that the atmosphere is only affected by the earth's diurnal rotation intermediately, or by friction; the velocity of motion imparted to it by the earth being less considerable than the velocity of the earth itself. We shall hereafter find, when we come to treat of the trade-winds, that those permanent aërial currents are not altogether referable to atmospheric motion, but in some degree depend upon the diurnal motion of the earth.

Determination of the Weight of a given Volume of Atmospheric Air.—Nothing can be more easy than the theoretical means of solving this problem; and, though certain practical difficulties do interpose, we had better, for the sake of theoretical explanation, consider them absent.

Fig. 19.



The case under consideration is general, not specific. The determination of the weight of a given volume of atmospheric air, is accomplished similarly to the determination of the weight of a given volume of any other gas; nor does it differ in principle from the process had recourse to when solids or fluids are concerned.

If, to take the simplest practical case, without reference to cohesive state, it were desired to ascertain the weight of a given bulk of copper or brass—say one hundred cubic inches—the operator's first care would be to obtain a solid of copper or brass having these cubic dimensions; which having been obtained, no vessel for the purpose of weighing it in would be necessary. This is the simplest case of bulk-weighing which can occur; nevertheless, a condition has to be regarded which the superficial observer might forget, or, perhaps, not be aware of. The copper or brass alters its dimensions for every variation of temperature. If heated, it will expand; if cooled, it will contract; so that, practically, under no two degrees of temperature has the mass of brass or copper the same size. Practically, so long as solids are concerned, these variations of size, dependent upon variations of temperature, are not of much consequence in ordinary operations of weighing. It is necessary, however, to estimate them for other reasons, and to tabulate these variations. We shall have occasion to refer to this tabulation hereafter.

Let it now be assumed that the problem before us is to determine the weight of a given bulk of liquid—say water. In this case we must have recourse to some vessel of capacity, for the purpose of holding the water to be weighed; and further elements of complexity, in addition to that of temperature, are introduced. Firstly, the vessel employed has its own laws of expansion and contraction; secondly, the water, when poured into the vessel, will not have a perfectly flat surface; so that, except the mouth of the vessel be small, an error of considerable magnitude will be imparted; neither must it be too small, or the functions of capillary attraction will come into play, and then the water will present a higher level than properly belongs to it.

The chief source of inaccuracy, however, which the operator meets with in operating upon solids and liquids, is that dependent on the variations in bulk referable to thermal increments and decrements; any alteration due to variations in pressure being practically ignored. So far as atmospheric pressure is concerned—which is the only kind of pressure we need take cognizance of as affecting our subject—it exercises so little influence on the dimensions of solids and liquids, that we may put it altogether out of consideration. Far different, however, is it when gases are concerned. Their attenuation and elasticity are such, that variations of atmospheric pressure exercise the most powerful influence over them; so that the degree of atmospheric pressure operating at the time of the experiment, is, at least, of equal consequence with the degree of temperature.

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* See Pneumatics, ante, p. 772.

We are now in a position to trace the theoretical steps necessary to be followed in obtaining the weight of a given volume of any gas.

Necessarily, as in the previous case, a vessel of capacity is required; but, inasmuch as a gas does not admit of being poured into the vessel like water, some practical expedient for accomplishing this must be devised. We had better omit all consideration of this for the present, and assume that the vessel (which will be a globe or flask having a neck with stopcock attached) is already filled with the gas to be weighed, at a definite temperature and definite pressure. This accomplished, the operator has only to weigh his flask full of gas, deduct the weight of the flask from the total weight of flask and gas, and the result is gained. Practically, however, many points have to be considered.

The Gas must be Pure.—Whether gas, liquid, or solid, any body, the weight of which we desire to know, must be pure; but this precaution applies in the highest degree to gases.

The Gas must be either Dry, or its Amount of Moisture must be Definite.—The property which gases have of taking up vapours, especially aqueous vapour, is well known; and it will readily be seen, that to the extent of the presence of such vapour, will the weight of a given bulk of gas, and vapour mixed, fluctuate. One of two processes has now to be followed: either the gas must be artificially dried by exposure to one of the hygroscopic bodies used by chemists for that purpose; or, it must be saturated with moisture to the fullest capacity at some given temperature.

The further steps of the calculation are based upon a consideration of the ratio between the specific gravity of steam or vapour, and the specific gravity of dry gas; and, lastly, the amount of vapour which a gas absorbs at a definite temperature. According to Gay Lussac, the ratio between the specific gravity of aqueous vapour and air, under similar conditions of temperature and pressure, is

$$\frac{0{\cdot}620}{1} = \frac{\text{vapour}}{= \text{atmospheric air}}.$$

The amount of aqueous vapour which a unit volume of gas can absorb at given temperatures, has been ascertained and tabulated.

Applying this knowledge to practice, let us assume that 100 cubic inches of moist air, at 60° Fah. and 30 inches barometer, weigh 31 grains, it is required to know how much 100 cubic inches of dry air would weigh.

We begin by turning to a table indicating the quantity of vapour present in a gas saturated with vapour at any given temperature. Dalton's table gives this quantity for 60° Fah. as 0·524. We next perform the following calculation:—

30 : 0·524 : : 100 : 1·747 = the volume of vapour in 100 cubic inches of moist air at 60° Fah.

And as 100 cubic inches of aqueous vapour weigh 19 grains, 1·747 cubic inches weigh 0·332 of a grain.

Weight of 100 cubic inches of moist air .	31
Deduct	0·332
	30·668

Therefore the weight of 100 − 1·747 = 98·253 cubic inches of dry air = 30·668 grains, and 98·253 : 30·668 : : 100 : 31·213 grains.

Whence it follows, according to the foregoing calculation, that the weight of 100 cubic inches of dry air, at 30 inches barometer and 60° Fah., is 31·213 grains.

Such, then, are the practical operations by which the weight of a known volume of gas is determined. We have chosen atmospheric air as the subject of illustration, but the processes are identical whatever the gas may be.

Although the result of the calculation just effected gives 31·213 grains as the weight of 100 cubic inches of atmospheric air at 30 inches barometer and 60 Fah., and although the number may be accepted for all purposes of meteorologic calculation, nevertheless it must not be viewed in an implicit sense. In point of fact, the exact weight of atmospheric air is not yet made out. Probably the determinations of MM. Dumas and Boussingault are most reliable. According to their experiments, 1 litre or 61·02791 cubic inches of air, at 0° centigrade and 0·76 metres barometer, weigh 20·065 grains; whence it follows that 100 cubic inches, under the same conditions, must weigh 31·093 grains at 60° Fah.

Barometric Pressure at the time of Experiment must be an Element of the Calculation.—A consideration of the laws of pressure, as influencing the volume of gases and vapours, will have made the student aware that due allowance requires to be made for variations referable to this cause. Now we are acquainted with the amount of expansion and contraction dependent on variations of pressure. This information is conveyed by a study of the law of Marriotte, which proves that a rule of proportion will furnish the information required. Thus, for example, suppose we have 100 measures of any gas at a pressure of 29 inches of the mercurial barometric column, and it is required to ascertain what volume the gas will fill at 30 inches of the same—this being the normal pressure to which all calculations as to the volume of gases are reduced—then we say

As 30 : 29 : : 100 : 96·66.

In other words, the 100 volumes of gas, under those conditions, would contract into 96·66.

Temperature must be an Element of the Calculation.—When it is considered to what extent gases and vapour suffer expansion and contraction by variations of temperature, the necessity of this calculation will be obvious. We have already explained the ratio of thermal expansion, to which gases and vapours are subjected by variations of temperature. *Practically*, we have seen at page 1127, that this ratio may be considered identical for all of this class of bodies, and to be equal to $\frac{1}{491}$th part of their bulk at 32° Fah. and 30 inches barometer for every degree of temperature between 32° Fah. and 212° Fah. Let us now apply this information to practice.

If the temperature of the gas be above 32° Fah., multiply its total volume by 491, and divide the product by 491 *plus* the number of degrees that the temperature of the gas exceeds 32° Fah. The numeral result of this operation gives us the correct volume the gas in question would occupy at 32° Fah.

For example, we have 100 cubic inches of gas at 50° Fah.; it is required to know what volume this gas would occupy if raised to 60° Fah.: thus

$$\frac{100 \times 491}{491 + 18} = 96{\cdot}46 = \text{the volume at } 32^\circ \text{ Fah.}$$

And

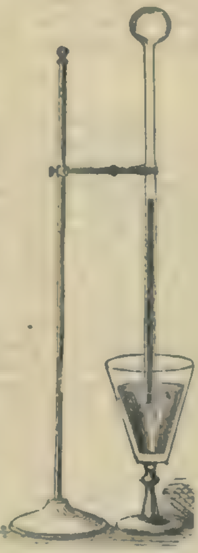
$$96{\cdot}46 + \frac{96{\cdot}46 \times 28}{491} = 101{\cdot}96 = \text{the volume at } 60.$$

Recapitulation and Deductions.—It appears, then, that the operation of weighing a gas demands in all cases that due allowance should be made for variations of heat and of pressure; and if the gas be charged with vapour, due allowance has to be made for moisture also.

THE THERMOMETER.—In the course of our preceding investigations relative to the atmosphere, we have seen that temperature is an important element. Not only are the chemical functions of the atmosphere intimately related with temperature, but, without allowing for the expansion produced by increments of heat, the meteorologic observer would be unable to comprehend some of the most ordinary physical conditions of the atmosphere. We have already seen that the general effect of heat, as regards alteration of dimensions, is expansion. If the amount of this expansion be determined for any particular range between fixed points, and the linear extension or space thus intercepted be divided into smaller spaces, each of these becomes a representative of heat or temperature. Supposing we assume the temperature at which water freezes to be our starting-point or first limit, and the temperature at which water boils to be our other limit; and supposing, furthermore, we assume the space between the two to be divided into any given number of parts—say, for example, 180—then we may describe any third body, the temperature of which is between the temperature of freezing and the temperature of boiling, to be one, two, or any number of parts above the former or below the latter. Thus, by applying these principles, we should have constituted the thermometer or heat-measurer. If the rate of expansion of any one body for given increments of temperature were regular and well determined, there would be no theoretical difficulties in the way of making a thermometer; practical difficulties there would be, but the hypothetical part of the task would be sufficiently easy. It so happens, however, that the number of expansive agents capable of employment for this purpose is limited.

The earliest thermometer was that of Sanctorini. Its construction is represented in the following diagram. A glass stem, open at one extremity, is terminated at the other by a bulb (Fig. 20). Into the bulb and a portion of the tube is poured a coloured fluid, which being done, the stem is inverted into the lower vessel. By virtue of the ordinary laws of hydrostatics, the level of the fluid in the stem will remain constant for every constant temperature; but inasmuch as every increment of heat will cause the air contained in the bulb to expand, so will it necessarily cause the coloured fluid—which latter merely serves as an index—to descend in such a manner, that were the ratios of successive equal linear measures of descent equal, the instrument would be a no less delicate measurer of variations of temperature than it is a delicate indicator of the same. For certain reasons, now to be described, it is not a delicate heat-measurer. Its successive linear columnar measurements are not comparable among themselves; whence it follows, that the instrument is not a thermometer or heat-measurer, but a thermoscope or heat-indicator.

Fig. 20.

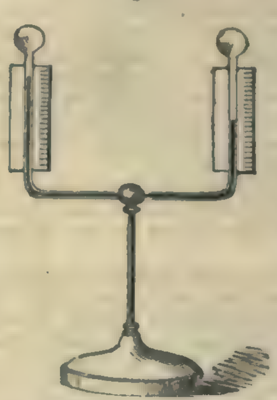


Concerning the reasons wherefore the instrument just described is not a perfect instrument, they readily admit of being made evident. They are immediately referable to the fact, that the coloured fluid, which, according to the necessities of the experiments should be dynamically passive, is really active. Its activity, moreover, is a variable quantity. If the coloured fluid were merely an index, having no dynamical power of its own, then the total increments of expansion and contraction of the air, contained in the bulb and part of the stem, would be proportionate to the increments of heat and cold within so small a deficit of the truth (see page 1127), that the error need not enter into calculation; but examination of the structure of the instrument will show that this cannot be so. *Actually*, the total expansion of the air in the bulb is the resultant of two forces—the force of aërial elasticity due to heat, and the force of pressure or downward tendency of the columnar liquid. Inasmuch, therefore, as the columnar height of the liquid in question varies for every temperature; and, moreover, as the rising and falling of the liquid in the reservoir, or lower receiving-vessel, also confuses the result, the heat-determining instrument of Sanctorini is not a correct measurer of temperature.

The Differential Thermometer.—Although various forms of air-thermometers are occasionally used in conducting certain specific experiments, their use is rare. Almost the only form of air-thermometer in frequent use is the differential thermometer, an instrument the function of which is to determine the difference between the temperature of any two adjacent bodies, or of the adjacent parts of any one body, without informing the observer concerning the actual temperature of either.

The differential thermometer is represented by the accompanying diagram (Fig. 21). It consists of a glass tube having a bulbular expansion at each extremity, and joined by a stem bent on itself twice at right angles, so that the two bulbs look upwards. During the process of manufacturing this instrument, whilst one of the bulbs is yet unclosed, liquid is poured into the instrument just enough in quantity to reach a little way up the vertical part of the stems; each of these vertical parts are supplied with a scale-division of equal lengths or degrees. Looking at this instrument, the observer will readily see that, supposing the tension or expansive force of the atmosphere in each bulb to be equal, the index fluid in the stem will stand at precisely the same elevation in both vertical stems; but supposing the air in one bulb to become heated to a higher degree than the air in the other; supposing, in other words, its expansion to be greater, the corresponding columnar height will be diminished, and necessarily, the opposite columnar height will be raised. Hence the difference between the two columnar heights will indicate that of the temperature of the two bulbs, expressed in equal parts of columnar measurement.

Fig. 21.



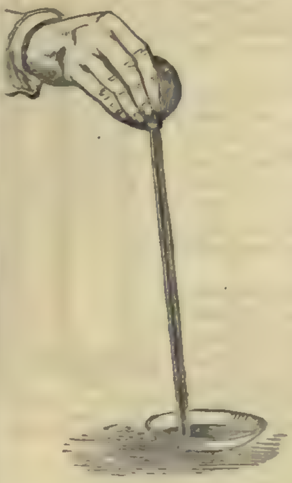
Thermometer Scales, and Ordinary Thermometers.—The liquids ordinarily employed in the manufacture of thermometers are mercury and alcohol. The former is preferred to all others, when its use is practicable, on account of the comparatively equal expansion to which it is subject for equal grades of temperature. In the manufacture of thermometers, however, intended to be employed for the measurement of temperatures below the freezing point of mercury, that fluid is necessarily inapplicable. Spirit, or alcohol, has never been frozen by the most intense cold yet produced, therefore it is substituted for mercury on such occasions. We shall now proceed to detail the successive steps in the manufacture and graduation of thermometers.

Under the head of *Barometer*, the inconvenience was pointed out of using a glass tube of small diameter, because of the interference resulting from the expansion and contraction of mercury by heat. Now that which is a cause of embarrassment in the barometer, is the function on which the action of the thermometer depends. It follows, therefore, that in proportion as the bore of a thermometer tube is smaller, so is the resulting instruments more delicate, because greater linear increments of expansion will be generated for given amounts of temperature. However, it happens in practice, that if the diameter of the tube, and the diameter of the mercurial column contained in the tube, be smaller than certain limits, it is difficult to be seen. It is hardly necessary to indicate, moreover, that the length of linear expansion may be increased to any given limit, by increasing the dimensions of the corresponding bulb, or mercurial reservoir. Practice alone can determine the proper relation which should subsist between the bore of a thermometer tube, and its corresponding bulb.

The following directions, for the manufacture of a thermometer, are not intended to enable the meteorological student to usurp the functions of the mathematical instrument-maker; on the contrary, they are intended to make known to him the defects which he should look for in a thermometer, and which, if discovered, should cause the thermometer to be rejected.

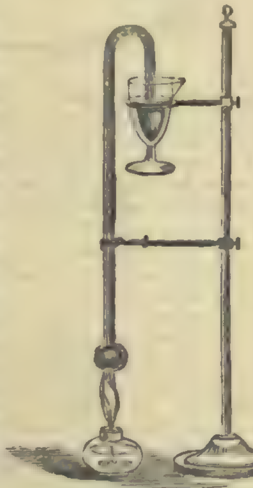
The Tube must be Equal in Bore throughout.—If the bore or diameter of a thermometer tube be not equal throughout, it is evident that the amount of linear expansion cannot be equal, and that the instrument will be absolutely worthless. A very easy means of gauging this equality is the following:—Having selected a piece of thermometer tube open at both ends, tie on to one extremity a rigid bottle of india-rubber, and dip the other end in mercury, thus (Fig. 22). Pressing the bottle, a portion of the atmospheric air will be expelled; then, allowing the bottle to expand, some mercury will enter, forming a mercurial column, the length of which admits of being measured. Let it be measured by means of a pair of compasses; and let the mercury be driven to various parts of the tube, and the measurements repeated. If the tube be of equal bore throughout, the mercurial column will necessarily be of equal length throughout; and if the tube stand this test, it may be considered good. Instead of the india-rubber bottle the breath might be employed, but it would be attended with the disadvantage of moistening the tube. However, it is possible to use the breath without prejudice, if the operator take the precaution of blowing through some material absorptive of moisture. The next step in the manufacture of a thermometer consists in fusing one end of the tube, and blowing it into a bulb; this, again, should be effected by means of the caoutchouc bottle, lest moisture be deposited. Mercury has now to be introduced, which is accomplished as follows:—

Fig. 22.



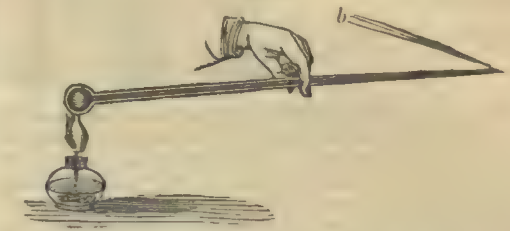
The thermometer tube having been bent as represented (Fig. 23), its open extremity is immersed in a vessel of mercury. Heat being now applied to the bulb, the air therein contained is expanded; and the heat being removed, a partial vacuum results, to fill which, mercury rushes in. By repeating the operation, the mercury already contained in the bulb is vaporised, and the vapour, expanding, drives out all the remaining atmospheric air; so that on the removal of heat, the whole—tube, bulb, stem and all—becomes filled with mercury. The bent part of the tube is now broken off, and the final quantity of mercury duly apportioned to the tube. The amount of this apportionment can be only determined by practice; but, in general terms, it may be described as being such a quantity, that, at the boiling point of mercury, it shall nearly extend to the extremity of the tube.[*]

Fig. 23.



The next process is one of extreme delicacy. It has for its object the sealing or melting the open extremity of the thermometer tube, without admitting the slightest portion of atmospheric air, the presence of which would materially interfere with the delicacy of the instrument. The operation is conducted as follows:—The open end of the tube having been melted in the blow-pipe flame, is drawn out to a fine termination, thereby diminishing still further the internal bore of the tube, and rendering the final closing of its orifice more easy. The mercury in the bulb is heated once more; and at the very instant when it is seen to fill the tube and to be in the act of overflowing the capillary extremity, the tube is closed by the fine jet of blow-pipe flame *b* (Fig. 24). It only now remains to graduate the thermometer.

Fig. 24.



Graduation.—The instrument just described is, in its present condition, a thermoscope, or indicator of heat; but it is by no means a heat-measurer—to convert it into which, it requires to be graduated into a determinate number of equal parts. These parts are usually called degrees; but it must be remembered that the so-called degrees have no ratio whatever to any particular unit. They are not like the degrees on the circumference of a circle, each one of which is related to the trigonometrical ratio, or the ratio of the circumference to the diameter. Thermometric degrees are altogether arbitrary, except insomuch as certain usages in this respect obtain. This much is, however, invariable and universal—the divisional parts or degrees must be established between certain limits, fixed by nature. The limits usually adopted in the manufacture of thermometers are the boiling and the freezing of water, which phenomena always, under similar conditions, take place at similar respective temperatures. Founded on the bases of these limits, three principal scales of graduation have been devised. They are the centigrade scale, or scale of Celsius; the scale of Reaumur, and the scale of Fahrenheit. The latter is mostly used in this country; the former is chiefly adopted on the continent.

Let us commence our illustrations with the centigrade scale, as being most easy. The term centigrade seems to be significant of a hundred divisions. In point of fact, the centigrade scale has for its 0, or zero, the temperature at which water freezes, and, for its 100, the temperature at which water boils; the intermediate space being divided into 100 equal parts. And if it be desired to carry the graduations above or below the gauge limits, this is accomplished by measuring off equal parts by means of a pair of compasses.

Reaumur's scale has also its zero point, at the elevation of mercurial column corresponding with the freezing point of water; but at the other extremity of its scale the boiling point of water is considered to indicate 80; hence the intermediate space is divided into 80 equal parts. The following diagram (Fig. 25) will render evident the peculiarities of the ordinarily employed thermometric scales.

Conversion of One System of Graduation to Another.—This conversion of thermometric scales is frequently necessary. The rules for effecting the conversion are evident on reflection; nevertheless it is well to reduce them to general formulæ. If all these scales counted their zero from the same point, the method of converting one scheme of graduation into another would be still easier than we find it. Actually, some little confusion at first arises from the circumstance that Fahrenheit's zero is placed not at, but below the freezing point of water.

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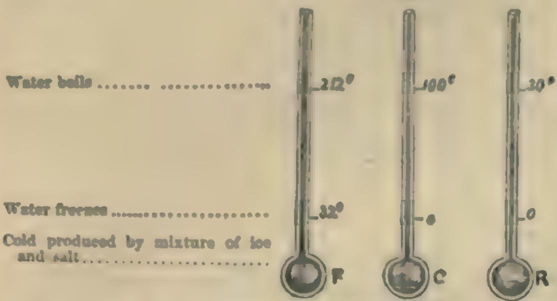
[*] Care is required, in this process, that the cold metal shall not suddenly impinge on the hot bulb, as that would certainly break it.—ED.

Fig. 25.



Conversion of Fahrenheit to Centigrade Degrees.—The proposition is evident that one Fahrenheit degree is equal to five-ninths of a centigrade degree, inasmuch as the number of Fahrenheit degrees, between the freezing and the boiling point, is to the number of centigrade degrees for the same space as unity to five-ninths, as is seen to be established by the following proportion :—

$$\overset{F.}{(212 - 32)} = 180 : \overset{C.}{100} :: \overset{F.}{1} : \overset{C.}{\tfrac{100}{180}} = \tfrac{5}{9},$$

whence it appears that we may convert Fahrenheit degrees into their centigrade equivalents, by first subtracting 32°, which leaves 180 ; then multiplying this number by five, and dividing by nine, as in the following proportion :—

$$212 - 32 = 180 \text{ and } \frac{180 \times 5}{9} = \frac{900}{9} = 100.$$

Conversion of Centigrade to Fahrenheit Degrees.—Inasmuch as each centigrade degree is longer than a degree of Fahrenheit in the ratio of $\frac{9}{5}$ to 1, therefore the former may be reduced to the latter by multiplying by 9, dividing by 5, and adding 32. The truth of this operation may be readily demonstrated by working on the number 100, which should give, if the rule just enumerated be correct, 212 :—

$$\frac{100 \times 9}{5} = 180, \text{ and } 180 + 32 = 212.$$

The examples just given, illustrate the process of calculation when positive degrees, or degrees above zero, thus (+), are concerned. Exactly the same rule has to be followed when negative degrees (—) are in question, although the rule, when stated in common terms, appears to be different, inasmuch as following the diction of arithmetic, the operator must be told to subtract 32. An example will render this more evident. Suppose we require to represent 5° below zero, or — 5° of C, by its equivalent F. Now the number 32, with 9 subtracted, gives 23 for remainder. Viewing all the steps of the calculation involved algebraically, it will be found that the rule of adding 32 has been implicitly followed; a negative 5 however (— 5) yields in the following operation a negative 9 (— 9), which, being added to + 32, is equivalent to subtracting a positive 9 (+ 9). For example—

$$\frac{-5 \times 9}{5} = \frac{-45}{5} = -9, \text{ and } -9 + 32 = 23.$$

The various steps for the reduction of one system of thermometric degrees to another, are comprehended in the appended formulæ :—

Fahrenheit to Centigrade, $\frac{F. - 32 \times 5}{9} = C.$

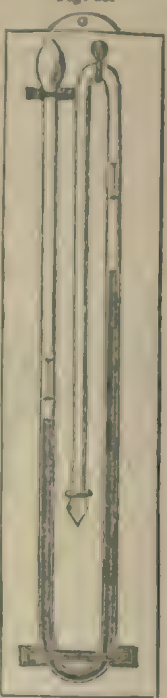
Centigrade to Fahrenheit, $\frac{C. \times 9}{5} + 32 = F.$

Fahrenheit to Reaumur, $\frac{F. - 32 \times 4}{9} = R.$

Reaumur to Fahrenheit, $\frac{R. \times 9}{4} + 32 = F.$

The Register Thermometer.—The greatest difficulty attendant upon the use of the thermometer for meteorologic observations, is referable to the necessity of frequent examination. To obviate in some measure the necessity for this, the instrument called the *Register Thermometer* has been devised—an instrument which depends for its action on the traversing of two steel bars in the bore of the tube, each pressed forward by the expansion of a liquid column. An instrument of this kind is represented in Fig. 26. The register thermometer is accurate enough in its indications for some rough purposes, but it is by no means adapted to supersede the use of thermometers of ordinary construction.

Fig. 26.



The Thermometer of Breguet.—A very delicate thermometer has been invented by M. Breguet. It differs from all which we have hitherto described, in the fact of its dispensing altogether with mercury, or other expansive liquid, and utilising the expansion or uncoiling of a compound metallic bar.

The illustration of the different rates of expansion by heat of two different metals (for instance, iron and brass) is often made by the following contrivance :—*a* (Fig. 27) is a compound bar of this kind, perfectly straight when cold ; but if this same bar be heated, it becomes curved, as represented by Fig. 28.

Fig. 27.

Breguet's thermometer is merely an amplification of the preceding experiment ; instead of a straight compound bar, a compound bar twisted into the spiral form is employed. One end of the spiral is fixed ; the other end is free, and is attached to an index. The mode of action of the instrument will be obvious. Variations of temperature producing variations of curve, will cause the spiral to unfold, or contract, according as the variations are towards the direction of increased heat or increased cold. The metals employed in making the spiral of Breguet's thermometers, are platina, gold, and silver. Experiment has demonstrated that the needle of this intrument travels over equal arcs for equal increments of temperature ; hence Breguet's thermometer is not only comparable with itself, but with all other instruments on the same construction. Unfortunately, this delicate instrument has no great range of application, its indications being limited between the freezing and the boiling points of water (Fig. 29).

Fig. 28.

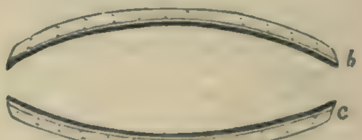
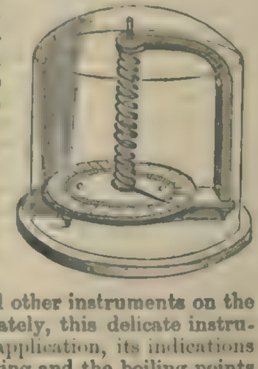


Fig. 29.



The Thermoscope of Nobili.—By far the most delicate indicator of minute increments and decrements of temperature, is an instrument founded on principles totally different to any already described. The electrical thermoscope of Nobili admits of being thus described :—The action of the instrument is based upon the fundamental fact of electro-magnetism ; viz., that a magnetic needle, freely suspended and placed in the vicinity of an electric current, finally arranges itself at right angles to that current. Hence the deflection of a magnetic needle becomes indicative of the existence of such current. Founded on the consideration of this fact, we have the instrument termed the galvanometer, which, in its simplest form, is represented by Fig. 30 ; and a still more delicate construction of which is represented in the following engraving (Fig. 31).

Fig. 30.

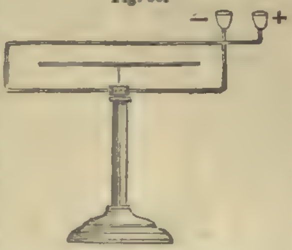
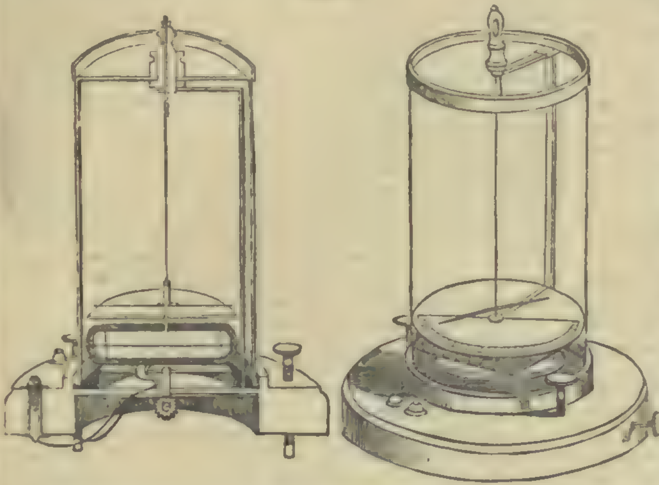
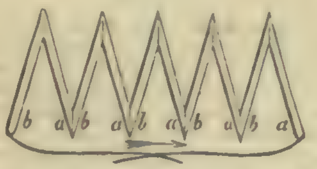


Fig. 31.



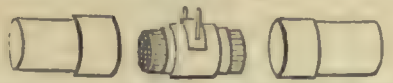
It remains now to remark that heat is a fruitful source of electricity, especially when heat is applied to one end of a mechanical arrangement of alternate bars of two metals, such as bismuth and antimony, as represented in the accompanying diagram (Fig. 32).

Fig. 32.



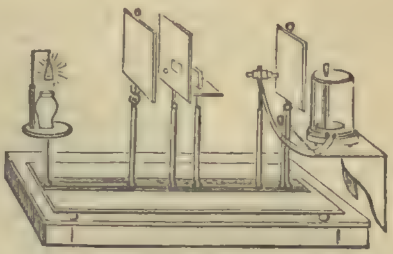
In order to render the combination of metallic bars more compact, they are usually arranged in a bundle, as represented below (Fig. 33).

Fig. 33.



Very few words will now render comprehensible to the reader the structure and functions of the thermoscope of Nobili. A bundle of bismuth and antimony reduplications, as just described, being placed in communication with a galvanometer, the magnetic needle of the latter is ready to be deflected on the first occurrence of an electric current, and such electric current is a direct consequence of the application of heat to one extremity of the system of compound metallic bars. The following diagram (Fig. 34) represents the thermoscope of Nobili, as employed to indicate small increments of radiant heat evolved from the little lamp on the left, and transmitted through the central diaphragm.*

Fig. 34.



Considerations respecting the Use of Thermometers.—Much that is incorrect passes current respecting the functions of the thermometer—the accuracy to which it is susceptible, and the spontaneous deteriorations to which the instrument is subject. A few words, therefore, on these matters may be advisable.

Let us assume the thermometer under consideration to be a mercurial thermometer, and that the mercury used was absolutely pure ; that the tube was proved to be of equal bore throughout ; that the process of sealing was accomplished without permitting the ingress of the slightest amount of atmospheric air ; let us, finally, assume that the tube has been accurately graduated—such an instrument may be regarded as free from all errors of construction.

Nevertheless, it is found that a thermometer thus unexceptionable at first, is liable to deteriorate by time, the gauge points—*i.e.*, 32° and 212°—corresponding with higher portions of the tube than they should do, and, necessarily, all other degrees similarly change. Most probably this result is referable to a gradual contraction of the sides of the tube and of the bulb, the contraction being determined by continuous atmospheric pressure. Looking at this contraction, the propriety is suggested of retaining thermometer tubes filled some time previous to graduation.

The kind of Heat indicated by Thermometers.—The term *heat* is commonly held to be synonymous with *temperature* ; but philosophy accepts it in a more extended sense, as comprehending not merely one effect (temperature), but the cause of many effects. The philosophy of latent and specific heat is almost too purely physical for extended examination here ; hence a slight reference to these conditions will suffice.† The thermometer is not adapted to take cognizance of heat in this latent or specific form. It is only indicative of evident heat or temperature ; nor are its indications in this narrow field so complete, nor the information it conveys so extensive, as is frequently supposed. The thermometer does not even profess to indicate the *quantity* of heat, but only its *degree* ; terms which are totally distinct, as will soon be perceived. Let us take the following as an illustration of the difference :—A pint of boiling water is as hot as a quart of boiling water, the temperature of both being 212° Fah. ; hence the thermometer, if appealed to, will indicate this identity of temperature. But, necessarily, a quart of boiling water must contain twice as much calorific force as a pint of the same : the deduction is too obvious for comment.

Again, strictly speaking, the thermometer cannot be said to present us with the *correct* temperature of anything, inasmuch as the degree of columnar expansion is not the degree corresponding with that of the thing, with which the thermometer is brought into contact, but the mean of the thing touched, and the bulb which touches it. This objection attains its minimum when the atmosphere itself is the medium, the temperature of which we desire to investigate; but it becomes of practical importance in all other cases; and its consideration teaches the thermometric observer the necessity for employing instruments, the bulbs of which are as small as compatible with well-marked amounts of columnar expansion. This remark especially applies to cases in which the bulk of liquid or solid operated upon is small. As concerns the graduation of thermometers, the most correct instruments are those of which the graduations are effected on the tube of the glass itself. If the graduation be made on a scale of brass, the resulting indications will be very incorrect, except the relative expansibility of brass and glass be taken into consideration, and duly allowed for; this, however, is so troublesome, that it will be too frequently evaded, and errors will creep in. Far better than brass is box-wood; and slate and ivory are better still.

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* For more detailed descriptions of these instruments, the reader may consult the Sections on *Electricity* and *Magnetism* generally, in which these principles of action are investigated.—Ed.

† These subjects have been fully discussed in the Section on *Heat.*

Correspondence between Thermometers.—Scarcely any two thermometers exactly correspond, even though the same materials be employed in their construction, so numerous are the points which require attention. Between thermometers constructed with different materials—air thermometers and mercurial thermometers, for instance, or either of these, and spirit thermometers—the discrepancies are still more considerable.

The experiments of M. Regnault relative to the discrepancies subsisting between mercurial and air thermometers, are amongst the latest on this important subject. He found, that between 32° and 212° Fahrenheit, there is an almost absolute coincidence between the degrees of the air and the mercurial thermometer. From 212° to 482° Fahrenheit, the mercurial and air thermometers remain pretty equal; but after the latter point the mercurial gains on the air thermometers.

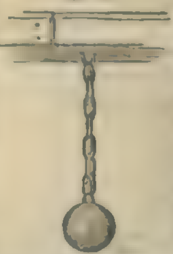
Hitherto we have treated of the atmosphere, statically, that is, in a condition of repose; but perfect atmospheric quiescence is unknown in nature—it is always agitated, more or less; hence we are led to the consideration of atmospheric currents or *winds*.

So variable are winds in these northern latitudes, that their incertitude has passed into a proverb; primary atmospheric currents, nevertheless, are constant in their direction, and are referable to variations of temperature simultaneously existing in different parts of the world.

Heat, in its non-latent condition, or, in other words, that condition of heat which is recognisable by the thermometer, has a tendency to equalise itself. Hence, if two bodies *a* and *b*, of which *a* is hotter than *b*, be situated in proximity to each other, there is an immediate tendency to equalisation of temperature as between the two; but the thermal conditions which regulate the production of winds, will be most readily appreciated by reflecting on what takes place when a heated solid is suspended in the atmosphere by a small chain or wire; we shall find, on investigation, that a heated solid thus circumstanced gradually becomes cool by the operation of three distributive influences—*conduction*, *radiation*, and *convection*. It will be unnecessary for us to do more than give a very general illustration of these terms, as the subjects are fully investigated in the first section of this volume.

Conduction.—If the chain by which the heated cannon-ball is represented to be suspended in the annexed diagram (Fig. 35), be examined from time to time by the thermometer, or even by the fingers, it will be found to increase in heat, evidently because it removes a portion of temperature from the heated cannon-ball by conducting it away. This function of heat is so well understood, and so commonly exemplified, that no further consideration of it will be necessary here.

Fig 35.



Radiation.—If a thermometer be held even at the distance of some feet from the heated cannon-ball, the mercurial column will be sensibly affected, thus demonstrating the transmission of heat. But how transmitted? By contact with the atmosphere? Clearly not, inasmuch as the result just indicated, still occurs if the cannon-ball be placed in the vacuum of an air-pump. By experiment, it has been determined that the temperature, in the case under consideration, has been given off in the condition of rays, precisely as light is evolved, and hence the propriety of the term *radiant heat*.

Convection.—Independently of the two processes of heat-distribution already described, there is yet a third. Referring to the suspended hot cannon-ball, we shall find that, if suspended in a room from the ceiling, the air near the ceiling becomes hotter than the air below: hence a portion of the temperature of the hot cannon-ball must have become accumulated there, by reason of some cause besides those of conduction and radiation, both of which distribute the temperature equally in all directions, through a homogeneous medium—such as the atmosphere for example—in our assumed experiment. The process of heat-distribution known as convection, is the necessary result of the expansion of liquids and fluids by heat; for when expanded they are specifically lighter; and when specifically lighter, they must necessarily ascend. It is by virtue of the process of heat-distribution, termed convection, that the child's soap-blown bubbles ascend instead of at once falling to the surface of the earth.

Temperature and pressure being equal, breath evolved from the lungs is heavier than atmospheric air, because it holds more carbonic acid. Nevertheless, inasmuch as it is evolved from the lungs hotter than the surrounding atmosphere, soap-bubbles blown with it ascend. Presently, however, they descend, because the heat acquired from the lungs being evolved, and equalisation of temperature with that of the surrounding atmosphere having ensued, the great specific gravity of the gas wherewith the bubble is blown, causes the latter to sink to the surface of the earth.

It is to the process of heat—convection—that we owe the salutary draughts in our chimneys, and also unsalutary draughts in our apartments. No sooner is fuel lighted in a fire-place, than the superincumbent air becomes higher and specifically lighter; it therefore ascends, and cold air rushes in to fill its place. Thus we have, in point of fact, a local wind; and the causes which determine that wind are exactly comparable to the causes of winds which take place in the grand economy of nature, as will soon be rendered manifest.

CHAPTER II.

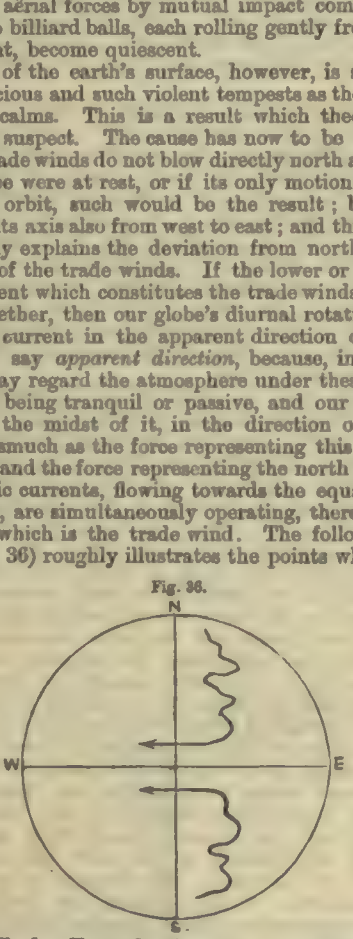
ATMOSPHERIC PHENOMENA.

THE TRADE WINDS.—Applying the facts already developed, let us now regard the surface of our globe in the aggregate, with reference to the localities of maximum and minimum temperature, and the consequence of such difference of temperature in originating an aërial current. It is evident that the hottest portions of our globe's surface are comprehended within the tropics, and the coldest portions are the arctic regions—north and south.

These circumstances being premised, we are now in a condition to anticipate the direction of the aërial currents or winds which must necessarily ensue. Firstly, an ordinary current of heated air should rise aloft in the tropical regions, then diverge and pass north and south to either arctic circle, thus constituting what may be termed the upper trade current. This current, as it proceeds north and south, gradually becomes cold, in which condition it is rendered specifically heavier, falls to the surface of the earth, and floats along towards the equator, thus generating two principal currents—one north, the other south. Whilst yet in the frigid and the temperate zones, these primary currents encounter so many interferences, that their primary or fundamental direction is masked or veiled: but still proceeding north and south, the persistent directive tendency of the trade winds is at length developed. But the currents no longer flow directly north and south. By the operation of a cause which will presently be rendered evident, the directive tendency of either current has acquired a certain impulse towards the west; or, in other words, both currents come more or less from the east. But at length they blow almost from due east, and finally cease altogether; so that the equator, and a certain space north and south of the equator, are comprehended within what is termed the region of calms. The north trade wind meeting the south trade wind, they are mutually destructive of each other. Two conflicting aërial forces by mutual impact come to rest, just as two billiard balls, each rolling gently from an opposite point, become quiescent.

No part of the earth's surface, however, is subject to such capricious and such violent tempests as the so-called region of calms. This is a result which theory would lead us to suspect. The cause has now to be explained why the trade winds do not blow directly north and south. If our globe were at rest, or if its only motion were motion in its orbit, such would be the result; but it revolves on its axis also from west to east; and this circumstance fully explains the deviation from northness and southness of the trade winds. If the lower or returning aërial current which constitutes the trade winds ceased to exist altogether, then our globe's diurnal rotation would generate a current in the apparent direction of east to west. We say *apparent direction*, because, in point of fact, we may regard the atmosphere under these circumstances as being tranquil or passive, and our globe revolving in the midst of it, in the direction of west to east. Inasmuch as the force representing this apparent east wind, and the force representing the north and south atmospheric currents, flowing towards the equator from either pole, are simultaneously operating, there occurs a resultant, which is the trade wind. The following diagram (Fig. 36) roughly illustrates the points which have been described. Towards the extreme north and extreme south, the great aerial currents, ultimately destined to become the trade winds, are represented fluctuating and variable; gradually, however, they acquire a northern and a southern directive tendency respectively; lastly, they come from the point of almost due east, and then cease altogether.

Fig. 36.



A consideration of the influences which determine trade winds, leads to a facile explanation of many aërial currents of less extent. The heating influence referable to the geographical position of the tropics is not the only influence of this kind A peculiar condition of the earth's surface, taken conjointly with a favourable condition of the sun's rays, may bring about similar results. In this manner the Mediterranean etesian winds, or aërial currents from the north, may be accounted for. Looking at the geographical condition of localities south of the Mediterranean, we find the Sahara, or Great Desert, a region whose surface is strewed with masses of pebbles and sand, and almost totally devoid of vegetation. Such a surface must necessarily become elevated by the solar rays to a high temperature, an atmospheric column must ascend, travel to the north, then fall, and at length return from the north to the Sahara, from whence it came.

LAND AND SEA BREEZES.—Many countries near the sea are subjected to winds of diurnal periodicity, known as land and sea-breezes. About eight or nine A.M., an aërial current begins to flow from the sea towards the land, and persists until about three P.M., when a current in the reverse direction, or from the land towards the sea, takes its place, and continues throughout the night until sunrise next morning, when it ceases, and a calm ensues until the completion of a period of twenty-four hours from the occurrence of the preceding land-breeze. These currents, in reverse directions, can be easily accounted for when we consider the heating agency of the sun. Necessarily, land becomes hotter than water under an equal power of calorific rays; whence it follows, that the surface of the ground, becoming heated after sunrise, determines the ascent of an atmospheric current vertically; thence, proceeding oceanward, the same current returns from the sea to the land. No sooner does the sun set, than this current is reversed.

In the preceding explanations of the cause of winds as being owing to inequalities of temperature, it will be observed that reference has alone been made to the calorific effects of the sun's rays. This is, in point of fact, the only source of heat which has to be noticed in meteoric considerations; for, although the earth's own temperature gradually increases as we pierce downwards below the surface, so imperfect are the conducting powers of the materials of which the crust of our planet is composed, that all consideration of them may be safely omitted in accounting for the present phenomenon.

PROCESS OF REVERSE ATMOSPHERIC CURRENTS.—Although the existence of atmospheric currents, proceeding in a direction reverse to those we meet with on the earth's surface, is forced upon the mind by inferential reasoning, and the fact must be accepted, even though no further evidence of it could be adduced, nevertheless direct proofs are not wanting. The direction of upper layers of clouds afford their testimony to the truth of the opinion. Where the trade winds prevail, the higher strata of clouds may be seen taking a direction opposite to that of the wind itself; and travellers, during their ascent of high mountains, have frequently proved the existence of a superior wind pursuing a course opposite to the wind below. This has been especially obvious on the Peak of Teneriffe, a mountain situated in the belt of the trade winds. On the summit of this peak, it has been always found that a south-west wind prevails; whereas the trade wind, at the base of the same mountain, blows from the north-east.

A similar remark has been made by travellers who have ascended Mona Kea, in Owyhee, the height of which is 18,000 feet. But, perhaps, the most striking illustration is the following:—The island of Barbadoes lies eastward of St. Vincent, and between the two the trade wind continually blows, and so forcibly, that it is only with difficulty, and by making a long circuit, that a ship can sail between the latter and the former. Nevertheless, on one occasion, during an eruption at St. Vincent, dense clouds formed over Barbadoes, and large quantities of ashes fell on the island. A similar result was observed after an eruption of the volcano of Cosoguina, on the shores of the Pacific, in Guatemala, in January, 1835, some of the volcanic ashes falling in Jamaica, more than 800 miles in a direct line distant, and directly opposed to the prevailing lower current. At the same time, another portion of ashes was carried westward, or in an opposite direction, falling on Her Majesty's ship *Conway*, in the Pacific, more than 1,200 miles distant.

The trade winds are, for the most part, only recognisable at sea; the solid material of land developing local aërial currents of their own. The extent of prevalence of the trade wind is various. In the Atlantic it prevails from 8° to 28° or 30°, but in the Pacific only to 25° N.L. In the southern hemisphere, the extent of the trade wind has been less accurately determined. When first the phenomena of trade winds were noticed by Columbus and his associates, they caused the greatest consternation. Accustomed to the fluctuating and irregular breezes of Europe, they regarded the continuance of a wind from the east as emblematic of their perpetual banishment from their native shores. The early Spanish navigators, however, very soon learned to appreciate the value of trade winds, by the aid of which treasure-laden galleons could, setting out from Acapulco, manage to arrive at Manilla almost without changing a sail.

As respects the upper current proceeding from the equator to either pole, it varies, as might be anticipated, in different localities. Travellers, who have ascended the Peak of Teneriffe, inform us that this upper current is found in that locality, at an elevation of 9,000 feet; but Humboldt, during his explorations on the Andes, discovered the eastern trade wind to be blowing at an elevation of 8,000 feet above the level of the sea. As the upper or equatorial current loses its heat, its specific gravity becomes greater, and it sinks lower and lower, no longer manifesting any well-marked directive tendency.

On the ocean, and between 30° and 40°, there is a prevalence of westerly winds, especially in the southern hemisphere. In the Atlantic this tendency is manifest; whence it follows, that the voyage from Europe to America occupies more time than a voyage in the reverse direction. It is difficult to say what may be regarded as the prevailing wind in these isles—probably, however, a south-western wind, as stated in the following table :—

Table representing the Relative Prevalence of Winds in different Countries for a Period of 1,000 *days.*

Countries.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.
England . .	82	111	99	81	111	225	171	120
France . .	126	140	84	76	117	192	155	110
Germany . .	84	98	119	87	97	185	198	131
Denmark .	65	99	100	129	92	198	161	156
Sweden . .	102	104	80	110	128	210	159	106
Russia . .	99	191	81	130	98	143	166	192
North America	96	116	49	108	123	197	101	201

The direction of winds is found, taking the average of many years, to vary according to the season. In Europe, south winds are more prevalent than any others during winter; east winds belong more especially to the spring; west and north winds to the summer; and towards October the wind usually veers round to the south. Usually, the wind is more strong in February and March than at any other time; and at all seasons the wind is usually strongest at noon.

STORMS.—Whenever the air, from any cause, is thrown into violent commotion, the result will be a storm. The philosophy of storms, notwithstanding the attention which has been devoted to the subject, is by no means well understood. In point of fact, the causes of storms are numerous and complex. If we reflect on the agency of temperature on the air, one prevalent cause of storms will at least become manifest. If one portion of the atmosphere be suddenly heated, violent commotion must arise—there must follow a storm. The laws of latent heat demonstrate, that whenever water is suddenly condensed, the surrounding air must be raised in temperature; and thus we have one of the most frequent local causes of storms. According to modern observations, storms are, for the most part, circular whirlwinds progressing in a north-eastern direction from the south to the north of the tropic of Cancer. In proportion as a locality is devoid of mountains and near the sea, so is it more liable to be subject to storms. Perhaps the most violent of all European storms are those which occur in the south of France during the prevalence of the north-east wind, termed *mistral;* but the most violent storms occur in and near the tropics, and are termed *tornadoes, trovadoes, hurricanes, typhoons,* &c. Hurricanes are essentially tropical; the West Indies suffer from them more than any other region. Hurricanes are of yearly occurrence in the West Indies; but the islands of Trinidad and Tobago, being protected by mountain elevations, usually escape them altogether. The wind, during a hurricane, frequently makes an entire circuit, blowing from every point of the compass; and it is by no means an unusual occurrence for the wind to cease awhile altogether, and then commence blowing again. Perhaps the most violent hurricane on record is the one which occurred in 1780. It destroyed the fleet of Lord Rodney, and a vast number of merchant ships. It killed no less than 9,000 individuals in Martinique; 6,000 in St. Lucia; and did immense damage at other places.

The eastern, western, and southern coasts of Africa are also subject to storms of almost equal violence with the West Indian hurricanes: these storms, however, in the localities under consideration, are called tornadoes. At Sierra Leone and the adjacent parts, two or three tornadoes usually usher in the dry season; they are sometimes accompanied with rain, and sometimes without: when of the latter kind, they are called white tornadoes.

The term *cyclone* is now applied to those storms, which are produced by a rotatory motion of the air. In the Northern Hemisphere, the rotation is contrary to the motion of the hands of a watch; whilst, in the Southern Hemisphere, it proceeds in the same direction as they move. The knowledge of the nature and motions of cyclones is now so complete, that, in many cases, the navigator, by watching the barometer and the direction of the wind, may so guide his vessel's course as to escape the violence of the storm. It is found, that near the centre, and the circumference of a cyclone, a calm usually exists, the force of the tempest being produced between those positions.

Typhoons may be described as hurricanes of the Chinese and Japanese seas; like hurricanes, they have a rotatory motion, but they are more localised in their action, having no distinct rectilineal progression.

Hot Winds.—Although heat may be regarded as primarily the cause of all winds, it does not follow that all winds must be hot; indeed, we know that the result is the direct opposite—that many winds are very cold. The temperature of a wind is for the most part totally independent of the temperature which caused it, and is determined by the nature of the surface over which it blows. The principal hot winds are those denominated the simoom, the harmattan, the chamsin, the sirocco, and the solano. The term simoom, or *samiel,* means poisonous, and is derived from a belief of the Arabs, that the devastating effects of this wind are attributable to some poisonous emanation which it bears. There is no foundation, however, for this notion. The terms chamsin and harmattan, are little else than Egyptian and negro appellations respectively for the simoom. The Egyptian term *chamsin* means fifty, and has reference to the duration of the wind fifty days—from April 27th to June 18th. The simoom is the terror of desert caravans. At its approach the horizon grows dark; the sun's rays scarcely penetrate with lurid gleam the atmosphere charged with particles of burning sand. The wind blows with fitful violence, scattering death and desolation in its track, withering the trees and shrubs which it encounters, suffocating animals, and burying them under waves of sand. The camels no sooner perceive the advent of the simoom, than rushing to the nearest tree or bush, or seeking the spur of some projecting rock, they place their heads in the direction opposite to which the wind blows, and endeavour to screen themselves from its violence. The traveller throws himself on the ground on the lee-side of the camel, and screens his head from the fiery blast within the folds of his robe. Too frequently, all these precautions are unavailing, both man and beast falling a prey to the terrible simoom. In the western part of Asia, more especially in Arabia, the simoom only blows in the summer months, and with maximum violence in July. It occurs only in the day-time, and for the most part only lasts a few hours. In Lower Egypt, the direction of the simoom is from the south-west; in Mecca, it comes from the east; in Surat, from the north; in Bassoria, from the north-west; in Bagdad, from the west; and in Syria, from the south-east; in every case, proceeding from the neighbouring desert, where the air has suffered rarefaction. The simoom, far from being poisonous, is in some localities beneficial to health, by drying up aqueous exhalations, which, if not removed, would give rise to fevers and other diseases. This is particularly the case on the western coast of Africa. It is, nevertheless, always injurious to vegetation.

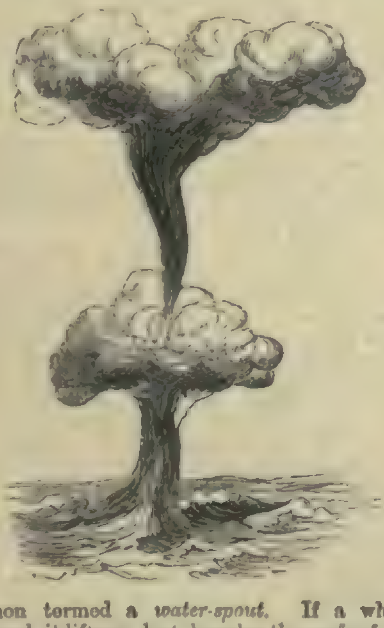
The Italian sirocco and the solano of Spain may be regarded as European continuations of the harmattan or simoom of Western Africa. The sirocco, although usually restricted to Malta, Sicily, and Southern Italy, sometimes extends into Germany and Switzerland: in the latter locality it is denominated the *föhn*. The föhn, although prejudicial to trees, develops, to a surprising degree, the vegetation of young plants, and can hardly be regarded as a calamity. It is most prevalent in Switzerland, near the Lake of the Four Cantons. Its period of duration does not usually exceed a few hours, though sometimes this period is exceeded; and it rarely occurs in winter.

The southern part of Australia is subject to a hot north wind, presenting such a marked resemblance to the sirocco, that geographers are led to the natural inference, that the unknown interior of the Australian continent is a desert of sand and rock, like the Sahara and the wilds of Arabia Petræa.

Cold Winds.—These winds are less noticeable and fewer in number than those already mentioned. Their low temperature is usually referable to the circumstance of their passing over mountain ranges covered with snow. The most considerable winds of this kind exist in Mongolia, Beloochistan, and the Russian steppes. To this class also belongs the *mistral*, a north-east wind prevalent in southern France, and which is exceedingly prejudicial to vegetable life.

Whirlwinds.—When two violent winds meet, the result is a whirlwind, so called from its rotatory character. If a whirlwind occur at sea, or over water, it elevates a large column of water aloft, sometimes to the height of many hundred feet, thus giving rise to the meteoric phenomenon termed a *water-spout*. If a whirlwind occur on land, it lifts up dust, boughs, the roofs of houses, and other solid matters, producing a column of well-defined shape. These whirlwind columns, whether they consist of water or solids, present the same general formation and contour. They consist of a hollow cone, sometimes straight, but more frequently curved or horn-shaped, its upper portion proceeding from a cloud; its lower part consisting of an aggregation of water or of sand and dust, according to the locality. The upper and lower portions of these columns are so much denser than the remainder, that they are generally opaque, whereas the middle portion is generally transparent. The tint of these colours is various—sometimes grey, sometimes brown or nearly black, and occasionally fiery red.

Fig. 37.

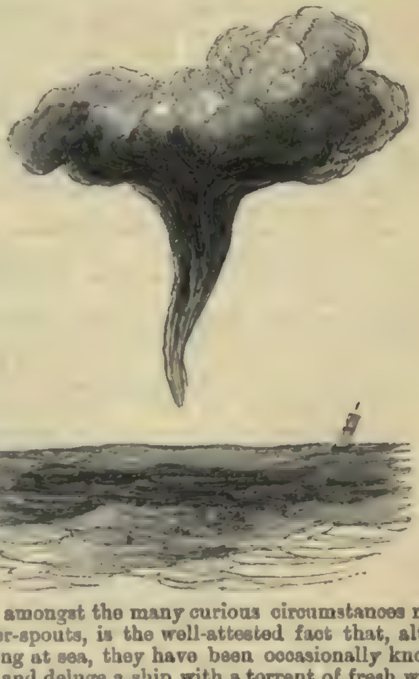


Independent of the circular or axial motion of these whirlwind columns, they pursue an onward course, sometimes straight, at other times curved. The velocity of this course differs within wide limits. Sometimes a man on foot can readily keep pace with it, whilst at other times they proceed at the rate of nine or ten miles an hour; sometimes more.

Whirlwind columns, whether they eventually become water-spouts or not, always originate on land, or in the vicinity of land where the winds and temperature are mutable. They are usually attended with thunder, lightning, and other electrical phenomena; and they constitute the centre of an aërial commotion, all around the focus of which, a profound calm prevails. Bodies which they have taken up are not readily deposited, but carried along in their onward course. Sometimes they are quite in the clouds, at other times on the surface of the earth or water; and their formation may be prevented. Even when already formed, they may frequently be destroyed by some violent aërial commotion, such as that produced by the discharge of a piece of ordnance—a fact well known to seafaring men. The size and height of these whirlwind currents are various; occasionally they present a diameter of no more than two feet; while the diameter of some has been estimated at two hundred, or even more. Again, the height of some is no more than thirty feet; whereas others have been known, the height of which was no less than three thousand feet.

WATER-SPOUTS.—Of these columnar whirls, the water-spout is less damaging than the dry-whirl, probably because the weight of fluid which it carries diminishes the violence of its rotatory motion. Not the least extraordinary amongst the many curious circumstances relative to water-spouts, is the well-attested fact that, although occurring at sea, they have been occasionally known to break, and deluge a ship with a torrent of fresh water.

Fig. 38.



Influence of Wind on the Barometer.—Although the barometer has hitherto been considered in reference only to the pressure of a tranquil column of air, its variations are influenced by many other circumstances, which we must not omit to consider. Amongst the most important of these are winds. Having regard to the ultimate cause of winds, it will be evident that the existence of a wind bespeaks the condition of different temperature in two different places. Hence, every wind necessarily varies to some degree the temperature which would have subsisted at any given place under a perfectly tranquil atmosphere. Now, as the atmosphere expands by heat and contracts by cold, varying, to a corresponding degree, its density or specific gravity, so it follows that the height of the barometric column will be influenced by winds. In Europe, it will be generally found that a fall of the barometer corresponds with a rise in the thermometer; this rule also prevails for the tropics: nevertheless it is subject to many variations. The barometer may rise and fall without any corresponding change in the thermometer, or both may rise and fall together.

The application of the barometer as a weather glass is altogether collateral and secondary; nevertheless its indications in this respect are, for the most part, worthy of confidence: generally, the barometric column sinks the day before rain occurs, and rises during its prevalence. The barometric column is much agitated during the existence of a storm, owing to the conflict which then ensues between atmospheric currents tending towards opposite directions.

Other Variations in the Barometer.—Besides the elevation of barometric mercury due to direct atmospheric pressure and to aërial currents, there exist other fluctuating causes, both diurnal and annual. The former is scarcely noticeable in temperate, but very conspicuous in the torrid zone. Every day the barometric column twice attains a maximum, and as often a minimum. The two periods of maximum elevation occur between 8½ and 10½ A.M. (say an average of 9h. 37m.); and between 9 and 11 P.M. (say an average of 10h. 11m.) The two periods of minimum elevation are between 3 and 5 A.M. (average 3¾); and between 3 and 5 P.M. (average 4h. 5m.) During winter and the rainy tropical season, the diurnal elevations of the barometer are least, and they assume their maximum in April. The variations are much less on elevated mountains than in the plains below.

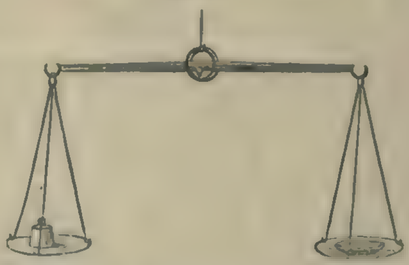
Mean Barometric Condition of a Place.—It was formerly assumed that everywhere, at the level of the sea, the barometric condition for the same time was identical. This opinion is fallacious, latitude having a well-determined influence in this respect. It is least of all at the equator, whence it increases north and south, attaining its maximum about 30° or 43° of latitude: it then decreases to between 60° and 70°. Within the polar circle it would appear to reascend; but further experiments for this locality are a *desideratum*.

Longitude also appears to exert some influence over the elevation of the barometric current. It is greater in the Atlantic than in the Pacific, by a small but readily perceptible quantity.

Causes of Periodical Barometric Variations.—Various opinions have been advanced to account for these periodical barometric variations. To say that they are attributable to difference of temperature, is to advance a cause too remote from the result. Many philosophers have attributed these variations to the existence of veritable atmospheric tides; but the most plausible explanation of diurnal barometric variations would seem to be that of Dove, who assumes them to depend upon the varying amount of aqueous vapour. Aqueous vapour and atmospheric air are possessed of different specific gravities; and the barometric height of a column of mercury for any time, will be the sum of pressure of dry atmospheric air and associated moisture; as the relative amount of the two varies, so will vary the height of the barometric column.

ATMOSPHERIC MOISTURE AND ITS DERIVATIVES.—When treating, at page 1129, of the means to be employed for weighing a gas, the facility wherewith gaseous bodies absorb moisture, was adverted to. Some idea then may be gained of the amount of moisture present in the atmosphere, seeing that the latter is ever in contact with large expanses of water. The atmosphere, in point of fact, is never dry, or in any way near dryness. Even when the air seems parching hot, drying the skin and withering vegetables, it is easy to demonstrate, by the aid of chemical agents, the existence of aqueous moisture; without the presence of which, neither the functions of animal or of vegetable life could be maintained. Even when the air approaches the condition of dryness, within very remote limits, breathing is difficult, and symptoms of feverish restlessness speedily set in. The natural craving of the lungs for moisture is demonstrated by the presence of a close stove in a small room. The sensations, which are very unpleasant, can always be alleviated by placing a small dish of water on the stove, so that evaporation may go on continuously. It is of the utmost importance, then, to be enabled, not only to demonstrate the existence of atmospheric moisture, but to determine its quantity. A few experiments for effecting this demonstration we shall now detail.

Fig. 39.



Experiment 1.—The accompanying diagram (Fig. 39) represents a balance, or pair of scales, into one pair of which there has been placed a small dish of oil of vitriol, and into the other a counterpoise. If the apparatus be exposed to the air, even when the earth is hottest and dryest, nevertheless, the equilibrium of the pair of scales will soon be destroyed. Some ponderable increase will have been acquired by the pan containing the oil of vitriol; and analogy demonstrates the increase in question to be due to the absorption of water. Founded on this property, oil of vitriol is frequently employed by the chemist for desiccating substances which could not be heated without damage. Accordingly, if a pan of oil of vitriol and a moistened sheet of paper be enclosed together, under an inverted glass, the paper will in course of time become dry. Far more rapid and powerful is the operation of the oil of vitriol when, instead of being placed together with the substance to be dried, and in a mere bell-glass, the two are placed under the receiver of an air-pump, and the air exhausted. Under these circumstances, an atmosphere of aqueous vapour soon fills the air-pump receiver, and the absorptive operation of the oil of vitriol being continuous, the water is speedily evaporated.

Experiment 2.—Instead of oil of vitriol, carbonate of soda, or chloride of calcium, and, in an inferior degree, impure common salt (chloride of sodium) may be used; for these bodies are all hygrometric—that is to say, they have the property of absorbing water from the air. Of the three bodies mentioned, chloride of calcium is the most hygrometric, and is of constant application by the chemist. Founded on the hygrometric quality of common salt, and other saline materials contained in sea-water, is the property which certain sea-weeds have of becoming moist in damp weather, and of indicating, by their dry crispness, an opposite atmospheric condition.

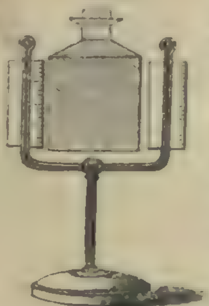
Although aqueous vapour is always present in the atmosphere, it is not always visible. Frequently it is quite transparent, and only demonstrable by the process of extracting it by such means as we have described; but at other times it aggregates, becoming vesicular, and forming clouds, fog, dew, rain, snow, hail, or sleet.

DEW.—Although the philosophy of dew is now perfectly well understood, no atmospheric phenomenon, before the happy researches of Dr. Wells, was more imperfectly explained and involved in greater mystery. The formation of dew is immediately referable to the function of radiation, concerning which it will be proper to make a short explanation, in addition to that which has been already stated.

The general indication only has been made that a heated body—for example, a cannon-ball—if suspended in space, darts off heat cognisable under the condition of rays. It remains now to be stated that the function of radiation is determined as to its extent by the surface of bodies: rough metallic surfaces radiate more than those which are smooth; glass surfaces radiate more than metallic surfaces; plants radiate more than the earth; grass and leaves more than bushes and trees; loose gravelly land more than hard soil.

To demonstrate the effect of surface on radiation, many instructive experiments may be performed by means of the differential thermometer and a cubical canister of tin plate. If such a canister be taken, and one side of it scratched, another polished smooth, another painted white, and the fourth black—a mixture of lamp-black and size being used by preference for the latter purpose: if the canister be now filled with hot water, and held between the two bulbs of a differential thermometer, as represented in the accompanying diagram (Fig. 40), each side of the canister will represent and indicate a different amount of radiating influence, as shown by the complementary disturbance of the two mercurial columns. It will be found that the polished side has the minimum, and the blackened side the maximum, radiating effect. It will soon be perceived that these deductions concerning the property of radiation are intimately connected with the philosophy of dew.*

Fig. 40.



Until the experiments of Dr. Wells, which will be soon adverted to, the most erroneous notions prevailed concerning the theory of dew. According to some it fell from the sky; according to others it rose from the ground; both which theories are altogether untenable.

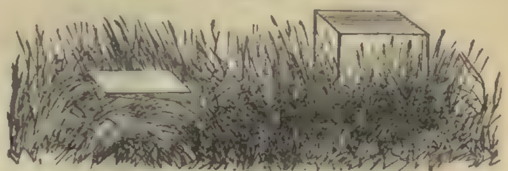
It is a sufficient answer to the proposition that dew falls from the sky, to say that dew never occurs when nights are cloudy; and it is a sufficient answer to the statement that it rises from the ground, to remark, that a slight screen thrown on the ground, or elevated above the ground, is incompatible with the formation of dew.

The theory of dew is easily explained by a consideration of the laws of radiant heat. The starting-point of the investigation is the atmosphere. Now the atmosphere always contains moisture, as we have already explained; and the amount of this moisture will, *cæteris paribus*, be correlative with the degree of atmospheric heat at the time. If, then, the atmosphere being raised to its fullest point of saturation for any given degree of temperature, that temperature should by any chance fall, the result will necessarily be a deposition of moisture. Let us now apply these principles to the conditions of a heat-radiating surface of the earth and a clouded sky. In this case no dew occurs, nor, according to theory, should any occur, inasmuch as the clouds perform the functions of a second radiating surface. The earth radiates heat owing to the clouds; but the clouds, in their turn, radiate heat back again to the earth; whence it follows that the earth practically does not lose heat, and its temperature not falling below the temperature of the circumambient atmosphere, no atmospheric moisture can be deposited; in other words, no dew can occur.

For these facts we are indebted to the late Dr. Wells. They are demonstrated by thousands of natural conditions, and bear the test of any properly-devised experiment.

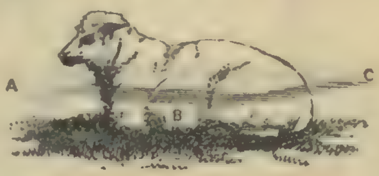
The following diagram (Fig. 41) is intended to show the manner in which a screen will prevent the occurrence of dew. Two plates of glass are represented as supported over an expanse of grass. Underneath the glass plate not the slightest dew will be found, though the grass around will be dewed heavily.

Fig. 41.



A very pretty illustration of the conditions which regulate the formation of dew, will frequently be supplied by a sheep lying down on the grass, on a clear, tranquil, cloudless night, when, to use a popular but incorrect expression, dew is falling; it will be found that the upper part or aspect of the wool of a sheep, is completely drenched with dew, although the under part or aspect of the animal is dry, as represented in the accompanying diagram (Fig. 42).

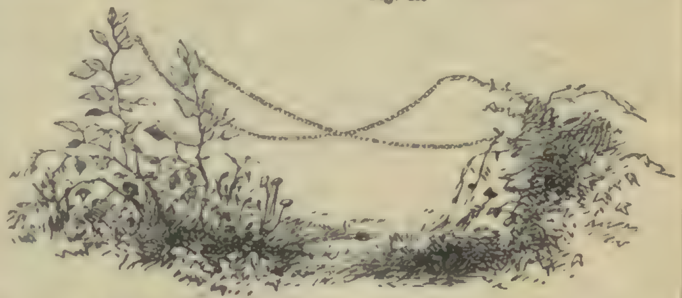
Fig. 42.



The explanation of this phenomenon will be so obvious that no further remark concerning it is necessary.

Consideration of the laws of radiant heat will render manifest the reason wherefore some surfaces are more bedewed than others. The amount of dew will depend, *cæteris paribus*, on two circumstances—firstly, on the kind of surface; and secondly, on its angle of inclination. Reference has already been made to the comparative facility wherewith certain bodies found in nature, favour the deposition of dew upon them; and the most casual observer cannot fail to be struck with the difference. In all cases, the bodies which radiate heat are the most favourable to the deposition of dew upon them. Few, if any, objects, naturally occurring, are so solicitous of dew as spiders' webs; and no object presents the phenomena of dew under a guise so beautiful. Not unfrequently, a thin filament of cobweb, so small that it would be invisible to the naked eye, presents itself to the vision on a dewy morning as if it were strung with little pearls (Figs. 43, 44).

Fig. 43.



That angular inclination of a body should influence, and be intimately connected with, the function of dew formation, directly follows from a consideration of the laws of radiant matter; and it may be readily illustrated by a diagram. Let us suppose that a screen of glass be supported over the radiating surface of the earth—in one case horizontally, in the other case at an angular inclination—as represented by the accompanying diagrams (Figs. 45, 46).

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* The student may refer to the article "Radiation," in the Section on *Heat*, in this volume.—Ed.

Fig. 44.

Fig. 45.

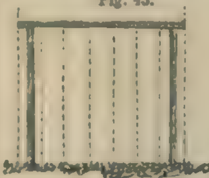
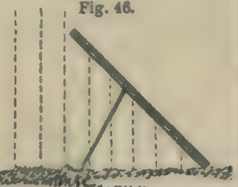


Fig. 46.



It will be evident that the horizontal glass in Fig. 45 will radiate more heat than the diagonal glass in Fig. 46.

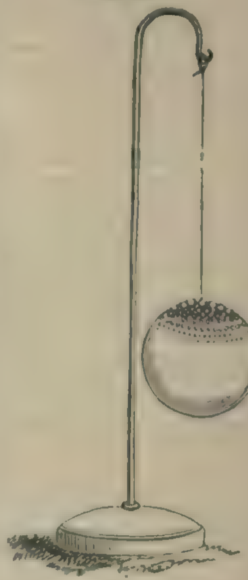
Determination of the Amount of Dew.—Although the laws which regulate the formation of dew are perfectly well known, and a rough method of determining the amount of dew deposition not difficult, yet no correct means of estimating its actual amount has yet been devised. Instruments for determining the amount of dew are called drosometers. A drosometer is a balance, suspended to one arm of which is a plate, to the other a pan containing weights, exactly proportionate to the weight of the plate, so that both may be in equilibrio. Supposing dew to be deposited on the plate, the latter will evidently increase in weight by the amount of such deposition, and a deviation of the beam from the horizontal will ensue. The principle of the instrument is unimpeachable, but in practice it is imperfect. Instead of the plate as described, recourse may be had to a lock of wool or eider-down, or one of a large choice of hygrometric materials. Bodies of this kind were employed by Wells. Wilson and Flangergue had recourse to a plate; but it seems that the materials employed by Dr. Wells should have the preference.

A very instructive experiment relating to a dew formation, and one which may be regarded as presenting a summary of the whole matter, is as follows:—

If, on a clear, cloudless night, when dew is being deposited, a glass ball (Fig. 47) be suspended in the open air, some height from the ground, dew-drops will form on the ball; not, however, equally on all portions of its surface. Its upper aspect will be bedewed, then its sides; but only rarely, and in extreme cases, the inferior aspect of the glass globe becomes covered with dew-drops; and when they do occur, they are very small; indeed, a complete gradation of size is manifest, the dew-drops decreasing in size as the upper aspect of the globe is departed from.

Generalisations.—The following generalisations relating to the phenomena of dew may now be appended; they will, for the most part, be seen to be directly deducible from a consideration of the laws of radiant heat:—Clouds and brisk winds are both inimical to the formation of dew; the former, because of their own radiating power; the latter, because of the removal of cool air, its place being supplied by air already warmed. If the night be cloudy, and the wind still, very little dew results. If clouds and wind occur together, dew is totally absent. Screen-like objects, interposed between the sky and the radiating surface, produce an effect identical with clouds; hence bodies freely exposed to the atmosphere, *cæteris paribus*, are most freely bedewed. Morning and evening are not the times, as commonly supposed, when dew is formed most copiously. It is deposited at all hours of the night, but most copiously rather after midnight. It sometimes occurs even before night, late in the afternoon. Dew is not deposited with equal readiness in all parts of the world, but attains its maximum in warm lands near the margin of the sea, rivers, or lakes; as, for example, near the Red Sea, the Persian Gulf, the coast of Coromandel, at Alexandria, and in Chili. It is quite absent in very arid regions, in the interior of continents; such, for example, as central Brazil, the Sahara, and Nubia; neither does it frequently occur at sea, because of the bad radiating quality of a surface of water.

Fig. 47.



The imperfect radiation of a surface of water is well illustrated by the following striking experiment:—Glass is a good radiating surface; whence a piece of glass freely exposed in an atmosphere when dew is forming, soon becomes covered with dew. If, however, the glass have its surface wetted previously to exposure, instead of becoming more wet it becomes dry, simply because radiation is impeded, and evaporation takes place unchecked. *Cæteris paribus*, the amount of dew produced, will be proportionate to the amount of aqueous vapour present in the atmosphere; and this readily explains the fact, that a copious production of dew is frequently the precursor of rain.

Honey-Dew.—Occasionally a sweet, damp, sticky moisture attaches itself to leaves during the night, and does not disappear throughout the day. The term honey-dew has commonly been applied to it, though, in point of fact, it is not dew at all, being merely an excretion from certain insects termed *aphides*.

Hoar-Frost.—Hoar-frost differs only from dew in the circumstance of temperature. One is deposited, and remains uncongealed; the other, becoming consolidated by the agency of freezing cold, is converted into ice. Every meteorological observer knows that the existence of hoar-frost is held to be indicative of coming rain, and in most instances the opinion is verified. The phenomena of hoar-frost are even more beautiful than the corresponding ones of unfrozen dew. Upon leaves and vegetable stems, the deposition of hoar-frost is particularly beautiful. If hoar-frost be examined microscopically, or sometimes even attentively by the naked eye, a crystalline structure will be evident. The crystals belong to the same crystallographic system (the rhombohedric) as those of snow, but their general appearance is somewhat different.

FOGS.—Fogs may be regarded as clouds, which form close to the earth's surface; hence we might discuss their peculiarities under the general head of Clouds. They are characterised by some peculiarities, chiefly dependent, however, upon the vesicular aqueous vapour of which they are composed, embracing and retaining volatile particles evolved naturally, or from the operations of man. In this way it is well known that London fog is anything but pure aqueous matter. One of its very important constituents is the condensible part of smoke. As regards the production of fog, it is usually referable to one of two circumstances: either non-visible aqueous vapour may be converted into the visible or vesicular form, by decrease of atmospheric temperature immediately, or by the cooling agency of the earth's surface, depressing the temperature of the layer of air next to it, below the dew point. In this country, and in Europe generally, fogs are of most frequent occurrence in spring and autumn. There are regions, however, in which fogs prevail throughout the year. The coasts of California are almost constantly veiled in fog; and the same remark applies, in a minor degree, to the western coast of the American continent, even so far south as Peru. Newfoundland, Nova Scotia, and Hudson's Bay, are all subject to frequent and dense fogs, attributable, in these localities, to the condensation of vapour, which arises from the hot gulf-stream, by contact with neighbouring and colder air. Fogs do not occur so frequently on level plains as in mountainous regions. In Arabia and the arid table-land of Persia they are almost altogether wanting. London and Amsterdam have acquired a somewhat evil character for fogs; but this meteoric condition applies to many other European localities with an almost equal amount of propriety. At Antwerp, fogs are very prevalent; and the navigation of the lower and middle Rhine is sometimes impeded for weeks together by the occurrence of this pest of the sailor. Neither can Paris boast of much immunity from fogs; they are somewhat less dense and less frequent than our own London fogs, it is true, but are, nevertheless, far from contemptible. Amongst the regions which are likely in future to be celebrated for the prevalence of fogs, the Black Sea may be enumerated. Until recent events, that locality was comparatively unknown to us; but during the time of the Crimean war, which necessitated a constant navigation of the Black Sea, the embarrassment of fogs became but too apparent.

Dry Fogs.—Under this name has been described a dull opaque appearance which the atmosphere of certain regions occasionally assumes, deadening the fiery beams of the sun, and dulling that luminary so that he may be looked at without pain by the naked eye, and embarrassing respiration.

The dry fog is most common in certain parts of North America, during the period known as the *Indian Summer.* It also occurs in Germany, and more rarely in England. There can be no doubt that many atmospheric opacities, different in character as well as in cause, have been summarily classed under the denomination of *dry fog.* When the phenomenon occurs locally, it can generally be traced to such causes as the burning of extensive districts of turf or of forests. When its prevalence is more general, the most rational explanation would seem to be that which attributes it to volcanic eruptions. Some meteorologists have invoked electricity as the cause of this phenomenon; but it is not easy to see in what way the assumed cause could produce the effect in question. Electricity has long been to meteorologists what the class *radiata* was to Cuvier; namely, the receptacle for things unknown or unexplained.*

Clouds.—Clouds are perhaps the most beautiful of all aërial phenomena. All the charms of changeful variety of colour, of form, and of motion are theirs; nor is their utility inferior to their beauty. Without clouds there would be neither rain, nor snow, nor hail; the consequences of this deprivation may be anticipated, or they may be readily learned, by turning to the geography of countries where rain, and snow, and hail are unknown. All regions thus circumstanced, provided irrigation be impossible, and the altogether exceptional condition of copious dews be absent, are, despite the most favourable conditions of climate and of soil, barren wastes.

Notwithstanding the thousandfold varieties of clouds—their Protean shapes, their manifold colours, and other distinctions—when the observer comes to regard them with a scrutinising eye, he will not fail to recognise broad distinctions between them, admitting of being made the basis of a philosophic classification. Thus, some clouds are devoid of outline, their edges merging away into circumambient air; some are black and massive, almost conveying the idea of a hard substance; some are white and fleecy; others extended like a pennon. All these are forms of cloud which present manifest distinctions amongst themselves. Mr. Howard was the first who effected a regular classification of clouds. This classification is now generally adopted. We shall, therefore, present the reader with an outline of his system. According to this meteorologist, there are three elementary and four secondary forms of cloud.

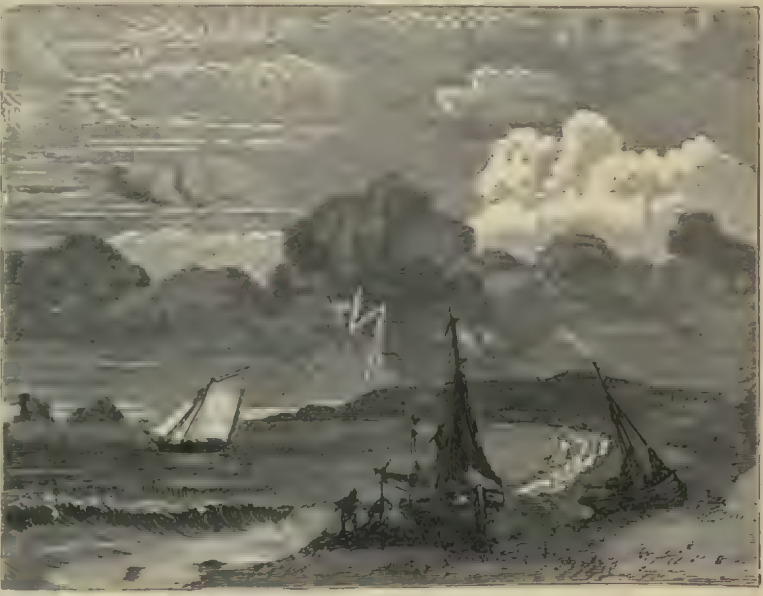
Primary Forms.—The first primary form is the *cirrus*, consisting of feathery expansions, and which is only seen in clear weather.

The second primary form of cloud is the *cumulus*, composed of large hemispheroidal masses, superiorly, and apparently resting below on a horizontal base. This form of cloud chiefly occurs in summer.

The third primary form of cloud is *stratus*, composed of horizontal layers, the smaller layers being underneath. It is this form of cloud, more than any other, which presents itself under a variety of beautiful colours. It chiefly appears at sunset.

Secondary Forms.—The secondary forms of clouds are —(1) *Cirro cumulus.* It is a mixture, as its name indicates, of cirrus with cumulus, and is made of an aggregation of small white clouds, which have been compared to a flock of sheep. (2) *Cirro stratus.* A compound cloud, which is formed of the two primary clouds embodied in its name. (3) *Cumulostratus.* This compound cloud chiefly appears towards night, in dry windy weather, and is of a leaden colour. (4) *Nimbus*, or *rain cloud.* This cloud is seen in greatest perfection during a thunder-storm. All the varieties of clouds described are represented in the above diagram (Fig. 48).

Fig. 48.



It will be readily anticipated that clouds are frequently so mingled and confounded, that they are not always susceptible of the precise classification just announced; nevertheless, a prevalence of one type of cloud over another will be generally seen to prevail.

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* Some have broached the curious theory that such fogs are caused by the immersion of the earth in the tail of a comet.—Ed.

Relation between Clouds and the Weather.—People who are in the habit of narrowly studying the phenomena of clouds, are enabled to draw conclusions of much accuracy respecting the coming weather. Thus cirro cumuli are for the most part indicative of serene, fair weather; the prevalence of wind subsequently to the appearance of much-extended and highly-coloured stratus clouds is a matter of popular experience. The appearance of nimbus clouds proclaims the advent of rain; and the cirro stratus, which sometimes colours the sky as with a veil, all well-defined form being absent, is almost a sure forerunner of bad weather.

Composition of Clouds.—That clouds are composed of water in some condition does not require to be demonstrated; but some explanation must be given of the circumstances enabling water, a material so much heavier than atmospheric air, to remain suspended frequently in very elevated regions, where the atmosphere, thin though it be on the earth's surface, is still more attenuated. This is a matter which, it must be confessed, is still veiled in considerable obscurity; but, perhaps, the most rational explanation of cloud formation is the following:—Firstly, aqueous vapour is diffused, or rather absorbed, invisibly throughout the air. The laws of the absorption or diffusion are perfectly well known. The amount differs, as we have already seen, for different temperatures, being proportionate to the temperature. Assuming, then, that the upper regions of the atmosphere at any time are saturated with atmospheric moisture in its invisible condition, let us now contemplate the effect of cooling that atmosphere by any cause. It is not difficult to furnish reasons for these cooling agencies; they are numerous and varied—such as sudden variations of electric condition, sudden variations in the direction of winds. If, then, from any cause, the atmosphere is cooled below its capacity for holding vapour in the invisible form, aqueous deposition occurs. If this deposition take place on the earth's surface, the result is dew; if aloft in the air, we have a cloud.

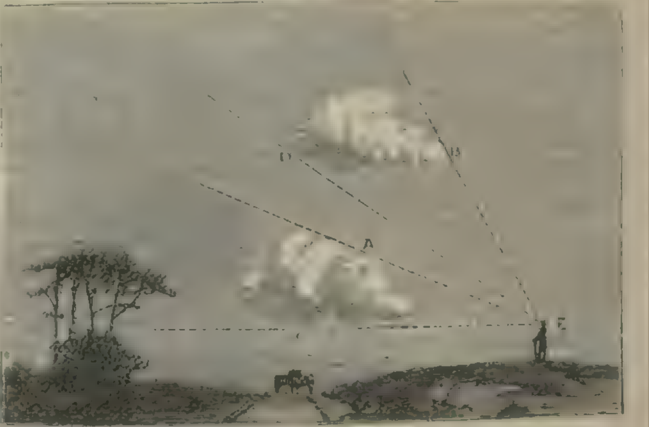
Thus far the steps of each succeeding change are evident; but the remaining points of cloud-formation are more obscure, the circumstances of chief difficulty being to find an explanation of the aërial permanence of clouds, seeing that the material of which they are composed is so much heavier than air. Probably the most consistent explanation is this:—Atmospheric moisture, when it changes from the invisible to the visible form, assumes the physical condition of spheroids or vesicles—minute bubbles of water, in point of fact, each bubble filled with air. If these vesicles be exposed to the sun's rays, it is evident, from consideration of known laws, that they must become specifically lighter than the surrounding medium; and thus affected, they would float, for the same reason that a soap-bubble floats whilst it is yet warm, notwithstanding that the air, which it contains, is heavier than the surrounding atmosphere. We seem, therefore, in a position enabling us to account for the upper part of cloud strata. Directing our attention now to the lower part of these strata, it seems rational to assume that the vesicles of which they are composed should descend. Probably they do so; continuing their descent until they come in contact with an atmosphere sufficiently warm to dissolve their aqueous coating, and convert their water once more into the vaporous or invisible form. Thus it may be, and most probably is, that a cloud, which looks permanent to the eye, is really exposed to the continued operation of resolution and reformation: or, rather, that the two opposing causes, which are here spoken of as producing active changes, balance themselves, and give rise to a condition of equipoise.

Although it has hitherto been taken for granted that clouds are formed of unfrozen water, we know that such is not invariably the case. If the temperature of a nimbus cloud sink to freezing point, or 32° F., its contents freeze, and snow is the result. Many philosophers, indeed, but more especially Kaemtz, are of opinion that the cirrus—the cloud which soars in the highest regions, frequently at an elevation not less than 30,000 feet above the earth's surface—consists of particles of snow or ice. Assuming this to be the case, it is not easy to advance the reason of such molecules remaining aloft in a medium so attenuated as is atmospheric air in a position so elevated.

Position of Clouds.—It is somewhat remarkable that every known form of cloud assumes more or less the horizontal position. Vertical clouds are very rare, and, if we choose to except water-spouts, not recognising them as clouds, perhaps we may say unknown. The horizontality of clouds, warrants their being spoken of as composed of strata; generally, indeed, these strata are very well defined. MM. Paytier and Hoffard have carefully examined the thickness of these cloud-strata on the Pyrenees, and have found their average variation to be between the limits of 3,400 and 1,000 feet. Mr. Glaisher, during his balloon ascents in 1862-'63, passed through clouds of about 8,000 feet thickness. In the fifth chapter of this section, the reader will find the observations of that meteorologist fully detailed.

Height of Clouds.—Several meteorologists, amongst whom Riccioli, Wrede, Kaemtz, and Arago must be particularly mentioned, have set themselves to the task of discovering the height of clouds. The methods by which these investigations were made have been various. Riccioli determined their height by placing two observers a certain and known distance apart; Wrede by making use of their shadows, and then reducing the computation of height to the solution of a problem in trigonometry. Riccioli stated the maximum height of clouds to be 25 000 feet; and Lambert gave their minimum height at 13,000: whilst their maximum height, according to the same, is from 15,000 to 20,000 feet. Gay Lussac, when he acquired in his balloon an elevation of 21,600 feet, perceived small clouds floating much above him. Kaemtz believed that the usual range of cumulus runs from 3.000 to 10,000 feet; of cirrus, from 10,000 to 24,000 feet; of nimbus, or thunder-cloud, between 1,500 and 5,000 feet. That very accurate physicist, Pouillet, as the result of certain experiments performed in 1840, states that he has proved the existence of clouds at an elevation from about 22,300 to 38,000 feet: and Mr. Glaisher, in 1862-'63 observed them at those heights.

Fig. 49.

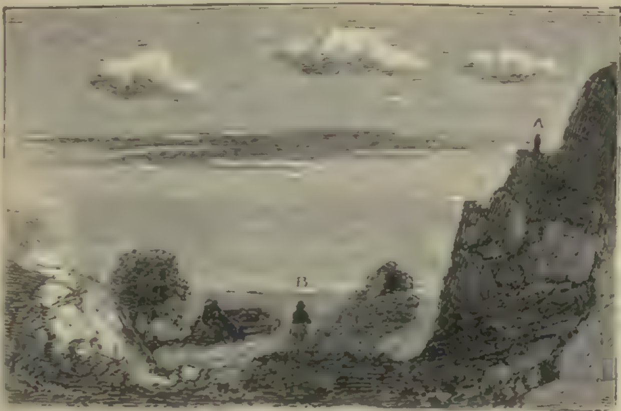


As regards the elevation of clouds, we may arrive at the conclusion, that cirrus does not descend below 2,000 or 3 000 feet; whilst nimbus occasionally descends so low that it approaches the earth within a few hundred feet of surface. The maximum mean elevation of clouds seems to be in low latitudes, evidently on account of the greater capacity of the atmosphere to absorb and dissolve aqueous vapour. It should here be remarked that the elevation of a cloud cannot be determined by reference to its apparent place in the sky; and, except the distance be known, neither can its actual size. These remarks are illustrated by the above diagram (Fig. 49), where the same cloud will be seen, at different times, under very different angles by the same observer, as reference to the angles A E C and B E D will testify, whence the height would differ for two different statures.

And again, as in Fig. 50, it is demonstrated that the observer at A will see clouds which are quite invisible to another observer at B.

RAIN.—When, from any cause, cloud-vesicles aggregate into drops, and these drops fall, the result is rain. Some years ago, Mr. Green, the aeronaut, stated that rain only fell when the cloud producing it was covered by another floating directly over it. This seems to be confirmed by Mr. Glaisher, during his balloon ascents of 1862–'63. Still, the phenomenon of rain without clouds is well attested.

Fig. 50.



The amount of rain which falls is measured by a pluviameter, or rain-gauge. The principles on which this instrument is founded are of the simplest kind. If, for example, a cup or basin, of known cubic capacity and known orifice, be exposed so that it may receive the fallen drops of rain, the cup or basin would be a rough rain-gauge. Were it not that the collected water thus exposed would be continually evaporating, thus apparently diminishing the total fall of rain, no better rain-gauge need be desired; but in practice it is necessary to provide against such evaporation; and it is usually accomplished by forming an instrument of such construction, that one part may be destined to collect the fallen rain, and another part to diminish, within the smallest limits, the amount of evaporation. In expressing the amount of rain which falls at any particular spot, it is necessary to express the height also at which the observation is made. The annual fall of rain increases as land acquires elevation; nevertheless, at one and the same place, the amount of rain decreases with elevation, for the reason that each drop of rain throughout its descent goes on collecting moisture, and becomes larger. Dalton appears to have been the first to notice this fact. He proved, on comparing two sets of observations—one set made at the base, the other at the summit, of a high tower—that the amount of rain at the top to that at the bottom was as two to three. Similar observations made at the Paris Observatory have led to similar results. This variation between the amount of rain at different elevations in one locality, is the more considerable as the point of aërial saturation is more nearly attained; for which reason, it is less in summer than in winter. The heaviest rains usually occur in the tropics, and during the hot season. There the fall of rain is enormous, sometimes amounting to an inch per hour; nay, Humboldt has related, that in South America, no less than five inches of rain fell in one hour. In these islands, we can hardly say that any one season merits the designation of the rainy season; but in tropical regions, except the belt of calms, and in the sub-tropical regions, the separation between the dry and rainy seasons is well marked. In the continental portions of the torrid zone, the rainy season sets in when the summer heat attains its maximum, and continues during four or five months, the atmosphere being clear and bright throughout the remaining portion of the year. Near the equator there are two wet seasons, sometimes separated from each other by a totally rainless period; but at other times demarcated only by periods of maximum and minimum fall of rain. Dutch Guiana furnishes the well-known illustration of a country having two well-marked rainy seasons; one, and that the chief, commences in April, and lasts till June; the other, or minor rainy season, commencing in the middle of December, lasts till the middle of February. The drops of tropical rain attain a magnitude never seen in the tamer showers of these northern regions; their weight is so considerable, and the force with which they descend so great, that their splash or stroke leaves a smarting sensation on the skin. The region situated between the influence of the two trade winds, and commonly known as the region of calms, is devoid of periodic rains, although the fall of rain there is frequent and heavy.

Rainless Portions of the Earth.—There are some localities in which rain never occurs: for example, Egypt, the Desert of Sahara, the table-lands of Persia and Mongolia, the rocky flat of Arabia Petræa, &c. Rain is generally the most abundant near mountain ranges; but there are exceptions, one of the most remarkable of which is presented by the part of Spain south of the Sierra Nevada.

Condition of Europe with regard to Rain.—Europe, considered in relation to the prevalence of rain, admits of being divided into three districts—the South European, the Middle European, and the Swedish. In Portugal, and the larger portion of southern and central Spain, there is an almost total absence of rain during summer; but north of the Pyrenees, rain occurs at variable times throughout the whole year. All the portions of Europe, north of the Alps and Pyrenees, are subject to the Middle European and the Swedish pluvial conditions. The characteristic of the Middle European climate, as regards rain, is, that the latter chiefly occurs during westerly winds; whereas the Swedish climate is characterised by the prevalence of rain during both easterly winds and westerly winds, which bring rain to the whole of Central Europe, and deluge our isles with wet, leaving the bulk of their moisture by the mountainous Scandinavian range, which separates Norway from Sweden. St. Petersburg and Moscow cannot be said to belong either to the Central or Northern European climate; these places lie on the confines of both; hence neither westerly nor easterly winds are there prevalent. In England, the maximum number of rainy days throughout the year occurs in Cornwall and Devonshire; passing thence east into Central Europe, the total number of rainy days per annum continually declines. If we assume the annual amount of rain which falls at St. Petersburg to be nearly three, the corresponding annual amount for the West of England will be 2·1;[*] in Central England, 1·4; in Central Germany, 1·2. This statement assumes an average of some special localities to have been taken into consideration: special places present many deviations. In describing any place as subject to rain, or *rainy*, distinction must be made between the actual quantity of rain per annum which falls, and the total average number of rainy days. Understanding by the latter term every day on which rain, much or little, falls, the number of rainy days increases in Europe from south to north. The mean average for Southern Europe may be taken as 120, in Central Europe as 146, and in Northern Europe as 180. The following statement indicates the total number of rainy days per annum for a few places specifically named:—

Buda	112	Ratisbon	115
Warsaw	138	Rotterdam	137
Germany, average of	150	Paris	160
Carlsruhe	174	Poictiers	99
Tagernsee	170	St. Petersburg . .	168
Munich	149	Moscow	205
Stuttgardt	127		

Annual Distribution of Rain.—The time of year at which rain is most prevalent, is subject to much variation for different countries. Throughout Central Europe rains are most prevalent in summer; but in Southern Europe the preponderance is on the side of winter rains. Norway is subject to copious winter rains; whilst in Sweden they are almost entirely wanting. Sweden, in point of fact, although placed so near the sea, has a climate altogether like that prevalent in continental regions The reason why Norway is subject to winter rains, and Sweden is deprived of the same, hinges upon the explanation already made, that western winds (which predominate in Scandinavia during winter) lose their moisture in passing over the Scandinavian range. Although summer rains are in many places rare, yet when they occur they are generally more copious than rains at any other period.

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[*] The position of our Western Coasts, in reference to the Gulf Stream, which washes them, must be borne in mind.—ED.

Rain which falls in summer at different places, taking the rain which falls on a winter's day at the corresponding place as unity—is in,

England	1·07
Western France	1·03
Central France	1·57
Germany	1·76
St. Petersburg	2·17

From which statement it appears that the prevalence of summer rain increases towards the east.

Peculiarities of Rain-Water.—Fresh-falling rain-water, collected far from towns or other sources of local contamination, is very nearly pure; nevertheless, modern chemical observation has succeeded in discovering the presence of many substances, in small quantities. Nitric acid and nitrate of ammonia are by no means unusual constituents; and iodine has been frequently recognised. As regards the sources of these and other extraneous bodies, much still remains to be discovered. Nitric acid is most probably formed in the atmosphere by the agency of electricity; and the ammonia may be referable to exhalations from decomposing matters on the earth's surface. Many of the extraneous bodies, especially salts, are unquestionably due to the action of winds upon finely-divided ocean-spray.

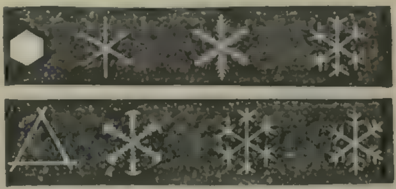
Showers of Fishes, Stones, &c.—Instances are on record of whole shoals of fishes, and numerous collections of other animals—also stones, &c.—being cast on the earth by showers. At one time these phenomena were regarded mysteriously, and referred to occult causes. At present they are deprived of their mystery, and referred to the previous elevation of the fishes, &c., by aërial currents, whirlwinds, and water-spouts.

SNOW.—If the temperature of a cloud should fall at any time to 32° F., or lower, instead of rain the result is snow. Much that is beautiful and beneficent is seen in this divided form of frozen water. In our own temperate clime we do not comprehend, except by reflection, the true value of snow in the economy of nature. Its fall amongst us is uncertain and exceptional; we know not when it is to come, or how long it is to remain. We therefore make no provision for it—regard it as a condition to be tolerated—regret that it interferes with our locomotion—that it impedes our railway trains, and wets our feet, and wish it away. Nevertheless, even in these isles, the farmer, from experience, is not insensible to the value of snow. He says it keeps his winter crops *warm*; and the thoughtless passer-by, wrapped in his own self-conceit, laughs at him for making a statement so apparently grotesque. The philosopher, however, who is aware of the low heat-conducting power of snow, and who can appreciate the evil consequences of frost on vegetation, endorses the farmer's statement.

If we would desire to recognise the full benefits of snow, we must direct our attention to northern climes—to Sweden, Russia, and Canada. There, the advent of snow is looked forward to as a blessing; and when it comes, the period of its duration admits of being predicted with tolerable accuracy. No sooner is the ground covered with sufficient snow, than wheeled carriages, which but yesterday were sticking up to the axletree in mud and wet, are put aside, and sledges supplied in their stead. Market-places, which, before the snow had fallen, were naked and unworthy, now teem with good things brought from hundreds of miles away. Snow has, all at once, laid down a far-stretching track, over which the sledges glide almost with the ease and velocity of a railway train.

Form of Snow-Flakes.—In certain conditions of temperature snow falls as a pulverulent body; in other conditions as a flaky amorphous mass; but if very dry snow be microscopically examined before it has been broken up, indications of crystalline structure will be recognisable. Sometimes these crystalline snow-flakes attain such large dimensions, that they are quite evident to the naked eye. The crystalline forms thus developed are numerous, but they are all referable to one crystalline system, the *rhombohedric* or *rhombohedral*; the characteristic of which is, that crystals belonging to it have three axes crossing each other at the angle of sixty degrees, and one axis at right angles to these. Scoresby, who has minutely examined these snow-flakes describes five principal forms of snow-crystals:—1st, crystals having the form of thin plates, which are the most abundant; 2nd, surfaces or spherical nuclei, with ramifying branches in different planes; 3rd, fine points, or six-sided prisms; 4th, six-sided pyramids. The latter form is the least frequent of all. The above diagram (Fig. 51) represents the principal crystalline varieties of snow-flakes.

Fig. 51.



The Snow-Line.—Inasmuch as the upper regions of the atmosphere are intensely cold, there is an elevation for every latitude at which atmospheric moisture is changed into snow. This elevation corresponds with what is termed the *snow-line*. At the equator, the snow-line is elevated from 12,000 to 15,000 feet above the sea-level. As we proceed towards the north, the elevation of the snow-line will evidently be lower. Snow does not fall on level ground in Europe, farther south than Central Italy; but in Asia and America the region extends nearer to the equator. Through Florence, passes the isothermal line of 59° F., and it may be regarded as the southern limit of the region, in which snow falls on level places. Snow does not usually fall at the time of maximum cold; some meteorologists say it never does—but this is an error. After snow has fallen, the weather generally increases in severity. We are usually in the habit of assuming that the total quantity of snow which falls, increases as we reach either pole—an assumption, however, which is only correct within certain limits. Thus, taking the northern hemisphere, for instance, the fall of snow increases from the isothermal of 59° F., to the isothermal of 41° F., which latter cuts the town of Drontheim, in Norway. Passing still further north, the quantity of snow goes on diminishing, evidently because, in the polar regions, the temperature of the air is too cold to retain much moisture, and atmospheric moisture must necessarily be the antecedent to either rain or snow.

The atmospheric condition during the fall of snow, may vary from the limits of almost complete tranquillity, to the other extreme of most violent perturbations. In Germany, and other countries having a corresponding latitude, the fall of snow is usually tranquil, except during the months of February and March. In high latitudes, snow usually occurs during violent tempest-gusts, almost equal, sometimes, to the West Indian hurricane, or the Chinese typhoon. In Norway, these storms are very frequent, also in Kamtschatka; in which latter region they are called *purga*. They are veritable thunder-storms, as is completely proved by the intense electrical condition of the atmosphere. On mountainous elevations, snow-storms are commonly prevalent, irrespective of latitude.

Coloured Snow.—Red and green snow have been frequently described by travellers. The cause of these phenomena is now referred to the presence of minute algæ—the *protococcus nivalis* in the so-called red snow, and the *protococcus viridis* in the green variety.

Hail.—Frequently, cloud vesicles become aggregated and frozen into lumps of various sizes and shapes, sometimes opaque, sometimes transparent, and occasionally, though not very often, containing neuclei of solid foreign matters. Meteorologically, aggregations of this kind constitute hail. In most parts of the world where rain occurs, hail is known; but certain localities are particularly subject to hail-storms. Generally hail falls by day; indeed an opinion prevails that hail-storms are unknown by night. This supposition is, however, erroneous. The form of hail-stones is various, though for the most part they assume a spherical, spheroidal, paraboloidal, or pyriform contour; and still more frequently they are rounded, flattened, or angular. According to Delcross, the most common form of hail is that of a three-sided spherical segment, resulting from the comminution of larger spheres.

The diameter of hail-stones at a mean latitude is, according to Muncke, not usually greater than one and a-half, or one and three-fourths of an inch, although, on some occasions, blocks of ice of enormous dimensions have fallen. For example, in 1719, there fell at Kremo, hail-stones weighing not less than six pounds; and at Namur, in 1717, others weighing not less than eight pounds. Again, it is stated that, in 1680, masses of ice fell in the Orkneys, twelve inches thick; and in 1795, hail-stones fell in New Holland, from six to eight inches long, and two fingers thick. It is recorded, but on doubtful testimony, that there fell in Hungary, on May 28th, 1802, a piece of ice three feet square by two feet thick, and the weight of which was 1,100 pounds. But even this is much exceeded by a statement that, in the latter part of the reign of Tippoo Saib, a lump of ice fell at Seringapatam as large as an elephant. The *size* of the elephant, however, is not mentioned.

With regard to foreign substances existing in hail-stones, they are described as various. In 1755, there fell, in Iceland, hail-stones containing sand and volcanic ashes; others which fell in Ireland, in 1821, contained a metallic nucleus, which proved, on analysis, to be iron pyrites (sulphide of iron); a similar phenomenon occurred in Siberia in the year 1824. The presence of small pieces of straw in hail has been frequently demonstrated.

The largest hail-stones fall in summer, during thunder-storms. Storms of this kind are most frequent in June and July; they are more rare in May, August, and September, and still more so in April and October. They usually occur at the close of long periods of calm, sultry weather. Hail-clouds are much lower in the sky than rain clouds; and are generally recognisable by a peculiar ragged or jagged contour, and by their lower portions being marked with white streaks, the other portions of the cloud being inky black. Previous to the occurrence of hail, the barometer sinks very low; and, what is unusual before rain, the thermometric column suffers a corresponding depression. The thermometer, during a hail-storm, has even been known to sink through 77° F. A peculiar rustling sound in the air is also indicative of speedy hail, accompanied by a darkness resembling that dependent on an eclipse of the sun. Hail-storms are very seldom of long duration, usually lasting a few moments only—seldom longer than a quarter of an hour. The rapidity with which hail-storms travel is very great: one which occurred in Central France, in 1788, travelled at the rate of forty miles an hour. The force of hail-stones is sometimes dangerously great, not only breaking windows and shattering tiles, but killing men and animals, cutting off branches of trees and herbage, and, in short, desolating all save the largest vegetable growths. The hail-storm in France of 1788, already adverted to, extended its devastations over 1,039 parishes, destroying property to the amount of 25,000,000 of francs. Although hail-storms often extend very far in a linear direction, their breadth is usually inconsiderable—often but a few hundred, or at most a few thousand feet; while the linear extension has been known to exceed four hundred miles.

It has been already mentioned, that wherever rain-clouds rest, hail may occur; nevertheless, latitude and local conditions determine the frequency of the phenomenon. Rain seldom occurs on the level land of tropical countries; and it is rare in the extreme north. The hail-belt, pre-eminently so considered, is comprehended between 30° and 60°, and to elevations less than 6,000 feet. Even within this belt, and below the limit of elevation just assigned, there are certain localities where the occurrence of hail is a very rare phenomenon. Certain of the Swiss valleys may be cited as a well-known illustration of this remark; more especially in the Valais and its allied dales. It has also been well determined that hail more rarely occurs at the base of mountains than in localities a short distance removed. Perhaps no country, upon the whole, is more subject than France to the ravages of hail-storms, and in no country are the effects more serious. It has been ascertained that the average annual number of hail-storms in France is about fifteen. They are especially prejudicial to the vine and the olive, sometimes laying whole districts under desolation. Having regard to the highly excited electrical condition of the atmosphere, as the rule, during the occurrence of hail-storms, great hopes were once entertained that they might be prevented, or their ravaging power diminished, by means of suspended conductors. The idea of using such conductors appears to have been first suggested by Guenaut de Montbeillard, in 1776; and hail-conductors have been extensively tried, but hitherto without any amount of practical benefit to justify their longer continuance. In 1820, a peculiar kind of hail-preventer was suggested by La Postolle, and subsequently by Thollard. The instruments consisted of straw ropes, in which a metallic wire was interwoven, and suspended by means of pointed rods similar to lightning-conductors; but, like instruments having a similar object, and which preceded them, they were found to be unavailing.

Methods of Determining the Amount of Atmospheric Moisture.—Having described the numerous forms under which aqueous moisture may exist in the atmosphere, it now remains to indicate and describe certain instruments which have been devised by different experimenters for determining its amount. These instruments, founded on different principles, as will be seen, are termed hygrometers.

It is a matter of common experience that many bodies are affected as regards their dimensions, more particularly their linear dimensions, by mutations of atmospheric moisture. Wood is a very common example of this property; more particularly a stick of wood cut transversely to the grain. Founded on this property of wood, the late Mr. Edgeworth constructed a very ingenious toy, which, though not a hygrometer, inasmuch as it does not measure the amount of atmosphere prevalent in the air, is at any rate a *hygroscope.* It is related, that the somewhat eccentric philosopher, just named, once laid a wager that a certain toy—a wooden horse—constructed by himself, should, after the lapse of some time, walk across his room. The horse was accordingly made, and placed at one end of a chamber; the door of the chamber was then locked, and the key deposited in safe keeping. In process of time the horse did indeed arrive at the other end of the room; and the manner in which this was accomplished will now be made evident.

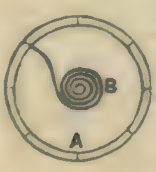
Underneath each hoof was a claw, long enough to stick into the flooring, and there take hold. The horse itself was made out of a piece of wood cut transversely to the grain; the consequence was, that when the weather was dry the linear dimensions of his back contracted, and when the weather was wet his back again elongated. Now, bearing in mind the construction of the feet of this toy-horse, it is evident that these alternate contractions and expansions must necessarily result in a forward motion.

Again, the condition of human hair illustrates the effect of varying amounts of moisture in the atmosphere. Every lady knows that she cannot retain her hair in curl during wet weather, so well as when the weather is dry, because of the moisture present, which causes the hair spirals to relax and unfold. In point of fact, each hair contracts and elongates alternately, by every mutation of dryness and moisture; so that if only the exact ratio of contraction and expansion could be determined and applied, the meteorologist might construct a hygrometer, having for its basis of actuation, a human hair. The hair-hygrometer of Saussure takes advantage of this principle.

To construct this hygrometer, a soft human hair is boiled for a short time in a solution of sulphate of soda; afterwards for a few minutes in pure water; it is then well washed to free it from all adhering salt, and dried in a shady place. Next, one extremity of the hair is fastened to the extremity of a little tongue, and the other end is wound round a small pulley having two grooves. The second groove is for the purpose of retaining a filament of silk, from which a weight is suspended for the purpose of retaining the hair in a constant state of tension. To the pulley is fastened an index, traversing a graduated arc, whenever the pulley turns in any direction by the contraction or elongation of the hair. The graduation of the instrument is thus effected:—It is placed in a receiver holding chloride of calcium or concentrated oil of vitriol; the air is exhausted from the receiver, and the place where the index then stands is marked. This mark corresponds with the point of greatest dryness, or 0 of the scale. We have next to determine the point of greatest saturation, which is thus effected:—The instrument is next placed in a receiver containing a dish of water; so that as the hair elongates the index turns, and finally coming to rest, the point at which it stands is marked. This mark corresponds to the maximum of moisture, which of course may be indicated by any number arbitrarily selected; this number being usually 100. Finally, it remains to divide the interval between the two extremes into 100 equal parts, and the instrument is complete. Notwithstanding all the care which may be devoted to the construction of this instrument, it is, after all, scarcely deserving the name of a hygrometer—it is little better than a hygroscope.

Occasionally, certain helical vegetable fibres have been used as hygrometers, in a way which the accompanying diagram will render manifest.

Fig. 52.



Let A (Fig. 52) represent a circular card or other flat disc, and B a vegetable filament helically coiled—it is evident that the helix will unravel in a damp, and tighten its coil in a dry, atmosphere. An instrument of this kind has been sometimes employed for the purpose of determining whether a bed be moist or dry. The instrument is a very good hygroscope; but, inasmuch as the coiling and uncoiling of the helix is most comparable with equal arcs, the instrument can hardly be termed a hygrometer.

We are under obligations to the ingenuity of the Dutch for another hygrometer, or rather *hygroscope*—for, like the instrument just described, it is not a true indicator of the *quantity* of moisture present in the atmosphere. The instrument is of this kind:—A piece of catgut is suspended from one extremity, and to the other, or lower extremity, is fixed, transversely, a little horizontal bar. On one extremity of the bar, a lady, in gay summer attire, is represented; on the other a man, dressed appropriately for a rainy day: finally, the catgut and its toy-appendages are surrounded with a case, having two openings, and in such a fashion that only one of the toy-images can be visible at a time. Now, it is evident that the fibres of the suspended catgut will partially untwist under the influence of moisture, and re-twist as the atmosphere becomes dry; whence it follows that the lady will appear under the latter circumstances, and the man under the former. This instrument, though something more than ingenious—for it is a good *hygroscope*—does not merit the dignified term of *hygrometer*.

The Dew-point Hygrometer of Daniell.—If a wine-bottle be taken from its bin, it will frequently be found covered with moisture; and in proportion as the air is saturated with moisture, so will the depression of temperature be at which this moisture begins to be deposited on the bottle. In this manner, if we had the means of regulating the temperature of the bottle at our will, depressing it at pleasure, we might ascertain the exact temperature at which moisture would begin to be deposited; and thus, noticing the variations of temperature at different times, we might establish and tabulate a correspondence between each particular temperature at which moisture was deposited, and the corresponding amount of moisture contained in the atmosphere. Now, the degree of temperature at which moisture begins to be deposited in this way, is called the *dew-point*; and hence the propriety of the appellation, *dew-point hygrometer*, which has been given to the instrument presently to be described. It consists of a doubly-bent, exhausted glass tube, each end terminating in a bulb. One bulb is covered with a coating of thin gold or platinum foil, the other with a fine linen rag. The former bulb is partially filled with ether, and holds a small thermometer, the graduated portion of which passes up the tube. If ether be dropped on the second bulb, evaporation rapidly ensues, and the bulb is cooled, thereby condensing the vapour of ether which it contains, and permitting a new evolution from the ether in the bulb. This evolution of ether cools the bulb, and causes dew to be deposited on its surface. The inclosed scale indicates the dew-point. The following reasoning explains how the determination of the dew-point can indicate the amount of aqueous vapour in the atmosphere. In proportion as the temperature of the air is elevated, will it be capable of holding more moisture. Hence, by cooling the air, its power of holding moisture is diminished; a portion of moisture, therefore, becomes condensed in the form of dew-vesicles. The greater the moisture contained in air, the more readily will condensation ensue for a given reduction of temperature.

The dew-point hygrometer of Daniell, though a great advance upon the rude instruments just described, is attended with some imperfections. Its construction has been improved upon by Dobereiner and by Regnault; but the instrument, in its most perfect form, still leaves much to be desired. Not only does its employment necessitate the use of a large amount of ether; but, what is of more consequence, when the weather is extremely dry, the deposition only takes place with great difficulty. A far more effective instrument, though based on different conditions, is that now to be described.

The Psychrometer.—The psychrometer consists of two thermometers mounted on the same frame, the bulb of one thermometer being naked, whilst the bulb of the other is enveloped in muslin or other similar absorbent texture, from which there extends a wick-like absorbent stem, terminating in a cistern of water. From a consideration of the structure of this compound instrument, it will be evident that the mercurial column of the naked or uncoated bulb will stand higher than the mercurial column of the second or wetted bulb. The reason of this is obvious. The process of evaporation lowers the temperature: and it follows that, under one condition, and *only one*, can the readings of the pair of thermometers which constitute the psychrometer correspond—namely, when the atmosphere is saturated with moisture, to such an extent that it is unable to take up more. By an extension of this reasoning it will be now evident that the mercurial readings of the pair of thermometers will continually vary, according to the amount of dryness or moisture of the surrounding atmosphere. The variation, in point of fact, is in an inverse ratio to the amount of moisture; so that by means of formulæ we can easily connect the indications of the psychrometer with the dew-point.

Diurnal Variation of Atmospheric Moisture.—The amount of moisture present in air varies at different times of the day. There appear to be two maxima and two minima. The first maximum occurs about 9 A.M., the second at 9 P.M.; the first minimum shortly before sunset, the second about 4 A.M. Popularly, the air is said to be most damp at sunrise; and in the sense of dew or palpable moisture, the popular expression is correct; but, provided the air be hot enough, we have already seen that it can absorb large quantities of moisture, retain it invisibly, and impart no sensation of moisture; indeed, pure steam is no more wet than a pure gas.

Monthly Variation of Atmospheric Moisture.—The fact needs no comment, that all months throughout the year are not equally moist. It appears that at London, Paris, Geneva, and Great St. Bernard, the absolute amount of vapour in these places attains its maximum in January, and its minimum at the end of July or the beginning of August; but the relative moisture is greatest at London, Paris, and Geneva, in December, and least in May.

Like the true aërial atmosphere, atmospheric aqueous vapour continually varies as to its amount of tension or elasticity. According to Dove, the amount of tension is less during north and south winds than during eastern and western winds, an observation which has also been confirmed by Kaemtz. Necessarily, too, the direction whence the wind blows must influence the quantity present of aërial vapour. North and north-east winds are, at least in these latitudes, less moist than winds blowing from the opposite direction. Mr. Glaisher found, during his balloon ascents in 1862, that the aqueous vapour rapidly decreased as increased elevation was attained. (See page 1174).

Inasmuch as the aqueous vapour dissolved invisibly in air, assumes the condition of vapour whenever the air is cooled below the dew-point, the influence which mountain ranges exercise in robbing winds of their moisture and producing rain, will be readily evident. By an extension of the same reasoning, it is evident that winds which have reached into continents far distant from the ocean, and lost considerable bulks of water, must be necessarily dry.

CHAPTER III.

ATMOSPHERIC PHENOMENA—(*Continued*).

PHENOMENA OF ATMOSPHERIC REFRACTION. — The amount of atmospheric refraction is due to inequality of the density of the air. But inequality of density, and therefore inequality of refractive power, may be the result of varying amounts of expansion, referable to the operation of varying degrees of heat; and thus arise what may be termed the abnormal effects of atmospheric refraction.

Every one must have noticed the peculiar, tremulous condition of the air in summer-time over an ignited brick-kiln, or near a red-hot bar of metal, or even on the surface of the ground, provided the weather be sufficiently hot. This tremulous appearance is referable primarily to the expansion of the air near a hot surface, and immediately to the diminished refrangibility attendant on such expansion. These local sources of heat set up local currents, each being composed of air of a different density from that of neighbouring currents, whence each has a different refractive power. That which an ignited brick-kiln, or a glowing metal-bar, can accomplish on the small scale, is accomplished on a larger scale by many natural causes, giving rise to phenomena both striking and delusive. Pictures of ships and towns inverted, the vain semblance of lakes of water in the midst of burning sands where no water really exists, aërial cities, spectral forms of men and animals—all these, and many more—are the phenomena of atmospheric refraction and reflection.

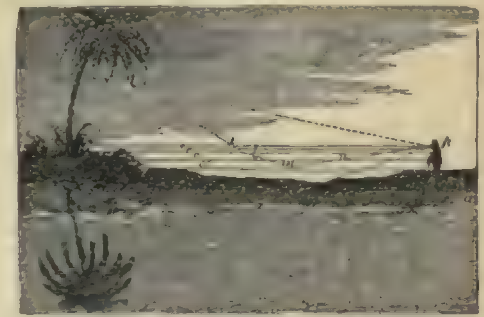
One of the most common effects of irregular atmospheric refraction is the twinkling of the stars. This appearance is strictly conformable with all the teachings of theory in reference to the laws of refraction, and is due to the fluctuations of variously-heated currents of air. When these small aërial currents, having different temperatures, are numerous, bad weather is likely to supervene; hence, an explanation of the increased twinkling of stars before bad weather sets in—a phenomenon which has been very commonly noticed.

Extreme examples of atmospheric phenomena are, for the most part, only seen in hot climates; but there they are frequent. The mirage is an atmospheric phenomenon, in part attributable to refraction, and in part to reflection; it occurs in Egypt, and gives rise to the impression in a stranger's mind, of a lake or tranquil expanse of water, though the region is only a waste of sand. The explanation of the phenomenon is this:—The villages throughout Lower Egypt are usually built on elevated mounds; hence the houses are to some extent raised above the general level of the earth, which level becoming intensely hot, imparts heat to the atmosphere placed in contact with it, and alters the refractive power of that portion of atmosphere. An optical illusion now ensues—the lower or heated atmospheric layer assumes a tremulous appearance, like the surface of a lake, on which the images of the buildings of the village are seen reflected, whilst the direct view of the village is still evident in its true position. The Egyptian mirage is so deceptive, that a stranger seeing it for the first time, can hardly be convinced that the semblance of water is only an optical delusion. The term *mirage* is peculiar to India; yet the phenomenon to which it refers is common in many other hot regions, especially in Central India and the Sahara.

The visual inversion of objects, a phenomenon not at all uncommon in hot places, is partly due to refraction and partly to reflection—for, in point of fact, the function of reflection may be demonstrated to be only an extreme case of refraction.

The *locale* is supposed to be a hot, sandy region, and a date-palm is the object seen inverted; the explanation of which phenomenon is as follows:—The eyes of the observer being at p (Fig. 53), will first see a direct image of the palm-tree by rays which come straight in the direction of the line $h\,p$; simultaneously he will see an inverted image of the palm-tree. Let us examine how this happens. Referring to the illustration, several parallel lines will be seen, $c\,c'\,c''\,c'''$. These are intended to denominate atmospheric layers of different amounts of density. Tracing the ray of light $h\,i$, let us now examine what becomes of it. First, it impinges on the upper atmospheric layer, which is hotter, and consequently more expanded, than the next layer above; the ray $h\,i$ is, therefore, refracted *from* the perpendicular. Passing on to the next layer, it is refracted still more *from* the perpendicular; and this refractive gradation is repeated on the ray $h\,i$ until it arrives at m, at which point, the ten-

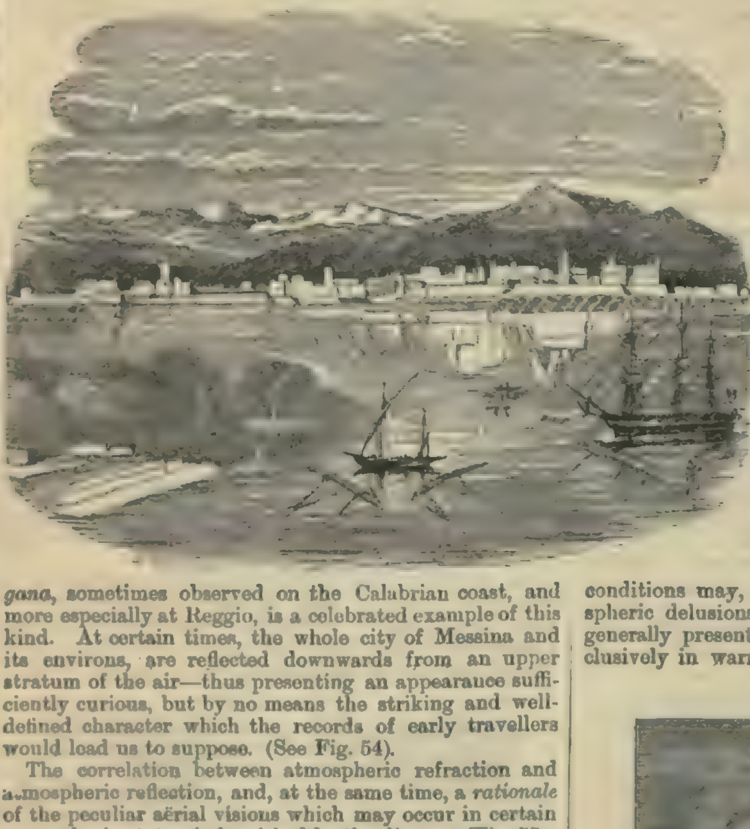
Fig. 53.



dency to fly from the perpendicular still remaining, this tendency is manifested, not under the condition of refraction, but reflection; consequently, the ray *h m* is directed towards the eye of the observer, and, along with the other rays upon which a similar operation has been effected, gives rise to the appearance of an inverted object. In order that the phenomenon should occur, however, there must be the following conditions besides those already mentioned:—Not only must the solar heat be considerable, but the air must be calm, so that the lower atmospheric layers may retain a density less considerable than the density of those above them.

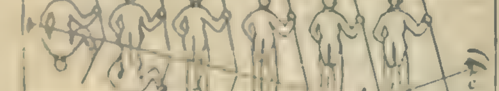
As, under the peculiar circumstances just mentioned, refraction may pass into reflection, and the reflection may be excited upwards from below, so may the operation be reversed; in which case the inverted images of terrestrial objects will be seen in the air. The celebrated *fata morgana*, sometimes observed on the Calabrian coast, and more especially at Reggio, is a celebrated example of this kind. At certain times, the whole city of Messina and its environs, are reflected downwards from an upper stratum of the air—thus presenting an appearance sufficiently curious, but by no means the striking and well-defined character which the records of early travellers would lead us to suppose. (See Fig. 54).

Fig. 54.



The correlation between atmospheric refraction and atmospheric reflection, and, at the same time, a *rationale* of the peculiar aërial visions which may occur in certain atmospheric states, is furnished by the diagram (Fig. 55), suggested by M. Biot. The line *b t c* is supposed to be a ray of light proceeding from *b*, passing thence downwards to the point *t*, whence it is reflected to the observer's eye at *c*. Now the optical conditions of this arrangement are such, that any rays proceeding from *b* below the ray *b t c* represented, would be invisible to the observer at *c*, whilst two images will be seen of all objects above this line. Supposing the object in question to be a man—suppose, further, the man to be walking *from* the observer, he would be presented to the latter under the successive forms seen in Fig. 55.

Fig. 55.

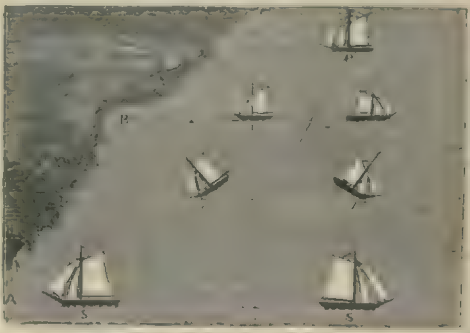


In these atmospheric optical delusions, involving the appearance of two images, one of them inverted, both natural and inverted images have occupied a horizontal plane. Occasionally, however, the reduplication of image has been projected on vertical planes, of which phenomenon the following is an example. On June 17th, 1820, whilst MM. Soret and Turine were in the second storey of a house on the lake of Geneva, they looked towards a ship two miles off, and making for the harbour. Immediately the vessel in question arrived at *q* (Fig. 56), she appeared reduplicated on a vertical plane; when she came to *r* the reduplication still continued, but the second image was further removed than before, and both were distorted; lastly, when the vessel arrived at the point *s*, the reduplicated image had receded to a distance still farther away, and both images, though distorted, presented an appearance of distortion very different from before; they were apparently drawn out, elongated both as to the hull and rigging, as represented in the diagram (Fig. 56). The following is the explanation of the phenomenon, as suggested by the two observers above-mentioned; and there seems no reason to doubt its correctness.

The letters, A B C, represent an outline of the eastern bank of the lake of Geneva; the air over that bank had, at the time of observation, been long under the shadow thrown upon it by the mountains of Savoy, whilst, contemporaneously, the western bank had been strongly heated by the sun; hence, from the conjoint operation of these two causes, there were two vertical layers of atmosphere of different temperature, and consequently of different densities; hence, they were of two different refractive and reflective powers.

All that is necessary to determine aërial reflections, is a sufficient difference between the temperatures of any two adjacent atmospheric layers. In the instance already mentioned, this difference had been occasioned by portions of the ground being hotter than the strata of air with which they were in contiguity. The reverse of these conditions may, however, obtain, and fantastic atmospheric delusions may be the result. This latter case generally presents itself at sea, and by no means exclusively in warm localities. Thus, for instance, it is prevalent enough in the Northern Ocean. Sometimes the atmospheric delusion has merely the effect of prolonging the appearance of an object really below the horizon; sometimes not only is the appearance prolonged, but the body is seen double. Whatever the appearance, the atmospheric delusions now under consideration are usually seen near the horizon—a position where the optical powers of the atmosphere attain their greatest intensity (Fig. 57).

Fig. 56.



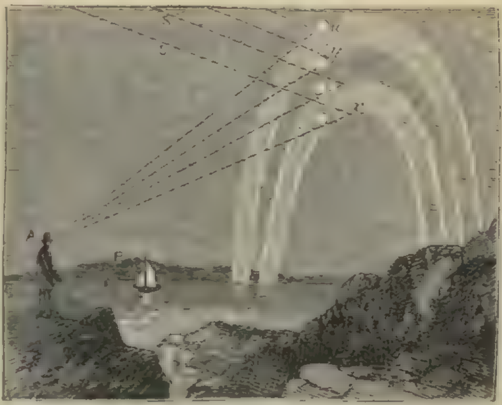
The Rainbow.—The most beautiful of all luminous meteorologic phenomena is the rainbow, which results from the decomposition of light by refraction through drops of rain, and subsequent reflection. Rainbows are of two classes, solar and lunar; the latter, however, are rare; and even when they do occur, the bow is seldom coloured.

Fig. 57.

The chief conditions under which a solar rainbow may occur, are the following:—The sun must not have less than 42° of angular elevation; the back of the spectator must be towards the sun, and rain must be falling from a highly illuminated cloud. The rainbow is usually double, and the theory of its formation may be thus explained. Let it be assumed that a straight line passes from the eye of the observer through the sun; then this line will constitute the axis of a cone, the base of which will be the rainbow, and its vertex the eye of the observer. If the bow be at the horizon, and the place of observation be a level plain, then the rainbow will appear as a perfect circle constituting the base of the cone. This complete circular appearance is, however, rare, the rainbow being far more generally, as its name implies, a mere arc of coloured light. The explanation is now evident of the fact that the rainbow cannot appear when the sun has attained an elevation greater than 45°. The rainbow, though still depicted, is depicted below the horizon, and is, therefore, invisible. It follows, then, that the size of the visible rainbow is inversely as the elevation of the sun above the horizon.

The following diagram (Fig. 58) illustrates the formation of the rainbow. The straight line A P is supposed to be drawn between the sun and the eye of the observer, passing through the latter. Through this line a vertical plane is supposed to be drawn. If the line A x be drawn through A so that the angle P A x shall amount to 42°, the rain-drops will reflect coloured drops to the eye. Assuming the line A x to rotate, a cone will evidently be generated, part of which, lying below the horizon, will be invisible. It is the surface of the cône thus generated which is the reflecting surface, and to which, therefore, the rainbow is due. Inasmuch as every colour has a refractive quality peculiar to itself, each drop only represents one tint to the eye. An arc having the breadth of about 2° is sufficient to include all the prismatic colours; 2° therefore is about the breadth of the rainbow.

Fig. 58.



The colours of the rainbow are partly due to refraction and partly to reflection, as has been observed. The first effect of light on the drops of rain is refraction, by the operation of which, white light arrives at the posterior side of each drop, and is decomposed or dissected into the primitive colours of which it is composed. At the posterior aspect of each drop of rain the dissected colours are reflected unto the eye, and a coloured image is presented.

Such is an explanation of the theory of the primary rainbow—besides which, the surrounding rainbow requires to be noticed. The secondary rainbow is outside the primary, and is larger than it, but also much fainter. Its angular position is defined by the limits 50° 59′ and 54° 9′, measured with reference to the axis A x. The secondary rainbow has all the colours of the primary, but less completely defined, and in a reverse order. Its existence may be explained by the statement that it is the result of light twice decomposed, whereas the true rainbow is the result of light only once decomposed. The secondary rainbow, then, is produced by drops of water very far off.

Inasmuch as any angular elevation of the sun above 45° is incompatible with the existence of a rainbow, it is evident that this beautiful meteor can never occur in the south. It may occur, however, either in the east, west, or north.

As concerns lunar rainbows, they are, as we have before remarked, exceedingly rare, and are very seldom coloured. Nevertheless, in northern latitudes, where the moon shines with a brilliancy unknown to us, coloured lunar rainbows are occasionally seen.

HALOS AND PARHELIA.—These meteoric phenomena are far more rare with us than in more northern latitudes, where they are continuously visible for long periods of time, and give rise to phenomena of extraordinary beauty. The term *halo* is applied to a luminous circle occasionally seen around luminous bodies, more particularly the sun and moon, and is partly due to the refraction of light by vaporous water, sometimes in the form of true clouds, sometimes not; and partly to the properties of light termed *diffraction* and *interference*. As respects diffraction, the circumstance has been already announced, that when light passes through a minute orifice—such, for example, as a small aperture punctured in a card—the edges of such light bend: this is termed diffraction. The term, *interference of light*, is used to explain the phenomena of colour, or alterations of luminous condition generally, which result from the assumed jarring impact of luminous waves meeting in different phases of their vibration.[*]

Solar halos frequently exist, though unnoticed; the sun's light being so powerful that the eye of the observer cannot withstand its impressions. By the aid of a sheet of glass rendered dull by smoke, these halos are frequently rendered visible (Fig. 59).

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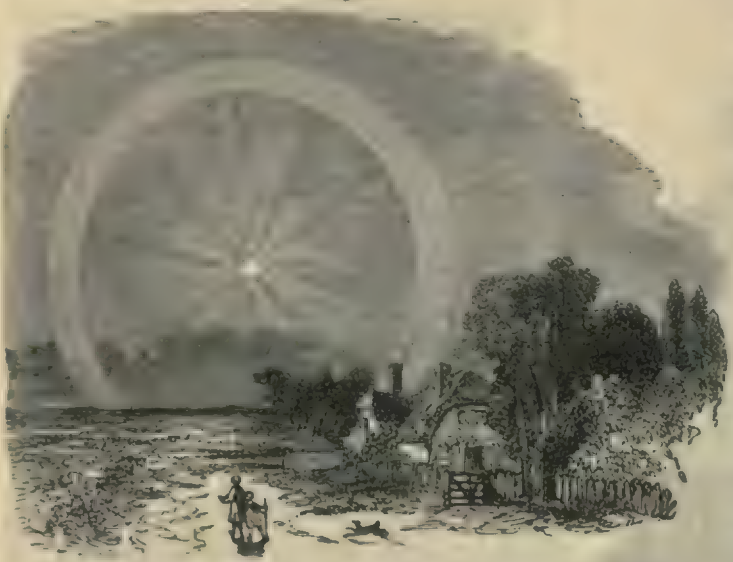
[*] See the Section on *Light*; articles "Interference of Light," and "Diffraction"—ED.

Fig. 59.



Although halos, and also the phenomena next to be described, are referable to the action of atmospheric moisture on luminous rays, yet it is evident that the condition of that moisture will vary according to temperature; in other words, the aërial moisture which would be mere cloud-vesicles at temperatures above the freezing point, would, if depressed below 32° F., be converted into snow, or spiculæ of ice. The alteration which these are capable of effecting on luminous rays being far greater than mere uncongealed vesicular water can effect, the resulting optical phenomena are far more brilliant and remarkable. Hence, in northern latitudes, the phenomena of halos and parhelia—as arcs of light appearing near the sun, and sometimes intersecting each other, are called—are brilliant and impressive beyond anything which corresponding phenomena occurring in this region would lead us to conceive. These luminous arcs, sometimes intersecting each other, are as often occasioned by the moon as by the sun. As the phenomena in question, when referable to the latter cause, are demonstrated *parhelia*, so, when dependent on the former cause, they are termed *paraselenæ*.

Frequently parhelia and paraselenæ consist, not only of the intersecting arcs just mentioned, but of circular luminous meteors, to which the term *mock suns* are especially applicable. Associated with parhelia, and sometimes included under the same name, is a luminous band, passing horizontally through the sun, and not unfrequently making a circuit of the whole heavens. Where this luminous band and the inner parhelion cross, a mock sun usually appears, as represented in the following diagram (Fig 60).

Aurora Borealis.—It has already been indicated, in the section on Electricity, that various circumstances materially operate to produce the condition termed electrical. By far the best-studied of atmospherical electrical phenomena are thunder-storms; but it is an error to suppose that the atmosphere contains, at the time of a thunder-storm, its maximum of electricity; the experiments of Faraday have sufficiently made out this point.

Reserving the consideration of thunder-storms for the present, we shall introduce here the subject of electrical phenomena by a description of the aurora borealis—a phenomenon sometimes said to be magnetic, inasmuch as the magnetic needle is strongly affected during its prevalence, but which, nevertheless, seems more naturally to belong to electricity.

The term *aurora borealis*, or northern light, is applied when the phenomenon presently to be described occurs in the north; and the term *aurora australis* is applied when it occurs in the south. But the former has been seen as far south as 45° of southern latitude, and the latter has, more than once, been visible in Britain. Nevertheless, the beautiful phenomena of northern and southern lights are most prevalent towards the north and south poles respectively.

Northern and southern lights, when in their greatest perfection, consist of a well-defined arc of white light, and luminous streams of coloured light flowing therefrom. The arc is not permanent, as in the rainbow, but bends and twists in all directions like a ribbon agitated by the wind. The intensity of the aurora varies within extensive limits: when faint, the light is only recognisable at night by careful examination; but when highly developed, the aurora borealis, or australis, can be seen during broad sunshine.

These phenomena may occur at any season, but they are most prevalent in the months of March, September, and October, or about the period of the equinoxes. The aurora borealis and australis are sometimes said to be magnetic storms; a more reasonable foundation is required for the remark than is supplied by the turbulent agitations of the magnetic needle with which they are attended; but we have already seen that the functions of magnetism and electricity are so nearly connected, that it is impossible in some cases to distinguish between the two.

Fig. 60.

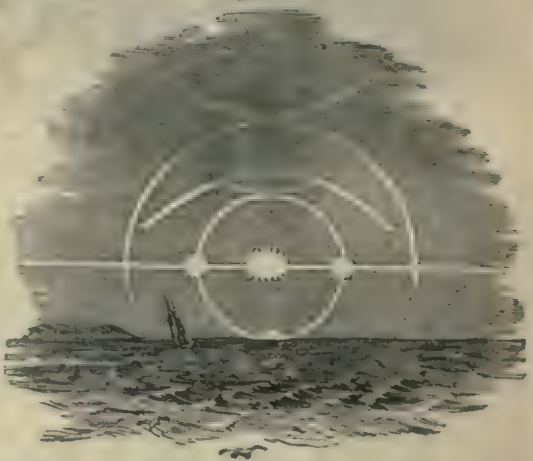
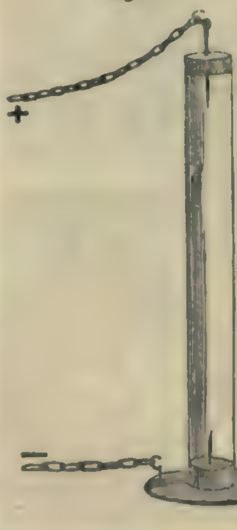


Fig. 61.



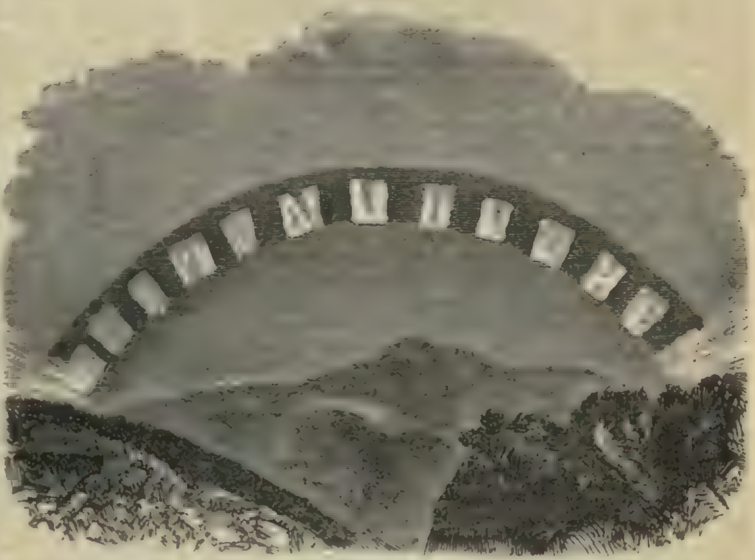
There is a very common electrical experiment which furnishes an artificial phenomenon very nearly resembling the northern and southern light, in all respects except in the shape of the illuminated body, which is a luminous arc.

The experiment consists in exhausting, by means of an air-pump, all the air out of a glass tube furnished with a metallic point at each end, looking internally, and placed in the electric current, as represented in Fig. 61. When a stream of electricity, of adequate intensity, is passed through the apparatus from + to —, the whole interior of the tube becomes illuminated with flashes of light, very similar in appearance to the flashes of the aurora.

The phenomenon of the aurora borealis was noticed by Aristotle and Pliny, although neither philosopher could have seen it to advantage. Gassendi first originated the term aurora borealis, to indicate the phenomena of this kind observed by him on September 12th, 1621. These phenomena appear to be subject to some laws of secular variation not yet understood. That they have appeared in certain years, and certain groups of years more than others, is certain. According to De Mairan, twenty-six occurred between A.D. 583 and 1354; thirty-four between 1446 and 1560; sixty-nine between 1561 and 1592; seventy between 1593 and 1633; thirty-four between 1634 and 1684; two hundred and nineteen between 1685 and 1721; nine hundred and sixty-one between 1722 and 1745; and twenty-eight between 1746 and 1751.

After 1790, auroras became unfrequent, but since 1825 they have been on the increase. A very remarkable aurora borealis occurred in the autumn of 1847: it was conspicuous not only in England, but even so far south as Italy and Spain (See Fig. 62).

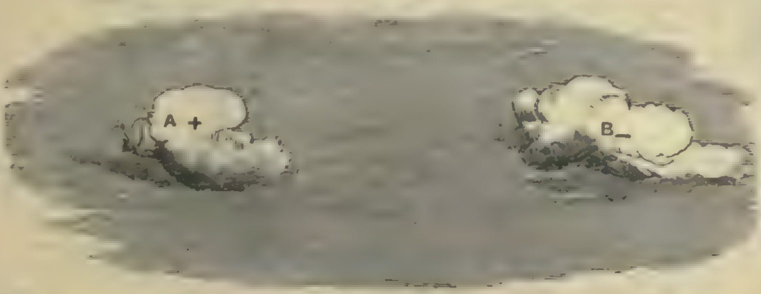
Fig. 62.



Height of the Aurora.—As a proof of the doubt which exists concerning the height of the aurora, they have been variously estimated from 3,000 or 4,000 feet to several miles. The probability is, that the conditions on which the aurora depends vary in the altitude of their operation; but the truth is, that, notwithstanding the electrician, by his artificial experiments, can imitate the light of the aurora borealis and australis—notwithstanding the prevalence of the phenomena in question near the magnetic poles, seems to point to magnetic agency as the cause—our real knowledge concerning the aurora borealis and australis is very slight.

It may be as well here to present the reader with a summary of the various opinions which have prevailed at different times relative to the phenomenon in question. Many early writers referred the appearances presented to mere optical causes, considering them to be due to the reflection of the sun's light thrown upwards by a mirror of snow and ice, and a subsequent reflection downwards by atmospheric agencies. De Mairan, the observer who, perhaps more than any one else, is entitled to be considered the chronicler *par excellence* of the phenomena of the aurora, attributed them to the penetration of our planet at certain periods into the solar atmosphere. On this supposition it will be remarked, that the solar atmosphere must be assumed to extend to the orbit of our planet, an hypothesis totally irreconcilable with the teachings of optics and astronomy. The celebrated Euler, the philosopher who could deal so satisfactorily with the abstractions of number and quantity, seems to have offered a most crude and improbable theory explanatory of the aurora. Adopting the molecular theory of light, he assumed that the solar rays, striking against the particles of our atmosphere, actually carried particles of the latter up into the heavens to a height of more than four thousand miles—the height at which Euler believed auroras to exist. Some philosophers, of whom Volta may be regarded the Coryphæus, adopted a chemical theory of auroras, referring them to the ignition of hydrogen gas spontaneously generated on the earth, and rising, by its light specific gravity, to the higher atmospheric regions. It was assumed by these philosophers that the evolution in question took place in the tropical regions chiefly, and that it was wafted by the upper current of air—treated of in connection with the trade wind—to the north and south poles respectively.

Fig. 63.



Halley was the first, we believe, who suggested that the phenomena of aurora borealis and australis might be due to the passage of magnetism from one magnetic pole to the other; and the theory of Halley is so far retained, that the aurora is assumed to be in some way connected with electricity and magnetism, but in what manner is beyond the competence of observers to decide.

ON THE PHENOMENA OF THUNDER AND LIGHTNING.—Perhaps no meteorologic phenomena are now so well understood as these; though, before electrical science had been studied by the philosophers of the last century, and the crowning experiment of Franklin performed, the phenomena of thunder and lightning were so mysterious, that even philosophers were content to refer them to the operation of an occult cause.

The intimate study of electrical science opens a field of somewhat abstruse matters for consideration; the field is far too wide and too abstruse to be dealt with satisfactorily here. In the section on Electricity it has been treated in detail; and to this we must refer the reader who desires to know more on this subject than strictly belongs to the necessities of what we may term *practical meteorology*.

With this explanation we shall not hesitate to adopt the term *electric fluid*, although the reader has already been made aware that no such fluid is at all likely to exist. Let us now contemplate the phenomenon of that electrical excitation, the solution of which is lightning, under the simplest conditions that the phenomenon can assume. Let A and B (Fig. 63) represent two clouds, which, being made up of watery vesicles, are electrical conductors; and being surrounded by the atmosphere on all sides, are necessarily insulated. For the sake of our illustration, it will now suffice to assume that neither of the clouds here represented is electrically excited at this period of the description, and hence that the marks + and — are for the present misplaced. Let it now be assumed that the cloud A becomes positively electrified, —that is to say, charged with positive electricity, owing to some natural cause unnecessary here to explain; and let the results of this condition be traced out. Firstly, there is not in all nature, and there *cannot be*, such a condition as that of independent electric excitation; in other words, there cannot be one body *positively* excited without the co-existence of another body *negatively* excited. Hence, if cloud B were away, and cloud A positively excited, the air circumadjacent to A would assume the second or negative function; but if the cloud B is present, it therefore becomes negative, and the two clouds A and B are mutually attracted, because opposite electricities attract each other. Hence, they approach until the space of air, between the two, is insufficient to restrain their mutual electric tension: this condition having arrived, a discharge takes place, precisely analogous to the discharge of a Leyden jar. Under the postulates of our experiment, the discharge, or lightning flash, takes place between the two clouds A and B.

It follows, however, from the consideration of known electrical laws, that just as the two oppositely electrified bodies may be two clouds, as assumed, so also may they be one cloud, and the surface of earth or water, or conductors placed upon either one or the other; under which conditions a downward discharge will take place; and generally, electricity will always take the nearest path between any two bodies oppositely charged, the conducting facilities being equal.

LIGHTNING-CONDUCTORS.—It is almost unnecessary, in these days, to announce that Franklin, in the year 1752, first demonstrated the nature of lightning by drawing electric sparks from the string of a kite, previously caused to ascend into the region of a thunder-cloud. This experiment performed, the connection between lightning and electricity could no longer be doubted, and a means of drawing off a surcharge of the electric fluid by lightning-conductors was immediately suggested. The most important instruments were not adopted, however, until after numerous and varied conflicts. Firstly, the argument was adduced by some, that lightning conductors could not be adopted without impiety, being intended to contravene the will of Providence. An argument so fallacious was no sooner abandoned than lightning-conductors were exposed to another ordeal, founded on an erroneous practical estimation of a truth in theoretical electricity. We allude, as the electrician will perceive, to the contest between the advocates of spherical, and of pointed, terminations for electrical conductors.

Now, regarding the question of points or spheres abstractedly, it is easy to see that preference should be given to the former, inasmuch as points draw off and give issue to the electric fluid in silence; whereas spheres draw off and give issue to electricity in sparks; but, inasmuch as the largest spherical termination ever used, or ever likely to be used, for the upper extremity of a lightning-conductor, is virtually a point in comparison with the enormous surface of the smallest thunder-cloud, the dispute, though violent and prolonged, never had the practical significance which was at one time taken for granted.

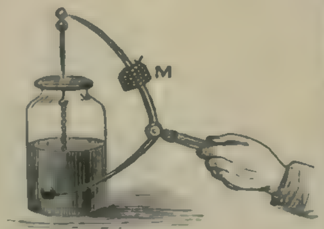
The history of lightning-conductors furnishes a remarkable illustration of the difference between the mere knowledge of a fact, and the confidence or conviction resulting from that knowledge, and justifying its practical application. The whole theory of lightning-conductors was almost as well known half a century ago as now; yet it is only within the last few years, and owing to the unflagging perseverance of Sir W. Snow Harris, that due effect has been given to the theory, and lightning-rods have been fearlessly applied.

Electrical Principles connected with Lightning-Conductors.—The electrical principles on which the efficiency of lightning-conductors depends are few and simple; they all admit of being readily demonstrated by electrical experiments artificially performed, and they have been universally justified by the result of three practical applications:—

(1.) *Electricity, cæteris paribus, follows its course through the best Conductors which happen to be in its path.*—Thus, for example, if a Leyden jar be charged, and the electric connection between its external and its internal coating be completed by three linear substances of equal length—say, for example, silk, wire, and linen thread—thin wire, being the best electrical conductor of the three, will transmit the whole of the electricity, to the exclusion of the silk and the linen thread. It is assumed, however, in the performance of this experiment, that the wire is sufficiently large to convey the whole electric discharge.

(2.) *Provided the Conductor be good, and its Sectional Area adequate, the Electric Fluid or Discharge is conveyed harmlessly away.*—No point in the whole of electrical science can be more satisfactorily established than this. It admits of being illustrated by numerous experiments, among which the following may suffice:—

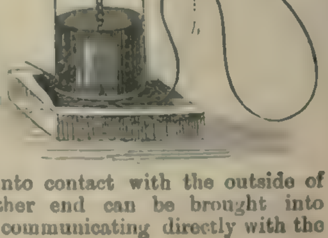
Fig. 64.



The diagram (Fig. 64) represents a common Leyden jar, in the act of being discharged by the ordinary discharging instrument. This is, as usual, of brass—all save the handle, which is of glass, and therefore a non-conductor of electricity. Those who are conversant with the form and construction of the discharger, are aware that its two terminal balls admit of being unscrewed. Assuming one of the balls, viz., the upper one in the diagram, to have been unscrewed, the liberated brass stem to be passed through a marroon, or box, holding gunpowder, and the ball to have been again replaced, the conditions will have been fulfilled which the diagram represents. It is evident that the Leyden jar, as represented, will be discharged. It is, moreover, evident that the whole of the charge will be transmitted through the gunpowder contained in the marroon, yet that gunpowder will not inflame. If, however, instead of the conditions of the last experiment, a very fine metal wire (a steel wire by preference) be passed through the marroon, or rather through some combinations of explosive materials less potent than a marroon, which would now be dangerous, and electricity transmitted as before, the wire, not presenting a sufficient amount of transverse area of surface to convey the electricity, would melt, and the explosive compound be inflamed.

(3.) *Lateral Discharge must be provided against.*—The meaning of lateral discharge will be illustrated by the following experiment:—The diagram (Fig. 65) represents, as before, a Leyden jar ready arranged for being discharged through a metallic wire, one end of which has already been brought into contact with the outside of the jar, while the other end can be brought into contact with the knob communicating directly with the

Fig. 65.



inside of the jar. The hand is represented in the act of holding a glass rod, around which one end of the wire is coiled, and the extremity of the wire is finished off with a ball. All these arrangements, the electrician will perceive, are necessary for giving effect to the efficient discharge of the Leyden jar through the conducting wire. The chief point for observation, however, is the bend of the wire, by means of which one part is caused very closely to approach another part, as represented at *a b*. Now, it is possible by choosing a wire sufficiently small, and causing the two bands to pass sufficiently near, to determine the passage of an electrical spark from *a* to *b*, instead of proceeding through the entire length of the wire, from the internal to the external coating of the jar. This is what electricians call the lateral discharge, and it requires to be studiously guarded against in the construction and arrangement of lightning-conductors.

Application of the foregoing Deductions.—Perhaps the deductions already arrived at, will suffice for practical guidance in the matter of lightning-rods, though these deductions by no means exhaust the science of the subject. The fact may be considered as proved, that all bodies, even the most dangerous and inflammable—all edifices, all living beings, may be shielded from the evil consequences of lightning, by the safeguard of lightning-conductors. The conductors, however, must present a sufficient external area; and the point has been made out by numerous trials, that a copper rod, a square inch in sectional diameter, will convey away the utmost fury of the most highly-charged thunder-cloud ever proved to exist. Copper is one of the best electrical conductors amongst metals; but, by providing a sufficiently increasing sectional area to compensate for inferior conducting power, any metal may be made to perform the function of copper. Whatever be the conductor, its upper extremity should project considerably above the edifice to be protected; and if pointed *theoretically*, all the better; though, practically, the bluntest termination could be only as a point by comparison with the enormous mass of a thunder-cloud. Far from preventing contact between the building to be protected and the conductor, as is sometimes done by the interposition of glass or earthenware guards, a lightning-conductor cannot be brought into too intimate metallic connection with every part of the edifice to be protected. The conductor should branch and ramify over the surface of the building, and should be brought into contact with every important system of metal line work, such as the iron pipe which frequently runs down the side of a wall; finally, the conductor should, at its lower extremity, be brought into contact as efficiently as possible with some good electric conductor, such as the system of gas or water-pipes which run underneath most houses, especially under the streets of most civilised towns. As regards the number of conductors necessary to be supplied to one building, that will depend on the shape of the building, whether it be composed of many elevations, or whether, like a column, it has only one. Perhaps the best practical testimony on this point is gleaned from the fact, that in a ship having three masts, one of which only was protected by a lightning-conductor, the unprotected masts have been shattered by the effects of a thunder-storm, while the other remained untouched. If a column be surmounted by a metallic statue, it is worse than useless to disfigure the head of the statue by a projecting metallic spike as the beginning of a lightning-conductor; nothing more is requisite, in this case, than to provide sufficient metallic conduction for the electricity downwards into the ground. Lightning-conductors, it should be remembered, *do not*, as they are commonly said to do, *attract* electricity. They no more attract electricity, than a gutter attracts water. They merely open a channel for electricity to pass through. Before the demonstrations of Sir William Snow Harris had taken effect, marine lightning-conductors were something more dangerous than lightning itself: consisting merely of chains, which were only elevated aloft after the thunder-storm had come on. Marine lightning-conductors are now fixtures on the masts; they are made of copper, running band-like down the mast, and imbedded in the latter in such a manner, that, whether the masts be elevated or lowered, perfect metallic contact, between any two pair of masts, remains uninterrupted.

RELATIVE PREVALENCE OF THUNDER-STORMS.—The phenomena of thunder and lightning are nowhere so violent or so frequent as in the so-called region of calms; but they are very prevalent throughout the torrid zone, more especially during the rainy season. Thunder and lightning are almost always absent in the polar regions; and even at a spot no farther north than Bergen, in Norway, the annual number of thunder-storms does not average more than six. As the rule, thunder-storms chiefly occur in hot weather, winter thunder-storms being comparatively rare. Iceland and the western coast of North America, however, are remarkable for the predominance of thunder-storms in winter. In Sweden, winter thunder-storms are almost unknown; thus furnishing another example of the circumstance already noted, that the Scandinavian range of mountains effects a remarkable difference between the general climate of Sweden and Norway, though the two, geographically considered, are so close together.

AEROLITES—SHOOTING STARS—METEORIC STONES.*—The beautiful phenomenon of shooting stars is common enough; but at certain periods it is peculiarly remarkable, the whole sky being filled with these fleeting meteors. The beginning of August and the beginning of November are noticeable for their connection with shooting stars; more especially have they been recorded between the 9th and 14th of August. The *bouquet* of shooting stars, observed at this period in North America, has been sometimes called the Shower of St. Lawrence.

For our knowledge respecting the periodicity of the phenomenon of falling stars, we are indebted to Quetelet, Besenberg, and others; but Muschenbroek, so long back as 1762, first directed attention to the so-called *Shower of St. Lawrence*. In addition to the August and November phenomena of the kind under consideration, other periods have been noticed—for instance, in April, and from the 6th to the 12th of December; but these *bouquets* of shooting stars, which thus occur, are less considerable and less regular than the former. Besides these showers, the periodicity of which is well attested, single meteors of this kind are frequently noticed, and they occur at all seasons; therefore, whatever may be the cause of shooting stars, it must be regarded as continuously operating.

Sometimes the luminous meteor attains a large magnitude; observers then speak of it as a fire-ball. The identity of shooting stars and fire-balls is now well established, though formerly they were treated of as distinct. Fire-balls are sometimes seen alone, but more frequently in connection with shooting stars. Their light and their bulk are frequently so considerable, that they can be seen in broad daylight. Their velocity through the heavens, or rather through the upper layers of our atmosphere, is various; but it generally exceeds that of the earth. As regards the nature of shooting stars, we have only theory and analogy for our guidance; but our knowledge of fire-balls is far more accurate, and there seems no reason to doubt that they quite illustrate the nature of falling stars. Numerous fire-balls dart through our atmosphere, become luminous, and disappear, no one knows whither. Others, though passing near to our atmosphere, fail to enter it, and therefore are not rendered visible. A third division not only come within our atmosphere, shine, and burst with a loud report, but they fall; yet, falling either into the sea or upon an unfrequented spot of land, the locality of their fall remains unknown. Whilst a fourth division of fire-balls may be seen to fall, be dug out, and examined.

Masses of this kind are termed aërolites, and their connection with fire-balls has been placed beyond all doubt. Fire-balls have been seen to fall, and aërolites have been extracted from the place whereon they have fallen. Nevertheless, some aërolites have fallen upon the earth without the assumption of a previous appearance of luminosity. Cases, though rare, are well attested of an aërolite suddenly falling from a small cloud, attended with a noise resembling the discharge of cannon; others, again, have fallen silently, and out of the clear air, not the slightest trace of cloud being visible at the time.*

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* Further remarks on this subject will be found in Chapter V., at the end of this section.

Testimony concerning showers of stars and the fall of aërolites has been handed down to us from all periods; but it is only since the time of Chladni that the occurrence of these phenomena has been placed beyond doubt.

Amongst the best-attested examples of the fall of aërolites are the following:—On the 16th of June, 1794, a shower of stars fell at Sienna; and in the following year, December 13, an aërolite, weighing no less than fifty-six pounds, fell in England. Three years afterwards, and, remarkably enough, also on December 13th, a fire-ball split up and discharged round stones. A very large shower of stones fell April 26th, 1803, near Aigle, in France; the occurrence is particularly interesting on account of its having been noticed and verified by M. Biot. Ten such meteoric showers were observed in France in twenty-six years—*i. e.*, between 1790 and 1815. The meteoric shower at Aigle in 1803, which poured its contents over a surface of two-and-a-half French miles long, by one in breadth, consisted of 2,000 fragments of different sizes, some weighing not more than two drachms, others near twenty pounds. Aërolites are sometimes much larger than this; one fell at Agram on the 26th of May, 1751, weighing seventy-one pounds; but the largest known aërolite fell in Mexico, and weighed between 30,000 and 40,000 pounds.

As to shape, aërolites are generally prismatic, or angular—rarely smooth; and almost always sheathed in a crust of pitchy blackness. Their specific gravity is various, some being sufficiently porous to absorb water with rapidity, others being dense and metallic. Looking at the specific gravity of aërolites in the aggregate, it may be said to vary between 1·94 to 4·28, presenting a mean of about 3·5. All the heavier varieties of aërolites are made up of iron, holding a little nickel; traces also of cobalt, manganese, chromium, copper, arsenic, tin, and other well-known elementary bodies, are found.

ORIGIN OF FIRE-BALLS AND SHOOTING STARS.—Various opinions have been advanced to account for these bodies. One of the earliest, if not the very earliest, of these hypotheses, originated in 1660, and assumed fallen aërolites to be mineral masses originally projected from lunar volcanoes; and calculations were made, having for their object to demonstrate what volcanic force might be sufficient to project aërolites of a given mass into the sphere of attraction of the earth's atmosphere. Unfortunately for the probability of this theory, the moon's surface appears to be altogether devoid of active volcanoes. Then followed the chemical hypotheses, according to which it was assumed that aërolites were nothing more than aggregations of metallic vapours, which had risen to the upper region of the atmosphere, aggregated there, and fallen. The opinion of Chladni is now, however, generally received; he regards aërolites to be of cosmical origin—to be so many planets, or planetary fragments, which revolve in orbits of their own, variously inclined to the orbit of the earth; that our planet encounters periodic shoals of these little worlds, some of which, becoming entangled in the earth's gravitating system, pass into our atmosphere, become heated by friction against its particles, and ultimately fall to the ground. Although the discovery of aërolites is comparatively rare, the meteors, of which they are the final result, are by no means so. It has been calculated that the average annual fall of aërolites is not less than 700, or about two daily.

METEOROLOGIC RESULT OF OCCULT EMANATIONS.—When the utmost powers of a refined chemistry have been applied to the analysis of atmospheric constituents and conditions, much still remains to be unveiled. There are atmospheric causes, whatever they may be, of epidemic and endemic diseases, and perhaps other agencies which our philosophy little suspects or dreams of. We have ventured to include these undetermined agencies under the general expression *occult emanations*. It is not difficult to point out objections to this designation in some of its applications. Perhaps it is not strictly philosophical to speak of emanations thus hypothetically; perhaps this may be only a repetition of the error of assuming the existence of an electric fluid; perhaps the number of influences due to allotropism and to polarity is greater than we imagine; but, at any rate, the term "occult emanations" may be accepted as a rallying-point for a certain class of facts, until the time arrives when their true significance shall be correctly made out.

Without invoking the hypotheses of allotropism and polarity, there are undoubtedly some atmospheric agencies to which the expression occult emanations is applicable, and concerning which, the only thing occult about them is the insufficiency of ordinary chemical examinations to demonstrate their existence, though that existence admits of being demonstrated by extraordinary chemical means. Thus, for example, it is a well-authenticated fact, that the atmosphere of localities in which fever is endemic, usually contains minute traces of hydrosulphuric acid, and an odorous animal matter—substances which ordinary chemical processes fail to detect, but which, nevertheless, by the adoption of refined methods of investigation, can be proved to exist. The late Professor Daniell was of opinion that the much-dreaded fever of Western Africa was augmented by the diffusion, through the atmosphere of that coast, of minute traces of sulphuretted hydrogen. And the circumstances under which African fever originates, are perfectly consonant with the above theory. The disease only prevails now on the coast, its ravages being limited to a small belt, partly of land and partly of sea, Central Africa being comparatively exempt from its inflictions. Now Professor Daniell assumes the hydrosulphuric acid to be the result of decomposition, under a powerful sun, of matter borne seawards by the Niger and other great rivers, in connection with certain sulphates of sea-water. Be this theory true or the contrary, there can be no doubt as to the truth of the assumption which refers fever, when endemic, to a vitiated condition of atmosphere; which vitiation may be generally summed up as consisting of minute traces of hydrosulphuric acid, and of undetermined animalised matter.

Amongst other occult emanations, we can hardly refuse to admit the cause, whatever that cause may be, of intermittent fevers. The ultimate, if not the proximate, cause of this class of disease is so well known, that we may almost produce or banish intermittent fevers at pleasure. Given, heat and moisture continuously; ague almost invariably sets in, and continues its ravages as long as the conditions of heat and moisture co-exist.

What is the occult emanation here? What is the proximate cause of intermittent fever? Are we to attribute the disease to the conjoined evaporation of moisture, and heat *directly*, or to some further emanation to which these conditions give rise? Some pathologists have assumed that light carburetted hydrogen, or marsh-gas, as it is called, determines the disease; but the notion hardly coincides with known facts. The horizontal demarcation of altitude, above which the influence of ague cannot extend, is one of the most remarkable circumstances in connection with the disease; a difference of no more than ten feet in perpendicular height frequently corresponding with the region of fever and the region of salubrity respectively.

The mention of light marsh-gas naturally suggests the curious meteorologic phenomenon called Will-o'-the-wisp, or Jack-o'-lantern; of which gas, ignited, or, according to some, phosphuretted hydrogen gas, it is believed to consist. The Will-o'-the-wisp is not a very frequent phenomenon anywhere; but it is chiefly seen in marshes and churchyards—the latter locality apparently adding to the hypothesis that it is nothing more than ignited phosphuretted hydrogen gas. (See Fig. 66).

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* A large collection of these curious substances may be seen in the British Museum.—ED.

Fig. 66.

Scarcely less accurately demonstrated than the locality of ague, is the locality of yellow fever, the focus of which may be considered to be Vera Cruz. Strange to say, the yellow fever is totally unknown on the Pacific coast of Mexico; it extends north as far as New Orleans, but rarely further. This condensation, so to speak, of febrile energy, points to some local cause—most probably of atmospheric origin; but chemistry has been unable to determine its nature.

Reflections similar to the above are suggested by the contemplation of several contagious diseases; of which the plague is a notable example. This scourge, although capable of spreading, from its normal focus, into regions wide apart, is in certain spots determined conditionally by the existence of some unknown circumstances; and these are most probably atmospheric. The plague never originates in very hot or very cold regions. Its focus of development is Egypt, Turkey, and the Levant, from which spots it can never be said to be absent altogether; but it only appears with violence at intervals of eight or ten years. Still more extraordinary than any of the preceding, are the varied conditions which give rise to Asiatic cholera. Unlike the plague and the yellow fever, and intermittents, this fell destroyer seems independent of region, recognisable limits, or other conditions of demarcation. From east to west it has extended its ravages, under every vicissitude of season and of clime. Much as there is mysterious in this—absolutely ignorant though we be of the proximate influences by the operation of which cholera and other epidemics are generated, important facts have, nevertheless, been made out; and their consideration tends to allay the extreme fear wherewith epidemics were formerly regarded. The most important fact in connection with this subject is, that epidemic influences, whatever their nature may be, only as a rule prevail over the weak, exhausted, ill-fed, or mentally broken-down individuals of a community; whence it happens, that in our metropolis the medical statician is enabled to lay his finger upon the regions of epidemic virulence, as he would on the locality of a mountain or a coal-field; and with equal satisfaction can he point to localities where epidemics formerly raged, but whence they have been banished by the art of man.

CLIMATOLOGY.—Many of the effects of heat, in its meteorological relations, have already been incidentally considered; but reference has not yet been made to the sources of heat, and to the means of its distribution over the surface of our planet. The term *heat*, as applied to the matter now under investigation, may be regarded as synonymous with elevation of temperature; and inasmuch as such elevation necessarily pre-supposes a condition of antecedent depression, we may, without impropriety, comprehend the meteorological effects of high and low temperatures (heat and cold) under one and the same generalisation.

Central Heat of the Earth.—The hypothesis was first propounded by Leibnitz, that the whole of our planet was once a molten mass, which, by the operation of cooling, uninterruptedly going on in successive ages, has become superficially encrusted over, the crust having become adapted to the necessities of animal and vegetable life. Various circumstances may be adduced in favour of this notion, more particularly the gradual increase of terrestrial heat downwards,* the heat of deep springs, and the evidences of fusion, in what geologists term the igneous rocks. Whether the idea of Leibnits hold good in its entire acceptation—that is to say, whether the centre of our planet be one molten mass or not—there can be little doubt that all positions of the globe, at a sufficient distance below the surface, have at some period been submitted to fusion. Nevertheless, at this time, the earth's central heat may be altogether ignored as tending to influence, in any manner, the climatic temperature of our globe. Primarily, the sun's direct rays determine the climatic temperature; those portions of the world's surface being most strongly heated on which the sun shines vertically—a remark which of course applies to the tropics; while those are least heated on which the direction of solar rays is most oblique—a remark which of course applies to the arctic and antarctic regions. But the latitude of a region has less connection with its climatology than might at first seem probable. The varying conditions of insular and continental sea-level, or elevated table-land, valley, or mountain, and still more the influence of thermal oceanic currents, have much to do with the climatic result. The high table-land of Mexico is strongly illustrative of the effect of mere elevation. The traveller who disembarks first on the Atlantic coast, and wanders inland, soon finds himself amidst all the luxuriance of a tropical forest, and surrounded by all the dangers of tropical existence. Still wending his way inland, he ascends a mountain elevation, well adapted to the growth of wheat, which refuses to grow anywhere in tropical lowlands.

Of all the causes which influence the climate of a region, that attributable to oceanic currents has been hitherto least studied; yet there is none which deserves to be scrutinised more narrowly. Looking at the enormous amount of oceanic surface in comparison with that of the land—taking into consideration the mobility of water, its susceptibility of thermal impressions, and the effect of the configuration of capes, headlands, and lines of coasts—the contemplative observer soon arrives at the deduction, that the ocean presents to him, at least, as wide a field for investigation as the atmosphere, and one scarcely less interesting. "The fauna and the flora," says Maury, "of the sea are as much the creatures of climate, and are as dependent for their well-being upon temperature, as are the fauna and the flora of dry land. Were it not so, we should find the fish and algæ, the marine insect and the coral, distributed equally and alike in all

* Careful observations have not confirmed this idea, as it has been found, in mines not at work, that there is no indication of an increased temperature as a descent is made. It is probable that the increase observed, in mines at work, may be due to the animal heat, lamps, &c., of the miners.—ED.

parts of the ocean. The polar whale would delight in the torrid zone, and the habitat of the pearl oyster would be also under the iceberg, or in frigid waters colder than the melting ice." The particles of water being mobile, the ocean, and indeed aqueous collections generally, are amenable to the same law of convection as was described when treating of the cause of winds. Hot water, being specifically lighter than cold water, must necessarily come to the surface; and for every current in one direction there must be a counter-current in the reverse direction, precisely in the same manner as occurs in the development of a wind.

Contemplating the ocean in its relation to the effects of heat and motion, our original ideas concerning that vast collection of waters are modified and expanded. Instead of regarding the ocean as one shapeless aggregation of briny water, it presents itself to us as an assemblage of many streams—a network of mighty rivers, each following its own course, each having its own temperature, its own flora, its own animals; and though devoid of palpable banks, scarcely less accurately defined on that account. Amongst all these oceanic currents, that denominated the gulf stream is the largest in size—the most important in the functions it subserves. The gulf stream is so far from being an imagining of mere theory, that the dark blue alone of its waters suffices to point out its limits and define its course.

All hypotheses as to the cause of the gulf stream are as unsatisfactory as the direction of the stream itself, and its benign influences are evident. The first idea relative to the gulf stream was, that it originated in the impetus given to the ocean by the disemboguement of the Mississippi; but placing out of consideration the inadequacy of this assumed cause, on account of the comparatively small amount of water which even a river so vast as the Mississippi can pour forth, it follows that, if really the cause of the gulf stream, the whole Gulf of Mexico should, in process of time, be found to contain only fresh, or, at the most, brackish water. This, it is scarcely necessary to remark, is not the case. Franklin advanced the theory, that the gulf stream is referable to the pressure of an inordinate amount of water against the coast of the Gulf of Mexico by the trade winds—an idea which is scarcely more tenable than the last.

Whatever the cause of the gulf stream may be, the direction of its current is obvious. Setting out from the hot regions of the Mexican Gulf and the Caribbean Sea, it proceeds northward to the great fishing-bank of Newfoundland, and thence to the shores of Europe, yielding up its heat to the genial west winds, and thus transferring a portion of the superfluous heat of the tropics to our colder shores. The greatest heat of the oceanic water of the Mexican Gulf is about 86°, or about 9° above the ocean temperature due to latitude alone. After it has ascended to 10° of north latitude, the gulf stream has still only lost 2° of the original heat with which it set out. Ascending northwards a distance of three thousand miles from its first origin, this mighty oceanic river still preserves the heat of summer even in winter time. It now crosses in an easterly direction, in a line coincident with about the fortieth degree of north latitude, spreads itself out, and imparts to Europe a genial temperature; which mere latitude could never give. The gulf stream now pauses in its course; it is split into two divisions by the British Isles, and two gulf streams are formed. Of these, one ends northward in the direction of Spitzbergen, while the second enters the Bay of Biscay—imparting temperature to each, and causing a soft mantle of vapour to arise, which, wafted landward, in its turn disperses the heat of the gulf stream far inland. Very little is known concerning the depth to which the gulf stream extends.

Lieut. Maury, of the United States naval service, assumes that depth to be two hundred fathoms; and, arguing on this assumption, he calculates that the amount of heat led away from the Gulf of Mexico by this oceanic torrent, raises, on a winter's day, the whole atmosphere which hovers over France and the British Isles, from the temperature of 32° F. to about 79°; in other words, from winter-cold to summer heat. But the genial influence of the gulf stream on the British Isles is more than this. Every western breeze that blows towards us crosses the mighty gulf stream, robs it of a portion of its heat, and comes towards our shores charged with warmth and laden with balmy moisture, clothing Ireland in a suit of green, and imparting a mildness to both England and Ireland which can be best appreciated when we consider that the coasts of Labrador, on the American side, and under the same parallel of latitude as England, are rigid with ice. In 1831, the harbour of St. John's, Newfoundland, was closed with ice as late as the month of June; yet the harbour of Liverpool, though 2° further north, is never ice-locked, even in the severest winters. By referring to any chart of isothermal lines, the current of the gulf stream, as just described, may be readily traced.

Although the climatic effect of the gulf stream is so advantageous to Western Europe, more especially to these isles, it is scarcely less advantageous to the regions whence it originates. If the gulf stream be the channel along which an amount of heat so considerable passes eastward, we may speculate on the consequences that would have arisen had the amount of temperature, now conveyed away, remained in the gulf itself. Even now the coast-line of this region is extremely hot and unhealthy; how much more hot and unhealthy would it have been had the gulf stream not existed!

Under-Current of the Gulf Stream.—As the trade winds are only an under atmospheric current, passing in an opposite direction to a current above; so the gulf stream is only the counterpart of an inferior current of cold water flowing back, to compensate for that which has departed. Not only does theory proclaim this, but it is borne out by experiment. At a mean depth of two hundred and forty fathoms, an under-current of water flows into the Caribbean Sea; and the temperature of this current has been found as low as 48°, whilst the surface-water had a temperature of 85°. At the depth of three hundred and eighty-six fathoms the temperature had fallen to 43°; and at the very bottom of the gulf stream the temperature was only 38°; hence the existence of the returning cold current is fully borne out. It comes, there is little reason to doubt, from the arctic circle; presenting the closest analogy to the lower aërial current which constitutes the trade wind.

The course and extent of the gulf stream were not generally known until the celebrated Dr. Franklin drew attention to the subject. The history of this event is worthy of narration, illustrating as it does the discriminating and logical mind of that extraordinary individual.

Happening to be in London in 1770, his opinion was demanded respecting a memorial presented by the Board of Customs at Boston to the Lords of the Treasury, stating that the Falmouth packets were generally a fortnight longer on their voyage to Boston than common traders were from London to Providence, Rhode Island: whence their request that the Falmouth packets might be sent to Providence instead of to Boston. "Franklin could not understand the reasonableness of this request, inasmuch as London was much further than Falmouth; and from Falmouth the routes were the same; so that the difference should have been the other way. Desiring a solution of his difficulty, he consulted Captain Folger, a Nantucket whaler, who chanced to be in London at the time. The whaler explained that the difference arose from the circumstance that Rhode-Island captains were acquainted with the gulf stream, while those of the English packets were not. The latter kept in it, and were driven back sixty or seventy miles a-day; while the former avoided it altogether."—*Maury.*

The manner in which the old whaling captain had been made acquainted with the existence, the extent, and the direction of this gulf stream, is curious enough in its way. His instructors were the objects of his search, *the arctic whales*—animals which, having a dislike to warm water, never enter the gulf stream, though they swim close up to it on both sides.

Franklin having extracted this intelligence from the whaling captain, got him to draw a chart of the gulf stream to the best of his ability. The chart was drawn; Franklin had it printed; and copies were sent to the Falmouth captains. They, however, were foolish enough to pay no heed to its teachings; nor did they profit for many years after by the knowledge of the gulf stream. Though the date of Franklin's discovery was 1775, yet a knowledge of the gulf stream was not generally diffused and acted upon until fifteen years later. Not the least extraordinary fact in connection with the gulf stream, is the sharpness of its line of demarcation. No river imprisoned between two scarped rocky banks could flow in channel more defined. "If," remarked the American author, Jonathan Williams, "these strips of water had been distinguished by colours of red, white, and blue, they could not be more distinctly discovered than they are by the thermometer."—"And he might have added," remarks Maury, "nor could they have marked the position of the ship more clearly."

The notion prevails among sailors that the gulf stream is the great storm breeder of the Atlantic—the father of storms; and, indeed, the tempests which follow in its course, or on its borders, warrant that designation. What are the indications of theory in this respect? Had the Atlantic been still an untravelled waste, and the existence of a gulf stream, such as we now know it, been propounded as the basis of discussion, would not the theorist have predicted that storms must originate in the meeting of the hot, moist atmosphere, which hovers over the ocean tract of seething waters, from the fiery shores of Mexico and the Caribbean Sea, mixed with the chilling blasts of the north? What torrents of water must result from the condensation of the tepid mists—what stupendous electrical force must be brought into operation!

If the gulf stream, by its impulsive flow, sometimes impedes the mariner, and drives his ship from the desired course, it nevertheless affords a compensation, not only in assisting to propel ships sailing in the direction of its course, but in affording a genial climate to the weather-beaten mariner, frozen and benumbed by the shivering blasts of the regions outside its channel. "No part of the world," says the writer, to whom we are largely indebted for much that in these pages concerns ocean currents and ocean climatology—"no part of the world," remarks Maury, "affords a more difficult or dangerous navigation than the approaches of our (the American) coast in winter." Before the warmth of the gulf stream was known, a voyage for this reason from Europe to New England, New York, and even to the capes of the Delaware or Chesapeake, was many times more trying, difficult, and dangerous than it now is. In making this part of the coast, vessels are frequently met by shore-storms and gales which mock the seaman's strength, and set at nought his skill. In a little while, his bark becomes a mass of ice, and his crew frosted and helpless. She remains obedient only to her helm, and is kept away for the gulf stream. After a few hours' run she reaches its edge, and almost at the next bound, passes from the midst of winter into a sea at summer-heat. Now the ice disappears from his apparel: the sailor bathes his stiffened limbs in tepid water; feeling himself invigorated and refreshed with the genial warmth about him, he realises out at sea, the fable of Antæus and his mother Earth. He rises up and attempts to make his port again; and is again as rudely met, and beaten back from the north-west; but each time that he is driven off from the contest, he arises forth from this stream, like the ancient son of Neptune, stronger and stronger; until after many days his freshened strength prevails, and he at last may enter his haven in safety; but in the contest he sometimes falls to rise no more, for it is often terrible. Many ships annually founder in these gales; and we might name instances, for they are not uncommon, in which vessels bound to Norfolk or Baltimore, with their crews enervated in tropical climates, have encountered, as far down as the Cape of Virginia, snow-storms that have driven them back into the gulf stream, times and again; and have kept them thus out for forty, fifty, and even for sixty days, trying to make an anchorage.

Nevertheless, the presence of the warm waters in the gulf stream, with their summer-heat in mid-winter, off the shores of New England, is a great boon to navigation. At this season of the year especially, the number of wrecks and loss of life along the Atlantic sea-board are frightful. The month's average of wrecks has been as high as three a-day. How many escape by seeking refuge from the cold, in the warm waters of the gulf stream, is a matter of conjecture. Suffice it to say, that before this temperature was known, vessels thus distressed knew of no place of refuge short of the West Indies; and the newspapers of that day—Franklin's *Pennsylvania Gazette* among them—inform us, that it was no uncommon occurrence for vessels bound to the capes of Delaware in winter to be blown off, and to go to the West Indies, and there wait for the return of spring, before they would attempt another approach towards their destination.

The gulf stream is the largest known oceanic current; it has been perhaps more fully studied than any other, and its teachings may, therefore, be appropriately regarded as the type of the rest. We have seen how powerful and extensive are its effects; we have seen a few of the purposes to which it ministers. The ocean is full of streams similar to this, each taking its well-defined course, carrying its own temperature, clad with its own fauna and flora, peopled with its own denizens. Thoughts like these prove how false and unfounded is the expression, *ocean waste*, so commonly applied. The ocean has its regions, its valleys, and its mountains—climate and varied inhabitants—no less than the earth.

OTHER OCEANIC CURRENTS.—*The Mediterranean.*—It has long been known that an upper or sailing current constantly sets into the Mediterranean through the Straits of Gibraltar. What, then, becomes of the water of the current? That water must either be dissipated by evaporation, or there must be a second or back-current.

The existence of this under-current was first demonstrated very curiously in 1712. At that time, France being at war with Holland, M. L'Aigle commanded a French privateer, called the *Phœnix of Marseilles.* Near Ceuta, this privateer gave chase to a Dutch ship bound to Holland—came up with her, delivered one broadside, when the Dutch ship went down. A few days later, the sunken ship, with all her cargo of brandy and oil, came to light again; but this took place on the coast of Tangier, at least four leagues westward of the place where the ship went down, and in a direction quite opposite to that of the upper or navigable current. This well-authenticated case was communicated to the Royal Society in 1724.

Currents of the Red Sea.—Precisely similar to the Mediterranean currents, just described, are those of the Red Sea. The necessity of a free change of waters here is even more decided than in the Mediterranean; the sea is not only shallower, and subjected to a more powerful evaporation, but its waters are not freshened by the afflux of any rivers. It has been calculated by Dr. Buist, that, taking into consideration the mean evaporation on every part of the surface of the Red Sea, a sheet of water, eight feet thick, and equal in superficial area to the whole extent of the surface of that sea, is raised in vapour annually. When this enormous rate of evaporation is considered, the necessity for a continuous interchange of water between the Red Sea and the external ocean will be evident. If the Red Sea outlet were choked up, so that ingress and egress were no longer possible, the evaporation of about a thousand years would, it is calculated, completely dry it up.

Currents of the Indian Ocean.—Many thermal currents originate in the Indian Ocean. Amongst the foremost of these is the Mozambique current; another of these currents, first escaping from the Straits of Malacca, and being swollen by warm streams from the Java and Chinese Seas, flows out into the Pacific between the Philippines and the Asiatic shores. Passing thence towards the Aleutian Isles, it ultimately loses itself on the north-west coast of America. Meteoric conditions like those which mark the course of the gulf stream, also mark the course of this. Fogs and mists follow in its track, and storms are also generated on the bank of this oceanic river.

For a fuller account of oceanic currents than is consistent with the limits and objects of these pages, we must refer the reader to treatises which deal with the special matter, and, above all, to the work of Lieutenant Maury, of the United States' service, to whom meteorologists are under deep obligations, for his contributions to their knowledge of ocean phenomena.

In contemplating these oceanic currents, of which the gulf stream may appropriately be considered the type, we cannot fail to be impressed with an evidence of design, where, at first, no design would seem to exist. These oceanic currents originate in, and are determined by, a peculiar conformation of the land. Now, what can be more seemingly irregular or capricious than the shape of land? If dropped down into the ocean at random, or elevated by a subterranean power equally capricious, the crust line of islands and continents, the solid blocks of our planetary crust, could not well be more irregular than they are; and yet how practically harmonious is the relation between land and water: how well adjusted the powers of each—how well adapted to the mutual benefit of mankind! It appears an unimportant matter when, in the map of the world, we skim our eye over the southern hemisphere of the terrestrial globe; there we behold the limits of the African and American continents in their furthest extent; but how terribly would the locomotive faculty of mankind have been impeded had either continent expanded itself to the south pole, or even traversed much further south than is actually the case!

In connection with the subject of ocean currents, the effect of aqueous temperature on the ocean's denizens, deserves a passing word of remark further than has already been devoted to it. In the Caribbean Sea, and other ocean cauldrons, where stores of heat are accumulated for distribution in regions far away—in the hot currents which originate in these tepid sources of oceanic rivers, there are a fauna and a flora: the fauna no less marked than we see on tropical lands. There grows the coral, there swims the shark; and thousands of shelled mollusca, of gorgeous colour and enormous size, revelling in oceanic forests of rank and bulky growth, represent the land forests, and their denizens of corresponding climes. But, though grandeur and beauty be the characteristics of these warm ocean spots—the thermal ocean-tropic (if the propriety of that expression be allowed)—it is the colder oceanic currents which support the form of animal life most useful to man. Strange though the circumstance may appear, it is no less true, that the fish of the hot parts of the world are always indifferent as food. This fact is strikingly illustrated in the Mediterranean. The temperature of the Mediterranean water is usually four or five degrees above the temperature of the external ocean; and what a difference in the fish! Whoever has compared the edible fish of the Atlantic with those of the Mediterranean, will be at no loss to admit the vast superiority of the former. The naturalist does not require to be informed, that not only are Mediterranean fish inferior to those of the Atlantic, but they are, for the most part, of different species.

Let us now take a glance at the locality of the principal fishing regions. Limiting ourselves to two of these, they would unquestionably be the fishing-grounds of Newfoundland and Japan. The former is the better known of the two, though, perhaps, the latter is the more considerable, seeing that the natives of Japan are debarred from the use of animal food by their religion, although the eating of fish being permitted, they are all ichthyophagi; and, notwithstanding the dense population of the Japanese islands, their inhabitants, though fish-eaters, are abundantly fed.

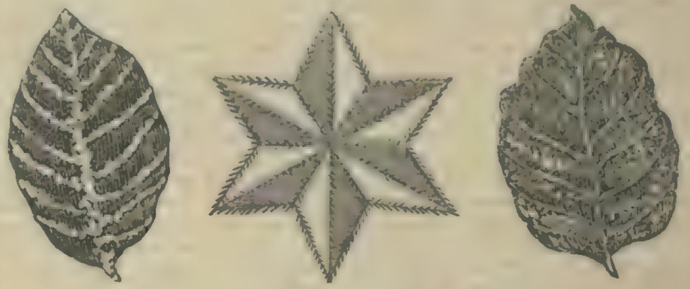
Between the gulf stream and the coast is a narrow band of cold water: here are the fisheries of Newfoundland. Between the China current, as it is denominated, and flowing in an opposite direction, is the cold current, in which the Japanese fisheries are prosecuted. These are the two most prominent examples, and they will be found to present the type of many others.

COLD, AND ITS EFFECTS.—Under the heads of Snow, Hail, and Hoar-Frost, some of the meteorologic effects of cold have been already described. The phenomena due to this powerful agency are, however, so numerous and so important, that it may be well to make the subject of cold a matter of special contemplation. If, casting our eye over the field of nature, we endeavour to select the most prominent results of cold, they will be found to relate to the departure from an ordinary law, which Nature has made in the expansion of water during the act of freezing.

The term *freezing* is but a general expression for the act of solidification by cold. Popularly, the term is only applied to solidified water; but this is altogether a conventional acceptation of the word. We are justified, then, in comparing the act of solidification of water with the act of solidification of any other fluid, and seeing to what extent the conditions which regulate one also regulate the other.

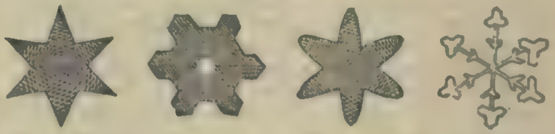
When the temperature falls to 32° F., water ceases to be liquid, and becomes ice; the weather is said to be frosty, and water is said to be frozen. Whatever water be contained in the atmosphere at the freezing temperature, is deposited in the solid form of hoar-frost, the particles not being irregular, but bounded by definite mathematical outlines—frequently giving rise to forms of great beauty, especially on window-panes, blades of grass, and leaves (Fig. 67). These forms are similar, in general contour, to those of well-formed snow-flakes (Fig. 68), but far more beautiful, and, like snow-flakes, prove that frozen water is a crystalline body, and that it crystallises in forms belonging to the rhombohedral system.

Fig. 67.



But the most important point connected with the freezing of water, and without which our globe would cease to be habitable, is this:—Water, during the act of freezing, expands, and thus becomes specifically lighter. Ice, therefore, swims on water; it cannot sink. How stupendous are the consequences of this departure from the law of freezing! Had it so happened that frozen water, like frozen mercury, were heavier than the corresponding liquid material, each frozen sheet of water would sink as soon as formed; and thus, being far removed from the melting influence of solar rays, the production of ice would have been accumulative, and the ocean, long ere this, would have been completely ice-locked.

Fig. 68.



In tracing, hypothetically, an assumed aberration of Nature to its consequences, the mind is speedily entangled and lost in the chaos of jarring effects. The speculative meteorologist finds himself unable to thread the labyrinth of causation here involved. Having shown that the ocean would have been full of ice, had the ordinary law of solidification not been departed from in the case of water, it is perhaps unnecessary to follow the development of our hypothetical case further. That ocean life must have been destroyed is evident; that the sea's liquid highway would have ceased to be, is only a figurative expression for a frozen ocean. But would what is now the solid land have then served the purposes of animal life? Where could the rivers have flown, had the ocean been a block of ice? or would not the rivers have remained frozen too, seeing the vast cooling power of a frozen ocean? It is easy to see that, under such circumstances, our planet would have been totally unfit to be a resting-place for its present denizens had the freezing of water not assumed a departure from a law, though it be impossible to imagine all the consequences that would have resulted.

The same expansive force of water during the act of freezing, by the operation of which it is rendered specifically lighter than water, subserves many important purposes in the world's economy, besides floating the ice. Water percolating into the fissures of rocks, and being subsequently frozen there, displays its irresistible force by splitting large rocks into fragments, and disintegrating their fragments. In this way hard and sterile districts become covered with useful soil, in which low forms of vegetable life can take root; and by their subsequent decay, contribute the elements necessary to the support of higher forms of vegetation.

Relation of Climate to Organic Development.—The naturalist, who, in his desire to see, as in a dioramic picture, the wonderful characteristics of animal and vegetable types, sighs to think that a vision so glorious will never pass in reality before his gaze, may at least console himself with the assurance of that great master of philosophic travel, Humboldt, that it is in the power of man's creative faculty, aided by philosophy, to imagine those striking types in the vividness of their truth, gladdening the closet with ideal images of the living features of Nature.

Even in the narrow region of European travel, the intelligent observer will not fail to see distinctive physiognomies. Passing from the cold green-sward and modest vegetation of our own Isles to the Mediterranean shore, a striking change in the aspect of nature meets the view. The sturdy oaks and elms of our own forests disappear; the absence of smaller grasses removes the green carpet of our meadows; tall graminaceæ spring up; the aloe and the prickly pear bespeak a mixed condition of heat and drought; and the date-palm, barely acclimatised, gives some faint notion of what the characteristics of a tropical forest must be. "It would be an enterprise worthy of a great artist," says Humboldt, "to study the aspect and the character of all these vegetable groups, not merely in hot-houses, or in the description of botanists, but in their native grandeur in the tropical zone. How interesting and instructive to the landscape painter would be a work which should present to the eye, first separately, and then in combination and contrast, their leading forms! How picturesque is the aspect of tree ferns, spreading their delicate fronds above the laurel-oaks of Mexico; or the groups of plantains overshadowed by arborescent grasses! It is the artist's privilege, having studied these groups, to analyse them: and thus, in his hands, the grand and beautiful form of nature which he would portray, resolves itself, like the written works of men, into a few simple elements."

When the meteorologist has exhausted his knowledge in the laying out of climatic groups—when he has placed in correlation, conditions identical, as he thinks, in every respect—the growth of vegetable forms demonstrates his inability to comprehend many hidden secrets of nature, which their delicate organisation makes known. European olive-trees grow luxuriantly at Quito, but they bear neither fruit nor flowers; and a similar remark applies to walnut-trees and hazel-nuts in the Isle of France. In India, the bamboo flowers luxuriantly; but in South America, where it flourishes equally well, so far as general aspect of growth is concerned, so rare an event is the infloration of the bamboo, that during a four years' residence in South America, Humboldt was only enabled to obtain blossoms once. But, perhaps, a still more remarkable example of luxuriant growth, without inflorescence, is furnished by the sugar-cane. The West Indies have come to be considered as the region *par excellence* of the sugar-cane; yet it seldom bears flowers there—nor indeed does it in any part of the American continent; thus furnishing a strong presumptive argument in favour of the theory, which asserts, that no variety of the sugar-cane is indigenous to the New World.

CHAPTER IV.

PRACTICAL METEOROLOGY.

So many amateur meteorologists are now springing up in every direction, that it may be desirable to point out the precautions and reductions necessary, in order to render the raw observations useful to science. The present paper is, therefore, written to enable the amateur, with as little trouble as possible, to reduce his observations to useful results.

To the non-meteorologist, such terms as "adopted mean temperature," "elastic force of vapour," "reduced barometric," &c., unexplained, serve rather to perplex than to enlighten, or point out that care has been bestowed upon the observations, and that the requisite reductions and precautions had been adopted.

The first duty of an observer is to procure good instruments, and to place them in such positions as may be deemed least likely to be affected by local circumstances, such as heat from a fire, draughts round a building, &c. Much time is frequently thrown away by using imperfect instruments. When a person purchases a thermometer, he is too apt to consider it correct; whereas, in many instances, it is far from being accurate. Mr. Hartnup, of the Liverpool Observatory, in his last report, mentions, that among the thermometers used by the captains of the merchant service, an error of 4° or 5° is quite common; even a thermometer fitted up for taking the temperature of water at different depths, and professing to have been made with care, was found to be 8½° in error in one part of its scale. The most curious instance, however, was a barometer which had the following errors:—

At 28	inches	was	— 2·22	inches	of the standard.
At 28½	,,	,,	— 1·88	,,	,,
At 29	,,	,,	— 0·73	,,	,,
At 29½	,,	,,	— 0·30	,,	,,
At 30	,,	,,	— 0·02	,,	,,
At 30½	,,	,,	+ 0·33	,,	,,
At 31	,,	,,	+ 1·07	,,	,,

The whole yearly range of pressure seldom reaches two inches, whilst the range in error of this instrument was 3·29 inches.

Meteorological Observations.—In order to place this portion of the subject in as clear a light as possible, let us suppose that daily observations are made of the barometer and its attached thermometer, of the temperature of the air, of the wet bulb thermometer, of self-registering maximum and minimum thermometers, of the amount of rain, amount of ozone, temperature on the grass and sunshine, state of electricity, amount and class of cloud, direction and force of winds, and state of the weather. Let us further suppose that two observations are taken daily, the hours of observation being 9 A.M. and 10 P.M.

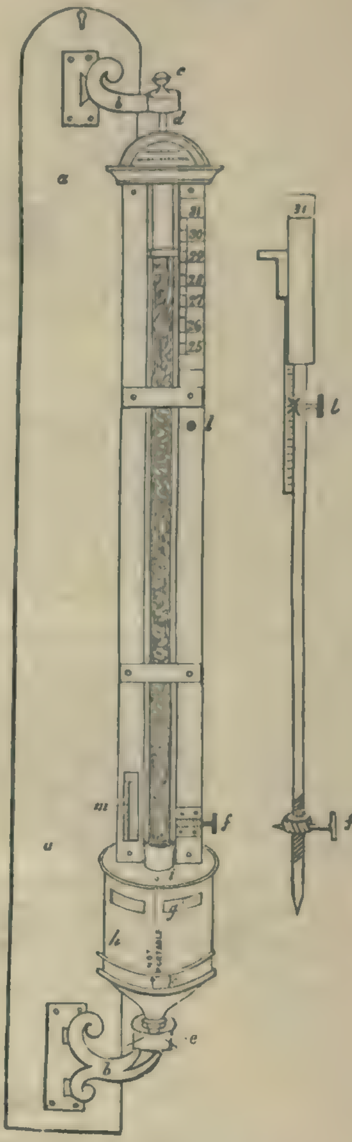
The Pressure of the Air.—The barometer is the instrument by which the alterations in the weight of the air are ascertained. The gravitation of the earth exerts a certain pressure, which would always be alike, were it not that lateral disturbances had the power of removing a portion of that pressure, and adding it elsewhere. Thus, suppose the average pressure to be 29½ inches; if the barometer be seen to rise to 30 inches, it is certain that, in another portion of the earth, a corresponding fall has taken place to account for this change: however, as it is not our object to enter into the physical and local changes of the weather here, we shall at once proceed to describe the reductions that are requisite.

It is essential to correct observation, that the barometer used be a standard instrument. The ordinary instruments are useless, owing to the friction of the mercury against the sides of the glass in small tubes; the impossibility of applying the reduction necessary to *correct*, for the alteration of the height of mercury in the cistern, with the common wheel barometer, owing to a rise and fall in the tube; and, further, because if errors occur in this barometer, they are increased by the circumstance of the index of the wheel barometer being a long arm worked by a small wheel, thus multiplying all the errors.

The Standard Barometer should have a tube of not less than $\frac{4}{10}$ths of an inch in diameter; and the nearer it approaches to $\frac{6}{10}$ths the better, as the correction requisite for the friction against the sides of the glass, rapidly diminishes with an increase in the size of the tube. This is the correction for capillarity, which is additive. The base of the tube should be plunged into a cistern of pure mercury, and the scale of inches should have a rod attached, terminating in an ivory point, so contrived as to be moved by rack-work until the point *just* touches the surface of the mercury. This is requisite, in order that the measurement may be made from the surface of the quicksilver in the cistern. Suppose we either neglect this operation, or that the barometer is one not capable of having it applied; and let us further take a reading without reference to this correction, after a sudden change in the barometer; this reading may be represented as 30·036 inches. On bringing the ivory point down to the mercury, this second reading may be 30·074 inches: thus exhibiting an error, from the want of this precaution, of ·038, or nearly four hundredths of an inch. The thermometer fitted for use should have its bulb plunged into the cistern of mercury, otherwise it will not give the temperature of the mercury itself, but merely show the heat of the apartment in which it is placed. Such an instrument is produced by many of the leading London makers: of similar kind are those made for the Royal Society, Greenwich Observatory, Admiralty, and the British Colonial Observatories.

Description.—*a* (Fig. 69) is the mahogany board to affix against the wall; *b*, brackets which support the barometer, between which it is capable of being revolved so as to observe the light on the surface of the mercury; *c*, a vase, which unscrews, to allow of the socket *d* being removed, to receive the upper end of the barometer; *e*, the adjusting screws for shifting the lower centre, by which the barometer is to be made exactly perpendicular: to accomplish this, the ivory point is to be adjusted to the surface of the mercury; the barometer gently turned between the two brackets; and if, in any position, the point should be elevated from the surface, or depressed into the mercury, the screws must be altered accordingly, until the point coincides in every position; *f*, the key by which the ivory point is adjusted—the ivory point being a termination of the brass scale marked off at the temperature of 32°, and which is adjusted by means of a tangent screw; *g*, the glass part of the cistern, through which the surface of the mercury and ivory point are seen; *h*, the cistern; *i*, the screw, which is to be loosened when the barometer is fixed, to admit the atmospheric pressure; *k*, the movable part of the cistern, on which the index ↗ is engraved; *l*, the key by which the Vernier is adjusted; *m*, the thermometer dipping into, and showing the temperature of the mercury.

Fig. 69.

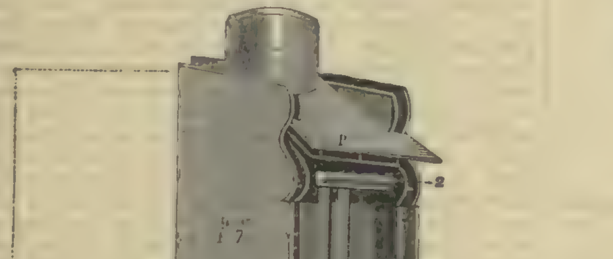


This barometer is necessarily expensive. Modifications of this standard are made by many London instrument-makers; and these are more reasonable in price, answering remarkably well, though possibly not equal to Mr. Newman's standard. Having procured a barometer, it should be compared with the Standard, in order to ascertain its index error. The apartment in which it is kept must not be subject to great changes of temperature; one with a window facing the north is to be preferred; and the instrument should hang where there is a strong light, but an outer wall should be avoided.

THERMOMETERS.—There are various constructions of instruments for ascertaining the temperature of the air; of these the mercurial ones are the best. Thermometers for comparison require to be placed upon a proper stand, as the errors arising from peculiarity of situation, from radiation, absorption of heat, &c., will alter the reading very materially. There are two forms of stand—the one constructed by the late Henry Lawson, Esq., F.R.S.; the other by James Glaisher, Esq., F.R.S.

Thermometer-Stands.—In comparing the readings of one thermometer with any other, it is requisite that each instrument should be placed, as much as possible, in a similar manner; without this uniformity, no deductions can be drawn with any claim to accuracy. In looking to the situation of instruments used by persons who have not been aware of the necessity of providing themselves with a thermometer-stand, some will be found facing the north, others the south, south-east, north-west, and, in short, every point of the compass. Again, some are placed from one to five feet from the ground, others ten to twenty feet; some in the angle of a large building; others exposed to the sun's rays, either during the morning or the evening; some touching a wall, others at a distance from one; in short, the situations vary in as great a degree as the temperature deduced from the observations made by them. To obviate these sources of inaccuracy, the late Mr. Lawson constructed his "Thermometer-Stand," a sketch of which is given below (Fig. 70).

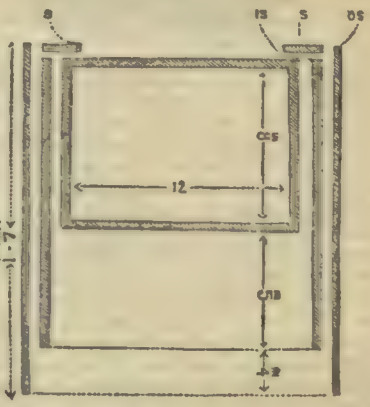
Fig. 70.



This stand consists of a frame, which has been found to answer the intended purpose very well. It is composed of *white deal* boards, and can be constructed by any carpenter. It consists of an oblong trunk T, 12 inches by 8 inches outside measure; to the opposite side of which are nailed boards *b b*, at the distance of three-quarters of an inch, and projecting about six inches from the trunk towards the north. Outside of these are nailed other thin boards *c c*, full half an inch distant, and projecting about four inches beyond the last-mentioned boards, also towards the north. These sides or shades being multiple, prevent the sun from heating the interior of the stand, where the thermometers are placed. The top, or pent-board, P, is made double; and the boards are placed full three-quarters of an inch distant from each other, and come forward so as to overhang, by a full inch, the Night Index Thermometer, placed immediately beneath, for the purpose of preventing rain or dew from falling perpendicularly upon the bulb of the thermometer. The legs, L L, of the stand, are merely the continuation of the sides of the trunk. The board or feet, F F, are loaded or fixed to the ground, to sustain the force of the wind. The interior, T, is blackened to prevent strong reflections of light.

Fig. 71 is a ground-plan of the machine, which will prove sufficiently clear to any intelligent workman for its construction. The sides (and wood-work generally) are of half-inch white deal. The distance or space between the sides of the trunk T and the board or inner side, *i s*, is three-quarters of an inch; and the distance from that board to the outer side, *o s*, is full half an inch. The narrow boards, *s s*, are to be nailed, with studs intervening, to the middle board or side, *i s*, and are for the purpose of preventing the sun from shining between the trunk, and the sides *o s* and *i s*, when near the meridian. The sides are fixed, one upon the other, at the required distance (viz., three-quarters of an inch and half an inch), by numerous wooden studs, about three-quarters of an inch in diameter; and the nails or screws passed through the sides and studs, fix the whole firmly together. The whole is to be painted white, and no other colour, except the face of the trunk T, which may be black, as mentioned already.

Fig. 71.

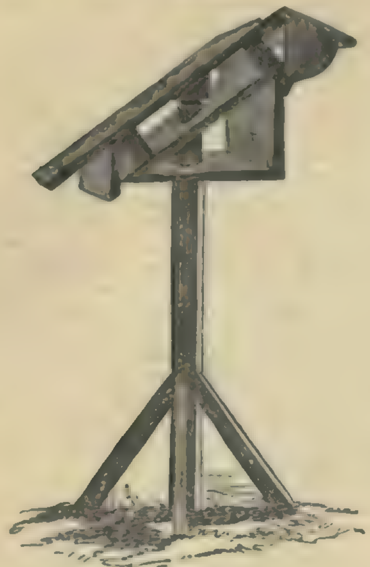


This thermometer-stand can be placed in *any* eligible spot that may suit the convenience of its owner; its four sides should face the cardinal points, commanding therefore a true north and south aspect. It can be visited on every side, and be free from all surrounding objects. The thermometers used can be read off with the greatest facility, and the whole will be at a known distance from the ground. Those instruments placed on the south face will have the meridian sun, and those on the north face will be always in the shade, in consequence of the projecting wings. The arrangement can be employed by any meteorologist, wherever residing; it is of a determinate form, height, and size; it is not costly, but firm, and can be placed on any open spot that may be thought eligible for its use. The instruments may be read off with the greatest promptitude, so as to prevent or reduce errors arising from the person of the observer being too long in the vicinity of the thermometers. By the general adoption of this stand, instruments placed upon it will all be used or observed under similar circumstances, and deductions from them be more correctly drawn than where there is no other used. It follows, that observations made by individuals wherever residing, either in Europe, Asia, Africa, or America, if drawn from instruments thus similarly placed, can be compared one with the other, with far less chance of error than has hitherto been the case.

Mr. Glaisher, F.R.S., has likewise constructed a stand, differing from the one just described, but which is also an excellent contrivance. (See Fig. 72.)

Rutherford's Thermometer.—The mercurial thermometer of Rutherford's construction, pushes a registering-pin before it as the mercury expands by heat; on the

Fig. 72.



air becoming cooler, the mercury contracts, leaving the registering-pin at the point of maximum heat. One objection to this instrument will always be felt—*i. e.*, the pin is not unlikely to become entangled with the mercury, especially if the mercury should oxidise.

It is very desirable to incline Rutherford's maximum thermometer from the horizontal position, so that the bulb shall be slightly the highest, in order that the thermometer may have the assistance of gravitation in pushing the pin forwards. Within the last fifteen years, many of Rutherford's construction have, at the Highfield House Observatory, become useless, owing to the registering-needle becoming entangled with the mercury. Two thermometers, however, of this construction deserve to be noticed—the one purchased of Dollond, and the other of Bennett: they have done their duty satisfactorily, and are at the present time in perfect working order.

Negretti's Patent Maximum Thermometer.—Too much praise cannot be given to this ingenious invention, which, from the circumstance of doing away with the registering-needle, prevents the possibility of the instrument getting out of order. Since the invention, half-a-dozen of these thermometers have been constantly employed by us without any derangement. The principle of the thermometer is this:—A small piece of enamel is pushed into the thermometer tube near the bulb, and the tube is then bent so as to secure it in its place; if the thermometer act once, it will continue to act until the instrument is broken. As the temperature rises, the mercury will flow over this enamel; yet, on the air becoming colder, the contraction will only take place below this point, the whole column of mercury in the tube being left to mark the maximum heat. It is only requisite to turn the thermometer in a vertical position, and give it a gentle shake, when the mercury will descend, and flow over the obstruction, which, in the horizontal position, had prevented it from returning into the bulb of the thermometer.

Phillips's Construction of the Maximum Thermometer.—This, the invention, of Professor J. Phillips, F.R.S., also records without a registering-needle. A small bubble of air is passed down the tube, and a portion of mercury is made to remain above this bubble; as the air increases in temperature the whole column rises; yet, when it cools, the mercury only falls from below where the bubble of air is situated. It is an instrument liable to get out of order in travelling; yet, when once properly fixed, does its work well. In consequence of the bubble of air expanding with an increase of temperature, a slight error will be occasioned, and the thermometer will read a little too high in warm weather, and the reverse in cold.

The Minimum Thermometer, until lately, has been filled with *spirit* instead of *mercury*, with a slender glass pin floating in the liquid. This pin is carried down with it to the lowest point; on the temperature rising, this pin is left behind, and thus marks the coldest point. Like the maximum thermometer, gravitation should be allowed to assist the descent of the pin, which is accomplished by slightly lowering the bulb from the horizontal position. The best that we have received have been from Messrs. Negretti and Zambra; in these the pin is much longer and more slender, and they very rarely get out of order. A disadvantage will always be felt with spirit thermometers, for their action differs from those filled with mercury.

The Mercurial Minimum Thermometer.—Meteorologists have for some time urged the opticians to invent a mercurial minimum thermometer, an invention which long seemed almost to be an impossibility. However, such an instrument now exists—thanks to Messrs. Negretti and Zambra. A thermometer, with a large tube, is placed in a vertical position; in it is a slender, pointed needle, which is brought down to the surface of the mercury; quicksilver, being heavier than the needle, the latter is held above it; yet, the needle being pointed, plunges a small, but sensible, distance in the mercury, which it invariably does by the side of the glass of the thermometer tube. This being the case, the needle will descend to the lowest degree of cold; on the thermometer rising, the mercury presses the needle to the glass, and rises up by the side of it, instead of raising the needle. Four months' working of this thermometer has proved it to be a valuable invention. The instrument must come into general use among meteorologists.

The Solar and Terrestrial Radiation Thermometers are made entirely of glass, the scale being engraven upon the bulb itself. For thermometers continually exposed to damp, the swelling of the wood, and the obliteration of the index thereon, have been felt a great annoyance; consequently, the substitution of glass for wood has been hailed with pleasure by meteorologists, independently of its great improvement in a scientific point of view.

The white enamel placed along the back of the thermometer tube—an invention of Negretti and Zambra's—is now becoming generally adopted; the improvement is at once manifest on inspecting two instruments, the one with, and the other without the enamel.

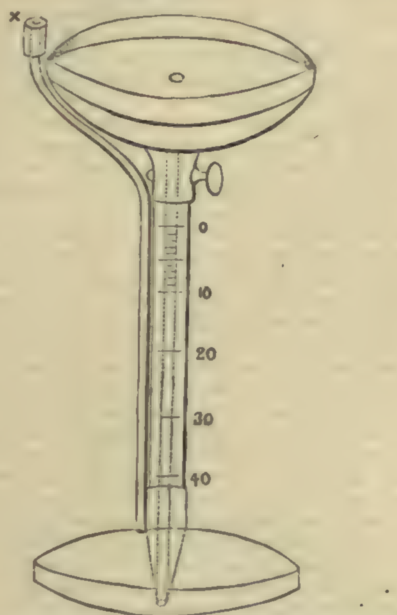
The Wet and Dry Bulb Thermometer.—The dry bulb is the ordinary thermometer; and the wet bulb differs only in having the bulb enclosed in a muslin bag, with a cotton-wick conductor to a cup of water, so that it shall always be wet from the capillary action of the cotton conveying the water constantly to it. If the muslin bag were attached to self-registering, instead of ordinary thermometers, the greatest heat and cold of the wet bulb would be obtained—an important addition to the meteorological instruments, yet one almost unknown. The muslin and cotton should be changed every month.

Bennett's Photographic Wet and Dry Bulb Thermometer is an additional evidence of the close relationship between heat and light. In this most ingenious contrivance, *light* is incessantly writing down the *heat* of the air. It is simply requisite to place a sheet of photographic paper upon the roller once a-day, in order to record every change in temperature of the wet and dry bulb for the twenty-four hours. This is, indeed, a triumph of science.

EVAPORATORS.—Having had constructed gauges of various sizes, we are enabled to speak with confidence as to their working. The water in gauges under eight inches in diameter becomes too warm, owing to the small quantity that can be contained in them; consequently, an excess of evaporation results. There are two gauges to be recommended: the one Newman's, and the other Negretti's. These gauges work well. Newman's is a very convenient and ornamental instrument, having a graduated glass tube. It consists of a short cylinder, twelve inches in diameter, having connected with it, by means of a stopcock, a glass tube graduated to hundredths, and terminating in a lower vessel, which will contain a sufficient quantity of water, to be raised by artificial pressure into the upper one, for exposure to the atmosphere.

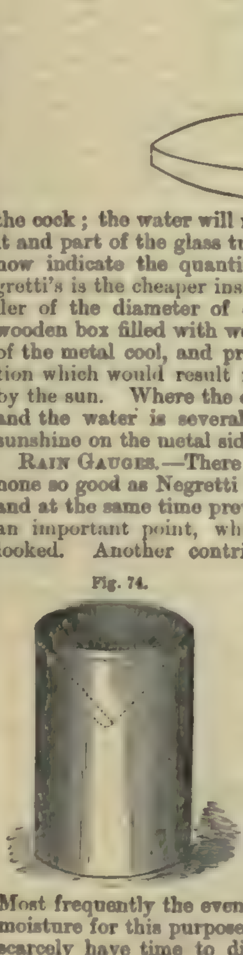
To use the apparatus, pour water into it until it rises to the zero in the glass tube; then, by means of a syringe, force the air through the tube × (Fig. 73) into the lower vessel, so as to raise the water into the upper one to any height you please. Now shut off the stopcock beneath to retain the water in the upper vessel; then, having exposed the apparatus for any length of time required, open the cock; the water will run into the lower vessel, filling it and part of the glass tube, the divisions of which will now indicate the quantity of water evaporated. Negretti's is the cheaper instrument, being simply a cylinder of the diameter of eight inches, which fits into a wooden box filled with wet sand; this keeps the outside of the metal cool, and prevents that excessive evaporation which would result from the heating of the metal by the sun. Where the diameter of the gauge is large, and the water is several inches deep, the effect of the sunshine on the metal sides is not felt.

Fig. 73.



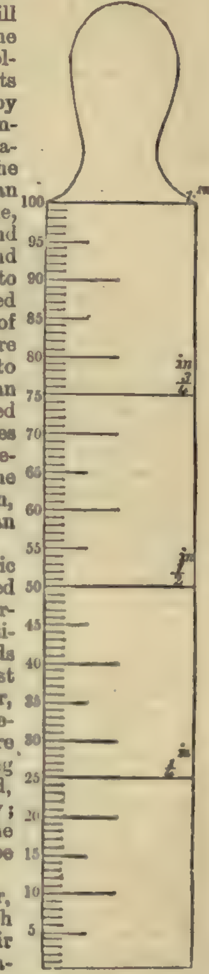
Rain Gauges.—There are several constructions, yet none so good as Negretti and Zambra's, which is simple, and at the same time prevents any loss by evaporation—an important point, which has been too much overlooked. Another contrivance is that of a cylindrical vessel of brass or zinc (Fig. 74); the latter would be the cheapest, and answer all purposes equally well. Into this cylinder, a funnel, with its tube bent, fits tightly: the diameter should be eight inches, and the tube about an inch in length. The object of this bent tube is to prevent evaporation taking place from the surface of rain collected in the rain gauge, for a few drops of water will hermetically seal the opening from the escape of vapour. Most frequently the evening dews will deposit sufficient moisture for this purpose, which the heat of the day will scarcely have time to dissipate, before night brings a fresh supply.

Fig. 74.



The readiest mode of measuring the amount deposited in the gauge, is by procuring another cylindrical vessel, or measure, which is exactly four inches in diameter, and four inches deep: this, when quite full, will just contain an amount equal to the deposit of an inch of rain as collected in the eight-inch gauge. Parts of an inch can be measured by plunging a rule (Fig. 75) perpendicularly to the bottom of the measure, the portion wetted by the water being the decimal part of an inch required. Having made a rule, exactly four inches in length, and divided it into ten equal parts, and each division being subdivided into ten others, a measure is obtained which will read off the hundredth of an inch; and, as the divisions are tolerably wide, it is not difficult to estimate even to thousandths of an inch. Thus, for example, as used in the measure, the rule four inches long, divided into 100 parts, represents one inch of rain fallen; the score at twenty-five, or one inch, represents a fall of a quarter of an inch, and so on.

Fig. 75.



Electrometers. — Atmospheric electricity has been much neglected by meteorologists; it is an important item of meteorological investigation. There are several methods of studying the subject: the most simple is Glaisher's electrometer, which, being portable, should become generally adopted. Where there are the conveniences for having exploring wires, properly insulated, the results are more satisfactory; and when this plan is adopted, the following electrometers should be used:—

1. De Saussure's Electrometer, which consists of two fine wires, each terminated by a small pith ball, their expansion being measured by a graduated scale.

2. Volta's Electrometer, consisting of two thin stems of about two inches in length, and fitted to a metal rod by small rings.

3. Singer's Electrometer, consisting of two slips of gold-leaf. For stronger electricity, a pair of Dutch-gold leaves become a useful addition.

4. Zamboni's Dry-pile Electrometer. It is a single gold-leaf suspended from the conducting-rod between two dry piles, the negative pole of the one and the positive of the other being uppermost. This shows whether the electricity is positive or negative.

For powerful electric storms, the self-registering apparatus belonging to the atmospheric recorder is very useful; whilst a nicely-arranged electrical bell will be a means of warning the observer that his presence is required in the electric-room.

The effect of a thunder-storm, when three or four miles distant, as shown on these electrometers, is exceedingly interesting. It is not only possible to witness the instantaneous convulsion caused by a flash of lightning, at least twenty seconds before the peal of thunder occasioned by it is heard, but it is also possible to know, several seconds before a flash takes place, that one is about to occur.

The beneficial effects of electricity on the vegetable kingdom are of a character so apparent, that any extended researches upon the branch of Meteorology calculated to throw additional light upon the subject is very desirable.

The Gimbal Vane.—This is a wind-vane exactly balanced and hung on gimbals, having vertical fans to carry it in the direction of the wind, and horizontal fans to enable it to tip *up* or *down*, to show the angle at which the air is blowing.

The Rain Angle is a simple contrivance for showing the angle at which the rain descends. It consists of a rod, which moves along a graduated arc.

The Distance Measurer.—A modification of the carpenter's rule, having an elevated eye-hole, fixed vertically in the joint, and a slender pin screwed vertically on each leg of the rule. The rule itself slides on a graduated arc. In order to measure approximately the distance of a peculiar cloud, or other celestial phenomenon, from any fixed object, the legs of the rule are opened until one of the vertical pins covers the object to be measured, and the other the object to which it is referred. The distance will be seen on the graduated arc.

Ozonometer.—A chemical contrivance for ascertaining the amount of ozone in the atmosphere. Ozone was discovered by Dr. Schönbein of the University of Bâle, in the year 1848. Test-papers are hung in a situation where there is a free current of air, but not in sunshine;—the north face of a thermometer-stand is a convenient situation. These papers will remain colourless if there be no ozone present, and will be more or less tinged with blue, as the ozone is more or less powerful. A scale of different tints, marked from 0 to 10, is furnished with the test-papers, to which the slips are referred after exposure to the atmosphere, immediately after immersion in water for the space of one minute.

These test-papers are made in the following manner:—

200 parts of distilled water,
10 „ of starch,
1 „ of iodide of potassium,

boiled together for a few seconds, and then slips of bibulous paper dipped into the solution; when dry, they are ready for use. If there be any ozone present in the air, it seizes on the potassium—the blue colour left being iodide of starch. Other test-papers have been prepared by Dr. Moffat of Hawarden, which do not require to be plunged into water, in order to bring out the proper colour. Dr. Moffat's test-papers must be kept in the dark. They are placed in a box, the bottom of which is perforated with holes for the free passage of air. These test-papers, if kept in the dark, will last for several years, retaining the amount of discoloration.

The Wind-Vane should be so contrived as to move with the least amount of friction; otherwise, in calm weather, the changes will not be seen to take place in the direction of the wind at the proper time.

The Atmospheric Recorder, although too expensive for the ordinary meteorologist, still cannot be passed over in silence. Having had one in work for a year, we can speak of its value. Every change in the atmosphere is written down by pencils at the precise moment of occurrence. It is merely requisite to supply the cylinders with sufficient paper; after which, if the clock be occasionally wound up, and the pencils kept capable of drawing a clearly-defined line, no further care or trouble is needed. This machine is incessantly writing down the force of every gust of wind, the extent of every change in its direction, the commencement and termination of every shower, the quantity of rain which has fallen, and the amount of evaporation and electricity. The temperature of the air, the pressure and hygrometrical condition, are recorded every quarter of an hour. The discussion of these records cannot fail in being productive of much good to meteorologists. Mr. Dollond makes this machine.

The Actinometer.—An instrument for ascertaining the force of solar radiation. It consists of a large hollow cylinder of glass, soldered at one end to a thermometer tube, terminated at the other end by a ball drawn out to a point, and broken off so as to leave the end open. The cylinder is closed at the other end. It is filled with a deep blue liquid (ammonio-sulphate of copper). The cylinder is enclosed in a chamber, which is blackened on three sides. This instrument requires careful manipulation, and is but little used.

Intensity of Light.—The gauge for ascertaining this consists of a long narrow box, with a window at one end facing the north. The box is painted black inside, with a white belt of a couple of inches in width, graduated up to 100. The brighter the day, the further can the figures be seen up this box.

The Transit Instrument is a useful addition. It has been well described by Mr. Breen in the Section on Astronomy. A very close approximation to correct time may be obtained by the use of Dollond's Portable Transit.

A good astronomical clock, with a mercurial pendulum, ought to be found in every observatory, and a good watch in the pocket of every observer; each should be provided with the seconds-hand.

Whewell's Anemometer, invented by the Rev. Dr. Whewell, is an ingenious self-registering anemometer, which gives the amount of horizontal movement in the air. A system of wheels is worked by windmill-sails, according to the velocity of the wind; and these carry a pencil, which is constantly recording the direction and velocity of the wind.

Osler's Anemometer constantly records the force and direction of the wind, and also the amount of rain. An admirable invention. Unfortunately, such instruments as this and Dr. Whewell's are too expensive for the majority of meteorological observers.

The Electrical Observatory at Kew, under the able management of Mr. Ronalds, has become a most important meteorological station, being, in fact, a collection of all requisite electrometers; a brief description of which will be found in Dr. Drew's *Practical Meteorology.*

Daniell's Hygrometer.—An instrument for ascertaining, from direct observation, the temperature of the dew-point. This is an interesting contrivance, as a ring of dew is precipitated at the temperature of the dew-point. It consists of two glass balls, communicating with each other by means of a bent tube. The one ball is of black glass, and the other transparent. A thermometer is fixed with its bulb within the blackened ball; and as soon as three-fourths of this ball is filled with sulphuric ether, it is immersed. The air having been exhausted, the tube is hermetically sealed. The transparent ball is covered with muslin. A duplicate thermometer for ascertaining the temperature of the air, is attached to the stem of the instrument. To find the temperature of the dew-point, all the ether must be made to run into the black ball; ether is then poured from a phial on the muslin: this produces rapid evaporation, and the temperature is cooled down to that of the dew-point; at this temperature, a ring of dew is formed round the black bulb, and at this instant the immersed thermometer must be read off; rapidity of observation being necessary, as the temperature will continue to fall below this point, and the ring of dew also to increase in width. In a few seconds the ring will gradually disappear, and at this moment the thermometer should be read a second time. The mean of the two readings will give the temperature of the dew-point. It is necessary that the ether should be very good.

The results obtained by actual observation and by calculation (from the wet and dry-bulb thermometers) are very nearly identical, as the following illustration will show:—

By Observation.

Daniell's hygrometer, temperature of air . .	56·0°
Daniell's hygrometer, temperature of dew-point when ring was formed	48·1
Daniell's hygrometer, temperature of dew-point when ring disappeared	48·2
Daniell's hygrometer, mean temperature of dew-point	48·1
Daniell's hygrometer, temperature of dew-point below temperature of air	7·9

By Calculation.

Temperature of dry bulb	56·0
Temperature of wet bulb	52·0
Calculated dew-point	48·2
Temperature of dew-point below temperature of air	7·8

Regnault's Hygrometer.—Another instrument for ascertaining the dew-point from direct observation. It differs considerably in its construction. Some meteorologists prefer it to Daniell's hygrometer. Messrs. Negretti and Zambra have constructed a modification of Regnault's hygrometer, and from them it can be procured at the same price as that usually charged for Daniell's hygrometer.

Connell's Hygrometer is a third instrument for ascertaining the dew-point from observation. In this, the temperature is lowered to the dew-point by means of an exhausting syringe. The wet and dry bulb thermometer, however, answers every purpose, and, to the ordinary observer, is not so liable to be incorrectly read off.

The Cyanometer consists of a flat ring, divided into 53 equal parts, and numbered from 0 to 52, the 0 being white, and 52 very dark blue; the other numbers are painted every intermediate tinge from nearly white to deep blue. By its use, the colour of the sky can be comparatively recorded.

GENERAL DIRECTIONS.—The following occasional phenomena require a few words. The most uninstructed observer may give useful information on these subjects with comparatively little trouble to himself, by making known the particular features to be observed. It is desirable to note, in

Thunder-storms, the direction in which they move, the point of the horizon in which they were first noticed, and that in which they disappeared; the time when thunder was first and last heard; the colour of the lightning; the number of seconds elapsing after a flash before thunder became audible (noted at different periods during the storm's continuance); the commencement and termination of rain; the direction of the wind before, after, and during its continuance; the time when the electrical breeze springs up (this is a peculiarly violent breeze noticed in most thunder-storms); whether hail falls; and any other feature that may appear remarkable or be deemed desirable to be recorded.

Aurora Borealis.—Its position amongst the stars, whether merely a low auroral arch, or accompanied by coruscations. If a brilliant display, whether a cupola or dome is formed, a little south of the zenith; and if formed, whether oscillatory among the stars. The hour of its occurrence when seen as a diffused light; if there be floating patches of luminous haze or cloud, &c.

Solar and Lunar Halos.—When visible, the quarter of the heavens where they appeared, and how long they remained visible.

Mock Suns and Complicated Circles of Light.—Their form and position with respect to the sun or moon; whether prismatic.

Meteors or Falling Stars.—Their apparent size, shape, colour, path amongst the stars, velocity and duration; whether accompanied by a streak of light, or separate fragments; if large, whether a streak of light remained after the meteor itself had disappeared; and after bursting, whether any noise of explosion be heard—if so, how many seconds after the meteor itself had burst; the time of appearance, &c. These observations should more especially be attended to from the 6th to the 16th of August, and from the 9th to the 14th of November.*

Gales of Wind.—Their direction, and estimated force; when they commenced and terminated; the height of the barometer during their continuance.

Snow.—When it fell, how deep in inches on the ground, and the form of the snow-crystals. To sketch the crystals, a magnifying glass is requisite.

Hail.—The shape of the stones, &c.

The times of breaking up of long dry periods, and frosts; the termination of rainy periods; the commencement and duration of fogs, wind-changes, &c.

Solar Eclipses.—During their continuance, and before and after, record the temperature in sun and shade repeatedly. Expose for ten seconds, every five minutes, slips of sensitive paper, to ascertain the effect of the diminution of sunlight.

REQUISITE TABLES OF REDUCTION.—1st. *Glaisher's Hygrometrical Tables* (2nd edition). These splendid tables enable the observer, with comparatively little labour, to calculate the temperature of the dew-point; the elastic force of vapour; the weight of vapour in a cubic foot of air; the additional weight of vapour required to saturate a cubic foot of air; the degree of humidity; the whole amount of water in a vertical column of the atmosphere; the weight of a cubic foot of air; and also to separate the pressure due to vapour from that due to the gases, for any temperature from 10° to 100° F. There is also a table for reducing the readings of the barometer to the level of the sea. Mr. Glaisher has calculated this table from the fact determined by M. Regnault, that air expands $\frac{1}{491\cdot13}$th part for every increase of 1° of heat.

2nd. *Tables of the Corrections for Temperature to Reduce Observations to 32° F. for Barometers with Brass Scales*, by J. Glaisher, Esq. It is absolutely requisite to reduce the readings of the barometer to a certain acknowledged temperature, otherwise the true pressure of the air could not be ascertained; for it must be remembered that, besides the pressure of the air on the mercury, the mercury itself obeys the same law which is pointed out to us by the thermometer—*i.e.*, it expands by heat, and contracts by cold. Thus, suppose the actual pressure of the air to be stationary, the barometer will be seen to rise or fall, if there be an increase or decrease in the temperature of the air. In like manner, the metal scale of the barometer is subject to expansion and contraction by an increase of heat or cold; and this, as Mr. Glaisher says, explains the apparent anomaly that, although the readings are said to be reduced to 32° (or the freezing-point), the point of no correction is 28½°. As metal bars will vary in length with every degree of temperature, it is apparent that a certain temperature should be determined upon at which the standard unit of measure must be referred to.

Now this temperature has been fixed at 62° F.; therefore above 62°, as the metal will expand, this will make the divisions on the scale of the barometer too large, and consequently the barometer will read lower than it should do; on the contrary, below 62°, the metal contracting, will bring the index divisions closer together, and the barometer will read too high, unless corrected. Thus, owing to this cause alone, at the temperature of 32°, the barometer is made to reach ·009 of an inch too high.

The great use of the reduction for temperature will be at once apparent, when an example is given: thus, suppose a person has two barometers, one in a room heated artificially, and the other as cold as possible;—

Reading of barometer at temperature of 90° = 30·200
Correction for temperature . . . — 0·165

Reading corrected for temperature . . 30·035 in.

Reading of barometer at temperature of 40° = 30.066
Correction for temperature . . . — ·031

Reading corrected for temperature . . 30·035 in.

The difference ·134 inch between the two readings, being due to the expansive action of heat.

These tables have been calculated trom Schumacher's formula, which is here copied:—

$$-z \times \frac{m\,(t-32^\circ)-s\,(t-62^\circ)}{1+m\,(t-32^\circ)}$$

z = reading of barometer.
m = the expansion in volume of mercury for 1° F. = 0·0001001.
t = the temperature of the mercury and the scale.
s = the expansion of the brass scale in length for 1° F. = 0·000010434 (the normal temperature being 62°).

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* During these periods, the appearance of "falling stars" is always found to be more frequent than during other times of the year.—ED.

Thus the formula becomes—

$$s \times \frac{0{\cdot}0001001 \times (t-32) - 0{\cdot}000010434 \times (t-62)}{1 + 0{\cdot}0001001 \times (t-32)}$$

3rd. *Table of Corrections to be applied to Meteorological Observations for Diurnal Range*, prepared by the Council of the British Meteorological Society. These tables are of the utmost importance, as they enable an observer, from one, two, or three readings daily, to find from them the true monthly means; in fact, to make his observations represent a reading taken every hour, day and night. Thus, if a reading of the barometer be made daily at 3 A.M. in March, the mean will be ·023 too low, or, at 11 A.M., ·015 too high. The necessity of this reduction becomes very evident from hourly readings of the thermometer; for, suppose the readings are made in June, at 4 A.M., the mean will be 9·3° too low, or, if at 2 P.M., 8·6° too high.

The only correction requisite for the reduction of meteorological observations not found in the three above-mentioned tables is that for capillarity—the capillary action of the tube of a barometer depressing the mercury by a quantity inversely proportional to the diameter of the tube. The following table will be found sufficient for this reduction: it is copied from the work published by the Committee of Physics and Meteorology of the Royal Society:—

Correction to be added to Barometer Readings for Capillary Action.

Diameter of tube. Inch.	Correction for unboiled tubes. Inch.	Correction for boiled tubes. Inch.
0·60	+ 0·004	+ 0·002
0·50	0·007	0·003
0·45	0·010	0·005
0·40	0·014	0·007
0·35	0·020	0·010
0·30	0·028	0·014
0·25	0·040	0·020
0·20	0·060	0·029
0·15	0·088	0·044
0·10	0·142	0·070

The following reductions for meteorological observations will supply examples of every reduction necessary:—

BAROMETER REDUCTIONS.—To find the mean pressure of the barometer for the month of February, 1856 (height above the sea-level, 281 feet).

Sum of all the readings made at 9 A.M., 867640;
at 10 P.M., 867528.
Divide the above by the number of observations.

29)867640(29·918 inches.	29)867528(29·915 inches.
58	58
287	287
261	261
266	265
261	261
54	42
29	29
250	138
232	145
−18	—7

Sum of all the readings of the attached thermometer at 9 A.M., 13435; at 10 P.M., 13745.

29)13435)46·3° temp. of mercury.
116
183
174
95
87
8

29)13745(47·4° temp. of mercury.
116
214
203
115
116
−1

Mean pressure at 9 A.M.	= 29·918
Correction for temperature of 46·3° . .	= —·047
Mean pressure corrected for temperature .	= 29·871
Index error	— ·002
Mean pressure further corrected for index error	29·869
Corrections for capillarity, the mercury being boiled	+ ·002
Correct reading for 9 A.M.	= 29·871
Correction for diurnal range for February .	— ·008
Approximate mean pressure . . .	= 29·863
Mean pressure at 10 P.M.	= 29·915
Correction for temperature of 47·4 . .	= —·050
Mean pressure corrected for temperature .	29·865
Index error	— 002
Mean pressure further corrected for index error	= 29·863
Correction for capillarity, the mercury being boiled	+ ·002
Correct reading for 10 P.M.	29·865
Correction for diurnal range for February .	— ·007
Approximate mean pressure . . .	29·858
Ditto. ditto. . . .	29·863
Sum of the two observations . . .	2)59·721
Adopted mean pressure for the month .	= 29·8605

To reduce the mean pressure of the month to the sea-level, the adopted mean temperature of the air being 36 0°, and the cistern of the barometer 281 feet above the level, the adopted mean pressure = 29·860 inches.

In Table 2 of *Glaisher's Hygrometrical Tables* (page v.), showing the volume of a mass of dry air, after expansion from heat, for each degree of Fahrenheit's scale, it will be seen that a stratum of air 90 feet in thickness, will balance a column of mercury 0·1 inch in height.

The factor for 36° is 1·008
Multiply this by 90 ft. 90

feet.
90·720 = 90·7)281·0(·3098 of an in.
2721
8900
8163
7370
7256
114

Adopted mean pressure for altitude of 281 feet	29·860
Correction to reduce to sea-level . . .	+ ·310
At sea-level the mean pressure is . .	30·170 in.

$$\text{As } 90{\cdot}7 : \overset{\text{in.}}{0{\cdot}1} :: \overset{\text{feet.}}{281} : = \frac{281 \times 0{\cdot}1}{90{\cdot}7} = 0{\cdot}310.$$

Another method, based upon the theorem of Sir George Shuckburgh and the calculations of Regnault, has been described by Dr. Drew of Southampton, who has constructed a table showing the height, in feet, of a column of air equivalent in weight to a column of mercury one inch in height, at different temperatures, under a pressure of thirty inches of mercury. This table is copied from Dr. Drew's *Practical Meteorology*.

Temp.	Feet.	Temp.	Feet.	Temp.	Feet.
30	865·1	47	895·2	64	925·2
31	866·8	48	897·0	65	927·0
32	868·5	49	898·8	66	928·8
33	870·3	50	900·5	67	930·6
34	872·1	51	902·2	68	932·4
35	873·9	52	903·9	69	934·1
36	875·7	53	905·7	70	935·8
37	877·5	54	907·5	71	937·5
38	879·3	55	909·3	72	939·3
39	881·1	56	911·1	73	941·1
40	882·8	57	912·9	74	942·9
41	884·5	58	914·7	75	944·7
42	886·2	59	916·5	76	946·5
43	888·0	60	918·2	77	948·3
44	889·8	61	919·9	78	950·1
45	891·6	62	921·6	79	951·8
46	893·4	63	923·4	80	953·5

For any temperature above that given in the table, the addition of 1·7 feet for every degree; and for any temperature below that given in the table, the subtraction of 1·7 feet for every degree, will give the factor required.

To work out the addition required to reduce to sea-level, let T represent the tabular number opposite the temperature of the air, s the reading of the barometer at f feet above the sea-level, and x the correction required; then

$$x = \frac{f}{T} \times \frac{s}{30}$$

Dr. Drew gives the following practical example:—The barometer being 29·500 inches, the temperature being 50°, and the height above sea-level 60 feet, what correction will be required?

$$x = \frac{60}{900 \cdot 5} \times \frac{29 \cdot 5}{30} = 0 \cdot 065 \text{ correction required.}$$

∴ 29·500 + 0·065 = 29·565 inches (the pressure reduced to the sea-level).

Thermometer Reductions.—To find the mean temperature for the month of December, 1855, from observations made at 9 A.M. and 10 P.M., and from self-registering thermometers.

Sum of all the readings made at 9 A.M, 10915; and at 10 P.M., 11017.

31)10915(35·2° mean at 9 A.M.
93 + 0·9 cor. for diurnal range.
161 36·1 approx. mean for month.
155
65
62
3

31)11017(35·5° mean at 10 P.M.
93 + 0·5 cor. for diurnal range.
171 36·0 approx. mean for month.
155
167
155
12

Sum of all the readings of a self-registering minimum thermometer, 9365; and maximum thermometer, 12759.

31)9365(30·2° mean minimum temp.
93 + 0·2 index error.
65 30·4 corrected for index error.
62

31)12759(41·2° mean maximum temp.
124 − 0·2 index error.
35 41·0 corrected for index error.
31
49
62
−13

30·4° mean minimum corrected.
41·0 mean maximum corrected.

2)71·4 sum of the two series.

35·7 mean of the two series.
0·0 correction of diurnal range.

35·7 approx. mean from self-registering instruments.
36·1 approx. mean from hourly observations at 9 A.M.
36·0 approx. mean from hourly observations at 10 P.M.

3)107·8 sum of the three series.

35·9 adopted mean temperature for December, 1855.

To find the mean temperature of the wet bulb thermometer, for the month of December, 1855, from daily observations made at 9 A.M. and 10 P.M.

Sum of all the readings at 9 A.M., = 10509; at 10 P.M., = 10658.

31)10509(33·9° mean at 9 A.M.
93 + 0·6 cor. for diurnal range.
120 34·5 approximate mean.
93
279
279

31)10658(34·4° mean at 10 P.M.
93 + 0·2 cor. for diurnal range.
135 34·6 approximate mean.
124
118
124
−6

Approximate mean from observations made at 9 A.M. . . . . . . = 34·5°
Approximate mean from observations made at 10 P.M. . . . . . . = 34·6

Sum of the series . . . . . . 2)69·1

Adopted mean temperature of the wet bulb = 34·5

To find the mean temperature of evaporation.

Adopted mean temperature of the air from dry bulb . . . . . . = 36·0°
Adopted mean temperature of the wet bulb = 34·5

Difference . . . . = 01·5−
Adopted mean temperature of air . . = 35·9

Difference, or mean temperature of evaporation . . . . . . 34·4

To find the mean temperature of the dew-point, the mean dry bulb* being 35·8°, and the mean wet bulb 34·5°.

Example 1.—In the hygrometrical table for 35°.

The dew-point opposite to 34° wet bulb, is	32·4 –	32·4
The dew-point opposite to 35° wet bulb, is	35·0	
Difference on the increase in dew-point for an increase of 1° in wet	2·6.	
Proportional part of increase for 0·5, is		+ 1·3
Temperature of the dew-point corresponding to 35° dry, and 34·5° wet, is		33·7
In the fourth column, the decrease of dew-point for an increase of 1° in the dry bulb, is — 1·5°; the proportional part for 0·8°, is		— 1·2
The adopted temperature of the dew-point for 35·8° dry, and 34·5° wet, is		= 32·5

Example 2.—To find the mean temperature of the dew-point for the month of December, 1855, dry bulb being 36·0°, and wet bulb 34·5°.

In the hygrometrical table for 36°,

The dew-point opposite to 34° wet bulb, is	31·0 –	31·0
The dew-point opposite to 35° wet bulb, is	33·5	
Difference	2·5	
Proportional part of the increase for 0·5°, is		+ 1·2
Mean temperature of dew-point		= 32·2

The same tables, and the same manner of reduction, will find "the elastic force of vapour," "the weight of vapour in a cubic foot of air," "the additional weight required to saturate a cubic foot of air," and the "degree of humidity." To render the tables clear, one example of each will be worked out for a temperature of 35·8° dry bulb, and 34·5° wet bulb.

To find the elastic force of vapour of the temperature of 35·8°, the wet bulb being 34·5°.

In the hygrometrical table for 35°,

	Inch.	Inch.
The elastic force of vapour opposite to 34° wet bulb, is	0·184 –	0·184
The elastic force of vapour opposite to 35° wet bulb, is	0·204	
Difference, or the increase for an increase of 1° in wet	0·020	
Proportional part of the increase for 0·5, is		+ 0·010
Elastic force of vapour corresponding to 35° dry, and 35·5° wet, is		0·194
In the 6th column, the decrease of the elastic force of vapour, for an increase of 1° in the dry bulb, is — 0·010; the proportional part for 0·8°, is		— 0·008
Mean elastic force of vapour		= 0·186

To find the whole amount of water in a vertical column of the atmosphere—*i. e.*, from the surface of the earth to the top of the atmosphere.

This is found by multiplying the elastic force of vapour by the constant 1383, which would give, in this instance, for the whole amount of water in a vertical column of the atmosphere, if precipitated on the earth at one time, a depth of 2½ inches.

To find the weight of vapour in a cubic foot of air. In the hygrometrical table of 35°,

	Grains.	Grains.
The weight of vapour in a cubic foot of air, opposite 34° wet bulb, is	2·1 –	2·1
The weight of vapour in a cubic foot of air opposite 35° wet bulb, is	2·4	
Difference, or the increase in weight of vapour for an increase of 1° in wet, is	0·3	
Proportional part of the increase for 0·5°		+ 0·1
Weight of vapour in a cubic foot of air corresponding to 35° dry and 34·5° wet, is		2·2
In the 8th column, the decrease of weight of vapour for an increase of 1° in the dry bulb, is 0·2; the proportional part for 0·8°, is		— 0·2
The adopted weight of vapour for 35·8° dry, and 34·5° wet, is		2·0

To find the additional weight of vapour required to saturate a cubic foot of air.

In the hygrometrical table for 35°,

The dry bulb being 35·8°, and the wet bulb 34·5°.

	Grain.	Grain.
Additional weight of vapour required to saturate a cubic foot of air opposite 34° wet bulb, is	0·3 –	0·3
Additional weight of vapour required to saturate a cubic foot of air opposite 35° wet bulb, is	0·0	
Difference, or the decrease in amount required to saturate a cubic foot of air for an increase in 1° in wet bulb, is	0·3	
Proportional part of the decrease for 3·5°		— 0·2
Additional weight of vapour required to saturate a cubic foot of air corresponding to 35° dry and 34·5° wet, is		= 0·1
In the 10th column, the increase in the weight of vapour required to saturate a cubic foot of air for an increase in 1° dry, is + 0·3; the proportional part for 0 8°, is		+ 0·2
The adopted additional weight of vapour required to saturate a cubic foot of air for 35 8° dry and 3·45° wet, is		= 0·3

To find the degree of humidity, the temperature being 35·8° and the wet bulb 34·5°, complete saturation = 100.

In the hygrometrical table for 35°.

Degree of humidity opposite 34° wet bulb, is	90 –	90
Degree of humidity opposite 35° wet bulb, is	100	
Difference, or the increase of humidity for an increase of 1° in wet, is	10	
Proportional part of the increase for 0 5°, is		+ 5
Degree of humidity corresponding to 35° dry and 34·5° wet, is		95
In the 12th column, the increase in the degree of humidity for an increase of 1° in the dry bulb, is 9; the proportional part for 0 8°, is		— 7
Adopted degree of humidity for 35·8° dry and 34·5° wet, is		= 88

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* The hourly readings of the thermometer, which are made at the same time as those of the wet bulb, are called the "dry bulb."

To find the weight of a cubic foot of air, the mean pressure of the barometer being 29·742 inches dry bulb 35·8°, and wet bulb 34·5°,

	Grain.
In the hygrometrical table for 35°.	
In column 13, the weight of a cubic foot of air (the barometer being 29·0 inches) opposite 34° wet, is	543·4
In column 14, the decrease of weight for an increase of 1° in dry, is −1·1°; the proportional part for 0·8°, is	−0·9
	542·5
In column 14, the increase of weight for an increase in the reading of the barometer for one inch, is + 18·7° grains.	
Opposite 7 in the table, under 18·7° in last column, is	13·1
Opposite ·04 in the table, in right-hand corner of the page, is	0·7
The adopted weight of a cubic foot of air is .	556·3

To find the mean pressure of dry air, or that due to the gases when separated from the water contained in the air.

Subtract the adopted elastic force of vapour (which is the pressure of the water contained in the air) from the adopted pressure of the barometer.

Example.

If the mean pressure of barometer be	= 29·742 inches,
And the elastic force of vapour is .	= 0·186 inch,
The pressure of dry air (of that due to the gases) will be	= 29·556 inches.

To find the monthly range of temperature, &c., deduct the coldest temperature from the hottest observed during the month. Thus—

Hottest temperature . . .	50·8°
Coldest temperature . . .	12·0
Monthly range of temperature .	38·8

To find the mean daily range of temperature, deduct the mean of all the readings of the minimum thermometer, from the mean of all the readings of the maximum thermometer.

Mean maximum temperature . .	41·0°
Mean minimum temperature . .	30·4
Mean daily range of temperature . =	10·6

To find the amount of terrestrial radiation, deduct the reading of a minimum thermometer, placed on the grass, from that of a minimum thermometer, placed four feet above the grass. Thus—

Greatest cold four feet above the grass	36·5°
Greatest cold on the grass . .	27·8
Amount of terrestrial radiation . . =	8·7

To find the amount of solar radiation, deduct the reading of a maximum thermometer (with a blackened bulb) placed in full sunshine from the greatest heat in shade. Thus—

Greatest heat in sunshine . . =	59·3°
Greatest heat in shade . . . =	34·1
Amount of solar radiation . .	25·2

To find the greatest heat and greatest cold of the wet bulb thermometer. This is obtained by attaching the muslin and cotton conductor to *self-registering* thermometers, instead of to the ordinary thermometer. Thus—

Greatest cold of dry bulb . .	34·0°
Greatest cold of wet bulb . .	32·5
Difference	1·5
Greatest heat of dry bulb . .	46·7
Greatest heat of wet bulb . .	42·8
Difference	3·9

To find the range of temperature of the wet bulb thermometer, deduct the greatest heat from the greatest cold recorded. Thus—

Greatest heat.	42·8°
Greatest cold	32·5
Range of wet.	10·3

To find the mean amount of cloud.

This, by practice, is accomplished very accurately by estimation, 10 being considered an overcast sky, and 0 a cloudless sky. It must be remembered, in making the estimate, that, with a partially covered sky, the forms of the clouds, and the space they cover, are only correctly seen in the zenith; the nearer the horizon we approach, the more obliquely do we look upon them; and, consequently, near the horizon, the sky will appear to be more overcast than it is in reality. The observer's judgment should, therefore, be more especially confined to the upper half of the sky.

Observations made at 9 A.M. and 10 P.M., December, 1855.

Sum of all the estimates at 9 A.M., 2158; and at 10 P.M., 2024.

31)2158(6·96° mean at 9 A.M.
186 − 0·10 cor. for diurnal range.
298 6·86 corrected amount.
279
190
186
. . 4

31)2024(6·5° mean at 10 P.M.
186 + 0·2 cor. for diurnal range.
164 6·7 corrected amount.
155
. . 9

Corrected reading at 9 A.M.	6·9°
Corrected reading at 10 P.M.	6·7
Sum of the two series	2)13·6
Adopted mean amount of cloud . .	6·8

CLASSES OF CLOUDS.—Mr. Luke Howard, the well-known meteorologist, was the first to classify the clouds; and these classes have been very conveniently abbreviated by Mr. Glaisher. They are—

Cirrus	= ci. . . .	feathery-looking, the most lofty cloud.
Cumulus	= cu. . . .	mountainous looking.
Stratus	= st. . . .	the ground-cloud—forms at sunset, and disappears at sunrise.
Cirro-cumulus	= ci.-cu. .	rounded masses, or woolly tufts.
Cirro-stratus	= ci.-st. .	horizontal masses.
Cumulo-stratus	= cu.-st. .	an accumulation of cumuli, sometimes fungus-shaped.
Nimbus	= ni. . . .	rain-cloud.
Scud	= sc. . . .	broken, flying nimbi.

The cut at page 1141 gives an idea of the general appearance of each kind of cloud just mentioned.

In recording the class of clouds, the interest is increased by also recording the direction in which the clouds are moving, the colour of the clouds, and their height and velocity. The height may be conveniently estimated by supposing 6 to represent very high clouds, and 0 those floating along the ground; the velocity, by supposing 6 to represent those moving at the greatest speed, and 0 those motionless.

To find the mean daily amount of rain. Whole amount of rain collected during the month of December, 1855, 0·764 of an inch.

31)0·764(0·025 of an inch being the mean daily amount.
62
——
144
155
——
— 11

To find the mean daily amount of evaporation for December, 1855. Whole amount evaporated, 0·610 inch.

31)0·610(0·0197
31
——
300
279
——
210
217
——
— 7 Therefore, ·0197, or, in even numbers, 0·020 inch is the daily amount.

To find the amount of evaporation in rainy weather.

Suppose 1·000 inch of water is placed in the evaporator, and that a rain-gauge, of the same diameter as the evaporator, and placed at the same level, has recorded 0·510 inch of rain since last observation, whilst the evaporator is found to contain 1·444 inches. Then

1·444 inches the amount in evaporator.
0·510 amount of rain.
——
— 0·934 difference.
1·000 amount of water placed in the evaporator.
——
0·066 of an inch is the amount due to evaporation.

To estimate the force of wind.

Anemometers being expensive, the majority of meteorologists estimate the wind's force. It is recommended that 0 represent a calm and 6 a hurricane, for this estimate is easily converted into 1 lb. pressure on the square foot; the square of the estimate will represent the lb. pressure on the square foot; viz.—

0 = 0 lb.
1 = 1
2 = 4
3 = 9
4 = 16
5 = 25
6 = 36

By practice, a near approximation to the truth can be obtained.

Mr. Belville, in his *Manual of the Barometer*, gives the following concise table of factors for deducing the temperature of the dew-point, from the temperature of the air and that of the temperature of evaporation: this table originally appeared in the *Greenwich Magnetical and Meteorological Observations for* 1844. The dew-point, deduced in this manner, is a close approximation to that obtained by Glaisher's hygrometrical tables.

Temperature.	Factor.	Temperature.	Factor.
28° to 29°	5·7	40° to 45°	2·3
29 „ 30	5·0	45 „ 50	2·2
30 „ 31	4·6	50 „ 55	2·1
31 „ 32	3·6	55 „ 60	1·9
32 „ 33	3·1	60 „ 70	1·8
33 „ 34	2·8	70 „ 80	1·7
34 „ 35	2·6	80 „ 85	1·6
35 „ 40	2·4	85 „ 90	1·5

Multiply the difference between the wet and dry bulb, by the factor corresponding to the temperature of the dry bulb, and subtract the product from the latter, which will give the temperature of the dew-point. Thus, let T be the temperature, W wet bulb, f factor, x the product, and D the dew-point, then

$$x = T - W \times f$$
$$\text{and } D = T - x$$

Example.—Temperature of dry bulb 50°, and wet bulb 45°,

$$50 - 45 = 5$$
$$5 \times 2{\cdot}1 = 10{\cdot}5$$
$$50 - 10{\cdot}5 = 39{\cdot}5^\circ \text{ the dew-point.}$$

By Mr. Glaisher's tables this dew-point is 39·7°.

RECOMMENDATIONS AND PRECAUTIONS.—Preserve, as much as possible, the continuity of the observations. It is desirable not to change the positions of the different instruments, nor even to alter the method of reading and registering. As it is probable, when two persons are employed in taking observations, that each will read slightly different, a series of simultaneous readings should be made, in order to find the personal error of the observers. If, from any cause, the continuity of the register should be broken, on no account attempt to fill the blank so caused by estimation. Be punctual to the hours determined upon for observations, and read off the instruments in the same way every day; by doing this, it is less likely that an observation of any one instrument shall be overlooked. It is convenient to rule and mark off the form of observation upon a slate, to be transferred to the observatory book, after the whole observations have been made. Before calculating the means, it is recommended to examine each column, to ascertain that no evident error of entry has been made;—an inch in the reading of the barometer is a common error. The maximum reading of the thermometer is also sometimes entered in the wrong column, being placed in that of the minimum; and *vice versâ*. Decimal arithmetic should always be used.

Before fixing the barometer, it should be ascertained that the space above the mercury is free from air. Incline the instrument slightly from its vertical position: if the mercury, in striking the upper end of the tube, produce a sharp rap, the vacuum is perfect; if the tap be dull, or not heard, there will be air above the mercury, which must be driven into the cistern by inverting the instrument and then tapping it gently. If the observer do not succeed in producing this sharp report by tapping, the instrument will require the aid of the maker. In fixing the barometer, adjust the tube vertically by the aid of a plumb-line. In reading the instrument, place the eye on the exact level of the top of the column of mercury, so that each side of the index, and the top of the column, shall be in the same horizontal plane.

The thermometers should be protected from rain; and in making a reading, the observer should do it quickly; and whilst doing so, avoid touching, breathing on, or in any way warming the thermometer by the near approach of his person.

Sir John Herschel recommends that every meteorologist should take an observation every hour throughout the twenty-four, on four stated days in the year; viz., March 21, June 21, September 21, and December 21, excepting when one of these days occurred on Sunday, then to substitute for this date the 22nd. The observations to commence at 6 A.M., and terminate at 6 A.M. next morning.

Thermometers should be frequently compared with a standard instrument, in order to ascertain whether the freezing-point has remained at the temperature as marked off on the scale. It is a well-known fact, that the zero point moves; this has been ascertained from the circumstance that, after a great change in temperature, the glass requires a considerable time to enable it to return to its normal condition.

NEWMAN'S STANDARD BAROMETER form of Rutherford's thermometer differs in the following manner from that already described:—A platinum tube is drawn over a steel wire, which is said to prevent the index from fixing by oxidation. Not having had this instrument at work, we are unable to speak of its advantages from practice. A good improvement has also been accomplished with Negretti and Zambra's rain-gauge. We conceived it was liable to have the water in the canal surrounding it emptied into the graduated glass with the rain to be measured. At our suggestion this has been altered; the canal is now placed much lower down the gauge, and there seems no possibility of an erroneous measurement. The gauge may, therefore, be now said to be perfect.

Of late years two new barometers have appeared—viz., the *Aneroid* and *Burdon's*. They are both good indicators of atmospheric pressure, but cannot take the place of a Standard barometer. In the first place, the metal is influenced by temperature; and in the second, there is a chance of the box of the *Aneroid* and the tube of the *Bourdon* losing their vacuum. As a household instrument, when extreme accuracy is not essential, they are preferred to the common wheel barometer.*

At the time of the Great Exhibition of 1851, when the jury were examining the meteorological instruments, a remark was made that a more perfect maximum and minimum thermometer was required, both of which should be *mercurial*; and on the council medal being presented to Messrs. Negretti and Zambra, their attention was called to this suggestion. Not many months had elapsed before the *patent maximum* was produced, and within four years the *patent minimum*. The latter was sent for trial to a few meteorologists about that time, and one of these instruments came to the Beeston Observatory. The experiments made with it have been perfectly successful; indeed, so much so, as to have astonished all who have used it. The ordinary minimum thermometers do not work uniformly with this new instrument, owing to the alcoholic vapour contained in the upper portion of their tube, and which is more or less developed according to the temperature. Two such important improvements have not taken place since the invention of the thermometer; and it is creditable to them that both should have emanated from the same persons who invented the enamelling of fine tubes.

Professor C. P. Smyth, the Astronomer Royal of Scotland, has caused the electric telegraph to work in meteorology. A wind dial at the one extremity of a wire, is made to turn another simultaneously at the other extremity. The time will come when all large towns will have buildings devoted to these observations, and in which dials will be seen in every direction, some labelled "Edinburgh," others "Liverpool," "Dublin," "London," "Paris," "York," &c., and where the public will be enabled to see the direction of the wind at the same instant at most remote places. The benefit to the farmer and the navigator will be great from such an arrangement. Were such stations to be thickly scattered throughout the country, every change of wind, and every shower, could be traced and recorded, and a knowledge imparted, the benefit of which could not be sufficiently appreciated.

To be enabled to announce the approach of a thunderstorm, however, at a time when the sky is free from clouds, and to ascertain its speed so as to foretel when it may be expected in any given place, would afford the farmer an opportunity of so benefiting by the information, that he would gladly pay a small rate in order to take advantage of it. The world is slow in appreciating any new invention until its usefulness is experienced: let us, however, hope that such invaluable stations may speedily be established, and we venture to predict that, at no very distant period, every conspicuous eminence will have its *road station*, to impart the information collected at the principal establishments, where the electric wires are made to record the changes as they occur. Of late years considerable progress has been made in this direction. Admiral Fitzroy has accomplished much by the "forecasts" he has daily published; and has largely benefited our seafaring population, by "warnings" communicated to the various seaports. (See following chapter).

It is a common expression that nothing is more changeable than the weather; yet that all these changes are governed by certain laws, is as certain as that gravitation binds the heavenly bodies together. Were there no laws to keep the changes within certain limits, we should at one period experience cold, and at another heat so intense, that existence would be intolerable; the earth would be deluged with rain, and anon parched up with drought; the sky would be cloudy for months, perhaps years together, and then cloudless for as long a period; in short, the laws which govern the weather, keep the extreme changes within proper bounds.

As the more complicated machinery of a meteorological observatory cannot be expected to be found except in our principal establishments, it will be requisite to mention what instruments are absolutely necessary for the ordinary observer. These are—A standard barometer; a wet and dry bulb thermometer; a maximum and a minimum thermometer; a rain-gauge; an evaporator; a thermometer-stand; a wind-vane. These would cost from £17 to £30, according to which barometer is selected. The following additional instruments are also desirable (the expense would be £4 or £5):—The solar and terrestrial radiation thermometers; Glaisher's electrometer; an ozonometer; and an extra rain-gauge.

SNOW-GAUGE.—The gauge used at Beeston observatory consists of a thin metal cylinder, eight inches in diameter, and twelve inches deep, graduated upon one side to a quarter of an inch. This cylinder will penetrate through the snow, scarcely disturbing it, and the depth in inches is at once seen. By careful manipulation, if the cylinder be turned round, all the enclosed snow can be lifted from the ground. It is desirable to melt it in a wide-mouthed bath, the gauge being previously kept corked to prevent evaporation. It frequently happens that snow is blown out of the mouth of the rain-gauge before it has had time to melt; consequently, the result of melted snow, as shown by the rain-gauge, will be too little in amount.

CALENDAR OF NATURE.—Every meteorologist should endeavour, as much as possible, to record the arrival and departure of migratory birds, the dates of trees coming into and losing their leaves, the blooming of plants, the ripening of fruit and seeds, the building of birds'-nests, the first appearance of various insects, diseases amongst animals and plants, the appearance of abundance or otherwise of crops of fruit, corn, &c. If such registers were extensively kept and carefully recorded, the effect of the weather upon the animal and vegetable kingdoms would be well seen. It is extremely desirable that every precaution should be taken, in order that, year after year, the same object should be the special one on which the remarks are based, and that one species be not mistaken for another. The following examples will show that it is essential to use the utmost care:—

First, "the elm is said to lose its leaves on a certain date." Such an observation is useless. It is requisite to mention the particular kind of elm; thus, the broad-leaf elm is the first tree to become leafless, which it frequently does in September; the Siberian elm, on the contrary, will remain green after all other trees have become leafless—sometimes it is in leaf as late as December. It will also be found that the same species, in a group whose boughs touch each other, will come into leaf at different dates. In no tree is this more strikingly exhibited than in the beech: two beeches growing close together may be seen to vary a couple of weeks in their period of coming into leaf. The age of the tree also causes a difference to occur. In the different kinds of lilacs and laburnums, there will be a range of some days in their time of coming into bloom. Amongst herbaceous plants, none that have been transplanted should be the objects of record. Respecting the observations on the ripening of fruits, the strawberry will serve as an example: the variety called Black Prince will be ripe before Kean's seedling; Kean's seedling before British Queen; and British Queen before the Elton Pine.

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* See Mr. Glaisher's report in the next chapter.

CHAPTER V.

WEATHER-FORECASTS—BALLOON OBSERVATIONS—CLIMATOLOGY, ETC.

[THIS Chapter has been added by the Editor, as in part supplementary to subjects already treated on, and also to afford information on various matters, pertaining to Meteorology, which have not hitherto been noticed. Amongst such are—Admiral Fitzroy's Weather-Forecasts; Polar and Equatorial Currents; Cyclones; Rules for predicting the Weather; Animal Instincts; Free Electricity, and its Phenomena; Mr. Glaisher's Balloon Ascents; Shooting-Stars and the Aurora; Meteorological influence of Drainage; Ozone; Determination of Heights; Weather Registration; Gaseous Radiation; Snow-line; Distribution of Heat; Rainfall; Plants and Animals as affected by Climate; Meteorology and Ethnology; Endemic and Epidemic Diseases; Tables of Mean Temperature, &c.]

IN the preceding pages, the student, or general reader, will find an ample disquisition on the laws of Meteorological Phenomena, and some useful advice and aid, should he be desirous of pursuing the subject practically. Within the last few years, an idea has prevailed in scientific circles, that it is possible to predict the probable kind of weather which may occur within certain limitations. Such an object is of the highest importance to us as a seafaring nation, losing, as we do, annually, an immense sum by the wreck of vessels near our coasts. The Meteorological Department of the Board of Trade is devoted to obtaining observations, and in generalising them, with the hope that the laws governing excessive changes of the weather may thus be discovered. To Admiral Fitzroy we are indebted for a great amount of untiring exertion and unequivocal success. Under his direction, telegrams are daily forwarded to all our ports, intimating the weather which may be expected to occur; and a system of signals has been arranged, by which vessels, near our coasts, can be warned of an impending storm.

We present our readers with the following epitome of the results arrived at by Admiral Fitzroy, and the principle on which he proceeds; and are indebted to the pages of the *Athenæum* for the *résumé*. The remarks are founded on a Report which was furnished by the admiral, during the summer of 1862:—

"The Report is highly valuable, for it contains the first-fruits of the measures taken by government to utilise our knowledge of meteorology, with reference more particularly to storm telegraphy. The system at present in use was organised in 1857; when it was arranged that simultaneous observations should be made daily at various localities in the British Islands, also in and around the Atlantic, and at stations on the European continental coasts. By combining these observations in synchronous charts and otherwise, it was discovered, that though the weather and wind appeared to be quite irregular, yet that there was much uniformity and similarity of character in successive variations. It was further ascertained, that by means of a comparatively small number of observations made daily at a few selected stations sufficiently far apart, and by the use of a self-registering barometer at a central station—to which meteorological telegrams are sent from the outlying stations—distinct intimation of marked changes of weather, and warning of dangerous storms, might be communicated to the centre, and from thence to all other points of any telegraphic combination.

"Some years were spent in bringing the system to working order; and on the 6th of February, 1861, the first warnings of portending storms were given. The value of this notice was particularly marked by the circumstance that a fleet of vessels became disabled or wrecked on the 8th or 9th, which were lying off Shields, and whose captains disregarded the timely warning. The misfortune was, however, a singularly pertinent example of the truth of the homely proverb, that 'it is an ill wind that blows no one good;' for since that event, Admiral Fitzroy states that, 'whether from having appreciated these storm signals, or from other reasons, the *fact is*, that very few wrecks occurred on our coasts during the notoriously tempestuous weather of February and March, 1861; and comparatively few since.'

"It is gratifying to find that the aid afforded by the government offices is not confined to British ships, and that a considerable demand for the publications emanating from Admiral Fitzroy's department, has been made by foreign nations. It has been truly observed, that one of the greatest evils of meteorology, hitherto, has been the practice of incessantly making observations without any very definite objects in view, with the vague hope that eventually they may become of value. The aim of the Government Meteorological Department is to correct this evil, by not only instituting observations of a strictly scientific nature, made with scientific instruments, but also in turning the observations to practical account.

"Admiral Fitzroy considers that the phenomena accompanying storms are now so readily explicable by the light thrown upon them by Dove's theory of polar and equatorial currents, which has been translated and published by the Board of Trade, and they are so marked by barometers, that it is quite inexcusable to undervalue their warnings. The principal phenomena connected with storms may be thus summed up. Great and important changes of weather and wind are preceded, as well as accompanied, by notable alterations in the state of the atmosphere; approaching storms are usually shown by falls in the barometer exceeding one inch, and by differences of temperature exceeding about fifteen degrees. A tenth of an inch an hour, is a fall indicating a storm or very heavy rain. The more rapidly such changes occur, the more risk there is of dangerous atmospheric commotion. These circumstances show the necessity of frequently observing the barometer, and the advantages of a self-registering instrument.

"As all barometers often, if not usually, show what may be expected a day, or even days, in advance, rather than the weather of the present or next few hours, and as wind or its direction affects them much more than rain or snow, due allowance should always be made for days as well as for hours to come. Admiral Fitzroy recommends that the following table of average temperatures between 8 and 9 o'clock A.M., near London, should be used to assist in foretelling the direction and nature of coming wind and weather:—

January . . .	37°	July . . .	62°
February . .	39	August . . .	61
March . . .	41	September . .	57
April . . .	46	October . .	50
May . . .	53	November . .	43
June . . .	59	December . .	39

"The thermometer (shaded and in open air), when much higher between 8 and 9 o'clock A.M. than the average, indicates southerly or westerly wind; but when considerably lower, the reverse, or northerly currents of air *approaching*, if not present. It is important to observe that the foregoing table of temperatures must be corrected for other places.

"Careful observations have now proved, that, although England is generally visited by one or more storms during the year, each of which is accompanied by terrible local disasters, yet all these tempests (be they ever so remarkable) are so much alike in character, and have been preceded by such similar warnings, as to warrant our reasoning inductively from their well-ascertained facts, and thence deducing laws. Indeed, all investigations into the characteristics of storms show the same results. Gales from the south and west are followed by dangerous storms from the north and east. Those from the west and south come on gradually; whereas those from the north and east begin suddenly, and often with extraordinary force. The barometer, with these north-eastern storms, does not give direct warning upon this coast, because it ranges higher than when the wind is in the opposite quarter. But though the barometer does not give much indication of a north-east storm, the thermometer does; and the known average temperature of the locality affords the means of knowing, by comparison, whether the wind will be northerly or southerly. Besides the familiar weather signs, Admiral Fitzroy considers that the following special peculiarities, previous to great atmospheric commotions, should be observed:—A great fall of rain or snow, much lightning, unusual cold, or excessive heat, invariably precedes high winds or strong gales.

"Admiral Fitzroy is cautious in advancing any opinion respecting the connection between electricity and storms. It is certain that, although many eminent men do not think that magnetic storms, or even auroras, are directly connected with atmospheric currents, or have any special relation to storms, many facts on record seem to point to a different conclusion. We can only notice one; but this is most important. On the evening previous to the *Royal Charter* storm, a brilliant meteor was seen; and during the previous week, blood-red streamers of aurora were very prevalent.

"We need hardly say that Admiral Fitzroy regards any faith in change of weather at the moon's quartering as utterly illusory, being based merely on deductions from coincidences, and not on any philosophical basis. It is, however, most remarkable how this unsound theory keeps its place among men even of high education. An amusing instance recurs to us. At one of the horticultural *fêtes* this summer, we met a gentleman who was loud in his indignation because, it being a wet day, he had ascertained that the Council of the Horticultural Society had fixed their shows regardless of the moon's quarters; whereas, as he stoutly maintained, had proper attention been paid to the lunar changes, wet weather would have been avoided. We apprehend that the indignant member has seen by this time, that the weather, during this summer, at all events, is not influenced by the moon.

"Although the great working instruments of the meteorologist are the barometer and thermometer, which should be of the highest order of excellence, we are glad to hear that Admiral Fitzroy makes use of an instrument called a storm-glass, which was much more common formerly than it is now. We have long been acquainted with this glass, and can speak confidently of its value in aiding, with the barometer and thermometer, in forecasting weather. Admiral Fitzroy has observed this instrument with great closeness, and these are his conclusions respecting its indications:—When a storm-glass, filled with the proper chemical mixture (camphor, nitrate of potass and sal-ammoniac, partly dissolved by alcohol, with water and air in an hermetically sealed glass bottle), is fixed undisturbed in free air, and not exposed to radiation, fire, or sun, the mixture varies in character with the *direction* of the wind. As the atmospheric current veers toward, comes from, or is approaching only from the polar direction, the mixture grows like fir or fern-leaves, hoar-frost, or crystallisations. As it tends to the opposite quarter, the lines or spikes—all regular, hard, and crisp features—gradually vanish. Before, and in a continued southerly wind, the mixture sinks gradually downwards in the bottle till it becomes shapeless, like melted sugar. Before or during the continuance of a northerly wind the crystallisations are beautiful. The glass should be wiped gently occasionally, care being taken not to shake the contents. While any hard or crisp features are visible below, above, or at the top of the liquid (where they form for much north wind), there is *plus* electricity in the air; a mixture of polar current co-existing in that locality with the opposite or southerly. When nothing but soft, melting, sugary substance is seen, the atmospheric current is southerly, unmixed with, and uninfluenced by the contrary wind.

"In August, 1861, Admiral Fitzroy commenced publishing single 'forecasts' of the weather. Twenty weather reports are received each morning, and ten each afternoon, besides five from the continent. Double 'forecasts' (two days in advance) are now published, and are sent to the daily papers. As these 'forecasts' have created considerable interest, Admiral Fitzroy explains the principles on which they are made.

"A barometrical height of 29·50, with about 60° of temperature, may be considered to indicate (if continued a certain time) fresh south-west wind and mild air, probably cloudy, perhaps rainy; about 30·50 inches, with 50° temperature, or less, would foretel and accompany north-easterly wind, dry weather, and usually a clear sky. Again, a high or rising barometer in the north-west, with low or falling temperature, while in the south and east the pressure is low or diminishing, indicates a north-west wind, either extending generally, or checked and deflected according to the diminution or increase of the temperature in the south-east.

"Objection has been taken to such forecasts, because they cannot be always exactly correct for all places in one district. It is, however, considered that general comprehensive expressions, in aid of local observers, who can form independent judgments from the latter, and their own instruments respecting their immediate vicinity, may be very useful as well as interesting; while, to a person unprovided with instruments, an idea of the kind of weather thought probable cannot be otherwise than acceptable.

"Certain it is, that although our conclusions may be incorrect, and our judgment erroneous, the laws of nature and the signs afforded to man are invariably true. Accurate interpretation is the real deficiency. There is no doubt, that attentive observation to the meteorological laws will enable us to interpret them better, and that, with increased knowledge and experience, 'forecasts' of the weather will be even more worthy of credence than they are now. But it is quite evident that enough is already known to enable an attentive observer, with good instruments, to acquire a useful foreknowledge of the weather during the coming day or two, or even three following days; and the utility of Admiral Fitzroy's weather warnings is amply illustrated by the fact, that notwithstanding the extremely stormy weather that prevailed in the North Sea during the first three months of 1862, the coasts of Durham and Northumberland have had a fortunate immunity from shipwreck, there having been no serious shipping disasters on that line of coast since November, 1861."

The preceding account of the progress made by the Admiral and his staff in 1862, will doubtless prove interesting to many of our readers: and we shall next call attention to the more developed and mature results, obtained since that date; detailing, also, more particularly the laws which are supposed to govern the production of storms, or unusual atmospheric disturbances.

In a communication with which we were favoured by Admiral Fitzroy, he especially draws attention, as the basis of his plans, to the following meteorological facts and phenomena: viz.—"A limited depth of atmosphere," as the region in which the phenomena occur, "not exceeding fifteen miles; horizontal motion of its several superposed or intermixed currents (like streams of water); their opposition, or, at times, combination; and their perennial progress, bodily (in mass), toward the east, in the temperate zone." These he considers as the most useful facts on which inferences may be formed, and the discovery of law can be educed, in dealing with questions relating to the forecasts and warnings to which we have already alluded.

With respect to the movements of the atmosphere, "accumulated facts have induced the conclusion that winds move in parallel currents, or circulate around a centrical area;" and that a uniform law of circulation or gyration exists, equally on the large scale as in the polar and tropical currents, and descending to the slightest aërial disturbance of any kind, with but rare and limited exceptions. The distinction between perennial trade-winds and local cyclones, seems to arise simply in the very circular direction of wind in the cyclone storm; whilst in the ordinary trade and similar winds, the direction is of a less curved, and consequently more constantly direct, character. The direction and force of the polar current are, of course, affected by the rotation of the earth, it becoming equatorial as that region is arrived at. The current, in its return, moves eastward towards each pole, and loses its momentum gradually, from gravitation and various local causes. It has been proved that storms, and all the greater circulations of the air, have an eastward motion bodily, while circulating around a centrical area, between the tropics and polar regions; whilst, within the tropics, the direction is westward until the currents recurve. To the circulation of these polar and tropical currents, often in parallel directions, and the effects of heat, moisture, &c., on the atmosphere generally, are assigned the cause of what are called "atmospheric waves," which are attended with a rise and fall of the mercury in the barometer, under certain limitations.

"Such currents, prolonged and excessively broad, are always flowing in nearly opposite directions, and always moving eastward, at an average rate of about five miles an hour, in our latitude." If near the earth's surface, they may be side by side or parallel; but if overlapping, or entirely superposed, they then cross in various directions, and more or less impinge on, or intermingle with, each other. Hence arise intermingling cyclones, which, affecting each other, apparently in opposition, but really in accordance with these laws, have, from our previous ignorance, much hindered the progress of our deductions, and frequently thrown doubt on the anticipatory predictions which have been made as to their occurrence.

Without detailing the entire known chain of causes which tend to produce the results, and which have, more or less, been already considered in the previous chapters of this section—it may be remarked, with respect to the polar current, that "the sensible result on the whole system of circulation, must be a continual easterly progression in the temperate zones; a general motion of the atmosphere towards the east, over the earth's surface; while, in the lower latitudes, and (perennial) trade-winds, its motion is different from that in higher latitudes, being towards the west. . . . . The normal state of our atmosphere appears to be a regular alternation, or circulation of currents, between polar and tropical regions—the polar usually, but not always, advancing along the earth's surface; the return current (*i. e.*, from the equator) "generally above, at higher elevations." The electrical characteristic of the polar current is generally that of positive excitement, with cold, great specific gravity, and dryness, except over a warm ocean; whilst the tropical current is generally negatively electrical, the positive state being rarely, if ever, observed.

These remarks are intended to explain what we may term the normal state of our atmosphere. The phenomena to which we have drawn attention may be considered as the natural condition of the gaseous envelope of the surface of the earth, undisturbed by local and abnormal influences. When, however, we descend to regions of comparatively slight elevation above the earth, then we find a vast number of other causes coming into operation, such as radiation, the physical character of country, as mountains, plains, swamps, seas, &c.; all of which tend to break-up the main currents of which we have been speaking, and thus to produce that diversity of wind and weather so proverbial in many parts; but especially in insular places, like our own country. In such situations, the study of atmospheric currents becomes a question of great complexity; and our readers may more fully understand its extent, when we remind them, that it is by no means an uncommon occurrence to find the wind blowing in every possible direction of the compass, at the same moment, in different parts, round and within our own coasts. It will be easily seen, after a careful perusal of our Section on Heat, and the earlier chapters of the present one, that an even-flowing current arriving from any direction at our shores, must immediately impinge, in some places, against cold mountain heights; in others, on warm and moist plains, &c.; and thus be broken into an immense number of smaller currents, which, re-acting on each other, afford about as difficult a problem for study as any within the range of philosophical investigation. The establishment of the primary currents, however, tends to assist us in explaining the cause of those of a secondary nature; and as we are assured that nothing occurs by chance, but follows certain definite, although possibly obscure laws, we are constantly encouraged to pursue these investigations, and, by observations and deductions, gradually arrive at the object of our researches.

In explaining the causes of barometric disturbance, Admiral Fitzroy relies chiefly—1st, on the effects of a rapid horizontal current, which tends, by producing a partial vacuum at the place of observation, to depress the mercury: 2ndly, on the presence of aqueous vapour, which diminishes the specific gravity of the air; and, 3rdly, on the expansion of the air by heat, which has precisely the same effect. As regards the influence of atmospheric waves on the rise and fall of the barometer, he is disposed to consider that their action is chiefly of a secondary character. It will hence be seen, that any change of wind and weather, whilst depending on a vast variety of causes, must *always* be attended by an increase or decrease of temperature, and moisture; and, consequently, accompanied with a variation in the height of the mercurial column. The barometer, thermometer, and hygrometer, are, therefore, *par excellence*, the instruments, taken together, on which the meteorologist essentially depends for unravelling the complex character of the winds, &c.

A popular illustration of these statements may be readily presented by referring to the experience of weather-changes in our own country. With a north-east wind, which is always dry, of great specific gravity, and low temperature, a rise of the mercury occurs, both at the moment, and shortly preceding the advent of this wind. On the other hand, the south-west wind, loaded with moisture, of a comparatively high temperature, and consequently low specific gravity, is always ushered in, and attended, by a fall of the mercury. Between this point and the north-east, as the wind veers northerly, the barometric column generally rises; whilst, as the wind travels southerly, from north-east, a barometric fall gradually occurs in most cases. These facts are palpable to the most cursory observer of the barometer; and those who object to the truth of forecasts, admit them tacitly, and condemn themselves practically, whenever they rely on barometric changes to predict, for a few hours, a change of the weather. The philosophic observer differs from such in arguing on a broader basis: he not only notes the barometric variations, but associates with them the *antecedent* changes of temperature and hygrometric condition, which, by experience, he has found invariably to occur. Thus his deductions have reason and fact for their basis, whilst the hasty weather-wise simply follow an empiric and uncertain method of indication. The use of the telegraph, in assisting weather prognostications, will be easily made apparent. Supposing, for instance, at a central station in England—say Wolverhampton—telegrams are received, respectively, from Bristol and Shields, the one noting the existence of a strong S.W. wind, and the other one from the N.E. It is evident, that if these currents meet, the hot, moist S.W. will, on coming in contact with the cold dry N.E., be condensed; and, in some places, where the difference of the temperature and moisture is greatest, their mutual impact *must* cause rain. On either side of this position the winds may modify each other's character; the S.W. will part with a portion of its moisture to the dry N.E., and the latter will, of course, become more hygrometric. Under such conditions clouds may be formed, but rain may not fall; and thus the weather may promise a change, whilst none of importance immediately occurs. It will thus be evident that the rain-tract, produced from causes just mentioned, may be surrounded with another, in which the moisture is not precipitated. By extending the same reasoning for the whole of our country, after being in full possession, by telegraphic reports, at the same moment, of the actual direction of the winds, their temperature and moisture, and remembering that our storms are usually of a cyclonic character—it will be plain, to the most unpractised observer, that forecasts *may* be made with every probability of a successful result, provided that the reports furnished afford data of all the various circumstances, on which, as we have already shown, the occurrence of the phenomena depend. With respect to the method of observing barometric and thermometric changes, in combination with each other, the reader will find a table of temperature averages at a previous page, which will guide him for each month in the course of the year.*

Few persons, without carefully reflecting, can gain an adequate idea of the multitude of minor occurrences, which, unheeded in most cases, give warning of impending atmospheric changes. The indications to which we are about to refer, depend mostly on the increase or decrease of atmospheric moisture. Thus, the long ribbon sea-weed becomes damp as moist weather approaches; so does common salt, to a slight extent: the painted walls of our apartments, and the iron rails in the garden, likewise become moist, for precisely the same reason as does the wet bulb of the hygrometer†—namely, the deposition of moisture from the heated air on a cooler surface. Stairs creak or are quiet as the hygrometric state of the atmosphere varies. With a dry wind, the back of the cat affords bright electric sparks when stroked; but none when the air is moist, because the damp conducts away the electricity. The approach of the S.W. wind, in winter, after a cold N.E. wind, is immediately indicated by the paving of our streets, and the tiles and slates of our roofs becoming moist; our window-frames and doors "stick" for similar reasons; ropes, loosely stretched, become tight, owing to the absorption of moisture, and the consequent contraction of their fibres: hence, the frequent annoyance to the housewife from the sudden break-down of the posts, between which clothes have been hung out to dry. The steam from the locomotive blast-pipe seems abundant in the moist air on a damp day; whilst in fine dry weather it is scarcely visible, being rapidly absorbed by the surrounding air; and, for similar reasons, any wet article hung out in damp weather, dries but slowly, because the air already surcharged with moisture can absorb no more. Our space does not permit us to enlarge on these interesting subjects; and we have only suggested the above as instances of which almost every one has some knowledge, and frequently use, who have any pretension to the name of what is commonly termed a weather-prophet.

In concluding these remarks on the laws of storms, &c., we cannot do better than transfer to our pages the "resulting impressions" which Admiral Fitzroy affords. They are as follows:—

1. The gyratory movements of wind, usually called cyclones, are consequences of the meeting of great air-currents.

2. When so caused, in any part of the world, they rotate, as eddies, during a certain time—more or less limited; not exceeding four complete days and nights without interruption, but usually for a much shorter period.

3. That cyclones originate on one side of the Atlantic, and traverse to the other, is a fallacy, which has arisen from an *insufficiency of facts*, and consequent erroneous combination of the details of various successive gyrations, *since* proved to be *frequent*.

4. When such an atmospheric combination happens, it is not usually an isolated occurrence, but one of many such, similar in nature and origin, though unequal in extent, duration, and force; not taking place at exactly the same time, necessarily, but prevalent in a certain zone or region of the world during a few days or weeks, or a season.

5. The conflicting action of two currents, opposed in much peculiarity as well as in *direction* (a feature connected with electricity?), not only originates a cyclone, but tends to continue its striking qualities, of a wet warm side and a dry cold one, owing to the continued access and addition of air from each of the currents (between which it is the eddy), as place is made by immense precipitation of vapour in rain, hail, or snow, supplied from the vaporous side as speedily as precipitated, or absorbed by the chilling and drying influence of the antagonistic current from a polar direction.

6. These cyclones, originating in *opposition* of currents otherwise caused, are different from local whirlwinds, occasioned by rarefaction or by *electrical* action, as in the sand-columns of the desert, and water-spouts.

7. When opposing currents meet, their masses must *continue* in motion a certain time, either rotating or ascending, or going onward horizontally, in *combination*.

8. Masses of air, either of polar or tropical origin (so to speak), *returning*, when driven back by stronger opposition, at first, and for a certain time, retain the characteristics of their peculiar and very different natures.

9. During the prevalence of northerly (polar) currents of air, electricity, positive or plus, is more active or developed. On the contrary, with a southerly (tropical) current there is no such action, no electricity in excess, no positive or plus, and but little, if any, of negative or minus free electricity.

This *résumé* of the, at present ascertained, laws on which the system of forecasts is founded, will enable the student to arrive at some general idea of the insight which we now possess into those previously hidden laws by which, so far as we can now judge, so many atmospheric phenomena are governed. It must be admitted that our knowledge is still extremely limited, chiefly because our means of observation are confined to within so short a distance from the surface of the earth. In a subsequent portion of this chapter, we shall give an account of many interesting particulars which have been arrived at by means of balloon ascents, undertaken at the instance and expense of the British Association, by Mr. Glaisher. Perhaps, to a certain extent, the preceding remarks and deduced laws may be modified by the results that Mr. Glaisher has obtained, and which certainly do not encourage, at first sight, that assumption of *regular* current forces to which we have so frequently alluded. It must be borne in mind, however, that at the utmost height which has been attained by the balloon, the observers have barely been beyond the sphere of action of purely local causes. Hence, the results necessarily cannot be relied on as bases on which to build a safe theoretical superstructure. Repeated observations, under constantly varying circumstances, will be needful, before we can safely trust ourselves to an unreserved acknowledgment of the truth of many deductions hitherto made. In every branch of philosophical investigations, dependent on experiment, it is safer to doubt than hastily to credit; but in meteorology, which is as yet the most uncertain branch of science, such a principle must be acted on without the least hesitation, if we would not commit ourselves to crude theories, and, possibly, to positively dangerous inferences.

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* See *ante*, p. 1172.

† See *ante*, p. 1146.

To the general inquiry, "On what meteorologic conditions or changes are the forecasts based?" Admiral Fitzroy replies in the following terms:*—"They depend on considering the atmosphere as a lighter ocean, having currents; elastic expansibility; equilibrium; momentum or inertia; chemical alterations, and extreme sensibility to heat or cold, its *chief motors;* and on knowing the statical condition of the air, in the oceanic envelope at many places simultaneously; likewise, again similarly after certain intervals of time, by which inter-comparisons are made, showing the relative conditions and causations, whence dynamic effects originate. These dynamic motions are proportional to disturbances of level, like those caused by a head of water; to inequalities of temperature, and consequent *chemical* changes, with more or less electric action. They are our winds, and may be soft or gentle, or as heavily boisterous as in a *tempest;* of which differences, through all degrees, instrumental means and telegraphy now give available information. To utilise their indications adequately, a central office should know the natural and general atmospheric movements, with their disturbing causes, even as a pilot knows the varieties of streams and eddies in a wide estuary.

"The whole map of a region (say the British Islands) should be outlined in the mind, as the estuary, with its shoals, is mentally visible to the pilot. The normal tendency of the whole atmosphere (in our latitudes) to *move eastward*, while crossed or variously interfered with by polar or tropical currents, that in course of seasons cause every variety of wind and weather, should be always considered; and then, with due allowances made for gradual advances from westward, for effects of land and differences of temperature, good forecasts may be generally drawn."

Hitherto we have referred solely to the *principles* on which the meteorologist should be guided; but we shall now afford a number of practical instructions, for the use of those who desire to undertake weather-forecasting. The following rules are chiefly selected from the *Barometer Manual*, published by the Board of Trade; a work which, whilst of trifling cost, will be found an admirable compendium, and a guide for making observations and registering results. It is highly possible that some who peruse these pages, reside in localities where the observations of an intelligent person may be of great use to those whose livelihood is obtained at sea, &c.; and a careful attendance to the following suggestions, with the possession of a good barometer and thermometer, may be of essential service. Care must be taken to correct the results obtained, for elevation above the sea-level, temperature, &c., if they are to be compared with those of other observers. Rules for these corrections have been already given in the preceding chapter.

RULES FOR PREDICTING CHANGES IN THE WEATHER.

1. If the mercury, standing at 30 inches, rise gradually whilst the thermometer falls, and dampness becomes less, N.W., N., or N.E. wind; less wind; or less rain and snow may be expected.

2. If a fall take place, with a rising thermometer, and increasing dampness, wind and rain may be expected from S.E., S., or S.W.: a fall in winter, with a low thermometer, foretels snow.

3. An impending N. wind, before which *the barometer often rises*, may be accompanied with rain, hail, or snow, and so forms an apparent *exception* to the above rules, for the barometer always rises with a north wind.

4. The barometer being at 29½ inches, a rise foretels less wind, or a change of it northward, or less wet. But if at 29 inches, a fast *first* rise precedes strong winds or squalls from N.W., N., or N.E.; after which, a gradual rise, with falling thermometer, indicates improving weather; with a steady or rising thermometer, a S. or S.W. wind will follow, especially if the rise of the barometer has been sudden.

5. A rapid barometric rise indicates unsettled, and a rapid fall, stormy weather, with rain or snow; whilst a steady barometer, with dryness, indicates continued fine weather.

6. The greatest barometric depressions indicate gales from the S.E., S., or S.W.; the greatest elevations foretel wind from N.W., N., or N.E., or calm weather.

7. A sudden fall of the barometer, with a westerly wind, is sometimes followed with a violent storm from the N.W., N., or N.E.

8. If the wind veer to the S., during a gale from E. to S.E., the barometer will continue to fall until the wind is near a marked change, when a lull *may* occur. The gale may afterwards be renewed, perhaps suddenly and violently; and if the wind then veer to the N.W., N., or N.E., the barometer will rise, and the thermometer fall.

9. The maximum height of the barometer occurs during a N.E. wind, and the minimum during one from the S.W.; hence these points may be considered as the poles of the wind. The range between these two heights depends on the direction of the wind, which causes, on an average, a change of half an inch; on the moisture of the air, which produces, in extreme cases, a change of half an inch; and on the strength of the wind, which may influence the barometer to the extent of two inches. These causes, separately or conjointly with the temperature, produce either steady or rapid barometric variations, according to their force.

Allowances should invariably be made for the state of the barometer and thermometer, for *some days* as well as *some hours previously* to forecasting, because sudden changes are always indicated antecedently to their occurrence. The longer and steadier a change is in coming, the longer the expected weather, either wet or fine, will last.

Sometimes severe weather from the S., *not lasting long*, may cause no great fall, because followed by a duration of N. winds; and, occasionally, the barometer may fall during N. winds and fine weather, because a continuance of S. wind is about to follow.

The table of average temperatures, *corrected for the place of observation*, but given for *Greenwich* at page 1172, is to form the standard of comparison with the actual temperature at the time of observation, in connection with the barometrical indications. If, at 8 or 9 A.M., the thermometer is much higher than the corrected average of the place, a southerly or westerly wind (tropical current) may be expected; if considerably lower, then a northerly wind (polar current) is indicated.

The preceding rules, of course, solely refer to observations with instruments; but the following external signs (chiefly selected from the *Manual*) of impending changes should be taken collaterally, because they will frequently serve as interpreters of obscure indications given by the barometer, &c. In connection with the following, our remarks on well-known phenomena, depending on the moisture of the atmosphere,† may also be read; and the general observer can employ both, when the ordinary meteorological instruments are not accessible. It will be seen that the objects are purely natural, and that the effects noticed really depend on certain influences, which, affording the phenomena as signs, are also their producing causes.

Clouds.—The following appearances depend chiefly on the refractive power of vapour, either of the cloud, or suspended invisibly in the air,‡ and, occasionally, on its comparative or nearly entire absence.

A rosy sky, at sunset, presages fine weather; an

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* See Report of the Meteorologic Office, 1863.

† See *ante*, p. 1175.

‡ The student may read, with this, the articles on Refraction and Colour, in the Section on Light. See pp. 45, 49, *et seq.*

Indian-red tint, rain; a red sky in the morning, bad weather, or much wind or rain; but a grey sky, fine weather; a dawn high in the sky, wind; if low, fair weather; soft or delicate-looking clouds indicate coming fine weather; hard-edged, oily-looking clouds, wind; a dark, gloomy, blue sky, wind; a bright blue, fine weather. The softer-looking the clouds, the less wind may be expected; the harder, more rolled, tufted or ragged, the stronger the coming wind. Inky-looking clouds foretel rain; light scud clouds, driving across heavy masses, foretel wind and rain; but if alone, wind only. High upper clouds, crossing the apparent face of the sun, moon, or stars, in a different direction to that of the lower clouds, or the wind felt below at the moment, foretel a change of wind toward their direction; streaks, curls, wisps, &c., of cloud, in a clear sky, increasing into general cloudiness, indicate coming rain or wind—perhaps both. The higher and more distant such clouds may be, the more gradual, but general and lasting, will be the coming change. Fine weather is usually accompanied with undefined clouds (if any), of delicate, quiet tints or colours; whilst hard, defined clouds, with gaudy or unusual colours, foretel rain, and probably strong wind. Misty clouds, forming or hanging on heights (commonly seen about bushes on the tops of hills, &c.), show wind and rain coming, if they remain, increase, or descend. If they rise, or disperse, fine weather may be expected. Dew and fog generally indicate fine weather; whilst remarkable clearness of the air, unusual twinkling of the stars, indistinctness of the moon's horn, haloes, the rainbow, &c., are, more or less, significant of increasing wind, and possibly of rain, either with or without wind.

It often surprises persons accustomed to reside in towns, how accurately those living in an open but especially hilly country, can predict the weather. In numerous instances, however, certain local phenomena interfere with regular predictions, of which we have met with several instances in various parts of England and Scotland. In journeying, for instance, northward from London, for about 100 miles, a great breadth of land extends between the German Ocean on the east and the Irish Sea on the west: proceeding still further northward, the breadth of coast rapidly narrows, until the width across Cumberland and Durham is little more than 80 miles. In the former case, the currents of air, arising from the surrounding seas, spread over a greater area of land, and more gradually part with their moisture in the form of rain; whilst in the north, the warm westerly and cold easterly winds speedily meet, and give rise to frequent and rapid showers or storms of rain. In the mountainous tracts of Scotland, &c., sudden storms of wind and rain are of constant occurrence; and, owing to the rapid eddies or small cyclones, and the narrowness of the land, the traveller may soon find his knowledge, acquired in other districts, of no practical value whatever. On the west of Scotland the Gulf-stream impinges; and we speak from an experience of some years, in remarking, that whilst staying, during the winter, at Rothesay, in Buteshire, we have never seen snow on the ground, at nearly the sea-level, more than an inch in depth, and no ice; whilst in London, the winter was, at the same moment, and had been for some time previously, most severe, with ice from eight to ten inches in thickness. From Hull, in Yorkshire, to Wick, in the north of Scotland, the winter is generally far more severe than between the same points on the western coasts, owing, doubtless, to the absence of the warm water of the Gulf-stream, and consequently of the heated superincumbent air accompanying it. Instances of this kind might be greatly multiplied; but we mention these only as cautions for persons who, living at places similarly circumstanced, may attempt, without modification, to avail themselves of the rules we have laid down in the preceding pages.

Animal Instincts.—That animals are by some means made acquainted with coming changes of the weather, is a fact supported by universal experience. The annual migration of birds alone is sufficient proof; but those creatures who are domesticated with us, afford more constant and evident signs of possessing a certain amount of weather-wisdom; and we may profitably instance a few facts respecting them, as indications to be used collaterally with those already mentioned. There can be little doubt that temperature, moisture, barometric variations, and electrical influences, may be taken, collectively, as the causes of uneasiness frequently experienced by ourselves, as well as by the lower creatures; for the similarity of the animal constitution, functions, and organism, can point out no other solution of the question.

Without trenching on the subject of physiology, we may shortly investigate the relation subsisting between animal sensation and meteorological phenomena; and, for this purpose, we must avail ourselves of the effects noticed on our own bodies, to explain the indications which the habits and instincts of animals afford us. Commencing with the lungs, it is evident that a decrease of atmospheric pressure must be attended with a diminished supply of the oxygen taken in at each inspiration, because the volume of air is inversely as the pressure. A diminished supply of oxygen immediately influences the process of digestion, by diminishing the rapidity with which the carbon and hydrogen of our food are converted, by combination with atmospheric oxygen, into carbonic acid and water. The blood consequently becomes, to a certain extent, vitiated, and the various humours suffer simultaneously. The common complaint of "dullness," experienced during wet weather with a southerly direction of wind, is thus readily accounted for; whilst precisely the opposite feeling, experienced during northerly winds and dry weather, is explained as the result of an increased atmospheric pressure, a quicker and larger supply of oxygen, and a more rapid oxygenation of the blood. With a southerly warm and moist wind the appetite decreases, to be renewed with full vigour when the brisk breezes of the northerly currents set in. Again, the fluids permeate all parts of the body, which is pressed on, on every side, by the weight of the atmosphere, amounting to some tons in the case of a man of ordinary size. If the barometric column diminish, the pressure is diminished on these fluids also, and hence the sense of "fulness" experienced under such circumstances, and the reverse when the atmospheric pressure again is increased. The skin of the body is alike sensitive to the effects of variable temperature and moisture; extending with an increase of them, and contracting with a decrease; and thus influencing our sensations to an extraordinary extent, as is well known by comparing our feelings experienced respectively during winds blowing from the S.W. and N.E. Electrical action, although more obscure in its effect, is not less decisive; and those who possess thin fine hair, may readily perceive a decided action on the organs of sight, and the nerves of the face, when positive electricity is present or absent: in the latter case, dimness of sight and neuralgia become exceedingly prevalent.

Now, whatever changes we experience, we can readily provide for, in various ways; for extra clothing, fires, &c., may, to a large extent, modify the ill effects to which we have alluded. But the lower class of animals, deficient, or perhaps destitute, of what we call "reason," can make no such provision; and hence, although feeling all the inconveniences we experience, they are compelled to bear with them until a favourable change occurs. Unprovided with external clothing, except that of Nature's affording, and which itself is part of their bodies, they are very sensitive of atmospheric changes, and therefore *sooner* feel them. They thus become indications to us of the coming bad weather, and their uneasiness continues until, again perceiving, *sooner* than ourselves, a coming but more favourable change, they equally foretel its advent. Besides, the hair, wool, and feathers of quadrupeds, and birds, rapidly radiate heat and electricity, and thus accelerate the *prevision* (if we may so call it) which we have just named. What we here *reason* upon is a matter of *blind experience* to persons of limited education, but necessarily of extended observing powers; and thus shepherds, farm labourers, seamen, and others, often depend on the instinct of the lower animals for what is, generally, indubitable evidence of an approaching change in the weather.

It would be beyond our limits to enumerate the various methods by which birds and animals indicate a sense of atmospheric change. Sea-birds, such as gulls, invariably fly inland in search of food on the approach of stormy weather; and some years ago, during a very severe winter, they were seen flying between London and Blackfriars bridges. Wild water-birds, of the geese and duck kind, forsake their marsh haunts, and approach nearer to towns, under similar circumstances. Swallows and rooks fly low, and keep near home, both preceding and during bad weather; and the petrel is always associated with the idea of its predicting coming storms, by its presence. Frogs are unusually noisy before rain; worms and leeches generally appear at the surface of the ground; moles throw up much earth; and ants close up their nests, to preserve their larvæ. The four-footed animals all exhibit considerable restlessness before rain; sheep and cattle huddle together, near bushes and trees; whilst horses and asses frequently assume a ludicrously thoughtful or meditative aspect. We need not, however, further extend these illustrations, as the experience of many of our readers will readily furnish them with additional instances; but the observance of the habits of animals, under such circumstances, is well worthy of the student's attention.

Many indications of coming change for wet or stormy weather have been already noticed; but there is one which we have repeatedly observed, and have long regarded as an infallible sign. It is that peculiar blue tint which distant objects in the landscape gradually assume; perhaps more perceptible to some persons than others. It is also remarkable, that better photographs can often be taken, in such circumstances, than when the air is clear—possibly owing to the presence of this "blueness." We have already noticed the vast variety of colours observable in clouds; and it may be safely presumed, that some peculiarity of action of the invisible aqueous vapour on the rays of light, may be the cause of the predominance of this colour: possibly it may arise from a species of epipolic dispersion.* At times, the gradual approach of the blue tint may be distinctly noticed, especially in a change from frosty weather to damp or thaw; and, on the contrary, as the weather improves, a greyish colour, succeeded by a brownish tint in the distance, is equally observable. As a last instance, drawn from inanimate nature, we may mention, that still water in ponds, &c., generally becomes muddy as wet weather approaches, owing to the diminished atmospheric pressure permitting the escape of gas, which, in ascending, carries up a portion of the fine dirt with each bubble from the bottom.

ELECTRIC PHENOMENA.—Before proceeding to examine the interesting results which Mr. Glaisher has afforded by the balloon ascents made in different parts of England, for meteorological purposes, we may call attention to some curious facts in reference to free electricity, occasionally present in great excess, under certain circumstances. It will have been noticed, that Admiral Fitzroy has been extremely cautious about admitting the electric action into any share in producing ordinary atmospheric phenomena. There is much prudence in such a course, because we know so little on the subject; and what information we do possess has been almost entirely derived from, and confined to, the extraordinary case of thunder-storms. It has, however, been already stated, that a northerly, or polar current, with cold and dryness, is generally associated with the presence of free positive electricity.† This is by no means a matter of surprise to the electrician, for he at once will remember that such circumstances are exactly those which would favour the development of free electricity. Whilst experimenting with the hydro-electric machine (described at page 172, in the Section on Electricity), we have been frequently put to extreme inconvenience by the large amount of free force pervading the surrounding atmosphere during a dry N.E. wind. Occasionally, the hair and whiskers of our audience, seated at a distance of thirty or forty feet from the machine, have been acted on to an extraordinary extent by electric repulsion. Under just such circumstances—only on a much larger scale—the following curious effects of redundant free electricity were noticed by several persons whilst visiting the Jungfrau, in 1863.

Mr. Watson, who headed the party, gives the following relation:—"On the 10th of July, I visited the Col de la Jungfrau, from the Æggisch-horn, in company with my wife, and Messrs. John Sowerby and W. G. Adams, of Marlborough College. We had with us, as guides, J. M. Claret, of Chamouni, and a young man from the hotel. The early morning was bright, and gave promise of a fine day; but, as we approached the Col, clouds settled down upon it; and, on reaching it, we encountered so severe a storm of wind, snow, and hail, that we were unable to stay more than a few minutes. As we descended, the snow continued to fall so densely that we lost our way, and for some time we were wandering up the Lötsch Sattel. We had hardly discovered our mistake, when a loud peal of thunder was heard; and shortly after, I observed that a strange singing sound, like that of a kettle" [the usual noise of a powerful electric discharge from a point], "was issuing from our alpen-stocks. We halted, and finding that all the axes and stocks emitted the same sound, stuck them into the snow. The guide from the hotel now pulled off his cap, shouting that his head burned; and his hair was seen to have a similar appearance to that which it would have presented had he been on an insulated stool, in connection with a powerful electrical machine.‡ We all of us experienced the sensation of pricking or burning, in some part of the body, more especially on the head and face; my hair, also, standing on an end, in an uncomfortable, but most amusing, manner. The snow gave out a hissing, as though a heavy shower of hail were falling; the veil on the 'wide-awake' of one of the party stood upright in the air; and, on waving our hands, the singing sound issued loudly from the fingers. Whenever a peal of thunder was heard, the phenomena ceased, to be resumed before the echoes had died away. At these times we felt shocks, more or less violent, in those portions of the body which were most affected. By one of these my right arm was paralysed so completely, that I could neither use nor raise it for several minutes, nor, indeed, until it had been severely rubbed by Claret; and I suffered much pain in it, at the shoulder-joint, for several hours. At half-past twelve the clouds began to pass away, and the phenomena finally ceased, having lasted twenty-five minutes. We saw no lightning; and were puzzled, at first, as to whether we should be afraid or amused. The young guide was very much alarmed; but Claret, who is devoid of fear, and who had twice before heard the singing noise (though without any of the other symptoms), laughed so heartily, that we joined him. No evil effects were felt afterwards, beyond the inconvenience arising from the burning of our faces, which, though we had no sun, were almost livid in hue when we arrived at the Æggisch-horn."

The above narrative, whilst exceedingly interesting, is of much importance in a meteorological point of view, as showing an instance in which free electricity was observed, in a surprising quantity, at a great elevation, with all the usual characteristics of its presence. In no balloon ascent, to a similar height (10,000 to 12,000 feet), have such phenomena been noticed; but this may readily be explained on account of the non-connection of the machine with any conducting body. It, in fact, would be isolated in the midst of such a development; whilst the travellers on the mountain would act as conductors, by means of their bodies standing on the ground. The great shower of hail and snow is another very noticeable feature; and should such observations be repeated, under similar circumstances, it is probable that some confirmation would be given to the theory which ascribes electricity as the cause of those phenomena. It is, however, just as possible that snow and hail may be the cause of the electrical development; for we may satisfactorily and independently account for them as arising from the loss of heat and moisture which damp air would sustain in such elevated regions, where the capacity of the atmosphere for heat is so much greater than near the sea-level. The moisture would be suddenly frozen into hail or snow; and the rapid transition, and loss of heat, thereby occasioned, may be supposed to be attended with a disturbance of the electric equilibrium, as can be readily demonstrated to occur by experiments on the small scale, conducted on similar principles.

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* See *ante*, p. 54.

† See *ante*, p. 1174.

‡ See *ante*, p. 175.

Numerous attempts have been made to collect free atmospheric electricity, developed during storms, &c.; and perhaps the most successful have been those of Mr. Crosse, to which we have alluded in the Section on Electricity.* On the small scale, this is constantly done by the electrometer used at observatories, but the amount thus collected is insignificant, and not more than sufficient to affect pith-balls or gold leaves. An electric kite† may be employed: it has, however, the disadvantage of inconstancy of flight, and may be productive of danger to, if not the cause of the death of, the operator. We have often purposed to employ a small silken balloon, well varnished, and attached to a hempen string, enclosing a thin metallic wire to act as a conductor. This string should run through the balloon, and terminate at the top with a bunch of metallic points. Such an apparatus, if filled with hydrogen,‡ and properly ballasted, so as to ascend vertically, might be arranged, for the purpose of observation, in a similar manner to that we have suggested for the electric kite, and almost any degree of elevation might be attained; whilst the operator could pursue his experiments safely on the electricity as conducted to him by the metallic thread. We but suggest such an arrangement, having never tried it; yet we believe, with a little precaution, it might be usefully employed. Of course, there would be no chance of the hydrogen igniting, and thus putting an end to the experiment, because the conducting wire would pass in unbroken continuity through the balloon, to the apparatus below: the only annoyance would be that of strong winds blowing away the balloon; this, however, might easily be provided against. Such an experiment should be conducted in a large field, free from trees, and the balloon should be left aloft until every sign of dangerous cloud, mist, &c., had disappeared. For experiments on the presence of electricity, when expected in but small quantities at moderate elevations, a fine wire, or string with a fine copper wire inclosed in it, may be attached to an arrow, whilst the other end is in contact with an electroscope. If the arrow be then shot from a bow, as it ascends, the gold leaves or pith-balls will diverge, more or less rapidly, in proportion to the amount of free electricity present. A metallic ball, if connected with the string, and thrown upwards by the hand, may be used instead of the arrow, and is often a convenient arrangement.

With respect to the cause of free atmospheric electricity, many theories have been proposed; but the question is still involved in much obscurity. The air is, perhaps, never absolutely destitute of its presence; hence this constant evidence of its existence must be connected with a chain of causes, equally constant in their action. If we inquire into these, we must depend chiefly for explanation on the variable quantities of atmospheric moisture, change of temperature, and friction of masses of air in motion. There seem, however, no exact indications afforded as to when we may expect a redundant or deficient state of electrical action. Thus, in winter time, it is well known that free positive electricity is most abundantly noticed, especially during cold, dry, N.E. winds; and yet, during the moist and hot periods of summer, we observe the most extraordinary instances of electrical disturbance—as in the thunder-storms, which attain their maximum effects in the hot regions of our globe. These facts, at first sight, present many difficulties. It must be remembered, however, that although the surface of our earth may be heated and moist, as in summer, there is always over us, at certain elevations, strata of air, of a temperature far below freezing point—perfectly dry, and, therefore, most probably surcharged with free electricity. As we progress from summer to winter, or from low to high latitudes, the intervening stratum of hot and moist air gets thinner and thinner, until the region of perpetual snow and free electricity becomes on a level equal to that of the sea. Now, in summer time, the earth, hot air, and superincumbent cold air, present an analogous case to that of the Leyden jar, which we have already explained;§ whilst in winter, the intervening hot air is absent, and the conditions just named are also wanting. From these facts we may thus see how it is that thunder-storms are most prevalent in hot weather; and, at the same time, why free electricity may be more readily noticed in winter than in summer; the former (or winter) involving a question of *conduction*, and the latter of *induction*, in respect to the laws of electrical science.

In assigning any probable cause of the constant presence of free electricity, we are in the predicament of not being able to reproduce on the small scale, in our laboratories, the same circumstances as transpire on the large scale in nature. Nothing is easier than to afford free electricity by the friction of glass, or other solid bodies; but to perform the same operation on either dry or moist air, so as to be able to draw intelligible and decisive inferences, has yet to be accomplished. By analogy, we may suppose that two currents of air passing parallel, either horizontally or vertically, and in opposite directions or together, at unequal velocities, may, by their friction, like that of silk and glass,|| develop free electricity: or again, that the difference of heat between two currents, in impact or parallel contact, may produce it in a manner similar to that of the Thermo-electric battery.¶ Separately, or conjointly, both these causes may be called into operation; for we frequently notice, during a thunder-storm, that the clouds move in contrary directions, indicating the existence of opposed air-currents, whilst, at the same time, we find that extremes of temperature simultaneously exist, as shown by the hail-shower in the midst of the hottest summer day.

As meteorological science becomes better known, and more extensively cultivated, we may hope for the establishment of observatories, where the electrical state of the atmosphere at different heights, at the same time and place, may be observed and registered, after the manner that is now adopted with the barometer and thermometer. We shall then have another and most important element added, in enabling us to study atmospheric conditions now involved in obscurity; and perhaps may succeed in unravelling the connection which subsists between them.

MR. GLAISHER'S BALLOON ASCENTS, Etc.

If our atmosphere were ever at rest, and unaffected by heat, moisture, &c.; or, in other words, were it merely a mixture of the two gases, oxygen and nitrogen, and solely acted on by the laws of gravitation, then its density would gradually diminish upwards, its capacity for heat would increase, and the sensible temperature fall. To ascertain how far these laws are disturbed by the abnormal conditions into which the lower mass of our atmosphere is thrown, becomes, therefore, a matter of deep scientific interest.

Some time ago, the British Association voted a sum for the purpose of defraying the expense of balloon ascents, by which meteorological observations, at various heights above the surface of the earth, might be carried on. We need scarcely remind our readers that no other mode exists of carrying out such observations, except that of ascending high mountains; and, in this way, the difficulties met with, and the inconvenience to which the observer is subjected, are almost insuperable. No such objection can arise when balloons are employed. In the course of a few minutes after leaving the earth, the

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* See *ante*, p. 183.

† *Ibid.*

‡ See *ante*, p. 320.

§ See *ante*, p. 183.

|| See *ante*, p. 170.

¶ See *ante*, p. 241.

observer can reach any altitude he may desire; he can ascend and descend as he pleases; his attention need be diverted by no object; and he is put to no personal inconvenience. Several ascents have been made; and we select the following account (furnished by Mr. Glaisher, of the Royal Observatory, Greenwich) of one made on July 30, 1862.

"Mr. Coxwell having planned an ascent with his large balloon, from the Crystal Palace, I availed myself of the offer of a seat in the car, for the purpose of taking a series of observations upon the variations of temperature, and the several hygrometrical states of the atmosphere at moderate elevations, and which I had failed to do in my recent ascent from Wolverhampton.

"The instruments necessary for these investigations were in the hands of Messrs. Negretti and Zambra, and were those which were broken and injured in my recent ascent with the same balloon, and confided to them for renewal and repair, and which they had executed in a very satisfactory manner, these gentlemen having, at all times, expressed an earnest desire to promote the experiments required by the British Association. All the other instruments are at Wolverhampton, ready for the next high ascent. The instruments used on this occasion were two aneroid barometers, dry and wet bulb thermometers, and a Daniell's dew-point hygrometer. As it is of essential importance that the thermometers should be so sensitive that they readily acquire the temperature of the surrounding air, they were of necessity very delicate.

"The air was in very gentle motion, which enabled Messrs. Negretti and Zambra carefully to fix the instruments before starting, so that I at once began to observe, which in my former ascent I was unable to do until I was the greater part of a mile high.

"The balloon left the earth at 4h. 40m., laden with thirteen gentlemen, and a large amount of sand for ballast, and, under the influence of a moderate breeze, bore away slowly nearly S.E., passing successively Eltham, Dartford, to the village of Singlewell, near Gravesend.

"The heights of the balloon, successively, were as follows:—At 4h. 43m. it had attained the elevation of 1,340 feet above the level of the sea; at 4h. 49m. that of 3,700 feet, increasing slowly, till, at 5h. 17m., it was about 5,300 feet. It then, by 5h. 20m., fell 200 feet. On throwing out sand it rose to 5,500 feet by 5h. 24m.; and to 6,600 feet by 5h. 43m. Some gas was let out, and it sank to 5,700 feet by 5h. 47m. Some sand was then thrown out, and it rose to 7,350—the highest point reached—at 6h. 1m. At 6h. 6m. it fell to 6,700. Some sand was thrown out, and it rose to 7,100 feet by 6h. 12m. At twenty minutes past six it descended to 5,300 feet, and slowly to 2,100 feet at 6h. 25m., and to the earth a little after half-past six o'clock.

"Between 4h. 46m. and 5h. 1m., the altitude and azimuth of the balloon were observed at the Royal Observatory, Greenwich, by E. J. Stone, Esq., M.A. If any similar observations have been taken at another place, I should be glad to be favoured with them, as furnishing the means of an independent determination of the elevations of the balloon, and thus checking those made by means of the variations of temperature and barometric pressure.

"The temperature of the air was 68° at the Crystal Palace. At 1,000 feet high it was 62°; at 3,700 feet it was 51°; it continued at this reading nearly till the height of 4,500 feet was reached. At 5h. 4m. it varied between 48° and 50°, till the height of 5,700; it decreased from 48° to 43½° between 5h. 31m. and 5h. 38m.; the height at the latter time being 6,100 feet. At the height of 7,350 feet the temperature was 41°, being 27° lower than on the surface of the earth. After this the temperature rose gradually as the balloon descended, and was 47° at 6h. 20m., 50° at 6h. 24m., and 68° on reaching the ground.

"The temperature of the dew-point, or that temperature at which the moisture in the air, in the invisible shape of vapour, is deposited as water, upon the objects cooled down to this temperature, were successively as follows:—In the gardens of the Crystal Palace this deposit of water took place when the temperature of the bulk of the hygrometer was reduced, by the action of ether, to 50°; at the height of 1,300 feet the bulb was bedewed at 43°; between 3,000 and 4,500 feet, at 40°; then up to 6,000 feet, at temperatures gradually decreasing; at about 7,300 feet, at a little below 32°; and it afterwards increased to 47° on reaching the surface of the earth. When the air is saturated with moisture, as in a wetting fog, the temperature both of the air and dew-points is the same. On this occasion, on the earth, the latter temperature was 18° below the former, and at the highest elevation was 9°, so that the air was nowhere saturated. From a knowledge of the temperature of the dew-point, we know the amount of water then present in a certain mass of air; this amounted, in the grounds of the Crystal Palace, to 4 grains in a cubic foot; at 1,300 feet high, to 3 grains; at 5,000 feet, but 2½ grains; and at 7,300 feet, there were about 2 grains in a cubic foot of air.

"As the amount of aqueous vapour in the air necessarily decreases with the temperature, the changes of the hygrometric condition of the air, at different elevations, may be better understood in speaking of the relative humidity of the atmosphere, by considering air as saturated with moisture represented by 100, and air without moisture by 0.

"The humidity of the air thus expressed was, on the surface of the ground, 50°, showing an unusual degree of dryness. It increased as we left the earth to 63° at 3,000 feet, to 66° at 4,000, and to 70° at 7,300; and the air gradually became less humid on approaching the earth. At no point, therefore, was complete saturation met with, as before noticed. The weight of a cubic foot of air varied from 526 grains on the ground, to 429 grains at the greatest height reached.

"Similar and simultaneous observations were taken at the Royal Observatory, Greenwich, from which we learn that the temperature of the air, on the surface of the ground, varied from 67° to 68°; that of the dew-point from 49° to 51°; that there were about 4 grains of water in a cubic foot of air; the degree of humidity was about 50; and the weight of a cubic foot of air was 526 grains: therefore the differences at the highest elevations, from these values, are due to the elevation alone.

"Test ozone papers were not coloured at all; and no ozone was noticed in the ascent from Wolverhampton.* We were never in the clouds, although rocky cumuli—which, it will be remembered, are the fine massive clouds of day—were at a lower elevation all round, and cumulostrati at a higher elevation than ourselves. The sky was free from clouds in the zenith, and of a deeper blue than as seen from the surface of the earth. At times there was a great mist, and, generally, the horizon was hazy and obscure; the shadow of the balloon was seen both on the ground, as well as on the surface of the clouds.

"A horizontal magnet occupied a somewhat longer time to perform a certain number of vibrations, both in this as well as in the previous ascent, than it did on the surface of the earth. This is contrary to the result obtained by Guy Lussac in 1804.

"At 5h. 24m. a gun was heard with a sharp sound; at 5h. 25m. a drum was heard; at 5h. 26m. a band was heard; at 5h. 38m. a gun was heard; and at 6h. 10m. a dog barking was heard; and the working of the engines on the Dover and Chatham Railway was distinctly heard.

"I have to thank W. F. Ingelow, Esq., for kindly reading one barometer for me, and for rendering considerable assistance in noticing the first appearance of dew on the black bulb of Daniell's hygrometer, enabling me thereby greatly to increase the number of my own observations. The gas furnished must have been of good quality to have raised so great a weight to such an elevation."

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* Mr. Glaisher states, subsequently, that this was owing to the paper-tests being of bad quality.

The preceding extracts, from Mr. Glaisher's description of his aërial voyage, afford our readers a general idea of the scope and objects which the British Association had in view when they first adopted balloon ascents, for the investigation of meteorological phenomena. Before stating the results which Mr. Glaisher has obtained, we may properly call attention to the variety of subjects which might be elucidated by observations carried on at a considerable height from the surface of the earth, and under circumstances in which numerous locally interfering causes would be absent.

In astronomical observations, it is of the highest importance that the proper refraction for any altitude of a star should be ascertained. The nearer the horizon, the greater is the refractive effect of the atmosphere on the apparent position of a heavenly body. Again, the *amount* of refraction depends not only on the altitude of the body, but also on the varying density of the atmosphere, its humidity, temperature, &c. Some of the effects of these accidents on terrestrial objects, have been already described in connection with the phenomena of the *Mirage*, &c.;* and the consideration of them will point out how essential an accurate estimation of their nature and amount is to the practical astronomer.

In estimating the force of gravity at any point on the surface of the earth, the pendulum is universally employed; and according to the number of its vibrations, in a given time, so the intensity of gravitating force is indicated for the place. In a similar manner, the vibrations of the magnetic needle may be made a measure of the increase or decrease of magnetic force, at any two altitudes when compared together. This subject is of extreme importance in connection with the philosophy of the Undulatory Forces, and one to which we have already directed attention in the Section on Magnetism. Our knowledge of the force of *gravitation*, may be enlarged by considering the attracting power, or perturbing forces, which each and all of the planets mutually exercise; whilst with respect to the *Magnetic forces*, our investigations are necessarily limited to the phenomena exhibited, on, a little beneath, or a short distance above, the surface of our globe.

The *theoretical* constitution of our atmosphere has been determined with comparative precision; but its *actual* composition, at various altitudes, must necessarily depend on local disturbing causes. It is but reasonable to suppose, that the neighbourhood of large towns would present an atmosphere tainted with the impurities produced by manufacturing operations, furnaces, and a crowded population. The *fact* of the air, in such localities, being deteriorated, has long been familiarly known; but the *amount* of impurity dissolved in the air, under such circumstances, and at varying heights, has not been even approximately measured until recently. The presence or absence of ozone, affords us a means of testing the respirable qualities of the air, as has been already pointed out at a previous page.†

In the article on Photography, we explained the peculiar effects of the actinic rays of light on salts of silver, &c.‡ Now the daily experience of every operator teaches him, that considerable difference in the power of the actinic rays is constantly occurring. A change of atmospheric humidity, amount of cloud, &c., is instantly attended with an increase or decrease of actinic force; and hence the varying time required to take a good photograph, especially in landscape effects. We also find, that within the tropics, at the level of the sea, it is difficult to get a good impression on the sensitive plate in the camera, although, in the same region, excellent views are produced at considerable altitudes.

Thus, in connection with Astronomy, Magnetism, Chemistry, and Photography, we must perceive, that investigations carried on in the upper strata of our atmosphere, can but result in an accession to our knowledge; for, at a height of a few thousand feet, we are, to a large extent, removed from irregular and accidental disturbing causes, the presence of which vitiate accurate observations and successful experiments in any of the branches of science to which we have referred. It will be unnecessary to urge similar considerations on those who have read the previous pages in this section, in connection with meteorological phenomena, which are almost invariably produced at heights above 1,000 feet from the surface of the earth. Clouds, hail, snow, rain, lightning, meteors or aërolites, &c., all owe their origin to atmospheric disturbances, occurring, in many cases, far above the reach of man, if unaided by the balloon. Science has provided us, in this machine, with a means by which, Dædalus-like, we may fly in the air, and, at our leisure, note phenomena of the utmost importance to the existence of our race. The *cui-bono* inquiry usually entertained in reference to most investigations of a purely scientific nature, may be readily answered, in the case of balloon ascents, by remarking, that if, through our present limited knowledge, obtained by observations on the surface of the earth, we have been enabled to forewarn the mariner of a coming storm, and save him and his vessel from shipwreck,§ how much more successful may we hope to become, if we venture into those winds, clouds, and storms which he dreads, and investigate their causes, connection, &c., in their own regions, under circumstances so essentially fitted for our purpose.

Having thus pointed out the value of balloon ascents, in a scientific and practical point of view, we proceed to give an analysis of the results which Mr. Glaisher has obtained.

The chief objects which were sought after in the balloon ascents of 1862, were as follows:—

1. The ratio of increase or decrease of the temperature, and the amount of aqueous vapour in the air, at heights within five miles (26,400 feet) above the sea-level, in this country.
2. To compare the readings of aneroid and mercurial barometers.
3. To ascertain the electrical state of the atmosphere at various elevations.
4. To discover the amount of free ozone.
5. To register the number of vibrations of a magnetic needle at increasing heights.
6. To find the relation of the temperature of the air and that of the dew-point, at heights corresponding with those of high lands in India, and other places; and to compare the indications of the dry and wet bulb-thermometers with those of Daniell's and Regnault's hygrometers.
7. To collect air at different elevations.
8. To note the height, kind, thickness, density, &c., of various strata of clouds.
9. To ascertain the rate and direction of atmospheric currents.
10. To make observations on the intensity, &c., of sound.

Besides the above particulars, it was intended to note any phenomena which might casually present themselves during the progress of the observations indicated above. The usual kind of meteorological instruments were employed—each, however, having been specially prepared for the purpose. Owing to occasional accidents, many of them were broken during descents.

Eight ascents, in all, were made in 1862; viz., three from Wolverhampton, which was chosen as a central position, and because it afforded a good breadth of land, without the danger of a voyage over the sea; four from the Crystal Palace at Sydenham; and one from Hendon, which was, in fact, a re-ascent from the spot at which the voyagers had alighted on the previous evening. The dates of the ascents were all in the summer months, being from July 17 to September 8, inclusive.

The greatest height attained was on September 5, in an ascent from Wolverhampton. This was estimated to be at least 36,000 feet; but as Mr. Glaisher became unconscious, owing to the high degree of atmospheric rarefaction, the precise elevation was undetermined. In the ascent of July 17, a height of 26,177 feet was attained; and "in the descent, a mass of vapour, 8,000 feet thick, was passed through. This was so dense that the balloon was not visible from the car."

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* See *ante*, pp. 1147, 1148. † See *ante*, p. 1122. ‡ See also *ante*, p. 57.

§ See *ante*, p. 1173.

Mr. Glaisher prepared the following table, which shows the mean temperature of the air at each 5,000 feet above the level of the sea, on the dates named, selecting those in which the highest ascents were made.

Height above the Level of the Sea.	Mean Temperature of the Air.					Decrease of temperature for an Increase of Height of 5,000 feet.
	July 17.	August 18.	August 21.	September 8.	Mean.	
Feet.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.
0	61·2	69·6	62·0	62·2	63·8	...
5,000	39·7	48·0	43·3	41·4	43·1	20·7
10,000	28·0	40·7	32·0	31·0	32·9	10·2
15,000	31·0	31·1	19·0	21·0	25·7	7·2
19,500	42·2	...	...	...	...	...
20,000	33·0	25·9	...	10·6	23·2	2·5
25,000	16·0	23·9	...	0·0	13·3	9·9
30,000	...	...	...	–5·3	...	...
Decrease of Temperature for an Increase of Height of 25,000 feet.	44·9	45·7	...	62·2	50·5	...

On these facts Mr. Glaisher remarks—"The results of July 17 are perfectly anomalous. Up to 10,000 feet the decrease accords with the other days of experiments; but from 10,000 feet, the results are perfectly surprising, and continue so up to more than 20,000 feet. Above 25,000 feet, they are again accordant. If we suppose that up to 10,000, and again at 25,000 feet, the results are not abnormal, by continuing the curve joining these two portions, and then reading, we should have the following readings: viz.—

"At 0 feet, mean temperature = 61°·2
" 5,000 " " " = 39°·7
" 10,000 " " " = 27°·5
" 15,000 " " " = 22°·7
" 19,500 " " " = 20°·0
" 20,000 " " " = 19°·5
" 25,000 " " " = 16°·3

"Then the measure of disturbance would be as follows: viz.—

"At 10,000 feet . . . . = 0°·5 in excess.
" 15,000 " . . . . = 8°·3 "
" 19,500 " . . . . = 22°·2 "
" 20,000 " . . . . = 13°·5 "
" 25,000 " . . . . = 0°·3 in defect.

"The numbers, in the last column of the table, show that the average decrease of temperature, in the first 5,000 feet, exceeds 20°, and, in the next 5,000 feet, is but little more than 10°. The numbers, in the lowest line of the table, show that the average decrease of temperature for 25,000 feet, is 51° nearly. From these numbers, it seems that two-fifths of the whole decrease of temperature, in five miles, takes place in the *first* mile; and, therefore, that the decrement is not uniform with the increment of elevation."

So far, our attention has been confined to the results which Mr. Glaisher obtained during the highest ascents he effected. In the following table the temperature is given for each 1,000 feet of elevation, between the earth and 5,000 above it, for each ascent made during the summer of 1862. It will be seen that the temperature decreases 21° in the 5,000 feet, and that the rate is not uniform. (See also Tables at pp. 1215 and 1216).

Height.	July 17.	July 30.	August 18.	August 20.	August 21.	Sept. 1.	Sept. 5.	Sept. 8.	Mean.	Effect of 1,000 feet.
Feet.	Degrees.	Degrees.	Degrees.	Degrees.	Degrees.	Degrees.	Degrees.	Degrees.	Degrees.	Degrees.
0	61·2	70·0	69·6	66·8	62·0	67·0	62·2	69·7	66·1	...
1,000	57·0	63·0	62·0	62·0	58·0	59·8	57·8	65·0	60·6	5·5
2,000	52·6	56·8	56·5	57·0	54·0	53·2	53·0	60·4	55·4	5·2
3,000	48·3	52·2	53·3	52·7	50·3	49·2	48·7	55·7	51·3	4·1
4,000	44·0	49·8	50·4	49·0	46·9	47·7	45·0	51·5	48·0	3·3
5,000	39·7	47·0	48·0	45·0	43·3	46·0	41·4	48·4	44·8	3·2
Effect of an Elevation of 5,000 feet	21·5	23·0	21·6	21·8	18·7	21·0	20·8	21·3	21·3	...

With respect to other objects of inquiry, it was found, that the electricity of the air was generally positive, diminishing in amount as the elevation increased; that ozone was present in large quantities; that the vibration of the magnet was longer at high elevations than on the earth; and that sounds varied, in the distance at which they could be heard—the barking of a dog being noticed at a height of two miles, whilst the shouting of a multitude was not heard at one mile high.

The general results deduced were—

1. That the temperature of the air does *not* decrease uniformly with an increasing elevation above the surface of the earth; a fact bearing strongly on the refractive powers of the atmosphere, in relation to astronomical observations.

2. That an aneroid barometer may be correctly read to one, and probably to two, places of decimals, when indicating a pressure equal to five inches of mercury in the barometer. (See Table at page 1214.)

3. That the atmospheric humidity decreases rapidly to a height of five miles, where its amount is extremely small.

4. That the readily accepted theory, as to the prevalence of a settled west or north-west wind, was not confirmed.

5. That the opinion of the clouds assuming a counterpart of the earth's surface, and that they rise and fall like hills and dales, was not countenanced by the appearance of their upper surface.

The following interesting description of the appearance of the clouds, as observed on the date named, is quoted from the *Athenæum*, to which periodical we are indebted for many of the facts we have adduced. The balloon had descended on the previous evening at Hendon, and had been anchored for the night, preparatory to an ascent on the following morning.

"Before sunrise, on August 21, all the instruments were replaced, and the balloon left the earth: it was a warm, dull, cloudy, morning; clouds were reached at the height of 5,000 feet; the light rapidly increased, and gradually the balloon emerged from dense clouds into a basin surrounded by immense black mountains of cloud, rising far above. Shortly afterwards there were deep ravines of grand proportions below, bounded with beautiful curved lines. The sky was blue, with cirri. The tops of the mountain-like clouds became silver and golden: at the height of 8,000 feet we were on their level, and the sun appeared, flooding, with golden light, all space for many degrees both right and left, tinting with orange and silver all the remaining space. It was a glorious sight. As the sun's rays fell on the balloon we rose more rapidly, each instant opening to us ravines of wonderful extent, and presenting elsewhere a mighty sea of cloud. Here there were shining masses in mountain chains, some rising perpendicularly from the plains, dark on one side, and silvery and bright on the other, with summits of dazzling whiteness: some there were of a pyramidal form; a large portion undulatory; and, in the horizon, Alpine ranges bounded the view. A height of nearly three miles was reached."

In reference to the ascent of September 1, Mr. Glaisher remarks—"When at the height of three-quarters of a mile over London, the whole course of the river Thames was visible from its mouth; and parallel to it, and bounded by its banks, a cloud or fog-bank extended the whole distance, following all its sinuosities. For half-an-hour before its descent, near Woking, in Surrey, the balloon was under one stratum of cloud, and above another: the upper surface of the latter was remarked as bluish white; the middle portion the pure white of the cumulus; and the lower surface a blackish white, and from it, rain was falling on the earth. The balloon descended to a height of 1,300 feet, but was still above the clouds. It was afterwards learnt that rain had been falling from these clouds all the afternoon."

In the following year (1863), Mr. Glaisher made fifteen ascents; and the results obtained agree, in the main, with those already detailed. The weather, during the summer, was generally calm, and comparatively little rain fell for some months. It will be unnecessary for us to present, in a tabular form, the results obtained; but we shall conclude our notice of the ascents by giving some of the most interesting particulars of the last, which was made on September 28. The object in this one was chiefly to observe the effect due to the influence (if any) of the autumnal equinox on atmospheric phenomena.

The balloon rose from the earth at 7h. 43m.; in two minutes afterwards it was at a height of 500 feet; in four minutes, at 1,000 feet; in nine minutes, at 2,000 feet; 2 miles high at 8h. 30m.;. 2½ miles at from 8h. 45m. to 9h. 8m.; 2¾ miles from 9h. 15m. to 9h. 23m.; and at 3½ miles, the highest point, at 9h. 33m.: it then descended to 2¾ miles at 9h. 52m.; 2½ miles at 9h. 59m.; 2 miles at 10h. 4m.; 1½ mile at 10h. 9m.; 1 mile at 10h. 13m.; 3,000 feet at 10h. 15½m.; 1,000 feet at 10h. 23m.; reaching the earth at 10h. 30m. The temperature of the air at 7 o'clock was 43°, and it became 48° when the balloon left the ground at 7h. 43m.; at 1,000 feet elevation it was 45°; at 3,000 feet, 40°; at 5,200 feet, 34°; at 8,000 feet, 27°; at 10,500 feet, 21°; and at 13,000 feet, 12°. The temperature varied irregularly with the rising sun, increasing to 16° at a height a little above 2½ miles. On descending, the temperature, which varied from 7° to 15° at the height of 2¾ miles, gradually increased to 35° at 5,000 feet; 40° at 3,000 feet; 53° at 1,000 feet; and the same amount was noticed on reaching the ground.

The temperature of the dew-point* was 44° on the ground; 35° at 2,000 feet high; 25° at 5,200 feet; 20° at 13,000 feet; ranging, at this level, between 20° and 12°; sometimes being at the same temperature as that of the air, and, therefore, indicating complete saturation in some places. Above this level, the dew-point decreased to 10° below zero, approaching more nearly to saturation than had been noticed, at that height, in previous ascents. On descending, the dew-point rose till it attained 46°, which was the temperature at which it was produced at the time of reaching the earth. On the ground, the air contained 3½ grains of aqueous vapour in the cubic foot; at 2,000 feet elevation there were 2½ grains; at 5,200 feet, 1½ grain; and at the height of 17,000 feet, there was less than half a grain in a cubic foot. Presuming the greatest amount of moisture which could be present at the then temperature of the air, to be divided into 100 parts, it would have been, on the ground, and up to a height of 1,000 feet, equal to 90 parts; at 5,200 feet, 72; and varied, between 70 and 100, up to the highest point reached.

In comparing the decrements of temperature during the ascent, and the increments as the descent was effected, it will be evident that no regularity could be noticed. If the results, again, are compared with those given at page 1182, the discrepancy is still more apparent. But this might have been expected, from the circumstances under which the observations were made. The ground was below the ordinary temperature at the early hour of starting; for, immediately the sun rose, the temperature was also raised. Of course, during the ascent, every stratum of air passed through would be similarly affected by the absorption of the heating solar rays; and hence any observations would be incomparable, in a practical point of view, with others preceding or following them. Instead, therefore, of the temperature decreasing, in a regular ratio, as a greater elevation was attained, the solar influence quite prevented such a result; and, in the descent, similar irregularities would be met with. Comparing the observations up to one mile elevation, we have a temperature at—

		Ascent.		Descent.
0 feet	. of .	41°	. of .	53°
1,000 „	. „ .	45°	. „ .	53°
3,000 „	. „ .	40°	. „ .	40°
5,200 „	. „ .	34°	. „ .	35°

Whence it would appear that the influence of local causes, such as terrestrial radiation, convection of heat upward from the surface of the earth, &c., greatly affected all the observations made up to a height of about one mile—a result similar to that mentioned at page 1182.

Although, for purely scientific purposes, these observations afford no certain data, we may yet gather valuable hints from them in a sanitary point of view. It is plainly evident that our atmosphere will be considerably and unequally affected at times, with respect to its purity, if it be so in regard to its temperature; because a normal atmosphere, acted on by the simple physical laws of pressure and expansion alone, would certainly decrease in its sensible heat in definite ratios depending on the altitude. But, from actual observations in this island, we find that there exists an extensive strata, just touching the earth's surface, which cannot be proved to obey this law in an equable manner; and this seeming exception arises solely because the effects are masked by accidental, and, therefore, local causes. Mr. Glaisher has, more than once, noticed the smoke, &c., from large towns (especially London), to reach a mile in height; just, in fact, the elevation beneath which the law of the temperature, being in the inverse ratio of the altitude, appears to be stopped in its action. From the numerous observations made, it has also been shown, that the stratum of air, within a mile over the earth, is that most readily affected by storms—a result which we should fully expect, from the abnormal state into which it is thrown by the variety of disturbing causes to which its lower mass is subject.

Let us, for a moment, direct attention to the various affections to which the atmosphere existing over large towns is liable. Thousands of fires in private dwellings; furnaces in the various manufactories; heat from the bodies of the inhabitants; radiation of previously absorbed solar heat, from the materials of the houses, paved roads, &c., &c.;—all tend to cause an enormous up-draught of heated air, which rises as if produced from an immense furnace. A spectator, situated at a short distance from any of our large towns (especially London), if on an elevated position, and in the absence of all wind, will observe an immense body of smoke, &c., rising vertically, and spreading itself, like a canopy of darkness, over the under-lying city. This smoke, or vapour, embodies every volatile impurity produced by all possible causes; and so long as it has a higher temperature than the atmosphere, it remains suspended over-head. If a purely normal atmosphere, uninfluenced by external disturbing causes, were perpetually surrounding such a district, every gas, and particle, of greater specific gravity than itself, would, on gaining the temperature of the air, be precipitated on to the place whence it had arisen—a very common occurrence, indeed, as is seen in the little "blacks" that so abundantly fall during heavy weather, in large towns, and which result from the aggregation of the carbon, &c., in the smoke. Hence, the inhabitants of such a place would live in an atmosphere completely stagnant, and daily becoming more impure; the culminating effect of which would be disease and death. *Locally*, on the small scale, this is constantly occurring in narrow streets, alleys, and crowded dwellings; and, *generally*, whenever the air has become still, and unaffected by moving causes.

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* See ante, p. 1146.

But the very irregularity in the physical constitution of an atmosphere such as ours, is productive of the most beneficial results. Never at rest in itself—which it would be if in its normal condition—it is ever susceptible of external influences; hence the refreshing breezes that course down our streets, or the boisterous March and November winds, which, scavenger-like, cleanse our fever-stricken courts and alleys. The very characteristic which seems to prove it an exception to that regularity of law prevalent in nature, is that which forms one of its most valued qualities, making it life-giving in its energy, whilst its stability of position would be synonymous with death.

Reverting, after this brief digression, to the results obtained by Mr. Glaisher, we may notice the actinic observations made during the ascent. By Herschel's actinometer, the force of the sun's rays was shown, at 11 A.M. (at the Wrottesley Observatory), by 48 divisions in one minute; and at 5 P.M., by 25 divisions, the sun being then low. At 9h. 8m., during the ascent, the increase was, with a bright sun, 5 divisions in one minute; at the height of 13,000 feet (at 9h. 29m.), it was 7 divisions; at 16,000 feet, the increase was about 7 or 8 divisions; at 10h. 7m., or 8,000 feet elevation, it was 20 divisions; at 10h. 19m., and at a height of a little over 1,000 feet, it was 25 divisions; and on the ground, at 10h. 45m., the number was 48. With respect to solar radiation, a delicate blackened-bulb thermometer, at heights varying from 13,500 to 1,000 feet, in the descent, showed a variation not exceeding 4° to 8° beyond that of the shaded thermometers: at the height of three miles, although the sun shone brightly, the blackened bulb showed an excess of but 2° to 4°; whilst, on the ground, that became 30°. The observations, during previous ascents, had proved that a self-registering blackened-bulb thermometer, in vacuo, with its bulb always in the beams of the sun, never read so high as before leaving the earth; and on being re-set, indicated a reading but little beyond that of the shaded thermometers. This result very possibly arose from the great capacity that rarefied air has for heat, which would cause the immediate loss, by radiation, of nearly all the caloric absorbed by the bulb during its exposure.

A sky spectrum, with the lines visible from B to G,* was observed, from 7h. 58m. to 9h. 4m., at heights varying from 3,000 to 13,000 feet; from 9h. 22m. to 9h. 43m., and at heights between 15,000 and 18,000 feet, the spectrum extended from A to H, at which latter point it became extremely dark, whilst the lines in the violet end were very numerous. We have already remarked that the actinic rays were most active at high elevations; and Professor Piazzi Smith, during the British Association Meeting of 1863, furnished some very interesting facts, which we may conveniently give, as verifying the observations of Mr. Glaisher. He stated that, in 1856, the chief object of astronomical experiments made on the Peak of Teneriffe, which has a height of 12,000 feet, was to ascertain the degree of improvement in definition, obtained whilst using the telescope at great elevations. It was found—the air being so dry and clear—that stars, when viewed by powerful instruments, always gave clear and well-defined discs, surrounded with regularly formed rings, even when a high magnifying power was used. He had recently magnified some of the photographs taken on Teneriffe, in 1856, at various elevations, and found that their clearness and definition depended on the elevation, and that they remarkably confirmed the results of telescopic observation. Near the sea-level a photograph could never be taken which would show clearly the detail on the side of a distant hill, however brightly the rocks or cliffs were illuminated by strong sun-light; the microscope only brought out a broad, faint, and nearly uniform tint. On the other hand, photographs taken at high elevations, exhibited clearly, and well-defined, the most minute detail; even bushes on a hill-side, distant four-and-a-half miles, were distinctly marked.

With respect to the amount of ozone, which was observed in the last ascent of Mr. Glaisher, it appeared that none was shown by papers freely distributed about the car; but ozone powders, made of wheat-starch and iodide of potassium, were slightly tinged when first exposed; were unchanged at a height of one mile; but when noticed at 9h. 57m., at a height of about 13,500 feet, they indicated an amount of colour equal to four, when compared with a scale whose maximun is ten.

We have thus put our readers in possession of the leading facts obtained during the scientific balloon ascents of late years. Whilst they have largely added to our knowledge of meteorological conditions, with regard to the temperature, moisture, quantity of ozone, actinism, &c., of the atmosphere immediately over the earth, they have entirely dispelled the illusive theory which was long indulged in—that the decrement of temperature was in inverse ratio of the altitude. In a purely theoretical point of view, such can only be the case in an atmosphere entirely unaffected by solar radiation and the convection of heat, and in the absence of purely local disturbing causes. These conditions cannot be present in our atmosphere, within any elevation approachable by man, above the surface of the earth. Beyond a distance of five or six miles, it is impossible that the observer could exist: indeed, in one of the ascents, this elevation was attained, and nearly proved fatal both to Mr. Glaisher and Mr. Coxwell; the former entirely losing all consciousness, whilst Mr. C. lost the control of muscular action, and was all but a passive observer of the nearly inevitable destruction of himself and companion.

There could scarcely have been chosen any situation in Europe in which so many disturbing causes to normal atmospheric conditions are presented. Surrounded by the sea; of narrow breadth; situated between the warm Gulf-stream on the west, and the cold polar currents on the north and east—our island presents almost a uniqueness of condition in respect to meteorological phenomena. If circumstances permitted, ascents made in regions less liable to such extraordinary interfering causes, might give some interesting results; and many parts of continental Europe afford such opportunities. At present, the proper mode of managing the balloon is but little known, and confined to but a few individuals, who possess sufficient practical skill. On frequent occasions, the lives of both the aeronauts were in imminent danger; and it is a matter of great surprise, that so many valuable observations could have been made under circumstances in which the majority of persons would have only exerted themselves for their own safety, and regardless of the interests of science. There can be little doubt that a constant repetition of these experiments will afford a much larger quota of results; for as, on each ascent, the external conditions of our ever-changeable atmosphere may be expected to vary, new phenomena will present themselves, whilst previously acquired experience will be prepared to seize on each novelty, and assign to it a proper place in the departments of meteorological science.

LUMINOUS METEORS, AND THE AURORA BOREALIS.—During the last few years, considerable attention has been devoted to the study of meteors, the appearance of which has become more frequent, or, perhaps, more noticed. In 1862 and 1863, several large luminous bodies were seen; and, amongst them, one was observed, simultaneously, in the middle and southern districts of England, and in the north and centre of France—an evidence that it must have travelled at a considerable height above the surface of the earth. In the early part of August, 1863, the usual annual meteoric display was exceedingly great, attaining the maximum on or about the 11th of the month, when as many as 200 "shooting stars" were noticed in about three hours. In the constellation of Cassiopeia, we perceived, at 9 P. M., an incessant discharge *south-westward*, lasting for above an hour, none of the balls having a great size, and the distance travelled by each being from three to five degrees: the elevation above the surface of the earth seemed to be great, judging from the comparatively small velocity and apparent magnitude of each meteor. Most observers remarked, that the paths of the meteors, about that date, chiefly lay from north-east to south-west. We observed but few instances in which any tail was shown, although the evenings were generally favourable for observation.

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* See *ante* p. 56.

The extremely rapid course of these bodies renders it very difficult either to calculate their elevation, or to use means of testing the nature of the light they emit, as can be done with the sun, &c., by spectrum analysis.* Two observers at distant stations, may, by chance, simultaneously observe one; but such an occurrence seldom presents itself; hence the varying estimates which are made respecting their height. There can be little doubt, from their periodicity, that they are solid bodies, and not of electric origin, as was formerly supposed—on, perhaps, the slightest possible grounds. The different theories which have been suggested, to account for these meteoric phenomena, have been already alluded to in an earlier portion of this section; and we regret that, at present, nothing of a more definite nature has been arrived at.†

With respect to the Aurora Borealis, which has also been spoken of, there can scarcely be a doubt as to its origin. Its great altitude, fleeting nature, and general appearance, all bespeak an electric origin. Few persons residing southward of Scotland can have any idea of the beauty of this phenomenon, which may be seen in great perfection, even in our own isles, during the winter season. In the western and northern districts of that country, we have often observed the entire heavens illuminated; the north, north-western, and north-easterly parts shining, of course, with the greatest brilliancy. Occasionally the coruscations proceed from those quarters extending far beyond the zenith, in a southerly direction, so that the whole sphere overhead is brilliantly lit up. At other times, but more rarely, the zenith itself seems to be the centre of action, and, ray-like, thousands of broad sheets of flame, of many colours, dart on all sides. Generally speaking, when an Aurora is thus developed, the flames assume a lambent or phosphorescent appearance, are comparatively ill-defined, and rarely more dense in one place than another. At times we have seen a lake, five miles long, and four broad, illuminated from the northern side by an Aurora, to an extent quite equal to that it usually presented under the full moon, and the shadows from the over-hanging mountains were deeply cast on the water. During several years' observations, often carried on nightly, we noticed that the Aurora was always better seen inland than near the sea; our places of observation ranging between the east and west coast of Scotland, on a line extending north of the Clyde and Forth rivers, as far as the counties of Ross and Cromarty.

In the experiments which we undertook, some years ago, on the spark in vacuo, from the Rhumkorf coil, and which are related at a previous page,‡ we frequently noticed the peculiar effect resulting from the introduction of a minute quantity of various vapours into the highly exhausted receiver. So long as the remaining atmospheric air was pure, the colour of the spark was a rich reddish purple, becoming bluer in proportion as the exhaustion was increased. A more dense state of the air, effected by its gradual admission, only made the spark of a redder tint. If, however, the vapour of bisulphide of carbon were passed in, in exceedingly minute quantities, a sudden flash of delicately whitish-yellow light was produced, exactly resembling that of the diffused Aurora we have shortly before alluded to. Without describing a great variety of experiments, we may remark, that every tint of the Aurora may thus be produced by choosing proper vapours (chiefly of ethers), and simultaneously regulating the vacuum, which should vary, in the air-pump, from about 27 to 29 inches, the external air having a barometric height of not less than 29·30.

Now a repetition of such experiments cannot fail to suggest, to any one who has seen the Auroral phenomena we have just described, their great similarity, or, to us, identity of appearance. If electricity be assumed as the cause of Auroræ, precisely such circumstances may occur, in the elevated regions of the atmosphere, as we refer to in the above remarks—namely, a minute quantity of solid colouring matter, diffused in a highly attenuated atmosphere. Rapid chemical combination would constantly occur, attended with the production of flame, between the solid matter and the oxygen of the atmosphere; and if it be objected that the quantity of oxygen present would be excessively small, we reply, that the results obtained in the air-pump receiver, are produced under exactly similar conditions; for the vacuum very nearly approaches that existing in the regions in which the Aurora is supposed to exist, assuming that to lie within the limits of our atmosphere.

Whilst offering this as an explanation of certain Auroral appearances, we do so under the conviction that it can be but a suggestion, the value of which is dependent on analogy alone. It is by no means certain that the Aurora is at all caused by electric action; indeed, such is only inferred from the disturbance of the magnetic needle generally attending the displays of Auroræ; and, again, the relation of this fact to electrical science is also one of inference, depending on the knowledge we possess of the mutual production of the electric and magnetic forces, so far as our experimental determinations have carried us.

THE METEOROLOGICAL INFLUENCE OF DRAINAGE.—We have incidentally spoken of the various local causes which influence the meteorological character of some districts, and we shall devote a short space to the consideration of that subject; more especially in reference to the great advances which have been made in the drainage, &c., of some parts of this country, and to the permanent improvement of extensive tracts of land, in a meteorological, agricultural, and sanitary point of view.

It is familiar, in the experience of every one, that a low, flat, and marshy tract of land is generally a region of fog, and often of rain. In England, for example, the land between the Thames and Boston Wash—a length of about 100 miles, and of variable width—is chiefly of this kind. The superincumbent air is loaded with moisture, and, as far as our experience has gone, there is a greater frequency of thunder-storms, and, for the moment, heavier showers of rain, than in the districts flanking it. The same remarks are equally applicable to extensive tracts in France, Holland, Russia, India; Africa, as at Sierra Leone; Florida, Georgia, and various places in North and South America; with many others too numerous to mention here. The meteorological characteristics we have just referred to, of course vary, according to the annual average temperature; being in excess in low latitudes and about the equator, and gradually diminishing as we approach the pole, where the moisture of the air arrives at its minimum. If these local peculiarities were due solely to the presence of water, then the ocean should be equally liable to them; but that such is not the case is well known; and the cause of the difference is easily explained.

In the Section on Heat, we called attention to the different radiating powers which bodies possess; pointing out that those of a rough nature radiate quickly, whilst polished surfaces are poor radiators.§ Now just such a difference subsists between the surface of the ocean, or any extensive sheet of water, and that of land saturated with moisture; the constituent particles of the latter radiating heat rapidly, whilst the former does so but very slightly. Generally speaking, good radiators greatly promote rapid evaporation, just as we find in the case of the ordinary porous wine-coolers, from the surface of which moisture is quickly carried off. In precisely a similar manner, the extensive surface of sodden earth, in swampy and marshy lands, absorbs heat during the daytime, and as readily radiates it at night. Evaporation is quickly carried on; but being attended with a cooling effect, the moisture conveyed into the air becomes partly condensed, and hence the mists and fogs, so common in the damp and fenny portions of Essex, Cambridgeshire, Lincolnshire, and other parts of our own country. For similar reasons, and arising from the same causes, we may often, in summer time, notice a deep haze coating a valley beneath us, as soon as the sun sets, if we are standing on some moderate elevation. The depth of the haze or mist will depend on the amount of moisture which has been evaporated during the day, and the fall of temperature at night—presuming, of course, that the air is perfectly still, and that various circumstances, to which we need not here allude, are favourable to radiation.*

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* See *ante*, p. 168.

† See *ante*, p. 1150.

‡ See the Section on Magnetism, p. 256, Exp. 15, *et seq.*

§ *Ante*, p. 26.

Such, therefore, being the causes of the superabundant moisture, haze, fog, &c., of damp grounds, we may proceed to notice their general effect on animal and vegetable life; and here we may observe, that the natural constitution of the subject will materially influence the effect, prejudicially or beneficially. Many plants can only vegetate in an exceedingly hot and moist air; and their healthy existence, in any place, is highly indicative of the surrounding climate. Even human beings can be so inured to a particular region, as to live, without serious inconvenience, in places where others would find an untimely grave. Many parts of the coasts of Africa, North and South America, &c., illustrate this; and even our own fenny districts, in which ague is so prevalent, are inhabited by individuals whose families have long resided there, and being acclimatised, are comparatively free from diseases which a casual or temporary visitor is sure to suffer from. It is not only, however, in the districts themselves, in which such meteorological phenomena are common, that danger to health may arise; the surrounding country is equally liable to be affected: and hence every method should be adopted which may possibly remedy the evil. The usual effect of too moist a climate, is that of a general depression of the nervous system, and, consequently, a liability to impression of diseases, especially those of the low febrile class, as the typhoid, &c. Most endemics partake of this character; but they are frequently accompanied with others, in which diarrhœa is usually a characteristic. In some cases, a peculiar fever becomes endemical: as an instance of which we may cite New Orleans, where yellow-fever is a regular and periodical visitant. It is stated, however, that the rate of mortality is not above the average, for those born and brought up in the city; and that the fever is chiefly fatal to visitors. This most likely arises from want of precaution on the part of the latter, who are likely to expose themselves to the dangerous influence of the climate, which the regular inhabitants prudently avoid.

In most marshy places, but especially in warm or hot climates, vegetable growth and decay proceed rapidly, being stimulated by the heat and moisture together. This is remarkably the case on the west coast of Africa, and in many parts of Italy, of which the Pontine Marshes are a well-known example. In such districts pestilence is always to be found, arising from the abundant miasmatic vapours which are generated. The Niger is too well known as a most dangerous locality for Europeans; and its pestilential effects are readily traced to the presence, in immense quantities, of poisonous gases, which are given off from the rich tropical vegetation as it undergoes decomposition, caused, and rapidly promoted, by the hot and moist air.

Of late years these facts have received the earnest attention of all classes, and the principles of meteorology have been studied in reference to the best method of detecting, avoiding, or ameliorating such conditions. The hygrometer has been of great use for the examination of the atmosphere in suspected localities; because, as the dew-point is formed at a temperature which differs from that of the air, just in proportion to the dryness of the latter,† it follows, that the lower the dew-point average is in any place, the drier, and therefore the more salubrious, is the country for a permanent residence. One of the objects which Mr. Glaisher carried out in his balloon ascents, was that of acquiring information on this matter, which might subsequently be of service in directing the choice of stations for our troops in India and other tropical countries.

In attempting to improve the climate of marshy places, of course drainage must be had recourse to; and as the practical part of this belongs more especially to engineering science, it will not fall within our scope to discuss it in its mechanical details. Not only does the withdrawal of the surface-water of swamps, marshes, &c., lessen, and, if complete, expel standing fogs and mists, but it also diminishes the rain-fall, on the average, throughout the year. The reason of this is twofold: first, there occurs a diminution of superincumbent moisture, which, in part, causes the rain; and, secondly, through that, the addition to rain falling from high regions (due to the presence of moisture at a lower level), is avoided; for, as we have already seen, more rain falls, in any spot, as we approach the surface of the earth, because of the drops gathering additional moisture from the air they pass through.‡ The application of a good drainage system has, therefore, the effect of removing the moisture itself, and, simultaneously also, the cause of its continued production. Occasionally, even the geological character of a country may be judged of by its meteorological characteristics. Thus we generally find that clayey soils, which will not allow the water on their surface to drain away, are covered with mists; and the temperature is usually lower than that of a surrounding but better drained country. Such soils are generally termed "cold" by the farmer, who, apart from any scientific theories, has discovered, by long experience, that their temperature, superabundant moisture, &c., unfit them for producing many kinds of crops. A few years ago, such places were left in their natural state, but that enterprise which commenced in our manufacturing districts, has infused itself gradually amongst the agricultural portion of the community; and many thousand acres, considered by our forefathers as irreclaimable wastes, now teem with excellent crops; and also have become places fit for residence, where formerly they were little better than sources of pestilence.

It has been discovered, that where land is in course of being well drained, and cleared of timber and brushwood, the annual fall of rain soon diminishes in amount. Of all other countries, America has furnished the most striking examples of this fact. There nature has done everything on the large scale; immense forests spread over surfaces exceeding in area some of the larger states of the old world: but as civilisation has gradually spread, the necessities of the colonists have encroached on the domains of forest-land; trees have been felled over extensive areas; and it has been found that, gradually, the amount of precipitated moisture is lessened. This result proceeds from the removal of those objects which radiated the heat of the earth, &c., and tended to produce rapid evaporation and subsequent condensation; for the branches and twigs of trees have an astonishing effect in precipitating rain, and of holding passing mists. We have often watched such flying over an extent of plain country, yet parting with but little, if any, of their moisture. The moment, however, they came in contact with a plantation of trees, they were stopped in their course, and the previously suspended moisture fell in copious showers, owing to the radiating and cooling action of the trees amongst which the mists had been caught. This singular and interesting phenomenon may be frequently noticed in Derbyshire, Yorkshire, and in some of the southern counties, where plains or downs, and moderate elevations, covered with trees, alternate on the face of the country. A familiar instance of this action on suspended moisture is seen in our gardens, during a November mist, when each twig is dripping with water: this may also be noticed on the edges of the hair and whiskers of any one hastily walking in the open air, under similar circumstances. So universal is the action of the radiating power of plants, that we may consider the moss, lichens, heather, grass, brushwood, shrubs, and trees, as specially adapted to rob the air of its suspended vapour, and thus to become the chief means by which our rills, lakes, and rivers are constantly fed. We may add, that the blades of grass, straw, and similar objects, have been found to rank amongst the best radiators of heat and electricity. Perhaps from some partial knowledge of this fact, has arisen the idea, prevalent in many country places, that a straw-wisp is an excellent "lightning conductor;" and it is often employed by farmers at the gable ends of thatched cottages, and on wheat and haystacks, for that purpose; whilst by others, it is, perhaps, merely added as a finishing touch.

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* See our remarks on radiation, the formation of dew, &c., at p. 26, in the Section on Heat.

† See *ante*, p. 1146.

‡ See *ante*, p. 1143.

Before concluding these remarks on the influence of drainage on meteorological conditions, we must distinctly state that they have referred solely to local influences. We need scarcely observe, that fogs may be so general and extensive as to cover vast areas both of land and ocean surface: indeed, the major part of these islands may frequently be enveloped in one general mist, which even yet falls far short of those that are seen near the island of Newfoundland, and in many other parts of the Northern Atlantic ocean. Such proceed from entirely different causes, and are due to the intermixture of currents of hot-moist, and cold air, which, immediately on impact, cause a condensation of aqueous vapour, on a scale commensurate with their extent of area. In a paper read by Dr. Gladstone, before the British Association in 1862, it was stated, that there was greater uniformity of distribution of fogs over the surface of the sea, than on the land around the British Isles; and that certain years were decidedly more foggy than others, in nearly all localities. These facts, however, do not affect the truth of the observations which we have made in reference to the production of *local* fogs and mists.

OZONE, &c.—It is but recently that the presence of ozone in the atmosphere has been made a subject of registration, in connection with wind and weather reports: in fact, so little is known of this element (if such it be), that it has rarely attracted the attention which its importance as a chemical agent demands. Of late years, many investigations have been made into its general characters and relations; and we may here add a few facts to those already given respecting it.[*]

The action of oxygen on many bodies is a matter of constant experience, as in the case of combustion, breathing, rusting, &c. Now the results of these processes are considered to arise simply from the chemical union of the oxygen with such bases as carbon and hydrogen, in the two former instances, and with metals in that of metallic oxidation. But ozone is evidently so far connected with oxygen, that they are only simultaneously developed; the former having never been detected, separately, from oxygen in any way whatever. The artificial generation of ozone we have already mentioned at the pages previously referred to; and we have also found that its effects generally resemble, or are just such as those we should, from our previous knowledge of the properties of oxygen, expect to arise when that element enters into combination. But ozone, also, somewhat supplements these: thus, putrefaction goes on more rapidly when free ozone is present with oxygen; it acts as an irritant on the animal tissues, which oxygen does not; and, moreover, it seems to have an affinity to free electricity, which is singularly indicative of its character. This is amply illustrated during the decomposition of water by voltaic electricity; the excitation of electricity by friction, as with the ordinary electrical machine, &c. To these we may add another and singular instance, which we noticed during a violent thunder-storm that visited the neighbourhood of London in the early part of September, 1863. Whilst watching the storm, and noting the interval between the lightning and the claps of thunder, a brilliant flash occurred, apparently just over the spot of observation, and accompanied with a loud peal of thunder. Less than half a second of time intervened between the two; hence the locality of the discharge could not have been more than 600 feet, if so much, in distance.[†] Almost immediately afterwards, the air smelt so strongly of ozone as to produce pain in the nostrils, showing that a large amount must have been set free, evidently by electrical action. Precisely the same result was noticed during the storm described at page 184, in the Section on Electricity: in that case, however, the entire body of air in the house which was "struck," was affected. These are, certainly, extraordinary instances of the development of free ozone; but others frequently occur, and may be especially noticed in large towns, where the smell of ozone is easily recognised, in contrast with the smoke usually so prevalent in such places. For instance, at certain periods of the year, we have noticed, in the centre of some of the largest squares in the metropolis (Russell Square, Lincoln's Inn, &c.), a strong smell of ozone, accompanied with a temporary cool and refreshing air, which unscientific companions have denominated, at the moment, a rich "country smell." Such instances, doubtless, arise from sudden down-draughts of cooler air, which open spaces frequently permit, even in crowded neighbourhoods. These facts would indicate that ozone is present continuously in the higher regions of the atmosphere, when positive electricity is redundant—a circumstance of frequent occurrence, as we have already pointed out.[‡] We looked with great interest for the results of Mr. Glaisher's observations on the presence of ozone in the higher regions of the atmosphere; but, generally speaking, whilst they noted an increased amount of this agent at great elevations, the indications have been so varied and uncertain, as to have added but little to our knowledge of the subject.[§]

The causes and presence of free ozone are matters of extreme interest in a sanitary point of view. It is remarked, that its redundancy in a free state in the air, occurs chiefly with cool northerly breezes, and is attended, in some respects, with a more salubrious state of the atmosphere. But, whilst it evidently has the tendency of purifying the air, by, perhaps, removing the animal and other noxious gaseous impurities, and thus of improving the general health of the community in respect to diseases of a zymotic character—such as the typhoid, &c.—yet a large amount of ozone may prove extremely harmful to those who suffer from irritation of the bronchial tubes, affections of the lungs, and other complaints of that nature. We are, indeed, almost tempted to suggest, whether the increased mortality, which frequently accompanies north-easterly winds (so prejudicial to the health of persons suffering from diseases of the organs of respiration), may not in some degree be due to this cause. We offer this as a suggestion only, recommending it as worthy of close observation to those whom circumstances may place in situations for carefully acquiring information on the subject.

The detection of ozone by prepared papers and powders, containing the iodide of potassium and starch, has been already described. But for those who may wish to become accustomed to its smell, we may also suggest, that if a few sticks of phosphorus be placed in a bottle containing water, so that the phosphorus may be only in part covered with the liquid, and the bottle be corked, the smell of ozone may be recognised on removing the cork after a few minutes have elapsed; and an occasional smelling of it, as produced after this method, will enable any one to detect it, even in minute quantities, when it occurs in the atmosphere in a free state.

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[*] See *ante*, Section on *Chemistry*, p. 315; and *Meteorology*, p. 1122.

[†] See *ante*, p. 276.

[‡] See *ante*, p. 1178.

[§] See *ante*, pp. 1130 and 1164.

We have alluded, at page 315, to the presence or absence of ozone in crowded localities, as a test of their sanitary state; and we may urge on our readers the great advantage which will arise from an extension of experiments of that kind. We there stated, that ozone tests were variously affected, according to the proximity, or otherwise, of crowded dwellings. The presence of animal matter in the atmosphere seems to entirely prevent any action on the test-papers; and this would confirm the idea that a chemical relationship, or affinity, subsists between ozone and the carbon, hydrogen, or nitrogen which gases, emitted from the animal body, contain. For such purposes, the salts of manganese[*] are more delicate as ozone tests, and are, therefore, to be preferred, in many cases, to the iodide of potassium and starch papers. In observing the effect of ozone on such tests, the depth of colour produced, as compared with a standard, is to be noticed: it is, therefore, desirable that specimen papers should first be procured; and they are sold at most of the instrument-makers. Any quantity of the test-paper may afterwards be easily made, by following the directions given at page 1164; and in using them, the direction of the wind, whether it may have blown over a town before being tested, or arrived in an unchanged and non-vitiated state, with all other circumstances which might modify the action of the free ozone, should be carefully noticed, otherwise the observations would be worse than useless, and be only calculated to mislead. We have no doubt that many curious particulars might be gathered by the medical profession, in relation to endemic and epidemic diseases, if this interesting subject were more fully understood. Such persons would have frequent opportunities, both in the localities and the sick room they visit, of making valuable additions to the facts we already possess.

BAROMETRIC DETERMINATION OF HEIGHTS.—Rules for determining the heights of mountains, &c., by the barometer, have been already given;[†] which, for some comparative results, will suffice: we may, however, here mention certain points which require careful attention, if extreme accuracy be required.

After a perusal of the pages devoted to a description of the ascents of Mr. Glaisher, the reader will perceive that the ordinary rule, of an equal decrement of temperature occurring at equally increased elevations, does not always hold good; and therefore, that heights, determined on the assumption of the correctness of that rule, must necessarily be erroneous. There are also other causes of error, which must be taken into account, and which were ably pointed out by Mr. Ball, in a paper read before the Physical Section of the British Association, in 1862. We shall avail ourselves of the report given in the *Athenæum*, for the purpose of laying the leading facts before our readers.

The author remarked, that serious errors are involved in the ordinary process of reducing barometric observations, for hypsometrical purposes. This process involved two assumptions; first, that the volume of a column of air, unequally heated, is nearly the same as that of an equal weight of air of the same mean temperature; secondly, that the mean temperature of the column or stratum of air, between the stations of observation, corresponds to the mean of the readings of thermometers standing in the shade, at each station. The error involved in the first assumption is not very considerable; that arising from the second, is, on the contrary, highly important. M. Bravais was the first to propose a practical plan for applying a correction to the assumed mean temperatures of the air, depending upon the hour of the day, and the season of the year, at which observations are made; but it is to M. Plantamour, the distinguished astronomer of Geneva, that we owe the fullest investigation of this important subject. Having ascertained, by careful levelling, the true height of the Great St. Bernard above Geneva, M. Plantamour finds that the mean of all the barometric observations made during eighteen years, deviates by fourteen feet, English, from the true height; and he attributes this deviation, with great apparent probability, to an abnormal depression of the mean temperature of Geneva, owing to the neighbourhood of the lake. The readings of the barometer and thermometer, at the observatories of Geneva and the St. Bernard, are taken daily at nine hours or epochs. M. Plantamour assumes that, on an average of a long period of years, the mean of the observations taken at any one epoch in the twenty-four hours, should give the true difference of height between the two stations, with an error due to the difference between the mean of the readings of the thermometer, at both stations, at the same epoch, and the true mean temperature of the air in the intervening stratum. Calculating, then, the height of the St. Bernard by the elements corresponding to each epoch of the day, during the four summer months, from June to September, he obtains a series of measures differing from the true height; those corresponding to the hottest hours being in excess, and those appertaining to the coldest, in defect of the true height. He then ascertains the amount of correction, which being applied to the mean sum of the readings of the thermometer, at each epoch in each of those months, would bring out the true height. In this manner he obtains a table, showing what he calls the *normal correction* for each of the epochs of the day, during the four summer months.

There is good reason to believe, that in reducing barometric observations, which are to be compared with those of Geneva and the St. Bernard, the application of the normal correction, ascertained in the manner above stated, will, in general, give truer results than those where it is not applied; but as it is obvious that the conditions of temperature, at the moment when a given observation is made, are constantly varying from the mean of the corresponding day and hour—it follows that a further supplemental correction should be made on this account. To apply this further correction is a matter of no small difficulty; but the method employed by M. Plantamour is as follows:—He obtains from the observations at Geneva and the St. Bernard, by interpolation when necessary, the elements corresponding to the day and hour of the observation which is sought to be reduced; and from these he calculates the height of the St. Bernard. The height so obtained, when compared with the measure, which is derived from the mean of the readings of the same day and hour (as shown in his table of normal corrections), furnishes a criterion by which to judge of the conditions, with respect to temperature, of the moment when the observations to be reduced were made. M. Plantamour thinks it not difficult to infer, from the observations themselves, and from the general state of the weather of the time, whether the moment was one of atmospheric equilibrium, or the reverse. In the latter case, the observation is treated as one of inferior utility, to which a lower value should be assigned in the final calculation. Supposing, on the contrary, the observations not to betray a disturbance of equilibrium between the two stations, the deviation of the height, as calculated for that particular moment, from the height derived from the corresponding means, is the measure of the amount, and sign of the supplemental correction corresponding to the moment of observation.

These facts acquaint us with a peculiar and local cause of error, which adds another difficulty to the method of measuring heights by means of the barometer. To some extent the sources of error are analogous to those we have already pointed out, when speaking of the variety of causes which affect a normal atmosphere in the neighbourhood of, or immediately over, large towns; but they differ, in being more constant, and therefore more easily determinable in their character. Many of our readers may imagine that the method of finding the elevation of accessible heights by the rules of trigonometry,[‡] would be sufficient for all purposes, and render us independent of the barometrical method; but the trigonometrical plan is not at all times possible; for,

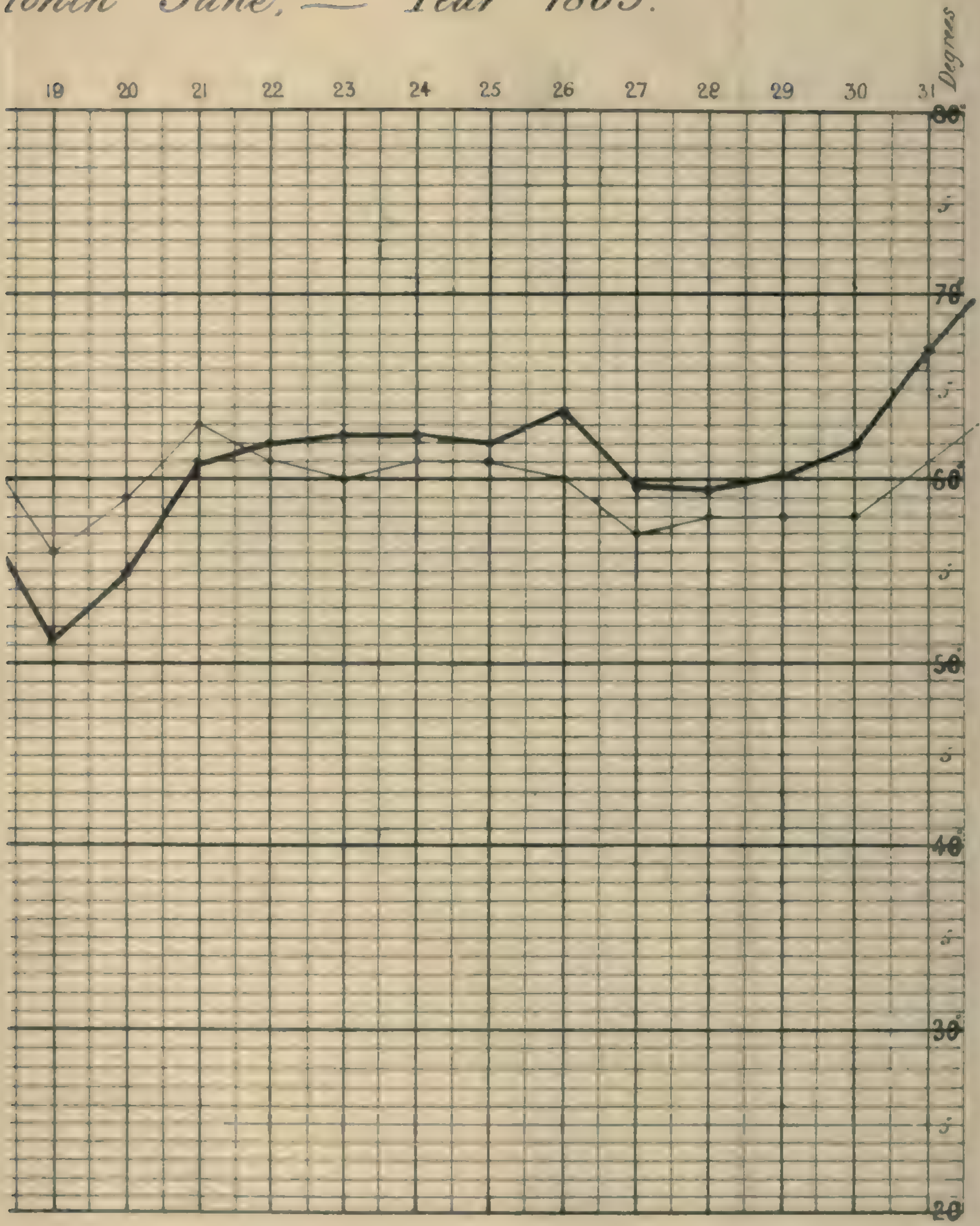
[*] See *ante*, p. 384.

[†] See *ante*, *Pneumatics*, p. 771; and *Meteorology*, pp. 1133, 1134.

[‡] See *ante*, p. 643.

See Pages 1189 & 1190.

Month June, — Year 1863.



frequently, the highest point of a mountain range is so completely hidden by surrounding but lower peaks, that an observation for taking the requisite angles is not possible. It also often happens that the summit of a mountain is almost constantly covered with mist—or, at a portion of its height, such may be constant—which equally prevent the necessary trigonometrical observations. Independently of these considerations, the barometer affords a check on, and sometimes a verification of, results obtained by other methods. At the present time, our estimates of the height of many of the most extensive ranges of mountains, in various parts of the world, depend upon barometrical observations; and, perhaps, many of them require considerable modification before a correct altitude can be obtained. Much difficulty arises in the use of the ordinary mercurial instrument, owing to its want of portability, and the necessity of the corrections which have been pointed out: accordingly, some of our leading instrument-makers have turned their attention to the perfecting of the aneroid. One of these instruments, manufactured by Messrs. Negretti and Zambra, has been used by Mr. Glaisher in his balloon ascents; and he considered that it might be depended on at a pressure equal to one indicated by five inches of mercury. As our highest mountains have an elevation at which a much taller mercurial column is supported, it is probable that such instruments may be of great service for the purpose, as substitutes for the barometer.

The following table will be of great utility, both for the purpose of measuring heights by the barometer, and for adjusting the level between two places where barometrical observations are being carried on. The barometric column is supposed to stand at a height of 31 inches, at the sea-level. For any other barometric height at the sea-level, at the moment of observation, an allowance for the difference above or below 31 inches must be made; which is, of course, readily done by the rules of common proportion. The table is copied (but corrected) from the *Barometer Manual* of the Board of Trade.

DIFFERENCE TABLE FOR ELEVATIONS.

Barometer Inches.	Height in Feet.	Barometer Inches.	Height in Feet.	Barometer Inches.	Height in Feet.	Barometer Inches.	Height in Feet.	Barometer Inches.	Height in Feet.	Barometer Inches.	Height in Feet.
31·0	0	28·9	1844	26·8	3829	24·7	5976	22·6	8317	20·5	10889
30·9	85	28·8	1935	26·7	3927	24·6	6083	22·5	8434	20·4	11018
30·8	170	28·7	2027	26·6	4025	24·5	6190	22·4	8551	20·3	11148
30·7	255	28·6	2119	26·5	4124	24·4	6297	22·3	8669	20·2	11278
30·6	341	28·5	2211	26·4	4223	24·3	6405	22·2	8787	20·1	11409
30·5	427	28·4	2303	26·3	4323	24·2	6514	22·1	8906	20·0	11541
30·4	513	28·3	2396	26·2	4423	24·1	6623	22·0	9025	19·9	11673
30·3	600	28·2	2489	26·1	4524	24·0	6733	21·9	9145	19·8	11805
30·2	687	28·1	2582	26·0	4625	23·9	6843	21·8	9266	19·7	11939
30·1	774	28·0	2675	25·9	4726	23·8	6953	21·7	9388	19·6	12074
30·0	862	27·9	2769	25·8	4828	23·7	7064	21·6	9510	19·5	12210
29·9	950	27·8	2864	25·7	4930	23·6	7175	21·5	9632	19·4	12346
29·8	1038	27·7	2959	25·6	5033	23·5	7287	21·4	9755	19·3	12483
29·7	1126	27·6	3054	25·5	5136	23·4	7399	21·3	9878	19·2	12620
29·6	1215	27·5	3149	25·4	5240	23·3	7512	21·2	10002	19·1	12757
29·5	1304	27·4	3245	25·3	5344	23·2	7625	21·1	10127	19·0	12894
29·4	1393	27·3	3341	25·2	5448	23·1	7739	21·0	10253	18·9	13032
29·3	1482	27·2	3438	25·1	5553	23·0	7854	20·9	10379	18·8	13170
29·2	1572	27·1	3535	25·0	5658	22·9	7969	20·8	10506	18·7	13309
29·1	1662	27·0	3633	24·9	5763	22·8	8085	20·7	10633	18·6	13448
29·0	1753	26·9	3731	24·8	5869	22·7	8201	20·6	10760	18·5	13588

CHARTS FOR WEATHER REGISTRATION.—Instructions, of a very detailed character, have already been given in Chapter IV., as to the method of making observations by the barometer and thermometer; the precautions which must be taken; corrections to be made, &c. Many of our readers, however, may be disposed not only to register their observations in a book, for subsequent reference, but also feel desirous of adopting some plan by which they may possess a general and combined view of the barometric and thermometric changes that occur monthly; so that, by mere inspection, they may be able to trace the variations which have occurred. One of the best plans which we have yet seen, is that recommended and used by the Meteorological Department of the Board of Trade, and of which a specimen is given in the folio plate. By an ingenious contrivance, both the barometric and thermometric ranges are marked down on one diagram.

The sheet is divided by vertical and horizontal lines, between which the daily ranges are to be marked with dots and lines. On the left hand, a scale ascends, which is divided so as to indicate the height of the barometer from 28 to 31 inches. This is sub-divided into tenths; and these, again, may be easily reduced to halves, &c., in marking the height. On the right, the thermometric range is denoted, running from 20°, or 12° below freezing point of Fahrenheit's scale, to 80°. The horizontal lines are crossed by vertical dark lines, with an intermediate lighter one—each entire division being intended to denote a day. Thirty-one and a-half are provided; so that the first three half divisions may be used for the last day and a-half of the previous month—a method which is useful for "forecasting," for reasons mentioned at a previous page.* It will be seen that two observations, at least, both of the barometer and the thermometer, may thus be marked on the lines daily; and by dividing the intermediate spaces, even more may be mapped down.

The method of using the diagram is as follows:—Supposing that the barometer stands at 29·50, on the first day of the month. Look to the left hand scale, and trace, by the eye, the horizontal line from 29·5 to the vertical *dark* line running downwards from the figure 1, at the top of the scale. Make a dot at the point where they intersect—the height of the barometer will thus be indicated at the time the observation is made, which should be at 8 A.M., that affording the best daily average. If another observation be made at 8 P.M.—that is, at an interval of twelve hours—proceed in the same way, for any height, and mark a dot where the horizontal line intersects with the *light* vertical line, which will be half-way between 1 and 2, as marked on the top of the scale. Draw a straight line, connecting the dot of the morning and that of the evening observations—thus the dots will indicate the actual heights, whilst the line will show the course of variation in twelve hours. On the 2nd of the month, at 8 A.M., make a dot, corresponding to the barometric height, on the dark vertical line, as on the previous day, and then draw a line to it from the dot of the previous evening or morning observation, according as to whether a single or double one has been made. Proceeding in this manner, a line will daily appear to progress from the left to the right of the scale, indicating the changes which have occurred throughout the month. The *barometric* line should be made both *dark* and *thick*, to distinguish it from the *thermometric* line, which should be *fine*, for the same reason.

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* See *ante*, p. 1176.

The indications of the thermometer are to be similarly marked. Supposing, for instance, that on the 1st of the month the thermometer stood at 62°—look on the right hand of the diagram for 60°, and, at two lines above it, will be found the division corresponding to 62°. By the eye, trace this line until it comes under figure 1 of the vertical scale, and there make a dot. Continue in the same manner as directed for the barometer, only making a *fine* line between the morning and evening, or morning and morning, divisions from day to day. Thus, the indications are to be traced by lines throughout the month, as is done with the barometer, and the two scales will be simultaneously marked for each day, or each twelve hours, as the observations are made and registered.

The first division and a-half, on the *left* hand of the diagram, are intended to indicate the respective heights of both instruments for the preceding 36 hours; that is, an entire division is given for the last day of the preceding month, and half a one for the day previous. The lines are to be connected with those of the current month, in the manner already described, and thus the observations of the two months will run on consecutively; and, from an inspection of the diagram, the probable weather for the commencement of the month may be predicted, without further reference to the plan of the past one. The place, month, and year of observation should be filled up at the top of the diagram; and space may be left at the bottom, to receive notes of any special meteorological occurrences.

To still further illustrate this method of registering, we may trace a portion of the folio plate. The indications have been brought from the previous month, and on the 1st of the current one, we notice that the height of the barometer (dark line) stands at 30·30; it then passes to 30·17, on the 2nd; the line then progresses to 30·05, and so on. Similarly, the thermometric fine line indicates 55° on the 1st of the month; on the 2nd, 59°, &c.; the heights of each instrument being simultaneously marked, and their variations clearly seen by the relative position of the dots and connecting lines on the diagram.

This plan is remarkable for its simplicity, and the ease with which any one may map out the observations made daily. For forecasting it has great advantages; for instead of appealing solely to the mind for the calculation of numerical differences, the eye is at once instructed, and, at a glance, the observer can see whether the barometer and thermometer are rising and falling together, or oppositely; whilst the rules for predicting weather, given at page 1176, may be immediately applied in connection with the observations on the direction of the wind blowing at the moment, or previously.

Although a full description has been given, in our previous pages, of the qualities which should characterise a good barometer, it may be useful here to mention some particulars in reference to the Standard Barometer, as supplied to the Navy by the Board of Trade; and the care which is shown in their construction, and subsequent verification, will be best understood by the following account of the mode of testing them, as described in the *Barometer Manual*. Our readers may thus learn some valuable lessons in respect to the purchase of instruments for their own use.

The barometers to be tested, are placed, together with a Standard,* in an air-tight chamber, to which an air-pump is applied, so that by partially exhausting the air, the Standard can be made to read much lower than the lowest pressure to which marine barometers are likely to be exposed; and by compressing the air, the mercury may also be read higher than it ever stands at the level of the sea. The tube of the Standard is contracted similarly to that of the marine barometer,† but a provision is made for adjusting the mercury in the cistern to the zero point. Glass windows are inserted in the upper part of the iron air-chamber, through which the scales of the barometers may be seen; but as the verniers cannot be moved, in the usual way, from the outside of the chamber, another method is adopted for reading the exact height of the mercury, independently of the verniers attached to the scales of the respective barometers. At the distance of some five or six feet from the air-tight chamber, a vertical scale is fixed; and the divisions on this scale correspond exactly with those on the tube of the Standard barometer. A vernier and telescope are made to slide on the scale, by means of a rack and pinion. The telescope has two horizontal wires—one fixed, and the other movable by a micrometer screw; so that the difference between the height of the column of mercury, and the nearest division on the scale of the Standard, and also of all the other barometers placed by the side of it for comparison, can be measured either with the vertical scale and vernier, or the micrometer wire. The means are thus possessed of testing barometers for index-error in any part of the scale, through the whole range of atmospheric pressure to which they are likely to be exposed; and the usual practice is to test them at every half-inch from 27·50 to 31 inches, which is about the extreme average range of the barometer in almost any latitude.

In this way various other descriptions of barometers have been tested; and some errors were found to be so large, that a few instruments read half an inch and upwards too high, while others read as much too low.‡ In some cases, those which were correct in one part of the scale, were found to be from half an inch to an inch wrong in other parts. In some the mercury would not descend lower than about 29 inches, owing to a fault very general in the construction of many common barometers—till lately, in frequent use—namely, *the cistern was not large enough* to hold the mercury which descended from the tube during a *low atmospheric pressure*.

In the Government Marine Barometers, the diameter of the *cistern* is about an inch and a quarter, and that of the *tube* about a quarter of an inch. The scale, instead of being divided into inches, in the usual way, is shortened, in the proportion of about 0·04 of an inch for every inch. The object of this is to avoid the necessity of applying a correction for the difference of capacity between the cistern and the tube. The perfection with which this is done may be judged of from the fact, that of the first twelve barometers tested at the Liverpool Observatory, with an apparatus exactly similar to that used at Kew (whence these instruments were sent by railway, after being tested and certified), the index corrections, in the pressures from 28 to 31 inches, of three of them, were the same; two differed 0·001 of an inch; and, for the remainder, the differences ranged from 0·002 to 0·006 of an inch. The corrections for capacity were therefore considered perfect, and, with one unimportant exception, agreed with those given at Kew.

From the above account, it will be seen how frequently common barometers may lead the observer astray—an occurrence which, whilst of comparatively slight importance on land, might cause the destruction of a vessel, and serious loss of life, if such indications of an imperfect instrument were relied on at sea. The peculiar nature and sudden advent of cyclonic storms,§ which are always accompanied by a barometric fall, render it of the utmost importance that the barometer on board of a vessel should be sensitive, and in good order; yet we have just seen that some instruments are sold, and doubtless "warranted," which would be incapable of showing a great mercurial depression, generally the only sign of the coming storm that could warn the mariner of impending danger. There is no difficulty whatever in having a good barometer properly tested; and as our knowledge of the science of storms is now so extensive, the commander of a vessel would be guilty of an equally great neglect of his duties and interests, if proceeding to sea without a good and certified instrument, as he would be were he to do so without either a compass or chronometer on board.

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* One that has been already fully tested, in every respect, at the Kew Observatory.

† This contraction of the marine barometer-tube prevents what is called "pumping;" that is, the extreme oscillation caused by the motion of the ship in a rough sea.

‡ See *ante*, p. 1159.

§ See *ante*, p. 1175.

Whilst referring to methods of registering observations, we may observe that the Board of Trade lend barometers, &c., on certain conditions, to properly qualified commanders in the merchant service. They are expected to make continuous observations, and to report them at head-quarters as occasion will permit; honorary rewards being distributed when the instructions are fully carried out, or when special diligence has been shown. This plan will prove highly beneficial to the interests of science; for, by it, an immense amount of data will be gathered from all parts of the world, and thus our means of induction will be proportionately increased. Besides the registration of the heights of the barometer, &c., the observers are desired to notice all concomitant occurrences, so as to increase our knowledge of matters which, although apparently of minor importance, are really efficient aids to the more exact processes of scientific investigations.[*] It may not be unreasonable to hope, that, in the course of time, simultaneous observations will be undertaken throughout the world, on a plan similar to that adopted at our observatories in this and other countries. The labour of reducing them would, of course, be enormous; but the information obtained would be invaluable. Carried on during a number of years, we might, eventually, be able to form from them so complete a weather-chart for each ocean and coast, as would result in a general diminution of shipwrecks, which now cause, annually, some millions sterling of loss. To many, such a work would seem Utopian, and utterly beyond human power; but, after all, it would scarcely equal the labours of astronomers, who have mapped out the place of each star, and who, from data similarly acquired, can assign its place, even for years hence, within an error of but a few seconds of time. In this labour we are encouraged by knowing that no event in nature depends on chance, and that every phenomenon is the result of some law which has been impressed by the Creator. In its operation, it may be so intricate as to baffle, for years, and even ages, the deepest penetration on our part; but still it stands before the eye of the philosopher, presented as a great prize for his grasp—the only condition imposed on him for success being, an intelligent, persevering, and discreet use of each fact as it arises, and of correct induction as light bursts on his mental vision. For some thousands of years, ingenuity was puzzled to account for the motions of the solar system; the most intricate schemes were brought forth to explain them; but when human pride failed, nature's simplicity conquered; and one broad comprehensive law—that of gravitation—was at last understood and completely established; a result which adequately rewarded years of unremitting and patient toil.

GASEOUS RADIATION AND ABSORPTION OF HEAT, AND THEIR EFFECTS ON CLIMATE.—In the Section on Heat, we have called attention to a variety of phenomena resulting from the radiation and absorption of caloric; and in the Addenda to that section, some interesting experiments and results, obtained by Professor Tyndall, are briefly noticed. We shall here extend this subject, so as to point out its bearings on the science of meteorology, in connection with climatic differences, and may find that many phenomena, otherwise almost inexplicable, receive a happy solution through the inductions which may be properly made from the facts that will be brought forward.

Casting a glance at a map, from the poles to the equator, the intelligent student will not fail to remember how various is the climate of the different countries delineated on it; and, consequently, how greatly the natural productions vary. Whilst experience teaches us that certain forms of both animal and vegetable existence are confined within definite regions or zones; that, in some places, rain, snow, frost, and hail are unknown; whilst in others they are characteristic of the country; and, in some, that there is experienced, during the day, a tropical heat, followed by an almost Arctic severity of climate as soon as the night sets in—we naturally seek for some general law, which may embrace a broad and definite exposition of the causes operating in each instance; not being content with an approximative or probable solution of our inquiries.

What an almost infinite variety exists of the evidences of vitality in different portions of our globe. Commencing at the polar regions, we find vegetation—the immediate characteristic of climate—in its lowest forms; the lichens being predominant in all respects. Gradually approaching the equator, we first see hardy plants and trees, such as the willow, birch, alder, &c.; and grass also becomes common. Flowers of various kinds, with cereal crops, next appear as we arrive in countries such as our own; thus the luxuriance of vegetable nature, and its profusion, develop, until we arrive at tropical regions, where the heat and moisture combine to produce the richest and most gorgeous of floral beauties, and sugar, coffee, spices, fruits, &c., which can only flourish in such countries. Animal life equally sympathises with the peculiarities of climate, if we confine ourselves to those forms that are below the human species, to which alone is accorded the power of accommodating itself to every climate, no matter how severe or sudden the transition.

The general laws which influence this climatic variety have been already inquired into in the previous pages; and, in numerous instances, the particular or local disturbing causes have also been pointed out. There are, however, other influences of both kinds, to which we shall now direct attention, and which have been, till lately, almost unknown, or scarcely allowed for, to the extent which their importance demands.

We may first notice the effect of the obliquity of the sun's rays, as incident at various parts of the world, being well aware, that where the sun is vertical, those regions, as at the equator, are the hottest of any on the globe. At the outer limits of the atmosphere, it is considered that the area bounded north and south of the equator, by the parallels of latitude of 23° 45′, receives as much solar heat and light as does all the rest of the surface of the earth. Whilst an obliquity of 25°, in the incidence of a solar ray, causes a loss of one-half of its intensity at that angle before reaching the earth; an inclination of only 5° permits only $\frac{1}{20}$th of the maximum to be transmitted to us. It hence follows, that the remaining portion of our globe must, on areas equal to any given one at the equator, receive an exceedingly small proportion of the total calorific rays emanating from the sun.

From this we may plainly see, that the great breadth of land extending between the eastern and western limits of Africa, and the parallels of 23° 45′ north and south of the equator, must act as an enormous absorber of solar heat; which it afterwards radiates, and from which, by convection, the remaining portion of the earth receives a great accession beyond what would be its normal temperature. The upward current of hot air rising from this surface, spreads itself northward and southward, and its place is supplied by returning currents of cold air, which, by this compensating action, and influenced by the rotation of the earth, are productive of the trade winds, as already pointed out in our previous pages.[†]

But it is evident, that unless some special provision had been made for the retention of the heat thus absorbed about the equatorial regions, and subsequently carried, by convection, to the various parts of the globe, a large proportion would be again radiated into space, and be lost, so far as our earth is concerned. This provision is twofold: namely, that of the absorption effected by the solid crust of the earth, and that by the aqueous vapour which floats over its surface. It is particularly the latter provision that we wish here to investigate.

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[*] See *ante*, p. 1177.

[†] See *ante*, Section on *Heat*, p. 30; and in this Section, p. 1184.

In our article on the Refraction of Heat, we pointed out that certain bodies have a property by virtue of which they either absorb or transmit that force; and that they vary much in the amount of this power: in fact, just as we noticed, that, with respect to light, some bodies are transparent, translucent, and opaque; so, similarly, some bodies have a precisely identical, or, at all events, analogous effect on radiant heat.* The general law of absorption was fully investigated in the article on that subject, a re-perusal of which will be useful to the student in connection with these remarks.†

It must be borne in mind, that considerable difference subsists between these two classes of absorbers. Those which refract heat have a general relation to certain similar properties in respect to light. Thus, rock-salt and alum will transmit and refract light; yet, whilst the former permits the passage of heat-rays with the greatest facility (exceeding that of plate-glass), a plate of alum, transparent to light, entirely obstructs the passage of non-luminous heat-rays. Similarly, we find that solid, good absorbers, such as lamp-black, &c., have a mutual relation to heat and light; but in this case they agree, for they absorb equally the luminous and calorific rays impinging on them.

Now, in nature, these two classes of absorbers and radiators are met with on the large scale. Every tract of land more or less absorbs the rays of heat, and also radiates them again into space. But above this solid matter is the atmosphere, which, by its heterogeneous nature, exercises a most important action on the heat reflected and radiated from the surface of the earth. Referring to the account of Dr. Tyndall's experiments (given at page 38, in the Section on Heat), we find that the mixture—not a chemical compound—forming our atmosphere, the carbonic acid generally found in it, and the aqueous vapour (also usually present), have each a definite and greatly varying relationship to, and action on, the rays of heat incident on, and transmitted by them. Here, again, we notice that certain qualities of bodies, such as transparency and translucency, are in connection alike with the forces of both light and heat; and the law of diathermancy seems to comprise a particular condition in regard to caloric, equivalent to transparency in relation to light.‡

Aqueous vapour possesses a wonderful power of absorbing the rays of heat, and thus preventing their passage from the earth into space. We have already noticed this when speaking of dew;§ as we then stated that a cloudy night entirely prevented its deposition; the clouds, of course, being composed of aqueous vapour in a partially condensed state, but still in minute divisions. But, in this case, the comparative opacity of the clouds would naturally suggest a certain amount of interference; and a supposition might be encouraged that they must act as a screen between the radiating surface of the earth and the open space beyond the limits of the atmosphere.

It is not, however, in the form of clouds alone, or even specially, that aqueous vapour becomes a retainer of terrestrial heat. Suspended invisibly, as in the clear air of a hot summer's day, it acts powerfully in absorbing the heat transmitted from the sun, and reflected or radiated from the earth, and that to an extent enormously beyond the similar power of our atmosphere when in a normal state; that is, considered as a mixture of oxygen and nitrogen gases simply. The moisture constantly present in the atmosphere has thus a most important influence on the temperature of most climates; indeed, all in which aqueous vapour is generally prevalent in the atmosphere.

This will be readily noticed if we compare the higher and lower latitudes of the earth, or even its mountain elevations. The nearer the poles, or the higher the elevation above the surface of the earth, the less the moisture that is constantly present in the superincumbent atmosphere;|| and on the other hand, the nearer we approach the equator, the greater becomes the constituent moisture: as a general and consequent law, therefore, the temperature is directly as the moisture in the air, under certain circumstances. In our own country, the contrast between the dry, cold, north-east wind, and that from the south-west, which is of a warm and genial temperature, and flowing from the Gulf-stream, is constantly recognised. For similar reasons, we shall find, in most countries, whilst ascending lofty mountains, which, at their base, have a climate of a high temperature, that we experience increasing cold until we arrive at a region where every particle of moisture is always congealed into the solid state, which will be at a point situated near what is called the snow-line. The folio plates give a general view of the chief elevations in the eastern and western hemispheres, and are accompanied with a scale, indicating the heights of mountain ranges, &c. There are also marked certain altitudes attained by Mr. Glaisher, Gay Lussac, and Humboldt, whose observations verify the remarks just made. A little above and below the snow-line, the annual variation of temperature either melts the congealed water in summer, or freezes it as winter comes on. The former process, together with the precipitation of moisture (as rain), gives rise to rivers, the chief of which are also noted, with a scale of their lengths, in the plates just referred to.

Thus we find, that when the atmosphere is destitute of suspended moisture, intense cold prevails, chiefly owing to the immediate radiation of the heat received with the luminous solar rays. On this principle may be further explained the fact noticed by Mr. Glaisher, that, as he ascended in the air, a blackened-bulb thermometer ceased to show an increased heat, when exposed to the full rays of the sun, beyond that indicated by those kept in the shade. We have already anticipated, in some respects, the solution of this at a previous page;¶ but may here add, that a blackened-bulb thermometer, similarly exposed at the sea-level, will occasionally rise even 40° to 60° above the temperature shown by a thermometer shaded from the rays of the sun at the same time and place. In the latter case, the absorbed heat will, according to the researches of Dr. Tyndall, be prevented from immediate radiation by the moisture resident in the surrounding hot atmosphere.

The average altitude of the snow-line does not depend entirely on latitude, but is greatly affected by the amount of moisture, &c., in the atmosphere. Thus, some portions of the Himalayas have the snow-line at 1,500 feet greater elevation than that common at the equator: and comparing, again, Sierra Nevada in Spain, in 35° north latitude, with some of the Himalayan mountains in Thibet, in the same latitude, we find that the snow-line of the Spanish mountain is at 9,500 feet, whilst that of the Himalayan ranges from 15,000 to 20,000 feet, above the sea-level—varying, in a distance of sixty miles, to the extent of 5,000 feet. This is chiefly due to the absence of moisture—a circumstance that prevents the formation of the snow, simply because there is literally nothing of watery matter to freeze. For similar reasons, the altitude of the snow-range on these mountains rises uniformly, in proceeding northward from India, and consequently from the equator, up to a north latitude of 35°. The rapid radiation of heat, and consequent equally rapid evaporation even of snow, in some parts of the Himalayan range, is the chief cause of this increased average altitude of the snow-line, which is frequently greater on the north than on the south side.

The following table gives its height above the sea-level, for many latitudes, both north and south of the equator, and will form a kind of supplement to the illustrations afforded in the folio plates. It will be noticed, that,

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* See Section on *Heat*, p. 29.

† *Ibid.*, p. 26.

‡ *Ibid.*, p. 28.

§ *Ibid.*, p. 26.

|| See *ante*, pp. 1180, and 1182, 1183.

¶ See *ante*, p. 1184.

generally speaking, as the sea is approached, the snow-level falls; but still numerous other local interfering causes must also be taken into account; and such may easily be studied by reference to an ordinary map, in conjunction with the latitudes given in the table. Another instance of curious variation of the snow-level, in respect of the latitude, is furnished by the Carpathian range, the highest of which, exceeding 9,500 feet, and in north latitude 49°, is still below the snow-line, although the Alps, in 45°, have it at an elevation of about 8,500 feet. The increased altitude in respect to the Carpathian range, is due to the great convection of heat from the plains below, which, of course, are powerful radiators and absorbers.

ELEVATION OF THE SNOW-LINE ABOVE THE LEVEL OF THE SEA.

Country or Range.	Situation.	Height in Feet of Snow-line.	Latitude.	Longitude.	Season.
South Shetland, in the Southern Ocean ...	Coast	0	62° S.	52° to 64° W.	Annual average.
Andes, South America, near Cape Horn ...	Near the Sea	3,500	55° S.	67° W.	"
Andes, Chilian	Inland	8,300	40° S.	72° W.	"
" "	"	12,780	33° S.	70° W.	"
" Copiapo... ...	"	13,800	27° S.	68° W.	"
" Cochamba Range	"	17,000	17° S.	70° W.	"
" Cuzco	"	20,000	14° S.	71° W.	"
At the Equator	——	15,500	0°	——	"
Himalayas, Asia, near Sikim	Far Inland	16,000	28° N.	88° E.	"
Himalayas, Tibetan ...	" "	15,000 to 20,000	35° N.	75° to 78° E.	"
Mount Ararat	" "	14,080	39¾° N.	44½° E.	"
Alps, Europe	Near Lakes	8,000 to 8,500	45° N.	7° to 8° E.	"
Carpathians, Europe...	Inland	10,000	49° N.	20° to 27° E.	"
Norway, Europe ...	Coast	4,450	60° N.	5° to 8° E.	"
	Inland	5,500			
" " ...	Coast	3,250	66° N.	12° to 15° E.	"
	Inland	3,700			
" " ...	Coast	2,900	70° N.	20° to 22° E.	"
	Inland	3,350			
North Cape, Europe...	Coast	2,000	71° N.	25° E.	"
Spitzbergen " ...	"	0	80° N.	10° to 20° E.	Winter.
" " ...	"	300			Summer.

We have frequently remarked that extreme dryness is characteristic of certain regions; and Thibet furnishes an instance of it, and also of excessive temperature; for the climate is exceedingly cold and dry, owing to the great elevation of the plateau on which it is situated (10,000 to 15,000 feet); yet, although its latitude is from 30° to 38° north of the equator, the snow-line (20,000 feet) is higher by nearly 5,000 feet. So deficient is the moisture, that few vegetables of any kind can be cultivated; and in most parts, trees and herbage are entirely absent. The radiation from the earth is enormous; for whilst, during the day, the thermometer will frequently indicate a temperature of 150°, at night it falls below freezing point. This affords us another proof of the truth of Dr. Tyndall's deduction—that to the power possessed by suspended aqueous vapour of retaining heat by absorption, is due the warmth and genial temperature of such climates as our own.

The causes of excessive climates, and excessive changes occurring in comparatively uniform climates, may be readily traced, in many cases, to arise from rapid radiation of heat into space, uninterfered with by strata of superincumbent aqueous vapour suspended in the atmosphere. Amongst the excessive class, we have already instanced Thibet; but we may add another illustration in that of the Sahara, the great desert of northern Africa. It is comprised between latitudes 16° and 33° N., and longitudes 17° W. and 23° E., and chiefly consists of table-lands, varying in height from 1,000 to 2,000 feet above the sea-level, with occasional mountain groups attaining an elevation of from 3,000 to 6,000 feet. The temperature varies from 50° to 130°, during the day; whilst, at night, water freezes when exposed to the wind. In some parts no dew falls, and snow is unknown, owing to the entire absence of aqueous vapour. Rain falls only, in quantity, at intervals of some years; for during nine months, annually, easterly winds prevail, varied, for three months only, by those from the west, or the regions bordering on the Atlantic. Generally, no plant, except an occasional palm, is met with, which derives its nourishment from springs resting beneath the surface; the only variation from a perpetual death-like aspect being the oases so sparsely scattered, and situated in depressions below the ordinary level of the country. The simoom here spreads its destructive effects with the fullest energy, loaded, as it is, with fine sand, and possessing an intense and suffocating heat, which is destructive of both animal and vegetable life.

A great portion of the Russian empire, in Europe and Asia, has a climate of the excessive type. Situated remote from extensive ocean tracts, the entire area experiences only winds which have been robbed of their moisture before arriving in most parts of the country. Again, the cold, dry, Arctic winds freely course over the surface, from the Frozen Ocean to the southern limits of the empire, unchecked by any mountain ranges of sufficient height to afford due protection to the interior. Hence, so far north as St. Petersburg, the heat in summer is great;* whilst in the Crimea, which has the same latitude (45°) as the south of France and the north of Italy, snow lies often for five months on the ground; and the Sea of Azov, the northern parts of the Black Sea, and the adjacent rivers, are all frozen. The sufferings of our troops during the Russian war (1854—'56), in this region, afforded a painful illustration of the severity of the climate in that country. Located on the same parallel as Bordeaux, Milan, Venice, &c., where a genial warmth is rarely disturbed by great variations of temperature, our armies were decimated, near Sebastopol, by a winter scarcely inferior to an Arctic one in its rigour. At Odessa, one of the chief Crimean ports on the Black Sea, the thermometer frequently falls so low as — 24° Fah. in the winter; the heat in summer being exceedingly intense.

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* The annual range of the thermometer, at St. Petersburg, is between 23° and 90° Fah.

In our own country we sometimes experience severe changes in the weather during the winter season; and, occasionally, the thermometer ranges very high in the height of summer. An extraordinary instance of the former kind occurred in 1860, when the mean *daily* average, for some days, fell surprisingly below the monthly average. In December of that year, the *mean* temperature of the 24th was 22°·4; on the 25th it fell so low as 20°·2. In many parts of the country the mercury fell far below zero; indeed, we noticed it, at about four miles north of London, at 2° below zero, or 34° below freezing point, early in the morning of the 25th; and at 4 A.M. of that day, on leaving a hot room and passing into the open air, the entire dress was *instantly* coated with fine particles like snow, arising from the congelation of the moisture which had permeated the clothing as perspiration. In some parts, a temperature so low as — 14°, or 46° below freezing point, was registered. Amongst other instances of the extreme severity of the weather, the breaking of the tires of railway carriage-wheels, owing to the contraction produced by the cold, was one of the most noticeable and common.

DISTRIBUTION OF HEAT.—The distribution of heat over the globe, in respect to latitudes and climates, is a subject of deep interest in many respects; and it has frequently, although perhaps incidentally, been noticed in the preceding pages. We shall devote a short space to its consideration, as being a question intimately connected with that which we have just been discussing.

It has been already remarked, that the regions near the equator may be considered as the chief heating surface on the globe. To the absorption of an immense amount of caloric by that portion of the continent of Africa, and its subsequent radiation, the genial temperature of Western Europe is due. On comparing the climate of temperate climes, in the latter continent, with those of others, we find fewer instances of excessive temperature; the heat and cold, at no season of the year, being intense, except under extraordinary and very rare occasions. Taking the area, including Britain, Denmark, Holland, France, Spain, and Portugal, we find climates in which every necessity of man, with respect to food, clothing, comfort, and luxury, can be produced; and to this cause, and also to the absence of positive noxious influences, Europe is chiefly indebted for its power as the centre of commerce, arts, manufactures, &c. This area is included between 35° and 60°, north of the equator; and if we refer to a map on Mercator's projection, bearing in mind the remarks already made on the climate of other countries, as Russia, Thibet, Tartary, &c., which are also included within the same parallels, we shall at once be impressed with a sense of the advantages that such portions of our continent possess.

One of the most accurate indications of the climate of any country is obtained from the monthly average of its temperature; and if this be noted, for each political or natural division, a series of lines may be drawn, which will intersect, on the map, places having an equal annual temperature. These have been called *isothermal lines*, and are generally drawn for each month of the year, besides that of the year itself. From the immense variety of local interfering causes, such lines are not coincident with the latitude of any two or more places. It is impossible, indeed, that they could be, unless the globe were entirely covered with water; for, as we have already seen, elevation, proximity to the sea, radiation, &c., all tend to produce an unceasing diversity of climatic characteristics, where land of even a moderate extent of area is found.

It was formerly considered that a mean between the average yearly temperatures, for a number of years, would indicate what we may term the climatic average. But that such would not be the true average will be plainly seen, if we consider the variety of temperatures which are observed in our own country, for example, monthly, in following years. Thus it is by no means a rare occurrence for us to experience, in succession, cold and damp springs and summers, which, for the years in which they occur, would seriously diminish the annual average. If, however, the same months are compared with each other, in successive years, and a mean struck between; and if the *monthly* average be taken, as indicative of the proper mean for our climate, we shall obtain a much more accurate idea of the mean which governs the year. This, therefore, is the method which is now universally adopted by meteorologists; and, at page 1172, the monthly means for Greenwich is given, which will illustrate these remarks.

To give some idea of the inflection of an isothermal line from any parallel of latitude, we may take that of the freezing point, or 32° Fahr. In January, commencing at Philadelphia in the United States, it crosses the banks of Newfoundland, touches the south of Iceland, and, descending to Holland, is extended by the Balkan to the Corea in Tartary, passing as far as the Aleutian Islands, in the North Pacific Ocean; the north latitudes being respectively, in round numbers, and commencing at Philadelphia—40°, 48°, 64°, 52°, 42°, 40°, 52°, for each of the positions we have named. Connecting this range of temperature with the north-easterly progress of the Gulf-stream,* we shall notice an immense wave of heat proceeding so far north as 64°, and producing a temperature as high as that found in a latitude of 40° (Philadelphia), which, although so much further south, has not the advantage of the heating effect arising from the Gulf-stream. The western shores of Europe, therefore, are situated under isothermal lines which have a great northerly inflection; and these contrast greatly, in direction, with those passing over extensive ranges of land. Thus we find, from the Balkan, where the Gulf-stream has no influence, to the Corea, that the isothermal line is nearly parallel with the latitude (42° to 40°) for a distance of several thousand miles; whilst between Philadelphia and Holland, a distance of 75° of longitude, there is a northerly inflection of 24°—a surprising instance of the influence which extensive surfaces of land and water possess in modifying climatic indications; the latter, in this instance, having an action in raising the temperature to an extent which has no parallel in any other portion of the globe. The warmer portion of this heat-ocean lies within 45° west of the meridian of Greenwich; whilst for 120° to the east, the monthly mean in the same latitude is much lower, and the isothermal inflection curves proportionally nearer to the equator.

Hitherto we have confined our remarks to latitudes north of the equator; but on the south of that line we find an entirely different condition to subsist. To 40° south, the average temperature is colder than up to the same parallel north of the equator; and South Shetland, Georgia, &c., although situated in a latitude as far south, as Scotland, in our island, is north of the equinoctial, have a climate in which the snow-line is level with the sea for the whole year, and they are destitute of all signs of vitality, except in the lowest forms of vegetable productions, as mosses, lichens, &c.

RAINFALL.—We need here only glance at another subject, which has already been repeatedly under review in this section—viz., the annual fall of rain in different regions. In our climate, where excessive ranges of temperature are rare, the amount of rain, although greater in some months than others, is pretty equally distributed throughout the year; whilst in tropical climates, in which the air is loaded with moisture at certain seasons, the fall of rain is chiefly confined to definite periods—a circumstance which has given rise to the terms "dry" and "rainy" season. In some such districts the amount of rainfall is prodigious. Thus, in French Guiana, about 130 inches fall annually; in Senegambia, 190; and at Paramaibo, 230 inches fall during the same period. At Cayenne, 21 inches have been measured in *one day*—an amount about equal to the *annual* rainfall at Paris, or of Essex in our own country.* At the equator, the entire fall for the year is produced during 80 days; whilst, in this country, from 150 to 200 wet days give the period during which the quantity for the year is afforded.

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* See *ante*, p. 1156.

A great variety of circumstances must be taken into account in connection with the differences which various parts of the earth exhibit in respect to rainfall; and it will be exceedingly difficult to predicate, according to any known law, the amount which is due at any particular place. As in the case of the average annual temperature, and for similar reasons, the moisture, elevation, and isothermal characteristics are combined as influencing causes. In Europe, for instance, we may notice two extremes; one presented by the west coast, extending from Bergen, in Norway, to Gibraltar, in Spain, on the whole of which a mild temperature, considerable rainfall, and a large number of rainy days, are indicative of the yearly average of climate; whilst from the North Cape, through St. Petersburg, to Hungary and Turkey, the annual rainfall is only from one-third to a-half of that common to the range of country just referred to. The former region flanks the ocean and the Gulf-stream, whilst the interior of Russia is entirely protected from moist air. In Norway, again, we find that whilst, at the coast (Bergen, see following table), no less than 77 inches fall, yet, in the same latitude, but about 150 miles more inland, the annual average is not more than 21 inches—diminishing still more, until, at St. Petersburg, it arrives at a minimum.

The average annual direction of the wind has a great influence on the climate of all places. In the north of Africa, across the Sahara, easterly winds prevail for about nine months out of the year, and, together with the great heat, parch every object of either animal or vegetable life. Throughout the western part of the Old World, an easterly wind produces precisely such effects on a smaller scale; for it diminishes the rainfall, lessens the suspended moisture, and, in certain years, affects the climate most seriously, and interferes, harmfully, with many agricultural pursuits. In the West Indies, the north of South America, and Hindostan, we find an opposite extreme. Situated in the tropics, surrounded by the ocean, traversed by large rivers, and crossed by mountain-chains whose summits are crowned with perpetual snow, the air, in those regions, at the sea-level, is loaded with aqueous vapour, maintained at a high temperature by the excessive tropical heat. A sudden down-draught of cold air, caused by periodical disturbances in the upper regions, and the return of both polar currents, causes a heavy fall of rain within a short period; and thus the thirsty earth is suddenly compensated for the constant loss it had sustained by rapid evaporation. The transition from an arid desert into a luxuriant verdure of vegetation is almost as sudden; whilst the rivers, swollen by the enormous and sudden accession of water, overflow their banks, and, as with the Nile, which traverses an almost rainless district, spread fertility on all sides, even to a great extent beyond their natural boundaries.

Our antipodes, in Southern Australia, New Zealand, &c., have a greater fall of rain, on an average, annually, than we experience, north of the equator, in the same latitude. This is chiefly owing to their insular position, and their being situated in the midst of oceans extending for thousands of miles in all directions. Yet the interior of Australia, far removed from such ocean influences, is in a condition very similar to that of Africa, being generally a dry sandy desert; and many places, as Sydney, &c., are visited at certain seasons by the hot winds arising from the north-west. These, occasionally, indicate a temperature so high as 120° to 130° in the shade, and destroy every vestige of vegetable life: in a moment the wheat-fields and fruit trees are blasted, and the human subject suffers acutely from the suffocating atmosphere. These winds are the analogues of the sirocco of the Sahara and northern districts of Africa; and, like it, are loaded with fine sand or dust, which is deposited in prodigious quantities. In either case the causes are precisely the same—viz., the drying of a large body of air by its passing over extensive sandy regions; an increase of its temperature, owing to the radiation of the solar heat from the underlying ground; and the absence of mountain-ranges which usually protect the adjacent country, and cool the hot air passing over their snow-covered peaks.

Some portions of the earth are almost entirely unvisited by rain: amongst which may be named the greater part of Thibet and Tartary; many parts of Central India; Arabia; Egypt; Central Africa; part of Chili, &c.; and, in each case, absence of ocean influences is the cause. As might be expected, both animal and vegetable life suffer accordingly; but, in numerous instances, abundant dews compensate for this deficiency of rainfall; or the overflow of rivers fed from distant mountains, periodically supplies the place of rain—as in the case of the Nile, already named; the Amazon, Mississippi, Ganges, Indus, the Australian rivers, &c. Some parts of our colonies in South Australia and Victoria, suffer seriously in the absence of such periodical inundations, most of the rivers in that country becoming little better than pools in the dry season; the lakes even being almost desiccated in summer. Before the setting-in of the rainy season in many countries, the atmosphere, which had, perhaps for weeks or months, been clear and quite free from clouds, &c., gives an indication of the coming change. The wind attains a settled direction, which is invariable in each year for that season; hurricanes and fearful thunder-storms suddenly come on, and are followed by rains, during which as much as one inch per hour falls at some places. It is a most providential circumstance that the advent of the rainy season is thus forewarned, and its periodicity so regular; as the torrents of water, soon generated, would speedily wash away the products of a year's growth. In some parts of the West Indies the heavy rains fall in October, November, and December; but, during August, the islands are generally visited by fearful hurricanes, some of which have almost ruined the colonists, by the universal havoc which has been occasioned. These storms, occasionally, spread over the two following months, as at Jamaica, which has a wet season from May to August, and in October and November; August, September, and October being the hurricane season.

Although in several degrees of more southerly latitude, China participates in a climate resembling, respectively, those of England, France, and Northern Italy; and the productions of that interesting country are also in many cases similar. It is to some extent affected by the Pacific Ocean washing it on the east coast, as Europe is, on the west, by the Atlantic; the circumstances being similar in many respects—the direction of the wind, however, being varied. Shanghae, in 31° N. lat., possesses one of the finest climates in the world; and although extremes of temperature are met with throughout the country, still the air is generally salubrious, and extremely healthy even for Europeans. At Canton the heat of summer is very great, and the yearly average is high, being 70°; yet whilst in a latitude of 23° N., or nearly 30° further *south* than London, fires are required in the houses during winter, and snow and frost are occasionally seen in the streets.

North America presents some curious peculiarities of climate, which are rarely paralleled in any other part of the globe. Generally speaking, the annual average is about 10° or 15° lower, for the same latitudes, than in Europe, and the ranges of temperature are very excessive. In Canada the thermometer stands frequently at 20° below zero in winter, or 52° below freezing-point; whilst in summer, a heat of 100°, or 105°, is commonly experienced. Quebec, in lat. 47° N., has an *average mean*, in winter, of 14°, and in summer, of 68°—showing a very excessive type of climate; whilst so far south as Georgia (lat. 32° N.), the extreme ranges are from 13° to 100°. Compared with these, we may observe, that in lat. 32° S., at Perth, in Western Australia, the range is from 55° of winter, to 75° of summer *mean;* and in the same latitude, north, in Egypt, at the delta of the Nile, the *range* is from 50° to 60° in winter, and from 80° to 90° in summer.

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* Excessive rainfalls are sometimes, and in certain places, noticed in England. Thus, on Spinkling-fell (height 948 feet), near Borrowdale, Cumberland, the mean of four years gave no less than 163 inches, and it has been nearly 200 in some years; whilst the total annual fall does not exceed, on an average, 31 inches for the whole country.

A great variety of circumstances combine to produce the low mean temperature of the northern portion of the American continent, a brief investigation of which may prove instructive. So far as is at present ascertained, one almost unbroken tract of frozen land or water extends between 60° N. lat. and the North Pole, a considerable portion being at a great elevation above the sea-level. The atmosphere over the whole of this region is, of course, cold, and free from aqueous vapour, and rarely, if ever, intermixes with warm currents of air from the south, to an extent sufficient to permanently modify its average temperature. Hence, whenever a N., N.W., or N.E. wind sets in, the greater portion of North America is exposed to a most inclement and chilling blast; and the immense quantities of icebergs floating southward from the Frozen Ocean, at certain seasons of the year, conduce most powerfully to increase this condition. The north-western portion of the continent is hedged in by a long range of hills—the "Rocky Mountains"—which extend from 70° to about 35° north latitude, with scarcely a break, and are continued to the extremity of the southern portion of the continent at Cape Horn, in latitude 55° S. The greater part of North America, comprised within the parallels of 35° and 70° N., is, therefore, completely shut out from the influence of the warm Pacific Ocean, the hill ranges being too high to permit of the passage of warm aërial currents over their generally frozen summits. On the east, the Gulf-stream, instead of impinging on its shores, passes over the Atlantic in a north-easterly direction, to Europe; and on the south, the high table-land of Mexico partly prevents the access of the hot tropical currents. About California, however, on the west coast, and beyond the Rocky Mountains, a more equable and mild climate subsists, because there the interfering causes, above described, are absent, and the genial air and moisture, arriving from the Pacific, exert their full influence in tempering the climate of that country.

Not only is the climate of North America of a very excessive type, but the range of temperature undergoes most remarkably sudden variations. In some parts of Canada, for instance, the thermometer will fall as much as 60° in 36 hours; and a variation of 30° or 40°, in the course of 24 hours, is by no means an uncommon occurrence during the winter season. The effect of such sudden changes—which, in a moist climate, would be positively injurious to animal life—is, to a great extent, moderated by the extreme dryness which prevails; the intense cold, of course, freeing the atmosphere from every vestige of suspended aqueous vapour. To such an extent does this occur, that, in the Arctic regions, the breath, as driven from the lungs, has its moisture instantly converted into minute particles of snow. The mechanical power of this extreme frost is often sufficient to rend trees and rocks, by the expansion of the ice, as if they had been blasted with gunpowder; metals, if touched by the hand, &c., blister the skin equally as when red-hot; and even hydrophobia is common amongst dogs, at Quebec, in December and January, just as would be, with more likelihood, expected during the hot months of summer.

The southern part of Mexico, between 15° and 20° north latitude, owing to the varying elevation of the land, presents some singular peculiarities of climate. On the low-lying coast the heat is very great, the mean annual temperature reaching an average of 76°. On the elevated table-land the mean is reduced to 64°. The climates range *hot*, beneath a height of 900 feet from the sea-level; *temperate*, at an elevation of from 4,000 to 5,000 feet; and become positively *cold*, at above 7,000 feet.

In a similar manner, and arising from like causes, the climate of Central America, extending between Mexico and the Isthmus of Panama, or between latitudes 18° and 8° N., varies considerably. On both sides the shores are washed by the ocean; the western by the Pacific, and the eastern by the Caribbean sea; and the temperature of the winter of each reaches nearly a mean of 80°. The temperature of the low-lying districts varies between 60° and 80°; but a range of mountains of considerable height (12,000 to 15,000 feet), being an intermediate link joining the Andes of South America, and the Rocky chain of the northern portion of the continent, must tend to modify considerably the heat, which would otherwise be very great. The rains that fall from May to October temper the climate on the Pacific coast; whilst those on the east side, falling between October and May, have there a similar effect, and render the country as healthy as any in the same latitude; fostering, at the same time, the greatest luxuriance and profusion amongst the vegetable productions of the country.

In South America every extreme of climate is found, this portion of the continent comprising all possible variety of elevation, &c. In the north, near the equator, and on the east coast, the level is low, the air moist and of a high temperature, and the rainfall, consequently, in some seasons, enormous, always exceeding in Guiana 100 inches annually, and occasionally reaching to more than double that quantity at places between 8° N. and 2° S. of the equator. On the west coast, the Andes, at an average distance of about 100 miles inland, rob the air of its moisture, as brought by the N.E. and S.E. trade-winds; and *northwards*, between the hills and the shore, from lat. 30° S. to 10° S., the chief parts of Peru and Chili present an almost rainless tract. To the *south* of 30° the air becomes more humid, the land being much narrower than in the northern part; and the prevalent winds are from the west, bringing with them moisture from the Pacific. The country, in fact, gradually assumes a condition, southward, resembling that of an island, so far as climate is concerned, extending thus as far as the south of Patagonia, in latitude 55° S. The Andes shut off the westerly breeze between 30° S. and 55° S., just as, above the parallel of 30° S. to 10° S., they exclude the easterly winds from the western coast; and thus another rainless tract is formed in the interior, occupying most of the southern portion of South America. The east and south-east coasts of that continent have the constant trade-winds from either the N.E. or S.E.; and the level plains of the interior, from 30° S. to the equator, are thus generally well watered: large rivers, including the mighty Amazon, flow through them, and a hot-moist climate tends to produce an unequalled profusion of most forms of animal and vegetable life. In these regions nature revels in her wildest and richest luxuriance; and on no other part of the globe are the mineral, vegetable, and animal kingdoms so fully developed: yet this tract is, at present, almost entirely unexplored, and its treasures jealously locked up by a narrow and selfish policy, that can only be paralleled in the darkest pages of the history of civilisation. It requires but little prophetic inspiration to foretel, that when the resources of the country become expanded, Brazil must hold a place in the world's history, which its extraordinary natural advantages will possibly make the most brilliant of any epoch. It may, perhaps, be now kept in reserve, to take, at some future time, the *status* of present flourishing kingdoms, when their power and riches have dwindled away, owing to the exhaustion of those natural capabilities they at this time possess—a result which is suggested by the present condition of many parts of Asia Minor and India.

We have thus passed in review the leading particulars which have reference to the rainfall, mean and excessive temperatures, &c., of the chief divisions of the earth; and it now only remains to notice a few facts in connection with the ocean-tracts, lakes, &c., which cover the rest of its surface. It has been explained, in previous pages, how much the climate of any place depends on a proximity to, or distance from, the sea. In the equatorial regions, the Pacific, Indian, and Atlantic oceans are the cause of the moist climates of tropical countries; whilst, on the other hand, the Arctic and Antarctic oceans tend to diminish the temperature of the land, not only in their immediate neighbourhood, but even for many degrees beyond their limits. Siberia, in North Asia; Lapland, Russia, and Norway in Europe; Greenland, and British and Russian America, are, to a great extent, frozen and inhospitable climes, owing to the extreme cold induced by the Frozen Ocean of the Arctic zone. From the Antarctic circle, northwards, to Patagonia, Tasmania, and New Zealand, the Southern Ocean reduces the mean temperature to an even greater extent than does the northern one in corresponding latitudes, owing, chiefly, to the greater area which is unoccupied by land, and hence not under the influence of the surface-absorption of the solar rays of heat. The difference of temperature between the two oceans is very remarkable; for we find, that whilst the Arctic has a heat of 40°, in lat. 70° N., the Antarctic, in lat. 68½° S., has one of but 30°·8; the difference for corresponding latitudes, within certain limits, being generally about 10°.

The area of inland seas and lakes is small, when compared with that of the oceans; but being mostly found in regions far removed from the sea, it must have a decided *local* effect on the country surrounding their shores. In Europe; the Baltic, Black, and Mediterranean seas; the Russian and Swiss lakes, &c.—in Asia; the Caspian, Aral, Baikal, and Black seas, &c.—in Africa; the Red Sea, Lakes Tchad, Nyassa, &c.—and, in America, the extensive system of lakes intersecting Canada, the United States, &c.—all have a direct influence on the adjacent country, either in reducing or raising its average temperature.* That areas of enclosed water should have these two opposite effects, in the same localities, at different seasons, may at first sight appear anomalous; but it can be easily explained. In the case of any lake or *nearly enclosed* sea—the Caspian being of the former, and the Black Sea of the latter type—it is plain that, during summer, the surrounding lands, owing to the absorption of solar heat, will be of a higher temperature than the waters of the lake, which reflect, but barely absorb, heat. On the contrary, during the winter, the water will be warmer than the land, because whilst the latter radiates, and therefore loses its heat acquired during summer, the lake radiates very slowly, and therefore as slowly loses its acquired temperature. Inland seas and lakes thus tend to equalise the amount of heat, and, like the fly-wheel in Mechanics, whilst they add no force, they regulate its distribution. As winter, however, passes away, the surface of such as are frozen will tend to maintain a low temperature in the surrounding atmosphere, just as we find in the great area of the North Atlantic, the icebergs, floating southwards, lower the temperature of the atmosphere; and coming into contact with the warmer air, in part produce fogs, so common about the banks of Newfoundland.†

The islands, spread throughout the oceans and seas, must have an effect analogous to that which we have described in respect to the lakes; and for the reasons already assigned, so far as the absorbing and radiating power of dry land is concerned. It is true that, individually, like the lakes, &c., they are extremely small, when compared with continents; but their great number, especially in the Pacific, compensates for their trifling area. They form so many surfaces from which currents of hot air are continually ascending, that are replaced by constant in-draughts of the cool sea-air. Hence islands generally possess those delightful land and sea-breezes, which, beneath the tropics, are often the only modifying agents that would permit many such places to be inhabited. In the Section on Heat this subject has been fully discussed, and some interesting observations, by Captain Maury, are also quoted, fully illustrating the nature of these land and water aërial currents.

In some islands the opposite coasts possess an entirely different climate—a circumstance which is generally attributable to extensive mountain ranges, having a considerable height above the sea-level; or to the setting-in of atmospheric currents from the ocean. Iceland presents a remarkable instance of this kind, owing to the different temperatures of the water by which it is surrounded. On the south side the Gulf-stream impinges, and has so great a heating effect, that the average annual temperature is 40°—an amount equal to that of Philadelphia, which is in a latitude of nearly 25° nearer the equator. But on the north side, where no such warming influences prevail, and at a distance of from 100 to 200 miles from the south coast, a region of perpetual snow and ice is found, which, when a north wind blows, often blasts, by its chilling influences, the remainder of the island, and even drives away the fish that otherwise abound in the surrounding ocean. England, Scotland, Denmark, and Norway, are similarly affected from the same cause; for on the west coast of each the climate is generally moist and mild; but the eastern side of each country, being exposed to the dry cold winds arriving from the interior of Russia, is generally bleak, and, in the northern districts, very cold and dry.‡ The Canaries afford a remarkable instance of this kind. Situated off the west coast of the African continent, and a little to the north of a line passing through the Sahara desert, they experience, at most times, a healthy climate; are refreshed by the sea-breezes, and possess a highly fertile and productive soil. When, however, an easterly or south-easterly wind prevails, the hot and dry winds from the desert reach the islands, accompanied at times with swarms of locusts; and from these causes, in conjoined operation, every vestige of vegetation is destroyed, the earth is parched up, and a desert, arid waste replaces the former luxuriant verdure. Ceylon presents instances of a similar kind; but in that island the variations are chiefly due to the differences of elevation of its various parts; and the mountain chain running through the island, has a remarkable effect on the climate of the east and west sides. When the S.W. monsoon sets in, the west side will receive copious rains, and be covered with a luxuriant tropical growth; whilst, on the east side, a great scarcity of water will be felt, and even an almost complete suspension of vegetable growth occurs, because the suspended aqueous moisture, arriving with the monsoon, is arrested in its course by the hills, and confined entirely to the opposite side of the island. We believe that similar circumstances affect the climate of Madagascar; but its interior is but little known, and, therefore, no accurate data have been obtained. Borneo has an almost south-European climate on its north side; whilst, on the south, that of a tropical character subsists. In the Mediterranean, Sicily presents another illustration; the mean annual temperature of Palermo, on the north, being 6° less than that of Catania on the south-east, the difference of latitude being only about one degree. In the West Indies, Guadeloupe has a low and hot west coast; whilst the east, which is elevated, is much more temperate; yet they are but a few miles apart. The same observations may also be applied to Jamaica, and many of the adjacent islands; and, generally speaking, each possesses a mountain range or tract, which, as we have already remarked, has so powerful an influence in tempering the excessive heat common to all tropical countries.

We have thus pointed out the chief causes, both of a general and special nature, which often affect the climate of continents, oceans, &c.; and our remarks will serve as an introduction to a table,§ in which we have included a variety of facts, carefully selected from various sources, and relating to subjects which have been discussed in the last few pages. The names of the chief countries, cities, &c., in each division of the world, are given in the first column; and, when important, several parts of the same country are named, so as to show the results of local disturbing causes. Wherever possible, and when the results could be relied on, the annual average fall of rain, running over several years, is marked down in the second column. In the succeeding one, the average mean temperature, taken from the *monthly mean* in most cases, is afforded;[*] but occasionally two "means" are given, because either an uncertainty existed, or the climate is subject to extraordinary variations in certain years—a by no means uncommon case with that of our own country. In the fourth column, either the extreme range of temperature, occurring in the year (*i. e.*, the greatest heat and cold), the mean of winter and summer, or the entire range is afforded. In attempting to give these particulars as accurately as possible, we can but remark, that the accounts of various observers differ so greatly, as to make it a matter of much difficulty to obtain information that can be relied on; and it is far more than likely, that the thermometers employed for this purpose have been often incorrect, for reasons already assigned at an earlier page.[†] Observations of this kind, if carelessly made, or with improper instruments, are worse than useless; we believe, however, that the figures in the following table will be found as nearly accurate as possible, because no one set of observations has been adopted, but rather independent data, so that they might act as a check on each other. If not otherwise stated, the temperatures are those of *nearly* the sea-level, or that distance above it on which the majority of towns are situated; which is generally not of any elevation worthy of notice, or likely to interfere, comparatively, with ordinary observations. For the sake of saving space, the letter E has been employed to indicate the *extreme* points between which the thermometer ranges; W shows the mean winter, and S the mean summer, average temperature; and R indicates the total range in degrees, which occurs between the highest and lowest readings in the course of the year.

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* See, also, remarks on the temperature near the Lake of Geneva; *ante*, p. 1188.

† The folio plates, at page 1192, give the extent of the chief lakes in each hemisphere, and of the islands in the oceans; from which an idea of their influence, with respect to climate, may be gathered.

‡ See *ante*, pp. 1177 and 1193.

§ See end of this Section.

In the column devoted to "moisture," of course every remark is but comparative or suggestive. The use of the hygrometer has been but recently adopted beyond the precincts of our observatories, and hence, the character of a climate in this respect, must be, at present, often a matter of guess-work, or represented in somewhat ideal expressions. The reader, however, must bear in mind the remarks which have been frequently made respecting temperature and moisture; and wherever the annual mean of the former is high, with the rainfall large, and the thermometric range not *very* excessive, an atmosphere containing much suspended aqueous vapour may be expected. This is illustrated in the case of the Guianas, Senegambia, Ceylon, &c. On the other hand, an excessive range, little rainfall, and a comparatively high or low annual mean temperature, indicate a dry climate, as those of Canada, Thibet, the Sahara, Russia, &c. The last two columns give the latitudes and longitudes; in some cases, *nearly*—that is, within a few miles, for the purpose of including a small district surrounding the place of observation.

The table, as thus arranged, may be employed for the purpose of marking down, on a map, isothermal lines for any mean annual temperature which may be taken from column 3. The latitude corresponding to it will be found in column 6, and the longitudes in column 7. Thus, choosing from column 3, the average of 52°, and commencing in Ireland, we pass to the middle of England; through a part of France, to the north of Paris; Prussia; the north of Turkey, or south of Russia (possibly the N. of the Black Sea); the north of India, or perhaps some part of Thibet; the north of China; and thence to New Zealand and Tasmania. All these countries have, in some part, the mean annual average of 52°; and, accordingly, the isothermal *yearly* line will be included in them. If we next turn to column 4, it will be seen that the yearly average is struck from very varying *monthly* or *six-monthly* means, for the *ranges* of temperature are greatly different in each country. Thus, England has a *total annual* range of 85°; Prussia has an *average mean*, between winter and summer, of about 38°; whilst, in the north of China, the similar range of winter and summer means is 53°; and in New Zealand and Tasmania, it is only about 20°. By taking six winter and six summer months, respectively, according to the table of Greenwich average monthly temperatures (given at page 1172), we shall obtain a *mean range, between winter and summer*, of about 15° (which is, however, too small); so that each country named above, although having the same *mean annual* temperature, has a remarkably different *monthly* mean, from which that annual average has been deduced. The reader will therefore perceive why *two* sets of isothermal lines are needed—one for the year, and another for the month—as we have already pointed out at a previous page.[‡] By such a plan only, can a proper average climatic mean be obtained, which may be correctly compared with that of any other place.

The remarks which have been made about climates of the excessive type, will receive a complete illustration in the table; and the student will be able to trace the connection subsisting between rainfall, suspended moisture, and range of temperature, as affecting various places, but especially those of a tropical character. Amongst the extraordinary inflections produced by special disturbing causes, that of the Gulf-stream may be particularly noticed, both in respect to latitude and longitude, and from its origin, till it is lost in the North Polar seas. It will also be observed how much less remote the extremes of the annual range of temperature become as land-surfaces diminish in breadth, and tend towards an insular nature, except under extraordinary local circumstances (as in Iceland, &c.); and the influence of ocean currents, or ocean static conditions, may thus be completely and satisfactorily traced, in connection with latitudes on either side of the equator, and compared with the effect produced by extensive land-surfaces in other regions. The temperature of the ocean in different latitudes has been given, and will be found useful in many inquiries affecting climate, &c.

DISTRIBUTION OF PLANTS AND ANIMALS.—In connection with the science of Meteorology, we may make a few remarks on the distribution of plants and animals; and thus recommend to our readers, as a most interesting study, the comparison of the natural productions of each region, taken in relation to its climatic characteristics.

Whatever theories may be adopted, in ethnology, with respect to the races or varieties of men; in zoology, regarding animals; in botany, as to plants, &c.; and the inferences we may draw, geologically, from specimens of extinct species, respecting their former regions of existence;—it is patent to the most cursory observer, that, with the exception of man, all classes and families of animals and plants may be so arranged as to be considered indigenous to certain climates. The varieties of species are certainly very numerous; and normal characteristics are often modified by various circumstances. Thus the breed of cattle and horses, the quality of fruit, &c., are respectively the subjects of change when those are removed from the country in which they exist in a wild or uncultivated state. The art of training both plants and animals to undergo and bear this change, has, of late years, been carefully studied, and exceedingly interesting results have accrued; and the subject is one of great importance in a commercial point of view, as it is now being constantly attempted to introduce animals, &c., into our colonies, not only to improve the existing stock, but also to add other and useful forms to it. The alpaca, salmon, poultry, pheasants, &c., have thus been transported to our Australian settlements, and are likely to be extremely productive, and of great service; and the cinchona plant has similarly been carried to India, and promises soon to afford an abundant supply of quinine, so essential in cases of debility, ague, &c., in tropical climates. These afford striking instances of the value which is attached to the study of climatic meteorology: and to those so engaged, the following table will doubtless prove of great assistance. We may, however, give here some additional hints, by pointing out the productions which characterise the separate zones, both of latitude and elevation.

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[*] See reasons for this, mentioned *ante*, p. 1194.

[†] See *ante*, remarks on the manufacture of thermometers, p. 1131.

[‡] See *ante*, p. 1194.

The varieties of climate may, as we have already seen, arise from either of two causes—latitude and elevation—or from their conjoint influence. Thus, at the equator, on the sea-level, we find an annual mean temperature of about 80°; but, ascending to a height of about 16,000 feet, we pass to a line of perpetual snow, equivalent to the condition of the polar regions. Now between these two extremes, it is found that, at every 2,000 feet, a zone is obtained as we ascend, nearly corresponding in productions with other zones, given by dividing the interval between the equator and the poles into eight parts, proceeding north or south from the equator. The *latitudinal* zones can only be limited approximately, because of the irregular inflections of the isothermal lines,[*] which rise towards the poles, or fall nearer the equator, according to various local affecting causes. The succeeding table, however, affords both the latitudes and longitudes of the various places, having the same yearly isothermal; and any map may thus be used in connection with the following observations, the line being temporarily traced by the pencil, from place to place, as required. To be explicit, we again remark, that elevation and latitude may be considered as convertible terms, in studying the distribution of living and extinct creatures of both kingdoms.

The following are the names of the zones, and their *approximate* range in latitude, north or south of the equator—subject to the modification of local causes.

1. Equatorial . .	extending	from	0° to 15°	lat.
2. Tropical . . .	„	„	15° to 24°	„
3. Sub-Tropical .	„	„	24° to 34°	„
4. Warm-Temperate	„	„	34° to 45°	„
5. Cool-Temperate	„	„	45° to 58°	„
6. Sub-Arctic . .	„	„	58° to 67°	„
7. Arctic . . .	„	„	67° to 75°	„
8. Polar	„	„	75° to 90°	„ (?)

1. Commencing with the equatorial regions, we find a climate exceedingly hot and moist, and just that in which vegetation has every circumstance favourable for its full development. There grow in perfection, palms, bananas, ginger, spices, gums, the beautiful water-lily called the Victoria Regia, rafflesia, the sugar-cane, coffee, dye-woods, cabinet woods, cotton, &c.

2. At about 15° north or south of the equator the tropical zone commences, and it extends to about 24° of latitude. Tree-ferns, convolvuli, pepper, tropical fruits of many kinds, the cocoa-nut, rice, coffee, sugar, dye-woods, cabinet woods, indigo, spices, gums, tobacco, and cotton, are extensively grown.

3. From 24° to 34°, which includes India, North Africa, the West Indies, &c., is the sub-tropical zone, the productions of which much resemble the former; but there may be added opium and tea, up to 31° N. This zone embraces the chief cotton-growing countries, both in India and the United States, and also of tobacco and sugar. The characteristic plants are the myrtle tribe, magnolias, large forest-trees, rich flowering plants, luscious fruits, date-palms, &c.

4 From 34° to 45° constitutes the warm-temperate zone, including Portugal, Spain, Greece, Italy, the south of France, &c. The vine, and the mulberry for the silk-worm, are characteristic. The trees are the cork-tree, oak, chesnut, beech, &c.; the myrtle, heaths, &c. The cereals flourish; rice is grown in some parts; amongst fruits are oranges, lemons, nuts of many kinds, grapes in great abundance for wine-making, or to be sold dried as raisins; currants for drying, grown in Greece; peaches, &c.;—such taking the place of sub-tropical fruits, which here disappear.

5. From 45° to 58° constitutes the cool-temperate zone; and the productions of our own country may, to some extent, be taken as the type. Amongst trees are the oak, elm, beech, ash, birch, &c.; cereals grow abundantly in the lower and middle latitudes; grasses also abound for pasture; flax and hemp are cultivated; apples, pears, plums, cherries, &c., are the fruits; the grape not ripening so as to be fit for producing wine, except in the lower division, where it is extensively produced. Great Britain, Ireland, Mid-France, Switzerland, Prussia, Austria, &c., are in this zone; and thus some of the most civilised portions of the world are comprised within its limits. In our country wheat will scarcely grow in the extreme north; hence oats and barley replace it to a large extent in Scotland, and also in Norway, Sweden, &c. The hop, ivy, honey-suckle, &c., are amongst the creeping plants, the two latter being common in the forests. Most of the colonised portions of Australia, Tasmania, and New Zealand are within this zone, in south latitude; and hence their vegetable products resemble those of the countries in Europe, similarly situated in north latitude; but are varied by the presence of trees peculiar to the Australian continent, such as the *Eucalyptus*, or gum-tree, a kind of acacia, numerous forest evergreens, tree-nettles, &c.

6. In the sub-Arctic zone, from 58° to 67°, the *Coniferæ* appear, and flourish well; hence the large pine-forests in Norway, and parts of North America which, however, from being so much cooler than in the same latitudes in Europe, have this kind of tree much nearer the equator. There are also the birch, willow, alder, grasses; rarely wheat; but oats, rye, and barley, lichens, and mosses.

7 & 8. The Arctic zone, extending from 67° to 75°, gradually loses all trace of trees, which are replaced by shrubs of rhododendra, bog-myrtle, lichens, and mosses; and, in the Polar zone, reaching hence to the poles, saxifrage, ranunculi, cochleariæ, and abundance of lichens, mark the extreme limits of vegetative power.

So far we have observed the distribution of plants, in respect to latitude, north or south of the equator—taken certainly on an extremely broad and general scale; for some portions of the earth's surface must be entirely omitted from any of the limits thus pointed out. Thibet, for example, whilst nominally in the sub-tropical zone, has really a climate almost, at times, resembling an Arctic one; but this is owing to its great elevation, of which we shall presently have to speak more fully. The Sahara desert, Arabia, much of Central Africa and South America, equally afford exceptions, the nature of which has been explained previously.[†]

With respect to zones of altitude, similar allowances have to be made, and for the same reasons. Commencing at the sea-level at the equator, we shall have, of course, a climate of the equatorial type, in which, up to a height of 2,000 feet, palms, bananas, ginger, &c., flourish. From 2,000 to 4,000 feet, the tree-ferns, &c., exist; from 4,000 to 6,000 feet, myrtles, laurels, magnolias, heaths, &c.; from 6,000 to 8,000 feet, evergreens, olives, &c.; from 8,000 to 10,000 feet, oaks, beeches, &c.; from 10,000 to 12,000 feet, firs, &c. (on Etna, so low as 7,000 feet, and in the Pyrenees, 5,500 feet); from 12,000 to 14,000 feet, rhododendra, and Alpine plants; and from 14,000 to 16,000 feet, Arctic plants of various kinds, mosses, and lichens in abundance—the latter, as in high latitudes, being the last form of vegetation that is met with.

It would be beyond the scope of our present object if we were to enquire minutely into the specific differences which subsist between the plants of the various zones, and the relation which they have with the climate in which any of them best flourish. This is a subject which would require considerable botanical knowledge on the part of our readers, and we should therefore have to presume on an acquaintance with that science, which has to be treated in detail in the succeeding volume. We may observe generally, however, that the existence of a plant depends more essentially on the *temperature* of any locality, than on the complete suitability of its food. This, of some kind, it can generally find anywhere, to an extent sufficient for bare existence, if not for luxuriant growth; but if the climate be too cold or hot, then many plants will not even grow; or, if they do, they never bear fruit which will ripen. This is a matter of constant experience, especially with the cereals, on which the food of man so greatly depends, and the growth of which is confined within the parallels of about 30° to 58° of latitude, or a corresponding elevation in respect to zones, above the equatorial sea-level. The same holds good with grapes, their limits, however, being somewhat more narrowed.

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[*] See *ante*, p. 1194.

[†] See *ante*, p. 1193.

The distribution of plants is a matter of great importance, not only for the vegetable food of man, but as also affecting those animals on which he depends for sustenance, carriage, farm-labour, &c.; and hence every civilised country has certain features, both physical and social, which more or less distinguish it from others. It is true that the extensive application of steam in railways, vessels, machinery, &c., is fast obliterating these distinctive characteristics in many places; but the progress of such improvements is slow, compared with the extent over which they must spread, before the revolution they have to effect is completed. In our country such progress is rapid, as also in many others in Europe and North America; but most of the colonies still have to depend on brute force for the saving of human labour.

In the particular case of cattle, sheep, and horses, we have an illustration of this. These animals depend entirely for sustenance on the grasses or inferior cereals; and where such can be obtained of good quality, and in sufficient abundance, the breed of each is not only supported in quantity, but the quality is simultaneously improved. In England, particular attention has been paid to this; and the breeds have been so improved, that the horses, cattle, and wool of this country equal any other, and, in many cases, exceed in quality those of the very countries from which the original stock was first imported. This has arisen chiefly from our climate being peculiarly suitable for such purposes, aided by judicious training, and careful attention to the requirements of each animal.

Turning to Lapland, we find that cattle and horses are there replaced by an entirely different animal, the Reindeer. An almost total absence of grass is compensated for by an abundance of lichen, which is the favourite food of that useful creature; and thus the exigencies of the climate are completely met by a peculiar adaptation of the vegetable to the animal kingdom, brought about even by its extreme severity; whilst the human being has, in one animal, a source of food and clothing, and a beast of draught. The camel in Africa and Arabia, the elephant in India, the llama in Peru and Chili, &c., are instances of a similar kind—all tending to show how climatic variety is compensated for, in a manner as complete as the circumstances of the case require.

The subjects of food, temperature, and *clothing*, must be simultaneously considered in connection with the distribution of animals; for these three points are those on which their existence depends; the skin and its covering being an additional and most important matter in the economy of animal life. In the vegetable kingdom, the organs of respiration and digestion are of so simple a character as to be more easily adapted to varieties of change, and the food is usually present. On the other hand, the animal has nothing but its instincts to guide it in the search for food: and from this arise all the circumstances which affect its longevity, productive powers, &c.; the accidents of its life being induced by its cravings for sustenance, and, in fact, struggles for existence.

Commencing with one of the lower forms of animal life, as the *Mollusca*, we find them pretty equally distributed throughout the waters of the ocean; but always in respect to the peculiarities of each climate; for certain kinds are found only within well-defined limits of latitude, beyond which they rarely exist. Amongst the lowest class of creatures, the coral *Polypi* are almost exclusively confined to certain parts of the Pacific Ocean, where, for ages past, they have been slowly but surely rearing islands, now rapidly becoming the centres of civilisation. Amongst *Fish*, again, we find that some are confined to certain zones, beyond which they are rarely seen: instances of this are found in the salmon tribe, which is limited to the cool, temperate, or sub-arctic zones. But many of the tribes, apparently indigenous to our northern seas and rivers, have their representatives or substitutes in those of low latitudes, the species corresponding, except in some physiological adaptation required by altered circumstances of existence; the type being retained, whilst the exact identity is somewhat affected.

Amongst the *Reptilia*, as the lizards, or saurian tribe, the *Loricata*, &c.—of which our common field-lizard is typical, and the analogue of the crocodiles and alligators of the Nile, India, and America—the development proceeds rapidly as to size and strength, as lower latitudes are reached. The variety of these classes is very extensive about the equatorial and tropical regions, where they abound both on land and in water. Equally, amongst *Birds*, also, the same climatic law is influential: the eagle of Scotland and Norway has its family represented in the Himalayas, and by the condor in the Andes; and we may observe the same fact throughout the whole class. In some islands, however, exceptions exist, as in the apteryx and the dinornis of New Zealand, the emeu of Australia, the dodo of Madagascar, &c.; but still, comparative anatomists can find connecting links for this apparently broken chain.

With respect to *Mammalia*, the distinction of climate is very remarkable. The *Quadrumana* (monkeys, &c.) are invariably inhabitants of tropical countries; as are the *Carnivora* (the lion, tiger, &c.), with comparatively few exceptions. The *Pachydermata* (elephant, rhinoceros, hippopotamus, &c.) are nearly limited to these regions. The *Rodentia* (beavers, rats, hares, &c.) are more extensively distributed, for their food is more generally diffused in its growth. A similar remark applies to the *Insectivora*. The *Edentata* are almost exclusively confined to tropical countries, especially on the south of the equator: they include the ant-eater, pangolin, sloth, &c. The *Ruminantia*, in which our domestic animals (of which we have already spoken) are comprised, embrace a vast variety, as the camel, llama, deer, giraffe, goat, bison, buffalo, &c. Amongst the horse tribe, or *Solidungula*, we may notice how much the ass is degenerated in our climate, compared with his appearance, temper, &c., in regions more southerly, as in Arabia, Asia Minor, &c., where he appears to great advantage; and, indeed, there some species are of great beauty, and even swiftness, as is seen in the djiggetai of Central Asia. Possibly the zebra may also be allied to it, for it partakes of many of its characteristics. The *Sirenia*, as the manatees and dugongs, resemble, in many respects, our *Cetacea* (whales) and *Pinnipedia* (seals); whilst the *Marsupiala* (kangaroos) are allied to the opossum of America.

We have specially remarked on the clothing of animals, as being peculiarly related to the climate in which any species is indigenous. Several remarkable instances of this may be cited. The Arctic bear, *Thalassarctos maritimus* of naturalists, exclusively a native of the polar regions, is clothed with a profusion of hair, the texture and colour of which prevents the rapid conduction and radiation of heat from its body.* Even the feet are covered with hair, which is supposed to give it a firm hold when walking over the snow and ice. The Arctic fox, *Vulpes lagopus*, is similarly clothed with abundant fur; but what is remarkable, the hair, which is of a dark-brown colour in summer, becomes quite white in winter, increasing rapidly in length and thickness, just as the exigency of the severe climate necessitates such a protection. Some species of the hare, as the *Lepus variabilis*, and of the weasels, as the ermine (*Mustela erminea*), which are confined to the colder climates, have the colour of the fur changed similarly, as winter approaches: this may frequently be noticed in both animals in the northern parts of our own country. Amongst the dog tribe, or *Canidæ*, as the Newfoundland and St. Bernard breed, we find a similar provision against the extreme cold, contrasting strongly with the clothing of the greyhound, &c., which are indigenous to milder climates. Numerous instances of this kind might be adduced; but the above will be sufficient to show how much the natural clothing, &c., of the animal becomes self-adaptable to climatic extremes, and excessive ranges in the temperature of the seasons.

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* See *ante*, Section on Heat, pp. 22 and 26.

Considerable analogy subsists, in general, between the external and protecting coat of trees and animals. The *exogenous* kind of plants, such as all the trees of this and other cool countries, are furnished with a thick bark, which affords a most essential protection to the tree during its growth, and the removal of which would cause its destruction. In *endogenous** plants, as the palms, bamboo, sugar-cane, &c., which are confined to tropical countries, this cuticle, or bark, if any exist (which has been disputed), is of the most elementary kind; and, after a certain period, not only does it not increase, but seems to prevent the stem also from increasing in width; hence the endogens are almost invariably tall and tapering plants, bearing flowers and fruits only at their upper extremities. It has been considered, indeed, that the "bark" of the endogens is nothing more than the hardened external cells of the stem; whilst that of the exogens is a true and complete sheath, regularly produced during the growth of the tree, and essentially connected with its vitality.

With respect to the growth of food specially adapted to certain species of animals, and which we have already noticed in respect to cattle, horses, sheep, &c., we may add a few more instances that are well worthy of notice, as indicative of the climate of many regions. The deer tribe are very fond of certain kinds of lichens, and both the plants and animals begin to be simultaneously developed as the climate becomes suitable for each. In the north of Scotland, for example, wild deer are scattered sparsely, and so is their lichen-food; but as we proceed further north, as in Lapland, Iceland, and Greenland, both the animal and vegetable—which are, comparatively speaking, a curiosity in our country—become essential to the existence of the inhabitants; the *lichen islandicus*, or Iceland moss, for example, being an abundant and characteristic production of that country.

Taking an extreme case of a similar nature, let us transfer our observations to a climate of an entirely opposite character—namely, that of the parched deserts of Northern Africa and Arabia. The herbage produced is at once scanty and coarse, and utterly unfit to support horses; but the camel feeds well on it, and is also enabled, by its peculiar constitution, even to accumulate a kind of stock of food, in the shape of the fat which forms its humps, and which are gradually assimilated into the system whenever its ordinary food cannot be obtained. Its stomach is so arranged in deep cells, as to enable it to retain a store of water when necessary, the cells being expanded or contracted by the voluntary or instinctive action of the quadruped. If taken far northward, it is removed from its natural sphere, and will die, in places where the horse tribe are produced in the greatest numbers, and in the best condition; yet the llama, which is the camel of South America, is impatient of heat, and, being a native of the Andes, frequently retreats, in warm weather, to the snow-line—generally living in flocks, at a great altitude above the sea-level. In Peru and Chili, the llama is employed as a beast of draught; as is the camel in Arabia and Africa; and, like it, affords food and clothing to the natives of those rocky and mountainous countries. Singularly enough, it varies from the camel (whose feet are adapted for walking on sand) in having completely divided toes, which are thus so suitable for climbing the rocks; and also in the absence of humps, which its more abundant, or perhaps extensively diffused, food renders unnecessary. Its clothing is a woolly hair, remarkable for its warmth, and it effectually protects the animal from the sudden changes of climate that occur in travelling from the plains to the mountains, so universal near the western coasts of South America.†

It will have been gathered from the preceding remarks, that certain species of animals are only found in definite tracts or zones. Such localities have been termed *specific centres*; and it is from these that animals of certain species have been spread, accidentally or otherwise, in countries to which they do not naturally belong—in a like manner, indeed, to the transplantation of objects of vegetable growth into climates similar to those in which they grow naturally. In judging of the original place of any species, it must be borne in mind, that influences besides those of climate are constantly active in effecting changes; and none, perhaps, more so than the progress of civilisation. Man has driven before him many animals from their ancient haunts; as, in our own country, the wolf, bear, hyæna, beaver, &c., have long been extinct. In geology, we are often able to judge of the climate which must have existed ages ago, in many places, by finding fossils of certain animals, comparing them with existing species, and inferring, from the habits of such, what must have been those of their ancient families. An instance of this kind is presented by the fossil elk of Ireland, which has undoubtedly existed since the creation of man. It is, therefore, highly essential that the exact specific centre of any recent and existing species should be carefully ascertained, as otherwise, in the comparison of them with fossil remains, serious errors may arise. Occasionally we meet with animals whose specific centres are so exactly shown, as to leave no reasonable doubt of their being confined to one region. Of this kind are the *Ornithorhyncidæ* and *Echidnidæ*, natives exclusively of Australia and Tasmania. These two families resemble each other in some respects: the ornithorhynchus, however, is distinguished by having the jaws precisely like the bill of the duck, with a body resembling that of the ordinary quadruped; many of the organs, &c., presenting analogies to that of the birds. In the echidna, the snout is produced into a long cylindrical organ, the body being thick and short, and covered with hairs. These two families are certainly paradoxes in natural history, and afford remarkable instances of the restriction of animals within certain specific centres.‡

Perhaps we may be indebted for the discovery of these singular creatures (and entertain the hope of many more in the Australian continent), to the fact, that man has, as yet, done little there in "improving" its immense extent of uncultivated, and, indeed, unexplored territory. Not only does civilisation wage war with the brute creation—but even our own species, when found as aborigines in any country, soon undergo a similar fate, as may be witnessed in nearly the whole of the northern portion of the American continent, but especially in that called the United States. In many parts, wholesale extermination, or extradition, has been resorted to, especially in the southern districts; and the destruction of the aborigines by the Spaniards, in other parts of the continent, are too fresh in the minds of our readers to require recital here. In most instances, the original inhabitants, where undisturbed and completely acclimatised, live happily and healthily in regions which are almost surely the grave of the intruding white man. The coasts of South America, Western Africa, &c., afford illustrations of this fact, and astonishingly declare, even in the case of the almost ever-accommodating constitution of our species, that we are subject to the law of specific centres, the removal from which may be productive of danger, or extinction in some cases—only, under any circumstances, to be provided against by an intelligence of which the lower kinds of animals are entirely destitute.

But although quadrupeds seem unable to exist in other than climates similar to that in which they are native, except under special circumstances—as when under the care of man—and, further, cannot shift from place to place, in cases of sudden changes of temperature, for any great distance; we find amongst the birds a powerful instinct, by which they remove themselves from one country to another at certain seasons, and so escape the inconveniences or death which they would otherwise have to suffer. In some birds, as the swallow, this migratory instinct is most strongly manifested, even so far as to cause them to forget their progeny, if it have been hatched late in the summer season, and but just previously to the usual time of their autumnal flight for other regions. This instinct is not confined solely to one order of birds, nor to any particular climate, at least within the limits of the temperate and arctic zones, but is almost universal. Amongst the *Natatores*, some of the auks, which are arctic birds, leave their native regions, and visit our coasts—going, as winter advances, even so far south as Italy and Sicily: so do the puffins, terns, teals, widgeons, pochards, wild swans and geese, flamingoes, &c. Of the *Grallatores*, the crakes, snipes, tatlers, spoonbills, storks, herons, cranes, &c., similarly migrate; some of this order spending the winter in Africa and South Asia, returning to the far north to breed in spring. The *Cursores*, ostriches, &c., possessing wings of but a rudimentary nature, are, like the quadrupeds, confined to one region throughout their lives; and their physical characters accord with this law of their existence. Amongst the *Rasores*, some are migratory, as the quails, &c.; but, generally speaking, they remain in their native country, having but little powers of flight. It is from this order that man has selected, out of the class of birds, those which are now domesticated; amongst them being the fowl tribe, pheasants, partridges, the peacock, &c. The *Columbæ*, or doves, in this, as in most respects, resemble the *Rasores*: some, however, as the turtle-dove, *Columba turtur*, are migratory. In this order is included the extinct dodo. The *Scansores*, comprising the cuckoos, woodpeckers, parrots, and toucans, have at least the first-named bird migratory, as is well known by its welcome visit to this country in early spring. Amongst the *Passeres*—one of the largest orders of birds, including our chief songsters—the nightingale is another welcome visitor to England, extending its passage to Sweden, Denmark, and Russia, but rarely going so far north with us as even the south of Scotland: the migration of the swallow tribe is familiar to all. The *Raptores*, or birds of prey, comprising the owls, eagles, falcons, &c., like the *Carnivora* amongst the mammalia, confine themselves to their own specific centres, having a ready supply of food generally at their command, and their clothing being abundantly provided in the thick covering of feathers which protect them in all seasons.

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* The terms "exogenous" and "exogens" are applied to such trees as the oak, elm, apple, &c., which increase in bulk by a growth *outwardly*; whilst "endogenous" and "endogens" are referred to such as grow *inwardly*, as the palm-tree, common wheat, grass, &c. The former increase constantly *both* in width and height; but the latter, after a certain period, grow in height alone.

† See *ante*, p. 1196.

‡ See pp. 412, 413—Vol. II.

We thus find, throughout each department of the vegetable and animal creations, from the lowest lichen to man himself, that the nature of climate is an essential element of existence, being at all times either permissive or exclusive in its operation. This arises, primarily, from the fact, that on the atmosphere depends the existence of all animate creation. Its chemical constitution is such, that plants and animals must equally breathe it, availing themselves of one (oxygen), or perhaps the other also (nitrogen), of its essential constituents. Its apparently accidental addition (carbonic acid), however, whilst deleterious to the animal, is of the utmost importance to the vegetable; for as that breathes, it assimilates to its organisation the carbon, and thus increases gradually in bulk and height. The purely physical qualities of the atmosphere are of equal importance, and have as complete an influence on vitality; for a certain density, temperature, moisture, &c., must be maintained, to carry on vegetable and animal growth; and the extent of this, with respect to the maturity of each individual, depends conjointly on a proper adaptation of both the chemical and physical qualities of the air to that purpose. Even descending from animated existence to the mineral kingdom, we notice that meteorological phenomena act precisely as essential a part; and this is really prevenient to, and therefore preparatory for, the subsequent processes of vegetation and animalisation. The bare and unfruitful rock is exposed, year after year, to the disintegrating power of air and moisture; its surface slowly crumbles into dust, and its chemical constituents are gradually eliminated; a spore of moss falls on to that surface, vegetates, and decays, and a small amount of "mould" is thus formed; a stray seed, dropped perhaps by a bird in its flight, alights thereon, germinates, and produces seeds, which falling on the rock, bring forth more, and, gradually, the naked surface becomes coated with verdure; birds and animals feed on this produce; and, lastly, man claims the ground as his heritage. Such a sketch is no flight of scientific romance—it is simply a history of every part of the globe that now bears a vital appearance. Indeed, by such means alone, have nearly the whole of the islands of the Pacific been converted from coral beds to abodes of civilised humanity; their surface, now teeming with every necessity and comfort of life, having once been a bare rock, changed, in the lapse of ages, by operations which have just been so incompletely described. In many parts of our own country this process may be watched from year to year; and in the experience of most of us, there may be remembered spots which, once hard and unfruitful, now reward the labours of the husbandman with abundant, and frequently overflowing harvests.

METEOROLOGY, AND ITS ETHNOLOGIC RELATIONS.—In the preceding article we have incidentally referred to the influence of climate on mankind, in respect to the varieties of our species; and without entering deeply into Ethnology (which will form the subject of a separate section in the second volume), we shall here draw attention to a few circumstances well worthy of notice; and that may, in part, serve as an introduction to the section referred to.

A little reflection reminds us that our world is peopled with a vast variety of human beings, whose differences are at once physical, mental, and moral. These three qualities of our race almost always vary simultaneously; in fact, there is evidently an established law which governs moral and mental development, in connection with that of the body. In the same tribes of any nation, the difference of the sexes is due, not so much to an original mental and moral cast, as to the difference of physical organisation. It hence follows, that whatever influence acts on the body, reacts on the sentient principle; and, by a repetition of action, a permanent habit is created, which at last becomes a distinctive feature of the individual of the tribe, and, consequently, of that variety of the human species to which it belongs. With the broad causes, and their specific effects, we cannot here deal, but must confine ourselves entirely to such as arise from the action of climate, as of heat, moisture, &c., and the ordinary food that the climate in which the individual resides, affords for his sustenance.

Commencing with the northern regions, we find the Esquimaux and Laplanders, inhabitants of the arctic zones, generally speaking, to be of exceedingly small stature, and ordinarily deficient of mental and moral culture. They mostly lead a travelling life, obtaining their food by the chase; some of it, as the seal, &c., being of the coarsest kind: the habitations, with the Esquimaux, are generally snow-burrows. For a lengthened period of the year they live in darkness, owing to the long night which the absence of the sun occasions during winter; and hence their climate cuts them off from most associations of the beautiful in nature, which, to more favoured people in the south, are sources of education, and refinement of mind and taste. The entire duties of their existence seem, in fact, to be comprised in two branches—one, of seeking their food; and the other, of eating it. Frequently, owing to the depth of snow, supply fails them, and thus another misfortune is added to the peculiarities of their condition, by the nature of the climate in which they constantly live.

To those dwelling in the temperate zones, it may seem impossible to picture a more forlorn state than that possessed by such people. Yet they seem to find an existence not only tolerable, but even have enjoyments, which are certainly peculiar to themselves. Habit and nature have so far acclimatised them, that they rarely, if ever, stray into more southerly climes; the reindeer, dogs, fish, seals, &c., which are, like them, indigenous to the country, provide them in food, with clothing, and the means of moving from place to place; and the oil of the whale, with such drift-wood as is cast on the shores of the lakes and seas, affords fuel and artificial light. In fact, they are as much adapted to what we must call the "specific centre" of their existence, as are any of the lower forms of creation to which we have previously devoted our enquiries. In person, both Esquimaux and Laplander, although of short stature, are strong, active, and hardy: they are dirty in their habits, addicted to the use of ardent spirits when they can obtain them, and the Esquimaux are extravagantly fond of oil as an essential article of food.

The latter particulars are, however, explicable on chemical and physiological principles. The intense cold of the climate requires a constant supply of carbonaceous matter in the food, which, by slow combustion, during the process of respiration, may educe animal heat. That the craving for fatty matter is not confined to them, under such circumstances, has been proved by the experience of all arctic navigators; and we were informed, as an instance of the kind, by an eminent commander of one of the expeditions in those regions, that tallow candles were often eaten by our own seamen whilst they were journeying with sledges over the snow—so powerful does our instinct lead us to its gratification, even in objects which, under other circumstances, would be turned from with extreme disgust. With respect to the use of ardent spirits in cold climates, we may notice that, in Sweden, eight gallons are consumed by each individual, on an average for the whole population, annually; and in the northern parts of Great Britain, it is well known that stimulants may be drunk, with impunity, to an extent which would be exceedingly injurious, if not fatal, in warmer climates.

Although anticipating subsequent observations, we must here contrast the necessities of the inhabitants, their tastes, and the natural productions of the country, with those of regions of a different climate, for the purpose of impressing on the minds of our readers the fact, that each zone is, in every respect, peculiarly adapted for the wants of the individuals whose "specific centre" of existence is found within it. Turning, for this purpose, to Spain, Portugal, South-France, Italy, and Greece, we notice that the characteristic vegetable productions (the only sure indications of climate) are just such as the exigencies of those climates demand. The fruits, &c.—as the grape, melon, orange—the salads, leguminous products, &c., are all chiefly composed of water, which has an exactly opposite alimentary effect on the animal system to that of the carbonaceous food, so abundantly provided in the northern regions. In the former countries, the external temperature leaves little to be required in the way of heat-giving food; whereas in the arctic climes such is absolutely essential. Even the natural starch-food of hot countries has but little heating power, as is seen in the rice of Egypt, India, America, &c.

The remarks which we have made respecting the Esquimaux, may be, within certain limitations, applied to all tribes living northward of the parallel of 68° to 70° north latitude, in both hemispheres. They have, so far as is yet known, no analogues in the southern or Antarctic circle, as the inhabited land in those regions does not extend northward beyond about 60° S.; or, perhaps, Tierra del Fuego, in lat. 55° S., is the extreme inhabitable limit.

We must now take a broad zone—say from 65° to 40° of north latitude, between which are comprised what are called the temperate regions. With but comparatively trifling exceptions, in respect to number, the whole of the civilised world may be considered to be comprised within these limits, excluding, of course, the Chinese empire. Nearly the whole of inhabited Europe, and North America, containing together about 350 millions of people, is embraced between these parallels; and here art, science, literature, commerce, &c., have their origin and home. Without being too minute in our enquiries, it is impossible not to notice the peculiarities impressed on each variety of the human species living within these limits, by the circumstances of climate, which, in many cases, corresponds in its boundary to the political and social divisions. Moisture and dryness as distinctly mark their traces, both physically and mentally, on each people, as they do on the productions of the earth, amongst the vegetable kingdom. Contrast, for instance, the phlegmatic Dutchman with the volatile and vivacious Frenchman; or the plodding and steady-going Briton, even with himself, when transplanted into the drier climate of the Canadas and the United States. The latter country, although peopled by immigrants from every civilised nation in Europe, has, during the last seventy years, produced a variety of humanity, as distinctly marked in physical, mental, and social peculiarities, and, as contrasted with the original stock, as are the changes which have ever been produced by the acclimatisation of any plant. Indeed, it will come within the experience of any thoughtful person, that removal from a moist to a dry climate, produces, in the course of years, a great effect on any individual, which is typical of the change of species-characteristics, to which we have just called attention. Generally speaking, the perception becomes keener; the interval between volition and action shorter; and hence, boldness of enterprise, whether political or commercial, arises as a natural result; the adage, "*Cœlum, non animan mutant,*" &c., of the Latin poet, might have held good when only the narrowest sea had to be crossed, but loses its force in our time, when oceans are less thought of than were lakes in former days.

The Chinese empire presents as marked a peculiarity in human history, as the paradoxical Australian animal[*]—to which reference has already been made—does in natural history; the specific centre, in either case, being as decidedly marked. Secluded, until within recent years, almost completely from the rest of the world by a national policy, the Chinese have preserved for ages the individual peculiarities which denote their physical, mental, and moral condition. Their language—oral and written—habits, customs, dress, religion, &c., all seem *sui-generis*, and afford a most interesting study to the ethnologist. The greater portion of their country is comprised between the parallels of 20° and 40° of north latitude, and hence they are nearer the equator by several degrees than is any portion of Europe. But, as we have pointed out at a previous page,[†] the climate does not greatly differ from that of the south of France, Spain, Portugal, &c., owing to causes already explained; and, coincident with that fact, we notice that, in many respects, the productions of the earth resemble such as are found in the countries just named. The Chinese territories, therefore, although situated, geographically, in the subtropical zone, really belong, climatically, to that of the warm and cool-temperate; and it is remarkable how many of the mental characteristics and acquirements of the people correspond to those possessed by nations similarly situated in Europe. Their ingenuity is surprising; the arts and sciences have made great progress; and, in many respects, they excel Europeans. All the appliances of agriculture and commerce are of a very advanced character; and in works both of utility and ornament, they are frequently copied or imitated by western rivals. The Japanese, whom we include with the Chinese, have shown astonishing skill in imitating articles of European manufacture; and some optical instruments, as telescopes, &c., which they have produced, have astonished us by their excellency of construction. Although but a few years have elapsed since a steam-vessel was first seen by them, they have been able to produce engines, boilers, &c., so complete, as to run vessels of their own construction on their waters. They contrast wonderfully with the nomadic tribes in Central Asia, who, within the same parallels of latitude, but living at a great elevation, and in a severe climate,[*] are still in a semi-savage state, devoid of all the arts, and chiefly exist to gratify the instincts of animal nature.

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[*] See remarks on the ornithorhynchus, &c., at page 1201.

[†] See *ante*, p. 1195.

With certain modifications, these remarks might be extended to Hindostan, Persia, Mexico, Brazil, &c.; but we shall refrain from further illustrations of the kind, referring our readers to the Section on Ethnology for fuller details.

Africa, owing to the intense heat, and the unhealthiness of its climate, has been, with but trifling exceptions, little explored. Around its coasts various settlements have been established by many nations, since the fall of the Egyptian and Carthaginian empires, up to the present day; but every glimpse into its interior has revealed human nature in its darkest forms, both literally and metaphorically. The Africans of the interior are completely isolated, both by the climate and difficulties of access; and seem to be split up into so many varieties, that scarcely anything but the physical characteristic has been arrived at, respecting them, with any degree of certainty. The negroes are distinguished from every other variety of the human race by the darkness of the skin, the woolliness or cotton-like character of the hair, protruding and thick lips, a yellow sclerotica of the eye,[†] &c. They contrast greatly with the more intelligent tribes inhabiting Nubia and Abyssinia, of which we shall presently speak. Dr. Latham remarks—"*A negro is an intertropical African, in a humid, alluvian locality.* . . . . In respect to descent, the negro of Sennaar has his closest relations, in the way of language, manners, and blood, with the African of Nubia, Abyssinia, and the parts about his own country. Not so, however, his physical conformation. This is with the Africans of Senegambia and Guinea—a fact brought about by the common conditions of *heat, moisture, and a low sea-level.*" This quotation of the opinion of one of our most intelligent ethnologists, gives a strong confirmation to the statements we have already advanced respecting the influence of climate on the characteristics of the varieties of our race. The same author observes, respecting the Hottentot—"The Hottentot stock has a better claim to be considered as forming a second species of the *genus homo* than any other section of mankind. . . . . Stature low, with slight limbs; colour, more brown or yellow than black (that of new-born children said to be nearly white); cheek-bones prominent; nasal profile depressed; hair in tuft, rather than equally distributed over the head." In contrast with these we may describe the Nubians, who, although dwelling between latitudes 13° and 24° north, are yet situated in a climate strikingly different from that of the negro and Hottentot country—that of the Nubians being from 3,000 *to* 4,000 *feet of elevation above the sea-level.* Rüppell thus speaks of them:—"A long oval countenance; beautifully curved nose, somewhat rounded towards the top; proportionally thick lips, but not protruding excessively; a retreating chin; scanty beard; lively eyes; strongly frizzled, *but never woolly*, hair; remarkably beautiful figures, generally of middle size; and a bronze colour." Of the inhabitants in the neighbourhood of Lake Tshad, it may be remarked, that they are perhaps the lowest specimens of humanity, and the most complete savages, in every respect, that the world can afford.

We have described three varieties of Africans, sufficiently to show how much human characteristics may vary with the circumstances of latitude, elevation, temperature, moisture, &c.; and, therefore, how much physical, mental, and moral development are dependent on the same. It is no part of our business to enter into the delicate question, of how far the degraded portion of our race may be elevated by removal into more genial climes, and by association with ameliorated conditions? But the successful instances of the kind are too numerous to require specifying. The recently established colony of Liberia has given wonderful testimony to the power of such influences in Africa itself; and our own and other countries will afford thousands of examples in which the neglected sons of Africa have shown a mental and moral aptitude, which places them in no way inferior to the rest of mankind.

In North and South America, situated between 70° of north and 55° of south latitude, great variety subsists. The Esquimaux we have already described;[‡] and, between their country and Mexico, numerous tribes of Indians roam, which are now rapidly becoming extinct. In Mexico, between 15° and 30° of north latitude—yet, generally speaking, at about 6,000 *or* 8,000 *feet of elevation above the sea-level*, and therefore, although situated, geographically, in the tropics, climatically nearly in the temperate zone—civilisation formerly existed to an astonishing extent. The pyramids, temples, grottoes, aqueducts, bridges, and other ruins, attest the skill of the former inhabitants, not only in architecture, but also in sculpture, picture-writing, &c. Passing to the Carib islands, we meet with the analogues of the African; whilst in Patagonia, between 39° and 55° of south latitude, we notice a fine, tall, robust, active, and warlike race, terminating, in a cool climate, the last vestige of humanity to the south of the equator.

In this rapid, and, therefore, exceedingly incomplete notice of some of the leading varieties of the human species, we have been desirous of directing attention to the *general influence* which latitude or elevation,[§] temperature, moisture, &c., exercise on physical, mental, and moral development: and that the student may more distinctly understand the tendency of our observations, we recommend a re-perusal of the description of the climatic characteristics, given, for each country, at page 1195, and the pages succeeding it. By reading the two articles, in their mutual connection, he will be able to comprehend the relationship and importance of the subjects. We must, however, guard our readers against accounting, *solely* on climatic grounds, for the great differences which subsist between the physical, intellectual, and moral condition of varieties of the human race. Such would be pushing the argument too far, and would prove in itself its own refutation. The object we have held in view, has been to show the important and ever active *accessory influence* of climate, and not to place it beyond other and far more powerful causes—to which we cannot here even allude, as being beyond the scope of our present enquiry. It would be ridiculous to argue, because any tribe is placed in a climate either excessively hot or cold, tropic or arctic, high or low in respect to the sea-level, moist or dry, &c., that it must necessarily be distinguished by depressed or elevated qualities in body and mind. The influencing tendency is alone to be regarded, and allowed for; and comparison is only to be made between two tribes similar in every respect, with the exception of climate, if we would gain an absolute knowledge of its effects. No condition of this kind can be found existing on the face of our globe: our enquiries and conclusions are, therefore, solely of a generally relative character, and afford an aid for arriving at a correct judgment, rather than being pronunciatory of the judgment itself. If possible, extensive tracts of land should be chosen, because islands frequently, if not invariably, are subject to many causes producing great uncertainty of climate. A bold contrast is afforded, for instance, by the great Sahara in Africa, and the plains of North America; some portions of Thibet, in Asia, and the north-eastern parts of South America; the interior of Africa and Australia generally, and their coasts, &c.; in each of which distinctions of all kinds are so decidedly marked.

METEOROLOGY IN ITS CONNECTION WITH ENDEMIC AND EPIDEMIC DISEASES.—That certain peculiarities or changes of climate are connected with endemic and epidemic diseases, is a matter of no doubt whatever; instances of which, in the yellow fever of the West Indies, the direful cholera of India, the plague of the East, &c., are familiar to all; but the chain of causation is by no means satisfactorily understood. If, in each case, a series of identical and progressive changes in the atmosphere of an infected place could be determined as invariably preceding the outbreak of the disease, the difficulties of this interesting subject would be lessened, if not entirely removed. But no such direct connection has yet been established; which may be partly owing to the deficient method of observation adopted until within late years, and also to the great obscurity of the causes which always precede, but often escape our notice, despite of every endeavour to discover them. These difficulties have already been named by Dr. Scoffern at a previous page;* and we only re-enter on the subject for the purpose of pointing out, more definitely, the facts of the case, which have hitherto been but incidentally mentioned, and to supplement, in some particulars, the remarks which have been offered.

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[*] See remarks on Thibet, &c., at page 1198, *ante*.

[†] See *ante*, p. 48, Section on *Light*.

[‡] See *ante*, p. 1202.

[§] See remarks respecting these being convertible terms, at page 1199.

Wherever an endemic or epidemic prevails, it will be generally found that the temperature and moisture of the district have much to do with it, and that they are either in excess or deficiency, according to the nature of the complaint. Such diseases are generally of two kinds; namely, those attacking either the organs of respiration or digestion. Amongst the former may be instanced influenza, bronchitis, and diseases of the lungs in general; and, of the latter kind, are diarrhœa, cholera, &c.: another class, that of the purely febrile type, has specific characters of its own.

In a hot and moist atmosphere, the entire human system generally suffers from debility; but a dry air, if either very hot or cold, and not long continued in, produces, comparatively speaking, but little inconvenience. Thus it is no uncommon occurrence for the temperature in Canada to fall to 20° or 30°, and, in the Arctic regions, to 60° or 70° *below zero*, where, so long as the air is still, no harm ensues; on the other hand, a temperature of 200° and 300°, with *dry* air, is quite harmless to the human body exposed to it. In India, Africa, and the West Indies, where, however, *great moisture* generally, but not always, accompanies a high temperature, the utmost danger exists to Europeans; the liver, organs of digestion, &c., becoming affected; and hence a predisposition to diseases of all kinds is generated; the body, in fact, enters, in respect to nearly all its organs and functions, into abnormal conditions, and disease is, generally speaking, the universal consequent.

In our own country, the sudden changes of temperature, with a comparatively moist atmosphere, are frequently productive of serious consequences, although those changes are slight in comparison with such as constantly take place, even within a few hours, in the Canadas. But with us, as in some parts of India and other countries, the conducting power of the moisture is the cause of injury; hence a hot summer in England is more intolerable to a West Indian, and a cold winter to a Russian, than such seasons are in their own countries; because there, on the average, the air is much drier, and therefore conducts heat, both to and from the body, much more slowly. The most healthy climates are, generally speaking—*all other things being equal*—those that are the driest: hence, in some parts of Chili, on the plains in the Andes, where the air is very dry and bracing, an age of about 100 years is by no means uncommon in individuals; and even that is very often greatly exceeded.

In numerous cases, therefore, we may partly account for the prevalence of disease in districts, or at certain seasons, by a rapid change in the conditions of temperature and moisture. In some parts of the tropical and sub-tropical zones, the duration of the rainy and hot seasons, and their changes, afford just such circumstances as are likely to produce disease in the human subject; for the skin, ever casting off, by its millions of pores, the watery humours and effete matter from the system, is often suddenly checked, or unduly stimulated in its action; and hence febrile symptoms, and inflammation of the organs of respiration and digestion, are immediately induced, and dysentery, in its worst forms, becomes prevalent and fatal.

In the remarks which we have already made on the meteorological influence of drainage, at page 1186, we have pointed out some of the evils arising from a moist climate, either when hot or cold, and also from local conditions of the same kind: several countries are also named in which these conditions are always present. In such places, the increase or check of perspiration so commonly occurring, is quite sufficient to account for the various diseases endemic in them. But, in many places, there can be no doubt that the outbreak of either an endemic or epidemic, may be due, directly, to the presence in the air of poisonous exhalations, which have an immediate action of their own on the human system, or may act conjointly with changes of temperature and moisture. The visitations of cholera in this country, during the last thirty years, have been highly instructive to the scientific observer, in respect to such causes; and perhaps, incidentally, have done more to improve the *general health* of the nation, than would have resulted had not their fatal effect stimulated us to adopt universal sanitary measures.

A careful observation of meteorological conditions which prevail preceding to, and during an outbreak of cholera, generally records a comparatively high temperature, and a moist and stagnant atmosphere. Diseases of the diarrhœa kind are rarely, if ever, prevalent during periods of high wind; and, on the other hand, they are *always* most virulent and fatal in places where a constant change of the atmosphere of the locality does not take place. During the last visitation of cholera in London, Lambeth and Southwark—a great portion of which districts is below the level of the Thames at high water, and constantly very damp—suffered most seriously and fatally from the disease; and, generally speaking, those districts which, like the above, were badly drained, and not exposed to the natural ventilation produced by free access of natural aërial currents, were plainly marked as infected and dangerous localities.

Another and important quality of moist air, to which but little attention has been directed, is the power of aqueous vapour, in a suspended state, of holding in solution gases, &c., and of thus carrying with it exhalations of all kinds, in a form ready for assimilation with the human system. Dry air may be mixed with other gases, which, in most cases, will readily diffuse themselves through it; but moist air, by reason of the water it contains, becomes a much more effective poison-carrier. Hence arises the peculiar smell of marsh lands in autumn; of water in which "greens" have been boiled, &c.: and, in each of these instances, the gases evolved during the chemical decomposition of the vegetable substances, are carried off in a volatile form, and are thus frequently detected by the sense of smell. With respect to the nature of these volatile poisons thus evolved, little or nothing is known.—We have already referred to this subject in the Section on Chemistry;† stating that it has hitherto been impossible to detect the active poison thus floating in the air, with the exception of the sulphuretted hydrogen, to which some have attributed, together with a carburetted hydrogen, the chief influence of miasmatic effluvia. The great attention which has lately been paid to detecting the presence of free ozone in the air,‡ may eventually be rewarded by a further insight into this interesting but obscure subject; at least, it is in that direction alone that any hope for a solution of our difficulties can be at present cherished.

In tropical countries, where a hot-moist air always prevails, the climate at the mouths of rivers, which generally enter the sea by a "delta" consisting of low-lying marsh lands, is almost invariably unhealthy, and sometimes uninhabitable. The periodical overflow of the rivers during the rainy season, inundates the land, often over a very extensive area. Vegetation progresses rapidly when the stream subsides; and, eventually, the decomposition of the plants produces malaria of an exceedingly fatal character. The atmosphere is sometimes poisoned even far into the interior, by the sea-breeze carrying the miasmatic air; and thus the baleful influence is largely spread over the adjacent country.* The mouths of the Mississippi, Amazon, Ganges, &c., afford instances of this kind.

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* See *ante*, p. 1154.

† See *ante*, pp. 428, 429.

‡ See *ante*, pp. 1187, 1188.

So far, we have suggested some of the causes which induce certain endemic and epidemic diseases, that chiefly attack the liver, stomach, &c.; but in our own country we have a constant and fearful source of disease and death, arising from affections of the lungs, including consumption, and complaints of a phthisical character in general. A very considerable proportion of the mortality (about one-sixth) arises from this cause; and after making due allowance for unhealthy manufactures, sedentary occupations, &c., still much is left, as the cause of that class of disease, to our variable, cold, and damp climate. There is, no doubt, much to be said for the prevalence of phthisis amongst us on the ground of hereditary tendency; but this frequently disappears in the course of successive descendants; whilst the mortality, in general, is not lessened, but rather has an annual increase.

We shall learn much respecting the influence of meteorological conditions on this disease, by comparing the indications afforded by different localities and seasons. In Holland, for example—which, of all other European countries, is the most damp; where a continual exhalation of moisture rises from the land, on which an almost constant fog settles at many periods of the year; a country, in fact, which is nothing better than an immense reclaimed sea-marsh—we find that phthisis is alarmingly common. Not only does a moist climate thus predispose to the disease, but a low elevation is equally as dangerous—perhaps on account of want of drainage, and consequent dampness. All these conditions meet in Holland, and their results are accordingly fatal to an extraordinary extent.

But, turning to drier climes, we find the opposite conditions to obtain. In Northern Russia, where the range of temperature is excessive, but the air exceedingly dry, consumptive diseases are very rare. Even within the tropics, where the influence of excessive heat induces debility—a constant source of the disease—yet the dryness of the air in some places prevents its occurrence. But we find, that if the inhabitants of those countries, or even the animals common in them, are removed to our climate, the diseases most commonly fatal to both are those of the phthisical type. Elevation above the sea-level, and, consequently, above a stagnant and moist atmosphere, is equally preventive of the disease; and to patients in the early, or even late stages, the removal to a dry and equable climate is the only chance left for possible restoration to health.

Having noticed the localities which most favour phthisis, we may remark on the influence of the seasons of the year. Winter and spring are generally the most predisposing and fatal; and in this fact we introduce another element, besides that of moisture—namely, the falling temperature of the atmosphere. As autumn approaches, the suspended moisture of the air, held invisible during the heat of summer, becomes condensed by the loss of caloric which the earth sustains, or rather, perhaps, by the failing supply, owing to the gradually increasing obliquity of the rays of the sun.† The atmosphere then holds the water in the form of minute vesicles, which, as we have seen, may have gases in solution, and prove a source of irritation to the organs of respiration. The increased sensible moisture rapidly conducts away the normal heat of the body; often checks perspiration; produces catarrhal affections, or constant irritation of the mucous membrane; debilitates the system; and thus gradually induces those conditions in which tuberculous or scrofulous affections are sure to make rapid progress. Sudden changes of temperature, together with the dampness of the air, ensue; and thus the disease, already planted, is fostered and matured. Possibly, as we have already hinted, the prevalence of free ozone, with a northerly or easterly wind, also accelerates the irritation of the bronchial tubes.‡ The effects of the advancing winter season are similarly developed, and constantly-recurring atmospheric changes prevail, until the cold winds of early spring complete the work of destruction. Such is an outline of the various causes which produce and accelerate a disease that carries off about one person in every five or six of our population; and of which, perhaps, scarcely a household in our land has not had painful experience.

Our remarks have hitherto been directed to general climatic causes, as inducive of phthisical complaints; and we can here only mention, that impure air, formed in close apartments, factories, workshops, &c., especially amongst millinery and such trades, is another and fatal cause, for reasons similar to those already assigned. The organs of respiration are kept, under such circumstances, for hours breathing air of a high temperature, and loaded with moisture, and various impurities. The individual, on passing into the open air, instantly encounters a lower temperature, varying with the changes of the season, &c.; and, gradually, this chain of influences acts precisely in the manner already described, and similar dangerous and fatal results follow.

Our space, and the limits of our subject, forbid us discussing the effects of climate in producing diseases of a scrofulous character, which are nearly allied to, if not identical with those we have been mentioning. Such are often the direct consequences of exposure to cold, moisture, &c.: and, in every case, a predisposition to those affections is almost certain to become developed in climates of a moist and variable nature; hence their prevalence in this country.

Having glanced at those diseases of the organs of respiration and digestion which result from climatic influences, we may, lastly, make a few observations on some of the most fatal of all endemics or epidemics—namely, fevers of the typhoid class. It is familiarly known of these, that they are most common in places where impure air and want of cleanliness are found; poverty, filth, intemperance, and vice, being their hotbeds. Yet climate has a most marked influence on these diseases, both with regard to the countries and seasons of their appearance. Typhus is completely unknown in the tropical regions, where a profuse and continued perspiration quickly carries off the effete matters of the body; and it is invariably prevalent in countries which have a cold and moist climate—becoming, in fact, an endemic in them. This may arise in a twofold manner; namely, in part, from the nature of the climate, by its sudden changes checking the perspiration, and inducing internal inflammatory symptoms, with deterioration of the blood; and also because the moisture of the atmosphere always assists in the spread of a disease, for reasons already given. But another and universal cause is found in the depressing influences on the animal system that such climates exercise; and typhus, of all others, is that which is most attendant on a debilitated state of the functions. Again, in moist weather, the skin often has a greater absorbing power; and hence a re-entry of excreted humours into the blood, or of infective and contagious matter, is more likely to take place. It is also noticed, that diseases of the typhoid class are always more prevalent during cold and rainy weather, than in that of a dry and bracing nature, in any locality, whether it be endemically or epidemically affected.

The report on the health of the navy, for 1860, contains some valuable statistics, showing the influence which climate has on the outbreak and prevalence of disease: and the following summary of the accounts from the East Indian, China, and Australian stations, affords ample verification of the statements we have advanced in the preceding pages.

The average ratio of sickness differed very considerably in the different vessels of the squadron: but the variety of employment to which they were subjected, prevents a fair comparison between them. The crews of those vessels most at sea, were much healthier than those employed in the rivers or harbours, or on the coast, for the obvious reason that they were not exposed, in the same degree, to the various diseases which are always produced by free communication with the shore. The total force was 8,540 men, of whom 615 were invalided; and the number of deaths was 244; making, together, a total loss to the service of about 10 per cent. Of the 244 deaths, no less than 116 were caused by dysentery*—the intractable disease of the Chinese coast. Experience has shown, that fever and dysentery follow the seasons in the order of their origin and decline; that they almost entirely disappear during the winter months; reappear in spring; and increase rapidly, in virulence and fatality, throughout the warm months of summer and autumn, until the cold of the approaching winter again puts them out of existence.

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* See *ante*, p. 1186. † See *ante*, p. 1191. ‡ See *ante*, p. 1187.

The returns show, that *dysenteric attacks were twice as numerous during the warm, as they were during the cold season;* and it appears that the greater portion of them were consequent upon exposure, in boats and on the shore, during August and September, when the force was employed in the Gulf of Pechili, but more especially when off the estuary of the Pehtang and Peiho rivers, or in about latitude 39° N. The experience of the last few years has shown, that the causes which give rise to fever and dysentery on the main-land of China, south of 32° latitude, are either altogether inoperative on the coast of Japan, or exist in a far less potent form. This suggests the obvious necessity of running northward, out of the dysenteric latitudes, immediately on the appearance of this fatal disease amongst the men. This, however, could not be done, owing to the military and naval operations which, in 1860, confined the squadron to the above-named waters.

The health of the Australian squadron contrasts most favourably with the above. The total number of the force was about 940 men; and of these, 22 were invalided, and 13 died; making a total loss of less than 4 per cent., or under one-half of that occurring on the East India and China station. The loss to the service, by invaliding, was greatest on the last-named station, where their ratio rose to 72 per 1,000 men; and least on the Australian, where it amounted to 24 per 1,000 men. The deaths by diseases were greatest on the North American and West India station, where their ratio was 38·7 per 1,000 men; and least on the Pacific station, where it amounted only to 3·7 per 1,000 men. The high number of deaths on the first-named station is attributable to the outbreak of yellow fever.

Yellow fever and dysentery were the chief instruments of death, and dysentery and pulmonary complaints, of invaliding. They are all preventible, and essentially climatic; but, generally speaking, pulmonary complaints were found to increase directly as attention to proper ventilation was neglected; for, in the *St. Jean d'Acre*, where there was no ventilation, there were 285 of such cases, out of a crew of 815; whilst in the *Marlborough*, under circumstances otherwise identical, out of an average complement of 1,145 men, and where particular attention was paid to the ventilation of the decks where the crew slept, there were only seven cases.

The general result gathered from the report is, that even in the case of yellow fever, dysentery, and other fatal diseases, proper attention and care, if exercised duly, will, on the most unhealthy stations—as on the west coast of Africa—secure a comparative immunity from disease; whilst on a healthy station, like the Mediterranean, their neglect is sure to be followed by invaliding, and numerous fatal cases. We thus see, that despite the malevolent influences of climate, still we have left in our hands a means of prevention in nearly every case.

It is no part of our province to point out the different characteristics of the diseases already referred to, or to descend into particulars respecting the symptoms attending the various forms or stages of them. It has been solely our object to give a general view of the connection which subsists between them and the meteorological conditions of our atmosphere; and in doing so, we have confined ourselves to matters familiarly known to most of our readers. We might have instanced a vast variety of affections to which the human subject is liable, arising from sudden changes of temperature, or its excessive range; but we should have trenched considerably on the domains of medical science, and perhaps lessened the force of the arguments and proofs already adduced. We may add generally, however, that almost every disease of the organs of respiration and digestion may be identified with climatic considerations, directly or remotely; and, therefore, the study of meteorology, in its pathological aspects, is one of the highest importance, and deserving of the deepest attention from the general and professional observer.

Whilst science points out, and explains, the causes, progress, &c., of such diseases, it is also its office to suggest such remedies as may tend to prevent them; but, as we have remarked in reference to our knowledge of the causes, so we must own, in respect to its application, that both are limited, owing to the paucity and obscurity of the facts which we possess. It is only of late years that the science of meteorology has been brought to bear on sanitary questions, and hence a proper estimate of the extent of their connection has yet to be made. In previous pages† we have mentioned the influence which a thorough system of drainage has in improving the climatic and sanitary conditions of low-lying and marshy places. In most of our large cities and towns, great attention is now paid to local sanitary matters; surface-water, which is so fertile a source of disease, is now quickly removed; drains are applied to each house; sewage-matter is carried to a distance from its places of production; and various other details, which we need not enumerate, are effected—all tending to render more pure, dry, and therefore healthy, the atmosphere of even the worst localities. By such means an immense amount of good has been effected, the mortality has been decreased, and the public health greatly benefited; although we have not yet attained, by a great length, to the maximum of improvement placed within our power by engineering and chemical science.

Despite every advance, however, there must still be certain localities which, on account of the peculiarities of their position, will always remain far below the standard at which others have arrived. We need but instance Hull, Liverpool, Bristol, &c., which, from their low elevation above, and proximity to, the sea, must necessarily have a moist and variable climate. In each case the drainage mixes with the sea-water running close to them; and as the sewage-matter comes in contact with the saline particles of the ocean, decomposition takes place, and the evolution of poisonous gases is the result. It is to be hoped, however, that the great drainage scheme, inaugurated by the metropolis, will gradually spread in the provinces, and that the refuse from private houses, factories, &c., will in time be everywhere removed far from its sources—a result than which nothing can be more essential for the health of the inhabitants. Then the moist superincumbent atmosphere will at least have less addition to its harmful qualities, arising from the suspension and solution of gases—to which we have frequently alluded, and which constant experience has proved to be so active a cause of disease; and the average moisture will also be itself diminished in many cases.

Our possessions in India have ever been a source of enormous expense, in connection with the civil and military departments, owing to the excessive mortality which constantly occurs. Of late years, the extraordinary fatality which has happened there, even to the highest in power—whose intelligence, means, and position would lead them to the use of every art to prolong life—shows, that however much other causes may operate prejudicially to the health of Europeans, still to the climatic peculiarities its existence must be chiefly due. The subject has received the earnest attention of many scientific men; and experience has shown, that the only remedy, or rather relief, which can be obtained at present, is that of frequent removal up to the elevated plains in the interior—

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* See *ante*, p. 1205.

† See *ante*, pp. 1183 and 1186.

as at Simla. This station is about 8,000 feet above the sea-level—a height that, according to the laws we explained at a previous page,* should afford a climate of the temperate character. It has now become a place of general resort for most Europeans resident in India; and its cool bracing air soon restores the sufferer, if he be not already too far debilitated. Referring to the following table of mean temperatures, &c., we find that the annual mean of Calcutta, Bombay, and Madras, is nearly 83°; whilst, at an elevation of 7,000 feet, it does not exceed 57°; hence, by simply ascending to a comparatively moderate elevation, the cool climate is readily reached. There are numerous other stations of this kind, in various parts of the peninsula, equally adapted for the alleviation of the effects of a climate almost intolerable, at times, to all but the natives.

A very ingenious method of purifying the air of any place—especially calculated for India, for the hospitals of which it was first proposed by Dr. Forbes Watson—has been already described in the Section on Chemistry.† Charcoal broken into pieces of the size of a pea, or a little larger, is distributed on trays enclosed in an air-tight case. The whole supply of air required in the establishment, is driven or drawn through the charcoal by suitable mechanical power. The moisture of the air is removed by caustic lime, which is substituted for this purpose, in another case, for charcoal; but the two arrangements can be employed, side by side, in combined operation. This method we have repeatedly and continuedly tested, and have found it exceedingly effective, the air being both purified and dried to any extent that may be desired. Arrangements of this kind would doubtless be of essential service in every tropical climate, where excessive moisture and miasmatic exhalations are usually so prevalent. We refer our readers to the extracts, from the descriptive pamphlet published by the inventors (given at the page just named) for a description of the *modus operandi*, and the benefits derived from this method. Of course such plans are limited in their operation, and can only be used in buildings, &c.: but in a city like Calcutta—which, from its situation amongst lagunes and pools of standing water, can scarcely be made more unhealthy than it is—a contrivance of this kind, or a similar adaptation of charcoal and lime, so applied, cannot but prove of great benefit as a preventive of disease generally.

Although the connection of meteorology with questions of health and sanitary measures, is one of great interest and importance, we cannot, for want of space, pursue it further. The various matters urged already for the consideration of our readers, have been selected with a view of calling attention to palpable instances; and others, of less evident character, but of almost equal consequence, we are compelled to leave untouched. It would be almost impossible, in fact, to state the limits of the subject, seeing, as we have, that so much of the physical, intellectual, and even moral character of a community, is so intimately connected with the condition of the atmosphere, and meteorological phenomena in general. It must be a matter of astonishment to many reflecting persons, that so common and well-known objects as air and moisture, should have such an astounding influence on our individual and social welfare; but to one thoroughly acquainted with the ordinary laws of nature, that cause of surprise at once loses its singularity; for it is but one out of thousands of instances in which a comparatively simple agent is productive of numerous and all-important results.

CONCLUDING REMARKS.—Perhaps it may not be out of place to make a few remarks in reference to the desirability of extending meteorological observations. Our readers will have gathered from the earlier part of the section, by Dr. Scoffern, a general knowledge of the science, immediately followed by practical instructions from Mr. Lowe, so well known by his ability as a scientific observer. In this chapter, it has been our endeavour to supplement the teachings of those gentlemen, by an account of all the new discoveries in the science, together with results, which a somewhat lengthened personal experience has enabled us to gather; so that, taken together, the reader will obtain full instructions, and an adequate knowledge of the science as it exists at the present day. We have enlarged on the popular branch of the subject, in the hope of inducing many of our readers to commence making observations; and have shown how much each individual has a personal interest in it, both in respect to his own health and comfort, and of the welfare of the community at large. Towards the close of the year 1863, perhaps the most terrific storm which has visited these islands during this century, spread its ravages both on land and water, and many valuable lives were sacrificed. It was one of the peculiar cyclone class,‡ in which no skill of our seamen seemed to be of avail: but, disastrous as have been its effects, they are trifling to what would have occurred, had not an almost universal deference been paid at our ports to the warnings given by Admiral Fitzroy, some time preceding its commencement; and by which many fleets of vessels were consequently detained in our harbours. Now, so valuable a result is simply the consequence of careful deductions, from an extended series of *individual* observations of meteorological phenomena, carried on, as yet, for a few years only. The more numerous these become, and the more widely-spread the observers, the nearer shall we arrive at a proper estimate of the changes in the weather, so incessant in our variable climate; because all correct deductions must be drawn, not from any one set of observations, but must depend, rather, on the check against error, which a multiplicity of them alone can ensure.

It hence follows, that all who possess a degree of intelligence and education beyond many of their fellows, have morally imposed on them a duty, which, from its nature, they can share, but never justly resign exclusively to others, provided they have the means and knowledge sufficient for its performance. The amount of knowledge required has been pointed out in the various parts of this section; and, from the other portions of this volume, it is believed that such may be easily acquired. In the Section on Heat, for example, the laws of conduction, radiation, absorption, temperature, &c., are fully explained: in that of Light, refraction, &c., may be studied: the nature and causes of natural electric phenomena may be examined in the Section on Electricity: of magnetic disturbances, in that on Magnetism: and the conduction, &c., of sound, in the Section on Acoustics. In the Section on Chemistry, the chemical properties of air and water, ozone, the various gases found in the atmosphere, derived from all sources, &c., are discussed: and the chapter on the sanitary applications of the science, will be found to have a connection with many of the subjects treated of in the present section. In the Mathematical Section, decimal notation, &c., may be studied, as required in setting down meteorological facts. In the Mechanical Sections, Statics and Pneumatics will be found to have an intimate connection with the science of meteorology: and in the Section on Astronomy, some facts as to the places of the constellations will be highly useful, in watching and noting the direction of meteoric bodies, the Aurora Borealis, &c.

Thus the intelligent reader may readily gain sufficient information from this volume, to enable him to become fully competent to undertake a series of meteorological observations, in all their different branches. The instruments he will at first require, are the barometer, thermometer, and hygrometer; others of minor importance can be afterwards added. Instructions for the purchase and management of, and for making observations by them, are plentifully supplied in this section, together with hints as to gathering various collateral indications, afforded constantly around us. The best form of registration for the barometer and thermometer, is that described and illustrated at page 1189; but a ruled book should also be kept, in which the figures obtained at each observation are to be duly entered, with any other particulars which may deserve remembrance. Various minutiæ are fully explained by Mr. Lowe, in the fourth chapter of the section, which will serve as a guide in all departments of practical meteorology; so that, altogether, even the most inexperienced need no further help than is thus afforded.

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* See *ante*, p. 1199.

† See *ante*, p. 437.

‡ See *ante*, p. 1175.

After such observations have been taken with due care, and properly registered, copies should be sent either to the Meteorological Society of London, or to the Meteorological Department of the Board of Trade.

The following pages afford tables of average rainfall, mean temperature, &c., to which reference has been made in this chapter. We have also appended, at foot, an account of a winter ascent, made by Mr. Glaisher on January 12, 1864; the results of which our readers may compare with others already described at page 1180, *et seq.*

[To Mr. Glaisher, for whose labours science is indebted for the reduction and generalisation of the observations, recorded and published by the Meteorological Society (of which he is the esteemed Honorary Secretary) since its foundation in 1850; and to Admiral Fitzroy, at the head of the Meteorological Department of the Board of Trade, we are under a debt of the greatest obligation, on account of the kind assistance they have afforded us by supplying the fullest particulars in their possession. Of these we have made great use in the preceding pages of this chapter; and have much pleasure in thus acknowledging the sources of our information. —EDITOR.]

MR. GLAISHER'S WINTER ASCENT, ON JANUARY 12, 1864.—The balloon left the Royal Arsenal, Woolwich, at seven minutes after two, with nearly a south wind, and passed over the river Thames; but, when at the height of 400 feet, the wind changed to a point with a great deal of west in it, and we found ourselves passing down the centre of the river. At 2h. 9m., at the height of 550 feet, we fell into a strong S.W. current; we crossed Tilbury Railway at 2h. 14m.; Hainault Forest was west at 2h. 21m., and our course was N.E., or almost directly towards the Blackwater River. Under these circumstances, we did not dare to rise as we intended. At 2h. 28m. we were 3,700 feet high; the direction of the wind changing to S. nearly. At 2h. 43m. we saw a large town east of us, which we thought was Chelmsford, and inferred our course to be nearly N. At 2h. 54m. we saw Maldon, the Blackwater River, and the sea-coast, which we estimated at a distance of fifteen to twenty miles east; we were then about 7,500 feet high. The wind continuing S. nearly, we determined to go above the clouds. At 3h. 8m. we were 8,500 feet high, and the wind veered to S.S.W. At 3h. 16m. we were 9,500 feet high, and the wind changed to S.S.E., and continued in this direction, we believed, to our highest point, which was 13,000 feet at 3h. 31m. There the earth was quite obscured, and we had nothing whatever to guide us as to our rate of motion, or of our locality. As the sun would set in little more than half-an-hour, we gave up the hope of reaching five miles, resolving to pass downwards through the clouds, and ascertain our position. At 3h. 37m. we were 10,500 feet high; in five minutes we descended to 8,500 feet, and beneath the clouds, being still 6,500 feet from the earth, which looked barren; and, as there was no place that any of us knew, we could not determine our locality at all. At four o'clock we saw a forest of pines. We still descended slowly to 1,250 feet, by 4h. 7m. We saw a village; and after passing this the earth looked barren—there were no villages, and scarcely any habitations. The mist was so thick we could not see two miles ahead, but we came to the ground at 4h. 15m., near Brandon.

On the ground the temperature was 42°; decreased to 38½° by 2h. 10m.; but it increased to 45° by 2h. 25m., and the sensation was then one of great warmth: at 2h. 28m. it decreased to 41½°; was 38° at 2h. 30m., and 31° at 2h. 35m. at 6,200 feet high; it was then nearly constant about 30° till 3h., when our height was 7,500 feet; it then decreased to 20° by 3h. 16m. at the height of 9,500 feet, and continued to decrease, and was 11° at 3h. 31m., at the height of 13,000 feet; it increased to 16° in six minutes on descending to 10,000 feet, and constantly increased with decrease of elevation to 40° at the height of 1,000 feet, and to 42° on the ground. Although it was probable that warm currents of air might be met with, it was quite unexpected to encounter such within 1,000 feet of the earth, and extending to nearly 4,000 feet high; the thickness of this warm S.W. wind being fully 3,000 feet. At the height of half a mile the temperature was nearly 4° warmer than on the ground; at this height, hitherto, we have always found the temperature from 12° to 16° colder.

Dew was deposited at the temperature of 35° before starting; at 36° between the heights of 1,500 and 3,000 feet; at 26° at 4,000 feet, decreasing to zero at 9,500 feet; and the dew-point was 4°, 5°, and 6° below zero at heights exceeding 10,000 feet: on passing below 10,000 feet the dew-point was the same as the temperature of the air, and therefore the air was saturated with moisture, and continued so till we very nearly reached the earth.

On leaving, the earth was dull, and the air very misty. When afterwards viewed, its appearance was very different from that in summer: all looked barren, bleak, and ice-bound; every sheet of water was covered with ice; and Mr. Coxwell, notwithstanding his 502 ascents, said he never before had seen sheets of water in their frozen aspect with those peculiar white-fringed edges which attracted our special attention. At 2h. 45m., at the height of 7,000 feet, we heard a railway train, and then the tick-tick of a threshing machine; and afterwards, at the height of 7,200 feet, we heard people's voices; and these were the only sounds we caught. At 3h. 29m., on looking down, we saw snow falling upon a fine cloud, the surface of which was unbroken and even, being free from irregularities, and with boundary lines very sharply defined. At this time dense clouds were far above us, and the air seemed to be full of snow. At 3h. 31m. we were about 13,000 feet high; and as we are quite ignorant of all meteorological conditions in the higher atmosphere at this time of the year, it was with great regret that I heard Mr. Coxwell's decision to descend; we were in fine but thick snow; we had lost sight of the earth, with, we believed, no great length of land before us, and even that being towards the sea. Had we ascended another mile, our descent must have been very close to the coast; and considering that the sun was setting, with large sheets of vapour obscuring the view, I felt that Mr. Coxwell's judgment was again wisely exercised, and I abstained from influencing him in any way whatever.

From 3h. 32m. fine granular snow covered my note-book, and continued till 3h. 47m., when we were about entering a cloud in descending, having a thickness of about 600 feet, through which space we passed in two minutes. The balloon must have been laden with snow. The cloud was saturated with moisture, as was the air below the cloud till within 1,000 feet of the earth. At 3h. 39m. Mr Coxwell was darker than usual; Mr. Norris was reddish-bluish; and I was redder than usual, with both eyes much suffused. At 4h. 9m. it became so dark that it was with difficulty I could read the instruments. The sun had set, a dense mist was in all directions, and we could not see far ahead. Mr. Coxwell then began the descent. The place was quite barren, and, to all appearances, uninhabited; the grapnel passed through light sand for some little time, but the descent was managed so admirably that we came down without causing injury to a single instrument.

Ozone papers were somewhat coloured, but those exposed at the Royal Observatory, Greenwich, remained colourless. The anemometer at the Royal Observatory, Greenwich, registered six miles as the horizontal movement of the air during the time the balloon passed more than 100 miles. The sun was never seen, and there was no opportunity of using the actinometer, the spectroscope, or the polariscope.

It will be seen that the above results widely differ from those afforded during ascents in the summer season.

A TABLE

OF THE RAINFALL, MEAN ANNUAL TEMPERATURE, THERMOMETRIC RANGE, Etc., OF VARIOUS COUNTRIES, OCEANS, Etc.

Division, Country, &c.	Inches of Annual Rainfall.	Mean Annual Temperature. Fah.	Temperature. Fah. Extremes = E, E W, and E S. Mean Winter = W. Mean Summer = S. Total Range = R.	Comparative Moisture.	Latitude.	Longitude.
EUROPE.	Inches.	Degrees.	Degrees.		Degrees.	Degrees.
England (General)	31	50 to 53	0 to 85 E	Moderate	50 N to 55¾ N	1¾ E to 5¾ W
Scotland (West)	35 to 45	48 to 49	10 to 75 E	Moist	54½ N to 58¾ N	1¾ W to 6 W
,, (East)	22 to 26	47	0 to 75 E	Dry		
,, (Central)	Variable	48	38 W to 57 S	Variable		
,, The Orkneys	Ditto	46	39¾ W to 54 S	Moist	59 N	3 W
Ireland	36	——	Milder than England	Moist	51½ N to 55½ N	5⅓ W to 10½ W
,, (South)	40	52	Ditto			
,, (Central)	31	50	Ditto			
,, (North)	35	48	Ditto			
France (North)	22	50	0 to 80 or 90 E	Variable	43½ N to 51 N	5 W to 7½ E
,, (South)	23	60 to 62				
,, (Middle)	24 ∠ 40 N.W	58				
Switzerland—Geneva	——	52¾	34 W to 66½ S	Variable	46¼ N	6¼ E
Portugal—Lisbon	Varies greatly	61 to 62	52½ W to 80 S	Moderate	38⅔ N	9⅒ W
Spain	12 to 36	62	14 to 100 E	Various	36 N to 44 N	9 W to 3½ E
,, Madrid	Little	58	43 W to 76 S	Dry	40⅓ N	3⅓ W
,, Cadiz	——	62	52¾ W to 70 S	——	36½ N	6¼ W
,, Gibraltar	25 to 30	67½	50 E W to 78 E S	——	36 N	5⅓ W
Russia—58° N. Latitude	15 to 20	32 to 40	——	Dry	N of 58 N	——
,, In 60° ditto (St. Petersburg)		——	22 to 90 E		60 N	30 E
,, Central parts		40 to 55	——		50 N to 55 N	——
,, Moscow		40	15 W to 64 S		55¾ N	37½ E
,, Astrakan		48	13 W to 70 S		45 N to 50 N	44 E to 50 E
,, Cracow		47	27 W to 66 S		50 N	19½ E
Greece	——	64	28 to 104 E	Various	36⅓ N to 40 N	20¼ E to 26 E
Turkey—Constantinople	Moderate	56⅓	40 W to 71 S	Dry	41 N	29 E
Prussia (General)	20 to 25	52	32 W to 70 S	Various	50 N to 55 N	6 E to 22 E
,, (Berlin)	——	48	31 W to 64 S	——	52½ N	13½ E
Bavaria	20 to 25	47	30 W to 63 S	——	47 N to 50½ N	9 E to 13¾ E
Italy (North)	35	53 to 55	40 W to 70 S	Moderate	43⅓ N to 46 N	8 E to 13 E
,, (Middle)	——	56 to 60	42 W to 72 S		41½ N to 43½ N	12 E to 16 E
,, (South)	——	60 to 63	47 W to 73 S		37 N to 41½ N	14 E to 18 E

Hungary	16	54½	43 W to 66 S	Dry	45 N to 49½ N	16 E to 25 E
Sicily—Catania	——	67½	53 W to 84½ S	Moderate	37 N	15 E
„ Palermo	——	61	52½ W to 74½ S		38 N	13½ E
Sardinia (Island)	——	67¾	34 to 90 R	Moist	39 N to 41½ N	8 E to 10 E
Norway—Christiania	21	41⅓	23 W to 60 ∠ 100 E	Dry	59¾ N	10¾ E
„ Bergen	77	Mild	——	Moist	60⅓ N	5⅓ E
Sweden—Stockholm	16 (Upsala)	42	30 W to 64 S	Dry	59½ N	18 E
Denmark	35	46½	7 to 90 E	Moist	53 N to 57¾ N	7 E to 12 E
Belgium	30 to 40	50	3 to 90 E	Very moist	49½ N to 51½ N	2½ E to 6 E
„ Brussels	——	49·6	38 W to 64 S	Moist	50⅚ N	4½ E
Madeira	20 to 25	64 to 65	53 to 73 E	Moderate	32 N to 33 N	16¼ W to 16¾ W
Azores	——	60 W	45 to 86 E	Medium	37 N to 40 N	25 W to 31 W
Greenland	Snow	27½	— 40 to + 59 E	Dry	N of 60 N	40 W to 50 W
Arctic range of temperature	——	Nearly zero	— 65 to + 50 E	——	81 N to 82 N	——
Iceland—Reikiavik	Variable	40	——	Moist	64 N	23 W
Temperature of the Water of the Arctic Ocean	——	Lat. 60 N = 49 „ 70 N = 40 „ 80 N = 34	——	——	N of 70 N	——
Asia.						
India—Upper Ganges Plain	——	——	32 to 100 E	Tropical	25 N	89 E
„ Calcutta	64	82½	67 to 120 E		22½ N	88⅓ E
„ Bombay	76	82	76 to 87 E		19 N	73 E
„ Madras	——	84	73 to 94 E		13 N	80¼ E
„ Neilgherry Hills	60	57	42 to 70 E		11 N	77 E
„ Cawnpoor	——	——	42 to 70 to 98 ∠ 104 E		27 N	80 E
„ Delhi	10 to 15	78¾	60 W to 88 S		28⅔ N	77⅓ E
„ Mysore, elevation of 7,000 feet	——	56	——	Variable	12 N	76 E
Burmah	150 to 200	80 (?)	75 W to 90 S ∠		16 N to 27 N	93 E to 99 E
Ceylon (General)	——	81 (?)	——		6 N to 9¾ N	80 E to 82 E
„ Kandy (1,467 feet elevation)	85	80	66 to 86 E		7½ N	80⅔ E
„ South-west coast	75 to 80	81	76 to 87 E	Tropical	*cir.* 7 N	*cir.* 80 E
„ Colombo	Ditto	80·7	79 to 81 E		7 N	80 E
„ Trincomalee	——	82 s	75 to 92 E		8½ N	81¼ E
„ Hills (6,200 feet)	——	——	35½ to 80½ E		7 N	81 E
China—Pekin	Variable	54¾	26 W to 81 S	Variable	40 N	116¾ E
„ Shanghae	Ditto	60 (?)	10 to 24 ∠ 90 E		31⅓ N	121 E
„ Canton	Ditto	70	54 W to 90 or 100 E		23 N	113¼ E
Thibet, &c.	Rainless	Excessive climate; see p. 1193.		Very dry	28 N to 38 N	74 E to 103 E
Afghanistan—Cabool	Variable	75 to 85 s	— 10 to + 110 E	Variable	34 N	69 E
Russia—Siberia	Much snow	Excessive	— 58 to + 77 E	Dry	47 N to 78 N	60 E to 170 W

A TABLE *of the Rainfall, Mean Annual Temperature, Thermometric Range, &c., of various Countries, Oceans, &c.—(Continued.)*

Division, Country, &c.	Inches of Annual Rainfall.	Mean Annual Temperature. Fah.	Temperature. Fah. Extremes = E, E W, and E S. Mean Winter = W. Mean Summer = S. Total Range = R.	Comparative Moisture.	Latitude	Longitude.
ASIA—*Continued.*	Inches.	Degrees.	Degrees.		Degrees.	Degrees.
Persia	But little known; climate excessive; 125 E			Dry	26 N to 40 N	44 E to 61½ E
Siam	Variable	83	(?)	Tropical	4 N to 22 N	98 E to 105 E
Penang	—	80	70 to 90 E		5½ N	100 E
Australia	Tropical in the north; temperate in the south			Very dry	10 S to 30 S	113 E to 154 E
„ New South Wales—Sydney	50	62	57 W to 68 S	Dry	34 S	151 E
„ Victoria—Melbourne	30	59	54 W to 70 S		37¾ S	145 E
„ South Australia—Adelaide	17 to 26	65	55 W to 73 S		35 S	138½ E
„ West Australia—Perth	45 (?)	65	55 W to 75 S		32 S	116 E
Tasmania—Hobart Town	40	52	42 W to 63 S	Like S. W of England.	43 S	147½ E
New Zealand—Wellington	29	52	44 W to 64 S	Moderate	41¼ S	174¾ E
„ Auckland	52	60	50 W to 70 S		36 S	174¾ E
Sandwich Isles	—	—	60 to 90 E		19 N to 22 N	155 W to 161 W
Temperature of the Antarctic Ocean	—	Lat. 68° 30′ S = 30°·8 „ 56° 26′ S = 39°·5	—	—	S of 55 S	—
AFRICA.						
Cape Colony (General)	23	68	58 W to 76 S to 105 E	Very dry	28 S to 35 S	17 E to 27 E
„ Cape Town	—	67 to 68	58 W to 76 S	Dry	34 S	18 E
Natal	Considerable	65	55 W to 76 S		27 S to 30 S	29 E to 31 E
Senegambia (Coast)	189	—	Excessively hot; temperature ∠ 131 E	Very moist	10 N to 18 N	7 W to 16 W
„ Senegal	50 to 60	—			15½ N	16½ W
Sierra Leone	190 to 310	82	10 daily R	Moist	8½ N	13½ W
Sahara Desert	Rainless	—	32 to 50 night ∠ 130 day	Very dry	16 N to 33 N	17 W to 28 E
Equator	95	79·8	—	—	—	—
Temperature of the Ocean under the Equator	—	78 to 82	—	—	—	—
Ditto ditto in Lat. 5¾ N to 6¼ N	—	82 to 84	—	—	5¾ N to 6¼ N	—
Egypt	Generally rainless	—	37 to 110 E	Dry	24 N to 32 N	25 E to 34 E
„ Delta	Occasional	—	50 to 60 to 80 to 90		32 N	32 E
„ Cairo	Trifling	72	58 to 64 to 90 to 100		30 N	31 E
„ Thebaid	Rainless	—	60 to 75 to 95 to 110		25 N to 26 N	33 E
Waters of the Red Sea	—	—	65 to 85 E	—	12¾ N to 30 N	34 E to 44 E
Isle of Bourbon	—	77	61 to 90 E	Variable	21 S	55½ E

Mauritius	40	76	70 to 96 E	—	20 s	57½ E
Aden	12	85	79 to 103 E	Dry	12¾ N	45¼ E
AMERICA.						
AMERICA (North)	From 10° to 15° cooler than same latitudes in Europe			—	20 N to 71 N	54 W to 166 W
Canada (West)	22	48	− 20 to + 103 E	Dry	42 N to 50 N	74 W to 82 W
,, (East)	—	42	− 29 to + 103 E		45 N to 50 N	64 W to 74 W
,, Quebec	—	—	− 30 to + 103 E		46¾ N	71 W
,, ,,	—	—	+ 14 W to 68 S		—	—
Chippewayan Fort	—	31¾	+ 6 W to 51¾ S	—	58¾ N	112 W
Newfoundland	—	40	+ 22 W to 58 S	Moist, foggy	48 N to 52 N	52 W to 59 W
United States—New York State	—	—	—	Variable	40½ N to 45 N	72 W to 80 W
,, Rochester City	33	47⅓	− 9 to + 102 E	—	43 N	78 W
,, Vermont State	—	—	− 17 to + 92 E	—	43 N to 45 N	72 W to 74 W
,, Massachusetts State	—	—	− 8 to + 98 E	—	42 N to 43 N	70 W to 74 W
,, Iowa State	72	—	− 18 to + 92 E	—	41 N to 44 N	91 W to 97 W
,, Minnesota State	—	(July) 69	− 48 to + 96 E	—	44 N to 49 N	92 W to 97 W
,, Georgia (South of)	—	—	+ 13 to 99 E	N cold, dry S hot, moist	32 N	82 W to 85 W
,, Utah State	Trifling	—	− 5 to + 90 ∠ 105 E	Dry	37 N to 42 N	105 W to 120 W
Mexico (Coast)	—	76	—	—	15 N to 20 N	95 W to 105 W
,, Table-land	—	64	—	—		
,, Vera Cruz	185	—	—	—	19⅕ N	96⅙ W
,, Gulf-stream	—	86	—	—	25 N	80 W
AMERICA (Central)	Moderate	—	60 to 120 E	—	8 N to 21 N	83 W to 92 W
,, ,, Guatemala	—	—	60 to 80 E	—	14 N to 17 N	88 W to 93 W
West Indies—Barbadoes	72	81	75 W to 87 S	Moist	13 N	60 W
,, Jamaica (Kingston)	65 to 70	80	70 to 100 ∠ E	Variable	17½ N to 18½ N	76 W to 78 W
,, ,, 1,400 feet elevation	—	78¾	76 W to 81 S			
,, Antigua	45	80	73 to 84 E	Moderate	17 N	61¾ W
,, Bermuda	70 (?)	71 to 75	60 to 86 E	Moist	32½ N	64¾ W
,, Guadeloupe	86	81	70 to 99 or 130 E		16 N	61¾ W
,, Trinidad	75	80	70 to 85 or 90 E		10½ N	61 W
AMERICA (South)	—	—	—	—	12½ N to 55 S	35 W to 81¼ W
Venezuela—Caraccas	Large	80	68 W to 72 S	Moist	10½ N	67 W
,, La Guayra	—		—		10⅗ N	67 W
Guiana (British)—Georgetown	103	80	77 to 81 E	Moist	1 N to 9 N	57 W to 61 W
,, ,, Hills		70 (?)	60 to 95 E			
,, (French)—Cayenne	128	80	—	Very moist	2 N to 6 N	51 W to 54 W
Buenos Ayres	—	—	40 to 90 E	Variable	37 S to 40 S	55 W to 65 W
Peru	Part rainless	65 to 70	60 to 85 E		3½ S to 21½ S	68 W to 81 W
Chili	Irregular in all respects, from its mountainous character, &c. N of 30° it is hot and dry; but S of 30° humid and cooler			—	25⅓ S to 43⅓ S	68 W to 74 W

TABLE *of the Rainfall, Mean Annual Temperature, Thermometric Range, &c., of various Countries, Oceans, &c.—(Continued.)*

Division, Country, &c.	Inches of Annual Rainfall.	Mean Annual Temperature. Fah.	Temperature. Fah. Extremes = E, E W, and E S. Mean Winter = W. Mean Summer = S. Total Range = R.	Comparative Moisture.	Latitude.	Longitude.
AMERICA—(South)—*Continued.*	Inches.	Degrees.	Degrees.		Degrees.	Degrees.
Patagonia and Tierra del Fuego	—	Winter mean 33	24 to 41 and 52 E	—	40 s to 55 s	64 w to 75 w
Islands—Falkland	30	47	39 W to 55 S	Moist	52 s	60 w
„ South Georgia	Snow-line at the sea-level			—	54 s	38 w
„ South Shetland				—	61 s to 64 s	52 w to 64 w
Mean Temperature of the Earth	—	58·2	—	—	—	—
„ „ „ Northern Hemisphere .	—	59·9	—	—	—	—
„ „ „ Southern Hemisphere .	—	56·5	—	—	—	—
Annual range of Temperature in the Tropics .	—	—	2 R	—	—	—
„ „ „ „ Temperate Zones	—	—	60 R	—	—	—
„ „ „ „ Arctic Zones .	—	—	114 R	—	—	—

TABLE *of Comparison of the Readings of Mercurial and Aneroid Barometers, in the Ascents on July 17, and August 18, 1862.* (See p. 1182.)

Month, Day, Hour, and Minute.				Readings of Barometers.		Excess of Reading of Aneroid above Siphon Barometer.
				Siphon.	Aneroid.	
	h.	m.	s.	In.	In.	In.
July 17,	9	49	0	25·22	25·32	+ 0·10
	9	51	0	24·14	24·30	+ 0·16
	9	53	0	22·42	22·65	+ 0·23
	9	54	0	22·02	22·20	+ 0·18
	9	55	0	21·58	21·80	+ 0.22
	9	56	0	20·93	21·10	+ 0·17
	9	58	0	19·63	20·09	+ 0·46
	10	2	0	19·28	19·60	+ 0·32
	10	3	0	18·63	18·90	+ 0·27
	10	5	0	18·14	18·40	+ 0·26
	10	8	0	17·24	17·52	+ 0·28
	10	11	0	16·74	17·10	+ 0·36
	10	15	0	16·04	16·25	+ 0·21
	10	25	0	14·94	15·15	+ 0·21
	10	27	0	14·64	15·30	+ 0 66
July 17,	10	29	0	14·64	15·30	+ 0·66
	10	30	0	14·64	15·30	+ 0·66
	10	35	0	14·64	15·00	+ 0·36
	10	44	0	14·63	15·10	+ 0·47
	10	47	0	14·13	14·70	+ 0·57
	10	50	0	13·64	14·20	+ 0·56
	10	54	0	13·14	13·60	+ 0·46
	10	57	0	12·14	12·60	+ 0·46
	11	3	0	11·64	12·10	+ 0·46
	11	7	0	11·65	12·10	+ 0·45
	11	12	0	11·95	12·40	+ 0·45
	11	20	0	12·65	13·20	+ 0·55
	11	25	0	13·14	13·60	+ 0·46
	11	38	0	18·94	19·00	+ 0·06
	11	39	0	20·04	20·40	+ 0·36
July 17,	11	40	0	20·54	20·80	+ 0·26
Aug. 18,	0	56	0	29·34	29·51	+ 0·17
	1	6	0	28·55	28·78	+ 0·23
	1	8	0	26·67	26·90	+ 0·23
	1	10	0	25·86	26·08	+ 0·22
	1	14	0	23·64	23·82	+ 0·18
	1	15	0	22·69	22·68	— 0·01
	1	20	0	19·90	20·05	+ 0·15
	1	24	15	20·90	21·28	+ 0·38
	1	27	0	22·62	22·90	+ 0·28
	1	32	0	22·80	22·85	+ 0·05
	1	38	0	24·46	24·60	+ 0·14
	1	41	0	25·08	25·30	+ 0·22
	1	52	0	25·80	25·82	+ 0·02
	1	55	0	25·08	25·25	+ 0·17
Aug. 18,	2	0	0	23·93	24·10	+ 0·17
	2	9	0	22·58	22·71	+ 0·13
	2	17	0	20·24	20·50	+ 0·26
	2	21	0	19·11	19·30	+ 0·19
	2	25	20	17·61	17·85	+ 0·24
	2	29	0	16·41	16·50	+ 0·09
	2	32	20	15·84	16·00	+ 0·16
	2	49	50	13·70	13·60	— 0·10
	3	5	0	12·93	13·20	+ 0·27
	3	18	30	13·45	13·55	+ 0·10
	3	25	0	13·65	13·72	+ 0·07
	3	34	+	17·53	17·42	— 0·11
	3	36	0	18·63	18·65	+ 0·02
	3	39	0	20·02	20·05	+ 0·03
	3	49	0	24·28	24·42	+ 0·14

July 17.—The differences between the readings are shown in the last column, and exhibit an increasing difference, increasing in amount with decreasing readings, till at the highest point the discordance between the readings amounted to half an inch. It is presumed that the aneroid barometer was in error to these amounts; but both instruments were broken in the descent, and no more information can be given.

August 18.—The differences between the readings of the mercurial and aneroid barometers in this ascent were as constant as could be expected, as the readings could seldom be strictly simultaneous. Upon the whole, the readings of the aneroid are as good as those of the siphon.

TABLE—*Showing the Decrease of Temperature in every 1,000 feet of elevation up to 29,000 feet, during Mr. Glaisher's Balloon Ascents in 1862.*—(See, also, p. 1182).

Height above the level of the Sea.		July 17.		July 30.		August 18.				August 20.			August 21.		September 1.		September 5.		September 8.		Mean (omitting July 17).			
		Cloudy.		Clear.	Cloudy.	Clear.				Cloudy.			Cloudy.		Partially clear.	Cloudy.	Partially clear.	Cloudy.	Cloudy.		Cloudy.	Clear.	Clear.	
From	To	Ascending.	Descending.	Ascending.	Descending.	Ascending.	Descending.	Ascending.	Descending	Ascending.	Descending.	Ascending.	Ascending.	Descending.	Ascending.	Descending.	Ascending.	Descending.	Ascending.	Descending.	At heights less than 5,000 feet.		At heights exceeding 5,000 feet.	Number of experiments.
ft.	ft.	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	
28,000	29,000	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	0·8	…	…	…	…	…	0·8	1
27,000	28,000	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	0·9	…	…	…	…	…	0·9	1
26,000	27,000	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	1·0	…	…	…	…	…	1·0	1
25,000	26,000	0·0	1·2	…	…	…	…	…	…	…	…	…	…	…	…	…	1·0	…	…	…	…	…	1·0	1
24,000	25,000	0·0	1·3	…	…	…	…	…	…	…	…	…	…	…	…	…	1·1	1·1	…	…	…	…	1·1	2
23,000	24,000	0·1	1·3	…	…	…	…	…	…	…	…	…	…	…	…	…	1·5	2·1	…	…	…	…	1·3	2
22,000	23,000	0·1	1·2	…	…	…	…	0·2	0·4	…	…	…	…	…	…	…	1·5	2·1	…	…	…	…	1·0	4
21,000	22,000	0·6	1·5	…	…	…	…	0·2	0·8	…	…	…	…	…	…	…	2·0	2·2	…	…	…	…	1·3	4
20,000	21,000	0·2	1·4	…	…	…	…	0·2	0·8	…	…	…	…	…	…	…	2·0	2·2	…	…	…	…	1·3	4
19,000	20,000	0·3	1·1	…	…	…	…	0·4	1·0	…	…	…	…	…	…	…	2·5	2·3	…	…	…	…	1·5	4
18,000	19,000	0·5	1·2	…	…	…	…	1·0	1·0	…	…	…	…	…	…	…	2·5	2·0	…	…	…	…	1·8	4
17,000	18,000	0·2	1·6	…	…	…	…	1·2	1·0	…	…	…	…	…	…	…	2·5	2·8	…	…	…	…	1·8	4
16,000	17,000	0·5	1·2	…	…	…	…	1·6	1·0	…	…	…	…	…	…	…	2·5	2·7	…	…	…	…	1·9	4
15,000	16,000	1·0	1·5	…	…	…	…	2·0	1·0	…	…	…	…	…	…	…	3·0	2·5	…	…	…	…	2·1	4
14,000	15,000	0·6	1·0	…	…	…	…	2·0	1·2	…	…	…	…	…	…	…	2·5	2·5	…	…	…	…	2·1	4
13,000	14,000	1·4	1·5	…	…	…	…	2·1	1·8	…	…	…	1·3	2·3	…	…	2·5	2·5	…	…	…	…	2·2	6
12,000	13,000	0·8	1·5	…	…	…	…	2·1	2·5	…	…	…	2·2	3·2	…	…	2·5	2·8	…	…	…	…	2·5	6
11,000	12,000	1·7	1·5	…	…	…	…	1·9	2·7	…	…	…	2·8	2·9	…	…	2·8	2·7	…	…	…	…	2·6	6
10,000	11,000	2·0	1·5	…	…	1·5	2·0	2·1	2·8	…	…	…	2·8	3·5	…	…	2·4	2·5	…	…	…	…	2·6	8
9,000	10,000	3·0	…	…	…	1·3	1·8	2·2	2·8	…	…	…	2·5	3·5	…	…	2·8	31	…	…	…	…	2·6	8
8,000	9,000	3·0	…	…	…	1·2	2·2	2·0	2·4	…	…	…	3·1	3·8	…	…	3·5	2·6	…	…	…	…	2·6	8
7,000	8,000	4·5	…	…	…	1·8	2·0	2·6	3·3	…	…	…	3·8	3·6	…	…	3·5	2·7	…	…	…	…	2·7	8
6,000	7,000	4·5	…	2·7	2·0	1·2	2·0	2·0	3·3	…	…	…	3·8	3·5	…	…	4·0	3·5	…	…	…	…	2·8	8
5,000	6,000	4·2	…	2·7	1·4	1·5	3·0	2·2	4·0	…	…	…	4·0	4·0	…	…	4·0	2·7	…	…	…	…	2·8	10
4,000	5,000	4·8	…	2·6	1·8	2·7	2·5	2·1	4·0	…	…	3·6	4·0	4·0	…	…	3·5	2·8	2·7	…	3·1	2·9	…	6 & 5
3,000	4,000	4·8	…	2·4	4·9	3·6	2·5	1·9	4·5	4·0	2·8	4·0	4·1	4·6	3·2	2·0	5·5	3·0	3·7	2·7	3·7	3·4	…	10 & 7
2,000	3,000	5·0	…	3·8	5·1	4·7	…	…	5·1	5·0	3·0	3·6	4·6	5·0	3·8	3·0	5·0	3·0	4·6	3·6	4·1	4·6	…	10 & 5
1,000	2,000	4·3	…	5·9	4·0	5·4	…	…	4·9	4·5	…	…	4·3	5·4	4·7	…	5·0	3·2	4·1	3·7	4·2	5·3	…	7 & 5
0	1,000	5·9	…	7·8	4·8	8·0	…	…	6·5	4·9	…	…	4·0	6·0	6·5	…	7·2	3·4	3·9	4·6	4·5	7·2	…	7 & 5

THE MEAN DECREASE OF TEMPERATURE OF THE AIR WITH A CLOUDY SKY.

Up to 1,000 feet was . . . . 4°·5 from 7 experiments, or 1° in 222 feet.
From 1,000 to 2,000 ,, ... 4°·2 ,, 7 ,, ,, 239 feet.
From 2,000 to 3,000 ,, ... 4°·1 ,, 10 ,, ,, 244 feet.
From 3,000 to 4,000 ,, ... 3°·7 ,, 10 ,, ,, 271 feet.
From 4,000 to 5,000 ,, ... 3°·1 ,, 6 ,, 323 feet.

THE MEAN DECREASE OF TEMPERATURE OF THE AIR WITH A PARTIALLY CLEAR SKY.

Up to 1,000 feet was . . . . 7°·2 from 5 experiments, or 1° in 139 feet.
From 1,000 to 2,000 ,, ... 5°·3 ,, 5 ,, ,, 189 feet.
From 2,000 to 3,000 ,, ... 4°·6 ,, 5 ,, ,, 254 feet.
From 3,000 to 4,000 ,, ... 3°·4 ,, 7 ,, ,, 295 feet.
From 4,000 to 5,000 ,, ... 2°·9 ,, 6 ,, ,, 345 feet.

TABLE, *showing the adopted Reading of the Barometer, reduced to 32° Fah.; calculated Height above the Sea; Temperature of the Air; and Dew-point, during Mr. Glaisher's Ascent on August 18th, 1862.* (See, also, p. 1182.)

Time—P.M.	Barometer.	Height.	Temp. of Air.	Temp. of Dew-point.	Time—P.M.	Barometer.	Height.	Temp. of Air.	Temp of Dew-point.
h. m. s.	Inches.	Feet.	Degrees.	Degrees.	h. m. s.	Inches.	Feet.	Degrees.	Degrees.
0 56 0	29·34	490	67·8	54·6	2 20 0	19·60	11,748	——	21·2
1 5 0	28·84	1,130	62·5	52·5	21 0	19·11	12,364	39·2	25·6
6 0	28·55	1,419	60·0	——	22 0	——	(12,595)	38·5	——
6 20	28·25	1,713	58·2	53·1	22 30	18·86	12,708	——	——
6 30	——	(1,795)	57·2	——	23 0	18·71	12,942	38·0	——
7 0	27·90	2,042	55·5	50·5	24 0	18·11	13,852	34·1	——
8 0	26·67	3,347	——	——	25 20	17·61	14,434	——	23·1
8 20	——	(3,466)	52·5	——	29 0	16·41	16,339	27·8	6·0
9 0	26·27	3,705	50·0	49·6	31 0	——	(16,885)	——	——
10 0	25·86	4,138	49·9	——	32 0	15·93	17,157	——	——
10 25	——	(4,440)	49·8	44·2	32 10	——	(17,240)	——	± 5·0
11 0	25·30	4,767	48·8	42·2	32 20	15·84	17,321	28·1	——
11 30	——	(5,140)	48·6	——	34 0	——	(17,770)	30·5	5·0 no dew.
12 0	24·60	5,509	48·2	——	34 20	——	(17,860)	——	3·5 no dew.
12 30	24·60	5,510	47·8	38·8	35 0	——	(18,039)	31·5	1·2
13 20	——	(6,155)	48·0	37·7	36 10	——	(18,445)	27·5	— 5·0
14 0	23·64	6,585	46·5	——	36 30	15·03	18,560	——	——
15 0	22·69	7,706	45·7	——	36 40	——	(18,605)	24·8	— 2·0
17 0	21·69	8,935	44·0	32·0	36 50	——	(18,650)	25·5	——
18 45	20·90	9,954	43·0	29·6	37 55	——	(18,935)	——	——
18 55	——	(10,129)	40·5	28·6	38 10	14·87	19,000	——	——
20 0	19·90	11,267	38·5	——	38 30	——	(19,200)	——	— 5·0
20 5	——	(11,285)	37·2	24·9	39 0	14·62	19,461	——	——
20 35	19·80	11,399	36·0	24·8	39 10	——	(19,604)	——	——
21 0	19·75	11,470	39·5	22·2	39 20	——	(19,800)	26·1	——
22 0	20·30	10,840	41·8	25·2	42 0	——	(20,350)	——	— 8·5
24 0	20·90	9,884	45·0	26·9	42 10	14·12	20,359	25·1	——
24 15	20·90	9,884	——	——	45 0	——	(20,665)	——	— 5·0
24 50	21·38	9,120	46·2	32·0	——	——	(20,888)	23·5	——
25 0	——	(9,040)	45·8	——	49 0	13·62	21,111	——	——
25 10	——	(8,960)	47·2	33·1	49 50	——	(21,200)	25·4	— 10·0
26 0	——	(8,575)	——	28·2	59 0	12·83	23,164	——	— 12·0
26 30	22·21	8,342	——	30·5	59 10	12·71	23,215	24·2	——
27 0	22·62	7,836	51·0	37·9	59 20	12·61	23,377	——	— 8·0
32 0	22·80	7,650	49·2	37·0	59 40	12·93	22,705	24·0	— 10·0
33 0	22·80	7,650	53·8	36·6	3 0 0	13·13	22,295	——	——
34 0	——	(7,265)	——	36·0	3 20	13·13	22,295	24·5	— 9·0
37 30	24·25	5,919	——	——	——	——	(22,295)	24·8	— 10·0
38 0	24·46	5,820	53·8	37·8	4 30	13·13	22,295	24·0	— 9·0
41 0	25·08	5,028	53·5	——	5 0	12·93	22,705	24·1	——
41 30	25·56	4,530	——	43·5	5 30	——	(22,705)	——	— 10·0
43 0	25·58	4,480	——	46·0	7 0	12·93	22,705	24·0	— 8·0
46 0	26·56	3,438	——	47·5	12 30	12·93	22,705	24·0	——
48 0	26·76	3,219	56·0	——	13 0	——	(22,160)	——	— 9·0
52 0	25·79	4,233	55·0	——	13 13	13·63	21,977	24·0	——
52 30	25·59	4,448	——	——	13 20	——	(22,000)	——	— 10·0
53 0	——	(4,562)	52·2	40·0	13 40	13·58	22,008	24·0	——
55 0	25·08	5,019	51·0	42·0	18 30	13·45	22,107	24·0	——
56 0	——	(5,273)	——	39·5	19 20	——	(21,757)	——	— 10·0
57 40	——	(5,695)	50·0	40·9	19 30	——	(21,685)	24·0	——
58 0	24·39	5,780	——	——	19 40	——	(21,610)	24·8	——
58 30	——	(5,913)	54·8	——	32 0	——	(16,405)	——	0·0
58 40	——	(5,958)	——	40·1	33 0	16·78	15,984	——	——
2 0 0	23·93	6,313	——	——	34 0	17·53	13,320	32·8	11·1
1 0	23·78	6,491	——	——	36 0	18·63	12,453	38·0	13·3
1 30	——	(6,580)	54·0	41·7	39 0	20·02	10,624	40·7	14·4
9 0	22·58	7,886	50·5	38·6	40 0	20·72	10,224	45·5	14·4
10 0	——	(8,342)	——	37·5	41 30	21·62	8,764	——	——
10 30	22·18	8,571	51·0	39·8	43 0	22·77	8,144	——	——
11 0	——	(8,660)	51·0	38·4	43 10	——	(7,910)	50·5	36·7
11 40	21·88	8,771	51·0	37·6	43 30	22·74	7,438	——	——
12 20	——	(8,771)	——	39·5	45 0	——	(6,943)	——	38·6
13 0	21·88	8,771	——	——	47 0	——	(6,282)	——	49·0
13 40	——	(9,327)	50·5	——	49 0	24·28	5,621	50·0	——
13 50	——	(9,715)	——	34·9	50 20	25·08	4,821	52·1	43·9
15 0	21·14	9,695	50·5	34·1	51 0	25·36	4,521	51·5	44·8
15 30	——	(9,987)	48·1	——	53 0	——	(3,900)	51·0	48·9
17 0	20·24	10,864	——	29·3	4 5 0	——	On the ground	67·0	——

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